Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.2.2-g-cos-^p-a+b-sin-^m-c+d-sin-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1563]. This is test number [74].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.81 (1560)	% 0.19 (3)
Mathematica	% 97.18 (1519)	% 2.82 (44)
Maple	% 88.29 (1380)	% 11.71 (183)
Maxima	% 61.29 (958)	% 38.71 (605)
Fricas	% 77.8 (1216)	% 22.2 (347)
Sympy	% 12.99 (203)	% 87.01 (1360)
Giac	% 71.4 (1116)	% 28.6 (447)

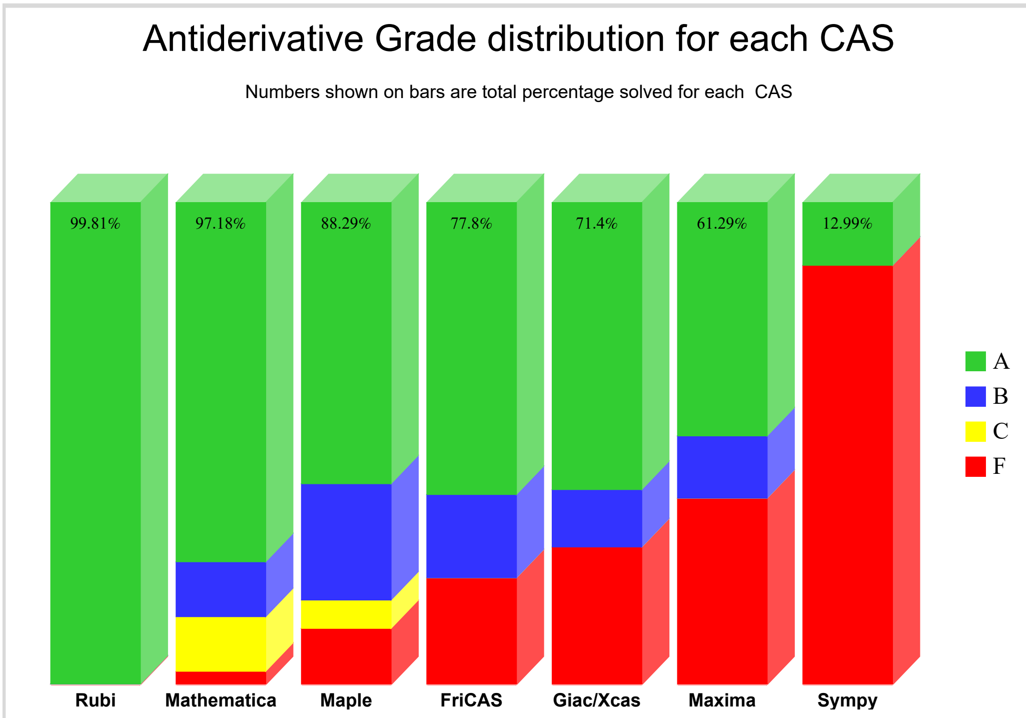
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

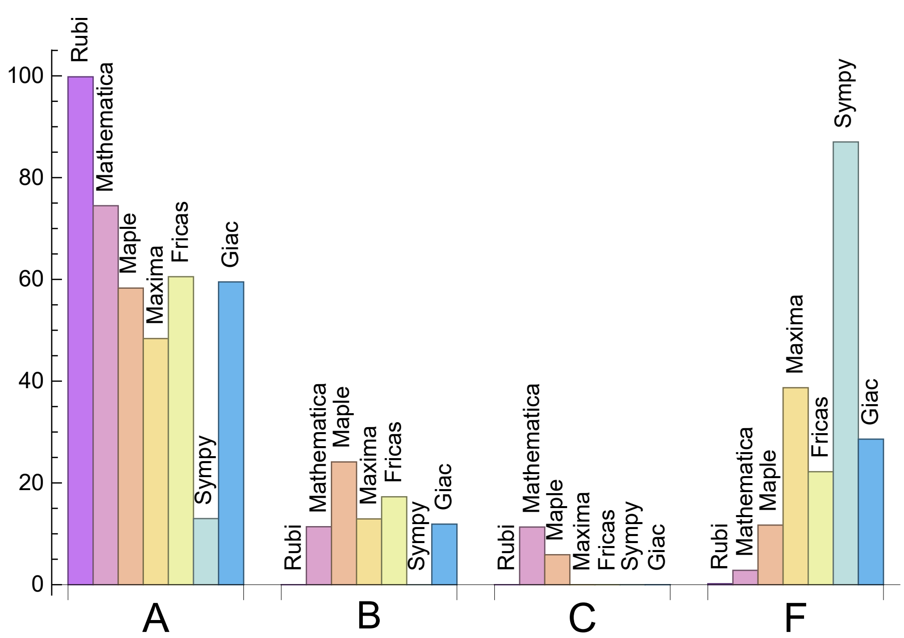
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.81	0.	0.	0.19
Mathematica	74.47	11.39	11.32	2.82
Maple	58.29	24.12	5.89	11.71
Maxima	48.37	12.92	0.	38.71
Fricas	60.52	17.27	0.	22.2
Sympy	12.99	0.	0.	87.01
Giac	59.5	11.9	0.	28.6

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.38	163.2	1.	125.	1.
Mathematica	3.45	440.33	2.64	145.	1.06
Maple	0.43	539.54	2.35	182.	1.44
Maxima	1.19	218.19	1.89	160.	1.42
Fricas	1.9	597.5	4.22	378.5	3.55
Sympy	32.89	631.23	6.46	294.	2.69
Giac	1.4	281.86	2.22	204.	1.87

1.4 list of integrals that has no closed form antiderivative

{1191, 1192, 1193, 1194, 1519, 1520, 1521, 1522, 1526, 1527, 1560}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {65, 70, 77, 78, 81, 84, 85, 86, 87, 164, 165, 167, 168, 349, 478, 488, 818, 911, 937, 938, 942, 945, 1030, 1042, 1049, 1074, 1075, 1084, 1091, 1111, 1121, 1122, 1133, 1134, 1141, 1157, 1159, 1165, 1167, 1188, 1189, 1190, 1247, 1248, 1262, 1263, 1272, 1276, 1292, 1293, 1307, 1308, 1326, 1327, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1479, 1480, 1516, 1517, 1518, 1550, 1551, 1561, 1562, 1563}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
```

```
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

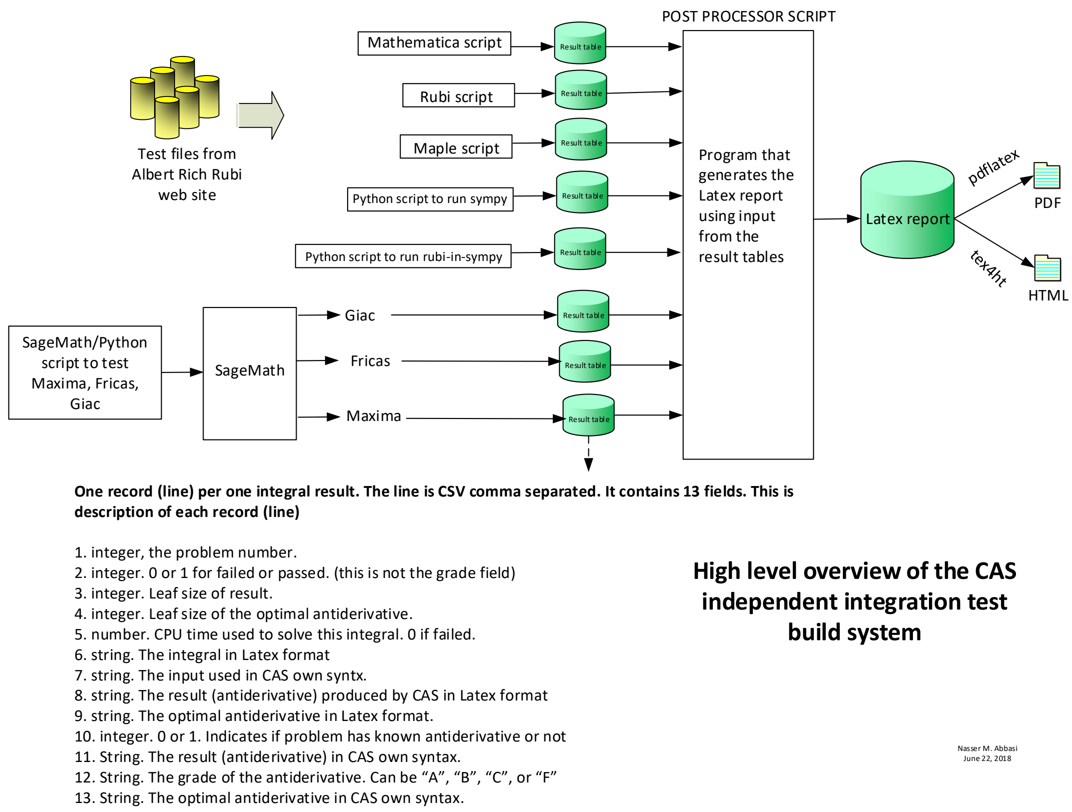
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563 }

B grade: { }

C grade: { }

F grade: { 1479, 1480, 1515}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 14, 15, 16, 18, 19, 20, 21, 22, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 69, 71, 72, 74, 75, 76, 79, 80, 81, 82, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 155, 156, 157, 158, 168, 173, 175, 176, 177, 178, 179, 180, 181, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 299, 301, 303, 304, 305, 306, 307, 308, 309, 316, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 336, 337, 338, 339, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 413, 414, 415, 417, 418, 419, 421, 425, 427, 428, 429, 430, 431, 432, 434, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 464, 465, 466, 467, 471, 472, 473, 474, 475, 480, 490, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 602, 603, 604, 605, 606, 607, 608, 609, 610, 612, 613, 614, 615, 616, 617, 619, 620, 621, 622, 623, 628, 629, 630, 631, 637, 638, 639, 640, 641, 642, 643, 647, 650, 651, 652, 653, 654, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702, 703, 704, 710, 711, 712, 713, 714, 715, 721, 727, 728, 729, 730, 731, 733, 735, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 759, 762, 763, 764, 765, 766, 768, 769, 770, 772, 773, 776, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 791, 792, 795, 796, 797, 798, 799, 800, 801, 802, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 819, 820, 821, 822, 823, 824, 825, 826, 829, 830, 831, 832, 833, 834, 835, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 938, 942, 945, 953, 954, 955, 956, 957, 961, 962, 963, 965, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1033, 1034, 1035, 1037, 1038, 1039, 1040, 1041, 1050, 1051, 1052, 1053, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1101, 1102, 1105, 1106, 1107, 1108, 1109, 1110, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1151, 1152, 1160, 1168, 1169, 1170, 1176, 1177, 1178, 1184, 1185, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1411, 1432, 1433, 1434, 1443, 1444, 1445, 1446, 1447, 1448, 1450, 1451, 1452, 1453, 1454, 1456, 1457, 1458, 1459, 1460, 1461, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1492, 1493, 1494, 1495, 1496, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1517, 1519, 1520, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560 }

B grade: { 13, 17, 23, 28, 35, 41, 42, 43, 44, 45, 77, 83, 167, 174, 188, 257, 282, 292, 293, 297, 298, 300, 302, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 326, 327, 328, 334, 335, 340, 341, 342, 376, 377, 388, 403, 410, 411, 412, 416, 420, 422, 423, 424, 426, 433, 435, 436, 437, 438, 439, 440, 441, 449, 463, 468, 469, 470, 476, 477, 478, 479, 492, 567, 585, 586, 587, 600, 601, 611, 618, 624, 625, 626, 627, 632, 633, 634, 635, 636, 644, 645, 646, 648, 649, 699, 705, 706, 707, 708, 709, 716, 717, 718, 719, 720, 722, 723, 724, 725, 726, 732, 734, 739, 740, 741, 757, 758, 760, 761, 767, 771, 774, 775, 777, 785, 786, 793, 794, 803, 818, 827, 828, 836, 837, 911, 964, 966, 967, 968, 1042, 1049, 1065, 1066, 1074, 1075, 1091, 1103, 1104, 1111, 1122, 1135, 1136, 1183, 1250, 1266, 1268, 1292, 1293, 1294, 1307, 1308, 1309, 1327, 1328, 1357, 1358, 1449, 1455, 1462, 1479, 1480, 1497, 1505, 1516, 1518, 1561, 1562, 1563 }

C grade: { 6, 7, 53, 61, 65, 70, 73, 78, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 108, 118, 129, 137, 146, 164, 165, 184, 270, 271, 343, 344, 345, 346, 347, 348, 349, 372, 373, 481, 482, 483, 484, 485, 486, 487, 488, 489, 581, 582, 858, 859, 860, 936, 937, 958, 959, 960, 998, 1030, 1036, 1054, 1055, 1099, 1100, 1145, 1146, 1147, 1148, 1149, 1150, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1171, 1172, 1173, 1174, 1175, 1179, 1180, 1181, 1182, 1186, 1187, 1188, 1189, 1190, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1489, 1490, 1491, 1515 }

F grade: { 66, 67, 68, 152, 153, 154, 159, 160, 161, 162, 163, 166, 169, 170, 171, 172, 182, 183, 491, 655, 939, 940, 941, 943, 944, 946, 947, 948, 949, 950, 951, 952, 1031, 1032, 1043, 1044, 1045, 1046, 1047, 1048, 1514, 1523, 1524, 1525}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 9, 10, 11, 12, 14, 15, 18, 19, 20, 21, 22, 24, 25, 26, 30, 31, 32, 33, 34, 36, 37, 38, 39, 43, 47, 49, 50, 51, 52, 53, 57, 58, 59, 60, 62, 64, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 301, 302, 303, 304, 305, 306, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 414, 417, 418, 425, 427, 429, 430, 431, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 631, 632, 639, 642, 648, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 668, 669, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 715, 731, 736, 745, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 766, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 815, 816, 817, 818, 821, 822, 825, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 866, 867, 868, 869, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 935, 936, 953, 954, 955, 956, 957, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 972, 977, 978, 979, 995, 996, 997, 1003, 1004, 1005, 1006, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1081, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1146, 1154, 1162, 1172, 1180, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1215, 1216, 1217, 1218, 1219, 1221, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1249, 1250, 1251, 1253, 1255, 1279, 1280, 1281, 1282, 1283, 1284, 1289, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1374, 1376, 1380, 1382, 1386, 1388, 1393, 1395, 1400, 1406, 1410, 1418, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1468, 1469, 1470, 1471, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1497, 1498, 1499, 1500, 1504, 1506, 1507, 1508, 1519, 1520, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1535, 1536, 1537, 1538, 1539, 1540, 1546, 1547, 1548, 1549, 1554, 1555, 1556, 1557, 1558, 1560 }

B grade: { 5, 7, 8, 13, 16, 17, 23, 27, 28, 29, 35, 40, 41, 42, 44, 45, 46, 48, 54, 55, 56, 61, 63, 297, 298, 299, 300, 307, 308, 309, 315, 385, 409, 410, 411, 412, 413, 415, 416, 419, 420, 421, 422, 423, 424, 426, 428, 432, 433, 434, 506, 507, 508, 509, 510, 521, 522, 597, 624, 625, 626, 627, 628, 629, 630, 633, 634, 635, 636, 637, 638, 640, 641, 643, 644, 645, 646, 647, 667, 670, 671, 672, 673, 674, 675, 676, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 732, 733, 734, 735, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 749, 750, 765, 767, 775, 811, 812, 813, 814, 819, 820, 823, 824, 826, 861, 862, 863, 864, 865, 870, 871, 872, 873, 874, 875, 877, 894, 937, 958, 970, 971, 973, 974, 975, 976, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1007, 1008, 1015, 1078, 1079, 1080, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1210, 1211, 1212, 1213, 1214, 1220, 1222, 1248, 1252, 1254, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1285, 1286, 1287, 1288, 1290, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1355, 1408, 1409, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1467, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1496, 1501, 1502, 1503, 1505, 1515, 1533, 1534, 1541, 1542, 1543, 1544, 1545, 1550, 1551, 1552, 1553, 1559 }

C grade: { 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 174, 1370, 1371, 1372, 1373, 1375, 1377, 1378, 1379, 1381, 1383, 1384, 1385, 1387, 1389, 1390, 1391, 1392, 1394, 1396, 1397, 1398, 1399, 1401, 1402, 1403, 1404, 1405, 1407 }

F grade: { 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 258, 259, 260, 261, 262, 263, 264, 265, 490, 491, 492, 493, 565, 566, 567, 568, 569, 570, 571, 653, 654, 655, 697, 698, 699, 700, 701, 702, 703, 704, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1195, 1196, 1197, 1235, 1236, 1237, 1238, 1509, 1510, 1511, 1512, 1513, 1514, 1516, 1517, 1518, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.4 Maxima

A grade: { 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 311, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 435, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 568, 569, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 700, 701, 702, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 774, 776, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 928, 929, 930, 931, 935, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 1000, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1279, 1280, 1281, 1282, 1283, 1284, 1295, 1296, 1297, 1298, 1299, 1300, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1332, 1333, 1334, 1335, 1336, 1337, 1346, 1347, 1348, 1349, 1350, 1351, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1519, 1520, 1521, 1522, 1526, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1560

B grade: { 5, 14, 25, 38, 46, 52, 59, 73, 74, 75, 175, 176, 177, 185, 252, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 772, 773, 775, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 999, 1001, 1002, 1559 }

C grade: { }

F grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 257, 258, 259, 260, 261, 262, 263, 264, 265, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 565, 566, 567, 570, 571, 653, 654, 655, 697, 698, 699, 703, 704, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1235, 1236, 1237, 1238, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1523, 1524, 1525, 1527, 1561, 1562, 1563 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 29, 30, 31, 32, 33, 34, 43, 45, 46, 47, 49, 55, 56, 62, 63, 64, 73, 74, 75, 81, 82, 83, 174, 181, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 253, 254, 255, 256, 257, 260, 261, 266, 267, 268, 269, 275, 276, 277, 278, 279, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 315, 316, 321, 322, 323, 324, 329, 330, 331, 336, 337, 338, 343, 344, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 377, 378, 379, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 443, 444, 453, 454, 455, 464, 465, 472, 473, 481, 482, 483, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 568, 569, 572, 573, 574, 575, 576, 577, 578, 579, 580, 588, 589, 590, 591, 592, 593, 594, 595, 596, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 700, 701, 702, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 759, 760, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 807, 819, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 857, 858, 861, 862, 863, 865, 866, 870, 871, 872, 873, 874, 875, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 916, 922, 923, 928, 929, 930, 931, 935, 953, 954, 955, 956, 957, 959, 960, 961, 962, 963, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 977, 978, 979, 981, 982, 983, 984, 985, 986, 987, 988, 990, 991, 992, 995, 996, 997, 998, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1017, 1018, 1021, 1022, 1023, 1033, 1034, 1035, 1040, 1041, 1050, 1051, 1052, 1053, 1059, 1060, 1061, 1062, 1063, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1085, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1135, 1136, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1256, 1257, 1258, 1259, 1260, 1266, 1267, 1268, 1269, 1270, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1307, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1320, 1321, 1322, 1323, 1324, 1326, 1330, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1346, 1347, 1348, 1349, 1350, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1471, 1472, 1473, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1560 }

B grade: { 17, 23, 28, 35, 41, 42, 44, 76, 79, 80, 175, 176, 177, 213, 220, 248, 252, 258, 259, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 290, 291, 292, 293, 302, 311, 312, 313, 314, 317, 318, 319, 320, 325, 326, 327, 328, 332, 333, 334, 335, 339, 340, 341, 342, 345, 346, 347, 348, 349, 372, 373, 374, 375, 376, 385, 386, 387, 401, 416, 435, 436, 437, 438, 439, 440, 441, 442, 445, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 466, 467, 468, 469, 470, 471, 474, 475, 476, 477, 478, 479, 480, 484, 485, 486, 487, 488, 489, 563, 564, 565, 566, 567, 581, 582, 583, 584, 585, 586, 587, 597, 598, 599, 601, 603, 616, 633, 697, 698, 699, 717, 753, 754, 755, 756, 757, 758, 761, 762, 763, 764, 768, 769, 770, 771, 802, 803, 804, 805, 806, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 820, 856, 859, 860, 864, 867, 868, 869, 877, 913, 914, 915, 920, 921, 936, 937, 958, 964, 975, 976, 980, 989, 993, 994, 999, 1015, 1016, 1020, 1054, 1055, 1056, 1057, 1058, 1064, 1081, 1082, 1083, 1084, 1086, 1087, 1088, 1089, 1090, 1099, 1100, 1101, 1102, 1103, 1132, 1133, 1134, 1137, 1138, 1139, 1140, 1141, 1142, 1231, 1232, 1233, 1234, 1235, 1236, 1250, 1251, 1252, 1253, 1254, 1255, 1261, 1262, 1263, 1264, 1265, 1271, 1272, 1273, 1274, 1275, 1293, 1306, 1308, 1309, 1318, 1319, 1325, 1327, 1328, 1329, 1331, 1344, 1345, 1351, 1369, 1468, 1469, 1470, 1474, 1475, 1476, 1477, 1556, 1557, 1558, 1559 }

C grade: { }

F grade: { 6, 7, 14, 15, 16, 24, 25, 26, 27, 36, 37, 38, 39, 40, 48, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 178, 179, 180, 182, 183, 184, 189, 262, 263, 264, 265, 490, 491, 492, 493, 570, 571, 653, 654, 655, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1036, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1091, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1237, 1238, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.6 Sympy

A grade: { 190, 191, 197, 198, 206, 207, 208, 217, 218, 219, 220, 224, 225, 226, 227, 232, 233, 234, 235, 240, 241, 242, 243, 244, 249, 250, 251, 252, 253, 258, 259, 260, 261, 266, 267, 268, 275, 276, 277, 285, 286, 294, 297, 298, 299, 300, 307, 308, 309, 310, 315, 316, 317, 350, 351, 352, 353, 357, 358, 359, 365, 366, 367, 368, 378, 379, 380, 381, 392, 393, 394, 395, 407, 409, 410, 411, 412, 422, 423, 424, 433, 434, 494, 495, 496, 497, 498, 511, 512, 513, 521, 522, 533, 534, 535, 545, 546, 556, 567, 572, 573, 574, 575, 588, 589, 590, 591, 605, 606, 607, 608, 626, 627, 636, 656, 657, 658, 659, 660, 661, 682, 683, 914, 915, 916, 921, 922, 923, 929, 930, 931, 935, 953, 954, 955, 956, 961, 962, 963, 969, 970, 971, 972, 977, 978, 979, 986, 987, 988, 989, 995, 996, 997, 1003, 1004, 1005, 1006, 1012, 1013, 1014, 1023, 1050, 1051, 1052, 1059, 1060, 1061, 1069, 1070, 1092, 1093, 1094, 1095, 1105, 1106, 1107, 1116, 1117, 1198, 1199, 1200, 1201, 1202, 1215, 1216, 1239, 1240, 1241, 1242, 1243, 1279, 1280, 1281, 1528, 1529, 1530, 1531, 1536, 1537, 1538, 1539, 1547, 1555 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 209, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 228, 229, 230, 231, 236, 237, 238, 239, 245, 246, 247, 248, 254, 255, 256, 257, 262, 263, 264, 265, 269, 270, 271, 272, 273, 274, 278, 279, 280, 281, 282, 283, 284, 287, 288, 289, 290, 291, 292, 293, 295, 296, 301, 302, 303, 304, 305, 306, 311, 312, 313, 314, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 354, 355, 356, 360, 361, 362, 363, 364, 369, 370, 371, 372, 373, 374, 375, 376, 377, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 413, 414, 415, 416, 417, 418, 419, 420, 421, 425, 426, 427, 428, 429, 430, 431, 432, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 514, 515, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 536, 537, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 550, 551, 552, 553, 554, 555, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 628, 629, 630, 631, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 917, 918, 919, 920, 924, 925, 926, 927, 928, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 957, 958, 959, 960, 964, 965, 966, 967, 968, 973, 974, 975, 976, 980, 981, 982, 983, 984, 985, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1007, 1008, 1009, 1010, 1011, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1053, 1054, 1055, 1056, 1057, 1058, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 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2.1.7 Giac

A grade: { 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 278, 280, 281, 283, 285, 286, 287, 288, 290, 291, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 336, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 472, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 559, 560, 561, 562, 563, 564, 568, 569, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 700, 701, 702, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 719, 720, 722, 723, 724, 725, 726, 727, 728, 729, 731, 732, 733, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 748, 749, 750, 751, 752, 754, 755, 756, 757, 758, 759, 760, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 825, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 935, 953, 954, 955, 956, 958, 959, 960, 961, 962, 963, 964, 969, 971, 972, 975, 976, 977, 978, 979, 981, 982, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1050, 1051, 1052, 1053, 1055, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1086, 1087, 1088, 1089, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1104, 1105, 1106, 1107, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1131, 1132, 1133, 1134, 1135, 1136, 1138, 1139, 1140, 1141, 1142, 1191, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1239, 1240, 1241, 1242, 1243, 1249, 1256, 1257, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1282, 1283, 1284, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1295, 1296, 1297, 1298, 1299, 1300, 1304, 1306, 1307, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1323, 1324, 1326, 1329, 1330, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1443, 1444, 1445, 1446, 1447, 1449, 1450, 1451, 1452, 1453, 1455, 1456, 1457, 1458, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1519, 1520, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1552, 1553, 1554, 1555, 1556, 1557 }

B grade: { 174, 177, 270, 272, 279, 282, 284, 289, 292, 293, 302, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 374, 375, 376, 385, 399, 408, 414, 416, 426, 428, 464, 465, 466, 467, 468, 469, 470, 471, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 554, 555, 556, 557, 558, 565, 566, 567, 583, 584, 585, 587, 597, 601, 603, 627, 633, 641, 648, 697, 698, 699, 709, 717, 718, 721, 730, 734, 747, 753, 761, 824, 826, 870, 871, 872, 913, 914, 915, 916, 920, 921, 922, 923, 928, 929, 930, 931, 957, 965, 966, 967, 968, 970, 973, 974, 980, 983, 984, 985, 986, 987, 988, 989, 990, 991, 1001, 1002, 1020, 1021, 1022, 1023, 1041, 1054, 1056, 1071, 1072, 1073, 1085, 1090, 1101, 1102, 1103, 1108, 1121, 1130, 1137, 1235, 1236, 1244, 1245, 1246, 1247, 1248, 1250, 1251, 1252, 1253, 1254, 1255, 1258, 1268, 1285, 1286, 1294, 1301, 1302, 1303, 1305, 1308, 1309, 1320, 1321, 1322, 1325, 1327, 1328, 1331, 1344, 1369, 1448, 1454, 1459, 1469, 1477, 1550, 1551, 1558, 1559 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 257, 262, 263, 264, 265, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 346, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 485, 490, 491, 492, 493, 570, 571, 653, 654, 655, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1091, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1192, 1195, 1196, 1197, 1237, 1238, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1521, 1523, 1524, 1525, 1560, 1561, 1562, 1563 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	104	133	0	274	0	0
normalized size	1	1.	1.13	1.45	0.	2.98	0.	0.
time (sec)	N/A	0.398	0.571	0.274	0.	1.784	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	94	106	0	235	0	0
normalized size	1	1.	1.02	1.15	0.	2.55	0.	0.
time (sec)	N/A	0.391	0.481	0.251	0.	1.702	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	83	90	0	193	0	0
normalized size	1	1.	0.9	0.98	0.	2.1	0.	0.
time (sec)	N/A	0.39	0.404	0.243	0.	1.686	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	59	55	0	144	0	0
normalized size	1	1.	0.64	0.6	0.	1.57	0.	0.
time (sec)	N/A	0.372	0.169	0.241	0.	1.722	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	62	94	522	154	0	0
normalized size	1	1.	1.38	2.09	11.6	3.42	0.	0.
time (sec)	N/A	0.284	0.308	0.236	1.737	1.604	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	115	138	0	0	0	0
normalized size	1	1.	1.16	1.39	0.	0.	0.	0.
time (sec)	N/A	0.425	1.073	0.243	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	104	194	0	0	0	0
normalized size	1	1.	1.07	2.	0.	0.	0.	0.
time (sec)	N/A	0.431	0.838	0.247	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	90	96	0	200	0	0
normalized size	1	1.	1.88	2.	0.	4.17	0.	0.
time (sec)	N/A	0.319	0.388	0.224	0.	1.706	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	166	133	0	292	0	0
normalized size	1	1.	1.19	0.95	0.	2.09	0.	0.
time (sec)	N/A	0.524	1.246	0.228	0.	1.822	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	156	116	0	251	0	0
normalized size	1	1.	1.11	0.83	0.	1.79	0.	0.
time (sec)	N/A	0.529	0.971	0.214	0.	1.811	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	82	67	0	190	0	0
normalized size	1	1.	0.59	0.48	0.	1.36	0.	0.
time (sec)	N/A	0.521	0.587	0.183	0.	1.689	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	83	90	0	194	0	0
normalized size	1	1.	0.9	0.98	0.	2.11	0.	0.
time (sec)	N/A	0.39	0.399	0.216	0.	1.699	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	111	141	0	193	0	0
normalized size	1	1.	2.47	3.13	0.	4.29	0.	0.
time (sec)	N/A	0.311	0.546	0.212	0.	1.634	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	130	173	1139	0	0	0
normalized size	1	1.	0.88	1.18	7.75	0.	0.	0.
time (sec)	N/A	0.546	1.068	0.184	1.954	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	169	222	0	0	0	0
normalized size	1	1.	1.17	1.54	0.	0.	0.	0.
time (sec)	N/A	0.547	1.052	0.195	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	191	276	0	0	0	0
normalized size	1	1.	1.3	1.88	0.	0.	0.	0.
time (sec)	N/A	0.566	1.413	0.188	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	110	127	0	259	0	0
normalized size	1	1.	2.29	2.65	0.	5.4	0.	0.
time (sec)	N/A	0.337	1.475	0.181	0.	1.684	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	118	152	0	320	0	0
normalized size	1	1.	1.22	1.57	0.	3.3	0.	0.
time (sec)	N/A	0.436	2.008	0.192	0.	1.78	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	176	143	0	306	0	0
normalized size	1	1.	0.94	0.76	0.	1.63	0.	0.
time (sec)	N/A	0.624	3.211	0.24	0.	2.008	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	87	77	0	244	0	0
normalized size	1	1.	0.46	0.41	0.	1.3	0.	0.
time (sec)	N/A	0.62	0.652	0.202	0.	2.434	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	152	116	0	252	0	0
normalized size	1	1.	1.09	0.83	0.	1.8	0.	0.
time (sec)	N/A	0.517	1.017	0.2	0.	2.413	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	106	0	236	0	0
normalized size	1	1.	1.	1.15	0.	2.57	0.	0.
time (sec)	N/A	0.395	0.494	0.227	0.	2.297	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	119	195	0	234	0	0
normalized size	1	1.	2.64	4.33	0.	5.2	0.	0.
time (sec)	N/A	0.307	0.92	0.227	0.	1.71	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	140	218	0	0	0	0
normalized size	1	1.	0.73	1.13	0.	0.	0.	0.
time (sec)	N/A	0.642	2.58	0.203	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	181	272	1512	0	0	0
normalized size	1	1.	0.94	1.42	7.88	0.	0.	0.
time (sec)	N/A	0.646	2.334	0.215	1.906	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	209	329	0	0	0	0
normalized size	1	1.	1.07	1.69	0.	0.	0.	0.
time (sec)	N/A	0.657	2.757	0.211	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	234	401	0	0	0	0
normalized size	1	1.	1.21	2.08	0.	0.	0.	0.
time (sec)	N/A	0.663	4.215	0.208	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	117	157	0	309	0	0
normalized size	1	1.	2.44	3.27	0.	6.44	0.	0.
time (sec)	N/A	0.339	4.33	0.181	0.	1.89	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	130	191	0	402	0	0
normalized size	1	1.	1.34	1.97	0.	4.14	0.	0.
time (sec)	N/A	0.438	6.18	0.206	0.	1.861	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	209	169	0	351	0	0
normalized size	1	1.	0.89	0.72	0.	1.49	0.	0.
time (sec)	N/A	0.725	5.701	0.31	0.	2.135	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	97	87	0	289	0	0
normalized size	1	1.	0.41	0.37	0.	1.22	0.	0.
time (sec)	N/A	0.735	1.311	0.231	0.	1.933	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	127	143	0	308	0	0
normalized size	1	1.	0.68	0.76	0.	1.64	0.	0.
time (sec)	N/A	0.62	1.904	0.23	0.	1.918	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	115	133	0	293	0	0
normalized size	1	1.	0.82	0.95	0.	2.09	0.	0.
time (sec)	N/A	0.515	1.609	0.228	0.	1.859	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	104	133	0	273	0	0
normalized size	1	1.	1.13	1.45	0.	2.97	0.	0.
time (sec)	N/A	0.386	0.54	0.249	0.	1.751	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	142	245	0	270	0	0
normalized size	1	1.	3.16	5.44	0.	6.	0.	0.
time (sec)	N/A	0.307	1.441	0.231	0.	1.74	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	473	253	0	0	0	0
normalized size	1	1.	1.96	1.05	0.	0.	0.	0.
time (sec)	N/A	0.76	6.456	0.216	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	196	307	0	0	0	0
normalized size	1	1.	0.82	1.29	0.	0.	0.	0.
time (sec)	N/A	0.754	4.823	0.241	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	223	365	1885	0	0	0
normalized size	1	1.	0.93	1.53	7.89	0.	0.	0.
time (sec)	N/A	0.764	6.502	0.237	2.383	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	442	435	0	0	0	0
normalized size	1	1.	1.83	1.8	0.	0.	0.	0.
time (sec)	N/A	0.781	6.638	0.234	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	437	490	0	0	0	0
normalized size	1	1.	1.8	2.02	0.	0.	0.	0.
time (sec)	N/A	0.785	6.662	0.243	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	412	188	0	371	0	0
normalized size	1	1.	8.58	3.92	0.	7.73	0.	0.
time (sec)	N/A	0.339	6.718	0.204	0.	1.836	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	419	233	0	478	0	0
normalized size	1	1.	4.32	2.4	0.	4.93	0.	0.
time (sec)	N/A	0.444	6.759	0.225	0.	1.912	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	419	243	0	517	0	0
normalized size	1	1.	2.89	1.68	0.	3.57	0.	0.
time (sec)	N/A	0.539	6.836	0.251	0.	1.995	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	134	195	0	235	0	0
normalized size	1	1.	2.98	4.33	0.	5.22	0.	0.
time (sec)	N/A	0.306	0.903	0.231	0.	1.708	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	120	141	0	192	0	0
normalized size	1	1.	2.67	3.13	0.	4.27	0.	0.
time (sec)	N/A	0.309	0.519	0.219	0.	1.663	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	62	90	524	153	0	0
normalized size	1	1.	1.38	2.	11.64	3.4	0.	0.
time (sec)	N/A	0.284	0.28	0.211	1.857	1.629	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	44	42	0	116	0	0
normalized size	1	1.	1.02	0.98	0.	2.7	0.	0.
time (sec)	N/A	0.288	0.296	0.21	0.	1.663	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	104	141	0	0	0	0
normalized size	1	1.	1.93	2.61	0.	0.	0.	0.
time (sec)	N/A	0.34	0.413	0.214	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	79	51	0	151	0	0
normalized size	1	1.	1.88	1.21	0.	3.6	0.	0.
time (sec)	N/A	0.321	0.5	0.21	0.	1.682	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	471	252	0	0	0	0
normalized size	1	1.	1.97	1.05	0.	0.	0.	0.
time (sec)	N/A	0.745	6.482	0.238	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	138	213	0	0	0	0
normalized size	1	1.	0.73	1.12	0.	0.	0.	0.
time (sec)	N/A	0.639	2.419	0.202	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	134	172	1139	0	0	0
normalized size	1	1.	0.92	1.19	7.86	0.	0.	0.
time (sec)	N/A	0.529	1.09	0.183	1.887	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	113	133	0	0	0	0
normalized size	1	1.	1.18	1.39	0.	0.	0.	0.
time (sec)	N/A	0.412	1.057	0.215	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	102	136	0	0	0	0
normalized size	1	1.	2.	2.67	0.	0.	0.	0.
time (sec)	N/A	0.333	0.432	0.208	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	103	173	0	404	0	0
normalized size	1	1.	1.98	3.33	0.	7.77	0.	0.
time (sec)	N/A	0.34	0.536	0.177	0.	1.952	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	163	245	0	814	0	0
normalized size	1	1.	1.57	2.36	0.	7.83	0.	0.
time (sec)	N/A	0.436	0.774	0.187	0.	2.009	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	553	347	0	0	0	0
normalized size	1	1.	1.94	1.22	0.	0.	0.	0.
time (sec)	N/A	0.865	6.67	0.199	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	179	305	0	0	0	0
normalized size	1	1.	0.76	1.29	0.	0.	0.	0.
time (sec)	N/A	0.752	5.05	0.238	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	164	268	1512	0	0	0
normalized size	1	1.	0.86	1.4	7.92	0.	0.	0.
time (sec)	N/A	0.639	2.43	0.217	2.007	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	153	228	0	0	0	0
normalized size	1	1.	1.07	1.59	0.	0.	0.	0.
time (sec)	N/A	0.538	1.059	0.19	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	100	190	0	0	0	0
normalized size	1	1.	1.03	1.96	0.	0.	0.	0.
time (sec)	N/A	0.423	0.888	0.213	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	80	50	0	151	0	0
normalized size	1	1.	1.86	1.16	0.	3.51	0.	0.
time (sec)	N/A	0.321	0.513	0.204	0.	1.685	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	163	244	0	814	0	0
normalized size	1	1.	1.57	2.35	0.	7.83	0.	0.
time (sec)	N/A	0.444	0.775	0.187	0.	2.012	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	163	196	0	676	0	0
normalized size	1	1.	1.07	1.29	0.	4.45	0.	0.
time (sec)	N/A	0.54	0.863	0.198	0.	2.114	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	3426	0	0	0	0	0
normalized size	1	1.	30.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.301	14.178	2.313	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	180.023	3.823	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	180.038	3.296	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	180.003	1.707	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.11	0.924	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	6442	0	0	0	0	0
normalized size	1	1.	83.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	19.933	1.348	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	88	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	0.205	0.652	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	91	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	0.153	0.732	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	695	0	753	979	0	0
normalized size	1	1.	2.85	0.	3.09	4.01	0.	0.
time (sec)	N/A	0.615	6.576	0.362	2.324	2.03	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	149	0	571	622	0	0
normalized size	1	1.	0.87	0.	3.32	3.62	0.	0.
time (sec)	N/A	0.467	3.113	0.349	2.3	1.876	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	111	0	421	402	0	0
normalized size	1	1.	1.04	0.	3.93	3.76	0.	0.
time (sec)	N/A	0.354	0.56	0.348	2.143	1.785	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	85	0	0	279	0	0
normalized size	1	1.	1.7	0.	0.	5.58	0.	0.
time (sec)	N/A	0.262	0.38	0.376	0.	1.716	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	218	0	0	0	0	0
normalized size	1	1.	2.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.362	6.503	0.376	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	3157	0	0	0	0	0
normalized size	1	1.	39.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.372	41.46	0.375	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	85	0	0	279	0	0
normalized size	1	1.	1.7	0.	0.	5.58	0.	0.
time (sec)	N/A	0.254	0.395	0.006	0.	1.727	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	85	0	0	279	0	0
normalized size	1	1.	1.7	0.	0.	5.58	0.	0.
time (sec)	N/A	0.254	0.368	0.407	0.	1.691	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	176	0	0	261	0	0
normalized size	1	1.	0.97	0.	0.	1.43	0.	0.
time (sec)	N/A	0.448	17.217	0.911	0.	1.796	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	142	0	0	186	0	0
normalized size	1	1.	1.25	0.	0.	1.63	0.	0.
time (sec)	N/A	0.335	12.662	0.817	0.	1.768	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	109	0	0	113	0	0
normalized size	1	1.	2.02	0.	0.	2.09	0.	0.
time (sec)	N/A	0.251	5.66	0.735	0.	1.761	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	589	0	0	0	0	0
normalized size	1	1.	5.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.38	21.394	0.79	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	1045	0	0	0	0	0
normalized size	1	1.	9.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.371	29.376	0.472	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	1519	0	0	0	0	0
normalized size	1	1.	13.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.323	22.287	1.009	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	4270	0	0	0	0	0
normalized size	1	1.	36.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.368	25.891	0.648	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	311	425	0	0	0	0
normalized size	1	1.	0.91	1.24	0.	0.	0.	0.
time (sec)	N/A	1.743	10.167	0.43	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	281	392	0	0	0	0
normalized size	1	1.	0.97	1.35	0.	0.	0.	0.
time (sec)	N/A	1.428	2.378	0.368	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	255	372	0	0	0	0
normalized size	1	1.	1.09	1.58	0.	0.	0.	0.
time (sec)	N/A	1.129	1.875	0.346	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	249	346	0	0	0	0
normalized size	1	1.	1.4	1.94	0.	0.	0.	0.
time (sec)	N/A	0.803	2.436	0.385	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	197	361	0	0	0	0
normalized size	1	1.	1.61	2.96	0.	0.	0.	0.
time (sec)	N/A	0.57	2.001	0.354	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	211	2835	0	0	0	0
normalized size	1	1.	1.72	23.05	0.	0.	0.	0.
time (sec)	N/A	0.582	1.765	0.398	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	229	2040	0	0	0	0
normalized size	1	1.	1.26	11.21	0.	0.	0.	0.
time (sec)	N/A	0.879	2.145	0.342	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	256	966	0	0	0	0
normalized size	1	1.	1.08	4.08	0.	0.	0.	0.
time (sec)	N/A	1.165	2.275	0.361	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	291	1126	0	0	0	0
normalized size	1	1.	1.	3.86	0.	0.	0.	0.
time (sec)	N/A	1.46	2.862	0.385	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	193	382	0	0	0	0
normalized size	1	1.	0.55	1.09	0.	0.	0.	0.
time (sec)	N/A	1.727	1.385	0.386	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	113	356	0	0	0	0
normalized size	1	1.	0.38	1.21	0.	0.	0.	0.
time (sec)	N/A	1.503	0.742	0.322	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	257	372	0	0	0	0
normalized size	1	1.	1.09	1.58	0.	0.	0.	0.
time (sec)	N/A	1.139	7.752	0.327	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	148	384	0	0	0	0
normalized size	1	1.	0.82	2.13	0.	0.	0.	0.
time (sec)	N/A	0.85	0.794	0.309	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	207	2892	0	0	0	0
normalized size	1	1.	1.14	15.89	0.	0.	0.	0.
time (sec)	N/A	0.876	1.766	0.339	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	191	3499	0	0	0	0
normalized size	1	1.	1.03	18.81	0.	0.	0.	0.
time (sec)	N/A	0.887	1.444	0.337	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	218	2684	0	0	0	0
normalized size	1	1.	0.9	11.05	0.	0.	0.	0.
time (sec)	N/A	1.194	2.406	0.356	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	464	1138	0	0	0	0
normalized size	1	1.	1.55	3.79	0.	0.	0.	0.
time (sec)	N/A	1.497	6.488	0.355	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	532	1298	0	0	0	0
normalized size	1	1.	1.49	3.64	0.	0.	0.	0.
time (sec)	N/A	1.808	6.539	0.41	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	120	366	0	0	0	0
normalized size	1	1.	0.3	0.9	0.	0.	0.	0.
time (sec)	N/A	2.089	1.195	0.372	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	189	382	0	0	0	0
normalized size	1	1.	0.54	1.09	0.	0.	0.	0.
time (sec)	N/A	1.757	1.365	0.394	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	281	394	0	0	0	0
normalized size	1	1.	0.97	1.36	0.	0.	0.	0.
time (sec)	N/A	1.42	2.484	0.365	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	158	415	0	0	0	0
normalized size	1	1.	0.68	1.77	0.	0.	0.	0.
time (sec)	N/A	1.142	1.869	0.338	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	278	2945	0	0	0	0
normalized size	1	1.	1.15	12.22	0.	0.	0.	0.
time (sec)	N/A	1.14	6.441	0.361	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	245	3549	0	0	0	0
normalized size	1	1.	1.01	14.6	0.	0.	0.	0.
time (sec)	N/A	1.175	2.588	0.375	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	246	4183	0	0	0	0
normalized size	1	1.	1.01	17.21	0.	0.	0.	0.
time (sec)	N/A	1.197	2.794	0.366	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	464	3455	0	0	0	0
normalized size	1	1.	1.55	11.52	0.	0.	0.	0.
time (sec)	N/A	1.483	6.597	0.335	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	532	1313	0	0	0	0
normalized size	1	1.	1.49	3.68	0.	0.	0.	0.
time (sec)	N/A	1.787	6.716	0.385	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	600	1473	0	0	0	0
normalized size	1	1.	1.45	3.56	0.	0.	0.	0.
time (sec)	N/A	2.08	6.691	0.454	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	226	392	0	0	0	0
normalized size	1	1.	0.49	0.85	0.	0.	0.	0.
time (sec)	N/A	2.39	3.687	0.464	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	212	404	0	0	0	0
normalized size	1	1.	0.52	0.99	0.	0.	0.	0.
time (sec)	N/A	2.042	3.144	0.41	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	333	425	0	0	0	0
normalized size	1	1.	0.97	1.24	0.	0.	0.	0.
time (sec)	N/A	1.72	5.506	0.394	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	181	436	0	0	0	0
normalized size	1	1.	0.63	1.51	0.	0.	0.	0.
time (sec)	N/A	1.437	3.945	0.352	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	284	2997	0	0	0	0
normalized size	1	1.	0.97	10.19	0.	0.	0.	0.
time (sec)	N/A	1.455	6.535	0.395	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	267	3601	0	0	0	0
normalized size	1	1.	0.9	12.08	0.	0.	0.	0.
time (sec)	N/A	1.488	4.865	0.398	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	406	4237	0	0	0	0
normalized size	1	1.	1.35	14.12	0.	0.	0.	0.
time (sec)	N/A	1.541	6.625	0.404	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	464	4829	0	0	0	0
normalized size	1	1.	1.55	16.1	0.	0.	0.	0.
time (sec)	N/A	1.534	6.692	0.402	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	532	3910	0	0	0	0
normalized size	1	1.	1.49	10.95	0.	0.	0.	0.
time (sec)	N/A	1.84	6.769	0.363	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	600	1484	0	0	0	0
normalized size	1	1.	1.45	3.58	0.	0.	0.	0.
time (sec)	N/A	2.171	6.818	0.417	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	471	668	1644	0	0	0	0
normalized size	1	1.	1.42	3.49	0.	0.	0.	0.
time (sec)	N/A	2.487	6.887	0.486	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	174	415	0	0	0	0
normalized size	1	1.	0.74	1.77	0.	0.	0.	0.
time (sec)	N/A	1.121	1.537	0.335	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	157	382	0	0	0	0
normalized size	1	1.	0.87	2.12	0.	0.	0.	0.
time (sec)	N/A	0.838	0.724	0.31	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	215	361	0	0	0	0
normalized size	1	1.	1.76	2.96	0.	0.	0.	0.
time (sec)	N/A	0.54	2.564	0.343	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	111	334	0	0	0	0
normalized size	1	1.	1.63	4.91	0.	0.	0.	0.
time (sec)	N/A	0.279	0.385	0.305	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	148	925	0	0	0	0
normalized size	1	1.	1.22	7.64	0.	0.	0.	0.
time (sec)	N/A	0.57	0.764	0.376	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	204	778	0	0	0	0
normalized size	1	1.	1.14	4.35	0.	0.	0.	0.
time (sec)	N/A	0.86	1.623	0.346	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	240	955	0	0	0	0
normalized size	1	1.	1.03	4.1	0.	0.	0.	0.
time (sec)	N/A	1.149	2.305	0.332	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	282	2994	0	0	0	0
normalized size	1	1.	0.96	10.18	0.	0.	0.	0.
time (sec)	N/A	1.42	6.559	0.394	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	238	2947	0	0	0	0
normalized size	1	1.	0.99	12.23	0.	0.	0.	0.
time (sec)	N/A	1.136	5.031	0.346	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	200	2890	0	0	0	0
normalized size	1	1.	1.1	15.88	0.	0.	0.	0.
time (sec)	N/A	0.844	1.647	0.333	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	213	2839	0	0	0	0
normalized size	1	1.	1.73	23.08	0.	0.	0.	0.
time (sec)	N/A	0.561	1.929	0.384	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	170	925	0	0	0	0
normalized size	1	1.	1.4	7.64	0.	0.	0.	0.
time (sec)	N/A	0.564	0.715	0.375	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	92	361	0	0	0	0
normalized size	1	1.	0.52	2.05	0.	0.	0.	0.
time (sec)	N/A	0.879	0.75	0.353	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	134	877	0	0	0	0
normalized size	1	1.	0.57	3.7	0.	0.	0.	0.
time (sec)	N/A	1.159	1.183	0.332	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	155	1177	0	0	0	0
normalized size	1	1.	0.53	4.	0.	0.	0.	0.
time (sec)	N/A	1.474	1.542	0.359	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	356	3655	0	0	0	0
normalized size	1	1.	1.	10.24	0.	0.	0.	0.
time (sec)	N/A	1.728	6.765	0.401	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	250	3598	0	0	0	0
normalized size	1	1.	0.84	12.07	0.	0.	0.	0.
time (sec)	N/A	1.424	5.056	0.401	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	230	3551	0	0	0	0
normalized size	1	1.	0.95	14.61	0.	0.	0.	0.
time (sec)	N/A	1.136	2.467	0.362	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	180	3497	0	0	0	0
normalized size	1	1.	0.97	18.8	0.	0.	0.	0.
time (sec)	N/A	0.852	1.216	0.327	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	230	2040	0	0	0	0
normalized size	1	1.	1.26	11.21	0.	0.	0.	0.
time (sec)	N/A	0.832	2.181	0.322	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	189	778	0	0	0	0
normalized size	1	1.	1.06	4.35	0.	0.	0.	0.
time (sec)	N/A	0.842	1.6	0.341	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	133	877	0	0	0	0
normalized size	1	1.	0.56	3.7	0.	0.	0.	0.
time (sec)	N/A	1.161	1.13	0.342	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	104	395	0	0	0	0
normalized size	1	1.	0.36	1.36	0.	0.	0.	0.
time (sec)	N/A	1.472	1.195	0.346	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	171	947	0	0	0	0
normalized size	1	1.	0.49	2.71	0.	0.	0.	0.
time (sec)	N/A	1.794	6.162	0.391	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	126	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.293	2.18	0.227	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.285	180.007	0.603	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.276	180.003	0.512	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	168.115	0.283	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.156	0.125	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.275	0.12	0.253	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	96	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.281	0.203	0.453	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	96	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.283	0.207	0.533	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.357	180.037	0.243	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.351	180.004	0.244	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.319	180.002	0.233	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.326	180.097	0.234	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.367	81.242	0.234	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	3845	0	0	0	0	0
normalized size	1	1.	33.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.357	52.3	0.23	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	832	0	0	0	0	0
normalized size	1	1.	7.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	13.425	0.007	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.313	180.135	0.286	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	382	0	0	0	0	0
normalized size	1	1.	3.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.37	17.709	0.385	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	202	0	0	0	0	0
normalized size	1	1.	1.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.367	6.168	0.338	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.359	103.719	0.261	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	15.996	0.214	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.341	175.476	0.244	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.353	84.49	0.297	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	133	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.287	40.633	5.265	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	132	9871	0	69	0	1314
normalized size	1	1.	2.32	173.18	0.	1.21	0.	23.05
time (sec)	N/A	0.228	80.914	5.367	0.	1.728	0.	27.221

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	210	0	1320	1609	0	0
normalized size	1	1.	1.03	0.	6.5	7.93	0.	0.
time (sec)	N/A	0.678	6.658	11.848	2.982	2.016	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	143	0	655	741	0	0
normalized size	1	1.	1.13	0.	5.16	5.83	0.	0.
time (sec)	N/A	0.408	1.323	11.848	2.217	1.887	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	96	0	279	320	0	6257
normalized size	1	1.	1.66	0.	4.81	5.52	0.	107.88
time (sec)	N/A	0.163	0.673	20.304	1.718	1.821	0.	29.699

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	68.242	10.299	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	135	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	45.015	11.248	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	136	0	0	0	0	0
normalized size	1	1.	1.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	37.122	11.237	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	94	0	0	113	0	0
normalized size	1	1.	1.84	0.	0.	2.22	0.	0.
time (sec)	N/A	0.172	0.953	0.446	0.	1.771	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.382	145.704	0.615	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.374	138.543	0.56	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	207	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.369	25.637	29.86	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	196	205	0	0
normalized size	1	1.	1.	0.	3.56	3.73	0.	0.
time (sec)	N/A	0.171	0.715	0.48	1.856	1.824	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	132	0	0	324	0	0
normalized size	1	1.	1.06	0.	0.	2.59	0.	0.
time (sec)	N/A	0.411	26.008	0.53	0.	1.843	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	183	0	0	460	0	0
normalized size	1	1.	0.9	0.	0.	2.25	0.	0.
time (sec)	N/A	0.665	30.039	0.636	0.	1.907	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	2681	0	0	664	0	0
normalized size	1	1.	9.24	0.	0.	2.29	0.	0.
time (sec)	N/A	0.937	37.399	0.645	0.	2.012	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	139	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.46	40.128	5.358	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	38	122	65	38
normalized size	1	1.	1.	0.85	1.15	3.7	1.97	1.15
time (sec)	N/A	0.045	0.014	0.016	1.407	1.62	2.51	1.16

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	28	38	93	41	38
normalized size	1	1.	0.91	0.85	1.15	2.82	1.24	1.15
time (sec)	N/A	0.036	0.091	0.013	1.299	1.645	1.35	1.29

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	26	25	30	62	0	31
normalized size	1	1.	1.08	1.04	1.25	2.58	0.	1.29
time (sec)	N/A	0.022	0.035	0.023	1.241	1.669	0.	1.146

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	28	34	82	0	35
normalized size	1	1.	1.32	1.12	1.36	3.28	0.	1.4
time (sec)	N/A	0.036	0.039	0.03	1.26	1.636	0.	1.286

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	27	32	69	0	32
normalized size	1	1.	0.97	0.9	1.07	2.3	0.	1.07
time (sec)	N/A	0.042	0.019	0.033	1.244	1.566	0.	1.293

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	35	92	0	35
normalized size	1	1.	1.	0.82	1.06	2.79	0.	1.06
time (sec)	N/A	0.043	0.024	0.034	1.115	1.544	0.	1.257

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	35	103	0	35
normalized size	1	1.	1.	0.82	1.06	3.12	0.	1.06
time (sec)	N/A	0.043	0.019	0.031	1.242	1.615	0.	1.146

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	45	61	173	85	61
normalized size	1	1.	0.96	0.82	1.11	3.15	1.55	1.11
time (sec)	N/A	0.067	0.326	0.019	1.161	1.666	4.905	1.244

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	45	61	134	87	61
normalized size	1	1.	0.69	0.82	1.11	2.44	1.58	1.11
time (sec)	N/A	0.048	0.051	0.017	1.148	1.648	2.097	1.139

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	46	55	108	0	57
normalized size	1	1.	1.	0.98	1.17	2.3	0.	1.21
time (sec)	N/A	0.039	0.024	0.028	1.171	1.702	0.	1.321

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	38	46	54	112	0	55
normalized size	1	1.	0.88	1.07	1.26	2.6	0.	1.28
time (sec)	N/A	0.055	0.023	0.036	1.181	1.666	0.	1.299

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	48	58	146	0	59
normalized size	1	1.	0.89	1.02	1.23	3.11	0.	1.26
time (sec)	N/A	0.065	0.017	0.04	1.184	1.662	0.	1.313

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	20	39	55	130	0	55
normalized size	1	1.	0.67	1.3	1.83	4.33	0.	1.83
time (sec)	N/A	0.058	0.023	0.035	1.145	1.557	0.	1.163

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	39	58	138	0	58
normalized size	1	1.	1.	0.71	1.05	2.51	0.	1.05
time (sec)	N/A	0.066	0.029	0.038	1.231	1.629	0.	1.199

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	39	58	162	0	58
normalized size	1	1.	1.	0.71	1.05	2.95	0.	1.05
time (sec)	N/A	0.067	0.03	0.039	1.234	1.608	0.	1.257

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	39	58	171	0	58
normalized size	1	1.	1.	0.71	1.05	3.11	0.	1.05
time (sec)	N/A	0.066	0.032	0.037	1.151	1.633	0.	1.287

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	58	78	244	104	78
normalized size	1	1.	1.1	0.79	1.07	3.34	1.42	1.07
time (sec)	N/A	0.074	0.323	0.021	0.997	1.71	13.147	1.275

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	58	78	209	107	78
normalized size	1	1.	0.96	0.79	1.07	2.86	1.47	1.07
time (sec)	N/A	0.071	0.285	0.02	1.122	1.647	7.554	1.202

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	30	57	78	169	102	78
normalized size	1	1.	0.67	1.27	1.73	3.76	2.27	1.73
time (sec)	N/A	0.045	0.118	0.016	1.156	1.696	4.398	1.27

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	62	74	146	0	76
normalized size	1	1.	1.	0.95	1.14	2.25	0.	1.17
time (sec)	N/A	0.044	0.028	0.033	0.987	1.751	0.	1.266

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	63	73	193	0	74
normalized size	1	1.	1.	1.02	1.18	3.11	0.	1.19
time (sec)	N/A	0.062	0.029	0.036	1.109	1.76	0.	1.173

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	62	73	180	0	74
normalized size	1	1.	0.87	1.02	1.2	2.95	0.	1.21
time (sec)	N/A	0.069	0.021	0.039	1.069	1.736	0.	1.21

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	64	78	221	0	80
normalized size	1	1.	0.88	0.98	1.2	3.4	0.	1.23
time (sec)	N/A	0.071	0.019	0.04	1.088	1.71	0.	1.238

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	20	49	73	170	0	73
normalized size	1	1.	0.67	1.63	2.43	5.67	0.	2.43
time (sec)	N/A	0.057	0.023	0.04	1.017	1.594	0.	1.19

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	71	49	76	197	0	76
normalized size	1	1.	1.16	0.8	1.25	3.23	0.	1.25
time (sec)	N/A	0.065	0.03	0.041	1.138	1.61	0.	1.233

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	76	208	0	76
normalized size	1	1.	1.	0.67	1.04	2.85	0.	1.04
time (sec)	N/A	0.072	0.03	0.041	1.235	1.773	0.	1.268

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	76	228	0	76
normalized size	1	1.	1.	0.67	1.04	3.12	0.	1.04
time (sec)	N/A	0.073	0.03	0.041	1.072	2.042	0.	1.279

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	100	71	96	324	97	96
normalized size	1	1.	1.1	0.78	1.05	3.56	1.07	1.05
time (sec)	N/A	0.085	0.813	0.023	1.101	1.98	24.931	1.29

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	90	70	96	281	119	96
normalized size	1	1.	1.02	0.8	1.09	3.19	1.35	1.09
time (sec)	N/A	0.081	0.529	0.021	1.113	2.01	18.559	1.223

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	80	70	96	247	119	96
normalized size	1	1.	1.19	1.04	1.43	3.69	1.78	1.43
time (sec)	N/A	0.075	0.333	0.022	1.125	1.954	10.28	1.187

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	30	71	96	207	121	96
normalized size	1	1.	0.67	1.58	2.13	4.6	2.69	2.13
time (sec)	N/A	0.045	0.095	0.019	1.09	1.369	5.227	1.258

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	78	92	180	0	93
normalized size	1	1.	1.	0.96	1.14	2.22	0.	1.15
time (sec)	N/A	0.047	0.037	0.032	1.142	1.453	0.	1.195

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	79	90	224	0	92
normalized size	1	1.	1.	1.01	1.15	2.87	0.	1.18
time (sec)	N/A	0.066	0.037	0.037	1.153	1.534	0.	1.261

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	54	79	89	232	0	90
normalized size	1	1.	0.68	0.99	1.11	2.9	0.	1.12
time (sec)	N/A	0.076	0.072	0.044	1.068	1.417	0.	1.174

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	60	80	85	157	102	103
normalized size	1	1.	0.71	0.94	1.	1.85	1.2	1.21
time (sec)	N/A	0.087	0.129	0.024	1.071	1.495	5.595	1.295

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	64	72	127	66	86
normalized size	1	1.	0.75	0.96	1.07	1.9	0.99	1.28
time (sec)	N/A	0.078	0.104	0.023	1.126	1.484	3.403	1.237

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	38	48	55	97	53	61
normalized size	1	1.	0.78	0.98	1.12	1.98	1.08	1.24
time (sec)	N/A	0.07	0.059	0.02	0.976	1.33	1.456	1.25

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	32	41	63	37	42
normalized size	1	1.	0.81	1.03	1.32	2.03	1.19	1.35
time (sec)	N/A	0.046	0.02	0.02	1.078	1.36	0.968	1.325

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	33	42	74	0	45
normalized size	1	1.	1.	1.03	1.31	2.31	0.	1.41
time (sec)	N/A	0.036	0.016	0.033	1.065	1.469	0.	1.278

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	49	58	134	0	61
normalized size	1	1.	1.	1.07	1.26	2.91	0.	1.33
time (sec)	N/A	0.062	0.033	0.037	1.036	1.562	0.	1.227

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	64	74	198	0	77
normalized size	1	1.	1.	1.02	1.17	3.14	0.	1.22
time (sec)	N/A	0.079	0.036	0.04	1.098	1.588	0.	1.302

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	81	88	281	0	90
normalized size	1	1.	1.	0.99	1.07	3.43	0.	1.1
time (sec)	N/A	0.082	0.044	0.043	1.106	1.473	0.	1.193

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	73	83	95	220	238	144
normalized size	1	1.	0.84	0.95	1.09	2.53	2.74	1.66
time (sec)	N/A	0.089	0.565	0.034	1.122	1.468	4.72	1.261

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	66	80	190	209	122
normalized size	1	1.	1.01	0.94	1.14	2.71	2.99	1.74
time (sec)	N/A	0.08	0.178	0.035	1.139	1.418	2.676	1.335

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	55	50	63	146	185	95
normalized size	1	1.	1.06	0.96	1.21	2.81	3.56	1.83
time (sec)	N/A	0.073	0.187	0.033	1.086	1.495	1.553	1.296

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	27	35	46	104	95	76
normalized size	1	1.	0.73	0.95	1.24	2.81	2.57	2.05
time (sec)	N/A	0.048	0.028	0.03	1.405	1.431	1.029	1.286

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	36	50	62	162	0	61
normalized size	1	1.	0.69	0.96	1.19	3.12	0.	1.17
time (sec)	N/A	0.049	0.054	0.043	1.09	1.447	0.	1.249

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	45	68	92	269	0	93
normalized size	1	1.	0.66	1.	1.35	3.96	0.	1.37
time (sec)	N/A	0.074	0.153	0.046	1.094	1.41	0.	1.289

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	61	83	108	389	0	117
normalized size	1	1.	0.72	0.98	1.27	4.58	0.	1.38
time (sec)	N/A	0.087	0.189	0.052	1.158	1.499	0.	1.204

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	98	99	122	508	0	139
normalized size	1	1.	0.97	0.98	1.21	5.03	0.	1.38
time (sec)	N/A	0.099	2.212	0.055	1.111	1.537	0.	1.245

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	101	128	317	760	120
normalized size	1	1.	0.95	0.91	1.15	2.86	6.85	1.08
time (sec)	N/A	0.106	0.7	0.039	1.108	1.506	12.059	1.173

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	78	85	109	284	456	99
normalized size	1	1.	0.84	0.91	1.17	3.05	4.9	1.06
time (sec)	N/A	0.096	2.134	0.039	1.132	1.509	7.043	1.199

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	70	68	96	251	303	76
normalized size	1	1.	0.95	0.92	1.3	3.39	4.09	1.03
time (sec)	N/A	0.09	0.403	0.036	1.122	1.413	3.691	1.26

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	65	54	81	197	257	61
normalized size	1	1.	1.08	0.9	1.35	3.28	4.28	1.02
time (sec)	N/A	0.081	0.66	0.034	1.093	1.379	2.178	1.236

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	59	111	99	38
normalized size	1	1.	1.	1.1	1.97	3.7	3.3	1.27
time (sec)	N/A	0.046	0.029	0.032	1.068	1.331	2.097	1.257

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	68	97	284	0	80
normalized size	1	1.	0.7	0.92	1.31	3.84	0.	1.08
time (sec)	N/A	0.058	0.17	0.048	0.999	1.482	0.	1.306

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	61	86	123	404	0	104
normalized size	1	1.	0.68	0.96	1.37	4.49	0.	1.16
time (sec)	N/A	0.084	0.302	0.049	1.016	1.518	0.	1.321

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	71	102	139	522	0	116
normalized size	1	1.	0.66	0.94	1.29	4.83	0.	1.07
time (sec)	N/A	0.102	0.577	0.055	1.007	1.439	0.	1.291

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	81	118	153	641	0	131
normalized size	1	1.	0.64	0.94	1.21	5.09	0.	1.04
time (sec)	N/A	0.113	5.723	0.062	0.984	1.544	0.	1.27

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	119	103	142	381	796	113
normalized size	1	1.	1.03	0.89	1.22	3.28	6.86	0.97
time (sec)	N/A	0.104	0.843	0.04	1.004	1.601	14.559	1.194

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	127	86	127	346	527	89
normalized size	1	1.	1.34	0.91	1.34	3.64	5.55	0.94
time (sec)	N/A	0.097	6.553	0.039	1.006	1.548	9.818	1.216

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	61	72	112	292	466	74
normalized size	1	1.	0.73	0.87	1.35	3.52	5.61	0.89
time (sec)	N/A	0.093	0.357	0.037	1.111	1.539	4.086	1.234

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	53	43	90	174	76	51
normalized size	1	1.	1.77	1.43	3.	5.8	2.53	1.7
time (sec)	N/A	0.065	0.184	0.034	1.085	1.367	3.983	1.186

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	30	33	77	147	129	38
normalized size	1	1.	0.65	0.72	1.67	3.2	2.8	0.83
time (sec)	N/A	0.052	0.028	0.03	1.088	1.415	3.863	1.18

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	62	86	128	405	0	93
normalized size	1	1.	0.64	0.89	1.32	4.18	0.	0.96
time (sec)	N/A	0.064	0.339	0.046	1.118	1.485	0.	1.223

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	73	104	154	525	0	117
normalized size	1	1.	0.66	0.94	1.39	4.73	0.	1.05
time (sec)	N/A	0.094	0.985	0.05	1.052	1.442	0.	1.246

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	85	120	170	643	0	131
normalized size	1	1.	0.65	0.92	1.3	4.91	0.	1.
time (sec)	N/A	0.118	3.695	0.055	1.085	1.548	0.	1.27

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	118	42	0	232	0	0
normalized size	1	1.	2.31	0.82	0.	4.55	0.	0.
time (sec)	N/A	0.062	0.145	0.103	0.	1.443	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	80	0	0	701	1856	171
normalized size	1	1.	0.7	0.	0.	6.15	16.28	1.5
time (sec)	N/A	0.116	0.258	3.113	0.	1.626	134.487	1.214

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	65	0	0	468	1061	136
normalized size	1	1.	0.71	0.	0.	5.14	11.66	1.49
time (sec)	N/A	0.095	0.149	2.313	0.	1.858	77.423	1.187

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	0	0	292	530	101
normalized size	1	1.	0.74	0.	0.	4.29	7.79	1.49
time (sec)	N/A	0.086	0.202	2.069	0.	1.744	18.31	1.163

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	0	0	139	190	61
normalized size	1	1.	0.93	0.	0.	3.39	4.63	1.49
time (sec)	N/A	0.052	0.268	1.553	0.	1.717	7.212	1.252

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.034	0.562	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.03	0.819	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.034	0.917	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.033	0.982	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	71	95	88	193	192	124
normalized size	1	1.	0.68	0.9	0.84	1.84	1.83	1.18
time (sec)	N/A	0.166	0.194	0.029	1.211	1.712	5.159	1.378

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	54	77	70	162	144	84
normalized size	1	1.	0.67	0.95	0.86	2.	1.78	1.04
time (sec)	N/A	0.128	0.103	0.027	1.297	1.659	2.679	1.369

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	57	53	128	119	63
normalized size	1	1.	0.65	0.88	0.82	1.97	1.83	0.97
time (sec)	N/A	0.093	0.098	0.024	1.241	1.707	1.328	1.275

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	74	63	77	174	0	117
normalized size	1	1.	1.45	1.24	1.51	3.41	0.	2.29
time (sec)	N/A	0.064	0.072	0.05	1.232	1.713	0.	1.233

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	73	236	0	146
normalized size	1	1.	1.83	1.39	1.78	5.76	0.	3.56
time (sec)	N/A	0.055	0.043	0.046	1.806	1.753	0.	1.402

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	109	81	89	300	0	128
normalized size	1	1.	2.1	1.56	1.71	5.77	0.	2.46
time (sec)	N/A	0.077	0.042	0.056	1.685	1.741	0.	1.404

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	95	80	82	316	0	155
normalized size	1	1.	1.83	1.54	1.58	6.08	0.	2.98
time (sec)	N/A	0.108	0.038	0.053	1.058	1.648	0.	1.355

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	135	102	108	377	0	157
normalized size	1	1.	1.82	1.38	1.46	5.09	0.	2.12
time (sec)	N/A	0.126	0.051	0.056	1.112	1.654	0.	1.329

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	177	124	124	466	0	194
normalized size	1	1.	1.97	1.38	1.38	5.18	0.	2.16
time (sec)	N/A	0.129	0.07	0.059	1.068	1.783	0.	1.367

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	86	151	142	250	275	166
normalized size	1	1.	0.64	1.12	1.05	1.85	2.04	1.23
time (sec)	N/A	0.248	0.48	0.04	1.133	1.723	21.2	1.368

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	76	142	126	211	309	143
normalized size	1	1.	0.74	1.38	1.22	2.05	3.	1.39
time (sec)	N/A	0.168	0.43	0.034	1.133	1.656	11.815	1.35

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	105	57	95	93	174	172	97
normalized size	1	1.15	0.63	1.04	1.02	1.91	1.89	1.07
time (sec)	N/A	0.13	0.197	0.033	1.079	1.67	4.647	1.293

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	86	101	231	0	136
normalized size	1	1.	1.	1.21	1.42	3.25	0.	1.92
time (sec)	N/A	0.12	0.367	0.065	1.117	1.733	0.	1.333

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	94	89	107	281	0	193
normalized size	1	1.	1.27	1.2	1.45	3.8	0.	2.61
time (sec)	N/A	0.104	0.557	0.064	1.599	1.791	0.	1.337

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	102	93	140	352	0	173
normalized size	1	1.	1.4	1.27	1.92	4.82	0.	2.37
time (sec)	N/A	0.13	0.651	0.073	1.724	1.735	0.	1.452

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	140	117	112	410	0	190
normalized size	1	1.	1.92	1.6	1.53	5.62	0.	2.6
time (sec)	N/A	0.221	0.538	0.072	1.64	1.721	0.	1.401

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	209	112	176	406	0	221
normalized size	1	1.	2.55	1.37	2.15	4.95	0.	2.7
time (sec)	N/A	0.201	0.114	0.075	1.094	1.81	0.	1.42

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	189	136	147	493	0	221
normalized size	1	1.	1.89	1.36	1.47	4.93	0.	2.21
time (sec)	N/A	0.21	0.729	0.074	1.039	1.817	0.	1.332

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	229	160	252	567	0	308
normalized size	1	1.	1.85	1.29	2.03	4.57	0.	2.48
time (sec)	N/A	0.256	0.684	0.08	1.397	1.745	0.	1.459

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	86	194	174	248	379	166
normalized size	1	1.	0.65	1.47	1.32	1.88	2.87	1.26
time (sec)	N/A	0.305	0.647	0.039	1.115	1.787	9.707	1.347

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	133	76	156	143	215	328	143
normalized size	1	1.14	0.65	1.33	1.22	1.84	2.8	1.22
time (sec)	N/A	0.181	0.397	0.036	1.1	1.709	5.616	1.374

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	82	111	134	267	0	194
normalized size	1	1.	0.83	1.12	1.35	2.7	0.	1.96
time (sec)	N/A	0.17	0.619	0.072	1.107	1.763	0.	1.328

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	106	105	126	321	0	219
normalized size	1	1.	1.15	1.14	1.37	3.49	0.	2.38
time (sec)	N/A	0.138	1.06	0.073	1.739	1.794	0.	1.4

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	112	113	167	397	0	248
normalized size	1	1.	1.14	1.15	1.7	4.05	0.	2.53
time (sec)	N/A	0.149	1.039	0.077	1.625	1.881	0.	1.402

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	148	117	158	451	0	217
normalized size	1	1.	1.63	1.29	1.74	4.96	0.	2.38
time (sec)	N/A	0.171	0.396	0.079	1.752	1.781	0.	1.33

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	133	141	198	497	0	235
normalized size	1	1.	1.33	1.41	1.98	4.97	0.	2.35
time (sec)	N/A	0.221	0.53	0.083	1.734	1.7	0.	1.392

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	267	136	209	501	0	265
normalized size	1	1.	2.67	1.36	2.09	5.01	0.	2.65
time (sec)	N/A	0.24	0.13	0.082	1.137	1.732	0.	1.377

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	252	160	270	575	0	308
normalized size	1	1.	2.03	1.29	2.18	4.64	0.	2.48
time (sec)	N/A	0.282	3.535	0.086	1.094	1.814	0.	1.43

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	151	182	173	215	381	143
normalized size	1	1.	1.1	1.33	1.26	1.57	2.78	1.04
time (sec)	N/A	0.157	0.433	0.055	1.129	1.649	6.287	1.399

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	95	135	169	306	0	244
normalized size	1	1.	0.81	1.15	1.44	2.62	0.	2.09
time (sec)	N/A	0.199	0.852	0.078	1.142	1.787	0.	1.197

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	136	127	158	360	0	262
normalized size	1	1.	1.17	1.09	1.36	3.1	0.	2.26
time (sec)	N/A	0.168	1.528	0.073	1.628	1.757	0.	1.402

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	281	245	348	182	1360	154
normalized size	1	1.	2.7	2.36	3.35	1.75	13.08	1.48
time (sec)	N/A	0.136	5.104	0.091	1.558	1.72	74.381	1.341

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	271	245	320	149	1222	154
normalized size	1	1.	3.11	2.82	3.68	1.71	14.05	1.77
time (sec)	N/A	0.13	1.652	0.08	1.608	1.658	43.514	1.318

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	46	141	211	116	563	101
normalized size	1	1.	0.74	2.27	3.4	1.87	9.08	1.63
time (sec)	N/A	0.117	0.083	0.072	1.657	1.62	19.689	1.274

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	161	142	180	85	366	97
normalized size	1	1.	3.58	3.16	4.	1.89	8.13	2.16
time (sec)	N/A	0.072	0.561	0.06	1.595	1.575	9.314	1.283

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	37	37	70	109	0	42
normalized size	1	1.	1.68	1.68	3.18	4.95	0.	1.91
time (sec)	N/A	0.072	0.089	0.088	2.098	1.793	0.	1.361

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	69	56	95	173	0	88
normalized size	1	1.	2.38	1.93	3.28	5.97	0.	3.03
time (sec)	N/A	0.055	0.215	0.104	1.154	1.596	0.	1.374

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	94	94	155	247	0	127
normalized size	1	1.	1.77	1.77	2.92	4.66	0.	2.4
time (sec)	N/A	0.103	0.399	0.114	1.109	1.931	0.	1.367

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	126	132	207	336	0	173
normalized size	1	1.	1.75	1.83	2.88	4.67	0.	2.4
time (sec)	N/A	0.123	0.575	0.135	1.109	1.652	0.	1.308

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	125	170	263	396	0	212
normalized size	1	1.	1.32	1.79	2.77	4.17	0.	2.23
time (sec)	N/A	0.134	1.079	0.134	1.174	1.705	0.	1.384

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	189	208	316	487	0	252
normalized size	1	1.	1.66	1.82	2.77	4.27	0.	2.21
time (sec)	N/A	0.138	0.624	0.154	1.103	1.704	0.	1.428

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	209	300	537	386	3766	196
normalized size	1	1.	1.88	2.7	4.84	3.48	33.93	1.77
time (sec)	N/A	0.245	1.433	0.112	1.684	1.666	86.745	1.339

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	165	198	421	313	2263	143
normalized size	1	1.	1.99	2.39	5.07	3.77	27.27	1.72
time (sec)	N/A	0.222	0.922	0.104	1.725	1.666	48.8	1.281

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	163	305	259	1358	123
normalized size	1	1.	1.	2.36	4.42	3.75	19.68	1.78
time (sec)	N/A	0.272	0.147	0.095	1.614	1.692	28.227	1.425

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	117	64	188	201	479	105
normalized size	1	1.	2.49	1.36	4.	4.28	10.19	2.23
time (sec)	N/A	0.072	0.317	0.081	1.702	1.67	14.53	1.255

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	115	40	74	301	0	51
normalized size	1	1.	2.88	1.	1.85	7.52	0.	1.27
time (sec)	N/A	0.157	0.148	0.128	1.111	1.621	0.	1.349

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	216	77	157	433	0	122
normalized size	1	1.	4.	1.43	2.91	8.02	0.	2.26
time (sec)	N/A	0.096	0.762	0.145	1.161	1.709	0.	1.371

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	364	114	217	659	0	157
normalized size	1	1.	4.67	1.46	2.78	8.45	0.	2.01
time (sec)	N/A	0.22	0.75	0.16	1.193	1.703	0.	1.365

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	472	153	269	801	0	197
normalized size	1	1.	5.19	1.68	2.96	8.8	0.	2.16
time (sec)	N/A	0.251	1.248	0.171	1.196	1.718	0.	1.341

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	197	205	424	423	2263	158
normalized size	1	1.	2.03	2.11	4.37	4.36	23.33	1.63
time (sec)	N/A	0.263	1.057	0.115	1.779	1.676	111.301	1.352

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	96	106	308	365	1246	108
normalized size	1	1.	1.26	1.39	4.05	4.8	16.39	1.42
time (sec)	N/A	0.181	0.665	0.104	1.792	1.715	60.818	1.307

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	145	83	192	308	602	81
normalized size	1	1.	2.38	1.36	3.15	5.05	9.87	1.33
time (sec)	N/A	0.109	0.401	0.101	1.727	1.614	35.121	1.323

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	185	82	193	528	0	89
normalized size	1	1.	2.72	1.21	2.84	7.76	0.	1.31
time (sec)	N/A	0.193	0.378	0.147	1.166	1.712	0.	1.322

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	255	119	273	745	0	147
normalized size	1	1.	3.11	1.45	3.33	9.09	0.	1.79
time (sec)	N/A	0.208	1.477	0.166	1.035	1.734	0.	1.444

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	308	157	333	969	0	193
normalized size	1	1.	2.91	1.48	3.14	9.14	0.	1.82
time (sec)	N/A	0.269	5.632	0.178	1.215	1.784	0.	1.429

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	171	130	479	624	0	162
normalized size	1	1.	1.19	0.9	3.33	4.33	0.	1.12
time (sec)	N/A	0.162	0.92	0.132	1.132	1.654	0.	1.324

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	109	85	0	432	0	0
normalized size	1	1.	0.56	0.44	0.	2.24	0.	0.
time (sec)	N/A	0.568	1.197	0.774	0.	1.699	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	99	75	0	346	0	0
normalized size	1	1.	0.8	0.6	0.	2.79	0.	0.
time (sec)	N/A	0.358	0.591	0.771	0.	1.594	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	65	0	302	0	0
normalized size	1	1.	0.97	0.71	0.	3.28	0.	0.
time (sec)	N/A	0.188	0.403	0.688	0.	1.637	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	143	103	0	693	0	0
normalized size	1	1.	1.54	1.11	0.	7.45	0.	0.
time (sec)	N/A	0.339	0.214	0.801	0.	1.689	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	206	125	0	749	0	0
normalized size	1	1.	2.31	1.4	0.	8.42	0.	0.
time (sec)	N/A	0.199	0.943	0.865	0.	1.777	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	249	126	0	853	0	0
normalized size	1	1.	2.47	1.25	0.	8.45	0.	0.
time (sec)	N/A	0.397	0.748	0.939	0.	1.746	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	285	144	0	961	0	0
normalized size	1	1.	2.08	1.05	0.	7.01	0.	0.
time (sec)	N/A	0.48	1.359	1.122	0.	1.738	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	120	97	0	562	0	0
normalized size	1	1.	0.52	0.42	0.	2.41	0.	0.
time (sec)	N/A	0.72	3.768	0.723	0.	1.65	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	110	87	0	468	0	0
normalized size	1	1.	0.71	0.56	0.	3.	0.	0.
time (sec)	N/A	0.432	1.917	0.868	0.	1.631	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	100	77	0	396	0	0
normalized size	1	1.	0.81	0.62	0.	3.19	0.	0.
time (sec)	N/A	0.255	1.45	0.75	0.	1.565	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	145	123	0	764	0	0
normalized size	1	1.	1.18	1.	0.	6.21	0.	0.
time (sec)	N/A	0.459	0.267	0.825	0.	1.809	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	233	144	0	833	0	0
normalized size	1	1.	1.93	1.19	0.	6.88	0.	0.
time (sec)	N/A	0.308	0.768	0.872	0.	1.736	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	271	151	0	936	0	0
normalized size	1	1.	2.07	1.15	0.	7.15	0.	0.
time (sec)	N/A	0.518	0.659	0.987	0.	1.747	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	286	144	0	996	0	0
normalized size	1	1.	2.06	1.04	0.	7.17	0.	0.
time (sec)	N/A	0.573	0.882	0.897	0.	1.747	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	97	74	0	373	0	282
normalized size	1	1.	0.61	0.47	0.	2.36	0.	1.78
time (sec)	N/A	0.404	1.235	0.67	0.	1.625	0.	2.532

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	87	64	0	320	0	277
normalized size	1	1.	0.95	0.7	0.	3.48	0.	3.01
time (sec)	N/A	0.343	0.339	1.152	0.	1.633	0.	2.347

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	77	54	0	251	0	198
normalized size	1	1.	1.28	0.9	0.	4.18	0.	3.3
time (sec)	N/A	0.127	0.42	0.648	0.	1.585	0.	2.747

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	116	87	0	647	0	348
normalized size	1	1.	1.84	1.38	0.	10.27	0.	5.52
time (sec)	N/A	0.216	0.13	0.836	0.	1.705	0.	2.376

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	138	103	0	706	0	486
normalized size	1	1.	2.23	1.66	0.	11.39	0.	7.84
time (sec)	N/A	0.101	0.3	0.898	0.	1.698	0.	2.349

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	272	124	0	861	0	682
normalized size	1	1.	2.72	1.24	0.	8.61	0.	6.82
time (sec)	N/A	0.317	1.857	0.947	0.	1.741	0.	2.467

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	975	0	737
normalized size	1	1.	2.16	1.07	0.	7.22	0.	5.46
time (sec)	N/A	0.403	0.696	1.062	0.	1.733	0.	2.443

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	201	148	0	725	0	486
normalized size	1	1.	1.09	0.8	0.	3.94	0.	2.64
time (sec)	N/A	0.589	1.809	0.967	0.	1.768	0.	2.399

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	150	114	0	647	0	410
normalized size	1	1.	1.07	0.81	0.	4.62	0.	2.93
time (sec)	N/A	0.348	0.284	1.32	0.	1.772	0.	2.463

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	149	112	0	595	0	332
normalized size	1	1.	1.38	1.04	0.	5.51	0.	3.07
time (sec)	N/A	0.158	0.87	0.723	0.	1.755	0.	2.292

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	130	97	0	801	0	0
normalized size	1	1.	1.53	1.14	0.	9.42	0.	0.
time (sec)	N/A	0.259	0.211	0.862	0.	1.75	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	206	134	0	1133	0	636
normalized size	1	1.	1.82	1.19	0.	10.03	0.	5.63
time (sec)	N/A	0.224	2.106	0.959	0.	1.904	0.	2.368

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	309	164	0	1353	0	837
normalized size	1	1.	2.02	1.07	0.	8.84	0.	5.47
time (sec)	N/A	0.544	3.589	0.985	0.	1.854	0.	2.522

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	1501	0	938
normalized size	1	1.	1.74	0.95	0.	7.86	0.	4.91
time (sec)	N/A	0.714	2.231	1.032	0.	1.934	0.	2.588

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	92	68	188	90	68
normalized size	1	1.	0.78	1.42	1.05	2.89	1.38	1.05
time (sec)	N/A	0.069	0.3	0.032	0.985	1.773	7.576	1.387

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	74	68	155	90	68
normalized size	1	1.	0.78	1.14	1.05	2.38	1.38	1.05
time (sec)	N/A	0.072	0.214	0.029	0.974	1.709	4.299	1.294

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	58	54	68	127	66	68
normalized size	1	1.	1.18	1.1	1.39	2.59	1.35	1.39
time (sec)	N/A	0.081	0.1	0.023	0.969	1.752	2.24	1.335

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	36	65	97	60	65
normalized size	1	1.	0.98	0.8	1.44	2.16	1.33	1.44
time (sec)	N/A	0.033	0.015	0.033	1.007	1.991	1.189	1.295

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	60	63	132	0	65
normalized size	1	1.	1.	1.07	1.12	2.36	0.	1.16
time (sec)	N/A	0.055	0.029	0.051	1.017	1.926	0.	1.277

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	82	62	176	0	63
normalized size	1	1.	1.	1.55	1.17	3.32	0.	1.19
time (sec)	N/A	0.058	0.034	0.052	0.988	1.876	0.	1.226

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	60	83	61	167	0	62
normalized size	1	1.	1.11	1.54	1.13	3.09	0.	1.15
time (sec)	N/A	0.034	0.113	0.056	1.001	1.455	0.	1.341

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	28	30	39	120	277	39
normalized size	1	1.	0.76	0.81	1.05	3.24	7.49	1.05
time (sec)	N/A	0.097	0.123	0.023	0.987	1.419	27.475	1.382

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	28	30	39	93	224	39
normalized size	1	1.	0.76	0.81	1.05	2.51	6.05	1.05
time (sec)	N/A	0.08	0.093	0.02	0.982	1.24	14.811	1.385

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	28	34	61	245	34
normalized size	1	1.	0.75	0.88	1.06	1.91	7.66	1.06
time (sec)	N/A	0.042	0.037	0.023	0.982	1.391	8.148	1.418

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	33	36	62	0	38
normalized size	1	1.	0.79	1.14	1.24	2.14	0.	1.31
time (sec)	N/A	0.077	0.033	0.036	0.997	1.522	0.	1.302

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	22	30	39	84	0	41
normalized size	1	1.	0.73	1.	1.3	2.8	0.	1.37
time (sec)	N/A	0.079	0.037	0.034	0.97	1.261	0.	1.394

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	25	35	73	0	35
normalized size	1	1.	0.75	0.78	1.09	2.28	0.	1.09
time (sec)	N/A	0.065	0.03	0.036	1.185	1.257	0.	1.344

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	28	29	35	93	0	35
normalized size	1	1.	0.76	0.78	0.95	2.51	0.	0.95
time (sec)	N/A	0.081	0.053	0.038	1.129	1.287	0.	1.398

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	28	29	35	104	0	35
normalized size	1	1.	0.76	0.78	0.95	2.81	0.	0.95
time (sec)	N/A	0.098	0.044	0.039	1.112	1.326	0.	1.434

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	84	124	96	270	272	144
normalized size	1	1.	0.59	0.87	0.67	1.89	1.9	1.01
time (sec)	N/A	0.169	0.274	0.034	1.015	1.509	23.588	1.372

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	71	106	82	230	248	124
normalized size	1	1.	0.56	0.83	0.65	1.81	1.95	0.98
time (sec)	N/A	0.165	0.204	0.033	1.097	1.499	13.015	1.421

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	81	88	88	198	192	144
normalized size	1	1.	0.79	0.85	0.85	1.92	1.86	1.4
time (sec)	N/A	0.133	0.201	0.03	1.	1.531	7.721	1.349

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	71	68	70	163	167	124
normalized size	1	1.	0.82	0.78	0.8	1.87	1.92	1.43
time (sec)	N/A	0.099	0.153	0.023	1.05	1.5	4.361	1.354

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	81	97	109	252	0	196
normalized size	1	1.	0.91	1.09	1.22	2.83	0.	2.2
time (sec)	N/A	0.087	0.147	0.05	1.148	1.545	0.	1.468

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	77	119	122	300	0	192
normalized size	1	1.	0.93	1.43	1.47	3.61	0.	2.31
time (sec)	N/A	0.114	0.414	0.053	1.574	1.668	0.	1.388

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	143	136	365	0	220
normalized size	1	1.	1.	1.52	1.45	3.88	0.	2.34
time (sec)	N/A	0.109	0.795	0.059	1.557	1.572	0.	1.414

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	124	425	0	190
normalized size	1	1.	1.52	1.29	1.51	5.18	0.	2.32
time (sec)	N/A	0.075	0.051	0.056	1.612	1.675	0.	1.407

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	153	128	144	494	0	207
normalized size	1	1.	1.74	1.45	1.64	5.61	0.	2.35
time (sec)	N/A	0.099	0.047	0.06	1.592	1.669	0.	1.397

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	135	116	116	441	0	234
normalized size	1	1.	1.82	1.57	1.57	5.96	0.	3.16
time (sec)	N/A	0.118	0.034	0.06	1.087	1.509	0.	1.522

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	175	138	143	497	0	271
normalized size	1	1.	1.79	1.41	1.46	5.07	0.	2.77
time (sec)	N/A	0.152	0.043	0.06	1.094	1.607	0.	1.573

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	239	160	159	593	0	309
normalized size	1	1.	2.1	1.4	1.39	5.2	0.	2.71
time (sec)	N/A	0.154	0.089	0.068	1.093	1.607	0.	1.252

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	279	182	186	656	0	271
normalized size	1	1.	2.05	1.34	1.37	4.82	0.	1.99
time (sec)	N/A	0.171	0.077	0.064	1.124	1.52	0.	1.443

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	116	218	166	333	554	235
normalized size	1	1.	0.63	1.18	0.9	1.8	2.99	1.27
time (sec)	N/A	0.339	0.702	0.041	1.071	1.621	35.374	1.42

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	86	162	136	290	335	166
normalized size	1	1.	0.54	1.02	0.86	1.82	2.11	1.04
time (sec)	N/A	0.255	0.697	0.042	1.099	1.603	22.024	1.426

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	96	164	138	258	420	189
normalized size	1	1.	0.68	1.16	0.98	1.83	2.98	1.34
time (sec)	N/A	0.258	0.501	0.039	1.078	1.576	12.697	1.437

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	86	106	111	213	223	166
normalized size	1	1.	0.67	0.82	0.86	1.65	1.73	1.29
time (sec)	N/A	0.139	0.324	0.033	1.111	1.536	7.516	1.332

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	96	127	132	309	0	244
normalized size	1	1.	0.81	1.07	1.11	2.6	0.	2.05
time (sec)	N/A	0.143	0.828	0.07	1.101	1.52	0.	1.33

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	83	137	173	360	0	284
normalized size	1	1.	0.72	1.18	1.49	3.1	0.	2.45
time (sec)	N/A	0.197	0.477	0.072	1.679	1.545	0.	1.372

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	158	161	204	427	0	240
normalized size	1	1.	1.61	1.64	2.08	4.36	0.	2.45
time (sec)	N/A	0.157	2.127	0.081	1.656	1.611	0.	1.384

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	191	190	188	474	0	282
normalized size	1	1.	1.95	1.94	1.92	4.84	0.	2.88
time (sec)	N/A	0.155	5.389	0.078	1.749	1.436	0.	1.313

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	215	149	225	566	0	219
normalized size	1	1.	1.85	1.28	1.94	4.88	0.	1.89
time (sec)	N/A	0.183	1.18	0.08	1.698	1.679	0.	1.327

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	200	170	167	621	0	279
normalized size	1	1.	1.69	1.44	1.42	5.26	0.	2.36
time (sec)	N/A	0.191	0.505	0.08	1.616	1.53	0.	1.275

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	267	152	244	537	0	309
normalized size	1	1.	2.02	1.15	1.85	4.07	0.	2.34
time (sec)	N/A	0.262	0.103	0.081	1.113	1.523	0.	1.35

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	291	200	315	709	0	396
normalized size	1	1.	1.65	1.14	1.79	4.03	0.	2.25
time (sec)	N/A	0.32	0.912	0.086	1.124	1.647	0.	1.495

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	313	224	239	795	0	352
normalized size	1	1.	1.86	1.33	1.42	4.73	0.	2.1
time (sec)	N/A	0.274	1.299	0.086	1.181	1.617	0.	1.408

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	353	248	382	887	0	482
normalized size	1	1.	1.62	1.14	1.75	4.07	0.	2.21
time (sec)	N/A	0.353	1.179	0.085	1.154	1.541	0.	1.486

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	126	288	228	379	648	258
normalized size	1	1.	0.62	1.42	1.12	1.87	3.19	1.27
time (sec)	N/A	0.393	1.022	0.046	1.078	1.592	61.152	1.452

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	116	252	201	327	595	235
normalized size	1	1.	0.64	1.38	1.1	1.8	3.27	1.29
time (sec)	N/A	0.382	1.046	0.046	1.12	1.689	35.757	1.392

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	106	216	186	300	486	212
normalized size	1	1.	0.67	1.36	1.17	1.89	3.06	1.33
time (sec)	N/A	0.323	0.816	0.04	1.063	1.567	22.427	1.441

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	96	178	155	254	440	189
normalized size	1	1.	0.61	1.13	0.99	1.62	2.8	1.2
time (sec)	N/A	0.194	0.451	0.038	1.123	1.335	12.888	1.256

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	102	149	182	350	0	309
normalized size	1	1.	0.71	1.04	1.27	2.45	0.	2.16
time (sec)	N/A	0.202	0.981	0.078	1.078	1.216	0.	1.348

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	148	152	190	394	0	305
normalized size	1	1.	1.13	1.16	1.45	3.01	0.	2.33
time (sec)	N/A	0.196	1.964	0.076	1.557	1.174	0.	1.529

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	164	161	247	459	0	325
normalized size	1	1.	1.2	1.18	1.8	3.35	0.	2.37
time (sec)	N/A	0.183	3.07	0.086	1.67	1.234	0.	1.494

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	201	190	250	524	0	338
normalized size	1	1.	1.5	1.42	1.87	3.91	0.	2.52
time (sec)	N/A	0.178	3.058	0.085	1.501	1.224	0.	1.382

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	215	215	282	593	0	325
normalized size	1	1.	1.56	1.56	2.04	4.3	0.	2.36
time (sec)	N/A	0.192	1.2	0.088	1.64	1.261	0.	1.486

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	216	173	243	655	0	305
normalized size	1	1.	1.64	1.31	1.84	4.96	0.	2.31
time (sec)	N/A	0.216	0.496	0.092	1.623	1.246	0.	1.37

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	217	194	290	745	0	323
normalized size	1	1.	1.29	1.15	1.73	4.43	0.	1.92
time (sec)	N/A	0.275	0.766	0.09	1.695	1.176	0.	1.519

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	363	176	278	629	0	352
normalized size	1	1.	2.42	1.17	1.85	4.19	0.	2.35
time (sec)	N/A	0.284	0.132	0.089	1.001	1.349	0.	1.808

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	313	200	332	707	0	396
normalized size	1	1.	1.78	1.14	1.89	4.02	0.	2.25
time (sec)	N/A	0.338	5.16	0.101	1.116	1.592	0.	1.617

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	313	224	362	803	0	439
normalized size	1	1.	1.61	1.15	1.87	4.14	0.	2.26
time (sec)	N/A	0.351	1.327	0.092	1.157	1.252	0.	1.368

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	366	248	416	871	0	482
normalized size	1	1.	1.69	1.15	1.93	4.03	0.	2.23
time (sec)	N/A	0.39	2.148	0.096	1.099	1.274	0.	1.536

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	116	306	251	343	746	235
normalized size	1	1.	0.62	1.64	1.34	1.83	3.99	1.26
time (sec)	N/A	0.234	1.176	0.044	1.11	1.239	36.57	1.34

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	209	190	294	560	0	370
normalized size	1	1.	1.49	1.36	2.1	4.	0.	2.64
time (sec)	N/A	0.228	5.291	0.091	1.642	1.264	0.	1.505

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	86	381	513	217	3048	224
normalized size	1	1.	0.64	2.82	3.8	1.61	22.58	1.66
time (sec)	N/A	0.198	0.256	0.085	1.689	1.135	125.481	1.441

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	377	347	458	182	2429	207
normalized size	1	1.	3.22	2.97	3.91	1.56	20.76	1.77
time (sec)	N/A	0.191	4.843	0.074	1.704	1.153	75.944	1.368

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	258	279	375	155	1464	171
normalized size	1	1.	2.84	3.07	4.12	1.7	16.09	1.88
time (sec)	N/A	0.162	2.406	0.067	1.692	1.015	42.927	1.329

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	219	279	347	123	1221	171
normalized size	1	1.	3.	3.82	4.75	1.68	16.73	2.34
time (sec)	N/A	0.112	1.653	0.06	1.665	1.072	23.916	1.6

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	159	211	167	0	119
normalized size	1	1.	1.02	2.69	3.58	2.83	0.	2.02
time (sec)	N/A	0.098	0.178	0.088	1.663	1.129	0.	1.377

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	93	97	208	225	0	153
normalized size	1	1.	1.9	1.98	4.24	4.59	0.	3.12
time (sec)	N/A	0.116	0.407	0.104	1.658	1.132	0.	1.44

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	102	112	186	288	0	139
normalized size	1	1.	1.76	1.93	3.21	4.97	0.	2.4
time (sec)	N/A	0.106	0.462	0.112	1.622	1.127	0.	1.422

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	124	132	209	311	0	171
normalized size	1	1.	2.14	2.28	3.6	5.36	0.	2.95
time (sec)	N/A	0.088	0.476	0.116	1.132	1.118	0.	1.446

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	125	132	208	365	0	174
normalized size	1	1.	1.52	1.61	2.54	4.45	0.	2.12
time (sec)	N/A	0.148	1.125	0.128	1.126	1.104	0.	1.376

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	189	170	263	452	0	212
normalized size	1	1.	1.89	1.7	2.63	4.52	0.	2.12
time (sec)	N/A	0.172	0.586	0.135	1.135	1.103	0.	1.353

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	229	246	370	513	0	292
normalized size	1	1.	1.85	1.98	2.98	4.14	0.	2.35
time (sec)	N/A	0.18	0.554	0.15	1.119	1.134	0.	1.44

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	418	381	535	248	0	224
normalized size	1	1.	2.84	2.59	3.64	1.69	0.	1.52
time (sec)	N/A	0.224	4.704	0.121	1.686	1.119	0.	1.3

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	76	347	477	215	0	207
normalized size	1	1.	0.59	2.69	3.7	1.67	0.	1.6
time (sec)	N/A	0.225	0.248	0.109	1.654	1.152	0.	1.31

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	308	279	392	181	1836	171
normalized size	1	1.	3.02	2.74	3.84	1.77	18.	1.68
time (sec)	N/A	0.199	1.419	0.099	1.721	1.127	127.725	1.354

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	258	245	333	155	1343	154
normalized size	1	1.	2.97	2.82	3.83	1.78	15.44	1.77
time (sec)	N/A	0.198	1.364	0.089	1.669	1.072	77.659	1.381

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	204	177	248	115	694	119
normalized size	1	1.	2.91	2.53	3.54	1.64	9.91	1.7
time (sec)	N/A	0.11	0.748	0.089	1.677	1.096	44.273	1.335

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	46	60	111	135	0	70
normalized size	1	1.	1.28	1.67	3.08	3.75	0.	1.94
time (sec)	N/A	0.129	0.133	0.121	1.739	1.159	0.	1.319

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	98	74	126	193	0	99
normalized size	1	1.	2.8	2.11	3.6	5.51	0.	2.83
time (sec)	N/A	0.149	0.357	0.138	1.703	1.159	0.	1.373

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	86	93	155	258	0	132
normalized size	1	1.	1.59	1.72	2.87	4.78	0.	2.44
time (sec)	N/A	0.151	0.521	0.158	1.13	1.078	0.	1.377

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	121	132	207	342	0	173
normalized size	1	1.	1.83	2.	3.14	5.18	0.	2.62
time (sec)	N/A	0.126	0.91	0.164	1.121	1.149	0.	1.375

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	116	170	263	406	0	212
normalized size	1	1.	1.21	1.77	2.74	4.23	0.	2.21
time (sec)	N/A	0.187	1.467	0.171	1.112	1.128	0.	1.355

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	189	208	315	491	0	251
normalized size	1	1.	1.69	1.86	2.81	4.38	0.	2.24
time (sec)	N/A	0.241	0.736	0.183	1.093	1.159	0.	1.387

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	229	246	371	555	0	290
normalized size	1	1.	1.66	1.78	2.69	4.02	0.	2.1
time (sec)	N/A	0.248	0.806	0.196	1.112	1.194	0.	1.393

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	195	300	537	381	0	196
normalized size	1	1.	1.79	2.75	4.93	3.5	0.	1.8
time (sec)	N/A	0.259	1.338	0.119	1.712	1.159	0.	1.352

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	181	198	421	328	2412	143
normalized size	1	1.	2.08	2.28	4.84	3.77	27.72	1.64
time (sec)	N/A	0.227	1.118	0.106	1.692	1.109	130.34	1.329

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	143	163	304	259	1357	123
normalized size	1	1.	1.79	2.04	3.8	3.24	16.96	1.54
time (sec)	N/A	0.139	0.722	0.098	1.674	1.144	79.959	1.373

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	122	58	105	338	0	63
normalized size	1	1.	2.71	1.29	2.33	7.51	0.	1.4
time (sec)	N/A	0.183	0.271	0.145	1.527	1.137	0.	1.363

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	156	77	157	451	0	122
normalized size	1	1.	2.89	1.43	2.91	8.35	0.	2.26
time (sec)	N/A	0.236	0.611	0.16	1.132	1.132	0.	1.428

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	213	115	217	662	0	157
normalized size	1	1.	2.73	1.47	2.78	8.49	0.	2.01
time (sec)	N/A	0.249	5.835	0.174	1.122	1.098	0.	1.409

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	251	153	269	810	0	197
normalized size	1	1.	2.61	1.59	2.8	8.44	0.	2.05
time (sec)	N/A	0.155	4.821	0.193	1.105	1.241	0.	1.4

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	601	191	325	1040	0	235
normalized size	1	1.	5.14	1.63	2.78	8.89	0.	2.01
time (sec)	N/A	0.3	6.16	0.204	1.147	1.096	0.	1.386

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	143	100	363	495	0	124
normalized size	1	1.	2.47	1.72	6.26	8.53	0.	2.14
time (sec)	N/A	0.106	1.225	0.127	1.178	1.099	0.	1.348

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	131	293	115	452	636	0	143
normalized size	1	1.47	3.29	1.29	5.08	7.15	0.	1.61
time (sec)	N/A	0.459	2.811	0.132	1.07	1.069	0.	1.407

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	195	130	541	775	0	162
normalized size	1	1.	1.24	0.83	3.45	4.94	0.	1.03
time (sec)	N/A	0.57	3.075	0.156	1.25	1.058	0.	1.441

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	109	85	0	509	0	0
normalized size	1	1.	0.7	0.54	0.	3.26	0.	0.
time (sec)	N/A	0.424	3.735	0.632	0.	1.132	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	99	75	0	427	0	0
normalized size	1	1.	0.8	0.6	0.	3.44	0.	0.
time (sec)	N/A	0.257	1.952	0.766	0.	1.046	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	195	141	0	817	0	0
normalized size	1	1.	1.23	0.89	0.	5.14	0.	0.
time (sec)	N/A	0.492	0.34	1.048	0.	1.176	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	258	162	0	867	0	0
normalized size	1	1.	1.74	1.09	0.	5.86	0.	0.
time (sec)	N/A	0.477	0.733	1.055	0.	1.276	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	297	178	0	961	0	0
normalized size	1	1.	1.9	1.14	0.	6.16	0.	0.
time (sec)	N/A	0.412	0.874	0.981	0.	1.188	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	309	170	0	1025	0	0
normalized size	1	1.	1.9	1.04	0.	6.29	0.	0.
time (sec)	N/A	0.389	1.398	1.127	0.	1.117	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	367	162	0	1141	0	0
normalized size	1	1.	2.12	0.94	0.	6.6	0.	0.
time (sec)	N/A	0.538	2.667	1.003	0.	1.262	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	403	180	0	1261	0	0
normalized size	1	1.	1.93	0.86	0.	6.03	0.	0.
time (sec)	N/A	0.691	4.371	1.2	0.	1.308	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	485	198	0	1423	0	0
normalized size	1	1.	1.98	0.81	0.	5.81	0.	0.
time (sec)	N/A	0.812	7.578	1.151	0.	1.262	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	191	216	0	1565	0	0
normalized size	1	1.	0.68	0.77	0.	5.57	0.	0.
time (sec)	N/A	0.948	2.052	1.28	0.	1.227	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	120	97	0	597	0	0
normalized size	1	1.	0.64	0.52	0.	3.18	0.	0.
time (sec)	N/A	0.515	8.85	0.693	0.	1.159	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	110	87	0	533	0	0
normalized size	1	1.	0.71	0.56	0.	3.42	0.	0.
time (sec)	N/A	0.325	5.03	0.777	0.	1.054	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	219	159	0	903	0	0
normalized size	1	1.	1.1	0.8	0.	4.54	0.	0.
time (sec)	N/A	0.711	0.484	0.864	0.	1.254	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	283	180	0	968	0	0
normalized size	1	1.	1.59	1.01	0.	5.44	0.	0.
time (sec)	N/A	0.649	1.31	0.948	0.	1.263	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	322	178	0	1061	0	0
normalized size	1	1.	1.73	0.96	0.	5.7	0.	0.
time (sec)	N/A	0.571	1.054	1.034	0.	1.212	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	334	196	0	1123	0	0
normalized size	1	1.	1.7	0.99	0.	5.7	0.	0.
time (sec)	N/A	0.5	1.27	1.03	0.	1.239	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	392	188	0	1250	0	0
normalized size	1	1.	1.91	0.92	0.	6.1	0.	0.
time (sec)	N/A	0.703	1.393	1.05	0.	1.236	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	404	180	0	1307	0	0
normalized size	1	1.	1.88	0.84	0.	6.08	0.	0.
time (sec)	N/A	0.808	1.708	1.161	0.	1.275	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	486	198	0	1503	0	0
normalized size	1	1.	1.92	0.78	0.	5.94	0.	0.
time (sec)	N/A	0.919	2.429	1.198	0.	1.307	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	522	216	0	1632	0	0
normalized size	1	1.	1.79	0.74	0.	5.61	0.	0.
time (sec)	N/A	1.061	4.764	1.213	0.	1.336	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	2303	234	0	1840	0	0
normalized size	1	1.	7.	0.71	0.	5.59	0.	0.
time (sec)	N/A	1.187	6.265	1.224	0.	1.468	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	143	74	0	443	0	428
normalized size	1	1.	1.15	0.6	0.	3.57	0.	3.45
time (sec)	N/A	0.406	1.674	0.68	0.	1.118	0.	3.148

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	87	64	0	365	0	351
normalized size	1	1.	0.95	0.7	0.	3.97	0.	3.82
time (sec)	N/A	0.193	1.569	0.756	0.	1.053	0.	2.378

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	169	123	0	761	0	498
normalized size	1	1.	1.3	0.95	0.	5.85	0.	3.83
time (sec)	N/A	0.564	0.252	0.894	0.	1.251	0.	2.363

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	190	126	0	818	0	649
normalized size	1	1.	1.6	1.06	0.	6.87	0.	5.45
time (sec)	N/A	0.503	0.361	0.967	0.	1.214	0.	2.248

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	296	150	0	922	0	748
normalized size	1	1.	2.37	1.2	0.	7.38	0.	5.98
time (sec)	N/A	0.543	3.927	1.013	0.	1.136	0.	2.396

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	987	0	787
normalized size	1	1.	2.16	1.07	0.	7.31	0.	5.83
time (sec)	N/A	0.597	0.578	1.1	0.	1.167	0.	2.32

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	374	162	0	1161	0	994
normalized size	1	1.	2.2	0.95	0.	6.83	0.	5.85
time (sec)	N/A	0.887	0.888	1.016	0.	1.193	0.	2.414

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	410	180	0	1273	0	1083
normalized size	1	1.	2.	0.88	0.	6.21	0.	5.28
time (sec)	N/A	1.143	0.996	1.147	0.	1.302	0.	2.484

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	102	77	0	456	0	358
normalized size	1	1.	0.5	0.38	0.	2.22	0.	1.75
time (sec)	N/A	0.776	5.405	0.838	0.	1.087	0.	2.327

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	67	0	386	0	351
normalized size	1	1.	1.	0.73	0.	4.2	0.	3.82
time (sec)	N/A	0.37	3.61	0.736	0.	1.081	0.	2.269

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	82	57	0	320	0	274
normalized size	1	1.	1.37	0.95	0.	5.33	0.	4.57
time (sec)	N/A	0.151	1.856	0.8	0.	1.046	0.	2.229

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	147	103	0	709	0	423
normalized size	1	1.	1.5	1.05	0.	7.23	0.	4.32
time (sec)	N/A	0.362	0.271	0.882	0.	1.177	0.	2.372

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	220	123	0	774	0	572
normalized size	1	1.	2.34	1.31	0.	8.23	0.	6.09
time (sec)	N/A	0.402	0.685	0.846	0.	1.121	0.	2.295

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	274	126	0	886	0	693
normalized size	1	1.	2.58	1.19	0.	8.36	0.	6.54
time (sec)	N/A	0.488	1.894	0.991	0.	1.165	0.	2.356

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	294	144	0	1003	0	795
normalized size	1	1.	2.04	1.	0.	6.97	0.	5.52
time (sec)	N/A	0.539	0.767	1.036	0.	1.159	0.	2.566

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	376	162	0	1172	0	995
normalized size	1	1.	2.07	0.89	0.	6.44	0.	5.47
time (sec)	N/A	0.727	0.964	0.987	0.	1.181	0.	2.268

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	412	180	0	1301	0	1091
normalized size	1	1.	1.87	0.82	0.	5.91	0.	4.96
time (sec)	N/A	0.876	1.362	1.076	0.	1.233	0.	2.588

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	224	166	0	846	0	637
normalized size	1	1.	0.86	0.64	0.	3.25	0.	2.45
time (sec)	N/A	1.359	1.056	1.031	0.	1.196	0.	2.674

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	225	166	0	786	0	562
normalized size	1	1.	1.01	0.75	0.	3.54	0.	2.53
time (sec)	N/A	1.08	3.236	1.003	0.	1.184	0.	2.429

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	201	132	0	711	0	486
normalized size	1	1.	1.19	0.78	0.	4.21	0.	2.88
time (sec)	N/A	0.428	2.301	0.878	0.	1.176	0.	2.474

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	177	130	0	659	0	409
normalized size	1	1.	1.29	0.95	0.	4.81	0.	2.99
time (sec)	N/A	0.235	1.493	0.844	0.	1.118	0.	2.411

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	154	116	0	1040	0	0
normalized size	1	1.	1.36	1.03	0.	9.2	0.	0.
time (sec)	N/A	0.391	0.4	0.874	0.	1.249	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	170	133	0	1133	0	637
normalized size	1	1.	1.5	1.18	0.	10.03	0.	5.64
time (sec)	N/A	0.524	2.616	0.922	0.	1.263	0.	2.453

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	309	164	0	1355	0	837
normalized size	1	1.	2.02	1.07	0.	8.86	0.	5.47
time (sec)	N/A	0.733	3.613	0.994	0.	1.21	0.	2.539

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	1504	0	938
normalized size	1	1.	1.74	0.95	0.	7.87	0.	4.91
time (sec)	N/A	0.947	2.339	1.19	0.	1.297	0.	2.639

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	414	200	0	1751	0	1141
normalized size	1	1.	1.81	0.87	0.	7.65	0.	4.98
time (sec)	N/A	1.314	5.016	1.201	0.	1.286	0.	2.657

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	164	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	0.285	5.112	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.257	2.973	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	441	0	0	0	0	0
normalized size	1	1.	3.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	11.18	1.718	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	312	0	0	0	0	0
normalized size	1	1.	1.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.242	4.651	1.661	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	138	97	297	136	180
normalized size	1	1.	1.	1.42	1.	3.06	1.4	1.86
time (sec)	N/A	0.089	0.447	0.033	1.014	1.161	57.032	1.239

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	87	120	97	257	136	159
normalized size	1	1.	0.9	1.24	1.	2.65	1.4	1.64
time (sec)	N/A	0.084	0.331	0.037	1.121	1.136	33.152	1.273

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	97	102	97	223	136	180
normalized size	1	1.	1.2	1.26	1.2	2.75	1.68	2.22
time (sec)	N/A	0.127	0.354	0.033	1.169	1.146	21.075	1.304

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	87	84	97	192	136	159
normalized size	1	1.	1.07	1.04	1.2	2.37	1.68	1.96
time (sec)	N/A	0.126	0.277	0.029	1.055	1.142	12.231	1.24

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	78	64	97	161	90	139
normalized size	1	1.	1.2	0.98	1.49	2.48	1.38	2.14
time (sec)	N/A	0.091	0.156	0.025	1.021	1.128	6.958	1.266

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	94	93	197	0	95
normalized size	1	1.	1.	1.09	1.08	2.29	0.	1.1
time (sec)	N/A	0.067	0.036	0.054	1.043	1.186	0.	1.215

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	116	93	250	0	107
normalized size	1	1.	1.	1.4	1.12	3.01	0.	1.29
time (sec)	N/A	0.079	0.042	0.05	1.099	1.157	0.	1.287

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	139	92	258	0	111
normalized size	1	1.	0.9	1.62	1.07	3.	0.	1.29
time (sec)	N/A	0.079	0.109	0.06	1.154	1.157	0.	1.299

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	159	93	300	0	109
normalized size	1	1.	0.89	1.87	1.09	3.53	0.	1.28
time (sec)	N/A	0.072	0.154	0.059	1.022	1.184	0.	1.32

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	93	292	0	111
normalized size	1	1.	1.07	1.68	1.15	3.6	0.	1.37
time (sec)	N/A	0.044	0.2	0.059	1.093	1.172	0.	1.28

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	92	160	97	332	0	113
normalized size	1	1.	1.07	1.86	1.13	3.86	0.	1.31
time (sec)	N/A	0.071	0.165	0.061	1.075	1.125	0.	1.315

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	110	95	254	0	95
normalized size	1	1.	1.	1.8	1.56	4.16	0.	1.56
time (sec)	N/A	0.104	0.024	0.061	1.136	1.06	0.	1.334

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	128	95	273	0	95
normalized size	1	1.	1.	1.97	1.46	4.2	0.	1.46
time (sec)	N/A	0.116	0.026	0.062	1.069	1.069	0.	1.297

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	88	148	95	289	0	95
normalized size	1	1.	1.09	1.83	1.17	3.57	0.	1.17
time (sec)	N/A	0.121	0.155	0.066	1.082	1.096	0.	1.22

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	88	166	95	308	0	95
normalized size	1	1.	1.09	2.05	1.17	3.8	0.	1.17
time (sec)	N/A	0.122	0.113	0.064	1.133	1.102	0.	1.23

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	184	95	321	0	95
normalized size	1	1.	0.91	1.9	0.98	3.31	0.	0.98
time (sec)	N/A	0.082	0.167	0.063	1.032	1.12	0.	1.33

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	202	95	344	0	95
normalized size	1	1.	0.91	2.08	0.98	3.55	0.	0.98
time (sec)	N/A	0.082	0.184	0.063	1.151	1.121	0.	1.267

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	110	158	131	279	189	227
normalized size	1	1.	0.87	1.24	1.03	2.2	1.49	1.79
time (sec)	N/A	0.125	0.796	0.04	1.173	1.124	32.761	1.306

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	99	156	131	246	214	204
normalized size	1	1.	0.91	1.43	1.2	2.26	1.96	1.87
time (sec)	N/A	0.126	0.731	0.037	1.11	1.154	20.818	1.227

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	90	102	131	209	163	181
normalized size	1	1.	1.01	1.15	1.47	2.35	1.83	2.03
time (sec)	N/A	0.084	0.344	0.034	1.063	1.121	11.875	1.286

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	78	122	127	247	0	128
normalized size	1	1.	0.66	1.03	1.07	2.08	0.	1.08
time (sec)	N/A	0.101	0.084	0.07	1.05	1.168	0.	1.302

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	114	130	127	306	0	144
normalized size	1	1.	1.	1.14	1.11	2.68	0.	1.26
time (sec)	N/A	0.118	0.055	0.069	1.124	1.17	0.	1.362

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	76	155	126	315	0	147
normalized size	1	1.	0.66	1.34	1.09	2.72	0.	1.27
time (sec)	N/A	0.118	0.167	0.079	1.07	1.318	0.	1.28

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	74	97	126	273	0	144
normalized size	1	1.	0.67	0.88	1.15	2.48	0.	1.31
time (sec)	N/A	0.101	0.176	0.08	1.093	1.431	0.	1.298

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	76	211	127	377	0	146
normalized size	1	1.	0.66	1.82	1.09	3.25	0.	1.26
time (sec)	N/A	0.067	0.469	0.082	1.064	1.496	0.	1.333

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	76	178	127	389	0	147
normalized size	1	1.	0.68	1.59	1.13	3.47	0.	1.31
time (sec)	N/A	0.102	0.127	0.081	1.135	1.537	0.	1.332

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	102	202	131	408	0	150
normalized size	1	1.	0.86	1.7	1.1	3.43	0.	1.26
time (sec)	N/A	0.121	0.041	0.083	1.031	1.585	0.	1.301

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	110	208	149	277	255	227
normalized size	1	1.	0.99	1.87	1.34	2.5	2.3	2.05
time (sec)	N/A	0.128	0.909	0.043	1.158	1.48	33.764	1.306

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	100	170	149	240	228	204
normalized size	1	1.	1.12	1.91	1.67	2.7	2.56	2.29
time (sec)	N/A	0.083	0.659	0.039	1.197	1.437	21.525	1.279

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	88	144	144	292	0	146
normalized size	1	1.	0.64	1.05	1.05	2.13	0.	1.07
time (sec)	N/A	0.106	0.105	0.079	1.209	1.656	0.	1.297

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	86	147	144	339	0	162
normalized size	1	1.	0.65	1.11	1.08	2.55	0.	1.22
time (sec)	N/A	0.123	0.165	0.082	1.08	1.608	0.	1.343

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	86	154	143	363	0	162
normalized size	1	1.	0.65	1.16	1.08	2.73	0.	1.22
time (sec)	N/A	0.128	0.148	0.092	1.178	1.574	0.	1.337

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	86	179	146	396	0	165
normalized size	1	1.	0.66	1.37	1.11	3.02	0.	1.26
time (sec)	N/A	0.11	0.257	0.086	1.037	1.538	0.	1.363

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	86	211	146	397	0	163
normalized size	1	1.	0.66	1.61	1.11	3.03	0.	1.24
time (sec)	N/A	0.07	0.523	0.091	1.14	1.555	0.	1.261

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	86	234	144	456	0	165
normalized size	1	1.	0.65	1.76	1.08	3.43	0.	1.24
time (sec)	N/A	0.11	0.168	0.091	1.1	1.446	0.	1.314

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	113	203	146	447	0	165
normalized size	1	1.	0.85	1.53	1.1	3.36	0.	1.24
time (sec)	N/A	0.126	0.04	0.093	1.032	1.569	0.	1.411

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	96	179	161	433	0	182
normalized size	1	1.	0.66	1.23	1.11	2.99	0.	1.26
time (sec)	N/A	0.121	0.156	0.095	1.18	1.491	0.	1.355

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	96	129	162	371	0	181
normalized size	1	1.	0.65	0.87	1.09	2.51	0.	1.22
time (sec)	N/A	0.079	0.148	0.095	1.127	1.6	0.	1.382

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	96	235	162	482	0	181
normalized size	1	1.	0.66	1.61	1.11	3.3	0.	1.24
time (sec)	N/A	0.116	0.175	0.096	1.048	1.612	0.	1.284

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	66	177	1520	66
normalized size	1	1.	0.66	0.67	0.9	2.42	20.82	0.9
time (sec)	N/A	0.113	0.335	0.074	1.08	1.425	129.939	1.151

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	66	149	862	66
normalized size	1	1.	0.66	0.67	0.9	2.04	11.81	0.9
time (sec)	N/A	0.156	0.208	0.069	1.133	1.663	79.205	1.149

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	66	122	741	66
normalized size	1	1.	0.87	0.89	1.2	2.22	13.47	1.2
time (sec)	N/A	0.107	0.154	0.058	1.071	1.499	45.032	1.172

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	62	69	127	0	82
normalized size	1	1.	0.75	0.95	1.06	1.95	0.	1.26
time (sec)	N/A	0.089	0.05	0.089	1.115	1.315	0.	1.219

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	45	63	70	167	0	88
normalized size	1	1.	0.73	1.02	1.13	2.69	0.	1.42
time (sec)	N/A	0.108	0.065	0.107	1.168	1.148	0.	1.265

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	45	61	70	165	0	85
normalized size	1	1.	0.75	1.02	1.17	2.75	0.	1.42
time (sec)	N/A	0.106	0.086	0.113	1.007	1.133	0.	1.276

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	63	68	198	0	84
normalized size	1	1.	0.75	0.98	1.06	3.09	0.	1.31
time (sec)	N/A	0.092	0.078	0.119	1.073	1.119	0.	1.24

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	30	49	62	162	0	62
normalized size	1	1.	0.59	0.96	1.22	3.18	0.	1.22
time (sec)	N/A	0.088	0.044	0.128	1.067	1.033	0.	1.203

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	62	185	0	62
normalized size	1	1.	0.87	0.89	1.13	3.36	0.	1.13
time (sec)	N/A	0.136	0.111	0.132	1.146	1.058	0.	1.296

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	62	193	0	62
normalized size	1	1.	0.66	0.67	0.85	2.64	0.	0.85
time (sec)	N/A	0.111	0.103	0.142	1.123	1.017	0.	1.324

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	62	217	0	62
normalized size	1	1.	0.66	0.67	0.85	2.97	0.	0.85
time (sec)	N/A	0.109	0.103	0.155	1.039	1.079	0.	1.255

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	53	180	0	53
normalized size	1	1.	0.69	0.71	0.96	3.27	0.	0.96
time (sec)	N/A	0.106	0.528	0.083	1.209	1.06	0.	1.292

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	53	155	588	53
normalized size	1	1.	0.69	0.71	0.96	2.82	10.69	0.96
time (sec)	N/A	0.104	0.673	0.075	1.125	1.095	132.556	1.225

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	53	123	672	53
normalized size	1	1.	0.69	0.71	0.96	2.24	12.22	0.96
time (sec)	N/A	0.068	0.267	0.062	1.085	1.043	81.13	1.196

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	36	46	53	100	0	63
normalized size	1	1.	0.77	0.98	1.13	2.13	0.	1.34
time (sec)	N/A	0.082	0.04	0.094	1.012	1.102	0.	1.198

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	46	55	107	0	72
normalized size	1	1.	0.74	1.07	1.28	2.49	0.	1.67
time (sec)	N/A	0.099	0.047	0.116	1.014	1.115	0.	1.267

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	38	48	54	140	0	70
normalized size	1	1.	0.81	1.02	1.15	2.98	0.	1.49
time (sec)	N/A	0.101	0.045	0.123	1.097	1.066	0.	1.277

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	37	49	124	0	49
normalized size	1	1.	0.65	1.19	1.58	4.	0.	1.58
time (sec)	N/A	0.079	0.044	0.132	1.059	1.002	0.	1.22

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	49	138	0	49
normalized size	1	1.	0.69	0.71	0.89	2.51	0.	0.89
time (sec)	N/A	0.046	0.07	0.139	1.053	0.995	0.	1.2

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	49	162	0	49
normalized size	1	1.	0.69	0.71	0.89	2.95	0.	0.89
time (sec)	N/A	0.085	0.071	0.146	1.001	1.043	0.	1.204

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	49	177	0	49
normalized size	1	1.	0.69	0.71	0.89	3.22	0.	0.89
time (sec)	N/A	0.103	0.074	0.157	1.101	1.039	0.	1.217

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	71	97	99	196	0	261
normalized size	1	1.	0.7	0.95	0.97	1.92	0.	2.56
time (sec)	N/A	0.126	0.962	0.119	1.128	1.136	0.	1.346

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	59	81	82	155	0	225
normalized size	1	1.	0.72	0.99	1.	1.89	0.	2.74
time (sec)	N/A	0.118	0.933	0.102	1.079	1.126	0.	1.326

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	65	72	132	1243	190
normalized size	1	1.	0.75	0.96	1.06	1.94	18.28	2.79
time (sec)	N/A	0.076	0.342	0.084	1.021	1.083	129.94	1.331

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	32	46	58	100	0	139
normalized size	1	1.	0.71	1.02	1.29	2.22	0.	3.09
time (sec)	N/A	0.086	0.038	0.124	0.983	1.091	0.	1.345

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	35	50	59	142	0	136
normalized size	1	1.	0.74	1.06	1.26	3.02	0.	2.89
time (sec)	N/A	0.106	0.05	0.139	1.102	1.14	0.	1.271

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	66	74	204	0	155
normalized size	1	1.	0.75	1.02	1.14	3.14	0.	2.38
time (sec)	N/A	0.113	0.082	0.155	1.101	1.123	0.	1.213

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	59	82	88	292	0	196
normalized size	1	1.	0.71	0.99	1.06	3.52	0.	2.36
time (sec)	N/A	0.1	0.116	0.161	1.049	1.12	0.	1.262

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	69	97	101	348	0	235
normalized size	1	1.	0.72	1.01	1.05	3.62	0.	2.45
time (sec)	N/A	0.068	0.319	0.181	1.19	1.131	0.	1.255

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	79	114	115	446	0	275
normalized size	1	1.	0.68	0.97	0.98	3.81	0.	2.35
time (sec)	N/A	0.119	0.127	0.184	1.075	1.191	0.	1.333

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	81	116	135	632	0	294
normalized size	1	1.	0.68	0.97	1.12	5.27	0.	2.45
time (sec)	N/A	0.083	0.729	0.197	1.091	1.197	0.	1.349

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	91	131	149	749	0	335
normalized size	1	1.	0.67	0.97	1.1	5.55	0.	2.48
time (sec)	N/A	0.132	0.289	0.208	1.224	1.188	0.	1.293

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	123	0	0	1554	0	1038
normalized size	1	1.	0.68	0.	0.	8.59	0.	5.73
time (sec)	N/A	0.181	0.564	11.083	0.	1.49	0.	1.232

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	110	0	0	1143	0	778
normalized size	1	1.	0.69	0.	0.	7.14	0.	4.86
time (sec)	N/A	0.167	0.389	7.947	0.	1.417	0.	1.254

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	345	0	0	713	8534	512
normalized size	1	1.	2.8	0.	0.	5.8	69.38	4.16
time (sec)	N/A	0.119	1.36	4.082	0.	1.277	173.843	1.143

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	74	0	167	332	0	124
normalized size	1	1.	0.81	0.	1.84	3.65	0.	1.36
time (sec)	N/A	0.139	0.69	1.957	1.352	1.148	0.	1.335

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	0	109	248	0	93
normalized size	1	1.	0.74	0.	1.6	3.65	0.	1.37
time (sec)	N/A	0.127	0.12	1.755	1.255	1.145	0.	1.351

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.094	1.217	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	72	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.106	1.464	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	121	134	116	315	318	225
normalized size	1	1.	0.73	0.81	0.7	1.91	1.93	1.36
time (sec)	N/A	0.2	0.556	0.034	1.02	1.193	56.563	1.276

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	101	116	103	269	294	185
normalized size	1	1.	0.68	0.78	0.69	1.81	1.97	1.24
time (sec)	N/A	0.206	0.337	0.033	1.06	1.185	32.666	1.277

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	91	98	103	232	248	165
normalized size	1	1.	0.73	0.78	0.82	1.86	1.98	1.32
time (sec)	N/A	0.155	0.275	0.03	1.042	1.202	20.561	1.261

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	91	78	85	200	223	165
normalized size	1	1.	0.83	0.72	0.78	1.83	2.05	1.51
time (sec)	N/A	0.12	0.262	0.025	1.033	1.192	11.585	1.167

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	100	131	143	321	0	271
normalized size	1	1.	0.79	1.03	1.13	2.53	0.	2.13
time (sec)	N/A	0.109	0.118	0.053	1.009	1.178	0.	1.239

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	98	153	163	375	0	267
normalized size	1	1.	0.81	1.26	1.35	3.1	0.	2.21
time (sec)	N/A	0.134	0.26	0.052	1.503	1.19	0.	1.184

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	117	177	177	435	0	289
normalized size	1	1.	0.87	1.32	1.32	3.25	0.	2.16
time (sec)	N/A	0.144	2.855	0.062	1.63	1.203	0.	1.364

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	174	199	165	491	0	297
normalized size	1	1.	1.34	1.53	1.27	3.78	0.	2.28
time (sec)	N/A	0.14	6.099	0.062	1.572	1.177	0.	1.207

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	138	221	184	562	0	288
normalized size	1	1.	1.03	1.65	1.37	4.19	0.	2.15
time (sec)	N/A	0.126	1.363	0.067	1.537	1.311	0.	1.293

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	164	159	169	620	0	269
normalized size	1	1.	1.34	1.3	1.39	5.08	0.	2.2
time (sec)	N/A	0.099	0.075	0.062	1.516	1.204	0.	1.203

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	193	181	185	694	0	281
normalized size	1	1.	1.51	1.41	1.45	5.42	0.	2.2
time (sec)	N/A	0.132	0.05	0.068	1.497	1.225	0.	1.383

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	175	152	143	562	0	308
normalized size	1	1.	1.82	1.58	1.49	5.85	0.	3.21
time (sec)	N/A	0.14	0.046	0.068	1.023	1.262	0.	1.188

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	215	174	170	625	0	346
normalized size	1	1.	1.76	1.43	1.39	5.12	0.	2.84
time (sec)	N/A	0.184	0.059	0.066	1.181	1.167	0.	1.348

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	301	196	186	713	0	346
normalized size	1	1.	2.18	1.42	1.35	5.17	0.	2.51
time (sec)	N/A	0.199	0.084	0.069	1.077	1.276	0.	1.316

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	341	218	213	798	0	383
normalized size	1	1.	2.13	1.36	1.33	4.99	0.	2.39
time (sec)	N/A	0.209	0.092	0.066	1.038	1.197	0.	1.36

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	363	240	227	894	0	459
normalized size	1	1.	2.06	1.36	1.29	5.08	0.	2.61
time (sec)	N/A	0.218	0.099	0.07	1.093	1.315	0.	1.309

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	136	238	186	389	656	281
normalized size	1	1.	0.65	1.14	0.89	1.86	3.14	1.34
time (sec)	N/A	0.396	1.345	0.04	1.045	1.28	89.785	1.259

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	126	172	157	335	384	258
normalized size	1	1.	0.69	0.94	0.86	1.83	2.1	1.41
time (sec)	N/A	0.266	0.994	0.041	1.017	1.296	60.745	1.24

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	106	184	173	300	529	212
normalized size	1	1.	0.64	1.12	1.05	1.82	3.21	1.28
time (sec)	N/A	0.296	0.636	0.04	1.025	1.223	36.583	1.296

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	106	116	126	251	282	212
normalized size	1	1.	0.69	0.76	0.82	1.64	1.84	1.39
time (sec)	N/A	0.154	0.645	0.036	1.031	1.21	22.336	1.228

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	112	165	166	386	0	331
normalized size	1	1.	0.7	1.02	1.03	2.4	0.	2.06
time (sec)	N/A	0.164	0.414	0.073	1.156	1.248	0.	1.301

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	110	175	234	439	0	370
normalized size	1	1.	0.7	1.11	1.48	2.78	0.	2.34
time (sec)	N/A	0.226	0.307	0.072	1.762	1.22	0.	1.254

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	174	199	258	497	0	329
normalized size	1	1.	1.24	1.42	1.84	3.55	0.	2.35
time (sec)	N/A	0.209	6.069	0.081	1.71	1.202	0.	1.285

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	209	223	257	554	0	370
normalized size	1	1.	1.37	1.46	1.68	3.62	0.	2.42
time (sec)	N/A	0.215	3.249	0.082	1.611	1.235	0.	1.287

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	227	247	278	636	0	350
normalized size	1	1.	1.48	1.61	1.82	4.16	0.	2.29
time (sec)	N/A	0.197	1.207	0.082	1.581	1.256	0.	1.341

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	264	293	248	686	0	367
normalized size	1	1.	1.9	2.11	1.78	4.94	0.	2.64
time (sec)	N/A	0.249	1.328	0.086	1.641	1.236	0.	1.294

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	270	205	297	782	0	350
normalized size	1	1.	1.72	1.31	1.89	4.98	0.	2.23
time (sec)	N/A	0.235	1.494	0.083	1.645	1.252	0.	1.357

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	262	229	208	833	0	365
normalized size	1	1.	1.62	1.41	1.28	5.14	0.	2.25
time (sec)	N/A	0.228	1.058	0.094	1.566	1.232	0.	1.295

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	401	192	298	668	0	351
normalized size	1	1.	2.2	1.05	1.64	3.67	0.	1.93
time (sec)	N/A	0.313	0.107	0.086	1.047	1.195	0.	1.396

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	313	216	209	756	0	437
normalized size	1	1.	2.06	1.42	1.38	4.97	0.	2.88
time (sec)	N/A	0.285	1.423	0.09	1.028	1.213	0.	1.242

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	353	240	369	855	0	437
normalized size	1	1.	1.55	1.05	1.62	3.75	0.	1.92
time (sec)	N/A	0.391	1.234	0.086	1.069	1.213	0.	1.387

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	187	264	266	940	0	524
normalized size	1	1.	0.96	1.36	1.37	4.85	0.	2.7
time (sec)	N/A	0.302	3.036	0.087	1.067	1.286	0.	1.364

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	197	288	436	1044	0	567
normalized size	1	1.	0.73	1.07	1.61	3.87	0.	2.1
time (sec)	N/A	0.426	4.595	0.089	1.06	1.336	0.	1.365

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	146	308	248	428	748	304
normalized size	1	1.	0.65	1.38	1.11	1.91	3.34	1.36
time (sec)	N/A	0.423	2.168	0.046	1.022	1.387	127.425	1.394

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	136	272	221	390	699	281
normalized size	1	1.	0.65	1.3	1.06	1.87	3.34	1.34
time (sec)	N/A	0.413	1.514	0.041	1.065	1.385	86.804	1.313

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	126	236	221	347	597	258
normalized size	1	1.	0.69	1.29	1.21	1.9	3.26	1.41
time (sec)	N/A	0.334	1.143	0.043	1.165	1.293	59.468	1.271

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	116	198	190	296	542	235
normalized size	1	1.	0.64	1.09	1.05	1.64	2.99	1.3
time (sec)	N/A	0.204	0.925	0.038	1.084	1.237	34.385	1.225

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	122	187	231	439	0	374
normalized size	1	1.	0.66	1.01	1.25	2.37	0.	2.02
time (sec)	N/A	0.236	0.685	0.081	1.129	1.274	0.	1.259

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	168	190	251	475	0	392
normalized size	1	1.	0.97	1.1	1.45	2.75	0.	2.27
time (sec)	N/A	0.244	1.88	0.079	1.825	1.232	0.	1.286

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	664	199	323	544	0	413
normalized size	1	1.	3.67	1.1	1.78	3.01	0.	2.28
time (sec)	N/A	0.253	6.372	0.086	1.717	1.235	0.	1.379

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	219	223	332	601	0	394
normalized size	1	1.	1.24	1.27	1.89	3.41	0.	2.24
time (sec)	N/A	0.211	1.381	0.089	1.682	1.266	0.	1.366

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	235	247	362	664	0	423
normalized size	1	1.	1.32	1.39	2.03	3.73	0.	2.38
time (sec)	N/A	0.219	0.853	0.096	1.733	1.321	0.	1.351

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	271	293	338	738	0	373
normalized size	1	1.	1.55	1.67	1.93	4.22	0.	2.13
time (sec)	N/A	0.299	1.637	0.095	1.79	1.289	0.	1.249

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	289	316	371	817	0	414
normalized size	1	1.	1.59	1.74	2.04	4.49	0.	2.27
time (sec)	N/A	0.248	1.819	0.102	1.636	1.243	0.	1.363

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	292	228	315	871	0	393
normalized size	1	1.	1.7	1.33	1.83	5.06	0.	2.28
time (sec)	N/A	0.29	1.355	0.096	1.609	1.224	0.	1.326

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	279	253	358	983	0	408
normalized size	1	1.	1.17	1.06	1.5	4.13	0.	1.71
time (sec)	N/A	0.355	1.172	0.096	1.67	1.235	0.	1.265

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	459	216	332	765	0	437
normalized size	1	1.	2.3	1.08	1.66	3.82	0.	2.18
time (sec)	N/A	0.359	0.141	0.096	1.084	1.228	0.	1.417

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	365	240	386	855	0	481
normalized size	1	1.	1.6	1.05	1.69	3.75	0.	2.11
time (sec)	N/A	0.427	2.046	0.094	1.059	1.264	0.	1.462

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	187	264	416	957	0	524
normalized size	1	1.	0.76	1.07	1.69	3.89	0.	2.13
time (sec)	N/A	0.436	3.447	0.098	1.082	1.335	0.	1.347

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	197	288	470	1053	0	567
normalized size	1	1.	0.73	1.07	1.74	3.9	0.	2.1
time (sec)	N/A	0.464	4.709	0.1	1.04	1.34	0.	1.56

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	283	312	497	1137	0	610
normalized size	1	1.	0.99	1.09	1.74	3.98	0.	2.13
time (sec)	N/A	0.462	6.49	0.098	1.064	1.368	0.	1.414

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	229	223	397	641	0	437
normalized size	1	1.	1.29	1.25	2.23	3.6	0.	2.46
time (sec)	N/A	0.281	1.641	0.096	1.567	1.289	0.	1.494

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	429	517	678	252	0	294
normalized size	1	1.	2.7	3.25	4.26	1.58	0.	1.85
time (sec)	N/A	0.216	8.587	0.106	1.552	1.197	0.	1.314

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	375	483	622	216	0	277
normalized size	1	1.	2.66	3.43	4.41	1.53	0.	1.96
time (sec)	N/A	0.215	8.755	0.09	1.585	1.151	0.	1.3

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	715	415	540	189	3048	242
normalized size	1	1.	6.22	3.61	4.7	1.64	26.5	2.1
time (sec)	N/A	0.177	11.528	0.081	1.555	1.113	125.808	1.25

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	377	415	512	155	2428	242
normalized size	1	1.	3.89	4.28	5.28	1.6	25.03	2.49
time (sec)	N/A	0.126	5.192	0.069	1.531	1.04	76.375	1.304

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	86	296	378	239	0	193
normalized size	1	1.	0.85	2.93	3.74	2.37	0.	1.91
time (sec)	N/A	0.123	0.376	0.11	1.495	1.15	0.	1.305

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	122	230	374	284	0	198
normalized size	1	1.	1.28	2.42	3.94	2.99	0.	2.08
time (sec)	N/A	0.156	0.781	0.119	1.547	1.133	0.	1.348

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	152	234	352	347	0	225
normalized size	1	1.	1.43	2.21	3.32	3.27	0.	2.12
time (sec)	N/A	0.161	0.492	0.126	1.533	1.169	0.	1.379

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	138	173	324	409	0	212
normalized size	1	1.	1.47	1.84	3.45	4.35	0.	2.26
time (sec)	N/A	0.138	0.916	0.139	1.555	1.261	0.	1.327

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	232	188	293	474	0	225
normalized size	1	1.	2.27	1.84	2.87	4.65	0.	2.21
time (sec)	N/A	0.133	0.659	0.138	1.526	1.134	0.	1.377

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	189	208	316	431	0	252
normalized size	1	1.	2.3	2.54	3.85	5.26	0.	3.07
time (sec)	N/A	0.111	0.757	0.145	1.017	1.099	0.	1.338

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	418	415	562	220	0	242
normalized size	1	1.	3.1	3.07	4.16	1.63	0.	1.79
time (sec)	N/A	0.347	3.087	0.098	1.569	1.108	0.	1.322

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	362	347	477	188	0	207
normalized size	1	1.	3.48	3.34	4.59	1.81	0.	1.99
time (sec)	N/A	0.268	1.954	0.085	1.536	1.063	0.	1.246

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	262	313	419	155	1834	189
normalized size	1	1.	2.62	3.13	4.19	1.55	18.34	1.89
time (sec)	N/A	0.126	1.245	0.076	1.543	1.079	131.729	1.42

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	69	160	254	207	0	123
normalized size	1	1.	0.95	2.19	3.48	2.84	0.	1.68
time (sec)	N/A	0.201	0.32	0.118	1.563	1.202	0.	1.434

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	116	196	273	254	0	177
normalized size	1	1.	1.57	2.65	3.69	3.43	0.	2.39
time (sec)	N/A	0.203	0.524	0.152	1.506	1.201	0.	1.337

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	134	134	275	319	0	173
normalized size	1	1.	1.84	1.84	3.77	4.37	0.	2.37
time (sec)	N/A	0.225	0.576	0.145	1.535	1.168	0.	1.383

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	124	149	238	378	0	185
normalized size	1	1.	1.7	2.04	3.26	5.18	0.	2.53
time (sec)	N/A	0.315	1.25	0.148	1.519	1.161	0.	1.256

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	116	170	262	378	0	213
normalized size	1	1.	1.41	2.07	3.2	4.61	0.	2.6
time (sec)	N/A	0.3	1.327	0.152	1.022	1.114	0.	1.411

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	189	170	263	460	0	212
normalized size	1	1.	1.89	1.7	2.63	4.6	0.	2.12
time (sec)	N/A	0.147	0.554	0.161	1.036	1.15	0.	1.384

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	229	246	370	524	0	292
normalized size	1	1.	1.85	1.98	2.98	4.23	0.	2.35
time (sec)	N/A	0.332	0.672	0.176	1.004	1.15	0.	1.346

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	366	381	504	219	0	224
normalized size	1	1.	2.84	2.95	3.91	1.7	0.	1.74
time (sec)	N/A	0.242	2.183	0.117	1.564	1.095	0.	1.311

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	310	279	392	189	0	171
normalized size	1	1.	2.95	2.66	3.73	1.8	0.	1.63
time (sec)	N/A	0.219	1.757	0.107	1.548	1.126	0.	1.265

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	105	255	279	360	150	0	171
normalized size	1	1.25	3.04	3.32	4.29	1.79	0.	2.04
time (sec)	N/A	0.166	1.402	0.109	1.54	1.065	0.	1.375

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	63	159	217	173	0	120
normalized size	1	1.	1.05	2.65	3.62	2.88	0.	2.
time (sec)	N/A	0.147	0.217	0.151	1.522	1.116	0.	1.287

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	106	97	213	235	0	150
normalized size	1	1.	2.16	1.98	4.35	4.8	0.	3.06
time (sec)	N/A	0.163	0.483	0.165	1.56	1.169	0.	1.375

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	126	112	186	301	0	146
normalized size	1	1.	2.1	1.87	3.1	5.02	0.	2.43
time (sec)	N/A	0.175	0.472	0.174	1.509	1.104	0.	1.37

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	115	132	207	347	0	173
normalized size	1	1.	1.6	1.83	2.88	4.82	0.	2.4
time (sec)	N/A	0.19	1.275	0.178	1.039	1.109	0.	1.352

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	125	170	263	406	0	211
normalized size	1	1.	1.34	1.83	2.83	4.37	0.	2.27
time (sec)	N/A	0.205	2.153	0.193	1.009	1.09	0.	1.398

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	189	208	316	502	0	252
normalized size	1	1.	1.66	1.82	2.77	4.4	0.	2.21
time (sec)	N/A	0.177	1.811	0.2	1.031	1.147	0.	1.322

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	188	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.285	0.6	14.569	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	164	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.242	0.283	10.043	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	0.298	5.675	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	117	166	127	369	184	220
normalized size	1	1.	0.91	1.29	0.98	2.86	1.43	1.71
time (sec)	N/A	0.101	1.032	0.035	1.054	1.682	175.577	1.39

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	137	148	127	336	182	261
normalized size	1	1.	1.21	1.31	1.12	2.97	1.61	2.31
time (sec)	N/A	0.14	0.724	0.034	1.044	1.641	117.722	1.267

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	127	130	127	294	182	240
normalized size	1	1.	1.12	1.15	1.12	2.6	1.61	2.12
time (sec)	N/A	0.137	0.559	0.036	0.999	1.608	82.113	1.319

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	117	112	127	259	138	220
normalized size	1	1.	1.21	1.15	1.31	2.67	1.42	2.27
time (sec)	N/A	0.132	0.605	0.036	1.024	1.602	59.825	1.26

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	94	127	224	138	180
normalized size	1	1.	1.	0.97	1.31	2.31	1.42	1.86
time (sec)	N/A	0.128	0.465	0.029	1.058	1.367	33.787	1.151

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	60	74	127	193	114	159
normalized size	1	1.	0.74	0.91	1.57	2.38	1.41	1.96
time (sec)	N/A	0.09	0.381	0.025	1.008	1.481	21.028	1.199

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	106	128	123	263	0	124
normalized size	1	1.	0.9	1.08	1.04	2.23	0.	1.05
time (sec)	N/A	0.077	0.139	0.053	1.003	1.547	0.	1.177

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	102	150	123	312	0	136
normalized size	1	1.	0.89	1.32	1.08	2.74	0.	1.19
time (sec)	N/A	0.087	0.137	0.052	1.048	1.633	0.	1.171

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	100	173	122	328	0	140
normalized size	1	1.	0.87	1.5	1.06	2.85	0.	1.22
time (sec)	N/A	0.087	0.125	0.061	1.023	1.672	0.	1.21

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	103	195	124	369	0	140
normalized size	1	1.	0.87	1.65	1.05	3.13	0.	1.19
time (sec)	N/A	0.086	0.22	0.068	1.024	1.747	0.	1.318

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	105	217	124	375	0	139
normalized size	1	1.	0.89	1.84	1.05	3.18	0.	1.18
time (sec)	N/A	0.089	0.572	0.063	1.038	1.7	0.	1.187

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	102	239	123	419	0	139
normalized size	1	1.	0.89	2.08	1.07	3.64	0.	1.21
time (sec)	N/A	0.082	0.186	0.063	1.045	1.385	0.	1.186

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	111	195	123	412	0	140
normalized size	1	1.	0.97	1.7	1.07	3.58	0.	1.22
time (sec)	N/A	0.053	0.383	0.069	1.031	1.162	0.	1.306

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	115	217	127	458	0	143
normalized size	1	1.	0.97	1.82	1.07	3.85	0.	1.2
time (sec)	N/A	0.085	0.391	0.065	1.04	1.207	0.	1.337

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	138	124	351	0	124
normalized size	1	1.	1.	1.86	1.68	4.74	0.	1.68
time (sec)	N/A	0.107	0.033	0.068	1.042	1.134	0.	1.359

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	156	124	370	0	124
normalized size	1	1.	1.	1.93	1.53	4.57	0.	1.53
time (sec)	N/A	0.119	0.068	0.066	1.03	1.124	0.	1.336

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	86	176	124	386	0	124
normalized size	1	1.	0.89	1.81	1.28	3.98	0.	1.28
time (sec)	N/A	0.125	0.184	0.062	1.036	1.165	0.	1.298

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	86	194	124	405	0	124
normalized size	1	1.	0.89	2.	1.28	4.18	0.	1.28
time (sec)	N/A	0.124	0.139	0.066	1.033	1.222	0.	1.339

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	86	212	124	421	0	124
normalized size	1	1.	0.76	1.88	1.1	3.73	0.	1.1
time (sec)	N/A	0.133	0.231	0.06	1.036	1.256	0.	1.363

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	86	230	124	446	0	124
normalized size	1	1.	0.76	2.04	1.1	3.95	0.	1.1
time (sec)	N/A	0.135	0.215	0.066	1.054	1.258	0.	1.335

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	86	248	124	460	0	124
normalized size	1	1.	0.67	1.92	0.96	3.57	0.	0.96
time (sec)	N/A	0.099	0.233	0.063	1.014	1.28	0.	1.374

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	93	311	0	93
normalized size	1	1.	0.62	0.63	0.85	2.85	0.	0.85
time (sec)	N/A	0.128	0.598	0.115	1.049	1.212	0.	1.312

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	93	278	0	93
normalized size	1	1.	0.62	0.63	0.85	2.55	0.	0.85
time (sec)	N/A	0.127	0.908	0.105	1.031	1.15	0.	1.28

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	93	239	0	93
normalized size	1	1.	0.62	0.63	0.85	2.19	0.	0.85
time (sec)	N/A	0.127	0.415	0.1	1.123	1.167	0.	1.291

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	93	209	0	93
normalized size	1	1.	0.75	0.76	1.02	2.3	0.	1.02
time (sec)	N/A	0.16	0.566	0.088	1.116	1.155	0.	1.284

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	93	182	0	93
normalized size	1	1.	0.75	0.76	1.02	2.	0.	1.02
time (sec)	N/A	0.162	0.291	0.082	1.011	1.106	0.	1.221

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	68	69	93	153	1794	93
normalized size	1	1.	0.93	0.95	1.27	2.1	24.58	1.27
time (sec)	N/A	0.115	0.268	0.065	1.025	1.152	120.397	1.299

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	65	90	122	1096	90
normalized size	1	1.	0.97	0.96	1.32	1.79	16.12	1.32
time (sec)	N/A	0.063	0.188	0.072	1.034	1.091	74.035	1.29

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	68	94	96	186	0	119
normalized size	1	1.	0.69	0.95	0.97	1.88	0.	1.2
time (sec)	N/A	0.1	0.072	0.101	1.033	1.165	0.	1.256

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	66	94	100	234	0	128
normalized size	1	1.	0.69	0.99	1.05	2.46	0.	1.35
time (sec)	N/A	0.12	0.129	0.118	1.043	1.159	0.	1.305

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	66	94	100	244	0	127
normalized size	1	1.	0.68	0.97	1.03	2.52	0.	1.31
time (sec)	N/A	0.118	0.109	0.128	1.023	1.156	0.	1.338

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	66	94	99	289	0	117
normalized size	1	1.	0.68	0.97	1.02	2.98	0.	1.21
time (sec)	N/A	0.12	0.177	0.129	1.029	1.165	0.	1.29

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	66	93	97	281	0	112
normalized size	1	1.	0.7	0.99	1.03	2.99	0.	1.19
time (sec)	N/A	0.119	0.315	0.133	1.004	1.155	0.	1.321

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	68	97	95	321	0	111
normalized size	1	1.	0.68	0.97	0.95	3.21	0.	1.11
time (sec)	N/A	0.104	0.103	0.144	1.046	1.158	0.	1.295

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	67	89	248	0	89
normalized size	1	1.	0.9	0.99	1.31	3.65	0.	1.31
time (sec)	N/A	0.09	0.135	0.149	0.991	1.1	0.	1.336

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	68	69	89	270	0	89
normalized size	1	1.	0.93	0.95	1.22	3.7	0.	1.22
time (sec)	N/A	0.139	0.152	0.158	1.017	1.139	0.	1.326

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	89	286	0	89
normalized size	1	1.	0.75	0.76	0.98	3.14	0.	0.98
time (sec)	N/A	0.16	0.14	0.167	1.032	1.203	0.	1.321

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	89	308	0	89
normalized size	1	1.	0.75	0.76	0.98	3.38	0.	0.98
time (sec)	N/A	0.16	0.156	0.177	1.013	1.131	0.	1.201

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	89	321	0	89
normalized size	1	1.	0.62	0.63	0.82	2.94	0.	0.82
time (sec)	N/A	0.121	0.106	0.19	0.995	1.136	0.	1.414

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	89	347	0	89
normalized size	1	1.	0.62	0.63	0.82	3.18	0.	0.82
time (sec)	N/A	0.123	0.108	0.201	1.046	1.102	0.	1.282

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	89	363	0	89
normalized size	1	1.	0.62	0.63	0.82	3.33	0.	0.82
time (sec)	N/A	0.122	0.108	0.215	1.026	1.241	0.	1.325

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	126	0	0	1751	0	1836
normalized size	1	1.	0.68	0.	0.	9.52	0.	9.98
time (sec)	N/A	0.178	0.904	20.262	0.	1.71	0.	1.366

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	126	0	0	1567	0	1376
normalized size	1	1.	0.68	0.	0.	8.52	0.	7.48
time (sec)	N/A	0.176	0.739	13.977	0.	1.642	0.	1.322

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	659	0	0	1219	0	910
normalized size	1	1.	3.95	0.	0.	7.3	0.	5.45
time (sec)	N/A	0.138	3.155	8.074	0.	1.571	0.	1.207

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	95	0	325	639	0	188
normalized size	1	1.	0.69	0.	2.37	4.66	0.	1.37
time (sec)	N/A	0.164	0.307	3.677	1.203	1.292	0.	1.526

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	117	0	170	414	0	127
normalized size	1	1.	1.27	0.	1.85	4.5	0.	1.38
time (sec)	N/A	0.143	0.341	2.848	1.204	1.228	0.	1.583

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	66	0	170	389	0	127
normalized size	1	1.	0.72	0.	1.85	4.23	0.	1.38
time (sec)	N/A	0.137	0.188	2.116	1.211	1.182	0.	1.643

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	104	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	0.254	1.497	0.	0.	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	108	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.206	1.239	0.	0.	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	518	755	952	324	0	417
normalized size	1	1.	2.48	3.61	4.56	1.55	0.	2.
time (sec)	N/A	0.282	14.444	0.128	1.637	1.208	0.	1.291

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	573	653	842	294	0	365
normalized size	1	1.	3.13	3.57	4.6	1.61	0.	1.99
time (sec)	N/A	0.24	12.231	0.119	1.726	1.229	0.	1.321

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	533	619	787	252	0	347
normalized size	1	1.	3.23	3.75	4.77	1.53	0.	2.1
time (sec)	N/A	0.237	15.162	0.11	1.759	1.175	0.	1.295

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	479	551	705	220	0	312
normalized size	1	1.	3.45	3.96	5.07	1.58	0.	2.24
time (sec)	N/A	0.197	9.543	0.106	1.614	1.197	0.	1.176

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	481	551	676	189	0	312
normalized size	1	1.	3.98	4.55	5.59	1.56	0.	2.58
time (sec)	N/A	0.145	11.375	0.082	1.564	1.132	0.	1.183

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	102	432	543	302	0	263
normalized size	1	1.	0.71	3.02	3.8	2.11	0.	1.84
time (sec)	N/A	0.146	0.271	0.124	1.573	1.153	0.	1.207

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	146	367	512	354	0	269
normalized size	1	1.	1.07	2.68	3.74	2.58	0.	1.96
time (sec)	N/A	0.174	0.705	0.135	1.57	1.151	0.	1.216

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	179	371	517	413	0	292
normalized size	1	1.	1.19	2.47	3.45	2.75	0.	1.95
time (sec)	N/A	0.189	0.524	0.148	1.545	1.218	0.	1.212

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	197	306	489	470	0	308
normalized size	1	1.	1.35	2.1	3.35	3.22	0.	2.11
time (sec)	N/A	0.18	0.79	0.148	1.542	1.182	0.	1.273

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	252	310	459	536	0	302
normalized size	1	1.	1.68	2.07	3.06	3.57	0.	2.01
time (sec)	N/A	0.177	0.733	0.148	1.595	1.175	0.	1.518

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	264	249	431	593	0	293
normalized size	1	1.	1.91	1.8	3.12	4.3	0.	2.12
time (sec)	N/A	0.151	0.995	0.154	1.523	1.166	0.	1.382

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	317	264	402	660	0	302
normalized size	1	1.	2.23	1.86	2.83	4.65	0.	2.13
time (sec)	N/A	0.179	0.944	0.156	1.536	1.203	0.	1.36

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	284	284	425	545	0	329
normalized size	1	1.	2.68	2.68	4.01	5.14	0.	3.1
time (sec)	N/A	0.148	0.91	0.163	1.031	1.189	0.	1.361

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	291	322	478	602	0	370
normalized size	1	1.	2.17	2.4	3.57	4.49	0.	2.76
time (sec)	N/A	0.218	0.998	0.174	1.009	1.165	0.	1.436

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	313	322	479	689	0	369
normalized size	1	1.	2.06	2.12	3.15	4.53	0.	2.43
time (sec)	N/A	0.235	1.352	0.179	1.022	1.239	0.	1.331

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	386	360	532	768	0	409
normalized size	1	1.	2.19	2.05	3.02	4.36	0.	2.32
time (sec)	N/A	0.248	1.509	0.188	1.051	1.24	0.	1.391

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	187	436	641	861	0	486
normalized size	1	1.	0.96	2.25	3.3	4.44	0.	2.51
time (sec)	N/A	0.253	2.937	0.209	1.138	1.234	0.	1.274

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	1453	653	875	332	0	365
normalized size	1	1.	7.16	3.22	4.31	1.64	0.	1.8
time (sec)	N/A	0.412	10.336	0.137	1.56	1.191	0.	1.32

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	585	619	817	292	0	347
normalized size	1	1.	3.16	3.35	4.42	1.58	0.	1.88
time (sec)	N/A	0.458	8.439	0.125	1.581	1.187	0.	1.29

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	430	551	732	255	0	312
normalized size	1	1.	2.7	3.47	4.6	1.6	0.	1.96
time (sec)	N/A	0.363	6.466	0.121	1.617	1.21	0.	1.272

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	481	483	647	230	0	277
normalized size	1	1.	3.41	3.43	4.59	1.63	0.	1.96
time (sec)	N/A	0.374	4.131	0.097	1.649	1.123	0.	1.27

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	418	449	589	189	0	259
normalized size	1	1.	3.37	3.62	4.75	1.52	0.	2.09
time (sec)	N/A	0.138	4.865	0.089	1.679	1.151	0.	1.285

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	93	329	450	274	0	211
normalized size	1	1.	0.78	2.76	3.78	2.3	0.	1.77
time (sec)	N/A	0.237	0.701	0.132	1.564	1.153	0.	1.312

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	128	333	470	323	0	251
normalized size	1	1.	1.1	2.87	4.05	2.78	0.	2.16
time (sec)	N/A	0.302	1.509	0.148	1.618	1.201	0.	1.317

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	158	234	446	383	0	227
normalized size	1	1.	1.63	2.41	4.6	3.95	0.	2.34
time (sec)	N/A	0.253	2.044	0.152	1.63	1.228	0.	1.336

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	184	272	413	431	0	262
normalized size	1	1.	1.9	2.8	4.26	4.44	0.	2.7
time (sec)	N/A	0.243	2.366	0.158	1.538	1.302	0.	1.326

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	219	173	355	513	0	215
normalized size	1	1.	1.89	1.49	3.06	4.42	0.	1.85
time (sec)	N/A	0.281	1.837	0.167	1.541	1.148	0.	1.295

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	254	226	348	567	0	263
normalized size	1	1.	2.15	1.92	2.95	4.81	0.	2.23
time (sec)	N/A	0.321	1.07	0.163	1.537	1.151	0.	1.35

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	145	246	371	501	0	290
normalized size	1	1.	1.1	1.86	2.81	3.8	0.	2.2
time (sec)	N/A	0.342	1.838	0.169	1.067	1.148	0.	1.356

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	251	284	424	585	0	331
normalized size	1	1.	2.02	2.29	3.42	4.72	0.	2.67
time (sec)	N/A	0.26	1.083	0.183	1.049	1.134	0.	1.325

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	291	322	479	655	0	369
normalized size	1	1.	1.65	1.83	2.72	3.72	0.	2.1
time (sec)	N/A	0.407	0.895	0.194	1.097	1.196	0.	1.373

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	313	284	424	736	0	331
normalized size	1	1.	1.86	1.69	2.52	4.38	0.	1.97
time (sec)	N/A	0.386	1.791	0.203	1.041	1.208	0.	1.349

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	353	398	587	817	0	447
normalized size	1	1.	1.62	1.83	2.69	3.75	0.	2.05
time (sec)	N/A	0.487	1.606	0.214	1.046	1.206	0.	1.402

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	186	436	640	910	0	487
normalized size	1	1.	0.89	2.08	3.05	4.33	0.	2.32
time (sec)	N/A	0.432	4.038	0.239	1.041	1.277	0.	1.369

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	482	517	674	263	0	294
normalized size	1	1.	2.99	3.21	4.19	1.63	0.	1.83
time (sec)	N/A	0.477	4.185	0.106	1.59	1.126	0.	1.311

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	429	415	562	217	0	242
normalized size	1	1.	3.23	3.12	4.23	1.63	0.	1.82
time (sec)	N/A	0.401	9.82	0.102	1.567	1.113	0.	1.281

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	366	415	531	190	0	242
normalized size	1	1.	2.79	3.17	4.05	1.45	0.	1.85
time (sec)	N/A	0.17	2.029	0.089	1.559	1.144	0.	1.272

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	80	239	363	238	0	174
normalized size	1	1.	0.81	2.41	3.67	2.4	0.	1.76
time (sec)	N/A	0.243	0.391	0.135	1.586	1.173	0.	1.31

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	126	230	385	289	0	198
normalized size	1	1.	1.37	2.5	4.18	3.14	0.	2.15
time (sec)	N/A	0.222	0.958	0.155	1.567	1.129	0.	1.294

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	144	234	360	358	0	232
normalized size	1	1.	1.47	2.39	3.67	3.65	0.	2.37
time (sec)	N/A	0.247	0.906	0.157	1.565	1.197	0.	1.336

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	132	173	327	413	0	212
normalized size	1	1.	1.43	1.88	3.55	4.49	0.	2.3
time (sec)	N/A	0.274	2.521	0.164	1.569	1.165	0.	1.286

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	165	188	294	452	0	224
normalized size	1	1.	1.7	1.94	3.03	4.66	0.	2.31
time (sec)	N/A	0.318	2.374	0.171	1.553	1.179	0.	1.342

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	189	208	317	468	0	251
normalized size	1	1.	1.89	2.08	3.17	4.68	0.	2.51
time (sec)	N/A	0.337	1.697	0.171	1.022	1.155	0.	1.319

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	242	246	370	532	0	292
normalized size	1	1.	1.95	1.98	2.98	4.29	0.	2.35
time (sec)	N/A	0.364	0.971	0.187	1.068	1.13	0.	1.345

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	251	284	425	612	0	329
normalized size	1	1.	1.79	2.03	3.04	4.37	0.	2.35
time (sec)	N/A	0.231	0.963	0.198	1.015	1.176	0.	1.373

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	317	322	478	691	0	370
normalized size	1	1.	1.91	1.94	2.88	4.16	0.	2.23
time (sec)	N/A	0.406	4.964	0.206	1.071	1.175	0.	1.287

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	104	101	333	0	142
normalized size	1	1.	1.	1.27	1.23	4.06	0.	1.73
time (sec)	N/A	0.131	0.301	0.05	1.538	1.117	0.	1.137

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	63	94	84	263	0	122
normalized size	1	1.	0.97	1.45	1.29	4.05	0.	1.88
time (sec)	N/A	0.102	0.102	0.05	1.55	1.083	0.	1.282

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	47	59	53	194	0	109
normalized size	1	1.	1.21	1.51	1.36	4.97	0.	2.79
time (sec)	N/A	0.104	0.033	0.046	1.579	1.049	0.	1.252

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	36	32	43	143	0	39
normalized size	1	1.	1.33	1.19	1.59	5.3	0.	1.44
time (sec)	N/A	0.047	0.02	0.037	1.651	1.061	0.	1.286

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	56	47	65	302	0	46
normalized size	1	1.	1.56	1.31	1.81	8.39	0.	1.28
time (sec)	N/A	0.074	0.032	0.066	1.064	1.083	0.	1.263

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	68	69	80	446	0	117
normalized size	1	1.	1.42	1.44	1.67	9.29	0.	2.44
time (sec)	N/A	0.109	0.075	0.073	1.095	1.11	0.	1.251

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	172	93	113	660	0	138
normalized size	1	1.	2.29	1.24	1.51	8.8	0.	1.84
time (sec)	N/A	0.13	1.486	0.086	1.13	1.133	0.	1.293

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	205	116	132	817	0	176
normalized size	1	1.	2.25	1.27	1.45	8.98	0.	1.93
time (sec)	N/A	0.131	4.833	0.084	1.083	1.128	0.	1.272

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	161	148	132	362	0	161
normalized size	1	1.	1.81	1.66	1.48	4.07	0.	1.81
time (sec)	N/A	0.208	0.514	0.062	1.598	1.087	0.	1.282

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	145	117	113	296	0	138
normalized size	1	1.	2.04	1.65	1.59	4.17	0.	1.94
time (sec)	N/A	0.086	0.391	0.056	1.638	1.087	0.	1.202

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	90	76	77	230	0	120
normalized size	1	1.	2.09	1.77	1.79	5.35	0.	2.79
time (sec)	N/A	0.057	0.369	0.049	1.708	1.09	0.	1.163

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	69	55	88	327	0	51
normalized size	1	1.	1.57	1.25	2.	7.43	0.	1.16
time (sec)	N/A	0.126	0.114	0.089	1.081	1.123	0.	1.267

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	96	92	97	473	0	132
normalized size	1	1.	1.66	1.59	1.67	8.16	0.	2.28
time (sec)	N/A	0.223	0.38	0.094	1.161	1.121	0.	1.272

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	124	104	167	718	0	157
normalized size	1	1.	1.44	1.21	1.94	8.35	0.	1.83
time (sec)	N/A	0.21	1.134	0.111	1.183	1.13	0.	1.329

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	125	212	219	440	0	225
normalized size	1	1.	1.13	1.91	1.97	3.96	0.	2.03
time (sec)	N/A	0.17	0.802	0.069	1.705	1.141	0.	1.283

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	115	167	158	378	0	161
normalized size	1	1.	1.29	1.88	1.78	4.25	0.	1.81
time (sec)	N/A	0.124	0.473	0.061	1.653	1.116	0.	1.306

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	145	130	131	297	0	138
normalized size	1	1.	2.16	1.94	1.96	4.43	0.	2.06
time (sec)	N/A	0.064	0.5	0.053	1.515	1.087	0.	1.265

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	74	70	113	382	0	66
normalized size	1	1.	1.54	1.46	2.35	7.96	0.	1.38
time (sec)	N/A	0.104	0.133	0.096	1.475	1.125	0.	1.277

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	96	93	119	490	0	132
normalized size	1	1.	1.71	1.66	2.12	8.75	0.	2.36
time (sec)	N/A	0.135	0.358	0.106	0.987	1.108	0.	1.275

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	124	117	182	721	0	157
normalized size	1	1.	1.55	1.46	2.28	9.01	0.	1.96
time (sec)	N/A	0.156	1.131	0.119	1.044	1.155	0.	1.306

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	211	128	216	887	0	200
normalized size	1	1.	2.15	1.31	2.2	9.05	0.	2.04
time (sec)	N/A	0.175	6.142	0.128	1.018	1.182	0.	1.246

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	148	126	319	215	0	169
normalized size	1	1.	1.78	1.52	3.84	2.59	0.	2.04
time (sec)	N/A	0.142	0.377	0.081	1.498	1.055	0.	1.168

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	111	104	208	190	0	104
normalized size	1	1.	1.59	1.49	2.97	2.71	0.	1.49
time (sec)	N/A	0.111	0.351	0.075	1.691	1.137	0.	1.156

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	106	70	122	127	0	92
normalized size	1	1.	2.12	1.4	2.44	2.54	0.	1.84
time (sec)	N/A	0.09	0.139	0.068	1.016	1.021	0.	1.286

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	104	70	149	126	0	77
normalized size	1	1.	2.81	1.89	4.03	3.41	0.	2.08
time (sec)	N/A	0.093	0.131	0.056	1.023	1.025	0.	1.23

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	149	103	184	333	0	112
normalized size	1	1.	1.89	1.3	2.33	4.22	0.	1.42
time (sec)	N/A	0.12	0.606	0.08	1.02	1.121	0.	1.299

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	245	139	290	440	0	180
normalized size	1	1.	2.63	1.49	3.12	4.73	0.	1.94
time (sec)	N/A	0.194	0.586	0.092	1.026	1.158	0.	1.281

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	191	267	568	350	0	216
normalized size	1	1.	1.28	1.79	3.81	2.35	0.	1.45
time (sec)	N/A	0.306	0.573	0.105	1.556	1.158	0.	1.293

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	148	169	452	325	0	204
normalized size	1	1.	1.23	1.41	3.77	2.71	0.	1.7
time (sec)	N/A	0.278	0.563	0.109	1.573	1.104	0.	1.288

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	143	146	336	297	0	139
normalized size	1	1.	1.35	1.38	3.17	2.8	0.	1.31
time (sec)	N/A	0.282	0.547	0.089	1.52	1.094	0.	1.196

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	84	100	221	197	0	127
normalized size	1	1.	1.27	1.52	3.35	2.98	0.	1.92
time (sec)	N/A	0.259	0.271	0.088	1.03	1.122	0.	1.192

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	86	100	248	198	0	127
normalized size	1	1.	1.18	1.37	3.4	2.71	0.	1.74
time (sec)	N/A	0.19	0.237	0.089	1.027	1.053	0.	1.216

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	82	100	275	204	0	127
normalized size	1	1.	1.15	1.41	3.87	2.87	0.	1.79
time (sec)	N/A	0.128	0.226	0.082	1.031	1.013	0.	1.259

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	196	145	338	471	0	147
normalized size	1	1.	1.7	1.26	2.94	4.1	0.	1.28
time (sec)	N/A	0.266	0.479	0.105	1.03	1.164	0.	1.281

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	289	182	417	591	0	217
normalized size	1	1.	2.22	1.4	3.21	4.55	0.	1.67
time (sec)	N/A	0.31	0.732	0.13	1.074	1.141	0.	1.276

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	328	219	478	699	0	252
normalized size	1	1.	2.08	1.39	3.03	4.42	0.	1.59
time (sec)	N/A	0.346	0.699	0.141	1.057	1.149	0.	1.337

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	224	211	568	431	0	239
normalized size	1	1.	1.48	1.4	3.76	2.85	0.	1.58
time (sec)	N/A	0.341	0.651	0.12	1.569	1.143	0.	1.196

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	214	187	452	408	0	174
normalized size	1	1.	1.51	1.32	3.18	2.87	0.	1.23
time (sec)	N/A	0.34	0.823	0.117	1.629	1.15	0.	1.283

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	104	130	311	266	0	162
normalized size	1	1.	1.02	1.27	3.05	2.61	0.	1.59
time (sec)	N/A	0.329	0.477	0.107	1.05	1.123	0.	1.252

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	104	130	338	263	0	162
normalized size	1	1.	1.18	1.48	3.84	2.99	0.	1.84
time (sec)	N/A	0.315	0.376	0.102	1.07	1.108	0.	1.206

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	130	365	265	0	162
normalized size	1	1.	1.01	1.26	3.54	2.57	0.	1.57
time (sec)	N/A	0.243	0.339	0.104	1.048	1.142	0.	1.243

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	104	130	392	270	0	162
normalized size	1	1.	1.05	1.31	3.96	2.73	0.	1.64
time (sec)	N/A	0.142	0.329	0.096	1.031	1.081	0.	1.311

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	341	187	454	602	0	182
normalized size	1	1.	2.26	1.24	3.01	3.99	0.	1.21
time (sec)	N/A	0.293	0.408	0.133	1.07	1.16	0.	1.247

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	351	224	533	713	0	252
normalized size	1	1.	2.17	1.38	3.29	4.4	0.	1.56
time (sec)	N/A	0.347	0.848	0.15	1.06	1.539	0.	1.309

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	84	164	130	285	0	246
normalized size	1	1.	0.72	1.4	1.11	2.44	0.	2.1
time (sec)	N/A	0.135	0.411	0.074	1.518	1.463	0.	1.283

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	76	154	117	258	0	181
normalized size	1	1.	0.75	1.52	1.16	2.55	0.	1.79
time (sec)	N/A	0.13	0.245	0.069	1.697	1.408	0.	1.266

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	88	224	0	167
normalized size	1	1.	1.12	1.36	1.22	3.11	0.	2.32
time (sec)	N/A	0.096	0.041	0.063	1.539	1.365	0.	1.263

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	69	88	74	196	0	100
normalized size	1	1.	1.15	1.47	1.23	3.27	0.	1.67
time (sec)	N/A	0.098	0.042	0.064	1.533	1.425	0.	1.233

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	82	53	127	0	90
normalized size	1	1.	1.	1.82	1.18	2.82	0.	2.
time (sec)	N/A	0.104	0.037	0.055	1.015	1.531	0.	1.263

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	36	35	127	0	72
normalized size	1	1.	1.	1.09	1.06	3.85	0.	2.18
time (sec)	N/A	0.084	0.021	0.047	1.04	1.567	0.	1.24

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	85	82	99	348	0	109
normalized size	1	1.	1.25	1.21	1.46	5.12	0.	1.6
time (sec)	N/A	0.087	0.116	0.082	1.17	1.674	0.	1.289

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	109	106	112	458	0	174
normalized size	1	1.	1.35	1.31	1.38	5.65	0.	2.15
time (sec)	N/A	0.124	0.057	0.098	1.013	1.615	0.	1.281

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	359	138	143	575	0	200
normalized size	1	1.	3.26	1.25	1.3	5.23	0.	1.82
time (sec)	N/A	0.136	6.086	0.106	1.167	1.468	0.	1.211

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	120	159	186	162	468	0	182
normalized size	1	1.19	1.57	1.84	1.6	4.63	0.	1.8
time (sec)	N/A	0.205	1.235	0.083	1.68	1.402	0.	1.23

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	131	162	128	400	0	116
normalized size	1	1.	1.52	1.88	1.49	4.65	0.	1.35
time (sec)	N/A	0.246	1.034	0.073	1.643	1.387	0.	1.273

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	79	114	96	329	0	90
normalized size	1	1.	1.25	1.81	1.52	5.22	0.	1.43
time (sec)	N/A	0.207	0.051	0.074	1.653	1.41	0.	1.22

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	72	99	76	236	0	51
normalized size	1	1.	1.2	1.65	1.27	3.93	0.	0.85
time (sec)	N/A	0.086	0.282	0.059	1.125	1.41	0.	1.272

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	142	92	122	578	0	99
normalized size	1	1.	1.95	1.26	1.67	7.92	0.	1.36
time (sec)	N/A	0.187	0.501	0.103	1.026	1.411	0.	1.27

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	135	156	144	810	0	159
normalized size	1	1.	1.55	1.79	1.66	9.31	0.	1.83
time (sec)	N/A	0.274	0.933	0.122	1.156	1.424	0.	1.321

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	190	168	216	1062	0	203
normalized size	1	1.	1.52	1.34	1.73	8.5	0.	1.62
time (sec)	N/A	0.308	2.112	0.135	1.191	1.468	0.	1.343

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	177	266	223	539	0	252
normalized size	1	1.	1.49	2.24	1.87	4.53	0.	2.12
time (sec)	N/A	0.194	2.226	0.098	1.567	1.468	0.	1.327

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	159	246	196	471	0	182
normalized size	1	1.	1.57	2.44	1.94	4.66	0.	1.8
time (sec)	N/A	0.162	1.46	0.088	1.7	1.426	0.	1.218

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	133	184	144	404	0	117
normalized size	1	1.	1.73	2.39	1.87	5.25	0.	1.52
time (sec)	N/A	0.219	1.253	0.08	1.686	1.373	0.	1.298

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	107	126	113	335	0	90
normalized size	1	1.	1.67	1.97	1.77	5.23	0.	1.41
time (sec)	N/A	0.139	0.719	0.071	1.691	1.355	0.	1.254

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	144	115	139	582	0	99
normalized size	1	1.	2.	1.6	1.93	8.08	0.	1.38
time (sec)	N/A	0.141	0.626	0.121	1.153	1.479	0.	1.243

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	135	155	166	819	0	159
normalized size	1	1.	1.57	1.8	1.93	9.52	0.	1.85
time (sec)	N/A	0.17	0.955	0.136	1.101	1.567	0.	1.323

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	190	202	246	1067	0	203
normalized size	1	1.	1.73	1.84	2.24	9.7	0.	1.85
time (sec)	N/A	0.19	2.111	0.244	1.181	1.478	0.	1.203

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	287	214	277	1314	0	262
normalized size	1	1.	2.24	1.67	2.16	10.27	0.	2.05
time (sec)	N/A	0.206	6.183	0.164	1.208	1.522	0.	1.214

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	252	360	321	620	0	270
normalized size	1	1.	1.76	2.52	2.24	4.34	0.	1.89
time (sec)	N/A	0.204	1.577	0.101	1.684	1.519	0.	1.301

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	158	268	213	477	0	182
normalized size	1	1.	1.56	2.65	2.11	4.72	0.	1.8
time (sec)	N/A	0.163	1.912	0.101	1.62	1.451	0.	1.318

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	224	210	540	288	0	201
normalized size	1	1.	1.91	1.79	4.62	2.46	0.	1.72
time (sec)	N/A	0.158	0.675	0.096	1.586	1.507	0.	1.299

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	191	166	429	258	0	176
normalized size	1	1.	1.82	1.58	4.09	2.46	0.	1.68
time (sec)	N/A	0.129	0.643	0.09	1.692	1.398	0.	1.307

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	106	130	289	194	0	162
normalized size	1	1.	1.54	1.88	4.19	2.81	0.	2.35
time (sec)	N/A	0.096	0.269	0.086	1.037	1.376	0.	1.289

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	106	115	316	190	0	162
normalized size	1	1.	1.93	2.09	5.75	3.45	0.	2.95
time (sec)	N/A	0.138	0.288	0.081	1.069	1.346	0.	1.245

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	106	130	343	188	0	147
normalized size	1	1.	1.45	1.78	4.7	2.58	0.	2.01
time (sec)	N/A	0.16	0.333	0.074	1.075	1.374	0.	1.259

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	106	130	370	189	0	162
normalized size	1	1.	1.93	2.36	6.73	3.44	0.	2.95
time (sec)	N/A	0.108	0.259	0.067	1.181	1.315	0.	1.216

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	267	187	432	413	0	184
normalized size	1	1.	2.32	1.63	3.76	3.59	0.	1.6
time (sec)	N/A	0.124	0.639	0.098	1.032	1.396	0.	1.202

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	341	223	512	514	0	240
normalized size	1	1.	2.71	1.77	4.06	4.08	0.	1.9
time (sec)	N/A	0.166	0.596	0.102	1.1	1.571	0.	1.286

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	267	253	684	404	0	236
normalized size	1	1.	1.72	1.63	4.41	2.61	0.	1.52
time (sec)	N/A	0.29	0.779	0.144	1.702	1.717	0.	1.324

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	257	230	568	373	0	209
normalized size	1	1.	1.84	1.64	4.06	2.66	0.	1.49
time (sec)	N/A	0.3	0.559	0.119	1.769	1.687	0.	1.231

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	126	145	400	263	0	197
normalized size	1	1.	1.48	1.71	4.71	3.09	0.	2.32
time (sec)	N/A	0.272	0.264	0.114	1.114	1.64	0.	1.279

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	126	160	427	267	0	197
normalized size	1	1.	1.38	1.76	4.69	2.93	0.	2.16
time (sec)	N/A	0.157	0.247	0.122	1.086	1.633	0.	1.196

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	126	130	454	267	0	162
normalized size	1	1.	1.38	1.43	4.99	2.93	0.	1.78
time (sec)	N/A	0.288	0.325	0.107	1.238	1.614	0.	1.281

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	126	160	481	263	0	197
normalized size	1	1.	1.38	1.76	5.29	2.89	0.	2.16
time (sec)	N/A	0.306	0.374	0.111	1.153	1.615	0.	1.258

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	134	160	508	267	0	197
normalized size	1	1.	1.44	1.72	5.46	2.87	0.	2.12
time (sec)	N/A	0.115	0.302	0.105	1.164	1.613	0.	1.192

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	352	229	570	549	0	217
normalized size	1	1.	2.36	1.54	3.83	3.68	0.	1.46
time (sec)	N/A	0.253	0.601	0.131	1.065	1.812	0.	1.28

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	442	266	649	671	0	275
normalized size	1	1.	2.7	1.62	3.96	4.09	0.	1.68
time (sec)	N/A	0.33	6.09	0.135	1.067	1.814	0.	1.31

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	277	303	710	786	0	321
normalized size	1	1.	1.43	1.56	3.66	4.05	0.	1.65
time (sec)	N/A	0.366	0.582	0.162	1.16	1.909	0.	1.336

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	273	272	657	478	0	244
normalized size	1	1.	1.53	1.53	3.69	2.69	0.	1.37
time (sec)	N/A	0.368	0.511	0.16	1.789	1.883	0.	1.297

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	185	190	489	328	0	232
normalized size	1	1.	1.53	1.57	4.04	2.71	0.	1.92
time (sec)	N/A	0.347	0.376	0.142	1.201	1.696	0.	1.195

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	185	190	516	323	0	232
normalized size	1	1.	1.76	1.81	4.91	3.08	0.	2.21
time (sec)	N/A	0.338	0.219	0.139	1.159	1.766	0.	1.304

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	185	175	543	335	0	215
normalized size	1	1.	1.46	1.38	4.28	2.64	0.	1.69
time (sec)	N/A	0.223	0.279	0.141	1.068	1.703	0.	1.26

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	185	190	570	323	0	217
normalized size	1	1.	1.76	1.81	5.43	3.08	0.	2.07
time (sec)	N/A	0.335	0.316	0.131	1.1	1.681	0.	1.273

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	185	190	597	333	0	232
normalized size	1	1.	1.46	1.5	4.7	2.62	0.	1.83
time (sec)	N/A	0.362	0.328	0.125	1.161	1.654	0.	1.292

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	185	190	597	328	0	232
normalized size	1	1.	1.5	1.54	4.85	2.67	0.	1.89
time (sec)	N/A	0.161	0.277	0.113	1.187	1.687	0.	1.319

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	204	271	686	676	0	252
normalized size	1	1.	1.09	1.45	3.67	3.61	0.	1.35
time (sec)	N/A	0.36	1.398	0.158	1.104	2.049	0.	1.25

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	230	308	765	794	0	311
normalized size	1	1.	1.15	1.54	3.82	3.97	0.	1.56
time (sec)	N/A	0.393	0.654	0.171	1.156	1.875	0.	1.308

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	166	190	659	409	0	232
normalized size	1	1.	1.14	1.31	4.54	2.82	0.	1.6
time (sec)	N/A	0.313	0.461	0.135	1.217	1.718	0.	1.343

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	166	220	686	402	0	267
normalized size	1	1.	1.14	1.52	4.73	2.77	0.	1.84
time (sec)	N/A	0.409	0.457	0.131	1.153	1.742	0.	1.337

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	184	166	218	713	390	0	267
normalized size	1	1.29	1.16	1.52	4.99	2.73	0.	1.87
time (sec)	N/A	0.356	0.492	0.144	1.256	1.622	0.	1.359

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	133	205	143	440	0	153
normalized size	1	1.	1.	1.54	1.08	3.31	0.	1.15
time (sec)	N/A	0.111	0.44	0.072	1.115	1.544	0.	1.393

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	123	147	128	410	0	136
normalized size	1	1.	1.07	1.28	1.11	3.57	0.	1.18
time (sec)	N/A	0.068	0.267	0.071	1.196	1.574	0.	1.424

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	106	133	116	351	0	126
normalized size	1	1.	1.01	1.27	1.1	3.34	0.	1.2
time (sec)	N/A	0.094	0.23	0.069	1.011	1.533	0.	1.495

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	114	116	350	0	122
normalized size	1	1.	1.	1.36	1.38	4.17	0.	1.45
time (sec)	N/A	0.102	0.201	0.067	1.027	1.353	0.	1.33

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	74	100	113	343	0	123
normalized size	1	1.	0.88	1.19	1.35	4.08	0.	1.46
time (sec)	N/A	0.098	0.031	0.059	1.114	1.447	0.	1.32

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	74	92	113	343	0	123
normalized size	1	1.	1.21	1.51	1.85	5.62	0.	2.02
time (sec)	N/A	0.067	0.029	0.053	1.059	1.499	0.	1.312

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	99	100	128	456	0	140
normalized size	1	1.	0.85	0.85	1.09	3.9	0.	1.2
time (sec)	N/A	0.107	0.209	0.085	1.013	1.445	0.	1.328

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	76	120	154	593	0	163
normalized size	1	1.	0.59	0.93	1.19	4.6	0.	1.26
time (sec)	N/A	0.121	0.189	0.109	1.081	1.536	0.	1.349

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	86	151	171	733	0	169
normalized size	1	1.	0.6	1.06	1.2	5.13	0.	1.18
time (sec)	N/A	0.132	0.754	0.112	1.133	1.586	0.	1.345

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	90	173	186	892	0	201
normalized size	1	1.	0.56	1.07	1.15	5.51	0.	1.24
time (sec)	N/A	0.141	1.211	0.123	1.035	1.608	0.	1.372

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	75	261	130	412	0	138
normalized size	1	1.	0.63	2.19	1.09	3.46	0.	1.16
time (sec)	N/A	0.088	0.247	0.091	1.119	1.584	0.	1.273

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	67	213	112	377	0	119
normalized size	1	1.	0.66	2.11	1.11	3.73	0.	1.18
time (sec)	N/A	0.131	0.12	0.088	0.986	1.467	0.	1.292

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	91	173	97	312	0	105
normalized size	1	1.	1.05	1.99	1.11	3.59	0.	1.21
time (sec)	N/A	0.139	0.381	0.079	1.084	1.523	0.	1.284

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	39	174	97	308	0	104
normalized size	1	1.	0.61	2.72	1.52	4.81	0.	1.62
time (sec)	N/A	0.118	0.107	0.075	1.082	1.389	0.	1.262

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	126	86	297	0	128
normalized size	1	1.	0.56	1.97	1.34	4.64	0.	2.
time (sec)	N/A	0.087	0.08	0.067	1.152	1.519	0.	1.22

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	66	112	113	412	0	123
normalized size	1	1.	0.65	1.11	1.12	4.08	0.	1.22
time (sec)	N/A	0.115	0.293	0.109	1.046	1.582	0.	1.24

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	74	176	140	582	0	155
normalized size	1	1.	0.64	1.52	1.21	5.02	0.	1.34
time (sec)	N/A	0.142	0.262	0.128	1.054	1.481	0.	1.242

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	84	199	161	732	0	169
normalized size	1	1.	0.63	1.49	1.2	5.46	0.	1.26
time (sec)	N/A	0.148	1.185	0.141	0.992	1.541	0.	1.27

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	133	215	180	906	0	192
normalized size	1	1.	0.89	1.43	1.2	6.04	0.	1.28
time (sec)	N/A	0.164	6.057	0.152	1.134	1.652	0.	1.258

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	73	325	130	351	0	327
normalized size	1	1.	0.64	2.85	1.14	3.08	0.	2.87
time (sec)	N/A	0.084	0.306	0.1	1.072	1.444	0.	1.257

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	61	309	111	311	0	282
normalized size	1	1.	0.64	3.22	1.16	3.24	0.	2.94
time (sec)	N/A	0.111	0.236	0.095	1.006	1.493	0.	1.209

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	53	237	95	269	0	240
normalized size	1	1.	0.68	3.04	1.22	3.45	0.	3.08
time (sec)	N/A	0.119	0.22	0.091	0.992	1.446	0.	1.213

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	45	220	80	211	0	169
normalized size	1	1.	0.7	3.44	1.25	3.3	0.	2.64
time (sec)	N/A	0.11	0.144	0.083	1.084	1.445	0.	1.273

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	154	57	104	0	43
normalized size	1	1.	0.97	4.97	1.84	3.35	0.	1.39
time (sec)	N/A	0.059	0.03	0.073	1.102	1.323	0.	1.192

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	54	172	95	312	0	166
normalized size	1	1.	0.7	2.23	1.23	4.05	0.	2.16
time (sec)	N/A	0.105	0.221	0.121	1.086	1.428	0.	1.313

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	63	176	122	441	0	224
normalized size	1	1.	0.68	1.89	1.31	4.74	0.	2.41
time (sec)	N/A	0.158	0.221	0.136	1.129	1.457	0.	1.207

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	73	241	139	567	0	267
normalized size	1	1.	0.66	2.17	1.25	5.11	0.	2.41
time (sec)	N/A	0.148	0.779	0.149	1.142	1.53	0.	1.275

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	153	208	282	585	0	242
normalized size	1	1.	0.65	0.88	1.19	2.48	0.	1.03
time (sec)	N/A	0.252	6.124	0.113	1.021	1.864	0.	1.362

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	143	192	266	554	0	217
normalized size	1	1.	0.65	0.87	1.21	2.52	0.	0.99
time (sec)	N/A	0.229	6.141	0.112	1.037	1.896	0.	1.391

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	133	175	251	527	0	198
normalized size	1	1.	0.67	0.88	1.26	2.65	0.	0.99
time (sec)	N/A	0.212	6.131	0.105	1.058	1.641	0.	1.344

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	117	162	236	466	0	184
normalized size	1	1.	0.62	0.86	1.26	2.48	0.	0.98
time (sec)	N/A	0.184	3.95	0.104	1.135	1.59	0.	1.366

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	101	162	236	464	0	184
normalized size	1	1.	0.78	1.25	1.82	3.57	0.	1.42
time (sec)	N/A	0.172	0.923	0.095	1.065	1.681	0.	1.324

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	101	162	236	460	0	184
normalized size	1	1.	0.75	1.21	1.76	3.43	0.	1.37
time (sec)	N/A	0.236	0.916	0.091	1.067	1.468	0.	1.355

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	92	144	234	460	0	184
normalized size	1	1.	0.61	0.95	1.54	3.03	0.	1.21
time (sec)	N/A	0.245	1.175	0.089	1.026	1.63	0.	1.367

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	101	162	236	452	0	184
normalized size	1	1.	0.67	1.08	1.57	3.01	0.	1.23
time (sec)	N/A	0.226	0.632	0.086	1.117	1.461	0.	1.331

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	92	144	236	451	0	184
normalized size	1	1.	0.61	0.96	1.57	3.01	0.	1.23
time (sec)	N/A	0.225	0.879	0.082	1.261	1.553	0.	1.409

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	92	144	236	455	0	184
normalized size	1	1.	0.62	0.97	1.59	3.07	0.	1.24
time (sec)	N/A	0.199	0.523	0.081	1.267	1.584	0.	1.346

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	92	144	236	456	0	184
normalized size	1	1.	0.71	1.11	1.82	3.51	0.	1.42
time (sec)	N/A	0.148	0.871	0.068	1.274	1.656	0.	1.287

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	145	162	236	462	0	184
normalized size	1	1.	0.88	0.98	1.43	2.8	0.	1.12
time (sec)	N/A	0.126	0.527	0.073	1.061	1.511	0.	1.324

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	189	176	252	564	0	201
normalized size	1	1.	0.94	0.87	1.25	2.79	0.	1.
time (sec)	N/A	0.201	6.13	0.089	1.022	1.641	0.	1.344

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	201	193	277	694	0	230
normalized size	1	1.	0.93	0.89	1.28	3.2	0.	1.06
time (sec)	N/A	0.239	6.139	0.103	1.028	1.673	0.	1.37

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	213	208	293	830	0	246
normalized size	1	1.	0.92	0.9	1.26	3.58	0.	1.06
time (sec)	N/A	0.246	6.164	0.108	1.083	1.728	0.	1.393

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	231	225	306	979	0	252
normalized size	1	1.	0.91	0.89	1.21	3.87	0.	1.
time (sec)	N/A	0.264	6.139	0.114	1.062	1.708	0.	1.393

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	139	248	123	284	0	186
normalized size	1	1.	1.53	2.73	1.35	3.12	0.	2.04
time (sec)	N/A	0.214	0.985	0.089	1.037	1.334	0.	1.282

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	169	227	319	626	0	244
normalized size	1	1.	0.64	0.86	1.21	2.37	0.	0.92
time (sec)	N/A	0.283	6.163	0.112	1.04	1.977	0.	1.354

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	159	212	304	602	0	225
normalized size	1	1.	0.64	0.86	1.23	2.44	0.	0.91
time (sec)	N/A	0.261	6.177	0.113	1.052	1.896	0.	1.32

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	137	198	289	537	0	211
normalized size	1	1.	0.59	0.85	1.24	2.3	0.	0.91
time (sec)	N/A	0.235	4.915	0.109	1.059	2.135	0.	1.356

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	122	198	289	529	0	211
normalized size	1	1.	0.79	1.29	1.88	3.44	0.	1.37
time (sec)	N/A	0.193	2.521	0.105	1.053	2.047	0.	1.334

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	121	198	289	533	0	211
normalized size	1	1.	0.76	1.24	1.81	3.33	0.	1.32
time (sec)	N/A	0.271	2.449	0.107	1.029	2.025	0.	1.401

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	124	180	289	529	0	211
normalized size	1	1.	0.7	1.01	1.62	2.97	0.	1.19
time (sec)	N/A	0.28	1.667	0.096	1.048	2.337	0.	1.363

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	122	198	289	518	0	211
normalized size	1	1.	0.69	1.12	1.64	2.94	0.	1.2
time (sec)	N/A	0.266	2.766	0.095	1.036	2.562	0.	1.332

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	116	180	289	521	0	211
normalized size	1	1.	0.6	0.93	1.49	2.69	0.	1.09
time (sec)	N/A	0.273	5.853	0.089	1.017	2.338	0.	1.281

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	116	180	289	517	0	211
normalized size	1	1.	0.6	0.94	1.51	2.69	0.	1.1
time (sec)	N/A	0.253	5.726	0.087	1.093	2.078	0.	1.41

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	104	162	289	512	0	211
normalized size	1	1.	0.6	0.93	1.66	2.94	0.	1.21
time (sec)	N/A	0.242	2.905	0.084	1.058	2.157	0.	1.357

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	122	198	289	518	0	211
normalized size	1	1.	0.71	1.15	1.68	3.01	0.	1.23
time (sec)	N/A	0.219	2.592	0.079	1.036	2.132	0.	1.262

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	116	180	289	520	0	211
normalized size	1	1.	0.75	1.17	1.88	3.38	0.	1.37
time (sec)	N/A	0.165	5.87	0.071	1.071	2.185	0.	1.316

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	165	198	289	520	0	211
normalized size	1	1.	0.79	0.94	1.38	2.48	0.	1.
time (sec)	N/A	0.167	1.431	0.073	1.045	2.105	0.	1.336

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	228	212	305	636	0	228
normalized size	1	1.	0.92	0.86	1.23	2.57	0.	0.92
time (sec)	N/A	0.244	6.197	0.1	1.024	2.224	0.	1.332

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	240	229	331	774	0	257
normalized size	1	1.	0.92	0.87	1.26	2.95	0.	0.98
time (sec)	N/A	0.291	6.203	0.104	1.025	2.191	0.	1.233

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	254	244	347	910	0	273
normalized size	1	1.	0.91	0.87	1.24	3.26	0.	0.98
time (sec)	N/A	0.302	6.227	0.112	1.037	2.338	0.	1.282

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	347	0	0	0	0	0
normalized size	1	1.	2.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.349	3.263	7.592	0.	0.	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.14	0.33	0.	0.	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	130	0	0	1897	0	2484
normalized size	1	1.	0.74	0.	0.	10.84	0.	14.19
time (sec)	N/A	0.206	0.596	0.565	0.	2.766	0.	1.215

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	105	0	0	1125	5722	1351
normalized size	1	1.	0.76	0.	0.	8.09	41.17	9.72
time (sec)	N/A	0.165	0.32	0.458	0.	2.369	173.645	1.223

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	0	0	599	2278	625
normalized size	1	1.	0.77	0.	0.	5.93	22.55	6.19
time (sec)	N/A	0.15	0.359	0.466	0.	2.019	29.733	1.137

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	52	0	0	250	586	211
normalized size	1	1.	0.85	0.	0.	4.1	9.61	3.46
time (sec)	N/A	0.096	0.487	0.269	0.	1.947	10.714	1.262

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.069	1.207	0.	0.	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	61	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.064	0.884	0.	0.	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.062	1.088	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	143	0	0	1706	0	2491
normalized size	1	1.	0.84	0.	0.	10.04	0.	14.65
time (sec)	N/A	0.182	0.659	7.776	0.	2.474	0.	1.322

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	0	905	4308	1354
normalized size	1	1.	0.85	0.	0.	6.8	32.39	10.18
time (sec)	N/A	0.14	0.382	3.055	0.	2.42	109.548	1.205

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	83	0	0	427	1686	624
normalized size	1	1.	0.86	0.	0.	4.45	17.56	6.5
time (sec)	N/A	0.112	0.393	2.322	0.	2.27	28.418	1.234

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	51	0	0	169	428	211
normalized size	1	1.	0.86	0.	0.	2.86	7.25	3.58
time (sec)	N/A	0.066	0.12	1.645	0.	2.05	9.607	1.218

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.097	1.079	0.	0.	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.103	1.174	0.	0.	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.09	1.373	0.	0.	0.	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	61	61	0	0	0	0	0
normalized size	1	1.13	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.08	3.917	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	150	0	215	501	0	1068
normalized size	1	1.	1.12	0.	1.6	3.74	0.	7.97
time (sec)	N/A	0.119	1.404	2.483	1.077	1.854	0.	1.27

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	94	0	161	336	1547	686
normalized size	1	1.	0.87	0.	1.49	3.11	14.32	6.35
time (sec)	N/A	0.097	0.571	1.652	1.038	1.8	70.741	1.248

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	113	217	756	387
normalized size	1	1.	0.96	0.	1.41	2.71	9.45	4.84
time (sec)	N/A	0.084	0.187	1.47	1.04	1.832	21.482	1.251

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	0	76	126	248	162
normalized size	1	1.	0.8	0.	1.41	2.33	4.59	3.
time (sec)	N/A	0.053	0.027	0.974	1.031	1.909	7.615	1.221

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.055	0.971	0.	0.	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.055	0.668	0.	0.	0.	0.

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.06	0.714	0.	0.	0.	0.

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	84	64	96	100	177	199	117
normalized size	1	1.06	0.81	1.22	1.27	2.24	2.52	1.48
time (sec)	N/A	0.092	0.608	0.053	1.038	1.715	1.319	1.271

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	220	160	0	1661	0	0
normalized size	1	1.	1.79	1.3	0.	13.5	0.	0.
time (sec)	N/A	0.551	2.729	1.283	0.	2.732	0.	0.

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	208404	4463	0	5009	0	0
normalized size	1	1.	1478.04	31.65	0.	35.52	0.	0.
time (sec)	N/A	0.672	33.288	0.319	0.	6.686	0.	0.

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	158	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	0.676	0.437	0.	0.	0.	0.

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	113.736	0.467	0.	0.	0.	0.

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	74.362	0.431	0.	0.	0.	0.

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	16.066	0.346	0.	0.	0.	0.

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	229	0	0	0	0	0
normalized size	1	1.	1.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	0.995	0.414	0.	0.	0.	0.

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	8.586	0.5	0.	0.	0.	0.

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	16.398	0.72	0.	0.	0.	0.

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	160	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	1.29	0.495	0.	0.	0.	0.

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.177	1.599	0.539	0.	0.	0.	0.

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.738	0.432	0.	0.	0.	0.

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	29.228	0.403	0.	0.	0.	0.

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	16.978	0.57	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	18.076	0.658	0.	0.	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	17.705	0.927	0.	0.	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	2.241	1.125	0.	0.	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	194	128	181	261	228	246
normalized size	1	1.	1.45	0.96	1.35	1.95	1.7	1.84
time (sec)	N/A	0.142	0.792	0.058	1.024	1.879	22.875	1.323

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	130	108	140	215	178	196
normalized size	1	1.	1.27	1.06	1.37	2.11	1.75	1.92
time (sec)	N/A	0.108	0.672	0.059	1.051	1.791	7.579	1.272

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	88	97	167	128	135
normalized size	1	1.	1.	1.13	1.24	2.14	1.64	1.73
time (sec)	N/A	0.094	0.804	0.054	0.973	1.66	2.448	1.274

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	46	44	57	120	75	70
normalized size	1	1.	0.94	0.9	1.16	2.45	1.53	1.43
time (sec)	N/A	0.063	0.394	0.026	1.009	1.64	0.63	1.265

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	68	47	39	78	0	154
normalized size	1	1.	2.	1.38	1.15	2.29	0.	4.53
time (sec)	N/A	0.07	0.034	0.065	1.011	1.774	0.	1.376

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	260	129	74	221	0	113
normalized size	1	1.	5.53	2.74	1.57	4.7	0.	2.4
time (sec)	N/A	0.086	0.614	0.089	0.994	1.836	0.	1.369

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	357	173	155	439	0	205
normalized size	1	1.	3.57	1.73	1.55	4.39	0.	2.05
time (sec)	N/A	0.122	1.527	0.096	1.024	1.813	0.	1.346

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	451	217	231	533	0	271
normalized size	1	1.	2.87	1.38	1.47	3.39	0.	1.73
time (sec)	N/A	0.168	1.69	0.104	1.083	1.859	0.	1.426

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	164	138	167	267	416	238
normalized size	1	1.	1.19	1.	1.21	1.93	3.01	1.72
time (sec)	N/A	0.141	0.88	0.056	1.037	1.916	13.828	1.327

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	120	118	132	217	306	180
normalized size	1	1.	1.08	1.06	1.19	1.95	2.76	1.62
time (sec)	N/A	0.113	0.619	0.055	1.015	1.775	4.836	1.292

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	64	96	100	169	199	112
normalized size	1	1.	0.76	1.14	1.19	2.01	2.37	1.33
time (sec)	N/A	0.095	0.662	0.052	1.011	1.764	1.342	1.321

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	85	54	76	184	0	49
normalized size	1	1.	2.93	1.86	2.62	6.34	0.	1.69
time (sec)	N/A	0.049	0.34	0.064	1.516	1.827	0.	1.36

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	97	72	80	166	0	127
normalized size	1	1.	1.94	1.44	1.6	3.32	0.	2.54
time (sec)	N/A	0.068	0.588	0.084	1.026	1.623	0.	1.322

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	223	102	116	259	0	304
normalized size	1	1.	3.05	1.4	1.59	3.55	0.	4.16
time (sec)	N/A	0.073	1.238	0.099	1.046	1.74	0.	1.287

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	315	130	144	350	0	466
normalized size	1	1.	3.28	1.35	1.5	3.65	0.	4.85
time (sec)	N/A	0.08	2.023	0.105	1.013	1.787	0.	1.263

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	407	158	170	436	0	628
normalized size	1	1.	3.42	1.33	1.43	3.66	0.	5.28
time (sec)	N/A	0.087	4.279	0.119	1.023	1.916	0.	1.366

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	86	231	227	328	389	323
normalized size	1	1.	0.64	1.72	1.69	2.45	2.9	2.41
time (sec)	N/A	0.178	1.185	0.069	1.045	2.083	37.79	1.297

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	70	201	192	277	335	273
normalized size	1	1.	0.67	1.91	1.83	2.64	3.19	2.6
time (sec)	N/A	0.149	0.344	0.067	1.018	1.925	14.024	1.366

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	171	130	224	228	157
normalized size	1	1.	0.85	2.19	1.67	2.87	2.92	2.01
time (sec)	N/A	0.109	0.406	0.068	0.979	1.793	4.965	1.288

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	75	92	177	143	119
normalized size	1	1.	0.96	1.47	1.8	3.47	2.8	2.33
time (sec)	N/A	0.07	0.093	0.032	1.064	1.832	1.36	1.37

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	127	70	135	0	297
normalized size	1	1.	0.85	2.12	1.17	2.25	0.	4.95
time (sec)	N/A	0.096	0.079	0.075	1.015	1.841	0.	1.346

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	189	50	123	0	151
normalized size	1	1.	0.95	4.4	1.16	2.86	0.	3.51
time (sec)	N/A	0.09	0.074	0.103	1.082	1.738	0.	1.336

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	281	117	385	0	176
normalized size	1	1.	0.97	3.65	1.52	5.	0.	2.29
time (sec)	N/A	0.119	0.137	0.112	1.039	1.815	0.	1.367

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	90	379	200	640	0	282
normalized size	1	1.	0.68	2.87	1.52	4.85	0.	2.14
time (sec)	N/A	0.157	0.656	0.115	1.029	1.776	0.	1.37

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	216	245	281	343	719	317
normalized size	1	1.	1.1	1.25	1.43	1.75	3.67	1.62
time (sec)	N/A	0.211	5.057	0.069	1.036	2.026	26.096	1.353

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	171	215	231	290	539	259
normalized size	1	1.	1.04	1.3	1.4	1.76	3.27	1.57
time (sec)	N/A	0.189	1.702	0.073	1.06	2.038	9.312	1.306

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	133	182	181	235	371	176
normalized size	1	1.	0.99	1.36	1.35	1.75	2.77	1.31
time (sec)	N/A	0.168	0.955	0.064	1.02	1.998	3.052	1.329

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	91	123	140	302	0	169
normalized size	1	1.	1.65	2.24	2.55	5.49	0.	3.07
time (sec)	N/A	0.093	0.242	0.079	1.529	2.092	0.	1.338

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	121	162	146	294	0	105
normalized size	1	1.	1.66	2.22	2.	4.03	0.	1.44
time (sec)	N/A	0.116	0.018	0.095	1.015	1.926	0.	1.331

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	178	231	198	273	0	259
normalized size	1	1.	1.71	2.22	1.9	2.62	0.	2.49
time (sec)	N/A	0.125	0.02	0.106	1.024	1.836	0.	1.243

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	130	295	240	377	0	439
normalized size	1	1.	1.01	2.29	1.86	2.92	0.	3.4
time (sec)	N/A	0.134	0.345	0.119	1.03	1.618	0.	1.291

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	156	359	279	475	0	622
normalized size	1	1.	1.01	2.33	1.81	3.08	0.	4.04
time (sec)	N/A	0.141	0.448	0.131	1.045	1.847	0.	1.314

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	181	423	321	583	0	806
normalized size	1	1.	1.01	2.36	1.79	3.26	0.	4.5
time (sec)	N/A	0.151	0.909	0.236	1.042	1.891	0.	1.444

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	86	345	246	400	530	382
normalized size	1	1.	0.64	2.57	1.84	2.99	3.96	2.85
time (sec)	N/A	0.182	1.547	0.07	1.018	2.027	65.602	1.381

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	70	305	213	321	471	311
normalized size	1	1.	0.67	2.9	2.03	3.06	4.49	2.96
time (sec)	N/A	0.152	0.435	0.072	1.135	1.924	24.665	1.326

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	53	265	170	286	313	232
normalized size	1	1.	0.68	3.4	2.18	3.67	4.01	2.97
time (sec)	N/A	0.134	0.24	0.069	1.022	1.798	8.326	1.342

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	36	98	113	224	204	157
normalized size	1	1.	0.71	1.92	2.22	4.39	4.	3.08
time (sec)	N/A	0.062	0.088	0.033	1.055	1.686	2.702	1.323

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	68	161	99	188	0	390
normalized size	1	1.	0.84	1.99	1.22	2.32	0.	4.81
time (sec)	N/A	0.095	0.126	0.087	1.031	1.787	0.	1.383

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	48	290	70	207	0	308
normalized size	1	1.	0.77	4.68	1.13	3.34	0.	4.97
time (sec)	N/A	0.109	0.115	0.116	1.023	1.745	0.	1.435

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	312	63	117	0	111
normalized size	1	1.	0.86	7.26	1.47	2.72	0.	2.58
time (sec)	N/A	0.082	0.044	0.123	1.034	1.578	0.	1.406

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	95	521	166	570	0	213
normalized size	1	1.	0.9	4.96	1.58	5.43	0.	2.03
time (sec)	N/A	0.135	0.253	0.129	1.064	1.775	0.	1.409

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	151	669	250	846	0	320
normalized size	1	1.	0.93	4.13	1.54	5.22	0.	1.98
time (sec)	N/A	0.186	0.631	0.223	1.065	1.865	0.	1.504

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	344	363	383	410	1042	369
normalized size	1	1.	1.49	1.57	1.66	1.77	4.51	1.6
time (sec)	N/A	0.268	6.057	0.072	1.087	2.107	46.532	1.401

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	183	323	313	342	823	293
normalized size	1	1.	0.92	1.62	1.56	1.71	4.12	1.46
time (sec)	N/A	0.239	2.184	0.07	1.105	1.92	16.362	1.331

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	146	279	269	285	588	223
normalized size	1	1.	0.92	1.75	1.69	1.79	3.7	1.4
time (sec)	N/A	0.217	1.365	0.069	1.138	1.837	5.755	1.315

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	82	219	225	414	0	198
normalized size	1	1.	0.9	2.41	2.47	4.55	0.	2.18
time (sec)	N/A	0.105	0.243	0.087	1.617	1.686	0.	1.304

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	121	248	221	402	0	126
normalized size	1	1.	1.75	3.59	3.2	5.83	0.	1.83
time (sec)	N/A	0.154	1.058	0.126	1.65	1.604	0.	1.33

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	94	333	254	464	0	197
normalized size	1	1.	0.88	3.11	2.37	4.34	0.	1.84
time (sec)	N/A	0.154	0.164	0.119	1.068	1.657	0.	1.374

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	135	435	308	350	0	351
normalized size	1	1.	1.17	3.78	2.68	3.04	0.	3.05
time (sec)	N/A	0.146	0.457	0.123	1.06	1.666	0.	1.367

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	176	535	365	436	0	531
normalized size	1	1.	1.26	3.82	2.61	3.11	0.	3.79
time (sec)	N/A	0.151	0.581	0.138	1.08	1.948	0.	1.318

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	69	107	140	204	3896	188
normalized size	1	1.	0.66	1.02	1.33	1.94	37.1	1.79
time (sec)	N/A	0.147	0.217	0.108	1.069	1.826	138.232	1.292

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	72	75	97	159	1703	128
normalized size	1	1.	0.91	0.95	1.23	2.01	21.56	1.62
time (sec)	N/A	0.116	0.149	0.095	1.102	1.662	46.584	1.343

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	44	43	59	113	588	69
normalized size	1	1.	0.77	0.75	1.04	1.98	10.32	1.21
time (sec)	N/A	0.095	0.094	0.085	1.047	1.729	13.664	1.31

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	51	46	76	60	47
normalized size	1	1.	0.86	1.42	1.28	2.11	1.67	1.31
time (sec)	N/A	0.082	0.032	0.032	1.061	1.799	0.677	1.359

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	112	78	207	0	107
normalized size	1	1.	0.98	2.49	1.73	4.6	0.	2.38
time (sec)	N/A	0.094	0.061	0.092	1.017	1.711	0.	1.305

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	75	169	153	423	0	198
normalized size	1	1.	0.82	1.86	1.68	4.65	0.	2.18
time (sec)	N/A	0.138	0.254	0.134	1.018	1.503	0.	1.34

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	105	245	223	512	0	259
normalized size	1	1.	0.72	1.68	1.53	3.51	0.	1.77
time (sec)	N/A	0.189	0.523	0.099	1.09	1.645	0.	1.391

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	142	321	297	602	0	319
normalized size	1	1.	0.69	1.57	1.45	2.94	0.	1.56
time (sec)	N/A	0.249	0.836	0.109	1.051	1.606	0.	1.438

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	52	82	112	204	0	128
normalized size	1	1.	0.66	1.04	1.42	2.58	0.	1.62
time (sec)	N/A	0.125	0.167	0.162	1.025	1.4	0.	1.296

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	34	58	82	161	1545	99
normalized size	1	1.	0.67	1.14	1.61	3.16	30.29	1.94
time (sec)	N/A	0.103	0.063	0.109	1.039	1.452	85.049	1.338

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	51	85	73	126	1428	124
normalized size	1	1.	0.77	1.29	1.11	1.91	21.64	1.88
time (sec)	N/A	0.107	0.093	0.109	1.029	1.474	27.582	1.421

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	56	58	112	121	103
normalized size	1	1.	0.93	1.27	1.32	2.55	2.75	2.34
time (sec)	N/A	0.068	0.073	0.048	1.023	1.445	1.017	1.333

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	69	150	113	358	0	140
normalized size	1	1.	0.97	2.11	1.59	5.04	0.	1.97
time (sec)	N/A	0.107	0.12	0.115	1.043	1.537	0.	1.384

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	87	207	188	597	0	228
normalized size	1	1.	0.71	1.68	1.53	4.85	0.	1.85
time (sec)	N/A	0.155	0.679	0.138	1.037	1.524	0.	1.361

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	123	283	279	687	0	289
normalized size	1	1.	0.69	1.58	1.56	3.84	0.	1.61
time (sec)	N/A	0.206	0.664	0.135	1.058	1.659	0.	1.446

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	160	359	340	782	0	348
normalized size	1	1.	0.68	1.52	1.44	3.31	0.	1.47
time (sec)	N/A	0.279	1.374	0.141	1.06	1.788	0.	1.695

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.269	0.448	4.148	0.	0.	0.	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	132	0	0	867	0	1893
normalized size	1	1.	0.83	0.	0.	5.45	0.	11.91
time (sec)	N/A	0.166	0.73	13.852	0.	2.004	0.	1.361

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	103	0	0	563	0	1162
normalized size	1	1.	0.84	0.	0.	4.58	0.	9.45
time (sec)	N/A	0.135	0.382	5.923	0.	1.719	0.	1.291

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	333	0	618
normalized size	1	1.	1.	0.	0.	3.58	0.	6.65
time (sec)	N/A	0.118	0.298	2.612	0.	1.558	0.	1.275

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	51	0	0	169	428	211
normalized size	1	1.	0.86	0.	0.	2.86	7.25	3.58
time (sec)	N/A	0.071	0.123	1.661	0.	1.494	8.306	1.321

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.111	1.	0.	0.	0.	0.

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	82	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.164	0.283	0.	0.	0.	0.

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	76	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.167	0.341	0.	0.	0.	0.

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.865	10.54	0.	0.	0.	0.

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	0.463	3.684	0.	0.	0.	0.

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	111	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.319	1.891	0.	0.	0.	0.

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	6104	0	0	0	0	0
normalized size	1	1.	49.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	24.989	0.272	0.	0.	0.	0.

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	1.551	0.306	0.	0.	0.	0.

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	3.509	0.404	0.	0.	0.	0.

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	160	0	0	478	0	0
normalized size	1	1.	0.67	0.	0.	2.	0.	0.
time (sec)	N/A	0.441	0.526	0.726	0.	1.654	0.	0.

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	119	0	0	317	0	0
normalized size	1	1.	0.71	0.	0.	1.89	0.	0.
time (sec)	N/A	0.306	0.262	0.632	0.	1.436	0.	0.

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	83	0	0	200	0	0
normalized size	1	1.	0.81	0.	0.	1.96	0.	0.
time (sec)	N/A	0.212	0.147	0.595	0.	1.5	0.	0.

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	300	0	0	0	0	0
normalized size	1	1.	1.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.264	3.445	0.432	0.	0.	0.	0.

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	144	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	0.54	1.438	0.	0.	0.	0.

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	150	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.272	0.638	0.468	0.	0.	0.	0.

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	155	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.276	0.87	0.515	0.	0.	0.	0.

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	0	0	81	0	0
normalized size	1	1.	1.03	0.	0.	2.53	0.	0.
time (sec)	N/A	0.117	0.172	4.252	0.	1.553	0.	0.

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	35	0	0	84	0	2515
normalized size	1	1.	1.03	0.	0.	2.47	0.	73.97
time (sec)	N/A	0.115	0.061	4.184	0.	1.429	0.	32.017

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	798	0	0	0	0	0
normalized size	1	1.	4.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.28	8.342	2.763	0.	0.	0.	0.

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0
normalized size	1	1.03	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	19.807	0.616	0.	0.	0.	0.

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	151	0	0	0	0	0	0
normalized size	1	1.04	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	3.823	0.394	0.	0.	0.	0.

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	155	0	0	0	0	0	0
normalized size	1	1.04	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.247	6.912	0.326	0.	0.	0.	0.

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0
normalized size	1	1.03	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	10.325	0.53	0.	0.	0.	0.

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0
normalized size	1	1.03	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	14.133	0.628	0.	0.	0.	0.

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0
normalized size	1	1.03	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.234	18.643	0.76	0.	0.	0.	0.

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	175	175	852	0	0	0	0	0
normalized size	1	1.	4.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.429	9.44	2.746	0.	0.	0.	0.

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	77	95	88	193	192	124
normalized size	1	1.	0.73	0.9	0.84	1.84	1.83	1.18
time (sec)	N/A	0.164	0.187	0.03	1.101	1.526	4.258	1.307

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	59	77	70	162	144	84
normalized size	1	1.	0.73	0.95	0.86	2.	1.78	1.04
time (sec)	N/A	0.131	0.096	0.029	1.042	1.437	2.203	1.254

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	57	53	128	119	63
normalized size	1	1.	0.94	0.88	0.82	1.97	1.83	0.97
time (sec)	N/A	0.094	0.111	0.026	1.124	1.467	1.137	1.266

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	74	63	77	174	0	117
normalized size	1	1.	1.45	1.24	1.51	3.41	0.	2.29
time (sec)	N/A	0.069	0.054	0.048	1.037	1.472	0.	1.321

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	73	236	0	146
normalized size	1	1.	1.83	1.39	1.78	5.76	0.	3.56
time (sec)	N/A	0.056	0.031	0.046	1.563	1.507	0.	1.38

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	109	81	89	300	0	128
normalized size	1	1.	2.1	1.56	1.71	5.77	0.	2.46
time (sec)	N/A	0.08	0.044	0.052	1.734	1.483	0.	1.31

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	95	80	82	316	0	155
normalized size	1	1.	1.83	1.54	1.58	6.08	0.	2.98
time (sec)	N/A	0.109	0.036	0.054	1.103	1.353	0.	1.262

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	135	102	108	377	0	157
normalized size	1	1.	1.82	1.38	1.46	5.09	0.	2.12
time (sec)	N/A	0.128	0.041	0.061	1.081	1.404	0.	1.35

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	177	124	124	466	0	194
normalized size	1	1.	1.97	1.38	1.38	5.18	0.	2.16
time (sec)	N/A	0.133	0.069	0.059	1.048	1.514	0.	1.29

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	132	150	140	274	275	190
normalized size	1	1.	0.69	0.79	0.74	1.44	1.45	1.
time (sec)	N/A	0.382	0.551	0.043	1.122	1.493	7.762	1.192

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	120	141	124	252	309	155
normalized size	1	1.	0.74	0.87	0.76	1.55	1.9	0.95
time (sec)	N/A	0.382	0.213	0.042	1.162	1.49	4.699	1.215

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	77	94	92	185	172	111
normalized size	1	1.	0.73	0.89	0.87	1.75	1.62	1.05
time (sec)	N/A	0.159	0.365	0.037	1.153	1.389	2.363	1.207

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	91	83	100	231	0	180
normalized size	1	1.	1.01	0.92	1.11	2.57	0.	2.
time (sec)	N/A	0.253	0.209	0.072	1.129	1.431	0.	1.331

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	116	102	107	308	0	200
normalized size	1	1.	1.49	1.31	1.37	3.95	0.	2.56
time (sec)	N/A	0.098	0.389	0.066	1.724	1.373	0.	1.361

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	155	126	139	414	0	200
normalized size	1	1.	1.74	1.42	1.56	4.65	0.	2.25
time (sec)	N/A	0.306	0.9	0.079	1.683	1.445	0.	1.371

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	538	114	111	423	0	225
normalized size	1	1.	5.6	1.19	1.16	4.41	0.	2.34
time (sec)	N/A	0.39	6.17	0.077	1.564	1.514	0.	1.268

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	579	173	174	504	0	246
normalized size	1	1.	4.71	1.41	1.41	4.1	0.	2.
time (sec)	N/A	0.365	6.171	0.082	1.066	1.489	0.	1.224

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	236	156	146	518	0	300
normalized size	1	1.	1.59	1.05	0.99	3.5	0.	2.03
time (sec)	N/A	0.386	0.819	0.082	1.083	1.394	0.	1.282

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	296	244	251	697	0	373
normalized size	1	1.	1.74	1.44	1.48	4.1	0.	2.19
time (sec)	N/A	0.396	0.777	0.083	1.114	1.585	0.	1.252

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	157	196	177	343	394	224
normalized size	1	1.	0.68	0.84	0.76	1.48	1.7	0.97
time (sec)	N/A	0.572	0.808	0.049	1.198	1.576	8.269	1.186

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	158	146	281	340	188
normalized size	1	1.	0.85	0.97	0.9	1.72	2.09	1.15
time (sec)	N/A	0.295	0.765	0.046	1.135	1.479	4.755	1.198

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	129	150	136	297	0	396
normalized size	1	1.	0.95	1.1	1.	2.18	0.	2.91
time (sec)	N/A	0.413	0.29	0.091	1.154	1.632	0.	1.245

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	125	128	370	0	269
normalized size	1	1.	1.4	1.23	1.25	3.63	0.	2.64
time (sec)	N/A	0.142	1.29	0.084	1.621	1.558	0.	1.185

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	192	171	173	510	0	367
normalized size	1	1.	1.39	1.24	1.25	3.7	0.	2.66
time (sec)	N/A	0.465	1.337	0.098	1.63	1.692	0.	1.277

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	615	159	161	562	0	300
normalized size	1	1.	4.46	1.15	1.17	4.07	0.	2.17
time (sec)	N/A	0.483	6.199	0.101	1.645	1.609	0.	1.309

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	690	207	201	666	0	316
normalized size	1	1.	4.54	1.36	1.32	4.38	0.	2.08
time (sec)	N/A	0.511	6.215	0.098	1.632	1.632	0.	1.351

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	344	227	212	675	0	392
normalized size	1	1.	1.88	1.24	1.16	3.69	0.	2.14
time (sec)	N/A	0.569	1.245	0.105	1.163	1.536	0.	1.275

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	369	276	273	755	0	478
normalized size	1	1.	1.74	1.3	1.29	3.56	0.	2.25
time (sec)	N/A	0.598	2.029	0.106	1.123	1.502	0.	1.329

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	246	460	0	1381	0	352
normalized size	1	1.	1.31	2.45	0.	7.35	0.	1.87
time (sec)	N/A	0.742	2.48	0.125	0.	1.867	0.	1.32

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	129	353	0	1237	0	285
normalized size	1	1.	0.84	2.31	0.	8.08	0.	1.86
time (sec)	N/A	0.492	0.377	0.117	0.	1.663	0.	1.305

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	130	229	0	1025	0	258
normalized size	1	1.	1.23	2.16	0.	9.67	0.	2.43
time (sec)	N/A	0.153	0.853	0.107	0.	1.639	0.	1.217

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	153	0	1111	0	176
normalized size	1	1.	1.05	1.66	0.	12.08	0.	1.91
time (sec)	N/A	0.242	0.211	0.141	0.	2.026	0.	1.229

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	139	245	0	1681	0	294
normalized size	1	1.	1.21	2.13	0.	14.62	0.	2.56
time (sec)	N/A	0.435	0.727	0.166	0.	2.294	0.	1.245

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	196	307	0	2500	0	347
normalized size	1	1.	1.25	1.96	0.	15.92	0.	2.21
time (sec)	N/A	0.768	3.097	0.19	0.	3.375	0.	1.337

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	385	390	0	3228	0	444
normalized size	1	1.	1.99	2.02	0.	16.73	0.	2.3
time (sec)	N/A	1.038	6.342	0.176	0.	2.836	0.	1.388

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	288	845	0	2309	0	722
normalized size	1	1.	1.08	3.18	0.	8.68	0.	2.71
time (sec)	N/A	0.878	5.708	0.149	0.	2.112	0.	1.366

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	159	711	0	1975	0	408
normalized size	1	1.	0.88	3.95	0.	10.97	0.	2.27
time (sec)	N/A	0.562	1.133	0.139	0.	1.906	0.	1.383

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	289	576	0	1702	0	346
normalized size	1	1.	1.73	3.45	0.	10.19	0.	2.07
time (sec)	N/A	0.279	1.897	0.132	0.	1.686	0.	1.288

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	632	0	2190	0	374
normalized size	1	1.	1.	4.1	0.	14.22	0.	2.43
time (sec)	N/A	0.477	1.09	0.18	0.	3.696	0.	1.37

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	195	729	0	3044	0	458
normalized size	1	1.	0.97	3.61	0.	15.07	0.	2.27
time (sec)	N/A	0.795	5.471	0.197	0.	4.554	0.	1.299

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	330	803	0	4301	0	710
normalized size	1	1.	1.23	2.99	0.	15.99	0.	2.64
time (sec)	N/A	1.151	6.317	0.207	0.	6.449	0.	1.332

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	347	347	3348	4546	0	0	0	0
normalized size	1	1.	9.65	13.1	0.	0.	0.	0.
time (sec)	N/A	0.774	21.513	0.409	0.	0.	0.	0.

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	92	124	96	270	272	144
normalized size	1	1.	0.64	0.87	0.67	1.89	1.9	1.01
time (sec)	N/A	0.191	0.25	0.035	1.006	1.512	24.772	1.282

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	77	106	82	230	248	124
normalized size	1	1.	0.61	0.83	0.65	1.81	1.95	0.98
time (sec)	N/A	0.187	0.188	0.032	1.088	1.479	15.02	1.403

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	88	88	88	198	192	144
normalized size	1	1.	0.85	0.85	0.85	1.92	1.86	1.4
time (sec)	N/A	0.148	0.186	0.03	1.156	1.463	8.066	1.363

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	68	70	163	167	124
normalized size	1	1.	0.89	0.78	0.8	1.87	1.92	1.43
time (sec)	N/A	0.111	0.162	0.027	1.107	1.38	4.689	1.315

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	109	97	109	252	0	196
normalized size	1	1.	1.22	1.09	1.22	2.83	0.	2.2
time (sec)	N/A	0.1	0.122	0.055	1.111	1.681	0.	1.357

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	105	119	123	300	0	192
normalized size	1	1.	1.27	1.43	1.48	3.61	0.	2.31
time (sec)	N/A	0.134	0.328	0.053	1.695	1.799	0.	1.388

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	132	143	136	365	0	220
normalized size	1	1.	1.4	1.52	1.45	3.88	0.	2.34
time (sec)	N/A	0.122	1.533	0.056	1.71	1.84	0.	1.448

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	124	425	0	190
normalized size	1	1.	1.52	1.29	1.51	5.18	0.	2.32
time (sec)	N/A	0.085	0.044	0.062	1.644	1.818	0.	1.417

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	153	128	144	494	0	207
normalized size	1	1.	1.74	1.45	1.64	5.61	0.	2.35
time (sec)	N/A	0.111	0.051	0.063	1.639	1.815	0.	1.437

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	135	116	116	441	0	234
normalized size	1	1.	1.82	1.57	1.57	5.96	0.	3.16
time (sec)	N/A	0.133	0.035	0.063	1.124	1.73	0.	1.4

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	175	138	143	497	0	271
normalized size	1	1.	1.79	1.41	1.46	5.07	0.	2.77
time (sec)	N/A	0.165	0.043	0.066	1.15	1.818	0.	1.428

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	239	160	159	593	0	309
normalized size	1	1.	2.1	1.4	1.39	5.2	0.	2.71
time (sec)	N/A	0.173	0.083	0.066	1.16	1.785	0.	1.277

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	279	182	186	656	0	271
normalized size	1	1.	2.05	1.34	1.37	4.82	0.	1.99
time (sec)	N/A	0.184	0.079	0.066	1.001	1.859	0.	1.387

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	144	161	135	315	335	192
normalized size	1	1.	0.48	0.53	0.45	1.05	1.11	0.64
time (sec)	N/A	0.654	0.957	0.043	0.999	1.864	23.89	1.359

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	141	163	136	316	420	203
normalized size	1	1.	0.51	0.59	0.49	1.14	1.51	0.73
time (sec)	N/A	0.627	0.581	0.044	1.01	1.856	13.583	1.224

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	132	105	109	224	223	190
normalized size	1	1.	1.02	0.81	0.84	1.74	1.73	1.47
time (sec)	N/A	0.181	0.422	0.037	0.999	1.793	8.164	1.227

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	190	125	123	131	309	0	288
normalized size	1	1.64	1.08	1.06	1.13	2.66	0.	2.48
time (sec)	N/A	0.447	0.517	0.083	1.001	1.943	0.	1.306

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	167	191	171	398	0	370
normalized size	1	1.	0.92	1.06	0.94	2.2	0.	2.04
time (sec)	N/A	0.521	0.707	0.073	1.514	1.885	0.	1.315

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	191	208	203	518	0	340
normalized size	1	1.	1.01	1.1	1.07	2.74	0.	1.8
time (sec)	N/A	0.484	3.567	0.089	1.491	1.854	0.	1.339

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	293	199	186	539	0	325
normalized size	1	1.	2.2	1.5	1.4	4.05	0.	2.44
time (sec)	N/A	0.162	6.183	0.087	1.518	1.864	0.	1.361

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	270	223	224	666	0	329
normalized size	1	1.	1.52	1.25	1.26	3.74	0.	1.85
time (sec)	N/A	0.463	2.718	0.088	1.612	1.832	0.	1.371

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	285	165	166	637	0	355
normalized size	1	1.	1.36	0.79	0.79	3.05	0.	1.7
time (sec)	N/A	0.516	1.457	0.089	1.515	1.823	0.	1.359

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	319	253	243	675	0	417
normalized size	1	1.	1.35	1.07	1.03	2.86	0.	1.77
time (sec)	N/A	0.604	0.826	0.089	1.151	1.901	0.	1.37

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	322	194	181	653	0	468
normalized size	1	1.	1.23	0.74	0.69	2.5	0.	1.79
time (sec)	N/A	0.629	1.294	0.106	1.112	1.857	0.	1.33

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	204	218	189	405	505	275
normalized size	1	1.	0.58	0.62	0.53	1.14	1.43	0.78
time (sec)	N/A	0.928	1.211	0.052	1.123	1.944	27.001	1.389

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	189	180	158	335	456	248
normalized size	1	1.	0.97	0.93	0.81	1.73	2.35	1.28
time (sec)	N/A	0.327	0.839	0.047	1.161	1.859	14.935	1.322

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	191	211	185	401	0	576
normalized size	1	1.	0.76	0.84	0.74	1.6	0.	2.3
time (sec)	N/A	0.659	0.512	0.098	1.159	1.965	0.	1.308

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	194	216	193	464	0	466
normalized size	1	1.	0.85	0.94	0.84	2.03	0.	2.03
time (sec)	N/A	0.677	2.962	0.099	1.541	1.926	0.	1.317

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	252	279	251	616	0	540
normalized size	1	1.	1.09	1.21	1.09	2.67	0.	2.34
time (sec)	N/A	0.692	6.16	0.103	1.54	1.995	0.	1.324

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	355	264	252	699	0	568
normalized size	1	1.	1.83	1.36	1.3	3.6	0.	2.93
time (sec)	N/A	0.223	6.219	0.106	1.668	1.943	0.	1.36

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	381	316	286	807	0	463
normalized size	1	1.	2.04	1.69	1.53	4.32	0.	2.48
time (sec)	N/A	0.656	6.254	0.112	1.679	1.99	0.	1.297

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	405	260	246	832	0	481
normalized size	1	1.	1.78	1.15	1.08	3.67	0.	2.12
time (sec)	N/A	0.711	1.247	0.109	1.682	1.925	0.	1.435

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	408	302	293	941	0	539
normalized size	1	1.	1.48	1.1	1.07	3.42	0.	1.96
time (sec)	N/A	0.756	1.774	0.112	1.646	1.908	0.	1.456

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	324	309	281	837	0	616
normalized size	1	1.	1.07	1.02	0.93	2.76	0.	2.03
time (sec)	N/A	0.86	0.935	0.116	1.057	1.884	0.	1.29

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	268	358	335	957	0	617
normalized size	1	1.	0.8	1.07	1.	2.87	0.	1.85
time (sec)	N/A	0.907	1.53	0.117	1.05	1.917	0.	1.378

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	378	1119	0	1451	0	724
normalized size	1	1.	1.23	3.64	0.	4.73	0.	2.36
time (sec)	N/A	1.016	4.112	0.139	0.	2.342	0.	1.232

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	325	938	0	1391	0	606
normalized size	1	1.	1.22	3.51	0.	5.21	0.	2.27
time (sec)	N/A	0.758	3.566	0.128	0.	2.204	0.	1.311

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	247	627	0	1141	0	405
normalized size	1	1.	1.52	3.85	0.	7.	0.	2.48
time (sec)	N/A	0.306	2.17	0.119	0.	2.073	0.	1.201

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	161	380	0	1226	0	386
normalized size	1	1.	1.18	2.77	0.	8.95	0.	2.82
time (sec)	N/A	0.274	0.702	0.151	0.	3.028	0.	1.386

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	182	396	0	1557	0	351
normalized size	1	1.	1.18	2.57	0.	10.11	0.	2.28
time (sec)	N/A	0.304	1.767	0.161	0.	2.931	0.	1.375

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	180	191	339	0	1891	0	371
normalized size	1	1.14	1.21	2.15	0.	11.97	0.	2.35
time (sec)	N/A	0.45	4.161	0.171	0.	2.613	0.	1.282

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	238	238	403	527	0	2639	0	481
normalized size	1	1.	1.69	2.21	0.	11.09	0.	2.02
time (sec)	N/A	0.703	6.367	0.178	0.	2.983	0.	1.316

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	496	634	0	3688	0	622
normalized size	1	1.	1.7	2.17	0.	12.63	0.	2.13
time (sec)	N/A	1.047	6.26	0.189	0.	4.662	0.	1.395

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	1250	1193	0	2415	0	729
normalized size	1	1.	3.78	3.6	0.	7.3	0.	2.2
time (sec)	N/A	1.008	10.228	0.16	0.	2.887	0.	1.365

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	1030	880	0	2152	0	531
normalized size	1	1.	3.63	3.1	0.	7.58	0.	1.87
time (sec)	N/A	0.738	6.254	0.152	0.	2.557	0.	1.258

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	274	639	0	1800	0	579
normalized size	1	1.	1.58	3.69	0.	10.4	0.	3.35
time (sec)	N/A	0.275	3.473	0.139	0.	2.356	0.	1.242

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	176	600	0	2211	0	371
normalized size	1	1.	1.01	3.43	0.	12.63	0.	2.12
time (sec)	N/A	0.288	1.75	0.184	0.	4.54	0.	1.33

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	184	489	0	2341	0	369
normalized size	1	1.	1.01	2.69	0.	12.86	0.	2.03
time (sec)	N/A	0.472	2.482	0.197	0.	2.923	0.	1.372

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	319	642	0	3416	0	533
normalized size	1	1.	1.46	2.94	0.	15.67	0.	2.44
time (sec)	N/A	0.772	6.185	0.213	0.	3.748	0.	1.427

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	289	289	459	780	0	4578	0	609
normalized size	1	1.	1.59	2.7	0.	15.84	0.	2.11
time (sec)	N/A	1.099	6.201	0.221	0.	5.473	0.	1.299

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	347	889	0	5862	0	743
normalized size	1	1.	1.02	2.61	0.	17.24	0.	2.19
time (sec)	N/A	1.459	4.508	0.223	0.	7.435	0.	1.387

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	327	1619	0	0	0	0
normalized size	1	1.	0.71	3.5	0.	0.	0.	0.
time (sec)	N/A	1.048	4.85	1.677	0.	0.	0.	0.

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	326	1356	0	0	0	0
normalized size	1	1.	0.98	4.08	0.	0.	0.	0.
time (sec)	N/A	0.634	3.903	1.645	0.	0.	0.	0.

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	435	1155	0	0	0	0
normalized size	1	1.	1.29	3.42	0.	0.	0.	0.
time (sec)	N/A	0.892	3.319	1.514	0.	0.	0.	0.

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	422	657	0	0	0	0
normalized size	1	1.	1.31	2.03	0.	0.	0.	0.
time (sec)	N/A	0.876	3.499	1.615	0.	0.	0.	0.

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	450	1364	0	0	0	0
normalized size	1	1.	1.3	3.95	0.	0.	0.	0.
time (sec)	N/A	0.898	3.367	1.819	0.	0.	0.	0.

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	473	1495	0	0	0	0
normalized size	1	1.	1.35	4.26	0.	0.	0.	0.
time (sec)	N/A	0.862	5.509	1.838	0.	0.	0.	0.

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	643	1761	0	0	0	0
normalized size	1	1.	1.56	4.27	0.	0.	0.	0.
time (sec)	N/A	1.235	6.591	1.859	0.	0.	0.	0.

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	702	2075	0	0	0	0
normalized size	1	1.	1.45	4.29	0.	0.	0.	0.
time (sec)	N/A	1.622	6.55	2.067	0.	0.	0.	0.

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	528	382	1801	0	0	0	0
normalized size	1	1.	0.72	3.41	0.	0.	0.	0.
time (sec)	N/A	1.208	14.723	1.694	0.	0.	0.	0.

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	382	1619	0	0	0	0
normalized size	1	1.	0.97	4.11	0.	0.	0.	0.
time (sec)	N/A	0.859	11.391	1.836	0.	0.	0.	0.

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	477	1405	0	0	0	0
normalized size	1	1.	1.22	3.6	0.	0.	0.	0.
time (sec)	N/A	1.159	3.509	1.644	0.	0.	0.	0.

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	452	726	0	0	0	0
normalized size	1	1.	1.21	1.94	0.	0.	0.	0.
time (sec)	N/A	1.165	3.933	1.58	0.	0.	0.	0.

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	434	1379	0	0	0	0
normalized size	1	1.	1.13	3.6	0.	0.	0.	0.
time (sec)	N/A	1.165	3.137	1.774	0.	0.	0.	0.

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	486	1511	0	0	0	0
normalized size	1	1.	1.26	3.91	0.	0.	0.	0.
time (sec)	N/A	1.1	6.297	1.836	0.	0.	0.	0.

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	408	408	641	1760	0	0	0	0
normalized size	1	1.	1.57	4.31	0.	0.	0.	0.
time (sec)	N/A	1.237	6.574	1.83	0.	0.	0.	0.

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	700	2075	0	0	0	0
normalized size	1	1.	1.45	4.29	0.	0.	0.	0.
time (sec)	N/A	1.652	6.618	2.04	0.	0.	0.	0.

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	551	551	771	2458	0	0	0	0
normalized size	1	1.	1.4	4.46	0.	0.	0.	0.
time (sec)	N/A	1.984	6.64	2.666	0.	0.	0.	0.

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	450	1801	0	0	0	0
normalized size	1	1.	1.	3.99	0.	0.	0.	0.
time (sec)	N/A	1.075	21.194	1.499	0.	0.	0.	0.

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	447	521	1573	0	0	0	0
normalized size	1	1.	1.17	3.52	0.	0.	0.	0.
time (sec)	N/A	1.422	3.792	1.829	0.	0.	0.	0.

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	496	864	0	0	0	0
normalized size	1	1.	1.16	2.03	0.	0.	0.	0.
time (sec)	N/A	1.422	4.507	1.6	0.	0.	0.	0.

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	460	1520	0	0	0	0
normalized size	1	1.	1.07	3.53	0.	0.	0.	0.
time (sec)	N/A	1.423	5.21	1.63	0.	0.	0.	0.

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	466	1526	0	0	0	0
normalized size	1	1.	1.09	3.56	0.	0.	0.	0.
time (sec)	N/A	1.355	3.43	1.86	0.	0.	0.	0.

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	655	1777	0	0	0	0
normalized size	1	1.	1.46	3.96	0.	0.	0.	0.
time (sec)	N/A	1.511	6.593	1.879	0.	0.	0.	0.

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	482	700	2075	0	0	0	0
normalized size	1	1.	1.45	4.3	0.	0.	0.	0.
time (sec)	N/A	1.623	6.732	2.108	0.	0.	0.	0.

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	551	551	771	2458	0	0	0	0
normalized size	1	1.	1.4	4.46	0.	0.	0.	0.
time (sec)	N/A	1.971	6.696	2.677	0.	0.	0.	0.

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	471	382	1619	0	0	0	0
normalized size	1	1.	0.81	3.44	0.	0.	0.	0.
time (sec)	N/A	1.185	5.422	1.55	0.	0.	0.	0.

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	326	1356	0	0	0	0
normalized size	1	1.	0.8	3.35	0.	0.	0.	0.
time (sec)	N/A	0.886	4.118	1.609	0.	0.	0.	0.

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	275	1190	0	0	0	0
normalized size	1	1.	0.97	4.2	0.	0.	0.	0.
time (sec)	N/A	0.453	3.05	1.483	0.	0.	0.	0.

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	408	1018	0	0	0	0
normalized size	1	1.	1.42	3.53	0.	0.	0.	0.
time (sec)	N/A	0.655	3.555	1.494	0.	0.	0.	0.

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	416	704	0	0	0	0
normalized size	1	1.	1.46	2.47	0.	0.	0.	0.
time (sec)	N/A	0.673	3.466	2.826	0.	0.	0.	0.

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	443	913	0	0	0	0
normalized size	1	1.	1.44	2.97	0.	0.	0.	0.
time (sec)	N/A	0.665	3.367	3.119	0.	0.	0.	0.

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	475	1496	0	0	0	0
normalized size	1	1.	1.35	4.24	0.	0.	0.	0.
time (sec)	N/A	0.881	5.461	1.951	0.	0.	0.	0.

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	647	1761	0	0	0	0
normalized size	1	1.	1.57	4.27	0.	0.	0.	0.
time (sec)	N/A	1.244	6.559	1.953	0.	0.	0.	0.

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	466	326	1356	0	0	0	0
normalized size	1	1.	0.7	2.91	0.	0.	0.	0.
time (sec)	N/A	1.214	6.579	1.581	0.	0.	0.	0.

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	275	1190	0	0	0	0
normalized size	1	1.	0.69	2.97	0.	0.	0.	0.
time (sec)	N/A	0.904	5.067	1.694	0.	0.	0.	0.

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	222	943	0	0	0	0
normalized size	1	1.	0.85	3.61	0.	0.	0.	0.
time (sec)	N/A	0.424	3.917	1.546	0.	0.	0.	0.

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	419	1010	0	0	0	0
normalized size	1	1.	1.42	3.41	0.	0.	0.	0.
time (sec)	N/A	0.678	3.679	1.5	0.	0.	0.	0.

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	433	620	0	0	0	0
normalized size	1	1.	1.47	2.11	0.	0.	0.	0.
time (sec)	N/A	0.705	3.421	1.711	0.	0.	0.	0.

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	435	1349	0	0	0	0
normalized size	1	1.	1.19	3.69	0.	0.	0.	0.
time (sec)	N/A	0.928	4.814	1.832	0.	0.	0.	0.

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	468	1496	0	0	0	0
normalized size	1	1.	1.12	3.6	0.	0.	0.	0.
time (sec)	N/A	1.155	5.565	1.809	0.	0.	0.	0.

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	1044	2033	0	0	0	0
normalized size	1	1.	2.23	4.33	0.	0.	0.	0.
time (sec)	N/A	1.181	9.298	1.77	0.	0.	0.	0.

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	257	1642	0	0	0	0
normalized size	1	1.	0.63	4.	0.	0.	0.	0.
time (sec)	N/A	0.918	7.603	1.618	0.	0.	0.	0.

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	211	1430	0	0	0	0
normalized size	1	1.	0.83	5.63	0.	0.	0.	0.
time (sec)	N/A	0.44	5.889	1.565	0.	0.	0.	0.

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	443	1375	0	0	0	0
normalized size	1	1.	1.42	4.39	0.	0.	0.	0.
time (sec)	N/A	0.681	4.88	5.919	0.	0.	0.	0.

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	445	2112	0	0	0	0
normalized size	1	1.	1.29	6.1	0.	0.	0.	0.
time (sec)	N/A	0.988	5.261	1.895	0.	0.	0.	0.

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	407	407	622	2617	0	0	0	0
normalized size	1	1.	1.53	6.43	0.	0.	0.	0.
time (sec)	N/A	1.265	6.614	2.039	0.	0.	0.	0.

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	458	458	680	2870	0	0	0	0
normalized size	1	1.	1.48	6.27	0.	0.	0.	0.
time (sec)	N/A	1.564	6.727	2.266	0.	0.	0.	0.

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	510	510	1670	24365	0	0	0	0
normalized size	1	1.	3.27	47.77	0.	0.	0.	0.
time (sec)	N/A	1.91	6.526	0.898	0.	0.	0.	0.

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	23.261	0.5	0.	0.	0.	0.

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	5.835	0.389	0.	0.	0.	0.

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	4.571	0.411	0.	0.	0.	0.

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	5.142	0.398	0.	0.	0.	0.

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	623	623	195	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	1.759	0.802	12.201	0.	0.	0.	0.

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	167	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	1.12	0.315	7.588	0.	0.	0.	0.

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.157	3.066	0.	0.	0.	0.

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	105	138	97	297	136	180
normalized size	1	1.	1.08	1.42	1.	3.06	1.4	1.86
time (sec)	N/A	0.13	0.401	0.032	1.	2.01	58.405	1.199

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	94	120	97	257	136	159
normalized size	1	1.	0.97	1.24	1.	2.65	1.4	1.64
time (sec)	N/A	0.12	0.302	0.033	0.993	2.029	33.551	1.218

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	105	102	97	223	136	180
normalized size	1	1.	1.3	1.26	1.2	2.75	1.68	2.22
time (sec)	N/A	0.136	0.276	0.033	0.976	1.791	21.508	1.217

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	94	84	97	192	136	159
normalized size	1	1.	1.16	1.04	1.2	2.37	1.68	1.96
time (sec)	N/A	0.139	0.294	0.03	0.988	1.746	12.514	1.396

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	86	64	97	161	90	139
normalized size	1	1.	1.32	0.98	1.49	2.48	1.38	2.14
time (sec)	N/A	0.098	0.211	0.027	0.955	1.654	7.314	1.22

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	94	93	197	0	95
normalized size	1	1.	1.	1.09	1.08	2.29	0.	1.1
time (sec)	N/A	0.081	0.03	0.053	0.982	1.773	0.	1.331

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	116	93	250	0	107
normalized size	1	1.	1.	1.4	1.12	3.01	0.	1.29
time (sec)	N/A	0.093	0.03	0.055	0.972	1.759	0.	1.298

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	139	92	258	0	111
normalized size	1	1.	0.9	1.62	1.07	3.	0.	1.29
time (sec)	N/A	0.092	0.196	0.056	0.978	1.727	0.	1.235

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	159	93	300	0	109
normalized size	1	1.	0.89	1.87	1.09	3.53	0.	1.28
time (sec)	N/A	0.085	0.146	0.059	1.002	1.737	0.	1.195

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	93	292	0	111
normalized size	1	1.	1.07	1.68	1.15	3.6	0.	1.37
time (sec)	N/A	0.053	0.244	0.063	0.996	1.71	0.	1.193

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	92	160	97	332	0	113
normalized size	1	1.	1.07	1.86	1.13	3.86	0.	1.31
time (sec)	N/A	0.084	0.173	0.065	1.003	1.744	0.	1.195

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	110	95	254	0	95
normalized size	1	1.	1.	1.8	1.56	4.16	0.	1.56
time (sec)	N/A	0.112	0.026	0.063	0.998	1.584	0.	1.202

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	128	95	273	0	95
normalized size	1	1.	1.	1.97	1.46	4.2	0.	1.46
time (sec)	N/A	0.122	0.026	0.064	0.988	1.623	0.	1.178

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	88	148	95	289	0	95
normalized size	1	1.	1.09	1.83	1.17	3.57	0.	1.17
time (sec)	N/A	0.129	0.1	0.067	0.972	1.609	0.	1.286

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	88	166	95	308	0	95
normalized size	1	1.	1.09	2.05	1.17	3.8	0.	1.17
time (sec)	N/A	0.131	0.112	0.066	0.984	1.687	0.	1.224

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	184	95	321	0	95
normalized size	1	1.	0.91	1.9	0.98	3.31	0.	0.98
time (sec)	N/A	0.095	0.103	0.066	0.99	1.737	0.	1.248

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	202	95	344	0	95
normalized size	1	1.	0.91	2.08	0.98	3.55	0.	0.98
time (sec)	N/A	0.095	0.099	0.067	0.97	1.713	0.	1.303

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	169	155	146	297	214	234
normalized size	1	1.	1.22	1.12	1.06	2.15	1.55	1.7
time (sec)	N/A	0.188	0.741	0.046	1.066	1.77	22.796	1.234

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	138	101	146	220	163	205
normalized size	1	1.	1.	0.73	1.06	1.59	1.18	1.49
time (sec)	N/A	0.128	0.654	0.04	0.993	1.743	12.861	1.215

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	105	119	142	247	0	159
normalized size	1	1.	0.81	0.92	1.09	1.9	0.	1.22
time (sec)	N/A	0.134	0.156	0.08	0.967	1.824	0.	1.276

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	142	185	142	346	0	171
normalized size	1	1.	1.14	1.48	1.14	2.77	0.	1.37
time (sec)	N/A	0.159	0.05	0.076	0.98	1.783	0.	1.248

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	103	197	140	381	0	189
normalized size	1	1.	0.81	1.55	1.1	3.	0.	1.49
time (sec)	N/A	0.157	0.283	0.087	0.996	1.798	0.	1.303

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	103	255	139	382	0	171
normalized size	1	1.	0.86	2.12	1.16	3.18	0.	1.42
time (sec)	N/A	0.141	0.274	0.087	0.992	1.78	0.	1.199

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	107	220	142	437	0	186
normalized size	1	1.	0.85	1.75	1.13	3.47	0.	1.48
time (sec)	N/A	0.092	0.76	0.088	1.029	1.764	0.	1.272

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	105	279	142	425	0	177
normalized size	1	1.	0.85	2.25	1.15	3.43	0.	1.43
time (sec)	N/A	0.14	0.162	0.089	0.991	1.812	0.	1.235

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	107	196	146	448	0	181
normalized size	1	1.	0.82	1.51	1.12	3.45	0.	1.39
time (sec)	N/A	0.158	0.166	0.095	0.989	1.804	0.	1.244

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	104	218	143	362	0	159
normalized size	1	1.	0.81	1.69	1.11	2.81	0.	1.23
time (sec)	N/A	0.159	0.182	0.095	0.99	1.66	0.	1.215

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	108	173	143	379	0	159
normalized size	1	1.	0.78	1.25	1.04	2.75	0.	1.15
time (sec)	N/A	0.161	0.218	0.092	0.984	1.711	0.	1.24

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	264	342	294	671	0	405
normalized size	1	1.	1.12	1.46	1.25	2.86	0.	1.72
time (sec)	N/A	0.281	2.122	0.095	0.996	2.203	0.	1.189

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	225	285	248	590	0	336
normalized size	1	1.	1.17	1.48	1.28	3.06	0.	1.74
time (sec)	N/A	0.239	1.462	0.094	0.994	2.146	0.	1.234

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	188	229	200	482	0	262
normalized size	1	1.	1.2	1.46	1.27	3.07	0.	1.67
time (sec)	N/A	0.155	1.001	0.078	1.026	1.948	0.	1.202

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	111	169	159	425	0	208
normalized size	1	1.	0.92	1.41	1.32	3.54	0.	1.73
time (sec)	N/A	0.159	0.489	0.123	0.993	2.201	0.	1.219

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	95	151	162	473	0	177
normalized size	1	1.	0.87	1.39	1.49	4.34	0.	1.62
time (sec)	N/A	0.17	0.593	0.131	0.997	2.139	0.	1.215

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	116	189	198	743	0	257
normalized size	1	1.	0.89	1.44	1.51	5.67	0.	1.96
time (sec)	N/A	0.2	0.717	0.154	1.019	2.193	0.	1.223

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	127	209	213	859	0	285
normalized size	1	1.	0.86	1.42	1.45	5.84	0.	1.94
time (sec)	N/A	0.206	1.941	0.152	1.023	2.007	0.	1.194

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	187	282	255	1241	0	375
normalized size	1	1.	0.99	1.5	1.36	6.6	0.	1.99
time (sec)	N/A	0.171	6.137	0.155	1.013	2.141	0.	1.235

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	220	343	304	1553	0	448
normalized size	1	1.	0.97	1.52	1.35	6.87	0.	1.98
time (sec)	N/A	0.261	3.135	0.159	0.99	2.183	0.	1.211

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	139	0	0	1408	0	776
normalized size	1	1.	0.82	0.	0.	8.28	0.	4.56
time (sec)	N/A	0.213	0.661	10.642	0.	2.344	0.	1.182

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	97	0	0	713	0	512
normalized size	1	1.	0.79	0.	0.	5.8	0.	4.16
time (sec)	N/A	0.137	0.196	4.69	0.	1.952	0.	1.173

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	133	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.526	0.802	0.	0.	0.	0.

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	143	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.365	0.398	1.326	0.	0.	0.	0.

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	210	225	182	456	488	289
normalized size	1	1.	0.88	0.95	0.76	1.92	2.05	1.21
time (sec)	N/A	0.372	2.157	0.049	0.996	1.999	130.965	1.302

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	202	237	185	486	656	305
normalized size	1	1.	0.81	0.95	0.74	1.94	2.62	1.22
time (sec)	N/A	0.355	1.386	0.05	1.02	1.995	90.796	1.308

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	197	171	155	360	384	293
normalized size	1	1.	1.05	0.91	0.83	1.93	2.05	1.57
time (sec)	N/A	0.315	1.086	0.045	0.997	1.967	61.78	1.258

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	193	183	171	379	529	255
normalized size	1	1.	0.96	0.91	0.85	1.89	2.63	1.27
time (sec)	N/A	0.267	0.881	0.044	1.004	1.863	39.166	1.198

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	161	115	124	261	282	238
normalized size	1	1.	1.06	0.76	0.82	1.72	1.86	1.57
time (sec)	N/A	0.212	0.999	0.042	0.986	1.822	22.454	1.206

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	166	160	165	386	0	393
normalized size	1	1.	1.06	1.02	1.05	2.46	0.	2.5
time (sec)	N/A	0.207	0.28	0.086	0.997	1.831	0.	1.228

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	220	250	232	489	0	497
normalized size	1	1.	1.24	1.4	1.3	2.75	0.	2.79
time (sec)	N/A	0.463	0.4	0.078	1.487	1.887	0.	1.254

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	250	261	257	610	0	467
normalized size	1	1.	1.39	1.45	1.43	3.39	0.	2.59
time (sec)	N/A	0.305	6.175	0.092	1.516	1.881	0.	1.272

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	336	321	255	635	0	494
normalized size	1	1.	1.9	1.81	1.44	3.59	0.	2.79
time (sec)	N/A	0.44	6.25	0.089	1.524	1.902	0.	1.25

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	337	334	277	776	0	467
normalized size	1	1.	1.94	1.92	1.59	4.46	0.	2.68
time (sec)	N/A	0.278	6.181	0.092	1.523	1.859	0.	1.269

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	351	302	247	786	0	455
normalized size	1	1.	1.74	1.5	1.22	3.89	0.	2.25
time (sec)	N/A	0.188	1.083	0.095	1.54	1.845	0.	1.214

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	384	318	296	919	0	455
normalized size	1	1.	2.19	1.82	1.69	5.25	0.	2.6
time (sec)	N/A	0.261	1.022	0.093	1.476	1.884	0.	1.292

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	280	222	207	849	0	481
normalized size	1	1.	1.77	1.41	1.31	5.37	0.	3.04
time (sec)	N/A	0.414	1.373	0.108	1.463	1.862	0.	1.24

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	282	333	297	853	0	543
normalized size	1	1.	1.77	2.09	1.87	5.36	0.	3.42
time (sec)	N/A	0.319	0.797	0.099	1.006	1.809	0.	1.32

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	204	232	208	782	0	551
normalized size	1	1.	1.35	1.54	1.38	5.18	0.	3.65
time (sec)	N/A	0.405	1.12	0.096	0.991	1.898	0.	1.2

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	244	404	367	1138	0	632
normalized size	1	1.	1.16	1.92	1.75	5.42	0.	3.01
time (sec)	N/A	0.345	1.432	0.101	0.992	1.968	0.	1.305

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	250	303	265	990	0	678
normalized size	1	1.	1.26	1.53	1.34	5.	0.	3.42
time (sec)	N/A	0.457	1.713	0.101	1.001	2.021	0.	1.287

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	531	2076	0	2032	0	1303
normalized size	1	1.	1.01	3.95	0.	3.87	0.	2.48
time (sec)	N/A	1.909	8.87	0.141	0.	2.637	0.	1.276

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	471	462	1817	0	1898	0	1127
normalized size	1	1.	0.98	3.86	0.	4.03	0.	2.39
time (sec)	N/A	1.55	8.432	0.151	0.	2.472	0.	1.272

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	371	1321	0	1619	0	801
normalized size	1	1.	1.61	5.72	0.	7.01	0.	3.47
time (sec)	N/A	0.513	3.997	0.134	0.	2.261	0.	1.243

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	207	778	0	1598	0	477
normalized size	1	1.	0.78	2.92	0.	6.01	0.	1.79
time (sec)	N/A	0.351	1.686	0.17	0.	4.954	0.	1.224

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	215	680	0	2010	0	518
normalized size	1	1.	0.85	2.68	0.	7.91	0.	2.04
time (sec)	N/A	0.342	2.745	0.172	0.	4.672	0.	1.264

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	315	618	0	2700	0	625
normalized size	1	1.	1.25	2.46	0.	10.76	0.	2.49
time (sec)	N/A	0.336	6.19	0.19	0.	5.473	0.	1.276

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	428	678	0	3244	0	539
normalized size	1	1.	1.49	2.36	0.	11.3	0.	1.88
time (sec)	N/A	0.377	6.274	0.19	0.	4.752	0.	1.276

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	487	718	0	3613	0	641
normalized size	1	1.	1.61	2.37	0.	11.92	0.	2.12
time (sec)	N/A	1.211	6.204	0.187	0.	4.289	0.	1.296

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	361	897	0	4674	0	805
normalized size	1	1.	0.85	2.12	0.	11.02	0.	1.9
time (sec)	N/A	1.521	1.567	0.19	0.	5.22	0.	1.245

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	480	447	1048	0	6056	0	994
normalized size	1	1.	0.93	2.18	0.	12.62	0.	2.07
time (sec)	N/A	1.959	1.619	0.198	0.	8.746	0.	1.345

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	2015	2174	0	2642	0	1307
normalized size	1	1.	3.76	4.06	0.	4.93	0.	2.44
time (sec)	N/A	2.199	14.42	0.161	0.	2.936	0.	1.348

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	517	1676	0	2326	0	977
normalized size	1	1.	1.07	3.46	0.	4.8	0.	2.01
time (sec)	N/A	1.717	12.815	0.171	0.	2.694	0.	1.402

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	1250	1325	0	1864	0	784
normalized size	1	1.	5.27	5.59	0.	7.86	0.	3.31
time (sec)	N/A	0.465	8.288	0.157	0.	2.911	0.	1.308

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	243	988	0	2257	0	857
normalized size	1	1.	0.61	2.48	0.	5.66	0.	2.15
time (sec)	N/A	0.517	1.976	0.204	0.	5.84	0.	1.287

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	332	903	0	2634	0	622
normalized size	1	1.	1.06	2.88	0.	8.39	0.	1.98
time (sec)	N/A	0.493	6.204	0.21	0.	4.011	0.	1.323

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	384	943	0	3688	0	691
normalized size	1	1.	0.97	2.39	0.	9.34	0.	1.75
time (sec)	N/A	0.523	6.26	0.221	0.	5.219	0.	1.307

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	490	873	0	3528	0	645
normalized size	1	1.	1.49	2.65	0.	10.72	0.	1.96
time (sec)	N/A	1.329	6.269	0.229	0.	2.782	0.	1.284

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	363	1070	0	4537	0	814
normalized size	1	1.	1.02	3.01	0.	12.78	0.	2.29
time (sec)	N/A	1.682	1.339	0.228	0.	3.549	0.	1.36

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	492	448	1252	0	6091	0	987
normalized size	1	1.	0.91	2.54	0.	12.38	0.	2.01
time (sec)	N/A	2.155	1.714	0.239	0.	5.09	0.	1.277

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	600	600	728	1576	0	8743	0	1374
normalized size	1	1.	1.21	2.63	0.	14.57	0.	2.29
time (sec)	N/A	3.22	2.96	0.252	0.	6.903	0.	1.323

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	712	712	1906	58449	0	0	0	0
normalized size	1	1.	2.68	82.09	0.	0.	0.	0.
time (sec)	N/A	2.645	6.934	2.328	0.	0.	0.	0.

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	161	127	371	0	0	0	0
normalized size	1	1.01	0.8	2.33	0.	0.	0.	0.
time (sec)	N/A	0.428	0.441	0.375	0.	0.	0.	0.

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	105	616	0	0	0	0
normalized size	1	1.	0.54	3.19	0.	0.	0.	0.
time (sec)	N/A	0.474	0.653	0.398	0.	0.	0.	0.

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	73	90	167	128	92
normalized size	1	1.	0.87	0.96	1.18	2.2	1.68	1.21
time (sec)	N/A	0.092	0.201	0.027	0.976	1.469	2.567	1.17

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	54	66	119	87	68
normalized size	1	1.	0.89	0.98	1.2	2.16	1.58	1.24
time (sec)	N/A	0.08	0.103	0.027	0.987	1.51	1.504	1.205

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	35	45	74	66	46
normalized size	1	1.	0.97	1.03	1.32	2.18	1.94	1.35
time (sec)	N/A	0.05	0.023	0.026	0.988	1.525	0.869	1.193

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	35	45	80	0	47
normalized size	1	1.	1.	1.03	1.32	2.35	0.	1.38
time (sec)	N/A	0.041	0.017	0.036	0.974	1.471	0.	1.178

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	35	63	143	0	66
normalized size	1	1.	1.	0.7	1.26	2.86	0.	1.32
time (sec)	N/A	0.072	0.042	0.037	0.972	1.38	0.	1.231

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	73	89	234	0	96
normalized size	1	1.	1.	1.01	1.24	3.25	0.	1.33
time (sec)	N/A	0.091	0.047	0.046	0.977	1.593	0.	1.196

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	177	871	0	996	0	630
normalized size	1	1.	0.75	3.71	0.	4.24	0.	2.68
time (sec)	N/A	0.913	1.745	0.092	0.	1.792	0.	1.249

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	146	657	0	875	0	494
normalized size	1	1.	0.76	3.44	0.	4.58	0.	2.59
time (sec)	N/A	0.664	1.086	0.087	0.	1.564	0.	1.187

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	130	318	0	736	0	279
normalized size	1	1.	0.88	2.15	0.	4.97	0.	1.89
time (sec)	N/A	0.459	0.259	0.082	0.	1.601	0.	1.197

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	104	214	0	637	0	215
normalized size	1	1.	1.04	2.14	0.	6.37	0.	2.15
time (sec)	N/A	0.165	0.232	0.072	0.	1.502	0.	1.171

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	90	137	0	640	0	127
normalized size	1	1.	1.2	1.83	0.	8.53	0.	1.69
time (sec)	N/A	0.185	0.096	0.096	0.	1.761	0.	1.219

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	108	155	0	801	0	174
normalized size	1	1.	1.35	1.94	0.	10.01	0.	2.17
time (sec)	N/A	0.253	0.232	0.101	0.	1.779	0.	1.214

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	181	166	0	1121	0	267
normalized size	1	1.	1.59	1.46	0.	9.83	0.	2.34
time (sec)	N/A	0.456	0.847	0.108	0.	1.947	0.	1.222

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	351	250	0	1420	0	365
normalized size	1	1.	2.29	1.63	0.	9.28	0.	2.39
time (sec)	N/A	0.674	6.231	0.112	0.	1.898	0.	1.201

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	430	315	0	1901	0	454
normalized size	1	1.	2.22	1.62	0.	9.8	0.	2.34
time (sec)	N/A	0.953	6.261	0.113	0.	2.83	0.	1.246

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	506	439	0	2261	0	599
normalized size	1	1.	2.13	1.84	0.	9.5	0.	2.52
time (sec)	N/A	1.227	1.82	0.119	0.	2.809	0.	1.219

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	127	182	177	297	0	201
normalized size	1	1.	0.85	1.22	1.19	1.99	0.	1.35
time (sec)	N/A	0.199	0.279	0.052	1.013	1.627	0.	1.171

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	104	144	142	230	0	158
normalized size	1	1.	0.87	1.21	1.19	1.93	0.	1.33
time (sec)	N/A	0.172	0.348	0.049	0.981	1.581	0.	1.228

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	79	106	107	185	0	115
normalized size	1	1.	0.89	1.19	1.2	2.08	0.	1.29
time (sec)	N/A	0.114	0.201	0.046	0.985	1.499	0.	1.175

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	68	73	131	0	76
normalized size	1	1.	0.9	1.15	1.24	2.22	0.	1.29
time (sec)	N/A	0.117	0.069	0.082	0.993	1.545	0.	1.222

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	72	77	166	0	80
normalized size	1	1.	0.9	1.2	1.28	2.77	0.	1.33
time (sec)	N/A	0.122	0.078	0.079	0.982	1.663	0.	1.198

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	65	106	104	271	0	119
normalized size	1	1.	0.77	1.26	1.24	3.23	0.	1.42
time (sec)	N/A	0.092	0.156	0.088	0.983	1.645	0.	1.206

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	274	1501	0	1204	0	980
normalized size	1	1.	0.97	5.32	0.	4.27	0.	3.48
time (sec)	N/A	1.001	2.28	0.103	0.	1.746	0.	1.181

Problem 1302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	186	941	0	1056	0	618
normalized size	1	1.	0.79	4.	0.	4.49	0.	2.63
time (sec)	N/A	0.717	1.985	0.096	0.	1.906	0.	1.185

Problem 1303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	155	760	0	945	0	501
normalized size	1	1.	0.97	4.78	0.	5.94	0.	3.15
time (sec)	N/A	0.325	1.045	0.087	0.	1.649	0.	1.203

Problem 1304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	143	334	0	853	0	247
normalized size	1	1.	1.15	2.69	0.	6.88	0.	1.99
time (sec)	N/A	0.288	0.263	0.109	0.	2.281	0.	1.206

Problem 1305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	146	249	0	981	0	298
normalized size	1	1.	1.4	2.39	0.	9.43	0.	2.87
time (sec)	N/A	0.27	0.758	0.112	0.	2.523	0.	1.192

Problem 1306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	204	286	0	1337	0	293
normalized size	1	1.	1.66	2.33	0.	10.87	0.	2.38
time (sec)	N/A	0.301	1.533	0.12	0.	2.655	0.	1.249

Problem 1307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	350	348	0	1507	0	369
normalized size	1	1.	2.27	2.26	0.	9.79	0.	2.4
time (sec)	N/A	0.446	6.125	0.121	0.	2.508	0.	1.207

Problem 1308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	433	455	0	2068	0	506
normalized size	1	1.	2.19	2.3	0.	10.44	0.	2.56
time (sec)	N/A	0.761	6.213	0.128	0.	3.223	0.	1.261

Problem 1309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	507	583	0	2437	0	653
normalized size	1	1.	2.08	2.39	0.	9.99	0.	2.68
time (sec)	N/A	1.05	1.815	0.141	0.	3.516	0.	1.26

Problem 1310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	180	329	277	466	0	352
normalized size	1	1.	0.85	1.55	1.31	2.2	0.	1.66
time (sec)	N/A	0.238	1.248	0.052	0.989	1.728	0.	1.222

Problem 1311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	153	273	232	377	0	288
normalized size	1	1.	0.85	1.52	1.29	2.09	0.	1.6
time (sec)	N/A	0.207	0.721	0.049	0.983	1.637	0.	1.204

Problem 1312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	128	215	188	328	0	223
normalized size	1	1.	0.86	1.45	1.27	2.22	0.	1.51
time (sec)	N/A	0.141	0.611	0.043	1.002	1.503	0.	1.176

Problem 1313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	101	140	134	250	0	143
normalized size	1	1.	0.94	1.31	1.25	2.34	0.	1.34
time (sec)	N/A	0.14	0.14	0.075	0.989	1.758	0.	1.194

Problem 1314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	86	124	123	317	0	142
normalized size	1	1.	0.9	1.29	1.28	3.3	0.	1.48
time (sec)	N/A	0.155	0.176	0.085	1.009	1.73	0.	1.191

Problem 1315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	97	140	134	382	0	176
normalized size	1	1.	0.92	1.33	1.28	3.64	0.	1.68
time (sec)	N/A	0.18	0.388	0.086	1.002	1.742	0.	1.216

Problem 1316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	110	163	153	455	0	204
normalized size	1	1.	0.92	1.36	1.27	3.79	0.	1.7
time (sec)	N/A	0.174	0.283	0.093	0.975	1.697	0.	1.22

Problem 1317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	115	216	188	629	0	271
normalized size	1	1.	0.78	1.46	1.27	4.25	0.	1.83
time (sec)	N/A	0.133	3.746	0.093	0.999	1.546	0.	1.275

Problem 1318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	179	274	230	807	0	339
normalized size	1	1.	1.	1.53	1.28	4.51	0.	1.89
time (sec)	N/A	0.205	6.128	0.098	1.	1.527	0.	1.21

Problem 1319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	165	330	278	1031	0	406
normalized size	1	1.	0.78	1.56	1.31	4.86	0.	1.92
time (sec)	N/A	0.241	2.867	0.105	0.991	1.675	0.	1.357

Problem 1320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	467	467	403	2587	0	1659	0	1679
normalized size	1	1.	0.86	5.54	0.	3.55	0.	3.6
time (sec)	N/A	1.814	3.32	0.099	0.	2.002	0.	1.248

Problem 1321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	324	1808	0	1436	0	1165
normalized size	1	1.	0.79	4.43	0.	3.52	0.	2.86
time (sec)	N/A	1.458	2.994	0.102	0.	1.879	0.	1.242

Problem 1322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	275	1551	0	1310	0	992
normalized size	1	1.	1.21	6.8	0.	5.75	0.	4.35
time (sec)	N/A	0.515	2.273	0.094	0.	1.85	0.	1.218

Problem 1323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	220	827	0	1212	0	537
normalized size	1	1.	0.87	3.28	0.	4.81	0.	2.13
time (sec)	N/A	0.286	0.542	0.112	0.	3.803	0.	1.274

Problem 1324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	208	557	0	1311	0	408
normalized size	1	1.	1.14	3.04	0.	7.16	0.	2.23
time (sec)	N/A	0.255	1.38	0.119	0.	4.043	0.	1.259

Problem 1325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	259	483	0	1739	0	582
normalized size	1	1.	1.49	2.78	0.	9.99	0.	3.34
time (sec)	N/A	0.385	5.442	0.124	0.	4.592	0.	1.25

Problem 1326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	379	442	0	1855	0	428
normalized size	1	1.	1.92	2.24	0.	9.42	0.	2.17
time (sec)	N/A	0.275	6.191	0.124	0.	4.419	0.	1.228

Problem 1327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	275	448	523	0	2427	0	535
normalized size	1	1.41	2.3	2.68	0.	12.45	0.	2.74
time (sec)	N/A	0.29	6.183	0.125	0.	4.565	0.	1.26

Problem 1328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	307	504	629	0	2566	0	662
normalized size	1	1.27	2.09	2.61	0.	10.65	0.	2.75
time (sec)	N/A	1.12	1.363	0.126	0.	4.377	0.	1.26

Problem 1329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	356	780	0	3362	0	846
normalized size	1	1.	0.98	2.15	0.	9.26	0.	2.33
time (sec)	N/A	1.484	1.507	0.127	0.	6.774	0.	1.293

Problem 1330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	442	952	0	3822	0	1048
normalized size	1	1.	1.06	2.28	0.	9.17	0.	2.51
time (sec)	N/A	1.833	1.973	0.129	0.	5.285	0.	1.231

Problem 1331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	593	1143	0	4911	0	1280
normalized size	1	1.	1.25	2.4	0.	10.32	0.	2.69
time (sec)	N/A	2.215	3.574	0.135	0.	8.866	0.	1.308

Problem 1332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	83	95	111	223	0	115
normalized size	1	1.	0.89	1.02	1.19	2.4	0.	1.24
time (sec)	N/A	0.181	0.211	0.064	0.987	1.207	0.	1.257

Problem 1333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	81	92	174	0	96
normalized size	1	1.	0.9	1.01	1.15	2.17	0.	1.2
time (sec)	N/A	0.156	0.065	0.062	0.989	1.327	0.	1.214

Problem 1334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	87	76	88	157	0	96
normalized size	1	1.	1.18	1.03	1.19	2.12	0.	1.3
time (sec)	N/A	0.064	0.079	0.055	0.998	1.268	0.	1.226

Problem 1335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	84	95	108	225	0	116
normalized size	1	1.	0.9	1.02	1.16	2.42	0.	1.25
time (sec)	N/A	0.146	0.106	0.076	1.014	1.722	0.	1.24

Problem 1336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	113	128	348	0	153
normalized size	1	1.	0.88	1.03	1.16	3.16	0.	1.39
time (sec)	N/A	0.187	0.235	0.082	0.982	2.158	0.	1.209

Problem 1337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	144	154	501	0	200
normalized size	1	1.	1.	1.09	1.17	3.8	0.	1.52
time (sec)	N/A	0.202	0.534	0.092	0.994	3.161	0.	1.24

Problem 1338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	221	283	0	1138	0	281
normalized size	1	1.	0.82	1.06	0.	4.25	0.	1.05
time (sec)	N/A	0.396	1.525	0.108	0.	1.817	0.	1.2

Problem 1339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	186	162	0	954	0	234
normalized size	1	1.	1.02	0.89	0.	5.21	0.	1.28
time (sec)	N/A	0.273	1.032	0.102	0.	1.684	0.	1.186

Problem 1340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	152	138	0	822	0	177
normalized size	1	1.	1.14	1.04	0.	6.18	0.	1.33
time (sec)	N/A	0.178	0.77	0.093	0.	1.624	0.	1.198

Problem 1341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	152	117	0	684	0	144
normalized size	1	1.	1.58	1.22	0.	7.12	0.	1.5
time (sec)	N/A	0.107	0.198	0.088	0.	1.539	0.	1.181

Problem 1342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	151	116	0	687	0	143
normalized size	1	1.	1.84	1.41	0.	8.38	0.	1.74
time (sec)	N/A	0.099	0.174	0.082	0.	1.59	0.	1.198

Problem 1343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	191	130	0	1084	0	182
normalized size	1	1.	1.62	1.1	0.	9.19	0.	1.54
time (sec)	N/A	0.23	0.362	0.101	0.	3.058	0.	1.183

Problem 1344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	150	205	169	0	1370	0	350
normalized size	1	1.17	1.6	1.32	0.	10.7	0.	2.73
time (sec)	N/A	0.288	0.934	0.107	0.	2.925	0.	1.237

Problem 1345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	212	261	227	0	1931	0	331
normalized size	1	1.17	1.44	1.25	0.	10.67	0.	1.83
time (sec)	N/A	0.36	3.005	0.136	0.	5.029	0.	1.192

Problem 1346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	117	164	192	367	0	239
normalized size	1	1.	0.93	1.3	1.52	2.91	0.	1.9
time (sec)	N/A	0.202	0.505	0.079	0.975	2.008	0.	1.241

Problem 1347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	108	123	178	369	0	227
normalized size	1	1.	0.93	1.06	1.53	3.18	0.	1.96
time (sec)	N/A	0.231	0.455	0.075	0.981	2.094	0.	1.235

Problem 1348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	162	123	178	370	0	230
normalized size	1	1.	1.38	1.05	1.52	3.16	0.	1.97
time (sec)	N/A	0.165	0.374	0.067	0.994	1.847	0.	1.216

Problem 1349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	151	181	211	502	0	284
normalized size	1	1.	0.97	1.16	1.35	3.22	0.	1.82
time (sec)	N/A	0.242	0.682	0.089	1.	4.553	0.	1.242

Problem 1350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	174	199	270	672	0	377
normalized size	1	1.	1.02	1.16	1.58	3.93	0.	2.2
time (sec)	N/A	0.27	0.748	0.099	1.057	6.021	0.	1.295

Problem 1351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	168	231	329	973	0	371
normalized size	1	1.	0.85	1.17	1.67	4.94	0.	1.88
time (sec)	N/A	0.312	1.422	0.111	1.028	8.902	0.	1.3

Problem 1352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	195	269	0	1054	0	325
normalized size	1	1.	1.1	1.52	0.	5.95	0.	1.84
time (sec)	N/A	0.229	1.399	0.111	0.	1.595	0.	1.196

Problem 1353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	184	222	0	1027	0	306
normalized size	1	1.	1.3	1.56	0.	7.23	0.	2.15
time (sec)	N/A	0.239	1.418	0.105	0.	1.636	0.	1.226

Problem 1354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	200	224	0	1033	0	309
normalized size	1	1.	1.21	1.36	0.	6.26	0.	1.87
time (sec)	N/A	0.238	1.262	0.102	0.	1.747	0.	1.21

Problem 1355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	203	272	0	1027	0	324
normalized size	1	1.	1.47	1.97	0.	7.44	0.	2.35
time (sec)	N/A	0.223	1.267	0.095	0.	1.629	0.	1.196

Problem 1356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	334	279	0	1546	0	416
normalized size	1	1.	1.72	1.44	0.	7.97	0.	2.14
time (sec)	N/A	0.409	4.844	0.118	0.	5.346	0.	1.212

Problem 1357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	247	450	317	0	1891	0	482
normalized size	1	1.12	2.05	1.44	0.	8.6	0.	2.19
time (sec)	N/A	0.474	6.443	0.123	0.	5.393	0.	1.257

Problem 1358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	947	376	0	2596	0	563
normalized size	1	1.	2.85	1.13	0.	7.82	0.	1.7
time (sec)	N/A	0.548	6.231	0.138	0.	10.282	0.	1.303

Problem 1359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	212	338	427	946	0	544
normalized size	1	1.	0.88	1.41	1.78	3.94	0.	2.27
time (sec)	N/A	0.618	3.037	0.095	1.039	3.754	0.	1.263

Problem 1360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	198	321	409	780	0	518
normalized size	1	1.	0.9	1.45	1.85	3.53	0.	2.34
time (sec)	N/A	0.539	2.427	0.095	1.038	3.474	0.	1.261

Problem 1361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	187	308	390	687	0	501
normalized size	1	1.	0.9	1.48	1.88	3.3	0.	2.41
time (sec)	N/A	0.512	1.739	0.089	1.031	2.937	0.	1.243

Problem 1362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	184	304	389	594	0	463
normalized size	1	1.	0.9	1.49	1.91	2.91	0.	2.27
time (sec)	N/A	0.351	1.415	0.088	1.029	2.333	0.	1.292

Problem 1363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	169	260	373	589	0	450
normalized size	1	1.	0.89	1.37	1.96	3.1	0.	2.37
time (sec)	N/A	0.431	1.597	0.086	1.006	2.354	0.	1.277

Problem 1364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	166	261	360	579	0	440
normalized size	1	1.	0.91	1.43	1.98	3.18	0.	2.42
time (sec)	N/A	0.373	1.431	0.082	1.008	2.418	0.	1.256

Problem 1365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	163	262	358	583	0	439
normalized size	1	1.	0.92	1.47	2.01	3.28	0.	2.47
time (sec)	N/A	0.38	1.221	0.079	1.006	2.279	0.	1.252

Problem 1366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	244	259	360	574	0	436
normalized size	1	1.	1.38	1.46	2.03	3.24	0.	2.46
time (sec)	N/A	0.243	0.918	0.072	1.019	2.201	0.	1.217

Problem 1367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	220	321	404	790	0	528
normalized size	1	1.	0.94	1.38	1.73	3.39	0.	2.27
time (sec)	N/A	0.365	2.814	0.098	1.024	9.771	0.	1.283

Problem 1368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	257	340	487	980	0	564
normalized size	1	1.	1.03	1.36	1.95	3.92	0.	2.26
time (sec)	N/A	0.405	6.2	0.102	1.023	14.238	0.	1.292

Problem 1369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	281	371	570	1418	0	795
normalized size	1	1.	1.03	1.35	2.08	5.18	0.	2.9
time (sec)	N/A	0.474	6.243	0.111	1.076	21.035	0.	1.288

Problem 1370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	500	500	816	1674	0	0	0	0
normalized size	1	1.	1.63	3.35	0.	0.	0.	0.
time (sec)	N/A	1.237	26.945	5.607	0.	0.	0.	0.

Problem 1371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	448	448	789	2363	0	0	0	0
normalized size	1	1.	1.76	5.27	0.	0.	0.	0.
time (sec)	N/A	0.96	26.859	6.549	0.	0.	0.	0.

Problem 1372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	369	369	372	924	0	0	0	0
normalized size	1	1.	1.01	2.5	0.	0.	0.	0.
time (sec)	N/A	0.872	17.611	5.386	0.	0.	0.	0.

Problem 1373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	341	341	351	884	0	0	0	0
normalized size	1	1.	1.03	2.59	0.	0.	0.	0.
time (sec)	N/A	0.735	20.064	6.25	0.	0.	0.	0.

Problem 1374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	355	355	534	188	0	0	0	0
normalized size	1	1.	1.5	0.53	0.	0.	0.	0.
time (sec)	N/A	0.833	14.952	2.449	0.	0.	0.	0.

Problem 1375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	1550	1266	0	0	0	0
normalized size	1	1.	3.58	2.92	0.	0.	0.	0.
time (sec)	N/A	0.939	27.015	10.674	0.	0.	0.	0.

Problem 1376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	544	544	1582	307	0	0	0	0
normalized size	1	1.	2.91	0.56	0.	0.	0.	0.
time (sec)	N/A	1.057	29.023	2.48	0.	0.	0.	0.

Problem 1377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	621	621	1991	3600	0	0	0	0
normalized size	1	1.	3.21	5.8	0.	0.	0.	0.
time (sec)	N/A	1.541	27.505	8.401	0.	0.	0.	0.

Problem 1378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	514	514	1953	1778	0	0	0	0
normalized size	1	1.	3.8	3.46	0.	0.	0.	0.
time (sec)	N/A	1.214	27.636	7.529	0.	0.	0.	0.

Problem 1379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	426	426	1909	2432	0	0	0	0
normalized size	1	1.	4.48	5.71	0.	0.	0.	0.
time (sec)	N/A	0.983	26.953	6.761	0.	0.	0.	0.

Problem 1380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	439	439	484	216	0	0	0	0
normalized size	1	1.	1.1	0.49	0.	0.	0.	0.
time (sec)	N/A	1.138	5.304	2.729	0.	0.	0.	0.

Problem 1381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	2099	2324	0	0	0	0
normalized size	1	1.	4.48	4.96	0.	0.	0.	0.
time (sec)	N/A	1.257	27.439	11.012	0.	0.	0.	0.

Problem 1382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	574	574	2129	312	0	0	0	0
normalized size	1	1.	3.71	0.54	0.	0.	0.	0.
time (sec)	N/A	1.37	30.617	2.707	0.	0.	0.	0.

Problem 1383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	610	610	867	4548	0	0	0	0
normalized size	1	1.	1.42	7.46	0.	0.	0.	0.
time (sec)	N/A	1.359	27.156	7.449	0.	0.	0.	0.

Problem 1384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	501	501	824	1937	0	0	0	0
normalized size	1	1.	1.64	3.87	0.	0.	0.	0.
time (sec)	N/A	1.194	27.133	6.252	0.	0.	0.	0.

Problem 1385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	413	413	737	2612	0	0	0	0
normalized size	1	1.	1.78	6.32	0.	0.	0.	0.
time (sec)	N/A	0.972	24.847	6.952	0.	0.	0.	0.

Problem 1386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	425	425	484	259	0	0	0	0
normalized size	1	1.	1.14	0.61	0.	0.	0.	0.
time (sec)	N/A	1.149	24.565	2.695	0.	0.	0.	0.

Problem 1387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	462	462	1556	1987	0	0	0	0
normalized size	1	1.	3.37	4.3	0.	0.	0.	0.
time (sec)	N/A	1.261	27.33	11.355	0.	0.	0.	0.

Problem 1388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	557	557	1590	318	0	0	0	0
normalized size	1	1.	2.85	0.57	0.	0.	0.	0.
time (sec)	N/A	1.343	29.149	2.873	0.	0.	0.	0.

Problem 1389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	509	509	1953	1455	0	0	0	0
normalized size	1	1.	3.84	2.86	0.	0.	0.	0.
time (sec)	N/A	1.506	27.22	6.24	0.	0.	0.	0.

Problem 1390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	457	457	1915	2015	0	0	0	0
normalized size	1	1.	4.19	4.41	0.	0.	0.	0.
time (sec)	N/A	1.18	26.964	5.85	0.	0.	0.	0.

Problem 1391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	1326	855	0	0	0	0
normalized size	1	1.	3.49	2.25	0.	0.	0.	0.
time (sec)	N/A	0.931	25.56	5.814	0.	0.	0.	0.

Problem 1392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	546	1181	0	0	0	0
normalized size	1	1.	1.55	3.36	0.	0.	0.	0.
time (sec)	N/A	0.725	6.287	5.057	0.	0.	0.	0.

Problem 1393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	369	369	698	186	0	0	0	0
normalized size	1	1.	1.89	0.5	0.	0.	0.	0.
time (sec)	N/A	0.818	20.933	2.322	0.	0.	0.	0.

Problem 1394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	448	448	2093	1217	0	0	0	0
normalized size	1	1.	4.67	2.72	0.	0.	0.	0.
time (sec)	N/A	0.975	30.114	10.609	0.	0.	0.	0.

Problem 1395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	557	557	2129	315	0	0	0	0
normalized size	1	1.	3.82	0.57	0.	0.	0.	0.
time (sec)	N/A	1.032	30.507	3.082	0.	0.	0.	0.

Problem 1396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	584	584	820	1990	0	0	0	0
normalized size	1	1.	1.4	3.41	0.	0.	0.	0.
time (sec)	N/A	1.276	26.953	8.009	0.	0.	0.	0.

Problem 1397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	509	509	793	1613	0	0	0	0
normalized size	1	1.	1.56	3.17	0.	0.	0.	0.
time (sec)	N/A	1.067	26.879	9.466	0.	0.	0.	0.

Problem 1398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	453	453	785	1105	0	0	0	0
normalized size	1	1.	1.73	2.44	0.	0.	0.	0.
time (sec)	N/A	0.881	16.688	7.4	0.	0.	0.	0.

Problem 1399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	413	413	783	1938	0	0	0	0
normalized size	1	1.	1.9	4.69	0.	0.	0.	0.
time (sec)	N/A	0.953	16.369	8.036	0.	0.	0.	0.

Problem 1400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	507	507	1587	425	0	0	0	0
normalized size	1	1.	3.13	0.84	0.	0.	0.	0.
time (sec)	N/A	1.361	27.592	3.954	0.	0.	0.	0.

Problem 1401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	627	627	1635	3469	0	0	0	0
normalized size	1	1.	2.61	5.53	0.	0.	0.	0.
time (sec)	N/A	1.493	28.307	14.586	0.	0.	0.	0.

Problem 1402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	601	601	1958	1268	0	0	0	0
normalized size	1	1.	3.26	2.11	0.	0.	0.	0.
time (sec)	N/A	1.398	27.261	9.641	0.	0.	0.	0.

Problem 1403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	528	528	1193	2331	0	0	0	0
normalized size	1	1.	2.26	4.41	0.	0.	0.	0.
time (sec)	N/A	1.086	24.43	12.217	0.	0.	0.	0.

Problem 1404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	468	468	1184	1089	0	0	0	0
normalized size	1	1.	2.53	2.33	0.	0.	0.	0.
time (sec)	N/A	0.901	24.039	9.725	0.	0.	0.	0.

Problem 1405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	432	432	1183	2322	0	0	0	0
normalized size	1	1.	2.74	5.38	0.	0.	0.	0.
time (sec)	N/A	0.921	23.974	11.181	0.	0.	0.	0.

Problem 1406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	527	527	2136	627	0	0	0	0
normalized size	1	1.	4.05	1.19	0.	0.	0.	0.
time (sec)	N/A	1.314	30.176	4.605	0.	0.	0.	0.

Problem 1407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	651	651	2183	2312	0	0	0	0
normalized size	1	1.	3.35	3.55	0.	0.	0.	0.
time (sec)	N/A	1.464	27.865	16.944	0.	0.	0.	0.

Problem 1408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	926	926	1626	4649	0	0	0	0
normalized size	1	1.	1.76	5.02	0.	0.	0.	0.
time (sec)	N/A	1.842	27.314	0.391	0.	0.	0.	0.

Problem 1409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	578	578	176	3290	0	0	0	0
normalized size	1	1.	0.3	5.69	0.	0.	0.	0.
time (sec)	N/A	1.134	17.965	0.269	0.	0.	0.	0.

Problem 1410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	509	509	178	744	0	0	0	0
normalized size	1	1.	0.35	1.46	0.	0.	0.	0.
time (sec)	N/A	0.835	6.896	0.316	0.	0.	0.	0.

Problem 1411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	182	590	0	0	0	0
normalized size	1	1.	0.88	2.84	0.	0.	0.	0.
time (sec)	N/A	0.415	7.182	0.303	0.	0.	0.	0.

Problem 1412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	1622	2497	0	0	0	0
normalized size	1	1.	5.07	7.8	0.	0.	0.	0.
time (sec)	N/A	0.748	24.534	0.319	0.	0.	0.	0.

Problem 1413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	366	366	1648	2672	0	0	0	0
normalized size	1	1.	4.5	7.3	0.	0.	0.	0.
time (sec)	N/A	1.029	21.953	0.368	0.	0.	0.	0.

Problem 1414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	513	513	1729	6120	0	0	0	0
normalized size	1	1.	3.37	11.93	0.	0.	0.	0.
time (sec)	N/A	1.454	23.476	0.342	0.	0.	0.	0.

Problem 1415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	598	598	1771	6505	0	0	0	0
normalized size	1	1.	2.96	10.88	0.	0.	0.	0.
time (sec)	N/A	1.824	22.632	0.418	0.	0.	0.	0.

Problem 1416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	982	982	1901	2513	0	0	0	0
normalized size	1	1.	1.94	2.56	0.	0.	0.	0.
time (sec)	N/A	1.691	28.728	0.364	0.	0.	0.	0.

Problem 1417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	611	611	604	1926	0	0	0	0
normalized size	1	1.	0.99	3.15	0.	0.	0.	0.
time (sec)	N/A	1.03	21.551	0.264	0.	0.	0.	0.

Problem 1418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	577	577	178	930	0	0	0	0
normalized size	1	1.	0.31	1.61	0.	0.	0.	0.
time (sec)	N/A	0.972	11.556	0.273	0.	0.	0.	0.

Problem 1419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	321	321	1092	2587	0	0	0	0
normalized size	1	1.	3.4	8.06	0.	0.	0.	0.
time (sec)	N/A	0.702	20.37	0.281	0.	0.	0.	0.

Problem 1420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	435	435	1135	3014	0	0	0	0
normalized size	1	1.	2.61	6.93	0.	0.	0.	0.
time (sec)	N/A	1.016	20.442	0.251	0.	0.	0.	0.

Problem 1421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	525	525	1162	5828	0	0	0	0
normalized size	1	1.	2.21	11.1	0.	0.	0.	0.
time (sec)	N/A	1.353	20.498	0.312	0.	0.	0.	0.

Problem 1422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	688	688	1207	6707	0	0	0	0
normalized size	1	1.	1.75	9.75	0.	0.	0.	0.
time (sec)	N/A	1.779	21.116	0.402	0.	0.	0.	0.

Problem 1423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	936	936	1618	6219	0	0	0	0
normalized size	1	1.	1.73	6.64	0.	0.	0.	0.
time (sec)	N/A	1.6	26.986	0.404	0.	0.	0.	0.

Problem 1424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	572	572	1402	5148	0	0	0	0
normalized size	1	1.	2.45	9.	0.	0.	0.	0.
time (sec)	N/A	1.023	25.765	0.303	0.	0.	0.	0.

Problem 1425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	616	616	1614	5212	0	0	0	0
normalized size	1	1.	2.62	8.46	0.	0.	0.	0.
time (sec)	N/A	1.136	26.797	0.349	0.	0.	0.	0.

Problem 1426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	359	359	1659	4668	0	0	0	0
normalized size	1	1.	4.62	13.	0.	0.	0.	0.
time (sec)	N/A	0.812	26.699	0.264	0.	0.	0.	0.

Problem 1427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	519	519	1737	10138	0	0	0	0
normalized size	1	1.	3.35	19.53	0.	0.	0.	0.
time (sec)	N/A	1.21	24.522	0.339	0.	0.	0.	0.

Problem 1428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	612	612	1779	10704	0	0	0	0
normalized size	1	1.	2.91	17.49	0.	0.	0.	0.
time (sec)	N/A	1.584	24.007	0.438	0.	0.	0.	0.

Problem 1429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	822	822	1853	17102	0	0	0	0
normalized size	1	1.	2.25	20.81	0.	0.	0.	0.
time (sec)	N/A	2.141	24.984	0.552	0.	0.	0.	0.

Problem 1430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	616	616	1321	2313	0	0	0	0
normalized size	1	1.	2.14	3.75	0.	0.	0.	0.
time (sec)	N/A	1.141	27.244	0.307	0.	0.	0.	0.

Problem 1431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	508	508	518	941	0	0	0	0
normalized size	1	1.	1.02	1.85	0.	0.	0.	0.
time (sec)	N/A	0.772	16.505	0.28	0.	0.	0.	0.

Problem 1432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	170	520	0	0	0	0
normalized size	1	1.	0.81	2.49	0.	0.	0.	0.
time (sec)	N/A	0.354	4.208	0.224	0.	0.	0.	0.

Problem 1433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	213	631	0	0	0	0
normalized size	1	1.	0.78	2.31	0.	0.	0.	0.
time (sec)	N/A	0.586	8.52	0.269	0.	0.	0.	0.

Problem 1434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	229	2286	0	0	0	0
normalized size	1	1.	0.72	7.14	0.	0.	0.	0.
time (sec)	N/A	0.835	9.285	0.293	0.	0.	0.	0.

Problem 1435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	424	424	1137	2987	0	0	0	0
normalized size	1	1.	2.68	7.04	0.	0.	0.	0.
time (sec)	N/A	1.164	20.3	0.331	0.	0.	0.	0.

Problem 1436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1064	1064	1296	4622	0	0	0	0
normalized size	1	1.	1.22	4.34	0.	0.	0.	0.
time (sec)	N/A	1.624	80.914	0.329	0.	0.	0.	0.

Problem 1437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	379	1651	2540	0	0	0	0
normalized size	1	1.	4.36	6.7	0.	0.	0.	0.
time (sec)	N/A	0.827	23.904	0.305	0.	0.	0.	0.

Problem 1438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	1280	2539	0	0	0	0
normalized size	1	1.	3.42	6.79	0.	0.	0.	0.
time (sec)	N/A	0.911	23.063	0.344	0.	0.	0.	0.

Problem 1439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	1285	2523	0	0	0	0
normalized size	1	1.	3.38	6.64	0.	0.	0.	0.
time (sec)	N/A	0.92	22.095	0.3	0.	0.	0.	0.

Problem 1440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	568	568	1710	3104	0	0	0	0
normalized size	1	1.	3.01	5.46	0.	0.	0.	0.
time (sec)	N/A	1.413	24.471	0.323	0.	0.	0.	0.

Problem 1441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	673	673	1730	3315	0	0	0	0
normalized size	1	1.	2.57	4.93	0.	0.	0.	0.
time (sec)	N/A	1.811	23.368	0.352	0.	0.	0.	0.

Problem 1442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	331	331	717	3490	0	0	0	0
normalized size	1	1.	2.17	10.54	0.	0.	0.	0.
time (sec)	N/A	0.793	16.055	0.422	0.	0.	0.	0.

Problem 1443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	104	101	185	0	161
normalized size	1	1.	1.	1.27	1.23	2.26	0.	1.96
time (sec)	N/A	0.137	0.409	0.043	1.538	2.145	0.	1.199

Problem 1444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	63	94	84	153	0	140
normalized size	1	1.	0.97	1.45	1.29	2.35	0.	2.15
time (sec)	N/A	0.107	0.185	0.04	1.481	2.175	0.	1.233

Problem 1445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	47	59	53	108	0	78
normalized size	1	1.	1.24	1.55	1.39	2.84	0.	2.05
time (sec)	N/A	0.064	0.03	0.037	1.535	1.7	0.	1.232

Problem 1446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	36	32	43	82	0	58
normalized size	1	1.	1.33	1.19	1.59	3.04	0.	2.15
time (sec)	N/A	0.049	0.016	0.031	1.519	1.551	0.	1.193

Problem 1447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	56	47	65	188	0	65
normalized size	1	1.	1.56	1.31	1.81	5.22	0.	1.81
time (sec)	N/A	0.08	0.035	0.051	1.011	1.669	0.	1.192

Problem 1448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	68	69	80	269	0	139
normalized size	1	1.	1.42	1.44	1.67	5.6	0.	2.9
time (sec)	N/A	0.111	0.076	0.053	1.04	1.661	0.	1.214

Problem 1449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	172	93	113	333	0	157
normalized size	1	1.	2.29	1.24	1.51	4.44	0.	2.09
time (sec)	N/A	0.137	0.338	0.06	0.987	1.667	0.	1.23

Problem 1450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	104	147	131	221	0	232
normalized size	1	1.	1.11	1.56	1.39	2.35	0.	2.47
time (sec)	N/A	0.183	0.427	0.05	1.548	1.669	0.	1.196

Problem 1451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	112	190	0	185
normalized size	1	1.	0.82	1.23	1.19	2.02	0.	1.97
time (sec)	N/A	0.139	0.416	0.048	1.496	1.636	0.	1.206

Problem 1452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	66	75	76	132	0	111
normalized size	1	1.	1.57	1.79	1.81	3.14	0.	2.64
time (sec)	N/A	0.064	0.307	0.042	1.489	1.685	0.	1.196

Problem 1453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	70	58	68	86	209	0	77
normalized size	1	1.52	1.26	1.48	1.87	4.54	0.	1.67
time (sec)	N/A	0.168	0.197	0.075	1.012	1.953	0.	1.215

Problem 1454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	102	90	96	293	0	173
normalized size	1	1.	1.73	1.53	1.63	4.97	0.	2.93
time (sec)	N/A	0.281	0.339	0.071	1.004	1.933	0.	1.171

Problem 1455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	124	238	140	166	441	0	212
normalized size	1	1.24	2.38	1.4	1.66	4.41	0.	2.12
time (sec)	N/A	0.251	0.478	0.089	1.002	1.921	0.	1.233

Problem 1456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	196	162	166	489	0	275
normalized size	1	1.	1.88	1.56	1.6	4.7	0.	2.64
time (sec)	N/A	0.231	0.993	0.082	1.015	1.75	0.	1.234

Problem 1457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	147	214	221	319	0	454
normalized size	1	1.	0.75	1.09	1.12	1.62	0.	2.3
time (sec)	N/A	0.261	0.76	0.059	1.492	1.67	0.	1.227

Problem 1458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	113	169	161	271	0	279
normalized size	1	1.	0.77	1.16	1.1	1.86	0.	1.91
time (sec)	N/A	0.209	0.727	0.056	1.542	1.64	0.	1.206

Problem 1459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	91	132	134	211	0	200
normalized size	1	1.	1.21	1.76	1.79	2.81	0.	2.67
time (sec)	N/A	0.072	0.537	0.049	1.512	1.639	0.	1.217

Problem 1460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	83	100	116	262	0	116
normalized size	1	1.	1.06	1.28	1.49	3.36	0.	1.49
time (sec)	N/A	0.152	0.299	0.087	1.495	1.716	0.	1.237

Problem 1461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	114	111	122	342	0	200
normalized size	1	1.	1.31	1.28	1.4	3.93	0.	2.3
time (sec)	N/A	0.195	0.39	0.082	1.002	1.66	0.	1.247

Problem 1462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	267	161	185	479	0	242
normalized size	1	1.	2.02	1.22	1.4	3.63	0.	1.83
time (sec)	N/A	0.228	0.564	0.098	0.999	1.731	0.	1.276

Problem 1463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	287	209	219	608	0	331
normalized size	1	1.	1.75	1.27	1.34	3.71	0.	2.02
time (sec)	N/A	0.262	1.343	0.099	1.003	1.693	0.	1.257

Problem 1464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	236	303	0	1577	0	356
normalized size	1	1.	1.06	1.36	0.	7.1	0.	1.6
time (sec)	N/A	0.364	2.034	0.131	0.	2.018	0.	1.218

Problem 1465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	162	219	0	1203	0	300
normalized size	1	1.	0.76	1.03	0.	5.67	0.	1.42
time (sec)	N/A	0.298	0.965	0.122	0.	1.873	0.	1.2

Problem 1466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	169	282	0	1261	0	339
normalized size	1	1.	0.84	1.41	0.	6.3	0.	1.7
time (sec)	N/A	0.308	0.894	0.117	0.	1.872	0.	1.22

Problem 1467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	169	280	0	1238	0	328
normalized size	1	1.	1.27	2.11	0.	9.31	0.	2.47
time (sec)	N/A	0.211	0.841	0.109	0.	1.867	0.	1.234

Problem 1468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	203	304	0	2133	0	424
normalized size	1	1.	0.89	1.33	0.	9.31	0.	1.85
time (sec)	N/A	0.31	2.219	0.15	0.	7.802	0.	1.267

Problem 1469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	254	346	0	2925	0	706
normalized size	1	1.	1.02	1.4	0.	11.79	0.	2.85
time (sec)	N/A	0.372	3.234	0.159	0.	7.636	0.	1.324

Problem 1470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	356	404	0	3976	0	571
normalized size	1	1.	1.21	1.37	0.	13.48	0.	1.94
time (sec)	N/A	0.395	6.491	0.167	0.	14.649	0.	1.259

Problem 1471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	195	590	0	2005	0	474
normalized size	1	1.	0.5	1.52	0.	5.17	0.	1.22
time (sec)	N/A	0.58	3.163	0.145	0.	2.132	0.	1.253

Problem 1472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	204	702	0	2030	0	509
normalized size	1	1.	0.56	1.92	0.	5.55	0.	1.39
time (sec)	N/A	0.491	3.268	0.145	0.	2.167	0.	1.286

Problem 1473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	212	766	0	2067	0	518
normalized size	1	1.	0.61	2.19	0.	5.91	0.	1.48
time (sec)	N/A	0.561	3.238	0.142	0.	2.144	0.	1.272

Problem 1474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	206	643	0	1979	0	493
normalized size	1	1.	1.01	3.15	0.	9.7	0.	2.42
time (sec)	N/A	0.364	2.972	0.132	0.	2.12	0.	1.251

Problem 1475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	322	787	0	3615	0	555
normalized size	1	1.	0.8	1.96	0.	8.99	0.	1.38
time (sec)	N/A	0.51	6.589	0.178	0.	19.155	0.	1.252

Problem 1476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	379	829	0	4748	0	855
normalized size	1	1.	0.89	1.96	0.	11.2	0.	2.02
time (sec)	N/A	0.59	6.35	0.194	0.	20.927	0.	1.275

Problem 1477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	470	432	897	0	6075	0	1215
normalized size	1	1.	0.92	1.91	0.	12.93	0.	2.59
time (sec)	N/A	0.608	6.78	0.198	0.	34.976	0.	1.332

Problem 1478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	198	650	0	0	0	0
normalized size	1	1.	1.25	4.11	0.	0.	0.	0.
time (sec)	N/A	0.265	6.182	0.481	0.	0.	0.	0.

Problem 1479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	B	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	312	0	6059	2313	0	0	0	0
normalized size	1	0.	19.42	7.41	0.	0.	0.	0.
time (sec)	N/A	0.19	26.654	0.381	0.	0.	0.	0.

Problem 1480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	B	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	366	0	6142	2426	0	0	0	0
normalized size	1	0.	16.78	6.63	0.	0.	0.	0.
time (sec)	N/A	0.391	26.756	0.457	0.	0.	0.	0.

Problem 1481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	219	178	401	0	182
normalized size	1	1.	1.01	1.41	1.15	2.59	0.	1.17
time (sec)	N/A	0.169	0.589	0.05	0.981	2.101	0.	1.267

Problem 1482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	133	205	163	360	0	167
normalized size	1	1.	0.99	1.52	1.21	2.67	0.	1.24
time (sec)	N/A	0.133	0.353	0.05	0.996	2.061	0.	1.258

Problem 1483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	123	147	146	302	0	146
normalized size	1	1.	1.06	1.27	1.26	2.6	0.	1.26
time (sec)	N/A	0.109	0.305	0.047	0.967	2.093	0.	1.228

Problem 1484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	106	133	135	270	0	135
normalized size	1	1.	1.03	1.29	1.31	2.62	0.	1.31
time (sec)	N/A	0.12	0.309	0.049	1.002	1.982	0.	1.244

Problem 1485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	84	114	120	247	0	109
normalized size	1	1.	1.14	1.54	1.62	3.34	0.	1.47
time (sec)	N/A	0.133	0.266	0.049	0.977	1.98	0.	1.222

Problem 1486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	100	116	240	0	105
normalized size	1	1.	1.	1.35	1.57	3.24	0.	1.42
time (sec)	N/A	0.139	0.025	0.043	0.965	2.027	0.	1.267

Problem 1487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	92	101	212	0	90
normalized size	1	1.	1.	1.24	1.36	2.86	0.	1.22
time (sec)	N/A	0.105	0.024	0.037	1.003	1.982	0.	1.235

Problem 1488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	99	100	147	328	0	153
normalized size	1	1.	1.	1.01	1.48	3.31	0.	1.55
time (sec)	N/A	0.105	1.257	0.059	1.	2.05	0.	1.223

Problem 1489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	76	120	170	429	0	181
normalized size	1	1.	0.66	1.04	1.48	3.73	0.	1.57
time (sec)	N/A	0.143	0.378	0.062	1.004	2.141	0.	1.253

Problem 1490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	86	151	189	532	0	180
normalized size	1	1.	0.64	1.12	1.4	3.94	0.	1.33
time (sec)	N/A	0.157	0.606	0.066	1.009	2.173	0.	1.279

Problem 1491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	90	173	204	649	0	216
normalized size	1	1.	0.58	1.12	1.32	4.19	0.	1.39
time (sec)	N/A	0.161	0.859	0.068	0.978	2.147	0.	1.273

Problem 1492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	186	355	243	502	0	267
normalized size	1	1.	0.98	1.88	1.29	2.66	0.	1.41
time (sec)	N/A	0.359	1.587	0.077	0.996	2.521	0.	1.271

Problem 1493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	164	270	212	436	0	236
normalized size	1	1.	1.01	1.67	1.31	2.69	0.	1.46
time (sec)	N/A	0.27	2.159	0.076	0.994	2.448	0.	1.253

Problem 1494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	151	262	200	374	0	212
normalized size	1	1.	1.01	1.75	1.33	2.49	0.	1.41
time (sec)	N/A	0.285	1.031	0.073	0.97	2.229	0.	1.238

Problem 1495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	129	168	166	313	0	176
normalized size	1	1.	1.11	1.45	1.43	2.7	0.	1.52
time (sec)	N/A	0.223	0.364	0.067	1.007	2.115	0.	1.234

Problem 1496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	85	209	162	309	0	167
normalized size	1	1.	0.91	2.25	1.74	3.32	0.	1.8
time (sec)	N/A	0.194	0.747	0.062	0.977	2.03	0.	1.272

Problem 1497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	215	122	131	266	0	120
normalized size	1	1.	2.99	1.69	1.82	3.69	0.	1.67
time (sec)	N/A	0.099	2.738	0.053	0.99	1.969	0.	1.245

Problem 1498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	137	125	176	360	0	181
normalized size	1	1.	1.09	0.99	1.4	2.86	0.	1.44
time (sec)	N/A	0.217	0.917	0.089	0.983	2.027	0.	1.239

Problem 1499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	162	195	220	516	0	251
normalized size	1	1.	0.96	1.16	1.31	3.07	0.	1.49
time (sec)	N/A	0.354	2.877	0.088	1.011	2.112	0.	1.271

Problem 1500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	182	209	247	683	0	257
normalized size	1	1.	0.98	1.13	1.34	3.69	0.	1.39
time (sec)	N/A	0.384	3.754	0.098	0.992	2.253	0.	1.276

Problem 1501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	199	420	293	583	0	339
normalized size	1	1.	0.99	2.08	1.45	2.89	0.	1.68
time (sec)	N/A	0.341	1.059	0.091	1.004	2.245	0.	1.244

Problem 1502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	174	385	257	508	0	298
normalized size	1	1.	0.98	2.18	1.45	2.87	0.	1.68
time (sec)	N/A	0.368	0.576	0.086	0.998	2.235	0.	1.284

Problem 1503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	147	297	234	423	0	254
normalized size	1	1.	1.04	2.09	1.65	2.98	0.	1.79
time (sec)	N/A	0.304	0.452	0.085	0.991	2.148	0.	1.268

Problem 1504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	140	263	204	375	0	227
normalized size	1	1.	0.97	1.83	1.42	2.6	0.	1.58
time (sec)	N/A	0.253	0.394	0.074	1.019	2.192	0.	1.233

Problem 1505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	370	231	189	340	0	192
normalized size	1	1.	4.11	2.57	2.1	3.78	0.	2.13
time (sec)	N/A	0.111	1.451	0.066	0.993	2.013	0.	1.369

Problem 1506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	157	216	216	416	0	236
normalized size	1	1.	0.95	1.31	1.31	2.52	0.	1.43
time (sec)	N/A	0.247	0.571	0.105	1.	2.187	0.	1.314

Problem 1507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	161	221	254	560	0	284
normalized size	1	1.	0.94	1.29	1.49	3.27	0.	1.66
time (sec)	N/A	0.371	1.246	0.105	1.011	2.182	0.	1.356

Problem 1508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	190	285	293	791	0	324
normalized size	1	1.	0.86	1.29	1.33	3.58	0.	1.47
time (sec)	N/A	0.434	3.324	0.12	1.003	2.288	0.	1.387

Problem 1509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	164	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.53	0.197	1.954	0.	0.	0.	0.

Problem 1510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	158	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.296	0.162	1.816	0.	0.	0.	0.

Problem 1511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	158	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.258	0.219	1.642	0.	0.	0.	0.

Problem 1512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.111	0.885	0.	0.	0.	0.

Problem 1513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	241	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.541	0.452	1.071	0.	0.	0.	0.

Problem 1514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	487	487	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.566	11.955	1.485	0.	0.	0.	0.

Problem 1515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	B	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	502	0	1600	5578	0	0	0	0
normalized size	1	0.	3.19	11.11	0.	0.	0.	0.
time (sec)	N/A	0.37	23.975	0.783	0.	0.	0.	0.

Problem 1516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	458	458	3522	0	0	0	0	0
normalized size	1	1.	7.69	0.	0.	0.	0.	0.
time (sec)	N/A	1.242	7.046	0.376	0.	0.	0.	0.

Problem 1517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	341	341	398	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.651	5.069	0.264	0.	0.	0.	0.

Problem 1518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	301	0	0	0	0	0
normalized size	1	1.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	2.104	0.167	0.	0.	0.	0.

Problem 1519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	60.839	0.24	0.	0.	0.	0.

Problem 1520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	45.222	0.517	0.	0.	0.	0.

Problem 1521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	7.07	0.384	0.	0.	0.	0.

Problem 1522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	35.849	0.256	0.	0.	0.	0.

Problem 1523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	552	552	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.512	8.32	0.464	0.	0.	0.	0.

Problem 1524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	375	373	0	0	0	0	0	0
normalized size	1	0.99	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.635	3.306	0.368	0.	0.	0.	0.

Problem 1525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.31	0.234	0.	0.	0.	0.

Problem 1526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	3.034	0.697	0.	0.	0.	0.

Problem 1527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	4.457	0.872	0.	0.	0.	0.

Problem 1528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	151	128	204	274	228	246
normalized size	1	1.	0.8	0.68	1.09	1.46	1.21	1.31
time (sec)	N/A	0.242	0.774	0.088	1.018	1.531	21.918	1.248

Problem 1529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	116	108	157	224	178	196
normalized size	1	1.	0.81	0.76	1.1	1.57	1.24	1.37
time (sec)	N/A	0.171	0.291	0.063	1.005	1.348	7.337	1.237

Problem 1530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	80	88	108	174	128	135
normalized size	1	1.	0.82	0.91	1.11	1.79	1.32	1.39
time (sec)	N/A	0.115	0.25	0.059	0.99	1.411	2.234	1.279

Problem 1531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	44	61	123	75	70
normalized size	1	1.	0.87	0.85	1.17	2.37	1.44	1.35
time (sec)	N/A	0.054	0.08	0.029	0.97	1.382	0.601	1.227

Problem 1532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	68	83	86	170	0	90
normalized size	1	1.	1.06	1.3	1.34	2.66	0.	1.41
time (sec)	N/A	0.108	0.029	0.063	0.982	1.502	0.	1.286

Problem 1533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	129	105	232	0	113
normalized size	1	1.	0.92	2.19	1.78	3.93	0.	1.92
time (sec)	N/A	0.076	0.21	0.077	0.989	1.434	0.	1.309

Problem 1534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	82	173	151	286	0	154
normalized size	1	1.	0.93	1.97	1.72	3.25	0.	1.75
time (sec)	N/A	0.09	0.557	0.08	0.986	1.494	0.	1.277

Problem 1535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	104	217	193	342	0	188
normalized size	1	1.	0.88	1.84	1.64	2.9	0.	1.59
time (sec)	N/A	0.105	0.871	0.082	1.218	1.573	0.	1.286

Problem 1536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	295	229	321	425	389	377
normalized size	1	1.	0.85	0.66	0.92	1.22	1.11	1.08
time (sec)	N/A	0.392	1.468	0.078	1.	1.734	36.322	1.383

Problem 1537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	227	199	248	356	335	312
normalized size	1	1.	0.98	0.86	1.07	1.54	1.45	1.35
time (sec)	N/A	0.259	0.499	0.074	0.988	1.648	13.342	1.289

Problem 1538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	111	169	173	284	228	227
normalized size	1	1.	0.84	1.28	1.31	2.15	1.73	1.72
time (sec)	N/A	0.17	0.246	0.078	0.992	1.473	4.735	1.288

Problem 1539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	41	73	100	213	143	116
normalized size	1	1.	0.76	1.35	1.85	3.94	2.65	2.15
time (sec)	N/A	0.076	0.067	0.033	0.976	1.381	1.309	1.233

Problem 1540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	81	161	147	273	0	174
normalized size	1	1.	0.86	1.71	1.56	2.9	0.	1.85
time (sec)	N/A	0.174	0.198	0.079	0.988	1.543	0.	1.371

Problem 1541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	174	231	165	329	0	197
normalized size	1	1.	1.55	2.06	1.47	2.94	0.	1.76
time (sec)	N/A	0.18	1.46	0.095	0.987	1.556	0.	1.31

Problem 1542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	186	299	231	414	0	252
normalized size	1	1.	1.52	2.45	1.89	3.39	0.	2.07
time (sec)	N/A	0.156	1.721	0.097	0.991	1.525	0.	1.406

Problem 1543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	242	396	285	493	0	309
normalized size	1	1.	1.51	2.48	1.78	3.08	0.	1.93
time (sec)	N/A	0.205	1.487	0.097	1.027	1.565	0.	1.439

Problem 1544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	218	689	494	833	0	690
normalized size	1	1.	0.69	2.19	1.57	2.64	0.	2.19
time (sec)	N/A	0.356	0.876	0.075	1.013	1.907	0.	1.304

Problem 1545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	148	397	297	498	0	386
normalized size	1	1.	0.73	1.97	1.47	2.47	0.	1.91
time (sec)	N/A	0.248	0.451	0.072	0.998	1.698	0.	1.334

Problem 1546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	89	186	151	255	0	174
normalized size	1	1.	0.8	1.68	1.36	2.3	0.	1.57
time (sec)	N/A	0.163	0.376	0.066	0.985	1.532	0.	1.218

Problem 1547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	56	54	89	104	55
normalized size	1	1.	0.95	1.37	1.32	2.17	2.54	1.34
time (sec)	N/A	0.07	0.045	0.034	0.972	1.484	0.819	1.225

Problem 1548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	99	156	107	213	0	117
normalized size	1	1.	1.1	1.73	1.19	2.37	0.	1.3
time (sec)	N/A	0.148	0.177	0.076	0.966	1.879	0.	1.187

Problem 1549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	197	297	236	531	0	351
normalized size	1	1.	1.24	1.87	1.48	3.34	0.	2.21
time (sec)	N/A	0.29	0.759	0.1	1.002	3.516	0.	1.249

Problem 1550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	321	586	495	929	0	728
normalized size	1	1.	1.22	2.23	1.88	3.53	0.	2.77
time (sec)	N/A	0.448	1.279	0.106	1.016	9.245	0.	1.294

Problem 1551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	383	383	565	990	853	1454	0	1224
normalized size	1	1.	1.48	2.58	2.23	3.8	0.	3.2
time (sec)	N/A	0.681	2.593	0.109	1.111	23.361	0.	1.455

Problem 1552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	396	721	509	1220	0	770
normalized size	1	1.	1.22	2.23	1.57	3.77	0.	2.38
time (sec)	N/A	0.399	1.661	0.126	0.987	2.132	0.	1.271

Problem 1553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	234	422	309	763	0	443
normalized size	1	1.	1.14	2.05	1.5	3.7	0.	2.15
time (sec)	N/A	0.272	2.11	0.125	0.974	1.87	0.	1.531

Problem 1554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	111	202	159	408	0	254
normalized size	1	1.	0.98	1.79	1.41	3.61	0.	2.25
time (sec)	N/A	0.169	0.505	0.121	0.976	1.554	0.	1.189

Problem 1555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	42	63	65	128	178	108
normalized size	1	1.	0.88	1.31	1.35	2.67	3.71	2.25
time (sec)	N/A	0.077	0.087	0.056	0.992	1.42	1.273	1.36

Problem 1556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	178	240	198	671	0	277
normalized size	1	1.	1.32	1.78	1.47	4.97	0.	2.05
time (sec)	N/A	0.194	1.272	0.135	0.994	3.38	0.	1.339

Problem 1557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	246	388	467	1337	0	452
normalized size	1	1.	1.08	1.7	2.05	5.86	0.	1.98
time (sec)	N/A	0.326	1.644	0.17	1.004	7.894	0.	1.326

Problem 1558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	370	675	890	1997	0	1027
normalized size	1	1.	0.99	1.81	2.39	5.37	0.	2.76
time (sec)	N/A	0.564	4.125	0.175	1.079	21.023	0.	1.359

Problem 1559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	766	1080	1462	2850	0	1600
normalized size	1	1.	1.39	1.96	2.66	5.18	0.	2.91
time (sec)	N/A	0.928	6.202	0.184	1.183	56.115	0.	1.388

Problem 1560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	4.736	0.451	0.	0.	0.	0.

Problem 1561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	5087	0	0	0	0	0
normalized size	1	1.	15.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.416	34.335	0.72	0.	0.	0.	0.

Problem 1562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	508	508	38759	0	0	0	0	0
normalized size	1	1.	76.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.524	35.854	1.034	0.	0.	0.	0.

Problem 1563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	308	308	5101	0	0	0	0	0
normalized size	1	1.	16.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.616	28.984	0.807	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1382] had the largest ratio of [0.5455]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.	38	0.079
2	A	3	3	1.	38	0.079
3	A	3	3	1.	38	0.079
4	A	3	3	1.	38	0.079
5	A	2	2	1.	38	0.053
6	A	5	5	1.	38	0.132
7	A	5	5	1.	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	2	2	1.	38	0.053
9	A	4	3	1.	38	0.079
10	A	4	3	1.	38	0.079
11	A	4	3	1.	38	0.079
12	A	3	3	1.	38	0.079
13	A	2	2	1.	38	0.053
14	A	6	5	1.	38	0.132
15	A	6	6	1.	38	0.158
16	A	6	5	1.	38	0.132
17	A	2	2	1.	38	0.053
18	A	3	3	1.	38	0.079
19	A	5	3	1.	38	0.079
20	A	5	3	1.	38	0.079
21	A	4	3	1.	38	0.079
22	A	3	3	1.	38	0.079
23	A	2	2	1.	38	0.053
24	A	7	5	1.	38	0.132
25	A	7	6	1.	38	0.158
26	A	7	6	1.	38	0.158
27	A	7	5	1.	38	0.132
28	A	2	2	1.	38	0.053
29	A	3	3	1.	38	0.079
30	A	6	3	1.	38	0.079
31	A	6	3	1.	38	0.079
32	A	5	3	1.	38	0.079
33	A	4	3	1.	38	0.079
34	A	3	3	1.	38	0.079
35	A	2	2	1.	38	0.053
36	A	8	5	1.	38	0.132
37	A	8	6	1.	38	0.158
38	A	8	6	1.	38	0.158
39	A	8	6	1.	38	0.158
40	A	8	5	1.	38	0.132
41	A	2	2	1.	38	0.053
42	A	3	3	1.	38	0.079
43	A	4	3	1.	38	0.079
44	A	2	2	1.	38	0.053
45	A	2	2	1.	38	0.053
46	A	2	2	1.	38	0.053
47	A	2	2	1.	38	0.053
48	A	4	4	1.	38	0.105
49	A	2	2	1.	38	0.053
50	A	8	5	1.	38	0.132
51	A	7	5	1.	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	6	5	1.	38	0.132
53	A	5	5	1.	38	0.132
54	A	4	4	1.	38	0.105
55	A	3	3	1.	38	0.079
56	A	4	4	1.	38	0.105
57	A	9	6	1.	38	0.158
58	A	8	6	1.	38	0.158
59	A	7	6	1.	38	0.158
60	A	6	6	1.	38	0.158
61	A	5	5	1.	38	0.132
62	A	2	2	1.	38	0.053
63	A	4	4	1.	38	0.105
64	A	5	4	1.	38	0.105
65	A	5	5	1.	34	0.147
66	A	4	4	1.	34	0.118
67	A	4	4	1.	34	0.118
68	A	4	4	1.	32	0.125
69	A	3	3	1.	21	0.143
70	A	3	3	1.	34	0.088
71	A	4	4	1.	34	0.118
72	A	4	4	1.	34	0.118
73	A	5	3	1.	36	0.083
74	A	4	3	1.	36	0.083
75	A	3	3	1.	36	0.083
76	A	2	2	1.	36	0.056
77	A	4	4	1.	36	0.111
78	A	4	4	1.	36	0.111
79	A	2	2	1.	36	0.056
80	A	2	2	1.	36	0.056
81	A	4	3	1.	38	0.079
82	A	3	3	1.	38	0.079
83	A	2	2	1.	38	0.053
84	A	5	5	1.	38	0.132
85	A	5	5	1.	38	0.132
86	A	5	5	1.	36	0.139
87	A	5	5	1.	38	0.132
88	A	8	4	1.	42	0.095
89	A	7	4	1.	42	0.095
90	A	6	4	1.	42	0.095
91	A	5	4	1.	42	0.095
92	A	4	4	1.	42	0.095
93	A	4	4	1.	42	0.095
94	A	5	5	1.	42	0.119
95	A	6	5	1.	42	0.119

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	7	5	1.	42	0.119
97	A	8	4	1.	42	0.095
98	A	7	4	1.	42	0.095
99	A	6	4	1.	42	0.095
100	A	5	4	1.	42	0.095
101	A	5	5	1.	42	0.119
102	A	5	4	1.	42	0.095
103	A	6	5	1.	42	0.119
104	A	7	5	1.	42	0.119
105	A	8	5	1.	42	0.119
106	A	9	4	1.	42	0.095
107	A	8	4	1.	42	0.095
108	A	7	4	1.	42	0.095
109	A	6	4	1.	42	0.095
110	A	6	5	1.	42	0.119
111	A	6	5	1.	42	0.119
112	A	6	4	1.	42	0.095
113	A	7	5	1.	42	0.119
114	A	8	5	1.	42	0.119
115	A	9	5	1.	42	0.119
116	A	10	4	1.	42	0.095
117	A	9	4	1.	42	0.095
118	A	8	4	1.	42	0.095
119	A	7	4	1.	42	0.095
120	A	7	5	1.	42	0.119
121	A	7	5	1.	42	0.119
122	A	7	5	1.	42	0.119
123	A	7	4	1.	42	0.095
124	A	8	5	1.	42	0.119
125	A	9	5	1.	42	0.119
126	A	10	5	1.	42	0.119
127	A	6	4	1.	42	0.095
128	A	5	4	1.	42	0.095
129	A	4	4	1.	42	0.095
130	A	3	3	1.	42	0.071
131	A	4	4	1.	42	0.095
132	A	5	4	1.	42	0.095
133	A	6	4	1.	42	0.095
134	A	7	5	1.	42	0.119
135	A	6	5	1.	42	0.119
136	A	5	5	1.	42	0.119
137	A	4	4	1.	42	0.095
138	A	4	4	1.	42	0.095
139	A	5	4	1.	42	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	6	4	1.	42	0.095
141	A	7	4	1.	42	0.095
142	A	8	5	1.	42	0.119
143	A	7	5	1.	42	0.119
144	A	6	5	1.	42	0.119
145	A	5	4	1.	42	0.095
146	A	5	5	1.	42	0.119
147	A	5	4	1.	42	0.095
148	A	6	4	1.	42	0.095
149	A	7	4	1.	42	0.095
150	A	8	4	1.	42	0.095
151	A	4	4	1.	38	0.105
152	A	4	4	1.	38	0.105
153	A	4	4	1.	38	0.105
154	A	4	4	1.	36	0.111
155	A	3	3	1.	25	0.12
156	A	4	4	1.	38	0.105
157	A	4	4	1.	38	0.105
158	A	4	4	1.	38	0.105
159	A	4	4	1.	40	0.1
160	A	4	4	1.	40	0.1
161	A	4	4	1.	40	0.1
162	A	4	4	1.	40	0.1
163	A	4	4	1.	40	0.1
164	A	4	4	1.	40	0.1
165	A	4	4	1.	40	0.1
166	A	4	4	1.	40	0.1
167	A	4	4	1.	42	0.095
168	A	4	4	1.	42	0.095
169	A	4	4	1.	42	0.095
170	A	4	4	1.	40	0.1
171	A	4	4	1.	42	0.095
172	A	4	4	1.	42	0.095
173	A	4	4	1.	36	0.111
174	A	4	4	1.	42	0.095
175	A	3	2	1.	40	0.05
176	A	2	2	1.	40	0.05
177	A	1	1	1.	40	0.025
178	A	4	4	1.	40	0.1
179	A	4	4	1.	40	0.1
180	A	4	4	1.	40	0.1
181	A	3	3	1.	40	0.075
182	A	4	4	1.	45	0.089
183	A	4	4	1.	45	0.089

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	4	4	1.	45	0.089
185	A	1	1	1.	43	0.023
186	A	2	2	1.	45	0.044
187	A	3	2	1.	45	0.044
188	A	4	2	1.	45	0.044
189	A	5	5	1.	36	0.139
190	A	4	3	1.	25	0.12
191	A	4	3	1.	23	0.13
192	A	3	2	1.	17	0.118
193	A	4	3	1.	23	0.13
194	A	3	3	1.	25	0.12
195	A	4	3	1.	25	0.12
196	A	4	3	1.	25	0.12
197	A	4	3	1.	27	0.111
198	A	4	3	1.	25	0.12
199	A	3	2	1.	19	0.105
200	A	4	3	1.	25	0.12
201	A	4	3	1.	27	0.111
202	A	3	3	1.	27	0.111
203	A	4	3	1.	27	0.111
204	A	4	3	1.	27	0.111
205	A	4	3	1.	27	0.111
206	A	4	3	1.	27	0.111
207	A	4	3	1.	27	0.111
208	A	4	3	1.	25	0.12
209	A	3	2	1.	19	0.105
210	A	4	3	1.	25	0.12
211	A	4	3	1.	27	0.111
212	A	4	3	1.	27	0.111
213	A	3	3	1.	27	0.111
214	A	4	4	1.	27	0.148
215	A	4	3	1.	27	0.111
216	A	4	3	1.	27	0.111
217	A	4	3	1.	27	0.111
218	A	4	3	1.	27	0.111
219	A	4	3	1.	27	0.111
220	A	4	3	1.	25	0.12
221	A	3	2	1.	19	0.105
222	A	4	3	1.	25	0.12
223	A	4	3	1.	27	0.111
224	A	4	3	1.	27	0.111
225	A	4	3	1.	27	0.111
226	A	4	3	1.	27	0.111
227	A	4	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
228	A	4	4	1.	19	0.21
229	A	4	3	1.	25	0.12
230	A	4	3	1.	27	0.111
231	A	4	3	1.	27	0.111
232	A	4	3	1.	27	0.111
233	A	4	3	1.	27	0.111
234	A	4	3	1.	27	0.111
235	A	4	3	1.	25	0.12
236	A	3	2	1.	19	0.105
237	A	4	3	1.	25	0.12
238	A	4	3	1.	27	0.111
239	A	4	3	1.	27	0.111
240	A	4	3	1.	27	0.111
241	A	4	3	1.	27	0.111
242	A	4	3	1.	27	0.111
243	A	4	3	1.	27	0.111
244	A	3	3	1.	25	0.12
245	A	3	2	1.	19	0.105
246	A	4	3	1.	25	0.12
247	A	4	3	1.	27	0.111
248	A	4	3	1.	27	0.111
249	A	4	3	1.	27	0.111
250	A	4	3	1.	27	0.111
251	A	4	3	1.	27	0.111
252	A	3	3	1.	27	0.111
253	A	4	3	1.	25	0.12
254	A	3	2	1.	19	0.105
255	A	4	3	1.	25	0.12
256	A	4	3	1.	27	0.111
257	A	4	4	1.	21	0.19
258	A	3	2	1.	27	0.074
259	A	3	2	1.	27	0.074
260	A	3	2	1.	27	0.074
261	A	3	2	1.	25	0.08
262	A	2	2	1.	27	0.074
263	A	2	2	1.	27	0.074
264	A	2	2	1.	27	0.074
265	A	2	2	1.	27	0.074
266	A	8	6	1.	27	0.222
267	A	7	6	1.	27	0.222
268	A	6	6	1.	25	0.24
269	A	6	6	1.	23	0.261
270	A	7	6	1.	19	0.316
271	A	5	5	1.	25	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	5	5	1.	27	0.185
273	A	6	6	1.	27	0.222
274	A	7	6	1.	27	0.222
275	A	12	7	1.	29	0.241
276	A	5	4	1.	29	0.138
277	A	5	5	1.15	27	0.185
278	A	9	8	1.	25	0.32
279	A	8	6	1.	21	0.286
280	A	7	6	1.	27	0.222
281	A	8	7	1.	29	0.241
282	A	9	6	1.	29	0.207
283	A	10	7	1.	29	0.241
284	A	12	6	1.	29	0.207
285	A	15	7	1.	29	0.241
286	A	6	5	1.14	27	0.185
287	A	12	9	1.	25	0.36
288	A	10	7	1.	21	0.333
289	A	10	7	1.	27	0.259
290	A	10	6	1.	29	0.207
291	A	11	8	1.	29	0.276
292	A	12	7	1.	29	0.241
293	A	14	7	1.	29	0.241
294	A	6	4	1.	21	0.19
295	A	15	10	1.	25	0.4
296	A	12	6	1.	21	0.286
297	A	6	4	1.	29	0.138
298	A	6	4	1.	29	0.138
299	A	5	4	1.	29	0.138
300	A	4	4	1.	27	0.148
301	A	3	3	1.	25	0.12
302	A	4	4	1.	21	0.19
303	A	5	5	1.	27	0.185
304	A	5	4	1.	29	0.138
305	A	6	4	1.	29	0.138
306	A	6	4	1.	29	0.138
307	A	12	7	1.	29	0.241
308	A	9	7	1.	29	0.241
309	A	8	7	1.	29	0.241
310	A	3	2	1.	27	0.074
311	A	5	4	1.	25	0.16
312	A	7	5	1.	21	0.238
313	A	9	7	1.	27	0.259
314	A	11	7	1.	29	0.241
315	A	9	7	1.	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
316	A	4	4	1.	29	0.138
317	A	3	3	1.	27	0.111
318	A	7	5	1.	25	0.2
319	A	10	7	1.	21	0.333
320	A	11	8	1.	27	0.296
321	A	5	4	1.	27	0.148
322	A	7	7	1.	31	0.226
323	A	4	4	1.	31	0.129
324	A	3	3	1.	29	0.103
325	A	5	5	1.	27	0.185
326	A	4	4	1.	23	0.174
327	A	5	5	1.	29	0.172
328	A	6	6	1.	31	0.194
329	A	8	7	1.	31	0.226
330	A	5	4	1.	31	0.129
331	A	4	3	1.	29	0.103
332	A	6	5	1.	27	0.185
333	A	5	5	1.	23	0.217
334	A	6	5	1.	29	0.172
335	A	6	5	1.	31	0.161
336	A	6	6	1.	31	0.194
337	A	4	4	1.	31	0.129
338	A	2	2	1.	29	0.069
339	A	4	4	1.	27	0.148
340	A	4	4	1.	23	0.174
341	A	5	5	1.	29	0.172
342	A	6	5	1.	31	0.161
343	A	8	7	1.	31	0.226
344	A	5	5	1.	31	0.161
345	A	4	4	1.	29	0.138
346	A	6	5	1.	27	0.185
347	A	6	5	1.	23	0.217
348	A	8	6	1.	29	0.207
349	A	9	6	1.	31	0.194
350	A	4	3	1.	27	0.111
351	A	4	3	1.	27	0.111
352	A	6	5	1.	25	0.2
353	A	3	2	1.	19	0.105
354	A	4	3	1.	25	0.12
355	A	4	3	1.	25	0.12
356	A	3	2	1.	19	0.105
357	A	4	3	1.	29	0.103
358	A	5	3	1.	27	0.111
359	A	2	1	1.	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	4	3	1.	27	0.111
361	A	4	3	1.	27	0.111
362	A	5	4	1.	21	0.19
363	A	4	3	1.	27	0.111
364	A	4	3	1.	29	0.103
365	A	9	6	1.	27	0.222
366	A	9	6	1.	27	0.222
367	A	8	6	1.	27	0.222
368	A	7	6	1.	25	0.24
369	A	8	6	1.	25	0.24
370	A	9	8	1.	27	0.296
371	A	9	7	1.	25	0.28
372	A	9	7	1.	19	0.368
373	A	7	5	1.	25	0.2
374	A	6	5	1.	27	0.185
375	A	7	6	1.	27	0.222
376	A	8	6	1.	27	0.222
377	A	9	6	1.	27	0.222
378	A	16	6	1.	29	0.207
379	A	13	7	1.	29	0.241
380	A	14	6	1.	29	0.207
381	A	6	5	1.	27	0.185
382	A	11	8	1.	27	0.296
383	A	13	7	1.	29	0.241
384	A	12	8	1.	27	0.296
385	A	12	7	1.	21	0.333
386	A	13	6	1.	27	0.222
387	A	10	7	1.	29	0.241
388	A	11	6	1.	29	0.207
389	A	14	6	1.	29	0.207
390	A	13	7	1.	29	0.241
391	A	16	6	1.	29	0.207
392	A	19	6	1.	29	0.207
393	A	19	7	1.	29	0.241
394	A	17	7	1.	29	0.241
395	A	7	5	1.	27	0.185
396	A	15	9	1.	27	0.333
397	A	15	7	1.	29	0.241
398	A	15	8	1.	27	0.296
399	A	14	8	1.	21	0.381
400	A	15	7	1.	27	0.259
401	A	15	6	1.	29	0.207
402	A	14	8	1.	29	0.276
403	A	14	7	1.	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
404	A	16	7	1.	29	0.241
405	A	17	7	1.	29	0.241
406	A	19	7	1.	29	0.241
407	A	8	5	1.	29	0.172
408	A	17	8	1.	21	0.381
409	A	8	6	1.	29	0.207
410	A	8	6	1.	29	0.207
411	A	7	6	1.	29	0.207
412	A	6	6	1.	27	0.222
413	A	6	6	1.	27	0.222
414	A	6	6	1.	29	0.207
415	A	5	5	1.	27	0.185
416	A	5	5	1.	21	0.238
417	A	6	6	1.	27	0.222
418	A	7	6	1.	29	0.207
419	A	8	6	1.	29	0.207
420	A	11	5	1.	29	0.172
421	A	12	5	1.	29	0.172
422	A	10	5	1.	29	0.172
423	A	10	5	1.	29	0.172
424	A	4	4	1.	27	0.148
425	A	4	4	1.	27	0.148
426	A	6	5	1.	29	0.172
427	A	8	6	1.	27	0.222
428	A	9	6	1.	21	0.286
429	A	10	5	1.	27	0.185
430	A	10	5	1.	29	0.172
431	A	12	5	1.	29	0.172
432	A	12	7	1.	29	0.241
433	A	9	7	1.	29	0.241
434	A	4	4	1.	27	0.148
435	A	5	4	1.	27	0.148
436	A	7	6	1.	29	0.207
437	A	9	7	1.	27	0.259
438	A	11	6	1.	21	0.286
439	A	14	7	1.	27	0.259
440	A	2	2	1.	27	0.074
441	A	18	4	1.47	29	0.138
442	A	24	4	1.	29	0.138
443	A	5	4	1.	31	0.129
444	A	4	3	1.	29	0.103
445	A	9	9	1.	29	0.31
446	A	8	8	1.	31	0.258
447	A	7	7	1.	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
448	A	7	7	1.	23	0.304
449	A	9	6	1.	29	0.207
450	A	11	6	1.	31	0.194
451	A	13	6	1.	31	0.194
452	A	15	6	1.	31	0.194
453	A	6	4	1.	31	0.129
454	A	5	3	1.	29	0.103
455	A	12	12	1.	29	0.414
456	A	10	10	1.	31	0.323
457	A	9	9	1.	29	0.31
458	A	8	8	1.	23	0.348
459	A	11	9	1.	29	0.31
460	A	12	9	1.	31	0.29
461	A	14	9	1.	31	0.29
462	A	16	9	1.	31	0.29
463	A	18	9	1.	31	0.29
464	A	5	4	1.	31	0.129
465	A	3	3	1.	29	0.103
466	A	13	10	1.	29	0.345
467	A	11	8	1.	31	0.258
468	A	11	8	1.	29	0.276
469	A	11	7	1.	23	0.304
470	A	15	8	1.	29	0.276
471	A	17	8	1.	31	0.258
472	A	12	7	1.	31	0.226
473	A	4	4	1.	31	0.129
474	A	2	2	1.	29	0.069
475	A	6	6	1.	29	0.207
476	A	9	6	1.	31	0.194
477	A	8	6	1.	29	0.207
478	A	10	6	1.	23	0.261
479	A	12	6	1.	29	0.207
480	A	14	6	1.	31	0.194
481	A	18	9	1.	31	0.29
482	A	16	9	1.	31	0.29
483	A	6	5	1.	31	0.161
484	A	5	4	1.	29	0.138
485	A	9	6	1.	29	0.207
486	A	12	7	1.	31	0.226
487	A	14	8	1.	29	0.276
488	A	16	8	1.	23	0.348
489	A	18	8	1.	29	0.276
490	A	5	2	1.	29	0.069
491	A	3	2	1.	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
492	A	3	2	1.	29	0.069
493	A	5	4	1.	29	0.138
494	A	4	3	1.	27	0.111
495	A	4	3	1.	27	0.111
496	A	7	5	1.	27	0.185
497	A	7	5	1.	27	0.185
498	A	6	5	1.	25	0.2
499	A	4	3	1.	25	0.12
500	A	4	3	1.	27	0.111
501	A	4	3	1.	27	0.111
502	A	4	3	1.	25	0.12
503	A	3	2	1.	19	0.105
504	A	4	3	1.	25	0.12
505	A	6	5	1.	27	0.185
506	A	6	5	1.	27	0.185
507	A	7	5	1.	27	0.185
508	A	7	5	1.	27	0.185
509	A	4	3	1.	27	0.111
510	A	4	3	1.	27	0.111
511	A	4	3	1.	29	0.103
512	A	4	3	1.	29	0.103
513	A	4	3	1.	27	0.111
514	A	4	3	1.	27	0.111
515	A	4	3	1.	29	0.103
516	A	4	3	1.	29	0.103
517	A	4	3	1.	27	0.111
518	A	3	2	1.	21	0.095
519	A	4	3	1.	27	0.111
520	A	4	3	1.	29	0.103
521	A	4	3	1.	29	0.103
522	A	4	3	1.	27	0.111
523	A	4	3	1.	27	0.111
524	A	4	3	1.	29	0.103
525	A	4	3	1.	29	0.103
526	A	4	3	1.	27	0.111
527	A	3	2	1.	21	0.095
528	A	4	3	1.	27	0.111
529	A	4	3	1.	29	0.103
530	A	4	3	1.	27	0.111
531	A	3	2	1.	21	0.095
532	A	4	3	1.	27	0.111
533	A	4	3	1.	29	0.103
534	A	7	3	1.	29	0.103
535	A	6	5	1.	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
536	A	4	3	1.	27	0.111
537	A	4	3	1.	29	0.103
538	A	4	3	1.	29	0.103
539	A	4	3	1.	27	0.111
540	A	5	4	1.	21	0.19
541	A	6	5	1.	27	0.185
542	A	4	3	1.	29	0.103
543	A	4	3	1.	29	0.103
544	A	4	3	1.	29	0.103
545	A	4	3	1.	29	0.103
546	A	4	3	1.	27	0.111
547	A	4	3	1.	27	0.111
548	A	4	3	1.	29	0.103
549	A	4	3	1.	29	0.103
550	A	3	3	1.	27	0.111
551	A	3	2	1.	21	0.095
552	A	4	3	1.	27	0.111
553	A	4	3	1.	29	0.103
554	A	4	3	1.	29	0.103
555	A	4	3	1.	29	0.103
556	A	4	3	1.	27	0.111
557	A	4	3	1.	27	0.111
558	A	4	3	1.	29	0.103
559	A	4	3	1.	29	0.103
560	A	4	3	1.	27	0.111
561	A	3	2	1.	21	0.095
562	A	4	3	1.	27	0.111
563	A	3	2	1.	21	0.095
564	A	4	3	1.	27	0.111
565	A	3	2	1.	29	0.069
566	A	3	2	1.	29	0.069
567	A	3	2	1.	27	0.074
568	A	3	2	1.	29	0.069
569	A	3	2	1.	29	0.069
570	A	4	3	1.	29	0.103
571	A	4	4	1.	29	0.138
572	A	10	6	1.	27	0.222
573	A	10	6	1.	27	0.222
574	A	9	6	1.	27	0.222
575	A	8	6	1.	25	0.24
576	A	9	6	1.	25	0.24
577	A	10	8	1.	27	0.296
578	A	11	8	1.	27	0.296
579	A	11	7	1.	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
580	A	11	8	1.	25	0.32
581	A	11	7	1.	19	0.368
582	A	9	5	1.	25	0.2
583	A	7	5	1.	27	0.185
584	A	8	6	1.	27	0.222
585	A	9	6	1.	27	0.222
586	A	10	6	1.	27	0.222
587	A	10	6	1.	27	0.222
588	A	18	6	1.	29	0.207
589	A	14	7	1.	29	0.241
590	A	16	6	1.	29	0.207
591	A	7	5	1.	27	0.185
592	A	12	8	1.	27	0.296
593	A	17	7	1.	29	0.241
594	A	16	7	1.	29	0.241
595	A	17	8	1.	29	0.276
596	A	16	8	1.	27	0.296
597	A	15	7	1.	21	0.333
598	A	17	6	1.	27	0.222
599	A	12	7	1.	29	0.241
600	A	13	6	1.	29	0.207
601	A	12	7	1.	29	0.241
602	A	16	6	1.	29	0.207
603	A	14	7	1.	29	0.241
604	A	18	6	1.	29	0.207
605	A	21	6	1.	29	0.207
606	A	21	7	1.	29	0.241
607	A	19	7	1.	29	0.241
608	A	8	5	1.	27	0.185
609	A	17	9	1.	27	0.333
610	A	19	7	1.	29	0.241
611	A	17	8	1.	29	0.276
612	A	15	8	1.	29	0.276
613	A	16	8	1.	27	0.296
614	A	16	7	1.	21	0.333
615	A	18	7	1.	27	0.259
616	A	18	6	1.	29	0.207
617	A	17	8	1.	29	0.276
618	A	16	7	1.	29	0.241
619	A	18	7	1.	29	0.241
620	A	19	7	1.	29	0.241
621	A	21	7	1.	29	0.241
622	A	21	6	1.	29	0.207
623	A	22	7	1.	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
624	A	9	6	1.	29	0.207
625	A	9	6	1.	29	0.207
626	A	8	6	1.	29	0.207
627	A	7	6	1.	27	0.222
628	A	8	6	1.	27	0.222
629	A	9	8	1.	29	0.276
630	A	9	7	1.	29	0.241
631	A	8	7	1.	29	0.241
632	A	7	5	1.	27	0.185
633	A	6	5	1.	21	0.238
634	A	13	8	1.	29	0.276
635	A	6	5	1.	29	0.172
636	A	5	4	1.	27	0.148
637	A	10	9	1.	27	0.333
638	A	9	7	1.	29	0.241
639	A	8	7	1.	29	0.241
640	A	9	8	1.	29	0.276
641	A	10	7	1.	27	0.259
642	A	11	5	1.	21	0.238
643	A	13	7	1.	27	0.259
644	A	14	5	1.	29	0.172
645	A	12	5	1.	29	0.172
646	A	5	5	1.25	27	0.185
647	A	7	6	1.	27	0.222
648	A	7	6	1.	29	0.207
649	A	8	6	1.	29	0.207
650	A	10	6	1.	29	0.207
651	A	12	6	1.	27	0.222
652	A	12	5	1.	21	0.238
653	A	6	2	1.	29	0.069
654	A	5	2	1.	29	0.069
655	A	3	2	1.	27	0.074
656	A	4	3	1.	27	0.111
657	A	8	6	1.	27	0.222
658	A	8	6	1.	27	0.222
659	A	7	5	1.	27	0.185
660	A	7	5	1.	27	0.185
661	A	6	5	1.	25	0.2
662	A	4	3	1.	25	0.12
663	A	4	3	1.	27	0.111
664	A	4	3	1.	27	0.111
665	A	4	3	1.	27	0.111
666	A	4	3	1.	27	0.111
667	A	4	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
668	A	3	2	1.	19	0.105
669	A	4	3	1.	25	0.12
670	A	6	5	1.	27	0.185
671	A	6	5	1.	27	0.185
672	A	7	5	1.	27	0.185
673	A	7	5	1.	27	0.185
674	A	8	6	1.	27	0.222
675	A	8	6	1.	27	0.222
676	A	4	3	1.	27	0.111
677	A	4	3	1.	29	0.103
678	A	4	3	1.	29	0.103
679	A	4	3	1.	29	0.103
680	A	7	5	1.	29	0.172
681	A	7	5	1.	29	0.172
682	A	6	5	1.	27	0.185
683	A	3	2	1.	21	0.095
684	A	4	3	1.	27	0.111
685	A	4	3	1.	29	0.103
686	A	4	3	1.	29	0.103
687	A	4	3	1.	29	0.103
688	A	4	3	1.	29	0.103
689	A	4	3	1.	27	0.111
690	A	6	5	1.	21	0.238
691	A	6	5	1.	27	0.185
692	A	7	5	1.	29	0.172
693	A	7	5	1.	29	0.172
694	A	4	3	1.	29	0.103
695	A	4	3	1.	29	0.103
696	A	4	3	1.	29	0.103
697	A	3	2	1.	29	0.069
698	A	3	2	1.	29	0.069
699	A	3	2	1.	27	0.074
700	A	3	2	1.	29	0.069
701	A	3	2	1.	29	0.069
702	A	3	2	1.	29	0.069
703	A	8	4	1.	29	0.138
704	A	4	4	1.	29	0.138
705	A	11	6	1.	29	0.207
706	A	10	6	1.	29	0.207
707	A	10	6	1.	29	0.207
708	A	9	6	1.	29	0.207
709	A	8	6	1.	27	0.222
710	A	9	6	1.	27	0.222
711	A	10	8	1.	29	0.276

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
712	A	11	8	1.	29	0.276
713	A	11	7	1.	29	0.241
714	A	11	8	1.	29	0.276
715	A	10	7	1.	29	0.241
716	A	9	5	1.	27	0.185
717	A	7	5	1.	21	0.238
718	A	8	6	1.	27	0.222
719	A	9	6	1.	29	0.207
720	A	10	6	1.	29	0.207
721	A	10	6	1.	29	0.207
722	A	15	7	1.	29	0.241
723	A	17	7	1.	29	0.241
724	A	14	8	1.	29	0.276
725	A	15	7	1.	29	0.241
726	A	6	4	1.	27	0.148
727	A	12	9	1.	27	0.333
728	A	14	8	1.	29	0.276
729	A	13	9	1.	29	0.31
730	A	13	8	1.	29	0.276
731	A	14	7	1.	29	0.241
732	A	11	8	1.	29	0.276
733	A	12	7	1.	27	0.259
734	A	19	5	1.	21	0.238
735	A	15	7	1.	27	0.259
736	A	14	8	1.	29	0.276
737	A	17	7	1.	29	0.241
738	A	15	7	1.	29	0.241
739	A	18	8	1.	29	0.276
740	A	16	8	1.	29	0.276
741	A	6	5	1.	27	0.185
742	A	13	10	1.	27	0.37
743	A	11	8	1.	29	0.276
744	A	11	8	1.	29	0.276
745	A	11	7	1.	29	0.241
746	A	12	9	1.	29	0.31
747	A	13	8	1.	29	0.276
748	A	15	8	1.	27	0.296
749	A	17	4	1.	21	0.19
750	A	18	8	1.	27	0.296
751	A	8	7	1.	27	0.259
752	A	8	7	1.	25	0.28
753	A	5	5	1.	19	0.263
754	A	5	4	1.	23	0.174
755	A	6	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
756	A	7	6	1.	27	0.222
757	A	8	7	1.	27	0.259
758	A	8	7	1.	27	0.259
759	A	12	8	1.	27	0.296
760	A	6	5	1.	21	0.238
761	A	3	2	1.	25	0.08
762	A	9	7	1.	27	0.259
763	A	10	8	1.	29	0.276
764	A	12	7	1.	29	0.241
765	A	11	6	1.	27	0.222
766	A	8	6	1.	21	0.286
767	A	2	2	1.	25	0.08
768	A	4	3	1.	27	0.111
769	A	6	5	1.	29	0.172
770	A	8	6	1.	29	0.207
771	A	10	6	1.	29	0.207
772	A	7	5	1.	29	0.172
773	A	6	4	1.	27	0.148
774	A	5	4	1.	21	0.19
775	A	5	4	1.	25	0.16
776	A	7	5	1.	27	0.185
777	A	8	6	1.	29	0.207
778	A	15	10	1.	29	0.345
779	A	13	8	1.	29	0.276
780	A	12	8	1.	29	0.276
781	A	11	7	1.	27	0.259
782	A	10	5	1.	21	0.238
783	A	4	4	1.	25	0.16
784	A	11	8	1.	27	0.296
785	A	12	8	1.	29	0.276
786	A	15	8	1.	29	0.276
787	A	16	10	1.	29	0.345
788	A	16	9	1.	29	0.31
789	A	14	8	1.	29	0.276
790	A	14	7	1.	27	0.259
791	A	14	5	1.	21	0.238
792	A	5	4	1.	25	0.16
793	A	14	10	1.	27	0.37
794	A	14	10	1.	29	0.345
795	A	9	7	1.	27	0.259
796	A	9	7	1.	25	0.28
797	A	8	5	1.	19	0.263
798	A	6	4	1.	25	0.16
799	A	5	4	1.	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
800	A	5	4	1.	25	0.16
801	A	7	5	1.	25	0.2
802	A	8	6	1.	27	0.222
803	A	9	7	1.	27	0.259
804	A	4	4	1.19	21	0.19
805	A	7	7	1.	27	0.259
806	A	4	4	1.	29	0.138
807	A	4	4	1.	27	0.148
808	A	5	5	1.	27	0.185
809	A	7	7	1.	29	0.241
810	A	8	8	1.	29	0.276
811	A	10	7	1.	21	0.333
812	A	8	6	1.	27	0.222
813	A	5	5	1.	29	0.172
814	A	4	4	1.	27	0.148
815	A	6	4	1.	27	0.148
816	A	8	6	1.	29	0.207
817	A	10	7	1.	29	0.241
818	A	12	7	1.	29	0.241
819	A	13	7	1.	21	0.333
820	A	8	6	1.	29	0.207
821	A	8	5	1.	29	0.172
822	A	8	5	1.	27	0.185
823	A	6	5	1.	21	0.238
824	A	6	5	1.	27	0.185
825	A	7	4	1.	29	0.138
826	A	6	5	1.	27	0.185
827	A	7	5	1.	27	0.185
828	A	8	6	1.	29	0.207
829	A	14	8	1.	29	0.276
830	A	13	8	1.	29	0.276
831	A	11	7	1.	27	0.259
832	A	10	6	1.	21	0.286
833	A	12	6	1.	27	0.222
834	A	12	6	1.	29	0.207
835	A	4	3	1.	27	0.111
836	A	11	8	1.	27	0.296
837	A	12	8	1.	29	0.276
838	A	15	8	1.	29	0.276
839	A	17	9	1.	29	0.31
840	A	14	8	1.	29	0.276
841	A	14	7	1.	27	0.259
842	A	14	5	1.	21	0.238
843	A	15	6	1.	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
844	A	15	6	1.	29	0.207
845	A	5	3	1.	27	0.111
846	A	14	10	1.	27	0.37
847	A	14	10	1.	29	0.345
848	A	17	5	1.	21	0.238
849	A	18	6	1.	27	0.222
850	A	8	4	1.29	29	0.138
851	A	4	3	1.	25	0.12
852	A	3	2	1.	19	0.105
853	A	4	3	1.	25	0.12
854	A	5	4	1.	27	0.148
855	A	5	4	1.	27	0.148
856	A	5	4	1.	25	0.16
857	A	4	3	1.	25	0.12
858	A	4	3	1.	27	0.111
859	A	4	3	1.	27	0.111
860	A	4	3	1.	27	0.111
861	A	3	2	1.	21	0.095
862	A	4	3	1.	27	0.111
863	A	4	3	1.	29	0.103
864	A	5	4	1.	29	0.138
865	A	5	4	1.	27	0.148
866	A	4	3	1.	27	0.111
867	A	4	3	1.	29	0.103
868	A	4	3	1.	29	0.103
869	A	4	3	1.	29	0.103
870	A	3	2	1.	21	0.095
871	A	4	3	1.	27	0.111
872	A	4	3	1.	29	0.103
873	A	4	3	1.	29	0.103
874	A	3	3	1.	27	0.111
875	A	4	3	1.	27	0.111
876	A	4	3	1.	29	0.103
877	A	4	3	1.	29	0.103
878	A	4	3	1.	29	0.103
879	A	4	3	1.	29	0.103
880	A	4	3	1.	29	0.103
881	A	4	3	1.	27	0.111
882	A	8	5	1.	21	0.238
883	A	8	6	1.	27	0.222
884	A	9	6	1.	29	0.207
885	A	9	6	1.	29	0.207
886	A	9	6	1.	29	0.207
887	A	9	6	1.	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
888	A	8	6	1.	27	0.222
889	A	4	3	1.	21	0.143
890	A	4	3	1.	27	0.111
891	A	4	3	1.	29	0.103
892	A	4	3	1.	29	0.103
893	A	4	3	1.	29	0.103
894	A	11	5	1.	29	0.172
895	A	4	3	1.	29	0.103
896	A	4	3	1.	29	0.103
897	A	4	3	1.	27	0.111
898	A	9	5	1.	21	0.238
899	A	9	6	1.	27	0.222
900	A	10	6	1.	29	0.207
901	A	10	6	1.	29	0.207
902	A	11	7	1.	29	0.241
903	A	11	7	1.	29	0.241
904	A	10	6	1.	29	0.207
905	A	10	6	1.	29	0.207
906	A	9	6	1.	27	0.222
907	A	4	3	1.	21	0.143
908	A	4	3	1.	27	0.111
909	A	4	3	1.	29	0.103
910	A	4	3	1.	29	0.103
911	A	5	4	1.	33	0.121
912	A	3	3	1.	31	0.097
913	A	3	2	1.	31	0.065
914	A	3	2	1.	31	0.065
915	A	3	2	1.	31	0.065
916	A	3	2	1.	29	0.069
917	A	2	2	1.	31	0.065
918	A	2	2	1.	31	0.065
919	A	2	2	1.	31	0.065
920	A	3	2	1.	31	0.065
921	A	3	2	1.	31	0.065
922	A	3	2	1.	31	0.065
923	A	3	2	1.	29	0.069
924	A	2	2	1.	31	0.065
925	A	2	2	1.	31	0.065
926	A	2	2	1.	31	0.065
927	A	3	3	1.13	27	0.111
928	A	4	3	1.	27	0.111
929	A	4	3	1.	27	0.111
930	A	4	3	1.	27	0.111
931	A	4	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
932	A	2	2	1.	19	0.105
933	A	3	3	1.	25	0.12
934	A	3	3	1.	27	0.111
935	A	4	4	1.06	29	0.138
936	A	6	6	1.	35	0.171
937	A	6	6	1.	37	0.162
938	A	4	4	1.	33	0.121
939	A	3	3	1.	33	0.091
940	A	3	3	1.	33	0.091
941	A	3	3	1.	31	0.097
942	A	4	4	1.	33	0.121
943	A	3	3	1.	33	0.091
944	A	3	3	1.	33	0.091
945	A	4	4	1.	33	0.121
946	A	3	3	1.	33	0.091
947	A	3	3	1.	31	0.097
948	A	3	3	1.	33	0.091
949	A	4	4	1.	33	0.121
950	A	3	3	1.	33	0.091
951	A	3	3	1.	33	0.091
952	A	3	3	1.	33	0.091
953	A	3	2	1.	29	0.069
954	A	3	2	1.	29	0.069
955	A	3	2	1.	29	0.069
956	A	3	2	1.	27	0.074
957	A	3	2	1.	27	0.074
958	A	4	3	1.	29	0.103
959	A	4	3	1.	29	0.103
960	A	4	3	1.	29	0.103
961	A	6	4	1.	29	0.138
962	A	5	4	1.	29	0.138
963	A	4	4	1.	29	0.138
964	A	2	2	1.	29	0.069
965	A	3	3	1.	29	0.103
966	A	3	2	1.	29	0.069
967	A	3	2	1.	29	0.069
968	A	3	2	1.	29	0.069
969	A	3	2	1.	31	0.065
970	A	3	2	1.	31	0.065
971	A	3	2	1.	31	0.065
972	A	3	2	1.	29	0.069
973	A	3	2	1.	29	0.069
974	A	3	2	1.	31	0.065
975	A	4	3	1.	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
976	A	4	3	1.	31	0.097
977	A	7	5	1.	31	0.161
978	A	6	5	1.	31	0.161
979	A	5	5	1.	31	0.161
980	A	3	2	1.	31	0.065
981	A	4	4	1.	31	0.129
982	A	4	3	1.	31	0.097
983	A	4	3	1.	31	0.097
984	A	4	3	1.	31	0.097
985	A	4	3	1.	31	0.097
986	A	3	2	1.	31	0.065
987	A	3	2	1.	31	0.065
988	A	3	2	1.	31	0.065
989	A	3	2	1.	29	0.069
990	A	3	2	1.	29	0.069
991	A	3	2	1.	31	0.065
992	A	2	2	1.	31	0.065
993	A	4	3	1.	31	0.097
994	A	4	3	1.	31	0.097
995	A	8	5	1.	31	0.161
996	A	7	5	1.	31	0.161
997	A	6	5	1.	31	0.161
998	A	2	2	1.	31	0.065
999	A	4	4	1.	31	0.129
1000	A	4	4	1.	31	0.129
1001	A	4	3	1.	31	0.097
1002	A	4	3	1.	31	0.097
1003	A	3	2	1.	31	0.065
1004	A	3	2	1.	31	0.065
1005	A	3	2	1.	31	0.065
1006	A	3	2	1.	29	0.069
1007	A	4	3	1.	29	0.103
1008	A	4	3	1.	31	0.097
1009	A	4	3	1.	31	0.097
1010	A	4	3	1.	31	0.097
1011	A	3	2	1.	31	0.065
1012	A	3	2	1.	31	0.065
1013	A	3	2	1.	31	0.065
1014	A	3	2	1.	29	0.069
1015	A	4	3	1.	29	0.103
1016	A	4	3	1.	31	0.097
1017	A	4	3	1.	31	0.097
1018	A	4	3	1.	31	0.097
1019	A	4	4	1.	33	0.121

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1020	A	3	2	1.	31	0.065
1021	A	3	2	1.	31	0.065
1022	A	3	2	1.	31	0.065
1023	A	3	2	1.	29	0.069
1024	A	3	3	1.	29	0.103
1025	A	3	3	1.	31	0.097
1026	A	3	3	1.	31	0.097
1027	A	4	4	1.	31	0.129
1028	A	4	4	1.	31	0.129
1029	A	4	4	1.	31	0.129
1030	A	4	4	1.	31	0.129
1031	A	4	4	1.	31	0.129
1032	A	4	4	1.	31	0.129
1033	A	4	3	1.	38	0.079
1034	A	3	3	1.	38	0.079
1035	A	2	2	1.	38	0.053
1036	A	4	4	1.	38	0.105
1037	A	4	4	1.	36	0.111
1038	A	4	4	1.	38	0.105
1039	A	4	4	1.	38	0.105
1040	A	1	1	1.	40	0.025
1041	A	1	1	1.	40	0.025
1042	A	4	4	1.	35	0.114
1043	A	3	3	1.03	35	0.086
1044	A	3	3	1.04	33	0.091
1045	A	3	3	1.04	35	0.086
1046	A	3	3	1.03	35	0.086
1047	A	3	3	1.03	35	0.086
1048	A	3	3	1.03	35	0.086
1049	A	5	5	1.	35	0.143
1050	A	8	6	1.	27	0.222
1051	A	7	6	1.	27	0.222
1052	A	6	6	1.	25	0.24
1053	A	6	6	1.	23	0.261
1054	A	7	6	1.	19	0.316
1055	A	5	5	1.	25	0.2
1056	A	5	5	1.	27	0.185
1057	A	6	6	1.	27	0.222
1058	A	7	6	1.	27	0.222
1059	A	10	8	1.	29	0.276
1060	A	9	8	1.	29	0.276
1061	A	5	4	1.	27	0.148
1062	A	6	6	1.	25	0.24
1063	A	9	7	1.	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1064	A	6	6	1.	27	0.222
1065	A	6	6	1.	29	0.207
1066	A	8	8	1.	29	0.276
1067	A	9	9	1.	29	0.31
1068	A	9	8	1.	29	0.276
1069	A	10	9	1.	29	0.31
1070	A	6	4	1.	27	0.148
1071	A	7	7	1.	25	0.28
1072	A	11	9	1.	21	0.429
1073	A	7	7	1.	27	0.259
1074	A	7	7	1.	29	0.241
1075	A	7	7	1.	29	0.241
1076	A	9	9	1.	29	0.31
1077	A	10	10	1.	29	0.345
1078	A	9	9	1.	29	0.31
1079	A	8	8	1.	29	0.276
1080	A	5	5	1.	27	0.185
1081	A	8	8	1.	25	0.32
1082	A	8	7	1.	21	0.333
1083	A	9	8	1.	27	0.296
1084	A	10	8	1.	29	0.276
1085	A	9	8	1.	29	0.276
1086	A	8	8	1.	29	0.276
1087	A	6	6	1.	27	0.222
1088	A	8	7	1.	25	0.28
1089	A	9	8	1.	21	0.381
1090	A	10	8	1.	27	0.296
1091	A	5	5	1.	35	0.143
1092	A	9	6	1.	27	0.222
1093	A	9	6	1.	27	0.222
1094	A	8	6	1.	27	0.222
1095	A	7	6	1.	25	0.24
1096	A	8	6	1.	25	0.24
1097	A	9	8	1.	27	0.296
1098	A	9	7	1.	25	0.28
1099	A	9	7	1.	19	0.368
1100	A	7	5	1.	25	0.2
1101	A	6	5	1.	27	0.185
1102	A	7	6	1.	27	0.222
1103	A	8	6	1.	27	0.222
1104	A	9	6	1.	27	0.222
1105	A	10	8	1.	29	0.276
1106	A	9	8	1.	29	0.276
1107	A	6	4	1.	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1108	A	6	6	1.64	27	0.222
1109	A	6	6	1.	29	0.207
1110	A	6	6	1.	27	0.222
1111	A	13	9	1.	21	0.429
1112	A	6	6	1.	27	0.222
1113	A	6	6	1.	29	0.207
1114	A	8	8	1.	29	0.276
1115	A	9	9	1.	29	0.31
1116	A	10	8	1.	29	0.276
1117	A	7	4	1.	27	0.148
1118	A	7	6	1.	27	0.222
1119	A	7	6	1.	29	0.207
1120	A	7	6	1.	27	0.222
1121	A	17	10	1.	21	0.476
1122	A	7	6	1.	27	0.222
1123	A	7	6	1.	29	0.207
1124	A	7	6	1.	29	0.207
1125	A	9	8	1.	29	0.276
1126	A	10	9	1.	29	0.31
1127	A	9	7	1.	29	0.241
1128	A	8	7	1.	29	0.241
1129	A	6	6	1.	27	0.222
1130	A	6	6	1.	27	0.222
1131	A	6	6	1.	29	0.207
1132	A	7	7	1.14	27	0.259
1133	A	8	7	1.	21	0.333
1134	A	9	7	1.	27	0.259
1135	A	9	7	1.	29	0.241
1136	A	8	7	1.	29	0.241
1137	A	6	5	1.	27	0.185
1138	A	6	6	1.	27	0.222
1139	A	7	7	1.	29	0.241
1140	A	8	7	1.	27	0.259
1141	A	9	7	1.	21	0.333
1142	A	10	7	1.	27	0.259
1143	A	10	9	1.	31	0.29
1144	A	8	7	1.	29	0.241
1145	A	10	10	1.	29	0.345
1146	A	10	10	1.	31	0.323
1147	A	10	10	1.	29	0.345
1148	A	10	10	1.	23	0.435
1149	A	11	11	1.	29	0.379
1150	A	12	11	1.	31	0.355
1151	A	11	9	1.	31	0.29

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1152	A	9	7	1.	29	0.241
1153	A	11	10	1.	29	0.345
1154	A	11	10	1.	31	0.323
1155	A	11	10	1.	29	0.345
1156	A	11	11	1.	23	0.478
1157	A	11	10	1.	29	0.345
1158	A	12	11	1.	31	0.355
1159	A	13	11	1.	31	0.355
1160	A	10	7	1.	29	0.241
1161	A	12	10	1.	29	0.345
1162	A	12	10	1.	31	0.323
1163	A	12	10	1.	29	0.345
1164	A	12	11	1.	23	0.478
1165	A	12	11	1.	29	0.379
1166	A	12	10	1.	31	0.323
1167	A	13	11	1.	31	0.355
1168	A	10	8	1.	31	0.258
1169	A	9	8	1.	31	0.258
1170	A	7	6	1.	29	0.207
1171	A	9	9	1.	29	0.31
1172	A	9	9	1.	31	0.29
1173	A	9	9	1.	29	0.31
1174	A	10	10	1.	23	0.435
1175	A	11	10	1.	29	0.345
1176	A	10	8	1.	31	0.258
1177	A	9	8	1.	31	0.258
1178	A	7	7	1.	29	0.241
1179	A	9	9	1.	29	0.31
1180	A	9	9	1.	31	0.29
1181	A	10	10	1.	29	0.345
1182	A	11	10	1.	23	0.435
1183	A	10	8	1.	31	0.258
1184	A	9	8	1.	31	0.258
1185	A	7	6	1.	29	0.207
1186	A	9	9	1.	29	0.31
1187	A	10	10	1.	31	0.323
1188	A	11	10	1.	29	0.345
1189	A	12	10	1.	23	0.435
1190	A	8	7	1.	35	0.2
1191	A	0	0	0.	0	0.
1192	A	0	0	0.	0	0.
1193	A	0	0	0.	0	0.
1194	A	0	0	0.	0	0.
1195	A	8	6	1.	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1196	A	7	6	1.	29	0.207
1197	A	3	2	1.	27	0.074
1198	A	4	3	1.	27	0.111
1199	A	4	3	1.	27	0.111
1200	A	7	5	1.	27	0.185
1201	A	7	5	1.	27	0.185
1202	A	6	5	1.	25	0.2
1203	A	4	3	1.	25	0.12
1204	A	4	3	1.	27	0.111
1205	A	4	3	1.	27	0.111
1206	A	4	3	1.	25	0.12
1207	A	3	2	1.	19	0.105
1208	A	4	3	1.	25	0.12
1209	A	6	5	1.	27	0.185
1210	A	6	5	1.	27	0.185
1211	A	7	5	1.	27	0.185
1212	A	7	5	1.	27	0.185
1213	A	4	3	1.	27	0.111
1214	A	4	3	1.	27	0.111
1215	A	4	3	1.	29	0.103
1216	A	4	3	1.	27	0.111
1217	A	4	3	1.	27	0.111
1218	A	4	3	1.	29	0.103
1219	A	4	3	1.	29	0.103
1220	A	4	3	1.	27	0.111
1221	A	3	2	1.	21	0.095
1222	A	4	3	1.	27	0.111
1223	A	4	3	1.	29	0.103
1224	A	4	3	1.	29	0.103
1225	A	4	3	1.	29	0.103
1226	A	4	3	1.	29	0.103
1227	A	4	3	1.	29	0.103
1228	A	4	3	1.	27	0.111
1229	A	4	3	1.	27	0.111
1230	A	4	3	1.	29	0.103
1231	A	4	3	1.	29	0.103
1232	A	4	3	1.	27	0.111
1233	A	3	2	1.	21	0.095
1234	A	4	3	1.	27	0.111
1235	A	3	2	1.	29	0.069
1236	A	3	2	1.	27	0.074
1237	A	5	4	1.	29	0.138
1238	A	5	4	1.	29	0.138
1239	A	12	7	1.	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1240	A	12	9	1.	29	0.31
1241	A	11	7	1.	29	0.241
1242	A	11	8	1.	29	0.276
1243	A	7	4	1.	27	0.148
1244	A	9	5	1.	27	0.185
1245	A	12	8	1.	29	0.276
1246	A	11	8	1.	29	0.276
1247	A	12	9	1.	29	0.31
1248	A	12	9	1.	27	0.333
1249	A	16	10	1.	21	0.476
1250	A	11	9	1.	27	0.333
1251	A	9	5	1.	29	0.172
1252	A	9	9	1.	29	0.31
1253	A	9	5	1.	29	0.172
1254	A	11	10	1.	29	0.345
1255	A	10	5	1.	29	0.172
1256	A	11	8	1.	29	0.276
1257	A	10	8	1.	29	0.276
1258	A	7	6	1.	27	0.222
1259	A	16	11	1.	27	0.407
1260	A	16	11	1.	29	0.379
1261	A	16	11	1.	29	0.379
1262	A	17	10	1.	29	0.345
1263	A	9	7	1.	27	0.259
1264	A	10	7	1.	21	0.333
1265	A	11	7	1.	27	0.259
1266	A	11	8	1.	29	0.276
1267	A	10	8	1.	29	0.276
1268	A	7	6	1.	27	0.222
1269	A	20	11	1.	27	0.407
1270	A	20	11	1.	29	0.379
1271	A	21	11	1.	29	0.379
1272	A	9	7	1.	29	0.241
1273	A	10	7	1.	27	0.259
1274	A	11	7	1.	21	0.333
1275	A	13	7	1.	29	0.241
1276	A	8	7	1.	35	0.2
1277	A	9	9	1.01	37	0.243
1278	A	6	6	1.	37	0.162
1279	A	4	3	1.	27	0.111
1280	A	4	3	1.	27	0.111
1281	A	4	3	1.	25	0.12
1282	A	4	4	1.	19	0.21
1283	A	4	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1284	A	4	3	1.	27	0.111
1285	A	10	8	1.	29	0.276
1286	A	9	8	1.	29	0.276
1287	A	8	8	1.	29	0.276
1288	A	5	5	1.	27	0.185
1289	A	6	6	1.	25	0.24
1290	A	7	7	1.	21	0.333
1291	A	8	8	1.	27	0.296
1292	A	9	8	1.	29	0.276
1293	A	10	8	1.	29	0.276
1294	A	11	8	1.	29	0.276
1295	A	4	3	1.	29	0.103
1296	A	4	3	1.	29	0.103
1297	A	4	3	1.	27	0.111
1298	A	4	3	1.	27	0.111
1299	A	4	3	1.	27	0.111
1300	A	3	2	1.	21	0.095
1301	A	9	7	1.	29	0.241
1302	A	8	7	1.	29	0.241
1303	A	6	5	1.	27	0.185
1304	A	6	6	1.	27	0.222
1305	A	6	6	1.	29	0.207
1306	A	6	6	1.	27	0.222
1307	A	7	7	1.	21	0.333
1308	A	8	7	1.	27	0.259
1309	A	9	7	1.	29	0.241
1310	A	4	3	1.	29	0.103
1311	A	4	3	1.	29	0.103
1312	A	4	3	1.	27	0.111
1313	A	4	3	1.	27	0.111
1314	A	4	3	1.	29	0.103
1315	A	4	3	1.	29	0.103
1316	A	4	3	1.	27	0.111
1317	A	3	2	1.	21	0.095
1318	A	4	3	1.	27	0.111
1319	A	4	3	1.	29	0.103
1320	A	11	7	1.	29	0.241
1321	A	10	7	1.	29	0.241
1322	A	7	5	1.	27	0.185
1323	A	14	9	1.	27	0.333
1324	A	13	10	1.	29	0.345
1325	A	6	6	1.	29	0.207
1326	A	13	9	1.	29	0.31
1327	A	15	8	1.41	27	0.296

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1328	A	9	7	1.27	21	0.333
1329	A	10	7	1.	27	0.259
1330	A	11	7	1.	29	0.241
1331	A	12	7	1.	29	0.241
1332	A	4	3	1.	27	0.111
1333	A	4	3	1.	25	0.12
1334	A	3	2	1.	19	0.105
1335	A	4	3	1.	25	0.12
1336	A	4	3	1.	27	0.111
1337	A	4	3	1.	27	0.111
1338	A	14	13	1.	29	0.448
1339	A	12	11	1.	29	0.379
1340	A	9	8	1.	27	0.296
1341	A	8	7	1.	21	0.333
1342	A	5	5	1.	25	0.2
1343	A	10	9	1.	27	0.333
1344	A	13	11	1.17	29	0.379
1345	A	17	12	1.17	29	0.414
1346	A	4	3	1.	21	0.143
1347	A	5	4	1.	27	0.148
1348	A	5	4	1.	27	0.148
1349	A	4	3	1.	27	0.111
1350	A	4	3	1.	29	0.103
1351	A	4	3	1.	29	0.103
1352	A	13	9	1.	21	0.429
1353	A	10	9	1.	27	0.333
1354	A	10	9	1.	29	0.31
1355	A	6	5	1.	27	0.185
1356	A	12	10	1.	27	0.37
1357	A	15	12	1.12	29	0.414
1358	A	20	13	1.	29	0.448
1359	A	6	4	1.	29	0.138
1360	A	6	4	1.	29	0.138
1361	A	6	4	1.	27	0.148
1362	A	5	3	1.	21	0.143
1363	A	6	4	1.	27	0.148
1364	A	6	5	1.	29	0.172
1365	A	6	5	1.	29	0.172
1366	A	6	4	1.	27	0.148
1367	A	4	3	1.	27	0.111
1368	A	4	3	1.	29	0.103
1369	A	4	3	1.	29	0.103
1370	A	21	14	1.	33	0.424
1371	A	18	13	1.	33	0.394

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1372	A	15	12	1.	33	0.364
1373	A	12	10	1.	31	0.323
1374	A	16	11	1.	31	0.355
1375	A	19	14	1.	33	0.424
1376	A	25	15	1.	33	0.454
1377	A	24	16	1.	33	0.485
1378	A	20	15	1.	33	0.454
1379	A	13	11	1.	31	0.355
1380	A	21	16	1.	31	0.516
1381	A	24	17	1.	33	0.515
1382	A	30	18	1.	33	0.546
1383	A	24	16	1.	33	0.485
1384	A	20	15	1.	33	0.454
1385	A	13	11	1.	31	0.355
1386	A	21	16	1.	31	0.516
1387	A	24	17	1.	33	0.515
1388	A	30	18	1.	33	0.546
1389	A	23	15	1.	33	0.454
1390	A	19	14	1.	33	0.424
1391	A	15	13	1.	33	0.394
1392	A	12	10	1.	31	0.323
1393	A	16	11	1.	31	0.355
1394	A	19	14	1.	33	0.424
1395	A	25	15	1.	33	0.454
1396	A	22	15	1.	33	0.454
1397	A	18	14	1.	33	0.424
1398	A	15	13	1.	33	0.394
1399	A	13	11	1.	31	0.355
1400	A	21	16	1.	31	0.516
1401	A	25	18	1.	33	0.546
1402	A	22	16	1.	33	0.485
1403	A	18	14	1.	33	0.424
1404	A	15	13	1.	33	0.394
1405	A	13	11	1.	31	0.355
1406	A	21	16	1.	31	0.516
1407	A	25	18	1.	33	0.546
1408	A	31	15	1.	37	0.405
1409	A	19	14	1.	37	0.378
1410	A	16	12	1.	37	0.324
1411	A	5	4	1.	37	0.108
1412	A	9	8	1.	37	0.216
1413	A	11	9	1.	37	0.243
1414	A	16	9	1.	37	0.243
1415	A	19	9	1.	37	0.243

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1416	A	31	16	1.	37	0.432
1417	A	19	15	1.	37	0.405
1418	A	18	14	1.	37	0.378
1419	A	8	7	1.	37	0.189
1420	A	12	9	1.	37	0.243
1421	A	15	9	1.	37	0.243
1422	A	20	9	1.	37	0.243
1423	A	31	17	1.	37	0.46
1424	A	19	15	1.	37	0.405
1425	A	20	16	1.	37	0.432
1426	A	10	9	1.	37	0.243
1427	A	15	10	1.	37	0.27
1428	A	18	10	1.	37	0.27
1429	A	24	10	1.	37	0.27
1430	A	19	14	1.	37	0.378
1431	A	15	11	1.	37	0.297
1432	A	4	3	1.	37	0.081
1433	A	7	6	1.	37	0.162
1434	A	9	7	1.	37	0.189
1435	A	13	8	1.	37	0.216
1436	A	31	17	1.	37	0.46
1437	A	10	9	1.	37	0.243
1438	A	11	10	1.	37	0.27
1439	A	11	10	1.	37	0.27
1440	A	16	12	1.	37	0.324
1441	A	19	12	1.	37	0.324
1442	A	8	7	1.	37	0.189
1443	A	8	7	1.	27	0.259
1444	A	8	7	1.	25	0.28
1445	A	7	5	1.	19	0.263
1446	A	5	4	1.	23	0.174
1447	A	6	6	1.	25	0.24
1448	A	7	6	1.	27	0.222
1449	A	8	7	1.	27	0.259
1450	A	8	7	1.	27	0.259
1451	A	11	9	1.	21	0.429
1452	A	4	3	1.	25	0.12
1453	A	8	8	1.52	27	0.296
1454	A	7	6	1.	29	0.207
1455	A	10	9	1.24	29	0.31
1456	A	8	7	1.	29	0.241
1457	A	17	8	1.	27	0.296
1458	A	14	10	1.	21	0.476
1459	A	3	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1460	A	11	8	1.	27	0.296
1461	A	12	9	1.	29	0.31
1462	A	14	9	1.	29	0.31
1463	A	15	8	1.	29	0.276
1464	A	12	7	1.	29	0.241
1465	A	12	7	1.	27	0.259
1466	A	12	7	1.	21	0.333
1467	A	6	6	1.	25	0.24
1468	A	13	8	1.	27	0.296
1469	A	15	10	1.	29	0.345
1470	A	17	11	1.	29	0.379
1471	A	18	8	1.	29	0.276
1472	A	18	8	1.	27	0.296
1473	A	18	8	1.	21	0.381
1474	A	7	6	1.	25	0.24
1475	A	19	9	1.	27	0.333
1476	A	21	11	1.	29	0.379
1477	A	23	12	1.	29	0.414
1478	A	2	2	1.	35	0.057
1479	F	0	0	N/A	0	N/A
1480	F	0	0	N/A	0	N/A
1481	A	11	8	1.	27	0.296
1482	A	10	8	1.	25	0.32
1483	A	7	5	1.	19	0.263
1484	A	7	5	1.	25	0.2
1485	A	6	5	1.	27	0.185
1486	A	6	6	1.	27	0.222
1487	A	6	6	1.	25	0.24
1488	A	8	6	1.	25	0.24
1489	A	10	8	1.	27	0.296
1490	A	10	8	1.	27	0.296
1491	A	11	8	1.	27	0.296
1492	A	9	6	1.	27	0.222
1493	A	8	5	1.	21	0.238
1494	A	9	6	1.	27	0.222
1495	A	7	5	1.	29	0.172
1496	A	5	5	1.	29	0.172
1497	A	6	5	1.	27	0.185
1498	A	6	5	1.	27	0.185
1499	A	6	4	1.	29	0.138
1500	A	6	4	1.	29	0.138
1501	A	8	5	1.	21	0.238
1502	A	9	6	1.	27	0.222
1503	A	8	6	1.	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1504	A	7	6	1.	29	0.207
1505	A	5	5	1.	27	0.185
1506	A	6	5	1.	27	0.185
1507	A	6	4	1.	29	0.138
1508	A	6	4	1.	29	0.138
1509	A	6	4	1.	29	0.138
1510	A	5	4	1.	29	0.138
1511	A	5	4	1.	29	0.138
1512	A	4	3	1.	27	0.111
1513	A	10	3	1.	29	0.103
1514	A	17	5	1.	29	0.172
1515	F	0	0	N/A	0	N/A
1516	A	11	8	1.	35	0.229
1517	A	10	7	1.	33	0.212
1518	A	2	2	1.	23	0.087
1519	A	0	0	0.	0	0.
1520	A	0	0	0.	0	0.
1521	A	0	0	0.	0	0.
1522	A	0	0	0.	0	0.
1523	A	11	8	1.	33	0.242
1524	A	10	7	0.99	31	0.226
1525	A	2	2	1.	21	0.095
1526	A	0	0	0.	0	0.
1527	A	0	0	0.	0	0.
1528	A	3	2	1.	29	0.069
1529	A	3	2	1.	29	0.069
1530	A	3	2	1.	29	0.069
1531	A	3	2	1.	27	0.074
1532	A	5	4	1.	27	0.148
1533	A	3	3	1.	29	0.103
1534	A	4	4	1.	29	0.138
1535	A	5	4	1.	29	0.138
1536	A	3	2	1.	31	0.065
1537	A	3	2	1.	31	0.065
1538	A	3	2	1.	31	0.065
1539	A	3	2	1.	29	0.069
1540	A	6	4	1.	29	0.138
1541	A	5	4	1.	31	0.129
1542	A	4	4	1.	31	0.129
1543	A	5	5	1.	31	0.161
1544	A	3	2	1.	31	0.065
1545	A	3	2	1.	31	0.065
1546	A	3	2	1.	31	0.065
1547	A	3	2	1.	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1548	A	3	2	1.	29	0.069
1549	A	4	3	1.	31	0.097
1550	A	5	3	1.	31	0.097
1551	A	6	3	1.	31	0.097
1552	A	3	2	1.	31	0.065
1553	A	3	2	1.	31	0.065
1554	A	3	2	1.	31	0.065
1555	A	3	2	1.	29	0.069
1556	A	3	2	1.	29	0.069
1557	A	4	3	1.	31	0.097
1558	A	5	3	1.	31	0.097
1559	A	6	3	1.	31	0.097
1560	A	0	0	0.	0	0.
1561	A	4	2	1.	35	0.057
1562	A	5	2	1.	35	0.057
1563	A	5	3	1.	35	0.086

Chapter 3

Listing of integrals

3.1 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{6cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(15*c*f*Sqrt[a + a*Sin[e + f*x
]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2))/(6
*c*f)
```

Rubi [A] time = 0.397736, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{6cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(15*c*f*Sqrt[a + a*Sin[e + f*x
]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2))/(6
*c*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
```

```
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}{6cf} + \frac{\int \sqrt{\ldots}}{\ldots}$$
$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{a+a\sin\ldots}}{\ldots}$$

Mathematica [A] time = 0.571301, size = 104, normalized size = 1.13

$$\frac{c^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(1080\sin(e+fx)+20\sin(3(e+fx))-36\sin(5(e+fx))+405\cos(\ldots}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(405*
Cos[2*(e + f*x)] + 90*Cos[4*(e + f*x)] - 5*Cos[6*(e + f*x)] + 1080*Sin[e +
f*x] + 20*Sin[3*(e + f*x)] - 36*Sin[5*(e + f*x)]))/(960*f)
```

Maple [A] time = 0.274, size = 133, normalized size = 1.5

$$\frac{\sin(fx+e)\left(5\left(\cos(fx+e)\right)^8+3\left(\cos(fx+e)\right)^6\sin(fx+e)+4\left(\cos(fx+e)\right)^6+7\sin(fx+e)\left(\cos(fx+e)\right)^4+7\ldots}{30f\left(\cos(fx+e)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/30/f*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(5*co
s(f*x+e)^8+3*cos(f*x+e)^6*sin(f*x+e)+4*cos(f*x+e)^6+7*sin(f*x+e)*cos(f*x+e)
^4+7*cos(f*x+e)^2*sin(f*x+e)-7*cos(f*x+e)^2+28*sin(f*x+e)+28)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(fx+e)+a}\left(-c\sin(fx+e)+c\right)^{\frac{7}{2}}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)
^2, x)
```

Fricas [A] time = 1.78385, size = 274, normalized size = 2.98

$$-\frac{\left(5\,c^3\cos\left(fx+e\right)^6-30\,c^3\cos\left(fx+e\right)^4+25\,c^3+2\left(9\,c^3\cos\left(fx+e\right)^4-8\,c^3\cos\left(fx+e\right)^2-16\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{a}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(5*c^3*cos(f*x + e)^6 - 30*c^3*cos(f*x + e)^4 + 25*c^3 + 2*(9*c^3*cos
(f*x + e)^4 - 8*c^3*cos(f*x + e)^2 - 16*c^3)*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.2 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(10*c*f*Sqrt[a + a*Sin[e + f*x
]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))/(5
*c*f)
```

Rubi [A] time = 0.390901, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2),x]
```


```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(10*c*f*Sqrt[a + a*Sin[e + f*x
]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))/(5
*c*f)
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{5cf} + \frac{2}{}$$
$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{a+a}}{}$$

Mathematica [A] time = 0.48096, size = 94, normalized size = 1.02

$$\frac{c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(70\sin(e+fx)+5\sin(3(e+fx))-\sin(5(e+fx))+20\cos(2(e+}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(20*C
os[2*(e + f*x)] + 5*Cos[4*(e + f*x)] + 70*Sin[e + f*x] + 5*Sin[3*(e + f*x)]
 - Sin[5*(e + f*x)]))/(80*f)
```

Maple [A] time = 0.251, size = 106, normalized size = 1.2

$$\frac{\sin(fx+e)\left(2\left(\cos(fx+e)\right)^6+\sin(fx+e)\left(\cos(fx+e)\right)^4+2\left(\cos(fx+e)\right)^4+3\left(\cos(fx+e)\right)^2\sin(fx+e)+}{10f\left(\cos(fx+e)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/10/f*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(2*co
s(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)
+6*sin(f*x+e)+6)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(fx+e)+a}\left(-c\sin(fx+e)+c\right)^{\frac{5}{2}}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)
^2, x)
```

Fricas [A] time = 1.70249, size = 235, normalized size = 2.55

$$\frac{\left(5\,c^{2}\cos\left(fx+e\right)^{4}-5\,c^{2}-2\left(c^{2}\cos\left(fx+e\right)^{4}-2\,c^{2}\cos\left(fx+e\right)^{2}-4\,c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/10*(5*c^2*cos(f*x + e)^4 - 5*c^2 - 2*(c^2*cos(f*x + e)^4 - 2*c^2*cos(f*x
+ e)^2 - 4*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.3 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*c*f*Sqrt[a + a*Sin[e + f*x]
]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))/(4*
c*f)
```

Rubi [A] time = 0.390051, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*c*f*Sqrt[a + a*Sin[e + f*x]
]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))/(4*
c*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{4cf} + \frac{\int \sqrt{}}{}$$
$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{a+a\sin}}{}$$

Mathematica [A] time = 0.403659, size = 83, normalized size = 0.9

$$\frac{c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(8(9\sin(e+fx)+\sin(3(e+fx)))+12\cos(2(e+fx))+3\cos(4(e+f}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(12*Cos
[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3*(e + f*x)]))
)/(96*f)
```

Maple [A] time = 0.243, size = 90, normalized size = 1.

$$\frac{\sin(fx+e)\left(3\left(\cos(fx+e)\right)^4+\left(\cos(fx+e)\right)^2\sin(fx+e)+4\left(\cos(fx+e)\right)^2+5\sin(fx+e)+5\right)}{12f\left(\cos(fx+e)\right)^3}\left(-c\left(-1+\sin\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/f*(-c*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(3*co
s(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)+4*cos(f*x+e)^2+5*sin(f*x+e)+5)/cos(f*x+e
)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(fx+e)+a}\left(-c\sin(fx+e)+c\right)^{\frac{3}{2}}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)
^2, x)
```

Fricas [A] time = 1.68568, size = 193, normalized size = 2.1

$$\frac{\left(3\,c\cos\left(fx+e\right)^{4}+4\left(c\cos\left(fx+e\right)^{2}+2\,c\right)\sin\left(fx+e\right)-3\,c\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/12*(3*c*cos(f*x + e)^4 + 4*(c*cos(f*x + e)^2 + 2*c)*sin(f*x + e) - 3*c)*s
qrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.4 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*c*f*Sqrt[a + a*Sin[e + f*x]
]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(3*
c*f)
```

Rubi [A] time = 0.371708, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*c*f*Sqrt[a + a*Sin[e + f*x]
]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(3*
c*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}{3cf} + \frac{2\int}{}$$
$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{a+a\,\mathrm{si}}}{}$$

Mathematica [A] time = 0.169298, size = 59, normalized size = 0.64

$$\frac{(9\sin(e+fx)+\sin(3(e+fx)))\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(9*Sin[e
+ f*x] + Sin[3*(e + f*x)]))/(12*f)
```

Maple [A] time = 0.241, size = 55, normalized size = 0.6

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{2}+2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^2+2)*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+
e)))^(1/2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
, x)
```

Fricas [A] time = 1.72177, size = 144, normalized size = 1.57

$$\frac{\left(\cos\left(fx+e\right)^2+2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*(cos(f*x + e)^2 + 2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x
)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
, x)
```

3.5 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.283645, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.307567, size = 62, normalized size = 1.38

$$-\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(\cos(2(e+fx))-4\sin(e+fx))}{4cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -(Sec[e + f*x]*(Cos[2*(e + f*x)] - 4*Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x]
)]*Sqrt[c - c*Sin[e + f*x]])/(4*c*f)
```

Maple [B] time = 0.236, size = 94, normalized size = 2.1

$$\frac{\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}+\sin\left(fx+e\right)+2\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}{2f\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\frac{}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2+sin(f*x+e)+2*cos(f*x+e)-1)*sin(f*
x+e)*(a*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(-1+sin(f*x+e))
)^(1/2)
```

Maxima [B] time = 1.73736, size = 522, normalized size = 11.6

$$-\frac{\frac{2\sqrt{a}\sqrt{c}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}{c+\frac{2c\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{c\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}-\frac{2\sqrt{a}\sqrt{c}-\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}{c+\frac{2c\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{c\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}+\frac{2\left(\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+\right.}{\left(\cos\left(fx+e\right)+1\right.}\right.}{c+\frac{2c\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{c}{\left(\cos\right.}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*((2*sqrt(a)*sqrt(c) + sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1)
+ 3*sqrt(a)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3)/(c + 2*c*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - (2*sqrt(a)*sqrt(c) - sqrt(
a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(a)*sqrt(c)*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - sqrt(a)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
)/(c + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + c*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4) + 2*(sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(a)
*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3)/(c + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + c*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Fricas [A] time = 1.60429, size = 154, normalized size = 3.42

$$-\frac{\left(\cos\left(fx+e\right)^{2}-2\sin\left(fx+e\right)-1\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/2*(cos(f*x + e)^2 - 2*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c
*sin(f*x + e) + c)/(c*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\cos^2\left(e+fx\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x) -
 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\cos\left(fx+e\right)^2}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) + c)
, x)
```

3.6 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-2*a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[
c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.425, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{(2a\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{(2a\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin\ldots\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+\ldots}}$$

$$= -\frac{2a\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 1.07287, size = 115, normalized size = 1.16

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(4\log\left(i-e^{i(e+fx)}\right)+\sin(e+fx)-2ifx\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*((-2*
I)*f*x + 4*Log[I - E^(I*(e + f*x))] + Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(3/2)))
```

Maple [A] time = 0.243, size = 138, normalized size = 1.4

$$\frac{\sin(fx+e)\cos(fx+e)-(\cos(fx+e))^2-2\sin(fx+e)-\cos(fx+e)+2}{f\left(1-\cos(fx+e)+\sin(fx+e)\right)}\left(\sin(fx+e)+4\ln\left(-\frac{-1+\cos(fx+\ldots}{\sin(\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(sin(f*x+e)+4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*ln(2/(cos(f*
x+e)+1)))*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a
```

```
*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(3/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^{2}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\cos^{2}\left(e+fx\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cos(e + f*x)**2/(-c*(sin(e + f*x) - 1))
**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(3/
2), x)
```

3.7 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f*x])^(3/2)) +
(a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.430997, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f*x])^(3/2)) +
(a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac} \\
&= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2} \\
&= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{(a\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{c\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{(a\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e\right.}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}} \\
&= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{a\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.837657, size = 104, normalized size = 1.07

$$\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(2\log\left(i-e^{i(e+fx)}\right)+\left(ifx-2\log\left(i-e^{i(e+fx)}\right)\right)\sin(e+fx)-ifx+2\right)}{c^2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(2 - I*f*x + 2*Log[I - E^(I*(e + f
*x))] + (I*f*x - 2*Log[I - E^(I*(e + f*x))])*Sin[e + f*x]))/(c^2*f*Sqrt[c -
 c*Sin[e + f*x]])
```

Maple [B] time = 0.247, size = 194, normalized size = 2.

$$-\frac{\sin(fx+e)\cos(fx+e)-(\cos(fx+e))^2-2\sin(fx+e)-\cos(fx+e)+2}{f(1-\cos(fx+e)+\sin(fx+e))}\left(\sin(fx+e)\ln\left(2(\cos(fx+e)+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-ln(2/(cos(f*x+e)+1))+2*sin(f*x+e)+2*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e)))*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-co
```

```
s(f*x+e)+2)*(a*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(-1+sin(
f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(5/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^{2}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(5/
2), x)
```

3.8 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4acf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*a*c*f*(c - c*Sin[e + f*x])^(5/
2))
```

Rubi [A] time = 0.318728, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4acf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*a*c*f*(c - c*Sin[e + f*x])^(5/
2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4acf(c-c\sin(e+fx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.387789, size = 90, normalized size = 1.88

$$\frac{\sin(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(c^4*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
))
```

Maple [B] time = 0.224, size = 96, normalized size = 2.

$$-\frac{\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)}{f\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/f*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*sin(f*x
+e)*(a*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(-1+sin(f*x+e)))
^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\sin\left(fx+e\right)+a}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(7/
2), x)
```

Fricas [A] time = 1.70599, size = 200, normalized size = 4.17

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{c^{4}f\cos\left(fx+e\right)^{3}+2\,c^{4}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^{4}f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(c^4*f*cos
(f*x + e)^3 + 2*c^4*f*cos(f*x + e)*sin(f*x + e) - 2*c^4*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(7/
2), x)
```

3.9 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=140

$$-\frac{4a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{105cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{7cf}-\frac{2a\cos(e+fx)\sqrt{a\sin}}{}$$

```
[Out] (-4*a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(105*c*f*Sqrt[a + a*Sin[e
+ f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(9/2))/(21*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(9/2))/(7*c*f)
```

Rubi [A] time = 0.524338, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{4a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{105cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{7cf}-\frac{2a\cos(e+fx)\sqrt{a\sin}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-4*a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(105*c*f*Sqrt[a + a*Sin[e
+ f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(9/2))/(21*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(9/2))/(7*c*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{7cf} +$$
$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}{21cf} -$$
$$= -\frac{4a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{105cf\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a}}{}$$

Mathematica [A] time = 1.24586, size = 166, normalized size = 1.19

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}(4725\sin(e+fx)+665\sin(3(e+fx))+21\sin(5(e+fx))}{6720f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
 f*x]]*(1050*Cos[2*(e + f*x)] + 420*Cos[4*(e + f*x)] + 70*Cos[6*(e + f*x)]
+ 4725*Sin[e + f*x] + 665*Sin[3*(e + f*x)] + 21*Sin[5*(e + f*x)] - 15*Sin[7
*(e + f*x)]))/(6720*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A] time = 0.228, size = 133, normalized size = 1.

$$\frac{\sin\left(fx+e\right)\left(15\left(\cos\left(fx+e\right)\right)^8+5\left(\cos\left(fx+e\right)\right)^6\sin\left(fx+e\right)+16\left(\cos\left(fx+e\right)\right)^6+13\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4\right.}{105\,f\left(\cos\left(fx+e\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/105/f*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(15*
cos(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)+16*cos(f*x+e)^6+13*sin(f*x+e)*cos(f*
x+e)^4+16*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+58*sin(f*x+e)+58)/cos(f*x
+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x +
e)^2, x)
```

Fricas [A] time = 1.8217, size = 292, normalized size = 2.09

$$\frac{\left(35\,ac^3\cos\left(fx+e\right)^6-35\,ac^3-\left(15\,ac^3\cos\left(fx+e\right)^6-24\,ac^3\cos\left(fx+e\right)^4-32\,ac^3\cos\left(fx+e\right)^2-64\,ac^3\right)\sin\left(fx+\right.\right.}{105\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/105*(35*a*c^3*cos(f*x + e)^6 - 35*a*c^3 - (15*a*c^3*cos(f*x + e)^6 - 24*a
*c^3*cos(f*x + e)^4 - 32*a*c^3*cos(f*x + e)^2 - 64*a*c^3)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.10 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6cf}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+}}{1}$$

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*c*f*Sqrt[a + a*Sin[e + f
*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/
2))/(15*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(7/2))/(6*c*f)
```

Rubi [A] time = 0.529182, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6cf}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*c*f*Sqrt[a + a*Sin[e + f
*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/
2))/(15*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(7/2))/(6*c*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{6cf}$$
$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/}}{15cf}$$
$$= -\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15cf\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)}{}$$

Mathematica [A] time = 0.97086, size = 156, normalized size = 1.11

$$\frac{c^2(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}(600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+fx))}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(75*Cos[2*(e + f*x)] + 30*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)] + 600
*Sin[e + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(960*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A] time = 0.214, size = 116, normalized size = 0.8

$$\frac{\sin(fx+e)\left(5\left(\cos(fx+e)\right)^6+\sin(fx+e)\left(\cos(fx+e)\right)^4+6\left(\cos(fx+e)\right)^4+3\left(\cos(fx+e)\right)^2\sin(fx+e)+\right.}{30f\left(\cos(fx+e)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/30/f*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(5*co
s(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)
+8*cos(f*x+e)^2+11*sin(f*x+e)+11)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(fx+e)+a\right)^{\frac{3}{2}}\left(-c\sin(fx+e)+c\right)^{\frac{5}{2}}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

Fricas [A] time = 1.81095, size = 251, normalized size = 1.79

$$\frac{\left(5\,ac^2\cos\left(fx+e\right)^6-5\,ac^2+2\left(3\,ac^2\cos\left(fx+e\right)^4+4\,ac^2\cos\left(fx+e\right)^2+8\,ac^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\,s}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/30*(5*a*c^2*cos(f*x + e)^6 - 5*a*c^2 + 2*(3*a*c^2*cos(f*x + e)^4 + 4*a*c^
2*cos(f*x + e)^2 + 8*a*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*
sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.11 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{2a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5cf}-\frac{a\cos(e+fx)\sqrt{a\sin(}}{}$$

```
[Out] (-2*a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2))/(5*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2))/(5*c*f)
```

Rubi [A] time = 0.520556, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{2a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5cf}-\frac{a\cos(e+fx)\sqrt{a\sin(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2))/(5*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2))/(5*c*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5cf} +$$
$$= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{5cf} -$$
$$= -\frac{2a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15cf\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a}}{}$$

Mathematica [A] time = 0.587113, size = 82, normalized size = 0.59

$$-\frac{c(\sin(e+fx)-1)(150\sin(e+fx)+25\sin(3(e+fx))+3\sin(5(e+fx)))\sec^3(e+fx)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c
- c*Sin[e + f*x]]*(150*Sin[e + f*x] + 25*Sin[3*(e + f*x)] + 3*Sin[5*(e + f*
x)]))/(240*f)
```

Maple [A] time = 0.183, size = 67, normalized size = 0.5

$$\frac{\left(3\left(\cos\left(fx+e\right)\right)^4+4\left(\cos\left(fx+e\right)\right)^2+8\right)\sin\left(fx+e\right)}{15f\left(\cos\left(fx+e\right)\right)^3}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/15/f*(3*cos(f*x+e)^4+4*cos(f*x+e)^2+8)*(-c*(-1+sin(f*x+e)))^(3/2)*sin(f*x
+e)*(a*(1+sin(f*x+e)))^(3/2)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x +
e)^2, x)
```

Fricas [A] time = 1.68888, size = 190, normalized size = 1.36

$$\frac{\left(3\,ac\cos\left(fx+e\right)^{4}+4\,ac\cos\left(fx+e\right)^{2}+8\,ac\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(3*a*c*cos(f*x + e)^4 + 4*a*c*cos(f*x + e)^2 + 8*a*c)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.12 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*a*f*Sqrt[c - c*Sin[e + f*x]]
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(4*a
*f)
```

Rubi [A] time = 0.390201, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*a*f*Sqrt[c - c*Sin[e + f*x]]
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(4*a
*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{4af} + \frac{\int (}{}$$
$$= \frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6af\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx)(a+a\,\mathrm{si}}{}$$

Mathematica [A] time = 0.398599, size = 83, normalized size = 0.9

$$\frac{a\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(8(9\sin(e+fx)+\sin(3(e+fx)))-12\cos(2(e+fx))-3\cos(4(e}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (a*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-12*Co
s[2*(e + f*x)] - 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3*(e + f*x)])
))/(96*f)
```

Maple [A] time = 0.216, size = 90, normalized size = 1.

$$-\frac{\sin(fx+e)\left(-3\left(\cos(fx+e)\right)^4+\left(\cos(fx+e)\right)^2\sin(fx+e)-4\left(\cos(fx+e)\right)^2+5\sin(fx+e)-5\right)}{12f\left(\cos(fx+e)\right)^3}\sqrt{-c\left(-1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(-3*
cos(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^2+5*sin(f*x+e)-5)/cos(f*x
+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(fx+e)+a\right)^{\frac{3}{2}}\sqrt{-c\sin(fx+e)+c}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

Fricas [A] time = 1.6993, size = 194, normalized size = 2.11

$$-\frac{\left(3\,a\cos\left(fx+e\right)^{4}-4\left(a\cos\left(fx+e\right)^{2}+2\,a\right)\sin\left(fx+e\right)-3\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/12*(3*a*cos(f*x + e)^4 - 4*(a*cos(f*x + e)^2 + 2*a)*sin(f*x + e) - 3*a)*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.13 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.310588, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [B] time = 0.546324, size = 111, normalized size = 2.47

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(15\sin(e+fx)-\sin(3(e+fx))-6\cos(2(e+fx)))}{12f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-6*Cos
[2*(e + f*x)] + 15*Sin[e + f*x] - Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.212, size = 141, normalized size = 3.1

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{3}-3\sin\left(fx+e\right)\cos\left(fx+e\right)+2\left(\cos\left(fx+e\right)\right)^{2}-\sin\left(fx+e\right)-4\cos\left(\right.\right.}{3f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3-3*sin(f*x+e)*cos(f*x+e)+2*cos(f
*x+e)^2-sin(f*x+e)-4*cos(f*x+e)+1)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(sin
(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+
e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [A] time = 1.63408, size = 193, normalized size = 4.29

$$-\frac{\left(3\,a\cos\left(fx+e\right)^{2}+\left(a\cos\left(fx+e\right)^{2}-4\,a\right)\sin\left(fx+e\right)-3\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/3*(3*a*cos(f*x + e)^2 + (a*cos(f*x + e)^2 - 4*a)*sin(f*x + e) - 3*a)*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^{\frac{3}{2}}\cos(fx+e)^{2}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

3.14 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-4*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f
*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c
*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.546371, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$-\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-4*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f
*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c
*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} + \cdots$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} + \cdots$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} - \cdots$$

$$= -\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} \cdots$$

Mathematica [A] time = 1.06778, size = 130, normalized size = 0.88

$$-\frac{(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12\sin(e+fx)-\cos(2(e+fx))+32\log\left(\cos\left(\frac{1}{2}(e+fx\cdots\right.\right.\right.}{4cf\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-Cos[
2*(e + f*x)] + 32*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 12*Sin[e + f*x
]))/(4*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]
)
```

Maple [A] time = 0.184, size = 173, normalized size = 1.2

$$-\frac{\sin(fx+e)\cos(fx+e)-(\cos(fx+e))^2-2\sin(fx+e)-\cos(fx+e)+2}{2f\left(\sin(fx+e)\cos(fx+e)+(\cos(fx+e))^2-2\sin(fx+e)+\cos(fx+e)-2\right)}\left(-(\cos(fx+e))^2+6\sin(fx\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(-cos(f*x+e)^2+6*sin(f*x+e)+16*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))-8*ln(2/(cos(f*x+e)+1))+1)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f
*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x
+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [B] time = 1.95406, size = 1139, normalized size = 7.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/2*(16*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 8*a^(3/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + (10*a^(3/2) - 11*a
^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 15*a^(3/2)*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 20*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^(3/2)*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 7*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5)/(c^(3/2) - 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*c^(3/2)*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)
 - (10*a^(3/2) - 13*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 25*a^(3/2)*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 20*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 15*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 9*a^(3/2)*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/(c^(3/2) - 2*c^(3/2)*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^(3/2)*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 2*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6) + 2*(5*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) - 5*a
^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^(3/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 - 5*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^(3/2)
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^(3/2) - 2*c^(3/2)*sin(f*x + e)/(co
s(f*x + e) + 1) + 3*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^(3/2)
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 - 2*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+a\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x +
e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
3/2), x)
```

3.15 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (4*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) + (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.54727, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (4*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) + (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
```

```
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{2\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(4}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} - \frac{}{c\sqrt{}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(4}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}$$

Mathematica [A] time = 1.05235, size = 169, normalized size = 1.17

$$\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos(2(e+fx))+16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right.\right.\right.}{2c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\mathrm{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(7 +
Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (2 - 16*Lo
```

```
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c^2*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.195, size = 222, normalized size = 1.5

$$-\frac{\left(\cos\left(fx+e\right)\right)^{2}-\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(fx+e\right)+2\sin\left(fx+e\right)-2}{f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2\right)}\left(\left(\cos\left(fx+e\right)\right)^{2}-8\sin\left(fx+e\right)\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(cos(f*x+e)^2-8*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+
4*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+5*sin(f*x+e)+8*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-4*ln(2/(cos(f*x+e)+1))-1)*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x
+e)+cos(f*x+e)+2*sin(f*x+e)-2)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x
+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+a\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*c
os(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
5/2), x)
```

3.16 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c - c*Sin[e + f*x])^(
3/2)) - (a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.56556, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$-\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c - c*Sin[e + f*x])^(
3/2)) - (a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a\int}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} + \frac{}{c^2\sqrt{}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} - \frac{(a^2}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} - \frac{a}{c^3f}$$

Mathematica [A] time = 1.41336, size = 191, normalized size = 1.3

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos(2(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\right.}{c^3f(\sin(e+fx)-1)^3\sqrt{c-c\sin(e+fx)}\left(\mathrm{s}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -((a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-2
 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]] + 4*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]])*Sin[e + f*x]))/(c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [B] time = 0.188, size = 276, normalized size = 1.9

$$\frac{\sin(fx+e)\cos(fx+e)-(\cos(fx+e))^2-2\sin(fx+e)-\cos(fx+e)+2}{f\left(\sin(fx+e)\cos(fx+e)+(\cos(fx+e))^2-2\sin(fx+e)+\cos(fx+e)-2\right)}\left(2(\cos(fx+e))^2\ln\left(-\frac{-1+\cos(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/f*(2*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^2
*ln(2/(cos(f*x+e)+1))+4*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)^2-4*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))+2*ln(2/(cos(f*x+e)+1))+2)*(sin(f*x+e)*cos(f*x+e)-cos(
f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*co
s(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+a\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^{4}\cos\left(fx+e\right)^{4}-8\,c^{4}\cos\left(fx+e\right)^{2}+8\,c^{4}+4\left(c^{4}\cos\left(fx+e\right)^{2}-2\,c^{4}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral((a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e
)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
7/2), x)
```

3.17 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6acf(c-c\sin(e+fx))^{7/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*a*c*f*(c - c*Sin[e + f*x])^(7/
2))
```

Rubi [A] time = 0.337432, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6acf(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(9/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*a*c*f*(c - c*Sin[e + f*x])^(7/
2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6acf(c-c\sin(e+fx))^{7/2}}\end{aligned}$$

Mathematica [B] time = 1.47514, size = 110, normalized size = 2.29

$$-\frac{a(3\cos(2(e+fx))-5)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{6c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] -(a*(-5 + 3*Cos[2*(e + f*x)])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[
a*(1 + Sin[e + f*x])])/(6*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
 Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.181, size = 127, normalized size = 2.7

$$-\frac{\left(\left(\cos\left(fx+e\right)\right)^{2}-4\right)\sin\left(fx+e\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)}{3f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/3/f*(cos(f*x+e)^2-4)*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(sin(f*x+e)*cos
(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(sin(f*x+e)*cos(f*x+e)+cos(
f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

Fricas [B] time = 1.68417, size = 259, normalized size = 5.4

$$\frac{\left(3\,a\cos\left(fx+e\right)^{2}-4\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(3\,c^{5}f\cos\left(fx+e\right)^{3}-4\,c^{5}f\cos\left(fx+e\right)-\left(c^{5}f\cos\left(fx+e\right)^{3}-4\,c^{5}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*(3*a*cos(f*x + e)^2 - 4*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(3*c^5*f*cos(f*x + e)^3 - 4*c^5*f*cos(f*x + e) - (c^5*f*cos(f*x + e)
```

```
^3 - 4*c^5*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

3.18 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48ac^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*a*c*f*(c - c*Sin[e + f*x])^(9/
2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*a*c^2*f*(c - c*Sin[e +
f*x])^(7/2))
```

Rubi [A] time = 0.435671, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48ac^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(11/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*a*c*f*(c - c*Sin[e + f*x])^(9/
2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*a*c^2*f*(c - c*Sin[e +
f*x])^(7/2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{8ac^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48ac^2f(c-c\sin(e+fx))^{7/2}}$$

Mathematica [A] time = 2.00804, size = 118, normalized size = 1.22

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(4\sin(e+fx)-3\cos(2(e+fx))+5)}{12c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(5 -
 3*Cos[2*(e + f*x)] + 4*Sin[e + f*x]))/(12*c^5*f*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.192, size = 152, normalized size = 1.6

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-4\left(\cos\left(fx+e\right)\right)^2-4\sin\left(fx+e\right)+10\right)\sin\left(fx+e\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(f\right.\right.\right.}{6f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^2-2\sin\left(fx+e\right)+\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/6/f*(cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^2-4*sin(f*x+e)+10)*(a*(1+sin(f*
x+e)))^(3/2)*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-co
s(f*x+e)+2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/
(-c*(-1+sin(f*x+e)))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

Fricas [A] time = 1.78047, size = 320, normalized size = 3.3

$$-\frac{\left(3\,a\cos\left(fx+e\right)^{2}-2\,a\sin\left(fx+e\right)-4\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^{6}f\cos\left(fx+e\right)^{5}-8\,c^{6}f\cos\left(fx+e\right)^{3}+8\,c^{6}f\cos\left(fx+e\right)+4\left(c^{6}f\cos\left(fx+e\right)^{3}-2\,c^{6}f\cos\left(fx+e\right)\right)\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -1/6*(3*a*cos(f*x + e)^2 - 2*a*sin(f*x + e) - 4*a)*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c)/(c^6*f*cos(f*x + e)^5 - 8*c^6*f*cos(f*x + e)^3 +
 8*c^6*f*cos(f*x + e) + 4*(c^6*f*cos(f*x + e)^3 - 2*c^6*f*cos(f*x + e))*sin
(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

3.19 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=188

$$-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{14cf}-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)}{}$$

```
[Out] -(a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(35*c*f*Sqrt[a + a*Sin[e + f
*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/
2))/(14*c*f) - (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(9/2))/(28*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(9/2))/(8*c*f)
```

Rubi [A] time = 0.624161, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{14cf}-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(35*c*f*Sqrt[a + a*Sin[e + f
*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/
2))/(14*c*f) - (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(9/2))/(28*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(9/2))/(8*c*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{8cf}$$
$$= -\frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))}{28cf}$$
$$= -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}{14cf}$$
$$= -\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35cf\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\sqrt{}}{}$$

Mathematica [A] time = 3.21113, size = 176, normalized size = 0.94

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}(19600\sin(e+fx)+3920\sin(3(e+fx))+784\sin(5(e}{35840f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e +
 f*x]]*(1960*Cos[2*(e + f*x)] + 980*Cos[4*(e + f*x)] + 280*Cos[6*(e + f*x)]
 + 35*Cos[8*(e + f*x)] + 19600*Sin[e + f*x] + 3920*Sin[3*(e + f*x)] + 784*S
in[5*(e + f*x)] + 80*Sin[7*(e + f*x)]))/(35840*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.24, size = 143, normalized size = 0.8

$$\frac{\sin(fx+e)\left(35\left(\cos(fx+e)\right)^8+5\left(\cos(fx+e)\right)^6\sin(fx+e)+40\left(\cos(fx+e)\right)^6+13\sin(fx+e)\left(\cos(fx+e\right.\right.}{280f\left(\cos(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/280/f*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(35*
cos(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)+40*cos(f*x+e)^6+13*sin(f*x+e)*cos(f*
x+e)^4+48*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+64*cos(f*x+e)^2+93*sin(f*
x+e)+93)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(fx+e)+a\right)^{\frac{5}{2}}\left(-c\sin(fx+e)+c\right)^{\frac{7}{2}}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x +
e)^2, x)
```

Fricas [A] time = 2.00811, size = 306, normalized size = 1.63

$$\frac{\left(35\,a^2c^3\cos\left(fx+e\right)^8-35\,a^2c^3+8\left(5\,a^2c^3\cos\left(fx+e\right)^6+6\,a^2c^3\cos\left(fx+e\right)^4+8\,a^2c^3\cos\left(fx+e\right)^2+16\,a^2c^3\right)\sin\left(fx\right.}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/280*(35*a^2*c^3*cos(f*x + e)^8 - 35*a^2*c^3 + 8*(5*a^2*c^3*cos(f*x + e)^6
 + 6*a^2*c^3*cos(f*x + e)^4 + 8*a^2*c^3*cos(f*x + e)^2 + 16*a^2*c^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.20 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=188

$$-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{35cf}-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e}{}$$

```
[Out] (-2*a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (4*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(7/2))/(35*c*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e
+ f*x])^(7/2))/(7*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Si
n[e + f*x])^(7/2))/(7*c*f)
```

Rubi [A] time = 0.620187, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{35cf}-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (4*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(7/2))/(35*c*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e
+ f*x])^(7/2))/(7*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Si
n[e + f*x])^(7/2))/(7*c*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7cf} +$$
$$= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{7cf} -$$
$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{35cf} -$$
$$= -\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35cf\sqrt{a+a\sin(e+fx)}} - \frac{4a^2\cos(e+fx)\sqrt{}}{}$$

Mathematica [A] time = 0.652245, size = 87, normalized size = 0.46

$$\frac{a^2c^2(1225\sin(e+fx)+245\sin(3(e+fx))+49\sin(5(e+fx))+5\sin(7(e+fx)))\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c}}{2240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
1225*Sin[e + f*x] + 245*Sin[3*(e + f*x)] + 49*Sin[5*(e + f*x)] + 5*Sin[7*(e
 + f*x)]))/(2240*f)
```

Maple [A] time = 0.202, size = 77, normalized size = 0.4

$$\frac{\left(5\left(\cos\left(fx+e\right)\right)^6+6\left(\cos\left(fx+e\right)\right)^4+8\left(\cos\left(fx+e\right)\right)^2+16\right)\sin\left(fx+e\right)}{35f\left(\cos\left(fx+e\right)\right)^5}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(a\left(1+\sin\left(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/35/f*(5*cos(f*x+e)^6+6*cos(f*x+e)^4+8*cos(f*x+e)^2+16)*(-c*(-1+sin(f*x+e)
))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

Fricas [A] time = 2.43427, size = 244, normalized size = 1.3

$$\frac{\left(5\,a^2c^2\cos\left(fx+e\right)^6+6\,a^2c^2\cos\left(fx+e\right)^4+8\,a^2c^2\cos\left(fx+e\right)^2+16\,a^2c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{35\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/35*(5*a^2*c^2*cos(f*x + e)^6 + 6*a^2*c^2*cos(f*x + e)^4 + 8*a^2*c^2*cos(f
*x + e)^2 + 16*a^2*c^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*
sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.21 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6af}+\frac{2c\cos(e+fx)(a\sin(e+f}{15}$$

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(15*a*f*Sqrt[c - c*Sin[e + f*
x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
])/(15*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])
^(3/2))/(6*a*f)
```

Rubi [A] time = 0.51661, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6af}+\frac{2c\cos(e+fx)(a\sin(e+f}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(15*a*f*Sqrt[c - c*Sin[e + f*
x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
])/(15*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])
^(3/2))/(6*a*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6af}$$
$$= \frac{2c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15af}$$
$$= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{15af\sqrt{c-c\sin(e+fx)}} + \frac{2c\cos(e+fx)(a}{}$$

Mathematica [A] time = 1.01715, size = 152, normalized size = 1.09

$$-\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}(600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+fx))}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x
]]*(-75*Cos[2*(e + f*x)] - 30*Cos[4*(e + f*x)] - 5*Cos[6*(e + f*x)] + 600*S
in[e + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(960*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.2, size = 116, normalized size = 0.8

$$-\frac{\sin(fx+e)\left(-5(\cos(fx+e))^6+\sin(fx+e)(\cos(fx+e))^4-6(\cos(fx+e))^4+3(\cos(fx+e))^2\sin(fx+e)}{30f(\cos(fx+e))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/30/f*(-c*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-5*
cos(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4-6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+
e)-8*cos(f*x+e)^2+11*sin(f*x+e)-11)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x +
e)^2, x)
```

Fricas [A] time = 2.41294, size = 252, normalized size = 1.8

$$-\frac{\left(5\,a^{2}c\cos\left(fx+e\right)^{6}-5\,a^{2}c-2\left(3\,a^{2}c\cos\left(fx+e\right)^{4}+4\,a^{2}c\cos\left(fx+e\right)^{2}+8\,a^{2}c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(5*a^2*c*cos(f*x + e)^6 - 5*a^2*c - 2*(3*a^2*c*cos(f*x + e)^4 + 4*a^2
*c*cos(f*x + e)^2 + 8*a^2*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c
*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.22 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*a*f*Sqrt[c - c*Sin[e + f*x]
]) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(5*
a*f)
```

Rubi [A] time = 0.395089, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*a*f*Sqrt[c - c*Sin[e + f*x]
]) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(5*
a*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{5af} + \frac{2\int(a}{}$$
$$= \frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10af\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx)(a+a\sin(e}{}$$

Mathematica [A] time = 0.493871, size = 92, normalized size = 1.

$$-\frac{a^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(-70\sin(e+fx)-5\sin(3(e+fx))+\sin(5(e+fx))+20\cos(2(e+}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -(a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(20*
Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] - 70*Sin[e + f*x] - 5*Sin[3*(e + f*x)
] + Sin[5*(e + f*x)]))/(80*f)
```

Maple [A] time = 0.227, size = 106, normalized size = 1.2

$$-\frac{\sin(fx+e)\left(-2\left(\cos(fx+e)\right)^6+\sin(fx+e)\left(\cos(fx+e)\right)^4-2\left(\cos(fx+e)\right)^4+3\left(\cos(fx+e)\right)^2\sin(fx+e)+6}{10f\left(\cos(fx+e)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/10/f*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-2*
cos(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4-2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+
e)+6*sin(f*x+e)-6)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sin(fx+e)+c}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

Fricas [A] time = 2.29749, size = 236, normalized size = 2.57

$$-\frac{\left(5\,a^{2}\cos\left(fx+e\right)^{4}-5\,a^{2}+2\left(a^{2}\cos\left(fx+e\right)^{4}-2\,a^{2}\cos\left(fx+e\right)^{2}-4\,a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/10*(5*a^2*cos(f*x + e)^4 - 5*a^2 + 2*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x
 + e)^2 - 4*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.23 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.306869, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [B] time = 0.920495, size = 119, normalized size = 2.64

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(56\sin(e+fx)-8\sin(3(e+fx))-28\cos(2(e+fx))+\cos(4}{32f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(-28*Co
s[2*(e + f*x)] + Cos[4*(e + f*x)] + 56*Sin[e + f*x] - 8*Sin[3*(e + f*x)]))/
(32*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.227, size = 195, normalized size = 4.3

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{4}-\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}-4\left(\cos\left(fx+e\right)\right)^{3}-3\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-4\left(\cos\left(fx+e\right)\right.}{4f\left(\left(\cos\left(fx+e\right)\right)^{3}-\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-3\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*(cos(f*x+e)^4-sin(f*x+e)*cos(f*x+e)^3-4*cos(f*x+e)^3-3*cos(f*x+e)^2*s
in(f*x+e)-4*cos(f*x+e)^2+7*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)+sin(f*x+e)-1)
*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-
3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(-1
+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [B] time = 1.70971, size = 234, normalized size = 5.2

$$\frac{\left(a^{2}\cos\left(fx+e\right)^{4}-8\,a^{2}\cos\left(fx+e\right)^{2}+7\,a^{2}-4\left(a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(f\right.}}{4\,cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/4*(a^2*cos(f*x + e)^4 - 8*a^2*cos(f*x + e)^2 + 7*a^2 - 4*(a^2*cos(f*x + e
)^2 - 2*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
```

```
c)/(c*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

3.24 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}} \, dx$

Optimal. Leaf size=193

$$-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{8a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] (-8*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (4*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c
*f*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/
(c*f*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/
(3*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.64229, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{8a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf\sqrt{c-c\sin(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(3/2),
x]
```


```
[Out] (-8*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (4*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c
*f*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/
(c*f*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/
(3*c*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}} + \frac{(4a}{}$$

$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{c}{}$$

$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{c}{}$$

$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{c}{}$$

$$= -\frac{8a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx}}{cf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 2.57986, size = 140, normalized size = 0.73

$$-\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(87\sin(e+fx)-\sin(3(e+fx))-12\cos(2(e+fx))+192\,\mathrm{lo}\right.}{12cf\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(-12*C
os[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 87*Sin[e +
 f*x] - Sin[3*(e + f*x)]))/(12*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*
Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.203, size = 218, normalized size = 1.1

$$\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2}{3f\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{3}+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^{2}-4\sin\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^2*sin(f*x+e)+6*cos(f*x+e)^2-22*sin(f*x+e)-48*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))+24*ln(2/(cos(f*x+e)+1))-6)*(sin(f*x+e)*cos(f
*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(5/2)/(cos
(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*
sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2}\cos\left(fx+e\right)^{4}-2\,a^{2}\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-2\,a^{2}\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right.}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f
*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
3/2), x)
```

3.25 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=192

$$\frac{6a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{12a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (12*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + (6*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
)/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.646381, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$\frac{6a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{12a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (12*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + (6*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
)/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{3\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(6a$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{6a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} + \frac{3a\,c$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{6a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} + \frac{3a\,c$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{6a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} + \frac{3a\,c$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{12a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 2.33449, size = 181, normalized size = 0.94

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(3(e+fx))+18\cos(2(e+fx))+192\log\left(\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{8c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(44
 + 18*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (39
 - 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e +
f*x)]))/(8*c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^
2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.215, size = 272, normalized size = 1.4

$$\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2}{2f\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{3}+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^{2}-4\sin\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/2/f*(cos(f*x+e)^2*sin(f*x+e)+24*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*sin(f*
x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+9*cos(f*x+e)^2-24*ln(2/(cos
(f*x+e)+1))+25*sin(f*x+e)+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-9)*
(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*
x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+
3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [B] time = 1.90576, size = 1512, normalized size = 7.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] -1/6*(144*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 72*a^(
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) + (46*a^(5/2) - 1
21*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 149*a^(5/2)*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 179*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 148*a
^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 43*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 33*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15*a^(5
/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(5/2) - 4*c^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 8*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 12*c^(
5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*c^(5/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 12*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*c^(5/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 4*c^(5/2)*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7 + c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) - (46*a^(5/2) - 19
9*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 335*a^(5/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 509*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 496*a^
(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 373*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 219*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 63*a^(
5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(5/2) - 4*c^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 8*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 12*c^
(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*c^(5/2)*sin(f*x + e)^4/(cos(
```

```
f*x + e) + 1)^4 - 12*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*c^(5/2
)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 4*c^(5/2)*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) + 6*(13*a^(5/2)*s
in(f*x + e)/(cos(f*x + e) + 1) - 39*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 55*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 74*a^(5/2)*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 55*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 - 39*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 13*a^(5/2)*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7)/(c^(5/2) - 4*c^(5/2)*sin(f*x + e)/(cos(f*x + e) +
 1) + 8*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 12*c^(5/2)*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 + 14*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 12*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*c^(5/2)*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 4*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(
5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,a^2\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos
(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
5/2), x)
```

3.26 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{3a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{6a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*(c - c*Sin[e + f
*x])^(3/2)) - (6*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^2*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.656914, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{3a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{6a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*(c - c*Sin[e + f
*x])^(3/2)) - (6*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^2*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{2c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{3/2}} + \frac{(3a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{3/2}} - \frac{3a^2}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{3/2}} - \frac{3a^2}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{3/2}} - \frac{3a^2}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{3/2}} - \frac{6}{c^3f}$$

Mathematica [A] time = 2.7571, size = 209, normalized size = 1.07

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(3(e+fx))-72\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{4c^3f(\sin(e+fx)-1)^3\sqrt{c-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-
28 - 72*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Cos[2*(e + f*x)]*(-1 +
 6*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (41 + 96*Log[Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*c^3*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]
)
```

Maple [A] time = 0.211, size = 329, normalized size = 1.7

$$\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2}{f\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{3}+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^{2}-4\sin\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/f*(12*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^
2*sin(f*x+e)-6*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+24*sin(f*x+e)*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))-12*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-10*cos(f*
x+e)^2-24*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*sin(f*x+e)+12*ln(2/(
cos(f*x+e)+1))+10)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x
+e)+2)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin
(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(
f*x+e)))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^{2}\cos\left(fx+e\right)^{4}-2\,a^{2}\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-2\,a^{2}\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+\right.}}{c^{4}\cos\left(fx+e\right)^{4}-8\,c^{4}\cos\left(fx+e\right)^{2}+8\,c^{4}+4\left(c^{4}\cos\left(fx+e\right)^{2}-2\,c^{4}\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*c
os(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(
f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*
sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
7/2), x)
```

3.27 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=193

$$\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}+$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e + f*x])^(7/2)
) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*(c - c*Sin[e + f*x
])^(5/2)) + (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^3*f*(c - c*Sin[e
 + f*x])^(3/2)) + (a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^4*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.662676, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(9/2),
x]
```


```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e + f*x])^(7/2)
) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*(c - c*Sin[e + f*x
])^(5/2)) + (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^3*f*(c - c*Sin[e
 + f*x])^(3/2)) + (a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^4*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} + \frac{a\int \cdots}{\cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} + \frac{a^2\cos\cdots}{\cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} + \frac{a^2\cos\cdots}{\cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} + \frac{a^2\cos\cdots}{\cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} + \frac{a^2\cos\cdots}{\cdots}$$

Mathematica [A] time = 4.21542, size = 234, normalized size = 1.21

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(3\sin(3(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+3\cdots\right.}{6c^4f(\sin(e\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(34
 + 30*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 18*Cos[2*(e + f*x)]*(1 + L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) - 9*(4 + 5*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]])*Sin[e + f*x] + 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
]*Sin[3*(e + f*x)]))/(6*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.208, size = 401, normalized size = 2.1

$$\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2}{3f\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{3}+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^{2}-4\sin\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/3/f*(6*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
-3*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-8*cos(f*x+e)^2*sin(f*x+e)-1
8*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+9*cos(f*x+e)^2*ln
(2/(cos(f*x+e)+1))-24*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
+12*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+6*cos(f*x+e)^2+14*sin(f*x+e)+24*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*ln(2/(cos(f*x+e)+1))-6)*(sin(f*x+e)*
cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(5/2)
/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)
^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^{2}\cos\left(fx+e\right)^{4}-2\,a^{2}\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-2\,a^{2}\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+\right.}}{5\,c^{5}\cos\left(fx+e\right)^{4}-20\,c^{5}\cos\left(fx+e\right)^{2}+16\,c^{5}-\left(c^{5}\cos\left(fx+e\right)^{4}-12\,c^{5}\cos\left(fx+e\right)^{2}+16\,c^{5}\right)\sin\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*c
os(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*co
s(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^
5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

3.28 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*a*c*f*(c - c*Sin[e + f*x])^(9/2))

Rubi [A] time = 0.339403, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(11/2),x]

[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*a*c*f*(c - c*Sin[e + f*x])^(9/2))

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8acf(c-c\sin(e+fx))^{9/2}}\end{aligned}$$

Mathematica [B] time = 4.33023, size = 117, normalized size = 2.44

$$\frac{a^2(\sin(3(e+fx))-7\sin(e+fx))\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{4c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-7
*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.181, size = 157, normalized size = 3.3

$$-\frac{\left(\left(\cos\left(fx+e\right)\right)^{2}-2\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)\sin\left(\right.}{f\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{3}+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^{2}-4\sin\left(fx+e\right)+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/f*(cos(f*x+e)^2-2)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(
f*x+e)+2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(
f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4
)/(-c*(-1+sin(f*x+e)))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

Fricas [B] time = 1.88982, size = 309, normalized size = 6.44

$$-\frac{\left(a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{c^{6}f\cos\left(fx+e\right)^{5}-8\,c^{6}f\cos\left(fx+e\right)^{3}+8\,c^{6}f\cos\left(fx+e\right)+4\left(c^{6}f\cos\left(fx+e\right)^{3}-2\,c^{6}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -(a^2*cos(f*x + e)^2 - 2*a^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e)/(c^6*f*cos(f*x + e)^5 - 8*c^6*f*cos(f*x + e)^3 + 8*c^6*f
```

```
*cos(f*x + e) + 4*(c^6*f*cos(f*x + e)^3 - 2*c^6*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

3.29 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80ac^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*a*c*f*(c - c*Sin[e + f*x])^(1
1/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*a*c^2*f*(c - c*Sin[e
+ f*x])^(9/2))
```

Rubi [A] time = 0.437702, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80ac^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(13/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*a*c*f*(c - c*Sin[e + f*x])^(1
1/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*a*c^2*f*(c - c*Sin[e
+ f*x])^(9/2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10ac^2}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80ac^2f(c-c\sin(e+fx))^{9/2}}$$

Mathematica [A] time = 6.18012, size = 130, normalized size = 1.34

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(35\sin(e+fx)-5\sin(3(e+fx))-10\cos(2(e+fx))+14}{40c^6f(\sin(e+fx)-1)^6\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(14
 - 10*Cos[2*(e + f*x)] + 35*Sin[e + f*x] - 5*Sin[3*(e + f*x)]))/(40*c^6*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^6*Sqrt[c - c*Sin[e
 + f*x]])
```

Maple [B] time = 0.206, size = 191, normalized size = 2.

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^4+5\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-17\left(\cos\left(fx+e\right)\right)^2-10\sin\left(fx+e\right)+26\right)\sin\left(fx+e\right)\left(\sin\left(fx+\right.\right.}{10f\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^3+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\cos\left(f\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] 1/10/f*(cos(f*x+e)^4+5*cos(f*x+e)^2*sin(f*x+e)-17*cos(f*x+e)^2-10*sin(f*x+e
)+26)*(a*(1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)
^2-2*sin(f*x+e)-cos(f*x+e)+2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f
*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*
x+e)))^(13/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="maxima")
```

[Out] Timed out

Fricas [A] time = 1.86123, size = 402, normalized size = 4.14

$$-\frac{\left(5\,a^{2}\cos\left(fx+e\right)^{2}-6\,a^{2}+5\left(a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx\right.}}{10\left(5\,c^{7}f\cos\left(fx+e\right)^{5}-20\,c^{7}f\cos\left(fx+e\right)^{3}+16\,c^{7}f\cos\left(fx+e\right)-\left(c^{7}f\cos\left(fx+e\right)^{5}-12\,c^{7}f\cos\left(fx+e\right)^{3}+16\,c^{7}f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="fricas")
```

```
[Out] -1/10*(5*a^2*cos(f*x + e)^2 - 6*a^2 + 5*(a^2*cos(f*x + e)^2 - 2*a^2)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^7*f*cos(f*x
 + e)^5 - 20*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e) - (c^7*f*cos(f*x
+ e)^5 - 12*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
13/2), x)
```

3.30 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx$

Optimal. Leaf size=236

$$-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{11/2}}{15cf}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{11/2}}{105cf}$$

```
[Out] (-4*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(11/2))/(315*c*f*Sqrt[a + a*Sin[e
 + f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(11/2))/(105*c*f) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*
Sin[e + f*x])^(11/2))/(15*c*f) - (4*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/
2)*(c - c*Sin[e + f*x])^(11/2))/(45*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x
])^(7/2)*(c - c*Sin[e + f*x])^(11/2))/(10*c*f)
```

Rubi [A] time = 0.724664, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{11/2}}{15cf}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{11/2}}{105cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2),x]
```


```
[Out] (-4*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(11/2))/(315*c*f*Sqrt[a + a*Sin[e
 + f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(11/2))/(105*c*f) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*
Sin[e + f*x])^(11/2))/(15*c*f) - (4*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/
2)*(c - c*Sin[e + f*x])^(11/2))/(45*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x
])^(7/2)*(c - c*Sin[e + f*x])^(11/2))/(10*c*f)
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
```

```
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{11/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{11/2}}{10cf} +$$
$$= -\frac{4a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{11/2}}{45cf}$$
$$= -\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{11/2}}{15cf}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{11/2}}{105cf}$$
$$= -\frac{4a^4\cos(e+fx)(c-c\sin(e+fx))^{11/2}}{315cf\sqrt{a+a\sin(e+fx)}} - \frac{4a^3\cos(e+fx)\cdots}{}$$

Mathematica [A] time = 5.70095, size = 209, normalized size = 0.89

$$\frac{a^3c^4(\sin(e+fx)-1)^4(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(158760\sin(e+fx)+35280\sin(3(e+fx)\cdots}{322560f\left(\text{co}\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9
/2),x]
```

```
[Out] (a^3*c^4*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(13230*Cos[2*(e + f*x)] + 7560*Cos[4*(e + f*x)
] + 2835*Cos[6*(e + f*x)] + 630*Cos[8*(e + f*x)] + 63*Cos[10*(e + f*x)] + 1
58760*Sin[e + f*x] + 35280*Sin[3*(e + f*x)] + 9072*Sin[5*(e + f*x)] + 1620*
Sin[7*(e + f*x)] + 140*Sin[9*(e + f*x)]))/(322560*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A] time = 0.31, size = 169, normalized size = 0.7

$$\frac{\sin(fx+e)\left(63\left(\cos(fx+e)\right)^{10}+7\sin(fx+e)\left(\cos(fx+e)\right)^{8}+70\left(\cos(fx+e)\right)^{8}+17\left(\cos(fx+e)\right)^{6}\sin(fx+e\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/630/f*(-c*(-1+sin(f*x+e)))^(9/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(63*
cos(f*x+e)^10+7*sin(f*x+e)*cos(f*x+e)^8+70*cos(f*x+e)^8+17*cos(f*x+e)^6*sin
(f*x+e)+80*cos(f*x+e)^6+33*sin(f*x+e)*cos(f*x+e)^4+96*cos(f*x+e)^4+65*cos(f
*x+e)^2*sin(f*x+e)+128*cos(f*x+e)^2+193*sin(f*x+e)+193)/cos(f*x+e)^9
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.13492, size = 351, normalized size = 1.49

$$\frac{\left(63\,a^3c^4\cos\left(fx+e\right)^{10} - 63\,a^3c^4 + 2\left(35\,a^3c^4\cos\left(fx+e\right)^8 + 40\,a^3c^4\cos\left(fx+e\right)^6 + 48\,a^3c^4\cos\left(fx+e\right)^4 + 64\,a^3c^4\,c\right.}{630\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] 1/630*(63*a^3*c^4*cos(f*x + e)^10 - 63*a^3*c^4 + 2*(35*a^3*c^4*cos(f*x + e)
^8 + 40*a^3*c^4*cos(f*x + e)^6 + 48*a^3*c^4*cos(f*x + e)^4 + 64*a^3*c^4*cos
(f*x + e)^2 + 128*a^3*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*s
in(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.31 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=236

$$-\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{21cf}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{63cf}-8$$

```
[Out] (-8*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(315*c*f*Sqrt[a + a*Sin[e
+ f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(9/2))/(63*c*f) - (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*S
in[e + f*x])^(9/2))/(21*c*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(
c - c*Sin[e + f*x])^(9/2))/(9*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/
2)*(c - c*Sin[e + f*x])^(9/2))/(9*c*f)
```

Rubi [A] time = 0.734853, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$-\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{21cf}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{63cf}-8$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2),x]
```


```
[Out] (-8*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(315*c*f*Sqrt[a + a*Sin[e
+ f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(9/2))/(63*c*f) - (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*S
in[e + f*x])^(9/2))/(21*c*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(
c - c*Sin[e + f*x])^(9/2))/(9*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/
2)*(c - c*Sin[e + f*x])^(9/2))/(9*c*f)
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
```

```
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}}{9cf}$$
$$= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9}}{9cf}$$
$$= -\frac{2a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx}{21cf}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9}}{63cf}$$
$$= -\frac{8a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315cf\sqrt{a+a\sin(e+fx)}} - \frac{4a^3\cos(e+f}{}$$

Mathematica [A] time = 1.31127, size = 97, normalized size = 0.41

$$\frac{a^3c^3(39690\sin(e+fx)+8820\sin(3(e+fx))+2268\sin(5(e+fx))+405\sin(7(e+fx))+35\sin(9(e+fx)))\sec(e+}{80640f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
39690*Sin[e + f*x] + 8820*Sin[3*(e + f*x)] + 2268*Sin[5*(e + f*x)] + 405*Si
n[7*(e + f*x)] + 35*Sin[9*(e + f*x)]))/(80640*f)
```

Maple [A] time = 0.231, size = 87, normalized size = 0.4

$$\frac{\left(35\left(\cos\left(fx+e\right)\right)^8+40\left(\cos\left(fx+e\right)\right)^6+48\left(\cos\left(fx+e\right)\right)^4+64\left(\cos\left(fx+e\right)\right)^2+128\right)\sin\left(fx+e\right)}{315f\left(\cos\left(fx+e\right)\right)^7}\left(-c\left(-1+\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/315/f*(35*cos(f*x+e)^8+40*cos(f*x+e)^6+48*cos(f*x+e)^4+64*cos(f*x+e)^2+12
8)*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/cos(f*x+e
)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x +
e)^2, x)
```

Fricas [A] time = 1.93308, size = 289, normalized size = 1.22

$$\frac{\left(35\,a^3c^3\cos\left(fx+e\right)^8+40\,a^3c^3\cos\left(fx+e\right)^6+48\,a^3c^3\cos\left(fx+e\right)^4+64\,a^3c^3\cos\left(fx+e\right)^2+128\,a^3c^3\right)\sqrt{a\sin\left(fx+e\right)}}{315\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/315*(35*a^3*c^3*cos(f*x + e)^8 + 40*a^3*c^3*cos(f*x + e)^6 + 48*a^3*c^3*c
os(f*x + e)^4 + 64*a^3*c^3*cos(f*x + e)^2 + 128*a^3*c^3)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x +
e)^2, x)
```

3.32 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=188

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{14af}+\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{35af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+}{}$$

```
[Out] (c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(35*a*f*Sqrt[c - c*Sin[e + f*
x]]) + (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f*x]
])/(14*a*f) + (3*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[e + f
*x])^(3/2))/(28*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[
e + f*x])^(5/2))/(8*a*f)
```

Rubi [A] time = 0.619513, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{14af}+\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{35af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(35*a*f*Sqrt[c - c*Sin[e + f*
x]]) + (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f*x]
])/(14*a*f) + (3*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[e + f
*x])^(3/2))/(28*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[
e + f*x])^(5/2))/(8*a*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{5/2}}{8af} + \frac{3}{}$$
$$= \frac{3c\cos(e+fx)(a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{3/2}}{28af} +$$
$$= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}}{14af} + \frac{3}{}$$
$$= \frac{c^3\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{35af\sqrt{c-c\sin(e+fx)}} + \frac{c^2\cos(e+fx)(a+}{}$$

Mathematica [A] time = 1.90371, size = 127, normalized size = 0.68

$$\frac{a^3c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(19600\sin(e+fx)+3920\sin(3(e+fx))+784\sin(5(e+fx))+8}{35840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a^3*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-1960*Cos[2*(e + f*x)] - 980*Cos[4*(e + f*x)] - 280*Cos[6*(e + f*x)] - 35*C
os[8*(e + f*x)] + 19600*Sin[e + f*x] + 3920*Sin[3*(e + f*x)] + 784*Sin[5*(e
 + f*x)] + 80*Sin[7*(e + f*x)]))/(35840*f)
```

Maple [A] time = 0.23, size = 143, normalized size = 0.8

$$-\frac{\sin(fx+e)\left(-35\left(\cos(fx+e)\right)^8+5\left(\cos(fx+e)\right)^6\sin(fx+e)-40\left(\cos(fx+e)\right)^6+13\sin(fx+e)\left(\cos(fx+e\right.}{280f\left(\cos(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/280/f*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-3
5*cos(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)-40*cos(f*x+e)^6+13*sin(f*x+e)*cos(
f*x+e)^4-48*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)-64*cos(f*x+e)^2+93*sin(
f*x+e)-93)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(fx+e)+a\right)^{\frac{7}{2}}\left(-c\sin(fx+e)+c\right)^{\frac{5}{2}}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

Fricas [A] time = 1.91788, size = 308, normalized size = 1.64

$$-\frac{\left(35\,a^3c^2\cos\left(fx+e\right)^8-35\,a^3c^2-8\left(5\,a^3c^2\cos\left(fx+e\right)^6+6\,a^3c^2\cos\left(fx+e\right)^4+8\,a^3c^2\cos\left(fx+e\right)^2+16\,a^3c^2\right)\sin}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/280*(35*a^3*c^2*cos(f*x + e)^8 - 35*a^3*c^2 - 8*(5*a^3*c^2*cos(f*x + e)^
6 + 6*a^3*c^2*cos(f*x + e)^4 + 8*a^3*c^2*cos(f*x + e)^2 + 16*a^3*c^2)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

3.33 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{4c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{105af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}(c-c\sin(e+fx))^{3/2}}{7af}+\frac{2c\cos(e+fx)(a\sin(e+$$

```
[Out] (4*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(105*a*f*Sqrt[c - c*Sin[e +
 f*x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f
*x]])/(21*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[e + f*
x])^(3/2))/(7*a*f)
```

Rubi [A] time = 0.515013, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{4c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{105af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}(c-c\sin(e+fx))^{3/2}}{7af}+\frac{2c\cos(e+fx)(a\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (4*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(105*a*f*Sqrt[c - c*Sin[e +
 f*x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f
*x]])/(21*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[e + f*
x])^(3/2))/(7*a*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{3/2}}{7af}$$
$$= \frac{2c\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}}{21af}$$
$$= \frac{4c^2\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{105af\sqrt{c-c\sin(e+fx)}} + \frac{2c\cos(e+fx)(}{}$$

Mathematica [A] time = 1.60899, size = 115, normalized size = 0.82

$$\frac{a^3c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(4725\sin(e+fx)+665\sin(3(e+fx))+21\sin(5(e+fx))-15}{6720f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a^3*c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-1
050*Cos[2*(e + f*x)] - 420*Cos[4*(e + f*x)] - 70*Cos[6*(e + f*x)] + 4725*Si
n[e + f*x] + 665*Sin[3*(e + f*x)] + 21*Sin[5*(e + f*x)] - 15*Sin[7*(e + f*x
)]))/(6720*f)
```

Maple [A] time = 0.228, size = 133, normalized size = 1.

$$-\frac{\sin(fx+e)\left(-15\left(\cos(fx+e)\right)^8+5\left(\cos(fx+e)\right)^6\sin(fx+e)-16\left(\cos(fx+e)\right)^6+13\sin(fx+e)\left(\cos(fx+\right.\right.}{105f\left(\cos(fx+e)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/105/f*(-c*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-1
5*cos(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)-16*cos(f*x+e)^6+13*sin(f*x+e)*cos(
f*x+e)^4-16*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+58*sin(f*x+e)-58)/cos(f
*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(fx+e)+a\right)^{\frac{7}{2}}\left(-c\sin(fx+e)+c\right)^{\frac{3}{2}}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x +
e)^2, x)
```

Fricas [A] time = 1.85931, size = 293, normalized size = 2.09

$$-\frac{\left(35\,a^3c\cos\left(fx+e\right)^6-35\,a^3c+\left(15\,a^3c\cos\left(fx+e\right)^6-24\,a^3c\cos\left(fx+e\right)^4-32\,a^3c\cos\left(fx+e\right)^2-64\,a^3c\right)\sin\left(fx+e\right.}{105\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/105*(35*a^3*c*cos(f*x + e)^6 - 35*a^3*c + (15*a^3*c*cos(f*x + e)^6 - 24*
a^3*c*cos(f*x + e)^4 - 32*a^3*c*cos(f*x + e)^2 - 64*a^3*c)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.34 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{6af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{15af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(15*a*f*Sqrt[c - c*Sin[e + f*x]
]) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f*x]])/(6*
a*f)
```

Rubi [A] time = 0.386481, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2740, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{6af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{15af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(15*a*f*Sqrt[c - c*Sin[e + f*x]
]) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f*x]])/(6*
a*f)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}}{6af} + \frac{\int (a+$$
$$= \frac{c\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{15af\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx)(a+a\sin(e$$

Mathematica [A] time = 0.539982, size = 104, normalized size = 1.13

$$\frac{a^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(1080\sin(e+fx)+20\sin(3(e+fx))-36\sin(5(e+fx))-405\cos($$
$$960f$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-405
*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)] + 1080*Sin[e +
 f*x] + 20*Sin[3*(e + f*x)] - 36*Sin[5*(e + f*x)]))/(960*f)
```

Maple [A] time = 0.249, size = 133, normalized size = 1.5

$$-\frac{\sin(fx+e)\left(-5\left(\cos(fx+e)\right)^8+3\left(\cos(fx+e)\right)^6\sin(fx+e)-4\left(\cos(fx+e)\right)^6+7\sin(fx+e)\left(\cos(fx+e)\right)^4 -}{30f\left(\cos(fx+e)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/30/f*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-5*
cos(f*x+e)^8+3*cos(f*x+e)^6*sin(f*x+e)-4*cos(f*x+e)^6+7*sin(f*x+e)*cos(f*x+
e)^4+7*cos(f*x+e)^2*sin(f*x+e)+7*cos(f*x+e)^2+28*sin(f*x+e)-28)/cos(f*x+e)^
7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(fx+e)+a\right)^{\frac{7}{2}}\sqrt{-c\sin(fx+e)+c}\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

Fricas [A] time = 1.75089, size = 273, normalized size = 2.97

$$\frac{\left(5\,a^3\cos\left(fx+e\right)^6-30\,a^3\cos\left(fx+e\right)^4+25\,a^3-2\left(9\,a^3\cos\left(fx+e\right)^4-8\,a^3\cos\left(fx+e\right)^2-16\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/30*(5*a^3*cos(f*x + e)^6 - 30*a^3*cos(f*x + e)^4 + 25*a^3 - 2*(9*a^3*cos(
f*x + e)^4 - 8*a^3*cos(f*x + e)^2 - 16*a^3)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

3.35 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.307475, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [B] time = 1.44084, size = 142, normalized size = 3.16

$$\frac{a^3(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(210\sin(e+fx)-45\sin(3(e+fx))+\sin(5}{80f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 +
 Sin[e + f*x])]*(-120*Cos[2*(e + f*x)] + 10*Cos[4*(e + f*x)] + 210*Sin[e +
f*x] - 45*Sin[3*(e + f*x)] + Sin[5*(e + f*x)]))/(80*f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.231, size = 245, normalized size = 5.4

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{5}+\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+4\left(\cos\left(fx+e\right)\right)^{4}-5\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}-12\left(\cos\left(fx+e\right.\right.\right.}{5f\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}+\left(\cos\left(fx+e\right)\right)^{4}-4\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/5/f*(cos(f*x+e)^5+sin(f*x+e)*cos(f*x+e)^4+4*cos(f*x+e)^4-5*sin(f*x+e)*cos
(f*x+e)^3-12*cos(f*x+e)^3-7*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2+15*sin(f
*x+e)*cos(f*x+e)+16*cos(f*x+e)+sin(f*x+e)-1)*sin(f*x+e)*(a*(1+sin(f*x+e)))^
(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos
(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+
8)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [B] time = 1.7402, size = 270, normalized size = 6.

$$\frac{\left(5\,a^{3}\cos\left(fx+e\right)^{4}-20\,a^{3}\cos\left(fx+e\right)^{2}+15\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{4}-12\,a^{3}\cos\left(fx+e\right)^{2}+16\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin}}{5\,cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/5*(5*a^3*cos(f*x + e)^4 - 20*a^3*cos(f*x + e)^2 + 15*a^3 + (a^3*cos(f*x +
 e)^4 - 12*a^3*cos(f*x + e)^2 + 16*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c)/(c*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

3.36 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{8a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{16a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-16*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (8*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(c*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(5/2))/(3*c*f*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*
x])^(7/2))/(4*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.759596, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$-\frac{8a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{16a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(3/2),
x]
```


```
[Out] (-16*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (8*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(c*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(5/2))/(3*c*f*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*
x])^(7/2))/(4*c*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf\sqrt{c-c\sin(e+fx)}} + \frac{(4}{}$$

$$= -\frac{2a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} -$$

$$= -\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} -$$

$$= -\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} -$$

$$= -\frac{16a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx}}{cf\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 6.45611, size = 473, normalized size = 1.96

$$\frac{5\sin(3(e+fx))(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{12f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} - \frac{65\sin(e+fx)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\right.\right.}{4f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (23*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e
+ f*x]))^(7/2))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
```

```
f*x])^(3/2)) - (Cos[4*(e + f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(
a*(1 + Sin[e + f*x]))^(7/2))/(32*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*
(c - c*Sin[e + f*x])^(3/2)) - (32*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2)) - (65*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sin[e + f*x]*(a*(1 + Sin[e + f*x]))^(7
/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2
)) + (5*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2
)*Sin[3*(e + f*x)])/(12*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Si
n[e + f*x])^(3/2))
```

Maple [A] time = 0.216, size = 253, normalized size = 1.1

$$-\frac{\sin(fx+e)\cos(fx+e)-(\cos(fx+e))^2-2\sin(fx+e}{12f\left(\sin(fx+e)(\cos(fx+e))^3+(\cos(fx+e))^4-4(\cos(fx+e))^2\sin(fx+e)+3(\cos(fx+e))^3-4\sin(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/12/f*(-3*cos(f*x+e)^4+20*cos(f*x+e)^2*sin(f*x+e)+72*cos(f*x+e)^2-200*sin
(f*x+e)+192*ln(2/(cos(f*x+e)+1))-384*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))-69)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*
(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*
sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+
e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e)+a)^{\frac{7}{2}}\cos(fx+e)^2}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3a^3\cos(fx+e)^4-4a^3\cos(fx+e)^2+\left(a^3\cos(fx+e)^4-4a^3\cos(fx+e)^2\right)\sin(fx+e)\right)\sqrt{a\sin(fx+}}{c^2\cos(fx+e)^2+2c^2\sin(fx+e)-2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^4
 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
3/2), x)
```

3.37 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=238

$$\frac{16a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{4a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{32a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (32*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + (16*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (4*a^2*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(3/2))/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (4*a*Cos[e + f*x]*(a + a*Sin[
e + f*x])^(5/2))/(3*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.753604, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$\frac{16a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{4a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{32a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(5/2),
x]
```


```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (32*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + (16*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (4*a^2*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(3/2))/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (4*a*Cos[e + f*x]*(a + a*Sin[
e + f*x])^(5/2))/(3*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
```

```
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{4\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{4a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(8a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{4a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{c^2f\sqrt{c-c\sin(e+fx)}} + \frac{4a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{16a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} + \frac{4a^2}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{16a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} + \frac{4a^2}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{16a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} + \frac{4a^2}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{32a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 4.82273, size = 196, normalized size = 0.82

$$-\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-396\sin(e+fx)-16\sin(3(e+fx))-172\cos(2(e+f\right.}{24c^2f(\sin(e+fx)-1)^2\sqrt{c-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-
177 - 172*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] - 1536*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] - 396*Sin[e + f*x] + 1536*Log[Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]]*Sin[e + f*x] - 16*Sin[3*(e + f*x)]))/(24*c^2*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.241, size = 307, normalized size = 1.3

$$-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^2-2\sin\left(fx+e\right.}{3f\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3+\left(\cos\left(fx+e\right)\right)^4-4\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^3-4\sin\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/3/f*(-cos(f*x+e)^4+8*cos(f*x+e)^2*sin(f*x+e)+96*sin(f*x+e)*ln(2/(cos(f*x
+e)+1))-192*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+44*cos(f*
x+e)^2+91*sin(f*x+e)-96*ln(2/(cos(f*x+e)+1))+192*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-43)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f
*x+e)+2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*c
os(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^
2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2+\left(a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)\cdots}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^4
 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)
*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
5/2), x)
```

3.38 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{12a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3 f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^3 f\sqrt{c-c\sin(e+fx)}}-\frac{24a^4\cos(e+fx)\log(1-\sin(e+fx)}{c^3 f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+f}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(c^2*f*(c - c*Sin[e + f*x
])^(3/2)) - (24*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (12*a^3*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^3*f*Sqrt[c - c*Sin[e + f*x]]) - (3*a^2*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(3/2))/(c^3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.763642, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{12a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3 f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^3 f\sqrt{c-c\sin(e+fx)}}-\frac{24a^4\cos(e+fx)\log(1-\sin(e+fx)}{c^3 f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(c^2*f*(c - c*Sin[e + f*x
])^(3/2)) - (24*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (12*a^3*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^3*f*Sqrt[c - c*Sin[e + f*x]]) - (3*a^2*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(3/2))/(c^3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
```

```
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{c^2f(c-c\sin(e+fx))^{3/2}} + \frac{(6a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{c^2f(c-c\sin(e+fx))^{3/2}} - \frac{3a^2}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{c^2f(c-c\sin(e+fx))^{3/2}} - \frac{12a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{c^2f(c-c\sin(e+fx))^{3/2}} - \frac{12a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{c^2f(c-c\sin(e+fx))^{3/2}} - \frac{12a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{c^2f(c-c\sin(e+fx))^{3/2}} - \frac{2}{c^3f}$$

Mathematica [A] time = 6.50178, size = 223, normalized size = 0.93

$$\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-320\sin(e+fx)-24\sin(3(e+fx))+\cos(4(e+fx))+\right.}{16c^3f(\text{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(27
3 + Cos[4*(e + f*x)] + Cos[2*(e + f*x)]*(106 - 384*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]) + 1152*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 320*Sin
[e + f*x] - 1536*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 24
*Sin[3*(e + f*x)]))/(16*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.237, size = 365, normalized size = 1.5

$$-\frac{\sin(fx+e)\cos(fx+e)-(\cos(fx+e))^2-2\sin(fx+e)}{2f\left(\sin(fx+e)(\cos(fx+e))^3+(\cos(fx+e))^4-4(\cos(fx+e))^2\sin(fx+e)+3(\cos(fx+e))^3-4\sin(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/2/f*(-cos(f*x+e)^4+12*cos(f*x+e)^2*sin(f*x+e)+96*cos(f*x+e)^2*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))-48*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+192*s
in(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-96*sin(f*x+e)*ln(2/(co
s(f*x+e)+1))-73*cos(f*x+e)^2-58*sin(f*x+e)-192*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+96*ln(2/(cos(f*x+e)+1))+74)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e
)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x
+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*co
s(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(
7/2)
```

Maxima [B] time = 2.38308, size = 1885, normalized size = 7.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/30*(1440*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 720*a
^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + (334*a^(7/2)
- 1449*a^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 2693*a^(7/2)*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 3278*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 3199*a^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2014*a^(7/2)*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 315*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
```

```
 + 10*a^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 525*a^(7/2)*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 + 75*a^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/
(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 17*c^(7/2)*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 32*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 46*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 52*c^(7/2)*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 + 46*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
32*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(7/2)*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8 - 6*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + c^(7
/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10) - (334*a^(7/2) - 2079*a^(7/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 6203*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 10698*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15049*a^(7/2)*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 15354*a^(7/2)*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 12165*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 7410*a^(7
/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 2985*a^(7/2)*sin(f*x + e)^8/(cos(
f*x + e) + 1)^8 - 555*a^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/(c^(7/2)
 - 6*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 17*c^(7/2)*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 32*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 46*c^
(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 52*c^(7/2)*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 46*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32*c^(7/
2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(7/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - 6*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + c^(7/2)*sin(
f*x + e)^10/(cos(f*x + e) + 1)^10) + 10*(75*a^(7/2)*sin(f*x + e)/(cos(f*x +
 e) + 1) - 375*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 854*a^(7/2)*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1257*a^(7/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 1534*a^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1257*a^(7/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 854*a^(7/2)*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 - 375*a^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 75*a^(7/2)*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos
(f*x + e) + 1) + 17*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 32*c^(7/2
)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 46*c^(7/2)*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 52*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 46*c^(7/2)*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 17*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 6*c^(7/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2+\left(a^3\cos\left(fx+e\right)^4-4\,a^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^
4 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4
*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
7/2), x)
```

3.39 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=241

$$\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(3*c*f*(c - c*Sin[e + f*x])^(7/2)
) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c^2*f*(c - c*Sin[e + f
*x])^(5/2)) + (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(c^3*f*(c - c
*Sin[e + f*x])^(3/2)) + (8*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^4*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*a^3*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(c^4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.78103, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(9/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(3*c*f*(c - c*Sin[e + f*x])^(7/2)
) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c^2*f*(c - c*Sin[e + f
*x])^(5/2)) + (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(c^3*f*(c - c
*Sin[e + f*x])^(3/2)) + (8*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^4*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*a^3*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(c^4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
```

```
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{4\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{3c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{5/2}} +$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{5/2}} +$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{5/2}} +$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{5/2}} +$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{5/2}} +$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{5/2}} +$$

Mathematica [A] time = 6.6383, size = 442, normalized size = 1.83

$$\frac{\sin(e+fx)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}{f(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{24(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}{f(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+c\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] (16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(
3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2)) - (
16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2)) + (24*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2)) + (16*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(9/2)) + ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*Sin
[e + f*x]*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^7*(c - c*Sin[e + f*x])^(9/2))
```

Maple [A] time = 0.234, size = 435, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/3/f*(24*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-48*sin(f*x+e)*cos(f*
x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*cos(f*x+e)^4+49*cos(f*x
+e)^2*sin(f*x+e)-72*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+144*cos(f*x+e)^2*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-96*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+1
92*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-63*cos(f*x+e)^2-76
*sin(f*x+e)+96*ln(2/(cos(f*x+e)+1))-192*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))+60)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*
(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)
^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f
*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,a^{3}\cos\left(fx+e\right)^{4}-4\,a^{3}\cos\left(fx+e\right)^{2}+\left(a^{3}\cos\left(fx+e\right)^{4}-4\,a^{3}\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx\cdot\right.}}{5\,c^{5}\cos\left(fx+e\right)^{4}-20\,c^{5}\cos\left(fx+e\right)^{2}+16\,c^{5}-\left(c^{5}\cos\left(fx+e\right)^{4}-12\,c^{5}\cos\left(fx+e\right)^{2}+16\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^
4 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c
^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

3.40 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^3f(c-c\sin(e+fx))^{5/2}}-\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*c*f*(c - c*Sin[e + f*x])^(9/2)
) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c^2*f*(c - c*Sin[e + f*x
])^(7/2)) + (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^3*f*(c - c*S
in[e + f*x])^(5/2)) - (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^4*f*(c
 - c*Sin[e + f*x])^(3/2)) - (a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^5*f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.784987, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^3f(c-c\sin(e+fx))^{5/2}}-\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(11/2)
,x]
```


```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*c*f*(c - c*Sin[e + f*x])^(9/2)
) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c^2*f*(c - c*Sin[e + f*x
])^(7/2)) + (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^3*f*(c - c*S
in[e + f*x])^(5/2)) - (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^4*f*(c
 - c*Sin[e + f*x])^(3/2)) - (a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^5*f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}} + \frac{a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}} + \frac{a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}} + \frac{a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}} + \frac{a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}} + \frac{a}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}} + \frac{a}{}$$

Mathematica [A] time = 6.66159, size = 437, normalized size = 1.8

$$-\frac{8(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}{f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{12(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.}{f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] (4*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (32
```

```
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + (12
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (8*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (2*Log[C
os[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^1
1*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*
(c - c*Sin[e + f*x])^(11/2))
```

Maple [B] time = 0.243, size = 490, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/3/f*(6*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*cos(f*x
+e)^4*ln(2/(cos(f*x+e)+1))-8*cos(f*x+e)^4+24*sin(f*x+e)*cos(f*x+e)^2*ln(-(-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f
*x+e)+1))-8*cos(f*x+e)^2*sin(f*x+e)-48*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))+24*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+28*cos(f*x+e)^2-48
*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*sin(f*x+e)*ln(2/(
cos(f*x+e)+1))+8*sin(f*x+e)+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2
4*ln(2/(cos(f*x+e)+1))-20)*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2
*sin(f*x+e)-2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)
^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f
*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin\left(fx+e\right)+a\right)^{\frac{7}{2}} \cos\left(fx+e\right)^{2}}{\left(-c \sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,a^{3} \cos\left(fx+e\right)^{4}-4\,a^{3} \cos\left(fx+e\right)^{2}+\left(a^{3} \cos\left(fx+e\right)^{4}-4\,a^{3} \cos\left(fx+e\right)^{2}\right) \sin\left(fx+e\right)\right) \sqrt{a \sin\left(fx+e\right)\cdot}}{c^{6} \cos\left(fx+e\right)^{6}-18\,c^{6} \cos\left(fx+e\right)^{4}+48\,c^{6} \cos\left(fx+e\right)^{2}-32\,c^{6}+2\left(3\,c^{6} \cos\left(fx+e\right)^{4}-16\,c^{6} \cos\left(fx+e\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^4
 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(c^6*cos(f*x + e)^6 - 18*c^6*cos(f*x + e)^4 + 48*c^6*cos(f*x
 + e)^2 - 32*c^6 + 2*(3*c^6*cos(f*x + e)^4 - 16*c^6*cos(f*x + e)^2 + 16*c^6
)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

3.41 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}} \, dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(10*a*c*f*(c - c*Sin[e + f*x])^(1
1/2))
```

Rubi [A] time = 0.338952, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(13/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(10*a*c*f*(c - c*Sin[e + f*x])^(1
1/2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}} \, dx &= \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{11/2}} \, dx}{ac} \\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{10acf(c-c\sin(e+fx))^{11/2}}\end{aligned}$$

Mathematica [B] time = 6.71833, size = 412, normalized size = 8.58

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}{f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^{7}} - \frac{4(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right. \cdots}{f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right. \cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] (16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(
5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) -
(8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + (8*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) - (4*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + ((Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

Maple [B] time = 0.204, size = 188, normalized size = 3.9

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{4}-12\left(\cos\left(fx+e\right)\right)^{2}+16\right)\sin\left(fx+e\right)\left(\left(\cos\left(fx+e\right)\right)^{2}-\sin\left(fx+e\right)\cos\right.}{5f\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}+\left(\cos\left(fx+e\right)\right)^{4}-4\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^{3}-4\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] 1/5/f*(cos(f*x+e)^4-12*cos(f*x+e)^2+16)*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e)
*(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(sin(f*x+e)
*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f
*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*
x+e)))^(13/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.83634, size = 371, normalized size = 7.73

$$\frac{\left(5a^{3}\cos\left(fx+e\right)^{4}-20a^{3}\cos\left(fx+e\right)^{2}+16a^{3}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{5\left(5c^{7}f\cos\left(fx+e\right)^{5}-20c^{7}f\cos\left(fx+e\right)^{3}+16c^{7}f\cos\left(fx+e\right)-\left(c^{7}f\cos\left(fx+e\right)^{5}-12c^{7}f\cos\left(fx+e\right)^{3}+16c^{7}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="fricas")
```

```
[Out] 1/5*(5*a^3*cos(f*x + e)^4 - 20*a^3*cos(f*x + e)^2 + 16*a^3)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^7*f*cos(f*x + e)^5 - 20*c^7*f*cos(
f*x + e)^3 + 16*c^7*f*cos(f*x + e) - (c^7*f*cos(f*x + e)^5 - 12*c^7*f*cos(f
*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
13/2), x)
```

3.42 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{120ac^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(12*a*c*f*(c - c*Sin[e + f*x])^(1
3/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(120*a*c^2*f*(c - c*Sin[e
 + f*x])^(11/2))
```

Rubi [A] time = 0.444469, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{120ac^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(15/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(12*a*c*f*(c - c*Sin[e + f*x])^(1
3/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(120*a*c^2*f*(c - c*Sin[e
 + f*x])^(11/2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{13/2}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{12ac^2}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{120ac^2f(c-c\sin(e+fx))^{11/2}}$$

Mathematica [B] time = 6.75914, size = 419, normalized size = 4.32

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}{2f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} - \frac{8(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.}{3f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(15/2),x]
```

```
[Out] (8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(3
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (
32*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(5
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (
6*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (8*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + ((Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))
```

Maple [B] time = 0.225, size = 233, normalized size = 2.4

$$\frac{\left(3\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4-18\left(\cos\left(fx+e\right)\right)^4-36\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+116\left(\cos\left(fx+e\right)\right)^2+48\sin\left(f\right.\right.}{30f\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3+\left(\cos\left(fx+e\right)\right)^4-4\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x)
```

```
[Out] 1/30/f*(3*sin(f*x+e)*cos(f*x+e)^4-18*cos(f*x+e)^4-36*cos(f*x+e)^2*sin(f*x+e
)+116*cos(f*x+e)^2+48*sin(f*x+e)-128)*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(
sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(sin(f*x+e)*c
os(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x
+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+
e)))^(15/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, al
gorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.91221, size = 478, normalized size = 4.93

$$-\frac{\left(15\,a^{3}\cos\left(fx+e\right)^{4}-60\,a^{3}\cos\left(fx+e\right)^{2}+48\,a^{3}-4\left(5\,a^{3}\cos\left(fx+e\right)^{2}-8\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a}}{30\left(c^{8}f\cos\left(fx+e\right)^{7}-18\,c^{8}f\cos\left(fx+e\right)^{5}+48\,c^{8}f\cos\left(fx+e\right)^{3}-32\,c^{8}f\cos\left(fx+e\right)+2\left(3\,c^{8}f\cos\left(fx+e\right)^{5}-1\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, al
gorithm="fricas")
```

```
[Out] -1/30*(15*a^3*cos(f*x + e)^4 - 60*a^3*cos(f*x + e)^2 + 48*a^3 - 4*(5*a^3*co
s(f*x + e)^2 - 8*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c)/(c^8*f*cos(f*x + e)^7 - 18*c^8*f*cos(f*x + e)^5 + 48*c^8*f*cos(
f*x + e)^3 - 32*c^8*f*cos(f*x + e) + 2*(3*c^8*f*cos(f*x + e)^5 - 16*c^8*f*c
os(f*x + e)^3 + 16*c^8*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, al
gorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
15/2), x)
```

3.43 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal. Leaf size=145

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{840ac^3f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(14*a*c*f*(c - c*Sin[e + f*x])^(1
5/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(84*a*c^2*f*(c - c*Sin[e
+ f*x])^(13/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(840*a*c^3*f*(c
 - c*Sin[e + f*x])^(11/2))
```

Rubi [A] time = 0.538983, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{840ac^3f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(17/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(14*a*c*f*(c - c*Sin[e + f*x])^(1
5/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(84*a*c^2*f*(c - c*Sin[e
+ f*x])^(13/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(840*a*c^3*f*(c
 - c*Sin[e + f*x])^(11/2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{15/2}}dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{13/2}}dx}{7ac^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}} + \frac{\int}{}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}} + \frac{\cos}{8}$$

Mathematica [B] time = 6.83592, size = 419, normalized size = 2.89

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}{3f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} - \frac{2(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}{f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(17/2),x]
```

```
[Out] (16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(
7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
(16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(
3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) +
(24*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(
5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
(2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) + ((C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2))
```

Maple [A] time = 0.251, size = 243, normalized size = 1.7

$$\frac{\left(9\left(\cos(fx+e)\right)^6+63\sin(fx+e)\left(\cos(fx+e)\right)^4-216\left(\cos(fx+e)\right)^4-406\left(\cos(fx+e)\right)^2\sin(fx+e)+790\right.}{105f\left(\sin(fx+e)\left(\cos(fx+e)\right)^3+\left(\cos(fx+e)\right)^4-4\left(\cos(fx+e)\right)^2\sin(fx+e)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x)
```

```
[Out] 1/105/f*(9*cos(f*x+e)^6+63*sin(f*x+e)*cos(f*x+e)^4-216*cos(f*x+e)^4-406*cos
(f*x+e)^2*sin(f*x+e)+790*cos(f*x+e)^2+448*sin(f*x+e)-688)*(a*(1+sin(f*x+e))
)^(7/2)*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x
+e)+2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*co
s(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)
+8)/(-c*(-1+sin(f*x+e)))^(17/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, al
gorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.99511, size = 517, normalized size = 3.57

$$-\frac{\left(35\,a^{3}\cos\left(fx+e\right)^{4}-154\,a^{3}\cos\left(fx+e\right)^{2}+128\,a^{3}-14\left(5\,a^{3}\cos\left(fx+e\right)^{2}-8\,a^{3}\right)\sin\left(fx\right.}{105\left(7\,c^{9}f\cos\left(fx+e\right)^{7}-56\,c^{9}f\cos\left(fx+e\right)^{5}+112\,c^{9}f\cos\left(fx+e\right)^{3}-64\,c^{9}f\cos\left(fx+e\right)-\left(c^{9}f\cos\left(fx+e\right)^{7}-24\,c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, al
gorithm="fricas")
```

```
[Out] -1/105*(35*a^3*cos(f*x + e)^4 - 154*a^3*cos(f*x + e)^2 + 128*a^3 - 14*(5*a^
3*cos(f*x + e)^2 - 8*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(7*c^9*f*cos(f*x + e)^7 - 56*c^9*f*cos(f*x + e)^5 + 112*c^9
*f*cos(f*x + e)^3 - 64*c^9*f*cos(f*x + e) - (c^9*f*cos(f*x + e)^7 - 24*c^9*
f*cos(f*x + e)^5 + 80*c^9*f*cos(f*x + e)^3 - 64*c^9*f*cos(f*x + e))*sin(f*x
 + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(17/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, al
gorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
17/2), x)
```

3.44 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.305721, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx}{ac}\\ &= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [B] time = 0.902693, size = 134, normalized size = 2.98

$$\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(56\sin(e+fx)-8\sin(3(e+fx))+28\cos}{32f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c
*Sin[e + f*x]]*(28*Cos[2*(e + f*x)] - Cos[4*(e + f*x)] + 56*Sin[e + f*x] -
8*Sin[3*(e + f*x)]))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*Sqrt[a*(
1 + Sin[e + f*x])])
```

Maple [B] time = 0.231, size = 195, normalized size = 4.3

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{4}+\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}-4\left(\cos\left(fx+e\right)\right)^{3}+3\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-4\left(\cos\left(fx+e\right)\right)^{2}\cdots}{4f\left(\left(\cos\left(fx+e\right)\right)^{3}+\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-3\left(\cos\left(fx+e\right)\right)^{2}+2\sin\left(fx+e\right)\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*(cos(f*x+e)^4+sin(f*x+e)*cos(f*x+e)^3-4*cos(f*x+e)^3+3*cos(f*x+e)^2*s
in(f*x+e)-4*cos(f*x+e)^2-7*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)-sin(f*x+e)-1)
*sin(f*x+e)*(-c*(-1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e
)-3*cos(f*x+e)^2+2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1
+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)^2/sqrt(a*sin(f*x + e) +
a), x)
```

Fricas [B] time = 1.70762, size = 235, normalized size = 5.22

$$-\frac{\left(c^{2}\cos\left(fx+e\right)^{4}-8\,c^{2}\cos\left(fx+e\right)^{2}+7\,c^{2}+4\left(c^{2}\cos\left(fx+e\right)^{2}-2\,c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx\cdots\right.}}{4\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(c^2*cos(f*x + e)^4 - 8*c^2*cos(f*x + e)^2 + 7*c^2 + 4*(c^2*cos(f*x +
e)^2 - 2*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
```

```
c)/(a*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)^2/sqrt(a*sin(f*x + e) +
a), x)
```

3.45 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.309111, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx}{ac}\\ &= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [B] time = 0.519238, size = 120, normalized size = 2.67

$$-\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(15\sin(e+fx)-\sin(3(e+fx))+6\cos(2(e+}{12f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] -(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Si
n[e + f*x]]*(6*Cos[2*(e + f*x)] + 15*Sin[e + f*x] - Sin[3*(e + f*x)]))/(12*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] time = 0.219, size = 141, normalized size = 3.1

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{3}-\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+2\left(\cos\left(fx+e\right)\right)^{2}+3\sin\left(fx+e\right)\cos\left(fx+e\right)-4\cos\left(fx+e\right)+s\right.}{3f\left(\left(\cos\left(fx+e\right)\right)^{2}-\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(fx+e\right)+2\sin\left(fx+e\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)+2*cos(f*x+e)^2+3*sin(f*x+e)*cos
(f*x+e)-4*cos(f*x+e)+sin(f*x+e)+1)*(-c*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)/(c
os(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a*(1+sin(f*x+
e)))^(1/2)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)^2/sqrt(a*sin(f*x + e) +
a), x)
```

Fricas [A] time = 1.66322, size = 192, normalized size = 4.27

$$\frac{\left(3\,c\cos\left(fx+e\right)^{2}-\left(c\cos\left(fx+e\right)^{2}-4\,c\right)\sin\left(fx+e\right)-3\,c\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*(3*c*cos(f*x + e)^2 - (c*cos(f*x + e)^2 - 4*c)*sin(f*x + e) - 3*c)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)^2/sqrt(a*sin(f*x + e) +
a), x)
```

3.46 $\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.283838, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx}{ac}\\ &= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.279765, size = 62, normalized size = 1.38

$$\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(4\sin(e+fx)+\cos(2(e+fx)))}{4af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(Cos[2*(e + f*x)] + 4*Sin[e + f*x]
)*Sqrt[c - c*Sin[e + f*x]])/(4*a*f)
```

Maple [B] time = 0.211, size = 90, normalized size = 2.

$$\frac{\left(\left(\cos\left(fx+e\right)\right)^{2}+\sin\left(fx+e\right)\cos\left(fx+e\right)-2\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{2f\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\frac{\quad}{\sqrt{\quad}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+sin(f*x+e)+1)*sin(f*
x+e)*(-c*(-1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)
))^(1/2)
```

Maxima [B] time = 1.857, size = 524, normalized size = 11.64

$$-\frac{\frac{2\sqrt{a}\sqrt{c}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}{a+\frac{2a\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}-\frac{2\sqrt{a}\sqrt{c}-\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}{a+\frac{2a\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}+\frac{2\left(\frac{\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sqrt{a}\sqrt{c}\sin\left(fx+\right.}{\left(\cos\left(fx+e\right)+1\right.}\right.}{a+\frac{2a\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a}{\left(\cos\right.}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*((2*sqrt(a)*sqrt(c) + sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1)
+ sqrt(a)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - (2*sqrt(a)*sqrt(c) - sqrt(a)
*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(a)*sqrt(c)*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - sqrt(a)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
)/(a + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4) + 2*(sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) - sqrt(a)
*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3)/(a + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Fricas [A] time = 1.62901, size = 153, normalized size = 3.4

$$\frac{\left(\cos\left(fx+e\right)^{2}+2\sin\left(fx+e\right)-1\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/2*(cos(f*x + e)^2 + 2*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*
sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^2\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)**2/sqrt(a*(sin(e + f*x) +
 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^2}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/sqrt(a*sin(f*x + e) + a)
, x)
```

3.47 $\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]))
```

Rubi [A] time = 0.288419, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.296315, size = 44, normalized size = 1.02

$$\frac{\sin(2(e+fx))}{2f\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
),x]
```

```
[Out] Sin[2*(e + f*x)]/(2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.21, size = 42, normalized size = 1.

$$\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{f}\frac{1}{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}}\frac{1}{\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/f*cos(f*x+e)*sin(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(-1+sin(f*x+e)))^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)), x)
```

Fricas [A] time = 1.66291, size = 116, normalized size = 2.7

$$\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(a*c*f*cos(
f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{2}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x)
- 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)), x)
```

3.48 $\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \, dx$

Optimal. Leaf size=54

$$-\frac{\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]))
```

Rubi [A] time = 0.339794, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2737, 2667, 31}

$$-\frac{\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]
```



```
[Out] -((Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]))
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= -\frac{\cos(e+fx)\operatorname{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= -\frac{\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.412908, size = 104, normalized size = 1.93

$$-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/
2)),x]
```

```
[Out] (-2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Sqrt[a*(1 + Sin[e + f*x
])]*(c - c*Sin[e + f*x])^(3/2))
```

Maple [B] time = 0.214, size = 141, normalized size = 2.6

$$-\frac{\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^2-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)}{2f\left(-1+\cos\left(fx+e\right)\right)}\left(2\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(2/(cos(f*x+e)+1)))*
(1-cos(f*x+e)+sin(f*x+e))*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-
cos(f*x+e)+2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/(-c*(-1+sin(f*x+e)))
^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3
/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{ac^2\sin\left(fx+e\right)-ac^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c^2*sin(f*x
 + e) - a*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1)
)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3
/2)), x)
```

3.49 $\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

```
[Out] Cos[e + f*x]/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.321385, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$\frac{\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] Cos[e + f*x]/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx &= \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}\\ &= \frac{\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.499594, size = 79, normalized size = 1.88

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x])^(5/2))
```

Maple [A] time = 0.21, size = 51, normalized size = 1.2

$$-\frac{\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}{f}\frac{1}{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(-1+sin(f*x+e))*cos(f*x+e)*sin(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(-1
+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5
/2)), x)
```

Fricas [A] time = 1.68165, size = 151, normalized size = 3.6

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{3}f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c^3*f*cos(f*x + e)*s
in(f*x + e) - a*c^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5
/2)), x)
```

3.50 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=239

$$\frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a\sin(e+fx)+a}}+\frac{16c^4\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+$$

```
[Out] (16*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) + (8*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/
2))/(a*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*(c - c*Sin[e + f*x])
^(5/2))/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x
])^(7/2))/(4*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.744696, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$\frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a\sin(e+fx)+a}}+\frac{16c^4\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(3/2),
x]
```


```
[Out] (16*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) + (8*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/
2))/(a*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*(c - c*Sin[e + f*x])
^(5/2))/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x
])^(7/2))/(4*a*f*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{9/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af\sqrt{a+a\sin(e+fx)}} + \frac{(4c)}{}$$

$$= \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a+a\sin(e+fx)}} + \frac{c}{}$$

$$= \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c}{}$$

$$= \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c}{}$$

$$= \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c}{}$$

$$= \frac{16c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} +$$

Mathematica [A] time = 6.4823, size = 471, normalized size = 1.97

$$\frac{5\sin(3(e+fx))(c-c\sin(e+fx))^{7/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{12f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7} - \frac{23\cos(2(e+fx))(c-c\sin(e+fx))^{7/2}\left(\sin\left(\right.\right.}{8f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-23*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e
+ f*x])^(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e +
```

```
f*x]))^(3/2)) + (Cos[4*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*
(c - c*Sin[e + f*x])^(7/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(
a*(1 + Sin[e + f*x]))^(3/2)) + (32*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(7/2))/(f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/2)) - (65*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[e + f*x]*(c - c*Sin[e + f*x])^(7/
2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/
2)) + (5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(7/2)
*Sin[3*(e + f*x)])/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Si
n[e + f*x]))^(3/2))
```

Maple [A] time = 0.238, size = 252, normalized size = 1.1

$$\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)}{12f\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}-\left(\cos\left(fx+e\right)\right)^{4}-4\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-3\left(\cos\left(fx+e\right)\right)^{3}-4\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/12/f*(-3*cos(f*x+e)^4-20*cos(f*x+e)^2*sin(f*x+e)+72*cos(f*x+e)^2+192*ln(2
/(cos(f*x+e)+1))+200*sin(f*x+e)-384*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)-69)*(-c*(-1+sin(f*x+e)))^(7/2)*(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(
f*x+e)+cos(f*x+e)-2)/(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-4*cos(f*x+e)^2*s
in(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)^2+8*sin(f*x+e
)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3c^{3}\cos\left(fx+e\right)^{4}-4c^{3}\cos\left(fx+e\right)^{2}-\left(c^{3}\cos\left(fx+e\right)^{4}-4c^{3}\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right.}}{a^{2}\cos\left(fx+e\right)^{2}-2a^{2}\sin\left(fx+e\right)-2a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*c^3*cos(f*x + e)^4 - 4*c^3*cos(f*x + e)^2 - (c^3*cos(f*x + e)^4
 - 4*c^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
3/2), x)
```

3.51 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{8c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a\sin(e+fx)+a}}+$$

```
[Out] (8*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (4*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*
f*Sqrt[a + a*Sin[e + f*x]]) + (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(
a*f*Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(
3*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.639499, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$\frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{8c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a\sin(e+fx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (4*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*
f*Sqrt[a + a*Sin[e + f*x]]) + (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(
a*f*Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(
3*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(4c)\int}{}$$

$$= \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \cos($$

$$= \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \cos($$

$$= \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \cos($$

$$= \frac{8c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} +$$

Mathematica [A] time = 2.41881, size = 138, normalized size = 0.73

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-87\sin(e+fx)+\sin(3(e+fx))-12\cos(2(e+fx))+192\log\right.}{12f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-12*
Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 87*Sin[e
+ f*x] + Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*
(1 + Sin[e + f*x]))^(3/2))
```

Maple [A] time = 0.202, size = 213, normalized size = 1.1

$$\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2}{3f\left(\left(\cos\left(fx+e\right)\right)^{3}+\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-3\left(\cos\left(fx+e\right)\right)^{2}+2\sin\left(fx+e\right)\cos\left(fx+e\right)-2\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)^2*sin(f*x+e)-6*cos(f*x+e)^2-22*sin(f*x+e)+48*ln((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))-24*ln(2/(cos(f*x+e)+1))+6)*(-c*(-1+sin(f*x+e))
)^(5/2)*(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(cos
(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+2*sin(f*x+e)*cos(f*x+e)-2*
cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^{2}\cos\left(fx+e\right)^{4}+2\,c^{2}\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-2\,c^{2}\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((c^2*cos(f*x + e)^4 + 2*c^2*cos(f*x + e)^2*sin(f*x + e) - 2*c^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f
*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
3/2), x)
```

3.52 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (4*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*
Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*a*
f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.529118, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (4*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*
Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*a*
f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} + \frac{(4c)\int}{}$$

$$= \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} + \frac{(4c^2 c}{\sqrt{a+a}}$$

$$= \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} + \frac{(4c^2 co}{a}$$

$$= \frac{4c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} +$$

Mathematica [A] time = 1.08957, size = 134, normalized size = 0.92

$$\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(12\sin(e+fx)+\cos(2(e+fx))-32\log\left(\sin\right.\right.}{4f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e + f*x])*(Cos[2*(e +
f*x)] - 32*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 12*Sin[e + f*x])*Sqrt
[c - c*Sin[e + f*x]])/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 +
Sin[e + f*x]))^(3/2))
```

Maple [A] time = 0.183, size = 172, normalized size = 1.2

$$-\frac{\sin(fx+e)\cos(fx+e)+(\cos(fx+e))^2-2\sin(fx+e)+\cos(fx+e)-2}{2f\left(\sin(fx+e)\cos(fx+e)-(\cos(fx+e))^2-2\sin(fx+e)-\cos(fx+e)+2\right)}\left(\left((\cos(fx+e))^2+6\sin(fx+e)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)^2+6*sin(f*x+e)-16*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))+8*ln(2/(cos(f*x+e)+1))-1)*(-c*(-1+sin(f*x+e)))^(3/2)*(sin(f*x+e)*cos(f*x
+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x+
e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [B] time = 1.88745, size = 1139, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*(16*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 8*c^(3/
2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - (10*c^(3/2) + 13*
c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 25*c^(3/2)*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 20*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*c^(3/2)
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9*c^(3/2)*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5)/(a^(3/2) + 2*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^(3/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^(3/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 2*a^(3/2)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + a^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6) + (10*c^(3/2) + 11*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 15*c^(3/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*c^(3/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7*c^(3/2)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(3/2) + 2*a^(3/2)*sin(f*x + e)/(cos(f*x
+ e) + 1) + 3*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^(3/2)*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 2*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^(3/2)*sin(f*x + e)^
6/(cos(f*x + e) + 1)^6) - 2*(5*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5*
c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*c^(3/2)*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*c^(3/2
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(3/2) + 2*a^(3/2)*sin(f*x + e)/(c
os(f*x + e) + 1) + 3*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^(3/2
)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^(3/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 2*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^(3/2)*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-c\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((c*cos(f*x + e)^2*sin(f*x + e) - c*cos(f*x + e)^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e
) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
3/2), x)
```

3.53 $\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*Sqrt[a
 + a*Sin[e + f*x]])
```

Rubi [A] time = 0.411528, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2740, 2737, 2667, 31}

$$\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3/2),x]
```


```
[Out] (2*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*Sqrt[a
 + a*Sin[e + f*x]])
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{(2c\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{(2c\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+f}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{2c\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 1.0568, size = 113, normalized size = 1.18

$$-\frac{\sqrt{c-c\sin(e+fx)}\left(-4\log\left(e^{i(e+fx)}+i\right)+\sin(e+fx)+2ifx\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((2*I)*f*x - 4*Log[I + E^(I*(e +
 f*x))] + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)))
```

Maple [A] time = 0.215, size = 133, normalized size = 1.4

$$\frac{\sin(fx+e)\cos(fx+e)+\left(\cos(fx+e)\right)^2-2\sin(fx+e)+\cos(fx+e)-2}{f\left(-1+\cos(fx+e)+\sin(fx+e)\right)}\left(\sin(fx+e)-4\ln\left(\frac{1-\cos(fx+e)+s}{\sin(fx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(sin(f*x+e)-4*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*ln(2/(cos(f*x+
e)+1)))*(-c*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*si
```

```
n(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(3/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^{2}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)**2/(a*(sin(e + f*x) + 1))
**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(3/
2), x)
```

3.54 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rubi [A] time = 0.333223, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2737, 2667, 31}

$$\frac{\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.432335, size = 102, normalized size = 2.

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] (2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(f*(a*(1 + Sin[e + f*x]))^(
3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.208, size = 136, normalized size = 2.7

$$\frac{\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^2-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2\right)}{2f\left(-1+\cos\left(fx+e\right)\right)}\left(-\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(-ln(2/(cos(f*x+e)+1))+2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))*(-
1+cos(f*x+e)+sin(f*x+e))*(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+c
os(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)/(-c*(-1+sin(f*x+e)))^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^{2}c\sin\left(fx+e\right)+a^{2}c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*c*sin(f*x
+ e) + a^2*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*
x) - 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

3.55 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{acf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])
```

Rubi [A] time = 0.339742, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{acf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)\int \sec(e+fx)\,dx}{ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{acf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.535731, size = 103, normalized size = 1.98

$$\frac{\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{cf(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (Cos[e + f*x]^3*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))
^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.177, size = 173, normalized size = 3.3

$$\frac{\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^2-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(f\right.\right.\right.}{2f\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e)))*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e
)+2)*(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos
(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)

Fricas [A] time = 1.95157, size = 404, normalized size = 7.77

$$\left[\frac{\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^3-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^3}\right)}{2\,a^2c^2f}, -\frac{\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)}{a^2c^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [1/2*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)/(a^2*c^2*f), -sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))/(a^2*c^2*f)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)

3.56 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} dx$

Optimal. Leaf size=104

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2acf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

```
[Out] Cos[e + f*x]/(2*a*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
+ (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c^2*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.435843, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2acf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] Cos[e + f*x]/(2*a*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
+ (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c^2*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}}{2ac^2}$$

$$= \frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{\cos(e+fx)\int \mathrm{se}}{2ac^2\sqrt{a+a\sin(e+fx}}$$

$$= \frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{\tanh^{-1}(\sin(e+f}{2ac^2f\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 0.773807, size = 163, normalized size = 1.57

$$\frac{\cos^3(e+fx)\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx)\left(\log\left(\mathrm{co}\right.\right.\right.}{2c^2f(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (Cos[e + f*x]^3*(1 - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] -
 Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c^2*f*(-1 + Si
n[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.187, size = 245, normalized size = 2.4

$$-\frac{\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^2-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx\right.\right.\right.}{4f\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/4/f*(sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+sin(f*x+e)-ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))*(sin(f*x+e)*cos
(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(sin(f*x+e)*cos(f*x+e)+cos(
f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(3/2
)/(-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

Fricas [A] time = 2.00882, size = 814, normalized size = 7.83

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)}{4\left(a^{2}c^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-a^{2}c^{3}f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)*sin(f*x + e) - a^2*c
^3*f*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x +
 e))*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(
a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c))/(a^2*c^3*f*cos(f*x + e)*sin(f*x + e) - a^2*c^3*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

3.57 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}} dx$

Optimal. Leaf size=285

$$-\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{10c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-80*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (40*c^4*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]]
)/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (10*c^3*Cos[e + f*x]*(c - c*Sin[e + f*
x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (10*c^2*Cos[e + f*x]*(c - c*S
in[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (5*c*Cos[e + f*x]*
(c - c*Sin[e + f*x])^(7/2))/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f
*x]*(c - c*Sin[e + f*x])^(9/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.864944, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{10c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```


```
[Out] (-80*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (40*c^4*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]]
)/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (10*c^3*Cos[e + f*x]*(c - c*Sin[e + f*
x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (10*c^2*Cos[e + f*x]*(c - c*S
in[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (5*c*Cos[e + f*x]*
(c - c*Sin[e + f*x])^(7/2))/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f
*x]*(c - c*Sin[e + f*x])^(9/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{11/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{5\int \frac{(c-c\sin(e+fx))^{9/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{5c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(10c$$

$$= -\frac{10c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} - \frac{5c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{80c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+f}{a^2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 6.66968, size = 553, normalized size = 1.94

$$\frac{203\sin(e+fx)(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{4f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}+\frac{47\cos(2(e+fx))(c-c\sin(e+fx))^{9/2}\left(\mathrm{si}\right.}{8f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (-32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(9/2))/(f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (4
7*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f
*x])^(9/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*
x]))^(5/2)) - (Cos[4*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c
- c*Sin[e + f*x])^(9/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(
1 + Sin[e + f*x]))^(5/2)) - (160*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2))/(f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (203*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*x]*(c - c*Sin[e + f*x])^(9/2
))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2
)) - (7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2)*
Sin[3*(e + f*x)])/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin
[e + f*x]))^(5/2))
```

Maple [A] time = 0.199, size = 347, normalized size = 1.2

$$\frac{\sin(fx+e)\cos(fx+e)+(\cos\ldots}{12f\left(\left(\cos(fx+e)\right)^5+\sin(fx+e)\left(\cos(fx+e)\right)^4-5\left(\cos(fx+e)\right)^4+4\sin(fx+e)\left(\cos(fx+e)\right)^3-8\left(\cos(fx\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/12/f*(3*sin(f*x+e)*cos(f*x+e)^4-25*cos(f*x+e)^4-116*cos(f*x+e)^2*sin(f*x+
e)+500*cos(f*x+e)^2+1920*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e
)-960*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-859*sin(f*x+e)+1920*ln((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))-960*ln(2/(cos(f*x+e)+1))-475)*(-c*(-1+sin(f*x+e)))^
(9/2)*(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(cos(f
*x+e)^5+sin(f*x+e)*cos(f*x+e)^4-5*cos(f*x+e)^4+4*sin(f*x+e)*cos(f*x+e)^3-8*
cos(f*x+e)^3-12*cos(f*x+e)^2*sin(f*x+e)+20*cos(f*x+e)^2-8*sin(f*x+e)*cos(f*
x+e)+8*cos(f*x+e)+16*sin(f*x+e)-16)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(9/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^4\cos\left(fx+e\right)^6-8\,c^4\cos\left(fx+e\right)^4+8\,c^4\cos\left(fx+e\right)^2+4\left(c^4\cos\left(fx+e\right)^4-2\,c^4\cos\left(fx+e\right)^2\right)\sin\left(fx+\right.\right.}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(c^4*cos(f*x + e)^6 - 8*c^4*cos(f*x + e)^4 + 8*c^4*cos(f*x + e)^2
 + 4*(c^4*cos(f*x + e)^4 - 2*c^4*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3
*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(9/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

3.58 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{32c^4\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+f}}$$

```
[Out] (-32*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (16*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]]
)/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (4*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (4*c*Cos[e + f*x]*(c - c*Sin[e
 + f*x])^(5/2))/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*S
in[e + f*x])^(7/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.751905, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{32c^4\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```


```
[Out] (-32*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (16*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]]
)/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (4*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (4*c*Cos[e + f*x]*(c - c*Sin[e
 + f*x])^(5/2))/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*S
in[e + f*x])^(7/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
```

```
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{4\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{4c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(8c)}{}$$

$$= -\frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{32c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+f}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 5.05005, size = 179, normalized size = 0.76

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(396\sin(e+fx)+16\sin(3(e+fx))-172\cos(2(e+fx))+\dots\right.}{24f(a(\sin(e+fx)+1))^{5/\dots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-177
 - 172*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] - 1536*Log[Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]] + 396*Sin[e + f*x] - 1536*Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]]*Sin[e + f*x] + 16*Sin[3*(e + f*x)]))/(24*f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A] time = 0.238, size = 305, normalized size = 1.3

$$-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^2-2\sin\left(fx+e\right)\dots}{3f\left(\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3-\left(\cos\left(fx+e\right)\right)^4-4\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-3\left(\cos\left(fx+e\right)\right)^3-4\sin\left(fx\dots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/3/f*(cos(f*x+e)^4+8*cos(f*x+e)^2*sin(f*x+e)+96*sin(f*x+e)*ln(2/(cos(f*x+
e)+1))-192*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-44*cos(f*x+e
)^2+91*sin(f*x+e)+96*ln(2/(cos(f*x+e)+1))-192*ln((1-cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))+43)*(-c*(-1+sin(f*x+e)))^(7/2)*(sin(f*x+e)*cos(f*x+e)+cos(f*x+e
)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-4*cos(
f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)^2+8
*sin(f*x+e)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,c^3\cos\left(fx+e\right)^4-4\,c^3\cos\left(fx+e\right)^2-\left(c^3\cos\left(fx+e\right)^4-4\,c^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*c^3*cos(f*x + e)^4 - 4*c^3*cos(f*x + e)^2 - (c^3*cos(f*x + e)^4
 - 4*c^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)
*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

3.59 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}} dx$

Optimal. Leaf size=191

$$-\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{12c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2a^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-12*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (6*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])
/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (3*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*Sin[e + f*
x])^(5/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.6388, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{12c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2a^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```


```
[Out] (-12*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (6*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])
/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (3*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*Sin[e + f*
x])^(5/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{3\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{3c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(6c)}{}$$

$$= -\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{3c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2a^2f\sqrt{a+a\sin(e+fx)}} - \text{co}$$

$$= -\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{3c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2a^2f\sqrt{a+a\sin(e+fx)}} - \text{co}$$

$$= -\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{3c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2a^2f\sqrt{a+a\sin(e+fx)}} - \text{co}$$

$$= -\frac{12c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 2.42963, size = 164, normalized size = 0.86

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(3(e+fx))-18\cos(2(e+fx))-192\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{8f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\text{s}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-44
- 18*Cos[2*(e + f*x)] - 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (39
- 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f
*x)]))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5
/2))
```

Maple [A] time = 0.217, size = 268, normalized size = 1.4

$$\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2}{2f\left(\left(\cos\left(fx+e\right)\right)^{3}+\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-3\left(\cos\left(fx+e\right)\right)^{2}+2\sin\left(fx+e\right)\cos\left(fx+e\right)-2\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/2/f*(cos(f*x+e)^2*sin(f*x+e)-9*cos(f*x+e)^2+24*sin(f*x+e)*ln(2/(cos(f*x+e
)+1))-48*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+25*sin(f*x+e)+
24*ln(2/(cos(f*x+e)+1))-48*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+9)*(-c*
(-1+sin(f*x+e)))^(5/2)*(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos
(f*x+e)-2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+2*sin(f*x+e
)*cos(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [B] time = 2.00674, size = 1512, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/6*(144*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 72*c^(5
/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - (46*c^(5/2) + 19
9*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 335*c^(5/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 509*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 496*c^
(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 373*c^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 219*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*c^(
5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 8*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*a^
(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a^(5/2)*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 12*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*a^(5/2
)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a^(5/2)*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) + (46*c^(5/2) + 1
21*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 149*c^(5/2)*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 179*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 148*c
^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 43*c^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 33*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15*c^(5
/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 8*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*a^(
5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a^(5/2)*sin(f*x + e)^4/(cos(f
```

```
*x + e) + 1)^4 + 12*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*a^(5/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a^(5/2)*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7 + a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) - 6*(13*c^(5/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 39*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 55*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 74*c^(5/2)*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 55*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 + 39*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 13*c^(5/2)*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)/(cos(f*x + e) +
1) + 8*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*a^(5/2)*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 14*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 12*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*a^(5/2)*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 + 4*a^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + a^(5
/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2\cos\left(fx+e\right)^4+2\,c^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,c^2\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((c^2*cos(f*x + e)^4 + 2*c^2*cos(f*x + e)^2*sin(f*x + e) - 2*c^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos
(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

3.60 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}} dx$

Optimal. Leaf size=143

$$-\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] (-4*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) - (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/
(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.538421, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2841, 2739, 2740, 2737, 2667, 31}

$$-\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-4*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) - (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/
(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
```

```
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{2\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(4c)\int}{}$$

$$= -\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(4c^2}{a\sqrt{a+}}$$

$$= -\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(4c^2 c}{}$$

$$= -\frac{4c^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2 f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2 f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 1.05932, size = 153, normalized size = 1.07

$$-\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos(2(e+fx))+16\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\right.}{2f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(7 + C
os[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(-1 + 8*L
```

```
og[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A] time = 0.19, size = 228, normalized size = 1.6

$$\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2}{f\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos\left(fx+e\right)\right)^{2}-2\sin\left(fx+e\right)-\cos\left(fx+e\right)+2\right)}\left(4\sin\left(fx+e\right)\ln\left(2\left(\cos\left(fx+e\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(4*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-8*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))*sin(f*x+e)-cos(f*x+e)^2+5*sin(f*x+e)+4*ln(2/(cos(f*x+e)+1))-8*ln((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+1)*(-c*(-1+sin(f*x+e)))^(3/2)*(sin(f*x+e
)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(sin(f*x+e)*cos(f*x+e)
-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-c\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((c*cos(f*x + e)^2*sin(f*x + e) - c*cos(f*x + e)^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*co
s(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

3.61 $\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -((c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])) - (Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*(a
+ a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.423393, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2739, 2737, 2667, 31}

$$-\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])) - (Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*(a
+ a*Sin[e + f*x])^(3/2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx &= \frac{\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac} \\
&= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2} \\
&= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(c\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{a\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(c\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+f\right.}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{c\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.887886, size = 100, normalized size = 1.03

$$\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}\left(-2\log\left(e^{i(e+fx)}+i\right)+\left(ifx-2\log\left(e^{i(e+fx)}+i\right)\right)\sin(e+fx)+ifx-2\right)}{a^2f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]]*(-2 + I*f*x - 2*Log[I + E^(I*(e + f*
x))] + (I*f*x - 2*Log[I + E^(I*(e + f*x))])*Sin[e + f*x]))/(a^2*f*Sqrt[a*(1
 + Sin[e + f*x])])
```

Maple [B] time = 0.213, size = 190, normalized size = 2.

$$\frac{\sin(fx+e)\cos(fx+e)+(\cos(fx+e))^2-2\sin(fx+e)+\cos(fx+e)-2}{f(-1+\cos(fx+e)+\sin(fx+e))}\left(2\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\text{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-sin(f*x+e)*ln(2/
(cos(f*x+e)+1))-2*sin(f*x+e)+2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(
2/(cos(f*x+e)+1)))*(-c*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*cos(f*x+e)+cos(f*
```

```
x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+
e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(5/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^2}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(5/
2), x)
```

3.62 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -(Cos[e + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Rubi [A] time = 0.320839, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2738}

$$-\frac{\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -(Cos[e + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}\\ &= -\frac{\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.513351, size = 80, normalized size = 1.86

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{f(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3)/(f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [A] time = 0.204, size = 50, normalized size = 1.2

$$\frac{\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}{f}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{-\frac{5}{2}}\frac{1}{\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(1+sin(f*x+e))*cos(f*x+e)*sin(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(-1+s
in(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

Fricas [A] time = 1.68484, size = 151, normalized size = 3.51

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^3cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^3cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^3*c*f*cos(f*x + e)*s
in(f*x + e) + a^3*c*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

3.63 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2acf(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -Cos[e + f*x]/(2*a*c*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])
 + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.443844, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2acf(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] -Cos[e + f*x]/(2*a*c*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])
 + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-}}}{2a^2c}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx}{2a^2c\sqrt{a+a\sin(e}}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\tanh^{-1}(\sin(}{2a^2cf\sqrt{a+a\sin(}}$$

Mathematica [A] time = 0.775292, size = 163, normalized size = 1.57

$$\frac{\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx)\left(\log\left(c\right.\right.\right.}{2cf(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (Cos[e + f*x]^3*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] -
 Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c*f*(-1 + Sin[
e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.187, size = 244, normalized size = 2.4

$$-\frac{\left(\left(\cos\left(fx+e\right)\right)^2-\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(fx+e\right)+2\sin\left(fx+e\right)-2\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\left(\cos\left(\right.\right.\right.}{4f\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/f*(sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-sin(f*x+e)+ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))-ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))*(cos(f*x+e)^2-s
in(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)*(sin(f*x+e)*cos(f*x+e)+cos(
f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(5/2
)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

Fricas [A] time = 2.01209, size = 814, normalized size = 7.83

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^3-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^3}\right)-}{4\left(a^3c^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^3c^2f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)*sin(f*x + e) + a^3*c
^2*f*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x +
 e))*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(
a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c))/(a^3*c^2*f*cos(f*x + e)*sin(f*x + e) + a^3*c^2*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

3.64 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+}{2acf(a\sin(e+fx)+a)^{3/}}$$

```
[Out] -Cos[e + f*x]/(2*a*c*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2
)) + Cos[e + f*x]/(2*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a^2*c^2*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.540404, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2841, 2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+}{2acf(a\sin(e+fx)+a)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)),
x]
```



```
[Out] -Cos[e + f*x]/(2*a*c*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2
)) + Cos[e + f*x]/(2*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a^2*c^2*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}(\cdots}}{a\cdots}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\cdots}{2a^2cf\sqrt{a+a\sin\cdots}}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\cdots}{2a^2cf\sqrt{a+a\sin\cdots}}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\cdots}{2a^2cf\sqrt{a+a\sin\cdots}}$$

Mathematica [A] time = 0.863374, size = 163, normalized size = 1.07

$$\frac{\sec(e+fx)\left(2\sin(e+fx)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\cos(\cdots\right)}{4a^2c^2f\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (Sec[e + f*x]*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*(-Log[Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Sin[e + f*x]))/
(4*a^2*c^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.198, size = 196, normalized size = 1.3

$$-\frac{\left(\sin(fx+e)\cos(fx+e)-(\cos(fx+e))^2-2\sin(fx+e)-\cos(fx+e)+2\right)\left(\sin(fx+e)\cos(fx+e)+(\cos(\cdots\right)}{4f\left(-1+\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/4/f*(-cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln((1-cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+sin(f*x+e))*(sin(f*x+e)*cos(f*x
+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(sin(f*x+e)*cos(f*x+e)+cos(f*x+
e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(5/2)/(-
c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

Fricas [A] time = 2.114, size = 676, normalized size = 4.45

$$\left[\frac{\sqrt{ac}\cos\left(fx+e\right)^3\log\left(-\frac{ac\cos\left(fx+e\right)^3-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^3}\right)+2\,\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin}}{4\,a^3c^3f\cos\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e
) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x
+ e))/cos(f*x + e)^3) + 2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)*sin(f*x + e))/(a^3*c^3*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*arctan(sqrt(-a
*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*si
n(f*x + e)))*cos(f*x + e)^3 - sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e))/(a^3*c^3*f*cos(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

3.65 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=114

$$\frac{c^2 2^{n+\frac{3}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(-2n-1);\frac{1}{2}(2m+5);\right.}{f(2m+3)}$$

```
[Out] (2^(3/2 + n)*c^2*Cos[e + f*x]^3*Hypergeometric2F1[(3 + 2*m)/2, (-1 - 2*n)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 - n)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n))/(f*(3 + 2*m))
```

Rubi [A] time = 0.300745, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{c^2 2^{n+\frac{3}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(-2n-1);\frac{1}{2}(2m+5);\right.}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(3/2 + n)*c^2*Cos[e + f*x]^3*Hypergeometric2F1[(3 + 2*m)/2, (-1 - 2*n)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 - n)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n))/(f*(3 + 2*m))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{1+n}\,dx}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}+n}c^3\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c-c\right.}{}$$

$$= \frac{2^{\frac{3}{2}+n}c^2\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(3+2m),\frac{1}{2}(-1-2n);\frac{1}{2}(5+2\right.}{}$$

Mathematica [C] time = 14.1785, size = 3426, normalized size = 30.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (-64*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m +
n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*A
ppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/2
 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + P
i/2 - f*x)/2]^(2 + 2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n))*Sin[(-e +
Pi/2 - f*x)/2]^(2 + 2*n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*Tan[
(-e + Pi/2 - f*x)/4])/(f*(1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(
(16*m*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x
)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m +
 n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*
AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/
2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e +
Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*(m + n))*Sin[(-e +
 Pi/2 - f*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)
/4]^2)^(-1 - 2*m))/(1 + 2*n) + (4*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3
```

/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*Appell
F1[1/2 + n, -2*m, 2*(2 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2
+ n, -2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^
2)^(1 + 2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n))/((1 + 2*n)*(1 - Tan[(-e
+ Pi/2 - f*x)/4]^2)^(2*m)) + (16*n*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n),
3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*Appel
lF1[1/2 + n, -2*m, 2*(2 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n
, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2
 + n, -2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Sec[(-e + Pi/2 - f*x
)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(-1 + 2*n)*Tan[(-e + Pi/2 - f*
x)/4])/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (16*m*(AppellF1
[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m + n), 3/2 + n, T
an[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 +
n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^
(-1 + 2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f*x)/2
]^(1 + 2*n)*Tan[(-e + Pi/2 - f*x)/4])/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)
/4]^2)^(2*m)) + (16*(m + n)*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n
, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2
 + n, -2*m, 2*(2 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -
2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*
(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]^2)/((1 + 2
n)(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (16*Cos[(-e + Pi/2 - f*x)/2]^
(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*
n)*Tan[(-e + Pi/2 - f*x)/4]*(-((m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 2*(1
 + m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 +
 n)*(1 + m + n)*AppellF1[3/2 + n, -2*m, 1 + 2*(1 + m + n), 5/2 + n, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^
2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 + n) - 5*(-((m*(1/2 + n)*AppellF1[3/2 + n,
 1 - 2*m, 3 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 +
 n)) - ((1/2 + n)*(3 + 2*(m + n))*AppellF1[3/2 + n, -2*m, 4 + 2*(m + n), 5/
2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + P
i/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 + n))) - 4*(-((m*(1/2 + n
)*AppellF1[3/2 + n, 1 - 2*m, 5 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/
4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/
2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(5 + 2*(m + n))*AppellF1[3/2 + n, -2*m
, 6 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 + n)))
 + 8*(-((m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 2*(2 + m + n), 5/2 + n, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)
/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(2 + m + n)*AppellF
1[3/2 + n, -2*m, 1 + 2*(2 + m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -T
an[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)
/4])/(3/2 + n))))/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))))

Maple [F] time = 2.313, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2} \left(a+a\sin\left(fx+e\right)\right)^{m} \left(c-c\sin\left(fx+e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m} \left(-c\sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m} \left(-c\sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m} \left(-c\sin\left(fx+e\right)+c\right)^{n} \cos\left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

3.66 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{a^4c^3 2^{m+\frac{3}{2}}\cos^9(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{9}{2},-m-\frac{1}{2};\frac{11}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{9f}$$

```
[Out] -(2^(3/2 + m)*a^4*c^3*Cos[e + f*x]^9*Hypergeometric2F1[9/2, -1/2 - m, 11/2,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(
-4 + m))/(9*f)
```

Rubi [A] time = 0.211901, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^4c^3 2^{m+\frac{3}{2}}\cos^9(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{9}{2},-m-\frac{1}{2};\frac{11}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -(2^(3/2 + m)*a^4*c^3*Cos[e + f*x]^9*Hypergeometric2F1[9/2, -1/2 - m, 11/2,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(
-4 + m))/(9*f)
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^8(e+fx)(a+a\sin(e+fx))^{-3+m}\,dx$$

$$= \frac{\left(a^5c^3\cos^9(e+fx)\right)\text{Subst}\left(\int (a-ax)^{7/2}(a+ax)^{\frac{1}{2}+m}\,dx,x,\text{si}\right.}{f(a-a\sin(e+fx))^{9/2}(a+a\sin(e+fx))^{9/2}}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^5c^3\cos^9(e+fx)(a+a\sin(e+fx))^{-4+m}\left(\frac{a+a\sin(e+fx)}{a}\right.\right.}{f(a-a\sin(}$$

$$= -\frac{2^{\frac{3}{2}+m}a^4c^3\cos^9(e+fx)\,{}_2F_1\left(\frac{9}{2},-\frac{1}{2}-m;\frac{11}{2};\frac{1}{2}(1-\sin(e+fx\right.}{9f}$$

Mathematica [F] time = 180.023, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] $Aborted
```

Maple [F] time = 3.823, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,c^3\cos\left(fx+e\right)^4-4\,c^3\cos\left(fx+e\right)^2-\left(c^3\cos\left(fx+e\right)^4-4\,c^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(3*c^3*cos(f*x + e)^4 - 4*c^3*cos(f*x + e)^2 - (c^3*cos(f*x + e)^
4 - 4*c^3*cos(f*x + e)^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(c\sin\left(fx+e\right)-c\right)^3\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

3.67 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=86

$$-\frac{a^3c^2 2^{m+\frac{3}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{1}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

```
[Out] -(2^(3/2 + m)*a^3*c^2*Cos[e + f*x]^7*Hypergeometric2F1[7/2, -1/2 - m, 9/2,
(1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-
3 + m))/(7*f)
```

Rubi [A] time = 0.209284, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^3c^2 2^{m+\frac{3}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{1}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -(2^(3/2 + m)*a^3*c^2*Cos[e + f*x]^7*Hypergeometric2F1[7/2, -1/2 - m, 9/2,
(1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-
3 + m))/(7*f)
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-2+m}\,dx$$
$$= \frac{\left(a^4c^2\cos^7(e+fx)\right)\operatorname{Subst}\left(\int (a-ax)^{5/2}(a+ax)^{\frac{1}{2}+m}\,dx, x$$
$$f(a-a\sin(e+fx))^{7/2}(a+a\sin(e+fx))^{7/2}$$
$$= \frac{\left(2^{\frac{1}{2}+m}a^4c^2\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}\left(\frac{a+a\sin(e+}{a}\right.\right.}{f(a-a\,\mathrm{s}}$$
$$= -\frac{2^{\frac{3}{2}+m}a^3c^2\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},-\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+\right.}{7f}$$

Mathematica [F] time = 180.038, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] $Aborted
```

Maple [F] time = 3.296, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(fx+e\right)-c\right)^2\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c^2\cos\left(fx+e\right)^4+2\,c^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,c^2\cos\left(fx+e\right)^2\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(c^2*cos(f*x + e)^4 + 2*c^2*cos(f*x + e)^2*sin(f*x + e) - 2*c^2*c
os(f*x + e)^2)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(fx+e\right)-c\right)^2\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

3.68 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=84

$$-\frac{a^2c2^{m+\frac{3}{2}}\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{1}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

```
[Out] -(2^(3/2 + m)*a^2*c*Cos[e + f*x]^5*Hypergeometric2F1[5/2, -1/2 - m, 7/2, (1
 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-2
+ m))/(5*f)
```

Rubi [A] time = 0.155804, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^2c2^{m+\frac{3}{2}}\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{1}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(2^(3/2 + m)*a^2*c*Cos[e + f*x]^5*Hypergeometric2F1[5/2, -1/2 - m, 7/2, (1
 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-2
+ m))/(5*f)
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx = (ac)\int \cos^4(e+fx)(a+a\sin(e+fx))^{-1+m}\,dx$$

$$= \frac{\left(a^3c\cos^5(e+fx)\right)\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{\frac{1}{2}+m}\,dx, x, \sin(\right.}{f(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^3c\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-}\right.}{f(a-a\sin(e}$$

$$= -\frac{2^{\frac{3}{2}+m}a^2c\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},-\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

Mathematica [F] time = 180.003, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] $Aborted
```

Maple [F] time = 1.707, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -integrate((c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-c\cos\left(fx+e\right)^{2}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral(-(c*cos(f*x + e)^2*sin(f*x + e) - c*cos(f*x + e)^2)*(a*sin(f*x + e
) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

3.69 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

```
[Out] -(2^(3/2 + m)*a*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 - S
in[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-1 + m)
)/(3*f)
```

Rubi [A] time = 0.0687709, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(2^(3/2 + m)*a*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 - S
in[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-1 + m)
)/(3*f)
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos^3(e+fx)\right)\text{Subst}\left(\int \sqrt{a-ax}(a+ax)^{\frac{1}{2}+m}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^2\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{2}-m}\right)\text{Subst}\left(\int\left(\right.\right.}{f(a-a\sin(e+fx))^{3/2}}$$

$$= -\frac{2^{\frac{3}{2}+m}a\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx)}{3f}$$

Mathematica [A] time = 0.110177, size = 78, normalized size = 0.96

$$-\frac{2^{m+\frac{3}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{3}{2}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(2^(3/2 + m)*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 - Sin
[e + f*x])/2]*(1 + Sin[e + f*x])^(-3/2 - m)*(a*(1 + Sin[e + f*x]))^m)/(3*f)
```

Maple [F] time = 0.924, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^m \cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m \cos\left(fx+e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

3.70 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=77

$$-\frac{2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

```
[Out] -((2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sin[
e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(c*f))
```

Rubi [A] time = 0.164882, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2840, 2652, 2651}

$$-\frac{2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sin[
e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(c*f))
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}\,dx}{ac}$$
$$= \frac{\left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^{1+m}\,dx}{c}$$
$$= -\frac{2^{\frac{3}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}}{cf}$$

Mathematica [C] time = 19.9326, size = 6442, normalized size = 83.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.348, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m}{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2}{c\sin\left(fx+e\right)-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c), x)
```

3.71 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=81

$$\frac{2^{m+\frac{3}{2}}\sec(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{1}{2},-m-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{ac^2f}$$

```
[Out] (2^(3/2 + m)*Hypergeometric2F1[-1/2, -1/2 - m, 1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(a*
c^2*f)
```

Rubi [A] time = 0.205679, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2840, 2689, 70, 69}

$$\frac{2^{m+\frac{3}{2}}\sec(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{1}{2},-m-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(3/2 + m)*Hypergeometric2F1[-1/2, -1/2 - m, 1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(a*
c^2*f)
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{2+m}\,dx}{a^2c^2}$$
$$= \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}}{(a-ax)^{3/2}}\,dx, x\right.}{c^2f}$$
$$= \frac{\left(2^{\frac{1}{2}+m}\sec(e+fx)\sqrt{a-a\sin(e+fx)}(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{2}}\right.}{c^2f}$$
$$= \frac{2^{\frac{3}{2}+m}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}}}{ac^2f}$$

Mathematica [A] time = 0.205215, size = 88, normalized size = 1.09

$$\frac{2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(-\frac{1}{2},-m-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{c^2f(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -1/2 - m, 1/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a*(1 + Sin[e + f*x]))^m)/(c^2*f*
(1 - Sin[e + f*x]))
```

Maple [F] time = 0.652, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2}{\left(c\sin\left(fx+e\right)-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c^2*cos(f*x + e)^2 + 2*c^2
*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^2, x)
```

3.72 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{2^{m+\frac{3}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{3}{2},-m-\frac{1}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3a^2c^3f}$$

```
[Out] (2^(3/2 + m)*Hypergeometric2F1[-3/2, -1/2 - m, -1/2, (1 - Sin[e + f*x])/2]*
Sec[e + f*x]^3*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/
(3*a^2*c^3*f)
```

Rubi [A] time = 0.212883, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2840, 2689, 70, 69}

$$\frac{2^{m+\frac{3}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{3}{2},-m-\frac{1}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(3/2 + m)*Hypergeometric2F1[-3/2, -1/2 - m, -1/2, (1 - Sin[e + f*x])/2]*
Sec[e + f*x]^3*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/
(3*a^2*c^3*f)
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{3+m}\,dx}{a^3c^3}$$

$$= \frac{\left(\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{2}+m}}{(a-ax)^{5/2}}\,dx\right.}{ac^3f}$$

$$= \frac{\left(2^{\frac{1}{2}+m}\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{2+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{2}}\right.}{ac^3f}$$

$$= \frac{2^{\frac{3}{2}+m}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}}}{3a^2c^3f}$$

Mathematica [A] time = 0.152698, size = 91, normalized size = 1.06

$$\frac{2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(-\frac{3}{2},-m-\frac{1}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3c^3f(1-\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[-3/2, -1/2 - m, -1/2, (1 - Sin[
e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a*(1 + Sin[e + f*x]))^m)/(3*c^3
*f*(1 - Sin[e + f*x])^2)
```

Maple [F] time = 0.732, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2}{\left(c\sin\left(fx+e\right)-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(3*c^3*cos(f*x + e)^2 - 4*c
^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^3, x)
```

3.73 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=244

$$\frac{192c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^{m+1}}{af(2m+9)\left(4m^2+24m+35\right)}+\frac{768c^3\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)(2m+9)\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (768*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(7 + 2*m)*(9 + 2*m
)*(15 + 16*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) + (192*c^2*Cos[e + f*x]*(a
+ a*Sin[e + f*x])^(1 + m)*Sqrt[c - c*Sin[e + f*x]])/(a*f*(9 + 2*m)*(35 + 24
*m + 4*m^2)) + (24*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e
 + f*x])^(3/2))/(a*f*(63 + 32*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*Sin[e +
f*x])^(1 + m)*(c - c*Sin[e + f*x])^(5/2))/(a*f*(9 + 2*m))
```

Rubi [A] time = 0.615432, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2841, 2740, 2738}

$$\frac{192c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^{m+1}}{af(2m+9)\left(4m^2+24m+35\right)}+\frac{768c^3\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)(2m+9)\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (768*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(7 + 2*m)*(9 + 2*m
)*(15 + 16*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) + (192*c^2*Cos[e + f*x]*(a
+ a*Sin[e + f*x])^(1 + m)*Sqrt[c - c*Sin[e + f*x]])/(a*f*(9 + 2*m)*(35 + 24
*m + 4*m^2)) + (24*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e
 + f*x])^(3/2))/(a*f*(63 + 32*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*Sin[e +
f*x])^(1 + m)*(c - c*Sin[e + f*x])^(5/2))/(a*f*(9 + 2*m))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
```

```
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{5/2}}{af(9+2m)}$$
$$= \frac{24c\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))}{af\left(63+32m+4m^2\right)}$$
$$= \frac{192c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+f}}{af(5+2m)\left(63+32m+4m^2\right)}$$
$$= \frac{768c^3\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)(5+2m)\left(63+32m+4m^2\right)\sqrt{c-c\sin(e+f}}$$

Mathematica [C] time = 6.57638, size = 695, normalized size = 2.85

$$(c-c\sin(e+fx))^{5/2}(a(\sin(e+fx)+1))^m\left(\frac{\left(8m^3+108m^2+590m+2205\right)\left(\left(\frac{3}{8}-\frac{3i}{8}\right)\sin\left(\frac{1}{2}(e+fx)\right)+\left(\frac{3}{8}+\frac{3i}{8}\right)\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(2m+3)(2m+5)(2m+7)(2m+9)}+\frac{\left(8m^3+108m^2+590\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(5/2)*(((2205 + 590*m + 108*
m^2 + 8*m^3)*((3/8 + (3*I)/8)*Cos[(e + f*x)/2] + (3/8 - (3*I)/8)*Sin[(e + f
*x)/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((2205 + 590*m + 108*m
^2 + 8*m^3)*((3/8 - (3*I)/8)*Cos[(e + f*x)/2] + (3/8 + (3*I)/8)*Sin[(e + f*
x)/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((191*m + 48*m^2 + 4*m^
3)*((1 - I)*Cos[(3*(e + f*x))/2] - (1 + I)*Sin[(3*(e + f*x))/2]))/((3 + 2*m
)*(5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((191*m + 48*m^2 + 4*m^3)*((1 + I)*Cos[(
3*(e + f*x))/2] - (1 - I)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)*(7 +
2*m)*(9 + 2*m)) + ((21 + 2*m)*((3/2 + (3*I)/2)*Cos[(5*(e + f*x))/2] + (3/2
- (3*I)/2)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((21 +
2*m)*((3/2 - (3*I)/2)*Cos[(5*(e + f*x))/2] + (3/2 + (3*I)/2)*Sin[(5*(e + f*
x))/2]))/((5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((15 + 2*m)*((3/16 - (3*I)/16)*C
os[(7*(e + f*x))/2] - (3/16 + (3*I)/16)*Sin[(7*(e + f*x))/2]))/((7 + 2*m)*(
9 + 2*m)) + ((15 + 2*m)*((3/16 + (3*I)/16)*Cos[(7*(e + f*x))/2] - (3/16 - (
3*I)/16)*Sin[(7*(e + f*x))/2]))/((7 + 2*m)*(9 + 2*m)) + ((-1/16 + I/16)*Cos
[(9*(e + f*x))/2] - (1/16 + I/16)*Sin[(9*(e + f*x))/2])/(9 + 2*m) + ((-1/16
 - I/16)*Cos[(9*(e + f*x))/2] - (1/16 - I/16)*Sin[(9*(e + f*x))/2])/(9 + 2*
m)))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [B] time = 2.32389, size = 753, normalized size = 3.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorit
hm="maxima")
```

```
[Out] -2*((8*m^3 + 108*m^2 + 526*m + 957)*a^m*c^(5/2) - 3*(8*m^3 + 76*m^2 + 142*m
 - 315)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) - 24*(4*m^2 + 16*m - 81
)*a^m*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 16*(4*m^3 + 36*m^2 + 95
*m + 315)*a^m*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*(8*m^3 + 60*m
^2 + 206*m - 567)*a^m*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*(8*m^
3 + 60*m^2 + 206*m - 567)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 16*(4*m^3 + 36*m^2 + 95*m + 315)*a^m*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - 24*(4*m^2 + 16*m - 81)*a^m*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 3*(8*m^3 + 76*m^2 + 142*m - 315)*a^m*c^(5/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + (8*m^3 + 108*m^2 + 526*m + 957)*a^m*c^(5/2)*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9)*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((16*m^4 + 192*m^3 + 824*m^2 +
1488*m + 2*(16*m^4 + 192*m^3 + 824*m^2 + 1488*m + 945)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + (16*m^4 + 192*m^3 + 824*m^2 + 1488*m + 945)*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 945)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
5/2))
```

Fricas [A] time = 2.02952, size = 979, normalized size = 4.01

$$-\frac{2\left(\left(8\,c^2m^3+60\,c^2m^2+142\,c^2m+105\,c^2\right)\cos\left(fx+e\right)^5-\left(8\,c^2m^3+108\,c^2m^2+334\,c^2m+285\,c^2\right)\cos\left(fx+e\right)^4-2\left(8\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorit
hm="fricas")
```

```
[Out] -2*((8*c^2*m^3 + 60*c^2*m^2 + 142*c^2*m + 105*c^2)*cos(f*x + e)^5 - (8*c^2*
m^3 + 108*c^2*m^2 + 334*c^2*m + 285*c^2)*cos(f*x + e)^4 - 2*(8*c^2*m^3 + 84
*c^2*m^2 + 334*c^2*m + 339*c^2)*cos(f*x + e)^3 - 384*c^2*cos(f*x + e) - 96*
(2*c^2*m - c^2)*cos(f*x + e)^2 - 768*c^2 + ((8*c^2*m^3 + 60*c^2*m^2 + 142*c
^2*m + 105*c^2)*cos(f*x + e)^4 + 2*(8*c^2*m^3 + 84*c^2*m^2 + 238*c^2*m + 19
5*c^2)*cos(f*x + e)^3 - 384*c^2*cos(f*x + e) - 96*(2*c^2*m + 3*c^2)*cos(f*x
 + e)^2 - 768*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e)
+ a)^m/(16*f*m^4 + 192*f*m^3 + 824*f*m^2 + 1488*f*m + (16*f*m^4 + 192*f*m^3
 + 824*f*m^2 + 1488*f*m + 945*f)*cos(f*x + e) - (16*f*m^4 + 192*f*m^3 + 824
*f*m^2 + 1488*f*m + 945*f)*sin(f*x + e) + 945*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.74 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{64c^2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{16c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+24m+35\right)}+\frac{2\cos(e+$$

```
[Out] (64*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(7 + 2*m)*(15 + 16*
m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) + (16*c*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(1 + m)*Sqrt[c - c*Sin[e + f*x]])/(a*f*(35 + 24*m + 4*m^2)) + (2*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(3/2))/(a*f*(7 + 2
*m))
```

Rubi [A] time = 0.467299, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2841, 2740, 2738}

$$\frac{64c^2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{16c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+24m+35\right)}+\frac{2\cos(e+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (64*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(7 + 2*m)*(15 + 16*
m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) + (16*c*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(1 + m)*Sqrt[c - c*Sin[e + f*x]])/(a*f*(35 + 24*m + 4*m^2)) + (2*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(3/2))/(a*f*(7 + 2
*m))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{3/2}}{af(7+2m)}$$
$$= \frac{16c\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}}{af\left(35+24m+4m^2\right)}$$
$$= \frac{64c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\left(35+24m+4m^2\right)\sqrt{c-c\sin(e+fx)}} + 16c$$

Mathematica [A] time = 3.11294, size = 149, normalized size = 0.87

$$-\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(a(\sin(e+fx)+1))^m\left(4\left(4m^2+24m+27\right)\sin(e+fx)+(4\right.}{f(2m+3)(2m+5)(2m+7)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -((c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^m*Sqrt[
c - c*Sin[e + f*x]]*(-157 - 80*m - 12*m^2 + (15 + 16*m + 4*m^2)*Cos[2*(e +
f*x)] + 4*(27 + 24*m + 4*m^2)*Sin[e + f*x]))/(f*(3 + 2*m)*(5 + 2*m)*(7 + 2*
m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [B] time = 2.30023, size = 571, normalized size = 3.32

$$-\frac{2\left(\left(4m^2+32m+71\right)a^mc^{\frac{3}{2}}-\frac{\left(4m^2-105\right)a^mc^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\left(12m^2+64m-91\right)a^mc^{\frac{3}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{\left(12m^2+32m+245\right)a^mc^{\frac{3}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\right.}{\left(8m^3+60m^2+142m+\frac{2\left(8m^3+60m^2+14\right.}{\left(\cos\left(f\right.\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorit
hm="maxima")
```

```
[Out] -2*((4*m^2 + 32*m + 71)*a^m*c^(3/2) - (4*m^2 - 105)*a^m*c^(3/2)*sin(f*x + e
)/(cos(f*x + e) + 1) - (12*m^2 + 64*m - 91)*a^m*c^(3/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + (12*m^2 + 32*m + 245)*a^m*c^(3/2)*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + (12*m^2 + 32*m + 245)*a^m*c^(3/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - (12*m^2 + 64*m - 91)*a^m*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 - (4*m^2 - 105)*a^m*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4
*m^2 + 32*m + 71)*a^m*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*e^(2*m*l
og(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 1))/((8*m^3 + 60*m^2 + 142*m + 2*(8*m^3 + 60*m^2 + 142*m + 105)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (8*m^3 + 60*m^2 + 142*m + 105)*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 105)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)^(3/2))
```

Fricas [A] time = 1.87571, size = 622, normalized size = 3.62

$$\frac{2\left(\left(4\,cm^2+16\,cm+15\,c\right)\cos\left(fx+e\right)^4+\left(4\,cm^2+32\,cm+39\,c\right)\cos\left(fx+e\right)^3+8\,(2\,cm-c)\cos\left(fx+e\right)^2+32\,c\cos\left(f\right.}{8\,fm^3+60\,fm^2+142\,fm+\left(8\,fm^3+60\,fm^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorit
hm="fricas")
```

```
[Out] 2*((4*c*m^2 + 16*c*m + 15*c)*cos(f*x + e)^4 + (4*c*m^2 + 32*c*m + 39*c)*cos
(f*x + e)^3 + 8*(2*c*m - c)*cos(f*x + e)^2 + 32*c*cos(f*x + e) - ((4*c*m^2
+ 16*c*m + 15*c)*cos(f*x + e)^3 - 8*(2*c*m + 3*c)*cos(f*x + e)^2 - 32*c*cos
(f*x + e) - 64*c)*sin(f*x + e) + 64*c)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x
 + e) + a)^m/(8*f*m^3 + 60*f*m^2 + 142*f*m + (8*f*m^3 + 60*f*m^2 + 142*f*m
+ 105*f)*cos(f*x + e) - (8*f*m^3 + 60*f*m^2 + 142*f*m + 105*f)*sin(f*x + e)
 + 105*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.75 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=107

$$\frac{8c\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{2\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^{m+1}}{af(2m+5)}$$

```
[Out] (8*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(15 + 16*m + 4*m^2)*Sq
rt[c - c*Sin[e + f*x]]) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt
[c - c*Sin[e + f*x]])/(a*f*(5 + 2*m))
```

Rubi [A] time = 0.354111, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2841, 2740, 2738}

$$\frac{8c\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{2\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^{m+1}}{af(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (8*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(15 + 16*m + 4*m^2)*Sq
rt[c - c*Sin[e + f*x]]) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt
[c - c*Sin[e + f*x]])/(a*f*(5 + 2*m))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}}{af(5+2m)} + \frac{4}{}$$
$$= \frac{8c\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af\left(15+16m+4m^2\right)\sqrt{c-c\sin(e+fx)}} + \frac{2\cos(e+fx)(}{}$$

Mathematica [A] time = 0.560205, size = 111, normalized size = 1.04

$$-\frac{2\sqrt{c-c\sin(e+fx)}((2m+3)\sin(e+fx)-2m-7)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(a(\sin(e+fx)+1))^m}{f(2m+3)(2m+5)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^m*Sqrt[c
 - c*Sin[e + f*x]]*(-7 - 2*m + (3 + 2*m)*Sin[e + f*x]))/(f*(3 + 2*m)*(5 + 2
*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\sqrt{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [B] time = 2.14321, size = 421, normalized size = 3.93

$$-\frac{2\left(a^m\sqrt{c}(2\,m+7)+\frac{a^m\sqrt{c}(2\,m+15)\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{2\,a^m\sqrt{c}(2\,m-5)\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{2\,a^m\sqrt{c}(2\,m-5)\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{a^m\sqrt{c}(2\,m+15)\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{a^m}{}\right.}{\left(4\,m^2+16\,m+\frac{2\left(4\,m^2+16\,m+15\right)\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{\left(4\,m^2+16\,m+15\right)\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+15\right)f\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] -2*(a^m*sqrt(c)*(2*m + 7) + a^m*sqrt(c)*(2*m + 15)*sin(f*x + e)/(cos(f*x +
e) + 1) - 2*a^m*sqrt(c)*(2*m - 5)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a
^m*sqrt(c)*(2*m - 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^m*sqrt(c)*(2*m
 + 15)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^m*sqrt(c)*(2*m + 7)*sin(f*x
```

```
+ e)^5/(cos(f*x + e) + 1)^5)*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1
) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 16*m + 2*(4*m
^2 + 16*m + 15)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (4*m^2 + 16*m + 15)*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15)*f*sqrt(sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 1))
```

Fricas [A] time = 1.78459, size = 402, normalized size = 3.76

$$\frac{2\left((2\,m+3)\cos\left(fx+e\right)^{3}+(2\,m-1)\cos\left(fx+e\right)^{2}+\left((2\,m+3)\cos\left(fx+e\right)^{2}+4\,\cos\left(fx+e\right)+8\right)\sin\left(fx+e\right)+4\,\cos}{4\,fm^{2}+16\,fm+\left(4\,fm^{2}+16\,fm+15\,f\right)\cos\left(fx+e\right)-\left(4\,fm^{2}+16\,fm+15\,f\right)\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 2*((2*m + 3)*cos(f*x + e)^3 + (2*m - 1)*cos(f*x + e)^2 + ((2*m + 3)*cos(f*x
 + e)^2 + 4*cos(f*x + e) + 8)*sin(f*x + e) + 4*cos(f*x + e) + 8)*sqrt(-c*si
n(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 16*f*m + (4*f*m^2 + 16*f*
m + 15*f)*cos(f*x + e) - (4*f*m^2 + 16*f*m + 15*f)*sin(f*x + e) + 15*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.76 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 + 2*m)*Sqrt[c - c*Sin
[e + f*x]])
```

Rubi [A] time = 0.261661, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2841, 2738}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 + 2*m)*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.380026, size = 85, normalized size = 1.7

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(a(\sin(e+fx)+1))^m}{f(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3*(a*(1 + Sin[e + f*x]))^m)/(f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F] time = 0.376, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{2} \left(a+a\sin \left(fx+e\right)\right)^{m} \frac{1}{\sqrt{c-c\sin \left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.71622, size = 279, normalized size = 5.58

$$-\frac{2\left(\cos \left(fx+e\right)^{2}-\left(\cos \left(fx+e\right)+2\right)\sin \left(fx+e\right)-\cos \left(fx+e\right)-2\right)\sqrt{-c\sin \left(fx+e\right)+c}\left(a\sin \left(fx+e\right)+a\right)^{m}}{2\,cfm+3\,cf+\left(2\,cfm+3\,cf\right)\cos \left(fx+e\right)-\left(2\,cfm+3\,cf\right)\sin \left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] -2*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)*sq
rt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(2*c*f*m + 3*c*f + (2*c*f*m
+ 3*c*f)*cos(f*x + e) - (2*c*f*m + 3*c*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sin \left(e+fx\right)+1\right)\right)^{m}\cos^{2}\left(e+fx\right)}{\sqrt{-c\left(\sin \left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x) -
1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) + c),
x)
```

3.77 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{acf(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^(1 + m))/(a*c*f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.361743, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2841, 2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{acf(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^(1 + m))/(a*c*f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{1+m}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{3}{2}+m}\,dx}{ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(a+x)^{\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{3}{2}+m;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{1+m}}{acf(3+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 6.50331, size = 218, normalized size = 2.87

$$-\frac{2^{-2m-\frac{5}{2}}\cos^2\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a\sin(e+fx)+a)^m\left(\sec^4\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\sec\right.}{f(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((2^(-5/2 - 2*m)*Cos[(-e + Pi/2 - f*x)/2]^2*(-(4^(1 + m)*Hypergeometric2F1
[1, 2 + 2*m, 3 + 2*m, Cos[(-e + Pi/2 - f*x)/2]]) + Hypergeometric2F1[2 + 2*
m, 2 + 2*m, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2]*Sec[(-e + Pi/2 - f
*x)/4]^4*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^3*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(c - c*Sin[e + f*x])^(3/2)))
```

Maple [F] time = 0.376, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(3/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(
c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(3/2)
, x)
```

3.78 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2ac^2f(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[2, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^(1 + m))/(2*a*c^2*f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]]
)
```

Rubi [A] time = 0.372359, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2841, 2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2ac^2f(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[2, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^(1 + m))/(2*a*c^2*f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]]
)
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{1+m}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}\\
&= \frac{\cos(e+fx)\int \sec^3(e+fx)(a+a\sin(e+fx))^{\frac{5}{2}+m}\,dx}{a^2c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= \frac{(a\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+x)^{\frac{1}{2}+m}}{(a-x)^2}\,dx, x, a\sin(e+fx)\right)}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= \frac{\cos(e+fx)\,{}_2F_1\left(2,\frac{3}{2}+m;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{1+m}}{2ac^2f(3+2m)\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 41.4602, size = 3157, normalized size = 39.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a + a*Sin[e + f*x])^m*((Cos[(-e
+ Pi/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e - P
i/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^3 + (2*Cos[(-e + Pi/2 - f*x
)/2]^(2*m)*Cos[Pi/4 + (e - Pi/2 + f*x)/2]*Sin[Pi/4 + (e - Pi/2 + f*x)/2])/(
Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^3 + (Cos[(
-e + Pi/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e
- Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^3)*(-((AppellF1[1, -2*m,
 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e +
 Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 -
 f*x)/4]^2)^(2*m)) + ((Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(4^m*(1 + 2*m)*App
ellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4
]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 + 2*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 -
 Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(1 - Tan[(-
e + Pi/2 - f*x)/4]^2)^(1 + 2*m)))/((1 + 2*m)*(2 - 2*Tan[(-e + Pi/2 - f*x)/4
]^2)^(2*m))))/(4*Sqrt[2]*f*(c - c*Sin[e + f*x])^(5/2)*(-(m*Cos[(-e + Pi/2 -
 f*x)/2]^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]*(-((AppellF1[1, -2*m, 2*m, 2,
Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f
*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^
2)^(2*m)) + ((Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(4^m*(1 + 2*m)*AppellF1[1,
-2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[
(-e + Pi/2 - f*x)/4]^2 + 2*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e
+ Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(1 - Tan[(-e + Pi/2
- f*x)/4]^2)^(1 + 2*m)))/((1 + 2*m)*(2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m
))))/(4*Sqrt[2]) + (Cos[(-e + Pi/2 - f*x)/2]^(2*m)*((m*AppellF1[1, -2*m, 2*
m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi
/2 - f*x)/4]^3*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*
x)/4]^2)^(2*m) + m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -C
ot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(1 - Cot[(-e + Pi/2 -
 f*x)/4]^2)^(-1 - 2*m)*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m) + (AppellF1[1
```

```
, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Co
t[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m))/(2*(1 - Cot[
(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/
2 - f*x)/4]^2)^(2*m)*((m*AppellF1[2, 1 - 2*m, 2*m, 3, Cot[(-e + Pi/2 - f*x)
/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2
 - f*x)/4]^2)/2 + (m*AppellF1[2, -2*m, 1 + 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]
^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 -
f*x)/4]^2)/2))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2*m*(Sec[(-e + Pi/
2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4]*(2 - 2*Tan[(-e + Pi/2 - f
*x)/4]^2)^(-1 - 2*m)*(4^m*(1 + 2*m)*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 + 2*
AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 -
Tan[(-e + Pi/2 - f*x)/4]^2]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)))/(1
 + 2*m) + (m*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]*(4
^m*(1 + 2*m)*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 + 2*AppellF1[1 + 2*m, 2*m,
1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4
]^2]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)))/((1 + 2*m)*(2 - 2*Tan[(-e
 + Pi/2 - f*x)/4]^2)^(2*m)) + ((Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(2^(-1 +
2*m)*(1 + 2*m)*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]
+ 4^m*(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4]^2*(-(m*AppellF1[2, 1 - 2*m, 2*m, 3
, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 -
 f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2 - (m*AppellF1[2, -2*m, 1 + 2*m, 3, T
an[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*
x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) - (1 + 2*m)*AppellF1[1 + 2*m, 2*m, 1,
2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2
]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 -
 f*x)/4]^2)^(2*m) + 2*(-((1 + 2*m)*AppellF1[2 + 2*m, 2*m, 2, 3 + 2*m, (1 -
Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi
/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(2 + 2*m)) - (m*(1 + 2*m)*Appel
lF1[2 + 2*m, 1 + 2*m, 1, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - T
an[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)
/4])/(2*(2 + 2*m)))*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)))/((1 + 2*m)
*(2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))))/(4*Sqrt[2])))
```

Maple [F] time = 0.375, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(5/2)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(
3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(5/2)
, x)
```

3.79 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 + 2*m)*Sqrt[c - c*Sin
[e + f*x]])
```

Rubi [A] time = 0.254303, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2841, 2738}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 + 2*m)*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.395079, size = 85, normalized size = 1.7

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(a(\sin(e+fx)+1))^m}{f(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3*(a*(1 + Sin[e + f*x]))^m)/(f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{2} \left(a+a\sin \left(fx+e\right)\right)^{m} \frac{1}{\sqrt{c-c\sin \left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.72712, size = 279, normalized size = 5.58

$$-\frac{2 \left(\cos \left(fx+e\right)^{2} - \left(\cos \left(fx+e\right)+2\right) \sin \left(fx+e\right) - \cos \left(fx+e\right) - 2\right) \sqrt{-c\sin \left(fx+e\right)+c} \left(a\sin \left(fx+e\right)+a\right)^{m}}{2\,cfm+3\,cf+\left(2\,cfm+3\,cf\right)\cos \left(fx+e\right) - \left(2\,cfm+3\,cf\right)\sin \left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] -2*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)*sq
rt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(2*c*f*m + 3*c*f + (2*c*f*m
+ 3*c*f)*cos(f*x + e) - (2*c*f*m + 3*c*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \left(\sin \left(e+fx\right)+1\right)\right)^{m} \cos^{2}\left(e+fx\right)}{\sqrt{-c \left(\sin \left(e+fx\right)-1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x) -
1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) + c),
x)
```

3.80 $\int \frac{\cos^2(e+fx)(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\cos(e+fx)(c\sin(e+fx)+c)^{m+1}}{cf(2m+3)\sqrt{a-a\sin(e+fx)}}$$

```
[Out] (2*Cos[e + f*x]*(c + c*Sin[e + f*x])^(1 + m))/(c*f*(3 + 2*m)*Sqrt[a - a*Sin
[e + f*x]])
```

Rubi [A] time = 0.253745, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2841, 2738}

$$\frac{2\cos(e+fx)(c\sin(e+fx)+c)^{m+1}}{cf(2m+3)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*(c + c*Sin[e + f*x])^(1 + m))/(c*f*(3 + 2*m)*Sqrt[a - a*Sin
[e + f*x]])
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{1+m}\,dx}{ac}\\ &= \frac{2\cos(e+fx)(c+c\sin(e+fx))^{1+m}}{cf(3+2m)\sqrt{a-a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.368477, size = 85, normalized size = 1.7

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(c(\sin(e+fx)+1))^m}{f(2m+3)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3*(c*(1 + Sin[e + f*x]))^m)/(f*(3 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

Maple [F] time = 0.407, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2}\left(c+c\sin\left(fx+e\right)\right)^{m}\frac{1}{\sqrt{a-a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.69054, size = 279, normalized size = 5.58

$$-\frac{2\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{m}}{2\,afm+3\,af+\left(2\,afm+3\,af\right)\cos\left(fx+e\right)-\left(2\,afm+3\,af\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] -2*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)*sq
rt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^m/(2*a*f*m + 3*a*f + (2*a*f*m
+ 3*a*f)*cos(f*x + e) - (2*a*f*m + 3*a*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{m}\cos^{2}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/sqrt(-a*(sin(e + f*x) -
1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(fx+e\right)+c\right)^{m}\cos\left(fx+e\right)^{2}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((c*sin(f*x + e) + c)^m*cos(f*x + e)^2/sqrt(-a*sin(f*x + e) + a),
x)
```

3.81 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-5-m}\,dx$

Optimal. Leaf size=182

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-3}}{ac^2f\left(4m^2+24m+35\right)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m}}{ac^3f(2m+7)\left(4m^2+16m+15\right)}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-4 - m))/(
a*c*f*(7 + 2*m)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[
e + f*x])^(-3 - m))/(a*c^2*f*(35 + 24*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-2 - m))/(a*c^3*f*(7 + 2*m)*(15
 + 16*m + 4*m^2))
```

Rubi [A] time = 0.448437, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-3}}{ac^2f\left(4m^2+24m+35\right)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m}}{ac^3f(2m+7)\left(4m^2+16m+15\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-5 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-4 - m))/(
a*c*f*(7 + 2*m)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[
e + f*x])^(-3 - m))/(a*c^2*f*(35 + 24*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-2 - m))/(a*c^3*f*(7 + 2*m)*(15
 + 16*m + 4*m^2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
```

```
Q[m, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-5-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-4-m}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-4-m}}{acf(7+2m)}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-4-m}}{acf(7+2m)}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-4-m}}{acf(7+2m)}$$

Mathematica [A] time = 17.2171, size = 176, normalized size = 0.97

$$\frac{2^{-m-2}\cos^3\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m-7}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-5}\left(\cos\left(\frac{1}{2}(e+fx)\right)-s\right.}{f(2m+3)(2m+5)(2m+7)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-5 -
m),x]
```

```
[Out] (2^(-2 - m)*Cos[(-e + Pi/2 - f*x)/2]^3*Sin[(-e + Pi/2 - f*x)/2]^(-7 - 2*m)*
(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-5 - m)*(4*(6 + 5*m + m^2) + C
os[2*(-e + Pi/2 - f*x)] - 2*(5 + 2*m)*Sin[e + f*x]))/(f*(3 + 2*m)*(5 + 2*m)
*(7 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-5 - m)))
```

Maple [F] time = 0.911, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{-5-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x, algori
thm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.79568, size = 261, normalized size = 1.43

$$-\frac{\left(2 \cos\left(fx+e\right)^{5}+2\,(2\,m+5)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-\left(4\,m^{2}+20\,m+25\right)\cos\left(fx+e\right)^{3}\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(\cdots\right.}{8\,fm^{3}+60\,fm^{2}+142\,fm+105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x, algori
thm="fricas")
```

```
[Out] -(2*cos(f*x + e)^5 + 2*(2*m + 5)*cos(f*x + e)^3*sin(f*x + e) - (4*m^2 + 20*
m + 25)*cos(f*x + e)^3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m -
5)/(8*f*m^3 + 60*f*m^2 + 142*f*m + 105*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-5-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-5}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 5)*cos(f*x + e
)^2, x)
```

3.82 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}\,dx$

Optimal. Leaf size=114

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{ac^2f\left(4m^2+16m+15\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-3}}{acf(2m+5)}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-3 - m))/(
a*c*f*(5 + 2*m)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e
+ f*x])^(-2 - m))/(a*c^2*f*(15 + 16*m + 4*m^2))
```

Rubi [A] time = 0.334502, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2841, 2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{ac^2f\left(4m^2+16m+15\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-3}}{acf(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-3 - m))/(
a*c*f*(5 + 2*m)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e
+ f*x])^(-2 - m))/(a*c^2*f*(15 + 16*m + 4*m^2))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-3-m}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-3}}{acf(5+2m)}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-3}}{acf(5+2m)}$$

Mathematica [A] time = 12.662, size = 142, normalized size = 1.25

$$-\frac{2^{-m-1}\cos^3\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m-5}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(\sin(e+fx)-2(m+2))(a\sin(e+fx)+a)^m(c-c\sin(e+}{f(2m+3)(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 -
m),x]
```

```
[Out] -((2^(-1 - m)*Cos[(-e + Pi/2 - f*x)/2]^3*Sin[(-e + Pi/2 - f*x)/2]^(-5 - 2*m
)*(-2*(2 + m) + Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(
-4 - m))/(f*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-
4 - m))))
```

Maple [F] time = 0.817, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2 \left(a+a\sin\left(fx+e\right)\right)^m \left(c-c\sin\left(fx+e\right)\right)^{-4-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m \left(-c\sin\left(fx+e\right)+c\right)^{-m-4}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 4)*cos(f*x + e
)^2, x)
```

Fricas [A] time = 1.76837, size = 186, normalized size = 1.63

$$\frac{\left(2\,(m+2)\cos\left(fx+e\right)^3-\cos\left(fx+e\right)^3\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-4}}{4\,fm^2+16\,fm+15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x, algori
thm="fricas")
```

```
[Out] (2*(m + 2)*cos(f*x + e)^3 - cos(f*x + e)^3*sin(f*x + e))*(a*sin(f*x + e) +
a)^m*(-c*sin(f*x + e) + c)^(-m - 4)/(4*f*m^2 + 16*f*m + 15*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-4-m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x, algori
thm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.83 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=54

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{acf(2m+3)}$$

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-2 - m))/(
a*c*f*(3 + 2*m))
```

Rubi [A] time = 0.251078, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2841, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{acf(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-2 - m))/(
a*c*f*(3 + 2*m))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-2-m}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-2}}{acf(3+2m)}$$

Mathematica [B] time = 5.66044, size = 109, normalized size = 2.02

$$\frac{2^{-m}\sin^3\left(\frac{1}{4}(2e+2fx+\pi)\right)\cos^{-2m-3}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^{-m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}{c^3f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 -
m),x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-3 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4]^3)/(2^m*c^3*f*(3
+ 2*m)*(c - c*Sin[e + f*x])^m)
```

Maple [F] time = 0.735, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{2} \left(a+a\sin \left(fx+e\right)\right)^{m} \left(c-c\sin \left(fx+e\right)\right)^{-3-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(a \sin \left(f x + e\right) + a\right)}^{m} {\left(-c \sin \left(f x + e\right) + c\right)}^{-m - 3} \cos \left(f x + e\right)^{2}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)*cos(f*x + e
)^2, x)
```

Fricas [A] time = 1.76124, size = 113, normalized size = 2.09

$$\frac{{\left(a \sin \left(f x + e\right) + a\right)}^{m} {\left(-c \sin \left(f x + e\right) + c\right)}^{-m - 3} \cos \left(f x + e\right)^{3}}{2 \, f m + 3 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algori
thm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)*cos(f*x + e)^3/(2*f*m
 + 3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-3-m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algori
thm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.84 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=113

$$\frac{2^{-m-\frac{1}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5\right)}{f(2m+3)}$$

```
[Out] (2^(-1/2 - m)*Cos[e + f*x]^3*Hypergeometric2F1[(3 + 2*m)/2, (3 + 2*m)/2, (5
 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e
+ f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rubi [A] time = 0.379798, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{2^{-m-\frac{1}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5\right)}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m),x]
```

```
[Out] (2^(-1/2 - m)*Cos[e + f*x]^3*Hypergeometric2F1[(3 + 2*m)/2, (3 + 2*m)/2, (5
 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e
+ f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-1-m}\,dx}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx)\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{s}\right.}{}$$

$$= \frac{\left(2^{-\frac{3}{2}-m}c\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c\right.}{}$$

$$= \frac{2^{-\frac{1}{2}-m}\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(3+2m),\frac{1}{2}(3+2m);\frac{1}{2}(5+\right.}{}$$

Mathematica [C] time = 21.3943, size = 589, normalized size = 5.21

$$-\frac{2^{1-m}(2m-3)\cos^2\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\cot\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\csc^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\,\mathrm{s}}{f\left(4m^2-1\right)\left(8(m+1)F_1\left(\frac{3}{2}-m;-2m-1,1;\frac{5}{2}-m;\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right),-\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\right)+4F_1\left(\frac{3}{2}-n\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 -
m),x]
```

```
[Out] -((2^(1 - m)*(-3 + 2*m)*Cos[(-e + Pi/2 - f*x)/2]^2*Cot[(-e + Pi/2 - f*x)/4]
*Csc[(-e + Pi/2 - f*x)/4]^2*(-((1 + 2*m)*AppellF1[1/2 - m, -2*(1 + m), 1, 3
/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]) + (-1 + 2
*m)*Cot[(-e + Pi/2 - f*x)/4]^2*Hypergeometric2F1[-1/2 - m, -2*(1 + m), 1/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2])*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-2 - m))/(f*(-1 + 4*m^2)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^(2*(-2 - m))*(8*(1 + m)*AppellF1[3/2 - m, -1 - 2*m,
1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*Ap
pellF1[3/2 - m, -2*(1 + m), 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + (-3 + 2*m)*(2*AppellF1[1/2 - m, -2*(1 + m), 1, 3/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/
2 - f*x)/4]^2 - Csc[(-e + Pi/2 - f*x)/4]^4*(1 + Sin[e + f*x])*(1 - Tan[(-e
+ Pi/2 - f*x)/4]^2)^(2*m)))))
```

Maple [F] time = 0.79, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)*cos(f*x + e
)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algori
thm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)*cos(f*x + e)
^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-2-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)*cos(f*x + e
)^2, x)
```

3.85 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=114

$$\frac{c2^{\frac{1}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5)\right.}{f(2m+3)}$$

```
[Out] (2^(1/2 - m)*c*Cos[e + f*x]^3*Hypergeometric2F1[(1 + 2*m)/2, (3 + 2*m)/2, (
5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e
 + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rubi [A] time = 0.371424, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{c2^{\frac{1}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5)\right.}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2^(1/2 - m)*c*Cos[e + f*x]^3*Hypergeometric2F1[(1 + 2*m)/2, (3 + 2*m)/2, (
5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e
 + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-m}\,dx}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx)\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c-c\,\text{s}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-m}c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(\right.}{}$$

$$= \frac{2^{\frac{1}{2}-m}c\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(3+2m);\frac{1}{2}(5+\right.}{}$$

Mathematica [C] time = 29.3758, size = 1045, normalized size = 9.17

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] (2^(3 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 - m, -2*m, 2, 3/2
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*AppellF1
[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/4]^3*Cos[(-e + Pi/2 - f*x)/2]^2*Sin[(-e +
Pi/2 - f*x)/4]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(-1
 + 2*m)*((-3 + 2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 - 4*(-3 +
2*m)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 + 4*(-3 + 2*m)*AppellF1[1
/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 + 8*AppellF1[3/2 - m, -2*m, 3, 5/2 - m,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Cos[(-e + Pi
/2 - f*x)/2]) + 2*(2*m*AppellF1[3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*m*AppellF1[3/2 - m, 1 - 2*m
, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*
m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
```

```
-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 12*AppellF1[3/2 - m, -2*m, 4,
5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sin[(-e
+ Pi/2 - f*x)/4]^2)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^(2*(-1 - m)))
```

Maple [F] time = 0.472, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e
)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algori
thm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e)
^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e
)^2, x)
```

3.86 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=116

$$\frac{c^2 2^{\frac{3}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5\right.}{f(2m+3)}$$

```
[Out] (2^(3/2 - m)*c^2*Cos[e + f*x]^3*Hypergeometric2F1[(-1 + 2*m)/2, (3 + 2*m)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rubi [A] time = 0.323419, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{c^2 2^{\frac{3}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5\right.}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(3/2 - m)*c^2*Cos[e + f*x]^3*Hypergeometric2F1[(-1 + 2*m)/2, (3 + 2*m)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{1-m}\,dx}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c-c\sin\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}-m}c^3\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c-\right.}{}$$

$$= \frac{2^{\frac{3}{2}-m}c^2\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(-1+2m),\frac{1}{2}(3+2m);\frac{1}{2}(5+\right.}{}$$

Mathematica [C] time = 22.287, size = 1519, normalized size = 13.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(6 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 - m, -2*m, 3, 3/2
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1
[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2] - 4*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^
2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/4]^5*Cos[(-e + Pi/2
- f*x)/2]^2*Sin[(-e + Pi/2 - f*x)/4]^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^(2*m)*(a + a*Sin[e + f*x])^m)/(f*(-1 + 2*m)*((-3 + 2*m)*AppellF1[1/2 - m,
 -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
*Cos[(-e + Pi/2 - f*x)/4]^2 - 5*(-3 + 2*m)*AppellF1[1/2 - m, -2*m, 3, 3/2 -
 m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2
 - f*x)/4]^2 + 8*(-3 + 2*m)*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + P
i/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 +
12*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 - 8*m*AppellF1[1/2 - m, -2*
m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos
[(-e + Pi/2 - f*x)/4]^2 + 4*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-
e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]
```

```
^2 - 20*m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2
, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 + 32*m*AppellF1[3
/2 - m, 1 - 2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 - 16*m*AppellF1[3/2 - m, 1 - 2*m, 5,
5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e +
 Pi/2 - f*x)/4]^2 + 4*AppellF1[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 - 30*App
ellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 + 64*AppellF1[3/2 - m, -2*m, 5, 5
/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e +
Pi/2 - f*x)/4]^2 - 40*AppellF1[3/2 - m, -2*m, 6, 5/2 - m, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2)*Sin[(-e
 + Pi/2 - f*x)/2]^(2*m)*(c - c*Sin[e + f*x])^m)
```

Maple [F] time = 1.009, size = 0, normalized size = 0.

$$\int \frac{\left(\cos \left(fx+e\right)\right)^{2}\left(a+a\sin \left(fx+e\right)\right)^{m}}{\left(c-c\sin \left(fx+e\right)\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin \left(fx+e\right)+a\right)^{m}\cos \left(fx+e\right)^{2}}{\left(-c\sin \left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm
="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a\sin \left(fx+e\right)+a\right)^{m}\cos \left(fx+e\right)^{2}}{\left(-c\sin \left(fx+e\right)+c\right)^{m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm
="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/((c-c*sin(f*x+e))**m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm
="giac")
```

```
[Out] Exception raised: AttributeError
```

3.87 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=116

$$\frac{c^3 2^{\frac{5}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5\right.}{f(2m+3)}$$

```
[Out] (2^(5/2 - m)*c^3*Cos[e + f*x]^3*Hypergeometric2F1[(-3 + 2*m)/2, (3 + 2*m)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rubi [A] time = 0.36766, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2841, 2745, 2689, 70, 69}

$$\frac{c^3 2^{\frac{5}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5\right.}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] (2^(5/2 - m)*c^3*Cos[e + f*x]^3*Hypergeometric2F1[(-3 + 2*m)/2, (3 + 2*m)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 2841

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{2-m}\,dx}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{m}\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c-c\sin\right.}{}$$

$$= \frac{\left(2^{\frac{3}{2}-m}c^4\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))^m(c\right.}{}$$

$$= \frac{2^{\frac{5}{2}-m}c^3\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(-3+2m),\frac{1}{2}(3+2m);\frac{1}{2}(5\right.}{}$$

Mathematica [C] time = 25.891, size = 4270, normalized size = 36.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m
),x]
```

```
[Out] (2^(9 - m)*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF1[1/2 - m,
-2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
- 12*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*(a + a*Sin[e + f*x])^m*(c - c
*Sin[e + f*x])^(1 - m)*((Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/
2 + f*x)/2]^6)/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x
)/2])^(2*m) - (2*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/2 + f*x)
/2]^5*Sin[Pi/4 + (e - Pi/2 + f*x)/2])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin
[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) - (Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi
/4 + (e - Pi/2 + f*x)/2]^4*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e
 - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (4*Cos[(-e + Pi
/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/2 + f*x)/2]^3*Sin[Pi/4 + (e - Pi/2 +
f*x)/2]^3)/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2]
)^(2*m) - (Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/2 + f*x)/2]^2*
```

Sin[Pi/4 + (e - Pi/2 + f*x)/2]^4)/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) - (2*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[Pi/4 + (e - Pi/2 + f*x)/2]*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^5)/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^6)/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m))*Tan[(-e + Pi/2 - f*x)/4])/(f*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(1 - m))*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-((2^(9 - m)*m*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 12*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/((-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m))) - (2^(7 - m)*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 12*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2)/((-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(9 - m)*m*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 12*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(-1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(9 - m)*m*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 12*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (2^(9 - m)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Tan[(-e + Pi/2 - f*x)/4]*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (3*(1/2 - m)*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 - m)) - 6*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (2*(1/2 - m)*AppellF1[3/2 - m, -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) + 13*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (5*(1/2 - m)*AppellF1[3/2 - m, -2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 - m))) - 12*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/

```
2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 -
m)) - (3*(1/2 - m)*AppellF1[3/2 - m, -2*m, 7, 5/2 - m, Tan[(-e + Pi/2 - f*x
)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + P
i/2 - f*x)/4])/(3/2 - m)) + 4*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 7,
 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e
+ Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (7*(1/2 - m)*Appe
llF1[3/2 - m, -2*m, 8, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2
- m)))))/((-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f
*x)/4]^2)^(2*m))))
```

Maple [F] time = 0.648, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1)*cos(f*x + e
)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorit
hm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1)*cos(f*x + e)
^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1)*cos(f*x + e
)^2, x)
```

3.88 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=343

$$\frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2ac^3\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{7fg\sqrt{a\sin(e+fx)+a}}+\frac{10ac^2(c-c\sin(e+fx))^{3/2}(g\,c}{77fg\sqrt{a\sin(e+fx)}}$$

```
[Out] (2*a*c^4*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (2*a*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
+ (2*a*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(7*f*g*Sqrt[a +
 a*Sin[e + f*x]]) + (10*a*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2))/(77*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a*c*(g*Cos[e + f*x])^(5/2)*(c
- c*Sin[e + f*x])^(5/2))/(33*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e
+ f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(11*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 1.74316, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2ac^3\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{7fg\sqrt{a\sin(e+fx)+a}}+\frac{10ac^2(c-c\sin(e+fx))^{3/2}(g\,c}{77fg\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (2*a*c^4*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (2*a*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
+ (2*a*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(7*f*g*Sqrt[a +
 a*Sin[e + f*x]]) + (10*a*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2))/(77*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a*c*(g*Cos[e + f*x])^(5/2)*(c
- c*Sin[e + f*x])^(5/2))/(33*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e
+ f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(11*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
```

```
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx &= -\frac{2a(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{11fg\sqrt{a+a\sin(e+fx)}}+\frac{1}{11}(3a)\int \\ &= \frac{2ac(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{33fg\sqrt{a+a\sin(e+fx)}}-\frac{2a(g\cos(e}{11} \\ &= \frac{10ac^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{77fg\sqrt{a+a\sin(e+fx)}}+\frac{2ac(g\cos}{} \\ &= \frac{2ac^3(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{7fg\sqrt{a+a\sin(e+fx)}}+\frac{10ac^2(g\cos(}{77} \\ &= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2ac^3(g\cos(}{7f} \\ &= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2ac^3(g\cos(}{7f} \\ &= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2ac^3(g\cos(}{7f} \\ &= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2ac^4g\sqrt{\cos}}{f\sqrt{a}}\end{aligned}$$

Mathematica [C] time = 10.1668, size = 311, normalized size = 0.91

$$\frac{c^4ge^{-5i(e+fx)}\left(e^{i(e+fx)}-i\right)\left(4928e^{7i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(154e^{i(e+fx)}+423ie^{2i(e+fx)}-308e^{3i(e+fx)}+\right.\right.}{3696f\left(e^{i(e+fx}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (c^4*(-I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e +
f*x))]*(-21*I + 154*E^(I*(e + f*x)) + (423*I)*E^((2*I)*(e + f*x)) - 308*E^(
(3*I)*(e + f*x)) + (1374*I)*E^((4*I)*(e + f*x)) - 7392*E^((5*I)*(e + f*x))
+ (1374*I)*E^((6*I)*(e + f*x)) + 308*E^((7*I)*(e + f*x)) + (423*I)*E^((8*I)
*(e + f*x)) - 154*E^((9*I)*(e + f*x)) - (21*I)*E^((10*I)*(e + f*x))) + 4928
*E^((7*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))]
)*Sqrt[a*(1 + Sin[e + f*x])])/(3696*E^((5*I)*(e + f*x))*(I + E^(I*(e + f*x)
```

```
))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.43, size = 425, normalized size = 1.2

$$-\frac{2}{231 f \left(\left(\cos \left(f x+e\right)\right)^{2} \sin \left(f x+e\right)-3 \left(\cos \left(f x+e\right)\right)^{2}-4 \sin \left(f x+e\right)+4\right) \sin \left(f x+e\right) \left(\cos \left(f x+e\right)\right)^{3}} \left(-c \left(-1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/231/f*(-c*(-1+sin(f*x+e)))^(7/2)*(21*cos(f*x+e)^6*sin(f*x+e)+231*I*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)-77*cos(f*x+e)^6+231*I*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-132*sin(f*x+e)*cos(f*x+e)^
4+154*cos(f*x+e)^4+154*cos(f*x+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a
*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)
+4)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g \cos \left(f x+e\right)\right)^{\frac{3}{2}} \sqrt{a \sin \left(f x+e\right)+a}\left(-c \sin \left(f x+e\right)+c\right)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3 c^{3} g \cos \left(f x+e\right)^{3}-4 c^{3} g \cos \left(f x+e\right)-\left(c^{3} g \cos \left(f x+e\right)^{3}-4 c^{3} g \cos \left(f x+e\right)\right) \sin \left(f x+e\right)\right) \sqrt{g \cos \left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(3*c^3*g*cos(f*x + e)^3 - 4*c^3*g*cos(f*x + e) - (c^3*g*cos(f*x +
 e)^3 - 4*c^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.89 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=290

$$\frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22ac^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{105fg\sqrt{a\sin(e+fx)+a}}+\frac{22ac^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{15f\sqrt{a\sin(e+fx)+}}$$

```
[Out] (22*a*c^3*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]]) + (22*a*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]]) + (22*a*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(105*f*g
*Sqrt[a + a*Sin[e + f*x]]) + (2*a*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2))/(21*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)
*(c - c*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 1.42838, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22ac^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{105fg\sqrt{a\sin(e+fx)+a}}+\frac{22ac^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{15f\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (22*a*c^3*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]]) + (22*a*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]]) + (22*a*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(105*f*g
*Sqrt[a + a*Sin[e + f*x]]) + (2*a*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2))/(21*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)
*(c - c*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}}+\frac{1}{3}a\int \frac{(g\,c\ldots}{}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{21fg\sqrt{a+a\sin(e+fx)}}-\frac{2a(g\cos(e\ldots}{9f\ldots}$$

$$= \frac{22ac^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{105fg\sqrt{a+a\sin(e+fx)}}+\frac{2ac(g\cos(e\ldots}{21\ldots}$$

$$= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22ac^2(g\,c\ldots}{10\ldots}$$

$$= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22ac^2(g\,c\ldots}{10\ldots}$$

$$= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22ac^2(g\,c\ldots}{10\ldots}$$

$$= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22ac^3g\sqrt{c\ldots}}{15f\ldots}$$

Mathematica [C] time = 2.37835, size = 281, normalized size = 0.97

$$-\frac{c^3ge^{-4i(e+fx)}\left(e^{i(e+fx)}-i\right)\left(\sqrt{1+e^{2i(e+fx)}}\left(-180ie^{i(e+fx)}+238e^{2i(e+fx)}-540ie^{3i(e+fx)}+3696e^{4i(e+fx)}-540ie^{5i(e+fx)}-238e\ldots\right.\right.}{2520f\left(e^{i(e+fx)}+i\right)\sqrt{1+e^{2i\ldots}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] -(c^3*(-I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e +
 f*x))]*(-35 - (180*I)*E^(I*(e + f*x)) + 238*E^((2*I)*(e + f*x)) - (540*I)*
E^((3*I)*(e + f*x)) + 3696*E^((4*I)*(e + f*x)) - (540*I)*E^((5*I)*(e + f*x)
) - 238*E^((6*I)*(e + f*x)) - (180*I)*E^((7*I)*(e + f*x)) + 35*E^((8*I)*(e
+ f*x))) - 2464*E^((6*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(2520*E^((4*I)*(e + f*x))*(I +
 E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.368, size = 392, normalized size = 1.4

$$-\frac{2}{315f\left(\left(\cos\left(fx+e\right)\right)^2+2\sin\left(fx+e\right)-2\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(231\,i\sin\left(fx+e\right)c\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/315/f*(-c*(-1+sin(f*x+e)))^(5/2)*(231*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)-231*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+35*co
s(f*x+e)^6+90*sin(f*x+e)*cos(f*x+e)^4+231*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)-231*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-112*cos(f*x+e)^4-154*cos(f
*x+e)^2+231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)/(cos(
f*x+e)^2+2*sin(f*x+e)-2)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c^2g\cos\left(fx+e\right)^3+2\,c^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.90 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=235

$$\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6ac^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a(c-c\sin(e+fx}{7fg\sqrt{a\sin}}$$

```
[Out] (2*a*c^2*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (6*a*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) + (6*a*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35*f*g*Sqrt[a
+ a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/(7*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 1.12907, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6ac^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a(c-c\sin(e+fx}{7fg\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (2*a*c^2*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (6*a*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) + (6*a*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35*f*g*Sqrt[a
+ a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/(7*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7fg\sqrt{a+a\sin(e+fx)}}+\frac{1}{7}(3a)\int$$

$$=\frac{6ac(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{35fg\sqrt{a+a\sin(e+fx)}}-\frac{2a(g\cos(e+}{7fg}$$

$$=\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6ac(g\cos(e}{35f}$$

$$=\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6ac(g\cos(e}{35f}$$

$$=\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6ac(g\cos(e}{35f}$$

$$=\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6ac^2g\sqrt{\cos}}{5f\sqrt{a}}$$

Mathematica [C] time = 1.87459, size = 255, normalized size = 1.09

$$\frac{c^2ge^{-3i(e+fx)}\left(e^{i(e+fx)}-i\right)\left(112e^{5i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(-14e^{i(e+fx)}+15ie^{2i(e+fx)}-168e^{3i(e+fx)}+15i\right.\right.}{140f\left(e^{i(e+fx)}+i\right)\sqrt{1+e^{2i(e+fx)}}\sqrt{c-c\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] (c^2*(-I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e +
f*x))]*(5*I - 14*E^(I*(e + f*x)) + (15*I)*E^((2*I)*(e + f*x)) - 168*E^((3*I
)*(e + f*x)) + (15*I)*E^((4*I)*(e + f*x)) + 14*E^((5*I)*(e + f*x)) + (5*I)*
E^((6*I)*(e + f*x))) + 112*E^((5*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4,
7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(140*E^((3*I)*(e +
f*x))*(I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[c - c*Sin[
e + f*x]])
```

Maple [C] time = 0.346, size = 372, normalized size = 1.6

$$-\frac{2}{35f\left(-1+\sin\left(fx+e\right)\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(21\,i\sin\left(fx+e\right)\cos\left(fx+e\right)\mathrm{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/35/f*(-c*(-1+sin(f*x+e)))^(3/2)*(21*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)-21*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*sin(f*
x+e)*cos(f*x+e)^4+21*I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*sin(f*x+e)
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)-7*cos(f*x+e)^4-14*cos(f*x+e)^2+21*cos(f*x+e))*(a
*(1+sin(f*x+e)))^(1/2)*(g*cos(f*x+e))^(3/2)/(-1+sin(f*x+e))/sin(f*x+e)/cos(
f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.91 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=178

$$-\frac{2a\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}}+\frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6acg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx\right.}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c}}$$

```
[Out] (2*a*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]]) + (6*a*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[
(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) -
(2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*Sqrt[a + a*Sin
[e + f*x]])
```

Rubi [A] time = 0.802853, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{2a\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}}+\frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6acg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx\right.}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (2*a*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]]) + (6*a*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[
(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) -
(2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*Sqrt[a + a*Sin
[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5fg\sqrt{a+a\sin(e+fx)}} + \frac{1}{5}(3a)\int \frac{(g\cos$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+}{5fg\sqrt{}}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+}{5fg\sqrt{}}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+}{5fg\sqrt{}}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{6acg\sqrt{\cos(e+}}{5f\sqrt{a+}}$$

Mathematica [C] time = 2.43584, size = 249, normalized size = 1.4

$$\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\sec^3(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(12(\cos(fx)-i\sin(fx))\sqrt{i\sin(2(e+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Csc[e/2]*Sec[e/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sin[e
+ f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-11*Cos[f*x] - 13*Cos[2*e + f*x] + Cos[2
*e + 3*f*x] - Cos[4*e + 3*f*x] + 12*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^(
(2*I)*f*x)*(Cos[e] + I*Sin[e])^2)]*(Cos[f*x] - I*Sin[f*x])*Sqrt[1 + Cos[2*(
e + f*x)] + I*Sin[2*(e + f*x)]] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((
2*I)*f*x)*(Cos[e] + I*Sin[e])^2)]*(Cos[f*x] + I*Sin[f*x])*Sqrt[1 + Cos[2*(e
 + f*x)] + I*Sin[2*(e + f*x)]]))/(40*f)
```

Maple [C] time = 0.385, size = 346, normalized size = 1.9

$$\frac{2}{5f\sin(fx+e)(\cos(fx+e))^3}\sqrt{-c(-1+\sin(fx+e))}\left(3i\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{(\cos(fx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/5/f*(-c*(-1+sin(f*x+e)))^(1/2)*(3*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
```

```
sin(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x
+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f
*x+e)^4-2*cos(f*x+e)^2+3*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e))
)^(1/2)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (g\cos(fx+e))^{\frac{3}{2}} \sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}g\cos(fx+e),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.92 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=122

$$\frac{2ag\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-2*a*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Si
n[e + f*x]]) + (2*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.569529, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{2ag\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*
x]],x]
```


```
[Out] (-2*a*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Si
n[e + f*x]]) + (2*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}dx = -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + a\int \frac{(g\cos(e+fx))}{\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

$$= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(ag\cos(e+fx))\int\sqrt{g\cos(e+}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

$$= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{\left(ag\sqrt{\cos(e+fx)}\sqrt{g\cos(e+}\right.}{\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

$$= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2ag\sqrt{\cos(e+fx)}\sqrt{g\cos(e+}}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

Mathematica [C] time = 2.00108, size = 197, normalized size = 1.61

$$-\frac{ig\sqrt{ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2}\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(4e^{3i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)-i\sqrt{1+e^{2i(e+fx)}}\left(-6ie^{i(e+fx)}+e^{2i(}\right.\right.}{3cf\left(1+e^{2i(e+fx)}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] ((-I/3)*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*g*Sqrt[((1 + E
^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((-I)*Sqrt[1 + E^((2*I)*(e + f*x))]
*(1 - (6*I)*E^(I*(e + f*x)) + E^((2*I)*(e + f*x))) + 4*E^((3*I)*(e + f*x))*
Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e +
 f*x])])/(c*(1 + E^((2*I)*(e + f*x)))^(3/2)*f)
```

Maple [C] time = 0.354, size = 361, normalized size = 3.

$$-\frac{2}{3f\left(1+\sin\left(fx+e\right)\right)\sin\left(fx+e\right)\cos\left(fx+e\right)}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\left(3i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{c}{\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)*(3*I*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)+cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-3*cos(f*x+e))/(1+sin(
f*x+e))/sin(f*x+e)/cos(f*x+e)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}\sqrt{a\sin(fx+e)+a}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}g\cos(fx+e)}{c\sin(fx+e)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}\sqrt{a\sin(fx+e)+a}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x +
 e) + c), x)
```

3.93 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{6ag\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2)) - (6*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.581852, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$\frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{6ag\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2)) - (6*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{(3a)\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}}{c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{(3ag\cos(e+fx))\int\sqrt{g\cos(e}}{c\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{\left(3ag\sqrt{\cos(e+fx)}\sqrt{g\cos(e+}\right.}{c\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{6ag\sqrt{\cos(e+fx)}\sqrt{g\cos(e+f}}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

Mathematica [C] time = 1.7648, size = 211, normalized size = 1.72

$$-\frac{2g\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(2e^{2i(e+fx)}\left(e^{i(e+fx)}-i\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(i-5e^{i(e+fx)}\right)\right)\sqrt{a(\sin(e+fx}}{cf\left(e^{i(e+fx)}+i\right)\sqrt{1+e^{2i(e+fx)}}\sqrt{ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (-2*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((I - 5*E^(I*(e +
 f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))] + 2*E^((2*I)*(e + f*x))*(-I + E^(I*(e
 + f*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1
 + Sin[e + f*x])])/(c*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*
(I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f)
```

Maple [C] time = 0.398, size = 2835, normalized size = 23.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))*(-(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)-10*cos(f*x+e)^2+2*cos(f*x+e)^3-cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+c
os(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+
```

```
e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+
e)^2)+4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-6*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)+6*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*sin(f*x+e)-ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-4*cos(f*x+e)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln
(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+2*cos(f*x+e)^
2*sin(f*x+e)+sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*l
n(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-sin(f*x+e)
*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)-3*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+3*si
n(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*sin(f*x+e)*cos(f*x+e)*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)+6*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)-6*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*sin(f*x+e)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*s
in(f*x+e)-12*I*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+12*I*cos(f*x+e)^2*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*cos(f*x
+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)^3*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*
I*cos(f*x+e)^3*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*
x+e)))^(1/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)
/(-c*(-1+sin(f*x+e)))^(3/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\,g\cos(fx+e)}{c^2\cos(fx+e)^2+2c^2\sin(fx+e)-2c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

3.94 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{6ag\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{6a(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{4a(g\,c}{5fg\sqrt{a\sin(e+fx}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2)) - (6*a*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e + f*
x]]*(c - c*Sin[e + f*x])^(3/2)) + (6*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e +
f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rubi [A] time = 0.878637, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{6ag\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{6a(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{4a(g\,c}{5fg\sqrt{a\sin(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2)) - (6*a*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e + f*
x]]*(c - c*Sin[e + f*x])^(3/2)) + (6*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e +
f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} - \frac{(3a)\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+f}}{5c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} - \frac{6a(g\cos(e+fx)}{5cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} - \frac{6a(g\cos(e+fx)}{5cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} - \frac{6a(g\cos(e+fx)}{5cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} - \frac{6a(g\cos(e+fx)}{5cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

Mathematica [C] time = 2.14511, size = 229, normalized size = 1.26

$$\frac{4ig\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(e^{i(e+fx)}\left(e^{i(e+fx)}-i\right)^3 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\left(4ie^{i(e+fx)}-3e^{2i(e+fx)}+5\right)\sqrt{1+e^{2i(e+fx)}}\right)\sqrt{a(}}{5cf\left(e^{i(e+fx)}+i\right)\sqrt{1+e^{2i(e+fx)}}\left(ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (((4*I)/5)*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((5 + (4*I
)*E^(I*(e + f*x)) - 3*E^((2*I)*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))] +
E^(I*(e + f*x))*(-I + E^(I*(e + f*x)))^3*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(c*((I*c*(-I + E^(I*(e +
f*x)))^2)/E^(I*(e + f*x)))^(3/2)*(I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e
 + f*x))]*f)
```

Maple [C] time = 0.342, size = 2040, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/10/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)*(-1+cos(f*x+e))^3*(-1
+sin(f*x+e))*(-4*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
+20*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-5*ln(-2*(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3+5*ln(-(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3+5*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)-5*cos(f*x
+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+8*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-20*cos(f*x+e)^3*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)+8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-8*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+e)+5*cos(f*x+e)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-12*sin(f*x
+e)*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+24*I*sin(f*x+e)*cos(f
*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*s
in(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)+12*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)-12*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-12*I*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+12*I*EllipticF(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4-12*I*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4+24*I*cos(f*x+e)^3*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*cos(f*x
+e)^3*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))/(1+si
n(f*x+e))/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(-1+sin(f*x+e)))^(5/2)/s
in(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g\cos\left(fx+e\right)}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 -
 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

3.95 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{2a(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{2ag\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a(g}{15cfg\sqrt{a\sin(e+}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(7/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e + f
*x]]*(c - c*Sin[e + f*x])^(5/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(15*c^2*f*g
*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (2*a*g*Sqrt[Cos[e +
 f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.16495, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{2a(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{2ag\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a(g}{15cfg\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(7/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(7/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e + f
*x]]*(c - c*Sin[e + f*x])^(5/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(15*c^2*f*g
*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (2*a*g*Sqrt[Cos[e +
 f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{a\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+f}}{3c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\cos(e+}{15cfg\sqrt{a+a\sin(e+fx)}(}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\cos(e+}{15cfg\sqrt{a+a\sin(e+fx)}(}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\cos(e+}{15cfg\sqrt{a+a\sin(e+fx)}(}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\cos(e+}{15cfg\sqrt{a+a\sin(e+fx)}(}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{2a(g\cos(e+}{15cfg\sqrt{a+a\sin(e+fx)}(}$$

Mathematica [C] time = 2.27547, size = 256, normalized size = 1.08

$$\frac{4e^{3i(e+fx)}\left(ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^{3/2}\left(\left(e^{i(e+fx)}-i\right)^5 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(e^{i(e+fx)}+15ie^{2i(e+fx)}-3e^{3}\right.\right.}{45c^3f\left(e^{i(e+fx)}-i\right)^4\left(e^{i(e+fx)}+i\right)\left(1+e^{2i(e+fx)}\right)^{3/2}\sqrt{ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] (4*E^((3*I)*(e + f*x))*(((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x)))^(3/2
)*(Sqrt[1 + E^((2*I)*(e + f*x))]*(-29*I + E^(I*(e + f*x)) + (15*I)*E^((2*I)
*(e + f*x)) - 3*E^((3*I)*(e + f*x))) + (-I + E^(I*(e + f*x)))^5*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(4
5*c^3*(-I + E^(I*(e + f*x)))^4*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e
```

```
+ f*x))]*(I + E^(I*(e + f*x)))*(1 + E^((2*I)*(e + f*x)))^(3/2)*f)
```

Maple [C] time = 0.361, size = 966, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/45/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*cos(f*x+e)
-sin(f*x+e)-cos(f*x+e)+1)*(6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-12*I*co
s(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+
e)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4
+12*I*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+9*I*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
sin(f*x+e)*cos(f*x+e)^2+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4+6*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4
+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4-9*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2+3*cos(f*x+e)^4+6*cos(f*x+e)^2*sin(f*x+e)+
10*cos(f*x+e)^3-16*sin(f*x+e)*cos(f*x+e)-19*cos(f*x+e)^2+10*sin(f*x+e)-4*co
s(f*x+e)+10)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(1+sin(f*x+e))/(-c*(-1+sin(f*x+e
)))^(7/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g\cos\left(fx+e\right)}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g*cos(f*x + e)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4
*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

3.96 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{2a(g\cos(e+fx))^{5/2}}{65c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2a(g\cos(e+fx))^{5/2}}{65c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{2ag\sqrt{\cos(e+fx)}E\left(\right.}{65c^4f\sqrt{a\sin(e+}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(13*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(9/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(39*c*f*g*Sqrt[a + a*Sin[e +
f*x]]*(c - c*Sin[e + f*x])^(7/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(65*c^2*f*
g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (2*a*(g*Cos[e + f*
x])^(5/2))/(65*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
 + (2*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
)/(65*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.45982, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{2a(g\cos(e+fx))^{5/2}}{65c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2a(g\cos(e+fx))^{5/2}}{65c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{2ag\sqrt{\cos(e+fx)}E\left(\right.}{65c^4f\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(9/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(13*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(9/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(39*c*f*g*Sqrt[a + a*Sin[e +
f*x]]*(c - c*Sin[e + f*x])^(7/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(65*c^2*f*
g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (2*a*(g*Cos[e + f*
x])^(5/2))/(65*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
 + (2*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
)/(65*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
```

```
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}} - \frac{(3a)\int \frac{(g\cos(e+fx))^{3}}{\sqrt{a+a\sin(e+fx)}(c-c\sin}}{13c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}} - \frac{2a(g\cos(e}{39cfg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}} - \frac{2a(g\cos(e}{39cfg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}} - \frac{2a(g\cos(e}{39cfg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}} - \frac{2a(g\cos(e}{39cfg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}} - \frac{2a(g\cos(e}{39cfg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}} - \frac{2a(g\cos(e}{39cfg\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 2.86199, size = 291, normalized size = 1.

$$\frac{4e^{3i(e+fx)}\left(ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^{3/2}\left(\sqrt{1+e^{2i(e+fx)}}\left(149ie^{i(e+fx)}+44e^{2i(e+fx)}-64ie^{3i(e+fx)}+21e^{4i(e+fx)}+3ie^{5i(e+fx)}-\right.\right.}{195c^4f\left(1-ie^{i(e+fx)}\right)\left(e^{i(e+fx)}-i\right)^6\left(1+e^{2i(e+fx)}\right)^{3/2}\sqrt{ice^{-i(e+fx)}\left(e\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (4*E^((3*I)*(e + f*x))*(((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x)))^(3/2
)*(Sqrt[1 + E^((2*I)*(e + f*x))]*(-1 + (149*I)*E^(I*(e + f*x)) + 44*E^((2*I
)*(e + f*x)) - (64*I)*E^((3*I)*(e + f*x)) + 21*E^((4*I)*(e + f*x)) + (3*I)*
E^((5*I)*(e + f*x))) - I*(-I + E^(I*(e + f*x)))^7*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(195*c^4*(1 - I*
E^(I*(e + f*x)))*(-I + E^(I*(e + f*x)))^6*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^
2)/E^(I*(e + f*x))]*(1 + E^((2*I)*(e + f*x)))^(3/2)*f)
```

Maple [C] time = 0.385, size = 1126, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -2/195/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*cos(f*x+
e)-sin(f*x+e)-cos(f*x+e)+1)*(27*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21
*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-18*I*EllipticE(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)+9*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x
+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
-9*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-12*I*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*
x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-27*I*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)
^6-12*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+12*I*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)+12*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*sin(f*x+e)*cos(f*x+e)^4+18*I*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x
+e)^6-5*sin(f*x+e)*cos(f*x+e)^3-9*cos(f*x+e)^4-10*cos(f*x+e)^2*sin(f*x+e)-2
4*cos(f*x+e)^3+42*sin(f*x+e)*cos(f*x+e)+45*cos(f*x+e)^2-30*sin(f*x+e)+18*co
s(f*x+e)-30)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(1+sin(f*x+e))/(-c*(-1+sin(f*x+e
)))^(9/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g\cos\left(fx+e\right)}{5\,c^5\cos\left(fx+e\right)^4-20\,c^5\cos\left(fx+e\right)^2+16\,c^5-\left(c^5\cos\left(fx+e\right)^4-12\,c^5\cos\left(fx+e\right)^2+16\,c^5\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g*cos(f*x + e)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5
 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

3.97 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=352

$$\frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}}+\frac{14a^2c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{15f\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

```
[Out] (14*a^2*c^3*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (14*a^2*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (2*a^2*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(15
*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(3/2))/(33*f*g*Sqrt[a + a*Sin[e + f*x]]) - (14*a^2*(g*Cos[e + f*
x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(99*f*g*Sqrt[a + a*Sin[e + f*x]]) - (
2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2))/(11*f*g)
```

Rubi [A] time = 1.72694, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}}+\frac{14a^2c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{15f\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (14*a^2*c^3*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (14*a^2*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (2*a^2*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(15
*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(3/2))/(33*f*g*Sqrt[a + a*Sin[e + f*x]]) - (14*a^2*(g*Cos[e + f*
x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(99*f*g*Sqrt[a + a*Sin[e + f*x]]) - (
2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2))/(11*f*g)
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
```

```
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}(c-c\sin(\ldots}{11fg}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{99fg\sqrt{a+a\sin(e+fx)}} - \frac{2a(\ldots}{\ldots}$$

$$= \frac{2a^2c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{33fg\sqrt{a+a\sin(e+fx)}} - \frac{14a^2\ldots}{\ldots}$$

$$= \frac{2a^2c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{15fg\sqrt{a+a\sin(e+fx)}} + \frac{2a^2c(\ldots}{\ldots}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c\ldots}{\ldots}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c\ldots}{\ldots}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c\ldots}{\ldots}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{14a^2\ldots}{\ldots}$$

Mathematica [A] time = 1.38476, size = 193, normalized size = 0.55

$$\frac{c^2(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(3696E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}(\ldots\right.}{3960f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (c^2*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3
/2)*Sqrt[c - c*Sin[e + f*x]]*(3696*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e +
 f*x]]*(450*Cos[e + f*x] + 225*Cos[3*(e + f*x)] + 45*Cos[5*(e + f*x)] + 836
*Sin[2*(e + f*x)] + 110*Sin[4*(e + f*x)])))/(3960*f*Cos[e + f*x]^(3/2)*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
```

)

Maple [C] time = 0.386, size = 382, normalized size = 1.1

$$\frac{2}{495 f \left(-1+\sin \left(f x+e\right)\right) \sin \left(f x+e\right) \left(\cos \left(f x+e\right)\right)^{5}} \left(-c \left(-1+\sin \left(f x+e\right)\right)\right)^{\frac{5}{2}} \left(-45 \left(\cos \left(f x+e\right)\right)^{6} \sin \left(f x+e\right)+231\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/495/f*(-c*(-1+sin(f*x+e)))^(5/2)*(-45*cos(f*x+e)^6*sin(f*x+e)+231*I*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)+55*cos(f*x+e)^6+231*I*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+22*cos(f*x+e)^4+154*cos(f*
x+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)/(-1+si
n(f*x+e))/sin(f*x+e)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g \cos \left(f x+e\right)\right)^{\frac{3}{2}} \left(a \sin \left(f x+e\right)+a\right)^{\frac{3}{2}} \left(-c \sin \left(f x+e\right)+c\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a c^{2} g \cos \left(f x+e\right)^{3} \sin \left(f x+e\right)-a c^{2} g \cos \left(f x+e\right)^{3}\right) \sqrt{g \cos \left(f x+e\right)} \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c \sin \left(f x+e\right)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*g*cos(f*x + e)^3*sin(f*x + e) - a*c^2*g*cos(f*x + e)^3)*sq
rt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.98 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=295

$$\frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2(c-c\sin(e+f\ldots}{9fg\sqrt{a\sin\ldots}}$$

```
[Out] (14*a^2*c^2*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (14*a^2*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (2*a^2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(15*f
*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(3/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2
)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(9*f*g)
```

Rubi [A] time = 1.50321, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2(c-c\sin(e+f\ldots}{9fg\sqrt{a\sin\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(3/2),x]
```


```
[Out] (14*a^2*c^2*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (14*a^2*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (2*a^2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(15*f
*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(3/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2
)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(9*f*g)
```


Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}(c-c\sin(}{9fg}$$
$$= -\frac{2a^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{9fg\sqrt{a+a\sin(e+fx)}} - \frac{2a(g}{}$$
$$= \frac{2a^2c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{15fg\sqrt{a+a\sin(e+fx)}} - \frac{2a^2(g\,c}{}$$
$$= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c}{}$$
$$= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c}{}$$
$$= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c}{}$$
$$= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{14a^2}{}$$

Mathematica [A] time = 0.742305, size = 113, normalized size = 0.38

$$-\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(168E\left(\frac{1}{2}(e+fx)\middle|2\right)+(38\sin(2(e+fx}{180f\cos^{\frac{9}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -(c*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)
*Sqrt[c - c*Sin[e + f*x]]*(168*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x
]]*(38*Sin[2*(e + f*x)] + 5*Sin[4*(e + f*x)])))/(180*f*Cos[e + f*x]^(9/2))
```

Maple [C] time = 0.322, size = 356, normalized size = 1.2

$$\frac{2}{45f\sin(fx+e)(\cos(fx+e))^5}\left(-c\left(-1+\sin(fx+e)\right)\right)^{\frac{3}{2}}\left(21i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos(fx\right.}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/45/f*(-c*(-1+sin(f*x+e)))^(3/2)*(21*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)-21*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5*cos(f*x
+e)^6+21*I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*sin(f*x+e)*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)-2*cos(f*x+e)^4-14*cos(f*x+e)^2+21*cos(f*x+e))*(g*cos(f*x+e))
^(3/2)*(a*(1+sin(f*x+e)))^(3/2)/sin(f*x+e)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}acg\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*a*c*g*cos(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.99 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=235

$$-\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6a^2cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c(a\sin(e+fx)+a)}{7fg\sqrt{c-c\mathrm{s}}}$$

```
[Out] (-2*a^2*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (6*a^2*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]) - (6*a*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(35*f*g*Sqrt[c
 - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(7*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.13886, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6a^2cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c(a\sin(e+fx)+a)}{7fg\sqrt{c-c\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*
x]],x]
```


```
[Out] (-2*a^2*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (6*a^2*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]) - (6*a*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(35*f*g*Sqrt[c
 - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(7*f*g*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{7fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{7}(3c)\int$$

$$= -\frac{6ac(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{35fg\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6ac(g\,c$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6ac(g\,c$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6ac(g\,c$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{6a^2cg}{5}$$

Mathematica [C] time = 7.75171, size = 257, normalized size = 1.09

$$\frac{ia^2ge^{-3i(e+fx)}\left(e^{i(e+fx)}+i\right)\left(112ie^{5i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(-14ie^{i(e+fx)}+15e^{2i(e+fx)}-168ie^{3i(e+fx)}\right.\right.}{140f\left(e^{i(e+fx)}-i\right)\sqrt{1+e^{2i(e+fx)}}\sqrt{a(\sin(e+fx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] ((I/140)*a^2*(I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I
)*(e + f*x))]*(5 - (14*I)*E^(I*(e + f*x)) + 15*E^((2*I)*(e + f*x)) - (168*I
)*E^((3*I)*(e + f*x)) + 15*E^((4*I)*(e + f*x)) + (14*I)*E^((5*I)*(e + f*x))
 + 5*E^((6*I)*(e + f*x))) + (112*I)*E^((5*I)*(e + f*x))*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[c - c*Sin[e + f*x]])/(E^((3*I)*(e
 + f*x))*(-I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[a*(1 +
 Sin[e + f*x])])
```

Maple [C] time = 0.327, size = 372, normalized size = 1.6

$$-\frac{2}{35f\left(1+\sin\left(fx+e\right)\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\left(21\,i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx\right.}{\cos\left(fx+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/35/f*(-c*(-1+sin(f*x+e)))^(1/2)*(21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
+5*sin(f*x+e)*cos(f*x+e)^4+7*cos(f*x+e)^4+14*cos(f*x+e)^2-21*cos(f*x+e))*(g
*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)/(1+sin(f*x+e))/sin(f*x+e)/cos(f
*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral((a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.100 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=180

$$-\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\sqrt{a\sin(e+fx)+}}{5fg\sqrt{c-c\,s}}$$

```
[Out] (-14*a^2*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) - (2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*f*g*Sqrt[c - c
*Sin[e + f*x]])
```

Rubi [A] time = 0.849926, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\sqrt{a\sin(e+fx)+}}{5fg\sqrt{c-c\,s}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Sin[e +
f*x]],x]
```


```
[Out] (-14*a^2*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) - (2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*f*g*Sqrt[c - c
*Sin[e + f*x]])
```


Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{5}(7a)\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{c-c\,\mathrm{s}}}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+fx))^{5/2}\sqrt{a}}{5fg\sqrt{c-c\sin(}}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+fx))^{5/2}\sqrt{a}}{5fg\sqrt{c-c\sin(}}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+fx))^{5/2}\sqrt{a}}{5fg\sqrt{c-c\sin(}}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{14a^2g\sqrt{\cos(e+fx)}\sqrt{g}}{5f\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 0.793908, size = 148, normalized size = 0.82

$$-\frac{(a(\sin(e+fx)+1))^{3/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}(3\sin(2(e+fx))+20\cos\right.}{15f\cos^{\frac{3}{2}}(e+fx)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Si
n[e + f*x]],x]
```

```
[Out] -((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(3/2)*(-42*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(20*Co
s[e + f*x] + 3*Sin[2*(e + f*x)])))/(15*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.309, size = 384, normalized size = 2.1

$$-\frac{2}{15f\left(-\left(\cos\left(fx+e\right)\right)^2+2\sin\left(fx+e\right)+2\right)\sin\left(fx+e\right)\cos\left(fx+e\right)}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)*(-21*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
```

```
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)-3*cos(f*x+e)^4+10*cos(f*x+e)^2*sin(f*x+e)+24*cos(f*x+e
)^2-21*cos(f*x+e))/(-cos(f*x+e)^2+2*sin(f*x+e)+2)/sin(f*x+e)/cos(f*x+e)/(-c
*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ag\cos(fx+e)\sin(fx+e)+ag\cos(fx+e))\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{c\sin(fx+e)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) -
 c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

3.101 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{3cfg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{14a^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{4a\sqrt{a\sin(e+fx)+a}}{fg(c-c\sin(e}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(f*g*(c - c*Sin[e + f
*x])^(3/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(3*c*f*g*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e
+ f*x]]*EllipticE[(e + f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]])
```

Rubi [A] time = 0.875732, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{3cfg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{14a^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{4a\sqrt{a\sin(e+fx)+a}}{fg(c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(f*g*(c - c*Sin[e + f
*x])^(3/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(3*c*f*g*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e
+ f*x]]*EllipticE[(e + f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]])
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e\cdots}}{\sqrt{c-c\sin(e+fx)}}}{c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a^2(g\cos(e+fx\cdots}{3cfg\sqrt{a+a\sin(e+fx)}\sqrt{c-\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a^2(g\cos(e+fx\cdots}{3cfg\sqrt{a+a\sin(e+fx)}\sqrt{c-\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a^2(g\cos(e+fx\cdots}{3cfg\sqrt{a+a\sin(e+fx)}\sqrt{c-\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a^2(g\cos(e+fx\cdots}{3cfg\sqrt{a+a\sin(e+fx)}\sqrt{c-\cdots}}$$

Mathematica [A] time = 1.76575, size = 207, normalized size = 1.14

$$-\frac{2(a(\sin(e+fx)+1))^{3/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)(\cos(\cdots\right.\right.}{3cf(\sin(e+fx)-1)\cos^{\frac{3}{2}}(e+fx)\sqrt{c-c\sin(e+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(-21*Ell
ipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + Sqrt[Cos[e +
 f*x]]*(Cos[(e + f*x)/2]*(12 + Cos[e + f*x]) - (-12 + Cos[e + f*x])*Sin[(e
+ f*x)/2]))*(a*(1 + Sin[e + f*x]))^(3/2))/(3*c*f*Cos[e + f*x]^(3/2)*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [C] time = 0.339, size = 2892, normalized size = 15.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)+33*cos(f*x+e)^2-8*cos(f*x+e)^3-sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^
4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+12*c
os(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)+18*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-18*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)-3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+3*ln(-(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(3/2)*sin(f*x+e)+12*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*l
n(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-12*cos(f*x
+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)-9*cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^4-3*sin(f*x+e)*cos(f*x+e)^3*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)+3*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^
2)-9*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+9*sin(f*x+e)*cos(f
*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-9*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(3/2)+9*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+21*I*sin(f*x+e
)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(
```

```
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)-42*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+42*I*cos(f*x
+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+2
1*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)+21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^
(3/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos
(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(-1+sin(f*x+e)))^(3/2)/sin(f*x+e)/
cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)
^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

3.102 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{42a^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{28a^2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}} + \frac{4a\sqrt{a\sin(e+f}}{5fg(c-}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*f*g*(c - c*Sin[e +
 f*x])^(5/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (42*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Co
s[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.8868, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$\frac{42a^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{28a^2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}} + \frac{4a\sqrt{a\sin(e+f}}{5fg(c-}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(5/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*f*g*(c - c*Sin[e +
 f*x])^(5/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (42*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Co
s[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}}{5c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a^2(g\cos(e+fx))^{5}}{5cfg\sqrt{a+a\sin(e+fx)}(c-c\sin}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a^2(g\cos(e+fx))^{5}}{5cfg\sqrt{a+a\sin(e+fx)}(c-c\sin}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a^2(g\cos(e+fx))^{5}}{5cfg\sqrt{a+a\sin(e+fx)}(c-c\sin}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a^2(g\cos(e+fx))^{5}}{5cfg\sqrt{a+a\sin(e+fx)}(c-c\sin}$$

Mathematica [A] time = 1.44415, size = 191, normalized size = 1.03

$$-\frac{a\sqrt{\cos(e+fx)}\sqrt{a(\sin(e+fx)+1)}(g\cos(e+fx))^{3/2}\left(8\sqrt{\cos(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)-2\sin\left(\frac{3}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.\right.}{5c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] -(a*Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*Sqrt[a*(1 + Sin[e + f*x])]*(-
42*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 8*Sq
rt[Cos[e + f*x]]*(Cos[(e + f*x)/2] + 2*Cos[(3*(e + f*x))/2] + Sin[(e + f*x)
/2] - 2*Sin[(3*(e + f*x))/2])))/(5*c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.337, size = 3499, normalized size = 18.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/5/f*(-1+cos(f*x+e))*(5*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-10*(-cos(
```

```
f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)+10*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-38*cos(f*x+e)^2+5*sin(f*
x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5*sin(f*x+e)*cos(f*x+e)^4*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)+9*cos(f*x+e)^3-5*sin(f*x+e)*cos(f*x+e)^3+10*cos(f*x+
e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-10*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x
+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-
10*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+10*cos(f*x+e)^3*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)-40*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+40*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)+10*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-10*ln(-(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*sin(f*x+e)-35*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+35*c
os(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+46*cos(f*x+e)^2*sin(f*x+e)+5*cos(f*x+e)
^4+25*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-25*sin(f*x+e)*cos
(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+45*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)-45*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+35*sin
(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-35*sin(f*x+e)*cos(f*x+e)*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)-21*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos
(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+63*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)-63*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
```

```
^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+42*I*cos(f*x+e)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)-42*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-5*cos(f*x+e)^5*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)+21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x
+e)-63*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+63*I*sin(f*x+e)*
cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-42*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*sin(f*x+e)+42*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-21*I*
cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*
(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*co
s(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(-1+sin(f*x+e)))^(
5/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x +
e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

3.103 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=243

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{14a^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{28a^2(}{45cfg\sqrt{a\sin(e+}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(9*f*g*(c - c*Sin[e +
 f*x])^(7/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(45*c*f*g*Sqrt[a + a*Sin[e
+ f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(15*c
^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (14*a^2*g*Sqr
t[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.19428, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{14a^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{28a^2(}{45cfg\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(7/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(9*f*g*(c - c*Sin[e +
 f*x])^(7/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(45*c*f*g*Sqrt[a + a*Sin[e
+ f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(15*c
^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (14*a^2*g*Sqr
t[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e\cdots}}{(c-c\sin(e+fx))^{5/2}}}{9c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos(e+f\cdots}{45cfg\sqrt{a+a\sin(e+fx)}(c-\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos(e+f\cdots}{45cfg\sqrt{a+a\sin(e+fx)}(c-\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos(e+f\cdots}{45cfg\sqrt{a+a\sin(e+fx)}(c-\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos(e+f\cdots}{45cfg\sqrt{a+a\sin(e+fx)}(c-\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{28a^2(g\cos(e+f\cdots}{45cfg\sqrt{a+a\sin(e+fx)}(c-\cdots}$$

Mathematica [A] time = 2.40562, size = 218, normalized size = 0.9

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{a(\sin(e+fx)+1)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}\left(-74\sin\left(\frac{1}{2}(e+fx)\right)+15\sin\left(\frac{3}{2}(e+fx)\right)+21\sin\cdots\right.\right.}{90c^3f(\sin(e+fx)-1)^3\sqrt{c-c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(7/2),x]
```

```
[Out] (a*Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*Sqrt[a*(1 + Sin[e + f*x])]*(84
*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + Sqrt[C
os[e + f*x]]*(-74*Cos[(e + f*x)/2] - 15*Cos[(3*(e + f*x))/2] + 21*Cos[(5*(e
 + f*x))/2] - 74*Sin[(e + f*x)/2] + 15*Sin[(3*(e + f*x))/2] + 21*Sin[(5*(e
+ f*x))/2])))/(90*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e
```

```
 + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.356, size = 2684, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/90/f*(cos(f*x+e)+1)*(-1+cos(f*x+e))^4*(336*I*cos(f*x+e)^3*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-168*I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+248*sin(f*x+e)*cos(f*x
+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-88*cos(f*x+e)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)+90*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*cos(f*x+e)^3-90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)*cos(f*x+e)^3-90*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)+90*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+80*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*sin(f*x+e)+88*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+80*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-164*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)+90*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)*cos(f*x+e)*sin(f*x+e)-90*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-12*sin(f*x+e)*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)+84*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-45*sin(f*x+e)*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)+45*sin(f*x+e)*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-180*sin(f*x+e)*cos(f*x+e)^3*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+168*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-336*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+168*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-168*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+168*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+336*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-336*I*cos(f*x+e)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-168*I*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+84*I*sin(f*x+e)*cos(
f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-84*
```

```
I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)+168*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-168*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-84*I*sin(f*x+e)*cos(f*x+e
)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+84*I*sin
(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)-336*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+c
os(f*x+e))/sin(f*x+e),I)+336*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(-1+sin(f*x+e))*(a*(1+sin(f*x+e))
)^(3/2)*(g*cos(f*x+e))^(3/2)/sin(f*x+e)^9/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)/(-c*(-1+sin(f*x+e)))^(7/2)/(-cos(f*x+e)^2+2*sin(f*x+e)+2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral((a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^
4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x +
 e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

3.104 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=300

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{195c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{195c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{14a^2g\sqrt{\cos(e+}}{195c^4f\sqrt{a\,\mathrm{si}}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(13*f*g*(c - c*Sin[e
+ f*x])^(9/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(117*c*f*g*Sqrt[a + a*Sin[
e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(19
5*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (14*a^2*(g
*Cos[e + f*x])^(5/2))/(195*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
f*x])^(3/2)) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[
(e + f*x)/2, 2])/(195*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x
]])
```

Rubi [A] time = 1.4969, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{195c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{195c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{14a^2g\sqrt{\cos(e+}}{195c^4f\sqrt{a\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(9/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(13*f*g*(c - c*Sin[e
+ f*x])^(9/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(117*c*f*g*Sqrt[a + a*Sin[
e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(19
5*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (14*a^2*(g
*Cos[e + f*x])^(5/2))/(195*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
f*x])^(3/2)) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[
(e + f*x)/2, 2])/(195*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x
]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
```

```
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}}{13c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos(e+fx)}{117cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos(e+fx)}{117cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos(e+fx)}{117cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos(e+fx)}{117cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos(e+fx)}{117cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{28a^2(g\cos(e+fx)}{117cfg\sqrt{a+a\sin(e+fx)}(c-c}$$

Mathematica [A] time = 6.48803, size = 464, normalized size = 1.55

$$\sec(e+fx)(a(\sin(e+fx)+1))^{3/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9\left(\frac{28\sin\left(\frac{1}{2}(e+fx)\right)}{195\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{}{19}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(9/2),x]
```

```
[Out] (-14*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(3/2))/(195*f*Cos[e + f*x]^(3/2)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(9/2)) + ((g*C
os[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(14
/195 + 8/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) - 64/(117*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^4) + 14/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2) + (16*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
- (128*Sin[(e + f*x)/2])/(117*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (2
8*Sin[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (28*Sin
[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e +
 f*x]))^(3/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*
x])^(9/2))
```

Maple [C] time = 0.355, size = 1138, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -2/585/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)*(sin(f*x+e)*cos(f*x+
e)-sin(f*x+e)-cos(f*x+e)+1)*(-189*I*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-
126*I*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-84*I*sin(f*x+e)*EllipticE(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)+147*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6-63*I*sin(f*x+e)*cos(f*x+e)^4*EllipticE
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^6+189*I*cos(f*x+
e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)-147*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)-84*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+84*I*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+8
4*I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*sin(f*x+e)*cos(f*x+e)^4+63*I*si
n(f*x+e)*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+126*I*cos(f*x+e)^4*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)-160*sin(f*x+e)*cos(f*x+e)^3+63*cos(f*x+e)^4+265*cos(f*x+e
)^2*sin(f*x+e)-222*cos(f*x+e)^3+96*sin(f*x+e)*cos(f*x+e)+75*cos(f*x+e)^2-18
0*sin(f*x+e)+264*cos(f*x+e)-180)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(-cos(f*x+e)
^2+2*sin(f*x+e)+2)/(-c*(-1+sin(f*x+e)))^(9/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{5\,c^{5}\cos\left(fx+e\right)^{4}-20\,c^{5}\cos\left(fx+e\right)^{2}+16\,c^{5}-\left(c^{5}\cos\left(fx+e\right)^{4}-12\,c^{5}\cos\left(fx+e\right)^{2}+16\,c^{5}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] integral((a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e
)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x
 + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

3.105 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=357

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{1105c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{1105c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{14a^2}{663c^2fg\sqrt{a\sin(e}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(17*f*g*(c - c*Sin[e
+ f*x])^(11/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(221*c*f*g*Sqrt[a + a*Sin
[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(6
63*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*a^2*(
g*Cos[e + f*x])^(5/2))/(1105*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(5/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(1105*c^4*f*g*Sqrt[a + a*S
in[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(1105*c^5*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.8083, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{1105c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{1105c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{14a^2}{663c^2fg\sqrt{a\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(11/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(17*f*g*(c - c*Sin[e
+ f*x])^(11/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(221*c*f*g*Sqrt[a + a*Sin
[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(6
63*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*a^2*(
g*Cos[e + f*x])^(5/2))/(1105*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(5/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(1105*c^4*f*g*Sqrt[a + a*S
in[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(1105*c^5*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
```

```
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{(7a)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e\cdot}}{(c-c\sin(e+fx))^{9/2}}}{17c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos(e+}{221cfg\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos(e+}{221cfg\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos(e+}{221cfg\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos(e+}{221cfg\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos(e+}{221cfg\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos(e+}{221cfg\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{28a^2(g\cos(e+}{221cfg\sqrt{a+a\sin(e+fx)}(c}$$

Mathematica [A] time = 6.53852, size = 532, normalized size = 1.49

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{3/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}\left(\frac{28\sin\left(\frac{1}{2}(e+fx)\right)}{1105\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(11/2),x]
```

```
[Out] (-14*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(3/2))/(1105*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(11/2)) + ((
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11
*(14/1105 + 8/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 80/(221*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^6) + 14/(663*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^4) + 14/(1105*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (16*Sin[(e
 + f*x)/2])/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (160*Sin[(e + f*
x)/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (28*Sin[(e + f*x)/2]
)/(663*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (28*Sin[(e + f*x)/2])/(11
05*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (28*Sin[(e + f*x)/2])/(1105*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(3/2))/(f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(11/2))
```

Maple [C] time = 0.41, size = 1298, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -2/3315/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)*(sin(f*x+e)*cos(f*x
+e)-sin(f*x+e)-cos(f*x+e)+1)*(-780-780*sin(f*x+e)+948*cos(f*x+e)+612*sin(f*
x+e)*cos(f*x+e)+605*cos(f*x+e)^2+35*cos(f*x+e)^5-941*cos(f*x+e)^3-523*sin(f
*x+e)*cos(f*x+e)^3-84*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6+84*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)^6+336*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^
4-336*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4-420*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*cos(f*x+e)^2+420*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2-168*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*sin(f*x+e)+168*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-84
*sin(f*x+e)*cos(f*x+e)^4+775*cos(f*x+e)^2*sin(f*x+e)-21*cos(f*x+e)^6+154*co
s(f*x+e)^4-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^6+21*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^6+168*I*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
```

```
I)-168*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-189*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+
e)*cos(f*x+e)^4+189*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4+336
*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2-336*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(-cos(f*x+
e)^2+2*sin(f*x+e)+2)/(-c*(-1+sin(f*x+e)))^(11/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ag\cos\left(fx+e\right)\sin\left(fx+e\right)+ag\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin}}{c^6\cos\left(fx+e\right)^6-18\,c^6\cos\left(fx+e\right)^4+48\,c^6\cos\left(fx+e\right)^2-32\,c^6+2\left(3\,c^6\cos\left(fx+e\right)^4-16\,c^6\cos\left(fx-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="fricas")
```

```
[Out] integral(-(a*g*cos(f*x + e)*sin(f*x + e) + a*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^6*cos(f*x + e)
^6 - 18*c^6*cos(f*x + e)^4 + 48*c^6*cos(f*x + e)^2 - 32*c^6 + 2*(3*c^6*cos(
f*x + e)^4 - 16*c^6*cos(f*x + e)^2 + 16*c^6)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

3.106 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=406

$$\frac{22a^3c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{195fg\sqrt{a\sin(e+fx)+a}}+\frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154a^3c^3g\sqrt{\cos(e+fx)}E(}{195f\sqrt{a\sin(e+fx}}$$

```
[Out] (154*a^3*c^3*(g*Cos[e + f*x])^(5/2))/(585*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (154*a^3*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*
x]]*EllipticE[(e + f*x)/2, 2])/(195*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]]) + (22*a^3*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]
)/(195*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a^3*c*(g*Cos[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(3/2))/(39*f*g*Sqrt[a + a*Sin[e + f*x]]) - (14*a^3*(g*Cos[
e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(117*f*g*Sqrt[a + a*Sin[e + f*x
]]) - (2*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2))/(13*f*g) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(
3/2)*(c - c*Sin[e + f*x])^(5/2))/(13*f*g)
```

Rubi [A] time = 2.08878, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{22a^3c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{195fg\sqrt{a\sin(e+fx)+a}}+\frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154a^3c^3g\sqrt{\cos(e+fx)}E(}{195f\sqrt{a\sin(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (154*a^3*c^3*(g*Cos[e + f*x])^(5/2))/(585*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (154*a^3*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*
x]]*EllipticE[(e + f*x)/2, 2])/(195*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]]) + (22*a^3*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]
)/(195*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a^3*c*(g*Cos[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(3/2))/(39*f*g*Sqrt[a + a*Sin[e + f*x]]) - (14*a^3*(g*Cos[
e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(117*f*g*Sqrt[a + a*Sin[e + f*x
]]) - (2*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2))/(13*f*g) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(
3/2)*(c - c*Sin[e + f*x])^(5/2))/(13*f*g)
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e}{13fg}$$
$$= -\frac{2a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+}{13fg}$$
$$= -\frac{14a^3(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{117fg\sqrt{a+a\sin(e+fx)}} - \frac{2a^2(g}{}$$
$$= \frac{2a^3c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{39fg\sqrt{a+a\sin(e+fx)}} - \frac{14a^3(g}{}$$
$$= \frac{22a^3c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{195fg\sqrt{a+a\sin(e+fx)}} + \frac{2a^3c(g}{}$$
$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22a^3c^2}{}$$
$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22a^3c^2}{}$$
$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22a^3c^2}{}$$
$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{154a^3c}{1}$$

Mathematica [A] time = 1.19533, size = 120, normalized size = 0.3

$$\frac{a^2c^2\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(7392E\left(\frac{1}{2}(e+fx)\middle|2\right)+(1897\sin(2(e+fx))+400\sin(4(e}{9360f\cos^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (a^2*c^2*(g*Cos[e + f*x])^(3/2)*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e
 + f*x]]*(7392*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(1897*Sin[2*(
```

```
e + f*x)] + 400*Sin[4*(e + f*x)] + 45*Sin[6*(e + f*x)])))/(9360*f*Cos[e + f
*x]^(5/2))
```

Maple [C] time = 0.372, size = 366, normalized size = 0.9

$$-\frac{2}{585 f \sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{7}}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(45\left(\cos\left(fx+e\right)\right)^{8}+231\,i\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx\right.\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/585/f*(-c*(-1+sin(f*x+e)))^(5/2)*(45*cos(f*x+e)^8+231*I*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)+10*cos(f*x+e)^6+231*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I
*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+22*cos(f*x+e)^4+154*cos(f*x+e)^2-231*
cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)/sin(f*x+e)/cos(f*
x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}a^{2}c^{2}g\cos\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*a^2*c^2*g*cos(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.107 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=352

$$-\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2c^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{15fg\sqrt{c-c\sin(e+fx)}}+\frac{14a^3c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}\right.}{15f\sqrt{a\sin(e+fx)}}$$

```
[Out] (-14*a^3*c^2*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (14*a^3*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]
]*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) - (2*a^2*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(1
5*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Si
n[e + f*x])^(3/2))/(33*f*g*Sqrt[c - c*Sin[e + f*x]]) + (14*c^2*(g*Cos[e + f
*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(99*f*g*Sqrt[c - c*Sin[e + f*x]]) +
(2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f
*x]])/(11*f*g)
```

Rubi [A] time = 1.75728, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2c^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{15fg\sqrt{c-c\sin(e+fx)}}+\frac{14a^3c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}\right.}{15f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-14*a^3*c^2*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (14*a^3*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]
]*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) - (2*a^2*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(1
5*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Si
n[e + f*x])^(3/2))/(33*f*g*Sqrt[c - c*Sin[e + f*x]]) + (14*c^2*(g*Cos[e + f
*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(99*f*g*Sqrt[c - c*Sin[e + f*x]]) +
(2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f
*x]])/(11*f*g)
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
```

```
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e\cdots}}{11fg}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{99fg\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\,\mathrm{co}\cdots}{}$$

$$= -\frac{2ac^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{33fg\sqrt{c-c\sin(e+fx)}} + \frac{14c^2(g\cdots}{}$$

$$= -\frac{2a^2c^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{15fg\sqrt{c-c\sin(e+fx)}} - \frac{2ac^2(g\cdots}{}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2c^2(\cdots}{}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2c^2(\cdots}{}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2c^2(\cdots}{}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{14a^3c^2\cdots}{\cdots}$$

Mathematica [A] time = 1.36524, size = 189, normalized size = 0.54

$$\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}(-836\sin(2(e+fx))-110\sin\cdots\right.}{3960f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\mathrm{s}\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (c*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*
Sqrt[c - c*Sin[e + f*x]]*(-3696*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*
x]]*(450*Cos[e + f*x] + 225*Cos[3*(e + f*x)] + 45*Cos[5*(e + f*x)] - 836*Si
n[2*(e + f*x)] - 110*Sin[4*(e + f*x)])))/(3960*f*Cos[e + f*x]^(3/2)*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [C] time = 0.394, size = 382, normalized size = 1.1

$$\frac{2}{495 f \left(1+\sin \left(f x+e\right)\right) \sin \left(f x+e\right) \left(\cos \left(f x+e\right)\right)^{5}} \left(-c \left(-1+\sin \left(f x+e\right)\right)\right)^{\frac{3}{2}} \left(-45 \left(\cos \left(f x+e\right)\right)^{6} \sin \left(f x+e\right)+23\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/495/f*(-c*(-1+sin(f*x+e)))^(3/2)*(-45*cos(f*x+e)^6*sin(f*x+e)+231*I*sin(f
*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*sin(f*x+e)*cos(f*x+e)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)-55*cos(f*x+e)^6+231*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)-231*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-22*cos(f*x+e)^4-154*cos(f*
x+e)^2+231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)/(1+sin
(f*x+e))/sin(f*x+e)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g \cos \left(f x+e\right)\right)^{\frac{3}{2}} \left(a \sin \left(f x+e\right)+a\right)^{\frac{5}{2}} \left(-c \sin \left(f x+e\right)+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(a^{2} c g \cos \left(f x+e\right)^{3} \sin \left(f x+e\right)+a^{2} c g \cos \left(f x+e\right)^{3}\right) \sqrt{g \cos \left(f x+e\right)} \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c \sin \left(f x+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*g*cos(f*x + e)^3*sin(f*x + e) + a^2*c*g*cos(f*x + e)^3)*sqr
t(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.108 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=290

$$-\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2c\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{105fg\sqrt{c-c\sin(e+fx)}}+\frac{22a^3cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}\right.}{15f\sqrt{a\sin(e+fx)}}$$

```
[Out] (-22*a^3*c*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]]) + (22*a^3*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
f*x]]) - (22*a^2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(105*f*
g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e +
f*x])^(3/2))/(21*f*g*Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2
)*(a + a*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.42011, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2c\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{105fg\sqrt{c-c\sin(e+fx)}}+\frac{22a^3cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}\right.}{15f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-22*a^3*c*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]]) + (22*a^3*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
f*x]]) - (22*a^2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(105*f*
g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e +
f*x])^(3/2))/(21*f*g*Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2
)*(a + a*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg\sqrt{c-c\sin(e+fx)}}+\frac{1}{3}c\int \frac{(g\cos$$

$$= -\frac{2ac(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg\sqrt{c-c\sin(e+fx)}}+\frac{2c(g\cos(}{}$$

$$= -\frac{22a^2c(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{105fg\sqrt{c-c\sin(e+fx)}}-\frac{2ac(g\cos}{}$$

$$= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2c(g}{}$$

$$= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2c(g}{}$$

$$= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2c(g}{}$$

$$= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22a^3cg\sqrt{}}{15}$$

Mathematica [C] time = 2.48427, size = 281, normalized size = 0.97

$$\frac{a^3ge^{-4i(e+fx)}\left(e^{i(e+fx)}+i\right)\left(\sqrt{1+e^{2i(e+fx)}}\left(180ie^{i(e+fx)}+238e^{2i(e+fx)}+540ie^{3i(e+fx)}+3696e^{4i(e+fx)}+540ie^{5i(e+fx)}-238e^{6i(e}\right.\right.}{2520f\left(e^{i(e+fx)}-i\right)\sqrt{1+e^{2i(e+f}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] (a^3*(I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e + f
*x))]*(-35 + (180*I)*E^(I*(e + f*x)) + 238*E^((2*I)*(e + f*x)) + (540*I)*E^
((3*I)*(e + f*x)) + 3696*E^((4*I)*(e + f*x)) + (540*I)*E^((5*I)*(e + f*x))
- 238*E^((6*I)*(e + f*x)) + (180*I)*E^((7*I)*(e + f*x)) + 35*E^((8*I)*(e +
f*x))) - 2464*E^((6*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(e + f*x))])*Sqrt[c - c*Sin[e + f*x]])/(2520*E^((4*I)*(e + f*x))*(-I + E^
(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [C] time = 0.365, size = 394, normalized size = 1.4

$$\frac{2}{315f\left(-\left(\cos\left(fx+e\right)\right)^2+2\sin\left(fx+e\right)+2\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\left(231\,i\sin\left(fx+e\right)\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/315/f*(-c*(-1+sin(f*x+e)))^(1/2)*(231*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)-231*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+35*cos
(f*x+e)^6+231*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*sin(f*x+e)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)-90*sin(f*x+e)*cos(f*x+e)^4-112*cos(f*x+e)^4-154*cos(f*
x+e)^2+231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)/(-cos(
f*x+e)^2+2*sin(f*x+e)+2)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (g\cos(fx+e))^{\frac{3}{2}} (a\sin(fx+e)+a)^{\frac{5}{2}} \sqrt{-c\sin(fx+e)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2 g\cos(fx+e)^3 - 2a^2 g\cos(fx+e)\sin(fx+e) - 2a^2 g\cos(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e) -}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.109 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=234

$$-\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{35fg\sqrt{c-c\sin(e+fx)}}+\frac{22a^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{5f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-22*a^3*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (22*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) - (22*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(35*f*g*Sqrt[c
 - c*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(7*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.14248, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{35fg\sqrt{c-c\sin(e+fx)}}+\frac{22a^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{5f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (-22*a^3*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (22*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) - (22*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(35*f*g*Sqrt[c
 - c*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(7*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{7fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{7}(11a)\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{c-c}}$$

$$= -\frac{22a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{35fg\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+fx))^{5/2}(a+a\sin}{7fg\sqrt{c-c\sin(e+}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{35fg\sqrt{c-c\sin(e}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{35fg\sqrt{c-c\sin(e}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{35fg\sqrt{c-c\sin(e}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22a^3g\sqrt{\cos(e+fx)}\sqrt{g\cos}}{5f\sqrt{a+a\sin(e+fx}}$$

Mathematica [A] time = 1.86945, size = 158, normalized size = 0.68

$$-\frac{(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}(126\sin(2(e+fx))+515\cos\right.}{210f\cos^{\frac{3}{2}}(e+fx)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Si
n[e + f*x]],x]
```

```
[Out] -((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(5/2)*(-924*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(515*
Cos[e + f*x] - 15*Cos[3*(e + f*x)] + 126*Sin[2*(e + f*x)])))/(210*f*Cos[e +
 f*x]^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]
])
```

Maple [C] time = 0.338, size = 415, normalized size = 1.8

$$-\frac{2}{105f\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^{2}-4\sin\left(fx+e\right)-4\right)\sin\left(fx+e\right)\cos\left(fx+e\right)}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/105/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)*(-231*I*sin(f*x+e)*c
os(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)+15*sin(f*x+e)*cos(f*x+e)^4-231*I*sin(f*x+e)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)+231*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+63*cos(f*x+e)^4-140*co
s(f*x+e)^2*sin(f*x+e)-294*cos(f*x+e)^2+231*cos(f*x+e))/(cos(f*x+e)^2*sin(f*
x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/sin(f*x+e)/cos(f*x+e)/(-c*(-1+sin(f*x+e
)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2}g\cos\left(fx+e\right)^{3}-2\,a^{2}g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{2}g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+}}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

3.110 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$\frac{154a^3(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22a^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{5cfg\sqrt{c-c\sin(e+fx)}}-\frac{154a^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{5cf\sqrt{a\sin(e+fx)+}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(f*g*(c - c*Sin[e +
 f*x])^(3/2)) + (154*a^3*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (154*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*C
os[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) + (22*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e +
f*x]])/(5*c*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.14029, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{154a^3(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22a^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{5cfg\sqrt{c-c\sin(e+fx)}}-\frac{154a^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{5cf\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(3/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(f*g*(c - c*Sin[e +
 f*x])^(3/2)) + (154*a^3*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (154*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*C
os[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) + (22*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e +
f*x]])/(5*c*f*g*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+}{\sqrt{c-c\sin(e+fx)}}}{c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{5cfg\sqrt{c-c\sin(e+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{154a^3(g\cos(e+fx}{15cfg\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{154a^3(g\cos(e+fx}{15cfg\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{154a^3(g\cos(e+fx}{15cfg\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{154a^3(g\cos(e+fx}{15cfg\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

Mathematica [A] time = 6.44126, size = 278, normalized size = 1.15

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\frac{1}{5}\sin(2(e+fx))+\frac{8}{3}\cos(e+fx\right.}{f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(5/2))/(5*f*Cos[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(3/2)) + ((g*Co
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(
```

```
1 + Sin[e + f*x]))^(5/2)*(16 + (8*Cos[e + f*x])/3 + (32*Sin[(e + f*x)/2])/(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + Sin[2*(e + f*x)]/5))/(f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(3/2))
```

Maple [C] time = 0.361, size = 2945, normalized size = 12.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/15/f*(-1+cos(f*x+e))*(-30*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+30*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)-351*cos(f*x+e)^2-3*cos(f*x+e)^5+94*cos(f*x+e)^3+17*sin(f*x+e)*cos
(f*x+e)^3+231*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*sin(f*x
+e)*cos(f*x+e)^4-30*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2
*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+30*cos(f*x+e)^
4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)-120*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+12
0*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-180*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)+180*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+30*ln(-2*(2*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)*sin(f*x+e)-30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-120*cos(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)+120*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+111*c
os(f*x+e)^2*sin(f*x+e)-20*cos(f*x+e)^4+30*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)-30*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+90*
sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-90*sin(f*x+e)*cos(f*x+e
)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f
```

```
*x+e)/(cos(f*x+e)+1)^2)^(3/2)+90*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)-90*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+231*I*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)-462*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x
+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+
231*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)^3*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/s
in(f*x+e),I)+231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*sin(f*x+e)*
cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+
e)))^(5/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+
3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*
x+e)+8)/cos(f*x+e)/(-c*(-1+sin(f*x+e)))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2g\cos\left(fx+e\right)^3-2\,a^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

3.111 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154a^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{44a^2\sqrt{a\sin(e+}}{5cfg(c-}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(5*f*g*(c - c*Sin[e
 + f*x])^(5/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(5*c*f*g*(c - c*Sin[e + f*x])^(3/2)) - (154*a^3*(g*Cos[e + f*x])^(5/2))/(15
*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (154*a^3*g*Sq
rt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.17454, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154a^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{44a^2\sqrt{a\sin(e+}}{5cfg(c-}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(5/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(5*f*g*(c - c*Sin[e
 + f*x])^(5/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(5*c*f*g*(c - c*Sin[e + f*x])^(3/2)) - (154*a^3*(g*Cos[e + f*x])^(5/2))/(15
*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (154*a^3*g*Sq
rt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin}{(c-c\sin(e+fx))}}{5c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{5cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{5cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{5cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{5cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{5cfg(c-c\sin(e+}$$

Mathematica [A] time = 2.58756, size = 245, normalized size = 1.01

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(226\sin\left(\frac{1}{2}(e+fx)\right)-3\right.\right.}{30c^2f(\sin(e+fx)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] -(a^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[a
*(1 + Sin[e + f*x])]*(-924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(226*Cos[(e + f*x)/2] + 327*Cos[(3*(
e + f*x))/2] - 5*Cos[(5*(e + f*x))/2] + 226*Sin[(e + f*x)/2] - 327*Sin[(3*(
e + f*x))/2] - 5*Sin[(5*(e + f*x))/2])))/(30*c^2*f*Cos[e + f*x]^(3/2)*(Cos[
```

```
(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f
*x]])
```

Maple [C] time = 0.375, size = 3549, normalized size = 14.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/15/f*(-1+cos(f*x+e))*(45*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-90*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)+90*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-438*cos(f*x+e)^2+45*s
in(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-45*sin(f*x+e)*cos(f*x+e)
^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*s
in(f*x+e)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-462*I*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)+462*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*cos(f*x+e)*sin(f*x+e)+5*cos(f*x+e)^5+89*cos(f*x+e)^3-65*sin(f*x+e)*cos(f*
x+e)^3+5*sin(f*x+e)*cos(f*x+e)^4+90*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)-90*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-90*cos(f*x+e)^3*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)+90*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-360*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)+360*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+90*ln(-
2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-
315*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+315*cos(f*x+e)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
```

```
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)+486*cos(f*x+e)^2*sin(f*x+e)+70*cos(f*x+e)^4+225*sin(f*x+e)*cos(f*x
+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-225*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)+405*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-405*si
n(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+315*sin(f*x+e)*cos(f*x+e)*l
n(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)-315*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)-231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-45*cos(f*x+e)^5
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin
(f*x+e)-231*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^4*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos
(f*x+e))/sin(f*x+e),I)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-693*I*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)-462*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(
f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(a*(
1+sin(f*x+e)))^(5/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*s
in(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e
)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(5/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2g\cos\left(fx+e\right)^3-2\,a^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{\cdots}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*s
in(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

3.112 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=243

$$\frac{308a^3(g\cos(e+fx))^{5/2}}{45c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{154a^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{44a^2\sqrt{a\sin(}}{45cf}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(9*f*g*(c - c*Sin[e
 + f*x])^(7/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(45*c*f*g*(c - c*Sin[e + f*x])^(5/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))/(4
5*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (154*a^3*g
*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3
*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.19684, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$\frac{308a^3(g\cos(e+fx))^{5/2}}{45c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{154a^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{44a^2\sqrt{a\sin(}}{45cf}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(7/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(9*f*g*(c - c*Sin[e
 + f*x])^(7/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(45*c*f*g*(c - c*Sin[e + f*x])^(5/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))/(4
5*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (154*a^3*g
*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3
*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+}{(c-c\sin(e+fx))^{5/2}}}{9c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{45cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{45cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{45cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{45cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{45cfg(c-c\sin(e+f}$$

Mathematica [A] time = 2.79423, size = 246, normalized size = 1.01

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2\sqrt{\cos(e+fx)}\left(182\sin\left(\frac{1}{2}(e+fx)\right)-195\right.\right.}{90c^3f(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(7/2),x]
```

```
[Out] -(a^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[a
*(1 + Sin[e + f*x])]*(-924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^5 + 2*Sqrt[Cos[e + f*x]]*(182*Cos[(e + f*x)/2] + 195*Cos[(3
*(e + f*x))/2] - 93*Cos[(5*(e + f*x))/2] + 182*Sin[(e + f*x)/2] - 195*Sin[(
3*(e + f*x))/2] - 93*Sin[(5*(e + f*x))/2])))/(90*c^3*f*Cos[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e
 + f*x]])
```

Maple [C] time = 0.366, size = 4183, normalized size = 17.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/45/f*(-1+cos(f*x+e))*(810*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-540*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)+540*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-462*I*sin(f*x+e)*co
s(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-1928*cos(f*x+e)^2-135*sin(f*x+e)*cos(f
*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+135*sin(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)+135*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^6-135*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)*cos(f*x+e)^6-90*cos(f*x+e)^5+268*cos(f*x+e)^3-348*sin(f*x+e)
*cos(f*x+e)^3-90*sin(f*x+e)*cos(f*x+e)^4+1350*cos(f*x+e)^4*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)-1350*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-2025*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+2025*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+540*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-540*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*
x+e)-1890*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1890*cos(f*x+e)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)+1768*cos(f*x+e)^2*sin(f*x+e)+1182*cos(f*x+e)^4+135*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^5-135*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*s
in(f*x+e)*cos(f*x+e)^5+945*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)
^2)-945*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+2295*sin(f*x+e)*c
os(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^
2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-2295*sin(f*x+e)*cos(f*x+e)^2*ln(-(2
```

```
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)+1890*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-
1890*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-1848*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*cos(f*x+e)+1848*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+462*I*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^5-462*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x
+e)^5+1848*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-1848*I*cos(f*x+e)^4*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-462*I*c
os(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-2772*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)+2772*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-810*cos(f*x+e)^5*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)-2772*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)+1848*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)-1848*I*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)-462*I*sin(f*x+e)*cos(f*x+e)^4*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)+462*I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)+462*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+2772*I*sin(f*x+e)
*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x
+e)))^(5/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)
+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f
*x+e)+8)/(-c*(-1+sin(f*x+e)))^(7/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2g\cos\left(fx+e\right)^3-2\,a^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*c
os(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

3.113 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{195c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{308a^3(g\cos(e+fx))^{5/2}}{585c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{154a^3g\sqrt{\cos(e+\ldots}}{195c^4f\sqrt{a\sin\ldots}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(13*f*g*(c - c*Sin[
e + f*x])^(9/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])
/(117*c*f*g*(c - c*Sin[e + f*x])^(7/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))/
(585*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (154*a^
3*(g*Cos[e + f*x])^(5/2))/(195*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[
e + f*x])^(3/2)) + (154*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(195*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]])
```

Rubi [A] time = 1.48314, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{195c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{308a^3(g\cos(e+fx))^{5/2}}{585c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{154a^3g\sqrt{\cos(e+\ldots}}{195c^4f\sqrt{a\sin\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(9/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(13*f*g*(c - c*Sin[
e + f*x])^(9/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])
/(117*c*f*g*(c - c*Sin[e + f*x])^(7/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))/
(585*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (154*a^
3*(g*Cos[e + f*x])^(5/2))/(195*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[
e + f*x])^(3/2)) + (154*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(195*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
```

```
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin}{(c-c\sin(e+fx))}}{13c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{117cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{117cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{117cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{117cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{117cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{117cfg(c-c\sin(e}$$

Mathematica [A] time = 6.59689, size = 464, normalized size = 1.55

$$\frac{154E\left(\frac{1}{2}(e+fx)\middle|2\right)(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}{195f\cos^{\frac{3}{2}}(e+fx)(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5} + \frac{\sec(e+fx)(a(\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(9/2),x]
```

```
[Out] (154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))/(195*f*Cos[e + f*x]^(3/2)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2)) + ((g*C
os[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(-1
54/195 + 16/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) - 232/(117*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^4) + 236/(195*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^2) + (32*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^7) - (464*Sin[(e + f*x)/2])/(117*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)
 + (472*Sin[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (
308*Sin[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 +
Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin
[e + f*x])^(9/2))
```

Maple [C] time = 0.335, size = 3455, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/1170/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)*(-1+cos(f*x+e))^4*(
-1+sin(f*x+e))*(cos(f*x+e)+1)*(-2256*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)+5136*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-2
925*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f
*x+e)^3+2925*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)*cos(f*x+e)^3+2340*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*cos(f*x+e)-2340*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+1440*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*
x+e)-7476*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+1440*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1008*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-2340*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^
2)*cos(f*x+e)*sin(f*x+e)+2340*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-2752*sin(f*x+e)*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3696*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)-3696*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)+7392*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+924*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6-924*I*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6+1755*sin(f*x+e)*cos(f*
x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-17
55*sin(f*x+e)*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
```

```
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)+1868*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)+924*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*cos(f*x+e)^4+585
*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+
e)^5-585*cos(f*x+e)^5*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)-432*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2340*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^5+1848*I*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^5-1848*I*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^5-3696*I*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4+3696*I
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^
4-9240*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos
(f*x+e)^3+9240*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*cos(f*x+e)^3-924*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*cos(f*x+e)^2+924*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)^2-3696*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+7392*I*(1/(cos(f*x+e)+1))^(1/2)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+3696*I*(1/(cos(f*x+e)+1))^(1/2)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-7392*I*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+2772*I*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4-
2772*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f
*x+e)*cos(f*x+e)^4+5544*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^3-5544*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^3-924*I*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2+
924*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*
x+e)*cos(f*x+e)^2-7392*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/s
in(f*x+e),I)*sin(f*x+e)*cos(f*x+e))/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2
-4*sin(f*x+e)-4)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(-1+sin(f*x+e)))^
(9/2)/sin(f*x+e)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2g\cos\left(fx+e\right)^3-2\,a^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}}{5\,c^5\cos\left(fx+e\right)^4-20\,c^5\cos\left(fx+e\right)^2+16\,c^5-\left(c^5\cos\left(fx+e\right)^4-12\,c^5\cos\left(fx+e\right)^2+16\,c^5\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c^5
*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

3.114 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=357

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{154a^3(g\cos(e+fx))^{5/2}}{3315c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{30}{1989c^2fg\sqrt{a\sin}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(17*f*g*(c - c*Sin[
e + f*x])^(11/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]
)/(221*c*f*g*(c - c*Sin[e + f*x])^(9/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))
/(1989*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) - (154*
a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2)) - (154*a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a
 + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (154*a^3*g*Sqrt[Cos[e + f*
x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(3315*c^5*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.78659, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{154a^3(g\cos(e+fx))^{5/2}}{3315c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{30}{1989c^2fg\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(11/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(17*f*g*(c - c*Sin[
e + f*x])^(11/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]
)/(221*c*f*g*(c - c*Sin[e + f*x])^(9/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))
/(1989*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) - (154*
a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2)) - (154*a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a
 + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (154*a^3*g*Sqrt[Cos[e + f*
x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(3315*c^5*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
```

```
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin}{(c-c\sin(e+fx))}}{17c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a}}{221cfg(c-c\sin(e}$$

Mathematica [A] time = 6.71596, size = 532, normalized size = 1.49

$$\frac{154E\left(\frac{1}{2}(e+fx)\middle|2\right)(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}{3315f\cos^{\frac{3}{2}}(e+fx)(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^{5}}+\frac{\sec(e+fx)(a(\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(11/2),x]
```

```
[Out] (154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(5/2))/(3315*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(11/2)) + ((
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11
*(-154/3315 + 16/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 296/(221*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^6) + 1172/(1989*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^4) - 154/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (3
2*Sin[(e + f*x)/2])/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (592*Sin
[(e + f*x)/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (2344*Sin[(e
 + f*x)/2])/(1989*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) - (308*Sin[(e +
f*x)/2])/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (308*Sin[(e + f*x
)/2])/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^
(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(11/
2))
```

Maple [C] time = 0.385, size = 1313, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 2/9945/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)*(sin(f*x+e)*cos(f*x+
e)-sin(f*x+e)-cos(f*x+e)+1)*(-4680+1848*I*sin(f*x+e)*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)-924*I*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+924*I*cos(f*x+e)^6*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)+3696*I*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3696*I*cos
(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-4620*I*cos(f*x+e)^2*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)+4620*I*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-1848*I*sin(f*x+e)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)+1848*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-1848*I*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)-4680*sin(f*x+e)+2832*cos(f*x+e)+6528*sin(f*x+e)*cos(f*x+e)+992
0*cos(f*x+e)^2+2930*cos(f*x+e)^5-6224*cos(f*x+e)^3-4192*sin(f*x+e)*cos(f*x+
e)^3+231*I*sin(f*x+e)*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*sin(f*x+
```

```
e)*cos(f*x+e)^6*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-2079*I*sin(f*x+e)*cos(f*x+e)^4*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)+2079*I*sin(f*x+e)*cos(f*x+e)^4*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)+924*sin(f*x+e)*cos(f*x+e)^4+1420*cos(f*x+e)^2*sin(f*x+e)+231*cos(f*x+
e)^6-5009*cos(f*x+e)^4+3696*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)-3696*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(cos(f*x+e)^
2+2*cos(f*x+e)+1)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/(
-c*(-1+sin(f*x+e)))^(11/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2g\cos\left(fx+e\right)^3-2\,a^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+}}{c^6\cos\left(fx+e\right)^6-18\,c^6\cos\left(fx+e\right)^4+48\,c^6\cos\left(fx+e\right)^2-32\,c^6+2\left(3\,c^6\cos\left(fx+e\right)^4-16\,c^6\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c^6*cos(f*x + e)^6 - 18*c^6*cos(f*x + e)^4 + 48*c^6*cos(f*x +
 e)^2 - 32*c^6 + 2*(3*c^6*cos(f*x + e)^4 - 16*c^6*cos(f*x + e)^2 + 16*c^6)*
sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

3.115 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=414

$$-\frac{22a^3(g\cos(e+fx))^{5/2}}{3315c^5fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{22a^3(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{}{1989c^3fg\sqrt{a}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(21*f*g*(c - c*Sin[
e + f*x])^(13/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]
)/(357*c*f*g*(c - c*Sin[e + f*x])^(11/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2))
/(663*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)) - (22*a^
3*(g*Cos[e + f*x])^(5/2))/(1989*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin
[e + f*x])^(7/2)) - (22*a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a +
a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (22*a^3*(g*Cos[e + f*x])^(5/2
))/(3315*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (22
*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(
3315*c^6*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 2.07998, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$-\frac{22a^3(g\cos(e+fx))^{5/2}}{3315c^5fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{22a^3(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{}{1989c^3fg\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(13/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(21*f*g*(c - c*Sin[
e + f*x])^(13/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]
)/(357*c*f*g*(c - c*Sin[e + f*x])^(11/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2))
/(663*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)) - (22*a^
3*(g*Cos[e + f*x])^(5/2))/(1989*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin
[e + f*x])^(7/2)) - (22*a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a +
a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (22*a^3*(g*Cos[e + f*x])^(5/2
))/(3315*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (22
*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(
3315*c^6*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{(11a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+}{(c-c\sin(e+fx))^{11/2}}}{21c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{357cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{357cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{357cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{357cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{357cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{357cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{357cfg(c-c\sin(e+f}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a}}{357cfg(c-c\sin(e+f}$$

Mathematica [A] time = 6.69057, size = 600, normalized size = 1.45

$$\frac{22E\left(\frac{1}{2}(e+fx)\middle|2\right)(a(\sin(e+fx)+1))^{5/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{13}}{3315f\cos^{\frac{3}{2}}(e+fx)(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^{5}}+\frac{\sec(e+fx)(a(\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(13/2),x]
```

```
[Out] (22*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^13*(a*(1 + Sin[e + f*x]))^(5/2))/(3315*f*Cos[e + f*x]^(3/2)
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(13/2)) + ((g
*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^13*
(-22/3315 + 16/(21*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10) - 120/(119*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^8) + 84/(221*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^6) - 22/(1989*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) - 22/(331
5*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (32*Sin[(e + f*x)/2])/(21*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^11) - (240*Sin[(e + f*x)/2])/(119*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^9) + (168*Sin[(e + f*x)/2])/(221*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^7) - (44*Sin[(e + f*x)/2])/(1989*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^5) - (44*Sin[(e + f*x)/2])/(3315*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3) - (44*Sin[(e + f*x)/2])/(3315*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^5*(c - c*Sin[e + f*x])^(13/2))
```

Maple [C] time = 0.454, size = 1473, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] -2/69615/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)*(sin(f*x+e)*cos(f*
x+e)-sin(f*x+e)-cos(f*x+e)+1)*(26520+26520*sin(f*x+e)-22824*cos(f*x+e)-3021
6*sin(f*x+e)*cos(f*x+e)-43600*cos(f*x+e)^2-11998*cos(f*x+e)^5+35284*cos(f*x
+e)^3+24488*sin(f*x+e)*cos(f*x+e)^3-385*sin(f*x+e)*cos(f*x+e)^5-2618*sin(f*
x+e)*cos(f*x+e)^4+231*cos(f*x+e)^6*sin(f*x+e)-18020*cos(f*x+e)^2*sin(f*x+e)
-1155*cos(f*x+e)^6+17773*cos(f*x+e)^4+3696*I*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3696
*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)+1155*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)-1155*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(
f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+231*I*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)-3234*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3234*I*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)+9471*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*c
os(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-9471
```

```
*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-10164*I*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)+10164*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^
2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3696*I*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)+3696*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5
775*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5775*I*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+8316*I*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)-8316*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*
x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
))*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*
sin(f*x+e)-4)/(-c*(-1+sin(f*x+e)))^(13/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2g\cos\left(fx+e\right)^3-2\,a^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(f\right.}}{7\,c^7\cos\left(fx+e\right)^6-56\,c^7\cos\left(fx+e\right)^4+112\,c^7\cos\left(fx+e\right)^2-64\,c^7-\left(c^7\cos\left(fx+e\right)^6-24\,c^7\cos\left(fx+e\right)^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="fricas")
```

```
[Out] integral((a^2*g*cos(f*x + e)^3 - 2*a^2*g*cos(f*x + e)*sin(f*x + e) - 2*a^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(7*c^7*cos(f*x + e)^6 - 56*c^7*cos(f*x + e)^4 + 112*c^7*cos(f*
x + e)^2 - 64*c^7 - (c^7*cos(f*x + e)^6 - 24*c^7*cos(f*x + e)^4 + 80*c^7*co
s(f*x + e)^2 - 64*c^7)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(13/2), x)
```

3.116 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=463

$$-\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^3c^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{195fg\sqrt{c-c\sin(e+fx)}}-\frac{2a^2c^3(a\sin(e+fx)+a)^{3/2}}{39fg\sqrt{c-c\sin(}$$

```
[Out] (-154*a^4*c^3*(g*Cos[e + f*x])^(5/2))/(585*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) + (154*a^4*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(195*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*
Sin[e + f*x]]) - (22*a^3*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]
])/(195*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*c^3*(g*Cos[e + f*x])^(5/2)*(
a + a*Sin[e + f*x])^(3/2))/(39*f*g*Sqrt[c - c*Sin[e + f*x]]) - (14*a*c^3*(g
*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(585*f*g*Sqrt[c - c*Sin[e
+ f*x]]) + (14*c^3*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(195*
f*g*Sqrt[c - c*Sin[e + f*x]]) + (22*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e
 + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(195*f*g) + (2*c*(g*Cos[e + f*x])^
(5/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(15*f*g)
```

Rubi [A] time = 2.38991, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^3c^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{195fg\sqrt{c-c\sin(e+fx)}}-\frac{2a^2c^3(a\sin(e+fx)+a)^{3/2}}{39fg\sqrt{c-c\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^
(5/2),x]
```


```
[Out] (-154*a^4*c^3*(g*Cos[e + f*x])^(5/2))/(585*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) + (154*a^4*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(195*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*
Sin[e + f*x]]) - (22*a^3*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]
])/(195*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*c^3*(g*Cos[e + f*x])^(5/2)*(
a + a*Sin[e + f*x])^(3/2))/(39*f*g*Sqrt[c - c*Sin[e + f*x]]) - (14*a*c^3*(g
*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(585*f*g*Sqrt[c - c*Sin[e
+ f*x]]) + (14*c^3*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(195*
f*g*Sqrt[c - c*Sin[e + f*x]]) + (22*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e
 + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(195*f*g) + (2*c*(g*Cos[e + f*x])^
(5/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(15*f*g)
```


Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] && !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(}{15fg}$$

$$= \frac{22c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}\sqrt{c-c\,s}}{195fg}$$

$$= \frac{14c^3(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}}{195fg\sqrt{c-c\sin(e+fx)}} + \frac{22c^2}{}$$

$$= -\frac{14ac^3(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{585fg\sqrt{c-c\sin(e+fx)}} + \frac{14}{}$$

$$= -\frac{2a^2c^3(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{39fg\sqrt{c-c\sin(e+fx)}} - \frac{14}{}$$

$$= -\frac{22a^3c^3(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{195fg\sqrt{c-c\sin(e+fx)}} - \frac{2a}{}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22}{}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22}{}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22}{}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{15}{}$$

Mathematica [A] time = 3.68715, size = 226, normalized size = 0.49

$$-\frac{a^3c^2(\sin(e+fx)-1)^2(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}(-379\right.}{18720f\cos^{\frac{3}{2}}(e+fx)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] -(a^3*c^2*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3
*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-14784*EllipticE[(e +
 f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(1365*Cos[e + f*x] + 819*Cos[3*(e + f*x)]
+ 273*Cos[5*(e + f*x)] + 39*Cos[7*(e + f*x)] - 3794*Sin[2*(e + f*x)] - 800*
Sin[4*(e + f*x)] - 90*Sin[6*(e + f*x)]])))/(18720*f*Cos[e + f*x]^(3/2)*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [C] time = 0.464, size = 392, normalized size = 0.9

$$-\frac{2}{585 f \left(1+\sin \left(f x+e\right)\right) \left(\cos \left(f x+e\right)\right)^{7} \sin \left(f x+e\right)} \left(-c \left(-1+\sin \left(f x+e\right)\right)\right)^{\frac{5}{2}} \left(39 \sin \left(f x+e\right) \left(\cos \left(f x+e\right)\right)^{8}+45 \left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/585/f*(-c*(-1+sin(f*x+e)))^(5/2)*(39*sin(f*x+e)*cos(f*x+e)^8+45*cos(f*x+
e)^8+10*cos(f*x+e)^6+231*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*
x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-
231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+231*I*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f
*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+22*cos(f*x
+e)^4+154*cos(f*x+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e
)))^(7/2)/(1+sin(f*x+e))/cos(f*x+e)^7/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g \cos \left(f x+e\right)\right)^{\frac{3}{2}} \left(a \sin \left(f x+e\right)+a\right)^{\frac{7}{2}} \left(-c \sin \left(f x+e\right)+c\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(a^{3} c^{2} g \cos \left(f x+e\right)^{5} \sin \left(f x+e\right)+a^{3} c^{2} g \cos \left(f x+e\right)^{5}\right) \sqrt{g \cos \left(f x+e\right)} \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c \sin \left(f x+e\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((a^3*c^2*g*cos(f*x + e)^5*sin(f*x + e) + a^3*c^2*g*cos(f*x + e)^5)
*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.117 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=409

$$-\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{13fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2c^2(a\sin(e+fx)+a)^{3/2}(g}{143fg\sqrt{c-c\sin(e}$$

```
[Out] (-14*a^4*c^2*(g*Cos[e + f*x])^(5/2))/(39*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (14*a^4*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]
]*EllipticE[(e + f*x)/2, 2])/(13*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) - (2*a^3*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(1
3*f*g*Sqrt[c - c*Sin[e + f*x]]) - (10*a^2*c^2*(g*Cos[e + f*x])^(5/2)*(a + a
*Sin[e + f*x])^(3/2))/(143*f*g*Sqrt[c - c*Sin[e + f*x]]) - (14*a*c^2*(g*Cos
[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(429*f*g*Sqrt[c - c*Sin[e + f*
x]]) + (14*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(143*f*g*
Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x]
)^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(13*f*g)
```

Rubi [A] time = 2.04229, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{13fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2c^2(a\sin(e+fx)+a)^{3/2}(g}{143fg\sqrt{c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^
(3/2),x]
```


```
[Out] (-14*a^4*c^2*(g*Cos[e + f*x])^(5/2))/(39*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (14*a^4*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]
]*EllipticE[(e + f*x)/2, 2])/(13*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) - (2*a^3*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(1
3*f*g*Sqrt[c - c*Sin[e + f*x]]) - (10*a^2*c^2*(g*Cos[e + f*x])^(5/2)*(a + a
*Sin[e + f*x])^(3/2))/(143*f*g*Sqrt[c - c*Sin[e + f*x]]) - (14*a*c^2*(g*Cos
[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(429*f*g*Sqrt[c - c*Sin[e + f*
x]]) + (14*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(143*f*g*
Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x]
)^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(13*f*g)
```


Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin}}{13fg}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}}{143fg\sqrt{c-c\sin(e+fx)}} + \frac{2c(g}{}$$

$$= -\frac{14ac^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{429fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{}$$

$$= -\frac{10a^2c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{143fg\sqrt{c-c\sin(e+fx)}} - \frac{1}{}$$

$$= -\frac{2a^3c^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg\sqrt{c-c\sin(e+fx)}} - \frac{10a}{}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^3}{}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^3}{}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^3}{}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{14a}{}$$

Mathematica [A] time = 3.14435, size = 212, normalized size = 0.52

$$\frac{a^3c(\sin(e+fx)-1)(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}(-1507\,\mathrm{si}\right.}{6864f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (a^3*c*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3*Sqrt
[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-7392*EllipticE[(e + f*x)/
```

```
2, 2] + Sqrt[Cos[e + f*x]]*(1560*Cos[e + f*x] + 780*Cos[3*(e + f*x)] + 156*
Cos[5*(e + f*x)] - 1507*Sin[2*(e + f*x)] - 88*Sin[4*(e + f*x)] + 33*Sin[6*(
e + f*x)])))/(6864*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [C] time = 0.41, size = 404, normalized size = 1.

$$-\frac{2}{429 f\left(-\left(\cos\left(fx+e\right)\right)^{2}+2\sin\left(fx+e\right)+2\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{5}}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(-33\left(\cos\left(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/429/f*(-c*(-1+sin(f*x+e)))^(3/2)*(-33*cos(f*x+e)^8+78*cos(f*x+e)^6*sin(f
*x+e)+231*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+88*cos(f*x+e)^6+231*I*EllipticE(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(
f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+22*cos(f*
x+e)^4+154*cos(f*x+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+
e)))^(7/2)/(-cos(f*x+e)^2+2*sin(f*x+e)+2)/sin(f*x+e)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^{3}cg\cos\left(fx+e\right)^{5}-2\,a^{3}cg\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-2\,a^{3}cg\cos\left(fx+e\right)^{3}\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*c*g*cos(f*x + e)^5 - 2*a^3*c*g*cos(f*x + e)^3*sin(f*x + e) -
 2*a^3*c*g*cos(f*x + e)^3)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.118 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=343

$$-\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{7fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2c(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{77fg\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-2*a^4*c*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (2*a^4*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (2*a^3*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(7*f*g*Sqrt[c
- c*Sin[e + f*x]]) - (10*a^2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^
(3/2))/(77*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c*(g*Cos[e + f*x])^(5/2)*(a
 + a*Sin[e + f*x])^(5/2))/(33*f*g*Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e
 + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(11*f*g*Sqrt[c - c*Sin[e + f*x]]
)
```

Rubi [A] time = 1.71989, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{7fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2c(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{77fg\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*
x]],x]
```


```
[Out] (-2*a^4*c*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (2*a^4*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (2*a^3*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(7*f*g*Sqrt[c
- c*Sin[e + f*x]]) - (10*a^2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^
(3/2))/(77*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c*(g*Cos[e + f*x])^(5/2)*(a
 + a*Sin[e + f*x])^(5/2))/(33*f*g*Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e
 + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(11*f*g*Sqrt[c - c*Sin[e + f*x]]
)
```


Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
```

```
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{7/2}}{11fg\sqrt{c-c\sin(e+fx)}}+\frac{1}{11}(3c)\int$$

$$= -\frac{2ac(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{33fg\sqrt{c-c\sin(e+fx)}}+\frac{2c(g\,c}{}$$

$$= -\frac{10a^2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{77fg\sqrt{c-c\sin(e+fx)}}-\frac{2ac(}{}$$

$$= -\frac{2a^3c(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{7fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2c(g}{}$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c(g}{}$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c(g}{}$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c(g}{}$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2a^4cg}{}$$

Mathematica [C] time = 5.50583, size = 333, normalized size = 0.97

$$-\frac{a^3g^2\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(1374\cos(e+fx)+423\cos(3(e+fx))-7(44\sin(2(e+fx))-22\sin(4(e}{1848f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] (4*a^4*(I + E^(I*(e + f*x)))*Sqrt[(1 + E^((2*I)*(e + f*x)))/E^(I*(e + f*x))
]*(g*Cos[e + f*x])^(3/2)*(Sqrt[1 + E^((2*I)*(e + f*x))] + (-1 + E^((2*I)*e)
)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(e + f*x))])*Sqrt[c - c*Sin[e
 + f*x]])/((-1 + E^((2*I)*e))*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*a*(I + E^(I
*(e + f*x)))^2)/E^(I*(e + f*x))]*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Cos[e + f*
x]^(3/2)) - (a^3*g^2*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(1
374*Cos[e + f*x] + 423*Cos[3*(e + f*x)] - 7*(3*Cos[5*(e + f*x)] - 528*Cot[e
```

```
] + 44*Sin[2*(e + f*x)] - 22*Sin[4*(e + f*x)]))/(1848*f*Sqrt[g*Cos[e + f*x
]])
```

Maple [C] time = 0.394, size = 425, normalized size = 1.2

$$\frac{2}{231 f\left(\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^{2}-4\sin\left(fx+e\right)-4\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}}\sqrt{-c\left(-1+\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/231/f*(-c*(-1+sin(f*x+e)))^(1/2)*(-21*cos(f*x+e)^6*sin(f*x+e)+231*I*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)-77*cos(f*x+e)^6+231*I*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+132*sin(f*x+e)*cos(f*x+e)^
4+154*cos(f*x+e)^4+154*cos(f*x+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a
*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)
-4)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3a^{3}g\cos\left(fx+e\right)^{3}-4a^{3}g\cos\left(fx+e\right)+\left(a^{3}g\cos\left(fx+e\right)^{3}-4a^{3}g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
 e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.119 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=288

$$-\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{21fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2(a\sin(e+fx)+a)^{3/2}(g\cos}{21fg\sqrt{c-c\sin(e+f}}$$

```
[Out] (-22*a^4*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (22*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(21*f*g*Sqrt[c
- c*Sin[e + f*x]]) - (10*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3
/2))/(21*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a
*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.43713, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$-\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{21fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2(a\sin(e+fx)+a)^{3/2}(g\cos}{21fg\sqrt{c-c\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (-22*a^4*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (22*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(21*f*g*Sqrt[c
- c*Sin[e + f*x]]) - (10*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3
/2))/(21*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a
*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{3}(5a)\int \frac{(g\cos(e+fx))}{\sqrt{c-}}$$

$$= -\frac{10a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos(e+fx))^{5/2}(a}{9fg\sqrt{c-c\sin}}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{21fg\sqrt{c-c\sin(e+fx)}} - \frac{10a^2(g\cos(e+fx))^{5/2}(a}{21fg\sqrt{c-c\sin}}$$

$$= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^3(g\cos(e+fx))^{5/2}\sqrt{}}{21fg\sqrt{c-c\sin}}$$

$$= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^3(g\cos(e+fx))^{5/2}\sqrt{}}{21fg\sqrt{c-c\sin}}$$

$$= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22a^3(g\cos(e+fx))^{5/2}\sqrt{}}{21fg\sqrt{c-c\sin}}$$

$$= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22a^4g\sqrt{\cos(e+fx)}\sqrt{g\,c}}{3f\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 3.94527, size = 181, normalized size = 0.63

$$-\frac{a^3(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}(350\sin\right.}{252f\cos^{\frac{3}{2}}(e+fx)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Si
n[e + f*x]],x]
```

```
[Out] -(a^3*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin
[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-1848*EllipticE[(e + f*x)/2, 2] +
Sqrt[Cos[e + f*x]]*(1128*Cos[e + f*x] - 72*Cos[3*(e + f*x)] + 350*Sin[2*(e
+ f*x)] - 7*Sin[4*(e + f*x)])))/(252*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.352, size = 436, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/63/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)*(231*I*sin(f*x+e)*cos
(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/s
in(f*x+e),I)-7*cos(f*x+e)^6+36*sin(f*x+e)*cos(f*x+e)^4+231*I*sin(f*x+e)*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)-231*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+98*cos(f*
x+e)^4-168*cos(f*x+e)^2*sin(f*x+e)-322*cos(f*x+e)^2+231*cos(f*x+e))/(-cos(f
*x+e)^4+4*cos(f*x+e)^2*sin(f*x+e)+8*cos(f*x+e)^2-8*sin(f*x+e)-8)/sin(f*x+e)
/cos(f*x+e)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)+\left(a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+\right.}}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

3.120 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=294

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{cfg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{66a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{7cfg\sqrt{c-c\sin(e+fx)}}+\frac{30a^2(a\sin(e+fx)+a)^{3/2}(g\cos(}{7cfg\sqrt{c-c\sin(e+fx}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(f*g*(c - c*Sin[e +
 f*x])^(3/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(c*f*g*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (66*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e
+ f*x]]*EllipticE[(e + f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (66*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(7*c*f*g*Sqrt[c - c*Sin[e + f*x]]) + (30*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*
Sin[e + f*x])^(3/2))/(7*c*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.45529, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{cfg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{66a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{7cfg\sqrt{c-c\sin(e+fx)}}+\frac{30a^2(a\sin(e+fx)+a)^{3/2}(g\cos(}{7cfg\sqrt{c-c\sin(e+fx}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(3/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(f*g*(c - c*Sin[e +
 f*x])^(3/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(c*f*g*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (66*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e
+ f*x]]*EllipticE[(e + f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (66*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(7*c*f*g*Sqrt[c - c*Sin[e + f*x]]) + (30*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*
Sin[e + f*x])^(3/2))/(7*c*f*g*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
```

```
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} - \frac{(15a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin}{\sqrt{c-c\sin(e+f}}}{c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{30a^2(g\cos(e+fx))^{5/2}(a+}{7cfg\sqrt{c-c\sin(}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{66a^3(g\cos(e+fx))^{5/2}\sqrt{a}}{7cfg\sqrt{c-c\sin(e}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^4(g\cos(e+f}{cfg\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^4(g\cos(e+f}{cfg\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^4(g\cos(e+f}{cfg\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^4(g\cos(e+f}{cfg\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

Mathematica [A] time = 6.53516, size = 284, normalized size = 0.97

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(2(e+fx))+\frac{109}{14}\cos(e+\right.}{f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-66*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*Cos[e + f*x]^(3/2)*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2)) + ((g*Cos[e
 + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 +
 Sin[e + f*x]))^(7/2)*(32 + (109*Cos[e + f*x])/14 - Cos[3*(e + f*x)]/14 + (
64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + Sin[2*(e + f*x
)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2))
```

Maple [C] time = 0.395, size = 2997, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/7/f*(-1+cos(f*x+e))*(-28*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+28*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)-343*cos(f*x+e)^2-6*cos(f*x+e)^5+98*cos(f*x+e)^3+21*sin(f*x+e)*cos(
f*x+e)^3-sin(f*x+e)*cos(f*x+e)^5+231*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)-7*sin(f*x+e)*cos(f*x+e)^4-28*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)+28*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-112*cos(f*x+e)^3*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)+112*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-168*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+168*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)+28*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-28*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*si
n(f*x+e)-112*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f
*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+112*cos(f*x+e)*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)+119*cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^6-28*cos(f*x+e)^4+
28*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-28*sin(f*x+e)*cos(f*
```

```
x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+84*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)-84*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+84*sin(f*
x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-84*sin(f*x+e)*cos(f*x+e)*ln(-(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)+231*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x
+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-2
31*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-462*I*cos(f*x+e)^2*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos
(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*c
os(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)-231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^3*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)-231*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))
*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^4-cos
(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3+5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+
e)+8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)-20*cos(f*x+e)^2+16*sin(f*x+e)-8*c
os(f*x+e)+16)/(-c*(-1+sin(f*x+e)))^(3/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^{3}g\cos\left(fx+e\right)^{3}-4\,a^{3}g\cos\left(fx+e\right)+\left(a^{3}g\cos\left(fx+e\right)^{3}-4\,a^{3}g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(f\right.}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x
 + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

3.121 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=298

$$-\frac{154a^4(g\cos(e+fx))^{5/2}}{5c^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{66a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{5c^2fg\sqrt{c-c\sin(e+fx)}}+\frac{462a^4g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}\right.}{5c^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(5*f*g*(c - c*Sin[e
 + f*x])^(5/2)) - (12*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2)
)/(c*f*g*(c - c*Sin[e + f*x])^(3/2)) - (154*a^4*(g*Cos[e + f*x])^(5/2))/(5*
c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (462*a^4*g*Sqr
t[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sq
rt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (66*a^3*(g*Cos[e + f*x])
^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*c^2*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.48816, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{154a^4(g\cos(e+fx))^{5/2}}{5c^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{66a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{5c^2fg\sqrt{c-c\sin(e+fx)}}+\frac{462a^4g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}\right.}{5c^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(5*f*g*(c - c*Sin[e
 + f*x])^(5/2)) - (12*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2)
)/(c*f*g*(c - c*Sin[e + f*x])^(3/2)) - (154*a^4*(g*Cos[e + f*x])^(5/2))/(5*
c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (462*a^4*g*Sqr
t[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sq
rt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (66*a^3*(g*Cos[e + f*x])
^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*c^2*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
```

```
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{(3a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+f}{(c-c\sin(e+fx))^{3/2}}}{c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/2}(a+a\,\mathrm{s}}{cfg(c-c\sin(e+f\mathrm{x}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/2}(a+a\,\mathrm{s}}{cfg(c-c\sin(e+f\mathrm{x}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/2}(a+a\,\mathrm{s}}{cfg(c-c\sin(e+f\mathrm{x}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/2}(a+a\,\mathrm{s}}{cfg(c-c\sin(e+f\mathrm{x}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/2}(a+a\,\mathrm{s}}{cfg(c-c\sin(e+f\mathrm{x}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e+fx))^{5/2}(a+a\,\mathrm{s}}{cfg(c-c\sin(e+f\mathrm{x}}$$

Mathematica [A] time = 4.86451, size = 267, normalized size = 0.9

$$-\frac{a^3\sqrt{a(\sin(e+fx)+1)}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(487\sin\left(\frac{1}{2}(e+fx)\right)-633\right.\right.}{20c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] -(a^3*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[a
*(1 + Sin[e + f*x])]*(-1848*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(487*Cos[(e + f*x)/2] + 633*Cos[(3*
(e + f*x))/2] - 17*Cos[(5*(e + f*x))/2] + Cos[(7*(e + f*x))/2] + 487*Sin[(e
 + f*x)/2] - 633*Sin[(3*(e + f*x))/2] - 17*Sin[(5*(e + f*x))/2] - Sin[(7*(e
 + f*x))/2])))/(20*c^2*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.398, size = 3601, normalized size = 12.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/5/f*(-1+cos(f*x+e))*(-40*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+80*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)-80*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+446*cos(f*x+e)^2-40*si
n(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+40*sin(f*x+e)*cos(f*x+e)^
4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)-8*cos(f*x+e)^5-85*cos(f*x+e)^3+69*sin(f*x+e)*co
s(f*x+e)^3-sin(f*x+e)*cos(f*x+e)^5+231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*sin(f*x+e)-9*sin(f*x+e)*cos(f*x+e)^4-80*cos(f*x+e)^4*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)+80*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+80*cos(f*x+e)^3*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)-80*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+320*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-320*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)-80*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+80*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*si
n(f*x+e)+280*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f
*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-280*cos(f*x+e)*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
```

```
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)-478*cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^6-78*cos(f*x+e)^4-
200*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+200*sin(f*x+e)*cos(
f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-360*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)+360*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-280*s
in(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+280*sin(f*x+e)*cos(f*x+e)*
ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)+40*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-231*I*c
os(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+462*I*(1/(cos(f*x+e)+1))^(
1/2)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)+231*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x
+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-462*I*(1/(cos(f*x+e)+1))^(1/
2)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-693*
I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+462*I*(1/(cos(f*x+e)+1)
)^(1/2)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*sin(f*x+e)-462*I*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*s
in(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*
x+e)^4-cos(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3+5*cos(f*x+e)^4-12*cos(f*x+e)^
2*sin(f*x+e)+8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)-20*cos(f*x+e)^2+16*sin(
f*x+e)-8*cos(f*x+e)+16)/cos(f*x+e)/(-c*(-1+sin(f*x+e)))^(5/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^{3}g\cos\left(fx+e\right)^{3}-4\,a^{3}g\cos\left(fx+e\right)+\left(a^{3}g\cos\left(fx+e\right)^{3}-4\,a^{3}g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(f\right.}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^
3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

3.122 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=300

$$\frac{154a^4(g\cos(e+fx))^{5/2}}{9c^3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{44a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{3c^2fg(c-c\sin(e+fx))^{3/2}}-\frac{154a^4g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{3c^3f\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(9*f*g*(c - c*Sin[e
 + f*x])^(7/2)) - (4*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))
/(3*c*f*g*(c - c*Sin[e + f*x])^(5/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt
[a + a*Sin[e + f*x]])/(3*c^2*f*g*(c - c*Sin[e + f*x])^(3/2)) + (154*a^4*(g*
Cos[e + f*x])^(5/2))/(9*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (154*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(3*c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.54104, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{154a^4(g\cos(e+fx))^{5/2}}{9c^3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{44a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{3c^2fg(c-c\sin(e+fx))^{3/2}}-\frac{154a^4g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{3c^3f\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(9*f*g*(c - c*Sin[e
 + f*x])^(7/2)) - (4*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))
/(3*c*f*g*(c - c*Sin[e + f*x])^(5/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt
[a + a*Sin[e + f*x]])/(3*c^2*f*g*(c - c*Sin[e + f*x])^(3/2)) + (154*a^4*(g*
Cos[e + f*x])^(5/2))/(9*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (154*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(3*c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
```

```
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] && !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{(5a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(}{(c-c\sin(e+fx))^{5/}}}{3c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{3cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{3cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{3cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{3cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{3cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{3cfg(c-c\sin(e+}$$

Mathematica [A] time = 6.62458, size = 406, normalized size = 1.35

$$\frac{\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\frac{2}{3}\cos(e+fx)+\frac{224\sin(}{3\left(\cos\left(\frac{1}{2}(e+fx)\right.\right.}\right.}{f(c-c\sin(e+fx))^{7/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(7/2),x]
```

```
[Out] (-154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*Cos[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2)) + ((g*Co
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(112
/3 + (2*Cos[e + f*x])/3 + 32/(9*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) -
32/(3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Sin[(e + f*x)/2])/(9*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) - (64*Sin[(e + f*x)/2])/(3*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^3) + (224*Sin[(e + f*x)/2])/(3*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

Maple [C] time = 0.404, size = 4237, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/9/f*(-1+cos(f*x+e))*(-54*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e
)^5+324*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-216*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+
e)^2)+216*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-940*cos(f*x+e)^2-54*sin(f*x+e)*cos(f*x+
e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+54*sin(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)-54*cos(f*x+e)^5+146*cos(f*x+e)^3-162*sin(f*x+e)*cos(f*x+e)^3-3*s
in(f*x+e)*cos(f*x+e)^5+231*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*sin(f*x
+e)-57*sin(f*x+e)*cos(f*x+e)^4-231*I*cos(f*x+e)^5*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
+231*I*cos(f*x+e)^5*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-924*I*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*cos(f*x+e)^4+924*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x+
e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
+1386*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2-1386*I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*cos(f*x+e)^2+924*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-924*I*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*cos(f*x+e)+540*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5
```

```
40*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-810*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)+810*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+216*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(3/2)*sin(f*x+e)-216*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-756*cos(f*x
+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+756*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+90
8*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^6+567*cos(f*x+e)^4+378*sin(f*x+e)*co
s(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-378*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)+918*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-9
18*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+756*sin(f*x+e)*cos(f*x
+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-756*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)-231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-924*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)+924*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*cos(f*x+e)*sin(f*x+e)+54*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^6-54*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^6-324*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)+231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-231*I*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*EllipticE(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-1386*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos
(f*x+e)^2*sin(f*x+e)+1386*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+
e)+54*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^5)*(g*cos(f*x+e))^(3/
2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^4-cos(f*x+e)^5+4*sin(f*x
```

```
+e)*cos(f*x+e)^3+5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)+8*cos(f*x+e)^3-8
*sin(f*x+e)*cos(f*x+e)-20*cos(f*x+e)^2+16*sin(f*x+e)-8*cos(f*x+e)+16)/(-c*(
-1+sin(f*x+e)))^(7/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(3\,a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)+\left(a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx\right.}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
 e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*
x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

3.123 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{308a^4(g\cos(e+fx))^{5/2}}{39c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{44a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{39c^2fg(c-c\sin(e+fx))^{5/2}}+\frac{154a^4g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}\right.}{13c^4f\sqrt{a\sin(e+fx)}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(13*f*g*(c - c*Sin[
e + f*x])^(9/2)) - (20*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(39*c*f*g*(c - c*Sin[e + f*x])^(7/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*S
qrt[a + a*Sin[e + f*x]])/(39*c^2*f*g*(c - c*Sin[e + f*x])^(5/2)) - (308*a^4
*(g*Cos[e + f*x])^(5/2))/(39*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (154*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(13*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]])
```

Rubi [A] time = 1.53357, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$-\frac{308a^4(g\cos(e+fx))^{5/2}}{39c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{44a^3\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{39c^2fg(c-c\sin(e+fx))^{5/2}}+\frac{154a^4g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}\right.}{13c^4f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(9/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(13*f*g*(c - c*Sin[
e + f*x])^(9/2)) - (20*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(39*c*f*g*(c - c*Sin[e + f*x])^(7/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*S
qrt[a + a*Sin[e + f*x]])/(39*c^2*f*g*(c - c*Sin[e + f*x])^(5/2)) - (308*a^4
*(g*Cos[e + f*x])^(5/2))/(39*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (154*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(13*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
```

```
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{(15a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin}{(c-c\sin(e+fx))}}{13c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{39cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{39cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{39cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{39cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{39cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{39cfg(c-c\sin(e}$$

Mathematica [A] time = 6.69192, size = 464, normalized size = 1.55

$$\frac{154E\left(\frac{1}{2}(e+fx)\middle|2\right)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}{13f\cos^{\frac{3}{2}}(e+fx)(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{\sec(e+fx)(a(\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(9/2),x]
```

```
[Out] (154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(13*f*Cos[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2)) + ((g*Co
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(-12
8/13 + 32/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) - 224/(39*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^4) + 80/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2) + (64*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

```
- (448*Sin[(e + f*x)/2])/(39*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (16
0*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (256*Sin
[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e +
f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x
])^(9/2))
```

Maple [C] time = 0.402, size = 4829, normalized size = 16.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -2/39/f*(-1+cos(f*x+e))*(-2772*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)+1848*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)-1848*I*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)-231*I*sin(f*x+e)*cos(f*x+e)^5*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(
1/(cos(f*x+e)+1))^(1/2)+1155*I*sin(f*x+e)*cos(f*x+e)^4*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^
(1/2)-1155*I*sin(f*x+e)*cos(f*x+e)^4*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(
cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-546*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^5+1092*cos(f*x+e)^5*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)-624*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+624*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)-1800*cos(f*x+e)^2-858*sin(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
+858*sin(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+78*sin(f*x+e)*cos(f*
x+e)^6*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-78*sin(f*x+e)*cos(f*x+e)^6*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)-237*cos(f*x+e)^5+332*cos(f*x+e)^3-284*sin(f*x+e)*cos(f*x+e)^3+
39*sin(f*x+e)*cos(f*x+e)^5-660*sin(f*x+e)*cos(f*x+e)^4+2184*cos(f*x+e)^4*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)-2184*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+546*c
os(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-546*cos(f*x+e)^3*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

```
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*
x+e)^2)-2184*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+2184*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)+624*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-624*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-2184*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
+2184*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1896*cos(f*x+e)^2*sin(f*x+e)-39*
cos(f*x+e)^6+1472*cos(f*x+e)^4+546*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)-546*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+2496*sin(
f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-2496*sin(f*x+e)*cos(f*x+e)^
2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)+2184*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)-2184*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+924*I*cos(f*x+
e)^3*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*(1/(cos(f*x+e)+1))^(1/2)+78*cos(f*x+e)^7*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-78
*cos(f*x+e)^7*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-1092*cos(f*x+e)^5*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)+231*I*sin(f*x+e)*cos(f*x+e)^5*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)+2772*I*s
in(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-1848*I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*cos(f*x+e)+1848*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+462*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)^5-462*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^
5-2772*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+2772*I*cos(f*x+e)^2*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^6*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)-231*I*cos(f
*x+e)^6*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f
```

```
*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)-2541*I*cos(f*x+e)^4*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(
1/2)+2541*I*cos(f*x+e)^4*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)-924*I*cos(f*x+e)^3*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(
cos(f*x+e)+1))^(1/2)+546*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)*cos(f*x+e)^5)
*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^4-cos
(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3+5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+
e)+8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)-20*cos(f*x+e)^2+16*sin(f*x+e)-8*c
os(f*x+e)+16)/(-c*(-1+sin(f*x+e)))^(9/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)+\left(a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx\right.}}{5\,c^5\cos\left(fx+e\right)^4-20\,c^5\cos\left(fx+e\right)^2+16\,c^5-\left(c^5\cos\left(fx+e\right)^4-12\,c^5\cos\left(fx+e\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
 e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos
(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5
)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

3.124 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=357

$$\frac{154a^4(g\cos(e+fx))^{5/2}}{221c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}} - \frac{308a^4(g\cos(e+fx))^{5/2}}{663c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}} + \frac{220a^3\sqrt{a\sin(e+f}}{663c^2fg(c-}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(17*f*g*(c - c*Sin[
e + f*x])^(11/2)) - (60*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/
2))/(221*c*f*g*(c - c*Sin[e + f*x])^(9/2)) + (220*a^3*(g*Cos[e + f*x])^(5/2
)*Sqrt[a + a*Sin[e + f*x]])/(663*c^2*f*g*(c - c*Sin[e + f*x])^(7/2)) - (308
*a^4*(g*Cos[e + f*x])^(5/2))/(663*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2)) + (154*a^4*(g*Cos[e + f*x])^(5/2))/(221*c^4*f*g*Sqrt[a
+ a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (154*a^4*g*Sqrt[Cos[e + f*x
]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(221*c^5*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.83963, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{154a^4(g\cos(e+fx))^{5/2}}{221c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}} - \frac{308a^4(g\cos(e+fx))^{5/2}}{663c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}} + \frac{220a^3\sqrt{a\sin(e+f}}{663c^2fg(c-}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(17*f*g*(c - c*Sin[
e + f*x])^(11/2)) - (60*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/
2))/(221*c*f*g*(c - c*Sin[e + f*x])^(9/2)) + (220*a^3*(g*Cos[e + f*x])^(5/2
)*Sqrt[a + a*Sin[e + f*x]])/(663*c^2*f*g*(c - c*Sin[e + f*x])^(7/2)) - (308
*a^4*(g*Cos[e + f*x])^(5/2))/(663*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2)) + (154*a^4*(g*Cos[e + f*x])^(5/2))/(221*c^4*f*g*Sqrt[a
+ a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (154*a^4*g*Sqrt[Cos[e + f*x
]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(221*c^5*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
```

```
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{(15a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin}{(c-c\sin(e+fx))}}{17c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/2}(a+}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/2}(a+}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/2}(a+}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/2}(a+}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/2}(a+}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/2}(a+}{221cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+fx))^{5/2}(a+}{221cfg(c-c\sin(e}$$

Mathematica [A] time = 6.76856, size = 532, normalized size = 1.49

$$\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}\left(\frac{308\sin\left(\frac{1}{2}(e+fx)\right)}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}-\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(11/2),x]
```

```
[Out] (-154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(221*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + ((
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11
*(154/221 + 32/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 864/(221*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^6) + 2096/(663*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4) - 288/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Si
n[(e + f*x)/2])/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (1728*Sin[(e
 + f*x)/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (4192*Sin[(e +
f*x)/2])/(663*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) - (576*Sin[(e + f*x)
/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (308*Sin[(e + f*x)/2])
/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(7/2))
/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2))
```

Maple [C] time = 0.363, size = 3910, normalized size = 11.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/1326/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)*(-1+cos(f*x+e))^4*(-
1+sin(f*x+e))*(cos(f*x+e)+1)*(-2652*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*si
n(f*x+e)*cos(f*x+e)^5-663*cos(f*x+e)^5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+663*cos(f*x+e)^5*ln(-(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-9888*sin(f*x+e)*c
os(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4896*cos(f*x+e)*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-3696*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*c
os(f*x+e)^6*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-7956*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3+7956*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3+5304*ln(-2*(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)-5304*cos(f*x+e)*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-2496*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-7856*cos(f*x+e)^3*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-2496*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+6928*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-5304*ln(-2*(2*cos(f*x+e)^2*(-cos(f
```

```
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+e)+5304*cos(f*x+e)*
ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+
752*sin(f*x+e)*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3696*I*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+53
04*sin(f*x+e)*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)-5304*sin(f*x+e)*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+11840*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)+1044*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f
*x+e)*cos(f*x+e)^4+2652*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)*cos(f*x+e)^5-2652*cos(f*x+e)^5*ln(-(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5356*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)+2960*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^5+924*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^6-7392*I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+7392*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-7392*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^5+7392*I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^5+7392*I*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-7392*I*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2217
6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+
e)^3-22176*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*cos(f*x+e)^3+3696*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)-3696*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*cos(f*x+e)^2+7392*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-14784*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)-7392*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+14784*I*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+924*I*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^6-9
24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x
+e)*cos(f*x+e)^6+1848*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^5-1848*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*cos(f*x+e)^5-6468*I*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
```

```
)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4+6
468*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*
x+e)*cos(f*x+e)^4-14784*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^3+14784*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*cos(f*x+e)^3+14784*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)
-14784*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2))/(-cos(f*x+e)^4+4*cos(f*x+e)^2*sin(f*x+e)+8*cos(f*x+e)^2
-8*sin(f*x+e)-8)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(-1+sin(f*x+e)))^
(11/2)/sin(f*x+e)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)+\left(a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+\right.}}{c^6\cos\left(fx+e\right)^6-18\,c^6\cos\left(fx+e\right)^4+48\,c^6\cos\left(fx+e\right)^2-32\,c^6+2\left(3\,c^6\cos\left(fx+e\right)^4-16\,c^6\,\mathrm{co}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^6*cos(f*x + e)^6 - 18*c^6*cos(f*
x + e)^4 + 48*c^6*cos(f*x + e)^2 - 32*c^6 + 2*(3*c^6*cos(f*x + e)^4 - 16*c^
6*cos(f*x + e)^2 + 16*c^6)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

3.125 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=414

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{663c^5fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{663c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{220a^4}{1989c^3fg\sqrt{a\sin(e}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(21*f*g*(c - c*Sin[
e + f*x])^(13/2)) - (20*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/
2))/(119*c*f*g*(c - c*Sin[e + f*x])^(11/2)) + (220*a^3*(g*Cos[e + f*x])^(5/
2)*Sqrt[a + a*Sin[e + f*x]])/(1547*c^2*f*g*(c - c*Sin[e + f*x])^(9/2)) - (2
20*a^4*(g*Cos[e + f*x])^(5/2))/(1989*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(7/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(663*c^4*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (22*a^4*(g*Cos[e + f*x])^
(5/2))/(663*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) -
(22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
)/(663*c^6*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 2.17137, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{663c^5fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{663c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{220a^4}{1989c^3fg\sqrt{a\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(13/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(21*f*g*(c - c*Sin[
e + f*x])^(13/2)) - (20*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/
2))/(119*c*f*g*(c - c*Sin[e + f*x])^(11/2)) + (220*a^3*(g*Cos[e + f*x])^(5/
2)*Sqrt[a + a*Sin[e + f*x]])/(1547*c^2*f*g*(c - c*Sin[e + f*x])^(9/2)) - (2
20*a^4*(g*Cos[e + f*x])^(5/2))/(1989*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(7/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(663*c^4*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (22*a^4*(g*Cos[e + f*x])^
(5/2))/(663*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) -
(22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
)/(663*c^6*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{(5a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(}{(c-c\sin(e+fx))^{11}}}{7c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{119cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{119cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{119cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{119cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{119cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{119cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{119cfg(c-c\sin(e}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e+fx))^{5/2}(a+}{119cfg(c-c\sin(e}$$

Mathematica [A] time = 6.81841, size = 600, normalized size = 1.45

$$\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{13}\left(\frac{44\sin\left(\frac{1}{2}(e+fx)\right)}{663\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(13/2),x]
```

```
[Out] (-22*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^13*(a*(1 + Sin[e + f*x]))^(7/2))/(663*f*Cos[e + f*x]^(3/2)
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + ((g
*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^13*
(22/663 + 32/(21*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10) - 352/(119*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^8) + 464/(221*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^6) - 1216/(1989*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) + 22/(66
3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Sin[(e + f*x)/2])/(21*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^11) - (704*Sin[(e + f*x)/2])/(119*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^9) + (928*Sin[(e + f*x)/2])/(221*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^7) - (2432*Sin[(e + f*x)/2])/(1989*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^5) + (44*Sin[(e + f*x)/2])/(663*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^3) + (44*Sin[(e + f*x)/2])/(663*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

Maple [C] time = 0.417, size = 1484, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] 2/13923/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)*(sin(f*x+e)*cos(f*x
+e)-sin(f*x+e)-cos(f*x+e)+1)*(-10608-10608*sin(f*x+e)+14304*cos(f*x+e)+6912
*sin(f*x+e)*cos(f*x+e)+12092*cos(f*x+e)^2+6566*cos(f*x+e)^5-20408*cos(f*x+e
)^3-12640*sin(f*x+e)*cos(f*x+e)^3+4256*sin(f*x+e)*cos(f*x+e)^5-7259*sin(f*x
+e)*cos(f*x+e)^4+231*cos(f*x+e)^6*sin(f*x+e)+19108*cos(f*x+e)^2*sin(f*x+e)-
1155*cos(f*x+e)^6-791*cos(f*x+e)^4+3696*I*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3696*I*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)+1155*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)-1155*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x
+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+231*I*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)-3234*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3234*I*EllipticF(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)+9471*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(
f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-9471*I*
```

```
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-10164*I*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)+10164*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3696*I*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)+3696*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5775
*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5775*I*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+8316*I*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)-8316*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e
)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*
(cos(f*x+e)^2+2*cos(f*x+e)+1)/(-cos(f*x+e)^4+4*cos(f*x+e)^2*sin(f*x+e)+8*co
s(f*x+e)^2-8*sin(f*x+e)-8)/(-c*(-1+sin(f*x+e)))^(13/2)/sin(f*x+e)^5/cos(f*x
+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3\,a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)+\left(a^3 g\cos\left(fx+e\right)^3-4\,a^3 g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(f\right.}}{7\,c^7\cos\left(fx+e\right)^6-56\,c^7\cos\left(fx+e\right)^4+112\,c^7\cos\left(fx+e\right)^2-64\,c^7-\left(c^7\cos\left(fx+e\right)^6-24\,c^7\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^7*cos(f*x + e)^6 - 56*c^7*cos(
f*x + e)^4 + 112*c^7*cos(f*x + e)^2 - 64*c^7 - (c^7*cos(f*x + e)^6 - 24*c^7
*cos(f*x + e)^4 + 80*c^7*cos(f*x + e)^2 - 64*c^7)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(13/2), x)
```

3.126 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=471

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{5525c^6fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{5525c^5fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{}{3315c^4fg\sqrt{a}}$$

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(25*f*g*(c - c*Sin[
e + f*x])^(15/2)) - (4*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(35*c*f*g*(c - c*Sin[e + f*x])^(13/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*
Sqrt[a + a*Sin[e + f*x]])/(595*c^2*f*g*(c - c*Sin[e + f*x])^(11/2)) - (44*a
^4*(g*Cos[e + f*x])^(5/2))/(1105*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Si
n[e + f*x])^(9/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a +
 a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (22*a^4*(g*Cos[e + f*x])^(5/
2))/(5525*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (2
2*a^4*(g*Cos[e + f*x])^(5/2))/(5525*c^6*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(3/2)) - (22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5525*c^7*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]])
```

Rubi [A] time = 2.48679, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{5525c^6fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{5525c^5fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{}{3315c^4fg\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(15/2),x]
```


```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(25*f*g*(c - c*Sin[
e + f*x])^(15/2)) - (4*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(35*c*f*g*(c - c*Sin[e + f*x])^(13/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*
Sqrt[a + a*Sin[e + f*x]])/(595*c^2*f*g*(c - c*Sin[e + f*x])^(11/2)) - (44*a
^4*(g*Cos[e + f*x])^(5/2))/(1105*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Si
n[e + f*x])^(9/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a +
 a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (22*a^4*(g*Cos[e + f*x])^(5/
2))/(5525*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (2
2*a^4*(g*Cos[e + f*x])^(5/2))/(5525*c^6*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(3/2)) - (22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5525*c^7*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
```

```
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{(3a)\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(}{(c-c\sin(e+fx))^{13}}}{5c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+fx))^{5/2}(a+}{35cfg(c-c\sin(e+}$$

Mathematica [A] time = 6.88652, size = 668, normalized size = 1.42

$$\sec(e+fx)(a(\sin(e+fx)+1))^{7/2}(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{15}\left(\frac{44\sin\left(\frac{1}{2}(e+fx)\right)}{5525\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
 + f*x])^(15/2),x]
```

```
[Out] (-22*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^15*(a*(1 + Sin[e + f*x]))^(7/2))/(5525*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + ((
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^15
*(22/5525 + 32/(25*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^12) - 416/(175*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^10) + 4656/(2975*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^8) - 2144/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) + 2
2/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) + 22/(5525*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2) + (64*Sin[(e + f*x)/2])/(25*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^13) - (832*Sin[(e + f*x)/2])/(175*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^11) + (9312*Sin[(e + f*x)/2])/(2975*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^9) - (4288*Sin[(e + f*x)/2])/(5525*(Cos[(e + f*x)/2] - Sin[(e
```

```
+ f*x)/2])^7) + (44*Sin[(e + f*x)/2])/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5) + (44*Sin[(e + f*x)/2])/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3) + (44*Sin[(e + f*x)/2])/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))
*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(
c - c*Sin[e + f*x])^(15/2))
```

Maple [C] time = 0.486, size = 1644, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x)
```

```
[Out] 2/116025/f*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)*(sin(f*x+e)*cos(f*
x+e)-sin(f*x+e)-cos(f*x+e)+1)*(-74256-74256*sin(f*x+e)+81648*cos(f*x+e)+668
64*sin(f*x+e)*cos(f*x+e)+112324*cos(f*x+e)^2+50003*cos(f*x+e)^5-130804*cos(
f*x+e)^3-85052*sin(f*x+e)*cos(f*x+e)^3+20895*sin(f*x+e)*cos(f*x+e)^5-29673*
sin(f*x+e)*cos(f*x+e)^4-7392*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+7392*I*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)-385*cos(f*x+e)^7+1386*cos(f*x+e)^6*sin(f*x+e)+99836*cos(f*x+
e)^2*sin(f*x+e)-4004*cos(f*x+e)^6-34757*cos(f*x+e)^4+231*cos(f*x+e)^8+23562
*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+22176*I*EllipticF(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)-22176*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^
2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+7392*I*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)-7392*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2
31*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^8*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticE(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-1386*I*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*cos(f*x+e)^8*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)+1386*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^8*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+10164*I*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)-10164*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-235
62*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-4389*I*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)+4389*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
sin(f*x+e)*cos(f*x+e)^6*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)+15246*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*
x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-15246*I*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-18480*I*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)+18480*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(-cos(f*x+e)^4+4*cos(f*x+e)^2
*sin(f*x+e)+8*cos(f*x+e)^2-8*sin(f*x+e)-8)/(-c*(-1+sin(f*x+e)))^(15/2)/sin(
f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)+\left(a^3g\cos\left(fx+e\right)^3-4\,a^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right.}{c^8\cos\left(fx+e\right)^8-32\,c^8\cos\left(fx+e\right)^6+160\,c^8\cos\left(fx+e\right)^4-256\,c^8\cos\left(fx+e\right)^2+128\,c^8+8\left(c^8\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/
2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*g*cos(f*x + e)^3 - 4*a^3*g*cos(f*x + e) + (a^3*g*cos(f*x +
 e)^3 - 4*a^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^8*cos(f*x + e)^8 - 32*c^8*cos(f
*x + e)^6 + 160*c^8*cos(f*x + e)^4 - 256*c^8*cos(f*x + e)^2 + 128*c^8 + 8*(
c^8*cos(f*x + e)^6 - 10*c^8*cos(f*x + e)^4 + 24*c^8*cos(f*x + e)^2 - 16*c^8
)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/
2),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(15/2), x)
```

3.127 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=234

$$\frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{35fg\sqrt{a\sin(e+fx)+a}}+\frac{22c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{5f\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

```
[Out] (22*c^3*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (22*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 + (22*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35*f*g*Sqrt[a
+ a*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/(7*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 1.12086, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{35fg\sqrt{a\sin(e+fx)+a}}+\frac{22c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{5f\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (22*c^3*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (22*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 + (22*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35*f*g*Sqrt[a
+ a*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/(7*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7fg\sqrt{a+a\sin(e+fx)}} + \frac{1}{7}(11c)\int \frac{(g\cos(e+fx))^{3/2}(c\cdots}{\sqrt{a+a\sin\cdots}}$$

$$= \frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{35fg\sqrt{a+a\sin(e+fx)}} + \frac{2c(g\cos(e+fx))^{5/2}(c-c\sin(\cdots}{7fg\sqrt{a+a\sin(e+f\cdots}}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\cdots}}{35fg\sqrt{a+a\sin(e+\cdots}}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\cdots}}{35fg\sqrt{a+a\sin(e+\cdots}}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\cdots}}{35fg\sqrt{a+a\sin(e+\cdots}}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^3g\sqrt{\cos(e+fx)}\sqrt{g\cos(e\cdots}}{5f\sqrt{a+a\sin(e+fx)}\cdots}$$

Mathematica [A] time = 1.53681, size = 174, normalized size = 0.74

$$\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(924E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos\cdots}\right.}{210f\cos^{\frac{3}{2}}(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Si
n[e + f*x]],x]
```

```
[Out] (c^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin
[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]*(924*EllipticE[(e + f*x)/2, 2] + Sqrt
[Cos[e + f*x]]*(515*Cos[e + f*x] - 3*(5*Cos[3*(e + f*x)] + 42*Sin[2*(e + f*
x)]))))/(210*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*S
qrt[a*(1 + Sin[e + f*x])])
```

Maple [C] time = 0.335, size = 415, normalized size = 1.8

$$-\frac{2}{105f\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-3\left(\cos\left(fx+e\right)\right)^2-4\sin\left(fx+e\right)+4\right)\sin\left(fx+e\right)\cos\left(fx+e\right)}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/105/f*(g*cos(f*x+e))^(3/2)*(231*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*
x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+15*sin(f*x+
e)*cos(f*x+e)^4+231*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)-63*cos(f*x+e)^4-140*cos(f*x+e)^2*sin(f*x+e)+294*
cos(f*x+e)^2-231*cos(f*x+e))*(-c*(-1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f
*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)
))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x
 + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^2g\cos\left(fx+e\right)^3+2\,c^2g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^2g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(
f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x
 + e) + a), x)
```

3.128 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=180

$$\frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c\sqrt{c-c\sin(e+f}}{5fg\sqrt{a\sin}}$$

```
[Out] (14*c^2*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (14*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 + (2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*Sqrt[a + a*
Sin[e + f*x]])
```

Rubi [A] time = 0.838002, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c\sqrt{c-c\sin(e+f}}{5fg\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (14*c^2*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (14*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 + (2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*Sqrt[a + a*
Sin[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{2c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5fg\sqrt{a+a\sin(e+fx)}} + \frac{1}{5}(7c)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-}}{\sqrt{a+a\sin(e+}}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos(e+fx))^{5/2}\sqrt{c-c\,\mathrm{si}}}{5fg\sqrt{a+a\sin(e+f}}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos(e+fx))^{5/2}\sqrt{c-c\,\mathrm{si}}}{5fg\sqrt{a+a\sin(e+f}}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos(e+fx))^{5/2}\sqrt{c-c\,\mathrm{si}}}{5fg\sqrt{a+a\sin(e+f}}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{14c^2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e}}{5f\sqrt{a+a\sin(e+fx)}\sqrt{}}$$

Mathematica [A] time = 0.724199, size = 157, normalized size = 0.87

$$-\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(42E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(}\right.}{15f\cos^{\frac{3}{2}}(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Si
n[e + f*x]],x]
```

```
[Out] -(c*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[
e + f*x])*Sqrt[c - c*Sin[e + f*x]]*(42*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos
[e + f*x]]*(20*Cos[e + f*x] - 3*Sin[2*(e + f*x)])))/(15*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [C] time = 0.31, size = 382, normalized size = 2.1

$$\frac{2}{15f\left(\left(\cos\left(fx+e\right)\right)^2+2\sin\left(fx+e\right)-2\right)\sin\left(fx+e\right)\cos\left(fx+e\right)}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(21i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\mathrm{co}}{\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/15/f*(g*cos(f*x+e))^(3/2)*(21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(
```

```
f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*cos(
f*x+e)^4-10*cos(f*x+e)^2*sin(f*x+e)+24*cos(f*x+e)^2-21*cos(f*x+e))*(-c*(-1+
sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/(a*(1+sin(f*x+e)))^(1/2)/s
in(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x
 + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x
 + e) + a), x)
```

3.129 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=122

$$\frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*c*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin
[e + f*x]]) + (2*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.539677, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2851, 2842, 2640, 2639}

$$\frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (2*c*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin
[e + f*x]]) + (2*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}dx = \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + c\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\,s}}$$

$$= \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(cg\cos(e+fx))\int\sqrt{g\cos(e+f}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{\left(cg\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx}\right.}{\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

$$= \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2cg\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx}}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c}}$$

Mathematica [C] time = 2.56402, size = 215, normalized size = 1.76

$$\frac{\sqrt{e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\sqrt{-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}\left(12ie^{i(e+fx)}{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(e+fx)}\right)+\sqrt{1+e^{2i(e+fx)}}\left(-6ie^{i(e+fx)}+e^{2i(e}\right.\right.}{3af\left(1+e^{2i(e+fx)}\right)^{3/2}\cos^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] (Sqrt[((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*Sqrt[(1 + E^((2*I)*
(e + f*x)))/E^(I*(e + f*x))]*(g*Cos[e + f*x])^(3/2)*(Sqrt[1 + E^((2*I)*(e +
 f*x))]*(1 - (6*I)*E^(I*(e + f*x)) + E^((2*I)*(e + f*x))) + (12*I)*E^(I*(e
+ f*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(e + f*x))])*Sqrt[c - c
*Sin[e + f*x]])/(3*a*(1 + E^((2*I)*(e + f*x)))^(3/2)*f*Cos[e + f*x]^(3/2))
```

Maple [C] time = 0.343, size = 361, normalized size = 3.

$$-\frac{2}{3f\left(-1+\sin\left(fx+e\right)\right)\sin\left(fx+e\right)\cos\left(fx+e\right)}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\left(3i\cos\left(fx+e\right)\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3/f*(g*cos(f*x+e))^(3/2)*(-c*(-1+sin(f*x+e)))^(1/2)*(3*I*cos(f*x+e)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)+3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-3*I*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+3*cos(f*x+e))/(-1+s
in(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x +
 e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}g\cos\left(fx+e\right)}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*g*cos(f*x + e)/sqrt
(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x +
 e) + a), x)
```

3.130 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=68

$$\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.279441, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2842, 2640, 2639}

$$\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x
]]),x]
```


```
[Out] (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int\sqrt{g\cos(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\left(g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\right)\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.384678, size = 111, normalized size = 1.63

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f\cos^{\frac{3}{2}}(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Cos[e + f*x]^(3/2)
*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.305, size = 334, normalized size = 4.9

$$2\frac{\left(g\cos\left(fx+e\right)\right)^{3/2}}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\sin\left(fx+e\right)\cos\left(fx+e\right)}\left(i\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{(\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/f*(g*cos(f*x+e))^(3/2)*(I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)
-I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*sin(f*x+e)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)^2+cos(f*x+
e))/(a*(1+sin(f*x+e)))^(1/2)/(-c*(-1+sin(f*x+e)))^(1/2)/sin(f*x+e)/cos(f*x+
e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{ac\cos\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g/(a*c*cos(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

3.131 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x
)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.570069, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x
)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}dx = \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}}}{c}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{(g\cos(e+fx))\int\sqrt{g\,c}}{c\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{\left(g\sqrt{\cos(e+fx)}\sqrt{g\cos(}\right.}{c\sqrt{a+a\sin(e+f}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{2g\sqrt{\cos(e+fx)}\sqrt{g\cos(}}{cf\sqrt{a+a\sin(e+fx}}$$

Mathematica [A] time = 0.764301, size = 148, normalized size = 1.22

$$\frac{2(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+E\left(\frac{1}{2}(e+\right.\right.}{cf\cos^{\frac{3}{2}}(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(EllipticE[
(e + f*x)/2, 2]*(-Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sqrt[Cos[e + f*x]]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(c*f*Cos[e + f*x]^(3/2)*Sqrt[a*(1
+ Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.376, size = 925, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(-1+sin(f*x+e))*(4*I*sin(f*x+
e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-4*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)+8*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-8*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*I*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-4*I*(1/(cos(f*x+e)+1))^(
```

```
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-4*sin(f*x+e)*cos(f*x+
e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+e)+cos(f*x+e)*ln(-(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+4*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*sin(f*x+e)-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))/(cos(f*x+e)+1)/sin(
f*x+e)^5/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(-1+sin(f*x+e)))^(3/2)/(a
*(1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{ac^2\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^2\cos\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a*c^2*cos(f*x + e)*sin(f*x + e) - a*c^2*cos(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

3.132 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{2(g\cos(}{5fg\sqrt{a\sin(e+fx)+}}$$

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e + f*x]]*
(c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*
EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin
[e + f*x]])
```

Rubi [A] time = 0.859745, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{2(g\cos(}{5fg\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e + f*x]]*
(c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*
EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}dx = \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e\cdots}}{5c}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{2(g\cos(e\cdots}{5cfg\sqrt{a+a\sin(e+f\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{2(g\cos(e\cdots}{5cfg\sqrt{a+a\sin(e+f\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{2(g\cos(e\cdots}{5cfg\sqrt{a+a\sin(e+f\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{2(g\cos(e\cdots}{5cfg\sqrt{a+a\sin(e+f\cdots}}$$

Mathematica [A] time = 1.6232, size = 204, normalized size = 1.14

$$\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}\left(4\sin^3\left(\frac{1}{2}(e\cdots\right.\right.\right.}{5c^2f(\sin(e+fx)-1)^2\cos^{\frac{3}{2}}(e+fx)\sqrt{a(\sin\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(-2*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(3*Cos[(e + f*x)/2] + Cos[(3*(e
+ f*x))/2] + 4*Sin[(e + f*x)/2]^3)))/(5*c^2*f*Cos[e + f*x]^(3/2)*(-1 + Sin[
e + f*x])^2*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.346, size = 778, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/5/f*(g*cos(f*x+e))^(3/2)*(sin(f*x+e)*cos(f*x+e)-sin(f*x+e)-cos(f*x+e)+1)*
(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
```

```
*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)+I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-I*sin(f*x+e)*co
s(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x
+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*sin(f*x+e)+I*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+I*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)-I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^2+sin(f*
x+e)-2*cos(f*x+e)+1)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)
/(-c*(-1+sin(f*x+e)))^(5/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}g}{ac^3\cos(fx+e)^3+2\,ac^3\cos(fx+e)\sin(fx+e)-2\,ac^3\cos(fx+e)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a*c^3*cos(f*x + e)^3 + 2*a*c^3*cos(f*x + e)*sin(f*x + e) - 2*a*c^
3*cos(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

3.133 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=233

$$\frac{2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos}{15cfg\sqrt{a\sin(e+fx}}$$

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(7/2)) + (2*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e + f*x]]
*(c - c*Sin[e + f*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2))/(15*c^2*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*S
qrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.14926, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos}{15cfg\sqrt{a\sin(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
7/2)),x]
```


```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(7/2)) + (2*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e + f*x]]
*(c - c*Sin[e + f*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2))/(15*c^2*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*S
qrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}dx = \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} + \frac{\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e\cdots}}{3c}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} + \frac{2(g\cos(e\cdots}{15cfg\sqrt{a+a\sin(e+f\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} + \frac{2(g\cos(e\cdots}{15cfg\sqrt{a+a\sin(e+f\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} + \frac{2(g\cos(e\cdots}{15cfg\sqrt{a+a\sin(e+f\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} + \frac{2(g\cos(e\cdots}{15cfg\sqrt{a+a\sin(e+f\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} + \frac{2(g\cos(e\cdots}{15cfg\sqrt{a+a\sin(e+f\cdots}}$$

Mathematica [A] time = 2.3048, size = 240, normalized size = 1.03

$$\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{\cos(e+fx)}\left(-32\sin\left(\frac{1}{2}\cdots\right.\right.\right.}{90c^3f(\sin\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(7/2)),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(12*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5 + Sqrt[Cos[e + f*x]]*(-32*Cos[(e + f*x)/2] - 15*Cos[(
3*(e + f*x))/2] + 3*Cos[(5*(e + f*x))/2] - 32*Sin[(e + f*x)/2] + 15*Sin[(3*
(e + f*x))/2] + 3*Sin[(5*(e + f*x))/2])))/(90*c^3*f*Cos[e + f*x]^(3/2)*(-1
+ Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.332, size = 955, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/45/f*(g*cos(f*x+e))^(3/2)*(sin(f*x+e)*cos(f*x+e)-sin(f*x+e)-cos(f*x+e)+1)
*(-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-12*I*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^4*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)
^4+6*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-9*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)+9*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*sin(f*x+e)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
+6*I*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)-3*cos(f*x+e)^4-6*cos(f*x+e)^2*sin(f*x+e)+5*cos(f*x+e)^3+sin(f*x+e
)*cos(f*x+e)+4*cos(f*x+e)^2+5*sin(f*x+e)-11*cos(f*x+e)+5)*(cos(f*x+e)^2+2*c
os(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(-1+sin(f*x+e)))^(7/2)/sin(f*x+e)
^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{3\,ac^4\cos\left(fx+e\right)^3-4\,ac^4\cos\left(fx+e\right)-\left(ac^4\cos\left(fx+e\right)^3-4\,ac^4\cos\left(fx+e\right)\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(3*a*c^4*cos(f*x + e)^3 - 4*a*c^4*cos(f*x + e) - (a*c^4*cos(f*x +
```

```
e)^3 - 4*a*c^4*cos(f*x + e))*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(7/2)), x)
```

3.134 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=294

$$-\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{66c^3\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{7afg\sqrt{a\sin(e+fx)+a}}-\frac{30c^2(c-c\sin(e+fx))^{3/2}(g\cos(}{7afg\sqrt{a\sin(e+fx)+}}$$

```
[Out] (-22*c^4*(g*Cos[e + f*x])^(5/2))/(a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) - (66*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (66*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(7*a*f*g*Sqrt[a
 + a*Sin[e + f*x]]) - (30*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2))/(7*a*f*g*Sqrt[a + a*Sin[e + f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 1.41985, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{66c^3\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{7afg\sqrt{a\sin(e+fx)+a}}-\frac{30c^2(c-c\sin(e+fx))^{3/2}(g\cos(}{7afg\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]
)^(3/2),x]
```


```
[Out] (-22*c^4*(g*Cos[e + f*x])^(5/2))/(a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) - (66*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (66*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(7*a*f*g*Sqrt[a
 + a*Sin[e + f*x]]) - (30*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2))/(7*a*f*g*Sqrt[a + a*Sin[e + f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2))
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
```

```
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}} - \frac{(15c)\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin}{\sqrt{a+a\sin(e+f}}}{a}$$

$$= -\frac{30c^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7afg\sqrt{a+a\sin(e+fx)}} - \frac{4c(g\cos(e+fx))^{5/2}(c-}{fg(a+a\sin(e+}$$

$$= -\frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{7afg\sqrt{a+a\sin(e+fx)}} - \frac{30c^2(g\cos(e+fx))^{5/2}(c-}{7afg\sqrt{a+a\sin(}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c}}{7afg\sqrt{a+a\sin(}}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c}}{7afg\sqrt{a+a\sin(}}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c}}{7afg\sqrt{a+a\sin(}}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^4g\sqrt{\cos(e+fx)}\sqrt{g\cos}}{af\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 6.55942, size = 282, normalized size = 0.96

$$\frac{\sec(e+fx)(c-c\sin(e+fx))^{7/2}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin(2(e+fx))-\frac{109}{14}\cos(e+f\right.}{f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-66*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(7/2))/(f*Cos[e + f*x]^(3/2)*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/2)) + ((g*Cos[e
 + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*
Sin[e + f*x])^(7/2)*(-32 - (109*Cos[e + f*x])/14 + Cos[3*(e + f*x)]/14 + (6
4*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sin[2*(e + f*x)
]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/2)
)
```

Maple [C] time = 0.394, size = 2994, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/7/f*(-1+cos(f*x+e))*(-28*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+28*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)-343*cos(f*x+e)^2-6*cos(f*x+e)^5+98*cos(f*x+e)^3-21*sin(f*x+e)*cos
(f*x+e)^3+sin(f*x+e)*cos(f*x+e)^5+231*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)+7*sin(f*x+e)*cos(f*x+e)^4-231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)+231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-28*cos(f*x+e)^4*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)+28*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-112*cos(
f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+112*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e
)^2)-168*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+168*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)-28*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+28*ln(-(2*cos(f
*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(3/2)*sin(f*x+e)-112*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+112*cos
(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
```

```
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-119*cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^6-
28*cos(f*x+e)^4-28*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+28*s
in(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-84*sin(f*x+e)*cos(f*x+e)^2
*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(3/2)+84*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)-84*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+84*sin(f*x+e)*c
os(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+231*I*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)-231*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*c
os(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*
sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-462*I*cos(f*x+e)^2*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(
f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
)*(g*cos(f*x+e))^(3/2)*(-c*(-1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^5+sin(f*x+e)*
cos(f*x+e)^4-5*cos(f*x+e)^4+4*sin(f*x+e)*cos(f*x+e)^3-8*cos(f*x+e)^3-12*cos
(f*x+e)^2*sin(f*x+e)+20*cos(f*x+e)^2-8*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)+1
6*sin(f*x+e)-16)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,c^3 g\cos(fx+e)^3-4\,c^3 g\cos(fx+e)-\left(c^3 g\cos(fx+e)^3-4\,c^3 g\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{g\cos(fx}}{a^2\cos(fx+e)^2-2\,a^2\sin(fx+e)-2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((3*c^3*g*cos(f*x + e)^3 - 4*c^3*g*cos(f*x + e) - (c^3*g*cos(f*x +
e)^3 - 4*c^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x
 + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

3.135 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5afg\sqrt{a\sin(e+fx)+a}}-\frac{154c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(\right.}{5af\sqrt{a\sin(e+fx)+}}$$

```
[Out] (-154*c^3*(g*Cos[e + f*x])^(5/2))/(15*a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) - (154*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(5*a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (22*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a*f*g
*Sqrt[a + a*Sin[e + f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x
])^(3/2))/(f*g*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 1.13633, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5afg\sqrt{a\sin(e+fx)+a}}-\frac{154c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(\right.}{5af\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]
)^(3/2),x]
```


```
[Out] (-154*c^3*(g*Cos[e + f*x])^(5/2))/(15*a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) - (154*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(5*a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (22*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a*f*g
*Sqrt[a + a*Sin[e + f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x
])^(3/2))/(f*g*(a + a*Sin[e + f*x])^(3/2))
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{fg(a+a\sin(e+fx))^{3/2}} - \frac{(11c)\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+}{\sqrt{a+a\sin(e+fx)}}}{a}$$

$$= -\frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5afg\sqrt{a+a\sin(e+fx)}} - \frac{4c(g\cos(e+fx))^{5/2}(c-c\sin}{fg(a+a\sin(e+fx)}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c}}{5afg\sqrt{a+a\sin(e}}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c}}{5afg\sqrt{a+a\sin(e}}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c}}{5afg\sqrt{a+a\sin(e}}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{154c^3g\sqrt{\cos(e+fx)}\sqrt{g\,c}}{5af\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 5.03136, size = 238, normalized size = 0.99

$$-\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(-520\sin\left(\right.\right.\right.}{30f\cos^{\frac{3}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] -(c^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(-1 +
Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]*(924*EllipticE[(e + f*x)/2, 2]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sqrt[Cos[e + f*x]]*(520*Cos[(e + f*x)/
2] + 37*Cos[(3*(e + f*x))/2] + 3*Cos[(5*(e + f*x))/2] - 520*Sin[(e + f*x)/2
] + 37*Sin[(3*(e + f*x))/2] - 3*Sin[(5*(e + f*x))/2])))/(30*f*Cos[e + f*x]^
```

```
(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [C] time = 0.346, size = 2947, normalized size = 12.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/15/f*(-1+cos(f*x+e))*(-30*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+30*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)-351*cos(f*x+e)^2-3*cos(f*x+e)^5+94*cos(f*x+e)^3-17*sin(f*x+e)*co
s(f*x+e)^3+231*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*sin(f*
x+e)*cos(f*x+e)^4-231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*
x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(I*(-1+cos(f*x+e))/sin(f*x+e),I)-30*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)+30*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-120*cos(f*x+e)^3*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)+120*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-180*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)+180*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-30*ln
(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e
)-120*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+120*cos(f*x+e)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)-111*cos(f*x+e)^2*sin(f*x+e)-20*cos(f*x+e)^4-30*sin(f*x+e)*cos(f*
x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+30*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)-90*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+90*sin(
f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
```

```
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-90*sin(f*x+e)*cos(f*x+e)*ln(-
2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)+90*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+
231*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-231*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/
sin(f*x+e),I)-462*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)
^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231
*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(-c*(-1+sin(f
*x+e)))^(5/2)/(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+
e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)^2+8*sin(f*x+e)+4*cos
(f*x+e)-8)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2 g\cos\left(fx+e\right)^3+2\,c^2 g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^2 g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

3.136 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{14c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\sin(e+fx}}{fg(a\sin(e+}$$

```
[Out] (-14*c^2*(g*Cos[e + f*x])^(5/2))/(3*a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]]) - (14*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]) - (4*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(f*g*(a + a*Sin[
e + f*x])^(3/2))
```

Rubi [A] time = 0.844361, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$-\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{14c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\sin(e+fx}}{fg(a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]
)^(3/2),x]
```


```
[Out] (-14*c^2*(g*Cos[e + f*x])^(5/2))/(3*a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]]) - (14*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]) - (4*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(f*g*(a + a*Sin[
e + f*x])^(3/2))
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{fg(a+a\sin(e+fx))^{3/2}} - \frac{(7c)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+}}{\sqrt{a+a\sin(e+fx)}}}{a}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx))^{5/2}\sqrt{c}}{fg(a+a\sin(e+}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx))^{5/2}\sqrt{c}}{fg(a+a\sin(e+}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx))^{5/2}\sqrt{c}}{fg(a+a\sin(e+}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{14c^2g\sqrt{\cos(e+fx)}\sqrt{g}}{af\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 1.6466, size = 200, normalized size = 1.1

$$\frac{2c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e\right.\right.\right.}{3f\cos^{\frac{3}{2}}(e+fx)(a(\sin(e+fx)+1))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (2*c*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(21*Ell
ipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sqrt[Cos[e +
 f*x]]*(Cos[(e + f*x)/2]*(12 + Cos[e + f*x]) + (-12 + Cos[e + f*x])*Sin[(e
+ f*x)/2]))*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*f*Cos[e + f*x]
^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(3/2)
)
```

Maple [C] time = 0.333, size = 2890, normalized size = 15.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)+33*cos(f*x+e)^2-8*cos(f*x+e)^3+sin(f*x+e)*cos(f*x+e)^3+42*I*cos(f*x+
e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),I)-42*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+
21*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)+21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)^3*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)+3*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*l
n(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*cos(f*x+
e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x
+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-
12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+18*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)-18*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*ln(-2*(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)*sin(f*x+e)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+12*cos(f*x+e)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)-12*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*I*sin(f
*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+9*cos(f*x+e)^2*sin(f*x+e)+cos(f*
x+e)^4+3*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2
*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*sin(f*x+e)*c
os(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+9*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*
```

```
x+e)+1)^2)^(3/2)-9*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+9*sin(
f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-9*sin(f*x+e)*cos(f*x+e)*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)+21*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2
1*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(g*cos(f*x+e))^(3/2)*(-c*(-1+sin(f*x+e))
)^(3/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+2*sin(f*x+e)*c
os(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+
e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral((c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^
2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

3.137 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{4c(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{6cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-4*c*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Si
n[e + f*x]]) - (6*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.560933, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$-\frac{4c(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{6cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^
(3/2),x]
```


```
[Out] (-4*c*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Si
n[e + f*x]]) - (6*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(3c)\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}}}{a}$$
$$= -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(3cg\cos(e+fx))\int \sqrt{g\cos(e}}{a\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$
$$= -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{\left(3cg\sqrt{\cos(e+fx)}\sqrt{g\cos(e+}\right.}{a\sqrt{a+a\sin(e+fx)}\sqrt{}}$$
$$= -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{6cg\sqrt{\cos(e+fx)}\sqrt{g\cos(e+}}{af\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

Mathematica [C] time = 1.92901, size = 213, normalized size = 1.73

$$\frac{2g\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(2e^{2i(e+fx)}\left(e^{i(e+fx)}+i\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)-\left(5e^{i(e+fx)}+i\right)\sqrt{1+e^{2i(e+fx)}}\right)\sqrt{c-c\sin(e+f}}{af\left(e^{i(e+fx)}-i\right)\sqrt{1+e^{2i(e+fx)}}\sqrt{-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (2*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*(-((I + 5*E^(I*(e
+ f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]) + 2*E^((2*I)*(e + f*x))*(I + E^(I*(
e + f*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[c -
 c*Sin[e + f*x]])/(a*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*a*(I + E^(I*(e + f*x
)))^2)/E^(I*(e + f*x))]*Sqrt[1 + E^((2*I)*(e + f*x))]*f)
```

Maple [C] time = 0.384, size = 2839, normalized size = 23.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))*((-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
+10*cos(f*x+e)^2+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-2*cos(
f*x+e)^3+cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-cos(f*x+e)^4*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
```

```
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)+4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-4*cos(f*x+e)^3*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)+6*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-6*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-4
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+2*cos(f*x+e)^2*sin(f*x+e)+sin(f*x+e)*
cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1
)/sin(f*x+e)^2)+3*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*sin
(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+3*sin(f*x+e)*cos(f*x+e)*ln(-
2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)-3*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-6
*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*El
lipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-12*I*cos(f*x+e)^2*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)+12*I*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x
+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*
cos(f*x+e)^3*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*cos(f*x+e)^3*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)-6*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2+6*I*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2)*(-c*(-1+sin(f*x+e)))^(1/2)*(g*cos(
f*x+e))^(3/2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2
)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}g\cos(fx+e)}{a^2\cos(fx+e)^2-2a^2\sin(fx+e)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

3.138 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=121

$$-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[
e + f*x]]) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*
x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.563675, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[
e + f*x]]) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*
x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}}}{a}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(g\cos(e+fx))\int\sqrt{g\cos}}{a\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{\left(g\sqrt{\cos(e+fx)}\sqrt{g\cos(e}\right.}{a\sqrt{a+a\sin(e+fx}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e}}{af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.715092, size = 170, normalized size = 1.4

$$-\frac{2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e+\right.\right.}{f\cos^{\frac{3}{2}}(e+fx)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin
[e + f*x]]),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2*(Sqrt[Cos[e + f*x]]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]) + EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])))/(f*Cos[e + f*x]^(3/2)*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
 f*x]])
```

Maple [C] time = 0.375, size = 925, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e))*(4*I*sin(f*x+e)
*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-4*I*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)+8*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-8*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*Elli
```

```
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-4*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+4*sin(f*x+e)*cos(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+ln(-2*(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+e)+4*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*sin(f*x+e)-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))/(cos(f*x+e)+1)/sin(f*
x+e)^5/(a*(1+sin(f*x+e)))^(3/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(-c*(-
1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{a^2c\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2c\cos\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g/(a^2*c*cos(f*x + e)*sin(f*x + e) + a^2*c*cos(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

3.139 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{acf\sqrt{a\sin(e+fx)+}}$$

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(3/2)) + (2*(g*Cos[e + f*x])^(5/2))/(a*f*g*Sqrt[a + a*Sin[e + f*x]]*(
c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]])
```

Rubi [A] time = 0.878735, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{2(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{acf\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(3/2)),x]
```


```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(3/2)) + (2*(g*Cos[e + f*x])^(5/2))/(a*f*g*Sqrt[a + a*Sin[e + f*x]]*(
c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]])
```


Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}(c-c\sin}}{a}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{2(g\cos(}{afg\sqrt{a+a\sin(e+f}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{2(g\cos(}{afg\sqrt{a+a\sin(e+f}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{2(g\cos(}{afg\sqrt{a+a\sin(e+f}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{2(g\cos(}{afg\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 0.75021, size = 92, normalized size = 0.52

$$-\frac{2(g\cos(e+fx))^{5/2}\left(\sin(e+fx)-\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{cfg(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(3/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2)*(-(Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
 + Sin[e + f*x]))/(c*f*g*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*S
qrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.353, size = 361, normalized size = 2.1

$$2\,\frac{\left(\cos\left(fx+e\right)+1\right)^{2}\left(g\cos\left(fx+e\right)\right)^{3/2}\left(-1+\cos\left(fx+e\right)\right)^{2}\left(1+\sin\left(fx+e\right)\right)\left(-1+\sin\left(fx+e\right)\right)}{f\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{3/2}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{3/2}\left(\sin\left(fx+e\right)\right)^{5}\cos\left(fx+e\right)}\left(i\mathrm{EllipticF}\left(\frac{i\left(-1+\right.}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e))*
(-1+sin(f*x+e))*(I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)-I*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
```

```
os(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f
*x+e)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e)-1)/(a*(1+sin(f*x+e)
))^(3/2)/(-c*(-1+sin(f*x+e)))^(3/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{a^{2}c^{2}\cos\left(fx+e\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g/(a^2*c^2*cos(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(3/2)), x)
```

3.140 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{6g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5acfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{6(g}{5afg\sqrt{a\sin(e+}}$$

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*Sqrt[a + a*Sin[e + f*x]]
*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a*c*f*g*Sqrt[a
 + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (6*g*Sqrt[Cos[e + f*x]]*Sq
rt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a*c^2*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.15884, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{6g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5acfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{6(g}{5afg\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2)),x]
```


```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*Sqrt[a + a*Sin[e + f*x]]
*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a*c*f*g*Sqrt[a
 + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (6*g*Sqrt[Cos[e + f*x]]*Sq
rt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a*c^2*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\int \frac{(g\cos(e+fx))}{\sqrt{a+a\sin(e+fx)}(c-c\,\mathrm{si}}}{a}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{6(g\cos(}{5afg\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{6(g\cos(}{5afg\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{6(g\cos(}{5afg\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{6(g\cos(}{5afg\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{6(g\cos(}{5afg\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 1.18282, size = 134, normalized size = 0.57

$$-\frac{\sqrt{\cos(e+fx)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}(-6\sin(e+fx)-3\cos(2(e+fx))+1)+E\left(\frac{1}{2}(e+fx)\middle|2\right)(6\cos(e+fx)\right.}{5c^2f(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(5/2)),x]
```

```
[Out] -(Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(Sqrt[Cos[e + f*x]]*(1 - 3*Cos[
2*(e + f*x)] - 6*Sin[e + f*x]) + EllipticE[(e + f*x)/2, 2]*(6*Cos[e + f*x]
- 3*Sin[2*(e + f*x)])))/(5*c^2*f*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]
))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.332, size = 877, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/5/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e
))*(-1+sin(f*x+e))*(3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)-3*I
*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f
*x+e)+3*I*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*cos(f*x+e)^2*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*cos(f*x+e)^3*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/s
in(f*x+e),I)+3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*sin(f*x+e)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)-3*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*cos(f*x+e)
^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),I)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+3*sin
(f*x+e)*cos(f*x+e)-2*sin(f*x+e)-3*cos(f*x+e)+3)/(a*(1+sin(f*x+e)))^(3/2)/(-
c*(-1+sin(f*x+e)))^(5/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{a^2c^3\cos\left(fx+e\right)^3\sin\left(fx+e\right)-a^2c^3\cos\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a^2*c^3*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*cos(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

3.141 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=294

$$\frac{2(g\cos(e+fx))^{5/2}}{3ac^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3ac^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2(g}{3acfg\sqrt{a\sin(e+}$$

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(7/2)) + (10*(g*Cos[e + f*x])^(5/2))/(9*a*f*g*Sqrt[a + a*Sin[e + f*x]
]*(c - c*Sin[e + f*x])^(7/2)) + (2*(g*Cos[e + f*x])^(5/2))/(3*a*c*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2)
)/(3*a*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (2*g*
Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(3*a*c^3
*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.47401, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{2(g\cos(e+fx))^{5/2}}{3ac^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3ac^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2(g}{3acfg\sqrt{a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(7/2)),x]
```


```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(7/2)) + (10*(g*Cos[e + f*x])^(5/2))/(9*a*f*g*Sqrt[a + a*Sin[e + f*x]
]*(c - c*Sin[e + f*x])^(7/2)) + (2*(g*Cos[e + f*x])^(5/2))/(3*a*c*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2)
)/(3*a*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (2*g*
Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(3*a*c^3
*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{5\int \frac{(g\cos(e+fx))}{\sqrt{a+a\sin(e+fx)}(c-c\,\mathrm{si}}}{a}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{10(g\cos}{9afg\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{10(g\cos}{9afg\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{10(g\cos}{9afg\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{10(g\cos}{9afg\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{10(g\cos}{9afg\sqrt{a+a\sin(e+}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{10(g\cos}{9afg\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 1.5424, size = 155, normalized size = 0.53

$$\frac{\sqrt{\cos(e+fx)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}(-17\sin(e+fx)+3\sin(3(e+fx))-12\cos(2(e+fx))+4)+3E\left(\frac{1}{2}(e+f\right.\right.}{18c^3f(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(7/2)),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(3*EllipticE[(e + f*x)/2, 2]*(5*
Cos[e + f*x] - Cos[3*(e + f*x)] - 4*Sin[2*(e + f*x)]) + Sqrt[Cos[e + f*x]]*
(4 - 12*Cos[2*(e + f*x)] - 17*Sin[e + f*x] + 3*Sin[3*(e + f*x)])))/(18*c^3*
f*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [C] time = 0.359, size = 1177, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/9/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e)
)*(-1+sin(f*x+e))*(-6*I*cos(f*x+e)^3*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*cos(f*x+
e)^3*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)-6*I
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)-3*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*
x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+3*I*sin(f*x+e)*co
s(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-3*I*sin(f*x+e)*cos(f*x+e)^2*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)-6*I*cos(f*x
+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+6*
I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(
f*x+e)+3*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*cos(f*x+e)^3
-6*sin(f*x+e)*cos(f*x+e)+2*cos(f*x+e)^2+4*sin(f*x+e)+6*cos(f*x+e)-5)/(a*(1+
sin(f*x+e)))^(3/2)/(-c*(-1+sin(f*x+e)))^(7/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{a^2c^4\cos\left(fx+e\right)^5+2\,a^2c^4\cos\left(fx+e\right)^3\sin\left(fx+e\right)-2\,a^2c^4\cos\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a^2*c^4*cos(f*x + e)^5 + 2*a^2*c^4*cos(f*x + e)^3*sin(f*x + e) -
2*a^2*c^4*cos(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

3.142 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=357

$$\frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{1254c^4\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{35a^2fg\sqrt{a\sin(e+fx)+a}}+\frac{114c^3(c-c\sin(e+fx))^{3/2}}{7a^2fg\sqrt{a\sin(e+}}$$

```
[Out] (418*c^5*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (1254*c^5*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) + (1254*c^4*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35
*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]) + (114*c^3*(g*Cos[e + f*x])^(5/2)*(c - c
*Sin[e + f*x])^(3/2))/(7*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]) + (76*c^2*(g*Cos
[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^
(3/2)) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(5*f*g*(a
+ a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 1.72833, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{1254c^4\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{35a^2fg\sqrt{a\sin(e+fx)+a}}+\frac{114c^3(c-c\sin(e+fx))^{3/2}}{7a^2fg\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```


```
[Out] (418*c^5*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (1254*c^5*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) + (1254*c^4*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35
*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]) + (114*c^3*(g*Cos[e + f*x])^(5/2)*(c - c
*Sin[e + f*x])^(3/2))/(7*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]) + (76*c^2*(g*Cos
[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^
(3/2)) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(5*f*g*(a
+ a*Sin[e + f*x])^(5/2))
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
```

```
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(19c)\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+}{(a+a\sin(e+fx))^{3/2}}}{5a}$$

$$= \frac{76c^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}} - \frac{4c(g\cos(e+fx))^{5/2}(c-c\sin}{5fg(a+a\sin(e+fx}$$

$$= \frac{114c^3(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7a^2fg\sqrt{a+a\sin(e+fx)}} + \frac{76c^2(g\cos(e+fx))^{5/2}(c-}{5afg(a+a\sin(e+}$$

$$= \frac{1254c^4(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{35a^2fg\sqrt{a+a\sin(e+fx)}} + \frac{114c^3(g\cos(e+fx))^{5/2}(c-}{7a^2fg\sqrt{a+a\sin(e}}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^4(g\cos(e+fx))^{5/2}\sqrt{c}}{35a^2fg\sqrt{a+a\sin(}}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^4(g\cos(e+fx))^{5/2}\sqrt{c}}{35a^2fg\sqrt{a+a\sin(}}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^4(g\cos(e+fx))^{5/2}\sqrt{c}}{35a^2fg\sqrt{a+a\sin(}}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^5g\sqrt{\cos(e+fx)}\sqrt{g\,co}}{5a^2f\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 6.76515, size = 356, normalized size = 1.

$$\frac{1254E\left(\frac{1}{2}(e+fx)\middle|2\right)(c-c\sin(e+fx))^{9/2}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{5f\cos^{\frac{3}{2}}(e+fx)(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}+\frac{\sec(e+fx)(c-c\,s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (1254*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2))/(5*f*Cos[e + f*x]^(3/2)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + ((g*Co
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c -
 c*Sin[e + f*x])^(9/2)*(736/5 + (221*Cos[e + f*x])/14 - Cos[3*(e + f*x)]/14
 + (128*Sin[(e + f*x)/2])/(5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) - 64/
(5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) - (1472*Sin[(e + f*x)/2])/(5*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])) - (7*Sin[2*(e + f*x)])/5))/(f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [C] time = 0.401, size = 3655, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/35/f*(-1+cos(f*x+e))*(-700*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1400*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)-1400*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+8554*cos(f*x+e)^
2+700*sin(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-700*sin(f*x+e)*co
s(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-192*cos(f*x+e)^5-1575*cos(f*x+e)^3-135
1*sin(f*x+e)*cos(f*x+e)^3+39*sin(f*x+e)*cos(f*x+e)^5-4389*I*cos(f*x+e)^4*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)+4389*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+13167
*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-13167*I*cos(f*x+e)^2*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)+8778*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-8778*I*cos(f*x+e)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)+231*sin(f*x+e)*cos(f*x+e)^4+5*cos(f*x+e)^7-1400*
cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-c
```

os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1400*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1400*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-1400*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5600*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5600*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+1400*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-1400*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+4900*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-4900*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)^6*sin(f*x+e)-13167*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+8778*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-8778*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+4389*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+9002*cos(f*x+e)^2*sin(f*x+e)+44*cos(f*x+e)^6-1582*cos(f*x+e)^4+3500*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3500*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+6300*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-6300*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4900*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-4900*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-4389*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+700*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+13167*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e))*(g*cos(f*x+e))^(3/2)*(-c*(-1+sin(f*x+e)))^(9/2)/(sin(f*x+e)*cos(f*x+e)^5-cos(f*x+e)^6-6*sin(f*x+e)*cos(f*x+e)^4-5*cos(f*x+e)^5-12*sin(f*x+e)*cos(f*x+e)^3+18*cos(f*x+e)^4+32*cos(f*x+e)^2*sin(f*x+e)+20*cos(f*x+e)^3+16*sin(f*x+e)*cos(f*x+e)-48*cos(f*x+e)^2-32*s

```
in(f*x+e)-16*cos(f*x+e)+32)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{9}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c^4g\cos(fx+e)^5-8\,c^4g\cos(fx+e)^3+8\,c^4g\cos(fx+e)+4\left(c^4g\cos(fx+e)^3-2\,c^4g\cos(fx+e)\right)\sin\right.}{3\,a^3\cos(fx+e)^2-4\,a^3+\left(a^3\cos(fx+e)^2-4\,a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-(c^4*g*cos(f*x + e)^5 - 8*c^4*g*cos(f*x + e)^3 + 8*c^4*g*cos(f*x
+ e) + 4*(c^4*g*cos(f*x + e)^3 - 2*c^4*g*cos(f*x + e))*sin(f*x + e))*sqrt(g
*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*co
s(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{9}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

3.143 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{66c^3\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}}+\frac{462c^4g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{5a^2f\sqrt{a\sin(e+fx)+}}$$

```
[Out] (154*c^4*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (462*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (66*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a^2
*f*g*Sqrt[a + a*Sin[e + f*x]]) + (12*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[
e + f*x])^(3/2))/(a*f*g*(a + a*Sin[e + f*x])^(3/2)) - (4*c*(g*Cos[e + f*x])
^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 1.42436, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{66c^3\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}}+\frac{462c^4g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{5a^2f\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```


```
[Out] (154*c^4*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (462*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (66*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a^2
*f*g*Sqrt[a + a*Sin[e + f*x]]) + (12*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[
e + f*x])^(3/2))/(a*f*g*(a + a*Sin[e + f*x])^(3/2)) - (4*c*(g*Cos[e + f*x])
^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2))
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
```

```
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(3c)\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+f}{(a+a\sin(e+fx))^{3/2}}}{a}$$

$$= \frac{12c^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{afg(a+a\sin(e+fx))^{3/2}} - \frac{4c(g\cos(e+fx))^{5/2}(c-c\sin}{5fg(a+a\sin(e+fx}$$

$$= \frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5a^2fg\sqrt{a+a\sin(e+fx)}} + \frac{12c^2(g\cos(e+fx))^{5/2}(c-c\sin}{afg(a+a\sin(e+fx)}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c-}}{5a^2fg\sqrt{a+a\sin(e}}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c-}}{5a^2fg\sqrt{a+a\sin(e}}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c-}}{5a^2fg\sqrt{a+a\sin(e}}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{462c^4g\sqrt{\cos(e+fx)}\sqrt{g\cos}}{5a^2f\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 5.05606, size = 250, normalized size = 0.84

$$\frac{c^3\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(-487\sin\left(\frac{1}{2}(e+fx)\right)+633\,\mathrm{si}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (c^3*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c
- c*Sin[e + f*x]]*(1848*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(487*Cos[(e + f*x)/2] + 633*Cos[(3*(e +
 f*x))/2] - 17*Cos[(5*(e + f*x))/2] + Cos[(7*(e + f*x))/2] - 487*Sin[(e + f
*x)/2] + 633*Sin[(3*(e + f*x))/2] + 17*Sin[(5*(e + f*x))/2] + Sin[(7*(e + f
*x))/2])))/(20*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(
a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [C] time = 0.401, size = 3598, normalized size = 12.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/5/f*(-1+cos(f*x+e))*(-40*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+80*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)-80*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+446*cos(f*x+e)^2+40*s
in(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-40*sin(f*x+e)*cos(f*x+e)
^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-8*cos(f*x+e)^5-85*cos(f*x+e)^3-69*sin(f*x+e)*c
os(f*x+e)^3+sin(f*x+e)*cos(f*x+e)^5+231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*sin(f*x+e)+9*sin(f*x+e)*cos(f*x+e)^4-80*cos(f*x+e)^4*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)+80*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+80*cos(f*x+e)^3*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)-80*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+320*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-320*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^
2)+80*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-80*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*s
in(f*x+e)+280*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-280*cos(f*x+e)*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
```

```
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)+478*cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^6-78*cos(f*x+e)^4
+200*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-200*sin(f*x+e)*cos
(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+360*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(3/2)-360*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+280*
sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-280*sin(f*x+e)*cos(f*x+e)
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)+40*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-231*I*
cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-231*I*cos(f*x+e)^4*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+693*I*cos(
f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-693*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)+462*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f
*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-462*I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-693
*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+462*I*(1/(cos(f*x+e)+1
))^(1/2)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*sin(f*x+e)-462*I*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
sin(f*x+e))*(g*cos(f*x+e))^(3/2)*(-c*(-1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^5+s
in(f*x+e)*cos(f*x+e)^4-5*cos(f*x+e)^4+4*sin(f*x+e)*cos(f*x+e)^3-8*cos(f*x+e
)^3-12*cos(f*x+e)^2*sin(f*x+e)+20*cos(f*x+e)^2-8*sin(f*x+e)*cos(f*x+e)+8*co
s(f*x+e)+16*sin(f*x+e)-16)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,c^3g\cos\left(fx+e\right)^3-4\,c^3g\cos\left(fx+e\right)-\left(c^3g\cos\left(fx+e\right)^3-4\,c^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx\right.}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((3*c^3*g*cos(f*x + e)^3 - 4*c^3*g*cos(f*x + e) - (c^3*g*cos(f*x +
e)^3 - 4*c^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^
3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

3.144 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=243

$$\frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{44c^2\sqrt{c-c\sin(e}{5afg(a\sin}$$

```
[Out] (154*c^3*(g*Cos[e + f*x])^(5/2))/(15*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (154*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) + (44*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a*
f*g*(a + a*Sin[e + f*x])^(3/2)) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(3/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 1.13576, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2851, 2842, 2640, 2639}

$$\frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{44c^2\sqrt{c-c\sin(e}{5afg(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```


```
[Out] (154*c^3*(g*Cos[e + f*x])^(5/2))/(15*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (154*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) + (44*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a*
f*g*(a + a*Sin[e + f*x])^(3/2)) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(3/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2))
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2851

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m,
 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(11c)\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin}{(a+a\sin(e+fx))}}{5a}$$

$$= \frac{44c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5afg(a+a\sin(e+fx))^{3/2}} - \frac{4c(g\cos(e+fx))^{5/2}(c-c\sin}{5fg(a+a\sin(e+f}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{44c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{44c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{44c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{154c^3g\sqrt{\cos(e+fx)}\sqrt{}}{5a^2f\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 2.46736, size = 230, normalized size = 0.95

$$\frac{c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(-226\sin\left(\frac{1}{2}(e+fx)\right)+32\right.\right.}{30f\cos^{\frac{3}{2}}(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (c^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c
- c*Sin[e + f*x]]*(924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(226*Cos[(e + f*x)/2] + 327*Cos[(3*(e +
f*x))/2] - 5*Cos[(5*(e + f*x))/2] - 226*Sin[(e + f*x)/2] + 327*Sin[(3*(e +
f*x))/2] + 5*Sin[(5*(e + f*x))/2])))/(30*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x
```

```
)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [C] time = 0.362, size = 3551, normalized size = 14.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/15/f*(-1+cos(f*x+e))*(45*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-90*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)+90*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-438*cos(f*x+e)^2-45*si
n(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+45*sin(f*x+e)*cos(f*x+e)^
4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)-693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*si
n(f*x+e)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-462*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)+462*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*cos(f*x+e)*sin(f*x+e)+5*cos(f*x+e)^5+89*cos(f*x+e)^3+65*sin(f*x+e)*cos(f*x
+e)^3-5*sin(f*x+e)*cos(f*x+e)^4+90*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
-90*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-90*cos(f*x+e)^3*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)+90*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-360*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)+360*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-90*ln(-2
*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-3
15*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+315*cos(f*x+e)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*
```

```
x+e)^2)-486*cos(f*x+e)^2*sin(f*x+e)+70*cos(f*x+e)^4-225*sin(f*x+e)*cos(f*x+
e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+225*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)-405*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+405*sin
(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-315*sin(f*x+e)*cos(f*x+e)*ln
(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)+315*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)-231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-45*cos(f*x+e)^5*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)+231*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(
f*x+e)+462*I*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^4*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-693*I*cos(
f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+693*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)-462*I*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(g*cos(f*x+e))^(3/2)*(-c*(
-1+sin(f*x+e)))^(5/2)/(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-4*cos(f*x+e)^2*
sin(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)^2+8*sin(f*x+
e)+4*cos(f*x+e)-8)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^{2}g\cos\left(fx+e\right)^{3}+2\,c^{2}g\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^{2}g\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2*
g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*s
in(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

3.145 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{42c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\sin(e\ldots}}{5fg(a\sin\ldots}$$

```
[Out] (28*c^2*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c
- c*Sin[e + f*x]]) + (42*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Elli
pticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*(a +
 a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.851811, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2850, 2842, 2640, 2639}

$$\frac{42c^2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\sin(e\ldots}}{5fg(a\sin\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```


```
[Out] (28*c^2*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c
- c*Sin[e + f*x]]) + (42*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Elli
pticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*(a +
 a*Sin[e + f*x])^(5/2))
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(7c)\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}}{5a}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx))^{5/2}\sqrt{c-c}}{5fg(a+a\sin(e+f}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx))^{5/2}\sqrt{c-c}}{5fg(a+a\sin(e+f}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos(e+fx))^{5/2}\sqrt{c-c}}{5fg(a+a\sin(e+f}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{42c^2g\sqrt{\cos(e+fx)}\sqrt{g\cos}}{5a^2f\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 1.21603, size = 180, normalized size = 0.97

$$\frac{c\sqrt{\cos(e+fx)}\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(8\sqrt{\cos(e+fx)}\left(-\sin\left(\frac{1}{2}(e+fx)\right)+2\sin\left(\frac{3}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{5f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (c*Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]*(42*E
llipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 8*Sqrt[C
os[e + f*x]]*(Cos[(e + f*x)/2] + 2*Cos[(3*(e + f*x))/2] - Sin[(e + f*x)/2]
+ 2*Sin[(3*(e + f*x))/2])))/(5*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a
*(1 + Sin[e + f*x]))^(5/2))
```

Maple [C] time = 0.327, size = 3497, normalized size = 18.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/5/f*(-1+cos(f*x+e))*(-5*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+10*(-cos
```

(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-10*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+38*cos(f*x+e)^2+5*sin(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5*sin(f*x+e)*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-9*cos(f*x+e)^3-5*sin(f*x+e)*cos(f*x+e)^3-21*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-10*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+10*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+10*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-10*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+40*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-40*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+10*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)-10*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sin(f*x+e)+35*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-35*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+46*cos(f*x+e)^2*sin(f*x+e)-5*cos(f*x+e)^4+25*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-25*sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+45*sin(f*x+e)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-45*sin(f*x+e)*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+35*sin(f*x+e)*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-35*sin(f*x+e)*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-21*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+63*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co

```
s(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-63*I*cos(f*x+e
)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)+42*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-42*
I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)+5*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+63*I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2-63*I*sin(f*x+e)*cos(f*x+e)^2*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)+42*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-42
*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e))*(g*cos(f*x+e))^(3/2)*(-
c*(-1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)
^2+2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/sin(f*x+e)/cos(f*x+
e)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral((c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e
)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

3.146 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{6cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6c(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c(g\cos(\cdot}{5fg(a\sin(e+fx)+a}$$

```
[Out] (-4*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*
Sin[e + f*x]]) + (6*c*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])
^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (6*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e +
 f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]])
```

Rubi [A] time = 0.831772, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2850, 2852, 2842, 2640, 2639}

$$\frac{6cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6c(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c(g\cos(\cdot}{5fg(a\sin(e+fx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^
(5/2),x]
```



```
[Out] (-4*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*
Sin[e + f*x]]) + (6*c*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])
^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (6*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e +
 f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]])
```


Rule 2850

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*
x])^n)/(f*g*(2*n + p + 1)), x] - Dist[(b*(2*m + p - 1))/(d*(2*n + p + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(3c)\int \frac{(g\cos(e+fx))^{3}}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c}}}{5a}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{6c(g\cos(e+}{5afg(a+a\sin(e+fx))^{3}}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{6c(g\cos(e+}{5afg(a+a\sin(e+fx))^{3}}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{6c(g\cos(e+}{5afg(a+a\sin(e+fx))^{3}}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{6c(g\cos(e+}{5afg(a+a\sin(e+fx))^{3}}$$

Mathematica [C] time = 2.18137, size = 230, normalized size = 1.26

$$\frac{4ig\sqrt{ge^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}\left(e^{i(e+fx)}\left(e^{i(e+fx)}+i\right)^3 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)+\left(-4ie^{i(e+fx)}-3e^{2i(e+fx)}+5\right)\sqrt{1+e^{2i(e+fx)}}}{5af\left(e^{i(e+fx)}-i\right)\sqrt{1+e^{2i(e+fx)}}\left(-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (((4*I)/5)*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((5 - (4*I
)*E^(I*(e + f*x)) - 3*E^((2*I)*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))] +
E^(I*(e + f*x))*(I + E^(I*(e + f*x)))^3*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(e + f*x))])*Sqrt[c - c*Sin[e + f*x]])/(a*(-I + E^(I*(e + f*x)))*((
(-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*Sqrt[1 + E^((2*I)*(e
 + f*x))]*f)
```

Maple [C] time = 0.322, size = 2040, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/10/f*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^3*(1+sin(f*x+e))*(-c*(-1+sin(f*
x+e)))^(1/2)*(12*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)-12*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+24*I*sin(f*x+e)*cos(f*x+e)*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*sin(f*x
+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)-4*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-20*cos(f*x+
e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3-5*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3-5*ln(-2*(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)+5*cos(f*x+e)*ln(-(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+8*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+20*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+8*cos(f*x+e)^2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+e)+5*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-12*sin(f*x+e)*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-24*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-12*I*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+12*I*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4-12*I*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4+24*I*cos(f*x+e)^3*Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*cos(f*x+e)^3
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-12*I*(1/(co
s(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e))/(-1+si
n(f*x+e))/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(a*(1+sin(f*x+e)))^(5/2)/sin
(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g\cos\left(fx+e\right)}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g*cos(f*x + e)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 -
 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

3.147 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=179

$$-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2(g\cos}{5fg(a\sin(e+fx)}$$

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Si
n[e + f*x]]) - (2*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^(3/
2)*Sqrt[c - c*Sin[e + f*x]]) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Si
n[e + f*x]])
```

Rubi [A] time = 0.842057, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{2g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2(g\cos}{5fg(a\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f
*x]]),x]
```


```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Si
n[e + f*x]]) - (2*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^(3/
2)*Sqrt[c - c*Sin[e + f*x]]) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Si
n[e + f*x]])
```


Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

x]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}dx = -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(\cdots}}}{5a}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{2(g\cos(e+\cdots}{5afg(a+a\sin(e+fx))^{\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{2(g\cos(e+\cdots}{5afg(a+a\sin(e+fx))^{\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{2(g\cos(e+\cdots}{5afg(a+a\sin(e+fx))^{\cdots}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{2(g\cos(e+\cdots}{5afg(a+a\sin(e+fx))^{\cdots}}$$

Mathematica [A] time = 1.59954, size = 189, normalized size = 1.06

$$-\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(-4\sin^3\left(\frac{1}{2}(e\cdots\right.\right.\right.}{5f\cos^{\frac{3}{2}}(e+fx)(a(\sin(e+fx)+1)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin
[e + f*x]]),x]
```

```
[Out] -((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2*(2*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(3*Cos[(e + f*x)/2] + Cos[(3*(e
+ f*x))/2] - 4*Sin[(e + f*x)/2]^3)))/(5*f*Cos[e + f*x]^(3/2)*(a*(1 + Sin[e
+ f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.341, size = 778, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/5/f*(g*cos(f*x+e))^(3/2)*(sin(f*x+e)*cos(f*x+e)-sin(f*x+e)+cos(f*x+e)-1)*
(-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(
```

```
f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)+I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^
2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-I*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+
e),I)-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*
x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*sin(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-I*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)+I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^2+sin(f
*x+e)+2*cos(f*x+e)-1)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(a*(1+sin(f*x+e)))^(5/2
)/(-c*(-1+sin(f*x+e)))^(1/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{a^{3}c\cos\left(fx+e\right)^{3}-2\,a^{3}c\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}c\cos\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a^3*c*cos(f*x + e)^3 - 2*a^3*c*cos(f*x + e)*sin(f*x + e) - 2*a^3*
c*cos(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

3.148 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{6g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{6(g\cos(e+fx))^{5/2}}{5acfg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{6(g}{5cfg(a\sin(e+f}$$

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)) - (6*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*(a + a*Sin[e + f*x])^(5/2
)*Sqrt[c - c*Sin[e + f*x]]) - (6*(g*Cos[e + f*x])^(5/2))/(5*a*c*f*g*(a + a*
Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) - (6*g*Sqrt[Cos[e + f*x]]*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*c*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.16096, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{6g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{6(g\cos(e+fx))^{5/2}}{5acfg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{6(g}{5cfg(a\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)) - (6*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*(a + a*Sin[e + f*x])^(5/2
)*Sqrt[c - c*Sin[e + f*x]]) - (6*(g*Cos[e + f*x])^(5/2))/(5*a*c*f*g*(a + a*
Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) - (6*g*Sqrt[Cos[e + f*x]]*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*c*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} + \frac{3\int \frac{(g\cos(e+fx))^{3/}}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c}}}{c}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6(g\cos(e}{5cfg(a+a\sin(e+fx)}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6(g\cos(e}{5cfg(a+a\sin(e+fx)}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6(g\cos(e}{5cfg(a+a\sin(e+fx)}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6(g\cos(e}{5cfg(a+a\sin(e+fx)}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{6(g\cos(e}{5cfg(a+a\sin(e+fx)}$$

Mathematica [A] time = 1.12999, size = 133, normalized size = 0.56

$$\frac{\sqrt{\cos(e+fx)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}(-6\sin(e+fx)+3\cos(2(e+fx))-1)+3E\left(\frac{1}{2}(e+fx)\middle|2\right)(\sin(2(e+fx))}{5cf(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(Sqrt[Cos[e + f*x]]*(-1 + 3*Cos[
2*(e + f*x)] - 6*Sin[e + f*x]) + 3*EllipticE[(e + f*x)/2, 2]*(2*Cos[e + f*x
] + Sin[2*(e + f*x)])))/(5*c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(
5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.342, size = 877, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/5/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(1+sin(f*x+e))*(-1+cos(f*x+e))^
2*(-1+sin(f*x+e))*(3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+3*I*cos(f*x+e)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-3*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)+3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*cos(f*x+e)-3*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)+3*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*sin(
f*x+e)*cos(f*x+e)+3*cos(f*x+e)-2*sin(f*x+e)-3)/(a*(1+sin(f*x+e)))^(5/2)/(-c
*(-1+sin(f*x+e)))^(3/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{a^{3}c^{2}\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+a^{3}c^{2}\cos\left(fx+e\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g/(a^3*c^2*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*cos(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(3/2)), x)
```

3.149 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{6g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5a^2cfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{6(}{5a^2fg\sqrt{a\sin(e}}$$

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
 + f*x])^(5/2)) - (2*(g*Cos[e + f*x])^(5/2))/(a*f*g*(a + a*Sin[e + f*x])^(3/
2)*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt
[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2
))/(5*a^2*c*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (6*g
*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*
c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 1.47239, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$-\frac{6g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5a^2cfg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{6(}{5a^2fg\sqrt{a\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])
^(5/2)),x]
```


```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
 + f*x])^(5/2)) - (2*(g*Cos[e + f*x])^(5/2))/(a*f*g*(a + a*Sin[e + f*x])^(3/
2)*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt
[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2
))/(5*a^2*c*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (6*g
*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*
c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{(g\cos(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c}}{a}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{2(g\cos}{afg(a+a\sin(e+f}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{2(g\cos}{afg(a+a\sin(e+f}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{2(g\cos}{afg(a+a\sin(e+f}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{2(g\cos}{afg(a+a\sin(e+f}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{2(g\cos}{afg(a+a\sin(e+f}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{2(g\cos}{afg(a+a\sin(e+f}$$

Mathematica [A] time = 1.19474, size = 104, normalized size = 0.36

$$\frac{\sec^3(e+fx)(g\cos(e+fx))^{3/2}\left(7\sin(e+fx)+3\sin(3(e+fx))-12\cos^{\frac{5}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{10a^2c^2f\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(5/2)),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]^3*(-12*Cos[e + f*x]^(5/2)*EllipticE[(e
 + f*x)/2, 2] + 7*Sin[e + f*x] + 3*Sin[3*(e + f*x)]))/(10*a^2*c^2*f*Sqrt[a*
(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.346, size = 395, normalized size = 1.4

$$-\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)^2\left(1+\sin\left(fx+e\right)\right)\left(-1+\sin\left(fx+e\right)\right)}{5f\left(\sin\left(fx+e\right)\right)^5\cos\left(fx+e\right)}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(3i\sin\left(fx+e\right)\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/5/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e
))*(-1+sin(f*x+e))*(3*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3
*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*sin(f*x+e)*cos(f*x
+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)-3*cos(f*x+e)^3+2*cos(f*x+e)^2+1)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(-
1+sin(f*x+e)))^(5/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{a^{3}c^{3}\cos\left(fx+e\right)^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*g/(a^3*c^3*cos(f*x + e)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

3.150 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=350

$$\frac{14(g\cos(e+fx))^{5/2}}{15a^2c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{14g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15a^2c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{}{15a^2cfg\sqrt{a\sin}}$$

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(7/2)) - (14*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^
(3/2)*(c - c*Sin[e + f*x])^(7/2)) + (14*(g*Cos[e + f*x])^(5/2))/(9*a^2*f*g*
Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*(g*Cos[e + f*x])
^(5/2))/(15*a^2*c*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))
+ (14*(g*Cos[e + f*x])^(5/2))/(15*a^2*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c -
 c*Sin[e + f*x])^(3/2)) - (14*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(15*a^2*c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]])
```

Rubi [A] time = 1.79434, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2852, 2842, 2640, 2639}

$$\frac{14(g\cos(e+fx))^{5/2}}{15a^2c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{14g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15a^2c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{}{15a^2cfg\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])
^(7/2)),x]
```


```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(7/2)) - (14*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^
(3/2)*(c - c*Sin[e + f*x])^(7/2)) + (14*(g*Cos[e + f*x])^(5/2))/(9*a^2*f*g*
Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*(g*Cos[e + f*x])
^(5/2))/(15*a^2*c*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))
+ (14*(g*Cos[e + f*x])^(5/2))/(15*a^2*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c -
 c*Sin[e + f*x])^(3/2)) - (14*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(15*a^2*c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]])
```


Rule 2852

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 2842

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(g
```

```
*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} + \frac{7\int \frac{(g\cos(e+f}{(a+a\sin(e+fx))^{3/2}(c}}{5a}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{14(g\,c}{5afg(a+a\sin(e+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{14(g\,c}{5afg(a+a\sin(e+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{14(g\,c}{5afg(a+a\sin(e+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{14(g\,c}{5afg(a+a\sin(e+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{14(g\,c}{5afg(a+a\sin(e+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{14(g\,c}{5afg(a+a\sin(e+}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{14(g\,c}{5afg(a+a\sin(e+}$$

Mathematica [A] time = 6.16182, size = 171, normalized size = 0.49

$$-\frac{\sqrt{\cos(e+fx)}(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}(98\sin(e+fx)+42\sin(3(e+fx))+28\cos(2(e+fx))+21\cos(4(e+fx}{180c^3f(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{5/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(7/2)),x]
```

```
[Out] -(Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(42*EllipticE[(e + f*x)/2, 2]*(
-3*Cos[e + f*x] - Cos[3*(e + f*x)] + 4*Cos[e + f*x]^3*Sin[e + f*x]) + Sqrt[
Cos[e + f*x]]*(-9 + 28*Cos[2*(e + f*x)] + 21*Cos[4*(e + f*x)] + 98*Sin[e +
f*x] + 42*Sin[3*(e + f*x)])))/(180*c^3*f*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[
```

```
e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] time = 0.391, size = 947, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/45/f*(cos(f*x+e)+1)^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e
))*(-1+sin(f*x+e))*(21*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*Ellipt
icF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+21*I*cos(f*x+e)^5*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)-21*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)^5*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)-21*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+
e),I)+21*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*cos(f*x+e)^4-21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)-21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^3*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos
(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*sin(f
*x+e)*cos(f*x+e)^3+14*cos(f*x+e)^2*sin(f*x+e)+21*cos(f*x+e)^3-14*cos(f*x+e)
^2+2*sin(f*x+e)-7)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(-1+sin(f*x+e)))^(7/2)/cos(
f*x+e)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}g}{a^3c^4\cos\left(fx+e\right)^5\sin\left(fx+e\right)-a^3c^4\cos\left(fx+e\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)*g/(a^3*c^4*cos(f*x + e)^5*sin(f*x + e) - a^3*c^4*cos(f*x + e)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

3.151 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$\frac{c2^{n+\frac{9}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{-n-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(-4n-1);\frac{1}{4}(4\right.}{fg(4m+5)}$$

```
[Out] (2^(9/4 + n)*c*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (-1 -
4*n)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 - n)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*g*(5 + 4*m))
```

Rubi [A] time = 0.292605, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2853, 2689, 70, 69}

$$\frac{c2^{n+\frac{9}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{-n-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(-4n-1);\frac{1}{4}(4\right.}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(9/4 + n)*c*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (-1 -
4*n)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 - n)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*g*(5 + 4*m))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\sin(e+f}{}$$

$$= \frac{\Big(2^{\frac{1}{4}+n}c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\sin(e}{}$$

$$= \frac{2^{\frac{9}{4}+n}c(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\frac{1}{4}(-1-4n);\frac{1}{4}(9}{}$$

Mathematica [A] time = 2.1797, size = 126, normalized size = 1.06

$$-\frac{8g\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\sqrt{g\cos(e+fx)}(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n\csc^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{m+n+\frac{3}{2}}\,{}_2F_1\Big(n}{f(4n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^n,x]
```

```
[Out] (-8*g*Sqrt[g*Cos[e + f*x]]*Cos[(2*e + Pi + 2*f*x)/4]^2*(Csc[(2*e + Pi + 2*f
*x)/4]^2)^(3/2 + m + n)*Hypergeometric2F1[5/4 + n, 5/2 + m + n, 9/4 + n, -T
an[(2*e - Pi + 2*f*x)/4]^2]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n
)/(f*(5 + 4*n))
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
n*g*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] sage2
```

3.152 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=93

$$-\frac{a^4c^3 2^{m+\frac{9}{4}}(g\cos(e+fx))^{17/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{17}{4},-m-\frac{1}{4};\frac{21}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{17fg^7}$$

```
[Out] -(2^(9/4 + m)*a^4*c^3*(g*Cos[e + f*x])^(17/2)*Hypergeometric2F1[17/4, -1/4
- m, 21/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e
 + f*x])^(-4 + m))/(17*f*g^7)
```

Rubi [A] time = 0.284945, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^4c^3 2^{m+\frac{9}{4}}(g\cos(e+fx))^{17/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{17}{4},-m-\frac{1}{4};\frac{21}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{17fg^7}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -(2^(9/4 + m)*a^4*c^3*(g*Cos[e + f*x])^(17/2)*Hypergeometric2F1[17/4, -1/4
- m, 21/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e
 + f*x])^(-4 + m))/(17*f*g^7)
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx = \frac{\left(a^3c^3\right)\int (g\cos(e+fx))^{15/2}(a+a\sin(e+fx))^{-3+m}\,dx}{g^6}$$

$$= \frac{\left(a^5c^3(g\cos(e+fx))^{17/2}\right)\text{Subst}\left(\int (a-ax)^{13/4}(a+ax\ldots\right.}{fg^7(a-a\sin(e+fx))^{17/4}(a+a\sin(e\ldots}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^5c^3(g\cos(e+fx))^{17/2}(a+a\sin(e+fx))^{-4+m}\ldots\right.}{fg\ldots}$$

$$= -\frac{2^{\frac{9}{4}+m}a^4c^3(g\cos(e+fx))^{17/2}\,{}_2F_1\left(\frac{17}{4},-\frac{1}{4}-m;\frac{21}{4};\frac{1}{2}(\ldots\right.}{\ldots}$$

Mathematica [F] time = 180.007, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^3,x]

[Out] $Aborted
```

Maple [F] time = 0.603, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)

[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)-c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) +
a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,c^3g\cos\left(fx+e\right)^3-4\,c^3g\cos\left(fx+e\right)-\left(c^3g\cos\left(fx+e\right)^3-4\,c^3g\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] integral(-(3*c^3*g*cos(f*x + e)^3 - 4*c^3*g*cos(f*x + e) - (c^3*g*cos(f*x +
 e)^3 - 4*c^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*(a*sin(f*x
 + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] sage2
```

3.153 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=93

$$-\frac{a^3c^2 2^{m+\frac{9}{4}}(g\cos(e+fx))^{13/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{13}{4},-m-\frac{1}{4};\frac{17}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{13fg^5}$$

```
[Out] -(2^(9/4 + m)*a^3*c^2*(g*Cos[e + f*x])^(13/2)*Hypergeometric2F1[13/4, -1/4
- m, 17/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e
 + f*x])^(-3 + m))/(13*f*g^5)
```

Rubi [A] time = 0.275592, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^3c^2 2^{m+\frac{9}{4}}(g\cos(e+fx))^{13/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{13}{4},-m-\frac{1}{4};\frac{17}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{13fg^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -(2^(9/4 + m)*a^3*c^2*(g*Cos[e + f*x])^(13/2)*Hypergeometric2F1[13/4, -1/4
- m, 17/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e
 + f*x])^(-3 + m))/(13*f*g^5)
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx = \frac{\left(a^2c^2\right)\int (g\cos(e+fx))^{11/2}(a+a\sin(e+fx))^{-2+m}\,dx}{g^4}$$

$$= \frac{\left(a^4c^2(g\cos(e+fx))^{13/2}\right)\text{Subst}\left(\int (a-ax)^{9/4}(a+ax)^{\frac{1}{4}+}\right.}{fg^5(a-a\sin(e+fx))^{13/4}(a+a\sin(e+f}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^4c^2(g\cos(e+fx))^{13/2}(a+a\sin(e+fx))^{-3+m}\left(\frac{a+}{}\right.\right.}{fg^5(a}$$

$$= -\frac{2^{\frac{9}{4}+m}a^3c^2(g\cos(e+fx))^{13/2}\,{}_2F_1\left(\frac{13}{4},-\frac{1}{4}-m;\frac{17}{4};\frac{1}{2}(1-\right.}{1}$$

Mathematica [F] time = 180.003, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^2,x]

[Out] $Aborted
```

Maple [F] time = 0.512, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)

[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)-c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a
)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c^{2} g \cos\left(fx+e\right)^{3}+2\, c^{2} g \cos\left(fx+e\right) \sin\left(fx+e\right)-2\, c^{2} g \cos\left(fx+e\right)\right) \sqrt{g \cos\left(fx+e\right)} \left(a \sin\left(fx+e\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.154 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=91

$$-\frac{a^2c2^{m+\frac{9}{4}}(g\cos(e+fx))^{9/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{9}{4},-m-\frac{1}{4};\frac{13}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{9fg^3}$$

```
[Out] -(2^(9/4 + m)*a^2*c*(g*Cos[e + f*x])^(9/2)*Hypergeometric2F1[9/4, -1/4 - m,
 13/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f
*x])^(-2 + m))/(9*f*g^3)
```

Rubi [A] time = 0.207681, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2840, 2689, 70, 69}

$$-\frac{a^2c2^{m+\frac{9}{4}}(g\cos(e+fx))^{9/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{9}{4},-m-\frac{1}{4};\frac{13}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{9fg^3}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(2^(9/4 + m)*a^2*c*(g*Cos[e + f*x])^(9/2)*Hypergeometric2F1[9/4, -1/4 - m,
 13/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f
*x])^(-2 + m))/(9*f*g^3)
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx = \frac{(ac)\int (g\cos(e+fx))^{7/2}(a+a\sin(e+fx))^{-1+m}\,dx}{g^2}$$

$$= \frac{\left(a^3c(g\cos(e+fx))^{9/2}\right)\operatorname{Subst}\left(\int (a-ax)^{5/4}(a+ax)^{\frac{1}{4}+}\right.}{fg^3(a-a\sin(e+fx))^{9/4}(a+a\sin(e+f}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^3c(g\cos(e+fx))^{9/2}(a+a\sin(e+fx))^{-2+m}\left(\frac{a+}{}\right.\right.}{fg^3(}$$

$$= -\frac{2^{\frac{9}{4}+m}a^2c(g\cos(e+fx))^{9/2}\,{}_2F_1\left(\frac{9}{4},-\frac{1}{4}-m;\frac{13}{4};\frac{1}{2}(1-\right.}{}$$

Mathematica [F] time = 168.115, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
), x]
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)
^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] sage2
```

3.155 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{a2^{m+\frac{9}{4}}(g\cos(e+fx))^{5/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5fg}$$

```
[Out] -(2^(9/4 + m)*a*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9/4
, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^
(-1 + m))/(5*f*g)
```

Rubi [A] time = 0.0855542, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}(g\cos(e+fx))^{5/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(2^(9/4 + m)*a*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9/4
, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^
(-1 + m))/(5*f*g)
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2(g\cos(e+fx))^{5/2}\right)\text{Subst}\left(\int \sqrt[4]{a-ax}(a+ax)^{\frac{1}{4}+m}\,dx, x, \sin(e+fx)\right)}{fg(a-a\sin(e+fx))^{5/4}(a+a\sin(e+fx))^{5/4}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{-1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{4}-m}\right)\text{Subs}}{fg(a-a\sin(e+fx))^{5/4}}$$

$$= -\frac{2^{\frac{9}{4}+m}a(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e-}{5fg}$$

Mathematica [A] time = 0.155589, size = 85, normalized size = 0.97

$$-\frac{2^{m+\frac{9}{4}}(g\cos(e+fx))^{5/2}(\sin(e+fx)+1)^{-m-\frac{5}{4}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5fg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(2^(9/4 + m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9/4,
(1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-5/4 - m)*(a*(1 + Sin[e + f*x]))^
m)/(5*f*g)
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^m g\cos\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.156 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{4},-m-\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

```
[Out] -((2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[1/4, -1/4 - m, 5/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^m
)/(c*f))
```

Rubi [A] time = 0.274795, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$-\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{4},-m-\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[1/4, -1/4 - m, 5/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^m
)/(c*f))
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx = \frac{g^2\int \frac{(a+a\sin(e+fx))^{1+m}}{\sqrt{g\cos(e+fx)}}\,dx}{ac}$$

$$= \frac{\left(ag\sqrt{g\cos(e+fx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{3/4}}\,dx, x, \sin(e+fx)\right)}{cf\sqrt[4]{a-a\sin(e+fx)}\sqrt[4]{a+a\sin(e+fx)}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}ag\sqrt{g\cos(e+fx)}(a+a\sin(e+fx))^m\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{4}-m}\right)\text{Subst}}{cf\sqrt[4]{a-a\sin(e+fx)}}$$

$$= -\frac{2^{\frac{9}{4}+m}g\sqrt{g\cos(e+fx)}\,{}_2F_1\left(\frac{1}{4},-\frac{1}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(}{cf}$$

Mathematica [A] time = 0.11968, size = 84, normalized size = 1.

$$-\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{4},-m-\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x]),x]
```

```
[Out] -((2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[1/4, -1/4 - m, 5/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a*(1 + Sin[e + f*x]))
^m)/(c*f))
```

Maple [F] time = 0.253, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{c-c\sin\left(fx+e\right)}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(c*sin
(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c), x)
```

3.157 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=93

$$\frac{g^3 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{4},-m-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f(g\cos(e+fx))^{3/2}}$$

```
[Out] (2^(9/4 + m)*g^3*Hypergeometric2F1[-3/4, -1/4 - m, 1/4, (1 - Sin[e + f*x])/
2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3*a*c^2*f*(
g*Cos[e + f*x])^(3/2))
```

Rubi [A] time = 0.280569, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$\frac{g^3 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{4},-m-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^2,
x]
```

```
[Out] (2^(9/4 + m)*g^3*Hypergeometric2F1[-3/4, -1/4 - m, 1/4, (1 - Sin[e + f*x])/
2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3*a*c^2*f*(
g*Cos[e + f*x])^(3/2))
```

Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx = \frac{g^4\int \frac{(a+a\sin(e+fx))^{2+m}}{(g\cos(e+fx))^{5/2}}\,dx}{a^2c^2}$$

$$= \frac{\left(g^3(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{3/4}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{7/4}}\,dx,x,\text{si}\right.}{c^2f(g\cos(e+fx))^{3/2}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}g^3(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{4}-m}\right)\text{Su}}{c^2f(g\cos(e+fx))^{3/2}}$$

$$= \frac{2^{\frac{9}{4}+m}g^3\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+}{3ac^2f(g\cos(e+fx))^{3/2}}$$

Mathematica [A] time = 0.202824, size = 96, normalized size = 1.03

$$-\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(-\frac{3}{4},-m-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3c^2f(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^2,x]
```

```
[Out] -(2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[-3/4, -1/4 - m, 1/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a*(1 + Sin[e + f*x]))
^m)/(3*c^2*f*(-1 + Sin[e + f*x]))
```

Maple [F] time = 0.453, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^2}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m}{\left(c\sin\left(fx+e\right)-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(c^2*c
os(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
)^2, x)
```

3.158 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{g^5 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{7}{4},-m-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{7a^2c^3f(g\cos(e+fx))^{7/2}}$$

```
[Out] (2^(9/4 + m)*g^5*Hypergeometric2F1[-7/4, -1/4 - m, -3/4, (1 - Sin[e + f*x])
/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(2 + m))/(7*a^2*c^3*
f*(g*Cos[e + f*x])^(7/2))
```

Rubi [A] time = 0.282566, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2840, 2689, 70, 69}

$$\frac{g^5 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{7}{4},-m-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{7a^2c^3f(g\cos(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^3,
x]
```


```
[Out] (2^(9/4 + m)*g^5*Hypergeometric2F1[-7/4, -1/4 - m, -3/4, (1 - Sin[e + f*x])
/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(2 + m))/(7*a^2*c^3*
f*(g*Cos[e + f*x])^(7/2))
```


Rule 2840

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
(a^m*c^m)/g^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
```

```
a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx = \frac{g^6\int \frac{(a+a\sin(e+fx))^{3+m}}{(g\cos(e+fx))^{9/2}}\,dx}{a^3c^3}$$

$$= \frac{\left(g^5(a-a\sin(e+fx))^{7/4}(a+a\sin(e+fx))^{7/4}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{11/4}}\,dx, x\right.}{ac^3f(g\cos(e+fx))^{7/2}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}g^5(a-a\sin(e+fx))^{7/4}(a+a\sin(e+fx))^{2+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{4}-m}\right.}{ac^3f(g\cos(e+fx))^{7/2}}$$

$$= \frac{2^{\frac{9}{4}+m}g^5\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4}-m;-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}}{7a^2c^3f(g\cos(e+fx))^{7/2}}$$

Mathematica [A] time = 0.206877, size = 96, normalized size = 1.03

$$\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(-\frac{7}{4},-m-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{7c^3f(\sin(e+fx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^3,x]
```

```
[Out] (2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[-7/4, -1/4 - m, -3/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a*(1 + Sin[e + f*x]))
^m)/(7*c^3*f*(-1 + Sin[e + f*x])^2)
```

Maple [F] time = 0.533, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^3}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m}{\left(c\sin\left(fx+e\right)-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(3*c^3
*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c)^3, x)
```

3.159 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=114

$$-\frac{a^3c^2 2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{15/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{15}{4},-m-\right.}{15fg^6}$$

```
[Out] -(2^(9/4 + m)*a^3*c^2*(g*Cos[e + f*x])^(15/2)*Hypergeometric2F1[15/4, -1/4
- m, 19/4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)
*(a + a*Sin[e + f*x])^(-3 + m)*Sqrt[c - c*Sin[e + f*x]])/(15*f*g^6)
```

Rubi [A] time = 0.356974, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 2689, 70, 69}

$$-\frac{a^3c^2 2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{15/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{15}{4},-m-\right.}{15fg^6}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2
),x]
```


```
[Out] -(2^(9/4 + m)*a^3*c^2*(g*Cos[e + f*x])^(15/2)*Hypergeometric2F1[15/4, -1/4
- m, 19/4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)
*(a + a*Sin[e + f*x])^(-3 + m)*Sqrt[c - c*Sin[e + f*x]])/(15*f*g^6)
```


Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx = \frac{\left(a^2c^2\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\right.}{g^5}$$

$$= \frac{\left(a^4c^2(g\cos(e+fx))^{15/2}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\right)}{fg^6(a-a\sin(e+fx))^{15/4}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^4c^2(g\cos(e+fx))^{15/2}\sec(e+fx)(a+a\sin(e+\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a^3c^2(g\cos(e+fx))^{15/2}\,{}_2F_1\left(\frac{15}{4},-\frac{1}{4}-m;\frac{19}{4};\frac{1}{2}(1-\right.}{}$$

Mathematica [F] time = 180.037, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e
) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(c^{2} g \cos\left(fx+e\right)^{3}+2\,c^{2} g \cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^{2} g \cos\left(fx+e\right)\right)\sqrt{g \cos\left(fx+e\right)}\sqrt{-c \sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x
+ e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.160 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=112

$$-\frac{a^2c2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{11/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{11}{4},-m-\frac{1}{4};\frac{1}{4}\right.}{11fg^4}$$

```
[Out] -(2^(9/4 + m)*a^2*c*(g*Cos[e + f*x])^(11/2)*Hypergeometric2F1[11/4, -1/4 -
m, 15/4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)*(
a + a*Sin[e + f*x])^(-2 + m)*Sqrt[c - c*Sin[e + f*x]])/(11*f*g^4)
```

Rubi [A] time = 0.350661, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 2689, 70, 69}

$$-\frac{a^2c2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{11/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{11}{4},-m-\frac{1}{4};\frac{1}{4}\right.}{11fg^4}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2
),x]
```


```
[Out] -(2^(9/4 + m)*a^2*c*(g*Cos[e + f*x])^(11/2)*Hypergeometric2F1[11/4, -1/4 -
m, 15/4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)*(
a + a*Sin[e + f*x])^(-2 + m)*Sqrt[c - c*Sin[e + f*x]])/(11*f*g^4)
```


Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx = \frac{\left(ac\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx}\right.}{g^3}$$

$$= \frac{\left(a^3c(g\cos(e+fx))^{11/2}\sec(e+fx)\sqrt{c-c\sin(e+f\right.}{fg^4(a-a\sin(e+fx))}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^3c(g\cos(e+fx))^{11/2}\sec(e+fx)(a+a\sin(e\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a^2c(g\cos(e+fx))^{11/2}\,{}_2F_1\left(\frac{11}{4},-\frac{1}{4}-m;\frac{15}{4};\frac{1}{2}(\right.}{}$$

Mathematica [F] time = 180.004, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(3/2),x]

[Out] $Aborted
```

Maple [F] time = 0.244, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)

[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e
) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(cg\cos\left(fx+e\right)\sin\left(fx+e\right)-cg\cos\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e
) + a)^m, x)
```

3.161 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=109

$$-\frac{a2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{7/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\frac{1}{4};\frac{11}{4}\right.}{7fg^2}$$

```
[Out] -(2^(9/4 + m)*a*(g*Cos[e + f*x])^(7/2)*Hypergeometric2F1[7/4, -1/4 - m, 11/
4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*
Sin[e + f*x])^(-1 + m)*Sqrt[c - c*Sin[e + f*x]])/(7*f*g^2)
```

Rubi [A] time = 0.318776, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{7/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\frac{1}{4};\frac{11}{4}\right.}{7fg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] -(2^(9/4 + m)*a*(g*Cos[e + f*x])^(7/2)*Hypergeometric2F1[7/4, -1/4 - m, 11/
4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*
Sin[e + f*x])^(-1 + m)*Sqrt[c - c*Sin[e + f*x]])/(7*f*g^2)
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx = \frac{\left(\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\right)\int (g\,c}{g}$$

$$= \frac{\left(a^2(g\cos(e+fx))^{7/2}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\right)\text{Sub}}{fg^2(a-a\sin(e+fx))^{7/4}(a-}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2(g\cos(e+fx))^{7/2}\sec(e+fx)(a+a\sin(e+fx))\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a(g\cos(e+fx))^{7/2}\,{}_2F_1\left(\frac{7}{4},-\frac{1}{4}-m;\frac{11}{4};\frac{1}{2}(1-\sin(e\right.}{}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.233, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\sqrt{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e)
+ a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m*g*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.162 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -(2^(9/4 + m)*a*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
-1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*S
in[e + f*x])^(-1 + m))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.325717, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -(2^(9/4 + m)*a*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
-1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*S
in[e + f*x])^(-1 + m))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int \sqrt{g\cos(e+fx)}(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(g\cos(e+fx))^{3/2}\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{\sqrt[4]{a-ax}}\,dx, x, \sin(e+fx)\right.}{f(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{5/4}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2\cos(e+fx)(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{-1+m}\left(\frac{a+a\sin(e+}{a}\right.\right.}{f(a-a\sin(e+fx))^{3/4}\sqrt{c-c\,\mathrm{si}}}$$

$$= -\frac{2^{\frac{9}{4}+m}a\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(e+\right.}{3f\sqrt{c-c\sin(e+fx}}$$

Mathematica [F] time = 180.097, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

3.163 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{g^2 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{4},-m-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf\sqrt{c-c\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

```
[Out] (2^(9/4 + m)*g^2*Cos[e + f*x]*Hypergeometric2F1[-1/4, -1/4 - m, 3/4, (1 - S
in[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^m)/(c*f*
Sqrt[g*Cos[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.366558, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 2689, 70, 69}

$$\frac{g^2 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{4},-m-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf\sqrt{c-c\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(3
/2),x]
```


```
[Out] (2^(9/4 + m)*g^2*Cos[e + f*x]*Hypergeometric2F1[-1/4, -1/4 - m, 3/4, (1 - S
in[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^m)/(c*f*
Sqrt[g*Cos[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(g^3\cos(e+fx)\right)\int \frac{(a+a\sin(e+fx))^{\frac{3}{2}+m}}{(g\cos(e+fx))^{3/2}}\,dx}{ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(ag^2\cos(e+fx)\sqrt[4]{a-a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{5/4}}\,dx,x,\sin(e+fx)\right.}{cf\sqrt{g\cos(e+fx)}\sqrt[4]{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}ag^2\cos(e+fx)\sqrt[4]{a-a\sin(e+fx)}(a+a\sin(e+fx))^m\left(\frac{a+a\sin(e+fx)}{a}\right.\right.}{cf\sqrt{g\cos(e+fx)}\sqrt{c-c\sin(e+}}$$

$$= \frac{2^{\frac{9}{4}+m}g^2\cos(e+fx)\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+f}{cf\sqrt{g\cos(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [F] time = 81.2422, size = 0, normalized size = 0.

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^(3/2), x]
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c-c\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(3/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^(3/2), x)
```

3.164 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{g^4 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{5}{4},-m-\frac{1}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5ac^2f\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}$$

```
[Out] (2^(9/4 + m)*g^4*Cos[e + f*x]*Hypergeometric2F1[-5/4, -1/4 - m, -1/4, (1 -
Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(1 + m)
)/(5*a*c^2*f*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.357219, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 2689, 70, 69}

$$\frac{g^4 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{5}{4},-m-\frac{1}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5ac^2f\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (2^(9/4 + m)*g^4*Cos[e + f*x]*Hypergeometric2F1[-5/4, -1/4 - m, -1/4, (1 -
Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(1 + m)
)/(5*a*c^2*f*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(g^5\cos(e+fx)\right)\int \frac{(a+a\sin(e+fx))^{\frac{5}{2}+m}}{(g\cos(e+fx))^{7/2}}\,dx}{a^2c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(g^4\cos(e+fx)(a-a\sin(e+fx))^{5/4}(a+a\sin(e+fx))^{3/4}\right)\text{Subst}\left(\int \frac{(a\cdots}{(a\cdots}\right.}{c^2f(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}g^4\cos(e+fx)(a-a\sin(e+fx))^{5/4}(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin\cdots}{a\cdots}\right.\right.}{c^2f(g\cos(e+fx))^{5/2}\sqrt{c-c\sin\cdots}}$$

$$= \frac{2^{\frac{9}{4}+m}g^4\cos(e+fx)\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4}-m;-\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e\cdots}{5ac^2f(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+f\cdots}}$$

Mathematica [C] time = 52.3, size = 3845, normalized size = 33.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] -(((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a + a*Si
n[e + f*x])^m*((-5*Cos[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*Sqrt[Cos[e + f*x]]*(
-1 + Cot[(-e + Pi/2 - f*x)/4]^2)*(-3*AppellF1[1/4, -1/2 - 2*m, 2*m, 5/4, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[-3/4, -1/
2 - 2*m, 2*m, 1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
*Cot[(-e + Pi/2 - f*x)/4]^2)*Csc[(-e + Pi/2 - f*x)/4]^2)/(12*Sqrt[2]*(8*m*A
ppellF1[5/4, -1/2 - 2*m, 1 + 2*m, 9/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] + (2 + 8*m)*AppellF1[5/4, 1/2 - 2*m, 2*m, 9/4, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*Cot[(-e + Pi/2 - f*x)
/4]^2*(AppellF1[1/4, -1/2 - 2*m, 2*m, 5/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2] - 8*m*AppellF1[1/4, -1/2 - 2*m, 1 + 2*m, 5/4, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 2*AppellF1[1/4, 1/2
- 2*m, 2*m, 5/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] -
 8*m*AppellF1[1/4, 1/2 - 2*m, 2*m, 5/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + AppellF1[-3/4, -1/2 - 2*m, 2*m, 1/4, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2))) -
(10*Sqrt[2]*Cos[(-e + Pi/2 - f*x)/4]^2*Cos[(-e + Pi/2 - f*x)/2]^(-2 + 2*m)*
Cos[e + f*x]^(3/2)*(-1 + Cot[(-e + Pi/2 - f*x)/4]^2)*(-3*AppellF1[1/4, -1/2
 - 2*m, 2*m, 5/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
+ AppellF1[-3/4, -1/2 - 2*m, 2*m, 1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2)*Csc[(-e + Pi/2 - f*x)/2]^3
*Sin[(-e + Pi/2 - f*x)/4]^4)/(3*(40*AppellF1[-3/4, -1/2 - 2*m, 2*m, 1/4, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x
```

```
)/4]^4 + 4*Sin[(-e + Pi/2 - f*x)/4]^2*(5*AppellF1[1/4, -1/2 - 2*m, 2*m, 5/4
, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(1 + Cos[(-e + P
i/2 - f*x)/2]) - 2*(10*(4*m*AppellF1[1/4, -1/2 - 2*m, 1 + 2*m, 5/4, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + (1 + 4*m)*AppellF1[1/4,
 1/2 - 2*m, 2*m, 5/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2])*Cos[(-e + Pi/2 - f*x)/4]^2 + 8*m*AppellF1[5/4, -1/2 - 2*m, 1 + 2*m, 9/
4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2
- f*x)/4]^2 + 2*(1 + 4*m)*AppellF1[5/4, 1/2 - 2*m, 2*m, 9/4, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2)))) -
 ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*Cos
[e + f*x]*(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2
, -Tan[(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x
)/4]^4)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4])/(6
0*Sqrt[2]*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])
^4*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2]*(-((Cos[(-e + Pi/2 - f*x)/4]^2)^(
2*m)*Sqrt[Cos[e + f*x]]*(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m,
 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-
e + Pi/2 - f*x)/4]^4)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2 + 2*m)*Tan[(-e + Pi/2
 - f*x)/4]^2)/(240*(2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2)^(3/2)) - ((Cos[(-e +
Pi/2 - f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x]]*(5*AppellF1[3/4, -1/2 - 2*m, 2*m
, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 3*AppellF
1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
- f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2 +
2*m))/(480*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2]) + (m*(Cos[(-e + Pi/2 - f
*x)/4]^2)^(-1 + 2*m)*Sqrt[Cos[e + f*x]]*(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7
/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[-
5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m
)*Sin[(-e + Pi/2 - f*x)/4]^2)/(120*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2])
- ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4
, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/
4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*
Sin[e + f*x]*Tan[(-e + Pi/2 - f*x)/4])/(240*Sqrt[Cos[e + f*x]]*Sqrt[2 - 2*T
an[(-e + Pi/2 - f*x)/4]^2]) - ((1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)
*Sqrt[Cos[e + f*x]]*(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m
, -1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e +
Pi/2 - f*x)/4]^4)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f
*x)/4]^2)/(240*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2]) - ((Cos[(-e + Pi/2 -
 f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x]]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)
*Tan[(-e + Pi/2 - f*x)/4]*(-3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^
3*Csc[(-e + Pi/2 - f*x)/4]^2 + 3*Cot[(-e + Pi/2 - f*x)/4]^4*(-5*m*AppellF1[
-1/4, -1/2 - 2*m, 1 + 2*m, 3/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + (5*(-1/
2 - 2*m)*AppellF1[-1/4, 1/2 - 2*m, 2*m, 3/4, Tan[(-e + Pi/2 - f*x)/4]^2, -T
an[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)
/4])/2) + 5*((-3*m*AppellF1[7/4, -1/2 - 2*m, 1 + 2*m, 11/4, Tan[(-e + Pi/2
- f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-
e + Pi/2 - f*x)/4])/7 + (3*(-1/2 - 2*m)*AppellF1[7/4, 1/2 - 2*m, 2*m, 11/4,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 -
f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/14)))/(120*Sqrt[2 - 2*Tan[(-e + Pi/2 -
f*x)/4]^2])))))/(f*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[e + f*x]^(3/2)*(c - c
*Sin[e + f*x])^(5/2)))
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c-c\sin\left(fx+e\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4
*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^(5/2), x)
```

3.165 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+fx)\right.}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -(2^(9/4 + m)*a*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
-1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*S
in[e + f*x])^(-1 + m))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.310918, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+fx)\right.}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]
],x]
```



```
[Out] -(2^(9/4 + m)*a*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
-1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*S
in[e + f*x])^(-1 + m))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int\sqrt{g\cos(e+fx)}(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(g\cos(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{(a+ax)^{\frac{1}{4}+m}}{\sqrt[4]{a-ax}}\,dx,x,\sin(e+fx)\right)}{f(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{5/4}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2\cos(e+fx)(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{-1+m}\left(\frac{a+a\sin(e+fx)}{a}\right.\right.}{f(a-a\sin(e+fx))^{3/4}\sqrt{c-c\sin(e}}$$

$$= -\frac{2^{\frac{9}{4}+m}a\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(e+fx)\right.}{3f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 13.4247, size = 832, normalized size = 7.85

$$-\frac{}{3f\left(616F_1\left(\frac{3}{4};-2m-\frac{1}{2},2m+3;\frac{7}{4};\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right),-\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\right)\cos^4\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)-4\sin^2\left(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (-88*Sqrt[2]*Cos[(-e + Pi/2 - f*x)/4]^6*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)
*Csc[(-e + Pi/2 - f*x)/2]*Sec[(-e + Pi/2 - f*x)/2]^2*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])*(a + a*Sin[e + f*x])^m*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(-
7*AppellF1[3/4, -1/2 - 2*m, 3 + 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[7/4, -1/2 - 2*m, 3 + 2*m, 11/4, Tan[(-
e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]
^2))/(3*f*(616*AppellF1[3/4, -1/2 - 2*m, 3 + 2*m, 7/4, Tan[(-e + Pi/2 - f*x
)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^4 - 4*Sin[(-e
 + Pi/2 - f*x)/4]^2*(88*(3 + 2*m)*AppellF1[7/4, -1/2 - 2*m, 4 + 2*m, 11/4,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f
*x)/4]^2 + 44*(1 + 4*m)*AppellF1[7/4, 1/2 - 2*m, 3 + 2*m, 11/4, Tan[(-e + P
i/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 +
77*AppellF1[7/4, -1/2 - 2*m, 3 + 2*m, 11/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Ta
n[(-e + Pi/2 - f*x)/4]^2]*(1 + Cos[(-e + Pi/2 - f*x)/2]) - 28*((6 + 4*m)*Ap
pellF1[11/4, -1/2 - 2*m, 4 + 2*m, 15/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + (1 + 4*m)*AppellF1[11/4, 1/2 - 2*m, 3 + 2*m, 15/4,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sin[(-e + Pi/2 -
f*x)/4]^2))*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}} \frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}} \left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m} g\cos\left(fx+e\right)}{c\sin\left(fx+e\right)-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

3.166 $\int \frac{(g\cos(e+fx))^{3/2}(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{c2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(c\sin(e+fx)+c)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3f\sqrt{a-a\sin(e+fx)}}$$

```
[Out] -(2^(9/4 + m)*c*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
-1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(c + c*S
in[e + f*x])^(-1 + m))/(3*f*Sqrt[a - a*Sin[e + f*x]])
```

Rubi [A] time = 0.313294, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 2689, 70, 69}

$$-\frac{c2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(c\sin(e+fx)+c)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3f\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]
],x]
```

```
[Out] -(2^(9/4 + m)*c*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
-1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(c + c*S
in[e + f*x])^(-1 + m))/(3*f*Sqrt[a - a*Sin[e + f*x]])
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int\sqrt{g\cos(e+fx)}(c+c\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\left(c^2\cos(e+fx)(g\cos(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{(c+cx)^{\frac{1}{4}+m}}{\sqrt[4]{c-cx}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}(c-c\sin(e+fx))^{3/4}(c+c\sin(e+fx))^{5/4}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}c^2\cos(e+fx)(g\cos(e+fx))^{3/2}(c+c\sin(e+fx))^{-1+m}\left(\frac{c+c\sin(e+fx)}{c}\right)\right.}{f\sqrt{a-a\sin(e+fx)}(c-c\sin(e+f}$$

$$= -\frac{2^{\frac{9}{4}+m}c\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right.}{3f\sqrt{a-a\sin(e+fx)}}$$

Mathematica [F] time = 180.135, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e
+ f*x]],x]

[Out] $Aborted
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int\left(c+c\sin\left(fx+e\right)\right)^m\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt{a-a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)

[Out] int((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)+c\right)^m}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e
) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{m}g\cos\left(fx+e\right)}{a\sin\left(fx+e\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) +
c)^m*g*cos(f*x + e)/(a*sin(f*x + e) - a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(c\sin\left(fx+e\right)+c\right)^{m}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e
) + a), x)
```

3.167 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=123

$$\frac{2^{-m-\frac{3}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+11);\frac{1}{4}(4m\right.}{c^2fg(4m+5)}$$

```
[Out] (2^(-3/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (11 + 4
*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a
+ a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^2*f*g*(5 + 4*m))
```

Rubi [A] time = 0.369605, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{2^{-m-\frac{3}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+11);\frac{1}{4}(4m\right.}{c^2fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3
- m),x]
```


```
[Out] (2^(-3/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (11 + 4
*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a
+ a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^2*f*g*(5 + 4*m))
```


Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\sin\right.}{}$$

$$= \frac{\left(2^{-\frac{11}{4}-m}(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-\right.}{}$$

$$= \frac{2^{-\frac{3}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\frac{1}{4}(11+4\right.}{}$$

Mathematica [B] time = 17.7088, size = 382, normalized size = 3.11

$$\frac{2^{-m-4}\sec^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\sec(e+fx)(g\cos(e+fx))^{3/2}\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\left(1-\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\right)^{-2m}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-3 - m),x]
```

```
[Out] (2^(-4 - m)*(g*Cos[e + f*x])^(3/2)*((-3 + 8*m + 16*m^2)*Cot[(-e + Pi/2 - f*
x)/4]^4*Hypergeometric2F1[-3/2 - 2*m, -7/4 - m, -3/4 - m, Tan[(-e + Pi/2 -
f*x)/4]^2] + (7 + 4*m)*(2*(-1 + 4*m)*Cot[(-e + Pi/2 - f*x)/4]^2*Hypergeomet
ric2F1[-3/2 - 2*m, -3/4 - m, 1/4 - m, Tan[(-e + Pi/2 - f*x)/4]^2] + (3 + 4*
m)*Hypergeometric2F1[-3/2 - 2*m, 1/4 - m, 5/4 - m, Tan[(-e + Pi/2 - f*x)/4]
^2]))*Sec[(-e + Pi/2 - f*x)/4]^2*Sec[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c
*Sin[e + f*x])^(-3 - m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1/2 - 2*m))/(f*(
-1 + 4*m)*(3 + 4*m)*(7 + 4*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))
```

Maple [F] time = 0.385, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}} \left(a\sin\left(fx+e\right)+a\right)^{m} \left(-c\sin\left(fx+e\right)+c\right)^{-m-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x
, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 3)*g*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-3-m
),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x
, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.168 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=123

$$\frac{2^{\frac{1}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+7);\frac{1}{4}(4$$
$$cfg(4m+5)$$

```
[Out] (2^(1/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (7 + 4*m
)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c*f*g*(5 + 4*m))
```

Rubi [A] time = 0.36663, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{2^{\frac{1}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+7);\frac{1}{4}(4$$
$$cfg(4m+5)$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2
- m),x]
```

```
[Out] (2^(1/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (7 + 4*m
)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c*f*g*(5 + 4*m))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\sin(e\right.}{}$$

$$= \frac{\left(2^{-\frac{7}{4}-m}(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\,\mathrm{si}\right.}{}$$

$$= \frac{2^{\frac{1}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\frac{1}{4}(7+4m);\frac{1}{4}\right.}{}$$

Mathematica [A] time = 6.16755, size = 202, normalized size = 1.64

$$\frac{g2^{-m-1}\csc^2\left(\frac{1}{8}(-2e-2fx+\pi)\right)\sqrt{g\cos(e+fx)}\cos^{-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(1-\tan^2\left(\frac{1}{8}(2e+2fx-\pi)\right)\right)^{-2m-\frac{1}{2}}(a(\sin(e+}{c^2f(4m+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-2 - m),x]
```

```
[Out] (2^(-1 - m)*g*Sqrt[g*Cos[e + f*x]]*Csc[(-2*e + Pi - 2*f*x)/8]^2*Hypergeomet
ric2F1[-3/2 - 2*m, -3/4 - m, 1/4 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^(2*(2 + m))*(a*(1 + Sin[e + f*x]))^m*(1 - Ta
n[(2*e - Pi + 2*f*x)/8]^2)^(-1/2 - 2*m))/(c^2*f*(3 + 4*m)*Cos[(2*e + Pi + 2
*f*x)/4]^(2*m)*(-1 + Sin[e + f*x])^2*(c - c*Sin[e + f*x])^m)
```

Maple [F] time = 0.338, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x
, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 2)*g*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-2-m
),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x
, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.169 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=120

$$\frac{2^{\frac{5}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+3),\frac{1}{4}(4m+5);\frac{1}{4}(4m+\right.}{fg(4m+5)}$$

```
[Out] (2^(5/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(3 + 4*m)/4, (5 + 4*m
)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rubi [A] time = 0.359251, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{2^{\frac{5}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+3),\frac{1}{4}(4m+5);\frac{1}{4}(4m+\right.}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1
- m),x]
```

```
[Out] (2^(5/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(3 + 4*m)/4, (5 + 4*m
)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\sin\right.}{}$$

$$= \frac{\left(2^{-\frac{3}{4}-m}c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-\right.}{}$$

$$= \frac{2^{\frac{5}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(3+4m),\frac{1}{4}(5+4m\right.}{}$$

Mathematica [F] time = 103.719, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-1 - m),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-1 - m), x]
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x
, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 1)*g*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1-m
),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x
, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.170 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=121

$$\frac{c2^{\frac{9}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-1),\frac{1}{4}(4m+5);\frac{1}{4}(\right.}{fg(4m+5)}$$

```
[Out] (2^(9/4 - m)*c*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-1 + 4*m)/4, (5 +
4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rubi [A] time = 0.310735, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 2689, 70, 69}

$$\frac{c2^{\frac{9}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-1),\frac{1}{4}(4m+5);\frac{1}{4}(\right.}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^m,
x]
```

```
[Out] (2^(9/4 - m)*c*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-1 + 4*m)/4, (5 +
4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+f$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\sin(e+}{}$$

$$= \frac{\Big(2^{\frac{1}{4}-m}c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\sin}{}$$

$$= \frac{2^{\frac{9}{4}-m}c(g\cos(e+fx))^{5/2}\,{}_2F_1\Big(\frac{1}{4}(-1+4m),\frac{1}{4}(5+4m);\frac{1}{4}}{}$$

Mathematica [F] time = 15.9956, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^m,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^m, x]
```

Maple [F] time = 0.214, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^m}\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m}{\left(-c\sin\left(fx+e\right)+c\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}g\cos\left(fx+e\right)}{\left(-c\sin\left(fx+e\right)+c\right)^{m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(-c*sin
(f*x + e) + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/((c-c*sin(f*x+e))**m),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, a
lgorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^m, x)
```

3.171 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=123

$$\frac{c^2 2^{\frac{13}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-5),\frac{1}{4}(4m+5);\frac{1}{4}(4\right.}{fg(4m+5)}$$

```
[Out] (2^(13/4 - m)*c^2*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-5 + 4*m)/4, (5
 + 4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)
*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rubi [A] time = 0.341085, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{c^2 2^{\frac{13}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-5),\frac{1}{4}(4m+5);\frac{1}{4}(4\right.}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 -
 m),x]
```

```
[Out] (2^(13/4 - m)*c^2*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-5 + 4*m)/4, (5
 + 4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)
*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\sin(\right.}{}$$

$$= \frac{\left(2^{\frac{5}{4}-m}c^3(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-\right.}{}$$

$$= \frac{2^{\frac{13}{4}-m}c^2(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(-5+4m),\frac{1}{4}(5+\right.}{}$$

Mathematica [F] time = 175.476, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(1 - m),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(1 - m), x]
```

Maple [F] time = 0.244, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x,
 algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 1)*g*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1-m)
,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x,
 algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.172 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$

Optimal. Leaf size=123

$$\frac{c^3 2^{\frac{17}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-9),\frac{1}{4}(4m+5);\frac{1}{4}\right.}{fg(4m+5)}$$

```
[Out] (2^(17/4 - m)*c^3*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-9 + 4*m)/4, (5
 + 4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)
*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rubi [A] time = 0.353497, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2853, 2689, 70, 69}

$$\frac{c^3 2^{\frac{17}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-9),\frac{1}{4}(4m+5);\frac{1}{4}\right.}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 -
 m),x]
```


```
[Out] (2^(17/4 - m)*c^3*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-9 + 4*m)/4, (5
 + 4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)
*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```


Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+\ldots\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\sin(e+\ldots\right.}{\ldots}$$

$$= \frac{\left(2^{\frac{9}{4}-m}c^4(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^m(c-c\,\mathrm{si}\ldots\right.}{\ldots}$$

$$= \frac{2^{\frac{17}{4}-m}c^3(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(-9+4m),\frac{1}{4}(5+4m\ldots\right.}{\ldots}$$

Mathematica [F] time = 84.4901, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(2 - m),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(2 - m), x]
```

Maple [F] time = 0.297, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{g\cos\left(fx+e\right)}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}g\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x,
 algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 2)*g*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(2-m)
,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x,
 algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.173 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=135

$$\frac{c2^{n+\frac{p}{2}+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{p+1}(1-\sin(e+fx))^{\frac{1}{2}(-2n-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2n-p+1),\frac{1}{2}(2m$$
$$fg(2m+p+1)$$

```
[Out] (2^(1/2 + n + p/2)*c*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 - 2*n -
p)/2, (1 + 2*m + p)/2, (3 + 2*m + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e +
f*x])^((1 - 2*n - p)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n
))/(f*g*(1 + 2*m + p))
```

Rubi [A] time = 0.286558, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2853, 2689, 70, 69}

$$\frac{c2^{n+\frac{p}{2}+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{p+1}(1-\sin(e+fx))^{\frac{1}{2}(-2n-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2n-p+1),\frac{1}{2}(2m$$
$$fg(2m+p+1)$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```


```
[Out] (2^(1/2 + n + p/2)*c*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 - 2*n -
p)/2, (1 + 2*m + p)/2, (3 + 2*m + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e +
f*x])^((1 - 2*n - p)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n
))/(f*g*(1 + 2*m + p))
```


Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+f\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m(c-c\sin(e+\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+n+\frac{p}{2}}c^2(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m(c-\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n+\frac{p}{2}}c(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1-2n-p),\frac{1}{2}(1+2\right.}{}$$

Mathematica [A] time = 40.6333, size = 133, normalized size = 0.99

$$\frac{2\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^p\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m+n+p}\,{}_2F_1\left(n\right.}{f(2n+p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,
x]
```

```
[Out] (2*(g*Cos[e + f*x])^p*Hypergeometric2F1[1 + m + n + p, (1 + 2*n + p)/2, (3
+ 2*n + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(
m + n + p)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n*Tan[(2*e - Pi +
2*f*x)/4])/(f*(1 + 2*n + p))
```

Maple [F] time = 5.265, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^p\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^p\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

3.174 $\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+m}\,dx$

Optimal. Leaf size=57

$$-\frac{g\log(1-\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{cf}$$

```
[Out] -((g*Log[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^m)/(
c*f*(g*Cos[e + f*x])^(2*m)))
```

Rubi [A] time = 0.227553, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2843, 12, 2667, 31}

$$-\frac{g\log(1-\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
(-1 + m),x]
```

```
[Out] -((g*Log[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^m)/(
c*f*(g*Cos[e + f*x])^(2*m)))
```

Rule 2843

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*
m + p - 1, 0] && EqQ[m - n - 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+m}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e\right.$$
$$= \left(g(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(\right.$$
$$= -\frac{\left(g(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin\right.}{c}$$
$$= -\frac{g(g\cos(e+fx))^{-2m}\log(1-\sin(e+fx))(a+a\sin}{cf}$$

Mathematica [B] time = 80.9138, size = 132, normalized size = 2.32

$$\frac{g2^{m+1}\cos^{2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^(-1 + m),x]
```

```
[Out] (2^(1 + m)*g*Cos[(2*e + Pi + 2*f*x)/4]^(2*m)*(Log[Csc[(2*e + 3*Pi + 2*f*x)/
8]^2] - Log[Tan[(-2*e + Pi - 2*f*x)/8]])*(a*(1 + Sin[e + f*x]))^m*(c - c*Si
n[e + f*x])^m)/(c*f*(g*Cos[e + f*x])^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^(2*m))
```

Maple [C] time = 5.367, size = 9871, normalized size = 173.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (g\cos(fx+e))^{-2m+1}(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m + 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^(m - 1), x)
```

Fricas [A] time = 1.72801, size = 69, normalized size = 1.21

$$-\frac{a\left(\frac{ac}{g^2}\right)^{m-1}\log\left(-\sin\left(fx+e\right)+1\right)}{fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m)
,x, algorithm="fricas")
```

```
[Out] -a*(a*c/g^2)^(m - 1)*log(-sin(f*x + e) + 1)/(f*g)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1
+m),x)
```

```
[Out] Timed out
```

Giac [B] time = 27.2214, size = 1314, normalized size = 23.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m)
,x, algorithm="giac")
```

```
[Out] 1/2*(4*pi*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs(g)) - log(abs(c))
+ log(abs(g)))*floor(1/4*(pi + 2*f*x - 4*pi*floor(1/2*(pi + f*x + e)/pi) +
2*e)/pi)*tan(1/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*m*floor(-1/4*sgn(c
) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi*m - p
i*floor(-1/4*sgn(c) + 1) - 1/4*pi*sgn(c) + 1/4*pi*sgn(g))^2 + 4*pi*e^(m*log
(abs(a)) + m*log(abs(c)) - 2*m*log(abs(g)) - log(abs(c)) + log(abs(g)))*flo
or(1/2*(pi + f*x + e)/pi)*tan(1/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*m
*floor(-1/4*sgn(c) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c) - 1/2*pi*m*sgn(
g) + 1/4*pi*m - pi*floor(-1/4*sgn(c) + 1) - 1/4*pi*sgn(c) + 1/4*pi*sgn(g))^
2 + 2*pi*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs(g)) - log(abs(c)) +
 log(abs(g)))*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*tan(1/4*pi + pi*m*floor(-1/4*
sgn(a) + 1/2) + pi*m*floor(-1/4*sgn(c) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sg
n(c) - 1/2*pi*m*sgn(g) + 1/4*pi*m - pi*floor(-1/4*sgn(c) + 1) - 1/4*pi*sgn(
c) + 1/4*pi*sgn(g))^2 + 3*pi*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs
(g)) - log(abs(c)) + log(abs(g)))*tan(1/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2
) + pi*m*floor(-1/4*sgn(c) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c) - 1/2*p
i*m*sgn(g) + 1/4*pi*m - pi*floor(-1/4*sgn(c) + 1) - 1/4*pi*sgn(c) + 1/4*pi*
sgn(g))^2 - 4*pi*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs(g)) - log(a
bs(c)) + log(abs(g)))*floor(1/4*(pi + 2*f*x - 4*pi*floor(1/2*(pi + f*x + e)
/pi) + 2*e)/pi) - 4*pi*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs(g)) -
 log(abs(c)) + log(abs(g)))*floor(1/2*(pi + f*x + e)/pi) - 2*pi*e^(m*log(ab
```

```
s(a)) + m*log(abs(c)) - 2*m*log(abs(g)) - log(abs(c)) + log(abs(g)))*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1) - 4*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(ab
s(g)) - log(abs(c)) + log(abs(g)))*log(2*(tan(1/2*f*x + 1/2*e)^2 - 2*tan(1/
2*f*x + 1/2*e) + 1)/(tan(1/2*f*x + 1/2*e)^2 + 1))*tan(1/4*pi + pi*m*floor(-
1/4*sgn(a) + 1/2) + pi*m*floor(-1/4*sgn(c) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*
m*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi*m - pi*floor(-1/4*sgn(c) + 1) - 1/4*pi*
sgn(c) + 1/4*pi*sgn(g)) - 2*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs(
g)) - log(abs(c)) + log(abs(g)) + 1)*tan(1/4*pi + pi*m*floor(-1/4*sgn(a) +
1/2) + pi*m*floor(-1/4*sgn(c) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c) - 1/
2*pi*m*sgn(g) + 1/4*pi*m - pi*floor(-1/4*sgn(c) + 1) - 1/4*pi*sgn(c) + 1/4*
pi*sgn(g))^2 - 3*pi*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs(g)) - lo
g(abs(c)) + log(abs(g))) + 2*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs
(g)) - log(abs(c)) + log(abs(g)) + 1))/(f*tan(1/4*pi + pi*m*floor(-1/4*sgn(
a) + 1/2) + pi*m*floor(-1/4*sgn(c) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c)
 - 1/2*pi*m*sgn(g) + 1/4*pi*m - pi*floor(-1/4*sgn(c) + 1) - 1/4*pi*sgn(c) +
 1/4*pi*sgn(g))^2 + f)
```

3.175 $\int (g\cos(e+fx))^{5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=203

$$-\frac{8a^3(a\sin(e+fx)+a)^{m-3}(c-c\sin(e+fx))^n(g\cos(e+fx))^{6-2m}}{fg(-m+n+3)(-m+n+4)(-m+n+5)}-\frac{4a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(g\cos(e+fx))^{6-2m}}{fg(-m+n+4)(-m+n+5)}$$

```
[Out] (-8*a^3*(g*Cos[e + f*x])^(6 - 2*m)*(a + a*Sin[e + f*x])^(-3 + m)*(c - c*Sin
[e + f*x])^n)/(f*g*(3 - m + n)*(4 - m + n)*(5 - m + n)) - (4*a^2*(g*Cos[e +
 f*x])^(6 - 2*m)*(a + a*Sin[e + f*x])^(-2 + m)*(c - c*Sin[e + f*x])^n)/(f*g
*(4 - m + n)*(5 - m + n)) - (a*(g*Cos[e + f*x])^(6 - 2*m)*(a + a*Sin[e + f*
x])^(-1 + m)*(c - c*Sin[e + f*x])^n)/(f*g*(5 - m + n))
```

Rubi [A] time = 0.678372, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2846, 2844}

$$-\frac{8a^3(a\sin(e+fx)+a)^{m-3}(c-c\sin(e+fx))^n(g\cos(e+fx))^{6-2m}}{fg(-m+n+3)(-m+n+4)(-m+n+5)}-\frac{4a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(g\cos(e+fx))^{6-2m}}{fg(-m+n+4)(-m+n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] (-8*a^3*(g*Cos[e + f*x])^(6 - 2*m)*(a + a*Sin[e + f*x])^(-3 + m)*(c - c*Sin
[e + f*x])^n)/(f*g*(3 - m + n)*(4 - m + n)*(5 - m + n)) - (4*a^2*(g*Cos[e +
 f*x])^(6 - 2*m)*(a + a*Sin[e + f*x])^(-2 + m)*(c - c*Sin[e + f*x])^n)/(f*g
*(4 - m + n)*(5 - m + n)) - (a*(g*Cos[e + f*x])^(6 - 2*m)*(a + a*Sin[e + f*
x])^(-1 + m)*(c - c*Sin[e + f*x])^n)/(f*g*(5 - m + n))
```

Rule 2846

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[Simplify[m + p/2 - 1/2], 0] &&  !LtQ[n, -1] &&  !(IGtQ[Simpli
fy[n + p/2 - 1/2], 0] && GtQ[m - n, 0]) &&  !(ILtQ[Simplify[m + n + p], 0]
&& GtQ[Simplify[2*m + n + (3*p)/2 + 1], 0])
```

Rule 2844

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])
^n)/(f*g*(m - n - 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m - n -
1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^{5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = -\frac{a(g\cos(e+fx))^{6-2m}(a+a\sin(e+fx))^{-1+m}(c-c\sin}{fg(5-m+n)}$$
$$= -\frac{4a^2(g\cos(e+fx))^{6-2m}(a+a\sin(e+fx))^{-2+m}(c-c}{fg(4-m+n)(5-m+n)}$$
$$= -\frac{8a^3(g\cos(e+fx))^{6-2m}(a+a\sin(e+fx))^{-3+m}(c-c}{fg(3-m+n)(4-m+n)(5-m+n)}$$

Mathematica [A] time = 6.65832, size = 210, normalized size = 1.03

$$\frac{g^5\cos^{2n}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(g\cos(e+fx))^{-2m}(a(\sin(e+fx)+1))^{m-n}\left(-4\left(m^2-2mn-9m+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n,x]
```

```
[Out] (E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e +
 f*x]]))*g^5*Cos[e + f*x]^(2*n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(a*
(1 + Sin[e + f*x]))^(m - n)*(-76 + 29*m - 3*m^2 - 29*n + 6*m*n - 3*n^2 + (1
2 + m^2 + 7*n + n^2 - m*(7 + 2*n))*Cos[2*(e + f*x)] - 4*(18 - 9*m + m^2 + 9
*n - 2*m*n + n^2)*Sin[e + f*x]))/(2*f*(3 - m + n)*(4 - m + n)*(5 - m + n)*(
g*Cos[e + f*x])^(2*m))
```

Maple [F] time = 11.848, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{5-2m}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [B] time = 2.98214, size = 1320, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="maxima")
```

```
[Out] ((m^2 - m*(2*n + 11) + n^2 + 11*n + 32)*a^m*c^n*g^5 - 2*(m^2 - m*(2*n + 15)
 + n^2 + 15*n + 60)*a^m*c^n*g^5*sin(f*x + e)/(cos(f*x + e) + 1) - (3*m^2 -
m*(6*n + 1) + 3*n^2 + n - 160)*a^m*c^n*g^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 8*(m^2 - m*(2*n + 7) + n^2 + 7*n - 20)*a^m*c^n*g^5*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 2*(m^2 - m*(2*n - 5) + n^2 - 5*n + 160)*a^m*c^n*g^5*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*(3*m^2 - m*(6*n + 13) + 3*n^2 + 13*n +
```

```
 116)*a^m*c^n*g^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*(m^2 - m*(2*n - 5
) + n^2 - 5*n + 160)*a^m*c^n*g^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8*(m
^2 - m*(2*n + 7) + n^2 + 7*n - 20)*a^m*c^n*g^5*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 - (3*m^2 - m*(6*n + 1) + 3*n^2 + n - 160)*a^m*c^n*g^5*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 2*(m^2 - m*(2*n + 15) + n^2 + 15*n + 60)*a^m*c^n*g
^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + (m^2 - m*(2*n + 11) + n^2 + 11*n +
 32)*a^m*c^n*g^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*e^(2*n*log(sin(f*x
+ e)/(cos(f*x + e) + 1) - 1) - 2*m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1
) + m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1) - n*log(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1))/(((m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47
)*m - 12*n^2 - 47*n - 60)*g^(2*m) + 5*(m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 +
 24*n + 47)*m - 12*n^2 - 47*n - 60)*g^(2*m)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 10*(m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47)*m - 12*n^2 - 47*
n - 60)*g^(2*m)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*(m^3 - 3*m^2*(n +
4) - n^3 + (3*n^2 + 24*n + 47)*m - 12*n^2 - 47*n - 60)*g^(2*m)*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 5*(m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47
)*m - 12*n^2 - 47*n - 60)*g^(2*m)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + (m^
3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47)*m - 12*n^2 - 47*n - 60)*g^(2*
m)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*f)
```

Fricas [B] time = 2.01563, size = 1609, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="fricas")
```

```
[Out] -((m^2 - (2*m - 7)*n + n^2 - 7*m + 12)*cos(f*x + e)^3 - (m^2 - (2*m - 11)*n
 + n^2 - 11*m + 24)*cos(f*x + e)^2 - 2*(m^2 - (2*m - 9)*n + n^2 - 9*m + 22)
*cos(f*x + e) - ((m^2 - (2*m - 7)*n + n^2 - 7*m + 12)*cos(f*x + e)^2 + 2*(m
^2 - (2*m - 9)*n + n^2 - 9*m + 18)*cos(f*x + e) - 8)*sin(f*x + e) - 8)*(g*c
os(f*x + e))^(-2*m + 5)*(a*sin(f*x + e) + a)^m*e^(2*n*log(g*cos(f*x + e)) -
 n*log(a*sin(f*x + e) + a) + n*log(a*c/g^2))/(4*f*m^3 - 4*f*n^3 - (f*m^3 -
f*n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m + 47*f)*n
 - 60*f)*cos(f*x + e)^3 - 48*f*m^2 + 12*(f*m - 4*f)*n^2 - 3*(f*m^3 - f*n^3
- 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m + 47*f)*n - 60*
f)*cos(f*x + e)^2 + 188*f*m - 4*(3*f*m^2 - 24*f*m + 47*f)*n + 2*(f*m^3 - f*
n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m + 47*f)*n -
 60*f)*cos(f*x + e) + (4*f*m^3 - 4*f*n^3 - 48*f*m^2 + 12*(f*m - 4*f)*n^2 -
(f*m^3 - f*n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m
+ 47*f)*n - 60*f)*cos(f*x + e)^2 + 188*f*m - 4*(3*f*m^2 - 24*f*m + 47*f)*n
+ 2*(f*m^3 - f*n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*
f*m + 47*f)*n - 60*f)*cos(f*x + e) - 240*f)*sin(f*x + e) - 240*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x
)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.176 $\int (g\cos(e+fx))^{3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=127

$$-\frac{2a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(g\cos(e+fx))^{4-2m}}{fg(-m+n+2)(-m+n+3)}-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g\cos(e+fx))^{4-2m}}{fg(-m+n+3)}$$

```
[Out] (-2*a^2*(g*Cos[e + f*x])^(4 - 2*m)*(a + a*Sin[e + f*x])^(-2 + m)*(c - c*Sin
[e + f*x])^n)/(f*g*(2 - m + n)*(3 - m + n)) - (a*(g*Cos[e + f*x])^(4 - 2*m)
*(a + a*Sin[e + f*x])^(-1 + m)*(c - c*Sin[e + f*x])^n)/(f*g*(3 - m + n))
```

Rubi [A] time = 0.407658, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2846, 2844}

$$-\frac{2a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(g\cos(e+fx))^{4-2m}}{fg(-m+n+2)(-m+n+3)}-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g\cos(e+fx))^{4-2m}}{fg(-m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
n,x]
```



```
[Out] (-2*a^2*(g*Cos[e + f*x])^(4 - 2*m)*(a + a*Sin[e + f*x])^(-2 + m)*(c - c*Sin
[e + f*x])^n)/(f*g*(2 - m + n)*(3 - m + n)) - (a*(g*Cos[e + f*x])^(4 - 2*m)
*(a + a*Sin[e + f*x])^(-1 + m)*(c - c*Sin[e + f*x])^n)/(f*g*(3 - m + n))
```


Rule 2846

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(
b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x]
)^n)/(f*g*(m + n + p)), x] + Dist[(a*(2*m + p - 1))/(m + n + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[Simplify[m + p/2 - 1/2], 0] &&  !LtQ[n, -1] &&  !(IGtQ[Simpli
fy[n + p/2 - 1/2], 0] && GtQ[m - n, 0]) &&  !(ILtQ[Simplify[m + n + p], 0]
&& GtQ[Simplify[2*m + n + (3*p)/2 + 1], 0])
```

Rule 2844

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])
^n)/(f*g*(m - n - 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m - n -
1, 0]
```

Rubi steps

$$\begin{aligned}\int (g\cos(e+fx))^{3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx &= -\frac{a(g\cos(e+fx))^{4-2m}(a+a\sin(e+fx))^{-1+m}(c-c\sin(e+fx))^n}{fg(3-m+n)} \\ &= -\frac{2a^2(g\cos(e+fx))^{4-2m}(a+a\sin(e+fx))^{-2+m}(c-c\sin(e+fx))^n}{fg(2-m+n)(3-m+n)}\end{aligned}$$

Mathematica [A] time = 1.32339, size = 143, normalized size = 1.13

$$-\frac{g^3\cos^{2n}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(g\cos(e+fx))^{-2m}((-m+n+2)\sin(e+fx)-m+n+4)(a(\sin(e}{f(-m+n+2)(-m+n+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n,x]
```

```
[Out] -((E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e
 + f*x]]))*g^3*Cos[e + f*x]^(2*n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(
a*(1 + Sin[e + f*x]))^(m - n)*(4 - m + n + (2 - m + n)*Sin[e + f*x]))/(f*(2
 - m + n)*(3 - m + n)*(g*Cos[e + f*x])^(2*m)))
```

Maple [F] time = 11.848, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{3-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [B] time = 2.21669, size = 655, normalized size = 5.16

$$\frac{\left(a^m c^n g^3(m-n-4)-\frac{2\,a^m c^n g^3(m-n-6)\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{a^m c^n g^3(m-n+12)\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{4\,a^m c^n g^3(m-n+2)\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{a^m c^n g^3(m-n+12)\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right.}{\left(\left(m^2-m(2\,n+5)+n^2+5\,n+6\right)g^{2\,m}+\frac{3\left(m^2-m(2\,n+5)+n^2+5\,n+6\right)g^{2\,m}\sin(}{\left(\cos\left(fx+e\right)+1\right)^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="maxima")
```

```
[Out] (a^m*c^n*g^3*(m - n - 4) - 2*a^m*c^n*g^3*(m - n - 6)*sin(f*x + e)/(cos(f*x
+ e) + 1) - a^m*c^n*g^3*(m - n + 12)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
4*a^m*c^n*g^3*(m - n + 2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^m*c^n*g^3
*(m - n + 12)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*a^m*c^n*g^3*(m - n -
6)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^m*c^n*g^3*(m - n - 4)*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6)*e^(2*n*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
- 2*m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1) + m*log(sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 1) - n*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/(((
m^2 - m*(2*n + 5) + n^2 + 5*n + 6)*g^(2*m) + 3*(m^2 - m*(2*n + 5) + n^2 + 5
*n + 6)*g^(2*m)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*(m^2 - m*(2*n + 5)
+ n^2 + 5*n + 6)*g^(2*m)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + (m^2 - m*(2*
n + 5) + n^2 + 5*n + 6)*g^(2*m)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*f)
```

Fricas [B] time = 1.88668, size = 741, normalized size = 5.83

$$\frac{\left((m-n-2)\cos\left(fx+e\right)^2+(m-n-4)\cos\left(fx+e\right)+\left((m-n-2\right.\right.}{2\,fm^2+2\,fn^2-\left(fm^2+fn^2-5\,fm-\left(2\,fm-5\,f\right)n+6\,f\right)\cos\left(fx+e\right)^2-10\,fm-2\left(2\,fm-5\,f\right)n+\left(fm^2+fn^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="fricas")
```

```
[Out] ((m - n - 2)*cos(f*x + e)^2 + (m - n - 4)*cos(f*x + e) + ((m - n - 2)*cos(f
*x + e) + 2)*sin(f*x + e) - 2)*(g*cos(f*x + e))^(-2*m + 3)*(a*sin(f*x + e)
+ a)^m*e^(2*n*log(g*cos(f*x + e)) - n*log(a*sin(f*x + e) + a) + n*log(a*c/g
^2))/(2*f*m^2 + 2*f*n^2 - (f*m^2 + f*n^2 - 5*f*m - (2*f*m - 5*f)*n + 6*f)*c
os(f*x + e)^2 - 10*f*m - 2*(2*f*m - 5*f)*n + (f*m^2 + f*n^2 - 5*f*m - (2*f*
m - 5*f)*n + 6*f)*cos(f*x + e) + (2*f*m^2 + 2*f*n^2 - 10*f*m - 2*(2*f*m - 5
*f)*n + (f*m^2 + f*n^2 - 5*f*m - (2*f*m - 5*f)*n + 6*f)*cos(f*x + e) + 12*f
)*sin(f*x + e) + 12*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x
)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.177 $\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=58

$$-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g\cos(e+fx))^{2-2m}}{fg(-m+n+1)}$$

```
[Out] -((a*(g*Cos[e + f*x])^(2 - 2*m)*(a + a*Sin[e + f*x])^(-1 + m)*(c - c*Sin[e
+ f*x])^n)/(f*g*(1 - m + n)))
```

Rubi [A] time = 0.163097, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2844}

$$-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g\cos(e+fx))^{2-2m}}{fg(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] -((a*(g*Cos[e + f*x])^(2 - 2*m)*(a + a*Sin[e + f*x])^(-1 + m)*(c - c*Sin[e
+ f*x])^n)/(f*g*(1 - m + n)))
```

Rule 2844

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])
^n)/(f*g*(m - n - 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m - n -
1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = -\frac{a(g\cos(e+fx))^{2-2m}(a+a\sin(e+fx))^{-1+m}(c-c\sin}{fg(1-m+n)}$$

Mathematica [A] time = 0.672585, size = 96, normalized size = 1.66

$$\frac{g(\sin(e+fx)-1)\cos^{2n}(e+fx)(g\cos(e+fx))^{-2m}(a(\sin(e+fx)+1))^{m-n}\exp(n(\log(a(\sin(e+fx)+1))+\log(c-c\sin(}{f(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n,x]
```

```
[Out] (E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e +
 f*x]]))*g*Cos[e + f*x]^(2*n)*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(m
```

```
- n))/(f*(1 - m + n)*(g*Cos[e + f*x])^(2*m))
```

Maple [F] time = 20.304, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{1-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [B] time = 1.71773, size = 279, normalized size = 4.81

$$\frac{\left(a^{m}c^{n}g-\frac{2\,a^{m}c^{n}g\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{a^{m}c^{n}g\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)e^{\left(2\,n\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)-2\,m\log\left(-\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)+m\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)-n\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)\right)}}{\left(g^{2\,m}(m-n-1)+\frac{g^{2\,m}(m-n-1)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="maxima")
```

```
[Out] (a^m*c^n*g - 2*a^m*c^n*g*sin(f*x + e)/(cos(f*x + e) + 1) + a^m*c^n*g*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*e^(2*n*log(sin(f*x + e)/(cos(f*x + e) + 1) -
 1) - 2*m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1) + m*log(sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 1) - n*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
/((g^(2*m)*(m - n - 1) + g^(2*m)*(m - n - 1)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*f)
```

Fricas [B] time = 1.82133, size = 320, normalized size = 5.52

$$\frac{\left(g\cos\left(fx+e\right)\right)^{-2\,m+1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)e^{\left(2\,n\log\left(g\cos\left(fx+e\right)\right)-n\log\left(a\sin\left(fx+e\right)+a\right)+n\log\left(\frac{ac}{g^{2}}\right)\right)}}{fm-fn+\left(fm-fn-f\right)\cos\left(fx+e\right)+\left(fm-fn-f\right)\sin\left(fx+e\right)-f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="fricas")
```

```
[Out] (g*cos(f*x + e))^(-2*m + 1)*(a*sin(f*x + e) + a)^m*(cos(f*x + e) - sin(f*x
 + e) + 1)*e^(2*n*log(g*cos(f*x + e)) - n*log(a*sin(f*x + e) + a) + n*log(a*
c/g^2))/(f*m - f*n + (f*m - f*n - f)*cos(f*x + e) + (f*m - f*n - f)*sin(f*x
 + e) - f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x
)
```

```
[Out] Timed out
```

Giac [B] time = 29.6986, size = 6257, normalized size = 107.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="giac")
```

```
[Out] -(e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(t
an(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(-1/8*pi + 1/4*f*x
 + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(
abs(c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*
e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + log(abs(g)))*tan(1/8*pi - pi*
m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) +
 pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/
4) - pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*n*floor(1/2*f*x/pi + 1/2*
e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*n*floor(-1/4*sgn(c) + 1) -
 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*
e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*n*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi*n
+ 1/4*f*x + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/p
i + 1/2) + 1/4) + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4*pi*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*sgn(g) + 1/4*e)^2*tan(1/2*f*x + 1/2*e)^2
 - 2*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))
/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(-1/8*pi + 1/4*
f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a)) + n*l
og(abs(c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(-1/8*pi + 1/4*f*x + 1
/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + log(abs(g)))*tan(1/8*pi -
pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4
) + pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) +
 1/4) - pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*n*floor(1/2*f*x/pi + 1
/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*n*floor(-1/4*sgn(c) + 1
) - 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/2*pi*n*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*n*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi
*n + 1/4*f*x + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*
e/pi + 1/2) + 1/4) + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4*pi*sgn
(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*sgn(g) + 1/4*e)^2*tan(1/2*f*x + 1/2*e
) + 2*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e)
)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(-1/8*pi + 1/4
*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a)) + n*
log(abs(c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(-1/8*pi + 1/4*f*x +
1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + log(abs(g)))*tan(1/8*pi -
 pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/
4) + pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2)
+ 1/4) - pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*n*floor(1/2*f*x/pi +
```

```
1/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*n*floor(-1/4*sgn(c) +
1) - 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/2*pi*n*sgn(tan(1/2*f*x +
1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*n*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*p
i*n + 1/4*f*x + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2
*e/pi + 1/2) + 1/4) + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4*pi*sg
n(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*sgn(g) + 1/4*e)*tan(1/2*f*x + 1/2*e)
^2 + e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))
/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(-1/8*pi + 1/4*
f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a)) + n*l
og(abs(c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(-1/8*pi + 1/4*f*x + 1
/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + log(abs(g)))*tan(1/8*pi -
pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4
) + pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) +
 1/4) - pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*n*floor(1/2*f*x/pi + 1
/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*n*floor(-1/4*sgn(c) + 1
) - 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/2*pi*n*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*n*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi
*n + 1/4*f*x + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*
e/pi + 1/2) + 1/4) + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4*pi*sgn
(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*sgn(g) + 1/4*e)^2 - e^(m*log(2) - n*l
og(2) - 2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*
x + 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/
8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*log(a
bs(g)) - log(2) + log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi +
1/4*f*x + 1/4*e)^2 + 1)) + log(abs(g)))*tan(1/2*f*x + 1/2*e)^2 - 2*e^(m*log
(2) - n*log(2) - 2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))
/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) -
2*m*log(abs(g)) - log(2) + log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-
1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + log(abs(g)))*tan(1/8*pi - pi*m*floor(1/
2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*n*floo
r(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*m*
floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2
) + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*n*floor(-1/4*sgn(c) + 1) - 1/2*pi*m*
sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
+ 1/4*pi*m*sgn(a) + 1/4*pi*n*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi*n + 1/4*f*x
+ 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) +
 1/4) + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4*pi*sgn(tan(1/2*f*x
+ 1/2*e)^2 - 1) + 1/4*pi*sgn(g) + 1/4*e) + 2*e^(m*log(2) - n*log(2) - 2*m*l
og(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2
+ 1)) + 2*n*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*
x + 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*log(abs(g)) - log(
2) + log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4
*e)^2 + 1)) + log(abs(g)))*tan(1/2*f*x + 1/2*e) - e^(m*log(2) - n*log(2) -
2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*
e)^2 + 1)) + 2*n*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1
/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*log(abs(g)) -
 log(2) + log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x
+ 1/4*e)^2 + 1)) + log(abs(g))))/(f*m*tan(1/8*pi - pi*m*floor(1/2*f*x/pi +
1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*n*floor(1/2*f*x/p
i + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*m*floor(1/2*f
*x/pi + 1/2*e/pi + 1/2) + pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*fl
oor(-1/4*sgn(a) + 1/2) + pi*n*floor(-1/4*sgn(c) + 1) - 1/2*pi*m*sgn(tan(1/2
*f*x + 1/2*e)^2 - 1) + 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*
sgn(a) + 1/4*pi*n*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi*n + 1/4*f*x + 1/2*pi*fl
oor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1/2
*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2
- 1) + 1/4*pi*sgn(g) + 1/4*e)^2*tan(1/2*f*x + 1/2*e)^2 - f*n*tan(1/8*pi - p
i*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4)
```

```
 + pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) +
1/4) - pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*n*floor(1/2*f*x/pi + 1/
2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*n*floor(-1/4*sgn(c) + 1)
 - 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/2*pi*n*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*n*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi*
n + 1/4*f*x + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e
/pi + 1/2) + 1/4) + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4*pi*sgn(
tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*sgn(g) + 1/4*e)^2*tan(1/2*f*x + 1/2*e)
^2 - f*tan(1/8*pi - pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1
/2*e/pi + 1/2) + 1/4) + pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi
 + 1/2*e/pi + 1/2) + 1/4) - pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*n*
floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*n*f
loor(-1/4*sgn(c) + 1) - 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/2*pi*n
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*n*sgn(c) - 1/2*
pi*m*sgn(g) + 1/4*pi*n + 1/4*f*x + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - flo
or(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi
 + 1/2) + 1/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*sgn(g) + 1/4*e)^2
*tan(1/2*f*x + 1/2*e)^2 + f*m*tan(1/8*pi - pi*m*floor(1/2*f*x/pi + 1/2*e/pi
 - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*n*floor(1/2*f*x/pi + 1/2*
e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*m*floor(1/2*f*x/pi +
1/2*e/pi + 1/2) + pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4
*sgn(a) + 1/2) + pi*n*floor(-1/4*sgn(c) + 1) - 1/2*pi*m*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1) + 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) +
 1/4*pi*n*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi*n + 1/4*f*x + 1/2*pi*floor(1/2*
f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1/2*pi*floo
r(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1
/4*pi*sgn(g) + 1/4*e)^2 - f*n*tan(1/8*pi - pi*m*floor(1/2*f*x/pi + 1/2*e/pi
 - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*n*floor(1/2*f*x/pi + 1/2*
e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*m*floor(1/2*f*x/pi +
1/2*e/pi + 1/2) + pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4
*sgn(a) + 1/2) + pi*n*floor(-1/4*sgn(c) + 1) - 1/2*pi*m*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1) + 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) +
 1/4*pi*n*sgn(c) - 1/2*pi*m*sgn(g) + 1/4*pi*n + 1/4*f*x + 1/2*pi*floor(1/2*
f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1/2*pi*floo
r(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1
/4*pi*sgn(g) + 1/4*e)^2 + f*m*tan(1/2*f*x + 1/2*e)^2 - f*n*tan(1/2*f*x + 1/
2*e)^2 - f*tan(1/8*pi - pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi
 + 1/2*e/pi + 1/2) + 1/4) + pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*
x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + p
i*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1/2) + pi
*n*floor(-1/4*sgn(c) + 1) - 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/2*
pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*n*sgn(c) -
1/2*pi*m*sgn(g) + 1/4*pi*n + 1/4*f*x + 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi -
 floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1/2*pi*floor(1/2*f*x/pi + 1/2*
e/pi + 1/2) + 1/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*sgn(g) + 1/4*
e)^2 - f*tan(1/2*f*x + 1/2*e)^2 + f*m - f*n - f)
```

3.178 $\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=81

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\,{}_2F_1\left(1,n-m;-m+n+1;\frac{1}{2}(1-\sin(e+fx))\right)}{2fg(m-n)}$$

```
[Out] (Hypergeometric2F1[1, -m + n, 1 - m + n, (1 - Sin[e + f*x])/2]*(a + a*Sin[e
 + f*x])^m*(c - c*Sin[e + f*x])^n)/(2*f*g*(m - n)*(g*Cos[e + f*x])^(2*m))
```

Rubi [A] time = 0.230153, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 12, 2667, 68}

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\,{}_2F_1\left(1,n-m;-m+n+1;\frac{1}{2}(1-\sin(e+fx))\right)}{2fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^n,x]
```

```
[Out] (Hypergeometric2F1[1, -m + n, 1 - m + n, (1 - Sin[e + f*x])/2]*(a + a*Sin[e
 + f*x])^m*(c - c*Sin[e + f*x])^n)/(2*f*g*(m - n)*(g*Cos[e + f*x])^(2*m))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+\right.$$

$$= \frac{\left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+\right.}{g}$$

$$= -\frac{\left(c(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e\right.}{f}$$

$$= \frac{(g\cos(e+fx))^{-2m}\,{}_2F_1\left(1,-m+n;1-m+n;\frac{1}{2}(1-\mathrm{s}\right.}{2fg(m}$$

Mathematica [A] time = 68.2423, size = 115, normalized size = 1.42

$$\frac{(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{n-m}\,{}_2F_1\left(n-m,n-m;-m+n+1;-\right.}{2fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[-m + n, -m + n, 1 - m + n, -Tan[(2*e - Pi + 2*f*x)/4]^2]
*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(-m + n)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin
[e + f*x])^n)/(2*f*g*(m - n)*(g*Cos[e + f*x])^(2*m))
```

Maple [F] time = 10.299, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-2m}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-2m-1}\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-2\,m-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e
) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] sage2
```

3.179 $\int (g\cos(e+fx))^{-3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=85

$$\frac{c(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(2,-m+n-1;n-m;\frac{1}{2}(1-\sin(e+fx))\right)}{4fg^3(m-n+1)}$$

```
[Out] (c*Hypergeometric2F1[2, -1 - m + n, -m + n, (1 - Sin[e + f*x])/2]*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(4*f*g^3*(1 + m - n)*(g*Cos[e
+ f*x])^(2*m))
```

Rubi [A] time = 0.244095, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 12, 2667, 68}

$$\frac{c(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(2,-m+n-1;n-m;\frac{1}{2}(1-\sin(e+fx))\right)}{4fg^3(m-n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^n,x]
```

```
[Out] (c*Hypergeometric2F1[2, -1 - m + n, -m + n, (1 - Sin[e + f*x])/2]*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(4*f*g^3*(1 + m - n)*(g*Cos[e
+ f*x])^(2*m))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (g\cos(e+fx))^{-3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(\right.$$

$$= \frac{\left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(\right.}{}$$

$$= -\frac{\left(c^3(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\,s\right.}{}$$

$$= \frac{c(g\cos(e+fx))^{-2m}\,{}_2F_1\left(2,-1-m+n;-m+n;\frac{1}{2}\right.}{4f}$$

Mathematica [A] time = 45.0147, size = 135, normalized size = 1.59

$$\frac{\cot^2\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{n-m}\,{}_2F_1\left(-\right.}{8fg^3(m-n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^n,x]
```

```
[Out] (Cot[(2*e - Pi + 2*f*x)/4]^2*Hypergeometric2F1[-2 - m + n, -1 - m + n, -m +
 n, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(-m + n)*(a
*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/(8*f*g^3*(1 + m - n)*(g*Cos[
e + f*x])^(2*m))
```

Maple [F] time = 11.248, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-3-2m}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-2m-3}\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-2\,m-3}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-2*m - 3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e
) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-3-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\cos\left(fx+e\right)\right)^{-2\,m-3}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

3.180 $\int (g\cos(e+fx))^{-5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=88

$$\frac{c^2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(3,-m+n-2;-m+n-1;\frac{1}{2}(1-\sin(e+fx))\right)}{8fg^5(m-n+2)}$$

```
[Out] (c^2*Hypergeometric2F1[3, -2 - m + n, -1 - m + n, (1 - Sin[e + f*x])/2]*(a
+ a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n))/(8*f*g^5*(2 + m - n)*(g*
Cos[e + f*x])^(2*m))
```

Rubi [A] time = 0.244216, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2853, 12, 2667, 68}

$$\frac{c^2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(3,-m+n-2;-m+n-1;\frac{1}{2}(1-\sin(e+fx))\right)}{8fg^5(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^n,x]
```

```
[Out] (c^2*Hypergeometric2F1[3, -2 - m + n, -1 - m + n, (1 - Sin[e + f*x])/2]*(a
+ a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n))/(8*f*g^5*(2 + m - n)*(g*
Cos[e + f*x])^(2*m))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (g\cos(e+fx))^{-5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+\right.$$

$$= \frac{\left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+\right.}{g^5}$$

$$= -\frac{\left(c^5(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(\right.}{f}$$

$$= \frac{c^2(g\cos(e+fx))^{-2m}\,{}_2F_1\left(3,-2-m+n;-1-m+n;\right.}{8fg}$$

Mathematica [A] time = 37.122, size = 136, normalized size = 1.55

$$\frac{\cot^4\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{n-m}\,{}_2F_1\left(-m+\right.}{32fg^5(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^n,x]
```

```
[Out] (Cot[(2*e - Pi + 2*f*x)/4]^4*Hypergeometric2F1[-4 - m + n, -2 - m + n, -1 -
 m + n, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(-m + n
)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/(32*f*g^5*(2 + m - n)*(g
*Cos[e + f*x])^(2*m))
```

Maple [F] time = 11.237, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-5-2m}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-2m-5}\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 5)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-2m-5}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-2*m - 5)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e
) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-5-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\cos\left(fx+e\right)\right)^{-2m-5}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 5)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

3.181 $\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}(\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{fg}$$

```
[Out] (ArcTanh[Sin[e + f*x]]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^m)/(f*g*
(g*Cos[e + f*x])^(2*m))
```

Rubi [A] time = 0.172097, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2847, 12, 3770}

$$\frac{\tanh^{-1}(\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^m,x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^m)/(f*g*
(g*Cos[e + f*x])^(2*m))
```

Rule 2847

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+\right.$$

$$= \frac{\left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+\right.}{g}$$

$$= \frac{\tanh^{-1}(\sin(e+fx))(g\cos(e+fx))^{-2m}(a+a\sin(e+}{fg}$$

Mathematica [A] time = 0.95267, size = 94, normalized size = 1.84

$$\frac{\sqrt{-\tan^2(e+fx)}\csc(e+fx)\cos^{2(m+1)}(e+fx)\sin^{-1}(\sec(e+fx))(g\cos(e+fx))^{-2m-1}\exp(m(\log(a(\sin(e+fx)+1))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^m,x]
```

```
[Out] (E^(m*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e +
 f*x]]))*ArcSin[Sec[e + f*x]]*Cos[e + f*x]^(2*(1 + m))*(g*Cos[e + f*x])^(-1
 - 2*m)*Csc[e + f*x]*Sqrt[-Tan[e + f*x]^2])/f
```

Maple [F] time = 0.446, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x)
```

```
[Out] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-2m-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^m, x)
```

Fricas [A] time = 1.77073, size = 113, normalized size = 2.22

$$\frac{\left(\frac{ac}{g^2}\right)^m\log\left(\sin\left(fx+e\right)+1\right)-\left(\frac{ac}{g^2}\right)^m\log\left(-\sin\left(fx+e\right)+1\right)}{2\,fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x,
algorithm="fricas")
```

```
[Out] 1/2*((a*c/g^2)^m*log(sin(f*x + e) + 1) - (a*c/g^2)^m*log(-sin(f*x + e) + 1)
)/(f*g)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**m,
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-2\,m-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^m, x)
```

3.182 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3+n}\,dx$

Optimal. Leaf size=134

$$\frac{c^3 2^{-\frac{m}{2}+\frac{n}{2}+3}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-4),\frac{m-n}{2};\frac{1}{2}(\ldots\right.}{fg(m-n)}$$

```
[Out] (2^(3 - m/2 + n/2)*c^3*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(-4 + m
- n)/2, (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^
((m - n)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rubi [A] time = 0.382441, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2853, 2689, 70, 69}

$$\frac{c^3 2^{-\frac{m}{2}+\frac{n}{2}+3}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-4),\frac{m-n}{2};\frac{1}{2}(\ldots\right.}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(3 + n),x]
```

```
[Out] (2^(3 - m/2 + n/2)*c^3*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(-4 + m
- n)/2, (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^
((m - n)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3+n}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\right.}{}$$

$$= \frac{\left(2^{2-\frac{m}{2}+\frac{n}{2}}c^4(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))\right.}{}$$

$$= \frac{2^{3-\frac{m}{2}+\frac{n}{2}}c^3(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(-4+m-n)\right.}{}$$

Mathematica [F] time = 145.704, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3+n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(3 + n),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(3 + n), x]
```

Maple [F] time = 0.615, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{3+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{n+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n)
,x, algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
 e) + c)^(n + 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3
+n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n)
,x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 3), x)
```

3.183 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2+n}\,dx$

Optimal. Leaf size=134

$$\frac{c^2 2^{-\frac{m}{2}+\frac{n}{2}+2}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-2),\frac{m-n}{2};\frac{1}{2}(m-\right.}{fg(m-n)}$$

```
[Out] (2^(2 - m/2 + n/2)*c^2*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(-2 + m
- n)/2, (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^
((m - n)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rubi [A] time = 0.373765, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2853, 2689, 70, 69}

$$\frac{c^2 2^{-\frac{m}{2}+\frac{n}{2}+2}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-2),\frac{m-n}{2};\frac{1}{2}(m-\right.}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(2 + n),x]
```

```
[Out] (2^(2 - m/2 + n/2)*c^2*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(-2 + m
- n)/2, (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^
((m - n)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2+n}\,dx = \Big((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\,\text{s}$$

$$= \frac{\Big(c^2(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-}{}$$

$$= \frac{\Big(2^{1-\frac{m}{2}+\frac{n}{2}}c^3(g\cos(e+fx))^{-m-n}(a+a\sin(e+f}{}$$

$$= \frac{2^{2-\frac{m}{2}+\frac{n}{2}}c^2(g\cos(e+fx))^{-m-n}\,{}_2F_1\Big(\frac{1}{2}(-2+m-}{}$$

Mathematica [F] time = 138.543, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2+n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(2 + n),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(2 + n), x]
```

Maple [F] time = 0.56, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{2+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{n+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n)
,x, algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
 e) + c)^(n + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(2
+n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n)
,x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 2), x)
```

3.184 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1+n}\,dx$

Optimal. Leaf size=131

$$\frac{c2^{-\frac{m}{2}+\frac{n}{2}+1}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{m-n}{2},\frac{m-n}{2};\frac{1}{2}(m-n+2)\right)}{fg(m-n)}$$

```
[Out] (2^(1 - m/2 + n/2)*c*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(m - n)/2,
 (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((m - n
)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rubi [A] time = 0.36937, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2853, 2689, 70, 69}

$$\frac{c2^{-\frac{m}{2}+\frac{n}{2}+1}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{m-n}{2},\frac{m-n}{2};\frac{1}{2}(m-n+2)\right)}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(1 + n),x]
```

```
[Out] (2^(1 - m/2 + n/2)*c*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(m - n)/2,
 (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((m - n
)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1+n}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\,\mathrm{s}\right.}{}$$

$$= \frac{\left(2^{-\frac{m}{2}+\frac{n}{2}}c^2(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m\right.}{}$$

$$= \frac{2^{1-\frac{m}{2}+\frac{n}{2}}c(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{m-n}{2},\frac{m-n}{2};\frac{1}{2}(2+\right.}{}$$

Mathematica [C] time = 25.637, size = 207, normalized size = 1.58

$$\frac{ic(\sin(e+fx)-1)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}\left({}_2F_1\left(1,-m+n+1;-m+n+2;-\frac{i\left(\tan\left(\frac{1}{2}\right.\right.}{\tan\left(\frac{1}{2}\right.}\right.\right.}{fg(m-n-1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\mathrm{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(1 + n),x]
```

```
[Out] (I*c*(g*Cos[e + f*x])^(-m - n)*(Hypergeometric2F1[1, 1 - m + n, 2 - m + n,
((-I)*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])] - Hypergeometric2F1[
1, 1 - m + n, 2 - m + n, (I*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])
])*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/(f*
g*(-1 + m - n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]))
```

Maple [F] time = 29.86, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{1+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n)
,x, algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
 e) + c)^(n + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1
+n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n)
,x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 1), x)
```

3.185 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=55

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{fg(m-n)}$$

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(
f*g*(m - n))
```

Rubi [A] time = 0.171202, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2848}

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^n,x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(
f*g*(m - n))
```

Rule 2848

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(m - n)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\sin(e}{fg(m-n)}$$

Mathematica [A] time = 0.715047, size = 55, normalized size = 1.

$$\frac{(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^n,x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)
/(f*g*(m - n))
```

Maple [F] time = 0.48, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [B] time = 1.85559, size = 196, normalized size = 3.56

$$\frac{a^{m}c^{n}g^{-m-n-1}e^{\left(m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-n\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)+2\,n\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)-m\log\left(-\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-n\log\left(-\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)\right)}}{f(m-n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] a^m*c^n*g^(-m - n - 1)*e^(m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) + 1) + 2*n*log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1) - m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*log(-sin(f*x +
 e)/(cos(f*x + e) + 1) + 1))/(f*(m - n))
```

Fricas [A] time = 1.82405, size = 205, normalized size = 3.73

$$\frac{\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)e^{\left(2\,n\log\left(g\cos\left(fx+e\right)\right)-n\log\left(a\sin\left(fx+e\right)+a\right)+n\log\left(\frac{ac}{g^{2}}\right)\right)}}{fm-fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] (g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*cos(f*x + e)*e^(2*n*lo
g(g*cos(f*x + e)) - n*log(a*sin(f*x + e) + a) + n*log(a*c/g^2))/(f*m - f*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^n, x)
```

3.186 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+n}\,dx$

Optimal. Leaf size=125

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{fg(m-n+2)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx)}{cfg(m-n)(m-n+2)}$$

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1
+ n))/(f*g*(2 + m - n)) + ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m
*(c - c*Sin[e + f*x])^n)/(c*f*g*(m - n)*(2 + m - n))
```

Rubi [A] time = 0.410686, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2849, 2848}

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{fg(m-n+2)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx)}{cfg(m-n)(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-1 + n),x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1
+ n))/(f*g*(2 + m - n)) + ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m
*(c - c*Sin[e + f*x])^n)/(c*f*g*(m - n)*(2 + m - n))
```

Rule 2849

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2
- b^2, 0] && ILtQ[Simplify[m + n + p + 1], 0] && NeQ[2*m + p + 1, 0] && (Su
mSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 2848

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(m - n)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+n}\,dx=\frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c}{fg(2+m-n)}$$

$$=\frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c}{fg(2+m-n)}$$

Mathematica [A] time = 26.008, size = 132, normalized size = 1.06

$$-\frac{2^{n-1}\cos^{2(n-1)}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^{n-1}(\sin(e+fx)-m+n-1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}{fg(m-n)(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(-1 + n),x]
```

```
[Out] -((2^(-1 + n)*(g*Cos[e + f*x])^(-m - n)*Cos[(2*e + Pi + 2*f*x)/4]^(2*(-1 +
n))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2 - 2*n)*(a*(1 + Sin[e + f*x]))^
m*(-1 - m + n + Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 + n))/(f*g*(m - n)*(
2 + m - n)))
```

Maple [F] time = 0.53, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-1+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n
),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.84344, size = 324, normalized size = 2.59

$$\frac{\left((m-n+1)\cos\left(fx+e\right)-\cos\left(fx+e\right)\sin\left(fx+e\right)\right)\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}e^{\left(2\,(n-1)\log\left(g\cos\left(fx+e\right)\right)-\right.}}{fm^2+fn^2+2\,fm-2\left(fm+f\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n
),x, algorithm="fricas")
```

```
[Out] ((m - n + 1)*cos(f*x + e) - cos(f*x + e)*sin(f*x + e))*(g*cos(f*x + e))^(-m
 - n - 1)*(a*sin(f*x + e) + a)^m*e^(2*(n - 1)*log(g*cos(f*x + e)) - (n - 1)
*log(a*sin(f*x + e) + a) + (n - 1)*log(a*c/g^2))/(f*m^2 + f*n^2 + 2*f*m - 2
```

```
*(f*m + f)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-
1+n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (g\cos(fx+e))^{-m-n-1}(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n - 1), x)
```

3.187 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2+n}\,dx$

Optimal. Leaf size=204

$$\frac{2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{c^2fg(m-n)(m-n+2)(m-n+4)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e+fx))^{-}}{fg(m-n+4)}$$

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2
+ n))/(f*g*(4 + m - n)) + (2*(g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-1 + n))/(c*f*g*(2 + m - n)*(4 + m - n)) + (2*(g*C
os[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(c^2*f
*g*(m - n)*(2 + m - n)*(4 + m - n))
```

Rubi [A] time = 0.665382, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2849, 2848}

$$\frac{2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{c^2fg(m-n)(m-n+2)(m-n+4)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e+fx))^{-}}{fg(m-n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-2 + n),x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2
+ n))/(f*g*(4 + m - n)) + (2*(g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-1 + n))/(c*f*g*(2 + m - n)*(4 + m - n)) + (2*(g*C
os[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(c^2*f
*g*(m - n)*(2 + m - n)*(4 + m - n))
```

Rule 2849

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2
- b^2, 0] && ILtQ[Simplify[m + n + p + 1], 0] && NeQ[2*m + p + 1, 0] && (Su
mSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 2848

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(m - n)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2+n}\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c}{fg(4+m-n)}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c}{fg(4+m-n)}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c}{fg(4+m-n)}$$

Mathematica [A] time = 30.0394, size = 183, normalized size = 0.9

$$\frac{2^{n-2}\cos(e+fx)\sin^{2n-4}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx\right.\right.}{f(m-n)(m-n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(-2 + n),x]
```

```
[Out] (2^(-2 + n)*Cos[e + f*x]*(g*Cos[e + f*x])^(-1 - m - n)*Sin[(-e + Pi/2 - f*x
)/2]^(-4 + 2*n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n)*(3 + 4
*m + m^2 - 4*n - 2*m*n + n^2 + Cos[2*(-e + Pi/2 - f*x)] - 2*(2 + m - n)*Sin
[e + f*x]))/(f*(m - n)*(2 + m - n)*(4 + m - n)*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^(2*(-2 + n)))
```

Maple [F] time = 0.636, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{-2+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n
),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.90677, size = 460, normalized size = 2.25

$$-\frac{\left(2\cos\left(fx+e\right)^3+2(m-n+2)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(m^2-2(m+2)n+n^2+4m+4\right)\cos\left(fx+e\right)\right)\left(g\cos\left(\right.\right.}{fm^3-fn^3+6fm^2+3\left(fm+2f\right)n^2+8fm-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n
),x, algorithm="fricas")
```

```
[Out] -(2*cos(f*x + e)^3 + 2*(m - n + 2)*cos(f*x + e)*sin(f*x + e) - (m^2 - 2*(m
+ 2)*n + n^2 + 4*m + 4)*cos(f*x + e))*(g*cos(f*x + e))^(-m - n - 1)*(a*sin(
f*x + e) + a)^m*e^(2*(n - 2)*log(g*cos(f*x + e)) - (n - 2)*log(a*sin(f*x +
e) + a) + (n - 2)*log(a*c/g^2))/(f*m^3 - f*n^3 + 6*f*m^2 + 3*(f*m + 2*f)*n^
2 + 8*f*m - (3*f*m^2 + 12*f*m + 8*f)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-
2+n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n - 2), x)
```

3.188 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3+n}\,dx$

Optimal. Leaf size=290

$$\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{c^2fg(m-n+2)(m-n+4)(m-n+6)}+\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+f}{c^3fg(m-n)(m-n+2)(m-n+4)(m-n+6}$$

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3
+ n))/(f*g*(6 + m - n)) + (3*(g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-2 + n))/(c*f*g*(4 + m - n)*(6 + m - n)) + (6*(g*C
os[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))
/(c^2*f*g*(2 + m - n)*(4 + m - n)*(6 + m - n)) + (6*(g*Cos[e + f*x])^(-m -
n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(c^3*f*g*(m - n)*(2 + m -
 n)*(4 + m - n)*(6 + m - n))
```

Rubi [A] time = 0.937238, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2849, 2848}

$$\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{c^2fg(m-n+2)(m-n+4)(m-n+6)}+\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+f}{c^3fg(m-n)(m-n+2)(m-n+4)(m-n+6}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-3 + n),x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3
+ n))/(f*g*(6 + m - n)) + (3*(g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-2 + n))/(c*f*g*(4 + m - n)*(6 + m - n)) + (6*(g*C
os[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))
/(c^2*f*g*(2 + m - n)*(4 + m - n)*(6 + m - n)) + (6*(g*Cos[e + f*x])^(-m -
n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(c^3*f*g*(m - n)*(2 + m -
 n)*(4 + m - n)*(6 + m - n))
```

Rule 2849

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(2*m + p + 1)), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2
- b^2, 0] && ILtQ[Simplify[m + n + p + 1], 0] && NeQ[2*m + p + 1, 0] && (Su
mSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 2848

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b
*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a
*f*g*(m - n)), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3+n}\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\sin}{fg(6+m-n)}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\sin}{fg(6+m-n)}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\sin}{fg(6+m-n)}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\sin}{fg(6+m-n)}$$

Mathematica [B] time = 37.399, size = 2681, normalized size = 9.24

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(-3 + n),x]
```

```
[Out] -((2^(-4 - m + 2*n)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Cos[e + f*x]*(g*Cos[e +
f*x])^(-1 - m - n)*Csc[(-e + Pi/2 - f*x)/2]^6*(Cos[(-e + Pi/2 - f*x)/8]*(-S
in[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^(2*n)*(Cos[(-e + P
i/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8] - S
in[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 - f*x))/8]))^(-m - n)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 + n)*(-30 - 46*m - 18*m^2 - 2*m^
3 + 46*n + 36*m*n + 6*m^2*n - 18*n^2 - 6*m*n^2 + 2*n^3 - 6*(3 + m - n)*Cos[
2*(-e + Pi/2 - f*x)] + 3*Cos[3*(-e + Pi/2 - f*x)] + 3*(15 + 2*m^2 - 4*m*(-3
 + n) - 12*n + 2*n^2)*Sin[e + f*x])*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[P
i/4 + (e - Pi/2 + f*x)/2])^(-7 + 2*n))/(f*(m - n)*(2 + m - n)*(4 + m - n)*(
6 + m - n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-3 + n))*((2^(-4 - m +
 2*n)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Csc[(-e + Pi/2 - f*x)/2]^6*(Cos[(-e +
Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^
(2*n)*(Cos[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + P
i/2 - f*x))/8] - Sin[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 - f*x))/8
]))^(-m - n)*(-3*(15 + 2*m^2 - 4*m*(-3 + n) - 12*n + 2*n^2)*Cos[e + f*x] +
12*(3 + m - n)*Sin[2*(-e + Pi/2 - f*x)] - 9*Sin[3*(-e + Pi/2 - f*x)]))/((m
- n)*(2 + m - n)*(4 + m - n)*(6 + m - n)) - (2^(-4 - m + 2*n)*m*Cos[(-e + P
i/2 - f*x)/2]^(-1 + 2*m)*Csc[(-e + Pi/2 - f*x)/2]^5*(Cos[(-e + Pi/2 - f*x)/
8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^(2*n)*(Cos[(
-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/
8] - Sin[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 - f*x))/8]))^(-m - n)
*(-30 - 46*m - 18*m^2 - 2*m^3 + 46*n + 36*m*n + 6*m^2*n - 18*n^2 - 6*m*n^2
+ 2*n^3 - 6*(3 + m - n)*Cos[2*(-e + Pi/2 - f*x)] + 3*Cos[3*(-e + Pi/2 - f*x
)] + 3*(15 + 2*m^2 - 4*m*(-3 + n) - 12*n + 2*n^2)*Sin[e + f*x]))/((m - n)*(
2 + m - n)*(4 + m - n)*(6 + m - n)) - (3*2^(-4 - m + 2*n)*Cos[(-e + Pi/2 -
f*x)/2]^(1 + 2*m)*Csc[(-e + Pi/2 - f*x)/2]^7*(Cos[(-e + Pi/2 - f*x)/8]*(-Si
n[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^(2*n)*(Cos[(-e + Pi
/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8] - Si
n[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 - f*x))/8]))^(-m - n)*(-30 -
 46*m - 18*m^2 - 2*m^3 + 46*n + 36*m*n + 6*m^2*n - 18*n^2 - 6*m*n^2 + 2*n^3
 - 6*(3 + m - n)*Cos[2*(-e + Pi/2 - f*x)] + 3*Cos[3*(-e + Pi/2 - f*x)] + 3*
(15 + 2*m^2 - 4*m*(-3 + n) - 12*n + 2*n^2)*Sin[e + f*x]))/((m - n)*(2 + m -
 n)*(4 + m - n)*(6 + m - n)) + (2^(-3 - m + 2*n)*n*Cos[(-e + Pi/2 - f*x)/2]
^(2*m)*Csc[(-e + Pi/2 - f*x)/2]^6*(Cos[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/
2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8]))^(-1 + 2*n)*(Cos[(-e + Pi/2 - f
*x)/8]*(-Cos[(-e + Pi/2 - f*x)/8]/8 + (3*Cos[(3*(-e + Pi/2 - f*x))/8])/8) -
```

```
(Sin[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 -
 f*x))/8]))/8)*(Cos[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(
3*(-e + Pi/2 - f*x))/8] - Sin[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2
- f*x))/8]))^(-m - n)*(-30 - 46*m - 18*m^2 - 2*m^3 + 46*n + 36*m*n + 6*m^2*
n - 18*n^2 - 6*m*n^2 + 2*n^3 - 6*(3 + m - n)*Cos[2*(-e + Pi/2 - f*x)] + 3*C
os[3*(-e + Pi/2 - f*x)] + 3*(15 + 2*m^2 - 4*m*(-3 + n) - 12*n + 2*n^2)*Sin[
e + f*x]))/((m - n)*(2 + m - n)*(4 + m - n)*(6 + m - n)) + (2^(-4 - m + 2*n
)*(-m - n)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Csc[(-e + Pi/2 - f*x)/2]^6*(Cos[(
-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/
8]))^(2*n)*(Cos[(-e + Pi/2 - f*x)/8]*(-Sin[(-e + Pi/2 - f*x)/8] + Sin[(3*(-
e + Pi/2 - f*x))/8] - Sin[(5*(-e + Pi/2 - f*x))/8] + Sin[(7*(-e + Pi/2 - f*
x))/8]))^(-1 - m - n)*(Cos[(-e + Pi/2 - f*x)/8]*(-Cos[(-e + Pi/2 - f*x)/8]/
8 + (3*Cos[(3*(-e + Pi/2 - f*x))/8])/8 - (5*Cos[(5*(-e + Pi/2 - f*x))/8])/8
 + (7*Cos[(7*(-e + Pi/2 - f*x))/8])/8) - (Sin[(-e + Pi/2 - f*x)/8]*(-Sin[(-
e + Pi/2 - f*x)/8] + Sin[(3*(-e + Pi/2 - f*x))/8] - Sin[(5*(-e + Pi/2 - f*x
))/8] + Sin[(7*(-e + Pi/2 - f*x))/8]))/8)*(-30 - 46*m - 18*m^2 - 2*m^3 + 46
*n + 36*m*n + 6*m^2*n - 18*n^2 - 6*m*n^2 + 2*n^3 - 6*(3 + m - n)*Cos[2*(-e
+ Pi/2 - f*x)] + 3*Cos[3*(-e + Pi/2 - f*x)] + 3*(15 + 2*m^2 - 4*m*(-3 + n)
- 12*n + 2*n^2)*Sin[e + f*x]))/((m - n)*(2 + m - n)*(4 + m - n)*(6 + m - n)
))*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] + Sin[Pi/4 + (e - Pi/2 + f*x)/2])))
```

Maple [F] time = 0.645, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m-n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-3+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n
),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.01214, size = 664, normalized size = 2.29

$$-\frac{\left(6\left(m-n+3\right)\cos\left(fx+e\right)^{3}-\left(m^{3}+3\left(m+3\right)n^{2}-n^{3}+9\,m^{2}-\left(3\,m^{2}+18\,m+26\right)n+26\,m+24\right)\cos\left(fx+e\right)-3}{fm^{4}+fn^{4}+12\,fm^{3}-4\left(fm+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n
),x, algorithm="fricas")
```

```
[Out] -(6*(m - n + 3)*cos(f*x + e)^3 - (m^3 + 3*(m + 3)*n^2 - n^3 + 9*m^2 - (3*m^
2 + 18*m + 26)*n + 26*m + 24)*cos(f*x + e) - 3*(2*cos(f*x + e)^3 - (m^2 - 2
*(m + 3)*n + n^2 + 6*m + 8)*cos(f*x + e))*sin(f*x + e))*(g*cos(f*x + e))^(-
m - n - 1)*(a*sin(f*x + e) + a)^m*e^(2*(n - 3)*log(g*cos(f*x + e)) - (n - 3
)*log(a*sin(f*x + e) + a) + (n - 3)*log(a*c/g^2))/(f*m^4 + f*n^4 + 12*f*m^3
 - 4*(f*m + 3*f)*n^3 + 44*f*m^2 + 2*(3*f*m^2 + 18*f*m + 22*f)*n^2 + 48*f*m
- 4*(f*m^3 + 9*f*m^2 + 22*f*m + 12*f)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-
3+n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n - 3), x)
```

3.189 $\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=138

$$\frac{c2^{n-\frac{p}{2}+\frac{1}{2}}\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\sec(e+fx))^p(1-\sin(e+fx))^{\frac{1}{2}(-2n+p+1)}\,{}_2F_1\left(\frac{1}{2}(2m-\right.}{f(2m-p+1)}$$

```
[Out] (2^(1/2 + n - p/2)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m - p)/2, (1 - 2
*n + p)/2, (3 + 2*m - p)/2, (1 + Sin[e + f*x])/2]*(g*Sec[e + f*x])^p*(1 - S
in[e + f*x])^((1 - 2*n + p)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
(-1 + n))/(f*(1 + 2*m - p))
```

Rubi [A] time = 0.460227, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2926, 2853, 2689, 70, 69}

$$\frac{c2^{n-\frac{p}{2}+\frac{1}{2}}\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\sec(e+fx))^p(1-\sin(e+fx))^{\frac{1}{2}(-2n+p+1)}\,{}_2F_1\left(\frac{1}{2}(2m-\right.}{f(2m-p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n - p/2)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m - p)/2, (1 - 2
*n + p)/2, (3 + 2*m - p)/2, (1 + Sin[e + f*x])/2]*(g*Sec[e + f*x])^p*(1 - S
in[e + f*x])^((1 - 2*n + p)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
(-1 + n))/(f*(1 + 2*m - p))
```

Rule 2926

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[((a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(g*Cos[e + f*x])^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] &&  !IntegerQ[p]
```

Rule 2853

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*
x])^FracPart[m])/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m])), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int (g\cos(e+fx))^{-p}(a+$$

$$= \left((g\cos(e+fx))^{-2m+p}(g\sec(e+fx))^p(a+a\sin(e+fx))^m$$

$$= \frac{\left(c^2\cos(e+fx)(g\sec(e+fx))^p(a+a\sin(e+fx))^m(c-c\,\mathrm{s}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+n-\frac{p}{2}}c^2\cos(e+fx)(g\sec(e+fx))^p(a+a\sin(e+fx)\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n-\frac{p}{2}}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m-p),\frac{1}{2}(1-2n+p);\frac{1}{2}(\right.}{}$$

Mathematica [A] time = 40.1278, size = 139, normalized size = 1.01

$$\frac{2\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n(g\sec(e+fx))^p\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m+n-p}\,{}_2F_1\left(m+\right.}{f(2n-p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,
x]
```

```
[Out] (2*Hypergeometric2F1[1 + m + n - p, 1/2 + n - p/2, 3/2 + n - p/2, -Tan[(2*e
 - Pi + 2*f*x)/4]^2]*(g*Sec[e + f*x])^p*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(m +
n - p)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n*Tan[(2*e - Pi + 2*f*
x)/4])/(f*(1 + 2*n - p))
```

Maple [F] time = 5.358, size = 0, normalized size = 0.

$$\int \left(g\sec\left(fx+e\right)\right)^p\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

3.190 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0454494, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+x)}{a^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int x^2(a+x)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(ax^2+x^3\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.0140133, size = 33, normalized size = 1.

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.016, size = 28, normalized size = 0.9

$$\frac{1}{d}\left(\frac{(\sin(dx+c))^4 a}{4}+\frac{(\sin(dx+c))^3 a}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/4*sin(d*x+c)^4*a+1/3*sin(d*x+c)^3*a)
```

Maxima [A] time = 1.4071, size = 38, normalized size = 1.15

$$\frac{3\,a\sin(dx+c)^4+4\,a\sin(dx+c)^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.62002, size = 122, normalized size = 3.7

$$\frac{3\,a\cos(dx+c)^4-6\,a\cos(dx+c)^2-4\left(a\cos(dx+c)^2-a\right)\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^4 - 6*a*cos(d*x + c)^2 - 4*(a*cos(d*x + c)^2 - a)*si
n(d*x + c))/d
```

Sympy [A] time = 2.51002, size = 65, normalized size = 1.97

$$\begin{cases}\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)\cos^2(c+dx)}{2d}-\frac{a\cos^4(c+dx)}{4d} & \text{for } d\neq 0\\ x\,(a\sin(c)+a)\sin^2(c)\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**3/(3*d) - a*sin(c + d*x)**2*cos(c + d*x)**2/(2*d
) - a*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c),
 True))
```

Giac [A] time = 1.16031, size = 38, normalized size = 1.15

$$\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3)/d
```

3.191 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}$$

```
[Out] (a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0358782, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2833, 12, 43}

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/(3*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+x)}{a}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int x(a+x)\,dx,x,a\sin(c+dx)\right)}{a^2d}\\ &= \frac{\text{Subst}\left(\int \left(ax+x^2\right)dx,x,a\sin(c+dx)\right)}{a^2d}\\ &= \frac{a\sin^2(c+dx)}{2d}+\frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.090569, size = 30, normalized size = 0.91

$$\frac{4a\sin^3(c+dx)-3a\cos(2(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Cos[2*(c + d*x)] + 4*a*Sin[c + d*x]^3)/(12*d)
```

Maple [A] time = 0.013, size = 28, normalized size = 0.9

$$\frac{1}{d}\left(\frac{(\sin(dx+c))^3 a}{3}+\frac{(\sin(dx+c))^2 a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*sin(d*x+c)^3*a+1/2*sin(d*x+c)^2*a)
```

Maxima [A] time = 1.29919, size = 38, normalized size = 1.15

$$\frac{2\,a\sin(dx+c)^3+3\,a\sin(dx+c)^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.64546, size = 93, normalized size = 2.82

$$-\frac{3\,a\cos(dx+c)^2+2\left(a\cos(dx+c)^2-a\right)\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a*cos(d*x + c)^2 + 2*(a*cos(d*x + c)^2 - a)*sin(d*x + c))/d
```

Sympy [A] time = 1.3498, size = 41, normalized size = 1.24

$$\begin{cases} \frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^2(c+dx)}{2d} & \text{for } d\neq 0 \\ x\,(a\sin(c)+a)\sin(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**3/(3*d) - a*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x
*(a*sin(c) + a)*sin(c)*cos(c), True))
```

Giac [A] time = 1.29005, size = 38, normalized size = 1.15

$$\frac{2\, a \sin\left(dx + c\right)^{3} + 3\, a \sin\left(dx + c\right)^{2}}{6\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2)/d
```

3.192 $\int \cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0218486, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2707, 43}

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a+x}{x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0350826, size = 26, normalized size = 1.08

$$\frac{a(\sin(c+dx)+\log(\tan(c+dx))+\log(\cos(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]] + Sin[c + d*x]))/d
```

Maple [A] time = 0.023, size = 25, normalized size = 1.

$$\frac{a\ln(\sin(dx+c))}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

Maxima [A] time = 1.24067, size = 30, normalized size = 1.25

$$\frac{a\log(\sin(dx+c))+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sin(d*x + c)) + a*sin(d*x + c))/d
```

Fricas [A] time = 1.66884, size = 62, normalized size = 2.58

$$\frac{a\log\left(\frac{1}{2}\sin(dx+c)\right)+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*log(1/2*sin(d*x + c)) + a*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\cos(c+dx)\csc(c+dx)\,dx+\int\sin(c+dx)\cos(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c + d
*x)*csc(c + d*x), x))
```

Giac [A] time = 1.1463, size = 31, normalized size = 1.29

$$\frac{a\log\left(|\sin\left(dx+c\right)|\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(sin(d*x + c))) + a*sin(d*x + c))/d
```

3.193 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=25

$$\frac{a\log(\sin(c+dx))}{d}-\frac{a\csc(c+dx)}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0360492, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2833, 12, 43}

$$\frac{a\log(\sin(c+dx))}{d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a^2(a+x)}{x^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{a\,\text{Subst}\left(\int \frac{a+x}{x^2}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a\,\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{1}{x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0392552, size = 33, normalized size = 1.32

$$\frac{a(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A] time = 0.03, size = 28, normalized size = 1.1

$$-\frac{a}{d\sin(dx+c)}+\frac{a\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)+a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.25986, size = 34, normalized size = 1.36

$$\frac{a\log(\sin(dx+c))-\frac{a}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sin(d*x + c)) - a/sin(d*x + c))/d
```

Fricas [A] time = 1.63557, size = 82, normalized size = 3.28

$$\frac{a\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-a}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*log(1/2*sin(d*x + c))*sin(d*x + c) - a)/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\cos(c+dx)\csc^2(c+dx)\,dx+\int\sin(c+dx)\cos(c+dx)\csc^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x)**2, x) + Integral(sin(c + d*x)*cos(c
+ d*x)*csc(c + d*x)**2, x))
```

Giac [A] time = 1.28595, size = 35, normalized size = 1.4

$$\frac{a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{a}{\sin\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(sin(d*x + c))) - a/sin(d*x + c))/d
```

3.194 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^2}{2ad}$$

```
[Out] -(Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2)/(2*a*d)
```

Rubi [A] time = 0.0417927, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 37}

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^2}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2)/(2*a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a^3(a+x)}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{a^2\,\text{Subst}\left(\int \frac{a+x}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{\csc^2(c+dx)(a+a\sin(c+dx))^2}{2ad}\end{aligned}$$

Mathematica [A] time = 0.0191811, size = 29, normalized size = 0.97

$$-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.033, size = 27, normalized size = 0.9

$$\frac{a}{d}\left(-(\sin(dx+c))^{-1}-\frac{1}{2(\sin(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*a*(-1/sin(d*x+c)-1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.24401, size = 32, normalized size = 1.07

$$-\frac{2a\sin(dx+c)+a}{2d\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*sin(d*x + c) + a)/(d*sin(d*x + c)^2)
```

Fricas [A] time = 1.5656, size = 69, normalized size = 2.3

$$\frac{2a\sin(dx+c)+a}{2\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*sin(d*x + c) + a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29335, size = 32, normalized size = 1.07

$$-\frac{2\,a\sin\left(dx+c\right)+a}{2\,d\sin\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*sin(d*x + c) + a)/(d*sin(d*x + c)^2)
```

3.195 $\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}$$

```
[Out] -(a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0427132, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a^4(a+x)}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{a^3\,\text{Subst}\left(\int \frac{a+x}{x^4}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\,\text{Subst}\left(\int \left(\frac{a}{x^4}+\frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= -\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0240431, size = 33, normalized size = 1.

$$-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.034, size = 27, normalized size = 0.8

$$\frac{a}{d}\left(-\frac{1}{3\left(\sin\left(dx+c\right)\right)^{3}}-\frac{1}{2\left(\sin\left(dx+c\right)\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*a*(-1/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.11486, size = 35, normalized size = 1.06

$$-\frac{3\,a\sin\left(dx+c\right)+2\,a}{6\,d\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*sin(d*x + c) + 2*a)/(d*sin(d*x + c)^3)
```

Fricas [A] time = 1.54374, size = 92, normalized size = 2.79

$$\frac{3\,a\sin\left(dx+c\right)+2\,a}{6\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*sin(d*x + c) + 2*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25737, size = 35, normalized size = 1.06

$$-\frac{3\,a\sin\left(dx+c\right)+2\,a}{6\,d\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*a*sin(d*x + c) + 2*a)/(d*sin(d*x + c)^3)
```

3.196 $\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -(a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0426549, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^4)/(4*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a^5(a+x)}{x^5}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{a^4\,\text{Subst}\left(\int \frac{a+x}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^4\,\text{Subst}\left(\int \left(\frac{a}{x^5}+\frac{1}{x^4}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.0186258, size = 33, normalized size = 1.

$$-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.031, size = 27, normalized size = 0.8

$$\frac{a}{d}\left(-\frac{1}{4\left(\sin\left(dx+c\right)\right)^{4}}-\frac{1}{3\left(\sin\left(dx+c\right)\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*a*(-1/4/sin(d*x+c)^4-1/3/sin(d*x+c)^3)
```

Maxima [A] time = 1.24179, size = 35, normalized size = 1.06

$$-\frac{4\,a\sin\left(dx+c\right)+3\,a}{12\,d\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*sin(d*x + c) + 3*a)/(d*sin(d*x + c)^4)
```

Fricas [A] time = 1.61533, size = 103, normalized size = 3.12

$$-\frac{4\,a\sin\left(dx+c\right)+3\,a}{12\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(4*a*sin(d*x + c) + 3*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14554, size = 35, normalized size = 1.06

$$-\frac{4 a \sin(dx+c) + 3 a}{12 d \sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(4*a*sin(d*x + c) + 3*a)/(d*sin(d*x + c)^4)
```

3.197 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^3(c+dx)}{3d}$$

```
[Out] (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^4)/(2*d) + (a^2*Sin[c + d*x]
^5)/(5*d)
```

Rubi [A] time = 0.0668741, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^4)/(2*d) + (a^2*Sin[c + d*x]
^5)/(5*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+x)^2}{a^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int x^2(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(a^2x^2+2ax^3+x^4\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.32562, size = 53, normalized size = 0.96

$$\frac{a^2\left(104\sin^3(c+dx)+15\cos(4(c+dx))-12\left(2\sin^3(c+dx)+5\right)\cos(2(c+dx))\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(15*Cos[4*(c + d*x)] + 104*Sin[c + d*x]^3 - 12*Cos[2*(c + d*x)]*(5 + 2
*Sin[c + d*x]^3)))/(240*d)
```

Maple [A] time = 0.019, size = 45, normalized size = 0.8

$$\frac{1}{d}\left(\frac{(\sin(dx+c))^5 a^2}{5}+\frac{(\sin(dx+c))^4 a^2}{2}+\frac{a^2(\sin(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*sin(d*x+c)^5*a^2+1/2*sin(d*x+c)^4*a^2+1/3*a^2*sin(d*x+c)^3)
```

Maxima [A] time = 1.16116, size = 61, normalized size = 1.11

$$\frac{6\,a^2\sin(dx+c)^5+15\,a^2\sin(dx+c)^4+10\,a^2\sin(dx+c)^3}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3)
/d
```

Fricas [A] time = 1.666, size = 173, normalized size = 3.15

$$\frac{15\,a^2\cos(dx+c)^4-30\,a^2\cos(dx+c)^2+2\left(3\,a^2\cos(dx+c)^4-11\,a^2\cos(dx+c)^2+8\,a^2\right)\sin(dx+c)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a^2*cos(d*x + c)^4 - 30*a^2*cos(d*x + c)^2 + 2*(3*a^2*cos(d*x + c)
^4 - 11*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 4.90546, size = 85, normalized size = 1.55

$$\begin{cases} \frac{a^2\sin^5(c+dx)}{5d} + \frac{a^2\sin^3(c+dx)}{3d} - \frac{a^2\sin^2(c+dx)\cos^2(c+dx)}{d} - \frac{a^2\cos^4(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a\sin(c) + a\right)^2\sin^2(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)**5/(5*d) + a**2*sin(c + d*x)**3/(3*d) - a**2*s
in(c + d*x)**2*cos(c + d*x)**2/d - a**2*cos(c + d*x)**4/(2*d), Ne(d, 0)), (
x*(a*sin(c) + a)**2*sin(c)**2*cos(c), True))
```

Giac [A] time = 1.2436, size = 61, normalized size = 1.11

$$\frac{6\,a^2\sin(dx+c)^5 + 15\,a^2\sin(dx+c)^4 + 10\,a^2\sin(dx+c)^3}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3)
/d
```

3.198 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{2d}$$

```
[Out] (a^2*Sin[c + d*x]^2)/(2*d) + (2*a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(4*d)
```

Rubi [A] time = 0.0482555, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^2)/(2*d) + (2*a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(4*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{a}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int x(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^2d}\\ &= \frac{\text{Subst}\left(\int \left(a^2x+2ax^2+x^3\right)dx,x,a\sin(c+dx)\right)}{a^2d}\\ &= \frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^4(c+dx)}{4d}\end{aligned}$$

Mathematica [A] time = 0.0506013, size = 38, normalized size = 0.69

$$\frac{a^2\sin^2(c+dx)\left(3\sin^2(c+dx)+8\sin(c+dx)+6\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^2*(6 + 8*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(12*d)
```

Maple [A] time = 0.017, size = 45, normalized size = 0.8

$$\frac{1}{d}\left(\frac{(\sin(dx+c))^4a^2}{4}+\frac{2a^2(\sin(dx+c))^3}{3}+\frac{(\sin(dx+c))^2a^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(1/4*sin(d*x+c)^4*a^2+2/3*a^2*sin(d*x+c)^3+1/2*sin(d*x+c)^2*a^2)
```

Maxima [A] time = 1.14817, size = 61, normalized size = 1.11

$$\frac{3a^2\sin(dx+c)^4+8a^2\sin(dx+c)^3+6a^2\sin(dx+c)^2}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.64804, size = 134, normalized size = 2.44

$$\frac{3a^2\cos(dx+c)^4-12a^2\cos(dx+c)^2-8\left(a^2\cos(dx+c)^2-a^2\right)\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*cos(d*x + c)^4 - 12*a^2*cos(d*x + c)^2 - 8*(a^2*cos(d*x + c)^2
- a^2)*sin(d*x + c))/d
```

Sympy [A] time = 2.09734, size = 87, normalized size = 1.58

$$\begin{cases}\frac{2a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)\cos^2(c+dx)}{2d}-\frac{a^2\cos^4(c+dx)}{4d}-\frac{a^2\cos^2(c+dx)}{2d} & \text{for } d\neq 0\\ x\left(a\sin(c)+a\right)^2\sin(c)\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) - a**2*sin(c + d*x)**2*cos(c + d*x)
**2/(2*d) - a**2*cos(c + d*x)**4/(4*d) - a**2*cos(c + d*x)**2/(2*d), Ne(d,
0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c), True))
```

Giac [A] time = 1.13873, size = 61, normalized size = 1.11

$$\frac{3\,a^2\sin\left(dx+c\right)^4+8\,a^2\sin\left(dx+c\right)^3+6\,a^2\sin\left(dx+c\right)^2}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2)/d
```

3.199 $\int \cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=47

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(
2*d)
```

Rubi [A] time = 0.0385452, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 43}

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(
2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2}{x}+x\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0242228, size = 47, normalized size = 1.

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(
2*d)
```

Maple [A] time = 0.028, size = 46, normalized size = 1.

$$\frac{a^2 \ln(\sin(dx+c))}{d} + 2\,\frac{a^2 \sin(dx+c)}{d} + \frac{(\sin(dx+c))^2 a^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] a^2*ln(sin(d*x+c))/d+2*a^2*sin(d*x+c)/d+1/2*a^2*sin(d*x+c)^2/d
```

Maxima [A] time = 1.17073, size = 55, normalized size = 1.17

$$\frac{a^2 \sin(dx+c)^2 + 2\,a^2 \log(\sin(dx+c)) + 4\,a^2 \sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*sin(d*x + c)^2 + 2*a^2*log(sin(d*x + c)) + 4*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.70214, size = 108, normalized size = 2.3

$$-\frac{a^2 \cos(dx+c)^2 - 2\,a^2 \log\left(\frac{1}{2}\sin(dx+c)\right) - 4\,a^2 \sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c)) - 4*a^2*sin(d*x + c)
)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \cos(c+dx)\csc(c+dx)\,dx + \int 2\sin(c+dx)\cos(c+dx)\csc(c+dx)\,dx + \int \sin^2(c+dx)\cos(c+dx)\csc(c+d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(2*sin(c + d*x)*cos(
c + d*x)*csc(c + d*x), x) + Integral(sin(c + d*x)**2*cos(c + d*x)*csc(c + d
*x), x))
```

Giac [A] time = 1.32148, size = 57, normalized size = 1.21

$$\frac{a^2 \sin (dx+c)^2 + 2\,a^2 \log (|\sin (dx+c)|) + 4\,a^2 \sin (dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c))) + 4*a^2*sin(d*x + c)
)/d
```

3.200 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=43

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/
d
```

Rubi [A] time = 0.0546732, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/
d
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{a^2(a+x)^2}{x^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^2}{x^2}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a\,\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}+\frac{2a}{x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0228808, size = 38, normalized size = 0.88

$$a^2\left(\frac{\sin(c+dx)}{d}-\frac{\csc(c+dx)}{d}+\frac{2\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(-(Csc[c + d*x]/d) + (2*Log[Sin[c + d*x]])/d + Sin[c + d*x]/d)
```

Maple [A] time = 0.036, size = 46, normalized size = 1.1

$$\frac{a^2\sin(dx+c)}{d}-\frac{a^2}{d\sin(dx+c)}+2\frac{a^2\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] a^2*sin(d*x+c)/d-1/d*a^2/sin(d*x+c)+2*a^2*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.18064, size = 54, normalized size = 1.26

$$\frac{2a^2\log(\sin(dx+c))+a^2\sin(dx+c)-\frac{a^2}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(sin(d*x + c)) + a^2*sin(d*x + c) - a^2/sin(d*x + c))/d
```

Fricas [A] time = 1.6657, size = 112, normalized size = 2.6

$$-\frac{a^2\cos(dx+c)^2-2a^2\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c))*sin(d*x + c))/(d*sin(d*x
 + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29898, size = 55, normalized size = 1.28

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+a^2\sin\left(dx+c\right)-\frac{a^2}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*log(abs(sin(d*x + c))) + a^2*sin(d*x + c) - a^2/sin(d*x + c))/d
```

3.201 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=47

$$-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) + (a^2*Log[Sin[c + d*x
]])/d
```

Rubi [A] time = 0.0648706, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) + (a^2*Log[Sin[c + d*x
]])/d
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{a^3(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{a^2\,\text{Subst}\left(\int \frac{(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= -\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}+\frac{a^2\log(\sin(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.0169466, size = 42, normalized size = 0.89

$$a^2\left(-\frac{\csc^2(c+dx)}{2d}-\frac{2\csc(c+dx)}{d}+\frac{\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*((-2*Csc[c + d*x])/d - Csc[c + d*x]^2/(2*d) + Log[Sin[c + d*x]]/d)
```

Maple [A] time = 0.04, size = 48, normalized size = 1.

$$-2\frac{a^2}{d\sin(dx+c)}+\frac{a^2\ln(\sin(dx+c))}{d}-\frac{a^2}{2d(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/d*a^2/sin(d*x+c)+a^2*ln(sin(d*x+c))/d-1/2/d*a^2/sin(d*x+c)^2
```

Maxima [A] time = 1.18434, size = 58, normalized size = 1.23

$$\frac{2a^2\log(\sin(dx+c))-\frac{4a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*log(sin(d*x + c)) - (4*a^2*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.66186, size = 146, normalized size = 3.11

$$\frac{4a^2\sin(dx+c)+a^2+2\left(a^2\cos(dx+c)^2-a^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)}{2\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*sin(d*x + c) + a^2 + 2*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*
x + c)))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31304, size = 59, normalized size = 1.26

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{4\,a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(sin(d*x + c))) - (4*a^2*sin(d*x + c) + a^2)/sin(d*x + c)
^2)/d
```

3.202 $\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^3(c+dx)(a\sin(c+dx)+a)^3}{3ad}$$

```
[Out] -(Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*a*d)
```

Rubi [A] time = 0.0581122, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 37}

$$-\frac{\csc^3(c+dx)(a\sin(c+dx)+a)^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{a^4(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{a^3\,\text{Subst}\left(\int \frac{(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{\csc^3(c+dx)(a+a\sin(c+dx))^3}{3ad}\end{aligned}$$

Mathematica [A] time = 0.0233936, size = 20, normalized size = 0.67

$$-\frac{a^2(\csc(c+dx)+1)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(1 + Csc[c + d*x])^3)/(3*d)
```

Maple [A] time = 0.035, size = 39, normalized size = 1.3

$$\frac{a^2}{d}\left(-(\sin(dx+c))^{-1}-\frac{1}{3(\sin(dx+c))^3}-(\sin(dx+c))^{-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*(-1/sin(d*x+c)-1/3/sin(d*x+c)^3-1/sin(d*x+c)^2)
```

Maxima [A] time = 1.14513, size = 55, normalized size = 1.83

$$-\frac{3a^2\sin(dx+c)^2+3a^2\sin(dx+c)+a^2}{3d\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*sin(d*x + c)^2 + 3*a^2*sin(d*x + c) + a^2)/(d*sin(d*x + c)^3)
```

Fricas [A] time = 1.55655, size = 130, normalized size = 4.33

$$-\frac{3a^2\cos(dx+c)^2-3a^2\sin(dx+c)-4a^2}{3\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(d*x + c)^2 - 3*a^2*sin(d*x + c) - 4*a^2)/((d*cos(d*x + c)^2
 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16261, size = 55, normalized size = 1.83

$$-\frac{3\,a^2\sin\left(dx+c\right)^2+3\,a^2\sin\left(dx+c\right)+a^2}{3\,d\sin\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*sin(d*x + c)^2 + 3*a^2*sin(d*x + c) + a^2)/(d*sin(d*x + c)^3)
```

3.203 $\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{2d}$$

```
[Out] -(a^2*Csc[c + d*x]^2)/(2*d) - (2*a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d
*x]^4)/(4*d)
```

Rubi [A] time = 0.0663529, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^2)/(2*d) - (2*a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d
*x]^4)/(4*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{a^5(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{a^4\,\text{Subst}\left(\int \frac{(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^4\,\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2a}{x^4}+\frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}\end{aligned}$$

Mathematica [A] time = 0.0290794, size = 55, normalized size = 1.

$$-\frac{a^2 \csc^4(c+dx)}{4d}-\frac{2a^2 \csc^3(c+dx)}{3d}-\frac{a^2 \csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^2)/(2*d) - (2*a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d
*x]^4)/(4*d)
```

Maple [A] time = 0.038, size = 39, normalized size = 0.7

$$\frac{a^2}{d}\left(-\frac{1}{4\left(\sin(dx+c)\right)^4}-\frac{2}{3\left(\sin(dx+c)\right)^3}-\frac{1}{2\left(\sin(dx+c)\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*(-1/4/sin(d*x+c)^4-2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.23087, size = 58, normalized size = 1.05

$$-\frac{6\,a^2\sin(dx+c)^2+8\,a^2\sin(dx+c)+3\,a^2}{12\,d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(6*a^2*sin(d*x + c)^2 + 8*a^2*sin(d*x + c) + 3*a^2)/(d*sin(d*x + c)^4
)
```

Fricas [A] time = 1.62856, size = 138, normalized size = 2.51

$$\frac{6\,a^2\cos(dx+c)^2-8\,a^2\sin(dx+c)-9\,a^2}{12\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^2*cos(d*x + c)^2 - 8*a^2*sin(d*x + c) - 9*a^2)/(d*cos(d*x + c)^4
- 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19858, size = 58, normalized size = 1.05

$$-\frac{6\,a^2 \sin\left(dx+c\right)^2 + 8\,a^2 \sin\left(dx+c\right) + 3\,a^2}{12\,d \sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(6*a^2*sin(d*x + c)^2 + 8*a^2*sin(d*x + c) + 3*a^2)/(d*sin(d*x + c)^4
)
```

3.204 $\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}-\frac{a^2\csc^3(c+dx)}{3d}$$

```
[Out] -(a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x
]^5)/(5*d)
```

Rubi [A] time = 0.0666974, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}-\frac{a^2\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x
]^5)/(5*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a^6(a+x)^2}{x^6}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{a^5\operatorname{Subst}\left(\int \frac{(a+x)^2}{x^6}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^5\operatorname{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{2a}{x^5}+\frac{1}{x^4}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{2d}-\frac{a^2\csc^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.0299349, size = 55, normalized size = 1.

$$-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}-\frac{a^2\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x
]^5)/(5*d)
```

Maple [A] time = 0.039, size = 39, normalized size = 0.7

$$\frac{a^2}{d}\left(-\frac{1}{5\left(\sin\left(dx+c\right)\right)^5}-\frac{1}{2\left(\sin\left(dx+c\right)\right)^4}-\frac{1}{3\left(\sin\left(dx+c\right)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*(-1/5/sin(d*x+c)^5-1/2/sin(d*x+c)^4-1/3/sin(d*x+c)^3)
```

Maxima [A] time = 1.23421, size = 58, normalized size = 1.05

$$-\frac{10\,a^2\sin\left(dx+c\right)^2+15\,a^2\sin\left(dx+c\right)+6\,a^2}{30\,d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(10*a^2*sin(d*x + c)^2 + 15*a^2*sin(d*x + c) + 6*a^2)/(d*sin(d*x + c)
^5)
```

Fricas [A] time = 1.60771, size = 162, normalized size = 2.95

$$\frac{10\,a^2\cos\left(dx+c\right)^2-15\,a^2\sin\left(dx+c\right)-16\,a^2}{30\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*a^2*cos(d*x + c)^2 - 15*a^2*sin(d*x + c) - 16*a^2)/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25664, size = 58, normalized size = 1.05

$$-\frac{10\,a^2\sin\left(dx+c\right)^2+15\,a^2\sin\left(dx+c\right)+6\,a^2}{30\,d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(10*a^2*sin(d*x + c)^2 + 15*a^2*sin(d*x + c) + 6*a^2)/(d*sin(d*x + c)
^5)
```

3.205 $\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{4d}$$

```
[Out] -(a^2*Csc[c + d*x]^4)/(4*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d
*x]^6)/(6*d)
```

Rubi [A] time = 0.0660291, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^4)/(4*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d
*x]^6)/(6*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a^7(a+x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{a^6\operatorname{Subst}\left(\int \frac{(a+x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^6\operatorname{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2a}{x^6}+\frac{1}{x^5}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= -\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^6(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] time = 0.0320567, size = 55, normalized size = 1.

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^4)/(4*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d
*x]^6)/(6*d)
```

Maple [A] time = 0.037, size = 39, normalized size = 0.7

$$\frac{a^2}{d}\left(-\frac{2}{5\left(\sin(dx+c)\right)^5}-\frac{1}{4\left(\sin(dx+c)\right)^4}-\frac{1}{6\left(\sin(dx+c)\right)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*(-2/5/sin(d*x+c)^5-1/4/sin(d*x+c)^4-1/6/sin(d*x+c)^6)
```

Maxima [A] time = 1.15056, size = 58, normalized size = 1.05

$$-\frac{15\,a^2\sin(dx+c)^2+24\,a^2\sin(dx+c)+10\,a^2}{60\,d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(15*a^2*sin(d*x + c)^2 + 24*a^2*sin(d*x + c) + 10*a^2)/(d*sin(d*x + c
)^6)
```

Fricas [A] time = 1.63264, size = 171, normalized size = 3.11

$$-\frac{15\,a^2\cos(dx+c)^2-24\,a^2\sin(dx+c)-25\,a^2}{60\left(d\cos(dx+c)^6-3\,d\cos(dx+c)^4+3\,d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(15*a^2*cos(d*x + c)^2 - 24*a^2*sin(d*x + c) - 25*a^2)/(d*cos(d*x + c
)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2871, size = 58, normalized size = 1.05

$$-\frac{15\,a^2\sin\left(dx+c\right)^2+24\,a^2\sin\left(dx+c\right)+10\,a^2}{60\,d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15*a^2*sin(d*x + c)^2 + 24*a^2*sin(d*x + c) + 10*a^2)/(d*sin(d*x + c
)^6)
```

3.206 $\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}$$

```
[Out] (a^3*Sin[c + d*x]^4)/(4*d) + (3*a^3*Sin[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*
x]^6)/(2*d) + (a^3*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0738102, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^4)/(4*d) + (3*a^3*Sin[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*
x]^6)/(2*d) + (a^3*Sin[c + d*x]^7)/(7*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx&=\frac{\text{Subst}\left(\int \frac{x^3(a+x)^3}{a^3}\,dx,x,a\sin(c+dx)\right)}{ad}\\&=\frac{\text{Subst}\left(\int x^3(a+x)^3\,dx,x,a\sin(c+dx)\right)}{a^4d}\\&=\frac{\text{Subst}\left(\int \left(a^3x^3+3a^2x^4+3ax^5+x^6\right)dx,x,a\sin(c+dx)\right)}{a^4d}\\&=\frac{a^3\sin^4(c+dx)}{4d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{a^3\sin^7(c+d}{7d}\end{aligned}$$

Mathematica [A] time = 0.322901, size = 80, normalized size = 1.1

$$-\frac{a^3(-1015\sin(c+dx)+525\sin(3(c+dx))-119\sin(5(c+dx))+5\sin(7(c+dx))+805\cos(2(c+dx))-280\cos(4(c}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(-350 + 805*Cos[2*(c + d*x)] - 280*Cos[4*(c + d*x)] + 35*Cos[6*(c + d
*x)] - 1015*Sin[c + d*x] + 525*Sin[3*(c + d*x)] - 119*Sin[5*(c + d*x)] + 5*
Sin[7*(c + d*x)]))/(2240*d)
```

Maple [A] time = 0.021, size = 58, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a^3(\sin(dx+c))^7}{7}+\frac{a^3(\sin(dx+c))^6}{2}+\frac{3a^3(\sin(dx+c))^5}{5}+\frac{a^3(\sin(dx+c))^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/7*a^3*sin(d*x+c)^7+1/2*a^3*sin(d*x+c)^6+3/5*a^3*sin(d*x+c)^5+1/4*a^3
*sin(d*x+c)^4)
```

Maxima [A] time = 0.996887, size = 78, normalized size = 1.07

$$\frac{20a^3\sin(dx+c)^7+70a^3\sin(dx+c)^6+84a^3\sin(dx+c)^5+35a^3\sin(dx+c)^4}{140d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(20*a^3*sin(d*x + c)^7 + 70*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)^
5 + 35*a^3*sin(d*x + c)^4)/d
```

Fricas [A] time = 1.71006, size = 244, normalized size = 3.34

$$-\frac{70a^3\cos(dx+c)^6-245a^3\cos(dx+c)^4+280a^3\cos(dx+c)^2+4\left(5a^3\cos(dx+c)^6-36a^3\cos(dx+c)^4+57a^3c\right.}{140d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/140*(70*a^3*cos(d*x + c)^6 - 245*a^3*cos(d*x + c)^4 + 280*a^3*cos(d*x +
c)^2 + 4*(5*a^3*cos(d*x + c)^6 - 36*a^3*cos(d*x + c)^4 + 57*a^3*cos(d*x + c
)^2 - 26*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 13.1465, size = 104, normalized size = 1.42

$$\begin{cases} \frac{a^3\sin^7(c+dx)}{7d} + \frac{a^3\sin^6(c+dx)}{2d} + \frac{3a^3\sin^5(c+dx)}{5d} - \frac{a^3\sin^2(c+dx)\cos^2(c+dx)}{2d} - \frac{a^3\cos^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a\sin(c)+a\right)^3\sin^3(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**7/(7*d) + a**3*sin(c + d*x)**6/(2*d) + 3*a**3
*sin(c + d*x)**5/(5*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**2/(2*d) - a**3*
cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**3*cos(c), Tr
ue))
```

Giac [A] time = 1.27538, size = 78, normalized size = 1.07

$$\frac{20\,a^3\sin(dx+c)^7 + 70\,a^3\sin(dx+c)^6 + 84\,a^3\sin(dx+c)^5 + 35\,a^3\sin(dx+c)^4}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/140*(20*a^3*sin(d*x + c)^7 + 70*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)^
5 + 35*a^3*sin(d*x + c)^4)/d
```

3.207 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}$$

```
[Out] (a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^4)/(4*d) + (3*a^3*Sin[c +
d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.0706032, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^4)/(4*d) + (3*a^3*Sin[c +
d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(6*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+x)^3}{a^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int x^2(a+x)^3\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(a^3x^2+3a^2x^3+3ax^4+x^5\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^6(c+dx)}{6d}\end{aligned}$$

Mathematica [A] time = 0.284775, size = 70, normalized size = 0.96

$$-\frac{a^3(-1200\sin(c+dx)+520\sin(3(c+dx))-72\sin(5(c+dx))+870\cos(2(c+dx))-240\cos(4(c+dx))+10\cos(6(c+}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(-45 + 870*Cos[2*(c + d*x)] - 240*Cos[4*(c + d*x)] + 10*Cos[6*(c + d*
x)] - 1200*Sin[c + d*x] + 520*Sin[3*(c + d*x)] - 72*Sin[5*(c + d*x)]))/(192
0*d)
```

Maple [A] time = 0.02, size = 58, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a^3(\sin(dx+c))^6}{6}+\frac{3a^3(\sin(dx+c))^5}{5}+\frac{3a^3(\sin(dx+c))^4}{4}+\frac{a^3(\sin(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/6*a^3*sin(d*x+c)^6+3/5*a^3*sin(d*x+c)^5+3/4*a^3*sin(d*x+c)^4+1/3*a^3
*sin(d*x+c)^3)
```

Maxima [A] time = 1.12206, size = 78, normalized size = 1.07

$$\frac{10a^3\sin(dx+c)^6+36a^3\sin(dx+c)^5+45a^3\sin(dx+c)^4+20a^3\sin(dx+c)^3}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4
 + 20*a^3*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.64717, size = 209, normalized size = 2.86

$$-\frac{10a^3\cos(dx+c)^6-75a^3\cos(dx+c)^4+120a^3\cos(dx+c)^2-4\left(9a^3\cos(dx+c)^4-23a^3\cos(dx+c)^2+14a^3\right)\sin(}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(10*a^3*cos(d*x + c)^6 - 75*a^3*cos(d*x + c)^4 + 120*a^3*cos(d*x + c)
^2 - 4*(9*a^3*cos(d*x + c)^4 - 23*a^3*cos(d*x + c)^2 + 14*a^3)*sin(d*x + c)
)/d
```

Sympy [A] time = 7.55424, size = 107, normalized size = 1.47

$$\begin{cases} \frac{a^3\sin^6(c+dx)}{6d} + \frac{3a^3\sin^5(c+dx)}{5d} + \frac{a^3\sin^3(c+dx)}{3d} - \frac{3a^3\sin^2(c+dx)\cos^2(c+dx)}{2d} - \frac{3a^3\cos^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a\sin(c)+a\right)^3\sin^2(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**6/(6*d) + 3*a**3*sin(c + d*x)**5/(5*d) + a**3
*sin(c + d*x)**3/(3*d) - 3*a**3*sin(c + d*x)**2*cos(c + d*x)**2/(2*d) - 3*a
**3*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**2*cos(c)
, True))
```

Giac [A] time = 1.20233, size = 78, normalized size = 1.07

$$\frac{10\,a^3\sin(dx+c)^6 + 36\,a^3\sin(dx+c)^5 + 45\,a^3\sin(dx+c)^4 + 20\,a^3\sin(dx+c)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4
 + 20*a^3*sin(d*x + c)^3)/d
```

3.208 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=45

$$\frac{(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^4}{4ad}$$

```
[Out] -(a + a*Sin[c + d*x])^4/(4*a*d) + (a + a*Sin[c + d*x])^5/(5*a^2*d)
```

Rubi [A] time = 0.0453911, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a + a*Sin[c + d*x])^4/(4*a*d) + (a + a*Sin[c + d*x])^5/(5*a^2*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{a}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int x(a+x)^3\,dx,x,a\sin(c+dx)\right)}{a^2d}\\ &= \frac{\text{Subst}\left(\int \left(-a(a+x)^3+(a+x)^4\right)dx,x,a\sin(c+dx)\right)}{a^2d}\\ &= -\frac{(a+a\sin(c+dx))^4}{4ad}+\frac{(a+a\sin(c+dx))^5}{5a^2d}\end{aligned}$$

Mathematica [A] time = 0.118371, size = 30, normalized size = 0.67

$$\frac{a^3(\sin(c+dx)+1)^4(4\sin(c+dx)-1)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^4*(-1 + 4*Sin[c + d*x]))/(20*d)
```

Maple [A] time = 0.016, size = 57, normalized size = 1.3

$$\frac{1}{d}\left(\frac{a^3(\sin(dx+c))^5}{5}+\frac{3a^3(\sin(dx+c))^4}{4}+a^3(\sin(dx+c))^3+\frac{a^3(\sin(dx+c))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*a^3*sin(d*x+c)^5+3/4*a^3*sin(d*x+c)^4+a^3*sin(d*x+c)^3+1/2*a^3*sin
(d*x+c)^2)
```

Maxima [A] time = 1.15551, size = 78, normalized size = 1.73

$$\frac{4a^3\sin(dx+c)^5+15a^3\sin(dx+c)^4+20a^3\sin(dx+c)^3+10a^3\sin(dx+c)^2}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(4*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
+ 10*a^3*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.69621, size = 169, normalized size = 3.76

$$\frac{15a^3\cos(dx+c)^4-40a^3\cos(dx+c)^2+4\left(a^3\cos(dx+c)^4-7a^3\cos(dx+c)^2+6a^3\right)\sin(dx+c)}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/20*(15*a^3*cos(d*x + c)^4 - 40*a^3*cos(d*x + c)^2 + 4*(a^3*cos(d*x + c)^4
 - 7*a^3*cos(d*x + c)^2 + 6*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 4.39767, size = 102, normalized size = 2.27

$$\begin{cases}\frac{a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^3(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)\cos^2(c+dx)}{2d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a^3\cos^2(c+dx)}{2d} & \text{for } d\neq 0\\ x(a\sin(c)+a)^3\sin(c)\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**5/(5*d) + a**3*sin(c + d*x)**3/d - 3*a**3*sin
(c + d*x)**2*cos(c + d*x)**2/(2*d) - 3*a**3*cos(c + d*x)**4/(4*d) - a**3*co
s(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)*cos(c), True))
```

Giac [A] time = 1.27019, size = 78, normalized size = 1.73

$$\frac{4\,a^3\sin\left(dx+c\right)^5+15\,a^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3+10\,a^3\sin\left(dx+c\right)^2}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/20*(4*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
+ 10*a^3*sin(d*x + c)^2)/d
```

3.209 $\int \cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=65

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)
/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0438446, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 43}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)
/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^3}{x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(3a^2+\frac{a^3}{x}+3ax+x^2\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0281867, size = 65, normalized size = 1.

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)
/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.033, size = 62, normalized size = 1.

$$\frac{a^3\ln\left(\sin\left(dx+c\right)\right)}{d}+3\,\frac{a^3\sin\left(dx+c\right)}{d}+\frac{3\,a^3\left(\sin\left(dx+c\right)\right)^2}{2\,d}+\frac{a^3\left(\sin\left(dx+c\right)\right)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3*sin(
d*x+c)^3/d
```

Maxima [A] time = 0.986767, size = 74, normalized size = 1.14

$$\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2+6\,a^3\log\left(\sin\left(dx+c\right)\right)+18\,a^3\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c))
+ 18*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.75093, size = 146, normalized size = 2.25

$$-\frac{9\,a^3\cos\left(dx+c\right)^2-6\,a^3\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+2\left(a^3\cos\left(dx+c\right)^2-10\,a^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a^3*cos(d*x + c)^2 - 6*a^3*log(1/2*sin(d*x + c)) + 2*(a^3*cos(d*x +
 c)^2 - 10*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26631, size = 76, normalized size = 1.17

$$\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2+6\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+18\,a^3\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x +
c))) + 18*a^3*sin(d*x + c))/d
```

3.210 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=62

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x]
)/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0618724, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x]
)/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a^2(a+x)^3}{x^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{a\operatorname{Subst}\left(\int \frac{(a+x)^3}{x^2}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a\operatorname{Subst}\left(\int \left(3a+\frac{a^3}{x^2}+\frac{3a^2}{x}+x\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0292881, size = 62, normalized size = 1.

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x]
)/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.036, size = 63, normalized size = 1.

$$\frac{a^3(\sin(dx+c))^2}{2d}+3\frac{a^3\sin(dx+c)}{d}-\frac{a^3}{d\sin(dx+c)}+3\frac{a^3\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sin(d*x+c)^2/d+3*a^3*sin(d*x+c)/d-1/d*a^3/sin(d*x+c)+3*a^3*ln(sin(d
*x+c))/d
```

Maxima [A] time = 1.10877, size = 73, normalized size = 1.18

$$\frac{a^3\sin(dx+c)^2+6a^3\log(\sin(dx+c))+6a^3\sin(dx+c)-\frac{2a^3}{\sin(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^3*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c)) + 6*a^3*sin(d*x + c) - 2*
a^3/sin(d*x + c))/d
```

Fricas [A] time = 1.75988, size = 193, normalized size = 3.11

$$-\frac{12a^3\cos(dx+c)^2-12a^3\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-8a^3+\left(2a^3\cos(dx+c)^2-a^3\right)\sin(dx+c)}{4d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a^3*cos(d*x + c)^2 - 12*a^3*log(1/2*sin(d*x + c))*sin(d*x + c) - 8
*a^3 + (2*a^3*cos(d*x + c)^2 - a^3)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17326, size = 74, normalized size = 1.19

$$\frac{a^3\sin(dx+c)^2 + 6\,a^3\log(|\sin(dx+c)|) + 6\,a^3\sin(dx+c) - \frac{2\,a^3}{\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x + c))) + 6*a^3*sin(d*x + c)
 - 2*a^3/sin(d*x + c))/d
```

3.211 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=61

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (3*a^3*Log[Sin[c + d
*x]])/d + (a^3*Sin[c + d*x])/d
```

Rubi [A] time = 0.0691863, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (3*a^3*Log[Sin[c + d
*x]])/d + (a^3*Sin[c + d*x])/d
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{a^3(a+x)^3}{x^3}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{a^2\,\text{Subst}\left(\int \frac{(a+x)^3}{x^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^2\,\text{Subst}\left(\int \left(1+\frac{a^3}{x^3}+\frac{3a^2}{x^2}+\frac{3a}{x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{3a^3\log(\sin(c+dx))}{d}+\frac{a^3 s}{}\end{aligned}$$

Mathematica [A] time = 0.020793, size = 53, normalized size = 0.87

$$a^3\left(\frac{\sin(c+dx)}{d}-\frac{\csc^2(c+dx)}{2d}-\frac{3\csc(c+dx)}{d}+\frac{3\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-3*Csc[c + d*x])/d - Csc[c + d*x]^2/(2*d) + (3*Log[Sin[c + d*x]])/d +
 Sin[c + d*x]/d)
```

Maple [A] time = 0.039, size = 62, normalized size = 1.

$$\frac{a^3\sin(dx+c)}{d}-3\,\frac{a^3}{d\sin(dx+c)}+3\,\frac{a^3\ln(\sin(dx+c))}{d}-\frac{a^3}{2\,d\,(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3*sin(d*x+c)/d-3/d*a^3/sin(d*x+c)+3*a^3*ln(sin(d*x+c))/d-1/2/d*a^3/sin(d*
x+c)^2
```

Maxima [A] time = 1.06912, size = 73, normalized size = 1.2

$$\frac{6\,a^3\log(\sin(dx+c))+2\,a^3\sin(dx+c)-\frac{6\,a^3\sin(dx+c)+a^3}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^3*log(sin(d*x + c)) + 2*a^3*sin(d*x + c) - (6*a^3*sin(d*x + c) + a
^3)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.73561, size = 180, normalized size = 2.95

$$\frac{a^3+6\left(a^3\cos(dx+c)^2-a^3\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+2\left(a^3\cos(dx+c)^2+2\,a^3\right)\sin(dx+c)}{2\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3 + 6*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 2*(a^3*cos(
d*x + c)^2 + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21023, size = 74, normalized size = 1.21

$$\frac{6\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+2\,a^3\sin\left(dx+c\right)-\frac{6\,a^3\sin(dx+c)+a^3}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^3*log(abs(sin(d*x + c))) + 2*a^3*sin(d*x + c) - (6*a^3*sin(d*x + c
) + a^3)/sin(d*x + c)^2)/d
```

3.212 $\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=65

$$-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

```
[Out] (-3*a^3*Csc[c + d*x])/d - (3*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^
3)/(3*d) + (a^3*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0707239, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (3*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^
3)/(3*d) + (a^3*Log[Sin[c + d*x]])/d
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{a^4(a+x)^3}{x^4}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{a^3\,\text{Subst}\left(\int \frac{(a+x)^3}{x^4}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^3\,\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2}{x^3}+\frac{3a}{x^2}+\frac{1}{x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{3a^3\csc(c+dx)}{d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}+\frac{a^3\log(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0191501, size = 57, normalized size = 0.88

$$a^3\left(-\frac{\csc^3(c+dx)}{3d}-\frac{3\csc^2(c+dx)}{2d}-\frac{3\csc(c+dx)}{d}+\frac{\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-3*Csc[c + d*x])/d - (3*Csc[c + d*x]^2)/(2*d) - Csc[c + d*x]^3/(3*d)
+ Log[Sin[c + d*x]]/d)
```

Maple [A] time = 0.04, size = 64, normalized size = 1.

$$-3\,\frac{a^3}{d\sin(dx+c)}+\frac{a^3\ln(\sin(dx+c))}{d}-\frac{a^3}{3\,d\,(\sin(dx+c))^3}-\frac{3\,a^3}{2\,d\,(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -3/d*a^3/sin(d*x+c)+a^3*ln(sin(d*x+c))/d-1/3/d*a^3/sin(d*x+c)^3-3/2/d*a^3/s
in(d*x+c)^2
```

Maxima [A] time = 1.0883, size = 78, normalized size = 1.2

$$\frac{6\,a^3\log(\sin(dx+c))-\frac{18\,a^3\sin(dx+c)^2+9\,a^3\sin(dx+c)+2\,a^3}{\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*log(sin(d*x + c)) - (18*a^3*sin(d*x + c)^2 + 9*a^3*sin(d*x + c)
+ 2*a^3)/sin(d*x + c)^3)/d
```

Fricas [A] time = 1.70971, size = 221, normalized size = 3.4

$$-\frac{18\,a^3\cos(dx+c)^2-9\,a^3\sin(dx+c)-20\,a^3-6\left(a^3\cos(dx+c)^2-a^3\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)}{6\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*cos(d*x + c)^2 - 9*a^3*sin(d*x + c) - 20*a^3 - 6*(a^3*cos(d*x
+ c)^2 - a^3)*log(1/2*sin(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*s
in(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23809, size = 80, normalized size = 1.23

$$\frac{6\,a^3\log\left(|\sin\left(dx+c\right)|\right)-\frac{18\,a^3\sin(dx+c)^2+9\,a^3\sin(dx+c)+2\,a^3}{\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(abs(sin(d*x + c))) - (18*a^3*sin(d*x + c)^2 + 9*a^3*sin(d*x
+ c) + 2*a^3)/sin(d*x + c)^3)/d
```

3.213 $\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{4ad}$$

```
[Out] -(Csc[c + d*x]^4*(a + a*Sin[c + d*x])^4)/(4*a*d)
```

Rubi [A] time = 0.0568223, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 37}

$$-\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]^4*(a + a*Sin[c + d*x])^4)/(4*a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{a^5(a+x)^3}{x^5}\,dx, x, a\sin(c+dx)\right)}{ad} \\
&= \frac{a^4\,\text{Subst}\left(\int \frac{(a+x)^3}{x^5}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= -\frac{\csc^4(c+dx)(a+a\sin(c+dx))^4}{4ad}
\end{aligned}$$

Mathematica [A] time = 0.0234424, size = 20, normalized size = 0.67

$$-\frac{a^3(\csc(c+dx)+1)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Csc[c + d*x])^4)/(4*d)
```

Maple [A] time = 0.04, size = 49, normalized size = 1.6

$$\frac{a^3}{d}\left(-(\sin(dx+c))^{-1}-\frac{1}{4(\sin(dx+c))^4}-(\sin(dx+c))^{-3}-\frac{3}{2(\sin(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*(-1/sin(d*x+c)-1/4/sin(d*x+c)^4-1/sin(d*x+c)^3-3/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.01684, size = 73, normalized size = 2.43

$$-\frac{4a^3\sin(dx+c)^3+6a^3\sin(dx+c)^2+4a^3\sin(dx+c)+a^3}{4d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 + 4*a^3*sin(d*x + c) + a^
3)/(d*sin(d*x + c)^4)
```

Fricas [B] time = 1.59412, size = 170, normalized size = 5.67

$$\frac{6a^3\cos(dx+c)^2-7a^3+4\left(a^3\cos(dx+c)^2-2a^3\right)\sin(dx+c)}{4\left(d\cos(dx+c)^4-2d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^3*cos(d*x + c)^2 - 7*a^3 + 4*(a^3*cos(d*x + c)^2 - 2*a^3)*sin(d*x
+ c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18985, size = 73, normalized size = 2.43

$$-\frac{4a^3\sin(dx+c)^3+6a^3\sin(dx+c)^2+4a^3\sin(dx+c)+a^3}{4d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 + 4*a^3*sin(d*x + c) + a^
3)/(d*sin(d*x + c)^4)
```

3.214 $\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=61

$$\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{20ad}-\frac{\csc^5(c+dx)(a\sin(c+dx)+a)^4}{5ad}$$

```
[Out] (Csc[c + d*x]^4*(a + a*Sin[c + d*x])^4)/(20*a*d) - (Csc[c + d*x]^5*(a + a*S
in[c + d*x])^4)/(5*a*d)
```

Rubi [A] time = 0.0649166, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2833, 12, 45, 37}

$$\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{20ad}-\frac{\csc^5(c+dx)(a\sin(c+dx)+a)^4}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^4*(a + a*Sin[c + d*x])^4)/(20*a*d) - (Csc[c + d*x]^5*(a + a*S
in[c + d*x])^4)/(5*a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^5(c+dx)(a+a\sin(c+dx))^4}{5ad} - \frac{a^4\,\text{Subst}\left(\int \frac{(a+x)^3}{x^5}\,dx, x, a\sin\right.}{5d}$$
$$= \frac{\csc^4(c+dx)(a+a\sin(c+dx))^4}{20ad} - \frac{\csc^5(c+dx)(a+a\sin(c+dx)}{5ad}$$

Mathematica [A] time = 0.0295041, size = 71, normalized size = 1.16

$$-\frac{a^3\csc^5(c+dx)}{5d} - \frac{3a^3\csc^4(c+dx)}{4d} - \frac{a^3\csc^3(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^3)/d - (3*a^3*Csc[c + d*x]^
4)/(4*d) - (a^3*Csc[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.041, size = 49, normalized size = 0.8

$$\frac{a^3}{d}\left(-\frac{1}{5\left(\sin(dx+c)\right)^5} - \frac{3}{4\left(\sin(dx+c)\right)^4} - \left(\sin(dx+c)\right)^{-3} - \frac{1}{2\left(\sin(dx+c)\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*(-1/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4-1/sin(d*x+c)^3-1/2/sin(d*x+c)^2
)
```

Maxima [A] time = 1.138, size = 76, normalized size = 1.25

$$-\frac{10\,a^3\sin(dx+c)^3 + 20\,a^3\sin(dx+c)^2 + 15\,a^3\sin(dx+c) + 4\,a^3}{20\,d\sin(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/20*(10*a^3*sin(d*x + c)^3 + 20*a^3*sin(d*x + c)^2 + 15*a^3*sin(d*x + c)
+ 4*a^3)/(d*sin(d*x + c)^5)
```

Fricas [A] time = 1.60963, size = 197, normalized size = 3.23

$$\frac{20\,a^{3}\cos\left(dx+c\right)^{2}-24\,a^{3}+5\left(2\,a^{3}\cos\left(dx+c\right)^{2}-5\,a^{3}\right)\sin\left(dx+c\right)}{20\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/20*(20*a^3*cos(d*x + c)^2 - 24*a^3 + 5*(2*a^3*cos(d*x + c)^2 - 5*a^3)*sin
(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23292, size = 76, normalized size = 1.25

$$-\frac{10\,a^{3}\sin\left(dx+c\right)^{3}+20\,a^{3}\sin\left(dx+c\right)^{2}+15\,a^{3}\sin\left(dx+c\right)+4\,a^{3}}{20\,d\sin\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(10*a^3*sin(d*x + c)^3 + 20*a^3*sin(d*x + c)^2 + 15*a^3*sin(d*x + c)
+ 4*a^3)/(d*sin(d*x + c)^5)
```

3.215 $\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}$$

```
[Out] -(a^3*Csc[c + d*x]^3)/(3*d) - (3*a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c +
 d*x]^5)/(5*d) - (a^3*Csc[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.0719408, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[c + d*x]^3)/(3*d) - (3*a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c +
 d*x]^5)/(5*d) - (a^3*Csc[c + d*x]^6)/(6*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx&=\frac{\text{Subst}\left(\int \frac{a^7(a+x)^3}{x^7}\,dx,x,a\sin(c+dx)\right)}{ad}\\&=\frac{a^6\,\text{Subst}\left(\int \frac{(a+x)^3}{x^7}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{a^6\,\text{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2}{x^6}+\frac{3a}{x^5}+\frac{1}{x^4}\right)dx,x,a\sin(c+dx)\right)}{d}\\&=-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^6}{}\end{aligned}$$

Mathematica [A] time = 0.0303022, size = 73, normalized size = 1.

$$-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[c + d*x]^3)/(3*d) - (3*a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c +
 d*x]^5)/(5*d) - (a^3*Csc[c + d*x]^6)/(6*d)
```

Maple [A] time = 0.041, size = 49, normalized size = 0.7

$$\frac{a^3}{d}\left(-\frac{3}{5\left(\sin\left(dx+c\right)\right)^5}-\frac{3}{4\left(\sin\left(dx+c\right)\right)^4}-\frac{1}{6\left(\sin\left(dx+c\right)\right)^6}-\frac{1}{3\left(\sin\left(dx+c\right)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*(-3/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4-1/6/sin(d*x+c)^6-1/3/sin(d*x+c)
^3)
```

Maxima [A] time = 1.23501, size = 76, normalized size = 1.04

$$-\frac{20\,a^3\sin\left(dx+c\right)^3+45\,a^3\sin\left(dx+c\right)^2+36\,a^3\sin\left(dx+c\right)+10\,a^3}{60\,d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(20*a^3*sin(d*x + c)^3 + 45*a^3*sin(d*x + c)^2 + 36*a^3*sin(d*x + c)
+ 10*a^3)/(d*sin(d*x + c)^6)
```

Fricas [A] time = 1.77253, size = 208, normalized size = 2.85

$$-\frac{45\,a^3\cos\left(dx+c\right)^2-55\,a^3+4\left(5\,a^3\cos\left(dx+c\right)^2-14\,a^3\right)\sin\left(dx+c\right)}{60\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(45*a^3*cos(d*x + c)^2 - 55*a^3 + 4*(5*a^3*cos(d*x + c)^2 - 14*a^3)*s
in(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26843, size = 76, normalized size = 1.04

$$-\frac{20\,a^{3}\sin\left(dx+c\right)^{3}+45\,a^{3}\sin\left(dx+c\right)^{2}+36\,a^{3}\sin\left(dx+c\right)+10\,a^{3}}{60\,d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(20*a^3*sin(d*x + c)^3 + 45*a^3*sin(d*x + c)^2 + 36*a^3*sin(d*x + c)
+ 10*a^3)/(d*sin(d*x + c)^6)
```

3.216 $\int \cot(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$-\frac{a^3\csc^7(c+dx)}{7d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}$$

```
[Out] -(a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a^3*Csc[c + d
*x]^6)/(2*d) - (a^3*Csc[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0728913, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{a^3\csc^7(c+dx)}{7d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a^3*Csc[c + d
*x]^6)/(2*d) - (a^3*Csc[c + d*x]^7)/(7*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{a^8(a+x)^3}{x^8}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{a^7\,\text{Subst}\left(\int \frac{(a+x)^3}{x^8}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^7\,\text{Subst}\left(\int \left(\frac{a^3}{x^8}+\frac{3a^2}{x^7}+\frac{3a}{x^6}+\frac{1}{x^5}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a^3\csc^4(c+dx)}{4d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{a^3\csc^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] time = 0.0297715, size = 73, normalized size = 1.

$$-\frac{a^3\csc^7(c+dx)}{7d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a^3*Csc[c + d
*x]^6)/(2*d) - (a^3*Csc[c + d*x]^7)/(7*d)
```

Maple [A] time = 0.041, size = 49, normalized size = 0.7

$$\frac{a^3}{d}\left(-\frac{1}{7\,(\sin(dx+c))^7}-\frac{3}{5\,(\sin(dx+c))^5}-\frac{1}{4\,(\sin(dx+c))^4}-\frac{1}{2\,(\sin(dx+c))^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*(-1/7/sin(d*x+c)^7-3/5/sin(d*x+c)^5-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)
^6)
```

Maxima [A] time = 1.07243, size = 76, normalized size = 1.04

$$-\frac{35\,a^3\sin(dx+c)^3+84\,a^3\sin(dx+c)^2+70\,a^3\sin(dx+c)+20\,a^3}{140\,d\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/140*(35*a^3*sin(d*x + c)^3 + 84*a^3*sin(d*x + c)^2 + 70*a^3*sin(d*x + c)
 + 20*a^3)/(d*sin(d*x + c)^7)
```

Fricas [A] time = 2.04225, size = 228, normalized size = 3.12

$$-\frac{84\,a^3\cos(dx+c)^2-104\,a^3+35\left(a^3\cos(dx+c)^2-3\,a^3\right)\sin(dx+c)}{140\left(d\cos(dx+c)^6-3\,d\cos(dx+c)^4+3\,d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/140*(84*a^3*cos(d*x + c)^2 - 104*a^3 + 35*(a^3*cos(d*x + c)^2 - 3*a^3)*s
in(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27914, size = 76, normalized size = 1.04

$$-\frac{35 a^3 \sin(dx+c)^3 + 84 a^3 \sin(dx+c)^2 + 70 a^3 \sin(dx+c) + 20 a^3}{140 d \sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/140*(35*a^3*sin(d*x + c)^3 + 84*a^3*sin(d*x + c)^2 + 70*a^3*sin(d*x + c)
 + 20*a^3)/(d*sin(d*x + c)^7)
```

3.217 $\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=91

$$\frac{a^4\sin^9(c+dx)}{9d}+\frac{a^4\sin^8(c+dx)}{2d}+\frac{6a^4\sin^7(c+dx)}{7d}+\frac{2a^4\sin^6(c+dx)}{3d}+\frac{a^4\sin^5(c+dx)}{5d}$$

```
[Out] (a^4*Sin[c + d*x]^5)/(5*d) + (2*a^4*Sin[c + d*x]^6)/(3*d) + (6*a^4*Sin[c +
d*x]^7)/(7*d) + (a^4*Sin[c + d*x]^8)/(2*d) + (a^4*Sin[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0849619, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^4\sin^9(c+dx)}{9d}+\frac{a^4\sin^8(c+dx)}{2d}+\frac{6a^4\sin^7(c+dx)}{7d}+\frac{2a^4\sin^6(c+dx)}{3d}+\frac{a^4\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^5)/(5*d) + (2*a^4*Sin[c + d*x]^6)/(3*d) + (6*a^4*Sin[c +
d*x]^7)/(7*d) + (a^4*Sin[c + d*x]^8)/(2*d) + (a^4*Sin[c + d*x]^9)/(9*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{x^4(a+x)^4}{a^4}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int x^4(a+x)^4\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(a^4x^4+4a^3x^5+6a^2x^6+4ax^7+x^8\right)dx,x,a\sin(c+dx)\right.}{a^5d}\\ &= \frac{a^4\sin^5(c+dx)}{5d}+\frac{2a^4\sin^6(c+dx)}{3d}+\frac{6a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^8(c}{2a}\end{aligned}$$

Mathematica [A] time = 0.812519, size = 100, normalized size = 1.1

$$\frac{a^4(52290\sin(c+dx)-30660\sin(3(c+dx))+9828\sin(5(c+dx))-1395\sin(7(c+dx))+35\sin(9(c+dx))-42840\cos}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(4095 - 42840*Cos[2*(c + d*x)] + 18900*Cos[4*(c + d*x)] - 4200*Cos[6*(
c + d*x)] + 315*Cos[8*(c + d*x)] + 52290*Sin[c + d*x] - 30660*Sin[3*(c + d*
x)] + 9828*Sin[5*(c + d*x)] - 1395*Sin[7*(c + d*x)] + 35*Sin[9*(c + d*x)]))
/(80640*d)
```

Maple [A] time = 0.023, size = 71, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a^4(\sin(dx+c))^9}{9}+\frac{a^4(\sin(dx+c))^8}{2}+\frac{6a^4(\sin(dx+c))^7}{7}+\frac{2a^4(\sin(dx+c))^6}{3}+\frac{a^4(\sin(dx+c))^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/9*a^4*sin(d*x+c)^9+1/2*a^4*sin(d*x+c)^8+6/7*a^4*sin(d*x+c)^7+2/3*a^4
*sin(d*x+c)^6+1/5*a^4*sin(d*x+c)^5)
```

Maxima [A] time = 1.10056, size = 96, normalized size = 1.05

$$\frac{70a^4\sin(dx+c)^9+315a^4\sin(dx+c)^8+540a^4\sin(dx+c)^7+420a^4\sin(dx+c)^6+126a^4\sin(dx+c)^5}{630d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/630*(70*a^4*sin(d*x + c)^9 + 315*a^4*sin(d*x + c)^8 + 540*a^4*sin(d*x + c
)^7 + 420*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)^5)/d
```

Fricas [A] time = 1.98013, size = 324, normalized size = 3.56

$$\frac{315a^4\cos(dx+c)^8-1680a^4\cos(dx+c)^6+3150a^4\cos(dx+c)^4-2520a^4\cos(dx+c)^2+2\left(35a^4\cos(dx+c)^8-410\right.}{630d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/630*(315*a^4*cos(d*x + c)^8 - 1680*a^4*cos(d*x + c)^6 + 3150*a^4*cos(d*x
+ c)^4 - 2520*a^4*cos(d*x + c)^2 + 2*(35*a^4*cos(d*x + c)^8 - 410*a^4*cos(d
*x + c)^6 + 1083*a^4*cos(d*x + c)^4 - 1076*a^4*cos(d*x + c)^2 + 368*a^4)*si
n(d*x + c))/d
```

Sympy [A] time = 24.9305, size = 97, normalized size = 1.07

$$\begin{cases} \frac{a^4 \sin^9 (c+dx)}{9d} + \frac{a^4 \sin^8 (c+dx)}{2d} + \frac{6a^4 \sin^7 (c+dx)}{7d} + \frac{2a^4 \sin^6 (c+dx)}{3d} + \frac{a^4 \sin^5 (c+dx)}{5d} & \text{for } d \neq 0 \\ x\,(a \sin (c) + a)^4 \sin^4 (c) \cos (c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**9/(9*d) + a**4*sin(c + d*x)**8/(2*d) + 6*a**4
*sin(c + d*x)**7/(7*d) + 2*a**4*sin(c + d*x)**6/(3*d) + a**4*sin(c + d*x)**
5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**4*cos(c), True))
```

Giac [A] time = 1.28994, size = 96, normalized size = 1.05

$$\frac{70\,a^4 \sin (dx+c)^9 + 315\,a^4 \sin (dx+c)^8 + 540\,a^4 \sin (dx+c)^7 + 420\,a^4 \sin (dx+c)^6 + 126\,a^4 \sin (dx+c)^5}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/630*(70*a^4*sin(d*x + c)^9 + 315*a^4*sin(d*x + c)^8 + 540*a^4*sin(d*x + c
)^7 + 420*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)^5)/d
```

3.218 $\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=88

$$\frac{a^4\sin^8(c+dx)}{8d}+\frac{4a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^6(c+dx)}{d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{4d}$$

```
[Out] (a^4*Sin[c + d*x]^4)/(4*d) + (4*a^4*Sin[c + d*x]^5)/(5*d) + (a^4*Sin[c + d*
x]^6)/d + (4*a^4*Sin[c + d*x]^7)/(7*d) + (a^4*Sin[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.0809407, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^4\sin^8(c+dx)}{8d}+\frac{4a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^6(c+dx)}{d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^4)/(4*d) + (4*a^4*Sin[c + d*x]^5)/(5*d) + (a^4*Sin[c + d*
x]^6)/d + (4*a^4*Sin[c + d*x]^7)/(7*d) + (a^4*Sin[c + d*x]^8)/(8*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+x)^4}{a^3}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int x^3(a+x)^4\,dx,x,a\sin(c+dx)\right)}{a^4d}\\ &= \frac{\text{Subst}\left(\int \left(a^4x^3+4a^3x^4+6a^2x^5+4ax^6+x^7\right)dx,x,a\sin(c+dx)\right)}{a^4d}\\ &= \frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^6(c+dx)}{d}+\frac{4a^4\sin^7(c+}{7d}\end{aligned}$$

Mathematica [A] time = 0.529192, size = 90, normalized size = 1.02

$$\frac{a^4(87360\sin(c+dx)-47040\sin(3(c+dx))+12096\sin(5(c+dx))-960\sin(7(c+dx))-69720\cos(2(c+dx))+26}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(36400 - 69720*Cos[2*(c + d*x)] + 26460*Cos[4*(c + d*x)] - 4200*Cos[6*
(c + d*x)] + 105*Cos[8*(c + d*x)] + 87360*Sin[c + d*x] - 47040*Sin[3*(c + d
*x)] + 12096*Sin[5*(c + d*x)] - 960*Sin[7*(c + d*x)]))/(107520*d)
```

Maple [A] time = 0.021, size = 70, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a^4(\sin(dx+c))^8}{8}+\frac{4a^4(\sin(dx+c))^7}{7}+a^4(\sin(dx+c))^6+\frac{4a^4(\sin(dx+c))^5}{5}+\frac{a^4(\sin(dx+c))^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/8*a^4*sin(d*x+c)^8+4/7*a^4*sin(d*x+c)^7+a^4*sin(d*x+c)^6+4/5*a^4*sin
(d*x+c)^5+1/4*a^4*sin(d*x+c)^4)
```

Maxima [A] time = 1.11293, size = 96, normalized size = 1.09

$$\frac{35\,a^4\sin(dx+c)^8+160\,a^4\sin(dx+c)^7+280\,a^4\sin(dx+c)^6+224\,a^4\sin(dx+c)^5+70\,a^4\sin(dx+c)^4}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*a^4*sin(d*x + c)^8 + 160*a^4*sin(d*x + c)^7 + 280*a^4*sin(d*x + c
)^6 + 224*a^4*sin(d*x + c)^5 + 70*a^4*sin(d*x + c)^4)/d
```

Fricas [A] time = 2.00981, size = 281, normalized size = 3.19

$$\frac{35\,a^4\cos(dx+c)^8-420\,a^4\cos(dx+c)^6+1120\,a^4\cos(dx+c)^4-1120\,a^4\cos(dx+c)^2-32\left(5\,a^4\cos(dx+c)^6-22\,a\right.}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/280*(35*a^4*cos(d*x + c)^8 - 420*a^4*cos(d*x + c)^6 + 1120*a^4*cos(d*x +
c)^4 - 1120*a^4*cos(d*x + c)^2 - 32*(5*a^4*cos(d*x + c)^6 - 22*a^4*cos(d*x
+ c)^4 + 29*a^4*cos(d*x + c)^2 - 12*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 18.5593, size = 119, normalized size = 1.35

$$\begin{cases} \frac{a^4\sin^8(c+dx)}{8d} + \frac{4a^4\sin^7(c+dx)}{7d} + \frac{a^4\sin^6(c+dx)}{d} + \frac{4a^4\sin^5(c+dx)}{5d} - \frac{a^4\sin^2(c+dx)\cos^2(c+dx)}{2d} - \frac{a^4\cos^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a\sin(c)+a\right)^4\sin^3(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**8/(8*d) + 4*a**4*sin(c + d*x)**7/(7*d) + a**4
*sin(c + d*x)**6/d + 4*a**4*sin(c + d*x)**5/(5*d) - a**4*sin(c + d*x)**2*co
s(c + d*x)**2/(2*d) - a**4*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) +
 a)**4*sin(c)**3*cos(c), True))
```

Giac [A] time = 1.22331, size = 96, normalized size = 1.09

$$\frac{35\,a^4\sin(dx+c)^8 + 160\,a^4\sin(dx+c)^7 + 280\,a^4\sin(dx+c)^6 + 224\,a^4\sin(dx+c)^5 + 70\,a^4\sin(dx+c)^4}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(35*a^4*sin(d*x + c)^8 + 160*a^4*sin(d*x + c)^7 + 280*a^4*sin(d*x + c
)^6 + 224*a^4*sin(d*x + c)^5 + 70*a^4*sin(d*x + c)^4)/d
```

3.219 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=67

$$\frac{(a\sin(c+dx)+a)^7}{7a^3d}-\frac{(a\sin(c+dx)+a)^6}{3a^2d}+\frac{(a\sin(c+dx)+a)^5}{5ad}$$

```
[Out] (a + a*Sin[c + d*x])^5/(5*a*d) - (a + a*Sin[c + d*x])^6/(3*a^2*d) + (a + a*
Sin[c + d*x])^7/(7*a^3*d)
```

Rubi [A] time = 0.0753, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{(a\sin(c+dx)+a)^7}{7a^3d}-\frac{(a\sin(c+dx)+a)^6}{3a^2d}+\frac{(a\sin(c+dx)+a)^5}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a + a*Sin[c + d*x])^5/(5*a*d) - (a + a*Sin[c + d*x])^6/(3*a^2*d) + (a + a*
Sin[c + d*x])^7/(7*a^3*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+x)^4}{a^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int x^2(a+x)^4\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(a^2(a+x)^4-2a(a+x)^5+(a+x)^6\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{(a+a\sin(c+dx))^5}{5ad}-\frac{(a+a\sin(c+dx))^6}{3a^2d}+\frac{(a+a\sin(c+dx))^7}{7a^3d}
\end{aligned}$$

Mathematica [A] time = 0.332869, size = 80, normalized size = 1.19

$$-\frac{a^4(-7245\sin(c+dx)+3395\sin(3(c+dx))-609\sin(5(c+dx))+15\sin(7(c+dx))+5460\cos(2(c+dx))-1680\cos(4}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(a^4*(-630 + 5460*Cos[2*(c + d*x)] - 1680*Cos[4*(c + d*x)] + 140*Cos[6*(c
+ d*x)] - 7245*Sin[c + d*x] + 3395*Sin[3*(c + d*x)] - 609*Sin[5*(c + d*x)]
+ 15*Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.022, size = 70, normalized size = 1.

$$\frac{1}{d}\left(\frac{a^4(\sin(dx+c))^7}{7}+\frac{2a^4(\sin(dx+c))^6}{3}+\frac{6a^4(\sin(dx+c))^5}{5}+a^4(\sin(dx+c))^4+\frac{a^4(\sin(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/7*a^4*sin(d*x+c)^7+2/3*a^4*sin(d*x+c)^6+6/5*a^4*sin(d*x+c)^5+a^4*sin
(d*x+c)^4+1/3*a^4*sin(d*x+c)^3)
```

Maxima [A] time = 1.12522, size = 96, normalized size = 1.43

$$\frac{15a^4\sin(dx+c)^7+70a^4\sin(dx+c)^6+126a^4\sin(dx+c)^5+105a^4\sin(dx+c)^4+35a^4\sin(dx+c)^3}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/105*(15*a^4*sin(d*x + c)^7 + 70*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)
^5 + 105*a^4*sin(d*x + c)^4 + 35*a^4*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.95417, size = 247, normalized size = 3.69

$$-\frac{70a^4\cos(dx+c)^6-315a^4\cos(dx+c)^4+420a^4\cos(dx+c)^2+\left(15a^4\cos(dx+c)^6-171a^4\cos(dx+c)^4+332a^4\text{co}\right.}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(70*a^4*cos(d*x + c)^6 - 315*a^4*cos(d*x + c)^4 + 420*a^4*cos(d*x +
c)^2 + (15*a^4*cos(d*x + c)^6 - 171*a^4*cos(d*x + c)^4 + 332*a^4*cos(d*x +
c)^2 - 176*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 10.2798, size = 119, normalized size = 1.78

$$\begin{cases} \frac{a^4 \sin^7(c+dx)}{7d} + \frac{2a^4 \sin^6(c+dx)}{3d} + \frac{6a^4 \sin^5(c+dx)}{5d} + \frac{a^4 \sin^3(c+dx)}{3d} - \frac{2a^4 \sin^2(c+dx)\cos^2(c+dx)}{d} - \frac{a^4 \cos^4(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^4 \sin^2(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**7/(7*d) + 2*a**4*sin(c + d*x)**6/(3*d) + 6*a*
*4*sin(c + d*x)**5/(5*d) + a**4*sin(c + d*x)**3/(3*d) - 2*a**4*sin(c + d*x)
**2*cos(c + d*x)**2/d - a**4*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a*sin(c) + a
)**4*sin(c)**2*cos(c), True))
```

Giac [A] time = 1.18746, size = 96, normalized size = 1.43

$$\frac{15\,a^4 \sin(dx+c)^7 + 70\,a^4 \sin(dx+c)^6 + 126\,a^4 \sin(dx+c)^5 + 105\,a^4 \sin(dx+c)^4 + 35\,a^4 \sin(dx+c)^3}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*a^4*sin(d*x + c)^7 + 70*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)
^5 + 105*a^4*sin(d*x + c)^4 + 35*a^4*sin(d*x + c)^3)/d
```

3.220 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=45

$$\frac{(a\sin(c+dx)+a)^6}{6a^2d}-\frac{(a\sin(c+dx)+a)^5}{5ad}$$

```
[Out] -(a + a*Sin[c + d*x])^5/(5*a*d) + (a + a*Sin[c + d*x])^6/(6*a^2*d)
```

Rubi [A] time = 0.0448015, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{(a\sin(c+dx)+a)^6}{6a^2d}-\frac{(a\sin(c+dx)+a)^5}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(a + a*Sin[c + d*x])^5/(5*a*d) + (a + a*Sin[c + d*x])^6/(6*a^2*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+x)^4}{a}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int x(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(-a(a+x)^4+(a+x)^5\right)dx, x, a\sin(c+dx)\right)}{a^2d}\\
&= -\frac{(a+a\sin(c+dx))^5}{5ad}+\frac{(a+a\sin(c+dx))^6}{6a^2d}
\end{aligned}$$

Mathematica [A] time = 0.0946878, size = 30, normalized size = 0.67

$$\frac{a^4(\sin(c+dx)+1)^5(5\sin(c+dx)-1)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^5*(-1 + 5*Sin[c + d*x]))/(30*d)
```

Maple [A] time = 0.019, size = 71, normalized size = 1.6

$$\frac{1}{d}\left(\frac{a^4(\sin(dx+c))^6}{6}+\frac{4a^4(\sin(dx+c))^5}{5}+\frac{3a^4(\sin(dx+c))^4}{2}+\frac{4a^4(\sin(dx+c))^3}{3}+\frac{a^4(\sin(dx+c))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/6*a^4*sin(d*x+c)^6+4/5*a^4*sin(d*x+c)^5+3/2*a^4*sin(d*x+c)^4+4/3*a^4
*sin(d*x+c)^3+1/2*a^4*sin(d*x+c)^2)
```

Maxima [A] time = 1.09017, size = 96, normalized size = 2.13

$$\frac{5a^4\sin(dx+c)^6+24a^4\sin(dx+c)^5+45a^4\sin(dx+c)^4+40a^4\sin(dx+c)^3+15a^4\sin(dx+c)^2}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/30*(5*a^4*sin(d*x + c)^6 + 24*a^4*sin(d*x + c)^5 + 45*a^4*sin(d*x + c)^4
+ 40*a^4*sin(d*x + c)^3 + 15*a^4*sin(d*x + c)^2)/d
```

Fricas [B] time = 1.36852, size = 207, normalized size = 4.6

$$-\frac{5a^4\cos(dx+c)^6-60a^4\cos(dx+c)^4+120a^4\cos(dx+c)^2-8\left(3a^4\cos(dx+c)^4-11a^4\cos(dx+c)^2+8a^4\right)\sin(}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/30*(5*a^4*cos(d*x + c)^6 - 60*a^4*cos(d*x + c)^4 + 120*a^4*cos(d*x + c)^
2 - 8*(3*a^4*cos(d*x + c)^4 - 11*a^4*cos(d*x + c)^2 + 8*a^4)*sin(d*x + c))/
d
```

Sympy [A] time = 5.22655, size = 121, normalized size = 2.69

$$\begin{cases} \frac{a^4\sin^6(c+dx)}{6d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{4a^4\sin^3(c+dx)}{3d}-\frac{3a^4\sin^2(c+dx)\cos^2(c+dx)}{d}-\frac{3a^4\cos^4(c+dx)}{2d}-\frac{a^4\cos^2(c+dx)}{2d} & \text{for } d\neq 0 \\ x\left(a\sin(c)+a\right)^4\sin(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**6/(6*d) + 4*a**4*sin(c + d*x)**5/(5*d) + 4*a*
*4*sin(c + d*x)**3/(3*d) - 3*a**4*sin(c + d*x)**2*cos(c + d*x)**2/d - 3*a**
4*cos(c + d*x)**4/(2*d) - a**4*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(
c) + a)**4*sin(c)*cos(c), True))
```

Giac [A] time = 1.25758, size = 96, normalized size = 2.13

$$\frac{5\,a^4\sin(dx+c)^6+24\,a^4\sin(dx+c)^5+45\,a^4\sin(dx+c)^4+40\,a^4\sin(dx+c)^3+15\,a^4\sin(dx+c)^2}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(5*a^4*sin(d*x + c)^6 + 24*a^4*sin(d*x + c)^5 + 45*a^4*sin(d*x + c)^4
+ 40*a^4*sin(d*x + c)^3 + 15*a^4*sin(d*x + c)^2)/d
```

3.221 $\int \cot(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=81

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\log(\sin(c+dx))}{d}$$

```
[Out] (a^4*Log[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (3*a^4*Sin[c + d*x]^2)
/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0472288, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 43}

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Log[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (3*a^4*Sin[c + d*x]^2)
/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(4a^3+\frac{a^4}{x}+6a^2x+4ax^2+x^3\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^4\log(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}+
\end{aligned}$$

Mathematica [A] time = 0.0365896, size = 81, normalized size = 1.

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Log[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (3*a^4*Sin[c + d*x]^2)
/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.032, size = 78, normalized size = 1.

$$\frac{a^4 \ln(\sin(dx+c))}{d} + 4\frac{a^4 \sin(dx+c)}{d} + 3\frac{a^4 (\sin(dx+c))^2}{d} + \frac{4a^4 (\sin(dx+c))^3}{3d} + \frac{a^4 (\sin(dx+c))^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x)
```

```
[Out] a^4*ln(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+3*a^4*sin(d*x+c)^2/d+4/3*a^4*sin(d*
x+c)^3/d+1/4*a^4*sin(d*x+c)^4/d
```

Maxima [A] time = 1.14212, size = 92, normalized size = 1.14

$$\frac{3a^4 \sin(dx+c)^4 + 16a^4 \sin(dx+c)^3 + 36a^4 \sin(dx+c)^2 + 12a^4 \log(\sin(dx+c)) + 48a^4 \sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 36*a^4*sin(d*x + c)^2
+ 12*a^4*log(sin(d*x + c)) + 48*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.45256, size = 180, normalized size = 2.22

$$\frac{3a^4 \cos(dx+c)^4 - 42a^4 \cos(dx+c)^2 + 12a^4 \log\left(\frac{1}{2}\sin(dx+c)\right) - 16\left(a^4 \cos(dx+c)^2 - 4a^4\right)\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*cos(d*x + c)^4 - 42*a^4*cos(d*x + c)^2 + 12*a^4*log(1/2*sin(d*x
 + c)) - 16*(a^4*cos(d*x + c)^2 - 4*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19535, size = 93, normalized size = 1.15

$$\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+36\,a^4\sin\left(dx+c\right)^2+12\,a^4\log\left(|\sin\left(dx+c\right)|\right)+48\,a^4\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 36*a^4*sin(d*x + c)^2
+ 12*a^4*log(abs(sin(d*x + c))) + 48*a^4*sin(d*x + c))/d
```

3.222 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=78

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{6a^4\sin(c+dx)}{d}-\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}$$

```
[Out] -((a^4*Csc[c + d*x])/d) + (4*a^4*Log[Sin[c + d*x]])/d + (6*a^4*Sin[c + d*x]
)/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0662441, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{6a^4\sin(c+dx)}{d}-\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((a^4*Csc[c + d*x])/d) + (4*a^4*Log[Sin[c + d*x]])/d + (6*a^4*Sin[c + d*x]
)/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*Sin[c + d*x]^3)/(3*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{a^2(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a\,\text{Subst}\left(\int \left(6a^2+\frac{a^4}{x^2}+\frac{4a^3}{x}+4ax+x^2\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= -\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}+\frac{6a^4\sin(c+dx)}{d}+\frac{2a^4\sin^2(}{d}
\end{aligned}$$

Mathematica [A] time = 0.0368254, size = 78, normalized size = 1.

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{6a^4\sin(c+dx)}{d}-\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((a^4*Csc[c + d*x])/d) + (4*a^4*Log[Sin[c + d*x]])/d + (6*a^4*Sin[c + d*x]
)/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.037, size = 79, normalized size = 1.

$$\frac{a^4(\sin(dx+c))^3}{3d}+2\frac{a^4(\sin(dx+c))^2}{d}+6\frac{a^4\sin(dx+c)}{d}-\frac{a^4}{d\sin(dx+c)}+4\frac{a^4\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/3*a^4*sin(d*x+c)^3/d+2*a^4*sin(d*x+c)^2/d+6*a^4*sin(d*x+c)/d-1/d*a^4/sin(
d*x+c)+4*a^4*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.15289, size = 90, normalized size = 1.15

$$\frac{a^4\sin(dx+c)^3+6a^4\sin(dx+c)^2+12a^4\log(\sin(dx+c))+18a^4\sin(dx+c)-\frac{3a^4}{\sin(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 12*a^4*log(sin(d*x + c)) +
 18*a^4*sin(d*x + c) - 3*a^4/sin(d*x + c))/d
```

Fricas [A] time = 1.53351, size = 224, normalized size = 2.87

$$\frac{a^4\cos(dx+c)^4-20a^4\cos(dx+c)^2+12a^4\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+16a^4-3\left(2a^4\cos(dx+c)^2-a^4\right)\sin(}{3d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*cos(d*x + c)^4 - 20*a^4*cos(d*x + c)^2 + 12*a^4*log(1/2*sin(d*x +
c))*sin(d*x + c) + 16*a^4 - 3*(2*a^4*cos(d*x + c)^2 - a^4)*sin(d*x + c))/(d
*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26127, size = 92, normalized size = 1.18

$$\frac{a^4 \sin(dx+c)^3 + 6\,a^4 \sin(dx+c)^2 + 12\,a^4 \log(|\sin(dx+c)|) + 18\,a^4 \sin(dx+c) - \frac{3\,a^4}{\sin(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 12*a^4*log(abs(sin(d*x + c
))) + 18*a^4*sin(d*x + c) - 3*a^4/sin(d*x + c))/d
```

3.223 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=80

$$\frac{a^4\sin^2(c+dx)}{2d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{6a^4\log(\sin(c+dx))}{d}$$

```
[Out] (-4*a^4*Csc[c + d*x])/d - (a^4*Csc[c + d*x]^2)/(2*d) + (6*a^4*Log[Sin[c + d
*x]])/d + (4*a^4*Sin[c + d*x])/d + (a^4*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0761459, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^4\sin^2(c+dx)}{2d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{6a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a^4*Csc[c + d*x])/d - (a^4*Csc[c + d*x]^2)/(2*d) + (6*a^4*Log[Sin[c + d
*x]])/d + (4*a^4*Sin[c + d*x])/d + (a^4*Sin[c + d*x]^2)/(2*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{a^3(a+x)^4}{x^3}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{a^2\,\text{Subst}\left(\int \frac{(a+x)^4}{x^3}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^2\,\text{Subst}\left(\int \left(4a+\frac{a^4}{x^3}+\frac{4a^3}{x^2}+\frac{6a^2}{x}+x\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{4a^4\csc(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}+\frac{6a^4\log(\sin(c+dx))}{d}+\frac{4a^4}{}
\end{aligned}$$

Mathematica [A] time = 0.0722376, size = 54, normalized size = 0.68

$$-\frac{a^4\left(-\sin^2(c+dx)-8\sin(c+dx)+\csc^2(c+dx)+8\csc(c+dx)-12\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(a^4*(8*Csc[c + d*x] + Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] - 8*Sin[c + d
*x] - Sin[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.044, size = 79, normalized size = 1.

$$\frac{a^4(\sin(dx+c))^2}{2d}+4\frac{a^4\sin(dx+c)}{d}-4\frac{a^4}{d\sin(dx+c)}+6\frac{a^4\ln(\sin(dx+c))}{d}-\frac{a^4}{2d(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*sin(d*x+c)^2/d+4*a^4*sin(d*x+c)/d-4/d*a^4/sin(d*x+c)+6*a^4*ln(sin(d
*x+c))/d-1/2/d*a^4/sin(d*x+c)^2
```

Maxima [A] time = 1.06831, size = 89, normalized size = 1.11

$$\frac{a^4\sin(dx+c)^2+12a^4\log(\sin(dx+c))+8a^4\sin(dx+c)-\frac{8a^4\sin(dx+c)+a^4}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*(a^4*sin(d*x + c)^2 + 12*a^4*log(sin(d*x + c)) + 8*a^4*sin(d*x + c) - (
8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.41718, size = 232, normalized size = 2.9

$$-\frac{2a^4\cos(dx+c)^4-16a^4\cos(dx+c)^2\sin(dx+c)-3a^4\cos(dx+c)^2-a^4-24\left(a^4\cos(dx+c)^2-a^4\right)\log\left(\frac{1}{2}\sin(dx+\right.}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^4*cos(d*x + c)^4 - 16*a^4*cos(d*x + c)^2*sin(d*x + c) - 3*a^4*cos
(d*x + c)^2 - a^4 - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*sin(d*x + c)))/(d
*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17421, size = 90, normalized size = 1.12

$$\frac{a^4\sin\left(dx+c\right)^2+12\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)+8\,a^4\sin\left(dx+c\right)-\frac{8\,a^4\sin(dx+c)+a^4}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(a^4*sin(d*x + c)^2 + 12*a^4*log(abs(sin(d*x + c))) + 8*a^4*sin(d*x + c
) - (8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2)/d
```

3.224 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] Log[1 + Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d) -
 Sin[c + d*x]^3/(3*a*d) + Sin[c + d*x]^4/(4*a*d)
```

Rubi [A] time = 0.0866874, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d) -
 Sin[c + d*x]^3/(3*a*d) + Sin[c + d*x]^4/(4*a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{a^4(a+x)}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x^4}{a+x}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\operatorname{Subst}\left(\int \left(-a^3+a^2x-ax^2+x^3+\frac{a^4}{a+x}\right)dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\log(1+\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^4(c+dx)}{4ad}
\end{aligned}$$

Mathematica [A] time = 0.129097, size = 60, normalized size = 0.71

$$\frac{3\sin^4(c+dx)-4\sin^3(c+dx)+6\sin^2(c+dx)-12\sin(c+dx)+12\log(\sin(c+dx)+1)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (12*Log[1 + Sin[c + d*x]] - 12*Sin[c + d*x] + 6*Sin[c + d*x]^2 - 4*Sin[c +
d*x]^3 + 3*Sin[c + d*x]^4)/(12*a*d)
```

Maple [A] time = 0.024, size = 80, normalized size = 0.9

$$\frac{\ln(1+\sin(dx+c))}{da}-\frac{\sin(dx+c)}{da}+\frac{(\sin(dx+c))^2}{2da}-\frac{(\sin(dx+c))^3}{3da}+\frac{(\sin(dx+c))^4}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(1+sin(d*x+c))/a/d-sin(d*x+c)/d/a+1/2*sin(d*x+c)^2/d/a-1/3*sin(d*x+c)^3/d
/a+1/4*sin(d*x+c)^4/d/a
```

Maxima [A] time = 1.07133, size = 85, normalized size = 1.

$$\frac{\frac{3\sin(dx+c)^4-4\sin(dx+c)^3+6\sin(dx+c)^2-12\sin(dx+c)}{a}+\frac{12\log(\sin(dx+c)+1)}{a}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 12*sin(d*x
+ c))/a + 12*log(sin(d*x + c) + 1)/a)/d
```

Fricas [A] time = 1.49545, size = 157, normalized size = 1.85

$$\frac{3\cos(dx+c)^4-12\cos(dx+c)^2+4\left(\cos(dx+c)^2-4\right)\sin(dx+c)+12\log(\sin(dx+c)+1)}{12ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 4)*sin(d*x
 + c) + 12*log(sin(d*x + c) + 1))/(a*d)
```

Sympy [A] time = 5.59529, size = 102, normalized size = 1.2

$$\begin{cases} \frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad} - \frac{\sin^{3}\left(c+dx\right)}{3ad} - \frac{\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{2ad} - \frac{\sin\left(c+dx\right)}{ad} - \frac{\cos^{4}\left(c+dx\right)}{4ad} - \frac{\cos^{2}\left(c+dx\right)}{2ad} & \text{for}\ d \neq 0 \\ \frac{x\sin^{4}\left(c\right)\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d) - sin(c + d*x)**3/(3*a*d) - sin(c +
d*x)**2*cos(c + d*x)**2/(2*a*d) - sin(c + d*x)/(a*d) - cos(c + d*x)**4/(4*a
*d) - cos(c + d*x)**2/(2*a*d), Ne(d, 0)), (x*sin(c)**4*cos(c)/(a*sin(c) + a
), True))
```

Giac [A] time = 1.29529, size = 103, normalized size = 1.21

$$\frac{\frac{12\ \log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a} + \frac{3\,a^{3}\sin\left(dx+c\right)^{4} - 4\,a^{3}\sin\left(dx+c\right)^{3} + 6\,a^{3}\sin\left(dx+c\right)^{2} - 12\,a^{3}\sin\left(dx+c\right)}{a^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*log(abs(sin(d*x + c) + 1))/a + (3*a^3*sin(d*x + c)^4 - 4*a^3*sin(d
*x + c)^3 + 6*a^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c))/a^4)/d
```

3.225 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] -(Log[1 + Sin[c + d*x]]/(a*d)) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d
) + Sin[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0781027, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Log[1 + Sin[c + d*x]]/(a*d)) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d
) + Sin[c + d*x]^3/(3*a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \frac{x^3}{a+x}\,dx,x,a\sin(c+dx)\right)}{a^4d}\\
&= \frac{\text{Subst}\left(\int \left(a^2-ax+x^2-\frac{a^3}{a+x}\right)dx,x,a\sin(c+dx)\right)}{a^4d}\\
&= -\frac{\log(1+\sin(c+dx))}{ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] time = 0.103932, size = 50, normalized size = 0.75

$$\frac{2\sin^3(c+dx)-3\sin^2(c+dx)+6\sin(c+dx)-6\log(\sin(c+dx)+1)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-6*Log[1 + Sin[c + d*x]] + 6*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 2*Sin[c + d
*x]^3)/(6*a*d)
```

Maple [A] time = 0.023, size = 64, normalized size = 1.

$$-\frac{\ln(1+\sin(dx+c))}{da}+\frac{\sin(dx+c)}{da}-\frac{(\sin(dx+c))^2}{2da}+\frac{(\sin(dx+c))^3}{3da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -ln(1+sin(d*x+c))/a/d+sin(d*x+c)/d/a-1/2*sin(d*x+c)^2/d/a+1/3*sin(d*x+c)^3/
d/a
```

Maxima [A] time = 1.1263, size = 72, normalized size = 1.07

$$\frac{\frac{2\sin(dx+c)^3-3\sin(dx+c)^2+6\sin(dx+c)}{a}-\frac{6\log(\sin(dx+c)+1)}{a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 6*sin(d*x + c))/a - 6*log(sin(d
*x + c) + 1)/a)/d
```

Fricas [A] time = 1.48379, size = 127, normalized size = 1.9

$$\frac{3\cos(dx+c)^2-2\left(\cos(dx+c)^2-4\right)\sin(dx+c)-6\log(\sin(dx+c)+1)}{6ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 4)*sin(d*x + c) - 6*log(sin(d*x
 + c) + 1))/(a*d)
```

Sympy [A] time = 3.40311, size = 66, normalized size = 0.99

$$\begin{cases} -\frac{\log(\sin(c+dx)+1)}{ad} + \frac{\sin^3(c+dx)}{3ad} + \frac{\sin(c+dx)}{ad} + \frac{\cos^2(c+dx)}{2ad} & \text{for } d \neq 0 \\ \frac{x\sin^3(c)\cos(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**3/(3*a*d) + sin(c +
 d*x)/(a*d) + cos(c + d*x)**2/(2*a*d), Ne(d, 0)), (x*sin(c)**3*cos(c)/(a*si
n(c) + a), True))
```

Giac [A] time = 1.23687, size = 86, normalized size = 1.28

$$-\frac{\frac{6\log(|\sin(dx+c)+1|)}{a} - \frac{2a^2\sin(dx+c)^3 - 3a^2\sin(dx+c)^2 + 6a^2\sin(dx+c)}{a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(sin(d*x + c) + 1))/a - (2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*
x + c)^2 + 6*a^2*sin(d*x + c))/a^3)/d
```

3.226 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] Log[1 + Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.0699884, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{a+x}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(-a+x+\frac{a^2}{a+x}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\log(1+\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.0591142, size = 38, normalized size = 0.78

$$\frac{\sin^2(c+dx)-2\sin(c+dx)+2\log(\sin(c+dx)+1)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*Log[1 + Sin[c + d*x]] - 2*Sin[c + d*x] + Sin[c + d*x]^2)/(2*a*d)
```

Maple [A] time = 0.02, size = 48, normalized size = 1.

$$\frac{\ln(1+\sin(dx+c))}{da}-\frac{\sin(dx+c)}{da}+\frac{(\sin(dx+c))^2}{2\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(1+sin(d*x+c))/a/d-sin(d*x+c)/d/a+1/2*sin(d*x+c)^2/d/a
```

Maxima [A] time = 0.976022, size = 55, normalized size = 1.12

$$\frac{\frac{\sin(dx+c)^2-2\,\sin(dx+c)}{a}+\frac{2\,\log(\sin(dx+c)+1)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((sin(d*x + c)^2 - 2*sin(d*x + c))/a + 2*log(sin(d*x + c) + 1)/a)/d
```

Fricas [A] time = 1.33008, size = 97, normalized size = 1.98

$$-\frac{\cos(dx+c)^2-2\,\log(\sin(dx+c)+1)+2\,\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(d*x + c)^2 - 2*log(sin(d*x + c) + 1) + 2*sin(d*x + c))/(a*d)
```

Sympy [A] time = 1.45616, size = 53, normalized size = 1.08

$$\begin{cases}\frac{\log(\sin(c+dx)+1)}{ad}-\frac{\sin(c+dx)}{ad}-\frac{\cos^2(c+dx)}{2ad} & \text{for } d\neq 0\\ \frac{x\sin^2(c)\cos(c)}{a\sin(c)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d) - sin(c + d*x)/(a*d) - cos(c + d*x)*
*2/(2*a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a), True))
```

Giac [A] time = 1.25012, size = 61, normalized size = 1.24

$$\frac{\frac{2\,\log\left(|\sin\left(dx+c\right)+1|\right)}{a}+\frac{a\sin\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)}{a^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c) + 1))/a + (a*sin(d*x + c)^2 - 2*a*sin(d*x + c))
/a^2)/d
```

3.227 $\int \frac{\cos(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\sin(c+dx)}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] -(Log[1 + Sin[c + d*x]]/(a*d)) + Sin[c + d*x]/(a*d)
```

Rubi [A] time = 0.0460874, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Log[1 + Sin[c + d*x]]/(a*d)) + Sin[c + d*x]/(a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a(a+x)}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \frac{x}{a+x}\,dx,x,a\sin(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a+x}\right)dx,x,a\sin(c+dx)\right)}{a^2d}\\
&= -\frac{\log(1+\sin(c+dx))}{ad}+\frac{\sin(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0202694, size = 25, normalized size = 0.81

$$\frac{\sin(c+dx)-\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-Log[1 + Sin[c + d*x]] + Sin[c + d*x])/(a*d)
```

Maple [A] time = 0.02, size = 32, normalized size = 1.

$$-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{da}+\frac{\sin\left(dx+c\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -ln(1+sin(d*x+c))/a/d+sin(d*x+c)/d/a
```

Maxima [A] time = 1.07792, size = 41, normalized size = 1.32

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a}-\frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c) + 1)/a - sin(d*x + c)/a)/d
```

Fricas [A] time = 1.36042, size = 63, normalized size = 2.03

$$-\frac{\log\left(\sin\left(dx+c\right)+1\right)-\sin\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(log(sin(d*x + c) + 1) - sin(d*x + c))/(a*d)
```

Sympy [A] time = 0.967933, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin\left(c+dx\right)}{ad} & \text{for } d\neq 0 \\ \frac{x\sin\left(c\right)\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)/(a*d), Ne(d, 0)), (x
*sin(c)*cos(c)/(a*sin(c) + a), True))
```

Giac [A] time = 1.32545, size = 42, normalized size = 1.35

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(sin(d*x + c) + 1))/a - sin(d*x + c)/a)/d
```

3.228 $\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0359275, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2707, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a\sin(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0155676, size = 32, normalized size = 1.

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Maple [A] time = 0.033, size = 33, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{da}-\frac{\ln(1+\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.06519, size = 42, normalized size = 1.31

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a}-\frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a)/d
```

Fricas [A] time = 1.46932, size = 74, normalized size = 2.31

$$\frac{\log\left(\frac{1}{2}\sin(dx+c)\right)-\log(\sin(dx+c)+1)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (log(1/2*sin(d*x + c)) - log(sin(d*x + c) + 1))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos(c+dx)\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.27787, size = 45, normalized size = 1.41

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a)/d
```

3.229 $\int \frac{\cot(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=46

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*
d)
```

Rubi [A] time = 0.0617029, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 44}

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*
d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= -\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(1+\sin(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] time = 0.032843, size = 46, normalized size = 1.

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*
d)
```

Maple [A] time = 0.037, size = 49, normalized size = 1.1

$$\frac{\ln(1+\sin(dx+c))}{da}-\frac{1}{da\sin(dx+c)}-\frac{\ln(\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(1+sin(d*x+c))/a/d-1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.03582, size = 58, normalized size = 1.26

$$\frac{\frac{\log(\sin(dx+c)+1)}{a}-\frac{\log(\sin(dx+c))}{a}-\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a - 1/(a*sin(d*x + c)))/d
```

Fricas [A] time = 1.5621, size = 134, normalized size = 2.91

$$-\frac{\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-\log(\sin(dx+c)+1)\sin(dx+c)+1}{ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(log(1/2*sin(d*x + c))*sin(d*x + c) - log(sin(d*x + c) + 1)*sin(d*x + c) +
 1)/(a*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos(c+dx)\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.22712, size = 61, normalized size = 1.33

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)|)}{a} - \frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a - 1/(a*sin(d*x + c
)))/d
```

3.230 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=63

$$-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) + Log[Sin[c + d*x]]/(a*d) - Log
[1 + Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0786564, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) + Log[Sin[c + d*x]]/(a*d) - Log
[1 + Sin[c + d*x]]/(a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{a^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{a^2x^2}+\frac{1}{a^3x}-\frac{1}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0356259, size = 63, normalized size = 1.

$$-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) + Log[Sin[c + d*x]]/(a*d) - Log
[1 + Sin[c + d*x]]/(a*d)
```

Maple [A] time = 0.04, size = 64, normalized size = 1.

$$-\frac{\ln(1+\sin(dx+c))}{da}-\frac{1}{2\,da\,(\sin(dx+c))^2}+\frac{\ln(\sin(dx+c))}{da}+\frac{1}{da\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -ln(1+sin(d*x+c))/a/d-1/2/d/a/sin(d*x+c)^2+ln(sin(d*x+c))/a/d+1/d/a/sin(d*x
+c)
```

Maxima [A] time = 1.09842, size = 74, normalized size = 1.17

$$-\frac{\frac{2\log(\sin(dx+c)+1)}{a}-\frac{2\log(\sin(dx+c))}{a}-\frac{2\sin(dx+c)-1}{a\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c) + 1)/a - 2*log(sin(d*x + c))/a - (2*sin(d*x + c) -
 1)/(a*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.58783, size = 198, normalized size = 3.14

$$\frac{2\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-2\left(\cos(dx+c)^2-1\right)\log(\sin(dx+c)+1)-2\sin(dx+c)+1}{2\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 2*(cos(d*x + c)^2 - 1)*
log(sin(d*x + c) + 1) - 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos(c+dx)\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.30154, size = 77, normalized size = 1.22

$$-\frac{\frac{2\log(|\sin(dx+c)+1|)}{a}-\frac{2\log(|\sin(dx+c)|)}{a}-\frac{2\sin(dx+c)-1}{a\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a - 2*log(abs(sin(d*x + c)))/a - (2*sin(
d*x + c) - 1)/(a*sin(d*x + c)^2))/d
```

3.231 $\int \frac{\cot(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) - L
og[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.082485, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) - L
og[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{a^4}{x^4(a+x)}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{a^3\,\text{Subst}\left(\int \frac{1}{x^4(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{1}{a^2x^3}+\frac{1}{a^3x^2}-\frac{1}{a^4x}+\frac{1}{a^4(a+x)}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(1+\sin(c+d
\end{aligned}$$

Mathematica [A] time = 0.0440093, size = 82, normalized size = 1.

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) - L
og[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*d)
```

Maple [A] time = 0.043, size = 81, normalized size = 1.

$$\frac{\ln(1+\sin(dx+c))}{da}-\frac{1}{3\,da\,(\sin(dx+c))^3}-\frac{1}{da\sin(dx+c)}+\frac{1}{2\,da\,(\sin(dx+c))^2}-\frac{\ln(\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(1+sin(d*x+c))/a/d-1/3/d/a/sin(d*x+c)^3-1/d/a/sin(d*x+c)+1/2/d/a/sin(d*x+
c)^2-ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.10643, size = 88, normalized size = 1.07

$$\frac{\frac{6\log(\sin(dx+c)+1)}{a}-\frac{6\log(\sin(dx+c))}{a}-\frac{6\sin(dx+c)^2-3\sin(dx+c)+2}{a\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(sin(d*x + c) + 1)/a - 6*log(sin(d*x + c))/a - (6*sin(d*x + c)^2
- 3*sin(d*x + c) + 2)/(a*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.47274, size = 281, normalized size = 3.43

$$-\frac{6\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-6\left(\cos(dx+c)^2-1\right)\log(\sin(dx+c)+1)\sin(dx+c)+6\cos(d}{6\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*sin(d*x + c) - 6*(cos(d*
x + c)^2 - 1)*log(sin(d*x + c) + 1)*sin(d*x + c) + 6*cos(d*x + c)^2 + 3*sin
(d*x + c) - 8)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos(c+dx)\csc^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.19256, size = 90, normalized size = 1.1

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a} - \frac{6\log(|\sin(dx+c)|)}{a} - \frac{6\sin(dx+c)^2-3\sin(dx+c)+2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c)))/a - (6*sin(d
*x + c)^2 - 3*sin(d*x + c) + 2)/(a*sin(d*x + c)^3))/d
```

3.232 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{\sin^3(c+dx)}{3a^2d}-\frac{\sin^2(c+dx)}{a^2d}+\frac{3\sin(c+dx)}{a^2d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^2*d) + (3*Sin[c + d*x])/(a^2*d) - Sin[c + d*x
]^2/(a^2*d) + Sin[c + d*x]^3/(3*a^2*d) - 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0885047, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^3(c+dx)}{3a^2d}-\frac{\sin^2(c+dx)}{a^2d}+\frac{3\sin(c+dx)}{a^2d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^2*d) + (3*Sin[c + d*x])/(a^2*d) - Sin[c + d*x
]^2/(a^2*d) + Sin[c + d*x]^3/(3*a^2*d) - 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2 - 2ax + x^2 + \frac{a^4}{(a+x)^2} - \frac{4a^3}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^2 d} + \frac{3\sin(c+dx)}{a^2 d} - \frac{\sin^2(c+dx)}{a^2 d} + \frac{\sin^3(c+dx)}{3a^2 d} - \frac{}{d\left(a^2 + a^2 \right.}$$

Mathematica [A] time = 0.564839, size = 73, normalized size = 0.84

$$\frac{\sin^3(c+dx) - 3\sin^2(c+dx) + 9\sin(c+dx) - 12\log(\sin(c+dx)+1) - \frac{3}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}}{3a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-12*Log[1 + Sin[c + d*x]] - 3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 9*
Sin[c + d*x] - 3*Sin[c + d*x]^2 + Sin[c + d*x]^3)/(3*a^2*d)
```

Maple [A] time = 0.034, size = 83, normalized size = 1.

$$\frac{(\sin(dx+c))^3}{3a^2 d} - \frac{(\sin(dx+c))^2}{a^2 d} + 3\frac{\sin(dx+c)}{a^2 d} - \frac{1}{a^2 d(1+\sin(dx+c))} - 4\frac{\ln(1+\sin(dx+c))}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/3*sin(d*x+c)^3/a^2/d-sin(d*x+c)^2/a^2/d+3*sin(d*x+c)/a^2/d-1/d/a^2/(1+sin
(d*x+c))-4*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.12249, size = 95, normalized size = 1.09

$$-\frac{\frac{3}{a^2\sin(dx+c)+a^2} - \frac{\sin(dx+c)^3 - 3\sin(dx+c)^2 + 9\sin(dx+c)}{a^2} + \frac{12\log(\sin(dx+c)+1)}{a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3/(a^2*sin(d*x + c) + a^2) - (sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 9*s
in(d*x + c))/a^2 + 12*log(sin(d*x + c) + 1)/a^2)/d
```

Fricas [A] time = 1.46774, size = 220, normalized size = 2.53

$$\frac{2\cos(dx+c)^4-16\cos(dx+c)^2-24(\sin(dx+c)+1)\log(\sin(dx+c)+1)+\left(4\cos(dx+c)^2+17\right)\sin(dx+c)+11}{6\left(a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c)^4 - 16*cos(d*x + c)^2 - 24*(sin(d*x + c) + 1)*log(sin(d
*x + c) + 1) + (4*cos(d*x + c)^2 + 17)*sin(d*x + c) + 11)/(a^2*d*sin(d*x +
c) + a^2*d)
```

Sympy [A] time = 4.7202, size = 238, normalized size = 2.74

$$\begin{cases} -\frac{12\log(\sin(c+dx)+1)\sin(c+dx)}{3a^2d\sin(c+dx)+3a^2d} - \frac{12\log(\sin(c+dx)+1)}{3a^2d\sin(c+dx)+3a^2d} + \frac{\sin^4(c+dx)}{3a^2d\sin(c+dx)+3a^2d} + \frac{8\sin^2(c+dx)}{3a^2d\sin(c+dx)+3a^2d} + \frac{2\sin(c+dx)\cos^2(c+dx)}{3a^2d\sin(c+dx)+3a^2d} + \frac{2\cos^2(c+dx)}{3a^2d\sin(c+dx)+3a^2d} \\ \frac{x\sin^4(c)\cos(c)}{(a\sin(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-12*log(sin(c + d*x) + 1)*sin(c + d*x)/(3*a**2*d*sin(c + d*x) +
3*a**2*d) - 12*log(sin(c + d*x) + 1)/(3*a**2*d*sin(c + d*x) + 3*a**2*d) + s
in(c + d*x)**4/(3*a**2*d*sin(c + d*x) + 3*a**2*d) + 8*sin(c + d*x)**2/(3*a*
*2*d*sin(c + d*x) + 3*a**2*d) + 2*sin(c + d*x)*cos(c + d*x)**2/(3*a**2*d*si
n(c + d*x) + 3*a**2*d) + 2*cos(c + d*x)**2/(3*a**2*d*sin(c + d*x) + 3*a**2*
d) - 12/(3*a**2*d*sin(c + d*x) + 3*a**2*d), Ne(d, 0)), (x*sin(c)**4*cos(c)/
(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.26145, size = 144, normalized size = 1.66

$$-\frac{\frac{(a\sin(dx+c)+a)^3\left(\frac{6a}{a\sin(dx+c)+a}-\frac{18a^2}{(a\sin(dx+c)+a)^2}-1\right)}{a^5}-\frac{12\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}+\frac{3}{(a\sin(dx+c)+a)a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*((a*sin(d*x + c) + a)^3*(6*a/(a*sin(d*x + c) + a) - 18*a^2/(a*sin(d*x
+ c) + a)^2 - 1)/a^5 - 12*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)
^2*abs(a)))/a^2 + 3/((a*sin(d*x + c) + a)*a))/d
```

3.233 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] (3*Log[1 + Sin[c + d*x]])/(a^2*d) - (2*Sin[c + d*x])/(a^2*d) + Sin[c + d*x]
^2/(2*a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0801053, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*Log[1 + Sin[c + d*x]])/(a^2*d) - (2*Sin[c + d*x])/(a^2*d) + Sin[c + d*x]
^2/(2*a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+x-\frac{a^3}{(a+x)^2}+\frac{3a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{3\log(1+\sin(c+dx))}{a^2 d} - \frac{2\sin(c+dx)}{a^2 d} + \frac{\sin^2(c+dx)}{2a^2 d} + \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.17847, size = 71, normalized size = 1.01

$$\frac{\sin^3(c+dx)-3\sin^2(c+dx)+\sin(c+dx)(6\log(\sin(c+dx)+1)-4)+6\log(\sin(c+dx)+1)+2}{2a^2 d(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2 + 6*Log[1 + Sin[c + d*x]] + (-4 + 6*Log[1 + Sin[c + d*x]])*Sin[c + d*x]
- 3*Sin[c + d*x]^2 + Sin[c + d*x]^3)/(2*a^2*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.035, size = 66, normalized size = 0.9

$$\frac{(\sin(dx+c))^2}{2a^2 d} - 2\frac{\sin(dx+c)}{a^2 d} + \frac{1}{a^2 d(1+\sin(dx+c))} + 3\frac{\ln(1+\sin(dx+c))}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2*sin(d*x+c)^2/a^2/d-2*sin(d*x+c)/a^2/d+1/d/a^2/(1+sin(d*x+c))+3*ln(1+sin
(d*x+c))/a^2/d
```

Maxima [A] time = 1.13884, size = 80, normalized size = 1.14

$$\frac{\frac{2}{a^2\sin(dx+c)+a^2} + \frac{\sin(dx+c)^2-4\sin(dx+c)}{a^2} + \frac{6\log(\sin(dx+c)+1)}{a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2/(a^2*sin(d*x + c) + a^2) + (sin(d*x + c)^2 - 4*sin(d*x + c))/a^2 + 6
*log(sin(d*x + c) + 1)/a^2)/d
```

Fricas [A] time = 1.41799, size = 190, normalized size = 2.71

$$\frac{6\cos\left(dx+c\right)^{2}+12\left(\sin\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(2\cos\left(dx+c\right)^{2}+7\right)\sin\left(dx+c\right)-3}{4\left(a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(d*x + c)^2 + 12*(sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (2*co
s(d*x + c)^2 + 7)*sin(d*x + c) - 3)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A] time = 2.67568, size = 209, normalized size = 2.99

$$\begin{cases} \frac{6\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{2a^{2}d\sin\left(c+dx\right)+2a^{2}d}+\frac{6\log\left(\sin\left(c+dx\right)+1\right)}{2a^{2}d\sin\left(c+dx\right)+2a^{2}d}+\frac{4\sin^{3}\left(c+dx\right)}{2a^{2}d\sin\left(c+dx\right)+2a^{2}d}+\frac{3\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{2a^{2}d\sin\left(c+dx\right)+2a^{2}d}+\frac{3\cos^{2}\left(c+dx\right)}{2a^{2}d\sin\left(c+dx\right)+2a^{2}d}+\frac{6}{2a^{2}d\sin\left(c+\right.} \\ \frac{x\sin^{3}\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((6*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**2*d*sin(c + d*x) + 2*
a**2*d) + 6*log(sin(c + d*x) + 1)/(2*a**2*d*sin(c + d*x) + 2*a**2*d) + 4*si
n(c + d*x)**3/(2*a**2*d*sin(c + d*x) + 2*a**2*d) + 3*sin(c + d*x)*cos(c + d
*x)**2/(2*a**2*d*sin(c + d*x) + 2*a**2*d) + 3*cos(c + d*x)**2/(2*a**2*d*sin
(c + d*x) + 2*a**2*d) + 6/(2*a**2*d*sin(c + d*x) + 2*a**2*d), Ne(d, 0)), (x
*sin(c)**3*cos(c)/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.33541, size = 122, normalized size = 1.74

$$-\frac{\frac{\left(a\sin\left(dx+c\right)+a\right)^{2}\left(\frac{6a}{a\sin\left(dx+c\right)+a}-1\right)}{a^{4}}+\frac{6\log\left(\frac{\left|a\sin\left(dx+c\right)+a\right|}{\left(a\sin\left(dx+c\right)+a\right)^{2}\left|a\right|}\right)}{a^{2}}-\frac{2}{\left(a\sin\left(dx+c\right)+a\right)a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((a*sin(d*x + c) + a)^2*(6*a/(a*sin(d*x + c) + a) - 1)/a^4 + 6*log(abs
(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a^2 - 2/((a*sin(d*x +
 c) + a)*a))/d
```

3.234 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$\frac{\sin(c+dx)}{a^2d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] (-2*Log[1 + Sin[c + d*x]])/(a^2*d) + Sin[c + d*x]/(a^2*d) - 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rubi [A] time = 0.0725106, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{a^2d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Log[1 + Sin[c + d*x]])/(a^2*d) + Sin[c + d*x]/(a^2*d) - 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{(a+x)^2}-\frac{2a}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= -\frac{2\log(1+\sin(c+dx))}{a^2d} + \frac{\sin(c+dx)}{a^2d} - \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.187079, size = 55, normalized size = 1.06

$$\frac{4\sin(c+dx) - 8\log(\sin(c+dx)+1) - \frac{4}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-8*Log[1 + Sin[c + d*x]] - 4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 4*S
in[c + d*x])/(4*a^2*d)
```

Maple [A] time = 0.033, size = 50, normalized size = 1.

$$\frac{\sin(dx+c)}{a^2d} - \frac{1}{a^2d(1+\sin(dx+c))} - 2\frac{\ln(1+\sin(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] sin(d*x+c)/a^2/d-1/d/a^2/(1+sin(d*x+c))-2*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.08563, size = 63, normalized size = 1.21

$$-\frac{\frac{1}{a^2\sin(dx+c)+a^2} + \frac{2\log(\sin(dx+c)+1)}{a^2} - \frac{\sin(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(1/(a^2*sin(d*x + c) + a^2) + 2*log(sin(d*x + c) + 1)/a^2 - sin(d*x + c)/a
^2)/d
```

Fricas [A] time = 1.4947, size = 146, normalized size = 2.81

$$-\frac{\cos\left(dx+c\right)^{2}+2\left(\sin\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-\sin\left(dx+c\right)}{a^{2}d\sin\left(dx+c\right)+a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 + 2*(sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - sin(d*x + c
))/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A] time = 1.55332, size = 185, normalized size = 3.56

$$\begin{cases} -\frac{2\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}-\frac{2\log\left(\sin\left(c+dx\right)+1\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}-\frac{\sin^{3}\left(c+dx\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}-\frac{\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}-\frac{\cos^{2}\left(c+dx\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}-\frac{2}{a^{2}d\sin\left(c+dx\right)+a^{2}d} \\ \frac{x\sin^{2}\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-2*log(sin(c + d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**
2*d) - 2*log(sin(c + d*x) + 1)/(a**2*d*sin(c + d*x) + a**2*d) - sin(c + d*x
)**3/(a**2*d*sin(c + d*x) + a**2*d) - sin(c + d*x)*cos(c + d*x)**2/(a**2*d*
sin(c + d*x) + a**2*d) - cos(c + d*x)**2/(a**2*d*sin(c + d*x) + a**2*d) - 2
/(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) +
 a)**2, True))
```

Giac [A] time = 1.29634, size = 95, normalized size = 1.83

$$\frac{\frac{2\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^{2}|a|}\right)}{a^{2}}+\frac{a\sin(dx+c)+a}{a^{3}}-\frac{1}{(a\sin(dx+c)+a)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a^2 + (a*si
n(d*x + c) + a)/a^3 - 1/((a*sin(d*x + c) + a)*a))/d
```

3.235 $\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=37

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.048023, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{(a+x)^2}+\frac{1}{a+x}\right)dx,x,a\sin(c+dx)\right)}{a^2d}\\
&= \frac{\log(1+\sin(c+dx))}{a^2d}+\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.0278844, size = 27, normalized size = 0.73

$$\frac{\frac{1}{\sin(c+dx)+1}+\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*d)
```

Maple [A] time = 0.03, size = 35, normalized size = 1.

$$\frac{1}{da^2\left(1+\sin\left(dx+c\right)\right)}+\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2/(1+sin(d*x+c))+ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.40492, size = 46, normalized size = 1.24

$$\frac{\frac{1}{a^2\sin(dx+c)+a^2}+\frac{\log(\sin(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*sin(d*x + c) + a^2) + log(sin(d*x + c) + 1)/a^2)/d
```

Fricas [A] time = 1.43138, size = 104, normalized size = 2.81

$$\frac{\left(\sin\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)+1}{a^2d\sin\left(dx+c\right)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A] time = 1.02931, size = 95, normalized size = 2.57

$$\begin{cases}\frac{\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{a^2d\sin\left(c+dx\right)+a^2d}+\frac{\log\left(\sin\left(c+dx\right)+1\right)}{a^2d\sin\left(c+dx\right)+a^2d}+\frac{1}{a^2d\sin\left(c+dx\right)+a^2d} & \text{for}\ d\neq 0\\ \frac{x\sin\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d
) + log(sin(c + d*x) + 1)/(a**2*d*sin(c + d*x) + a**2*d) + 1/(a**2*d*sin(c
+ d*x) + a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.28639, size = 76, normalized size = 2.05

$$-\frac{\frac{\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a}-\frac{1}{a\sin(dx+c)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a - 1/(a*sin
(d*x + c) + a))/(a*d)
```

3.236 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rubi [A] time = 0.0485757, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```


Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{1}{a(a+x)^2}-\frac{1}{a^2(a+x)}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(1+\sin(c+dx))}{a^2d}+\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.0542632, size = 36, normalized size = 0.69

$$\frac{\frac{1}{\sin(c+dx)+1}+\log(\sin(c+dx))-\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Log[Sin[c + d*x]] - Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*
d)
```

Maple [A] time = 0.043, size = 50, normalized size = 1.

$$\frac{1}{da^2(1+\sin(dx+c))}-\frac{\ln(1+\sin(dx+c))}{da^2}+\frac{\ln(\sin(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^2/d+ln(sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.08983, size = 62, normalized size = 1.19

$$\frac{\frac{1}{a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*sin(d*x + c) + a^2) - log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c))
/a^2)/d
```

Fricas [A] time = 1.44698, size = 162, normalized size = 3.12

$$\frac{(\sin(dx+c)+1)\log\left(\frac{1}{2}\sin(dx+c)\right)-(\sin(dx+c)+1)\log(\sin(dx+c)+1)+1}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) - (sin(d*x + c) + 1)*log(sin(d*x
+ c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos(c+dx)\csc(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1),
x)/a**2
```

Giac [A] time = 1.24873, size = 61, normalized size = 1.17

$$\frac{a\left(\frac{\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^3}+\frac{1}{(a\sin(dx+c)+a)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a*(log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^3 + 1/((a*sin(d*x + c) + a)*a^2)
)/d
```

3.237 $\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}+\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*Log[Sin[c + d*x]])/(a^2*d) + (2*Log[1 + Sin[c
+ d*x]])/(a^2*d) - 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0741999, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 44}

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}+\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*Log[Sin[c + d*x]])/(a^2*d) + (2*Log[1 + Sin[c
+ d*x]])/(a^2*d) - 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2}{a^3x} + \frac{1}{a^2(a+x)^2} + \frac{2}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{\csc(c+dx)}{a^2d} - \frac{2\log(\sin(c+dx))}{a^2d} + \frac{2\log(1+\sin(c+dx))}{a^2d} - \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.153453, size = 45, normalized size = 0.66

$$-\frac{\frac{1}{\sin(c+dx)+1} + \csc(c+dx) + 2\log(\sin(c+dx)) - 2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Csc[c + d*x] + 2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (1 + Sin[
c + d*x])^(-1))/(a^2*d))
```

Maple [A] time = 0.046, size = 68, normalized size = 1.

$$-\frac{1}{da^2(1+\sin(dx+c))} + 2\frac{\ln(1+\sin(dx+c))}{da^2} - \frac{1}{da^2\sin(dx+c)} - 2\frac{\ln(\sin(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+sin(d*x+c))+2*ln(1+sin(d*x+c))/a^2/d-1/d/a^2/sin(d*x+c)-2*ln(si
n(d*x+c))/a^2/d
```

Maxima [A] time = 1.09386, size = 92, normalized size = 1.35

$$-\frac{\frac{2\sin(dx+c)+1}{a^2\sin(dx+c)^2+a^2\sin(dx+c)} - \frac{2\log(\sin(dx+c)+1)}{a^2} + \frac{2\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((2*sin(d*x + c) + 1)/(a^2*sin(d*x + c)^2 + a^2*sin(d*x + c)) - 2*log(sin(
d*x + c) + 1)/a^2 + 2*log(sin(d*x + c))/a^2)/d
```

Fricas [A] time = 1.41018, size = 269, normalized size = 3.96

$$-\frac{2\left(\cos\left(dx+c\right)^{2}-\sin\left(dx+c\right)-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-2\left(\cos\left(dx+c\right)^{2}-\sin\left(dx+c\right)-1\right)\log\left(\sin\left(dx+c\right)+1\right)-2}{a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\sin\left(dx+c\right)-a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*(cos(d*x + c)^2 - sin(d*x + c) - 1)*log(1/2*sin(d*x + c)) - 2*(cos(d*x
+ c)^2 - sin(d*x + c) - 1)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) - 1)/(a^2
*d*cos(d*x + c)^2 - a^2*d*sin(d*x + c) - a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos\left(c+dx\right)\csc^{2}\left(c+dx\right)}{\sin^{2}\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A] time = 1.28909, size = 93, normalized size = 1.37

$$-\frac{\frac{2\log\left(\left|-\frac{a}{a\sin\left(dx+c\right)+a}+1\right|\right)}{a^{2}}+\frac{1}{\left(a\sin\left(dx+c\right)+a\right)a}-\frac{1}{a^{2}\left(\frac{a}{a\sin\left(dx+c\right)+a}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^2 + 1/((a*sin(d*x + c) + a)*a)
- 1/(a^2*(a/(a*sin(d*x + c) + a) - 1)))/d
```

3.238 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{3\log(\sin(c+dx))}{a^2d}-\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (3*Log[Sin[c + d*x]])
/(a^2*d) - (3*Log[1 + Sin[c + d*x]])/(a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]
))
```

Rubi [A] time = 0.0874987, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{3\log(\sin(c+dx))}{a^2d}-\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (3*Log[Sin[c + d*x]])
/(a^2*d) - (3*Log[1 + Sin[c + d*x]])/(a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]
))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{a^2x^3} - \frac{2}{a^3x^2} + \frac{3}{a^4x} - \frac{1}{a^3(a+x)^2} - \frac{3}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2a^2d} + \frac{3\log(\sin(c+dx))}{a^2d} - \frac{3\log(1+\sin(c+dx))}{a^2d} + \frac{}{d\left(a^2\right.}$$

Mathematica [A] time = 0.188544, size = 61, normalized size = 0.72

$$\frac{\frac{2}{\sin(c+dx)+1} - \csc^2(c+dx) + 4\csc(c+dx) + 6\log(\sin(c+dx)) - 6\log(\sin(c+dx)+1)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 6*Log[Sin[c + d*x]] - 6*Log[1 + Sin[c + d*x]] + 2/(1 + Sin[c + d*x]))/(2*a^2*d)
```

Maple [A] time = 0.052, size = 83, normalized size = 1.

$$\frac{1}{da^2(1+\sin(dx+c))} - 3\frac{\ln(1+\sin(dx+c))}{da^2} - \frac{1}{2da^2(\sin(dx+c))^2} + 2\frac{1}{da^2\sin(dx+c)} + 3\frac{\ln(\sin(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2/(1+sin(d*x+c))-3*ln(1+sin(d*x+c))/a^2/d-1/2/d/a^2/sin(d*x+c)^2+2/d/a^2/sin(d*x+c)+3*ln(sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.15811, size = 108, normalized size = 1.27

$$\frac{\frac{6\sin(dx+c)^2+3\sin(dx+c)-1}{a^2\sin(dx+c)^3+a^2\sin(dx+c)^2} - \frac{6\log(\sin(dx+c)+1)}{a^2} + \frac{6\log(\sin(dx+c))}{a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((6*sin(d*x + c)^2 + 3*sin(d*x + c) - 1)/(a^2*sin(d*x + c)^3 + a^2*sin(d*x + c)^2) - 6*log(sin(d*x + c) + 1)/a^2 + 6*log(sin(d*x + c))/a^2)/d
```

Fricas [A] time = 1.49914, size = 389, normalized size = 4.58

$$\frac{6 \cos(dx+c)^2 + 6\left(\cos(dx+c)^2 + \left(\cos(dx+c)^2 - 1\right)\sin(dx+c) - 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 6\left(\cos(dx+c)^2 + \left(\cos(dx\right.\right.}{2\left(a^2 d\cos(dx+c)^2 - a^2 d + \left(a^2 d\cos(dx+c)^2 - a^2 d\right)\sin(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*cos(d*x + c)^2 + 6*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - 1)*log(1/2*sin(d*x + c)) - 6*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*si
n(d*x + c) - 1)*log(sin(d*x + c) + 1) - 3*sin(d*x + c) - 5)/(a^2*d*cos(d*x
+ c)^2 - a^2*d + (a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos(c+dx)\csc^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A] time = 1.2036, size = 117, normalized size = 1.38

$$\frac{\frac{6\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^2} + \frac{2}{(a\sin(dx+c)+a)a} - \frac{\frac{6a}{a\sin(dx+c)+a}-5}{a^2\left(\frac{a}{a\sin(dx+c)+a}-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(6*log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^2 + 2/((a*sin(d*x + c) + a)*
a) - (6*a/(a*sin(d*x + c) + a) - 5)/(a^2*(a/(a*sin(d*x + c) + a) - 1)^2))/d
```

3.239 $\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{a^2d}-\frac{3\csc(c+dx)}{a^2d}-\frac{4\log(\sin(c+dx))}{a^2d}+\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] (-3*Csc[c + d*x])/(a^2*d) + Csc[c + d*x]^2/(a^2*d) - Csc[c + d*x]^3/(3*a^2*
d) - (4*Log[Sin[c + d*x]])/(a^2*d) + (4*Log[1 + Sin[c + d*x]])/(a^2*d) - 1/
(d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0986431, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{a^2d}-\frac{3\csc(c+dx)}{a^2d}-\frac{4\log(\sin(c+dx))}{a^2d}+\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*Csc[c + d*x])/(a^2*d) + Csc[c + d*x]^2/(a^2*d) - Csc[c + d*x]^3/(3*a^2*
d) - (4*Log[Sin[c + d*x]])/(a^2*d) + (4*Log[1 + Sin[c + d*x]])/(a^2*d) - 1/
(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^4}{x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{1}{x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{a^2x^4} - \frac{2}{a^3x^3} + \frac{3}{a^4x^2} - \frac{4}{a^5x} + \frac{1}{a^4(a+x)^2} + \frac{4}{a^5(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3\csc(c+dx)}{a^2d} + \frac{\csc^2(c+dx)}{a^2d} - \frac{\csc^3(c+dx)}{3a^2d} - \frac{4\log(\sin(c+dx))}{a^2d} + \frac{4\log(1+\sin(c+}{a^2d}$$

Mathematica [A] time = 2.21165, size = 98, normalized size = 0.97

$$-\frac{1}{a^2d(\sin(c+dx)+1)} - \frac{\csc^3(c+dx)}{3a^2d} + \frac{\csc^2(c+dx)}{a^2d} - \frac{3\csc(c+dx)}{a^2d} - \frac{4\log(\sin(c+dx))}{a^2d} + \frac{4\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*Csc[c + d*x])/(a^2*d) + Csc[c + d*x]^2/(a^2*d) - Csc[c + d*x]^3/(3*a^2*
d) - (4*Log[Sin[c + d*x]])/(a^2*d) + (4*Log[1 + Sin[c + d*x]])/(a^2*d) - 1/
(a^2*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.055, size = 99, normalized size = 1.

$$-\frac{1}{da^2(1+\sin(dx+c))} + 4\frac{\ln(1+\sin(dx+c))}{da^2} - \frac{1}{3da^2(\sin(dx+c))^3} + \frac{1}{da^2(\sin(dx+c))^2} - 3\frac{1}{da^2\sin(dx+c)} - 4\frac{\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+sin(d*x+c))+4*ln(1+sin(d*x+c))/a^2/d-1/3/d/a^2/sin(d*x+c)^3+1/d
/a^2/sin(d*x+c)^2-3/d/a^2/sin(d*x+c)-4*ln(sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.11096, size = 122, normalized size = 1.21

$$-\frac{\frac{12\sin(dx+c)^3+6\sin(dx+c)^2-2\sin(dx+c)+1}{a^2\sin(dx+c)^4+a^2\sin(dx+c)^3} - \frac{12\log(\sin(dx+c)+1)}{a^2} + \frac{12\log(\sin(dx+c))}{a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*((12*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 2*sin(d*x + c) + 1)/(a^2*sin(
d*x + c)^4 + a^2*sin(d*x + c)^3) - 12*log(sin(d*x + c) + 1)/a^2 + 12*log(si
n(d*x + c))/a^2)/d
```

Fricas [A] time = 1.53725, size = 508, normalized size = 5.03

$$\frac{6 \cos(dx+c)^2 - 12\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 - \left(\cos(dx+c)^2 - 1\right)\sin(dx+c) + 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) + 12\left(\mathrm{c}\right.}{3\left(a^2 d\cos(dx+c)^4 - 2a^2 d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x +
c)^2 - 1)*sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) + 12*(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 - (cos(d*x + c)^2 - 1)*sin(d*x + c) + 1)*log(sin(d*x + c) +
1) + 2*(6*cos(d*x + c)^2 - 5)*sin(d*x + c) - 7)/(a^2*d*cos(d*x + c)^4 - 2*a
^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24538, size = 139, normalized size = 1.38

$$-\frac{\frac{12\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^2} + \frac{3}{(a\sin(dx+c)+a)a} + \frac{\frac{30a}{a\sin(dx+c)+a} - \frac{18a^2}{(a\sin(dx+c)+a)^2} - 13}{a^2\left(\frac{a}{a\sin(dx+c)+a} - 1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^2 + 3/((a*sin(d*x + c) + a
)*a) + (30*a/(a*sin(d*x + c) + a) - 18*a^2/(a*sin(d*x + c) + a)^2 - 13)/(a^
2*(a/(a*sin(d*x + c) + a) - 1)^3))/d
```

3.240 $\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=111

$$\frac{\sin^3(c+dx)}{3a^3d} - \frac{3\sin^2(c+dx)}{2a^3d} + \frac{6\sin(c+dx)}{a^3d} - \frac{5}{d\left(a^3\sin(c+dx)+a^3\right)} - \frac{10\log(\sin(c+dx)+1)}{a^3d} + \frac{1}{2ad(a\sin(c+dx)+}$$

```
[Out] (-10*Log[1 + Sin[c + d*x]])/(a^3*d) + (6*Sin[c + d*x])/(a^3*d) - (3*Sin[c +
 d*x]^2)/(2*a^3*d) + Sin[c + d*x]^3/(3*a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x
])^2) - 5/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.10643, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^3(c+dx)}{3a^3d} - \frac{3\sin^2(c+dx)}{2a^3d} + \frac{6\sin(c+dx)}{a^3d} - \frac{5}{d\left(a^3\sin(c+dx)+a^3\right)} - \frac{10\log(\sin(c+dx)+1)}{a^3d} + \frac{1}{2ad(a\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-10*Log[1 + Sin[c + d*x]])/(a^3*d) + (6*Sin[c + d*x])/(a^3*d) - (3*Sin[c +
 d*x]^2)/(2*a^3*d) + Sin[c + d*x]^3/(3*a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x
])^2) - 5/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule steps

$$\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{a^5(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(6a^2-3ax+x^2-\frac{a^5}{(a+x)^3}+\frac{5a^4}{(a+x)^2}-\frac{10a^3}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= -\frac{10\log(1+\sin(c+dx))}{a^3 d}+\frac{6\sin(c+dx)}{a^3 d}-\frac{3\sin^2(c+dx)}{2a^3 d}+\frac{\sin^3(c+dx)}{3a^3 d}+\frac{}{2ad(a+}$$

Mathematica [A] time = 0.700328, size = 106, normalized size = 0.95

$$\frac{32\sin^5(c+dx)-80\sin^4(c+dx)+320\sin^3(c+dx)+\sin^2(c+dx)(1023-960\log(\sin(c+dx)+1))-6\sin(c+dx)(3}{96a^3 d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-417 - 960*Log[1 + Sin[c + d*x]] - 6*(-21 + 320*Log[1 + Sin[c + d*x]])*Sin
[c + d*x] + (1023 - 960*Log[1 + Sin[c + d*x]])*Sin[c + d*x]^2 + 320*Sin[c +
 d*x]^3 - 80*Sin[c + d*x]^4 + 32*Sin[c + d*x]^5)/(96*a^3*d*(1 + Sin[c + d*x
])^2)
```

Maple [A] time = 0.039, size = 101, normalized size = 0.9

$$\frac{(\sin(dx+c))^3}{3a^3 d}-\frac{3(\sin(dx+c))^2}{2a^3 d}+6\frac{\sin(dx+c)}{a^3 d}+\frac{1}{2a^3 d(1+\sin(dx+c))^2}-5\frac{1}{a^3 d(1+\sin(dx+c))}-10\frac{\ln(1+\text{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/3*sin(d*x+c)^3/a^3/d-3/2*sin(d*x+c)^2/a^3/d+6*sin(d*x+c)/a^3/d+1/2/d/a^3/
(1+sin(d*x+c))^2-5/d/a^3/(1+sin(d*x+c))-10*ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.108, size = 128, normalized size = 1.15

$$-\frac{\frac{3(10\sin(dx+c)+9)}{a^3\sin(dx+c)^2+2a^3\sin(dx+c)+a^3}-\frac{2\sin(dx+c)^3-9\sin(dx+c)^2+36\sin(dx+c)}{a^3}+\frac{60\log(\sin(dx+c)+1)}{a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(10*sin(d*x + c) + 9)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^
3) - (2*sin(d*x + c)^3 - 9*sin(d*x + c)^2 + 36*sin(d*x + c))/a^3 + 60*log(s
in(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.50639, size = 317, normalized size = 2.86

$$\frac{10\cos(dx+c)^4+115\cos(dx+c)^2-120\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(\sin(dx+c)+1)-2\left(2\cos(dx+c)^4-\right.}{12\left(a^3d\cos(dx+c)^2-2a^3d\sin(dx+c)-2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(10*cos(d*x + c)^4 + 115*cos(d*x + c)^2 - 120*(cos(d*x + c)^2 - 2*sin(
d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*(2*cos(d*x + c)^4 - 24*cos(d*x + c)
^2 + 37)*sin(d*x + c) - 80)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) -
2*a^3*d)
```

Sympy [A] time = 12.0587, size = 760, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-60*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(6*a**3*d*sin(c + d*x)
**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 120*log(sin(c + d*x) + 1)*sin(c
+ d*x)/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 60*
log(sin(c + d*x) + 1)/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d*x) +
6*a**3*d) - sin(c + d*x)**6/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d
*x) + 6*a**3*d) - 2*sin(c + d*x)**4*cos(c + d*x)**2/(6*a**3*d*sin(c + d*x)*
*2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 16*sin(c + d*x)**4/(6*a**3*d*sin(
c + d*x)**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 4*sin(c + d*x)**3*cos(c
+ d*x)**2/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) -
sin(c + d*x)**2*cos(c + d*x)**4/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c
 + d*x) + 6*a**3*d) - 12*sin(c + d*x)**2*cos(c + d*x)**2/(6*a**3*d*sin(c +
d*x)**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 2*sin(c + d*x)*cos(c + d*x)*
*4/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 20*sin(
c + d*x)*cos(c + d*x)**2/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d*x)
 + 6*a**3*d) - 100*sin(c + d*x)/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c
 + d*x) + 6*a**3*d) - cos(c + d*x)**4/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d
*sin(c + d*x) + 6*a**3*d) - 10*cos(c + d*x)**2/(6*a**3*d*sin(c + d*x)**2 +
12*a**3*d*sin(c + d*x) + 6*a**3*d) - 80/(6*a**3*d*sin(c + d*x)**2 + 12*a**3
*d*sin(c + d*x) + 6*a**3*d), Ne(d, 0)), (x*sin(c)**5*cos(c)/(a*sin(c) + a)*
*3, True))
```

Giac [A] time = 1.17292, size = 120, normalized size = 1.08

$$-\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^3}+\frac{3(10\sin(dx+c)+9)}{a^3(\sin(dx+c)+1)^2}-\frac{2a^6\sin(dx+c)^3-9a^6\sin(dx+c)^2+36a^6\sin(dx+c)}{a^9}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(60*log(abs(sin(d*x + c) + 1))/a^3 + 3*(10*sin(d*x + c) + 9)/(a^3*(sin
(d*x + c) + 1)^2) - (2*a^6*sin(d*x + c)^3 - 9*a^6*sin(d*x + c)^2 + 36*a^6*s
in(d*x + c))/a^9)/d
```

3.241 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{6\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] (6*Log[1 + Sin[c + d*x]])/(a^3*d) - (3*Sin[c + d*x])/(a^3*d) + Sin[c + d*x]
^2/(2*a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 4/(d*(a^3 + a^3*Sin[c + d
*x]))
```

Rubi [A] time = 0.0963241, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{6\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Log[1 + Sin[c + d*x]])/(a^3*d) - (3*Sin[c + d*x])/(a^3*d) + Sin[c + d*x]
^2/(2*a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 4/(d*(a^3 + a^3*Sin[c + d
*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a+x+\frac{a^4}{(a+x)^3}-\frac{4a^3}{(a+x)^2}+\frac{6a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{6\log(1+\sin(c+dx))}{a^3 d} - \frac{3\sin(c+dx)}{a^3 d} + \frac{\sin^2(c+dx)}{2a^3 d} - \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{}{d}$$

Mathematica [A] time = 2.13367, size = 78, normalized size = 0.84

$$\frac{8\sin^2(c+dx)+\left(\frac{64}{(\sin(c+dx)+1)^2}-48\right)\sin(c+dx)+96\log(\sin(c+dx)+1)+\frac{56}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}}{16a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (96*Log[1 + Sin[c + d*x]] + 56/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 8*
Sin[c + d*x]^2 + Sin[c + d*x]*(-48 + 64/(1 + Sin[c + d*x])^2))/(16*a^3*d)
```

Maple [A] time = 0.039, size = 85, normalized size = 0.9

$$\frac{(\sin(dx+c))^2}{2a^3 d} - 3\frac{\sin(dx+c)}{a^3 d} - \frac{1}{2a^3 d(1+\sin(dx+c))^2} + 4\frac{1}{a^3 d(1+\sin(dx+c))} + 6\frac{\ln(1+\sin(dx+c))}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2*sin(d*x+c)^2/a^3/d-3*sin(d*x+c)/a^3/d-1/2/d/a^3/(1+sin(d*x+c))^2+4/d/a^
3/(1+sin(d*x+c))+6*ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.13208, size = 109, normalized size = 1.17

$$\frac{\frac{8\sin(dx+c)+7}{a^3\sin(dx+c)^2+2a^3\sin(dx+c)+a^3}+\frac{\sin(dx+c)^2-6\sin(dx+c)}{a^3}+\frac{12\log(\sin(dx+c)+1)}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((8*sin(d*x + c) + 7)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) +
 (sin(d*x + c)^2 - 6*sin(d*x + c))/a^3 + 12*log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.50938, size = 284, normalized size = 3.05

$$-\frac{2\cos(dx+c)^4+19\cos(dx+c)^2-24\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(\sin(dx+c)+1)+2\left(4\cos(dx+c)^2-3\right.}{4\left(a^3d\cos(dx+c)^2-2a^3d\sin(dx+c)-2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(d*x + c)^4 + 19*cos(d*x + c)^2 - 24*(cos(d*x + c)^2 - 2*sin(d*x
 + c) - 2)*log(sin(d*x + c) + 1) + 2*(4*cos(d*x + c)^2 - 3)*sin(d*x + c) -
8)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [A] time = 7.04268, size = 456, normalized size = 4.9

$$\begin{cases}\frac{12\log(\sin(c+dx)+1)\sin^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}+\frac{24\log(\sin(c+dx)+1)\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}+\frac{12\log(\sin(c+dx)+1)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}+\frac{3\sin^4(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin} \\ \frac{x\sin^4(c)\cos(c)}{(a\sin(c)+a)^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((12*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 24*log(sin(c + d*x) + 1)*sin(c + d
*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 12*log(
sin(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**
3*d) + 3*sin(c + d*x)**4/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x)
+ 2*a**3*d) + 2*sin(c + d*x)**2*cos(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 +
 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 4*sin(c + d*x)*cos(c + d*x)**2/(2*a**3
*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 20*sin(c + d*x)/(2
*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*cos(c + d*x
)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 16/(2*
a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Ne(d, 0)), (x*s
in(c)**4*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.19862, size = 99, normalized size = 1.06

$$\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^3}+\frac{8\sin(dx+c)+7}{a^3(\sin(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^2-6a^3\sin(dx+c)}{a^6}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(12*log(abs(sin(d*x + c) + 1))/a^3 + (8*sin(d*x + c) + 7)/(a^3*(sin(d*x
 + c) + 1)^2) + (a^3*sin(d*x + c)^2 - 6*a^3*sin(d*x + c))/a^6)/d
```

3.242 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=74

$$\frac{\sin(c+dx)}{a^3d}-\frac{3}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{3\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] (-3*Log[1 + Sin[c + d*x]])/(a^3*d) + Sin[c + d*x]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) - 3/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.0896497, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{a^3d}-\frac{3}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{3\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-3*Log[1 + Sin[c + d*x]])/(a^3*d) + Sin[c + d*x]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) - 3/(d*(a^3 + a^3*Sin[c + d*x]))
```


Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a^3}{(a+x)^3}+\frac{3a^2}{(a+x)^2}-\frac{3a}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= -\frac{3\log(1+\sin(c+dx))}{a^3 d}+\frac{\sin(c+dx)}{a^3 d}+\frac{1}{2ad(a+a\sin(c+dx))^2}-\frac{3}{d\left(a^3+a^3\sin(c+dx\right.}$$

Mathematica [A] time = 0.402829, size = 70, normalized size = 0.95

$$\frac{\frac{\sin^2(c+dx)}{(\sin(c+dx)+1)^2}+4\sin(c+dx)+\frac{-10\sin(c+dx)-9}{(\sin(c+dx)+1)^2}-12\log(\sin(c+dx)+1)}{4a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-12*Log[1 + Sin[c + d*x]] + 4*Sin[c + d*x] + (-9 - 10*Sin[c + d*x])/(1 + S
in[c + d*x])^2 + Sin[c + d*x]^2/(1 + Sin[c + d*x])^2)/(4*a^3*d)
```

Maple [A] time = 0.036, size = 68, normalized size = 0.9

$$\frac{\sin(dx+c)}{a^3 d}+\frac{1}{2a^3 d(1+\sin(dx+c))^2}-3\frac{1}{a^3 d(1+\sin(dx+c))}-3\frac{\ln(1+\sin(dx+c))}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] sin(d*x+c)/a^3/d+1/2/d/a^3/(1+sin(d*x+c))^2-3/d/a^3/(1+sin(d*x+c))-3*ln(1+s
in(d*x+c))/a^3/d
```

Maxima [A] time = 1.12152, size = 96, normalized size = 1.3

$$-\frac{\frac{6\sin(dx+c)+5}{a^3\sin(dx+c)^2+2a^3\sin(dx+c)+a^3}+\frac{6\log(\sin(dx+c)+1)}{a^3}-\frac{2\sin(dx+c)}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((6*sin(d*x + c) + 5)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3)
+ 6*log(sin(d*x + c) + 1)/a^3 - 2*sin(d*x + c)/a^3)/d
```

Fricas [A] time = 1.41297, size = 251, normalized size = 3.39

$$\frac{4\,\cos\left(dx+c\right)^{2}-6\,\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)+2\,\left(\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)+1}{2\,\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\sin\left(dx+c\right)-2\,a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x
 + c) + 1) + 2*(cos(d*x + c)^2 + 1)*sin(d*x + c) + 1)/(a^3*d*cos(d*x + c)^2
 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [A] time = 3.69133, size = 303, normalized size = 4.09

$$\begin{cases} -\frac{6\log\left(\sin\left(c+dx\right)+1\right)\sin^{2}\left(c+dx\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}-\frac{12\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}-\frac{6\log\left(\sin\left(c+dx\right)+1\right)}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d}+\frac{2\sin^{3}\left(\ldots\right.}{2a^{3}d\sin^{2}\left(c+dx\right)+4a\ldots} \\ \frac{x\sin^{3}\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-6*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 12*log(sin(c + d*x) + 1)*sin(c + d
*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 6*log(s
in(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3
*d) + 2*sin(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) +
 2*a**3*d) - 12*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d
*x) + 2*a**3*d) - 9/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a
**3*d), Ne(d, 0)), (x*sin(c)**3*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.2596, size = 76, normalized size = 1.03

$$-\frac{\frac{6\,\log\left(|\sin(dx+c)+1|\right)}{a^{3}}-\frac{2\,\sin(dx+c)}{a^{3}}+\frac{6\,\sin(dx+c)+5}{a^{3}(\sin(dx+c)+1)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*log(abs(sin(d*x + c) + 1))/a^3 - 2*sin(d*x + c)/a^3 + (6*sin(d*x +
c) + 5)/(a^3*(sin(d*x + c) + 1)^2))/d
```

3.243 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] Log[1 + Sin[c + d*x]]/(a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 2/(d*(a^
3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.0805542, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 2/(d*(a^
3 + a^3*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{(a+x)^3} - \frac{2a}{(a+x)^2} + \frac{1}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\log(1+\sin(c+dx))}{a^3d} - \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{2}{d\left(a^3+a^3\sin(c+dx)\right)}$$

Mathematica [A] time = 0.659864, size = 65, normalized size = 1.08

$$\frac{\frac{16\sin(c+dx)}{(\sin(c+dx)+1)^2} + 8\log(\sin(c+dx)+1) + \frac{12}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (8*Log[1 + Sin[c + d*x]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (16
*Sin[c + d*x])/(1 + Sin[c + d*x])^2)/(8*a^3*d)
```

Maple [A] time = 0.034, size = 54, normalized size = 0.9

$$-\frac{1}{2\,a^3d\,(1+\sin(dx+c))^2} + 2\,\frac{1}{a^3d\,(1+\sin(dx+c))} + \frac{\ln(1+\sin(dx+c))}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d/a^3/(1+sin(d*x+c))^2+2/d/a^3/(1+sin(d*x+c))+ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.09314, size = 81, normalized size = 1.35

$$\frac{\frac{4\,\sin(dx+c)+3}{a^3\sin(dx+c)^2+2\,a^3\sin(dx+c)+a^3} + \frac{2\,\log(\sin(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) +
 2*log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.37867, size = 197, normalized size = 3.28

$$\frac{2\left(\cos(dx+c)^2 - 2\sin(dx+c) - 2\right)\log(\sin(dx+c)+1) - 4\sin(dx+c) - 3}{2\left(a^3d\cos(dx+c)^2 - 2a^3d\sin(dx+c) - 2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 4*sin(
d*x + c) - 3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [A] time = 2.17808, size = 257, normalized size = 4.28

$$\begin{cases} \frac{2\log(\sin(c+dx)+1)\sin^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} + \frac{4\log(\sin(c+dx)+1)\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} + \frac{2\log(\sin(c+dx)+1)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} + \frac{4\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin} \\ \frac{x\sin^2(c)\cos(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**
2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 4*log(sin(c + d*x) + 1)*sin(c + d*x
)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*log(sin
(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d
) + 4*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a*
*3*d) + 3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Ne
(d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.2356, size = 61, normalized size = 1.02

$$\frac{\frac{2\log(|\sin(dx+c)+1|)}{a^3} + \frac{4\sin(dx+c)+3}{a^3(\sin(dx+c)+1)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 + (4*sin(d*x + c) + 3)/(a^3*(sin(d*x
+ c) + 1)^2))/d
```

3.244 $\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=30

$$\frac{\sin^2(c+dx)}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] Sin[c + d*x]^2/(2*a*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0458973, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 37}

$$\frac{\sin^2(c+dx)}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]^2/(2*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}\\
&= \frac{\sin^2(c+dx)}{2ad(a+a\sin(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 0.0289561, size = 30, normalized size = 1.

$$\frac{\sin^2(c+dx)}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]^2/(2*a*d*(a + a*Sin[c + d*x])^2)
```

Maple [A] time = 0.032, size = 33, normalized size = 1.1

$$\frac{1}{a^3d}\left(\frac{1}{2\,(1+\sin(dx+c))^2}-(1+\sin(dx+c))^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(1/2/(1+sin(d*x+c))^2-1/(1+sin(d*x+c)))
```

Maxima [A] time = 1.06835, size = 59, normalized size = 1.97

$$-\frac{2\sin(dx+c)+1}{2\left(a^3\sin(dx+c)^2+2\,a^3\sin(dx+c)+a^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*sin(d*x + c) + 1)/((a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3)*
d)
```

Fricas [A] time = 1.33102, size = 111, normalized size = 3.7

$$\frac{2\sin(dx+c)+1}{2\left(a^3d\cos(dx+c)^2-2\,a^3d\sin(dx+c)-2\,a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*sin(d*x + c) + 1)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a
^3*d)
```

Sympy [A] time = 2.0969, size = 99, normalized size = 3.3

$$\begin{cases} -\frac{2\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}-\frac{1}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} & \text{for } d\neq 0\\ \frac{x\sin(c)\cos(c)}{(a\sin(c)+a)^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-2*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x
) + 2*a**3*d) - 1/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**
3*d), Ne(d, 0)), (x*sin(c)*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.25707, size = 38, normalized size = 1.27

$$-\frac{2\sin(dx+c)+1}{2a^3d(\sin(dx+c)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*sin(d*x + c) + 1)/(a^3*d*(sin(d*x + c) + 1)^2)
```

3.245 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=74

$$\frac{1}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.0576459, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x}-\frac{1}{a(a+x)^3}-\frac{1}{a^2(a+x)^2}-\frac{1}{a^3(a+x)}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(1+\sin(c+dx))}{a^3d}+\frac{1}{2ad(a+a\sin(c+dx))^2}+\frac{1}{d\left(a^3+a^3\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.169574, size = 52, normalized size = 0.7

$$\frac{\frac{2\sin(c+dx)+3}{(\sin(c+dx)+1)^2}+2\log(\sin(c+dx))-2\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (3 + 2*Sin[c + d*x])/(1 +
Sin[c + d*x])^2)/(2*a^3*d)
```

Maple [A] time = 0.048, size = 68, normalized size = 0.9

$$\frac{1}{2\,a^{3}d\left(1+\sin\left(dx+c\right)\right)^{2}}+\frac{1}{a^{3}d\left(1+\sin\left(dx+c\right)\right)}-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{a^{3}d}+\frac{\ln\left(\sin\left(dx+c\right)\right)}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3/(1+sin(d*x+c))^2+1/d/a^3/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^3/d+ln
(sin(d*x+c))/a^3/d
```

Maxima [A] time = 0.998509, size = 97, normalized size = 1.31

$$\frac{\frac{2\,\sin(dx+c)+3}{a^{3}\sin(dx+c)^{2}+2\,a^{3}\sin(dx+c)+a^{3}}-\frac{2\,\log(\sin(dx+c)+1)}{a^{3}}+\frac{2\,\log(\sin(dx+c))}{a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((2*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) -
 2*log(sin(d*x + c) + 1)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

Fricas [A] time = 1.48187, size = 284, normalized size = 3.84

$$\frac{2\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-2\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)}{2\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\sin\left(dx+c\right)-2\,a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(1/2*sin(d*x + c)) - 2*(cos
(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) -
3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos\left(c+dx\right)\csc\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3
*sin(c + d*x) + 1), x)/a**3
```

Giac [A] time = 1.30571, size = 80, normalized size = 1.08

$$-\frac{\frac{2\log\left(|\sin(dx+c)+1|\right)}{a^3}-\frac{2\log\left(|\sin(dx+c)|\right)}{a^3}-\frac{2\sin(dx+c)+3}{a^3(\sin(dx+c)+1)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (2*
sin(d*x + c) + 3)/(a^3*(sin(d*x + c) + 1)^2))/d
```

3.246 $\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=90

$$-\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{3\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] -(Csc[c + d*x]/(a^3*d)) - (3*Log[Sin[c + d*x]])/(a^3*d) + (3*Log[1 + Sin[c
+ d*x]])/(a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) - 2/(d*(a^3 + a^3*Sin[c
 + d*x]))
```

Rubi [A] time = 0.0836685, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 44}

$$-\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{3\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]/(a^3*d)) - (3*Log[Sin[c + d*x]])/(a^3*d) + (3*Log[1 + Sin[c
+ d*x]])/(a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) - 2/(d*(a^3 + a^3*Sin[c
 + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3}{a^4x} + \frac{1}{a^2(a+x)^3} + \frac{2}{a^3(a+x)^2} + \frac{3}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{a^3d} - \frac{3\log(\sin(c+dx))}{a^3d} + \frac{3\log(1+\sin(c+dx))}{a^3d} - \frac{1}{2ad(a+a\sin(c+dx))^2} -$$

Mathematica [A] time = 0.301958, size = 61, normalized size = 0.68

$$-\frac{\frac{4}{\sin(c+dx)+1} + \frac{1}{(\sin(c+dx)+1)^2} + 2\csc(c+dx) + 6\log(\sin(c+dx)) - 6\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(2*Csc[c + d*x] + 6*Log[Sin[c + d*x]] - 6*Log[1 + Sin[c + d*x]] + (1 + Sin
[c + d*x])^(-2) + 4/(1 + Sin[c + d*x]))/(2*a^3*d)
```

Maple [A] time = 0.049, size = 86, normalized size = 1.

$$-\frac{1}{2\,a^3d\,(1+\sin(dx+c))^2} - 2\,\frac{1}{a^3d\,(1+\sin(dx+c))} + 3\,\frac{\ln(1+\sin(dx+c))}{a^3d} - \frac{1}{a^3d\sin(dx+c)} - 3\,\frac{\ln(\sin(dx+c))}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d/a^3/(1+sin(d*x+c))^2-2/d/a^3/(1+sin(d*x+c))+3*ln(1+sin(d*x+c))/a^3/d
-1/d/a^3/sin(d*x+c)-3*ln(sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.01597, size = 123, normalized size = 1.37

$$-\frac{\frac{6\,\sin(dx+c)^2+9\,\sin(dx+c)+2}{a^3\sin(dx+c)^3+2\,a^3\sin(dx+c)^2+a^3\sin(dx+c)} - \frac{6\,\log(\sin(dx+c)+1)}{a^3} + \frac{6\,\log(\sin(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((6*sin(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3 + 2*a^3*s
in(d*x + c)^2 + a^3*sin(d*x + c)) - 6*log(sin(d*x + c) + 1)/a^3 + 6*log(sin
(d*x + c))/a^3)/d
```

Fricas [A] time = 1.51805, size = 404, normalized size = 4.49

$$-\frac{6\cos\left(dx+c\right)^2+6\left(2\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)-2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-6\left(2\cos\left(dx+c\right)^2+\right.}{2\left(2a^3d\cos\left(dx+c\right)^2-2a^3d+\left(a^3d\cos\left(dx+c\right)^2-2a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cos(d*x + c)^2 + 6*(2*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2)*sin(d*x
 + c) - 2)*log(1/2*sin(d*x + c)) - 6*(2*cos(d*x + c)^2 + (cos(d*x + c)^2 -
2)*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 9*sin(d*x + c) - 8)/(2*a^3*d*c
os(d*x + c)^2 - 2*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos\left(c+dx\right)\csc^2\left(c+dx\right)}{\sin^3\left(c+dx\right)+3\sin^2\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A] time = 1.32079, size = 104, normalized size = 1.16

$$\frac{\frac{6\log\left(|\sin(dx+c)+1|\right)}{a^3}-\frac{6\log\left(|\sin(dx+c)|\right)}{a^3}-\frac{6\sin(dx+c)^2+9\sin(dx+c)+2}{a^3(\sin(dx+c)+1)^2\sin(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c)))/a^3 - (6*s
in(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*(sin(d*x + c) + 1)^2*sin(d*x + c))
)/d
```

3.247 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=108

$$\frac{3}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{6\log(\sin(c+dx))}{a^3d}-\frac{6\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)$$

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (6*Log[Sin[c + d*x]])
/(a^3*d) - (6*Log[1 + Sin[c + d*x]])/(a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x]
)^2) + 3/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.101641, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$\frac{3}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{6\log(\sin(c+dx))}{a^3d}-\frac{6\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (6*Log[Sin[c + d*x]])
/(a^3*d) - (6*Log[1 + Sin[c + d*x]])/(a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x]
)^2) + 3/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{a^3x^3} - \frac{3}{a^4x^2} + \frac{6}{a^5x} - \frac{1}{a^3(a+x)^3} - \frac{3}{a^4(a+x)^2} - \frac{6}{a^5(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{3\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^3d} + \frac{6\log(\sin(c+dx))}{a^3d} - \frac{6\log(1+\sin(c+dx))}{a^3d} + \frac{}{2ad(a}$$

Mathematica [A] time = 0.577214, size = 71, normalized size = 0.66

$$\frac{\frac{6}{\sin(c+dx)+1} + \frac{1}{(\sin(c+dx)+1)^2} - \csc^2(c+dx) + 6\csc(c+dx) + 12\log(\sin(c+dx)) - 12\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] - 12*Log[1 + Sin[c
+ d*x]] + (1 + Sin[c + d*x])^(-2) + 6/(1 + Sin[c + d*x]))/(2*a^3*d)
```

Maple [A] time = 0.055, size = 102, normalized size = 0.9

$$\frac{1}{2a^3d(1+\sin(dx+c))^2} + 3\frac{1}{a^3d(1+\sin(dx+c))} - 6\frac{\ln(1+\sin(dx+c))}{a^3d} - \frac{1}{2a^3d(\sin(dx+c))^2} + 3\frac{1}{a^3d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3/(1+sin(d*x+c))^2+3/d/a^3/(1+sin(d*x+c))-6*ln(1+sin(d*x+c))/a^3/d-
1/2/d/a^3/sin(d*x+c)^2+3/d/a^3/sin(d*x+c)+6*ln(sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.00719, size = 139, normalized size = 1.29

$$\frac{\frac{12\sin(dx+c)^3+18\sin(dx+c)^2+4\sin(dx+c)-1}{a^3\sin(dx+c)^4+2a^3\sin(dx+c)^3+a^3\sin(dx+c)^2} - \frac{12\log(\sin(dx+c)+1)}{a^3} + \frac{12\log(\sin(dx+c))}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((12*sin(d*x + c)^3 + 18*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(
d*x + c)^4 + 2*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2) - 12*log(sin(d*x +
c) + 1)/a^3 + 12*log(sin(d*x + c))/a^3)/d
```

Fricas [A] time = 1.43931, size = 522, normalized size = 4.83

$$-\frac{18\cos(dx+c)^2-12\left(\cos(dx+c)^4-3\cos(dx+c)^2-2\left(\cos(dx+c)^2-1\right)\sin(dx+c)+2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+12\left(\right.}{2\left(a^3d\cos(dx+c)^4-3a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(18*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 2*(cos(d*
x + c)^2 - 1)*sin(d*x + c) + 2)*log(1/2*sin(d*x + c)) + 12*(cos(d*x + c)^4
- 3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 2)*log(sin(d*x +
 c) + 1) + 4*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) - 17)/(a^3*d*cos(d*x + c)^
4 - 3*a^3*d*cos(d*x + c)^2 + 2*a^3*d - 2*(a^3*d*cos(d*x + c)^2 - a^3*d)*sin
(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A] time = 1.2906, size = 116, normalized size = 1.07

$$-\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^3}-\frac{12\log(|\sin(dx+c)|)}{a^3}-\frac{12\sin(dx+c)^3+18\sin(dx+c)^2+4\sin(dx+c)-1}{\left(\sin(dx+c)^2+\sin(dx+c)\right)^2a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(12*log(abs(sin(d*x + c) + 1))/a^3 - 12*log(abs(sin(d*x + c)))/a^3 - (
12*sin(d*x + c)^3 + 18*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/((sin(d*x + c)^
2 + sin(d*x + c))^2*a^3))/d
```

3.248 $\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$-\frac{4}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{6\csc(c+dx)}{a^3d}-\frac{10\log(\sin(c+dx))}{a^3d}+\frac{10\log(\sin(c+dx)}{a^3d}$$

```
[Out] (-6*Csc[c + d*x])/(a^3*d) + (3*Csc[c + d*x]^2)/(2*a^3*d) - Csc[c + d*x]^3/(
3*a^3*d) - (10*Log[Sin[c + d*x]])/(a^3*d) + (10*Log[1 + Sin[c + d*x]])/(a^3
*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) - 4/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.112726, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$-\frac{4}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{6\csc(c+dx)}{a^3d}-\frac{10\log(\sin(c+dx))}{a^3d}+\frac{10\log(\sin(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-6*Csc[c + d*x])/(a^3*d) + (3*Csc[c + d*x]^2)/(2*a^3*d) - Csc[c + d*x]^3/(
3*a^3*d) - (10*Log[Sin[c + d*x]])/(a^3*d) + (10*Log[1 + Sin[c + d*x]])/(a^3
*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) - 4/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^4}{x^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{1}{x^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{a^3x^4} - \frac{3}{a^4x^3} + \frac{6}{a^5x^2} - \frac{10}{a^6x} + \frac{1}{a^4(a+x)^3} + \frac{4}{a^5(a+x)^2} + \frac{10}{a^6(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{6\csc(c+dx)}{a^3d} + \frac{3\csc^2(c+dx)}{2a^3d} - \frac{\csc^3(c+dx)}{3a^3d} - \frac{10\log(\sin(c+dx))}{a^3d} + \frac{10\log(1+\sin(}{a^3d}$$

Mathematica [A] time = 5.7232, size = 81, normalized size = 0.64

$$-\frac{\frac{3(8\sin(c+dx)+9)}{(\sin(c+dx)+1)^2} + 2\csc^3(c+dx) - 9\csc^2(c+dx) + 36\csc(c+dx) + 60\log(\sin(c+dx)) - 60\log(\sin(c+dx)+1)}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(36*Csc[c + d*x] - 9*Csc[c + d*x]^2 + 2*Csc[c + d*x]^3 + 60*Log[Sin[c + d*
x]] - 60*Log[1 + Sin[c + d*x]] + (3*(9 + 8*Sin[c + d*x]))/(1 + Sin[c + d*x]
)^2)/(6*a^3*d)
```

Maple [A] time = 0.062, size = 118, normalized size = 0.9

$$-\frac{1}{2\,a^3d\,(1+\sin(dx+c))^2} - 4\,\frac{1}{a^3d\,(1+\sin(dx+c))} + 10\,\frac{\ln(1+\sin(dx+c))}{a^3d} - \frac{1}{3\,a^3d\,(\sin(dx+c))^3} + \frac{3}{2\,a^3d\,(\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d/a^3/(1+sin(d*x+c))^2-4/d/a^3/(1+sin(d*x+c))+10*ln(1+sin(d*x+c))/a^3/
d-1/3/d/a^3/sin(d*x+c)^3+3/2/d/a^3/sin(d*x+c)^2-6/d/a^3/sin(d*x+c)-10*ln(si
n(d*x+c))/a^3/d
```

Maxima [A] time = 0.984024, size = 153, normalized size = 1.21

$$-\frac{\frac{60\,\sin(dx+c)^4+90\,\sin(dx+c)^3+20\,\sin(dx+c)^2-5\,\sin(dx+c)+2}{a^3\sin(dx+c)^5+2\,a^3\sin(dx+c)^4+a^3\sin(dx+c)^3} - \frac{60\,\log(\sin(dx+c)+1)}{a^3} + \frac{60\,\log(\sin(dx+c))}{a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*((60*sin(d*x + c)^4 + 90*sin(d*x + c)^3 + 20*sin(d*x + c)^2 - 5*sin(d*
x + c) + 2)/(a^3*sin(d*x + c)^5 + 2*a^3*sin(d*x + c)^4 + a^3*sin(d*x + c)^3
) - 60*log(sin(d*x + c) + 1)/a^3 + 60*log(sin(d*x + c))/a^3)/d
```

Fricas [B] time = 1.54369, size = 641, normalized size = 5.09

$$-\frac{60\cos(dx+c)^4-140\cos(dx+c)^2+60\left(2\cos(dx+c)^4-4\cos(dx+c)^2+\left(\cos(dx+c)^4-3\cos(dx+c)^2+2\right)\mathrm{s}}{6\left(2a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(60*cos(d*x + c)^4 - 140*cos(d*x + c)^2 + 60*(2*cos(d*x + c)^4 - 4*cos
(d*x + c)^2 + (cos(d*x + c)^4 - 3*cos(d*x + c)^2 + 2)*sin(d*x + c) + 2)*log
(1/2*sin(d*x + c)) - 60*(2*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c
)^4 - 3*cos(d*x + c)^2 + 2)*sin(d*x + c) + 2)*log(sin(d*x + c) + 1) - 5*(18
*cos(d*x + c)^2 - 17)*sin(d*x + c) + 82)/(2*a^3*d*cos(d*x + c)^4 - 4*a^3*d*
cos(d*x + c)^2 + 2*a^3*d + (a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^2 +
 2*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2696, size = 131, normalized size = 1.04

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^3}-\frac{60\log(|\sin(dx+c)|)}{a^3}-\frac{60\sin(dx+c)^4+90\sin(dx+c)^3+20\sin(dx+c)^2-5\sin(dx+c)+2}{a^3(\sin(dx+c)+1)^2\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c)))/a^3 - (6
0*sin(d*x + c)^4 + 90*sin(d*x + c)^3 + 20*sin(d*x + c)^2 - 5*sin(d*x + c) +
 2)/(a^3*(sin(d*x + c) + 1)^2*sin(d*x + c)^3))/d
```

3.249 $\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=116

$$\frac{\sin^2(c+dx)}{2a^4d}-\frac{4\sin(c+dx)}{a^4d}+\frac{10}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{5}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{10\log(\sin(c+dx)+1)}{a^4d}+\frac{}{3ad(a\sin}$$

```
[Out] (10*Log[1 + Sin[c + d*x]])/(a^4*d) - (4*Sin[c + d*x])/(a^4*d) + Sin[c + d*x
]^2/(2*a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 5/(2*d*(a^2 + a^2*Sin[c
+ d*x])^2) + 10/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.104187, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin^2(c+dx)}{2a^4d}-\frac{4\sin(c+dx)}{a^4d}+\frac{10}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{5}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{10\log(\sin(c+dx)+1)}{a^4d}+\frac{}{3ad(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (10*Log[1 + Sin[c + d*x]])/(a^4*d) - (4*Sin[c + d*x])/(a^4*d) + Sin[c + d*x
]^2/(2*a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 5/(2*d*(a^2 + a^2*Sin[c
+ d*x])^2) + 10/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{a^5(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a+x-\frac{a^5}{(a+x)^4}+\frac{5a^4}{(a+x)^3}-\frac{10a^3}{(a+x)^2}+\frac{10a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{10\log(1+\sin(c+dx))}{a^4 d} - \frac{4\sin(c+dx)}{a^4 d} + \frac{\sin^2(c+dx)}{2a^4 d} + \frac{1}{3ad(a+a\sin(c+dx))^3} -$$

Mathematica [A] time = 0.842507, size = 119, normalized size = 1.03

$$\frac{3\sin^5(c+dx) - 15\sin^4(c+dx) + \sin^3(c+dx)(60\log(\sin(c+dx)+1)-63) + 9\sin^2(c+dx)(20\log(\sin(c+dx)+1)}{6a^4 d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (47 + 60*Log[1 + Sin[c + d*x]] + 9*(9 + 20*Log[1 + Sin[c + d*x]])*Sin[c + d
*x] + 9*(-1 + 20*Log[1 + Sin[c + d*x]])*Sin[c + d*x]^2 + (-63 + 60*Log[1 +
Sin[c + d*x]])*Sin[c + d*x]^3 - 15*Sin[c + d*x]^4 + 3*Sin[c + d*x]^5)/(6*a^
4*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.04, size = 103, normalized size = 0.9

$$\frac{(\sin(dx+c))^2}{2a^4 d} - 4\frac{\sin(dx+c)}{a^4 d} - \frac{5}{2a^4 d(1+\sin(dx+c))^2} + \frac{1}{3a^4 d(1+\sin(dx+c))^3} + 10\frac{1}{a^4 d(1+\sin(dx+c))} + 10$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/2*sin(d*x+c)^2/a^4/d-4*sin(d*x+c)/a^4/d-5/2/d/a^4/(1+sin(d*x+c))^2+1/3/d/
a^4/(1+sin(d*x+c))^3+10/d/a^4/(1+sin(d*x+c))+10*ln(1+sin(d*x+c))/a^4/d
```

Maxima [A] time = 1.00361, size = 142, normalized size = 1.22

$$\frac{\frac{60\sin(dx+c)^2+105\sin(dx+c)+47}{a^4\sin(dx+c)^3+3a^4\sin(dx+c)^2+3a^4\sin(dx+c)+a^4} + \frac{3\left(\sin(dx+c)^2-8\sin(dx+c)\right)}{a^4} + \frac{60\log(\sin(dx+c)+1)}{a^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((60*sin(d*x + c)^2 + 105*sin(d*x + c) + 47)/(a^4*sin(d*x + c)^3 + 3*a^
4*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) + 3*(sin(d*x + c)^2 - 8*sin(d*
```

```
x + c))/a^4 + 60*log(sin(d*x + c) + 1)/a^4)/d
```

Fricas [A] time = 1.60126, size = 381, normalized size = 3.28

$$\frac{30 \cos (dx+c)^4 - 87 \cos (dx+c)^2 + 120 \left(3 \cos (dx+c)^2 + \left(\cos (dx+c)^2 - 4\right) \sin (dx+c) - 4\right) \log (\sin (dx+c) + 1) - 3}{12 \left(3 a^4 d \cos (dx+c)^2 - 4 a^4 d + \left(a^4 d \cos (dx+c)^2 - 4 a^4 d\right) \sin (dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(30*cos(d*x + c)^4 - 87*cos(d*x + c)^2 + 120*(3*cos(d*x + c)^2 + (cos(
d*x + c)^2 - 4)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) - 3*(2*cos(d*x + c)
^4 + 39*cos(d*x + c)^2 + 10)*sin(d*x + c) - 34)/(3*a^4*d*cos(d*x + c)^2 - 4
*a^4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [A] time = 14.5586, size = 796, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((60*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)*
*3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 180*l
og(sin(c + d*x) + 1)*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*
sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 180*log(sin(c + d*x)
 + 1)*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 +
18*a**4*d*sin(c + d*x) + 6*a**4*d) + 60*log(sin(c + d*x) + 1)/(6*a**4*d*sin
(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*
d) - 24*sin(c + d*x)**4/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)*
*2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) - 3*sin(c + d*x)**3*cos(c + d*x)**2
/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c +
d*x) + 6*a**4*d) - 9*sin(c + d*x)**2*cos(c + d*x)**2/(6*a**4*d*sin(c + d*x)
**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 204*
sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*
a**4*d*sin(c + d*x) + 6*a**4*d) - 9*sin(c + d*x)*cos(c + d*x)**2/(6*a**4*d*
sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a*
*4*d) + 297*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)
**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) - 3*cos(c + d*x)**2/(6*a**4*d*sin(
c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d
) + 119/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*s
in(c + d*x) + 6*a**4*d), Ne(d, 0)), (x*sin(c)**5*cos(c)/(a*sin(c) + a)**4,
True))
```

Giac [A] time = 1.19353, size = 113, normalized size = 0.97

$$\frac{\frac{60 \log (|\sin (dx+c)+1|)}{a^4} + \frac{60 \sin (dx+c)^2 + 105 \sin (dx+c) + 47}{a^4 (\sin (dx+c)+1)^3} + \frac{3 \left(a^4 \sin (dx+c)^2 - 8 a^4 \sin (dx+c)\right)}{a^8}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(60*log(abs(sin(d*x + c) + 1))/a^4 + (60*sin(d*x + c)^2 + 105*sin(d*x +
 c) + 47)/(a^4*(sin(d*x + c) + 1)^3) + 3*(a^4*sin(d*x + c)^2 - 8*a^4*sin(d*
x + c))/a^8)/d
```

3.250 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=95

$$\frac{\sin(c+dx)}{a^4d}-\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4\log(\sin(c+dx)+1)}{a^4d}-\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^4*d) + Sin[c + d*x]/(a^4*d) - 1/(3*a*d*(a + a
*Sin[c + d*x])^3) + 2/(d*(a^2 + a^2*Sin[c + d*x])^2) - 6/(d*(a^4 + a^4*Sin[
c + d*x]))
```

Rubi [A] time = 0.0965907, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{a^4d}-\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4\log(\sin(c+dx)+1)}{a^4d}-\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^4*d) + Sin[c + d*x]/(a^4*d) - 1/(3*a*d*(a + a
*Sin[c + d*x])^3) + 2/(d*(a^2 + a^2*Sin[c + d*x])^2) - 6/(d*(a^4 + a^4*Sin[
c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^4}{(a+x)^4}-\frac{4a^3}{(a+x)^3}+\frac{6a^2}{(a+x)^2}-\frac{4a}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^4 d}+\frac{\sin(c+dx)}{a^4 d}-\frac{1}{3ad(a+a\sin(c+dx))^3}+\frac{2}{d\left(a^2+a^2\sin(c+\right.}$$

Mathematica [A] time = 6.55278, size = 127, normalized size = 1.34

$$-\frac{3(2\sin(c+dx)+1)^2}{16a^4d(\sin(c+dx)+1)^3}-\frac{\frac{252\sin^2(c+dx)+444\sin(c+dx)+197}{(\sin(c+dx)+1)^3}-48\sin(c+dx)+192\log(\sin(c+dx)+1)}{48a^4d}-\frac{}{24a^4d\left(\sin\left(\frac{1}{2}(c\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/(24*a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) - (3*(1 + 2*Sin[c + d
*x])^2)/(16*a^4*d*(1 + Sin[c + d*x])^3) - (192*Log[1 + Sin[c + d*x]] - 48*S
in[c + d*x] + (197 + 444*Sin[c + d*x] + 252*Sin[c + d*x]^2)/(1 + Sin[c + d*
x])^3)/(48*a^4*d)
```

Maple [A] time = 0.039, size = 86, normalized size = 0.9

$$\frac{\sin(dx+c)}{a^4d}+2\frac{1}{a^4d(1+\sin(dx+c))^2}-\frac{1}{3a^4d(1+\sin(dx+c))^3}-6\frac{1}{a^4d(1+\sin(dx+c))}-4\frac{\ln(1+\sin(dx+c))}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x)
```

```
[Out] sin(d*x+c)/a^4/d+2/d/a^4/(1+sin(d*x+c))^2-1/3/d/a^4/(1+sin(d*x+c))^3-6/d/a^
4/(1+sin(d*x+c))-4*ln(1+sin(d*x+c))/a^4/d
```

Maxima [A] time = 1.00581, size = 127, normalized size = 1.34

$$-\frac{\frac{18\sin(dx+c)^2+30\sin(dx+c)+13}{a^4\sin(dx+c)^3+3a^4\sin(dx+c)^2+3a^4\sin(dx+c)+a^4}+\frac{12\log(\sin(dx+c)+1)}{a^4}-\frac{3\sin(dx+c)}{a^4}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((18*sin(d*x + c)^2 + 30*sin(d*x + c) + 13)/(a^4*sin(d*x + c)^3 + 3*a^
4*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) + 12*log(sin(d*x + c) + 1)/a^4
```

```
- 3*sin(d*x + c)/a^4)/d
```

Fricas [A] time = 1.54789, size = 346, normalized size = 3.64

$$-\frac{3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 + 12\left(3 \cos(dx+c)^2 + \left(\cos(dx+c)^2 - 4\right)\sin(dx+c) - 4\right)\log(\sin(dx+c)+1) - 9\left(\right.}{3\left(3 a^4 d \cos(dx+c)^2 - 4 a^4 d + \left(a^4 d \cos(dx+c)^2 - 4 a^4 d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 + 12*(3*cos(d*x + c)^2 + (cos(d*x
 + c)^2 - 4)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) - 9*(cos(d*x + c)^2 +
2)*sin(d*x + c) - 19)/(3*a^4*d*cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x +
c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [A] time = 9.81753, size = 527, normalized size = 5.55

$$\begin{cases} -\frac{12\log(\sin(c+dx)+1)\sin^3(c+dx)}{3a^4d\sin^3(c+dx)+9a^4d\sin^2(c+dx)+9a^4d\sin(c+dx)+3a^4d} - \frac{36\log(\sin(c+dx)+1)\sin^2(c+dx)}{3a^4d\sin^3(c+dx)+9a^4d\sin^2(c+dx)+9a^4d\sin(c+dx)+3a^4d} - \frac{36\log(\sin(c+dx)+}{3a^4d\sin^3(c+dx)+9a^4d\sin^2(c+d} \\ \frac{x\sin^4(c)\cos(c)}{(a\sin(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-12*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(3*a**4*d*sin(c + d*x)
**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log
(sin(c + d*x) + 1)*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin
(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)
*sin(c + d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4
*d*sin(c + d*x) + 3*a**4*d) - 12*log(sin(c + d*x) + 1)/(3*a**4*d*sin(c + d*
x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 3*si
n(c + d*x)**4/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4
*d*sin(c + d*x) + 3*a**4*d) - 36*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3
+ 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 54*sin(c +
 d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c
 + d*x) + 3*a**4*d) - 22/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)*
*2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d), Ne(d, 0)), (x*sin(c)**4*cos(c)/(a*s
in(c) + a)**4, True))
```

Giac [A] time = 1.21594, size = 89, normalized size = 0.94

$$-\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^4} - \frac{3\sin(dx+c)}{a^4} + \frac{18\sin(dx+c)^2+30\sin(dx+c)+13}{a^4(\sin(dx+c)+1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*log(abs(sin(d*x + c) + 1))/a^4 - 3*sin(d*x + c)/a^4 + (18*sin(d*x
+ c)^2 + 30*sin(d*x + c) + 13)/(a^4*(sin(d*x + c) + 1)^3))/d
```

3.251 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=83

$$\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{3}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

```
[Out] Log[1 + Sin[c + d*x]]/(a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 3/(2*d*(
a^2 + a^2*Sin[c + d*x])^2) + 3/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.0928894, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{3}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 3/(2*d*(
a^2 + a^2*Sin[c + d*x])^2) + 3/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a^3}{(a+x)^4}+\frac{3a^2}{(a+x)^3}-\frac{3a}{(a+x)^2}+\frac{1}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\log(1+\sin(c+dx))}{a^4d}+\frac{1}{3ad(a+a\sin(c+dx))^3}-\frac{3}{2d\left(a^2+a^2\sin(c+dx)\right)^2}+\frac{\ }{d\left(a^4+a^4\right.}$$

Mathematica [A] time = 0.357063, size = 61, normalized size = 0.73

$$\frac{18\sin^2(c+dx)+27\sin(c+dx)+6(\sin(c+dx)+1)^3\log(\sin(c+dx)+1)+11}{6a^4d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (11 + 27*Sin[c + d*x] + 18*Sin[c + d*x]^2 + 6*Log[1 + Sin[c + d*x]]*(1 + Si
n[c + d*x])^3)/(6*a^4*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.037, size = 72, normalized size = 0.9

$$-\frac{3}{2\,da^4\,(1+\sin(dx+c))^2}+\frac{1}{3\,da^4\,(1+\sin(dx+c))^3}+3\,\frac{1}{da^4\,(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -3/2/d/a^4/(1+sin(d*x+c))^2+1/3/d/a^4/(1+sin(d*x+c))^3+3/d/a^4/(1+sin(d*x+c
))+ln(1+sin(d*x+c))/a^4/d
```

Maxima [A] time = 1.11051, size = 112, normalized size = 1.35

$$\frac{\frac{18\sin(dx+c)^2+27\sin(dx+c)+11}{a^4\sin(dx+c)^3+3\,a^4\sin(dx+c)^2+3\,a^4\sin(dx+c)+a^4}+\frac{6\log(\sin(dx+c)+1)}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((18*sin(d*x + c)^2 + 27*sin(d*x + c) + 11)/(a^4*sin(d*x + c)^3 + 3*a^4
*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) + 6*log(sin(d*x + c) + 1)/a^4)/
d
```

Fricas [A] time = 1.53932, size = 292, normalized size = 3.52

$$\frac{18 \cos(dx+c)^2 + 6\left(3 \cos(dx+c)^2 + \left(\cos(dx+c)^2 - 4\right)\sin(dx+c) - 4\right)\log(\sin(dx+c)+1) - 27 \sin(dx+c) - 2}{6\left(3 a^4 d\cos(dx+c)^2 - 4 a^4 d + \left(a^4 d\cos(dx+c)^2 - 4 a^4 d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*cos(d*x + c)^2 + 6*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x
 + c) - 4)*log(sin(d*x + c) + 1) - 27*sin(d*x + c) - 29)/(3*a^4*d*cos(d*x +
 c)^2 - 4*a^4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [A] time = 4.08603, size = 466, normalized size = 5.61

$$\begin{cases} \frac{6\log(\sin(c+dx)+1)\sin^3(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d} + \frac{18\log(\sin(c+dx)+1)\sin^2(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d} + \frac{18\log(\sin}{6a^4d\sin^3(c+dx)+18a^4d} \\ \frac{x\sin^3(c)\cos(c)}{(a\sin(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((6*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)**
3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*log
(sin(c + d*x) + 1)*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*si
n(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*log(sin(c + d*x) +
1)*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*
a**4*d*sin(c + d*x) + 6*a**4*d) + 6*log(sin(c + d*x) + 1)/(6*a**4*d*sin(c +
 d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) -
 6*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 +
18*a**4*d*sin(c + d*x) + 6*a**4*d) + 9*sin(c + d*x)/(6*a**4*d*sin(c + d*x)*
*3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 5/(6*
a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x)
 + 6*a**4*d), Ne(d, 0)), (x*sin(c)**3*cos(c)/(a*sin(c) + a)**4, True))
```

Giac [A] time = 1.23416, size = 74, normalized size = 0.89

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^4} + \frac{18\sin(dx+c)^2+27\sin(dx+c)+11}{a^4(\sin(dx+c)+1)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c) + 1))/a^4 + (18*sin(d*x + c)^2 + 27*sin(d*x + c
) + 11)/(a^4*(sin(d*x + c) + 1)^3))/d
```

3.252 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=30

$$\frac{\sin^3(c+dx)}{3ad(a\sin(c+dx)+a)^3}$$

```
[Out] Sin[c + d*x]^3/(3*a*d*(a + a*Sin[c + d*x])^3)
```

Rubi [A] time = 0.065052, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 37}

$$\frac{\sin^3(c+dx)}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Sin[c + d*x]^3/(3*a*d*(a + a*Sin[c + d*x])^3)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\sin^3(c+dx)}{3ad(a+a\sin(c+dx))^3}
\end{aligned}$$

Mathematica [A] time = 0.183911, size = 53, normalized size = 1.77

$$\frac{-6\sin(c+dx)+3\cos(2(c+dx))-5}{6a^4d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-5 + 3*Cos[2*(c + d*x)] - 6*Sin[c + d*x])/(6*a^4*d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^6)
```

Maple [A] time = 0.034, size = 43, normalized size = 1.4

$$\frac{1}{da^4}\left((1+\sin(dx+c))^{-2}-\frac{1}{3(1+\sin(dx+c))^3}-(1+\sin(dx+c))^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(1/(1+sin(d*x+c))^2-1/3/(1+sin(d*x+c))^3-1/(1+sin(d*x+c)))
```

Maxima [B] time = 1.08461, size = 90, normalized size = 3.

$$-\frac{3\sin(dx+c)^2+3\sin(dx+c)+1}{3\left(a^4\sin(dx+c)^3+3a^4\sin(dx+c)^2+3a^4\sin(dx+c)+a^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 + 3*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^3 + 3*a^4*s
in(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4)*d)
```

Fricas [B] time = 1.36662, size = 174, normalized size = 5.8

$$-\frac{3\cos(dx+c)^2-3\sin(dx+c)-4}{3\left(3a^4d\cos(dx+c)^2-4a^4d+\left(a^4d\cos(dx+c)^2-4a^4d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 - 3*sin(d*x + c) - 4)/(3*a^4*d*cos(d*x + c)^2 - 4*a^
4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [A] time = 3.98311, size = 76, normalized size = 2.53

$$\begin{cases} \frac{\sin^3(c+dx)}{3a^4d\sin^3(c+dx)+9a^4d\sin^2(c+dx)+9a^4d\sin(c+dx)+3a^4d} & \text{for } d \neq 0 \\ \frac{x\sin^2(c)\cos(c)}{(a\sin(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((sin(c + d*x)**3/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x
)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d), Ne(d, 0)), (x*sin(c)**2*cos(c)/(a
*sin(c) + a)**4, True))
```

Giac [A] time = 1.18622, size = 51, normalized size = 1.7

$$-\frac{3\sin(dx+c)^2+3\sin(dx+c)+1}{3a^4d(\sin(dx+c)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 + 3*sin(d*x + c) + 1)/(a^4*d*(sin(d*x + c) + 1)^3)
```

3.253 $\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=46

$$\frac{1}{3ad(a\sin(c+dx)+a)^3}-\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}$$

```
[Out] 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 1/(2*d*(a^2 + a^2*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0520895, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{1}{3ad(a\sin(c+dx)+a)^3}-\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 1/(2*d*(a^2 + a^2*Sin[c + d*x])^2)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a(a+x)^4}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^4}\,dx,x,a\sin(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{(a+x)^4}+\frac{1}{(a+x)^3}\right)dx,x,a\sin(c+dx)\right)}{a^2d}\\
&= \frac{1}{3ad(a+a\sin(c+dx))^3}-\frac{1}{2d\left(a^2+a^2\sin(c+dx)\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.028238, size = 30, normalized size = 0.65

$$-\frac{3\sin(c+dx)+1}{6a^4d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(1 + 3*Sin[c + d*x])/(6*a^4*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.03, size = 33, normalized size = 0.7

$$\frac{1}{da^4}\left(-\frac{1}{2\left(1+\sin\left(dx+c\right)\right)^2}+\frac{1}{3\left(1+\sin\left(dx+c\right)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(-1/2/(1+sin(d*x+c))^2+1/3/(1+sin(d*x+c))^3)
```

Maxima [A] time = 1.08795, size = 77, normalized size = 1.67

$$-\frac{3\sin\left(dx+c\right)+1}{6\left(a^4\sin\left(dx+c\right)^3+3\,a^4\sin\left(dx+c\right)^2+3\,a^4\sin\left(dx+c\right)+a^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^3 + 3*a^4*sin(d*x + c)^2 + 3*a
^4*sin(d*x + c) + a^4)*d)
```

Fricas [A] time = 1.41514, size = 147, normalized size = 3.2

$$\frac{3\sin\left(dx+c\right)+1}{6\left(3\,a^4d\cos\left(dx+c\right)^2-4\,a^4d+\left(a^4d\cos\left(dx+c\right)^2-4\,a^4d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*sin(d*x + c) + 1)/(3*a^4*d*cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x
 + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [A] time = 3.86334, size = 129, normalized size = 2.8

$$\begin{cases} -\frac{3\sin\left(c+dx\right)}{6a^4d\sin^3\left(c+dx\right)+18a^4d\sin^2\left(c+dx\right)+18a^4d\sin\left(c+dx\right)+6a^4d}-\frac{1}{6a^4d\sin^3\left(c+dx\right)+18a^4d\sin^2\left(c+dx\right)+18a^4d\sin\left(c+dx\right)+6a^4d} & \text{for}\ d\neq 0 \\ \frac{x\sin\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-3*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*
x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) - 1/(6*a**4*d*sin(c + d*x)**3 +
18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d), Ne(d, 0)),
(x*sin(c)*cos(c)/(a*sin(c) + a)**4, True))
```

Giac [A] time = 1.1799, size = 38, normalized size = 0.83

$$-\frac{3 \sin(dx+c)+1}{6 a^4 d (\sin(dx+c)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*sin(d*x + c) + 1)/(a^4*d*(sin(d*x + c) + 1)^3)
```

3.254 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=97

$$\frac{1}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\log(\sin(c+dx))}{a^4d}-\frac{\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

```
[Out] Log[Sin[c + d*x]]/(a^4*d) - Log[1 + Sin[c + d*x]]/(a^4*d) + 1/(3*a*d*(a + a
*Sin[c + d*x])^3) + 1/(2*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(d*(a^4 + a^4*Si
n[c + d*x]))
```

Rubi [A] time = 0.064456, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\log(\sin(c+dx))}{a^4d}-\frac{\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^4*d) - Log[1 + Sin[c + d*x]]/(a^4*d) + 1/(3*a*d*(a + a
*Sin[c + d*x])^3) + 1/(2*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(d*(a^4 + a^4*Si
n[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^4}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^4x}-\frac{1}{a(a+x)^4}-\frac{1}{a^2(a+x)^3}-\frac{1}{a^3(a+x)^2}-\frac{1}{a^4(a+x)}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\log(\sin(c+dx))}{a^4d}-\frac{\log(1+\sin(c+dx))}{a^4d}+\frac{1}{3ad(a+a\sin(c+dx))^3}+\frac{1}{2d\left(a^2+a^2\sin(c+dx\right)}\end{aligned}$$

Mathematica [A] time = 0.338893, size = 62, normalized size = 0.64

$$\frac{\frac{6\sin^2(c+dx)+15\sin(c+dx)+11}{(\sin(c+dx)+1)^3}+6\log(\sin(c+dx))-6\log(\sin(c+dx)+1)}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (6*Log[Sin[c + d*x]] - 6*Log[1 + Sin[c + d*x]] + (11 + 15*Sin[c + d*x] + 6*
Sin[c + d*x]^2)/(1 + Sin[c + d*x])^3)/(6*a^4*d)
```

Maple [A] time = 0.046, size = 86, normalized size = 0.9

$$\frac{1}{3\,da^4\,(1+\sin(dx+c))^3}+\frac{1}{2\,da^4\,(1+\sin(dx+c))^2}+\frac{1}{da^4\,(1+\sin(dx+c))}-\frac{\ln(1+\sin(dx+c))}{da^4}+\frac{\ln(\sin(dx+c))}{da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/3/d/a^4/(1+sin(d*x+c))^3+1/2/d/a^4/(1+sin(d*x+c))^2+1/d/a^4/(1+sin(d*x+c)
)-ln(1+sin(d*x+c))/a^4/d+ln(sin(d*x+c))/a^4/d
```

Maxima [A] time = 1.11793, size = 128, normalized size = 1.32

$$\frac{\frac{6\,\sin(dx+c)^2+15\,\sin(dx+c)+11}{a^4\sin(dx+c)^3+3\,a^4\sin(dx+c)^2+3\,a^4\sin(dx+c)+a^4}-\frac{6\,\log(\sin(dx+c)+1)}{a^4}+\frac{6\,\log(\sin(dx+c))}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((6*sin(d*x + c)^2 + 15*sin(d*x + c) + 11)/(a^4*sin(d*x + c)^3 + 3*a^4*
sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) - 6*log(sin(d*x + c) + 1)/a^4 +
6*log(sin(d*x + c))/a^4)/d
```

Fricas [A] time = 1.48474, size = 405, normalized size = 4.18

$$\frac{6\,\cos(dx+c)^2+6\left(3\,\cos(dx+c)^2+\left(\cos(dx+c)^2-4\right)\sin(dx+c)-4\right)\log\left(\frac{1}{2}\,\sin(dx+c)\right)-6\left(3\,\cos(dx+c)^2+\left(\right.\right.}{6\left(3\,a^4d\cos(dx+c)^2-4\,a^4d+\left(a^4d\cos(dx+c)^2-4\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*cos(d*x + c)^2 + 6*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x
+ c) - 4)*log(1/2*sin(d*x + c)) - 6*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4
)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) - 15*sin(d*x + c) - 17)/(3*a^4*d*
```

```
cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos(c+dx)\csc(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6
*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.223, size = 93, normalized size = 0.96

$$-\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^4}-\frac{6\log(|\sin(dx+c)|)}{a^4}-\frac{6\sin(dx+c)^2+15\sin(dx+c)+11}{a^4(\sin(dx+c)+1)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(sin(d*x + c) + 1))/a^4 - 6*log(abs(sin(d*x + c)))/a^4 - (6*
sin(d*x + c)^2 + 15*sin(d*x + c) + 11)/(a^4*(sin(d*x + c) + 1)^3))/d
```

3.255 $\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=111

$$-\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{\csc(c+dx)}{a^4d}-\frac{4\log(\sin(c+dx))}{a^4d}+\frac{4\log(\sin(c+dx)+1)}{a^4d}-\frac{}{3ad}$$

```
[Out] -(Csc[c + d*x]/(a^4*d)) - (4*Log[Sin[c + d*x]])/(a^4*d) + (4*Log[1 + Sin[c
+ d*x]])/(a^4*d) - 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 1/(d*(a^2 + a^2*Sin[c
 + d*x])^2) - 3/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.0936986, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 44}

$$-\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{\csc(c+dx)}{a^4d}-\frac{4\log(\sin(c+dx))}{a^4d}+\frac{4\log(\sin(c+dx)+1)}{a^4d}-\frac{}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(Csc[c + d*x]/(a^4*d)) - (4*Log[Sin[c + d*x]])/(a^4*d) + (4*Log[1 + Sin[c
+ d*x]])/(a^4*d) - 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 1/(d*(a^2 + a^2*Sin[c
 + d*x])^2) - 3/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{a^4x^2} - \frac{4}{a^5x} + \frac{1}{a^2(a+x)^4} + \frac{2}{a^3(a+x)^3} + \frac{3}{a^4(a+x)^2} + \frac{4}{a^5(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{a^4d} - \frac{4\log(\sin(c+dx))}{a^4d} + \frac{4\log(1+\sin(c+dx))}{a^4d} - \frac{1}{3ad(a+a\sin(c+dx))^3} -$$

Mathematica [A] time = 0.985034, size = 73, normalized size = 0.66

$$-\frac{\frac{9}{\sin(c+dx)+1} + \frac{3}{(\sin(c+dx)+1)^2} + \frac{1}{(\sin(c+dx)+1)^3} + 3\csc(c+dx) + 12\log(\sin(c+dx)) - 12\log(\sin(c+dx)+1)}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(3*Csc[c + d*x] + 12*Log[Sin[c + d*x]] - 12*Log[1 + Sin[c + d*x]] + (1 + S
in[c + d*x])^(-3) + 3/(1 + Sin[c + d*x])^2 + 9/(1 + Sin[c + d*x]))/(3*a^4*d
)
```

Maple [A] time = 0.05, size = 104, normalized size = 0.9

$$-\frac{1}{3\,da^4\,(1+\sin(dx+c))^3} - \frac{1}{da^4\,(1+\sin(dx+c))^2} - 3\,\frac{1}{da^4\,(1+\sin(dx+c))} + 4\,\frac{\ln(1+\sin(dx+c))}{da^4} - \frac{1}{da^4\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/3/d/a^4/(1+sin(d*x+c))^3-1/d/a^4/(1+sin(d*x+c))^2-3/d/a^4/(1+sin(d*x+c))
+4*ln(1+sin(d*x+c))/a^4/d-1/d/a^4/sin(d*x+c)-4*ln(sin(d*x+c))/a^4/d
```

Maxima [A] time = 1.05201, size = 154, normalized size = 1.39

$$-\frac{\frac{12\sin(dx+c)^3+30\sin(dx+c)^2+22\sin(dx+c)+3}{a^4\sin(dx+c)^4+3\,a^4\sin(dx+c)^3+3\,a^4\sin(dx+c)^2+a^4\sin(dx+c)} - \frac{12\log(\sin(dx+c)+1)}{a^4} + \frac{12\log(\sin(dx+c))}{a^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((12*sin(d*x + c)^3 + 30*sin(d*x + c)^2 + 22*sin(d*x + c) + 3)/(a^4*si
n(d*x + c)^4 + 3*a^4*sin(d*x + c)^3 + 3*a^4*sin(d*x + c)^2 + a^4*sin(d*x +
c)) - 12*log(sin(d*x + c) + 1)/a^4 + 12*log(sin(d*x + c))/a^4)/d
```

Fricas [A] time = 1.44199, size = 525, normalized size = 4.73

$$\frac{30\cos(dx+c)^2-12\left(\cos(dx+c)^4-5\cos(dx+c)^2-\left(3\cos(dx+c)^2-4\right)\sin(dx+c)+4\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+1}{3\left(a^4d\cos(dx+c)^4-5a^4d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(30*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 5*cos(d*x + c)^2 - (3*cos(d*x
 + c)^2 - 4)*sin(d*x + c) + 4)*log(1/2*sin(d*x + c)) + 12*(cos(d*x + c)^4 -
 5*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 4)*sin(d*x + c) + 4)*log(sin(d*x +
c) + 1) + 2*(6*cos(d*x + c)^2 - 17)*sin(d*x + c) - 33)/(a^4*d*cos(d*x + c)^
4 - 5*a^4*d*cos(d*x + c)^2 + 4*a^4*d - (3*a^4*d*cos(d*x + c)^2 - 4*a^4*d)*s
in(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos(c+dx)\csc^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3
+ 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.2458, size = 117, normalized size = 1.05

$$\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^4}-\frac{12\log(|\sin(dx+c)|)}{a^4}-\frac{12\sin(dx+c)^3+30\sin(dx+c)^2+22\sin(dx+c)+3}{a^4(\sin(dx+c)+1)^3\sin(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(12*log(abs(sin(d*x + c) + 1))/a^4 - 12*log(abs(sin(d*x + c)))/a^4 - (1
2*sin(d*x + c)^3 + 30*sin(d*x + c)^2 + 22*sin(d*x + c) + 3)/(a^4*(sin(d*x +
 c) + 1)^3*sin(d*x + c)))/d
```

3.256 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=131

$$\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{3}{2d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{10\log(\sin(c+dx))}{a^4d}-\frac{10\log(\sin(c\cdots}{a^4d}$$

```
[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (10*Log[Sin[c + d*x]]
)/(a^4*d) - (10*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*
x])^3) + 3/(2*d*(a^2 + a^2*Sin[c + d*x])^2) + 6/(d*(a^4 + a^4*Sin[c + d*x])
)
```

Rubi [A] time = 0.117688, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{3}{2d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{10\log(\sin(c+dx))}{a^4d}-\frac{10\log(\sin(c\cdots}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (10*Log[Sin[c + d*x]]
)/(a^4*d) - (10*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*
x])^3) + 3/(2*d*(a^2 + a^2*Sin[c + d*x])^2) + 6/(d*(a^4 + a^4*Sin[c + d*x])
)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{a^4x^3} - \frac{4}{a^5x^2} + \frac{10}{a^6x} - \frac{1}{a^3(a+x)^4} - \frac{3}{a^4(a+x)^3} - \frac{6}{a^5(a+x)^2} - \frac{10}{a^6(a+x)}\right) dx, x, a\sin(c$$
$$= \frac{4\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{10\log(\sin(c+dx))}{a^4d} - \frac{10\log(1+\sin(c+dx))}{a^4d} + \frac{}{3ad}$$

Mathematica [A] time = 3.69481, size = 85, normalized size = 0.65

$$\frac{\frac{36}{\sin(c+dx)+1} + \frac{9}{(\sin(c+dx)+1)^2} + \frac{2}{(\sin(c+dx)+1)^3} - 3\csc^2(c+dx) + 24\csc(c+dx) + 60\log(\sin(c+dx)) - 60\log(\sin(c+dx)}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (24*Csc[c + d*x] - 3*Csc[c + d*x]^2 + 60*Log[Sin[c + d*x]] - 60*Log[1 + Sin
[c + d*x]] + 2/(1 + Sin[c + d*x])^3 + 9/(1 + Sin[c + d*x])^2 + 36/(1 + Sin[
c + d*x]))/(6*a^4*d)
```

Maple [A] time = 0.055, size = 120, normalized size = 0.9

$$\frac{1}{3\,da^4\,(1+\sin(dx+c))^3} + \frac{3}{2\,da^4\,(1+\sin(dx+c))^2} + 6\,\frac{1}{da^4\,(1+\sin(dx+c))} - 10\,\frac{\ln(1+\sin(dx+c))}{da^4} - \frac{1}{2\,da^4\,(\sin(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/3/d/a^4/(1+sin(d*x+c))^3+3/2/d/a^4/(1+sin(d*x+c))^2+6/d/a^4/(1+sin(d*x+c)
)-10*ln(1+sin(d*x+c))/a^4/d-1/2/d/a^4/sin(d*x+c)^2+4/d/a^4/sin(d*x+c)+10*ln
(sin(d*x+c))/a^4/d
```

Maxima [A] time = 1.08489, size = 170, normalized size = 1.3

$$\frac{\frac{60\,\sin(dx+c)^4+150\,\sin(dx+c)^3+110\,\sin(dx+c)^2+15\,\sin(dx+c)-3}{a^4\sin(dx+c)^5+3\,a^4\sin(dx+c)^4+3\,a^4\sin(dx+c)^3+a^4\sin(dx+c)^2} - \frac{60\,\log(\sin(dx+c)+1)}{a^4} + \frac{60\,\log(\sin(dx+c))}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((60*sin(d*x + c)^4 + 150*sin(d*x + c)^3 + 110*sin(d*x + c)^2 + 15*sin(
d*x + c) - 3)/(a^4*sin(d*x + c)^5 + 3*a^4*sin(d*x + c)^4 + 3*a^4*sin(d*x +
c)^3 + a^4*sin(d*x + c)^2) - 60*log(sin(d*x + c) + 1)/a^4 + 60*log(sin(d*x
```

```
+ c))/a^4)/d
```

Fricas [A] time = 1.54763, size = 643, normalized size = 4.91

$$\frac{60\cos(dx+c)^4 - 230\cos(dx+c)^2 + 60\left(3\cos(dx+c)^4 - 7\cos(dx+c)^2 + \left(\cos(dx+c)^4 - 5\cos(dx+c)^2 + 4\right)\sin(d}{6\left(3a^4d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*cos(d*x + c)^4 - 230*cos(d*x + c)^2 + 60*(3*cos(d*x + c)^4 - 7*cos(
d*x + c)^2 + (cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 4)*sin(d*x + c) + 4)*log(
1/2*sin(d*x + c)) - 60*(3*cos(d*x + c)^4 - 7*cos(d*x + c)^2 + (cos(d*x + c)
^4 - 5*cos(d*x + c)^2 + 4)*sin(d*x + c) + 4)*log(sin(d*x + c) + 1) - 15*(10
*cos(d*x + c)^2 - 11)*sin(d*x + c) + 167)/(3*a^4*d*cos(d*x + c)^4 - 7*a^4*d
*cos(d*x + c)^2 + 4*a^4*d + (a^4*d*cos(d*x + c)^4 - 5*a^4*d*cos(d*x + c)^2
+ 4*a^4*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos(c+dx)\csc^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3
+ 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.26951, size = 131, normalized size = 1.

$$-\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^4} - \frac{60\log(|\sin(dx+c)|)}{a^4} - \frac{60\sin(dx+c)^4+150\sin(dx+c)^3+110\sin(dx+c)^2+15\sin(dx+c)-3}{a^4(\sin(dx+c)+1)^3\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(60*log(abs(sin(d*x + c) + 1))/a^4 - 60*log(abs(sin(d*x + c)))/a^4 - (
60*sin(d*x + c)^4 + 150*sin(d*x + c)^3 + 110*sin(d*x + c)^2 + 15*sin(d*x +
c) - 3)/(a^4*(sin(d*x + c) + 1)^3*sin(d*x + c)^2))/d
```

3.257 $\int \cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a\sin(c+dx)+a}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Si
n[c + d*x]])/d
```

Rubi [A] time = 0.0616903, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2707, 50, 63, 207}

$$\frac{2\sqrt{a\sin(c+dx)+a}}{d}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Si
n[c + d*x]])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2\sqrt{a+a\sin(c+dx)}}{d} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+a\sin(c+dx)}\right)}{d}$$
$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sin(c+dx)}}{d}$$

Mathematica [B] time = 0.145009, size = 118, normalized size = 2.31

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((2*Cos[(c + d*x)/2] + Log[1 - Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1
 + Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2])*Sqrt[a*(1 + S
in[c + d*x])])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.103, size = 42, normalized size = 0.8

$$\frac{1}{d}\left(2\sqrt{a+a\sin(dx+c)}-2\sqrt{a}\text{Artanh}\left(\frac{\sqrt{a+a\sin(dx+c)}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(2*(a+a*sin(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+a*sin(d*x+c))^(1/2)/a^(1
/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44299, size = 232, normalized size = 4.55

$$\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^2+4\sqrt{a\sin(dx+c)+a}\sqrt{a}(\sin(dx+c)+2)-8a\sin(dx+c)-9a}{\cos(dx+c)^2-1}\right)+4\sqrt{a\sin(dx+c)+a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(a)*log((a*cos(d*x + c)^2 + 4*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(si
n(d*x + c) + 2) - 8*a*sin(d*x + c) - 9*a)/(cos(d*x + c)^2 - 1)) + 4*sqrt(a*
sin(d*x + c) + a))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\sin(c+dx)+1)}\cos(c+dx)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)*csc(c + d*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.258 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=114

$$\frac{a^4\sin^{n+1}(c+dx)}{d(n+1)}+\frac{4a^4\sin^{n+2}(c+dx)}{d(n+2)}+\frac{6a^4\sin^{n+3}(c+dx)}{d(n+3)}+\frac{4a^4\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a^4\sin^{n+5}(c+dx)}{d(n+5)}$$

```
[Out] (a^4*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (4*a^4*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (6*a^4*Sin[c + d*x]^(3 + n))/(d*(3 + n)) + (4*a^4*Sin[c + d*x]^(4
+ n))/(d*(4 + n)) + (a^4*Sin[c + d*x]^(5 + n))/(d*(5 + n))
```

Rubi [A] time = 0.115689, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{a^4\sin^{n+1}(c+dx)}{d(n+1)}+\frac{4a^4\sin^{n+2}(c+dx)}{d(n+2)}+\frac{6a^4\sin^{n+3}(c+dx)}{d(n+3)}+\frac{4a^4\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a^4\sin^{n+5}(c+dx)}{d(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (4*a^4*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (6*a^4*Sin[c + d*x]^(3 + n))/(d*(3 + n)) + (4*a^4*Sin[c + d*x]^(4
+ n))/(d*(4 + n)) + (a^4*Sin[c + d*x]^(5 + n))/(d*(5 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^4\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(a^4\left(\frac{x}{a}\right)^n+4a^4\left(\frac{x}{a}\right)^{1+n}+6a^4\left(\frac{x}{a}\right)^{2+n}+4a^4\left(\frac{x}{a}\right)^{3+n}+a^4\left(\frac{x}{a}\right)^{4}\right.}{ad}\\ &= \frac{a^4\sin^{1+n}(c+dx)}{d(1+n)}+\frac{4a^4\sin^{2+n}(c+dx)}{d(2+n)}+\frac{6a^4\sin^{3+n}(c+dx)}{d(3+n)}+\frac{4a^4 s}{}\end{aligned}$$

Mathematica [A] time = 0.257733, size = 80, normalized size = 0.7

$$\frac{a^4\sin^{n+1}(c+dx)\left(\frac{\sin^4(c+dx)}{n+5}+\frac{4\sin^3(c+dx)}{n+4}+\frac{6\sin^2(c+dx)}{n+3}+\frac{4\sin(c+dx)}{n+2}+\frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (4*Sin[c + d*x])/(2 + n) + (6*Sin
[c + d*x]^2)/(3 + n) + (4*Sin[c + d*x]^3)/(4 + n) + Sin[c + d*x]^4/(5 + n))
)/d
```

Maple [F] time = 3.113, size = 0, normalized size = 0.

$$\int \cos(dx+c)(\sin(dx+c))^n(a+a\sin(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62567, size = 701, normalized size = 6.15

$$\frac{\left(8a^4n^4 + 96a^4n^3 + 400a^4n^2 + 672a^4n + 4\left(a^4n^4 + 11a^4n^3 + 41a^4n^2 + 61a^4n + 30a^4\right)\cos(dx+c)^4 + 360a^4 - 4\left(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] (8*a^4*n^4 + 96*a^4*n^3 + 400*a^4*n^2 + 672*a^4*n + 4*(a^4*n^4 + 11*a^4*n^3
 + 41*a^4*n^2 + 61*a^4*n + 30*a^4)*cos(d*x + c)^4 + 360*a^4 - 4*(3*a^4*n^4
+ 35*a^4*n^3 + 141*a^4*n^2 + 229*a^4*n + 120*a^4)*cos(d*x + c)^2 + (8*a^4*n
^4 + 96*a^4*n^3 + 400*a^4*n^2 + 672*a^4*n + (a^4*n^4 + 10*a^4*n^3 + 35*a^4*
n^2 + 50*a^4*n + 24*a^4)*cos(d*x + c)^4 + 384*a^4 - 4*(2*a^4*n^4 + 23*a^4*n
^3 + 91*a^4*n^2 + 142*a^4*n + 72*a^4)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x
 + c)^n/(d*n^5 + 15*d*n^4 + 85*d*n^3 + 225*d*n^2 + 274*d*n + 120*d)
```

Sympy [A] time = 134.487, size = 1856, normalized size = 16.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**4*sin(c)**n*cos(c), Eq(d, 0)), (a**4*log(sin(c
 + d*x))/d - 4*a**4/(d*sin(c + d*x)) - 3*a**4/(d*sin(c + d*x)**2) - 4*a**4/
(3*d*sin(c + d*x)**3) - a**4/(4*d*sin(c + d*x)**4), Eq(n, -5)), (4*a**4*log
(sin(c + d*x))/d + a**4*sin(c + d*x)/d - 6*a**4/(d*sin(c + d*x)) - 2*a**4/(
d*sin(c + d*x)**2) - a**4/(3*d*sin(c + d*x)**3), Eq(n, -4)), (6*a**4*log(si
n(c + d*x))/d + 4*a**4*sin(c + d*x)/d - a**4*cos(c + d*x)**2/(2*d) - 4*a**4
/(d*sin(c + d*x)) - a**4/(2*d*sin(c + d*x)**2), Eq(n, -3)), (4*a**4*log(sin
(c + d*x))/d + a**4*sin(c + d*x)**3/(3*d) + 6*a**4*sin(c + d*x)/d - 2*a**4*
cos(c + d*x)**2/d - a**4/(d*sin(c + d*x)), Eq(n, -2)), (a**4*log(sin(c + d*
x))/d + 4*a**4*sin(c + d*x)**3/(3*d) - a**4*sin(c + d*x)**2*cos(c + d*x)**2
/(2*d) + 4*a**4*sin(c + d*x)/d - a**4*cos(c + d*x)**4/(4*d) - 3*a**4*cos(c
+ d*x)**2/d, Eq(n, -1)), (a**4*n**4*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**5
 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 4*a**4*n**4*sin(
c + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 +
274*d*n + 120*d) + 6*a**4*n**4*sin(c + d*x)**3*sin(c + d*x)**n/(d*n**5 + 15
*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 4*a**4*n**4*sin(c + d
*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d
*n + 120*d) + a**4*n**4*sin(c + d*x)*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 +
85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 10*a**4*n**3*sin(c + d*x)**5*si
n(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*
d) + 44*a**4*n**3*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*
d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 72*a**4*n**3*sin(c + d*x)**3*sin(c
 + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d)
+ 52*a**4*n**3*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n
**3 + 225*d*n**2 + 274*d*n + 120*d) + 14*a**4*n**3*sin(c + d*x)*sin(c + d*x
)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 35*a
**4*n**2*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 +
225*d*n**2 + 274*d*n + 120*d) + 164*a**4*n**2*sin(c + d*x)**4*sin(c + d*x)*
*n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 294*a*
*4*n**2*sin(c + d*x)**3*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 2
25*d*n**2 + 274*d*n + 120*d) + 236*a**4*n**2*sin(c + d*x)**2*sin(c + d*x)**
n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 71*a**4
*n**2*sin(c + d*x)*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*
n**2 + 274*d*n + 120*d) + 50*a**4*n*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**5
 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 244*a**4*n*sin(c
 + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 2
74*d*n + 120*d) + 468*a**4*n*sin(c + d*x)**3*sin(c + d*x)**n/(d*n**5 + 15*d
*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 428*a**4*n*sin(c + d*x)
**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n
+ 120*d) + 154*a**4*n*sin(c + d*x)*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85
*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 24*a**4*sin(c + d*x)**5*sin(c + d
*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 12
0*a**4*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 22
5*d*n**2 + 274*d*n + 120*d) + 240*a**4*sin(c + d*x)**3*sin(c + d*x)**n/(d*n
**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 240*a**4*sin(
c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 +
274*d*n + 120*d) + 120*a**4*sin(c + d*x)*sin(c + d*x)**n/(d*n**5 + 15*d*n**
4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d), True))
```

Giac [A] time = 1.2136, size = 171, normalized size = 1.5

$$\frac{\frac{a^4 \sin(dx+c)^n \sin(dx+c)^5}{n+5} + \frac{4\,a^4 \sin(dx+c)^n \sin(dx+c)^4}{n+4} + \frac{6\,a^4 \sin(dx+c)^n \sin(dx+c)^3}{n+3} + \frac{4\,a^4 \sin(dx+c)^n \sin(dx+c)^2}{n+2} + \frac{a^4 \sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (a^4*sin(d*x + c)^n*sin(d*x + c)^5/(n + 5) + 4*a^4*sin(d*x + c)^n*sin(d*x +
 c)^4/(n + 4) + 6*a^4*sin(d*x + c)^n*sin(d*x + c)^3/(n + 3) + 4*a^4*sin(d*x
 + c)^n*sin(d*x + c)^2/(n + 2) + a^4*sin(d*x + c)^(n + 1)/(n + 1))/d
```

3.259 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{3a^3\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a^3\sin^{n+4}(c+dx)}{d(n+4)}$$

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (3*a^3*Sin[c + d*x]^(3 + n))/(d*(3 + n)) + (a^3*Sin[c + d*x]^(4 +
n))/(d*(4 + n))
```

Rubi [A] time = 0.0953491, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{3a^3\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a^3\sin^{n+4}(c+dx)}{d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (3*a^3*Sin[c + d*x]^(3 + n))/(d*(3 + n)) + (a^3*Sin[c + d*x]^(4 +
n))/(d*(4 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx&=\frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n(a+x)^3\,dx,x,a\sin(c+dx)\right)}{ad}\\&=\frac{\text{Subst}\left(\int \left(a^3\left(\frac{x}{a}\right)^n+3a^3\left(\frac{x}{a}\right)^{1+n}+3a^3\left(\frac{x}{a}\right)^{2+n}+a^3\left(\frac{x}{a}\right)^{3+n}\right)dx,x,a\sin(c+dx)\right)}{ad}\\&=\frac{a^3\sin^{1+n}(c+dx)}{d(1+n)}+\frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)}+\frac{3a^3\sin^{3+n}(c+dx)}{d(3+n)}+\frac{a^3\sin^{4+n}(c+dx)}{d(4+n)}\end{aligned}$$

Mathematica [A] time = 0.148885, size = 65, normalized size = 0.71

$$\frac{a^3\sin^{n+1}(c+dx)\left(\frac{\sin^3(c+dx)}{n+4}+\frac{3\sin^2(c+dx)}{n+3}+\frac{3\sin(c+dx)}{n+2}+\frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (3*Sin[c + d*x])/(2 + n) + (3*Sin
[c + d*x]^2)/(3 + n) + Sin[c + d*x]^3/(4 + n)))/d
```

Maple [F] time = 2.313, size = 0, normalized size = 0.

$$\int \cos(dx+c)(\sin(dx+c))^n (a+a\sin(dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85824, size = 468, normalized size = 5.14

$$\frac{\left(4a^3n^3 + 30a^3n^2 + \left(a^3n^3 + 6a^3n^2 + 11a^3n + 6a^3\right)\cos(dx+c)^4 + 68a^3n + 42a^3 - \left(5a^3n^3 + 36a^3n^2 + 79a^3n + 48\right.\right.}{dn^4 + 10dn^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (4*a^3*n^3 + 30*a^3*n^2 + (a^3*n^3 + 6*a^3*n^2 + 11*a^3*n + 6*a^3)*cos(d*x
+ c)^4 + 68*a^3*n + 42*a^3 - (5*a^3*n^3 + 36*a^3*n^2 + 79*a^3*n + 48*a^3)*c
os(d*x + c)^2 + (4*a^3*n^3 + 30*a^3*n^2 + 68*a^3*n + 48*a^3 - 3*(a^3*n^3 +
7*a^3*n^2 + 14*a^3*n + 8*a^3)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x + c)^n/
(d*n^4 + 10*d*n^3 + 35*d*n^2 + 50*d*n + 24*d)
```

Sympy [A] time = 77.423, size = 1061, normalized size = 11.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**3*sin(c)**n*cos(c), Eq(d, 0)), (a**3*log(sin(c
 + d*x))/d - 3*a**3/(d*sin(c + d*x)) - 3*a**3/(2*d*sin(c + d*x)**2) - a**3/
(3*d*sin(c + d*x)**3), Eq(n, -4)), (3*a**3*log(sin(c + d*x))/d + a**3*sin(c
 + d*x)/d - 3*a**3/(d*sin(c + d*x)) - a**3/(2*d*sin(c + d*x)**2), Eq(n, -3)
), (3*a**3*log(sin(c + d*x))/d + 3*a**3*sin(c + d*x)/d - a**3*cos(c + d*x)*
*2/(2*d) - a**3/(d*sin(c + d*x)), Eq(n, -2)), (a**3*log(sin(c + d*x))/d + a
**3*sin(c + d*x)**3/(3*d) + 3*a**3*sin(c + d*x)/d - 3*a**3*cos(c + d*x)**2/
(2*d), Eq(n, -1)), (a**3*n**3*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 10*
d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 3*a**3*n**3*sin(c + d*x)**3*sin(c + d
*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 3*a**3*n**3*sin(c
 + d*x)**2*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d)
 + a**3*n**3*sin(c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 +
 50*d*n + 24*d) + 6*a**3*n**2*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 10*
d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 21*a**3*n**2*sin(c + d*x)**3*sin(c +
d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 24*a**3*n**2*sin
(c + d*x)**2*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*
d) + 9*a**3*n**2*sin(c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n*
*2 + 50*d*n + 24*d) + 11*a**3*n*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 1
0*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 42*a**3*n*sin(c + d*x)**3*sin(c + d
*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 57*a**3*n*sin(c +
 d*x)**2*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) +
 26*a**3*n*sin(c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 5
0*d*n + 24*d) + 6*a**3*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 10*d*n**3
+ 35*d*n**2 + 50*d*n + 24*d) + 24*a**3*sin(c + d*x)**3*sin(c + d*x)**n/(d*n
**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 36*a**3*sin(c + d*x)**2*sin(
c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 24*a**3*sin(
c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d),
True))
```

Giac [A] time = 1.18669, size = 136, normalized size = 1.49

$$\frac{\frac{a^3 \sin(dx+c)^n \sin(dx+c)^4}{n+4} + \frac{3\,a^3 \sin(dx+c)^n \sin(dx+c)^3}{n+3} + \frac{3\,a^3 \sin(dx+c)^n \sin(dx+c)^2}{n+2} + \frac{a^3 \sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (a^3*sin(d*x + c)^n*sin(d*x + c)^4/(n + 4) + 3*a^3*sin(d*x + c)^n*sin(d*x +
 c)^3/(n + 3) + 3*a^3*sin(d*x + c)^n*sin(d*x + c)^2/(n + 2) + a^3*sin(d*x +
 c)^(n + 1)/(n + 1))/d
```

3.260 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=68

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}$$

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n))
```

Rubi [A] time = 0.0862026, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^2\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(a^2\left(\frac{x}{a}\right)^n+2a^2\left(\frac{x}{a}\right)^{1+n}+a^2\left(\frac{x}{a}\right)^{2+n}\right)dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)}+\frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)}+\frac{a^2\sin^{3+n}(c+dx)}{d(3+n)}\end{aligned}$$

Mathematica [A] time = 0.201845, size = 50, normalized size = 0.74

$$\frac{a^2\sin^{n+1}(c+dx)\left(\frac{\sin^2(c+dx)}{n+3}+\frac{2\sin(c+dx)}{n+2}+\frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (2*Sin[c + d*x])/(2 + n) + Sin[c
+ d*x]^2/(3 + n)))/d
```

Maple [F] time = 2.069, size = 0, normalized size = 0.

$$\int \cos(dx+c)(\sin(dx+c))^n(a+a\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.744, size = 292, normalized size = 4.29

$$\frac{\left(2a^2n^2+8a^2n-2\left(a^2n^2+4a^2n+3a^2\right)\cos(dx+c)^2+6a^2+\left(2a^2n^2+8a^2n-\left(a^2n^2+3a^2n+2a^2\right)\cos(dx+c)^2+8\right.}{dn^3+6dn^2+11dn+6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*n^2 + 8*a^2*n - 2*(a^2*n^2 + 4*a^2*n + 3*a^2)*cos(d*x + c)^2 + 6*a^2
 + (2*a^2*n^2 + 8*a^2*n - (a^2*n^2 + 3*a^2*n + 2*a^2)*cos(d*x + c)^2 + 8*a^
2)*sin(d*x + c))*sin(d*x + c)^n/(d*n^3 + 6*d*n^2 + 11*d*n + 6*d)
```

Sympy [A] time = 18.3099, size = 530, normalized size = 7.79

$$\begin{cases} x(a\sin(c)+a)^2\sin^n(c)\cos(c) \\ \frac{a^2\log(\sin(c+dx))}{d}-\frac{2a^2}{d\sin(c+dx)}-\frac{a^2}{2d\sin^2(c+dx)} \\ \frac{2a^2\log(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2}{d\sin(c+dx)} \\ \frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos^2(c+dx)}{2d} \\ \frac{a^2n^2\sin^3(c+dx)\sin^n(c+dx)}{dn^3+6dn^2+11dn+6d}+\frac{2a^2n^2\sin^2(c+dx)\sin^n(c+dx)}{dn^3+6dn^2+11dn+6d}+\frac{a^2n^2\sin(c+dx)\sin^n(c+dx)}{dn^3+6dn^2+11dn+6d}+\frac{3a^2n\sin^3(c+dx)\sin^n(c+dx)}{dn^3+6dn^2+11dn+6d}+\frac{8a^2n\sin^2(c+dx)\sin^n(c+dx)}{dn^3+6dn^2+11dn+6d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**2*sin(c)**n*cos(c), Eq(d, 0)), (a**2*log(sin(c
 + d*x))/d - 2*a**2/(d*sin(c + d*x)) - a**2/(2*d*sin(c + d*x)**2), Eq(n, -3
)), (2*a**2*log(sin(c + d*x))/d + a**2*sin(c + d*x)/d - a**2/(d*sin(c + d*x
)), Eq(n, -2)), (a**2*log(sin(c + d*x))/d + 2*a**2*sin(c + d*x)/d - a**2*co
s(c + d*x)**2/(2*d), Eq(n, -1)), (a**2*n**2*sin(c + d*x)**3*sin(c + d*x)**n
/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 2*a**2*n**2*sin(c + d*x)**2*sin(c + d
*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + a**2*n**2*sin(c + d*x)*sin(c +
d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 3*a**2*n*sin(c + d*x)**3*sin(c
 + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 8*a**2*n*sin(c + d*x)**2*si
n(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 5*a**2*n*sin(c + d*x)*si
n(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 2*a**2*sin(c + d*x)**3*s
in(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 6*a**2*sin(c + d*x)**2*
sin(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 6*a**2*sin(c + d*x)*si
n(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d), True))
```

Giac [A] time = 1.16253, size = 101, normalized size = 1.49

$$\frac{\frac{a^2\sin(dx+c)^n\sin(dx+c)^3}{n+3}+\frac{2\,a^2\sin(dx+c)^n\sin(dx+c)^2}{n+2}+\frac{a^2\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*sin(d*x + c)^n*sin(d*x + c)^3/(n + 3) + 2*a^2*sin(d*x + c)^n*sin(d*x +
 c)^2/(n + 2) + a^2*sin(d*x + c)^(n + 1)/(n + 1))/d
```

3.261 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}$$

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
```

Rubi [A] time = 0.0524414, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2833, 43}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(a\left(\frac{x}{a}\right)^n + a\left(\frac{x}{a}\right)^{1+n}\right)dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{a\sin^{1+n}(c+dx)}{d(1+n)}+\frac{a\sin^{2+n}(c+dx)}{d(2+n)}\end{aligned}$$

Mathematica [A] time = 0.268006, size = 38, normalized size = 0.93

$$\frac{a\sin^{n+1}(c+dx)((n+1)\sin(c+dx)+n+2)}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n)*(2 + n + (1 + n)*Sin[c + d*x]))/(d*(1 + n)*(2 + n))
```

Maple [F] time = 1.553, size = 0, normalized size = 0.

$$\int \cos(dx+c)\left(\sin(dx+c)\right)^{n}\left(a+a\sin(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71689, size = 139, normalized size = 3.39

$$-\frac{\left((an+a)\cos(dx+c)^{2}-an-(an+2\,a)\sin(dx+c)-a\right)\sin(dx+c)^{n}}{dn^{2}+3\,dn+2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a*n + a)*cos(d*x + c)^2 - a*n - (a*n + 2*a)*sin(d*x + c) - a)*sin(d*x +
c)^n/(d*n^2 + 3*d*n + 2*d)
```

Sympy [A] time = 7.2123, size = 190, normalized size = 4.63

$$\begin{cases} x\left(a\sin(c)+a\right)\sin^{n}(c)\cos(c) & \text{for}\: d=0\\ \frac{a\log(\sin(c+dx))}{d}-\frac{a}{d\sin(c+dx)} & \text{for}\: n=-2\\ \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d} & \text{for}\: n=-1\\ \frac{an\sin^{2}(c+dx)\sin^{n}(c+dx)}{dn^{2}+3dn+2d}+\frac{an\sin(c+dx)\sin^{n}(c+dx)}{dn^{2}+3dn+2d}+\frac{a\sin^{2}(c+dx)\sin^{n}(c+dx)}{dn^{2}+3dn+2d}+\frac{2a\sin(c+dx)\sin^{n}(c+dx)}{dn^{2}+3dn+2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)*sin(c)**n*cos(c), Eq(d, 0)), (a*log(sin(c + d*x
))/d - a/(d*sin(c + d*x)), Eq(n, -2)), (a*log(sin(c + d*x))/d + a*sin(c + d
*x)/d, Eq(n, -1)), (a*n*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**2 + 3*d*n + 2
*d) + a*n*sin(c + d*x)*sin(c + d*x)**n/(d*n**2 + 3*d*n + 2*d) + a*sin(c + d
*x)**2*sin(c + d*x)**n/(d*n**2 + 3*d*n + 2*d) + 2*a*sin(c + d*x)*sin(c + d*
x)**n/(d*n**2 + 3*d*n + 2*d), True))
```

Giac [A] time = 1.25171, size = 61, normalized size = 1.49

$$\frac{\frac{a\sin(dx+c)^n\sin(dx+c)^2}{n+2} + \frac{a\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*sin(d*x + c)^n*sin(d*x + c)^2/(n + 2) + a*sin(d*x + c)^(n + 1)/(n + 1))/
d
```

3.262 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{ad(n+1)}$$

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
*d*(1 + n))
```

Rubi [A] time = 0.0748402, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 64}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
*d*(1 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{ad(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0342522, size = 38, normalized size = 1.

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
*d*(1 + n))
```

Maple [F] time = 0.562, size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)(\sin(dx+c))^n}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)^n\cos(dx+c)}{a\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

3.263 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=38

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(2,n+1;n+2;-\sin(c+dx))}{a^2d(n+1)}$$

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^2*d*(1 + n))
```

Rubi [A] time = 0.0738427, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 64}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(2,n+1;n+2;-\sin(c+dx))}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^2*d*(1 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{{}_2F_1(2,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^2d(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0300204, size = 38, normalized size = 1.

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(2,n+1;n+2;-\sin(c+dx))}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^2*d*(1 + n))
```

Maple [F] time = 0.819, size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)(\sin(dx+c))^n}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sin(dx+c)^n\cos(dx+c)}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x +
 c) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^2, x)
```

3.264 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=38

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}$$

```
[Out] (Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^3*d*(1 + n))
```

Rubi [A] time = 0.0741083, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 64}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^3*d*(1 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{{}_2F_1(3,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^3d(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0337427, size = 38, normalized size = 1.

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^3*d*(1 + n))
```

Maple [F] time = 0.917, size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)(\sin(dx+c))^n}{(a+a\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sin(dx+c)^n\cos(dx+c)}{3a^3\cos(dx+c)^2-4a^3+\left(a^3\cos(dx+c)^2-4a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*
cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^3, x)
```

3.265 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=38

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(4,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}$$

```
[Out] (Hypergeometric2F1[4, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^4*d*(1 + n))
```

Rubi [A] time = 0.0751362, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 64}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(4,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Hypergeometric2F1[4, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^4*d*(1 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{{}_2F_1(4,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0330643, size = 38, normalized size = 1.

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(4,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Hypergeometric2F1[4, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^4*d*(1 + n))
```

Maple [F] time = 0.982, size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)(\sin(dx+c))^n}{(a+a\sin(dx+c))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)^n\cos(dx+c)}{a^4\cos(dx+c)^4-8a^4\cos(dx+c)^2+8a^4-4\left(a^4\cos(dx+c)^2-2a^4\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x +
c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^4, x)
```

3.266 $\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}$$

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a*Cos
[c + d*x]^3*Sin[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.165903, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a*Cos
[c + d*x]^3*Sin[c + d*x]^3)/(6*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin^3(c+dx)\,dx + a\int \cos^2(c+dx)\sin^4(c+dx)\,dx$$

$$= -\frac{a\cos^3(c+dx)\sin^3(c+dx)}{6d} + \frac{1}{2}a\int \cos^2(c+dx)\sin^2(c+dx)\,dx -$$

$$= -\frac{a\cos^3(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin^3(c+dx)}{6d} + \frac{1}{8}a\int \text{co}$$

$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\,\text{co}}{}$$

$$= \frac{ax}{16} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} -$$

Mathematica [A] time = 0.194146, size = 71, normalized size = 0.68

$$\frac{a(-15\sin(2(c+dx)) - 15\sin(4(c+dx)) + 5\sin(6(c+dx)) - 120\cos(c+dx) - 20\cos(3(c+dx)) + 12\cos(5(c+dx)) + 6}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(60*d*x - 120*Cos[c + d*x] - 20*Cos[3*(c + d*x)] + 12*Cos[5*(c + d*x)] -
 15*Sin[2*(c + d*x)] - 15*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.029, size = 95, normalized size = 0.9

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^3}{6} - \frac{(\cos(dx+c))^3\sin(dx+c)}{8} + \frac{\cos(dx+c)\sin(dx+c)}{16} + \frac{dx}{16} + \frac{c}{16}\right) + a\left(-\frac{(\sin(dx}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos
(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+a*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*
cos(d*x+c)^3))
```

Maxima [A] time = 1.21096, size = 88, normalized size = 0.84

$$\frac{64\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a - 5\left(4\sin(2\,dx+2\,c)^3 - 12\,dx - 12\,c + 3\sin(4\,dx+4\,c)\right)a}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a - 5*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.71204, size = 193, normalized size = 1.84

$$\frac{48\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+15\,adx+5\left(8\,a\cos\left(dx+c\right)^{5}-14\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*a*d*x + 5*(8*a*cos(d*
x + c)^5 - 14*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 5.1592, size = 192, normalized size = 1.83

$$\begin{cases}\frac{ax\sin^{6}(c+dx)}{16}+\frac{3ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{ax\cos^{6}(c+dx)}{16}+\frac{a\sin^{5}(c+dx)\cos(c+dx)}{16d}-\frac{a\sin^{3}(c+dx)\cos^{3}(c+dx)}{6d} \\ x\left(a\sin(c)+a\right)\sin^{3}(c)\cos^{2}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) - a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a*sin(c + d*x)*cos(c + d*x)**
5/(16*d) - 2*a*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*
*3*cos(c)**2, True))
```

Giac [A] time = 1.3779, size = 124, normalized size = 1.18

$$\frac{1}{16}\,ax+\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{a\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x + 1/80*a*cos(5*d*x + 5*c)/d - 1/48*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d + 1/192*a*sin(6*d*x + 6*c)/d - 1/64*a*sin(4*d*x + 4*c)/d - 1/6
4*a*sin(2*d*x + 2*c)/d
```

3.267 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

```
[Out] (a*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
d*x]*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.128165, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
d*x]*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin^2(c+dx)\,dx + a\int \cos^2(c+dx)\sin^3(c+dx)$$

$$= -\frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\,(\int$$

$$= \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}a\int 1\,d$$

$$= \frac{ax}{8} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{8d}$$

Mathematica [A] time = 0.103072, size = 54, normalized size = 0.67

$$\frac{a(-15\sin(4(c+dx)) - 60\cos(c+dx) - 10\cos(3(c+dx)) + 6\cos(5(c+dx)) + 60c + 60dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(60*c + 60*d*x - 60*Cos[c + d*x] - 10*Cos[3*(c + d*x)] + 6*Cos[5*(c + d*
x)] - 15*Sin[4*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.027, size = 77, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^3}{5} - \frac{2(\cos(dx+c))^3}{15}\right) + a\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4} + \frac{\cos(dx+c)\sin(dx+c}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+a*(-1/4*cos(d*x+c
)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c))
```

Maxima [A] time = 1.2968, size = 70, normalized size = 0.86

$$\frac{32\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a + 15(4dx + 4c - \sin(4dx+4c))a}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

[Out] 1/480*(32*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a + 15*(4*d*x + 4*c - sin(4
*d*x + 4*c))*a)/d

Fricas [A] time = 1.65869, size = 162, normalized size = 2.

$$\frac{24\,a\cos\left(dx+c\right)^5-40\,a\cos\left(dx+c\right)^3+15\,adx-15\left(2\,a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(24*a*cos(d*x + c)^5 - 40*a*cos(d*x + c)^3 + 15*a*d*x - 15*(2*a*cos(d
*x + c)^3 - a*cos(d*x + c))*sin(d*x + c))/d

Sympy [A] time = 2.67902, size = 144, normalized size = 1.78

$$\begin{cases}\frac{ax\sin^4(c+dx)}{8}+\frac{ax\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{ax\cos^4(c+dx)}{8}+\frac{a\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{a\sin^2(c+dx)\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{2a\cos\end{cases}$$
$$x\left(a\sin\left(c\right)+a\right)\sin^2\left(c\right)\cos^2\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)

[Out] Piecewise((a*x*sin(c + d*x)**4/8 + a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
a*x*cos(c + d*x)**4/8 + a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a*sin(c + d*
x)**2*cos(c + d*x)**3/(3*d) - a*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a*co
s(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**2, Tru
e))

Giac [A] time = 1.36946, size = 84, normalized size = 1.04

$$\frac{1}{8}\,ax+\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/8*a*x + 1/80*a*cos(5*d*x + 5*c)/d - 1/48*a*cos(3*d*x + 3*c)/d - 1/8*a*cos
(d*x + c)/d - 1/32*a*sin(4*d*x + 4*c)/d

3.268 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

```
[Out] (a*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(8*d) -
(a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0929874, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(8*d) -
(a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin(c+dx)\,dx + a\int \cos^2(c+dx)\sin^2(c+dx)\,dx$$
$$= -\frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int x^2 \right.}{}$$
$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin(c+}{4d}$$
$$= \frac{ax}{8} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin(}{4d}$$

Mathematica [A] time = 0.0978183, size = 42, normalized size = 0.65

$$-\frac{a(3(\sin(4(c+dx))-4dx)+24\cos(c+dx)+8\cos(3(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(24*Cos[c + d*x] + 8*Cos[3*(c + d*x)] + 3*(-4*d*x + Sin[4*(c + d*x)])))
/(96*d)
```

Maple [A] time = 0.024, size = 57, normalized size = 0.9

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{a(\cos(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*
c)-1/3*a*cos(d*x+c)^3)
```

Maxima [A] time = 1.24129, size = 53, normalized size = 0.82

$$-\frac{32\,a\cos(dx+c)^3-3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*a*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.70671, size = 128, normalized size = 1.97

$$-\frac{8\,a\cos\left(dx+c\right)^{3}-3\,adx+3\,\left(2\,a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*a*cos(d*x + c)^3 - 3*a*d*x + 3*(2*a*cos(d*x + c)^3 - a*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [A] time = 1.32814, size = 119, normalized size = 1.83

$$\begin{cases}\frac{ax\sin^{4}(c+dx)}{8}+\frac{ax\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{ax\cos^{4}(c+dx)}{8}+\frac{a\sin^{3}(c+dx)\cos(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^{3}(c+dx)}{8d}-\frac{a\cos^{3}(c+dx)}{3d} & \text{for } d\neq 0\\ x\left(a\sin(c)+a\right)\sin(c)\cos^{2}(c) & \text{otherwis}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**4/8 + a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
a*x*cos(c + d*x)**4/8 + a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a*sin(c + d*
x)*cos(c + d*x)**3/(8*d) - a*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c)
 + a)*sin(c)*cos(c)**2, True))
```

Giac [A] time = 1.27498, size = 63, normalized size = 0.97

$$\frac{1}{8}\,ax-\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{a\cos\left(dx+c\right)}{4\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*a*x - 1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/32*a*sin(4*d
*x + 4*c)/d
```

3.269 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{ax}{2}$$

```
[Out] (a*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0637439, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2838, 2592, 321, 206, 2635, 8}

$$\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\,dx + a\int \cos(c+dx)\cot(c+dx)\,dx$$
$$= \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx - \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos\ldots\right.}{d}$$
$$= \frac{ax}{2} + \frac{a\cos(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\ldots\right.}{\ldots}$$
$$= \frac{ax}{2} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c\ldots}{2d}$$

Mathematica [A] time = 0.0717706, size = 74, normalized size = 1.45

$$\frac{a(c+dx)}{2d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\cos(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a
*Log[Sin[(c + d*x)/2]])/d + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.05, size = 63, normalized size = 1.2

$$\frac{\cos(dx+c)\,a\sin(dx+c)}{2\,d} + \frac{ax}{2} + \frac{ca}{2\,d} + \frac{\cos(dx+c)\,a}{d} + \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*a*x+1/2/d*c*a+a*cos(d*x+c)/d+1/d*a*ln(csc
(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.23225, size = 77, normalized size = 1.51

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a + 2\,a(2\,\cos(dx+c) - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 2*a*(2*cos(d*x + c) - log(cos(d*x
 + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.71267, size = 174, normalized size = 3.41

$$\frac{adx + a\cos\left(dx+c\right)\sin\left(dx+c\right) + 2\,a\cos\left(dx+c\right) - a\log\left(\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right) + a\log\left(-\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + a*cos(d*x + c)*sin(d*x + c) + 2*a*cos(d*x + c) - a*log(1/2*cos
(d*x + c) + 1/2) + a*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \cos^2\left(c+dx\right)\csc\left(c+dx\right)dx + \int \sin\left(c+dx\right)\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**2*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c
+ d*x)**2*csc(c + d*x), x))
```

Giac [A] time = 1.23311, size = 117, normalized size = 2.29

$$\frac{(dx+c)a + 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(a*tan(1/2*d*x +
1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.270 $\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

```
[Out] -(a*x) - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rubi [A] time = 0.0550534, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2710, 2592, 321, 206, 3473, 8}

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx &= \int \left(a\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx\\ &= a\int \cos(c+dx)\cot(c+dx)\,dx+a\int \cot^2(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)}{d}-a\int 1\,dx-\frac{a\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -ax+\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -ax-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}\end{aligned}$$

Mathematica [C] time = 0.0431169, size = 75, normalized size = 1.83

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d
```

Maple [A] time = 0.046, size = 57, normalized size = 1.4

$$-ax-\frac{a\cot(dx+c)}{d}+\frac{\cos(dx+c)\,a}{d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d}-\frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] -a*x-a*cot(d*x+c)/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a
```

Maxima [A] time = 1.80564, size = 73, normalized size = 1.78

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-a(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.75282, size = 236, normalized size = 5.76

$$-\frac{a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)+2\,(adx-a\cos(\ldots}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - a*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40176, size = 146, normalized size = 3.56

$$-\frac{6\,(dx+c)a-6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*a*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

3.271 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$-\frac{a\cot(c+dx)}{d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-ax$$

```
[Out] -(a*x) + (a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d - (a*Cot[c +
d*x]*Csc[c + d*x])/(2*d)
```

Rubi [A] time = 0.0771847, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2838, 2611, 3770, 3473, 8}

$$-\frac{a\cot(c+dx)}{d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) + (a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d - (a*Cot[c +
d*x]*Csc[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\,dx + a\int \cot^2(c+dx)\csc(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}a\int \csc(c+dx)\,dx$$
$$= -ax + \frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot(c+dx)\csc(}{2d}$$

Mathematica [C] time = 0.0424215, size = 109, normalized size = 2.1

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, -Tan[c + d*x]^2])/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Sin[(c
+ d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.056, size = 81, normalized size = 1.6

$$-ax - \frac{a\cot(dx+c)}{d} - \frac{ca}{d} - \frac{a(\cos(dx+c))^3}{2d(\sin(dx+c))^2} - \frac{\cos(dx+c)a}{2d} - \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -a*x-a*cot(d*x+c)/d-1/d*c*a-1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a*cos(d*x
+c)/d-1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.68494, size = 89, normalized size = 1.71

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a - a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} + \log(\cos(dx+c)+1) - \log(\cos(dx+c)-1)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c)/(cos(d*x + c)^2 -
1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.74136, size = 300, normalized size = 5.77

$$-\frac{4adx\cos(dx+c)^2 - 4adx - 4a\cos(dx+c)\sin(dx+c) - 2a\cos(dx+c) - \left(a\cos(dx+c)^2 - a\right)\log\left(\frac{1}{2}\cos(dx+c\right.}{4\left(d\cos(dx+c)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 - 4*a*d*x - 4*a*cos(d*x + c)*sin(d*x + c) - 2*
a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + (a*co
s(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4035, size = 128, normalized size = 2.46

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,(dx+c)a-4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 8*(d*x + c)*a - 4*a*log(abs(tan(1/2*d*x + 1
/2*c))) + 4*a*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^2 - 4*a*tan(
1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.272 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]*Csc[c + d*x])/(2*d)
```

Rubi [A] time = 0.108351, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]*Csc[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc(c+dx)\,dx + a\int \cot^2(c+dx)\csc^2(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}a\int \csc(c+dx)\,dx + \frac{a\,\text{Subst}\left(\int x^2 d\right.}{}$$
$$= \frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

Mathematica [A] time = 0.0379738, size = 95, normalized size = 1.83

$$-\frac{a\cot^3(c+dx)}{3d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^3)/(3*d) - (a*Csc[(c + d*x)/2]^2)/(8*d) + (a*Log[Cos[(c +
d*x)/2]])/(2*d) - (a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/
(8*d)
```

Maple [A] time = 0.053, size = 80, normalized size = 1.5

$$-\frac{a(\cos(dx+c))^3}{2d(\sin(dx+c))^2} - \frac{\cos(dx+c)a}{2d} - \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d} - \frac{a(\cos(dx+c))^3}{3d(\sin(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a*cos(d*x+c)/d-1/2/d*a*ln(csc(d*x+c)
-cot(d*x+c))-1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^3
```

Maxima [A] time = 1.05787, size = 82, normalized size = 1.58

$$\frac{3a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} + \log(\cos(dx+c)+1) - \log(\cos(dx+c)-1)\right) - \frac{4a}{\tan(dx+c)^3}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - lo
g(cos(d*x + c) - 1)) - 4*a/tan(d*x + c)^3)/d
```

Fricas [B] time = 1.64813, size = 316, normalized size = 6.08

$$\frac{4a\cos(dx+c)^3 + 6a\cos(dx+c)\sin(dx+c) + 3\left(a\cos(dx+c)^2 - a\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 3\left(a\cos(d\right.}{12\left(d\cos(dx+c)^2 - d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*cos(d*x + c)^3 + 6*a*cos(d*x + c)*sin(d*x + c) + 3*(a*cos(d*x + c
)^2 - a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a*cos(d*x + c)^2 - a
)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35528, size = 155, normalized size = 2.98

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{22\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a}{\mathrm{ta}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*log(abs(
tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x + 1/2*c) + (22*a*tan(1/2*d*x + 1/2
*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*
d*x + 1/2*c)^3)/d
```

3.273 $\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) + (a*Cot[c + d*x
]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.126206, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) + (a*Cot[c + d*x
]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^2(c+dx)\,dx + a\int \cot^2(c+dx)\csc^3(c+dx)$$
$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a\int \csc^3(c+dx)\,dx + \frac{a\,\mathrm{Subst}\big(\int$$
$$= -\frac{a\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(}{4d}$$
$$= \frac{a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc(c+d}{8d}$$

Mathematica [A] time = 0.0509982, size = 135, normalized size = 1.82

$$-\frac{a\cot^3(c+dx)}{3d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^3)/(3*d) + (a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Log[Cos[(c + d*x)/2]])/(8*d) - (a*Log[Sin[(c + d*x)/2]]
)/(8*d) - (a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.056, size = 102, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^3}{3d(\sin(dx+c))^3} - \frac{a(\cos(dx+c))^3}{4d(\sin(dx+c))^4} - \frac{a(\cos(dx+c))^3}{8d(\sin(dx+c))^2} - \frac{\cos(dx+c)a}{8d} - \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^3-1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*
a/sin(d*x+c)^2*cos(d*x+c)^3-1/8*a*cos(d*x+c)/d-1/8/d*a*ln(csc(d*x+c)-cot(d*
x+c))
```

Maxima [A] time = 1.11194, size = 108, normalized size = 1.46

$$-\frac{3a\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) + \frac{16a}{\tan(dx+c)^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/48*(3*a*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x +
 c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*a/tan(d*x
+ c)^3)/d
```

Fricas [B] time = 1.65404, size = 377, normalized size = 5.09

$$-\frac{16\,a\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+6\,a\cos\left(dx+c\right)^{3}+6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\cos\right.}{48\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(16*a*cos(d*x + c)^3*sin(d*x + c) + 6*a*cos(d*x + c)^3 + 6*a*cos(d*x
+ c) - 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) +
 1/2) + 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c)
 + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.329, size = 157, normalized size = 2.12

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{50\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+24\,a}{}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*a*tan(1/2*d*x + 1/2*c)^3 - 24*a*log(a
bs(tan(1/2*d*x + 1/2*c))) - 24*a*tan(1/2*d*x + 1/2*c) + (50*a*tan(1/2*d*x +
 1/2*c)^4 + 24*a*tan(1/2*d*x + 1/2*c)^3 - 8*a*tan(1/2*d*x + 1/2*c) - 3*a)/t
an(1/2*d*x + 1/2*c)^4)/d
```

3.274 $\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]^5)/(5*d) + (a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^3)/(4*d)
```

Rubi [A] time = 0.128629, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]^5)/(5*d) + (a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^3)/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^3(c+dx)\,dx + a\int \cot^2(c+dx)\csc^4(c+dx)\,dx$$

$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a\int \csc^3(c+dx)\,dx + \frac{a\,\mathrm{Subst}\left(\int x^2\right.}{}$$

$$= \frac{a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{8}a\int \csc(c$$

$$= \frac{a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} + \frac{a\cot(c+}{}$$

Mathematica [A] time = 0.0698738, size = 177, normalized size = 1.97

$$\frac{2a\cot(c+dx)}{15d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+d\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(15*d) + (a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*Cot[c + d*x]*C
sc[c + d*x]^4)/(5*d) + (a*Log[Cos[(c + d*x)/2]])/(8*d) - (a*Log[Sin[(c + d*
x)/2]])/(8*d) - (a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*
d)
```

Maple [A] time = 0.059, size = 124, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^3}{4d(\sin(dx+c))^4} - \frac{a(\cos(dx+c))^3}{8d(\sin(dx+c))^2} - \frac{\cos(dx+c)a}{8d} - \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{8d} - \frac{a(\cos(dx+c))^3}{5d(\sin(dx+c))^5} - \frac{2a(}{15d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/8*a*
cos(d*x+c)/d-1/8/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*a/sin(d*x+c)^5*cos(d*x
+c)^3-2/15/d*a/sin(d*x+c)^3*cos(d*x+c)^3
```

Maxima [A] time = 1.06786, size = 124, normalized size = 1.38

$$-\frac{15\,a\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\left(5\,\tan(dx+c)^2+3\right)a}{\tan(dx+c)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(15*a*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*(5*tan(d
*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Fricas [B] time = 1.78324, size = 466, normalized size = 5.18

$$\frac{32\,a\cos\left(dx+c\right)^5-80\,a\cos\left(dx+c\right)^3+15\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right.}{240\left(d\cos\left(dx+c\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*(a*cos(d*x + c)^4 - 2
*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a*cos
(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 30*(a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36653, size = 194, normalized size = 2.16

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-60\,a\tan\left(\frac{1}{2}\,d\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(
1/2*d*x + 1/2*c)^3 - 120*a*log(abs(tan(1/2*d*x + 1/2*c))) - 60*a*tan(1/2*d*
x + 1/2*c) + (274*a*tan(1/2*d*x + 1/2*c)^5 + 60*a*tan(1/2*d*x + 1/2*c)^4 -
10*a*tan(1/2*d*x + 1/2*c)^2 - 15*a*tan(1/2*d*x + 1/2*c) - 6*a)/tan(1/2*d*x
+ 1/2*c)^5)/d
```

3.275 $\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=135

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{3a^2\cos^5(c+dx)}{5d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{a^2\sin^3(c+dx)\cos^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

```
[Out] (a^2*x)/8 - (2*a^2*Cos[c + d*x]^3)/(3*d) + (3*a^2*Cos[c + d*x]^5)/(5*d) - (
a^2*Cos[c + d*x]^7)/(7*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^2*Co
s[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x]^3)/(3*d
)
```

Rubi [A] time = 0.248058, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{3a^2\cos^5(c+dx)}{5d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{a^2\sin^3(c+dx)\cos^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (a^2*x)/8 - (2*a^2*Cos[c + d*x]^3)/(3*d) + (3*a^2*Cos[c + d*x]^5)/(5*d) - (
a^2*Cos[c + d*x]^7)/(7*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^2*Co
s[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x]^3)/(3*d
)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^2(c+dx)\sin^3(c+dx)+2a^2\cos^2(c+dx)\sin^4(c+dx)+a^2\right.$$

$$= a^2\int \cos^2(c+dx)\sin^3(c+dx)\,dx + a^2\int \cos^2(c+dx)\sin^5(c+dx)\,d$$

$$= -\frac{a^2\cos^3(c+dx)\sin^3(c+dx)}{3d} + a^2\int \cos^2(c+dx)\sin^2(c+dx)\,dx -$$

$$= -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a^2\cos^3(c+dx)\sin^3(c+dx)}{3d} + \frac{1}{4}a^2$$

$$= -\frac{2a^2\cos^3(c+dx)}{3d} + \frac{3a^2\cos^5(c+dx)}{5d} - \frac{a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos(c}{}$$

$$= \frac{a^2x}{8} - \frac{2a^2\cos^3(c+dx)}{3d} + \frac{3a^2\cos^5(c+dx)}{5d} - \frac{a^2\cos^7(c+dx)}{7d} + \frac{a^2}{}$$

Mathematica [A] time = 0.480042, size = 86, normalized size = 0.64

$$\frac{a^2(-210\sin(2(c+dx))-210\sin(4(c+dx))+70\sin(6(c+dx))-1365\cos(c+dx)-175\cos(3(c+dx))+147\cos(5(c+}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*Cos[
5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(c +
d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.04, size = 151, normalized size = 1.1

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^3}{7}-\frac{4(\sin(dx+c))^2(\cos(dx+c))^3}{35}-\frac{8(\cos(dx+c))^3}{105}\right)+2a^2\left(-1/6(\sin(dx+c))^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/1
05*cos(d*x+c)^3)+2*a^2*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin
(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*(-1/5*sin(d*x+c)^2*
cos(d*x+c)^3-2/15*cos(d*x+c)^3))
```

Maxima [A] time = 1.13296, size = 142, normalized size = 1.05

$$-\frac{32\left(15\cos\left(dx+c\right)^{7}-42\cos\left(dx+c\right)^{5}+35\cos\left(dx+c\right)^{3}\right)a^{2}-224\left(3\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{3}\right)a^{2}+35\left(4\sin\right.}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3360*(32*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^2
 - 224*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2 + 35*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A] time = 1.72279, size = 250, normalized size = 1.85

$$-\frac{120\,a^{2}\cos\left(dx+c\right)^{7}-504\,a^{2}\cos\left(dx+c\right)^{5}+560\,a^{2}\cos\left(dx+c\right)^{3}-105\,a^{2}dx-35\left(8\,a^{2}\cos\left(dx+c\right)^{5}-14\,a^{2}\cos\left(dx+\right.\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/840*(120*a^2*cos(d*x + c)^7 - 504*a^2*cos(d*x + c)^5 + 560*a^2*cos(d*x +
 c)^3 - 105*a^2*d*x - 35*(8*a^2*cos(d*x + c)^5 - 14*a^2*cos(d*x + c)^3 + 3*
a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 21.1997, size = 275, normalized size = 2.04

$$\begin{cases}\frac{a^{2}x\sin^{6}\left(c+dx\right)}{8}+\frac{3a^{2}x\sin^{4}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}+\frac{3a^{2}x\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{8}+\frac{a^{2}x\cos^{6}\left(c+dx\right)}{8}+\frac{a^{2}\sin^{5}\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{a^{2}\sin^{4}\left(c+dx\right)\cos^{3}}{3d}\\ x\left(a\sin\left(c\right)+a\right)^{2}\sin^{3}\left(c\right)\cos^{2}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/8 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x)
**2/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + a**2*x*cos(c + d*x)**6
/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) - a**2*sin(c + d*x)**4*cos(c +
 d*x)**3/(3*d) - a**2*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 4*a**2*sin(c
+ d*x)**2*cos(c + d*x)**5/(15*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*
d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 8*a**2*cos(c + d*x)**7/(105*
d) - 2*a**2*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*
*3*cos(c)**2, True))
```

Giac [A] time = 1.36784, size = 166, normalized size = 1.23

$$\frac{1}{8}a^2x - \frac{a^2\cos(7dx+7c)}{448d} + \frac{7a^2\cos(5dx+5c)}{320d} - \frac{5a^2\cos(3dx+3c)}{192d} - \frac{13a^2\cos(dx+c)}{64d} + \frac{a^2\sin(6dx+6c)}{96d} - \frac{a^2\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x - 1/448*a^2*cos(7*d*x + 7*c)/d + 7/320*a^2*cos(5*d*x + 5*c)/d - 5
/192*a^2*cos(3*d*x + 3*c)/d - 13/64*a^2*cos(d*x + c)/d + 1/96*a^2*sin(6*d*x
 + 6*c)/d - 1/32*a^2*sin(4*d*x + 4*c)/d - 1/32*a^2*sin(2*d*x + 2*c)/d
```

3.276 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=103

$$-\frac{a^2\cos^5(c+dx)}{10d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{3a^2x}{16}-\frac{\cos^3(c+dx)(a\sin(c+dx)+a}{6ad}$$

```
[Out] (3*a^2*x)/16 - (a^2*Cos[c + d*x]^5)/(10*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*
x])/(16*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (Cos[c + d*x]^3*(a +
 a*Sin[c + d*x])^3)/(6*a*d)
```

Rubi [A] time = 0.167851, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2870, 2669, 2635, 8}

$$-\frac{a^2\cos^5(c+dx)}{10d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{3a^2x}{16}-\frac{\cos^3(c+dx)(a\sin(c+dx)+a}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/16 - (a^2*Cos[c + d*x]^5)/(10*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*
x])/(16*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (Cos[c + d*x]^3*(a +
 a*Sin[c + d*x])^3)/(6*a*d)
```

Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^3}{6ad}+\frac{1}{2}a\int \cos^4(c+dx)(a+a\sin(c$$
$$= -\frac{a^2\cos^5(c+dx)}{10d}-\frac{\cos^3(c+dx)(a+a\sin(c+dx))^3}{6ad}+\frac{1}{2}a^2\int \cos^4($$
$$= -\frac{a^2\cos^5(c+dx)}{10d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{\cos^3(c+dx)(a+}{6ad}$$
$$= -\frac{a^2\cos^5(c+dx)}{10d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^2\cos^3(c+dx)\mathrm{si}}{8d}$$
$$= \frac{3a^2x}{16}-\frac{a^2\cos^5(c+dx)}{10d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^2\cos^3(c+}{}$$

Mathematica [A] time = 0.429726, size = 76, normalized size = 0.74

$$\frac{a^2(-15\sin(2(c+dx))-45\sin(4(c+dx))+5\sin(6(c+dx))-240\cos(c+dx)-40\cos(3(c+dx))+24\cos(5(c+dx))+}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[5*(
c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)])
)/(960*d)
```

Maple [A] time = 0.034, size = 142, normalized size = 1.4

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^3}{6}-\frac{(\cos(dx+c))^3\sin(dx+c)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2a^2\left(-1/5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+2*a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3
-2/15*cos(d*x+c)^3)+a^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*
x+c)+1/8*d*x+1/8*c))
```

Maxima [A] time = 1.13309, size = 126, normalized size = 1.22

$$\frac{128\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)a^2-5\left(4\sin(2dx+2c)^3-12dx-12c+3\sin(4dx+4c)\right)a^2+30(4dx+4c-\mathrm{s}}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/960*(128*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2 - 5*(4*sin(2*d*x + 2*c
)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2 + 30*(4*d*x + 4*c - sin(4*d*x
```

```
+ 4*c))*a^2)/d
```

Fricas [A] time = 1.65558, size = 211, normalized size = 2.05

$$\frac{96 a^2 \cos(dx+c)^5 - 160 a^2 \cos(dx+c)^3 + 45 a^2 dx + 5\left(8 a^2 \cos(dx+c)^5 - 26 a^2 \cos(dx+c)^3 + 9 a^2 \cos(dx+c)\right) \sin}{240 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/240*(96*a^2*cos(d*x + c)^5 - 160*a^2*cos(d*x + c)^3 + 45*a^2*d*x + 5*(8*a
^2*cos(d*x + c)^5 - 26*a^2*cos(d*x + c)^3 + 9*a^2*cos(d*x + c))*sin(d*x + c
))/d
```

Sympy [A] time = 11.8154, size = 309, normalized size = 3.

$$\begin{cases} \frac{a^2 x \sin^6(c+dx)}{16} + \frac{3a^2 x \sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{a^2 x \sin^4(c+dx)}{8} + \frac{3a^2 x \sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{a^2 x \sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^2 x \cos^6(c+dx)}{16} \\ x\left(a\sin(c)+a\right)^2 \sin^2(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/16 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)*
*4/16 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**2*x*cos(c + d*x)**6/1
6 + a**2*x*cos(c + d*x)**4/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) - a
**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + a**2*sin(c + d*x)**3*cos(c + d*
x)/(8*d) - 2*a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a**2*sin(c + d*x)
*cos(c + d*x)**5/(16*d) - a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 4*a**2*
cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**2
, True))
```

Giac [A] time = 1.34954, size = 143, normalized size = 1.39

$$\frac{3}{16} a^2 x + \frac{a^2 \cos(5dx+5c)}{40d} - \frac{a^2 \cos(3dx+3c)}{24d} - \frac{a^2 \cos(dx+c)}{4d} + \frac{a^2 \sin(6dx+6c)}{192d} - \frac{3a^2 \sin(4dx+4c)}{64d} - \frac{a^2 \sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/16*a^2*x + 1/40*a^2*cos(5*d*x + 5*c)/d - 1/24*a^2*cos(3*d*x + 3*c)/d - 1/
4*a^2*cos(d*x + c)/d + 1/192*a^2*sin(6*d*x + 6*c)/d - 3/64*a^2*sin(4*d*x +
4*c)/d - 1/64*a^2*sin(2*d*x + 2*c)/d
```

3.277 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=91

$$\frac{a^2\cos^5(c+dx)}{5d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{a^2x}{4}$$

```
[Out] (a^2*x)/4 - (2*a^2*Cos[c + d*x]^3)/(3*d) + (a^2*Cos[c + d*x]^5)/(5*d) + (a^
2*Cos[c + d*x]*Sin[c + d*x])/(4*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d
)
```

Rubi [A] time = 0.130063, antiderivative size = 105, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2\cos^3(c+dx)}{6d}-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{10d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{a^2x}{4}-\frac{\cos^3(c+dx)(a\sin(c+dx)+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*x)/4 - (a^2*Cos[c + d*x]^3)/(6*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) - (Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(5*d) - (Cos[c + d*x]^3*(a^2
+ a^2*Sin[c + d*x]))/(10*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}+\frac{2}{5}\int \cos^2(c+dx)(a+a\sin(c$$
$$= -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{\cos^3(c+dx)\left(a^2+a^2\sin(c+\right.}{10d}$$
$$= -\frac{a^2\cos^3(c+dx)}{6d}-\frac{\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{\cos^3(c+d}{}$$
$$= -\frac{a^2\cos^3(c+dx)}{6d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{4d}-\frac{\cos^3(c+dx)(a+}{5}$$
$$= \frac{a^2x}{4}-\frac{a^2\cos^3(c+dx)}{6d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{4d}-\frac{\cos^3(c+dx}{}$$

Mathematica [A] time = 0.19693, size = 57, normalized size = 0.63

$$\frac{a^2(-90\cos(c+dx)-25\cos(3(c+dx))+3(-5\sin(4(c+dx))+\cos(5(c+dx))+20c+20dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-90*Cos[c + d*x] - 25*Cos[3*(c + d*x)] + 3*(20*c + 20*d*x + Cos[5*(c
+ d*x)] - 5*Sin[4*(c + d*x)])))/(240*d)
```

Maple [A] time = 0.033, size = 95, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^3}{5}-\frac{2(\cos(dx+c))^3}{15}\right)+2a^2\left(-1/4(\cos(dx+c))^3\sin(dx+c)+1/8\cos(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+2*a^2*(-1/4*cos
(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-1/3*a^2*cos(d
*x+c)^3)
```

Maxima [A] time = 1.07884, size = 93, normalized size = 1.02

$$-\frac{80\,a^2\cos(dx+c)^3-16\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)a^2-15\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(80*a^2*cos(d*x + c)^3 - 16*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^
2 - 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A] time = 1.67019, size = 174, normalized size = 1.91

$$\frac{12\,a^2\cos\left(dx+c\right)^5-40\,a^2\cos\left(dx+c\right)^3+15\,a^2dx-15\left(2\,a^2\cos\left(dx+c\right)^3-a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^2*cos(d*x + c)^5 - 40*a^2*cos(d*x + c)^3 + 15*a^2*d*x - 15*(2*a^
2*cos(d*x + c)^3 - a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 4.64726, size = 172, normalized size = 1.89

$$\begin{cases}\frac{a^2x\sin^4\left(c+dx\right)}{4}+\frac{a^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{2}+\frac{a^2x\cos^4\left(c+dx\right)}{4}+\frac{a^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{a^2\sin^2\left(c+dx\right)\cos^3\left(c+dx\right)}{3d}-\frac{a^2\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{4d}- \\ x\left(a\sin\left(c\right)+a\right)^2\sin\left(c\right)\cos^2\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**4/4 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**
2/2 + a**2*x*cos(c + d*x)**4/4 + a**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) -
a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a**2*sin(c + d*x)*cos(c + d*x)
**3/(4*d) - 2*a**2*cos(c + d*x)**5/(15*d) - a**2*cos(c + d*x)**3/(3*d), Ne(
d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c)**2, True))
```

Giac [A] time = 1.29283, size = 97, normalized size = 1.07

$$\frac{1}{4}\,a^2x+\frac{a^2\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{5\,a^2\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{3\,a^2\cos\left(dx+c\right)}{8\,d}-\frac{a^2\sin\left(4\,dx+4\,c\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*x + 1/80*a^2*cos(5*d*x + 5*c)/d - 5/48*a^2*cos(3*d*x + 3*c)/d - 3/8
*a^2*cos(d*x + c)/d - 1/16*a^2*sin(4*d*x + 4*c)/d
```

3.278 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=71

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+a^2x$$

```
[Out] a^2*x - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d - (a^2*Cos[c +
 d*x]^3)/(3*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/d
```

Rubi [A] time = 0.12029, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2873, 2635, 8, 2592, 321, 206, 2565, 30}

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d - (a^2*Cos[c +
 d*x]^3)/(3*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
 + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(2a^2\cos^2(c+dx)+a^2\cos(c+dx)\cot(c+dx)+a^2\cos^2(c+dx)\sin\right.\\ &= a^2\int \cos(c+dx)\cot(c+dx)\,dx+a^2\int \cos^2(c+dx)\sin(c+dx)\,dx+\\ &= \frac{a^2\cos(c+dx)\sin(c+dx)}{d}+a^2\int 1\,dx-\frac{a^2\,\mathrm{Subst}\left(\int x^2\,dx,x,\cos(c+\right.}{d}\\ &= a^2x+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{d}\\ &= a^2x-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\cos^3(c+dx)}{3d}+\end{aligned}$$

Mathematica [A] time = 0.367208, size = 71, normalized size = 1.

$$\frac{a^2\left(9\cos(c+dx)-\cos(3(c+dx))+6\left(\sin(2(c+dx))+2\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+c+dx\right)\right)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(9*Cos[c + d*x] - Cos[3*(c + d*x)] + 6*(2*(c + d*x - Log[Cos[(c + d*x)
/2]] + Log[Sin[(c + d*x)/2]]) + Sin[2*(c + d*x)])))/(12*d)
```

Maple [A] time = 0.065, size = 86, normalized size = 1.2

$$-\frac{a^2(\cos(dx+c))^3}{3d}+\frac{a^2\cos(dx+c)\sin(dx+c)}{d}+a^2x+\frac{ca^2}{d}+\frac{a^2\cos(dx+c)}{d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)*sin(d*x+c)/d+a^2*x+1/d*c*a^2+a^2*cos
(d*x+c)/d+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.11724, size = 101, normalized size = 1.42

$$-\frac{2\,a^2\cos\left(dx+c\right)^3-3\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2-3\,a^2\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right.\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 3*a^2
*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.73264, size = 231, normalized size = 3.25

$$-\frac{2\,a^2\cos\left(dx+c\right)^3-6\,a^2dx-6\,a^2\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,a^2\cos\left(dx+c\right)+3\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,a^2\,\mathrm{lo}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^2*cos(d*x + c)^3 - 6*a^2*d*x - 6*a^2*cos(d*x + c)*sin(d*x + c) -
6*a^2*cos(d*x + c) + 3*a^2*log(1/2*cos(d*x + c) + 1/2) - 3*a^2*log(-1/2*cos
(d*x + c) + 1/2))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33272, size = 136, normalized size = 1.92

$$\frac{3\,(dx+c)a^2+3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*a^2*tan(
1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2
*c) - 2*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.279 $\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=74

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

```
[Out] -(a^2*x)/2 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.104235, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 2638, 2635}

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] -(a^2*x)/2 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```


Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\sin(c+dx)-a^4\sin^2(c+dx)\right)dx}{a^2}$$
$$= a^2\int \csc^2(c+dx)\,dx - a^2\int \sin^2(c+dx)\,dx + \left(2a^2\right)\int \csc(c+dx)\,dx - \left(2a^2\right)\int$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}a^2$$
$$= -\frac{a^2x}{2} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cos(c$$

Mathematica [A] time = 0.556689, size = 94, normalized size = 1.27

$$-\frac{a^2\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(7\cos(c+dx)+\cos(3(c+dx))+4\sin(c+dx)\left(-4\cos(c+dx)-4\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(7*Cos[c + d*x] + Cos[3*(c + d*x)]
+ 4*(c + d*x - 4*Cos[c + d*x] + 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*
x)/2]])*Sin[c + d*x]))/(16*d)
```

Maple [A] time = 0.064, size = 89, normalized size = 1.2

$$\frac{a^2\cos(dx+c)\sin(dx+c)}{2d} - \frac{a^2x}{2} - \frac{ca^2}{2d} + 2\frac{a^2\cos(dx+c)}{d} + 2\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d} - \frac{a^2\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*cos(d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*c*a^2+2*a^2*cos(d*x+c)/d+2/
d*a^2*ln(csc(d*x+c)-cot(d*x+c))-a^2*cot(d*x+c)/d
```

Maxima [A] time = 1.59901, size = 107, normalized size = 1.45

$$\frac{(2dx+2c+\sin(2dx+2c))a^2 - 4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2 + 4a^2(2\cos(dx+c) - \log(\cos(dx+c)+1) + \log(\cos(dx+c$$
$$4d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 4*(d*x + c + 1/tan(d*x + c))*a^
2 + 4*a^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
```

```
/d
```

Fricas [A] time = 1.79059, size = 281, normalized size = 3.8

$$-\frac{a^2\cos(dx+c)^3+2\,a^2\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,a^2\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+a^2\cos(dx}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(a^2*cos(d*x + c)^3 + 2*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + a^2*cos(d*x + c) + (a^2*
d*x - 4*a^2*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33705, size = 193, normalized size = 2.61

$$-\frac{(dx+c)a^2-4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - a^2*tan(1/2*d*
x + 1/2*c) + (4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) + 2*(a
^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x
+ 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.280 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=73

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-2a^2x$$

```
[Out] -2*a^2*x - (a^2*ArcTanh[Cos[c + d*x]])/(2*d) + (a^2*Cos[c + d*x])/d - (2*a^
2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rubi [A] time = 0.129619, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 3767, 8, 3768, 3770, 2638}

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -2*a^2*x - (a^2*ArcTanh[Cos[c + d*x]])/(2*d) + (a^2*Cos[c + d*x])/d - (2*a^
2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int\left(-2a^4+2a^4\csc^2(c+dx)+a^4\csc^3(c+dx)-a^4\sin(c+dx)\right)dx}{a^2}$$
$$= -2a^2x+a^2\int\csc^3(c+dx)\,dx-a^2\int\sin(c+dx)\,dx+\left(2a^2\right)\int\text{cs}$$
$$= -2a^2x+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a^2\int\text{csc}$$
$$= -2a^2x-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+}{d}$$

Mathematica [A] time = 0.65086, size = 102, normalized size = 1.4

$$\frac{a^2\left(8\cos(c+dx)+8\tan\left(\frac{1}{2}(c+dx)\right)-8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-16*c - 16*d*x + 8*Cos[c + d*x] - 8*Cot[(c + d*x)/2] - Csc[(c + d*x)/
2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]
^2 + 8*Tan[(c + d*x)/2]))/(8*d)
```

Maple [A] time = 0.073, size = 93, normalized size = 1.3

$$\frac{a^2\cos(dx+c)}{2d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-2a^2x-2\frac{a^2\cot(dx+c)}{d}-2\frac{ca^2}{d}-\frac{a^2(\cos(dx+c))^3}{2d(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*cos(d*x+c)/d+1/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2*a^2*x-2*a^2*cot(
d*x+c)/d-2/d*c*a^2-1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3
```

Maxima [A] time = 1.72393, size = 140, normalized size = 1.92

$$-\frac{8\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)-2a^2(2\cos(dx+c)-\log}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(8*(d*x + c + 1/tan(d*x + c))*a^2 - a^2*(2*cos(d*x + c)/(cos(d*x + c)^
2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 2*a^2*(2*cos(d*x
+ c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.73496, size = 352, normalized size = 4.82

$$-\frac{8\,a^2dx\cos\left(dx+c\right)^2-4\,a^2\cos\left(dx+c\right)^3-8\,a^2dx-8\,a^2\cos\left(dx+c\right)\sin\left(dx+c\right)+2\,a^2\cos\left(dx+c\right)+\left(a^2\cos\left(dx+c\right)^2-\right.}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/4*(8*a^2*d*x*cos(d*x + c)^2 - 4*a^2*cos(d*x + c)^3 - 8*a^2*d*x - 8*a^2*c
os(d*x + c)*sin(d*x + c) + 2*a^2*cos(d*x + c) + (a^2*cos(d*x + c)^2 - a^2)*
log(1/2*cos(d*x + c) + 1/2) - (a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x +
 c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45154, size = 173, normalized size = 2.37

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,(dx+c)a^2+4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{16\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{6\,a^2\tan\left(\frac{1}{2}\,dx\cdots\right.}{\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*a^2 + 4*a^2*log(abs(tan(1/2*
d*x + 1/2*c))) + 8*a^2*tan(1/2*d*x + 1/2*c) + 16*a^2/(tan(1/2*d*x + 1/2*c)^
2 + 1) - (6*a^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*tan(1/2*d*x + 1/2*c) + a^2)/
tan(1/2*d*x + 1/2*c)^2)/d
```

3.281 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=73

$$-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-a^2x$$

```
[Out] -(a^2*x) + (a^2*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d - (a^2*Cot[
c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d
```

Rubi [A] time = 0.221475, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (a^2*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d - (a^2*Cot[
c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx)+2a^2\cot^2(c+dx)\csc(c+dx)+a^2\cot^2(c+dx)\right.$$
$$= a^2\int \cot^2(c+dx)\,dx + a^2\int \cot^2(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right)\int$$
$$= -\frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot(c+dx)\csc(c+dx)}{d} - a^2\int 1\,dx - a^2\int \mathrm{cs}$$
$$= -a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d}$$

Mathematica [A] time = 0.537671, size = 140, normalized size = 1.92

$$-\frac{a^2\left(-8\tan\left(\frac{1}{2}(c+dx)\right)+8\cot\left(\frac{1}{2}(c+dx)\right)+6\csc^2\left(\frac{1}{2}(c+dx)\right)-6\sec^2\left(\frac{1}{2}(c+dx)\right)+24\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-24\log\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 - 24*Log[C
os[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2 - 8*Csc[
c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 - 8*Tan
[(c + d*x)/2]))/(24*d)
```

Maple [A] time = 0.072, size = 117, normalized size = 1.6

$$-a^2x - \frac{a^2\cot(dx+c)}{d} - \frac{ca^2}{d} - \frac{a^2(\cos(dx+c))^3}{d(\sin(dx+c))^2} - \frac{a^2\cos(dx+c)}{d} - \frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d} - \frac{a^2(\cos(dx+c}{3d(\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -a^2*x-a^2*cot(d*x+c)/d-1/d*c*a^2-1/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-a^2*cos
(d*x+c)/d-1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*a^2/sin(d*x+c)^3*cos(d*x+
c)^3
```

Maxima [A] time = 1.63969, size = 112, normalized size = 1.53

$$-\frac{6\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2 - 3a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)+\frac{2a^2}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6*(6*(d*x + c + 1/tan(d*x + c))*a^2 - 3*a^2*(2*cos(d*x + c)/(cos(d*x + c
)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 2*a^2/tan(d*x +
 c)^3)/d
```

Fricas [B] time = 1.72115, size = 410, normalized size = 5.62

$$-\frac{4\,a^2\cos\left(dx+c\right)^3-6\,a^2\cos\left(dx+c\right)-3\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(a^2\cos\left(dx+\right.\right.}{6\left(d\cos\left(dx+c\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(4*a^2*cos(d*x + c)^3 - 6*a^2*cos(d*x + c) - 3*(a^2*cos(d*x + c)^2 - a
^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^2*cos(d*x + c)^2 - a^2)
*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(a^2*d*x*cos(d*x + c)^2 - a^
2*d*x - a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40057, size = 190, normalized size = 2.6

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\left(dx+c\right)a^2-24\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*(d*x +
 c)*a^2 - 24*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 9*a^2*tan(1/2*d*x + 1/2*c
) + (44*a^2*tan(1/2*d*x + 1/2*c)^3 - 9*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*t
an(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.282 $\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=82

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (3*a^2
*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.200896, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (3*a^2
*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx)\csc(c+dx)+2a^2\cot^2(c+dx)\csc^2(c+dx)+\right.$$
$$= a^2\int \cot^2(c+dx)\csc(c+dx)\,dx + a^2\int \cot^2(c+dx)\csc^3(c+d$$
$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a^2$$
$$= \frac{a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{3a^2\cot(c+dx)\csc}{8d}$$
$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{3a^2\cot(c+dx)\csc}{8d}$$

Mathematica [B] time = 0.113833, size = 209, normalized size = 2.55

$$a^2\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3\sec^2\left(\frac{1}{2}(c+\right.}{32d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Cos[(c
 + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x)/2]^
2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))
```

Maple [A] time = 0.075, size = 112, normalized size = 1.4

$$-\frac{5a^2(\cos(dx+c))^3}{8d(\sin(dx+c))^2}-\frac{5a^2\cos(dx+c)}{8d}-\frac{5a^2\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{2a^2(\cos(dx+c))^3}{3d(\sin(dx+c))^3}-\frac{a^2(\cos(dx+c}{4d(\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-5/8*a^2*cos(d*x+c)/d-5/8/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))-2/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1/4/d*a^2/sin(d*x+c)
^4*cos(d*x+c)^3
```

Maxima [A] time = 1.09377, size = 176, normalized size = 2.15

$$-\frac{3a^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(3*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*a^2*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c)
 - 1)) + 32*a^2/tan(d*x + c)^3)/d
```

Fricas [B] time = 1.8103, size = 406, normalized size = 4.95

$$-\frac{32\,a^2\cos\left(dx+c\right)^3\sin\left(dx+c\right)-18\,a^2\cos\left(dx+c\right)^3+30\,a^2\cos\left(dx+c\right)-15\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(32*a^2*cos(d*x + c)^3*sin(d*x + c) - 18*a^2*cos(d*x + c)^3 + 30*a^2*
cos(d*x + c) - 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2
*cos(d*x + c) + 1/2) + 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)
*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42004, size = 221, normalized size = 2.7

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-48\,a^2\tan\left(\frac{1}{2}\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*a^
2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 48*a^2*
tan(1/2*d*x + 1/2*c) + (250*a^2*tan(1/2*d*x + 1/2*c)^4 + 48*a^2*tan(1/2*d*x
 + 1/2*c)^3 - 24*a^2*tan(1/2*d*x + 1/2*c)^2 - 16*a^2*tan(1/2*d*x + 1/2*c) -
 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.283 $\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=100

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^2\cot(c+dx)\csc(c+}{4d}$$

```
[Out] (a^2*ArcTanh[Cos[c + d*x]])/(4*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot
[c + d*x]^5)/(5*d) + (a^2*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (a^2*Cot[c + d
*x]*Csc[c + d*x]^3)/(2*d)
```

Rubi [A] time = 0.209662, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^2\cot(c+dx)\csc(c+}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Cos[c + d*x]])/(4*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot
[c + d*x]^5)/(5*d) + (a^2*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (a^2*Cot[c + d
*x]*Csc[c + d*x]^3)/(2*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx)\csc^2(c+dx)+2a^2\cot^2(c+dx)\csc^3(c+dx)+a\right.$$
$$= a^2\int \cot^2(c+dx)\csc^2(c+dx)\,dx + a^2\int \cot^2(c+dx)\csc^4(c+dx)$$
$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d} - \frac{1}{2}a^2\int \csc^3(c+dx)\,dx + \frac{a^2\,\mathrm{Subst}\,(}{}$$
$$= -\frac{a^2\cot^3(c+dx)}{3d} + \frac{a^2\cot(c+dx)\csc(c+dx)}{4d} - \frac{a^2\cot(c+dx)\csc^{3}}{2d}$$
$$= \frac{a^2\tanh^{-1}(\cos(c+dx))}{4d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2 c}{}$$

Mathematica [A] time = 0.729497, size = 189, normalized size = 1.89

$$-\frac{a^2\csc^5(c+dx)\left(180\sin(2(c+dx))+30\sin(4(c+dx))+200\cos(c+dx)+20\cos(3(c+dx))-28\cos(5(c+dx))+150\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^5*(200*Cos[c + d*x] + 20*Cos[3*(c + d*x)] - 28*Cos[5*(c
+ d*x)] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d*x)/2]
]*Sin[c + d*x] + 180*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
 d*x)] - 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 30*Sin[4*(c + d*x)] -
15*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*Sin[5*
(c + d*x)]))/(960*d)
```

Maple [A] time = 0.074, size = 136, normalized size = 1.4

$$-\frac{7a^2(\cos(dx+c))^3}{15d(\sin(dx+c))^3} - \frac{a^2(\cos(dx+c))^3}{2d(\sin(dx+c))^4} - \frac{a^2(\cos(dx+c))^3}{4d(\sin(dx+c))^2} - \frac{a^2\cos(dx+c)}{4d} - \frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{4d} - \frac{a^2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -7/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1/2/d*a^2/sin(d*x+c)^4*cos(d*x+c)^3-1
/4/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-1/4*a^2*cos(d*x+c)/d-1/4/d*a^2*ln(csc(d*
x+c)-cot(d*x+c))-1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^3
```

Maxima [A] time = 1.03934, size = 147, normalized size = 1.47

$$-\frac{15\,a^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{40\,a^2}{\tan(dx+c)^3}+\frac{8\left(5\,\tan(dx+c)^2+3\right)a^2}{\tan(dx+c)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*(15*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 40*a^2/ta
n(d*x + c)^3 + 8*(5*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Fricas [B] time = 1.81746, size = 493, normalized size = 4.93

$$\frac{56\,a^2\cos\left(dx+c\right)^5-80\,a^2\cos\left(dx+c\right)^3+15\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin(\ldots}{120\left(d\cos\left(dx\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(56*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c)^3 + 15*(a^2*cos(d*x + c)
^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 30*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c))*sin(d*x +
 c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33177, size = 221, normalized size = 2.21

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-90\,a^2\tan\ldots}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 + 25*a^
2*tan(1/2*d*x + 1/2*c)^3 - 120*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 90*a^2*
tan(1/2*d*x + 1/2*c) + (274*a^2*tan(1/2*d*x + 1/2*c)^5 + 90*a^2*tan(1/2*d*x
 + 1/2*c)^4 - 25*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) -
 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.284 $\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=124

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^2\cot(c+dx)\csc^3}{24d}$$

```
[Out] (3*a^2*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (2*a^
2*Cot[c + d*x]^5)/(5*d) + (3*a^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (5*a^2
*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(6
*d)
```

Rubi [A] time = 0.255639, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 14}

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^2\cot(c+dx)\csc^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (2*a^
2*Cot[c + d*x]^5)/(5*d) + (3*a^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (5*a^2
*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(6
*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx)\csc^3(c+dx)+2a^2\cot^2(c+dx)\csc^4(c+dx)+a\right.$$

$$= a^2\int \cot^2(c+dx)\csc^3(c+dx)\,dx + a^2\int \cot^2(c+dx)\csc^5(c+dx)$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d} - \frac{1}{6}a^2\int$$

$$= \frac{a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{5a^2\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a^2\cot(}{}$$

$$= \frac{a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} + \frac{3a}{}$$

$$= \frac{3a^2\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} + \frac{3}{}$$

Mathematica [A] time = 0.68412, size = 229, normalized size = 1.85

$$\frac{a^2\csc^6(c+dx)\left(-960\sin(2(c+dx))-384\sin(4(c+dx))+64\sin(6(c+dx))-1500\cos(c+dx)+130\cos(3(c+dx))+9\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c + d*x]^6*(-1500*Cos[c + d*x] + 130*Cos[3*(c + d*x)] + 90*Cos[5*(
c + d*x)] + 450*Log[Cos[(c + d*x)/2]] - 675*Cos[2*(c + d*x)]*Log[Cos[(c + d
*x)/2]] + 270*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 45*Cos[6*(c + d*x)]*
Log[Cos[(c + d*x)/2]] - 450*Log[Sin[(c + d*x)/2]] + 675*Cos[2*(c + d*x)]*Lo
g[Sin[(c + d*x)/2]] - 270*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 45*Cos[6
*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 960*Sin[2*(c + d*x)] - 384*Sin[4*(c + d
*x)] + 64*Sin[6*(c + d*x)]))/(7680*d)
```

Maple [A] time = 0.08, size = 160, normalized size = 1.3

$$-\frac{3a^2(\cos(dx+c))^3}{8d(\sin(dx+c))^4} - \frac{3a^2(\cos(dx+c))^3}{16d(\sin(dx+c))^2} - \frac{3a^2\cos(dx+c)}{16d} - \frac{3a^2\ln(\csc(dx+c)-\cot(dx+c))}{16d} - \frac{2a^2(\cos(dx+c))^{}}{5d(\sin(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/8/d*a^2/sin(d*x+c)^4*cos(d*x+c)^3-3/16/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-3
/16*a^2*cos(d*x+c)/d-3/16/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/5/d*a^2/sin(d*x
+c)^5*cos(d*x+c)^3-4/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1/6/d*a^2/sin(d*x+c
)^6*cos(d*x+c)^3
```

Maxima [A] time = 1.39654, size = 252, normalized size = 2.03

$$-\frac{5\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+30\,a^2\left(\frac{2\left(\cos(dx+c)^3+\cos\right.}{\cos(dx+c)^4-2\,\cos(}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^2*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 30*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c
))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(co
s(d*x + c) - 1)) + 64*(5*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Fricas [B] time = 1.74528, size = 567, normalized size = 4.57

$$-\frac{90\,a^2\cos\left(dx+c\right)^5-80\,a^2\cos\left(dx+c\right)^3-90\,a^2\cos\left(dx+c\right)-45\left(a^2\cos\left(dx+c\right)^6-3\,a^2\cos\left(dx+c\right)^4+3\,a^2\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(90*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c)^3 - 90*a^2*cos(d*x + c)
 - 45*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a
^2)*log(1/2*cos(d*x + c) + 1/2) + 45*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x +
c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2) + 64*(2*a^2
*cos(d*x + c)^5 - 5*a^2*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3
*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45891, size = 308, normalized size = 2.48

$$\frac{5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+24\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+45\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+40\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*tan(1/2*d*x + 1/2*c)^5 + 45*a
^2*tan(1/2*d*x + 1/2*c)^4 + 40*a^2*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*
d*x + 1/2*c)^2 - 360*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 240*a^2*tan(1/2*d
*x + 1/2*c) + (882*a^2*tan(1/2*d*x + 1/2*c)^6 + 240*a^2*tan(1/2*d*x + 1/2*c
)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 - 40*a^2*tan(1/2*d*x + 1/2*c)^3 - 45*a^
2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*tan(1/2*d*x + 1/2*c) - 5*a^2)/tan(1/2*d*x
 + 1/2*c)^6)/d
```

3.285 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=132

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{d}-\frac{4a^3\cos^3(c+dx)}{3d}-\frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{2d}-\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d}$$

```
[Out] (5*a^3*x)/16 - (4*a^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/d - (a^3
*Cos[c + d*x]^7)/(7*d) + (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^3*
Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a^3*Cos[c + d*x]^3*Sin[c + d*x]^3)/(2
*d)
```

Rubi [A] time = 0.30451, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2568, 2635, 8, 2565, 14, 270}

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{d}-\frac{4a^3\cos^3(c+dx)}{3d}-\frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{2d}-\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/16 - (4*a^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/d - (a^3
*Cos[c + d*x]^7)/(7*d) + (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^3*
Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a^3*Cos[c + d*x]^3*Sin[c + d*x]^3)/(2
*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^2(c+dx)\sin^2(c+dx) + 3a^3\cos^2(c+dx)\sin^3(c+dx) + 3\right.$$

$$= a^3\int \cos^2(c+dx)\sin^2(c+dx)\,dx + a^3\int \cos^2(c+dx)\sin^5(c+dx)\,$$

$$= -\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a^3\cos^3(c+dx)\sin^3(c+dx)}{2d} + \frac{1}{4}a^3$$

$$= \frac{a^3\cos(c+dx)\sin(c+dx)}{8d} - \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{8d} - \frac{a^3\cos^3}{}$$

$$= \frac{a^3x}{8} - \frac{4a^3\cos^3(c+dx)}{3d} + \frac{a^3\cos^5(c+dx)}{d} - \frac{a^3\cos^7(c+dx)}{7d} + \frac{5a^3}{}$$

$$= \frac{5a^3x}{16} - \frac{4a^3\cos^3(c+dx)}{3d} + \frac{a^3\cos^5(c+dx)}{d} - \frac{a^3\cos^7(c+dx)}{7d} + \frac{5a^3}{}$$

Mathematica [A] time = 0.647043, size = 86, normalized size = 0.65

$$\frac{a^3(-63\sin(2(c+dx)) - 105\sin(4(c+dx)) + 21\sin(6(c+dx)) - 609\cos(c+dx) - 91\cos(3(c+dx)) + 63\cos(5(c+dx))}{1344d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(420*c + 420*d*x - 609*Cos[c + d*x] - 91*Cos[3*(c + d*x)] + 63*Cos[5*(
c + d*x)] - 3*Cos[7*(c + d*x)] - 63*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)]
 + 21*Sin[6*(c + d*x)]))/(1344*d)
```

Maple [A] time = 0.039, size = 194, normalized size = 1.5

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^3}{7} - \frac{4(\sin(dx+c))^2(\cos(dx+c))^3}{35} - \frac{8(\cos(dx+c))^3}{105}\right) + 3a^3\left(-1/6(\sin(dx+c))^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/1
05*cos(d*x+c)^3)+3*a^3*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin
(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+3*a^3*(-1/5*sin(d*x+c)^
2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos
(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c))
```

Maxima [A] time = 1.11474, size = 174, normalized size = 1.32

$$-\frac{64\left(15\cos(dx+c)^7-42\cos(dx+c)^5+35\cos(dx+c)^3\right)a^3-1344\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)a^3+105\left(4\ \mathrm{s}\right.}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6720*(64*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^3
 - 1344*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^3 + 105*(4*sin(2*d*x + 2*c)
^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^3 - 210*(4*d*x + 4*c - sin(4*d*x
 + 4*c))*a^3)/d
```

Fricas [A] time = 1.7867, size = 248, normalized size = 1.88

$$-\frac{48\,a^3\cos(dx+c)^7-336\,a^3\cos(dx+c)^5+448\,a^3\cos(dx+c)^3-105\,a^3dx-21\left(8\,a^3\cos(dx+c)^5-18\,a^3\cos(dx+\right.}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/336*(48*a^3*cos(d*x + c)^7 - 336*a^3*cos(d*x + c)^5 + 448*a^3*cos(d*x +
c)^3 - 105*a^3*d*x - 21*(8*a^3*cos(d*x + c)^5 - 18*a^3*cos(d*x + c)^3 + 5*a
^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 9.70729, size = 379, normalized size = 2.87

$$\begin{cases}\frac{3a^3x\sin^6(c+dx)}{16}+\frac{9a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{a^3x\sin^4(c+dx)}{8}+\frac{9a^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^3x\cos^6(c+dx}{16} \\ x\left(a\sin(c)+a\right)^3\sin^2(c)\cos^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**6/16 + 9*a**3*x*sin(c + d*x)**4*cos(c + d
*x)**2/16 + a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**3*x*cos(c + d*x)*
*6/16 + a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*
d) - a**3*sin(c + d*x)**4*cos(c + d*x)**3/(3*d) - a**3*sin(c + d*x)**3*cos(
c + d*x)**3/(2*d) + a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 4*a**3*sin(c
+ d*x)**2*cos(c + d*x)**5/(15*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**3/d -
```

```
3*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a**3*sin(c + d*x)*cos(c + d*x
)**3/(8*d) - 8*a**3*cos(c + d*x)**7/(105*d) - 2*a**3*cos(c + d*x)**5/(5*d),
 Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**2*cos(c)**2, True))
```

Giac [A] time = 1.34728, size = 166, normalized size = 1.26

$$\frac{5}{16}a^3x - \frac{a^3\cos(7\,dx+7\,c)}{448\,d} + \frac{3\,a^3\cos(5\,dx+5\,c)}{64\,d} - \frac{13\,a^3\cos(3\,dx+3\,c)}{192\,d} - \frac{29\,a^3\cos(dx+c)}{64\,d} + \frac{a^3\sin(6\,dx+6\,c)}{64\,d} - \frac{5\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/16*a^3*x - 1/448*a^3*cos(7*d*x + 7*c)/d + 3/64*a^3*cos(5*d*x + 5*c)/d - 1
3/192*a^3*cos(3*d*x + 3*c)/d - 29/64*a^3*cos(d*x + c)/d + 1/64*a^3*sin(6*d*
x + 6*c)/d - 5/64*a^3*sin(4*d*x + 4*c)/d - 3/64*a^3*sin(2*d*x + 2*c)/d
```

3.286 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=117

$$\frac{3a^3\cos^5(c+dx)}{5d}-\frac{4a^3\cos^3(c+dx)}{3d}-\frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{7a^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{7a^3\sin(c+dx)\,c}{16d}$$

```
[Out] (7*a^3*x)/16 - (4*a^3*Cos[c + d*x]^3)/(3*d) + (3*a^3*Cos[c + d*x]^5)/(5*d)
+ (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (7*a^3*Cos[c + d*x]^3*Sin[c +
d*x])/(8*d) - (a^3*Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.181315, antiderivative size = 133, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{7a^3\cos^3(c+dx)}{24d}-\frac{7\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{40d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{7a^3x}{16}-\frac{a\cos^3(c+dx)(a\sin}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*x)/16 - (7*a^3*Cos[c + d*x]^3)/(24*d) + (7*a^3*Cos[c + d*x]*Sin[c +
d*x])/(16*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(10*d) - (Cos[c +
d*x]^3*(a + a*Sin[c + d*x])^3)/(6*d) - (7*Cos[c + d*x]^3*(a^3 + a^3*Sin[c +
 d*x]))/(40*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} + \frac{1}{2}\int \cos^2(c+dx)(a+a\sin(c+$$
$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d} - \frac{\cos^3(c+dx)(a+a\sin(c+dx)}{6d}$$
$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d} - \frac{\cos^3(c+dx)(a+a\sin(c+dx)}{6d}$$
$$= -\frac{7a^3\cos^3(c+dx)}{24d} - \frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d} - \frac{\cos^3(c+d}{}$$
$$= -\frac{7a^3\cos^3(c+dx)}{24d} + \frac{7a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^3(c+dx)(a}{10}$$
$$= \frac{7a^3x}{16} - \frac{7a^3\cos^3(c+dx)}{24d} + \frac{7a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^3(c+}{}$$

Mathematica [A] time = 0.396885, size = 76, normalized size = 0.65

$$\frac{a^3(-15\sin(2(c+dx)) - 105\sin(4(c+dx)) + 5\sin(6(c+dx)) - 600\cos(c+dx) - 140\cos(3(c+dx)) + 36\cos(5(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(450*c + 420*d*x - 600*Cos[c + d*x] - 140*Cos[3*(c + d*x)] + 36*Cos[5*
(c + d*x)] - 15*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)
]))/(960*d)
```

Maple [A] time = 0.036, size = 156, normalized size = 1.3

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^3}{6} - \frac{(\cos(dx+c))^3\sin(dx+c)}{8} + \frac{\cos(dx+c)\sin(dx+c)}{16} + \frac{dx}{16} + \frac{c}{16}\right) + 3a^3\left(-1/5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+3*a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3
-2/15*cos(d*x+c)^3)+3*a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(
d*x+c)+1/8*d*x+1/8*c)-1/3*a^3*cos(d*x+c)^3)
```

Maxima [A] time = 1.09966, size = 143, normalized size = 1.22

$$-\frac{320a^3\cos(dx+c)^3 - 192\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a^3 + 5\left(4\sin(2dx+2c)^3 - 12dx - 12c + 3\sin(4dx+4c\right.}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(320*a^3*cos(d*x + c)^3 - 192*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*
a^3 + 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^3 - 9
0*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A] time = 1.70915, size = 215, normalized size = 1.84

$$\frac{144\,a^3\cos\left(dx+c\right)^5-320\,a^3\cos\left(dx+c\right)^3+105\,a^3dx+5\left(8\,a^3\cos\left(dx+c\right)^5-50\,a^3\cos\left(dx+c\right)^3+21\,a^3\cos\left(dx+c\right)\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(144*a^3*cos(d*x + c)^5 - 320*a^3*cos(d*x + c)^3 + 105*a^3*d*x + 5*(8
*a^3*cos(d*x + c)^5 - 50*a^3*cos(d*x + c)^3 + 21*a^3*cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [A] time = 5.61595, size = 328, normalized size = 2.8

$$\begin{cases} \frac{a^3x\sin^6(c+dx)}{16}+\frac{3a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3a^3x\sin^4(c+dx)}{8}+\frac{3a^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{a^3x\cos^6(c+dx)}{16} \\ x\left(a\sin\left(c\right)+a\right)^3\sin\left(c\right)\cos^2\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**6/16 + 3*a**3*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**3*x*cos(c + d*x)*
*6/16 + 3*a**3*x*cos(c + d*x)**4/8 + a**3*sin(c + d*x)**5*cos(c + d*x)/(16*
d) - a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 3*a**3*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**3/d - a**3*sin(c + d*
x)*cos(c + d*x)**5/(16*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a
**3*cos(c + d*x)**5/(5*d) - a**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*si
n(c) + a)**3*sin(c)*cos(c)**2, True))
```

Giac [A] time = 1.37411, size = 143, normalized size = 1.22

$$\frac{7}{16}\,a^3x+\frac{3\,a^3\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{7\,a^3\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{5\,a^3\cos\left(dx+c\right)}{8\,d}+\frac{a^3\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{7\,a^3\sin\left(4\,dx+4\,c\right)}{64\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 7/16*a^3*x + 3/80*a^3*cos(5*d*x + 5*c)/d - 7/48*a^3*cos(3*d*x + 3*c)/d - 5/
8*a^3*cos(d*x + c)/d + 1/192*a^3*sin(6*d*x + 6*c)/d - 7/64*a^3*sin(4*d*x +
4*c)/d - 1/64*a^3*sin(2*d*x + 2*c)/d
```

3.287 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=99

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}-\frac{a^3\tanh^{-1}(\cos(c+dx)}{d}$$

```
[Out] (13*a^3*x)/8 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d - (a^3*
Cos[c + d*x]^3)/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^3*Cos[c +
 d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.169719, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2873, 2635, 8, 2592, 321, 206, 2565, 30, 2568}

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}-\frac{a^3\tanh^{-1}(\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*a^3*x)/8 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d - (a^3*
Cos[c + d*x]^3)/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^3*Cos[c +
 d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(3a^3\cos^2(c+dx)+a^3\cos(c+dx)\cot(c+dx)+3a^3\cos^2(c+d\right.$$

$$= a^3\int \cos(c+dx)\cot(c+dx)\,dx + a^3\int \cos^2(c+dx)\sin^2(c+dx)\,d$$

$$= \frac{3a^3\cos(c+dx)\sin(c+dx)}{2d} - \frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}a^3\int$$

$$= \frac{3a^3x}{2} + \frac{a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c}{8d}$$

$$= \frac{13a^3x}{8} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+d}{d}$$

Mathematica [A] time = 0.619033, size = 82, normalized size = 0.83

$$\frac{a^3\left(24\sin(2(c+dx))-\sin(4(c+dx))+8\cos(c+dx)-8\cos(3(c+dx))+32\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-32\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(52*c + 52*d*x + 8*Cos[c + d*x] - 8*Cos[3*(c + d*x)] - 32*Log[Cos[(c +
 d*x)/2]] + 32*Log[Sin[(c + d*x)/2]] + 24*Sin[2*(c + d*x)] - Sin[4*(c + d*x
)]))/(32*d)
```

Maple [A] time = 0.072, size = 111, normalized size = 1.1

$$-\frac{a^3 (\cos(dx+c))^3 \sin(dx+c)}{4d} + \frac{13 a^3 \cos(dx+c)\sin(dx+c)}{8d} + \frac{13 a^3 x}{8} + \frac{13 a^3 c}{8d} - \frac{a^3 (\cos(dx+c))^3}{d} + \frac{a^3 \cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d+13/8*a^3*cos(d*x+c)*sin(d*x+c)/d+13/8*a^
3*x+13/8/d*a^3*c-a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-
cot(d*x+c))
```

Maxima [A] time = 1.10688, size = 134, normalized size = 1.35

$$-\frac{32 a^3 \cos(dx+c)^3 - (4 dx + 4 c - \sin(4 dx + 4 c))a^3 - 24 (2 dx + 2 c + \sin(2 dx + 2 c))a^3 - 16 a^3(2 \cos(dx+c) - \log(c}{32 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*a^3*cos(d*x + c)^3 - (4*d*x + 4*c - sin(4*d*x + 4*c))*a^3 - 24*(2
*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 16*a^3*(2*cos(d*x + c) - log(cos(d*x +
 c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.76301, size = 267, normalized size = 2.7

$$-\frac{8 a^3 \cos(dx+c)^3 - 13 a^3 dx - 8 a^3 \cos(dx+c) + 4 a^3 \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) - 4 a^3 \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + \left(2 a^3 c\right.}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^3*cos(d*x + c)^3 - 13*a^3*d*x - 8*a^3*cos(d*x + c) + 4*a^3*log(1/
2*cos(d*x + c) + 1/2) - 4*a^3*log(-1/2*cos(d*x + c) + 1/2) + (2*a^3*cos(d*x
 + c)^3 - 13*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3275, size = 194, normalized size = 1.96

$$\frac{13\,(dx+c)a^3+8\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\bigg(11\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+16\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+19\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-19\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(13*(d*x + c)*a^3 + 8*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(11*a^3*ta
n(1/2*d*x + 1/2*c)^7 + 16*a^3*tan(1/2*d*x + 1/2*c)^6 + 19*a^3*tan(1/2*d*x +
 1/2*c)^5 - 19*a^3*tan(1/2*d*x + 1/2*c)^3 - 16*a^3*tan(1/2*d*x + 1/2*c)^2 -
 11*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.288 $\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=92

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

```
[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3
*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c +
 d*x])/(2*d)
```

Rubi [A] time = 0.137939, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3
*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c +
 d*x])/(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(2a^5+3a^5\csc(c+dx)+a^5\csc^2(c+dx)-2a^5\sin(c+dx)-3a^5\sin^2(c+d\right.}{a^2}$$

$$= 2a^3x + a^3\int \csc^2(c+dx)\,dx - a^3\int \sin^3(c+dx)\,dx - \left(2a^3\right)\int \sin(c+dx)\,dx$$

$$= 2a^3x - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^3\cos(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+}{2d}$$

$$= \frac{a^3x}{2} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+dx)}{3d} - \frac{a^3\cot}{}$$

Mathematica [A] time = 1.05953, size = 106, normalized size = 1.15

$$-\frac{a^3\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((15-66\sin(c+dx))\cos(c+dx)+(2\sin(c+dx)+9)\cos(3(c+dx))-12\sin(c+\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x]*(15 - 66*Sin[c + d*x]
) - 12*(c + d*x - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]])*Sin[c
+ d*x] + Cos[3*(c + d*x)]*(9 + 2*Sin[c + d*x])))/(48*d)
```

Maple [A] time = 0.073, size = 105, normalized size = 1.1

$$-\frac{a^3(\cos(dx+c))^3}{3d} + \frac{3a^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{a^3x}{2} + \frac{a^3c}{2d} + 3\frac{a^3\cos(dx+c)}{d} + 3\frac{a^3\ln(\csc(dx+c)-\cot(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cos(d*x+c)^3/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3
*c+3*a^3*cos(d*x+c)/d+3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-a^3*cot(d*x+c)/d
```

Maxima [A] time = 1.73871, size = 126, normalized size = 1.37

$$-\frac{4a^3\cos(dx+c)^3 - 9(2dx+2c+\sin(2dx+2c))a^3 + 12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3 - 18a^3(2\cos(dx+c)-\log(\cos(d}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(4*a^3*cos(d*x + c)^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(
d*x + c + 1/tan(d*x + c))*a^3 - 18*a^3*(2*cos(d*x + c) - log(cos(d*x + c) +
 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.79395, size = 321, normalized size = 3.49

$$-\frac{9\,a^3\cos\left(dx+c\right)^3+9\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,a^3\cos\left(dx\right.}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(9*a^3*cos(d*x + c)^3 + 9*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 - 9*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^3*cos(d*x + c) + (
2*a^3*cos(d*x + c)^3 - 3*a^3*d*x - 18*a^3*cos(d*x + c))*sin(d*x + c))/(d*si
n(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40009, size = 219, normalized size = 2.38

$$\frac{3\left(dx+c\right)a^3+18\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^3\tan\left(\frac{1}{2}\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*a^3 + 18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/
2*d*x + 1/2*c) - 3*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)
- 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^3*
tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 16*a^3)/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

3.289 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=98

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{5a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+d$$

```
[Out] (-5*a^3*x)/2 - (5*a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (3*a^3*Cos[c + d*x])/d
 - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (a^3*Co
s[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.149492, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635}

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{5a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-5*a^3*x)/2 - (5*a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (3*a^3*Cos[c + d*x])/d
 - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (a^3*Co
s[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-2a^5+2a^5\csc(c+dx)+3a^5\csc^2(c+dx)+a^5\csc^3(c+dx)-3a^5}{a^2}$$

$$= -2a^3x + a^3\int \csc^3(c+dx)\,dx - a^3\int \sin^2(c+dx)\,dx + \left(2a^3\right)\int \csc($$

$$= -2a^3x - \frac{2a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cot(c+dx}{2}$$

$$= -\frac{5a^3x}{2} - \frac{5a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{3a^3\cos(c+dx)}{d} - \frac{3a^3\cot(c+}{d}$$

Mathematica [A] time = 1.03916, size = 112, normalized size = 1.14

$$\frac{a^3\left(2\sin(2(c+dx))+24\cos(c+dx)+12\tan\left(\frac{1}{2}(c+dx)\right)-12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+2}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-20*c - 20*d*x + 24*Cos[c + d*x] - 12*Cot[(c + d*x)/2] - Csc[(c + d*x
)/2]^2 - 20*Log[Cos[(c + d*x)/2]] + 20*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x
)/2]^2 + 2*Sin[2*(c + d*x)] + 12*Tan[(c + d*x)/2]))/(8*d)
```

Maple [A] time = 0.077, size = 113, normalized size = 1.2

$$\frac{a^3\cos(dx+c)\sin(dx+c)}{2d} - \frac{5a^3x}{2} - \frac{5a^3c}{2d} + \frac{5a^3\cos(dx+c)}{2d} + \frac{5a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d} - 3\frac{a^3\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d-5/2*a^3*x-5/2/d*a^3*c+5/2*a^3*cos(d*x+c)/d+
5/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3*a^3*cot(d*x+c)/d-1/2/d*a^3/sin(d*x+c)
^2*cos(d*x+c)^3
```

Maxima [A] time = 1.6246, size = 167, normalized size = 1.7

$$\frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^3-12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3+a^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right.\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 12*(d*x + c + 1/tan(d*x + c))*a
^3 + a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log
(cos(d*x + c) - 1)) + 6*a^3*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)))/d
```

Fricas [A] time = 1.88082, size = 397, normalized size = 4.05

$$-\frac{10\,a^3dx\cos\left(dx+c\right)^2-12\,a^3\cos\left(dx+c\right)^3-10\,a^3dx+10\,a^3\cos\left(dx+c\right)+5\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\cos\left(dx+\right.\right.}{4\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(10*a^3*d*x*cos(d*x + c)^2 - 12*a^3*cos(d*x + c)^3 - 10*a^3*d*x + 10*a
^3*cos(d*x + c) + 5*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2)
- 5*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*cos(d*
x + c)^3 + 5*a^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40233, size = 248, normalized size = 2.53

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-20\,(dx+c)a^3+20\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+20\,a^3\,\mathrm{t}}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 20*(d*x + c)*a^3 + 20*a^3*log(abs(tan(1/2
*d*x + 1/2*c))) + 12*a^3*tan(1/2*d*x + 1/2*c) - (10*a^3*tan(1/2*d*x + 1/2*c
)^6 + 20*a^3*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*a^
3*tan(1/2*d*x + 1/2*c)^3 - 36*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d
*x + 1/2*c) + a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2)/d
```

3.290 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}-3a^3x$$

```
[Out] -3*a^3*x + (a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*Cos[c + d*x])/d - (3*a^
3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c
+ d*x])/(2*d)
```

Rubi [A] time = 0.170831, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 3768, 2638}

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}-3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] -3*a^3*x + (a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*Cos[c + d*x])/d - (3*a^
3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c
+ d*x])/(2*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-3a^5-2a^5\csc(c+dx)+2a^5\csc^2(c+dx)+3a^5\csc^3(c+dx)+ \right.}{a^2}$$

$$= -3a^3x + a^3\int \csc^4(c+dx)\,dx - a^3\int \sin(c+dx)\,dx - \left(2a^3\right)\int \csc($$

$$= -3a^3x + \frac{2a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} - \frac{3a^3\cot(c+d}{2}$$

$$= -3a^3x + \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3\cos(c+dx)}{d} - \frac{3a^3\cot(c+dx}{d}$$

Mathematica [A] time = 0.396047, size = 148, normalized size = 1.63

$$\frac{a^3\left(24\cos(c+dx)+32\tan\left(\frac{1}{2}(c+dx)\right)-32\cot\left(\frac{1}{2}(c+dx)\right)-9\csc^2\left(\frac{1}{2}(c+dx)\right)+9\sec^2\left(\frac{1}{2}(c+dx)\right)-12\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-72*c - 72*d*x + 24*Cos[c + d*x] - 32*Cot[(c + d*x)/2] - 9*Csc[(c + d
*x)/2]^2 + 12*Log[Cos[(c + d*x)/2]] - 12*Log[Sin[(c + d*x)/2]] + 9*Sec[(c +
 d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[(c + d*x)/2]^4*Sin[
c + d*x])/2 + 32*Tan[(c + d*x)/2]))/(24*d)
```

Maple [A] time = 0.079, size = 117, normalized size = 1.3

$$-\frac{a^3\cos(dx+c)}{2d} - \frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d} - 3a^3x - 3\frac{a^3\cot(dx+c)}{d} - 3\frac{a^3c}{d} - \frac{3a^3(\cos(dx+c))^3}{2d(\sin(dx+c))^2} - \frac{a^3(\cos(}{3d(\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2*a^3*cos(d*x+c)/d-1/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3*a^3*x-3*a^3*cot
(d*x+c)/d-3/d*a^3*c-3/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-1/3/d*a^3/sin(d*x+c
)^3*cos(d*x+c)^3
```

Maxima [A] time = 1.75237, size = 158, normalized size = 1.74

$$-\frac{36\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3 - 9a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right) - 6a^3(2\cos(dx+c)-\log}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(36*(d*x + c + 1/tan(d*x + c))*a^3 - 9*a^3*(2*cos(d*x + c)/(cos(d*x +
 c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 6*a^3*(2*cos(
d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 4*a^3/tan(d*x +
 c)^3)/d
```

Fricas [B] time = 1.78067, size = 451, normalized size = 4.96

$$-\frac{32\,a^3\cos\left(dx+c\right)^3-36\,a^3\cos\left(dx+c\right)-3\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(a^3\cos\left(d\right.\right.}{12\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(32*a^3*cos(d*x + c)^3 - 36*a^3*cos(d*x + c) - 3*(a^3*cos(d*x + c)^2
- a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^3*cos(d*x + c)^2 - a
^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(6*a^3*d*x*cos(d*x + c)^2
 - 2*a^3*cos(d*x + c)^3 - 6*a^3*d*x - a^3*cos(d*x + c))*sin(d*x + c))/((d*c
os(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32951, size = 217, normalized size = 2.38

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\left(dx+c\right)a^3-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+33\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x +
 c)*a^3 - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 33*a^3*tan(1/2*d*x + 1/2*
c) + 48*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) + (22*a^3*tan(1/2*d*x + 1/2*c)^3 -
 33*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*
d*x + 1/2*c)^3)/d
```

3.291 $\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=100

$$-\frac{a^3\cot^3(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{11a^3\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] -(a^3*x) + (13*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (a^3*Cot[c + d*x])/d - (a
^3*Cot[c + d*x]^3)/d - (11*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*Cot[
c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.22124, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{a^3\cot^3(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{11a^3\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (13*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (a^3*Cot[c + d*x])/d - (a
^3*Cot[c + d*x]^3)/d - (11*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*Cot[
c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx)+3a^3\cot^2(c+dx)\csc(c+dx)+3a^3\cot^2(c+\right.$$

$$= a^3\int \cot^2(c+dx)\,dx + a^3\int \cot^2(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right.$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{3a^3\cot(c+dx)\csc(c+dx)}{2d} - \frac{a^3\cot(c+dx)\,c}{4d}$$

$$= -a^3x + \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+}{d}$$

$$= -a^3x + \frac{13a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+}{d}$$

Mathematica [A] time = 0.530498, size = 133, normalized size = 1.33

$$\frac{a^3\left(-22\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+22\sec^2\left(\frac{1}{2}(c+dx)\right)-(4\sin(c+dx)+1)\csc^4\left(\frac{1}{2}(c+dx)\right)-8\left(13\log\left(\mathrm{s}\right.\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-22*Csc[(c + d*x)/2]^2 + 22*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 -
 8*(8*c + 8*d*x - 13*Log[Cos[(c + d*x)/2]] + 13*Log[Sin[(c + d*x)/2]] - 8*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4) - Csc[(c + d*x)/2]^4*(1 + 4*Sin[c + d*x])
))/(64*d)
```

Maple [A] time = 0.083, size = 141, normalized size = 1.4

$$-a^3x - \frac{a^3\cot(dx+c)}{d} - \frac{a^3c}{d} - \frac{13\,a^3(\cos(dx+c))^3}{8\,d(\sin(dx+c))^2} - \frac{13\,a^3\cos(dx+c)}{8\,d} - \frac{13\,a^3\ln(\csc(dx+c)-\cot(dx+c))}{8\,d} - \frac{a^3(}{d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -a^3*x-a^3*cot(d*x+c)/d-1/d*a^3*c-13/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-13/8
*a^3*cos(d*x+c)/d-13/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/d*a^3/sin(d*x+c)^3
*cos(d*x+c)^3-1/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^3
```

Maxima [A] time = 1.73384, size = 198, normalized size = 1.98

$$-\frac{16\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3+a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-12a^3\left(\frac{2\cos(dx+\cdots}{\cos(dx+c)^2\cdots}\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*(d*x + c + 1/tan(d*x + c))*a^3 + a^3*(2*(cos(d*x + c)^3 + cos(d*x
 + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + lo
g(cos(d*x + c) - 1)) - 12*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(co
s(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 16*a^3/tan(d*x + c)^3)/d
```

Fricas [B] time = 1.69957, size = 497, normalized size = 4.97

$$-\frac{16a^3dx\cos\left(dx+c\right)^4-32a^3dx\cos\left(dx+c\right)^2-22a^3\cos\left(dx+c\right)^3+16a^3dx+16a^3\cos\left(dx+c\right)\sin\left(dx+c\right)+26a^3\cos\cdots}{16\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(16*a^3*d*x*cos(d*x + c)^4 - 32*a^3*d*x*cos(d*x + c)^2 - 22*a^3*cos(d
*x + c)^3 + 16*a^3*d*x + 16*a^3*cos(d*x + c)*sin(d*x + c) + 26*a^3*cos(d*x
+ c) - 13*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x
 + c) + 1/2) + 13*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/
2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39205, size = 235, normalized size = 2.35

$$\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+72a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-192\left(dx+c\right)a^3-312a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.\right.\cdots}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*tan(1/2*d*x + 1/2*c)^4 + 24*a^3*tan(1/2*d*x + 1/2*c)^3 + 72*a^
3*tan(1/2*d*x + 1/2*c)^2 - 192*(d*x + c)*a^3 - 312*a^3*log(abs(tan(1/2*d*x
+ 1/2*c))) + 24*a^3*tan(1/2*d*x + 1/2*c) + (650*a^3*tan(1/2*d*x + 1/2*c)^4
- 24*a^3*tan(1/2*d*x + 1/2*c)^3 - 72*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*a^3*ta
n(1/2*d*x + 1/2*c) - 3*a^3)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.292 $\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=100

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] (7*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (4*a^3*Cot[c + d*x]^3)/(3*d) - (a^3*C
ot[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (3*a^3*Cot[c
 + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.240212, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (4*a^3*Cot[c + d*x]^3)/(3*d) - (a^3*C
ot[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (3*a^3*Cot[c
 + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\csc^2(c+dx)+\right.$$
$$= a^3\int \cot^2(c+dx)\csc(c+dx)\,dx + a^3\int \cot^2(c+dx)\csc^4(c+d$$
$$= -\frac{a^3\cot(c+dx)\csc(c+dx)}{2d} - \frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{2}a$$
$$= \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3\cot^3(c+dx)}{d} - \frac{a^3\cot(c+dx)\csc(c}{8d}$$
$$= \frac{7a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{4a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} -$$

Mathematica [B] time = 0.130221, size = 267, normalized size = 2.67

$$a^3\left(-\frac{17\tan\left(\frac{1}{2}(c+dx)\right)}{30d}+\frac{17\cot\left(\frac{1}{2}(c+dx)\right)}{30d}-\frac{3\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^2\left(\frac{1}{2}\right.}{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((17*Cot[(c + d*x)/2])/(30*d) - Csc[(c + d*x)/2]^2/(32*d) - (59*Cot[(c
+ d*x)/2]*Csc[(c + d*x)/2]^2)/(480*d) - (3*Csc[(c + d*x)/2]^4)/(64*d) - (Co
t[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(160*d) + (7*Log[Cos[(c + d*x)/2]])/(8*d
) - (7*Log[Sin[(c + d*x)/2]])/(8*d) + Sec[(c + d*x)/2]^2/(32*d) + (3*Sec[(c
 + d*x)/2]^4)/(64*d) - (17*Tan[(c + d*x)/2])/(30*d) + (59*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(480*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(160*d)
)
```

Maple [A] time = 0.082, size = 136, normalized size = 1.4

$$-\frac{7a^3(\cos(dx+c))^3}{8d(\sin(dx+c))^2}-\frac{7a^3\cos(dx+c)}{8d}-\frac{7a^3\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{17a^3(\cos(dx+c))^3}{15d(\sin(dx+c))^3}-\frac{3a^3(\cos(dx-}{4d(\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -7/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-7/8*a^3*cos(d*x+c)/d-7/8/d*a^3*ln(csc(
d*x+c)-cot(d*x+c))-17/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^3-3/4/d*a^3/sin(d*x+
c)^4*cos(d*x+c)^3-1/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^3
```

Maxima [A] time = 1.13744, size = 209, normalized size = 2.09

$$-\frac{45\,a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-60\,a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/240*(45*a^3*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 60*a^3*(2
*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x +
c) - 1)) + 240*a^3/tan(d*x + c)^3 + 16*(5*tan(d*x + c)^2 + 3)*a^3/tan(d*x +
 c)^5)/d
```

Fricas [B] time = 1.73164, size = 501, normalized size = 5.01

$$\frac{272\,a^3\cos\left(dx+c\right)^5-320\,a^3\cos\left(dx+c\right)^3+105\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(\right.}{240\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(272*a^3*cos(d*x + c)^5 - 320*a^3*cos(d*x + c)^3 + 105*(a^3*cos(d*x +
 c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) - 105*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + 30*(a^3*cos(d*x + c)^3 - 7*a^3*cos(d*x + c))*sin
(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37693, size = 265, normalized size = 2.65

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+130\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-840\,a^3\log\left(\left|\mathrm{ta}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*tan(1/2*d*x + 1/2*c)^4 + 130*a
^3*tan(1/2*d*x + 1/2*c)^3 + 120*a^3*tan(1/2*d*x + 1/2*c)^2 - 840*a^3*log(ab
s(tan(1/2*d*x + 1/2*c))) - 420*a^3*tan(1/2*d*x + 1/2*c) + (1918*a^3*tan(1/2
*d*x + 1/2*c)^5 + 420*a^3*tan(1/2*d*x + 1/2*c)^4 - 120*a^3*tan(1/2*d*x + 1/
2*c)^3 - 130*a^3*tan(1/2*d*x + 1/2*c)^2 - 45*a^3*tan(1/2*d*x + 1/2*c) - 6*a
^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.293 $\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=124

$$-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{17a^3\cot(c+dx)\csc^3(c}{24d}$$

```
[Out] (7*a^3*ArcTanh[Cos[c + d*x]])/(16*d) - (4*a^3*Cot[c + d*x]^3)/(3*d) - (3*a^
3*Cot[c + d*x]^5)/(5*d) + (7*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (17*a^
3*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(
6*d)
```

Rubi [A] time = 0.281918, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{17a^3\cot(c+dx)\csc^3(c}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*ArcTanh[Cos[c + d*x]])/(16*d) - (4*a^3*Cot[c + d*x]^3)/(3*d) - (3*a^
3*Cot[c + d*x]^5)/(5*d) + (7*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (17*a^
3*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(
6*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx)\csc^2(c+dx)+3a^3\cot^2(c+dx)\csc^3(c+dx)\right.$$

$$= a^3\int \cot^2(c+dx)\csc^2(c+dx)\,dx + a^3\int \cot^2(c+dx)\csc^5(c+$$

$$= -\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{a^3\cot(c+dx)\csc^5(c+dx)}{6d} - \frac{1}{6}$$

$$= -\frac{a^3\cot^3(c+dx)}{3d} + \frac{3a^3\cot(c+dx)\csc(c+dx)}{8d} - \frac{17a^3\cot(c+}{2}$$

$$= \frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{4a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^5(c+dx)}{5d}$$

$$= \frac{7a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^5(c+dx)}{5d}$$

Mathematica [B] time = 3.53536, size = 252, normalized size = 2.03

$$-\frac{a^3\sin(c+dx)(\sin(c+dx)+1)^3\left(\csc^6\left(\frac{1}{2}(c+dx)\right)(5\csc(c+dx)+18)+\csc^4\left(\frac{1}{2}(c+dx)\right)(90\csc(c+dx)+34)-2\,\mathrm{cs}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(Csc[(c + d*x)/2]^6*(18 + 5*Csc[c + d*x]) + Csc[(c + d*x)/2]^4*(34 +
90*Csc[c + d*x]) - 2*Csc[(c + d*x)/2]^2*(176 + 105*Csc[c + d*x]) - 840*Csc[
c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + (97 + 159*Cos[c
+ d*x] + 44*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^6 + 840*Csc[c + d*x]^3*Sin[(
c + d*x)/2]^2 - 1440*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4 - 320*Csc[c + d*x]^7
*Sin[(c + d*x)/2]^6)*Sin[c + d*x]*(1 + Sin[c + d*x])^3)/(1920*d*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.086, size = 160, normalized size = 1.3

$$-\frac{11\,a^3(\cos(dx+c))^3}{15\,d(\sin(dx+c))^3} - \frac{7\,a^3(\cos(dx+c))^3}{8\,d(\sin(dx+c))^4} - \frac{7\,a^3(\cos(dx+c))^3}{16\,d(\sin(dx+c))^2} - \frac{7\,a^3\cos(dx+c)}{16\,d} - \frac{7\,a^3\ln(\csc(dx+c)-\cot(dx}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -11/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^3-7/8/d*a^3/sin(d*x+c)^4*cos(d*x+c)^3-
7/16/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-7/16*a^3*cos(d*x+c)/d-7/16/d*a^3*ln(cs
c(d*x+c)-cot(d*x+c))-3/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^3-1/6/d*a^3/sin(d*x+
c)^6*cos(d*x+c)^3
```

Maxima [A] time = 1.09389, size = 270, normalized size = 2.18

$$-\frac{5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+90\,a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx\right.}{\cos(dx+c)^4-2\,\cos(dx+}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^3*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 90*a^3*(2*(cos(d*x + c)^3 + cos(d*x + c
))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(co
s(d*x + c) - 1)) + 160*a^3/tan(d*x + c)^3 + 96*(5*tan(d*x + c)^2 + 3)*a^3/t
an(d*x + c)^5)/d
```

Fricas [B] time = 1.81423, size = 575, normalized size = 4.64

$$-\frac{210\,a^3\cos\left(dx+c\right)^5-80\,a^3\cos\left(dx+c\right)^3-210\,a^3\cos\left(dx+c\right)-105\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/480*(210*a^3*cos(d*x + c)^5 - 80*a^3*cos(d*x + c)^3 - 210*a^3*cos(d*x +
c) - 105*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2
- a^3)*log(1/2*cos(d*x + c) + 1/2) + 105*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*
x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) + 32*(1
1*a^3*cos(d*x + c)^5 - 20*a^3*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)
^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42961, size = 308, normalized size = 2.48

$$5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+140\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^3\tan\left(\frac{1}{2}\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)^5 + 105*
a^3*tan(1/2*d*x + 1/2*c)^4 + 140*a^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/
2*d*x + 1/2*c)^2 - 840*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 600*a^3*tan(1/2
*d*x + 1/2*c) + (2058*a^3*tan(1/2*d*x + 1/2*c)^6 + 600*a^3*tan(1/2*d*x + 1/
2*c)^5 + 15*a^3*tan(1/2*d*x + 1/2*c)^4 - 140*a^3*tan(1/2*d*x + 1/2*c)^3 - 1
05*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a^3*tan(1/2*d*x + 1/2*c) - 5*a^3)/tan(1/
2*d*x + 1/2*c)^6)/d
```

3.294 $\int \cos^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=137

$$-\frac{7a^4\cos^3(c+dx)}{8d}-\frac{3\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)^2}{10d}-\frac{21\cos^3(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{40d}+\frac{21a^4\sin(c+dx)\cos(}{16d}$$

```
[Out] (21*a^4*x)/16 - (7*a^4*Cos[c + d*x]^3)/(8*d) + (21*a^4*Cos[c + d*x]*Sin[c +
 d*x])/(16*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(6*d) - (3*Cos[c
+ d*x]^3*(a^2 + a^2*Sin[c + d*x])^2)/(10*d) - (21*Cos[c + d*x]^3*(a^4 + a^4
*Sin[c + d*x]))/(40*d)
```

Rubi [A] time = 0.157458, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2678, 2669, 2635, 8}

$$-\frac{7a^4\cos^3(c+dx)}{8d}-\frac{3\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)^2}{10d}-\frac{21\cos^3(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{40d}+\frac{21a^4\sin(c+dx)\cos(}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (21*a^4*x)/16 - (7*a^4*Cos[c + d*x]^3)/(8*d) + (21*a^4*Cos[c + d*x]*Sin[c +
 d*x])/(16*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(6*d) - (3*Cos[c
+ d*x]^3*(a^2 + a^2*Sin[c + d*x])^2)/(10*d) - (21*Cos[c + d*x]^3*(a^4 + a^4
*Sin[c + d*x]))/(40*d)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} + \frac{1}{2}(3a)\int \cos^2(c+dx)(a+a\sin(c+dx)$$

$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} - \frac{3\cos^3(c+dx)\left(a^2+a^2\sin(c+dx)\right)^2}{10d} +$$

$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} - \frac{3\cos^3(c+dx)\left(a^2+a^2\sin(c+dx)\right)^2}{10d} -$$

$$= -\frac{7a^4\cos^3(c+dx)}{8d} - \frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} - \frac{3\cos^3(c+dx)\left(a^2\right.}{10d}$$

$$= -\frac{7a^4\cos^3(c+dx)}{8d} + \frac{21a^4\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^3(c+dx)(a+a\sin(}{6d}$$

$$= \frac{21a^4x}{16} - \frac{7a^4\cos^3(c+dx)}{8d} + \frac{21a^4\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^3(c+dx)(a}{6d}$$

Mathematica [A] time = 0.432605, size = 151, normalized size = 1.1

$$-\frac{a^4\left(630\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\left(40\sin^6(c+dx)+152\sin^5(c+dx)+158\sin^4(c+d\right.\right.}{240d(\sin(c+dx)-1)^2(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(a^4*Cos[c + d*x]^3*(630*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - S
in[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(448 - 373*Sin[c + d*x] - 331*Sin[c +
 d*x]^2 - 94*Sin[c + d*x]^3 + 158*Sin[c + d*x]^4 + 152*Sin[c + d*x]^5 + 40*
Sin[c + d*x]^6)))/(240*d*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.055, size = 182, normalized size = 1.3

$$\frac{1}{d}\left(a^4\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^3}{6} - \frac{(\cos(dx+c))^3\sin(dx+c)}{8} + \frac{\cos(dx+c)\sin(dx+c)}{16} + \frac{dx}{16} + \frac{c}{16}\right) + 4a^4\left(-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+4*a^4*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3
-2/15*cos(d*x+c)^3)+6*a^4*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(
d*x+c)+1/8*d*x+1/8*c)-4/3*a^4*cos(d*x+c)^3+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c))
```

Maxima [A] time = 1.12893, size = 173, normalized size = 1.26

$$-\frac{1280\,a^4\cos(dx+c)^3 - 256\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a^4 + 5\left(4\sin(2\,dx+2\,c)^3 - 12\,dx - 12\,c + 3\sin(4\,dx\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/960*(1280*a^4*cos(d*x + c)^3 - 256*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)
*a^4 + 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^4 -
180*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 240*(2*d*x + 2*c + sin(2*d*x + 2
*c))*a^4)/d
```

Fricas [A] time = 1.64891, size = 215, normalized size = 1.57

$$\frac{192\,a^4\cos\left(dx+c\right)^5 - 640\,a^4\cos\left(dx+c\right)^3 + 315\,a^4dx + 5\left(8\,a^4\cos\left(dx+c\right)^5 - 86\,a^4\cos\left(dx+c\right)^3 + 63\,a^4\cos\left(dx+c\right)\right)\sin}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(192*a^4*cos(d*x + c)^5 - 640*a^4*cos(d*x + c)^3 + 315*a^4*d*x + 5*(8
*a^4*cos(d*x + c)^5 - 86*a^4*cos(d*x + c)^3 + 63*a^4*cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [A] time = 6.28719, size = 381, normalized size = 2.78

$$\begin{cases} \frac{a^4x\sin^6(c+dx)}{16} + \frac{3a^4x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^4x\sin^4(c+dx)}{4} + \frac{3a^4x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{a^4x\sin^2(c+dx)}{2} + \\ x\left(a\sin\left(c\right)+a\right)^4\cos^2\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x*sin(c + d*x)**6/16 + 3*a**4*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**4*x*sin(c + d*x)**4/4 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + a**4*x*sin(c + d*x)*
*2/2 + a**4*x*cos(c + d*x)**6/16 + 3*a**4*x*cos(c + d*x)**4/4 + a**4*x*cos(
c + d*x)**2/2 + a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - a**4*sin(c + d*x
)**3*cos(c + d*x)**3/(6*d) + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) - 4*
a**4*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a**4*sin(c + d*x)*cos(c + d*x)
**5/(16*d) - 3*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a**4*sin(c + d*x)*
cos(c + d*x)/(2*d) - 8*a**4*cos(c + d*x)**5/(15*d) - 4*a**4*cos(c + d*x)**3
/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*cos(c)**2, True))
```

Giac [A] time = 1.39885, size = 143, normalized size = 1.04

$$\frac{21}{16}\,a^4x + \frac{a^4\cos\left(5\,dx+5\,c\right)}{20\,d} - \frac{5\,a^4\cos\left(3\,dx+3\,c\right)}{12\,d} - \frac{3\,a^4\cos\left(dx+c\right)}{2\,d} + \frac{a^4\sin\left(6\,dx+6\,c\right)}{192\,d} - \frac{13\,a^4\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{15\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 21/16*a^4*x + 1/20*a^4*cos(5*d*x + 5*c)/d - 5/12*a^4*cos(3*d*x + 3*c)/d - 3
/2*a^4*cos(d*x + c)/d + 1/192*a^4*sin(6*d*x + 6*c)/d - 13/64*a^4*sin(4*d*x
+ 4*c)/d + 15/64*a^4*sin(2*d*x + 2*c)/d
```

3.295 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=117

$$\frac{a^4\cos^5(c+dx)}{5d}-\frac{7a^4\cos^3(c+dx)}{3d}+\frac{a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{5a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^4}{}$$

```
[Out] (5*a^4*x)/2 - (a^4*ArcTanh[Cos[c + d*x]])/d + (a^4*Cos[c + d*x])/d - (7*a^4
*Cos[c + d*x]^3)/(3*d) + (a^4*Cos[c + d*x]^5)/(5*d) + (5*a^4*Cos[c + d*x]*S
in[c + d*x])/(2*d) - (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d
```

Rubi [A] time = 0.199499, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2873, 2635, 8, 2592, 321, 206, 2565, 30, 2568, 14}

$$\frac{a^4\cos^5(c+dx)}{5d}-\frac{7a^4\cos^3(c+dx)}{3d}+\frac{a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{5a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (5*a^4*x)/2 - (a^4*ArcTanh[Cos[c + d*x]])/d + (a^4*Cos[c + d*x])/d - (7*a^4
*Cos[c + d*x]^3)/(3*d) + (a^4*Cos[c + d*x]^5)/(5*d) + (5*a^4*Cos[c + d*x]*S
in[c + d*x])/(2*d) - (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^4\,dx = \int \left(4a^4\cos^2(c+dx)+a^4\cos(c+dx)\cot(c+dx)+6a^4\cos^2(c+dx)\,\mathrm{s}\right.$$
$$= a^4\int \cos(c+dx)\cot(c+dx)\,dx + a^4\int \cos^2(c+dx)\sin^3(c+dx)\,dx$$
$$= \frac{2a^4\cos(c+dx)\sin(c+dx)}{d} - \frac{a^4\cos^3(c+dx)\sin(c+dx)}{d} + a^4\int \cos$$
$$= 2a^4x + \frac{a^4\cos(c+dx)}{d} - \frac{2a^4\cos^3(c+dx)}{d} + \frac{5a^4\cos(c+dx)\sin(c+d}{2d}$$
$$= \frac{5a^4x}{2} - \frac{a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^4\cos(c+dx)}{d} - \frac{7a^4\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 0.851788, size = 95, normalized size = 0.81

$$\frac{a^4\left(-150\cos(c+dx)-125\cos(3(c+dx))+3\cos(5(c+dx))+30\left(8\sin(2(c+dx))-\sin(4(c+dx))+8\log\left(\sin\left(\frac{1}{2}(c+d\right.\right.\right.\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(-150*Cos[c + d*x] - 125*Cos[3*(c + d*x)] + 3*Cos[5*(c + d*x)] + 30*(2
0*c + 20*d*x - 8*Log[Cos[(c + d*x)/2]] + 8*Log[Sin[(c + d*x)/2]] + 8*Sin[2*
(c + d*x)] - Sin[4*(c + d*x)])))/(240*d)
```

Maple [A] time = 0.078, size = 135, normalized size = 1.2

$$-\frac{a^4(\cos(dx+c))^3(\sin(dx+c))^2}{5d}-\frac{32\,a^4(\cos(dx+c))^3}{15\,d}-\frac{a^4(\cos(dx+c))^3\sin(dx+c)}{d}+\frac{5\,a^4\cos(dx+c)\sin(dx+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/5/d*a^4*cos(d*x+c)^3*sin(d*x+c)^2-32/15*a^4*cos(d*x+c)^3/d-a^4*cos(d*x+c
)^3*sin(d*x+c)/d+5/2*a^4*cos(d*x+c)*sin(d*x+c)/d+5/2*a^4*x+5/2/d*a^4*c+a^4*
cos(d*x+c)/d+1/d*a^4*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.14235, size = 169, normalized size = 1.44

$$-\frac{240\,a^4\cos(dx+c)^3-8\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)a^4-15\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^4-120\,(2\,dx+2\,c+}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/120*(240*a^4*cos(d*x + c)^3 - 8*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^
4 - 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 120*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^4 - 60*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x
+ c) - 1)))/d
```

Fricas [A] time = 1.78666, size = 306, normalized size = 2.62

$$\frac{6\,a^4\cos(dx+c)^5-70\,a^4\cos(dx+c)^3+75\,a^4dx+30\,a^4\cos(dx+c)-15\,a^4\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15\,a^4\log\left(-\frac{1}{2}\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(6*a^4*cos(d*x + c)^5 - 70*a^4*cos(d*x + c)^3 + 75*a^4*d*x + 30*a^4*co
s(d*x + c) - 15*a^4*log(1/2*cos(d*x + c) + 1/2) + 15*a^4*log(-1/2*cos(d*x +
 c) + 1/2) - 15*(2*a^4*cos(d*x + c)^3 - 5*a^4*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19736, size = 244, normalized size = 2.09

$$\frac{75\,(dx+c)a^4 + 30\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - \frac{2\left(45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+150\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+210\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+300\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(75*(d*x + c)*a^4 + 30*a^4*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(45*a^4*
tan(1/2*d*x + 1/2*c)^9 + 150*a^4*tan(1/2*d*x + 1/2*c)^8 + 210*a^4*tan(1/2*d
*x + 1/2*c)^7 + 300*a^4*tan(1/2*d*x + 1/2*c)^6 + 40*a^4*tan(1/2*d*x + 1/2*c
)^4 - 210*a^4*tan(1/2*d*x + 1/2*c)^3 + 20*a^4*tan(1/2*d*x + 1/2*c)^2 - 45*a
^4*tan(1/2*d*x + 1/2*c) + 34*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.296 $\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=116

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}-$$

```
[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (
4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*S
in[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.168284, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 2635, 2633}

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (
4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*S
in[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(5a^6+4a^6\csc(c+dx)+a^6\csc^2(c+dx)-5a^6\sin^2(c+dx)-4a^6\sin^3(c+dx)\right.}{a^2}$$

$$= 5a^4x + a^4\int \csc^2(c+dx)\,dx - a^4\int \sin^4(c+dx)\,dx + \left(4a^4\right)\int \csc(c+dx)\,dx - ($$

$$= 5a^4x - \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{5a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos(c+dx)}{4d}$$

$$= \frac{5a^4x}{2} - \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+dx)}{3d} - \frac{a^4\cot(}{}$$

$$= \frac{17a^4x}{8} - \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+dx)}{3d} - \frac{a^4\text{co}}{}$$

Mathematica [A] time = 1.52819, size = 136, normalized size = 1.17

$$\frac{a^4\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(408c\sin(c+dx)+408dx\sin(c+dx)+320\sin(2(c+dx))-32\sin(4(c+dx))-48\,\text{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-48*Cos[c + d*x] - 147*Cos[3*(c + d
*x)] + 3*Cos[5*(c + d*x)] + 408*c*Sin[c + d*x] + 408*d*x*Sin[c + d*x] - 768
*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 768*Log[Sin[(c + d*x)/2]]*Sin[c + d*x
] + 320*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(384*d)
```

Maple [A] time = 0.073, size = 127, normalized size = 1.1

$$-\frac{a^4(\cos(dx+c))^3\sin(dx+c)}{4d} + \frac{25a^4\cos(dx+c)\sin(dx+c)}{8d} + \frac{17a^4x}{8} + \frac{17a^4c}{8d} - \frac{4a^4(\cos(dx+c))^3}{3d} + 4\frac{a^4\cos(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/4*a^4*cos(d*x+c)^3*sin(d*x+c)/d+25/8*a^4*cos(d*x+c)*sin(d*x+c)/d+17/8*a^
4*x+17/8/d*a^4*c-4/3*a^4*cos(d*x+c)^3/d+4*a^4*cos(d*x+c)/d+4/d*a^4*ln(csc(d
*x+c)-cot(d*x+c))-a^4*cot(d*x+c)/d
```

Maxima [A] time = 1.62824, size = 158, normalized size = 1.36

$$-\frac{128a^4\cos(dx+c)^3 - 3(4dx+4c-\sin(4dx+4c))a^4 - 144(2dx+2c+\sin(2dx+2c))a^4 + 96\left(dx+c+\frac{1}{\tan(dx+c)}\right)a}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] -1/96*(128*a^4*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 14
4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 96*(d*x + c + 1/tan(d*x + c))*a^4
- 192*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

Fricas [A] time = 1.75748, size = 360, normalized size = 3.1

$$\frac{6\,a^4\cos\left(dx+c\right)^5-81\,a^4\cos\left(dx+c\right)^3-48\,a^4\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+48\,a^4\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\mathrm{s}}{24\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*a^4*cos(d*x + c)^5 - 81*a^4*cos(d*x + c)^3 - 48*a^4*log(1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) + 48*a^4*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 51*a^4*cos(d*x + c) - (32*a^4*cos(d*x + c)^3 - 51*a^4*d*x - 96*a^4*cos(
d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40173, size = 262, normalized size = 2.26

$$\frac{51\,(dx+c)a^4+96\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{12\left(8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(69\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+93\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(51*(d*x + c)*a^4 + 96*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^4*tan
(1/2*d*x + 1/2*c) - 12*(8*a^4*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2
*c) - 2*(69*a^4*tan(1/2*d*x + 1/2*c)^7 + 93*a^4*tan(1/2*d*x + 1/2*c)^5 - 19
2*a^4*tan(1/2*d*x + 1/2*c)^4 - 93*a^4*tan(1/2*d*x + 1/2*c)^3 - 256*a^4*tan(
1/2*d*x + 1/2*c)^2 - 69*a^4*tan(1/2*d*x + 1/2*c) - 64*a^4)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^4)/d
```

3.297 $\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$\frac{\cos^5(c+dx)}{5ad}-\frac{2\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

```
[Out] (3*x)/(8*a) + Cos[c + d*x]/(a*d) - (2*Cos[c + d*x]^3)/(3*a*d) + Cos[c + d*x
]^5/(5*a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - (Cos[c + d*x]*Sin[c +
 d*x]^3)/(4*a*d)
```

Rubi [A] time = 0.135656, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 2635, 8, 2633}

$$\frac{\cos^5(c+dx)}{5ad}-\frac{2\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) + Cos[c + d*x]/(a*d) - (2*Cos[c + d*x]^3)/(3*a*d) + Cos[c + d*x
]^5/(5*a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - (Cos[c + d*x]*Sin[c +
 d*x]^3)/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sin^4(c+dx)\,dx}{a} - \frac{\int \sin^5(c+dx)\,dx}{a}$$
$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{4ad} + \frac{3\int \sin^2(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, c\right.}{ad}$$
$$= \frac{\cos(c+dx)}{ad} - \frac{2\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{3\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\cos(c+}{}$$
$$= \frac{3x}{8a} + \frac{\cos(c+dx)}{ad} - \frac{2\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{3\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\text{co}}{}$$

Mathematica [B] time = 5.10352, size = 281, normalized size = 2.7

$$\frac{1}{480}\left(\frac{60\sin^2\left(\frac{1}{2}(c+dx)\right)}{d(a\sin(c+dx)+a)} - \frac{300\sin(c)\sin(dx)}{ad} + \frac{50\sin(3c)\sin(3dx)}{ad} - \frac{6\sin(5c)\sin(5dx)}{ad} + \frac{30\sin(c+dx)}{ad(\sin(c+dx)+1)} + 30\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((180*x)/a + (300*Cos[c]*Cos[d*x])/(a*d) - (50*Cos[3*c]*Cos[3*d*x])/(a*d) +
 (6*Cos[5*c]*Cos[5*d*x])/(a*d) - (120*Cos[2*d*x]*Sin[2*c])/(a*d) + (15*Cos[
4*d*x]*Sin[4*c])/(a*d) - (300*Sin[c]*Sin[d*x])/(a*d) - (120*Cos[2*c]*Sin[2*
d*x])/(a*d) + (50*Sin[3*c]*Sin[3*d*x])/(a*d) + (15*Cos[4*c]*Sin[4*d*x])/(a*
d) - (6*Sin[5*c]*Sin[5*d*x])/(a*d) - (60*Sin[(d*x)/2])/(a*d*(Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (30*Sin[c + d*x])/(a*d*(1 +
 Sin[c + d*x])) + (60*Sin[(c + d*x)/2]^2)/(d*(a + a*Sin[c + d*x])))/480
```

Maple [B] time = 0.091, size = 245, normalized size = 2.4

$$\frac{3}{4da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5} + \frac{7}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5} + \frac{32}{3da}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+7/2/d/a/(1+tan(1/2*
d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7+32/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*ta
n(1/2*d*x+1/2*c)^4-7/2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+
16/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2-3/4/d/a/(1+tan(1/2
*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)+16/15/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5+3/
4/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.55767, size = 348, normalized size = 3.35

$$-\frac{\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{210\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{640\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{210\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{45\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-64}{a+\frac{5a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((45*sin(d*x + c)/(cos(d*x + c) + 1) - 320*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 210*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 640*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 210*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 45*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 - 64)/(a + 5*a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 10*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a*sin(d*x + c)^10
/(cos(d*x + c) + 1)^10) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.72003, size = 182, normalized size = 1.75

$$\frac{24\cos(dx+c)^5 - 80\cos(dx+c)^3 + 45\,dx + 15\left(2\cos(dx+c)^3 - 5\cos(dx+c)\right)\sin(dx+c) + 120\cos(dx+c)}{120\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*cos(d*x + c)^5 - 80*cos(d*x + c)^3 + 45*d*x + 15*(2*cos(d*x + c)^
3 - 5*cos(d*x + c))*sin(d*x + c) + 120*cos(d*x + c))/(a*d)
```

Sympy [A] time = 74.3806, size = 1360, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((45*d*x*tan(c/2 + d*x/2)**10/(120*a*d*tan(c/2 + d*x/2)**10 + 600*
a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 +
 d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 225*d*x*tan(c/2 + d*x
/2)**8/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a
*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d
*x/2)**2 + 120*a*d) + 450*d*x*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)
**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*
d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 450*d*x*ta
n(c/2 + d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)*
*8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*
tan(c/2 + d*x/2)**2 + 120*a*d) + 225*d*x*tan(c/2 + d*x/2)**2/(120*a*d*tan(c
/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**
6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) +
 45*d*x/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*
a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 +
d*x/2)**2 + 120*a*d) + 90*tan(c/2 + d*x/2)**9/(120*a*d*tan(c/2 + d*x/2)**10
 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*ta
n(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 420*tan(c/2 +
d*x/2)**7/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 120
0*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2
+ d*x/2)**2 + 120*a*d) + 1280*tan(c/2 + d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)
**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*
```

```
d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 420*tan(c/
2 + d*x/2)**3/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 +
 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(
c/2 + d*x/2)**2 + 120*a*d) + 640*tan(c/2 + d*x/2)**2/(120*a*d*tan(c/2 + d*x
/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200
*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 90*tan(
c/2 + d*x/2)/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 +
1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c
/2 + d*x/2)**2 + 120*a*d) + 128/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan
(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)
**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d), Ne(d, 0)), (x*sin(c)**4*cos(c
)**2/(a*sin(c) + a), True))
```

Giac [A] time = 1.34131, size = 154, normalized size = 1.48

$$\frac{\frac{45\,(dx+c)}{a}+\frac{2\left(45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+64\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(45*(d*x + c)/a + 2*(45*tan(1/2*d*x + 1/2*c)^9 + 210*tan(1/2*d*x + 1/
2*c)^7 + 640*tan(1/2*d*x + 1/2*c)^4 - 210*tan(1/2*d*x + 1/2*c)^3 + 320*tan(
1/2*d*x + 1/2*c)^2 - 45*tan(1/2*d*x + 1/2*c) + 64)/((tan(1/2*d*x + 1/2*c)^2
 + 1)^5*a))/d
```

3.298 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=87

$$\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}+\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{3x}{8a}$$

```
[Out] (-3*x)/(8*a) - Cos[c + d*x]/(a*d) + Cos[c + d*x]^3/(3*a*d) + (3*Cos[c + d*x
]*Sin[c + d*x])/(8*a*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a*d)
```

Rubi [A] time = 0.129538, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 2633, 2635, 8}

$$\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}+\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(8*a) - Cos[c + d*x]/(a*d) + Cos[c + d*x]^3/(3*a*d) + (3*Cos[c + d*x
]*Sin[c + d*x])/(8*a*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sin^3(c+dx)\,dx}{a} - \frac{\int \sin^4(c+dx)\,dx}{a}$$
$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4ad} - \frac{3\int \sin^2(c+dx)\,dx}{4a} - \frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(c+d\right.}{ad}$$
$$= -\frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{3\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos(c+dx)\sin^3(c+dx}{4ad}$$
$$= -\frac{3x}{8a} - \frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{3\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos(c+dx)\sin^3(}{4ad}$$

Mathematica [B] time = 1.65153, size = 271, normalized size = 3.11

$$\frac{-72dx\sin\left(\frac{c}{2}\right) + 72\sin\left(\frac{c}{2}+dx\right) - 72\sin\left(\frac{3c}{2}+dx\right) + 24\sin\left(\frac{3c}{2}+2dx\right) + 24\sin\left(\frac{5c}{2}+2dx\right) - 8\sin\left(\frac{5c}{2}+3dx\right) + 8\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (24*(c - 3*d*x)*Cos[c/2] - 72*Cos[c/2 + d*x] - 72*Cos[(3*c)/2 + d*x] + 24*C
os[(3*c)/2 + 2*d*x] - 24*Cos[(5*c)/2 + 2*d*x] + 8*Cos[(5*c)/2 + 3*d*x] + 8*
Cos[(7*c)/2 + 3*d*x] - 3*Cos[(7*c)/2 + 4*d*x] + 3*Cos[(9*c)/2 + 4*d*x] - 48
*Sin[c/2] + 24*c*Sin[c/2] - 72*d*x*Sin[c/2] + 72*Sin[c/2 + d*x] - 72*Sin[(3
*c)/2 + d*x] + 24*Sin[(3*c)/2 + 2*d*x] + 24*Sin[(5*c)/2 + 2*d*x] - 8*Sin[(5
*c)/2 + 3*d*x] + 8*Sin[(7*c)/2 + 3*d*x] - 3*Sin[(7*c)/2 + 4*d*x] - 3*Sin[(9
*c)/2 + 4*d*x])/(192*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.08, size = 245, normalized size = 2.8

$$-\frac{3}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4} - \frac{11}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4} - 4\frac{(\tan(1/}{da\left(1+(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-11/4/d/a/(1+tan(1/
2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan
(1/2*d*x+1/2*c)^4+11/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-
16/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+3/4/d/a/(1+tan(1/2
*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-4/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4-3/4/
a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.60839, size = 320, normalized size = 3.68

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1} - \frac{64\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{48\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - 16}{a + \frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} - \frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 64*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 - 16)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*
a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 9*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.6584, size = 149, normalized size = 1.71

$$\frac{8\cos(dx+c)^3 - 9\,dx - 3\left(2\cos(dx+c)^3 - 5\cos(dx+c)\right)\sin(dx+c) - 24\cos(dx+c)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*cos(d*x + c)^3 - 9*d*x - 3*(2*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(
d*x + c) - 24*cos(d*x + c))/(a*d)
```

Sympy [A] time = 43.5137, size = 1222, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-45*d*x*tan(c/2 + d*x/2)**8/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a
*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*
x/2)**2 + 120*a*d) - 180*d*x*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)*
*8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*ta
n(c/2 + d*x/2)**2 + 120*a*d) - 270*d*x*tan(c/2 + d*x/2)**4/(120*a*d*tan(c/2
 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 +
480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 180*d*x*tan(c/2 + d*x/2)**2/(120*a
*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*
x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 45*d*x/(120*a*d*tan(c/2
+ d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 4
80*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 18*tan(c/2 + d*x/2)**8/(120*a*d*tan
(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**
4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 90*tan(c/2 + d*x/2)**7/(120*a*
d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x
/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 72*tan(c/2 + d*x/2)**6/(1
20*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2
+ d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 330*tan(c/2 + d*x/2)
**5/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*ta
n(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 372*tan(c/2 +
d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*
a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 330*tan(
c/2 + d*x/2)**3/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6
```

```
+ 720*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 56
8*tan(c/2 + d*x/2)**2/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/
2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d
) + 90*tan(c/2 + d*x/2)/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*
x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a
*d) - 142/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*
a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d), Ne(d, 0))
, (x*sin(c)**3*cos(c)**2/(a*sin(c) + a), True))
```

Giac [A] time = 1.31751, size = 154, normalized size = 1.77

$$-\frac{\frac{9\,(dx+c)}{a}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+64\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+16\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(9*(d*x + c)/a + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 33*tan(1/2*d*x + 1/2*c
)^5 + 48*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2*c)^3 + 64*tan(1/2*d*
x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 16)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4
*a))/d
```

3.299 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=62

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

```
[Out] x/(2*a) + Cos[c + d*x]/(a*d) - Cos[c + d*x]^3/(3*a*d) - (Cos[c + d*x]*Sin[c
 + d*x])/(2*a*d)
```

Rubi [A] time = 0.117336, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 2635, 8, 2633}

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(2*a) + Cos[c + d*x]/(a*d) - Cos[c + d*x]^3/(3*a*d) - (Cos[c + d*x]*Sin[c
 + d*x])/(2*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sin^2(c+dx)\,dx}{a} - \frac{\int \sin^3(c+dx)\,dx}{a}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int 1\,dx}{2a} + \frac{\text{Subst}\left(\int \left(1-x^2\right)dx, x, \cos(c+dx)\right)}{ad}$$
$$= \frac{x}{2a} + \frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

Mathematica [A] time = 0.082831, size = 46, normalized size = 0.74

$$\frac{-3\sin(2(c+dx)) + 9\cos(c+dx) - \cos(3(c+dx)) + 6c + 6dx}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*c + 6*d*x + 9*Cos[c + d*x] - Cos[3*(c + d*x)] - 3*Sin[2*(c + d*x)])/(12*
a*d)
```

Maple [B] time = 0.072, size = 141, normalized size = 2.3

$$\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-3} + 4\frac{(\tan(1/2\,dx+c/2))^2}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^3} - \frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+4/d/a/(1+tan(1/2*d*x+
1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d
*x+1/2*c)+4/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3+1/a/d*arctan(tan(1/2*d*x+1/2*c
))
```

Maxima [B] time = 1.65713, size = 211, normalized size = 3.4

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1} - \frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - 4}{a + \frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*((3*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4)/(a + 3*a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
)/d
```

Fricas [A] time = 1.62002, size = 116, normalized size = 1.87

$$-\frac{2 \cos(dx+c)^3 - 3\,dx + 3\,\cos(dx+c)\sin(dx+c) - 6\,\cos(dx+c)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*cos(d*x + c)^3 - 3*d*x + 3*cos(d*x + c)*sin(d*x + c) - 6*cos(d*x +
c))/(a*d)
```

Sympy [A] time = 19.6895, size = 563, normalized size = 9.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*ta
n(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d
*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*t
an(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*
x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d)
+ 3*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d) + 6*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)*
*6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 24*
tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4
 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 6*tan(c/2 + d*x/2)/(6*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a
*d) + 8/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**2/(a*sin(c) + a
), True))
```

Giac [A] time = 1.27396, size = 101, normalized size = 1.63

$$\frac{\frac{3(dx+c)}{a} + \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 12*tan(1/2*d*x + 1/2*c)^
2 - 3*tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

3.300 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(c+dx)}{ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

```
[Out] -x/(2*a) - Cos[c + d*x]/(a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.072349, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2839, 2638, 2635, 8}

$$-\frac{\cos(c+dx)}{ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(2*a) - Cos[c + d*x]/(a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx&=\frac{\int \sin(c+dx)\,dx}{a}-\frac{\int \sin^2(c+dx)\,dx}{a}\\&=-\frac{\cos(c+dx)}{ad}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\int 1\,dx}{2a}\\&=-\frac{x}{2a}-\frac{\cos(c+dx)}{ad}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [B] time = 0.561115, size = 161, normalized size = 3.58

$$\frac{-4dx\sin\left(\frac{c}{2}\right)+4\sin\left(\frac{c}{2}+dx\right)-4\sin\left(\frac{3c}{2}+dx\right)+\sin\left(\frac{3c}{2}+2dx\right)+\sin\left(\frac{5c}{2}+2dx\right)+2\cos\left(\frac{c}{2}\right)(c-2dx)-4\cos\left(\frac{c}{2}+dx\right)\cdots}{8ad\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(c - 2*d*x)*Cos[c/2] - 4*Cos[c/2 + d*x] - 4*Cos[(3*c)/2 + d*x] + Cos[(3*
c)/2 + 2*d*x] - Cos[(5*c)/2 + 2*d*x] - 4*Sin[c/2] + 2*c*Sin[c/2] - 4*d*x*Si
n[c/2] + 4*Sin[c/2 + d*x] - 4*Sin[(3*c)/2 + d*x] + Sin[(3*c)/2 + 2*d*x] + S
in[(5*c)/2 + 2*d*x])/(8*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.06, size = 142, normalized size = 3.2

$$-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-2\,\frac{(\tan(1/2\,dx+c/2))^2}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/a/(1+tan(1/2*d*x
+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*
d*x+1/2*c)-2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2-1/a/d*arctan(tan(1/2*d*x+1/2*c)
)
```

Maxima [B] time = 1.59499, size = 180, normalized size = 4.

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-2}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2)/(a + 2*a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.57502, size = 85, normalized size = 1.89

$$-\frac{dx-\cos(dx+c)\sin(dx+c)+2\cos(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*x - cos(d*x + c)*sin(d*x + c) + 2*cos(d*x + c))/(a*d)
```

Sympy [A] time = 9.31351, size = 366, normalized size = 8.13

$$\begin{cases} -\frac{dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} - \frac{2dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} - \frac{dx}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} + \frac{2}{2ad\tan^4\left(\frac{c}{2}+\right.} \\ \frac{x\sin(c)\cos^2(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(
c/2 + d*x/2)**2 + 2*a*d) - 2*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2
)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - d*x/(2*a*d*tan(c/2 + d*x/2)**4
+ 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2
 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*tan(c/2 + d*x/2)**3/(
2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*tan(c/2
+ d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) -
2/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d), Ne(d, 0)
), (x*sin(c)*cos(c)**2/(a*sin(c) + a), True))
```

Giac [A] time = 1.28341, size = 97, normalized size = 2.16

$$-\frac{\frac{dx+c}{a} + \frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a + 2*(tan(1/2*d*x + 1/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 -
tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.301 $\int \frac{\cos(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

```
[Out] -(x/a) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0715916, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2839, 3770, 8}

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int 1\,dx}{a}+\frac{\int \csc(c+dx)\,dx}{a}\\ &= -\frac{x}{a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}\end{aligned}$$

Mathematica [A] time = 0.088778, size = 37, normalized size = 1.68

$$-\frac{-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+c+dx}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])/(a*d))
```

Maple [A] time = 0.088, size = 37, normalized size = 1.7

$$-2\,{\frac {\arctan \left(\tan \left(1/2\,dx+c/2 \right) \right) }{da}}+{\frac{1}{da}\ln \left(\tan \left({\frac{dx}{2}}+{\frac{c}{2}} \right) \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/a/d*arctan(tan(1/2*d*x+1/2*c))+1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 2.09784, size = 70, normalized size = 3.18

$$-\frac{\frac{2 \arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a} - \frac{\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x +
 c) + 1))/a)/d
```

Fricas [A] time = 1.79255, size = 109, normalized size = 4.95

$$-\frac{2 \, d x + \log\left(\frac{1}{2} \, \cos\left(d x + c\right) + \frac{1}{2}\right) - \log\left(-\frac{1}{2} \, \cos\left(d x + c\right) + \frac{1}{2}\right)}{2 \, a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*x + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d*x + c) + 1/2))/(
a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^{2}{\left (c + d x \right)} \csc{\left (c + d x \right)}}{\sin{\left (c + d x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.36141, size = 42, normalized size = 1.91

$$-\frac{\frac{dx+c}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c)))/a)/d
```

3.302 $\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)

Rubi [A] time = 0.0549483, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2706, 3767, 8, 3770}

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]

[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc(c+dx)\,dx}{a}+\frac{\int \csc^2(c+dx)\,dx}{a}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{ad}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}\end{aligned}$$

Mathematica [B] time = 0.215274, size = 69, normalized size = 2.38

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)+\sin(c+dx)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x] + (-Log[Cos[(c + d*x)/2]]
 + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]))/(2*a*d)
```

Maple [A] time = 0.104, size = 56, normalized size = 1.9

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+
1/2*c))
```

Maxima [B] time = 1.15449, size = 95, normalized size = 3.28

$$-\frac{\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\cos(dx+c)+1}{a\sin(dx+c)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (cos(d*x + c) + 1)/(a*sin(
d*x + c)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [B] time = 1.59607, size = 173, normalized size = 5.97

$$\frac{\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)}{2\,ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [B] time = 1.37431, size = 88, normalized size = 3.03

$$-\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (2*tan(
1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)))/d
```

3.303 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{\cot(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) + Cot[c + d*x]/(a*d) - (Cot[c + d*x]*Csc[c +
 d*x])/(2*a*d)
```

Rubi [A] time = 0.102679, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 3768, 3770, 3767, 8}

$$\frac{\cot(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) + Cot[c + d*x]/(a*d) - (Cot[c + d*x]*Csc[c +
 d*x])/(2*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc^2(c+dx)\,dx}{a} + \frac{\int \csc^3(c+dx)\,dx}{a}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int \csc(c+dx)\,dx}{2a} + \frac{\text{Subst}(\int 1\,dx, x, \cot(c+dx))}{ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{\cot(c+dx)}{ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Mathematica [A] time = 0.398791, size = 94, normalized size = 1.77

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\sin(2(c+dx))-\cos(c+dx)+\sin^2(c+dx)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}\right.\right.\right.\right.}{8ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(-Cos[c + d*x] + (-Log[Cos[(c + d*
x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^2 + Sin[2*(c + d*x)]))/(8*a*d*
(1 + Sin[c + d*x]))
```

Maple [A] time = 0.114, size = 94, normalized size = 1.8

$$\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}+\frac{1}{2da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/2/d/a*tan(1/2*d*x+1/2*c)+1/2/d/a/tan(1/2*d*x
+1/2*c)+1/2/d/a*ln(tan(1/2*d*x+1/2*c))-1/8/d/a/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.10902, size = 155, normalized size = 2.92

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a}-\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - 4*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (4*sin(d*x + c)/(cos(d*
x + c) + 1) - 1)*(cos(d*x + c) + 1)^2/(a*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.93126, size = 247, normalized size = 4.66

$$-\frac{\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\cos\left(dx+c\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+4\cos\left(dx+c\right)\sin\left(dx+c\right)}{4\left(ad\cos\left(dx+c\right)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 -
1)*log(-1/2*cos(d*x + c) + 1/2) + 4*cos(d*x + c)*sin(d*x + c) - 2*cos(d*x +
 c))/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.36731, size = 127, normalized size = 2.4

$$\frac{\frac{4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(4*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x + 1/2*c)^2 - 4*a*t
an(1/2*d*x + 1/2*c))/a^2 - (6*tan(1/2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*
c) + 1)/(a*tan(1/2*d*x + 1/2*c)^2))/d
```

3.304 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=72

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]/(a*d) - Cot[c + d*x]^3/(3*a*d)
 + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.123105, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 3767, 3768, 3770}

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]/(a*d) - Cot[c + d*x]^3/(3*a*d)
 + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc^3(c+dx)\,dx}{a} + \frac{\int \csc^4(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\int \csc(c+dx)\,dx}{2a} - \frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(c+dx)\right)}{ad}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Mathematica [A] time = 0.574997, size = 126, normalized size = 1.75

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-12(\sin(c+dx)-1)\cos(c+dx)-4\left(\cos(3(c+dx\right.\right.}{192ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2
*(-12*Cos[c + d*x]*(-1 + Sin[c + d*x]) - 4*(Cos[3*(c + d*x)] + 3*(Log[Cos[(
c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3)))/(192*a*d*(1 + Sin[c
 + d*x]))
```

Maple [A] time = 0.135, size = 132, normalized size = 1.8

$$\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + \frac{3}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{3}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1} - \frac{1}{2\,da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a*tan(1/2*d*x+1/2*c)^2+3/8/d/a*tan(1/2*
d*x+1/2*c)-3/8/d/a/tan(1/2*d*x+1/2*c)-1/2/d/a*ln(tan(1/2*d*x+1/2*c))-1/24/d
/a/tan(1/2*d*x+1/2*c)^3+1/8/d/a/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.10887, size = 207, normalized size = 2.88

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a} - \frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(d
*x + c) + 1))/a + (3*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.65169, size = 336, normalized size = 4.67

$$-\frac{8\cos\left(dx+c\right)^{3}-3\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\cos\left(dx+\right.\right.}{12\left(ad\cos\left(dx+c\right)^{2}-ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(8*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 6*cos(d*x + c)*sin(d*x + c) - 12*cos(d*x + c))/((a*d*cos(d*x + c)^
2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30829, size = 173, normalized size = 2.4

$$-\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-3a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+9a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{3}}-\frac{22\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a - (a^2*tan(1/2*d*x + 1/2*c)^3 -
3*a^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/
2*d*x + 1/2*c)^3 - 9*tan(1/2*d*x + 1/2*c)^2 + 3*tan(1/2*d*x + 1/2*c) - 1)/(
a*tan(1/2*d*x + 1/2*c)^3))/d
```

3.305 $\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(8*a*d) + Cot[c + d*x]/(a*d) + Cot[c + d*x]^3/(3
*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^
3)/(4*a*d)
```

Rubi [A] time = 0.13442, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 3768, 3770, 3767}

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(8*a*d) + Cot[c + d*x]/(a*d) + Cot[c + d*x]^3/(3
*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^
3)/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc^4(c+dx)\,dx}{a} + \frac{\int \csc^5(c+dx)\,dx}{a}$$
$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{3\int \csc^3(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int \left(1+x^2\right)dx, x, \cot(c+\right.}{ad}$$
$$= \frac{\cot(c+dx)}{ad} + \frac{\cot^3(c+dx)}{3ad} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{\cot(c+dx)}{ad} + \frac{\cot^3(c+dx)}{3ad} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Mathematica [A] time = 1.07876, size = 125, normalized size = 1.32

$$-\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-48\sin(2(c+dx))+66\cos(c+dx)+2(16\sin(c+dx)-9)\cos(3\right.}{192ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(66*Cos[c + d*x] +
 72*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + 2*Cos[
3*(c + d*x)]*(-9 + 16*Sin[c + d*x]) - 48*Sin[2*(c + d*x)]))/(192*a*d*(1 + S
in[c + d*x]))
```

Maple [A] time = 0.134, size = 170, normalized size = 1.8

$$\frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 - \frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{3}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{3}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a*tan(1/2*d*x+1/2*c)^3+1/8/d/a*tan(1/2
*d*x+1/2*c)^2-3/8/d/a*tan(1/2*d*x+1/2*c)+3/8/d/a/tan(1/2*d*x+1/2*c)-1/64/d/
a/tan(1/2*d*x+1/2*c)^4+3/8/d/a*ln(tan(1/2*d*x+1/2*c))+1/24/d/a/tan(1/2*d*x+
1/2*c)^3-1/8/d/a/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.1745, size = 263, normalized size = 2.77

$$-\frac{\frac{\frac{72\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a} - \frac{72\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{72\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/192*((72*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^4/(cos(d
```

```
*x + c) + 1)^4)/a - 72*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (8*sin(d*x
+ c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 72*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a*sin(d*x + c)^4)
)/d
```

Fricas [A] time = 1.70459, size = 396, normalized size = 4.17

$$\frac{18\,\cos\left(dx+c\right)^{3}-9\,\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+9\,\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right.}{48\,\left(ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(18*cos(d*x + c)^3 - 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2
*cos(d*x + c) + 1/2) + 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*c
os(d*x + c) + 1/2) - 16*(2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c) -
30*cos(d*x + c))/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38365, size = 212, normalized size = 2.23

$$\frac{\frac{72\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+24\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-72\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{4}}-\frac{150\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-72\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{a\tan}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(72*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^3*tan(1/2*d*x + 1/2*c)^4
- 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c)^2 - 72*a^3*tan
(1/2*d*x + 1/2*c))/a^4 - (150*tan(1/2*d*x + 1/2*c)^4 - 72*tan(1/2*d*x + 1/2
*c)^3 + 24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*
d*x + 1/2*c)^4))/d
```

3.306 $\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=114

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\cot(c+dx)\operatorname{cs}}{8ad}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]/(a*d) - (2*Cot[c + d*x]^3)
/(3*a*d) - Cot[c + d*x]^5/(5*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) +
 (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rubi [A] time = 0.137707, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 3767, 3768, 3770}

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\cot(c+dx)\operatorname{cs}}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]/(a*d) - (2*Cot[c + d*x]^3)
/(3*a*d) - Cot[c + d*x]^5/(5*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) +
 (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc^5(c+dx)\,dx}{a} + \frac{\int \csc^6(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{3\int \csc^3(c+dx)\,dx}{4a} - \frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, \cot(c\right.}{ad}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{2\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{3\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot(c+}{}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{8ad} - \frac{\cot(c+dx)}{ad} - \frac{2\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{3\cot(c+dx)}{8a}$$

Mathematica [A] time = 0.624266, size = 189, normalized size = 1.66

$$\frac{\csc^5(c+dx)\left(420\sin(2(c+dx)) - 90\sin(4(c+dx)) - 640\cos(c+dx) + 320\cos(3(c+dx)) - 64\cos(5(c+dx)) - 450\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^5*(-640*Cos[c + d*x] + 320*Cos[3*(c + d*x)] - 64*Cos[5*(c + d
*x)] + 450*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 450*Log[Sin[(c + d*x)/2]]*S
in[c + d*x] + 420*Sin[2*(c + d*x)] - 225*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d
*x)] + 225*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 90*Sin[4*(c + d*x)] + 4
5*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 45*Log[Sin[(c + d*x)/2]]*Sin[5*(
c + d*x)]))/(1920*a*d)
```

Maple [A] time = 0.154, size = 208, normalized size = 1.8

$$\frac{1}{160\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + \frac{5}{96\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + \frac{5}{16\,da}\tan\left(\frac{dx}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a*tan(1/2*d*x+1/2*c)^4+5/96/d/a*tan(1
/2*d*x+1/2*c)^3-1/8/d/a*tan(1/2*d*x+1/2*c)^2+5/16/d/a*tan(1/2*d*x+1/2*c)-5/
16/d/a/tan(1/2*d*x+1/2*c)-1/160/d/a/tan(1/2*d*x+1/2*c)^5+1/64/d/a/tan(1/2*d
*x+1/2*c)^4-3/8/d/a*ln(tan(1/2*d*x+1/2*c))-5/96/d/a/tan(1/2*d*x+1/2*c)^3+1/
8/d/a/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.10291, size = 316, normalized size = 2.77

$$\frac{\frac{\frac{300\sin(dx+c)}{\cos(dx+c)+1} - \frac{120\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{50\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{6\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a} - \frac{360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1} - \frac{50\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{300\sin(}{(\cos(dx+}\right.}{a\sin(dx+c)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*((300*sin(d*x + c)/(cos(d*x + c) + 1) - 120*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 360*log(sin(
d*x + c)/(cos(d*x + c) + 1))/a + (15*sin(d*x + c)/(cos(d*x + c) + 1) - 50*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 300*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1)^5/(a*si
n(d*x + c)^5))/d
```

Fricas [A] time = 1.70407, size = 487, normalized size = 4.27

$$-\frac{128\cos(dx+c)^5 - 320\cos(dx+c)^3 - 45\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c)}{240\left(ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(128*cos(d*x + c)^5 - 320*cos(d*x + c)^3 - 45*(cos(d*x + c)^4 - 2*co
s(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 45*(cos(d*x +
c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30
*(3*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(d*x + c) + 240*cos(d*x + c))/((a*d
*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42755, size = 252, normalized size = 2.21

$$-\frac{\frac{360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{6a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 50a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 120a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 300a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^5} - 822}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/960*(360*log(abs(tan(1/2*d*x + 1/2*c)))/a - (6*a^4*tan(1/2*d*x + 1/2*c)^
5 - 15*a^4*tan(1/2*d*x + 1/2*c)^4 + 50*a^4*tan(1/2*d*x + 1/2*c)^3 - 120*a^4
*tan(1/2*d*x + 1/2*c)^2 + 300*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (822*tan(1/2*
d*x + 1/2*c)^5 - 300*tan(1/2*d*x + 1/2*c)^4 + 120*tan(1/2*d*x + 1/2*c)^3 -
50*tan(1/2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c) - 6)/(a*tan(1/2*d*x + 1
/2*c)^5))/d
```

3.307 $\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=111

$$\frac{2\cos^3(c+dx)}{3a^2d}-\frac{4\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{11\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{27x}{8a^2}$$

```
[Out] (-27*x)/(8*a^2) - (4*Cos[c + d*x])/(a^2*d) + (2*Cos[c + d*x]^3)/(3*a^2*d) +
 (11*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(
4*a^2*d) - (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.245498, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2966, 2638, 2635, 8, 2633, 2648}

$$\frac{2\cos^3(c+dx)}{3a^2d}-\frac{4\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{11\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{27x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-27*x)/(8*a^2) - (4*Cos[c + d*x])/(a^2*d) + (2*Cos[c + d*x]^3)/(3*a^2*d) +
 (11*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(
4*a^2*d) - (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\sin^4(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{\int \left(-2+2\sin(c+dx)-2\sin^2(c+dx)+2\sin^3(c+dx)-\sin^4(c+dx)+\frac{2}{1+\sin(c+dx)}\right.}{a^2}$$

$$= -\frac{2x}{a^2}-\frac{\int \sin^4(c+dx)\,dx}{a^2}+\frac{2\int \sin(c+dx)\,dx}{a^2}-\frac{2\int \sin^2(c+dx)\,dx}{a^2}+\frac{2\int \sin^3(c+\ldots}{a^2}$$

$$= -\frac{2x}{a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d}-\frac{2\,\ldots}{a^2d(1\ldots}$$

$$= -\frac{3x}{a^2}-\frac{4\cos(c+dx)}{a^2d}+\frac{2\cos^3(c+dx)}{3a^2d}+\frac{11\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos(c+dx)\,\mathrm{s}\ldots}{4a^2\ldots}$$

$$= -\frac{27x}{8a^2}-\frac{4\cos(c+dx)}{a^2d}+\frac{2\cos^3(c+dx)}{3a^2d}+\frac{11\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos(c+dx)\ldots}{4a\ldots}$$

Mathematica [A] time = 1.43328, size = 209, normalized size = 1.88

$$\frac{-648dx\sin\left(c+\frac{dx}{2}\right)+4\sin\left(c+\frac{dx}{2}\right)-264\sin\left(2c+\frac{3dx}{2}\right)+56\sin\left(2c+\frac{5dx}{2}\right)+13\sin\left(4c+\frac{7dx}{2}\right)-3\sin\left(4c+\frac{9dx}{2}\right)-\ldots}{192a^2d\left(\sin\left(\frac{c}{2}\right)+\cos\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((4 - 648*d*x)*Cos[(d*x)/2] - 340*Cos[c + (d*x)/2] - 264*Cos[c + (3*d*x)/2]
 - 56*Cos[3*c + (5*d*x)/2] + 13*Cos[3*c + (7*d*x)/2] + 3*Cos[5*c + (9*d*x)/
2] + 1100*Sin[(d*x)/2] + 4*Sin[c + (d*x)/2] - 648*d*x*Sin[c + (d*x)/2] - 26
4*Sin[2*c + (3*d*x)/2] + 56*Sin[2*c + (5*d*x)/2] + 13*Sin[4*c + (7*d*x)/2]
- 3*Sin[4*c + (9*d*x)/2])/(192*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]))
```

Maple [B] time = 0.112, size = 300, normalized size = 2.7

$$-\frac{11}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}-4\,\frac{(\tan(1/2\,dx+c/2))^6}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^4}-\frac{19}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -11/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-4/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6-19/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)^5-20/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^4+19/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-68/3/d/a^
2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+11/4/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-20/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4-27/4/
d/a^2*arctan(tan(1/2*d*x+1/2*c))-4/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.68375, size = 537, normalized size = 4.84

$$-\frac{\frac{\frac{47 \sin(dx+c)}{\cos(dx+c)+1}+\frac{431 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{215 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{471 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{297 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{297 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{81 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{81 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+128}{a^2+\frac{a^2 \sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^2 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6a^2 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4a^2 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{81 \arctan(\cdots}{\cdots}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*((47*sin(d*x + c)/(cos(d*x + c) + 1) + 431*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 215*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 471*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 297*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 297*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 81*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 81*s
in(d*x + c)^8/(cos(d*x + c) + 1)^8 + 128)/(a^2 + a^2*sin(d*x + c)/(cos(d*x
+ c) + 1) + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^2*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 6*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*a^2*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + 4*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^2*sin(d*x + c)^8/(cos(
d*x + c) + 1)^8 + a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9) + 81*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.66638, size = 386, normalized size = 3.48

$$\frac{6 \cos(dx+c)^5+16 \cos(dx+c)^4-29 \cos(dx+c)^3-81\,dx-3\,(27\,dx+35)\cos(dx+c)-96 \cos(dx+c)^2-\left(6 \cos(d\cdots\right.}{24\left(a^2 d \cos(dx+c)+a^2 d \sin(dx+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*cos(d*x + c)^5 + 16*cos(d*x + c)^4 - 29*cos(d*x + c)^3 - 81*d*x - 3
*(27*d*x + 35)*cos(d*x + c) - 96*cos(d*x + c)^2 - (6*cos(d*x + c)^4 - 10*co
s(d*x + c)^3 + 81*d*x - 39*cos(d*x + c)^2 + 57*cos(d*x + c) - 48)*sin(d*x +
 c) - 48)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A] time = 86.7452, size = 3766, normalized size = 33.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-81*d*x*tan(c/2 + d*x/2)**9/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*
a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(
c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x
/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24
*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 81*d*x*tan(c/2 + d*x/2)**8/(24*a**2
*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2
+ d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**
5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**
2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 324*d*x
*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d
*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 +
144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d
*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 +
d*x/2) + 24*a**2*d) - 324*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/2 + d*x/
2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*
a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*ta
n(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x
/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 486*d*x*tan(c/2 + d*x/2)
**5/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**
2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/
2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)
**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*
d) - 486*d*x*tan(c/2 + d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d
*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 +
d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4
 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*
d*tan(c/2 + d*x/2) + 24*a**2*d) - 324*d*x*tan(c/2 + d*x/2)**3/(24*a**2*d*ta
n(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x
/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 1
44*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*t
an(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 324*d*x*tan(
c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)
**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a
**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(
c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2
) + 24*a**2*d) - 81*d*x*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2 + d*x/2)**9 + 2
4*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*ta
n(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d
*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 +
24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 81*d*x/(24*a**2*d*tan(c/2 + d*x/2
)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a
**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan
(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/
2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) + 54*tan(c/2 + d*x/2)**9/(2
4*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*ta
n(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*
x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 +
96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 1
08*tan(c/2 + d*x/2)**8/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 +
 d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6
+ 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2
```

```
*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2
+ d*x/2) + 24*a**2*d) + 54*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)*
*9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**
2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c
/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)
**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 378*tan(c/2 + d*x/2)**6/(24
*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan
(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x
/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 9
6*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 27
0*tan(c/2 + d*x/2)**5/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 +
d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 +
 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*
d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 +
 d*x/2) + 24*a**2*d) - 618*tan(c/2 + d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)*
*9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**
2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c
/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)
**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 214*tan(c/2 + d*x/2)**3/(24
*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan
(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x
/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 9
6*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 64
6*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 +
d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 +
 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*
d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 +
 d*x/2) + 24*a**2*d) - 40*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2 + d*x/2)**9 +
 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*
tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 +
 d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2
+ 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 202/(24*a**2*d*tan(c/2 + d*x/2)
**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a*
*2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(
c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2
)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d), Ne(d, 0)), (x*sin(c)**4*cos
(c)**2/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.33877, size = 196, normalized size = 1.77

$$-\frac{\frac{81\,(dx+c)}{a^2}+\frac{96}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+57\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-57\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+272\,\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(81*(d*x + c)/a^2 + 96/(a^2*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(33*tan(1
/2*d*x + 1/2*c)^7 + 48*tan(1/2*d*x + 1/2*c)^6 + 57*tan(1/2*d*x + 1/2*c)^5 +
 240*tan(1/2*d*x + 1/2*c)^4 - 57*tan(1/2*d*x + 1/2*c)^3 + 272*tan(1/2*d*x +
 1/2*c)^2 - 33*tan(1/2*d*x + 1/2*c) + 80)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a
^2))/d
```

3.308 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{3\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3x}{a^2}$$

```
[Out] (3*x)/a^2 + (3*Cos[c + d*x])/(a^2*d) - Cos[c + d*x]^3/(3*a^2*d) - (Cos[c +
d*x]*Sin[c + d*x])/(a^2*d) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.221578, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2966, 2638, 2635, 8, 2633, 2648}

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{3\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/a^2 + (3*Cos[c + d*x])/(a^2*d) - Cos[c + d*x]^3/(3*a^2*d) - (Cos[c +
d*x]*Sin[c + d*x])/(a^2*d) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\sin^3(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{\int \left(2-2\sin(c+dx)+2\sin^2(c+dx)-\sin^3(c+dx)-\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}$$
$$= \frac{2x}{a^2}-\frac{\int \sin^3(c+dx)\,dx}{a^2}-\frac{2\int \sin(c+dx)\,dx}{a^2}+\frac{2\int \sin^2(c+dx)\,dx}{a^2}-\frac{2\int \frac{1}{1+\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{2x}{a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}+\frac{\int 1\,dx}{a^2}+\text{Subs}$$
$$= \frac{3x}{a^2}+\frac{3\cos(c+dx)}{a^2d}-\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}$$

Mathematica [A] time = 0.92207, size = 165, normalized size = 1.99

$$-\frac{-72dx\sin\left(c+\frac{dx}{2}\right)-27\sin\left(2c+\frac{3dx}{2}\right)+5\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(4c+\frac{7dx}{2}\right)-31\cos\left(c+\frac{dx}{2}\right)-27\cos\left(c+\frac{3dx}{2}\right)-5\cos(}{24a^2d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(-72*d*x*Cos[(d*x)/2] - 31*Cos[c + (d*x)/2] - 27*Cos[c + (3*d*x)/2] - 5*Co
s[3*c + (5*d*x)/2] + Cos[3*c + (7*d*x)/2] + 131*Sin[(d*x)/2] - 72*d*x*Sin[c
 + (d*x)/2] - 27*Sin[2*c + (3*d*x)/2] + 5*Sin[2*c + (5*d*x)/2] + Sin[4*c +
(7*d*x)/2])/(24*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]))
```

Maple [B] time = 0.104, size = 198, normalized size = 2.4

$$2\frac{(\tan(1/2\,dx+c/2))^5}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+4\frac{(\tan(1/2\,dx+c/2))^4}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+12\frac{(\tan(1/2\,dx+c/2))^2}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}-2\frac{\tan}{da^2\left(1+(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+4/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)^2-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+16
/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3+6/d/a^2*arctan(tan(1/2*d*x+1/2*c))+4/d/
a^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.72474, size = 421, normalized size = 5.07

$$\frac{2\left(\frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{33\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{24\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{9\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+14}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 2/3*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 33*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 18*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 24*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 9*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 9*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6 + 14)/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*a^2*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2
)/d
```

Fricas [A] time = 1.66617, size = 313, normalized size = 3.77

$$-\frac{\cos(dx+c)^4 - 2\cos(dx+c)^3 - 9\,dx - 3\,(3\,dx+4)\cos(dx+c) - 9\cos(dx+c)^2 + \left(\cos(dx+c)^3 - 9\,dx + 3\cos(d\right.}{3\left(a^2d\cos(dx+c) + a^2d\sin(dx+c) + a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 9*d*x - 3*(3*d*x + 4)*cos(d*x + c
) - 9*cos(d*x + c)^2 + (cos(d*x + c)^3 - 9*d*x + 3*cos(d*x + c)^2 - 6*cos(d
*x + c) + 6)*sin(d*x + c) - 6)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a
^2*d)
```

Sympy [A] time = 48.7996, size = 2263, normalized size = 27.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((9*d*x*tan(c/2 + d*x/2)**7/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2
*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 +
d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3
*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*
tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x
/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a*
*2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 27*d*x*t
an(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2
)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2
*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 +
d*x/2) + 3*a**2*d) + 27*d*x*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)*
*7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d
*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*
x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 27*d*x*tan(c/2 + d*x/2)**
3/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*t
an(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/
2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d
) + 27*d*x*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan
(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)
**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*
d*tan(c/2 + d*x/2) + 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 +
 d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 +
9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(
c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 9*d*x/(3*a**2*d*t
an(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**
2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 18*tan(c/
2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*ta
n(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2
) + 3*a**2*d) + 18*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2
+ d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 +
 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 48*tan(c/2 + d*x/2)**4/(3*a**2*d*t
an(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**
2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 36*tan(c/
2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*ta
n(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2
) + 3*a**2*d) + 66*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2
+ d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 +
 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 10*tan(c/2 + d*x/2)/(3*a**2*d*tan(
c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)*
*5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d
*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 28/(3*a**2*d
*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*
x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a
**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d), Ne(d, 0)
), (x*sin(c)**3*cos(c)**2/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.28067, size = 143, normalized size = 1.72

$$\frac{\frac{9\,(dx+c)}{a^2}+\frac{12}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(9*(d*x + c)/a^2 + 12/(a^2*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(3*tan(1/2*d
*x + 1/2*c)^5 + 6*tan(1/2*d*x + 1/2*c)^4 + 18*tan(1/2*d*x + 1/2*c)^2 - 3*ta
n(1/2*d*x + 1/2*c) + 8)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2))/d
```

3.309 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=69

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5x}{2a^2}$$

```
[Out] (-5*x)/(2*a^2) - (2*Cos[c + d*x])/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*
a^2*d) - (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.271632, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2950, 2709, 2638, 2635, 8, 2648}

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*x)/(2*a^2) - (2*Cos[c + d*x])/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*
a^2*d) - (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \frac{\sin^2(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}\\ &= \frac{\int (a-a\sin(c+dx))^2\tan^2(c+dx)\,dx}{a^4}\\ &= \frac{\int \left(-2+2\sin(c+dx)-\sin^2(c+dx)+\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}\\ &= -\frac{2x}{a^2}-\frac{\int \sin^2(c+dx)\,dx}{a^2}+\frac{2\int \sin(c+dx)\,dx}{a^2}+\frac{2\int \frac{1}{1+\sin(c+dx)}\,dx}{a^2}\\ &= -\frac{2x}{a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}-\frac{\int 1\,dx}{2a^2}\\ &= -\frac{5x}{2a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.146948, size = 69, normalized size = 1.

$$\frac{-10(c+dx)+\sin(2(c+dx))-8\cos(c+dx)+\frac{16\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-10*(c + d*x) - 8*Cos[c + d*x] + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]) + Sin[2*(c + d*x)])/(4*a^2*d)
```

Maple [B] time = 0.095, size = 163, normalized size = 2.4

$$-\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-4\frac{(\tan(1/2\,dx+c/2))^2}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+\frac{1}{da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-4/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*ta
n(1/2*d*x+1/2*c)-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2-5/d/a^2*arctan(tan(1/2*
d*x+1/2*c))-4/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.61373, size = 305, normalized size = 4.42

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+8}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{5\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -((3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 8)/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a^2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 2*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^2*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5) + 5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.69242, size = 259, normalized size = 3.75

$$-\frac{\cos(dx+c)^3+5\,dx+(5\,dx+7)\cos(dx+c)+4\,\cos(dx+c)^2+\left(5\,dx-\cos(dx+c)^2+3\,\cos(dx+c)-4\right)\sin(dx+c)}{2\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(cos(d*x + c)^3 + 5*d*x + (5*d*x + 7)*cos(d*x + c) + 4*cos(d*x + c)^2
+ (5*d*x - cos(d*x + c)^2 + 3*cos(d*x + c) - 4)*sin(d*x + c) + 4)/(a^2*d*co
s(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A] time = 28.2271, size = 1358, normalized size = 19.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-35*d*x*tan(c/2 + d*x/2)**5/(14*a**2*d*tan(c/2 + d*x/2)**5 + 14*
a**2*d*tan(c/2 + d*x/2)**4 + 28*a**2*d*tan(c/2 + d*x/2)**3 + 28*a**2*d*tan(
c/2 + d*x/2)**2 + 14*a**2*d*tan(c/2 + d*x/2) + 14*a**2*d) - 35*d*x*tan(c/2
+ d*x/2)**4/(14*a**2*d*tan(c/2 + d*x/2)**5 + 14*a**2*d*tan(c/2 + d*x/2)**4
+ 28*a**2*d*tan(c/2 + d*x/2)**3 + 28*a**2*d*tan(c/2 + d*x/2)**2 + 14*a**2*d
```

```
*tan(c/2 + d*x/2) + 14*a**2*d) - 70*d*x*tan(c/2 + d*x/2)**3/(14*a**2*d*tan(
c/2 + d*x/2)**5 + 14*a**2*d*tan(c/2 + d*x/2)**4 + 28*a**2*d*tan(c/2 + d*x/2
)**3 + 28*a**2*d*tan(c/2 + d*x/2)**2 + 14*a**2*d*tan(c/2 + d*x/2) + 14*a**2
*d) - 70*d*x*tan(c/2 + d*x/2)**2/(14*a**2*d*tan(c/2 + d*x/2)**5 + 14*a**2*d
*tan(c/2 + d*x/2)**4 + 28*a**2*d*tan(c/2 + d*x/2)**3 + 28*a**2*d*tan(c/2 +
d*x/2)**2 + 14*a**2*d*tan(c/2 + d*x/2) + 14*a**2*d) - 35*d*x*tan(c/2 + d*x/
2)/(14*a**2*d*tan(c/2 + d*x/2)**5 + 14*a**2*d*tan(c/2 + d*x/2)**4 + 28*a**2
*d*tan(c/2 + d*x/2)**3 + 28*a**2*d*tan(c/2 + d*x/2)**2 + 14*a**2*d*tan(c/2
+ d*x/2) + 14*a**2*d) - 35*d*x/(14*a**2*d*tan(c/2 + d*x/2)**5 + 14*a**2*d*t
an(c/2 + d*x/2)**4 + 28*a**2*d*tan(c/2 + d*x/2)**3 + 28*a**2*d*tan(c/2 + d*
x/2)**2 + 14*a**2*d*tan(c/2 + d*x/2) + 14*a**2*d) + 30*tan(c/2 + d*x/2)**5/
(14*a**2*d*tan(c/2 + d*x/2)**5 + 14*a**2*d*tan(c/2 + d*x/2)**4 + 28*a**2*d*
tan(c/2 + d*x/2)**3 + 28*a**2*d*tan(c/2 + d*x/2)**2 + 14*a**2*d*tan(c/2 + d
*x/2) + 14*a**2*d) - 40*tan(c/2 + d*x/2)**4/(14*a**2*d*tan(c/2 + d*x/2)**5
+ 14*a**2*d*tan(c/2 + d*x/2)**4 + 28*a**2*d*tan(c/2 + d*x/2)**3 + 28*a**2*d
*tan(c/2 + d*x/2)**2 + 14*a**2*d*tan(c/2 + d*x/2) + 14*a**2*d) - 10*tan(c/2
 + d*x/2)**3/(14*a**2*d*tan(c/2 + d*x/2)**5 + 14*a**2*d*tan(c/2 + d*x/2)**4
 + 28*a**2*d*tan(c/2 + d*x/2)**3 + 28*a**2*d*tan(c/2 + d*x/2)**2 + 14*a**2*
d*tan(c/2 + d*x/2) + 14*a**2*d) - 94*tan(c/2 + d*x/2)**2/(14*a**2*d*tan(c/2
 + d*x/2)**5 + 14*a**2*d*tan(c/2 + d*x/2)**4 + 28*a**2*d*tan(c/2 + d*x/2)**
3 + 28*a**2*d*tan(c/2 + d*x/2)**2 + 14*a**2*d*tan(c/2 + d*x/2) + 14*a**2*d)
 - 12*tan(c/2 + d*x/2)/(14*a**2*d*tan(c/2 + d*x/2)**5 + 14*a**2*d*tan(c/2 +
 d*x/2)**4 + 28*a**2*d*tan(c/2 + d*x/2)**3 + 28*a**2*d*tan(c/2 + d*x/2)**2
+ 14*a**2*d*tan(c/2 + d*x/2) + 14*a**2*d) - 82/(14*a**2*d*tan(c/2 + d*x/2)*
*5 + 14*a**2*d*tan(c/2 + d*x/2)**4 + 28*a**2*d*tan(c/2 + d*x/2)**3 + 28*a**
2*d*tan(c/2 + d*x/2)**2 + 14*a**2*d*tan(c/2 + d*x/2) + 14*a**2*d), Ne(d, 0)
), (x*sin(c)**2*cos(c)**2/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.42527, size = 123, normalized size = 1.78

$$-\frac{\frac{5\,(dx+c)}{a^2}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{8}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)/a^2 + 2*(tan(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^
2 - tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + 8/(a^2
*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.310 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{2x}{a^2}$$

```
[Out] (2*x)/a^2 + Cos[c + d*x]/(a^2*d) + (2*Cos[c + d*x])/(d*(a^2 + a^2*Sin[c + d
*x]))
```

Rubi [A] time = 0.0724305, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2857, 2638}

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + Cos[c + d*x]/(a^2*d) + (2*Cos[c + d*x])/(d*(a^2 + a^2*Sin[c + d
*x]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\int(-2a+a\sin(c+dx))\,dx}{a^3}\\ &= \frac{2x}{a^2}+\frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\int\sin(c+dx)\,dx}{a^2}\\ &= \frac{2x}{a^2}+\frac{\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [B] time = 0.317242, size = 117, normalized size = 2.49

$$\frac{12dx\sin\left(c+\frac{dx}{2}\right)+3\sin\left(2c+\frac{3dx}{2}\right)+2\cos\left(c+\frac{dx}{2}\right)+3\cos\left(c+\frac{3dx}{2}\right)-28\sin\left(\frac{dx}{2}\right)+12dx\cos\left(\frac{dx}{2}\right)}{6a^2d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (12*d*x*Cos[(d*x)/2] + 2*Cos[c + (d*x)/2] + 3*Cos[c + (3*d*x)/2] - 28*Sin[(
d*x)/2] + 12*d*x*Sin[c + (d*x)/2] + 3*Sin[2*c + (3*d*x)/2])/(6*a^2*d*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.081, size = 64, normalized size = 1.4

$$2\,\frac{1}{da^2\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}+4\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2}+4\,\frac{1}{da^2\left(\tan\left(1/2\,dx+c/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)+4/d/a^2*arctan(tan(1/2*d*x+1/2*c))+4/d/a^2
/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.70188, size = 188, normalized size = 4.

$$\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+3}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*((sin(d*x + c)/(cos(d*x + c) + 1) + 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3)/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1) + a^2*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 2*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.66963, size = 201, normalized size = 4.28

$$\frac{2\,dx+(2\,dx+3)\cos\left(dx+c\right)+\cos\left(dx+c\right)^2+\left(2\,dx+\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)+2}{a^2d\cos\left(dx+c\right)+a^2d\sin\left(dx+c\right)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*d*x + (2*d*x + 3)*cos(d*x + c) + cos(d*x + c)^2 + (2*d*x + cos(d*x + c)
- 2)*sin(d*x + c) + 2)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A] time = 14.5296, size = 479, normalized size = 10.19

$$\begin{cases} \frac{2dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2dx\tan\left(\right.}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\right.} \\ \frac{x\sin(c)\cos^2(c)}{(a\sin(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*d*x*tan(c/2 + d*x/2)**3/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*t
an(c/2 + d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d) + 2*d*x*tan(c/2 + d*
x/2)**2/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*t
an(c/2 + d*x/2) + a**2*d) + 2*d*x*tan(c/2 + d*x/2)/(a**2*d*tan(c/2 + d*x/2)
**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d) + 2*d*
x/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*tan(c/2
 + d*x/2) + a**2*d) + 4*tan(c/2 + d*x/2)**2/(a**2*d*tan(c/2 + d*x/2)**3 + a
**2*d*tan(c/2 + d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d) + 2*tan(c/2 +
 d*x/2)/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*t
an(c/2 + d*x/2) + a**2*d) + 6/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2
+ d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d), Ne(d, 0)), (x*sin(c)*cos(c
)**2/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.25503, size = 105, normalized size = 2.23

$$\frac{2\left(\frac{dx+c}{a^2}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((d*x + c)/a^2 + (2*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 3)/((
tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 1)
*a^2))/d
```

3.311 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=40

$$\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x
]))
```

Rubi [A] time = 0.157168, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2874, 2966, 3770, 2648}

$$\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x
]))
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{\int \left(\csc(c+dx) - \frac{2}{1+\sin(c+dx)}\right)dx}{a^2}$$
$$= \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{2\int \frac{1}{1+\sin(c+dx)}\,dx}{a^2}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}$$

Mathematica [B] time = 0.148496, size = 115, normalized size = 2.88

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right.}{a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2]*(Log[Cos[(c +
d*x)/2]] - Log[Sin[(c + d*x)/2]]) + (4 + Log[Cos[(c + d*x)/2]] - Log[Sin[(c
 + d*x)/2]])*Sin[(c + d*x)/2]))/(a^2*d*(1 + Sin[c + d*x])^2))
```

Maple [A] time = 0.128, size = 40, normalized size = 1.

$$4\,\frac{1}{da^2\left(\tan(1/2\,dx+c/2)+1\right)}+\frac{1}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 4/d/a^2/(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.11075, size = 74, normalized size = 1.85

$$\frac{\frac{4}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (4/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1)) + log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^2)/d
```

Fricas [B] time = 1.62081, size = 301, normalized size = 7.52

$$-\frac{(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+\sin(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right.}{2\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c) + sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(
d*x + c) + sin(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) - 4*cos(d*x + c)
+ 4*sin(d*x + c) - 4)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^2(c+dx)\csc(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A] time = 1.34909, size = 51, normalized size = 1.27

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{4}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 4/(a^2*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.312 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d(\csc(c+dx)+1)}$$

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Cot[c + d*x])
/(a^2*d*(1 + Csc[c + d*x]))
```

Rubi [A] time = 0.0962669, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 3770, 3767, 8, 3777}

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d(\csc(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Cot[c + d*x])
/(a^2*d*(1 + Csc[c + d*x]))
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \left(2-2\csc(c+dx)+\csc^2(c+dx)-\frac{2}{1+\csc(c+dx)}\right)dx}{a^2}$$
$$= \frac{2x}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}-\frac{2\int \frac{1}{1+\csc(c+dx)}\,dx}{a^2}$$
$$= \frac{2x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d(1+\csc(c+dx))}+\frac{2\int -1\,dx}{a^2}-\frac{\text{Subst}(\int 1\,dx,x,\cot(}{a^2d}$$
$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d(1+\csc(c+dx))}$$

Mathematica [B] time = 0.761603, size = 216, normalized size = 4.

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{3}{2}(c+dx)\right)\left(-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\,l\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3
*(Cos[(3*(c + d*x))/2]*(5 + 2*Log[Cos[(c + d*x)/2]] - 2*Log[Sin[(c + d*x)/2
]]) + Cos[(c + d*x)/2]*(-3 - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/
2]]) + 2*(-2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + Cos[c + d*x]
*(1 - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]]))*Sin[(c + d*x)/2])
)/(4*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.145, size = 77, normalized size = 1.4

$$\frac{1}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\frac{1}{da^2\left(\tan\left(1/2\,dx+c/2\right)+1\right)}-\frac{1}{2da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-2\frac{\ln\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-4/d/a^2/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^2/tan(1
/2*d*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.16115, size = 157, normalized size = 2.91

$$-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+1}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a^2*sin(d*x + c)/(cos(d*x +
c) + 1) + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 4*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

Fricas [B] time = 1.70917, size = 433, normalized size = 8.02

$$\frac{3 \cos(dx+c)^2 + \left(\cos(dx+c)^2 - (\cos(dx+c)+1)\sin(dx+c) - 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - \left(\cos(dx+c)^2 - (\cos(dx\right.}{a^2 d \cos(dx+c)^2 - a^2 d - \left(a^2 d \cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (3*cos(d*x + c)^2 + (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*
log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x
+ c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + (3*cos(d*x + c) + 2)*sin(d*x + c)
+ cos(d*x + c) - 2)/(a^2*d*cos(d*x + c)^2 - a^2*d - (a^2*d*cos(d*x + c) + a
^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A] time = 1.37124, size = 122, normalized size = 2.26

$$-\frac{\frac{4 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a^2 - (2*
tan(1/2*d*x + 1/2*c)^2 - 7*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)
^2 + tan(1/2*d*x + 1/2*c))*a^2))/d
```

3.313 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=78

$$\frac{2\cot(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*d) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.220133, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2874, 2966, 3770, 3767, 8, 3768, 2648}

$$\frac{2\cot(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*d) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}\\ &= \frac{\int \left(2\csc(c+dx)-2\csc^2(c+dx)+\csc^3(c+dx)-\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}\\ &= \frac{\int \csc^3(c+dx)\,dx}{a^2}+\frac{2\int \csc(c+dx)\,dx}{a^2}-\frac{2\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \frac{1}{1+\sin(c+dx)}\,dx}{a^2}\\ &= -\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}+\frac{\int \csc(c+}{2a^2}\\ &= -\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+d}\end{aligned}$$

Mathematica [B] time = 0.750099, size = 364, normalized size = 4.67

$$-\frac{4\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{d(a\sin(c+dx)+a)^2}-\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}{2d(a\sin(c+dx)+a)^2}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(d*(a + a*Sin
[c + d*x])^2) + (Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/
(d*(a + a*Sin[c + d*x])^2) - (Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^4)/(8*d*(a + a*Sin[c + d*x])^2) - (5*Log[Cos[(c + d*x)/2]]*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(2*d*(a + a*Sin[c + d*x])^2) + (5*Log[
Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(2*d*(a + a*Sin[
c + d*x])^2) + (Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
/(8*d*(a + a*Sin[c + d*x])^2) - ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Ta
n[(c + d*x)/2])/(d*(a + a*Sin[c + d*x])^2)
```

Maple [A] time = 0.16, size = 114, normalized size = 1.5

$$\frac{1}{8da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\,\frac{1}{da^2\left(\tan(1/2\,dx+c/2)+1\right)}-\frac{1}{8da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}+\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)+4/d/a^2/(tan(1/2*
d*x+1/2*c)+1)-1/8/d/a^2/tan(1/2*d*x+1/2*c)^2+1/d/a^2/tan(1/2*d*x+1/2*c)+5/2
/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.19286, size = 217, normalized size = 2.78

$$\frac{\frac{\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}-\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{20\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*((7*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 1)/(a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3) - (8*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(
cos(d*x + c) + 1)^2)/a^2 + 20*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [B] time = 1.703, size = 659, normalized size = 8.45

$$\frac{16\cos(dx+c)^3+10\cos(dx+c)^2-5\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)\cdots\right.}{4(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(16*cos(d*x + c)^3 + 10*cos(d*x + c)^2 - 5*(cos(d*x + c)^3 + cos(d*x +
c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*log(1/2*cos(d*
x + c) + 1/2) + 5*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*s
in(d*x + c) - cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*(8*cos(d*x
 + c)^2 + 3*cos(d*x + c) - 4)*sin(d*x + c) - 14*cos(d*x + c) - 8)/(a^2*d*co
s(d*x + c)^3 + a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d + (a^2*d*c
os(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36502, size = 157, normalized size = 2.01

$$\frac{\frac{20 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}+\frac{32}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(20*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 -
8*a^2*tan(1/2*d*x + 1/2*c))/a^4 + 32/(a^2*(tan(1/2*d*x + 1/2*c) + 1)) - (30
*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/
2*c)^2))/d
```

3.314 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^2*d) - (3*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]
^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d) - (2*Cos[c + d*x])/(a^2*
d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.251215, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2966, 3770, 3767, 8, 3768, 2648}

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^2*d) - (3*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]
^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d) - (2*Cos[c + d*x])/(a^2*
d*(1 + Sin[c + d*x]))
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\csc^4(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{\int \left(-2\csc(c+dx)+2\csc^2(c+dx)-2\csc^3(c+dx)+\csc^4(c+dx)+\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}$$

$$= \frac{\int \csc^4(c+dx)\,dx}{a^2} - \frac{2\int \csc(c+dx)\,dx}{a^2} + \frac{2\int \csc^2(c+dx)\,dx}{a^2} - \frac{2\int \csc^3(c+dx)\,dx}{a^2} + $$

$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{a^2d} - \frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))} - \frac{\int \csc(c+}{a^2}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{3\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{a^2d} - $$

Mathematica [B] time = 1.24768, size = 472, normalized size = 5.19

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^3\left(12\sin\left(\frac{1}{2}(c+dx)\right)-6\sin\left(\frac{3}{2}(c+dx)\right)-2\sin\left(\frac{5}{2}(c+dx)\right)+8\sin\left(\frac{7}{2}(c+dx)\right)-10\,c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^3*(-10*Cos[(5*(c + d*x))/2] + 20*Cos
[(7*(c + d*x))/2] - 9*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*x)/2]] + 9*Cos[(7
*(c + d*x))/2]*Log[Cos[(c + d*x)/2]] + 3*Cos[(c + d*x)/2]*(8 + 9*Log[Cos[(c
 + d*x)/2]] - 9*Log[Sin[(c + d*x)/2]]) - 3*Cos[(3*(c + d*x))/2]*(14 + 9*Log
[Cos[(c + d*x)/2]] - 9*Log[Sin[(c + d*x)/2]]) + 9*Cos[(5*(c + d*x))/2]*Log[
Sin[(c + d*x)/2]] - 9*Cos[(7*(c + d*x))/2]*Log[Sin[(c + d*x)/2]] + 12*Sin[(
c + d*x)/2] + 27*Log[Cos[(c + d*x)/2]]*Sin[(c + d*x)/2] - 27*Log[Sin[(c + d
*x)/2]]*Sin[(c + d*x)/2] - 6*Sin[(3*(c + d*x))/2] + 27*Log[Cos[(c + d*x)/2]
]*Sin[(3*(c + d*x))/2] - 27*Log[Sin[(c + d*x)/2]]*Sin[(3*(c + d*x))/2] - 2*
Sin[(5*(c + d*x))/2] - 9*Log[Cos[(c + d*x)/2]]*Sin[(5*(c + d*x))/2] + 9*Log
[Sin[(c + d*x)/2]]*Sin[(5*(c + d*x))/2] + 8*Sin[(7*(c + d*x))/2] - 9*Log[Co
s[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] + 9*Log[Sin[(c + d*x)/2]]*Sin[(7*(c +
d*x))/2]))/(192*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.171, size = 153, normalized size = 1.7

$$\frac{1}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{11}{8\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\,\frac{1}{da^2\left(\tan\left(1/2\,dx+c/2\right)+1\right)}-\frac{1}{24\,da^2}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^2*tan(1/2*d*x+1/2*c)^2+11/8/d/a^2*t
an(1/2*d*x+1/2*c)-4/d/a^2/(tan(1/2*d*x+1/2*c)+1)-1/24/d/a^2/tan(1/2*d*x+1/2
*c)^3+1/4/d/a^2/tan(1/2*d*x+1/2*c)^2-11/8/d/a^2/tan(1/2*d*x+1/2*c)-3/d/a^2*
ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.19624, size = 269, normalized size = 2.96

$$\frac{\frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{27\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{129\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1}{\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{33\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{72\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((5*sin(d*x + c)/(cos(d*x + c) + 1) - 27*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 129*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a^2*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (33*sin(d*
x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 72*log(sin(d*x + c)/(cos(d*x + c) + 1))
/a^2)/d
```

Fricas [B] time = 1.71775, size = 801, normalized size = 8.8

$$28\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3-42\,\cos\left(dx+c\right)^2+9\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)^3+\cos\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(28*cos(d*x + c)^4 + 10*cos(d*x + c)^3 - 42*cos(d*x + c)^2 + 9*(cos(d*x
 + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c
) - 1)*sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 9*(cos(d*x + c)^4 -
2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sin
(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(14*cos(d*x + c)^3 + 9*cos(
d*x + c)^2 - 12*cos(d*x + c) - 6)*sin(d*x + c) - 12*cos(d*x + c) + 12)/(a^2
*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^3
+ a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34138, size = 197, normalized size = 2.16

$$-\frac{\frac{72 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^2}+\frac{96}{a^2\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}-\frac{132 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-33 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+6 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1}{a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3}-\frac{a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-6 a^4 \tan\left(\frac{1}{2} dx\cdots\right.}{a^6}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(72*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 96/(a^2*(tan(1/2*d*x + 1/2*c
) + 1)) - (132*tan(1/2*d*x + 1/2*c)^3 - 33*tan(1/2*d*x + 1/2*c)^2 + 6*tan(1
/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)^3) - (a^4*tan(1/2*d*x + 1/2*
c)^3 - 6*a^4*tan(1/2*d*x + 1/2*c)^2 + 33*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.315 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{19\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11x}{2a^3}$$

```
[Out] (-11*x)/(2*a^3) - (3*Cos[c + d*x])/(a^3*d) + (Cos[c + d*x]*Sin[c + d*x])/(2
*a^3*d) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x])^2) - (19*Cos[c + d*x
])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.263303, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2874, 2966, 2638, 2635, 8, 2650, 2648}

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{19\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) - (3*Cos[c + d*x])/(a^3*d) + (Cos[c + d*x]*Sin[c + d*x])/(2
*a^3*d) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x])^2) - (19*Cos[c + d*x
])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\sin^3(c+dx)(a-a\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx}{a^2}$$
$$= \frac{\int \left(-\frac{5}{a}+\frac{3\sin(c+dx)}{a}-\frac{\sin^2(c+dx)}{a}-\frac{2}{a(1+\sin(c+dx))^2}+\frac{7}{a(1+\sin(c+dx))}\right)dx}{a^2}$$
$$= -\frac{5x}{a^3}-\frac{\int \sin^2(c+dx)\,dx}{a^3}-\frac{2\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}+\frac{3\int \sin(c+dx)\,dx}{a^3}+\frac{7\int \frac{1}{1+\sin(c+dx)}\,d}{a^3}$$
$$= -\frac{5x}{a^3}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}-\frac{7\cos(c}{a^3d(1+\sin}$$
$$= -\frac{11x}{2a^3}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}-\frac{19\cos}{3a^3d(1+}$$

Mathematica [B] time = 1.05728, size = 197, normalized size = 2.03

$$-\frac{1980dx\sin\left(c+\frac{dx}{2}\right)+660dx\sin\left(c+\frac{3dx}{2}\right)+498\sin\left(2c+\frac{3dx}{2}\right)+135\sin\left(2c+\frac{5dx}{2}\right)+15\sin\left(4c+\frac{7dx}{2}\right)-1326\cos\left(c+\right.}{240a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(1980*d*x*Cos[(d*x)/2] - 1326*Cos[c + (d*x)/2] + 2012*Cos[c + (3*d*x)/2] -
 660*d*x*Cos[2*c + (3*d*x)/2] - 135*Cos[3*c + (5*d*x)/2] + 15*Cos[3*c + (7*
d*x)/2] - 3216*Sin[(d*x)/2] + 1980*d*x*Sin[c + (d*x)/2] + 660*d*x*Sin[c + (
3*d*x)/2] + 498*Sin[2*c + (3*d*x)/2] + 135*Sin[2*c + (5*d*x)/2] + 15*Sin[4*
c + (7*d*x)/2])/(240*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^3)
```

Maple [B] time = 0.115, size = 205, normalized size = 2.1

$$-\frac{1}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-6\,\frac{(\tan(1/2\,dx+c/2))^2}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+\frac{1}{da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*ta
n(1/2*d*x+1/2*c)-6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2-11/d/a^3*arctan(tan(1/2
*d*x+1/2*c))+8/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3-4/d/a^3/(tan(1/2*d*x+1/2*c)
+1)^2-10/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.7791, size = 424, normalized size = 4.37

$$-\frac{\frac{\frac{123 \sin(dx+c)}{\cos(dx+c)+1}+\frac{161 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{210 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{154 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{99 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{33 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+52}{a^3+\frac{3a^3 \sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{7a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{33 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/3*((123*sin(d*x + c)/(cos(d*x + c) + 1) + 161*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 210*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 154*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 99*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 33*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 52)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1
) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 7*a^3*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 7*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 5*a^3*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 3*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a
^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 33*arctan(sin(d*x + c)/(cos(d*x +
 c) + 1))/a^3)/d
```

Fricas [A] time = 1.67553, size = 423, normalized size = 4.36

$$\frac{3 \cos(dx+c)^4 - (33\,dx - 53)\cos(dx+c)^2 - 12 \cos(dx+c)^3 + 66\,dx + (33\,dx + 64)\cos(dx+c) + \left(3 \cos(dx+c)^3 + \right.}{6\left(a^3 d \cos(dx+c)^2 - a^3 d \cos(dx+c) - 2\,a^3 d - \left(a^3 d \cos(dx+c)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*cos(d*x + c)^4 - (33*d*x - 53)*cos(d*x + c)^2 - 12*cos(d*x + c)^3 +
66*d*x + (33*d*x + 64)*cos(d*x + c) + (3*cos(d*x + c)^3 + 66*d*x + (33*d*x
+ 68)*cos(d*x + c) + 15*cos(d*x + c)^2 + 4)*sin(d*x + c) - 4)/(a^3*d*cos(d*
x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)*si
n(d*x + c))
```

Sympy [A] time = 111.301, size = 2263, normalized size = 23.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-33*d*x*tan(c/2 + d*x/2)**7/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a
**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c
/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)
**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 99*d*x*tan(c/2 + d*x/2)**6/(
6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*ta
n(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x
/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**
3*d) - 165*d*x*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*
d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 +
 d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2
+ 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 231*d*x*tan(c/2 + d*x/2)**4/(6*a
**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c
/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)
**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d
) - 231*d*x*tan(c/2 + d*x/2)**3/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*t
an(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*
x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 1
8*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 165*d*x*tan(c/2 + d*x/2)**2/(6*a**3
*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2
+ d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3
 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) -
 99*d*x*tan(c/2 + d*x/2)/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2
+ d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4
 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*
d*tan(c/2 + d*x/2) + 6*a**3*d) - 33*d*x/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*
a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(
c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2
)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) + 22*tan(c/2 + d*x/2)**7/(6*a
**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c
/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)
**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d
) - 88*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/
2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)*
*4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**
3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 154*tan(c/2 + d*x/2)**4/(6*a**3*d*tan(c/
2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)*
*5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**
3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 266*tan(
c/2 + d*x/2)**3/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)*
*6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**
3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2
 + d*x/2) + 6*a**3*d) - 212*tan(c/2 + d*x/2)**2/(6*a**3*d*tan(c/2 + d*x/2)*
*7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**
3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2
 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 180*tan(c/2 + d*x/2
)/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d
*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 +
d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*
a**3*d) - 82/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6
+ 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d
*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 +
d*x/2) + 6*a**3*d), Ne(d, 0)), (x*sin(c)**3*cos(c)**2/(a*sin(c) + a)**3, Tr
ue))
```

Giac [A] time = 1.35208, size = 158, normalized size = 1.63

$$-\frac{\frac{33\,(dx+c)}{a^3}+\frac{6\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}+\frac{4\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)/a^3 + 6*(tan(1/2*d*x + 1/2*c)^3 + 6*tan(1/2*d*x + 1/2*c)
^2 - tan(1/2*d*x + 1/2*c) + 6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 4*(15
*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) + 17)/(a^3*(tan(1/2*d*x +
 1/2*c) + 1)^3))/d
```

3.316 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3x}{a^3}+\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

```
[Out] (3*x)/a^3 + (3*Cos[c + d*x])/(a^3*d) - Cos[c + d*x]^3/(3*d*(a + a*Sin[c + d
*x])^3) + (2*Cos[c + d*x]^3)/(a*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.181297, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2871, 2680, 2682, 8}

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3x}{a^3}+\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*x)/a^3 + (3*Cos[c + d*x])/(a^3*d) - Cos[c + d*x]^3/(3*d*(a + a*Sin[c + d
*x])^3) + (2*Cos[c + d*x]^3)/(a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2871

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] - Dist[1/g^2, Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && EqQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3} - \int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$

$$= -\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3} + \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2} + \frac{3\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{3\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3} + \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2} + \frac{3\int 1\,dx}{a^3}$$

$$= \frac{3x}{a^3} + \frac{3\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3} + \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}$$

Mathematica [A] time = 0.665049, size = 96, normalized size = 1.26

$$\frac{3\cos(c+dx) - \frac{2\sin\left(\frac{1}{2}(c+dx)\right)(13\sin(c+dx)+11)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} + 9c + 9dx}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (9*c + 9*d*x + 3*Cos[c + d*x] - 2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 -
 (2*Sin[(c + d*x)/2]*(11 + 13*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3)/(3*a^3*d)
```

Maple [A] time = 0.104, size = 106, normalized size = 1.4

$$2\,\frac{1}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)} + 6\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^3} - \frac{8}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3} + 4\,\frac{1}{da^3\,(\tan(1/2\,dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)+6/d/a^3*arctan(tan(1/2*d*x+1/2*c))-8/3/d/a
^3/(tan(1/2*d*x+1/2*c)+1)^3+4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+6/d/a^3/(tan(1
/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.79176, size = 308, normalized size = 4.05

$$\frac{2\left(\frac{\frac{33\sin(dx+c)}{\cos(dx+c)+1}+\frac{29\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{27\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+14}{a^3+\frac{3a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}} + \frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2/3*((33*sin(d*x + c)/(cos(d*x + c) + 1) + 29*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 27*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 14)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 4*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.71524, size = 365, normalized size = 4.8

$$\frac{(9\,dx-16)\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)^{3}-18\,dx-(9\,dx+17)\cos\left(dx+c\right)-\left(18\,dx+(9\,dx+19)\cos\left(dx+c\right)+3\,\cos\left(\right.\right.}{3\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\cos\left(dx+c\right)-2\,a^{3}d-\left(a^{3}d\cos\left(dx+c\right)+2\,a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/3*((9*d*x - 16)*cos(d*x + c)^2 + 3*cos(d*x + c)^3 - 18*d*x - (9*d*x + 17)
*cos(d*x + c) - (18*d*x + (9*d*x + 19)*cos(d*x + c) + 3*cos(d*x + c)^2 + 2)
*sin(d*x + c) + 2)/(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (
a^3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c))
```

Sympy [A] time = 60.8184, size = 1246, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*d*x*tan(c/2 + d*x/2)**5/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3
*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2
+ d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 27*d*x*tan(c/2 + d*x/
2)**4/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**
3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2
+ d*x/2) + 3*a**3*d) + 36*d*x*tan(c/2 + d*x/2)**3/(3*a**3*d*tan(c/2 + d*x/2
)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a*
*3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 36*d*x*t
an(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2
)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a*
*3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 27*d*x*tan(c/2 + d*x/2)/(3*a**3*d*tan(c
/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)*
*3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d)
+ 9*d*x/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a
**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/
2 + d*x/2) + 3*a**3*d) + 18*tan(c/2 + d*x/2)**4/(3*a**3*d*tan(c/2 + d*x/2)*
*5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3
*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 54*tan(c/2
 + d*x/2)**3/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 +
 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*t
an(c/2 + d*x/2) + 3*a**3*d) + 58*tan(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*
x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12
*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 66*ta
n(c/2 + d*x/2)/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4
```

```
 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d
*tan(c/2 + d*x/2) + 3*a**3*d) + 28/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d
*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 +
d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d), Ne(d, 0)), (x*sin(c)**2*
cos(c)**2/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.3067, size = 108, normalized size = 1.42

$$\frac{\frac{9\,(dx+c)}{a^3} + \frac{6}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} + \frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(9*(d*x + c)/a^3 + 6/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + 2*(9*tan(1/2*
d*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 11)/(a^3*(tan(1/2*d*x + 1/2*c) +
 1)^3))/d
```

3.317 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=61

$$-\frac{7\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}-\frac{x}{a^3}+\frac{2\cos(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

```
[Out] -(x/a^3) - (7*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x])) + (2*Cos[c + d*x])
/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.108683, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2857, 2735, 2648}

$$-\frac{7\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}-\frac{x}{a^3}+\frac{2\cos(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (7*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x])) + (2*Cos[c + d*x])
/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2}-\frac{\int \frac{-4a+3a\sin(c+dx)}{a+a\sin(c+dx)}\,dx}{3a^3}\\
&= -\frac{x}{a^3}+\frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2}+\frac{7\int \frac{1}{a+a\sin(c+dx)}\,dx}{3a^2}\\
&= -\frac{x}{a^3}+\frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2}-\frac{7\cos(c+dx)}{3d\left(a^3+a^3\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [B] time = 0.400779, size = 145, normalized size = 2.38

$$-\frac{180dx\sin\left(c+\frac{dx}{2}\right)+60dx\sin\left(c+\frac{3dx}{2}\right)+3\sin\left(2c+\frac{3dx}{2}\right)-351\cos\left(c+\frac{dx}{2}\right)+277\cos\left(c+\frac{3dx}{2}\right)-60dx\cos\left(2c+\frac{3}{}\right.}{120a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(180*d*x*Cos[(d*x)/2] - 351*Cos[c + (d*x)/2] + 277*Cos[c + (3*d*x)/2] - 60
*d*x*Cos[2*c + (3*d*x)/2] - 471*Sin[(d*x)/2] + 180*d*x*Sin[c + (d*x)/2] + 6
0*d*x*Sin[c + (3*d*x)/2] + 3*Sin[2*c + (3*d*x)/2])/(120*a^3*d*(Cos[c/2] + S
in[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A] time = 0.101, size = 83, normalized size = 1.4

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^3}+\frac{8}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-4\,\frac{1}{da^3\left(\tan\left(1/2\,dx+c/2\right)+1\right)^2}-2\,\frac{1}{da^3\left(\tan\left(1/2\,dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/d/a^3*arctan(tan(1/2*d*x+1/2*c))+8/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3-4/d/
a^3/(tan(1/2*d*x+1/2*c)+1)^2-2/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.72732, size = 192, normalized size = 3.15

$$-\frac{2\left(\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+5}{a^3+\frac{3a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/3*((12*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 5)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^3*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] time = 1.61444, size = 308, normalized size = 5.05

$$-\frac{(3\,dx-7)\cos\left(dx+c\right)^2-6\,dx-(3\,dx+5)\cos\left(dx+c\right)-(6\,dx+(3\,dx+7)\cos\left(dx+c\right)+2)\sin\left(dx+c\right)+2}{3\left(a^3d\cos\left(dx+c\right)^2-a^3d\cos\left(dx+c\right)-2\,a^3d-\left(a^3d\cos\left(dx+c\right)+2\,a^3d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*((3*d*x - 7)*cos(d*x + c)^2 - 6*d*x - (3*d*x + 5)*cos(d*x + c) - (6*d*
x + (3*d*x + 7)*cos(d*x + c) + 2)*sin(d*x + c) + 2)/(a^3*d*cos(d*x + c)^2 -
 a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c)
)
```

Sympy [A] time = 35.1207, size = 602, normalized size = 9.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-39*d*x*tan(c/2 + d*x/2)**3/(39*a**3*d*tan(c/2 + d*x/2)**3 + 117
*a**3*d*tan(c/2 + d*x/2)**2 + 117*a**3*d*tan(c/2 + d*x/2) + 39*a**3*d) - 11
7*d*x*tan(c/2 + d*x/2)**2/(39*a**3*d*tan(c/2 + d*x/2)**3 + 117*a**3*d*tan(c
/2 + d*x/2)**2 + 117*a**3*d*tan(c/2 + d*x/2) + 39*a**3*d) - 117*d*x*tan(c/2
 + d*x/2)/(39*a**3*d*tan(c/2 + d*x/2)**3 + 117*a**3*d*tan(c/2 + d*x/2)**2 +
 117*a**3*d*tan(c/2 + d*x/2) + 39*a**3*d) - 39*d*x/(39*a**3*d*tan(c/2 + d*x
/2)**3 + 117*a**3*d*tan(c/2 + d*x/2)**2 + 117*a**3*d*tan(c/2 + d*x/2) + 39*
a**3*d) + 36*tan(c/2 + d*x/2)**3/(39*a**3*d*tan(c/2 + d*x/2)**3 + 117*a**3*
d*tan(c/2 + d*x/2)**2 + 117*a**3*d*tan(c/2 + d*x/2) + 39*a**3*d) + 30*tan(c
/2 + d*x/2)**2/(39*a**3*d*tan(c/2 + d*x/2)**3 + 117*a**3*d*tan(c/2 + d*x/2)
**2 + 117*a**3*d*tan(c/2 + d*x/2) + 39*a**3*d) - 204*tan(c/2 + d*x/2)/(39*a
**3*d*tan(c/2 + d*x/2)**3 + 117*a**3*d*tan(c/2 + d*x/2)**2 + 117*a**3*d*tan
(c/2 + d*x/2) + 39*a**3*d) - 94/(39*a**3*d*tan(c/2 + d*x/2)**3 + 117*a**3*d
*tan(c/2 + d*x/2)**2 + 117*a**3*d*tan(c/2 + d*x/2) + 39*a**3*d), Ne(d, 0)),
 (x*sin(c)*cos(c)**2/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.3235, size = 81, normalized size = 1.33

$$-\frac{\frac{3\,(dx+c)}{a^3}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^3 + 2*(3*tan(1/2*d*x + 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*
c) + 5)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.318 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=68

$$\frac{5\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d
*x])^2) + (5*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.193275, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2874, 2966, 3770, 2650, 2648}

$$\frac{5\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d
*x])^2) + (5*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx}{a^2}$$
$$= \frac{\int \left(\frac{\csc(c+dx)}{a} - \frac{2}{a(1+\sin(c+dx))^2} - \frac{1}{a(1+\sin(c+dx))}\right)dx}{a^2}$$
$$= \frac{\int \csc(c+dx)\,dx}{a^3} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} - \frac{2\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2} + \frac{\cos(c+dx)}{a^3d(1+\sin(c+dx))} - \frac{2\int \frac{1}{1+\sin(c+dx)}}{3a^3}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2} + \frac{5\cos(c+dx)}{3a^3d(1+\sin(c+dx))}$$

Mathematica [B] time = 0.37831, size = 185, normalized size = 2.72

$$\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)^3\left(-4\sin\left(\tfrac{1}{2}(c+dx)\right)-10\sin\left(\tfrac{1}{2}(c+dx)\right)\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)^2+2\left(s\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-4*Sin[(c + d*x)/2] + 2*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]) - 10*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 - 3*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3 + 3*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/
(3*d*(a + a*Sin[c + d*x])^3)
```

Maple [A] time = 0.147, size = 82, normalized size = 1.2

$$\frac{8}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-4\,\frac{1}{da^3\,(\tan(1/2\,dx+c/2)+1)^2}+6\,\frac{1}{da^3\,(\tan(1/2\,dx+c/2)+1)}+\frac{1}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 8/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3-4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+6/d/a^3
/(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.16581, size = 193, normalized size = 2.84

$$\frac{\frac{2\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+7\right)}{a^3+\frac{3a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(2*(12*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 7)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^3*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*l
og(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] time = 1.71214, size = 528, normalized size = 7.76

$$-\frac{10\cos(dx+c)^2+3\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3(}{6\left(a^3d\cos(dx+c)^2-a^3d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x +
c) - cos(d*x + c) - 2)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 - (c
os(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)*log(-1/2*cos(d*x + c) + 1
/2) + 2*(5*cos(d*x + c) - 2)*sin(d*x + c) + 14*cos(d*x + c) + 4)/(a^3*d*cos
(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)
*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^2(c+dx)\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A] time = 1.32223, size = 89, normalized size = 1.31

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(9*tan(1/2*d*x + 1/2*c)^2 + 1
2*tan(1/2*d*x + 1/2*c) + 7)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.319 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13\cot(c+dx)}{3a^3d(\csc(c+dx)+1)}+\frac{2\cot(c+dx)}{3a^3d(\csc(c+dx)+1)^2}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) + (2*Cot[c + d*x])
/(3*a^3*d*(1 + Csc[c + d*x])^2) - (13*Cot[c + d*x])/(3*a^3*d*(1 + Csc[c + d
*x]))
```

Rubi [A] time = 0.207704, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 3777, 3919, 3794}

$$-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13\cot(c+dx)}{3a^3d(\csc(c+dx)+1)}+\frac{2\cot(c+dx)}{3a^3d(\csc(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) + (2*Cot[c + d*x])
/(3*a^3*d*(1 + Csc[c + d*x])^2) - (13*Cot[c + d*x])/(3*a^3*d*(1 + Csc[c + d
*x]))
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
```

```
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(\frac{5}{a}-\frac{3\csc(c+dx)}{a}+\frac{\csc^2(c+dx)}{a}+\frac{2}{a(1+\csc(c+dx))^2}-\frac{7}{a(1+\csc(c+dx))}\right)dx}{a^2}$$

$$=\frac{5x}{a^3}+\frac{\int \csc^2(c+dx)\,dx}{a^3}+\frac{2\int \frac{1}{(1+\csc(c+dx))^2}\,dx}{a^3}-\frac{3\int \csc(c+dx)\,dx}{a^3}-\frac{7\int \frac{1}{1+\csc(c+dx)}\,dx}{a^3}$$

$$=\frac{5x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-\frac{7\cot(c+dx)}{a^3d(1+\csc(c+dx))}-\frac{2\int \frac{-3+c}{1+cs}}{3}$$

$$=\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-\frac{7\cot(c+dx)}{a^3d(1+\csc(c+dx))}-$$

$$=\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-\frac{13\cot(c+dx)}{3a^3d(1+\csc(c+dx))}$$

Mathematica [B] time = 1.47682, size = 255, normalized size = 3.11

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(8\sin\left(\frac{1}{2}(c+dx)\right)+44\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(8*Sin[(c + d*x)/2] - 4*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) + 44*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2 - 3*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3
 + 18*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 18*Lo
g[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 3*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^3*Tan[(c + d*x)/2]))/(6*d*(a + a*Sin[c + d*x])^3
)
```

Maple [A] time = 0.166, size = 119, normalized size = 1.5

$$\frac{1}{2da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{3da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+4\frac{1}{da^3\left(\tan(1/2\,dx+c/2)+1\right)^2}-10\frac{1}{da^3\left(\tan(1/2\,dx+c/2)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-8/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3+4/d/a^3/(ta
n(1/2*d*x+1/2*c)+1)^2-10/d/a^3/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^3/tan(1/2*d*x
+1/2*c)-3/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.03531, size = 273, normalized size = 3.33

$$-\frac{\frac{\frac{61\sin(dx+c)}{\cos(dx+c)+1}+\frac{105\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{63\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+3}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{18\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{3\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*((61*sin(d*x + c)/(cos(d*x + c) + 1) + 105*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3)/(a^3*sin(d*x + c)/(c
os(d*x + c) + 1) + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) +
18*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 3*sin(d*x + c)/(a^3*(cos(d*x
+ c) + 1)))/d
```

Fricas [B] time = 1.73355, size = 745, normalized size = 9.09

$$-\frac{28\cos(dx+c)^3-10\cos(dx+c)^2-9\left(\cos(dx+c)^3+2\cos(dx+c)^2+\left(\cos(dx+c)^2-\cos(dx+c)-2\right)\sin(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(28*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 9*(cos(d*x + c)^3 + 2*cos(d*x
 + c)^2 + (cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) - cos(d*x + c) -
 2)*log(1/2*cos(d*x + c) + 1/2) + 9*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + (c
os(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) - cos(d*x + c) - 2)*log(-1/2
*cos(d*x + c) + 1/2) - 2*(14*cos(d*x + c)^2 + 19*cos(d*x + c) + 2)*sin(d*x
+ c) - 34*cos(d*x + c) + 4)/(a^3*d*cos(d*x + c)^3 + 2*a^3*d*cos(d*x + c)^2
- a^3*d*cos(d*x + c) - 2*a^3*d + (a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c)
 - 2*a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A] time = 1.44403, size = 147, normalized size = 1.79

$$-\frac{\frac{18 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{3\left(6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{4\left(15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(18*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^3 -
3*(6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(15*tan(1/2*d
*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 13)/(a^3*(tan(1/2*d*x + 1/2*c) +
1)^3))/d
```

3.320 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=106

$$\frac{3\cot(c+dx)}{a^3d}+\frac{17\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c +
 d*x]*Csc[c + d*x])/(2*a^3*d) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]
)^2) + (17*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.268578, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2874, 2966, 3770, 3767, 8, 3768, 2650, 2648}

$$\frac{3\cot(c+dx)}{a^3d}+\frac{17\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c +
 d*x]*Csc[c + d*x])/(2*a^3*d) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]
)^2) + (17*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx}{a^2}$$

$$= \frac{\int \left(\frac{5\csc(c+dx)}{a} - \frac{3\csc^2(c+dx)}{a} + \frac{\csc^3(c+dx)}{a} - \frac{2}{a(1+\sin(c+dx))^2} - \frac{5}{a(1+\sin(c+dx))}\right)dx}{a^2}$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^3} - \frac{2\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} - \frac{3\int \csc^2(c+dx)\,dx}{a^3} + \frac{5\int \csc(c+dx)\,dx}{a^3}$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2} + \frac{5}{a^3d(1}$$

$$= -\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{3\cot(c+dx)}{a^3d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2\cos(c+}{3a^3d(1+\sin(}$$

Mathematica [B] time = 5.6316, size = 308, normalized size = 2.91

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-32\sin\left(\frac{1}{2}(c+dx)\right)-272\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-32*Sin[(c + d*x)/2] - 3*(1 + Cot
[(c + d*x)/2])^3*Sin[(c + d*x)/2] + 16*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
) - 272*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 36*Cot[(
c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 132*Log[Cos[(c + d*x)
/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 132*Log[Sin[(c + d*x)/2]]*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 36*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3*Tan[(c + d*x)/2] + 3*Cos[(c + d*x)/2]*(1 + Tan[(c + d*x)/2])^3))/(2
4*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.178, size = 157, normalized size = 1.5

$$\frac{1}{8\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{3}{2\,da^{3}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8}{3\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-4\,\frac{1}{da^{3}\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{2}}+14\,\frac{}{da^{3}\,(\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^3*tan(1/2*d*x+1/2*c)+8/3/d/a^3/(tan(
1/2*d*x+1/2*c)+1)^3-4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+14/d/a^3/(tan(1/2*d*x+
1/2*c)+1)-1/8/d/a^3/tan(1/2*d*x+1/2*c)^2+3/2/d/a^3/tan(1/2*d*x+1/2*c)+11/2/
d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.21528, size = 333, normalized size = 3.14

$$\frac{\frac{\frac{27\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{403\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{681\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{372\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-3}{\frac{a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}-\frac{3\left(\frac{12\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{a^{3}}+\frac{132\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((27*sin(d*x + c)/(cos(d*x + c) + 1) + 403*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 681*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 372*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 3)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) - 3*(12*sin(d*x + c)/(cos(d*x
+ c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 132*log(sin(d*x + c)
/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] time = 1.78354, size = 969, normalized size = 9.14

$$-\frac{104\,\cos(dx+c)^{4}+142\,\cos(dx+c)^{3}-90\,\cos(dx+c)^{2}+33\left(\cos(dx+c)^{4}-\cos(dx+c)^{3}-3\,\cos(dx+c)^{2}-\left(\cos(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(104*cos(d*x + c)^4 + 142*cos(d*x + c)^3 - 90*cos(d*x + c)^2 + 33*(co
s(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 - (cos(d*x + c)^3 + 2*cos(
d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(1/2*cos
(d*x + c) + 1/2) - 33*(cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 -
 (cos(d*x + c)^3 + 2*cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) + cos(
d*x + c) + 2)*log(-1/2*cos(d*x + c) + 1/2) + 2*(52*cos(d*x + c)^3 - 19*cos(
d*x + c)^2 - 64*cos(d*x + c) + 4)*sin(d*x + c) - 136*cos(d*x + c) - 8)/(a^3
```

```
*d*cos(d*x + c)^4 - a^3*d*cos(d*x + c)^3 - 3*a^3*d*cos(d*x + c)^2 + a^3*d*c
os(d*x + c) + 2*a^3*d - (a^3*d*cos(d*x + c)^3 + 2*a^3*d*cos(d*x + c)^2 - a^
3*d*cos(d*x + c) - 2*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42917, size = 193, normalized size = 1.82

$$\frac{\frac{132 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{3\left(66 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 12 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} + \frac{3\left(a^3 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 12 a^3 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^6} + \frac{16\left(21 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 3\cdots\right.}{a^3\left(\tan\left(\frac{1}{2}dx\cdots\right.\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(132*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*(66*tan(1/2*d*x + 1/2*c)^2
 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^2) + 3*(a^3*tan(1
/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16*(21*tan(1/2*d*x +
 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) + 19)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^3
))/d
```

3.321 $\int \frac{\cos^2(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx$

Optimal. Leaf size=144

$$\frac{4\cos(e+fx)}{315f\left(a^6\sin(e+fx)+a^6\right)}+\frac{4\cos(e+fx)}{315f\left(a^3\sin(e+fx)+a^3\right)^2}+\frac{2\cos(e+fx)}{105f\left(a^2\sin(e+fx)+a^2\right)^3}-\frac{19\cos(e+fx)}{63a^2f(a\sin(e+fx)+a)^4}+\frac{\cdots}{9\cdots}$$

```
[Out] (2*Cos[e + f*x])/(9*a*f*(a + a*Sin[e + f*x])^5) - (19*Cos[e + f*x])/(63*a^2
*f*(a + a*Sin[e + f*x])^4) + (2*Cos[e + f*x])/(105*f*(a^2 + a^2*Sin[e + f*x
])^3) + (4*Cos[e + f*x])/(315*f*(a^3 + a^3*Sin[e + f*x])^2) + (4*Cos[e + f*
x])/(315*f*(a^6 + a^6*Sin[e + f*x]))
```

Rubi [A] time = 0.162312, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2857, 2750, 2650, 2648}

$$\frac{4\cos(e+fx)}{315f\left(a^6\sin(e+fx)+a^6\right)}+\frac{4\cos(e+fx)}{315f\left(a^3\sin(e+fx)+a^3\right)^2}+\frac{2\cos(e+fx)}{105f\left(a^2\sin(e+fx)+a^2\right)^3}-\frac{19\cos(e+fx)}{63a^2f(a\sin(e+fx)+a)^4}+\frac{\cdots}{9\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] (2*Cos[e + f*x])/(9*a*f*(a + a*Sin[e + f*x])^5) - (19*Cos[e + f*x])/(63*a^2
*f*(a + a*Sin[e + f*x])^4) + (2*Cos[e + f*x])/(105*f*(a^2 + a^2*Sin[e + f*x
])^3) + (4*Cos[e + f*x])/(315*f*(a^3 + a^3*Sin[e + f*x])^2) + (4*Cos[e + f*
x])/(315*f*(a^6 + a^6*Sin[e + f*x]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx = \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{\int \frac{-10a+9a\sin(e+fx)}{(a+a\sin(e+fx))^4}\,dx}{9a^3}$$
$$= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4} - \frac{2\int \frac{1}{(a+a\sin(e+fx))^3}\,dx}{21a^3}$$
$$= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4} + \frac{2\cos(e+fx)}{105f\left(a^2+a^2\sin(e+fx)\right)^3}$$
$$= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4} + \frac{2\cos(e+fx)}{105f\left(a^2+a^2\sin(e+fx)\right)^3}$$
$$= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5} - \frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4} + \frac{2\cos(e+fx)}{105f\left(a^2+a^2\sin(e+fx)\right)^3}$$

Mathematica [A] time = 0.920331, size = 171, normalized size = 1.19

$$-\frac{2562\sin\left(2e+\frac{3fx}{2}\right)-900\sin\left(2e+\frac{5fx}{2}\right)-27\sin\left(4e+\frac{7fx}{2}\right)+25\sin\left(4e+\frac{9fx}{2}\right)+378\cos\left(e+\frac{fx}{2}\right)+210\cos\left(e+\frac{3}{}\right.}{13860a^6f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\text{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] -(378*Cos[e + (f*x)/2] + 210*Cos[e + (3*f*x)/2] - 108*Cos[3*e + (5*f*x)/2]
+ 225*Cos[3*e + (7*f*x)/2] + 3*Cos[5*e + (9*f*x)/2] + 3150*Sin[(f*x)/2] + 2
562*Sin[2*e + (3*f*x)/2] - 900*Sin[2*e + (5*f*x)/2] - 27*Sin[4*e + (7*f*x)/
2] + 25*Sin[4*e + (9*f*x)/2])/(13860*a^6*f*(Cos[e/2] + Sin[e/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^9)
```

Maple [A] time = 0.132, size = 130, normalized size = 0.9

$$4\frac{1}{fa^6}\left(\frac{16}{9\left(\tan\left(1/2\,fx+e/2\right)+1\right)^9}-9\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-4}-8\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-8}+\frac{116}{7\left(\tan\left(1/2\,fx+e\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x)
```

```
[Out] 4/f/a^6*(16/9/(tan(1/2*f*x+1/2*e)+1)^9-9/(tan(1/2*f*x+1/2*e)+1)^4-8/(tan(1/
2*f*x+1/2*e)+1)^8+116/7/(tan(1/2*f*x+1/2*e)+1)^7-62/3/(tan(1/2*f*x+1/2*e)+1
)^6-1/2/(tan(1/2*f*x+1/2*e)+1)^2+84/5/(tan(1/2*f*x+1/2*e)+1)^5+3/(tan(1/2*f
*x+1/2*e)+1)^3)
```

Maxima [B] time = 1.13211, size = 479, normalized size = 3.33

$$-\frac{2\left(\frac{99\sin(fx+e)}{\cos(fx+e)+1}+\frac{81\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{609\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{441\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{945\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{315\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{315\,\mathrm{s}}{(\cos(}\right.}{315\left(a^6+\frac{9a^6\sin(fx+e)}{\cos(fx+e)+1}+\frac{36a^6\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{84a^6\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{126a^6\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{126a^6\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{84a^6\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{36a^6\sin(fx+e)^7}{(\cos(fx+e)+1)^7}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/315*(99*sin(f*x + e)/(cos(f*x + e) + 1) + 81*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 609*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 441*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 945*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 315*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 315*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11)/
((a^6 + 9*a^6*sin(f*x + e)/(cos(f*x + e) + 1) + 36*a^6*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 84*a^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*a^6*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 126*a^6*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 84*a^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 36*a^6*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7 + 9*a^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^6*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9)*f)
```

Fricas [A] time = 1.65385, size = 624, normalized size = 4.33

$$\frac{4\cos(fx+e)^5-16\cos(fx+e)^4-50\cos(fx+e)^3-65\cos(fx+e)^2-\left(4\cos(fx+e)^4+20\cos\right.}{315\left(a^6f\cos(fx+e)^5+5a^6f\cos(fx+e)^4-8a^6f\cos(fx+e)^3-20a^6f\cos(fx+e)^2+8a^6f\cos(fx+e)+16a^6f+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/315*(4*cos(f*x + e)^5 - 16*cos(f*x + e)^4 - 50*cos(f*x + e)^3 - 65*cos(f*
x + e)^2 - (4*cos(f*x + e)^4 + 20*cos(f*x + e)^3 - 30*cos(f*x + e)^2 + 35*c
os(f*x + e) + 70)*sin(f*x + e) + 35*cos(f*x + e) + 70)/(a^6*f*cos(f*x + e)^
5 + 5*a^6*f*cos(f*x + e)^4 - 8*a^6*f*cos(f*x + e)^3 - 20*a^6*f*cos(f*x + e)
^2 + 8*a^6*f*cos(f*x + e) + 16*a^6*f + (a^6*f*cos(f*x + e)^4 - 4*a^6*f*cos(
f*x + e)^3 - 12*a^6*f*cos(f*x + e)^2 + 8*a^6*f*cos(f*x + e) + 16*a^6*f)*sin
(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*sin(f*x+e)/(a+a*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32431, size = 162, normalized size = 1.12

$$-\frac{2\left(315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}+945\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+441\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+609\tan\left(\frac{1}{2}fx\right.\right.}{315\,a^{6}f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/315*(315*tan(1/2*f*x + 1/2*e)^7 + 315*tan(1/2*f*x + 1/2*e)^6 + 945*tan(1
/2*f*x + 1/2*e)^5 + 441*tan(1/2*f*x + 1/2*e)^4 + 609*tan(1/2*f*x + 1/2*e)^3
 + 81*tan(1/2*f*x + 1/2*e)^2 + 99*tan(1/2*f*x + 1/2*e) + 11)/(a^6*f*(tan(1/
2*f*x + 1/2*e) + 1)^9)
```

3.322 $\int \cos^2(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=193

$$\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}+\frac{2a\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}-\frac{76\cos(c+d$$

```
[Out] (-76*a*Cos[c + d*x])/(495*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(693*d*Sqrt[a + a*Sin[c + d*x]]) + (2*a*Cos[c + d*x]*Sin[c
+ d*x]^4)/(99*d*Sqrt[a + a*Sin[c + d*x]]) + (152*Cos[c + d*x]*Sqrt[a + a*Si
n[c + d*x]])/(3465*d) + (2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d
*x]])/(11*d) - (76*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(1155*a*d)
```

Rubi [A] time = 0.568302, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2879, 2976, 2981, 2770, 2759, 2751, 2646}

$$\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}+\frac{2a\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}-\frac{76\cos(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-76*a*Cos[c + d*x])/(495*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(693*d*Sqrt[a + a*Sin[c + d*x]]) + (2*a*Cos[c + d*x]*Sin[c
+ d*x]^4)/(99*d*Sqrt[a + a*Sin[c + d*x]]) + (152*Cos[c + d*x]*Sqrt[a + a*Si
n[c + d*x]])/(3465*d) + (2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d
*x]])/(11*d) - (76*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(1155*a*d)
```

Rule 2879

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{2\int \sin^3(c+dx)\sqrt{}}{}$$

$$= \frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\,\text{si}}}{11d}$$

$$= -\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} + \frac{2}{}$$

$$= -\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} + \frac{2}{}$$

$$= -\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} + \frac{15}{}$$

$$= -\frac{76a\cos(c+dx)}{495d\sqrt{a+a\sin(c+dx)}} - \frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\,\text{co}}{99}$$

Mathematica [A] time = 1.19717, size = 109, normalized size = 0.56

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(7638\sin(c+dx)-1330\sin(3(c+dx))-3540\cos(2(c+dx))}{13860d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(5657
- 3540*Cos[2*(c + d*x)] + 315*Cos[4*(c + d*x)] + 7638*Sin[c + d*x] - 1330*S
in[3*(c + d*x)]))/(13860*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.774, size = 85, normalized size = 0.4

$$-\frac{(2+2\sin(dx+c))a(\sin(dx+c)-1)^2\left(315(\sin(dx+c))^4+665(\sin(dx+c))^3+570(\sin(dx+c))^2+456\sin(dx+\right.}{3465d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3465*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(315*sin(d*x+c)^4+665*sin(d*x+c)^
3+570*sin(d*x+c)^2+456*sin(d*x+c)+304)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin(dx+c)+a}\cos(dx+c)^2\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c)^3, x)
```

Fricas [A] time = 1.69922, size = 432, normalized size = 2.24

$$\frac{2\left(315\cos(dx+c)^6+350\cos(dx+c)^5-500\cos(dx+c)^4-586\cos(dx+c)^3+17\cos(dx+c)^2+\left(315\cos(dx+c)^5\right.\right.}{3465(d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/3465*(315*cos(d*x + c)^6 + 350*cos(d*x + c)^5 - 500*cos(d*x + c)^4 - 586*
cos(d*x + c)^3 + 17*cos(d*x + c)^2 + (315*cos(d*x + c)^5 - 35*cos(d*x + c)^
4 - 535*cos(d*x + c)^3 + 51*cos(d*x + c)^2 + 68*cos(d*x + c) + 136)*sin(d*x
```

```
 + c) - 68*cos(d*x + c) - 136)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d
*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^2\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c)^3, x)
```

3.323 $\int \cos^2(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{8a^2\cos^3(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9ad}+\frac{4\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3(c+dx)}{21d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*
x]^3)/(21*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]^3*Sqrt[a + a*Sin[c
+ d*x]])/(21*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(9*a*d)
```

Rubi [A] time = 0.358043, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2878, 2856, 2674, 2673}

$$-\frac{8a^2\cos^3(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9ad}+\frac{4\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3(c+dx)}{21d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*
x]^3)/(21*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]^3*Sqrt[a + a*Sin[c
+ d*x]])/(21*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(9*a*d)
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9ad} + \frac{2\int \cos^2(c+dx)\left(\frac{3a}{2} - 3a\right.\cdots}{\cdots}$$
$$= \frac{4\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \frac{2\cos^3(c+dx)(a+a\sin(c+\cdots}{9ad}$$
$$= -\frac{2a\cos^3(c+dx)}{21d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \cdots$$
$$= -\frac{8a^2\cos^3(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^3(c+dx)}{21d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos^3(c+\cdots}{\cdots}$$

Mathematica [A] time = 0.590976, size = 99, normalized size = 0.8

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(-69\sin(c+dx)+7\sin(3(c+dx))+30\cos(2(c+dx))-62\cdots}{126d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(-62 +
30*Cos[2*(c + d*x)] - 69*Sin[c + d*x] + 7*Sin[3*(c + d*x)]))/(126*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.771, size = 75, normalized size = 0.6

$$-\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)^2\left(7(\sin(dx+c))^3+15(\sin(dx+c))^2+12\sin(dx+c)+8\right)}{63\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/63*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(7*sin(d*x+c)^3+15*sin(d*x+c)^2+12*
sin(d*x+c)+8)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^2\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c)^2, x)
```

Fricas [A] time = 1.59411, size = 346, normalized size = 2.79

$$\frac{2\left(7\cos\left(dx+c\right)^{5}-\cos\left(dx+c\right)^{4}-11\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\left(7\cos\left(dx+c\right)^{4}+8\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)^{2}\right.}{63\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/63*(7*cos(d*x + c)^5 - cos(d*x + c)^4 - 11*cos(d*x + c)^3 + cos(d*x + c)^
2 - (7*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 4*cos(d*x + c
) - 8)*sin(d*x + c) - 4*cos(d*x + c) - 8)*sqrt(a*sin(d*x + c) + a)/(d*cos(d
*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**2*cos(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c)^2, x)
```

3.324 $\int \cos^2(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=92

$$-\frac{8a^2\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{2a\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d
*x]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + a*Sin[c
 + d*x]])/(7*d)
```

Rubi [A] time = 0.18834, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$-\frac{8a^2\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{2a\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d
*x]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + a*Sin[c
 + d*x]])/(7*d)
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d} + \frac{1}{7}\int \cos^2(c+dx)\sqrt{a+a\sin(c+\cdots}$$
$$= -\frac{2a\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d} + \frac{1}{35}(4a\cdots$$
$$= -\frac{8a^2\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx\cdots}{\cdots}$$

Mathematica [A] time = 0.402999, size = 89, normalized size = 0.97

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(66\sin(c+dx)-15\cos(2(c+dx))+59)}{105d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(59 -
15*Cos[2*(c + d*x)] + 66*Sin[c + d*x]))/(105*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]))
```

Maple [A] time = 0.688, size = 65, normalized size = 0.7

$$-\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)^2\left(15\,(\sin(dx+c))^2+33\sin(dx+c)+22\right)}{105\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/105*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(15*sin(d*x+c)^2+33*sin(d*x+c)+22)
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\,\cos(dx+c)^2\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c), x)
```

Fricas [A] time = 1.63686, size = 302, normalized size = 3.28

$$-\frac{2\left(15\cos(dx+c)^4+18\cos(dx+c)^3-\cos(dx+c)^2+\left(15\cos(dx+c)^3-3\cos(dx+c)^2-4\cos(dx+c)-8\right)\sin(dx\cdots}{105\,(d\cos(dx+c)+d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/105*(15*cos(d*x + c)^4 + 18*cos(d*x + c)^3 - cos(d*x + c)^2 + (15*cos(d*
x + c)^3 - 3*cos(d*x + c)^2 - 4*cos(d*x + c) - 8)*sin(d*x + c) + 4*cos(d*x
+ c) + 8)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)*cos(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c), x)
```

3.325 $\int \cos(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.339329, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2874, 2976, 2981, 2773, 206}

$$\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(3*d)
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \csc(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{2\int \csc(c+dx)\sqrt{a+a\sin(c+d}{3a^2}$$

$$= \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \int \csc($$

$$= \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{(2a)\,\mathrm{Su}}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+}{}$$

Mathematica [A] time = 0.213736, size = 143, normalized size = 1.54

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)-3\log\left(-\sin\left(\frac{1}{2}(c+\right.\right.}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(3*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2] - 3*
Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Log[1 - Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*d*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.801, size = 103, normalized size = 1.1

$$-\frac{2+2\sin(dx+c)}{3ad\cos(dx+c)}\sqrt{-a(\sin(dx+c)-1)}\left(3a^{3/2}\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)+(a-a\sin(dx+c))^{\frac{3}{2}}-3a\sqrt{a-a\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*a^(3/2)*arctanh((a-a*sin(d
*x+c))^(1/2)/a^(1/2))+(a-a*sin(d*x+c))^(3/2)-3*a*(a-a*sin(d*x+c))^(1/2))/a/
cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^2\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c), x)
```

Fricas [B] time = 1.68882, size = 693, normalized size = 7.45

$$\frac{3\sqrt{a}(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-}\right.}{6(d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4
*(cos(d*x + c)^2 + (cos(d*x + c) - 1)*sin(d*x + c) + 2*cos(d*x + c) + 1)*sq
rt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sin(c+dx)+1)}\cos^2(c+dx)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2*csc(c + d*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.326 $\int \cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{3a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) + (
3*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Sqrt[a + a*S
in[c + d*x]])/d
```

Rubi [A] time = 0.198787, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2716, 2981, 2773, 206}

$$\frac{3a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) + (
3*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Sqrt[a + a*S
in[c + d*x]])/d
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} + \frac{\int \csc(c+dx)\left(\frac{a}{2}-\frac{3}{2}a\sin(c+dx)\right)\sqrt{a+a\,s}}{a}$$
$$= \frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} + \frac{1}{2}\int \csc(c+dx)\sqrt{a+}$$
$$= \frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x\right.}{d}$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c}}{d}$$

Mathematica [B] time = 0.943243, size = 206, normalized size = 2.31

$$\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{3}{2}(c+dx)\right)-4\cos\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)-s\right.}{d\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(-4*Cos[(c + d*x)/2] + 2*Cos
[(3*(c + d*x))/2] + 4*Sin[(c + d*x)/2] - Log[1 + Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*Sin[c + d*x] + Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
c + d*x] + 2*Sin[(3*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x
)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*x)/4]))
```

Maple [A] time = 0.865, size = 125, normalized size = 1.4

$$\frac{1+\sin(dx+c)}{\cos(dx+c)\sin(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(2\sqrt{a-a\sin(dx+c)}a^{3/2}-\mathrm{Artanh}\left(\sqrt{a-a\sin(dx+c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] (1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(2*(a-a*sin(d*x+c))^(1
/2)*a^(3/2)-arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^2)-(a-a*sin(d*x+c))^(
1/2)*a^(3/2))/sin(d*x+c)/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^2\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c)^2, x)
```

Fricas [B] time = 1.77692, size = 749, normalized size = 8.42

$$\frac{\left(\cos\left(dx+c\right)^{2} - \left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right) - 1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3} - 7\,a\cos(dx+c)^{2} - 4\left(\cos(dx+c)^{2} + (\cos(dx+c)+3)\sin(dx+c) - 2\,\cos(dx\cdots\right.}{\cos(dx+c)^{3} + \cos(dx+c)^{2} + \left(\cos\cdots\right.}\right.}{4\left(d\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) - 4*(2*cos(d*x + c)^2 + (2*cos(d*x + c) + 3)*sin(d*x + c) -
 cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2 - (d*cos(d*x
 + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.327 $\int \cot^2(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

```
[Out] (5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
- (a*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d
*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rubi [A] time = 0.397162, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2874, 2975, 2980, 2773, 206}

$$-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
- (a*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d
*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{\int \csc^2(c+dx)\sqrt{a+a\,\mathrm{s}}$$

$$= -\frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}$$

$$= -\frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}$$

$$= \frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \cot(c+dx)\,\mathrm{cs}$$

Mathematica [B] time = 0.74775, size = 249, normalized size = 2.47

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+6\sin\left(\frac{3}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+6\cos\left(\frac{3}{2}(c+dx)\right)+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(c + d*x)/2] + 6*Cos
[(3*(c + d*x))/2] - 5*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 5*Cos[
2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 5*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 5*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2] + 6*Sin[(3*(c + d*x))/2]))/(4*d
*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

Maple [A] time = 0.939, size = 126, normalized size = 1.3

$$-\frac{1+\sin(dx+c)}{4(\sin(dx+c))^2\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(-5\,\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin(dx+c))^2a^2+5\sqrt{-a(\mathrm{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-5*arctanh((-a*(sin(d*x+c)-1
))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+5*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)-3*(-
a*(sin(d*x+c)-1))^(3/2)*a^(1/2))/a^(3/2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\csc\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c)^3, x)
```

Fricas [B] time = 1.7461, size = 853, normalized size = 8.45

$$\frac{5\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\,(\cdots}{\cdots}\right.}{16\left(d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(5*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2
+ (3*cos(d*x + c) + 1)*sin(d*x + c) + 2*cos(d*x + c) - 1)*sqrt(a*sin(d*x +
c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x
 + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.328 $\int \cot^2(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{3a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{a\cot(c+dx)\csc}{12d\sqrt{a\sin(c+}}$$

```
[Out] (3*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
+ (3*a*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c
 + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqr
t[a + a*Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.479941, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2874, 2975, 2980, 2772, 2773, 206}

$$\frac{3a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{a\cot(c+dx)\csc}{12d\sqrt{a\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
+ (3*a*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c
 + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqr
t[a + a*Sin[c + d*x]])/(3*d)
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
```

```
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{\int \csc^3(c+dx)\sqrt{a+ \cdots}}{\cdots}$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+ \cdots}}{3d}$$

$$= \frac{3a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^{\cdots}}{\cdots}$$

$$= \frac{3a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^{\cdots}}{\cdots}$$

$$= \frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} + \frac{3a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\cdots}{12d\sqrt{a+a\,\mathrm{s}\cdots}}$$

Mathematica [B] time = 1.35948, size = 285, normalized size = 2.08

$$-\frac{\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(12\sin\left(\frac{1}{2}(c+dx)\right)+58\sin\left(\frac{3}{2}(c+dx)\right)-18\sin\left(\frac{5}{2}(c+dx)\right)-12\cos\left(\frac{1}{2}(c+dx)\right)\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-12*Cos[(c + d*x)/2] + 58
*Cos[(3*(c + d*x))/2] + 18*Cos[(5*(c + d*x))/2] + 12*Sin[(c + d*x)/2] - 27*
```

```
Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 27*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 58*Sin[(3*(c + d*x))/2] - 1
8*Sin[(5*(c + d*x))/2] + 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] - 9*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d
*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^
2)^3)
```

Maple [A] time = 1.122, size = 144, normalized size = 1.1

$$-\frac{1+\sin(dx+c)}{24\,(\sin(dx+c))^3\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(9\sqrt{-a(\sin(dx+c)-1)}a^{7/2}+8\,(-a(\sin(dx+c)-1))^{3/2}a^{5/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(9*(-a*(sin(d*x+c)-1))^(1/2)
*a^(7/2)+8*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)-9*(-a*(sin(d*x+c)-1))^(5/2)*a^
(3/2)-9*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^3)/a^(7/2
)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^2\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c)^4, x)
```

Fricas [B] time = 1.73819, size = 961, normalized size = 7.01

$$\frac{9\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)
^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(9*cos(d*x + c)^3 + 19*cos(d*x + c)^2 - (9*cos(d*x + c)^2 - 10*cos(d*x + c
) - 11)*sin(d*x + c) - cos(d*x + c) - 11)*sqrt(a*sin(d*x + c) + a))/(d*cos(
```

```
d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*
cos(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.329 $\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=233

$$-\frac{38a^2\sin^4(c+dx)\cos(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{862a^2\sin^3(c+dx)\cos(c+dx)}{9009d\sqrt{a\sin(c+dx)+a}}-\frac{1724a^2\cos(c+dx)}{6435d\sqrt{a\sin(c+dx)+a}}+\frac{2\sin^4(c+dx)\cos(c+$$

```
[Out] (-1724*a^2*Cos[c + d*x])/(6435*d*Sqrt[a + a*Sin[c + d*x]]) - (862*a^2*Cos[c
 + d*x]*Sin[c + d*x]^3)/(9009*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a^2*Cos[c +
 d*x]*Sin[c + d*x]^4)/(1287*d*Sqrt[a + a*Sin[c + d*x]]) + (3448*a*Cos[c + d
*x]*Sqrt[a + a*Sin[c + d*x]])/(45045*d) + (6*a*Cos[c + d*x]*Sin[c + d*x]^4*
Sqrt[a + a*Sin[c + d*x]])/(143*d) - (1724*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(3/2))/(15015*d) + (2*Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^(3/
2))/(13*d)
```

Rubi [A] time = 0.72037, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2879, 2976, 2981, 2770, 2759, 2751, 2646}

$$-\frac{38a^2\sin^4(c+dx)\cos(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{862a^2\sin^3(c+dx)\cos(c+dx)}{9009d\sqrt{a\sin(c+dx)+a}}-\frac{1724a^2\cos(c+dx)}{6435d\sqrt{a\sin(c+dx)+a}}+\frac{2\sin^4(c+dx)\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-1724*a^2*Cos[c + d*x])/(6435*d*Sqrt[a + a*Sin[c + d*x]]) - (862*a^2*Cos[c
 + d*x]*Sin[c + d*x]^3)/(9009*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a^2*Cos[c +
 d*x]*Sin[c + d*x]^4)/(1287*d*Sqrt[a + a*Sin[c + d*x]]) + (3448*a*Cos[c + d
*x]*Sqrt[a + a*Sin[c + d*x]])/(45045*d) + (6*a*Cos[c + d*x]*Sin[c + d*x]^4*
Sqrt[a + a*Sin[c + d*x]])/(143*d) - (1724*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(3/2))/(15015*d) + (2*Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^(3/
2))/(13*d)
```


Rule 2879

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
```

```
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/2}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^{3/2}}{13d} + \frac{2\int \sin^3(c+$$

$$= \frac{6a\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{143d} + \frac{2\cos(c+dx)}{}$$

$$= -\frac{38a^2\cos(c+dx)\sin^4(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} + \frac{6a\cos(c+dx)\sin^4(c+dx)\sqrt{}}{143d}$$

$$= -\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} - \frac{38a^2\cos(c+dx)\sin^4(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} - \frac{38a^2\cos(c+dx)\sin^4(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} - \frac{38a^2\cos(c+dx)\sin^4(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{1724a^2\cos(c+dx)}{6435d\sqrt{a+a\sin(c+dx)}} - \frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [A] time = 3.76826, size = 120, normalized size = 0.52

$$-\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(381174\sin(c+dx)-77665\sin(3(c+dx))+3465\sin(5(}{360360d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(281
816 - 194160*Cos[2*(c + d*x)] + 22680*Cos[4*(c + d*x)] + 381174*Sin[c + d*x
] - 77665*Sin[3*(c + d*x)] + 3465*Sin[5*(c + d*x)]))/(360360*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.723, size = 97, normalized size = 0.4

$$-\frac{(2+2\sin(dx+c))a^2(\sin(dx+c)-1)^2\left(3465(\sin(dx+c))^5+11340(\sin(dx+c))^4+15085(\sin(dx+c))^3+129}{45045\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/45045*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(3465*sin(d*x+c)^5+11340*sin(d
*x+c)^4+15085*sin(d*x+c)^3+12930*sin(d*x+c)^2+10344*sin(d*x+c)+6896)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c)^3, x)
```

Fricas [A] time = 1.65028, size = 562, normalized size = 2.41

$$-\frac{2\left(3465\,a\cos(dx+c)^7-4410\,a\cos(dx+c)^6-14140\,a\cos(dx+c)^5+7330\,a\cos(dx+c)^4+15299\,a\cos(dx+c)^3-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] -2/45045*(3465*a*cos(d*x + c)^7 - 4410*a*cos(d*x + c)^6 - 14140*a*cos(d*x +
 c)^5 + 7330*a*cos(d*x + c)^4 + 15299*a*cos(d*x + c)^3 - 568*a*cos(d*x + c)
^2 + 2272*a*cos(d*x + c) - (3465*a*cos(d*x + c)^6 + 7875*a*cos(d*x + c)^5 -
 6265*a*cos(d*x + c)^4 - 13595*a*cos(d*x + c)^3 + 1704*a*cos(d*x + c)^2 + 2
272*a*cos(d*x + c) + 4544*a)*sin(d*x + c) + 4544*a)*sqrt(a*sin(d*x + c) + a
)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c)^3, x)
```

3.330 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=156

$$-\frac{48a^2\cos^3(c+dx)}{385d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^3(c+dx)}{385d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{11ad}+\frac{4\cos^3(c+dx)(a\sin(c}{33d}$$

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(385*d*(a + a*Sin[c + d*x])^(3/2)) - (48*a^2*Cos[c
 + d*x]^3)/(385*d*Sqrt[a + a*Sin[c + d*x]]) - (6*a*Cos[c + d*x]^3*Sqrt[a +
a*Sin[c + d*x]])/(77*d) + (4*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(33
*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2))/(11*a*d)
```

Rubi [A] time = 0.431542, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2878, 2856, 2674, 2673}

$$-\frac{48a^2\cos^3(c+dx)}{385d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^3(c+dx)}{385d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{11ad}+\frac{4\cos^3(c+dx)(a\sin(c}{33d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(385*d*(a + a*Sin[c + d*x])^(3/2)) - (48*a^2*Cos[c
 + d*x]^3)/(385*d*Sqrt[a + a*Sin[c + d*x]]) - (6*a*Cos[c + d*x]^3*Sqrt[a +
a*Sin[c + d*x]])/(77*d) + (4*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(33
*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2))/(11*a*d)
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^3(c+dx)(a+a\sin(c+dx))^{5/2}}{11ad} + \frac{2\int \cos^2(c+dx)\left(\frac{5a}{2}-3a\,s\right.}{}$$

$$= \frac{4\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{33d} - \frac{2\cos^3(c+dx)(a+a\sin(c+}{11ad}$$

$$= -\frac{6a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{77d} + \frac{4\cos^3(c+dx)(a+a\sin(c+}{33d}$$

$$= -\frac{48a^2\cos^3(c+dx)}{385d\sqrt{a+a\sin(c+dx)}} - \frac{6a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{77d} +$$

$$= -\frac{64a^3\cos^3(c+dx)}{385d(a+a\sin(c+dx))^{3/2}} - \frac{48a^2\cos^3(c+dx)}{385d\sqrt{a+a\sin(c+dx)}} - \frac{6a\cos^3(c}{}$$

Mathematica [A] time = 1.91679, size = 110, normalized size = 0.71

$$-\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(5076\sin(c+dx)-700\sin(3(c+dx))-2280\cos(2(c+dx))}{4620d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(415
9 - 2280*Cos[2*(c + d*x)] + 105*Cos[4*(c + d*x)] + 5076*Sin[c + d*x] - 700*
Sin[3*(c + d*x)]))/(4620*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.868, size = 87, normalized size = 0.6

$$-\frac{(2+2\sin(dx+c))\,a^2(\sin(dx+c)-1)^2\left(105(\sin(dx+c))^4+350(\sin(dx+c))^3+465(\sin(dx+c))^2+372\sin(dx\right.}{1155\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/1155*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(105*sin(d*x+c)^4+350*sin(d*x+c
)^3+465*sin(d*x+c)^2+372*sin(d*x+c)+248)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c)^2, x)
```

Fricas [A] time = 1.63129, size = 468, normalized size = 3.

$$\frac{2 \left(105 \, a \cos\left(dx + c\right)^{6} + 245 \, a \cos\left(dx + c\right)^{5} - 185 \, a \cos\left(dx + c\right)^{4} - 397 \, a \cos\left(dx + c\right)^{3} + 24 \, a \cos\left(dx + c\right)^{2} - 96 \, a \cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 2/1155*(105*a*cos(d*x + c)^6 + 245*a*cos(d*x + c)^5 - 185*a*cos(d*x + c)^4
- 397*a*cos(d*x + c)^3 + 24*a*cos(d*x + c)^2 - 96*a*cos(d*x + c) + (105*a*c
os(d*x + c)^5 - 140*a*cos(d*x + c)^4 - 325*a*cos(d*x + c)^3 + 72*a*cos(d*x
+ c)^2 + 96*a*cos(d*x + c) + 192*a)*sin(d*x + c) - 192*a)*sqrt(a*sin(d*x +
c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin\left(dx + c\right) + a\right)^{\frac{3}{2}} \cos\left(dx + c\right)^{2} \sin\left(dx + c\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c)^2, x)
```

3.331 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9d}-\frac{2a\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}$$

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(315*d*(a + a*Sin[c + d*x])^(3/2)) - (16*a^2*Cos[c
 + d*x]^3)/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^3*Sqrt[a +
a*Sin[c + d*x]])/(21*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(9*
d)
```

Rubi [A] time = 0.255076, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$-\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9d}-\frac{2a\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-64*a^3*Cos[c + d*x]^3)/(315*d*(a + a*Sin[c + d*x])^(3/2)) - (16*a^2*Cos[c
 + d*x]^3)/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^3*Sqrt[a +
a*Sin[c + d*x]])/(21*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(9*
d)
```


Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9d}+\frac{1}{3}\int \cos^2(c+dx)(a+a$$
$$= -\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2\cos^3(c+dx)(a+a\sin($$
$$= -\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-$$
$$= -\frac{64a^3\cos^3(c+dx)}{315d(a+a\sin(c+dx))^{3/2}}-\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^3}{}$$

Mathematica [A] time = 1.45006, size = 100, normalized size = 0.81

$$\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(-741\sin(c+dx)+35\sin(3(c+dx))+240\cos(2(c+dx))}{630d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(-664
 + 240*Cos[2*(c + d*x)] - 741*Sin[c + d*x] + 35*Sin[3*(c + d*x)]))/(630*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.75, size = 77, normalized size = 0.6

$$-\frac{(2+2\sin(dx+c))\,a^2\,(\sin(dx+c)-1)^2\left(35\,(\sin(dx+c))^3+120\,(\sin(dx+c))^2+159\,\sin(dx+c)+106\right)}{315\,d\cos(dx+c)}\frac{}{\sqrt{a+a\sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/315*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(35*sin(d*x+c)^3+120*sin(d*x+c)^
2+159*sin(d*x+c)+106)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c), x)
```

Fricas [A] time = 1.5655, size = 396, normalized size = 3.19

$$\frac{2\left(35\,a\cos\left(dx+c\right)^{5}-50\,a\cos\left(dx+c\right)^{4}-109\,a\cos\left(dx+c\right)^{3}+8\,a\cos\left(dx+c\right)^{2}-32\,a\cos\left(dx+c\right)-\left(35\,a\cos\left(dx+c\right)^{4}\right.\right.}{315\left(d\cos\left(dx+c\right)+d\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/315*(35*a*cos(d*x + c)^5 - 50*a*cos(d*x + c)^4 - 109*a*cos(d*x + c)^3 + 8
*a*cos(d*x + c)^2 - 32*a*cos(d*x + c) - (35*a*cos(d*x + c)^4 + 85*a*cos(d*x
 + c)^3 - 24*a*cos(d*x + c)^2 - 32*a*cos(d*x + c) - 64*a)*sin(d*x + c) - 64
*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c), x)
```

3.332 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=123

$$-\frac{2a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{2\cos(c+dx)(a\sin(c+a}{5d}$$

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
2*a^2*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*a*Cos[c + d*x]*Sqrt
[a + a*Sin[c + d*x]])/(5*d) + (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(
5*d)
```

Rubi [A] time = 0.459499, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2874, 2976, 2981, 2773, 206}

$$-\frac{2a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{2\cos(c+dx)(a\sin(c+a}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
2*a^2*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*a*Cos[c + d*x]*Sqrt
[a + a*Sin[c + d*x]])/(5*d) + (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(
5*d)
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \csc(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/2}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} + \frac{2\int \csc(c+dx)(a+a\sin(c+d}{5}$$

$$= \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{2\cos(c+dx)(a+a\sin(c+dx))}{5d}$$

$$= -\frac{2a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{2\cos}{}$$

$$= -\frac{2a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{2\cos}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+}{}$$

Mathematica [A] time = 0.266546, size = 145, normalized size = 1.18

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)+5\cos\left(\frac{3}{2}(c+dx)\right)-\cos\left(\frac{5}{2}(c+dx)\right)-10\log\left(-\sin\left(\frac{1}{2}(c+d\right.\right.}{10d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(5*Cos[(3*(c + d*x))/2] - Cos[(5*(c + d*x))/2
] - 10*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 10*Log[1 - Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2])
)/(10*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A] time = 0.825, size = 123, normalized size = 1.

$$-\frac{2+2\sin(dx+c)}{5ad\cos(dx+c)}\sqrt{-a(\sin(dx+c)-1)}\left(5a^{5/2}\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{5}{2}}+5(a-a\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/5*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(5*a^(5/2)*arctanh((a-a*sin(d
*x+c))^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(5/2)+5*(a-a*sin(d*x+c))^(3/2)*a-5*a
^2*(a-a*sin(d*x+c))^(1/2))/a/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c), x)
```

Fricas [B] time = 1.80858, size = 764, normalized size = 6.21

$$\frac{5\,(a\cos(dx+c)+a\sin(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin(\ldots}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/10*(5*(a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c)
 - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c)
+ (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)
^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)
) - 4*(a*cos(d*x + c)^3 - 2*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) - (a*cos(d*
x + c)^2 + 3*a*cos(d*x + c) + a)*sin(d*x + c) + a)*sqrt(a*sin(d*x + c) + a)
)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.333 $\int \cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=121

$$\frac{11a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{5a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{\cot(c+dx)(a\sin(c+dx)\cdot}{d}$$

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
11*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (5*a*Cos[c + d*x]*Sqr
t[a + a*Sin[c + d*x]])/(3*d) - (Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.308013, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2716, 2976, 2981, 2773, 206}

$$\frac{11a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{5a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{\cot(c+dx)(a\sin(c+dx)\cdot}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```



```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
11*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (5*a*Cos[c + d*x]*Sqr
t[a + a*Sin[c + d*x]])/(3*d) - (Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/d
```


Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{\cot(c+dx)(a+a\sin(c+dx))^{3/2}}{d} + \frac{\int \csc(c+dx)\left(\frac{3a}{2}-\frac{5}{2}a\sin(c+dx)\right)(a+a}{a}$$

$$= \frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\cot(c+dx)(a+a\sin(c+dx))^{3/2}}{d} + \frac{2\int \csc}{}$$

$$= \frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\cot(c+dx)(a+a}{d}$$

$$= \frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\cot(c+dx)(a+a}{d}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{5a\cos(c+dx)\sqrt{a+a}}{3d}$$

Mathematica [A] time = 0.767613, size = 233, normalized size = 1.93

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-14\sin\left(\frac{1}{2}(c+dx)\right)-9\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{5}{2}(c+dx)\right)+14\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{3d\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\csc\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(14*Cos[(c + d*x)/2] - 9*
Cos[(3*(c + d*x))/2] + Cos[(5*(c + d*x))/2] - 14*Sin[(c + d*x)/2] + 9*Log[1
 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 9*Log[1 - Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 9*Sin[(3*(c + d*x))/2] - Sin[(5*(
c + d*x))/2]))/(3*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x
)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*x)/4]))
```

Maple [A] time = 0.872, size = 144, normalized size = 1.2

$$-\frac{1+\sin(dx+c)}{3\cos(dx+c)\sin(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(2(a-a\sin(dx+c))^{3/2}\sqrt{a}-12\sqrt{a-a\sin(dx+c)}a^{3/2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(2*(a-a*sin(d*x+c
))^(3/2)*a^(1/2)-12*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+9*arctanh((a-a*sin(d*x+c
))^(1/2)/a^(1/2))*a^2)+3*(a-a*sin(d*x+c))^(1/2)*a^(3/2))/sin(d*x+c)/a^(1/2)
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c)^2, x)
```

Fricas [B] time = 1.73617, size = 833, normalized size = 6.88

$$\frac{9\left(a\cos(dx+c)^2-(a\cos(dx+c)+a)\sin(dx+c)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)\right.}{\cos(dx+c)^3+\cos(dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(9*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
 c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a
) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x +
c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)) + 4*(2*a*cos(d*x + c)^3 - 8*a*cos(d*x + c)^2 + a*cos
(d*x + c) - (2*a*cos(d*x + c)^2 + 10*a*cos(d*x + c) + 11*a)*sin(d*x + c) +
11*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*si
n(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.334 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=131

$$\frac{13a^2\cos(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{3a\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{\cot(c+dx)\csc(c+dx)(a\,\mathrm{si}}{2d}$$

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d) +
(13*a^2*Cos[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(4*d) - (Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d
*x])^(3/2))/(2*d)
```

Rubi [A] time = 0.518467, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2874, 2975, 2981, 2773, 206}

$$\frac{13a^2\cos(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{3a\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{\cot(c+dx)\csc(c+dx)(a\,\mathrm{si}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d) +
(13*a^2*Cos[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(4*d) - (Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d
*x])^(3/2))/(2*d)
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/2}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}}{2d} + \frac{\int \csc^2(c+dx)(a +$$

$$= -\frac{3a\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{\cot(c+dx)\csc(c+dx)(a+a}{2d}$$

$$= \frac{13a^2\cos(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{\cot(c}{}$$

$$= \frac{13a^2\cos(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{\cot(c}{}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} + \frac{13a^2\cos(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+d}{}$$

Mathematica [B] time = 0.659407, size = 271, normalized size = 2.07

$$-\frac{a\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(22\sin\left(\frac{1}{2}(c+dx)\right)+22\sin\left(\frac{3}{2}(c+dx)\right)-8\sin\left(\frac{5}{2}(c+dx)\right)-22\cos\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-22*Cos[(c + d*x)/2] + 2
2*Cos[(3*(c + d*x))/2] + 8*Cos[(5*(c + d*x))/2] - Log[1 + Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - Cos[2*(c + d*x)]*
Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 22*Sin[(c + d*x)/2] + 22*Sin
[(3*(c + d*x))/2] - 8*Sin[(5*(c + d*x))/2]))/(4*d*(1 + Cot[(c + d*x)/2])*(C
sc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

Maple [A] time = 0.987, size = 151, normalized size = 1.2

$$\frac{1+\sin(dx+c)}{4\,(\sin(dx+c))^2\cos(dx+c)\,d}\sqrt{-a\,(\sin(dx+c)-1)}\left(8\sqrt{-a\,(\sin(dx+c)-1)}\,(\sin(dx+c))^2\,a^{3/2}+\operatorname{Artanh}\left(\sqrt{-a\,(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(8*(-a*(sin(d*x+c)-1))^(1/2)*s
in(d*x+c)^2*a^(3/2)+arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2
*a^2+7*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)-9*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2
))/sin(d*x+c)^2/a^(1/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c)^3, x)
```

Fricas [B] time = 1.74725, size = 936, normalized size = 7.15

$$\frac{\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)+\left(a\cos(dx+c)^2-a\right)\sin(dx+c)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*((a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x +
c)^2 - a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c
)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c)
- 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^
2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^
2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(8*a*cos(d*x
 + c)^3 + 15*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - (8*a*cos(d*x + c)^2 - 7*
a*cos(d*x + c) - 13*a)*sin(d*x + c) - 13*a)*sqrt(a*sin(d*x + c) + a))/(d*co
s(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)^2 - d)*s
in(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.335 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=139

$$\frac{5a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{13a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{a\cot(c+dx)}{}$$

```
[Out] (13*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 + (5*a^2*Cot[c + d*x])/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*C
sc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d) - (Cot[c + d*x]*Csc[c + d*x]^2*
(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.573337, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2874, 2975, 2980, 2773, 206}

$$\frac{5a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{13a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{a\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (13*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 + (5*a^2*Cot[c + d*x])/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*C
sc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d) - (Cot[c + d*x]*Csc[c + d*x]^2*
(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
```

```
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/2}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} + \frac{\int \csc^3(c+dx)(a$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{\cot(c+dx)\csc^2(c$$

$$= \frac{5a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+}}{4d}$$

$$= \frac{5a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+}}{4d}$$

$$= \frac{13a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} + \frac{5a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c}{}$$

Mathematica [B] time = 0.881587, size = 286, normalized size = 2.06

$$-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-12\sin\left(\frac{1}{2}(c+dx)\right)+70\sin\left(\frac{3}{2}(c+dx)\right)+18\sin\left(\frac{5}{2}(c+dx)\right)+12\cos\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(12*Cos[(c + d*x)/2] + 7
0*Cos[(3*(c + d*x))/2] - 18*Cos[(5*(c + d*x))/2] - 12*Sin[(c + d*x)/2] - 11
7*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 117*Log[1 - C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 70*Sin[(3*(c + d*x))/2]
+ 18*Sin[(5*(c + d*x))/2] + 39*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[3*(c + d*x)] - 39*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(
c + d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x
)/4]^2)^3)
```

Maple [A] time = 0.897, size = 144, normalized size = 1.

$$\frac{1+\sin(dx+c)}{24(\sin(dx+c))^3\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(39\,\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^3(\sin(dx+c))^3-9\,(-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(39*arctanh((-a*(sin(d*x+c)-1
))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^3-9*(-a*(sin(d*x+c)-1))^(5/2)*a^(1/2)+40*(
-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)-39*(-a*(sin(d*x+c)-1))^(1/2)*a^(5/2))/a^(3
/2)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c)^4, x)
```

Fricas [B] time = 1.74652, size = 996, normalized size = 7.17

$$\frac{39\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2-\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a\right)\sin(dx+c)+a\right)\sqrt{a}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(39*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*cos
(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*
x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x
 + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*
x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c
) - 1)) + 4*(9*a*cos(d*x + c)^3 - 13*a*cos(d*x + c)^2 - 17*a*cos(d*x + c) -
 (9*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 5*a)*sin(d*x + c) + 5*a)*sqrt(a*
sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x + c)
^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.336 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=158

$$-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105a^2d}+\frac{2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}+\frac{8\cos(c+dx)\sqrt{a\sin(}}{315ad}$$

```
[Out] (-4*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[c +
 d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sin[c + d*x]^4)/
(9*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/
(315*a*d) - (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105*a^2*d)
```

Rubi [A] time = 0.403586, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2879, 2981, 2770, 2759, 2751, 2646}

$$-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105a^2d}+\frac{2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}+\frac{8\cos(c+dx)\sqrt{a\sin(}}{315ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[c +
 d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sin[c + d*x]^4)/
(9*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/
(315*a*d) - (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105*a^2*d)
```

Rule 2879

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{9a}$$
$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \sin^2(c+dx)\sqrt{a+a\sin}}{21a}$$
$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)(a+a\sin(c+}{105a^2d}$$
$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{a+a\sin(c}}{315ad}$$
$$= -\frac{4\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + 8$$

Mathematica [A] time = 1.23475, size = 97, normalized size = 0.61

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(-201\sin(c+dx)+35\sin(3(c+dx))+60\cos(2}{630d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])*(-124 + 60*Cos[2*(c + d*x)] - 201*Sin[c + d*x] + 35*Sin[3*(c + d*x)]))/
(630*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.67, size = 74, normalized size = 0.5

$$-\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^2\left(35(\sin(dx+c))^3+30(\sin(dx+c))^2+24\sin(dx+c)+16\right)}{315\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/315*(1+sin(d*x+c))*(sin(d*x+c)-1)^2*(35*sin(d*x+c)^3+30*sin(d*x+c)^2+24*
sin(d*x+c)+16)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2 \sin(dx+c)^3}{\sqrt{a \sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.62542, size = 373, normalized size = 2.36

$$\frac{2\left(35 \cos(dx+c)^5 + 40 \cos(dx+c)^4 - 64 \cos(dx+c)^3 - 82 \cos(dx+c)^2 - \left(35 \cos(dx+c)^4 - 5 \cos(dx+c)^3 - 69\right.\right.}{315\,(ad\cos(dx+c) + ad\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/315*(35*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - 64*cos(d*x + c)^3 - 82*cos(d
*x + c)^2 - (35*cos(d*x + c)^4 - 5*cos(d*x + c)^3 - 69*cos(d*x + c)^2 + 13*
cos(d*x + c) + 26)*sin(d*x + c) + 13*cos(d*x + c) + 26)*sqrt(a*sin(d*x + c)
 + a)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.53172, size = 282, normalized size = 1.78

$$-\frac{\frac{4\left(\left(\left(\left(\left(\left(\frac{2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^{11}}+\frac{9\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{11}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{63\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{11}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{63\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{11}}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{9}{2}}}}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/161280*(4*(((((2*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)^2/a^
11 + 9*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c)^2 - 63*sgn(
tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) + 63*sgn(tan(1/2*d*x +
 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c)^2 - 9*sgn(tan(1/2*d*x + 1/2*c) + 1)
/a^11)*tan(1/2*d*x + 1/2*c)^2 - 2*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)/(a*ta
n(1/2*d*x + 1/2*c)^2 + a)^(9/2) - 13*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/
a^(31/2))/d
```

3.337 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=92

$$-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7ad}+\frac{12\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{22a\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-22*a*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) + (12*Cos[c + d*x
]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + a*Sin[c +
 d*x]])/(7*a*d)
```

Rubi [A] time = 0.342793, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2877, 2856, 2674, 2673}

$$-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7ad}+\frac{12\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{22a\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-22*a*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) + (12*Cos[c + d*x
]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + a*Sin[c +
 d*x]])/(7*a*d)
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}dx &= \frac{\cos^3(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\left(-\frac{a}{2}-2a\sin(c+dx)\right)\sqrt{a+a\sin(c+dx)}\,dx}{2a^2}\\ &= \frac{\cos^3(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7ad} + \frac{11\int \cos^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{28a}\\ &= \frac{12\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7ad} + \frac{11}{35}\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}dx\\ &= -\frac{22a\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}} + \frac{12\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7ad}\end{aligned}$$

Mathematica [A] time = 0.338654, size = 87, normalized size = 0.95

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(24\sin(c+dx)-15\cos(2(c+dx))+31)}{105d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])*(31 - 15*Cos[2*(c + d*x)] + 24*Sin[c + d*x]))/(105*d*Sqrt[a*(1 + Sin[c
 + d*x])])
```

Maple [A] time = 1.152, size = 64, normalized size = 0.7

$$-\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^2\left(15(\sin(dx+c))^2+12\sin(dx+c)+8\right)}{105\,d\cos(dx+c)}\frac{1}{\sqrt{a(1+\sin(dx+c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(1+sin(d*x+c))*(sin(d*x+c)-1)^2*(15*sin(d*x+c)^2+12*sin(d*x+c)+8)/
cos(d*x+c)/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2\sin(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.63284, size = 320, normalized size = 3.48

$$-\frac{2\left(15\,\cos\left(dx+c\right)^{4}-3\,\cos\left(dx+c\right)^{3}-29\,\cos\left(dx+c\right)^{2}+\left(15\,\cos\left(dx+c\right)^{3}+18\,\cos\left(dx+c\right)^{2}-11\,\cos\left(dx+c\right)-22\right.}{105\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2/105*(15*cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 29*cos(d*x + c)^2 + (15*cos(
d*x + c)^3 + 18*cos(d*x + c)^2 - 11*cos(d*x + c) - 22)*sin(d*x + c) + 11*co
s(d*x + c) + 22)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*sin(d*x +
 c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**2*cos(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 2.34651, size = 277, normalized size = 3.01

$$-\frac{\frac{2\left(\left(\left(\left(\left(\frac{2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{9}}+\frac{7\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{9}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{35\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{9}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{35\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{9}}\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{7}{2}}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/13440*(2*(((((2*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)^2/a^9
 + 7*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^9)*tan(1/2*d*x + 1/2*c) - 35*sgn(tan(1
/2*d*x + 1/2*c) + 1)/a^9)*tan(1/2*d*x + 1/2*c) + 35*sgn(tan(1/2*d*x + 1/2*c
) + 1)/a^9)*tan(1/2*d*x + 1/2*c) - 7*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^9)*tan
(1/2*d*x + 1/2*c)^2 - 2*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^9)/(a*tan(1/2*d*x +
 1/2*c)^2 + a)^(7/2) + 11*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(25/2))/d
```

3.338 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2a\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (2*a*Cos[c + d*x]^3)/(15*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^3)
/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.126818, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2856, 2673}

$$\frac{2a\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*a*Cos[c + d*x]^3)/(15*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^3)
/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{2\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}-\frac{1}{5}\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx\\ &= \frac{2a\cos^3(c+dx)}{15d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.419678, size = 77, normalized size = 1.28

$$-\frac{2(3\sin(c+dx)+2)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{15d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])*(2 + 3*Sin[c + d*x]))/(15*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.648, size = 54, normalized size = 0.9

$$-\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^2(3\sin(dx+c)+2)}{15d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/15*(1+sin(d*x+c))*(sin(d*x+c)-1)^2*(3*sin(d*x+c)+2)/cos(d*x+c)/(a+a*sin(
d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2\sin(dx+c)}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.58472, size = 251, normalized size = 4.18

$$-\frac{2\left(3\cos(dx+c)^3+4\cos(dx+c)^2-\left(3\cos(dx+c)^2-\cos(dx+c)-2\right)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a\sin(dx+c)+a}}{15(ad\cos(dx+c)+ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/15*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - cos(d*x +
c) - 2)*sin(d*x + c) - cos(d*x + c) - 2)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(
d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)\cos^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)*cos(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 2.74696, size = 198, normalized size = 3.3

$$-\frac{\frac{\left(\left(\frac{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{7}}-\frac{5\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{7}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{5\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{7}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\frac{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{7}}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{5}{2}}}-\frac{\sqrt{2}\operatorname{sgn}(\mathrm{t}}{}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -1/30*((((sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)^2/a^7 - 5*sgn(
tan(1/2*d*x + 1/2*c) + 1)/a^7)*tan(1/2*d*x + 1/2*c) + 5*sgn(tan(1/2*d*x + 1
/2*c) + 1)/a^7)*tan(1/2*d*x + 1/2*c)^2 - sgn(tan(1/2*d*x + 1/2*c) + 1)/a^7)
/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2) - sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) +
1)/a^(19/2))/d
```

3.339 $\int \frac{\cos(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=63

$$\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.216204, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2874, 2981, 2773, 206}

$$\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\int \csc(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}\\
&= \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a}\\
&= \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d}+\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.130316, size = 116, normalized size = 1.84

$$
\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)-\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+\right.\right.}{d\sqrt{a(\sin(c+dx)+1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((2*Cos[(c + d*x)/2] - Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2])*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.836, size = 87, normalized size = 1.4

$$
2\,\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}}{\cos(dx+c)\,a\sqrt{a+a\sin(dx+c)}d}\left(\sqrt{a-a\sin(dx+c)}-\sqrt{a}\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a*((a-a*sin(d*x+c))^(1/2)-a^(1/2
)*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\cos(dx+c)^2\csc(dx+c)}{\sqrt{a\sin(dx+c)+a}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.70545, size = 647, normalized size = 10.27

$$\frac{\sqrt{a}(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c}\right.}{2(ad\cos(dx+c)+ad\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*s
qrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a*d*cos(d*x + c
) + a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 2.37636, size = 348, normalized size = 5.52

$$-\frac{\frac{2\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{1}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\frac{\left(2a\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{-a}\sqrt{a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+2\sqrt{2}\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}a}-\frac{2\arctan}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -(2*(tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 1/sgn(tan(1/2*d*x
 + 1/2*c) + 1))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + (2*a*arctan((sqrt(2)*s
qrt(a) + sqrt(a))/sqrt(-a)) - sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a
)) + 2*sqrt(2)*sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a)
 - 2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + log(abs(-sqrt(a)
*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(t
an(1/2*d*x + 1/2*c) + 1)))/d
```

3.340 $\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) - Cot[
c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.100631, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2716, 21, 2773, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) - Cot[
c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(-\frac{a}{2}-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}$$
$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a}$$
$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 0.299677, size = 138, normalized size = 2.23

$$\frac{\left(\tan\left(\frac{1}{2}(c+dx)\right)+1\right)\csc\left(\frac{1}{4}(c+dx)\right)\sec\left(\frac{1}{4}(c+dx)\right)\left(2\sin\left(\frac{1}{2}(c+dx)\right)-2\cos\left(\frac{1}{2}(c+dx)\right)+\sin(c+dx)\left(\log\left(-\sin\right.\right.\right.}{8d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/4]*Sec[(c + d*x)/4]*(-2*Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2
] + (Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]])*Sin[c + d*x])*(1 + Tan[(c + d*x)/2]))/(8*d*Sqrt[a*(1
 + Sin[c + d*x])])
```

Maple [A] time = 0.898, size = 103, normalized size = 1.7

$$-\frac{1+\sin(dx+c)}{\cos(dx+c)\sin(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(-\text{Artanh}\left(\sqrt{a-a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)\sin(dx+c)a+\sqrt{a-a\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-arctanh((a-a*sin(d*x+c))^(1/2)/
a^(1/2))*sin(d*x+c)*a+(a-a*sin(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/a^(3/2)/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2\csc(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.69811, size = 706, normalized size = 11.39

$$\frac{\left(\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx\cdots\right.}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos\cdots\right.}\right.}{4\left(ad\cos\left(dx+c\right)^2-ad-(ad\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) +
 1))/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2\left(c+dx\right)\csc^2\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 2.3487, size = 486, normalized size = 7.84

$$\frac{\frac{\left(2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{2}\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+2\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)-\sqrt{2}\sqrt{-a}-3\sqrt{-a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{2}\sqrt{-a}\sqrt{a}+\sqrt{-a}\sqrt{a}}-\frac{2\arctan\left(\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((2*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - sqrt
(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 2*sqrt(a)*arctan((sqrt(2)*sqr
t(a) + sqrt(a))/sqrt(-a)) - sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - sqrt(
2)*sqrt(-a) - 3*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2)*sqrt(-a)*s
qrt(a) + sqrt(-a)*sqrt(a)) - 2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
```

```
(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c
) + 1)) + log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*sqrt(a)/(((sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sgn(tan(1/2*d
*x + 1/2*c) + 1)))/d
```

3.341 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(4*Sqrt[a]*d) + Co
t[c + d*x]/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x])/(2*
d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.316527, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2874, 2980, 2772, 2773, 206}

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(4*Sqrt[a]*d) + Co
t[c + d*x]/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x])/(2*
d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{4a}$$
$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,d}{8a}$$
$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right.}{4d}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}d} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 1.85661, size = 272, normalized size = 2.72

$$\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\left(4\tan\left(\tfrac{1}{4}(c+dx)\right)+4\cot\left(\tfrac{1}{4}(c+dx)\right)-\csc^2\left(\tfrac{1}{4}(c+dx)\right)+\sec^2\left(\tfrac{1}{4}(c+dx)\right)-\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8 + 4*Cot[(c + d*x)/4] - Csc[(c +
d*x)/4]^2 + 4*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Cos[(c + d*x)/4]
 - Sin[(c + d*x)/4])^2 - (8*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] - Sin[(c +
d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (8*Sin[(c + d*x)/4])
/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 4*Tan[(c + d*x)/4]))/(32*d*Sqrt[a*
(1 + Sin[c + d*x])])
```

Maple [A] time = 0.947, size = 124, normalized size = 1.2

$$-\frac{1+\sin(dx+c)}{4(\sin(dx+c))^2\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left((-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{3}{2}}-\text{Artanh}\left(\sqrt{-a(\sin(dx+c)-1)}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*((-a*(sin(d*x+c)-1))^
(3/2)*a^(3/2)-arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2+(
-a*(sin(d*x+c)-1))^(1/2)*a^(5/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2 \csc(dx+c)^3}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.74132, size = 861, normalized size = 8.61

$$\frac{\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx\right.}{}\right.}{16\left(ad\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*((cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
- cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*
(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqr
t(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*
cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos
(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(cos(d*x + c)^2 + (c
os(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a
))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d
*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.46748, size = 682, normalized size = 6.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/
2*d*x + 1/2*c) + 1)) - 2/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) + (4*sqrt(2)*sq
rt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 2*sqrt(2)*sqrt(-a)*log
(sqrt(2)*sqrt(a) + sqrt(a)) + 6*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/
sqrt(-a)) - 3*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 14*sqrt(2)*sqrt(-a)
 + 18*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(2*sqrt(2)*sqrt(-a)*sqrt(a) +
 3*sqrt(-a)*sqrt(a)) - 2*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)
) + log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 - 2*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + 2*a^(3/2))/(((sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^2*sgn(tan(
1/2*d*x + 1/2*c) + 1)))/d
```

3.342 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(8*Sqrt[a]*d) + Co
t[c + d*x]/(8*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x])/(12
*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d*Sqrt[a +
a*Sin[c + d*x]])
```

Rubi [A] time = 0.403147, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2874, 2980, 2772, 2773, 206}

$$\frac{\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(8*Sqrt[a]*d) + Co
t[c + d*x]/(8*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x])/(12
*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d*Sqrt[a +
a*Sin[c + d*x]])
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{6a}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc^2(c+dx)\sqrt{a+a\sin(c+}}{8a}$$

$$= \frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \int \csc$$

$$= \frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \text{Subs}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{3d\sqrt{}}$$

Mathematica [B] time = 0.696361, size = 292, normalized size = 2.16

$$\csc^9\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(60\sin\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{3}{2}(c+dx)\right)+6\sin\left(\frac{5}{2}(c+dx)\right)-60\,\text{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-60*Cos[(c + d*x
)/2] + 2*Cos[(3*(c + d*x))/2] - 6*Cos[(5*(c + d*x))/2] + 60*Sin[(c + d*x)/2
] + 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 9*Log[1 -
 Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 2*Sin[(3*(c + d*x))/2]
 + 6*Sin[(5*(c + d*x))/2] - 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*
Sin[3*(c + d*x)] + 3*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c
+ d*x)]))/(24*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3*Sqrt[a*(1 + Sin
[c + d*x])])
```

Maple [A] time = 1.062, size = 144, normalized size = 1.1

$$-\frac{1+\sin(dx+c)}{24\left(\sin(dx+c)\right)^3\cos(dx+c)\,d}\sqrt{-a\left(\sin(dx+c)-1\right)}\left(3\sqrt{-a\left(\sin(dx+c)-1\right)}a^{9/2}+8\left(-a\left(\sin(dx+c)-1\right)\right)^{3/2}a^{7/2}-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-1))^(1/2)
*a^(9/2)+8*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)-3*(-a*(sin(d*x+c)-1))^(5/2)*a^
(5/2)-3*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x+c)^3)/a^(11/
2)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2\csc(dx+c)^4}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.73309, size = 975, normalized size = 7.22

$$\frac{3\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)
^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(3*cos(d*x + c)^3 + cos(d*x + c)^2 - (3*cos(d*x + c)^2 + 2*cos(d*x + c) +
7)*sin(d*x + c) + 5*cos(d*x + c) + 7)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*
x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d - (a*d*cos(d*x + c)^3 + a*d*cos(d*x +
 c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.44294, size = 737, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a*sgn(ta
n(1/2*d*x + 1/2*c) + 1)) - 3/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x
 + 1/2*c) + 2/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) + (30*sqrt(2)*a^(3/2)*arct
an((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 15*sqrt(2)*sqrt(-a)*a*log(sqrt(2
)*sqrt(a) + sqrt(a)) + 42*a^(3/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-
a)) - 21*sqrt(-a)*a*log(sqrt(2)*sqrt(a) + sqrt(a)) - 88*sqrt(2)*sqrt(-a)*a
- 126*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(5*sqrt(2)*sqrt(-a)*a^(3/2)
 + 7*sqrt(-a)*a^(3/2)) - 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) +
1)) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(3*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5 - 6*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(a) - 3*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 - 2*a^(5/2))/((
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^
3*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.343 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=184

$$\frac{76\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105a^2d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\sin^3(c+dx)\cos(c+dx)}{7ad\sqrt{a\sin(c+dx)+a}}-\frac{16\sin^2(c+dx)\cos(c+}{35ad\sqrt{a\sin(c+dx)-}}$$

```
[Out] (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - (344*Cos[c + d*x])/(105*a*d*Sqrt[a + a*Sin[c + d*x]]) - (
16*Cos[c + d*x]*Sin[c + d*x]^2)/(35*a*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[
c + d*x]*Sin[c + d*x]^3)/(7*a*d*Sqrt[a + a*Sin[c + d*x]]) + (76*Cos[c + d*x
]*Sqrt[a + a*Sin[c + d*x]])/(105*a^2*d)
```

Rubi [A] time = 0.589225, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2879, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{76\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105a^2d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\sin^3(c+dx)\cos(c+dx)}{7ad\sqrt{a\sin(c+dx)+a}}-\frac{16\sin^2(c+dx)\cos(c+}{35ad\sqrt{a\sin(c+dx)-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - (344*Cos[c + d*x])/(105*a*d*Sqrt[a + a*Sin[c + d*x]]) - (
16*Cos[c + d*x]*Sin[c + d*x]^2)/(35*a*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[
c + d*x]*Sin[c + d*x]^3)/(7*a*d*Sqrt[a + a*Sin[c + d*x]]) + (76*Cos[c + d*x
]*Sqrt[a + a*Sin[c + d*x]])/(105*a^2*d)
```

Rule 2879

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{\sin^3(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sin^2(c+dx)\left(-3a^2+4a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{7a^3}$$

$$= -\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{4\int \frac{\sin(c+dx)\left(8a^3-\frac{19}{2}a^3\sin\right.}{\sqrt{a+a\sin(c+dx)}}}{35a^4}$$

$$= -\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{4\int \frac{8a^3\sin(c+dx)-\frac{19}{2}a^3\sin^2}{\sqrt{a+a\sin(c+dx)}}}{35a^4}$$

$$= -\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{76\cos(c+dx)\sqrt{a+a\,s}}{105a^2d}$$

$$= -\frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}} - \frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+d}{7ad\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}} - \frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+d}{7ad\sqrt{a+a\sin(c+dx}}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}} - \frac{16\cos(c+dx)\sin^2(c+}{35ad\sqrt{a+a\sin(c+}}$$

Mathematica [C] time = 1.80943, size = 201, normalized size = 1.09

$$\frac{\sqrt{a}(\sin(c+dx)+1)\left(1365\sin\left(\frac{1}{2}(c+dx)\right)+245\sin\left(\frac{3}{2}(c+dx)\right)-63\sin\left(\frac{5}{2}(c+dx)\right)-15\sin\left(\frac{7}{2}(c+dx)\right)-1365\cos\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((1680 + 1680*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 1365*
Cos[(c + d*x)/2] + 245*Cos[(3*(c + d*x))/2] + 63*Cos[(5*(c + d*x))/2] - 15*
Cos[(7*(c + d*x))/2] + 1365*Sin[(c + d*x)/2] + 245*Sin[(3*(c + d*x))/2] - 6
3*Sin[(5*(c + d*x))/2] - 15*Sin[(7*(c + d*x))/2]))/(420*a^2*d*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.967, size = 148, normalized size = 0.8

$$\frac{2+2\sin(dx+c)}{105a^5\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(105a^{7/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-15(a-a\sin(dx+c))^{7/2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/105*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(105*a^(7/2)*2^(1/2)*arctanh
(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-15*(a-a*sin(d*x+c))^(7/2)+21*(
a-a*sin(d*x+c))^(5/2)*a-35*(a-a*sin(d*x+c))^(3/2)*a^2-105*a^3*(a-a*sin(d*x+
c))^(1/2))/a^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(dx+c)^2\sin(dx+c)^3}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 1.76751, size = 725, normalized size = 3.94

$$\frac{\frac{105\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-2\left(15\cos(dx+c)^4\right.}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/105*(105*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)
^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(
cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 2*(15*
cos(d*x + c)^4 - 24*cos(d*x + c)^3 - 92*cos(d*x + c)^2 + (15*cos(d*x + c)^3
 + 39*cos(d*x + c)^2 - 53*cos(d*x + c) - 211)*sin(d*x + c) + 158*cos(d*x +
c) + 211)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c
) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.39869, size = 486, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/26880*(107520*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(t
an(1/2*d*x + 1/2*c) + 1)) - 2*(((((((67*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1
/2*d*x + 1/2*c)/a^10 - 105*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^10)*tan(1/2*d*x
+ 1/2*c) + 287*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^10)*tan(1/2*d*x + 1/2*c) - 3
85*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^10)*tan(1/2*d*x + 1/2*c) + 385*sgn(tan(1
/2*d*x + 1/2*c) + 1)/a^10)*tan(1/2*d*x + 1/2*c) - 287*sgn(tan(1/2*d*x + 1/2
*c) + 1)/a^10)*tan(1/2*d*x + 1/2*c) + 105*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^1
0)*tan(1/2*d*x + 1/2*c) - 67*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^10)/(a*tan(1/2
*d*x + 1/2*c)^2 + a)^(7/2) - sqrt(2)*(107520*a^(27/2)*arctan(sqrt(a)/sqrt(-
a)) + 211*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^(29/2)))/d
```

3.344 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\cos^3(c+dx)}{5ad\sqrt{a\sin(c+dx)+a}}+\frac{18\cos(c+dx)}{5ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d) + (18*Cos[c + d*x])/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*
Cos[c + d*x]^3)/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(5*a^2*d)
```

Rubi [A] time = 0.348267, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2878, 2858, 2751, 2649, 206}

$$-\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\cos^3(c+dx)}{5ad\sqrt{a\sin(c+dx)+a}}+\frac{18\cos(c+dx)}{5ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d) + (18*Cos[c + d*x])/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*
Cos[c + d*x]^3)/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(5*a^2*d)
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2858

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(d*Cos[e + f*x]*(
a + b*Sin[e + f*x])^(m + 2))/(b^2*f*(m + 3)), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\cos^2(c+dx)\left(-\frac{a}{2}-3a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{5a}$$

$$= -\frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5a^2d} - \frac{4\int \frac{-\frac{3a^2}{4}+\frac{27}{4}a^2\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}}{15a^3}$$

$$= \frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5a^2d}$$

$$= \frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5a^2d}$$

$$= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.283815, size = 150, normalized size = 1.07

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-30\sin\left(\frac{1}{2}(c+dx)\right)-5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)+30\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{10d(a(\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*((40 + 40*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 30*Cos[(c + d*x)/2] - 5*Cos
[(3*(c + d*x))/2] - Cos[(5*(c + d*x))/2] - 30*Sin[(c + d*x)/2] - 5*Sin[(3*(
c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(10*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 1.32, size = 114, normalized size = 0.8

$$-\frac{2+2\sin(dx+c)}{5da^4\cos(dx+c)}\sqrt{-a(\sin(dx+c)-1)}\left(5a^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{5}{2}}-5a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(5*a^(5/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(5/2)-5*a^2*(a
-a*sin(d*x+c))^(1/2))/a^4/cos(d*x+c)/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2 \sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 1.77163, size = 647, normalized size = 4.62

$$\frac{\frac{5\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-2\left(\cos(dx+c)^3+3\right.}{5\left(a^2d\cos(dx+c)+a^2d\sin(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/5*(5*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 2*(cos(d*x
 + c)^3 + 3*cos(d*x + c)^2 - (cos(d*x + c)^2 - 2*cos(d*x + c) - 9)*sin(d*x
+ c) - 7*cos(d*x + c) - 9)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c) +
a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.46318, size = 410, normalized size = 2.93

$$\frac{\frac{1280\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{2\left(\left(\left(\left(\left(\frac{3\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^8}-\frac{5\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\right.\right.\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/320*(1280*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/
2*d*x + 1/2*c) + 1)) - 2*(((((3*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x +
 1/2*c)/a^8 - 5*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^8)*tan(1/2*d*x + 1/2*c) + 1
0*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^8)*tan(1/2*d*x + 1/2*c) - 10*sgn(tan(1/2*
d*x + 1/2*c) + 1)/a^8)*tan(1/2*d*x + 1/2*c) + 5*sgn(tan(1/2*d*x + 1/2*c) +
1)/a^8)*tan(1/2*d*x + 1/2*c) - 3*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^8)/(a*tan(
1/2*d*x + 1/2*c)^2 + a)^(5/2) - sqrt(2)*(1280*a^(21/2)*arctan(sqrt(a)/sqrt(
-a)) + 9*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^(23/2)))/d
```

3.345 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - (10*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) + (2*C
os[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a^2*d)
```

Rubi [A] time = 0.157564, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2858, 2751, 2649, 206}

$$\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - (10*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) + (2*C
os[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a^2*d)
```

Rule 2858

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(d*Cos[e + f*x]*(
a + b*Sin[e + f*x])^(m + 2))/(b^2*f*(m + 3)), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} - \frac{2\int \frac{\frac{a}{2}-\frac{5}{2}a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} - \frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}$$

$$= -\frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{ad}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c}{3a^2d}$$

Mathematica [C] time = 0.869575, size = 149, normalized size = 1.38

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)-9\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+(12+12i)(-1)^{3/4}\tan\right.}{3a^2d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((12 + 12*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 9*Cos[(c
+ d*x)/2] + Cos[(3*(c + d*x))/2] + 9*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2
]))/(3*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.723, size = 112, normalized size = 1.

$$\frac{2+2\sin(dx+c)}{3a^3\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(3a^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{3}{2}}-3a\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*a^(3/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(3/2)-3*a*(a-a*si
n(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.75489, size = 595, normalized size = 5.51

$$\frac{\frac{3\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+2\left(\cos(dx+c)^2+(c\cdots\right.}{3\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/3*(3*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 2*(cos(d*x
 + c)^2 + (cos(d*x + c) + 5)*sin(d*x + c) - 4*cos(d*x + c) - 5)*sqrt(a*sin(
d*x + c) + a))/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.29189, size = 332, normalized size = 3.07

$$\frac{2\left(\frac{2\left(\left(\left(\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}+\frac{\left(6\sqrt{2}a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\cdots\right.}{\cdots}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] 2/3*(2*(((2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 3/sgn(tan(
1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 3/sgn(tan(1/2*d*x + 1/2*c) +
1))*tan(1/2*d*x + 1/2*c) - 2/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x
+ 1/2*c)^2 + a)^(3/2) + (6*sqrt(2)*a*arctan(sqrt(a)/sqrt(-a)) + 5*sqrt(2)*s
qrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^2) - 6*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))
/d
```

3.346 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d)
```

Rubi [A] time = 0.258633, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2874, 2985, 2649, 206, 2773}

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d)
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}\\
&= \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a^2}-\frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad}+\frac{4\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}
\end{aligned}
$$

Mathematica [C] time = 0.211244, size = 130, normalized size = 1.53

$$
-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left((4+4i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(c+dx)\right)-1\right)\right)+\log\left(-\sin\left(\frac{1}{2}(\right.\right.\right.}{d(a(\sin(c+dx)+1))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((((4 + 4*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(d*(a*(1
 + Sin[c + d*x]))^(3/2)))
```

Maple [A] time = 0.862, size = 97, normalized size = 1.1

$$
2\,\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}}{a^{3/2}\cos(dx+c)\sqrt{a+a\sin(dx+c)}d}\left(\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-\text{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))-arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/a^(3/
2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\cos(dx+c)^2\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.75037, size = 801, normalized size = 9.42

$$\frac{2\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)\cdots}{\cdots}\right.}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x +
c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/s
qrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x +
 c) - cos(d*x + c) - 2)) + sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)))/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.347 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
(2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - Cot[c + d*x]/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.223829, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2715, 2985, 2649, 206, 2773}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
(2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - Cot[c + d*x]/(a*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2715

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Di
st[1/b^2, Int[((a + b*Sin[e + f*x])^(m + 1)*(b*m - a*(m + 1)*Sin[e + f*x]))
/Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(-\frac{3a}{2}+\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{3\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a^2} + \frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}$$

$$= -\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad} - \frac{4\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \ldots\right)}{ad}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 2.10568, size = 206, normalized size = 1.82

$$\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)^3\left(-\cot\left(\tfrac{1}{4}(c+dx)\right)+(16+16i)(-1)^{3/4}\tanh^{-1}\left(\left(\tfrac{1}{2}+\tfrac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\tfrac{1}{4}(c+dx)\right)-1\right)\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*((16 + 16*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - Cot[(c + d*x)/4] + 2*(3*Log
[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[1 - Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + Sec[(c + d*x)/2] + Csc[c + d*x]*Sin[(c + d*x)/4]^2 - Csc[
c + d*x]*Sin[(c + d*x)/4]*Sin[(3*(c + d*x))/4])))/(4*d*(a*(1 + Sin[c + d*x]
))^(3/2))
```

Maple [A] time = 0.959, size = 134, normalized size = 1.2

$$-\frac{1+\sin(dx+c)}{\cos(dx+c)\sin(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\,a^2\left(2\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-3\,\mathrm{Artanh}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/a^(7/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*a^2*(2*2^(1
/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-3*arctanh((a-a*sin(
d*x+c))^(1/2)/a^(1/2)))+(a-a*sin(d*x+c))^(1/2)*a^(3/2))/sin(d*x+c)/cos(d*x+
c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.90403, size = 1133, normalized size = 10.03

$$\frac{3\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,c\right.}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) +
 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9
*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - c
os(d*x + c) - 1)) + 4*sqrt(2)*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*
x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d - (a^2*d*cos(d*x + c) + a^2*d)*s
in(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{2}\left(c+dx\right)\csc^{2}\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 2.36795, size = 636, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((6*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 8*sq
rt(2)*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 3*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqr
t(a) + sqrt(a)) + 6*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) -
16*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 3*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt
(a)) - sqrt(2)*sqrt(-a) - 3*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2
)*sqrt(-a)*a^(3/2) + sqrt(-a)*a^(3/2)) + 8*sqrt(2)*arctan(-1/2*sqrt(2)*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/s
qrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 6*arctan(-(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a
)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c
) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c)
+ 1)) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*sgn(tan(1/2*d*x + 1/2*c) +
1)) + 2/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a)*sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.348 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d
) + (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d
*x]])])/(a^(3/2)*d) + (5*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (
Cot[c + d*x]*Csc[c + d*x])/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.544084, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2874, 2984, 2985, 2649, 206, 2773}

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d
) + (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d
*x]])])/(a^(3/2)*d) + (5*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (
Cot[c + d*x]*Csc[c + d*x])/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}}+\frac{\int \frac{\csc^2(c+dx)\left(-\frac{5a^2}{2}+\frac{3}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^3} \\
&= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}}+\frac{\int \frac{\csc(c+dx)\left(\frac{11a^3}{4}-\frac{5}{4}a^3\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^4} \\
&= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}}+\frac{11\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}}{8a^2} \\
&= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}}-\frac{11\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4ad} \\
&= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}}-
\end{aligned}$$

Mathematica [C] time = 3.58916, size = 309, normalized size = 2.02

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(12\tan\left(\frac{1}{4}(c+dx)\right)+12\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)-\frac{}{\cos}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-24 - (128 + 128*I)*(-1)^(3/4)*Ar
cTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 12*Cot[(c + d*x)/4]
```

```
 - Csc[(c + d*x)/4]^2 - 44*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4
4*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Co
s[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (24*Sin[(c + d*x)/4])/(Cos[(c + d*x)
/4] - Sin[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (24*S
in[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 12*Tan[(c + d*x)/4
]))/(32*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.985, size = 164, normalized size = 1.1

$$\frac{1+\sin(dx+c)}{4\left(\sin(dx+c)\right)^{2}\cos(dx+c)\,d}\sqrt{-a\left(\sin(dx+c)-1\right)}\left(3\sqrt{-a\left(\sin(dx+c)-1\right)}a^{7/2}-5\left(-a\left(\sin(dx+c)-1\right)\right)^{3/2}a^{5/2}+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-1))^(1/2)*a
^(7/2)-5*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)+8*2^(1/2)*arctanh(1/2*(-a*(sin(d
*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*sin(d*x+c)^2-11*a^4*arctanh((-a*(sin(d
*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2)/a^(11/2)/sin(d*x+c)^2/cos(d*x+c)/(a+
a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.85439, size = 1353, normalized size = 8.84

$$\frac{11\left(\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\ldots}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(11*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8
*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (
```

```
cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 16*sqrt(2)*(a*cos(d
*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x + c)^2 - a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) - 4*(5*cos(d*x + c)^2 + (5*cos(d*x + c) + 7)*sin(d*
x + c) - 2*cos(d*x + c) - 7)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^
3 + a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d + (a^2*d*cos(d*x + c)
^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.52186, size = 837, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(
1/2*d*x + 1/2*c) + 1)) - 6/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (44*sqrt(
2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 96*sqrt(2)*sqrt(a
)*arctan(sqrt(a)/sqrt(-a)) - 22*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt
(a)) + 66*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 128*sqrt(a
)*arctan(sqrt(a)/sqrt(-a)) - 33*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 3
0*sqrt(2)*sqrt(-a) - 38*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(2*sqrt(2)*
sqrt(-a)*a^(3/2) + 3*sqrt(-a)*a^(3/2)) - 32*sqrt(2)*arctan(-1/2*sqrt(2)*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/
sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 22*arctan(-(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(
-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 11*log(abs(-sqrt(a)*tan(1/2*d*x + 1/
2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*
c) + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^3 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))*a + 6*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.349 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}+\frac{7c}{12}$$

```
[Out] (23*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*a^(3/2)*d)
 - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(a^(3/2)*d) - (9*Cot[c + d*x])/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) + (7
*Cot[c + d*x]*Csc[c + d*x])/(12*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*
x]*Csc[c + d*x]^2)/(3*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.714488, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2874, 2984, 2985, 2649, 206, 2773}

$$\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}+\frac{7c}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (23*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*a^(3/2)*d)
 - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(a^(3/2)*d) - (9*Cot[c + d*x])/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) + (7
*Cot[c + d*x]*Csc[c + d*x])/(12*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*
x]*Csc[c + d*x]^2)/(3*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{\csc^4(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{7a^2}{2}+\frac{5}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a^3}$$

$$= \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{27a^3}{4}-\frac{21}{4}a^3\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,d\ldots}{6a^4}$$

$$= -\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \ldots$$

$$= -\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{23\ldots}{\ldots}$$

$$= -\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{23\ldots}{\ldots}$$

$$= \frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{3/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \ldots$$

Mathematica [C] time = 2.23112, size = 332, normalized size = 1.74

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-\frac{8\csc^9\left(\frac{1}{2}(c+dx)\right)\left(-228\sin\left(\frac{1}{2}(c+dx)\right)-110\sin\left(\frac{3}{2}(c+dx)\right)+54\sin\left(\frac{5}{2}(c+dx)\right)+228\cos\left(\frac{1}{2}(c+dx)\right)-110\cos\left(\frac{3}{2}(c\ldots\right.\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*((768 + 768*I)*(-1)^(3/4)*ArcTanh[
(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - (8*Csc[(c + d*x)/2]^9*(22
8*Cos[(c + d*x)/2] - 110*Cos[(3*(c + d*x))/2] - 54*Cos[(5*(c + d*x))/2] - 2
28*Sin[(c + d*x)/2] - 207*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[
c + d*x] + 207*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] -
110*Sin[(3*(c + d*x))/2] + 54*Sin[(5*(c + d*x))/2] + 69*Log[1 + Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 69*Log[1 - Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]))/(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]
^2)^3))/(192*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 1.032, size = 182, normalized size = 1.

$$-\frac{1+\sin(dx+c)}{24\,(\sin(dx+c))^3\cos(dx+c)\,d}\sqrt{-a\,(\sin(dx+c)-1)}\left(-69\,a^6\mathrm{Artanh}\left(\frac{\sqrt{-a\,(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin(dx+c))^3+27\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/24/a^(15/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-69*a^6*arctanh((-
a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3+27*(-a*(sin(d*x+c)-1))^(5/2)*
a^(7/2)-40*(-a*(sin(d*x+c)-1))^(3/2)*a^(9/2)+48*2^(1/2)*arctanh(1/2*(-a*(si
n(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^6*sin(d*x+c)^3+21*(-a*(sin(d*x+c)-1))
^(1/2)*a^(11/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.93417, size = 1501, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(69*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
```

```
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
96*sqrt(2)*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*c
os(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
- (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos
(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 4*(27*cos
(d*x + c)^3 + 41*cos(d*x + c)^2 - (27*cos(d*x + c)^2 - 14*cos(d*x + c) - 49
)*sin(d*x + c) - 35*cos(d*x + c) - 49)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos
(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^3 + a^2*
d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.58764, size = 938, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a^2*sgn(
tan(1/2*d*x + 1/2*c) + 1)) - 9/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2
*d*x + 1/2*c) + 38/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) + (690*sqrt(2)*sqrt
(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 1344*sqrt(2)*sqrt(a)*arc
tan(sqrt(a)/sqrt(-a)) - 345*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
 + 966*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 1920*sqrt(a)*
arctan(sqrt(a)/sqrt(-a)) - 483*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 49
6*sqrt(2)*sqrt(-a) - 714*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(5*sqrt(2)
*sqrt(-a)*a^(3/2) + 7*sqrt(-a)*a^(3/2)) + 192*sqrt(2)*arctan(-1/2*sqrt(2)*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a)
)/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 138*arctan(-(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sq
rt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 69*log(abs(-sqrt(a)*tan(1/2*d*x +
 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1
/2*c) + 1)) - 2*(9*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^5 - 42*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^4*sqrt(a) + 72*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*a^(3/2) - 9*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))*a^2 - 38*a^(5/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3*a*sgn(tan(1/2*d*x + 1/2*c)
 + 1)))/d
```

3.350 $\int \cos^3(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}$$

```
[Out] (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^6)/(6
*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0688219, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 75}

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^6)/(6
*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)x^3(a+x)^2}{a^3}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int (a-x)x^3(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^6d}\\
&= \frac{\text{Subst}\left(\int \left(a^3x^3+a^2x^4-ax^5-x^6\right)dx,x,a\sin(c+dx)\right)}{a^6d}\\
&= \frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] time = 0.300185, size = 51, normalized size = 0.78

$$\frac{a\left(-315\cos(2(c+dx))+35\cos(6(c+dx))+96\sin^5(c+dx)(5\cos(2(c+dx))+9)\right)}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-315*Cos[2*(c + d*x)] + 35*Cos[6*(c + d*x)] + 96*(9 + 5*Cos[2*(c + d*x)
])*Sin[c + d*x]^5))/(6720*d)
```

Maple [A] time = 0.032, size = 92, normalized size = 1.4

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^4}{7}-\frac{3\sin(dx+c)(\cos(dx+c))^4}{35}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{35}\right)+a\left(-\frac{(\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2
+cos(d*x+c)^2)*sin(d*x+c))+a*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c
)^4))
```

Maxima [A] time = 0.985143, size = 68, normalized size = 1.05

$$-\frac{60\,a\sin(dx+c)^7+70\,a\sin(dx+c)^6-84\,a\sin(dx+c)^5-105\,a\sin(dx+c)^4}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 84*a*sin(d*x + c)^5 - 1
05*a*sin(d*x + c)^4)/d
```

Fricas [A] time = 1.77282, size = 188, normalized size = 2.89

$$\frac{70\,a\cos(dx+c)^6-105\,a\cos(dx+c)^4+12\left(5\,a\cos(dx+c)^6-8\,a\cos(dx+c)^4+a\cos(dx+c)^2+2\,a\right)\sin(dx+c)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*a*cos(d*x + c)^6 - 105*a*cos(d*x + c)^4 + 12*(5*a*cos(d*x + c)^6
- 8*a*cos(d*x + c)^4 + a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A] time = 7.57609, size = 90, normalized size = 1.38

$$\begin{cases} \frac{2a\sin^7(c+dx)}{35d} + \frac{a\sin^6(c+dx)}{12d} + \frac{a\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{a\sin^4(c+dx)\cos^2(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a\sin(c) + a\right)\sin^3(c)\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**7/(35*d) + a*sin(c + d*x)**6/(12*d) + a*sin(c
+ d*x)**5*cos(c + d*x)**2/(5*d) + a*sin(c + d*x)**4*cos(c + d*x)**2/(4*d),
Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**3, True))
```

Giac [A] time = 1.38701, size = 68, normalized size = 1.05

$$-\frac{60\,a\sin(dx+c)^7 + 70\,a\sin(dx+c)^6 - 84\,a\sin(dx+c)^5 - 105\,a\sin(dx+c)^4}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 84*a*sin(d*x + c)^5 - 1
05*a*sin(d*x + c)^4)/d
```

3.351 $\int \cos^3(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) - (a*Sin[c + d*x]^5)/(5
*d) - (a*Sin[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.072234, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 75}

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) - (a*Sin[c + d*x]^5)/(5
*d) - (a*Sin[c + d*x]^6)/(6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)x^2(a+x)^2}{a^2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int (a-x)x^2(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left(a^3x^2+a^2x^3-ax^4-x^5\right)dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= \frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^6(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] time = 0.214164, size = 51, normalized size = 0.78

$$\frac{a\left(-45\cos(2(c+dx))+5\cos(6(c+dx))+32\sin^3(c+dx)(3\cos(2(c+dx))+7)\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-45*Cos[2*(c + d*x)] + 5*Cos[6*(c + d*x)] + 32*(7 + 3*Cos[2*(c + d*x)])
*Sin[c + d*x]^3))/(960*d)
```

Maple [A] time = 0.029, size = 74, normalized size = 1.1

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{6}-\frac{(\cos(dx+c))^4}{12}\right)+a\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin}{15}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+a*(-1/5*sin(d*x+c
)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 0.974002, size = 68, normalized size = 1.05

$$-\frac{10\,a\sin(dx+c)^6+12\,a\sin(dx+c)^5-15\,a\sin(dx+c)^4-20\,a\sin(dx+c)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 15*a*sin(d*x + c)^4 - 20
*a*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.70878, size = 155, normalized size = 2.38

$$\frac{10\,a\cos(dx+c)^6-15\,a\cos(dx+c)^4-4\left(3\,a\cos(dx+c)^4-a\cos(dx+c)^2-2\,a\right)\sin(dx+c)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(10*a*cos(d*x + c)^6 - 15*a*cos(d*x + c)^4 - 4*(3*a*cos(d*x + c)^4 - a
*cos(d*x + c)^2 - 2*a)*sin(d*x + c))/d
```

Sympy [A] time = 4.29856, size = 90, normalized size = 1.38

$$\begin{cases} \frac{a\sin^6(c+dx)}{12d} + \frac{2a\sin^5(c+dx)}{15d} + \frac{a\sin^4(c+dx)\cos^2(c+dx)}{4d} + \frac{a\sin^3(c+dx)\cos^2(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin^2(c)\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**6/(12*d) + 2*a*sin(c + d*x)**5/(15*d) + a*sin(c
+ d*x)**4*cos(c + d*x)**2/(4*d) + a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d),
Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**3, True))
```

Giac [A] time = 1.29402, size = 68, normalized size = 1.05

$$-\frac{10\,a\sin(dx+c)^6 + 12\,a\sin(dx+c)^5 - 15\,a\sin(dx+c)^4 - 20\,a\sin(dx+c)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 15*a*sin(d*x + c)^4 - 20
*a*sin(d*x + c)^3)/d
```

3.352 $\int \cos^3(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=49

$$-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}$$

```
[Out] -(a*Cos[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^3)/(3*d) - (a*Sin[c + d*x]^5)/(
5*d)
```

Rubi [A] time = 0.0811166, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2834, 2565, 30, 2564, 14}

$$-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^3)/(3*d) - (a*Sin[c + d*x]^5)/(
5*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^3(c+dx)\sin(c+dx)\,dx + a\int \cos^3(c+dx)\sin^2(c+dx)\,dx\\ &= -\frac{a\,\text{Subst}\left(\int x^3\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \sin\right.}{d}\\ &= -\frac{a\cos^4(c+dx)}{4d} + \frac{a\,\text{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, \sin(c+dx)\right)}{d}\\ &= -\frac{a\cos^4(c+dx)}{4d} + \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.099882, size = 58, normalized size = 1.18

$$-\frac{a(-60\sin(c+dx)+10\sin(3(c+dx))+6\sin(5(c+dx))+60\cos(2(c+dx))+15\cos(4(c+dx))+45)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(45 + 60*Cos[2*(c + d*x)] + 15*Cos[4*(c + d*x)] - 60*Sin[c + d*x] + 10*
Sin[3*(c + d*x)] + 6*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.023, size = 54, normalized size = 1.1

$$\frac{1}{d}\left(a\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)-\frac{a(\cos(dx+c))^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/4*
a*cos(d*x+c)^4)
```

Maxima [A] time = 0.968803, size = 68, normalized size = 1.39

$$-\frac{12\,a\sin(dx+c)^5+15\,a\sin(dx+c)^4-20\,a\sin(dx+c)^3-30\,a\sin(dx+c)^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30
*a*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.75243, size = 127, normalized size = 2.59

$$-\frac{15\,a\cos\left(dx+c\right)^{4}+4\left(3\,a\cos\left(dx+c\right)^{4}-a\cos\left(dx+c\right)^{2}-2\,a\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*a*cos(d*x + c)^4 + 4*(3*a*cos(d*x + c)^4 - a*cos(d*x + c)^2 - 2*a
)*sin(d*x + c))/d
```

Sympy [A] time = 2.24027, size = 66, normalized size = 1.35

$$\begin{cases} \frac{2a\sin^{5}\left(c+dx\right)}{15d}+\frac{a\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}-\frac{a\cos^{4}\left(c+dx\right)}{4d} & \text{for}\ d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\sin\left(c\right)\cos^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**5/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**2/(
3*d) - a*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*cos(c)*
*3, True))
```

Giac [A] time = 1.33475, size = 68, normalized size = 1.39

$$-\frac{12\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-30\,a\sin\left(dx+c\right)^{2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30
*a*sin(d*x + c)^2)/d
```

3.353 $\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=45

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

```
[Out] (2*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - (a + a*Sin[c + d*x])^4/(4*a^3*d)
```

Rubi [A] time = 0.0332209, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - (a + a*Sin[c + d*x])^4/(4*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a+x)^2-(a+x)^3\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{2(a+a\sin(c+dx))^3}{3a^2d}-\frac{(a+a\sin(c+dx))^4}{4a^3d}\end{aligned}$$

Mathematica [A] time = 0.0145817, size = 44, normalized size = 0.98

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^4)/(4*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.033, size = 36, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^4}{4}+\frac{a\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/4*a*cos(d*x+c)^4+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.00706, size = 65, normalized size = 1.44

$$-\frac{3\,a\sin(dx+c)^4+4\,a\sin(dx+c)^3-6\,a\sin(dx+c)^2-12\,a\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

Fricas [A] time = 1.99117, size = 97, normalized size = 2.16

$$-\frac{3\,a\cos(dx+c)^4-4\left(a\cos(dx+c)^2+2\,a\right)\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 - 4*(a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A] time = 1.18872, size = 60, normalized size = 1.33

$$\begin{cases} \frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d}-\frac{a\cos^4(c+dx)}{4d} & \text{for } d\neq 0 \\ x\,(a\sin(c)+a)\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - a
*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**3, True))
```

Giac [A] time = 1.29465, size = 65, normalized size = 1.44

$$-\frac{3 a \sin(dx+c)^4 + 4 a \sin(dx+c)^3 - 6 a \sin(dx+c)^2 - 12 a \sin(dx+c)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

3.354 $\int \cos^2(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=56

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) - (
a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0552297, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 75}

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) - (
a*Sin[c + d*x]^3)/(3*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a(a-x)(a+x)^2}{x}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x}\,dx,x,a\sin(c+dx)\right)}{a^2d}\\ &= \frac{\text{Subst}\left(\int \left(a^2+\frac{a^3}{x}-ax-x^2\right)dx,x,a\sin(c+dx)\right)}{a^2d}\\ &= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin^3(c+d}{3d}\end{aligned}$$

Mathematica [A] time = 0.0290646, size = 56, normalized size = 1.

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) - (
a*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.051, size = 60, normalized size = 1.1

$$\frac{(\cos(dx+c))^2\sin(dx+c)\,a}{3d}+\frac{2a\sin(dx+c)}{3d}+\frac{a(\cos(dx+c))^2}{2d}+\frac{a\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3/d*cos(d*x+c)^2*sin(d*x+c)*a+2/3*a*sin(d*x+c)/d+1/2*a*cos(d*x+c)^2/d+a*l
n(sin(d*x+c))/d
```

Maxima [A] time = 1.01705, size = 63, normalized size = 1.12

$$-\frac{2a\sin(dx+c)^3+3a\sin(dx+c)^2-6a\log(\sin(dx+c))-6a\sin(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 6*a*log(sin(d*x + c)) - 6*a
*sin(d*x + c))/d
```

Fricas [A] time = 1.92589, size = 132, normalized size = 2.36

$$\frac{3a\cos(dx+c)^2+6a\log\left(\frac{1}{2}\sin(dx+c)\right)+2\left(a\cos(dx+c)^2+2a\right)\sin(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^2 + 6*a*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2 +
 2*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27741, size = 65, normalized size = 1.16

$$-\frac{2\,a\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-6\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-6\,a\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 6*a*log(abs(sin(d*x + c)))
- 6*a*sin(d*x + c))/d
```

3.355 $\int \cos(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=53

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*S
in[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.057603, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 75}

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*S
in[c + d*x]^2)/(2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a^2(a-x)(a+x)^2}{x^2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(-a+\frac{a^3}{x^2}+\frac{a^2}{x}-x\right)dx,x,a\sin(c+dx)\right)}{ad}\\ &= -\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0335284, size = 53, normalized size = 1.

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*S
in[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.052, size = 82, normalized size = 1.6

$$\frac{a(\cos(dx+c))^2}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{a(\cos(dx+c))^4}{d\sin(dx+c)}-\frac{(\cos(dx+c))^2\sin(dx+c)a}{d}-2\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d-1/d*a/sin(d*x+c)*cos(d*x+c)^4-1/d*c
os(d*x+c)^2*sin(d*x+c)*a-2*a*sin(d*x+c)/d
```

Maxima [A] time = 0.987907, size = 62, normalized size = 1.17

$$-\frac{a\sin(dx+c)^2-2a\log(\sin(dx+c))+2a\sin(dx+c)+\frac{2a}{\sin(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*sin(d*x + c)^2 - 2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + 2*a/sin
(d*x + c))/d
```

Fricas [A] time = 1.87556, size = 176, normalized size = 3.32

$$\frac{4a\cos(dx+c)^2+4a\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+\left(2a\cos(dx+c)^2-a\right)\sin(dx+c)-8a}{4d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*cos(d*x + c)^2 + 4*a*log(1/2*sin(d*x + c))*sin(d*x + c) + (2*a*cos
(d*x + c)^2 - a)*sin(d*x + c) - 8*a)/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22565, size = 63, normalized size = 1.19

$$-\frac{a\sin\left(dx+c\right)^{2}-2\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a}{\sin\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*sin(d*x + c)^2 - 2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) + 2*
a/sin(d*x + c))/d
```

3.356 $\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0341207, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 75}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (a*Sin[c + d*x])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{x^3}+\frac{a^2}{x^2}-\frac{a}{x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.1128, size = 60, normalized size = 1.11

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}-\frac{a\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Ta
n[c + d*x]]))/(2*d) - (a*Sin[c + d*x])/d
```

Maple [A] time = 0.056, size = 83, normalized size = 1.5

$$-\frac{a(\cos(dx+c))^4}{d\sin(dx+c)}-\frac{(\cos(dx+c))^2\sin(dx+c)a}{d}-2\frac{a\sin(dx+c)}{d}-\frac{a(\cot(dx+c))^2}{2d}-\frac{a\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^4-1/d*cos(d*x+c)^2*sin(d*x+c)*a-2*a*sin(d*x+c)
/d-1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.00127, size = 61, normalized size = 1.13

$$-\frac{2a\log(\sin(dx+c))+2a\sin(dx+c)+\frac{2a\sin(dx+c)+a}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a)/sin
(d*x + c)^2)/d
```

Fricas [A] time = 1.45479, size = 167, normalized size = 3.09

$$-\frac{2\left(a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+2\left(a\cos(dx+c)^2-2a\right)\sin(dx+c)-a}{2\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2
- 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3406, size = 62, normalized size = 1.15

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a\sin(dx+c)+a}{\sin(dx+c)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a
)/sin(d*x + c)^2)/d
```

3.357 $\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^4(c+dx)}{4ad}$$

```
[Out] Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d)
```

Rubi [A] time = 0.0970588, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)x^2}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int (a-x)x^2\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left(ax^2-x^3\right)\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\sin^3(c+dx)}{3ad}-\frac{\sin^4(c+dx)}{4ad}
\end{aligned}$$

Mathematica [A] time = 0.123074, size = 28, normalized size = 0.76

$$\frac{(4-3\sin(c+dx))\sin^3(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((4 - 3*Sin[c + d*x])*Sin[c + d*x]^3)/(12*a*d)
```

Maple [A] time = 0.023, size = 30, normalized size = 0.8

$$-\frac{1}{da}\left(\frac{(\sin(dx+c))^4}{4}-\frac{(\sin(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a*(1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3)
```

Maxima [A] time = 0.98732, size = 39, normalized size = 1.05

$$-\frac{3\sin(dx+c)^4-4\sin(dx+c)^3}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3)/(a*d)
```

Fricas [A] time = 1.41939, size = 120, normalized size = 3.24

$$-\frac{3\cos(dx+c)^4-6\cos(dx+c)^2+4\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 1)*sin(d*x
 + c))/(a*d)
```

Sympy [A] time = 27.4755, size = 277, normalized size = 7.49

$$\begin{cases}\frac{8\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad}-\frac{12\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\,\text{ta}\ldots} \\ \frac{x\sin^2(c)\cos^3(c)}{a\sin(c)+a}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*tan(c/2 + d*x/2)**5/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/
2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3
*a*d) - 12*tan(c/2 + d*x/2)**4/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a
*d) + 8*tan(c/2 + d*x/2)**3/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d
*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d)
, Ne(d, 0)), (x*sin(c)**2*cos(c)**3/(a*sin(c) + a), True))
```

Giac [A] time = 1.38205, size = 39, normalized size = 1.05

$$-\frac{3 \sin(dx+c)^4 - 4 \sin(dx+c)^3}{12\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3)/(a*d)
```

3.358 $\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] Sin[c + d*x]^2/(2*a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0803932, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2835, 2564, 30}

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^2/(2*a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos(c+dx)\sin(c+dx)\,dx}{a}-\frac{\int \cos(c+dx)\sin^2(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}(\int x\,dx,x,\sin(c+dx))}{ad}-\frac{\text{Subst}\left(\int x^2\,dx,x,\sin(c+dx)\right)}{ad}\\ &= \frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.0927779, size = 28, normalized size = 0.76

$$\frac{(3-2\sin(c+dx))\sin^2(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((3 - 2*Sin[c + d*x])*Sin[c + d*x]^2)/(6*a*d)
```

Maple [A] time = 0.02, size = 30, normalized size = 0.8

$$-\frac{1}{da}\left(\frac{(\sin(dx+c))^3}{3}-\frac{(\sin(dx+c))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a*(1/3*sin(d*x+c)^3-1/2*sin(d*x+c)^2)
```

Maxima [A] time = 0.982102, size = 39, normalized size = 1.05

$$-\frac{2\sin(dx+c)^3-3\sin(dx+c)^2}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sin(d*x + c)^3 - 3*sin(d*x + c)^2)/(a*d)
```

Fricas [A] time = 1.24017, size = 93, normalized size = 2.51

$$-\frac{3\cos(dx+c)^2-2\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c))/(a*d)
```

Sympy [A] time = 14.8111, size = 224, normalized size = 6.05

$$\begin{cases}\frac{6\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad}-\frac{8\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad}+\frac{6\tan\cdots}{3ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^4\cdots} \\ \frac{x\sin(c)\cos^3(c)}{a\sin(c)+a}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((6*tan(c/2 + d*x/2)**4/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2
 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 8*tan(c/2 + d*x/2)**3/(
3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x
/2)**2 + 3*a*d) + 6*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*
tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d), Ne(d, 0)), (x*sin
(c)*cos(c)**3/(a*sin(c) + a), True))
```

Giac [A] time = 1.38533, size = 39, normalized size = 1.05

$$-\frac{2\,\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)^{2}}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(2*sin(d*x + c)^3 - 3*sin(d*x + c)^2)/(a*d)
```

3.359 $\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.0416463, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2667}

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)\,dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.036708, size = 24, normalized size = 0.75

$$-\frac{(\sin(c+dx)-2)\sin(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((-2 + Sin[c + d*x])*Sin[c + d*x])/(2*a*d)
```

Maple [A] time = 0.023, size = 28, normalized size = 0.9

$$-\frac{1}{da}\left(\frac{(\sin(dx+c))^2}{2}-\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a*(1/2*sin(d*x+c)^2-sin(d*x+c))
```

Maxima [A] time = 0.981905, size = 34, normalized size = 1.06

$$-\frac{\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

Fricas [A] time = 1.39142, size = 61, normalized size = 1.91

$$\frac{\cos\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(d*x + c)^2 + 2*sin(d*x + c))/(a*d)
```

Sympy [A] time = 8.14768, size = 245, normalized size = 7.66

$$\begin{cases} -\frac{2\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{2\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{6\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{2\tan\left(\frac{c}{2}\right.}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} \\ \frac{x\cos^{3}(c)}{a\sin(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-2*tan(c/2 + d*x/2)**4/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2
+ d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d
*tan(c/2 + d*x/2)**2 + a*d) - 6*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**
4 + 2*a*d*tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*
x/2)**4 + 2*a*d*tan(c/2 + d*x/2)**2 + a*d) - 2/(a*d*tan(c/2 + d*x/2)**4 + 2
*a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a), Tr
ue))
```

Giac [A] time = 1.41785, size = 34, normalized size = 1.06

$$-\frac{\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

3.360 $\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d)
```

Rubi [A] time = 0.0768426, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{a(a-x)}{x}\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \frac{a-x}{x}\,dx, x, a\sin(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(-1+\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{a^2d}\\
&= \frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0332479, size = 23, normalized size = 0.79

$$\frac{\log(\sin(c+dx))-\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Log[Sin[c + d*x]] - Sin[c + d*x])/(a*d)
```

Maple [A] time = 0.036, size = 33, normalized size = 1.1

$$-\frac{1}{da\csc(dx+c)}-\frac{\ln(\csc(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a/csc(d*x+c)-1/d/a*ln(csc(d*x+c))
```

Maxima [A] time = 0.996756, size = 36, normalized size = 1.24

$$\frac{\frac{\log(\sin(dx+c))}{a}-\frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c))/a - sin(d*x + c)/a)/d
```

Fricas [A] time = 1.52175, size = 62, normalized size = 2.14

$$\frac{\log\left(\frac{1}{2}\sin(dx+c)\right)-\sin(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (log(1/2*sin(d*x + c)) - sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30181, size = 38, normalized size = 1.31

$$\frac{\frac{\log(|\sin(dx+c)|)}{a} - \frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(sin(d*x + c)))/a - sin(d*x + c)/a)/d
```

3.361 $\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=30

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0792449, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{a^2(a-x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \frac{a-x}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{ad}\\
&= -\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0370506, size = 22, normalized size = 0.73

$$-\frac{\csc(c+dx)+\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Csc[c + d*x] + Log[Sin[c + d*x]])/(a*d))
```

Maple [A] time = 0.034, size = 30, normalized size = 1.

$$-\frac{\csc(dx+c)}{da}+\frac{\ln(\csc(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -csc(d*x+c)/d/a+1/d/a*ln(csc(d*x+c))
```

Maxima [A] time = 0.970025, size = 39, normalized size = 1.3

$$-\frac{\frac{\log(\sin(dx+c))}{a}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c))/a + 1/(a*sin(d*x + c)))/d
```

Fricas [A] time = 1.26051, size = 84, normalized size = 2.8

$$-\frac{\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+1}{ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(log(1/2*sin(d*x + c))*sin(d*x + c) + 1)/(a*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39437, size = 41, normalized size = 1.37

$$-\frac{\frac{\log(|\sin(dx+c)|)}{a}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(sin(d*x + c)))/a + 1/(a*sin(d*x + c)))/d
```

3.362 $\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.06456, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2706, 2606, 30, 8}

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int 1\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x\,dx,x,\csc(c+dx)\right)}{ad}\\ &= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.0301619, size = 24, normalized size = 0.75

$$-\frac{(\csc(c+dx)-2)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((-2 + Csc[c + d*x])*Csc[c + d*x])/(2*a*d)
```

Maple [A] time = 0.036, size = 25, normalized size = 0.8

$$\frac{1}{da}\left(-\frac{(\csc(dx+c))^2}{2}+\csc(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/2*csc(d*x+c)^2+csc(d*x+c))
```

Maxima [A] time = 1.18528, size = 35, normalized size = 1.09

$$\frac{2\sin(dx+c)-1}{2\,ad\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

Fricas [A] time = 1.25667, size = 73, normalized size = 2.28

$$-\frac{2\sin(dx+c)-1}{2\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34424, size = 35, normalized size = 1.09

$$\frac{2\sin(dx+c)-1}{2\,ad\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

3.363 $\int \frac{\cot^3(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0809194, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{a^4(a-x)}{x^4}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{a\,\text{Subst}\left(\int \frac{a-x}{x^4}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a\,\text{Subst}\left(\int \left(\frac{a}{x^4}-\frac{1}{x^3}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] time = 0.0529001, size = 28, normalized size = 0.76

$$\frac{(3\sin(c+dx)-2)\csc^3(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^3*(-2 + 3*Sin[c + d*x]))/(6*a*d)
```

Maple [A] time = 0.038, size = 29, normalized size = 0.8

$$\frac{1}{da}\left(-\frac{(\csc(dx+c))^3}{3}+\frac{(\csc(dx+c))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/3*csc(d*x+c)^3+1/2*csc(d*x+c)^2)
```

Maxima [A] time = 1.12923, size = 35, normalized size = 0.95

$$\frac{3\sin(dx+c)-2}{6\,ad\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c) - 2)/(a*d*sin(d*x + c)^3)
```

Fricas [A] time = 1.28677, size = 93, normalized size = 2.51

$$-\frac{3\sin(dx+c)-2}{6\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sin(d*x + c) - 2)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39771, size = 35, normalized size = 0.95

$$\frac{3\,\sin\left(dx+c\right)-2}{6\,ad\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*sin(d*x + c) - 2)/(a*d*sin(d*x + c)^3)
```

3.364 $\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}$$

```
[Out] Csc[c + d*x]^3/(3*a*d) - Csc[c + d*x]^4/(4*a*d)
```

Rubi [A] time = 0.0976363, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^3/(3*a*d) - Csc[c + d*x]^4/(4*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx&=\frac{\text{Subst}\left(\int \frac{a^5(a-x)}{x^5}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{a^2\,\text{Subst}\left(\int \frac{a-x}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{a^2\,\text{Subst}\left(\int \left(\frac{a}{x^5}-\frac{1}{x^4}\right)dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}\end{aligned}$$

Mathematica [A] time = 0.0439687, size = 28, normalized size = 0.76

$$\frac{(4\sin(c+dx)-3)\csc^4(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^4*(-3 + 4*Sin[c + d*x]))/(12*a*d)
```

Maple [A] time = 0.039, size = 29, normalized size = 0.8

$$\frac{1}{da}\left(-\frac{(\csc(dx+c))^4}{4}+\frac{(\csc(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/4*csc(d*x+c)^4+1/3*csc(d*x+c)^3)
```

Maxima [A] time = 1.11174, size = 35, normalized size = 0.95

$$\frac{4\sin(dx+c)-3}{12\,ad\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*sin(d*x + c) - 3)/(a*d*sin(d*x + c)^4)
```

Fricas [A] time = 1.32628, size = 104, normalized size = 2.81

$$\frac{4\sin(dx+c)-3}{12\left(ad\cos(dx+c)^4-2\,ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sin(d*x + c) - 3)/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43395, size = 35, normalized size = 0.95

$$\frac{4\sin(dx+c)-3}{12\,ad\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(4*sin(d*x + c) - 3)/(a*d*sin(d*x + c)^4)
```

3.365 $\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$-\frac{a\cos^9(c+dx)}{9d}+\frac{2a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c}{}$$

```
[Out] (3*a*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (2*a*Cos[c + d*x]^7)/(7*d) - (a*Co
s[c + d*x]^9)/(9*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c +
d*x]^3*Sin[c + d*x])/(64*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*C
os[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rubi [A] time = 0.169091, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 270}

$$-\frac{a\cos^9(c+dx)}{9d}+\frac{2a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (2*a*Cos[c + d*x]^7)/(7*d) - (a*Co
s[c + d*x]^9)/(9*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c +
d*x]^3*Sin[c + d*x])/(64*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*C
os[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^4(c+dx)\,dx + a\int \cos^4(c+dx)\sin^5(c+dx)$$

$$= -\frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3a)\int \cos^4(c+dx)\sin^2(c+dx)$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{16}a$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^7(c+dx)}{7d} - \frac{a\cos^9(c+dx)}{9d} + \frac{a\cos^3(c+}{}$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^7(c+dx)}{7d} - \frac{a\cos^9(c+dx)}{9d} + \frac{3a\cos(c+}{}$$

$$= \frac{3ax}{128} - \frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^7(c+dx)}{7d} - \frac{a\cos^9(c+dx)}{9d} + \frac{3a\cos}{}$$

Mathematica [A] time = 0.273619, size = 84, normalized size = 0.59

$$\frac{a(-2520\sin(4(c+dx))+315\sin(8(c+dx))-7560\cos(c+dx)-1680\cos(3(c+dx))+1008\cos(5(c+dx))+180\cos}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(7560*c + 7560*d*x - 7560*Cos[c + d*x] - 1680*Cos[3*(c + d*x)] + 1008*Co
s[5*(c + d*x)] + 180*Cos[7*(c + d*x)] - 140*Cos[9*(c + d*x)] - 2520*Sin[4*(
c + d*x)] + 315*Sin[8*(c + d*x)]))/(322560*d)
```

Maple [A] time = 0.034, size = 124, normalized size = 0.9

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^5}{9} - \frac{4(\sin(dx+c))^2(\cos(dx+c))^5}{63} - \frac{8(\cos(dx+c))^5}{315}\right) + a\left(-\frac{(\sin(dx+c))^3(\cos}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315
*cos(d*x+c)^5)+a*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)
^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

Maxima [A] time = 1.01538, size = 96, normalized size = 0.67

$$-\frac{1024\left(35\,\cos\left(dx+c\right)^{9}-90\,\cos\left(dx+c\right)^{7}+63\,\cos\left(dx+c\right)^{5}\right)a-315\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right.}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/322560*(1024*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a - 315*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.50918, size = 270, normalized size = 1.89

$$-\frac{4480\,a\cos\left(dx+c\right)^{9}-11520\,a\cos\left(dx+c\right)^{7}+8064\,a\cos\left(dx+c\right)^{5}-945\,adx-315\left(16\,a\cos\left(dx+c\right)^{7}-24\,a\cos\left(dx+c\right.\right.}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40320*(4480*a*cos(d*x + c)^9 - 11520*a*cos(d*x + c)^7 + 8064*a*cos(d*x +
 c)^5 - 945*a*d*x - 315*(16*a*cos(d*x + c)^7 - 24*a*cos(d*x + c)^5 + 2*a*co
s(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 23.5876, size = 272, normalized size = 1.9

$$\begin{cases}\frac{3ax\sin^{8}(c+dx)}{128}+\frac{3ax\sin^{6}(c+dx)\cos^{2}(c+dx)}{32}+\frac{9ax\sin^{4}(c+dx)\cos^{4}(c+dx)}{64}+\frac{3ax\sin^{2}(c+dx)\cos^{6}(c+dx)}{32}+\frac{3ax\cos^{8}(c+dx)}{128}+\frac{3a\sin^{7}(c+dx)\cos(c+d}{128d}\\ x\left(a\sin\left(c\right)+a\right)\sin^{4}\left(c\right)\cos^{4}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**8/128 + 3*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 9*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a*x*sin(c + d*x)**2*cos
(c + d*x)**6/32 + 3*a*x*cos(c + d*x)**8/128 + 3*a*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + 11*a*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - a*sin(c + d*x)
**4*cos(c + d*x)**5/(5*d) - 11*a*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) -
4*a*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 3*a*sin(c + d*x)*cos(c + d*x)*
*7/(128*d) - 8*a*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(
c)**4*cos(c)**4, True))
```

Giac [A] time = 1.37223, size = 144, normalized size = 1.01

$$\frac{3}{128}\,ax-\frac{a\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{a\cos\left(7\,dx+7\,c\right)}{1792\,d}+\frac{a\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{192\,d}-\frac{3\,a\cos\left(dx+c\right)}{128\,d}+\frac{a\sin\left(8\,dx+\right.}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*a*x - 1/2304*a*cos(9*d*x + 9*c)/d + 1/1792*a*cos(7*d*x + 7*c)/d + 1/3
20*a*cos(5*d*x + 5*c)/d - 1/192*a*cos(3*d*x + 3*c)/d - 3/128*a*cos(d*x + c)
/d + 1/1024*a*sin(8*d*x + 8*c)/d - 1/128*a*sin(4*d*x + 4*c)/d
```

3.366 $\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=127

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{64d}+$$

```
[Out] (3*a*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (3*a*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) -
 (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*Cos[c + d*x]^5*Sin[c + d*x]^3)
/(8*d)
```

Rubi [A] time = 0.164829, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{64d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (3*a*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) -
 (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*Cos[c + d*x]^5*Sin[c + d*x]^3)
/(8*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^3(c+dx)\,dx + a\int \cos^4(c+dx)\sin^4(c+dx)$$

$$= -\frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3a)\int \cos^4(c+dx)\sin^2(c+dx$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{16}a$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{64d} -$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3a\cos(c+dx)\sin(c+dx)}{128d} +$$

$$= \frac{3ax}{128} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3a\cos(c+dx)\sin(c+d}{128d}$$

Mathematica [A] time = 0.204121, size = 71, normalized size = 0.56

$$\frac{a(-280\sin(4(c+dx))+35\sin(8(c+dx))-1680\cos(c+dx)-560\cos(3(c+dx))+112\cos(5(c+dx))+80\cos(7(c+}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(840*d*x - 1680*Cos[c + d*x] - 560*Cos[3*(c + d*x)] + 112*Cos[5*(c + d*x
)] + 80*Cos[7*(c + d*x)] - 280*Sin[4*(c + d*x)] + 35*Sin[8*(c + d*x)]))/(35
840*d)
```

Maple [A] time = 0.033, size = 106, normalized size = 0.8

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^5}{8} - \frac{\sin(dx+c)(\cos(dx+c))^5}{16} + \frac{\sin(dx+c)}{64}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+a*(-1/7*sin(d*x+c
)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5))
```

Maxima [A] time = 1.09684, size = 82, normalized size = 0.65

$$\frac{1024 \left(5 \cos (dx+c)^7 - 7 \cos (dx+c)^5\right)a + 35 \left(24 dx + 24 c + \sin (8 dx + 8 c) - 8 \sin (4 dx + 4 c)\right)a}{35840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/35840*(1024*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a + 35*(24*d*x + 24*c +
 sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.49883, size = 230, normalized size = 1.81

$$\frac{640 a \cos (dx+c)^7 - 896 a \cos (dx+c)^5 + 105 adx + 35 \left(16 a \cos (dx+c)^7 - 24 a \cos (dx+c)^5 + 2 a \cos (dx+c)^3 + 3 a c\right.}{4480 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4480*(640*a*cos(d*x + c)^7 - 896*a*cos(d*x + c)^5 + 105*a*d*x + 35*(16*a*
cos(d*x + c)^7 - 24*a*cos(d*x + c)^5 + 2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [A] time = 13.0148, size = 248, normalized size = 1.95

$$\begin{cases} \frac{3ax\sin^8(c+dx)}{128} + \frac{3ax\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{9ax\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{3ax\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{3ax\cos^8(c+dx)}{128} + \frac{3a\sin^7(c+dx)\cos(c+d}{128d} \\ x\left(a\sin(c)+a\right)\sin^3(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**8/128 + 3*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 9*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a*x*sin(c + d*x)**2*cos
(c + d*x)**6/32 + 3*a*x*cos(c + d*x)**8/128 + 3*a*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + 11*a*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*a*sin(c + d
*x)**3*cos(c + d*x)**5/(128*d) - a*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) -
3*a*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 2*a*cos(c + d*x)**7/(35*d), Ne(d
, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**4, True))
```

Giac [A] time = 1.42067, size = 124, normalized size = 0.98

$$\frac{3}{128} ax + \frac{a \cos (7 dx + 7 c)}{448 d} + \frac{a \cos (5 dx + 5 c)}{320 d} - \frac{a \cos (3 dx + 3 c)}{64 d} - \frac{3 a \cos (dx + c)}{64 d} + \frac{a \sin (8 dx + 8 c)}{1024 d} - \frac{a \sin (4 dx +}{128 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*a*x + 1/448*a*cos(7*d*x + 7*c)/d + 1/320*a*cos(5*d*x + 5*c)/d - 1/64*
a*cos(3*d*x + 3*c)/d - 3/64*a*cos(d*x + c)/d + 1/1024*a*sin(8*d*x + 8*c)/d
- 1/128*a*sin(4*d*x + 4*c)/d
```

3.367 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=103

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+$$

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Co
s[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.132512, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```


```
[Out] (a*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Co
s[c + d*x]^5*Sin[c + d*x])/(6*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^2(c+dx)\,dx + a\int \cos^4(c+dx)\sin^3(c+dx)$$
$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}a\int \cos^4(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{8}a\int c$$
$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} + \frac{a}{}$$
$$= \frac{ax}{16} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d}$$

Mathematica [A] time = 0.201471, size = 81, normalized size = 0.79

$$\frac{a(105\sin(2(c+dx)) - 105\sin(4(c+dx)) - 35\sin(6(c+dx)) - 315\cos(c+dx) - 105\cos(3(c+dx)) + 21\cos(5(c+dx$$
$$6720d$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(420*d*x - 315*Cos[c + d*x] - 105*Cos[3*(c + d*x)] + 21*Cos[5*(c + d*x)]
 + 15*Cos[7*(c + d*x)] + 105*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)] - 35*S
in[6*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.03, size = 88, normalized size = 0.9

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7} - \frac{2(\cos(dx+c))^5}{35}\right) + a\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6} + \frac{\sin(dx+c)}{24}\left((\cos(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a*(-1/6*sin(d*x+c
)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*
c))
```

Maxima [A] time = 1.00037, size = 88, normalized size = 0.85

$$\frac{192\left(5\cos(dx+c)^7 - 7\cos(dx+c)^5\right)a + 35\left(4\sin(2dx+2c)^3 + 12dx + 12c - 3\sin(4dx+4c)\right)a}{6720d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6720*(192*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a + 35*(4*sin(2*d*x + 2*c
)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.53086, size = 198, normalized size = 1.92

$$\frac{240\,a\cos\left(dx+c\right)^{7}-336\,a\cos\left(dx+c\right)^{5}+105\,adx-35\left(8\,a\cos\left(dx+c\right)^{5}-2\,a\cos\left(dx+c\right)^{3}-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*cos(d*x + c)^7 - 336*a*cos(d*x + c)^5 + 105*a*d*x - 35*(8*a*c
os(d*x + c)^5 - 2*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 7.72064, size = 192, normalized size = 1.86

$$\begin{cases}\frac{ax\sin^{6}(c+dx)}{16}+\frac{3ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{ax\cos^{6}(c+dx)}{16}+\frac{a\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{a\sin^{3}(c+dx)\cos^{3}(c+dx)}{6d}- \\ x\left(a\sin\left(c\right)+a\right)\sin^{2}\left(c\right)\cos^{4}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - a*sin(c + d*x)*cos(c + d*x)**
5/(16*d) - 2*a*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*
*2*cos(c)**4, True))
```

Giac [A] time = 1.34899, size = 144, normalized size = 1.4

$$\frac{1}{16}\,ax+\frac{a\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{a\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\,a\cos\left(dx+c\right)}{64\,d}-\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+4\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x + 1/448*a*cos(7*d*x + 7*c)/d + 1/320*a*cos(5*d*x + 5*c)/d - 1/64*a
*cos(3*d*x + 3*c)/d - 3/64*a*cos(d*x + c)/d - 1/192*a*sin(6*d*x + 6*c)/d -
1/64*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2*d*x + 2*c)/d
```

3.368 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}$$

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(16*d)
+ (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d)
```

Rubi [A] time = 0.0993137, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(16*d)
+ (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin(c+dx)\,dx + a\int \cos^4(c+dx)\sin^2(c+dx)\,dx$$
$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}a\int \cos^4(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int x^4 \ldots\right.}{\ldots}$$
$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a\cos^5(c+dx)\sin(c+\ldots}{6d}$$
$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\cos^3(c+dx)\sin(c+\ldots}{24d}$$
$$= \frac{ax}{16} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\cos^3(c+dx)\sin(\ldots}{24d}$$

Mathematica [A] time = 0.153168, size = 71, normalized size = 0.82

$$-\frac{a(-15\sin(2(c+dx))+15\sin(4(c+dx))+5\sin(6(c+dx))+120\cos(c+dx)+60\cos(3(c+dx))+12\cos(5(c+dx))-\ldots}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(-60*d*x + 120*Cos[c + d*x] + 60*Cos[3*(c + d*x)] + 12*Cos[5*(c + d*x)]
 - 15*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.023, size = 68, normalized size = 0.8

$$\frac{1}{d}\left(a\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6}+\frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{dx}{16}+\frac{c}{16}\right)-\frac{a(\cos(dx+c))^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin
(d*x+c)+1/16*d*x+1/16*c)-1/5*a*cos(d*x+c)^5)
```

Maxima [A] time = 1.05019, size = 70, normalized size = 0.8

$$-\frac{192\,a\cos(dx+c)^5-5\left(4\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\sin(4\,dx+4\,c)\right)a}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/960*(192*a*cos(d*x + c)^5 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*
sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.50034, size = 163, normalized size = 1.87

$$-\frac{48\,a\cos\left(dx+c\right)^{5}-15\,adx+5\left(8\,a\cos\left(dx+c\right)^{5}-2\,a\cos\left(dx+c\right)^{3}-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(48*a*cos(d*x + c)^5 - 15*a*d*x + 5*(8*a*cos(d*x + c)^5 - 2*a*cos(d*
x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 4.36124, size = 167, normalized size = 1.92

$$\begin{cases} \frac{ax\sin^{6}(c+dx)}{16}+\frac{3ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{ax\cos^{6}(c+dx)}{16}+\frac{a\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{a\sin^{3}(c+dx)\cos^{3}(c+dx)}{6d} \\ x\left(a\sin\left(c\right)+a\right)\sin\left(c\right)\cos^{4}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a*cos(c + d*x)**5/(5*d), Ne(d,
0)), (x*(a*sin(c) + a)*sin(c)*cos(c)**4, True))
```

Giac [A] time = 1.35401, size = 124, normalized size = 1.43

$$\frac{1}{16}\,ax-\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}-\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{a\sin\left(2\,dx+\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x - 1/80*a*cos(5*d*x + 5*c)/d - 1/16*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d - 1/192*a*sin(6*d*x + 6*c)/d - 1/64*a*sin(4*d*x + 4*c)/d + 1/6
4*a*sin(2*d*x + 2*c)/d
```

3.369 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ax}{8}$$

```
[Out] (3*a*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d
*x]^3)/(3*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*Cos[c + d*x]^3*Si
n[c + d*x])/(4*d)
```

Rubi [A] time = 0.086916, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2592, 302, 206, 2635, 8}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d
*x]^3)/(3*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*Cos[c + d*x]^3*Si
n[c + d*x])/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\,dx + a\int \cos^3(c+dx)\cot(c+dx)\,dx$$

$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\big(}{}$$

$$= \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}(3a)\int$$

$$= \frac{3ax}{8} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{3a\cos(c+dx)\sin(c+dx}{8d}$$

$$= \frac{3ax}{8} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} +$$

Mathematica [A] time = 0.146551, size = 81, normalized size = 0.91

$$\frac{a\left(120\cos(c+dx)+8\cos(3(c+dx))+3\left(8\sin(2(c+dx))+\sin(4(c+dx))+32\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-32\log\left(\cos\left(\right.\right.\right.\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(120*Cos[c + d*x] + 8*Cos[3*(c + d*x)] + 3*(12*c + 12*d*x - 32*Log[Cos[(
c + d*x)/2]] + 32*Log[Sin[(c + d*x)/2]] + 8*Sin[2*(c + d*x)] + Sin[4*(c + d
*x)])))/(96*d)
```

Maple [A] time = 0.05, size = 97, normalized size = 1.1

$$\frac{a(\cos(dx+c))^3\sin(dx+c)}{4d} + \frac{3\cos(dx+c)\,a\sin(dx+c)}{8d} + \frac{3ax}{8} + \frac{3ca}{8d} + \frac{a(\cos(dx+c))^3}{3d} + \frac{\cos(dx+c)\,a}{d} + \frac{a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4*a*cos(d*x+c)^3*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+3/8*a*x+3/8/d
*c*a+1/3*a*cos(d*x+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.14849, size = 109, normalized size = 1.22

$$\frac{16\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a+3\left(12dx+12c+\sin(4dx\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a)/d
```

Fricas [A] time = 1.54472, size = 252, normalized size = 2.83

$$\frac{8\,a\cos\left(dx+c\right)^{3}+9\,adx+24\,a\cos\left(dx+c\right)-12\,a\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+12\,a\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(2\,a\cos\left(\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*a*d*x + 24*a*cos(d*x + c) - 12*a*log(1/2*cos(d
*x + c) + 1/2) + 12*a*log(-1/2*cos(d*x + c) + 1/2) + 3*(2*a*cos(d*x + c)^3
+ 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46771, size = 196, normalized size = 2.2

$$\frac{9\,(dx+c)a+24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-96\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+9\,a\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(15*a*tan(1/2
*d*x + 1/2*c)^7 - 48*a*tan(1/2*d*x + 1/2*c)^6 - 9*a*tan(1/2*d*x + 1/2*c)^5
- 96*a*tan(1/2*d*x + 1/2*c)^4 + 9*a*tan(1/2*d*x + 1/2*c)^3 - 80*a*tan(1/2*d
*x + 1/2*c)^2 - 15*a*tan(1/2*d*x + 1/2*c) - 32*a)/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

3.370 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ax}{2}$$

```
[Out] (-3*a*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
d*x]^3)/(3*d) - (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/
(2*d)
```

Rubi [A] time = 0.113601, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2838, 2591, 288, 321, 203, 2592, 302, 206}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
d*x]^3)/(3*d) - (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/
(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^3(c+dx)\cot(c+dx)\,dx + a\int \cos^2(c+dx)\cot^2(c+dx)\,dx$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(\ldots\right.}{d}$$

$$= \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cos(\ldots\right.}{d}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\cot(c+dx)}{2d} + \frac{a\cos^2(c+dx)\,\text{co}\ldots}{2d}$$

$$= -\frac{3ax}{2} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \ldots$$

Mathematica [A] time = 0.414031, size = 77, normalized size = 0.93

$$\frac{a\left(15\cos(c+dx)+\cos(3(c+dx))-3\left(\sin(2(c+dx))+4\cot(c+dx)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\ldots\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(15*Cos[c + d*x] + Cos[3*(c + d*x)] - 3*(6*c + 6*d*x + 4*Cot[c + d*x] +
4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)])))/(12
*d)
```

Maple [A] time = 0.053, size = 119, normalized size = 1.4

$$\frac{a(\cos(dx+c))^3}{3d} + \frac{\cos(dx+c)\,a}{d} + \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d} - \frac{a(\cos(dx+c))^5}{d\sin(dx+c)} - \frac{a(\cos(dx+c))^3\sin(dx+c)}{d} - \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3*a*cos(d*x+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*a/s
in(d*x+c)*cos(d*x+c)^5-a*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a*cos(d*x+c)*sin(d*x
+c)/d-3/2*a*x-3/2/d*c*a
```

Maxima [A] time = 1.57359, size = 122, normalized size = 1.47

$$\frac{\left(2\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a-3\left(3\,dx+3\,c+\frac{3\tan\left(dx+c\right)^{2}}{\tan\left(dx+c\right)^{3}+\tan}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(c
os(d*x + c) - 1))*a - 3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)
^3 + tan(d*x + c)))*a)/d
```

Fricas [A] time = 1.66776, size = 300, normalized size = 3.61

$$\frac{3\,a\cos\left(dx+c\right)^{3}-3\,a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\,a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a\cos\left(dx+\right.}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^3 - 3*a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*
a*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*a*cos(d*x + c) + (2*a*cos(d
*x + c)^3 - 9*a*d*x + 6*a*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38823, size = 192, normalized size = 2.31

$$-\frac{9\left(dx+c\right)a-6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,a\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + 3*(2*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c) - 2*(3*a*t
an(1/2*d*x + 1/2*c)^5 + 12*a*tan(1/2*d*x + 1/2*c)^4 + 12*a*tan(1/2*d*x + 1/
2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) + 8*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.371 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{3a\cos(c+dx)}{2d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx)}{2d}$$

```
[Out] (-3*a*x)/2 + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) -
 (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (a*Cos[
c + d*x]*Cot[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.109099, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2838, 2592, 288, 321, 206, 2591, 203}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```


```
[Out] (-3*a*x)/2 + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) -
 (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (a*Cos[
c + d*x]*Cot[c + d*x]^2)/(2*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^2(c+dx)\cot^2(c+dx)\,dx + a\int \cos(c+dx)\cot^3(c+dx)\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx,x,\cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx,x,\text{co}\right.}{d}\\ &= \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} + \frac{(3a)\,\text{Subst}}{}\\ &= -\frac{3a\cos(c+dx)}{2d} - \frac{3a\cot(c+dx)}{2d} + \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\,\text{co}}{}\\ &= -\frac{3ax}{2} + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} - \frac{3a\cot(c+dx)}{2d} + \end{aligned}$$

Mathematica [A] time = 0.795045, size = 94, normalized size = 1.

$$-\frac{a\left(2\sin(2(c+dx))+8\cos(c+dx)+8\cot(c+dx)+\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)+12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-12}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(12*c + 12*d*x + 8*Cos[c + d*x] + 8*Cot[c + d*x] + Csc[(c + d*x)/2]^2 -
 12*Log[Cos[(c + d*x)/2]] + 12*Log[Sin[(c + d*x)/2]] - Sec[(c + d*x)/2]^2 +
 2*Sin[2*(c + d*x)]))/(8*d)
```

Maple [A] time = 0.059, size = 143, normalized size = 1.5

$$-\frac{a\left(\cos\left(dx+c\right)\right)^5}{d\sin\left(dx+c\right)} - \frac{a\left(\cos\left(dx+c\right)\right)^3\sin\left(dx+c\right)}{d} - \frac{3\cos\left(dx+c\right)a\sin\left(dx+c\right)}{2d} - \frac{3ax}{2} - \frac{3ca}{2d} - \frac{a\left(\cos\left(dx+c\right)\right)^5}{2d\left(\sin\left(dx+c\right)\right)^2} - \frac{a\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^5-a*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a*cos(d*x+c)
*sin(d*x+c)/d-3/2*a*x-3/2/d*c*a-1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a*cos
(d*x+c)^3/d-3/2*a*cos(d*x+c)/d-3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.55659, size = 136, normalized size = 1.45

$$-\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a-a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)$$
$$4\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a - a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos
(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.57165, size = 365, normalized size = 3.88

$$-\frac{6\,adx\cos(dx+c)^2+4\,a\cos(dx+c)^3-6\,adx-6\,a\cos(dx+c)-3\left(a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+3}{4\left(d\cos(dx+c)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*d*x*cos(d*x + c)^2 + 4*a*cos(d*x + c)^3 - 6*a*d*x - 6*a*cos(d*x +
 c) - 3*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x +
 c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(a*cos(d*x + c)^3 - 3*a*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41398, size = 220, normalized size = 2.34

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,(dx+c)a-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x + c)*a - 12*a*log(abs(tan(1/2*d*x +
 1/2*c))) + 4*a*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^6 + 4*a*ta
n(1/2*d*x + 1/2*c)^5 - 5*a*tan(1/2*d*x + 1/2*c)^4 - 16*a*tan(1/2*d*x + 1/2*
c)^3 - 12*a*tan(1/2*d*x + 1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2
*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2)/d
```

3.372 $\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

```
[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rubi [A] time = 0.0746513, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx = \int \left(a\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)dx$$
$$= a\int \cos(c+dx)\cot^3(c+dx)\,dx + a\int \cot^4(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)}{3d} - a\int \cot^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + a\int 1\,dx + \frac{(3a)\,\mathrm{Su}}{}$$
$$= ax - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx}{3d}$$
$$= ax + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\,\mathrm{co}}{2d}$$

Mathematica [C] time = 0.0505445, size = 125, normalized size = 1.52

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{a\cos(c+dx)}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3a\log\left(\sin\left(\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*a*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2
)/(8*d)
```

Maple [A] time = 0.056, size = 106, normalized size = 1.3

$$-\frac{a(\cos(dx+c))^5}{2d(\sin(dx+c))^2} - \frac{a(\cos(dx+c))^3}{2d} - \frac{3\cos(dx+c)a}{2d} - \frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d} - \frac{a(\cot(dx+c))^3}{3d} + a\,\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a*cos(d*x+c)^3/d-3/2*a*cos(d*x+c)/d-
3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1
/d*c*a
```

Maxima [A] time = 1.6122, size = 124, normalized size = 1.51

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)-1)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*a*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.67519, size = 425, normalized size = 5.18

$$\frac{16\,a\cos(dx+c)^3+9\left(a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9\left(a\cos(dx+c)^2-a\right)\log\left(-\frac{1}{2}\cos(\right.}{12\left(d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*a*cos(d*x
+ c)^3 - 2*a*d*x + 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40703, size = 190, normalized size = 2.32

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,(dx+c)a-36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{}{\tan(}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*a*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*a/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.373 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=88

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] a*x - (3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (a*Cot[c + d*x])/d - (a*Cot[c + d
*x]^3)/(3*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Cs
c[c + d*x])/(4*d)
```

Rubi [A] time = 0.0994979, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2838, 2611, 3770, 3473, 8}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] a*x - (3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (a*Cot[c + d*x])/d - (a*Cot[c + d
*x]^3)/(3*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Cs
c[c + d*x])/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\,dx + a\int \cot^4(c+dx)\csc(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3a)\int \cot^2(c+dx)$$
$$= \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} + \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot^3(}{}$$
$$= ax - \frac{3a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} + \frac{3a\,c}{}$$

Mathematica [C] time = 0.046652, size = 153, normalized size = 1.74

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a\sec}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Cot[c
+ d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) - (3*a*Lo
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (5*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.06, size = 128, normalized size = 1.5

$$-\frac{a(\cot(dx+c))^3}{3d} + \frac{a\cot(dx+c)}{d} + ax + \frac{ca}{d} - \frac{a(\cos(dx+c))^5}{4d(\sin(dx+c))^4} + \frac{a(\cos(dx+c))^5}{8d(\sin(dx+c))^2} + \frac{a(\cos(dx+c))^3}{8d} + \frac{3\cos(dx+}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1/d*c*a-1/4/d*a/sin(d*x+c)^4*cos(d
*x+c)^5+1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/8*a*cos(d*x+c)^3/d+3/8*a*cos(d*
x+c)/d+3/8/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.59189, size = 144, normalized size = 1.64

$$\frac{16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a - 3\,a\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1} + 3\,\log(\cos(dx+c)+1) - 3\,\log(\cos(dx+c)-1)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a - 3*a*(2*(
5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)
+ 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.66913, size = 494, normalized size = 5.61

$$\frac{48\,adx\cos\left(dx+c\right)^4-96\,adx\cos\left(dx+c\right)^2-30\,a\cos\left(dx+c\right)^3+48\,adx+18\,a\cos\left(dx+c\right)-9\left(a\cos\left(dx+c\right)^4-2\,a\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 - 96*a*d*x*cos(d*x + c)^2 - 30*a*cos(d*x + c)
^3 + 48*a*d*x + 18*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^
2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) - 16*(4*a*cos(d*x + c)^3 - 3*a*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39688, size = 207, normalized size = 2.35

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+192\,(dx+c)a+72\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*a*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*(d*x + c)*a + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 120*a*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 - 120*a*tan(1/2
*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*a*tan(1/2*d*x + 1/2*c) +
3*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.374 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^5)/(5*d) + (3*a*Cot[c
+ d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Csc[c + d*x])/(4*d)
```

Rubi [A] time = 0.117673, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^5)/(5*d) + (3*a*Cot[c
+ d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Csc[c + d*x])/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc(c+dx)\,dx + a\int \cot^4(c+dx)\csc^2(c+dx)\,d$$
$$= -\frac{a\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3a)\int \cot^2(c+dx)\csc(c+dx)\,d$$
$$= -\frac{a\cot^5(c+dx)}{5d} + \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot^3(c+dx)\cs}{4d}$$
$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^5(c+dx)}{5d} + \frac{3a\cot(c+dx)\csc(c}{8d}$$

Mathematica [A] time = 0.0344219, size = 135, normalized size = 1.82

$$-\frac{a\cot^5(c+dx)}{5d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a\log\left(\sin\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^5)/(5*d) + (5*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d
*x)/2]^4)/(64*d) - (3*a*Log[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*
x)/2]])/(8*d) - (5*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(6
4*d)
```

Maple [A] time = 0.06, size = 116, normalized size = 1.6

$$-\frac{a(\cos(dx+c))^5}{4d(\sin(dx+c))^4} + \frac{a(\cos(dx+c))^5}{8d(\sin(dx+c))^2} + \frac{a(\cos(dx+c))^3}{8d} + \frac{3\cos(dx+c)a}{8d} + \frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{8d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/8*a*
cos(d*x+c)^3/d+3/8*a*cos(d*x+c)/d+3/8/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*a
/sin(d*x+c)^5*cos(d*x+c)^5
```

Maxima [A] time = 1.08749, size = 116, normalized size = 1.57

$$-\frac{5a\left(\frac{2\left(5\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1} + 3\log(\cos(dx+c)+1) - 3\log(\cos(dx+c)-1)\right) + \frac{16a}{\tan(dx+c)^5}}{80d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*a*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 16*a/
tan(d*x + c)^5)/d
```

Fricas [B] time = 1.50885, size = 441, normalized size = 5.96

$$-\frac{16\,a\cos(dx+c)^5+15\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-15\left(a\cos(dx+c)^4\right.}{80\left(d\cos(dx+c)^4-2\,d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*a*cos(d*x + c)^5 + 15*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a*cos(d*x + c)^4 - 2*a*cos(
d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(5*a*cos(d*x
 + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x
+ c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52155, size = 234, normalized size = 3.16

$$\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/320*(2*a*tan(1/2*d*x + 1/2*c)^5 + 5*a*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1
/2*d*x + 1/2*c)^3 - 40*a*tan(1/2*d*x + 1/2*c)^2 + 120*a*log(abs(tan(1/2*d*x
 + 1/2*c))) + 20*a*tan(1/2*d*x + 1/2*c) - (274*a*tan(1/2*d*x + 1/2*c)^5 + 2
0*a*tan(1/2*d*x + 1/2*c)^4 - 40*a*tan(1/2*d*x + 1/2*c)^3 - 10*a*tan(1/2*d*x
 + 1/2*c)^2 + 5*a*tan(1/2*d*x + 1/2*c) + 2*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.375 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=98

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\,\mathrm{c}}{16d}$$

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.152339, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\,\mathrm{c}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```


```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^3)/(6*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^2(c+dx)\,dx + a\int \cot^4(c+dx)\csc^3(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}a\int \cot^2(c+dx)\csc^3(c+dx)\,dx +$$
$$= -\frac{a\cot^5(c+dx)}{5d} + \frac{a\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a\cot^3(c+dx)\csc^3(c}{6d}$$
$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot(c+dx)\csc(c+dx)}{16d} + \frac{a\cot(c+dx)\csc^3(c+}{8d}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot(c+dx)\csc(c+dx}{16d}$$

Mathematica [A] time = 0.0434611, size = 175, normalized size = 1.79

$$-\frac{a\cot^5(c+dx)}{5d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^5)/(5*d) - (a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*x
)/2]^4)/(64*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) - (a*Log[Cos[(c + d*x)/2]])
/(16*d) + (a*Log[Sin[(c + d*x)/2]])/(16*d) + (a*Sec[(c + d*x)/2]^2)/(64*d)
- (a*Sec[(c + d*x)/2]^4)/(64*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A] time = 0.06, size = 138, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^5}{5d(\sin(dx+c))^5} - \frac{a(\cos(dx+c))^5}{6d(\sin(dx+c))^6} - \frac{a(\cos(dx+c))^5}{24d(\sin(dx+c))^4} + \frac{a(\cos(dx+c))^5}{48d(\sin(dx+c))^2} + \frac{a(\cos(dx+c))^3}{48d} + \frac{\cos(dx+c)a}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^5-1/6/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d
*a/sin(d*x+c)^4*cos(d*x+c)^5+1/48/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a*cos(
d*x+c)^3/d+1/16*a*cos(d*x+c)/d+1/16/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.09354, size = 143, normalized size = 1.46

$$\frac{5a\left(\frac{2\left(3\cos(dx+c)^5+8\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1} - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1)\right) - \frac{96a}{\tan(dx+c)^5}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/480*(5*a*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 96*a/tan(d*x + c)^5)/d

---

Fricas [B] time = 1.60651, size = 497, normalized size = 5.07

$$\frac{96\,a\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)+30\,a\cos\left(dx+c\right)^{5}+80\,a\cos\left(dx+c\right)^{3}-30\,a\cos\left(dx+c\right)-15\,\left(a\cos\left(dx+c\right)^{6}-3\,a\cos\cdots\right.}{480\,\left(d\cos\left(dx\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/480*(96*a*cos(d*x + c)^5*sin(d*x + c) + 30*a*cos(d*x + c)^5 + 80*a*cos(d*x + c)^3 - 30*a*cos(d*x + c) - 15*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 15*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [B] time = 1.57256, size = 271, normalized size = 2.77

$$\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-60\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/1920*(5*a*tan(1/2*d*x + 1/2*c)^6 + 12*a*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan(1/2*d*x + 1/2*c)^4 - 60*a*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*c)^2 + 120*a*log(abs(tan(1/2*d*x + 1/2*c))) + 120*a*tan(1/2*d*x + 1/2*c) - (294*a*tan(1/2*d*x + 1/2*c)^6 + 120*a*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan(1/2*d*x + 1/2*c)^4 - 60*a*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*c)^2 + 12*a*tan(1/2*d*x + 1/2*c) + 5*a)/tan(1/2*d*x + 1/2*c)^6)/d

3.376 $\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=114

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-$$

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]^7)/(7*d) - (a*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c
 + d*x]^3)/(8*d) - (a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.154384, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]^7)/(7*d) - (a*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c
 + d*x]^3)/(8*d) - (a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^3(c+dx)\,dx + a\int \cot^4(c+dx)\csc^4(c+dx)$$

$$= -\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}a\int \cot^2(c+dx)\csc^3(c+dx)\,dx$$

$$= \frac{a\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} + \frac{1}{8}a\int$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot(c+dx)\csc(c+dx)}{16d} + \frac{a}{}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\,\mathrm{co}}{}$$

Mathematica [B] time = 0.0890008, size = 239, normalized size = 2.1

$$-\frac{2a\cot(c+dx)}{35d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\sec^4\left(\frac{1}{2}(c+\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(35*d) - (a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*
x)/2]^4)/(64*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^2)/(35*d) + (8*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^6)/(7*d) - (a*Log[Cos[(c + d*x)/2]])/(16*d) + (a*Log[Sin[(c +
 d*x)/2]])/(16*d) + (a*Sec[(c + d*x)/2]^2)/(64*d) - (a*Sec[(c + d*x)/2]^4)/
(64*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A] time = 0.068, size = 160, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^5}{6d(\sin(dx+c))^6} - \frac{a(\cos(dx+c))^5}{24d(\sin(dx+c))^4} + \frac{a(\cos(dx+c))^5}{48d(\sin(dx+c))^2} + \frac{a(\cos(dx+c))^3}{48d} + \frac{\cos(dx+c)a}{16d} + \frac{a\ln(\csc(dx+}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/48/
d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a*cos(d*x+c)^3/d+1/16*a*cos(d*x+c)/d+1/1
6/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a/
sin(d*x+c)^5*cos(d*x+c)^5
```

Maxima [A] time = 1.09303, size = 159, normalized size = 1.39

$$\frac{35\,a\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\left(7\,\tan(dx+c)^2+5\right)a}{\tan(dx+c)^7}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3360*(35*a*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 96*(7*tan(d*x + c)^2 + 5)*a/tan(d*x + c)
^7)/d
```

Fricas [B] time = 1.60665, size = 593, normalized size = 5.2

$$-\frac{192\,a\cos\left(dx+c\right)^7-672\,a\cos\left(dx+c\right)^5+105\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3360*(192*a*cos(d*x + c)^7 - 672*a*cos(d*x + c)^5 + 105*(a*cos(d*x + c)^
6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 105*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x +
 c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*(3*a*cos(d*x + c)
^5 + 8*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25202, size = 309, normalized size = 2.71

$$\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 + 35*a*tan(1/2*d*x + 1/2*c)^6 - 21*a*t
an(1/2*d*x + 1/2*c)^5 - 105*a*tan(1/2*d*x + 1/2*c)^4 - 105*a*tan(1/2*d*x +
1/2*c)^3 - 105*a*tan(1/2*d*x + 1/2*c)^2 + 840*a*log(abs(tan(1/2*d*x + 1/2*c
))) + 315*a*tan(1/2*d*x + 1/2*c) - (2178*a*tan(1/2*d*x + 1/2*c)^7 + 315*a*t
an(1/2*d*x + 1/2*c)^6 - 105*a*tan(1/2*d*x + 1/2*c)^5 - 105*a*tan(1/2*d*x +
1/2*c)^4 - 105*a*tan(1/2*d*x + 1/2*c)^3 - 21*a*tan(1/2*d*x + 1/2*c)^2 + 35*
a*tan(1/2*d*x + 1/2*c) + 15*a)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.377 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=136

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-$$

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c
+ d*x]^7)/(7*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^3)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rubi [A] time = 0.171337, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 14}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c
+ d*x]^7)/(7*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^3)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^4(c+dx)\,dx + a\int \cot^4(c+dx)\csc^5(c+dx)$$

$$= -\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{1}{8}(3a)\int \cot^2(c+dx)\csc^5(c+dx$$

$$= \frac{a\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d} + \frac{1}{16}a\int$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{64d} +$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{3a\cot(c+dx)\csc(c+dx)}{128d} -$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{3a}{}$$

Mathematica [B] time = 0.0772301, size = 279, normalized size = 2.05

$$-\frac{2a\cot(c+dx)}{35d} - \frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} - \frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\sec^8\left(\frac{1}{2}(c+\right.}{2048d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(35*d) - (3*a*Csc[(c + d*x)/2]^2)/(512*d) + (a*Csc[(c +
 d*x)/2]^4)/(1024*d) + (a*Csc[(c + d*x)/2]^6)/(512*d) - (a*Csc[(c + d*x)/2]
^8)/(2048*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) + (8*a*Cot[c + d*x]*C
sc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]*Csc[c + d*x]^6)/(7*d) - (3*a*Log[Co
s[(c + d*x)/2]])/(128*d) + (3*a*Log[Sin[(c + d*x)/2]])/(128*d) + (3*a*Sec[(
c + d*x)/2]^2)/(512*d) - (a*Sec[(c + d*x)/2]^4)/(1024*d) - (a*Sec[(c + d*x)
/2]^6)/(512*d) + (a*Sec[(c + d*x)/2]^8)/(2048*d)
```

Maple [A] time = 0.064, size = 182, normalized size = 1.3

$$-\frac{a(\cos(dx+c))^5}{7d(\sin(dx+c))^7} - \frac{2a(\cos(dx+c))^5}{35d(\sin(dx+c))^5} - \frac{a(\cos(dx+c))^5}{8d(\sin(dx+c))^8} - \frac{a(\cos(dx+c))^5}{16d(\sin(dx+c))^6} - \frac{a(\cos(dx+c))^5}{64d(\sin(dx+c))^4} + \frac{a(\cos}{128d(\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a/sin(d*x+c)^5*cos(d*x+c)^5-1/8/d
*a/sin(d*x+c)^8*cos(d*x+c)^5-1/16/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/64/d*a/si
n(d*x+c)^4*cos(d*x+c)^5+1/128/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/128*a*cos(d*x
+c)^3/d+3/128*a*cos(d*x+c)/d+3/128/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.12362, size = 186, normalized size = 1.37

$$\frac{35\,a\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{256\left(7\,\tan(dx+c)^2+5\right)a}{\tan(dx+c)^7}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8960*(35*a*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 +
 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
256*(7*tan(d*x + c)^2 + 5)*a/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.51967, size = 656, normalized size = 4.82

$$\frac{210\,a\cos\left(dx+c\right)^7-770\,a\cos\left(dx+c\right)^5-770\,a\cos\left(dx+c\right)^3+210\,a\cos\left(dx+c\right)-105\left(a\cos\left(dx+c\right)^8-4\,a\cos\left(dx+c\right)^6+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8960*(210*a*cos(d*x + c)^7 - 770*a*cos(d*x + c)^5 - 770*a*cos(d*x + c)^3
+ 210*a*cos(d*x + c) - 105*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos
(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 105*(a*
cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 256*(2*a*cos(d*x + c)^7 - 7*a*cos(d*
x + c)^5)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*
x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44299, size = 271, normalized size = 1.99

$$35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-112\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-560\,a\tan\left(\frac{1}{2}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/71680*(35*a*tan(1/2*d*x + 1/2*c)^8 + 80*a*tan(1/2*d*x + 1/2*c)^7 - 112*a*
tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 560*a*tan(1/2*d*x +
 1/2*c)^3 + 1680*a*log(abs(tan(1/2*d*x + 1/2*c))) + 1680*a*tan(1/2*d*x + 1/
2*c) - (4566*a*tan(1/2*d*x + 1/2*c)^8 + 1680*a*tan(1/2*d*x + 1/2*c)^7 - 560
*a*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 112*a*tan(1/2*d*
x + 1/2*c)^3 + 80*a*tan(1/2*d*x + 1/2*c) + 35*a)/tan(1/2*d*x + 1/2*c)^8)/d
```

3.378 $\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=185

$$-\frac{2a^2\cos^9(c+dx)}{9d}+\frac{4a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{3a^2\sin^3(c+dx)\cos^5(c+dx)}{16d}$$

```
[Out] (9*a^2*x)/256 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (4*a^2*Cos[c + d*x]^7)/(7*d)
 - (2*a^2*Cos[c + d*x]^9)/(9*d) + (9*a^2*Cos[c + d*x]*Sin[c + d*x])/(256*d)
 + (3*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) - (3*a^2*Cos[c + d*x]^5*Sin[
c + d*x])/(32*d) - (3*a^2*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*d) - (a^2*Cos[
c + d*x]^5*Sin[c + d*x]^5)/(10*d)
```

Rubi [A] time = 0.339205, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 270}

$$-\frac{2a^2\cos^9(c+dx)}{9d}+\frac{4a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{3a^2\sin^3(c+dx)\cos^5(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (9*a^2*x)/256 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (4*a^2*Cos[c + d*x]^7)/(7*d)
 - (2*a^2*Cos[c + d*x]^9)/(9*d) + (9*a^2*Cos[c + d*x]*Sin[c + d*x])/(256*d)
 + (3*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) - (3*a^2*Cos[c + d*x]^5*Sin[
c + d*x])/(32*d) - (3*a^2*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*d) - (a^2*Cos[
c + d*x]^5*Sin[c + d*x]^5)/(10*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^4(c+dx)+2a^2\cos^4(c+dx)\sin^5(c+dx)\right.$$
$$= a^2\int \cos^4(c+dx)\sin^4(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin^6(c+$$
$$= -\frac{a^2\cos^5(c+dx)\sin^3(c+dx)}{8d} - \frac{a^2\cos^5(c+dx)\sin^5(c+dx)}{10d} +$$
$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{3a^2\cos^5(c+dx)\sin^3(c+dx)}{16d} -$$
$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c+dx)}{9d} +$$
$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c+dx)}{9d} +$$
$$= \frac{3a^2x}{128} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c+dx)}{9d}$$
$$= \frac{9a^2x}{256} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c+dx)}{9d}$$

Mathematica [A] time = 0.701515, size = 116, normalized size = 0.63

$$\frac{a^2(-1260\sin(2(c+dx)) - 7560\sin(4(c+dx)) + 630\sin(6(c+dx)) + 945\sin(8(c+dx)) - 126\sin(10(c+dx)) - 3024}{64}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(22680*c + 22680*d*x - 30240*Cos[c + d*x] - 6720*Cos[3*(c + d*x)] + 40
32*Cos[5*(c + d*x)] + 720*Cos[7*(c + d*x)] - 560*Cos[9*(c + d*x)] - 1260*Si
n[2*(c + d*x)] - 7560*Sin[4*(c + d*x)] + 630*Sin[6*(c + d*x)] + 945*Sin[8*(
c + d*x)] - 126*Sin[10*(c + d*x)]))/(645120*d)
```

Maple [A] time = 0.041, size = 218, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^5(\cos(dx+c))^5}{10} - \frac{(\sin(dx+c))^3(\cos(dx+c))^5}{16} - \frac{\sin(dx+c)(\cos(dx+c))^5}{32} + \frac{\sin(dx+c)}{128}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/
32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/256*d*x+3/256*c)+2*a^2*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*c
os(d*x+c)^5-8/315*cos(d*x+c)^5)+a^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*si
n(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d
*x+3/128*c))
```

Maxima [A] time = 1.07072, size = 166, normalized size = 0.9

$$-\frac{4096\left(35\cos\left(dx+c\right)^{9}-90\cos\left(dx+c\right)^{7}+63\cos\left(dx+c\right)^{5}\right)a^{2}+63\left(32\sin\left(2\,dx+2\,c\right)^{5}-120\,dx-120\,c-5\sin\left(8\,dx\right.\right.}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/645120*(4096*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^2 + 63*(32*sin(2*d*x + 2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x + 8*c) + 4
0*sin(4*d*x + 4*c))*a^2 - 630*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d
*x + 4*c))*a^2)/d
```

Fricas [A] time = 1.62081, size = 333, normalized size = 1.8

$$-\frac{17920\,a^{2}\cos\left(dx+c\right)^{9}-46080\,a^{2}\cos\left(dx+c\right)^{7}+32256\,a^{2}\cos\left(dx+c\right)^{5}-2835\,a^{2}dx+63\left(128\,a^{2}\cos\left(dx+c\right)^{9}-496\,a^{2}\right.}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/80640*(17920*a^2*cos(d*x + c)^9 - 46080*a^2*cos(d*x + c)^7 + 32256*a^2*c
os(d*x + c)^5 - 2835*a^2*d*x + 63*(128*a^2*cos(d*x + c)^9 - 496*a^2*cos(d*x
 + c)^7 + 488*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x + c)^3 - 45*a^2*cos(d*x +
 c))*sin(d*x + c))/d
```

Sympy [A] time = 35.374, size = 554, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/256 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 3*a**2*x*sin(c + d*x)**8/128 + 15*a**2*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 3*a**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**2*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 9*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**4/64 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a**2*x*sin(c +
d*x)**2*cos(c + d*x)**6/32 + 3*a**2*x*cos(c + d*x)**10/256 + 3*a**2*x*cos(c
 + d*x)**8/128 + 3*a**2*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**2*sin(c
```

```
 + d*x)**7*cos(c + d*x)**3/(128*d) + 3*a**2*sin(c + d*x)**7*cos(c + d*x)/(1
28*d) - a**2*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 11*a**2*sin(c + d*x)*
*5*cos(c + d*x)**3/(128*d) - 2*a**2*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) -
 7*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 11*a**2*sin(c + d*x)**3*c
os(c + d*x)**5/(128*d) - 8*a**2*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 3*
a**2*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 3*a**2*sin(c + d*x)*cos(c + d*x
)**7/(128*d) - 16*a**2*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*sin(c) + a
)**2*sin(c)**4*cos(c)**4, True))
```

Giac [A] time = 1.41972, size = 235, normalized size = 1.27

$$\frac{9}{256}a^2x - \frac{a^2\cos(9\,dx + 9\,c)}{1152\,d} + \frac{a^2\cos(7\,dx + 7\,c)}{896\,d} + \frac{a^2\cos(5\,dx + 5\,c)}{160\,d} - \frac{a^2\cos(3\,dx + 3\,c)}{96\,d} - \frac{3\,a^2\cos(dx + c)}{64\,d} - \frac{a^2 s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 9/256*a^2*x - 1/1152*a^2*cos(9*d*x + 9*c)/d + 1/896*a^2*cos(7*d*x + 7*c)/d
+ 1/160*a^2*cos(5*d*x + 5*c)/d - 1/96*a^2*cos(3*d*x + 3*c)/d - 3/64*a^2*cos
(d*x + c)/d - 1/5120*a^2*sin(10*d*x + 10*c)/d + 3/2048*a^2*sin(8*d*x + 8*c)
/d + 1/1024*a^2*sin(6*d*x + 6*c)/d - 3/256*a^2*sin(4*d*x + 4*c)/d - 1/512*a
^2*sin(2*d*x + 2*c)/d
```

3.379 $\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=159

$$-\frac{a^2\cos^9(c+dx)}{9d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{4d}-\frac{a^2\sin(c+dx)\cos^5(c+dx)}{8d}+\frac{a}{}$$

```
[Out] (3*a^2*x)/64 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (3*a^2*Cos[c + d*x]^7)/(7*d)
- (a^2*Cos[c + d*x]^9)/(9*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(64*d) + (
a^2*Cos[c + d*x]^3*Sin[c + d*x])/(32*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])
/(8*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.254925, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{a^2\cos^9(c+dx)}{9d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{4d}-\frac{a^2\sin(c+dx)\cos^5(c+dx)}{8d}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/64 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (3*a^2*Cos[c + d*x]^7)/(7*d)
- (a^2*Cos[c + d*x]^9)/(9*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(64*d) + (
a^2*Cos[c + d*x]^3*Sin[c + d*x])/(32*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])
/(8*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x]^3)/(4*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^3(c+dx)+2a^2\cos^4(c+dx)\sin^4(c+dx)\right.$$
$$= a^2\int \cos^4(c+dx)\sin^3(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin^5(c+$$
$$= -\frac{a^2\cos^5(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{4}\left(3a^2\right)\int \cos^4(c+dx)\sin^2(c+$$
$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{8d} - \frac{a^2\cos^5(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{8}$$
$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{3a^2\cos^7(c+dx)}{7d} - \frac{a^2\cos^9(c+dx)}{9d} + \frac{a^2\cos}{}$$
$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{3a^2\cos^7(c+dx)}{7d} - \frac{a^2\cos^9(c+dx)}{9d} + \frac{3a^2\cos}{}$$
$$= \frac{3a^2x}{64} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{3a^2\cos^7(c+dx)}{7d} - \frac{a^2\cos^9(c+dx)}{9d} +$$

Mathematica [A] time = 0.697071, size = 86, normalized size = 0.54

$$\frac{a^2(-2520\sin(4(c+dx))+315\sin(8(c+dx))-11340\cos(c+dx)-3360\cos(3(c+dx))+1008\cos(5(c+dx))+450\,}{161280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(7560*c + 7560*d*x - 11340*Cos[c + d*x] - 3360*Cos[3*(c + d*x)] + 1008
*Cos[5*(c + d*x)] + 450*Cos[7*(c + d*x)] - 70*Cos[9*(c + d*x)] - 2520*Sin[4
*(c + d*x)] + 315*Sin[8*(c + d*x)]))/(161280*d)
```

Maple [A] time = 0.042, size = 162, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^5}{9} - \frac{4(\sin(dx+c))^2(\cos(dx+c))^5}{63} - \frac{8(\cos(dx+c))^5}{315}\right) + 2a^2\left(-1/8(\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/3
15*cos(d*x+c)^5)+2*a^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(
d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+a
^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5))
```

Maxima [A] time = 1.09879, size = 136, normalized size = 0.86

$$-\frac{512\left(35\cos\left(dx+c\right)^{9}-90\cos\left(dx+c\right)^{7}+63\cos\left(dx+c\right)^{5}\right)a^{2}-4608\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a^{2}-315\,(24\,d\ldots}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/161280*(512*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*
a^2 - 4608*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 - 315*(24*d*x + 24*c +
 sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A] time = 1.60285, size = 290, normalized size = 1.82

$$-\frac{2240\,a^{2}\cos\left(dx+c\right)^{9}-8640\,a^{2}\cos\left(dx+c\right)^{7}+8064\,a^{2}\cos\left(dx+c\right)^{5}-945\,a^{2}dx-315\left(16\,a^{2}\cos\left(dx+c\right)^{7}-24\,a^{2}\cos\left(\ldots\right.\right.}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20160*(2240*a^2*cos(d*x + c)^9 - 8640*a^2*cos(d*x + c)^7 + 8064*a^2*cos(
d*x + c)^5 - 945*a^2*d*x - 315*(16*a^2*cos(d*x + c)^7 - 24*a^2*cos(d*x + c)
^5 + 2*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 22.0236, size = 335, normalized size = 2.11

$$\begin{cases}\frac{3a^{2}x\sin^{8}\left(c+dx\right)}{64}+\frac{3a^{2}x\sin^{6}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{16}+\frac{9a^{2}x\sin^{4}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{32}+\frac{3a^{2}x\sin^{2}\left(c+dx\right)\cos^{6}\left(c+dx\right)}{16}+\frac{3a^{2}x\cos^{8}\left(c+dx\right)}{64}+\frac{3a^{2}\sin^{7}\left(c+dx\right)\ldots}{64d}\ldots \\ x\left(a\sin\left(c\right)+a\right)^{2}\sin^{3}\left(c\right)\cos^{4}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**8/64 + 3*a**2*x*sin(c + d*x)**6*cos(c + d
*x)**2/16 + 9*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 3*a**2*x*sin(c +
d*x)**2*cos(c + d*x)**6/16 + 3*a**2*x*cos(c + d*x)**8/64 + 3*a**2*sin(c + d
*x)**7*cos(c + d*x)/(64*d) + 11*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(64*d)
 - a**2*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 11*a**2*sin(c + d*x)**3*cos
(c + d*x)**5/(64*d) - 4*a**2*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - a**2*
sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**7
/(64*d) - 8*a**2*cos(c + d*x)**9/(315*d) - 2*a**2*cos(c + d*x)**7/(35*d), N
e(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**3*cos(c)**4, True))
```

Giac [A] time = 1.42578, size = 166, normalized size = 1.04

$$\frac{3}{64}a^2x - \frac{a^2\cos(9dx+9c)}{2304d} + \frac{5a^2\cos(7dx+7c)}{1792d} + \frac{a^2\cos(5dx+5c)}{160d} - \frac{a^2\cos(3dx+3c)}{48d} - \frac{9a^2\cos(dx+c)}{128d} + \frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/64*a^2*x - 1/2304*a^2*cos(9*d*x + 9*c)/d + 5/1792*a^2*cos(7*d*x + 7*c)/d
+ 1/160*a^2*cos(5*d*x + 5*c)/d - 1/48*a^2*cos(3*d*x + 3*c)/d - 9/128*a^2*co
s(d*x + c)/d + 1/512*a^2*sin(8*d*x + 8*c)/d - 1/64*a^2*sin(4*d*x + 4*c)/d
```

3.380 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=141

$$\frac{2a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{11a^2\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{11a^2\sin(c+dx)\cos^3(c+dx)}{192d}$$

```
[Out] (11*a^2*x)/128 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (2*a^2*Cos[c + d*x]^7)/(7*d
) + (11*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(192*d) - (11*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a^2*Cos
[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rubi [A] time = 0.257826, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 14}

$$\frac{2a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{11a^2\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{11a^2\sin(c+dx)\cos^3(c+dx)}{192d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (11*a^2*x)/128 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (2*a^2*Cos[c + d*x]^7)/(7*d
) + (11*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(192*d) - (11*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a^2*Cos
[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^2(c+dx) + 2a^2\cos^4(c+dx)\sin^3(c+dx) \right.$$
$$= a^2\int \cos^4(c+dx)\sin^2(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin^4(c+$$
$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{a^2\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{6}$$
$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{11a^2\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{a^2}{}$$
$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos(c+dx)\sin(c+}{16d}$$
$$= \frac{a^2x}{16} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{2a^2\cos^7(c+dx)}{7d} + \frac{11a^2\cos(c+dx)\sin}{128d}$$
$$= \frac{11a^2x}{128} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{2a^2\cos^7(c+dx)}{7d} + \frac{11a^2\cos(c+dx)}{128d}$$

Mathematica [A] time = 0.500808, size = 96, normalized size = 0.68

$$\frac{a^2(1680\sin(2(c+dx)) - 2520\sin(4(c+dx)) - 560\sin(6(c+dx)) + 105\sin(8(c+dx)) - 10080\cos(c+dx) - 3360\cos}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(3360*c + 9240*d*x - 10080*Cos[c + d*x] - 3360*Cos[3*(c + d*x)] + 672*
Cos[5*(c + d*x)] + 480*Cos[7*(c + d*x)] + 1680*Sin[2*(c + d*x)] - 2520*Sin[
4*(c + d*x)] - 560*Sin[6*(c + d*x)] + 105*Sin[8*(c + d*x)]))/(107520*d)
```

Maple [A] time = 0.039, size = 164, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^5}{8} - \frac{\sin(dx+c)(\cos(dx+c))^5}{16} + \frac{\sin(dx+c)}{64}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+2*a^2*(-1/7*sin
(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5
```

```
+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A] time = 1.07777, size = 138, normalized size = 0.98

$$\frac{6144\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a^{2}+560\left(4\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\sin\left(4\,dx+4\,c\right)\right)a^{2}+105\,(24\,dx+}{107520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/107520*(6144*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 + 560*(4*sin(2*d*x
 + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2 + 105*(24*d*x + 24*c +
sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A] time = 1.57593, size = 258, normalized size = 1.83

$$\frac{3840\,a^{2}\cos\left(dx+c\right)^{7}-5376\,a^{2}\cos\left(dx+c\right)^{5}+1155\,a^{2}dx+35\left(48\,a^{2}\cos\left(dx+c\right)^{7}-136\,a^{2}\cos\left(dx+c\right)^{5}+22\,a^{2}\cos\left(dx\right.\right.}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/13440*(3840*a^2*cos(d*x + c)^7 - 5376*a^2*cos(d*x + c)^5 + 1155*a^2*d*x +
 35*(48*a^2*cos(d*x + c)^7 - 136*a^2*cos(d*x + c)^5 + 22*a^2*cos(d*x + c)^3
 + 33*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 12.6969, size = 420, normalized size = 2.98

$$\begin{cases}\frac{3a^{2}x\sin^{8}(c+dx)}{128}+\frac{3a^{2}x\sin^{6}(c+dx)\cos^{2}(c+dx)}{32}+\frac{a^{2}x\sin^{6}(c+dx)}{16}+\frac{9a^{2}x\sin^{4}(c+dx)\cos^{4}(c+dx)}{64}+\frac{3a^{2}x\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3a^{2}x\sin^{2}(c+dx)\cos}{32}\\ x\left(a\sin\left(c\right)+a\right)^{2}\sin^{2}\left(c\right)\cos^{4}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**8/128 + 3*a**2*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + a**2*x*sin(c + d*x)**6/16 + 9*a**2*x*sin(c + d*x)**4*cos(c + d
*x)**4/64 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*a**2*x*sin(c +
d*x)**2*cos(c + d*x)**6/32 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
3*a**2*x*cos(c + d*x)**8/128 + a**2*x*cos(c + d*x)**6/16 + 3*a**2*sin(c + d
*x)**7*cos(c + d*x)/(128*d) + 11*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(128*
d) + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 11*a**2*sin(c + d*x)**3*cos
(c + d*x)**5/(128*d) + a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 2*a**2*
sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**7
/(128*d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 4*a**2*cos(c + d*x)**
7/(35*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**4, True))
```

Giac [A] time = 1.43746, size = 189, normalized size = 1.34

$$\frac{11}{128}a^2x+\frac{a^2\cos(7\,dx+7\,c)}{224\,d}+\frac{a^2\cos(5\,dx+5\,c)}{160\,d}-\frac{a^2\cos(3\,dx+3\,c)}{32\,d}-\frac{3\,a^2\cos(dx+c)}{32\,d}+\frac{a^2\sin(8\,dx+8\,c)}{1024\,d}-\frac{a^2\text{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 11/128*a^2*x + 1/224*a^2*cos(7*d*x + 7*c)/d + 1/160*a^2*cos(5*d*x + 5*c)/d
- 1/32*a^2*cos(3*d*x + 3*c)/d - 3/32*a^2*cos(d*x + c)/d + 1/1024*a^2*sin(8*
d*x + 8*c)/d - 1/192*a^2*sin(6*d*x + 6*c)/d - 3/128*a^2*sin(4*d*x + 4*c)/d
+ 1/64*a^2*sin(2*d*x + 2*c)/d
```

3.381 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$-\frac{a^2\cos^5(c+dx)}{15d}-\frac{\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{21d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{12d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^2x}{8}-$$

```
[Out] (a^2*x)/8 - (a^2*Cos[c + d*x]^5)/(15*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(12*d) - (Cos[c + d*x]^5*(a + a*Si
n[c + d*x])^2)/(7*d) - (Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(21*d)
```

Rubi [A] time = 0.138963, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2\cos^5(c+dx)}{15d}-\frac{\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{21d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{12d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^2x}{8}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (a^2*x)/8 - (a^2*Cos[c + d*x]^5)/(15*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(12*d) - (Cos[c + d*x]^5*(a + a*Si
n[c + d*x])^2)/(7*d) - (Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(21*d)
```


Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d} + \frac{2}{7}\int \cos^4(c+dx)(a+a\sin(c$$

$$= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d} - \frac{\cos^5(c+dx)\left(a^2+a^2\sin(c+\right.}{21d}$$

$$= -\frac{a^2\cos^5(c+dx)}{15d} - \frac{\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d} - \frac{\cos^5(c+d}{}$$

$$= -\frac{a^2\cos^5(c+dx)}{15d} + \frac{a^2\cos^3(c+dx)\sin(c+dx)}{12d} - \frac{\cos^5(c+dx)(a}{7}$$

$$= -\frac{a^2\cos^5(c+dx)}{15d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+dx)\,\mathrm{s}}{12d}$$

$$= \frac{a^2x}{8} - \frac{a^2\cos^5(c+dx)}{15d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+}{}$$

Mathematica [A] time = 0.324188, size = 86, normalized size = 0.67

$$\frac{a^2(210\sin(2(c+dx)) - 210\sin(4(c+dx)) - 70\sin(6(c+dx)) - 1155\cos(c+dx) - 525\cos(3(c+dx)) - 63\cos(5(c+}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(840*c + 840*d*x - 1155*Cos[c + d*x] - 525*Cos[3*(c + d*x)] - 63*Cos[5
*(c + d*x)] + 15*Cos[7*(c + d*x)] + 210*Sin[2*(c + d*x)] - 210*Sin[4*(c + d
*x)] - 70*Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.033, size = 106, normalized size = 0.8

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7} - \frac{2(\cos(dx+c))^5}{35}\right) + 2a^2\left(-1/6\sin(dx+c)(\cos(dx+c))^5 + 1/24\left((\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+2*a^2*(-1/6*sin
(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x
+1/16*c)-1/5*a^2*cos(d*x+c)^5)
```

Maxima [A] time = 1.11102, size = 111, normalized size = 0.86

$$-\frac{672a^2\cos(dx+c)^5 - 96\left(5\cos(dx+c)^7 - 7\cos(dx+c)^5\right)a^2 - 35\left(4\sin(2dx+2c)^3 + 12dx + 12c - 3\sin(4dx+\right.}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(672*a^2*cos(d*x + c)^5 - 96*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*
a^2 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A] time = 1.53621, size = 213, normalized size = 1.65

$$\frac{120\,a^2\cos\left(dx+c\right)^7 - 336\,a^2\cos\left(dx+c\right)^5 + 105\,a^2dx - 35\left(8\,a^2\cos\left(dx+c\right)^5 - 2\,a^2\cos\left(dx+c\right)^3 - 3\,a^2\cos\left(dx+c\right)\right)\sin}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^2*cos(d*x + c)^7 - 336*a^2*cos(d*x + c)^5 + 105*a^2*d*x - 35*(
8*a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c))*sin(d*x +
 c))/d
```

Sympy [A] time = 7.51552, size = 223, normalized size = 1.73

$$\begin{cases} \frac{a^2x\sin^6(c+dx)}{8} + \frac{3a^2x\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3a^2x\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{a^2x\cos^6(c+dx)}{8} + \frac{a^2\sin^5(c+dx)\cos(c+dx)}{8d} + \frac{a^2\sin^3(c+dx)\cos^3(c+}{3d} \\ x\left(a\sin(c)+a\right)^2\sin(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/8 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x)
**2/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + a**2*x*cos(c + d*x)**6
/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) + a**2*sin(c + d*x)**3*cos(c +
 d*x)**3/(3*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - a**2*sin(c +
d*x)*cos(c + d*x)**5/(8*d) - 2*a**2*cos(c + d*x)**7/(35*d) - a**2*cos(c + d
*x)**5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c)**4, True))
```

Giac [A] time = 1.33229, size = 166, normalized size = 1.29

$$\frac{1}{8}\,a^2x + \frac{a^2\cos\left(7\,dx+7\,c\right)}{448\,d} - \frac{3\,a^2\cos\left(5\,dx+5\,c\right)}{320\,d} - \frac{5\,a^2\cos\left(3\,dx+3\,c\right)}{64\,d} - \frac{11\,a^2\cos\left(dx+c\right)}{64\,d} - \frac{a^2\sin\left(6\,dx+6\,c\right)}{96\,d} - \frac{a^2\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x + 1/448*a^2*cos(7*d*x + 7*c)/d - 3/320*a^2*cos(5*d*x + 5*c)/d - 5
/64*a^2*cos(3*d*x + 3*c)/d - 11/64*a^2*cos(d*x + c)/d - 1/96*a^2*sin(6*d*x
+ 6*c)/d - 1/32*a^2*sin(4*d*x + 4*c)/d + 1/32*a^2*sin(2*d*x + 2*c)/d
```

3.382 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=119

$$-\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}-a^2$$

```
[Out] (3*a^2*x)/4 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) - (a^2*Cos[c + d*x]^5)/(5*d) + (3*a^2*Cos[c + d*x]*Sin
[c + d*x])/(4*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.142898, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2873, 2635, 8, 2592, 302, 206, 2565, 30}

$$-\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}-a^2$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/4 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) - (a^2*Cos[c + d*x]^5)/(5*d) + (3*a^2*Cos[c + d*x]*Sin
[c + d*x])/(4*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(2a^2\cos^4(c+dx)+a^2\cos^3(c+dx)\cot(c+dx)+a^2\cos^4(c+dx)\right.$$

$$= a^2\int \cos^3(c+dx)\cot(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin(c+dx)\,dx$$

$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}\left(3a^2\right)\int \cos^2(c+dx)\,dx - \frac{a^2\,\mathrm{Subst}}{}$$

$$= -\frac{a^2\cos^5(c+dx)}{5d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^2\cos^3(c+dx)\sin}{2d}$$

$$= \frac{3a^2x}{4} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{a^2\cos^5(c+dx)}{5d} + \frac{3a^2\cos}{}$$

$$= \frac{3a^2x}{4} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 0.82845, size = 96, normalized size = 0.81

$$\frac{a^2\left(270\cos(c+dx)+5\cos(3(c+dx))-3\cos(5(c+dx))+15\left(8\sin(2(c+dx))+\sin(4(c+dx))+4\left(4\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(270*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - 3*Cos[5*(c + d*x)] + 15*(4*(3
*c + 3*d*x - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]]) + 8*Sin[2*(
c + d*x)] + Sin[4*(c + d*x)])))/(240*d)
```

Maple [A] time = 0.07, size = 127, normalized size = 1.1

$$-\frac{a^2(\cos(dx+c))^5}{5d} + \frac{a^2(\cos(dx+c))^3\sin(dx+c)}{2d} + \frac{3a^2\cos(dx+c)\sin(dx+c)}{4d} + \frac{3a^2x}{4} + \frac{3ca^2}{4d} + \frac{a^2(\cos(dx+c))^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/5*a^2*cos(d*x+c)^5/d+1/2*a^2*cos(d*x+c)^3*sin(d*x+c)/d+3/4*a^2*cos(d*x+c
)*sin(d*x+c)/d+3/4*a^2*x+3/4/d*c*a^2+1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)/
d+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.10069, size = 132, normalized size = 1.11

$$-\frac{48\,a^2\cos\left(dx+c\right)^5-40\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^2-15}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^5 - 40*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^2 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

Fricas [A] time = 1.52003, size = 309, normalized size = 2.6

$$-\frac{12\,a^2\cos\left(dx+c\right)^5-20\,a^2\cos\left(dx+c\right)^3-45\,a^2dx-60\,a^2\cos\left(dx+c\right)+30\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-30\,a^2\log\left(-\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a^2*cos(d*x + c)^5 - 20*a^2*cos(d*x + c)^3 - 45*a^2*d*x - 60*a^2*
cos(d*x + c) + 30*a^2*log(1/2*cos(d*x + c) + 1/2) - 30*a^2*log(-1/2*cos(d*x
 + c) + 1/2) - 15*(2*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c))*sin(d*x + c))
/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32986, size = 244, normalized size = 2.05

$$\frac{45\,(dx+c)a^2+60\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-360\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\right.}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*(d*x + c)*a^2 + 60*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(75*a^2*
tan(1/2*d*x + 1/2*c)^9 - 60*a^2*tan(1/2*d*x + 1/2*c)^8 + 30*a^2*tan(1/2*d*x
 + 1/2*c)^7 - 360*a^2*tan(1/2*d*x + 1/2*c)^6 - 320*a^2*tan(1/2*d*x + 1/2*c)
^4 - 30*a^2*tan(1/2*d*x + 1/2*c)^3 - 280*a^2*tan(1/2*d*x + 1/2*c)^2 - 75*a^
2*tan(1/2*d*x + 1/2*c) - 68*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.383 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}-\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}-\frac{2a^2}{}$$

```
[Out] (-9*a^2*x)/8 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d + (
2*a^2*Cos[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[
c + d*x])/(8*d) - (a^2*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.196879, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 2638, 2635, 2633}

$$\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}-\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}-\frac{2a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-9*a^2*x)/8 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d + (
2*a^2*Cos[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[
c + d*x])/(8*d) - (a^2*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-a^6+2a^6\csc(c+dx)+a^6\csc^2(c+dx)-4a^6\sin(c+dx)-a^6\,\text{si}\right.}{a^4}$$

$$= -a^2x+a^2\int \csc^2(c+dx)\,dx-a^2\int \sin^2(c+dx)\,dx+a^2\int \sin^4(c$$

$$= -a^2x-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^2\cos(c+dx)}{d}+\frac{a^2\cos(c+dx}{2}$$

$$= -\frac{3a^2x}{2}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos^3(c}{3d}$$

$$= -\frac{9a^2x}{8}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos^3(c}{3d}$$

Mathematica [A] time = 0.47703, size = 83, normalized size = 0.72

$$\frac{a^2\left(240\cos(c+dx)+16\cos(3(c+dx))-3\left(-\sin(4(c+dx))+32\cot(c+dx)-64\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+64\log\left(\cos\left(\frac{1}{2}\right.\right.\right.\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(240*Cos[c + d*x] + 16*Cos[3*(c + d*x)] - 3*(36*c + 36*d*x + 32*Cot[c
+ d*x] + 64*Log[Cos[(c + d*x)/2]] - 64*Log[Sin[(c + d*x)/2]] - Sin[4*(c + d
*x)])))/(96*d)
```

Maple [A] time = 0.072, size = 137, normalized size = 1.2

$$-\frac{3a^2(\cos(dx+c))^3\sin(dx+c)}{4d}-\frac{9a^2\cos(dx+c)\sin(dx+c)}{8d}-\frac{9a^2x}{8}-\frac{9ca^2}{8d}+\frac{2a^2(\cos(dx+c))^3}{3d}+2\frac{a^2\cos(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-9/8*a^2*cos(d*x+c)*sin(d*x+c)/d-9/8*a^2*
x-9/8/d*c*a^2+2/3*a^2*cos(d*x+c)^3/d+2*a^2*cos(d*x+c)/d+2/d*a^2*ln(csc(d*x+
c)-cot(d*x+c))-1/d*a^2/sin(d*x+c)*cos(d*x+c)^5
```

Maxima [A] time = 1.6788, size = 173, normalized size = 1.49

$$\frac{32\left(2\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a^{2}+3\left(12\,dx+12\,c+\sin\left(4\,d\right.\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/96*(32*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a^2 - 48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 +
 tan(d*x + c)))*a^2)/d
```

Fricas [A] time = 1.54529, size = 360, normalized size = 3.1

$$-\frac{6\,a^{2}\cos\left(dx+c\right)^{5}-9\,a^{2}\cos\left(dx+c\right)^{3}+24\,a^{2}\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-24\,a^{2}\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{24\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*a^2*cos(d*x + c)^5 - 9*a^2*cos(d*x + c)^3 + 24*a^2*log(1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) - 24*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 27*a^2*cos(d*x + c) - (16*a^2*cos(d*x + c)^3 - 27*a^2*d*x + 48*a^2*cos(
d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37223, size = 284, normalized size = 2.45

$$-\frac{27\left(dx+c\right)a^{2}-48\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-12\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\left(4\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{2}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-\cdots\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)*a^2 - 48*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 12*a^2*ta
n(1/2*d*x + 1/2*c) + 12*(4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/
2*c) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^7 - 96*a^2*tan(1/2*d*x + 1/2*c)^6 - 21
*a^2*tan(1/2*d*x + 1/2*c)^5 - 192*a^2*tan(1/2*d*x + 1/2*c)^4 + 21*a^2*tan(1
/2*d*x + 1/2*c)^3 - 160*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/
2*c) - 64*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.384 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=98

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -3*a^2*x + (a^2*ArcTanh[Cos[c + d*x]])/(2*d) + (a^2*Cos[c + d*x]^3)/(3*d) -
 (2*a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (a^2*Cos[
c + d*x]*Sin[c + d*x])/d
```

Rubi [A] time = 0.157049, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -3*a^2*x + (a^2*ArcTanh[Cos[c + d*x]])/(2*d) + (a^2*Cos[c + d*x]^3)/(3*d) -
 (2*a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (a^2*Cos[
c + d*x]*Sin[c + d*x])/d
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-4a^6 - a^6\csc(c+dx) + 2a^6\csc^2(c+dx) + a^6\csc^3(c+dx) - a^6 \text{si}\right.}{a^4}$$
$$= -4a^2x - a^2\int \csc(c+dx)\,dx + a^2\int \csc^3(c+dx)\,dx - a^2\int \sin(c+$$
$$= -4a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+dx)\,\text{c}}{2d}$$
$$= -3a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{2a^2\cot(c+dx}{d}$$

Mathematica [A] time = 2.1272, size = 158, normalized size = 1.61

$$\frac{a^2(\sin(c+dx)+1)^2\left(6\cos(c+dx)+2\cos(3(c+dx))+3\left(-4\sin(2(c+dx))+8\tan\left(\frac{1}{2}(c+dx)\right)-8\cot\left(\frac{1}{2}(c+dx)\right)-\text{cs}\right.\right.}{24d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(6*Cos[c + d*x] + 2*Cos[3*(c + d*x)] + 3*(-24*c -
 24*d*x - 8*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 + 4*Log[Cos[(c + d*x)/2]]
 - 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 - 4*Sin[2*(c + d*x)] + 8*Ta
n[(c + d*x)/2])))/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.081, size = 161, normalized size = 1.6

$$-\frac{a^2(\cos(dx+c))^3}{6d} - \frac{a^2\cos(dx+c)}{2d} - \frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d} - 2\frac{a^2(\cos(dx+c))^5}{d\sin(dx+c)} - 2\frac{a^2(\cos(dx+c))^3\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/6*a^2*cos(d*x+c)^3/d-1/2*a^2*cos(d*x+c)/d-1/2/d*a^2*ln(csc(d*x+c)-cot(d*
x+c))-2/d*a^2/sin(d*x+c)*cos(d*x+c)^5-2*a^2*cos(d*x+c)^3*sin(d*x+c)/d-3*a^2
*cos(d*x+c)*sin(d*x+c)/d-3*a^2*x-3/d*c*a^2-1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c
)^5
```

Maxima [A] time = 1.65571, size = 204, normalized size = 2.08

$$\frac{2\left(2\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a^{2}-12\left(3\,dx+3\,c+\frac{3\tan(d\cdots}{\tan(dx+c)^{3}\cdots}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*lo
g(cos(d*x + c) - 1))*a^2 - 12*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*
x + c)^3 + tan(d*x + c)))*a^2 + 3*a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1)
- 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.61137, size = 427, normalized size = 4.36

$$\frac{4\,a^{2}\cos\left(dx+c\right)^{5}-36\,a^{2}dx\cos\left(dx+c\right)^{2}-4\,a^{2}\cos\left(dx+c\right)^{3}+36\,a^{2}dx+6\,a^{2}\cos\left(dx+c\right)+3\left(a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\cdots}{12\left(d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(4*a^2*cos(d*x + c)^5 - 36*a^2*d*x*cos(d*x + c)^2 - 4*a^2*cos(d*x + c)
^3 + 36*a^2*d*x + 6*a^2*cos(d*x + c) + 3*(a^2*cos(d*x + c)^2 - a^2)*log(1/2
*cos(d*x + c) + 1/2) - 3*(a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) +
 1/2) - 12*(a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38414, size = 240, normalized size = 2.45

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)a^2-12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,a^2\tan}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x + c)*a^2 - 12*a^2*log(abs(tan(
1/2*d*x + 1/2*c))) + 24*a^2*tan(1/2*d*x + 1/2*c) + 3*(6*a^2*tan(1/2*d*x + 1
/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 16*(3*
a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a^2*tan(1/2*d*x + 1/2*c)^4 - 3*a^2*tan(1/2*d
*x + 1/2*c) + a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.385 $\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=98

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}$$

```
[Out] -(a^2*x)/2 + (3*a^2*ArcTanh[Cos[c + d*x]])/d - (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (a^2*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.155112, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635}

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x)/2 + (3*a^2*ArcTanh[Cos[c + d*x]])/d - (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (a^2*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-a^6-4a^6\csc(c+dx)-a^6\csc^2(c+dx)+2a^6\csc^3(c+dx)+a^6\csc^4(c+dx)\right.}{a^4}$$

$$= -a^2x-a^2\int \csc^2(c+dx)\,dx+a^2\int \csc^4(c+dx)\,dx+a^2\int \sin^2(c+dx)\,dx+\left(2\right.$$

$$= -a^2x+\frac{4a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}$$

$$= -\frac{a^2x}{2}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c}{}$$

Mathematica [A] time = 5.3888, size = 191, normalized size = 1.95

$$\frac{a^2(\sin(c+dx)+1)^2\left(-12(c+dx)-6\sin(2(c+dx))-48\cos(c+dx)-4\tan\left(\frac{1}{2}(c+dx)\right)+4\cot\left(\frac{1}{2}(c+dx)\right)-6\csc^2\left(\frac{1}{2}(\right.\right.}{24d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-12*(c + d*x) - 48*Cos[c + d*x] + 4*Cot[(c + d*x
)/2] - 6*Csc[(c + d*x)/2]^2 + 72*Log[Cos[(c + d*x)/2]] - 72*Log[Sin[(c + d*
x)/2]] + 6*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[
(c + d*x)/2]^4*Sin[c + d*x])/2 - 6*Sin[2*(c + d*x)] - 4*Tan[(c + d*x)/2]))/
(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [B] time = 0.078, size = 190, normalized size = 1.9

$$-\frac{a^2(\cos(dx+c))^5}{d\sin(dx+c)}-\frac{a^2(\cos(dx+c))^3\sin(dx+c)}{d}-\frac{3a^2\cos(dx+c)\sin(dx+c)}{2d}-\frac{a^2x}{2}-\frac{ca^2}{2d}-\frac{a^2(\cos(dx+c))^5}{d(\sin(dx+c))^2}-a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^5-a^2*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a^2*cos(
d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*c*a^2-1/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5
-a^2*cos(d*x+c)^3/d-3*a^2*cos(d*x+c)/d-3/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/
3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d
```

Maxima [A] time = 1.74867, size = 188, normalized size = 1.92

$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2-2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(c\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a^2 - 2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*a
^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x +
c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.43615, size = 474, normalized size = 4.84

$$\frac{3\,a^2\cos\left(dx+c\right)^5-4\,a^2\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)+9\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^5 - 4*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c) + 9*(
a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a^2
*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a^2*d
*x*cos(d*x + c)^2 + 4*a^2*cos(d*x + c)^3 - a^2*d*x - 6*a^2*cos(d*x + c))*si
n(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31259, size = 282, normalized size = 2.88

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\left(dx+c\right)a^2-72\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x +
 c)*a^2 - 72*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan
(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a^2*tan(1/
2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*tan(1/2*d*x + 1/2*c
) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.386 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\cot(c+dx)}{d}+\frac{9a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}+a^2 \cdots$$

```
[Out] 2*a^2*x + (9*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cos[c + d*x])/d + (2*a
^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*x]*Csc[c
 + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.182647, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 3770, 3767, 8, 3768, 2638}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\cot(c+dx)}{d}+\frac{9a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}+a^2 \cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (9*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cos[c + d*x])/d + (2*a
^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*x]*Csc[c
 + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^6 - a^6\csc(c+dx) - 4a^6\csc^2(c+dx) - a^6\csc^3(c+dx) + 2a^6\,\text{cs}\right.}{a^4}$$
$$= 2a^2x - a^2\int \csc(c+dx)\,dx - a^2\int \csc^3(c+dx)\,dx + a^2\int \csc^5(c+$$
$$= 2a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cot(c+dx)\,\text{cs}}{2d}$$
$$= 2a^2x + \frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^2\cos(c+dx)}{d} + \frac{2a^2\cot(c+dx)}{d}$$
$$= 2a^2x + \frac{9a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^2\cos(c+dx)}{d} + \frac{2a^2\cot(c+dx)}{d}$$

Mathematica [A] time = 1.18024, size = 215, normalized size = 1.85

$$-\frac{a^2\sin(c+dx)(\sin(c+dx)+1)^2\left(192\cot(c+dx)+\csc^4\left(\frac{1}{2}(c+dx)\right)(3\csc(c+dx)+8)-2\csc^2\left(\frac{1}{2}(c+dx)\right)(3\csc(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(192*Cot[c + d*x] + Csc[(c + d*x)/2]^4*(8 + 3*Csc[c + d*x]) - 2*Csc[(
c + d*x)/2]^2*(64 + 3*Csc[c + d*x]) - 24*Csc[c + d*x]*(16*(c + d*x) + 9*Log
[Cos[(c + d*x)/2]] - 9*Log[Sin[(c + d*x)/2]]) + 8*(7 + 8*Cos[c + d*x])*Sec[
(c + d*x)/2]^4 + 24*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 - 48*Csc[c + d*x]^5*S
in[(c + d*x)/2]^4)*Sin[c + d*x]*(1 + Sin[c + d*x])^2)/(192*d*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.08, size = 149, normalized size = 1.3

$$-\frac{3a^2(\cos(dx+c))^5}{8d(\sin(dx+c))^2} - \frac{3a^2(\cos(dx+c))^3}{8d} - \frac{9a^2\cos(dx+c)}{8d} - \frac{9a^2\ln(\csc(dx+c)-\cot(dx+c))}{8d} - \frac{2a^2(\cot(dx+c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5-3/8*a^2*cos(d*x+c)^3/d-9/8*a^2*cos(d*x
+c)/d-9/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/3*a^2*cot(d*x+c)^3/d+2*a^2*cot(
d*x+c)/d+2*a^2*x+2/d*c*a^2-1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5
```

Maxima [A] time = 1.69817, size = 225, normalized size = 1.94

$$\frac{32\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2 - 3\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1} + 3\,\log(\cos(dx+c)+1) - 3\,\log(\cos(dx+c)-1)\right) + 1}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/48*(32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*a^2*
(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 +
 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 12*a^2*(2*cos(d*
x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*
log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.67941, size = 566, normalized size = 4.88

$$\frac{96\,a^2dx\cos(dx+c)^4 - 48\,a^2\cos(dx+c)^5 - 192\,a^2dx\cos(dx+c)^2 + 90\,a^2\cos(dx+c)^3 + 96\,a^2dx - 54\,a^2\cos(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(96*a^2*d*x*cos(d*x + c)^4 - 48*a^2*cos(d*x + c)^5 - 192*a^2*d*x*cos(d
*x + c)^2 + 90*a^2*cos(d*x + c)^3 + 96*a^2*d*x - 54*a^2*cos(d*x + c) + 27*(
a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2
) - 27*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x +
 c) + 1/2) - 32*(4*a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32732, size = 219, normalized size = 1.89

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 384\,(dx+c)a^2 - 216\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - 240\,a^2\tan\left(\frac{1}{2}\,dx\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*tan(1/2*d*x + 1/2*c)^3 + 384*(
d*x + c)*a^2 - 216*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 240*a^2*tan(1/2*d*x
```

```
 + 1/2*c) - 384*a^2/(tan(1/2*d*x + 1/2*c)^2 + 1) + (450*a^2*tan(1/2*d*x + 1
/2*c)^4 + 240*a^2*tan(1/2*d*x + 1/2*c)^3 - 16*a^2*tan(1/2*d*x + 1/2*c) - 3*
a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.387 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=118

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{2d}+\frac{3a^2 c}{}$$

```
[Out] a^2*x - (3*a^2*ArcTanh[Cos[c + d*x]])/(4*d) + (a^2*Cot[c + d*x])/d - (a^2*C
ot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]^5)/(5*d) + (3*a^2*Cot[c + d*x]*Csc
[c + d*x])/(4*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x])/(2*d)
```

Rubi [A] time = 0.191195, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{2d}+\frac{3a^2 c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (3*a^2*ArcTanh[Cos[c + d*x]])/(4*d) + (a^2*Cot[c + d*x])/d - (a^2*C
ot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]^5)/(5*d) + (3*a^2*Cot[c + d*x]*Csc
[c + d*x])/(4*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x])/(2*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx) + 2a^2\cot^4(c+dx)\csc(c+dx) + a^2\cot^4(c+dx)\right.$$
$$= a^2\int \cot^4(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right)\int$$
$$= -\frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^3(c+dx)\csc(c+dx)}{2d} - a^2\int \cot^2(c+dx)$$
$$= \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{3a^2\cot(c+d}{4}$$
$$= a^2x - \frac{3a^2\tanh^{-1}(\cos(c+dx))}{4d} + \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d}$$

Mathematica [A] time = 0.504887, size = 200, normalized size = 1.69

$$\frac{a^2\left(-272\tan\left(\frac{1}{2}(c+dx)\right) + 272\cot\left(\frac{1}{2}(c+dx)\right) + 150\csc^2\left(\frac{1}{2}(c+dx)\right) + 15\sec^4\left(\frac{1}{2}(c+dx)\right) - 150\sec^2\left(\frac{1}{2}(c+dx)\right) + 36\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(480*c + 480*d*x + 272*Cot[(c + d*x)/2] + 150*Csc[(c + d*x)/2]^2 - 360
*Log[Cos[(c + d*x)/2]] + 360*Log[Sin[(c + d*x)/2]] - 150*Sec[(c + d*x)/2]^2
 + 15*Sec[(c + d*x)/2]^4 - 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 96*Csc[c +
 d*x]^5*Sin[(c + d*x)/2]^6 + (Csc[(c + d*x)/2]^4*(-30 + Sin[c + d*x]))/2 -
(3*Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 - 272*Tan[(c + d*x)/2]))/(480*d)
```

Maple [A] time = 0.08, size = 170, normalized size = 1.4

$$-\frac{a^2(\cot(dx+c))^3}{3d} + \frac{a^2\cot(dx+c)}{d} + a^2x + \frac{ca^2}{d} - \frac{a^2(\cos(dx+c))^5}{2d(\sin(dx+c))^4} + \frac{a^2(\cos(dx+c))^5}{4d(\sin(dx+c))^2} + \frac{a^2(\cos(dx+c))^3}{4d} + \frac{3a^2\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d+a^2*x+1/d*c*a^2-1/2/d*a^2/sin(d*x+
c)^4*cos(d*x+c)^5+1/4/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+1/4*a^2*cos(d*x+c)^3/
d+3/4*a^2*cos(d*x+c)/d+3/4/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*a^2/sin(d*
x+c)^5*cos(d*x+c)^5
```

Maxima [A] time = 1.61636, size = 167, normalized size = 1.42

$$\frac{40\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-15\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/120*(40*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 15*a^
2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) - 24*a^2/tan(d*x
 + c)^5)/d
```

Fricas [B] time = 1.53048, size = 621, normalized size = 5.26

$$136\,a^2\cos\left(dx+c\right)^5-280\,a^2\cos\left(dx+c\right)^3+120\,a^2\cos\left(dx+c\right)-45\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(136*a^2*cos(d*x + c)^5 - 280*a^2*cos(d*x + c)^3 + 120*a^2*cos(d*x +
c) - 45*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 45*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a
^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(4*a^2*d*x*cos(d*x + c)^
4 - 8*a^2*d*x*cos(d*x + c)^2 - 5*a^2*cos(d*x + c)^3 + 4*a^2*d*x + 3*a^2*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27501, size = 279, normalized size = 2.36

$$3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+480\,(dx+c)a^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 + 5*a^2
*tan(1/2*d*x + 1/2*c)^3 - 120*a^2*tan(1/2*d*x + 1/2*c)^2 + 480*(d*x + c)*a^
2 + 360*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 270*a^2*tan(1/2*d*x + 1/2*c) -
 (822*a^2*tan(1/2*d*x + 1/2*c)^5 - 270*a^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2
*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*tan(1/2*d*x
 + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.388 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=132

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{a^2\cot(c$$

```
[Out] (-7*a^2*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) + (5*a
^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x])/(4
*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^2*Cot[c + d*x]^3*Csc[c +
 d*x]^3)/(6*d)
```

Rubi [A] time = 0.261761, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{a^2\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-7*a^2*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) + (5*a
^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x])/(4
*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^2*Cot[c + d*x]^3*Csc[c +
 d*x]^3)/(6*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc(c+dx)+2a^2\cot^4(c+dx)\csc^2(c+dx)+a^2\right.$$

$$= a^2\int \cot^4(c+dx)\csc(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^3(c+dx)\,d$$

$$= -\frac{a^2\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}a^2$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} + \frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)}{4d}$$

$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2a^2\cot^5(c+dx)}{5d} + \frac{5a^2\cot(c+dx)\csc}{16d}$$

$$= -\frac{7a^2\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^2\cot^5(c+dx)}{5d} + \frac{5a^2\cot(c+dx)\csc}{16d}$$

Mathematica [B] time = 0.103325, size = 267, normalized size = 2.02

$$a^2\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{5d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{5d} - \frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{9\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{9\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(-Cot[(c + d*x)/2]/(5*d) + (9*Csc[(c + d*x)/2]^2)/(64*d) + (7*Cot[(c +
d*x)/2]*Csc[(c + d*x)/2]^2)/(80*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/
(80*d) - Csc[(c + d*x)/2]^6/(384*d) - (7*Log[Cos[(c + d*x)/2]])/(16*d) + (7
*Log[Sin[(c + d*x)/2]])/(16*d) - (9*Sec[(c + d*x)/2]^2)/(64*d) + Sec[(c + d
*x)/2]^6/(384*d) + Tan[(c + d*x)/2]/(5*d) - (7*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(80*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))
```

Maple [A] time = 0.081, size = 152, normalized size = 1.2

$$-\frac{7a^2(\cos(dx+c))^5}{24d(\sin(dx+c))^4} + \frac{7a^2(\cos(dx+c))^5}{48d(\sin(dx+c))^2} + \frac{7a^2(\cos(dx+c))^3}{48d} + \frac{7a^2\cos(dx+c)}{16d} + \frac{7a^2\ln(\csc(dx+c)-\cot(dx+c}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -7/24/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+7/48/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+
7/48*a^2*cos(d*x+c)^3/d+7/16*a^2*cos(d*x+c)/d+7/16/d*a^2*ln(csc(d*x+c)-cot(
```

```
d*x+c))-2/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1/6/d*a^2/sin(d*x+c)^6*cos(d*x+
c)^5
```

Maxima [A] time = 1.1125, size = 244, normalized size = 1.85

$$\frac{5\,a^{2}\left(\frac{2\left(3\,\cos(dx+c)^{5}+8\,\cos(dx+c)^{3}-3\,\cos(dx+c)\right)}{\cos(dx+c)^{6}-3\,\cos(dx+c)^{4}+3\,\cos(dx+c)^{2}-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-30\,a^{2}\left(\frac{2\left(5\,\cos(dx+c)^{3}-3\,\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\,\cos(d\right.}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/480*(5*a^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 30*a^2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x
+ c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3
*log(cos(d*x + c) - 1)) - 192*a^2/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.52329, size = 537, normalized size = 4.07

$$\frac{192\,a^{2}\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)-270\,a^{2}\cos\left(dx+c\right)^{5}+560\,a^{2}\cos\left(dx+c\right)^{3}-210\,a^{2}\cos\left(dx+c\right)-105\left(a^{2}\cos\left(dx+\right.\right.}{480\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(192*a^2*cos(d*x + c)^5*sin(d*x + c) - 270*a^2*cos(d*x + c)^5 + 560*a
^2*cos(d*x + c)^3 - 210*a^2*cos(d*x + c) - 105*(a^2*cos(d*x + c)^6 - 3*a^2*
cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) +
105*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2
)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*
d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35044, size = 309, normalized size = 2.34

$$5\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^6 + 24\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 15\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 - 120\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 255\,a^2\tan\left(\frac{1}{2}\,dx +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a
^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2*tan(1/2*d*x + 1/2*c)^3 - 255*a^2*tan(1/
2*d*x + 1/2*c)^2 + 840*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 240*a^2*tan(1/2
*d*x + 1/2*c) - (2058*a^2*tan(1/2*d*x + 1/2*c)^6 + 240*a^2*tan(1/2*d*x + 1/
2*c)^5 - 255*a^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2*tan(1/2*d*x + 1/2*c)^3 +
15*a^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*tan(1/2*d*x + 1/2*c) + 5*a^2)/tan(1/
2*d*x + 1/2*c)^6)/d
```

3.389 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=176

$$-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{11a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^2\cot^3(c+dx)\,\mathrm{cs}}{6d}$$

```
[Out] (-11*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2
*a^2*Cot[c + d*x]^7)/(7*d) - (11*a^2*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (
7*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x
]^3)/(6*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a^2*Cot[c + d*x]^3
*Csc[c + d*x]^5)/(8*d)
```

Rubi [A] time = 0.32037, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 14}

$$-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{11a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^2\cot^3(c+dx)\,\mathrm{cs}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2
*a^2*Cot[c + d*x]^7)/(7*d) - (11*a^2*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (
7*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x
]^3)/(6*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a^2*Cot[c + d*x]^3
*Csc[c + d*x]^5)/(8*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc^3(c+dx)+2a^2\cot^4(c+dx)\csc^4(c+dx)+a\right.$$

$$= a^2\int \cot^4(c+dx)\csc^3(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^5(c+dx)$$

$$= -\frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{1}{8}\left(3\right.$$

$$= \frac{a^2\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d} + \frac{a^2\cot}{}$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot(c+dx)\csc(c+dx)}{16d}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} - 1$$

$$= -\frac{11a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d}$$

Mathematica [A] time = 0.911599, size = 291, normalized size = 1.65

$$-\frac{a^2\csc^8(c+dx)\left(86016\sin(2(c+dx))+64512\sin(4(c+dx))+12288\sin(6(c+dx))-1536\sin(8(c+dx))+158270\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^8*(158270*Cos[c + d*x] + 77210*Cos[3*(c + d*x)] - 18130*
Cos[5*(c + d*x)] - 2310*Cos[7*(c + d*x)] + 40425*Log[Cos[(c + d*x)/2]] - 64
680*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 32340*Cos[4*(c + d*x)]*Log[Cos
[(c + d*x)/2]] - 9240*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 1155*Cos[8*(
c + d*x)]*Log[Cos[(c + d*x)/2]] - 40425*Log[Sin[(c + d*x)/2]] + 64680*Cos[2
*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 32340*Cos[4*(c + d*x)]*Log[Sin[(c + d*x
)/2]] + 9240*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 1155*Cos[8*(c + d*x)]
*Log[Sin[(c + d*x)/2]] + 86016*Sin[2*(c + d*x)] + 64512*Sin[4*(c + d*x)] +
12288*Sin[6*(c + d*x)] - 1536*Sin[8*(c + d*x)]))/(1720320*d)
```

Maple [A] time = 0.086, size = 200, normalized size = 1.1

$$-\frac{11a^2(\cos(dx+c))^5}{48d(\sin(dx+c))^6} - \frac{11a^2(\cos(dx+c))^5}{192d(\sin(dx+c))^4} + \frac{11a^2(\cos(dx+c))^5}{384d(\sin(dx+c))^2} + \frac{11a^2(\cos(dx+c))^3}{384d} + \frac{11a^2\cos(dx+c)}{128d} + \frac{11a^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -11/48/d*a^2/sin(d*x+c)^6*cos(d*x+c)^5-11/192/d*a^2/sin(d*x+c)^4*cos(d*x+c)
^5+11/384/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+11/384*a^2*cos(d*x+c)^3/d+11/128*
a^2*cos(d*x+c)/d+11/128/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/7/d*a^2/sin(d*x+c
)^7*cos(d*x+c)^5-4/35/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1/8/d*a^2/sin(d*x+c)^
8*cos(d*x+c)^5
```

Maxima [A] time = 1.12378, size = 315, normalized size = 1.79

$$\frac{105\,a^2\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+280\,a^2\left(\cdots\right.}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/26880*(105*a^2*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)
^3 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)
) + 280*a^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
+ 1) + 3*log(cos(d*x + c) - 1)) - 1536*(7*tan(d*x + c)^2 + 5)*a^2/tan(d*x +
 c)^7)/d
```

Fricas [A] time = 1.64732, size = 709, normalized size = 4.03

$$\frac{2310\,a^2\cos\left(dx+c\right)^7+490\,a^2\cos\left(dx+c\right)^5-8470\,a^2\cos\left(dx+c\right)^3+2310\,a^2\cos\left(dx+c\right)-1155\left(a^2\cos\left(dx+c\right)^8-4\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/26880*(2310*a^2*cos(d*x + c)^7 + 490*a^2*cos(d*x + c)^5 - 8470*a^2*cos(d*
x + c)^3 + 2310*a^2*cos(d*x + c) - 1155*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x
 + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*
x + c) + 1/2) + 1155*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos
(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c) + 1/2) + 15
36*(2*a^2*cos(d*x + c)^7 - 7*a^2*cos(d*x + c)^5)*sin(d*x + c))/(d*cos(d*x +
 c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.495, size = 396, normalized size = 2.25

$$\frac{105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+480\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-672\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2520\,a^2\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/215040*(105*a^2*tan(1/2*d*x + 1/2*c)^8 + 480*a^2*tan(1/2*d*x + 1/2*c)^7 +
 560*a^2*tan(1/2*d*x + 1/2*c)^6 - 672*a^2*tan(1/2*d*x + 1/2*c)^5 - 2520*a^2
*tan(1/2*d*x + 1/2*c)^4 - 3360*a^2*tan(1/2*d*x + 1/2*c)^3 - 1680*a^2*tan(1/
2*d*x + 1/2*c)^2 + 18480*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 10080*a^2*tan
(1/2*d*x + 1/2*c) - (50226*a^2*tan(1/2*d*x + 1/2*c)^8 + 10080*a^2*tan(1/2*d
*x + 1/2*c)^7 - 1680*a^2*tan(1/2*d*x + 1/2*c)^6 - 3360*a^2*tan(1/2*d*x + 1/
2*c)^5 - 2520*a^2*tan(1/2*d*x + 1/2*c)^4 - 672*a^2*tan(1/2*d*x + 1/2*c)^3 +
 560*a^2*tan(1/2*d*x + 1/2*c)^2 + 480*a^2*tan(1/2*d*x + 1/2*c) + 105*a^2)/t
an(1/2*d*x + 1/2*c)^8)/d
```

3.390 $\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{3a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d}+$$

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(64*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (3*a
^2*Cot[c + d*x]^7)/(7*d) - (a^2*Cot[c + d*x]^9)/(9*d) - (3*a^2*Cot[c + d*x]
*Csc[c + d*x])/(64*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (a^2*Cot
[c + d*x]*Csc[c + d*x]^5)/(8*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(4*d)
```

Rubi [A] time = 0.274119, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{3a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(64*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (3*a
^2*Cot[c + d*x]^7)/(7*d) - (a^2*Cot[c + d*x]^9)/(9*d) - (3*a^2*Cot[c + d*x]
*Csc[c + d*x])/(64*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (a^2*Cot
[c + d*x]*Csc[c + d*x]^5)/(8*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(4*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc^4(c+dx)+2a^2\cot^4(c+dx)\csc^5(c+dx)+a\right.$$

$$= a^2\int \cot^4(c+dx)\csc^4(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^6(c+dx)$$

$$= -\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d} - \frac{1}{4}\left(3a^2\right)\int \cot^2(c+dx)\csc^5(c+dx$$

$$= \frac{a^2\cot(c+dx)\csc^5(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d} + \frac{1}{8}a^2\int$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} - \frac{a^2\cot(c}{}$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} - \frac{3a^2\cot(c}{}$$

$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{64d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^7(c+dx)}{7d} -$$

Mathematica [A] time = 1.29892, size = 313, normalized size = 1.86

$$-\frac{a^2\csc^9(c+dx)\left(212940\sin(2(c+dx))+195300\sin(4(c+dx))+16380\sin(6(c+dx))-1890\sin(8(c+dx))+451584\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^9*(451584*Cos[c + d*x] + 155904*Cos[3*(c + d*x)] - 20736
*Cos[5*(c + d*x)] - 14976*Cos[7*(c + d*x)] + 1664*Cos[9*(c + d*x)] + 119070
*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 119070*Log[Sin[(c + d*x)/2]]*Sin[c +
d*x] + 212940*Sin[2*(c + d*x)] - 79380*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x
)] + 79380*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 195300*Sin[4*(c + d*x)]
 + 34020*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 34020*Log[Sin[(c + d*x)/2
]]*Sin[5*(c + d*x)] + 16380*Sin[6*(c + d*x)] - 8505*Log[Cos[(c + d*x)/2]]*S
in[7*(c + d*x)] + 8505*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 1890*Sin[8*
(c + d*x)] + 945*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 945*Log[Sin[(c +
d*x)/2]]*Sin[9*(c + d*x)]))/(5160960*d)
```

Maple [A] time = 0.086, size = 224, normalized size = 1.3

$$-\frac{13 a^2 (\cos (dx+c))^5}{63 d (\sin (dx+c))^7}-\frac{26 a^2 (\cos (dx+c))^5}{315 d (\sin (dx+c))^5}-\frac{a^2 (\cos (dx+c))^5}{4 d (\sin (dx+c))^8}-\frac{a^2 (\cos (dx+c))^5}{8 d (\sin (dx+c))^6}-\frac{a^2 (\cos (dx+c))^5}{32 d (\sin (dx+c))^4}+\frac{a^2 (\cos}{64 d (\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -13/63/d*a^2/sin(d*x+c)^7*cos(d*x+c)^5-26/315/d*a^2/sin(d*x+c)^5*cos(d*x+c)
^5-1/4/d*a^2/sin(d*x+c)^8*cos(d*x+c)^5-1/8/d*a^2/sin(d*x+c)^6*cos(d*x+c)^5-
1/32/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+1/64/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+1
/64*a^2*cos(d*x+c)^3/d+3/64*a^2*cos(d*x+c)/d+3/64/d*a^2*ln(csc(d*x+c)-cot(d
*x+c))-1/9/d*a^2/sin(d*x+c)^9*cos(d*x+c)^5
```

Maxima [A] time = 1.18087, size = 239, normalized size = 1.42

$$\frac{315 a^2\left(\frac{2\left(3 \cos(dx+c)^7-11 \cos(dx+c)^5-11 \cos(dx+c)^3+3 \cos(dx+c)\right)}{\cos(dx+c)^8-4 \cos(dx+c)^6+6 \cos(dx+c)^4-4 \cos(dx+c)^2+1}-3 \log (\cos (dx+c)+1)+3 \log (\cos (dx+c)-1)\right)-\frac{1152 (7 \mathrm{t}}{\mathrm{ta}}}{40320 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/40320*(315*a^2*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)
^3 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)
) - 1152*(7*tan(d*x + c)^2 + 5)*a^2/tan(d*x + c)^7 - 128*(63*tan(d*x + c)^4
 + 90*tan(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

Fricas [A] time = 1.61747, size = 795, normalized size = 4.73

$$-\frac{3328 a^2 \cos (dx+c)^9-14976 a^2 \cos (dx+c)^7+16128 a^2 \cos (dx+c)^5+945 \left(a^2 \cos (dx+c)^8-4 a^2 \cos (dx+c)^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/40320*(3328*a^2*cos(d*x + c)^9 - 14976*a^2*cos(d*x + c)^7 + 16128*a^2*co
s(d*x + c)^5 + 945*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d
*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 945*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)
^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
- 630*(3*a^2*cos(d*x + c)^7 - 11*a^2*cos(d*x + c)^5 - 11*a^2*cos(d*x + c)^3
 + 3*a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^
6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**10*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40843, size = 352, normalized size = 2.1

$$70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+450\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1008\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2520\,a^2\tan\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/322560*(70*a^2*tan(1/2*d*x + 1/2*c)^9 + 315*a^2*tan(1/2*d*x + 1/2*c)^8 +
450*a^2*tan(1/2*d*x + 1/2*c)^7 - 1008*a^2*tan(1/2*d*x + 1/2*c)^5 - 2520*a^2
*tan(1/2*d*x + 1/2*c)^4 - 3360*a^2*tan(1/2*d*x + 1/2*c)^3 + 15120*a^2*log(a
bs(tan(1/2*d*x + 1/2*c))) + 11340*a^2*tan(1/2*d*x + 1/2*c) - (42774*a^2*tan
(1/2*d*x + 1/2*c)^9 + 11340*a^2*tan(1/2*d*x + 1/2*c)^8 - 3360*a^2*tan(1/2*d
*x + 1/2*c)^6 - 2520*a^2*tan(1/2*d*x + 1/2*c)^5 - 1008*a^2*tan(1/2*d*x + 1/
2*c)^4 + 450*a^2*tan(1/2*d*x + 1/2*c)^2 + 315*a^2*tan(1/2*d*x + 1/2*c) + 70
*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.391 $\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=218

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{4a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{9a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^3(c+dx)\csc^7(c+dx)}{10d}-$$

```
[Out] (-9*a^2*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (4*
a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (9*a^2*Cot[c + d
*x]*Csc[c + d*x])/(256*d) - (3*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (
9*a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(160*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*
x]^5)/(8*d) + (3*a^2*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^2*Cot[c + d*x
]^3*Csc[c + d*x]^7)/(10*d)
```

Rubi [A] time = 0.352659, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 270}

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{4a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{9a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^3(c+dx)\csc^7(c+dx)}{10d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-9*a^2*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (4*
a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (9*a^2*Cot[c + d
*x]*Csc[c + d*x])/(256*d) - (3*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (
9*a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(160*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*
x]^5)/(8*d) + (3*a^2*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^2*Cot[c + d*x
]^3*Csc[c + d*x]^7)/(10*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc^5(c+dx)+2a^2\cot^4(c+dx)\csc^6(c+dx)+a\right.$$
$$= a^2\int \cot^4(c+dx)\csc^5(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^7(c+dx)$$
$$= -\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)\csc^7(c+dx)}{10d} - \frac{1}{10}($$
$$= \frac{a^2\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d} + \frac{3a^2\text{co}}{}$$
$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{a^2\cot(}{}$$
$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{3a^2\cot}{}$$
$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} -$$
$$= -\frac{9a^2\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} -$$

Mathematica [A] time = 1.1789, size = 353, normalized size = 1.62

$$-\frac{a^2\csc^{10}(c+dx)\left(1720320\sin(2(c+dx))+1228800\sin(4(c+dx))+184320\sin(6(c+dx))-40960\sin(8(c+dx))+40\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^10*(3219300*Cos[c + d*x] + 1237320*Cos[3*(c + d*x)] - 27
8712*Cos[5*(c + d*x)] - 54810*Cos[7*(c + d*x)] + 5670*Cos[9*(c + d*x)] + 35
7210*Log[Cos[(c + d*x)/2]] - 595350*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]]
+ 340200*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 127575*Cos[6*(c + d*x)]*L
og[Cos[(c + d*x)/2]] + 28350*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 2835*
Cos[10*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 357210*Log[Sin[(c + d*x)/2]] + 59
5350*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 340200*Cos[4*(c + d*x)]*Log[S
in[(c + d*x)/2]] + 127575*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 28350*Co
s[8*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 2835*Cos[10*(c + d*x)]*Log[Sin[(c +
d*x)/2]] + 1720320*Sin[2*(c + d*x)] + 1228800*Sin[4*(c + d*x)] + 184320*Sin
[6*(c + d*x)] - 40960*Sin[8*(c + d*x)] + 4096*Sin[10*(c + d*x)]))/(41287680
```

```
*d)
```

Maple [A] time = 0.085, size = 248, normalized size = 1.1

$$-\frac{3a^2(\cos(dx+c))^5}{16d(\sin(dx+c))^8}-\frac{3a^2(\cos(dx+c))^5}{32d(\sin(dx+c))^6}-\frac{3a^2(\cos(dx+c))^5}{128d(\sin(dx+c))^4}+\frac{3a^2(\cos(dx+c))^5}{256d(\sin(dx+c))^2}+\frac{3a^2(\cos(dx+c))^3}{256d}+\frac{9a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/16/d*a^2/sin(d*x+c)^8*cos(d*x+c)^5-3/32/d*a^2/sin(d*x+c)^6*cos(d*x+c)^5-
3/128/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+3/256/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5
+3/256*a^2*cos(d*x+c)^3/d+9/256*a^2*cos(d*x+c)/d+9/256/d*a^2*ln(csc(d*x+c)-
cot(d*x+c))-2/9/d*a^2/sin(d*x+c)^9*cos(d*x+c)^5-8/63/d*a^2/sin(d*x+c)^7*cos
(d*x+c)^5-16/315/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1/10/d*a^2/sin(d*x+c)^10*c
os(d*x+c)^5
```

Maxima [A] time = 1.15422, size = 382, normalized size = 1.75

$$\frac{63a^2\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7+128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/161280*(63*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 + 128*cos(d*x +
c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 630*a^2*(2*(3*cos(d*x +
c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 + 3*cos(d*x + c))/(cos(d*x + c
)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 3*log(c
os(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 1024*(63*tan(d*x + c)^4 + 90*
tan(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

Fricas [A] time = 1.54106, size = 887, normalized size = 4.07

$$\frac{5670a^2\cos(dx+c)^9-26460a^2\cos(dx+c)^7+16128a^2\cos(dx+c)^5+26460a^2\cos(dx+c)^3-5670a^2\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/161280*(5670*a^2*cos(d*x + c)^9 - 26460*a^2*cos(d*x + c)^7 + 16128*a^2*co
s(d*x + c)^5 + 26460*a^2*cos(d*x + c)^3 - 5670*a^2*cos(d*x + c) - 2835*(a^2
*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10*a^2*co
```

```
s(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) + 28
35*(a^2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10
*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1
/2) + 1024*(8*a^2*cos(d*x + c)^9 - 36*a^2*cos(d*x + c)^7 + 63*a^2*cos(d*x +
 c)^5)*sin(d*x + c))/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x
 + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**11*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48593, size = 482, normalized size = 2.21

$$126\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+945\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+720\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-630\,a^2\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(126*a^2*tan(1/2*d*x + 1/2*c)^10 + 560*a^2*tan(1/2*d*x + 1/2*c)^9
 + 945*a^2*tan(1/2*d*x + 1/2*c)^8 + 720*a^2*tan(1/2*d*x + 1/2*c)^7 - 630*a^
2*tan(1/2*d*x + 1/2*c)^6 - 4032*a^2*tan(1/2*d*x + 1/2*c)^5 - 7560*a^2*tan(1
/2*d*x + 1/2*c)^4 - 6720*a^2*tan(1/2*d*x + 1/2*c)^3 + 1260*a^2*tan(1/2*d*x
+ 1/2*c)^2 + 45360*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 30240*a^2*tan(1/2*d
*x + 1/2*c) - (132858*a^2*tan(1/2*d*x + 1/2*c)^10 + 30240*a^2*tan(1/2*d*x +
 1/2*c)^9 + 1260*a^2*tan(1/2*d*x + 1/2*c)^8 - 6720*a^2*tan(1/2*d*x + 1/2*c)
^7 - 7560*a^2*tan(1/2*d*x + 1/2*c)^6 - 4032*a^2*tan(1/2*d*x + 1/2*c)^5 - 63
0*a^2*tan(1/2*d*x + 1/2*c)^4 + 720*a^2*tan(1/2*d*x + 1/2*c)^3 + 945*a^2*tan
(1/2*d*x + 1/2*c)^2 + 560*a^2*tan(1/2*d*x + 1/2*c) + 126*a^2)/tan(1/2*d*x +
 1/2*c)^10)/d
```

3.392 $\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=203

$$\frac{a^3\cos^{11}(c+dx)}{11d}-\frac{2a^3\cos^9(c+dx)}{3d}+\frac{9a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{5a^3\sin^3}{}$$

```
[Out] (15*a^3*x)/256 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (9*a^3*Cos[c + d*x]^7)/(7*d
) - (2*a^3*Cos[c + d*x]^9)/(3*d) + (a^3*Cos[c + d*x]^11)/(11*d) + (15*a^3*C
os[c + d*x]*Sin[c + d*x])/(256*d) + (5*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(12
8*d) - (5*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(32*d) - (5*a^3*Cos[c + d*x]^5*S
in[c + d*x]^3)/(16*d) - (3*a^3*Cos[c + d*x]^5*Sin[c + d*x]^5)/(10*d)
```

Rubi [A] time = 0.393111, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 270}

$$\frac{a^3\cos^{11}(c+dx)}{11d}-\frac{2a^3\cos^9(c+dx)}{3d}+\frac{9a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{5a^3\sin^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (15*a^3*x)/256 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (9*a^3*Cos[c + d*x]^7)/(7*d
) - (2*a^3*Cos[c + d*x]^9)/(3*d) + (a^3*Cos[c + d*x]^11)/(11*d) + (15*a^3*C
os[c + d*x]*Sin[c + d*x])/(256*d) + (5*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(12
8*d) - (5*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(32*d) - (5*a^3*Cos[c + d*x]^5*S
in[c + d*x]^3)/(16*d) - (3*a^3*Cos[c + d*x]^5*Sin[c + d*x]^5)/(10*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^4(c+dx)\sin^4(c+dx)+3a^3\cos^4(c+dx)\sin^5(c+dx)+3a\right.$$
$$= a^3\int \cos^4(c+dx)\sin^4(c+dx)\,dx + a^3\int \cos^4(c+dx)\sin^7(c+dx)\,d$$
$$= -\frac{a^3\cos^5(c+dx)\sin^3(c+dx)}{8d} - \frac{3a^3\cos^5(c+dx)\sin^5(c+dx)}{10d} + \frac{1}{8}\Big($$
$$= -\frac{a^3\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{5a^3\cos^5(c+dx)\sin^3(c+dx)}{16d} - \frac{3a^3}{}$$
$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c+dx)}{3d} + \frac{a^3\cos^1}{1}$$
$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c+dx)}{3d} + \frac{a^3\cos^1}{1}$$
$$= \frac{3a^3x}{128} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c+dx)}{3d} + \frac{a}{}$$
$$= \frac{15a^3x}{256} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c+dx)}{3d} +$$

Mathematica [A] time = 1.02214, size = 126, normalized size = 0.62

$$\frac{a^3(-13860\sin(2(c+dx)) - 46200\sin(4(c+dx)) + 6930\sin(6(c+dx)) + 5775\sin(8(c+dx)) - 1386\sin(10(c+dx)) - 19}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(138600*c + 138600*d*x - 198660*Cos[c + d*x] - 41580*Cos[3*(c + d*x)]
+ 27258*Cos[5*(c + d*x)] + 3630*Cos[7*(c + d*x)] - 3850*Cos[9*(c + d*x)] +
210*Cos[11*(c + d*x)] - 13860*Sin[2*(c + d*x)] - 46200*Sin[4*(c + d*x)] + 6
930*Sin[6*(c + d*x)] + 5775*Sin[8*(c + d*x)] - 1386*Sin[10*(c + d*x)]))/(23
65440*d)
```

Maple [A] time = 0.046, size = 288, normalized size = 1.4

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^6(\cos(dx+c))^5}{11} - \frac{2(\sin(dx+c))^4(\cos(dx+c))^5}{33} - \frac{8(\sin(dx+c))^2(\cos(dx+c))^5}{231} - \frac{16(\cos(dx}{1155}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/11*sin(d*x+c)^6*cos(d*x+c)^5-2/33*sin(d*x+c)^4*cos(d*x+c)^5-8/
231*sin(d*x+c)^2*cos(d*x+c)^5-16/1155*cos(d*x+c)^5)+3*a^3*(-1/10*sin(d*x+c)
^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/32*sin(d*x+c)*cos(d*x+c)^5
+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+3*a^3*(-
1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+
c)^5)+a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64
*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

Maxima [A] time = 1.07797, size = 228, normalized size = 1.12

$$\frac{2048 \left(105 \cos(dx+c)^{11} - 385 \cos(dx+c)^{9} + 495 \cos(dx+c)^{7} - 231 \cos(dx+c)^{5}\right)a^3 - 22528 \left(35 \cos(dx+c)^{9} - \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2365440*(2048*(105*cos(d*x + c)^11 - 385*cos(d*x + c)^9 + 495*cos(d*x + c
)^7 - 231*cos(d*x + c)^5)*a^3 - 22528*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^
7 + 63*cos(d*x + c)^5)*a^3 - 693*(32*sin(2*d*x + 2*c)^5 - 120*d*x - 120*c -
 5*sin(8*d*x + 8*c) + 40*sin(4*d*x + 4*c))*a^3 + 2310*(24*d*x + 24*c + sin(
8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A] time = 1.59162, size = 379, normalized size = 1.87

$$\frac{26880 a^3 \cos(dx+c)^{11} - 197120 a^3 \cos(dx+c)^{9} + 380160 a^3 \cos(dx+c)^{7} - 236544 a^3 \cos(dx+c)^{5} + 17325 a^3 dx - \cdots}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/295680*(26880*a^3*cos(d*x + c)^11 - 197120*a^3*cos(d*x + c)^9 + 380160*a^
3*cos(d*x + c)^7 - 236544*a^3*cos(d*x + c)^5 + 17325*a^3*d*x - 231*(384*a^3
*cos(d*x + c)^9 - 1168*a^3*cos(d*x + c)^7 + 984*a^3*cos(d*x + c)^5 - 50*a^3
*cos(d*x + c)^3 - 75*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 61.1515, size = 648, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**10/256 + 45*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 3*a**3*x*sin(c + d*x)**8/128 + 45*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 3*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 45*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 9*a**3*x*sin(c + d*x)**4*cos(c + d*x
)**4/64 + 45*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a**3*x*sin(c +
```

```
d*x)**2*cos(c + d*x)**6/32 + 9*a**3*x*cos(c + d*x)**10/256 + 3*a**3*x*cos(c
 + d*x)**8/128 + 9*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 21*a**3*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + 3*a**3*sin(c + d*x)**7*cos(c + d*x)/(
128*d) - a**3*sin(c + d*x)**6*cos(c + d*x)**5/(5*d) - 3*a**3*sin(c + d*x)**
5*cos(c + d*x)**5/(10*d) + 11*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d)
- 6*a**3*sin(c + d*x)**4*cos(c + d*x)**7/(35*d) - 3*a**3*sin(c + d*x)**4*co
s(c + d*x)**5/(5*d) - 21*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 11*
a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 8*a**3*sin(c + d*x)**2*cos(c
 + d*x)**9/(105*d) - 12*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 9*a**
3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**
7/(128*d) - 16*a**3*cos(c + d*x)**11/(1155*d) - 8*a**3*cos(c + d*x)**9/(105
*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**4*cos(c)**4, True))
```

Giac [A] time = 1.45208, size = 258, normalized size = 1.27

$$\frac{15}{256}a^3x+\frac{a^3\cos(11\,dx+11\,c)}{11264\,d}-\frac{5\,a^3\cos(9\,dx+9\,c)}{3072\,d}+\frac{11\,a^3\cos(7\,dx+7\,c)}{7168\,d}+\frac{59\,a^3\cos(5\,dx+5\,c)}{5120\,d}-\frac{9\,a^3\cos(3\,dx+3}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 15/256*a^3*x + 1/11264*a^3*cos(11*d*x + 11*c)/d - 5/3072*a^3*cos(9*d*x + 9*
c)/d + 11/7168*a^3*cos(7*d*x + 7*c)/d + 59/5120*a^3*cos(5*d*x + 5*c)/d - 9/
512*a^3*cos(3*d*x + 3*c)/d - 43/512*a^3*cos(d*x + c)/d - 3/5120*a^3*sin(10*
d*x + 10*c)/d + 5/2048*a^3*sin(8*d*x + 8*c)/d + 3/1024*a^3*sin(6*d*x + 6*c)
/d - 5/256*a^3*sin(4*d*x + 4*c)/d - 3/512*a^3*sin(2*d*x + 2*c)/d
```

3.393 $\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=182

$$-\frac{a^3\cos^9(c+dx)}{3d}+\frac{a^3\cos^7(c+dx)}{d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{7a^3\sin^3(c+dx)\cos^5(c+dx)}{16d}$$

```
[Out] (21*a^3*x)/256 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (a^3*Cos[c + d*x]^7)/d - (a
^3*Cos[c + d*x]^9)/(3*d) + (21*a^3*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (7*
a^3*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) - (7*a^3*Cos[c + d*x]^5*Sin[c + d*
x])/(32*d) - (7*a^3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*d) - (a^3*Cos[c + d*
x]^5*Sin[c + d*x]^5)/(10*d)
```

Rubi [A] time = 0.382367, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{a^3\cos^9(c+dx)}{3d}+\frac{a^3\cos^7(c+dx)}{d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{7a^3\sin^3(c+dx)\cos^5(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (21*a^3*x)/256 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (a^3*Cos[c + d*x]^7)/d - (a
^3*Cos[c + d*x]^9)/(3*d) + (21*a^3*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (7*
a^3*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) - (7*a^3*Cos[c + d*x]^5*Sin[c + d*
x])/(32*d) - (7*a^3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*d) - (a^3*Cos[c + d*
x]^5*Sin[c + d*x]^5)/(10*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^4(c+dx)\sin^3(c+dx)+3a^3\cos^4(c+dx)\sin^4(c+dx)+3a\right.$$

$$= a^3\int \cos^4(c+dx)\sin^3(c+dx)\,dx + a^3\int \cos^4(c+dx)\sin^6(c+dx)\,d$$

$$= -\frac{3a^3\cos^5(c+dx)\sin^3(c+dx)}{8d} - \frac{a^3\cos^5(c+dx)\sin^5(c+dx)}{10d} + \frac{1}{2}a$$

$$= -\frac{3a^3\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{7a^3\cos^5(c+dx)\sin^3(c+dx)}{16d} - \frac{a^3}{}$$

$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+dx)}{3d} + \frac{3a^3\cos^3(}{}$$

$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+dx)}{3d} + \frac{9a^3\cos(c}{}$$

$$= \frac{9a^3x}{128} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+dx)}{3d} + \frac{21a}{}$$

$$= \frac{21a^3x}{256} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+dx)}{3d} + \frac{21}{}$$

Mathematica [A] time = 1.04624, size = 116, normalized size = 0.64

$$\frac{a^3(-60\sin(2(c+dx)) - 840\sin(4(c+dx)) + 30\sin(6(c+dx)) + 105\sin(8(c+dx)) - 6\sin(10(c+dx)) - 3600\cos(c+dx}{30720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(2700*c + 2520*d*x - 3600*Cos[c + d*x] - 960*Cos[3*(c + d*x)] + 384*Co
s[5*(c + d*x)] + 120*Cos[7*(c + d*x)] - 40*Cos[9*(c + d*x)] - 60*Sin[2*(c +
 d*x)] - 840*Sin[4*(c + d*x)] + 30*Sin[6*(c + d*x)] + 105*Sin[8*(c + d*x)]
- 6*Sin[10*(c + d*x)]))/(30720*d)
```

Maple [A] time = 0.046, size = 252, normalized size = 1.4

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^5(\cos(dx+c))^5}{10}-\frac{(\sin(dx+c))^3(\cos(dx+c))^5}{16}-\frac{\sin(dx+c)(\cos(dx+c))^5}{32}+\frac{\sin(dx+c)}{128}\left((\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/
32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/256*d*x+3/256*c)+3*a^3*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*c
os(d*x+c)^5-8/315*cos(d*x+c)^5)+3*a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*
sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128
*d*x+3/128*c)+a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5))
```

Maxima [A] time = 1.1205, size = 201, normalized size = 1.1

$$-\frac{2048\left(35\cos(dx+c)^9-90\cos(dx+c)^7+63\cos(dx+c)^5\right)a^3-6144\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^3+21\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/215040*(2048*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^3 - 6144*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^3 + 21*(32*sin(2*d*x +
2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x + 8*c) + 40*sin(4*d*x + 4*c))*a^3 -
630*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A] time = 1.6892, size = 327, normalized size = 1.8

$$-\frac{1280\,a^3\cos(dx+c)^9-3840\,a^3\cos(dx+c)^7+3072\,a^3\cos(dx+c)^5-315\,a^3dx+3\left(128\,a^3\cos(dx+c)^9-816\,a^3\cos\right.}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/3840*(1280*a^3*cos(d*x + c)^9 - 3840*a^3*cos(d*x + c)^7 + 3072*a^3*cos(d
*x + c)^5 - 315*a^3*d*x + 3*(128*a^3*cos(d*x + c)^9 - 816*a^3*cos(d*x + c)^
7 + 968*a^3*cos(d*x + c)^5 - 70*a^3*cos(d*x + c)^3 - 105*a^3*cos(d*x + c))*
sin(d*x + c))/d
```

Sympy [A] time = 35.7566, size = 595, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**10/256 + 15*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 9*a**3*x*sin(c + d*x)**8/128 + 15*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 9*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 27*a**3*x*sin(c + d*x)**4*cos(c + d*
x)**4/64 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 9*a**3*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 3*a**3*x*cos(c + d*x)**10/256 + 9*a**3*x*cos(
c + d*x)**8/128 + 3*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**3*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + 9*a**3*sin(c + d*x)**7*cos(c + d*x)/(
128*d) - a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 33*a**3*sin(c + d*x)
**5*cos(c + d*x)**3/(128*d) - 3*a**3*sin(c + d*x)**4*cos(c + d*x)**5/(5*d)
- 7*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 33*a**3*sin(c + d*x)**3*
cos(c + d*x)**5/(128*d) - 12*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) -
a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**3*sin(c + d*x)*cos(c + d*
x)**9/(256*d) - 9*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 8*a**3*cos(c
+ d*x)**9/(105*d) - 2*a**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c)
+ a)**3*sin(c)**3*cos(c)**4, True))
```

Giac [A] time = 1.39211, size = 235, normalized size = 1.29

$$\frac{21}{256}a^3x - \frac{a^3\cos(9\,dx+9\,c)}{768\,d} + \frac{a^3\cos(7\,dx+7\,c)}{256\,d} + \frac{a^3\cos(5\,dx+5\,c)}{80\,d} - \frac{a^3\cos(3\,dx+3\,c)}{32\,d} - \frac{15\,a^3\cos(dx+c)}{128\,d} - \frac{a^3\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 21/256*a^3*x - 1/768*a^3*cos(9*d*x + 9*c)/d + 1/256*a^3*cos(7*d*x + 7*c)/d
+ 1/80*a^3*cos(5*d*x + 5*c)/d - 1/32*a^3*cos(3*d*x + 3*c)/d - 15/128*a^3*co
s(d*x + c)/d - 1/5120*a^3*sin(10*d*x + 10*c)/d + 7/2048*a^3*sin(8*d*x + 8*c
)/d + 1/1024*a^3*sin(6*d*x + 6*c)/d - 7/256*a^3*sin(4*d*x + 4*c)/d - 1/512*
a^3*sin(2*d*x + 2*c)/d
```

3.394 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=159

$$-\frac{a^3\cos^9(c+dx)}{9d}+\frac{5a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{17a^3\sin(c+dx)\cos^5(c+a}{48d}$$

```
[Out] (17*a^3*x)/128 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (5*a^3*Cos[c + d*x]^7)/(7*d
) - (a^3*Cos[c + d*x]^9)/(9*d) + (17*a^3*Cos[c + d*x]*Sin[c + d*x])/(128*d)
 + (17*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (17*a^3*Cos[c + d*x]^5*Si
n[c + d*x])/(48*d) - (3*a^3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rubi [A] time = 0.323036, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2568, 2635, 8, 2565, 14, 270}

$$-\frac{a^3\cos^9(c+dx)}{9d}+\frac{5a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{17a^3\sin(c+dx)\cos^5(c+a}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (17*a^3*x)/128 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (5*a^3*Cos[c + d*x]^7)/(7*d
) - (a^3*Cos[c + d*x]^9)/(9*d) + (17*a^3*Cos[c + d*x]*Sin[c + d*x])/(128*d)
 + (17*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (17*a^3*Cos[c + d*x]^5*Si
n[c + d*x])/(48*d) - (3*a^3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^4(c+dx)\sin^2(c+dx) + 3a^3\cos^4(c+dx)\sin^3(c+dx) + 3a\right.$$

$$= a^3\int \cos^4(c+dx)\sin^2(c+dx)\,dx + a^3\int \cos^4(c+dx)\sin^5(c+dx)\,d$$

$$= -\frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{3a^3\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{6}a^3$$

$$= \frac{a^3\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{17a^3\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{3a^3 c}{}$$

$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{5a^3\cos^7(c+dx)}{7d} - \frac{a^3\cos^9(c+dx)}{9d} + \frac{a^3\cos(c}{}$$

$$= \frac{a^3x}{16} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{5a^3\cos^7(c+dx)}{7d} - \frac{a^3\cos^9(c+dx)}{9d} + \frac{17a}{}$$

$$= \frac{17a^3x}{128} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{5a^3\cos^7(c+dx)}{7d} - \frac{a^3\cos^9(c+dx)}{9d} + \frac{1}{}$$

Mathematica [A] time = 0.815539, size = 106, normalized size = 0.67

$$\frac{a^3(5040\sin(2(c+dx)) - 12600\sin(4(c+dx)) - 1680\sin(6(c+dx)) + 945\sin(8(c+dx)) - 52920\cos(c+dx) - 16800\cos}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(30240*c + 42840*d*x - 52920*Cos[c + d*x] - 16800*Cos[3*(c + d*x)] + 4
032*Cos[5*(c + d*x)] + 2340*Cos[7*(c + d*x)] - 140*Cos[9*(c + d*x)] + 5040*
Sin[2*(c + d*x)] - 12600*Sin[4*(c + d*x)] - 1680*Sin[6*(c + d*x)] + 945*Sin
[8*(c + d*x)]))/(322560*d)
```

Maple [A] time = 0.04, size = 216, normalized size = 1.4

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^5}{9} - \frac{4(\sin(dx+c))^2(\cos(dx+c))^5}{63} - \frac{8(\cos(dx+c))^5}{315}\right) + 3a^3\left(-1/8(\sin(dx+c))^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/3
15*cos(d*x+c)^5)+3*a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(
d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+3
*a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a^3*(-1/6*sin(d*x+c
)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*
c))
```

Maxima [A] time = 1.06261, size = 186, normalized size = 1.17

$$-\frac{1024\left(35\cos\left(dx+c\right)^{9}-90\cos\left(dx+c\right)^{7}+63\cos\left(dx+c\right)^{5}\right)a^{3}-27648\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a^{3}-168}{322}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/322560*(1024*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^3 - 27648*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^3 - 1680*(4*sin(2*d*x
+ 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3 - 945*(24*d*x + 24*c + s
in(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A] time = 1.56711, size = 300, normalized size = 1.89

$$-\frac{4480\,a^{3}\cos\left(dx+c\right)^{9}-28800\,a^{3}\cos\left(dx+c\right)^{7}+32256\,a^{3}\cos\left(dx+c\right)^{5}-5355\,a^{3}dx-105\left(144\,a^{3}\cos\left(dx+c\right)^{7}-28\right.}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40320*(4480*a^3*cos(d*x + c)^9 - 28800*a^3*cos(d*x + c)^7 + 32256*a^3*co
s(d*x + c)^5 - 5355*a^3*d*x - 105*(144*a^3*cos(d*x + c)^7 - 280*a^3*cos(d*x
 + c)^5 + 34*a^3*cos(d*x + c)^3 + 51*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 22.4269, size = 486, normalized size = 3.06

$$\begin{cases}\frac{9a^{3}x\sin^{8}(c+dx)}{128}+\frac{9a^{3}x\sin^{6}(c+dx)\cos^{2}(c+dx)}{32}+\frac{a^{3}x\sin^{6}(c+dx)}{16}+\frac{27a^{3}x\sin^{4}(c+dx)\cos^{4}(c+dx)}{64}+\frac{3a^{3}x\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{9a^{3}x\sin^{2}(c+}{3}\\ x\left(a\sin\left(c\right)+a\right)^{3}\sin^{2}\left(c\right)\cos^{4}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**8/128 + 9*a**3*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + a**3*x*sin(c + d*x)**6/16 + 27*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**4/64 + 3*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a**3*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
```

```
 9*a**3*x*cos(c + d*x)**8/128 + a**3*x*cos(c + d*x)**6/16 + 9*a**3*sin(c +
d*x)**7*cos(c + d*x)/(128*d) + 33*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128
*d) + a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - a**3*sin(c + d*x)**4*cos(c
 + d*x)**5/(5*d) - 33*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + a**3*s
in(c + d*x)**3*cos(c + d*x)**3/(6*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)*
*7/(35*d) - 3*a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 9*a**3*sin(c + d
*x)*cos(c + d*x)**7/(128*d) - a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 8*
a**3*cos(c + d*x)**9/(315*d) - 6*a**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x
*(a*sin(c) + a)**3*sin(c)**2*cos(c)**4, True))
```

Giac [A] time = 1.44074, size = 212, normalized size = 1.33

$$\frac{17}{128}a^3x - \frac{a^3\cos(9dx+9c)}{2304d} + \frac{13a^3\cos(7dx+7c)}{1792d} + \frac{a^3\cos(5dx+5c)}{80d} - \frac{5a^3\cos(3dx+3c)}{96d} - \frac{21a^3\cos(dx+c)}{128d} + \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 17/128*a^3*x - 1/2304*a^3*cos(9*d*x + 9*c)/d + 13/1792*a^3*cos(7*d*x + 7*c)
/d + 1/80*a^3*cos(5*d*x + 5*c)/d - 5/96*a^3*cos(3*d*x + 3*c)/d - 21/128*a^3
*cos(d*x + c)/d + 3/1024*a^3*sin(8*d*x + 8*c)/d - 1/192*a^3*sin(6*d*x + 6*c
)/d - 5/128*a^3*sin(4*d*x + 4*c)/d + 1/64*a^3*sin(2*d*x + 2*c)/d
```

3.395 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=157

$$-\frac{9a^3\cos^5(c+dx)}{80d}-\frac{9\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{9a^3\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{27a^3\sin(c+dx)\cos(c+dx)}{128d}$$

```
[Out] (27*a^3*x)/128 - (9*a^3*Cos[c + d*x]^5)/(80*d) + (27*a^3*Cos[c + d*x]*Sin[c
 + d*x])/(128*d) + (9*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) - (3*a*Cos[c
+ d*x]^5*(a + a*Sin[c + d*x])^2)/(56*d) - (Cos[c + d*x]^5*(a + a*Sin[c + d*
x])^3)/(8*d) - (9*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(112*d)
```

Rubi [A] time = 0.193872, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{9a^3\cos^5(c+dx)}{80d}-\frac{9\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{9a^3\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{27a^3\sin(c+dx)\cos(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*a^3*x)/128 - (9*a^3*Cos[c + d*x]^5)/(80*d) + (27*a^3*Cos[c + d*x]*Sin[c
 + d*x])/(128*d) + (9*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) - (3*a*Cos[c
+ d*x]^5*(a + a*Sin[c + d*x])^2)/(56*d) - (Cos[c + d*x]^5*(a + a*Sin[c + d*
x])^3)/(8*d) - (9*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(112*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^3}{8d}+\frac{3}{8}\int \cos^4(c+dx)(a+a\sin(c+$$

$$= -\frac{3a\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{\cos^5(c+dx)(a+a\sin(c+dx}{8d}$$

$$= -\frac{3a\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{\cos^5(c+dx)(a+a\sin(c+dx}{8d}$$

$$= -\frac{9a^3\cos^5(c+dx)}{80d}-\frac{3a\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\cos^5(c+$$

$$= -\frac{9a^3\cos^5(c+dx)}{80d}+\frac{9a^3\cos^3(c+dx)\sin(c+dx)}{64d}-\frac{3a\cos^5(c+dx)(}{5}$$

$$= -\frac{9a^3\cos^5(c+dx)}{80d}+\frac{27a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{9a^3\cos^3(c+dx}{64d}$$

$$= \frac{27a^3x}{128}-\frac{9a^3\cos^5(c+dx)}{80d}+\frac{27a^3\cos(c+dx)\sin(c+dx)}{128d}+9a^3\cos^3$$

Mathematica [A] time = 0.451391, size = 96, normalized size = 0.61

$$\frac{a^3(1680\sin(2(c+dx))-1960\sin(4(c+dx))-560\sin(6(c+dx))+35\sin(8(c+dx))-9520\cos(c+dx)-3920\cos(3(c}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(8400*c + 7560*d*x - 9520*Cos[c + d*x] - 3920*Cos[3*(c + d*x)] - 112*C
os[5*(c + d*x)] + 240*Cos[7*(c + d*x)] + 1680*Sin[2*(c + d*x)] - 1960*Sin[4
*(c + d*x)] - 560*Sin[6*(c + d*x)] + 35*Sin[8*(c + d*x)]))/(35840*d)
```

Maple [A] time = 0.038, size = 178, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^5}{8}-\frac{\sin(dx+c)(\cos(dx+c))^5}{16}+\frac{\sin(dx+c)}{64}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+3*a^3*(-1/7*sin
(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a^3*(-1/6*sin(d*x+c)*cos(d*x+c)
^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-1/5*a^3*c
os(d*x+c)^5)
```

Maxima [A] time = 1.12321, size = 155, normalized size = 0.99

$$-\frac{7168\,a^3\cos\left(dx+c\right)^5-3072\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)a^3-560\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\right.\right.}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35840*(7168*a^3*cos(d*x + c)^5 - 3072*(5*cos(d*x + c)^7 - 7*cos(d*x + c)
^5)*a^3 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a
^3 - 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A] time = 1.33474, size = 254, normalized size = 1.62

$$\frac{1920\,a^3\cos\left(dx+c\right)^7-3584\,a^3\cos\left(dx+c\right)^5+945\,a^3dx+35\left(16\,a^3\cos\left(dx+c\right)^7-88\,a^3\cos\left(dx+c\right)^5+18\,a^3\cos\left(dx\right.\right.}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4480*(1920*a^3*cos(d*x + c)^7 - 3584*a^3*cos(d*x + c)^5 + 945*a^3*d*x + 3
5*(16*a^3*cos(d*x + c)^7 - 88*a^3*cos(d*x + c)^5 + 18*a^3*cos(d*x + c)^3 +
27*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 12.8882, size = 440, normalized size = 2.8

$$\begin{cases}\frac{3a^3x\sin^8(c+dx)}{128}+\frac{3a^3x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{3a^3x\sin^6(c+dx)}{16}+\frac{9a^3x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{9a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3a^3x\sin^2(c+\right.}{} \\ x\left(a\sin\left(c\right)+a\right)^3\sin\left(c\right)\cos^4\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**8/128 + 3*a**3*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + 3*a**3*x*sin(c + d*x)**6/16 + 9*a**3*x*sin(c + d*x)**4*cos(c +
 d*x)**4/64 + 9*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*a**3*x*sin(c
+ d*x)**2*cos(c + d*x)**6/32 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16
+ 3*a**3*x*cos(c + d*x)**8/128 + 3*a**3*x*cos(c + d*x)**6/16 + 3*a**3*sin(c
 + d*x)**7*cos(c + d*x)/(128*d) + 11*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(
128*d) + 3*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 11*a**3*sin(c + d*x)*
*3*cos(c + d*x)**5/(128*d) + a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3
*a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**3*sin(c + d*x)*cos(c + d
*x)**7/(128*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 6*a**3*cos(c
+ d*x)**7/(35*d) - a**3*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)
**3*sin(c)*cos(c)**4, True))
```

Giac [A] time = 1.25603, size = 189, normalized size = 1.2

$$\frac{27}{128}\,a^3x+\frac{3\,a^3\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{a^3\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{7\,a^3\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{17\,a^3\cos\left(dx+c\right)}{64\,d}+\frac{a^3\sin\left(8\,dx+8\,c\right)}{1024\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 27/128*a^3*x + 3/448*a^3*cos(7*d*x + 7*c)/d - 1/320*a^3*cos(5*d*x + 5*c)/d
- 7/64*a^3*cos(3*d*x + 3*c)/d - 17/64*a^3*cos(d*x + c)/d + 1/1024*a^3*sin(8
*d*x + 8*c)/d - 1/64*a^3*sin(6*d*x + 6*c)/d - 7/128*a^3*sin(4*d*x + 4*c)/d
+ 3/64*a^3*sin(2*d*x + 2*c)/d
```

3.396 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=143

$$-\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{19a^3\sin(c+dx)\cos^3(c+dx)}{24d}+$$

```
[Out] (19*a^3*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d + (a^3
*Cos[c + d*x]^3)/(3*d) - (3*a^3*Cos[c + d*x]^5)/(5*d) + (19*a^3*Cos[c + d*x
]*Sin[c + d*x])/(16*d) + (19*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^3
*Cos[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.202189, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2873, 2635, 8, 2592, 302, 206, 2565, 30, 2568}

$$-\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{19a^3\sin(c+dx)\cos^3(c+dx)}{24d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (19*a^3*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d + (a^3
*Cos[c + d*x]^3)/(3*d) - (3*a^3*Cos[c + d*x]^5)/(5*d) + (19*a^3*Cos[c + d*x
]*Sin[c + d*x])/(16*d) + (19*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^3
*Cos[c + d*x]^5*Sin[c + d*x])/(6*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(3a^3\cos^4(c+dx) + a^3\cos^3(c+dx)\cot(c+dx) + 3a^3\cos^4(c+dx\right.$$

$$= a^3\int \cos^3(c+dx)\cot(c+dx)\,dx + a^3\int \cos^4(c+dx)\sin^2(c+dx)\,d$$

$$= \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}a^3\int$$

$$= -\frac{3a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{19a^3\cos^3(c+dx}{24d}$$

$$= \frac{9a^3x}{8} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{3a^3\cos^5(c+dx)}{5d} + \frac{19a^3}{}$$

$$= \frac{19a^3x}{16} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 0.980717, size = 102, normalized size = 0.71

$$\frac{a^3\left(735\sin(2(c+dx)) + 75\sin(4(c+dx)) - 5\sin(6(c+dx)) + 840\cos(c+dx) - 100\cos(3(c+dx)) - 36\cos(5(c+dx)) +\right.}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1140*c + 1140*d*x + 840*Cos[c + d*x] - 100*Cos[3*(c + d*x)] - 36*Cos[
5*(c + d*x)] - 960*Log[Cos[(c + d*x)/2]] + 960*Log[Sin[(c + d*x)/2]] + 735*
```

```
Sin[2*(c + d*x)] + 75*Sin[4*(c + d*x)] - 5*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.078, size = 149, normalized size = 1.

$$-\frac{a^3 (\cos (dx+c))^5 \sin (dx+c)}{6d}+\frac{19\,a^3 (\cos (dx+c))^3 \sin (dx+c)}{24d}+\frac{19\,a^3 \cos (dx+c) \sin (dx+c)}{16d}+\frac{19\,a^3x}{16}+\frac{19\,a^3c}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d+19/24*a^3*cos(d*x+c)^3*sin(d*x+c)/d+19/1
6*a^3*cos(d*x+c)*sin(d*x+c)/d+19/16*a^3*x+19/16/d*a^3*c-3/5*a^3*cos(d*x+c)^
5/d+1/3*a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c
))
```

Maxima [A] time = 1.07763, size = 182, normalized size = 1.27

$$-\frac{576\,a^3 \cos (dx+c)^5 - 160\left(2\,\cos (dx+c)^3 + 6\,\cos (dx+c) - 3\,\log (\cos (dx+c)+1) + 3\,\log (\cos (dx+c)-1)\right)a^3 -}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(576*a^3*cos(d*x + c)^5 - 160*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^3 - 5*(4*sin(2*d*x + 2*
c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3 - 90*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A] time = 1.21633, size = 350, normalized size = 2.45

$$-\frac{144\,a^3 \cos (dx+c)^5 - 80\,a^3 \cos (dx+c)^3 - 285\,a^3dx - 240\,a^3 \cos (dx+c) + 120\,a^3 \log \left(\frac{1}{2}\,\cos (dx+c)+\frac{1}{2}\right) - 120\,a^3}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a^3*cos(d*x + c)^5 - 80*a^3*cos(d*x + c)^3 - 285*a^3*d*x - 240*
a^3*cos(d*x + c) + 120*a^3*log(1/2*cos(d*x + c) + 1/2) - 120*a^3*log(-1/2*c
os(d*x + c) + 1/2) + 5*(8*a^3*cos(d*x + c)^5 - 38*a^3*cos(d*x + c)^3 - 57*a
^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34776, size = 309, normalized size = 2.16

$$285\,(dx+c)a^3 + 240\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{2\left(435\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 240\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} + 865\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 1200\,a^3\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(285*(d*x + c)*a^3 + 240*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(435*
a^3*tan(1/2*d*x + 1/2*c)^11 + 240*a^3*tan(1/2*d*x + 1/2*c)^10 + 865*a^3*tan
(1/2*d*x + 1/2*c)^9 - 1200*a^3*tan(1/2*d*x + 1/2*c)^8 - 210*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 1760*a^3*tan(1/2*d*x + 1/2*c)^6 + 210*a^3*tan(1/2*d*x + 1/2*c
)^5 - 1440*a^3*tan(1/2*d*x + 1/2*c)^4 - 865*a^3*tan(1/2*d*x + 1/2*c)^3 - 12
96*a^3*tan(1/2*d*x + 1/2*c)^2 - 435*a^3*tan(1/2*d*x + 1/2*c) - 176*a^3)/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.397 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$-\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{11a^3\sin(c+$$

```
[Out] (-3*a^3*x)/8 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d + (
a^3*Cos[c + d*x]^3)/d - (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x])/d +
 (11*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (3*a^3*Cos[c + d*x]*Sin[c + d*x
]^3)/(4*d)
```

Rubi [A] time = 0.196466, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{11a^3\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*x)/8 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d + (
a^3*Cos[c + d*x]^3)/d - (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x])/d +
 (11*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (3*a^3*Cos[c + d*x]*Sin[c + d*x
]^3)/(4*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(a^7+3a^7\csc(c+dx)+a^7\csc^2(c+dx)-5a^7\sin(c+dx)-5a^7\sin\right.}{}$$
$$= a^3x + a^3\int \csc^2(c+dx)\,dx + a^3\int \sin^3(c+dx)\,dx + a^3\int \sin^5(c+$$
$$= a^3x - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{5a^3\cos(c+dx)}{d} + \frac{5a^3\cos(c+dx)}{2d}$$
$$= -\frac{3a^3x}{2} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+}{d}$$
$$= -\frac{3a^3x}{8} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+}{d}$$

Mathematica [A] time = 1.96367, size = 148, normalized size = 1.13

$$\frac{(a\sin(c+dx)+a)^3\left(-60(c+dx)+80\sin(2(c+dx))+15\sin(4(c+dx))+580\cos(c+dx)+30\cos(3(c+dx))-2\cos(5\right.}{160d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-60*(c + d*x) + 580*Cos[c + d*x] + 30*Cos[3*(c + d
*x)] - 2*Cos[5*(c + d*x)] - 80*Cot[(c + d*x)/2] - 480*Log[Cos[(c + d*x)/2]]
 + 480*Log[Sin[(c + d*x)/2]] + 80*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)] +
80*Tan[(c + d*x)/2]))/(160*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.076, size = 152, normalized size = 1.2

$$-\frac{a^3(\cos(dx+c))^5}{5d} - \frac{a^3(\cos(dx+c))^3\sin(dx+c)}{4d} - \frac{3a^3\cos(dx+c)\sin(dx+c)}{8d} - \frac{3a^3x}{8} - \frac{3a^3c}{8d} + \frac{a^3(\cos(dx+c))^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/5*a^3*cos(d*x+c)^5/d-1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-3/8*a^3*cos(d*x+c
)*sin(d*x+c)/d-3/8*a^3*x-3/8/d*a^3*c+a^3*cos(d*x+c)^3/d+3*a^3*cos(d*x+c)/d+
```

```
3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/d*a^3/sin(d*x+c)*cos(d*x+c)^5
```

Maxima [A] time = 1.55699, size = 190, normalized size = 1.45

$$-\frac{32\,a^3\cos\left(dx+c\right)^5-80\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-15}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/160*(32*a^3*cos(d*x + c)^5 - 80*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^3 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 + 80*(3*d*x + 3*c + (3*tan(d*x +
c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a^3)/d
```

Fricas [A] time = 1.17434, size = 394, normalized size = 3.01

$$-\frac{30\,a^3\cos\left(dx+c\right)^5-5\,a^3\cos\left(dx+c\right)^3+60\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-60\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{40\,d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40*(30*a^3*cos(d*x + c)^5 - 5*a^3*cos(d*x + c)^3 + 60*a^3*log(1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) - 60*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) + 15*a^3*cos(d*x + c) + (8*a^3*cos(d*x + c)^5 - 40*a^3*cos(d*x + c)^3 +
15*a^3*d*x - 120*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.52869, size = 305, normalized size = 2.33

$$-\frac{15\left(dx+c\right)a^3-120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{20\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/40*(15*(d*x + c)*a^3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 20*a^3*t
an(1/2*d*x + 1/2*c) + 20*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1
/2*c) + 2*(55*a^3*tan(1/2*d*x + 1/2*c)^9 - 200*a^3*tan(1/2*d*x + 1/2*c)^8 -
 10*a^3*tan(1/2*d*x + 1/2*c)^7 - 720*a^3*tan(1/2*d*x + 1/2*c)^6 - 800*a^3*t
an(1/2*d*x + 1/2*c)^4 + 10*a^3*tan(1/2*d*x + 1/2*c)^3 - 560*a^3*tan(1/2*d*x
 + 1/2*c)^2 - 55*a^3*tan(1/2*d*x + 1/2*c) - 152*a^3)/(tan(1/2*d*x + 1/2*c)^
2 + 1)^5)/d
```

3.398 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=137

$$\frac{a^3\cos^3(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}-3a$$

```
[Out] (-33*a^3*x)/8 - (3*a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (2*a^3*Cos[c + d*x])/
d + (a^3*Cos[c + d*x]^3)/d - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]*Csc
[c + d*x])/(2*d) - (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^3*Cos[c + d
*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.182618, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{a^3\cos^3(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}-3a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-33*a^3*x)/8 - (3*a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (2*a^3*Cos[c + d*x])/
d + (a^3*Cos[c + d*x]^3)/d - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]*Csc
[c + d*x])/(2*d) - (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^3*Cos[c + d
*x]*Sin[c + d*x]^3)/(4*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-5a^7 + a^7\csc(c+dx) + 3a^7\csc^2(c+dx) + a^7\csc^3(c+dx) - 5a^7 \right.}{}$$

$$= -5a^3x + a^3\int \csc(c+dx)\,dx + a^3\int \csc^3(c+dx)\,dx + a^3\int \sin^2(c+$$

$$= -5a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{5a^3\cos(c+dx)}{d} - \frac{a^3\cot(c+dx)}{2d}$$

$$= -\frac{9a^3x}{2} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{2a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+}{d}$$

$$= -\frac{33a^3x}{8} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{2a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+}{d}$$

Mathematica [A] time = 3.06984, size = 164, normalized size = 1.2

$$\frac{(a\sin(c+dx)+a)^3\left(-132(c+dx) - 16\sin(2(c+dx)) + \sin(4(c+dx)) + 88\cos(c+dx) + 8\cos(3(c+dx)) + 48\tan\left(\frac{1}{2}(c\right.\right.}{32d\left(\sin\left(\frac{1}{2}(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-132*(c + d*x) + 88*Cos[c + d*x] + 8*Cos[3*(c + d*
x)] - 48*Cot[(c + d*x)/2] - 4*Csc[(c + d*x)/2]^2 - 48*Log[Cos[(c + d*x)/2]]
 + 48*Log[Sin[(c + d*x)/2]] + 4*Sec[(c + d*x)/2]^2 - 16*Sin[2*(c + d*x)] +
Sin[4*(c + d*x)] + 48*Tan[(c + d*x)/2]))/(32*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^6)
```

Maple [A] time = 0.086, size = 161, normalized size = 1.2

$$-\frac{11a^3(\cos(dx+c))^3\sin(dx+c)}{4d} - \frac{33a^3\cos(dx+c)\sin(dx+c)}{8d} - \frac{33a^3x}{8} - \frac{33a^3c}{8d} + \frac{a^3(\cos(dx+c))^3}{2d} + \frac{3a^3\cos(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -11/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-33/8*a^3*cos(d*x+c)*sin(d*x+c)/d-33/8*a
^3*x-33/8/d*a^3*c+1/2*a^3*cos(d*x+c)^3/d+3/2*a^3*cos(d*x+c)/d+3/2/d*a^3*ln(
csc(d*x+c)-cot(d*x+c))-3/d*a^3/sin(d*x+c)*cos(d*x+c)^5-1/2/d*a^3/sin(d*x+c)
^2*cos(d*x+c)^5
```

Maxima [A] time = 1.67029, size = 247, normalized size = 1.8

$$16\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a^3+(12\,dx+12\,c+\sin(4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/32*(16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^3 + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a^3 - 48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + t
an(d*x + c)))*a^3 + 8*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x
+ c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.23403, size = 459, normalized size = 3.35

$$8\,a^3\cos(dx+c)^5-33\,a^3dx\cos(dx+c)^2+8\,a^3\cos(dx+c)^3+33\,a^3dx-12\,a^3\cos(dx+c)-6\left(a^3\cos(dx+c)^2-a^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8*(8*a^3*cos(d*x + c)^5 - 33*a^3*d*x*cos(d*x + c)^2 + 8*a^3*cos(d*x + c)^
3 + 33*a^3*d*x - 12*a^3*cos(d*x + c) - 6*(a^3*cos(d*x + c)^2 - a^3)*log(1/2
*cos(d*x + c) + 1/2) + 6*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) +
 1/2) + (2*a^3*cos(d*x + c)^5 - 11*a^3*cos(d*x + c)^3 + 33*a^3*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49442, size = 325, normalized size = 2.37

$$a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 33\,(dx+c)a^3 + 12\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + 12\,a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{18\,a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 12\,a^3 \tan\left(\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 33*(d*x + c)*a^3 + 12*a^3*log(abs(tan(1/2
*d*x + 1/2*c))) + 12*a^3*tan(1/2*d*x + 1/2*c) - (18*a^3*tan(1/2*d*x + 1/2*c
)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2 + 2*(7*a^3*
tan(1/2*d*x + 1/2*c)^7 + 40*a^3*tan(1/2*d*x + 1/2*c)^6 + 15*a^3*tan(1/2*d*x
 + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 15*a^3*tan(1/2*d*x + 1/2*c)^3
 + 56*a^3*tan(1/2*d*x + 1/2*c)^2 - 7*a^3*tan(1/2*d*x + 1/2*c) + 24*a^3)/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.399 $\int \cot^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=134

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{2a^3\cot(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\mathrm{co}}{2d}$$

```
[Out] (-7*a^3*x)/2 + (7*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a^3*Cos[c + d*x])/d
 + (a^3*Cos[c + d*x]^3)/(3*d) - (2*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^
3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (3*a^3*Cos[c + d*x]*Si
n[c + d*x])/(2*d)
```

Rubi [A] time = 0.177603, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{2a^3\cot(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\mathrm{co}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-7*a^3*x)/2 + (7*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a^3*Cos[c + d*x])/d
 + (a^3*Cos[c + d*x]^3)/(3*d) - (2*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^
3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (3*a^3*Cos[c + d*x]*Si
n[c + d*x])/(2*d)
```


Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-5a^7-5a^7\csc(c+dx)+a^7\csc^2(c+dx)+3a^7\csc^3(c+dx)+a^7\csc^4(c+dx)\right.}{a^4}$$
$$= -5a^3x+a^3\int\csc^2(c+dx)\,dx+a^3\int\csc^4(c+dx)\,dx+a^3\int\sin(c+dx)\,dx+a^3$$
$$= -5a^3x+\frac{5a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)\csc(c+dx}{2d}$$
$$= -\frac{7a^3x}{2}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\text{co}}{}$$

Mathematica [A] time = 3.05767, size = 201, normalized size = 1.5

$$\frac{a^3(\sin(c+dx)+1)^3\left(-84(c+dx)-18\sin(2(c+dx))-42\cos(c+dx)+2\cos(3(c+dx))+20\tan\left(\frac{1}{2}(c+dx)\right)-20\cot\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-84*(c + d*x) - 42*Cos[c + d*x] + 2*Cos[3*(c + d
*x)] - 20*Cot[(c + d*x)/2] - 9*Csc[(c + d*x)/2]^2 + 84*Log[Cos[(c + d*x)/2]
] - 84*Log[Sin[(c + d*x)/2]] + 9*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[
(c + d*x)/2]^4 - (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 - 18*Sin[2*(c + d*x)]
+ 20*Tan[(c + d*x)/2]))/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.085, size = 190, normalized size = 1.4

$$-\frac{7a^3(\cos(dx+c))^3}{6d}-\frac{7a^3\cos(dx+c)}{2d}-\frac{7a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}-3\frac{a^3(\cos(dx+c))^5}{d\sin(dx+c)}-3\frac{a^3(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -7/6*a^3*cos(d*x+c)^3/d-7/2*a^3*cos(d*x+c)/d-7/2/d*a^3*ln(csc(d*x+c)-cot(d*
x+c))-3/d*a^3/sin(d*x+c)*cos(d*x+c)^5-3*a^3*cos(d*x+c)^3*sin(d*x+c)/d-9/2*a
^3*cos(d*x+c)*sin(d*x+c)/d-7/2*a^3*x-7/2/d*a^3*c-3/2/d*a^3/sin(d*x+c)^2*cos
(d*x+c)^5-1/3*a^3*cot(d*x+c)^3/d+a^3*cot(d*x+c)/d
```

Maxima [A] time = 1.50102, size = 250, normalized size = 1.87

$$\frac{2\left(2\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a^{3}-18\left(3\,dx+3\,c+\frac{3\tan(d\cdots}{\tan(dx+c)^{3}\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*lo
g(cos(d*x + c) - 1))*a^3 - 18*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*
x + c)^3 + tan(d*x + c)))*a^3 + 4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan
(d*x + c)^3)*a^3 + 9*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x +
 c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.22401, size = 524, normalized size = 3.91

$$\frac{18\,a^{3}\cos\left(dx+c\right)^{5}-56\,a^{3}\cos\left(dx+c\right)^{3}+42\,a^{3}\cos\left(dx+c\right)+21\left(a^{3}\cos\left(dx+c\right)^{2}-a^{3}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(18*a^3*cos(d*x + c)^5 - 56*a^3*cos(d*x + c)^3 + 42*a^3*cos(d*x + c) +
 21*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2
1*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*
(2*a^3*cos(d*x + c)^5 - 21*a^3*d*x*cos(d*x + c)^2 - 14*a^3*cos(d*x + c)^3 +
 21*a^3*d*x + 21*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3824, size = 338, normalized size = 2.52

$$3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-252\,(dx+c)a^3-252\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+63\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^3*tan(1/2*d*x + 1/2*c)^2 - 252*(d
*x + c)*a^3 - 252*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 63*a^3*tan(1/2*d*x +
 1/2*c) + (154*a^3*tan(1/2*d*x + 1/2*c)^9 + 153*a^3*tan(1/2*d*x + 1/2*c)^8
+ 291*a^3*tan(1/2*d*x + 1/2*c)^7 - 192*a^3*tan(1/2*d*x + 1/2*c)^6 - 195*a^3
*tan(1/2*d*x + 1/2*c)^5 - 414*a^3*tan(1/2*d*x + 1/2*c)^4 - 167*a^3*tan(1/2*
d*x + 1/2*c)^3 - 72*a^3*tan(1/2*d*x + 1/2*c)^2 - 27*a^3*tan(1/2*d*x + 1/2*c
) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^3)/d
```

3.400 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=138

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{2a^3\cot(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cdots}{}$$

```
[Out] (3*a^3*x)/2 + (33*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (3*a^3*Cos[c + d*x])/d
 + (2*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/d - (7*a^3*Cot[c + d*x]*Cs
c[c + d*x])/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) - (a^3*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.192219, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635}

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{2a^3\cot(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*a^3*x)/2 + (33*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (3*a^3*Cos[c + d*x])/d
 + (2*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/d - (7*a^3*Cot[c + d*x]*Cs
c[c + d*x])/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) - (a^3*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(a^7 - 5a^7\csc(c+dx) - 5a^7\csc^2(c+dx) + a^7\csc^3(c+dx) + 3a^7 \text{cs}\right.}{a}$$

$$= a^3x + a^3\int \csc^3(c+dx)\,dx + a^3\int \csc^5(c+dx)\,dx + a^3\int \sin^2(c+$$

$$= a^3x + \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cot(c+dx)\,\text{c}}{2d}$$

$$= \frac{3a^3x}{2} + \frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a^3\cos(c+dx)}{d} + \frac{2a^3\cot(c+dx}{d}$$

$$= \frac{3a^3x}{2} + \frac{33a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{3a^3\cos(c+dx)}{d} + \frac{2a^3\cot(c+}{d}$$

Mathematica [A] time = 1.20033, size = 215, normalized size = 1.56

$$a^3(\sin(c+dx)+1)^3\left(96(c+dx) - 16\sin(2(c+dx)) - 192\cos(c+dx) - 96\tan\left(\tfrac{1}{2}(c+dx)\right) + 96\cot\left(\tfrac{1}{2}(c+dx)\right) - \csc^4\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(96*(c + d*x) - 192*Cos[c + d*x] + 96*Cot[(c + d*
x)/2] - 14*Csc[(c + d*x)/2]^2 - Csc[(c + d*x)/2]^4 + 264*Log[Cos[(c + d*x)/
2]] - 264*Log[Sin[(c + d*x)/2]] + 14*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^
4 + 64*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 4*Csc[(c + d*x)/2]^4*Sin[c + d*x
] - 16*Sin[2*(c + d*x)] - 96*Tan[(c + d*x)/2]))/(64*d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^6)
```

Maple [A] time = 0.088, size = 215, normalized size = 1.6

$$-\frac{a^3(\cos(dx+c))^5}{d\sin(dx+c)} - \frac{a^3(\cos(dx+c))^3\sin(dx+c)}{d} - \frac{3a^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{3a^3x}{2} + \frac{3a^3c}{2d} - \frac{11a^3(\cos(dx+c))}{8d(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/d*a^3/sin(d*x+c)*cos(d*x+c)^5-a^3*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a^3*cos(
d*x+c)*sin(d*x+c)/d+3/2*a^3*x+3/2/d*a^3*c-11/8/d*a^3/sin(d*x+c)^2*cos(d*x+c
)^5-11/8*a^3*cos(d*x+c)^3/d-33/8*a^3*cos(d*x+c)/d-33/8/d*a^3*ln(csc(d*x+c)-
cot(d*x+c))-a^3*cot(d*x+c)^3/d+3*a^3*cot(d*x+c)/d-1/4/d*a^3/sin(d*x+c)^4*co
s(d*x+c)^5
```

Maxima [A] time = 1.63978, size = 282, normalized size = 2.04

$$-\frac{8\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^3-16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+a^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(8*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
c)))*a^3 - 16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + a
^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^
2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) - 12*a^3*(2*cos
(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) -
 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.26099, size = 593, normalized size = 4.3

$$\frac{24\,a^3dx\cos\left(dx+c\right)^4-48\,a^3\cos\left(dx+c\right)^5-48\,a^3dx\cos\left(dx+c\right)^2+110\,a^3\cos\left(dx+c\right)^3+24\,a^3dx-66\,a^3\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(24*a^3*d*x*cos(d*x + c)^4 - 48*a^3*cos(d*x + c)^5 - 48*a^3*d*x*cos(d*
x + c)^2 + 110*a^3*cos(d*x + c)^3 + 24*a^3*d*x - 66*a^3*cos(d*x + c) + 33*(
a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2
) - 33*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x +
 c) + 1/2) - 8*(a^3*cos(d*x + c)^5 + 4*a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48639, size = 325, normalized size = 2.36

$$a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 8a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 16a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 96(dx+c)a^3 - 264a^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 16*a^3*ta
n(1/2*d*x + 1/2*c)^2 + 96*(d*x + c)*a^3 - 264*a^3*log(abs(tan(1/2*d*x + 1/2
*c))) - 88*a^3*tan(1/2*d*x + 1/2*c) + 64*(a^3*tan(1/2*d*x + 1/2*c)^3 - 6*a^
3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/(tan(1/2*d*x +
 1/2*c)^2 + 1)^2 + (550*a^3*tan(1/2*d*x + 1/2*c)^4 + 88*a^3*tan(1/2*d*x + 1
/2*c)^3 - 16*a^3*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*tan(1/2*d*x + 1/2*c) - a^3)
/tan(1/2*d*x + 1/2*c)^4)/d
```

3.401 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=132

$$-\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{3a^3\cot(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)}{4d}$$

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (a^3*Cos[c + d*x])/d + (3*a
^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) +
(11*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]
^3)/(4*d)
```

Rubi [A] time = 0.216112, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 3768, 2638}

$$-\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{3a^3\cot(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (a^3*Cos[c + d*x])/d + (3*a
^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) +
(11*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]
^3)/(4*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(3a^7 + a^7\csc(c+dx) - 5a^7\csc^2(c+dx) - 5a^7\csc^3(c+dx) + a^7 c\right.}{}$$

$$= 3a^3x + a^3\int \csc(c+dx)\,dx + a^3\int \csc^4(c+dx)\,dx + a^3\int \csc^6(c+$$

$$= 3a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{5a^3\cot(c+dx)\,c}{2d}$$

$$= 3a^3x + \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3\cos(c+dx)}{d} + \frac{3a^3\cot(c+dx)}{d}$$

$$= 3a^3x + \frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^3\cos(c+dx)}{d} + \frac{3a^3\cot(c+dx)}{d}$$

Mathematica [A] time = 0.495615, size = 216, normalized size = 1.64

$$\frac{a^3\left(-320\cos(c+dx) - 608\tan\left(\frac{1}{2}(c+dx)\right) + 608\cot\left(\frac{1}{2}(c+dx)\right) - 15\csc^4\left(\frac{1}{2}(c+dx)\right) + 110\csc^2\left(\frac{1}{2}(c+dx)\right) + 15\sec^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(960*c + 960*d*x - 320*Cos[c + d*x] + 608*Cot[(c + d*x)/2] + 110*Csc[(
c + d*x)/2]^2 - 15*Csc[(c + d*x)/2]^4 + 120*Log[Cos[(c + d*x)/2]] - 120*Log
[Sin[(c + d*x)/2]] - 110*Sec[(c + d*x)/2]^2 + 15*Sec[(c + d*x)/2]^4 + 208*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 64*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - 1
3*Csc[(c + d*x)/2]^4*Sin[c + d*x] - Csc[(c + d*x)/2]^6*Sin[c + d*x] - 608*T
an[(c + d*x)/2]))/(320*d)
```

Maple [A] time = 0.092, size = 173, normalized size = 1.3

$$-\frac{a^3(\cos(dx+c))^5}{8d(\sin(dx+c))^2} - \frac{a^3(\cos(dx+c))^3}{8d} - \frac{3a^3\cos(dx+c)}{8d} - \frac{3a^3\ln(\csc(dx+c)-\cot(dx+c))}{8d} - \frac{a^3(\cot(dx+c))^3}{d} + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5-1/8*a^3*cos(d*x+c)^3/d-3/8*a^3*cos(d*x
+c)/d-3/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-a^3*cot(d*x+c)^3/d+3*a^3*x+3*a^3*
cot(d*x+c)/d+3/d*a^3*c-3/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5-1/5/d*a^3/sin(d*
x+c)^5*cos(d*x+c)^5
```

Maxima [A] time = 1.62311, size = 243, normalized size = 1.84

$$\frac{80\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3-15\,a^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/80*(80*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 15*a^3
*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2
+ 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 20*a^3*(2*cos(d
*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3
*log(cos(d*x + c) - 1)) - 16*a^3/tan(d*x + c)^5)/d
```

Fricas [B] time = 1.24622, size = 655, normalized size = 4.96

$$\frac{304\,a^3\cos\left(dx+c\right)^5-560\,a^3\cos\left(dx+c\right)^3+240\,a^3\cos\left(dx+c\right)+15\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/80*(304*a^3*cos(d*x + c)^5 - 560*a^3*cos(d*x + c)^3 + 240*a^3*cos(d*x + c
) + 15*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) - 15*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^
3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(24*a^3*d*x*cos(d*x + c)^
4 - 8*a^3*cos(d*x + c)^5 - 48*a^3*d*x*cos(d*x + c)^2 + 5*a^3*cos(d*x + c)^3
 + 24*a^3*d*x - 3*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*
cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37016, size = 305, normalized size = 2.31

$$\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+960\,(dx+c)a^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/320*(2*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*a^3*tan(1/2*d*x + 1/2*c)^4 + 30*a^
3*tan(1/2*d*x + 1/2*c)^3 - 80*a^3*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a^
3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 580*a^3*tan(1/2*d*x + 1/2*c) -
 640*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) + (274*a^3*tan(1/2*d*x + 1/2*c)^5 + 5
80*a^3*tan(1/2*d*x + 1/2*c)^4 + 80*a^3*tan(1/2*d*x + 1/2*c)^3 - 30*a^3*tan(
1/2*d*x + 1/2*c)^2 - 15*a^3*tan(1/2*d*x + 1/2*c) - 2*a^3)/tan(1/2*d*x + 1/2
*c)^5)/d
```

3.402 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=168

$$-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{19a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d}-3$$

```
[Out] a^3*x - (19*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (a^3*Cot[c + d*x])/d - (a^3
*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]^5)/(5*d) + (17*a^3*Cot[c + d*x
]*Csc[c + d*x])/(16*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x])/(4*d) + (a^3*C
ot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*
d)
```

Rubi [A] time = 0.274657, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{19a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d}-3$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```



```
[Out] a^3*x - (19*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (a^3*Cot[c + d*x])/d - (a^3
*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]^5)/(5*d) + (17*a^3*Cot[c + d*x
]*Csc[c + d*x])/(16*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x])/(4*d) + (a^3*C
ot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*
d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx) + 3a^3\cot^4(c+dx)\csc(c+dx) + 3a^3\cot^4(c+d\right.$$

$$= a^3\int \cot^4(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right)\int$$

$$= -\frac{a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{a^3\cot^3(c+dx)\,c}{6d}$$

$$= \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^5(c+dx)}{5d} + \frac{9a^3\cot(c+}{}$$

$$= a^3x - \frac{9a^3\tanh^{-1}(\cos(c+dx))}{8d} + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d}$$

$$= a^3x - \frac{19a^3\tanh^{-1}(\cos(c+dx))}{16d} + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d}$$

Mathematica [A] time = 0.766006, size = 217, normalized size = 1.29

$$\frac{a^3\left(-704\tan\left(\frac{1}{2}(c+dx)\right) + 704\cot\left(\frac{1}{2}(c+dx)\right) + 870\csc^2\left(\frac{1}{2}(c+dx)\right) + 5\sec^6\left(\frac{1}{2}(c+dx)\right) + 60\sec^4\left(\frac{1}{2}(c+dx)\right) - 870\,\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1920*c + 1920*d*x + 704*Cot[(c + d*x)/2] + 870*Csc[(c + d*x)/2]^2 - 2
280*Log[Cos[(c + d*x)/2]] + 2280*Log[Sin[(c + d*x)/2]] - 870*Sec[(c + d*x)/
2]^2 + 60*Sec[(c + d*x)/2]^4 + 5*Sec[(c + d*x)/2]^6 - 1376*Csc[c + d*x]^3*S
in[(c + d*x)/2]^4 - Csc[(c + d*x)/2]^6*(5 + 18*Sin[c + d*x]) + Csc[(c + d*x
)/2]^4*(-60 + 86*Sin[c + d*x]) - 704*Tan[(c + d*x)/2] + 36*Sec[(c + d*x)/2]
^4*Tan[(c + d*x)/2]))/(1920*d)
```

Maple [A] time = 0.09, size = 194, normalized size = 1.2

$$-\frac{a^3 (\cot (dx+c))^3}{3d}+\frac{a^3 \cot (dx+c)}{d}+a^3x+\frac{a^3c}{d}-\frac{19\,a^3 (\cos (dx+c))^5}{24\,d (\sin (dx+c))^4}+\frac{19\,a^3 (\cos (dx+c))^5}{48\,d (\sin (dx+c))^2}+\frac{19\,a^3 (\cos (dx+c))}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cot(d*x+c)^3/d+a^3*cot(d*x+c)/d+a^3*x+1/d*a^3*c-19/24/d*a^3/sin(d*
x+c)^4*cos(d*x+c)^5+19/48/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+19/48*a^3*cos(d*x
+c)^3/d+19/16*a^3*cos(d*x+c)/d+19/16/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/5/d*
a^3/sin(d*x+c)^5*cos(d*x+c)^5-1/6/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5
```

Maxima [A] time = 1.69469, size = 290, normalized size = 1.73

$$\frac{160\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(d\right.\right.\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/480*(160*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + 5*a^
3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^6
 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1)) - 90*a^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*
x + c) - 1)) - 288*a^3/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.17591, size = 745, normalized size = 4.43

$$\frac{480\,a^3dx\cos\left(dx+c\right)^6-1440\,a^3dx\cos\left(dx+c\right)^4-870\,a^3\cos\left(dx+c\right)^5+1440\,a^3dx\cos\left(dx+c\right)^2+1520\,a^3\cos\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(480*a^3*d*x*cos(d*x + c)^6 - 1440*a^3*d*x*cos(d*x + c)^4 - 870*a^3*c
os(d*x + c)^5 + 1440*a^3*d*x*cos(d*x + c)^2 + 1520*a^3*cos(d*x + c)^3 - 480
*a^3*d*x - 570*a^3*cos(d*x + c) - 285*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x +
 c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 285*(a^3*
cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(-1/
2*cos(d*x + c) + 1/2) - 32*(11*a^3*cos(d*x + c)^5 - 35*a^3*cos(d*x + c)^3 +
 15*a^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4
+ 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.51949, size = 323, normalized size = 1.92

$$5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+36a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+75a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-100a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-735a^3\tan\left(\frac{1}{2}dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)^5 + 75*a
^3*tan(1/2*d*x + 1/2*c)^4 - 100*a^3*tan(1/2*d*x + 1/2*c)^3 - 735*a^3*tan(1/
2*d*x + 1/2*c)^2 + 1920*(d*x + c)*a^3 + 2280*a^3*log(abs(tan(1/2*d*x + 1/2*
c))) - 840*a^3*tan(1/2*d*x + 1/2*c) - (5586*a^3*tan(1/2*d*x + 1/2*c)^6 - 84
0*a^3*tan(1/2*d*x + 1/2*c)^5 - 735*a^3*tan(1/2*d*x + 1/2*c)^4 - 100*a^3*tan
(1/2*d*x + 1/2*c)^3 + 75*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*tan(1/2*d*x +
1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.403 $\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=150

$$-\frac{a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)\csc(}{4d}$$

```
[Out] (-9*a^3*ArcTanh[Cos[c + d*x]])/(16*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (a^3
*Cot[c + d*x]^7)/(7*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (a^3*Co
t[c + d*x]^3*Csc[c + d*x])/(4*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d
) - (a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(2*d)
```

Rubi [A] time = 0.284279, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)\csc(}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9*a^3*ArcTanh[Cos[c + d*x]])/(16*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (a^3
*Cot[c + d*x]^7)/(7*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (a^3*Co
t[c + d*x]^3*Csc[c + d*x])/(4*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d
) - (a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(2*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc(c+dx)+3a^3\cot^4(c+dx)\csc^2(c+dx)+3a\right.$$
$$= a^3\int \cot^4(c+dx)\csc(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^4(c+dx)\,d$$
$$= -\frac{a^3\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d} - \frac{1}{4}(3a$$
$$= -\frac{3a^3\cot^5(c+dx)}{5d} + \frac{3a^3\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^3\cot^3(c+dx)}{4d}$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} +$$
$$= -\frac{9a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} +$$

Mathematica [B] time = 0.131935, size = 363, normalized size = 2.42

$$a^3\left(\frac{23\tan\left(\frac{1}{2}(c+dx)\right)}{70d} - \frac{23\cot\left(\frac{1}{2}(c+dx)\right)}{70d} - \frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{128d} + \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{7\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{\sec^6\left(\frac{1}{2}(c+d\right.}{128d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-23*Cot[(c + d*x)/2])/(70*d) + (7*Csc[(c + d*x)/2]^2)/(64*d) + (297*C
ot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(2240*d) + Csc[(c + d*x)/2]^4/(32*d) -
(31*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(2240*d) - Csc[(c + d*x)/2]^6/(128
*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^6)/(896*d) - (9*Log[Cos[(c + d*x)/
2]])/(16*d) + (9*Log[Sin[(c + d*x)/2]])/(16*d) - (7*Sec[(c + d*x)/2]^2)/(64
*d) - Sec[(c + d*x)/2]^4/(32*d) + Sec[(c + d*x)/2]^6/(128*d) + (23*Tan[(c +
 d*x)/2])/(70*d) - (297*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(2240*d) + (31
*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(2240*d) + (Sec[(c + d*x)/2]^6*Tan[(c
 + d*x)/2])/(896*d))
```

Maple [A] time = 0.089, size = 176, normalized size = 1.2

$$-\frac{3a^3(\cos(dx+c))^5}{8d(\sin(dx+c))^4} + \frac{3a^3(\cos(dx+c))^5}{16d(\sin(dx+c))^2} + \frac{3a^3(\cos(dx+c))^3}{16d} + \frac{9a^3\cos(dx+c)}{16d} + \frac{9a^3\ln(\csc(dx+c)-\cot(dx+c)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -3/8/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+3/16/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+3
/16*a^3*cos(d*x+c)^3/d+9/16*a^3*cos(d*x+c)/d+9/16/d*a^3*ln(csc(d*x+c)-cot(d
*x+c))-23/35/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-1/2/d*a^3/sin(d*x+c)^6*cos(d*x
+c)^5-1/7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^5
```

Maxima [A] time = 1.00123, size = 278, normalized size = 1.85

$$\frac{35\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-70\,a^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\right.}{\cos(dx+c)^4-2\,\cos\right.}}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/1120*(35*a^3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(c
os(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x +
c) + 1) + 3*log(cos(d*x + c) - 1)) - 70*a^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*
x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) -
 3*log(cos(d*x + c) - 1)) - 672*a^3/tan(d*x + c)^5 - 32*(7*tan(d*x + c)^2 +
 5)*a^3/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.34851, size = 629, normalized size = 4.19

$$-\frac{736\,a^3\cos\left(dx+c\right)^7-896\,a^3\cos\left(dx+c\right)^5+315\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1120*(736*a^3*cos(d*x + c)^7 - 896*a^3*cos(d*x + c)^5 + 315*(a^3*cos(d*x
 + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) - 315*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4
 + 3*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
70*(7*a^3*cos(d*x + c)^5 - 24*a^3*cos(d*x + c)^3 + 9*a^3*cos(d*x + c))*sin(
d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.80796, size = 352, normalized size = 2.35

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+77\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-455\,a^3\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*tan(1/2*d*x + 1/2*c)^7 + 35*a^3*tan(1/2*d*x + 1/2*c)^6 + 77*a
^3*tan(1/2*d*x + 1/2*c)^5 - 35*a^3*tan(1/2*d*x + 1/2*c)^4 - 455*a^3*tan(1/2
*d*x + 1/2*c)^3 - 665*a^3*tan(1/2*d*x + 1/2*c)^2 + 2520*a^3*log(abs(tan(1/2
*d*x + 1/2*c))) + 945*a^3*tan(1/2*d*x + 1/2*c) - (6534*a^3*tan(1/2*d*x + 1/
2*c)^7 + 945*a^3*tan(1/2*d*x + 1/2*c)^6 - 665*a^3*tan(1/2*d*x + 1/2*c)^5 -
455*a^3*tan(1/2*d*x + 1/2*c)^4 - 35*a^3*tan(1/2*d*x + 1/2*c)^3 + 77*a^3*tan
(1/2*d*x + 1/2*c)^2 + 35*a^3*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/
2*c)^7)/d
```

3.404 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=176

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{27a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^3\cot^3(c+dx)\,\text{cs}}{2d}$$

```
[Out] (-27*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (3
*a^3*Cot[c + d*x]^7)/(7*d) - (27*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (
23*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^3*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(2*d) + (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a^3*Cot[c + d*x]^
3*Csc[c + d*x]^5)/(8*d)
```

Rubi [A] time = 0.338176, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{27a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^3\cot^3(c+dx)\,\text{cs}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-27*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (3
*a^3*Cot[c + d*x]^7)/(7*d) - (27*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (
23*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^3*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(2*d) + (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a^3*Cot[c + d*x]^
3*Csc[c + d*x]^5)/(8*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc^2(c+dx)+3a^3\cot^4(c+dx)\csc^3(c+dx)+3\right.$$
$$= a^3\int \cot^4(c+dx)\csc^2(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^5(c+dx)$$
$$= -\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d} - \frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{1}{8}\left(3\right.$$
$$= -\frac{a^3\cot^5(c+dx)}{5d} + \frac{3a^3\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a^3\cot^3(c+dx)\,c}{2d}$$
$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c+dx)}{7d} - \frac{3a^3\cot(c+dx)\csc(c+dx)}{16d}$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c+dx)}{7d} -$$
$$= -\frac{27a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c+dx)}{7d}$$

Mathematica [A] time = 5.16025, size = 313, normalized size = 1.78

$$-\frac{a^3\sin(c+dx)(\sin(c+dx)+1)^3\left(10(7\csc(c+dx)+24)\csc^8\left(\frac{1}{2}(c+dx)\right)+8(105\csc(c+dx)-76)\csc^6\left(\frac{1}{2}(c+dx)\right)-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(10*Csc[(c + d*x)/2]^8*(24 + 7*Csc[c + d*x]) + 8*Csc[(c + d*x)/2]^6*(
-76 + 105*Csc[c + d*x]) + 8*Csc[(c + d*x)/2]^2*(1664 + 945*Csc[c + d*x]) -
4*Csc[(c + d*x)/2]^4*(856 + 1715*Csc[c + d*x]) - 4*(-7560*Csc[c + d*x]*(Log
[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + (703 + 1056*Cos[c + d*x] + 51
7*Cos[2*(c + d*x)] + 104*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^8 + 7560*Csc[c
+ d*x]^3*Sin[(c + d*x)/2]^2 - 27440*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4 + 134
40*Csc[c + d*x]^7*Sin[(c + d*x)/2]^6 + 4480*Csc[c + d*x]^9*Sin[(c + d*x)/2]
^8))*Sin[c + d*x]*(1 + Sin[c + d*x])^3)/(143360*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^6)
```

Maple [A] time = 0.101, size = 200, normalized size = 1.1

$$-\frac{13 a^{3} (\cos (dx+c))^{5}}{35 d (\sin (dx+c))^{5}}-\frac{9 a^{3} (\cos (dx+c))^{5}}{16 d (\sin (dx+c))^{6}}-\frac{9 a^{3} (\cos (dx+c))^{5}}{64 d (\sin (dx+c))^{4}}+\frac{9 a^{3} (\cos (dx+c))^{5}}{128 d (\sin (dx+c))^{2}}+\frac{9 a^{3} (\cos (dx+c))^{3}}{128 d}+\frac{27 a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -13/35/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-9/16/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5
-9/64/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+9/128/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5
+9/128*a^3*cos(d*x+c)^3/d+27/128*a^3*cos(d*x+c)/d+27/128/d*a^3*ln(csc(d*x+c
)-cot(d*x+c))-3/7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^5-1/8/d*a^3/sin(d*x+c)^8*co
s(d*x+c)^5
```

Maxima [A] time = 1.11625, size = 332, normalized size = 1.89

$$\frac{35 a^{3}\left(\frac{2\left(3 \cos(dx+c)^{7}-11 \cos(dx+c)^{5}-11 \cos(dx+c)^{3}+3 \cos(dx+c)\right)}{\cos(dx+c)^{8}-4 \cos(dx+c)^{6}+6 \cos(dx+c)^{4}-4 \cos(dx+c)^{2}+1}-3 \log (\cos (dx+c)+1)+3 \log (\cos (dx+c)-1)\right)+280 a^{3}\Big(}{89}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8960*(35*a^3*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3
 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))
+ 280*a^3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*
x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) +
1) + 3*log(cos(d*x + c) - 1)) - 1792*a^3/tan(d*x + c)^5 - 768*(7*tan(d*x +
c)^2 + 5)*a^3/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.59157, size = 707, normalized size = 4.02

$$\frac{1890 a^{3} \cos (dx+c)^{7}+2030 a^{3} \cos (dx+c)^{5}-6930 a^{3} \cos (dx+c)^{3}+1890 a^{3} \cos (dx+c)-945 \left(a^{3} \cos (dx+c)^{8}-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8960*(1890*a^3*cos(d*x + c)^7 + 2030*a^3*cos(d*x + c)^5 - 6930*a^3*cos(d*
x + c)^3 + 1890*a^3*cos(d*x + c) - 945*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x
+ c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x
 + c) + 1/2) + 945*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d
*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2) + 256*
(13*a^3*cos(d*x + c)^7 - 28*a^3*cos(d*x + c)^5)*sin(d*x + c))/(d*cos(d*x +
c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.61677, size = 396, normalized size = 2.25

$$\frac{35 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 240 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 560 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 112 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 1960 a^3 \tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/71680*(35*a^3*tan(1/2*d*x + 1/2*c)^8 + 240*a^3*tan(1/2*d*x + 1/2*c)^7 + 5
60*a^3*tan(1/2*d*x + 1/2*c)^6 + 112*a^3*tan(1/2*d*x + 1/2*c)^5 - 1960*a^3*t
an(1/2*d*x + 1/2*c)^4 - 3920*a^3*tan(1/2*d*x + 1/2*c)^3 - 1680*a^3*tan(1/2*
d*x + 1/2*c)^2 + 15120*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9520*a^3*tan(1/
2*d*x + 1/2*c) - (41094*a^3*tan(1/2*d*x + 1/2*c)^8 + 9520*a^3*tan(1/2*d*x +
 1/2*c)^7 - 1680*a^3*tan(1/2*d*x + 1/2*c)^6 - 3920*a^3*tan(1/2*d*x + 1/2*c)
^5 - 1960*a^3*tan(1/2*d*x + 1/2*c)^4 + 112*a^3*tan(1/2*d*x + 1/2*c)^3 + 560
*a^3*tan(1/2*d*x + 1/2*c)^2 + 240*a^3*tan(1/2*d*x + 1/2*c) + 35*a^3)/tan(1/
2*d*x + 1/2*c)^8)/d
```

3.405 $\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{5a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}$$

```
[Out] (-17*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (5
*a^3*Cot[c + d*x]^7)/(7*d) - (a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Cot[c + d
*x]*Csc[c + d*x])/(128*d) + (5*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a
^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^
5)/(16*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rubi [A] time = 0.350867, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2611, 3768, 3770, 2607, 14, 270}

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{5a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-17*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (5
*a^3*Cot[c + d*x]^7)/(7*d) - (a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Cot[c + d
*x]*Csc[c + d*x])/(128*d) + (5*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a
^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^
5)/(16*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(8*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc^3(c+dx)+3a^3\cot^4(c+dx)\csc^4(c+dx)+3\right.$$

$$= a^3\int \cot^4(c+dx)\csc^3(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^6(c+dx)$$

$$= -\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{1}{2}a$$

$$= \frac{a^3\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d} + \frac{3a^3\text{co}}{}$$

$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{5a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{9d} - \frac{a^3\cot(c}{}$$

$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{5a^3\cot^7(c+dx)}{7d} -$$

$$= -\frac{17a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{5a^3\cot^7(c+dx)}{7d} -$$

Mathematica [A] time = 1.32716, size = 313, normalized size = 1.61

$$-\frac{a^3\csc^9(c+dx)\left(669060\sin(2(c+dx))+676620\sin(4(c+dx))-14700\sin(6(c+dx))-10710\sin(8(c+dx))+116121\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[c + d*x]^9*(1161216*Cos[c + d*x] + 247296*Cos[3*(c + d*x)] - 1981
44*Cos[5*(c + d*x)] - 71424*Cos[7*(c + d*x)] + 7936*Cos[9*(c + d*x)] + 6747
30*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 674730*Log[Sin[(c + d*x)/2]]*Sin[c
+ d*x] + 669060*Sin[2*(c + d*x)] - 449820*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
d*x)] + 449820*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 676620*Sin[4*(c + d
*x)] + 192780*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 192780*Log[Sin[(c +
d*x)/2]]*Sin[5*(c + d*x)] - 14700*Sin[6*(c + d*x)] - 48195*Log[Cos[(c + d*x
)/2]]*Sin[7*(c + d*x)] + 48195*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 107
10*Sin[8*(c + d*x)] + 5355*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 5355*Lo
g[Sin[(c + d*x)/2]]*Sin[9*(c + d*x)]))/(10321920*d)
```

Maple [A] time = 0.092, size = 224, normalized size = 1.2

$$-\frac{17 a^3 (\cos (dx+c))^5}{48 d (\sin (dx+c))^6}-\frac{17 a^3 (\cos (dx+c))^5}{192 d (\sin (dx+c))^4}+\frac{17 a^3 (\cos (dx+c))^5}{384 d (\sin (dx+c))^2}+\frac{17 a^3 (\cos (dx+c))^3}{384 d}+\frac{17 a^3 \cos (dx+c)}{128 d}+\frac{17}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -17/48/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5-17/192/d*a^3/sin(d*x+c)^4*cos(d*x+c)
^5+17/384/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+17/384*a^3*cos(d*x+c)^3/d+17/128*
a^3*cos(d*x+c)/d+17/128/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-31/63/d*a^3/sin(d*x
+c)^7*cos(d*x+c)^5-62/315/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-3/8/d*a^3/sin(d*x
+c)^8*cos(d*x+c)^5-1/9/d*a^3/sin(d*x+c)^9*cos(d*x+c)^5
```

Maxima [A] time = 1.15675, size = 362, normalized size = 1.87

$$\frac{945 a^3 \left(\frac{2 \left(3 \cos(dx+c)^7-11 \cos(dx+c)^5-11 \cos(dx+c)^3+3 \cos(dx+c)\right)}{\cos(dx+c)^8-4 \cos(dx+c)^6+6 \cos(dx+c)^4-4 \cos(dx+c)^2+1}-3 \log (\cos (dx+c)+1)+3 \log (\cos (dx+c)-1)\right)+840 a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] 1/80640*(945*a^3*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)
^3 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)
) + 840*a^3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
+ 1) + 3*log(cos(d*x + c) - 1)) - 6912*(7*tan(d*x + c)^2 + 5)*a^3/tan(d*x +
 c)^7 - 256*(63*tan(d*x + c)^4 + 90*tan(d*x + c)^2 + 35)*a^3/tan(d*x + c)^9
)/d
```

Fricas [A] time = 1.25196, size = 803, normalized size = 4.14

$$-\frac{15872 a^3 \cos (dx+c)^9-71424 a^3 \cos (dx+c)^7+64512 a^3 \cos (dx+c)^5+5355 \left(a^3 \cos (dx+c)^8-4 a^3 \cos (dx+c)^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] -1/80640*(15872*a^3*cos(d*x + c)^9 - 71424*a^3*cos(d*x + c)^7 + 64512*a^3*c
os(d*x + c)^5 + 5355*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos
(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 5355*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x +
 c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) - 210*(51*a^3*cos(d*x + c)^7 - 59*a^3*cos(d*x + c)^5 - 187*a^3*cos(d*x +
```

```
c)^3 + 51*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x
 + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**10*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36776, size = 439, normalized size = 2.26

$$\frac{140\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+945\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+2340\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1680\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-4032\,a^3\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/645120*(140*a^3*tan(1/2*d*x + 1/2*c)^9 + 945*a^3*tan(1/2*d*x + 1/2*c)^8 +
 2340*a^3*tan(1/2*d*x + 1/2*c)^7 + 1680*a^3*tan(1/2*d*x + 1/2*c)^6 - 4032*a
^3*tan(1/2*d*x + 1/2*c)^5 - 12600*a^3*tan(1/2*d*x + 1/2*c)^4 - 16800*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 5040*a^3*tan(1/2*d*x + 1/2*c)^2 + 85680*a^3*log(abs(
tan(1/2*d*x + 1/2*c))) + 52920*a^3*tan(1/2*d*x + 1/2*c) - (242386*a^3*tan(1
/2*d*x + 1/2*c)^9 + 52920*a^3*tan(1/2*d*x + 1/2*c)^8 - 5040*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 16800*a^3*tan(1/2*d*x + 1/2*c)^6 - 12600*a^3*tan(1/2*d*x + 1/
2*c)^5 - 4032*a^3*tan(1/2*d*x + 1/2*c)^4 + 1680*a^3*tan(1/2*d*x + 1/2*c)^3
+ 2340*a^3*tan(1/2*d*x + 1/2*c)^2 + 945*a^3*tan(1/2*d*x + 1/2*c) + 140*a^3)
/tan(1/2*d*x + 1/2*c)^9)/d
```

3.406 $\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=216

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{a^3\cot^7(c+dx)}{d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{21a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^3(c+dx)\csc^7(c+dx)}{10d}-$$

```
[Out] (-21*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (a
^3*Cot[c + d*x]^7)/d - (a^3*Cot[c + d*x]^9)/(3*d) - (21*a^3*Cot[c + d*x]*Cs
c[c + d*x])/(256*d) - (7*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (29*a^3
*Cot[c + d*x]*Csc[c + d*x]^5)/(160*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x]^
5)/(8*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^3*Cot[c + d*x]^3
*Csc[c + d*x]^7)/(10*d)
```

Rubi [A] time = 0.389845, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{a^3\cot^7(c+dx)}{d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{21a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^3(c+dx)\csc^7(c+dx)}{10d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-21*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (a
^3*Cot[c + d*x]^7)/d - (a^3*Cot[c + d*x]^9)/(3*d) - (21*a^3*Cot[c + d*x]*Cs
c[c + d*x])/(256*d) - (7*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (29*a^3
*Cot[c + d*x]*Csc[c + d*x]^5)/(160*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x]^
5)/(8*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^3*Cot[c + d*x]^3
*Csc[c + d*x]^7)/(10*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc^4(c+dx)+3a^3\cot^4(c+dx)\csc^5(c+dx)+3\right.$$

$$= a^3\int \cot^4(c+dx)\csc^4(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^7(c+dx)$$

$$= -\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{a^3\cot^3(c+dx)\csc^7(c+dx)}{10d} - \frac{1}{10}$$

$$= \frac{3a^3\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d} + \frac{3a^3}{}$$

$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{d} - \frac{a^3\cot^9(c+dx)}{3d} - \frac{3a^3\cot(c}{}$$

$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{d} - \frac{a^3\cot^9(c+dx)}{3d} - \frac{9a^3\cot(c}{}$$

$$= -\frac{9a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{d} -$$

$$= -\frac{21a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{d} -$$

Mathematica [A] time = 2.14824, size = 366, normalized size = 1.69

$$a^3(\sin(c+dx)+1)^3\left(4096\tan\left(\tfrac{1}{2}(c+dx)\right)-4096\cot\left(\tfrac{1}{2}(c+dx)\right)-1260\csc^2\left(\tfrac{1}{2}(c+dx)\right)+6\sec^{10}\left(\tfrac{1}{2}(c+dx)\right)+75\sec\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-4096*Cot[(c + d*x)/2] - 1260*Csc[(c + d*x)/2]^2
 - 5040*Log[Cos[(c + d*x)/2]] + 5040*Log[Sin[(c + d*x)/2]] + 1260*Sec[(c +
d*x)/2]^2 - 180*Sec[(c + d*x)/2]^4 - 390*Sec[(c + d*x)/2]^6 + 75*Sec[(c + d
*x)/2]^8 + 6*Sec[(c + d*x)/2]^10 + 64*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 4
*Csc[(c + d*x)/2]^4*(-45 + Sin[c + d*x]) + 5*Csc[(c + d*x)/2]^8*(-15 + 4*Si
```

```
n[c + d*x]) - 2*Csc[(c + d*x)/2]^10*(3 + 10*Sin[c + d*x]) + 6*Csc[(c + d*x)
/2]^6*(65 + 42*Sin[c + d*x]) + 4096*Tan[(c + d*x)/2] - 504*Sec[(c + d*x)/2]
^4*Tan[(c + d*x)/2] - 40*Sec[(c + d*x)/2]^6*Tan[(c + d*x)/2] + 40*Sec[(c +
d*x)/2]^8*Tan[(c + d*x)/2]))/(61440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^6)
```

Maple [A] time = 0.096, size = 248, normalized size = 1.2

$$-\frac{a^3 (\cos(dx+c))^5}{3d(\sin(dx+c))^7}-\frac{2a^3(\cos(dx+c))^5}{15d(\sin(dx+c))^5}-\frac{7a^3(\cos(dx+c))^5}{16d(\sin(dx+c))^8}-\frac{7a^3(\cos(dx+c))^5}{32d(\sin(dx+c))^6}-\frac{7a^3(\cos(dx+c))^5}{128d(\sin(dx+c))^4}+\frac{7a^3(\cdots}{256d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d*a^3/sin(d*x+c)^7*cos(d*x+c)^5-2/15/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-7
/16/d*a^3/sin(d*x+c)^8*cos(d*x+c)^5-7/32/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5-7/
128/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+7/256/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+7
/256*a^3*cos(d*x+c)^3/d+21/256*a^3*cos(d*x+c)/d+21/256/d*a^3*ln(csc(d*x+c)-
cot(d*x+c))-1/3/d*a^3/sin(d*x+c)^9*cos(d*x+c)^5-1/10/d*a^3/sin(d*x+c)^10*co
s(d*x+c)^5
```

Maxima [A] time = 1.09919, size = 416, normalized size = 1.93

$$\frac{21a^3\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7+128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] 1/53760*(21*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 + 128*cos(d*x + c
)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x + c
)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*lo
g(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 630*a^3*(2*(3*cos(d*x + c
)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 + 3*cos(d*x + c))/(cos(d*x + c)
^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 1536*(7*tan(d*x + c)^2 + 5)*a^
3/tan(d*x + c)^7 - 512*(63*tan(d*x + c)^4 + 90*tan(d*x + c)^2 + 35)*a^3/tan
(d*x + c)^9)/d
```

Fricas [A] time = 1.27438, size = 871, normalized size = 4.03

$$\frac{630a^3\cos(dx+c)^9-2940a^3\cos(dx+c)^7+768a^3\cos(dx+c)^5+2940a^3\cos(dx+c)^3-630a^3\cos(dx+c)-315\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/7680*(630*a^3*cos(d*x + c)^9 - 2940*a^3*cos(d*x + c)^7 + 768*a^3*cos(d*x
+ c)^5 + 2940*a^3*cos(d*x + c)^3 - 630*a^3*cos(d*x + c) - 315*(a^3*cos(d*x
+ c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d*x + c
)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 315*(a^3*co
s(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d
*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) + 512*
(2*a^3*cos(d*x + c)^9 - 9*a^3*cos(d*x + c)^7 + 12*a^3*cos(d*x + c)^5)*sin(d
*x + c))/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10
*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**11*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.53611, size = 482, normalized size = 2.23

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-30\,a^3\tan\left(\frac{1}{2}\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/61440*(6*a^3*tan(1/2*d*x + 1/2*c)^10 + 40*a^3*tan(1/2*d*x + 1/2*c)^9 + 10
5*a^3*tan(1/2*d*x + 1/2*c)^8 + 120*a^3*tan(1/2*d*x + 1/2*c)^7 - 30*a^3*tan(
1/2*d*x + 1/2*c)^6 - 384*a^3*tan(1/2*d*x + 1/2*c)^5 - 840*a^3*tan(1/2*d*x +
 1/2*c)^4 - 960*a^3*tan(1/2*d*x + 1/2*c)^3 + 60*a^3*tan(1/2*d*x + 1/2*c)^2
+ 5040*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3600*a^3*tan(1/2*d*x + 1/2*c) -
 (14762*a^3*tan(1/2*d*x + 1/2*c)^10 + 3600*a^3*tan(1/2*d*x + 1/2*c)^9 + 60*
a^3*tan(1/2*d*x + 1/2*c)^8 - 960*a^3*tan(1/2*d*x + 1/2*c)^7 - 840*a^3*tan(1
/2*d*x + 1/2*c)^6 - 384*a^3*tan(1/2*d*x + 1/2*c)^5 - 30*a^3*tan(1/2*d*x + 1
/2*c)^4 + 120*a^3*tan(1/2*d*x + 1/2*c)^3 + 105*a^3*tan(1/2*d*x + 1/2*c)^2 +
 40*a^3*tan(1/2*d*x + 1/2*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^10)/d
```

3.407 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=187

$$-\frac{11a^4\cos^7(c+dx)}{112d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)^2}{18d}-\frac{11\cos^7(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{144d}+\frac{11a^4\sin(c+dx)\,\text{co}}{96d}$$

```
[Out] (55*a^4*x)/256 - (11*a^4*Cos[c + d*x]^7)/(112*d) + (55*a^4*Cos[c + d*x]*Sin
[c + d*x])/(256*d) + (55*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (11*a^4
*Cos[c + d*x]^5*Sin[c + d*x])/(96*d) - (Cos[c + d*x]^5*(a + a*Sin[c + d*x])
^5)/(10*a*d) - (Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x])^2)/(18*d) - (11*Cos
[c + d*x]^7*(a^4 + a^4*Sin[c + d*x]))/(144*d)
```

Rubi [A] time = 0.234233, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2870, 2678, 2669, 2635, 8}

$$-\frac{11a^4\cos^7(c+dx)}{112d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)^2}{18d}-\frac{11\cos^7(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{144d}+\frac{11a^4\sin(c+dx)\,\text{co}}{96d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```



```
[Out] (55*a^4*x)/256 - (11*a^4*Cos[c + d*x]^7)/(112*d) + (55*a^4*Cos[c + d*x]*Sin
[c + d*x])/(256*d) + (55*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (11*a^4
*Cos[c + d*x]^5*Sin[c + d*x])/(96*d) - (Cos[c + d*x]^5*(a + a*Sin[c + d*x])
^5)/(10*a*d) - (Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x])^2)/(18*d) - (11*Cos
[c + d*x]^7*(a^4 + a^4*Sin[c + d*x]))/(144*d)
```


Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}+\frac{1}{2}a\int \cos^6(c+dx)(a+a\sin(c$$

$$= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}-\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c+d\right.}{18d}$$

$$= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}-\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c+d\right.}{18d}$$

$$= -\frac{11a^4\cos^7(c+dx)}{112d}-\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}-\frac{\cos^7(c+d}{}$$

$$= -\frac{11a^4\cos^7(c+dx)}{112d}+\frac{11a^4\cos^5(c+dx)\sin(c+dx)}{96d}-\frac{\cos^5(c+dx)(}{}$$

$$= -\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos^3(c+dx)\sin(c+dx)}{384d}+\frac{11a^4\cos^5(c+}{}$$

$$= -\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos(c+dx)\sin(c+dx)}{256d}+\frac{55a^4\cos^3(c+}{38}$$

$$= \frac{55a^4x}{256}-\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos(c+dx)\sin(c+dx)}{256d}+\frac{55a^4\,c}{}$$

Mathematica [A] time = 1.17601, size = 116, normalized size = 0.62

$$a^4(8820\sin(2(c+dx))-42840\sin(4(c+dx))-2730\sin(6(c+dx))+4095\sin(8(c+dx))-126\sin(10(c+dx))-18144$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(136080*c + 138600*d*x - 181440*Cos[c + d*x] - 53760*Cos[3*(c + d*x)]
+ 16128*Cos[5*(c + d*x)] + 7200*Cos[7*(c + d*x)] - 1120*Cos[9*(c + d*x)] +
8820*Sin[2*(c + d*x)] - 42840*Sin[4*(c + d*x)] - 2730*Sin[6*(c + d*x)] + 40
95*Sin[8*(c + d*x)] - 126*Sin[10*(c + d*x)]))/(645120*d)
```

Maple [A] time = 0.044, size = 306, normalized size = 1.6

$$\frac{1}{d}\left(a^4\left(-\frac{(\sin(dx+c))^5(\cos(dx+c))^5}{10}-\frac{(\sin(dx+c))^3(\cos(dx+c))^5}{16}-\frac{\sin(dx+c)(\cos(dx+c))^5}{32}+\frac{\sin(dx+c)}{128}\left((\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/
32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/256*d*x+3/256*c)+4*a^4*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*c
os(d*x+c)^5-8/315*cos(d*x+c)^5)+6*a^4*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*
sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128
*d*x+3/128*c)+4*a^4*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a^4*
(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+1/16*d*x+1/16*c))
```

Maxima [A] time = 1.11004, size = 251, normalized size = 1.34

$$-\frac{8192\left(35\cos(dx+c)^9-90\cos(dx+c)^7+63\cos(dx+c)^5\right)a^4-73728\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^4+63\left(\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] -1/645120*(8192*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^4 - 73728*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^4 + 63*(32*sin(2*d*x +
 2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x + 8*c) + 40*sin(4*d*x + 4*c))*a^4 -
 3360*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^4 - 378
0*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^4)/d
```

Fricas [A] time = 1.23926, size = 343, normalized size = 1.83

$$-\frac{35840\,a^4\cos(dx+c)^9-138240\,a^4\cos(dx+c)^7+129024\,a^4\cos(dx+c)^5-17325\,a^4dx+21\left(384\,a^4\cos(dx+c)^9-\ldots\right.}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/80640*(35840*a^4*cos(d*x + c)^9 - 138240*a^4*cos(d*x + c)^7 + 129024*a^4
*cos(d*x + c)^5 - 17325*a^4*d*x + 21*(384*a^4*cos(d*x + c)^9 - 3888*a^4*cos
(d*x + c)^7 + 5704*a^4*cos(d*x + c)^5 - 550*a^4*cos(d*x + c)^3 - 825*a^4*co
s(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 36.57, size = 746, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**10/256 + 15*a**4*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 9*a**4*x*sin(c + d*x)**8/64 + 15*a**4*x*sin(c + d*x)**6*cos
(c + d*x)**4/128 + 9*a**4*x*sin(c + d*x)**6*cos(c + d*x)**2/16 + a**4*x*sin
(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 27*a**4*x
```

```
*sin(c + d*x)**4*cos(c + d*x)**4/32 + 3*a**4*x*sin(c + d*x)**4*cos(c + d*x)
**2/16 + 15*a**4*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 9*a**4*x*sin(c + d
*x)**2*cos(c + d*x)**6/16 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3
*a**4*x*cos(c + d*x)**10/256 + 9*a**4*x*cos(c + d*x)**8/64 + a**4*x*cos(c +
 d*x)**6/16 + 3*a**4*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**4*sin(c +
d*x)**7*cos(c + d*x)**3/(128*d) + 9*a**4*sin(c + d*x)**7*cos(c + d*x)/(64*d
) - a**4*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 33*a**4*sin(c + d*x)**5*c
os(c + d*x)**3/(64*d) + a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 4*a**4*s
in(c + d*x)**4*cos(c + d*x)**5/(5*d) - 7*a**4*sin(c + d*x)**3*cos(c + d*x)*
*7/(128*d) - 33*a**4*sin(c + d*x)**3*cos(c + d*x)**5/(64*d) + a**4*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) - 16*a**4*sin(c + d*x)**2*cos(c + d*x)**7/(35
*d) - 4*a**4*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**4*sin(c + d*x)*co
s(c + d*x)**9/(256*d) - 9*a**4*sin(c + d*x)*cos(c + d*x)**7/(64*d) - a**4*s
in(c + d*x)*cos(c + d*x)**5/(16*d) - 32*a**4*cos(c + d*x)**9/(315*d) - 8*a*
*4*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**2*cos(c)
**4, True))
```

Giac [A] time = 1.33955, size = 235, normalized size = 1.26

$$\frac{55}{256}a^4x - \frac{a^4\cos(9dx+9c)}{576d} + \frac{5a^4\cos(7dx+7c)}{448d} + \frac{a^4\cos(5dx+5c)}{40d} - \frac{a^4\cos(3dx+3c)}{12d} - \frac{9a^4\cos(dx+c)}{32d} - \frac{a^4\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 55/256*a^4*x - 1/576*a^4*cos(9*d*x + 9*c)/d + 5/448*a^4*cos(7*d*x + 7*c)/d
+ 1/40*a^4*cos(5*d*x + 5*c)/d - 1/12*a^4*cos(3*d*x + 3*c)/d - 9/32*a^4*cos(
d*x + c)/d - 1/5120*a^4*sin(10*d*x + 10*c)/d + 13/2048*a^4*sin(8*d*x + 8*c)
/d - 13/3072*a^4*sin(6*d*x + 6*c)/d - 17/256*a^4*sin(4*d*x + 4*c)/d + 7/512
*a^4*sin(2*d*x + 2*c)/d
```

3.408 $\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=140

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+$$

```
[Out] (-61*a^4*x)/8 + (2*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x]^3)/(3
*d) - (5*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]^3)/(3*d) - (2*a^4*Cot[c +
d*x]*Csc[c + d*x])/d - (19*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.228222, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (-61*a^4*x)/8 + (2*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x]^3)/(3
*d) - (5*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]^3)/(3*d) - (2*a^4*Cot[c +
d*x]*Csc[c + d*x])/d - (19*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```


Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(-10a^8-4a^8\csc(c+dx)+4a^8\csc^2(c+dx)+4a^8\csc^3(c+dx)+a^8\csc^4(c+d\right.}{a}$$

$$= -10a^4x+a^4\int \csc^4(c+dx)\,dx+a^4\int \sin^4(c+dx)\,dx-\left(4a^4\right)\int \csc(c+dx)\,dx$$

$$= -10a^4x+\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{2a^4\cot(c+dx)\csc(c+}{d}$$

$$= -8a^4x+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\mathrm{co}}{}$$

$$= -\frac{61a^4x}{8}+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\mathrm{c}}{}$$

Mathematica [A] time = 5.29066, size = 209, normalized size = 1.49

$$\frac{a^4(\sin(c+dx)+1)^4\left(-732(c+dx)-120\sin(2(c+dx))+3\sin(4(c+dx))+96\cos(c+dx)+32\cos(3(c+dx))+224\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(-732*(c + d*x) + 96*Cos[c + d*x] + 32*Cos[3*(c +
 d*x)] - 224*Cot[(c + d*x)/2] - 48*Csc[(c + d*x)/2]^2 + 192*Log[Cos[(c + d*
x)/2]] - 192*Log[Sin[(c + d*x)/2]] + 48*Sec[(c + d*x)/2]^2 + 32*Csc[c + d*x
]^3*Sin[(c + d*x)/2]^4 - 2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 120*Sin[2*(c +
 d*x)] + 3*Sin[4*(c + d*x)] + 224*Tan[(c + d*x)/2]))/(96*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^8)
```

Maple [A] time = 0.091, size = 190, normalized size = 1.4

$$-\frac{23\,a^4(\cos(dx+c))^3\sin(dx+c)}{4\,d}-\frac{69\,a^4\cos(dx+c)\sin(dx+c)}{8\,d}-\frac{61\,a^4x}{8}-\frac{61\,a^4c}{8\,d}-\frac{2\,a^4(\cos(dx+c))^3}{3\,d}-2\,\frac{a^4\cos(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -23/4*a^4*cos(d*x+c)^3*sin(d*x+c)/d-69/8*a^4*cos(d*x+c)*sin(d*x+c)/d-61/8*a
^4*x-61/8/d*a^4*c-2/3*a^4*cos(d*x+c)^3/d-2*a^4*cos(d*x+c)/d-2/d*a^4*ln(csc(
d*x+c)-cot(d*x+c))-6/d*a^4/sin(d*x+c)*cos(d*x+c)^5-2/d*a^4/sin(d*x+c)^2*cos
(d*x+c)^5-1/3*a^4*cot(d*x+c)^3/d+a^4*cot(d*x+c)/d
```

Maxima [A] time = 1.64189, size = 294, normalized size = 2.1

$$\frac{64\left(2\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a^{4}+3\left(12\,dx+12\,c+\sin\left(4\,d\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/96*(64*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a^4 - 288*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3
+ tan(d*x + c)))*a^4 + 32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c
)^3)*a^4 + 96*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3
*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.26403, size = 560, normalized size = 4.

$$-\frac{6\,a^{4}\cos\left(dx+c\right)^{7}-75\,a^{4}\cos\left(dx+c\right)^{5}+244\,a^{4}\cos\left(dx+c\right)^{3}-183\,a^{4}\cos\left(dx+c\right)-24\left(a^{4}\cos\left(dx+c\right)^{2}-a^{4}\right)\log\left(\frac{1}{2}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^7 - 75*a^4*cos(d*x + c)^5 + 244*a^4*cos(d*x + c)^
3 - 183*a^4*cos(d*x + c) - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) + 24*(a^4*cos(d*x + c)^2 - a^4)*log(-1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) - (32*a^4*cos(d*x + c)^5 - 183*a^4*d*x*cos(d*x + c)^2
 - 32*a^4*cos(d*x + c)^3 + 183*a^4*d*x + 48*a^4*cos(d*x + c))*sin(d*x + c))
/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50484, size = 370, normalized size = 2.64

$$a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 12\,a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 183\,(dx + c)a^4 - 48\,a^4 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + 57\,a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*tan(1/2*d*x + 1/2*c)^2 - 183*(d*x
 + c)*a^4 - 48*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 57*a^4*tan(1/2*d*x + 1/
2*c) + (88*a^4*tan(1/2*d*x + 1/2*c)^3 - 57*a^4*tan(1/2*d*x + 1/2*c)^2 - 12*
a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(57*a^4*tan(1/2*
d*x + 1/2*c)^7 + 96*a^4*tan(1/2*d*x + 1/2*c)^6 + 81*a^4*tan(1/2*d*x + 1/2*c
)^5 + 96*a^4*tan(1/2*d*x + 1/2*c)^4 - 81*a^4*tan(1/2*d*x + 1/2*c)^3 + 32*a^
4*tan(1/2*d*x + 1/2*c)^2 - 57*a^4*tan(1/2*d*x + 1/2*c) + 32*a^4)/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

3.409 $\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}+\frac{\sin(c+dx)\,\mathrm{c}}{16a\mathrm{c}}$$

```
[Out] x/(16*a) + Cos[c + d*x]^3/(3*a*d) - (2*Cos[c + d*x]^5)/(5*a*d) + Cos[c + d*
x]^7/(7*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) - (Cos[c + d*x]^3*Sin[c
 + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a*d)
```

Rubi [A] time = 0.197698, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 270}

$$\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}+\frac{\sin(c+dx)\,\mathrm{c}}{16a\mathrm{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(16*a) + Cos[c + d*x]^3/(3*a*d) - (2*Cos[c + d*x]^5)/(5*a*d) + Cos[c + d*
x]^7/(7*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) - (Cos[c + d*x]^3*Sin[c
 + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a}$$
$$= -\frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2 \right.}{ad}$$
$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\int \cos^2(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{\cos^3(c+dx)}{3ad} - \frac{2\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^3(c+dx}{8}$$
$$= \frac{x}{16a} + \frac{\cos^3(c+dx)}{3ad} - \frac{2\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^3}{}$$

Mathematica [A] time = 0.256363, size = 86, normalized size = 0.64

$$\frac{-105\sin(2(c+dx)) - 105\sin(4(c+dx)) + 35\sin(6(c+dx)) + 525\cos(c+dx) + 35\cos(3(c+dx)) - 63\cos(5(c+dx)) +}{6720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (420*c + 420*d*x + 525*Cos[c + d*x] + 35*Cos[3*(c + d*x)] - 63*Cos[5*(c + d
*x)] + 15*Cos[7*(c + d*x)] - 105*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)] +
35*Sin[6*(c + d*x)])/(6720*a*d)
```

Maple [B] time = 0.085, size = 381, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13+5/6/d/a/(1+tan(1/2
*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11-97/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7
*tan(1/2*d*x+1/2*c)^9+32/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c
)^8-16/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6+97/24/d/a/(1+t
an(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5+16/5/d/a/(1+tan(1/2*d*x+1/2*c)^
2)^7*tan(1/2*d*x+1/2*c)^4-5/6/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/
2*c)^3+16/15/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2-1/8/d/a/(1
+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)+16/105/d/a/(1+tan(1/2*d*x+1/2*c
)^2)^7+1/8/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.68898, size = 513, normalized size = 3.8

$$-\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{896\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{700\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2688\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3395\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4480\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{8960\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{3395\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{700\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{105\,\sin(dx}{(\cos(dx+c}}{a+\frac{7\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35\,a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21\,a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7\,a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 896*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 700*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2688*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 3395*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4480*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 8960*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 3395*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 700*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 105*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 128)/(a + 7*a*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 21*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 35*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 35*a*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8 + 21*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 7*a*sin(d*x + c
)^12/(cos(d*x + c) + 1)^12 + a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.13487, size = 217, normalized size = 1.61

$$\frac{240\,\cos(dx+c)^7-672\,\cos(dx+c)^5+560\,\cos(dx+c)^3+105\,dx+35\left(8\,\cos(dx+c)^5-14\,\cos(dx+c)^3+3\,\cos(d\right.}{1680\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*cos(d*x + c)^7 - 672*cos(d*x + c)^5 + 560*cos(d*x + c)^3 + 105*
d*x + 35*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x +
c))/(a*d)
```

Sympy [A] time = 125.481, size = 3048, normalized size = 22.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((210*d*x*tan(c/2 + d*x/2)**14/(3360*a*d*tan(c/2 + d*x/2)**14 + 23
520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*
tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 +
d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 1470*d*x*tan(c/2 +
d*x/2)**12/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12
+ 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*
a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2
 + d*x/2)**2 + 3360*a*d) + 4410*d*x*tan(c/2 + d*x/2)**10/(3360*a*d*tan(c/2
```

+ d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)*
*10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 705
60*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 73
50*d*x*tan(c/2 + d*x/2)**8/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c
/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x
/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 2
3520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 7350*d*x*tan(c/2 + d*x/2)**6/(33
60*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*ta
n(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 +
d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 +
3360*a*d) + 4410*d*x*tan(c/2 + d*x/2)**4/(3360*a*d*tan(c/2 + d*x/2)**14 +
23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*
d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2
+ d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 1470*d*x*tan(c/2
+ d*x/2)**2/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12
+ 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600
*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/
2 + d*x/2)**2 + 3360*a*d) + 210*d*x/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*
a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(
c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/
2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) - 115*tan(c/2 + d*x/2)**1
4/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a
*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c
/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)
2 + 3360*a*d) + 420*tan(c/2 + d*x/2)13/(3360*a*d*tan(c/2 + d*x/2)**14 +
23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a
*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2
+ d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) - 805*tan(c/2 + d*
x/2)**12/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 +
70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*
d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 +
d*x/2)**2 + 3360*a*d) + 2800*tan(c/2 + d*x/2)**11/(3360*a*d*tan(c/2 + d*x/
2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 +
117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d
*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) - 2415*tan
(c/2 + d*x/2)**10/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/
2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 +
117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*
tan(c/2 + d*x/2)**2 + 3360*a*d) - 13580*tan(c/2 + d*x/2)**9/(3360*a*d*tan(c
/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/
2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 +
70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) +
31815*tan(c/2 + d*x/2)**8/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c
/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x
/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 2
3520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) - 21945*tan(c/2 + d*x/2)**6/(3360*
a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c
/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x
/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 33
60*a*d) + 13580*tan(c/2 + d*x/2)**5/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*
a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(
c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/
2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 8337*tan(c/2 + d*x/2)**
4/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a
*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c
/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)
2 + 3360*a*d) - 2800*tan(c/2 + d*x/2)3/(3360*a*d*tan(c/2 + d*x/2)**14 +
23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a
*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2

```
 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 2779*tan(c/2 + d
*x/2)**2/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 +
70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*
d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 +
 d*x/2)**2 + 3360*a*d) - 420*tan(c/2 + d*x/2)/(3360*a*d*tan(c/2 + d*x/2)**1
4 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 11760
0*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(
c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 397/(3360*a*d
*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2
+ d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)
**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*
a*d), Ne(d, 0)), (x*sin(c)**4*cos(c)**4/(a*sin(c) + a), True))
```

Giac [A] time = 1.44052, size = 224, normalized size = 1.66

$$\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+700\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-3395\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+8960\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-4480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+3395\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{7}a}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1680*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 700*tan(1/2*d*x
+ 1/2*c)^11 - 3395*tan(1/2*d*x + 1/2*c)^9 + 8960*tan(1/2*d*x + 1/2*c)^8 - 4
480*tan(1/2*d*x + 1/2*c)^6 + 3395*tan(1/2*d*x + 1/2*c)^5 + 2688*tan(1/2*d*x
 + 1/2*c)^4 - 700*tan(1/2*d*x + 1/2*c)^3 + 896*tan(1/2*d*x + 1/2*c)^2 - 105
*tan(1/2*d*x + 1/2*c) + 128)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*a))/d
```

3.410 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

```
[Out] -x/(16*a) - Cos[c + d*x]^3/(3*a*d) + Cos[c + d*x]^5/(5*a*d) - (Cos[c + d*x]
*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(8*a*d) + (Cos[c +
d*x]^3*Sin[c + d*x]^3)/(6*a*d)
```

Rubi [A] time = 0.191169, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2565, 14, 2568, 2635, 8}

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(16*a) - Cos[c + d*x]^3/(3*a*d) + Cos[c + d*x]^5/(5*a*d) - (Cos[c + d*x]
*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(8*a*d) + (Cos[c +
d*x]^3*Sin[c + d*x]^3)/(6*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}dx = \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a}$$
$$= \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} - \frac{\mathrm{Subst}\left(\int x^2\left(1-x^2\right)\right.}{ad}$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\,dx}{8a} - \frac{\mathrm{Subst}}{}$$
$$= -\frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad}$$
$$= -\frac{x}{16a} - \frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(}{8ad}$$

Mathematica [B] time = 4.84269, size = 377, normalized size = 3.22

$$-120dx\sin\left(\frac{c}{2}\right)+120\sin\left(\frac{c}{2}+dx\right)-120\sin\left(\frac{3c}{2}+dx\right)+15\sin\left(\frac{3c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+2dx\right)+20\sin\left(\frac{5c}{2}+3dx\right)-$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (30*(3*c - 4*d*x)*Cos[c/2] - 120*Cos[c/2 + d*x] - 120*Cos[(3*c)/2 + d*x] +
15*Cos[(3*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 2*d*x] - 20*Cos[(5*c)/2 + 3*d*x]
 - 20*Cos[(7*c)/2 + 3*d*x] + 15*Cos[(7*c)/2 + 4*d*x] - 15*Cos[(9*c)/2 + 4*d
*x] + 12*Cos[(9*c)/2 + 5*d*x] + 12*Cos[(11*c)/2 + 5*d*x] - 5*Cos[(11*c)/2 +
 6*d*x] + 5*Cos[(13*c)/2 + 6*d*x] - 180*Sin[c/2] + 90*c*Sin[c/2] - 120*d*x*
Sin[c/2] + 120*Sin[c/2 + d*x] - 120*Sin[(3*c)/2 + d*x] + 15*Sin[(3*c)/2 + 2
*d*x] + 15*Sin[(5*c)/2 + 2*d*x] + 20*Sin[(5*c)/2 + 3*d*x] - 20*Sin[(7*c)/2
+ 3*d*x] + 15*Sin[(7*c)/2 + 4*d*x] + 15*Sin[(9*c)/2 + 4*d*x] - 12*Sin[(9*c)
/2 + 5*d*x] + 12*Sin[(11*c)/2 + 5*d*x] - 5*Sin[(11*c)/2 + 6*d*x] - 5*Sin[(1
3*c)/2 + 6*d*x])/(1920*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.074, size = 347, normalized size = 3.

$$-\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6}-\frac{17}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6}-4\,\frac{(\tan(}{da\left(1+(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

[Out] -1/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-17/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9-4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8+19/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7-8/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6-19/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+17/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3-8/5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2+1/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-4/15/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6-1/8/a/d*arctan(tan(1/2*d*x+1/2*c))

Maxima [B] time = 1.70372, size = 458, normalized size = 3.91

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{192\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{85\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{570\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{320\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{570\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{85\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{15\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-32}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 192*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 85*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 570*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 320*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 570*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 480*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 85*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 32)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d

---

Fricas [A] time = 1.15258, size = 182, normalized size = 1.56

$$\frac{48\cos(dx+c)^5-80\cos(dx+c)^3-15\,dx-5\left(8\cos(dx+c)^5-14\cos(dx+c)^3+3\cos(dx+c)\right)\sin(dx+c)}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/240*(48*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*d*x - 5*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x + c))/(a*d)

Sympy [A] time = 75.9437, size = 2429, normalized size = 20.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**12/(1680*a*d*tan(c/2 + d*x/2)**12 + 1
0080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*t
an(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*
x/2)**2 + 1680*a*d) - 630*d*x*tan(c/2 + d*x/2)**10/(1680*a*d*tan(c/2 + d*x/
2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 3
3600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*ta
n(c/2 + d*x/2)**2 + 1680*a*d) - 1575*d*x*tan(c/2 + d*x/2)**8/(1680*a*d*tan(
c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x
/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10
080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 2100*d*x*tan(c/2 + d*x/2)**6/(168
0*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan
(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/
2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 1575*d*x*tan(c/2 + d*x/
2)**4/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 252
00*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(
c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 630*d*x*tan(c
/2 + d*x/2)**2/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)*
*10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200
*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 105*
d*x/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200
*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/
2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 30*tan(c/2 + d*
x/2)**12/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 +
25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*t
an(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 210*tan(c/
2 + d*x/2)**11/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)*
*10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200
*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 180*
tan(c/2 + d*x/2)**10/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d
*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 +
 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d)
- 1190*tan(c/2 + d*x/2)**9/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c
/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2
)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680
*a*d) - 6270*tan(c/2 + d*x/2)**8/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d
*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 +
 d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2
+ 1680*a*d) + 7980*tan(c/2 + d*x/2)**7/(1680*a*d*tan(c/2 + d*x/2)**12 + 100
80*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan
(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/
2)**2 + 1680*a*d) - 3880*tan(c/2 + d*x/2)**6/(1680*a*d*tan(c/2 + d*x/2)**12
 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a
*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2
+ d*x/2)**2 + 1680*a*d) - 7980*tan(c/2 + d*x/2)**5/(1680*a*d*tan(c/2 + d*x/
2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 3
3600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*ta
n(c/2 + d*x/2)**2 + 1680*a*d) + 450*tan(c/2 + d*x/2)**4/(1680*a*d*tan(c/2 +
 d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**
8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a
*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 1190*tan(c/2 + d*x/2)**3/(1680*a*d*tan
(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*
x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 1
0080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 2508*tan(c/2 + d*x/2)**2/(1680*a
*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/
2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)*
*4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 210*tan(c/2 + d*x/2)/(1680
*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(
c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2
)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 418/(1680*a*d*tan(c/2 +
```

```
d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8
 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*
d*tan(c/2 + d*x/2)**2 + 1680*a*d), Ne(d, 0)), (x*sin(c)**3*cos(c)**4/(a*sin
(c) + a), True))
```

Giac [A] time = 1.3679, size = 207, normalized size = 1.77

$$-\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+85\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-570\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+570\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-85\,\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 85*tan(1/2*d*x + 1
/2*c)^9 + 480*tan(1/2*d*x + 1/2*c)^8 - 570*tan(1/2*d*x + 1/2*c)^7 + 320*tan
(1/2*d*x + 1/2*c)^6 + 570*tan(1/2*d*x + 1/2*c)^5 - 85*tan(1/2*d*x + 1/2*c)^
3 + 192*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 32)/((tan(1/2*d*
x + 1/2*c)^2 + 1)^6*a))/d
```

3.411 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{\sin(c+dx)\cos(c+dx)}{8ad}+\frac{x}{8a}$$

```
[Out] x/(8*a) + Cos[c + d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) + (Cos[c + d*x]*S
in[c + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.16189, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 14}

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{\sin(c+dx)\cos(c+dx)}{8ad}+\frac{x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(8*a) + Cos[c + d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) + (Cos[c + d*x]*S
in[c + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a}$$
$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int \cos^2(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \cos(c+d$$
$$= \frac{\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int 1\,dx}{8a} + \frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,$$
$$= \frac{x}{8a} + \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} + \frac{\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)\sin(c+dx}{4ad}$$

Mathematica [B] time = 2.4064, size = 258, normalized size = 2.84

$$\frac{120dx\sin\left(\frac{c}{2}\right) - 60\sin\left(\frac{c}{2}+dx\right) + 60\sin\left(\frac{3c}{2}+dx\right) - 10\sin\left(\frac{5c}{2}+3dx\right) + 10\sin\left(\frac{7c}{2}+3dx\right) - 15\sin\left(\frac{7c}{2}+4dx\right) - 15\sin\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (120*d*x*Cos[c/2] + 60*Cos[c/2 + d*x] + 60*Cos[(3*c)/2 + d*x] + 10*Cos[(5*c
)/2 + 3*d*x] + 10*Cos[(7*c)/2 + 3*d*x] - 15*Cos[(7*c)/2 + 4*d*x] + 15*Cos[(
9*c)/2 + 4*d*x] - 6*Cos[(9*c)/2 + 5*d*x] - 6*Cos[(11*c)/2 + 5*d*x] + 120*Si
n[c/2] + 120*d*x*Sin[c/2] - 60*Sin[c/2 + d*x] + 60*Sin[(3*c)/2 + d*x] - 10*
Sin[(5*c)/2 + 3*d*x] + 10*Sin[(7*c)/2 + 3*d*x] - 15*Sin[(7*c)/2 + 4*d*x] -
15*Sin[(9*c)/2 + 4*d*x] + 6*Sin[(9*c)/2 + 5*d*x] - 6*Sin[(11*c)/2 + 5*d*x])
/(960*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.067, size = 279, normalized size = 3.1

$$\frac{1}{4da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5} - \frac{3}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5} + 4\frac{(\tan(1/2\,dx}{da\left(1+(\tan(1/2\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9-3/2/d/a/(1+tan(1/2*
d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7+4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)^6-4/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4+3/2
/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+4/3/d/a/(1+tan(1/2*d*x
+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2-1/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/
2*d*x+1/2*c)+4/15/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5+1/4/a/d*arctan(tan(1/2*d*x
```

```
+1/2*c))
```

Maxima [B] time = 1.69164, size = 375, normalized size = 4.12

$$-\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{90\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{90\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-16}{a+\frac{5a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 80*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 90*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 80*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 240*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 90*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 15*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 16)/(a +
 5*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 10*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 + a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 15*arctan(s
in(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.01536, size = 155, normalized size = 1.7

$$-\frac{24\cos(dx+c)^5-40\cos(dx+c)^3-15\,dx+15\left(2\cos(dx+c)^3-\cos(dx+c)\right)\sin(dx+c)}{120\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/120*(24*cos(d*x + c)^5 - 40*cos(d*x + c)^3 - 15*d*x + 15*(2*cos(d*x + c)
^3 - cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [A] time = 42.9268, size = 1464, normalized size = 16.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**10/(120*a*d*tan(c/2 + d*x/2)**10 + 600*
a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 +
 d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 75*d*x*tan(c/2 + d*x/
2)**8/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*
d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*
x/2)**2 + 120*a*d) + 150*d*x*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)*
*10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d
*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 150*d*x*tan
(c/2 + d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**
8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*t
```

```
an(c/2 + d*x/2)**2 + 120*a*d) + 75*d*x*tan(c/2 + d*x/2)**2/(120*a*d*tan(c/2
 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6
+ 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 1
5*d*x/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*
d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*
x/2)**2 + 120*a*d) + 30*tan(c/2 + d*x/2)**9/(120*a*d*tan(c/2 + d*x/2)**10 +
 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(
c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 180*tan(c/2 + d*
x/2)**7/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*
a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 +
d*x/2)**2 + 120*a*d) + 480*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)**1
0 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*t
an(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 160*tan(c/2 +
 d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 12
00*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2
 + d*x/2)**2 + 120*a*d) + 180*tan(c/2 + d*x/2)**3/(120*a*d*tan(c/2 + d*x/2)
**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*
d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 160*tan(c/
2 + d*x/2)**2/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 +
 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(
c/2 + d*x/2)**2 + 120*a*d) - 30*tan(c/2 + d*x/2)/(120*a*d*tan(c/2 + d*x/2)*
*10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d
*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 32/(120*a*d
*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*
x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*
a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**4/(a*sin(c) + a), True))
```

Giac [A] time = 1.32905, size = 171, normalized size = 1.88

$$\frac{\frac{15(dx+c)}{a} + \frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 - 90\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 80\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 90\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 80\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 15\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^5 a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^9 - 90*tan(1/2*d*x + 1/2
*c)^7 + 240*tan(1/2*d*x + 1/2*c)^6 - 80*tan(1/2*d*x + 1/2*c)^4 + 90*tan(1/2
*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 16)
/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a))/d
```

3.412 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

```
[Out] -x/(8*a) - Cos[c + d*x]^3/(3*a*d) - (Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (
Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.112174, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2565, 30, 2568, 2635, 8}

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(8*a) - Cos[c + d*x]^3/(3*a*d) - (Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (
Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a}\\ &= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{\int \cos^2(c+dx)\,dx}{4a}-\frac{\operatorname{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{ad}\\ &= -\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{\int 1\,dx}{8a}\\ &= -\frac{x}{8a}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}\end{aligned}$$

Mathematica [B] time = 1.65278, size = 219, normalized size = 3.

$$-\frac{24dx\sin\left(\frac{c}{2}\right)-24\sin\left(\frac{c}{2}+dx\right)+24\sin\left(\frac{3c}{2}+dx\right)-8\sin\left(\frac{5c}{2}+3dx\right)+8\sin\left(\frac{7c}{2}+3dx\right)-3\sin\left(\frac{7c}{2}+4dx\right)-3\sin\left(\frac{9c}{2}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(-24*(c - d*x)*Cos[c/2] + 24*Cos[c/2 + d*x] + 24*Cos[(3*c)/2 + d*x] + 8*Co
s[(5*c)/2 + 3*d*x] + 8*Cos[(7*c)/2 + 3*d*x] - 3*Cos[(7*c)/2 + 4*d*x] + 3*Co
s[(9*c)/2 + 4*d*x] + 48*Sin[c/2] - 24*c*Sin[c/2] + 24*d*x*Sin[c/2] - 24*Sin
[c/2 + d*x] + 24*Sin[(3*c)/2 + d*x] - 8*Sin[(5*c)/2 + 3*d*x] + 8*Sin[(7*c)/
2 + 3*d*x] - 3*Sin[(7*c)/2 + 4*d*x] - 3*Sin[(9*c)/2 + 4*d*x])/(192*a*d*(Cos
[c/2] + Sin[c/2]))
```

Maple [B] time = 0.06, size = 279, normalized size = 3.8

$$-\frac{1}{4da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}-2\frac{(\tan(1/2\,dx+c/2))^6}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^4}+\frac{7}{4da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/d/a/(1+tan(1/2*d
*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6+7/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(
1/2*d*x+1/2*c)^5-2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4-7/4/
d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-2/3/d/a/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+1/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)-2/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4-1/4/a/d*arctan(tan(1/2*d*x+1
/2*c))
```

Maxima [B] time = 1.6647, size = 347, normalized size = 4.75

$$\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{21\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{24\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{24\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-8}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 21*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 24*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 24*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 8)/(a + 4*a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.07232, size = 123, normalized size = 1.68

$$-\frac{8\cos(dx+c)^3+3dx-3\left(2\cos(dx+c)^3-\cos(dx+c)\right)\sin(dx+c)}{24ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*cos(d*x + c)^3 + 3*d*x - 3*(2*cos(d*x + c)^3 - cos(d*x + c))*sin(d
*x + c))/(a*d)
```

Sympy [A] time = 23.9163, size = 1221, normalized size = 16.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**8/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a
*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*
x/2)**2 + 120*a*d) - 60*d*x*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)**
8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan
(c/2 + d*x/2)**2 + 120*a*d) - 90*d*x*tan(c/2 + d*x/2)**4/(120*a*d*tan(c/2 +
 d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 48
0*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 60*d*x*tan(c/2 + d*x/2)**2/(120*a*d*
tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2
)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 15*d*x/(120*a*d*tan(c/2 + d
*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 480*
a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 54*tan(c/2 + d*x/2)**8/(120*a*d*tan(c/
2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)**4 +
 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 30*tan(c/2 + d*x/2)**7/(120*a*d*t
```

```
an(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d*x/2)
**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 24*tan(c/2 + d*x/2)**6/(120*
a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c/2 + d
*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 210*tan(c/2 + d*x/2)**5
/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*tan(c
/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 84*tan(c/2 + d*x/
2)**4/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*
tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 210*tan(c/2
+ d*x/2)**3/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 72
0*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 136*ta
n(c/2 + d*x/2)**2/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**
6 + 720*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) +
30*tan(c/2 + d*x/2)/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)
**6 + 720*a*d*tan(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d)
- 26/(120*a*d*tan(c/2 + d*x/2)**8 + 480*a*d*tan(c/2 + d*x/2)**6 + 720*a*d*t
an(c/2 + d*x/2)**4 + 480*a*d*tan(c/2 + d*x/2)**2 + 120*a*d), Ne(d, 0)), (x*
sin(c)*cos(c)**4/(a*sin(c) + a), True))
```

Giac [A] time = 1.60002, size = 171, normalized size = 2.34

$$-\frac{\frac{3\,(dx+c)}{a}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^7 + 24*tan(1/2*d*x + 1/2*c
)^6 - 21*tan(1/2*d*x + 1/2*c)^5 + 24*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*
x + 1/2*c)^3 + 8*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 8)/((tan
(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.413 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{2a}$$

```
[Out] -x/(2*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) - (Cos[c + d*x]
*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.0980408, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2592, 321, 206, 2635, 8}

$$\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(2*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) - (Cos[c + d*x]
*Sin[c + d*x])/(2*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos^2(c+dx)\,dx}{a} + \frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a} \\
&= -\frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int 1\,dx}{2a} - \frac{\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad} \\
&= -\frac{x}{2a} + \frac{\cos(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad} \\
&= -\frac{x}{2a} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\cos(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [A] time = 0.178481, size = 60, normalized size = 1.02

$$
-\frac{\sin(2(c+dx)) - 4\cos(c+dx) + 2\left(-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + c + dx\right)}{4ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(-4*Cos[c + d*x] + 2*(c + d*x + 2*Log[Cos[(c + d*x)/2]] - 2*Log[Sin[(c + d
*x)/2]]) + Sin[2*(c + d*x)])/(4*a*d)
```

Maple [B] time = 0.088, size = 159, normalized size = 2.7

$$
\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2} + 2\,\frac{(\tan(1/2\,dx+c/2))^2}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} - \frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/d/a/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d
*x+1/2*c)+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2-1/a/d*arctan(tan(1/2*d*x+1/2*c))
+1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.66312, size = 211, normalized size = 3.58

$$
-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1} - \frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - 2}{a + \frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2)/(a + 2*a*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.12896, size = 167, normalized size = 2.83

$$-\frac{dx+\cos(dx+c)\sin(dx+c)-2\cos(dx+c)+\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*x + cos(d*x + c)*sin(d*x + c) - 2*cos(d*x + c) + log(1/2*cos(d*x +
c) + 1/2) - log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos^4(c+dx)\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.37715, size = 119, normalized size = 2.02

$$-\frac{\frac{dx+c}{a}-\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a - 2*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(tan(1/2*d*x + 1
/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*
x + 1/2*c)^2 + 1)^2*a))/d
```

3.414 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

```
[Out] -(x/a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cot[c + d*x]/(a
*d)
```

Rubi [A] time = 0.116347, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 3473, 8, 2592, 321, 206}

$$-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cot[c + d*x]/(a
*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\,dx}{a}\\ &= -\frac{\cot(c+dx)}{ad}-\frac{\int 1\,dx}{a}+\frac{\operatorname{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\ &= -\frac{x}{a}-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\ &= -\frac{x}{a}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.407286, size = 93, normalized size = 1.9

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^2\left(\cos(c+dx)+\sin(c+dx)\left(\cos(c+dx)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}\right.\right.\right.\right.}{2ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((1 + Cot[(c + d*x)/2])^2*(Cos[c + d*x] + (c + d*x + Cos[c + d*x] - Log[Co
s[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x])*Tan[(c + d*x)/2])/(2
*a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.104, size = 97, normalized size = 2.

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\frac{1}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\frac{\arctan(\tan(1/2\,dx+c/2))}{da}-\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{1}{da}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-2/a/d/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*arctan(tan(
1/2*d*x+1/2*c))-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.6583, size = 208, normalized size = 4.24

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((4*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3) + 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 2*log(sin(d*x + c
)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.13203, size = 225, normalized size = 4.59

$$-\frac{2\left(dx+\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\cos\left(dx+c\right)}{2\,ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(d*x + cos(d*x + c))*sin(d*x + c) - log(1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*cos(d*x + c))/(a*
d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43979, size = 153, normalized size = 3.12

$$-\frac{\frac{6\,(dx+c)}{a}+\frac{6\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)/a + 6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*tan(1/2*d*x +
1/2*c)/a - (2*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 10*tan(1/
2*d*x + 1/2*c) - 3)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a))/d
```

3.415 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{x}{a}$$

```
[Out] x/a + ArcTanh[Cos[c + d*x]]/(2*a*d) + Cot[c + d*x]/(a*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.105861, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2611, 3770, 3473, 8}

$$\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + ArcTanh[Cos[c + d*x]]/(2*a*d) + Cot[c + d*x]/(a*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(2*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^2(c+dx)\,dx}{a} + \frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)}{ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\int \csc(c+dx)\,dx}{2a} + \frac{\int 1\,dx}{a}$$
$$= \frac{x}{a} + \frac{\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{\cot(c+dx)}{ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Mathematica [A] time = 0.461766, size = 102, normalized size = 1.76

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left((2\sin(c+dx)-1)\cos(c+dx)+\sin^2(c+dx)\left(-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\right.\right.\right.\right.}{8ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*((2*c + 2*d*x + Log[Cos[(c + d*x)/
2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^2 + Cos[c + d*x]*(-1 + 2*Sin[c +
d*x])))/(8*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.112, size = 112, normalized size = 1.9

$$\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da} - \frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2} + \frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/2/d/a*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/
2*d*x+1/2*c))-1/8/d/a/tan(1/2*d*x+1/2*c)^2+1/2/d/a/tan(1/2*d*x+1/2*c)-1/2/d
/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.6215, size = 186, normalized size = 3.21

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a} - \frac{16\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - 16*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 4*log(sin(d*x + c)/
(cos(d*x + c) + 1))/a - (4*sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(cos(d*x +
c) + 1)^2/(a*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.12652, size = 288, normalized size = 4.97

$$\frac{4\,dx\cos\left(dx+c\right)^{2}-4\,dx+\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\left(ad\cos\left(dx+c\right)^{2}-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x*cos(d*x + c)^2 - 4*d*x + (cos(d*x + c)^2 - 1)*log(1/2*cos(d*x +
c) + 1/2) - (cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 4*cos(d*x +
 c)*sin(d*x + c) + 2*cos(d*x + c))/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42191, size = 139, normalized size = 2.4

$$\frac{\frac{8\,(dx+c)}{a}-\frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}+\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/a - 4*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x +
1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c))/a^2 + (6*tan(1/2*d*x + 1/2*c)^2 + 4*ta
n(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)^2))/d
```

3.416 $\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc
[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.0884282, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc
[c + d*x])/(2*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int \csc(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(c+dx)\right)}{ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Mathematica [B] time = 0.475656, size = 124, normalized size = 2.14

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\cos(3(c+dx))+(3-6\sin(c+dx))\cos(c+dx\right.}{96ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2
*(Cos[3*(c + d*x)] + Cos[c + d*x]*(3 - 6*Sin[c + d*x]) + 6*(Log[Cos[(c + d*
x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3))/(96*a*d*(1 + Sin[c + d*x])
)
```

Maple [B] time = 0.116, size = 132, normalized size = 2.3

$$\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{1}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1} + \frac{1}{2\,da}\ln\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/8/d/a*tan(1/2*
d*x+1/2*c)+1/8/d/a/tan(1/2*d*x+1/2*c)+1/2/d/a*ln(tan(1/2*d*x+1/2*c))-1/24/d
/a/tan(1/2*d*x+1/2*c)^3+1/8/d/a/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.13181, size = 209, normalized size = 3.6

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a} - \frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(
d*x + c) + 1))/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(c
```

```
os(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [B] time = 1.11849, size = 311, normalized size = 5.36

$$\frac{4 \cos\left(dx+c\right)^{3} - 3 \left(\cos\left(dx+c\right)^{2} - 1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right) + \frac{1}{2}\right) \sin\left(dx+c\right) + 3 \left(\cos\left(dx+c\right)^{2} - 1\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right) + \right.}{12 \left(ad \cos\left(dx+c\right)^{2} - ad\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 6*cos(d*x + c)*sin(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x +
c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44607, size = 171, normalized size = 2.95

$$\frac{\frac{12 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a} + \frac{a^{2} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{3} - 3 a^{2} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} - 3 a^{2} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^{3}} - \frac{22 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{3} - 3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} - 3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1}{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{3}}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3
*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/2
*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a
*tan(1/2*d*x + 1/2*c)^3))/d
```

3.417 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(8*a*d) + Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc[
c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rubi [A] time = 0.147818, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2611, 3768, 3770, 2607, 30}

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(8*a*d) + Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc[
c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a}\\ &= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\int \csc^3(c+dx)\,dx}{4a}-\frac{\text{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\int \csc(c+dx)\,dx}{8a}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^3(c+}{4ad}\end{aligned}$$

Mathematica [A] time = 1.12517, size = 125, normalized size = 1.52

$$\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-42\cos(c+dx)+2(8\sin(c+dx)-3)\cos(3(c+dx))+24\left(\sin(2(c+\right.\right.}{192ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(-42*Cos[c + d*x] +
 2*Cos[3*(c + d*x)]*(-3 + 8*Sin[c + d*x]) + 24*((Log[Cos[(c + d*x)/2]] - Lo
g[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + Sin[2*(c + d*x)])))/(192*a*d*(1 + Sin
[c + d*x]))
```

Maple [A] time = 0.128, size = 132, normalized size = 1.6

$$\frac{1}{64da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{24da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{8da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{1}{64da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a*tan(1/2*d*x+1/2*c)^3+1/8/d/a*tan(1/2
*d*x+1/2*c)-1/8/d/a/tan(1/2*d*x+1/2*c)-1/64/d/a/tan(1/2*d*x+1/2*c)^4-1/8/d/
a*ln(tan(1/2*d*x+1/2*c))+1/24/d/a/tan(1/2*d*x+1/2*c)^3
```

Maxima [B] time = 1.12598, size = 208, normalized size = 2.54

$$\frac{\frac{\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a}-\frac{24\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/192*((24*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/a - 24*log(sin(d*x + c)/(c
os(d*x + c) + 1))/a + (8*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a*sin(d*x + c)^4))/d
```

Fricas [A] time = 1.10376, size = 365, normalized size = 4.45

$$\frac{16 \cos(dx+c)^3 \sin(dx+c) - 6 \cos(dx+c)^3 + 3\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - 3(c}{48\left(ad\cos(dx+c)^4 - 2\,ad\cos(dx+c)^2 + a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*cos(d*x + c)^3*sin(d*x + c) - 6*cos(d*x + c)^3 + 3*(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^4 -
2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c))/(a*d*c
os(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37647, size = 174, normalized size = 2.12

$$-\frac{\frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4} - \frac{50\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 8\tan\left(\frac{1}{2}dx+\right.}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(24*log(abs(tan(1/2*d*x + 1/2*c)))/a - (3*a^3*tan(1/2*d*x + 1/2*c)^4
 - 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c))/a^4 - (50*ta
n(1/2*d*x + 1/2*c)^4 - 24*tan(1/2*d*x + 1/2*c)^3 + 8*tan(1/2*d*x + 1/2*c) -
 3)/(a*tan(1/2*d*x + 1/2*c)^4))/d
```

3.418 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(8*a*d) - Cot[c + d*x]^3/(3*a*d) - Cot[c + d*x]^5/(5
*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]*Csc[c + d*x]^3)
/(4*a*d)
```

Rubi [A] time = 0.171977, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2607, 14, 2611, 3768, 3770}

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(8*a*d) - Cot[c + d*x]^3/(3*a*d) - Cot[c + d*x]^5/(5
*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]*Csc[c + d*x]^3)
/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a} + \frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{\int \csc^3(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, -\cot(c$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{\int \csc(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\right.$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{8ad} - \frac{\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad} - \frac{\cot(c+dx)\csc(c+dx)}{8ad} +$$

Mathematica [A] time = 0.585639, size = 189, normalized size = 1.89

$$-\frac{\csc^5(c+dx)\left(-180\sin(2(c+dx)) - 30\sin(4(c+dx)) + 320\cos(c+dx) + 80\cos(3(c+dx)) - 16\cos(5(c+dx)) - 15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^5*(320*Cos[c + d*x] + 80*Cos[3*(c + d*x)] - 16*Cos[5*(c + d*
x)] + 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[Sin[(c + d*x)/2]]*Si
n[c + d*x] - 180*Sin[2*(c + d*x)] - 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x
)] + 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*Sin[4*(c + d*x)] + 15*L
og[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[5*(c +
 d*x)]))/(1920*a*d)
```

Maple [A] time = 0.135, size = 170, normalized size = 1.7

$$\frac{1}{160\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + \frac{1}{96\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{1}{16\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{1}{16\,da}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a*tan(1/2*d*x+1/2*c)^4+1/96/d/a*tan(1
/2*d*x+1/2*c)^3-1/16/d/a*tan(1/2*d*x+1/2*c)+1/16/d/a/tan(1/2*d*x+1/2*c)-1/1
60/d/a/tan(1/2*d*x+1/2*c)^5+1/64/d/a/tan(1/2*d*x+1/2*c)^4+1/8/d/a*ln(tan(1/
2*d*x+1/2*c))-1/96/d/a/tan(1/2*d*x+1/2*c)^3
```

Maxima [B] time = 1.13519, size = 263, normalized size = 2.63

$$-\frac{\frac{\frac{60 \sin(dx+c)}{\cos(dx+c)+1}-\frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}-\frac{120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1}-\frac{10 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-6\right)(\cos(dx+c)+1)^5}{a \sin(dx+c)^5}}{960 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/960*((60*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5)/a - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (15*sin(d
*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 60*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1)^5/(a*sin(d*x + c)
^5))/d
```

Fricas [A] time = 1.10251, size = 452, normalized size = 4.52

$$\frac{32 \cos(dx+c)^5 - 80 \cos(dx+c)^3 - 15\left(\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 15\,(\cdots}{240\left(ad \cos(dx+c)^4 - 2\,ad \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(32*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(c
os(d*x + c)^3 + cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*co
s(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35326, size = 212, normalized size = 2.12

$$\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a}+\frac{6 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5-15 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+10 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-60 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^5}-\frac{274 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5-60 \tan\left(\frac{1}{2} dx+\cdots\right.}{a \cdots}}{960 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a + (6*a^4*tan(1/2*d*x + 1/2*c)^5
 - 15*a^4*tan(1/2*d*x + 1/2*c)^4 + 10*a^4*tan(1/2*d*x + 1/2*c)^3 - 60*a^4*t
an(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*d*x + 1/2*c)^5 - 60*tan(1/2*d*x + 1
/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 6)/(a*tan(1
/2*d*x + 1/2*c)^5))/d
```

3.419 $\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}+\frac{\cot(c+dx)}{1\ldots}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(16*a*d) + Cot[c + d*x]^3/(3*a*d) + Cot[c + d*x]^5/(5
*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(16*a*d) + (Cot[c + d*x]*Csc[c + d*x]^3
)/(24*a*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d)
```

Rubi [A] time = 0.180328, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2611, 3768, 3770, 2607, 14}

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}+\frac{\cot(c+dx)}{1\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(16*a*d) + Cot[c + d*x]^3/(3*a*d) + Cot[c + d*x]^5/(5
*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(16*a*d) + (Cot[c + d*x]*Csc[c + d*x]^3
)/(24*a*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a}$$

$$= -\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\int \csc^5(c+dx)\,dx}{6a}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,-\cot(\right.}{ad}$$

$$= \frac{\cot(c+dx)\csc^3(c+dx)}{24ad}-\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\int \csc^3(c+dx)\,dx}{8a}-\frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}+\frac{\cot(c+dx)\csc(c+dx)}{16ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}+\frac{\cot(c+dx)\csc(c+dx)}{16ad}+\frac{c}{}$$

Mathematica [A] time = 0.553884, size = 229, normalized size = 1.85

$$-\frac{\csc^6(c+dx)\left(-480\sin(2(c+dx))-192\sin(4(c+dx))+32\sin(6(c+dx))+1140\cos(c+dx)+170\cos(3(c+dx))-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^6*(1140*Cos[c + d*x] + 170*Cos[3*(c + d*x)] - 30*Cos[5*(c +
d*x)] - 150*Log[Cos[(c + d*x)/2]] + 225*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/
2]] - 90*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 15*Cos[6*(c + d*x)]*Log[C
os[(c + d*x)/2]] + 150*Log[Sin[(c + d*x)/2]] - 225*Cos[2*(c + d*x)]*Log[Sin
[(c + d*x)/2]] + 90*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 15*Cos[6*(c +
d*x)]*Log[Sin[(c + d*x)/2]] - 480*Sin[2*(c + d*x)] - 192*Sin[4*(c + d*x)] +
 32*Sin[6*(c + d*x)]))/(7680*a*d)
```

Maple [B] time = 0.15, size = 246, normalized size = 2.

$$\frac{1}{384\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6-\frac{1}{160\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{1}{128\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{96\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{128\,da}\left(\text{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/384/d/a*tan(1/2*d*x+1/2*c)^6-1/160/d/a*tan(1/2*d*x+1/2*c)^5+1/128/d/a*tan
(1/2*d*x+1/2*c)^4-1/96/d/a*tan(1/2*d*x+1/2*c)^3-1/128/d/a*tan(1/2*d*x+1/2*c
```

```
)^2+1/16/d/a*tan(1/2*d*x+1/2*c)-1/16/d/a/tan(1/2*d*x+1/2*c)+1/160/d/a/tan(1
/2*d*x+1/2*c)^5-1/128/d/a/tan(1/2*d*x+1/2*c)^4-1/16/d/a*ln(tan(1/2*d*x+1/2*
c))-1/384/d/a/tan(1/2*d*x+1/2*c)^6+1/96/d/a/tan(1/2*d*x+1/2*c)^3+1/128/d/a/
tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.11909, size = 370, normalized size = 2.98

$$\frac{\frac{\frac{120 \sin(dx+c)}{\cos(dx+c)+1}-\frac{15 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{20 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{12 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a}-\frac{120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{12 \sin(dx+c)}{\cos(dx+c)+1}-\frac{15 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{20 \sin(d}{(\cos(dx+}\right.}{}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1920*((120*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 12*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6)/a - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (
12*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 120*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c) + 1)^6
/(a*sin(d*x + c)^6))/d
```

Fricas [A] time = 1.13406, size = 513, normalized size = 4.14

$$-\frac{30 \cos(dx+c)^5 - 80 \cos(dx+c)^3 - 15\left(\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{480\left(ad\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/480*(30*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*(cos(d*x + c)^6 - 3*cos(
d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*
x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) +
 1/2) - 32*(2*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*sin(d*x + c) - 30*cos(d*x
+ c))/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43961, size = 292, normalized size = 2.35

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} - \frac{5\,a^5 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 12\,a^5 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 15\,a^5 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 20\,a^5 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 15\,a^5 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 120\,a^5 \mathrm{t}}{a^6}}{1920\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1920*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (5*a^5*tan(1/2*d*x + 1/2*c)
^6 - 12*a^5*tan(1/2*d*x + 1/2*c)^5 + 15*a^5*tan(1/2*d*x + 1/2*c)^4 - 20*a^5
*tan(1/2*d*x + 1/2*c)^3 - 15*a^5*tan(1/2*d*x + 1/2*c)^2 + 120*a^5*tan(1/2*d
*x + 1/2*c))/a^6 - (294*tan(1/2*d*x + 1/2*c)^6 - 120*tan(1/2*d*x + 1/2*c)^5
 + 15*tan(1/2*d*x + 1/2*c)^4 + 20*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x +
 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*c) - 5)/(a*tan(1/2*d*x + 1/2*c)^6))/d
```

3.420 $\int \frac{\cos^4(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^5(c+dx)\cos(c+dx)}{3a^2d}+\frac{5\sin^3(c+dx)\cos(c+dx)}{12a^2d}$$

```
[Out] (-5*x)/(8*a^2) - (2*Cos[c + d*x])/(a^2*d) + (5*Cos[c + d*x]^3)/(3*a^2*d) -
(4*Cos[c + d*x]^5)/(5*a^2*d) + Cos[c + d*x]^7/(7*a^2*d) + (5*Cos[c + d*x]*S
in[c + d*x])/(8*a^2*d) + (5*Cos[c + d*x]*Sin[c + d*x]^3)/(12*a^2*d) + (Cos[
c + d*x]*Sin[c + d*x]^5)/(3*a^2*d)
```

Rubi [A] time = 0.223546, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2633, 2635, 8}

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^5(c+dx)\cos(c+dx)}{3a^2d}+\frac{5\sin^3(c+dx)\cos(c+dx)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*x)/(8*a^2) - (2*Cos[c + d*x])/(a^2*d) + (5*Cos[c + d*x]^3)/(3*a^2*d) -
(4*Cos[c + d*x]^5)/(5*a^2*d) + Cos[c + d*x]^7/(7*a^2*d) + (5*Cos[c + d*x]*S
in[c + d*x])/(8*a^2*d) + (5*Cos[c + d*x]*Sin[c + d*x]^3)/(12*a^2*d) + (Cos[
c + d*x]*Sin[c + d*x]^5)/(3*a^2*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sin^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sin^5(c+dx)-2a^2\sin^6(c+dx)+a^2\sin^7(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \sin^5(c+dx)\,dx}{a^2}+\frac{\int \sin^7(c+dx)\,dx}{a^2}-\frac{2\int \sin^6(c+dx)\,dx}{a^2}$$
$$= \frac{\cos(c+dx)\sin^5(c+dx)}{3a^2d}-\frac{5\int \sin^4(c+dx)\,dx}{3a^2}-\frac{\text{Subst}\left(\int\left(1-2x^2+x^4\right)dx,x,\cos\right.}{a^2d}$$
$$= -\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{5\cos(c+dx)\,\mathrm{s}}{12a^2}$$
$$= -\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{5\cos(c+dx)\,\mathrm{s}}{8a^2d}$$
$$= -\frac{5x}{8a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{5\cos(c}{}$$

Mathematica [B] time = 4.70445, size = 418, normalized size = 2.84

$$\frac{-8400dx\sin\left(\frac{c}{2}\right)+7875\sin\left(\frac{c}{2}+dx\right)-7875\sin\left(\frac{3c}{2}+dx\right)+3150\sin\left(\frac{3c}{2}+2dx\right)+3150\sin\left(\frac{5c}{2}+2dx\right)-1435\sin\left(\frac{5c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-210*(1 + 40*d*x)*Cos[c/2] - 7875*Cos[c/2 + d*x] - 7875*Cos[(3*c)/2 + d*x]
 + 3150*Cos[(3*c)/2 + 2*d*x] - 3150*Cos[(5*c)/2 + 2*d*x] + 1435*Cos[(5*c)/2
 + 3*d*x] + 1435*Cos[(7*c)/2 + 3*d*x] - 630*Cos[(7*c)/2 + 4*d*x] + 630*Cos[
(9*c)/2 + 4*d*x] - 231*Cos[(9*c)/2 + 5*d*x] - 231*Cos[(11*c)/2 + 5*d*x] + 7
0*Cos[(11*c)/2 + 6*d*x] - 70*Cos[(13*c)/2 + 6*d*x] + 15*Cos[(13*c)/2 + 7*d*
x] + 15*Cos[(15*c)/2 + 7*d*x] + 210*Sin[c/2] - 8400*d*x*Sin[c/2] + 7875*Sin
[c/2 + d*x] - 7875*Sin[(3*c)/2 + d*x] + 3150*Sin[(3*c)/2 + 2*d*x] + 3150*Si
n[(5*c)/2 + 2*d*x] - 1435*Sin[(5*c)/2 + 3*d*x] + 1435*Sin[(7*c)/2 + 3*d*x]
- 630*Sin[(7*c)/2 + 4*d*x] - 630*Sin[(9*c)/2 + 4*d*x] + 231*Sin[(9*c)/2 + 5
*d*x] - 231*Sin[(11*c)/2 + 5*d*x] + 70*Sin[(11*c)/2 + 6*d*x] + 70*Sin[(13*c
)/2 + 6*d*x] - 15*Sin[(13*c)/2 + 7*d*x] + 15*Sin[(15*c)/2 + 7*d*x])/(13440*
a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.121, size = 381, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13-25/3/d/a^2/(1+t
an(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11-283/12/d/a^2/(1+tan(1/2*d*x+1/
```

```
2*c)^2)^7*tan(1/2*d*x+1/2*c)^9-32/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/
2*d*x+1/2*c)^8-176/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6+
283/12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5-208/5/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4+25/3/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^7*tan(1/2*d*x+1/2*c)^3-208/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1
/2*d*x+1/2*c)^2+5/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)-208
/105/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7-5/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.68619, size = 535, normalized size = 3.64

$$\frac{\frac{\frac{525\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{5824\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3500\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{17472\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9905\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{24640\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4480\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{9905\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3500\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{525\,\sin(dx+c}{(\cos(dx+c)+1}}{a^2+\frac{7\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35\,a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21\,a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7\,a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/420*((525*sin(d*x + c)/(cos(d*x + c) + 1) - 5824*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 3500*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 17472*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 9905*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 24640*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4480*sin(d*x + c)^8/(cos(d*x + c) + 1
)^8 - 9905*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3500*sin(d*x + c)^11/(cos(
d*x + c) + 1)^11 - 525*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 832)/(a^2 +
7*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 21*a^2*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 35*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 35*a^2*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 + 21*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 7*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + a^2*sin(d*x + c)^14/(cos(d*x
 + c) + 1)^14) - 525*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.11901, size = 248, normalized size = 1.69

$$\frac{120\,\cos(dx+c)^7-672\,\cos(dx+c)^5+1400\,\cos(dx+c)^3-525\,dx+35\left(8\,\cos(dx+c)^5-26\,\cos(dx+c)^3+33\,\cos(d\right.}{840\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/840*(120*cos(d*x + c)^7 - 672*cos(d*x + c)^5 + 1400*cos(d*x + c)^3 - 525*
d*x + 35*(8*cos(d*x + c)^5 - 26*cos(d*x + c)^3 + 33*cos(d*x + c))*sin(d*x +
 c) - 1680*cos(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30012, size = 224, normalized size = 1.52

$$-\frac{\frac{525\,(dx+c)}{a^2}+\frac{2\left(525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+3500\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+9905\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+24640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9905\,\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{7}a^2}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(525*(d*x + c)/a^2 + 2*(525*tan(1/2*d*x + 1/2*c)^13 + 3500*tan(1/2*d
*x + 1/2*c)^11 + 9905*tan(1/2*d*x + 1/2*c)^9 + 4480*tan(1/2*d*x + 1/2*c)^8
+ 24640*tan(1/2*d*x + 1/2*c)^6 - 9905*tan(1/2*d*x + 1/2*c)^5 + 17472*tan(1/
2*d*x + 1/2*c)^4 - 3500*tan(1/2*d*x + 1/2*c)^3 + 5824*tan(1/2*d*x + 1/2*c)^
2 - 525*tan(1/2*d*x + 1/2*c) + 832)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*a^2))/d
```

3.421 $\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=129

$$\frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^5(c+dx)\cos(c+dx)}{6a^2d}-\frac{11\sin^3(c+dx)\cos(c+dx)}{24a^2d}-\frac{11\sin(c+dx}{16}$$

```
[Out] (11*x)/(16*a^2) + (2*Cos[c + d*x])/(a^2*d) - (4*Cos[c + d*x]^3)/(3*a^2*d) +
 (2*Cos[c + d*x]^5)/(5*a^2*d) - (11*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d) -
 (11*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5
)/(6*a^2*d)
```

Rubi [A] time = 0.225079, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2635, 8, 2633}

$$\frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^5(c+dx)\cos(c+dx)}{6a^2d}-\frac{11\sin^3(c+dx)\cos(c+dx)}{24a^2d}-\frac{11\sin(c+dx}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (11*x)/(16*a^2) + (2*Cos[c + d*x])/(a^2*d) - (4*Cos[c + d*x]^3)/(3*a^2*d) +
 (2*Cos[c + d*x]^5)/(5*a^2*d) - (11*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d) -
 (11*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5
)/(6*a^2*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sin^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sin^4(c+dx)-2a^2\sin^5(c+dx)+a^2\sin^6(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \sin^4(c+dx)\,dx}{a^2}+\frac{\int \sin^6(c+dx)\,dx}{a^2}-\frac{2\int \sin^5(c+dx)\,dx}{a^2}$$
$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d}-\frac{\cos(c+dx)\sin^5(c+dx)}{6a^2d}+\frac{3\int \sin^2(c+dx)\,dx}{4a^2}+\frac{5\int s$$
$$= \frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{8a^2d}-\frac{11\,c$$
$$= \frac{3x}{8a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{11\cos(c+dx)\sin(c+dx)}{16a^2d}$$
$$= \frac{11x}{16a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{11\cos(c+dx)\sin(c+d}{16a^2d}$$

Mathematica [A] time = 0.247646, size = 76, normalized size = 0.59

$$\frac{-465\sin(2(c+dx))+75\sin(4(c+dx))-5\sin(6(c+dx))+1200\cos(c+dx)-200\cos(3(c+dx))+24\cos(5(c+dx)}{960a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (660*c + 660*d*x + 1200*Cos[c + d*x] - 200*Cos[3*(c + d*x)] + 24*Cos[5*(c +
 d*x)] - 465*Sin[2*(c + d*x)] + 75*Sin[4*(c + d*x)] - 5*Sin[6*(c + d*x)])/(
960*a^2*d)
```

Maple [B] time = 0.109, size = 347, normalized size = 2.7

$$\frac{11}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6}+\frac{187}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6}+\frac{47}{4\,da^2}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 11/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+187/24/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+47/4/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^6*tan(1/2*d*x+1/2*c)^7+64/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2
*d*x+1/2*c)^6-47/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+32
/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4-187/24/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+64/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^2-11/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)+32/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6+11/8/d/a^2*arctan(tan(1/2*d*x
+1/2*c))
```

Maxima [B] time = 1.6535, size = 477, normalized size = 3.7

$$-\frac{\frac{\frac{165\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{1536\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{935\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3840\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1410\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2560\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{1410\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{935\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{165\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-256}{a^2+\frac{6\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\,a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6\,a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{165\,a}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 1536*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 935*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3840*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 1410*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2560*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1410*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 - 935*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 165*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 256)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a
^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*sin(d*x + c)^
10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 165
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.15177, size = 215, normalized size = 1.67

$$\frac{96\,\cos(dx+c)^5-320\,\cos(dx+c)^3+165\,dx-5\left(8\,\cos(dx+c)^5-38\,\cos(dx+c)^3+63\,\cos(dx+c)\right)\sin(dx+c)+48}{240\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/240*(96*cos(d*x + c)^5 - 320*cos(d*x + c)^3 + 165*d*x - 5*(8*cos(d*x + c)
^5 - 38*cos(d*x + c)^3 + 63*cos(d*x + c))*sin(d*x + c) + 480*cos(d*x + c))/
(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31003, size = 207, normalized size = 1.6

$$\frac{\frac{165\,(dx+c)}{a^2}+\frac{2\left(165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+935\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+1410\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+2560\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1410\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3840\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-93\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(165*(d*x + c)/a^2 + 2*(165*tan(1/2*d*x + 1/2*c)^11 + 935*tan(1/2*d*x
 + 1/2*c)^9 + 1410*tan(1/2*d*x + 1/2*c)^7 + 2560*tan(1/2*d*x + 1/2*c)^6 - 1
410*tan(1/2*d*x + 1/2*c)^5 + 3840*tan(1/2*d*x + 1/2*c)^4 - 935*tan(1/2*d*x
+ 1/2*c)^3 + 1536*tan(1/2*d*x + 1/2*c)^2 - 165*tan(1/2*d*x + 1/2*c) + 256)/
((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^2))/d
```

3.422 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{3x}{4a^2}$$

```
[Out] (-3*x)/(4*a^2) - (2*Cos[c + d*x])/(a^2*d) + Cos[c + d*x]^3/(a^2*d) - Cos[c
+ d*x]^5/(5*a^2*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*d) + (Cos[c + d*x
]*Sin[c + d*x]^3)/(2*a^2*d)
```

Rubi [A] time = 0.199063, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2633, 2635, 8}

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{3x}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(4*a^2) - (2*Cos[c + d*x])/(a^2*d) + Cos[c + d*x]^3/(a^2*d) - Cos[c
+ d*x]^5/(5*a^2*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*d) + (Cos[c + d*x
]*Sin[c + d*x]^3)/(2*a^2*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sin^3(c+dx)-2a^2\sin^4(c+dx)+a^2\sin^5(c+dx)\right)\,dx}{a^4}$$
$$= \frac{\int \sin^3(c+dx)\,dx}{a^2}+\frac{\int \sin^5(c+dx)\,dx}{a^2}-\frac{2\int \sin^4(c+dx)\,dx}{a^2}$$
$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2a^2d}-\frac{3\int \sin^2(c+dx)\,dx}{2a^2}-\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(c+d\right.}{a^2d}$$
$$= -\frac{2\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{5a^2d}+\frac{3\cos(c+dx)\sin(c+dx)}{4a^2d}+\cos(c$$
$$= -\frac{3x}{4a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{5a^2d}+\frac{3\cos(c+dx)\sin(c+dx)}{4a^2d}+$$

Mathematica [B] time = 1.41871, size = 308, normalized size = 3.02

$$-\frac{120dx\sin\left(\frac{c}{2}\right)-110\sin\left(\frac{c}{2}+dx\right)+110\sin\left(\frac{3c}{2}+dx\right)-40\sin\left(\frac{3c}{2}+2dx\right)-40\sin\left(\frac{5c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+3dx\right)-1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(5*(1 + 24*d*x)*Cos[c/2] + 110*Cos[c/2 + d*x] + 110*Cos[(3*c)/2 + d*x] - 4
0*Cos[(3*c)/2 + 2*d*x] + 40*Cos[(5*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 3*d*x]
- 15*Cos[(7*c)/2 + 3*d*x] + 5*Cos[(7*c)/2 + 4*d*x] - 5*Cos[(9*c)/2 + 4*d*x]
 + Cos[(9*c)/2 + 5*d*x] + Cos[(11*c)/2 + 5*d*x] - 5*Sin[c/2] + 120*d*x*Sin[
c/2] - 110*Sin[c/2 + d*x] + 110*Sin[(3*c)/2 + d*x] - 40*Sin[(3*c)/2 + 2*d*x
] - 40*Sin[(5*c)/2 + 2*d*x] + 15*Sin[(5*c)/2 + 3*d*x] - 15*Sin[(7*c)/2 + 3*
d*x] + 5*Sin[(7*c)/2 + 4*d*x] + 5*Sin[(9*c)/2 + 4*d*x] - Sin[(9*c)/2 + 5*d*
x] + Sin[(11*c)/2 + 5*d*x])/(160*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.099, size = 279, normalized size = 2.7

$$-\frac{3}{2\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5}-7\,\frac{(\tan(1/2\,dx+c/2))^7}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^5}-4\,\frac{(\tan(1/2\,dx+c/2))^6}{da^2\left(1+(\tan(1/2\,dx+c/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9-7/d/a^2/(1+tan(1
/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^6-20/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)
^4+7/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3-12/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2+3/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)-12/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5-3/2/d/a^2*arcta
n(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.72114, size = 392, normalized size = 3.84

$$\frac{\frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1}-\frac{120 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{70 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{200 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{40 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{70 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}-24}{a^2+\frac{5a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/10*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 120*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 70*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 200*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 40*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 70*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 15*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 24)/(a^2
 + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 10*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x
 + c)^8/(cos(d*x + c) + 1)^8 + a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) -
 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.12741, size = 181, normalized size = 1.77

$$-\frac{4 \cos(dx+c)^5-20 \cos(dx+c)^3+15\,dx+5\left(2 \cos(dx+c)^3-5 \cos(dx+c)\right)\sin(dx+c)+40 \cos(dx+c)}{20\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20*(4*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 15*d*x + 5*(2*cos(d*x + c)^3
- 5*cos(d*x + c))*sin(d*x + c) + 40*cos(d*x + c))/(a^2*d)
```

Sympy [A] time = 127.725, size = 1836, normalized size = 18.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-315*d*x*tan(c/2 + d*x/2)**10/(420*a**2*d*tan(c/2 + d*x/2)**10 +
 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a
**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) -
 1575*d*x*tan(c/2 + d*x/2)**8/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*
d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c
/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 3150*d*x*t
an(c/2 + d*x/2)**6/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 +
 d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)
**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 3150*d*x*tan(c/2 + d*
x/2)**4/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8
+ 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*
a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 1575*d*x*tan(c/2 + d*x/2)**2/(42
```

```
0*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2
*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(
c/2 + d*x/2)**2 + 420*a**2*d) - 315*d*x/(420*a**2*d*tan(c/2 + d*x/2)**10 +
2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a*
*2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) +
355*tan(c/2 + d*x/2)**10/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan
(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 +
d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 630*tan(c/2 + d
*x/2)**9/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8
 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100
*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) + 1775*tan(c/2 + d*x/2)**8/(420*a
**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*
tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2
 + d*x/2)**2 + 420*a**2*d) - 2940*tan(c/2 + d*x/2)**7/(420*a**2*d*tan(c/2 +
 d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2
)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 +
420*a**2*d) + 1870*tan(c/2 + d*x/2)**6/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2
100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**
2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 4
850*tan(c/2 + d*x/2)**4/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(
c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d
*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) + 2940*tan(c/2 + d
*x/2)**3/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8
 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100
*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 3265*tan(c/2 + d*x/2)**2/(420*a
**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*
tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2
 + d*x/2)**2 + 420*a**2*d) + 630*tan(c/2 + d*x/2)/(420*a**2*d*tan(c/2 + d*x
/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6
 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*
a**2*d) - 653/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/
2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 +
 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d), Ne(d, 0)), (x*sin(c)**3*cos
(c)**4/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.35405, size = 171, normalized size = 1.68

$$-\frac{\frac{15\,(dx+c)}{a^2}+\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+70\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+40\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+200\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\tan(\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5a^2}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(15*(d*x + c)/a^2 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 70*tan(1/2*d*x + 1
/2*c)^7 + 40*tan(1/2*d*x + 1/2*c)^6 + 200*tan(1/2*d*x + 1/2*c)^4 - 70*tan(1
/2*d*x + 1/2*c)^3 + 120*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) +
24)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^2))/d
```

3.423 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=87

$$-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}-\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

```
[Out] (7*x)/(8*a^2) + (2*Cos[c + d*x])/(a^2*d) - (2*Cos[c + d*x]^3)/(3*a^2*d) - (
7*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a
^2*d)
```

Rubi [A] time = 0.197573, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2635, 8, 2633}

$$-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}-\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*x)/(8*a^2) + (2*Cos[c + d*x])/(a^2*d) - (2*Cos[c + d*x]^3)/(3*a^2*d) - (
7*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a
^2*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

&& IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sin^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\sin^2(c+dx)-2a^2\sin^3(c+dx)+a^2\sin^4(c+dx)\right)dx}{a^4}\\
&= \frac{\int \sin^2(c+dx)\,dx}{a^2}+\frac{\int \sin^4(c+dx)\,dx}{a^2}-\frac{2\int \sin^3(c+dx)\,dx}{a^2}\\
&= -\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d}+\frac{\int 1\,dx}{2a^2}+\frac{3\int \sin^2(c+dx)\,dx}{4a^2}\\
&= \frac{x}{2a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}-\frac{\cos(c+dx)\sin}{4a^2d}\\
&= \frac{7x}{8a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}-\frac{\cos(c+dx)\sin}{4a^2d}
\end{aligned}$$

Mathematica [B] time = 1.36355, size = 258, normalized size = 2.97

$$\frac{168dx\sin\left(\frac{c}{2}\right)-144\sin\left(\frac{c}{2}+dx\right)+144\sin\left(\frac{3c}{2}+dx\right)-48\sin\left(\frac{3c}{2}+2dx\right)-48\sin\left(\frac{5c}{2}+2dx\right)+16\sin\left(\frac{5c}{2}+3dx\right)-16}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (168*d*x*Cos[c/2] + 144*Cos[c/2 + d*x] + 144*Cos[(3*c)/2 + d*x] - 48*Cos[(3
*c)/2 + 2*d*x] + 48*Cos[(5*c)/2 + 2*d*x] - 16*Cos[(5*c)/2 + 3*d*x] - 16*Cos
[(7*c)/2 + 3*d*x] + 3*Cos[(7*c)/2 + 4*d*x] - 3*Cos[(9*c)/2 + 4*d*x] + 8*Sin
[c/2] + 168*d*x*Sin[c/2] - 144*Sin[c/2 + d*x] + 144*Sin[(3*c)/2 + d*x] - 48
*Sin[(3*c)/2 + 2*d*x] - 48*Sin[(5*c)/2 + 2*d*x] + 16*Sin[(5*c)/2 + 3*d*x] -
 16*Sin[(7*c)/2 + 3*d*x] + 3*Sin[(7*c)/2 + 4*d*x] + 3*Sin[(9*c)/2 + 4*d*x])
/(192*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.089, size = 245, normalized size = 2.8

$$\frac{7}{4da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}+\frac{15}{4da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}+8\frac{(\tan(1}{da^2\left(1+(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 7/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+15/4/d/a^2/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5+8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^4-15/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^3+32/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2-7/4/d/a^2
/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+8/3/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^4+7/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.66931, size = 333, normalized size = 3.83

$$-\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{128\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{45\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{96\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{45\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{21\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-32}{a^2+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{21\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) - 128*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 45*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 96*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 45*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 21*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 32)/(a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 21*arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.07199, size = 155, normalized size = 1.78

$$-\frac{16\cos(dx+c)^3-21\,dx-3\left(2\cos(dx+c)^3-9\cos(dx+c)\right)\sin(dx+c)-48\cos(dx+c)}{24\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(16*cos(d*x + c)^3 - 21*d*x - 3*(2*cos(d*x + c)^3 - 9*cos(d*x + c))*s
in(d*x + c) - 48*cos(d*x + c))/(a^2*d)
```

Sympy [A] time = 77.6587, size = 1343, normalized size = 15.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((21*d*x*tan(c/2 + d*x/2)**8/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a
**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(
c/2 + d*x/2)**2 + 24*a**2*d) + 84*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/
2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)
**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 126*d*x*tan(c/2 + d*x/2)
**4/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a*
*2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 84*
d*x*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2
+ d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**
2 + 24*a**2*d) + 21*d*x/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2
+ d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**
```

```
2 + 24*a**2*d) - 14*tan(c/2 + d*x/2)**8/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96
*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*ta
n(c/2 + d*x/2)**2 + 24*a**2*d) + 42*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2
+ d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**
4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 56*tan(c/2 + d*x/2)**6/(24
*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*ta
n(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 90*tan(c/2
 + d*x/2)**5/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6
 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2
*d) + 108*tan(c/2 + d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*ta
n(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*
x/2)**2 + 24*a**2*d) - 90*tan(c/2 + d*x/2)**3/(24*a**2*d*tan(c/2 + d*x/2)**
8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**
2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 200*tan(c/2 + d*x/2)**2/(24*a**2*d*t
an(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d
*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 42*tan(c/2 + d*x/2)
/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*
d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 50/(24
*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*ta
n(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d), Ne(d, 0)),
(x*sin(c)**2*cos(c)**4/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.38077, size = 154, normalized size = 1.77

$$\frac{\frac{21\,(dx+c)}{a^2}+\frac{2\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+96\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+128\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+32\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*(d*x + c)/a^2 + 2*(21*tan(1/2*d*x + 1/2*c)^7 + 45*tan(1/2*d*x + 1/
2*c)^5 + 96*tan(1/2*d*x + 1/2*c)^4 - 45*tan(1/2*d*x + 1/2*c)^3 + 128*tan(1/
2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x + 1/2*c) + 32)/((tan(1/2*d*x + 1/2*c)^2 +
 1)^4*a^2))/d
```

3.424 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{x}{a^2}-\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^2}$$

```
[Out] -(x/a^2) - (2*Cos[c + d*x]^3)/(3*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(a^2*
d) - Cos[c + d*x]^5/(d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.109522, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2859, 2682, 2635, 8}

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{x}{a^2}-\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cos[c + d*x]^3)/(3*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(a^2*
d) - Cos[c + d*x]^5/(d*(a + a*Sin[c + d*x])^2)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2} - \frac{2\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{a}$$
$$= -\frac{2\cos^3(c+dx)}{3a^2d} - \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2} - \frac{2\int \cos^2(c+dx)\,dx}{a^2}$$
$$= -\frac{2\cos^3(c+dx)}{3a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2} - \frac{\int 1\,dx}{a^2}$$
$$= -\frac{x}{a^2} - \frac{2\cos^3(c+dx)}{3a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2}$$

Mathematica [B] time = 0.747531, size = 204, normalized size = 2.91

$$\frac{-24dx\sin\left(\frac{c}{2}\right) + 21\sin\left(\frac{c}{2}+dx\right) - 21\sin\left(\frac{3c}{2}+dx\right) + 6\sin\left(\frac{3c}{2}+2dx\right) + 6\sin\left(\frac{5c}{2}+2dx\right) - \sin\left(\frac{5c}{2}+3dx\right) + \sin\left(\frac{7c}{2}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(1 + 12*d*x)*Cos[c/2] - 21*Cos[c/2 + d*x] - 21*Cos[(3*c)/2 + d*x] + 6*C
os[(3*c)/2 + 2*d*x] - 6*Cos[(5*c)/2 + 2*d*x] + Cos[(5*c)/2 + 3*d*x] + Cos[(
7*c)/2 + 3*d*x] + 2*Sin[c/2] - 24*d*x*Sin[c/2] + 21*Sin[c/2 + d*x] - 21*Sin
[(3*c)/2 + d*x] + 6*Sin[(3*c)/2 + 2*d*x] + 6*Sin[(5*c)/2 + 2*d*x] - Sin[(5*
c)/2 + 3*d*x] + Sin[(7*c)/2 + 3*d*x])/(24*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.089, size = 177, normalized size = 2.5

$$-2\,{\frac{(\tan(1/2\,dx+c/2))^5}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}}-2\,{\frac{(\tan(1/2\,dx+c/2))^4}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}}-8\,{\frac{(\tan(1/2\,dx+c/2))^2}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}}+2\,\frac{}{da^2\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5-2/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)^2+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-10
/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.67655, size = 248, normalized size = 3.54

$$\frac{2\left(\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-5}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/3*((3*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - 5)/(a^2 + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.09594, size = 115, normalized size = 1.64

$$\frac{\cos\left(dx+c\right)^{3}-3\,dx+3\,\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,\cos\left(dx+c\right)}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 - 3*d*x + 3*cos(d*x + c)*sin(d*x + c) - 6*cos(d*x + c))
/(a^2*d)
```

Sympy [A] time = 44.2726, size = 694, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**
2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 9*d*x*
tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 9*d*x*tan(c/2 + d*x/2)**
2/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*t
an(c/2 + d*x/2)**2 + 3*a**2*d) - 3*d*x/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a*
*2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 6*tan
(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)*
*4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 6*tan(c/2 + d*x/2)**4/(3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
+ d*x/2)**2 + 3*a**2*d) - 24*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)
**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*
d) + 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 +
d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 10/(3*a**2*d*tan(c/2
 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2
+ 3*a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)**4/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.33507, size = 119, normalized size = 1.7

$$-\frac{\frac{3\,(dx+c)}{a^{2}}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a^{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 3*tan(1/2*d*x + 1/2*c
)^4 + 12*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 5)/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^3*a^2))/d
```

3.425 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2x}{a^2}$$

```
[Out] (-2*x)/a^2 - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cos[c + d*x]/(a^2*d)
```

Rubi [A] time = 0.129302, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2869, 2746, 2735, 3770}

$$-\frac{\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cos[c + d*x]/(a^2*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= -\frac{\cos(c+dx)}{a^2d} + \frac{\int \csc(c+dx)\left(a^2-2a^2\sin(c+dx)\right)dx}{a^4}$$
$$= -\frac{2x}{a^2} - \frac{\cos(c+dx)}{a^2d} + \frac{\int \csc(c+dx)\,dx}{a^2}$$
$$= -\frac{2x}{a^2} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{a^2d}$$

Mathematica [A] time = 0.133405, size = 46, normalized size = 1.28

$$-\frac{\cos(c+dx)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2c+2dx}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2*c + 2*d*x + Cos[c + d*x] + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2
]])/(a^2*d))
```

Maple [A] time = 0.121, size = 60, normalized size = 1.7

$$-2\,\frac{1}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)}-4\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^2}+\frac{1}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))+1/d/a^
2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.73861, size = 111, normalized size = 3.08

$$-\frac{\frac{2}{a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2/(a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 4*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2 - log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.15881, size = 135, normalized size = 3.75

$$-\frac{4\,dx+2\,\cos\left(dx+c\right)+\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d*x + 2*cos(d*x + c) + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d
*x + c) + 1/2))/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos^{4}\left(c+dx\right)\csc\left(c+dx\right)}{\sin^{2}\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A] time = 1.31897, size = 70, normalized size = 1.94

$$-\frac{\frac{2\,(dx+c)}{a^{2}}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(d*x + c)/a^2 - log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 2/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^2))/d
```

3.426 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=35

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{x}{a^2}$$

```
[Out] x/a^2 + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d)
```

Rubi [A] time = 0.148947, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 3770, 3767, 8}

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] x/a^2 + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2-2a^2\csc(c+dx)+a^2\csc^2(c+dx)\right)dx}{a^4}$$
$$= \frac{x}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}$$
$$= \frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^2d}$$
$$= \frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}$$

Mathematica [B] time = 0.356838, size = 98, normalized size = 2.8

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(2(c+dx)+\tan\left(\frac{1}{2}(c+dx)\right)-\cot\left(\frac{1}{2}(c+dx)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\mathrm{co}\right.\right.}{2d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(2*(c + d*x) - Cot[(c + d*x)/2] +
4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] + Tan[(c + d*x)/2]))/(2*d
*(a + a*Sin[c + d*x])^2)
```

Maple [B] time = 0.138, size = 74, normalized size = 2.1

$$\frac{1}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2}-\frac{1}{2\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-2\,\frac{\ln\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/2/d/a^2/t
an(1/2*d*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.70309, size = 126, normalized size = 3.6

$$\frac{\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\cos(dx+c)+1}{a^2\sin(dx+c)}+\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 4*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2 - (cos(d*x + c) + 1)/(a^2*sin(d*x + c)) + sin(d*x + c)
```

```
/(a^2*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.15895, size = 193, normalized size = 5.51

$$\frac{dx\sin(dx+c)+\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-\cos(dx+c)}{a^2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (d*x*sin(d*x + c) + log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos
(d*x + c) + 1/2)*sin(d*x + c) - cos(d*x + c))/(a^2*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37281, size = 99, normalized size = 2.83

$$\frac{\frac{2(dx+c)}{a^2}-\frac{4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}+\frac{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a^2 - 4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + tan(1/2*d*x +
 1/2*c)/a^2 + (4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)))/d
```

3.427 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=54

$$\frac{2\cot(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rubi [A] time = 0.151406, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2869, 2757, 3770, 3767, 8, 3768}

$$\frac{2\cot(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*d)
```


Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\csc(c+dx)-2a^2\csc^2(c+dx)+a^2\csc^3(c+dx)\right)\,dx}{a^4}\\
&= \frac{\int \csc(c+dx)\,dx}{a^2}+\frac{\int \csc^3(c+dx)\,dx}{a^2}-\frac{2\int \csc^2(c+dx)\,dx}{a^2}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{\int \csc(c+dx)\,dx}{2a^2}+\frac{2\,\mathrm{Subst}(\int 1
\end{aligned}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Mathematica [A] time = 0.521196, size = 86, normalized size = 1.59

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(\cot(c+dx)(\csc(c+dx)-4)+3\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{2a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Cot[c + d*x]*(-4 + Csc[c + d*x]) + 3*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c
 + d*x)/2]]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(2*a^2*d*(1 + Sin[c
+ d*x])^2)
```

Maple [A] time = 0.158, size = 93, normalized size = 1.7

$$\frac{1}{8da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}+\frac{3}{2da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{8da^2}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)+1/d/a^2/tan(1/2*d
*x+1/2*c)+3/2/d/a^2*ln(tan(1/2*d*x+1/2*c))-1/8/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.13041, size = 155, normalized size = 2.87

$$-\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a^2\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*((8*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (8*sin(d*x + c)/(c
os(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)^2/(a^2*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.07754, size = 258, normalized size = 4.78

$$-\frac{3\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+8\cos\left(dx+c\right)\sin\left(dx\cdots\right.}{4\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/4*(3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^
2 - 1)*log(-1/2*cos(d*x + c) + 1/2) + 8*cos(d*x + c)*sin(d*x + c) - 2*cos(d
*x + c))/(a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37688, size = 132, normalized size = 2.44

$$\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{2}}+\frac{a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-8a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{4}}-\frac{18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 -
8*a^2*tan(1/2*d*x + 1/2*c))/a^4 - (18*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*
x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/2*c)^2))/d
```

3.428 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(a^2*d) - (2*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(
3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d)
```

Rubi [A] time = 0.126169, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2708, 2757, 3767, 8, 3768, 3770}

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a^2*d) - (2*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(
3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d)
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\csc^2(c+dx)-2a^2\csc^3(c+dx)+a^2\csc^4(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \csc^2(c+dx)\,dx}{a^2}+\frac{\int \csc^4(c+dx)\,dx}{a^2}-\frac{2\int \csc^3(c+dx)\,dx}{a^2}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{\int \csc(c+dx)\,dx}{a^2}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^2d}-\frac{\text{Subst}\left(\int\left(1+\right.\right.}{}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

Mathematica [A] time = 0.909655, size = 121, normalized size = 1.83

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^4\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-9\cos(c+dx)+5\cos(3(c+dx))+6\left(\sin(2(c+dx))+2\sin^3(c\right.\right.}{96a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((1 + Cot[(c + d*x)/2])^4*Sec[(c + d*x)/2]^2*(-9*Cos[c + d*x] + 5*Cos[3*(c
+ d*x)] + 6*(2*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]
^3 + Sin[2*(c + d*x)]))*Tan[(c + d*x)/2])/(96*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [B] time = 0.164, size = 132, normalized size = 2.

$$\frac{1}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{7}{8\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{1}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^2*tan(1/2*d*x+1/2*c)^2+7/8/d/a^2*ta
n(1/2*d*x+1/2*c)-7/8/d/a^2/tan(1/2*d*x+1/2*c)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)
)-1/24/d/a^2/tan(1/2*d*x+1/2*c)^3+1/4/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.12086, size = 207, normalized size = 3.14

$$\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((21*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2 + (6*sin(d*x + c)/(cos(d*x + c) + 1) - 21*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.14882, size = 342, normalized size = 5.18

$$-\frac{10\cos(dx+c)^3-3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx\right.}{6\left(a^2d\cos(dx+c)^2-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(10*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 6*cos(d*x + c)*sin(d*x + c) - 12*cos(d*x + c))/((a^2*d*cos(d*x + c
)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37458, size = 173, normalized size = 2.62

$$-\frac{\frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{44\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+21a^4\tan\left(\frac{1}{2}dx+\right.}{a^6}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(24*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (44*tan(1/2*d*x + 1/2*c)^3 -
 21*tan(1/2*d*x + 1/2*c)^2 + 6*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x +
 1/2*c)^3) - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^4*tan(1/2*d*x + 1/2*c)^2 + 2
1*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.429 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=96

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{7\cot(c+dx)\csc(c+dx)}{8a^2d}$$

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(8*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (2*Cot[c
+ d*x]^3)/(3*a^2*d) - (7*Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rubi [A] time = 0.187241, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2869, 2757, 3768, 3770, 3767}

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{7\cot(c+dx)\csc(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(8*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (2*Cot[c
+ d*x]^3)/(3*a^2*d) - (7*Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\csc^3(c+dx)-2a^2\csc^4(c+dx)+a^2\csc^5(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \csc^3(c+dx)\,dx}{a^2}+\frac{\int \csc^5(c+dx)\,dx}{a^2}-\frac{2\int \csc^4(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{\int \csc(c+dx)\,dx}{2a^2}+\frac{3\int \csc^3(}{4}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc(c+dx)}{8a^2d}$$
$$= -\frac{7\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc(c+d}{8a^2d}$$

Mathematica [A] time = 1.46709, size = 116, normalized size = 1.21

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-48\sin(2(c+dx))+45\cos(c+dx)+(32\sin(c+dx)-21)\cos(3(c+dx))+8\right.}{1536a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^4*(45*Cos[c + d*x] + 84*(Log[Cos[(c
 + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + Cos[3*(c + d*x)]*(-21
 + 32*Sin[c + d*x]) - 48*Sin[2*(c + d*x)]))/(1536*a^2*d*(1 + Sin[c + d*x])^
2)
```

Maple [A] time = 0.171, size = 170, normalized size = 1.8

$$\frac{1}{64\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{12\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{3}{4\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{3}{4\,da^2}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-1/12/d/a^2*tan(1/2*d*x+1/2*c)^3+1/4/d/a^2*t
an(1/2*d*x+1/2*c)^2-3/4/d/a^2*tan(1/2*d*x+1/2*c)+3/4/d/a^2/tan(1/2*d*x+1/2*
c)-1/64/d/a^2/tan(1/2*d*x+1/2*c)^4+7/8/d/a^2*ln(tan(1/2*d*x+1/2*c))+1/12/d/
a^2/tan(1/2*d*x+1/2*c)^3-1/4/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.11237, size = 263, normalized size = 2.74

$$-\frac{\frac{\frac{144\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^2}-\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{144\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a^2\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/192*((144*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4)/a^2 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (16*
sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
144*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a^2*sin(
d*x + c)^4))/d
```

Fricas [A] time = 1.12775, size = 406, normalized size = 4.23

$$\frac{42\cos(dx+c)^3-21\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+21\left(\cos(dx+c)^4-2\cos(dx+c)^2\cdots\right.}{48\left(a^2d\cos(dx+c)^4-2a^2d\cos(d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(42*cos(d*x + c)^3 - 21*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/
2*cos(d*x + c) + 1/2) + 21*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2
*cos(d*x + c) + 1/2) - 32*(2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c)
- 54*cos(d*x + c))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35478, size = 212, normalized size = 2.21

$$\frac{\frac{168\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{350\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-144\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+48\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\frac{3a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-16a^6\tan\left(\frac{1}{2}dx+\cdots\right.}{\cdots}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(168*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (350*tan(1/2*d*x + 1/2*c)^4
 - 144*tan(1/2*d*x + 1/2*c)^3 + 48*tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x
+ 1/2*c) + 3)/(a^2*tan(1/2*d*x + 1/2*c)^4) + (3*a^6*tan(1/2*d*x + 1/2*c)^4
```

```
- 16*a^6*tan(1/2*d*x + 1/2*c)^3 + 48*a^6*tan(1/2*d*x + 1/2*c)^2 - 144*a^6*t
an(1/2*d*x + 1/2*c))/a^8)/d
```

3.430 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}+\frac{3\cot(c+dx)\csc(c}{4a^2d}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(4*a^2*d) - (2*Cot[c + d*x])/(a^2*d) - Cot[c + d*
x]^3/(a^2*d) - Cot[c + d*x]^5/(5*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(4*
a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(2*a^2*d)
```

Rubi [A] time = 0.240746, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 3767, 3768, 3770}

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}+\frac{3\cot(c+dx)\csc(c}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (3*ArcTanh[Cos[c + d*x]])/(4*a^2*d) - (2*Cot[c + d*x])/(a^2*d) - Cot[c + d*
x]^3/(a^2*d) - Cot[c + d*x]^5/(5*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(4*
a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(2*a^2*d)
```


Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\csc^4(c+dx)-2a^2\csc^5(c+dx)+a^2\csc^6(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \csc^4(c+dx)\,dx}{a^2}+\frac{\int \csc^6(c+dx)\,dx}{a^2}-\frac{2\int \csc^5(c+dx)\,dx}{a^2}$$
$$= \frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}-\frac{3\int \csc^3(c+dx)\,dx}{2a^2}-\frac{\text{Subst}\left(\int \left(1+x^2\right)dx,x,\cot(c+ \ldots}{a^2d}$$
$$= -\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot(c \ldots}{}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}-\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{3\cot(c+ \ldots}{}$$

Mathematica [A] time = 0.736342, size = 189, normalized size = 1.69

$$\frac{\csc^5(c+dx)\left(140\sin(2(c+dx))-30\sin(4(c+dx))-160\cos(c+dx)+120\cos(3(c+dx))-24\cos(5(c+dx))-150 \ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-160*Cos[c + d*x] + 120*Cos[3*(c + d*x)] - 24*Cos[5*(c + d
*x)] + 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[Sin[(c + d*x)/2]]*S
in[c + d*x] + 140*Sin[2*(c + d*x)] - 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*
x)] + 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*Sin[4*(c + d*x)] + 15*
Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[5*(c
+ d*x)]))/(320*a^2*d)
```

Maple [A] time = 0.183, size = 208, normalized size = 1.9

$$\frac{1}{160\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{32\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+\frac{3}{32\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{11}{16\,da^2}\ \text{ta}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/160/d/a^2*tan(1/2*d*x+1/2*c)^5-1/32/d/a^2*tan(1/2*d*x+1/2*c)^4+3/32/d/a^2
*tan(1/2*d*x+1/2*c)^3-1/4/d/a^2*tan(1/2*d*x+1/2*c)^2+11/16/d/a^2*tan(1/2*d*
x+1/2*c)-11/16/d/a^2/tan(1/2*d*x+1/2*c)-1/160/d/a^2/tan(1/2*d*x+1/2*c)^5+1/
32/d/a^2/tan(1/2*d*x+1/2*c)^4-3/4/d/a^2*ln(tan(1/2*d*x+1/2*c))-3/32/d/a^2/t
an(1/2*d*x+1/2*c)^3+1/4/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.0929, size = 315, normalized size = 2.81

$$\frac{\frac{\frac{110\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{110\sin(\ldots}{(\cos(dx+\ldots}\right.}{a^2\sin(dx+c)^5}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/160*((110*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 15*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 120*log(sin(d*
x + c)/(cos(d*x + c) + 1))/a^2 + (5*sin(d*x + c)/(cos(d*x + c) + 1) - 15*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- 110*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)*(cos(d*x + c) + 1)^5/(a^2*si
n(d*x + c)^5))/d
```

Fricas [A] time = 1.15899, size = 491, normalized size = 4.38

$$-\frac{48\cos(dx+c)^5-120\cos(dx+c)^3-15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+15\ldots}{40\left(a^2d\cos(dx+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/40*(48*cos(d*x + c)^5 - 120*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(
3*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(d*x + c) + 80*cos(d*x + c))/((a^2*d*
cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38664, size = 251, normalized size = 2.24

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{274\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-110\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\ldots\right.}{\ldots}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/160*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (274*tan(1/2*d*x + 1/2*c)^
5 - 110*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x
 + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)^5) - (a
^8*tan(1/2*d*x + 1/2*c)^5 - 5*a^8*tan(1/2*d*x + 1/2*c)^4 + 15*a^8*tan(1/2*d
*x + 1/2*c)^3 - 40*a^8*tan(1/2*d*x + 1/2*c)^2 + 110*a^8*tan(1/2*d*x + 1/2*c
))/a^10)/d
```

3.431 $\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{11\cot(c+dx)\csc}{24a^2d}$$

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (4*Cot[
c + d*x]^3)/(3*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) - (11*Cot[c + d*x]*Csc
[c + d*x])/(16*a^2*d) - (11*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^5)/(6*a^2*d)
```

Rubi [A] time = 0.248182, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 3768, 3770, 3767}

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{11\cot(c+dx)\csc}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (4*Cot[
c + d*x]^3)/(3*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) - (11*Cot[c + d*x]*Csc
[c + d*x])/(16*a^2*d) - (11*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^5)/(6*a^2*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^7(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\csc^5(c+dx)-2a^2\csc^6(c+dx)+a^2\csc^7(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \csc^5(c+dx)\,dx}{a^2}+\frac{\int \csc^7(c+dx)\,dx}{a^2}-\frac{2\int \csc^6(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}+\frac{3\int \csc^3(c+dx)\,dx}{4a^2}+\frac{5\int}{}$$
$$= \frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{11\,\mathrm{c}}{}$$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{11\,\mathrm{cc}}{}$$
$$= -\frac{11\tanh^{-1}(\cos(c+dx))}{16a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{11\,\mathrm{c}}{}$$

Mathematica [A] time = 0.806337, size = 229, normalized size = 1.66

$$\frac{\csc^6(c+dx)\left(3840\sin(2(c+dx))-1536\sin(4(c+dx))+256\sin(6(c+dx))-2820\cos(c+dx)+1870\cos(3(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^6*(-2820*Cos[c + d*x] + 1870*Cos[3*(c + d*x)] - 330*Cos[5*(c
+ d*x)] - 1650*Log[Cos[(c + d*x)/2]] + 2475*Cos[2*(c + d*x)]*Log[Cos[(c + d
*x)/2]] - 990*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 165*Cos[6*(c + d*x)]
*Log[Cos[(c + d*x)/2]] + 1650*Log[Sin[(c + d*x)/2]] - 2475*Cos[2*(c + d*x)]
*Log[Sin[(c + d*x)/2]] + 990*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 165*C
os[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 3840*Sin[2*(c + d*x)] - 1536*Sin[4*
(c + d*x)] + 256*Sin[6*(c + d*x)]))/(7680*a^2*d)
```

Maple [A] time = 0.196, size = 246, normalized size = 1.8

$$\frac{1}{384\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6-\frac{1}{80\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{5}{128\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{5}{48\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{31}{128\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/384/d/a^2*tan(1/2*d*x+1/2*c)^6-1/80/d/a^2*tan(1/2*d*x+1/2*c)^5+5/128/d/a^
2*tan(1/2*d*x+1/2*c)^4-5/48/d/a^2*tan(1/2*d*x+1/2*c)^3+31/128/d/a^2*tan(1/2
*d*x+1/2*c)^2-5/8/d/a^2*tan(1/2*d*x+1/2*c)+5/8/d/a^2/tan(1/2*d*x+1/2*c)+1/8
0/d/a^2/tan(1/2*d*x+1/2*c)^5-5/128/d/a^2/tan(1/2*d*x+1/2*c)^4+11/16/d/a^2*l
```

```
n(tan(1/2*d*x+1/2*c))-1/384/d/a^2/tan(1/2*d*x+1/2*c)^6+5/48/d/a^2/tan(1/2*d
*x+1/2*c)^3-31/128/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.1117, size = 371, normalized size = 2.69

$$-\frac{\frac{\frac{1200\sin(dx+c)}{\cos(dx+c)+1}-\frac{465\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{75\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^2}-\frac{1320\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{75\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200 \ldots}{(\cos \ldots}\right.}{\ldots}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1920*((1200*sin(d*x + c)/(cos(d*x + c) + 1) - 465*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 75*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6)/a^2 - 1320*log(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^2 - (24*sin(d*x + c)/(cos(d*x + c) + 1) - 75*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 465*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 1200*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x
 + c) + 1)^6/(a^2*sin(d*x + c)^6))/d
```

Fricas [A] time = 1.19363, size = 555, normalized size = 4.02

$$\frac{330\cos(dx+c)^5-880\cos(dx+c)^3-165\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\ldots\right.}{480\,(a \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(330*cos(d*x + c)^5 - 880*cos(d*x + c)^3 - 165*(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 165*(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c
) + 1/2) - 64*(8*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(
d*x + c) + 630*cos(d*x + c))/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4
 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39306, size = 290, normalized size = 2.1

$$\frac{\frac{1320 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{3234 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 1200 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 465 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 200 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 75 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 24 \tan\left(\frac{1}{2}\,d\right.}{a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(1320*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (3234*tan(1/2*d*x + 1/2*c
)^6 - 1200*tan(1/2*d*x + 1/2*c)^5 + 465*tan(1/2*d*x + 1/2*c)^4 - 200*tan(1/
2*d*x + 1/2*c)^3 + 75*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 5)
/(a^2*tan(1/2*d*x + 1/2*c)^6) + (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^10*ta
n(1/2*d*x + 1/2*c)^5 + 75*a^10*tan(1/2*d*x + 1/2*c)^4 - 200*a^10*tan(1/2*d*
x + 1/2*c)^3 + 465*a^10*tan(1/2*d*x + 1/2*c)^2 - 1200*a^10*tan(1/2*d*x + 1/
2*c))/a^12)/d
```

3.432 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{7\cos(c+dx)}{a^3d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{19\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{51x}{8a^3}$$

```
[Out] (51*x)/(8*a^3) + (7*Cos[c + d*x])/(a^3*d) - Cos[c + d*x]^3/(a^3*d) - (19*Co
s[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^3*d
) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.259208, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2872, 2638, 2635, 8, 2633, 2648}

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{7\cos(c+dx)}{a^3d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{19\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{51x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (51*x)/(8*a^3) + (7*Cos[c + d*x])/(a^3*d) - Cos[c + d*x]^3/(a^3*d) - (19*Co
s[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^3*d
) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sin(c+dx)(a-a\sin(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(4a-4a\sin(c+dx)+4a\sin^2(c+dx)-3a\sin^3(c+dx)+a\sin^4(c+dx)-\frac{4a}{1+\sin(c}\right.}{a^4}$$
$$= \frac{4x}{a^3}+\frac{\int \sin^4(c+dx)\,dx}{a^3}-\frac{3\int \sin^3(c+dx)\,dx}{a^3}-\frac{4\int \sin(c+dx)\,dx}{a^3}+\frac{4\int \sin^2(c+d}{a^3}$$
$$= \frac{4x}{a^3}+\frac{4\cos(c+dx)}{a^3d}-\frac{2\cos(c+dx)\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin^3(c+dx)}{4a^3d}+\frac{4\,c}{a^3d(1}$$
$$= \frac{6x}{a^3}+\frac{7\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{a^3d}-\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{\cos(c+dx)\sin^3}{4a^3d}$$
$$= \frac{51x}{8a^3}+\frac{7\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{a^3d}-\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{\cos(c+dx)\sin}{4a^3d}$$

Mathematica [A] time = 1.3382, size = 195, normalized size = 1.79

$$\frac{2040dx\sin\left(c+\frac{dx}{2}\right)+800\sin\left(2c+\frac{3dx}{2}\right)-160\sin\left(2c+\frac{5dx}{2}\right)-35\sin\left(4c+\frac{7dx}{2}\right)+5\sin\left(4c+\frac{9dx}{2}\right)+997\cos\left(c+\frac{dx}{2}\right.}{320a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2040*d*x*Cos[(d*x)/2] + 997*Cos[c + (d*x)/2] + 800*Cos[c + (3*d*x)/2] + 16
0*Cos[3*c + (5*d*x)/2] - 35*Cos[3*c + (7*d*x)/2] - 5*Cos[5*c + (9*d*x)/2] -
 3563*Sin[(d*x)/2] + 2040*d*x*Sin[c + (d*x)/2] + 800*Sin[2*c + (3*d*x)/2] -
 160*Sin[2*c + (5*d*x)/2] - 35*Sin[4*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)/2
])/(320*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [B] time = 0.119, size = 300, normalized size = 2.8

$$\frac{19}{4\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}+8\,\frac{(\tan(1/2\,dx+c/2))^6}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^4}+\frac{27}{4\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 19/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+8/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6+27/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)
^4*tan(1/2*d*x+1/2*c)^5+36/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2
*c)^4-27/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+40/d/a^3/(
1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2-19/4/d/a^3/(1+tan(1/2*d*x+1/
2*c)^2)^4*tan(1/2*d*x+1/2*c)+12/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4+51/4/d/a^3
*arctan(tan(1/2*d*x+1/2*c))+8/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.7116, size = 537, normalized size = 4.93

$$\frac{\frac{\frac{29 \sin(dx+c)}{\cos(dx+c)+1} + \frac{269 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{133 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{309 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{171 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{187 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{51 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{51 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + 80}{a^3 + \frac{a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{4 a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{4 a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{6 a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{6 a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{4 a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{4 a^3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{a^3 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{a^3 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}} + \frac{51 \arctan\left(\frac{s}{co}\right.}{a^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((29*sin(d*x + c)/(cos(d*x + c) + 1) + 269*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 133*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 309*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 171*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 187*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 51*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 51*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 80)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c
) + 1) + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^3*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*a^3*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 4*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^3*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8 + a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9) + 51*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.15889, size = 381, normalized size = 3.5

$$-\frac{2 \cos(dx+c)^5 + 8 \cos(dx+c)^4 - 15 \cos(dx+c)^3 - 51\,dx - (51\,dx + 67)\cos(dx+c) - 56 \cos(dx+c)^2 - \left(2 \cos(dx \right.}{8\left(a^3 d \cos(dx+c) + a^3 d \sin(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/8*(2*cos(d*x + c)^5 + 8*cos(d*x + c)^4 - 15*cos(d*x + c)^3 - 51*d*x - (5
1*d*x + 67)*cos(d*x + c) - 56*cos(d*x + c)^2 - (2*cos(d*x + c)^4 - 6*cos(d*
x + c)^3 + 51*d*x - 21*cos(d*x + c)^2 + 35*cos(d*x + c) - 32)*sin(d*x + c)
- 32)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35192, size = 196, normalized size = 1.8

$$\frac{\frac{51\,(dx+c)}{a^3}+\frac{64}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+32\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+144\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+160\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+48\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(51*(d*x + c)/a^3 + 64/(a^3*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(19*tan(1/2
*d*x + 1/2*c)^7 + 32*tan(1/2*d*x + 1/2*c)^6 + 27*tan(1/2*d*x + 1/2*c)^5 + 1
44*tan(1/2*d*x + 1/2*c)^4 - 27*tan(1/2*d*x + 1/2*c)^3 + 160*tan(1/2*d*x + 1
/2*c)^2 - 19*tan(1/2*d*x + 1/2*c) + 48)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3
))/d
```

3.433 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=87

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{5\cos(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{11x}{2a^3}$$

```
[Out] (-11*x)/(2*a^3) - (5*Cos[c + d*x])/(a^3*d) + Cos[c + d*x]^3/(3*a^3*d) + (3*
Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c +
 d*x]))
```

Rubi [A] time = 0.226797, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2709, 2638, 2635, 8, 2633, 2648}

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{5\cos(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{11x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) - (5*Cos[c + d*x])/(a^3*d) + Cos[c + d*x]^3/(3*a^3*d) + (3*
Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c +
 d*x]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int (a-a\sin(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(-4a+4a\sin(c+dx)-3a\sin^2(c+dx)+a\sin^3(c+dx)+\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4}$$

$$= -\frac{4x}{a^3}+\frac{\int \sin^3(c+dx)\,dx}{a^3}-\frac{3\int \sin^2(c+dx)\,dx}{a^3}+\frac{4\int \sin(c+dx)\,dx}{a^3}+\frac{4\int \frac{1}{1+\sin(c+}}{a^3}$$

$$= -\frac{4x}{a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}-\frac{3\int 1\,dx}{2a^3}$$

$$= -\frac{11x}{2a^3}-\frac{5\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{4\cos(c+d}{a^3d(1+\sin(c+}$$

Mathematica [B] time = 1.11834, size = 181, normalized size = 2.08

$$\frac{-660dx\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{dx}{2}\right)-240\sin\left(2c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{5dx}{2}\right)+5\sin\left(4c+\frac{7dx}{2}\right)-286\cos\left(c+\frac{dx}{2}\right)-24}{120a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((1 - 660*d*x)*Cos[(d*x)/2] - 286*Cos[c + (d*x)/2] - 240*Cos[c + (3*d*x)/2]
 - 40*Cos[3*c + (5*d*x)/2] + 5*Cos[3*c + (7*d*x)/2] + 1244*Sin[(d*x)/2] + S
in[c + (d*x)/2] - 660*d*x*Sin[c + (d*x)/2] - 240*Sin[2*c + (3*d*x)/2] + 40*
Sin[2*c + (5*d*x)/2] + 5*Sin[4*c + (7*d*x)/2])/(120*a^3*d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [B] time = 0.106, size = 198, normalized size = 2.3

$$-3\frac{(\tan(1/2\,dx+c/2))^5}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}-8\frac{(\tan(1/2\,dx+c/2))^4}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}-20\frac{(\tan(1/2\,dx+c/2))^2}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+3\frac{}{da^3(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5-8/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-20/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*t
an(1/2*d*x+1/2*c)^2+3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-2
8/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3-11/d/a^3*arctan(tan(1/2*d*x+1/2*c))-8/
d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.6924, size = 421, normalized size = 4.84

$$-\frac{\frac{\frac{19\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{123\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{96\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{33\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{33\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+52}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{33\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/3*((19*sin(d*x + c)/(cos(d*x + c) + 1) + 123*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 60*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 96*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 33*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 52)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*a^3*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 + a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7) + 33*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a^3)/d
```

Fricas [A] time = 1.10882, size = 328, normalized size = 3.77

$$\frac{2\,\cos\left(dx+c\right)^4-7\,\cos\left(dx+c\right)^3-33\,dx-3\,(11\,dx+15)\cos\left(dx+c\right)-30\,\cos\left(dx+c\right)^2+\left(2\,\cos\left(dx+c\right)^3-33\,dx+9\,\mathrm{c}\right.}{6\left(a^3d\cos\left(dx+c\right)+a^3d\sin\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*cos(d*x + c)^4 - 7*cos(d*x + c)^3 - 33*d*x - 3*(11*d*x + 15)*cos(d*x
 + c) - 30*cos(d*x + c)^2 + (2*cos(d*x + c)^3 - 33*d*x + 9*cos(d*x + c)^2 -
 21*cos(d*x + c) + 24)*sin(d*x + c) - 24)/(a^3*d*cos(d*x + c) + a^3*d*sin(d
*x + c) + a^3*d)
```

Sympy [A] time = 130.34, size = 2412, normalized size = 27.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-1320*d*x*tan(c/2 + d*x/2)**7/(240*a**3*d*tan(c/2 + d*x/2)**7 +
240*a**3*d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan(c/2 + d*x/2)**5 + 720*a**3*
d*tan(c/2 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3*d*tan(c/2
 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d*x/2) + 240*a**3*d) - 1320*d*x*tan(c/2
 + d*x/2)**6/(240*a**3*d*tan(c/2 + d*x/2)**7 + 240*a**3*d*tan(c/2 + d*x/2)*
*6 + 720*a**3*d*tan(c/2 + d*x/2)**5 + 720*a**3*d*tan(c/2 + d*x/2)**4 + 720*
a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d*ta
n(c/2 + d*x/2) + 240*a**3*d) - 3960*d*x*tan(c/2 + d*x/2)**5/(240*a**3*d*tan
(c/2 + d*x/2)**7 + 240*a**3*d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan(c/2 + d*
x/2)**5 + 720*a**3*d*tan(c/2 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*x/2)**3 +
 720*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d*x/2) + 240*a**3*d)
 - 3960*d*x*tan(c/2 + d*x/2)**4/(240*a**3*d*tan(c/2 + d*x/2)**7 + 240*a**3*
d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan(c/2 + d*x/2)**5 + 720*a**3*d*tan(c/2
 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3*d*tan(c/2 + d*x/2)
**2 + 240*a**3*d*tan(c/2 + d*x/2) + 240*a**3*d) - 3960*d*x*tan(c/2 + d*x/2)
**3/(240*a**3*d*tan(c/2 + d*x/2)**7 + 240*a**3*d*tan(c/2 + d*x/2)**6 + 720*
a**3*d*tan(c/2 + d*x/2)**5 + 720*a**3*d*tan(c/2 + d*x/2)**4 + 720*a**3*d*ta
n(c/2 + d*x/2)**3 + 720*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d
*x/2) + 240*a**3*d) - 3960*d*x*tan(c/2 + d*x/2)**2/(240*a**3*d*tan(c/2 + d*
x/2)**7 + 240*a**3*d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan(c/2 + d*x/2)**5 +
 720*a**3*d*tan(c/2 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3
*d*tan(c/2 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d*x/2) + 240*a**3*d) - 1320*d
*x*tan(c/2 + d*x/2)/(240*a**3*d*tan(c/2 + d*x/2)**7 + 240*a**3*d*tan(c/2 +
d*x/2)**6 + 720*a**3*d*tan(c/2 + d*x/2)**5 + 720*a**3*d*tan(c/2 + d*x/2)**4
 + 720*a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3*d*tan(c/2 + d*x/2)**2 + 240*a*
*3*d*tan(c/2 + d*x/2) + 240*a**3*d) - 1320*d*x/(240*a**3*d*tan(c/2 + d*x/2)
**7 + 240*a**3*d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan(c/2 + d*x/2)**5 + 720
*a**3*d*tan(c/2 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3*d*t
an(c/2 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d*x/2) + 240*a**3*d) + 1269*tan(c
/2 + d*x/2)**7/(240*a**3*d*tan(c/2 + d*x/2)**7 + 240*a**3*d*tan(c/2 + d*x/2
)**6 + 720*a**3*d*tan(c/2 + d*x/2)**5 + 720*a**3*d*tan(c/2 + d*x/2)**4 + 72
0*a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d*
tan(c/2 + d*x/2) + 240*a**3*d) - 1371*tan(c/2 + d*x/2)**6/(240*a**3*d*tan(c
/2 + d*x/2)**7 + 240*a**3*d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan(c/2 + d*x/
2)**5 + 720*a**3*d*tan(c/2 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*x/2)**3 + 7
20*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d*x/2) + 240*a**3*d) +
 1167*tan(c/2 + d*x/2)**5/(240*a**3*d*tan(c/2 + d*x/2)**7 + 240*a**3*d*tan(
c/2 + d*x/2)**6 + 720*a**3*d*tan(c/2 + d*x/2)**5 + 720*a**3*d*tan(c/2 + d*x
/2)**4 + 720*a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3*d*tan(c/2 + d*x/2)**2 +
240*a**3*d*tan(c/2 + d*x/2) + 240*a**3*d) - 3873*tan(c/2 + d*x/2)**4/(240*a
**3*d*tan(c/2 + d*x/2)**7 + 240*a**3*d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan
(c/2 + d*x/2)**5 + 720*a**3*d*tan(c/2 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*
x/2)**3 + 720*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d*x/2) + 24
0*a**3*d) - 993*tan(c/2 + d*x/2)**3/(240*a**3*d*tan(c/2 + d*x/2)**7 + 240*a
**3*d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan(c/2 + d*x/2)**5 + 720*a**3*d*tan
(c/2 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3*d*tan(c/2 + d*
x/2)**2 + 240*a**3*d*tan(c/2 + d*x/2) + 240*a**3*d) - 6033*tan(c/2 + d*x/2)
**2/(240*a**3*d*tan(c/2 + d*x/2)**7 + 240*a**3*d*tan(c/2 + d*x/2)**6 + 720*
a**3*d*tan(c/2 + d*x/2)**5 + 720*a**3*d*tan(c/2 + d*x/2)**4 + 720*a**3*d*ta
n(c/2 + d*x/2)**3 + 720*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d
*x/2) + 240*a**3*d) - 251*tan(c/2 + d*x/2)/(240*a**3*d*tan(c/2 + d*x/2)**7
+ 240*a**3*d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan(c/2 + d*x/2)**5 + 720*a**
3*d*tan(c/2 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*x/2)**3 + 720*a**3*d*tan(c
/2 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d*x/2) + 240*a**3*d) - 2891/(240*a**3
*d*tan(c/2 + d*x/2)**7 + 240*a**3*d*tan(c/2 + d*x/2)**6 + 720*a**3*d*tan(c/
2 + d*x/2)**5 + 720*a**3*d*tan(c/2 + d*x/2)**4 + 720*a**3*d*tan(c/2 + d*x/2
)**3 + 720*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d*tan(c/2 + d*x/2) + 240*a
**3*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**4/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.32942, size = 143, normalized size = 1.64

$$-\frac{\frac{33\,(dx+c)}{a^3}+\frac{48}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+28\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)/a^3 + 48/(a^3*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(9*tan(1/2
*d*x + 1/2*c)^5 + 24*tan(1/2*d*x + 1/2*c)^4 + 60*tan(1/2*d*x + 1/2*c)^2 - 9
*tan(1/2*d*x + 1/2*c) + 28)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

3.434 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=80

$$\frac{9\cos(c+dx)}{2a^3d}+\frac{3\cos^3(c+dx)}{2d\left(a^3\sin(c+dx)+a^3\right)}+\frac{9x}{2a^3}+\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^3}$$

```
[Out] (9*x)/(2*a^3) + (9*Cos[c + d*x])/(2*a^3*d) + Cos[c + d*x]^5/(d*(a + a*Sin[c
 + d*x])^3) + (3*Cos[c + d*x]^3)/(2*d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.139081, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2859, 2679, 2682, 8}

$$\frac{9\cos(c+dx)}{2a^3d}+\frac{3\cos^3(c+dx)}{2d\left(a^3\sin(c+dx)+a^3\right)}+\frac{9x}{2a^3}+\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (9*x)/(2*a^3) + (9*Cos[c + d*x])/(2*a^3*d) + Cos[c + d*x]^5/(d*(a + a*Sin[c
 + d*x])^3) + (3*Cos[c + d*x]^3)/(2*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{a}\\
&= \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\left(a^3+a^3\sin(c+dx)\right)}+\frac{9\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{2a^2}\\
&= \frac{9\cos(c+dx)}{2a^3d}+\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\left(a^3+a^3\sin(c+dx)\right)}+\frac{9\int 1\,dx}{2a^3}\\
&= \frac{9x}{2a^3}+\frac{9\cos(c+dx)}{2a^3d}+\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\left(a^3+a^3\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.722272, size = 143, normalized size = 1.79

$$\frac{180dx\sin\left(c+\frac{dx}{2}\right)+55\sin\left(2c+\frac{3dx}{2}\right)-5\sin\left(2c+\frac{5dx}{2}\right)+59\cos\left(c+\frac{dx}{2}\right)+55\cos\left(c+\frac{3dx}{2}\right)+5\cos\left(3c+\frac{5dx}{2}\right)-381\sin}{40a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (180*d*x*Cos[(d*x)/2] + 59*Cos[c + (d*x)/2] + 55*Cos[c + (3*d*x)/2] + 5*Cos
[3*c + (5*d*x)/2] - 381*Sin[(d*x)/2] + 180*d*x*Sin[c + (d*x)/2] + 55*Sin[2*
c + (3*d*x)/2] - 5*Sin[2*c + (5*d*x)/2])/(40*a^3*d*(Cos[c/2] + Sin[c/2])*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [B] time = 0.098, size = 163, normalized size = 2.

$$\frac{1}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}+6\,\frac{(\tan(1/2\,dx+c/2))^2}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-\frac{1}{da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+6/d/a^3/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan
(1/2*d*x+1/2*c)+6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2+9/d/a^3*arctan(tan(1/2*d
*x+1/2*c))+8/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.67427, size = 304, normalized size = 3.8

$$\frac{\frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+14}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] ((5*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 14)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.14407, size = 259, normalized size = 3.24

$$\frac{\cos\left(dx+c\right)^{3}+9\,dx+\left(9\,dx+13\right)\cos\left(dx+c\right)+6\,\cos\left(dx+c\right)^{2}+\left(9\,dx-\cos\left(dx+c\right)^{2}+5\,\cos\left(dx+c\right)-8\right)\sin\left(dx+\right.}{2\left(a^{3}d\cos\left(dx+c\right)+a^{3}d\sin\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(d*x + c)^3 + 9*d*x + (9*d*x + 13)*cos(d*x + c) + 6*cos(d*x + c)^2
+ (9*d*x - cos(d*x + c)^2 + 5*cos(d*x + c) - 8)*sin(d*x + c) + 8)/(a^3*d*co
s(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [A] time = 79.9592, size = 1357, normalized size = 16.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((135*d*x*tan(c/2 + d*x/2)**5/(30*a**3*d*tan(c/2 + d*x/2)**5 + 30*
a**3*d*tan(c/2 + d*x/2)**4 + 60*a**3*d*tan(c/2 + d*x/2)**3 + 60*a**3*d*tan(
c/2 + d*x/2)**2 + 30*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) + 135*d*x*tan(c/2
 + d*x/2)**4/(30*a**3*d*tan(c/2 + d*x/2)**5 + 30*a**3*d*tan(c/2 + d*x/2)**4
 + 60*a**3*d*tan(c/2 + d*x/2)**3 + 60*a**3*d*tan(c/2 + d*x/2)**2 + 30*a**3*
d*tan(c/2 + d*x/2) + 30*a**3*d) + 270*d*x*tan(c/2 + d*x/2)**3/(30*a**3*d*ta
n(c/2 + d*x/2)**5 + 30*a**3*d*tan(c/2 + d*x/2)**4 + 60*a**3*d*tan(c/2 + d*x
/2)**3 + 60*a**3*d*tan(c/2 + d*x/2)**2 + 30*a**3*d*tan(c/2 + d*x/2) + 30*a*
*3*d) + 270*d*x*tan(c/2 + d*x/2)**2/(30*a**3*d*tan(c/2 + d*x/2)**5 + 30*a**
3*d*tan(c/2 + d*x/2)**4 + 60*a**3*d*tan(c/2 + d*x/2)**3 + 60*a**3*d*tan(c/2
 + d*x/2)**2 + 30*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) + 135*d*x*tan(c/2 +
d*x/2)/(30*a**3*d*tan(c/2 + d*x/2)**5 + 30*a**3*d*tan(c/2 + d*x/2)**4 + 60*
a**3*d*tan(c/2 + d*x/2)**3 + 60*a**3*d*tan(c/2 + d*x/2)**2 + 30*a**3*d*tan(
c/2 + d*x/2) + 30*a**3*d) + 135*d*x/(30*a**3*d*tan(c/2 + d*x/2)**5 + 30*a**
3*d*tan(c/2 + d*x/2)**4 + 60*a**3*d*tan(c/2 + d*x/2)**3 + 60*a**3*d*tan(c/2
 + d*x/2)**2 + 30*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 86*tan(c/2 + d*x/2
)**5/(30*a**3*d*tan(c/2 + d*x/2)**5 + 30*a**3*d*tan(c/2 + d*x/2)**4 + 60*a*
*3*d*tan(c/2 + d*x/2)**3 + 60*a**3*d*tan(c/2 + d*x/2)**2 + 30*a**3*d*tan(c/
2 + d*x/2) + 30*a**3*d) + 184*tan(c/2 + d*x/2)**4/(30*a**3*d*tan(c/2 + d*x/
2)**5 + 30*a**3*d*tan(c/2 + d*x/2)**4 + 60*a**3*d*tan(c/2 + d*x/2)**3 + 60*
a**3*d*tan(c/2 + d*x/2)**2 + 30*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) + 38*t
an(c/2 + d*x/2)**3/(30*a**3*d*tan(c/2 + d*x/2)**5 + 30*a**3*d*tan(c/2 + d*x
/2)**4 + 60*a**3*d*tan(c/2 + d*x/2)**3 + 60*a**3*d*tan(c/2 + d*x/2)**2 + 30
```

```
*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) + 458*tan(c/2 + d*x/2)**2/(30*a**3*d*
tan(c/2 + d*x/2)**5 + 30*a**3*d*tan(c/2 + d*x/2)**4 + 60*a**3*d*tan(c/2 + d
*x/2)**3 + 60*a**3*d*tan(c/2 + d*x/2)**2 + 30*a**3*d*tan(c/2 + d*x/2) + 30*
a**3*d) + 64*tan(c/2 + d*x/2)/(30*a**3*d*tan(c/2 + d*x/2)**5 + 30*a**3*d*ta
n(c/2 + d*x/2)**4 + 60*a**3*d*tan(c/2 + d*x/2)**3 + 60*a**3*d*tan(c/2 + d*x
/2)**2 + 30*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) + 334/(30*a**3*d*tan(c/2 +
 d*x/2)**5 + 30*a**3*d*tan(c/2 + d*x/2)**4 + 60*a**3*d*tan(c/2 + d*x/2)**3
+ 60*a**3*d*tan(c/2 + d*x/2)**2 + 30*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d),
Ne(d, 0)), (x*sin(c)*cos(c)**4/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.37265, size = 123, normalized size = 1.54

$$\frac{\frac{9\,(dx+c)}{a^3}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}+\frac{16}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(9*(d*x + c)/a^3 + 2*(tan(1/2*d*x + 1/2*c)^3 + 6*tan(1/2*d*x + 1/2*c)^2
 - tan(1/2*d*x + 1/2*c) + 6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 16/(a^3
*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.435 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=45

$$\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{a^3}$$

```
[Out] x/a^3 - ArcTanh[Cos[c + d*x]]/(a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c
+ d*x]))
```

Rubi [A] time = 0.182899, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2875, 2872, 3770, 2648}

$$\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/a^3 - ArcTanh[Cos[c + d*x]]/(a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c
+ d*x]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(a+a\csc(c+dx)-\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4}$$
$$= \frac{x}{a^3}+\frac{\int \csc(c+dx)\,dx}{a^3}-\frac{4\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}$$
$$= \frac{x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

Mathematica [B] time = 0.271051, size = 122, normalized size = 2.71

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+c+dx\right)+\sin\left(\frac{1}{2}(c\right.\right.}{a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(Cos[(c + d*x)/2]*(c + d*x - Log[C
os[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]) + (-8 + c + d*x - Log[Cos[(c + d*
x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[(c + d*x)/2]))/(a^3*d*(1 + Sin[c + d*x]
)^3)
```

Maple [A] time = 0.145, size = 58, normalized size = 1.3

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^3}+8\,\frac{1}{da^3\left(\tan\left(1/2\,dx+c/2\right)+1\right)}+\frac{1}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*arctan(tan(1/2*d*x+1/2*c))+8/d/a^3/(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*l
n(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.52696, size = 105, normalized size = 2.33

$$\frac{\frac{8}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (8/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1)) + 2*arctan(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] time = 1.13666, size = 338, normalized size = 7.51

$$\frac{2\,dx + 2\,(dx + 4)\cos\left(dx + c\right) - \left(\cos\left(dx + c\right) + \sin\left(dx + c\right) + 1\right)\log\left(\frac{1}{2}\,\cos\left(dx + c\right) + \frac{1}{2}\right) + \left(\cos\left(dx + c\right) + \sin\left(dx + c\right.}{2\,\left(a^{3}d\cos\left(dx + c\right) + a^{3}d\sin\left(dx + c\right) + a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x + 2*(d*x + 4)*cos(d*x + c) - (cos(d*x + c) + sin(d*x + c) + 1)*l
og(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c) + sin(d*x + c) + 1)*log(-1/2*cos
(d*x + c) + 1/2) + 2*(d*x - 4)*sin(d*x + c) + 8)/(a^3*d*cos(d*x + c) + a^3*
d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36281, size = 63, normalized size = 1.4

$$\frac{\frac{dx+c}{a^{3}} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}} + \frac{8}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^3 + log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 8/(a^3*(tan(1/2*d*x +
 1/2*c) + 1)))/d
```

3.436 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=54

$$-\frac{\cot(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (4*Cos[c + d*x])
/(a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.235539, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2875, 2872, 3770, 3767, 8, 2648}

$$-\frac{\cot(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (4*Cos[c + d*x])
/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^2(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-3a\csc(c+dx)+a\csc^2(c+dx)+\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4}$$
$$= \frac{\int \csc^2(c+dx)\,dx}{a^3} - \frac{3\int \csc(c+dx)\,dx}{a^3} + \frac{4\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))} - \frac{\text{Subst}(\int 1\,dx, x, \cot(c+dx))}{a^3d}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{\cot(c+dx)}{a^3d} - \frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

Mathematica [B] time = 0.611471, size = 156, normalized size = 2.89

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\cot^2\left(\frac{1}{2}(c+dx)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.\right.}{2a^3d(\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((Cos[(c + d*x)/2]*(-17 + Cot[(c + d*x)/2]^2 - 6*Log[Cos[(c + d*x)/2]] + 6
*Log[Sin[(c + d*x)/2]] + Cot[(c + d*x)/2]*(1 - 6*Log[Cos[(c + d*x)/2]] + 6*
Log[Sin[(c + d*x)/2]])) - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^5*Tan[(c + d*x)/2])/(2*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.16, size = 77, normalized size = 1.4

$$\frac{1}{2da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\frac{1}{da^3(\tan(1/2\,dx+c/2)+1)}-\frac{1}{2da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-3\frac{\ln(\tan(1/2\,dx+c/2))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-8/d/a^3/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^3/tan(1
/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.13214, size = 157, normalized size = 2.91

$$-\frac{\frac{\frac{17\sin(dx+c)}{\cos(dx+c)+1}+1}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*((17*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a^3*sin(d*x + c)/(cos(d*x +
 c) + 1) + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 6*log(sin(d*x + c)/(c
os(d*x + c) + 1))/a^3 - sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [B] time = 1.13199, size = 451, normalized size = 8.35

$$\frac{10 \cos(dx+c)^2 + 3\left(\cos(dx+c)^2 - (\cos(dx+c)+1)\sin(dx+c) - 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - 3\left(\cos(dx+c)^2 - (\cos\right.}{2\left(a^3 d\cos(dx+c)^2 - a^3 d - \left(a^3 d\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c
) - 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)
*sin(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(5*cos(d*x + c) + 4)*si
n(d*x + c) + 2*cos(d*x + c) - 8)/(a^3*d*cos(d*x + c)^2 - a^3*d - (a^3*d*cos
(d*x + c) + a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42827, size = 122, normalized size = 2.26

$$-\frac{\frac{6 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3} - \frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 14\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - tan(1/2*d*x + 1/2*c)/a^3 - (3*
tan(1/2*d*x + 1/2*c)^2 - 14*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c
)^2 + tan(1/2*d*x + 1/2*c))*a^3))/d
```

3.437 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=78

$$\frac{3\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{9\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.248554, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2648}

$$\frac{3\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{9\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^3(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(4a\csc(c+dx)-3a\csc^2(c+dx)+a\csc^3(c+dx)-\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4} \\
&= \frac{\int \csc^3(c+dx)\,dx}{a^3}-\frac{3\int \csc^2(c+dx)\,dx}{a^3}+\frac{4\int \csc(c+dx)\,dx}{a^3}-\frac{4\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} \\
&= -\frac{4\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}+\frac{\int \csc(c+ \cdots}{2a^3 \cdots} \\
&= -\frac{9\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+d \cdots}
\end{aligned}$$

Mathematica [B] time = 5.83541, size = 213, normalized size = 2.73

$$-\frac{\sin^8\left(\frac{1}{2}(c+dx)\right)\sin^7(c+dx)\left(\csc^2\left(\frac{1}{2}(c+dx)\right)+2\csc(c+dx)\right)^5\left((\csc(c+dx)-6)\csc^6\left(\frac{1}{2}(c+dx)\right)-8(\csc(c+dx)-6 \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((Csc[(c + d*x)/2]^2 + 2*Csc[c + d*x])^5*(Csc[(c + d*x)/2]^6*(-6 + Csc[c +
 d*x]) - 8*(-6 + Csc[c + d*x])*Csc[c + d*x]^3 + 2*Csc[(c + d*x)/2]^4*Csc[c
+ d*x]*(-6 + Csc[c + d*x] + 18*Log[Cos[(c + d*x)/2]] - 18*Log[Sin[(c + d*x)
/2]]) - 4*Csc[(c + d*x)/2]^2*Csc[c + d*x]^2*(-38 + Csc[c + d*x] - 18*Log[Co
s[(c + d*x)/2]] + 18*Log[Sin[(c + d*x)/2]]))*Sin[(c + d*x)/2]^8*Sin[c + d*x
]^7)/(512*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.174, size = 115, normalized size = 1.5

$$\frac{1}{8da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{3}{2da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\frac{1}{da^3(\tan(1/2dx+c/2)+1)}-\frac{1}{8da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}+\frac{3}{2da^3}\left(\tan\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^3*tan(1/2*d*x+1/2*c)+8/d/a^3/(tan(1/
2*d*x+1/2*c)+1)-1/8/d/a^3/tan(1/2*d*x+1/2*c)^2+3/2/d/a^3/tan(1/2*d*x+1/2*c)
+9/2/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.12202, size = 217, normalized size = 2.78

$$\frac{\frac{\frac{11\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}-\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3}+\frac{36\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8*((11*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 1)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3) - (12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2
/(cos(d*x + c) + 1)^2)/a^3 + 36*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] time = 1.09795, size = 662, normalized size = 8.49

$$\frac{28\cos(dx+c)^3+18\cos(dx+c)^2-9\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(28*cos(d*x + c)^3 + 18*cos(d*x + c)^2 - 9*(cos(d*x + c)^3 + cos(d*x +
c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*log(1/2*cos(d*
x + c) + 1/2) + 9*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*s
in(d*x + c) - cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*(14*cos(d*
x + c)^2 + 5*cos(d*x + c) - 8)*sin(d*x + c) - 26*cos(d*x + c) - 16)/(a^3*d*
cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d + (a^3*d
*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40937, size = 157, normalized size = 2.01

$$\frac{\frac{36 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{64}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{54 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^3 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{a^3 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12 a^3 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(36*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 64/(a^3*(tan(1/2*d*x + 1/2*c)
+ 1)) - (54*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(
1/2*d*x + 1/2*c)^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/
2*c))/a^6)/d
```

3.438 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=96

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{5\cot(c+dx)}{a^3d}+\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{4\cot(c+dx)}{a^3d(\csc(c+dx)+1)}$$

```
[Out] (11*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (5*Cot[c + d*x])/(a^3*d) - Cot[c + d
*x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (4*Cot[c + d*x]
)/(a^3*d*(1 + Csc[c + d*x]))
```

Rubi [A] time = 0.154857, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 3768, 3777}

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{5\cot(c+dx)}{a^3d}+\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{4\cot(c+dx)}{a^3d(\csc(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (11*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (5*Cot[c + d*x])/(a^3*d) - Cot[c + d
*x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (4*Cot[c + d*x]
)/(a^3*d*(1 + Csc[c + d*x]))
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(4a-4a\csc(c+dx)+4a\csc^2(c+dx)-3a\csc^3(c+dx)+a\csc^4(c+dx)-\frac{4a}{1+\csc(c+dx)}\right)dx}{a^4}$$

$$= \frac{4x}{a^3}+\frac{\int\csc^4(c+dx)\,dx}{a^3}-\frac{3\int\csc^3(c+dx)\,dx}{a^3}-\frac{4\int\csc(c+dx)\,dx}{a^3}+\frac{4\int\csc^2(c+dx)\,dx}{a^3}-$$

$$= \frac{4x}{a^3}+\frac{4\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{4\cot(c+dx)}{a^3d(1+\csc(c+dx))}-\frac{3\int\csc(c}{2a}$$

$$= \frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{5\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{4}{a^3d(1}$$

Mathematica [B] time = 4.82126, size = 251, normalized size = 2.61

$$-\frac{\sin^2\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^5\csc^3(c+dx)\left(-4\sin^8\left(\frac{1}{2}(c+dx)\right)-8\sin(c+dx)(7\sin(c+dx)-2)\sin^6\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((1 + Cot[(c + d*x)/2])^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2*(-4*Sin[(c + d
*x)/2]^8 - 8*Sin[(c + d*x)/2]^6*Sin[c + d*x]*(-2 + 7*Sin[c + d*x]) + (Sin[c
 + d*x]^4*(-8 + Cot[(c + d*x)/2] + 28*Sin[c + d*x]))/4 - (Sin[(c + d*x)/2]^
2*Sin[c + d*x]^3*(9 + (-28 + 66*Log[Cos[(c + d*x)/2]] - 66*Log[Sin[(c + d*x
)/2]])*Sin[c + d*x]))/2 + Sin[(c + d*x)/2]^4*Sin[c + d*x]^2*(9 - 2*(62 + 33
*Log[Cos[(c + d*x)/2]] - 33*Log[Sin[(c + d*x)/2]])*Sin[c + d*x])))/(12*a^3*
d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.193, size = 153, normalized size = 1.6

$$\frac{1}{24da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3}{8da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{19}{8da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\frac{1}{da^3(\tan(1/2dx+c/2)+1)}-\frac{1}{24da^3}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^3*tan(1/2*d*x+1/2*c)^2+19/8/d/a^3*t
an(1/2*d*x+1/2*c)-8/d/a^3/(tan(1/2*d*x+1/2*c)+1)-1/24/d/a^3/tan(1/2*d*x+1/2
*c)^3+3/8/d/a^3/tan(1/2*d*x+1/2*c)^2-19/8/d/a^3/tan(1/2*d*x+1/2*c)-11/2/d/a
^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.10518, size = 269, normalized size = 2.8

$$\frac{\frac{\frac{8 \sin(dx+c)}{\cos(dx+c)+1} - \frac{48 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{249 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - 1}{\frac{a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{57 \sin(dx+c)}{\cos(dx+c)+1} - \frac{9 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3} - \frac{132 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((8*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 249*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a^3*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (57*sin(d*
x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3)/a^3 - 132*log(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^3)/d
```

Fricas [B] time = 1.24088, size = 810, normalized size = 8.44

$$104 \cos(dx+c)^4 + 38 \cos(dx+c)^3 - 156 \cos(dx+c)^2 + 33\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 - \left(\cos(dx+c)^3 + \cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(104*cos(d*x + c)^4 + 38*cos(d*x + c)^3 - 156*cos(d*x + c)^2 + 33*(cos
(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x
 + c) - 1)*sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 33*(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1
)*sin(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(52*cos(d*x + c)^3 + 3
3*cos(d*x + c)^2 - 45*cos(d*x + c) - 24)*sin(d*x + c) - 42*cos(d*x + c) + 4
8)/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d - (a^3*d*cos(d*x
+ c)^3 + a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3999, size = 197, normalized size = 2.05

$$-\frac{\frac{132 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} + \frac{192}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)} - \frac{242 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 57 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 9 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 1}{a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} - \frac{a^6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 9\,a^6 \tan(}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/24*(132*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 192/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)) - (242*tan(1/2*d*x + 1/2*c)^3 - 57*tan(1/2*d*x + 1/2*c)^2 + 9*tan
(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (a^6*tan(1/2*d*x + 1/
2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 + 57*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

3.439 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{7\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{51\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{19\cot(c$$

```
[Out] (-51*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (7*Cot[c + d*x])/(a^3*d) + Cot[c +
d*x]^3/(a^3*d) - (19*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x]*C
sc[c + d*x]^3)/(4*a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.300309, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2648}

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{7\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{51\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{19\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-51*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (7*Cot[c + d*x])/(a^3*d) + Cot[c +
d*x]^3/(a^3*d) - (19*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x]*C
sc[c + d*x]^3)/(4*a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^5(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(4a\csc(c+dx) - 4a\csc^2(c+dx) + 4a\csc^3(c+dx) - 3a\csc^4(c+dx) + a\csc^5(c+dx)\right.}{a^4}$$

$$= \frac{\int \csc^5(c+dx)\,dx}{a^3} - \frac{3\int \csc^4(c+dx)\,dx}{a^3} + \frac{4\int \csc(c+dx)\,dx}{a^3} - \frac{4\int \csc^2(c+dx)\,dx}{a^3} + \frac{4}{}$$

$$= -\frac{4\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d} + \frac{4}{a^3d(}$$

$$= -\frac{6\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{7\cot(c+dx)}{a^3d} + \frac{\cot^3(c+dx)}{a^3d} - \frac{19\cot(c+dx)\csc(c+dx)}{8a^3d} -$$

$$= -\frac{51\tanh^{-1}(\cos(c+dx))}{8a^3d} + \frac{7\cot(c+dx)}{a^3d} + \frac{\cot^3(c+dx)}{a^3d} - \frac{19\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Mathematica [B] time = 6.1605, size = 601, normalized size = 5.14

$$-\frac{8\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}{d(a\sin(c+dx)+a)^3} - \frac{51\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}{8d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-8*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(d*(a + a*Sin
[c + d*x])^3) + (3*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6
)/(d*(a + a*Sin[c + d*x])^3) - (19*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^6)/(32*d*(a + a*Sin[c + d*x])^3) + (Cot[(c + d*x)/2]*Csc[(
c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(8*d*(a + a*Sin[c +
d*x])^3) - (Csc[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(64
*d*(a + a*Sin[c + d*x])^3) - (51*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^6)/(8*d*(a + a*Sin[c + d*x])^3) + (51*Log[Sin[(c + d*x)/2
]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(8*d*(a + a*Sin[c + d*x])^3) +
(19*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(32*d*(a +
a*Sin[c + d*x])^3) + (Sec[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^6)/(64*d*(a + a*Sin[c + d*x])^3) - (3*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^6*Tan[(c + d*x)/2])/(d*(a + a*Sin[c + d*x])^3) - (Sec[(c + d*x)/2]^2*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*Tan[(c + d*x)/2])/(8*d*(a + a*Sin[c
```

```
+ d*x])^3)
```

Maple [A] time = 0.204, size = 191, normalized size = 1.6

$$\frac{1}{64\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{25}{8\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\,\frac{1}{da^3\,(\tan(1/2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/64/d/a^3*tan(1/2*d*x+1/2*c)^4-1/8/d/a^3*tan(1/2*d*x+1/2*c)^3+5/8/d/a^3*ta
n(1/2*d*x+1/2*c)^2-25/8/d/a^3*tan(1/2*d*x+1/2*c)+8/d/a^3/(tan(1/2*d*x+1/2*c
)+1)-1/64/d/a^3/tan(1/2*d*x+1/2*c)^4+1/8/d/a^3/tan(1/2*d*x+1/2*c)^3-5/8/d/a
^3/tan(1/2*d*x+1/2*c)^2+25/8/d/a^3/tan(1/2*d*x+1/2*c)+51/8/d/a^3*ln(tan(1/2
*d*x+1/2*c))
```

Maxima [B] time = 1.14717, size = 325, normalized size = 2.78

$$\frac{\frac{\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{32\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{160\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{712\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1}{\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}-\frac{\frac{200\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^3}+\frac{408\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/64*((7*sin(d*x + c)/(cos(d*x + c) + 1) - 32*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 160*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 712*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 1)/(a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5) - (200*sin(d*x + c)/(cos(d*x + c) + 1) - 40*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/a^3 + 408*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^3)/d
```

Fricas [B] time = 1.096, size = 1040, normalized size = 8.89

$$\frac{160\,\cos(dx+c)^5+102\,\cos(dx+c)^4-298\,\cos(dx+c)^3-170\,\cos(dx+c)^2-51\left(\cos(dx+c)^5+\cos(dx+c)^4-2\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(160*cos(d*x + c)^5 + 102*cos(d*x + c)^4 - 298*cos(d*x + c)^3 - 170*co
s(d*x + c)^2 - 51*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*c
os(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(
d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 51*(cos(d*x + c)^5 + cos(d*x +
```

```
c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x +
c)^2 + 1)*sin(d*x + c) + cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) - 2
*(80*cos(d*x + c)^4 + 29*cos(d*x + c)^3 - 120*cos(d*x + c)^2 - 35*cos(d*x +
 c) + 32)*sin(d*x + c) + 134*cos(d*x + c) + 64)/(a^3*d*cos(d*x + c)^5 + a^3
*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 + a^3*d
*cos(d*x + c) + a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^
3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38598, size = 235, normalized size = 2.01

$$\frac{\frac{408 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^3}+\frac{512}{a^3\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}-\frac{850 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4-200 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+40 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-8 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1}{a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4}+\frac{a^9 \tan\left(\frac{1}{2} dx+ \cdots\right.}{\cdots}}{64\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(408*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 512/(a^3*(tan(1/2*d*x + 1/2*
c) + 1)) - (850*tan(1/2*d*x + 1/2*c)^4 - 200*tan(1/2*d*x + 1/2*c)^3 + 40*ta
n(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c
)^4) + (a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*c)^3 + 40*a^9*
tan(1/2*d*x + 1/2*c)^2 - 200*a^9*tan(1/2*d*x + 1/2*c))/a^12)/d
```

3.440 $\int \frac{\cos^4(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx$

Optimal. Leaf size=58

$$\frac{\cos^5(e+fx)}{7f(a\sin(e+fx)+a)^6}-\frac{6\cos^5(e+fx)}{35af(a\sin(e+fx)+a)^5}$$

```
[Out] Cos[e + f*x]^5/(7*f*(a + a*Sin[e + f*x])^6) - (6*Cos[e + f*x]^5)/(35*a*f*(a
 + a*Sin[e + f*x])^5)
```

Rubi [A] time = 0.105719, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2859, 2671}

$$\frac{\cos^5(e+fx)}{7f(a\sin(e+fx)+a)^6}-\frac{6\cos^5(e+fx)}{35af(a\sin(e+fx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] Cos[e + f*x]^5/(7*f*(a + a*Sin[e + f*x])^6) - (6*Cos[e + f*x]^5)/(35*a*f*(a
 + a*Sin[e + f*x])^5)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx &= \frac{\cos^5(e+fx)}{7f(a+a\sin(e+fx))^6}+\frac{6\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^5}\,dx}{7a}\\ &= \frac{\cos^5(e+fx)}{7f(a+a\sin(e+fx))^6}-\frac{6\cos^5(e+fx)}{35af(a+a\sin(e+fx))^5}\end{aligned}$$

Mathematica [B] time = 1.22492, size = 143, normalized size = 2.47

$$\frac{1134\sin\left(2e+\frac{3fx}{2}\right)-224\sin\left(2e+\frac{5fx}{2}\right)+\sin\left(4e+\frac{7fx}{2}\right)+4585\cos\left(e+\frac{fx}{2}\right)-2982\cos\left(e+\frac{3fx}{2}\right)-1148\cos\left(3e+\right.}{4620a^6f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^4*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] (4585*Cos[e + (f*x)/2] - 2982*Cos[e + (3*f*x)/2] - 1148*Cos[3*e + (5*f*x)/2
] + 197*Cos[3*e + (7*f*x)/2] + 2275*Sin[(f*x)/2] + 1134*Sin[2*e + (3*f*x)/2
] - 224*Sin[2*e + (5*f*x)/2] + Sin[4*e + (7*f*x)/2])/(4620*a^6*f*(Cos[e/2]
+ Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A] time = 0.127, size = 100, normalized size = 1.7

$$4\frac{1}{fa^6}\left(-1/2\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-2}+\frac{56}{5\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{5}}-8\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-6}-8\left(\tan\left(1/2\,fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x)
```

```
[Out] 4/f/a^6*(-1/2/(tan(1/2*f*x+1/2*e)+1)^2+56/5/(tan(1/2*f*x+1/2*e)+1)^5-8/(tan
(1/2*f*x+1/2*e)+1)^6-8/(tan(1/2*f*x+1/2*e)+1)^4+3/(tan(1/2*f*x+1/2*e)+1)^3+
16/7/(tan(1/2*f*x+1/2*e)+1)^7)
```

Maximas [B] time = 1.1775, size = 363, normalized size = 6.26

$$-\frac{2\left(\frac{7\sin(fx+e)}{\cos(fx+e)+1}-\frac{14\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{70\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+1\right)}{35\left(a^6+\frac{7a^6\sin(fx+e)}{\cos(fx+e)+1}+\frac{21a^6\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35a^6\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{35a^6\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{21a^6\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{7a^6\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{a^6\sin(fx+e)^7}{(\cos(fx+e)+1)^7}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/35*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 14*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 70*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 35*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1)/((a^6 + 7*a^6*s
in(f*x + e)/(cos(f*x + e) + 1) + 21*a^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 35*a^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*a^6*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 21*a^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*a^6*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + a^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*f)
```

Fricas [B] time = 1.09916, size = 495, normalized size = 8.53

$$\frac{6\cos\left(fx+e\right)^4-11\cos\left(fx+e\right)^3-27\cos\left(fx+e\right)^2+\left(6\cos\left(fx+e\right)^3+17\cos\left(fx+e\right)^2-10\cos\left(fx+\right.\right.}{35\left(a^6f\cos\left(fx+e\right)^4-3\,a^6f\cos\left(fx+e\right)^3-8\,a^6f\cos\left(fx+e\right)^2+4\,a^6f\cos\left(fx+e\right)+8\,a^6f-\left(a^6f\cos\left(fx+e\right)^3+4\,a^6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/35*(6*cos(f*x + e)^4 - 11*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + (6*cos(f*x
 + e)^3 + 17*cos(f*x + e)^2 - 10*cos(f*x + e) - 20)*sin(f*x + e) + 10*cos(f
*x + e) + 20)/(a^6*f*cos(f*x + e)^4 - 3*a^6*f*cos(f*x + e)^3 - 8*a^6*f*cos(
f*x + e)^2 + 4*a^6*f*cos(f*x + e) + 8*a^6*f - (a^6*f*cos(f*x + e)^3 + 4*a^6
*f*cos(f*x + e)^2 - 4*a^6*f*cos(f*x + e) - 8*a^6*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*sin(f*x+e)/(a+a*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34783, size = 124, normalized size = 2.14

$$-\frac{2\left(35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-14\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+7\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{35\,a^{6}f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/35*(35*tan(1/2*f*x + 1/2*e)^5 - 35*tan(1/2*f*x + 1/2*e)^4 + 70*tan(1/2*f
*x + 1/2*e)^3 - 14*tan(1/2*f*x + 1/2*e)^2 + 7*tan(1/2*f*x + 1/2*e) + 1)/(a^
6*f*(tan(1/2*f*x + 1/2*e) + 1)^7)
```

3.441 $\int \frac{\cos^4(e+fx)\sin^2(e+fx)}{(a+a\sin(e+fx))^7}\,dx$

Optimal. Leaf size=89

$$-\frac{47\cos^5(e+fx)}{315a^2f(a\sin(e+fx)+a)^5}-\frac{a\cos^7(e+fx)}{18f(a\sin(e+fx)+a)^8}+\frac{25\cos^5(e+fx)}{126af(a\sin(e+fx)+a)^6}$$

```
[Out] -(a*Cos[e + f*x]^7)/(18*f*(a + a*Sin[e + f*x])^8) + (25*Cos[e + f*x]^5)/(12
6*a*f*(a + a*Sin[e + f*x])^6) - (47*Cos[e + f*x]^5)/(315*a^2*f*(a + a*Sin[e
 + f*x])^5)
```

Rubi [A] time = 0.458959, antiderivative size = 131, normalized size of antiderivative = 1.47, number of steps used = 18, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2875, 2872, 2650, 2648}

$$-\frac{47\cos(e+fx)}{315a^7f(\sin(e+fx)+1)}+\frac{268\cos(e+fx)}{315a^7f(\sin(e+fx)+1)^2}-\frac{181\cos(e+fx)}{105a^7f(\sin(e+fx)+1)^3}+\frac{92\cos(e+fx)}{63a^7f(\sin(e+fx)+1)^4}-\frac{4\cos(e+fx)}{9a^7f(\sin(e+fx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*Sin[e + f*x]^2)/(a + a*Sin[e + f*x])^7,x]
```

```
[Out] (-4*Cos[e + f*x])/(9*a^7*f*(1 + Sin[e + f*x])^5) + (92*Cos[e + f*x])/(63*a^
7*f*(1 + Sin[e + f*x])^4) - (181*Cos[e + f*x])/(105*a^7*f*(1 + Sin[e + f*x]
)^3) + (268*Cos[e + f*x])/(315*a^7*f*(1 + Sin[e + f*x])^2) - (47*Cos[e + f*
x])/(315*a^7*f*(1 + Sin[e + f*x]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)\sin^2(e+fx)}{(a+a\sin(e+fx))^7}\,dx = \frac{\int \sec^8(e+fx)(a-a\sin(e+fx))^7\tan^2(e+fx)\,dx}{a^{14}}$$

$$= \frac{\int \left(\frac{4}{a^3(1+\sin(e+fx))^5} - \frac{12}{a^3(1+\sin(e+fx))^4} + \frac{13}{a^3(1+\sin(e+fx))^3} - \frac{6}{a^3(1+\sin(e+fx))^2} + \frac{1}{a^3(1+\sin(e+fx))}\right.}{a^4}$$

$$= \frac{\int \frac{1}{1+\sin(e+fx)}\,dx}{a^7} + \frac{4\int \frac{1}{(1+\sin(e+fx))^5}\,dx}{a^7} - \frac{6\int \frac{1}{(1+\sin(e+fx))^2}\,dx}{a^7} - \frac{12\int \frac{1}{(1+\sin(e+fx))^4}\,dx}{a^7}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{12\cos(e+fx)}{7a^7f(1+\sin(e+fx))^4} - \frac{13\cos(e+fx)}{5a^7f(1+\sin(e+fx))^3} + \frac{}{a^7f}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{11\cos(e+fx)}{7a^7f(1+\sin(e+fx))^3} + \frac{}{15}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{181\cos(e+fx)}{105a^7f(1+\sin(e+fx))^3} +$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{181\cos(e+fx)}{105a^7f(1+\sin(e+fx))^3} +$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{181\cos(e+fx)}{105a^7f(1+\sin(e+fx))^3} +$$

Mathematica [B] time = 2.81116, size = 293, normalized size = 3.29

$$\frac{1890\sin\left(e+\frac{fx}{2}\right)+1260\sin\left(e+\frac{3fx}{2}\right)+659400\sin\left(2e+\frac{3fx}{2}\right)-303192\sin\left(2e+\frac{5fx}{2}\right)-540\sin\left(3e+\frac{5fx}{2}\right)-135\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^4*Sin[e + f*x]^2)/(a + a*Sin[e + f*x])^7,x]
```

```
[Out] (1890*Cos[(f*x)/2] + 718830*Cos[e + (f*x)/2] - 467208*Cos[e + (3*f*x)/2] -
1260*Cos[2*e + (3*f*x)/2] - 540*Cos[2*e + (5*f*x)/2] - 179640*Cos[3*e + (5*
f*x)/2] + 30753*Cos[3*e + (7*f*x)/2] + 135*Cos[4*e + (7*f*x)/2] + 15*Cos[4*
e + (9*f*x)/2] - 15*Cos[5*e + (9*f*x)/2] + 971082*Sin[(f*x)/2] + 1890*Sin[e
 + (f*x)/2] + 1260*Sin[e + (3*f*x)/2] + 659400*Sin[2*e + (3*f*x)/2] - 30319
2*Sin[2*e + (5*f*x)/2] - 540*Sin[3*e + (5*f*x)/2] - 135*Sin[3*e + (7*f*x)/2
] - 89955*Sin[4*e + (7*f*x)/2] + 13427*Sin[4*e + (9*f*x)/2] + 15*Sin[5*e +
(9*f*x)/2])/(720720*a^7*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^9)
```

Maple [A] time = 0.132, size = 115, normalized size = 1.3

$$8\,\frac{1}{fa^7}\left(5/2\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-4}-\frac{41}{5\left(\tan\left(1/2\,fx+e/2\right)+1\right)^5}+\frac{44}{3\left(\tan\left(1/2\,fx+e/2\right)+1\right)^6}+8\left(\tan\left(1/2\,fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x)
```

```
[Out] 8/f/a^7*(5/2/(tan(1/2*f*x+1/2*e)+1)^4-41/5/(tan(1/2*f*x+1/2*e)+1)^5+44/3/(t
an(1/2*f*x+1/2*e)+1)^6+8/(tan(1/2*f*x+1/2*e)+1)^8-1/3/(tan(1/2*f*x+1/2*e)+1
)^3-104/7/(tan(1/2*f*x+1/2*e)+1)^7-16/9/(tan(1/2*f*x+1/2*e)+1)^9)
```

Maxima [B] time = 1.07027, size = 452, normalized size = 5.08

$$-\frac{4\left(\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{36\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{126\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{441\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{315\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{210\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)}\right.}{315\left(a^7+\frac{9a^7\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{36a^7\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{84a^7\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{126a^7\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{126a^7\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{84a^7\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{36a^7\sin\left(fx+e\right)^7}{\left(\cos\left(fx+e\right)+1\right)^7}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x, algorithm="maxima
")
```

```
[Out] -4/315*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 36*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 441*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 210*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 1)/((a^7 + 9*a^7*sin(f*x + e)/(cos(f*x + e) + 1
) + 36*a^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*a^7*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 126*a^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 126*a^7*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*a^7*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 36*a^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*a^7*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8 + a^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)*f)
```

Fricas [B] time = 1.06934, size = 636, normalized size = 7.15

$$-\frac{47\cos\left(fx+e\right)^5+127\cos\left(fx+e\right)^4-115\cos\left(fx+e\right)^3-265\cos\left(fx+e\right)^2-\left(47\cos\left(fx+e\right)^4-80\,\mathrm{c}\right.}{315\left(a^7f\cos\left(fx+e\right)^5+5a^7f\cos\left(fx+e\right)^4-8a^7f\cos\left(fx+e\right)^3-20a^7f\cos\left(fx+e\right)^2+8a^7f\cos\left(fx+e\right)+16a^7f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x, algorithm="fricas
")
```

```
[Out] -1/315*(47*cos(f*x + e)^5 + 127*cos(f*x + e)^4 - 115*cos(f*x + e)^3 - 265*c
os(f*x + e)^2 - (47*cos(f*x + e)^4 - 80*cos(f*x + e)^3 - 195*cos(f*x + e)^2
 + 70*cos(f*x + e) + 140)*sin(f*x + e) + 70*cos(f*x + e) + 140)/(a^7*f*cos(
f*x + e)^5 + 5*a^7*f*cos(f*x + e)^4 - 8*a^7*f*cos(f*x + e)^3 - 20*a^7*f*cos
(f*x + e)^2 + 8*a^7*f*cos(f*x + e) + 16*a^7*f + (a^7*f*cos(f*x + e)^4 - 4*a
^7*f*cos(f*x + e)^3 - 12*a^7*f*cos(f*x + e)^2 + 8*a^7*f*cos(f*x + e) + 16*a
^7*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*sin(f*x+e)**2/(a+a*sin(f*x+e))**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40673, size = 143, normalized size = 1.61

$$-\frac{4\left(210\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+441\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-126\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+36\tan\left(\frac{1}{2}fx+\cdots\right.\right.}{315\,a^{7}f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x, algorithm="giac")
```

```
[Out] -4/315*(210*tan(1/2*f*x + 1/2*e)^6 - 315*tan(1/2*f*x + 1/2*e)^5 + 441*tan(1
/2*f*x + 1/2*e)^4 - 126*tan(1/2*f*x + 1/2*e)^3 + 36*tan(1/2*f*x + 1/2*e)^2
+ 9*tan(1/2*f*x + 1/2*e) + 1)/(a^7*f*(tan(1/2*f*x + 1/2*e) + 1)^9)
```

3.442 $\int \frac{\cos^4(e+fx)\sin^3(e+fx)}{(a+a\sin(e+fx))^8}\,dx$

Optimal. Leaf size=157

$$-\frac{152\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)}+\frac{1003\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)^2}-\frac{846\cos(e+fx)}{385a^8f(\sin(e+fx)+1)^3}+\frac{617\cos(e+fx)}{231a^8f(\sin(e+fx)+1)^4}-\frac{}{33a^8}$$

```
[Out] (4*Cos[e + f*x])/(11*a^8*f*(1 + Sin[e + f*x])^6) - (52*Cos[e + f*x])/(33*a^
8*f*(1 + Sin[e + f*x])^5) + (617*Cos[e + f*x])/(231*a^8*f*(1 + Sin[e + f*x]
)^4) - (846*Cos[e + f*x])/(385*a^8*f*(1 + Sin[e + f*x])^3) + (1003*Cos[e +
f*x])/(1155*a^8*f*(1 + Sin[e + f*x])^2) - (152*Cos[e + f*x])/(1155*a^8*f*(1
 + Sin[e + f*x]))
```

Rubi [A] time = 0.56958, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2875, 2872, 2650, 2648}

$$-\frac{152\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)}+\frac{1003\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)^2}-\frac{846\cos(e+fx)}{385a^8f(\sin(e+fx)+1)^3}+\frac{617\cos(e+fx)}{231a^8f(\sin(e+fx)+1)^4}-\frac{}{33a^8}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*Sin[e + f*x]^3)/(a + a*Sin[e + f*x])^8,x]
```

```
[Out] (4*Cos[e + f*x])/(11*a^8*f*(1 + Sin[e + f*x])^6) - (52*Cos[e + f*x])/(33*a^
8*f*(1 + Sin[e + f*x])^5) + (617*Cos[e + f*x])/(231*a^8*f*(1 + Sin[e + f*x]
)^4) - (846*Cos[e + f*x])/(385*a^8*f*(1 + Sin[e + f*x])^3) + (1003*Cos[e +
f*x])/(1155*a^8*f*(1 + Sin[e + f*x])^2) - (152*Cos[e + f*x])/(1155*a^8*f*(1
 + Sin[e + f*x]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)\sin^3(e+fx)}{(a+a\sin(e+fx))^8}\,dx = \frac{\int \sec^9(e+fx)(a-a\sin(e+fx))^8\tan^3(e+fx)\,dx}{a^{16}}$$

$$= \frac{\int \left(-\frac{4}{a^4(1+\sin(e+fx))^6}+\frac{16}{a^4(1+\sin(e+fx))^5}-\frac{25}{a^4(1+\sin(e+fx))^4}+\frac{19}{a^4(1+\sin(e+fx))^3}-\frac{7}{a^4(1+\sin(e+fx}\right.}{a^4}$$

$$= \frac{\int \frac{1}{1+\sin(e+fx)}\,dx}{a^8}-\frac{4\int \frac{1}{(1+\sin(e+fx))^6}\,dx}{a^8}-\frac{7\int \frac{1}{(1+\sin(e+fx))^2}\,dx}{a^8}+\frac{16\int \frac{1}{(1+\sin(e+fx))^5}\,dx}{a^8}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{16\cos(e+fx)}{9a^8f(1+\sin(e+fx))^5}+\frac{25\cos(e+fx)}{7a^8f(1+\sin(e+fx))^4}-\frac{}{5a^8}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{23\cos(e+fx)}{9a^8f(1+\sin(e+fx))^4}-\frac{}{35}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{617\cos(e+fx)}{231a^8f(1+\sin(e+fx))^4}-$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{617\cos(e+fx)}{231a^8f(1+\sin(e+fx))^4}-$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{617\cos(e+fx)}{231a^8f(1+\sin(e+fx))^4}-$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{617\cos(e+fx)}{231a^8f(1+\sin(e+fx))^4}-$$

Mathematica [A] time = 3.07504, size = 195, normalized size = 1.24

$$-\frac{-299970\sin\left(2e+\frac{3fx}{2}\right)+145695\sin\left(2e+\frac{5fx}{2}\right)+44990\sin\left(4e+\frac{7fx}{2}\right)-6710\sin\left(4e+\frac{9fx}{2}\right)+\sin\left(6e+\frac{11fx}{2}\right)-4}{240240a^8f\left(\mathrm{si}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^4*Sin[e + f*x]^3)/(a + a*Sin[e + f*x])^8,x]
```

```
[Out] -(-486024*Cos[e + (f*x)/2] + 351450*Cos[e + (3*f*x)/2] + 180015*Cos[3*e + (
5*f*x)/2] - 63580*Cos[3*e + (7*f*x)/2] - 15004*Cos[5*e + (9*f*x)/2] + 1975*
Cos[5*e + (11*f*x)/2] - 425964*Sin[(f*x)/2] - 299970*Sin[2*e + (3*f*x)/2] +
 145695*Sin[2*e + (5*f*x)/2] + 44990*Sin[4*e + (7*f*x)/2] - 6710*Sin[4*e +
(9*f*x)/2] + Sin[6*e + (11*f*x)/2])/(240240*a^8*f*(Cos[e/2] + Sin[e/2])*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^11)
```

Maple [A] time = 0.156, size = 130, normalized size = 0.8

$$16\,\frac{1}{fa^8}\left(\frac{11}{5\left(\tan\left(1/2\,fx+e/2\right)+1\right)^5}-1/4\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-4}-24\left(\tan\left(1/2\,fx+e/2\right)+1\right)^{-8}-8\left(\tan\left(1/2\,fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x)
```

```
[Out] 16/f/a^8*(11/5/(tan(1/2*f*x+1/2*e)+1)^5-1/4/(tan(1/2*f*x+1/2*e)+1)^4-24/(ta
n(1/2*f*x+1/2*e)+1)^8-8/(tan(1/2*f*x+1/2*e)+1)^10+129/7/(tan(1/2*f*x+1/2*e)
+1)^7+16/11/(tan(1/2*f*x+1/2*e)+1)^11-17/2/(tan(1/2*f*x+1/2*e)+1)^6+56/3/(t
an(1/2*f*x+1/2*e)+1)^9)
```

Maxima [B] time = 1.24958, size = 541, normalized size = 3.45

$$-\frac{4\left(\frac{11\sin(fx+e)}{\cos(fx+e)+1}+\frac{55\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{165\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{825\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2541\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\cdots\right.}{1155\left(a^8+\frac{11a^8\sin(fx+e)}{\cos(fx+e)+1}+\frac{55a^8\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{165a^8\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{330a^8\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{462a^8\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{462a^8\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{330a^8\sin(\cdots}{(\cos(fx+e\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x, algorithm="maxima
")
```

```
[Out] -4/1155*(11*sin(f*x + e)/(cos(f*x + e) + 1) + 55*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 165*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 825*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 2541*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2079*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
1)/((a^8 + 11*a^8*sin(f*x + e)/(cos(f*x + e) + 1) + 55*a^8*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 165*a^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*a^8
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 462*a^8*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 462*a^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 330*a^8*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 165*a^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 55
*a^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*a^8*sin(f*x + e)^10/(cos(f*x
+ e) + 1)^10 + a^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11)*f)
```

Fricas [B] time = 1.05793, size = 775, normalized size = 4.94

$$\frac{152\cos(fx+e)^6-243\cos(fx+e)^5-745\cos(fx+e)^4+455\cos(fx+e)^3+1015\cos(fx+e)^2+(1\cdots}{1155\left(a^8f\cos(fx+e)^6-5a^8f\cos(fx+e)^5-18a^8f\cos(fx+e)^4+20a^8f\cos(fx+e)^3+48a^8f\cos(fx+e)^2-16a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x, algorithm="fricas
")
```

```
[Out] 1/1155*(152*cos(f*x + e)^6 - 243*cos(f*x + e)^5 - 745*cos(f*x + e)^4 + 455*
cos(f*x + e)^3 + 1015*cos(f*x + e)^2 + (152*cos(f*x + e)^5 + 395*cos(f*x +
e)^4 - 350*cos(f*x + e)^3 - 805*cos(f*x + e)^2 + 210*cos(f*x + e) + 420)*si
n(f*x + e) - 210*cos(f*x + e) - 420)/(a^8*f*cos(f*x + e)^6 - 5*a^8*f*cos(f*
x + e)^5 - 18*a^8*f*cos(f*x + e)^4 + 20*a^8*f*cos(f*x + e)^3 + 48*a^8*f*cos
(f*x + e)^2 - 16*a^8*f*cos(f*x + e) - 32*a^8*f - (a^8*f*cos(f*x + e)^5 + 6*
a^8*f*cos(f*x + e)^4 - 12*a^8*f*cos(f*x + e)^3 - 32*a^8*f*cos(f*x + e)^2 +
16*a^8*f*cos(f*x + e) + 32*a^8*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*sin(f*x+e)**3/(a+a*sin(f*x+e))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44066, size = 162, normalized size = 1.03

$$-\frac{4\left(1155\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}-2079\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+2541\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-825\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+165\,\tan\left(\frac{1}{2}\right.\right.}{1155\,a^{8}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x, algorithm="giac")
```

```
[Out] -4/1155*(1155*tan(1/2*f*x + 1/2*e)^7 - 2079*tan(1/2*f*x + 1/2*e)^6 + 2541*t
an(1/2*f*x + 1/2*e)^5 - 825*tan(1/2*f*x + 1/2*e)^4 + 165*tan(1/2*f*x + 1/2*
e)^3 + 55*tan(1/2*f*x + 1/2*e)^2 + 11*tan(1/2*f*x + 1/2*e) + 1)/(a^8*f*(tan
(1/2*f*x + 1/2*e) + 1)^11)
```

3.443 $\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{368a^2\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{1472a^3\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{13ad}+\frac{20\cos^5(c+dx)\sqrt{a\,s}}{143d}$$

```
[Out] (-1472*a^3*Cos[c + d*x]^5)/(45045*d*(a + a*Sin[c + d*x])^(5/2)) - (368*a^2*
Cos[c + d*x]^5)/(9009*d*(a + a*Sin[c + d*x])^(3/2)) - (46*a*Cos[c + d*x]^5)
/(1287*d*Sqrt[a + a*Sin[c + d*x]]) + (20*Cos[c + d*x]^5*Sqrt[a + a*Sin[c +
d*x]])/(143*d) - (2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13*a*d)
```

Rubi [A] time = 0.424233, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2878, 2856, 2674, 2673}

$$-\frac{368a^2\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{1472a^3\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{13ad}+\frac{20\cos^5(c+dx)\sqrt{a\,s}}{143d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-1472*a^3*Cos[c + d*x]^5)/(45045*d*(a + a*Sin[c + d*x])^(5/2)) - (368*a^2*
Cos[c + d*x]^5)/(9009*d*(a + a*Sin[c + d*x])^(3/2)) - (46*a*Cos[c + d*x]^5)
/(1287*d*Sqrt[a + a*Sin[c + d*x]]) + (20*Cos[c + d*x]^5*Sqrt[a + a*Sin[c +
d*x]])/(143*d) - (2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13*a*d)
```


Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13ad} + \frac{2\int \cos^4(c+dx)\left(\frac{3a}{2} - 5a\right.}{}$$
$$= \frac{20\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+a\sin(c+}{13ad}$$
$$= -\frac{46a\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} + \frac{20\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d}$$
$$= -\frac{368a^2\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}} - \frac{46a\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} + \frac{20\,\mathrm{co}}{}$$
$$= -\frac{1472a^3\cos^5(c+dx)}{45045d(a+a\sin(c+dx))^{5/2}} - \frac{368a^2\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}} - \frac{}{12}$$

Mathematica [A] time = 3.73492, size = 109, normalized size = 0.7

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(119780\sin(c+dx)-21420\sin(3(c+dx))-62440\cos(2(c}{180180d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(81183
 - 62440*Cos[2*(c + d*x)] + 3465*Cos[4*(c + d*x)] + 119780*Sin[c + d*x] - 2
1420*Sin[3*(c + d*x)]))/(180180*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.632, size = 85, normalized size = 0.5

$$\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)^3\left(3465\,(\sin(dx+c))^4+10710\,(\sin(dx+c))^3+12145\,(\sin(dx+c))^2+6940\,\mathrm{s}\right.}{45045\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/45045*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^3*(3465*sin(d*x+c)^4+10710*sin(d*x+
c)^3+12145*sin(d*x+c)^2+6940*sin(d*x+c)+2776)/cos(d*x+c)/(a+a*sin(d*x+c))^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^4\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Fricas [A] time = 1.1321, size = 509, normalized size = 3.26

$$\frac{2\left(3465\cos\left(dx+c\right)^7-315\cos\left(dx+c\right)^6-4585\cos\left(dx+c\right)^5+115\cos\left(dx+c\right)^4-184\cos\left(dx+c\right)^3+368\cos\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/45045*(3465*cos(d*x + c)^7 - 315*cos(d*x + c)^6 - 4585*cos(d*x + c)^5 + 1
15*cos(d*x + c)^4 - 184*cos(d*x + c)^3 + 368*cos(d*x + c)^2 - (3465*cos(d*x
 + c)^6 + 3780*cos(d*x + c)^5 - 805*cos(d*x + c)^4 - 920*cos(d*x + c)^3 - 1
104*cos(d*x + c)^2 - 1472*cos(d*x + c) - 2944)*sin(d*x + c) - 1472*cos(d*x
+ c) - 2944)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^4\sin\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

3.444 $\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{16a^2\cos^5(c+dx)}{693d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}-\frac{2a\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+}}$$

```
[Out] (-64*a^3*Cos[c + d*x]^5)/(3465*d*(a + a*Sin[c + d*x])^(5/2)) - (16*a^2*Cos[
c + d*x]^5)/(693*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^5)/(99*d
*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(1
1*d)
```

Rubi [A] time = 0.256545, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$-\frac{16a^2\cos^5(c+dx)}{693d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}-\frac{2a\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^5)/(3465*d*(a + a*Sin[c + d*x])^(5/2)) - (16*a^2*Cos[
c + d*x]^5)/(693*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^5)/(99*d
*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(1
1*d)
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{1}{11}\int \cos^4(c+dx)\sqrt{a+a\sin(c$$
$$= -\frac{2a\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{1}{99}(8a$$
$$= -\frac{16a^2\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx$$
$$= -\frac{64a^3\cos^5(c+dx)}{3465d(a+a\sin(c+dx))^{5/2}} - \frac{16a^2\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos}{99d\sqrt{a+}}$$

Mathematica [A] time = 1.95219, size = 99, normalized size = 0.8

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(-5165\sin(c+dx)+315\sin(3(c+dx))+1960\cos(2(c+dx))-}{6930d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(-3648
+ 1960*Cos[2*(c + d*x)] - 5165*Sin[c + d*x] + 315*Sin[3*(c + d*x)]))/(6930*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.766, size = 75, normalized size = 0.6

$$\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)^3\left(315(\sin(dx+c))^3+980(\sin(dx+c))^2+1055\sin(dx+c)+422\right)}{3465\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3465*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^3*(315*sin(d*x+c)^3+980*sin(d*x+c)^2
+1055*sin(d*x+c)+422)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^4\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [A] time = 1.0462, size = 427, normalized size = 3.44

$$-\frac{2\left(315\cos\left(dx+c\right)^{6}+350\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{4}+8\cos\left(dx+c\right)^{3}-16\cos\left(dx+c\right)^{2}+\left(315\cos\left(dx+c\right)^{5}\right.\right.}{3465\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/3465*(315*cos(d*x + c)^6 + 350*cos(d*x + c)^5 - 5*cos(d*x + c)^4 + 8*cos
(d*x + c)^3 - 16*cos(d*x + c)^2 + (315*cos(d*x + c)^5 - 35*cos(d*x + c)^4 -
 40*cos(d*x + c)^3 - 48*cos(d*x + c)^2 - 64*cos(d*x + c) - 128)*sin(d*x + c
) + 64*cos(d*x + c) + 128)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin
(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

3.445 $\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{164\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105d}+\frac{8a\cos(c\cdots}{15d\sqrt{a\sin(c\cdots}}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
8*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[c
 + d*x]^3)/(7*d*Sqrt[a + a*Sin[c + d*x]]) + (164*Cos[c + d*x]*Sqrt[a + a*Si
n[c + d*x]])/(105*d) - (12*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(35*a*d
)
```

Rubi [A] time = 0.491639, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2881, 2770, 2759, 2751, 2646, 3046, 2981, 2773, 206}

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{164\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105d}+\frac{8a\cos(c\cdots}{15d\sqrt{a\sin(c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
8*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[c
 + d*x]^3)/(7*d*Sqrt[a + a*Sin[c + d*x]]) + (164*Cos[c + d*x]*Sqrt[a + a*Si
n[c + d*x]])/(105*d) - (12*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(35*a*d
)
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc(c+dx)\sqrt{a+a\sin(c+$$

$$= -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{6}{7}$$

$$= \frac{4a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{}}{}$$

$$= \frac{4a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{164\cos(c+dx}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{8a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+}{7d\sqrt{a+}}$$

Mathematica [A] time = 0.339744, size = 195, normalized size = 1.23

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(-525\sin\left(\frac{1}{2}(c+dx)\right)+175\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)+525\cos\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(525*Cos[(c + d*x)/2] + 175*Cos[(3*(c + d*x))/2
] + 21*Cos[(5*(c + d*x))/2] + 15*Cos[(7*(c + d*x))/2] - 420*Log[1 + Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 420*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 525*Sin[(c + d*x)/2] + 175*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*
x))/2] + 15*Sin[(7*(c + d*x))/2]))/(420*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]))
```

Maple [A] time = 1.048, size = 141, normalized size = 0.9

$$-\frac{2+2\sin(dx+c)}{105a^3\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(105a^{7/2}\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-15(a-a\sin(dx+c))^{7/2}+63(a-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/105*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(105*a^(7/2)*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2))-15*(a-a*sin(d*x+c))^(7/2)+63*(a-a*sin(d*x+c))^(5/
2)*a-35*(a-a*sin(d*x+c))^(3/2)*a^2-105*a^3*(a-a*sin(d*x+c))^(1/2))/a^3/cos(
d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^4\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c), x)
```

Fricas [B] time = 1.17624, size = 817, normalized size = 5.14

$$\frac{105\sqrt{a}(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(d}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/210*(105*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3
- 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c)
- 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) +
 (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^
3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1))
 + 4*(15*cos(d*x + c)^4 + 18*cos(d*x + c)^3 + 34*cos(d*x + c)^2 + (15*cos(d
*x + c)^3 - 3*cos(d*x + c)^2 + 31*cos(d*x + c) - 43)*sin(d*x + c) + 74*cos(
d*x + c) + 43)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*sin(d*x + c) +
 d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.446 $\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}+\frac{61a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+}}{d}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) + (
61*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt[a
 + a*Sin[c + d*x]])/(15*d) - (Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d - (2
*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*a*d)
```

Rubi [A] time = 0.477178, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2881, 2759, 2751, 2646, 3044, 2981, 2773, 206}

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}+\frac{61a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) + (
61*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt[a
 + a*Sin[c + d*x]])/(15*d) - (Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d - (2
*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*a*d)
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^2(c+dx)\sqrt{a+a\sin$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - \frac{2\cos(c+dx)(a+a\sin(c+dx}{5ad}$$

$$= \frac{5a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} - \cot(c$$

$$= \frac{61a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} - \cot($$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} + 4\cos(c$$

Mathematica [A] time = 0.73294, size = 258, normalized size = 1.74

$$\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(155\sin\left(\frac{1}{2}(c+dx)\right)+87\sin\left(\frac{3}{2}(c+dx)\right)-5\sin\left(\frac{5}{2}(c+dx)\right)+3\sin\left(\frac{7}{2}(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(-155*Cos[(c + d*x)/2] + 87*
Cos[(3*(c + d*x))/2] + 5*Cos[(5*(c + d*x))/2] + 3*Cos[(7*(c + d*x))/2] + 15
5*Sin[(c + d*x)/2] - 30*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c
+ d*x] + 30*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 87*
Sin[(3*(c + d*x))/2] - 5*Sin[(5*(c + d*x))/2] + 3*Sin[(7*(c + d*x))/2]))/(3
0*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c +
d*x)/4] + Sec[(c + d*x)/4]))
```

Maple [A] time = 1.055, size = 162, normalized size = 1.1

$$\frac{1+\sin(dx+c)}{15\cos(dx+c)\sin(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(30\sqrt{a-a\sin(dx+c)}a^{7/2}+20a^{5/2}(a-a\sin(dx+c))^{3/}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(30*(a-a*sin(d*x+
c))^(1/2)*a^(7/2)+20*a^(5/2)*(a-a*sin(d*x+c))^(3/2)-6*a^(3/2)*(a-a*sin(d*x+
c))^(5/2)-15*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^4)-15*(a-a*sin(d*x+c
))^(1/2)*a^(7/2))/a^(7/2)/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin(dx+c)+a}\cos(dx+c)^4\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^2, x)
```

Fricas [B] time = 1.27564, size = 867, normalized size = 5.86

$$15\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos\right.}{\cos(dx+c)^3+\cos(dx+c)^2+(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c)
```

```
+ 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) -
 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c)
- a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) -
 cos(d*x + c) - 1)) - 4*(6*cos(d*x + c)^4 + 8*cos(d*x + c)^3 + 40*cos(d*x +
 c)^2 + (6*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + 38*cos(d*x + c) + 61)*sin(d*
x + c) - 23*cos(d*x + c) - 61)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2
- (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.447 $\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{13\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\cot($$

```
[Out] (13*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x])/(4*
d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*
d) - (Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rubi [A] time = 0.411593, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2881, 2751, 2646, 3044, 2980, 2773, 206}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{13\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\cot($$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (13*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x])/(4*
d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*
d) - (Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \int \sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^3(c+dx)\sqrt{a+a\sin(c\\ &= -\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+}}{2d}\\ &= -\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{}}{3}\\ &= -\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{}}{3}\\ &= \frac{13\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(}{4d\sqrt{a+a}}\end{aligned}$$

Mathematica [A] time = 0.874459, size = 297, normalized size = 1.9

$$\csc^7\left(\tfrac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(26\sin\left(\tfrac{1}{2}(c+dx)\right) - 14\sin\left(\tfrac{3}{2}(c+dx)\right) - 12\sin\left(\tfrac{5}{2}(c+dx)\right) + 4\sin\left(\tfrac{7}{2}(c+dx)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-26*Cos[(c + d*x)/2] - 14*C
os[(3*(c + d*x))/2] + 12*Cos[(5*(c + d*x))/2] + 4*Cos[(7*(c + d*x))/2] + 39
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39*Cos[2*(c + d*x)]*Log[1 +
 Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] + 39*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] + 26*Sin[(c + d*x)/2] - 14*Sin[(3*(c + d*x))/2] - 12*Sin[(5*(c + d*x))
/2] + 4*Sin[(7*(c + d*x))/2]))/(12*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/
4]^2 - Sec[(c + d*x)/4]^2)^2)
```

Maple [A] time = 0.981, size = 178, normalized size = 1.1

$$\frac{1+\sin(dx+c)}{12\,(\sin(dx+c))^2\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(8\,(-a(\sin(dx+c)-1))^{3/2}(\sin(dx+c))^2\sqrt{a}-24\sqrt{-a(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/12*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(3/2)*(8*(-a*(sin(d*x+c)-1)
)^(3/2)*sin(d*x+c)^2*a^(1/2)-24*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c
)^2+39*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+9*(-a*(s
in(d*x+c)-1))^(3/2)*a^(1/2)-15*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))/sin(d*x+c
)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin(dx+c)+a}\cos(dx+c)^4\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^3, x)
```

Fricas [B] time = 1.18815, size = 961, normalized size = 6.16

$$\frac{39\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\,(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(39*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8
*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (
```

```
cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(8*cos(d*x + c)^4
 + 16*cos(d*x + c)^3 - 9*cos(d*x + c)^2 + (8*cos(d*x + c)^3 - 8*cos(d*x + c
)^2 - 17*cos(d*x + c) + 5)*sin(d*x + c) - 22*cos(d*x + c) - 5)*sqrt(a*sin(d
*x + c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*co
s(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.448 $\int \cot^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=163

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{11a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (2*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + (11*a*Cot[c + d*x])/(8
*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x])/(12*d*Sqrt[a +
 a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/
(3*d)
```

Rubi [A] time = 0.388503, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2718, 2646, 3044, 2980, 2772, 2773, 206}

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{11a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (2*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + (11*a*Cot[c + d*x])/(8
*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x])/(12*d*Sqrt[a +
 a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/
(3*d)
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sqrt{a+a\sin(c+dx)}\,dx + \int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\left(1-2\sin^2(c+\right.$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{\int \csc^3}{}$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d}$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{11a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{11a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{11a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [A] time = 1.39802, size = 309, normalized size = 1.9

$$\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-252\sin\left(\frac{1}{2}(c+dx)\right)-250\sin\left(\frac{3}{2}(c+dx)\right)+114\sin\left(\frac{5}{2}(c+dx)\right)+48\sin\left(\frac{7}{2}(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(252*Cos[(c + d*x)/2] - 250
*Cos[(3*(c + d*x))/2] - 114*Cos[(5*(c + d*x))/2] + 48*Cos[(7*(c + d*x))/2]
- 252*Sin[(c + d*x)/2] + 99*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Si
n[c + d*x] - 99*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] -
 250*Sin[(3*(c + d*x))/2] + 114*Sin[(5*(c + d*x))/2] - 33*Log[1 + Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 33*Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 48*Sin[(7*(c + d*x))/2]))/(24*d*(1 +
 Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3)
```

Maple [A] time = 1.127, size = 170, normalized size = 1.

$$\frac{1+\sin(dx+c)}{24\,(\sin(dx+c))^3\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(-48\sqrt{-a(\sin(dx+c)-1)}a^{7/2}(\sin(dx+c))^3+33\,(-a(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(-48*(-a*(sin(d*x+c)-
1))^(1/2)*a^(7/2)*sin(d*x+c)^3+33*(-a*(sin(d*x+c)-1))^(5/2)*a^(3/2)+33*arct
anh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^3-56*(-a*(sin(d*x+c)-
1))^(3/2)*a^(5/2)+15*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))/sin(d*x+c)^3/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin(dx+c)+a}\cos(dx+c)^4\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^4, x)
```

Fricas [B] time = 1.11681, size = 1025, normalized size = 6.29

$$\frac{33\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3}{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(33*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
4*(48*cos(d*x + c)^4 - 33*cos(d*x + c)^3 - 139*cos(d*x + c)^2 + (48*cos(d*x
 + c)^3 + 81*cos(d*x + c)^2 - 58*cos(d*x + c) - 83)*sin(d*x + c) + 25*cos(d
*x + c) + 83)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c
)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x +
c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.449 $\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=173

$$\frac{61a\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{67\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{a\cot(c+dx)\csc^2}{24d\sqrt{a\sin(c+}}$$

```
[Out] (-67*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*
d) + (61*a*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (61*a*Cot[c + d*
x]*Csc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c +
d*x]^2)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt
[a + a*Sin[c + d*x]])/(4*d)
```

Rubi [A] time = 0.538, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2881, 2773, 206, 3044, 2980, 2772}

$$\frac{61a\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{67\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{a\cot(c+dx)\csc^2}{24d\sqrt{a\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-67*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*
d) + (61*a*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (61*a*Cot[c + d*
x]*Csc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c +
d*x]^2)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt
[a + a*Sin[c + d*x]])/(4*d)
```


Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^5(c+dx)\sqrt{a+a\sin(c$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{\int \csc^4(c+dx)\left(\frac{a}{2}\right.}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{a\,c}{2}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} + \frac{61a\cot(}{96d\sqrt{}}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} + \frac{61a\cot(}{96d\sqrt{}}$$

$$= -\frac{67\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} + \frac{61a\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} + \frac{61a\cot}{96d}$$

Mathematica [B] time = 2.66687, size = 367, normalized size = 2.12

$$-\frac{\csc^{13}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-442\sin\left(\frac{1}{2}(c+dx)\right)-162\sin\left(\frac{3}{2}(c+dx)\right)-122\sin\left(\frac{5}{2}(c+dx)\right)+366\sin\left(\frac{7}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(Csc[(c + d*x)/2]^13*Sqrt[a*(1 + Sin[c + d*x])]*(442*Cos[(c + d*x)/2] - 16
2*Cos[(3*(c + d*x))/2] + 122*Cos[(5*(c + d*x))/2] + 366*Cos[(7*(c + d*x))/2
] + 603*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 804*Cos[2*(c + d*x)]
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 201*Cos[4*(c + d*x)]*Log[1
+ Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 603*Log[1 - Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + 804*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]] - 201*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
 - 442*Sin[(c + d*x)/2] - 162*Sin[(3*(c + d*x))/2] - 122*Sin[(5*(c + d*x))/
2] + 366*Sin[(7*(c + d*x))/2]))/(192*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x
)/4]^2 - Sec[(c + d*x)/4]^2)^4)
```

Maple [A] time = 1.003, size = 162, normalized size = 0.9

$$-\frac{1+\sin(dx+c)}{192\,(\sin(dx+c))^4\cos(dx+c)\,d}\sqrt{-a\,(\sin(dx+c)-1)}\left(201\,\mathrm{Artanh}\left(\frac{\sqrt{-a\,(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin(dx+c))^4\,a^4+183\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(201*arctanh((-a*(s
in(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^4*a^4+183*(-a*(sin(d*x+c)-1))^(7/2)
*a^(1/2)-671*(-a*(sin(d*x+c)-1))^(5/2)*a^(3/2)+737*(-a*(sin(d*x+c)-1))^(3/2
)*a^(5/2)-201*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))/sin(d*x+c)^4/cos(d*x+c)/(a
+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^4\csc(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^5, x)
```

Fricas [B] time = 1.26237, size = 1141, normalized size = 6.6

$$\frac{201\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\sin(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/768*(201*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x
+ c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x + c
) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)
^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x +
 c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
- a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 -
 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(183*cos(d*x + c)^4 + 122*cos(d*x
 + c)^3 - 188*cos(d*x + c)^2 + (183*cos(d*x + c)^3 + 61*cos(d*x + c)^2 - 12
7*cos(d*x + c) - 53)*sin(d*x + c) - 74*cos(d*x + c) + 53)*sqrt(a*sin(d*x +
c) + a))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 - 2*d*co
s(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.450 $\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=209

$$-\frac{31a\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{31\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{a\cot(c+dx)\csc}{40d\sqrt{a\sin(c+}}$$

```
[Out] (-31*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128
*d) - (31*a*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (97*a*Cot[c +
d*x]*Csc[c + d*x])/(192*d*Sqrt[a + a*Sin[c + d*x]]) + (97*a*Cot[c + d*x]*Cs
c[c + d*x]^2)/(240*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*
x]^3)/(40*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[a
 + a*Sin[c + d*x]])/(5*d)
```

Rubi [A] time = 0.690556, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2881, 2772, 2773, 206, 3044, 2980}

$$-\frac{31a\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{31\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{a\cot(c+dx)\csc}{40d\sqrt{a\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-31*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128
*d) - (31*a*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (97*a*Cot[c +
d*x]*Csc[c + d*x])/(192*d*Sqrt[a + a*Sin[c + d*x]]) + (97*a*Cot[c + d*x]*Cs
c[c + d*x]^2)/(240*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*
x]^3)/(40*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[a
 + a*Sin[c + d*x]])/(5*d)
```


Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^6(c+dx)\sqrt{a+a\sin}$$

$$= -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+}}{5d}$$

$$= -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{97a\cot(c+}{240d\sqrt{a}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{97a\cot(c+}{192d\sqrt{a}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{31a\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} + \frac{97a\cot}{192d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{31a\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} + \frac{97a\cot}{192d}$$

$$= -\frac{31\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128d} - \frac{31a\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} + \frac{97a}{192}$$

Mathematica [A] time = 4.37091, size = 403, normalized size = 1.93

$$-\frac{\csc^{16}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-10180\sin\left(\frac{1}{2}(c+dx)\right)-2240\sin\left(\frac{3}{2}(c+dx)\right)+1392\sin\left(\frac{5}{2}(c+dx)\right)+4810\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(Csc[(c + d*x)/2]^16*Sqrt[a*(1 + Sin[c + d*x])]*(10180*Cos[(c + d*x)/2] -
2240*Cos[(3*(c + d*x))/2] - 1392*Cos[(5*(c + d*x))/2] + 4810*Cos[(7*(c + d*
x))/2] + 930*Cos[(9*(c + d*x))/2] - 10180*Sin[(c + d*x)/2] + 4650*Log[1 + C
os[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 4650*Log[1 - Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 2240*Sin[(3*(c + d*x))/2] + 1392*S
in[(5*(c + d*x))/2] - 2325*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] + 2325*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c
+ d*x)] + 4810*Sin[(7*(c + d*x))/2] - 930*Sin[(9*(c + d*x))/2] + 465*Log[1
+ Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 465*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(1920*d*(1 + Cot[(c + d*
x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^5)
```

Maple [A] time = 1.2, size = 180, normalized size = 0.9

$$\frac{1+\sin(dx+c)}{1920\left(\sin(dx+c)\right)^{5}\cos(dx+c)\,d}\sqrt{-a\left(\sin(dx+c)-1\right)}\left(465\sqrt{-a\left(\sin(dx+c)-1\right)}a^{11/2}-2170\left(-a\left(\sin(dx+c)-1\right)\right)^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/1920*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(465*(-a*(sin(d*x+c)-1))^(1
/2)*a^(11/2)-2170*(-a*(sin(d*x+c)-1))^(3/2)*a^(9/2)+896*(-a*(sin(d*x+c)-1))
^(5/2)*a^(7/2)+890*(-a*(sin(d*x+c)-1))^(7/2)*a^(5/2)-465*(-a*(sin(d*x+c)-1)
)^(9/2)*a^(3/2)-465*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^6*sin(d*x+
c)^5)/a^(11/2)/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^{4}\csc(dx+c)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^6, x)
```

Fricas [B] time = 1.30777, size = 1261, normalized size = 6.03

$$\frac{465\left(\cos(dx+c)^{6}-3\cos(dx+c)^{4}+3\cos(dx+c)^{2}-\left(\cos(dx+c)^{5}+\cos(dx+c)^{4}-2\cos(dx+c)^{3}-2\cos(dx+c)^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/7680*(465*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*
x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
 c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(465*cos(d*x
+ c)^5 + 1435*cos(d*x + c)^4 - 154*cos(d*x + c)^3 - 1662*cos(d*x + c)^2 - (
465*cos(d*x + c)^4 - 970*cos(d*x + c)^3 - 1124*cos(d*x + c)^2 + 538*cos(d*x
 + c) + 611)*sin(d*x + c) + 73*cos(d*x + c) + 611)*sqrt(a*sin(d*x + c) + a)
)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - (d*cos(d*x
+ c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos
(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.451 $\int \cot^4(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=245

$$-\frac{55a\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}-\frac{55\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}-\frac{\cot(c+dx)\csc^5(c+dx)\sqrt{a\sin(c+dx)+a}}{6d}-\frac{a\cot(c+dx)\,\mathrm{c}}{60d\sqrt{a\sin(c}}$$

```
[Out] (-55*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(512
*d) - (55*a*Cot[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) - (55*a*Cot[c +
d*x]*Csc[c + d*x])/(768*d*Sqrt[a + a*Sin[c + d*x]]) + (329*a*Cot[c + d*x]*C
sc[c + d*x]^2)/(960*d*Sqrt[a + a*Sin[c + d*x]]) + (47*a*Cot[c + d*x]*Csc[c
+ d*x]^3)/(160*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^4
)/(60*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^5*Sqrt[a + a
*Sin[c + d*x]])/(6*d)
```

Rubi [A] time = 0.811591, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2881, 2772, 2773, 206, 3044, 2980}

$$-\frac{55a\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}-\frac{55\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}-\frac{\cot(c+dx)\csc^5(c+dx)\sqrt{a\sin(c+dx)+a}}{6d}-\frac{a\cot(c+dx)\,\mathrm{c}}{60d\sqrt{a\sin(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-55*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(512
*d) - (55*a*Cot[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) - (55*a*Cot[c +
d*x]*Csc[c + d*x])/(768*d*Sqrt[a + a*Sin[c + d*x]]) + (329*a*Cot[c + d*x]*C
sc[c + d*x]^2)/(960*d*Sqrt[a + a*Sin[c + d*x]]) + (47*a*Cot[c + d*x]*Csc[c
+ d*x]^3)/(160*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^4
)/(60*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^5*Sqrt[a + a
*Sin[c + d*x]])/(6*d)
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^7(c+dx)\sqrt{a+a\sin(c$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^5(c+dx)\sqrt{a+a\sin(c+}{6d}$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\,c}{60d\sqrt{a+a\,\mathrm{si}}}$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{47a\cot(c+dx}{160d\sqrt{a+a}}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx}{2d\sqrt{a+a}}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{55a\cot(c+}{768d\sqrt{a+}}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{55a\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} - \frac{55a\cot(c}{768d\sqrt{a}}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{55a\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} - \frac{55a\cot(c}{768d\sqrt{a}}$$

$$= -\frac{55\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{512d} - \frac{55a\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} - \frac{55a\cot(}{768d\sqrt{}}$$

Mathematica [A] time = 7.57846, size = 485, normalized size = 1.98

$$\csc^{19}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(24540\sin\left(\frac{1}{2}(c+dx)\right) - 25684\sin\left(\frac{3}{2}(c+dx)\right) + 14490\sin\left(\frac{5}{2}(c+dx)\right) - 15006\,\mathrm{si}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^19*Sqrt[a*(1 + Sin[c + d*x])]*(-24540*Cos[(c + d*x)/2] -
25684*Cos[(3*(c + d*x))/2] - 14490*Cos[(5*(c + d*x))/2] - 15006*Cos[(7*(c +
 d*x))/2] - 550*Cos[(9*(c + d*x))/2] - 1650*Cos[(11*(c + d*x))/2] - 8250*Lo
g[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12375*Cos[2*(c + d*x)]*Log[1 +
 Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4950*Cos[4*(c + d*x)]*Log[1 + Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 825*Cos[6*(c + d*x)]*Log[1 + Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] + 8250*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 12375*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4
950*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 825*Cos
[6*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24540*Sin[(c +
 d*x)/2] - 25684*Sin[(3*(c + d*x))/2] + 14490*Sin[(5*(c + d*x))/2] - 15006*
Sin[(7*(c + d*x))/2] + 550*Sin[(9*(c + d*x))/2] - 1650*Sin[(11*(c + d*x))/2
]))/(7680*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2
)^6)
```

Maple [A] time = 1.151, size = 198, normalized size = 0.8

$$\frac{1+\sin(dx+c)}{7680\,(\sin(dx+c))^6\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(825\,(-a(\sin(dx+c)-1))^{11/2}a^{5/2} - 4675\,(-a(\sin(dx+c)-1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/7680*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(15/2)*(825*(-a*(sin(d*x+
c)-1))^(11/2)*a^(5/2)-4675*(-a*(sin(d*x+c)-1))^(9/2)*a^(7/2)-825*arctanh((-
a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^8*sin(d*x+c)^6+7818*(-a*(sin(d*x+c)-1))^
(7/2)*a^(9/2)-1398*(-a*(sin(d*x+c)-1))^(5/2)*a^(11/2)-4675*(-a*(sin(d*x+c)-
1))^(3/2)*a^(13/2)+825*(-a*(sin(d*x+c)-1))^(1/2)*a^(15/2))/sin(d*x+c)^6/cos
(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^4\csc(dx+c)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^7, x)
```

Fricas [B] time = 1.2622, size = 1423, normalized size = 5.81

$$\frac{825\left(\cos(dx+c)^7+\cos(dx+c)^6-3\cos(dx+c)^5-3\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/30720*(825*(cos(d*x + c)^7 + cos(d*x + c)^6 - 3*cos(d*x + c)^5 - 3*cos(d*
x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^6 - 3*cos(d*
x + c)^4 + 3*cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c
) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
 - cos(d*x + c) - 1)) + 4*(825*cos(d*x + c)^6 + 550*cos(d*x + c)^5 + 707*co
s(d*x + c)^4 + 1156*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + (825*cos(d*x + c)
^5 + 275*cos(d*x + c)^4 + 982*cos(d*x + c)^3 - 174*cos(d*x + c)^2 - 399*cos
(d*x + c) + 27)*sin(d*x + c) - 426*cos(d*x + c) - 27)*sqrt(a*sin(d*x + c) +
 a))/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 - 3*d*cos(d*
x + c)^4 + 3*d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*co
s(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c) -
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.452 $\int \cot^4(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{61a\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}-\frac{61\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}-\frac{\cot(c+dx)\csc^6(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{a\cot(c+}{84d\sqrt{a\,\mathrm{si}}}$$

```
[Out] (-61*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(102
4*d) - (61*a*Cot[c + d*x])/(1024*d*Sqrt[a + a*Sin[c + d*x]]) - (61*a*Cot[c
+ d*x]*Csc[c + d*x])/(1536*d*Sqrt[a + a*Sin[c + d*x]]) - (61*a*Cot[c + d*x]
*Csc[c + d*x]^2)/(1920*d*Sqrt[a + a*Sin[c + d*x]]) + (579*a*Cot[c + d*x]*Cs
c[c + d*x]^3)/(2240*d*Sqrt[a + a*Sin[c + d*x]]) + (193*a*Cot[c + d*x]*Csc[c
 + d*x]^4)/(840*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^
5)/(84*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^6*Sqrt[a +
a*Sin[c + d*x]])/(7*d)
```

Rubi [A] time = 0.947658, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2881, 2772, 2773, 206, 3044, 2980}

$$-\frac{61a\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}-\frac{61\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}-\frac{\cot(c+dx)\csc^6(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{a\cot(c+}{84d\sqrt{a\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-61*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(102
4*d) - (61*a*Cot[c + d*x])/(1024*d*Sqrt[a + a*Sin[c + d*x]]) - (61*a*Cot[c
+ d*x]*Csc[c + d*x])/(1536*d*Sqrt[a + a*Sin[c + d*x]]) - (61*a*Cot[c + d*x]
*Csc[c + d*x]^2)/(1920*d*Sqrt[a + a*Sin[c + d*x]]) + (579*a*Cot[c + d*x]*Cs
c[c + d*x]^3)/(2240*d*Sqrt[a + a*Sin[c + d*x]]) + (193*a*Cot[c + d*x]*Csc[c
 + d*x]^4)/(840*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^
5)/(84*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^6*Sqrt[a +
a*Sin[c + d*x]])/(7*d)
```


Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^8(c+dx)\sqrt{a+a\sin$$

$$= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^6(c+dx)\sqrt{a+a\sin}}{7d}$$

$$= -\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot}{84a}$$

$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+}{3d\sqrt{a+}}$$

$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+}{3d\sqrt{a+}}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c}{12d\sqrt{a}}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{61a\cot(}{1536d\sqrt{}}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{61a\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}} - \frac{61a\,c}{153}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{61a\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}} - \frac{61a\,c}{153}$$

$$= -\frac{61\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{1024d} - \frac{61a\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}} - \frac{61a}{15}$$

Mathematica [A] time = 2.05157, size = 191, normalized size = 0.68

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(-102480\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)+102480\log\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(-102480*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 102480*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Csc[c + d*x]
^7*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(-201298 - 244533*Cos[2*(c + d*x)]
 - 52094*Cos[4*(c + d*x)] + 6405*Cos[6*(c + d*x)] + 49128*Sin[c + d*x] - 17
9636*Sin[3*(c + d*x)] - 8540*Sin[5*(c + d*x)])))/(3440640*d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 1.28, size = 216, normalized size = 0.8

$$-\frac{1+\sin(dx+c)}{107520\,(\sin(dx+c))^7\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(6405\,(-a(\sin(dx+c)-1))^{13/2}a^{7/2}-42700\,(-a(\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/107520*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(19/2)*(6405*(-a*(sin(
d*x+c)-1))^(13/2)*a^(7/2)-42700*(-a*(sin(d*x+c)-1))^(11/2)*a^(9/2)+120841*(
-a*(sin(d*x+c)-1))^(9/2)*a^(11/2)+6405*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^
(1/2))*a^10*sin(d*x+c)^7-156672*(-a*(sin(d*x+c)-1))^(7/2)*a^(13/2)+51191*(-
a*(sin(d*x+c)-1))^(5/2)*a^(15/2)+42700*(-a*(sin(d*x+c)-1))^(3/2)*a^(17/2)-6
405*(-a*(sin(d*x+c)-1))^(1/2)*a^(19/2))/sin(d*x+c)^7/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^4\csc(dx+c)^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^8, x)
```

Fricas [B] time = 1.22736, size = 1565, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/430080*(6405*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*co
s(d*x + c)^2 - (cos(d*x + c)^7 + cos(d*x + c)^6 - 3*cos(d*x + c)^5 - 3*cos(
d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 - cos(d*x + c) - 1)*sin(d*
x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(
d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x
+ c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c
)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(6405*cos(d*x + c)^7 + 2135*
cos(d*x + c)^6 - 22631*cos(d*x + c)^5 - 37613*cos(d*x + c)^4 + 1343*cos(d*x
 + c)^3 + 27477*cos(d*x + c)^2 - (6405*cos(d*x + c)^6 + 4270*cos(d*x + c)^5
 - 18361*cos(d*x + c)^4 + 19252*cos(d*x + c)^3 + 20595*cos(d*x + c)^2 - 688
2*cos(d*x + c) - 7359)*sin(d*x + c) - 477*cos(d*x + c) - 7359)*sqrt(a*sin(d
*x + c) + a))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 -
 4*d*cos(d*x + c)^2 - (d*cos(d*x + c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x +
c)^5 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos
(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.453 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=188

$$-\frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^5(c+dx)}{1287d(a\sin(c+dx)+a)^{3/2}}-\frac{256a^4\cos^5(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+a}{15ad}$$

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(6435*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(1287*d*(a + a*Sin[c + d*x])^(3/2)) - (56*a^2*Cos[c + d*x]^5)/
(1287*d*Sqrt[a + a*Sin[c + d*x]]) - (14*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c +
 d*x]])/(429*d) + (4*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(39*d) - (2
*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(15*a*d)
```

Rubi [A] time = 0.51479, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2878, 2856, 2674, 2673}

$$-\frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^5(c+dx)}{1287d(a\sin(c+dx)+a)^{3/2}}-\frac{256a^4\cos^5(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+a}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(6435*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(1287*d*(a + a*Sin[c + d*x])^(3/2)) - (56*a^2*Cos[c + d*x]^5)/
(1287*d*Sqrt[a + a*Sin[c + d*x]]) - (14*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c +
 d*x]])/(429*d) + (4*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(39*d) - (2
*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(15*a*d)
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^5(c+dx)(a+a\sin(c+dx))^{5/2}}{15ad} + \frac{2\int \cos^4(c+dx)\left(\frac{5a}{2} - \right.}{}$$

$$= \frac{4\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{39d} - \frac{2\cos^5(c+dx)(a+a\sin(}{15ad}$$

$$= -\frac{14a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{429d} + \frac{4\cos^5(c+dx)(a+a\sin}{39d}$$

$$= -\frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} - \frac{14a\cos^5(c+dx)\sqrt{a+a\sin(c+d}}{429d}$$

$$= -\frac{64a^3\cos^5(c+dx)}{1287d(a+a\sin(c+dx))^{3/2}} - \frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} - \frac{14a}{}$$

$$= -\frac{256a^4\cos^5(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}} - \frac{64a^3\cos^5(c+dx)}{1287d(a+a\sin(c+dx))^{3/2}} - \frac{}{1}$$

Mathematica [A] time = 8.84998, size = 120, normalized size = 0.64

$$-\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(66470\sin(c+dx)-14445\sin(3(c+dx))+429\sin(5(c+}{51480d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(431
22 - 36640*Cos[2*(c + d*x)] + 3630*Cos[4*(c + d*x)] + 66470*Sin[c + d*x] -
14445*Sin[3*(c + d*x)] + 429*Sin[5*(c + d*x)]))/(51480*d*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]))
```

Maple [A] time = 0.693, size = 97, normalized size = 0.5

$$\frac{(2+2\sin(dx+c))a^2(\sin(dx+c)-1)^3\left(429(\sin(dx+c))^5+1815(\sin(dx+c))^4+3075(\sin(dx+c))^3+2765(\mathrm{si}\right.}{6435\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/6435*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^3*(429*sin(d*x+c)^5+1815*sin(d*x+c
)^4+3075*sin(d*x+c)^3+2765*sin(d*x+c)^2+1580*sin(d*x+c)+632)/cos(d*x+c)/(a+
a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Fricas [A] time = 1.15855, size = 597, normalized size = 3.18

$$2\left(429\,a\cos(dx+c)^8+957\,a\cos(dx+c)^7-633\,a\cos(dx+c)^6-1301\,a\cos(dx+c)^5+20\,a\cos(dx+c)^4-32\,a\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 2/6435*(429*a*cos(d*x + c)^8 + 957*a*cos(d*x + c)^7 - 633*a*cos(d*x + c)^6
- 1301*a*cos(d*x + c)^5 + 20*a*cos(d*x + c)^4 - 32*a*cos(d*x + c)^3 + 64*a*
cos(d*x + c)^2 - 256*a*cos(d*x + c) + (429*a*cos(d*x + c)^7 - 528*a*cos(d*x
 + c)^6 - 1161*a*cos(d*x + c)^5 + 140*a*cos(d*x + c)^4 + 160*a*cos(d*x + c)
^3 + 192*a*cos(d*x + c)^2 + 256*a*cos(d*x + c) + 512*a)*sin(d*x + c) - 512*
a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

3.454 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=156

$$-\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^5(c+dx)}{1001d(a\sin(c+dx)+a)^{3/2}}-\frac{256a^4\cos^5(c+dx)}{5005d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+}{13d}$$

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(5005*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(1001*d*(a + a*Sin[c + d*x])^(3/2)) - (8*a^2*Cos[c + d*x]^5)/(
143*d*Sqrt[a + a*Sin[c + d*x]]) - (6*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*
x]])/(143*d) - (2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13*d)
```

Rubi [A] time = 0.324718, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$-\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^5(c+dx)}{1001d(a\sin(c+dx)+a)^{3/2}}-\frac{256a^4\cos^5(c+dx)}{5005d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(5005*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(1001*d*(a + a*Sin[c + d*x])^(3/2)) - (8*a^2*Cos[c + d*x]^5)/(
143*d*Sqrt[a + a*Sin[c + d*x]]) - (6*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*
x]])/(143*d) - (2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13*d)
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13d} + \frac{3}{13}\int \cos^4(c+dx)(a+a\sin$$
$$= -\frac{6a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+a\sin(c+}{13d}$$
$$= -\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a+a\sin(c+dx)}} - \frac{6a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d} - 2$$
$$= -\frac{64a^3\cos^5(c+dx)}{1001d(a+a\sin(c+dx))^{3/2}} - \frac{8a^2\cos^5(c+dx)}{143d\sqrt{a+a\sin(c+dx)}} - 6a\cos^5(c$$
$$= -\frac{256a^4\cos^5(c+dx)}{5005d(a+a\sin(c+dx))^{5/2}} - \frac{64a^3\cos^5(c+dx)}{1001d(a+a\sin(c+dx))^{3/2}} - \frac{8a}{143d}$$

Mathematica [A] time = 5.03024, size = 110, normalized size = 0.71

$$-\frac{a\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(28230\sin(c+dx)-3290\sin(3(c+dx))-12600\cos(2(c+d}{20020d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(195
59 - 12600*Cos[2*(c + d*x)] + 385*Cos[4*(c + d*x)] + 28230*Sin[c + d*x] - 3
290*Sin[3*(c + d*x)]))/(20020*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.777, size = 87, normalized size = 0.6

$$\frac{(2+2\sin(dx+c))\,a^2(\sin(dx+c)-1)^3\left(385(\sin(dx+c))^4+1645(\sin(dx+c))^3+2765(\sin(dx+c))^2+2295\sin(d}{5005\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5005*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^3*(385*sin(d*x+c)^4+1645*sin(d*x+c
)^3+2765*sin(d*x+c)^2+2295*sin(d*x+c)+918)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [A] time = 1.05441, size = 533, normalized size = 3.42

$$\frac{2\left(385\,a\cos\left(dx+c\right)^{7}-490\,a\cos\left(dx+c\right)^{6}-1015\,a\cos\left(dx+c\right)^{5}+20\,a\cos\left(dx+c\right)^{4}-32\,a\cos\left(dx+c\right)^{3}+64\,a\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/5005*(385*a*cos(d*x + c)^7 - 490*a*cos(d*x + c)^6 - 1015*a*cos(d*x + c)^5
 + 20*a*cos(d*x + c)^4 - 32*a*cos(d*x + c)^3 + 64*a*cos(d*x + c)^2 - 256*a*
cos(d*x + c) - (385*a*cos(d*x + c)^6 + 875*a*cos(d*x + c)^5 - 140*a*cos(d*x
 + c)^4 - 160*a*cos(d*x + c)^3 - 192*a*cos(d*x + c)^2 - 256*a*cos(d*x + c)
- 512*a)*sin(d*x + c) - 512*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d
*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

3.455 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=199

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{14a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+$$

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
14*a^2*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (34*a^2*Cos[c + d*x]
*Sin[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[
c + d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (388*a*Cos[c + d*x]*Sqrt[a + a
*Sin[c + d*x]])/(315*d) + (16*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105
*d)
```

Rubi [A] time = 0.710798, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2881, 2763, 21, 2770, 2759, 2751, 2646, 3046, 2976, 2981, 2773, 206}

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{14a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
14*a^2*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (34*a^2*Cos[c + d*x]
*Sin[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[
c + d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (388*a*Cos[c + d*x]*Sqrt[a + a
*Sin[c + d*x]])/(315*d) + (16*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105
*d)
```


Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc(c+dx)(a+a\sin($$

$$= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}$$

$$= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{4a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}$$

$$= \frac{6a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c}{9d\sqrt{a}}$$

$$= \frac{6a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c}{9d\sqrt{a}}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{6a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos(}{63d\sqrt{a}}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{14a^2\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos}{63d\sqrt{}}$$

Mathematica [A] time = 0.48403, size = 219, normalized size = 1.1

$$(a(\sin(c+dx)+1))^{3/2}\left(-1260\sin\left(\frac{1}{2}(c+dx)\right)+1470\sin\left(\frac{3}{2}(c+dx)\right)+126\sin\left(\frac{5}{2}(c+dx)\right)+135\sin\left(\frac{7}{2}(c+dx)\right)+35\sin$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(1260*Cos[(c + d*x)/2] + 1470*Cos[(3*(c + d*x
))/2] - 126*Cos[(5*(c + d*x))/2] + 135*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c
+ d*x))/2] - 2520*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2520*Log[1
```

```
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 1260*Sin[(c + d*x)/2] + 1470*Sin[
(3*(c + d*x))/2] + 126*Sin[(5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))/2] + 35
*Sin[(9*(c + d*x))/2]))/(2520*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A] time = 0.864, size = 159, normalized size = 0.8

$$-\frac{2+2\sin(dx+c)}{315\,a^3\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(315\,a^{9/2}\text{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)+35\,(a-a\sin(dx+c))^{9/2}-225\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/315*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(315*a^(9/2)*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2))+35*(a-a*sin(d*x+c))^(9/2)-225*a*(a-a*sin(d*x+c))^
(7/2)+441*a^2*(a-a*sin(d*x+c))^(5/2)-105*a^3*(a-a*sin(d*x+c))^(3/2)-315*a^4
*(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c), x)
```

Fricas [A] time = 1.25361, size = 903, normalized size = 4.54

$$315\,(a\cos(dx+c)+a\sin(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/630*(315*(a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c
)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x +
 c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x +
c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) -
 1)) - 4*(35*a*cos(d*x + c)^5 - 50*a*cos(d*x + c)^4 - 46*a*cos(d*x + c)^3 -
 118*a*cos(d*x + c)^2 - 158*a*cos(d*x + c) - (35*a*cos(d*x + c)^4 + 85*a*co
s(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 157*a*cos(d*x + c) - a)*sin(d*x + c) -
 a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.456 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=178

$$\frac{171a^2\cos(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin(c+$$

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
171*a^2*Cos[c + d*x])/(35*d*Sqrt[a + a*Sin[c + d*x]]) + (69*a*Cos[c + d*x]*
Sqrt[a + a*Sin[c + d*x]])/(35*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/
2))/(35*d) - (Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/d - (2*Cos[c + d*x]*
(a + a*Sin[c + d*x])^(5/2))/(7*a*d)
```

Rubi [A] time = 0.649182, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2881, 2759, 2751, 2647, 2646, 3044, 2976, 2981, 2773, 206}

$$\frac{171a^2\cos(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
171*a^2*Cos[c + d*x])/(35*d*Sqrt[a + a*Sin[c + d*x]]) + (69*a*Cos[c + d*x]*
Sqrt[a + a*Sin[c + d*x]])/(35*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/
2))/(35*d) - (Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/d - (2*Cos[c + d*x]*
(a + a*Sin[c + d*x])^(5/2))/(7*a*d)
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^2(c+dx)(a+a$$

$$= -\frac{\cot(c+dx)(a+a\sin(c+dx))^{3/2}}{d} - \frac{2\cos(c+dx)(a+a\sin(c}{7ad}$$

$$= \frac{7a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{4\cos(c+dx)(a+a\sin(c}{35d}$$

$$= \frac{19a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{69a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{35d} +$$

$$= \frac{171a^2\cos(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} + \frac{69a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{35d} +$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{171a^2\cos(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} + \frac{69a}{}$$

Mathematica [A] time = 1.30973, size = 283, normalized size = 1.59

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-840\sin\left(\frac{1}{2}(c+dx)\right)-574\sin\left(\frac{3}{2}(c+dx)\right)-30\sin\left(\frac{5}{2}(c+dx)\right)-21\sin\left(\frac{7}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(840*Cos[(c + d*x)/2] - 5
74*Cos[(3*(c + d*x))/2] + 30*Cos[(5*(c + d*x))/2] - 21*Cos[(7*(c + d*x))/2]
 + 5*Cos[(9*(c + d*x))/2] - 840*Sin[(c + d*x)/2] + 420*Log[1 + Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 420*Log[1 - Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]*Sin[c + d*x] - 574*Sin[(3*(c + d*x))/2] - 30*Sin[(5*(c + d*x)
)/2] - 21*Sin[(7*(c + d*x))/2] - 5*Sin[(9*(c + d*x))/2]))/(140*d*(1 + Cot[(
c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[
(c + d*x)/4]))
```

Maple [A] time = 0.948, size = 180, normalized size = 1.

$$\frac{1+\sin(dx+c)}{35\cos(dx+c)\sin(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(140\sqrt{a-a\sin(dx+c)}a^{7/2}+70a^{5/2}(a-a\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/35*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(sin(d*x+c)*(140*(a-a
*sin(d*x+c))^(1/2)*a^(7/2)+70*a^(5/2)*(a-a*sin(d*x+c))^(3/2)-56*a^(3/2)*(a-
a*sin(d*x+c))^(5/2)+10*a^(1/2)*(a-a*sin(d*x+c))^(7/2)-105*arctanh((a-a*sin(
d*x+c))^(1/2)/a^(1/2))*a^4)-35*(a-a*sin(d*x+c))^(1/2)*a^(7/2))/sin(d*x+c)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^2, x)
```

Fricas [B] time = 1.26272, size = 968, normalized size = 5.44

$$105\left(a\cos(dx+c)^2-(a\cos(dx+c)+a)\sin(dx+c)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-\right.}{\cos(dx+c)^3+\cos(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/140*(105*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(
a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*
x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqr
t(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x
 + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x
+ c) - cos(d*x + c) - 1)) + 4*(10*a*cos(d*x + c)^5 - 16*a*cos(d*x + c)^4 -
8*a*cos(d*x + c)^3 - 120*a*cos(d*x + c)^2 + 33*a*cos(d*x + c) - (10*a*cos(d
*x + c)^4 + 26*a*cos(d*x + c)^3 + 18*a*cos(d*x + c)^2 + 138*a*cos(d*x + c)
+ 171*a)*sin(d*x + c) + 171*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2
- (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.457 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=186

$$\frac{73a^2\cos(c+dx)}{20d\sqrt{a\sin(c+dx)+a}}+\frac{9a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c+d}{5d}$$

```
[Out] (9*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
+ (73*a^2*Cos[c + d*x])/(20*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]
*Sqrt[a + a*Sin[c + d*x]])/(5*d) - (3*a*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x
]])/(4*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*d) - (Cot[c + d*
x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(2*d)
```

Rubi [A] time = 0.571473, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2881, 2751, 2647, 2646, 3044, 2975, 2981, 2773, 206}

$$\frac{73a^2\cos(c+dx)}{20d\sqrt{a\sin(c+dx)+a}}+\frac{9a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c+d}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (9*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
+ (73*a^2*Cos[c + d*x])/(20*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]
*Sqrt[a + a*Sin[c + d*x]])/(5*d) - (3*a*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x
]])/(4*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*d) - (Cot[c + d*
x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(2*d)
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^3(c+dx)(a+a\,\mathrm{si}$$

$$= -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} - \frac{\cot(c+dx)\csc(c+dx)(a}{2d}$$

$$= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} - \frac{3a\cot(c+dx)\sqrt{a+a\sin(}}{4d}$$

$$= \frac{73a^2\cos(c+dx)}{20d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} - \frac{3a}{}$$

$$= \frac{73a^2\cos(c+dx)}{20d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} - \frac{3a}{}$$

$$= \frac{9a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} + \frac{73a^2\cos(c+dx)}{20d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(}{}$$

Mathematica [A] time = 1.05447, size = 322, normalized size = 1.73

$$-\frac{a\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(118\sin\left(\frac{1}{2}(c+dx)\right)+130\sin\left(\frac{3}{2}(c+dx)\right)-36\sin\left(\frac{5}{2}(c+dx)\right)-10\sin\left(\frac{7}{2}(c-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-118*Cos[(c + d*x)/2] +
130*Cos[(3*(c + d*x))/2] + 36*Cos[(5*(c + d*x))/2] - 10*Cos[(7*(c + d*x))/2
] + 2*Cos[(9*(c + d*x))/2] - 45*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] + 45*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*L
og[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 45*Cos[2*(c + d*x)]*Log[1 - C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + 118*Sin[(c + d*x)/2] + 130*Sin[(3*(c
+ d*x))/2] - 36*Sin[(5*(c + d*x))/2] - 10*Sin[(7*(c + d*x))/2] - 2*Sin[(9*(
c + d*x))/2]))/(20*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c +
d*x)/4]^2)^2)
```

Maple [A] time = 1.034, size = 178, normalized size = 1.

$$-\frac{1+\sin(dx+c)}{20\,(\sin(dx+c))^2\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(8\,(-a(\sin(dx+c)-1))^{5/2}(\sin(dx+c))^2\sqrt{a}-40\,(-a(\sin(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/20*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(3/2)*(8*(-a*(sin(d*x+c)-1
))^(5/2)*sin(d*x+c)^2*a^(1/2)-40*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)*sin(d*x+
c)^2-45*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^3-35*(-a*
(sin(d*x+c)-1))^(3/2)*a^(3/2)+45*(-a*(sin(d*x+c)-1))^(1/2)*a^(5/2))/sin(d*x
+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^3, x)
```

Fricas [B] time = 1.21174, size = 1061, normalized size = 5.7

$$45\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)+\left(a\cos(dx+c)^2-a\right)\sin(dx+c)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/80*(45*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x
 + c)^2 - a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
+ c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x +
c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x +
c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x +
c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(8*a*cos(
d*x + c)^5 - 16*a*cos(d*x + c)^4 + 16*a*cos(d*x + c)^3 + 99*a*cos(d*x + c)^
2 - 14*a*cos(d*x + c) - (8*a*cos(d*x + c)^4 + 24*a*cos(d*x + c)^3 + 40*a*co
s(d*x + c)^2 - 59*a*cos(d*x + c) - 73*a)*sin(d*x + c) - 73*a)*sqrt(a*sin(d*
x + c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos
(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.458 $\int \cot^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=197

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{29a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\cdots$$

```
[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (8*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (29*a^2*Cot[c + d*
x])/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sqrt[a + a*Sin[c +
d*x]])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d)
 - (Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.499766, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2718, 2647, 2646, 3044, 2975, 2980, 2773, 206}

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{29a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (8*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (29*a^2*Cot[c + d*
x])/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sqrt[a + a*Sin[c +
d*x]])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d)
 - (Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int (a+a\sin(c+dx))^{3/2}\,dx + \int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\left(1-2\sin^2(c\right.$$

$$= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx}{3d}$$

$$= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{a\cot(c+dx)\csc}{}$$

$$= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{29a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin(c}}{3d}$$

$$= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{29a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin(c}}{3d}$$

$$= \frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{29a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx}}$$

Mathematica [A] time = 1.26978, size = 334, normalized size = 1.7

$$-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(276\sin\left(\frac{1}{2}(c+dx)\right)+326\sin\left(\frac{3}{2}(c+dx)\right)-78\sin\left(\frac{5}{2}(c+dx)\right)-72\sin\left(\frac{7}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-276*Cos[(c + d*x)/2] +
 326*Cos[(3*(c + d*x))/2] + 78*Cos[(5*(c + d*x))/2] - 72*Cos[(7*(c + d*x))/
2] + 8*Cos[(9*(c + d*x))/2] + 276*Sin[(c + d*x)/2] - 333*Log[1 + Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 333*Log[1 - Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*Sin[c + d*x] + 326*Sin[(3*(c + d*x))/2] - 78*Sin[(5*(c + d*
x))/2] + 111*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]
- 111*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 72*Si
n[(7*(c + d*x))/2] - 8*Sin[(9*(c + d*x))/2]))/(24*d*(1 + Cot[(c + d*x)/2])*
(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3)
```

Maple [A] time = 1.03, size = 196, normalized size = 1.

$$-\frac{1+\sin(dx+c)}{24(\sin(dx+c))^3\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(96\sqrt{-a(\sin(dx+c)-1)}a^{5/2}(\sin(dx+c))^3-16(-a(\sin(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(96*(-a*(sin(d*x+c)-1))^(1/2
)*a^(5/2)*sin(d*x+c)^3-16*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)*sin(d*x+c)^3-11
1*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^3+15*(-a*(sin(d
*x+c)-1))^(1/2)*a^(5/2)+8*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)-15*(-a*(sin(d*x
+c)-1))^(5/2)*a^(1/2))/a^(3/2)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^4, x)
```

Fricas [B] time = 1.2388, size = 1123, normalized size = 5.7

$$111\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2-\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a\right)\sin(dx+c)+a\right)\sqrt{a}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(111*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*co
s(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d
*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*
x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d
*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x +
c) - 1)) - 4*(16*a*cos(d*x + c)^5 - 64*a*cos(d*x + c)^4 - 17*a*cos(d*x + c)
^3 + 165*a*cos(d*x + c)^2 + 9*a*cos(d*x + c) - (16*a*cos(d*x + c)^4 + 80*a*
cos(d*x + c)^3 + 63*a*cos(d*x + c)^2 - 102*a*cos(d*x + c) - 93*a)*sin(d*x +
 c) - 93*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c)
 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.459 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=205

$$-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{149a^2\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}+\frac{21a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}+\frac{19a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a\sin(c+dx)+a}}-\text{cot}$$

```
[Out] (21*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*d
) - (2*a^2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + (149*a^2*Cot[c + d*
x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (19*a^2*Cot[c + d*x]*Csc[c + d*x])/(3
2*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + a*S
in[c + d*x]])/(8*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/
2))/(4*d)
```

Rubi [A] time = 0.703011, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2881, 2763, 21, 2773, 206, 3044, 2975, 2980, 2772}

$$-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{149a^2\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}+\frac{21a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}+\frac{19a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a\sin(c+dx)+a}}-\text{cot}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (21*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*d
) - (2*a^2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + (149*a^2*Cot[c + d*
x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (19*a^2*Cot[c + d*x]*Csc[c + d*x])/(3
2*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + a*S
in[c + d*x]])/(8*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/
2))/(4*d)
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^5(c+dx)(a+a\sin$$

$$= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)(a+a\sin(c+$$

$$= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c}{8d}$$

$$= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{19a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c}{}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{149a^2}{64d\sqrt{a}}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{149a^2}{64d\sqrt{a}}$$

$$= \frac{21a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{149a^2}{64d\sqrt{a+}}$$

Mathematica [A] time = 1.39276, size = 392, normalized size = 1.91

$$-\frac{a\csc^{13}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-1486\sin\left(\frac{1}{2}(c+dx)\right)-1030\sin\left(\frac{3}{2}(c+dx)\right)+754\sin\left(\frac{5}{2}(c+dx)\right)+426\,\mathrm{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^13*Sqrt[a*(1 + Sin[c + d*x])]*(1486*Cos[(c + d*x)/2] -
 1030*Cos[(3*(c + d*x))/2] - 754*Cos[(5*(c + d*x))/2] + 426*Cos[(7*(c + d*x
))/2] + 128*Cos[(9*(c + d*x))/2] - 63*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 84*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- 21*Cos[4*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 63*Log
[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 84*Cos[2*(c + d*x)]*Log[1 - Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] + 21*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] - 1486*Sin[(c + d*x)/2] - 1030*Sin[(3*(c + d*x))/
2] + 754*Sin[(5*(c + d*x))/2] + 426*Sin[(7*(c + d*x))/2] - 128*Sin[(9*(c +
d*x))/2]))/(64*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)
/4]^2)^4)
```

Maple [A] time = 1.05, size = 188, normalized size = 0.9

$$-\frac{1+\sin(dx+c)}{64(\sin(dx+c))^4\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(128\sqrt{-a(\sin(dx+c)-1)}(\sin(dx+c))^4a^{7/2}-21\,\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/64*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(128*(-a*(sin(d*x+c)
-1))^(1/2)*sin(d*x+c)^4*a^(7/2)-21*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2
))*sin(d*x+c)^4*a^4+149*(-a*(sin(d*x+c)-1))^(7/2)*a^(1/2)-461*(-a*(sin(d*x+
c)-1))^(5/2)*a^(3/2)+435*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)-107*(-a*(sin(d*x
+c)-1))^(1/2)*a^(7/2))/sin(d*x+c)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^5, x)
```

Fricas [B] time = 1.23616, size = 1250, normalized size = 6.1

$$21\left(a\cos(dx+c)^5+a\cos(dx+c)^4-2\,a\cos(dx+c)^3-2\,a\cos(dx+c)^2+a\cos(dx+c)+\left(a\cos(dx+c)^4-2\,a\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/256*(21*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*c
os(d*x + c)^2 + a*cos(d*x + c) + (a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a
)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*
(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqr
t(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*
cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos
(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(128*a*cos(d*x + c)^
5 + 277*a*cos(d*x + c)^4 - 242*a*cos(d*x + c)^3 - 500*a*cos(d*x + c)^2 + 13
0*a*cos(d*x + c) - (128*a*cos(d*x + c)^4 - 149*a*cos(d*x + c)^3 - 391*a*cos
(d*x + c)^2 + 109*a*cos(d*x + c) + 239*a)*sin(d*x + c) + 239*a)*sqrt(a*sin(
d*x + c) + a))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 -
2*d*cos(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 + d)*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.460 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=215

$$\frac{91a^2\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{165a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}+\frac{31a^2\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a\sin(c+dx)+a}}+\frac{73a^2\cot(c+dx)\csc(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-165*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(12
8*d) + (91*a^2*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (73*a^2*Cot
[c + d*x]*Csc[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (31*a^2*Cot[c + d
*x]*Csc[c + d*x]^2)/(80*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[c + d*x]*Csc
[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(40*d) - (Cot[c + d*x]*Csc[c + d*x]^4
*(a + a*Sin[c + d*x])^(3/2))/(5*d)
```

Rubi [A] time = 0.807951, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2881, 2762, 21, 2773, 206, 3044, 2975, 2980, 2772}

$$\frac{91a^2\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{165a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}+\frac{31a^2\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a\sin(c+dx)+a}}+\frac{73a^2\cot(c+dx)\csc(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-165*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(12
8*d) + (91*a^2*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (73*a^2*Cot
[c + d*x]*Csc[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (31*a^2*Cot[c + d
*x]*Csc[c + d*x]^2)/(80*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[c + d*x]*Csc
[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(40*d) - (Cot[c + d*x]*Csc[c + d*x]^4
*(a + a*Sin[c + d*x])^(3/2))/(5*d)
```


Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^6(c+dx)(a+a\sin$$

$$= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx)}{5d}$$

$$= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+}}{40d}$$

$$= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{31a^2\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c}{}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{73a^2\cot(c}{64d\sqrt{a}}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{91a^2\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} + \frac{73a^2 c}{64d}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{91a^2\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} + \frac{73a^2 c}{64d}$$

$$= -\frac{165a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128d} + \frac{91a^2\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} + \frac{73a^2}{6}$$

Mathematica [A] time = 1.70755, size = 404, normalized size = 1.88

$$-\frac{a\csc^{16}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-1380\sin\left(\frac{1}{2}(c+dx)\right)+320\sin\left(\frac{3}{2}(c+dx)\right)-1296\sin\left(\frac{5}{2}(c+dx)\right)+2010\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^16*Sqrt[a*(1 + Sin[c + d*x])]*(1380*Cos[(c + d*x)/2] +
 320*Cos[(3*(c + d*x))/2] + 1296*Cos[(5*(c + d*x))/2] + 2010*Cos[(7*(c + d*
x))/2] - 910*Cos[(9*(c + d*x))/2] - 1380*Sin[(c + d*x)/2] + 8250*Log[1 + Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 8250*Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 320*Sin[(3*(c + d*x))/2] - 1296*Sin
[(5*(c + d*x))/2] - 4125*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3
*(c + d*x)] + 4125*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c +
d*x)] + 2010*Sin[(7*(c + d*x))/2] + 910*Sin[(9*(c + d*x))/2] + 825*Log[1 +
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 825*Log[1 - Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(640*d*(1 + Cot[(c + d*x)/
2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^5)
```

Maple [A] time = 1.161, size = 180, normalized size = 0.8

$$\frac{1+\sin(dx+c)}{640\,(\sin(dx+c))^5\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(-825\,\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^5(\sin(dx+c))^5+455\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/640*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(-825*arctanh((-a*(s
in(d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x+c)^5+455*(-a*(sin(d*x+c)-1))^(9/2)
*a^(1/2)-2550*(-a*(sin(d*x+c)-1))^(7/2)*a^(3/2)+4992*(-a*(sin(d*x+c)-1))^(5
/2)*a^(5/2)-3850*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+825*(-a*(sin(d*x+c)-1))^
(1/2)*a^(9/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^6, x)
```

Fricas [B] time = 1.27496, size = 1307, normalized size = 6.08

$$\frac{825\left(a\cos(dx+c)^6-3a\cos(dx+c)^4+3a\cos(dx+c)^2-\left(a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/2560*(825*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - (
a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*cos(d*x + c)
^2 + a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) -
 4*(455*a*cos(d*x + c)^5 - 275*a*cos(d*x + c)^4 - 982*a*cos(d*x + c)^3 + 17
4*a*cos(d*x + c)^2 + 399*a*cos(d*x + c) - (455*a*cos(d*x + c)^4 + 730*a*cos
(d*x + c)^3 - 252*a*cos(d*x + c)^2 - 426*a*cos(d*x + c) - 27*a)*sin(d*x + c
) - 27*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4
+ 3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.461 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=253

$$-\frac{179a^2\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}-\frac{179a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}+\frac{137a^2\cot(c+dx)\csc^3(c+dx)}{480d\sqrt{a\sin(c+dx)+a}}+\frac{239a^2\cot(c+dx)\csc^2}{320d\sqrt{a\sin(c+dx}}$$

```
[Out] (-179*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(51
2*d) - (179*a^2*Cot[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) + (111*a^2*C
ot[c + d*x]*Csc[c + d*x])/(256*d*Sqrt[a + a*Sin[c + d*x]]) + (239*a^2*Cot[c
 + d*x]*Csc[c + d*x]^2)/(320*d*Sqrt[a + a*Sin[c + d*x]]) + (137*a^2*Cot[c +
 d*x]*Csc[c + d*x]^3)/(480*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Cs
c[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]])/(20*d) - (Cot[c + d*x]*Csc[c + d*x]^
5*(a + a*Sin[c + d*x])^(3/2))/(6*d)
```

Rubi [A] time = 0.918546, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2881, 2762, 21, 2772, 2773, 206, 3044, 2975, 2980}

$$-\frac{179a^2\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}-\frac{179a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}+\frac{137a^2\cot(c+dx)\csc^3(c+dx)}{480d\sqrt{a\sin(c+dx)+a}}+\frac{239a^2\cot(c+dx)\csc^2}{320d\sqrt{a\sin(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-179*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(51
2*d) - (179*a^2*Cot[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) + (111*a^2*C
ot[c + d*x]*Csc[c + d*x])/(256*d*Sqrt[a + a*Sin[c + d*x]]) + (239*a^2*Cot[c
 + d*x]*Csc[c + d*x]^2)/(320*d*Sqrt[a + a*Sin[c + d*x]]) + (137*a^2*Cot[c +
 d*x]*Csc[c + d*x]^3)/(480*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Cs
c[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]])/(20*d) - (Cot[c + d*x]*Csc[c + d*x]^
5*(a + a*Sin[c + d*x])^(3/2))/(6*d)
```


Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
```

```
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^7(c+dx)(a+a$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^5(c+dx)(a+a\,\mathrm{si}}{6d}$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^4(c+dx)\sqrt{a+a}}{20d}$$

$$= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{137a^2\,\mathrm{c}}{480d}$$

$$= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{239a^2\,\mathrm{c}}{320d}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{111a^2}{256}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{179a^2\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} + \frac{111}{2}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{179a^2\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} + \frac{111}{2}$$

$$= -\frac{179a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{512d} - \frac{179a^2\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}} + \frac{1}{}$$

Mathematica [A] time = 2.4293, size = 486, normalized size = 1.92

$$\frac{a\csc^{19}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-25140\sin\left(\frac{1}{2}(c+dx)\right)-71972\sin\left(\frac{3}{2}(c+dx)\right)+42690\sin\left(\frac{5}{2}(c+dx)\right)-57\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^19*Sqrt[a*(1 + Sin[c + d*x])]*(25140*Cos[(c + d*x)/2] -
 71972*Cos[(3*(c + d*x))/2] - 42690*Cos[(5*(c + d*x))/2] - 5718*Cos[(7*(c +
 d*x))/2] + 18690*Cos[(9*(c + d*x))/2] - 5370*Cos[(11*(c + d*x))/2] - 26850
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 40275*Cos[2*(c + d*x)]*Log[
1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 16110*Cos[4*(c + d*x)]*Log[1 + C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2685*Cos[6*(c + d*x)]*Log[1 + Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 26850*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]] - 40275*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] + 16110*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
2685*Cos[6*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 25140*
Sin[(c + d*x)/2] - 71972*Sin[(3*(c + d*x))/2] + 42690*Sin[(5*(c + d*x))/2]
- 5718*Sin[(7*(c + d*x))/2] - 18690*Sin[(9*(c + d*x))/2] - 5370*Sin[(11*(c
+ d*x))/2]))/(7680*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c +
d*x)/4]^2)^6)
```

Maple [A] time = 1.198, size = 198, normalized size = 0.8

$$-\frac{1+\sin(dx+c)}{7680\,(\sin(dx+c))^{6}\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(2685\,\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^{7}(\sin(dx+c))^{6}-268\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/7680*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(2685*arctanh((-a*(sin(d*x
+c)-1))^(1/2)/a^(1/2))*a^7*sin(d*x+c)^6-2685*(-a*(sin(d*x+c)-1))^(1/2)*a^(1
3/2)+15215*(-a*(sin(d*x+c)-1))^(3/2)*a^(11/2)-10866*(-a*(sin(d*x+c)-1))^(5/
2)*a^(9/2)-7794*(-a*(sin(d*x+c)-1))^(7/2)*a^(7/2)+10095*(-a*(sin(d*x+c)-1))
^(9/2)*a^(5/2)-2685*(-a*(sin(d*x+c)-1))^(11/2)*a^(3/2))/a^(11/2)/sin(d*x+c)
^6/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{4}\csc(dx+c)^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^7, x)
```

Fricas [B] time = 1.30729, size = 1503, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/30720*(2685*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^5 - 3
*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - a*cos(d*x + c
) + (a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*sin(d*
x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(
d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x
+ c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c
)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(2685*a*cos(d*x + c)^6 - 333
0*a*cos(d*x + c)^5 - 5649*a*cos(d*x + c)^4 + 7188*a*cos(d*x + c)^3 + 6715*a
*cos(d*x + c)^2 - 2578*a*cos(d*x + c) + (2685*a*cos(d*x + c)^5 + 6015*a*cos
(d*x + c)^4 + 366*a*cos(d*x + c)^3 - 6822*a*cos(d*x + c)^2 - 107*a*cos(d*x
+ c) + 2471*a)*sin(d*x + c) - 2471*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x
+ c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 3*d*c
os(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)^6 - 3
*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.462 $\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=291

$$-\frac{171a^2\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}-\frac{171a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}+\frac{9a^2\cot(c+dx)\csc^4(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{1237a^2\cot(c+dx)\csc^3(c}{2240d\sqrt{a\sin(c+dx)+}}$$

```
[Out] (-171*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(10
24*d) - (171*a^2*Cot[c + d*x])/(1024*d*Sqrt[a + a*Sin[c + d*x]]) - (57*a^2*
Cot[c + d*x]*Csc[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) + (199*a^2*Cot[
c + d*x]*Csc[c + d*x]^2)/(640*d*Sqrt[a + a*Sin[c + d*x]]) + (1237*a^2*Cot[c
 + d*x]*Csc[c + d*x]^3)/(2240*d*Sqrt[a + a*Sin[c + d*x]]) + (9*a^2*Cot[c +
d*x]*Csc[c + d*x]^4)/(40*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[
c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(28*d) - (Cot[c + d*x]*Csc[c + d*x]^6*
(a + a*Sin[c + d*x])^(3/2))/(7*d)
```

Rubi [A] time = 1.06083, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2881, 2762, 21, 2772, 2773, 206, 3044, 2975, 2980}

$$-\frac{171a^2\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}-\frac{171a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}+\frac{9a^2\cot(c+dx)\csc^4(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{1237a^2\cot(c+dx)\csc^3(c}{2240d\sqrt{a\sin(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-171*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(10
24*d) - (171*a^2*Cot[c + d*x])/(1024*d*Sqrt[a + a*Sin[c + d*x]]) - (57*a^2*
Cot[c + d*x]*Csc[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) + (199*a^2*Cot[
c + d*x]*Csc[c + d*x]^2)/(640*d*Sqrt[a + a*Sin[c + d*x]]) + (1237*a^2*Cot[c
 + d*x]*Csc[c + d*x]^3)/(2240*d*Sqrt[a + a*Sin[c + d*x]]) + (9*a^2*Cot[c +
d*x]*Csc[c + d*x]^4)/(40*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[
c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(28*d) - (Cot[c + d*x]*Csc[c + d*x]^6*
(a + a*Sin[c + d*x])^(3/2))/(7*d)
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
```

```
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^8(c+dx)(a+a\sin$$

$$= -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^6(c+dx)(a+a\sin(}{7d}$$

$$= -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^5(c+dx)\sqrt{a+a\,\mathrm{s}}}{28d}$$

$$= -\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{9a^2}{}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c}{3d\sqrt{a}}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{199a^2\mathrm{co}}{640d}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{57a^2\cot}{512d}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{171a^2\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}} - \frac{57a^2}{51}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{171a^2\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}} - \frac{57a^2}{51}$$

$$= -\frac{171a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{1024d} - \frac{171a^2\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}} - \frac{57a}{5}$$

Mathematica [A] time = 4.76426, size = 522, normalized size = 1.79

$$a\csc^{22}\left(\tfrac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(306488\sin\left(\tfrac{1}{2}(c+dx)\right)-177170\sin\left(\tfrac{3}{2}(c+dx)\right)-6566\sin\left(\tfrac{5}{2}(c+dx)\right)-21954$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^22*Sqrt[a*(1 + Sin[c + d*x])]*(-306488*Cos[(c + d*x)/2]
 - 177170*Cos[(3*(c + d*x))/2] + 6566*Cos[(5*(c + d*x))/2] - 219540*Cos[(7*
(c + d*x))/2] + 33292*Cos[(9*(c + d*x))/2] - 3990*Cos[(11*(c + d*x))/2] + 1
1970*Cos[(13*(c + d*x))/2] + 306488*Sin[(c + d*x)/2] - 209475*Log[1 + Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 209475*Log[1 - Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 177170*Sin[(3*(c + d*x))/2] - 6566*S
in[(5*(c + d*x))/2] + 125685*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*S
in[3*(c + d*x)] - 125685*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3
*(c + d*x)] - 219540*Sin[(7*(c + d*x))/2] - 33292*Sin[(9*(c + d*x))/2] - 41
895*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 41895*L
og[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 3990*Sin[(11
```

```
*(c + d*x))/2] - 11970*Sin[(13*(c + d*x))/2] + 5985*Log[1 + Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 5985*Log[1 - Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(35840*d*(1 + Cot[(c + d*x)/2])*(Csc[(c
+ d*x)/4]^2 - Sec[(c + d*x)/4]^2)^7)
```

Maple [A] time = 1.213, size = 216, normalized size = 0.7

$$-\frac{1+\sin(dx+c)}{35840\,(\sin(dx+c))^7\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(5985\,(-a(\sin(dx+c)-1))^{13/2}a^{5/2}-39900\,(-a(\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/35840*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(15/2)*(5985*(-a*(sin(d
*x+c)-1))^(13/2)*a^(5/2)-39900*(-a*(sin(d*x+c)-1))^(11/2)*a^(7/2)+5985*arct
anh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^9*sin(d*x+c)^7+98581*(-a*(sin(d*x+
c)-1))^(9/2)*a^(9/2)-95232*(-a*(sin(d*x+c)-1))^(7/2)*a^(11/2)+1771*(-a*(sin
(d*x+c)-1))^(5/2)*a^(13/2)+39900*(-a*(sin(d*x+c)-1))^(3/2)*a^(15/2)-5985*(-
a*(sin(d*x+c)-1))^(1/2)*a^(17/2))/sin(d*x+c)^7/cos(d*x+c)/(a+a*sin(d*x+c))^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\csc(dx+c)^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^8, x)
```

Fricas [B] time = 1.33622, size = 1632, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/143360*(5985*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4
- 4*a*cos(d*x + c)^2 - (a*cos(d*x + c)^7 + a*cos(d*x + c)^6 - 3*a*cos(d*x +
 c)^5 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - a*co
s(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x
 + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x
 + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x
 + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(5985*
a*cos(d*x + c)^7 + 1995*a*cos(d*x + c)^6 - 6811*a*cos(d*x + c)^5 - 14633*a*
```

```
cos(d*x + c)^4 - 5997*a*cos(d*x + c)^3 + 10097*a*cos(d*x + c)^2 + 1703*a*co
s(d*x + c) - (5985*a*cos(d*x + c)^6 + 3990*a*cos(d*x + c)^5 - 2821*a*cos(d*
x + c)^4 + 11812*a*cos(d*x + c)^3 + 5815*a*cos(d*x + c)^2 - 4282*a*cos(d*x
+ c) - 2579*a)*sin(d*x + c) - 2579*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x
+ c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 - (d*
cos(d*x + c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4
 + 3*d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x +
c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.463 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=329

$$-\frac{1587a^2\cot(c+dx)}{16384d\sqrt{a\sin(c+dx)+a}}-\frac{1587a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{16384d}+\frac{83a^2\cot(c+dx)\csc^5(c+dx)}{448d\sqrt{a\sin(c+dx)+a}}+\frac{1957a^2\cot(c+dx)}{4480d\sqrt{a\sin(c}$$

```
[Out] (-1587*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(1
6384*d) - (1587*a^2*Cot[c + d*x])/(16384*d*Sqrt[a + a*Sin[c + d*x]]) - (529
*a^2*Cot[c + d*x]*Csc[c + d*x])/(8192*d*Sqrt[a + a*Sin[c + d*x]]) - (529*a^
2*Cot[c + d*x]*Csc[c + d*x]^2)/(10240*d*Sqrt[a + a*Sin[c + d*x]]) + (8653*a
^2*Cot[c + d*x]*Csc[c + d*x]^3)/(35840*d*Sqrt[a + a*Sin[c + d*x]]) + (1957*
a^2*Cot[c + d*x]*Csc[c + d*x]^4)/(4480*d*Sqrt[a + a*Sin[c + d*x]]) + (83*a^
2*Cot[c + d*x]*Csc[c + d*x]^5)/(448*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[
c + d*x]*Csc[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]])/(112*d) - (Cot[c + d*x]*C
sc[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2))/(8*d)
```

Rubi [A] time = 1.18735, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2881, 2762, 21, 2772, 2773, 206, 3044, 2975, 2980}

$$-\frac{1587a^2\cot(c+dx)}{16384d\sqrt{a\sin(c+dx)+a}}-\frac{1587a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{16384d}+\frac{83a^2\cot(c+dx)\csc^5(c+dx)}{448d\sqrt{a\sin(c+dx)+a}}+\frac{1957a^2\cot(c+dx)}{4480d\sqrt{a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-1587*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(1
6384*d) - (1587*a^2*Cot[c + d*x])/(16384*d*Sqrt[a + a*Sin[c + d*x]]) - (529
*a^2*Cot[c + d*x]*Csc[c + d*x])/(8192*d*Sqrt[a + a*Sin[c + d*x]]) - (529*a^
2*Cot[c + d*x]*Csc[c + d*x]^2)/(10240*d*Sqrt[a + a*Sin[c + d*x]]) + (8653*a
^2*Cot[c + d*x]*Csc[c + d*x]^3)/(35840*d*Sqrt[a + a*Sin[c + d*x]]) + (1957*
a^2*Cot[c + d*x]*Csc[c + d*x]^4)/(4480*d*Sqrt[a + a*Sin[c + d*x]]) + (83*a^
2*Cot[c + d*x]*Csc[c + d*x]^5)/(448*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[
c + d*x]*Csc[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]])/(112*d) - (Cot[c + d*x]*C
sc[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2))/(8*d)
```


Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^9(c+dx)(a+a$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^7(c+dx)(a+a\,\mathrm{s}}{8d}$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\csc^6(c+dx)\sqrt{a}}{112d}$$

$$= -\frac{5a^2\cot(c+dx)\csc^2(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{25a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} - \frac{5a^2\cot(c+dx)\csc^2(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{75a^2\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{25a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} - \frac{5a^2}{8}$$

$$= -\frac{75a^2\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{25a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} - \frac{529}{10}$$

$$= -\frac{75a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{75a^2\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{529}{8}$$

$$= -\frac{75a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{1587a^2\cot(c+dx)}{16384d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{75a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{1587a^2\cot(c+dx)}{16384d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{1587a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{16384d} - \frac{1587a^2\cot(c+dx)}{16384d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 6.26504, size = 2303, normalized size = 7.

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (6053*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3) - (6053*Cot[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(286720*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (179*Csc[(c + d*x)/4]^2*(a*(1
+ Sin[c + d*x]))^(3/2))/(131072*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
+ (107*Cot[(c + d*x)/4]*Csc[(c + d*x)/4]^2*(a*(1 + Sin[c + d*x]))^(3/2))/(5
73440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (113*Csc[(c + d*x)/4]^4*
(a*(1 + Sin[c + d*x]))^(3/2))/(262144*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
```

```
])^3) + (31*Cot[(c + d*x)/4]*Csc[(c + d*x)/4]^4*(a*(1 + Sin[c + d*x]))^(3/2
))/(143360*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (Csc[(c + d*x)/4]^6
*(a*(1 + Sin[c + d*x]))^(3/2))/(131072*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3) - (3*Cot[(c + d*x)/4]*Csc[(c + d*x)/4]^6*(a*(1 + Sin[c + d*x]))^(3/2
))/(229376*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (Csc[(c + d*x)/4]^8
*(a*(1 + Sin[c + d*x]))^(3/2))/(524288*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3) - (1587*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*(1 + Sin[c +
 d*x]))^(3/2))/(32768*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (1587*Lo
g[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*(1 + Sin[c + d*x]))^(3/2))/(3
2768*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (179*Sec[(c + d*x)/4]^2*(
a*(1 + Sin[c + d*x]))^(3/2))/(131072*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3) - (113*Sec[(c + d*x)/4]^4*(a*(1 + Sin[c + d*x]))^(3/2))/(262144*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (Sec[(c + d*x)/4]^6*(a*(1 + Sin[c +
d*x]))^(3/2))/(131072*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (Sec[(c
+ d*x)/4]^8*(a*(1 + Sin[c + d*x]))^(3/2))/(524288*d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^3) + (a*(1 + Sin[c + d*x]))^(3/2)/(32768*d*(Cos[(c + d*x)/4]
 - Sin[(c + d*x)/4])^8*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (5*(a*(1
+ Sin[c + d*x]))^(3/2))/(114688*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^6*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (5939*(a*(1 + Sin[c + d*x]))^(3/2
))/(2293760*d*(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^4*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^3) + (5409*(a*(1 + Sin[c + d*x]))^(3/2))/(2293760*d*(Cos[(
c + d*x)/4] - Sin[(c + d*x)/4])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
+ (3*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(14336*d*(Cos[(c + d*x)
/4] - Sin[(c + d*x)/4])^7*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (31*Si
n[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(17920*d*(Cos[(c + d*x)/4] - S
in[(c + d*x)/4])^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (107*Sin[(c +
 d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*d*(Cos[(c + d*x)/4] - Sin[(c
 + d*x)/4])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (6053*Sin[(c + d*x
)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*d*(Cos[(c + d*x)/4] - Sin[(c + d
*x)/4])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (a*(1 + Sin[c + d*x]))^(
3/2)/(32768*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^8*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^3) - (3*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(14
336*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^7*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3) + (19*(a*(1 + Sin[c + d*x]))^(3/2))/(114688*d*(Cos[(c + d*x)/4]
 + Sin[(c + d*x)/4])^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (31*Sin[(
c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(17920*d*(Cos[(c + d*x)/4] + Sin[
(c + d*x)/4])^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (1971*(a*(1 + Si
n[c + d*x]))^(3/2))/(2293760*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^4*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (107*Sin[(c + d*x)/4]*(a*(1 + Sin[c
+ d*x]))^(3/2))/(143360*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^3*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^3) - (7121*(a*(1 + Sin[c + d*x]))^(3/2))/(2293
760*d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3) - (6053*Sin[(c + d*x)/4]*(a*(1 + Sin[c + d*x]))^(3/2))/(143360*
d*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3) - (6053*(a*(1 + Sin[c + d*x]))^(3/2)*Tan[(c + d*x)/4])/(286720*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (107*Sec[(c + d*x)/4]^2*(a*(1 + Sin[
c + d*x]))^(3/2)*Tan[(c + d*x)/4])/(573440*d*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3) + (31*Sec[(c + d*x)/4]^4*(a*(1 + Sin[c + d*x]))^(3/2)*Tan[(c + d
*x)/4])/(143360*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - (3*Sec[(c + d*
x)/4]^6*(a*(1 + Sin[c + d*x]))^(3/2)*Tan[(c + d*x)/4])/(229376*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A] time = 1.224, size = 234, normalized size = 0.7

$$\frac{1+\sin(dx+c)}{573440\,(\sin(dx+c))^{8}\cos(dx+c)\,d}\sqrt{-a\,(\sin(dx+c)-1)}\left(55545\,(-a\,(\sin(dx+c)-1))^{15/2}a^{7/2}-425845\,(-a\,(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/573440*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(19/2)*(55545*(-a*(sin(
d*x+c)-1))^(15/2)*a^(7/2)-425845*(-a*(sin(d*x+c)-1))^(13/2)*a^(9/2)+1418249
*(-a*(sin(d*x+c)-1))^(11/2)*a^(11/2)-55545*arctanh((-a*(sin(d*x+c)-1))^(1/2
)/a^(1/2))*a^11*sin(d*x+c)^8-2509197*(-a*(sin(d*x+c)-1))^(9/2)*a^(13/2)+217
6627*(-a*(sin(d*x+c)-1))^(7/2)*a^(15/2)-416759*(-a*(sin(d*x+c)-1))^(5/2)*a^
(17/2)-425845*(-a*(sin(d*x+c)-1))^(3/2)*a^(19/2)+55545*(-a*(sin(d*x+c)-1))^
(1/2)*a^(21/2))/sin(d*x+c)^8/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.4684, size = 1840, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/2293760*(55545*(a*cos(d*x + c)^9 + a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^7
- 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^5 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*
x + c)^3 - 4*a*cos(d*x + c)^2 + a*cos(d*x + c) + (a*cos(d*x + c)^8 - 4*a*co
s(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*sin(d*x + c) +
a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 +
 (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c)
+ a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)
*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*
sin(d*x + c) - cos(d*x + c) - 1)) + 4*(55545*a*cos(d*x + c)^8 + 37030*a*cos
(d*x + c)^7 - 214774*a*cos(d*x + c)^6 + 27358*a*cos(d*x + c)^5 + 199004*a*c
os(d*x + c)^4 - 185006*a*cos(d*x + c)^3 - 153786*a*cos(d*x + c)^2 + 48938*a
*cos(d*x + c) + (55545*a*cos(d*x + c)^7 + 18515*a*cos(d*x + c)^6 - 196259*a
*cos(d*x + c)^5 - 223617*a*cos(d*x + c)^4 - 24613*a*cos(d*x + c)^3 + 160393
*a*cos(d*x + c)^2 + 6607*a*cos(d*x + c) - 42331*a)*sin(d*x + c) + 42331*a)*
sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^9 + d*cos(d*x + c)^8 - 4*d*cos(d*
x + c)^7 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^5 + 6*d*cos(d*x + c)^4 - 4
*d*cos(d*x + c)^3 - 4*d*cos(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^8
 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*
x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.464 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=124

$$-\frac{152a^2\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11ad}+\frac{20\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\cos^5(c+dx)}{693d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-152*a^2*Cos[c + d*x]^5)/(3465*d*(a + a*Sin[c + d*x])^(5/2)) - (38*a*Cos[c
 + d*x]^5)/(693*d*(a + a*Sin[c + d*x])^(3/2)) + (20*Cos[c + d*x]^5)/(99*d*S
qrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(11*
a*d)
```

Rubi [A] time = 0.405719, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2877, 2856, 2674, 2673}

$$-\frac{152a^2\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11ad}+\frac{20\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\cos^5(c+dx)}{693d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-152*a^2*Cos[c + d*x]^5)/(3465*d*(a + a*Sin[c + d*x])^(5/2)) - (38*a*Cos[c
 + d*x]^5)/(693*d*(a + a*Sin[c + d*x])^(3/2)) + (20*Cos[c + d*x]^5)/(99*d*S
qrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(11*
a*d)
```


Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\cos^5(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\int \cos^4(c+dx)\left(-\frac{a}{2}-4a\sin(c+dx)\right)\sqrt{a+a\sin(c+dx)}\,dx}{4a^2}$$

$$= \frac{\cos^5(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11ad} + \frac{19\int \cos^4(c+dx)\sqrt{a+a\,\mathrm{s}}}{88a}$$

$$= \frac{20\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11ad} + \frac{19}{99}\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{38a\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}} + \frac{20\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+}}{11ad}$$

$$= -\frac{152a^2\cos^5(c+dx)}{3465d(a+a\sin(c+dx))^{5/2}} - \frac{38a\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}} + \frac{20\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [A] time = 1.67413, size = 143, normalized size = 1.15

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(5773\sin\left(\frac{1}{2}(c+dx)\right)+3495\sin\left(\frac{3}{2}(c+dx)\right)-1505\sin\left(\frac{5}{2}(c+dx)\right)-315\sin\left(\frac{7}{2}(c\right.\right.}{13860d\sqrt{a(\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(5773*Cos[(c + d*x)/2] - 3495*Cos
[(3*(c + d*x))/2] - 1505*Cos[(5*(c + d*x))/2] + 315*Cos[(7*(c + d*x))/2] +
5773*Sin[(c + d*x)/2] + 3495*Sin[(3*(c + d*x))/2] - 1505*Sin[(5*(c + d*x))/
2] - 315*Sin[(7*(c + d*x))/2]))/(13860*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.68, size = 74, normalized size = 0.6

$$\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^3\left(315(\sin(dx+c))^3+595(\sin(dx+c))^2+340\sin(dx+c)+136\right)}{3465\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3465*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(315*sin(d*x+c)^3+595*sin(d*x+c)^2+3
40*sin(d*x+c)+136)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4\sin(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.11762, size = 443, normalized size = 3.57

$$-\frac{2\left(315\,\cos\left(dx+c\right)^{6}-35\,\cos\left(dx+c\right)^{5}-445\,\cos\left(dx+c\right)^{4}+19\,\cos\left(dx+c\right)^{3}-38\,\cos\left(dx+c\right)^{2}+\left(315\,\cos\left(dx+c\ldots\right.\right.\right.}{3465\,(a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2/3465*(315*cos(d*x + c)^6 - 35*cos(d*x + c)^5 - 445*cos(d*x + c)^4 + 19*c
os(d*x + c)^3 - 38*cos(d*x + c)^2 + (315*cos(d*x + c)^5 + 350*cos(d*x + c)^
4 - 95*cos(d*x + c)^3 - 114*cos(d*x + c)^2 - 152*cos(d*x + c) - 304)*sin(d*
x + c) + 152*cos(d*x + c) + 304)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c)
 + a*d*sin(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.14817, size = 428, normalized size = 3.45

$$-\frac{\left(\left(\left(\left(\left(\left(\left(17\left(\frac{2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{13}}+\frac{11\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{13}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{1155\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{13}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{1287\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,d\ldots\right.\right.}{a^{13}}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/3548160*(((((((((17*(2*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c
)^2/a^13 + 11*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) - 11
55*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) + 1287*sgn(tan(
1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) + 231*sgn(tan(1/2*d*x + 1/
2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) - 231*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^
```

```
13)*tan(1/2*d*x + 1/2*c) - 1287*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2
*d*x + 1/2*c) + 1155*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*
c) - 187*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c)^2 - 34*sg
n(tan(1/2*d*x + 1/2*c) + 1)/a^13)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(11/2) + 7
6*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(37/2))/d
```

3.465 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{8a^2\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}+\frac{2a\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (8*a^2*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) + (2*a*Cos[c + d*
x]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(9*d*Sqrt[a +
a*Sin[c + d*x]])
```

Rubi [A] time = 0.19254, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2856, 2674, 2673}

$$\frac{8a^2\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}+\frac{2a\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (8*a^2*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) + (2*a*Cos[c + d*
x]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(9*d*Sqrt[a +
a*Sin[c + d*x]])
```


Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{1}{9}\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= \frac{2a\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{1}{63}(4a)\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= \frac{8a^2\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}} + \frac{2a\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 1.56883, size = 87, normalized size = 0.95

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(130\sin(c+dx)-35\cos(2(c+dx))+87)}{315d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])*(87 - 35*Cos[2*(c + d*x)] + 130*Sin[c + d*x]))/(315*d*Sqrt[a*(1 + Sin[
c + d*x])])
```

Maple [A] time = 0.756, size = 64, normalized size = 0.7

$$\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^3\left(35(\sin(dx+c))^2+65\sin(dx+c)+26\right)}{315\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(35*sin(d*x+c)^2+65*sin(d*x+c)+26)/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4\sin(dx+c)}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.05287, size = 365, normalized size = 3.97

$$-\frac{2\left(35\cos(dx+c)^5+40\cos(dx+c)^4-\cos(dx+c)^3+2\cos(dx+c)^2-\left(35\cos(dx+c)^4-5\cos(dx+c)^3-6\cos(d\right.\right.}{315\left(ad\cos(dx+c)+ad\sin(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - cos(d*x + c)^3 + 2*cos(d*x
+ c)^2 - (35*cos(d*x + c)^4 - 5*cos(d*x + c)^3 - 6*cos(d*x + c)^2 - 8*cos(d
*x + c) - 16)*sin(d*x + c) - 8*cos(d*x + c) - 16)*sqrt(a*sin(d*x + c) + a)/
(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.37807, size = 351, normalized size = 3.82

$$-\frac{\left(\left(\left(\left(\left(\left(\frac{13\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{11}}-\frac{99\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{11}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{105\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{11}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{63\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\cdots}{a^{11}}\cdots}{(a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -1/630*((((((((13*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)^2/a^11
 - 99*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) + 105*sgn(ta
n(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) + 63*sgn(tan(1/2*d*x + 1
/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) - 63*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^
11)*tan(1/2*d*x + 1/2*c) - 105*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*
d*x + 1/2*c) + 99*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c)^
2 - 13*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(
9/2) - 8*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(31/2))/d
```

3.466 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=130

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}+\frac{32\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (32*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[c
+ d*x]^2)/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + a*Sin[c
 + d*x]])/(15*a*d)
```

Rubi [A] time = 0.564465, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2881, 2778, 2968, 3023, 2751, 2649, 206, 3046, 2985, 2773}

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}+\frac{32\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (32*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[c
+ d*x]^2)/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + a*Sin[c
 + d*x]])/(15*a*d)
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= \frac{4\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\sin(c+dx)(-4a+a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{5a} + \frac{2\int \cdots}{\cdots}$$

$$= \frac{4\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{-4a\sin(c+dx)+a\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{5a} + \frac{\int \csc \cdots}{\cdots}$$

$$= \frac{4\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15ad} - \cdots$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{32\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \cdots$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{32\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \cdots$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{32\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos \cdots}{\cdots}$$

Mathematica [A] time = 0.251687, size = 169, normalized size = 1.3

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-60\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)+60\cos\left(\frac{1}{2}(c+dx)\right)+5\cdots\right.}{30d\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(60*Cos[(c + d*x)/2] + 5*Cos[(3*(c +
 d*x))/2] + 3*Cos[(5*(c + d*x))/2] - 30*Log[1 + Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] + 30*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 60*Sin[(c + d
*x)/2] + 5*Sin[(3*(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2]))/(30*d*Sqrt[a*(1
+ Sin[c + d*x])])
```

Maple [A] time = 0.894, size = 123, normalized size = 1.

$$-\frac{2+2\sin(dx+c)}{15a^3\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(15a^{5/2}\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)+3(a-a\sin(dx+c))^{5/2}-5(a-a\sin\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*a^(5/2)*arctanh((a-a*sin
(d*x+c))^(1/2)/a^(1/2))+3*(a-a*sin(d*x+c))^(5/2)-5*(a-a*sin(d*x+c))^(3/2)*a
-15*a^2*(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \csc(dx+c)}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.25133, size = 761, normalized size = 5.85

$$15\sqrt{a}(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx-}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(15*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
 4*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - cos(d*x + c)
+ 13)*sin(d*x + c) + 14*cos(d*x + c) + 13)*sqrt(a*sin(d*x + c) + a))/(a*d*c
os(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.363, size = 498, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] 1/120*(240*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(((((17*
sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^7 - 15*sgn(tan(1/2*d*x
 + 1/2*c) + 1)/a^7)*tan(1/2*d*x + 1/2*c) + 20*sgn(tan(1/2*d*x + 1/2*c) + 1)
/a^7)*tan(1/2*d*x + 1/2*c) - 20*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^7)*tan(1/2*
d*x + 1/2*c) + 15*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^7)*tan(1/2*d*x + 1/2*c) -
 17*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^7)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2)
 - 15*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a)))*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(19/2) - (240*a^(19/2)*arctan((sqrt
(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 15*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a
)) + 26*sqrt(2)*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^(19/2))
)/d
```

3.467 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) + (4*C
os[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - Cot[c + d*x]/(d*Sqrt[a + a*Si
n[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a*d)
```

Rubi [A] time = 0.503369, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2881, 2759, 2751, 2649, 206, 3044, 2985, 2773}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) + (4*C
os[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - Cot[c + d*x]/(d*Sqrt[a + a*Si
n[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a*d)
```


Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^2(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} + \frac{2\int \frac{\frac{a}{2}-a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a} + \ldots$$

$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} - \ldots$$

$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} + \ldots$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\ldots}{\ldots}$$

Mathematica [A] time = 0.361274, size = 190, normalized size = 1.6

$$\left(\tan\left(\frac{1}{2}(c+dx)\right)+1\right)\csc\left(\frac{1}{4}(c+dx)\right)\sec\left(\frac{1}{4}(c+dx)\right)\left(10\sin\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{5}{2}(c+dx)\right)-10\cos\left(\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/4]*Sec[(c + d*x)/4]*(-10*Cos[(c + d*x)/2] + 3*Cos[(3*(c + d*
x))/2] + Cos[(5*(c + d*x))/2] + 10*Sin[(c + d*x)/2] + 3*Log[1 + Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 3*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]]*Sin[c + d*x] + 3*Sin[(3*(c + d*x))/2] - Sin[(5*(c + d*x))/2])*
(1 + Tan[(c + d*x)/2]))/(24*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.967, size = 126, normalized size = 1.1

$$-\frac{1+\sin(dx+c)}{3\cos(dx+c)\sin(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(-2\sqrt{a}\,(a-a\sin(dx+c))^{3/2}-3\,\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin}}{\sqrt{}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(-2*a^(1/2)*(a-a*
sin(d*x+c))^(3/2)-3*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^2)+3*(a-a*sin
(d*x+c))^(1/2)*a^(3/2))/a^(5/2)/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4\csc(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.21375, size = 818, normalized size = 6.87

$$3\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\right.}{\cos(dx+c)^3+\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c)
+ 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) -
9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
```

```
 a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) -
cos(d*x + c) - 1)) - 4*(2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - (2*cos(d*x +
c)^2 - 2*cos(d*x + c) - 7)*sin(d*x + c) - 5*cos(d*x + c) - 7)*sqrt(a*sin(d*
x + c) + a))/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) + a*d)*sin(d*x +
 c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.24795, size = 649, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/6*((6*sqrt(2)*a^(15/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) + 6*a
^(15/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3*sqrt(-a)*a^7 - 307
2*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 3072*sqrt(-a)*log(sqrt(
2)*sqrt(a) + sqrt(a)) - 11264*sqrt(2)*sqrt(-a) - 22528*sqrt(-a))*sgn(tan(1/
2*d*x + 1/2*c) + 1)/(sqrt(2)*sqrt(-a)*a^(15/2) + sqrt(-a)*a^(15/2)) + 1024*
((((3*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^6 - 4*sgn(tan(1/
2*d*x + 1/2*c) + 1)/a^6)*tan(1/2*d*x + 1/2*c) + 18*sgn(tan(1/2*d*x + 1/2*c)
 + 1)/a^6)*tan(1/2*d*x + 1/2*c) - 12*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^6)*tan
(1/2*d*x + 1/2*c) + 7*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^6)/(a*tan(1/2*d*x + 1
/2*c)^2 + a)^(3/2) - 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1))
+ 6*sqrt(a)/((((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^2 - a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 3072*log(abs(-sqrt(a)*tan(1/
2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))*sgn(tan(1/2*d*x + 1/2
*c) + 1)/a^(15/2))/d
```

3.468 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*Sqrt[a]*d)
- (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + Cot[c + d*x]/(4*d*Sqrt[a
+ a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*
x]])
```

Rubi [A] time = 0.543092, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2881, 2751, 2649, 206, 3044, 2984, 2985, 2773}

$$-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*Sqrt[a]*d)
- (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + Cot[c + d*x]/(4*d*Sqrt[a
+ a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*
x]])
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^3(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-\frac{a}{2}-\frac{5}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}dx}{2a} -$$

$$= -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \int$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(}{2d\sqrt{}}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(}{2d\sqrt{}}$$

$$= \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+}{2d\sqrt{a+}}$$

Mathematica [B] time = 3.92684, size = 296, normalized size = 2.37

$$\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\left(64\sin\left(\tfrac{1}{2}(c+dx)\right)-64\cos\left(\tfrac{1}{2}(c+dx)\right)+4\tan\left(\tfrac{1}{4}(c+dx)\right)+4\cot\left(\tfrac{1}{4}(c+dx)\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8 - 64*Cos[(c + d*x)/2] + 4*Cot[(c
 + d*x)/4] - Csc[(c + d*x)/4]^2 + 36*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - 36*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]
^2 + 2/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (8*Sin[(c + d*x)/4])/(Cos[
(c + d*x)/4] - Sin[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^
2 + (8*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 64*Sin[(c
+ d*x)/2] + 4*Tan[(c + d*x)/4]))/(32*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 1.013, size = 150, normalized size = 1.2

$$-\frac{1+\sin(dx+c)}{4\left(\sin(dx+c)\right)^2\cos(dx+c)\,d}\sqrt{-a\left(\sin(dx+c)-1\right)}\left(8\sqrt{-a\left(\sin(dx+c)-1\right)}a^{3/2}\left(\sin(dx+c)\right)^2-9\,\mathrm{Artanh}\left(\frac{\sqrt{-a(}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(8*(-a*(sin(d*x+c)-1)
)^(1/2)*a^(3/2)*sin(d*x+c)^2-9*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*s
in(d*x+c)^2*a^2+(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)+(-a*(sin(d*x+c)-1))^(1/2)
*a^(3/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \csc(dx+c)^3}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.13629, size = 922, normalized size = 7.38

$$9\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(9*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(8*cos(d*x + c)^3
+ 9*cos(d*x + c)^2 - (8*cos(d*x + c)^2 - cos(d*x + c) - 11)*sin(d*x + c) -
10*cos(d*x + c) - 11)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^3 + a*d*c
os(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d*cos(d*x + c)^2 - a*d)*sin(d*x
 + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.39612, size = 748, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*((36*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 18*
sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 54*sqrt(a)*arctan((sqrt(2
)*sqrt(a) + sqrt(a))/sqrt(-a)) - 27*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
 + 62*sqrt(2)*sqrt(-a) + 82*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(2*sqrt
(2)*sqrt(-a)*sqrt(a) + 3*sqrt(-a)*sqrt(a)) + (((tan(1/2*d*x + 1/2*c)/sgn(ta
n(1/2*d*x + 1/2*c) + 1) - 2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/
2*c) + 17/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 18/sgn(tan(
1/2*d*x + 1/2*c) + 1))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) - 18*arctan(-(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(
sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 9*log(abs(-sqrt(a)*tan(1/2*d*x +
1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/
2*c) + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^3 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))*a + 2*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 - a)^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.469 $\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{9\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*Sqrt[a]*d)
 + (9*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c +
d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d*
Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.596962, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2718, 2649, 206, 3044, 2984, 2985, 2773}

$$\frac{9\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*Sqrt[a]*d)
 + (9*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c +
d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d*
Sqrt[a + a*Sin[c + d*x]])
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
```

```
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^4(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos}{\sqrt{a+a\sin}}\right.}{d}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \int$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{3d\sqrt{}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{3d\sqrt{}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{3d\sqrt{}}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{a}d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+d}{3d\sqrt{a+}}$$

Mathematica [B] time = 0.577521, size = 292, normalized size = 2.16

$$\csc^9\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-36\sin\left(\frac{1}{2}(c+dx)\right)-46\sin\left(\frac{3}{2}(c+dx)\right)+54\sin\left(\frac{5}{2}(c+dx)\right)+36\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(36*Cos[(c + d*x)
/2] - 46*Cos[(3*(c + d*x))/2] - 54*Cos[(5*(c + d*x))/2] - 36*Sin[(c + d*x)/
2] - 63*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 63*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 46*Sin[(3*(c + d*x)
)/2] + 54*Sin[(5*(c + d*x))/2] + 21*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] - 21*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
n[3*(c + d*x)]))/(24*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3*Sqrt[a*(
1 + Sin[c + d*x])])
```

Maple [A] time = 1.1, size = 144, normalized size = 1.1

$$\frac{1+\sin(dx+c)}{24\,(\sin(dx+c))^3\cos(dx+c)\,d}\sqrt{-a\,(\sin(dx+c)-1)}\left(-21\,\mathrm{Artanh}\left(\frac{\sqrt{-a\,(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^3\,(\sin(dx+c))^3+27\,(-a\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-21*arctanh((-a*(sin(d*x+c)-
1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^3+27*(-a*(sin(d*x+c)-1))^(5/2)*a^(1/2)-56
*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)+21*(-a*(sin(d*x+c)-1))^(1/2)*a^(5/2))/a^
(7/2)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.16703, size = 987, normalized size = 7.31

$$21\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(21*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) -
4*(27*cos(d*x + c)^3 + 25*cos(d*x + c)^2 - (27*cos(d*x + c)^2 + 2*cos(d*x +
 c) - 17)*sin(d*x + c) - 19*cos(d*x + c) - 17)*sqrt(a*sin(d*x + c) + a))/(a
*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d - (a*d*cos(d*x + c)^3 + a*d*
cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.31981, size = 787, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a*sgn(ta
n(1/2*d*x + 1/2*c) + 1)) - 3/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x
 + 1/2*c) - 22/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (210*sqrt(2)*sqrt(a)*ar
ctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 105*sqrt(2)*sqrt(-a)*log(sqrt(
2)*sqrt(a) + sqrt(a)) + 294*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt
(-a)) - 147*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 128*sqrt(2)*sqrt(-a)
- 186*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(5*sqrt(2)*sqrt(-a)*sqrt(a) +
 7*sqrt(-a)*sqrt(a)) + 42*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1
)) - 21*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(3*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5 + 18*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(a) - 48*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) - 3*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 + 22*a
^(5/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a)^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.470 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=170

$$-\frac{11\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{53\cot(c+$$

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*Sqrt[a]*
d) - (11*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (53*Cot[c + d*x]*C
sc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x]^2
)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*d*Sqrt
[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.886759, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2881, 2780, 2649, 206, 2773, 3044, 2984, 2985}

$$-\frac{11\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{53\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*Sqrt[a]*
d) - (11*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (53*Cot[c + d*x]*C
sc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x]^2
)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*d*Sqrt
[a + a*Sin[c + d*x]])
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^5(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^4(c+dx)\left(-\frac{a}{2}-\frac{9}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a} + \frac{\int \csc(c+dx)\sqrt{a+a\sin(c}{a}$$

$$= \frac{\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{53a^2}{4}-\frac{5}{4}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{12a^2}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{53\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{11\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} + 5$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{11\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} + 5$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{11\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} + 5$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{a}d} - \frac{11\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} + \frac{53\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} + \frac{cc}{2}$$

Mathematica [B] time = 0.88838, size = 374, normalized size = 2.2

$$\csc^{12}\left(\tfrac{1}{2}(c+dx)\right)\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\left(-214\sin\left(\tfrac{1}{2}(c+dx)\right)-558\sin\left(\tfrac{3}{2}(c+dx)\right)+490\sin\left(\tfrac{5}{2}(c+dx)\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(214*Cos[(c + d*
x)/2] - 558*Cos[(3*(c + d*x))/2] - 490*Cos[(5*(c + d*x))/2] + 66*Cos[(7*(c
+ d*x))/2] - 99*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 132*Cos[2*(c
 + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 33*Cos[4*(c + d*x)]
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 99*Log[1 - Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] - 132*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + 33*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 214*Sin[(c + d*x)/2] - 558*Sin[(3*(c + d*x))/2] + 490*Sin[(5*(c + d
*x))/2] + 66*Sin[(7*(c + d*x))/2]))/(192*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d
*x)/4]^2)^4*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 1.016, size = 162, normalized size = 1.

$$\frac{1+\sin(dx+c)}{192\,(\sin(dx+c))^4\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(33\,(-a(\sin(dx+c)-1))^{7/2}a^{3/2}-33\,\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(dx+c)}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(33*(-a*(sin(d*x+c)
-1))^(7/2)*a^(3/2)-33*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*
x+c)^4+7*(-a*(sin(d*x+c)-1))^(5/2)*a^(5/2)-121*(-a*(sin(d*x+c)-1))^(3/2)*a^
(7/2)+33*(-a*(sin(d*x+c)-1))^(1/2)*a^(9/2))/sin(d*x+c)^4/cos(d*x+c)/(a+a*si
n(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.1929, size = 1161, normalized size = 6.83

$$\frac{33\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/768*(33*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x +
 c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x + c)
 + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^
2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x +
c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
 a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 -
1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(33*cos(d*x + c)^4 - 106*cos(d*x +
 c)^3 - 164*cos(d*x + c)^2 + (33*cos(d*x + c)^3 + 139*cos(d*x + c)^2 - 25*c
os(d*x + c) - 83)*sin(d*x + c) + 58*cos(d*x + c) + 83)*sqrt(a*sin(d*x + c)
+ a))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a
*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d + (a*d*cos(d*x + c)^4 - 2*a*d*co
s(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.41396, size = 994, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/384*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*(3*tan(1/2*d*x + 1/2*c)/(a*sg
n(tan(1/2*d*x + 1/2*c) + 1)) - 4/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2
*d*x + 1/2*c) - 33/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c)
+ 64/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (792*sqrt(2)*a^(3/2)*arctan((sqrt
(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 396*sqrt(2)*sqrt(-a)*a*log(sqrt(2)*sqrt(
a) + sqrt(a)) + 1122*a^(3/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) -
 561*sqrt(-a)*a*log(sqrt(2)*sqrt(a) + sqrt(a)) + 2054*sqrt(2)*sqrt(-a)*a +
2896*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(12*sqrt(2)*sqrt(-a)*a^(3/2)
 + 17*sqrt(-a)*a^(3/2)) + 66*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)
+ 1)) - 33*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(33*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 - 48*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 57*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a + 192*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2)
- 57*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*
a^2 - 208*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*a^(5/2) + 33*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))*a^3 + 64*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 - a)^4*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.471 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=205

$$-\frac{9\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{a}d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{29\cot(}{80d}$$

```
[Out] (-9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128*Sqrt[a]*
d) - (9*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) - (3*Cot[c + d*x]*Cs
c[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (29*Cot[c + d*x]*Csc[c + d*x]
^2)/(80*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x]^3)/(40*d*S
qrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*d*Sqrt[a + a*Si
n[c + d*x]])
```

Rubi [A] time = 1.14305, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2881, 2779, 2985, 2649, 206, 2773, 3044, 2984}

$$-\frac{9\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{a}d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{29\cot(}{80d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128*Sqrt[a]*
d) - (9*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) - (3*Cot[c + d*x]*Cs
c[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (29*Cot[c + d*x]*Csc[c + d*x]
^2)/(80*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x]^3)/(40*d*S
qrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*d*Sqrt[a + a*Si
n[c + d*x]])
```


Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^6(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^5(c+dx)\left(-\frac{a}{2}-\frac{11}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{5a}$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{29\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - $$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}d} - \frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{a}d} - \frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} - \frac{3\cot(c+dx)\csc(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} +$$

Mathematica [A] time = 0.996352, size = 410, normalized size = 2.

$$-\frac{\csc^{15}\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-820\sin\left(\frac{1}{2}(c+dx)\right)+1600\sin\left(\frac{3}{2}(c+dx)\right)-1616\sin\left(\frac{5}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(Csc[(c + d*x)/2]^15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(820*Cos[(c + d
*x)/2] + 1600*Cos[(3*(c + d*x))/2] + 1616*Cos[(5*(c + d*x))/2] - 30*Cos[(7*
(c + d*x))/2] + 90*Cos[(9*(c + d*x))/2] - 820*Sin[(c + d*x)/2] + 450*Log[1
+ Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 450*Log[1 - Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 1600*Sin[(3*(c + d*x))/2] - 1616
*Sin[(5*(c + d*x))/2] - 225*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Si
n[3*(c + d*x)] + 225*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c
+ d*x)] - 30*Sin[(7*(c + d*x))/2] - 90*Sin[(9*(c + d*x))/2] + 45*Log[1 + Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 45*Log[1 - Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(640*d*(Csc[(c + d*x)/4]^2 -
Sec[(c + d*x)/4]^2)^5*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 1.147, size = 180, normalized size = 0.9

$$-\frac{1+\sin(dx+c)}{640\,(\sin(dx+c))^5\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(45\,(-a(\sin(dx+c)-1))^{9/2}a^{5/2}-210\,(-a(\sin(dx+c)-1))^{7/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/640*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(15/2)*(45*(-a*(sin(d*x+c
)-1))^(9/2)*a^(5/2)-210*(-a*(sin(d*x+c)-1))^(7/2)*a^(7/2)+45*arctanh((-a*(s
in(d*x+c)-1))^(1/2)/a^(1/2))*a^7*sin(d*x+c)^5+128*(-a*(sin(d*x+c)-1))^(5/2)
*a^(9/2)+210*(-a*(sin(d*x+c)-1))^(3/2)*a^(11/2)-45*(-a*(sin(d*x+c)-1))^(1/2
)*a^(13/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.30192, size = 1273, normalized size = 6.21

$$\frac{45\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/2560*(45*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x
 + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(45*cos(d*x +
c)^5 + 15*cos(d*x + c)^4 + 142*cos(d*x + c)^3 + 186*cos(d*x + c)^2 - (45*co
s(d*x + c)^4 + 30*cos(d*x + c)^3 + 172*cos(d*x + c)^2 - 14*cos(d*x + c) - 7
3)*sin(d*x + c) - 59*cos(d*x + c) - 73)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(
d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d
*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^
2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.4837, size = 1083, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/1280*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*((4*tan(1/2*d*x + 1/2*c)/(a*
sgn(tan(1/2*d*x + 1/2*c) + 1)) - 5/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1
/2*d*x + 1/2*c) - 12/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c
) + 35/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) - 32/(a*sgn(
tan(1/2*d*x + 1/2*c) + 1))) - (2610*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a)
 + sqrt(a))/sqrt(-a)) - 1305*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)
) + 3690*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 1845*sqrt(-
a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 5058*sqrt(2)*sqrt(-a) - 7156*sqrt(-a))*
sgn(tan(1/2*d*x + 1/2*c) + 1)/(29*sqrt(2)*sqrt(-a)*sqrt(a) + 41*sqrt(-a)*sq
rt(a)) + 90*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 45*log(ab
s(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqr
t(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(35*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^9 - 80*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(a) - 110*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7*a + 240*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(3/2) - 80*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(5/2) + 110*(s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^3 + 8
0*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(
7/2) - 35*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))*a^4 - 32*a^(9/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - a)^5*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.472 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11a^2d}-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{385a^3d}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{1155a^2d}+$$

```
[Out] (-4*Cos[c + d*x])/(165*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[
c + d*x]^3)/(231*a*d*Sqrt[a + a*Sin[c + d*x]]) + (14*Cos[c + d*x]*Sin[c + d
*x]^4)/(33*a*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin[c
 + d*x]])/(1155*a^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + a*Sin[c +
d*x]])/(11*a^2*d) - (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(385*a^3*d)
```

Rubi [A] time = 0.776041, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2880, 2770, 2759, 2751, 2646, 3046, 2981}

$$-\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11a^2d}-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{385a^3d}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{1155a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4*Cos[c + d*x])/(165*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[
c + d*x]^3)/(231*a*d*Sqrt[a + a*Sin[c + d*x]]) + (14*Cos[c + d*x]*Sin[c + d
*x]^4)/(33*a*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin[c
 + d*x]])/(1155*a^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + a*Sin[c +
d*x]])/(11*a^2*d) - (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(385*a^3*d)
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \sin^4(c+dx)\sqrt{a+a\sin}}{a^2}$$

$$= \frac{4\cos(c+dx)\sin^4(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{11a^2d} + \frac{2\int \sin}{}$$

$$= \frac{32\cos(c+dx)\sin^3(c+dx)}{63ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+}{11a}$$

$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+}{11a}$$

$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}} - \frac{128\cos(c+dx)\sqrt{a+a}}{315a^2d}$$

$$= \frac{64\cos(c+dx)}{45ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{4\cos(c+dx)}{165ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+d}{33ad\sqrt{a+a\sin(c+dx}}$$

Mathematica [A] time = 5.40543, size = 102, normalized size = 0.5

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(-475\sin(c+dx)+105\sin(3(c+dx))+140\cos(2(c+dx))-2}{2310a^2d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(-204 +
 140*Cos[2*(c + d*x)] - 475*Sin[c + d*x] + 105*Sin[3*(c + d*x)]))/(2310*a^2
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.838, size = 77, normalized size = 0.4

$$\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^3\left(105(\sin(dx+c))^3+70(\sin(dx+c))^2+40\sin(dx+c)+16\right)}{1155\,ad\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/1155/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(105*sin(d*x+c)^3+70*sin(d*x+c)^2+
40*sin(d*x+c)+16)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(dx+c)^4\sin(dx+c)^3}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 1.08678, size = 456, normalized size = 2.22

$$-\frac{2\left(105\cos(dx+c)^6-140\cos(dx+c)^5-460\cos(dx+c)^4+274\cos(dx+c)^3+607\cos(dx+c)^2+\left(105\cos(dx+c\right.\right.}{1155\left(a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] -2/1155*(105*cos(d*x + c)^6 - 140*cos(d*x + c)^5 - 460*cos(d*x + c)^4 + 274
*cos(d*x + c)^3 + 607*cos(d*x + c)^2 + (105*cos(d*x + c)^5 + 245*cos(d*x +
```

```
c)^4 - 215*cos(d*x + c)^3 - 489*cos(d*x + c)^2 + 118*cos(d*x + c) + 236)*si
n(d*x + c) - 118*cos(d*x + c) - 236)*sqrt(a*sin(d*x + c) + a)/(a^2*d*cos(d*
x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.32672, size = 358, normalized size = 1.75

$$-\frac{2\left(\left(\left(\left(\left(\left(\frac{2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^{14}}+\frac{11\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{14}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{264\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{14}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{693\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\cdots\right.\right.}{a^{14}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/1182720*(2*(((((((2*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)^2
/a^14 + 11*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^14)*tan(1/2*d*x + 1/2*c)^2 - 264
*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^14)*tan(1/2*d*x + 1/2*c) + 693*sgn(tan(1/2
*d*x + 1/2*c) + 1)/a^14)*tan(1/2*d*x + 1/2*c) - 693*sgn(tan(1/2*d*x + 1/2*c
) + 1)/a^14)*tan(1/2*d*x + 1/2*c) + 264*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^14)
*tan(1/2*d*x + 1/2*c)^2 - 11*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^14)*tan(1/2*d*
x + 1/2*c)^2 - 2*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^14)/(a*tan(1/2*d*x + 1/2*c
)^2 + a)^(11/2) - 59*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(39/2))/d
```

3.473 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{2\cos^5(c+dx)}{9ad\sqrt{a\sin(c+dx)+a}}+\frac{20\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{46a\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-46*a*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) + (20*Cos[c + d*x
]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(9*a*d*Sqrt[a +
 a*Sin[c + d*x]])
```

Rubi [A] time = 0.369968, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2877, 2856, 2674, 2673}

$$-\frac{2\cos^5(c+dx)}{9ad\sqrt{a\sin(c+dx)+a}}+\frac{20\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{46a\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-46*a*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) + (20*Cos[c + d*x
]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(9*a*d*Sqrt[a +
 a*Sin[c + d*x]])
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\cos^5(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{\int \frac{\cos^4(c+dx)\left(-\frac{3a}{2}-2a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}\\ &= \frac{\cos^5(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^5(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}}+\frac{23\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{36a}\\ &= \frac{20\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^5(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}}+\frac{23}{63}\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx\\ &= -\frac{46a\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}}+\frac{20\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^5(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 3.61044, size = 92, normalized size = 1.

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(40\sin(c+dx)-35\cos(2(c+dx))+51)}{315a^2d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(51 -
35*Cos[2*(c + d*x)] + 40*Sin[c + d*x]))/(315*a^2*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))
```

Maple [A] time = 0.736, size = 67, normalized size = 0.7

$$\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^3\left(35(\sin(dx+c))^2+20\sin(dx+c)+8\right)}{315\,ad\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/315/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(35*sin(d*x+c)^2+20*sin(d*x+c)+8)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 1.08125, size = 386, normalized size = 4.2

$$-\frac{2\left(35\cos(dx+c)^5+85\cos(dx+c)^4-73\cos(dx+c)^3-169\cos(dx+c)^2-\left(35\cos(dx+c)^4-50\cos(dx+c)^3-1\ldots\right.\right.}{315\left(a^2d\cos(dx+c)+a^2d\sin(dx\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 + 85*cos(d*x + c)^4 - 73*cos(d*x + c)^3 - 169*cos
(d*x + c)^2 - (35*cos(d*x + c)^4 - 50*cos(d*x + c)^3 - 123*cos(d*x + c)^2 +
 46*cos(d*x + c) + 92)*sin(d*x + c) + 46*cos(d*x + c) + 92)*sqrt(a*sin(d*x
+ c) + a)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.26862, size = 351, normalized size = 3.82

$$-\frac{\left(\left(\left(\left(\left(\left(\frac{2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^{12}}+\frac{9\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{12}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{105\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{12}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{252\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{12}}\right)\mathrm{ta}\ldots}{\left(a\tan\left(\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/80640*((((((((2*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)^2/a^1
2 + 9*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*c) - 105*sgn(ta
n(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*c) + 252*sgn(tan(1/2*d*x +
1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*c) - 252*sgn(tan(1/2*d*x + 1/2*c) + 1)/
a^12)*tan(1/2*d*x + 1/2*c) + 105*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/
2*d*x + 1/2*c) - 9*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*c)
^2 - 2*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^12)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(
9/2) + 23*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(33/2))/d
```

3.474 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{6a\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (6*a*Cos[c + d*x]^5)/(35*d*(a + a*Sin[c + d*x])^(5/2)) - (2*Cos[c + d*x]^5)
/(7*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.150723, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2856, 2673}

$$\frac{6a\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (6*a*Cos[c + d*x]^5)/(35*d*(a + a*Sin[c + d*x])^(5/2)) - (2*Cos[c + d*x]^5)
/(7*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{2\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}-\frac{3}{7}\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx\\ &= \frac{6a\cos^5(c+dx)}{35d(a+a\sin(c+dx))^{5/2}}-\frac{2\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 1.85613, size = 82, normalized size = 1.37

$$-\frac{2(5\sin(c+dx)+2)\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}{35a^2d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(2 +
 5*Sin[c + d*x]))/(35*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.8, size = 57, normalized size = 1.

$$\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^3(5\sin(dx+c)+2)}{35ad\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/35/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(5*sin(d*x+c)+2)/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.04614, size = 320, normalized size = 5.33

$$\frac{2\left(5\cos(dx+c)^4-8\cos(dx+c)^3-19\cos(dx+c)^2+\left(5\cos(dx+c)^3+13\cos(dx+c)^2-6\cos(dx+c)-12\right)\sin(d}{35\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 - 8*cos(d*x + c)^3 - 19*cos(d*x + c)^2 + (5*cos(d*x
+ c)^3 + 13*cos(d*x + c)^2 - 6*cos(d*x + c) - 12)*sin(d*x + c) + 6*cos(d*x
+ c) + 12)*sqrt(a*sin(d*x + c) + a)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c
) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.22917, size = 274, normalized size = 4.57

$$-\frac{\frac{\left(\left(\left(\left(\frac{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{10}}-\frac{14\,\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{10}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{35\,\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{10}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{35\,\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{10}}\right)\mathrm{ta}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{7}{2}}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -1/840*((((((sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)^2/a^10 - 14
*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^10)*tan(1/2*d*x + 1/2*c) + 35*sgn(tan(1/2*
d*x + 1/2*c) + 1)/a^10)*tan(1/2*d*x + 1/2*c) - 35*sgn(tan(1/2*d*x + 1/2*c)
+ 1)/a^10)*tan(1/2*d*x + 1/2*c) + 14*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^10)*ta
n(1/2*d*x + 1/2*c)^2 - sgn(tan(1/2*d*x + 1/2*c) + 1)/a^10)/(a*tan(1/2*d*x +
 1/2*c)^2 + a)^(7/2) - 6*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(27/2))/d
```

3.475 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (10*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[
a + a*Sin[c + d*x]])/(3*a^2*d)
```

Rubi [A] time = 0.362325, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2880, 2646, 3046, 2981, 2773, 206}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (10*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[
a + a*Sin[c + d*x]])/(3*a^2*d)
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
```

```
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \sqrt{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{4\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{2\int \csc(c+dx)\left(\frac{3a}{2}+\frac{1}{2}a\right.}{}$$

$$= \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{\int \csc(c+dx)\sqrt{a+a\sin}}{a^2}$$

$$= \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\right.}{ad}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+a}}{3a^2d}$$

Mathematica [A] time = 0.271321, size = 147, normalized size = 1.5

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-9\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)+9\cos\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{3}{2}(c+dx)\right)-3\,\mathrm{l}\right.}{3d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(9*Cos[(c + d*x)/2] - Cos[(3*(c +
d*x))/2] - 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 9*Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]
))/(3*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.882, size = 103, normalized size = 1.1

$$\frac{2+2\sin(dx+c)}{3a^3\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(-3a^{3/2}\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)+(a-a\sin(dx+c))^{\frac{3}{2}}+3a\sqrt{a-a\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/3/a^3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-3*a^(3/2)*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2))+(a-a*sin(d*x+c))^(3/2)+3*a*(a-a*sin(d*x+c))^(1/2)
)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [B] time = 1.17738, size = 709, normalized size = 7.23

$$\frac{3\,\sqrt{a}(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1)\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\cdots}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cdots}\right.\cdots}{6\left(a^{2}d\cos\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(cos(d*x + c)^2 + (cos(d*x + c) + 5)*sin(d*x + c) - 4*cos(d*x + c) - 5)*sq
rt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.37194, size = 423, normalized size = 4.32

$$-\frac{\frac{\left(6\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+10\sqrt{2}\sqrt{-a}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}a^{\frac{3}{2}}}+\frac{4\left(\left(\left(\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{3}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}\cdots\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}d\cdots\right.\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -1/3*((6*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3*sqrt(-a)*
log(sqrt(2)*sqrt(a) + sqrt(a)) + 10*sqrt(2)*sqrt(-a))*sgn(tan(1/2*d*x + 1/2
*c) + 1)/(sqrt(-a)*a^(3/2)) + 4*(((2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x +
 1/2*c) + 1) - 3/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 3/sg
n(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 2/sgn(tan(1/2*d*x + 1/2
*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 6*arctan(-(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*s
gn(tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1))
)/d
```

3.476 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{a^2d}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
Cos[c + d*x]/(a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Sqrt[a + a*Sin[
c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.402422, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2880, 2773, 206, 3044, 21, 2763}

$$-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{a^2d}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
Cos[c + d*x]/(a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Sqrt[a + a*Sin[
c + d*x]])/(a^2*d)
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
```

```
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \csc(c+dx)\sqrt{a+a\,\mathrm{si}}}{a^2}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2 d} + \frac{\int \csc(c+dx)\left(\frac{a}{2}+\frac{1}{2}a\sin(c+dx)\right)\sqrt{a+a\sin(c}}{a^3}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2 d} + \frac{\int \csc(c+dx)(a+a\,\mathrm{si}}{2a^3}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2 d}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2 d}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2 d}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2 d}$$

Mathematica [B] time = 0.685073, size = 220, normalized size = 2.34

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(8\sin\left(\frac{1}{2}(c+dx)\right)-8\cos\left(\frac{1}{2}(c+dx)\right)-\tan\left(\frac{1}{4}(c+dx)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+\frac{}{\cos(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(2 - 8*Cos[(c + d*x)/2] - Cot[(c +
 d*x)/4] + 6*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 6*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] + (2*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] -
Sin[(c + d*x)/4]) - (2*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/
4]) + 8*Sin[(c + d*x)/2] - Tan[(c + d*x)/4]))/(4*d*(a*(1 + Sin[c + d*x]))^(
3/2))
```

Maple [A] time = 0.846, size = 123, normalized size = 1.3

$$-\frac{1+\sin(dx+c)}{\cos(dx+c)\sin(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(2\sqrt{a-a\sin(dx+c)}\sqrt{a}-3\,\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/a^(5/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(2*(a-a*sin
(d*x+c))^(1/2)*a^(1/2)-3*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a)+(a-a*si
n(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.12098, size = 774, normalized size = 8.23

$$\frac{3\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(\ldots\right.}{\cos(dx+c)^3+\cos(dx+c)^2+(\ldots}\right.}{4\left(a^2d\cos(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) +
 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9
*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - c
os(d*x + c) - 1)) + 4*(2*cos(d*x + c)^2 + (2*cos(d*x + c) + 1)*sin(d*x + c)
 + cos(d*x + c) - 1)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^2 - a^2*
d - (a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.295, size = 572, normalized size = 6.09

$$\frac{\left(6\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{2}\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+6\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-3\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+3\sqrt{2}\sqrt{-a}+5\sqrt{-a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{2}\sqrt{-a}a^{\frac{3}{2}}+\sqrt{-a}a^{\frac{3}{2}}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((6*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3*sq
rt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 6*sqrt(a)*arctan((sqrt(2)*s
qrt(a) + sqrt(a))/sqrt(-a)) - 3*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 3
*sqrt(2)*sqrt(-a) + 5*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2)*sqrt
(-a)*a^(3/2) + sqrt(-a)*a^(3/2)) + ((tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*
x + 1/2*c) + 1)) + 4/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c
) - 3/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
 - 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(-sqr
t(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(3/2)*s
gn(tan(1/2*d*x + 1/2*c) + 1)) + 2/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1)))
/d
```

3.477 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2a^2d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d)
 + (7*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c
+ d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*a^2*d)
```

Rubi [A] time = 0.487594, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2880, 2772, 2773, 206, 3044, 2980}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2a^2d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d)
 + (7*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c
+ d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*a^2*d)
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)\,dx}{a^2} - \frac{2\int \csc^2(c+dx)\sqrt{a+a\sin}}{a^2}$$

$$= \frac{2\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2a^2d} + \frac{\int \csc^2(c+dx)}{}$$

$$= \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2a^2d} + \frac{11\int \csc(c+}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a}}{2a^2d}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+}}{2a^2d}$$

Mathematica [B] time = 1.89442, size = 274, normalized size = 2.58

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(12\tan\left(\frac{1}{4}(c+dx)\right)+12\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-24 + 12*Cot[(c + d*x)/4] - Csc[(
c + d*x)/4]^2 - 12*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Cos[(c + d
*x)/4] - Sin[(c + d*x)/4])^2 - (24*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] - Si
```

```
n[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (24*Sin[(c +
d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 12*Tan[(c + d*x)/4]))/(32*
d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.991, size = 126, normalized size = 1.2

$$\frac{1+\sin(dx+c)}{4\,(\sin(dx+c))^2\cos(dx+c)\,d}\sqrt{-a\,(\sin(dx+c)-1)}\left(-3\,\mathrm{Artanh}\left(\frac{\sqrt{-a\,(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin(dx+c))^2a^2+3\sqrt{-a\,(\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-3*arctanh((-a*(sin(d*x+c)-1)
)^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+3*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)-5*(-a
*(sin(d*x+c)-1))^(3/2)*a^(1/2))/a^(7/2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.16465, size = 886, normalized size = 8.36

$$\frac{3\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(\right.}{}\right.}{16\left(a^2d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(5*cos(d*x + c)^2
+ (5*cos(d*x + c) + 7)*sin(d*x + c) - 2*cos(d*x + c) - 7)*sqrt(a*sin(d*x +
c) + a))/(a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c)
- a^2*d + (a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.35618, size = 693, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(
1/2*d*x + 1/2*c) + 1)) - 6/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (12*sqrt(
2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 6*sqrt(2)*sqrt(-a
)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 18*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqr
t(a))/sqrt(-a)) - 9*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 30*sqrt(2)*sq
rt(-a) - 38*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(2*sqrt(2)*sqrt(-a)*a^(
3/2) + 3*sqrt(-a)*a^(3/2)) + 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2
*c) + 1)) - 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 - 6*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) + (sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + 6*a^(3/2))/((
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^
2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.478 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}+\frac{11\cot(c+dx)\csc(c+\cdots}{12ad\sqrt{a\sin(c+dx)\cdots}}$$

```
[Out] -ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(8*a^(3/2)*d) - C
ot[c + d*x]/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) + (11*Cot[c + d*x]*Csc[c + d*x
])/(12*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a
+ a*Sin[c + d*x]])/(3*a^2*d)
```

Rubi [A] time = 0.539332, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2717, 2772, 2773, 206, 3044, 2980}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}+\frac{11\cot(c+dx)\csc(c+\cdots}{12ad\sqrt{a\sin(c+dx)\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(8*a^(3/2)*d) - C
ot[c + d*x]/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) + (11*Cot[c + d*x]*Csc[c + d*x
])/(12*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a
+ a*Sin[c + d*x]])/(3*a^2*d)
```

Rule 2717

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^
2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \csc^3(c+dx)\sqrt{a+a\sin(c}{a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{\int \csc^3(c+dx)}{}$$

$$= \frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\,\mathrm{s}}}{3a^2d}$$

$$= -\frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a}}{3a^2d}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2a^{3/2}d} - \frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{3/2}d} - \frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{}$$

Mathematica [B] time = 0.767357, size = 294, normalized size = 2.04

$$\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(132\sin\left(\frac{1}{2}(c+dx)\right)+62\sin\left(\frac{3}{2}(c+dx)\right)-6\sin\left(\frac{5}{2}(c+dx)\right)-1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Csc[(c + d*x)/2]^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-132*Cos[(c +
d*x)/2] + 62*Cos[(3*(c + d*x))/2] + 6*Cos[(5*(c + d*x))/2] + 132*Sin[(c + d
*x)/2] - 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 9*Lo
g[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 62*Sin[(3*(c + d*
x))/2] - 6*Sin[(5*(c + d*x))/2] + 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] - 3*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)]))/(24*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3*(a*(1 + S
in[c + d*x]))^(3/2))
```

Maple [A] time = 1.036, size = 144, normalized size = 1.

$$\frac{1+\sin(dx+c)}{24\,(\sin(dx+c))^3\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\Big(3\sqrt{-a(\sin(dx+c)-1)}a^{7/2}-8\,(-a(\sin(dx+c)-1))^{3/2}a^{5/2}-3\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-1))^(1/2)*
a^(7/2)-8*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)-3*(-a*(sin(d*x+c)-1))^(5/2)*a^(
3/2)-3*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^3)/a^(11/2
)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.15885, size = 1003, normalized size = 6.97

$$\frac{3\Big(\cos(dx+c)^4-2\cos(dx+c)^2-\big(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\big)\sin(dx+c)+1\Big)\sqrt{a}\log\Big(\frac{a\cos(dx+c)^3-}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)
^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4
```

```
*(3*cos(d*x + c)^3 + 17*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 14*cos(d*x + c
) - 25)*sin(d*x + c) - 11*cos(d*x + c) - 25)*sqrt(a*sin(d*x + c) + a))/(a^2
*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^3
+ a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.56644, size = 795, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a^2*sgn(
tan(1/2*d*x + 1/2*c) + 1)) - 9/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2
*d*x + 1/2*c) + 14/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (30*sqrt(2)*sqrt(
a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 15*sqrt(2)*sqrt(-a)*log(s
qrt(2)*sqrt(a) + sqrt(a)) + 42*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/s
qrt(-a)) - 21*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 280*sqrt(2)*sqrt(-a
) + 402*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(5*sqrt(2)*sqrt(-a)*a^(3/2)
 + 7*sqrt(-a)*a^(3/2)) + 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)
+ 1)) - 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(9*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5 - 18*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(a) + 24*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) - 9*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 - 14
*a^(5/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a)^3*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.479 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{3/2}d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4a^2d}-\frac{3\cot(c+dx)}{64ad\sqrt{a\sin(c+dx)+a}}+\frac{5\cot(c+dx)\csc^2(}{8ad\sqrt{a\sin(c+dx}$$

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*a^(3/2)*d
) - (3*Cot[c + d*x])/(64*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[
c + d*x])/(32*a*d*Sqrt[a + a*Sin[c + d*x]]) + (5*Cot[c + d*x]*Csc[c + d*x]^
2)/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[a +
 a*Sin[c + d*x]])/(4*a^2*d)
```

Rubi [A] time = 0.727433, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2880, 2772, 2773, 206, 3044, 2980}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{3/2}d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4a^2d}-\frac{3\cot(c+dx)}{64ad\sqrt{a\sin(c+dx)+a}}+\frac{5\cot(c+dx)\csc^2(}{8ad\sqrt{a\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*a^(3/2)*d
) - (3*Cot[c + d*x])/(64*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[
c + d*x])/(32*a*d*Sqrt[a + a*Sin[c + d*x]]) + (5*Cot[c + d*x]*Csc[c + d*x]^
2)/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[a +
 a*Sin[c + d*x]])/(4*a^2*d)
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^5(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \csc^4(c+dx)\sqrt{a+a\sin}}{a^2}$$

$$= \frac{2\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{\int \csc^4(c}{}$$

$$= \frac{5\cot(c+dx)\csc(c+dx)}{6ad\sqrt{a+a\sin(c+dx)}} + \frac{5\cot(c+dx)\csc^2(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a}}{4a^2d}$$

$$= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} + \frac{5\cot(c+dx)\csc^2(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} - \text{co}$$

$$= -\frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} + \frac{5\cot(c+dx)\csc^2(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} - \frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} + \frac{5\,\text{c}}{8}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64a^{3/2}d} - \frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} + 5$$

Mathematica [B] time = 0.964182, size = 376, normalized size = 2.07

$$-\frac{\csc^{12}\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-446\sin\left(\frac{1}{2}(c+dx)\right)-182\sin\left(\frac{3}{2}(c+dx)\right)+2\sin\left(\frac{5}{2}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(Csc[(c + d*x)/2]^12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(446*Cos[(c +
 d*x)/2] - 182*Cos[(3*(c + d*x))/2] - 2*Cos[(5*(c + d*x))/2] - 6*Cos[(7*(c
+ d*x))/2] + 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 12*Cos[2*(c +
 d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[4*(c + d*x)]*Lo
g[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 9*Log[1 - Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] + 12*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] - 3*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
- 446*Sin[(c + d*x)/2] - 182*Sin[(3*(c + d*x))/2] + 2*Sin[(5*(c + d*x))/2]
- 6*Sin[(7*(c + d*x))/2]))/(64*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^
4*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.987, size = 162, normalized size = 0.9

$$\frac{1+\sin(dx+c)}{64\,(\sin(dx+c))^4\cos(dx+c)\,d}\sqrt{-a\,(\sin(dx+c)-1)}\left(3\,(-a\,(\sin(dx+c)-1))^{7/2}\,a^{5/2}-11\,(-a\,(\sin(dx+c)-1))^{5/2}\,a^{7/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/64/a^(15/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-1
))^(7/2)*a^(5/2)-11*(-a*(sin(d*x+c)-1))^(5/2)*a^(7/2)-3*arctanh((-a*(sin(d*
x+c)-1))^(1/2)/a^(1/2))*a^6*sin(d*x+c)^4-11*(-a*(sin(d*x+c)-1))^(3/2)*a^(9/
2)+3*(-a*(sin(d*x+c)-1))^(1/2)*a^(11/2))/sin(d*x+c)^4/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.18098, size = 1172, normalized size = 6.44

$$\frac{3\left(\cos(dx+c)^5+\cos(dx+c)^4-2\,\cos(dx+c)^3-2\,\cos(dx+c)^2+\left(\cos(dx+c)^4-2\,\cos(dx+c)^2+1\right)\sin(dx+c)+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/256*(3*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x +
c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x + c)
```

```
+ 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c
) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1
)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^4 + 2*cos(d*x + c)^
3 + 20*cos(d*x + c)^2 + (3*cos(d*x + c)^3 + cos(d*x + c)^2 + 21*cos(d*x + c
) + 39)*sin(d*x + c) - 18*cos(d*x + c) - 39)*sqrt(a*sin(d*x + c) + a))/(a^2
*d*cos(d*x + c)^5 + a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d
*cos(d*x + c)^2 + a^2*d*cos(d*x + c) + a^2*d + (a^2*d*cos(d*x + c)^4 - 2*a^
2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.26825, size = 995, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/128*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*(tan(1/2*d*x + 1/2*c)/(a^2*sg
n(tan(1/2*d*x + 1/2*c) + 1)) - 4/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1
/2*d*x + 1/2*c) + 13/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2
*c) - 16/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (72*sqrt(2)*sqrt(a)*arctan(
(sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 36*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqr
t(a) + sqrt(a)) + 102*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a))
- 51*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 1134*sqrt(2)*sqrt(-a) - 1600
*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(12*sqrt(2)*sqrt(-a)*a^(3/2) + 17*
sqrt(-a)*a^(3/2)) + 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1))
 - 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(13*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 - 32*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 5*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a + 48*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) - 5*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 - 32*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/
2) + 13*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
*a^3 + 16*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 - a)^4*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.480 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128a^{3/2}d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{3\cot(c+dx)}{128ad\sqrt{a\sin(c+dx)+a}}+\frac{19\cot(c+dx)\csc}{40ad\sqrt{a\sin(c+}}$$

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128*a^(3/2)*
d) - (3*Cot[c + d*x])/(128*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Cs
c[c + d*x])/(64*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^
2)/(80*a*d*Sqrt[a + a*Sin[c + d*x]]) + (19*Cot[c + d*x]*Csc[c + d*x]^3)/(40
*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[a + a*Si
n[c + d*x]])/(5*a^2*d)
```

Rubi [A] time = 0.875979, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2880, 2772, 2773, 206, 3044, 2980}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128a^{3/2}d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{3\cot(c+dx)}{128ad\sqrt{a\sin(c+dx)+a}}+\frac{19\cot(c+dx)\csc}{40ad\sqrt{a\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128*a^(3/2)*
d) - (3*Cot[c + d*x])/(128*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Cs
c[c + d*x])/(64*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^
2)/(80*a*d*Sqrt[a + a*Sin[c + d*x]]) + (19*Cot[c + d*x]*Csc[c + d*x]^3)/(40
*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[a + a*Si
n[c + d*x]])/(5*a^2*d)
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^6(c+dx)\sqrt{a+a\sin(c+dx)}\left(1+\sin^2(c+dx)\right)dx}{a^2} - \frac{2\int \csc^5(c+dx)\sqrt{a+a\,\mathrm{s}}}{a^2}$$

$$= \frac{\cot(c+dx)\csc^3(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+dx)}}{5a^2d} + \frac{\int \csc^5(c}{}$$

$$= \frac{7\cot(c+dx)\csc^2(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} + \frac{19\cot(c+dx)\csc^3(c+dx)}{40ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)}{5a^2d}$$

$$= \frac{35\cot(c+dx)\csc(c+dx)}{48ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{80ad\sqrt{a+a\sin(c+dx)}} + \frac{19\cot(c+dx)\csc^3(c+dx)}{40ad\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{35\cot(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{80ad\sqrt{a+a\sin(c+dx)}} + \frac{19}{4}$$

$$= -\frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{80ad\sqrt{a+a\sin(c+dx)}} +$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{32a^{3/2}d} - \frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} -$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128a^{3/2}d} - \frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [A] time = 1.36168, size = 412, normalized size = 1.87

$$-\frac{\csc^{15}\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-7100\sin\left(\frac{1}{2}(c+dx)\right)-2880\sin\left(\frac{3}{2}(c+dx)\right)+144\sin\left(\frac{5}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(Csc[(c + d*x)/2]^15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(7100*Cos[(c
+ d*x)/2] - 2880*Cos[(3*(c + d*x))/2] - 144*Cos[(5*(c + d*x))/2] - 10*Cos[(
7*(c + d*x))/2] + 30*Cos[(9*(c + d*x))/2] - 7100*Sin[(c + d*x)/2] + 150*Log
[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[1 - Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 2880*Sin[(3*(c + d*x))/2] + 1
44*Sin[(5*(c + d*x))/2] - 75*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*S
in[3*(c + d*x)] + 75*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c
+ d*x)] - 10*Sin[(7*(c + d*x))/2] - 30*Sin[(9*(c + d*x))/2] + 15*Log[1 + Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[1 - Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(640*d*(Csc[(c + d*x)/4]^2 -
Sec[(c + d*x)/4]^2)^5*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 1.076, size = 180, normalized size = 0.8

$$-\frac{1+\sin(dx+c)}{640\,(\sin(dx+c))^5\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(15\,(-a(\sin(dx+c)-1))^{9/2}a^{7/2}-70\,(-a(\sin(dx+c)-1))^{7/2}a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/640/a^(19/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*(-a*(sin(d*x+c
)-1))^(9/2)*a^(7/2)-70*(-a*(sin(d*x+c)-1))^(7/2)*a^(9/2)+128*(-a*(sin(d*x+c
)-1))^(5/2)*a^(11/2)+15*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^8*sin(
d*x+c)^5+70*(-a*(sin(d*x+c)-1))^(3/2)*a^(13/2)-15*(-a*(sin(d*x+c)-1))^(1/2)
*a^(15/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.23256, size = 1301, normalized size = 5.91

$$\frac{15\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/2560*(15*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x
 + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(15*cos(d*x +
c)^5 + 5*cos(d*x + c)^4 - 38*cos(d*x + c)^3 - 194*cos(d*x + c)^2 - (15*cos(
d*x + c)^4 + 10*cos(d*x + c)^3 - 28*cos(d*x + c)^2 + 166*cos(d*x + c) + 317
)*sin(d*x + c) + 151*cos(d*x + c) + 317)*sqrt(a*sin(d*x + c) + a))/(a^2*d*c
os(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d - (
a^2*d*cos(d*x + c)^5 + a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^3 - 2*a^
2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.58785, size = 1091, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/1280*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*((4*tan(1/2*d*x + 1/2*c)/(a^
2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 15/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*
tan(1/2*d*x + 1/2*c) + 28/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x
+ 1/2*c) - 95/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) + 1
28/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (870*sqrt(2)*sqrt(a)*arctan((sqrt
(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 435*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a)
 + sqrt(a)) + 1230*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 6
15*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 22282*sqrt(2)*sqrt(-a) + 31524
*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(29*sqrt(2)*sqrt(-a)*a^(3/2) + 41*
sqrt(-a)*a^(3/2)) + 30*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)
) - 15*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*(95*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^9 - 240*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(a) - 70*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7*a + 560*(sqrt(
```

```
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(3/2) - 7
20*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^
(5/2) + 70*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^3*a^3 + 400*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*a^(7/2) - 95*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))*a^4 - 128*a^(9/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.481 $\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{2\sin^5(c+dx)\cos(c+dx)}{11a^2d\sqrt{a\sin(c+dx)+a}}+\frac{46\sin^4(c+dx)\cos(c+dx)}{99a^2d\sqrt{a\sin(c+dx)+a}}-\frac{424\sin^3(c+dx)\cos(c+dx)}{693a^2d\sqrt{a\sin(c+dx)+a}}+\frac{200\sin^2(c+dx)\cos(c+dx)}{231a^2d\sqrt{a\sin(c+dx)+\ldots}}$$

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (4496*Cos[c + d*x])/(693*a^2*d*Sqrt[a + a*Sin[c + d*x]])
 + (200*Cos[c + d*x]*Sin[c + d*x]^2)/(231*a^2*d*Sqrt[a + a*Sin[c + d*x]]) -
 (424*Cos[c + d*x]*Sin[c + d*x]^3)/(693*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (
46*Cos[c + d*x]*Sin[c + d*x]^4)/(99*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Co
s[c + d*x]*Sin[c + d*x]^5)/(11*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (1048*Cos[
c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(693*a^3*d)
```

Rubi [A] time = 1.35904, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2880, 2778, 2983, 2968, 3023, 2751, 2649, 206, 3046}

$$-\frac{2\sin^5(c+dx)\cos(c+dx)}{11a^2d\sqrt{a\sin(c+dx)+a}}+\frac{46\sin^4(c+dx)\cos(c+dx)}{99a^2d\sqrt{a\sin(c+dx)+a}}-\frac{424\sin^3(c+dx)\cos(c+dx)}{693a^2d\sqrt{a\sin(c+dx)+a}}+\frac{200\sin^2(c+dx)\cos(c+dx)}{231a^2d\sqrt{a\sin(c+dx)+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (4496*Cos[c + d*x])/(693*a^2*d*Sqrt[a + a*Sin[c + d*x]])
 + (200*Cos[c + d*x]*Sin[c + d*x]^2)/(231*a^2*d*Sqrt[a + a*Sin[c + d*x]]) -
 (424*Cos[c + d*x]*Sin[c + d*x]^3)/(693*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (
46*Cos[c + d*x]*Sin[c + d*x]^4)/(99*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Co
s[c + d*x]*Sin[c + d*x]^5)/(11*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (1048*Cos[
c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(693*a^3*d)
```


Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\sin^4(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}dx}{a^2} - \frac{2\int \frac{\sin^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}dx}{a^2}$$

$$= \frac{4\cos(c+dx)\sin^4(c+dx)}{9a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^5(c+dx)}{11a^2d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sin^4(c+dx)\left(\frac{21a}{2}-\frac{1}{2}a\sin(c+a\right.}{\sqrt{a+a\sin(c+dx)}}}{11a^3}$$

$$= -\frac{4\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+dx)\sin^4(c+dx)}{99a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^5(c+}{11a^2d\sqrt{a+a\sin(c+d}$$

$$= \frac{76\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+dx)\sin^4(c}{99a^2d\sqrt{a+a\sin(c+}$$

$$= \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+dx)\sin^4(}{99a^2d\sqrt{a+a\sin(c}$$

$$= \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+dx)\sin^4(}{99a^2d\sqrt{a+a\sin(c}$$

$$= \frac{1144\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c}{693a^2d\sqrt{a+a\sin(c+}$$

$$= \frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c}{693a^2d\sqrt{a+a\sin(c+}$$

$$= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+dx)\sin}{231a^2d\sqrt{a+a\sin(}$$

$$= -\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+dx)\sin}{231a^2d\sqrt{a+a\sin(}$$

Mathematica [C] time = 1.05643, size = 224, normalized size = 0.86

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-73458\sin\left(\frac{1}{2}(c+dx)\right)-15246\sin\left(\frac{3}{2}(c+dx)\right)+4851\sin\left(\frac{5}{2}(c+dx)\right)+1485\,\mathrm{s}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((88704 + 88704*I)*(-1)^(3/4)*ArcT
anh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 73458*Cos[(c + d*x)/2
] - 15246*Cos[(3*(c + d*x))/2] - 4851*Cos[(5*(c + d*x))/2] + 1485*Cos[(7*(c
 + d*x))/2] + 385*Cos[(9*(c + d*x))/2] - 63*Cos[(11*(c + d*x))/2] - 73458*S
in[(c + d*x)/2] - 15246*Sin[(3*(c + d*x))/2] + 4851*Sin[(5*(c + d*x))/2] +
1485*Sin[(7*(c + d*x))/2] - 385*Sin[(9*(c + d*x))/2] - 63*Sin[(11*(c + d*x)
)/2]))/(11088*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A] time = 1.031, size = 166, normalized size = 0.6

$$-\frac{2+2\sin(dx+c)}{693a^8\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(1386a^{11/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-63(a-a\sin(dx+c))^{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/693*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(1386*a^(11/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-63*(a-a*sin(d*x+c))^(11/2)+
154*a*(a-a*sin(d*x+c))^(9/2)-198*a^2*(a-a*sin(d*x+c))^(7/2)-231*a^4*(a-a*si
n(d*x+c))^(3/2)-1386*a^5*(a-a*sin(d*x+c))^(1/2))/a^8/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4\sin(dx+c)^4}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 1.19554, size = 846, normalized size = 3.25

$$2\left(\frac{693\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-\left(63\cos(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/693*(693*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)
^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(
cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - (63*co
s(d*x + c)^6 - 161*cos(d*x + c)^5 - 562*cos(d*x + c)^4 + 622*cos(d*x + c)^3
 + 1759*cos(d*x + c)^2 + (63*cos(d*x + c)^5 + 224*cos(d*x + c)^4 - 338*cos(
d*x + c)^3 - 960*cos(d*x + c)^2 + 799*cos(d*x + c) + 2984)*sin(d*x + c) - 2
185*cos(d*x + c) - 2984)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c) + a^
3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.67354, size = 637, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/709632*(5677056*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sg
n(tan(1/2*d*x + 1/2*c) + 1)) - ((((((((((431*sgn(tan(1/2*d*x + 1/2*c) + 1)
*tan(1/2*d*x + 1/2*c)/a^15 - 693*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/
2*d*x + 1/2*c) + 2717*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2
*c) - 3927*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) + 7326*
sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) - 8778*sgn(tan(1/2
*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) + 8778*sgn(tan(1/2*d*x + 1/2*
c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) - 7326*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^1
5)*tan(1/2*d*x + 1/2*c) + 3927*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*
d*x + 1/2*c) - 2717*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c
) + 693*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) - 431*sgn(
tan(1/2*d*x + 1/2*c) + 1)/a^15)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(11/2) - 2*s
qrt(2)*(2838528*a^(39/2)*arctan(sqrt(a)/sqrt(-a)) + 373*sqrt(-a)*a)*sgn(tan
(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^(43/2)))/d
```

3.482 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{2\sin^4(c+dx)\cos(c+dx)}{9a^2d\sqrt{a\sin(c+dx)+a}}+\frac{38\sin^3(c+dx)\cos(c+dx)}{63a^2d\sqrt{a\sin(c+dx)+a}}-\frac{92\sin^2(c+dx)\cos(c+dx)}{105a^2d\sqrt{a\sin(c+dx)+a}}+\frac{472\cos(c+dx)\sqrt{a\sin(c+dx)}}{315a^3d}$$

```
[Out] (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - (2048*Cos[c + d*x])/(315*a^2*d*Sqrt[a + a*Sin[c + d*x]])
- (92*Cos[c + d*x]*Sin[c + d*x]^2)/(105*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (
38*Cos[c + d*x]*Sin[c + d*x]^3)/(63*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Co
s[c + d*x]*Sin[c + d*x]^4)/(9*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (472*Cos[c
+ d*x]*Sqrt[a + a*Sin[c + d*x]])/(315*a^3*d)
```

Rubi [A] time = 1.07987, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2880, 2778, 2983, 2968, 3023, 2751, 2649, 206, 3046}

$$-\frac{2\sin^4(c+dx)\cos(c+dx)}{9a^2d\sqrt{a\sin(c+dx)+a}}+\frac{38\sin^3(c+dx)\cos(c+dx)}{63a^2d\sqrt{a\sin(c+dx)+a}}-\frac{92\sin^2(c+dx)\cos(c+dx)}{105a^2d\sqrt{a\sin(c+dx)+a}}+\frac{472\cos(c+dx)\sqrt{a\sin(c+dx)}}{315a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - (2048*Cos[c + d*x])/(315*a^2*d*Sqrt[a + a*Sin[c + d*x]])
- (92*Cos[c + d*x]*Sin[c + d*x]^2)/(105*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (
38*Cos[c + d*x]*Sin[c + d*x]^3)/(63*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Co
s[c + d*x]*Sin[c + d*x]^4)/(9*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (472*Cos[c
+ d*x]*Sqrt[a + a*Sin[c + d*x]])/(315*a^3*d)
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\sin^3(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\sin^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{4\cos(c+dx)\sin^3(c+dx)}{7a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)}{9a^2d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sin^3(c+dx)\left(\frac{17a}{2}-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}}{9a^3}$$

$$= -\frac{4\cos(c+dx)\sin^2(c+dx)}{35a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)}{9a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)}{9a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)}{9a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{296\cos(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)\sin^2(c+}{105a^2d\sqrt{a+a\sin(c+d}$$

$$= \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)\sin^2(c+}{105a^2d\sqrt{a+a\sin(c+d}$$

Mathematica [C] time = 3.23577, size = 225, normalized size = 1.01

$$\sqrt{a}(\sin(c+dx)+1)\left(16380\sin\left(\frac{1}{2}(c+dx)\right)+3150\sin\left(\frac{3}{2}(c+dx)\right)-882\sin\left(\frac{5}{2}(c+dx)\right)-225\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\right($$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((20160 + 20160*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/
2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 163
80*Cos[(c + d*x)/2] + 3150*Cos[(3*(c + d*x))/2] + 882*Cos[(5*(c + d*x))/2]
- 225*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c + d*x))/2] + 16380*Sin[(c + d*x)/
2] + 3150*Sin[(3*(c + d*x))/2] - 882*Sin[(5*(c + d*x))/2] - 225*Sin[(7*(c +
 d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*a^3*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]))
```

Maple [A] time = 1.003, size = 166, normalized size = 0.8

$$\frac{2+2\sin(dx+c)}{315a^7\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(630a^{9/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-35(a-a\sin(dx+c))^{9/2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(630*a^(9/2)*2^(1/2)*arctanh
(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-35*(a-a*sin(d*x+c))^(9/2)+45*a
*(a-a*sin(d*x+c))^(7/2)-63*a^2*(a-a*sin(d*x+c))^(5/2)-105*a^3*(a-a*sin(d*x+
c))^(3/2)-630*a^4*(a-a*sin(d*x+c))^(1/2))/a^7/cos(d*x+c)/(a+a*sin(d*x+c))^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^3}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 1.18415, size = 786, normalized size = 3.54

$$2\left(\frac{315\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}} - \left(35\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/315*(315*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)
^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(
cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - (35*co
s(d*x + c)^5 + 130*cos(d*x + c)^4 - 208*cos(d*x + c)^3 - 634*cos(d*x + c)^2
 - (35*cos(d*x + c)^4 - 95*cos(d*x + c)^3 - 303*cos(d*x + c)^2 + 331*cos(d*
x + c) + 1292)*sin(d*x + c) + 961*cos(d*x + c) + 1292)*sqrt(a*sin(d*x + c)
+ a))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.42919, size = 562, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] -1/80640*(645120*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn
(tan(1/2*d*x + 1/2*c) + 1)) - ((((((((197*sgn(tan(1/2*d*x + 1/2*c) + 1)*ta
n(1/2*d*x + 1/2*c)/a^13 - 315*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d
*x + 1/2*c) + 1044*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c)
 - 1470*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) + 2142*sgn
(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) - 2142*sgn(tan(1/2*d*
x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) + 1470*sgn(tan(1/2*d*x + 1/2*c)
+ 1)/a^13)*tan(1/2*d*x + 1/2*c) - 1044*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*
tan(1/2*d*x + 1/2*c) + 315*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x
+ 1/2*c) - 197*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)/(a*tan(1/2*d*x + 1/2*c)^
2 + a)^(9/2) - sqrt(2)*(645120*a^(33/2)*arctan(sqrt(a)/sqrt(-a)) + 323*sqrt
(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^(37/2)))/d
```

3.483 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{2\cos^5(c+dx)}{7ad(a\sin(c+dx)+a)^{3/2}}+\frac{4\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{5/2}}+\frac{}{3ad(}$$

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (4*Cos[c + d*x]^5)/(7*d*(a + a*Sin[c + d*x])^(5/2)) + (2
*Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(7
*a*d*(a + a*Sin[c + d*x])^(3/2)) + (4*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c
 + d*x]])
```

Rubi [A] time = 0.428343, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2878, 2860, 2679, 2649, 206}

$$\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{2\cos^5(c+dx)}{7ad(a\sin(c+dx)+a)^{3/2}}+\frac{4\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{5/2}}+\frac{}{3ad(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (4*Cos[c + d*x]^5)/(7*d*(a + a*Sin[c + d*x])^(5/2)) + (2
*Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(7
*a*d*(a + a*Sin[c + d*x])^(3/2)) + (4*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c
 + d*x]])
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
```

```
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{3/2}} + \frac{2\int \frac{\cos^4(c+dx)\left(-\frac{3a}{2}-5a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{5/2}}\,dx}{7a}$$

$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{3/2}} + \int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$$

$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{3/2}} + \frac{2\int}{}$$

$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{3/2}} + \frac{}{a^2d}$$

$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{3/2}} + \frac{}{a^2d}$$

$$= -\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 2.30096, size = 201, normalized size = 1.19

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(525\sin\left(\frac{1}{2}(c+dx)\right)+91\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)-3\sin\left(\frac{7}{2}(c+dx)\right)-525\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(Sqrt[a*(1 + Sin[c + d*x])]*((672 + 672*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*
(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 525*Co
s[(c + d*x)/2] + 91*Cos[(3*(c + d*x))/2] + 21*Cos[(5*(c + d*x))/2] - 3*Cos[
(7*(c + d*x))/2] + 525*Sin[(c + d*x)/2] + 91*Sin[(3*(c + d*x))/2] - 21*Sin[
(5*(c + d*x))/2] - 3*Sin[(7*(c + d*x))/2]))/(84*a^3*d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]))
```

Maple [A] time = 0.878, size = 132, normalized size = 0.8

$$-\frac{2+2\sin(dx+c)}{21a^6\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(42a^{7/2}\sqrt{2}\operatorname{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-3(a-a\sin(dx+c))^{7/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/21*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(42*a^(7/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-3*(a-a*sin(d*x+c))^(7/2)-7*(a-a
*sin(d*x+c))^(3/2)*a^2-42*a^3*(a-a*sin(d*x+c))^(1/2))/a^6/cos(d*x+c)/(a+a*s
in(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 1.17619, size = 711, normalized size = 4.21

$$\frac{2\left(\frac{21\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+\left(3\cos(dx+\right.\right.}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/21*(21*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(co
s(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^
2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + (3*cos(d
*x + c)^4 - 9*cos(d*x + c)^3 - 31*cos(d*x + c)^2 + (3*cos(d*x + c)^3 + 12*c
os(d*x + c)^2 - 19*cos(d*x + c) - 80)*sin(d*x + c) + 61*cos(d*x + c) + 80)*
sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.47393, size = 486, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/1344*(10752*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(ta
n(1/2*d*x + 1/2*c) + 1)) - (((((((13*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*
d*x + 1/2*c)/a^11 - 21*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/
2*c) + 56*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) - 70*sgn
(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) + 70*sgn(tan(1/2*d*x
+ 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) - 56*sgn(tan(1/2*d*x + 1/2*c) + 1)
/a^11)*tan(1/2*d*x + 1/2*c) + 21*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/
2*d*x + 1/2*c) - 13*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)/(a*tan(1/2*d*x + 1/
2*c)^2 + a)^(7/2) - 4*sqrt(2)*(2688*a^(27/2)*arctan(sqrt(a)/sqrt(-a)) + 5*s
qrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^(31/2)))/d
```

3.484 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{2\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - (2*Cos[c + d*x]^5)/(5*d*(a + a*Sin[c + d*x])^(5/2)) - (2*
Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (4*Cos[c + d*x])/(a^2*
d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.23456, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2679, 2649, 206}

$$-\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{2\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - (2*Cos[c + d*x]^5)/(5*d*(a + a*Sin[c + d*x])^(5/2)) - (2*
Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (4*Cos[c + d*x])/(a^2*
d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$$

$$= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{a}$$

$$= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{4\int \frac{}{\sqrt{a}}}{}$$

$$= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{8\,\mathrm{Sub}}{}$$

$$= \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} -$$

Mathematica [C] time = 1.49299, size = 177, normalized size = 1.29

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(180\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)-180\cos\left(\frac{1}{2}(c+dx)\right)+25\cos\left(\frac{3}{2}(c+d\right.\right.}{30a^3d\left(\sin\left(\frac{1}{2}(c+dx)\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((240 + 240*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(
-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 180*Cos
[(c + d*x)/2] + 25*Cos[(3*(c + d*x))/2] + 3*Cos[(5*(c + d*x))/2] + 180*Sin[
(c + d*x)/2] + 25*Sin[(3*(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2]))/(30*a^3*d
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.844, size = 130, normalized size = 1.

$$\frac{2+2\sin(dx+c)}{15a^5\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(30a^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-3(a-a\sin(dx+c))^{5/2}-5(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(30*a^(5/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-3*(a-a*sin(d*x+c))^(5/2)-5*(a-a*
sin(d*x+c))^(3/2)*a-30*a^2*(a-a*sin(d*x+c))^(1/2))/a^5/cos(d*x+c)/(a+a*sin(
d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 1.11816, size = 659, normalized size = 4.81

$$\frac{2\left(\frac{15\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+\left(3\cos(dx+\right.\right.}{15\left(a^3d\cos(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] 2/15*(15*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(co
s(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^
2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + (3*cos(d
*x + c)^3 + 14*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 11*cos(d*x + c) - 52)*s
in(d*x + c) - 41*cos(d*x + c) - 52)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*
x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.41112, size = 409, normalized size = 2.99

$$\frac{\left(\left(\left(\left(\frac{19\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^9}-\frac{30\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^9}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{55\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^9}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{55\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^9}\right)\tan\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] 1/30*((((((19*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^9 - 30*s
gn(tan(1/2*d*x + 1/2*c) + 1)/a^9)*tan(1/2*d*x + 1/2*c) + 55*sgn(tan(1/2*d*x
 + 1/2*c) + 1)/a^9)*tan(1/2*d*x + 1/2*c) - 55*sgn(tan(1/2*d*x + 1/2*c) + 1)
/a^9)*tan(1/2*d*x + 1/2*c) + 30*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^9)*tan(1/2*
d*x + 1/2*c) - 19*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^9)/(a*tan(1/2*d*x + 1/2*c
)^2 + a)^(5/2) - 240*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2
*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*sqrt(2)*(120*a^(21/2)*arctan(sqrt(a)/sq
rt(-a)) + 13*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^(25/2)))
/d
```

3.485 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) +
 (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) - (2*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.391326, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2880, 2649, 206, 3046, 2985, 2773}

$$-\frac{2\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) +
 (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) - (2*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\csc(c+dx)\left(\frac{a}{2}-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^3} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^2d}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a^3}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^2d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.400316, size = 154, normalized size = 1.36

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+(8+8i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\cdots\right.\right.\right.}{d(a(\sin(c+d\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((((8 + 8*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + 2*Cos[(c + d*x)/2] + Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2])*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^5)/(d*(a*(1 + Sin[c + d*x]))^(5/2)))
```

Maple [A] time = 0.874, size = 116, normalized size = 1.

$$-2\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}}{a^3\cos(dx+c)\sqrt{a+a\sin(dx+c)}d}\left(\sqrt{a-a\sin(dx+c)}+\sqrt{a}\mathrm{Artanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-2\sqrt{a}\sqrt{2}\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/a^3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*((a-a*sin(d*x+c))^(1/2)+a^(
1/2)*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))-2*a^(1/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [B] time = 1.24913, size = 1040, normalized size = 9.2

$$\frac{\sqrt{a}(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*s
qrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 - (cos(d*
x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c)
 - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d
*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*sqrt(a*sin(d*x +
 c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a^3*d*cos(d*x + c) + a^3*d*sin
(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.486 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{\cot(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}$$

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) -
(4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - Cot[c + d*x]/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.524005, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2880, 2780, 2649, 206, 2773, 3044, 2985}

$$-\frac{\cot(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) -
(4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - Cot[c + d*x]/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{\int \frac{\csc^2(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}\\
&= -\frac{\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(-\frac{a}{2}+\frac{3}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^3} - \frac{2\int \csc(c+dx)\sqrt{a+a\sin(c}{a^3}\\
&= -\frac{\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a^3} + \frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}\\
&= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} +\\
&= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 2.61593, size = 170, normalized size = 1.5

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-\tan\left(\frac{1}{4}(c+dx)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+2\sec\left(\frac{1}{2}(c+dx)\right)+(32+32i)(-1)^{3/4}\tanh^{-1}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((32 + 32*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - Cot[(c + d*x)/4] + 10*Log[1
 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 10*Log[1 - Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + 2*Sec[(c + d*x)/2] - Tan[(c + d*x)/4]))/(4*d*(a*(1 + Sin[c
 + d*x]))^(5/2))
```

Maple [A] time = 0.922, size = 133, normalized size = 1.2

$$-\frac{1+\sin(dx+c)}{\cos(dx+c)\sin(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(-\sin(dx+c)\,a\left(-4\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)+5\,\mathrm{A}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/a^(7/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-sin(d*x+c)*a*(-4*2^(1
/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+5*arctanh((a-a*sin(
d*x+c))^(1/2)/a^(1/2)))+(a-a*sin(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/cos(d*x+
c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.26301, size = 1133, normalized size = 10.03

$$\frac{5\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,c\right.}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(5*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) +
 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9
*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - c
os(d*x + c) - 1)) + 8*sqrt(2)*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sq
```

```
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*
x + c) + 1))/(a^3*d*cos(d*x + c)^2 - a^3*d - (a^3*d*cos(d*x + c) + a^3*d)*s
in(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.45329, size = 637, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((10*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 32*
sqrt(2)*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 5*sqrt(2)*sqrt(-a)*log(sqrt(2)*s
qrt(a) + sqrt(a)) + 20*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a))
 - 32*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 10*sqrt(-a)*log(sqrt(2)*sqrt(a) +
sqrt(a)) - 3*sqrt(2)*sqrt(-a) - 2*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(
sqrt(2)*sqrt(-a)*a^(5/2) + 2*sqrt(-a)*a^(5/2)) + 16*sqrt(2)*arctan(-1/2*sqr
t(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + s
qrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 10*arctan(
-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-
a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 5*log(abs(-sqrt(a)*tan(1
/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(tan(1/2
*d*x + 1/2*c) + 1)) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^3*sgn(tan(1/2*d
*x + 1/2*c) + 1)) + 2/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 - a)*a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.487 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$\frac{9\cot(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}-\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-23*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(5/2)*d
) + (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d
*x]])])/(a^(5/2)*d) + (9*Cot[c + d*x])/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]]) -
 (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.732941, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2880, 2779, 2985, 2649, 206, 2773, 3044, 2984}

$$\frac{9\cot(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}-\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-23*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(5/2)*d
) + (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d
*x]])])/(a^(5/2)*d) + (9*Cot[c + d*x])/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]]) -
 (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^3(c+dx)(1+\sin^2(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-\frac{a}{2}+\frac{7}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^3} +$$

$$= \frac{9\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(\frac{15a^2}{4}-\frac{1}{4}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^4}$$

$$= \frac{9\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a+a\sin(c+dx)}} + \frac{15\int \csc(c+dx)\sqrt{a+a\sin(c+}}{8a^3}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{9\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+d}}$$

$$= -\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d} + \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{9\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+}}$$

Mathematica [C] time = 3.61302, size = 309, normalized size = 2.02

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(20\tan\left(\frac{1}{4}(c+dx)\right)+20\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(-40 - (256 + 256*I)*(-1)^(3/4)*Ar
cTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 20*Cot[(c + d*x)/4]
 - Csc[(c + d*x)/4]^2 - 92*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9
2*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Co
s[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (40*Sin[(c + d*x)/4])/(Cos[(c + d*x)
/4] - Sin[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (40*S
in[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 20*Tan[(c + d*x)/4
]))/(32*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A] time = 0.994, size = 164, normalized size = 1.1

$$\frac{1+\sin(dx+c)}{4(\sin(dx+c))^2\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(16\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{-a(\sin(dx+c)-1)}\sqrt{2}}{\sqrt{a}}\right)a^3(\sin(dx+c))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(16*2^(1/2)*arctanh(1/2*(-a*(s
in(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2+7*(-a*(sin(d*x+c)-1))
^(1/2)*a^(5/2)-9*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)-23*arctanh((-a*(sin(d*x+
c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^3)/a^(11/2)/sin(d*x+c)^2/cos(d*x+c)/(a
```

```
+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.21041, size = 1355, normalized size = 8.86

$$23\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(23*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8
*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (
cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 32*sqrt(2)*(a*cos(d
*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x + c)^2 - a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) - 4*(9*cos(d*x + c)^2 + (9*cos(d*x + c) + 11)*sin(d
*x + c) - 2*cos(d*x + c) - 11)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c
)^3 + a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d + (a^3*d*cos(d*x +
c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.5394, size = 837, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a^3*sgn(tan(
1/2*d*x + 1/2*c) + 1)) - 10/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (138*sqr
t(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 256*sqrt(2)*sqr
t(a)*arctan(sqrt(a)/sqrt(-a)) - 69*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + s
qrt(a)) + 184*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 384*sq
rt(a)*arctan(sqrt(a)/sqrt(-a)) - 92*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
 - 58*sqrt(2)*sqrt(-a) - 92*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(3*sqrt
(2)*sqrt(-a)*a^(5/2) + 4*sqrt(-a)*a^(5/2)) - 64*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(
a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 46*arctan(-(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/
(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 23*log(abs(-sqrt(a)*tan(1/2*
d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(tan(1/2*d*
x + 1/2*c) + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^3 - 10*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))*a + 10*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^2*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))
/d
```

3.488 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{19\cot(c+dx)}{8a^2d\sqrt{a\sin(c+dx)+a}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a\sin(c+dx)+a}}+\frac{13\,c}{12a}$$

```
[Out] (45*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*a^(5/2)*d)
 - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(a^(5/2)*d) - (19*Cot[c + d*x])/(8*a^2*d*Sqrt[a + a*Sin[c + d*x]]) +
 (13*Cot[c + d*x]*Csc[c + d*x])/(12*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.947277, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2717, 2779, 2984, 2985, 2649, 206, 2773, 3044}

$$-\frac{19\cot(c+dx)}{8a^2d\sqrt{a\sin(c+dx)+a}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a\sin(c+dx)+a}}+\frac{13\,c}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (45*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*a^(5/2)*d)
 - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(a^(5/2)*d) - (19*Cot[c + d*x])/(8*a^2*d*Sqrt[a + a*Sin[c + d*x]]) +
 (13*Cot[c + d*x]*Csc[c + d*x])/(12*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2717

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^
2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
```

```
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^4(c+dx)(1+\sin^2(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{a}{2}+\frac{11}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a^3} + \frac{\int \text{c}}{}$$

$$= -\frac{\cot(c+dx)}{2a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2}{}}{}$$

$$= -\frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(}{}}{}$$

$$= -\frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}} - \frac{17\int \text{c}}{}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2a^{5/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\,\text{c}}{12a}$$

$$= \frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{5/2}d} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13}{12}$$

Mathematica [C] time = 2.3391, size = 332, normalized size = 1.74

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-\frac{8\csc^9\left(\frac{1}{2}(c+dx)\right)\left(-396\sin\left(\frac{1}{2}(c+dx)\right)-218\sin\left(\frac{3}{2}(c+dx)\right)+114\sin\left(\frac{5}{2}(c+dx)\right)+396\cos\left(\frac{1}{2}(c+dx)\right)-218\cos\left(\frac{3}{2}(\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((1536 + 1536*I)*(-1)^(3/4)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - (8*Csc[(c + d*x)/2]^9*(
396*Cos[(c + d*x)/2] - 218*Cos[(3*(c + d*x))/2] - 114*Cos[(5*(c + d*x))/2]
 - 396*Sin[(c + d*x)/2] - 405*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*S
in[c + d*x] + 405*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x]
 - 218*Sin[(3*(c + d*x))/2] + 114*Sin[(5*(c + d*x))/2] + 135*Log[1 + Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 135*Log[1 - Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]))/(Csc[(c + d*x)/4]^2 - Sec[(c + d
*x)/4]^2)^3))/(192*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A] time = 1.19, size = 182, normalized size = 1.

$$-\frac{1+\sin(dx+c)}{24(\sin(dx+c))^3\cos(dx+c)d}\sqrt{-a(\sin(dx+c)-1)}\left(-135a^5\text{Artanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin(dx+c))^3+57(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/24/a^(15/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-135*a^5*arctanh((
-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3+57*(-a*(sin(d*x+c)-1))^(5/2)
*a^(5/2)+96*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*
a^5*sin(d*x+c)^3-88*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+39*(-a*(sin(d*x+c)-1)
)^(1/2)*a^(9/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.29704, size = 1504, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(135*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x +
c)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
 192*sqrt(2)*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a
*cos(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(-(cos(d*x + c)^
2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(c
os(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)
^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 4*(57*c
os(d*x + c)^3 + 83*cos(d*x + c)^2 - (57*cos(d*x + c)^2 - 26*cos(d*x + c) -
91)*sin(d*x + c) - 65*cos(d*x + c) - 91)*sqrt(a*sin(d*x + c) + a))/(a^3*d*c
os(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d - (a^3*d*cos(d*x + c)^3 + a^
3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.63869, size = 938, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*tan(1/2*d*x + 1/2*c)/(a^3*sgn(
tan(1/2*d*x + 1/2*c) + 1)) - 15/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/
2*d*x + 1/2*c) + 74/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1))) + (1890*sqrt(2)*sq
rt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3840*sqrt(2)*sqrt(a)*a
rctan(sqrt(a)/sqrt(-a)) - 945*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a
)) + 2700*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 5376*sqrt(
a)*arctan(sqrt(a)/sqrt(-a)) - 1350*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
- 1302*sqrt(2)*sqrt(-a) - 1808*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(7*s
qrt(2)*sqrt(-a)*a^(5/2) + 10*sqrt(-a)*a^(5/2)) + 384*sqrt(2)*arctan(-1/2*sq
rt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) +
sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 270*arcta
n(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt
(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 135*log(abs(-sqrt(a)*t
an(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)*sgn(tan
(1/2*d*x + 1/2*c) + 1)) - 2*(15*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^5 - 78*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^4*sqrt(a) + 144*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) - 15*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 - 74*a^(5/2))/(((sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3*a^2*sgn(tan
(1/2*d*x + 1/2*c) + 1)))/d
```

3.489 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=229

$$\frac{149\cot(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}-\frac{363\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}+$$

```
[Out] (-363*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*a^(5/2)
*d) + (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c +
 d*x]])])/(a^(5/2)*d) + (149*Cot[c + d*x])/(64*a^2*d*Sqrt[a + a*Sin[c + d*x
]]) - (107*Cot[c + d*x]*Csc[c + d*x])/(96*a^2*d*Sqrt[a + a*Sin[c + d*x]]) +
 (17*Cot[c + d*x]*Csc[c + d*x]^2)/(24*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (Co
t[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 1.3136, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2880, 2779, 2984, 2985, 2649, 206, 2773, 3044}

$$\frac{149\cot(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}-\frac{363\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-363*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*a^(5/2)
*d) + (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c +
 d*x]])])/(a^(5/2)*d) + (149*Cot[c + d*x])/(64*a^2*d*Sqrt[a + a*Sin[c + d*x
]]) - (107*Cot[c + d*x]*Csc[c + d*x])/(96*a^2*d*Sqrt[a + a*Sin[c + d*x]]) +
 (17*Cot[c + d*x]*Csc[c + d*x]^2)/(24*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (Co
t[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2880

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^5(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^4(c+dx)\left(-\frac{a}{2}+\frac{15}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}}{4a^3}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{6a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\csc^2(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{7\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} - \frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\csc^2(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{149\cot(c+dx)}{64a^2d\sqrt{a+a\sin(c+dx)}} - \frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\csc^2(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{149\cot(c+dx)}{64a^2d\sqrt{a+a\sin(c+dx)}} - \frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\csc^2(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{149\cot(c+dx)}{64a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{363\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64a^{5/2}d} + \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{149\cot(c+dx)}{64a^2d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 5.01605, size = 414, normalized size = 1.81

$$\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)^5\left(-\frac{16\csc^{12}\left(\frac{1}{2}(c+dx)\right)\left(-6250\sin\left(\frac{1}{2}(c+dx)\right)-4626\sin\left(\frac{3}{2}(c+dx)\right)+1750\sin\left(\frac{5}{2}(c+dx)\right)+894\sin\left(\frac{7}{2}(c+dx)\right)+6\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((-24576 - 24576*I)*(-1)^(3/4)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - (16*Csc[(c + d*x)/2]
^12*(6250*Cos[(c + d*x)/2] - 4626*Cos[(3*(c + d*x))/2] - 1750*Cos[(5*(c + d
*x))/2] + 894*Cos[(7*(c + d*x))/2] + 3267*Log[1 + Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - 4356*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 1089*Cos[4*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- 3267*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4356*Cos[2*(c + d*x)]
*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 1089*Cos[4*(c + d*x)]*Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 6250*Sin[(c + d*x)/2] - 4626*Sin[
(3*(c + d*x))/2] + 1750*Sin[(5*(c + d*x))/2] + 894*Sin[(7*(c + d*x))/2]))/(
Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^4))/(3072*d*(a*(1 + Sin[c + d*x]))
^(5/2))
```

Maple [A] time = 1.201, size = 200, normalized size = 0.9

$$-\frac{1+\sin(dx+c)}{192\,(\sin(dx+c))^4\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(1089\,a^7\mathrm{Artanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin(dx+c))^4+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/192/a^(19/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(1089*a^7*arctanh(
(-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^4+447*(-a*(sin(d*x+c)-1))^(7/
2)*a^(7/2)-1127*(-a*(sin(d*x+c)-1))^(5/2)*a^(9/2)-768*2^(1/2)*arctanh(1/2*(
-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^7*sin(d*x+c)^4+1049*(-a*(sin(d*
x+c)-1))^(3/2)*a^(11/2)-321*(-a*(sin(d*x+c)-1))^(1/2)*a^(13/2))/sin(d*x+c)^
4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.28645, size = 1751, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/768*(1089*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x
 + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x +
c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c
)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x
+ c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
 - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2
- 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 1536*sqrt(2)*(a*cos(d*x + c)^5 + a
*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*cos(d*x + c)^2 + a*cos(d*x + c)
+ (a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*sin(d*x + c) + a)*log(-(cos(d
*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(
d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) -
 4*(447*cos(d*x + c)^4 - 214*cos(d*x + c)^3 - 1244*cos(d*x + c)^2 + (447*co
s(d*x + c)^3 + 661*cos(d*x + c)^2 - 583*cos(d*x + c) - 845)*sin(d*x + c) +
262*cos(d*x + c) + 845)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^5 + a
^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 + a^3
*d*cos(d*x + c) + a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 +
a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.65691, size = 1141, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/384*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*(3*tan(1/2*d*x + 1/2*c)/(a^3*
sgn(tan(1/2*d*x + 1/2*c) + 1)) - 20/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))*ta
n(1/2*d*x + 1/2*c) + 159/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x +
 1/2*c) - 640/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (37026*sqrt(2)*sqrt(a)
*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 73728*sqrt(2)*sqrt(a)*arcta
n(sqrt(a)/sqrt(-a)) - 18513*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
 + 52272*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 104448*sqrt
(a)*arctan(sqrt(a)/sqrt(-a)) - 26136*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)
) - 29680*sqrt(2)*sqrt(-a) - 42100*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/
(17*sqrt(2)*sqrt(-a)*a^(5/2) + 24*sqrt(-a)*a^(5/2)) - 3072*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 217
8*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 1089*log(abs(-s
qrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a^(5/2)
*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(159*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 - 720*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 135*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a + 1920*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) - 135*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 - 1840*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) +
159*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3
 + 640*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^2 - a)^4*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.490 $\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=200

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+\cdots\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*Cos[c
+ d*x]*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (a^2*Cos[c + d*x]*Hyperg
eometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 + n
))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.232272, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2873, 2577}

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+\cdots\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*Cos[c
+ d*x]*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (a^2*Cos[c + d*x]*Hyperg
eometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 + n
))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^n(c+dx)+2a^2\cos^4(c+dx)\sin^{1+n}(c+dx\right.$$
$$= a^2\int \cos^4(c+dx)\sin^n(c+dx)\,dx + a^2\int \cos^4(c+dx)\sin^{2+n}(c$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}} +$$

Mathematica [A] time = 0.284876, size = 164, normalized size = 0.82

$$\frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{n+1}(c+dx)\left(\left(n^2+5n+6\right){}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+(n+1)\sin(c+dx)\left(2(n\right.\right.}{d(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*((6 + 5*n + n^2
)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + (1 + n)*S
in[c + d*x]*(2*(3 + n)*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2] + (2 + n)*Hypergeometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin[c + d
*x]^2]*Sin[c + d*x])))/(d*(1 + n)*(2 + n)*(3 + n))
```

Maple [F] time = 5.112, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^4(\sin(dx+c))^n(a+a\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^2\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2\cos(dx+c)^6-2\,a^2\cos(dx+c)^4\sin(dx+c)-2\,a^2\cos(dx+c)^4\right)\sin(dx+c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-(a^2*cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*c
os(d*x + c)^4)*sin(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^2 \sin(dx+c)^n \cos(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

3.491 $\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+\ldots\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (a*Cos[c + d*x]
*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.134777, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2838, 2577}

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+\ldots\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (a*Cos[c + d*x]
*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^4(c+dx)\sin^n(c+dx)\,dx + a\int \cos^4(c+dx)\sin^{1+n}(c+d\ldots\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}+\ldots\end{aligned}$$

Mathematica [F] time = 0.256514, size = 0, normalized size = 0.

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x]), x]
```

Maple [F] time = 2.973, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^{4} (\sin (dx+c))^{n} (a+a\sin (dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin (dx+c)+a)\sin (dx+c)^{n}\cos (dx+c)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cos (dx+c)^{4}\sin (dx+c)+a\cos (dx+c)^{4}\right)\sin (dx+c)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^4*sin(d*x + c) + a*cos(d*x + c)^4)*sin(d*x + c)^n,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

3.492 $\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{ad(n+1)\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{ad(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]
*Sin[c + d*x]^(1 + n))/(a*d*(1 + n)*Sqrt[Cos[c + d*x]^2]) - (Cos[c + d*x]*H
ypergeometric2F1[-1/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(
2 + n))/(a*d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.176057, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2839, 2577}

$$\frac{\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{ad(n+1)\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{ad(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]
*Sin[c + d*x]^(1 + n))/(a*d*(1 + n)*Sqrt[Cos[c + d*x]^2]) - (Cos[c + d*x]*H
ypergeometric2F1[-1/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(
2 + n))/(a*d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^2(c+dx)\sin^n(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^{1+n}(c+dx)\,dx}{a}\\ &= \frac{\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{ad(1+n)\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{2+n}{2}\right.}{ad(2+}\end{aligned}$$

Mathematica [B] time = 11.1801, size = 441, normalized size = 3.29

$$\frac{2^{n+1}\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^n\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^n\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\frac{{}_2F_1\left(\frac{n+1}{2},n+4;\frac{n+3}{2};-\right.}{n+1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2^(1 + n)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Tan[(c + d*x)/2]*(Tan[(c
 + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^n*(1 + Tan[(c + d*x)/2]^2)^n*(Hypergeo
metric2F1[(1 + n)/2, 4 + n, (3 + n)/2, -Tan[(c + d*x)/2]^2]/(1 + n) + Tan[(
c + d*x)/2]*((-2*Hypergeometric2F1[(2 + n)/2, 4 + n, (4 + n)/2, -Tan[(c + d
*x)/2]^2])/(2 + n) + Tan[(c + d*x)/2]*(-(Hypergeometric2F1[(3 + n)/2, 4 + n
, (5 + n)/2, -Tan[(c + d*x)/2]^2]/(3 + n)) + (4*Hypergeometric2F1[(4 + n)/2
, 4 + n, (6 + n)/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2])/(4 + n) - (Hyper
geometric2F1[4 + n, (5 + n)/2, (7 + n)/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x
)/2]^2)/(5 + n) - (2*Hypergeometric2F1[4 + n, (6 + n)/2, (8 + n)/2, -Tan[(c
 + d*x)/2]^2]*Tan[(c + d*x)/2]^3)/(6 + n) + (Hypergeometric2F1[4 + n, (7 +
n)/2, (9 + n)/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^4)/(7 + n)))))/(d*(a
 + a*Sin[c + d*x]))
```

Maple [F] time = 1.718, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^4(\sin(dx+c))^n}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^4}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)^n\cos(dx+c)^4}{a\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^4}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

3.493 $\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=173

$$\frac{(2n+3)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{a^2d(n+1)(n+2)\sqrt{\cos^2(c+dx)}}-\frac{2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin\right.}{a^2d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(1 + n))/(a^2*d*(2 + n))) + ((3 + 2*n)*Cos[c +
 d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^(1 + n))/(a^2*d*(1 + n)*(2 + n)*Sqrt[Cos[c + d*x]^2]) - (2*Cos[c + d*x
]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(a^2*d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.242226, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2869, 2763, 2748, 2643}

$$\frac{(2n+3)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{a^2d(n+1)(n+2)\sqrt{\cos^2(c+dx)}}-\frac{2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin\right.}{a^2d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(1 + n))/(a^2*d*(2 + n))) + ((3 + 2*n)*Cos[c +
 d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^(1 + n))/(a^2*d*(1 + n)*(2 + n)*Sqrt[Cos[c + d*x]^2]) - (2*Cos[c + d*x
]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(a^2*d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sin^n(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= -\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)} + \frac{\int \sin^n(c+dx)\left(a^2(3+2n)-2a^2(2+n)\sin(c+dx)\right)dx}{a^4(2+n)}$$
$$= -\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)} - \frac{2\int \sin^{1+n}(c+dx)\,dx}{a^2} + \frac{(3+2n)\int \sin^n(c+dx)\,dx}{a^2(2+n)}$$
$$= -\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)} + \frac{(3+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^1}{a^2d(1+n)(2+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [A] time = 4.6507, size = 312, normalized size = 1.8

$$2\tan\left(\frac{1}{2}(c+dx)\right)\sin^n(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)^n\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(\frac{{}_2F_1\left(\frac{n+1}{2},n+3;\frac{n+3}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{n+1}+\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*(Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Sin[c + d
*x]^n*Tan[(c + d*x)/2]*(Hypergeometric2F1[(1 + n)/2, 3 + n, (3 + n)/2, -Tan
[(c + d*x)/2]^2]/(1 + n) + Tan[(c + d*x)/2]*((-4*Hypergeometric2F1[(2 + n)/
2, 3 + n, (4 + n)/2, -Tan[(c + d*x)/2]^2])/(2 + n) + Tan[(c + d*x)/2]*((6*H
ypergeometric2F1[(3 + n)/2, 3 + n, (5 + n)/2, -Tan[(c + d*x)/2]^2])/(3 + n)
 - (4*Hypergeometric2F1[3 + n, (4 + n)/2, (6 + n)/2, -Tan[(c + d*x)/2]^2]*T
an[(c + d*x)/2])/(4 + n) + (Hypergeometric2F1[3 + n, (5 + n)/2, (7 + n)/2,
-Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(5 + n)))))/(d*(a + a*Sin[c + d*x]
)^2)
```

Maple [F] time = 1.661, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^4(\sin(dx+c))^n}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^4}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sin(dx+c)^n \cos(dx+c)^4}{a^2\cos(dx+c)^2 - 2\,a^2\sin(dx+c) - 2\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)^4/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x
 + c) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^4}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a)^2, x)
```

3.494 $\int \cos^5(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^{10}(c+dx)}{10d}-\frac{2a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}$$

```
[Out] (a*Sin[c + d*x]^6)/(6*d) + (a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^8)/(4
*d) - (2*a*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*x]^10)/(10*d) + (a*Sin[c +
d*x]^11)/(11*d)
```

Rubi [A] time = 0.0894381, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^{10}(c+dx)}{10d}-\frac{2a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^6)/(6*d) + (a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^8)/(4
*d) - (2*a*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*x]^10)/(10*d) + (a*Sin[c +
d*x]^11)/(11*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^5(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^5(a+x)^3}{a^5}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^5(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^{10} d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^5+a^4x^6-2a^3x^7-2a^2x^8+ax^9+x^{10}\right)dx, x, a\sin(c+dx)\right)}{a^{10}d}$$
$$= \frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2a\sin^9(c+dx)}{9d}$$

Mathematica [A] time = 0.446772, size = 97, normalized size = 1.

$$-\frac{a(-34650\sin(c+dx)+11550\sin(3(c+dx))+3465\sin(5(c+dx))-2475\sin(7(c+dx))-385\sin(9(c+dx))+315}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(34650*Cos[2*(c + d*x)] - 5775*Cos[6*(c + d*x)] + 693*Cos[10*(c + d*x)]
 - 34650*Sin[c + d*x] + 11550*Sin[3*(c + d*x)] + 3465*Sin[5*(c + d*x)] - 24
75*Sin[7*(c + d*x)] - 385*Sin[9*(c + d*x)] + 315*Sin[11*(c + d*x)]))/(35481
60*d)
```

Maple [A] time = 0.033, size = 138, normalized size = 1.4

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^5(\cos(dx+c))^6}{11}-\frac{5(\sin(dx+c))^3(\cos(dx+c))^6}{99}-\frac{5\sin(dx+c)(\cos(dx+c))^6}{231}+\frac{\sin(dx+c)}{231}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x+c)^6-5/23
1*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))+a*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60
*cos(d*x+c)^6))
```

Maxima [A] time = 1.01364, size = 97, normalized size = 1.

$$\frac{1260\,a\sin(dx+c)^{11}+1386\,a\sin(dx+c)^{10}-3080\,a\sin(dx+c)^9-3465\,a\sin(dx+c)^8+1980\,a\sin(dx+c)^7+2310}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/13860*(1260*a*sin(d*x + c)^11 + 1386*a*sin(d*x + c)^10 - 3080*a*sin(d*x +
 c)^9 - 3465*a*sin(d*x + c)^8 + 1980*a*sin(d*x + c)^7 + 2310*a*sin(d*x + c)
^6)/d
```

Fricas [A] time = 1.1613, size = 297, normalized size = 3.06

$$-\frac{1386\,a\cos\left(dx+c\right)^{10}-3465\,a\cos\left(dx+c\right)^{8}+2310\,a\cos\left(dx+c\right)^{6}+20\left(63\,a\cos\left(dx+c\right)^{10}-161\,a\cos\left(dx+c\right)^{8}+113\,a\right.}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/13860*(1386*a*cos(d*x + c)^10 - 3465*a*cos(d*x + c)^8 + 2310*a*cos(d*x +
 c)^6 + 20*(63*a*cos(d*x + c)^10 - 161*a*cos(d*x + c)^8 + 113*a*cos(d*x + c
)^6 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

Sympy [A] time = 57.0322, size = 136, normalized size = 1.4

$$\begin{cases}\frac{8a\sin^{11}(c+dx)}{693d}+\frac{4a\sin^{9}(c+dx)\cos^{2}(c+dx)}{63d}+\frac{a\sin^{7}(c+dx)\cos^{4}(c+dx)}{7d}-\frac{a\sin^{4}(c+dx)\cos^{6}(c+dx)}{6d}-\frac{a\sin^{2}(c+dx)\cos^{8}(c+dx)}{12d}-\frac{a\cos^{10}(c+dx)}{60d} & \text{fo}\\ x\left(a\sin\left(c\right)+a\right)\sin^{5}\left(c\right)\cos^{5}\left(c\right) & \text{o}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**11/(693*d) + 4*a*sin(c + d*x)**9*cos(c + d*x)*
*2/(63*d) + a*sin(c + d*x)**7*cos(c + d*x)**4/(7*d) - a*sin(c + d*x)**4*cos
(c + d*x)**6/(6*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - a*cos(c + d
*x)**10/(60*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**5*cos(c)**5, True))
```

Giac [A] time = 1.23851, size = 180, normalized size = 1.86

$$-\frac{a\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{512\,d}-\frac{a\sin\left(11\,dx+11\,c\right)}{11264\,d}+\frac{a\sin\left(9\,dx+9\,c\right)}{9216\,d}+\frac{5\,a\sin\left(7\,dx\right.}{7168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/5120*a*cos(10*d*x + 10*c)/d + 5/3072*a*cos(6*d*x + 6*c)/d - 5/512*a*cos(
2*d*x + 2*c)/d - 1/11264*a*sin(11*d*x + 11*c)/d + 1/9216*a*sin(9*d*x + 9*c)
/d + 5/7168*a*sin(7*d*x + 7*c)/d - 1/1024*a*sin(5*d*x + 5*c)/d - 5/1536*a*s
in(3*d*x + 3*c)/d + 5/512*a*sin(d*x + c)/d
```

3.495 $\int \cos^5(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^{10}(c+dx)}{10d}+\frac{a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}$$

```
[Out] (a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^6)/(6*d) - (2*a*Sin[c + d*x]^7)/
(7*d) - (a*Sin[c + d*x]^8)/(4*d) + (a*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*
x]^10)/(10*d)
```

Rubi [A] time = 0.0844256, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a\sin^{10}(c+dx)}{10d}+\frac{a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^6)/(6*d) - (2*a*Sin[c + d*x]^7)/
(7*d) - (a*Sin[c + d*x]^8)/(4*d) + (a*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*
x]^10)/(10*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^5(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^4(a+x)^3}{a^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^4(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^9d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^4+a^4x^5-2a^3x^6-2a^2x^7+ax^8+x^9\right)dx, x, a\sin(c+dx)\right.}{a^9d}$$
$$= \frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^6(c+dx)}{6d}-\frac{2a\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}+$$

Mathematica [A] time = 0.330925, size = 87, normalized size = 0.9

$$-\frac{a(-7560\sin(c+dx)+1680\sin(3(c+dx))+1008\sin(5(c+dx))-180\sin(7(c+dx))-140\sin(9(c+dx))+3150\cos(2}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(3150*Cos[2*(c + d*x)] - 525*Cos[6*(c + d*x)] + 63*Cos[10*(c + d*x)] -
7560*Sin[c + d*x] + 1680*Sin[3*(c + d*x)] + 1008*Sin[5*(c + d*x)] - 180*Sin
[7*(c + d*x)] - 140*Sin[9*(c + d*x)]))/(322560*d)
```

Maple [A] time = 0.037, size = 120, normalized size = 1.2

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^6}{10}-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{20}-\frac{(\cos(dx+c))^6}{60}\right)+a\left(-\frac{(\sin(dx+c))^3(\cos(dx+}{9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60
*cos(d*x+c)^6)+a*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)
^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.12145, size = 97, normalized size = 1.

$$\frac{126\,a\sin(dx+c)^{10}+140\,a\sin(dx+c)^9-315\,a\sin(dx+c)^8-360\,a\sin(dx+c)^7+210\,a\sin(dx+c)^6+252\,a\sin(dx+}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1260*(126*a*sin(d*x + c)^10 + 140*a*sin(d*x + c)^9 - 315*a*sin(d*x + c)^8
 - 360*a*sin(d*x + c)^7 + 210*a*sin(d*x + c)^6 + 252*a*sin(d*x + c)^5)/d
```

Fricas [A] time = 1.13591, size = 257, normalized size = 2.65

$$-\frac{126\,a\cos\left(dx+c\right)^{10}-315\,a\cos\left(dx+c\right)^{8}+210\,a\cos\left(dx+c\right)^{6}-4\,\left(35\,a\cos\left(dx+c\right)^{8}-50\,a\cos\left(dx+c\right)^{6}+3\,a\cos\left(\right.\right.}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/1260*(126*a*cos(d*x + c)^10 - 315*a*cos(d*x + c)^8 + 210*a*cos(d*x + c)^
6 - 4*(35*a*cos(d*x + c)^8 - 50*a*cos(d*x + c)^6 + 3*a*cos(d*x + c)^4 + 4*a
*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d
```

Sympy [A] time = 33.1517, size = 136, normalized size = 1.4

$$\begin{cases}\frac{8a\sin^{9}(c+dx)}{315d}+\frac{4a\sin^{7}(c+dx)\cos^{2}(c+dx)}{35d}+\frac{a\sin^{5}(c+dx)\cos^{4}(c+dx)}{5d}-\frac{a\sin^{4}(c+dx)\cos^{6}(c+dx)}{6d}-\frac{a\sin^{2}(c+dx)\cos^{8}(c+dx)}{12d}-\frac{a\cos^{10}(c+dx)}{60d}\\ x\left(a\sin\left(c\right)+a\right)\sin^{4}\left(c\right)\cos^{5}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**9/(315*d) + 4*a*sin(c + d*x)**7*cos(c + d*x)**
2/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - a*sin(c + d*x)**4*cos(
c + d*x)**6/(6*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - a*cos(c + d*
x)**10/(60*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**4*cos(c)**5, True))
```

Giac [A] time = 1.27286, size = 159, normalized size = 1.64

$$-\frac{a\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{512\,d}+\frac{a\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{a\sin\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a\sin\left(5\,dx\right.}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/5120*a*cos(10*d*x + 10*c)/d + 5/3072*a*cos(6*d*x + 6*c)/d - 5/512*a*cos(
2*d*x + 2*c)/d + 1/2304*a*sin(9*d*x + 9*c)/d + 1/1792*a*sin(7*d*x + 7*c)/d
- 1/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 3/128*a*sin(d*x
 + c)/d
```

3.496 $\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^5)/(
5*d) - (2*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.126958, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2565, 14, 2564, 270}

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^5)/(
5*d) - (2*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^9)/(9*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^3(c+dx)\,dx + a\int \cos^5(c+dx)\sin^4(c+dx)$$
$$= -\frac{a\,\text{Subst}\left(\int x^5\left(1-x^2\right)dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^4\left(1-\right.\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int \left(x^5-x^7\right)dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \left(x^4-2x^6\right.\right.}{}$$
$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^8(c+dx)}{8d} + \frac{a\sin^5(c+dx)}{5d} - \frac{2a\sin^7(c+}{7d}$$

Mathematica [A] time = 0.354229, size = 97, normalized size = 1.2

$$\frac{a(7560\sin(c+dx)-1680\sin(3(c+dx))-1008\sin(5(c+dx))+180\sin(7(c+dx))+140\sin(9(c+dx))-7560\cos(2}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-7560*Cos[2*(c + d*x)] - 1260*Cos[4*(c + d*x)] + 840*Cos[6*(c + d*x)] +
 315*Cos[8*(c + d*x)] + 7560*Sin[c + d*x] - 1680*Sin[3*(c + d*x)] - 1008*Si
n[5*(c + d*x)] + 180*Sin[7*(c + d*x)] + 140*Sin[9*(c + d*x)]))/(322560*d)
```

Maple [A] time = 0.033, size = 102, normalized size = 1.3

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^6}{9} - \frac{\sin(dx+c)(\cos(dx+c))^6}{21} + \frac{\sin(dx+c)}{105}\left(\frac{8}{3} + (\cos(dx+c))^4 + \frac{4(\cos(dx+}{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/8*sin(d*x+c)^2*cos(d*x
+c)^6-1/24*cos(d*x+c)^6))
```

Maxima [A] time = 1.16918, size = 97, normalized size = 1.2

$$\frac{280\,a\sin(dx+c)^9 + 315\,a\sin(dx+c)^8 - 720\,a\sin(dx+c)^7 - 840\,a\sin(dx+c)^6 + 504\,a\sin(dx+c)^5 + 630\,a\sin(d}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2520*(280*a*sin(d*x + c)^9 + 315*a*sin(d*x + c)^8 - 720*a*sin(d*x + c)^7
- 840*a*sin(d*x + c)^6 + 504*a*sin(d*x + c)^5 + 630*a*sin(d*x + c)^4)/d
```

Fricas [A] time = 1.14556, size = 223, normalized size = 2.75

$$\frac{315\,a\cos\left(dx+c\right)^{8}-420\,a\cos\left(dx+c\right)^{6}+8\left(35\,a\cos\left(dx+c\right)^{8}-50\,a\cos\left(dx+c\right)^{6}+3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*cos(d*x + c)^8 - 420*a*cos(d*x + c)^6 + 8*(35*a*cos(d*x + c)^
8 - 50*a*cos(d*x + c)^6 + 3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a)*si
n(d*x + c))/d
```

Sympy [A] time = 21.0753, size = 136, normalized size = 1.68

$$\begin{cases}\frac{8a\sin^{9}(c+dx)}{315d}+\frac{a\sin^{8}(c+dx)}{24d}+\frac{4a\sin^{7}(c+dx)\cos^{2}(c+dx)}{35d}+\frac{a\sin^{6}(c+dx)\cos^{2}(c+dx)}{6d}+\frac{a\sin^{5}(c+dx)\cos^{4}(c+dx)}{5d}+\frac{a\sin^{4}(c+dx)\cos^{4}(c+dx)}{4d} & \text{fo}\\ x\left(a\sin\left(c\right)+a\right)\sin^{3}\left(c\right)\cos^{5}\left(c\right) & \text{ot}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**9/(315*d) + a*sin(c + d*x)**8/(24*d) + 4*a*sin
(c + d*x)**7*cos(c + d*x)**2/(35*d) + a*sin(c + d*x)**6*cos(c + d*x)**2/(6*
d) + a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + a*sin(c + d*x)**4*cos(c + d*
x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**5, True))
```

Giac [A] time = 1.30362, size = 180, normalized size = 2.22

$$\frac{a\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{a\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{3\,a\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{a\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{a\sin\left(7\,dx+7\,c\right)}{1792\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*a*cos(8*d*x + 8*c)/d + 1/384*a*cos(6*d*x + 6*c)/d - 1/256*a*cos(4*d*
x + 4*c)/d - 3/128*a*cos(2*d*x + 2*c)/d + 1/2304*a*sin(9*d*x + 9*c)/d + 1/1
792*a*sin(7*d*x + 7*c)/d - 1/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x +
 3*c)/d + 3/128*a*sin(d*x + c)/d
```

3.497 $\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(
3*d) - (2*a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.125666, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2564, 270, 2565, 14}

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(
3*d) - (2*a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^2(c+dx)\,dx + a\int \cos^5(c+dx)\sin^3(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^5\left(1-x^2\right)dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\left(1-x^2\right)\right.}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \sin(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(x^5-x^7\right.\right.}{}$$
$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^8(c+dx)}{8d} + \frac{a\sin^3(c+dx)}{3d} - \frac{2a\sin^5(c+dx)}{5d}$$

Mathematica [A] time = 0.276744, size = 87, normalized size = 1.07

$$-\frac{a(-8400\sin(c+dx)+560\sin(3(c+dx))+1008\sin(5(c+dx))+240\sin(7(c+dx))+2520\cos(2(c+dx))+420\cos(4}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(2520*Cos[2*(c + d*x)] + 420*Cos[4*(c + d*x)] - 280*Cos[6*(c + d*x)] -
105*Cos[8*(c + d*x)] - 8400*Sin[c + d*x] + 560*Sin[3*(c + d*x)] + 1008*Sin[
5*(c + d*x)] + 240*Sin[7*(c + d*x)]))/(107520*d)
```

Maple [A] time = 0.029, size = 84, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{8} - \frac{(\cos(dx+c))^6}{24}\right) + a\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7} + \frac{\sin(dx+c)}{35}\left(\frac{8}{3} + (\cos(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+a*(-1/7*sin(d*x+c
)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.05539, size = 97, normalized size = 1.2

$$\frac{105\,a\sin(dx+c)^8 + 120\,a\sin(dx+c)^7 - 280\,a\sin(dx+c)^6 - 336\,a\sin(dx+c)^5 + 210\,a\sin(dx+c)^4 + 280\,a\sin(dx+}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(105*a*sin(d*x + c)^8 + 120*a*sin(d*x + c)^7 - 280*a*sin(d*x + c)^6 -
 336*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 + 280*a*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.14169, size = 192, normalized size = 2.37

$$\frac{105\,a\cos(dx+c)^8-140\,a\cos(dx+c)^6-8\left(15\,a\cos(dx+c)^6-3\,a\cos(dx+c)^4-4\,a\cos(dx+c)^2-8\,a\right)\sin(dx+c}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/840*(105*a*cos(d*x + c)^8 - 140*a*cos(d*x + c)^6 - 8*(15*a*cos(d*x + c)^6
 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

Sympy [A] time = 12.2315, size = 136, normalized size = 1.68

$$\begin{cases}\frac{a\sin^8(c+dx)}{24d}+\frac{8a\sin^7(c+dx)}{105d}+\frac{a\sin^6(c+dx)\cos^2(c+dx)}{6d}+\frac{4a\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{a\sin^4(c+dx)\cos^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)\cos^4(c+dx)}{3d}\\ x\,(a\sin(c)+a)\sin^2(c)\cos^5(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**8/(24*d) + 8*a*sin(c + d*x)**7/(105*d) + a*sin(c
 + d*x)**6*cos(c + d*x)**2/(6*d) + 4*a*sin(c + d*x)**5*cos(c + d*x)**2/(15*
d) + a*sin(c + d*x)**4*cos(c + d*x)**4/(4*d) + a*sin(c + d*x)**3*cos(c + d*
x)**4/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**5, True))
```

Giac [A] time = 1.24026, size = 159, normalized size = 1.96

$$\frac{a\cos(8\,dx+8\,c)}{1024\,d}+\frac{a\cos(6\,dx+6\,c)}{384\,d}-\frac{a\cos(4\,dx+4\,c)}{256\,d}-\frac{3\,a\cos(2\,dx+2\,c)}{128\,d}-\frac{a\sin(7\,dx+7\,c)}{448\,d}-\frac{3\,a\sin(5\,dx+5}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*a*cos(8*d*x + 8*c)/d + 1/384*a*cos(6*d*x + 6*c)/d - 1/256*a*cos(4*d*
x + 4*c)/d - 3/128*a*cos(2*d*x + 2*c)/d - 1/448*a*sin(7*d*x + 7*c)/d - 3/32
0*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 5/64*a*sin(d*x + c)/d
```

3.498 $\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^6(c+dx)}{6d}$$

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Sin[c + d*x]^3)/(3*d) - (2*a*Sin[c + d*x]^5)
/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0908986, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2834, 2565, 30, 2564, 270}

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Sin[c + d*x]^3)/(3*d) - (2*a*Sin[c + d*x]^5)
/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin(c+dx)\,dx + a\int \cos^5(c+dx)\sin^2(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^5\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x$$
$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sin(c+dx)\right)}{d}$$
$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\sin^3(c+dx)}{3d} - \frac{2a\sin^5(c+dx)}{5d} + \frac{a\sin^7(c+d$$

Mathematica [A] time = 0.156388, size = 78, normalized size = 1.2

$$-\frac{a(-525\sin(c+dx)+35\sin(3(c+dx))+63\sin(5(c+dx))+15\sin(7(c+dx))+525\cos(2(c+dx))+210\cos(4(c+}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(350 + 525*Cos[2*(c + d*x)] + 210*Cos[4*(c + d*x)] + 35*Cos[6*(c + d*x)
] - 525*Sin[c + d*x] + 35*Sin[3*(c + d*x)] + 63*Sin[5*(c + d*x)] + 15*Sin[7
*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.025, size = 64, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)\right)-\frac{a(\cos(dx+c))^6}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))-1/6*a*cos(d*x+c)^6)
```

Maxima [A] time = 1.02143, size = 97, normalized size = 1.49

$$\frac{30\,a\sin(dx+c)^7+35\,a\sin(dx+c)^6-84\,a\sin(dx+c)^5-105\,a\sin(dx+c)^4+70\,a\sin(dx+c)^3+105\,a\sin(dx+c)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(30*a*sin(d*x + c)^7 + 35*a*sin(d*x + c)^6 - 84*a*sin(d*x + c)^5 - 10
5*a*sin(d*x + c)^4 + 70*a*sin(d*x + c)^3 + 105*a*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.12755, size = 161, normalized size = 2.48

$$-\frac{35\,a\cos\left(dx+c\right)^{6}+2\left(15\,a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}-8\,a\right)\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/210*(35*a*cos(d*x + c)^6 + 2*(15*a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 -
 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

Sympy [A] time = 6.95768, size = 90, normalized size = 1.38

$$\begin{cases} \frac{8a\sin^{7}\left(c+dx\right)}{105d}+\frac{4a\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{15d}+\frac{a\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{3d}-\frac{a\cos^{6}\left(c+dx\right)}{6d} & \text{for}\ d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\sin\left(c\right)\cos^{5}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**7/(105*d) + 4*a*sin(c + d*x)**5*cos(c + d*x)**
2/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - a*cos(c + d*x)**6/(6*d
), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*cos(c)**5, True))
```

Giac [A] time = 1.26566, size = 139, normalized size = 2.14

$$-\frac{a\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{64\,d}-\frac{a\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{3\,a\sin\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\sin\left(3\,dx+3\,c\right)}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*a*cos(6*d*x + 6*c)/d - 1/32*a*cos(4*d*x + 4*c)/d - 5/64*a*cos(2*d*x
+ 2*c)/d - 1/448*a*sin(7*d*x + 7*c)/d - 3/320*a*sin(5*d*x + 5*c)/d - 1/192*
a*sin(3*d*x + 3*c)/d + 5/64*a*sin(d*x + c)/d
```

3.499 $\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*a*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0666494, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 88}

$$\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*a*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)^3}{x}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x}\,dx, x, a\sin(c+dx)\right)}{a^4d}\\
&= \frac{\text{Subst}\left(\int \left(a^4+\frac{a^5}{x}-2a^3x-2a^2x^2+ax^3+x^4\right)dx, x, a\sin(c+dx)\right)}{a^4d}\\
&= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin^3(c+
\end{aligned}$$

Mathematica [A] time = 0.0357863, size = 86, normalized size = 1.

$$\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*a*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.054, size = 94, normalized size = 1.1

$$\frac{8a\sin(dx+c)}{15d}+\frac{\sin(dx+c)(\cos(dx+c))^4a}{5d}+\frac{4(\cos(dx+c))^2\sin(dx+c)a}{15d}+\frac{a(\cos(dx+c))^4}{4d}+\frac{a(\cos(dx+c))^2}{2d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 8/15*a*sin(d*x+c)/d+1/5/d*cos(d*x+c)^4*sin(d*x+c)*a+4/15/d*cos(d*x+c)^2*sin
(d*x+c)*a+1/4*a*cos(d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.0433, size = 93, normalized size = 1.08

$$\frac{12a\sin(dx+c)^5+15a\sin(dx+c)^4-40a\sin(dx+c)^3-60a\sin(dx+c)^2+60a\log(\sin(dx+c))+60a\sin(dx+c)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*a*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c))/d
```

Fricas [A] time = 1.18617, size = 197, normalized size = 2.29

$$\frac{15a\cos(dx+c)^4+30a\cos(dx+c)^2+60a\log\left(\frac{1}{2}\sin(dx+c)\right)+4\left(3a\cos(dx+c)^4+4a\cos(dx+c)^2+8a\right)\sin(dx+}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^2 + 60*a*log(1/2*sin(d*x + c)
) + 4*(3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21495, size = 95, normalized size = 1.1

$$\frac{12\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-40\,a\sin\left(dx+c\right)^{3}-60\,a\sin\left(dx+c\right)^{2}+60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a\sin\left(dx+\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*a*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c))/d
```

3.500 $\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.078624, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)^3}{x^2}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(-2a^3+\frac{a^5}{x^2}+\frac{a^4}{x}-2a^2x+ax^2+x^3\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= -\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0419195, size = 83, normalized size = 1.

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.05, size = 116, normalized size = 1.4

$$\frac{a(\cos(dx+c))^4}{4d}+\frac{a(\cos(dx+c))^2}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{a(\cos(dx+c))^6}{d\sin(dx+c)}-\frac{8a\sin(dx+c)}{3d}-\frac{\sin(dx+c)(\cos(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4*a*cos(d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d-1/d*a/sin(d*x+
c)*cos(d*x+c)^6-8/3*a*sin(d*x+c)/d-1/d*cos(d*x+c)^4*sin(d*x+c)*a-4/3/d*cos(
d*x+c)^2*sin(d*x+c)*a
```

Maxima [A] time = 1.09919, size = 93, normalized size = 1.12

$$\frac{3a\sin(dx+c)^4+4a\sin(dx+c)^3-12a\sin(dx+c)^2+12a\log(\sin(dx+c))-24a\sin(dx+c)-\frac{12a}{\sin(dx+c)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 12*a*
log(sin(d*x + c)) - 24*a*sin(d*x + c) - 12*a/sin(d*x + c))/d
```

Fricas [A] time = 1.15737, size = 250, normalized size = 3.01

$$\frac{32a\cos(dx+c)^4+128a\cos(dx+c)^2+96a\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+3\left(8a\cos(dx+c)^4+16a\cos(dx+c\right.}{96d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(32*a*cos(d*x + c)^4 + 128*a*cos(d*x + c)^2 + 96*a*log(1/2*sin(d*x + c
))*sin(d*x + c) + 3*(8*a*cos(d*x + c)^4 + 16*a*cos(d*x + c)^2 - 11*a)*sin(d
*x + c) - 256*a)/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28746, size = 107, normalized size = 1.29

$$\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-12\,a\sin\left(dx+c\right)^{2}+12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-24\,a\sin\left(dx+c\right)-\frac{12\,(a\sin(dx+c)+a)}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 12*a*
log(abs(sin(d*x + c))) - 24*a*sin(d*x + c) - 12*(a*sin(d*x + c) + a)/sin(d*
x + c))/d
```

3.501 $\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (2*a*Log[Sin[c + d*x]])/
d - (2*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/(3
*d)
```

Rubi [A] time = 0.0793945, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (2*a*Log[Sin[c + d*x]])/
d - (2*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/(3
*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2+\frac{a^5}{x^3}+\frac{a^4}{x^2}-\frac{2a^3}{x}+ax+x^2\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= -\frac{a\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{2a\log(\sin(c+dx))}{d} - \frac{2a\sin(c+d}{d}$$

Mathematica [A] time = 0.109221, size = 77, normalized size = 0.9

$$\frac{a\sin^3(c+dx)}{3d} - \frac{2a\sin(c+dx)}{d} - \frac{a\csc(c+dx)}{d} - \frac{a\left(-\sin^2(c+dx)+\csc^2(c+dx)+4\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (2*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^3)/(3*d) - (
a*(Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.06, size = 139, normalized size = 1.6

$$-\frac{a(\cos(dx+c))^6}{d\sin(dx+c)} - \frac{8a\sin(dx+c)}{3d} - \frac{\sin(dx+c)(\cos(dx+c))^4 a}{d} - \frac{4(\cos(dx+c))^2\sin(dx+c)a}{3d} - \frac{a(\cos(dx+c))^6}{2d(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^6-8/3*a*sin(d*x+c)/d-1/d*cos(d*x+c)^4*sin(d*x+
c)*a-4/3/d*cos(d*x+c)^2*sin(d*x+c)*a-1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^6-1/2*
a*cos(d*x+c)^4/d-a*cos(d*x+c)^2/d-2*a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.15379, size = 92, normalized size = 1.07

$$\frac{2a\sin(dx+c)^3 + 3a\sin(dx+c)^2 - 12a\log(\sin(dx+c)) - 12a\sin(dx+c) - \frac{3(2a\sin(dx+c)+a)}{\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(sin(d*x + c)) - 12*
a*sin(d*x + c) - 3*(2*a*sin(d*x + c) + a)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.15681, size = 258, normalized size = 3.

$$-\frac{6\,a\cos\left(dx+c\right)^{4}-9\,a\cos\left(dx+c\right)^{2}+24\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\left(a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+\right.\right.}{12\,\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*cos(d*x + c)^4 - 9*a*cos(d*x + c)^2 + 24*(a*cos(d*x + c)^2 - a)*
log(1/2*sin(d*x + c)) + 4*(a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 - 8*a)*sin
(d*x + c) - 3*a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29853, size = 111, normalized size = 1.29

$$\frac{2\,a\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-12\,a\sin\left(dx+c\right)+\frac{3\,\left(6\,a\sin(dx+c)^{2}-2\,a\sin(dx+c)-a\right)}{\sin(dx+c)^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(abs(sin(d*x + c)))
- 12*a*sin(d*x + c) + 3*(6*a*sin(d*x + c)^2 - 2*a*sin(d*x + c) - a)/sin(d*x
+ c)^2)/d
```

3.502 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{2a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (2*a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0720851, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 88}

$$\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{2a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (2*a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^3}{x^4}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^4}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(a+\frac{a^5}{x^4}+\frac{a^4}{x^3}-\frac{2a^3}{x^2}-\frac{2a^2}{x}+x\right)dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{2a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{2a\log(\sin(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.153568, size = 76, normalized size = 0.89

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}-\frac{a\left(-\sin^2(c+dx)+\csc^2(c+dx)+4\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*(
Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.059, size = 159, normalized size = 1.9

$$-\frac{a(\cos(dx+c))^6}{2d(\sin(dx+c))^2}-\frac{a(\cos(dx+c))^4}{2d}-\frac{a(\cos(dx+c))^2}{d}-2\frac{a\ln(\sin(dx+c))}{d}-\frac{a(\cos(dx+c))^6}{3d(\sin(dx+c))^3}+\frac{a(\cos(dx+c}{d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^6-1/2*a*cos(d*x+c)^4/d-a*cos(d*x+c)^2/d-2*
a*ln(sin(d*x+c))/d-1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a/sin(d*x+c)*cos(d
*x+c)^6+8/3*a*sin(d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*a+4/3/d*cos(d*x+c)^2
*sin(d*x+c)*a
```

Maxima [A] time = 1.02227, size = 93, normalized size = 1.09

$$\frac{3a\sin(dx+c)^2-12a\log(\sin(dx+c))+6a\sin(dx+c)+\frac{12a\sin(dx+c)^2-3a\sin(dx+c)-2a}{\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*sin(d*x + c)^2 - 12*a*log(sin(d*x + c)) + 6*a*sin(d*x + c) + (12*a
*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 2*a)/sin(d*x + c)^3)/d
```

Fricas [A] time = 1.18384, size = 300, normalized size = 3.53

$$-\frac{12a\cos(dx+c)^4-48a\cos(dx+c)^2+24\left(a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+3\left(2a\cos(dx+c)\right.}{12\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^4 - 48*a*cos(d*x + c)^2 + 24*(a*cos(d*x + c)^2 - a
)*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(2*a*cos(d*x + c)^4 - 3*a*cos(d*x
```

```
+ c)^2 - a)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32014, size = 109, normalized size = 1.28

$$\frac{3\,a\sin\left(dx+c\right)^2 - 12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right) + 6\,a\sin\left(dx+c\right) + \frac{22\,a\sin(dx+c)^3+12\,a\sin(dx+c)^2-3\,a\sin(dx+c)-2\,a}{\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*sin(d*x + c)^2 - 12*a*log(abs(sin(d*x + c))) + 6*a*sin(d*x + c) +
(22*a*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 2*a)/sin(d*
x + c)^3)/d
```

3.503 $\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0440546, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a^5}{x^5}+\frac{a^4}{x^4}-\frac{2a^3}{x^3}-\frac{2a^2}{x^2}+\frac{a}{x}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{2a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin
\end{aligned}$$

Mathematica [A] time = 0.200457, size = 87, normalized size = 1.07

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}+\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (a*Sin[c
 + d*x])/d
```

Maple [A] time = 0.059, size = 136, normalized size = 1.7

$$-\frac{a\left(\cos\left(dx+c\right)\right)^{6}}{3d\left(\sin\left(dx+c\right)\right)^{3}}+\frac{a\left(\cos\left(dx+c\right)\right)^{6}}{d\sin\left(dx+c\right)}+\frac{8a\sin\left(dx+c\right)}{3d}+\frac{\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{4}a}{d}+\frac{4\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a/sin(d*x+c)*cos(d*x+c)^6+8/3*a*sin(
d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*a+4/3/d*cos(d*x+c)^2*sin(d*x+c)*a-1/4/
d*a*cot(d*x+c)^4+1/2*a*cot(d*x+c)^2/d+a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.09274, size = 93, normalized size = 1.15

$$\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,a\sin\left(dx+c\right)+\frac{24\,a\sin\left(dx+c\right)^{3}+12\,a\sin\left(dx+c\right)^{2}-4\,a\sin\left(dx+c\right)-3\,a}{\sin\left(dx+c\right)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*a*sin(d*x + c) + (24*a*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*a*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.17238, size = 292, normalized size = 3.6

$$-\frac{12\,a\cos\left(dx+c\right)^{2}-12\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-4\left(3\,a\cos\left(dx+c\right)^{4}-12\,a\cos\left(dx+\right.\right.}{12\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 8*a)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28016, size = 111, normalized size = 1.37

$$\frac{12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a\sin\left(dx+c\right)-\frac{25\,a\sin(dx+c)^4-24\,a\sin(dx+c)^3-12\,a\sin(dx+c)^2+4\,a\sin(dx+c)+3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*a*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

3.504 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{2a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) + (a*Csc[c + d*x]^2)/d + (2*a*Csc[c + d*x]^3)/(3*d) -
 (a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (a*Log[Sin[c + d*x]]
)/d
```

Rubi [A] time = 0.0705625, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 88}

$$-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{2a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Csc[c + d*x]^2)/d + (2*a*Csc[c + d*x]^3)/(3*d) -
 (a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (a*Log[Sin[c + d*x]]
)/d
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^3}{x^6}dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^6}dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{a^4}{x^5}-\frac{2a^3}{x^4}-\frac{2a^2}{x^3}+\frac{a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx}{4d}$$

Mathematica [A] time = 0.164674, size = 92, normalized size = 1.07

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*
d) + (a*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Ta
n[c + d*x]]))/(4*d)
```

Maple [A] time = 0.061, size = 160, normalized size = 1.9

$$-\frac{a(\cot(dx+c))^4}{4d}+\frac{a(\cot(dx+c))^2}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{a(\cos(dx+c))^6}{5d(\sin(dx+c))^5}+\frac{a(\cos(dx+c))^6}{15d(\sin(dx+c))^3}-\frac{a(\cos(dx+c}{5d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a*cot(d*x+c)^4+1/2*a*cot(d*x+c)^2/d+a*ln(sin(d*x+c))/d-1/5/d*a/sin(d
*x+c)^5*cos(d*x+c)^6+1/15/d*a/sin(d*x+c)^3*cos(d*x+c)^6-1/5/d*a/sin(d*x+c)*
cos(d*x+c)^6-8/15*a*sin(d*x+c)/d-1/5/d*cos(d*x+c)^4*sin(d*x+c)*a-4/15/d*cos
(d*x+c)^2*sin(d*x+c)*a
```

Maxima [A] time = 1.07469, size = 97, normalized size = 1.13

$$\frac{60\,a\log(\sin(dx+c))-\frac{60\,a\sin(dx+c)^4-60\,a\sin(dx+c)^3-40\,a\sin(dx+c)^2+15\,a\sin(dx+c)+12\,a}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*a*log(sin(d*x + c)) - (60*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 -
 40*a*sin(d*x + c)^2 + 15*a*sin(d*x + c) + 12*a)/sin(d*x + c)^5)/d
```

Fricas [A] time = 1.1246, size = 332, normalized size = 3.86

$$-\frac{60\,a\cos\left(dx+c\right)^4-80\,a\cos\left(dx+c\right)^2-60\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+15}{60\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^4 - 80*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^4 - 2
*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c))*sin(d*x + c) + 15*(4*a*cos(d*x
 + c)^2 - 3*a)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2
 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.315, size = 113, normalized size = 1.31

$$\frac{60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{137\,a\sin\left(dx+c\right)^5+60\,a\sin\left(dx+c\right)^4-60\,a\sin\left(dx+c\right)^3-40\,a\sin\left(dx+c\right)^2+15\,a\sin\left(dx+c\right)+12\,a}{\sin\left(dx+c\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(sin(d*x + c))) - (137*a*sin(d*x + c)^5 + 60*a*sin(d*x +
c)^4 - 60*a*sin(d*x + c)^3 - 40*a*sin(d*x + c)^2 + 15*a*sin(d*x + c) + 12*a
)/sin(d*x + c)^5)/d
```

3.505 $\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=61

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}$$

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x])/d + (2*a*Csc[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.103967, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 30, 2606, 194}

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x])/d + (2*a*Csc[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x]^5)/(5*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc(c+dx)\,dx + a\int \cot^5(c+dx)\csc^2(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x,\right.}{d}$$
$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(c+dx)\right)}{d}$$
$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc(c+dx)}{d} + \frac{2a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d}$$

Mathematica [A] time = 0.0242166, size = 61, normalized size = 1.

$$-\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc^5(c+dx)}{5d} + \frac{2a\csc^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x])/d + (2*a*Csc[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.061, size = 110, normalized size = 1.8

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{5(\sin(dx+c))^5} + \frac{(\cos(dx+c))^6}{15(\sin(dx+c))^3} - \frac{(\cos(dx+c))^6}{5\sin(dx+c)} - \frac{\sin(dx+c)}{5}\left(\frac{8}{3} + (\cos(dx+c))^4 + \frac{4(\cos(dx+c))^2}{3}\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/s
in(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-
1/6*a/sin(d*x+c)^6*cos(d*x+c)^6)
```

Maxima [A] time = 1.13625, size = 95, normalized size = 1.56

$$-\frac{30\,a\sin(dx+c)^5 + 15\,a\sin(dx+c)^4 - 20\,a\sin(dx+c)^3 - 15\,a\sin(dx+c)^2 + 6\,a\sin(dx+c) + 5\,a}{30\,d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(30*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*a*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

Fricas [A] time = 1.06001, size = 254, normalized size = 4.16

$$\frac{15\,a\cos\left(dx+c\right)^{4}-15\,a\cos\left(dx+c\right)^{2}+2\left(15\,a\cos\left(dx+c\right)^{4}-20\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)+5\,a}{30\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*cos(d*x + c)^4 - 15*a*cos(d*x + c)^2 + 2*(15*a*cos(d*x + c)^4 -
20*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) + 5*a)/(d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33398, size = 95, normalized size = 1.56

$$-\frac{30\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-15\,a\sin\left(dx+c\right)^{2}+6\,a\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(30*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*a*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

3.506 $\int \cot^5(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.115577, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 30}

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^2(c+dx)\,dx + a\int \cot^5(c+dx)\csc^3(c+dx)$$
$$= -\frac{a\,\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^2}{d}$$
$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\,\text{Subst}\left(\int\left(x^2-2x^4+x^6\right)dx, x, \csc(c+dx)\right)}{d}$$
$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+}{7d}$$

Mathematica [A] time = 0.0256111, size = 65, normalized size = 1.

$$-\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc^7(c+dx)}{7d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Maple [B] time = 0.062, size = 128, normalized size = 2.

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^6}{6(\sin(dx+c))^6} + a\left(-\frac{(\cos(dx+c))^6}{7(\sin(dx+c))^7} - \frac{(\cos(dx+c))^6}{35(\sin(dx+c))^5} + \frac{(\cos(dx+c))^6}{105(\sin(dx+c))^3} - \frac{(\cos(dx+c))^6}{35\sin(dx+c)} - \frac{\sin(dx+}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*a/sin(d*x+c)^6*cos(d*x+c)^6+a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/3
5/sin(d*x+c)^5*cos(d*x+c)^6+1/105/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)
*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.06919, size = 95, normalized size = 1.46

$$-\frac{105\,a\sin(dx+c)^5 + 70\,a\sin(dx+c)^4 - 105\,a\sin(dx+c)^3 - 84\,a\sin(dx+c)^2 + 35\,a\sin(dx+c) + 30\,a}{210\,d\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(105*a*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*a*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*a*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

Fricas [A] time = 1.06936, size = 273, normalized size = 4.2

$$\frac{70\,a\cos\left(dx+c\right)^{4}-56\,a\cos\left(dx+c\right)^{2}+35\,\left(3\,a\cos\left(dx+c\right)^{4}-3\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)+16\,a}{210\,\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(70*a*cos(d*x + c)^4 - 56*a*cos(d*x + c)^2 + 35*(3*a*cos(d*x + c)^4 -
 3*a*cos(d*x + c)^2 + a)*sin(d*x + c) + 16*a)/((d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2966, size = 95, normalized size = 1.46

$$-\frac{105\,a\sin\left(dx+c\right)^{5}+70\,a\sin\left(dx+c\right)^{4}-105\,a\sin\left(dx+c\right)^{3}-84\,a\sin\left(dx+c\right)^{2}+35\,a\sin\left(dx+c\right)+30\,a}{210\,d\sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/210*(105*a*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*a*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*a*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

3.507 $\int \cot^5(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^3)/(
3*d) + (2*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.121172, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 14, 2606, 270}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^3)/(
3*d) + (2*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^3(c+dx)\,dx + a\int \cot^5(c+dx)\csc^4(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^2 dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x^5 (1+\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right) dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(x^5+\right.\right.}{}$$
$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(c+dx)}{5d}$$

Mathematica [A] time = 0.155217, size = 88, normalized size = 1.09

$$-\frac{a\csc^7(c+dx)}{7d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\left(3\csc^8(c+dx)-8\csc^6(c+dx)+6\csc^4(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)
/(7*d) - (a*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

Maple [B] time = 0.066, size = 148, normalized size = 1.8

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{7(\sin(dx+c))^7} - \frac{(\cos(dx+c))^6}{35(\sin(dx+c))^5} + \frac{(\cos(dx+c))^6}{105(\sin(dx+c))^3} - \frac{(\cos(dx+c))^6}{35\sin(dx+c)} - \frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105
/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c
)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/si
n(d*x+c)^6*cos(d*x+c)^6))
```

Maxima [A] time = 1.08151, size = 95, normalized size = 1.17

$$-\frac{280\,a\sin(dx+c)^5 + 210\,a\sin(dx+c)^4 - 336\,a\sin(dx+c)^3 - 280\,a\sin(dx+c)^2 + 120\,a\sin(dx+c) + 105\,a}{840\,d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*(280*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*a*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*a*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

Fricas [A] time = 1.0961, size = 289, normalized size = 3.57

$$-\frac{210\,a\cos\left(dx+c\right)^{4}-140\,a\cos\left(dx+c\right)^{2}+8\left(35\,a\cos\left(dx+c\right)^{4}-28\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)+35\,a}{840\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(210*a*cos(d*x + c)^4 - 140*a*cos(d*x + c)^2 + 8*(35*a*cos(d*x + c)^
4 - 28*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) + 35*a)/(d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21974, size = 95, normalized size = 1.17

$$-\frac{280\,a\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-336\,a\sin\left(dx+c\right)^{3}-280\,a\sin\left(dx+c\right)^{2}+120\,a\sin\left(dx+c\right)+105\,a}{840\,d\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/840*(280*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*a*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*a*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

3.508 $\int \cot^5(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}$$

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^5)/(
5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.121623, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 14}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^5)/(
5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^4(c+dx)\,dx + a\int \cot^5(c+dx)\csc^5(c+dx)$$
$$= -\frac{a\,\text{Subst}\left(\int x^4\left(-1+x^2\right)^2 dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x^5 (1\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int \left(x^5+x^7\right) dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(x^4-2\right.\right.}{}$$
$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^5(c+dx)}{5d} + \frac{2a\csc^7(c+}{7d}$$

Mathematica [A] time = 0.113413, size = 88, normalized size = 1.09

$$-\frac{a\csc^9(c+dx)}{9d} + \frac{2a\csc^7(c+dx)}{7d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\left(3\csc^8(c+dx) - 8\csc^6(c+dx) + 6\csc^4(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^5)/(5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)
/(9*d) - (a*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

Maple [B] time = 0.064, size = 166, normalized size = 2.1

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{8(\sin(dx+c))^8} - \frac{(\cos(dx+c))^6}{24(\sin(dx+c))^6}\right) + a\left(-\frac{(\cos(dx+c))^6}{9(\sin(dx+c))^9} - \frac{(\cos(dx+c))^6}{21(\sin(dx+c))^7} - \frac{(\cos(dx+c))^6}{105(\sin(dx+c))^5} + \frac{(}{31}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6)+a*(-
1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x+c)^7*cos(d*x+c)^6-1/105/sin(d*x+
c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d*x+c)^6-1/105/sin(d*x+c)*cos(d*x+
c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.13296, size = 95, normalized size = 1.17

$$-\frac{630\,a\sin(dx+c)^5 + 504\,a\sin(dx+c)^4 - 840\,a\sin(dx+c)^3 - 720\,a\sin(dx+c)^2 + 315\,a\sin(dx+c) + 280\,a}{2520\,d\sin(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2520*(630*a*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*a*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*a*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

Fricas [A] time = 1.10186, size = 308, normalized size = 3.8

$$-\frac{504\,a\cos\left(dx+c\right)^{4}-288\,a\cos\left(dx+c\right)^{2}+105\left(6\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)+64\,a}{2520\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(504*a*cos(d*x + c)^4 - 288*a*cos(d*x + c)^2 + 105*(6*a*cos(d*x + c
)^4 - 4*a*cos(d*x + c)^2 + a)*sin(d*x + c) + 64*a)/((d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**10*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22954, size = 95, normalized size = 1.17

$$-\frac{630\,a\sin\left(dx+c\right)^{5}+504\,a\sin\left(dx+c\right)^{4}-840\,a\sin\left(dx+c\right)^{3}-720\,a\sin\left(dx+c\right)^{2}+315\,a\sin\left(dx+c\right)+280\,a}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2520*(630*a*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*a*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*a*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

3.509 $\int \cot^5(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^{10}(c+dx)}{10d}-\frac{a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}$$

```
[Out] -(a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/(6*d) + (2*a*Csc[c + d*x]^7)
/(7*d) + (a*Csc[c + d*x]^8)/(4*d) - (a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d
*x]^10)/(10*d)
```

Rubi [A] time = 0.0817221, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\csc^{10}(c+dx)}{10d}-\frac{a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/(6*d) + (2*a*Csc[c + d*x]^7)
/(7*d) + (a*Csc[c + d*x]^8)/(4*d) - (a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d
*x]^10)/(10*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^{11}(a-x)^2(a+x)^3}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{a^4}{x^{10}}-\frac{2a^3}{x^9}-\frac{2a^2}{x^8}+\frac{a}{x^7}+\frac{1}{x^6}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}+\frac{2a\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+dx)}{4d}$$

Mathematica [A] time = 0.167413, size = 88, normalized size = 0.91

$$-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\left(6\csc^{10}(c+dx)-15\csc^8(c+dx)+10\csc^6(c+dx)\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^5)/(5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)
/(9*d) - (a*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/(6
0*d)
```

Maple [B] time = 0.063, size = 184, normalized size = 1.9

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{9(\sin(dx+c))^9}-\frac{(\cos(dx+c))^6}{21(\sin(dx+c))^7}-\frac{(\cos(dx+c))^6}{105(\sin(dx+c))^5}+\frac{(\cos(dx+c))^6}{315(\sin(dx+c))^3}-\frac{(\cos(dx+c))^6}{105\sin(dx+c)}-\frac{\sin(dx+c}{105}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x+c)^7*cos(d*x+c)^6-1/105
/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d*x+c)^6-1/105/sin(d*x+c)
*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/1
0/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x+c)^8*cos(d*x+c)^6-1/60/sin(d*x+c)
^6*cos(d*x+c)^6))
```

Maxima [A] time = 1.03153, size = 95, normalized size = 0.98

$$-\frac{252\,a\sin(dx+c)^5+210\,a\sin(dx+c)^4-360\,a\sin(dx+c)^3-315\,a\sin(dx+c)^2+140\,a\sin(dx+c)+126\,a}{1260\,d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/1260*(252*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*a*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*a*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

Fricas [A] time = 1.12034, size = 321, normalized size = 3.31

$$\frac{210\,a\cos\left(dx+c\right)^{4}-105\,a\cos\left(dx+c\right)^{2}+4\,\left(63\,a\cos\left(dx+c\right)^{4}-36\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)+21\,a}{1260\,\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1260*(210*a*cos(d*x + c)^4 - 105*a*cos(d*x + c)^2 + 4*(63*a*cos(d*x + c)^
4 - 36*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) + 21*a)/(d*cos(d*x + c)^10 - 5*
d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x
+ c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**11*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32973, size = 95, normalized size = 0.98

$$-\frac{252\,a\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-360\,a\sin\left(dx+c\right)^{3}-315\,a\sin\left(dx+c\right)^{2}+140\,a\sin\left(dx+c\right)+126\,a}{1260\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1260*(252*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*a*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*a*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

3.510 $\int \cot^5(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^{11}(c+dx)}{11d}-\frac{a\csc^{10}(c+dx)}{10d}+\frac{2a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}$$

```
[Out] -(a*Csc[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^8)/(
4*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^10)/(10*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rubi [A] time = 0.0818183, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\csc^{11}(c+dx)}{11d}-\frac{a\csc^{10}(c+dx)}{10d}+\frac{2a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^8)/(
4*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^10)/(10*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^{12}(a-x)^2(a+x)^3}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{a^5}{x^{12}}+\frac{a^4}{x^{11}}-\frac{2a^3}{x^{10}}-\frac{2a^2}{x^9}+\frac{a}{x^8}+\frac{1}{x^7}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+dx)}{4d}+\frac{2a\csc^9(c+$$

Mathematica [A] time = 0.184271, size = 88, normalized size = 0.91

$$-\frac{a\csc^{11}(c+dx)}{11d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\left(6\csc^{10}(c+dx)-15\csc^8(c+dx)+10\csc^6(c+dx)\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^7)/(7*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^11
)/(11*d) - (a*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/
(60*d)
```

Maple [B] time = 0.063, size = 202, normalized size = 2.1

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{10(\sin(dx+c))^{10}}-\frac{(\cos(dx+c))^6}{20(\sin(dx+c))^8}-\frac{(\cos(dx+c))^6}{60(\sin(dx+c))^6}\right)+a\left(-\frac{(\cos(dx+c))^6}{11(\sin(dx+c))^{11}}-\frac{5(\cos(dx+c))^6}{99(\sin(dx+c))^9}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x+c)^8*cos(d*x+c)^6-1/6
0/sin(d*x+c)^6*cos(d*x+c)^6)+a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^6-5/99/sin(d
*x+c)^9*cos(d*x+c)^6-5/231/sin(d*x+c)^7*cos(d*x+c)^6-1/231/sin(d*x+c)^5*cos
(d*x+c)^6+1/693/sin(d*x+c)^3*cos(d*x+c)^6-1/231/sin(d*x+c)*cos(d*x+c)^6-1/2
31*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.15121, size = 95, normalized size = 0.98

$$-\frac{2310\,a\sin(dx+c)^5+1980\,a\sin(dx+c)^4-3465\,a\sin(dx+c)^3-3080\,a\sin(dx+c)^2+1386\,a\sin(dx+c)+1260\,a}{13860\,d\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/13860*(2310*a*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*a*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*a*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
```

```
)^11)
```

Fricas [A] time = 1.12054, size = 344, normalized size = 3.55

$$\frac{1980\,a\cos\left(dx+c\right)^{4}-880\,a\cos\left(dx+c\right)^{2}+231\,\left(10\,a\cos\left(dx+c\right)^{4}-5\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)+160\,a}{13860\,\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/13860*(1980*a*cos(d*x + c)^4 - 880*a*cos(d*x + c)^2 + 231*(10*a*cos(d*x +
 c)^4 - 5*a*cos(d*x + c)^2 + a)*sin(d*x + c) + 160*a)/((d*cos(d*x + c)^10 -
 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d
*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**12*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26654, size = 95, normalized size = 0.98

$$-\frac{2310\,a\sin\left(dx+c\right)^{5}+1980\,a\sin\left(dx+c\right)^{4}-3465\,a\sin\left(dx+c\right)^{3}-3080\,a\sin\left(dx+c\right)^{2}+1386\,a\sin\left(dx+c\right)+1260\,a}{13860\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13860*(2310*a*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*a*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*a*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

3.511 $\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=127

$$\frac{a^2\sin^{10}(c+dx)}{10d}+\frac{2a^2\sin^9(c+dx)}{9d}-\frac{a^2\sin^8(c+dx)}{8d}-\frac{4a^2\sin^7(c+dx)}{7d}-\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin$$

```
[Out] (a^2*Sin[c + d*x]^4)/(4*d) + (2*a^2*Sin[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*
x]^6)/(6*d) - (4*a^2*Sin[c + d*x]^7)/(7*d) - (a^2*Sin[c + d*x]^8)/(8*d) + (
2*a^2*Sin[c + d*x]^9)/(9*d) + (a^2*Sin[c + d*x]^10)/(10*d)
```

Rubi [A] time = 0.125219, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^{10}(c+dx)}{10d}+\frac{2a^2\sin^9(c+dx)}{9d}-\frac{a^2\sin^8(c+dx)}{8d}-\frac{4a^2\sin^7(c+dx)}{7d}-\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^4)/(4*d) + (2*a^2*Sin[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*
x]^6)/(6*d) - (4*a^2*Sin[c + d*x]^7)/(7*d) - (a^2*Sin[c + d*x]^8)/(8*d) + (
2*a^2*Sin[c + d*x]^9)/(9*d) + (a^2*Sin[c + d*x]^10)/(10*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3(a+x)^4}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^3(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6x^3+2a^5x^4-a^4x^5-4a^3x^6-a^2x^7+2ax^8+x^9\right)dx, x, a\right.}{a^8d}$$
$$= \frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^6(c+dx)}{6d}-\frac{4a^2\sin^7(c+}{7d}$$

Mathematica [A] time = 0.795818, size = 110, normalized size = 0.87

$$-\frac{a^2(-15120\sin(c+dx)+3360\sin(3(c+dx))+2016\sin(5(c+dx))-360\sin(7(c+dx))-280\sin(9(c+dx))+10710\cos}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(-2625 + 10710*Cos[2*(c + d*x)] + 1260*Cos[4*(c + d*x)] - 1365*Cos[6*
(c + d*x)] - 315*Cos[8*(c + d*x)] + 63*Cos[10*(c + d*x)] - 15120*Sin[c + d*
x] + 3360*Sin[3*(c + d*x)] + 2016*Sin[5*(c + d*x)] - 360*Sin[7*(c + d*x)] -
 280*Sin[9*(c + d*x)]))/(322560*d)
```

Maple [A] time = 0.04, size = 158, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^6}{10}-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{20}-\frac{(\cos(dx+c))^6}{60}\right)+2a^2\left(-1/9(\sin(dx+c))^3(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/
60*cos(d*x+c)^6)+2*a^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(
d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/8*si
n(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

Maxima [A] time = 1.17302, size = 131, normalized size = 1.03

$$\frac{252a^2\sin(dx+c)^{10}+560a^2\sin(dx+c)^9-315a^2\sin(dx+c)^8-1440a^2\sin(dx+c)^7-420a^2\sin(dx+c)^6+1008a^2 s}{2520d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2520*(252*a^2*sin(d*x + c)^10 + 560*a^2*sin(d*x + c)^9 - 315*a^2*sin(d*x
+ c)^8 - 1440*a^2*sin(d*x + c)^7 - 420*a^2*sin(d*x + c)^6 + 1008*a^2*sin(d*
```

```
x + c)^5 + 630*a^2*sin(d*x + c)^4)/d
```

Fricas [A] time = 1.12418, size = 279, normalized size = 2.2

$$-\frac{252\,a^2\cos\left(dx+c\right)^{10}-945\,a^2\cos\left(dx+c\right)^8+840\,a^2\cos\left(dx+c\right)^6-16\left(35\,a^2\cos\left(dx+c\right)^8-50\,a^2\cos\left(dx+c\right)^6+3\,a}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2520*(252*a^2*cos(d*x + c)^10 - 945*a^2*cos(d*x + c)^8 + 840*a^2*cos(d*x
 + c)^6 - 16*(35*a^2*cos(d*x + c)^8 - 50*a^2*cos(d*x + c)^6 + 3*a^2*cos(d*x
 + c)^4 + 4*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 32.7615, size = 189, normalized size = 1.49

$$\begin{cases}\frac{16a^2\sin^9\left(c+dx\right)}{315d}+\frac{8a^2\sin^7\left(c+dx\right)\cos^2\left(c+dx\right)}{35d}+\frac{2a^2\sin^5\left(c+dx\right)\cos^4\left(c+dx\right)}{5d}-\frac{a^2\sin^4\left(c+dx\right)\cos^6\left(c+dx\right)}{6d}-\frac{a^2\sin^2\left(c+dx\right)\cos^8\left(c+dx\right)}{12d}-\frac{a^2\sin^2\left(c\right.}{} \\ x\left(a\sin\left(c\right)+a\right)^2\sin^3\left(c\right)\cos^5\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((16*a**2*sin(c + d*x)**9/(315*d) + 8*a**2*sin(c + d*x)**7*cos(c +
 d*x)**2/(35*d) + 2*a**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - a**2*sin(c
 + d*x)**4*cos(c + d*x)**6/(6*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**8/(12
*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**6/(6*d) - a**2*cos(c + d*x)**10/(6
0*d) - a**2*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*
*3*cos(c)**5, True))
```

Giac [A] time = 1.30571, size = 227, normalized size = 1.79

$$-\frac{a^2\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{a^2\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{13\,a^2\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{a^2\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{17\,a^2\cos\left(2\,dx+2\,c\right)}{512\,d}+\frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/5120*a^2*cos(10*d*x + 10*c)/d + 1/1024*a^2*cos(8*d*x + 8*c)/d + 13/3072*
a^2*cos(6*d*x + 6*c)/d - 1/256*a^2*cos(4*d*x + 4*c)/d - 17/512*a^2*cos(2*d*
x + 2*c)/d + 1/1152*a^2*sin(9*d*x + 9*c)/d + 1/896*a^2*sin(7*d*x + 7*c)/d -
 1/160*a^2*sin(5*d*x + 5*c)/d - 1/96*a^2*sin(3*d*x + 3*c)/d + 3/64*a^2*sin(
d*x + c)/d
```

3.512 $\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=109

$$\frac{(a\sin(c+dx)+a)^9}{9a^7d}-\frac{3(a\sin(c+dx)+a)^8}{4a^6d}+\frac{13(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

```
[Out] (4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(a + a*Sin[c + d*x])^6)/(a^4*d) +
 (13*(a + a*Sin[c + d*x])^7)/(7*a^5*d) - (3*(a + a*Sin[c + d*x])^8)/(4*a^6*
d) + (a + a*Sin[c + d*x])^9/(9*a^7*d)
```

Rubi [A] time = 0.125813, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{(a\sin(c+dx)+a)^9}{9a^7d}-\frac{3(a\sin(c+dx)+a)^8}{4a^6d}+\frac{13(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(a + a*Sin[c + d*x])^6)/(a^4*d) +
 (13*(a + a*Sin[c + d*x])^7)/(7*a^5*d) - (3*(a + a*Sin[c + d*x])^8)/(4*a^6*
d) + (a + a*Sin[c + d*x])^9/(9*a^7*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2(a+x)^4}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^2(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4(a+x)^4 - 12a^3(a+x)^5 + 13a^2(a+x)^6 - 6a(a+x\right.\right.}{a^7d}$$
$$= \frac{4(a+a\sin(c+dx))^5}{5a^3d} - \frac{2(a+a\sin(c+dx))^6}{a^4d} + \frac{13(a+a\sin(c+}{7a^5d}$$

Mathematica [A] time = 0.731221, size = 99, normalized size = 0.91

$$-\frac{a^2(-16380\sin(c+dx) + 1680\sin(3(c+dx)) + 2016\sin(5(c+dx)) + 270\sin(7(c+dx)) - 70\sin(9(c+dx)) + 7560\,c}{161280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(7560*Cos[2*(c + d*x)] + 1260*Cos[4*(c + d*x)] - 840*Cos[6*(c + d*x)]
 - 315*Cos[8*(c + d*x)] - 16380*Sin[c + d*x] + 1680*Sin[3*(c + d*x)] + 2016
*Sin[5*(c + d*x)] + 270*Sin[7*(c + d*x)] - 70*Sin[9*(c + d*x)]))/(161280*d)
```

Maple [A] time = 0.037, size = 156, normalized size = 1.4

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^6}{9} - \frac{\sin(dx+c)(\cos(dx+c))^6}{21} + \frac{\sin(dx+c)}{105}\left(\frac{8}{3} + (\cos(dx+c))^4 + \frac{4(\cos(dx}{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+2*a^2*(-1/8*sin(d*x+c)^2*c
os(d*x+c)^6-1/24*cos(d*x+c)^6)+a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.11049, size = 131, normalized size = 1.2

$$\frac{140\,a^2\sin(dx+c)^9 + 315\,a^2\sin(dx+c)^8 - 180\,a^2\sin(dx+c)^7 - 840\,a^2\sin(dx+c)^6 - 252\,a^2\sin(dx+c)^5 + 630\,a^2\,s}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1260*(140*a^2*sin(d*x + c)^9 + 315*a^2*sin(d*x + c)^8 - 180*a^2*sin(d*x +
 c)^7 - 840*a^2*sin(d*x + c)^6 - 252*a^2*sin(d*x + c)^5 + 630*a^2*sin(d*x +
 c)^4 + 420*a^2*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.15399, size = 246, normalized size = 2.26

$$\frac{315\,a^2\cos\left(dx+c\right)^8-420\,a^2\cos\left(dx+c\right)^6+4\left(35\,a^2\cos\left(dx+c\right)^8-95\,a^2\cos\left(dx+c\right)^6+12\,a^2\cos\left(dx+c\right)^4+16\,a^2\cos\left(\right.\right.}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/1260*(315*a^2*cos(d*x + c)^8 - 420*a^2*cos(d*x + c)^6 + 4*(35*a^2*cos(d*x
 + c)^8 - 95*a^2*cos(d*x + c)^6 + 12*a^2*cos(d*x + c)^4 + 16*a^2*cos(d*x +
c)^2 + 32*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 20.8176, size = 214, normalized size = 1.96

$$\begin{cases}\frac{8a^2\sin^9(c+dx)}{315d}+\frac{a^2\sin^8(c+dx)}{12d}+\frac{4a^2\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{8a^2\sin^7(c+dx)}{105d}+\frac{a^2\sin^6(c+dx)\cos^2(c+dx)}{3d}+\frac{a^2\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{4a^2\sin^5}{} \\ x\left(a\sin\left(c\right)+a\right)^2\sin^2\left(c\right)\cos^5\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**9/(315*d) + a**2*sin(c + d*x)**8/(12*d) + 4
*a**2*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 8*a**2*sin(c + d*x)**7/(105*
d) + a**2*sin(c + d*x)**6*cos(c + d*x)**2/(3*d) + a**2*sin(c + d*x)**5*cos(
c + d*x)**4/(5*d) + 4*a**2*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + a**2*si
n(c + d*x)**4*cos(c + d*x)**4/(2*d) + a**2*sin(c + d*x)**3*cos(c + d*x)**4/
(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**5, True))
```

Giac [A] time = 1.22668, size = 204, normalized size = 1.87

$$\frac{a^2\cos\left(8\,dx+8\,c\right)}{512\,d}+\frac{a^2\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{a^2\cos\left(4\,dx+4\,c\right)}{128\,d}-\frac{3\,a^2\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{a^2\sin\left(9\,dx+9\,c\right)}{2304\,d}-\frac{3\,a^2\sin\left(7\,dx\right.}{1792\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*a^2*cos(8*d*x + 8*c)/d + 1/192*a^2*cos(6*d*x + 6*c)/d - 1/128*a^2*cos
(4*d*x + 4*c)/d - 3/64*a^2*cos(2*d*x + 2*c)/d + 1/2304*a^2*sin(9*d*x + 9*c)
/d - 3/1792*a^2*sin(7*d*x + 7*c)/d - 1/80*a^2*sin(5*d*x + 5*c)/d - 1/96*a^2
*sin(3*d*x + 3*c)/d + 13/128*a^2*sin(d*x + c)/d
```

3.513 $\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=89

$$\frac{(a\sin(c+dx)+a)^8}{8a^6d}-\frac{5(a\sin(c+dx)+a)^7}{7a^5d}+\frac{4(a\sin(c+dx)+a)^6}{3a^4d}-\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

```
[Out] (-4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) + (4*(a + a*Sin[c + d*x])^6)/(3*a^4*d
) - (5*(a + a*Sin[c + d*x])^7)/(7*a^5*d) + (a + a*Sin[c + d*x])^8/(8*a^6*d)
```

Rubi [A] time = 0.0843447, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 77}

$$\frac{(a\sin(c+dx)+a)^8}{8a^6d}-\frac{5(a\sin(c+dx)+a)^7}{7a^5d}+\frac{4(a\sin(c+dx)+a)^6}{3a^4d}-\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) + (4*(a + a*Sin[c + d*x])^6)/(3*a^4*d
) - (5*(a + a*Sin[c + d*x])^7)/(7*a^5*d) + (a + a*Sin[c + d*x])^8/(8*a^6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x(a+x)^4}{a}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3(a+x)^4+8a^2(a+x)^5-5a(a+x)^6+(a+x)^7\right)dx, x$$
$$= -\frac{4(a+a\sin(c+dx))^5}{5a^3d}+\frac{4(a+a\sin(c+dx))^6}{3a^4d}-\frac{5(a+a\sin(c+dx)}{7a^5d}$$

Mathematica [A] time = 0.343968, size = 90, normalized size = 1.01

$$-\frac{a^2(-16800\sin(c+dx)+1120\sin(3(c+dx))+2016\sin(5(c+dx))+480\sin(7(c+dx))+10920\cos(2(c+dx))+3780}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(-2590 + 10920*Cos[2*(c + d*x)] + 3780*Cos[4*(c + d*x)] + 280*Cos[6*(
c + d*x)] - 105*Cos[8*(c + d*x)] - 16800*Sin[c + d*x] + 1120*Sin[3*(c + d*x
)] + 2016*Sin[5*(c + d*x)] + 480*Sin[7*(c + d*x)]))/(107520*d)
```

Maple [A] time = 0.034, size = 102, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{8}-\frac{(\cos(dx+c))^6}{24}\right)+2a^2\left(-1/7\sin(dx+c)(\cos(dx+c))^6+1/35\left(8/3+(\cos(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+2*a^2*(-1/7*sin
(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1
/6*a^2*cos(d*x+c)^6)
```

Maxima [A] time = 1.06281, size = 131, normalized size = 1.47

$$\frac{105\,a^2\sin(dx+c)^8+240\,a^2\sin(dx+c)^7-140\,a^2\sin(dx+c)^6-672\,a^2\sin(dx+c)^5-210\,a^2\sin(dx+c)^4+560\,a^2\sin(}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/840*(105*a^2*sin(d*x + c)^8 + 240*a^2*sin(d*x + c)^7 - 140*a^2*sin(d*x +
c)^6 - 672*a^2*sin(d*x + c)^5 - 210*a^2*sin(d*x + c)^4 + 560*a^2*sin(d*x +
c)^3 + 420*a^2*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.12122, size = 209, normalized size = 2.35

$$\frac{105\,a^2\cos\left(dx+c\right)^8-280\,a^2\cos\left(dx+c\right)^6-16\left(15\,a^2\cos\left(dx+c\right)^6-3\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2-8\,a^2\right)\sin}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*a^2*cos(d*x + c)^8 - 280*a^2*cos(d*x + c)^6 - 16*(15*a^2*cos(d*x
 + c)^6 - 3*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 - 8*a^2)*sin(d*x + c)
)/d
```

Sympy [A] time = 11.8753, size = 163, normalized size = 1.83

$$\begin{cases}\frac{a^2\sin^8(c+dx)}{24d}+\frac{16a^2\sin^7(c+dx)}{105d}+\frac{a^2\sin^6(c+dx)\cos^2(c+dx)}{6d}+\frac{8a^2\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{a^2\sin^4(c+dx)\cos^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)\cos^4(c}{3d}\\ x\left(a\sin\left(c\right)+a\right)^2\sin\left(c\right)\cos^5\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)**8/(24*d) + 16*a**2*sin(c + d*x)**7/(105*d) +
a**2*sin(c + d*x)**6*cos(c + d*x)**2/(6*d) + 8*a**2*sin(c + d*x)**5*cos(c +
 d*x)**2/(15*d) + a**2*sin(c + d*x)**4*cos(c + d*x)**4/(4*d) + 2*a**2*sin(c
 + d*x)**3*cos(c + d*x)**4/(3*d) - a**2*cos(c + d*x)**6/(6*d), Ne(d, 0)), (
x*(a*sin(c) + a)**2*sin(c)*cos(c)**5, True))
```

Giac [A] time = 1.28626, size = 181, normalized size = 2.03

$$\frac{a^2\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a^2\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{9\,a^2\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{13\,a^2\cos\left(2\,dx+2\,c\right)}{128\,d}-\frac{a^2\sin\left(7\,dx+7\,c\right)}{224\,d}-\frac{3\,a^2\sin(}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1024*a^2*cos(8*d*x + 8*c)/d - 1/384*a^2*cos(6*d*x + 6*c)/d - 9/256*a^2*co
s(4*d*x + 4*c)/d - 13/128*a^2*cos(2*d*x + 2*c)/d - 1/224*a^2*sin(7*d*x + 7*
c)/d - 3/160*a^2*sin(5*d*x + 5*c)/d - 1/96*a^2*sin(3*d*x + 3*c)/d + 5/32*a^
2*sin(d*x + c)/d
```

3.514 $\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=119

$$\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^4(c+dx)}{4d}-\frac{4a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log(\sin}{d}$$

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(
2*d) - (4*a^2*Sin[c + d*x]^3)/(3*d) - (a^2*Sin[c + d*x]^4)/(4*d) + (2*a^2*S
in[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.100646, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^4(c+dx)}{4d}-\frac{4a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(
2*d) - (4*a^2*Sin[c + d*x]^3)/(3*d) - (a^2*Sin[c + d*x]^4)/(4*d) + (2*a^2*S
in[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x]^6)/(6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a^5+\frac{a^6}{x}-a^4x-4a^3x^2-a^2x^3+2ax^4+x^5\right)dx, x, a\,\text{si}\right.}{a^4 d}$$
$$= \frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{4a^2\sin}{}$$

Mathematica [A] time = 0.0839573, size = 78, normalized size = 0.66

$$\frac{a^2\left(10\sin^6(c+dx)+24\sin^5(c+dx)-15\sin^4(c+dx)-80\sin^3(c+dx)-30\sin^2(c+dx)+120\sin(c+dx)+60\log(\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(60*Log[Sin[c + d*x]] + 120*Sin[c + d*x] - 30*Sin[c + d*x]^2 - 80*Sin[
c + d*x]^3 - 15*Sin[c + d*x]^4 + 24*Sin[c + d*x]^5 + 10*Sin[c + d*x]^6))/(6
0*d)
```

Maple [A] time = 0.07, size = 122, normalized size = 1.

$$-\frac{a^2(\cos(dx+c))^6}{6d}+\frac{16a^2\sin(dx+c)}{15d}+\frac{2a^2\sin(dx+c)(\cos(dx+c))^4}{5d}+\frac{8a^2\sin(dx+c)(\cos(dx+c))^2}{15d}+\frac{(\cos(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2*cos(d*x+c)^6+16/15*a^2*sin(d*x+c)/d+2/5/d*sin(d*x+c)*a^2*cos(d*x
+c)^4+8/15/d*sin(d*x+c)*a^2*cos(d*x+c)^2+1/4/d*cos(d*x+c)^4*a^2+1/2/d*a^2*c
os(d*x+c)^2+a^2*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.05009, size = 127, normalized size = 1.07

$$\frac{10a^2\sin(dx+c)^6+24a^2\sin(dx+c)^5-15a^2\sin(dx+c)^4-80a^2\sin(dx+c)^3-30a^2\sin(dx+c)^2+60a^2\log(\sin}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/60*(10*a^2*sin(d*x + c)^6 + 24*a^2*sin(d*x + c)^5 - 15*a^2*sin(d*x + c)^4
 - 80*a^2*sin(d*x + c)^3 - 30*a^2*sin(d*x + c)^2 + 60*a^2*log(sin(d*x + c))
 + 120*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.16847, size = 247, normalized size = 2.08

$$-\frac{10\,a^2\cos\left(dx+c\right)^6-15\,a^2\cos\left(dx+c\right)^4-30\,a^2\cos\left(dx+c\right)^2-60\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\left(3\,a^2\cos\left(dx+c\right)^4+4\,a^2\cos\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(10*a^2*cos(d*x + c)^6 - 15*a^2*cos(d*x + c)^4 - 30*a^2*cos(d*x + c)^
2 - 60*a^2*log(1/2*sin(d*x + c)) - 8*(3*a^2*cos(d*x + c)^4 + 4*a^2*cos(d*x
+ c)^2 + 8*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30226, size = 128, normalized size = 1.08

$$\frac{10\,a^2\sin\left(dx+c\right)^6+24\,a^2\sin\left(dx+c\right)^5-15\,a^2\sin\left(dx+c\right)^4-80\,a^2\sin\left(dx+c\right)^3-30\,a^2\sin\left(dx+c\right)^2+60\,a^2\log\left(|\sin\left(dx\right.\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*a^2*sin(d*x + c)^6 + 24*a^2*sin(d*x + c)^5 - 15*a^2*sin(d*x + c)^4
 - 80*a^2*sin(d*x + c)^3 - 30*a^2*sin(d*x + c)^2 + 60*a^2*log(abs(sin(d*x +
 c))) + 120*a^2*sin(d*x + c))/d
```

3.515 $\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=114

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}-\frac{a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/
d - (2*a^2*Sin[c + d*x]^2)/d - (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(2*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.118494, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}-\frac{a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/
d - (2*a^2*Sin[c + d*x]^2)/d - (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(2*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4+\frac{a^6}{x^2}+\frac{2a^5}{x}-4a^3x-a^2x^2+2ax^3+x^4\right)dx, x, a\sin(c$$
$$= -\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^2}{}$$

Mathematica [A] time = 0.0546058, size = 114, normalized size = 1.

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}-\frac{a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/
d - (2*a^2*Sin[c + d*x]^2)/d - (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(2*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.069, size = 130, normalized size = 1.1

$$-\frac{32\,a^2\sin(dx+c)}{15\,d}-\frac{4\,a^2\sin(dx+c)(\cos(dx+c))^4}{5\,d}-\frac{16\,a^2\sin(dx+c)(\cos(dx+c))^2}{15\,d}+\frac{(\cos(dx+c))^4a^2}{2\,d}+\frac{a^2(\cos(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -32/15*a^2*sin(d*x+c)/d-4/5/d*sin(d*x+c)*a^2*cos(d*x+c)^4-16/15/d*sin(d*x+c
)*a^2*cos(d*x+c)^2+1/2/d*cos(d*x+c)^4*a^2+1/d*a^2*cos(d*x+c)^2+2*a^2*ln(sin
(d*x+c))/d-1/d*a^2/sin(d*x+c)*cos(d*x+c)^6
```

Maxima [A] time = 1.12375, size = 127, normalized size = 1.11

$$\frac{6\,a^2\sin(dx+c)^5+15\,a^2\sin(dx+c)^4-10\,a^2\sin(dx+c)^3-60\,a^2\sin(dx+c)^2+60\,a^2\log(\sin(dx+c))-30\,a^2\sin(dx}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^3
- 60*a^2*sin(d*x + c)^2 + 60*a^2*log(sin(d*x + c)) - 30*a^2*sin(d*x + c) -
30*a^2/sin(d*x + c))/d
```

Fricas [A] time = 1.16981, size = 306, normalized size = 2.68

$$-\frac{48\,a^2\cos\left(dx+c\right)^6-64\,a^2\cos\left(dx+c\right)^4-256\,a^2\cos\left(dx+c\right)^2-480\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+512\,a^2-1}{240\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 - 64*a^2*cos(d*x + c)^4 - 256*a^2*cos(d*x + c
)^2 - 480*a^2*log(1/2*sin(d*x + c))*sin(d*x + c) + 512*a^2 - 15*(8*a^2*cos(
d*x + c)^4 + 16*a^2*cos(d*x + c)^2 - 11*a^2)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36175, size = 144, normalized size = 1.26

$$\frac{6\,a^2\sin\left(dx+c\right)^5+15\,a^2\sin\left(dx+c\right)^4-10\,a^2\sin\left(dx+c\right)^3-60\,a^2\sin\left(dx+c\right)^2+60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-30\,a^2\sin}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^3
- 60*a^2*sin(d*x + c)^2 + 60*a^2*log(abs(sin(d*x + c))) - 30*a^2*sin(d*x +
c) - 30*(2*a^2*sin(d*x + c) + a^2)/sin(d*x + c))/d
```

3.516 $\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{4a^2\sin(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\log(\sin(}{d}$$

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - (a^2*Log[Sin[c + d*x
]])/d - (4*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + (2*a^2*Sin[c
+ d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.117751, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{4a^2\sin(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\log(\sin(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - (a^2*Log[Sin[c + d*x
]])/d - (4*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + (2*a^2*Sin[c
+ d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^4)/(4*d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3 + \frac{a^6}{x^3} + \frac{2a^5}{x^2} - \frac{a^4}{x} - a^2x + 2ax^2 + x^3\right) dx, x, a\sin(\right.}{a^2 d}$$
$$= -\frac{2a^2\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{a^2\log(\sin(c+dx))}{d} - \frac{4a^2}{}$$

Mathematica [A] time = 0.167488, size = 76, normalized size = 0.66

$$-\frac{a^2\left(-3\sin^4(c+dx) - 8\sin^3(c+dx) + 6\sin^2(c+dx) + 48\sin(c+dx) + 6\csc^2(c+dx) + 24\csc(c+dx) + 12\log(\sin(\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(24*Csc[c + d*x] + 6*Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] + 48*Sin[c
 + d*x] + 6*Sin[c + d*x]^2 - 8*Sin[c + d*x]^3 - 3*Sin[c + d*x]^4))/(12*d)
```

Maple [A] time = 0.079, size = 155, normalized size = 1.3

$$-\frac{(\cos(dx+c))^4 a^2}{4d} - \frac{a^2(\cos(dx+c))^2}{2d} - \frac{a^2\ln(\sin(dx+c))}{d} - 2\frac{a^2(\cos(dx+c))^6}{d\sin(dx+c)} - \frac{16a^2\sin(dx+c)}{3d} - 2\frac{a^2\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*cos(d*x+c)^4*a^2-1/2/d*a^2*cos(d*x+c)^2-a^2*ln(sin(d*x+c))/d-2/d*a^2
/sin(d*x+c)*cos(d*x+c)^6-16/3*a^2*sin(d*x+c)/d-2/d*sin(d*x+c)*a^2*cos(d*x+c
)^4-8/3/d*sin(d*x+c)*a^2*cos(d*x+c)^2-1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^6
```

Maxima [A] time = 1.06963, size = 126, normalized size = 1.09

$$\frac{3a^2\sin(dx+c)^4 + 8a^2\sin(dx+c)^3 - 6a^2\sin(dx+c)^2 - 12a^2\log(\sin(dx+c)) - 48a^2\sin(dx+c) - \frac{6\left(4a^2\sin(dx+c)+\right.}{\sin(dx+c)^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 - 6*a^2*sin(d*x + c)^2 -
12*a^2*log(sin(d*x + c)) - 48*a^2*sin(d*x + c) - 6*(4*a^2*sin(d*x + c) + a^
2)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.31846, size = 315, normalized size = 2.72

$$\frac{24\,a^2\cos(dx+c)^6 - 24\,a^2\cos(dx+c)^4 - 9\,a^2\cos(dx+c)^2 + 57\,a^2 - 96\left(a^2\cos(dx+c)^2 - a^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 64}{96\left(d\cos(dx+c)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*a^2*cos(d*x + c)^6 - 24*a^2*cos(d*x + c)^4 - 9*a^2*cos(d*x + c)^2
+ 57*a^2 - 96*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x + c)) - 64*(a^2*co
s(d*x + c)^4 + 4*a^2*cos(d*x + c)^2 - 8*a^2)*sin(d*x + c))/(d*cos(d*x + c)^
2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28033, size = 147, normalized size = 1.27

$$\frac{3\,a^2\sin(dx+c)^4 + 8\,a^2\sin(dx+c)^3 - 6\,a^2\sin(dx+c)^2 - 12\,a^2\log(|\sin(dx+c)|) - 48\,a^2\sin(dx+c) + \frac{6\left(3\,a^2\sin(dx+c)^2 - 4\right.}{\sin(dx+}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 - 6*a^2*sin(d*x + c)^2 -
12*a^2*log(abs(sin(d*x + c))) - 48*a^2*sin(d*x + c) + 6*(3*a^2*sin(d*x + c)
^2 - 4*a^2*sin(d*x + c) - a^2)/sin(d*x + c)^2)/d
```

3.517 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=110

$$\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}-\frac{4a^2\log(\sin}{d}$$

```
[Out] (a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]^3)/(3*d)
- (4*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)
/d + (a^2*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.101402, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}-\frac{4a^2\log(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]^3)/(3*d)
- (4*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)
/d + (a^2*Sin[c + d*x]^3)/(3*d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2+\frac{a^6}{x^4}+\frac{2a^5}{x^3}-\frac{a^4}{x^2}-\frac{4a^3}{x}+2ax+x^2\right)dx, x, a\sin(c+dx\right)}{ad}$$
$$= \frac{a^2\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{d} - \frac{a^2\csc^3(c+dx)}{3d} - \frac{4a^2\log(\sin(c+}{d}$$

Mathematica [A] time = 0.176152, size = 74, normalized size = 0.67

$$\frac{a^2\left(\sin^3(c+dx)+3\sin^2(c+dx)-3\sin(c+dx)-\csc^3(c+dx)-3\csc^2(c+dx)+3\csc(c+dx)-12\log(\sin(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(3*Csc[c + d*x] - 3*Csc[c + d*x]^2 - Csc[c + d*x]^3 - 12*Log[Sin[c + d
*x]] - 3*Sin[c + d*x] + 3*Sin[c + d*x]^2 + Sin[c + d*x]^3))/(3*d)
```

Maple [A] time = 0.08, size = 97, normalized size = 0.9

$$-\frac{a^2(\cos(dx+c))^6}{d(\sin(dx+c))^2} - \frac{(\cos(dx+c))^4a^2}{d} - 2\frac{a^2(\cos(dx+c))^2}{d} - 4\frac{a^2\ln(\sin(dx+c))}{d} - \frac{a^2(\cos(dx+c))^6}{3d(\sin(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)^2*cos(d*x+c)^6-1/d*cos(d*x+c)^4*a^2-2/d*a^2*cos(d*x+c)^
2-4*a^2*ln(sin(d*x+c))/d-1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6
```

Maxima [A] time = 1.09278, size = 126, normalized size = 1.15

$$\frac{a^2\sin(dx+c)^3+3a^2\sin(dx+c)^2-12a^2\log(\sin(dx+c))-3a^2\sin(dx+c)+\frac{3a^2\sin(dx+c)^2-3a^2\sin(dx+c)-a^2}{\sin(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(a^2*sin(d*x + c)^3 + 3*a^2*sin(d*x + c)^2 - 12*a^2*log(sin(d*x + c)) -
 3*a^2*sin(d*x + c) + (3*a^2*sin(d*x + c)^2 - 3*a^2*sin(d*x + c) - a^2)/sin
(d*x + c)^3)/d
```

Fricas [A] time = 1.4313, size = 273, normalized size = 2.48

$$\frac{2\,a^2\cos\left(dx+c\right)^6-24\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-3\left(2\,a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right.}{6\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*a^2*cos(d*x + c)^6 - 24*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x +
 c))*sin(d*x + c) - 3*(2*a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 - a^2)*s
in(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2978, size = 144, normalized size = 1.31

$$\frac{a^2\sin\left(dx+c\right)^3+3\,a^2\sin\left(dx+c\right)^2-12\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-3\,a^2\sin\left(dx+c\right)+\frac{22\,a^2\sin(dx+c)^3+3\,a^2\sin(dx+c)^2-3\,a^2\sin(dx+}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*sin(d*x + c)^3 + 3*a^2*sin(d*x + c)^2 - 12*a^2*log(abs(sin(d*x + c
))) - 3*a^2*sin(d*x + c) + (22*a^2*sin(d*x + c)^3 + 3*a^2*sin(d*x + c)^2 -
3*a^2*sin(d*x + c) - a^2)/sin(d*x + c)^3)/d
```

3.518 $\int \cot^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}+\frac{4a^2\csc(c+dx)}{d}-\frac{a^2\log(\sin}{d}$$

```
[Out] (4*a^2*Csc[c + d*x])/d + (a^2*Csc[c + d*x]^2)/(2*d) - (2*a^2*Csc[c + d*x]^3
)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) - (a^2*Log[Sin[c + d*x]])/d + (2*a^2*S
in[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0671104, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}+\frac{4a^2\csc(c+dx)}{d}-\frac{a^2\log(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*a^2*Csc[c + d*x])/d + (a^2*Csc[c + d*x]^2)/(2*d) - (2*a^2*Csc[c + d*x]^3
)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) - (a^2*Log[Sin[c + d*x]])/d + (2*a^2*S
in[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(2a+\frac{a^6}{x^5}+\frac{2a^5}{x^4}-\frac{a^4}{x^3}-\frac{4a^3}{x^2}-\frac{a^2}{x}+x\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{4a^2\csc(c+dx)}{d}+\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}-\frac{a^2\log}{}\end{aligned}$$

Mathematica [A] time = 0.468678, size = 76, normalized size = 0.66

$$\frac{a^2\left(6\sin^2(c+dx)+24\sin(c+dx)-3\csc^4(c+dx)-8\csc^3(c+dx)+6\csc^2(c+dx)+48\csc(c+dx)-12\log(\sin(c+dx\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(48*Csc[c + d*x] + 6*Csc[c + d*x]^2 - 8*Csc[c + d*x]^3 - 3*Csc[c + d*x
]^4 - 12*Log[Sin[c + d*x]] + 24*Sin[c + d*x] + 6*Sin[c + d*x]^2))/(12*d)
```

Maple [A] time = 0.082, size = 211, normalized size = 1.8

$$-\frac{a^2 (\cos (dx+c))^6}{2 d (\sin (dx+c))^2}-\frac{(\cos (dx+c))^4 a^2}{2 d}-\frac{a^2 (\cos (dx+c))^2}{d}-\frac{a^2 \ln (\sin (dx+c))}{d}-\frac{2 a^2 (\cos (dx+c))^6}{3 d (\sin (dx+c))^3}+2 \frac{a^2 (\cos (d}{d \sin (d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*cos(d*x+c)^4*a^2-1/d*a^2*cos(d*x
+c)^2-a^2*ln(sin(d*x+c))/d-2/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6+2/d*a^2/sin(
d*x+c)*cos(d*x+c)^6+16/3*a^2*sin(d*x+c)/d+2/d*sin(d*x+c)*a^2*cos(d*x+c)^4+8
/3/d*sin(d*x+c)*a^2*cos(d*x+c)^2-1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c
)^2
```

Maxima [A] time = 1.06437, size = 127, normalized size = 1.09

$$\frac{6 a^2 \sin (dx+c)^2-12 a^2 \log (\sin (dx+c))+24 a^2 \sin (dx+c)+\frac{48 a^2 \sin (dx+c)^3+6 a^2 \sin (dx+c)^2-8 a^2 \sin (dx+c)-3 a^2}{\sin (dx+c)^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(6*a^2*sin(d*x + c)^2 - 12*a^2*log(sin(d*x + c)) + 24*a^2*sin(d*x + c)
 + (48*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2 - 8*a^2*sin(d*x + c) - 3*a
^2)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.49638, size = 377, normalized size = 3.25

$$-\frac{6 a^2 \cos (dx+c)^6-15 a^2 \cos (dx+c)^4+18 a^2 \cos (dx+c)^2-6 a^2+12 \left(a^2 \cos (dx+c)^4-2 a^2 \cos (dx+c)^2+a^2\right) \mathrm{lo}}{12 \left(d \cos (dx+c)^4-2 d \cos (dx+c)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/12*(6*a^2*cos(d*x + c)^6 - 15*a^2*cos(d*x + c)^4 + 18*a^2*cos(d*x + c)^2
 - 6*a^2 + 12*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*sin
(d*x + c)) - 8*(3*a^2*cos(d*x + c)^4 - 12*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d
```

```
*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33312, size = 146, normalized size = 1.26

$$\frac{6\,a^2\sin\left(dx+c\right)^2-12\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+24\,a^2\sin\left(dx+c\right)+\frac{25\,a^2\sin(dx+c)^4+48\,a^2\sin(dx+c)^3+6\,a^2\sin(dx+c)^2-8\,a^2\sin(dx+c)-3\,a^2}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*a^2*sin(d*x + c)^2 - 12*a^2*log(abs(sin(d*x + c))) + 24*a^2*sin(d*x
 + c) + (25*a^2*sin(d*x + c)^4 + 48*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)
^2 - 8*a^2*sin(d*x + c) - 3*a^2)/sin(d*x + c)^4)/d
```

3.519 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=112

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}+\frac{a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(s}{}$$

```
[Out] (a^2*Csc[c + d*x])/d + (2*a^2*Csc[c + d*x]^2)/d + (a^2*Csc[c + d*x]^3)/(3*d
) - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x]^5)/(5*d) + (2*a^2*Log[Si
n[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rubi [A] time = 0.101829, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}+\frac{a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(s}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c + d*x])/d + (2*a^2*Csc[c + d*x]^2)/d + (a^2*Csc[c + d*x]^3)/(3*d
) - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x]^5)/(5*d) + (2*a^2*Log[Si
n[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(1+\frac{a^6}{x^6}+\frac{2a^5}{x^5}-\frac{a^4}{x^4}-\frac{4a^3}{x^3}-\frac{a^2}{x^2}+\frac{2a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\csc(c+dx)}{d}+\frac{2a^2\csc^2(c+dx)}{d}+\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{2d}$$

Mathematica [A] time = 0.127337, size = 76, normalized size = 0.68

$$\frac{a^2\left(30\sin(c+dx)-6\csc^5(c+dx)-15\csc^4(c+dx)+10\csc^3(c+dx)+60\csc^2(c+dx)+30\csc(c+dx)+60\log(\sin(c\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(30*Csc[c + d*x] + 60*Csc[c + d*x]^2 + 10*Csc[c + d*x]^3 - 15*Csc[c +
d*x]^4 - 6*Csc[c + d*x]^5 + 60*Log[Sin[c + d*x]] + 30*Sin[c + d*x]))/(30*d)
```

Maple [A] time = 0.081, size = 178, normalized size = 1.6

$$-\frac{4a^2(\cos(dx+c))^6}{15d(\sin(dx+c))^3}+\frac{4a^2(\cos(dx+c))^6}{5d\sin(dx+c)}+\frac{32a^2\sin(dx+c)}{15d}+\frac{4a^2\sin(dx+c)(\cos(dx+c))^4}{5d}+\frac{16a^2\sin(dx+c)(\cos}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -4/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6+4/5/d*a^2/sin(d*x+c)*cos(d*x+c)^6+32/
15*a^2*sin(d*x+c)/d+4/5/d*sin(d*x+c)*a^2*cos(d*x+c)^4+16/15/d*sin(d*x+c)*a^
2*cos(d*x+c)^2-1/2/d*a^2*cot(d*x+c)^4+1/d*a^2*cot(d*x+c)^2+2*a^2*ln(sin(d*x
+c))/d-1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^6
```

Maxima [A] time = 1.13529, size = 127, normalized size = 1.13

$$\frac{60a^2\log(\sin(dx+c))+30a^2\sin(dx+c)+\frac{30a^2\sin(dx+c)^4+60a^2\sin(dx+c)^3+10a^2\sin(dx+c)^2-15a^2\sin(dx+c)-6a^2}{\sin(dx+c)^5}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(60*a^2*log(sin(d*x + c)) + 30*a^2*sin(d*x + c) + (30*a^2*sin(d*x + c)
^4 + 60*a^2*sin(d*x + c)^3 + 10*a^2*sin(d*x + c)^2 - 15*a^2*sin(d*x + c) -
```

```
6*a^2)/sin(d*x + c)^5)/d
```

Fricas [A] time = 1.53737, size = 389, normalized size = 3.47

$$-\frac{30\,a^2\cos\left(dx+c\right)^6-120\,a^2\cos\left(dx+c\right)^4+160\,a^2\cos\left(dx+c\right)^2-60\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\right.}{30\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/30*(30*a^2*cos(d*x + c)^6 - 120*a^2*cos(d*x + c)^4 + 160*a^2*cos(d*x + c
)^2 - 60*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*sin(d*x
+ c))*sin(d*x + c) - 64*a^2 + 15*(4*a^2*cos(d*x + c)^2 - 3*a^2)*sin(d*x + c
))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33154, size = 147, normalized size = 1.31

$$\frac{60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+30\,a^2\sin\left(dx+c\right)-\frac{137\,a^2\sin(dx+c)^5-30\,a^2\sin(dx+c)^4-60\,a^2\sin(dx+c)^3-10\,a^2\sin(dx+c)^2+15\,a^2\sin(dx+c)+6\,a^2}{\sin(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a^2*log(abs(sin(d*x + c))) + 30*a^2*sin(d*x + c) - (137*a^2*sin(d*
x + c)^5 - 30*a^2*sin(d*x + c)^4 - 60*a^2*sin(d*x + c)^3 - 10*a^2*sin(d*x +
 c)^2 + 15*a^2*sin(d*x + c) + 6*a^2)/sin(d*x + c)^5)/d
```

3.520 $\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=119

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{4d}+\frac{4a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log(s$$

```
[Out] (-2*a^2*Csc[c + d*x])/d + (a^2*Csc[c + d*x]^2)/(2*d) + (4*a^2*Csc[c + d*x]^
3)/(3*d) + (a^2*Csc[c + d*x]^4)/(4*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2
*Csc[c + d*x]^6)/(6*d) + (a^2*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.120996, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{4d}+\frac{4a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log(s$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d + (a^2*Csc[c + d*x]^2)/(2*d) + (4*a^2*Csc[c + d*x]^
3)/(3*d) + (a^2*Csc[c + d*x]^4)/(4*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2
*Csc[c + d*x]^6)/(6*d) + (a^2*Log[Sin[c + d*x]])/d
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^6}{x^7}+\frac{2a^5}{x^6}-\frac{a^4}{x^5}-\frac{4a^3}{x^4}-\frac{a^2}{x^3}+\frac{2a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\csc^2(c+dx)}{2d}+\frac{4a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc}{}$$

Mathematica [A] time = 0.0405315, size = 102, normalized size = 0.86

$$a^2\left(-\frac{\csc^6(c+dx)}{6d}-\frac{2\csc^5(c+dx)}{5d}+\frac{\csc^4(c+dx)}{4d}+\frac{4\csc^3(c+dx)}{3d}+\frac{\csc^2(c+dx)}{2d}-\frac{2\csc(c+dx)}{d}+\frac{\log(\sin(c+dx)}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*((-2*Csc[c + d*x])/d + Csc[c + d*x]^2/(2*d) + (4*Csc[c + d*x]^3)/(3*d)
+ Csc[c + d*x]^4/(4*d) - (2*Csc[c + d*x]^5)/(5*d) - Csc[c + d*x]^6/(6*d) +
Log[Sin[c + d*x]]/d)
```

Maple [A] time = 0.083, size = 202, normalized size = 1.7

$$-\frac{a^2(\cot(dx+c))^4}{4d}+\frac{a^2(\cot(dx+c))^2}{2d}+\frac{a^2\ln(\sin(dx+c))}{d}-\frac{2a^2(\cos(dx+c))^6}{5d(\sin(dx+c))^5}+\frac{2a^2(\cos(dx+c))^6}{15d(\sin(dx+c))^3}-\frac{2a^2(\cos}{5d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c)^2+a^2*ln(sin(d*x+c))/d-2/5/d*a
^2/sin(d*x+c)^5*cos(d*x+c)^6+2/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6-2/5/d*a^2
/sin(d*x+c)*cos(d*x+c)^6-16/15*a^2*sin(d*x+c)/d-2/5/d*sin(d*x+c)*a^2*cos(d*
x+c)^4-8/15/d*sin(d*x+c)*a^2*cos(d*x+c)^2-1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)
^6
```

Maxima [A] time = 1.03126, size = 131, normalized size = 1.1

$$\frac{60\,a^2\log(\sin(dx+c)) - \frac{120\,a^2\sin(dx+c)^5-30\,a^2\sin(dx+c)^4-80\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+24\,a^2\sin(dx+c)+10\,a^2}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(60*a^2*log(sin(d*x + c)) - (120*a^2*sin(d*x + c)^5 - 30*a^2*sin(d*x +
 c)^4 - 80*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*x + c)^2 + 24*a^2*sin(d*x + c)
```

```
+ 10*a^2)/sin(d*x + c)^6)/d
```

Fricas [A] time = 1.58512, size = 408, normalized size = 3.43

$$-\frac{30\,a^2\cos\left(dx+c\right)^4 - 75\,a^2\cos\left(dx+c\right)^2 + 35\,a^2 - 60\left(a^2\cos\left(dx+c\right)^6 - 3\,a^2\cos\left(dx+c\right)^4 + 3\,a^2\cos\left(dx+c\right)^2 - a^2\right)\log}{60\left(d\cos\left(dx+c\right)^6 - 3\,d\cos\left(dx+c\right)^4 + 3\,d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/60*(30*a^2*cos(d*x + c)^4 - 75*a^2*cos(d*x + c)^2 + 35*a^2 - 60*(a^2*cos
(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*si
n(d*x + c)) - 8*(15*a^2*cos(d*x + c)^4 - 20*a^2*cos(d*x + c)^2 + 8*a^2)*sin
(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3006, size = 150, normalized size = 1.26

$$\frac{60\,a^2\log\left(|\sin\left(dx+c\right)|\right) - \frac{147\,a^2\sin(dx+c)^6+120\,a^2\sin(dx+c)^5-30\,a^2\sin(dx+c)^4-80\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+24\,a^2\sin(dx+c)+10\,a^2}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(sin(d*x + c))) - (147*a^2*sin(d*x + c)^6 + 120*a^2*sin
(d*x + c)^5 - 30*a^2*sin(d*x + c)^4 - 80*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*
x + c)^2 + 24*a^2*sin(d*x + c) + 10*a^2)/sin(d*x + c)^6)/d
```

3.521 $\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{(a\sin(c+dx)+a)^{10}}{10a^7d}-\frac{2(a\sin(c+dx)+a)^9}{3a^6d}+\frac{13(a\sin(c+dx)+a)^8}{8a^5d}-\frac{12(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

```
[Out] (2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (12*(a + a*Sin[c + d*x])^7)/(7*a^4*d
) + (13*(a + a*Sin[c + d*x])^8)/(8*a^5*d) - (2*(a + a*Sin[c + d*x])^9)/(3*a
^6*d) + (a + a*Sin[c + d*x])^10/(10*a^7*d)
```

Rubi [A] time = 0.128079, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{(a\sin(c+dx)+a)^{10}}{10a^7d}-\frac{2(a\sin(c+dx)+a)^9}{3a^6d}+\frac{13(a\sin(c+dx)+a)^8}{8a^5d}-\frac{12(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (12*(a + a*Sin[c + d*x])^7)/(7*a^4*d
) + (13*(a + a*Sin[c + d*x])^8)/(8*a^5*d) - (2*(a + a*Sin[c + d*x])^9)/(3*a
^6*d) + (a + a*Sin[c + d*x])^10/(10*a^7*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2(a+x)^5}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$

$$= \frac{\text{Subst}\left(\int (a-x)^2x^2(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^7d}$$

$$= \frac{\text{Subst}\left(\int \left(4a^4(a+x)^5 - 12a^3(a+x)^6 + 13a^2(a+x)^7 - 6a(a+x)^8 + \right.\right.}{a^7d}$$

$$= \frac{2(a+a\sin(c+dx))^6}{3a^3d} - \frac{12(a+a\sin(c+dx))^7}{7a^4d} + \frac{13(a+a\sin(c+dx}{8a^5d}$$

Mathematica [A] time = 0.909289, size = 110, normalized size = 0.99

$$-\frac{a^3(-63840\sin(c+dx) + 8960\sin(3(c+dx)) + 8064\sin(5(c+dx)) + 240\sin(7(c+dx)) - 560\sin(9(c+dx)) + 34440\,\text{c}}{430080d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(-2835 + 34440*Cos[2*(c + d*x)] + 5040*Cos[4*(c + d*x)] - 4060*Cos[6*
(c + d*x)] - 1260*Cos[8*(c + d*x)] + 84*Cos[10*(c + d*x)] - 63840*Sin[c + d
*x] + 8960*Sin[3*(c + d*x)] + 8064*Sin[5*(c + d*x)] + 240*Sin[7*(c + d*x)]
- 560*Sin[9*(c + d*x)]))/(430080*d)
```

Maple [B] time = 0.043, size = 208, normalized size = 1.9

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^6}{10} - \frac{(\sin(dx+c))^2(\cos(dx+c))^6}{20} - \frac{(\cos(dx+c))^6}{60}\right) + 3a^3\left(-1/9\,(\sin(dx+c))^3\,(\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/
60*cos(d*x+c)^6)+3*a^3*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(
d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*(-1/8*
sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+a^3*(-1/7*sin(d*x+c)*cos(d*x+c
)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.15809, size = 149, normalized size = 1.34

$$\frac{84\,a^3\sin(dx+c)^{10} + 280\,a^3\sin(dx+c)^9 + 105\,a^3\sin(dx+c)^8 - 600\,a^3\sin(dx+c)^7 - 700\,a^3\sin(dx+c)^6 + 168\,a^3\sin(}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/840*(84*a^3*sin(d*x + c)^10 + 280*a^3*sin(d*x + c)^9 + 105*a^3*sin(d*x +
c)^8 - 600*a^3*sin(d*x + c)^7 - 700*a^3*sin(d*x + c)^6 + 168*a^3*sin(d*x +
```

```
c)^5 + 630*a^3*sin(d*x + c)^4 + 280*a^3*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.4803, size = 277, normalized size = 2.5

$$-\frac{84\,a^3\cos\left(dx+c\right)^{10}-525\,a^3\cos\left(dx+c\right)^8+560\,a^3\cos\left(dx+c\right)^6-8\left(35\,a^3\cos\left(dx+c\right)^8-65\,a^3\cos\left(dx+c\right)^6+6\,a^3\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/840*(84*a^3*cos(d*x + c)^10 - 525*a^3*cos(d*x + c)^8 + 560*a^3*cos(d*x +
 c)^6 - 8*(35*a^3*cos(d*x + c)^8 - 65*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x +
c)^4 + 8*a^3*cos(d*x + c)^2 + 16*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 33.7642, size = 255, normalized size = 2.3

$$\begin{cases}\frac{8a^3\sin^9(c+dx)}{105d}+\frac{12a^3\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{8a^3\sin^7(c+dx)}{105d}+\frac{3a^3\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{4a^3\sin^5(c+dx)\cos^2(c+dx)}{15d}-\frac{a^3\sin^4(c+dx)\cos^6}{6d} \\ x\left(a\sin\left(c\right)+a\right)^3\sin^2\left(c\right)\cos^5\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**9/(105*d) + 12*a**3*sin(c + d*x)**7*cos(c +
 d*x)**2/(35*d) + 8*a**3*sin(c + d*x)**7/(105*d) + 3*a**3*sin(c + d*x)**5*c
os(c + d*x)**4/(5*d) + 4*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) - a**3
*sin(c + d*x)**4*cos(c + d*x)**6/(6*d) + a**3*sin(c + d*x)**3*cos(c + d*x)*
*4/(3*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - a**3*sin(c + d*x)*
*2*cos(c + d*x)**6/(2*d) - a**3*cos(c + d*x)**10/(60*d) - a**3*cos(c + d*x)
**8/(8*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**2*cos(c)**5, True))
```

Giac [A] time = 1.30594, size = 227, normalized size = 2.05

$$-\frac{a^3\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{3\,a^3\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{29\,a^3\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{3\,a^3\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{41\,a^3\cos\left(2\,dx+2\,c\right)}{512\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/5120*a^3*cos(10*d*x + 10*c)/d + 3/1024*a^3*cos(8*d*x + 8*c)/d + 29/3072*
a^3*cos(6*d*x + 6*c)/d - 3/256*a^3*cos(4*d*x + 4*c)/d - 41/512*a^3*cos(2*d*
x + 2*c)/d + 1/768*a^3*sin(9*d*x + 9*c)/d - 1/1792*a^3*sin(7*d*x + 7*c)/d -
 3/160*a^3*sin(5*d*x + 5*c)/d - 1/48*a^3*sin(3*d*x + 3*c)/d + 19/128*a^3*si
n(d*x + c)/d
```

3.522 $\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=89

$$\frac{(a\sin(c+dx)+a)^9}{9a^6d}-\frac{5(a\sin(c+dx)+a)^8}{8a^5d}+\frac{8(a\sin(c+dx)+a)^7}{7a^4d}-\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

```
[Out] (-2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) + (8*(a + a*Sin[c + d*x])^7)/(7*a^4*d
) - (5*(a + a*Sin[c + d*x])^8)/(8*a^5*d) + (a + a*Sin[c + d*x])^9/(9*a^6*d)
```

Rubi [A] time = 0.0834011, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 77}

$$\frac{(a\sin(c+dx)+a)^9}{9a^6d}-\frac{5(a\sin(c+dx)+a)^8}{8a^5d}+\frac{8(a\sin(c+dx)+a)^7}{7a^4d}-\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) + (8*(a + a*Sin[c + d*x])^7)/(7*a^4*d
) - (5*(a + a*Sin[c + d*x])^8)/(8*a^5*d) + (a + a*Sin[c + d*x])^9/(9*a^6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x(a+x)^5}{a}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3(a+x)^5+8a^2(a+x)^6-5a(a+x)^7+(a+x)^8\right) dx\right.}{a^6d}$$
$$= -\frac{2(a+a\sin(c+dx))^6}{3a^3d}+\frac{8(a+a\sin(c+dx))^7}{7a^4d}-\frac{5(a+a\sin(c+}{8a^5d}$$

Mathematica [A] time = 0.658734, size = 100, normalized size = 1.12

$$\frac{a^3(16632\sin(c+dx)-1344\sin(3(c+dx))-2016\sin(5(c+dx))-396\sin(7(c+dx))+28\sin(9(c+dx))-9576\cos(}{64512d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(4662 - 9576*Cos[2*(c + d*x)] - 2772*Cos[4*(c + d*x)] + 168*Cos[6*(c +
 d*x)] + 189*Cos[8*(c + d*x)] + 16632*Sin[c + d*x] - 1344*Sin[3*(c + d*x)]
- 2016*Sin[5*(c + d*x)] - 396*Sin[7*(c + d*x)] + 28*Sin[9*(c + d*x)]))/(645
12*d)
```

Maple [B] time = 0.039, size = 170, normalized size = 1.9

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^6}{9}-\frac{\sin(dx+c)(\cos(dx+c))^6}{21}+\frac{\sin(dx+c)}{105}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx}{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*(-1/8*sin(d*x+c)^2*c
os(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a^3*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/
3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*a^3*cos(d*x+c)^6)
```

Maxima [A] time = 1.19729, size = 149, normalized size = 1.67

$$\frac{56\,a^3\sin(dx+c)^9+189\,a^3\sin(dx+c)^8+72\,a^3\sin(dx+c)^7-420\,a^3\sin(dx+c)^6-504\,a^3\sin(dx+c)^5+126\,a^3\sin}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/504*(56*a^3*sin(d*x + c)^9 + 189*a^3*sin(d*x + c)^8 + 72*a^3*sin(d*x + c)
^7 - 420*a^3*sin(d*x + c)^6 - 504*a^3*sin(d*x + c)^5 + 126*a^3*sin(d*x + c)
^4 + 504*a^3*sin(d*x + c)^3 + 252*a^3*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.43655, size = 240, normalized size = 2.7

$$\frac{189\,a^3\cos(dx+c)^8-336\,a^3\cos(dx+c)^6+8\left(7\,a^3\cos(dx+c)^8-37\,a^3\cos(dx+c)^6+6\,a^3\cos(dx+c)^4+8\,a^3\cos(dx+\right.}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/504*(189*a^3*cos(d*x + c)^8 - 336*a^3*cos(d*x + c)^6 + 8*(7*a^3*cos(d*x +
 c)^8 - 37*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x + c)^4 + 8*a^3*cos(d*x + c)^2
 + 16*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 21.5253, size = 228, normalized size = 2.56

$$\begin{cases}\frac{8a^3\sin^9(c+dx)}{315d}+\frac{a^3\sin^8(c+dx)}{8d}+\frac{4a^3\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{8a^3\sin^7(c+dx)}{35d}+\frac{a^3\sin^6(c+dx)\cos^2(c+dx)}{2d}+\frac{a^3\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{4a^3\sin^5}{} \\ x\,(a\sin(c)+a)^3\sin(c)\cos^5(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**9/(315*d) + a**3*sin(c + d*x)**8/(8*d) + 4*
a**3*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 8*a**3*sin(c + d*x)**7/(35*d)
 + a**3*sin(c + d*x)**6*cos(c + d*x)**2/(2*d) + a**3*sin(c + d*x)**5*cos(c
+ d*x)**4/(5*d) + 4*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 3*a**3*sin
(c + d*x)**4*cos(c + d*x)**4/(4*d) + a**3*sin(c + d*x)**3*cos(c + d*x)**4/d
 - a**3*cos(c + d*x)**6/(6*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)*cos(c
)**5, True))
```

Giac [A] time = 1.2792, size = 204, normalized size = 2.29

$$\frac{3\,a^3\cos(8\,dx+8\,c)}{1024\,d}+\frac{a^3\cos(6\,dx+6\,c)}{384\,d}-\frac{11\,a^3\cos(4\,dx+4\,c)}{256\,d}-\frac{19\,a^3\cos(2\,dx+2\,c)}{128\,d}+\frac{a^3\sin(9\,dx+9\,c)}{2304\,d}-\frac{11\,a^3\sin}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 3/1024*a^3*cos(8*d*x + 8*c)/d + 1/384*a^3*cos(6*d*x + 6*c)/d - 11/256*a^3*c
os(4*d*x + 4*c)/d - 19/128*a^3*cos(2*d*x + 2*c)/d + 1/2304*a^3*sin(9*d*x +
9*c)/d - 11/1792*a^3*sin(7*d*x + 7*c)/d - 1/32*a^3*sin(5*d*x + 5*c)/d - 1/4
8*a^3*sin(3*d*x + 3*c)/d + 33/128*a^3*sin(d*x + c)/d
```

3.523 $\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=137

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin}{}$$

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3*Sin[c + d*x]^2)/(
2*d) - (5*a^3*Sin[c + d*x]^3)/(3*d) - (5*a^3*Sin[c + d*x]^4)/(4*d) + (a^3*S
in[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(2*d) + (a^3*Sin[c + d*x]^7)/(7
*d)
```

Rubi [A] time = 0.106017, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3*Sin[c + d*x]^2)/(
2*d) - (5*a^3*Sin[c + d*x]^3)/(3*d) - (5*a^3*Sin[c + d*x]^4)/(4*d) + (a^3*S
in[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(2*d) + (a^3*Sin[c + d*x]^7)/(7
*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)^5}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(3a^6 + \frac{a^7}{x} + a^5x - 5a^4x^2 - 5a^3x^3 + a^2x^4 + 3ax^5 + x^6\right) dx, x\right.}{a^4 d}$$

$$= \frac{a^3\log(\sin(c+dx))}{d} + \frac{3a^3\sin(c+dx)}{d} + \frac{a^3\sin^2(c+dx)}{2d} - \frac{5a^3\sin^3(c}{3d}$$

Mathematica [A] time = 0.104836, size = 88, normalized size = 0.64

$$\frac{a^3\left(60\sin^7(c+dx) + 210\sin^6(c+dx) + 84\sin^5(c+dx) - 525\sin^4(c+dx) - 700\sin^3(c+dx) + 210\sin^2(c+dx) + 1260\right.}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(420*Log[Sin[c + d*x]] + 1260*Sin[c + d*x] + 210*Sin[c + d*x]^2 - 700*
Sin[c + d*x]^3 - 525*Sin[c + d*x]^4 + 84*Sin[c + d*x]^5 + 210*Sin[c + d*x]^
6 + 60*Sin[c + d*x]^7))/(420*d)
```

Maple [A] time = 0.079, size = 144, normalized size = 1.1

$$-\frac{a^3(\cos(dx+c))^6\sin(dx+c)}{7d} + \frac{176a^3\sin(dx+c)}{105d} + \frac{22a^3(\cos(dx+c))^4\sin(dx+c)}{35d} + \frac{88a^3(\cos(dx+c))^2\sin(dx+}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/7/d*a^3*cos(d*x+c)^6*sin(d*x+c)+176/105*a^3*sin(d*x+c)/d+22/35/d*a^3*cos
(d*x+c)^4*sin(d*x+c)+88/105/d*a^3*cos(d*x+c)^2*sin(d*x+c)-1/2/d*a^3*cos(d*x
+c)^6+1/4/d*cos(d*x+c)^4*a^3+1/2/d*a^3*cos(d*x+c)^2+a^3*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.20914, size = 144, normalized size = 1.05

$$\frac{60a^3\sin(dx+c)^7 + 210a^3\sin(dx+c)^6 + 84a^3\sin(dx+c)^5 - 525a^3\sin(dx+c)^4 - 700a^3\sin(dx+c)^3 + 210a^3\sin(dx}{420d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a^3*sin(d*x + c)^7 + 210*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)
^5 - 525*a^3*sin(d*x + c)^4 - 700*a^3*sin(d*x + c)^3 + 210*a^3*sin(d*x + c)
^2 + 420*a^3*log(sin(d*x + c)) + 1260*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.65629, size = 292, normalized size = 2.13

$$-\frac{210\,a^3\cos\left(dx+c\right)^6-105\,a^3\cos\left(dx+c\right)^4-210\,a^3\cos\left(dx+c\right)^2-420\,a^3\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\left(15\,a^3\cos\left(dx+c\right)^6\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/420*(210*a^3*cos(d*x + c)^6 - 105*a^3*cos(d*x + c)^4 - 210*a^3*cos(d*x +
 c)^2 - 420*a^3*log(1/2*sin(d*x + c)) + 4*(15*a^3*cos(d*x + c)^6 - 66*a^3*c
os(d*x + c)^4 - 88*a^3*cos(d*x + c)^2 - 176*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29708, size = 146, normalized size = 1.07

$$\frac{60\,a^3\sin\left(dx+c\right)^7+210\,a^3\sin\left(dx+c\right)^6+84\,a^3\sin\left(dx+c\right)^5-525\,a^3\sin\left(dx+c\right)^4-700\,a^3\sin\left(dx+c\right)^3+210\,a^3\sin}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/420*(60*a^3*sin(d*x + c)^7 + 210*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)
^5 - 525*a^3*sin(d*x + c)^4 - 700*a^3*sin(d*x + c)^3 + 210*a^3*sin(d*x + c)
^2 + 420*a^3*log(abs(sin(d*x + c))) + 1260*a^3*sin(d*x + c))/d
```

3.524 $\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+}{d}$$

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/
d - (5*a^3*Sin[c + d*x]^2)/(2*d) - (5*a^3*Sin[c + d*x]^3)/(3*d) + (a^3*Sin[
c + d*x]^4)/(4*d) + (3*a^3*Sin[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(6*
d)
```

Rubi [A] time = 0.123447, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/
d - (5*a^3*Sin[c + d*x]^2)/(2*d) - (5*a^3*Sin[c + d*x]^3)/(3*d) + (a^3*Sin[
c + d*x]^4)/(4*d) + (3*a^3*Sin[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(6*
d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)^5}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5+\frac{a^7}{x^2}+\frac{3a^6}{x}-5a^4x-5a^3x^2+a^2x^3+3ax^4+x^5\right) dx\right.}{a^3 d}$$
$$= -\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{5a^3\,\text{s}}{}$$

Mathematica [A] time = 0.164678, size = 86, normalized size = 0.65

$$-\frac{a^3\left(-10\sin^6(c+dx)-36\sin^5(c+dx)-15\sin^4(c+dx)+100\sin^3(c+dx)+150\sin^2(c+dx)-60\sin(c+dx)+60\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(60*Csc[c + d*x] - 180*Log[Sin[c + d*x]] - 60*Sin[c + d*x] + 150*Sin[
c + d*x]^2 + 100*Sin[c + d*x]^3 - 15*Sin[c + d*x]^4 - 36*Sin[c + d*x]^5 - 1
0*Sin[c + d*x]^6))/(60*d)
```

Maple [A] time = 0.082, size = 147, normalized size = 1.1

$$-\frac{a^3(\cos(dx+c))^6}{6d}-\frac{16a^3\sin(dx+c)}{15d}-\frac{2a^3(\cos(dx+c))^4\sin(dx+c)}{5d}-\frac{8a^3(\cos(dx+c))^2\sin(dx+c)}{15d}+\frac{3a^3(\text{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/6/d*a^3*cos(d*x+c)^6-16/15*a^3*sin(d*x+c)/d-2/5/d*a^3*cos(d*x+c)^4*sin(d
*x+c)-8/15/d*a^3*cos(d*x+c)^2*sin(d*x+c)+3/4/d*cos(d*x+c)^4*a^3+3/2/d*a^3*c
os(d*x+c)^2+3*a^3*ln(sin(d*x+c))/d-1/d*a^3/sin(d*x+c)*cos(d*x+c)^6
```

Maxima [A] time = 1.08016, size = 144, normalized size = 1.08

$$\frac{10a^3\sin(dx+c)^6+36a^3\sin(dx+c)^5+15a^3\sin(dx+c)^4-100a^3\sin(dx+c)^3-150a^3\sin(dx+c)^2+180a^3\log(}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 100*a^3*sin(d*x + c)^3 - 150*a^3*sin(d*x + c)^2 + 180*a^3*log(sin(d*x +
c)) + 60*a^3*sin(d*x + c) - 60*a^3/sin(d*x + c))/d
```

Fricas [A] time = 1.60819, size = 339, normalized size = 2.55

$$-\frac{144\,a^3\cos(dx+c)^6 - 32\,a^3\cos(dx+c)^4 - 128\,a^3\cos(dx+c)^2 - 720\,a^3\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c) + 256\,a^3 + 5\,(8}{240\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(144*a^3*cos(d*x + c)^6 - 32*a^3*cos(d*x + c)^4 - 128*a^3*cos(d*x +
c)^2 - 720*a^3*log(1/2*sin(d*x + c))*sin(d*x + c) + 256*a^3 + 5*(8*a^3*cos(
d*x + c)^6 - 36*a^3*cos(d*x + c)^4 - 72*a^3*cos(d*x + c)^2 + 47*a^3)*sin(d*
x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34305, size = 162, normalized size = 1.22

$$\frac{10\,a^3\sin(dx+c)^6 + 36\,a^3\sin(dx+c)^5 + 15\,a^3\sin(dx+c)^4 - 100\,a^3\sin(dx+c)^3 - 150\,a^3\sin(dx+c)^2 + 180\,a^3\log(|\sin}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 100*a^3*sin(d*x + c)^3 - 150*a^3*sin(d*x + c)^2 + 180*a^3*log(abs(sin(d*
x + c))) + 60*a^3*sin(d*x + c) - 60*(3*a^3*sin(d*x + c) + a^3)/sin(d*x + c)
)/d
```

3.525 $\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc}{}$$

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (a^3*Log[Sin[c + d*x
]])/d - (5*a^3*Sin[c + d*x])/d - (5*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c
+ d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^4)/(4*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.127574, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (a^3*Log[Sin[c + d*x
]])/d - (5*a^3*Sin[c + d*x])/d - (5*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c
+ d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^4)/(4*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)^5}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^4 + \frac{a^7}{x^3} + \frac{3a^6}{x^2} + \frac{a^5}{x} - 5a^3x + a^2x^2 + 3ax^3 + x^4\right) dx, x,$$
$$= -\frac{3a^3\csc(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d} + \frac{a^3\log(\sin(c+dx))}{d} - \frac{5a^3\sin$$

Mathematica [A] time = 0.147773, size = 86, normalized size = 0.65

$$-\frac{a^3\left(-12\sin^5(c+dx) - 45\sin^4(c+dx) - 20\sin^3(c+dx) + 150\sin^2(c+dx) + 300\sin(c+dx) + 30\csc^2(c+dx) + 180\,\text{cs}}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(180*Csc[c + d*x] + 30*Csc[c + d*x]^2 - 60*Log[Sin[c + d*x]] + 300*Si
n[c + d*x] + 150*Sin[c + d*x]^2 - 20*Sin[c + d*x]^3 - 45*Sin[c + d*x]^4 - 1
2*Sin[c + d*x]^5))/(60*d)
```

Maple [A] time = 0.092, size = 154, normalized size = 1.2

$$-\frac{112\,a^3\sin(dx+c)}{15\,d} - \frac{14\,a^3(\cos(dx+c))^4\sin(dx+c)}{5\,d} - \frac{56\,a^3(\cos(dx+c))^2\sin(dx+c)}{15\,d} + \frac{a^3(\cos(dx+c))^4}{4\,d} + \frac{a^3(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -112/15*a^3*sin(d*x+c)/d-14/5/d*a^3*cos(d*x+c)^4*sin(d*x+c)-56/15/d*a^3*cos
(d*x+c)^2*sin(d*x+c)+1/4/d*cos(d*x+c)^4*a^3+1/2/d*a^3*cos(d*x+c)^2+a^3*ln(s
in(d*x+c))/d-3/d*a^3/sin(d*x+c)*cos(d*x+c)^6-1/2/d*a^3/sin(d*x+c)^2*cos(d*x
+c)^6
```

Maxima [A] time = 1.17822, size = 143, normalized size = 1.08

$$\frac{12\,a^3\sin(dx+c)^5 + 45\,a^3\sin(dx+c)^4 + 20\,a^3\sin(dx+c)^3 - 150\,a^3\sin(dx+c)^2 + 60\,a^3\log(\sin(dx+c)) - 300\,a^3\sin(}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(12*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
 - 150*a^3*sin(d*x + c)^2 + 60*a^3*log(sin(d*x + c)) - 300*a^3*sin(d*x + c)
```

```
 - 30*(6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.57352, size = 363, normalized size = 2.73

$$\frac{360\,a^3\cos\left(dx+c\right)^6+120\,a^3\cos\left(dx+c\right)^4-855\,a^3\cos\left(dx+c\right)^2+615\,a^3+480\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\sin\left(dx\right.\right.}{480\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(360*a^3*cos(d*x + c)^6 + 120*a^3*cos(d*x + c)^4 - 855*a^3*cos(d*x +
c)^2 + 615*a^3 + 480*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 32*
(3*a^3*cos(d*x + c)^6 - 14*a^3*cos(d*x + c)^4 - 56*a^3*cos(d*x + c)^2 + 112
*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33728, size = 162, normalized size = 1.22

$$\frac{12\,a^3\sin\left(dx+c\right)^5+45\,a^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3-150\,a^3\sin\left(dx+c\right)^2+60\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)-300\,a^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(12*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
 - 150*a^3*sin(d*x + c)^2 + 60*a^3*log(abs(sin(d*x + c))) - 300*a^3*sin(d*x
 + c) - 30*(3*a^3*sin(d*x + c)^2 + 6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2
)/d
```

3.526 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$\frac{a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc(c+}{d}$$

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^
3)/(3*d) - (5*a^3*Log[Sin[c + d*x]])/d - (5*a^3*Sin[c + d*x])/d + (a^3*Sin[
c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/d + (a^3*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.109789, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^
3)/(3*d) - (5*a^3*Log[Sin[c + d*x]])/d - (5*a^3*Sin[c + d*x])/d + (a^3*Sin[
c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/d + (a^3*Sin[c + d*x]^4)/(4*d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^5}{x^4}\,dx,x,a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^4}\,dx,x,a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^3+\frac{a^7}{x^4}+\frac{3a^6}{x^3}+\frac{a^5}{x^2}-\frac{5a^4}{x}+a^2x+3ax^2+x^3\right)dx,x,a\right.}{ad}$$
$$= -\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{5a^3\log(s}{}$$

Mathematica [A] time = 0.25712, size = 86, normalized size = 0.66

$$-\frac{a^3\left(-3\sin^4(c+dx)-12\sin^3(c+dx)-6\sin^2(c+dx)+60\sin(c+dx)+4\csc^3(c+dx)+18\csc^2(c+dx)+12\csc(c\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(12*Csc[c + d*x] + 18*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 + 60*Log[Sin[
c + d*x]] + 60*Sin[c + d*x] - 6*Sin[c + d*x]^2 - 12*Sin[c + d*x]^3 - 3*Sin[
c + d*x]^4))/(12*d)
```

Maple [A] time = 0.086, size = 179, normalized size = 1.4

$$-\frac{5\,(\cos(dx+c))^4a^3}{4d}-\frac{5a^3(\cos(dx+c))^2}{2d}-5\frac{a^3\ln(\sin(dx+c))}{d}-2\frac{a^3(\cos(dx+c))^6}{d\sin(dx+c)}-\frac{16a^3\sin(dx+c)}{3d}-2\,\frac{(\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -5/4/d*cos(d*x+c)^4*a^3-5/2/d*a^3*cos(d*x+c)^2-5*a^3*ln(sin(d*x+c))/d-2/d*a
^3/sin(d*x+c)*cos(d*x+c)^6-16/3*a^3*sin(d*x+c)/d-2/d*a^3*cos(d*x+c)^4*sin(d
*x+c)-8/3/d*a^3*cos(d*x+c)^2*sin(d*x+c)-3/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^6
-1/3/d*a^3/sin(d*x+c)^3*cos(d*x+c)^6
```

Maxima [A] time = 1.03676, size = 146, normalized size = 1.11

$$\frac{3a^3\sin(dx+c)^4+12a^3\sin(dx+c)^3+6a^3\sin(dx+c)^2-60a^3\log(\sin(dx+c))-60a^3\sin(dx+c)-\frac{2\left(6a^3\sin(dx+c)\right.}{\text{si}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a^3*sin(d*x + c)^4 + 12*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 -
 60*a^3*log(sin(d*x + c)) - 60*a^3*sin(d*x + c) - 2*(6*a^3*sin(d*x + c)^2 +
```

```
9*a^3*sin(d*x + c) + 2*a^3)/sin(d*x + c)^3)/d
```

Fricas [A] time = 1.53772, size = 396, normalized size = 3.02

$$\frac{96\,a^3\cos\left(dx+c\right)^6+192\,a^3\cos\left(dx+c\right)^4-768\,a^3\cos\left(dx+c\right)^2+512\,a^3-480\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)}{96\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/96*(96*a^3*cos(d*x + c)^6 + 192*a^3*cos(d*x + c)^4 - 768*a^3*cos(d*x + c)
^2 + 512*a^3 - 480*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c))*sin(d*x
 + c) + 3*(8*a^3*cos(d*x + c)^6 - 40*a^3*cos(d*x + c)^4 + 45*a^3*cos(d*x +
c)^2 + 35*a^3)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36269, size = 165, normalized size = 1.26

$$\frac{3\,a^3\sin\left(dx+c\right)^4+12\,a^3\sin\left(dx+c\right)^3+6\,a^3\sin\left(dx+c\right)^2-60\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)-60\,a^3\sin\left(dx+c\right)+\frac{2\left(55\,a^3\sin(dx+c)^3\right.}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^3*sin(d*x + c)^4 + 12*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 -
 60*a^3*log(abs(sin(d*x + c))) - 60*a^3*sin(d*x + c) + 2*(55*a^3*sin(d*x +
c)^3 - 6*a^3*sin(d*x + c)^2 - 9*a^3*sin(d*x + c) - 2*a^3)/sin(d*x + c)^3)/d
```

3.527 $\int \cot^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc(c}{d}$$

```
[Out] (5*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^3)/
d - (a^3*Csc[c + d*x]^4)/(4*d) - (5*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c +
 d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.070055, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (5*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^3)/
d - (a^3*Csc[c + d*x]^4)/(4*d) - (5*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c +
 d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```


Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^2+\frac{a^7}{x^5}+\frac{3a^6}{x^4}+\frac{a^5}{x^3}-\frac{5a^4}{x^2}-\frac{5a^3}{x}+3ax+x^2\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{5a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}-5a^3\end{aligned}$$

Mathematica [A] time = 0.523199, size = 86, normalized size = 0.66

$$\frac{a^3\left(4\sin^3(c+dx)+18\sin^2(c+dx)+12\sin(c+dx)-3\csc^4(c+dx)-12\csc^3(c+dx)-6\csc^2(c+dx)+60\csc(c+d\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(60*Csc[c + d*x] - 6*Csc[c + d*x]^2 - 12*Csc[c + d*x]^3 - 3*Csc[c + d*
x]^4 - 60*Log[Sin[c + d*x]] + 12*Sin[c + d*x] + 18*Sin[c + d*x]^2 + 4*Sin[c
 + d*x]^3))/(12*d)
```

Maple [A] time = 0.091, size = 211, normalized size = 1.6

$$2\frac{a^3(\cos(dx+c))^6}{d\sin(dx+c)}+\frac{16a^3\sin(dx+c)}{3d}+2\frac{a^3(\cos(dx+c))^4\sin(dx+c)}{d}+\frac{8a^3(\cos(dx+c))^2\sin(dx+c)}{3d}-\frac{3a^3(\cos(\ldots}{2d(\sin(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d*a^3/sin(d*x+c)*cos(d*x+c)^6+16/3*a^3*sin(d*x+c)/d+2/d*a^3*cos(d*x+c)^4*
sin(d*x+c)+8/3/d*a^3*cos(d*x+c)^2*sin(d*x+c)-3/2/d*a^3/sin(d*x+c)^2*cos(d*x
+c)^6-3/2/d*cos(d*x+c)^4*a^3-3/d*a^3*cos(d*x+c)^2-5*a^3*ln(sin(d*x+c))/d-1/
d*a^3/sin(d*x+c)^3*cos(d*x+c)^6-1/4/d*a^3*cot(d*x+c)^4+1/2/d*a^3*cot(d*x+c)
^2
```

Maxima [A] time = 1.1401, size = 146, normalized size = 1.11

$$\frac{4a^3\sin(dx+c)^3+18a^3\sin(dx+c)^2-60a^3\log(\sin(dx+c))+12a^3\sin(dx+c)+\frac{3\left(20a^3\sin(dx+c)^3-2a^3\sin(dx+c)^2-4a^3\sin(\ldots}{\sin(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(4*a^3*sin(d*x + c)^3 + 18*a^3*sin(d*x + c)^2 - 60*a^3*log(sin(d*x + c
)) + 12*a^3*sin(d*x + c) + 3*(20*a^3*sin(d*x + c)^3 - 2*a^3*sin(d*x + c)^2
- 4*a^3*sin(d*x + c) - a^3)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.55544, size = 397, normalized size = 3.03

$$-\frac{18a^3\cos(dx+c)^6-45a^3\cos(dx+c)^4+30a^3\cos(dx+c)^2+60\left(a^3\cos(dx+c)^4-2a^3\cos(dx+c)^2+a^3\right)\log\left(\frac{1}{2}\sin\ldots\right.}{12\left(d\cos(dx+c)^4-2d\cos(dx+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(18*a^3*cos(d*x + c)^6 - 45*a^3*cos(d*x + c)^4 + 30*a^3*cos(d*x + c)^
2 + 60*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*sin(d*x +
c)) + 4*(a^3*cos(d*x + c)^6 - 6*a^3*cos(d*x + c)^4 + 24*a^3*cos(d*x + c)^2
```

```
- 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26071, size = 163, normalized size = 1.24

$$\frac{4\,a^3\sin\left(dx+c\right)^3+18\,a^3\sin\left(dx+c\right)^2-60\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a^3\sin\left(dx+c\right)+\frac{125\,a^3\sin(dx+c)^4+60\,a^3\sin(dx+c)^3-6\,a^3}{\sin(dx+c}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(4*a^3*sin(d*x + c)^3 + 18*a^3*sin(d*x + c)^2 - 60*a^3*log(abs(sin(d*x
 + c))) + 12*a^3*sin(d*x + c) + (125*a^3*sin(d*x + c)^4 + 60*a^3*sin(d*x +
c)^3 - 6*a^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c) - 3*a^3)/sin(d*x + c)^4)/
d
```

3.528 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc(c}{d}$$

```
[Out] (5*a^3*Csc[c + d*x])/d + (5*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^3
)/(3*d) - (3*a^3*Csc[c + d*x]^4)/(4*d) - (a^3*Csc[c + d*x]^5)/(5*d) + (a^3*
Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.110436, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*Csc[c + d*x])/d + (5*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^3
)/(3*d) - (3*a^3*Csc[c + d*x]^4)/(4*d) - (a^3*Csc[c + d*x]^5)/(5*d) + (a^3*
Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^5}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(3a+\frac{a^7}{x^6}+\frac{3a^6}{x^5}+\frac{a^5}{x^4}-\frac{5a^4}{x^3}-\frac{5a^3}{x^2}+\frac{a^2}{x}+x\right)dx, x, a\sin(\right.}{d}$$
$$= \frac{5a^3\csc(c+dx)}{d}+\frac{5a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^4(}{4d}$$

Mathematica [A] time = 0.16801, size = 86, normalized size = 0.65

$$\frac{a^3\left(30\sin^2(c+dx)+180\sin(c+dx)-12\csc^5(c+dx)-45\csc^4(c+dx)-20\csc^3(c+dx)+150\csc^2(c+dx)+300\,c\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(300*Csc[c + d*x] + 150*Csc[c + d*x]^2 - 20*Csc[c + d*x]^3 - 45*Csc[c
+ d*x]^4 - 12*Csc[c + d*x]^5 + 60*Log[Sin[c + d*x]] + 180*Sin[c + d*x] + 30
*Sin[c + d*x]^2))/(60*d)
```

Maple [A] time = 0.091, size = 234, normalized size = 1.8

$$-\frac{a^3(\cos(dx+c))^6}{2d(\sin(dx+c))^2}-\frac{(\cos(dx+c))^4a^3}{2d}-\frac{a^3(\cos(dx+c))^2}{d}+\frac{a^3\ln(\sin(dx+c))}{d}-\frac{14a^3(\cos(dx+c))^6}{15d(\sin(dx+c))^3}+\frac{14a^3(\cos}{5d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*cos(d*x+c)^4*a^3-1/d*a^3*cos(d*x
+c)^2+a^3*ln(sin(d*x+c))/d-14/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^6+14/5/d*a^3
/sin(d*x+c)*cos(d*x+c)^6+112/15*a^3*sin(d*x+c)/d+14/5/d*a^3*cos(d*x+c)^4*si
n(d*x+c)+56/15/d*a^3*cos(d*x+c)^2*sin(d*x+c)-3/4/d*a^3*cot(d*x+c)^4+3/2/d*a
^3*cot(d*x+c)^2-1/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^6
```

Maxima [A] time = 1.09972, size = 144, normalized size = 1.08

$$\frac{30a^3\sin(dx+c)^2+60a^3\log(\sin(dx+c))+180a^3\sin(dx+c)+\frac{300a^3\sin(dx+c)^4+150a^3\sin(dx+c)^3-20a^3\sin(dx+c)^2-45a^3\sin(}{\sin(dx+c)^5}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(30*a^3*sin(d*x + c)^2 + 60*a^3*log(sin(d*x + c)) + 180*a^3*sin(d*x +
c) + (300*a^3*sin(d*x + c)^4 + 150*a^3*sin(d*x + c)^3 - 20*a^3*sin(d*x + c)
```

```
^2 - 45*a^3*sin(d*x + c) - 12*a^3)/sin(d*x + c)^5)/d
```

Fricas [A] time = 1.44621, size = 456, normalized size = 3.43

$$-\frac{180\,a^3\cos(dx+c)^6 - 840\,a^3\cos(dx+c)^4 + 1120\,a^3\cos(dx+c)^2 - 448\,a^3 - 60\left(a^3\cos(dx+c)^4 - 2\,a^3\cos(dx+c)^2 + a\right.}{60\left(d\cos(dx+c)^4 - 2\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(180*a^3*cos(d*x + c)^6 - 840*a^3*cos(d*x + c)^4 + 1120*a^3*cos(d*x +
 c)^2 - 448*a^3 - 60*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(
1/2*sin(d*x + c))*sin(d*x + c) + 15*(2*a^3*cos(d*x + c)^6 - 5*a^3*cos(d*x +
 c)^4 + 14*a^3*cos(d*x + c)^2 - 8*a^3)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2
*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31364, size = 165, normalized size = 1.24

$$\frac{30\,a^3\sin(dx+c)^2 + 60\,a^3\log(|\sin(dx+c)|) + 180\,a^3\sin(dx+c) - \frac{137\,a^3\sin(dx+c)^5 - 300\,a^3\sin(dx+c)^4 - 150\,a^3\sin(dx+c)^3 + 20\,a^3\sin(dx}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(30*a^3*sin(d*x + c)^2 + 60*a^3*log(abs(sin(d*x + c))) + 180*a^3*sin(d
*x + c) - (137*a^3*sin(d*x + c)^5 - 300*a^3*sin(d*x + c)^4 - 150*a^3*sin(d*
x + c)^3 + 20*a^3*sin(d*x + c)^2 + 45*a^3*sin(d*x + c) + 12*a^3)/sin(d*x +
c)^5)/d
```

3.529 $\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}+\frac{5a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc}{}$$

```
[Out] -((a^3*Csc[c + d*x])/d) + (5*a^3*Csc[c + d*x]^2)/(2*d) + (5*a^3*Csc[c + d*x
]^3)/(3*d) - (a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a
^3*Csc[c + d*x]^6)/(6*d) + (3*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])
/d
```

Rubi [A] time = 0.126247, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}+\frac{5a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (5*a^3*Csc[c + d*x]^2)/(2*d) + (5*a^3*Csc[c + d*x
]^3)/(3*d) - (a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a
^3*Csc[c + d*x]^6)/(6*d) + (3*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])
/d
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2(a+x)^5}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(1+\frac{a^7}{x^7}+\frac{3a^6}{x^6}+\frac{a^5}{x^5}-\frac{5a^4}{x^4}-\frac{5a^3}{x^3}+\frac{a^2}{x^2}+\frac{3a}{x}\right)dx, x, a\sin(c\right.}{d}$$
$$= -\frac{a^3\csc(c+dx)}{d}+\frac{5a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc^3(c+dx)}{3d}-\frac{a^3\csc^4(c}{4d}$$

Mathematica [A] time = 0.0396926, size = 113, normalized size = 0.85

$$a^3\left(\frac{\sin(c+dx)}{d}-\frac{\csc^6(c+dx)}{6d}-\frac{3\csc^5(c+dx)}{5d}-\frac{\csc^4(c+dx)}{4d}+\frac{5\csc^3(c+dx)}{3d}+\frac{5\csc^2(c+dx)}{2d}-\frac{\csc(c+dx)}{d}+\frac{3\log(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*(-(Csc[c + d*x]/d) + (5*Csc[c + d*x]^2)/(2*d) + (5*Csc[c + d*x]^3)/(3*d
) - Csc[c + d*x]^4/(4*d) - (3*Csc[c + d*x]^5)/(5*d) - Csc[c + d*x]^6/(6*d)
+ (3*Log[Sin[c + d*x]])/d + Sin[c + d*x]/d)
```

Maple [A] time = 0.093, size = 203, normalized size = 1.5

$$-\frac{2a^3(\cos(dx+c))^6}{15d(\sin(dx+c))^3}+\frac{2a^3(\cos(dx+c))^6}{5d\sin(dx+c)}+\frac{16a^3\sin(dx+c)}{15d}+\frac{2a^3(\cos(dx+c))^4\sin(dx+c)}{5d}+\frac{8a^3(\cos(dx+c))^2 s}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^6+2/5/d*a^3/sin(d*x+c)*cos(d*x+c)^6+16/
15*a^3*sin(d*x+c)/d+2/5/d*a^3*cos(d*x+c)^4*sin(d*x+c)+8/15/d*a^3*cos(d*x+c)
^2*sin(d*x+c)-3/4/d*a^3*cot(d*x+c)^4+3/2/d*a^3*cot(d*x+c)^2+3*a^3*ln(sin(d*
x+c))/d-3/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^6-1/6/d*a^3/sin(d*x+c)^6*cos(d*x+
c)^6
```

Maxima [A] time = 1.0321, size = 146, normalized size = 1.1

$$\frac{180\,a^3\log(\sin(dx+c))+60\,a^3\sin(dx+c)-\frac{60\,a^3\sin(dx+c)^5-150\,a^3\sin(dx+c)^4-100\,a^3\sin(dx+c)^3+15\,a^3\sin(dx+c)^2+36\,a^3\sin(dx+c)+10\,a^3}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(180*a^3*log(sin(d*x + c)) + 60*a^3*sin(d*x + c) - (60*a^3*sin(d*x + c
)^5 - 150*a^3*sin(d*x + c)^4 - 100*a^3*sin(d*x + c)^3 + 15*a^3*sin(d*x + c)
```

```
^2 + 36*a^3*sin(d*x + c) + 10*a^3)/sin(d*x + c)^6)/d
```

Fricas [A] time = 1.56869, size = 447, normalized size = 3.36

$$-\frac{150\,a^3\cos\left(dx+c\right)^4 - 285\,a^3\cos\left(dx+c\right)^2 + 125\,a^3 - 180\left(a^3\cos\left(dx+c\right)^6 - 3\,a^3\cos\left(dx+c\right)^4 + 3\,a^3\cos\left(dx+c\right)^2 - \right.}{60\left(d\cos\left(dx+c\right)^6 - 3\,d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(150*a^3*cos(d*x + c)^4 - 285*a^3*cos(d*x + c)^2 + 125*a^3 - 180*(a^3
*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/
2*sin(d*x + c)) - 4*(15*a^3*cos(d*x + c)^6 - 30*a^3*cos(d*x + c)^4 + 40*a^3
*cos(d*x + c)^2 - 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c
)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4107, size = 165, normalized size = 1.24

$$\frac{180\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right) + 60\,a^3\sin\left(dx+c\right) - \frac{441\,a^3\sin(dx+c)^6 + 60\,a^3\sin(dx+c)^5 - 150\,a^3\sin(dx+c)^4 - 100\,a^3\sin(dx+c)^3 + 15\,a^3\sin(dx+c)^2 +}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(180*a^3*log(abs(sin(d*x + c))) + 60*a^3*sin(d*x + c) - (441*a^3*sin(d
*x + c)^6 + 60*a^3*sin(d*x + c)^5 - 150*a^3*sin(d*x + c)^4 - 100*a^3*sin(d*
x + c)^3 + 15*a^3*sin(d*x + c)^2 + 36*a^3*sin(d*x + c) + 10*a^3)/sin(d*x +
c)^6)/d
```

3.530 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=145

$$\frac{a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}-\frac{2a^4\sin^2(c+dx)}{d}-\frac{10a^4\sin(c+dx)}{d}-\frac{a^4\csc^3(c+dx)}{3d}-\frac{2a^4\csc^2(}{d}$$

```
[Out] (-4*a^4*Csc[c + d*x])/d - (2*a^4*Csc[c + d*x]^2)/d - (a^4*Csc[c + d*x]^3)/(
3*d) - (4*a^4*Log[Sin[c + d*x]])/d - (10*a^4*Sin[c + d*x])/d - (2*a^4*Sin[c
 + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/d + (a^4
*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.120683, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}-\frac{2a^4\sin^2(c+dx)}{d}-\frac{10a^4\sin(c+dx)}{d}-\frac{a^4\csc^3(c+dx)}{3d}-\frac{2a^4\csc^2(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a^4*Csc[c + d*x])/d - (2*a^4*Csc[c + d*x]^2)/d - (a^4*Csc[c + d*x]^3)/(
3*d) - (4*a^4*Log[Sin[c + d*x]])/d - (10*a^4*Sin[c + d*x])/d - (2*a^4*Sin[c
 + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/d + (a^4
*Sin[c + d*x]^5)/(5*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^6}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^6}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-10a^4 + \frac{a^8}{x^4} + \frac{4a^7}{x^3} + \frac{4a^6}{x^2} - \frac{4a^5}{x} - 4a^3x + 4a^2x^2 + 4ax^3 +\right.\right.}{ad}$$
$$= -\frac{4a^4\csc(c+dx)}{d} - \frac{2a^4\csc^2(c+dx)}{d} - \frac{a^4\csc^3(c+dx)}{3d} - \frac{4a^4\log(}{}$$

Mathematica [A] time = 0.155741, size = 96, normalized size = 0.66

$$-\frac{a^4\left(-3\sin^5(c+dx) - 15\sin^4(c+dx) - 20\sin^3(c+dx) + 30\sin^2(c+dx) + 150\sin(c+dx) + 5\csc^3(c+dx) + 30\csc\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(a^4*(60*Csc[c + d*x] + 30*Csc[c + d*x]^2 + 5*Csc[c + d*x]^3 + 60*Log[Sin[
c + d*x]] + 150*Sin[c + d*x] + 30*Sin[c + d*x]^2 - 20*Sin[c + d*x]^3 - 15*S
in[c + d*x]^4 - 3*Sin[c + d*x]^5))/(15*d)
```

Maple [A] time = 0.095, size = 179, normalized size = 1.2

$$-\frac{64\,a^4\sin(dx+c)}{5\,d} - \frac{24\,a^4\sin(dx+c)(\cos(dx+c))^4}{5\,d} - \frac{32\,a^4(\cos(dx+c))^2\sin(dx+c)}{5\,d} - \frac{a^4(\cos(dx+c))^4}{d} - 2\frac{a^4(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -64/5*a^4*sin(d*x+c)/d-24/5/d*a^4*sin(d*x+c)*cos(d*x+c)^4-32/5/d*a^4*cos(d*
x+c)^2*sin(d*x+c)-1/d*a^4*cos(d*x+c)^4-2/d*a^4*cos(d*x+c)^2-4*a^4*ln(sin(d*
x+c))/d-5/d*a^4/sin(d*x+c)*cos(d*x+c)^6-2/d*a^4/sin(d*x+c)^2*cos(d*x+c)^6-1
/3/d*a^4/sin(d*x+c)^3*cos(d*x+c)^6
```

Maxima [A] time = 1.18049, size = 161, normalized size = 1.11

$$\frac{3\,a^4\sin(dx+c)^5 + 15\,a^4\sin(dx+c)^4 + 20\,a^4\sin(dx+c)^3 - 30\,a^4\sin(dx+c)^2 - 60\,a^4\log(\sin(dx+c)) - 150\,a^4\sin}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/15*(3*a^4*sin(d*x + c)^5 + 15*a^4*sin(d*x + c)^4 + 20*a^4*sin(d*x + c)^3
- 30*a^4*sin(d*x + c)^2 - 60*a^4*log(sin(d*x + c)) - 150*a^4*sin(d*x + c) -
```

```
5*(12*a^4*sin(d*x + c)^2 + 6*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^3)/d
```

Fricas [A] time = 1.49109, size = 433, normalized size = 2.99

$$-\frac{24\,a^4\cos(dx+c)^8 - 256\,a^4\cos(dx+c)^6 - 576\,a^4\cos(dx+c)^4 + 2304\,a^4\cos(dx+c)^2 - 1536\,a^4 + 480\left(a^4\cos(dx+c)\right.}{120\left(d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/120*(24*a^4*cos(d*x + c)^8 - 256*a^4*cos(d*x + c)^6 - 576*a^4*cos(d*x +
c)^4 + 2304*a^4*cos(d*x + c)^2 - 1536*a^4 + 480*(a^4*cos(d*x + c)^2 - a^4)*
log(1/2*sin(d*x + c))*sin(d*x + c) - 15*(8*a^4*cos(d*x + c)^6 - 8*a^4*cos(d
*x + c)^4 - 3*a^4*cos(d*x + c)^2 + 19*a^4)*sin(d*x + c))/((d*cos(d*x + c)^2
 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35505, size = 182, normalized size = 1.26

$$\frac{3\,a^4\sin(dx+c)^5 + 15\,a^4\sin(dx+c)^4 + 20\,a^4\sin(dx+c)^3 - 30\,a^4\sin(dx+c)^2 - 60\,a^4\log(|\sin(dx+c)|) - 150\,a^4\sin(d}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^4*sin(d*x + c)^5 + 15*a^4*sin(d*x + c)^4 + 20*a^4*sin(d*x + c)^3
- 30*a^4*sin(d*x + c)^2 - 60*a^4*log(abs(sin(d*x + c))) - 150*a^4*sin(d*x +
 c) + 5*(22*a^4*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 - 6*a^4*sin(d*x + c)
 - a^4)/sin(d*x + c)^3)/d
```

3.531 $\int \cot^5(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=148

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}-\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^4(c+dx)}{4d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{2a^4 c\ldots}{}$$

```
[Out] (4*a^4*Csc[c + d*x])/d - (2*a^4*Csc[c + d*x]^2)/d - (4*a^4*Csc[c + d*x]^3)/
(3*d) - (a^4*Csc[c + d*x]^4)/(4*d) - (10*a^4*Log[Sin[c + d*x]])/d - (4*a^4*
Sin[c + d*x])/d + (2*a^4*Sin[c + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) +
 (a^4*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0788179, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}-\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^4(c+dx)}{4d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{2a^4 c\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*a^4*Csc[c + d*x])/d - (2*a^4*Csc[c + d*x]^2)/d - (4*a^4*Csc[c + d*x]^3)/
(3*d) - (a^4*Csc[c + d*x]^4)/(4*d) - (10*a^4*Log[Sin[c + d*x]])/d - (4*a^4*
Sin[c + d*x])/d + (2*a^4*Sin[c + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) +
 (a^4*Sin[c + d*x]^4)/(4*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^6}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-4a^3+\frac{a^8}{x^5}+\frac{4a^7}{x^4}+\frac{4a^6}{x^3}-\frac{4a^5}{x^2}-\frac{10a^4}{x}+4a^2x+4ax^2+x^3\right)dx, x, a\sin\ldots\right)}{d}\\ &= \frac{4a^4\csc(c+dx)}{d}-\frac{2a^4\csc^2(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{a^4\csc^4(c+dx)}{4d}-\frac{10\ldots}{}\end{aligned}$$

Mathematica [A] time = 0.148453, size = 96, normalized size = 0.65

$$\frac{a^4\left(3\sin^4(c+dx)+16\sin^3(c+dx)+24\sin^2(c+dx)-48\sin(c+dx)-3\csc^4(c+dx)-16\csc^3(c+dx)-24\csc^2(c+\ldots\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(48*Csc[c + d*x] - 24*Csc[c + d*x]^2 - 16*Csc[c + d*x]^3 - 3*Csc[c + d
*x]^4 - 120*Log[Sin[c + d*x]] - 48*Sin[c + d*x] + 24*Sin[c + d*x]^2 + 16*Si
n[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)
```

Maple [A] time = 0.095, size = 129, normalized size = 0.9

$$-\frac{11 a^4 (\cos(dx+c))^4}{4d}-\frac{11 a^4 (\cos(dx+c))^2}{2d}-10\frac{a^4 \ln(\sin(dx+c))}{d}-3\frac{a^4 (\cos(dx+c))^6}{d(\sin(dx+c))^2}-\frac{4a^4 (\cos(dx+c))^6}{3d(\sin(dx+c))^3}-\frac{a^4 (\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -11/4/d*a^4*cos(d*x+c)^4-11/2/d*a^4*cos(d*x+c)^2-10*a^4*ln(sin(d*x+c))/d-3/
d*a^4/sin(d*x+c)^2*cos(d*x+c)^6-4/3/d*a^4/sin(d*x+c)^3*cos(d*x+c)^6-1/4/d*a
^4*cot(d*x+c)^4+1/2/d*a^4*cot(d*x+c)^2
```

Maxima [A] time = 1.12739, size = 162, normalized size = 1.09

$$\frac{3a^4 \sin(dx+c)^4+16a^4 \sin(dx+c)^3+24a^4 \sin(dx+c)^2-120a^4 \log(\sin(dx+c))-48a^4 \sin(dx+c)+\frac{48a^4 \sin(dx+c)^3-}{}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 24*a^4*sin(d*x + c)^2
- 120*a^4*log(sin(d*x + c)) - 48*a^4*sin(d*x + c) + (48*a^4*sin(d*x + c)^3
- 24*a^4*sin(d*x + c)^2 - 16*a^4*sin(d*x + c) - 3*a^4)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.59987, size = 371, normalized size = 2.51

$$\frac{24a^4 \cos(dx+c)^8-128a^4 \cos(dx+c)^6 \sin(dx+c)-288a^4 \cos(dx+c)^6+615a^4 \cos(dx+c)^4-270a^4 \cos(dx+c)^2-}{96\left(d\cos(dx+c)^4-2d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*a^4*cos(d*x + c)^8 - 128*a^4*cos(d*x + c)^6*sin(d*x + c) - 288*a^4
*cos(d*x + c)^6 + 615*a^4*cos(d*x + c)^4 - 270*a^4*cos(d*x + c)^2 - 105*a^4
 - 960*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(1/2*sin(d*x +
c)))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38203, size = 181, normalized size = 1.22

$$\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+24\,a^4\sin\left(dx+c\right)^2-120\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)-48\,a^4\sin\left(dx+c\right)+\frac{250\,a^4\sin(d}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 24*a^4*sin(d*x + c)^2
- 120*a^4*log(abs(sin(d*x + c))) - 48*a^4*sin(d*x + c) + (250*a^4*sin(d*x +
 c)^4 + 48*a^4*sin(d*x + c)^3 - 24*a^4*sin(d*x + c)^2 - 16*a^4*sin(d*x + c)
 - 3*a^4)/sin(d*x + c)^4)/d
```

3.532 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=146

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^5(c+dx)}{5d}-\frac{a^4\csc^4(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}+\frac{2a^4\csc^2(c+dx)}{d}$$

```
[Out] (10*a^4*Csc[c + d*x])/d + (2*a^4*Csc[c + d*x]^2)/d - (4*a^4*Csc[c + d*x]^3)
/(3*d) - (a^4*Csc[c + d*x]^4)/d - (a^4*Csc[c + d*x]^5)/(5*d) - (4*a^4*Log[S
in[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*
Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.116307, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^5(c+dx)}{5d}-\frac{a^4\csc^4(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}+\frac{2a^4\csc^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (10*a^4*Csc[c + d*x])/d + (2*a^4*Csc[c + d*x]^2)/d - (4*a^4*Csc[c + d*x]^3)
/(3*d) - (a^4*Csc[c + d*x]^4)/d - (a^4*Csc[c + d*x]^5)/(5*d) - (4*a^4*Log[S
in[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*
Sin[c + d*x]^3)/(3*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^6}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)^6}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(4a^2+\frac{a^8}{x^6}+\frac{4a^7}{x^5}+\frac{4a^6}{x^4}-\frac{4a^5}{x^3}-\frac{10a^4}{x^2}-\frac{4a^3}{x}+4ax+x^2\right) d\right.}{d}$$
$$= \frac{10a^4\csc(c+dx)}{d}+\frac{2a^4\csc^2(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{a^4\csc^4}{}$$

Mathematica [A] time = 0.174539, size = 96, normalized size = 0.66

$$\frac{a^4\left(5\sin^3(c+dx)+30\sin^2(c+dx)+60\sin(c+dx)-3\csc^5(c+dx)-15\csc^4(c+dx)-20\csc^3(c+dx)+30\csc^2(c-\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(150*Csc[c + d*x] + 30*Csc[c + d*x]^2 - 20*Csc[c + d*x]^3 - 15*Csc[c +
 d*x]^4 - 3*Csc[c + d*x]^5 - 60*Log[Sin[c + d*x]] + 60*Sin[c + d*x] + 30*Si
n[c + d*x]^2 + 5*Sin[c + d*x]^3))/(15*d)
```

Maple [A] time = 0.096, size = 235, normalized size = 1.6

$$\frac{24\,a^4\,(\cos(dx+c))^6}{5\,d\sin(dx+c)}+\frac{64\,a^4\sin(dx+c)}{5\,d}+\frac{24\,a^4\sin(dx+c)(\cos(dx+c))^4}{5\,d}+\frac{32\,a^4(\cos(dx+c))^2\sin(dx+c)}{5\,d}-2\,\frac{a}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 24/5/d*a^4/sin(d*x+c)*cos(d*x+c)^6+64/5*a^4*sin(d*x+c)/d+24/5/d*a^4*sin(d*x
+c)*cos(d*x+c)^4+32/5/d*a^4*cos(d*x+c)^2*sin(d*x+c)-2/d*a^4/sin(d*x+c)^2*co
s(d*x+c)^6-2/d*a^4*cos(d*x+c)^4-4/d*a^4*cos(d*x+c)^2-4*a^4*ln(sin(d*x+c))/d
-29/15/d*a^4/sin(d*x+c)^3*cos(d*x+c)^6-1/d*a^4*cot(d*x+c)^4+2/d*a^4*cot(d*x
+c)^2-1/5/d*a^4/sin(d*x+c)^5*cos(d*x+c)^6
```

Maxima [A] time = 1.04787, size = 162, normalized size = 1.11

$$\frac{5\,a^4\sin(dx+c)^3+30\,a^4\sin(dx+c)^2-60\,a^4\log(\sin(dx+c))+60\,a^4\sin(dx+c)+\frac{150\,a^4\sin(dx+c)^4+30\,a^4\sin(dx+c)^3-20\,a^4}{\sin(dx+c)}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/15*(5*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2 - 60*a^4*log(sin(d*x + c
)) + 60*a^4*sin(d*x + c) + (150*a^4*sin(d*x + c)^4 + 30*a^4*sin(d*x + c)^3
```

```
- 20*a^4*sin(d*x + c)^2 - 15*a^4*sin(d*x + c) - 3*a^4)/sin(d*x + c)^5)/d
```

Fricas [A] time = 1.6116, size = 482, normalized size = 3.3

$$\frac{5\,a^4\cos\left(dx+c\right)^8 - 80\,a^4\cos\left(dx+c\right)^6 + 360\,a^4\cos\left(dx+c\right)^4 - 480\,a^4\cos\left(dx+c\right)^2 + 192\,a^4 - 60\left(a^4\cos\left(dx+c\right)^4 - 2\,a^4\right.}{15\left(d\cos\left(dx+c\right)^4 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/15*(5*a^4*cos(d*x + c)^8 - 80*a^4*cos(d*x + c)^6 + 360*a^4*cos(d*x + c)^4
 - 480*a^4*cos(d*x + c)^2 + 192*a^4 - 60*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*
x + c)^2 + a^4)*log(1/2*sin(d*x + c))*sin(d*x + c) - 15*(2*a^4*cos(d*x + c)
^6 - 5*a^4*cos(d*x + c)^4 + 6*a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c))/((d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28392, size = 181, normalized size = 1.24

$$\frac{5\,a^4\sin\left(dx+c\right)^3 + 30\,a^4\sin\left(dx+c\right)^2 - 60\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right) + 60\,a^4\sin\left(dx+c\right) + \frac{137\,a^4\sin(dx+c)^5+150\,a^4\sin(dx+c)^4+30\,a^4\,\mathrm{s}}{\mathrm{sir}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(5*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2 - 60*a^4*log(abs(sin(d*x
 + c))) + 60*a^4*sin(d*x + c) + (137*a^4*sin(d*x + c)^5 + 150*a^4*sin(d*x +
 c)^4 + 30*a^4*sin(d*x + c)^3 - 20*a^4*sin(d*x + c)^2 - 15*a^4*sin(d*x + c)
 - 3*a^4)/sin(d*x + c)^5)/d
```

3.533 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}$$

```
[Out] Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d) - Sin[c + d*x]^6/(6*a*d) +
Sin[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.113463, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$\frac{\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d) - Sin[c + d*x]^6/(6*a*d) +
Sin[c + d*x]^7/(7*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3(a+x)}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int (a-x)^2x^3(a+x)\,dx, x, a\sin(c+dx)\right)}{a^8d}\\
&= \frac{\text{Subst}\left(\int \left(a^3x^3-a^2x^4-ax^5+x^6\right)\,dx, x, a\sin(c+dx)\right)}{a^8d}\\
&= \frac{\sin^4(c+dx)}{4ad}-\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^6(c+dx)}{6ad}+\frac{\sin^7(c+dx)}{7ad}
\end{aligned}$$

Mathematica [A] time = 0.334576, size = 48, normalized size = 0.66

$$\frac{\sin^4(c+dx)\left(60\sin^3(c+dx)-70\sin^2(c+dx)-84\sin(c+dx)+105\right)}{420ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^4*(105 - 84*Sin[c + d*x] - 70*Sin[c + d*x]^2 + 60*Sin[c + d*x
]^3))/(420*a*d)
```

Maple [A] time = 0.074, size = 49, normalized size = 0.7

$$\frac{1}{da}\left(\frac{(\sin(dx+c))^7}{7}-\frac{(\sin(dx+c))^6}{6}-\frac{(\sin(dx+c))^5}{5}+\frac{(\sin(dx+c))^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/7*sin(d*x+c)^7-1/6*sin(d*x+c)^6-1/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4)
```

Maxima [A] time = 1.08004, size = 66, normalized size = 0.9

$$\frac{60\sin(dx+c)^7-70\sin(dx+c)^6-84\sin(dx+c)^5+105\sin(dx+c)^4}{420\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*(60*sin(d*x + c)^7 - 70*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin(
d*x + c)^4)/(a*d)
```

Fricas [A] time = 1.4254, size = 177, normalized size = 2.42

$$\frac{70\cos(dx+c)^6-105\cos(dx+c)^4-12\left(5\cos(dx+c)^6-8\cos(dx+c)^4+\cos(dx+c)^2+2\right)\sin(dx+c)}{420\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*cos(d*x + c)^6 - 105*cos(d*x + c)^4 - 12*(5*cos(d*x + c)^6 - 8*co
s(d*x + c)^4 + cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d)
```

Sympy [A] time = 129.939, size = 1520, normalized size = 20.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-15*tan(c/2 + d*x/2)**14/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d
*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 +
d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 7
35*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 105*tan(c/2 + d*x/2)**12/(105*a*d*t
an(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x
/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 220
5*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 105*ta
n(c/2 + d*x/2)**10/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)
**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*
a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 +
d*x/2)**2 + 105*a*d) - 672*tan(c/2 + d*x/2)**9/(105*a*d*tan(c/2 + d*x/2)**1
4 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d
*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*
x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 385*tan(c/2 + d*x/2)**8/
(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan
(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2
)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d
) + 576*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2
 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**
8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*t
an(c/2 + d*x/2)**2 + 105*a*d) - 385*tan(c/2 + d*x/2)**6/(105*a*d*tan(c/2 +
d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 +
 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan
(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 672*tan(c/2 + d
*x/2)**5/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 220
5*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/
2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2
+ 105*a*d) + 105*tan(c/2 + d*x/2)**4/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*
d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 +
 d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 +
735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 105*tan(c/2 + d*x/2)**2/(105*a*d*t
an(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x
/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 220
5*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 15/(10
5*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/
2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**
6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d),
Ne(d, 0)), (x*sin(c)**3*cos(c)**5/(a*sin(c) + a), True))
```

Giac [A] time = 1.15126, size = 66, normalized size = 0.9

$$\frac{60 \sin (dx+c)^7 - 70 \sin (dx+c)^6 - 84 \sin (dx+c)^5 + 105 \sin (dx+c)^4}{420\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(60*sin(d*x + c)^7 - 70*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin(
d*x + c)^4)/(a*d)
```

3.534 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d) +
Sin[c + d*x]^6/(6*a*d)
```

Rubi [A] time = 0.155688, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2835, 2564, 14}

$$\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d) +
Sin[c + d*x]^6/(6*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^3(c+dx)\sin^3(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)dx, x, \sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^3\left(1-x^2\right)dx, x, \sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(x^2-x^4\right)dx, x, \sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(x^3-x^5\right)dx, x, \sin(c+dx)\right)}{ad}$$
$$= \frac{\sin^3(c+dx)}{3ad} - \frac{\sin^4(c+dx)}{4ad} - \frac{\sin^5(c+dx)}{5ad} + \frac{\sin^6(c+dx)}{6ad}$$

Mathematica [A] time = 0.207874, size = 48, normalized size = 0.66

$$\frac{\sin^3(c+dx)\left(10\sin^3(c+dx)-12\sin^2(c+dx)-15\sin(c+dx)+20\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]

[Out] (Sin[c + d*x]^3*(20 - 15*Sin[c + d*x] - 12*Sin[c + d*x]^2 + 10*Sin[c + d*x]
^3))/(60*a*d)
```

Maple [A] time = 0.069, size = 49, normalized size = 0.7

$$\frac{1}{da}\left(\frac{(\sin(dx+c))^6}{6} - \frac{(\sin(dx+c))^5}{5} - \frac{(\sin(dx+c))^4}{4} + \frac{(\sin(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)

[Out] 1/d/a*(1/6*sin(d*x+c)^6-1/5*sin(d*x+c)^5-1/4*sin(d*x+c)^4+1/3*sin(d*x+c)^3)
```

Maxima [A] time = 1.13324, size = 66, normalized size = 0.9

$$\frac{10\sin(dx+c)^6 - 12\sin(dx+c)^5 - 15\sin(dx+c)^4 + 20\sin(dx+c)^3}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/60*(10*sin(d*x + c)^6 - 12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*
x + c)^3)/(a*d)
```

Fricas [A] time = 1.66327, size = 149, normalized size = 2.04

$$-\frac{10\cos(dx+c)^6 - 15\cos(dx+c)^4 + 4\left(3\cos(dx+c)^4 - \cos(dx+c)^2 - 2\right)\sin(dx+c)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(10*cos(d*x + c)^6 - 15*cos(d*x + c)^4 + 4*(3*cos(d*x + c)^4 - cos(d*
x + c)^2 - 2)*sin(d*x + c))/(a*d)
```

Sympy [A] time = 79.2048, size = 862, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((40*tan(c/2 + d*x/2)**9/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan
(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)*
*6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 6
0*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2
)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d
*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 24*tan(c/2 +
d*x/2)**7/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*
a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d
*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 40*tan(c/2 + d*x/2)**6/(1
5*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2
+ d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 9
0*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 24*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/
2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8
+ 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/
2 + d*x/2)**2 + 15*a*d) - 60*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**
12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*ta
n(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**
2 + 15*a*d) + 40*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*
tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/
2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d),
 Ne(d, 0)), (x*sin(c)**2*cos(c)**5/(a*sin(c) + a), True))
```

Giac [A] time = 1.14886, size = 66, normalized size = 0.9

$$\frac{10 \sin(dx+c)^6 - 12 \sin(dx+c)^5 - 15 \sin(dx+c)^4 + 20 \sin(dx+c)^3}{60\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(10*sin(d*x + c)^6 - 12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*
x + c)^3)/(a*d)
```

3.535 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^4(c+dx)}{4ad}$$

```
[Out] -Cos[c + d*x]^4/(4*a*d) - Sin[c + d*x]^3/(3*a*d) + Sin[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.107202, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2835, 2565, 30, 2564, 14}

$$\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cos[c + d*x]^4/(4*a*d) - Sin[c + d*x]^3/(3*a*d) + Sin[c + d*x]^5/(5*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^3(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int x^3\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \sin(c+dx)\right)}{ad}\\
&= -\frac{\cos^4(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, \sin(c+dx)\right)}{ad}\\
&= -\frac{\cos^4(c+dx)}{4ad} - \frac{\sin^3(c+dx)}{3ad} + \frac{\sin^5(c+dx)}{5ad}
\end{aligned}$$

Mathematica [A] time = 0.154122, size = 48, normalized size = 0.87

$$\frac{\sin^2(c+dx)\left(12\sin^3(c+dx)-15\sin^2(c+dx)-20\sin(c+dx)+30\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^2*(30 - 20*Sin[c + d*x] - 15*Sin[c + d*x]^2 + 12*Sin[c + d*x]
^3))/(60*a*d)
```

Maple [A] time = 0.058, size = 49, normalized size = 0.9

$$\frac{1}{da}\left(\frac{(\sin(dx+c))^5}{5} - \frac{(\sin(dx+c))^4}{4} - \frac{(\sin(dx+c))^3}{3} + \frac{(\sin(dx+c))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/5*sin(d*x+c)^5-1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2)
```

Maxima [A] time = 1.0715, size = 66, normalized size = 1.2

$$\frac{12\sin(dx+c)^5 - 15\sin(dx+c)^4 - 20\sin(dx+c)^3 + 30\sin(dx+c)^2}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 30*sin(d*
x + c)^2)/(a*d)
```

Fricas [A] time = 1.49926, size = 122, normalized size = 2.22

$$-\frac{15\cos(dx+c)^4 - 4\left(3\cos(dx+c)^4 - \cos(dx+c)^2 - 2\right)\sin(dx+c)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*cos(d*x + c)^4 - 4*(3*cos(d*x + c)^4 - cos(d*x + c)^2 - 2)*sin(d*
x + c))/(a*d)
```

Sympy [A] time = 45.0321, size = 741, normalized size = 13.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((30*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan
(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**
4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 40*tan(c/2 + d*x/2)**7/(15*a*d*t
an(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)
**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) +
30*tan(c/2 + d*x/2)**6/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/
2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*
tan(c/2 + d*x/2)**2 + 15*a*d) + 16*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*
x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a
*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*tan(c/2
+ d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150
*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d
*x/2)**2 + 15*a*d) - 40*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**10 +
75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2
+ d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*tan(c/2 + d*x/2)**2
/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/
2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 +
15*a*d), Ne(d, 0)), (x*sin(c)*cos(c)**5/(a*sin(c) + a), True))
```

Giac [A] time = 1.17198, size = 66, normalized size = 1.2

$$\frac{12 \sin(dx+c)^5 - 15 \sin(dx+c)^4 - 20 \sin(dx+c)^3 + 30 \sin(dx+c)^2}{60\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 30*sin(d*
x + c)^2)/(a*d)
```

3.536 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{\sin^3(c+dx)}{3ad} - \frac{\sin^2(c+dx)}{2ad} - \frac{\sin(c+dx)}{ad} + \frac{\log(\sin(c+dx))}{ad}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d) + Sin
[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0891006, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 75}

$$\frac{\sin^3(c+dx)}{3ad} - \frac{\sin^2(c+dx)}{2ad} - \frac{\sin(c+dx)}{ad} + \frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d) + Sin
[c + d*x]^3/(3*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2+\frac{a^3}{x}-ax+x^2\right)dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} - \frac{\sin^2(c+dx)}{2ad} + \frac{\sin^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.0497562, size = 49, normalized size = 0.75

$$\frac{2\sin^3(c+dx) - 3\sin^2(c+dx) - 6\sin(c+dx) + 6\log(\sin(c+dx)) - 2}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2 + 6*Log[Sin[c + d*x]] - 6*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 2*Sin[c + d
*x]^3)/(6*a*d)
```

Maple [A] time = 0.089, size = 62, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{da} - \frac{\sin(dx+c)}{da} - \frac{(\sin(dx+c))^2}{2\,da} + \frac{(\sin(dx+c))^3}{3\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/d/a-1/2*sin(d*x+c)^2/d/a+1/3*sin(d*x+c)^3/d/a
```

Maxima [A] time = 1.1146, size = 69, normalized size = 1.06

$$\frac{\frac{2\sin(dx+c)^3 - 3\sin(dx+c)^2 - 6\sin(dx+c)}{a} + \frac{6\log(\sin(dx+c))}{a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*sin(d*x + c)^3 - 3*sin(d*x + c)^2 - 6*sin(d*x + c))/a + 6*log(sin(d
*x + c))/a)/d
```

Fricas [A] time = 1.31532, size = 127, normalized size = 1.95

$$\frac{3\cos(dx+c)^2 - 2\left(\cos(dx+c)^2 + 2\right)\sin(dx+c) + 6\log\left(\frac{1}{2}\sin(dx+c)\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 + 2)*sin(d*x + c) + 6*log(1/2*sin
(d*x + c)))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21895, size = 82, normalized size = 1.26

$$\frac{\frac{6 \log\left(|\sin(dx+c)|\right)}{a} + \frac{2 a^2 \sin(dx+c)^3 - 3 a^2 \sin(dx+c)^2 - 6 a^2 \sin(dx+c)}{a^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c)))/a + (2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c
)^2 - 6*a^2*sin(d*x + c))/a^3)/d
```

3.537 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[
c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.108435, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[
c + d*x]^2/(2*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^3}{x^2}-\frac{a^2}{x}+x\right)dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} + \frac{\sin^2(c+dx)}{2ad}$$

Mathematica [A] time = 0.0648701, size = 45, normalized size = 0.73

$$\frac{\sin^2(c+dx) - 2\sin(c+dx) - 2\csc(c+dx) - 2\log(\sin(c+dx)) + 6}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6 - 2*Csc[c + d*x] - 2*Log[Sin[c + d*x]] - 2*Sin[c + d*x] + Sin[c + d*x]^2
)/(2*a*d)
```

Maple [A] time = 0.107, size = 63, normalized size = 1.

$$\frac{(\sin(dx+c))^2}{2\,da} - \frac{\sin(dx+c)}{da} - \frac{1}{da\sin(dx+c)} - \frac{\ln(\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2*sin(d*x+c)^2/d/a-sin(d*x+c)/d/a-1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.16758, size = 70, normalized size = 1.13

$$\frac{\frac{\sin(dx+c)^2-2\sin(dx+c)}{a} - \frac{2\log(\sin(dx+c))}{a} - \frac{2}{a\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((sin(d*x + c)^2 - 2*sin(d*x + c))/a - 2*log(sin(d*x + c))/a - 2/(a*sin
(d*x + c)))/d
```

Fricas [A] time = 1.14821, size = 167, normalized size = 2.69

$$\frac{4\cos(dx+c)^2 - \left(2\cos(dx+c)^2 - 1\right)\sin(dx+c) - 4\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c) - 8}{4\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*cos(d*x + c)^2 - (2*cos(d*x + c)^2 - 1)*sin(d*x + c) - 4*log(1/2*sin
(d*x + c))*sin(d*x + c) - 8)/(a*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26467, size = 88, normalized size = 1.42

$$-\frac{\frac{2\log\left(|\sin\left(dx+c\right)|\right)}{a}-\frac{a\sin\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)}{a^{2}}-\frac{2\,\left(\sin\left(dx+c\right)-1\right)}{a\sin\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c)))/a - (a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/a^
2 - 2*(sin(d*x + c) - 1)/(a*sin(d*x + c)))/d
```

3.538 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - Log[Sin[c + d*x]]/(a*d) + Sin
[c + d*x]/(a*d)
```

Rubi [A] time = 0.105576, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$\frac{\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - Log[Sin[c + d*x]]/(a*d) + Sin
[c + d*x]/(a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^3}{x^3}-\frac{a^2}{x^2}-\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad} - \frac{\log(\sin(c+dx))}{ad} + \frac{\sin(c+dx)}{ad}$$

Mathematica [A] time = 0.0862675, size = 45, normalized size = 0.75

$$-\frac{-2\sin(c+dx)+\csc^2(c+dx)-2\csc(c+dx)+2\log(\sin(c+dx))+3}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(3 - 2*Csc[c + d*x] + Csc[c + d*x]^2 + 2*Log[Sin[c + d*x]] - 2*Sin[c + d*x
])/(2*a*d)
```

Maple [A] time = 0.113, size = 61, normalized size = 1.

$$\frac{\sin(dx+c)}{da} + \frac{1}{da\sin(dx+c)} - \frac{\ln(\sin(dx+c))}{da} - \frac{1}{2\,da\,(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] sin(d*x+c)/d/a+1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d-1/2/d/a/sin(d*x+c)^2
```

Maxima [A] time = 1.00717, size = 70, normalized size = 1.17

$$-\frac{\frac{2\log(\sin(dx+c))}{a} - \frac{2\sin(dx+c)}{a} - \frac{2\sin(dx+c)-1}{a\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c))/a - 2*sin(d*x + c)/a - (2*sin(d*x + c) - 1)/(a*si
n(d*x + c)^2))/d
```

Fricas [A] time = 1.13329, size = 165, normalized size = 2.75

$$-\frac{2\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-2\left(\cos(dx+c)^2-2\right)\sin(dx+c)-1}{2\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 2*(cos(d*x + c)^2 - 2)
*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2756, size = 85, normalized size = 1.42

$$-\frac{\frac{2\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}-\frac{2\,\sin\left(dx+c\right)}{a}-\frac{3\,\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)-1}{a\sin\left(dx+c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c)))/a - 2*sin(d*x + c)/a - (3*sin(d*x + c)^2 + 2
*sin(d*x + c) - 1)/(a*sin(d*x + c)^2))/d
```

3.539 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=64

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] Csc[c + d*x]/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) + Log[
Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.092183, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 75}

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) + Log[
Sin[c + d*x]]/(a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)}{x^4}dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^4}dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^2}{x^3}-\frac{a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{\log(\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.0778566, size = 48, normalized size = 0.75

$$\frac{-2\csc^3(c+dx)+3\csc^2(c+dx)+6\csc(c+dx)+6\log(\sin(c+dx))}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 2*Csc[c + d*x]^3 + 6*Log[Sin[c + d*x]]
)/(6*a*d)
```

Maple [A] time = 0.119, size = 63, normalized size = 1.

$$\frac{1}{da\sin(dx+c)}+\frac{\ln(\sin(dx+c))}{da}-\frac{1}{3da(\sin(dx+c))^3}+\frac{1}{2da(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a/sin(d*x+c)+ln(sin(d*x+c))/a/d-1/3/d/a/sin(d*x+c)^3+1/2/d/a/sin(d*x+c)
^2
```

Maxima [A] time = 1.0733, size = 68, normalized size = 1.06

$$\frac{\frac{6\log(\sin(dx+c))}{a}+\frac{6\sin(dx+c)^2+3\sin(dx+c)-2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(sin(d*x + c))/a + (6*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin
(d*x + c)^3))/d
```

Fricas [A] time = 1.11885, size = 198, normalized size = 3.09

$$\frac{6\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+6\cos\left(dx+c\right)^{2}-3\sin\left(dx+c\right)-4}{6\left(ad\cos\left(dx+c\right)^{2}-ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*sin(d*x + c) + 6*cos(d*x
+ c)^2 - 3*sin(d*x + c) - 4)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24, size = 84, normalized size = 1.31

$$\frac{\frac{6\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}-\frac{11\sin\left(dx+c\right)^{3}-6\sin\left(dx+c\right)^{2}-3\sin\left(dx+c\right)+2}{a\sin\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c)))/a - (11*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 3
*sin(d*x + c) + 2)/(a*sin(d*x + c)^3))/d
```

3.540 $\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -Cot[c + d*x]^4/(4*a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0877663, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2706, 2607, 30, 2606}

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^4/(4*a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(3*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^3(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot^3(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= -\frac{\text{Subst}\left(\int x^3\,dx, x, -\cot(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)dx, x, \csc(c+dx)\right)}{ad}$$
$$= -\frac{\cot^4(c+dx)}{4ad} - \frac{\csc(c+dx)}{ad} + \frac{\csc^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.0440985, size = 30, normalized size = 0.59

$$-\frac{(\csc(c+dx)-1)^3(3\csc(c+dx)+5)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((-1 + Csc[c + d*x])^3*(5 + 3*Csc[c + d*x]))/(12*a*d)
```

Maple [A] time = 0.128, size = 49, normalized size = 1.

$$\frac{1}{da}\left(-(\sin(dx+c))^{-1} - \frac{1}{4(\sin(dx+c))^4} + \frac{1}{3(\sin(dx+c))^3} + \frac{1}{2(\sin(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/sin(d*x+c)-1/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.06731, size = 62, normalized size = 1.22

$$-\frac{12\sin(dx+c)^3 - 6\sin(dx+c)^2 - 4\sin(dx+c) + 3}{12\,ad\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

Fricas [A] time = 1.03288, size = 162, normalized size = 3.18

$$-\frac{6\cos(dx+c)^2 - 4\left(3\cos(dx+c)^2 - 2\right)\sin(dx+c) - 3}{12\left(ad\cos(dx+c)^4 - 2\,ad\cos(dx+c)^2 + ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^2 - 2)*sin(d*x + c) - 3)/(a*d*c
os(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20258, size = 62, normalized size = 1.22

$$-\frac{12 \sin (dx+c)^3 - 6 \sin (dx+c)^2 - 4 \sin (dx+c) + 3}{12\,ad \sin (dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

3.541 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\cot^4(c+dx)}{4ad}-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] Cot[c + d*x]^4/(4*a*d) + Csc[c + d*x]^3/(3*a*d) - Csc[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.135552, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2835, 2606, 14, 2607, 30}

$$\frac{\cot^4(c+dx)}{4ad}-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^4/(4*a*d) + Csc[c + d*x]^3/(3*a*d) - Csc[c + d*x]^5/(5*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^3(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot^3(c+dx)\csc^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^3\,dx,x,-\cot(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= \frac{\cot^4(c+dx)}{4ad}-\frac{\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= \frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.110942, size = 48, normalized size = 0.87

$$\frac{\csc^2(c+dx)\left(-12\csc^3(c+dx)+15\csc^2(c+dx)+20\csc(c+dx)-30\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^2*(-30 + 20*Csc[c + d*x] + 15*Csc[c + d*x]^2 - 12*Csc[c + d*x
]^3))/(60*a*d)
```

Maple [A] time = 0.132, size = 49, normalized size = 0.9

$$\frac{1}{da}\left(-\frac{1}{5\left(\sin\left(dx+c\right)\right)^5}+\frac{1}{4\left(\sin\left(dx+c\right)\right)^4}+\frac{1}{3\left(\sin\left(dx+c\right)\right)^3}-\frac{1}{2\left(\sin\left(dx+c\right)\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/5/sin(d*x+c)^5+1/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2
)
```

Maxima [A] time = 1.14586, size = 62, normalized size = 1.13

$$-\frac{30\sin\left(dx+c\right)^3-20\sin\left(dx+c\right)^2-15\sin\left(dx+c\right)+12}{60\,ad\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(30*sin(d*x + c)^3 - 20*sin(d*x + c)^2 - 15*sin(d*x + c) + 12)/(a*d*s
in(d*x + c)^5)
```

Fricas [A] time = 1.05824, size = 185, normalized size = 3.36

$$-\frac{20\cos(dx+c)^2-15\left(2\cos(dx+c)^2-1\right)\sin(dx+c)-8}{60\left(ad\cos(dx+c)^4-2\,ad\cos(dx+c)^2+ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(20*cos(d*x + c)^2 - 15*(2*cos(d*x + c)^2 - 1)*sin(d*x + c) - 8)/((a*
d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29617, size = 62, normalized size = 1.13

$$-\frac{30\sin(dx+c)^3-20\sin(dx+c)^2-15\sin(dx+c)+12}{60\,ad\sin(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(30*sin(d*x + c)^3 - 20*sin(d*x + c)^2 - 15*sin(d*x + c) + 12)/(a*d*s
in(d*x + c)^5)
```

3.542 $\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] -Csc[c + d*x]^3/(3*a*d) + Csc[c + d*x]^4/(4*a*d) + Csc[c + d*x]^5/(5*a*d) -
 Csc[c + d*x]^6/(6*a*d)
```

Rubi [A] time = 0.111392, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Csc[c + d*x]^3/(3*a*d) + Csc[c + d*x]^4/(4*a*d) + Csc[c + d*x]^5/(5*a*d) -
 Csc[c + d*x]^6/(6*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2(a+x)}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^3}{x^7}-\frac{a^2}{x^6}-\frac{a}{x^5}+\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^6(c+dx)}{6ad}$$

Mathematica [A] time = 0.10296, size = 48, normalized size = 0.66

$$\frac{\csc^3(c+dx)\left(-10\csc^3(c+dx)+12\csc^2(c+dx)+15\csc(c+dx)-20\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^3*(-20 + 15*Csc[c + d*x] + 12*Csc[c + d*x]^2 - 10*Csc[c + d*x
]^3))/(60*a*d)
```

Maple [A] time = 0.142, size = 49, normalized size = 0.7

$$\frac{1}{da}\left(\frac{1}{5\left(\sin(dx+c)\right)^5}+\frac{1}{4\left(\sin(dx+c)\right)^4}-\frac{1}{6\left(\sin(dx+c)\right)^6}-\frac{1}{3\left(\sin(dx+c)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/5/sin(d*x+c)^5+1/4/sin(d*x+c)^4-1/6/sin(d*x+c)^6-1/3/sin(d*x+c)^3)
```

Maxima [A] time = 1.12297, size = 62, normalized size = 0.85

$$-\frac{20\sin(dx+c)^3-15\sin(dx+c)^2-12\sin(dx+c)+10}{60\,ad\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(20*sin(d*x + c)^3 - 15*sin(d*x + c)^2 - 12*sin(d*x + c) + 10)/(a*d*s
in(d*x + c)^6)
```

Fricas [A] time = 1.01741, size = 193, normalized size = 2.64

$$\frac{15\cos(dx+c)^2-4\left(5\cos(dx+c)^2-2\right)\sin(dx+c)-5}{60\left(ad\cos(dx+c)^6-3\,ad\cos(dx+c)^4+3\,ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*cos(d*x + c)^2 - 4*(5*cos(d*x + c)^2 - 2)*sin(d*x + c) - 5)/(a*d*c
os(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32414, size = 62, normalized size = 0.85

$$-\frac{20 \sin\left(dx+c\right)^{3}-15 \sin\left(dx+c\right)^{2}-12 \sin\left(dx+c\right)+10}{60 \, ad \sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(20*sin(d*x + c)^3 - 15*sin(d*x + c)^2 - 12*sin(d*x + c) + 10)/(a*d*s
in(d*x + c)^6)
```

3.543 $\int \frac{\cot^5(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^4(c+dx)}{4ad}$$

```
[Out] -Csc[c + d*x]^4/(4*a*d) + Csc[c + d*x]^5/(5*a*d) + Csc[c + d*x]^6/(6*a*d) -
 Csc[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.10912, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 75}

$$-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Csc[c + d*x]^4/(4*a*d) + Csc[c + d*x]^5/(5*a*d) + Csc[c + d*x]^6/(6*a*d) -
 Csc[c + d*x]^7/(7*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^8(a-x)^2(a+x)}{x^8}\,dx,x,a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^8}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{a^3}{x^8}-\frac{a^2}{x^7}-\frac{a}{x^6}+\frac{1}{x^5}\right)dx,x,a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}$$

Mathematica [A] time = 0.10259, size = 48, normalized size = 0.66

$$\frac{\csc^4(c+dx)\left(-60\csc^3(c+dx)+70\csc^2(c+dx)+84\csc(c+dx)-105\right)}{420ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^4*(-105 + 84*Csc[c + d*x] + 70*Csc[c + d*x]^2 - 60*Csc[c + d*
x]^3))/(420*a*d)
```

Maple [A] time = 0.155, size = 49, normalized size = 0.7

$$\frac{1}{da}\left(-\frac{1}{7\left(\sin(dx+c)\right)^7}+\frac{1}{5\left(\sin(dx+c)\right)^5}-\frac{1}{4\left(\sin(dx+c)\right)^4}+\frac{1}{6\left(\sin(dx+c)\right)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/7/sin(d*x+c)^7+1/5/sin(d*x+c)^5-1/4/sin(d*x+c)^4+1/6/sin(d*x+c)^6
)
```

Maxima [A] time = 1.03947, size = 62, normalized size = 0.85

$$-\frac{105\sin(dx+c)^3-84\sin(dx+c)^2-70\sin(dx+c)+60}{420\,ad\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(105*sin(d*x + c)^3 - 84*sin(d*x + c)^2 - 70*sin(d*x + c) + 60)/(a*d
*sin(d*x + c)^7)
```

Fricas [A] time = 1.07927, size = 217, normalized size = 2.97

$$\frac{84\cos(dx+c)^2 - 35\left(3\cos(dx+c)^2 - 1\right)\sin(dx+c) - 24}{420\left(ad\cos(dx+c)^6 - 3\,ad\cos(dx+c)^4 + 3\,ad\cos(dx+c)^2 - ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(84*cos(d*x + c)^2 - 35*(3*cos(d*x + c)^2 - 1)*sin(d*x + c) - 24)/((a
*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)*sin(
d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25472, size = 62, normalized size = 0.85

$$-\frac{105\sin(dx+c)^3 - 84\sin(dx+c)^2 - 70\sin(dx+c) + 60}{420\,ad\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(105*sin(d*x + c)^3 - 84*sin(d*x + c)^2 - 70*sin(d*x + c) + 60)/(a*d
*sin(d*x + c)^7)
```

3.544 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^6(c+dx)}{6a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}+\frac{\sin^4(c+dx)}{4a^2d}$$

```
[Out] Sin[c + d*x]^4/(4*a^2*d) - (2*Sin[c + d*x]^5)/(5*a^2*d) + Sin[c + d*x]^6/(6
*a^2*d)
```

Rubi [A] time = 0.106426, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\sin^6(c+dx)}{6a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}+\frac{\sin^4(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] Sin[c + d*x]^4/(4*a^2*d) - (2*Sin[c + d*x]^5)/(5*a^2*d) + Sin[c + d*x]^6/(6
*a^2*d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-x)^2x^3}{a^3}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\operatorname{Subst}\left(\int (a-x)^2x^3\,dx,x,a\sin(c+dx)\right)}{a^8d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^2x^3-2ax^4+x^5\right)dx,x,a\sin(c+dx)\right)}{a^8d}\\ &= \frac{\sin^4(c+dx)}{4a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}+\frac{\sin^6(c+dx)}{6a^2d}\end{aligned}$$

Mathematica [A] time = 0.528169, size = 38, normalized size = 0.69

$$\frac{\sin^4(c+dx)\left(10\sin^2(c+dx)-24\sin(c+dx)+15\right)}{60a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^4*(15 - 24*Sin[c + d*x] + 10*Sin[c + d*x]^2))/(60*a^2*d)
```

Maple [A] time = 0.083, size = 39, normalized size = 0.7

$$\frac{1}{da^2}\left(\frac{(\sin(dx+c))^6}{6}-\frac{2(\sin(dx+c))^5}{5}+\frac{(\sin(dx+c))^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(1/6*sin(d*x+c)^6-2/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4)
```

Maxima [A] time = 1.20915, size = 53, normalized size = 0.96

$$\frac{10\sin(dx+c)^6-24\sin(dx+c)^5+15\sin(dx+c)^4}{60a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(10*sin(d*x + c)^6 - 24*sin(d*x + c)^5 + 15*sin(d*x + c)^4)/(a^2*d)
```

Fricas [A] time = 1.06009, size = 180, normalized size = 3.27

$$-\frac{10\cos(dx+c)^6-45\cos(dx+c)^4+60\cos(dx+c)^2+24\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\sin(dx+c)}{60a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/60*(10*cos(d*x + c)^6 - 45*cos(d*x + c)^4 + 60*cos(d*x + c)^2 + 24*(cos(
d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29208, size = 53, normalized size = 0.96

$$\frac{10 \sin(dx+c)^6 - 24 \sin(dx+c)^5 + 15 \sin(dx+c)^4}{60 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*sin(d*x + c)^6 - 24*sin(d*x + c)^5 + 15*sin(d*x + c)^4)/(a^2*d)
```

3.545 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^5(c+dx)}{5a^2d}-\frac{\sin^4(c+dx)}{2a^2d}+\frac{\sin^3(c+dx)}{3a^2d}$$

```
[Out] Sin[c + d*x]^3/(3*a^2*d) - Sin[c + d*x]^4/(2*a^2*d) + Sin[c + d*x]^5/(5*a^2
*d)
```

Rubi [A] time = 0.104194, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\sin^5(c+dx)}{5a^2d}-\frac{\sin^4(c+dx)}{2a^2d}+\frac{\sin^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^3/(3*a^2*d) - Sin[c + d*x]^4/(2*a^2*d) + Sin[c + d*x]^5/(5*a^2
*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int (a-x)^2x^2\,dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \left(a^2x^2-2ax^3+x^4\right)dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\sin^3(c+dx)}{3a^2d}-\frac{\sin^4(c+dx)}{2a^2d}+\frac{\sin^5(c+dx)}{5a^2d}
\end{aligned}$$

Mathematica [A] time = 0.672682, size = 38, normalized size = 0.69

$$-\frac{\sin^3(c+dx)(15\sin(c+dx)+3\cos(2(c+dx))-13)}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sin[c + d*x]^3*(-13 + 3*Cos[2*(c + d*x)] + 15*Sin[c + d*x]))/(30*a^2*d)
```

Maple [A] time = 0.075, size = 39, normalized size = 0.7

$$\frac{1}{da^2}\left(\frac{(\sin(dx+c))^5}{5}-\frac{(\sin(dx+c))^4}{2}+\frac{(\sin(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(1/5*sin(d*x+c)^5-1/2*sin(d*x+c)^4+1/3*sin(d*x+c)^3)
```

Maxima [A] time = 1.12476, size = 53, normalized size = 0.96

$$\frac{6\sin(dx+c)^5-15\sin(dx+c)^4+10\sin(dx+c)^3}{30a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 10*sin(d*x + c)^3)/(a^2*d)
```

Fricas [A] time = 1.09539, size = 155, normalized size = 2.82

$$-\frac{15\cos(dx+c)^4-30\cos(dx+c)^2-2\left(3\cos(dx+c)^4-11\cos(dx+c)^2+8\right)\sin(dx+c)}{30a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/30*(15*cos(d*x + c)^4 - 30*cos(d*x + c)^2 - 2*(3*cos(d*x + c)^4 - 11*cos
(d*x + c)^2 + 8)*sin(d*x + c))/(a^2*d)
```

Sympy [A] time = 132.556, size = 588, normalized size = 10.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((40*tan(c/2 + d*x/2)**7/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a**2
*d*tan(c/2 + d*x/2)**8 + 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan(c/
2 + d*x/2)**4 + 75*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 120*tan(c/2 +
d*x/2)**6/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a**2*d*tan(c/2 + d*x/2)**8 +
 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan(c/2 + d*x/2)**4 + 75*a**2*
d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 176*tan(c/2 + d*x/2)**5/(15*a**2*d*tan
(c/2 + d*x/2)**10 + 75*a**2*d*tan(c/2 + d*x/2)**8 + 150*a**2*d*tan(c/2 + d*
x/2)**6 + 150*a**2*d*tan(c/2 + d*x/2)**4 + 75*a**2*d*tan(c/2 + d*x/2)**2 +
15*a**2*d) - 120*tan(c/2 + d*x/2)**4/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a
**2*d*tan(c/2 + d*x/2)**8 + 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan
(c/2 + d*x/2)**4 + 75*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 40*tan(c/2
+ d*x/2)**3/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a**2*d*tan(c/2 + d*x/2)**8
 + 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan(c/2 + d*x/2)**4 + 75*a**
2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**5/(a*
sin(c) + a)**2, True))
```

Giac [A] time = 1.22469, size = 53, normalized size = 0.96

$$\frac{6\sin(dx+c)^5 - 15\sin(dx+c)^4 + 10\sin(dx+c)^3}{30a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 10*sin(d*x + c)^3)/(a^2*d)
```

3.546 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^4(c+dx)}{4a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^2(c+dx)}{2a^2d}$$

```
[Out] Sin[c + d*x]^2/(2*a^2*d) - (2*Sin[c + d*x]^3)/(3*a^2*d) + Sin[c + d*x]^4/(4
*a^2*d)
```

Rubi [A] time = 0.0675053, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$\frac{\sin^4(c+dx)}{4a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^2/(2*a^2*d) - (2*Sin[c + d*x]^3)/(3*a^2*d) + Sin[c + d*x]^4/(4
*a^2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2x}{a}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int (a-x)^2x\,dx, x, a\sin(c+dx)\right)}{a^6d}\\
&= \frac{\text{Subst}\left(\int \left(a^2x-2ax^2+x^3\right)dx, x, a\sin(c+dx)\right)}{a^6d}\\
&= \frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^4(c+dx)}{4a^2d}
\end{aligned}$$

Mathematica [A] time = 0.267495, size = 38, normalized size = 0.69

$$\frac{\sin^2(c+dx)\left(3\sin^2(c+dx)-8\sin(c+dx)+6\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^2*(6 - 8*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(12*a^2*d)
```

Maple [A] time = 0.062, size = 39, normalized size = 0.7

$$\frac{1}{da^2}\left(\frac{(\sin(dx+c))^4}{4}-\frac{2(\sin(dx+c))^3}{3}+\frac{(\sin(dx+c))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(1/4*sin(d*x+c)^4-2/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2)
```

Maxima [A] time = 1.08477, size = 53, normalized size = 0.96

$$\frac{3\sin(dx+c)^4-8\sin(dx+c)^3+6\sin(dx+c)^2}{12a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 8*sin(d*x + c)^3 + 6*sin(d*x + c)^2)/(a^2*d)
```

Fricas [A] time = 1.04333, size = 123, normalized size = 2.24

$$\frac{3\cos(dx+c)^4-12\cos(dx+c)^2+8\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{12a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 8*(cos(d*x + c)^2 - 1)*sin(d*x
 + c))/(a^2*d)
```

Sympy [A] time = 81.1295, size = 672, normalized size = 12.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-2*tan(c/2 + d*x/2)**8/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d
*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 +
d*x/2)**2 + 9*a**2*d) + 10*tan(c/2 + d*x/2)**6/(9*a**2*d*tan(c/2 + d*x/2)**
8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2
*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) - 48*tan(c/2 + d*x/2)**5/(9*a**2*d*tan(c
/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)
**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) + 60*tan(c/2 + d*x/2)**4/(9
*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan
(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) - 48*tan(c/2 +
 d*x/2)**3/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 +
54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) +
 10*tan(c/2 + d*x/2)**2/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 +
 d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2
+ 9*a**2*d) - 2/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)*
*6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2
*d), Ne(d, 0)), (x*sin(c)*cos(c)**5/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.19554, size = 53, normalized size = 0.96

$$\frac{3\sin(dx+c)^4 - 8\sin(dx+c)^3 + 6\sin(dx+c)^2}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 8*sin(d*x + c)^3 + 6*sin(d*x + c)^2)/(a^2*d)
```

3.547 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - (2*Sin[c + d*x])/(a^2*d) + Sin[c + d*x]^2/(2*a^
2*d)
```

Rubi [A] time = 0.0818374, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - (2*Sin[c + d*x])/(a^2*d) + Sin[c + d*x]^2/(2*a^
2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^2}{x}+x\right)dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\log(\sin(c+dx))}{a^2 d} - \frac{2\sin(c+dx)}{a^2 d} + \frac{\sin^2(c+dx)}{2a^2 d}$$

Mathematica [A] time = 0.0397337, size = 36, normalized size = 0.77

$$\frac{\sin^2(c+dx) - 4\sin(c+dx) + 2\log(\sin(c+dx))}{2a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 4*Sin[c + d*x] + Sin[c + d*x]^2)/(2*a^2*d)
```

Maple [A] time = 0.094, size = 46, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{da^2} - 2\frac{\sin(dx+c)}{da^2} + \frac{(\sin(dx+c))^2}{2\,da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] ln(sin(d*x+c))/a^2/d-2*sin(d*x+c)/a^2/d+1/2*sin(d*x+c)^2/a^2/d
```

Maxima [A] time = 1.01191, size = 53, normalized size = 1.13

$$\frac{\frac{\sin(dx+c)^2 - 4\sin(dx+c)}{a^2} + \frac{2\log(\sin(dx+c))}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(d*x + c)^2 - 4*sin(d*x + c))/a^2 + 2*log(sin(d*x + c))/a^2)/d
```

Fricas [A] time = 1.10183, size = 100, normalized size = 2.13

$$-\frac{\cos(dx+c)^2 - 2\log\left(\frac{1}{2}\sin(dx+c)\right) + 4\sin(dx+c)}{2\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(cos(d*x + c)^2 - 2*log(1/2*sin(d*x + c)) + 4*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1984, size = 63, normalized size = 1.34

$$\frac{\frac{2 \log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^2} + \frac{a^2 \sin\left(dx+c\right)^2 - 4 a^2 \sin\left(dx+c\right)}{a^4}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c)))/a^2 + (a^2*sin(d*x + c)^2 - 4*a^2*sin(d*x + c
))/a^4)/d
```

3.548 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=43

$$\frac{\sin(c+dx)}{a^2d}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}$$

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*Log[Sin[c + d*x]])/(a^2*d) + Sin[c + d*x]/(a^2
*d)
```

Rubi [A] time = 0.0993012, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{\sin(c+dx)}{a^2d}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*Log[Sin[c + d*x]])/(a^2*d) + Sin[c + d*x]/(a^2
*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}-\frac{2a}{x}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= -\frac{\csc(c+dx)}{a^2d} - \frac{2\log(\sin(c+dx))}{a^2d} + \frac{\sin(c+dx)}{a^2d}$$

Mathematica [A] time = 0.0468369, size = 32, normalized size = 0.74

$$-\frac{-\sin(c+dx)+\csc(c+dx)+2\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Csc[c + d*x] + 2*Log[Sin[c + d*x]] - Sin[c + d*x])/(a^2*d))
```

Maple [A] time = 0.116, size = 46, normalized size = 1.1

$$\frac{\sin(dx+c)}{da^2} - \frac{1}{da^2\sin(dx+c)} - 2\frac{\ln(\sin(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] sin(d*x+c)/a^2/d-1/d/a^2/sin(d*x+c)-2*ln(sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.01414, size = 55, normalized size = 1.28

$$-\frac{\frac{2\log(\sin(dx+c))}{a^2} - \frac{\sin(dx+c)}{a^2} + \frac{1}{a^2\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -(2*log(sin(d*x + c))/a^2 - sin(d*x + c)/a^2 + 1/(a^2*sin(d*x + c)))/d
```

Fricas [A] time = 1.11527, size = 107, normalized size = 2.49

$$-\frac{\cos(dx+c)^2 + 2\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)}{a^2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(cos(d*x + c)^2 + 2*log(1/2*sin(d*x + c))*sin(d*x + c))/(a^2*d*sin(d*x + c
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26741, size = 72, normalized size = 1.67

$$-\frac{\frac{2\log\left(|\sin(dx+c)|\right)}{a^2} - \frac{\sin(dx+c)}{a^2} - \frac{2\sin(dx+c)-1}{a^2\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*log(abs(sin(d*x + c)))/a^2 - sin(d*x + c)/a^2 - (2*sin(d*x + c) - 1)/(a
^2*sin(d*x + c)))/d
```

3.549 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + Log[Sin[c + d*x]]/(a^
2*d)
```

Rubi [A] time = 0.100767, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + Log[Sin[c + d*x]]/(a^
2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}-\frac{2a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{2\csc(c+dx)}{a^2 d} - \frac{\csc^2(c+dx)}{2a^2 d} + \frac{\log(\sin(c+dx))}{a^2 d}$$

Mathematica [A] time = 0.0451609, size = 38, normalized size = 0.81

$$\frac{-\csc^2(c+dx)+4\csc(c+dx)+2\log(\sin(c+dx))}{2a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 2*Log[Sin[c + d*x]])/(2*a^2*d)
```

Maple [A] time = 0.123, size = 48, normalized size = 1.

$$2\,\frac{1}{da^2\sin(dx+c)} + \frac{\ln(\sin(dx+c))}{da^2} - \frac{1}{2\,da^2(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/sin(d*x+c)+ln(sin(d*x+c))/a^2/d-1/2/d/a^2/sin(d*x+c)^2
```

Maxima [A] time = 1.09743, size = 54, normalized size = 1.15

$$\frac{\frac{2\log(\sin(dx+c))}{a^2} + \frac{4\sin(dx+c)-1}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(2*log(sin(d*x + c))/a^2 + (4*sin(d*x + c) - 1)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.06619, size = 140, normalized size = 2.98

$$\frac{2\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 4\sin(dx+c)+1}{2\left(a^2 d\cos(dx+c)^2 - a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 4*sin(d*x + c) + 1)/(a^
2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27659, size = 70, normalized size = 1.49

$$\frac{\frac{2\log\left(|\sin\left(dx+c\right)|\right)}{a^2}-\frac{3\sin\left(dx+c\right)^2-4\sin\left(dx+c\right)+1}{a^2\sin\left(dx+c\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c)))/a^2 - (3*sin(d*x + c)^2 - 4*sin(d*x + c) + 1)
/(a^2*sin(d*x + c)^2))/d
```

3.550 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=31

$$-\frac{\csc^3(c+dx)(a-a\sin(c+dx))^3}{3a^5d}$$

```
[Out] -(Csc[c + d*x]^3*(a - a*Sin[c + d*x])^3)/(3*a^5*d)
```

Rubi [A] time = 0.079296, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 37}

$$-\frac{\csc^3(c+dx)(a-a\sin(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^3*(a - a*Sin[c + d*x])^3)/(3*a^5*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a^4(a-x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(a-x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= -\frac{\csc^3(c+dx)(a-a\sin(c+dx))^3}{3a^5d}
\end{aligned}$$

Mathematica [A] time = 0.0440705, size = 20, normalized size = 0.65

$$-\frac{(\csc(c+dx)-1)^3}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(-1 + Csc[c + d*x])^3/(3*a^2*d)
```

Maple [A] time = 0.132, size = 37, normalized size = 1.2

$$\frac{1}{da^2}\left(-(\sin(dx+c))^{-1}-\frac{1}{3(\sin(dx+c))^3}+(\sin(dx+c))^{-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/sin(d*x+c)-1/3/sin(d*x+c)^3+1/sin(d*x+c)^2)
```

Maxima [A] time = 1.05884, size = 49, normalized size = 1.58

$$-\frac{3\sin(dx+c)^2-3\sin(dx+c)+1}{3a^2d\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 - 3*sin(d*x + c) + 1)/(a^2*d*sin(d*x + c)^3)
```

Fricas [A] time = 1.00157, size = 124, normalized size = 4.

$$-\frac{3\cos(dx+c)^2+3\sin(dx+c)-4}{3\left(a^2d\cos(dx+c)^2-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 + 3*sin(d*x + c) - 4)/((a^2*d*cos(d*x + c)^2 - a^2*d
)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22028, size = 49, normalized size = 1.58

$$-\frac{3\,\sin(dx+c)^2-3\,\sin(dx+c)+1}{3\,a^2d\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 - 3*sin(d*x + c) + 1)/(a^2*d*sin(d*x + c)^3)
```

3.551 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(
4*a^2*d)
```

Rubi [A] time = 0.0460195, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(
4*a^2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx&=\frac{\operatorname{Subst}\left(\int \frac{(a-x)^2}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{\operatorname{Subst}\left(\int \left(\frac{a^2}{x^5}-\frac{2a}{x^4}+\frac{1}{x^3}\right)dx,x,a\sin(c+dx)\right)}{d}\\&=-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4a^2d}\end{aligned}$$

Mathematica [A] time = 0.0702285, size = 38, normalized size = 0.69

$$\frac{\csc^4(c+dx)(8\sin(c+dx)+3\cos(2(c+dx))-6)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^4*(-6 + 3*Cos[2*(c + d*x)] + 8*Sin[c + d*x]))/(12*a^2*d)
```

Maple [A] time = 0.139, size = 39, normalized size = 0.7

$$\frac{1}{da^2}\left(-\frac{1}{4\left(\sin\left(dx+c\right)\right)^4}+\frac{2}{3\left(\sin\left(dx+c\right)\right)^3}-\frac{1}{2\left(\sin\left(dx+c\right)\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/4/sin(d*x+c)^4+2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.05331, size = 49, normalized size = 0.89

$$-\frac{6\sin\left(dx+c\right)^2-8\sin\left(dx+c\right)+3}{12\,a^2d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

Fricas [A] time = 0.9948, size = 138, normalized size = 2.51

$$\frac{6\cos\left(dx+c\right)^2+8\sin\left(dx+c\right)-9}{12\left(a^2d\cos\left(dx+c\right)^4-2\,a^2d\cos\left(dx+c\right)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(6*cos(d*x + c)^2 + 8*sin(d*x + c) - 9)/(a^2*d*cos(d*x + c)^4 - 2*a^2*
d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19987, size = 49, normalized size = 0.89

$$-\frac{6 \sin\left(dx+c\right)^{2} - 8 \sin\left(dx+c\right) + 3}{12 a^{2} d \sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

3.552 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{2a^2d}-\frac{\csc^3(c+dx)}{3a^2d}$$

```
[Out] -Csc[c + d*x]^3/(3*a^2*d) + Csc[c + d*x]^4/(2*a^2*d) - Csc[c + d*x]^5/(5*a^
2*d)
```

Rubi [A] time = 0.0849527, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$-\frac{\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{2a^2d}-\frac{\csc^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^3/(3*a^2*d) + Csc[c + d*x]^4/(2*a^2*d) - Csc[c + d*x]^5/(5*a^
2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a^2}{x^6}-\frac{2a}{x^5}+\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3a^2d}+\frac{\csc^4(c+dx)}{2a^2d}-\frac{\csc^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.0707545, size = 38, normalized size = 0.69

$$\frac{\csc^5(c+dx)(15\sin(c+dx)+5\cos(2(c+dx))-11)}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-11 + 5*Cos[2*(c + d*x)] + 15*Sin[c + d*x]))/(30*a^2*d)
```

Maple [A] time = 0.146, size = 39, normalized size = 0.7

$$\frac{1}{da^2}\left(-\frac{1}{5\,(\sin(dx+c))^5}+\frac{1}{2\,(\sin(dx+c))^4}-\frac{1}{3\,(\sin(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/5/sin(d*x+c)^5+1/2/sin(d*x+c)^4-1/3/sin(d*x+c)^3)
```

Maxima [A] time = 1.00123, size = 49, normalized size = 0.89

$$-\frac{10\sin(dx+c)^2-15\sin(dx+c)+6}{30\,a^2d\sin(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*(10*sin(d*x + c)^2 - 15*sin(d*x + c) + 6)/(a^2*d*sin(d*x + c)^5)
```

Fricas [A] time = 1.04253, size = 162, normalized size = 2.95

$$\frac{10\cos(dx+c)^2+15\sin(dx+c)-16}{30\left(a^2d\cos(dx+c)^4-2\,a^2d\cos(dx+c)^2+a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/30*(10*cos(d*x + c)^2 + 15*sin(d*x + c) - 16)/((a^2*d*cos(d*x + c)^4 - 2*
a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20421, size = 49, normalized size = 0.89

$$-\frac{10 \sin(dx+c)^2 - 15 \sin(dx+c) + 6}{30 a^2 d \sin(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(10*sin(d*x + c)^2 - 15*sin(d*x + c) + 6)/(a^2*d*sin(d*x + c)^5)
```

3.553 $\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

```
[Out] -Csc[c + d*x]^4/(4*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(
6*a^2*d)
```

Rubi [A] time = 0.102601, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^4/(4*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(
6*a^2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(a-x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^2}{x^7} - \frac{2a}{x^6} + \frac{1}{x^5}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^4(c+dx)}{4a^2d} + \frac{2\csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{6a^2d}$$

Mathematica [A] time = 0.0739833, size = 38, normalized size = 0.69

$$-\frac{\csc^4(c+dx)\left(10\csc^2(c+dx) - 24\csc(c+dx) + 15\right)}{60a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]

[Out] -(Csc[c + d*x]^4*(15 - 24*Csc[c + d*x] + 10*Csc[c + d*x]^2))/(60*a^2*d)
```

Maple [A] time = 0.157, size = 39, normalized size = 0.7

$$\frac{1}{da^2}\left(\frac{2}{5\left(\sin\left(dx+c\right)\right)^5} - \frac{1}{4\left(\sin\left(dx+c\right)\right)^4} - \frac{1}{6\left(\sin\left(dx+c\right)\right)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x)

[Out] 1/d/a^2*(2/5/sin(d*x+c)^5-1/4/sin(d*x+c)^4-1/6/sin(d*x+c)^6)
```

Maxima [A] time = 1.10102, size = 49, normalized size = 0.89

$$-\frac{15\sin\left(dx+c\right)^2 - 24\sin\left(dx+c\right) + 10}{60\,a^2d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")

[Out] -1/60*(15*sin(d*x + c)^2 - 24*sin(d*x + c) + 10)/(a^2*d*sin(d*x + c)^6)
```

Fricas [A] time = 1.03921, size = 177, normalized size = 3.22

$$-\frac{15\cos\left(dx+c\right)^2 + 24\sin\left(dx+c\right) - 25}{60\left(a^2d\cos\left(dx+c\right)^6 - 3\,a^2d\cos\left(dx+c\right)^4 + 3\,a^2d\cos\left(dx+c\right)^2 - a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/60*(15*cos(d*x + c)^2 + 24*sin(d*x + c) - 25)/(a^2*d*cos(d*x + c)^6 - 3*
a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2168, size = 49, normalized size = 0.89

$$-\frac{15 \sin(dx+c)^2 - 24 \sin(dx+c) + 10}{60\, a^2 d \sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15*sin(d*x + c)^2 - 24*sin(d*x + c) + 10)/(a^2*d*sin(d*x + c)^6)
```

3.554 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{\sin^5(c+dx)}{5a^3d}-\frac{3\sin^4(c+dx)}{4a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}-\frac{2\sin^2(c+dx)}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d) - (2*Sin[c +
d*x]^2)/(a^3*d) + (4*Sin[c + d*x]^3)/(3*a^3*d) - (3*Sin[c + d*x]^4)/(4*a^3*
d) + Sin[c + d*x]^5/(5*a^3*d)
```

Rubi [A] time = 0.125569, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{\sin^5(c+dx)}{5a^3d}-\frac{3\sin^4(c+dx)}{4a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}-\frac{2\sin^2(c+dx)}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d) - (2*Sin[c +
d*x]^2)/(a^3*d) + (4*Sin[c + d*x]^3)/(3*a^3*d) - (3*Sin[c + d*x]^4)/(4*a^3*
d) + Sin[c + d*x]^5/(5*a^3*d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3}{a^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4-4a^3x+4a^2x^2-3ax^3+x^4-\frac{4a^5}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{2\sin^2(c+dx)}{a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}-\frac{3\sin^4(}{4a}$$

Mathematica [A] time = 0.962285, size = 71, normalized size = 0.7

$$\frac{192\sin^5(c+dx)-720\sin^4(c+dx)+1280\sin^3(c+dx)-1920\sin^2(c+dx)+3840\sin(c+dx)-3840\log(\sin(c+dx}{960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (45 - 3840*Log[1 + Sin[c + d*x]] + 3840*Sin[c + d*x] - 1920*Sin[c + d*x]^2
+ 1280*Sin[c + d*x]^3 - 720*Sin[c + d*x]^4 + 192*Sin[c + d*x]^5)/(960*a^3*d
)
```

Maple [A] time = 0.119, size = 97, normalized size = 1.

$$-4\,\frac{\ln(1+\sin(dx+c))}{a^3d}+4\,\frac{\sin(dx+c)}{a^3d}-2\,\frac{(\sin(dx+c))^2}{a^3d}+\frac{4\,(\sin(dx+c))^3}{3\,a^3d}-\frac{3\,(\sin(dx+c))^4}{4\,a^3d}+\frac{(\sin(dx+c))^5}{5\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d+4*sin(d*x+c)/a^3/d-2*sin(d*x+c)^2/a^3/d+4/3*sin(d
*x+c)^3/a^3/d-3/4*sin(d*x+c)^4/a^3/d+1/5*sin(d*x+c)^5/a^3/d
```

Maxima [A] time = 1.1279, size = 99, normalized size = 0.97

$$\frac{\frac{12\sin(dx+c)^5-45\sin(dx+c)^4+80\sin(dx+c)^3-120\sin(dx+c)^2+240\sin(dx+c)}{a^3}-\frac{240\log(\sin(dx+c)+1)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*((12*sin(d*x + c)^5 - 45*sin(d*x + c)^4 + 80*sin(d*x + c)^3 - 120*sin(
d*x + c)^2 + 240*sin(d*x + c))/a^3 - 240*log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.13642, size = 196, normalized size = 1.92

$$-\frac{45\cos(dx+c)^4 - 210\cos(dx+c)^2 - 4\left(3\cos(dx+c)^4 - 26\cos(dx+c)^2 + 83\right)\sin(dx+c) + 240\log(\sin(dx+c)+}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(45*cos(d*x + c)^4 - 210*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^4 - 26*co
s(d*x + c)^2 + 83)*sin(d*x + c) + 240*log(sin(d*x + c) + 1))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34583, size = 261, normalized size = 2.56

$$\frac{\frac{60\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3} - \frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{137\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10} - 120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 + 805\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8 - 640\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 1910}{}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15*(60*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 120*log(abs(tan(1/2*d*x + 1/
2*c) + 1))/a^3 - (137*tan(1/2*d*x + 1/2*c)^10 - 120*tan(1/2*d*x + 1/2*c)^9
+ 805*tan(1/2*d*x + 1/2*c)^8 - 640*tan(1/2*d*x + 1/2*c)^7 + 1910*tan(1/2*d*
x + 1/2*c)^6 - 1136*tan(1/2*d*x + 1/2*c)^5 + 1910*tan(1/2*d*x + 1/2*c)^4 -
640*tan(1/2*d*x + 1/2*c)^3 + 805*tan(1/2*d*x + 1/2*c)^2 - 120*tan(1/2*d*x +
 1/2*c) + 137)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^3))/d
```

3.555 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{\sin^4(c+dx)}{4a^3d}-\frac{\sin^3(c+dx)}{a^3d}+\frac{2\sin^2(c+dx)}{a^3d}-\frac{4\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] (4*Log[1 + Sin[c + d*x]])/(a^3*d) - (4*Sin[c + d*x])/(a^3*d) + (2*Sin[c + d
*x]^2)/(a^3*d) - Sin[c + d*x]^3/(a^3*d) + Sin[c + d*x]^4/(4*a^3*d)
```

Rubi [A] time = 0.118477, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{\sin^4(c+dx)}{4a^3d}-\frac{\sin^3(c+dx)}{a^3d}+\frac{2\sin^2(c+dx)}{a^3d}-\frac{4\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (4*Log[1 + Sin[c + d*x]])/(a^3*d) - (4*Sin[c + d*x])/(a^3*d) + (2*Sin[c + d
*x]^2)/(a^3*d) - Sin[c + d*x]^3/(a^3*d) + Sin[c + d*x]^4/(4*a^3*d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2}{a^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3+4a^2x-3ax^2+x^3+\frac{4a^4}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{4\log(1+\sin(c+dx))}{a^3d} - \frac{4\sin(c+dx)}{a^3d} + \frac{2\sin^2(c+dx)}{a^3d} - \frac{\sin^3(c+dx)}{a^3d} + \frac{\sin^4(c+dx)}{4a^3d}$$

Mathematica [A] time = 0.932708, size = 59, normalized size = 0.72

$$\frac{-152\sin(c+dx)+8\sin(3(c+dx))-36\cos(2(c+dx))+\cos(4(c+dx))+128\log(\sin(c+dx)+1)+35}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (35 - 36*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + 128*Log[1 + Sin[c + d*x]] -
152*Sin[c + d*x] + 8*Sin[3*(c + d*x)])/(32*a^3*d)
```

Maple [A] time = 0.102, size = 81, normalized size = 1.

$$4\,\frac{\ln(1+\sin(dx+c))}{a^3d} - 4\,\frac{\sin(dx+c)}{a^3d} + 2\,\frac{(\sin(dx+c))^2}{a^3d} - \frac{(\sin(dx+c))^3}{a^3d} + \frac{(\sin(dx+c))^4}{4\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-4*sin(d*x+c)/a^3/d+2*sin(d*x+c)^2/a^3/d-sin(d*x+c)
^3/a^3/d+1/4*sin(d*x+c)^4/a^3/d
```

Maxima [A] time = 1.07935, size = 82, normalized size = 1.

$$\frac{\frac{\sin(dx+c)^4-4\sin(dx+c)^3+8\sin(dx+c)^2-16\sin(dx+c)}{a^3}+\frac{16\log(\sin(dx+c)+1)}{a^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 8*sin(d*x + c)^2 - 16*sin(d*x + c
))/a^3 + 16*log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.1258, size = 155, normalized size = 1.89

$$\frac{\cos(dx+c)^4 - 10\cos(dx+c)^2 + 4\left(\cos(dx+c)^2 - 5\right)\sin(dx+c) + 16\log(\sin(dx+c)+1)}{4a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 5)*sin(d*x +
c) + 16*log(sin(d*x + c) + 1))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32593, size = 225, normalized size = 2.74

$$-\frac{\frac{12\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3} - \frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8 - 24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 124\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 96\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 210}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(12*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 24*log(abs(tan(1/2*d*x + 1/2
*c) + 1))/a^3 - (25*tan(1/2*d*x + 1/2*c)^8 - 24*tan(1/2*d*x + 1/2*c)^7 + 12
4*tan(1/2*d*x + 1/2*c)^6 - 96*tan(1/2*d*x + 1/2*c)^5 + 210*tan(1/2*d*x + 1/
2*c)^4 - 96*tan(1/2*d*x + 1/2*c)^3 + 124*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/
2*d*x + 1/2*c) + 25)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

3.556 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=68

$$\frac{\sin^3(c+dx)}{3a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d) - (3*Sin[c +
d*x]^2)/(2*a^3*d) + Sin[c + d*x]^3/(3*a^3*d)
```

Rubi [A] time = 0.0763898, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 77}

$$\frac{\sin^3(c+dx)}{3a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d) - (3*Sin[c +
d*x]^2)/(2*a^3*d) + Sin[c + d*x]^3/(3*a^3*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x}{a(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2x}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2-3ax+x^2-\frac{4a^3}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{\sin^3(c+dx)}{3a^3d}$$

Mathematica [A] time = 0.341803, size = 51, normalized size = 0.75

$$\frac{32\sin^3(c+dx)-144\sin^2(c+dx)+384\sin(c+dx)-384\log(\sin(c+dx)+1)+15}{96a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (15 - 384*Log[1 + Sin[c + d*x]] + 384*Sin[c + d*x] - 144*Sin[c + d*x]^2 + 3
2*Sin[c + d*x]^3)/(96*a^3*d)
```

Maple [A] time = 0.084, size = 65, normalized size = 1.

$$-4\,\frac{\ln(1+\sin(dx+c))}{a^3d}+4\,\frac{\sin(dx+c)}{a^3d}-\frac{3\,(\sin(dx+c))^2}{2\,a^3d}+\frac{(\sin(dx+c))^3}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d+4*sin(d*x+c)/a^3/d-3/2*sin(d*x+c)^2/a^3/d+1/3*sin
(d*x+c)^3/a^3/d
```

Maxima [A] time = 1.0209, size = 72, normalized size = 1.06

$$\frac{\frac{2\sin(dx+c)^3-9\sin(dx+c)^2+24\sin(dx+c)}{a^3}-\frac{24\log(\sin(dx+c)+1)}{a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*((2*sin(d*x + c)^3 - 9*sin(d*x + c)^2 + 24*sin(d*x + c))/a^3 - 24*log(s
in(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.0833, size = 132, normalized size = 1.94

$$\frac{9 \cos(dx+c)^2 - 2\left(\cos(dx+c)^2 - 13\right)\sin(dx+c) - 24 \log(\sin(dx+c)+1)}{6a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(9*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 13)*sin(d*x + c) - 24*log(sin(d
*x + c) + 1))/(a^3*d)
```

Sympy [A] time = 129.94, size = 1243, normalized size = 18.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-480*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**6/(60*a**3*d*ta
n(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d
*x/2)**2 + 60*a**3*d) - 1440*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/
(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*
d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 1440*log(tan(c/2 + d*x/2) + 1)*tan(c/2
 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**
4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 480*log(tan(c/2 + d*x/2)
+ 1)/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*
a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 240*log(tan(c/2 + d*x/2)**2 + 1)*
tan(c/2 + d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d
*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 720*log(tan(c/2 +
d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**
3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 720
*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/
2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 6
0*a**3*d) + 240*log(tan(c/2 + d*x/2)**2 + 1)/(60*a**3*d*tan(c/2 + d*x/2)**6
 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**
3*d) - 157*tan(c/2 + d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*
tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 480*tan
(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/
2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 831*tan(c/2 + d*x/2)*
*4/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a*
*3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 1120*tan(c/2 + d*x/2)**3/(60*a**3*d
*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2
+ d*x/2)**2 + 60*a**3*d) - 831*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x
/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 +
60*a**3*d) + 480*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3
*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 157/
(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*
d*tan(c/2 + d*x/2)**2 + 60*a**3*d), Ne(d, 0)), (x*sin(c)*cos(c)**5/(a*sin(c
) + a)**3, True))
```

Giac [B] time = 1.33103, size = 190, normalized size = 2.79

$$\frac{2\left(\frac{6\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3}-\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{11\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-28\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+42\tan\ldots}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a^3}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(6*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 12*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^3 - (11*tan(1/2*d*x + 1/2*c)^6 - 12*tan(1/2*d*x + 1/2*c)^5 + 42*t
an(1/2*d*x + 1/2*c)^4 - 28*tan(1/2*d*x + 1/2*c)^3 + 42*tan(1/2*d*x + 1/2*c)
^2 - 12*tan(1/2*d*x + 1/2*c) + 11)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

3.557 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=45

$$\frac{\sin(c+dx)}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d) + Sin[c + d*x
]/(a^3*d)
```

Rubi [A] time = 0.08554, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 72}

$$\frac{\sin(c+dx)}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d) + Sin[c + d*x
]/(a^3*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{a(a-x)^2}{x(a+x)}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x(a+x)}\,dx,x,a\sin(c+dx)\right)}{a^4d}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}-\frac{4a}{a+x}\right)dx,x,a\sin(c+dx)\right)}{a^4d}\\
&= \frac{\log(\sin(c+dx))}{a^3d}-\frac{4\log(1+\sin(c+dx))}{a^3d}+\frac{\sin(c+dx)}{a^3d}
\end{aligned}$$

Mathematica [A] time = 0.0378115, size = 32, normalized size = 0.71

$$\frac{\sin(c+dx)+\log(\sin(c+dx))-4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Log[Sin[c + d*x]] - 4*Log[1 + Sin[c + d*x]] + Sin[c + d*x])/(a^3*d)
```

Maple [A] time = 0.124, size = 46, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{da^3}-4\,\frac{\ln(1+\sin(dx+c))}{da^3}+\frac{\sin(dx+c)}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d+sin(d*x+c)/a^3/d
```

Maxima [A] time = 0.982692, size = 58, normalized size = 1.29

$$-\frac{\frac{4\log(\sin(dx+c)+1)}{a^3}-\frac{\log(\sin(dx+c))}{a^3}-\frac{\sin(dx+c)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*log(sin(d*x + c) + 1)/a^3 - log(sin(d*x + c))/a^3 - sin(d*x + c)/a^3)/d
```

Fricas [A] time = 1.09148, size = 100, normalized size = 2.22

$$\frac{\log\left(\frac{1}{2}\sin(dx+c)\right)-4\log(\sin(dx+c)+1)+\sin(dx+c)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (log(1/2*sin(d*x + c)) - 4*log(sin(d*x + c) + 1) + sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34512, size = 139, normalized size = 3.09

$$\frac{\frac{3 \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+1\right)}{a^3}-\frac{8 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^3}+\frac{\log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^3}-\frac{3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+3}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+1\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 8*log(abs(tan(1/2*d*x + 1/2*c) + 1
))/a^3 + log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (3*tan(1/2*d*x + 1/2*c)^2 - 2
*tan(1/2*d*x + 1/2*c) + 3)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

3.558 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=47

$$-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] -(Csc[c + d*x]/(a^3*d)) - (3*Log[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c
+ d*x]])/(a^3*d)
```

Rubi [A] time = 0.105745, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]/(a^3*d)) - (3*Log[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c
+ d*x]])/(a^3*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{3}{x}+\frac{4}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{\csc(c+dx)}{a^3 d} - \frac{3\log(\sin(c+dx))}{a^3 d} + \frac{4\log(1+\sin(c+dx))}{a^3 d}$$

Mathematica [A] time = 0.050403, size = 35, normalized size = 0.74

$$-\frac{\csc(c+dx)+3\log(\sin(c+dx))-4\log(\sin(c+dx)+1)}{a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((Csc[c + d*x] + 3*Log[Sin[c + d*x]] - 4*Log[1 + Sin[c + d*x]])/(a^3*d))
```

Maple [A] time = 0.139, size = 50, normalized size = 1.1

$$4\,\frac{\ln(1+\sin(dx+c))}{da^3} - \frac{1}{da^3\sin(dx+c)} - 3\,\frac{\ln(\sin(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-1/d/a^3/sin(d*x+c)-3*ln(sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.10166, size = 59, normalized size = 1.26

$$\frac{\frac{4\log(\sin(dx+c)+1)}{a^3} - \frac{3\log(\sin(dx+c))}{a^3} - \frac{1}{a^3\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] (4*log(sin(d*x + c) + 1)/a^3 - 3*log(sin(d*x + c))/a^3 - 1/(a^3*sin(d*x + c
)))/d
```

Fricas [A] time = 1.14028, size = 142, normalized size = 3.02

$$-\frac{3\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c) - 4\log(\sin(dx+c)+1)\sin(dx+c) + 1}{a^3 d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -(3*log(1/2*sin(d*x + c))*sin(d*x + c) - 4*log(sin(d*x + c) + 1)*sin(d*x +
c) + 1)/(a^3*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27088, size = 136, normalized size = 2.89

$$-\frac{\frac{2\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)}{a^{3}}-\frac{16\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{3}}+\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{3}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{3}}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 16*log(abs(tan(1/2*d*x + 1/2*
c) + 1))/a^3 + 6*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + tan(1/2*d*x + 1/2*c)/
a^3 - (6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)))/d
```

3.559 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=65

$$-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (4*Log[Sin[c + d*x]])
/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.113156, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (4*Log[Sin[c + d*x]])
/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^3}-\frac{3}{x^2}+\frac{4}{ax}-\frac{4}{a(a+x)}\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{3\csc(c+dx)}{a^3 d} - \frac{\csc^2(c+dx)}{2a^3 d} + \frac{4\log(\sin(c+dx))}{a^3 d} - \frac{4\log(1+\sin(c+dx))}{a^3 d}$$

Mathematica [A] time = 0.0819546, size = 49, normalized size = 0.75

$$\frac{-\csc^2(c+dx)+6\csc(c+dx)+8\log(\sin(c+dx))-8\log(\sin(c+dx)+1)}{2a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 8*Log[Sin[c + d*x]] - 8*Log[1 + Sin[c +
d*x]])/(2*a^3*d)
```

Maple [A] time = 0.155, size = 66, normalized size = 1.

$$-4\,\frac{\ln(1+\sin(dx+c))}{da^3} - \frac{1}{2\,da^3(\sin(dx+c))^2} + 3\,\frac{1}{da^3\sin(dx+c)} + 4\,\frac{\ln(\sin(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d-1/2/d/a^3/sin(d*x+c)^2+3/d/a^3/sin(d*x+c)+4*ln(si
n(d*x+c))/a^3/d
```

Maxima [A] time = 1.10113, size = 74, normalized size = 1.14

$$-\frac{\frac{8\log(\sin(dx+c)+1)}{a^3} - \frac{8\log(\sin(dx+c))}{a^3} - \frac{6\sin(dx+c)-1}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(8*log(sin(d*x + c) + 1)/a^3 - 8*log(sin(d*x + c))/a^3 - (6*sin(d*x +
c) - 1)/(a^3*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.12308, size = 204, normalized size = 3.14

$$\frac{8\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-8\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\sin\left(dx+c\right)+1\right)-6\sin\left(dx+c\right)+1}{2\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(8*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 8*(cos(d*x + c)^2 - 1)*
log(sin(d*x + c) + 1) - 6*sin(d*x + c) + 1)/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21341, size = 155, normalized size = 2.38

$$-\frac{\frac{64\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{3}}-\frac{32\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{3}}+\frac{48\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}+\frac{a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-12a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{6}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(64*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 32*log(abs(tan(1/2*d*x +
1/2*c)))/a^3 + (48*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a
^3*tan(1/2*d*x + 1/2*c)^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d
*x + 1/2*c))/a^6)/d
```

3.560 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] (-4*Csc[c + d*x])/(a^3*d) + (3*Csc[c + d*x]^2)/(2*a^3*d) - Csc[c + d*x]^3/(
3*a^3*d) - (4*Log[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c + d*x]])/(a^3*d
)
```

Rubi [A] time = 0.10044, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Csc[c + d*x])/(a^3*d) + (3*Csc[c + d*x]^2)/(2*a^3*d) - Csc[c + d*x]^3/(
3*a^3*d) - (4*Log[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c + d*x]])/(a^3*d
)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{a^4(a-x)^2}{x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(a-x)^2}{x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^4}-\frac{3}{x^3}+\frac{4}{ax^2}-\frac{4}{a^2x}+\frac{4}{a^2(a+x)}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{4\csc(c+dx)}{a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{\csc^3(c+dx)}{3a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(1+\sin(c}{a^3d}$$

Mathematica [A] time = 0.115998, size = 59, normalized size = 0.71

$$-\frac{2\csc^3(c+dx)-9\csc^2(c+dx)+24\csc(c+dx)+24\log(\sin(c+dx))-24\log(\sin(c+dx)+1)}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(24*Csc[c + d*x] - 9*Csc[c + d*x]^2 + 2*Csc[c + d*x]^3 + 24*Log[Sin[c + d*
x]] - 24*Log[1 + Sin[c + d*x]])/(6*a^3*d)
```

Maple [A] time = 0.161, size = 82, normalized size = 1.

$$4\,\frac{\ln(1+\sin(dx+c))}{da^3}-\frac{1}{3\,da^3(\sin(dx+c))^3}+\frac{3}{2\,da^3(\sin(dx+c))^2}-4\,\frac{1}{da^3\sin(dx+c)}-4\,\frac{\ln(\sin(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-1/3/d/a^3/sin(d*x+c)^3+3/2/d/a^3/sin(d*x+c)^2-4/d/
a^3/sin(d*x+c)-4*ln(sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.04906, size = 88, normalized size = 1.06

$$\frac{\frac{24\log(\sin(dx+c)+1)}{a^3}-\frac{24\log(\sin(dx+c))}{a^3}-\frac{24\sin(dx+c)^2-9\sin(dx+c)+2}{a^3\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*(24*log(sin(d*x + c) + 1)/a^3 - 24*log(sin(d*x + c))/a^3 - (24*sin(d*x
+ c)^2 - 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.12039, size = 292, normalized size = 3.52

$$-\frac{24\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-24\left(\cos(dx+c)^2-1\right)\log(\sin(dx+c)+1)\sin(dx+c)+24}{6\left(a^3d\cos(dx+c)^2-a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(24*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*sin(d*x + c) - 24*(cos(
d*x + c)^2 - 1)*log(sin(d*x + c) + 1)*sin(d*x + c) + 24*cos(d*x + c)^2 + 9*
sin(d*x + c) - 26)/((a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26211, size = 196, normalized size = 2.36

$$\frac{\frac{192\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{96\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{176\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-51\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9a^6\mathrm{t}}{}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(192*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 96*log(abs(tan(1/2*d*x +
 1/2*c)))/a^3 + (176*tan(1/2*d*x + 1/2*c)^3 - 51*tan(1/2*d*x + 1/2*c)^2 + 9
*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (a^6*tan(1/2*d*x
+ 1/2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 + 51*a^6*tan(1/2*d*x + 1/2*c))/a^
9)/d
```

3.561 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=96

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3
*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 +
 Sin[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.0677812, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3
*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 +
 Sin[c + d*x]])/(a^3*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{x^5}-\frac{3}{x^4}+\frac{4}{ax^3}-\frac{4}{a^2x^2}+\frac{4}{a^3x}-\frac{4}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{4\csc(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{\csc^4(c+dx)}{4a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(1
\end{aligned}$$

Mathematica [A] time = 0.318515, size = 69, normalized size = 0.72

$$\frac{-\csc^4(c+dx)+4\csc^3(c+dx)-8\csc^2(c+dx)+16\csc(c+dx)+16\log(\sin(c+dx))-16\log(\sin(c+dx)+1)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (16*Csc[c + d*x] - 8*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 - Csc[c + d*x]^4 + 1
6*Log[Sin[c + d*x]] - 16*Log[1 + Sin[c + d*x]])/(4*a^3*d)
```

Maple [A] time = 0.181, size = 97, normalized size = 1.

$$-4\,\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{da^{3}}-\frac{1}{4\,da^{3}\left(\sin\left(dx+c\right)\right)^{4}}+\frac{1}{da^{3}\left(\sin\left(dx+c\right)\right)^{3}}-2\,\frac{1}{da^{3}\left(\sin\left(dx+c\right)\right)^{2}}+4\,\frac{1}{da^{3}\sin\left(dx+c\right)}+4\,\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d-1/4/d/a^3/sin(d*x+c)^4+1/d/a^3/sin(d*x+c)^3-2/d/a
^3/sin(d*x+c)^2+4/d/a^3/sin(d*x+c)+4*ln(sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.19002, size = 101, normalized size = 1.05

$$-\frac{\frac{16\,\log(\sin(dx+c)+1)}{a^{3}}-\frac{16\,\log(\sin(dx+c))}{a^{3}}-\frac{16\,\sin(dx+c)^{3}-8\,\sin(dx+c)^{2}+4\,\sin(dx+c)-1}{a^{3}\sin(dx+c)^{4}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(16*log(sin(d*x + c) + 1)/a^3 - 16*log(sin(d*x + c))/a^3 - (16*sin(d*x
 + c)^3 - 8*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^4))/d
```

Fricas [A] time = 1.13131, size = 348, normalized size = 3.62

$$\frac{8\,\cos\left(dx+c\right)^{2}+16\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-16\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right.}{4\left(a^{3}d\cos\left(dx+c\right)^{4}-2\,a^{3}d\cos\left(dx+c\right)^{2}+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(8*cos(d*x + c)^2 + 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*
sin(d*x + c)) - 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(sin(d*x + c)
 + 1) - 4*(4*cos(d*x + c)^2 - 5)*sin(d*x + c) - 9)/(a^3*d*cos(d*x + c)^4 -
2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25489, size = 235, normalized size = 2.45

$$-\frac{\frac{1536 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{768 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{1600 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-456 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+108 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}+}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/192*(1536*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 768*log(abs(tan(1/2*d
*x + 1/2*c)))/a^3 + (1600*tan(1/2*d*x + 1/2*c)^4 - 456*tan(1/2*d*x + 1/2*c)
^3 + 108*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2
*d*x + 1/2*c)^4) + 3*(a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*
c)^3 + 36*a^9*tan(1/2*d*x + 1/2*c)^2 - 152*a^9*tan(1/2*d*x + 1/2*c))/a^12)/
d
```

3.562 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$-\frac{\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{2\csc^2(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c}{a^3}$$

```
[Out] (-4*Csc[c + d*x])/(a^3*d) + (2*Csc[c + d*x]^2)/(a^3*d) - (4*Csc[c + d*x]^3)
/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - Csc[c + d*x]^5/(5*a^3*d) - (4*L
og[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.119461, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{2\csc^2(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-4*Csc[c + d*x])/(a^3*d) + (2*Csc[c + d*x]^2)/(a^3*d) - (4*Csc[c + d*x]^3)
/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - Csc[c + d*x]^5/(5*a^3*d) - (4*L
og[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c + d*x]])/(a^3*d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2}{x^6(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2}{x^6(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a}{x^6} - \frac{3}{x^5} + \frac{4}{ax^4} - \frac{4}{a^2x^3} + \frac{4}{a^3x^2} - \frac{4}{a^4x} + \frac{4}{a^4(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{4\csc(c+dx)}{a^3d} + \frac{2\csc^2(c+dx)}{a^3d} - \frac{4\csc^3(c+dx)}{3a^3d} + \frac{3\csc^4(c+dx)}{4a^3d} - \frac{\csc^5(c+dx)}{5a^3d} - 4\,\text{l}$$

Mathematica [A] time = 0.126769, size = 79, normalized size = 0.68

$$-\frac{12\csc^5(c+dx) - 45\csc^4(c+dx) + 80\csc^3(c+dx) - 120\csc^2(c+dx) + 240\csc(c+dx) + 240\log(\sin(c+dx)) - 240\,\text{lo}}{60a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(240*Csc[c + d*x] - 120*Csc[c + d*x]^2 + 80*Csc[c + d*x]^3 - 45*Csc[c + d*
x]^4 + 12*Csc[c + d*x]^5 + 240*Log[Sin[c + d*x]] - 240*Log[1 + Sin[c + d*x]
])/(60*a^3*d)
```

Maple [A] time = 0.184, size = 114, normalized size = 1.

$$4\frac{\ln(1+\sin(dx+c))}{da^3} - \frac{1}{5da^3(\sin(dx+c))^5} + \frac{3}{4da^3(\sin(dx+c))^4} - \frac{4}{3da^3(\sin(dx+c))^3} + 2\frac{1}{da^3(\sin(dx+c))^2} - 4\frac{}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-1/5/d/a^3/sin(d*x+c)^5+3/4/d/a^3/sin(d*x+c)^4-4/3/
d/a^3/sin(d*x+c)^3+2/d/a^3/sin(d*x+c)^2-4/d/a^3/sin(d*x+c)-4*ln(sin(d*x+c))
/a^3/d
```

Maxima [A] time = 1.07531, size = 115, normalized size = 0.98

$$\frac{\frac{240\log(\sin(dx+c)+1)}{a^3} - \frac{240\log(\sin(dx+c))}{a^3} - \frac{240\sin(dx+c)^4 - 120\sin(dx+c)^3 + 80\sin(dx+c)^2 - 45\sin(dx+c) + 12}{a^3\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(240*log(sin(d*x + c) + 1)/a^3 - 240*log(sin(d*x + c))/a^3 - (240*sin(
d*x + c)^4 - 120*sin(d*x + c)^3 + 80*sin(d*x + c)^2 - 45*sin(d*x + c) + 12)
/(a^3*sin(d*x + c)^5))/d
```

Fricas [A] time = 1.19092, size = 446, normalized size = 3.81

$$-\frac{240\cos(dx+c)^4+240\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-240\left(\cos(dx+c)^4-\right.}{60\left(a^3d\cos(dx+c)^4-2a^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(240*cos(d*x + c)^4 + 240*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log
(1/2*sin(d*x + c))*sin(d*x + c) - 240*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 +
1)*log(sin(d*x + c) + 1)*sin(d*x + c) - 560*cos(d*x + c)^2 + 15*(8*cos(d*x
+ c)^2 - 11)*sin(d*x + c) + 332)/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x +
 c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33299, size = 275, normalized size = 2.35

$$\frac{\frac{7680\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{3840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{8768\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-2460\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+660\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-190\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}}{960a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(7680*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 3840*log(abs(tan(1/2*d
*x + 1/2*c)))/a^3 + (8768*tan(1/2*d*x + 1/2*c)^5 - 2460*tan(1/2*d*x + 1/2*c
)^4 + 660*tan(1/2*d*x + 1/2*c)^3 - 190*tan(1/2*d*x + 1/2*c)^2 + 45*tan(1/2*
d*x + 1/2*c) - 6)/(a^3*tan(1/2*d*x + 1/2*c)^5) - (6*a^12*tan(1/2*d*x + 1/2*
c)^5 - 45*a^12*tan(1/2*d*x + 1/2*c)^4 + 190*a^12*tan(1/2*d*x + 1/2*c)^3 - 6
60*a^12*tan(1/2*d*x + 1/2*c)^2 + 2460*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.563 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{4}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^4(c+dx)}{4a^4d}+\frac{4\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{12\csc(c+dx)}{a^4d}+\frac{16\log(\sin(c+dx))}{a^4d}-\frac{16\log}{}$$

```
[Out] (12*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (4*Csc[c + d*x]^3)
/(3*a^4*d) - Csc[c + d*x]^4/(4*a^4*d) + (16*Log[Sin[c + d*x]])/(a^4*d) - (1
6*Log[1 + Sin[c + d*x]])/(a^4*d) + 4/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.0826289, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{4}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^4(c+dx)}{4a^4d}+\frac{4\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{12\csc(c+dx)}{a^4d}+\frac{16\log(\sin(c+dx))}{a^4d}-\frac{16\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (12*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (4*Csc[c + d*x]^3)
/(3*a^4*d) - Csc[c + d*x]^4/(4*a^4*d) + (16*Log[Sin[c + d*x]])/(a^4*d) - (1
6*Log[1 + Sin[c + d*x]])/(a^4*d) + 4/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x^5}-\frac{4}{ax^4}+\frac{8}{a^2x^3}-\frac{12}{a^3x^2}+\frac{16}{a^4x}-\frac{4}{a^3(a+x)^2}-\frac{16}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{12\csc(c+dx)}{a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{4\csc^3(c+dx)}{3a^4d}-\frac{\csc^4(c+dx)}{4a^4d}+\frac{16\log(\sin(c+dx))}{a^4d}-16\end{aligned}$$

Mathematica [A] time = 0.729173, size = 81, normalized size = 0.68

$$\frac{\frac{48}{\sin(c+dx)+1}-3\csc^4(c+dx)+16\csc^3(c+dx)-48\csc^2(c+dx)+144\csc(c+dx)+192\log(\sin(c+dx))-192\log(\sin(c}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (144*Csc[c + d*x] - 48*Csc[c + d*x]^2 + 16*Csc[c + d*x]^3 - 3*Csc[c + d*x]^
4 + 192*Log[Sin[c + d*x]] - 192*Log[1 + Sin[c + d*x]] + 48/(1 + Sin[c + d*x
]))/(12*a^4*d)
```

Maple [A] time = 0.197, size = 116, normalized size = 1.

$$4\,\frac{1}{da^4\left(1+\sin\left(dx+c\right)\right)}-16\,\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{da^4}-\frac{1}{4\,da^4\left(\sin\left(dx+c\right)\right)^4}+\frac{4}{3\,da^4\left(\sin\left(dx+c\right)\right)^3}-4\,\frac{1}{da^4\left(\sin\left(dx+c\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 4/d/a^4/(1+sin(d*x+c))-16*ln(1+sin(d*x+c))/a^4/d-1/4/d/a^4/sin(d*x+c)^4+4/3
/d/a^4/sin(d*x+c)^3-4/d/a^4/sin(d*x+c)^2+12/d/a^4/sin(d*x+c)+16*ln(sin(d*x+
c))/a^4/d
```

Maxima [A] time = 1.0906, size = 135, normalized size = 1.12

$$\frac{\frac{192\sin(dx+c)^4+96\sin(dx+c)^3-32\sin(dx+c)^2+13\sin(dx+c)-3}{a^4\sin(dx+c)^5+a^4\sin(dx+c)^4}-\frac{192\log(\sin(dx+c)+1)}{a^4}+\frac{192\log(\sin(dx+c))}{a^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/12*((192*sin(d*x + c)^4 + 96*sin(d*x + c)^3 - 32*sin(d*x + c)^2 + 13*sin(
d*x + c) - 3)/(a^4*sin(d*x + c)^5 + a^4*sin(d*x + c)^4) - 192*log(sin(d*x +
 c) + 1)/a^4 + 192*log(sin(d*x + c))/a^4)/d
```

Fricas [B] time = 1.19727, size = 632, normalized size = 5.27

$$\frac{192\,\cos\left(dx+c\right)^4-352\,\cos\left(dx+c\right)^2+192\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1\right)\sin}{12\left(a^4d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/12*(192*cos(d*x + c)^4 - 352*cos(d*x + c)^2 + 192*(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + 1)*log
(1/2*sin(d*x + c)) - 192*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + (cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (96*co
s(d*x + c)^2 - 109)*sin(d*x + c) + 157)/(a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos
```

```
(d*x + c)^2 + a^4*d + (a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^2 + a^4*
d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34919, size = 294, normalized size = 2.45

$$-\frac{\frac{6144\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{3072\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4} - \frac{1536\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+11\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^2} + \frac{6400\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1248\tan\left(\frac{1}{2}dx\cdots\right.}{a^4\cdots}}{192\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/192*(6144*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3072*log(abs(tan(1/2*
d*x + 1/2*c)))/a^4 - 1536*(6*tan(1/2*d*x + 1/2*c)^2 + 11*tan(1/2*d*x + 1/2*
c) + 6)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^2) + (6400*tan(1/2*d*x + 1/2*c)^4 -
 1248*tan(1/2*d*x + 1/2*c)^3 + 204*tan(1/2*d*x + 1/2*c)^2 - 32*tan(1/2*d*x
+ 1/2*c) + 3)/(a^4*tan(1/2*d*x + 1/2*c)^4) + (3*a^12*tan(1/2*d*x + 1/2*c)^4
 - 32*a^12*tan(1/2*d*x + 1/2*c)^3 + 204*a^12*tan(1/2*d*x + 1/2*c)^2 - 1248*
a^12*tan(1/2*d*x + 1/2*c))/a^16)/d
```

3.564 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=135

$$-\frac{4}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^5(c+dx)}{5a^4d}+\frac{\csc^4(c+dx)}{a^4d}-\frac{8\csc^3(c+dx)}{3a^4d}+\frac{6\csc^2(c+dx)}{a^4d}-\frac{16\csc(c+dx)}{a^4d}-\frac{20\log(s$$

```
[Out] (-16*Csc[c + d*x])/(a^4*d) + (6*Csc[c + d*x]^2)/(a^4*d) - (8*Csc[c + d*x]^3
)/(3*a^4*d) + Csc[c + d*x]^4/(a^4*d) - Csc[c + d*x]^5/(5*a^4*d) - (20*Log[S
in[c + d*x]])/(a^4*d) + (20*Log[1 + Sin[c + d*x]])/(a^4*d) - 4/(d*(a^4 + a^
4*Sin[c + d*x]))
```

Rubi [A] time = 0.132366, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{4}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^5(c+dx)}{5a^4d}+\frac{\csc^4(c+dx)}{a^4d}-\frac{8\csc^3(c+dx)}{3a^4d}+\frac{6\csc^2(c+dx)}{a^4d}-\frac{16\csc(c+dx)}{a^4d}-\frac{20\log(s$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (-16*Csc[c + d*x])/(a^4*d) + (6*Csc[c + d*x]^2)/(a^4*d) - (8*Csc[c + d*x]^3
)/(3*a^4*d) + Csc[c + d*x]^4/(a^4*d) - Csc[c + d*x]^5/(5*a^4*d) - (20*Log[S
in[c + d*x]])/(a^4*d) + (20*Log[1 + Sin[c + d*x]])/(a^4*d) - 4/(d*(a^4 + a^
4*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2}{x^6(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^2}{x^6(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{x^6}-\frac{4}{ax^5}+\frac{8}{a^2x^4}-\frac{12}{a^3x^3}+\frac{16}{a^4x^2}-\frac{20}{a^5x}+\frac{4}{a^4(a+x)^2}+\frac{20}{a^5(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{16\csc(c+dx)}{a^4d}+\frac{6\csc^2(c+dx)}{a^4d}-\frac{8\csc^3(c+dx)}{3a^4d}+\frac{\csc^4(c+dx)}{a^4d}-\frac{\csc^5(c+dx)}{5a^4d}-\frac{20}{}$$

Mathematica [A] time = 0.289052, size = 91, normalized size = 0.67

$$-\frac{\frac{60}{\sin(c+dx)+1}+3\csc^5(c+dx)-15\csc^4(c+dx)+40\csc^3(c+dx)-90\csc^2(c+dx)+240\csc(c+dx)+300\log(\sin(c+d}{15a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(240*Csc[c + d*x] - 90*Csc[c + d*x]^2 + 40*Csc[c + d*x]^3 - 15*Csc[c + d*x
]^4 + 3*Csc[c + d*x]^5 + 300*Log[Sin[c + d*x]] - 300*Log[1 + Sin[c + d*x]]
+ 60/(1 + Sin[c + d*x]))/(15*a^4*d)
```

Maple [A] time = 0.208, size = 131, normalized size = 1.

$$-4\,\frac{1}{da^4\,(1+\sin(dx+c))}+20\,\frac{\ln(1+\sin(dx+c))}{da^4}-\frac{1}{5\,da^4\,(\sin(dx+c))^5}+\frac{1}{da^4\,(\sin(dx+c))^4}-\frac{8}{3\,da^4\,(\sin(dx+c))^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -4/d/a^4/(1+sin(d*x+c))+20*ln(1+sin(d*x+c))/a^4/d-1/5/d/a^4/sin(d*x+c)^5+1/
d/a^4/sin(d*x+c)^4-8/3/d/a^4/sin(d*x+c)^3+6/d/a^4/sin(d*x+c)^2-16/d/a^4/sin
(d*x+c)-20*ln(sin(d*x+c))/a^4/d
```

Maxima [A] time = 1.2236, size = 149, normalized size = 1.1

$$-\frac{\frac{300\sin(dx+c)^5+150\sin(dx+c)^4-50\sin(dx+c)^3+25\sin(dx+c)^2-12\sin(dx+c)+3}{a^4\sin(dx+c)^6+a^4\sin(dx+c)^5}-\frac{300\log(\sin(dx+c)+1)}{a^4}+\frac{300\log(\sin(dx+c))}{a^4}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] -1/15*((300*sin(d*x + c)^5 + 150*sin(d*x + c)^4 - 50*sin(d*x + c)^3 + 25*si
n(d*x + c)^2 - 12*sin(d*x + c) + 3)/(a^4*sin(d*x + c)^6 + a^4*sin(d*x + c)^
```

```
5) - 300*log(sin(d*x + c) + 1)/a^4 + 300*log(sin(d*x + c))/a^4)/d
```

Fricas [B] time = 1.18772, size = 749, normalized size = 5.55

$$\frac{150 \cos(dx+c)^4 - 325 \cos(dx+c)^2 - 300\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - \left(\cos(dx+c)^4 - 2\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/15*(150*cos(d*x + c)^4 - 325*cos(d*x + c)^2 - 300*(cos(d*x + c)^6 - 3*cos
(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*si
n(d*x + c) - 1)*log(1/2*sin(d*x + c)) + 300*(cos(d*x + c)^6 - 3*cos(d*x + c
)^4 + 3*cos(d*x + c)^2 - (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x +
c) - 1)*log(sin(d*x + c) + 1) + 2*(150*cos(d*x + c)^4 - 275*cos(d*x + c)^2
+ 119)*sin(d*x + c) + 178)/(a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 +
 3*a^4*d*cos(d*x + c)^2 - a^4*d - (a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x +
 c)^2 + a^4*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29281, size = 335, normalized size = 2.48

$$\frac{\frac{19200 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{9600 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4} - \frac{1920\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+28\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+15\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^2} + \frac{21920\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-4350\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/480*(19200*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 9600*log(abs(tan(1/2*
d*x + 1/2*c)))/a^4 - 1920*(15*tan(1/2*d*x + 1/2*c)^2 + 28*tan(1/2*d*x + 1/2
*c) + 15)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^2) + (21920*tan(1/2*d*x + 1/2*c)^
5 - 4350*tan(1/2*d*x + 1/2*c)^4 + 840*tan(1/2*d*x + 1/2*c)^3 - 175*tan(1/2*
d*x + 1/2*c)^2 + 30*tan(1/2*d*x + 1/2*c) - 3)/(a^4*tan(1/2*d*x + 1/2*c)^5)
- (3*a^16*tan(1/2*d*x + 1/2*c)^5 - 30*a^16*tan(1/2*d*x + 1/2*c)^4 + 175*a^1
6*tan(1/2*d*x + 1/2*c)^3 - 840*a^16*tan(1/2*d*x + 1/2*c)^2 + 4350*a^16*tan(
1/2*d*x + 1/2*c))/a^20)/d
```

3.565 $\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=181

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^3\sin^{n+3}(c+dx)}{d(n+3)}-\frac{5a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{5a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^3\sin^{n+6}(c+d}{d(n+6)}$$

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (a^3*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (5*a^3*Sin[c + d*x]^(4 +
n))/(d*(4 + n)) - (5*a^3*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (a^3*Sin[c + d
*x]^(6 + n))/(d*(6 + n)) + (3*a^3*Sin[c + d*x]^(7 + n))/(d*(7 + n)) + (a^3*
Sin[c + d*x]^(8 + n))/(d*(8 + n))
```

Rubi [A] time = 0.180944, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^3\sin^{n+3}(c+dx)}{d(n+3)}-\frac{5a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{5a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^3\sin^{n+6}(c+d}{d(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (a^3*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (5*a^3*Sin[c + d*x]^(4 +
n))/(d*(4 + n)) - (5*a^3*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (a^3*Sin[c + d
*x]^(6 + n))/(d*(6 + n)) + (3*a^3*Sin[c + d*x]^(7 + n))/(d*(7 + n)) + (a^3*
Sin[c + d*x]^(8 + n))/(d*(8 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(a^7\left(\frac{x}{a}\right)^n+3a^7\left(\frac{x}{a}\right)^{1+n}+a^7\left(\frac{x}{a}\right)^{2+n}-5a^7\left(\frac{x}{a}\right)^{3+n}-5a^7\left(\frac{x}{a}\right)^{4}\right.\right.}{a^5d}\\ &= \frac{a^3\sin^{1+n}(c+dx)}{d(1+n)}+\frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)}+\frac{a^3\sin^{3+n}(c+dx)}{d(3+n)}-\frac{5a^3\text{s}}{}\end{aligned}$$

Mathematica [A] time = 0.564413, size = 123, normalized size = 0.68

$$\frac{a^3\sin^{n+1}(c+dx)\left(\frac{\sin^7(c+dx)}{n+8}+\frac{3\sin^6(c+dx)}{n+7}+\frac{\sin^5(c+dx)}{n+6}-\frac{5\sin^4(c+dx)}{n+5}-\frac{5\sin^3(c+dx)}{n+4}+\frac{\sin^2(c+dx)}{n+3}+\frac{3\sin(c+dx)}{n+2}+\frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (3*Sin[c + d*x])/(2 + n) + Sin[c
+ d*x]^2/(3 + n) - (5*Sin[c + d*x]^3)/(4 + n) - (5*Sin[c + d*x]^4)/(5 + n)
+ Sin[c + d*x]^5/(6 + n) + (3*Sin[c + d*x]^6)/(7 + n) + Sin[c + d*x]^7/(8 +
 n)))/d
```

Maple [F] time = 11.083, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^5 (\sin(dx+c))^n (a+a\sin(dx+c))^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.48964, size = 1554, normalized size = 8.59

$$\left(\left(a^3n^7+28\,a^3n^6+322\,a^3n^5+1960\,a^3n^4+6769\,a^3n^3+13132\,a^3n^2+13068\,a^3n+5040\,a^3\right)\cos(dx+c)^8+32\,a^3n^5+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] ((a^3*n^7 + 28*a^3*n^6 + 322*a^3*n^5 + 1960*a^3*n^4 + 6769*a^3*n^3 + 13132*
a^3*n^2 + 13068*a^3*n + 5040*a^3)*cos(d*x + c)^8 + 32*a^3*n^5 + 720*a^3*n^4
 - (5*a^3*n^7 + 142*a^3*n^6 + 1654*a^3*n^5 + 10180*a^3*n^4 + 35485*a^3*n^3
+ 69358*a^3*n^2 + 69416*a^3*n + 26880*a^3)*cos(d*x + c)^6 + 6080*a^3*n^3 +
23520*a^3*n^2 + 2*(2*a^3*n^7 + 49*a^3*n^6 + 470*a^3*n^5 + 2230*a^3*n^4 + 54
```

```
38*a^3*n^3 + 6361*a^3*n^2 + 2730*a^3*n)*cos(d*x + c)^4 + 39968*a^3*n + 2184
0*a^3 + 8*(2*a^3*n^6 + 45*a^3*n^5 + 380*a^3*n^4 + 1470*a^3*n^3 + 2498*a^3*n
^2 + 1365*a^3*n)*cos(d*x + c)^2 + (32*a^3*n^5 + 720*a^3*n^4 - 3*(a^3*n^7 +
29*a^3*n^6 + 343*a^3*n^5 + 2135*a^3*n^4 + 7504*a^3*n^3 + 14756*a^3*n^2 + 14
832*a^3*n + 5760*a^3)*cos(d*x + c)^6 + 6080*a^3*n^3 + 24000*a^3*n^2 + 2*(2*
a^3*n^7 + 53*a^3*n^6 + 566*a^3*n^5 + 3155*a^3*n^4 + 9908*a^3*n^3 + 17492*a^
3*n^2 + 15984*a^3*n + 5760*a^3)*cos(d*x + c)^4 + 44288*a^3*n + 30720*a^3 +
8*(2*a^3*n^6 + 47*a^3*n^5 + 425*a^3*n^4 + 1880*a^3*n^3 + 4268*a^3*n^2 + 468
8*a^3*n + 1920*a^3)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x + c)^n/(d*n^8 + 3
6*d*n^7 + 546*d*n^6 + 4536*d*n^5 + 22449*d*n^4 + 67284*d*n^3 + 118124*d*n^2
 + 109584*d*n + 40320*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23205, size = 1038, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^8 + 10*n*sin(d*x + c)^n*sin(d*x + c)^8 -
2*n^2*sin(d*x + c)^n*sin(d*x + c)^6 + 24*sin(d*x + c)^n*sin(d*x + c)^8 - 24
*n*sin(d*x + c)^n*sin(d*x + c)^6 + n^2*sin(d*x + c)^n*sin(d*x + c)^4 - 64*s
in(d*x + c)^n*sin(d*x + c)^6 + 14*n*sin(d*x + c)^n*sin(d*x + c)^4 + 48*sin(
d*x + c)^n*sin(d*x + c)^4)*a^3/(n^3 + 18*n^2 + 104*n + 192) + 3*(n^2*sin(d*
x + c)^n*sin(d*x + c)^7 + 8*n*sin(d*x + c)^n*sin(d*x + c)^7 - 2*n^2*sin(d*x
 + c)^n*sin(d*x + c)^5 + 15*sin(d*x + c)^n*sin(d*x + c)^7 - 20*n*sin(d*x +
c)^n*sin(d*x + c)^5 + n^2*sin(d*x + c)^n*sin(d*x + c)^3 - 42*sin(d*x + c)^n
*sin(d*x + c)^5 + 12*n*sin(d*x + c)^n*sin(d*x + c)^3 + 35*sin(d*x + c)^n*si
n(d*x + c)^3)*a^3/(n^3 + 15*n^2 + 71*n + 105) + 3*(n^2*sin(d*x + c)^n*sin(d
*x + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2*n^2*sin(d*x + c)^n*sin(d*
x + c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n*sin(d*x + c)^n*sin(d*x +
c)^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin(d*x + c)^n*sin(d*x + c)^4
 + 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*x + c)^n*sin(d*x + c)^2)*a
^3/(n^3 + 12*n^2 + 44*n + 48) + (n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 4*n*si
n(d*x + c)^n*sin(d*x + c)^5 - 2*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 3*sin(d
*x + c)^n*sin(d*x + c)^5 - 12*n*sin(d*x + c)^n*sin(d*x + c)^3 + n^2*sin(d*x
 + c)^n*sin(d*x + c) - 10*sin(d*x + c)^n*sin(d*x + c)^3 + 8*n*sin(d*x + c)^
n*sin(d*x + c) + 15*sin(d*x + c)^n*sin(d*x + c))*a^3/(n^3 + 9*n^2 + 23*n +
15))/d
```

3.566 $\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}-\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}-\frac{4a^2\sin^{n+4}(c+dx)}{d(n+4)}-\frac{a^2\sin^{n+5}(c+dx)}{d(n+5)}+\frac{2a^2\sin^{n+6}(c+dx)}{d(n+6)}$$

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (4*a^2*Sin[c + d*x]^(4 +
n))/(d*(4 + n)) - (a^2*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (2*a^2*Sin[c + d
*x]^(6 + n))/(d*(6 + n)) + (a^2*Sin[c + d*x]^(7 + n))/(d*(7 + n))
```

Rubi [A] time = 0.166758, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}-\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}-\frac{4a^2\sin^{n+4}(c+dx)}{d(n+4)}-\frac{a^2\sin^{n+5}(c+dx)}{d(n+5)}+\frac{2a^2\sin^{n+6}(c+dx)}{d(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (4*a^2*Sin[c + d*x]^(4 +
n))/(d*(4 + n)) - (a^2*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (2*a^2*Sin[c + d
*x]^(6 + n))/(d*(6 + n)) + (a^2*Sin[c + d*x]^(7 + n))/(d*(7 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx=\frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)^4\,dx,x,a\sin(c+dx)\right)}{a^5d}$$

$$=\frac{\text{Subst}\left(\int \left(a^6\left(\frac{x}{a}\right)^n+2a^6\left(\frac{x}{a}\right)^{1+n}-a^6\left(\frac{x}{a}\right)^{2+n}-4a^6\left(\frac{x}{a}\right)^{3+n}-a^6\left(\frac{x}{a}\right.\right.\right.}{a^5d}$$

$$=\frac{a^2\sin^{1+n}(c+dx)}{d(1+n)}+\frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)}-\frac{a^2\sin^{3+n}(c+dx)}{d(3+n)}-$$

Mathematica [A] time = 0.389313, size = 110, normalized size = 0.69

$$\frac{a^2 \sin^{n+1}(c+dx)\left(\frac{\sin^6(c+dx)}{n+7}+\frac{2\sin^5(c+dx)}{n+6}-\frac{\sin^4(c+dx)}{n+5}-\frac{4\sin^3(c+dx)}{n+4}-\frac{\sin^2(c+dx)}{n+3}+\frac{2\sin(c+dx)}{n+2}+\frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (2*Sin[c + d*x])/(2 + n) - Sin[c
+ d*x]^2/(3 + n) - (4*Sin[c + d*x]^3)/(4 + n) - Sin[c + d*x]^4/(5 + n) + (2
*Sin[c + d*x]^5)/(6 + n) + Sin[c + d*x]^6/(7 + n)))/d
```

Maple [F] time = 7.947, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^5 (\sin(dx+c))^n (a+a\sin(dx+c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.41732, size = 1143, normalized size = 7.14

$$-\frac{\left(2\left(a^2n^6+22\,a^2n^5+190\,a^2n^4+820\,a^2n^3+1849\,a^2n^2+2038\,a^2n+840\,a^2\right)\cos(dx+c)^6-16\,a^2n^4-256\,a^2n^3-2\left(a^2n\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(2*(a^2*n^6 + 22*a^2*n^5 + 190*a^2*n^4 + 820*a^2*n^3 + 1849*a^2*n^2 + 2038
*a^2*n + 840*a^2)*cos(d*x + c)^6 - 16*a^2*n^4 - 256*a^2*n^3 - 2*(a^2*n^6 +
18*a^2*n^5 + 118*a^2*n^4 + 348*a^2*n^3 + 457*a^2*n^2 + 210*a^2*n)*cos(d*x +
 c)^4 - 1376*a^2*n^2 - 2816*a^2*n - 8*(a^2*n^5 + 16*a^2*n^4 + 86*a^2*n^3 +
176*a^2*n^2 + 105*a^2*n)*cos(d*x + c)^2 - 1680*a^2 + ((a^2*n^6 + 21*a^2*n^5
 + 175*a^2*n^4 + 735*a^2*n^3 + 1624*a^2*n^2 + 1764*a^2*n + 720*a^2)*cos(d*x
```

```
 + c)^6 - 16*a^2*n^4 - 256*a^2*n^3 - 2*(a^2*n^6 + 20*a^2*n^5 + 159*a^2*n^4
+ 640*a^2*n^3 + 1364*a^2*n^2 + 1440*a^2*n + 576*a^2)*cos(d*x + c)^4 - 1472*
a^2*n^2 - 3584*a^2*n - 8*(a^2*n^5 + 17*a^2*n^4 + 108*a^2*n^3 + 316*a^2*n^2
+ 416*a^2*n + 192*a^2)*cos(d*x + c)^2 - 3072*a^2)*sin(d*x + c))*sin(d*x + c
)^n/(d*n^7 + 28*d*n^6 + 322*d*n^5 + 1960*d*n^4 + 6769*d*n^3 + 13132*d*n^2 +
 13068*d*n + 5040*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25401, size = 778, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^7 + 8*n*sin(d*x + c)^n*sin(d*x + c)^7 - 2
*n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 15*sin(d*x + c)^n*sin(d*x + c)^7 - 20*
n*sin(d*x + c)^n*sin(d*x + c)^5 + n^2*sin(d*x + c)^n*sin(d*x + c)^3 - 42*si
n(d*x + c)^n*sin(d*x + c)^5 + 12*n*sin(d*x + c)^n*sin(d*x + c)^3 + 35*sin(d
*x + c)^n*sin(d*x + c)^3)*a^2/(n^3 + 15*n^2 + 71*n + 105) + 2*(n^2*sin(d*x
+ c)^n*sin(d*x + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2*n^2*sin(d*x +
 c)^n*sin(d*x + c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n*sin(d*x + c)^
n*sin(d*x + c)^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin(d*x + c)^n*si
n(d*x + c)^4 + 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*x + c)^n*sin(d
*x + c)^2)*a^2/(n^3 + 12*n^2 + 44*n + 48) + (n^2*sin(d*x + c)^n*sin(d*x + c
)^5 + 4*n*sin(d*x + c)^n*sin(d*x + c)^5 - 2*n^2*sin(d*x + c)^n*sin(d*x + c)
^3 + 3*sin(d*x + c)^n*sin(d*x + c)^5 - 12*n*sin(d*x + c)^n*sin(d*x + c)^3 +
 n^2*sin(d*x + c)^n*sin(d*x + c) - 10*sin(d*x + c)^n*sin(d*x + c)^3 + 8*n*s
in(d*x + c)^n*sin(d*x + c) + 15*sin(d*x + c)^n*sin(d*x + c))*a^2/(n^3 + 9*n
^2 + 23*n + 15))/d
```

3.567 $\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{2a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a\sin^{n+6}(c+dx)}{d(n+6)}$$

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (2*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (2*a*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (a*Sin[c + d*x]^(6 + n))/(
d*(6 + n))
```

Rubi [A] time = 0.119184, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2836, 88}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{2a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a\sin^{n+6}(c+dx)}{d(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```


```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (2*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (2*a*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (a*Sin[c + d*x]^(6 + n))/(
d*(6 + n))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)^3\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(a^5\left(\frac{x}{a}\right)^n+a^5\left(\frac{x}{a}\right)^{1+n}-2a^5\left(\frac{x}{a}\right)^{2+n}-2a^5\left(\frac{x}{a}\right)^{3+n}+a^5\left(\frac{x}{a}\right)^{4+n}\right.\right.}{a^5d}\\ &= \frac{a\sin^{1+n}(c+dx)}{d(1+n)}+\frac{a\sin^{2+n}(c+dx)}{d(2+n)}-\frac{2a\sin^{3+n}(c+dx)}{d(3+n)}-\frac{2a\sin^{4+n}}{d(4+}\end{aligned}$$

Mathematica [B] time = 1.35988, size = 345, normalized size = 2.8

$$\frac{a\sin^{n+1}(c+dx)\left(2n^5\sin(c+dx)+3n^5\sin(3(c+dx))+n^5\sin(5(c+dx))+46n^4\sin(c+dx)+61n^4\sin(3(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n)*(8544 + 10520*n + 4888*n^2 + 1114*n^3 + 128*n^4 + 6
*n^5 + 8*(336 + 692*n + 484*n^2 + 147*n^3 + 20*n^4 + n^5)*Cos[2*(c + d*x)]
+ 2*(144 + 324*n + 260*n^2 + 95*n^3 + 16*n^4 + n^5)*Cos[4*(c + d*x)] + 2640
*Sin[c + d*x] + 4468*n*Sin[c + d*x] + 2258*n^2*Sin[c + d*x] + 474*n^3*Sin[c
 + d*x] + 46*n^4*Sin[c + d*x] + 2*n^5*Sin[c + d*x] + 840*Sin[3*(c + d*x)] +
 1798*n*Sin[3*(c + d*x)] + 1331*n^2*Sin[3*(c + d*x)] + 431*n^3*Sin[3*(c + d
*x)] + 61*n^4*Sin[3*(c + d*x)] + 3*n^5*Sin[3*(c + d*x)] + 120*Sin[5*(c + d*
x)] + 274*n*Sin[5*(c + d*x)] + 225*n^2*Sin[5*(c + d*x)] + 85*n^3*Sin[5*(c +
 d*x)] + 15*n^4*Sin[5*(c + d*x)] + n^5*Sin[5*(c + d*x)]))/(16*d*(1 + n)*(2
+ n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))
```

Maple [F] time = 4.082, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^5 (\sin(dx+c))^n (a+a\sin(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.27655, size = 713, normalized size = 5.8

$$-\frac{\left(\left(an^5+15\,an^4+85\,an^3+225\,an^2+274\,an+120\,a\right)\cos(dx+c)^6-\left(an^5+11\,an^4+41\,an^3+61\,an^2+30\,an\right)\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a*n^5 + 15*a*n^4 + 85*a*n^3 + 225*a*n^2 + 274*a*n + 120*a)*cos(d*x + c)^
6 - (a*n^5 + 11*a*n^4 + 41*a*n^3 + 61*a*n^2 + 30*a*n)*cos(d*x + c)^4 - 8*a*
```

```
n^3 - 72*a*n^2 - 4*(a*n^4 + 9*a*n^3 + 23*a*n^2 + 15*a*n)*cos(d*x + c)^2 - 1
84*a*n - ((a*n^5 + 16*a*n^4 + 95*a*n^3 + 260*a*n^2 + 324*a*n + 144*a)*cos(d
*x + c)^4 + 8*a*n^3 + 96*a*n^2 + 4*(a*n^4 + 13*a*n^3 + 56*a*n^2 + 92*a*n +
48*a)*cos(d*x + c)^2 + 352*a*n + 384*a)*sin(d*x + c) - 120*a)*sin(d*x + c)^
n/(d*n^6 + 21*d*n^5 + 175*d*n^4 + 735*d*n^3 + 1624*d*n^2 + 1764*d*n + 720*d
)
```

Sympy [A] time = 173.843, size = 8534, normalized size = 69.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)*sin(c)**n*cos(c)**5, Eq(d, 0)), (a*log(sin(c +
d*x))/d - 8*a/(15*d*sin(c + d*x)) + a*cos(c + d*x)**2/(2*d*sin(c + d*x)**2)
 + 4*a*cos(c + d*x)**2/(15*d*sin(c + d*x)**3) - a*cos(c + d*x)**4/(4*d*sin(
c + d*x)**4) - a*cos(c + d*x)**4/(5*d*sin(c + d*x)**5), Eq(n, -6)), (a*log(
sin(c + d*x))/d + 8*a*sin(c + d*x)/(3*d) + 4*a*cos(c + d*x)**2/(3*d*sin(c +
 d*x)) + a*cos(c + d*x)**2/(2*d*sin(c + d*x)**2) - a*cos(c + d*x)**4/(3*d*s
in(c + d*x)**3) - a*cos(c + d*x)**4/(4*d*sin(c + d*x)**4), Eq(n, -5)), (48*
a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**
7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 96*a*log(tan(c/2
 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(
c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 48*a*log(tan(c/2 + d*x/2)**2
+ 1)*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)*
*5 + 24*d*tan(c/2 + d*x/2)**3) - 48*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2
)**7/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 +
d*x/2)**3) - 96*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 +
 d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 48*a*lo
g(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)**7 + 48*d*ta
n(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - a*tan(c/2 + d*x/2)**10/(24*
d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3
) - 3*a*tan(c/2 + d*x/2)**9/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/
2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 19*a*tan(c/2 + d*x/2)**8/(24*d*tan(c/2
+ d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 27*a*t
an(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 2
4*d*tan(c/2 + d*x/2)**3) + 110*a*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)
**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 110*a*tan(c/2
+ d*x/2)**4/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan
(c/2 + d*x/2)**3) - 27*a*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)**7 + 48
*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 19*a*tan(c/2 + d*x/2)*
*2/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*
x/2)**3) - 3*a*tan(c/2 + d*x/2)/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 +
d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - a/(24*d*tan(c/2 + d*x/2)**7 + 48*d*
tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3), Eq(n, -4)), (48*a*log(tan(
c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(24*d*tan(c/2 + d*x/2)**8 + 72*d*t
an(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) +
 144*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x
/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2
 + d*x/2)**2) + 144*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(24*
d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4
 + 24*d*tan(c/2 + d*x/2)**2) + 48*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 +
d*x/2)**2/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c
/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 48*a*log(tan(c/2 + d*x/2))*tan
(c/2 + d*x/2)**8/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*
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d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 144*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 144*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 48*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 3*a*tan(c/2 + d*x/2)**10/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 12*a*tan(c/2 + d*x/2)**9/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 144*a*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 63*a*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 200*a*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 63*a*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 144*a*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 12*a*tan(c/2 + d*x/2)/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 3*a/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2), Eq(n, -3)), (-6*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**9/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 24*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**7/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 36*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 24*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**3/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 6*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) + 6*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**9/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) + 24*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**7/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) + 36*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**5/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) + 24*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**3/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) + 6*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 3*a*tan(c/2 + d*x/2)**10/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 39*a*tan(c/2 + d*x/2)**8/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 24*a*tan(c/2 + d*x/2)**7/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 86*a*tan(c/2 + d*x/2)**6/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 24*a*tan(c/2 + d*x/2)**5/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 86*a*tan(c/2 + d*x/2)**4/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 +

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d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 24*a*tan(c/
2 + d*x/2)**3/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*ta
n(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 39*a
*tan(c/2 + d*x/2)**2/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 +
36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2))
 - 3*a/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 +
 d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)), Eq(n, -2)),
(-15*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**10/(15*d*tan(c/2 + d*
x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan
(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) - 75*a*log(tan(c/2 + d*
x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2
+ d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*
tan(c/2 + d*x/2)**2 + 15*d) - 150*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 +
d*x/2)**6/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan
(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 1
5*d) - 150*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(15*d*tan(c/2
 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*
d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) - 75*a*log(tan(c/2
 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan
(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 +
75*d*tan(c/2 + d*x/2)**2 + 15*d) - 15*a*log(tan(c/2 + d*x/2)**2 + 1)/(15*d*
tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6
 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 15*a*log(
tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**10/(15*d*tan(c/2 + d*x/2)**10 + 75*d*ta
n(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 +
 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 75*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*
x/2)**8/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c
/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*
d) + 150*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(15*d*tan(c/2 + d*x/2)
**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2
 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 150*a*log(tan(c/2 + d*x/2
))*tan(c/2 + d*x/2)**4/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**
8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 +
d*x/2)**2 + 15*d) + 75*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(15*d*ta
n(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 +
 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 15*a*log(ta
n(c/2 + d*x/2))/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150
*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)*
*2 + 15*d) + 12*a*tan(c/2 + d*x/2)**10/(15*d*tan(c/2 + d*x/2)**10 + 75*d*ta
n(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 +
 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 30*a*tan(c/2 + d*x/2)**9/(15*d*tan(c/2
+ d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d
*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 40*a*tan(c/2 + d*
x/2)**7/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c
/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*
d) + 116*a*tan(c/2 + d*x/2)**5/(15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 +
d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*ta
n(c/2 + d*x/2)**2 + 15*d) + 40*a*tan(c/2 + d*x/2)**3/(15*d*tan(c/2 + d*x/2)
**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)**6 + 150*d*tan(c/2
 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 30*a*tan(c/2 + d*x/2)/(15
*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/2)
**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d) + 12*a/(
15*d*tan(c/2 + d*x/2)**10 + 75*d*tan(c/2 + d*x/2)**8 + 150*d*tan(c/2 + d*x/
2)**6 + 150*d*tan(c/2 + d*x/2)**4 + 75*d*tan(c/2 + d*x/2)**2 + 15*d), Eq(n,
 -1)), (a*n**5*sin(c + d*x)**2*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**6 + 21
*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + a*n**
5*sin(c + d*x)*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*
n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 4*a*n**4*sin(c + d*x)
**4*sin(c + d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*
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d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 4*a*n**4*sin(c + d*x)**3*sin(c +
 d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 16
24*d*n**2 + 1764*d*n + 720*d) + 19*a*n**4*sin(c + d*x)**2*sin(c + d*x)**n*c
os(c + d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2
+ 1764*d*n + 720*d) + 20*a*n**4*sin(c + d*x)*sin(c + d*x)**n*cos(c + d*x)**
4/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n +
720*d) + 8*a*n**3*sin(c + d*x)**6*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175
*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 8*a*n**3*sin(c + d
*x)**5*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624
*d*n**2 + 1764*d*n + 720*d) + 60*a*n**3*sin(c + d*x)**4*sin(c + d*x)**n*cos
(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 +
1764*d*n + 720*d) + 68*a*n**3*sin(c + d*x)**3*sin(c + d*x)**n*cos(c + d*x)*
*2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n +
 720*d) + 137*a*n**3*sin(c + d*x)**2*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**
6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) +
 155*a*n**3*sin(c + d*x)*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n**
5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 72*a*n**2*s
in(c + d*x)**6*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**
3 + 1624*d*n**2 + 1764*d*n + 720*d) + 96*a*n**2*sin(c + d*x)**5*sin(c + d*x
)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n
 + 720*d) + 308*a*n**2*sin(c + d*x)**4*sin(c + d*x)**n*cos(c + d*x)**2/(d*n
**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d)
 + 416*a*n**2*sin(c + d*x)**3*sin(c + d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*
d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 461*a*
n**2*sin(c + d*x)**2*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 +
175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 580*a*n**2*sin(
c + d*x)*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4 +
 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 184*a*n*sin(c + d*x)**6*sin
(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 +
1764*d*n + 720*d) + 352*a*n*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**6 + 21*d*
n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 612*a*n*
sin(c + d*x)**4*sin(c + d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d
*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 1072*a*n*sin(c + d*x
)**3*sin(c + d*x)**n*cos(c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735
*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 702*a*n*sin(c + d*x)**2*sin(c +
 d*x)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 16
24*d*n**2 + 1764*d*n + 720*d) + 1044*a*n*sin(c + d*x)*sin(c + d*x)**n*cos(c
 + d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 17
64*d*n + 720*d) + 120*a*sin(c + d*x)**6*sin(c + d*x)**n/(d*n**6 + 21*d*n**5
 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 384*a*sin(c
+ d*x)**5*sin(c + d*x)**n/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1
624*d*n**2 + 1764*d*n + 720*d) + 360*a*sin(c + d*x)**4*sin(c + d*x)**n*cos(
c + d*x)**2/(d*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1
764*d*n + 720*d) + 960*a*sin(c + d*x)**3*sin(c + d*x)**n*cos(c + d*x)**2/(d
*n**6 + 21*d*n**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*
d) + 360*a*sin(c + d*x)**2*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n
**5 + 175*d*n**4 + 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d) + 720*a*sin
(c + d*x)*sin(c + d*x)**n*cos(c + d*x)**4/(d*n**6 + 21*d*n**5 + 175*d*n**4
+ 735*d*n**3 + 1624*d*n**2 + 1764*d*n + 720*d), True))
```

Giac [B] time = 1.14342, size = 512, normalized size = 4.16

$$\frac{\frac{\left(n^2 \sin(dx+c)^n \sin(dx+c)^6 + 6\,n \sin(dx+c)^n \sin(dx+c)^6 - 2\,n^2 \sin(dx+c)^n \sin(dx+c)^4 + 8\,\sin(dx+c)^n \sin(dx+c)^6 - 16\,n \sin(dx+c)^n \sin(dx+c)^4 + n^2 \sin(dx+c)^n \sin}{n^3+12\,n^2+44\,n+48}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2
*n^2*sin(d*x + c)^n*sin(d*x + c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n
*sin(d*x + c)^n*sin(d*x + c)^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin
(d*x + c)^n*sin(d*x + c)^4 + 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*
x + c)^n*sin(d*x + c)^2)*a/(n^3 + 12*n^2 + 44*n + 48) + (n^2*sin(d*x + c)^n
*sin(d*x + c)^5 + 4*n*sin(d*x + c)^n*sin(d*x + c)^5 - 2*n^2*sin(d*x + c)^n*
sin(d*x + c)^3 + 3*sin(d*x + c)^n*sin(d*x + c)^5 - 12*n*sin(d*x + c)^n*sin(
d*x + c)^3 + n^2*sin(d*x + c)^n*sin(d*x + c) - 10*sin(d*x + c)^n*sin(d*x +
c)^3 + 8*n*sin(d*x + c)^n*sin(d*x + c) + 15*sin(d*x + c)^n*sin(d*x + c))*a/
(n^3 + 9*n^2 + 23*n + 15))/d
```

3.568 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)}-\frac{\sin^{n+2}(c+dx)}{ad(n+2)}-\frac{\sin^{n+3}(c+dx)}{ad(n+3)}+\frac{\sin^{n+4}(c+dx)}{ad(n+4)}$$

```
[Out] Sin[c + d*x]^(1 + n)/(a*d*(1 + n)) - Sin[c + d*x]^(2 + n)/(a*d*(2 + n)) - S
in[c + d*x]^(3 + n)/(a*d*(3 + n)) + Sin[c + d*x]^(4 + n)/(a*d*(4 + n))
```

Rubi [A] time = 0.139479, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 75}

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)}-\frac{\sin^{n+2}(c+dx)}{ad(n+2)}-\frac{\sin^{n+3}(c+dx)}{ad(n+3)}+\frac{\sin^{n+4}(c+dx)}{ad(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a*d*(1 + n)) - Sin[c + d*x]^(2 + n)/(a*d*(2 + n)) - S
in[c + d*x]^(3 + n)/(a*d*(3 + n)) + Sin[c + d*x]^(4 + n)/(a*d*(4 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)\,dx, x, a\sin(c+dx)\right)}{a^5 d}\\ &= \frac{\text{Subst}\left(\int \left(a^3\left(\frac{x}{a}\right)^n-a^3\left(\frac{x}{a}\right)^{1+n}-a^3\left(\frac{x}{a}\right)^{2+n}+a^3\left(\frac{x}{a}\right)^{3+n}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}\\ &= \frac{\sin^{1+n}(c+dx)}{ad(1+n)}-\frac{\sin^{2+n}(c+dx)}{ad(2+n)}-\frac{\sin^{3+n}(c+dx)}{ad(3+n)}+\frac{\sin^{4+n}(c+dx)}{ad(4+n)}\end{aligned}$$

Mathematica [A] time = 0.689956, size = 74, normalized size = 0.81

$$\frac{\sin^{n+1}(c+dx)\left(-\frac{(n+4)\sin^2(c+dx)}{n+3}-\frac{(n+4)\sin(c+dx)}{n+2}+\sin^3(c+dx)+\frac{n+4}{n+1}\right)}{ad(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((4 + n)/(1 + n) - ((4 + n)*Sin[c + d*x])/(2 + n) - (
(4 + n)*Sin[c + d*x]^2)/(3 + n) + Sin[c + d*x]^3))/(a*d*(4 + n))
```

Maple [F] time = 1.957, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^5 (\sin(dx+c))^n}{a + a\sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 1.35226, size = 167, normalized size = 1.84

$$\frac{\left(\left(n^3 + 6\,n^2 + 11\,n + 6\right)\sin(dx+c)^4 - \left(n^3 + 7\,n^2 + 14\,n + 8\right)\sin(dx+c)^3 - \left(n^3 + 8\,n^2 + 19\,n + 12\right)\sin(dx+c)^2 + \left(n^3 + \right.\right.}{\left(n^4 + 10\,n^3 + 35\,n^2 + 50\,n + 24\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*sin(d*x + c)^4 - (n^3 + 7*n^2 + 14*n + 8)*sin(d*x
 + c)^3 - (n^3 + 8*n^2 + 19*n + 12)*sin(d*x + c)^2 + (n^3 + 9*n^2 + 26*n +
24)*sin(d*x + c))*sin(d*x + c)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*a*d)
```

Fricas [A] time = 1.14804, size = 332, normalized size = 3.65

$$\frac{\left(\left(n^3 + 6\,n^2 + 11\,n + 6\right)\cos(dx+c)^4 - \left(n^3 + 4\,n^2 + 3\,n\right)\cos(dx+c)^2 - 2\,n^2 + \left(\left(n^3 + 7\,n^2 + 14\,n + 8\right)\cos(dx+c)^2 + 2\,n\right.\right.}{adn^4 + 10\,adn^3 + 35\,adn^2 + 50\,adn + 24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*cos(d*x + c)^4 - (n^3 + 4*n^2 + 3*n)*cos(d*x + c)
^2 - 2*n^2 + ((n^3 + 7*n^2 + 14*n + 8)*cos(d*x + c)^2 + 2*n^2 + 12*n + 16)*
sin(d*x + c) - 8*n - 6)*sin(d*x + c)^n/(a*d*n^4 + 10*a*d*n^3 + 35*a*d*n^2 +
 50*a*d*n + 24*a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33454, size = 124, normalized size = 1.36

$$\frac{\frac{\sin(dx+c)^n \sin(dx+c)^4}{n+4} - \frac{\sin(dx+c)^n \sin(dx+c)^3}{n+3} - \frac{\sin(dx+c)^n \sin(dx+c)^2}{n+2} + \frac{\sin(dx+c)^{n+1}}{n+1}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (sin(d*x + c)^n*sin(d*x + c)^4/(n + 4) - sin(d*x + c)^n*sin(d*x + c)^3/(n +
 3) - sin(d*x + c)^n*sin(d*x + c)^2/(n + 2) + sin(d*x + c)^(n + 1)/(n + 1))
/(a*d)
```

3.569 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)} - \frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)} + \frac{\sin^{n+3}(c+dx)}{a^2d(n+3)}$$

```
[Out] Sin[c + d*x]^(1 + n)/(a^2*d*(1 + n)) - (2*Sin[c + d*x]^(2 + n))/(a^2*d*(2 +
 n)) + Sin[c + d*x]^(3 + n)/(a^2*d*(3 + n))
```

Rubi [A] time = 0.127235, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 43}

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)} - \frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)} + \frac{\sin^{n+3}(c+dx)}{a^2d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^2*d*(1 + n)) - (2*Sin[c + d*x]^(2 + n))/(a^2*d*(2 +
 n)) + Sin[c + d*x]^(3 + n)/(a^2*d*(3 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left(a^2\left(\frac{x}{a}\right)^n - 2a^2\left(\frac{x}{a}\right)^{1+n} + a^2\left(\frac{x}{a}\right)^{2+n}\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\sin^{1+n}(c+dx)}{a^2d(1+n)} - \frac{2\sin^{2+n}(c+dx)}{a^2d(2+n)} + \frac{\sin^{3+n}(c+dx)}{a^2d(3+n)}
\end{aligned}$$

Mathematica [A] time = 0.120364, size = 50, normalized size = 0.74

$$\frac{\sin^{n+1}(c+dx)\left(\frac{\sin^2(c+dx)}{n+3} - \frac{2\sin(c+dx)}{n+2} + \frac{1}{n+1}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((1 + n)^(-1) - (2*Sin[c + d*x])/(2 + n) + Sin[c + d*
x]^2/(3 + n)))/(a^2*d)
```

Maple [F] time = 1.755, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^5 (\sin(dx+c))^n}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

Maxima [A] time = 1.25538, size = 109, normalized size = 1.6

$$\frac{\left(\left(n^2+3n+2\right)\sin(dx+c)^3-2\left(n^2+4n+3\right)\sin(dx+c)^2+\left(n^2+5n+6\right)\sin(dx+c)\right)\sin(dx+c)^n}{\left(n^3+6n^2+11n+6\right)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] ((n^2 + 3*n + 2)*sin(d*x + c)^3 - 2*(n^2 + 4*n + 3)*sin(d*x + c)^2 + (n^2 +
 5*n + 6)*sin(d*x + c))*sin(d*x + c)^n/((n^3 + 6*n^2 + 11*n + 6)*a^2*d)
```

Fricas [A] time = 1.14455, size = 248, normalized size = 3.65

$$\frac{\left(2\left(n^2+4n+3\right)\cos(dx+c)^2-2n^2-\left(\left(n^2+3n+2\right)\cos(dx+c)^2-2n^2-8n-8\right)\sin(dx+c)-8n-6\right)\sin(dx+}{a^2dn^3+6a^2dn^2+11a^2dn+6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (2*(n^2 + 4*n + 3)*cos(d*x + c)^2 - 2*n^2 - ((n^2 + 3*n + 2)*cos(d*x + c)^2
 - 2*n^2 - 8*n - 8)*sin(d*x + c) - 8*n - 6)*sin(d*x + c)^n/(a^2*d*n^3 + 6*a
^2*d*n^2 + 11*a^2*d*n + 6*a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35094, size = 93, normalized size = 1.37

$$\frac{\frac{\sin(dx+c)^n \sin(dx+c)^3}{n+3} - \frac{2 \sin(dx+c)^n \sin(dx+c)^2}{n+2} + \frac{\sin(dx+c)^{n+1}}{n+1}}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (sin(d*x + c)^n*sin(d*x + c)^3/(n + 3) - 2*sin(d*x + c)^n*sin(d*x + c)^2/(n
 + 2) + sin(d*x + c)^(n + 1)/(n + 1))/(a^2*d)
```

3.570 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=85

$$\frac{4\sin^{n+1}(c+dx)\,_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}-\frac{3\sin^{n+1}(c+dx)}{a^3d(n+1)}+\frac{\sin^{n+2}(c+dx)}{a^3d(n+2)}$$

```
[Out] (-3*Sin[c + d*x]^(1 + n))/(a^3*d*(1 + n)) + (4*Hypergeometric2F1[1, 1 + n,
2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^3*d*(1 + n)) + Sin[c + d*x]^
(2 + n)/(a^3*d*(2 + n))
```

Rubi [A] time = 0.144604, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 88, 64}

$$\frac{4\sin^{n+1}(c+dx)\,_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}-\frac{3\sin^{n+1}(c+dx)}{a^3d(n+1)}+\frac{\sin^{n+2}(c+dx)}{a^3d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-3*Sin[c + d*x]^(1 + n))/(a^3*d*(1 + n)) + (4*Hypergeometric2F1[1, 1 + n,
2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^3*d*(1 + n)) + Sin[c + d*x]^
(2 + n)/(a^3*d*(2 + n))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a\left(\frac{x}{a}\right)^n + a\left(\frac{x}{a}\right)^{1+n} + \frac{4a^2\left(\frac{x}{a}\right)^n}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= -\frac{3\sin^{1+n}(c+dx)}{a^3 d(1+n)} + \frac{\sin^{2+n}(c+dx)}{a^3 d(2+n)} + \frac{4\,\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$

$$= -\frac{3\sin^{1+n}(c+dx)}{a^3 d(1+n)} + \frac{4\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^3 d(1+n)} + \frac{\sin^{2+n}(c+d}{a^3 d(2+n)}$$

Mathematica [A] time = 0.0939923, size = 64, normalized size = 0.75

$$\frac{\sin^{n+1}(c+dx)(4(n+2)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx)) + (n+1)\sin(c+dx) - 3(n+2))}{a^3 d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(-3*(2 + n) + 4*(2 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, -Sin[c + d*x]] + (1 + n)*Sin[c + d*x]))/(a^3*d*(1 + n)*(2 + n))
```

Maple [F] time = 1.217, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^5(\sin(dx+c))^n}{(a+a\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^5}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sin(dx+c)^{n}\cos(dx+c)^{5}}{3\,a^{3}\cos(dx+c)^{2}-4\,a^{3}+\left(a^{3}\cos(dx+c)^{2}-4\,a^{3}\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)^5/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^
3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^{n}\cos(dx+c)^{5}}{(a\sin(dx+c)+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^3, x)
```

3.571 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=88

$$-\frac{4\sin^{n+1}(c+dx)\,_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d}+\frac{\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+1}(c+dx)}{d\left(a^4\sin(c+dx)+a^4\right)}$$

```
[Out] Sin[c + d*x]^(1 + n)/(a^4*d*(1 + n)) - (4*Hypergeometric2F1[1, 1 + n, 2 + n
, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^4*d) + (4*Sin[c + d*x]^(1 + n))/(
d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.137456, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2836, 89, 80, 64}

$$-\frac{4\sin^{n+1}(c+dx)\,_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d}+\frac{\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+1}(c+dx)}{d\left(a^4\sin(c+dx)+a^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^4*d*(1 + n)) - (4*Hypergeometric2F1[1, 1 + n, 2 + n
, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^4*d) + (4*Sin[c + d*x]^(1 + n))/(
d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2\left(\frac{x}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{4\sin^{1+n}(c+dx)}{d\left(a^4+a^4\sin(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{(a(3+4n)-x)\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{\sin^{1+n}(c+dx)}{a^4 d(1+n)} + \frac{4\sin^{1+n}(c+dx)}{d\left(a^4+a^4\sin(c+dx)\right)} - \frac{(4(1+n))\,\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+\right.}{a^4 d}$$

$$= \frac{\sin^{1+n}(c+dx)}{a^4 d(1+n)} - \frac{4\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4 d} + \frac{4\sin^{1+n}(\cdot}{d\left(a^4+a^4\sin\right.}$$

Mathematica [A] time = 0.105929, size = 72, normalized size = 0.82

$$\frac{\sin^{n+1}(c+dx)\left(-4(n+1)(\sin(c+dx)+1)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))+\sin(c+dx)+4n+5\right)}{a^4 d(n+1)(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(5 + 4*n + Sin[c + d*x] - 4*(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, -Sin[c + d*x]]*(1 + Sin[c + d*x])))/(a^4*d*(1 + n)*(1 + S
in[c + d*x]))
```

Maple [F] time = 1.464, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^5(\sin(dx+c))^n}{(a+a\sin(dx+c))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^5}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)^n\cos(dx+c)^5}{a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^5/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x
+ c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^5}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^4, x)
```

3.572 $\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=165

$$-\frac{a\cos^{11}(c+dx)}{11d}+\frac{2a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a\,\text{s}}{}$$

```
[Out] (3*a*x)/256 - (a*Cos[c + d*x]^7)/(7*d) + (2*a*Cos[c + d*x]^9)/(9*d) - (a*Co
s[c + d*x]^11)/(11*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (a*Cos[c
+ d*x]^3*Sin[c + d*x])/(128*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) -
(3*a*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]^3
)/(10*d)
```

Rubi [A] time = 0.200495, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 270}

$$-\frac{a\cos^{11}(c+dx)}{11d}+\frac{2a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a\,\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```


```
[Out] (3*a*x)/256 - (a*Cos[c + d*x]^7)/(7*d) + (2*a*Cos[c + d*x]^9)/(9*d) - (a*Co
s[c + d*x]^11)/(11*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (a*Cos[c
+ d*x]^3*Sin[c + d*x])/(128*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) -
(3*a*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]^3
)/(10*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin^4(c+dx)\,dx + a\int \cos^6(c+dx)\sin^5(c+dx)\,dx$$
$$= -\frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d} + \frac{1}{10}(3a)\int \cos^6(c+dx)\sin^2(c+dx)\,d$$
$$= -\frac{3a\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d} + \frac{1}{80}(3a)$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d} + \frac{a\cos^5(c+dx}{16}$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d} + \frac{a\cos^3(c+dx}{12}$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d} + \frac{3a\cos(c+d}{25}$$
$$= \frac{3ax}{256} - \frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d} + \frac{3a\cos(}{}$$

Mathematica [A] time = 0.556382, size = 121, normalized size = 0.73

$$\frac{a(13860\sin(2(c+dx)) - 27720\sin(4(c+dx)) - 6930\sin(6(c+dx)) + 3465\sin(8(c+dx)) + 1386\sin(10(c+dx)) - 6930}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(83160*d*x - 69300*Cos[c + d*x] - 23100*Cos[3*(c + d*x)] + 6930*Cos[5*(c
 + d*x)] + 4950*Cos[7*(c + d*x)] - 770*Cos[9*(c + d*x)] - 630*Cos[11*(c + d
*x)] + 13860*Sin[2*(c + d*x)] - 27720*Sin[4*(c + d*x)] - 6930*Sin[6*(c + d*
x)] + 3465*Sin[8*(c + d*x)] + 1386*Sin[10*(c + d*x)]))/(7096320*d)
```

Maple [A] time = 0.034, size = 134, normalized size = 0.8

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^7}{11} - \frac{4(\sin(dx+c))^2(\cos(dx+c))^7}{99} - \frac{8(\cos(dx+c))^7}{693}\right) + a\left(-\frac{(\sin(dx+c))^3(\cos(d}{10}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/69
3*cos(d*x+c)^7)+a*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+
```

```
c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256
*d*x+3/256*c))
```

Maxima [A] time = 1.01994, size = 116, normalized size = 0.7

$$-\frac{10240\left(63\cos\left(dx+c\right)^{11}-154\cos\left(dx+c\right)^{9}+99\cos\left(dx+c\right)^{7}\right)a-693\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\,\text{s}\right.}{7096320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7096320*(10240*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c
)^7)*a - 693*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c)
- 40*sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.19263, size = 315, normalized size = 1.91

$$-\frac{80640\,a\cos\left(dx+c\right)^{11}-197120\,a\cos\left(dx+c\right)^{9}+126720\,a\cos\left(dx+c\right)^{7}-10395\,adx-693\left(128\,a\cos\left(dx+c\right)^{9}-17\right.}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/887040*(80640*a*cos(d*x + c)^11 - 197120*a*cos(d*x + c)^9 + 126720*a*cos
(d*x + c)^7 - 10395*a*d*x - 693*(128*a*cos(d*x + c)^9 - 176*a*cos(d*x + c)^
7 + 8*a*cos(d*x + c)^5 + 10*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x +
 c))/d
```

Sympy [A] time = 56.5633, size = 318, normalized size = 1.93

$$\begin{cases}\frac{3ax\sin^{10}(c+dx)}{256}+\frac{15ax\sin^{8}(c+dx)\cos^{2}(c+dx)}{256}+\frac{15ax\sin^{6}(c+dx)\cos^{4}(c+dx)}{128}+\frac{15ax\sin^{4}(c+dx)\cos^{6}(c+dx)}{128}+\frac{15ax\sin^{2}(c+dx)\cos^{8}(c+dx)}{256}+\ldots\\ x\left(a\sin\left(c\right)+a\right)\sin^{4}\left(c\right)\cos^{6}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**10/256 + 15*a*x*sin(c + d*x)**8*cos(c + d*x)
**2/256 + 15*a*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a*x*sin(c + d*x)*
*4*cos(c + d*x)**6/128 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a*x
*cos(c + d*x)**10/256 + 3*a*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + a*sin(c + d*x)**5*cos(c + d*x)**5/(10
*d) - a*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 7*a*sin(c + d*x)**3*cos(c +
 d*x)**7/(128*d) - 4*a*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 3*a*sin(c +
 d*x)*cos(c + d*x)**9/(256*d) - 8*a*cos(c + d*x)**11/(693*d), Ne(d, 0)), (x
*(a*sin(c) + a)*sin(c)**4*cos(c)**6, True))
```

Giac [A] time = 1.27601, size = 225, normalized size = 1.36

$$\frac{3}{256}ax - \frac{a\cos(11\,dx + 11\,c)}{11264\,d} - \frac{a\cos(9\,dx + 9\,c)}{9216\,d} + \frac{5\,a\cos(7\,dx + 7\,c)}{7168\,d} + \frac{a\cos(5\,dx + 5\,c)}{1024\,d} - \frac{5\,a\cos(3\,dx + 3\,c)}{1536\,d} - \frac{5\,a\,\mathrm{co}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/256*a*x - 1/11264*a*cos(11*d*x + 11*c)/d - 1/9216*a*cos(9*d*x + 9*c)/d +
5/7168*a*cos(7*d*x + 7*c)/d + 1/1024*a*cos(5*d*x + 5*c)/d - 5/1536*a*cos(3*
d*x + 3*c)/d - 5/512*a*cos(d*x + c)/d + 1/5120*a*sin(10*d*x + 10*c)/d + 1/2
048*a*sin(8*d*x + 8*c)/d - 1/1024*a*sin(6*d*x + 6*c)/d - 1/256*a*sin(4*d*x
+ 4*c)/d + 1/512*a*sin(2*d*x + 2*c)/d
```

3.573 $\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=149

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{160d}$$

```
[Out] (3*a*x)/256 - (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c + d*x]^9)/(9*d) + (3*a*Co
s[c + d*x]*Sin[c + d*x])/(256*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(128*d)
+ (a*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) - (3*a*Cos[c + d*x]^7*Sin[c + d*x
])/(80*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*d)
```

Rubi [A] time = 0.206155, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{160d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/256 - (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c + d*x]^9)/(9*d) + (3*a*Co
s[c + d*x]*Sin[c + d*x])/(256*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(128*d)
+ (a*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) - (3*a*Cos[c + d*x]^7*Sin[c + d*x
])/(80*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin^3(c+dx)\,dx + a\int \cos^6(c+dx)\sin^4(c+dx)\,dx$$
$$= -\frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d} + \frac{1}{10}(3a)\int \cos^6(c+dx)\sin^2(c+dx)\,d$$
$$= -\frac{3a\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d} + \frac{1}{80}(3a)$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{160d} - \frac{3a}{}$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{128d} + \frac{a}{}$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{3a\cos(c+dx)\sin(c+dx)}{256d} + \frac{a}{}$$
$$= \frac{3ax}{256} - \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{3a\cos(c+dx)\sin(c+dx)}{256d}$$

Mathematica [A] time = 0.336643, size = 101, normalized size = 0.68

$$\frac{a(1260\sin(2(c+dx)) - 2520\sin(4(c+dx)) - 630\sin(6(c+dx)) + 315\sin(8(c+dx)) + 126\sin(10(c+dx)) - 15120\cos(}{645120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(7560*d*x - 15120*Cos[c + d*x] - 6720*Cos[3*(c + d*x)] + 1080*Cos[7*(c +
 d*x)] + 280*Cos[9*(c + d*x)] + 1260*Sin[2*(c + d*x)] - 2520*Sin[4*(c + d*x
)] - 630*Sin[6*(c + d*x)] + 315*Sin[8*(c + d*x)] + 126*Sin[10*(c + d*x)]))/
(645120*d)
```

Maple [A] time = 0.033, size = 116, normalized size = 0.8

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^7}{10} - \frac{3\sin(dx+c)(\cos(dx+c))^7}{80} + \frac{\sin(dx+c)}{160}\left((\cos(dx+c))^5 + \frac{5(\cos(dx+c))^3}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*
(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*
c)+a*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

Maxima [A] time = 1.06026, size = 103, normalized size = 0.69

$$\frac{10240\left(7\cos\left(dx+c\right)^{9}-9\cos\left(dx+c\right)^{7}\right)a+63\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\sin\left(8\,dx+8\,c\right)-40\sin\left(4\right.\right.}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/645120*(10240*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a + 63*(32*sin(2*d*x
+ 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.18549, size = 269, normalized size = 1.81

$$\frac{8960\,a\cos\left(dx+c\right)^{9}-11520\,a\cos\left(dx+c\right)^{7}+945\,adx+63\left(128\,a\cos\left(dx+c\right)^{9}-176\,a\cos\left(dx+c\right)^{7}+8\,a\cos\left(dx+c\right.\right.}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80640*(8960*a*cos(d*x + c)^9 - 11520*a*cos(d*x + c)^7 + 945*a*d*x + 63*(1
28*a*cos(d*x + c)^9 - 176*a*cos(d*x + c)^7 + 8*a*cos(d*x + c)^5 + 10*a*cos(
d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 32.6663, size = 294, normalized size = 1.97

$$\begin{cases}\frac{3ax\sin^{10}(c+dx)}{256}+\frac{15ax\sin^{8}(c+dx)\cos^{2}(c+dx)}{256}+\frac{15ax\sin^{6}(c+dx)\cos^{4}(c+dx)}{128}+\frac{15ax\sin^{4}(c+dx)\cos^{6}(c+dx)}{128}+\frac{15ax\sin^{2}(c+dx)\cos^{8}(c+dx)}{256}+\frac{3}{} \\ x\left(a\sin\left(c\right)+a\right)\sin^{3}\left(c\right)\cos^{6}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**10/256 + 15*a*x*sin(c + d*x)**8*cos(c + d*x)
**2/256 + 15*a*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a*x*sin(c + d*x)*
*4*cos(c + d*x)**6/128 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a*x
*cos(c + d*x)**10/256 + 3*a*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + a*sin(c + d*x)**5*cos(c + d*x)**5/(10
*d) - 7*a*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - a*sin(c + d*x)**2*cos(c
 + d*x)**7/(7*d) - 3*a*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 2*a*cos(c + d
*x)**9/(63*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**6, True))
```

Giac [A] time = 1.27674, size = 185, normalized size = 1.24

$$\frac{3}{256}\,ax+\frac{a\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{3\,a\cos\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{96\,d}-\frac{3\,a\cos\left(dx+c\right)}{128\,d}+\frac{a\sin\left(10\,dx+10\,c\right)}{5120\,d}+\frac{a\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/256*a*x + 1/2304*a*cos(9*d*x + 9*c)/d + 3/1792*a*cos(7*d*x + 7*c)/d - 1/9
6*a*cos(3*d*x + 3*c)/d - 3/128*a*cos(d*x + c)/d + 1/5120*a*sin(10*d*x + 10*
c)/d + 1/2048*a*sin(8*d*x + 8*c)/d - 1/1024*a*sin(6*d*x + 6*c)/d - 1/256*a*
sin(4*d*x + 4*c)/d + 1/512*a*sin(2*d*x + 2*c)/d
```

3.574 $\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=125

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+d\text{.}}{192d}$$

```
[Out] (5*a*x)/128 - (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c + d*x]^9)/(9*d) + (5*a*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d
) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]
)/(8*d)
```

Rubi [A] time = 0.154914, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+d\text{.}}{192d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/128 - (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c + d*x]^9)/(9*d) + (5*a*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d
) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]
)/(8*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin^2(c+dx)\,dx + a\int \cos^6(c+dx)\sin^3(c+dx)\,dx$$
$$= -\frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}a\int \cos^6(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int x^6\right.}{}$$
$$= \frac{a\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{48}(5a)\int$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{a}{}$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{5a\cos(c+dx)\sin(c+dx)}{128d} + \frac{5a}{}$$
$$= \frac{5ax}{128} - \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{5a\cos(c+dx)\sin(c+dx)}{128d}$$

Mathematica [A] time = 0.275032, size = 91, normalized size = 0.73

$$\frac{a(1008\sin(2(c+dx)) - 504\sin(4(c+dx)) - 336\sin(6(c+dx)) - 63\sin(8(c+dx)) - 1512\cos(c+dx) - 672\cos(3(c+dx)}{64512d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(2520*d*x - 1512*Cos[c + d*x] - 672*Cos[3*(c + d*x)] + 108*Cos[7*(c + d*
x)] + 28*Cos[9*(c + d*x)] + 1008*Sin[2*(c + d*x)] - 504*Sin[4*(c + d*x)] -
336*Sin[6*(c + d*x)] - 63*Sin[8*(c + d*x)]))/(64512*d)
```

Maple [A] time = 0.03, size = 98, normalized size = 0.8

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^7}{9} - \frac{2(\cos(dx+c))^7}{63}\right) + a\left(-\frac{\sin(dx+c)(\cos(dx+c))^7}{8} + \frac{\sin(dx+c)}{48}\left((\cos(dx+c)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+a*(-1/8*sin(d*x+c
)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x
+c)+5/128*d*x+5/128*c))
```

Maxima [A] time = 1.04192, size = 103, normalized size = 0.82

$$\frac{1024\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a+21\left(64\sin(2dx+2c)^3+120dx+120c-3\sin(8dx+8c)-24\sin(4dx\right.}{64512d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/64512*(1024*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a + 21*(64*sin(2*d*x +
2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.20244, size = 232, normalized size = 1.86

$$\frac{896a\cos(dx+c)^9-1152a\cos(dx+c)^7+315adx-21\left(48a\cos(dx+c)^7-8a\cos(dx+c)^5-10a\cos(dx+c)^3-1\right.}{8064d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8064*(896*a*cos(d*x + c)^9 - 1152*a*cos(d*x + c)^7 + 315*a*d*x - 21*(48*a
*cos(d*x + c)^7 - 8*a*cos(d*x + c)^5 - 10*a*cos(d*x + c)^3 - 15*a*cos(d*x +
 c))*sin(d*x + c))/d
```

Sympy [A] time = 20.5605, size = 248, normalized size = 1.98

$$\begin{cases}\frac{5ax\sin^8(c+dx)}{128}+\frac{5ax\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{15ax\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{5ax\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{5ax\cos^8(c+dx)}{128}+\frac{5a\sin^7(c+dx)\cos}{128d}\\ x(a\sin(c)+a)\sin^2(c)\cos^6(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**8/128 + 5*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*a*x*sin(c + d*x)**2*co
s(c + d*x)**6/32 + 5*a*x*cos(c + d*x)**8/128 + 5*a*sin(c + d*x)**7*cos(c +
d*x)/(128*d) + 55*a*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 73*a*sin(c +
d*x)**3*cos(c + d*x)**5/(384*d) - a*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) -
 5*a*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 2*a*cos(c + d*x)**9/(63*d), Ne(
d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**6, True))
```

Giac [A] time = 1.26139, size = 165, normalized size = 1.32

$$\frac{5}{128}ax+\frac{a\cos(9dx+9c)}{2304d}+\frac{3a\cos(7dx+7c)}{1792d}-\frac{a\cos(3dx+3c)}{96d}-\frac{3a\cos(dx+c)}{128d}-\frac{a\sin(8dx+8c)}{1024d}-\frac{a\sin(6}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/128*a*x + 1/2304*a*cos(9*d*x + 9*c)/d + 3/1792*a*cos(7*d*x + 7*c)/d - 1/9
6*a*cos(3*d*x + 3*c)/d - 3/128*a*cos(d*x + c)/d - 1/1024*a*sin(8*d*x + 8*c)
/d - 1/192*a*sin(6*d*x + 6*c)/d - 1/128*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2
*d*x + 2*c)/d
```

3.575 $\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=109

$$-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a\sin(c+$$

```
[Out] (5*a*x)/128 - (a*Cos[c + d*x]^7)/(7*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(1
28*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (a*Cos[c + d*x]^5*Sin[c
 + d*x])/(48*d) - (a*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rubi [A] time = 0.119503, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/128 - (a*Cos[c + d*x]^7)/(7*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(1
28*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (a*Cos[c + d*x]^5*Sin[c
 + d*x])/(48*d) - (a*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin(c+dx)\,dx + a\int \cos^6(c+dx)\sin^2(c+dx)\,dx$$
$$= -\frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}a\int \cos^6(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int x^6 \right.}{}$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{a\cos^7(c+dx)\sin(c+}{8d}$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{a\cos^5(c+dx)\sin(c}{48d}$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos(c+dx)\sin(c+dx)}{128d} + \frac{5a\cos^3(c+dx)\sin(c}{192d}$$
$$= \frac{5ax}{128} - \frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos(c+dx)\sin(c+dx)}{128d} + \frac{5a\cos^3(c+dx)}{192d}$$

Mathematica [A] time = 0.261617, size = 91, normalized size = 0.83

$$-\frac{a(-336\sin(2(c+dx))+168\sin(4(c+dx))+112\sin(6(c+dx))+21\sin(8(c+dx))+1680\cos(c+dx)+1008\cos(3(c}{21504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(-840*d*x + 1680*Cos[c + d*x] + 1008*Cos[3*(c + d*x)] + 336*Cos[5*(c +
d*x)] + 48*Cos[7*(c + d*x)] - 336*Sin[2*(c + d*x)] + 168*Sin[4*(c + d*x)] +
 112*Sin[6*(c + d*x)] + 21*Sin[8*(c + d*x)]))/(21504*d)
```

Maple [A] time = 0.025, size = 78, normalized size = 0.7

$$\frac{1}{d}\left(a\left(-\frac{\sin(dx+c)(\cos(dx+c))^7}{8}+\frac{\sin(dx+c)}{48}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{128}+\frac{5c}{128}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15
/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-1/7*a*cos(d*x+c)^7)
```

Maxima [A] time = 1.03327, size = 85, normalized size = 0.78

$$-\frac{3072a\cos(dx+c)^7-7\left(64\sin(2dx+2c)^3+120dx+120c-3\sin(8dx+8c)-24\sin(4dx+4c)\right)a}{21504d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/21504*(3072*a*cos(d*x + c)^7 - 7*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*
c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a)/d
```

Fricas [A] time = 1.19186, size = 200, normalized size = 1.83

$$-\frac{384\,a\cos\left(dx+c\right)^{7}-105\,adx+7\left(48\,a\cos\left(dx+c\right)^{7}-8\,a\cos\left(dx+c\right)^{5}-10\,a\cos\left(dx+c\right)^{3}-15\,a\cos\left(dx+c\right)\right)\sin\left(d\right.}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2688*(384*a*cos(d*x + c)^7 - 105*a*d*x + 7*(48*a*cos(d*x + c)^7 - 8*a*co
s(d*x + c)^5 - 10*a*cos(d*x + c)^3 - 15*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 11.5849, size = 223, normalized size = 2.05

$$\begin{cases} \frac{5ax\sin^{8}(c+dx)}{128}+\frac{5ax\sin^{6}(c+dx)\cos^{2}(c+dx)}{32}+\frac{15ax\sin^{4}(c+dx)\cos^{4}(c+dx)}{64}+\frac{5ax\sin^{2}(c+dx)\cos^{6}(c+dx)}{32}+\frac{5ax\cos^{8}(c+dx)}{128}+\frac{5a\sin^{7}(c+dx)\cos}{128d} \\ x\left(a\sin\left(c\right)+a\right)\sin\left(c\right)\cos^{6}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**8/128 + 5*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*a*x*sin(c + d*x)**2*co
s(c + d*x)**6/32 + 5*a*x*cos(c + d*x)**8/128 + 5*a*sin(c + d*x)**7*cos(c +
d*x)/(128*d) + 55*a*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 73*a*sin(c +
d*x)**3*cos(c + d*x)**5/(384*d) - 5*a*sin(c + d*x)*cos(c + d*x)**7/(128*d)
- a*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*cos(c)**6, T
rue))
```

Giac [A] time = 1.16715, size = 165, normalized size = 1.51

$$\frac{5}{128}\,ax-\frac{a\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{a\cos\left(5\,dx+5\,c\right)}{64\,d}-\frac{3\,a\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{5\,a\cos\left(dx+c\right)}{64\,d}-\frac{a\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a\sin\left(6\,d\right.}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/128*a*x - 1/448*a*cos(7*d*x + 7*c)/d - 1/64*a*cos(5*d*x + 5*c)/d - 3/64*a
*cos(3*d*x + 3*c)/d - 5/64*a*cos(d*x + c)/d - 1/1024*a*sin(8*d*x + 8*c)/d -
 1/192*a*sin(6*d*x + 6*c)/d - 1/128*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2*d*x
 + 2*c)/d
```

3.576 $\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=127

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+$$

```
[Out] (5*a*x)/16 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/
(16*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c
 + d*x])/(6*d)
```

Rubi [A] time = 0.109212, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2592, 302, 206, 2635, 8}

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```


```
[Out] (5*a*x)/16 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/
(16*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c
 + d*x])/(6*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\,dx + a\int \cos^5(c+dx)\cot(c+dx)\,dx$$

$$= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5a)\int \cos^4(c+dx)\,dx - \frac{a\,\mathrm{Subst}\big(}{}$$

$$= \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{8}(5a)$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{5a\cos(c+dx)}{16d}$$

$$= \frac{5ax}{16} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} +$$

Mathematica [A] time = 0.117986, size = 100, normalized size = 0.79

$$\frac{a\big(225\sin(2(c+dx)) + 45\sin(4(c+dx)) + 5\sin(6(c+dx)) + 1320\cos(c+dx) + 140\cos(3(c+dx)) + 12\cos(5(c+dx}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(300*c + 300*d*x + 1320*Cos[c + d*x] + 140*Cos[3*(c + d*x)] + 12*Cos[5*(
c + d*x)] - 960*Log[Cos[(c + d*x)/2]] + 960*Log[Sin[(c + d*x)/2]] + 225*Sin
[2*(c + d*x)] + 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.053, size = 131, normalized size = 1.

$$\frac{a(\cos(dx+c))^5\sin(dx+c)}{6d} + \frac{5a(\cos(dx+c))^3\sin(dx+c)}{24d} + \frac{5\cos(dx+c)a\sin(dx+c)}{16d} + \frac{5ax}{16} + \frac{5ca}{16d} + \frac{a(\cos(}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/6*a*cos(d*x+c)^5*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3*sin(d*x+c)/d+5/16*a*cos
(d*x+c)*sin(d*x+c)/d+5/16*a*x+5/16/d*c*a+1/5*a*cos(d*x+c)^5/d+1/3*a*cos(d*x
+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.00943, size = 143, normalized size = 1.13

$$\frac{32\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a-5\left(4\sin\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(32*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(
cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a - 5*(4*sin(2*d*x + 2*c)^3 -
 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 1.17841, size = 321, normalized size = 2.53

$$\frac{48\,a\cos(dx+c)^5+80\,a\cos(dx+c)^3+75\,adx+240\,a\cos(dx+c)-120\,a\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+120\,a\log\left(-\frac{1}{2}\cos\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^5 + 80*a*cos(d*x + c)^3 + 75*a*d*x + 240*a*cos(d*x
 + c) - 120*a*log(1/2*cos(d*x + c) + 1/2) + 120*a*log(-1/2*cos(d*x + c) + 1
/2) + 5*(8*a*cos(d*x + c)^5 + 10*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23933, size = 271, normalized size = 2.13

$$75(dx+c)a+240\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-\frac{2\left(165\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-720\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-25\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-2160\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(165*a*tan
(1/2*d*x + 1/2*c)^11 - 720*a*tan(1/2*d*x + 1/2*c)^10 - 25*a*tan(1/2*d*x + 1
/2*c)^9 - 2160*a*tan(1/2*d*x + 1/2*c)^8 + 450*a*tan(1/2*d*x + 1/2*c)^7 - 36
```

```
80*a*tan(1/2*d*x + 1/2*c)^6 - 450*a*tan(1/2*d*x + 1/2*c)^5 - 3360*a*tan(1/2
*d*x + 1/2*c)^4 + 25*a*tan(1/2*d*x + 1/2*c)^3 - 1488*a*tan(1/2*d*x + 1/2*c)
^2 - 165*a*tan(1/2*d*x + 1/2*c) - 368*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.577 $\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{15a\cot(c+dx)}{8d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}+\frac{5a\cos^2(c+dx)\cot(c}{8d}$$

```
[Out] (-15*a*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
 d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) - (15*a*Cot[c + d*x])/(8*d) + (5*
a*Cos[c + d*x]^2*Cot[c + d*x])/(8*d) + (a*Cos[c + d*x]^4*Cot[c + d*x])/(4*d
)
```

Rubi [A] time = 0.133799, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2838, 2591, 288, 321, 203, 2592, 302, 206}

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{15a\cot(c+dx)}{8d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}+\frac{5a\cos^2(c+dx)\cot(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```


```
[Out] (-15*a*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
 d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) - (15*a*Cot[c + d*x])/(8*d) + (5*
a*Cos[c + d*x]^2*Cot[c + d*x])/(8*d) + (a*Cos[c + d*x]^4*Cot[c + d*x])/(4*d
)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\cot(c+dx)\,dx + a\int \cos^4(c+dx)\cot^2(c+dx)\,\ldots$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\mathrm{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3}\,dx, x, \ldots\right.}{d}$$

$$= \frac{a\cos^4(c+dx)\cot(c+dx)}{4d} - \frac{a\,\mathrm{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)\ldots\right.}{d}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{5a\cos^2(c+d\ldots}{8\ldots}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos\ldots}{\ldots}$$

$$= -\frac{15ax}{8} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx\ldots}{3d}$$

Mathematica [A] time = 0.259947, size = 98, normalized size = 0.81

$$-\frac{a\left(240\sin(2(c+dx))+15\sin(4(c+dx))-660\cos(c+dx)-70\cos(3(c+dx))-6\cos(5(c+dx))+480\cot(c+dx)\ldots\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(900*c + 900*d*x - 660*Cos[c + d*x] - 70*Cos[3*(c + d*x)] - 6*Cos[5*(c
+ d*x)] + 480*Cot[c + d*x] + 480*Log[Cos[(c + d*x)/2]] - 480*Log[Sin[(c + d
*x)/2]] + 240*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.052, size = 153, normalized size = 1.3

$$\frac{a(\cos(dx+c))^5}{5d}+\frac{a(\cos(dx+c))^3}{3d}+\frac{\cos(dx+c)a}{d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d}-\frac{a(\cos(dx+c))^7}{d\sin(dx+c)}-\frac{a(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/5*a*cos(d*x+c)^5/d+1/3*a*cos(d*x+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c
)-cot(d*x+c))-1/d*a/sin(d*x+c)*cos(d*x+c)^7-a*cos(d*x+c)^5*sin(d*x+c)/d-5/4
*a*cos(d*x+c)^3*sin(d*x+c)/d-15/8*a*cos(d*x+c)*sin(d*x+c)/d-15/8*a*x-15/8/d
*c*a
```

Maxima [A] time = 1.50333, size = 163, normalized size = 1.35

$$\frac{4\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a-15\left(15a\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*(4*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(c
os(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a - 15*(15*d*x + 15*c + (15*ta
n(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 +
tan(d*x + c)))*a)/d
```

Fricas [A] time = 1.18991, size = 375, normalized size = 3.1

$$\frac{30a\cos(dx+c)^5+75a\cos(dx+c)^3-60a\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+60a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx}{120d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(30*a*cos(d*x + c)^5 + 75*a*cos(d*x + c)^3 - 60*a*log(1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) + 60*a*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 22
5*a*cos(d*x + c) + (24*a*cos(d*x + c)^5 + 40*a*cos(d*x + c)^3 - 225*a*d*x +
 120*a*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18444, size = 267, normalized size = 2.21

$$-\frac{225\,(dx+c)a - 120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - 60\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{60\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \frac{2\left(135\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+36\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(225*(d*x + c)*a - 120*a*log(abs(tan(1/2*d*x + 1/2*c))) - 60*a*tan(1
/2*d*x + 1/2*c) + 60*(2*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c) -
2*(135*a*tan(1/2*d*x + 1/2*c)^9 + 360*a*tan(1/2*d*x + 1/2*c)^8 + 150*a*tan(
1/2*d*x + 1/2*c)^7 + 720*a*tan(1/2*d*x + 1/2*c)^6 + 1120*a*tan(1/2*d*x + 1/
2*c)^4 - 150*a*tan(1/2*d*x + 1/2*c)^3 + 560*a*tan(1/2*d*x + 1/2*c)^2 - 135*
a*tan(1/2*d*x + 1/2*c) + 184*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.578 $\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{15a\cot(c+dx)}{8d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}+\frac{5a\,\mathrm{co}}{}$$

```
[Out] (-15*a*x)/8 + (5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a*Cos[c + d*x])/(2*d)
- (5*a*Cos[c + d*x]^3)/(6*d) - (15*a*Cot[c + d*x])/(8*d) + (5*a*Cos[c + d*x
]^2*Cot[c + d*x])/(8*d) + (a*Cos[c + d*x]^4*Cot[c + d*x])/(4*d) - (a*Cos[c
+ d*x]^3*Cot[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.143726, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2838, 2592, 288, 302, 206, 2591, 321, 203}

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{15a\cot(c+dx)}{8d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}+\frac{5a\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```


```
[Out] (-15*a*x)/8 + (5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a*Cos[c + d*x])/(2*d)
- (5*a*Cos[c + d*x]^3)/(6*d) - (15*a*Cot[c + d*x])/(8*d) + (5*a*Cos[c + d*x
]^2*Cot[c + d*x])/(8*d) + (a*Cos[c + d*x]^4*Cot[c + d*x])/(4*d) - (a*Cos[c
+ d*x]^3*Cot[c + d*x]^2)/(2*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^4(c+dx)\cot^2(c+dx)\,dx + a\int \cos^3(c+dx)\cot^3(c+dx)\\
&= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3}\,dx, x\right.}{d}\\
&= \frac{a\cos^4(c+dx)\cot(c+dx)}{4d} - \frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d} - \frac{(5a)\,\text{S}}{}\\
&= \frac{5a\cos^2(c+dx)\cot(c+dx)}{8d} + \frac{a\cos^4(c+dx)\cot(c+dx)}{4d} - \frac{a\cos}{}\\
&= -\frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3(c+dx)}{6d} - \frac{15a\cot(c+dx)}{8d} + \frac{5a\cos^2(}{}\\
&= -\frac{15ax}{8} + \frac{5a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3(c+}{6d}
\end{aligned}
$$

Mathematica [A] time = 2.85452, size = 117, normalized size = 0.87

$$
-\frac{a\left(216\cos(c+dx) + 8\cos(3(c+dx)) + 3\left(16\sin(2(c+dx)) + \sin(4(c+dx)) + 32\cot(c+dx) + 4\csc^2\left(\frac{1}{2}(c+dx)\right) -\right.\right.}{96d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(216*Cos[c + d*x] + 8*Cos[3*(c + d*x)] + 3*(60*c + 60*d*x + 32*Cot[c +
d*x] + 4*Csc[(c + d*x)/2]^2 - 80*Log[Cos[(c + d*x)/2]] + 80*Log[Sin[(c + d*
x)/2]] - 4*Sec[(c + d*x)/2]^2 + 16*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])))/(
```

```
96*d)
```

Maple [A] time = 0.062, size = 177, normalized size = 1.3

$$-\frac{a(\cos(dx+c))^7}{d\sin(dx+c)} - \frac{a(\cos(dx+c))^5\sin(dx+c)}{d} - \frac{5a(\cos(dx+c))^3\sin(dx+c)}{4d} - \frac{15\cos(dx+c)a\sin(dx+c)}{8d} - \frac{15a}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^7-a*cos(d*x+c)^5*sin(d*x+c)/d-5/4*a*cos(d*x+c)
^3*sin(d*x+c)/d-15/8*a*cos(d*x+c)*sin(d*x+c)/d-15/8*a*x-15/8/d*c*a-1/2/d*a/
sin(d*x+c)^2*cos(d*x+c)^7-1/2*a*cos(d*x+c)^5/d-5/6*a*cos(d*x+c)^3/d-5/2*a*c
os(d*x+c)/d-5/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.62985, size = 177, normalized size = 1.32

$$-\frac{2\left(4\cos(dx+c)^3 - \frac{6\cos(dx+c)}{\cos(dx+c)^2-1} + 24\cos(dx+c) - 15\log(\cos(dx+c)+1) + 15\log(\cos(dx+c)-1)\right)a + 3\left(15dx + \right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*(2*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a + 3*(15*d*
x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*
tan(d*x + c)^3 + tan(d*x + c)))*a)/d
```

Fricas [A] time = 1.20265, size = 435, normalized size = 3.25

$$-\frac{8a\cos(dx+c)^5 + 45adx\cos(dx+c)^2 + 40a\cos(dx+c)^3 - 45adx - 60a\cos(dx+c) - 30\left(a\cos(dx+c)^2 - a\right)\log\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*a*cos(d*x + c)^5 + 45*a*d*x*cos(d*x + c)^2 + 40*a*cos(d*x + c)^3 -
 45*a*d*x - 60*a*cos(d*x + c) - 30*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x +
 c) + 1/2) + 30*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 3*(2*
a*cos(d*x + c)^5 + 5*a*cos(d*x + c)^3 - 15*a*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36379, size = 289, normalized size = 2.16

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-45\,(dx+c)a-60\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(30\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,a\tan\left(\right.\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*a*tan(1/2*d*x + 1/2*c)^2 - 45*(d*x + c)*a - 60*a*log(abs(tan(1/2*d*
x + 1/2*c))) + 12*a*tan(1/2*d*x + 1/2*c) + 3*(30*a*tan(1/2*d*x + 1/2*c)^2 -
 4*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2 + 2*(27*a*tan(1/2*d*x
 + 1/2*c)^7 - 72*a*tan(1/2*d*x + 1/2*c)^6 + 3*a*tan(1/2*d*x + 1/2*c)^5 - 16
8*a*tan(1/2*d*x + 1/2*c)^4 - 3*a*tan(1/2*d*x + 1/2*c)^3 - 152*a*tan(1/2*d*x
 + 1/2*c)^2 - 27*a*tan(1/2*d*x + 1/2*c) - 56*a)/(tan(1/2*d*x + 1/2*c)^2 + 1
)^4)/d
```

3.579 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=130

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a\cos^2(c+dx)\,\mathrm{co}}{2d}$$

```
[Out] (5*a*x)/2 + (5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a*Cos[c + d*x])/(2*d) -
(5*a*Cos[c + d*x]^3)/(6*d) + (5*a*Cot[c + d*x])/(2*d) - (a*Cos[c + d*x]^3*C
ot[c + d*x]^2)/(2*d) - (5*a*Cot[c + d*x]^3)/(6*d) + (a*Cos[c + d*x]^2*Cot[c
 + d*x]^3)/(2*d)
```

Rubi [A] time = 0.139667, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2591, 288, 302, 203, 2592, 206}

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a\cos^2(c+dx)\,\mathrm{co}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```


```
[Out] (5*a*x)/2 + (5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a*Cos[c + d*x])/(2*d) -
(5*a*Cos[c + d*x]^3)/(6*d) + (5*a*Cot[c + d*x])/(2*d) - (a*Cos[c + d*x]^3*C
ot[c + d*x]^2)/(2*d) - (5*a*Cot[c + d*x]^3)/(6*d) + (a*Cos[c + d*x]^2*Cot[c
 + d*x]^3)/(2*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^3(c+dx)\cot^3(c+dx)\,dx + a\int \cos^2(c+dx)\cot^4(c+dx)$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\mathrm{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx, x\right.}{d}$$

$$= -\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d} + \frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d} + \frac{(5a}{}$$

$$= -\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d} + \frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d} + \frac{(5a}{}$$

$$= -\frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3(c+dx)}{6d} + \frac{5a\cot(c+dx)}{2d} - \frac{a\cos^3(c+}{}$$

$$= \frac{5ax}{2} + \frac{5a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3(c+dx}{6d}$$

Mathematica [A] time = 6.09878, size = 174, normalized size = 1.34

$$\frac{5a(c+dx)}{2d} + \frac{a\sin(2(c+dx))}{4d} - \frac{9a\cos(c+dx)}{4d} - \frac{a\cos(3(c+dx))}{12d} + \frac{7a\cot(c+dx)}{3d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*(c + d*x))/(2*d) - (9*a*Cos[c + d*x])/(4*d) - (a*Cos[3*(c + d*x)])/(12
*d) + (7*a*Cot[c + d*x])/(3*d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c +
d*x]*Csc[c + d*x]^2)/(3*d) + (5*a*Log[Cos[(c + d*x)/2]])/(2*d) - (5*a*Log[S
in[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d) + (a*Sin[2*(c + d*x)
])/(4*d)
```

Maple [A] time = 0.062, size = 199, normalized size = 1.5

$$-\frac{a(\cos(dx+c))^7}{2d(\sin(dx+c))^2} - \frac{a(\cos(dx+c))^5}{2d} - \frac{5a(\cos(dx+c))^3}{6d} - \frac{5\cos(dx+c)a}{2d} - \frac{5a\ln(\csc(dx+c)-\cot(dx+c))}{2d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^7-1/2*a*cos(d*x+c)^5/d-5/6*a*cos(d*x+c)^3/
d-5/2*a*cos(d*x+c)/d-5/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*a/sin(d*x+c)^3
*cos(d*x+c)^7+4/3/d*a/sin(d*x+c)*cos(d*x+c)^7+4/3*a*cos(d*x+c)^5*sin(d*x+c)
/d+5/3*a*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a*cos(d*x+c)*sin(d*x+c)/d+5/2*a*x+5/
2/d*c*a
```

Maxima [A] time = 1.57204, size = 165, normalized size = 1.27

$$-\frac{\left(4\cos\left(dx+c\right)^{3}-\frac{6\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{2}-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a-2\left(15\,dx+15\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x
 + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a - 2*(15*d*x
+ 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(
d*x + c)^3))*a)/d
```

Fricas [A] time = 1.1772, size = 491, normalized size = 3.78

$$-\frac{6\,a\cos\left(dx+c\right)^{5}-40\,a\cos\left(dx+c\right)^{3}-15\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+15\left(a\cos\left(dx+c\right)^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*cos(d*x + c)^5 - 40*a*cos(d*x + c)^3 - 15*(a*cos(d*x + c)^2 - a)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(a*cos(d*x + c)^2 - a)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*a*cos(d*x + c) + 2*(2*a*cos(d*x +
 c)^5 - 15*a*d*x*cos(d*x + c)^2 + 10*a*cos(d*x + c)^3 + 15*a*d*x - 15*a*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20706, size = 297, normalized size = 2.28

$$3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+180\,(dx+c)a-180\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-81\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/72*(3*a*tan(1/2*d*x + 1/2*c)^3 + 9*a*tan(1/2*d*x + 1/2*c)^2 + 180*(d*x +
c)*a - 180*a*log(abs(tan(1/2*d*x + 1/2*c))) - 81*a*tan(1/2*d*x + 1/2*c) + (
110*a*tan(1/2*d*x + 1/2*c)^9 + 9*a*tan(1/2*d*x + 1/2*c)^8 - 111*a*tan(1/2*d
*x + 1/2*c)^7 + 240*a*tan(1/2*d*x + 1/2*c)^6 - 273*a*tan(1/2*d*x + 1/2*c)^5
 + 306*a*tan(1/2*d*x + 1/2*c)^4 - 253*a*tan(1/2*d*x + 1/2*c)^3 + 72*a*tan(1
/2*d*x + 1/2*c)^2 - 9*a*tan(1/2*d*x + 1/2*c) - 3*a)/(tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))^3)/d
```

3.580 $\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$\frac{15a\cos(c+dx)}{8d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos}{}$$

```
[Out] (5*a*x)/2 - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d)
+ (5*a*Cot[c + d*x])/(2*d) + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) - (5*a
*Cot[c + d*x]^3)/(6*d) + (a*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*Cos[c
 + d*x]*Cot[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.126065, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2838, 2592, 288, 321, 206, 2591, 302, 203}

$$\frac{15a\cos(c+dx)}{8d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/2 - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d)
+ (5*a*Cot[c + d*x])/(2*d) + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) - (5*a
*Cot[c + d*x]^3)/(6*d) + (a*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*Cos[c
 + d*x]*Cot[c + d*x]^4)/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\cot^4(c+dx)\,dx + a\int \cos(c+dx)\cot^5(c+dx)\,d$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x,\right.}{d}$$

$$= \frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d} - \frac{a\cos(c+dx)\cot^4(c+dx)}{4d} + \frac{(5a)\,\text{Su}}{}$$

$$= \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d} - \frac{a\cos}{}$$

$$= \frac{15a\cos(c+dx)}{8d} + \frac{5a\cot(c+dx)}{2d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} -$$

$$= \frac{5ax}{2} - \frac{15a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15a\cos(c+dx)}{8d} + \frac{5a\cot(c+d}{2d}$$

Mathematica [A] time = 1.36301, size = 138, normalized size = 1.03

$$\frac{a\left(192\cos(c+dx) - 64\cot(c+dx)\left(\csc^2(c+dx)-7\right) + 3\left(16\sin(2(c+dx)) - \csc^4\left(\frac{1}{2}(c+dx)\right) + 18\csc^2\left(\frac{1}{2}(c+dx)\right)\right.}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(192*Cos[c + d*x] - 64*Cot[c + d*x]*(-7 + Csc[c + d*x]^2) + 3*(18*Csc[(c
 + d*x)/2]^2 - Csc[(c + d*x)/2]^4 + 40*(4*c + 4*d*x - 3*Log[Cos[(c + d*x)/2
]] + 3*Log[Sin[(c + d*x)/2]]) - 18*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4
+ 16*Sin[2*(c + d*x)])))/(192*d)
```

Maple [A] time = 0.067, size = 221, normalized size = 1.7

$$-\frac{a\left(\cos\left(dx+c\right)\right)^{7}}{3\,d\left(\sin\left(dx+c\right)\right)^{3}}+\frac{4\,a\left(\cos\left(dx+c\right)\right)^{7}}{3\,d\sin\left(dx+c\right)}+\frac{4\,a\left(\cos\left(dx+c\right)\right)^{5}\sin\left(dx+c\right)}{3\,d}+\frac{5\,a\left(\cos\left(dx+c\right)\right)^{3}\sin\left(dx+c\right)}{3\,d}+\frac{5\,\cos\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*a/sin(d*x+c)*cos(d*x+c)^7+4/3*a*co
s(d*x+c)^5*sin(d*x+c)/d+5/3*a*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a*cos(d*x+c)*si
n(d*x+c)/d+5/2*a*x+5/2/d*c*a-1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*a/sin(
d*x+c)^2*cos(d*x+c)^7+3/8*a*cos(d*x+c)^5/d+5/8*a*cos(d*x+c)^3/d+15/8*a*cos(
d*x+c)/d+15/8/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.53686, size = 184, normalized size = 1.37

$$\frac{8\left(15\,dx+15\,c+\frac{15\,\tan\left(dx+c\right)^{4}+10\,\tan\left(dx+c\right)^{2}-2}{\tan\left(dx+c\right)^{5}+\tan\left(dx+c\right)^{3}}\right)a-3\,a\left(\frac{2\left(9\,\cos\left(dx+c\right)^{3}-7\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+1\right)\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d
*x + c)^5 + tan(d*x + c)^3))*a - 3*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))
/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x
 + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.311, size = 562, normalized size = 4.19

$$\frac{120\,adx\cos\left(dx+c\right)^{4}+48\,a\cos\left(dx+c\right)^{5}-240\,adx\cos\left(dx+c\right)^{2}-150\,a\cos\left(dx+c\right)^{3}+120\,adx+90\,a\cos\left(dx+c\right)-45}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(120*a*d*x*cos(d*x + c)^4 + 48*a*cos(d*x + c)^5 - 240*a*d*x*cos(d*x +
c)^2 - 150*a*cos(d*x + c)^3 + 120*a*d*x + 90*a*cos(d*x + c) - 45*(a*cos(d*x
 + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 45*(a*cos(d
*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 8*(3*a*c
os(d*x + c)^5 - 20*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/(d*c
os(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29321, size = 288, normalized size = 2.15

$$3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+480\,(dx+c)a+360\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*a*tan(1/2*d*x + 1/2*c)^3 - 48*a*tan(1
/2*d*x + 1/2*c)^2 + 480*(d*x + c)*a + 360*a*log(abs(tan(1/2*d*x + 1/2*c)))
- 216*a*tan(1/2*d*x + 1/2*c) - 192*(a*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*
d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2 - (750*a*tan(1/2*d*x + 1/2*c)^4 - 216*a*tan(1/2*d*x + 1/2*c)^3 - 48*a*ta
n(1/2*d*x + 1/2*c)^2 + 8*a*tan(1/2*d*x + 1/2*c) + 3*a)/tan(1/2*d*x + 1/2*c)
^4)/d
```

3.581 $\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$\frac{15a\cos(c+dx)}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2(c+\cdots}{8d}$$

```
[Out] -(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rubi [A] time = 0.0991391, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$\frac{15a\cos(c+dx)}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2(c+\cdots}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx = \int \left(a\cos(c+dx)\cot^5(c+dx)+a\cot^6(c+dx)\right)dx$$
$$= a\int \cos(c+dx)\cot^5(c+dx)\,dx + a\int \cot^6(c+dx)\,dx$$
$$= -\frac{a\cot^5(c+dx)}{5d} - a\int \cot^4(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c+dx)\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + a\int \cot^2(c+$$
$$= -\frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c+dx}{}$$
$$= -ax + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3}{}$$
$$= -ax - \frac{15a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a\cos(c}{}$$

Mathematica [C] time = 0.0753375, size = 164, normalized size = 1.34

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} + \frac{a\cos(c+dx)}{d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{9a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d + (9*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^
4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x
]^2])/(5*d) - (15*a*Log[Cos[(c + d*x)/2]])/(8*d) + (15*a*Log[Sin[(c + d*x)/
2]])/(8*d) - (9*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d
)
```

Maple [A] time = 0.062, size = 159, normalized size = 1.3

$$-\frac{a(\cos(dx+c))^7}{4d(\sin(dx+c))^4}+\frac{3a(\cos(dx+c))^7}{8d(\sin(dx+c))^2}+\frac{3a(\cos(dx+c))^5}{8d}+\frac{5a(\cos(dx+c))^3}{8d}+\frac{15\cos(dx+c)a}{8d}+\frac{15a\ln(\csc(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*a/sin(d*x+c)^2*cos(d*x+c)^7+3/8*a*
cos(d*x+c)^5/d+5/8*a*cos(d*x+c)^3/d+15/8*a*cos(d*x+c)/d+15/8/d*a*ln(csc(d*x
+c)-cot(d*x+c))-1/5*a*cot(d*x+c)^5/d+1/3*a*cot(d*x+c)^3/d-a*cot(d*x+c)/d-a*
x-1/d*c*a
```

Maxima [A] time = 1.51615, size = 169, normalized size = 1.39

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+15\,a\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos(dx+c)+15\,\log(\cos(dx+c)+\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a + 15*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*
log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.20393, size = 620, normalized size = 5.08

$$-\frac{368\,a\cos(dx+c)^5-560\,a\cos(dx+c)^3+225\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(a*cos(d*x + c)^4
 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(
a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*a*cos(d*x +
 c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*a*cos(d*x + c)^3 + 8*a*d*x - 15*a*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20319, size = 269, normalized size = 2.2

$$6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-960\,(dx+c)a+180$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
a*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*a/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*a*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*a*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*a*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.582 $\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=128

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+dx)}{2\ldots}$$

```
[Out] -(a*x) + (5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x])/d + (a*Cot[c
 + d*x]^3)/(3*d) - (a*Cot[c + d*x]^5)/(5*d) - (5*a*Cot[c + d*x]*Csc[c + d*x
])/(16*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a*Cot[c + d*x]^5*Cs
c[c + d*x])/(6*d)
```

Rubi [A] time = 0.131858, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2838, 2611, 3770, 3473, 8}

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+dx)}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```


```
[Out] -(a*x) + (5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x])/d + (a*Cot[c
 + d*x]^3)/(3*d) - (a*Cot[c + d*x]^5)/(5*d) - (5*a*Cot[c + d*x]*Csc[c + d*x
])/(16*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a*Cot[c + d*x]^5*Cs
c[c + d*x])/(6*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\,dx + a\int \cot^6(c+dx)\csc(c+dx)\,dx$$
$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{1}{6}(5a)\int \cot^4(c+$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} + \frac{5a\cot^3(c+dx)\csc(c+dx)}{24d} - $$
$$= -\frac{a\cot(c+dx)}{d} + \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{5a\cot(c+dx)}{16}$$
$$= -ax + \frac{5a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot(c+dx)}{d} + \frac{a\cot^3(c+dx)}{3d} - $$

Mathematica [C] time = 0.050432, size = 193, normalized size = 1.51

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{11a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + $$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-11*a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*x)/2]^4)/(32*d) - (a*Csc[
(c + d*x)/2]^6)/(384*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2
, -Tan[c + d*x]^2])/(5*d) + (5*a*Log[Cos[(c + d*x)/2]])/(16*d) - (5*a*Log[S
in[(c + d*x)/2]])/(16*d) + (11*a*Sec[(c + d*x)/2]^2)/(64*d) - (a*Sec[(c + d
*x)/2]^4)/(32*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A] time = 0.068, size = 181, normalized size = 1.4

$$-\frac{a(\cot(dx+c))^5}{5d} + \frac{a(\cot(dx+c))^3}{3d} - \frac{a\cot(dx+c)}{d} - ax - \frac{ca}{d} - \frac{a(\cos(dx+c))^7}{6d(\sin(dx+c))^6} + \frac{a(\cos(dx+c))^7}{24d(\sin(dx+c))^4} - \frac{a(\cos}{16d(\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/5*a*cot(d*x+c)^5/d+1/3*a*cot(d*x+c)^3/d-a*cot(d*x+c)/d-a*x-1/d*c*a-1/6/d
*a/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*a/sin(d*x+c)^4*cos(d*x+c)^7-1/16/d*a/si
n(d*x+c)^2*cos(d*x+c)^7-1/16*a*cos(d*x+c)^5/d-5/48*a*cos(d*x+c)^3/d-5/16*a*
cos(d*x+c)/d-5/16/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.49655, size = 185, normalized size = 1.45

$$-\frac{32\left(15\,dx + 15\,c + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)a - 5\,a\left(\frac{2\left(33\,\cos(dx+c)^5 - 40\,\cos(dx+c)^3 + 15\,\cos(dx+c)\right)}{\cos(dx+c)^6 - 3\,\cos(dx+c)^4 + 3\,\cos(dx+c)^2 - 1} + 15\,\log(\cos(dx+c)+1\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/480*(32*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a - 5*a*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x
+ c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(c
os(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.22509, size = 694, normalized size = 5.42

$$-\frac{480\,adx\cos\left(dx+c\right)^{6}-1440\,adx\cos\left(dx+c\right)^{4}-330\,a\cos\left(dx+c\right)^{5}+1440\,adx\cos\left(dx+c\right)^{2}+400\,a\cos\left(dx+c\right)^{3}-48\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 - 1440*a*d*x*cos(d*x + c)^4 - 330*a*cos(d*
x + c)^5 + 1440*a*d*x*cos(d*x + c)^2 + 400*a*cos(d*x + c)^3 - 480*a*d*x - 1
50*a*cos(d*x + c) - 75*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x
 + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 75*(a*cos(d*x + c)^6 - 3*a*cos(d
*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) - 32*(23*a
*cos(d*x + c)^5 - 35*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3832, size = 281, normalized size = 2.2

$$\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-140\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+225\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*a*tan(1/2*d*x + 1/2*c)^6 + 12*a*tan(1/2*d*x + 1/2*c)^5 - 45*a*tan
(1/2*d*x + 1/2*c)^4 - 140*a*tan(1/2*d*x + 1/2*c)^3 + 225*a*tan(1/2*d*x + 1/
2*c)^2 - 1920*(d*x + c)*a - 600*a*log(abs(tan(1/2*d*x + 1/2*c))) + 1320*a*t
an(1/2*d*x + 1/2*c) + (1470*a*tan(1/2*d*x + 1/2*c)^6 - 1320*a*tan(1/2*d*x +
 1/2*c)^5 - 225*a*tan(1/2*d*x + 1/2*c)^4 + 140*a*tan(1/2*d*x + 1/2*c)^3 + 4
5*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*tan(1/2*d*x + 1/2*c) - 5*a)/tan(1/2*d*x +
 1/2*c)^6)/d
```

3.583 $\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=96

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{5a\cot(c+d}{16}$$

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^7)/(7*d) - (5*a*Cot[c
+ d*x]*Csc[c + d*x])/(16*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a
*Cot[c + d*x]^5*Csc[c + d*x])/(6*d)
```

Rubi [A] time = 0.140451, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{5a\cot(c+d}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^7)/(7*d) - (5*a*Cot[c
+ d*x]*Csc[c + d*x])/(16*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a
*Cot[c + d*x]^5*Csc[c + d*x])/(6*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc(c+dx)\,dx + a\int \cot^6(c+dx)\csc^2(c+dx)\,dx$$
$$= -\frac{a\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{1}{6}(5a)\int \cot^4(c+dx)\csc(c+dx)\,dx +$$
$$= -\frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot^3(c+dx)\csc(c+dx)}{24d} - \frac{a\cot^5(c+dx)\csc(}{6d}$$
$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{5a\cot(c+dx)\csc(c+dx)}{16d} + \frac{5a\cot^3(c+dx)\csc(}{24d}$$
$$= \frac{5a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^7(c+dx)}{7d} - \frac{5a\cot(c+dx)\csc(c+d}{16d}$$

Mathematica [A] time = 0.0461955, size = 175, normalized size = 1.82

$$-\frac{a\cot^7(c+dx)}{7d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{11a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\sec^4\left(\frac{1}{2}(c+d\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^7)/(7*d) - (11*a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c +
d*x)/2]^4)/(32*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) + (5*a*Log[Cos[(c + d*x)
/2]])/(16*d) - (5*a*Log[Sin[(c + d*x)/2]])/(16*d) + (11*a*Sec[(c + d*x)/2]^
2)/(64*d) - (a*Sec[(c + d*x)/2]^4)/(32*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A] time = 0.068, size = 152, normalized size = 1.6

$$-\frac{a(\cos(dx+c))^7}{6d(\sin(dx+c))^6} + \frac{a(\cos(dx+c))^7}{24d(\sin(dx+c))^4} - \frac{a(\cos(dx+c))^7}{16d(\sin(dx+c))^2} - \frac{a(\cos(dx+c))^5}{16d} - \frac{5a(\cos(dx+c))^3}{48d} - \frac{5\cos(dx+c)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/d*a/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*a/sin(d*x+c)^4*cos(d*x+c)^7-1/16/
d*a/sin(d*x+c)^2*cos(d*x+c)^7-1/16*a*cos(d*x+c)^5/d-5/48*a*cos(d*x+c)^3/d-5
/16*a*cos(d*x+c)/d-5/16/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/7/d*a/sin(d*x+c)^7*
cos(d*x+c)^7
```

Maxima [A] time = 1.02275, size = 143, normalized size = 1.49

$$\frac{7a\left(\frac{2\left(33\cos(dx+c)^5-40\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1} + 15\log(\cos(dx+c)+1) - 15\log(\cos(dx+c)-1)\right) - \frac{96a}{\tan(dx+c)^7}}{672d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/672*(7*a*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x +
c) + 1) - 15*log(cos(d*x + c) - 1)) - 96*a/tan(d*x + c)^7)/d
```

Fricas [B] time = 1.26202, size = 562, normalized size = 5.85

$$\frac{96\,a\cos\left(dx+c\right)^{7}+105\left(a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}+3\,a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+\cdots\right.}{672\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/672*(96*a*cos(d*x + c)^7 + 105*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3
*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(a*co
s(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 14*(33*a*cos(d*x + c)^5 - 40*a*cos(d*x + c)^3
+ 15*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4
+ 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18765, size = 308, normalized size = 3.21

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2688*(3*a*tan(1/2*d*x + 1/2*c)^7 + 7*a*tan(1/2*d*x + 1/2*c)^6 - 21*a*tan(
1/2*d*x + 1/2*c)^5 - 63*a*tan(1/2*d*x + 1/2*c)^4 + 63*a*tan(1/2*d*x + 1/2*c
)^3 + 315*a*tan(1/2*d*x + 1/2*c)^2 - 840*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 105*a*tan(1/2*d*x + 1/2*c) + (2178*a*tan(1/2*d*x + 1/2*c)^7 + 105*a*tan(1/
2*d*x + 1/2*c)^6 - 315*a*tan(1/2*d*x + 1/2*c)^5 - 63*a*tan(1/2*d*x + 1/2*c)
^4 + 63*a*tan(1/2*d*x + 1/2*c)^3 + 21*a*tan(1/2*d*x + 1/2*c)^2 - 7*a*tan(1/
2*d*x + 1/2*c) - 3*a)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.584 $\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{5a\cot(c+dx)}{64}$$

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^7)/(7*d) + (5*a*Cot[c
 + d*x]*Csc[c + d*x])/(128*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) +
(5*a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a*Cot[c + d*x]^5*Csc[c + d*x]
^3)/(8*d)
```

Rubi [A] time = 0.183989, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{5a\cot(c+dx)}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^7)/(7*d) + (5*a*Cot[c
 + d*x]*Csc[c + d*x])/(128*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) +
(5*a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a*Cot[c + d*x]^5*Csc[c + d*x]
^3)/(8*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc^2(c+dx)\,dx + a\int \cot^6(c+dx)\csc^3(c+dx)$$
$$= -\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{1}{8}(5a)\int \cot^4(c+dx)\csc^3(c+dx$$
$$= -\frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a\cot^5(c+dx)}{8d}$$
$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{64d} + \frac{5a\cot^3(c+dx)}{48d}$$
$$= -\frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot(c+dx)\csc(c+dx)}{128d} - \frac{5a\cot(c+dx)\csc}{64d}$$
$$= \frac{5a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot(c+dx)\csc(c}{128d}$$

Mathematica [A] time = 0.0585969, size = 215, normalized size = 1.76

$$-\frac{a\cot^7(c+dx)}{7d} - \frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{7a\csc^6\left(\frac{1}{2}(c+dx)\right)}{1536d} - \frac{15a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\sec^8\left(\frac{1}{2}\right.}{204}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^7)/(7*d) + (5*a*Csc[(c + d*x)/2]^2)/(512*d) - (15*a*Csc[(c
 + d*x)/2]^4)/(1024*d) + (7*a*Csc[(c + d*x)/2]^6)/(1536*d) - (a*Csc[(c + d*
x)/2]^8)/(2048*d) + (5*a*Log[Cos[(c + d*x)/2]])/(128*d) - (5*a*Log[Sin[(c +
 d*x)/2]])/(128*d) - (5*a*Sec[(c + d*x)/2]^2)/(512*d) + (15*a*Sec[(c + d*x)
/2]^4)/(1024*d) - (7*a*Sec[(c + d*x)/2]^6)/(1536*d) + (a*Sec[(c + d*x)/2]^8
)/(2048*d)
```

Maple [A] time = 0.066, size = 174, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^7}{7d(\sin(dx+c))^7} - \frac{a(\cos(dx+c))^7}{8d(\sin(dx+c))^8} - \frac{a(\cos(dx+c))^7}{48d(\sin(dx+c))^6} + \frac{a(\cos(dx+c))^7}{192d(\sin(dx+c))^4} - \frac{a(\cos(dx+c))^7}{128d(\sin(dx+c))^2} - \frac{a(\cos}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^7-1/8/d*a/sin(d*x+c)^8*cos(d*x+c)^7-1/48/d
*a/sin(d*x+c)^6*cos(d*x+c)^7+1/192/d*a/sin(d*x+c)^4*cos(d*x+c)^7-1/128/d*a/
```

```
sin(d*x+c)^2*cos(d*x+c)^7-1/128*a*cos(d*x+c)^5/d-5/384*a*cos(d*x+c)^3/d-5/1
28*a*cos(d*x+c)/d-5/128/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.18098, size = 170, normalized size = 1.39

$$-\frac{7\,a\left(\frac{2\left(15\,\cos(dx+c)^{7}+73\,\cos(dx+c)^{5}-55\,\cos(dx+c)^{3}+15\,\cos(dx+c)\right)}{\cos(dx+c)^{8}-4\,\cos(dx+c)^{6}+6\,\cos(dx+c)^{4}-4\,\cos(dx+c)^{2}+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{768\,a}{\tan(dx+c}}{5376\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/5376*(7*a*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3
+ 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)
) + 768*a/tan(d*x + c)^7)/d
```

Fricas [B] time = 1.16699, size = 625, normalized size = 5.12

$$-\frac{768\,a\cos\left(dx+c\right)^{7}\sin\left(dx+c\right)+210\,a\cos\left(dx+c\right)^{7}+1022\,a\cos\left(dx+c\right)^{5}-770\,a\cos\left(dx+c\right)^{3}+210\,a\cos\left(dx+c\right)-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/5376*(768*a*cos(d*x + c)^7*sin(d*x + c) + 210*a*cos(d*x + c)^7 + 1022*a*
cos(d*x + c)^5 - 770*a*cos(d*x + c)^3 + 210*a*cos(d*x + c) - 105*(a*cos(d*x
 + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)
*log(1/2*cos(d*x + c) + 1/2) + 105*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 +
 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2))
/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x
+ c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34801, size = 346, normalized size = 2.84

$$\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-112\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-336\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+168\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/43008*(21*a*tan(1/2*d*x + 1/2*c)^8 + 48*a*tan(1/2*d*x + 1/2*c)^7 - 112*a*
tan(1/2*d*x + 1/2*c)^6 - 336*a*tan(1/2*d*x + 1/2*c)^5 + 168*a*tan(1/2*d*x +
 1/2*c)^4 + 1008*a*tan(1/2*d*x + 1/2*c)^3 + 336*a*tan(1/2*d*x + 1/2*c)^2 -
1680*a*log(abs(tan(1/2*d*x + 1/2*c))) - 1680*a*tan(1/2*d*x + 1/2*c) + (4566
*a*tan(1/2*d*x + 1/2*c)^8 + 1680*a*tan(1/2*d*x + 1/2*c)^7 - 336*a*tan(1/2*d
*x + 1/2*c)^6 - 1008*a*tan(1/2*d*x + 1/2*c)^5 - 168*a*tan(1/2*d*x + 1/2*c)^
4 + 336*a*tan(1/2*d*x + 1/2*c)^3 + 112*a*tan(1/2*d*x + 1/2*c)^2 - 48*a*tan(
1/2*d*x + 1/2*c) - 21*a)/tan(1/2*d*x + 1/2*c)^8)/d
```

3.585 $\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=138

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d}$$

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c +
 d*x]^9)/(9*d) + (5*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (5*a*Cot[c + d*x
]*Csc[c + d*x]^3)/(64*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a*
Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rubi [A] time = 0.198958, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```


```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c +
 d*x]^9)/(9*d) + (5*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (5*a*Cot[c + d*x
]*Csc[c + d*x]^3)/(64*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a*
Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc^3(c+dx)\,dx + a\int \cot^6(c+dx)\csc^4(c+dx)$$

$$= -\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{1}{8}(5a)\int \cot^4(c+dx)\csc^3(c+dx$$

$$= \frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d} + \frac{1}{16}($$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{64d} +$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{5a\cot(c+dx)\csc(c+dx)}{128d} -$$

$$= \frac{5a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{5a\,c}{}$$

Mathematica [B] time = 0.0843953, size = 301, normalized size = 2.18

$$\frac{2a\cot(c+dx)}{63d} - \frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{7a\csc^6\left(\frac{1}{2}(c+dx)\right)}{1536d} - \frac{15a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\sec^8\left(\frac{1}{2}(c\right.}{2048}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(63*d) + (5*a*Csc[(c + d*x)/2]^2)/(512*d) - (15*a*Csc[(c
 + d*x)/2]^4)/(1024*d) + (7*a*Csc[(c + d*x)/2]^6)/(1536*d) - (a*Csc[(c + d*
x)/2]^8)/(2048*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(63*d) - (5*a*Cot[c + d
*x]*Csc[c + d*x]^4)/(21*d) + (19*a*Cot[c + d*x]*Csc[c + d*x]^6)/(63*d) - (a
*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) + (5*a*Log[Cos[(c + d*x)/2]])/(128*d) -
 (5*a*Log[Sin[(c + d*x)/2]])/(128*d) - (5*a*Sec[(c + d*x)/2]^2)/(512*d) + (
15*a*Sec[(c + d*x)/2]^4)/(1024*d) - (7*a*Sec[(c + d*x)/2]^6)/(1536*d) + (a*
Sec[(c + d*x)/2]^8)/(2048*d)
```

Maple [A] time = 0.069, size = 196, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^7}{8d(\sin(dx+c))^8} - \frac{a(\cos(dx+c))^7}{48d(\sin(dx+c))^6} + \frac{a(\cos(dx+c))^7}{192d(\sin(dx+c))^4} - \frac{a(\cos(dx+c))^7}{128d(\sin(dx+c))^2} - \frac{a(\cos(dx+c))^5}{128d} - \frac{5a(\cos}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/d*a/sin(d*x+c)^8*cos(d*x+c)^7-1/48/d*a/sin(d*x+c)^6*cos(d*x+c)^7+1/192
/d*a/sin(d*x+c)^4*cos(d*x+c)^7-1/128/d*a/sin(d*x+c)^2*cos(d*x+c)^7-1/128*a*
cos(d*x+c)^5/d-5/384*a*cos(d*x+c)^3/d-5/128*a*cos(d*x+c)/d-5/128/d*a*ln(csc
(d*x+c)-cot(d*x+c))-1/9/d*a/sin(d*x+c)^9*cos(d*x+c)^7-2/63/d*a/sin(d*x+c)^7
*cos(d*x+c)^7
```

Maxima [A] time = 1.07709, size = 186, normalized size = 1.35

$$-\frac{21\,a\left(\frac{2\left(15\,\cos(dx+c)^{7}+73\,\cos(dx+c)^{5}-55\,\cos(dx+c)^{3}+15\,\cos(dx+c)\right)}{\cos(dx+c)^{8}-4\,\cos(dx+c)^{6}+6\,\cos(dx+c)^{4}-4\,\cos(dx+c)^{2}+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{256\left(9\,\mathrm{ta}\right.}{\mathrm{ta}}}{16128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/16128*(21*a*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^
3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1)) + 256*(9*tan(d*x + c)^2 + 7)*a/tan(d*x + c)^9)/d
```

Fricas [B] time = 1.27641, size = 713, normalized size = 5.17

$$\frac{512\,a\cos\left(dx+c\right)^{9}-2304\,a\cos\left(dx+c\right)^{7}+315\left(a\cos\left(dx+c\right)^{8}-4\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/16128*(512*a*cos(d*x + c)^9 - 2304*a*cos(d*x + c)^7 + 315*(a*cos(d*x + c)
^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 315*(a*cos(d*x + c)^8 - 4*a*cos(d*x
+ c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 42*(15*a*cos(d*x + c)^7 + 73*a*cos(d*x + c)^5 - 55*a
*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31574, size = 346, normalized size = 2.51

$$28\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-108\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-336\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+504\,a\tan\left(\frac{1}{2}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/129024*(28*a*tan(1/2*d*x + 1/2*c)^9 + 63*a*tan(1/2*d*x + 1/2*c)^8 - 108*a
*tan(1/2*d*x + 1/2*c)^7 - 336*a*tan(1/2*d*x + 1/2*c)^6 + 504*a*tan(1/2*d*x
+ 1/2*c)^4 + 672*a*tan(1/2*d*x + 1/2*c)^3 + 1008*a*tan(1/2*d*x + 1/2*c)^2 -
 5040*a*log(abs(tan(1/2*d*x + 1/2*c))) - 1512*a*tan(1/2*d*x + 1/2*c) + (142
58*a*tan(1/2*d*x + 1/2*c)^9 + 1512*a*tan(1/2*d*x + 1/2*c)^8 - 1008*a*tan(1/
2*d*x + 1/2*c)^7 - 672*a*tan(1/2*d*x + 1/2*c)^6 - 504*a*tan(1/2*d*x + 1/2*c
)^5 + 336*a*tan(1/2*d*x + 1/2*c)^3 + 108*a*tan(1/2*d*x + 1/2*c)^2 - 63*a*ta
n(1/2*d*x + 1/2*c) - 28*a)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.586 $\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=160

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d}$$

```
[Out] (3*a*ArcTanh[Cos[c + d*x]])/(256*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c +
 d*x]^9)/(9*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(256*d) + (a*Cot[c + d*x]*
Csc[c + d*x]^3)/(128*d) - (a*Cot[c + d*x]*Csc[c + d*x]^5)/(32*d) + (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rubi [A] time = 0.209296, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 14}

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```


```
[Out] (3*a*ArcTanh[Cos[c + d*x]])/(256*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c +
 d*x]^9)/(9*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(256*d) + (a*Cot[c + d*x]*
Csc[c + d*x]^3)/(128*d) - (a*Cot[c + d*x]*Csc[c + d*x]^5)/(32*d) + (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc^4(c+dx)\,dx + a\int \cot^6(c+dx)\csc^5(c+dx)$$

$$= -\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{1}{2}a\int \cot^4(c+dx)\csc^5(c+dx)\,dx$$

$$= \frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} + \frac{1}{16}(3$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{32d} +$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{a\cot(c+dx)\csc^3(c+dx)}{128d} -$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{3a\cot(c+dx)\csc(c+dx)}{256d} +$$

$$= \frac{3a\tanh^{-1}(\cos(c+dx))}{256d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{3a\,c}{}$$

Mathematica [B] time = 0.0917636, size = 341, normalized size = 2.13

$$\frac{2a\cot(c+dx)}{63d} - \frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)}{10240d} + \frac{3a\csc^8\left(\frac{1}{2}(c+dx)\right)}{4096d} - \frac{3a\csc^6\left(\frac{1}{2}(c+dx)\right)}{2048d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} + \frac{3a\csc^2\left(\frac{1}{2}(c\right.}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(63*d) + (3*a*Csc[(c + d*x)/2]^2)/(1024*d) - (a*Csc[(c +
 d*x)/2]^4)/(1024*d) - (3*a*Csc[(c + d*x)/2]^6)/(2048*d) + (3*a*Csc[(c + d*
x)/2]^8)/(4096*d) - (a*Csc[(c + d*x)/2]^10)/(10240*d) + (a*Cot[c + d*x]*Csc
[c + d*x]^2)/(63*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^4)/(21*d) + (19*a*Cot[
c + d*x]*Csc[c + d*x]^6)/(63*d) - (a*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) + (
3*a*Log[Cos[(c + d*x)/2]])/(256*d) - (3*a*Log[Sin[(c + d*x)/2]])/(256*d) -
(3*a*Sec[(c + d*x)/2]^2)/(1024*d) + (a*Sec[(c + d*x)/2]^4)/(1024*d) + (3*a*
Sec[(c + d*x)/2]^6)/(2048*d) - (3*a*Sec[(c + d*x)/2]^8)/(4096*d) + (a*Sec[(
c + d*x)/2]^10)/(10240*d)
```

Maple [A] time = 0.066, size = 218, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^7}{9d(\sin(dx+c))^9} - \frac{2a(\cos(dx+c))^7}{63d(\sin(dx+c))^7} - \frac{a(\cos(dx+c))^7}{10d(\sin(dx+c))^{10}} - \frac{3a(\cos(dx+c))^7}{80d(\sin(dx+c))^8} - \frac{a(\cos(dx+c))^7}{160d(\sin(dx+c))^6} + \frac{a(c}{640d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x)

[Out] -1/9/d*a/sin(d*x+c)^9*cos(d*x+c)^7-2/63/d*a/sin(d*x+c)^7*cos(d*x+c)^7-1/10/d*a/sin(d*x+c)^10*cos(d*x+c)^7-3/80/d*a/sin(d*x+c)^8*cos(d*x+c)^7-1/160/d*a/sin(d*x+c)^6*cos(d*x+c)^7+1/640/d*a/sin(d*x+c)^4*cos(d*x+c)^7-3/1280/d*a/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*a*cos(d*x+c)^5/d-1/256*a*cos(d*x+c)^3/d-3/256*a*cos(d*x+c)/d-3/256/d*a*ln(csc(d*x+c)-cot(d*x+c))

Maxima [A] time = 1.03815, size = 213, normalized size = 1.33

$$-\frac{63\,a\left(\frac{2\,\left(15\,\cos\left(dx+c\right)^{9}-70\,\cos\left(dx+c\right)^{7}-128\,\cos\left(dx+c\right)^{5}+70\,\cos\left(dx+c\right)^{3}-15\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{10}-5\,\cos\left(dx+c\right)^{8}+10\,\cos\left(dx+c\right)^{6}-10\,\cos\left(dx+c\right)^{4}+5\,\cos\left(dx+c\right)^{2}-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-\right.\right.}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/161280*(63*a*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2560*(9*tan(d*x + c)^2 + 7)*a/tan(d*x + c)^9)/d

---

Fricas [B] time = 1.1965, size = 798, normalized size = 4.99

$$-\frac{1890\,a\cos\left(dx+c\right)^{9}-8820\,a\cos\left(dx+c\right)^{7}-16128\,a\cos\left(dx+c\right)^{5}+8820\,a\cos\left(dx+c\right)^{3}-1890\,a\cos\left(dx+c\right)-945\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/161280*(1890*a*cos(d*x + c)^9 - 8820*a*cos(d*x + c)^7 - 16128*a*cos(d*x + c)^5 + 8820*a*cos(d*x + c)^3 - 1890*a*cos(d*x + c) - 945*(a*cos(d*x + c)^10 - 5*a*cos(d*x + c)^8 + 10*a*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 945*(a*cos(d*x + c)^10 - 5*a*cos(d*x + c)^8 + 10*a*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2560*(2*a*cos(d*x + c)^9 - 9*a*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+a*sin(d*x+c)),x)

```
[Out] Timed out
```

Giac [A] time = 1.36023, size = 383, normalized size = 2.39

$$\frac{126\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1080\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-630\,a\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1290240*(126*a*tan(1/2*d*x + 1/2*c)^10 + 280*a*tan(1/2*d*x + 1/2*c)^9 - 3
15*a*tan(1/2*d*x + 1/2*c)^8 - 1080*a*tan(1/2*d*x + 1/2*c)^7 - 630*a*tan(1/2
*d*x + 1/2*c)^6 + 2520*a*tan(1/2*d*x + 1/2*c)^4 + 6720*a*tan(1/2*d*x + 1/2*
c)^3 + 1260*a*tan(1/2*d*x + 1/2*c)^2 - 15120*a*log(abs(tan(1/2*d*x + 1/2*c)
)) - 15120*a*tan(1/2*d*x + 1/2*c) + (44286*a*tan(1/2*d*x + 1/2*c)^10 + 1512
0*a*tan(1/2*d*x + 1/2*c)^9 - 1260*a*tan(1/2*d*x + 1/2*c)^8 - 6720*a*tan(1/2
*d*x + 1/2*c)^7 - 2520*a*tan(1/2*d*x + 1/2*c)^6 + 630*a*tan(1/2*d*x + 1/2*c
)^4 + 1080*a*tan(1/2*d*x + 1/2*c)^3 + 315*a*tan(1/2*d*x + 1/2*c)^2 - 280*a*
tan(1/2*d*x + 1/2*c) - 126*a)/tan(1/2*d*x + 1/2*c)^10)/d
```

3.587 $\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=176

$$-\frac{a\cot^{11}(c+dx)}{11d}-\frac{2a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a\cot^3(c$$

```
[Out] (3*a*ArcTanh[Cos[c + d*x]])/(256*d) - (a*Cot[c + d*x]^7)/(7*d) - (2*a*Cot[c
 + d*x]^9)/(9*d) - (a*Cot[c + d*x]^11)/(11*d) + (3*a*Cot[c + d*x]*Csc[c + d
*x])/(256*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) - (a*Cot[c + d*x]*Cs
c[c + d*x]^5)/(32*d) + (a*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
+ d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rubi [A] time = 0.217995, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 270, 2611, 3768, 3770}

$$-\frac{a\cot^{11}(c+dx)}{11d}-\frac{2a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a\cot^3(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```


```
[Out] (3*a*ArcTanh[Cos[c + d*x]])/(256*d) - (a*Cot[c + d*x]^7)/(7*d) - (2*a*Cot[c
 + d*x]^9)/(9*d) - (a*Cot[c + d*x]^11)/(11*d) + (3*a*Cot[c + d*x]*Csc[c + d
*x])/(256*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) - (a*Cot[c + d*x]*Cs
c[c + d*x]^5)/(32*d) + (a*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
+ d*x]^5*Csc[c + d*x]^5)/(10*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc^5(c+dx)\,dx + a\int \cot^6(c+dx)\csc^6(c+dx)$$

$$= -\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{1}{2}a\int \cot^4(c+dx)\csc^5(c+dx)\,dx$$

$$= \frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} + \frac{1}{16}(3$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\cot^{11}(c+dx)}{11d} - \frac{a\cot(c+}{}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\cot^{11}(c+dx)}{11d} + \frac{a\cot(c+}{}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\cot^{11}(c+dx)}{11d} + \frac{3a\cot(c-}{}$$

$$= \frac{3a\tanh^{-1}(\cos(c+dx))}{256d} - \frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\,c}{}$$

Mathematica [B] time = 0.0990616, size = 363, normalized size = 2.06

$$\frac{8a\cot(c+dx)}{693d} - \frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)}{10240d} + \frac{3a\csc^8\left(\frac{1}{2}(c+dx)\right)}{4096d} - \frac{3a\csc^6\left(\frac{1}{2}(c+dx)\right)}{2048d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} + \frac{3a\csc^2\left(\frac{1}{2}(c\right.}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (8*a*Cot[c + d*x])/(693*d) + (3*a*Csc[(c + d*x)/2]^2)/(1024*d) - (a*Csc[(c
+ d*x)/2]^4)/(1024*d) - (3*a*Csc[(c + d*x)/2]^6)/(2048*d) + (3*a*Csc[(c + d
*x)/2]^8)/(4096*d) - (a*Csc[(c + d*x)/2]^10)/(10240*d) + (4*a*Cot[c + d*x]*
Csc[c + d*x]^2)/(693*d) + (a*Cot[c + d*x]*Csc[c + d*x]^4)/(231*d) - (113*a*
Cot[c + d*x]*Csc[c + d*x]^6)/(693*d) + (23*a*Cot[c + d*x]*Csc[c + d*x]^8)/(
99*d) - (a*Cot[c + d*x]*Csc[c + d*x]^10)/(11*d) + (3*a*Log[Cos[(c + d*x)/2]
])/(256*d) - (3*a*Log[Sin[(c + d*x)/2]])/(256*d) - (3*a*Sec[(c + d*x)/2]^2)
/(1024*d) + (a*Sec[(c + d*x)/2]^4)/(1024*d) + (3*a*Sec[(c + d*x)/2]^6)/(204
8*d) - (3*a*Sec[(c + d*x)/2]^8)/(4096*d) + (a*Sec[(c + d*x)/2]^10)/(10240*d
)
```

Maple [A] time = 0.07, size = 240, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^7}{10d(\sin(dx+c))^{10}}-\frac{3a(\cos(dx+c))^7}{80d(\sin(dx+c))^8}-\frac{a(\cos(dx+c))^7}{160d(\sin(dx+c))^6}+\frac{a(\cos(dx+c))^7}{640d(\sin(dx+c))^4}-\frac{3a(\cos(dx+c))^7}{1280d(\sin(dx+c))^2}-3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/10/d*a/sin(d*x+c)^10*cos(d*x+c)^7-3/80/d*a/sin(d*x+c)^8*cos(d*x+c)^7-1/1
60/d*a/sin(d*x+c)^6*cos(d*x+c)^7+1/640/d*a/sin(d*x+c)^4*cos(d*x+c)^7-3/1280
/d*a/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*a*cos(d*x+c)^5/d-1/256*a*cos(d*x+c)^3
/d-3/256*a*cos(d*x+c)/d-3/256/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/11/d*a/sin(d*
x+c)^11*cos(d*x+c)^7-4/99/d*a/sin(d*x+c)^9*cos(d*x+c)^7-8/693/d*a/sin(d*x+c
)^7*cos(d*x+c)^7
```

Maxima [A] time = 1.09331, size = 227, normalized size = 1.29

$$-\frac{693a\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7-128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)\right.}{1774080d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/1774080*(693*a*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
 c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
 c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2560*(99*tan(d*x + c)^4
 + 154*tan(d*x + c)^2 + 63)*a/tan(d*x + c)^11)/d
```

Fricas [B] time = 1.31466, size = 894, normalized size = 5.08

$$\frac{20480a\cos(dx+c)^{11}-112640a\cos(dx+c)^9+253440a\cos(dx+c)^7+10395\left(a\cos(dx+c)^{10}-5a\cos(dx+c)^8+10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1774080*(20480*a*cos(d*x + c)^11 - 112640*a*cos(d*x + c)^9 + 253440*a*cos
(d*x + c)^7 + 10395*(a*cos(d*x + c)^10 - 5*a*cos(d*x + c)^8 + 10*a*cos(d*x
+ c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 10395*(a*cos(d*x + c)^10 - 5*a*cos(d*x + c)^8 + 10*a
*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a)*log(-1/2*co
s(d*x + c) + 1/2)*sin(d*x + c) - 1386*(15*a*cos(d*x + c)^9 - 70*a*cos(d*x +
 c)^7 - 128*a*cos(d*x + c)^5 + 70*a*cos(d*x + c)^3 - 15*a*cos(d*x + c))*sin
(d*x + c))/((d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 -
 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30875, size = 459, normalized size = 2.61

$$630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1386\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-770\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3465\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-4950\,a\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/14192640*(630*a*tan(1/2*d*x + 1/2*c)^11 + 1386*a*tan(1/2*d*x + 1/2*c)^10
- 770*a*tan(1/2*d*x + 1/2*c)^9 - 3465*a*tan(1/2*d*x + 1/2*c)^8 - 4950*a*tan
(1/2*d*x + 1/2*c)^7 - 6930*a*tan(1/2*d*x + 1/2*c)^6 + 6930*a*tan(1/2*d*x +
1/2*c)^5 + 27720*a*tan(1/2*d*x + 1/2*c)^4 + 23100*a*tan(1/2*d*x + 1/2*c)^3
+ 13860*a*tan(1/2*d*x + 1/2*c)^2 - 166320*a*log(abs(tan(1/2*d*x + 1/2*c)))
- 69300*a*tan(1/2*d*x + 1/2*c) + (502266*a*tan(1/2*d*x + 1/2*c)^11 + 69300*
a*tan(1/2*d*x + 1/2*c)^10 - 13860*a*tan(1/2*d*x + 1/2*c)^9 - 23100*a*tan(1/
2*d*x + 1/2*c)^8 - 27720*a*tan(1/2*d*x + 1/2*c)^7 - 6930*a*tan(1/2*d*x + 1/
2*c)^6 + 6930*a*tan(1/2*d*x + 1/2*c)^5 + 4950*a*tan(1/2*d*x + 1/2*c)^4 + 34
65*a*tan(1/2*d*x + 1/2*c)^3 + 770*a*tan(1/2*d*x + 1/2*c)^2 - 1386*a*tan(1/2
*d*x + 1/2*c) - 630*a)/tan(1/2*d*x + 1/2*c)^11)/d
```

3.588 $\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=209

$$-\frac{2a^2\cos^{11}(c+dx)}{11d}+\frac{4a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{17a^2\sin^3(c+dx)\cos^7(c+dx)}{120d}$$

```
[Out] (17*a^2*x)/1024 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (4*a^2*Cos[c + d*x]^9)/(9*
d) - (2*a^2*Cos[c + d*x]^11)/(11*d) + (17*a^2*Cos[c + d*x]*Sin[c + d*x])/(1
024*d) + (17*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(1536*d) + (17*a^2*Cos[c + d*
x]^5*Sin[c + d*x])/(1920*d) - (17*a^2*Cos[c + d*x]^7*Sin[c + d*x])/(320*d)
- (17*a^2*Cos[c + d*x]^7*Sin[c + d*x]^3)/(120*d) - (a^2*Cos[c + d*x]^7*Sin[
c + d*x]^5)/(12*d)
```

Rubi [A] time = 0.395716, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 270}

$$-\frac{2a^2\cos^{11}(c+dx)}{11d}+\frac{4a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{17a^2\sin^3(c+dx)\cos^7(c+dx)}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (17*a^2*x)/1024 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (4*a^2*Cos[c + d*x]^9)/(9*
d) - (2*a^2*Cos[c + d*x]^11)/(11*d) + (17*a^2*Cos[c + d*x]*Sin[c + d*x])/(1
024*d) + (17*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(1536*d) + (17*a^2*Cos[c + d*
x]^5*Sin[c + d*x])/(1920*d) - (17*a^2*Cos[c + d*x]^7*Sin[c + d*x])/(320*d)
- (17*a^2*Cos[c + d*x]^7*Sin[c + d*x]^3)/(120*d) - (a^2*Cos[c + d*x]^7*Sin[
c + d*x]^5)/(12*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^4(c+dx)+2a^2\cos^6(c+dx)\sin^5(c+dx)\right.$$

$$= a^2\int \cos^6(c+dx)\sin^4(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin^6(c+$$

$$= -\frac{a^2\cos^7(c+dx)\sin^3(c+dx)}{10d} - \frac{a^2\cos^7(c+dx)\sin^5(c+dx)}{12d} +$$

$$= -\frac{3a^2\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{17a^2\cos^7(c+dx)\sin^3(c+dx)}{120d}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+dx)}{11d} +$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+dx)}{11d} +$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+dx)}{11d} +$$

$$= \frac{3a^2x}{256} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+dx)}{11d}$$

$$= \frac{17a^2x}{1024} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+dx)}{11d}$$

Mathematica [A] time = 1.34498, size = 136, normalized size = 0.65

$$a^2(55440\sin(2(c+dx)) - 162855\sin(4(c+dx)) - 27720\sin(6(c+dx)) + 24255\sin(8(c+dx)) + 5544\sin(10(c+dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(166320*c + 471240*d*x - 554400*Cos[c + d*x] - 184800*Cos[3*(c + d*x)]
 + 55440*Cos[5*(c + d*x)] + 39600*Cos[7*(c + d*x)] - 6160*Cos[9*(c + d*x)]
- 5040*Cos[11*(c + d*x)] + 55440*Sin[2*(c + d*x)] - 162855*Sin[4*(c + d*x)]
 - 27720*Sin[6*(c + d*x)] + 24255*Sin[8*(c + d*x)] + 5544*Sin[10*(c + d*x)]
 - 1155*Sin[12*(c + d*x)]))/(28385280*d)
```

Maple [A] time = 0.04, size = 238, normalized size = 1.1

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^5(\cos(dx+c))^7}{12} - \frac{(\sin(dx+c))^3(\cos(dx+c))^7}{24} - \frac{\sin(dx+c)(\cos(dx+c))^7}{64} + \frac{\sin(dx+c)}{384}\right(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*cos(d*x+c)^7-1/
64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*
x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+2*a^2*(-1/11*sin(d*x+c)^4*cos(d*x+c)^
7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7)+a^2*(-1/10*sin(d*x+c)^
3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x
+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c))
```

Maxima [A] time = 1.04498, size = 186, normalized size = 0.89

$$-\frac{81920\left(63\cos(dx+c)^{11}-154\cos(dx+c)^{9}+99\cos(dx+c)^{7}\right)a^{2}-2772\left(32\sin(2dx+2c)^{5}+120dx+120c+5\sin\right.}{283}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/28385280*(81920*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x +
c)^7)*a^2 - 2772*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8
*c) - 40*sin(4*d*x + 4*c))*a^2 - 1155*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120
*c + 9*sin(8*d*x + 8*c) - 48*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A] time = 1.28049, size = 389, normalized size = 1.86

$$-\frac{645120a^{2}\cos(dx+c)^{11}-1576960a^{2}\cos(dx+c)^{9}+1013760a^{2}\cos(dx+c)^{7}-58905a^{2}dx+231\left(1280a^{2}\cos(dx+c\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3548160*(645120*a^2*cos(d*x + c)^11 - 1576960*a^2*cos(d*x + c)^9 + 10137
60*a^2*cos(d*x + c)^7 - 58905*a^2*d*x + 231*(1280*a^2*cos(d*x + c)^11 - 473
6*a^2*cos(d*x + c)^9 + 4272*a^2*cos(d*x + c)^7 - 136*a^2*cos(d*x + c)^5 - 1
70*a^2*cos(d*x + c)^3 - 255*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 89.7845, size = 656, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**12/1024 + 15*a**2*x*sin(c + d*x)**10*cos(
c + d*x)**2/512 + 3*a**2*x*sin(c + d*x)**10/256 + 75*a**2*x*sin(c + d*x)**8
```

```
*cos(c + d*x)**4/1024 + 15*a**2*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 25*
a**2*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 15*a**2*x*sin(c + d*x)**6*cos(
c + d*x)**4/128 + 75*a**2*x*sin(c + d*x)**4*cos(c + d*x)**8/1024 + 15*a**2*
x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**2*x*sin(c + d*x)**2*cos(c + d
*x)**10/512 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**2*x*cos(
c + d*x)**12/1024 + 3*a**2*x*cos(c + d*x)**10/256 + 5*a**2*sin(c + d*x)**11
*cos(c + d*x)/(1024*d) + 85*a**2*sin(c + d*x)**9*cos(c + d*x)**3/(3072*d) +
 3*a**2*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 33*a**2*sin(c + d*x)**7*cos(
c + d*x)**5/(512*d) + 7*a**2*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) - 33*a
**2*sin(c + d*x)**5*cos(c + d*x)**7/(512*d) + a**2*sin(c + d*x)**5*cos(c +
d*x)**5/(10*d) - 2*a**2*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 85*a**2*sin
(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 7*a**2*sin(c + d*x)**3*cos(c + d*x)
**7/(128*d) - 8*a**2*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 5*a**2*sin(c
+ d*x)*cos(c + d*x)**11/(1024*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**9/(256
*d) - 16*a**2*cos(c + d*x)**11/(693*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin
(c)**4*cos(c)**6, True))
```

Giac [A] time = 1.25871, size = 281, normalized size = 1.34

$$\frac{17}{1024}a^2x - \frac{a^2\cos(11\,dx + 11\,c)}{5632\,d} - \frac{a^2\cos(9\,dx + 9\,c)}{4608\,d} + \frac{5\,a^2\cos(7\,dx + 7\,c)}{3584\,d} + \frac{a^2\cos(5\,dx + 5\,c)}{512\,d} - \frac{5\,a^2\cos(3\,dx + 3\,c)}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 17/1024*a^2*x - 1/5632*a^2*cos(11*d*x + 11*c)/d - 1/4608*a^2*cos(9*d*x + 9*
c)/d + 5/3584*a^2*cos(7*d*x + 7*c)/d + 1/512*a^2*cos(5*d*x + 5*c)/d - 5/768
*a^2*cos(3*d*x + 3*c)/d - 5/256*a^2*cos(d*x + c)/d - 1/24576*a^2*sin(12*d*x
 + 12*c)/d + 1/5120*a^2*sin(10*d*x + 10*c)/d + 7/8192*a^2*sin(8*d*x + 8*c)/
d - 1/1024*a^2*sin(6*d*x + 6*c)/d - 47/8192*a^2*sin(4*d*x + 4*c)/d + 1/512*
a^2*sin(2*d*x + 2*c)/d
```

3.589 $\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=183

$$-\frac{a^2\cos^{11}(c+dx)}{11d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3a^2\sin(c+dx)\cos^7(c+dx)}{40d}+$$

```
[Out] (3*a^2*x)/128 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (a^2*Cos[c + d*x]^9)/(3*d) -
 (a^2*Cos[c + d*x]^11)/(11*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) +
 (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x
])/(80*d) - (3*a^2*Cos[c + d*x]^7*Sin[c + d*x])/(40*d) - (a^2*Cos[c + d*x]^
7*Sin[c + d*x]^3)/(5*d)
```

Rubi [A] time = 0.266328, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{a^2\cos^{11}(c+dx)}{11d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3a^2\sin(c+dx)\cos^7(c+dx)}{40d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (3*a^2*x)/128 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (a^2*Cos[c + d*x]^9)/(3*d) -
 (a^2*Cos[c + d*x]^11)/(11*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) +
 (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x
])/(80*d) - (3*a^2*Cos[c + d*x]^7*Sin[c + d*x])/(40*d) - (a^2*Cos[c + d*x]^
7*Sin[c + d*x]^3)/(5*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^3(c+dx)+2a^2\cos^6(c+dx)\sin^4(c+dx)\right.$$

$$= a^2\int \cos^6(c+dx)\sin^3(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin^5(c+$$

$$= -\frac{a^2\cos^7(c+dx)\sin^3(c+dx)}{5d} + \frac{1}{5}\left(3a^2\right)\int \cos^6(c+dx)\sin^2(c+$$

$$= -\frac{3a^2\cos^7(c+dx)\sin(c+dx)}{40d} - \frac{a^2\cos^7(c+dx)\sin^3(c+dx)}{5d} +$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos^9(c+dx)}{3d} - \frac{a^2\cos^{11}(c+dx)}{11d} + \frac{a^2\cos}{}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos^9(c+dx)}{3d} - \frac{a^2\cos^{11}(c+dx)}{11d} + \frac{a^2\cos}{}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos^9(c+dx)}{3d} - \frac{a^2\cos^{11}(c+dx)}{11d} + \frac{3a^2\cos}{}$$

$$= \frac{3a^2x}{128} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{a^2\cos^9(c+dx)}{3d} - \frac{a^2\cos^{11}(c+dx)}{11d} +$$

Mathematica [A] time = 0.994405, size = 126, normalized size = 0.69

$$\frac{a^2(4620\sin(2(c+dx)) - 9240\sin(4(c+dx)) - 2310\sin(6(c+dx)) + 1155\sin(8(c+dx)) + 462\sin(10(c+dx)) - 3927}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(27720*c + 27720*d*x - 39270*Cos[c + d*x] - 16170*Cos[3*(c + d*x)] + 1
155*Cos[5*(c + d*x)] + 2805*Cos[7*(c + d*x)] + 385*Cos[9*(c + d*x)] - 105*C
os[11*(c + d*x)] + 4620*Sin[2*(c + d*x)] - 9240*Sin[4*(c + d*x)] - 2310*Sin
[6*(c + d*x)] + 1155*Sin[8*(c + d*x)] + 462*Sin[10*(c + d*x)]))/(1182720*d)
```

Maple [A] time = 0.041, size = 172, normalized size = 0.9

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^7}{11}-\frac{4(\sin(dx+c))^2(\cos(dx+c))^7}{99}-\frac{8(\cos(dx+c))^7}{693}\right)+2a^2\left(-1/10(\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/
693*cos(d*x+c)^7)+2*a^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*co
s(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)
+3/256*d*x+3/256*c)+a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

Maxima [A] time = 1.01687, size = 157, normalized size = 0.86

$$-\frac{5120\left(63\cos(dx+c)^{11}-154\cos(dx+c)^9+99\cos(dx+c)^7\right)a^2-56320\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^2-693\left(\right.}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3548160*(5120*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)
^7)*a^2 - 56320*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2 - 693*(32*sin(2*d
*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a
^2)/d
```

Fricas [A] time = 1.29608, size = 335, normalized size = 1.83

$$-\frac{13440a^2\cos(dx+c)^{11}-49280a^2\cos(dx+c)^9+42240a^2\cos(dx+c)^7-3465a^2dx-231\left(128a^2\cos(dx+c)^9-176\right.}{147840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/147840*(13440*a^2*cos(d*x + c)^11 - 49280*a^2*cos(d*x + c)^9 + 42240*a^2
*cos(d*x + c)^7 - 3465*a^2*d*x - 231*(128*a^2*cos(d*x + c)^9 - 176*a^2*cos(
d*x + c)^7 + 8*a^2*cos(d*x + c)^5 + 10*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [A] time = 60.7454, size = 384, normalized size = 2.1

$$\begin{cases}\frac{3a^2x\sin^{10}(c+dx)}{128}+\frac{15a^2x\sin^8(c+dx)\cos^2(c+dx)}{128}+\frac{15a^2x\sin^6(c+dx)\cos^4(c+dx)}{64}+\frac{15a^2x\sin^4(c+dx)\cos^6(c+dx)}{64}+\frac{15a^2x\sin^2(c+dx)\cos^8(c+dx)}{128}+ \\ x(a\sin(c)+a)^2\sin^3(c)\cos^6(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/128 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/128 + 15*a**2*x*sin(c + d*x)**6*cos(c + d*x)**4/64 + 15*a**2*x*si
n(c + d*x)**4*cos(c + d*x)**6/64 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**
8/128 + 3*a**2*x*cos(c + d*x)**10/128 + 3*a**2*sin(c + d*x)**9*cos(c + d*x)
/(128*d) + 7*a**2*sin(c + d*x)**7*cos(c + d*x)**3/(64*d) + a**2*sin(c + d*x
)**5*cos(c + d*x)**5/(5*d) - a**2*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 7
*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(64*d) - 4*a**2*sin(c + d*x)**2*cos(c
 + d*x)**9/(63*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 3*a**2*sin
(c + d*x)*cos(c + d*x)**9/(128*d) - 8*a**2*cos(c + d*x)**11/(693*d) - 2*a**
2*cos(c + d*x)**9/(63*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**3*cos(c)*
*6, True))
```

Giac [A] time = 1.23996, size = 258, normalized size = 1.41

$$\frac{3}{128}a^2x - \frac{a^2\cos(11\,dx + 11\,c)}{11264\,d} + \frac{a^2\cos(9\,dx + 9\,c)}{3072\,d} + \frac{17\,a^2\cos(7\,dx + 7\,c)}{7168\,d} + \frac{a^2\cos(5\,dx + 5\,c)}{1024\,d} - \frac{7\,a^2\cos(3\,dx + 3}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/128*a^2*x - 1/11264*a^2*cos(11*d*x + 11*c)/d + 1/3072*a^2*cos(9*d*x + 9*c
)/d + 17/7168*a^2*cos(7*d*x + 7*c)/d + 1/1024*a^2*cos(5*d*x + 5*c)/d - 7/51
2*a^2*cos(3*d*x + 3*c)/d - 17/512*a^2*cos(d*x + c)/d + 1/2560*a^2*sin(10*d*
x + 10*c)/d + 1/1024*a^2*sin(8*d*x + 8*c)/d - 1/512*a^2*sin(6*d*x + 6*c)/d
- 1/128*a^2*sin(4*d*x + 4*c)/d + 1/256*a^2*sin(2*d*x + 2*c)/d
```

3.590 $\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=165

$$\frac{2a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{13a^2\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{13a^2\sin(c+dx)\cos}{480d}$$

```
[Out] (13*a^2*x)/256 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (2*a^2*Cos[c + d*x]^9)/(9*d
) + (13*a^2*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (13*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(384*d) + (13*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(480*d) - (13*a^2
*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (a^2*Cos[c + d*x]^7*Sin[c + d*x]^3)/
(10*d)
```

Rubi [A] time = 0.295518, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 14}

$$\frac{2a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{13a^2\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{13a^2\sin(c+dx)\cos}{480d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (13*a^2*x)/256 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (2*a^2*Cos[c + d*x]^9)/(9*d
) + (13*a^2*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (13*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(384*d) + (13*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(480*d) - (13*a^2
*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (a^2*Cos[c + d*x]^7*Sin[c + d*x]^3)/
(10*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^2(c+dx)+2a^2\cos^6(c+dx)\sin^3(c+dx)\right.$$

$$= a^2\int \cos^6(c+dx)\sin^2(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin^4(c+$$

$$= -\frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{a^2\cos^7(c+dx)\sin^3(c+dx)}{10d} + \frac{1}{8}$$

$$= \frac{a^2\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{13a^2\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{a^2}{}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{5a^2\cos^3(c+dx)\sin(c+}{192d}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{5a^2\cos(c+dx)\sin(c+}{128d}$$

$$= \frac{5a^2x}{128} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{13a^2\cos(c+dx)}{256d}$$

$$= \frac{13a^2x}{256} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{13a^2\cos(c+dx)}{256d}$$

Mathematica [A] time = 0.636394, size = 106, normalized size = 0.64

$$\frac{a^2(11340\sin(2(c+dx))-7560\sin(4(c+dx))-3990\sin(6(c+dx))-315\sin(8(c+dx))+126\sin(10(c+dx))-3024}{645120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(12600*c + 32760*d*x - 30240*Cos[c + d*x] - 13440*Cos[3*(c + d*x)] + 2
160*Cos[7*(c + d*x)] + 560*Cos[9*(c + d*x)] + 11340*Sin[2*(c + d*x)] - 7560
*Sin[4*(c + d*x)] - 3990*Sin[6*(c + d*x)] - 315*Sin[8*(c + d*x)] + 126*Sin[
10*(c + d*x)]))/(645120*d)
```

Maple [A] time = 0.04, size = 184, normalized size = 1.1

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^7}{10} - \frac{3\sin(dx+c)(\cos(dx+c))^7}{80} + \frac{\sin(dx+c)}{160}\left((\cos(dx+c))^5 + \frac{5(\cos(dx+c}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/16
0*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/25
6*c)+2*a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+a^2*(-1/8*sin
(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+5/128*d*x+5/128*c))
```

Maxima [A] time = 1.02504, size = 173, normalized size = 1.05

$$\frac{20480\left(7\cos\left(dx+c\right)^{9}-9\cos\left(dx+c\right)^{7}\right)a^{2}+63\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\sin\left(8\,dx+8\,c\right)-40\sin\left(4\,dx\right.\right.}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/645120*(20480*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2 + 63*(32*sin(2*d*
x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a^
2 + 210*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*
sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A] time = 1.22332, size = 300, normalized size = 1.82

$$\frac{17920\,a^{2}\cos\left(dx+c\right)^{9}-23040\,a^{2}\cos\left(dx+c\right)^{7}+4095\,a^{2}dx+21\left(384\,a^{2}\cos\left(dx+c\right)^{9}-1008\,a^{2}\cos\left(dx+c\right)^{7}+104\,a^{2}\mathrm{cc}\right.}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/80640*(17920*a^2*cos(d*x + c)^9 - 23040*a^2*cos(d*x + c)^7 + 4095*a^2*d*x
 + 21*(384*a^2*cos(d*x + c)^9 - 1008*a^2*cos(d*x + c)^7 + 104*a^2*cos(d*x +
 c)^5 + 130*a^2*cos(d*x + c)^3 + 195*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 36.5834, size = 529, normalized size = 3.21

$$\begin{cases}\frac{3a^{2}x\sin^{10}(c+dx)}{256}+\frac{15a^{2}x\sin^{8}(c+dx)\cos^{2}(c+dx)}{256}+\frac{5a^{2}x\sin^{8}(c+dx)}{128}+\frac{15a^{2}x\sin^{6}(c+dx)\cos^{4}(c+dx)}{128}+\frac{5a^{2}x\sin^{6}(c+dx)\cos^{2}(c+dx)}{32}+\frac{15a^{2}x\sin^{4}(c+}{1}\\ x\left(a\sin\left(c\right)+a\right)^{2}\sin^{2}\left(c\right)\cos^{6}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/256 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 5*a**2*x*sin(c + d*x)**8/128 + 15*a**2*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 5*a**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**2*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*
x)**4/64 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**2*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 3*a**2*x*cos(c + d*x)**10/256 + 5*a**2*x*cos(
c + d*x)**8/128 + 3*a**2*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**2*sin(
```

```
c + d*x)**7*cos(c + d*x)**3/(128*d) + 5*a**2*sin(c + d*x)**7*cos(c + d*x)/(
128*d) + a**2*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 55*a**2*sin(c + d*x)
**5*cos(c + d*x)**3/(384*d) - 7*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(128*d
) + 73*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) - 2*a**2*sin(c + d*x)**
2*cos(c + d*x)**7/(7*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 5*a
**2*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 4*a**2*cos(c + d*x)**9/(63*d), N
e(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**6, True))
```

Giac [A] time = 1.29614, size = 212, normalized size = 1.28

$$\frac{13}{256}a^2x + \frac{a^2\cos(9\,dx + 9\,c)}{1152\,d} + \frac{3\,a^2\cos(7\,dx + 7\,c)}{896\,d} - \frac{a^2\cos(3\,dx + 3\,c)}{48\,d} - \frac{3\,a^2\cos(dx + c)}{64\,d} + \frac{a^2\sin(10\,dx + 10\,c)}{5120\,d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 13/256*a^2*x + 1/1152*a^2*cos(9*d*x + 9*c)/d + 3/896*a^2*cos(7*d*x + 7*c)/d
 - 1/48*a^2*cos(3*d*x + 3*c)/d - 3/64*a^2*cos(d*x + c)/d + 1/5120*a^2*sin(1
0*d*x + 10*c)/d - 1/2048*a^2*sin(8*d*x + 8*c)/d - 19/3072*a^2*sin(6*d*x + 6
*c)/d - 3/256*a^2*sin(4*d*x + 4*c)/d + 9/512*a^2*sin(2*d*x + 2*c)/d
```

3.591 $\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=153

$$-\frac{a^2\cos^7(c+dx)}{28d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{36d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{96d}+\frac{5a^2}{}$$

```
[Out] (5*a^2*x)/64 - (a^2*Cos[c + d*x]^7)/(28*d) + (5*a^2*Cos[c + d*x]*Sin[c + d*
x])/(64*d) + (5*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(96*d) + (a^2*Cos[c + d*x]
^5*Sin[c + d*x])/(24*d) - (Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2)/(9*d) - (
Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(36*d)
```

Rubi [A] time = 0.153673, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2\cos^7(c+dx)}{28d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{36d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{96d}+\frac{5a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/64 - (a^2*Cos[c + d*x]^7)/(28*d) + (5*a^2*Cos[c + d*x]*Sin[c + d*
x])/(64*d) + (5*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(96*d) + (a^2*Cos[c + d*x]
^5*Sin[c + d*x])/(24*d) - (Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2)/(9*d) - (
Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(36*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx &= -\frac{\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}+\frac{2}{9}\int \cos^6(c+dx)(a+a\sin(c\\ &= -\frac{\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c+\right.}{36d}\\ &= -\frac{a^2\cos^7(c+dx)}{28d}-\frac{\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{\cos^7(c+d}{}\\ &= -\frac{a^2\cos^7(c+dx)}{28d}+\frac{a^2\cos^5(c+dx)\sin(c+dx)}{24d}-\frac{\cos^7(c+dx)(a}{9}\\ &= -\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos^3(c+dx)\sin(c+dx)}{96d}+\frac{a^2\cos^5(c+dx}{24}\\ &= -\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{64d}+\frac{5a^2\cos^3(c+dx}{96}\\ &= \frac{5a^2x}{64}-\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{64d}+\frac{5a^2\cos^3}{}\end{aligned}$$

Mathematica [A] time = 0.645294, size = 106, normalized size = 0.69

$$\frac{a^2(1008\sin(2(c+dx))-504\sin(4(c+dx))-336\sin(6(c+dx))-63\sin(8(c+dx))-3276\cos(c+dx)-1848\cos(3(}{32256d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(2520*c + 2520*d*x - 3276*Cos[c + d*x] - 1848*Cos[3*(c + d*x)] - 504*C
os[5*(c + d*x)] - 18*Cos[7*(c + d*x)] + 14*Cos[9*(c + d*x)] + 1008*Sin[2*(c
 + d*x)] - 504*Sin[4*(c + d*x)] - 336*Sin[6*(c + d*x)] - 63*Sin[8*(c + d*x)
]))/(32256*d)
```

Maple [A] time = 0.036, size = 116, normalized size = 0.8

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^7}{9}-\frac{2(\cos(dx+c))^7}{63}\right)+2a^2\left(-1/8\sin(dx+c)(\cos(dx+c))^7+1/48\left((\cos(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+2*a^2*(-1/8*sin
(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+5/128*d*x+5/128*c)-1/7*a^2*cos(d*x+c)^7)
```

Maxima [A] time = 1.03087, size = 126, normalized size = 0.82

$$-\frac{4608\,a^2\cos\left(dx+c\right)^7-512\left(7\,\cos\left(dx+c\right)^9-9\,\cos\left(dx+c\right)^7\right)a^2-21\left(64\,\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\,\sin\left(8\,d\right.\right.}{32256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32256*(4608*a^2*cos(d*x + c)^7 - 512*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^
7)*a^2 - 21*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) -
 24*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A] time = 1.21029, size = 251, normalized size = 1.64

$$\frac{448\,a^2\cos\left(dx+c\right)^9-1152\,a^2\cos\left(dx+c\right)^7+315\,a^2dx-21\left(48\,a^2\cos\left(dx+c\right)^7-8\,a^2\cos\left(dx+c\right)^5-10\,a^2\cos\left(dx+c\right)^3\right.}{4032\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(448*a^2*cos(d*x + c)^9 - 1152*a^2*cos(d*x + c)^7 + 315*a^2*d*x - 21
*(48*a^2*cos(d*x + c)^7 - 8*a^2*cos(d*x + c)^5 - 10*a^2*cos(d*x + c)^3 - 15
*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 22.3364, size = 282, normalized size = 1.84

$$\begin{cases}\frac{5a^2x\sin^8\left(c+dx\right)}{64}+\frac{5a^2x\sin^6\left(c+dx\right)\cos^2\left(c+dx\right)}{16}+\frac{15a^2x\sin^4\left(c+dx\right)\cos^4\left(c+dx\right)}{32}+\frac{5a^2x\sin^2\left(c+dx\right)\cos^6\left(c+dx\right)}{16}+\frac{5a^2x\cos^8\left(c+dx\right)}{64}+\frac{5a^2\sin^7\left(c+dx\right)}{64d} \\ x\left(a\sin\left(c\right)+a\right)^2\sin\left(c\right)\cos^6\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**8/64 + 5*a**2*x*sin(c + d*x)**6*cos(c + d
*x)**2/16 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 5*a**2*x*sin(c +
 d*x)**2*cos(c + d*x)**6/16 + 5*a**2*x*cos(c + d*x)**8/64 + 5*a**2*sin(c +
d*x)**7*cos(c + d*x)/(64*d) + 55*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(192*
d) + 73*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(192*d) - a**2*sin(c + d*x)**2
*cos(c + d*x)**7/(7*d) - 5*a**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) - 2*a**
2*cos(c + d*x)**9/(63*d) - a**2*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin
(c) + a)**2*sin(c)*cos(c)**6, True))
```

Giac [A] time = 1.22829, size = 212, normalized size = 1.39

$$\frac{5}{64}\,a^2x+\frac{a^2\cos\left(9\,dx+9\,c\right)}{2304\,d}-\frac{a^2\cos\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a^2\cos\left(5\,dx+5\,c\right)}{64\,d}-\frac{11\,a^2\cos\left(3\,dx+3\,c\right)}{192\,d}-\frac{13\,a^2\cos\left(dx+c\right)}{128\,d}-\frac{a^2\,\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/64*a^2*x + 1/2304*a^2*cos(9*d*x + 9*c)/d - 1/1792*a^2*cos(7*d*x + 7*c)/d
- 1/64*a^2*cos(5*d*x + 5*c)/d - 11/192*a^2*cos(3*d*x + 3*c)/d - 13/128*a^2*
cos(d*x + c)/d - 1/512*a^2*sin(8*d*x + 8*c)/d - 1/96*a^2*sin(6*d*x + 6*c)/d
 - 1/64*a^2*sin(4*d*x + 4*c)/d + 1/32*a^2*sin(2*d*x + 2*c)/d
```

3.592 $\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=161

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5a^2\sin(c+dx)\,c}{12d}$$

```
[Out] (5*a^2*x)/8 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) + (a^2*Cos[c + d*x]^5)/(5*d) - (a^2*Cos[c + d*x]^7)/(7
*d) + (5*a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a^2*Cos[c + d*x]^3*Sin[c
 + d*x])/(12*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.164433, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2873, 2635, 8, 2592, 302, 206, 2565, 30}

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5a^2\sin(c+dx)\,c}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/8 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) + (a^2*Cos[c + d*x]^5)/(5*d) - (a^2*Cos[c + d*x]^7)/(7
*d) + (5*a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a^2*Cos[c + d*x]^3*Sin[c
 + d*x])/(12*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(3*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(2a^2\cos^6(c+dx) + a^2\cos^5(c+dx)\cot(c+dx) + a^2\cos^6(c+\ldots\right.$$

$$= a^2\int \cos^5(c+dx)\cot(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin(c+dx)\ldots$$

$$= \frac{a^2\cos^5(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(5a^2\right)\int \cos^4(c+dx)\,dx - \frac{a^2\,\mathrm{Sul}\ldots}{}$$

$$= -\frac{a^2\cos^7(c+dx)}{7d} + \frac{5a^2\cos^3(c+dx)\sin(c+dx)}{12d} + \frac{a^2\cos^5(c+d\ldots}{3\ldots}$$

$$= \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} + \frac{a^2\cos^5(c+dx)}{5d} - \frac{a^2\cos^7(c+\ldots}{7d}$$

$$= \frac{5a^2x}{8} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+d\ldots}{3d}$$

Mathematica [A] time = 0.414069, size = 112, normalized size = 0.7

$$\frac{a^2\left(3150\sin(2(c+dx)) + 630\sin(4(c+dx)) + 70\sin(6(c+dx)) + 8715\cos(c+dx) + 665\cos(3(c+dx)) - 21\cos(5(c\ldots\right.}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(4200*c + 4200*d*x + 8715*Cos[c + d*x] + 665*Cos[3*(c + d*x)] - 21*Cos
[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 6720*Log[Cos[(c + d*x)/2]] + 6720*Log
[Sin[(c + d*x)/2]] + 3150*Sin[2*(c + d*x)] + 630*Sin[4*(c + d*x)] + 70*Sin[
6*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.073, size = 165, normalized size = 1.

$$-\frac{a^2(\cos(dx+c))^7}{7d} + \frac{a^2(\cos(dx+c))^5\sin(dx+c)}{3d} + \frac{5a^2(\cos(dx+c))^3\sin(dx+c)}{12d} + \frac{5a^2\cos(dx+c)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/7*a^2*cos(d*x+c)^7/d+1/3*a^2*cos(d*x+c)^5*sin(d*x+c)/d+5/12*a^2*cos(d*x+
c)^3*sin(d*x+c)/d+5/8*a^2*cos(d*x+c)*sin(d*x+c)/d+5/8*a^2*x+5/8/d*c*a^2+1/5
*a^2*cos(d*x+c)^5/d+1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)/d+1/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.15604, size = 166, normalized size = 1.03

$$-\frac{480\,a^2\cos\left(dx+c\right)^7-112\left(6\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^3+30\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\right.\right.}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(480*a^2*cos(d*x + c)^7 - 112*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3
 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a
^2 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*a^2)/d
```

Fricas [A] time = 1.24757, size = 386, normalized size = 2.4

$$-\frac{120\,a^2\cos\left(dx+c\right)^7-168\,a^2\cos\left(dx+c\right)^5-280\,a^2\cos\left(dx+c\right)^3-525\,a^2dx-840\,a^2\cos\left(dx+c\right)+420\,a^2\log\left(\frac{1}{2}\,\cos\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/840*(120*a^2*cos(d*x + c)^7 - 168*a^2*cos(d*x + c)^5 - 280*a^2*cos(d*x +
 c)^3 - 525*a^2*d*x - 840*a^2*cos(d*x + c) + 420*a^2*log(1/2*cos(d*x + c) +
 1/2) - 420*a^2*log(-1/2*cos(d*x + c) + 1/2) - 35*(8*a^2*cos(d*x + c)^5 + 1
0*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30114, size = 331, normalized size = 2.06

$$525\,(dx+c)a^2+840\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-1680\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+980\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-10080\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(525*(d*x + c)*a^2 + 840*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(1155
*a^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*tan(1/2*d*x + 1/2*c)^12 + 980*a^2*t
an(1/2*d*x + 1/2*c)^11 - 10080*a^2*tan(1/2*d*x + 1/2*c)^10 + 2975*a^2*tan(1
/2*d*x + 1/2*c)^9 - 16240*a^2*tan(1/2*d*x + 1/2*c)^8 - 24640*a^2*tan(1/2*d*
x + 1/2*c)^6 - 2975*a^2*tan(1/2*d*x + 1/2*c)^5 - 14448*a^2*tan(1/2*d*x + 1/
2*c)^4 - 980*a^2*tan(1/2*d*x + 1/2*c)^3 - 6496*a^2*tan(1/2*d*x + 1/2*c)^2 -
 1155*a^2*tan(1/2*d*x + 1/2*c) - 1168*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/
d
```

3.593 $\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=158

$$\frac{2a^2\cos^5(c+dx)}{5d}+\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{7a^2\sin^3(c+dx)}{24d}$$

```
[Out] (-25*a^2*x)/16 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d +
 (2*a^2*Cos[c + d*x]^3)/(3*d) + (2*a^2*Cos[c + d*x]^5)/(5*d) - (a^2*Cot[c +
 d*x])/d - (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (7*a^2*Cos[c + d*x]*S
in[c + d*x]^3)/(24*d) + (a^2*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.226387, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 2638, 2633, 2635}

$$\frac{2a^2\cos^5(c+dx)}{5d}+\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{7a^2\sin^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-25*a^2*x)/16 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d +
 (2*a^2*Cos[c + d*x]^3)/(3*d) + (2*a^2*Cos[c + d*x]^5)/(5*d) - (a^2*Cot[c +
 d*x])/d - (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (7*a^2*Cos[c + d*x]*S
in[c + d*x]^3)/(24*d) + (a^2*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-2a^8+2a^8\csc(c+dx)+a^8\csc^2(c+dx)-6a^8\sin(c+dx)+\right.}{}$$

$$= -2a^2x + a^2\int \csc^2(c+dx)\,dx - a^2\int \sin^6(c+dx)\,dx + \left(2a^2\right)\int$$

$$= -2a^2x - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{6a^2\cos(c+dx)}{d} - \frac{a^2\cos(c}{}$$

$$= -2a^2x - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{2a^2\cos^3(}{3d}$$

$$= -\frac{5a^2x}{4} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{2a^2\cos^3}{3}$$

$$= -\frac{25a^2x}{16} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{2a^2\cos}{}$$

Mathematica [A] time = 0.307121, size = 110, normalized size = 0.7

$$\frac{a^2\left(-255\sin(2(c+dx))+15\sin(4(c+dx))+5\sin(6(c+dx))+2640\cos(c+dx)+280\cos(3(c+dx))+24\cos(5(c+\right.}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-1500*c - 1500*d*x + 2640*Cos[c + d*x] + 280*Cos[3*(c + d*x)] + 24*Co
s[5*(c + d*x)] - 960*Cot[c + d*x] - 1920*Log[Cos[(c + d*x)/2]] + 1920*Log[S
in[(c + d*x)/2]] - 255*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)] + 5*Sin[6*(c
+ d*x)]))/(960*d)
```

Maple [A] time = 0.072, size = 175, normalized size = 1.1

$$-\frac{5a^2(\cos(dx+c))^5\sin(dx+c)}{6d} - \frac{25a^2(\cos(dx+c))^3\sin(dx+c)}{24d} - \frac{25a^2\cos(dx+c)\sin(dx+c)}{16d} - \frac{25a^2x}{16} - \frac{25ca}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-25/24*a^2*cos(d*x+c)^3*sin(d*x+c)/d-25/1
6*a^2*cos(d*x+c)*sin(d*x+c)/d-25/16*a^2*x-25/16/d*c*a^2+2/5*a^2*cos(d*x+c)^
```

```
5/d+2/3*a^2*cos(d*x+c)^3/d+2*a^2*cos(d*x+c)/d+2/d*a^2*ln(csc(d*x+c)-cot(d*x
+c))-1/d*a^2/sin(d*x+c)*cos(d*x+c)^7
```

Maxima [A] time = 1.76176, size = 234, normalized size = 1.48

$$\frac{64\left(6\cos\left(dx+c\right)^{5}+10\cos\left(dx+c\right)^{3}+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^{2}-5\left(4\sin\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/960*(64*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(
cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 - 5*(4*sin(2*d*x + 2*c)^3
 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - 120*(15*
d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
2*tan(d*x + c)^3 + tan(d*x + c)))*a^2)/d
```

Fricas [A] time = 1.22021, size = 439, normalized size = 2.78

$$-\frac{40\,a^{2}\cos\left(dx+c\right)^{7}-50\,a^{2}\cos\left(dx+c\right)^{5}-125\,a^{2}\cos\left(dx+c\right)^{3}+240\,a^{2}\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-240\,a^{2}\log\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(40*a^2*cos(d*x + c)^7 - 50*a^2*cos(d*x + c)^5 - 125*a^2*cos(d*x + c
)^3 + 240*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 240*a^2*log(-1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) + 375*a^2*cos(d*x + c) - (96*a^2*cos(d*x +
c)^5 + 160*a^2*cos(d*x + c)^3 - 375*a^2*d*x + 480*a^2*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25403, size = 370, normalized size = 2.34

$$-\frac{375\,(dx+c)a^{2}-480\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-120\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{120\left(4\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{2}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(105\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\ldots\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/240*(375*(d*x + c)*a^2 - 480*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 120*a^
2*tan(1/2*d*x + 1/2*c) + 120*(4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x
 + 1/2*c) - 2*(105*a^2*tan(1/2*d*x + 1/2*c)^11 + 1440*a^2*tan(1/2*d*x + 1/2
*c)^10 + 595*a^2*tan(1/2*d*x + 1/2*c)^9 + 4320*a^2*tan(1/2*d*x + 1/2*c)^8 -
 150*a^2*tan(1/2*d*x + 1/2*c)^7 + 7360*a^2*tan(1/2*d*x + 1/2*c)^6 + 150*a^2
*tan(1/2*d*x + 1/2*c)^5 + 6720*a^2*tan(1/2*d*x + 1/2*c)^4 - 595*a^2*tan(1/2
*d*x + 1/2*c)^3 + 2976*a^2*tan(1/2*d*x + 1/2*c)^2 - 105*a^2*tan(1/2*d*x + 1
/2*c) + 736*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.594 $\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=140

$$\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{9a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2\tan}{}$$

```
[Out] (-15*a^2*x)/4 + (3*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (a^2*Cos[c + d*x])/d
+ (a^2*Cos[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*C
sc[c + d*x])/(2*d) - (9*a^2*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^2*Cos[c +
 d*x]*Sin[c + d*x]^3)/(2*d)
```

Rubi [A] time = 0.209124, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 3768, 2635, 2633}

$$\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{9a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a^2*x)/4 + (3*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (a^2*Cos[c + d*x])/d
+ (a^2*Cos[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*C
sc[c + d*x])/(2*d) - (9*a^2*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^2*Cos[c +
 d*x]*Sin[c + d*x]^3)/(2*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-6a^8-2a^8\csc(c+dx)+2a^8\csc^2(c+dx)+a^8\csc^3(c+dx)\right.}{}$$

$$= -6a^2x+a^2\int \csc^3(c+dx)\,dx-a^2\int \sin^5(c+dx)\,dx-\left(2a^2\right)\int$$

$$= -6a^2x+\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}$$

$$= -3a^2x+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^5(c}{5d}$$

$$= -\frac{15a^2x}{4}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^5(c}{5d}$$

Mathematica [A] time = 6.06911, size = 174, normalized size = 1.24

$$\frac{(a\sin(c+dx)+a)^2\left(-300(c+dx)-80\sin(2(c+dx))-5\sin(4(c+dx))-70\cos(c+dx)+5\cos(3(c+dx))+\cos(5(c\right.}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^2*(-300*(c + d*x) - 70*Cos[c + d*x] + 5*Cos[3*(c + d*
x)] + Cos[5*(c + d*x)] - 80*Cot[(c + d*x)/2] - 10*Csc[(c + d*x)/2]^2 + 120*
Log[Cos[(c + d*x)/2]] - 120*Log[Sin[(c + d*x)/2]] + 10*Sec[(c + d*x)/2]^2 -
 80*Sin[2*(c + d*x)] - 5*Sin[4*(c + d*x)] + 80*Tan[(c + d*x)/2]))/(80*d*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.081, size = 199, normalized size = 1.4

$$-\frac{3a^2(\cos(dx+c))^5}{10d}-\frac{a^2(\cos(dx+c))^3}{2d}-\frac{3a^2\cos(dx+c)}{2d}-\frac{3a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-2\frac{a^2(\cos(dx+c}{d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/10*a^2*cos(d*x+c)^5/d-1/2*a^2*cos(d*x+c)^3/d-3/2*a^2*cos(d*x+c)/d-3/2/d*
a^2*ln(csc(d*x+c)-cot(d*x+c))-2/d*a^2/sin(d*x+c)*cos(d*x+c)^7-2*a^2*cos(d*x
+c)^5*sin(d*x+c)/d-5/2*a^2*cos(d*x+c)^3*sin(d*x+c)/d-15/4*a^2*cos(d*x+c)*si
n(d*x+c)/d-15/4*a^2*x-15/4/d*c*a^2-1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7
```

Maxima [A] time = 1.70991, size = 258, normalized size = 1.84

$$\frac{2\left(6 \cos(dx+c)^5 + 10 \cos(dx+c)^3 + 30 \cos(dx+c) - 15 \log(\cos(dx+c)+1) + 15 \log(\cos(dx+c)-1)\right)a^2 - 5\left(4 \cos \ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(2*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(co
s(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 - 5*(4*cos(d*x + c)^3 - 6*c
os(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c) +
1) + 15*log(cos(d*x + c) - 1))*a^2 - 15*(15*d*x + 15*c + (15*tan(d*x + c)^4
 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)
))*a^2)/d
```

Fricas [A] time = 1.20187, size = 497, normalized size = 3.55

$$\frac{4\,a^2 \cos(dx+c)^7 - 4\,a^2 \cos(dx+c)^5 - 75\,a^2dx \cos(dx+c)^2 - 20\,a^2 \cos(dx+c)^3 + 75\,a^2dx + 30\,a^2 \cos(dx+c) + 15\left(a^2 \ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/20*(4*a^2*cos(d*x + c)^7 - 4*a^2*cos(d*x + c)^5 - 75*a^2*d*x*cos(d*x + c)
^2 - 20*a^2*cos(d*x + c)^3 + 75*a^2*d*x + 30*a^2*cos(d*x + c) + 15*(a^2*cos
(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) - 15*(a^2*cos(d*x + c)^2 - a
^2)*log(-1/2*cos(d*x + c) + 1/2) - 5*(2*a^2*cos(d*x + c)^5 + 5*a^2*cos(d*x
+ c)^3 - 15*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28472, size = 329, normalized size = 2.35

$$5a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-150(dx+c)a^2-60a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+40a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{5\left(18a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\right.}{\tan\left(\frac{1}{2}dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40*(5*a^2*tan(1/2*d*x + 1/2*c)^2 - 150*(d*x + c)*a^2 - 60*a^2*log(abs(tan
(1/2*d*x + 1/2*c))) + 40*a^2*tan(1/2*d*x + 1/2*c) + 5*(18*a^2*tan(1/2*d*x +
 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 4*(4
5*a^2*tan(1/2*d*x + 1/2*c)^9 + 50*a^2*tan(1/2*d*x + 1/2*c)^7 - 80*a^2*tan(1
/2*d*x + 1/2*c)^6 - 80*a^2*tan(1/2*d*x + 1/2*c)^4 - 50*a^2*tan(1/2*d*x + 1/
2*c)^3 - 80*a^2*tan(1/2*d*x + 1/2*c)^2 - 45*a^2*tan(1/2*d*x + 1/2*c) - 16*a
^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.595 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=153

$$-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{4a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{5a^2\sin(c+dx)}{8d}$$

```
[Out] (5*a^2*x)/8 + (5*a^2*ArcTanh[Cos[c + d*x]])/d - (4*a^2*Cos[c + d*x])/d - (2
*a^2*Cos[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3
*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (5*a^2*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (a^2*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.215374, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{4a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{5a^2\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (5*a^2*x)/8 + (5*a^2*ArcTanh[Cos[c + d*x]])/d - (4*a^2*Cos[c + d*x])/d - (2
*a^2*Cos[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3
*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (5*a^2*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (a^2*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int\left(-6a^8\csc(c+dx)-2a^8\csc^2(c+dx)+2a^8\csc^3(c+dx)+a^8 \cdots\right.}{\cdots}$$

$$= a^2\int\csc^4(c+dx)\,dx - a^2\int\sin^4(c+dx)\,dx - \left(2a^2\right)\int\csc^2(c+\cdots$$

$$= \frac{6a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{6a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+dx)\csc\cdots}{d}$$

$$= a^2x + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^2\cos(c+dx)}{d} - \frac{2a^2\cos^3(c+\cdots}{3d}$$

$$= \frac{5a^2x}{8} + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^2\cos(c+dx)}{d} - \frac{2a^2\cos^3(\cdots}{3d}$$

Mathematica [A] time = 3.24934, size = 209, normalized size = 1.37

$$\frac{a^2(\sin(c+dx)+1)^2\left(60(c+dx)-24\sin(2(c+dx))-3\sin(4(c+dx))-432\cos(c+dx)-16\cos(3(c+dx))-64\tan\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(60*(c + d*x) - 432*Cos[c + d*x] - 16*Cos[3*(c +
d*x)] + 64*Cot[(c + d*x)/2] - 24*Csc[(c + d*x)/2]^2 + 480*Log[Cos[(c + d*x)
/2]] - 480*Log[Sin[(c + d*x)/2]] + 24*Sec[(c + d*x)/2]^2 + 32*Csc[c + d*x]^
3*Sin[(c + d*x)/2]^4 - 2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 24*Sin[2*(c + d*
x)] - 3*Sin[4*(c + d*x)] - 64*Tan[(c + d*x)/2]))/(96*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.082, size = 223, normalized size = 1.5

$$\frac{a^2(\cos(dx+c))^7}{3d\sin(dx+c)} + \frac{a^2(\cos(dx+c))^5\sin(dx+c)}{3d} + \frac{5a^2(\cos(dx+c))^3\sin(dx+c)}{12d} + \frac{5a^2\cos(dx+c)\sin(dx+c)}{8d} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/3/d*a^2/sin(d*x+c)*cos(d*x+c)^7+1/3*a^2*cos(d*x+c)^5*sin(d*x+c)/d+5/12*a^
2*cos(d*x+c)^3*sin(d*x+c)/d+5/8*a^2*cos(d*x+c)*sin(d*x+c)/d+5/8*a^2*x+5/8/d
*c*a^2-1/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-a^2*cos(d*x+c)^5/d-5/3*a^2*cos(d*x
+c)^3/d-5*a^2*cos(d*x+c)/d-5/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*a^2/sin(
d*x+c)^3*cos(d*x+c)^7
```

Maxima [A] time = 1.61065, size = 257, normalized size = 1.68

$$-\frac{4\left(4\cos\left(dx+c\right)^{3}-\frac{6\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^{2}+3\left(15\,dx+\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/24*(4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 + 3*(15*
d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
2*tan(d*x + c)^3 + tan(d*x + c)))*a^2 - 4*(15*d*x + 15*c + (15*tan(d*x + c)
^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^2)/d
```

Fricas [A] time = 1.23523, size = 554, normalized size = 3.62

$$\frac{6\,a^{2}\cos\left(dx+c\right)^{7}-3\,a^{2}\cos\left(dx+c\right)^{5}+20\,a^{2}\cos\left(dx+c\right)^{3}-15\,a^{2}\cos\left(dx+c\right)+60\left(a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\log\left(\frac{1}{2}\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*a^2*cos(d*x + c)^7 - 3*a^2*cos(d*x + c)^5 + 20*a^2*cos(d*x + c)^3 -
 15*a^2*cos(d*x + c) + 60*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 60*(a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) +
1/2)*sin(d*x + c) - (16*a^2*cos(d*x + c)^5 - 15*a^2*d*x*cos(d*x + c)^2 + 80
*a^2*cos(d*x + c)^3 + 15*a^2*d*x - 120*a^2*cos(d*x + c))*sin(d*x + c))/((d*
cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28678, size = 370, normalized size = 2.42

$$a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15(dx+c)a^2-120a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*(d*x +
 c)*a^2 - 120*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2
*c) + (220*a^2*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a
^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3 + 2*(15*a^2*tan(1/2*d
*x + 1/2*c)^7 - 144*a^2*tan(1/2*d*x + 1/2*c)^6 - 9*a^2*tan(1/2*d*x + 1/2*c)
^5 - 336*a^2*tan(1/2*d*x + 1/2*c)^4 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 304*a^
2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) - 112*a^2)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^4)/d
```

3.596 $\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=153

$$-\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{4a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos(}{8d}$$

```
[Out] 5*a^2*x + (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cos[c + d*x])/d - (a^2
*Cos[c + d*x]^3)/(3*d) + (4*a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3
*d) + (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d
*x]^3)/(4*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/d
```

Rubi [A] time = 0.197078, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2872, 3767, 8, 3768, 3770, 2638, 2635, 2633}

$$-\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{4a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos(}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] 5*a^2*x + (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cos[c + d*x])/d - (a^2
*Cos[c + d*x]^3)/(3*d) + (4*a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3
*d) + (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d
*x]^3)/(4*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(6a^8-6a^8\csc^2(c+dx)-2a^8\csc^3(c+dx)+2a^8\csc^4(c+dx)+\right.}{}$$

$$= 6a^2x + a^2\int \csc^5(c+dx)\,dx - a^2\int \sin^3(c+dx)\,dx - \left(2a^2\right)\int \mathrm{cs}$$

$$= 6a^2x - \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cot(c+dx)\csc(c+dx)}{d} - \frac{a^2\cot(c+}{}$$

$$= 5a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c+dx}{3d}$$

$$= 5a^2x + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c+}{3d}$$

Mathematica [A] time = 1.20725, size = 227, normalized size = 1.48

$$\frac{a^2(\sin(c+dx)+1)^2\left(960(c+dx)+96\sin(2(c+dx))-240\cos(c+dx)-16\cos(3(c+dx))-448\tan\left(\frac{1}{2}(c+dx)\right)+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(960*(c + d*x) - 240*Cos[c + d*x] - 16*Cos[3*(c +
 d*x)] + 448*Cot[(c + d*x)/2] + 30*Csc[(c + d*x)/2]^2 - 3*Csc[(c + d*x)/2]^
4 + 120*Log[Cos[(c + d*x)/2]] - 120*Log[Sin[(c + d*x)/2]] - 30*Sec[(c + d*x
)/2]^2 + 3*Sec[(c + d*x)/2]^4 + 128*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 8*C
sc[(c + d*x)/2]^4*Sin[c + d*x] + 96*Sin[2*(c + d*x)] - 448*Tan[(c + d*x)/2]
))/(192*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.082, size = 247, normalized size = 1.6

$$-\frac{a^2(\cos(dx+c))^7}{8d(\sin(dx+c))^2} - \frac{a^2(\cos(dx+c))^5}{8d} - \frac{5a^2(\cos(dx+c))^3}{24d} - \frac{5a^2\cos(dx+c)}{8d} - \frac{5a^2\ln(\csc(dx+c)-\cot(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/8*a^2*cos(d*x+c)^5/d-5/24*a^2*cos(d*
x+c)^3/d-5/8*a^2*cos(d*x+c)/d-5/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/3/d*a^2
/sin(d*x+c)^3*cos(d*x+c)^7+8/3/d*a^2/sin(d*x+c)*cos(d*x+c)^7+8/3*a^2*cos(d*
x+c)^5*sin(d*x+c)/d+10/3*a^2*cos(d*x+c)^3*sin(d*x+c)/d+5*a^2*cos(d*x+c)*sin
(d*x+c)/d+5*a^2*x+5/d*c*a^2-1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7
```

Maxima [A] time = 1.58052, size = 278, normalized size = 1.82

$$-\frac{4\left(4\cos(dx+c)^{3}-\frac{6\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^{2}-16\left(15\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 - 16*(15
*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 +
 tan(d*x + c)^3))*a^2 + 3*a^2*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d
*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) +
 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.25567, size = 636, normalized size = 4.16

$$-\frac{16\,a^{2}\cos(dx+c)^{7}-240\,a^{2}dx\cos(dx+c)^{4}+16\,a^{2}\cos(dx+c)^{5}+480\,a^{2}dx\cos(dx+c)^{2}-50\,a^{2}\cos(dx+c)^{3}-240\,a^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(16*a^2*cos(d*x + c)^7 - 240*a^2*d*x*cos(d*x + c)^4 + 16*a^2*cos(d*x
+ c)^5 + 480*a^2*d*x*cos(d*x + c)^2 - 50*a^2*cos(d*x + c)^3 - 240*a^2*d*x +
 30*a^2*cos(d*x + c) - 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)
*log(1/2*cos(d*x + c) + 1/2) + 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^
2 + a^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*(3*a^2*cos(d*x + c)^5 - 20*a^2*c
os(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*
cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34094, size = 350, normalized size = 2.29

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+960\,(dx+c)a^2-120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a^2 - 120*a^2*log(abs(tan(1/2*d*x
+ 1/2*c))) - 432*a^2*tan(1/2*d*x + 1/2*c) - 128*(3*a^2*tan(1/2*d*x + 1/2*c)
^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*ta
n(1/2*d*x + 1/2*c) + 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3 + (250*a^2*tan(1
/2*d*x + 1/2*c)^4 + 432*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*a^2*tan(1/2*d*x + 1
/2*c)^2 - 16*a^2*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.597 $\int \cot^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=139

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{15a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot(c\cdot}{}$$

```
[Out] (3*a^2*x)/2 - (15*a^2*ArcTanh[Cos[c + d*x]])/(4*d) + (2*a^2*Cos[c + d*x])/d
 + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^5)/(5*d) + (9*a^2*Cot[c + d*x]*
Csc[c + d*x])/(4*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(2*d) + (a^2*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.249414, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3768, 3767, 2638, 2635, 8}

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{15a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot(c\cdot}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 - (15*a^2*ArcTanh[Cos[c + d*x]])/(4*d) + (2*a^2*Cos[c + d*x])/d
 + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^5)/(5*d) + (9*a^2*Cot[c + d*x]*
Csc[c + d*x])/(4*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(2*d) + (a^2*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^8+6a^8\csc(c+dx)-6a^8\csc^3(c+dx)-2a^8\csc^4(c+dx)+2a^8\csc^5(c+\right.}{a^6}$$
$$= 2a^2x + a^2\int \csc^6(c+dx)\,dx - a^2\int \sin^2(c+dx)\,dx - \left(2a^2\right)\int \csc^4(c+dx)\,d$$
$$= 2a^2x - \frac{6a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{3a^2\cot(c+dx)\csc(c+}{d}$$
$$= \frac{3a^2x}{2} - \frac{3a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cot(c+dx)}{d} - \frac{a^2\text{co}}{}$$
$$= \frac{3a^2x}{2} - \frac{15a^2\tanh^{-1}(\cos(c+dx))}{4d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cot(c+dx)}{d} - \frac{a^2\text{c}}{}$$

Mathematica [A] time = 1.32833, size = 264, normalized size = 1.9

$$\frac{a^2(\sin(c+dx)+1)^2\left(240(c+dx)+40\sin(2(c+dx))+320\cos(c+dx)-64\tan\left(\frac{1}{2}(c+dx)\right)+64\cot\left(\frac{1}{2}(c+dx)\right)-5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(240*(c + d*x) + 320*Cos[c + d*x] + 64*Cot[(c + d
*x)/2] + 90*Csc[(c + d*x)/2]^2 - 5*Csc[(c + d*x)/2]^4 - 600*Log[Cos[(c + d*
x)/2]] + 600*Log[Sin[(c + d*x)/2]] - 90*Sec[(c + d*x)/2]^2 + 5*Sec[(c + d*x
)/2]^4 - 56*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (7*Csc[(c + d*x)/2]^4*Sin[c
 + d*x])/2 - (Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 + 40*Sin[2*(c + d*x)] - 64
*Tan[(c + d*x)/2] + Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(160*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [B] time = 0.086, size = 293, normalized size = 2.1

$$-\frac{a^2(\cos(dx+c))^7}{3d(\sin(dx+c))^3} + \frac{4a^2(\cos(dx+c))^7}{3d\sin(dx+c)} + \frac{4a^2(\cos(dx+c))^5\sin(dx+c)}{3d} + \frac{5a^2(\cos(dx+c))^3\sin(dx+c)}{3d} + \frac{5a^2\,\text{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*a^2/sin(d*x+c)*cos(d*x+c)^7+4/3*
a^2*cos(d*x+c)^5*sin(d*x+c)/d+5/3*a^2*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a^2*cos
(d*x+c)*sin(d*x+c)/d+3/2*a^2*x+3/2/d*c*a^2-1/2/d*a^2/sin(d*x+c)^4*cos(d*x+c
)^7+3/4/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7+3/4*a^2*cos(d*x+c)^5/d+5/4*a^2*cos(
d*x+c)^3/d+15/4*a^2*cos(d*x+c)/d+15/4/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/5*a
^2*cot(d*x+c)^5/d+1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)/d
```

Maxima [A] time = 1.64059, size = 248, normalized size = 1.78

$$\frac{20\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^2-8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-15\,a^2\left(\frac{2\left(9\,\cos(dx+c)^3-7\,c\right.}{\cos(dx+c)^4-2\,\cos(\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/120*(20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan
(d*x + c)^5 + tan(d*x + c)^3))*a^2 - 8*(15*d*x + 15*c + (15*tan(d*x + c)^4
- 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*a^2 - 15*a^2*(2*(9*cos(d*x + c)^3 -
 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
+ 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.23628, size = 686, normalized size = 4.94

$$-\frac{20\,a^2\cos(dx+c)^7-92\,a^2\cos(dx+c)^5+140\,a^2\cos(dx+c)^3-60\,a^2\cos(dx+c)+75\left(a^2\cos(dx+c)^4-2\,a^2\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/40*(20*a^2*cos(d*x + c)^7 - 92*a^2*cos(d*x + c)^5 + 140*a^2*cos(d*x + c)
^3 - 60*a^2*cos(d*x + c) + 75*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 +
a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 75*(a^2*cos(d*x + c)^4 - 2*
a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(6
*a^2*d*x*cos(d*x + c)^4 + 8*a^2*cos(d*x + c)^5 - 12*a^2*d*x*cos(d*x + c)^2
- 25*a^2*cos(d*x + c)^3 + 6*a^2*d*x + 15*a^2*cos(d*x + c))*sin(d*x + c))/((
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29388, size = 367, normalized size = 2.64

$$a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 5\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 - 5\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 80\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 240\,(dx + c)a^2 + 600$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/160*(a^2*tan(1/2*d*x + 1/2*c)^5 + 5*a^2*tan(1/2*d*x + 1/2*c)^4 - 5*a^2*ta
n(1/2*d*x + 1/2*c)^3 - 80*a^2*tan(1/2*d*x + 1/2*c)^2 + 240*(d*x + c)*a^2 +
600*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 70*a^2*tan(1/2*d*x + 1/2*c) - 160*
(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*
x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (1370*a^2*tan(1/2*d*x
+ 1/2*c)^5 - 70*a^2*tan(1/2*d*x + 1/2*c)^4 - 80*a^2*tan(1/2*d*x + 1/2*c)^3
- 5*a^2*tan(1/2*d*x + 1/2*c)^2 + 5*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*
d*x + 1/2*c)^5)/d
```

3.598 $\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=157

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{2a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{25a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\csc}{6d}$$

```
[Out] -2*a^2*x - (25*a^2*ArcTanh[Cos[c + d*x]])/(16*d) + (a^2*Cos[c + d*x])/d - (
2*a^2*Cot[c + d*x])/d + (2*a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^
5)/(5*d) + (7*a^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (7*a^2*Cot[c + d*x]*C
sc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.234526, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 3770, 3767, 8, 3768, 2638}

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{2a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{25a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\csc}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] -2*a^2*x - (25*a^2*ArcTanh[Cos[c + d*x]])/(16*d) + (a^2*Cos[c + d*x])/d - (
2*a^2*Cot[c + d*x])/d + (2*a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^
5)/(5*d) + (7*a^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (7*a^2*Cot[c + d*x]*C
sc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int\left(-2a^8+2a^8\csc(c+dx)+6a^8\csc^2(c+dx)-6a^8\csc^4(c+dx)\right.}{}$$
$$= -2a^2x+a^2\int\csc^7(c+dx)\,dx-a^2\int\sin(c+dx)\,dx+\left(2a^2\right)\int \mathrm{cs}$$
$$= -2a^2x-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cot(c+}{}$$
$$= -2a^2x-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+}{d}$$
$$= -2a^2x-\frac{5a^2\tanh^{-1}(\cos(c+dx))}{4d}+\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+}{d}$$
$$= -2a^2x-\frac{25a^2\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c}{d}$$

Mathematica [A] time = 1.49425, size = 270, normalized size = 1.72

$$-\frac{a^2\sin(c+dx)(\sin(c+dx)+1)^2\left(-1920\cot(c+dx)+\csc^2\left(\frac{1}{2}(c+dx)\right)(1472-210\csc(c+dx))+\csc^6\left(\frac{1}{2}(c+dx)\right)(5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(-1920*Cot[c + d*x] + Csc[(c + d*x)/2]^2*(1472 - 210*Csc[c + d*x]) +
Csc[(c + d*x)/2]^6*(12 + 5*Csc[c + d*x]) - 2*Csc[(c + d*x)/2]^4*(82 + 15*Cs
c[c + d*x]) + 120*Csc[c + d*x]*(32*(c + d*x) + 25*Log[Cos[(c + d*x)/2]] - 2
5*Log[Sin[(c + d*x)/2]]) - 2*(241 + 327*Cos[c + d*x] + 92*Cos[2*(c + d*x)])
*Sec[(c + d*x)/2]^6 + 840*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 + 480*Csc[c + d
*x]^5*Sin[(c + d*x)/2]^4 - 320*Csc[c + d*x]^7*Sin[(c + d*x)/2]^6)*Sin[c + d
*x]*(1 + Sin[c + d*x])^2)/(1920*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.083, size = 205, normalized size = 1.3

$$-\frac{5a^2(\cos(dx+c))^7}{24d(\sin(dx+c))^4}+\frac{5a^2(\cos(dx+c))^7}{16d(\sin(dx+c))^2}+\frac{5a^2(\cos(dx+c))^5}{16d}+\frac{25a^2(\cos(dx+c))^3}{48d}+\frac{25a^2\cos(dx+c)}{16d}+\frac{25a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/24/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7+5/16/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7+
5/16*a^2*cos(d*x+c)^5/d+25/48*a^2*cos(d*x+c)^3/d+25/16*a^2*cos(d*x+c)/d+25/
16/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/5*a^2*cot(d*x+c)^5/d+2/3*a^2*cot(d*x+c
```

```
)^3/d-2*a^2*cot(d*x+c)/d-2*a^2*x-2/d*c*a^2-1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c
)^7
```

Maxima [A] time = 1.64454, size = 297, normalized size = 1.89

$$-\frac{64\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-5\,a^2\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(64*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a^2 - 5*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(
d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*l
og(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 30*a^2*(2*(9*cos(d*x + c
)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x
+ c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.25213, size = 782, normalized size = 4.98

$$-\frac{960\,a^2dx\cos\left(dx+c\right)^6-480\,a^2\cos\left(dx+c\right)^7-2880\,a^2dx\cos\left(dx+c\right)^4+1650\,a^2\cos\left(dx+c\right)^5+2880\,a^2dx\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(960*a^2*d*x*cos(d*x + c)^6 - 480*a^2*cos(d*x + c)^7 - 2880*a^2*d*x*
cos(d*x + c)^4 + 1650*a^2*cos(d*x + c)^5 + 2880*a^2*d*x*cos(d*x + c)^2 - 20
00*a^2*cos(d*x + c)^3 - 960*a^2*d*x + 750*a^2*cos(d*x + c) + 375*(a^2*cos(d
*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(
d*x + c) + 1/2) - 375*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*co
s(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2) - 64*(23*a^2*cos(d*x + c)^
5 - 35*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35652, size = 350, normalized size = 2.23

$$5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+24\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-280\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-255\,a^{2}\tan\left(\frac{1}{2}\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*tan(1/2*d*x + 1/2*c)^5 - 15*a
^2*tan(1/2*d*x + 1/2*c)^4 - 280*a^2*tan(1/2*d*x + 1/2*c)^3 - 255*a^2*tan(1/
2*d*x + 1/2*c)^2 - 3840*(d*x + c)*a^2 + 3000*a^2*log(abs(tan(1/2*d*x + 1/2*
c))) + 2640*a^2*tan(1/2*d*x + 1/2*c) + 3840*a^2/(tan(1/2*d*x + 1/2*c)^2 + 1
) - (7350*a^2*tan(1/2*d*x + 1/2*c)^6 + 2640*a^2*tan(1/2*d*x + 1/2*c)^5 - 25
5*a^2*tan(1/2*d*x + 1/2*c)^4 - 280*a^2*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(
1/2*d*x + 1/2*c)^2 + 24*a^2*tan(1/2*d*x + 1/2*c) + 5*a^2)/tan(1/2*d*x + 1/2
*c)^6)/d
```

3.599 $\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=162

$$-\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot^5(c+dx)\csc(}{3d}$$

```
[Out] -(a^2*x) + (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cot[c + d*x])/d + (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)
/(7*d) - (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(8*d) + (5*a^2*Cot[c + d*x]^3*Cs
c[c + d*x])/(12*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x])/(3*d)
```

Rubi [A] time = 0.227897, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot^5(c+dx)\csc(}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cot[c + d*x])/d + (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)
/(7*d) - (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(8*d) + (5*a^2*Cot[c + d*x]^3*Cs
c[c + d*x])/(12*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x])/(3*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)+2a^2\cot^6(c+dx)\csc(c+dx)+a^2\cot^6(c+\right.$$
$$= a^2\int \cot^6(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right.$$
$$= -\frac{a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^5(c+dx)\csc(c+dx)}{3d} - a^2\int \cot^4(c+$$
$$= \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^7(c+dx)}{7d} + \frac{5a^2\cot^3(}{}$$
$$= -\frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^7(c}{7d}$$
$$= -a^2x + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+}{3d}$$

Mathematica [A] time = 1.05763, size = 262, normalized size = 1.62

$$\frac{a^2\left(9344\tan\left(\frac{1}{2}(c+dx)\right)-9344\cot\left(\frac{1}{2}(c+dx)\right)-4620\csc^2\left(\frac{1}{2}(c+dx)\right)+70\sec^6\left(\frac{1}{2}(c+dx)\right)-840\sec^4\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-13440*c - 13440*d*x - 9344*Cot[(c + d*x)/2] - 4620*Csc[(c + d*x)/2]^
2 + 8400*Log[Cos[(c + d*x)/2]] - 8400*Log[Sin[(c + d*x)/2]] + 4620*Sec[(c +
 d*x)/2]^2 - 840*Sec[(c + d*x)/2]^4 + 70*Sec[(c + d*x)/2]^6 - 4624*Csc[c +
d*x]^3*Sin[(c + d*x)/2]^4 - (15*Csc[(c + d*x)/2]^8*Sin[c + d*x])/2 + Csc[(c
 + d*x)/2]^6*(-70 + 33*Sin[c + d*x]) + Csc[(c + d*x)/2]^4*(840 + 289*Sin[c
+ d*x]) + 9344*Tan[(c + d*x)/2] - 66*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] +
15*Sec[(c + d*x)/2]^6*Tan[(c + d*x)/2]))/(13440*d)
```

Maple [A] time = 0.094, size = 229, normalized size = 1.4

$$-\frac{a^2(\cot(dx+c))^5}{5d}+\frac{a^2(\cot(dx+c))^3}{3d}-\frac{a^2\cot(dx+c)}{d}-a^2x-\frac{ca^2}{d}-\frac{a^2(\cos(dx+c))^7}{3d(\sin(dx+c))^6}+\frac{a^2(\cos(dx+c))^7}{12d(\sin(dx+c))^4}-\frac{a^2}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/5*a^2*cot(d*x+c)^5/d+1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)/d-a^2*x-1/d*c
*a^2-1/3/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/12/d*a^2/sin(d*x+c)^4*cos(d*x+c)
^7-1/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/8*a^2*cos(d*x+c)^5/d-5/24*a^2*cos(
d*x+c)^3/d-5/8*a^2*cos(d*x+c)/d-5/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/7/d*a
^2/sin(d*x+c)^7*cos(d*x+c)^7
```

Maxima [A] time = 1.56581, size = 208, normalized size = 1.28

$$-\frac{112\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-35\,a^2\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)+\right.\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1680*(112*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/ta
n(d*x + c)^5)*a^2 - 35*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*c
os(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 1
5*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 240*a^2/tan(d*x + c)^
7)/d
```

Fricas [B] time = 1.23233, size = 833, normalized size = 5.14

$$-\frac{2336\,a^2\cos\left(dx+c\right)^7-6496\,a^2\cos\left(dx+c\right)^5+5600\,a^2\cos\left(dx+c\right)^3-1680\,a^2\cos\left(dx+c\right)-525\left(a^2\cos\left(dx+c\right)^6-3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1680*(2336*a^2*cos(d*x + c)^7 - 6496*a^2*cos(d*x + c)^5 + 5600*a^2*cos(d
*x + c)^3 - 1680*a^2*cos(d*x + c) - 525*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x
 + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 525*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2
 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 70*(24*a^2*d*x*cos(d*x
+ c)^6 - 72*a^2*d*x*cos(d*x + c)^4 - 33*a^2*cos(d*x + c)^5 + 72*a^2*d*x*cos
(d*x + c)^2 + 40*a^2*cos(d*x + c)^3 - 24*a^2*d*x - 15*a^2*cos(d*x + c))*sin
(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d
)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+a*sin(d*x+c))**2,x)
```

[Out] Timed out

Giac [A] time = 1.29506, size = 365, normalized size = 2.25

$$15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-630\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-665\,a^2\tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 + 70*a^2*tan(1/2*d*x + 1/2*c)^6 - 21
*a^2*tan(1/2*d*x + 1/2*c)^5 - 630*a^2*tan(1/2*d*x + 1/2*c)^4 - 665*a^2*tan(
1/2*d*x + 1/2*c)^3 + 3150*a^2*tan(1/2*d*x + 1/2*c)^2 - 13440*(d*x + c)*a^2
- 8400*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 8715*a^2*tan(1/2*d*x + 1/2*c) +
 (21780*a^2*tan(1/2*d*x + 1/2*c)^7 - 8715*a^2*tan(1/2*d*x + 1/2*c)^6 - 3150
*a^2*tan(1/2*d*x + 1/2*c)^5 + 665*a^2*tan(1/2*d*x + 1/2*c)^4 + 630*a^2*tan(
1/2*d*x + 1/2*c)^3 + 21*a^2*tan(1/2*d*x + 1/2*c)^2 - 70*a^2*tan(1/2*d*x + 1
/2*c) - 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.600 $\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=182

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{45a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{a^2\cot^5(}{}$$

```
[Out] (45*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (35
*a^2*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x
])/(24*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x])/(6*d) - (5*a^2*Cot[c + d*x]*C
sc[c + d*x]^3)/(64*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a^2
*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rubi [A] time = 0.313335, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{45a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{a^2\cot^5(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (45*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (35
*a^2*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x
])/(24*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x])/(6*d) - (5*a^2*Cot[c + d*x]*C
sc[c + d*x]^3)/(64*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a^2
*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc(c+dx)+2a^2\cot^6(c+dx)\csc^2(c+dx)+\right.$$
$$= a^2\int \cot^6(c+dx)\csc(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^3(c+d$$
$$= -\frac{a^2\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{1}{8}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} + \frac{5a^2\cot^3(c+dx)\csc(c+dx)}{24d} - \frac{a^2\cot^5(c+}{}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{5a^2\cot(c+dx)\csc(c+dx)}{16d} + \frac{5a^2\cot^3(c+}{2}$$
$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{35a^2\cot(c+dx)}{128d}$$
$$= \frac{45a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{35a^2\cot(c+dx}{128}$$

Mathematica [B] time = 0.106805, size = 401, normalized size = 2.2

$$a^2\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{7d}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{7d}-\frac{\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d}+\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{17\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{83\csc^2\left(\frac{1}{2}(c+\right.}{512d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(Cot[(c + d*x)/2]/(7*d) - (83*Csc[(c + d*x)/2]^2)/(512*d) - (19*Cot[(c
+ d*x)/2]*Csc[(c + d*x)/2]^2)/(224*d) + (17*Csc[(c + d*x)/2]^4)/(1024*d) +
(5*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(224*d) + Csc[(c + d*x)/2]^6/(512*d
) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^6)/(448*d) - Csc[(c + d*x)/2]^8/(204
8*d) + (45*Log[Cos[(c + d*x)/2]])/(128*d) - (45*Log[Sin[(c + d*x)/2]])/(128
*d) + (83*Sec[(c + d*x)/2]^2)/(512*d) - (17*Sec[(c + d*x)/2]^4)/(1024*d) -
Sec[(c + d*x)/2]^6/(512*d) + Sec[(c + d*x)/2]^8/(2048*d) - Tan[(c + d*x)/2]
/(7*d) + (19*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(224*d) - (5*Sec[(c + d*x
)/2]^4*Tan[(c + d*x)/2])/(224*d) + (Sec[(c + d*x)/2]^6*Tan[(c + d*x)/2])/(4
48*d))
```

Maple [A] time = 0.086, size = 192, normalized size = 1.1

$$-\frac{3a^2(\cos(dx+c))^7}{16d(\sin(dx+c))^6}+\frac{3a^2(\cos(dx+c))^7}{64d(\sin(dx+c))^4}-\frac{9a^2(\cos(dx+c))^7}{128d(\sin(dx+c))^2}-\frac{9a^2(\cos(dx+c))^5}{128d}-\frac{15a^2(\cos(dx+c))^3}{128d}-\frac{45a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/16/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+3/64/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-
9/128/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-9/128*a^2*cos(d*x+c)^5/d-15/128*a^2*c
os(d*x+c)^3/d-45/128*a^2*cos(d*x+c)/d-45/128/d*a^2*ln(csc(d*x+c)-cot(d*x+c)
)-2/7/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/8/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7
```

Maxima [A] time = 1.04662, size = 298, normalized size = 1.64

$$-\frac{7\,a^2\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-56\,a^2\left(\cdots\right.}{5376\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/5376*(7*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^
3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1)) - 56*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x
+ c) + 1) - 15*log(cos(d*x + c) - 1)) + 1536*a^2/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.19501, size = 668, normalized size = 3.67

$$-\frac{512\,a^2\cos\left(dx+c\right)^7\sin\left(dx+c\right)-1162\,a^2\cos\left(dx+c\right)^7+3066\,a^2\cos\left(dx+c\right)^5-2310\,a^2\cos\left(dx+c\right)^3+630\,a^2\cos\left(dx\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1792*(512*a^2*cos(d*x + c)^7*sin(d*x + c) - 1162*a^2*cos(d*x + c)^7 + 30
66*a^2*cos(d*x + c)^5 - 2310*a^2*cos(d*x + c)^3 + 630*a^2*cos(d*x + c) - 31
5*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2
*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2) + 315*(a^2*cos(d*x + c)^
8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^
2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6
*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39621, size = 351, normalized size = 1.93

$$\frac{7\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+32\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-224\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-280\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+672\,a^2\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/14336*(7*a^2*tan(1/2*d*x + 1/2*c)^8 + 32*a^2*tan(1/2*d*x + 1/2*c)^7 - 224
*a^2*tan(1/2*d*x + 1/2*c)^5 - 280*a^2*tan(1/2*d*x + 1/2*c)^4 + 672*a^2*tan(
1/2*d*x + 1/2*c)^3 + 1792*a^2*tan(1/2*d*x + 1/2*c)^2 - 5040*a^2*log(abs(tan
(1/2*d*x + 1/2*c))) - 1120*a^2*tan(1/2*d*x + 1/2*c) + (13698*a^2*tan(1/2*d*
x + 1/2*c)^8 + 1120*a^2*tan(1/2*d*x + 1/2*c)^7 - 1792*a^2*tan(1/2*d*x + 1/2
*c)^6 - 672*a^2*tan(1/2*d*x + 1/2*c)^5 + 280*a^2*tan(1/2*d*x + 1/2*c)^4 + 2
24*a^2*tan(1/2*d*x + 1/2*c)^3 - 32*a^2*tan(1/2*d*x + 1/2*c) - 7*a^2)/tan(1/
2*d*x + 1/2*c)^8)/d
```

3.601 $\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=152

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5a^2\cot^3(c+dx)\csc^3(c}{24d}$$

```
[Out] (5*a^2*ArcTanh[Cos[c + d*x]])/(64*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (a^2*
Cot[c + d*x]^9)/(9*d) + (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(64*d) - (5*a^2*C
ot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x]^3)/
(24*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.285175, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5a^2\cot^3(c+dx)\csc^3(c}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (5*a^2*ArcTanh[Cos[c + d*x]])/(64*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (a^2*
Cot[c + d*x]^9)/(9*d) + (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(64*d) - (5*a^2*C
ot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x]^3)/
(24*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^3)/(4*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^2(c+dx)+2a^2\cot^6(c+dx)\csc^3(c+dx)\right.$$

$$= a^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^4(c+$$

$$= -\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}\left(5a^2\right)\int \cot^4(c+dx)\csc^3(c+$$

$$= -\frac{a^2\cot^7(c+dx)}{7d} + \frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{24d} - \frac{a^2\cot^5(c+$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} - \frac{5a^2\cot(c+dx)\csc^3(c+}{32d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} + \frac{5a^2\cot(c+dx)\csc(c+}{64d}$$

$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{64d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} +$$

Mathematica [B] time = 1.42304, size = 313, normalized size = 2.06

$$-\frac{a^2\csc^9(c+dx)\left(36540\sin(2(c+dx))+20916\sin(4(c+dx))+16044\sin(6(c+dx))+630\sin(8(c+dx))+72576\,\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^9*(72576*Cos[c + d*x] + 37632*Cos[3*(c + d*x)] + 6912*Co
s[5*(c + d*x)] - 1728*Cos[7*(c + d*x)] - 704*Cos[9*(c + d*x)] - 39690*Log[C
os[(c + d*x)/2]]*Sin[c + d*x] + 39690*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] +
36540*Sin[2*(c + d*x)] + 26460*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 264
60*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 20916*Sin[4*(c + d*x)] - 11340*
Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 11340*Log[Sin[(c + d*x)/2]]*Sin[5*
(c + d*x)] + 16044*Sin[6*(c + d*x)] + 2835*Log[Cos[(c + d*x)/2]]*Sin[7*(c +
 d*x)] - 2835*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] + 630*Sin[8*(c + d*x)]
 - 315*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] + 315*Log[Sin[(c + d*x)/2]]*S
in[9*(c + d*x)]))/(1032192*d)
```

Maple [A] time = 0.09, size = 216, normalized size = 1.4

$$-\frac{11\,a^2\left(\cos\left(dx+c\right)\right)^7}{63\,d\left(\sin\left(dx+c\right)\right)^7}-\frac{a^2\left(\cos\left(dx+c\right)\right)^7}{4\,d\left(\sin\left(dx+c\right)\right)^8}-\frac{a^2\left(\cos\left(dx+c\right)\right)^7}{24\,d\left(\sin\left(dx+c\right)\right)^6}+\frac{a^2\left(\cos\left(dx+c\right)\right)^7}{96\,d\left(\sin\left(dx+c\right)\right)^4}-\frac{a^2\left(\cos\left(dx+c\right)\right)^7}{64\,d\left(\sin\left(dx+c\right)\right)^2}-\frac{a^2\left(\cos\left(d\right.\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -11/63/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/4/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7-
1/24/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/96/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-1
/64/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/64*a^2*cos(d*x+c)^5/d-5/192*a^2*cos(d
*x+c)^3/d-5/64*a^2*cos(d*x+c)/d-5/64/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/9/d*
a^2/sin(d*x+c)^9*cos(d*x+c)^7
```

Maxima [A] time = 1.02776, size = 209, normalized size = 1.38

$$-\frac{21\,a^2\left(\frac{2\left(15\,\cos\left(dx+c\right)^7+73\,\cos\left(dx+c\right)^5-55\,\cos\left(dx+c\right)^3+15\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^8-4\,\cos\left(dx+c\right)^6+6\,\cos\left(dx+c\right)^4-4\,\cos\left(dx+c\right)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{1152}{\tan(dx}}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/8064*(21*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)
^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
 1)) + 1152*a^2/tan(d*x + c)^7 + 128*(9*tan(d*x + c)^2 + 7)*a^2/tan(d*x + c
)^9)/d
```

Fricas [B] time = 1.21343, size = 756, normalized size = 4.97

$$\frac{1408\,a^2\cos\left(dx+c\right)^9-2304\,a^2\cos\left(dx+c\right)^7+315\left(a^2\cos\left(dx+c\right)^8-4\,a^2\cos\left(dx+c\right)^6+6\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/8064*(1408*a^2*cos(d*x + c)^9 - 2304*a^2*cos(d*x + c)^7 + 315*(a^2*cos(d*
x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)
^2 + a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 315*(a^2*cos(d*x + c)^
8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^
2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 42*(15*a^2*cos(d*x + c)^7 +
73*a^2*cos(d*x + c)^5 - 55*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*
x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*
cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24185, size = 437, normalized size = 2.88

$$\frac{14\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-336\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-504\,a^2\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/64512*(14*a^2*tan(1/2*d*x + 1/2*c)^9 + 63*a^2*tan(1/2*d*x + 1/2*c)^8 + 18
*a^2*tan(1/2*d*x + 1/2*c)^7 - 336*a^2*tan(1/2*d*x + 1/2*c)^6 - 504*a^2*tan(
1/2*d*x + 1/2*c)^5 + 504*a^2*tan(1/2*d*x + 1/2*c)^4 + 1848*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 1008*a^2*tan(1/2*d*x + 1/2*c)^2 - 5040*a^2*log(abs(tan(1/2*d*x
 + 1/2*c))) - 3276*a^2*tan(1/2*d*x + 1/2*c) + (14258*a^2*tan(1/2*d*x + 1/2*
c)^9 + 3276*a^2*tan(1/2*d*x + 1/2*c)^8 - 1008*a^2*tan(1/2*d*x + 1/2*c)^7 -
1848*a^2*tan(1/2*d*x + 1/2*c)^6 - 504*a^2*tan(1/2*d*x + 1/2*c)^5 + 504*a^2*
tan(1/2*d*x + 1/2*c)^4 + 336*a^2*tan(1/2*d*x + 1/2*c)^3 - 18*a^2*tan(1/2*d*
x + 1/2*c)^2 - 63*a^2*tan(1/2*d*x + 1/2*c) - 14*a^2)/tan(1/2*d*x + 1/2*c)^9
)/d
```

3.602 $\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=228

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{13a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{a^2\cot^5(c+dx)\csc^3(}{8d}$$

```
[Out] (13*a^2*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2*
a^2*Cot[c + d*x]^9)/(9*d) + (13*a^2*Cot[c + d*x]*Csc[c + d*x])/(256*d) - (9
*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d
*x]^3)/(48*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d) - (a^2*Cot[c + d*
x]*Csc[c + d*x]^5)/(32*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a
^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rubi [A] time = 0.39092, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 14}

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{13a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{a^2\cot^5(c+dx)\csc^3(}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (13*a^2*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2*
a^2*Cot[c + d*x]^9)/(9*d) + (13*a^2*Cot[c + d*x]*Csc[c + d*x])/(256*d) - (9
*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d
*x]^3)/(48*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d) - (a^2*Cot[c + d*
x]*Csc[c + d*x]^5)/(32*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a
^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cot^6(c+dx)\csc^3(c+dx)+2a^2\cot^6(c+dx)\csc^4(c+dx)\right.\\
&= a^2\int \cot^6(c+dx)\csc^3(c+dx)\,dx+a^2\int \cot^6(c+dx)\csc^5(c+\\
&= -\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d}-\\
&= \frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d}+\\
&= -\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{5a^2\cot(c+dx)\csc^3(c+}{64d}\\
&= -\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}+\frac{5a^2\cot(c+dx)\csc(c+}{128d}\\
&= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}\\
&= \frac{13a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}
\end{aligned}$$

Mathematica [A] time = 1.23422, size = 353, normalized size = 1.55

$$-\frac{a^2\csc^{10}(c+dx)\left(1075200\sin(2(c+dx))+1044480\sin(4(c+dx))+414720\sin(6(c+dx))+51200\sin(8(c+dx))-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^10*(2732940*Cos[c + d*x] + 1151640*Cos[3*(c + d*x)] + 38
8248*Cos[5*(c + d*x)] - 135870*Cos[7*(c + d*x)] - 8190*Cos[9*(c + d*x)] - 5
15970*Log[Cos[(c + d*x)/2]] + 859950*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]]
 - 491400*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 184275*Cos[6*(c + d*x)]*
Log[Cos[(c + d*x)/2]] - 40950*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 4095
*Cos[10*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 515970*Log[Sin[(c + d*x)/2]] - 8
59950*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 491400*Cos[4*(c + d*x)]*Log[
Sin[(c + d*x)/2]] - 184275*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 40950*C
os[8*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 4095*Cos[10*(c + d*x)]*Log[Sin[(c +
 d*x)/2]] + 1075200*Sin[2*(c + d*x)] + 1044480*Sin[4*(c + d*x)] + 414720*Si
n[6*(c + d*x)] + 51200*Sin[8*(c + d*x)] - 5120*Sin[10*(c + d*x)]))/(4128768
```

```
0*d)
```

Maple [A] time = 0.086, size = 240, normalized size = 1.1

$$-\frac{13 a^2 (\cos (dx+c))^7}{80 d (\sin (dx+c))^8} - \frac{13 a^2 (\cos (dx+c))^7}{480 d (\sin (dx+c))^6} + \frac{13 a^2 (\cos (dx+c))^7}{1920 d (\sin (dx+c))^4} - \frac{13 a^2 (\cos (dx+c))^7}{1280 d (\sin (dx+c))^2} - \frac{13 a^2 (\cos (dx+c))^5}{1280 d} - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -13/80/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7-13/480/d*a^2/sin(d*x+c)^6*cos(d*x+c)
^7+13/1920/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-13/1280/d*a^2/sin(d*x+c)^2*cos(d
*x+c)^7-13/1280*a^2*cos(d*x+c)^5/d-13/768*a^2*cos(d*x+c)^3/d-13/256*a^2*cos
(d*x+c)/d-13/256/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/9/d*a^2/sin(d*x+c)^9*cos
(d*x+c)^7-4/63/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/10/d*a^2/sin(d*x+c)^10*cos
(d*x+c)^7
```

Maxima [A] time = 1.06939, size = 369, normalized size = 1.62

$$-\frac{63 a^2 \left(\frac{2 \left(15 \cos(dx+c)^9 - 70 \cos(dx+c)^7 - 128 \cos(dx+c)^5 + 70 \cos(dx+c)^3 - 15 \cos(dx+c)\right)}{\cos(dx+c)^{10} - 5 \cos(dx+c)^8 + 10 \cos(dx+c)^6 - 10 \cos(dx+c)^4 + 5 \cos(dx+c)^2 - 1} - 15 \log(\cos(dx+c)+1) + 15 \log(\cos(dx+c) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/161280*(63*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
 c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
 c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 210*a^2*(2*(15*cos(d*x
+ c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x
+ c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 5120*(9*tan(d*x + c)^2 +
 7)*a^2/tan(d*x + c)^9)/d
```

Fricas [A] time = 1.21268, size = 855, normalized size = 3.75

$$-\frac{8190 a^2 \cos(dx+c)^9 + 15540 a^2 \cos(dx+c)^7 - 69888 a^2 \cos(dx+c)^5 + 38220 a^2 \cos(dx+c)^3 - 8190 a^2 \cos(dx+c) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/161280*(8190*a^2*cos(d*x + c)^9 + 15540*a^2*cos(d*x + c)^7 - 69888*a^2*c
os(d*x + c)^5 + 38220*a^2*cos(d*x + c)^3 - 8190*a^2*cos(d*x + c) - 4095*(a^
2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10*a^2*c
```

```
os(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) + 4
095*(a^2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 1
0*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) +
1/2) + 5120*(2*a^2*cos(d*x + c)^9 - 9*a^2*cos(d*x + c)^7)*sin(d*x + c))/(d*
cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x +
 c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38668, size = 437, normalized size = 1.92

$$\frac{126\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-2160\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3990\,a^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(126*a^2*tan(1/2*d*x + 1/2*c)^10 + 560*a^2*tan(1/2*d*x + 1/2*c)^9
 + 315*a^2*tan(1/2*d*x + 1/2*c)^8 - 2160*a^2*tan(1/2*d*x + 1/2*c)^7 - 3990*
a^2*tan(1/2*d*x + 1/2*c)^6 + 7560*a^2*tan(1/2*d*x + 1/2*c)^4 + 13440*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 11340*a^2*tan(1/2*d*x + 1/2*c)^2 - 65520*a^2*log(abs
(tan(1/2*d*x + 1/2*c))) - 30240*a^2*tan(1/2*d*x + 1/2*c) + (191906*a^2*tan(
1/2*d*x + 1/2*c)^10 + 30240*a^2*tan(1/2*d*x + 1/2*c)^9 - 11340*a^2*tan(1/2*
d*x + 1/2*c)^8 - 13440*a^2*tan(1/2*d*x + 1/2*c)^7 - 7560*a^2*tan(1/2*d*x +
1/2*c)^6 + 3990*a^2*tan(1/2*d*x + 1/2*c)^4 + 2160*a^2*tan(1/2*d*x + 1/2*c)^
3 - 315*a^2*tan(1/2*d*x + 1/2*c)^2 - 560*a^2*tan(1/2*d*x + 1/2*c) - 126*a^2
)/tan(1/2*d*x + 1/2*c)^10)/d
```

3.603 $\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=194

$$-\frac{a^2\cot^{11}(c+dx)}{11d}-\frac{a^2\cot^9(c+dx)}{3d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d}+\frac{a^2 c}{}$$

```
[Out] (3*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (a^2
*Cot[c + d*x]^9)/(3*d) - (a^2*Cot[c + d*x]^11)/(11*d) + (3*a^2*Cot[c + d*x]
*Csc[c + d*x])/(128*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^2*Co
t[c + d*x]*Csc[c + d*x]^5)/(16*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(8*
d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.301631, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{a^2\cot^{11}(c+dx)}{11d}-\frac{a^2\cot^9(c+dx)}{3d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d}+\frac{a^2 c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (a^2
*Cot[c + d*x]^9)/(3*d) - (a^2*Cot[c + d*x]^11)/(11*d) + (3*a^2*Cot[c + d*x]
*Csc[c + d*x])/(128*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^2*Co
t[c + d*x]*Csc[c + d*x]^5)/(16*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(8*
d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(5*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^4(c+dx)+2a^2\cot^6(c+dx)\csc^5(c+dx)\right.$$
$$= a^2\int \cot^6(c+dx)\csc^4(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^6(c+$$
$$= -\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d} - a^2\int \cot^4(c+dx)\csc^5(c+dx)$$
$$= \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d} + \frac{1}{8}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} - \frac{a^2\cot^{11}(c+dx)}{11d} - \frac{a^2\cot}{}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} - \frac{a^2\cot^{11}(c+dx)}{11d} + \frac{a^2\cot}{}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} - \frac{a^2\cot^{11}(c+dx)}{11d} + \frac{3a^2c}{}$$
$$= \frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} -$$

Mathematica [A] time = 3.03577, size = 187, normalized size = 0.96

$$\frac{a^2(\sin(c+dx)+1)^2\left(887040\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{10}(c+dx)(1073226\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(887040*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x
)/2]]) - Cot[c + d*x]*Csc[c + d*x]^10*(1318400 + 1798400*Cos[2*(c + d*x)] +
 440320*Cos[4*(c + d*x)] - 81280*Cos[6*(c + d*x)] - 38400*Cos[8*(c + d*x)]
+ 3200*Cos[10*(c + d*x)] + 1073226*Sin[c + d*x] + 869484*Sin[3*(c + d*x)] +
 727188*Sin[5*(c + d*x)] + 40425*Sin[7*(c + d*x)] - 3465*Sin[9*(c + d*x)]))
```

```
)/(37847040*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.087, size = 264, normalized size = 1.4

$$-\frac{5\,a^2(\cos(dx+c))^7}{33\,d(\sin(dx+c))^9}-\frac{10\,a^2(\cos(dx+c))^7}{231\,d(\sin(dx+c))^7}-\frac{a^2(\cos(dx+c))^7}{5\,d(\sin(dx+c))^{10}}-\frac{3\,a^2(\cos(dx+c))^7}{40\,d(\sin(dx+c))^8}-\frac{a^2(\cos(dx+c))^7}{80\,d(\sin(dx+c))^6}+\frac{a^2(\cos}{320\,d(\si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -5/33/d*a^2/sin(d*x+c)^9*cos(d*x+c)^7-10/231/d*a^2/sin(d*x+c)^7*cos(d*x+c)^
7-1/5/d*a^2/sin(d*x+c)^10*cos(d*x+c)^7-3/40/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7
-1/80/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/320/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7
-3/640/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-3/640*a^2*cos(d*x+c)^5/d-1/128*a^2*c
os(d*x+c)^3/d-3/128*a^2*cos(d*x+c)/d-3/128/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-
1/11/d*a^2/sin(d*x+c)^11*cos(d*x+c)^7
```

Maxima [A] time = 1.06697, size = 266, normalized size = 1.37

$$-\frac{693\,a^2\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7-128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)\right.}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/887040*(693*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x
+ c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x
+ c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15
*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 14080*(9*tan(d*x + c)^
2 + 7)*a^2/tan(d*x + c)^9 + 1280*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2 +
63)*a^2/tan(d*x + c)^11)/d
```

Fricas [B] time = 1.28571, size = 940, normalized size = 4.85

$$\frac{12800\,a^2\cos(dx+c)^{11}-70400\,a^2\cos(dx+c)^9+84480\,a^2\cos(dx+c)^7+3465\left(a^2\cos(dx+c)^{10}-5\,a^2\cos(dx+c)^8+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/295680*(12800*a^2*cos(d*x + c)^11 - 70400*a^2*cos(d*x + c)^9 + 84480*a^2*
cos(d*x + c)^7 + 3465*(a^2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*
cos(d*x + c)^6 - 10*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(1/
2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3465*(a^2*cos(d*x + c)^10 - 5*a^2*cos(
d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x
```

```
+ c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 462*(15*a^2*cos(d
*x + c)^9 - 70*a^2*cos(d*x + c)^7 - 128*a^2*cos(d*x + c)^5 + 70*a^2*cos(d*x
 + c)^3 - 15*a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^10 - 5*d*cos(
d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2
 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+a*sin(d*x+c))**2,x)

[Out] Timed out
```

Giac [B] time = 1.36363, size = 524, normalized size = 2.7

$$\frac{105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+462\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+385\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-2805\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)

[Out] 1/2365440*(105*a^2*tan(1/2*d*x + 1/2*c)^11 + 462*a^2*tan(1/2*d*x + 1/2*c)^1
0 + 385*a^2*tan(1/2*d*x + 1/2*c)^9 - 1155*a^2*tan(1/2*d*x + 1/2*c)^8 - 2805
*a^2*tan(1/2*d*x + 1/2*c)^7 - 2310*a^2*tan(1/2*d*x + 1/2*c)^6 + 1155*a^2*ta
n(1/2*d*x + 1/2*c)^5 + 9240*a^2*tan(1/2*d*x + 1/2*c)^4 + 16170*a^2*tan(1/2*
d*x + 1/2*c)^3 + 4620*a^2*tan(1/2*d*x + 1/2*c)^2 - 55440*a^2*log(abs(tan(1/
2*d*x + 1/2*c))) - 39270*a^2*tan(1/2*d*x + 1/2*c) + (167422*a^2*tan(1/2*d*x
 + 1/2*c)^11 + 39270*a^2*tan(1/2*d*x + 1/2*c)^10 - 4620*a^2*tan(1/2*d*x + 1
/2*c)^9 - 16170*a^2*tan(1/2*d*x + 1/2*c)^8 - 9240*a^2*tan(1/2*d*x + 1/2*c)^
7 - 1155*a^2*tan(1/2*d*x + 1/2*c)^6 + 2310*a^2*tan(1/2*d*x + 1/2*c)^5 + 280
5*a^2*tan(1/2*d*x + 1/2*c)^4 + 1155*a^2*tan(1/2*d*x + 1/2*c)^3 - 385*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 462*a^2*tan(1/2*d*x + 1/2*c) - 105*a^2)/tan(1/2*d*x
+ 1/2*c)^11)/d
```

3.604 $\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=270

$$-\frac{2a^2\cot^{11}(c+dx)}{11d}-\frac{4a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{17a^2\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^2\cot^5(c+dx)\csc^7(c+dx)}{12d}-$$

```
[Out] (17*a^2*ArcTanh[Cos[c + d*x]])/(1024*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (4
*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Cot[c + d*x]^11)/(11*d) + (17*a^2*Cot[c
 + d*x]*Csc[c + d*x])/(1024*d) + (17*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(1536
*d) - (11*a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(384*d) + (a^2*Cot[c + d*x]^3*Cs
c[c + d*x]^5)/(16*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d) - (a^2*Co
t[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^7)/(24
*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^7)/(12*d)
```

Rubi [A] time = 0.426146, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 270}

$$-\frac{2a^2\cot^{11}(c+dx)}{11d}-\frac{4a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{17a^2\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^2\cot^5(c+dx)\csc^7(c+dx)}{12d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (17*a^2*ArcTanh[Cos[c + d*x]])/(1024*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (4
*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Cot[c + d*x]^11)/(11*d) + (17*a^2*Cot[c
 + d*x]*Csc[c + d*x])/(1024*d) + (17*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(1536
*d) - (11*a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(384*d) + (a^2*Cot[c + d*x]^3*Cs
c[c + d*x]^5)/(16*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d) - (a^2*Co
t[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^7)/(24
*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^7)/(12*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^5(c+dx)+2a^2\cot^6(c+dx)\csc^6(c+dx)\right.$$

$$= a^2\int \cot^6(c+dx)\csc^5(c+dx)\,dx + a^2\int \cot^6(c+dx)\csc^7(c+$$

$$= -\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{a^2\cot^5(c+dx)\csc^7(c+dx)}{12d} -$$

$$= \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d} +$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d} - \frac{2a^2\cot^{11}(c+dx)}{11d} -$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d} - \frac{2a^2\cot^{11}(c+dx)}{11d} +$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d} - \frac{2a^2\cot^{11}(c+dx)}{11d} +$$

$$= \frac{3a^2\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d}$$

$$= \frac{17a^2\tanh^{-1}(\cos(c+dx))}{1024d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d}$$

Mathematica [A] time = 4.59543, size = 197, normalized size = 0.73

$$a^2(\sin(c+dx)+1)^2\left(30159360\left(\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{11}(c+dx)(29655040\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(30159360*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d
*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^11*(65553642 + 67499586*Cos[2*(c + d*x
)] + 25966248*Cos[4*(c + d*x)] - 6944091*Cos[6*(c + d*x)] - 746130*Cos[8*(c
 + d*x)] + 58905*Cos[10*(c + d*x)] + 29655040*Sin[c + d*x] + 51445760*Sin[3
*(c + d*x)] + 25600000*Sin[5*(c + d*x)] + 3235840*Sin[7*(c + d*x)] - 532480
*Sin[9*(c + d*x)] + 40960*Sin[11*(c + d*x)])))/(1816657920*d*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.089, size = 288, normalized size = 1.1

$$-\frac{17 a^2 (\cos (dx+c))^7}{120 d (\sin (dx+c))^{10}} - \frac{17 a^2 (\cos (dx+c))^7}{320 d (\sin (dx+c))^8} - \frac{17 a^2 (\cos (dx+c))^7}{1920 d (\sin (dx+c))^6} + \frac{17 a^2 (\cos (dx+c))^7}{7680 d (\sin (dx+c))^4} - \frac{17 a^2 (\cos (dx+c))^7}{5120 d (\sin (dx+c))^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -17/120/d*a^2/sin(d*x+c)^10*cos(d*x+c)^7-17/320/d*a^2/sin(d*x+c)^8*cos(d*x+
c)^7-17/1920/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+17/7680/d*a^2/sin(d*x+c)^4*cos
(d*x+c)^7-17/5120/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-17/5120*a^2*cos(d*x+c)^5/
d-17/3072*a^2*cos(d*x+c)^3/d-17/1024*a^2*cos(d*x+c)/d-17/1024/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))-2/11/d*a^2/sin(d*x+c)^11*cos(d*x+c)^7-8/99/d*a^2/sin(d*x
+c)^9*cos(d*x+c)^7-16/693/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/12/d*a^2/sin(d*
x+c)^12*cos(d*x+c)^7
```

Maxima [A] time = 1.0597, size = 436, normalized size = 1.61

$$-\frac{1155 a^2 \left(\frac{2 (15 \cos(dx+c)^{11} - 85 \cos(dx+c)^9 + 198 \cos(dx+c)^7 + 198 \cos(dx+c)^5 - 85 \cos(dx+c)^3 + 15 \cos(dx+c))}{\cos(dx+c)^{12} - 6 \cos(dx+c)^{10} + 15 \cos(dx+c)^8 - 20 \cos(dx+c)^6 + 15 \cos(dx+c)^4 - 6 \cos(dx+c)^2 + 1} - 15 \log (\cos (dx+c)+1) + 15 \log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/7096320*(1155*a^2*(2*(15*cos(d*x + c)^11 - 85*cos(d*x + c)^9 + 198*cos(d
*x + c)^7 + 198*cos(d*x + c)^5 - 85*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(
d*x + c)^12 - 6*cos(d*x + c)^10 + 15*cos(d*x + c)^8 - 20*cos(d*x + c)^6 + 1
5*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*lo
g(cos(d*x + c) - 1)) + 2772*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 -
 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10
 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c
)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 20480*(99
*tan(d*x + c)^4 + 154*tan(d*x + c)^2 + 63)*a^2/tan(d*x + c)^11)/d
```

Fricas [A] time = 1.33592, size = 1044, normalized size = 3.87

$$-\frac{117810 a^2 \cos (dx+c)^{11} - 667590 a^2 \cos (dx+c)^9 + 135828 a^2 \cos (dx+c)^7 + 1555092 a^2 \cos (dx+c)^5 - 667590 a^2 \cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/7096320*(117810*a^2*cos(d*x + c)^11 - 667590*a^2*cos(d*x + c)^9 + 135828
*a^2*cos(d*x + c)^7 + 1555092*a^2*cos(d*x + c)^5 - 667590*a^2*cos(d*x + c)^
3 + 117810*a^2*cos(d*x + c) - 58905*(a^2*cos(d*x + c)^12 - 6*a^2*cos(d*x +
```

```
c)^10 + 15*a^2*cos(d*x + c)^8 - 20*a^2*cos(d*x + c)^6 + 15*a^2*cos(d*x + c)
^4 - 6*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2) + 58905*(a^2*c
os(d*x + c)^12 - 6*a^2*cos(d*x + c)^10 + 15*a^2*cos(d*x + c)^8 - 20*a^2*cos
(d*x + c)^6 + 15*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*
cos(d*x + c) + 1/2) + 20480*(8*a^2*cos(d*x + c)^11 - 44*a^2*cos(d*x + c)^9
+ 99*a^2*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^12 - 6*d*cos(d*x + c
)^10 + 15*d*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*
d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**13*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36471, size = 567, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/56770560*(1155*a^2*tan(1/2*d*x + 1/2*c)^12 + 5040*a^2*tan(1/2*d*x + 1/2*c
)^11 + 5544*a^2*tan(1/2*d*x + 1/2*c)^10 - 6160*a^2*tan(1/2*d*x + 1/2*c)^9 -
 24255*a^2*tan(1/2*d*x + 1/2*c)^8 - 39600*a^2*tan(1/2*d*x + 1/2*c)^7 - 2772
0*a^2*tan(1/2*d*x + 1/2*c)^6 + 55440*a^2*tan(1/2*d*x + 1/2*c)^5 + 162855*a^
2*tan(1/2*d*x + 1/2*c)^4 + 184800*a^2*tan(1/2*d*x + 1/2*c)^3 + 55440*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 942480*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 554400*a
^2*tan(1/2*d*x + 1/2*c) + (2924714*a^2*tan(1/2*d*x + 1/2*c)^12 + 554400*a^2
*tan(1/2*d*x + 1/2*c)^11 - 55440*a^2*tan(1/2*d*x + 1/2*c)^10 - 184800*a^2*t
an(1/2*d*x + 1/2*c)^9 - 162855*a^2*tan(1/2*d*x + 1/2*c)^8 - 55440*a^2*tan(1
/2*d*x + 1/2*c)^7 + 27720*a^2*tan(1/2*d*x + 1/2*c)^6 + 39600*a^2*tan(1/2*d*
x + 1/2*c)^5 + 24255*a^2*tan(1/2*d*x + 1/2*c)^4 + 6160*a^2*tan(1/2*d*x + 1/
2*c)^3 - 5544*a^2*tan(1/2*d*x + 1/2*c)^2 - 5040*a^2*tan(1/2*d*x + 1/2*c) -
1155*a^2)/tan(1/2*d*x + 1/2*c)^12)/d
```

3.605 $\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=224

$$\frac{a^3\cos^{13}(c+dx)}{13d}-\frac{6a^3\cos^{11}(c+dx)}{11d}+\frac{a^3\cos^9(c+dx)}{d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{4d}-\frac{9a^3\sin^3(c+$$

```
[Out] (27*a^3*x)/1024 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (a^3*Cos[c + d*x]^9)/d - (
6*a^3*Cos[c + d*x]^11)/(11*d) + (a^3*Cos[c + d*x]^13)/(13*d) + (27*a^3*Cos[
c + d*x]*Sin[c + d*x])/(1024*d) + (9*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(512*
d) + (9*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(640*d) - (27*a^3*Cos[c + d*x]^7*S
in[c + d*x])/(320*d) - (9*a^3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(40*d) - (a^3*
Cos[c + d*x]^7*Sin[c + d*x]^5)/(4*d)
```

Rubi [A] time = 0.422832, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2568, 2635, 8, 2565, 270}

$$\frac{a^3\cos^{13}(c+dx)}{13d}-\frac{6a^3\cos^{11}(c+dx)}{11d}+\frac{a^3\cos^9(c+dx)}{d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{4d}-\frac{9a^3\sin^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (27*a^3*x)/1024 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (a^3*Cos[c + d*x]^9)/d - (
6*a^3*Cos[c + d*x]^11)/(11*d) + (a^3*Cos[c + d*x]^13)/(13*d) + (27*a^3*Cos[
c + d*x]*Sin[c + d*x])/(1024*d) + (9*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(512*
d) + (9*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(640*d) - (27*a^3*Cos[c + d*x]^7*S
in[c + d*x])/(320*d) - (9*a^3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(40*d) - (a^3*
Cos[c + d*x]^7*Sin[c + d*x]^5)/(4*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^4(c+dx)+3a^3\cos^6(c+dx)\sin^5(c+dx)\right.$$

$$= a^3\int \cos^6(c+dx)\sin^4(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^7(c+$$

$$= -\frac{a^3\cos^7(c+dx)\sin^3(c+dx)}{10d} - \frac{a^3\cos^7(c+dx)\sin^5(c+dx)}{4d} +$$

$$= -\frac{3a^3\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{9a^3\cos^7(c+dx)\sin^3(c+dx)}{40d} -$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+dx)}{11d} +$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+dx)}{11d} +$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+dx)}{11d} +$$

$$= \frac{3a^3x}{256} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+dx)}{11d}$$

$$= \frac{27a^3x}{1024} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+dx)}{11d}$$

Mathematica [A] time = 2.16765, size = 146, normalized size = 0.65

$$a^3(80080\sin(2(c+dx)) - 385385\sin(4(c+dx)) - 40040\sin(6(c+dx)) + 65065\sin(8(c+dx)) + 8008\sin(10(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(720720*c + 1081080*d*x - 1401400*Cos[c + d*x] - 450450*Cos[3*(c + d*x
)] + 150150*Cos[5*(c + d*x)] + 94380*Cos[7*(c + d*x)] - 20020*Cos[9*(c + d*
x)] - 11830*Cos[11*(c + d*x)] + 770*Cos[13*(c + d*x)] + 80080*Sin[2*(c + d*
x)] - 385385*Sin[4*(c + d*x)] - 40040*Sin[6*(c + d*x)] + 65065*Sin[8*(c + d
*x)] + 8008*Sin[10*(c + d*x)] - 5005*Sin[12*(c + d*x)]))/(41000960*d)
```

Maple [A] time = 0.046, size = 308, normalized size = 1.4

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^6(\cos(dx+c))^7}{13} - \frac{6(\sin(dx+c))^4(\cos(dx+c))^7}{143} - \frac{8(\sin(dx+c))^2(\cos(dx+c))^7}{429} - \frac{16(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/13*sin(d*x+c)^6*cos(d*x+c)^7-6/143*sin(d*x+c)^4*cos(d*x+c)^7-8
/429*sin(d*x+c)^2*cos(d*x+c)^7-16/3003*cos(d*x+c)^7)+3*a^3*(-1/12*sin(d*x+c
)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*cos(d*x+c)^7-1/64*sin(d*x+c)*cos(d*x+c)^
7+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d
*x+5/1024*c)+3*a^3*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d
*x+c)^7-8/693*cos(d*x+c)^7)+a^3*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d
*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*si
n(d*x+c)+3/256*d*x+3/256*c))
```

Maxima [A] time = 1.02185, size = 248, normalized size = 1.11

$$\frac{40960\left(231\cos\left(dx+c\right)^{13}-819\cos\left(dx+c\right)^{11}+1001\cos\left(dx+c\right)^{9}-429\cos\left(dx+c\right)^{7}\right)a^{3}-532480\left(63\cos\left(dx+c\right)^{11}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/123002880*(40960*(231*cos(d*x + c)^13 - 819*cos(d*x + c)^11 + 1001*cos(d*
x + c)^9 - 429*cos(d*x + c)^7)*a^3 - 532480*(63*cos(d*x + c)^11 - 154*cos(d
*x + c)^9 + 99*cos(d*x + c)^7)*a^3 + 12012*(32*sin(2*d*x + 2*c)^5 + 120*d*x
 + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a^3 + 15015*(4*sin(4*d
*x + 4*c)^3 + 120*d*x + 120*c + 9*sin(8*d*x + 8*c) - 48*sin(4*d*x + 4*c))*a
^3)/d
```

Fricas [A] time = 1.38722, size = 428, normalized size = 1.91

$$\frac{394240\,a^{3}\cos\left(dx+c\right)^{13}-2795520\,a^{3}\cos\left(dx+c\right)^{11}+5125120\,a^{3}\cos\left(dx+c\right)^{9}-2928640\,a^{3}\cos\left(dx+c\right)^{7}+135135\,a^{3}d\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/5125120*(394240*a^3*cos(d*x + c)^13 - 2795520*a^3*cos(d*x + c)^11 + 51251
20*a^3*cos(d*x + c)^9 - 2928640*a^3*cos(d*x + c)^7 + 135135*a^3*d*x - 1001*
(1280*a^3*cos(d*x + c)^11 - 3712*a^3*cos(d*x + c)^9 + 2864*a^3*cos(d*x + c)
^7 - 72*a^3*cos(d*x + c)^5 - 90*a^3*cos(d*x + c)^3 - 135*a^3*cos(d*x + c))*
sin(d*x + c))/d
```

Sympy [A] time = 127.425, size = 748, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((15*a**3*x*sin(c + d*x)**12/1024 + 45*a**3*x*sin(c + d*x)**10*cos
(c + d*x)**2/512 + 3*a**3*x*sin(c + d*x)**10/256 + 225*a**3*x*sin(c + d*x)*
*8*cos(c + d*x)**4/1024 + 15*a**3*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 7
5*a**3*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 15*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 225*a**3*x*sin(c + d*x)**4*cos(c + d*x)**8/1024 + 15*a*
*3*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 45*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**10/512 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 15*a**3*x*
cos(c + d*x)**12/1024 + 3*a**3*x*cos(c + d*x)**10/256 + 15*a**3*sin(c + d*x
)**11*cos(c + d*x)/(1024*d) + 85*a**3*sin(c + d*x)**9*cos(c + d*x)**3/(1024
*d) + 3*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 99*a**3*sin(c + d*x)**7
*cos(c + d*x)**5/(512*d) + 7*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) -
 a**3*sin(c + d*x)**6*cos(c + d*x)**7/(7*d) - 99*a**3*sin(c + d*x)**5*cos(c
 + d*x)**7/(512*d) + a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) - 2*a**3*s
in(c + d*x)**4*cos(c + d*x)**9/(21*d) - 3*a**3*sin(c + d*x)**4*cos(c + d*x)
**7/(7*d) - 85*a**3*sin(c + d*x)**3*cos(c + d*x)**9/(1024*d) - 7*a**3*sin(c
 + d*x)**3*cos(c + d*x)**7/(128*d) - 8*a**3*sin(c + d*x)**2*cos(c + d*x)**1
1/(231*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)**9/(21*d) - 15*a**3*sin(c +
 d*x)*cos(c + d*x)**11/(1024*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*
d) - 16*a**3*cos(c + d*x)**13/(3003*d) - 8*a**3*cos(c + d*x)**11/(231*d), N
e(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**4*cos(c)**6, True))
```

Giac [A] time = 1.39393, size = 304, normalized size = 1.36

$$\frac{27}{1024}a^3x + \frac{a^3\cos(13dx+13c)}{53248d} - \frac{13a^3\cos(11dx+11c)}{45056d} - \frac{a^3\cos(9dx+9c)}{2048d} + \frac{33a^3\cos(7dx+7c)}{14336d} + \frac{15a^3\cos(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 27/1024*a^3*x + 1/53248*a^3*cos(13*d*x + 13*c)/d - 13/45056*a^3*cos(11*d*x
+ 11*c)/d - 1/2048*a^3*cos(9*d*x + 9*c)/d + 33/14336*a^3*cos(7*d*x + 7*c)/d
 + 15/4096*a^3*cos(5*d*x + 5*c)/d - 45/4096*a^3*cos(3*d*x + 3*c)/d - 35/102
4*a^3*cos(d*x + c)/d - 1/8192*a^3*sin(12*d*x + 12*c)/d + 1/5120*a^3*sin(10*
d*x + 10*c)/d + 13/8192*a^3*sin(8*d*x + 8*c)/d - 1/1024*a^3*sin(6*d*x + 6*c
)/d - 77/8192*a^3*sin(4*d*x + 4*c)/d + 1/512*a^3*sin(2*d*x + 2*c)/d
```

3.606 $\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=209

$$-\frac{3a^3\cos^{11}(c+dx)}{11d}+\frac{7a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{41a^3\sin^3(c+dx)\cos^7(c+dx)}{120d}$$

```
[Out] (41*a^3*x)/1024 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (7*a^3*Cos[c + d*x]^9)/(9*
d) - (3*a^3*Cos[c + d*x]^11)/(11*d) + (41*a^3*Cos[c + d*x]*Sin[c + d*x])/(1
024*d) + (41*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(1536*d) + (41*a^3*Cos[c + d*
x]^5*Sin[c + d*x])/(1920*d) - (41*a^3*Cos[c + d*x]^7*Sin[c + d*x])/(320*d)
- (41*a^3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(120*d) - (a^3*Cos[c + d*x]^7*Sin[
c + d*x]^5)/(12*d)
```

Rubi [A] time = 0.412502, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{3a^3\cos^{11}(c+dx)}{11d}+\frac{7a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{41a^3\sin^3(c+dx)\cos^7(c+dx)}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (41*a^3*x)/1024 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (7*a^3*Cos[c + d*x]^9)/(9*
d) - (3*a^3*Cos[c + d*x]^11)/(11*d) + (41*a^3*Cos[c + d*x]*Sin[c + d*x])/(1
024*d) + (41*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(1536*d) + (41*a^3*Cos[c + d*
x]^5*Sin[c + d*x])/(1920*d) - (41*a^3*Cos[c + d*x]^7*Sin[c + d*x])/(320*d)
- (41*a^3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(120*d) - (a^3*Cos[c + d*x]^7*Sin[
c + d*x]^5)/(12*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
```

```
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^3(c+dx)+3a^3\cos^6(c+dx)\sin^4(c+dx)\right.$$
$$= a^3\int \cos^6(c+dx)\sin^3(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^6(c+$$
$$= -\frac{3a^3\cos^7(c+dx)\sin^3(c+dx)}{10d} - \frac{a^3\cos^7(c+dx)\sin^5(c+dx)}{12d} +$$
$$= -\frac{9a^3\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{41a^3\cos^7(c+dx)\sin^3(c+dx)}{120d}$$
$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+dx)}{11d} + \frac{3a^3}{}$$
$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+dx)}{11d} + \frac{3a^3}{}$$
$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+dx)}{11d} + \frac{9a^3}{}$$
$$= \frac{9a^3x}{256} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+dx)}{11d}$$
$$= \frac{41a^3x}{1024} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+dx)}{11d}$$

Mathematica [A] time = 1.51411, size = 136, normalized size = 0.65

$$\frac{a^3(166320\sin(2(c+dx)) - 384615\sin(4(c+dx)) - 83160\sin(6(c+dx)) + 51975\sin(8(c+dx)) + 16632\sin(10(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1247400*c + 1136520*d*x - 1496880*Cos[c + d*x] - 572880*Cos[3*(c + d*
x)] + 83160*Cos[5*(c + d*x)] + 106920*Cos[7*(c + d*x)] + 3080*Cos[9*(c + d*
x)] - 7560*Cos[11*(c + d*x)] + 166320*Sin[2*(c + d*x)] - 384615*Sin[4*(c +
d*x)] - 83160*Sin[6*(c + d*x)] + 51975*Sin[8*(c + d*x)] + 16632*Sin[10*(c +
 d*x)] - 1155*Sin[12*(c + d*x)]))/(28385280*d)
```

Maple [A] time = 0.041, size = 272, normalized size = 1.3

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^5(\cos(dx+c))^7}{12}-\frac{(\sin(dx+c))^3(\cos(dx+c))^7}{24}-\frac{\sin(dx+c)(\cos(dx+c))^7}{64}+\frac{\sin(dx+c)}{384}\left((\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*cos(d*x+c)^7-1/
64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*
x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+3*a^3*(-1/11*sin(d*x+c)^4*cos(d*x+c)^
7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7)+3*a^3*(-1/10*sin(d*x+c
)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d
*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+a^3*(-1/9*sin(d*x+c)
^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

Maxima [A] time = 1.06464, size = 221, normalized size = 1.06

$$-\frac{122880\left(63\cos(dx+c)^{11}-154\cos(dx+c)^9+99\cos(dx+c)^7\right)a^3-450560\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^3-8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/28385280*(122880*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x +
 c)^7)*a^3 - 450560*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^3 - 8316*(32*si
n(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*
c))*a^3 - 1155*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120*c + 9*sin(8*d*x + 8*c)
 - 48*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A] time = 1.3848, size = 390, normalized size = 1.87

$$-\frac{967680\,a^3\cos(dx+c)^{11}-2759680\,a^3\cos(dx+c)^9+2027520\,a^3\cos(dx+c)^7-142065\,a^3dx+231\left(1280\,a^3\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/3548160*(967680*a^3*cos(d*x + c)^11 - 2759680*a^3*cos(d*x + c)^9 + 20275
20*a^3*cos(d*x + c)^7 - 142065*a^3*d*x + 231*(1280*a^3*cos(d*x + c)^11 - 78
08*a^3*cos(d*x + c)^9 + 8496*a^3*cos(d*x + c)^7 - 328*a^3*cos(d*x + c)^5 -
410*a^3*cos(d*x + c)^3 - 615*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 86.8045, size = 699, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(c + d*x)**12/1024 + 15*a**3*x*sin(c + d*x)**10*cos(
c + d*x)**2/512 + 9*a**3*x*sin(c + d*x)**10/256 + 75*a**3*x*sin(c + d*x)**8
*cos(c + d*x)**4/1024 + 45*a**3*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 25*
a**3*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 45*a**3*x*sin(c + d*x)**6*cos(
c + d*x)**4/128 + 75*a**3*x*sin(c + d*x)**4*cos(c + d*x)**8/1024 + 45*a**3*
x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**3*x*sin(c + d*x)**2*cos(c + d
*x)**10/512 + 45*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**3*x*cos(
c + d*x)**12/1024 + 9*a**3*x*cos(c + d*x)**10/256 + 5*a**3*sin(c + d*x)**11
*cos(c + d*x)/(1024*d) + 85*a**3*sin(c + d*x)**9*cos(c + d*x)**3/(3072*d) +
 9*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 33*a**3*sin(c + d*x)**7*cos(
c + d*x)**5/(512*d) + 21*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) - 33*
a**3*sin(c + d*x)**5*cos(c + d*x)**7/(512*d) + 3*a**3*sin(c + d*x)**5*cos(c
 + d*x)**5/(10*d) - 3*a**3*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 85*a**3*
sin(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 21*a**3*sin(c + d*x)**3*cos(c +
d*x)**7/(128*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)**9/(21*d) - a**3*sin(
c + d*x)**2*cos(c + d*x)**7/(7*d) - 5*a**3*sin(c + d*x)*cos(c + d*x)**11/(1
024*d) - 9*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 8*a**3*cos(c + d*x)*
*11/(231*d) - 2*a**3*cos(c + d*x)**9/(63*d), Ne(d, 0)), (x*(a*sin(c) + a)**
3*sin(c)**3*cos(c)**6, True))
```

Giac [A] time = 1.31328, size = 281, normalized size = 1.34

$$\frac{41}{1024}a^3x - \frac{3\,a^3\cos(11\,dx+11\,c)}{11264\,d} + \frac{a^3\cos(9\,dx+9\,c)}{9216\,d} + \frac{27\,a^3\cos(7\,dx+7\,c)}{7168\,d} + \frac{3\,a^3\cos(5\,dx+5\,c)}{1024\,d} - \frac{31\,a^3\cos(3\,d}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 41/1024*a^3*x - 3/11264*a^3*cos(11*d*x + 11*c)/d + 1/9216*a^3*cos(9*d*x + 9
*c)/d + 27/7168*a^3*cos(7*d*x + 7*c)/d + 3/1024*a^3*cos(5*d*x + 5*c)/d - 31
/1536*a^3*cos(3*d*x + 3*c)/d - 27/512*a^3*cos(d*x + c)/d - 1/24576*a^3*sin(
12*d*x + 12*c)/d + 3/5120*a^3*sin(10*d*x + 10*c)/d + 15/8192*a^3*sin(8*d*x
+ 8*c)/d - 3/1024*a^3*sin(6*d*x + 6*c)/d - 111/8192*a^3*sin(4*d*x + 4*c)/d
+ 3/512*a^3*sin(2*d*x + 2*c)/d
```

3.607 $\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=183

$$-\frac{a^3\cos^{11}(c+dx)}{11d}+\frac{5a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{3a^3\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{19a^3\sin(c+dx)\cos^7(c+dx)}{80d}$$

```
[Out] (19*a^3*x)/256 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (5*a^3*Cos[c + d*x]^9)/(9*d
) - (a^3*Cos[c + d*x]^11)/(11*d) + (19*a^3*Cos[c + d*x]*Sin[c + d*x])/(256*
d) + (19*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (19*a^3*Cos[c + d*x]^5*
Sin[c + d*x])/(480*d) - (19*a^3*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (3*a^
3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*d)
```

Rubi [A] time = 0.333503, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2568, 2635, 8, 2565, 14, 270}

$$-\frac{a^3\cos^{11}(c+dx)}{11d}+\frac{5a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{3a^3\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{19a^3\sin(c+dx)\cos^7(c+dx)}{80d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (19*a^3*x)/256 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (5*a^3*Cos[c + d*x]^9)/(9*d
) - (a^3*Cos[c + d*x]^11)/(11*d) + (19*a^3*Cos[c + d*x]*Sin[c + d*x])/(256*
d) + (19*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (19*a^3*Cos[c + d*x]^5*
Sin[c + d*x])/(480*d) - (19*a^3*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (3*a^
3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^2(c+dx)+3a^3\cos^6(c+dx)\sin^3(c+dx)\right.$$

$$= a^3\int \cos^6(c+dx)\sin^2(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^5(c+$$

$$= -\frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{3a^3\cos^7(c+dx)\sin^3(c+dx)}{10d} + $$

$$= \frac{a^3\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{19a^3\cos^7(c+dx)\sin(c+dx)}{80d} - $$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c+dx)}{11d} + $$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c+dx)}{11d} + $$

$$= \frac{5a^3x}{128} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c+dx)}{11d}$$

$$= \frac{19a^3x}{256} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c+dx)}{11d}$$

Mathematica [A] time = 1.14324, size = 126, normalized size = 0.69

$$a^3(152460\sin(2(c+dx)) - 138600\sin(4(c+dx)) - 57750\sin(6(c+dx)) + 3465\sin(8(c+dx)) + 4158\sin(10(c+dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(415800*c + 526680*d*x - 568260*Cos[c + d*x] - 244860*Cos[3*(c + d*x)]
 + 6930*Cos[5*(c + d*x)] + 40590*Cos[7*(c + d*x)] + 8470*Cos[9*(c + d*x)] -
 630*Cos[11*(c + d*x)] + 152460*Sin[2*(c + d*x)] - 138600*Sin[4*(c + d*x)]
- 57750*Sin[6*(c + d*x)] + 3465*Sin[8*(c + d*x)] + 4158*Sin[10*(c + d*x)]))
/(7096320*d)
```

Maple [A] time = 0.043, size = 236, normalized size = 1.3

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^7}{11} - \frac{4(\sin(dx+c))^2(\cos(dx+c))^7}{99} - \frac{8(\cos(dx+c))^7}{693}\right) + 3a^3\left(-1/10(\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/
693*cos(d*x+c)^7)+3*a^3*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*co
s(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)
+3/256*d*x+3/256*c)+3*a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7
)+a^3*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/
8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c))
```

Maxima [A] time = 1.16468, size = 221, normalized size = 1.21

$$-\frac{10240\left(63\cos(dx+c)^{11}-154\cos(dx+c)^{9}+99\cos(dx+c)^{7}\right)a^3-337920\left(7\cos(dx+c)^{9}-9\cos(dx+c)^{7}\right)a^3-20}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/7096320*(10240*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c
)^7)*a^3 - 337920*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^3 - 2079*(32*sin(
2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c)
)*a^3 - 2310*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
- 24*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A] time = 1.29344, size = 347, normalized size = 1.9

$$-\frac{80640\,a^3\cos(dx+c)^{11}-492800\,a^3\cos(dx+c)^{9}+506880\,a^3\cos(dx+c)^{7}-65835\,a^3dx-231\left(1152\,a^3\cos(dx+c)^{9}-\right.}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/887040*(80640*a^3*cos(d*x + c)^11 - 492800*a^3*cos(d*x + c)^9 + 506880*a
^3*cos(d*x + c)^7 - 65835*a^3*d*x - 231*(1152*a^3*cos(d*x + c)^9 - 2064*a^3
*cos(d*x + c)^7 + 152*a^3*cos(d*x + c)^5 + 190*a^3*cos(d*x + c)^3 + 285*a^3
*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 59.4676, size = 597, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**10/256 + 45*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 5*a**3*x*sin(c + d*x)**8/128 + 45*a**3*x*sin(c + d*x)**6*co
```

```
s(c + d*x)**4/128 + 5*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 45*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**3*x*sin(c + d*x)**4*cos(c + d*
x)**4/64 + 45*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**3*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 9*a**3*x*cos(c + d*x)**10/256 + 5*a**3*x*cos(
c + d*x)**8/128 + 9*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 21*a**3*sin
(c + d*x)**7*cos(c + d*x)**3/(128*d) + 5*a**3*sin(c + d*x)**7*cos(c + d*x)/
(128*d) + 3*a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 55*a**3*sin(c + d
*x)**5*cos(c + d*x)**3/(384*d) - a**3*sin(c + d*x)**4*cos(c + d*x)**7/(7*d)
 - 21*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) + 73*a**3*sin(c + d*x)**
3*cos(c + d*x)**5/(384*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) -
 3*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 9*a**3*sin(c + d*x)*cos(c +
 d*x)**9/(256*d) - 5*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 8*a**3*cos
(c + d*x)**11/(693*d) - 2*a**3*cos(c + d*x)**9/(21*d), Ne(d, 0)), (x*(a*sin
(c) + a)**3*sin(c)**2*cos(c)**6, True))
```

Giac [A] time = 1.27088, size = 258, normalized size = 1.41

$$\frac{19}{256}\,a^3x-\frac{a^3\cos\left(11\,dx+11\,c\right)}{11264\,d}+\frac{11\,a^3\cos\left(9\,dx+9\,c\right)}{9216\,d}+\frac{41\,a^3\cos\left(7\,dx+7\,c\right)}{7168\,d}+\frac{a^3\cos\left(5\,dx+5\,c\right)}{1024\,d}-\frac{53\,a^3\cos\left(3\,dx\right.}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 19/256*a^3*x - 1/11264*a^3*cos(11*d*x + 11*c)/d + 11/9216*a^3*cos(9*d*x + 9
*c)/d + 41/7168*a^3*cos(7*d*x + 7*c)/d + 1/1024*a^3*cos(5*d*x + 5*c)/d - 53
/1536*a^3*cos(3*d*x + 3*c)/d - 41/512*a^3*cos(d*x + c)/d + 3/5120*a^3*sin(1
0*d*x + 10*c)/d + 1/2048*a^3*sin(8*d*x + 8*c)/d - 25/3072*a^3*sin(6*d*x + 6
*c)/d - 5/256*a^3*sin(4*d*x + 4*c)/d + 11/512*a^3*sin(2*d*x + 2*c)/d
```

3.608 $\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=181

$$-\frac{33a^3\cos^7(c+dx)}{560d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{240d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{11a^3\sin(c+dx)\cos^3(c+d}{128d}$$

```
[Out] (33*a^3*x)/256 - (33*a^3*Cos[c + d*x]^7)/(560*d) + (33*a^3*Cos[c + d*x]*Sin
[c + d*x])/(256*d) + (11*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (11*a^3
*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) - (a*Cos[c + d*x]^7*(a + a*Sin[c + d*
x])^2)/(30*d) - (Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(10*d) - (11*Cos[c
+ d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(240*d)
```

Rubi [A] time = 0.203746, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{33a^3\cos^7(c+dx)}{560d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{240d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{11a^3\sin(c+dx)\cos^3(c+d}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (33*a^3*x)/256 - (33*a^3*Cos[c + d*x]^7)/(560*d) + (33*a^3*Cos[c + d*x]*Sin
[c + d*x])/(256*d) + (11*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (11*a^3
*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) - (a*Cos[c + d*x]^7*(a + a*Sin[c + d*
x])^2)/(30*d) - (Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(10*d) - (11*Cos[c
+ d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(240*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{\cos^7(c+dx)(a+a\sin(c+dx))^3}{10d}+\frac{3}{10}\int \cos^6(c+dx)(a+a\sin($$
$$= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{30d}-\frac{\cos^7(c+dx)(a+a\sin(c+$$
$$= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{30d}-\frac{\cos^7(c+dx)(a+a\sin(c+$$
$$= -\frac{33a^3\cos^7(c+dx)}{560d}-\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{30d}-\frac{\cos^7(c$$
$$= -\frac{33a^3\cos^7(c+dx)}{560d}+\frac{11a^3\cos^5(c+dx)\sin(c+dx)}{160d}-\frac{a\cos^7(c+$$
$$= -\frac{33a^3\cos^7(c+dx)}{560d}+\frac{11a^3\cos^3(c+dx)\sin(c+dx)}{128d}+\frac{11a^3\cos^5($$
$$= -\frac{33a^3\cos^7(c+dx)}{560d}+\frac{33a^3\cos(c+dx)\sin(c+dx)}{256d}+\frac{11a^3\cos^3(c$$
$$= \frac{33a^3x}{256}-\frac{33a^3\cos^7(c+dx)}{560d}+\frac{33a^3\cos(c+dx)\sin(c+dx)}{256d}+\frac{11a$$

Mathematica [A] time = 0.925399, size = 116, normalized size = 0.64

$$\frac{a^3(10500\sin(2(c+dx))-5880\sin(4(c+dx))-3570\sin(6(c+dx))-525\sin(8(c+dx))+42\sin(10(c+dx))-3192}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(31500*c + 27720*d*x - 31920*Cos[c + d*x] - 16800*Cos[3*(c + d*x)] - 3
360*Cos[5*(c + d*x)] + 600*Cos[7*(c + d*x)] + 280*Cos[9*(c + d*x)] + 10500*
Sin[2*(c + d*x)] - 5880*Sin[4*(c + d*x)] - 3570*Sin[6*(c + d*x)] - 525*Sin[
8*(c + d*x)] + 42*Sin[10*(c + d*x)]))/(215040*d)
```

Maple [A] time = 0.038, size = 198, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^7}{10}-\frac{3\sin(dx+c)(\cos(dx+c))^7}{80}+\frac{\sin(dx+c)}{160}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/16
0*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/25
6*c)+3*a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+3*a^3*(-1/8*s
in(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))
*sin(d*x+c)+5/128*d*x+5/128*c)-1/7*a^3*cos(d*x+c)^7)
```

Maxima [A] time = 1.08366, size = 190, normalized size = 1.05

$$-\frac{30720\,a^3\cos(dx+c)^7-10240\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^3-21\left(32\sin(2\,dx+2\,c)^5+120\,dx+120\,c+5\sin\right.}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/215040*(30720*a^3*cos(d*x + c)^7 - 10240*(7*cos(d*x + c)^9 - 9*cos(d*x +
 c)^7)*a^3 - 21*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*
c) - 40*sin(4*d*x + 4*c))*a^3 - 210*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*
c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A] time = 1.23707, size = 296, normalized size = 1.64

$$\frac{8960\,a^3\cos(dx+c)^9-15360\,a^3\cos(dx+c)^7+3465\,a^3dx+21\left(128\,a^3\cos(dx+c)^9-656\,a^3\cos(dx+c)^7+88\,a^3\cos(\right.}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/26880*(8960*a^3*cos(d*x + c)^9 - 15360*a^3*cos(d*x + c)^7 + 3465*a^3*d*x
+ 21*(128*a^3*cos(d*x + c)^9 - 656*a^3*cos(d*x + c)^7 + 88*a^3*cos(d*x + c)
^5 + 110*a^3*cos(d*x + c)^3 + 165*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 34.385, size = 542, normalized size = 2.99

$$\begin{cases} \frac{3a^3x\sin^{10}(c+dx)}{256}+\frac{15a^3x\sin^8(c+dx)\cos^2(c+dx)}{256}+\frac{15a^3x\sin^8(c+dx)}{128}+\frac{15a^3x\sin^6(c+dx)\cos^4(c+dx)}{128}+\frac{15a^3x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{15a^3x\sin^4(}{} \\ x\left(a\sin(c)+a\right)^3\sin(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**10/256 + 15*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 15*a**3*x*sin(c + d*x)**8/128 + 15*a**3*x*sin(c + d*x)**6*c
os(c + d*x)**4/128 + 15*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**3
*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 45*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**4/64 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 15*a**3*x*sin(
c + d*x)**2*cos(c + d*x)**6/32 + 3*a**3*x*cos(c + d*x)**10/256 + 15*a**3*x*
cos(c + d*x)**8/128 + 3*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**3*
sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + 15*a**3*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 55*a**3*sin(c +
```

```
 d*x)**5*cos(c + d*x)**3/(128*d) - 7*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(
128*d) + 73*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 3*a**3*sin(c + d
*x)**2*cos(c + d*x)**7/(7*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*d)
- 15*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 2*a**3*cos(c + d*x)**9/(21
*d) - a**3*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)*co
s(c)**6, True))
```

Giac [A] time = 1.2254, size = 235, normalized size = 1.3

$$\frac{33}{256}a^3x + \frac{a^3\cos(9\,dx+9\,c)}{768\,d} + \frac{5\,a^3\cos(7\,dx+7\,c)}{1792\,d} - \frac{a^3\cos(5\,dx+5\,c)}{64\,d} - \frac{5\,a^3\cos(3\,dx+3\,c)}{64\,d} - \frac{19\,a^3\cos(dx+c)}{128\,d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 33/256*a^3*x + 1/768*a^3*cos(9*d*x + 9*c)/d + 5/1792*a^3*cos(7*d*x + 7*c)/d
 - 1/64*a^3*cos(5*d*x + 5*c)/d - 5/64*a^3*cos(3*d*x + 3*c)/d - 19/128*a^3*c
os(d*x + c)/d + 1/5120*a^3*sin(10*d*x + 10*c)/d - 5/2048*a^3*sin(8*d*x + 8*
c)/d - 17/1024*a^3*sin(6*d*x + 6*c)/d - 7/256*a^3*sin(4*d*x + 4*c)/d + 25/5
12*a^3*sin(2*d*x + 2*c)/d
```

3.609 $\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=185

$$-\frac{3a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{25a^3\sin(c+dx}{48d}$$

```
[Out] (125*a^3*x)/128 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d + (a
^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/(5*d) - (3*a^3*Cos[c + d*x]
^7)/(7*d) + (125*a^3*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (125*a^3*Cos[c +
d*x]^3*Sin[c + d*x])/(192*d) + (25*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d)
- (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rubi [A] time = 0.235842, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2873, 2635, 8, 2592, 302, 206, 2565, 30, 2568}

$$-\frac{3a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{25a^3\sin(c+dx}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (125*a^3*x)/128 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d + (a
^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/(5*d) - (3*a^3*Cos[c + d*x]
^7)/(7*d) + (125*a^3*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (125*a^3*Cos[c +
d*x]^3*Sin[c + d*x])/(192*d) + (25*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d)
- (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \int \Big(3a^3\cos^6(c+dx)+a^3\cos^5(c+dx)\cot(c+dx)+3a^3\cos^6(c+$$

$$= a^3\int \cos^5(c+dx)\cot(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^2(c+dx$$

$$= \frac{a^3\cos^5(c+dx)\sin(c+dx)}{2d} - \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}a^3$$

$$= -\frac{3a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{8d} + \frac{25a^3\cos^5(c$$

$$= \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} + \frac{a^3\cos^5(c+dx)}{5d} - \frac{3a^3\cos^7(c}{7d}$$

$$= \frac{15a^3x}{16} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+}{3d}$$

$$= \frac{125a^3x}{128} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+}{3d}$$

Mathematica [A] time = 0.685172, size = 122, normalized size = 0.66

$$a^3\Big(77280\sin(2(c+dx))+14280\sin(4(c+dx))+1120\sin(6(c+dx))-105\sin(8(c+dx))+122640\cos(c+dx)+56$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(105000*c + 105000*d*x + 122640*Cos[c + d*x] + 560*Cos[3*(c + d*x)] -
3696*Cos[5*(c + d*x)] - 720*Cos[7*(c + d*x)] - 107520*Log[Cos[(c + d*x)/2]]
 + 107520*Log[Sin[(c + d*x)/2]] + 77280*Sin[2*(c + d*x)] + 14280*Sin[4*(c +
 d*x)] + 1120*Sin[6*(c + d*x)] - 105*Sin[8*(c + d*x)]))/(107520*d)
```

Maple [A] time = 0.081, size = 187, normalized size = 1.

$$-\frac{a^3 (\cos(dx+c))^7 \sin(dx+c)}{8d}+\frac{25 a^3 (\cos(dx+c))^5 \sin(dx+c)}{48d}+\frac{125 a^3 (\cos(dx+c))^3 \sin(dx+c)}{192d}+\frac{125 a^3 \cos(dx}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d+25/48*a^3*cos(d*x+c)^5*sin(d*x+c)/d+125/
192*a^3*cos(d*x+c)^3*sin(d*x+c)/d+125/128*a^3*cos(d*x+c)*sin(d*x+c)/d+125/1
28*a^3*x+125/128/d*a^3*c-3/7*a^3*cos(d*x+c)^7/d+1/5*a^3*cos(d*x+c)^5/d+1/3*
a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.12855, size = 231, normalized size = 1.25

$$-\frac{46080 a^3 \cos(dx+c)^7 - 3584 \left(6 \cos(dx+c)^5 + 10 \cos(dx+c)^3 + 30 \cos(dx+c) - 15 \log(\cos(dx+c)+1) + 15 \log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/107520*(46080*a^3*cos(d*x + c)^7 - 3584*(6*cos(d*x + c)^5 + 10*cos(d*x +
 c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1))*a^3 - 35*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
- 24*sin(4*d*x + 4*c))*a^3 + 1680*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A] time = 1.27421, size = 439, normalized size = 2.37

$$-\frac{5760 a^3 \cos(dx+c)^7 - 2688 a^3 \cos(dx+c)^5 - 4480 a^3 \cos(dx+c)^3 - 13125 a^3 dx - 13440 a^3 \cos(dx+c) + 6720 a^3 \log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/13440*(5760*a^3*cos(d*x + c)^7 - 2688*a^3*cos(d*x + c)^5 - 4480*a^3*cos(
d*x + c)^3 - 13125*a^3*d*x - 13440*a^3*cos(d*x + c) + 6720*a^3*log(1/2*cos(
d*x + c) + 1/2) - 6720*a^3*log(-1/2*cos(d*x + c) + 1/2) + 35*(48*a^3*cos(d*
x + c)^7 - 200*a^3*cos(d*x + c)^5 - 250*a^3*cos(d*x + c)^3 - 375*a^3*cos(d*
x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25884, size = 374, normalized size = 2.02

$$13125\,(dx+c)a^3 + 13440\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - \frac{2\left(27195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+65135\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-161280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/13440*(13125*(d*x + c)*a^3 + 13440*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2
*(27195*a^3*tan(1/2*d*x + 1/2*c)^15 + 65135*a^3*tan(1/2*d*x + 1/2*c)^13 - 1
61280*a^3*tan(1/2*d*x + 1/2*c)^12 + 63595*a^3*tan(1/2*d*x + 1/2*c)^11 - 286
720*a^3*tan(1/2*d*x + 1/2*c)^10 + 133175*a^3*tan(1/2*d*x + 1/2*c)^9 - 51968
0*a^3*tan(1/2*d*x + 1/2*c)^8 - 133175*a^3*tan(1/2*d*x + 1/2*c)^7 - 544768*a
^3*tan(1/2*d*x + 1/2*c)^6 - 63595*a^3*tan(1/2*d*x + 1/2*c)^5 - 254464*a^3*t
an(1/2*d*x + 1/2*c)^4 - 65135*a^3*tan(1/2*d*x + 1/2*c)^3 - 118784*a^3*tan(1
/2*d*x + 1/2*c)^2 - 27195*a^3*tan(1/2*d*x + 1/2*c) - 14848*a^3)/(tan(1/2*d*
x + 1/2*c)^2 + 1)^8)/d
```

3.610 $\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=173

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+dx)}{2d}$$

```
[Out] (-15*a^3*x)/16 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d +
 (a^3*Cos[c + d*x]^3)/d + (3*a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]^
7)/(7*d) - (a^3*Cot[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d)
 - (11*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(8*d) + (a^3*Cos[c + d*x]*Sin[c + d
*x]^5)/(2*d)
```

Rubi [A] time = 0.24395, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-15*a^3*x)/16 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d +
 (a^3*Cos[c + d*x]^3)/d + (3*a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]^
7)/(7*d) - (a^3*Cot[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d)
 - (11*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(8*d) + (a^3*Cos[c + d*x]*Sin[c + d
*x]^5)/(2*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(3a^9\csc(c+dx)+a^9\csc^2(c+dx)-8a^9\sin(c+dx)-6a^9\sin^2 \cdots\right.}{\cdots}$$

$$= a^3\int \csc^2(c+dx)\,dx - a^3\int \sin^7(c+dx)\,dx + \left(3a^3\right)\int \csc(c+ \cdots$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{8a^3\cos(c+dx)}{d} + \frac{3a^3\cos(c+dx)\cdots}{d}$$

$$= -3a^3x - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(\cdots}{d}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{d} \cdots$$

$$= -\frac{15a^3x}{16} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3\cos^3 \cdots}{\cdots}$$

Mathematica [A] time = 1.88001, size = 168, normalized size = 0.97

$$\frac{(a\sin(c+dx)+a)^3\left(-2100(c+dx)+455\sin(2(c+dx))+245\sin(4(c+dx))+35\sin(6(c+dx))+9065\cos(c+dx)\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-2100*(c + d*x) + 9065*Cos[c + d*x] + 875*Cos[3*(c
 + d*x)] + 49*Cos[5*(c + d*x)] - 5*Cos[7*(c + d*x)] - 1120*Cot[(c + d*x)/2]
 - 6720*Log[Cos[(c + d*x)/2]] + 6720*Log[Sin[(c + d*x)/2]] + 455*Sin[2*(c +
 d*x)] + 245*Sin[4*(c + d*x)] + 35*Sin[6*(c + d*x)] + 1120*Tan[(c + d*x)/2]
))/(2240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.079, size = 190, normalized size = 1.1

$$-\frac{a^3(\cos(dx+c))^7}{7d} - \frac{a^3(\cos(dx+c))^5\sin(dx+c)}{2d} - \frac{5a^3(\cos(dx+c))^3\sin(dx+c)}{8d} - \frac{15a^3\cos(dx+c)\sin(dx+c)}{16d} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/7*a^3*cos(d*x+c)^7/d-1/2*a^3*cos(d*x+c)^5*sin(d*x+c)/d-5/8*a^3*cos(d*x+c
)^3*sin(d*x+c)/d-15/16*a^3*cos(d*x+c)*sin(d*x+c)/d-15/16*a^3*x-15/16/d*a^3*
c+3/5*a^3*cos(d*x+c)^5/d+a^3*cos(d*x+c)^3/d+3*a^3*cos(d*x+c)/d+3/d*a^3*ln(c
sc(d*x+c)-cot(d*x+c))-1/d*a^3/sin(d*x+c)*cos(d*x+c)^7
```

Maxima [A] time = 1.82477, size = 251, normalized size = 1.45

$$-\frac{320\,a^3\cos(dx+c)^7 - 224\left(6\cos(dx+c)^5 + 10\cos(dx+c)^3 + 30\cos(dx+c) - 15\log(\cos(dx+c)+1) + 15\log(\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2240*(320*a^3*cos(d*x + c)^7 - 224*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3
 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a
^3 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*a^3 + 280*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x +
 c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^3)/d
```

Fricas [A] time = 1.23167, size = 475, normalized size = 2.75

$$-\frac{280\,a^3\cos(dx+c)^7 - 70\,a^3\cos(dx+c)^5 - 175\,a^3\cos(dx+c)^3 + 840\,a^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 840\,a^3\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/560*(280*a^3*cos(d*x + c)^7 - 70*a^3*cos(d*x + c)^5 - 175*a^3*cos(d*x +
c)^3 + 840*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 840*a^3*log(-1/2*
cos(d*x + c) + 1/2)*sin(d*x + c) + 525*a^3*cos(d*x + c) + (80*a^3*cos(d*x +
 c)^7 - 336*a^3*cos(d*x + c)^5 - 560*a^3*cos(d*x + c)^3 + 525*a^3*d*x - 168
0*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28641, size = 392, normalized size = 2.27

$$-\frac{525\,(dx+c)a^3 - 1680\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - 280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{280\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \frac{2\left(525\,a^3\tan\left(\frac{1}{2}\,d\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/560*(525*(d*x + c)*a^3 - 1680*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 280*a
^3*tan(1/2*d*x + 1/2*c) + 280*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*
x + 1/2*c) + 2*(525*a^3*tan(1/2*d*x + 1/2*c)^13 - 4480*a^3*tan(1/2*d*x + 1/
2*c)^12 - 980*a^3*tan(1/2*d*x + 1/2*c)^11 - 20160*a^3*tan(1/2*d*x + 1/2*c)^
10 + 945*a^3*tan(1/2*d*x + 1/2*c)^9 - 38080*a^3*tan(1/2*d*x + 1/2*c)^8 - 49
280*a^3*tan(1/2*d*x + 1/2*c)^6 - 945*a^3*tan(1/2*d*x + 1/2*c)^5 - 32256*a^3
*tan(1/2*d*x + 1/2*c)^4 + 980*a^3*tan(1/2*d*x + 1/2*c)^3 - 12992*a^3*tan(1/
2*d*x + 1/2*c)^2 - 525*a^3*tan(1/2*d*x + 1/2*c) - 2496*a^3)/(tan(1/2*d*x +
1/2*c)^2 + 1)^7)/d
```

3.611 $\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=181

$$\frac{3a^3\cos^5(c+dx)}{5d}+\frac{2a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3\sin^3(c+dx)}{24d}$$

```
[Out] (-85*a^3*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*Cos[c + d*x])/d +
 (2*a^3*Cos[c + d*x]^3)/(3*d) + (3*a^3*Cos[c + d*x]^5)/(5*d) - (3*a^3*Cot[c
 + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (43*a^3*Cos[c + d*x]*S
in[c + d*x])/(16*d) + (5*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a^3*Cos
[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.252971, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2872, 3767, 8, 3768, 3770, 2638, 2635, 2633}

$$\frac{3a^3\cos^5(c+dx)}{5d}+\frac{2a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3\sin^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-85*a^3*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*Cos[c + d*x])/d +
 (2*a^3*Cos[c + d*x]^3)/(3*d) + (3*a^3*Cos[c + d*x]^5)/(5*d) - (3*a^3*Cot[c
 + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (43*a^3*Cos[c + d*x]*S
in[c + d*x])/(16*d) + (5*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a^3*Cos
[c + d*x]*Sin[c + d*x]^5)/(6*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int\left(-8a^9+3a^9\csc^2(c+dx)+a^9\csc^3(c+dx)-6a^9\sin(c+dx)+\right.}{}$$

$$= -8a^3x+a^3\int\csc^3(c+dx)\,dx-a^3\int\sin^6(c+dx)\,dx+\left(3a^3\right)\int$$

$$= -8a^3x+\frac{6a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}-\frac{3a^3\cos}{}$$

$$= -5a^3x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^3\cos(c+dx)}{d}+\frac{2a^3\cos^3(c}{3d}$$

$$= -5a^3x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^3\cos(c+dx)}{d}+\frac{2a^3\cos^3(c}{3d}$$

$$= -\frac{85a^3x}{16}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^3\cos(c+dx)}{d}+\frac{2a^3\cos^3(}{3d}$$

Mathematica [B] time = 6.37225, size = 664, normalized size = 3.67

$$-\frac{81\sin(2(c+dx))(a\sin(c+dx)+a)^3}{64d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}-\frac{3\sin(4(c+dx))(a\sin(c+dx)+a)^3}{64d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}+\frac{\sin(6(c+dx))(a\sin(c+}{192d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-85*(c + d*x)*(a + a*Sin[c + d*x])^3)/(16*d*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^6) + (15*Cos[c + d*x]*(a + a*Sin[c + d*x])^3)/(8*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^6) + (17*Cos[3*(c + d*x)]*(a + a*Sin[c + d*x])^3)/(48
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (3*Cos[5*(c + d*x)]*(a + a*Si
n[c + d*x])^3)/(80*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) - (3*Cot[(c +
 d*x)/2]*(a + a*Sin[c + d*x])^3)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^6) - (Csc[(c + d*x)/2]^2*(a + a*Sin[c + d*x])^3)/(8*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^6) - (Log[Cos[(c + d*x)/2]]*(a + a*Sin[c + d*x])^3)/(2*d*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (Log[Sin[(c + d*x)/2]]*(a + a*Si
```

```
n[c + d*x])^3)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (Sec[(c + d*
x)/2]^2*(a + a*Sin[c + d*x])^3)/(8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
6) - (81*(a + a*Sin[c + d*x])^3*Sin[2*(c + d*x)])/(64*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^6) - (3*(a + a*Sin[c + d*x])^3*Sin[4*(c + d*x)])/(64*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + ((a + a*Sin[c + d*x])^3*Sin[6*(c
+ d*x)])/(192*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (3*(a + a*Sin[c
+ d*x])^3*Tan[(c + d*x)/2])/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.086, size = 199, normalized size = 1.1

$$-\frac{17\,a^3\left(\cos\left(dx+c\right)\right)^5\sin\left(dx+c\right)}{6\,d}-\frac{85\,a^3\left(\cos\left(dx+c\right)\right)^3\sin\left(dx+c\right)}{24\,d}-\frac{85\,a^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{16\,d}-\frac{85\,a^3x}{16}-\frac{85\,a^3c}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -17/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-85/24*a^3*cos(d*x+c)^3*sin(d*x+c)/d-85/
16*a^3*cos(d*x+c)*sin(d*x+c)/d-85/16*a^3*x-85/16/d*a^3*c+1/10*a^3*cos(d*x+c
)^5/d+1/6*a^3*cos(d*x+c)^3/d+1/2*a^3*cos(d*x+c)/d+1/2/d*a^3*ln(csc(d*x+c)-c
ot(d*x+c))-3/d*a^3/sin(d*x+c)*cos(d*x+c)^7-1/2/d*a^3/sin(d*x+c)^2*cos(d*x+c
)^7
```

Maxima [A] time = 1.71735, size = 323, normalized size = 1.78

$$\frac{96\left(6\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^3+30\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-80\left(4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/960*(96*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(
cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 80*(4*cos(d*x + c)^3 -
6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c)
 + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x -
60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 360*(15*d*x + 15*c +
 (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x +
c)^3 + tan(d*x + c)))*a^3)/d
```

Fricas [A] time = 1.23528, size = 544, normalized size = 3.01

$$\frac{144\,a^3\cos\left(dx+c\right)^7+16\,a^3\cos\left(dx+c\right)^5-1275\,a^3dx\cos\left(dx+c\right)^2+80\,a^3\cos\left(dx+c\right)^3+1275\,a^3dx-120\,a^3\cos\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(144*a^3*cos(d*x + c)^7 + 16*a^3*cos(d*x + c)^5 - 1275*a^3*d*x*cos(d*
x + c)^2 + 80*a^3*cos(d*x + c)^3 + 1275*a^3*d*x - 120*a^3*cos(d*x + c) - 60
*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 60*(a^3*cos(d*x +
 c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) + 5*(8*a^3*cos(d*x + c)^7 - 34*a^
3*cos(d*x + c)^5 - 85*a^3*cos(d*x + c)^3 + 255*a^3*cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37932, size = 413, normalized size = 2.28

$$\frac{30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1275\,(dx+c)a^3+120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+360\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{30\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{\mathrm{ta}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(30*a^3*tan(1/2*d*x + 1/2*c)^2 - 1275*(d*x + c)*a^3 + 120*a^3*log(abs
(tan(1/2*d*x + 1/2*c))) + 360*a^3*tan(1/2*d*x + 1/2*c) - 30*(6*a^3*tan(1/2*
d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2
+ 2*(645*a^3*tan(1/2*d*x + 1/2*c)^11 + 1440*a^3*tan(1/2*d*x + 1/2*c)^10 + 1
735*a^3*tan(1/2*d*x + 1/2*c)^9 + 3360*a^3*tan(1/2*d*x + 1/2*c)^8 + 450*a^3*
tan(1/2*d*x + 1/2*c)^7 + 5440*a^3*tan(1/2*d*x + 1/2*c)^6 - 450*a^3*tan(1/2*
d*x + 1/2*c)^5 + 4800*a^3*tan(1/2*d*x + 1/2*c)^4 - 1735*a^3*tan(1/2*d*x + 1
/2*c)^3 + 1824*a^3*tan(1/2*d*x + 1/2*c)^2 - 645*a^3*tan(1/2*d*x + 1/2*c) +
544*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.612 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=176

$$\frac{a^3\cos^5(c+dx)}{5d}-\frac{2a^3\cos^3(c+dx)}{3d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{4d}$$

```
[Out] (-25*a^3*x)/8 + (13*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a^3*Cos[c + d*x])
/d - (2*a^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cot[c
 + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])
/(2*d) - (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (3*a^3*Cos[c + d*x]*Sin
[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.211157, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2872, 3770, 3768, 3767, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^5(c+dx)}{5d}-\frac{2a^3\cos^3(c+dx)}{3d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-25*a^3*x)/8 + (13*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a^3*Cos[c + d*x])
/d - (2*a^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cot[c
 + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])
/(2*d) - (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (3*a^3*Cos[c + d*x]*Sin
[c + d*x]^3)/(4*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-6a^9-8a^9\csc(c+dx)+3a^9\csc^3(c+dx)+a^9\csc^4(c+dx) \cdots\right.}{\cdots}$$
$$= -6a^3x+a^3\int \csc^4(c+dx)\,dx-a^3\int \sin^5(c+dx)\,dx+\left(3a^3\right)\int \cdots$$
$$= -6a^3x+\frac{8a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{6a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c \cdots}{\cdots}$$
$$= -2a^3x+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5a^3\cos(c+dx)}{d}-\frac{2a^3\cos^{\cdots}}{3\cdots}$$
$$= -\frac{25a^3x}{8}+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5a^3\cos(c+dx)}{d}-\frac{2a^3\cos \cdots}{\cdots}$$

Mathematica [A] time = 1.38142, size = 219, normalized size = 1.24

$$\frac{a^3(\sin(c+dx)+1)^3\left(-1500(c+dx)-600\sin(2(c+dx))-45\sin(4(c+dx))-2580\cos(c+dx)-50\cos(3(c+dx))+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-1500*(c + d*x) - 2580*Cos[c + d*x] - 50*Cos[3*(
c + d*x)] + 6*Cos[5*(c + d*x)] - 160*Cot[(c + d*x)/2] - 180*Csc[(c + d*x)/2
]^2 + 3120*Log[Cos[(c + d*x)/2]] - 3120*Log[Sin[(c + d*x)/2]] + 180*Sec[(c
+ d*x)/2]^2 + 160*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 10*Csc[(c + d*x)/2]^4
*Sin[c + d*x] - 600*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 160*Tan[(c + d
*x)/2]))/(480*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.089, size = 223, normalized size = 1.3

$$-\frac{13\,a^3\,(\cos(dx+c))^5}{10\,d}-\frac{13\,a^3\,(\cos(dx+c))^3}{6\,d}-\frac{13\,a^3\cos(dx+c)}{2\,d}-\frac{13\,a^3\ln(\csc(dx+c)-\cot(dx+c))}{2\,d}-\frac{5\,a^3\,(\cos(dx}{3\,d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -13/10*a^3*cos(d*x+c)^5/d-13/6*a^3*cos(d*x+c)^3/d-13/2*a^3*cos(d*x+c)/d-13/
2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-5/3/d*a^3/sin(d*x+c)*cos(d*x+c)^7-5/3*a^3
*cos(d*x+c)^5*sin(d*x+c)/d-25/12*a^3*cos(d*x+c)^3*sin(d*x+c)/d-25/8*a^3*cos
(d*x+c)*sin(d*x+c)/d-25/8*a^3*x-25/8/d*a^3*c-3/2/d*a^3/sin(d*x+c)^2*cos(d*x
+c)^7-1/3/d*a^3/sin(d*x+c)^3*cos(d*x+c)^7
```

Maxima [A] time = 1.68217, size = 332, normalized size = 1.89

$$\frac{4\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^3-30\left(4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/120*(4*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(c
os(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 30*(4*cos(d*x + c)^3 - 6
*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1))*a^3 - 45*(15*d*x + 15*c + (15*tan(d*x + c)
^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x +
c)))*a^3 + 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/
(tan(d*x + c)^5 + tan(d*x + c)^3))*a^3)/d
```

Fricas [A] time = 1.26621, size = 601, normalized size = 3.41

$$\frac{90\,a^3\cos(dx+c)^7+75\,a^3\cos(dx+c)^5-500\,a^3\cos(dx+c)^3+375\,a^3\cos(dx+c)+390\left(a^3\cos(dx+c)^2-a^3\right)\log\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/120*(90*a^3*cos(d*x + c)^7 + 75*a^3*cos(d*x + c)^5 - 500*a^3*cos(d*x + c)
^3 + 375*a^3*cos(d*x + c) + 390*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) - 390*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + (24*a^3*cos(d*x + c)^7 - 104*a^3*cos(d*x + c)^5
- 375*a^3*d*x*cos(d*x + c)^2 - 520*a^3*cos(d*x + c)^3 + 375*a^3*d*x + 780*a
^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36642, size = 394, normalized size = 2.24

$$5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-375\,(dx+c)a^3-780\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+45\,a^3\tan\left(\frac{1}{2}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^3*tan(1/2*d*x + 1/2*c)^2 - 375*(
d*x + c)*a^3 - 780*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 45*a^3*tan(1/2*d*x
+ 1/2*c) + 5*(286*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c)^2
 - 9*a^3*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x + 1/2*c)^3 + 2*(345*a^3*ta
n(1/2*d*x + 1/2*c)^9 - 720*a^3*tan(1/2*d*x + 1/2*c)^8 + 330*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 2880*a^3*tan(1/2*d*x + 1/2*c)^6 - 3680*a^3*tan(1/2*d*x + 1/2*
c)^4 - 330*a^3*tan(1/2*d*x + 1/2*c)^3 - 2560*a^3*tan(1/2*d*x + 1/2*c)^2 - 3
45*a^3*tan(1/2*d*x + 1/2*c) - 656*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.613 $\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=178

$$-\frac{a^3\cos^3(c+dx)}{d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{5a^3\cot(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{3a^3\sin(c+dx)}{8d}$$

```
[Out] (45*a^3*x)/8 + (45*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (5*a^3*Cos[c + d*x])/
d - (a^3*Cos[c + d*x]^3)/d + (5*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/
d - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d*x
]^3)/(4*d) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*Cos[c + d*x]*Si
n[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.218743, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2633, 2635}

$$-\frac{a^3\cos^3(c+dx)}{d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{5a^3\cot(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{3a^3\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (45*a^3*x)/8 + (45*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (5*a^3*Cos[c + d*x])/
d - (a^3*Cos[c + d*x]^3)/d + (5*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/
d - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d*x
]^3)/(4*d) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*Cos[c + d*x]*Si
n[c + d*x]^3)/(4*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(6a^9 - 6a^9\csc(c+dx) - 8a^9\csc^2(c+dx) + 3a^9\csc^4(c+dx) + \right.}{}$$

$$= 6a^3x + a^3\int \csc^5(c+dx)\,dx - a^3\int \sin^4(c+dx)\,dx + \left(3a^3\right)\int \mathrm{cs}$$

$$= 6a^3x + \frac{6a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{8a^3\cos(c+dx)}{d} - \frac{a^3\cot(c+}{}$$

$$= 6a^3x + \frac{6a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{5a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+}{d}$$

$$= \frac{45a^3x}{8} + \frac{45a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{5a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(}{d}$$

Mathematica [A] time = 0.85342, size = 235, normalized size = 1.32

$$\frac{a^3(\sin(c+dx)+1)^3\left(360(c+dx)+16\sin(2(c+dx))-2\sin(4(c+dx))-368\cos(c+dx)-16\cos(3(c+dx))-192\,\mathrm{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(360*(c + d*x) - 368*Cos[c + d*x] - 16*Cos[3*(c +
 d*x)] + 192*Cot[(c + d*x)/2] - 6*Csc[(c + d*x)/2]^2 - Csc[(c + d*x)/2]^4 +
 360*Log[Cos[(c + d*x)/2]] - 360*Log[Sin[(c + d*x)/2]] + 6*Sec[(c + d*x)/2]
^2 + Sec[(c + d*x)/2]^4 + 64*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 4*Csc[(c +
 d*x)/2]^4*Sin[c + d*x] + 16*Sin[2*(c + d*x)] - 2*Sin[4*(c + d*x)] - 192*Ta
n[(c + d*x)/2]))/(64*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.096, size = 247, normalized size = 1.4

$$3\,{\frac{{a}^{3} \left(\cos \left(dx+c \right) \right) ^{7}}{d\sin \left(dx+c \right) }}+3\,{\frac{{a}^{3} \left(\cos \left(dx+c \right) \right) ^{5}\sin \left(dx+c \right) }{d}}+{\frac{15\,{a}^{3} \left(\cos \left(dx+c \right) \right) ^{3}\sin \left(dx+c \right) }{4\,d}}+{\frac{45\,{a}^{3}\cos \left(dx+c \right) \sin \left(dx+c \right)}{8\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 3/d*a^3/sin(d*x+c)*cos(d*x+c)^7+3*a^3*cos(d*x+c)^5*sin(d*x+c)/d+15/4*a^3*co
s(d*x+c)^3*sin(d*x+c)/d+45/8*a^3*cos(d*x+c)*sin(d*x+c)/d+45/8*a^3*x+45/8/d*
a^3*c-9/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-9/8*a^3*cos(d*x+c)^5/d-15/8*a^3*c
os(d*x+c)^3/d-45/8*a^3*cos(d*x+c)/d-45/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/
d*a^3/sin(d*x+c)^3*cos(d*x+c)^7-1/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7
```

Maxima [A] time = 1.73312, size = 362, normalized size = 2.03

$$-\frac{4\left(4\,\cos\left(dx+c\right)^{3}-\frac{6\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{2}-1}+24\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^{3}+2\left(15\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 + 2*(15*
d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
2*tan(d*x + c)^3 + tan(d*x + c)))*a^3 - 8*(15*d*x + 15*c + (15*tan(d*x + c)
^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^3 + a^3*(2
*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1
) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))
/d
```

Fricas [A] time = 1.32093, size = 664, normalized size = 3.73

$$-\frac{16\,a^{3}\cos\left(dx+c\right)^{7}-90\,a^{3}dx\cos\left(dx+c\right)^{4}+48\,a^{3}\cos\left(dx+c\right)^{5}+180\,a^{3}dx\cos\left(dx+c\right)^{2}-150\,a^{3}\cos\left(dx+c\right)^{3}-90\,a^{3}d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(16*a^3*cos(d*x + c)^7 - 90*a^3*d*x*cos(d*x + c)^4 + 48*a^3*cos(d*x +
 c)^5 + 180*a^3*d*x*cos(d*x + c)^2 - 150*a^3*cos(d*x + c)^3 - 90*a^3*d*x +
90*a^3*cos(d*x + c) - 45*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*
log(1/2*cos(d*x + c) + 1/2) + 45*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2
 + a^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*a^3*cos(d*x + c)^7 - 9*a^3*cos(
d*x + c)^5 + 60*a^3*cos(d*x + c)^3 - 45*a^3*cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)

[Out] Timed out
```

Giac [A] time = 1.35055, size = 423, normalized size = 2.38

$$a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+360(dx+c)a^3-360a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*tan
(1/2*d*x + 1/2*c)^2 + 360*(d*x + c)*a^3 - 360*a^3*log(abs(tan(1/2*d*x + 1/2
*c))) - 184*a^3*tan(1/2*d*x + 1/2*c) + (250*a^3*tan(1/2*d*x + 1/2*c)^12 + 1
36*a^3*tan(1/2*d*x + 1/2*c)^11 - 32*a^3*tan(1/2*d*x + 1/2*c)^10 + 552*a^3*t
an(1/2*d*x + 1/2*c)^9 - 837*a^3*tan(1/2*d*x + 1/2*c)^8 + 1248*a^3*tan(1/2*d
*x + 1/2*c)^7 - 1100*a^3*tan(1/2*d*x + 1/2*c)^6 + 736*a^3*tan(1/2*d*x + 1/2
*c)^5 - 556*a^3*tan(1/2*d*x + 1/2*c)^4 + 152*a^3*tan(1/2*d*x + 1/2*c)^3 - 1
2*a^3*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d
*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^4)/d
```

3.614 $\int \cot^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=175

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+dx)}{3d}+\frac{5a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] (13*a^3*x)/2 - (25*a^3*ArcTanh[Cos[c + d*x]])/(8*d) + (a^3*Cos[c + d*x])/d
- (a^3*Cos[c + d*x]^3)/(3*d) + (5*a^3*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]
^3)/(3*d) - (a^3*Cot[c + d*x]^5)/(5*d) + (23*a^3*Cot[c + d*x]*Csc[c + d*x])
/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (3*a^3*Cos[c + d*x]*Si
n[c + d*x])/(2*d)
```

Rubi [A] time = 0.298944, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 3768, 2635, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+dx)}{3d}+\frac{5a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```



```
[Out] (13*a^3*x)/2 - (25*a^3*ArcTanh[Cos[c + d*x]])/(8*d) + (a^3*Cos[c + d*x])/d
- (a^3*Cos[c + d*x]^3)/(3*d) + (5*a^3*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]
^3)/(3*d) - (a^3*Cot[c + d*x]^5)/(5*d) + (23*a^3*Cot[c + d*x]*Csc[c + d*x])
/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (3*a^3*Cos[c + d*x]*Si
n[c + d*x])/(2*d)
```


Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(8a^9+6a^9\csc(c+dx)-6a^9\csc^2(c+dx)-8a^9\csc^3(c+dx)+3a^9\csc^5(c+\right.}{a^6}$$

$$= 8a^3x+a^3\int \csc^6(c+dx)\,dx-a^3\int \sin^3(c+dx)\,dx+\left(3a^3\right)\int \csc^5(c+dx)\,d$$

$$= 8a^3x-\frac{6a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3\cot(c+dx)\csc(c+dx)}{d}-\frac{3a^3\cot(c+}{}$$

$$= \frac{13a^3x}{2}-\frac{2a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3 c}{}$$

$$= \frac{13a^3x}{2}-\frac{25a^3\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3}{}$$

Mathematica [A] time = 1.63668, size = 271, normalized size = 1.55

$$\frac{a^3(\sin(c+dx)+1)^3\left(6240(c+dx)+720\sin(2(c+dx))+720\cos(c+dx)-80\cos(3(c+dx))-2624\tan\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(6240*(c + d*x) + 720*Cos[c + d*x] - 80*Cos[3*(c
+ d*x)] + 2624*Cot[(c + d*x)/2] + 690*Csc[(c + d*x)/2]^2 - 45*Csc[(c + d*x)
/2]^4 - 3000*Log[Cos[(c + d*x)/2]] + 3000*Log[Sin[(c + d*x)/2]] - 690*Sec[(
c + d*x)/2]^2 + 45*Sec[(c + d*x)/2]^4 + 304*Csc[c + d*x]^3*Sin[(c + d*x)/2]
^4 - 19*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 3*Csc[(c + d*x)/2]^6*Sin[c + d*x]
 + 720*Sin[2*(c + d*x)] - 2624*Tan[(c + d*x)/2] + 6*Sec[(c + d*x)/2]^4*Tan[
(c + d*x)/2]))/(960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.095, size = 293, normalized size = 1.7

$$\frac{5a^3(\cos(dx+c))^7}{8d(\sin(dx+c))^2}+\frac{5a^3(\cos(dx+c))^5}{8d}+\frac{25a^3(\cos(dx+c))^3}{24d}+\frac{25a^3\cos(dx+c)}{8d}+\frac{25a^3\ln(\csc(dx+c)-\cot(d}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 5/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7+5/8*a^3*cos(d*x+c)^5/d+25/24*a^3*cos(d*
x+c)^3/d+25/8*a^3*cos(d*x+c)/d+25/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/d*a^3
/sin(d*x+c)^3*cos(d*x+c)^7+4/d*a^3/sin(d*x+c)*cos(d*x+c)^7+4*a^3*cos(d*x+c)
^5*sin(d*x+c)/d+5*a^3*cos(d*x+c)^3*sin(d*x+c)/d+15/2*a^3*cos(d*x+c)*sin(d*x
+c)/d+13/2*a^3*x+13/2/d*a^3*c-3/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7-1/5*a^3*c
ot(d*x+c)^5/d+1/3*a^3*cot(d*x+c)^3/d-a^3*cot(d*x+c)/d
```

Maxima [A] time = 1.7904, size = 338, normalized size = 1.93

$$-\frac{20\left(4\cos\left(dx+c\right)^{3}-\frac{6\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^{3}-120\left(15a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/240*(20*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos
(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 120*
(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^
5 + tan(d*x + c)^3))*a^3 + 16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d
*x + c)^2 + 3)/tan(d*x + c)^5)*a^3 + 45*a^3*(2*(9*cos(d*x + c)^3 - 7*cos(d*
x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(
cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.28919, size = 738, normalized size = 4.22

$$-\frac{360a^{3}\cos\left(dx+c\right)^{7}-2392a^{3}\cos\left(dx+c\right)^{5}+3640a^{3}\cos\left(dx+c\right)^{3}-1560a^{3}\cos\left(dx+c\right)+375\left(a^{3}\cos\left(dx+c\right)^{4}-2a^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(360*a^3*cos(d*x + c)^7 - 2392*a^3*cos(d*x + c)^5 + 3640*a^3*cos(d*x
 + c)^3 - 1560*a^3*cos(d*x + c) + 375*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x +
 c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 375*(a^3*cos(d*x +
c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 10*(8*a^3*cos(d*x + c)^7 - 156*a^3*d*x*cos(d*x + c)^4 - 40*a^3*cos(d*x
+ c)^5 + 312*a^3*d*x*cos(d*x + c)^2 + 125*a^3*cos(d*x + c)^3 - 156*a^3*d*x
- 75*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24863, size = 373, normalized size = 2.13

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+50\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6240\,(dx+c)a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*tan(1/2*d*x + 1/2*c)^4 + 50*a^
3*tan(1/2*d*x + 1/2*c)^3 - 600*a^3*tan(1/2*d*x + 1/2*c)^2 + 6240*(d*x + c)*
a^3 + 3000*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2580*a^3*tan(1/2*d*x + 1/2*
c) - 320*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*
a^3*tan(1/2*d*x + 1/2*c) - 4*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3 - (6850*a^
3*tan(1/2*d*x + 1/2*c)^5 - 2580*a^3*tan(1/2*d*x + 1/2*c)^4 - 600*a^3*tan(1/
2*d*x + 1/2*c)^3 + 50*a^3*tan(1/2*d*x + 1/2*c)^2 + 45*a^3*tan(1/2*d*x + 1/2
*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.615 $\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=182

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{85a^3\tanh^{-1}(\cos}{16d}$$

```
[Out] -(a^3*x)/2 - (85*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (3*a^3*Cos[c + d*x])/d
 - (a^3*Cot[c + d*x])/d + (2*a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x
]^5)/(5*d) + (43*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (5*a^3*Cot[c + d*x
]*Csc[c + d*x]^3)/(24*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d) + (a^3*C
os[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.248143, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2872, 3770, 3767, 8, 3768, 2638, 2635}

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{85a^3\tanh^{-1}(\cos}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x)/2 - (85*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (3*a^3*Cos[c + d*x])/d
 - (a^3*Cot[c + d*x])/d + (2*a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x
]^5)/(5*d) + (43*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (5*a^3*Cot[c + d*x
]*Csc[c + d*x]^3)/(24*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d) + (a^3*C
os[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(8a^9\csc(c+dx)+6a^9\csc^2(c+dx)-6a^9\csc^3(c+dx)-8a^9\csc\right.}{}$$

$$= a^3\int \csc^7(c+dx)\,dx - a^3\int \sin^2(c+dx)\,dx + \left(3a^3\right)\int \csc^6(c+$$

$$= -\frac{8a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{3a^3\cot(c+dx)\,\mathrm{c}}{d}$$

$$= -\frac{a^3x}{2} - \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cot(c+}{d}$$

$$= -\frac{a^3x}{2} - \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cot(c+}{d}$$

$$= -\frac{a^3x}{2} - \frac{85a^3\tanh^{-1}(\cos(c+dx))}{16d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cot(c+}{d}$$

Mathematica [A] time = 1.81905, size = 289, normalized size = 1.59

$$\frac{a^3(\sin(c+dx)+1)^3\left(-960(c+dx)+480\sin(2(c+dx))+5760\cos(c+dx)+2176\tan\left(\frac{1}{2}(c+dx)\right)-2176\cot\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-960*(c + d*x) + 5760*Cos[c + d*x] - 2176*Cot[(c
 + d*x)/2] + 1290*Csc[(c + d*x)/2]^2 - 30*Csc[(c + d*x)/2]^4 - 5*Csc[(c + d
*x)/2]^6 - 10200*Log[Cos[(c + d*x)/2]] + 10200*Log[Sin[(c + d*x)/2]] - 1290
*Sec[(c + d*x)/2]^2 + 30*Sec[(c + d*x)/2]^4 + 5*Sec[(c + d*x)/2]^6 - 3296*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 206*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 18
*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 2176*Tan[(c + d*x
)/2] + 36*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(1920*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.102, size = 316, normalized size = 1.7

$$\frac{4\,a^3(\cos(dx+c))^5\sin(dx+c)}{3\,d} + \frac{17\,a^3(\cos(dx+c))^5}{16\,d} - \frac{3\,a^3(\cot(dx+c))^5}{5\,d} + \frac{a^3(\cot(dx+c))^3}{d} - 3\,\frac{a^3\cot(dx+c)}{d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/3*a^3*cos(d*x+c)^5*sin(d*x+c)/d+17/16*a^3*cos(d*x+c)^5/d-3/5*a^3*cot(d*x+
c)^5/d+a^3*cot(d*x+c)^3/d-3*a^3*cot(d*x+c)/d-1/2/d*a^3*c+85/48*a^3*cos(d*x+
c)^3/d+85/16*a^3*cos(d*x+c)/d+85/16/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+17/16/d
*a^3/sin(d*x+c)^2*cos(d*x+c)^7-1/6/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7-1/2*a^3*
x-1/3/d*a^3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*a^3/sin(d*x+c)*cos(d*x+c)^7+5/3
*a^3*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a^3*cos(d*x+c)*sin(d*x+c)/d-17/24/d*a^3/
sin(d*x+c)^4*cos(d*x+c)^7
```

Maxima [A] time = 1.63581, size = 371, normalized size = 2.04

$$80\left(15\,dx + 15\,c + \frac{15\,\tan(dx+c)^4 + 10\,\tan(dx+c)^2 - 2}{\tan(dx+c)^5 + \tan(dx+c)^3}\right)a^3 - 96\left(15\,dx + 15\,c + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)a^3 + 5\,a^3\left(\frac{2\left(33\,\cos(dx+c)^5 - 4\cdots\right.}{\cos(dx+c)^6 - 3\,\cos\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/480*(80*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan
(d*x + c)^5 + tan(d*x + c)^3))*a^3 - 96*(15*d*x + 15*c + (15*tan(d*x + c)^4
 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*a^3 + 5*a^3*(2*(33*cos(d*x + c)^5
- 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 +
 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1
)) - 90*a^3*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d
*x + c) - 1)))/d
```

Fricas [A] time = 1.24269, size = 817, normalized size = 4.49

$$-\frac{240\,a^3 dx\cos(dx+c)^6 - 1440\,a^3\cos(dx+c)^7 - 720\,a^3 dx\cos(dx+c)^4 + 5610\,a^3\cos(dx+c)^5 + 720\,a^3 dx\cos(dx+c)^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/480*(240*a^3*d*x*cos(d*x + c)^6 - 1440*a^3*cos(d*x + c)^7 - 720*a^3*d*x*
cos(d*x + c)^4 + 5610*a^3*cos(d*x + c)^5 + 720*a^3*d*x*cos(d*x + c)^2 - 680
0*a^3*cos(d*x + c)^3 - 240*a^3*d*x + 2550*a^3*cos(d*x + c) + 1275*(a^3*cos(
d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos
(d*x + c) + 1/2) - 1275*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*
cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) - 16*(15*a^3*cos(d*x + c
)^7 + 23*a^3*cos(d*x + c)^5 - 35*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36252, size = 414, normalized size = 2.27

$$5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-340\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1215\,a^3\tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a
^3*tan(1/2*d*x + 1/2*c)^4 - 340*a^3*tan(1/2*d*x + 1/2*c)^3 - 1215*a^3*tan(1
/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a^3 + 10200*a^3*log(abs(tan(1/2*d*x + 1/2
*c))) + 1800*a^3*tan(1/2*d*x + 1/2*c) - 1920*(a^3*tan(1/2*d*x + 1/2*c)^3 -
6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/(tan(1/2*d
*x + 1/2*c)^2 + 1)^2 - (24990*a^3*tan(1/2*d*x + 1/2*c)^6 + 1800*a^3*tan(1/2
*d*x + 1/2*c)^5 - 1215*a^3*tan(1/2*d*x + 1/2*c)^4 - 340*a^3*tan(1/2*d*x + 1
/2*c)^3 + 45*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*tan(1/2*d*x + 1/2*c) + 5*a
^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.616 $\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=172

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d}-$$

```
[Out] -3*a^3*x - (15*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (a^3*Cos[c + d*x])/d - (
3*a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/(5*
d) - (a^3*Cot[c + d*x]^7)/(7*d) - (15*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d)
 + (11*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d
*x]^5)/(2*d)
```

Rubi [A] time = 0.28983, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3767, 8, 3768, 3770, 2638}

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] -3*a^3*x - (15*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (a^3*Cos[c + d*x])/d - (
3*a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/(5*
d) - (a^3*Cot[c + d*x]^7)/(7*d) - (15*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d)
 + (11*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d
*x]^5)/(2*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-3a^9+8a^9\csc^2(c+dx)+6a^9\csc^3(c+dx)-6a^9\csc^4(c+dx\right.}{}$$

$$= -3a^3x+a^3\int \csc^8(c+dx)\,dx-a^3\int \sin(c+dx)\,dx+\left(3a^3\right)\int c$$

$$= -3a^3x+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{d}+\frac{2a^3\cot}{}$$

$$= -3a^3x-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c}{d}$$

$$= -3a^3x+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{d}-\frac{3}{}$$

$$= -3a^3x-\frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c}{d}$$

Mathematica [A] time = 1.35453, size = 292, normalized size = 1.7

$$\frac{a^3\left(4480\cos(c+dx)+9984\tan\left(\frac{1}{2}(c+dx)\right)-9984\cot\left(\frac{1}{2}(c+dx)\right)-35\csc^6\left(\frac{1}{2}(c+dx)\right)+350\csc^4\left(\frac{1}{2}(c+dx)\right)-10\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-13440*c - 13440*d*x + 4480*Cos[c + d*x] - 9984*Cot[(c + d*x)/2] - 10
50*Csc[(c + d*x)/2]^2 + 350*Csc[(c + d*x)/2]^4 - 35*Csc[(c + d*x)/2]^6 - 42
00*Log[Cos[(c + d*x)/2]] + 4200*Log[Sin[(c + d*x)/2]] + 1050*Sec[(c + d*x)/
2]^2 - 350*Sec[(c + d*x)/2]^4 + 35*Sec[(c + d*x)/2]^6 - 7664*Csc[c + d*x]^3
*Sin[(c + d*x)/2]^4 + 479*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 17*Csc[(c + d*x
)/2]^6*Sin[c + d*x] - (5*Csc[(c + d*x)/2]^8*Sin[c + d*x])/2 + 9984*Tan[(c +
 d*x)/2] + 34*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] + 5*Sec[(c + d*x)/2]^6*Ta
n[(c + d*x)/2]))/(4480*d)
```

Maple [A] time = 0.096, size = 228, normalized size = 1.3

$$-\frac{a^3(\cos(dx+c))^7}{8d(\sin(dx+c))^4}+\frac{3a^3(\cos(dx+c))^7}{16d(\sin(dx+c))^2}+\frac{3a^3(\cos(dx+c))^5}{16d}+\frac{5a^3(\cos(dx+c))^3}{16d}+\frac{15a^3\cos(dx+c)}{16d}+\frac{15a^3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7+3/16/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7+3
/16*a^3*cos(d*x+c)^5/d+5/16*a^3*cos(d*x+c)^3/d+15/16*a^3*cos(d*x+c)/d+15/16
```

```
/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/5*a^3*cot(d*x+c)^5/d+a^3*cot(d*x+c)^3/d-
3*a^3*cot(d*x+c)/d-3*a^3*x-3/d*a^3*c-1/2/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7-1/
7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7
```

Maxima [A] time = 1.60888, size = 315, normalized size = 1.83

$$-\frac{224\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3-35\,a^3\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/1120*(224*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/ta
n(d*x + c)^5)*a^3 - 35*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*c
os(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 1
5*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 70*a^3*(2*(9*cos(d*x
+ c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d
*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 160*a^3/ta
n(d*x + c)^7)/d
```

Fricas [B] time = 1.22366, size = 871, normalized size = 5.06

$$-\frac{4992\,a^3\cos\left(dx+c\right)^7-12992\,a^3\cos\left(dx+c\right)^5+11200\,a^3\cos\left(dx+c\right)^3-3360\,a^3\cos\left(dx+c\right)+525\left(a^3\cos\left(dx+c\right)^6-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1120*(4992*a^3*cos(d*x + c)^7 - 12992*a^3*cos(d*x + c)^5 + 11200*a^3*cos
(d*x + c)^3 - 3360*a^3*cos(d*x + c) + 525*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d
*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 525*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)
^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 70*(48*a^3*d*x*cos(d*
x + c)^6 - 16*a^3*cos(d*x + c)^7 - 144*a^3*d*x*cos(d*x + c)^4 + 33*a^3*cos(
d*x + c)^5 + 144*a^3*d*x*cos(d*x + c)^2 - 40*a^3*cos(d*x + c)^3 - 48*a^3*d*
x + 15*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c
)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32594, size = 393, normalized size = 2.28

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+49\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-245\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-875\,a^3\tan\left(\frac{1}{2}\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*tan(1/2*d*x + 1/2*c)^7 + 35*a^3*tan(1/2*d*x + 1/2*c)^6 + 49*a
^3*tan(1/2*d*x + 1/2*c)^5 - 245*a^3*tan(1/2*d*x + 1/2*c)^4 - 875*a^3*tan(1/
2*d*x + 1/2*c)^3 + 455*a^3*tan(1/2*d*x + 1/2*c)^2 - 13440*(d*x + c)*a^3 + 4
200*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9065*a^3*tan(1/2*d*x + 1/2*c) + 89
60*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) - (10890*a^3*tan(1/2*d*x + 1/2*c)^7 + 9
065*a^3*tan(1/2*d*x + 1/2*c)^6 + 455*a^3*tan(1/2*d*x + 1/2*c)^5 - 875*a^3*t
an(1/2*d*x + 1/2*c)^4 - 245*a^3*tan(1/2*d*x + 1/2*c)^3 + 49*a^3*tan(1/2*d*x
 + 1/2*c)^2 + 35*a^3*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^7)/
d
```

3.617 $\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=238

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{125a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^5(c+dx)}{8d}$$

```
[Out] -(a^3*x) + (125*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (a^3*Cot[c + d*x])/d +
 (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a^3*Cot[c + d
*x]^7)/(7*d) - (115*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a^3*Cot[c +
 d*x]^3*Csc[c + d*x])/(8*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x])/(2*d) - (5*
a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) + (5*a^3*Cot[c + d*x]^3*Csc[c + d*x
]^3)/(48*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rubi [A] time = 0.355159, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2873, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{125a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^5(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (125*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (a^3*Cot[c + d*x])/d +
 (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a^3*Cot[c + d
*x]^7)/(7*d) - (115*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a^3*Cot[c +
 d*x]^3*Csc[c + d*x])/(8*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x])/(2*d) - (5*
a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) + (5*a^3*Cot[c + d*x]^3*Csc[c + d*x
]^3)/(48*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)+3a^3\cot^6(c+dx)\csc(c+dx)+3a^3\cot^6(c+\right.$$
$$= a^3\int \cot^6(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^3(c+dx)\,dx + \left(3a^3\right.$$
$$= -\frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^5(c+dx)\csc(c+dx)}{2d} - \frac{a^3\cot^5(c+dx}{8}$$
$$= \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c+dx)}{7d} + \frac{5a^3\cot^3}{}$$
$$= -\frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7}{7}$$
$$= -a^3x + \frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c-}{3d}$$
$$= -a^3x + \frac{125a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c}{3d}$$

Mathematica [A] time = 1.17163, size = 279, normalized size = 1.17

$$\frac{a^3\left(118784\tan\left(\frac{1}{2}(c+dx)\right)-118784\cot\left(\frac{1}{2}(c+dx)\right)-108780\csc^2\left(\frac{1}{2}(c+dx)\right)+105\sec^8\left(\frac{1}{2}(c+dx)\right)+700\sec^6\left(\frac{1}{2}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-215040*c - 215040*d*x - 118784*Cot[(c + d*x)/2] - 108780*Csc[(c + d*
x)/2]^2 + 210000*Log[Cos[(c + d*x)/2]] - 210000*Log[Sin[(c + d*x)/2]] + 108
780*Sec[(c + d*x)/2]^2 - 17010*Sec[(c + d*x)/2]^4 + 700*Sec[(c + d*x)/2]^6
+ 105*Sec[(c + d*x)/2]^8 + 71936*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + Csc[(c
 + d*x)/2]^4*(17010 - 4496*Sin[c + d*x]) - 15*Csc[(c + d*x)/2]^8*(7 + 24*Si
n[c + d*x]) + 4*Csc[(c + d*x)/2]^6*(-175 + 732*Sin[c + d*x]) + 118784*Tan[(
```

```
c + d*x)/2] - 5856*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] + 720*Sec[(c + d*x)/
2]^6*Tan[(c + d*x)/2]))/(215040*d)
```

Maple [A] time = 0.096, size = 253, normalized size = 1.1

$$-\frac{a^3(\cot(dx+c))^5}{5d}+\frac{a^3(\cot(dx+c))^3}{3d}-\frac{a^3\cot(dx+c)}{d}-a^3x-\frac{a^3c}{d}-\frac{25a^3(\cos(dx+c))^7}{48d(\sin(dx+c))^6}+\frac{25a^3(\cos(dx+c))^7}{192d(\sin(dx+c))^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/5*a^3*cot(d*x+c)^5/d+1/3*a^3*cot(d*x+c)^3/d-a^3*cot(d*x+c)/d-a^3*x-1/d*a
^3*c-25/48/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7+25/192/d*a^3/sin(d*x+c)^4*cos(d*
x+c)^7-25/128/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-25/128*a^3*cos(d*x+c)^5/d-125
/384*a^3*cos(d*x+c)^3/d-125/128*a^3*cos(d*x+c)/d-125/128/d*a^3*ln(csc(d*x+c
)-cot(d*x+c))-3/7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7-1/8/d*a^3/sin(d*x+c)^8*co
s(d*x+c)^7
```

Maxima [A] time = 1.67002, size = 358, normalized size = 1.5

$$-\frac{1792\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+35\,a^3\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\log(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/26880*(1792*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/
tan(d*x + c)^5)*a^3 + 35*a^3*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55
*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*c
os(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(c
os(d*x + c) - 1)) - 840*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*
cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) +
15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 11520*a^3/tan(d*x +
c)^7)/d
```

Fricas [A] time = 1.23547, size = 983, normalized size = 4.13

$$-\frac{26880\,a^3dx\cos(dx+c)^8-107520\,a^3dx\cos(dx+c)^6-54390\,a^3\cos(dx+c)^7+161280\,a^3dx\cos(dx+c)^4+127750\,a^3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/26880*(26880*a^3*d*x*cos(d*x + c)^8 - 107520*a^3*d*x*cos(d*x + c)^6 - 54
390*a^3*cos(d*x + c)^7 + 161280*a^3*d*x*cos(d*x + c)^4 + 127750*a^3*cos(d*x
```

```
 + c)^5 - 107520*a^3*d*x*cos(d*x + c)^2 - 96250*a^3*cos(d*x + c)^3 + 26880*
a^3*d*x + 26250*a^3*cos(d*x + c) - 13125*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*
x + c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d
*x + c) + 1/2) + 13125*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*c
os(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2) -
256*(116*a^3*cos(d*x + c)^7 - 406*a^3*cos(d*x + c)^5 + 350*a^3*cos(d*x + c)
^3 - 105*a^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x +
c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26472, size = 408, normalized size = 1.71

$$105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+720\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3696\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-14280\,a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/215040*(105*a^3*tan(1/2*d*x + 1/2*c)^8 + 720*a^3*tan(1/2*d*x + 1/2*c)^7 +
 1120*a^3*tan(1/2*d*x + 1/2*c)^6 - 3696*a^3*tan(1/2*d*x + 1/2*c)^5 - 14280*
a^3*tan(1/2*d*x + 1/2*c)^4 - 560*a^3*tan(1/2*d*x + 1/2*c)^3 + 77280*a^3*tan
(1/2*d*x + 1/2*c)^2 - 215040*(d*x + c)*a^3 - 210000*a^3*log(abs(tan(1/2*d*x
 + 1/2*c))) + 122640*a^3*tan(1/2*d*x + 1/2*c) + (570750*a^3*tan(1/2*d*x + 1
/2*c)^8 - 122640*a^3*tan(1/2*d*x + 1/2*c)^7 - 77280*a^3*tan(1/2*d*x + 1/2*c
)^6 + 560*a^3*tan(1/2*d*x + 1/2*c)^5 + 14280*a^3*tan(1/2*d*x + 1/2*c)^4 + 3
696*a^3*tan(1/2*d*x + 1/2*c)^3 - 1120*a^3*tan(1/2*d*x + 1/2*c)^2 - 720*a^3*
tan(1/2*d*x + 1/2*c) - 105*a^3)/tan(1/2*d*x + 1/2*c)^8)/d
```

3.618 $\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=200

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{55a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a^3\cot^3(c+dx)\csc}{16d}$$

```
[Out] (55*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a^
3*Cot[c + d*x]^9)/(9*d) - (25*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a
^3*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x])/
(6*d) - (15*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) + (5*a^3*Cot[c + d*x]^3
*Csc[c + d*x]^3)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rubi [A] time = 0.358502, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{55a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a^3\cot^3(c+dx)\csc}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (55*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a^
3*Cot[c + d*x]^9)/(9*d) - (25*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a
^3*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x])/
(6*d) - (15*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) + (5*a^3*Cot[c + d*x]^3
*Csc[c + d*x]^3)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc(c+dx) + 3a^3\cot^6(c+dx)\csc^2(c+dx) + \right.$$

$$= a^3\int \cot^6(c+dx)\csc(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^4(c+d$$

$$= -\frac{a^3\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d} -$$

$$= -\frac{3a^3\cot^7(c+dx)}{7d} + \frac{5a^3\cot^3(c+dx)\csc(c+dx)}{24d} - \frac{a^3\cot^5(c+$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{9d} - \frac{5a^3\cot(c+dx)\csc(c+d}{16d}$$

$$= \frac{5a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{9d} -$$

$$= \frac{55a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{9d}$$

Mathematica [B] time = 0.141289, size = 459, normalized size = 2.3

$$a^3\left(-\frac{29\tan\left(\frac{1}{2}(c+dx)\right)}{126d} + \frac{29\cot\left(\frac{1}{2}(c+dx)\right)}{126d} - \frac{3\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{17\csc^6\left(\frac{1}{2}(c+dx)\right)}{1536d} - \frac{13\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} - \frac{73}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((29*Cot[(c + d*x)/2])/(126*d) - (73*Csc[(c + d*x)/2]^2)/(512*d) - (416
3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(32256*d) - (13*Csc[(c + d*x)/2]^4)/
(1024*d) + (319*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(10752*d) + (17*Csc[(c
 + d*x)/2]^6)/(1536*d) - (53*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^6)/(32256*d)
 - (3*Csc[(c + d*x)/2]^8)/(2048*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^8)/
(4608*d) + (55*Log[Cos[(c + d*x)/2]])/(128*d) - (55*Log[Sin[(c + d*x)/2]])/
(128*d) + (73*Sec[(c + d*x)/2]^2)/(512*d) + (13*Sec[(c + d*x)/2]^4)/(1024*d
) - (17*Sec[(c + d*x)/2]^6)/(1536*d) + (3*Sec[(c + d*x)/2]^8)/(2048*d) - (2
9*Tan[(c + d*x)/2])/(126*d) + (4163*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3
2256*d) - (319*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(10752*d) + (53*Sec[(c
+ d*x)/2]^6*Tan[(c + d*x)/2])/(32256*d) + (Sec[(c + d*x)/2]^8*Tan[(c + d*x)
```

```
/2])/(4608*d))
```

Maple [A] time = 0.096, size = 216, normalized size = 1.1

$$-\frac{11\,a^3(\cos(dx+c))^7}{48\,d(\sin(dx+c))^6}+\frac{11\,a^3(\cos(dx+c))^7}{192\,d(\sin(dx+c))^4}-\frac{11\,a^3(\cos(dx+c))^7}{128\,d(\sin(dx+c))^2}-\frac{11\,a^3(\cos(dx+c))^5}{128\,d}-\frac{55\,a^3(\cos(dx+c))^3}{384\,d}-\frac{55\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -11/48/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7+11/192/d*a^3/sin(d*x+c)^4*cos(d*x+c)
^7-11/128/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-11/128*a^3*cos(d*x+c)^5/d-55/384*
a^3*cos(d*x+c)^3/d-55/128*a^3*cos(d*x+c)/d-55/128/d*a^3*ln(csc(d*x+c)-cot(d
*x+c))-29/63/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7-3/8/d*a^3/sin(d*x+c)^8*cos(d*x
+c)^7-1/9/d*a^3/sin(d*x+c)^9*cos(d*x+c)^7
```

Maxima [A] time = 1.08448, size = 332, normalized size = 1.66

$$-\frac{63\,a^3\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-1)\right)-168\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/16128*(63*a^3*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c
)^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^
4 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c)
- 1)) - 168*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c)
)/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d
*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 6912*a^3/tan(d*x + c)^7 + 256*(9
*tan(d*x + c)^2 + 7)*a^3/tan(d*x + c)^9)/d
```

Fricas [A] time = 1.22804, size = 765, normalized size = 3.82

$$\frac{7424\,a^3\cos(dx+c)^9-9216\,a^3\cos(dx+c)^7+3465\left(a^3\cos(dx+c)^8-4\,a^3\cos(dx+c)^6+6\,a^3\cos(dx+c)^4-4\,a^3\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/16128*(7424*a^3*cos(d*x + c)^9 - 9216*a^3*cos(d*x + c)^7 + 3465*(a^3*cos(
d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x +
c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3465*(a^3*cos(d*x +
c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 +
 a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 42*(219*a^3*cos(d*x + c)^
```

```
7 - 803*a^3*cos(d*x + c)^5 + 605*a^3*cos(d*x + c)^3 - 165*a^3*cos(d*x + c))
*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4
 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41675, size = 437, normalized size = 2.18

$$\frac{28\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+189\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+324\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-672\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3024\,a^3\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/129024*(28*a^3*tan(1/2*d*x + 1/2*c)^9 + 189*a^3*tan(1/2*d*x + 1/2*c)^8 +
324*a^3*tan(1/2*d*x + 1/2*c)^7 - 672*a^3*tan(1/2*d*x + 1/2*c)^6 - 3024*a^3*
tan(1/2*d*x + 1/2*c)^5 - 1512*a^3*tan(1/2*d*x + 1/2*c)^4 + 9744*a^3*tan(1/2
*d*x + 1/2*c)^3 + 18144*a^3*tan(1/2*d*x + 1/2*c)^2 - 55440*a^3*log(abs(tan(
1/2*d*x + 1/2*c))) - 16632*a^3*tan(1/2*d*x + 1/2*c) + (156838*a^3*tan(1/2*d
*x + 1/2*c)^9 + 16632*a^3*tan(1/2*d*x + 1/2*c)^8 - 18144*a^3*tan(1/2*d*x +
1/2*c)^7 - 9744*a^3*tan(1/2*d*x + 1/2*c)^6 + 1512*a^3*tan(1/2*d*x + 1/2*c)^
5 + 3024*a^3*tan(1/2*d*x + 1/2*c)^4 + 672*a^3*tan(1/2*d*x + 1/2*c)^3 - 324*
a^3*tan(1/2*d*x + 1/2*c)^2 - 189*a^3*tan(1/2*d*x + 1/2*c) - 28*a^3)/tan(1/2
*d*x + 1/2*c)^9)/d
```

3.619 $\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=228

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{3a^3\cot^5(c+dx)\csc^3(}{8d}$$

```
[Out] (33*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a^
3*Cot[c + d*x]^9)/(3*d) + (33*a^3*Cot[c + d*x]*Csc[c + d*x])/(256*d) - (29*
a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (5*a^3*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d
*x]*Csc[c + d*x]^5)/(32*d) + (a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (
a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rubi [A] time = 0.427153, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{3a^3\cot^5(c+dx)\csc^3(}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (33*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a^
3*Cot[c + d*x]^9)/(3*d) + (33*a^3*Cot[c + d*x]*Csc[c + d*x])/(256*d) - (29*
a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (5*a^3*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d
*x]*Csc[c + d*x]^5)/(32*d) + (a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (
a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^2(c+dx)+3a^3\cot^6(c+dx)\csc^3(c+dx)\right.$$
$$= a^3\int \cot^6(c+dx)\csc^2(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^5(c+$$
$$= -\frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} -$$
$$= -\frac{a^3\cot^7(c+dx)}{7d} + \frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{16d} - \frac{3a^3\cot^5(c+}{}$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{3d} - \frac{15a^3\cot(c+dx)\csc^3(c+}{64d}$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{3d} + \frac{15a^3\cot(c+dx)\csc(c+}{128d}$$
$$= \frac{15a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{3d}$$
$$= \frac{33a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{3d}$$

Mathematica [A] time = 2.04623, size = 365, normalized size = 1.6

$$\frac{a^3(\sin(c+dx)+1)^3\left(-51200\tan\left(\frac{1}{2}(c+dx)\right)+51200\cot\left(\frac{1}{2}(c+dx)\right)+13860\csc^2\left(\frac{1}{2}(c+dx)\right)+42\sec^{10}\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(51200*Cot[(c + d*x)/2] + 13860*Csc[(c + d*x)/2]^
2 + 55440*Log[Cos[(c + d*x)/2]] - 55440*Log[Sin[(c + d*x)/2]] - 13860*Sec[(
c + d*x)/2]^2 + 19320*Sec[(c + d*x)/2]^4 - 5250*Sec[(c + d*x)/2]^6 + 315*Se
c[(c + d*x)/2]^8 + 42*Sec[(c + d*x)/2]^10 + 164800*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + 3840*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + Csc[(c + d*x)/2]^6*(52
50 - 60*Sin[c + d*x]) - 14*Csc[(c + d*x)/2]^10*(3 + 10*Sin[c + d*x]) + 5*Cs
```

```
c[(c + d*x)/2]^8*(-63 + 172*Sin[c + d*x]) - 20*Csc[(c + d*x)/2]^4*(966 + 51
5*Sin[c + d*x]) - 51200*Tan[(c + d*x)/2] - 1720*Sec[(c + d*x)/2]^6*Tan[(c +
 d*x)/2] + 280*Sec[(c + d*x)/2]^8*Tan[(c + d*x)/2]))/(430080*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.094, size = 240, normalized size = 1.1

$$-\frac{5\,a^3\,(\cos(dx+c))^7}{21\,d\,(\sin(dx+c))^7}-\frac{33\,a^3\,(\cos(dx+c))^7}{80\,d\,(\sin(dx+c))^8}-\frac{11\,a^3\,(\cos(dx+c))^7}{160\,d\,(\sin(dx+c))^6}+\frac{11\,a^3\,(\cos(dx+c))^7}{640\,d\,(\sin(dx+c))^4}-\frac{33\,a^3\,(\cos(dx+c))^7}{1280\,d\,(\sin(dx+c))^2}-\frac{33\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -5/21/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7-33/80/d*a^3/sin(d*x+c)^8*cos(d*x+c)^7
-11/160/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7+11/640/d*a^3/sin(d*x+c)^4*cos(d*x+c
)^7-33/1280/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-33/1280*a^3*cos(d*x+c)^5/d-11/2
56*a^3*cos(d*x+c)^3/d-33/256*a^3*cos(d*x+c)/d-33/256/d*a^3*ln(csc(d*x+c)-co
t(d*x+c))-1/3/d*a^3/sin(d*x+c)^9*cos(d*x+c)^7-1/10/d*a^3/sin(d*x+c)^10*cos(
d*x+c)^7
```

Maxima [A] time = 1.05929, size = 386, normalized size = 1.69

$$-\frac{21\,a^3\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/53760*(21*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 210*a^3*(2*(15*cos(d*x +
 c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x +
 c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*lo
g(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 7680*a^3/tan(d*x + c)^7 +
 2560*(9*tan(d*x + c)^2 + 7)*a^3/tan(d*x + c)^9)/d
```

Fricas [A] time = 1.26405, size = 855, normalized size = 3.75

$$-\frac{6930\,a^3\cos(dx+c)^9+21420\,a^3\cos(dx+c)^7-59136\,a^3\cos(dx+c)^5+32340\,a^3\cos(dx+c)^3-6930\,a^3\cos(dx+c)-\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] -1/53760*(6930*a^3*cos(d*x + c)^9 + 21420*a^3*cos(d*x + c)^7 - 59136*a^3*co
s(d*x + c)^5 + 32340*a^3*cos(d*x + c)^3 - 6930*a^3*cos(d*x + c) - 3465*(a^3
*cos(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*co
s(d*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 34
65*(a^3*cos(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10
*a^3*cos(d*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1
/2) + 2560*(5*a^3*cos(d*x + c)^9 - 12*a^3*cos(d*x + c)^7)*sin(d*x + c))/(d*
cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x +
 c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4621, size = 481, normalized size = 2.11

$$\frac{42\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+525\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3570\,a^3\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/430080*(42*a^3*tan(1/2*d*x + 1/2*c)^10 + 280*a^3*tan(1/2*d*x + 1/2*c)^9 +
 525*a^3*tan(1/2*d*x + 1/2*c)^8 - 600*a^3*tan(1/2*d*x + 1/2*c)^7 - 3570*a^3
*tan(1/2*d*x + 1/2*c)^6 - 3360*a^3*tan(1/2*d*x + 1/2*c)^5 + 5880*a^3*tan(1/
2*d*x + 1/2*c)^4 + 16800*a^3*tan(1/2*d*x + 1/2*c)^3 + 10500*a^3*tan(1/2*d*x
 + 1/2*c)^2 - 55440*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 31920*a^3*tan(1/2*
d*x + 1/2*c) + (162382*a^3*tan(1/2*d*x + 1/2*c)^10 + 31920*a^3*tan(1/2*d*x
+ 1/2*c)^9 - 10500*a^3*tan(1/2*d*x + 1/2*c)^8 - 16800*a^3*tan(1/2*d*x + 1/2
*c)^7 - 5880*a^3*tan(1/2*d*x + 1/2*c)^6 + 3360*a^3*tan(1/2*d*x + 1/2*c)^5 +
 3570*a^3*tan(1/2*d*x + 1/2*c)^4 + 600*a^3*tan(1/2*d*x + 1/2*c)^3 - 525*a^3
*tan(1/2*d*x + 1/2*c)^2 - 280*a^3*tan(1/2*d*x + 1/2*c) - 42*a^3)/tan(1/2*d*
x + 1/2*c)^10)/d
```

3.620 $\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=246

$$-\frac{a^3\cot^{11}(c+dx)}{11d}-\frac{5a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{19a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}-$$

```
[Out] (19*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (5*
a^3*Cot[c + d*x]^9)/(9*d) - (a^3*Cot[c + d*x]^11)/(11*d) + (19*a^3*Cot[c +
d*x]*Csc[c + d*x])/(256*d) - (7*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) +
(5*a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a^3*Cot[c + d*x]^5*Csc[c +
d*x]^3)/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(32*d) + (3*a^3*Cot[c +
 d*x]^3*Csc[c + d*x]^5)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*
d)
```

Rubi [A] time = 0.435741, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2611, 3768, 3770, 2607, 14, 270}

$$-\frac{a^3\cot^{11}(c+dx)}{11d}-\frac{5a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{19a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (19*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (5*
a^3*Cot[c + d*x]^9)/(9*d) - (a^3*Cot[c + d*x]^11)/(11*d) + (19*a^3*Cot[c +
d*x]*Csc[c + d*x])/(256*d) - (7*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) +
(5*a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a^3*Cot[c + d*x]^5*Csc[c +
d*x]^3)/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(32*d) + (3*a^3*Cot[c +
 d*x]^3*Csc[c + d*x]^5)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*
d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^3(c+dx)+3a^3\cot^6(c+dx)\csc^4(c+dx)\right.$$
$$= a^3\int \cot^6(c+dx)\csc^3(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^6(c+$$
$$= -\frac{a^3\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} -$$
$$= \frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a^3\cot^5(c+dx)\csc^3(c+dx)}{8d} +$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c+dx)}{9d} - \frac{a^3\cot^{11}(c+dx)}{11d} - \frac{5a^3}{}$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c+dx)}{9d} - \frac{a^3\cot^{11}(c+dx)}{11d} + \frac{5a^3}{}$$
$$= \frac{5a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c+dx)}{9d}$$
$$= \frac{19a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c+dx)}{9d}$$

Mathematica [A] time = 3.44735, size = 187, normalized size = 0.76

$$a^3(\sin(c+dx)+1)^3\left(16853760\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{10}(c+dx)(14477694\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(16853760*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d
*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^10*(10050560 + 12423680*Cos[2*(c + d*x
)] + 839680*Cos[4*(c + d*x)] - 2149120*Cos[6*(c + d*x)] - 568320*Cos[8*(c +
 d*x)] + 47360*Cos[10*(c + d*x)] + 14477694*Sin[c + d*x] + 5875716*Sin[3*(c
```

```
 + d*x)] + 7902972*Sin[5*(c + d*x)] - 414645*Sin[7*(c + d*x)] - 65835*Sin[9
*(c + d*x)])))/(227082240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.098, size = 264, normalized size = 1.1

$$-\frac{19\,a^3(\cos(dx+c))^7}{80\,d(\sin(dx+c))^8}-\frac{19\,a^3(\cos(dx+c))^7}{480\,d(\sin(dx+c))^6}+\frac{19\,a^3(\cos(dx+c))^7}{1920\,d(\sin(dx+c))^4}-\frac{19\,a^3(\cos(dx+c))^7}{1280\,d(\sin(dx+c))^2}-\frac{19\,a^3(\cos(dx+c))^5}{1280\,d}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -19/80/d*a^3/sin(d*x+c)^8*cos(d*x+c)^7-19/480/d*a^3/sin(d*x+c)^6*cos(d*x+c)
^7+19/1920/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7-19/1280/d*a^3/sin(d*x+c)^2*cos(d
*x+c)^7-19/1280*a^3*cos(d*x+c)^5/d-19/768*a^3*cos(d*x+c)^3/d-19/256*a^3*cos
(d*x+c)/d-19/256/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-37/99/d*a^3/sin(d*x+c)^9*c
os(d*x+c)^7-74/693/d*a^3/sin(d*x+c)^7*cos(d*x+c)^7-3/10/d*a^3/sin(d*x+c)^10
*cos(d*x+c)^7-1/11/d*a^3/sin(d*x+c)^11*cos(d*x+c)^7
```

Maxima [A] time = 1.08224, size = 416, normalized size = 1.69

$$-\frac{2079\,a^3\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7-128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/1774080*(2079*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*
x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*
x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) -
15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2310*a^3*(2*(15*cos(
d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(
d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) -
15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 84480*(9*tan(d*x + c
)^2 + 7)*a^3/tan(d*x + c)^9 + 2560*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2
+ 63)*a^3/tan(d*x + c)^11)/d
```

Fricas [A] time = 1.33549, size = 957, normalized size = 3.89

$$\frac{189440\,a^3\cos(dx+c)^{11}-1041920\,a^3\cos(dx+c)^9+1013760\,a^3\cos(dx+c)^7+65835\left(a^3\cos(dx+c)^{10}-5\,a^3\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/1774080*(189440*a^3*cos(d*x + c)^11 - 1041920*a^3*cos(d*x + c)^9 + 101376
0*a^3*cos(d*x + c)^7 + 65835*(a^3*cos(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 +
```

```
10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 65835*(a^3*cos(d*x + c)^10 - 5*
a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d*x + c)^4 + 5*a^3*
cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 462*(285*
a^3*cos(d*x + c)^9 - 50*a^3*cos(d*x + c)^7 - 2432*a^3*cos(d*x + c)^5 + 1330
*a^3*cos(d*x + c)^3 - 285*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*c
os(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3471, size = 524, normalized size = 2.13

$$630\, a^3 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^{11} + 4158\, a^3 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^{10} + 8470\, a^3 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^{9} - 3465\, a^3 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^{8} - 4059$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/14192640*(630*a^3*tan(1/2*d*x + 1/2*c)^11 + 4158*a^3*tan(1/2*d*x + 1/2*c)
^10 + 8470*a^3*tan(1/2*d*x + 1/2*c)^9 - 3465*a^3*tan(1/2*d*x + 1/2*c)^8 - 4
0590*a^3*tan(1/2*d*x + 1/2*c)^7 - 57750*a^3*tan(1/2*d*x + 1/2*c)^6 + 6930*a
^3*tan(1/2*d*x + 1/2*c)^5 + 138600*a^3*tan(1/2*d*x + 1/2*c)^4 + 244860*a^3*
tan(1/2*d*x + 1/2*c)^3 + 152460*a^3*tan(1/2*d*x + 1/2*c)^2 - 1053360*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c))) - 568260*a^3*tan(1/2*d*x + 1/2*c) + (3181018*a
^3*tan(1/2*d*x + 1/2*c)^11 + 568260*a^3*tan(1/2*d*x + 1/2*c)^10 - 152460*a^
3*tan(1/2*d*x + 1/2*c)^9 - 244860*a^3*tan(1/2*d*x + 1/2*c)^8 - 138600*a^3*t
an(1/2*d*x + 1/2*c)^7 - 6930*a^3*tan(1/2*d*x + 1/2*c)^6 + 57750*a^3*tan(1/2
*d*x + 1/2*c)^5 + 40590*a^3*tan(1/2*d*x + 1/2*c)^4 + 3465*a^3*tan(1/2*d*x +
 1/2*c)^3 - 8470*a^3*tan(1/2*d*x + 1/2*c)^2 - 4158*a^3*tan(1/2*d*x + 1/2*c)
 - 630*a^3)/tan(1/2*d*x + 1/2*c)^11)/d
```

3.621 $\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=270

$$-\frac{3a^3\cot^{11}(c+dx)}{11d}-\frac{7a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{41a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+dx)\csc^7(c+dx)}{12d}-$$

```
[Out] (41*a^3*ArcTanh[Cos[c + d*x]])/(1024*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (7
*a^3*Cot[c + d*x]^9)/(9*d) - (3*a^3*Cot[c + d*x]^11)/(11*d) + (41*a^3*Cot[c
 + d*x]*Csc[c + d*x])/(1024*d) + (41*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(1536
*d) - (35*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(384*d) + (3*a^3*Cot[c + d*x]^3*
Csc[c + d*x]^5)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d) - (a^
3*Cot[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^3*Cot[c + d*x]^3*Csc[c + d*x]^7)
/(24*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^7)/(12*d)
```

Rubi [A] time = 0.464142, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{3a^3\cot^{11}(c+dx)}{11d}-\frac{7a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{41a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+dx)\csc^7(c+dx)}{12d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (41*a^3*ArcTanh[Cos[c + d*x]])/(1024*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (7
*a^3*Cot[c + d*x]^9)/(9*d) - (3*a^3*Cot[c + d*x]^11)/(11*d) + (41*a^3*Cot[c
 + d*x]*Csc[c + d*x])/(1024*d) + (41*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(1536
*d) - (35*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(384*d) + (3*a^3*Cot[c + d*x]^3*
Csc[c + d*x]^5)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d) - (a^
3*Cot[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^3*Cot[c + d*x]^3*Csc[c + d*x]^7)
/(24*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^7)/(12*d)
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^4(c+dx)+3a^3\cot^6(c+dx)\csc^5(c+dx)\right.$$
$$= a^3\int \cot^6(c+dx)\csc^4(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^7(c+$$
$$= -\frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{a^3\cot^5(c+dx)\csc^7(c+dx)}{12d} -$$
$$= \frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} +$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d} - \frac{3a^3\cot^{11}(c+dx)}{11d} - \frac{3a^3}{}$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d} - \frac{3a^3\cot^{11}(c+dx)}{11d} + \frac{3a^3}{}$$
$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d} - \frac{3a^3\cot^{11}(c+dx)}{11d} + \frac{9a^3}{}$$
$$= \frac{9a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d}$$
$$= \frac{41a^3\tanh^{-1}(\cos(c+dx))}{1024d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d}$$

Mathematica [A] time = 4.70877, size = 197, normalized size = 0.73

$$\frac{a^3(\sin(c+dx)+1)^3\left(72737280\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{11}(c+dx)(49776640\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(72737280*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d
*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^11*(91311066 + 62609778*Cos[2*(c + d*x
)] + 22551144*Cos[4*(c + d*x)] - 23426403*Cos[6*(c + d*x)] - 1799490*Cos[8*
(c + d*x)] + 142065*Cos[10*(c + d*x)] + 49776640*Sin[c + d*x] + 84039680*Si
n[3*(c + d*x)] + 38118400*Sin[5*(c + d*x)] + 2206720*Sin[7*(c + d*x)] - 153
0880*Sin[9*(c + d*x)] + 117760*Sin[11*(c + d*x)])))/(1816657920*d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.1, size = 288, normalized size = 1.1

$$-\frac{23\,a^3(\cos(dx+c))^7}{99\,d(\sin(dx+c))^9}-\frac{46\,a^3(\cos(dx+c))^7}{693\,d(\sin(dx+c))^7}-\frac{41\,a^3(\cos(dx+c))^7}{120\,d(\sin(dx+c))^{10}}-\frac{41\,a^3(\cos(dx+c))^7}{320\,d(\sin(dx+c))^8}-\frac{41\,a^3(\cos(dx+c))^7}{1920\,d(\sin(dx+c))^6}+\frac{4}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -23/99/d*a^3/sin(d*x+c)^9*cos(d*x+c)^7-46/693/d*a^3/sin(d*x+c)^7*cos(d*x+c)
^7-41/120/d*a^3/sin(d*x+c)^10*cos(d*x+c)^7-41/320/d*a^3/sin(d*x+c)^8*cos(d*
x+c)^7-41/1920/d*a^3/sin(d*x+c)^6*cos(d*x+c)^7+41/7680/d*a^3/sin(d*x+c)^4*c
os(d*x+c)^7-41/5120/d*a^3/sin(d*x+c)^2*cos(d*x+c)^7-41/5120*a^3*cos(d*x+c)^
5/d-41/3072*a^3*cos(d*x+c)^3/d-41/1024*a^3*cos(d*x+c)/d-41/1024/d*a^3*ln(cs
c(d*x+c)-cot(d*x+c))-3/11/d*a^3/sin(d*x+c)^11*cos(d*x+c)^7-1/12/d*a^3/sin(d
*x+c)^12*cos(d*x+c)^7
```

Maxima [A] time = 1.03957, size = 470, normalized size = 1.74

$$-\frac{1155\,a^3\left(\frac{2\left(15\cos(dx+c)^{11}-85\cos(dx+c)^9+198\cos(dx+c)^7+198\cos(dx+c)^5-85\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^{12}-6\cos(dx+c)^{10}+15\cos(dx+c)^8-20\cos(dx+c)^6+15\cos(dx+c)^4-6\cos(dx+c)^2+1}-15\log(\cos(dx+c)+1)+15\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/7096320*(1155*a^3*(2*(15*cos(d*x + c)^11 - 85*cos(d*x + c)^9 + 198*cos(d
*x + c)^7 + 198*cos(d*x + c)^5 - 85*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(
d*x + c)^12 - 6*cos(d*x + c)^10 + 15*cos(d*x + c)^8 - 20*cos(d*x + c)^6 + 1
5*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*lo
g(cos(d*x + c) - 1)) + 8316*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 -
 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10
 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c
)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 112640*(9
*tan(d*x + c)^2 + 7)*a^3/tan(d*x + c)^9 + 30720*(99*tan(d*x + c)^4 + 154*ta
n(d*x + c)^2 + 63)*a^3/tan(d*x + c)^11)/d
```

Fricas [A] time = 1.33993, size = 1053, normalized size = 3.9

$$-\frac{284130\,a^3\cos(dx+c)^{11}-1610070\,a^3\cos(dx+c)^9-507276\,a^3\cos(dx+c)^7+3750516\,a^3\cos(dx+c)^5-1610070\,a^3\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] -1/7096320*(284130*a^3*cos(d*x + c)^11 - 1610070*a^3*cos(d*x + c)^9 - 50727
6*a^3*cos(d*x + c)^7 + 3750516*a^3*cos(d*x + c)^5 - 1610070*a^3*cos(d*x + c
)^3 + 284130*a^3*cos(d*x + c) - 142065*(a^3*cos(d*x + c)^12 - 6*a^3*cos(d*x
 + c)^10 + 15*a^3*cos(d*x + c)^8 - 20*a^3*cos(d*x + c)^6 + 15*a^3*cos(d*x +
 c)^4 - 6*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2) + 142065*(a
^3*cos(d*x + c)^12 - 6*a^3*cos(d*x + c)^10 + 15*a^3*cos(d*x + c)^8 - 20*a^3
*cos(d*x + c)^6 + 15*a^3*cos(d*x + c)^4 - 6*a^3*cos(d*x + c)^2 + a^3)*log(-
1/2*cos(d*x + c) + 1/2) + 10240*(46*a^3*cos(d*x + c)^11 - 253*a^3*cos(d*x +
 c)^9 + 396*a^3*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^12 - 6*d*cos(
d*x + c)^10 + 15*d*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)
^4 - 6*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**13*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.56032, size = 567, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/56770560*(1155*a^3*tan(1/2*d*x + 1/2*c)^12 + 7560*a^3*tan(1/2*d*x + 1/2*c
)^11 + 16632*a^3*tan(1/2*d*x + 1/2*c)^10 + 3080*a^3*tan(1/2*d*x + 1/2*c)^9
- 51975*a^3*tan(1/2*d*x + 1/2*c)^8 - 106920*a^3*tan(1/2*d*x + 1/2*c)^7 - 83
160*a^3*tan(1/2*d*x + 1/2*c)^6 + 83160*a^3*tan(1/2*d*x + 1/2*c)^5 + 384615*
a^3*tan(1/2*d*x + 1/2*c)^4 + 572880*a^3*tan(1/2*d*x + 1/2*c)^3 + 166320*a^3
*tan(1/2*d*x + 1/2*c)^2 - 2273040*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 1496
880*a^3*tan(1/2*d*x + 1/2*c) + (7053722*a^3*tan(1/2*d*x + 1/2*c)^12 + 14968
80*a^3*tan(1/2*d*x + 1/2*c)^11 - 166320*a^3*tan(1/2*d*x + 1/2*c)^10 - 57288
0*a^3*tan(1/2*d*x + 1/2*c)^9 - 384615*a^3*tan(1/2*d*x + 1/2*c)^8 - 83160*a^
3*tan(1/2*d*x + 1/2*c)^7 + 83160*a^3*tan(1/2*d*x + 1/2*c)^6 + 106920*a^3*ta
n(1/2*d*x + 1/2*c)^5 + 51975*a^3*tan(1/2*d*x + 1/2*c)^4 - 3080*a^3*tan(1/2*
d*x + 1/2*c)^3 - 16632*a^3*tan(1/2*d*x + 1/2*c)^2 - 7560*a^3*tan(1/2*d*x +
1/2*c) - 1155*a^3)/tan(1/2*d*x + 1/2*c)^12)/d
```

3.622 $\int \cot^6(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=286

$$-\frac{a^3\cot^{13}(c+dx)}{13d}-\frac{6a^3\cot^{11}(c+dx)}{11d}-\frac{a^3\cot^9(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{27a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+d$$

```
[Out] (27*a^3*ArcTanh[Cos[c + d*x]])/(1024*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a
^3*Cot[c + d*x]^9)/d - (6*a^3*Cot[c + d*x]^11)/(11*d) - (a^3*Cot[c + d*x]^1
3)/(13*d) + (27*a^3*Cot[c + d*x]*Csc[c + d*x])/(1024*d) + (9*a^3*Cot[c + d*
x]*Csc[c + d*x]^3)/(512*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(128*d) +
(a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*
x]^5)/(10*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^3*Cot[c + d*
x]^3*Csc[c + d*x]^7)/(8*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^7)/(4*d)
```

Rubi [A] time = 0.462119, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2873, 2611, 3768, 3770, 2607, 270}

$$-\frac{a^3\cot^{13}(c+dx)}{13d}-\frac{6a^3\cot^{11}(c+dx)}{11d}-\frac{a^3\cot^9(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{27a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*a^3*ArcTanh[Cos[c + d*x]])/(1024*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a
^3*Cot[c + d*x]^9)/d - (6*a^3*Cot[c + d*x]^11)/(11*d) - (a^3*Cot[c + d*x]^1
3)/(13*d) + (27*a^3*Cot[c + d*x]*Csc[c + d*x])/(1024*d) + (9*a^3*Cot[c + d*
x]*Csc[c + d*x]^3)/(512*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(128*d) +
(a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*
x]^5)/(10*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^3*Cot[c + d*
x]^3*Csc[c + d*x]^7)/(8*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^7)/(4*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^5(c+dx)+3a^3\cot^6(c+dx)\csc^6(c+dx)\right.$$

$$= a^3\int \cot^6(c+dx)\csc^5(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^8(c+$$

$$= -\frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{a^3\cot^5(c+dx)\csc^7(c+dx)}{4d} -$$

$$= \frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} +$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} - \frac{6a^3\cot^{11}(c+dx)}{11d} -$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} - \frac{6a^3\cot^{11}(c+dx)}{11d} -$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} - \frac{6a^3\cot^{11}(c+dx)}{11d} -$$

$$= \frac{3a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} -$$

$$= \frac{27a^3\tanh^{-1}(\cos(c+dx))}{1024d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d}$$

Mathematica [A] time = 6.48981, size = 283, normalized size = 0.99

$$\frac{27(a\sin(c+dx)+a)^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{1024d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6} - \frac{27(a\sin(c+dx)+a)^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1024d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6} + \frac{\cot(c+dx)\csc^{12}(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*Log[Cos[(c + d*x)/2]]*(a + a*Sin[c + d*x])^3)/(1024*d*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^6) - (27*Log[Sin[(c + d*x)/2]]*(a + a*Sin[c + d*x])^3)
/(1024*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (Cot[c + d*x]*Csc[c + d
*x]^12*(a + a*Sin[c + d*x])^3*(-200294400 - 243712000*Cos[2*(c + d*x)] - 11
079680*Cos[4*(c + d*x)] + 43294720*Cos[6*(c + d*x)] + 9420800*Cos[8*(c + d*
x)] - 1433600*Cos[10*(c + d*x)] + 102400*Cos[12*(c + d*x)] - 194159966*Sin[
c + d*x] - 182107926*Sin[3*(c + d*x)] - 123736613*Sin[5*(c + d*x)] + 457156
```

```
7*Sin[7*(c + d*x)] + 1846845*Sin[9*(c + d*x)] - 135135*Sin[11*(c + d*x)]))/
(5248122880*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.098, size = 312, normalized size = 1.1

$$-\frac{a^3 (\cos (dx+c))^7}{4 d (\sin (dx+c))^{12}}-\frac{40 a^3 (\cos (dx+c))^7}{1001 d (\sin (dx+c))^7}-\frac{9 a^3 (\cos (dx+c))^3}{1024 d}-\frac{27 a^3 \cos (dx+c)}{1024 d}-\frac{27 a^3 \ln (\csc (dx+c)-\cot (dx}{1024 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d*a^3/sin(d*x+c)^12*cos(d*x+c)^7-40/1001/d*a^3/sin(d*x+c)^7*cos(d*x+c)
^7-9/1024*a^3*cos(d*x+c)^3/d-27/1024*a^3*cos(d*x+c)/d-27/1024/d*a^3*ln(csc(
d*x+c)-cot(d*x+c))-27/5120*a^3*cos(d*x+c)^5/d-45/143/d*a^3/sin(d*x+c)^11*co
s(d*x+c)^7-20/143/d*a^3/sin(d*x+c)^9*cos(d*x+c)^7-9/40/d*a^3/sin(d*x+c)^10*
cos(d*x+c)^7-27/320/d*a^3/sin(d*x+c)^8*cos(d*x+c)^7-9/640/d*a^3/sin(d*x+c)^
6*cos(d*x+c)^7+9/2560/d*a^3/sin(d*x+c)^4*cos(d*x+c)^7-27/5120/d*a^3/sin(d*x
+c)^2*cos(d*x+c)^7-1/13/d*a^3/sin(d*x+c)^13*cos(d*x+c)^7
```

Maxima [A] time = 1.06445, size = 497, normalized size = 1.74

$$-\frac{15015\, a^3 \left(\frac{2 \left(15 \cos(dx+c)^{11}-85 \cos(dx+c)^{9}+198 \cos(dx+c)^{7}+198 \cos(dx+c)^{5}-85 \cos(dx+c)^{3}+15 \cos(dx+c)\right)}{\cos(dx+c)^{12}-6 \cos(dx+c)^{10}+15 \cos(dx+c)^{8}-20 \cos(dx+c)^{6}+15 \cos(dx+c)^{4}-6 \cos(dx+c)^{2}+1}-15 \log (\cos (dx+c)+1)+15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/30750720*(15015*a^3*(2*(15*cos(d*x + c)^11 - 85*cos(d*x + c)^9 + 198*cos
(d*x + c)^7 + 198*cos(d*x + c)^5 - 85*cos(d*x + c)^3 + 15*cos(d*x + c))/(co
s(d*x + c)^12 - 6*cos(d*x + c)^10 + 15*cos(d*x + c)^8 - 20*cos(d*x + c)^6 +
 15*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*
log(cos(d*x + c) - 1)) + 12012*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^
7 - 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)
^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x
+ c)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 133120
*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2 + 63)*a^3/tan(d*x + c)^11 + 10240*
(429*tan(d*x + c)^6 + 1001*tan(d*x + c)^4 + 819*tan(d*x + c)^2 + 231)*a^3/t
an(d*x + c)^13)/d
```

Fricas [A] time = 1.36826, size = 1137, normalized size = 3.98

$$\frac{409600\, a^3 \cos (dx+c)^{13}-2662400\, a^3 \cos (dx+c)^{11}+7321600\, a^3 \cos (dx+c)^{9}-5857280\, a^3 \cos (dx+c)^{7}+135135 \left(a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/10250240*(409600*a^3*cos(d*x + c)^13 - 2662400*a^3*cos(d*x + c)^11 + 7321
600*a^3*cos(d*x + c)^9 - 5857280*a^3*cos(d*x + c)^7 + 135135*(a^3*cos(d*x +
 c)^12 - 6*a^3*cos(d*x + c)^10 + 15*a^3*cos(d*x + c)^8 - 20*a^3*cos(d*x + c
)^6 + 15*a^3*cos(d*x + c)^4 - 6*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 135135*(a^3*cos(d*x + c)^12 - 6*a^3*cos(d*x + c)^
10 + 15*a^3*cos(d*x + c)^8 - 20*a^3*cos(d*x + c)^6 + 15*a^3*cos(d*x + c)^4
- 6*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2
002*(135*a^3*cos(d*x + c)^11 - 765*a^3*cos(d*x + c)^9 + 758*a^3*cos(d*x + c
)^7 + 1782*a^3*cos(d*x + c)^5 - 765*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x +
c))*sin(d*x + c))/((d*cos(d*x + c)^12 - 6*d*cos(d*x + c)^10 + 15*d*cos(d*x
+ c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*d*cos(d*x + c)^2 + d
)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**14*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41425, size = 610, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/82001920*(770*a^3*tan(1/2*d*x + 1/2*c)^13 + 5005*a^3*tan(1/2*d*x + 1/2*c)
^12 + 11830*a^3*tan(1/2*d*x + 1/2*c)^11 + 8008*a^3*tan(1/2*d*x + 1/2*c)^10
- 20020*a^3*tan(1/2*d*x + 1/2*c)^9 - 65065*a^3*tan(1/2*d*x + 1/2*c)^8 - 943
80*a^3*tan(1/2*d*x + 1/2*c)^7 - 40040*a^3*tan(1/2*d*x + 1/2*c)^6 + 150150*a
^3*tan(1/2*d*x + 1/2*c)^5 + 385385*a^3*tan(1/2*d*x + 1/2*c)^4 + 450450*a^3*
tan(1/2*d*x + 1/2*c)^3 + 80080*a^3*tan(1/2*d*x + 1/2*c)^2 - 2162160*a^3*log
(abs(tan(1/2*d*x + 1/2*c))) - 1401400*a^3*tan(1/2*d*x + 1/2*c) + (6875958*a
^3*tan(1/2*d*x + 1/2*c)^13 + 1401400*a^3*tan(1/2*d*x + 1/2*c)^12 - 80080*a^
3*tan(1/2*d*x + 1/2*c)^11 - 450450*a^3*tan(1/2*d*x + 1/2*c)^10 - 385385*a^3
*tan(1/2*d*x + 1/2*c)^9 - 150150*a^3*tan(1/2*d*x + 1/2*c)^8 + 40040*a^3*tan
(1/2*d*x + 1/2*c)^7 + 94380*a^3*tan(1/2*d*x + 1/2*c)^6 + 65065*a^3*tan(1/2*
d*x + 1/2*c)^5 + 20020*a^3*tan(1/2*d*x + 1/2*c)^4 - 8008*a^3*tan(1/2*d*x +
1/2*c)^3 - 11830*a^3*tan(1/2*d*x + 1/2*c)^2 - 5005*a^3*tan(1/2*d*x + 1/2*c)
 - 770*a^3)/tan(1/2*d*x + 1/2*c)^13)/d
```

3.623 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=178

$$\frac{4a^4\cos^5(c+dx)}{5d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{4a^4\cot(c+dx)}{d}+\frac{a^4\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{23a^4\sin^3(c+dx)}{24d}$$

```
[Out] (-135*a^4*x)/16 + (6*a^4*ArcTanh[Cos[c + d*x]])/d - (4*a^4*Cos[c + d*x])/d
+ (4*a^4*Cos[c + d*x]^5)/(5*d) - (4*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]
^3)/(3*d) - (2*a^4*Cot[c + d*x]*Csc[c + d*x])/d - (89*a^4*Cos[c + d*x]*Sin[
c + d*x])/(16*d) + (23*a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a^4*Cos[c
 + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.281226, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 3768, 2635, 2633}

$$\frac{4a^4\cos^5(c+dx)}{5d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{4a^4\cot(c+dx)}{d}+\frac{a^4\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{23a^4\sin^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (-135*a^4*x)/16 + (6*a^4*ArcTanh[Cos[c + d*x]])/d - (4*a^4*Cos[c + d*x])/d
+ (4*a^4*Cos[c + d*x]^5)/(5*d) - (4*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]
^3)/(3*d) - (2*a^4*Cot[c + d*x]*Csc[c + d*x])/d - (89*a^4*Cos[c + d*x]*Sin[
c + d*x])/(16*d) + (23*a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a^4*Cos[c
 + d*x]*Sin[c + d*x]^5)/(6*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(-14a^{10}-8a^{10}\csc(c+dx)+3a^{10}\csc^2(c+dx)+4a^{10}\csc^3(c+\right.}{}$$

$$= -14a^4x + a^4\int \csc^4(c+dx)\,dx - a^4\int \sin^6(c+dx)\,dx + \left(3a^4\right)\int$$

$$= -14a^4x + \frac{8a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^4\cot(c+dx)\csc(c+dx}{d}$$

$$= -7a^4x + \frac{6a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d} + \frac{4a^4\cos^5(}{5d}$$

$$= -\frac{65a^4x}{8} + \frac{6a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d} + \frac{4a^4\cos}{}$$

$$= -\frac{135a^4x}{16} + \frac{6a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d} + \frac{4a^4\mathrm{co}}{}$$

Mathematica [A] time = 1.64108, size = 229, normalized size = 1.29

$$\frac{a^4(\sin(c+dx)+1)^4\left(-8100(c+dx)-2415\sin(2(c+dx))-135\sin(4(c+dx))+5\sin(6(c+dx))-3360\cos(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(-8100*(c + d*x) - 3360*Cos[c + d*x] + 240*Cos[3*
(c + d*x)] + 48*Cos[5*(c + d*x)] - 1760*Cot[(c + d*x)/2] - 480*Csc[(c + d*x
)/2]^2 + 5760*Log[Cos[(c + d*x)/2]] - 5760*Log[Sin[(c + d*x)/2]] + 480*Sec[
(c + d*x)/2]^2 + 320*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 20*Csc[(c + d*x)/2
]^4*Sin[c + d*x] - 2415*Sin[2*(c + d*x)] - 135*Sin[4*(c + d*x)] + 5*Sin[6*(
c + d*x)] + 1760*Tan[(c + d*x)/2]))/(960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^8)
```

Maple [A] time = 0.096, size = 223, normalized size = 1.3

$$-\frac{9a^4(\cos(dx+c))^5\sin(dx+c)}{2d} - \frac{6a^4(\cos(dx+c))^5}{5d} - \frac{45a^4(\cos(dx+c))^3\sin(dx+c)}{8d} - \frac{135a^4\cos(dx+c)\sin(d}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -9/2*a^4*cos(d*x+c)^5*sin(d*x+c)/d-6/5*a^4*cos(d*x+c)^5/d-45/8*a^4*cos(d*x+
c)^3*sin(d*x+c)/d-135/16*a^4*cos(d*x+c)*sin(d*x+c)/d-135/16/d*a^4*c-2*a^4*c
os(d*x+c)^3/d-6*a^4*cos(d*x+c)/d-6/d*a^4*ln(csc(d*x+c)-cot(d*x+c))-135/16*a
^4*x-1/3/d*a^4/sin(d*x+c)^3*cos(d*x+c)^7-14/3/d*a^4/sin(d*x+c)*cos(d*x+c)^7
-2/d*a^4/sin(d*x+c)^2*cos(d*x+c)^7
```

Maxima [A] time = 1.56699, size = 397, normalized size = 2.23

$$\frac{128\left(6\cos\left(dx+c\right)^5+10\cos\left(dx+c\right)^3+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^4-320}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/960*(128*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log
(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 - 320*(4*cos(d*x + c)^3
- 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x +
c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x
- 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^4 - 720*(15*d*x + 15*c
 + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x
+ c)^3 + tan(d*x + c)))*a^4 + 160*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*
tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^4)/d
```

Fricas [A] time = 1.28929, size = 641, normalized size = 3.6

$$-\frac{40\,a^4\cos\left(dx+c\right)^9-390\,a^4\cos\left(dx+c\right)^7-405\,a^4\cos\left(dx+c\right)^5+2700\,a^4\cos\left(dx+c\right)^3-2025\,a^4\cos\left(dx+c\right)-720\left(a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/240*(40*a^4*cos(d*x + c)^9 - 390*a^4*cos(d*x + c)^7 - 405*a^4*cos(d*x +
c)^5 + 2700*a^4*cos(d*x + c)^3 - 2025*a^4*cos(d*x + c) - 720*(a^4*cos(d*x +
 c)^2 - a^4)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 720*(a^4*cos(d*x +
c)^2 - a^4)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(64*a^4*cos(d*x +
 c)^7 - 64*a^4*cos(d*x + c)^5 - 675*a^4*d*x*cos(d*x + c)^2 - 320*a^4*cos(d*
x + c)^3 + 675*a^4*d*x + 480*a^4*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x +
c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49392, size = 437, normalized size = 2.46

$$10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2025\,(dx+c)a^4-1440\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+450\,a^4\tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(10*a^4*tan(1/2*d*x + 1/2*c)^3 + 120*a^4*tan(1/2*d*x + 1/2*c)^2 - 202
5*(d*x + c)*a^4 - 1440*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 450*a^4*tan(1/2
*d*x + 1/2*c) + 10*(264*a^4*tan(1/2*d*x + 1/2*c)^3 - 45*a^4*tan(1/2*d*x + 1
/2*c)^2 - 12*a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(13
35*a^4*tan(1/2*d*x + 1/2*c)^11 + 3085*a^4*tan(1/2*d*x + 1/2*c)^9 - 3840*a^4
*tan(1/2*d*x + 1/2*c)^8 + 1110*a^4*tan(1/2*d*x + 1/2*c)^7 - 7680*a^4*tan(1/
2*d*x + 1/2*c)^6 - 1110*a^4*tan(1/2*d*x + 1/2*c)^5 - 7680*a^4*tan(1/2*d*x +
 1/2*c)^4 - 3085*a^4*tan(1/2*d*x + 1/2*c)^3 - 4608*a^4*tan(1/2*d*x + 1/2*c)
^2 - 1335*a^4*tan(1/2*d*x + 1/2*c) - 768*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
6)/d
```

3.624 $\int \frac{\cos^6(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{\cos^9(c+dx)}{9ad}-\frac{2\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}+\frac{\sin(c+dx)\cos^3(}{64ad}$$

```
[Out] (3*x)/(128*a) + Cos[c + d*x]^5/(5*a*d) - (2*Cos[c + d*x]^7)/(7*a*d) + Cos[c
 + d*x]^9/(9*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(128*a*d) + (Cos[c + d*x]
^3*Sin[c + d*x])/(64*a*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(16*a*d) - (Cos[c
 + d*x]^5*Sin[c + d*x]^3)/(8*a*d)
```

Rubi [A] time = 0.215867, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 270}

$$\frac{\cos^9(c+dx)}{9ad}-\frac{2\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}+\frac{\sin(c+dx)\cos^3(}{64ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(128*a) + Cos[c + d*x]^5/(5*a*d) - (2*Cos[c + d*x]^7)/(7*a*d) + Cos[c
 + d*x]^9/(9*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(128*a*d) + (Cos[c + d*x]
^3*Sin[c + d*x])/(64*a*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(16*a*d) - (Cos[c
 + d*x]^5*Sin[c + d*x]^3)/(8*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^5(c+dx)\,dx}{a}\\
&= -\frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} + \frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^4\left(1-\right.\right.}{}\\
&= -\frac{\cos^5(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} + \frac{\int \cos^4(c+dx)\,dx}{16a} + \frac{\text{Subs}}{}\\
&= \frac{\cos^5(c+dx)}{5ad} - \frac{2\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{64ad} - \frac{\cos^5(c}{}\\
&= \frac{\cos^5(c+dx)}{5ad} - \frac{2\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} + \frac{3\cos(c+dx)\sin(c+dx)}{128ad} + \frac{\cos^3(c}{}\\
&= \frac{3x}{128a} + \frac{\cos^5(c+dx)}{5ad} - \frac{2\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} + \frac{3\cos(c+dx)\sin(c+dx)}{128ad}\end{aligned}$$

Mathematica [B] time = 8.5865, size = 429, normalized size = 2.7

$$\frac{15120dx\sin\left(\frac{c}{2}\right) - 7560\sin\left(\frac{c}{2}+dx\right) + 7560\sin\left(\frac{3c}{2}+dx\right) - 1680\sin\left(\frac{5c}{2}+3dx\right) + 1680\sin\left(\frac{7c}{2}+3dx\right) - 2520\sin\left(\frac{7c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2520*(5*c + 6*d*x)*Cos[c/2] + 7560*Cos[c/2 + d*x] + 7560*Cos[(3*c)/2 + d*x
] + 1680*Cos[(5*c)/2 + 3*d*x] + 1680*Cos[(7*c)/2 + 3*d*x] - 2520*Cos[(7*c)/
2 + 4*d*x] + 2520*Cos[(9*c)/2 + 4*d*x] - 1008*Cos[(9*c)/2 + 5*d*x] - 1008*C
os[(11*c)/2 + 5*d*x] - 180*Cos[(13*c)/2 + 7*d*x] - 180*Cos[(15*c)/2 + 7*d*x
] + 315*Cos[(15*c)/2 + 8*d*x] - 315*Cos[(17*c)/2 + 8*d*x] + 140*Cos[(17*c)/
2 + 9*d*x] + 140*Cos[(19*c)/2 + 9*d*x] + 12600*Sin[c/2] + 12600*c*Sin[c/2]
+ 15120*d*x*Sin[c/2] - 7560*Sin[c/2 + d*x] + 7560*Sin[(3*c)/2 + d*x] - 1680
*Sin[(5*c)/2 + 3*d*x] + 1680*Sin[(7*c)/2 + 3*d*x] - 2520*Sin[(7*c)/2 + 4*d*
x] - 2520*Sin[(9*c)/2 + 4*d*x] + 1008*Sin[(9*c)/2 + 5*d*x] - 1008*Sin[(11*c
)/2 + 5*d*x] + 180*Sin[(13*c)/2 + 7*d*x] - 180*Sin[(15*c)/2 + 7*d*x] + 315*
Sin[(15*c)/2 + 8*d*x] + 315*Sin[(17*c)/2 + 8*d*x] - 140*Sin[(17*c)/2 + 9*d*
x] + 140*Sin[(19*c)/2 + 9*d*x])/(645120*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.106, size = 517, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 16/315/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9-3/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*t
an(1/2*d*x+1/2*c)+16/35/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^2
-13/32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^3+64/35/d/a/(1+tan
(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^4+155/32/d/a/(1+tan(1/2*d*x+1/2*c)^
2)^9*tan(1/2*d*x+1/2*c)^5-32/5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1
/2*c)^6-169/32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^7+112/5/d/
a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^8-16/d/a/(1+tan(1/2*d*x+1/2
*c)^2)^9*tan(1/2*d*x+1/2*c)^10+169/32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/
2*d*x+1/2*c)^11+32/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^12-1
55/32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^13+13/32/d/a/(1+tan
(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^15+3/64/d/a/(1+tan(1/2*d*x+1/2*c)^2
)^9*tan(1/2*d*x+1/2*c)^17+3/64/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.55172, size = 678, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/20160*((945*sin(d*x + c)/(cos(d*x + c) + 1) - 9216*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 8190*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 36864*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 97650*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 12
9024*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 106470*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 - 451584*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 322560*sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 - 106470*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
- 215040*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 97650*sin(d*x + c)^13/(cos
(d*x + c) + 1)^13 - 8190*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 945*sin(d*
x + c)^17/(cos(d*x + c) + 1)^17 - 1024)/(a + 9*a*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 36*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 84*a*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 126*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 126*a*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 + 84*a*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 + 36*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 9*a*sin(d*x + c)^16/(c
os(d*x + c) + 1)^16 + a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18) - 945*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.19707, size = 252, normalized size = 1.58

$$\frac{4480 \cos(dx+c)^9 - 11520 \cos(dx+c)^7 + 8064 \cos(dx+c)^5 + 945\,dx + 315\left(16 \cos(dx+c)^7 - 24 \cos(dx+c)^5 + 2 \text{ c}\right.}{40320\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40320*(4480*cos(d*x + c)^9 - 11520*cos(d*x + c)^7 + 8064*cos(d*x + c)^5 +
 945*d*x + 315*(16*cos(d*x + c)^7 - 24*cos(d*x + c)^5 + 2*cos(d*x + c)^3 +
3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31384, size = 294, normalized size = 1.85

$$\frac{\frac{945\,(dx+c)}{a}+\frac{2\left(945\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+8190\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-97650\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+215040\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+106470\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-322560\tan\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40320*(945*(d*x + c)/a + 2*(945*tan(1/2*d*x + 1/2*c)^17 + 8190*tan(1/2*d*
x + 1/2*c)^15 - 97650*tan(1/2*d*x + 1/2*c)^13 + 215040*tan(1/2*d*x + 1/2*c)
^12 + 106470*tan(1/2*d*x + 1/2*c)^11 - 322560*tan(1/2*d*x + 1/2*c)^10 + 451
584*tan(1/2*d*x + 1/2*c)^8 - 106470*tan(1/2*d*x + 1/2*c)^7 - 129024*tan(1/2
*d*x + 1/2*c)^6 + 97650*tan(1/2*d*x + 1/2*c)^5 + 36864*tan(1/2*d*x + 1/2*c)
^4 - 8190*tan(1/2*d*x + 1/2*c)^3 + 9216*tan(1/2*d*x + 1/2*c)^2 - 945*tan(1/
2*d*x + 1/2*c) + 1024)/((tan(1/2*d*x + 1/2*c)^2 + 1)^9*a))/d
```

3.625 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{\cos^7(c+dx)}{7ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{3\sin(c$$

```
[Out] (-3*x)/(128*a) - Cos[c + d*x]^5/(5*a*d) + Cos[c + d*x]^7/(7*a*d) - (3*Cos[c
 + d*x]*Sin[c + d*x])/(128*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(64*a*d) +
(Cos[c + d*x]^5*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^5*Sin[c + d*x]^3)/(8
*a*d)
```

Rubi [A] time = 0.214673, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2565, 14, 2568, 2635, 8}

$$\frac{\cos^7(c+dx)}{7ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{3\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-3*x)/(128*a) - Cos[c + d*x]^5/(5*a*d) + Cos[c + d*x]^7/(7*a*d) - (3*Cos[c
 + d*x]*Sin[c + d*x])/(128*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(64*a*d) +
(Cos[c + d*x]^5*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^5*Sin[c + d*x]^3)/(8
*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a}$$
$$= \frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} - \frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^4\left(1-x^2\right.\right.}{a}$$
$$= \frac{\cos^5(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} - \frac{\int \cos^4(c+dx)\,dx}{16a} - \frac{\text{Subst}}{}$$
$$= -\frac{\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{\cos^5(c+dx)\sin(c+dx}{16ad}$$
$$= -\frac{\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} - \frac{3\cos(c+dx)\sin(c+dx)}{128ad} - \frac{\cos^3(c+dx)\sin(c+d}{64ad}$$
$$= -\frac{3x}{128a} - \frac{\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} - \frac{3\cos(c+dx)\sin(c+dx)}{128ad} - \frac{\cos^3(c+dx)\,\text{si}}{64ad}$$

Mathematica [B] time = 8.7552, size = 375, normalized size = 2.66

$$-1680dx\sin\left(\frac{c}{2}\right)+1680\sin\left(\frac{c}{2}+dx\right)-1680\sin\left(\frac{3c}{2}+dx\right)+560\sin\left(\frac{5c}{2}+3dx\right)-560\sin\left(\frac{7c}{2}+3dx\right)+280\sin\left(\frac{7c}{2}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (1680*(c - d*x)*Cos[c/2] - 1680*Cos[c/2 + d*x] - 1680*Cos[(3*c)/2 + d*x] -
560*Cos[(5*c)/2 + 3*d*x] - 560*Cos[(7*c)/2 + 3*d*x] + 280*Cos[(7*c)/2 + 4*d
*x] - 280*Cos[(9*c)/2 + 4*d*x] + 112*Cos[(9*c)/2 + 5*d*x] + 112*Cos[(11*c)/
2 + 5*d*x] + 80*Cos[(13*c)/2 + 7*d*x] + 80*Cos[(15*c)/2 + 7*d*x] - 35*Cos[(
15*c)/2 + 8*d*x] + 35*Cos[(17*c)/2 + 8*d*x] - 3360*Sin[c/2] + 1680*c*Sin[c/
2] - 1680*d*x*Sin[c/2] + 1680*Sin[c/2 + d*x] - 1680*Sin[(3*c)/2 + d*x] + 56
0*Sin[(5*c)/2 + 3*d*x] - 560*Sin[(7*c)/2 + 3*d*x] + 280*Sin[(7*c)/2 + 4*d*x
] + 280*Sin[(9*c)/2 + 4*d*x] - 112*Sin[(9*c)/2 + 5*d*x] + 112*Sin[(11*c)/2
+ 5*d*x] - 80*Sin[(13*c)/2 + 7*d*x] + 80*Sin[(15*c)/2 + 7*d*x] - 35*Sin[(15
*c)/2 + 8*d*x] - 35*Sin[(17*c)/2 + 8*d*x])/(71680*a*d*(Cos[c/2] + Sin[c/2])
)
```

Maple [B] time = 0.09, size = 483, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -4/35/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8+3/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*ta
n(1/2*d*x+1/2*c)-32/35/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2+
23/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3+4/5/d/a/(1+tan(1/
2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4-333/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^
8*tan(1/2*d*x+1/2*c)^5-32/5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*
c)^6+671/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7-4/d/a/(1+ta
n(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8-671/64/d/a/(1+tan(1/2*d*x+1/2*c)
^2)^8*tan(1/2*d*x+1/2*c)^9+333/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^11-4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12-23/64/d/
a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13-3/64/d/a/(1+tan(1/2*d*x+
1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-3/64/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.58512, size = 622, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2240*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 2048*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1792*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 11655*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14336*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 23485*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 - 8960*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 23485*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9 + 11655*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 8960*sin(d
*x + c)^12/(cos(d*x + c) + 1)^12 - 805*sin(d*x + c)^13/(cos(d*x + c) + 1)^1
3 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 256)/(a + 8*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 + 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12/(cos(
d*x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a*sin(d*x +
c)^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/
a)/d
```

Fricas [A] time = 1.15118, size = 216, normalized size = 1.53

$$\frac{640 \cos(dx+c)^7 - 896 \cos(dx+c)^5 - 105\,dx - 35\left(16 \cos(dx+c)^7 - 24 \cos(dx+c)^5 + 2 \cos(dx+c)^3 + 3 \cos(dx+\right.}{4480\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4480*(640*cos(d*x + c)^7 - 896*cos(d*x + c)^5 - 105*d*x - 35*(16*cos(d*x
+ c)^7 - 24*cos(d*x + c)^5 + 2*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x + c
))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29971, size = 277, normalized size = 1.96

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+805\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+8960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-11655\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+23485\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+8960\,\tan\left(\frac{1}{2}\,dx \cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4480*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 805*tan(1/2*d*x
 + 1/2*c)^13 + 8960*tan(1/2*d*x + 1/2*c)^12 - 11655*tan(1/2*d*x + 1/2*c)^11
 + 23485*tan(1/2*d*x + 1/2*c)^9 + 8960*tan(1/2*d*x + 1/2*c)^8 - 23485*tan(1
/2*d*x + 1/2*c)^7 + 14336*tan(1/2*d*x + 1/2*c)^6 + 11655*tan(1/2*d*x + 1/2*
c)^5 - 1792*tan(1/2*d*x + 1/2*c)^4 - 805*tan(1/2*d*x + 1/2*c)^3 + 2048*tan(
1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 256)/((tan(1/2*d*x + 1/2*c)
^2 + 1)^8*a))/d
```

3.626 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=115

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{\sin(c+dx)\cos(c+dx)}{16ad}+\frac{x}{16a}$$

```
[Out] x/(16*a) + Cos[c + d*x]^5/(5*a*d) - Cos[c + d*x]^7/(7*a*d) + (Cos[c + d*x]*
Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) - (Cos[c +
d*x]^5*Sin[c + d*x])/(6*a*d)
```

Rubi [A] time = 0.177171, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 14}

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{\sin(c+dx)\cos(c+dx)}{16ad}+\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(16*a) + Cos[c + d*x]^5/(5*a*d) - Cos[c + d*x]^7/(7*a*d) + (Cos[c + d*x]*
Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) - (Cos[c +
d*x]^5*Sin[c + d*x])/(6*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{6ad} + \frac{\int \cos^4(c+dx)\,dx}{6a} + \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx, x, \cos(c\right.}{ad}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} + \frac{\int \cos^2(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\cos^5(c+dx)}{5ad} - \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{24ad}$$

$$= \frac{x}{16a} + \frac{\cos^5(c+dx)}{5ad} - \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c}{24ad}$$

Mathematica [B] time = 11.5285, size = 715, normalized size = 6.22

$$\frac{5\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{64d(a\sin(c+dx)+a)} - \frac{-\frac{50\sin(c)\sin(dx)}{d} + \frac{10\sin(3c)\sin(3dx)}{d} - \frac{2\sin(5c)\sin(5dx)}{d} + \frac{50\cos(c)\cos(}{d}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(30*x + (50*Cos[c]*Cos[d*x])/d - (10*Cos[3*c]*Cos[3*d*x])/d + (2*Cos[5*c]*
Cos[5*d*x])/d - (20*Cos[2*d*x]*Sin[2*c])/d + (5*Cos[4*d*x]*Sin[4*c])/d - (5
0*Sin[c]*Sin[d*x])/d - (20*Cos[2*c]*Sin[2*d*x])/d + (10*Sin[3*c]*Sin[3*d*x]
)/d + (5*Cos[4*c]*Sin[4*d*x])/d - (2*Sin[5*c]*Sin[5*d*x])/d - (10*Sin[(d*x)
/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(160*
a) + (x + (Cos[c]*Cos[d*x])/d - (Sin[c]*Sin[d*x])/d - Sin[(d*x)/2]/(d*(Cos[
c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(16*a) - (-6*x - (
9*Cos[c]*Cos[d*x])/d + (Cos[3*c]*Cos[3*d*x])/d + (3*Cos[2*d*x]*Sin[2*c])/d
+ (9*Sin[c]*Sin[d*x])/d + (3*Cos[2*c]*Sin[2*d*x])/d - (Sin[3*c]*Sin[3*d*x])
/d + (3*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])))/(48*a) - (-420*x - (735*Cos[c]*Cos[d*x])/d + (175*Cos[3*c]*Cos[
3*d*x])/d - (63*Cos[5*c]*Cos[5*d*x])/d + (15*Cos[7*c]*Cos[7*d*x])/d + (315*
Cos[2*d*x]*Sin[2*c])/d - (105*Cos[4*d*x]*Sin[4*c])/d + (35*Cos[6*d*x]*Sin[6
*c])/d + (735*Sin[c]*Sin[d*x])/d + (315*Cos[2*c]*Sin[2*d*x])/d - (175*Sin[3
*c]*Sin[3*d*x])/d - (105*Cos[4*c]*Sin[4*d*x])/d + (63*Sin[5*c]*Sin[5*d*x])/
d + (35*Cos[6*c]*Sin[6*d*x])/d - (15*Sin[7*c]*Sin[7*d*x])/d + (105*Sin[(d*x
)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(672
0*a) + (5*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(64*d*(a
+ a*Sin[c + d*x]))
```

Maple [B] time = 0.081, size = 415, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13-11/6/d/a/(1+tan(1/
2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11+4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)^10+31/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)
^9-4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8+8/d/a/(1+tan(1/2*d
*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6-31/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)^5-8/5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4+
11/6/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3+4/5/d/a/(1+tan(1/2
*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2-1/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)+4/35/d/a/(1+tan(1/2*d*x+1/2*c)^2)^7+1/8/a/d*arctan(tan(1/2
*d*x+1/2*c))
```

Maxima [B] time = 1.55497, size = 540, normalized size = 4.7

$$-\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{672\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1540\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1344\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1085\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6720\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{3360\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{1085\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3360\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{1540\,\sin(dx+\cdots}{(\cos(dx+c)+1\cdots}}{a+\frac{7a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 672*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 1085*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6720*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 3360*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 - 1085*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3360*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 + 1540*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105*sin(d*x +
 c)^13/(cos(d*x + c) + 1)^13 - 96)/(a + 7*a*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 21*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*a*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 35*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a*sin(d*x +
 c)^10/(cos(d*x + c) + 1)^10 + 7*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 +
a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a)/d
```

Fricas [A] time = 1.11345, size = 189, normalized size = 1.64

$$-\frac{240\,\cos(dx+c)^7-336\,\cos(dx+c)^5-105\,dx+35\left(8\,\cos(dx+c)^5-2\,\cos(dx+c)^3-3\,\cos(dx+c)\right)\sin(dx+c)}{1680\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/1680*(240*cos(d*x + c)^7 - 336*cos(d*x + c)^5 - 105*d*x + 35*(8*cos(d*x
+ c)^5 - 2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [A] time = 125.808, size = 3048, normalized size = 26.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((210*d*x*tan(c/2 + d*x/2)**14/(3360*a*d*tan(c/2 + d*x/2)**14 + 23
520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*
tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 +
d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 1470*d*x*tan(c/2 +
d*x/2)**12/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12
+ 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*
a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2
 + d*x/2)**2 + 3360*a*d) + 4410*d*x*tan(c/2 + d*x/2)**10/(3360*a*d*tan(c/2
+ d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)*
*10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 705
60*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 73
50*d*x*tan(c/2 + d*x/2)**8/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c
/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x
/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 2
3520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 7350*d*x*tan(c/2 + d*x/2)**6/(33
60*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*ta
n(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 +
d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 +
 3360*a*d) + 4410*d*x*tan(c/2 + d*x/2)**4/(3360*a*d*tan(c/2 + d*x/2)**14 +
23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*
d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2
+ d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 1470*d*x*tan(c/2
+ d*x/2)**2/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12
 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600
*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/
2 + d*x/2)**2 + 3360*a*d) + 210*d*x/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*
a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(
c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/
2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) - 275*tan(c/2 + d*x/2)**1
4/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a
*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c
/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)
**2 + 3360*a*d) + 420*tan(c/2 + d*x/2)**13/(3360*a*d*tan(c/2 + d*x/2)**14 +
 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a
*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2
 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) - 1925*tan(c/2 + d
*x/2)**12/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 +
 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a
*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2
+ d*x/2)**2 + 3360*a*d) - 6160*tan(c/2 + d*x/2)**11/(3360*a*d*tan(c/2 + d*x
/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 +
 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*
d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 7665*ta
n(c/2 + d*x/2)**10/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x
/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 +
 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d
*tan(c/2 + d*x/2)**2 + 3360*a*d) + 4340*tan(c/2 + d*x/2)**9/(3360*a*d*tan(c
/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/
2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 +
70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) -
```

```
 23065*tan(c/2 + d*x/2)**8/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c
/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x
/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 2
3520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 17255*tan(c/2 + d*x/2)**6/(3360*
a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c
/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x
/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 33
60*a*d) - 4340*tan(c/2 + d*x/2)**5/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a
*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c
/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2
)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) - 11151*tan(c/2 + d*x/2)**
4/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a
*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c
/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)
**2 + 3360*a*d) + 6160*tan(c/2 + d*x/2)**3/(3360*a*d*tan(c/2 + d*x/2)**14 +
 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600*a
*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2
 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 763*tan(c/2 + d*
x/2)**2/(3360*a*d*tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 7
0560*a*d*tan(c/2 + d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d
*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 +
d*x/2)**2 + 3360*a*d) - 420*tan(c/2 + d*x/2)/(3360*a*d*tan(c/2 + d*x/2)**14
 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 + d*x/2)**10 + 117600
*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)**6 + 70560*a*d*tan(c
/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a*d) + 109/(3360*a*d*
tan(c/2 + d*x/2)**14 + 23520*a*d*tan(c/2 + d*x/2)**12 + 70560*a*d*tan(c/2 +
 d*x/2)**10 + 117600*a*d*tan(c/2 + d*x/2)**8 + 117600*a*d*tan(c/2 + d*x/2)*
*6 + 70560*a*d*tan(c/2 + d*x/2)**4 + 23520*a*d*tan(c/2 + d*x/2)**2 + 3360*a
*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**6/(a*sin(c) + a), True))
```

Giac [A] time = 1.24993, size = 242, normalized size = 2.1

$$\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-1540\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3360\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1085\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3360\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+6720\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{7}a}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1680*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^13 - 1540*tan(1/2*d*x
 + 1/2*c)^11 + 3360*tan(1/2*d*x + 1/2*c)^10 + 1085*tan(1/2*d*x + 1/2*c)^9 -
 3360*tan(1/2*d*x + 1/2*c)^8 + 6720*tan(1/2*d*x + 1/2*c)^6 - 1085*tan(1/2*d
*x + 1/2*c)^5 - 1344*tan(1/2*d*x + 1/2*c)^4 + 1540*tan(1/2*d*x + 1/2*c)^3 +
 672*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 96)/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^7*a))/d
```

3.627 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

```
[Out] -x/(16*a) - Cos[c + d*x]^5/(5*a*d) - (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) -
 (Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(6*
a*d)
```

Rubi [A] time = 0.126162, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2565, 30, 2568, 2635, 8}

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -x/(16*a) - Cos[c + d*x]^5/(5*a*d) - (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) -
 (Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(6*
a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^4(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a}$$
$$= \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \frac{\int \cos^4(c+dx)\,dx}{6a} - \frac{\mathrm{Subst}\left(\int x^4\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= -\frac{\cos^5(c+dx)}{5ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} + \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\,}{8a}$$
$$= -\frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} + \frac{\cos^5(c+dx)\sin(}{6ad}$$
$$= -\frac{x}{16a} - \frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} + \cos^5(c+$$

Mathematica [B] time = 5.19195, size = 377, normalized size = 3.89

$$-\frac{120dx\sin\left(\frac{c}{2}\right) - 120\sin\left(\frac{c}{2}+dx\right) + 120\sin\left(\frac{3c}{2}+dx\right) + 15\sin\left(\frac{3c}{2}+2dx\right) + 15\sin\left(\frac{5c}{2}+2dx\right) - 60\sin\left(\frac{5c}{2}+3dx\right) + 60\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(-30*(5*c - 4*d*x)*Cos[c/2] + 120*Cos[c/2 + d*x] + 120*Cos[(3*c)/2 + d*x]
+ 15*Cos[(3*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 2*d*x] + 60*Cos[(5*c)/2 + 3*d*
x] + 60*Cos[(7*c)/2 + 3*d*x] - 15*Cos[(7*c)/2 + 4*d*x] + 15*Cos[(9*c)/2 + 4
*d*x] + 12*Cos[(9*c)/2 + 5*d*x] + 12*Cos[(11*c)/2 + 5*d*x] - 5*Cos[(11*c)/2
 + 6*d*x] + 5*Cos[(13*c)/2 + 6*d*x] + 300*Sin[c/2] - 150*c*Sin[c/2] + 120*d
*x*Sin[c/2] - 120*Sin[c/2 + d*x] + 120*Sin[(3*c)/2 + d*x] + 15*Sin[(3*c)/2
+ 2*d*x] + 15*Sin[(5*c)/2 + 2*d*x] - 60*Sin[(5*c)/2 + 3*d*x] + 60*Sin[(7*c)
/2 + 3*d*x] - 15*Sin[(7*c)/2 + 4*d*x] - 15*Sin[(9*c)/2 + 4*d*x] - 12*Sin[(9
*c)/2 + 5*d*x] + 12*Sin[(11*c)/2 + 5*d*x] - 5*Sin[(11*c)/2 + 6*d*x] - 5*Sin
[(13*c)/2 + 6*d*x])/(1920*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.069, size = 415, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-2/d/a/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10+47/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*
```

```
tan(1/2*d*x+1/2*c)^9-2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8-
13/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7-4/d/a/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6+13/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)^5-4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4-47/
24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3-2/5/d/a/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2+1/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(
1/2*d*x+1/2*c)-2/5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6-1/8/a/d*arctan(tan(1/2*d*
x+1/2*c))
```

Maxima [B] time = 1.53122, size = 512, normalized size = 5.28

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{235\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{480\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{390\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{480\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{240\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{235\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{240\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 235*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 480*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 480*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 390*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 240*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 235*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 - 240*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 15*sin(d*x + c)^11/(cos(d*
x + c) + 1)^11 - 48)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/(cos(d*x
 + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.03995, size = 155, normalized size = 1.6

$$-\frac{48\cos\left(dx+c\right)^5+15\,dx-5\left(8\cos\left(dx+c\right)^5-2\cos\left(dx+c\right)^3-3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(48*cos(d*x + c)^5 + 15*d*x - 5*(8*cos(d*x + c)^5 - 2*cos(d*x + c)^3
 - 3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [A] time = 76.3752, size = 2428, normalized size = 25.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**12/(1680*a*d*tan(c/2 + d*x/2)**12 + 1
0080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*t
an(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*
x/2)**2 + 1680*a*d) - 630*d*x*tan(c/2 + d*x/2)**10/(1680*a*d*tan(c/2 + d*x/
2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 3
3600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*ta
n(c/2 + d*x/2)**2 + 1680*a*d) - 1575*d*x*tan(c/2 + d*x/2)**8/(1680*a*d*tan(
c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x
/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10
080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 2100*d*x*tan(c/2 + d*x/2)**6/(168
0*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan
(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/
2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 1575*d*x*tan(c/2 + d*x/
2)**4/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 252
00*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(
c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 630*d*x*tan(c
/2 + d*x/2)**2/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)*
*10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200
*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 105*
d*x/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200
*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/
2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 510*tan(c/2 + d
*x/2)**12/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 +
 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*
tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 210*tan(c
/2 + d*x/2)**11/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)
**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 2520
0*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 300
*tan(c/2 + d*x/2)**10/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 +
d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6
+ 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d)
 + 3290*tan(c/2 + d*x/2)**9/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(
c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/
2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 168
0*a*d) + 4290*tan(c/2 + d*x/2)**8/(1680*a*d*tan(c/2 + d*x/2)**12 + 10080*a*
d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2
+ d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2
 + 1680*a*d) - 5460*tan(c/2 + d*x/2)**7/(1680*a*d*tan(c/2 + d*x/2)**12 + 10
080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*a*d*ta
n(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2 + d*x
/2)**2 + 1680*a*d) + 3480*tan(c/2 + d*x/2)**6/(1680*a*d*tan(c/2 + d*x/2)**1
2 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 + 33600*
a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*tan(c/2
 + d*x/2)**2 + 1680*a*d) + 5460*tan(c/2 + d*x/2)**5/(1680*a*d*tan(c/2 + d*x
/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**8 +
33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a*d*t
an(c/2 + d*x/2)**2 + 1680*a*d) + 930*tan(c/2 + d*x/2)**4/(1680*a*d*tan(c/2
+ d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)*
*8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*
a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 3290*tan(c/2 + d*x/2)**3/(1680*a*d*ta
n(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d
*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 +
10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 2388*tan(c/2 + d*x/2)**2/(1680*
a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c
/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)
**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 210*tan(c/2 + d*x/2)/(168
0*a*d*tan(c/2 + d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan
(c/2 + d*x/2)**8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/
2)**4 + 10080*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 162/(1680*a*d*tan(c/2 +
```

```
d*x/2)**12 + 10080*a*d*tan(c/2 + d*x/2)**10 + 25200*a*d*tan(c/2 + d*x/2)**
8 + 33600*a*d*tan(c/2 + d*x/2)**6 + 25200*a*d*tan(c/2 + d*x/2)**4 + 10080*a
*d*tan(c/2 + d*x/2)**2 + 1680*a*d), Ne(d, 0)), (x*sin(c)*cos(c)**6/(a*sin(c
) + a), True))
```

Giac [B] time = 1.30425, size = 242, normalized size = 2.49

$$-\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-235\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-39}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 240*tan(1/2*d*x +
1/2*c)^10 - 235*tan(1/2*d*x + 1/2*c)^9 + 240*tan(1/2*d*x + 1/2*c)^8 + 390*t
an(1/2*d*x + 1/2*c)^7 + 480*tan(1/2*d*x + 1/2*c)^6 - 390*tan(1/2*d*x + 1/2*
c)^5 + 480*tan(1/2*d*x + 1/2*c)^4 + 235*tan(1/2*d*x + 1/2*c)^3 + 48*tan(1/2
*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 48)/((tan(1/2*d*x + 1/2*c)^2 +
1)^6*a))/d
```

3.628 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{8a}$$

```
[Out] (-3*x)/(8*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) + Cos[c + d
*x]^3/(3*a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin
[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.123102, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2592, 302, 206, 2635, 8}

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(8*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) + Cos[c + d
*x]^3/(3*a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin
[c + d*x])/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^4(c+dx)\,dx}{a} + \frac{\int \cos^3(c+dx)\cot(c+dx)\,dx}{a}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{3\int \cos^2(c+dx)\,dx}{4a} - \frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad}$$

$$= -\frac{3\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{3\int 1\,dx}{8a} - \frac{\text{Subst}\left(\int \left(-1-\right.\right.}{}$$

$$= -\frac{3x}{8a} + \frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} - \frac{3\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)\sin(c}{4ad}$$

$$= -\frac{3x}{8a} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} - \frac{3\cos(c+dx)\sin(c+d}{8ad}$$

Mathematica [A] time = 0.375653, size = 86, normalized size = 0.85

$$\frac{120\cos(c+dx)+8\cos(3(c+dx))-3\left(8\sin(2(c+dx))+\sin(4(c+dx))+4\left(-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8\log\left(\cos\left(\frac{1}{2}\right.\right.\right.\right.}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (120*Cos[c + d*x] + 8*Cos[3*(c + d*x)] - 3*(4*(3*c + 3*d*x + 8*Log[Cos[(c +
 d*x)/2]] - 8*Log[Sin[(c + d*x)/2]]) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)
]))/(96*a*d)
```

Maple [B] time = 0.11, size = 296, normalized size = 2.9

$$\frac{5}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}+4\,\frac{(\tan(1/2\,dx+c/2))^6}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^4}-\frac{3}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+4/d/a/(1+tan(1/2*d*
x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6-3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)^5+8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4+3/4/d
/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+20/3/d/a/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2-5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)+8/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4-3/4/a/d*arctan(tan(1/2*d*x+1
/2*c))+1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.49523, size = 378, normalized size = 3.74

$$-\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{96\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{48\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-32}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right.}{a}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 80*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 9*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 96*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 9*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 48*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 32)/(a + 4*
a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x
+ c) + 1)^8) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 12*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.1497, size = 239, normalized size = 2.37

$$\frac{8\cos(dx+c)^3-9dx-3\left(2\cos(dx+c)^3+3\cos(dx+c)\right)\sin(dx+c)+24\cos(dx+c)-12\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{24ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*cos(d*x + c)^3 - 9*d*x - 3*(2*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(
d*x + c) + 24*cos(d*x + c) - 12*log(1/2*cos(d*x + c) + 1/2) + 12*log(-1/2*c
os(d*x + c) + 1/2))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30466, size = 193, normalized size = 1.91

$$-\frac{\frac{9(dx+c)}{a}-\frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+48\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+96\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+80\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4a}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(9*(d*x + c)/a - 24*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(15*tan(1/2*
d*x + 1/2*c)^7 + 48*tan(1/2*d*x + 1/2*c)^6 - 9*tan(1/2*d*x + 1/2*c)^5 + 96*
tan(1/2*d*x + 1/2*c)^4 + 9*tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)
^2 - 15*tan(1/2*d*x + 1/2*c) + 32)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.629 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{3\cot(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{2a}$$

```
[Out] (-3*x)/(2*a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cos[c + d
*x]^3/(3*a*d) - (3*Cot[c + d*x])/(2*a*d) + (Cos[c + d*x]^2*Cot[c + d*x])/(2
*a*d)
```

Rubi [A] time = 0.156155, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2839, 2591, 288, 321, 203, 2592, 302, 206}

$$-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{3\cot(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-3*x)/(2*a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cos[c + d
*x]^3/(3*a*d) - (3*Cot[c + d*x])/(2*a*d) + (Cos[c + d*x]^2*Cot[c + d*x])/(2
*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^3(c+dx)\cot(c+dx)\,dx}{a} + \frac{\int \cos^2(c+dx)\cot^2(c+dx)\,dx}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= \frac{\cos^2(c+dx)\cot(c+dx)}{2ad} + \frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cos(c+dx)\right)}{ad} - \frac{3\,\text{Sub}}{}$$

$$= -\frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{3\cot(c+dx)}{2ad} + \frac{\cos^2(c+dx)\cot(c+dx)}{2ad} + \frac{\text{Subst}}{}$$

$$= -\frac{3x}{2a} + \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{3\cot(c+dx)}{2ad} + \frac{\cos^2(}{}$$

Mathematica [A] time = 0.780923, size = 122, normalized size = 1.28

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^2\left(27\cos(c+dx)+(2\sin(c+dx)-3)\cos(3(c+dx))+6\sin(c+dx)\left(5\cos(c+\right.\right.}{48ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((1 + Cot[(c + d*x)/2])^2*(27*Cos[c + d*x] + 6*(6*c + 6*d*x + 5*Cos[c + d*
x] - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]])*Sin[c + d*x] + Cos[
3*(c + d*x)]*(-3 + 2*Sin[c + d*x]))*Tan[(c + d*x)/2])/(48*a*d*(1 + Sin[c +
```

```
d*x]))
```

Maple [B] time = 0.119, size = 230, normalized size = 2.4

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-3}-4\frac{(\tan(1/2\,dx+c/2))^4}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}-4\frac{(\tan(1/2}{da\left(1+(\tan(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2
*c)^5-4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-4/d/a/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan
(1/2*d*x+1/2*c)-8/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3-3/a/d*arctan(tan(1/2*d*x
+1/2*c))-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.54743, size = 374, normalized size = 3.94

$$-\frac{\frac{\frac{16\sin(dx+c)}{\cos(dx+c)+1}+\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{24\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+3}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{18\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{3\sin(}{a(\cos(d}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((16*sin(d*x + c)/(cos(d*x + c) + 1) + 15*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 24*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 3)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7) + 18*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a + 6*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3*sin(d*x + c)/(a*(cos(
d*x + c) + 1)))/d
```

Fricas [A] time = 1.1328, size = 284, normalized size = 2.99

$$\frac{3\cos(dx+c)^3-\left(2\cos(dx+c)^3+9\,dx+6\cos(dx+c)\right)\sin(dx+c)+3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3\log\left(\right.}{6\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(d*x + c)^3 - (2*cos(d*x + c)^3 + 9*d*x + 6*cos(d*x + c))*sin(d*x
 + c) + 3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*log(-1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) - 9*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34769, size = 198, normalized size = 2.08

$$-\frac{\frac{9\,(dx+c)}{a}+\frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{3\left(2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)/a + 6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*tan(1/2*d*x +
1/2*c)/a - 3*(2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)) - 2*(3*t
an(1/2*d*x + 1/2*c)^5 - 12*tan(1/2*d*x + 1/2*c)^4 - 12*tan(1/2*d*x + 1/2*c)
^2 - 3*tan(1/2*d*x + 1/2*c) - 8)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

3.630 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{3\cos(c+dx)}{2ad}+\frac{3\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3x}{2a}$$

```
[Out] (3*x)/(2*a) + (3*ArcTanh[Cos[c + d*x]])/(2*a*d) - (3*Cos[c + d*x])/(2*a*d)
+ (3*Cot[c + d*x])/(2*a*d) - (Cos[c + d*x]^2*Cot[c + d*x])/(2*a*d) - (Cos[c
 + d*x]*Cot[c + d*x]^2)/(2*a*d)
```

Rubi [A] time = 0.161383, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2839, 2592, 288, 321, 206, 2591, 203}

$$-\frac{3\cos(c+dx)}{2ad}+\frac{3\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (3*x)/(2*a) + (3*ArcTanh[Cos[c + d*x]])/(2*a*d) - (3*Cos[c + d*x])/(2*a*d)
+ (3*Cot[c + d*x])/(2*a*d) - (Cos[c + d*x]^2*Cot[c + d*x])/(2*a*d) - (Cos[c
 + d*x]*Cot[c + d*x]^2)/(2*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^2(c+dx)\cot^2(c+dx)\,dx}{a} + \frac{\int \cos(c+dx)\cot^3(c+dx)\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)}{2ad} - \frac{\cos(c+dx)\cot^2(c+dx)}{2ad} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos\right.}{2ad}$$

$$= -\frac{3\cos(c+dx)}{2ad} + \frac{3\cot(c+dx)}{2ad} - \frac{\cos^2(c+dx)\cot(c+dx)}{2ad} - \frac{\cos(c+dx)\cot^2(c+}{2ad}$$

$$= \frac{3x}{2a} + \frac{3\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{3\cos(c+dx)}{2ad} + \frac{3\cot(c+dx)}{2ad} - \frac{\cos^2(c+dx)\cot(c}{2ad}$$

Mathematica [A] time = 0.491853, size = 152, normalized size = 1.43

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-10\sin(2(c+dx))+\sin(4(c+dx))+12\cos(c+dx)-4\cos(3(c+dx))+12\,\mathrm{lo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(-12*c - 12*d*x + 12*Cos[c + d*x]
 - 4*Cos[3*(c + d*x)] - 12*Log[Cos[(c + d*x)/2]] + 12*Cos[2*(c + d*x)]*(c +
 d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 12*Log[Sin[(c + d*x
)/2]] - 10*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(64*a*d*(1 + Sin[c + d*x])
)
```

Maple [B] time = 0.126, size = 234, normalized size = 2.2

$$\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{1}{2\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}-2\,\frac{(\tan(1/2\,dx+c/2)}{da\left(1+(\tan(1/2\,dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/2/d/a*tan(1/2*d*x+1/2*c)-1/d/a/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2
*d*x+1/2*c)^2+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-2/d/a/(1+
tan(1/2*d*x+1/2*c)^2)^2+3/a/d*arctan(tan(1/2*d*x+1/2*c))-1/8/d/a/tan(1/2*d*
x+1/2*c)^2+1/2/d/a/tan(1/2*d*x+1/2*c)-3/2/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.53292, size = 352, normalized size = 3.32

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}}{a}-\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{18\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{16\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{17\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{4\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-1}{\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right.}{a}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - (4*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 17*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 2*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6) - 24*arctan(sin(d*x + c)/(cos(d*x + c) +
1))/a + 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.16939, size = 347, normalized size = 3.27

$$\frac{6\,dx\cos(dx+c)^{2}-4\cos(dx+c)^{3}-6\,dx+3\left(\cos(dx+c)^{2}-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+c)^{2}-1\right)\log\left(-\frac{1}{2}\right.}{4\left(ad\cos(dx+c)^{2}-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(6*d*x*cos(d*x + c)^2 - 4*cos(d*x + c)^3 - 6*d*x + 3*(cos(d*x + c)^2 -
1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x +
c) + 1/2) + 2*(cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c) + 6*cos(d*x +
c))/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37917, size = 225, normalized size = 2.12

$$\frac{\frac{12\,(dx+c)}{a}-\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2 a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(12*(d*x + c)/a - 12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x
+ 1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c))/a^2 + (6*tan(1/2*d*x + 1/2*c)^6 - 4*
tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c)^4 + 16*tan(1/2*d*x + 1/2*c)
^3 - 12*tan(1/2*d*x + 1/2*c)^2 + 4*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x
+ 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2*a))/d
```

3.631 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=94

$$\frac{3\cos(c+dx)}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{x}{a}$$

```
[Out] x/a - (3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (3*Cos[c + d*x])/(2*a*d) + Cot[c
+ d*x]/(a*d) + (Cos[c + d*x]*Cot[c + d*x]^2)/(2*a*d) - Cot[c + d*x]^3/(3*a*
d)
```

Rubi [A] time = 0.137807, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2839, 3473, 8, 2592, 288, 321, 206}

$$\frac{3\cos(c+dx)}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a - (3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (3*Cos[c + d*x])/(2*a*d) + Cot[c
+ d*x]/(a*d) + (Cos[c + d*x]*Cot[c + d*x]^2)/(2*a*d) - Cot[c + d*x]^3/(3*a*
d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos(c+dx)\cot^3(c+dx)\,dx}{a}+\frac{\int \cot^4(c+dx)\,dx}{a}\\
&= -\frac{\cot^3(c+dx)}{3ad}-\frac{\int \cot^2(c+dx)\,dx}{a}+\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\cos(c+dx)\right)}{ad}\\
&= \frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\int 1\,dx}{a}-\frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\right.}{}\\
&= \frac{x}{a}+\frac{3\cos(c+dx)}{2ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{3\,\text{Su}}{}\\
&= \frac{x}{a}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3\cos(c+dx)}{2ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+}{2ad}
\end{aligned}$$

Mathematica [A] time = 0.915895, size = 138, normalized size = 1.47

$$\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(9\sin(2(c+dx))-2(3\sin(c+dx)+4)\cos(3(c+\right.}{192ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*
(12*(2*c + 2*d*x - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]])*Sin[c
 + d*x]^3 - 2*Cos[3*(c + d*x)]*(4 + 3*Sin[c + d*x]) + 9*Sin[2*(c + d*x)]))/
(192*a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.139, size = 173, normalized size = 1.8

$$\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{5}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{1}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)}+2\,\frac{\arctan(\text{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)

[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a*tan(1/2*d*x+1/2*c)^2-5/8/d/a*tan(1/2*d*x+1/2*c)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)+2/a/d*arctan(tan(1/2*d*x+1/2*c))-1/24/d/a/tan(1/2*d*x+1/2*c)^3+1/8/d/a/tan(1/2*d*x+1/2*c)^2+5/8/d/a/tan(1/2*d*x+1/2*c)+3/2/d/a*ln(tan(1/2*d*x+1/2*c))

Maxima [B] time = 1.55541, size = 324, normalized size = 3.45

$$-\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{51\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1}{\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}-\frac{48\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{36\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/24*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 51*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)/(a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) - 48*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 36*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d

---

Fricas [A] time = 1.26067, size = 409, normalized size = 4.35

$$\frac{16\cos(dx+c)^3-9\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+9\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+6\left(2dx\cos(dx+c)^2+2\cos(dx+c)^3-2dx-3\cos(dx+c)\right)\sin(dx+c)-12\cos(dx+c)}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/12*(16*cos(d*x + c)^3 - 9*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 9*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(2*d*x*cos(d*x + c)^2 + 2*cos(d*x + c)^3 - 2*d*x - 3*cos(d*x + c))*sin(d*x + c) - 12*cos(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [A] time = 1.32675, size = 212, normalized size = 2.26

$$\frac{\frac{24\,(dx+c)}{a}+\frac{36\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{48}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}-\frac{66\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)/a + 36*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d
*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c))
/a^3 + 48/((tan(1/2*d*x + 1/2*c)^2 + 1)*a) - (66*tan(1/2*d*x + 1/2*c)^3 - 1
5*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a*tan(1/2*d*x + 1/2
*c)^3))/d
```

3.632 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=102

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{x}{a}$$

```
[Out] -(x/a) - (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]/(a*d) + Cot[c + d
*x]^3/(3*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]^3*Csc
[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.133348, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2611, 3770, 3473, 8}

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]/(a*d) + Cot[c + d
*x]^3/(3*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]^3*Csc
[c + d*x])/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^4(c+dx)\,dx}{a} + \frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a}$$
$$= \frac{\cot^3(c+dx)}{3ad} - \frac{\cot^3(c+dx)\csc(c+dx)}{4ad} - \frac{3\int \cot^2(c+dx)\csc(c+dx)\,dx}{4a} + \frac{\int \cot^2$$
$$= -\frac{\cot(c+dx)}{ad} + \frac{\cot^3(c+dx)}{3ad} + \frac{3\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot^3(c+dx)\csc(c+dx}{4ad}$$
$$= -\frac{x}{a} - \frac{3\tanh^{-1}(\cos(c+dx))}{8ad} - \frac{\cot(c+dx)}{ad} + \frac{\cot^3(c+dx)}{3ad} + \frac{3\cot(c+dx)\csc(c+}{8ad}$$

Mathematica [B] time = 0.659336, size = 232, normalized size = 2.27

$$-\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(32\sin(2(c+dx))-32\sin(4(c+dx))+24c\cos(4(c+dx))+18\,\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(72*c + 72*d*x + 1
8*Cos[c + d*x] + 30*Cos[3*(c + d*x)] + 24*c*Cos[4*(c + d*x)] + 24*d*x*Cos[4
*(c + d*x)] + 27*Log[Cos[(c + d*x)/2]] + 9*Cos[4*(c + d*x)]*Log[Cos[(c + d*
x)/2]] - 12*Cos[2*(c + d*x)]*(8*c + 8*d*x + 3*Log[Cos[(c + d*x)/2]] - 3*Log
[Sin[(c + d*x)/2]]) - 27*Log[Sin[(c + d*x)/2]] - 9*Cos[4*(c + d*x)]*Log[Sin
[(c + d*x)/2]] + 32*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(192*a*d*(1 +
Sin[c + d*x]))
```

Maple [A] time = 0.138, size = 188, normalized size = 1.8

$$\frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{5}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\arctan(\tan(1/2}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a*tan(1/2
*d*x+1/2*c)^2+5/8/d/a*tan(1/2*d*x+1/2*c)-2/a/d*arctan(tan(1/2*d*x+1/2*c))-1
/64/d/a/tan(1/2*d*x+1/2*c)^4+1/24/d/a/tan(1/2*d*x+1/2*c)^3+1/8/d/a/tan(1/2*
d*x+1/2*c)^2-5/8/d/a/tan(1/2*d*x+1/2*c)+3/8/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.52618, size = 293, normalized size = 2.87

$$\frac{\frac{\frac{120\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a}-\frac{384\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{72\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}+\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{120\sin}{(\cos(dx}\right.}{a\sin(dx+c)}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/192*((120*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a - 384*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 72*log(
sin(d*x + c)/(cos(d*x + c) + 1))/a + (8*sin(d*x + c)/(cos(d*x + c) + 1) + 2
4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 - 3)*(cos(d*x + c) + 1)^4/(a*sin(d*x + c)^4))/d
```

Fricas [A] time = 1.13399, size = 474, normalized size = 4.65

$$-\frac{48\,dx\cos\left(dx+c\right)^4-96\,dx\cos\left(dx+c\right)^2+30\,\cos\left(dx+c\right)^3+48\,dx+9\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1\right)\log\left(\frac{1}{2}\,\cos \cdots\right.}{48\left(ad\,\cos \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(48*d*x*cos(d*x + c)^4 - 96*d*x*cos(d*x + c)^2 + 30*cos(d*x + c)^3 +
48*d*x + 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1
/2) - 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2
) - 16*(4*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c) - 18*cos(d*x + c))/
(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37655, size = 225, normalized size = 2.21

$$-\frac{\frac{192\,(dx+c)}{a}-\frac{72\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}+\frac{150\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 \cdots}{\cdots}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(192*(d*x + c)/a - 72*log(abs(tan(1/2*d*x + 1/2*c)))/a - (3*a^3*tan(
1/2*d*x + 1/2*c)^4 - 8*a^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/
2*c)^2 + 120*a^3*tan(1/2*d*x + 1/2*c))/a^4 + (150*tan(1/2*d*x + 1/2*c)^4 +
120*tan(1/2*d*x + 1/2*c)^3 - 24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/
2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^4))/d
```

3.633 $\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x
]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.110803, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x
]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a} \\
&= \frac{\cot^3(c+dx)\csc(c+dx)}{4ad}+\frac{3\int \cot^2(c+dx)\csc(c+dx)\,dx}{4a}+\frac{\text{Subst}\left(\int x^4\,dx,x,-\cot(c+dx)\right)}{ad} \\
&= -\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\int \csc(c+dx)\,dx}{8a} \\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}
\end{aligned}
$$

Mathematica [B] time = 0.757315, size = 189, normalized size = 2.3

$$
-\frac{\csc^5(c+dx)\left(20\sin(2(c+dx))-50\sin(4(c+dx))+80\cos(c+dx)+40\cos(3(c+dx))+8\cos(5(c+dx))+150\sin(c\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^5*(80*Cos[c + d*x] + 40*Cos[3*(c + d*x)] + 8*Cos[5*(c + d*x)
] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d*x)/2]]*Sin[
c + d*x] + 20*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)]
- 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 50*Sin[4*(c + d*x)] - 15*Log[
Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*
x)]))/(640*a*d)
```

Maple [B] time = 0.145, size = 208, normalized size = 2.5

$$
\frac{1}{160\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{32\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{1}{16\,da}\tan\left(\frac{dx}{2}+\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/32/d/a*tan(1
/2*d*x+1/2*c)^3+1/8/d/a*tan(1/2*d*x+1/2*c)^2+1/16/d/a*tan(1/2*d*x+1/2*c)-1/
16/d/a/tan(1/2*d*x+1/2*c)-1/160/d/a/tan(1/2*d*x+1/2*c)^5+1/64/d/a/tan(1/2*d
*x+1/2*c)^4-3/8/d/a*ln(tan(1/2*d*x+1/2*c))+1/32/d/a/tan(1/2*d*x+1/2*c)^3-1/
8/d/a/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.01695, size = 316, normalized size = 3.85

$$
\frac{\frac{\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{20\sin(dx}{(\cos(dx+c)}\right.}{a\sin(dx+c)^5}}{320\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/320*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 120*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a + (5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2)*(cos(d*x + c) + 1)^5/(a*sin(d*x
+ c)^5))/d
```

Fricas [B] time = 1.09888, size = 431, normalized size = 5.26

$$-\frac{16 \cos(dx+c)^5 - 15\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 15\left(\cos(dx+c)^4 - \cdots\right.}{80\left(ad\cos(dx+c)^4 - 2ad\cos(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*cos(d*x + c)^5 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(5*cos(d*x + c)^3 - 3*
cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a
*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33802, size = 252, normalized size = 3.07

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{2a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 5a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 10a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 40a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 20a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^5} - \frac{274\tan\cdots}{\cdots}}{320d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/320*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (2*a^4*tan(1/2*d*x + 1/2*c)^
5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 - 10*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^4*t
an(1/2*d*x + 1/2*c)^2 + 20*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*d*x
 + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x + 1/2*c)^3 + 10*ta
n(1/2*d*x + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 2)/(a*tan(1/2*d*x + 1/2*c)^
5))/d
```

3.634 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=135

$$-\frac{\cos^7(c+dx)}{7a^2d}+\frac{3\cos^5(c+dx)}{5a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}$$

```
[Out] -x/(8*a^2) - (2*Cos[c + d*x]^3)/(3*a^2*d) + (3*Cos[c + d*x]^5)/(5*a^2*d) -
Cos[c + d*x]^7/(7*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c +
 d*x]^3*Sin[c + d*x])/(4*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.347352, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{\cos^7(c+dx)}{7a^2d}+\frac{3\cos^5(c+dx)}{5a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(8*a^2) - (2*Cos[c + d*x]^3)/(3*a^2*d) + (3*Cos[c + d*x]^5)/(5*a^2*d) -
Cos[c + d*x]^7/(7*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c +
 d*x]^3*Sin[c + d*x])/(4*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x]^3)/(3*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^2(c+dx)\sin^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cos^2(c+dx)\sin^3(c+dx) - 2a^2\cos^2(c+dx)\sin^4(c+dx) + a^2\cos^2(c+dx)\sin\right.}{a^4}$$
$$= \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a^2} + \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a^2} - \frac{2\int \cos^2(c+dx)}{a^2}$$
$$= \frac{\cos^3(c+dx)\sin^3(c+dx)}{3a^2d} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a^2} - \frac{\mathrm{Subst}\left(\int x^2\left(1-x^2\right)\right.}{a^2d}$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{3a^2d} - \frac{\int \cos^2(c+dx)\,dx}{4a^2} - \mathrm{Subst}$$
$$= -\frac{2\cos^3(c+dx)}{3a^2d} + \frac{3\cos^5(c+dx)}{5a^2d} - \frac{\cos^7(c+dx)}{7a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{8a^2d} + \cos^3$$
$$= -\frac{x}{8a^2} - \frac{2\cos^3(c+dx)}{3a^2d} + \frac{3\cos^5(c+dx)}{5a^2d} - \frac{\cos^7(c+dx)}{7a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{8a^2d}$$

Mathematica [B] time = 3.0872, size = 418, normalized size = 3.1

$$-\frac{1680dx\sin\left(\frac{c}{2}\right) - 1365\sin\left(\frac{c}{2}+dx\right) + 1365\sin\left(\frac{3c}{2}+dx\right) - 210\sin\left(\frac{3c}{2}+2dx\right) - 210\sin\left(\frac{5c}{2}+2dx\right) - 175\sin\left(\frac{5c}{2}+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(210*(1 + 8*d*x)*Cos[c/2] + 1365*Cos[c/2 + d*x] + 1365*Cos[(3*c)/2 + d*x]
- 210*Cos[(3*c)/2 + 2*d*x] + 210*Cos[(5*c)/2 + 2*d*x] + 175*Cos[(5*c)/2 + 3
*d*x] + 175*Cos[(7*c)/2 + 3*d*x] - 210*Cos[(7*c)/2 + 4*d*x] + 210*Cos[(9*c)
/2 + 4*d*x] - 147*Cos[(9*c)/2 + 5*d*x] - 147*Cos[(11*c)/2 + 5*d*x] + 70*Cos
```

```
[(11*c)/2 + 6*d*x] - 70*Cos[(13*c)/2 + 6*d*x] + 15*Cos[(13*c)/2 + 7*d*x] +
15*Cos[(15*c)/2 + 7*d*x] - 210*Sin[c/2] + 1680*d*x*Sin[c/2] - 1365*Sin[c/2
+ d*x] + 1365*Sin[(3*c)/2 + d*x] - 210*Sin[(3*c)/2 + 2*d*x] - 210*Sin[(5*c)
/2 + 2*d*x] - 175*Sin[(5*c)/2 + 3*d*x] + 175*Sin[(7*c)/2 + 3*d*x] - 210*Sin
[(7*c)/2 + 4*d*x] - 210*Sin[(9*c)/2 + 4*d*x] + 147*Sin[(9*c)/2 + 5*d*x] - 1
47*Sin[(11*c)/2 + 5*d*x] + 70*Sin[(11*c)/2 + 6*d*x] + 70*Sin[(13*c)/2 + 6*d
*x] - 15*Sin[(13*c)/2 + 7*d*x] + 15*Sin[(15*c)/2 + 7*d*x])/(13440*a^2*d*(Co
s[c/2] + Sin[c/2]))
```

Maple [B] time = 0.098, size = 415, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13-5/3/d/a^2/(1+ta
n(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^7*tan(1/2*d*x+1/2*c)^10+97/12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*
x+1/2*c)^9-52/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8+8/3/d
/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6-97/12/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5-24/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7
*tan(1/2*d*x+1/2*c)^4+5/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*
c)^3-44/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2+1/4/d/a^2/
(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)-44/105/d/a^2/(1+tan(1/2*d*x+1
/2*c)^2)^7-1/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.56853, size = 562, normalized size = 4.16

$$\frac{\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1232 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{700 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{2016 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{3395 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{1120 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{7280 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{3395 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{1680 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} - \frac{700 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2 + \frac{7 a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{21 a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{35 a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{35 a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{21 a^2 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{7 a^2 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{a^2 \sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/420*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 1232*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 700*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2016*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 3395*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1120*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 7280*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 3395*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1680*sin(d*x + c)^10/(cos(d*x
 + c) + 1)^10 - 700*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105*sin(d*x + c
)^13/(cos(d*x + c) + 1)^13 - 176)/(a^2 + 7*a^2*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 21*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*a^2*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 35*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a^
2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 7*a^2*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12 + a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.10794, size = 220, normalized size = 1.63

$$-\frac{120\cos(dx+c)^7-504\cos(dx+c)^5+560\cos(dx+c)^3+105\,dx+35\left(8\cos(dx+c)^5-14\cos(dx+c)^3+3\cos\right.}{840\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/840*(120*cos(d*x + c)^7 - 504*cos(d*x + c)^5 + 560*cos(d*x + c)^3 + 105*
d*x + 35*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x +
c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32163, size = 242, normalized size = 1.79

$$-\frac{\frac{105\,(dx+c)}{a^2}+\frac{2\left(105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+700\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-3395\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+7280\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-1120\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right.}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(105*(d*x + c)/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 700*tan(1/2*d*
x + 1/2*c)^11 + 1680*tan(1/2*d*x + 1/2*c)^10 - 3395*tan(1/2*d*x + 1/2*c)^9
+ 7280*tan(1/2*d*x + 1/2*c)^8 - 1120*tan(1/2*d*x + 1/2*c)^6 + 3395*tan(1/2*
d*x + 1/2*c)^5 + 2016*tan(1/2*d*x + 1/2*c)^4 - 700*tan(1/2*d*x + 1/2*c)^3 +
 1232*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 176)/((tan(1/2*d*
x + 1/2*c)^2 + 1)^7*a^2))/d
```

3.635 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{\cos^5(c+dx)}{10a^2d}+\frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d}+\frac{\sin(c+dx)\cos^3(c+dx)}{8a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

```
[Out] (3*x)/(16*a^2) + Cos[c + d*x]^5/(10*a^2*d) + (3*Cos[c + d*x]*Sin[c + d*x])/
(16*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]^3*(a -
 a*Sin[c + d*x])^3)/(6*a^5*d)
```

Rubi [A] time = 0.26827, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2875, 2870, 2669, 2635, 8}

$$\frac{\cos^5(c+dx)}{10a^2d}+\frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d}+\frac{\sin(c+dx)\cos^3(c+dx)}{8a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/(16*a^2) + Cos[c + d*x]^5/(10*a^2*d) + (3*Cos[c + d*x]*Sin[c + d*x])/
(16*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]^3*(a -
 a*Sin[c + d*x])^3)/(6*a^5*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^2(c+dx)\sin^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d} + \frac{\int \cos^4(c+dx)(a-a\sin(c+dx))\,dx}{2a^3}$$
$$= \frac{\cos^5(c+dx)}{10a^2d} + \frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d} + \frac{\int \cos^4(c+dx)\,dx}{2a^2}$$
$$= \frac{\cos^5(c+dx)}{10a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d} + \frac{3\int c}{}$$
$$= \frac{\cos^5(c+dx)}{10a^2d} + \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3(c+dx}{}$$
$$= \frac{3x}{16a^2} + \frac{\cos^5(c+dx)}{10a^2d} + \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3}{}$$

Mathematica [B] time = 1.95445, size = 362, normalized size = 3.48

$$\frac{360dx\sin\left(\frac{c}{2}\right) - 240\sin\left(\frac{c}{2}+dx\right) + 240\sin\left(\frac{3c}{2}+dx\right) - 15\sin\left(\frac{3c}{2}+2dx\right) - 15\sin\left(\frac{5c}{2}+2dx\right) - 40\sin\left(\frac{5c}{2}+3dx\right) + 4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (360*d*x*Cos[c/2] + 240*Cos[c/2 + d*x] + 240*Cos[(3*c)/2 + d*x] - 15*Cos[(3
*c)/2 + 2*d*x] + 15*Cos[(5*c)/2 + 2*d*x] + 40*Cos[(5*c)/2 + 3*d*x] + 40*Cos
[(7*c)/2 + 3*d*x] - 45*Cos[(7*c)/2 + 4*d*x] + 45*Cos[(9*c)/2 + 4*d*x] - 24*
Cos[(9*c)/2 + 5*d*x] - 24*Cos[(11*c)/2 + 5*d*x] + 5*Cos[(11*c)/2 + 6*d*x] -
 5*Cos[(13*c)/2 + 6*d*x] + 50*Sin[c/2] + 360*d*x*Sin[c/2] - 240*Sin[c/2 + d
*x] + 240*Sin[(3*c)/2 + d*x] - 15*Sin[(3*c)/2 + 2*d*x] - 15*Sin[(5*c)/2 + 2
*d*x] - 40*Sin[(5*c)/2 + 3*d*x] + 40*Sin[(7*c)/2 + 3*d*x] - 45*Sin[(7*c)/2
+ 4*d*x] - 45*Sin[(9*c)/2 + 4*d*x] + 24*Sin[(9*c)/2 + 5*d*x] - 24*Sin[(11*c
)/2 + 5*d*x] + 5*Sin[(11*c)/2 + 6*d*x] + 5*Sin[(13*c)/2 + 6*d*x])/(1920*a^2
*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.085, size = 347, normalized size = 3.3

$$\frac{3}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6} - \frac{13}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6} + 8\frac{(\tan}{da^2\left(1+(t\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 3/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-13/24/d/a^2/(1+t
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^8-25/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
```

```
1/2*c)^7+16/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6+25/4/d/
a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+13/24/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+16/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^2-3/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)+8/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6+3/8/d/a^2*arctan(tan(1/2*d*x+1/2*c)
)
```

Maxima [B] time = 1.5361, size = 477, normalized size = 4.59

$$-\frac{\frac{\frac{45 \sin(dx+c)}{\cos(dx+c)+1}-\frac{384 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{65 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{750 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{640 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{750 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{960 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{65 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{45 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-64}{a^2+\frac{6 a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15 a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20 a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15 a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6 a^2 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{45 \arctan(\ldots}{\ldots}}{120\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*((45*sin(d*x + c)/(cos(d*x + c) + 1) - 384*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 65*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 750*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 - 640*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 750*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 960*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 65*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 45*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 - 64)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^2*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 45*arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.06272, size = 188, normalized size = 1.81

$$-\frac{96 \cos(dx+c)^5-160 \cos(dx+c)^3-45 dx-5\left(8 \cos(dx+c)^5-26 \cos(dx+c)^3+9 \cos(dx+c)\right)\sin(dx+c)}{240 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(96*cos(d*x + c)^5 - 160*cos(d*x + c)^3 - 45*d*x - 5*(8*cos(d*x + c)
^5 - 26*cos(d*x + c)^3 + 9*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24641, size = 207, normalized size = 1.99

$$\frac{\frac{45\,(dx+c)}{a^2}+\frac{2\left(45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-65\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-750\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+750\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+65\,\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(45*(d*x + c)/a^2 + 2*(45*tan(1/2*d*x + 1/2*c)^11 - 65*tan(1/2*d*x +
1/2*c)^9 + 960*tan(1/2*d*x + 1/2*c)^8 - 750*tan(1/2*d*x + 1/2*c)^7 + 640*ta
n(1/2*d*x + 1/2*c)^6 + 750*tan(1/2*d*x + 1/2*c)^5 + 65*tan(1/2*d*x + 1/2*c)
^3 + 384*tan(1/2*d*x + 1/2*c)^2 - 45*tan(1/2*d*x + 1/2*c) + 64)/((tan(1/2*d
*x + 1/2*c)^2 + 1)^6*a^2))/d
```

3.636 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$-\frac{2\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{6a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{x}{4a^2}-\frac{\cos^7(c+dx)}{3d(a\sin(c+dx)+a)^2}$$

```
[Out] -x/(4*a^2) - (2*Cos[c + d*x]^5)/(15*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(4
*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(6*a^2*d) - Cos[c + d*x]^7/(3*d*(a
+ a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.126212, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2859, 2682, 2635, 8}

$$-\frac{2\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{6a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{x}{4a^2}-\frac{\cos^7(c+dx)}{3d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(4*a^2) - (2*Cos[c + d*x]^5)/(15*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(4
*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(6*a^2*d) - Cos[c + d*x]^7/(3*d*(a
+ a*Sin[c + d*x])^2)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2} - \frac{2\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}dx}{3a}$$
$$= -\frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2} - \frac{2\int \cos^4(c+dx)\,dx}{3a^2}$$
$$= -\frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d} - \frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2} - \frac{\int \cos^2(c+d}{2a^2}$$
$$= -\frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{4a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d} - \frac{\cos^7(c}{3d(a+a\sin}$$
$$= -\frac{x}{4a^2} - \frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{4a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d} - \frac{c}{3d(a}$$

Mathematica [B] time = 1.24465, size = 262, normalized size = 2.62

$$\frac{-120dx\sin\left(\frac{c}{2}\right)+90\sin\left(\frac{c}{2}+dx\right)-90\sin\left(\frac{3c}{2}+dx\right)+25\sin\left(\frac{5c}{2}+3dx\right)-25\sin\left(\frac{7c}{2}+3dx\right)+15\sin\left(\frac{7c}{2}+4dx\right)+15}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*(5 + 24*d*x)*Cos[c/2] - 90*Cos[c/2 + d*x] - 90*Cos[(3*c)/2 + d*x] - 25*
Cos[(5*c)/2 + 3*d*x] - 25*Cos[(7*c)/2 + 3*d*x] + 15*Cos[(7*c)/2 + 4*d*x] -
15*Cos[(9*c)/2 + 4*d*x] + 3*Cos[(9*c)/2 + 5*d*x] + 3*Cos[(11*c)/2 + 5*d*x]
+ 25*Sin[c/2] - 120*d*x*Sin[c/2] + 90*Sin[c/2 + d*x] - 90*Sin[(3*c)/2 + d*x
] + 25*Sin[(5*c)/2 + 3*d*x] - 25*Sin[(7*c)/2 + 3*d*x] + 15*Sin[(7*c)/2 + 4*
d*x] + 15*Sin[(9*c)/2 + 4*d*x] - 3*Sin[(9*c)/2 + 5*d*x] + 3*Sin[(11*c)/2 +
5*d*x])/(480*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.076, size = 313, normalized size = 3.1

$$-\frac{1}{2\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5} - 2\,\frac{(\tan(1/2\,dx+c/2))^8}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^5} + 3\,\frac{(\tan(1/2\,dx+c/2))^7}{da^2\left(1+(\tan(1/2\,dx+c/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9-2/d/a^2/(1+tan(1
/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8+3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^7-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^
6-4/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4-3/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3-8/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)^2+1/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/
2*c)-14/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5-1/2/d/a^2*arctan(tan(1/2*d*x+1/
2*c))
```

Maxima [B] time = 1.54305, size = 419, normalized size = 4.19

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{90\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{40\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{90\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{60\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-28}{a^2+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 80*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 90*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 40*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 240*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 90*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 60*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15*sin(d*
x + c)^9/(cos(d*x + c) + 1)^9 - 28)/(a^2 + 5*a^2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^2*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^2*
sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 15*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^2)/d
```

Fricas [A] time = 1.07912, size = 155, normalized size = 1.55

$$\frac{12\cos(dx+c)^5-40\cos(dx+c)^3-15\,dx+15\left(2\cos(dx+c)^3-\cos(dx+c)\right)\sin(dx+c)}{60\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*cos(d*x + c)^5 - 40*cos(d*x + c)^3 - 15*d*x + 15*(2*cos(d*x + c)^3
 - cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [A] time = 131.729, size = 1834, normalized size = 18.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**10/(420*a**2*d*tan(c/2 + d*x/2)**10 +
 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a
**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) -
 525*d*x*tan(c/2 + d*x/2)**8/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d
*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/
2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 1050*d*x*ta
n(c/2 + d*x/2)**6/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 +
d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)*
*4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 1050*d*x*tan(c/2 + d*x
/2)**4/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 +
 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a
**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 525*d*x*tan(c/2 + d*x/2)**2/(420*
a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d
```

```
*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/
2 + d*x/2)**2 + 420*a**2*d) - 105*d*x/(420*a**2*d*tan(c/2 + d*x/2)**10 + 21
00*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2
*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) + 12
5*tan(c/2 + d*x/2)**10/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c
/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*
x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 210*tan(c/2 + d*x
/2)**9/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 +
 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a
**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 215*tan(c/2 + d*x/2)**8/(420*a**2
*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan
(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 +
d*x/2)**2 + 420*a**2*d) + 1260*tan(c/2 + d*x/2)**7/(420*a**2*d*tan(c/2 + d*
x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**
6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420
*a**2*d) - 2110*tan(c/2 + d*x/2)**6/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100
*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d
*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) + 690*
tan(c/2 + d*x/2)**4/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2
+ d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2
)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 1260*tan(c/2 + d*x/2
)**3/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4
200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**
2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d) - 495*tan(c/2 + d*x/2)**2/(420*a**2*d
*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c
/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*
x/2)**2 + 420*a**2*d) + 210*tan(c/2 + d*x/2)/(420*a**2*d*tan(c/2 + d*x/2)**
10 + 2100*a**2*d*tan(c/2 + d*x/2)**8 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 42
00*a**2*d*tan(c/2 + d*x/2)**4 + 2100*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*
d) - 267/(420*a**2*d*tan(c/2 + d*x/2)**10 + 2100*a**2*d*tan(c/2 + d*x/2)**8
 + 4200*a**2*d*tan(c/2 + d*x/2)**6 + 4200*a**2*d*tan(c/2 + d*x/2)**4 + 2100
*a**2*d*tan(c/2 + d*x/2)**2 + 420*a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)**6/(
a*sin(c) + a)**2, True))
```

Giac [A] time = 1.42032, size = 189, normalized size = 1.89

$$-\frac{\frac{15\,(dx+c)}{a^2}+\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-90\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+240\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+40\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15*(d*x + c)/a^2 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 60*tan(1/2*d*x + 1
/2*c)^8 - 90*tan(1/2*d*x + 1/2*c)^7 + 240*tan(1/2*d*x + 1/2*c)^6 + 40*tan(1
/2*d*x + 1/2*c)^4 + 90*tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 -
 15*tan(1/2*d*x + 1/2*c) + 28)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^2))/d
```

3.637 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{a^2}$$

```
[Out] -(x/a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a^2*d) - Cos[c + d
*x]^3/(3*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(a^2*d)
```

Rubi [A] time = 0.201429, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2875, 2873, 2635, 8, 2592, 321, 206, 2565, 30}

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] -(x/a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a^2*d) - Cos[c + d
*x]^3/(3*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cos(c+dx)\cot(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(-2a^2\cos^2(c+dx)+a^2\cos(c+dx)\cot(c+dx)+a^2\cos^2(c+dx)\sin(c+dx)\right)dx}{a^4} \\
&= \frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a^2}+\frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a^2}-\frac{2\int \cos^2(c+dx)\,dx}{a^2} \\
&= -\frac{\cos(c+dx)\sin(c+dx)}{a^2d}-\frac{\int 1\,dx}{a^2}-\frac{\text{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{a^2d}-\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\ldots\right.}{\ldots} \\
&= -\frac{x}{a^2}+\frac{\cos(c+dx)}{a^2d}-\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\ldots\right.}{a^2d} \\
&= -\frac{x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}
\end{aligned}$$

Mathematica [A] time = 0.319738, size = 69, normalized size = 0.95

$$-\frac{-9\cos(c+dx)+\cos(3(c+dx))+6\left(\sin(2(c+dx))+2\left(-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+c+dx\right)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(-9*Cos[c + d*x] + Cos[3*(c + d*x)] + 6*(2*(c + d*x + Log[Cos[(c + d*x)/2]
] - Log[Sin[(c + d*x)/2]]) + Sin[2*(c + d*x)]))/(12*a^2*d)
```

Maple [B] time = 0.118, size = 160, normalized size = 2.2

$$2\frac{(\tan(1/2\,dx+c/2))^5}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+4\frac{(\tan(1/2\,dx+c/2))^2}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}-2\frac{\tan(1/2\,dx+c/2)}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+\frac{4}{3\,da^2}\left(1+\left(\mathrm{t}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+4/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan
(1/2*d*x+1/2*c)+4/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3-2/d/a^2*arctan(tan(1/2
*d*x+1/2*c))+1/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.56294, size = 254, normalized size = 3.48

$$-\frac{\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-2\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a^2 - 3*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.20221, size = 207, normalized size = 2.84

$$-\frac{2\cos(dx+c)^3+6\,dx+6\cos(dx+c)\sin(dx+c)-6\cos(dx+c)+3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\log\left(-\frac{1}{2}\cos(dx+\right.}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*cos(d*x + c)^3 + 6*d*x + 6*cos(d*x + c)*sin(d*x + c) - 6*cos(d*x +
c) + 3*log(1/2*cos(d*x + c) + 1/2) - 3*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43382, size = 123, normalized size = 1.68

$$-\frac{\frac{3\,(dx+c)}{a^2}-\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 - 3*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 2*(3*tan(1/2
*d*x + 1/2*c)^5 + 6*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 2)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2))/d
```

3.638 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{2a^2}$$

```
[Out] -x/(2*a^2) + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - (2*Cos[c + d*x])/(a^2*d) -
 Cot[c + d*x]/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d)
```

Rubi [A] time = 0.203037, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2709, 3770, 3767, 8, 2638, 2635}

$$-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(2*a^2) + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - (2*Cos[c + d*x])/(a^2*d) -
 Cot[c + d*x]/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(-2a^4\csc(c+dx)+a^4\csc^2(c+dx)+2a^4\sin(c+dx)-a^4\sin^2(c+dx)\right)dx}{a^6}\\ &= \frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{\int \sin^2(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}+\frac{2\int \sin(c+dx)\,dx}{a^2}\\ &= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{\int 1\,dx}{2a^2}-\text{Subs}\\ &= -\frac{x}{2a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c}{2a^2d}\end{aligned}$$

Mathematica [A] time = 0.523539, size = 116, normalized size = 1.57

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-2(c+dx)+\sin(2(c+dx))-8\cos(c+dx)+2\tan\left(\frac{1}{2}(c+dx)\right)-2\cot\left(\frac{1}{2}(c+\right.\right.}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-2*(c + d*x) - 8*Cos[c + d*x] - 2
*Cot[(c + d*x)/2] + 8*Log[Cos[(c + d*x)/2]] - 8*Log[Sin[(c + d*x)/2]] + Sin
[2*(c + d*x)] + 2*Tan[(c + d*x)/2]))/(4*d*(a + a*Sin[c + d*x])^2)
```

Maple [B] time = 0.152, size = 196, normalized size = 2.7

$$\frac{1}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-4\frac{(\tan(1/2\,dx+c/2))^2}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+\frac{1}{da^2}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x
+1/2*c)^3-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d/a^2/(
1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^2-1/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^
2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.50557, size = 273, normalized size = 3.69

$$-\frac{\frac{\frac{8\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{8\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\,a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{2\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{4\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*((8*sin(d*x + c)/(cos(d*x + c) + 1) + 8*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)/(a^2*sin(d*x + c)/(cos(d
*x + c) + 1) + 2*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^2*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5) + 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 +
4*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*x + c
) + 1)))/d
```

Fricas [A] time = 1.20094, size = 254, normalized size = 3.43

$$-\frac{\cos\left(dx+c\right)^3+\left(dx+4\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-2\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)}{2\,a^2d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(cos(d*x + c)^3 + (d*x + 4*cos(d*x + c))*sin(d*x + c) - 2*log(1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) + 2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
+ cos(d*x + c))/(a^2*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33712, size = 177, normalized size = 2.39

$$-\frac{\frac{dx+c}{a^2}+\frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a^2 + 4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x +
1/2*c)/a^2 - (4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)) + 2*(t
an(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 4
)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.639 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2x}{a^2}$$

```
[Out] (2*x)/a^2 - ArcTanh[Cos[c + d*x]]/(2*a^2*d) + Cos[c + d*x]/(a^2*d) + (2*Cot
[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rubi [A] time = 0.224917, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2872, 3767, 8, 3768, 3770, 2638}

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 - ArcTanh[Cos[c + d*x]]/(2*a^2*d) + Cos[c + d*x]/(a^2*d) + (2*Cot
[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^2(c+dx)\csc(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(2a^4-2a^4\csc^2(c+dx)+a^4\csc^3(c+dx)-a^4\sin(c+dx)\right)dx}{a^6}\\ &= \frac{2x}{a^2}+\frac{\int \csc^3(c+dx)\,dx}{a^2}-\frac{\int \sin(c+dx)\,dx}{a^2}-\frac{2\int \csc^2(c+dx)\,dx}{a^2}\\ &= \frac{2x}{a^2}+\frac{\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{\int \csc(c+dx)\,dx}{2a^2}+\frac{2\,\text{Subst}(\int 1\,dx,}{a^2}\\ &= \frac{2x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{\cos(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx}{2a^2d}\end{aligned}$$

Mathematica [A] time = 0.575663, size = 134, normalized size = 1.84

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(16(c+dx)+8\cos(c+dx)-8\tan\left(\frac{1}{2}(c+dx)\right)+8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+\right.\right.}{8d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(16*(c + d*x) + 8*Cos[c + d*x] + 8
*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Si
n[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 - 8*Tan[(c + d*x)/2]))/(8*d*(a + a*Sin
[c + d*x])^2)
```

Maple [A] time = 0.145, size = 134, normalized size = 1.8

$$\frac{1}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{1}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)}+4\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^2}-\frac{1}{8\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)+2/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)+4/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/8/d/a^2/tan(1/2*d*x+1/
2*c)^2+1/d/a^2/tan(1/2*d*x+1/2*c)+1/2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.53461, size = 275, normalized size = 3.77

$$\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}+\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{32\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*((8*sin(d*x + c)/(cos(d*x + c) + 1) + 15*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a^2*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - (8*sin(d*x +
c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^2 + 32*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 4*log(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2)/d
```

Fricas [A] time = 1.16811, size = 319, normalized size = 4.37

$$\frac{8\,dx\cos(dx+c)^2+4\cos(dx+c)^3-8\,dx-\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\,\mathrm{co}\right.}{4\left(a^2d\cos(dx+c)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(8*d*x*cos(d*x + c)^2 + 4*cos(d*x + c)^3 - 8*d*x - (cos(d*x + c)^2 - 1)
*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) +
 1/2) - 8*cos(d*x + c)*sin(d*x + c) - 2*cos(d*x + c))/(a^2*d*cos(d*x + c)^2
 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38283, size = 173, normalized size = 2.37

$$\frac{\frac{16(dx+c)}{a^2}+\frac{4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}+\frac{16}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^2}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(16*(d*x + c)/a^2 + 4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (a^2*tan(1/2
*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 + 16/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a^2) - (6*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a
^2*tan(1/2*d*x + 1/2*c)^2))/d
```

3.640 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[Out] -(x/a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cot[c + d*x]/(a^2*d) - Cot[c + d
*x]^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d)
```

Rubi [A] time = 0.315257, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cot[c + d*x]/(a^2*d) - Cot[c + d
*x]^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^2(c+dx)\csc^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^2(c+dx)-2a^2\cot^2(c+dx)\csc(c+dx)+a^2\cot^2(c+dx)\csc^2(c+dx)\right)\,d}{a^4}$$

$$= \frac{\int \cot^2(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^2(c+dx)\csc(c+dx)}{a^2}$$

$$= -\frac{\cot(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{\int 1\,dx}{a^2}+\frac{\int \csc(c+dx)\,dx}{a^2}+\frac{\operatorname{Subst}\left(\int x\right.}{}$$

$$= -\frac{x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+d}{a^2d}$$

Mathematica [A] time = 1.24978, size = 124, normalized size = 1.7

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^4\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-6\sin(2(c+dx))+6\cos(c+dx)-2\cos(3(c+dx))+12\,\mathrm{si}\right.}{96a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((1 + Cot[(c + d*x)/2])^4*Sec[(c + d*x)/2]^2*(6*Cos[c + d*x] - 2*Cos[3*(c
+ d*x)] + 12*(c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[
c + d*x]^3 - 6*Sin[2*(c + d*x)])*Tan[(c + d*x)/2])/(96*a^2*d*(1 + Sin[c + d
*x])^2)
```

Maple [B] time = 0.148, size = 149, normalized size = 2.

$$\frac{1}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{3}{8\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2}-\frac{1}{24\,da^2}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^2*tan(1/2*d*x+1/2*c)^2+3/8/d/a^2*ta
n(1/2*d*x+1/2*c)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/24/d/a^2/tan(1/2*d*x+
1/2*c)^3+1/4/d/a^2/tan(1/2*d*x+1/2*c)^2-3/8/d/a^2/tan(1/2*d*x+1/2*c)+1/d/a^
2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.51876, size = 238, normalized size = 3.26

$$\frac{\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1}-\frac{6 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{48 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{24 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{6 \sin(dx+c)}{\cos(dx+c)+1}-\frac{9 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^2 \sin(dx+c)^3}}{24\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 48*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/a^2 + 24*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (6*s
in(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)
*(cos(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.16086, size = 378, normalized size = 5.18

$$-\frac{4 \cos(dx+c)^3 + 3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)\right.}{6\left(a^2 d \cos(dx+c)^2 - a^2 d\right)\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(4*cos(d*x + c)^3 + 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
*sin(d*x + c) - 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) + 6*(d*x*cos(d*x + c)^2 - d*x + cos(d*x + c))*sin(d*x + c) - 6*cos(d*
x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25638, size = 185, normalized size = 2.53

$$-\frac{\frac{24\,(dx+c)}{a^2}-\frac{24\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{44\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,a}{a^6}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/a^2 - 24*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (44*tan(1
/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 1)/
(a^2*tan(1/2*d*x + 1/2*c)^3) - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^4*tan(1/2*
d*x + 1/2*c)^2 + 9*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.641 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}$$

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(8*a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot
[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rubi [A] time = 0.300256, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2611, 3770, 2607, 30, 3768}

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(8*a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot
[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^2(c+dx)\csc^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(a^2\cot^2(c+dx)\csc(c+dx)-2a^2\cot^2(c+dx)\csc^2(c+dx)+a^2\cot^2(c+dx)\csc^3\right.}{a^4}\\ &= \frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a^2}-\frac{2\int \cot^2(c+dx)\,c}{a^2}\\ &= -\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{\int \csc^3(c+dx)\,dx}{4a^2}-\frac{\int \csc(c}{2}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+dx)\csc}{4a^2d}\\ &= \frac{5\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+dx)\,c}{4a^2}\end{aligned}$$

Mathematica [A] time = 1.32723, size = 116, normalized size = 1.41

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^4\left(24\sin(2(c+dx))-33\cos(c+dx)+(16\sin(c+dx)+9)\cos(3(c+dx))+60\sin\right.}{1536a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^4*(-33*Cos[c + d*x] + 60*(Log[Cos[(c
 + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + Cos[3*(c + d*x)]*(9 +
 16*Sin[c + d*x]) + 24*Sin[2*(c + d*x)]))/(1536*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [B] time = 0.152, size = 170, normalized size = 2.1

$$\frac{1}{64\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{12\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{1}{4\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

[Out] 1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-1/12/d/a^2*tan(1/2*d*x+1/2*c)^3+1/8/d/a^2*t
an(1/2*d*x+1/2*c)^2+1/4/d/a^2*tan(1/2*d*x+1/2*c)-1/4/d/a^2/tan(1/2*d*x+1/2*
c)-1/64/d/a^2/tan(1/2*d*x+1/2*c)^4-5/8/d/a^2*ln(tan(1/2*d*x+1/2*c))+1/12/d/
a^2/tan(1/2*d*x+1/2*c)^3-1/8/d/a^2/tan(1/2*d*x+1/2*c)^2

Maxima [B] time = 1.02228, size = 262, normalized size = 3.2

$$\frac{\frac{\frac{48\sin(dx+c)}{\cos(dx+c)+1}+\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^2}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{48\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a^2\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")

[Out] 1/192*((48*sin(d*x + c)/(cos(d*x + c) + 1) + 24*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a^2 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (16*si
n(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 48
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a^2*sin(d*x
 + c)^4))/d

Fricas [A] time = 1.11437, size = 378, normalized size = 4.61

$$\frac{32\cos(dx+c)^3\sin(dx+c)+18\cos(dx+c)^3+15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-15\,(\cdots}{48\left(a^2d\cos(dx+c)^4-2\,a^2d\cos(dx+c)^2+a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")

[Out] 1/48*(32*cos(d*x + c)^3*sin(d*x + c) + 18*cos(d*x + c)^3 + 15*(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 15*(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) - 30*cos(d*x + c))/(a
^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)

[Out] Timed out

Giac [B] time = 1.41124, size = 213, normalized size = 2.6

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} - \frac{250 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 48 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 24 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 16 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 3}{a^2 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4} - \frac{3a^6 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 16a^6 \tan\left(\frac{1}{2} \cdots\right.}{\cdots}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/192*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (250*tan(1/2*d*x + 1/2*c)^
4 - 48*tan(1/2*d*x + 1/2*c)^3 - 24*tan(1/2*d*x + 1/2*c)^2 + 16*tan(1/2*d*x
+ 1/2*c) - 3)/(a^2*tan(1/2*d*x + 1/2*c)^4) - (3*a^6*tan(1/2*d*x + 1/2*c)^4
- 16*a^6*tan(1/2*d*x + 1/2*c)^3 + 24*a^6*tan(1/2*d*x + 1/2*c)^2 + 48*a^6*ta
n(1/2*d*x + 1/2*c))/a^8)/d
```

3.642 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(4*a^2*d) - (2*Cot[c + d*x]^3)/(3*a^2*d) - Cot[c + d
*x]^5/(5*a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(4*a^2*d) + (Cot[c + d*x]*Csc
[c + d*x]^3)/(2*a^2*d)
```

Rubi [A] time = 0.14719, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 3767, 8, 3768, 3770}

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] -ArcTanh[Cos[c + d*x]]/(4*a^2*d) - (2*Cot[c + d*x]^3)/(3*a^2*d) - Cot[c + d
*x]^5/(5*a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(4*a^2*d) + (Cot[c + d*x]*Csc
[c + d*x]^3)/(2*a^2*d)
```


Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \left(-a^4\csc^2(c+dx)+2a^4\csc^3(c+dx)-2a^4\csc^5(c+dx)+a^4\csc^6(c+dx)\right)dx}{a^6}$$
$$= -\frac{\int \csc^2(c+dx)\,dx}{a^2}+\frac{\int \csc^6(c+dx)\,dx}{a^2}+\frac{2\int \csc^3(c+dx)\,dx}{a^2}-\frac{2\int \csc^5(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}+\frac{\int \csc(c+dx)\,dx}{a^2}-\frac{3\int \csc^3(c+a}{2a^2}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}+\cot($$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}+\cot($$

Mathematica [A] time = 0.553905, size = 189, normalized size = 1.89

$$-\frac{\csc^5(c+dx)\left(-180\sin(2(c+dx))-30\sin(4(c+dx))+200\cos(c+dx)+20\cos(3(c+dx))-28\cos(5(c+dx))-15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^5*(200*Cos[c + d*x] + 20*Cos[3*(c + d*x)] - 28*Cos[5*(c + d*
x)] + 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[Sin[(c + d*x)/2]]*Si
n[c + d*x] - 180*Sin[2*(c + d*x)] - 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x
)] + 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*Sin[4*(c + d*x)] + 15*L
og[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[5*(c +
 d*x)]))/(960*a^2*d)
```

Maple [A] time = 0.161, size = 170, normalized size = 1.7

$$\frac{1}{160\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{32\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+\frac{5}{96\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3}{16\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{3}{16\,da^2}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/160/d/a^2*tan(1/2*d*x+1/2*c)^5-1/32/d/a^2*tan(1/2*d*x+1/2*c)^4+5/96/d/a^2
*tan(1/2*d*x+1/2*c)^3-3/16/d/a^2*tan(1/2*d*x+1/2*c)+3/16/d/a^2/tan(1/2*d*x+
1/2*c)-1/160/d/a^2/tan(1/2*d*x+1/2*c)^5+1/32/d/a^2/tan(1/2*d*x+1/2*c)^4+1/4
/d/a^2*ln(tan(1/2*d*x+1/2*c))-5/96/d/a^2/tan(1/2*d*x+1/2*c)^3
```

Maxima [B] time = 1.03594, size = 263, normalized size = 2.63

$$-\frac{\frac{\frac{90\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{90\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-3\right)(\cos(dx+c)+1)^5}{a^2\sin(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*((90*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5)/a^2 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (15*s
in(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^2*sin(d*
x + c)^5))/d
```

Fricas [A] time = 1.14992, size = 460, normalized size = 4.6

$$\frac{56\cos(dx+c)^5 - 80\cos(dx+c)^3 - 15\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 15\,(\cdots}{120\left(a^2 d\cos(dx+c)^4 - 2\,a^2 d\,\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(56*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(c
os(d*x + c)^3 + cos(d*x + c))*sin(d*x + c))/((a^2*d*cos(d*x + c)^4 - 2*a^2*
d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38443, size = 212, normalized size = 2.12

$$\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} - \frac{274\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 90\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5} + \frac{3a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 15a^8\tan\left(\frac{1}{2}dx+\cdots\right.}{\cdots}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (274*tan(1/2*d*x + 1/2*c)^5
 - 90*tan(1/2*d*x + 1/2*c)^4 + 25*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x +
 1/2*c) + 3)/(a^2*tan(1/2*d*x + 1/2*c)^5) + (3*a^8*tan(1/2*d*x + 1/2*c)^5 -
 15*a^8*tan(1/2*d*x + 1/2*c)^4 + 25*a^8*tan(1/2*d*x + 1/2*c)^3 - 90*a^8*tan
(1/2*d*x + 1/2*c))/a^10)/d
```

3.643 $\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{5\cot(c+dx)\csc^3(c+dx)}{24a^2d}+\frac{3}{}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) + (2*Co
t[c + d*x]^5)/(5*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(16*a^2*d) - (5*Cot
[c + d*x]*Csc[c + d*x]^3)/(24*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^2
*d)
```

Rubi [A] time = 0.33197, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2611, 3768, 3770, 2607, 14}

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{5\cot(c+dx)\csc^3(c+dx)}{24a^2d}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (3*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) + (2*Co
t[c + d*x]^5)/(5*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(16*a^2*d) - (5*Cot
[c + d*x]*Csc[c + d*x]^3)/(24*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^2
*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^2(c+dx)\csc^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^2(c+dx)\csc^3(c+dx)-2a^2\cot^2(c+dx)\csc^4(c+dx)+a^2\cot^2(c+dx)\csc^5(c\right.}{a^4}$$
$$= \frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a^2}-\frac{2\int \cot^2(c+dx)\csc}{a^2}$$
$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{\int \csc^5(c+dx)\,dx}{6a^2}-\frac{\int \csc^3(c+}{4a^2}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{5\cot(c+dx)\csc^3(c+dx)}{24a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{\int \mathrm{cs}}{}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{16a^2d}-\frac{5}{}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{16a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{16a^2d}-$$

Mathematica [A] time = 0.671966, size = 229, normalized size = 1.85

$$-\frac{\csc^6(c+dx)\left(-960\sin(2(c+dx))-384\sin(4(c+dx))+64\sin(6(c+dx))+1500\cos(c+dx)-130\cos(3(c+dx))-90\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^6*(1500*Cos[c + d*x] - 130*Cos[3*(c + d*x)] - 90*Cos[5*(c +
d*x)] - 450*Log[Cos[(c + d*x)/2]] + 675*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/
2]] - 270*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 45*Cos[6*(c + d*x)]*Log[
Cos[(c + d*x)/2]] + 450*Log[Sin[(c + d*x)/2]] - 675*Cos[2*(c + d*x)]*Log[Si
n[(c + d*x)/2]] + 270*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 45*Cos[6*(c
+ d*x)]*Log[Sin[(c + d*x)/2]] - 960*Sin[2*(c + d*x)] - 384*Sin[4*(c + d*x)]
 + 64*Sin[6*(c + d*x)]))/(7680*a^2*d)
```

Maple [B] time = 0.176, size = 246, normalized size = 2.

$$\frac{1}{384\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6-\frac{1}{80\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3}{128\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{48\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{128\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/384/d/a^2*tan(1/2*d*x+1/2*c)^6-1/80/d/a^2*tan(1/2*d*x+1/2*c)^5+3/128/d/a^
2*tan(1/2*d*x+1/2*c)^4-1/48/d/a^2*tan(1/2*d*x+1/2*c)^3-1/128/d/a^2*tan(1/2*
d*x+1/2*c)^2+1/8/d/a^2*tan(1/2*d*x+1/2*c)-1/8/d/a^2/tan(1/2*d*x+1/2*c)+1/80
/d/a^2/tan(1/2*d*x+1/2*c)^5-3/128/d/a^2/tan(1/2*d*x+1/2*c)^4-3/16/d/a^2*ln(
tan(1/2*d*x+1/2*c))-1/384/d/a^2/tan(1/2*d*x+1/2*c)^6+1/48/d/a^2/tan(1/2*d*x
+1/2*c)^3+1/128/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.00403, size = 370, normalized size = 2.98

$$\frac{\frac{\frac{240\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^2}-\frac{360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{45\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{40 s}{(\cos}\right.}{}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1920*((240*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6)/a^2 - 360*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2
 + (24*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 240*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c) +
1)^6/(a^2*sin(d*x + c)^6))/d
```

Fricas [A] time = 1.14985, size = 524, normalized size = 4.23

$$-\frac{90\cos(dx+c)^5-80\cos(dx+c)^3-45\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\right.}{480\left(a^2d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(90*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 45*(cos(d*x + c)^6 - 3*cos(
d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 45*(cos(d*
x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) +
 1/2) - 64*(2*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*sin(d*x + c) - 90*cos(d*x
+ c))/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)
```

```
^2 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34646, size = 292, normalized size = 2.35

$$-\frac{\frac{360 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{882 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 240 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 40 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 45 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 24 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) -}{a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1920*(360*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (882*tan(1/2*d*x + 1/2*c)
^6 - 240*tan(1/2*d*x + 1/2*c)^5 + 15*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*
x + 1/2*c)^3 - 45*tan(1/2*d*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) - 5)/(a^
2*tan(1/2*d*x + 1/2*c)^6) - (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^10*tan(1/
2*d*x + 1/2*c)^5 + 45*a^10*tan(1/2*d*x + 1/2*c)^4 - 40*a^10*tan(1/2*d*x + 1
/2*c)^3 - 15*a^10*tan(1/2*d*x + 1/2*c)^2 + 240*a^10*tan(1/2*d*x + 1/2*c))/a
^12)/d
```

3.644 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{\sin^5(c+dx)\cos(c+dx)}{6a^3d}+\frac{23\sin^3(c+dx)\cos(c+dx)}{24a^3d}+\frac{23\sin(}{}$$

```
[Out] (-23*x)/(16*a^3) - (4*Cos[c + d*x])/(a^3*d) + (7*Cos[c + d*x]^3)/(3*a^3*d)
- (3*Cos[c + d*x]^5)/(5*a^3*d) + (23*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d)
+ (23*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^3*d) + (Cos[c + d*x]*Sin[c + d*x]^
5)/(6*a^3*d)
```

Rubi [A] time = 0.24209, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2633, 2635, 8}

$$-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{\sin^5(c+dx)\cos(c+dx)}{6a^3d}+\frac{23\sin^3(c+dx)\cos(c+dx)}{24a^3d}+\frac{23\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-23*x)/(16*a^3) - (4*Cos[c + d*x])/(a^3*d) + (7*Cos[c + d*x]^3)/(3*a^3*d)
- (3*Cos[c + d*x]^5)/(5*a^3*d) + (23*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d)
+ (23*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^3*d) + (Cos[c + d*x]*Sin[c + d*x]^
5)/(6*a^3*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sin^3(c+dx)-3a^3\sin^4(c+dx)+3a^3\sin^5(c+dx)-a^3\sin^6(c+dx)\right)dx}{a^6}$$
$$= \frac{\int \sin^3(c+dx)\,dx}{a^3}-\frac{\int \sin^6(c+dx)\,dx}{a^3}-\frac{3\int \sin^4(c+dx)\,dx}{a^3}+\frac{3\int \sin^5(c+dx)\,dx}{a^3}$$
$$= \frac{3\cos(c+dx)\sin^3(c+dx)}{4a^3d}+\frac{\cos(c+dx)\sin^5(c+dx)}{6a^3d}-\frac{5\int \sin^4(c+dx)\,dx}{6a^3}-\frac{9\int \sin^2}{}$$
$$= -\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{9\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{23\cos}{}$$
$$= -\frac{9x}{8a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{23\cos(c+dx)\sin(c+dx)}{16a^3d}$$
$$= -\frac{23x}{16a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{23\cos(c+dx)\sin(c+dx)}{16a^3d}$$

Mathematica [B] time = 2.18278, size = 366, normalized size = 2.84

$$\frac{-2760dx\sin\left(\frac{c}{2}\right)+2520\sin\left(\frac{c}{2}+dx\right)-2520\sin\left(\frac{3c}{2}+dx\right)+945\sin\left(\frac{3c}{2}+2dx\right)+945\sin\left(\frac{5c}{2}+2dx\right)-380\sin\left(\frac{5c}{2}+3dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(3 + 920*d*x)*Cos[c/2] - 2520*Cos[c/2 + d*x] - 2520*Cos[(3*c)/2 + d*x]
+ 945*Cos[(3*c)/2 + 2*d*x] - 945*Cos[(5*c)/2 + 2*d*x] + 380*Cos[(5*c)/2 + 3
*d*x] + 380*Cos[(7*c)/2 + 3*d*x] - 135*Cos[(7*c)/2 + 4*d*x] + 135*Cos[(9*c)
/2 + 4*d*x] - 36*Cos[(9*c)/2 + 5*d*x] - 36*Cos[(11*c)/2 + 5*d*x] + 5*Cos[(1
1*c)/2 + 6*d*x] - 5*Cos[(13*c)/2 + 6*d*x] + 9*Sin[c/2] - 2760*d*x*Sin[c/2]
+ 2520*Sin[c/2 + d*x] - 2520*Sin[(3*c)/2 + d*x] + 945*Sin[(3*c)/2 + 2*d*x]
+ 945*Sin[(5*c)/2 + 2*d*x] - 380*Sin[(5*c)/2 + 3*d*x] + 380*Sin[(7*c)/2 + 3
*d*x] - 135*Sin[(7*c)/2 + 4*d*x] - 135*Sin[(9*c)/2 + 4*d*x] + 36*Sin[(9*c)/
2 + 5*d*x] - 36*Sin[(11*c)/2 + 5*d*x] + 5*Sin[(11*c)/2 + 6*d*x] + 5*Sin[(13
*c)/2 + 6*d*x])/(1920*a^3*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.117, size = 381, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -23/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-391/24/d/a^3/(
1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9-4/d/a^3/(1+tan(1/2*d*x+1/2*c
)^2)^6*tan(1/2*d*x+1/2*c)^8-75/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d
*x+1/2*c)^7-136/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6+75/
```

```
4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-64/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4+391/24/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)
^6*tan(1/2*d*x+1/2*c)^3-136/5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)^2+23/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-68/15/d/a
^3/(1+tan(1/2*d*x+1/2*c)^2)^6-23/8/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.56375, size = 504, normalized size = 3.91

$$\frac{\frac{\frac{345 \sin(dx+c)}{\cos(dx+c)+1} - \frac{3264 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{1955 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{7680 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{2250 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5440 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{2250 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{480 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{1955 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{345 \sin(dx+}{(\cos(dx+c)+}}{a^3 + \frac{6 a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{15 a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{20 a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{15 a^3 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{6 a^3 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{a^3 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}}{120\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/120*((345*sin(d*x + c)/(cos(d*x + c) + 1) - 3264*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1955*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 7680*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 2250*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5440*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2250*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 - 480*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 1955*sin(d*x + c)^9/(cos(d*x
+ c) + 1)^9 - 345*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 544)/(a^3 + 6*a^3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 20*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*s
in(d*x + c)^12/(cos(d*x + c) + 1)^12) - 345*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^3)/d
```

Fricas [A] time = 1.09525, size = 219, normalized size = 1.7

$$-\frac{144 \cos(dx+c)^5 - 560 \cos(dx+c)^3 + 345\, dx - 5 \left(8 \cos(dx+c)^5 - 62 \cos(dx+c)^3 + 123 \cos(dx+c)\right) \sin(dx +}{240\, a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(144*cos(d*x + c)^5 - 560*cos(d*x + c)^3 + 345*d*x - 5*(8*cos(d*x +
c)^5 - 62*cos(d*x + c)^3 + 123*cos(d*x + c))*sin(d*x + c) + 960*cos(d*x + c
))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31117, size = 224, normalized size = 1.74

$$-\frac{\frac{345\,(dx+c)}{a^3}+\frac{2\left(345\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+2250\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+5440\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-2250\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7\cdots}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(345*(d*x + c)/a^3 + 2*(345*tan(1/2*d*x + 1/2*c)^11 + 1955*tan(1/2*d
*x + 1/2*c)^9 + 480*tan(1/2*d*x + 1/2*c)^8 + 2250*tan(1/2*d*x + 1/2*c)^7 +
5440*tan(1/2*d*x + 1/2*c)^6 - 2250*tan(1/2*d*x + 1/2*c)^5 + 7680*tan(1/2*d*
x + 1/2*c)^4 - 1955*tan(1/2*d*x + 1/2*c)^3 + 3264*tan(1/2*d*x + 1/2*c)^2 -
345*tan(1/2*d*x + 1/2*c) + 544)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^3))/d
```

3.645 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$\frac{\cos^5(c+dx)}{5a^3d}-\frac{5\cos^3(c+dx)}{3a^3d}+\frac{4\cos(c+dx)}{a^3d}-\frac{3\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{13x}{8a^3}$$

```
[Out] (13*x)/(8*a^3) + (4*Cos[c + d*x])/(a^3*d) - (5*Cos[c + d*x]^3)/(3*a^3*d) +
Cos[c + d*x]^5/(5*a^3*d) - (13*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (3*Co
s[c + d*x]*Sin[c + d*x]^3)/(4*a^3*d)
```

Rubi [A] time = 0.218723, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2869, 2757, 2635, 8, 2633}

$$\frac{\cos^5(c+dx)}{5a^3d}-\frac{5\cos^3(c+dx)}{3a^3d}+\frac{4\cos(c+dx)}{a^3d}-\frac{3\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{13x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*x)/(8*a^3) + (4*Cos[c + d*x])/(a^3*d) - (5*Cos[c + d*x]^3)/(3*a^3*d) +
Cos[c + d*x]^5/(5*a^3*d) - (13*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (3*Co
s[c + d*x]*Sin[c + d*x]^3)/(4*a^3*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sin^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sin^2(c+dx) - 3a^3\sin^3(c+dx) + 3a^3\sin^4(c+dx) - a^3\sin^5(c+dx)\right)dx}{a^6}$$
$$= \frac{\int \sin^2(c+dx)\,dx}{a^3} - \frac{\int \sin^5(c+dx)\,dx}{a^3} - \frac{3\int \sin^3(c+dx)\,dx}{a^3} + \frac{3\int \sin^4(c+dx)\,dx}{a^3}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{3\cos(c+dx)\sin^3(c+dx)}{4a^3d} + \frac{\int 1\,dx}{2a^3} + \frac{9\int \sin^2(c+dx)\,dx}{4a^3}$$
$$= \frac{x}{2a^3} + \frac{4\cos(c+dx)}{a^3d} - \frac{5\cos^3(c+dx)}{3a^3d} + \frac{\cos^5(c+dx)}{5a^3d} - \frac{13\cos(c+dx)\sin(c+dx)}{8a^3d} - 3$$
$$= \frac{13x}{8a^3} + \frac{4\cos(c+dx)}{a^3d} - \frac{5\cos^3(c+dx)}{3a^3d} + \frac{\cos^5(c+dx)}{5a^3d} - \frac{13\cos(c+dx)\sin(c+dx)}{8a^3d} - 3$$

Mathematica [B] time = 1.75688, size = 310, normalized size = 2.95

$$1560dx\sin\left(\frac{c}{2}\right) - 1380\sin\left(\frac{c}{2}+dx\right) + 1380\sin\left(\frac{3c}{2}+dx\right) - 480\sin\left(\frac{3c}{2}+2dx\right) - 480\sin\left(\frac{5c}{2}+2dx\right) + 170\sin\left(\frac{5c}{2}+3dx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (1560*d*x*Cos[c/2] + 1380*Cos[c/2 + d*x] + 1380*Cos[(3*c)/2 + d*x] - 480*Co
s[(3*c)/2 + 2*d*x] + 480*Cos[(5*c)/2 + 2*d*x] - 170*Cos[(5*c)/2 + 3*d*x] -
170*Cos[(7*c)/2 + 3*d*x] + 45*Cos[(7*c)/2 + 4*d*x] - 45*Cos[(9*c)/2 + 4*d*x
] + 6*Cos[(9*c)/2 + 5*d*x] + 6*Cos[(11*c)/2 + 5*d*x] + 10*Sin[c/2] + 1560*d
*x*Sin[c/2] - 1380*Sin[c/2 + d*x] + 1380*Sin[(3*c)/2 + d*x] - 480*Sin[(3*c)
/2 + 2*d*x] - 480*Sin[(5*c)/2 + 2*d*x] + 170*Sin[(5*c)/2 + 3*d*x] - 170*Sin
[(7*c)/2 + 3*d*x] + 45*Sin[(7*c)/2 + 4*d*x] + 45*Sin[(9*c)/2 + 4*d*x] - 6*S
in[(9*c)/2 + 5*d*x] + 6*Sin[(11*c)/2 + 5*d*x])/(960*a^3*d*(Cos[c/2] + Sin[c
/2]))
```

Maple [B] time = 0.107, size = 279, normalized size = 2.7

$$\frac{13}{4da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5} + \frac{25}{2da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5} + 12\frac{(\tan(1/2}{da^3\left(1+(\tan(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 13/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+25/2/d/a^3/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7+12/d/a^3/(1+tan(1/2*d*x+1/2*c)^2
)^5*tan(1/2*d*x+1/2*c)^6+116/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x
+1/2*c)^4-25/2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+76/3/d
/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2-13/4/d/a^3/(1+tan(1/2*
```

```
d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)+76/15/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5+1
3/4/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.54764, size = 392, normalized size = 3.73

$$-\frac{\frac{\frac{195\sin(dx+c)}{\cos(dx+c)+1}-\frac{1520\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2320\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{720\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{750\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{195\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-304}{a^3+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{195\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/60*((195*sin(d*x + c)/(cos(d*x + c) + 1) - 1520*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2320*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 720*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 750*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7 - 195*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
304)/(a^3 + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^
3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^10/(cos(d*x + c) +
 1)^10) - 195*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.12579, size = 189, normalized size = 1.8

$$\frac{24\cos(dx+c)^5-200\cos(dx+c)^3+195\,dx+15\left(6\cos(dx+c)^3-19\cos(dx+c)\right)\sin(dx+c)+480\cos(dx+c)}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/120*(24*cos(d*x + c)^5 - 200*cos(d*x + c)^3 + 195*d*x + 15*(6*cos(d*x + c
)^3 - 19*cos(d*x + c))*sin(d*x + c) + 480*cos(d*x + c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26507, size = 171, normalized size = 1.63

$$\frac{\frac{195\,(dx+c)}{a^3}+\frac{2\left(195\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+750\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+720\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+2320\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-750\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1520\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-195\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+304\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(195*(d*x + c)/a^3 + 2*(195*tan(1/2*d*x + 1/2*c)^9 + 750*tan(1/2*d*x
+ 1/2*c)^7 + 720*tan(1/2*d*x + 1/2*c)^6 + 2320*tan(1/2*d*x + 1/2*c)^4 - 750
*tan(1/2*d*x + 1/2*c)^3 + 1520*tan(1/2*d*x + 1/2*c)^2 - 195*tan(1/2*d*x + 1
/2*c) + 304)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^3))/d
```

3.646 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=84

$$\frac{\cos^3(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4a^3d}+\frac{15\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{15x}{8a^3}$$

```
[Out] (-15*x)/(8*a^3) - (4*Cos[c + d*x])/(a^3*d) + Cos[c + d*x]^3/(a^3*d) + (15*C
os[c + d*x]*Sin[c + d*x])/(8*a^3*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^3*
d)
```

Rubi [A] time = 0.165566, antiderivative size = 105, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2859, 2679, 2682, 2635, 8}

$$-\frac{5\cos^3(c+dx)}{4a^3d}-\frac{3\cos^5(c+dx)}{4d\left(a^3\sin(c+dx)+a^3\right)}-\frac{15\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{15x}{8a^3}-\frac{\cos^7(c+dx)}{d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-15*x)/(8*a^3) - (5*Cos[c + d*x]^3)/(4*a^3*d) - (15*Cos[c + d*x]*Sin[c + d
*x])/(8*a^3*d) - Cos[c + d*x]^7/(d*(a + a*Sin[c + d*x])^3) - (3*Cos[c + d*x
]^5)/(4*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{3\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{a}$$

$$= -\frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{3\cos^5(c+dx)}{4d\left(a^3+a^3\sin(c+dx)\right)} - \frac{15\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{4a^2}$$

$$= -\frac{5\cos^3(c+dx)}{4a^3d} - \frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{3\cos^5(c+dx)}{4d\left(a^3+a^3\sin(c+dx)\right)} - \frac{15\int \cos^2(c+dx)}{4a^3}$$

$$= -\frac{5\cos^3(c+dx)}{4a^3d} - \frac{15\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{3\cos^5(c+}{4d\left(a^3+a^3\sin(\right.}$$

$$= -\frac{15x}{8a^3} - \frac{5\cos^3(c+dx)}{4a^3d} - \frac{15\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3} - \frac{3\operatorname{co}}{4d\left(a^3+\right.}$$

Mathematica [B] time = 1.4017, size = 255, normalized size = 3.04

$$-\frac{120dx\sin\left(\frac{c}{2}\right) - 104\sin\left(\frac{c}{2}+dx\right) + 104\sin\left(\frac{3c}{2}+dx\right) - 32\sin\left(\frac{3c}{2}+2dx\right) - 32\sin\left(\frac{5c}{2}+2dx\right) + 8\sin\left(\frac{5c}{2}+3dx\right) - 8\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((1 + 120*d*x)*Cos[c/2] + 104*Cos[c/2 + d*x] + 104*Cos[(3*c)/2 + d*x] - 32
*Cos[(3*c)/2 + 2*d*x] + 32*Cos[(5*c)/2 + 2*d*x] - 8*Cos[(5*c)/2 + 3*d*x] -
8*Cos[(7*c)/2 + 3*d*x] + Cos[(7*c)/2 + 4*d*x] - Cos[(9*c)/2 + 4*d*x] - Sin[
c/2] + 120*d*x*Sin[c/2] - 104*Sin[c/2 + d*x] + 104*Sin[(3*c)/2 + d*x] - 32*
Sin[(3*c)/2 + 2*d*x] - 32*Sin[(5*c)/2 + 2*d*x] + 8*Sin[(5*c)/2 + 3*d*x] - 8
*Sin[(7*c)/2 + 3*d*x] + Sin[(7*c)/2 + 4*d*x] + Sin[(9*c)/2 + 4*d*x])/(64*a^
3*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.109, size = 279, normalized size = 3.3

$$-\frac{15}{4da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4} - 2\frac{(\tan(1/2\,dx+c/2))^6}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^4} - \frac{23}{4da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\operatorname{ta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -15/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6-23/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)^5-18/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^4+23/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-22/d/a^3/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+15/4/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)-6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4-15/4/d/a^3
*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.53987, size = 360, normalized size = 4.29

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{88\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{23\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{72\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{23\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-24}{a^3+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 88*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 23*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 72*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 23*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 24)/(a^3 + 4
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8) - 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.06535, size = 150, normalized size = 1.79

$$\frac{8\cos(dx+c)^3-15dx-\left(2\cos(dx+c)^3-17\cos(dx+c)\right)\sin(dx+c)-32\cos(dx+c)}{8a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*cos(d*x + c)^3 - 15*d*x - (2*cos(d*x + c)^3 - 17*cos(d*x + c))*sin(d
*x + c) - 32*cos(d*x + c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37456, size = 171, normalized size = 2.04

$$-\frac{\frac{15\,(dx+c)}{a^3}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+23\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+72\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-23\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+88\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(d*x + c)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^7 + 8*tan(1/2*d*x + 1/2
*c)^6 + 23*tan(1/2*d*x + 1/2*c)^5 + 72*tan(1/2*d*x + 1/2*c)^4 - 23*tan(1/2*
d*x + 1/2*c)^3 + 88*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 24)/
((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

3.647 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{7x}{2a^3}$$

```
[Out] (-7*x)/(2*a^3) - ArcTanh[Cos[c + d*x]]/(a^3*d) - (3*Cos[c + d*x])/(a^3*d) +
 (Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d)
```

Rubi [A] time = 0.147155, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2869, 2757, 3770, 2638, 2635, 8}

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{7x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-7*x)/(2*a^3) - ArcTanh[Cos[c + d*x]]/(a^3*d) - (3*Cos[c + d*x])/(a^3*d) +
 (Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d)
```


Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(-3a^3+a^3\csc(c+dx)+3a^3\sin(c+dx)-a^3\sin^2(c+dx)\right)dx}{a^6}\\ &= -\frac{3x}{a^3}+\frac{\int \csc(c+dx)\,dx}{a^3}-\frac{\int \sin^2(c+dx)\,dx}{a^3}+\frac{3\int \sin(c+dx)\,dx}{a^3}\\ &= -\frac{3x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{\int 1\,dx}{2a^3}\\ &= -\frac{7x}{2a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}\end{aligned}$$

Mathematica [A] time = 0.217078, size = 63, normalized size = 1.05

$$\frac{\sin(2(c+dx))-12\cos(c+dx)-2\left(-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+7c+7dx\right)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-12*Cos[c + d*x] - 2*(7*c + 7*d*x + 2*Log[Cos[(c + d*x)/2]] - 2*Log[Sin[(c
 + d*x)/2]]) + Sin[2*(c + d*x)])/(4*a^3*d)
```

Maple [B] time = 0.151, size = 159, normalized size = 2.7

$$-\frac{1}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-6\frac{(\tan(1/2\,dx+c/2))^2}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+\frac{1}{da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*ta
n(1/2*d*x+1/2*c)-6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2-7/d/a^3*arctan(tan(1/2*
d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.52224, size = 217, normalized size = 3.62

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-6}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{7\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 6)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 7*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)
/d
```

Fricas [A] time = 1.11585, size = 173, normalized size = 2.88

$$-\frac{7\,dx - \cos\left(dx+c\right)\sin\left(dx+c\right) + 6\,\cos\left(dx+c\right) + \log\left(\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right) - \log\left(-\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right)}{2\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(7*d*x - cos(d*x + c)*sin(d*x + c) + 6*cos(d*x + c) + log(1/2*cos(d*x
+ c) + 1/2) - log(-1/2*cos(d*x + c) + 1/2))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28735, size = 120, normalized size = 2.

$$-\frac{\frac{7\,(dx+c)}{a^3} - \frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} + \frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 6\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 1\right)^2 a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(7*(d*x + c)/a^3 - 2*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(tan(1/2*d
*x + 1/2*c)^3 + 6*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 6)/((tan(
1/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d
```

3.648 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=49

$$\frac{\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3x}{a^3}$$

```
[Out] (3*x)/a^3 + (3*ArcTanh[Cos[c + d*x]])/(a^3*d) + Cos[c + d*x]/(a^3*d) - Cot[
c + d*x]/(a^3*d)
```

Rubi [A] time = 0.163287, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2869, 2757, 3770, 3767, 8, 2638}

$$\frac{\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*x)/a^3 + (3*ArcTanh[Cos[c + d*x]])/(a^3*d) + Cos[c + d*x]/(a^3*d) - Cot[
c + d*x]/(a^3*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(3a^3-3a^3\csc(c+dx)+a^3\csc^2(c+dx)-a^3\sin(c+dx)\right)dx}{a^6}\\
&= \frac{3x}{a^3}+\frac{\int \csc^2(c+dx)\,dx}{a^3}-\frac{\int \sin(c+dx)\,dx}{a^3}-\frac{3\int \csc(c+dx)\,dx}{a^3}\\
&= \frac{3x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{a^3d}-\frac{\operatorname{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{a^3d}\\
&= \frac{3x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}
\end{aligned}$$

Mathematica [B] time = 0.482605, size = 106, normalized size = 2.16

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(6(c+dx)+2\cos(c+dx)+\tan\left(\frac{1}{2}(c+dx)\right)-\cot\left(\frac{1}{2}(c+dx)\right)-6\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{2d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(6*(c + d*x) + 2*Cos[c + d*x] - Co
t[(c + d*x)/2] + 6*Log[Cos[(c + d*x)/2]] - 6*Log[Sin[(c + d*x)/2]] + Tan[(c
 + d*x)/2]))/(2*d*(a + a*Sin[c + d*x])^3)
```

Maple [A] time = 0.165, size = 97, normalized size = 2.

$$\frac{1}{2da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{1}{da^3\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}+6\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^3}-\frac{1}{2da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)+6/d/a^3*arcta
n(tan(1/2*d*x+1/2*c))-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/2*d*x+1
/2*c))
```

Maxima [B] time = 1.55993, size = 213, normalized size = 4.35

$$\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + a^3*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3) + 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 6*log(sin(d
*x + c)/(cos(d*x + c) + 1))/a^3 + sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.16906, size = 235, normalized size = 4.8

$$\frac{2\,(3\,dx+\cos{(dx+c)})\sin{(dx+c)}+3\,\log\left(\frac{1}{2}\,\cos{(dx+c)}+\frac{1}{2}\right)\sin{(dx+c)}-3\,\log\left(-\frac{1}{2}\,\cos{(dx+c)}+\frac{1}{2}\right)\sin{(dx+c)}-2}{2\,a^3d\sin{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(3*d*x + cos(d*x + c))*sin(d*x + c) + 3*log(1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*cos(d*x + c)
)/(a^3*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37522, size = 150, normalized size = 3.06

$$\frac{\frac{6\,(dx+c)}{a^3}-\frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(d*x + c)/a^3 - 6*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + tan(1/2*d*x +
 1/2*c)/a^3 + (2*tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + 6*tan(1/
2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^3))/
d
```

3.649 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$\frac{3\cot(c+dx)}{a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{x}{a^3}$$

```
[Out] -(x/a^3) - (7*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) -
 (Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d)
```

Rubi [A] time = 0.175151, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2869, 2757, 3770, 3767, 8, 3768}

$$\frac{3\cot(c+dx)}{a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (7*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) -
 (Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(-a^3+3a^3\csc(c+dx)-3a^3\csc^2(c+dx)+a^3\csc^3(c+dx)\right)dx}{a^6}\\
&= -\frac{x}{a^3}+\frac{\int \csc^3(c+dx)\,dx}{a^3}+\frac{3\int \csc(c+dx)\,dx}{a^3}-\frac{3\int \csc^2(c+dx)\,dx}{a^3}\\
&= -\frac{x}{a^3}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{\int \csc(c+dx)\,dx}{2a^3}+\frac{3\,\text{Subst}}{}\\
&= -\frac{x}{a^3}-\frac{7\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}
\end{aligned}$$

Mathematica [B] time = 0.47218, size = 126, normalized size = 2.1

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-8(c+dx)-12\tan\left(\frac{1}{2}(c+dx)\right)+12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^2\left(\frac{1}{2}(\right.\right.}{8d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-8*(c + d*x) + 12*Cot[(c + d*x)/2
] - Csc[(c + d*x)/2]^2 - 28*Log[Cos[(c + d*x)/2]] + 28*Log[Sin[(c + d*x)/2]
] + Sec[(c + d*x)/2]^2 - 12*Tan[(c + d*x)/2]))/(8*d*(a + a*Sin[c + d*x])^3)
```

Maple [A] time = 0.174, size = 112, normalized size = 1.9

$$\frac{1}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{3}{2\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^3}-\frac{1}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}+\frac{3}{2\,da^3}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^3*tan(1/2*d*x+1/2*c)-2/d/a^3*arctan(
tan(1/2*d*x+1/2*c))-1/8/d/a^3/tan(1/2*d*x+1/2*c)^2+3/2/d/a^3/tan(1/2*d*x+1/
2*c)+7/2/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.50917, size = 186, normalized size = 3.1

$$-\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3}+\frac{16\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{28\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a^3\sin(dx+c)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/8*((12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) +
1)^2)/a^3 + 16*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 28*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3 - (12*sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(c
os(d*x + c) + 1)^2/(a^3*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.10356, size = 301, normalized size = 5.02

$$-\frac{4\,dx\cos\left(dx+c\right)^{2}-4\,dx+7\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-7\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)}{4\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(4*d*x*cos(d*x + c)^2 - 4*d*x + 7*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x
 + c) + 1/2) - 7*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) + 12*cos
(d*x + c)*sin(d*x + c) - 2*cos(d*x + c))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36977, size = 146, normalized size = 2.43

$$-\frac{\frac{8\,(dx+c)}{a^{3}}-\frac{28\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}+\frac{42\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}-\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(8*(d*x + c)/a^3 - 28*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + (42*tan(1/2
*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^2)
 - (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.650 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=72

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (4*Cot[c + d*x])/(a^3*d) - Cot[c + d*
x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d)
```

Rubi [A] time = 0.189961, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2869, 2757, 3770, 3767, 8, 3768}

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (4*Cot[c + d*x])/(a^3*d) - Cot[c + d*
x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\csc(c+dx)+3a^3\csc^2(c+dx)-3a^3\csc^3(c+dx)+a^3\csc^4(c+dx)\right)dx}{a^6}$$
$$= -\frac{\int \csc(c+dx)\,dx}{a^3}+\frac{\int \csc^4(c+dx)\,dx}{a^3}+\frac{3\int \csc^2(c+dx)\,dx}{a^3}-\frac{3\int \csc^3(c+dx)\,dx}{a^3}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3\int \csc(c+dx)\,dx}{2a^3}-\text{Subst}\left(\int\right.$$
$$= \frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Mathematica [A] time = 1.27493, size = 115, normalized size = 1.6

$$-\frac{\csc^3(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-18\sin(2(c+dx))+30\cos(c+dx)-22\cos(3(c+dx))-60\sin^3\right.}{24a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(30*Cos[c + d*x] -
 22*Cos[3*(c + d*x)] - 60*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*S
in[c + d*x]^3 - 18*Sin[2*(c + d*x)]))/(24*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.178, size = 132, normalized size = 1.8

$$\frac{1}{24\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{15}{8\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{15}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{5}{2\,da^3}\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^3*tan(1/2*d*x+1/2*c)^2+15/8/d/a^3*t
an(1/2*d*x+1/2*c)-15/8/d/a^3/tan(1/2*d*x+1/2*c)-5/2/d/a^3*ln(tan(1/2*d*x+1/
2*c))-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3+3/8/d/a^3/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.03858, size = 207, normalized size = 2.88

$$\frac{\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{45\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^3\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((45*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^3 - 60*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3 + (9*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^3*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.10883, size = 347, normalized size = 4.82

$$-\frac{44 \cos\left(dx+c\right)^{3}-15 \left(\cos\left(dx+c\right)^{2}-1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right) \sin\left(dx+c\right)+15 \left(\cos\left(dx+c\right)^{2}-1\right) \log\left(-\frac{1}{2} \cos\left(dx+\right.\right.}{12 \left(a^{3} d \cos\left(dx+c\right)^{2}-a^{3} d\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(44*cos(d*x + c)^3 - 15*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 15*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 18*cos(d*x + c)*sin(d*x + c) - 48*cos(d*x + c))/((a^3*d*cos(d*x
 + c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35172, size = 173, normalized size = 2.4

$$-\frac{\frac{60 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{3}}-\frac{110 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}-45 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+9 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1}{a^{3} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}}-\frac{a^{6} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}-9 a^{6} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+45 a^{6} \tan\left(\frac{1}{2} dx+\frac{1}{2}\right.}{a^{9}}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/24*(60*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (110*tan(1/2*d*x + 1/2*c)^3
- 45*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x
+ 1/2*c)^3) - (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 +
45*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

3.651 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{15\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{15\cot(c+dx)\csc(c+dx)}{8a^3d}$$

```
[Out] (-15*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (4*Cot[c + d*x])/(a^3*d) + Cot[c +
d*x]^3/(a^3*d) - (15*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x]*C
sc[c + d*x]^3)/(4*a^3*d)
```

Rubi [A] time = 0.204519, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2869, 2757, 3767, 8, 3768, 3770}

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{15\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{15\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-15*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (4*Cot[c + d*x])/(a^3*d) + Cot[c +
d*x]^3/(a^3*d) - (15*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x]*C
sc[c + d*x]^3)/(4*a^3*d)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^5(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\csc^2(c+dx)+3a^3\csc^3(c+dx)-3a^3\csc^4(c+dx)+a^3\csc^5(c+dx)\right)dx}{a^6}$$
$$= -\frac{\int \csc^2(c+dx)\,dx}{a^3}+\frac{\int \csc^5(c+dx)\,dx}{a^3}+\frac{3\int \csc^3(c+dx)\,dx}{a^3}-\frac{3\int \csc^4(c+dx)\,dx}{a^3}$$
$$= -\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{3\int \csc^3(c+dx)\,dx}{4a^3}+\frac{3\int \csc($$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{4\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{15\cot(c+dx)\csc(c+dx)}{8a^3d}-$$
$$= -\frac{15\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{4\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{15\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Mathematica [A] time = 2.15347, size = 125, normalized size = 1.34

$$-\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-56\sin(2(c+dx))+46\cos(c+dx)+6(8\sin(c+dx)-5)\cos(3(c+\right.}{64a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(46*Cos[c + d*x] +
 120*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + 6*Cos
[3*(c + d*x)]*(-5 + 8*Sin[c + d*x]) - 56*Sin[2*(c + d*x)]))/(64*a^3*d*(1 +
Sin[c + d*x])^3)
```

Maple [A] time = 0.193, size = 170, normalized size = 1.8

$$\frac{1}{64da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{8da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{2da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{13}{8da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{13}{8da^3}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/64/d/a^3*tan(1/2*d*x+1/2*c)^4-1/8/d/a^3*tan(1/2*d*x+1/2*c)^3+1/2/d/a^3*ta
n(1/2*d*x+1/2*c)^2-13/8/d/a^3*tan(1/2*d*x+1/2*c)+13/8/d/a^3/tan(1/2*d*x+1/2
*c)-1/64/d/a^3/tan(1/2*d*x+1/2*c)^4+15/8/d/a^3*ln(tan(1/2*d*x+1/2*c))+1/8/d
/a^3/tan(1/2*d*x+1/2*c)^3-1/2/d/a^3/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.00906, size = 263, normalized size = 2.83

$$-\frac{\frac{\frac{104\sin(dx+c)}{\cos(dx+c)+1}-\frac{32\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^3}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{32\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{104\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)(\cos(dx+c)+1)^4}{a^3\sin(dx+c)^4}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/64*((104*sin(d*x + c)/(cos(d*x + c) + 1) - 32*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4)/a^3 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (8*sin(d
*x + c)/(cos(d*x + c) + 1) - 32*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 104*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)*(cos(d*x + c) + 1)^4/(a^3*sin(d*x +
 c)^4))/d
```

Fricas [A] time = 1.09012, size = 406, normalized size = 4.37

$$\frac{30\cos(dx+c)^3-15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15\left(\cos(dx+c)^4-2\cos(dx+c\right.}{16\left(a^3d\cos(dx+c)^4-2a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(30*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/
2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2
*cos(d*x + c) + 1/2) - 16*(3*cos(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c)
- 34*cos(d*x + c))/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39788, size = 211, normalized size = 2.27

$$\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{250\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-104\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+32\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\frac{a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8a^9\tan\left(\frac{1}{2}dx+\right.}{}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (250*tan(1/2*d*x + 1/2*c)^4
- 104*tan(1/2*d*x + 1/2*c)^3 + 32*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x +
1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^4) + (a^9*tan(1/2*d*x + 1/2*c)^4 - 8*
a^9*tan(1/2*d*x + 1/2*c)^3 + 32*a^9*tan(1/2*d*x + 1/2*c)^2 - 104*a^9*tan(1/
2*d*x + 1/2*c))/a^12)/d
```

3.652 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=114

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{13\cot(c+}{8}$$

```
[Out] (13*ArcTanh[Cos[c + d*x]])/(8*a^3*d) - (4*Cot[c + d*x])/(a^3*d) - (5*Cot[c
+ d*x]^3)/(3*a^3*d) - Cot[c + d*x]^5/(5*a^3*d) + (13*Cot[c + d*x]*Csc[c + d
*x])/(8*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rubi [A] time = 0.17691, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2708, 2757, 3768, 3770, 3767}

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{13\cot(c+}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*ArcTanh[Cos[c + d*x]])/(8*a^3*d) - (4*Cot[c + d*x])/(a^3*d) - (5*Cot[c
+ d*x]^3)/(3*a^3*d) - Cot[c + d*x]^5/(5*a^3*d) + (13*Cot[c + d*x]*Csc[c + d
*x])/(8*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^6(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(-a^3\csc^3(c+dx)+3a^3\csc^4(c+dx)-3a^3\csc^5(c+dx)+a^3\csc^6(c+dx)\right)dx}{a^6}\\ &= -\frac{\int \csc^3(c+dx)\,dx}{a^3}+\frac{\int \csc^6(c+dx)\,dx}{a^3}+\frac{3\int \csc^4(c+dx)\,dx}{a^3}-\frac{3\int \csc^5(c+dx)\,dx}{a^3}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{\int \csc(c+dx)\,dx}{2a^3}-\frac{9\int \csc^3(c+dx)\,d}{4a^3}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{13\cot(c+dx)\csc(}{8a^3d}\\ &= \frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{13\cot(c+dx)\,\mathrm{c}}{8a^3d}\end{aligned}$$

Mathematica [A] time = 1.81144, size = 189, normalized size = 1.66

$$\frac{\csc^5(c+dx)\left(1500\sin(2(c+dx))-390\sin(4(c+dx))-1600\cos(c+dx)+1520\cos(3(c+dx))-304\cos(5(c+dx))-1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^5*(-1600*Cos[c + d*x] + 1520*Cos[3*(c + d*x)] - 304*Cos[5*(c
+ d*x)] + 1950*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 1950*Log[Sin[(c + d*x)/
2]]*Sin[c + d*x] + 1500*Sin[2*(c + d*x)] - 975*Log[Cos[(c + d*x)/2]]*Sin[3*
(c + d*x)] + 975*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 390*Sin[4*(c + d*
x)] + 195*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 195*Log[Sin[(c + d*x)/2]
]*Sin[5*(c + d*x)]))/(1920*a^3*d)
```

Maple [A] time = 0.2, size = 208, normalized size = 1.8

$$\frac{1}{160\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{3}{64\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+\frac{17}{96\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{23}{16\,da^3}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/160/d/a^3*tan(1/2*d*x+1/2*c)^5-3/64/d/a^3*tan(1/2*d*x+1/2*c)^4+17/96/d/a^
3*tan(1/2*d*x+1/2*c)^3-1/2/d/a^3*tan(1/2*d*x+1/2*c)^2+23/16/d/a^3*tan(1/2*d
*x+1/2*c)-23/16/d/a^3/tan(1/2*d*x+1/2*c)-1/160/d/a^3/tan(1/2*d*x+1/2*c)^5+3
/64/d/a^3/tan(1/2*d*x+1/2*c)^4-13/8/d/a^3*ln(tan(1/2*d*x+1/2*c))-17/96/d/a^
3/tan(1/2*d*x+1/2*c)^3+1/2/d/a^3/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.0308, size = 316, normalized size = 2.77

$$\frac{\frac{\frac{1380 \sin(dx+c)}{\cos(dx+c)+1} - \frac{480 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{170 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{45 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{6 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{1560 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{\left(\frac{45 \sin(dx+c)}{\cos(dx+c)+1} - \frac{170 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{480 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \cdots\right.}{a^3 \sin(dx+c)^5}}{960\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/960*((1380*sin(d*x + c)/(cos(d*x + c) + 1) - 480*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 170*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 45*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 1560*log
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + (45*sin(d*x + c)/(cos(d*x + c) + 1)
 - 170*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 480*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 1380*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1
)^5/(a^3*sin(d*x + c)^5))/d
```

Fricas [A] time = 1.14746, size = 502, normalized size = 4.4

$$-\frac{608 \cos(dx+c)^5 - 1520 \cos(dx+c)^3 - 195 \left(\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx + \cdots}{240 \left(a^3 d \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(608*cos(d*x + c)^5 - 1520*cos(d*x + c)^3 - 195*(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 195*(cos(d*x
 + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
 30*(13*cos(d*x + c)^3 - 19*cos(d*x + c))*sin(d*x + c) + 960*cos(d*x + c))/
((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32235, size = 252, normalized size = 2.21

$$-\frac{\frac{1560 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^3} - \frac{3562 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 1380 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 480 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 170 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 45 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 6}{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5} - \frac{6 a^{12} \cdots}{\cdots}}{960\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/960*(1560*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (3562*tan(1/2*d*x + 1/2*c
)^5 - 1380*tan(1/2*d*x + 1/2*c)^4 + 480*tan(1/2*d*x + 1/2*c)^3 - 170*tan(1/
2*d*x + 1/2*c)^2 + 45*tan(1/2*d*x + 1/2*c) - 6)/(a^3*tan(1/2*d*x + 1/2*c)^5
) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^12*tan(1/2*d*x + 1/2*c)^4 + 170*a
^12*tan(1/2*d*x + 1/2*c)^3 - 480*a^12*tan(1/2*d*x + 1/2*c)^2 + 1380*a^12*ta
n(1/2*d*x + 1/2*c))/a^15)/d
```

3.653 $\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=267

$$\frac{a^3\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*Cos[c
+ d*x]*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*Cos[c + d*x]*Hype
rgeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 +
 n))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2]) + (a^3*Cos[c + d*x]*Hypergeometric2F1
[-5/2, (4 + n)/2, (6 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(4 + n))/(d*(4 +
n)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.284884, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2873, 2577}

$$\frac{a^3\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*Cos[c
+ d*x]*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*Cos[c + d*x]*Hype
rgeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 +
 n))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2]) + (a^3*Cos[c + d*x]*Hypergeometric2F1
[-5/2, (4 + n)/2, (6 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(4 + n))/(d*(4 +
n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^n(c+dx) + 3a^3\cos^6(c+dx)\sin^{1+n}(c+dx) + \right.$$
$$= a^3\int \cos^6(c+dx)\sin^n(c+dx)\,dx + a^3\int \cos^6(c+dx)\sin^{3+n}(c+d$$
$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}} + \frac{3a^3}{}$$

Mathematica [A] time = 0.599743, size = 188, normalized size = 0.7

$$\frac{a^3\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{n+1}(c+dx)\left(\frac{{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{n+1} + \sin(c+dx)\left(\frac{3\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{n+2} + \sin(c+dx\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*(Hypergeometric
2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]/(1 + n) + Sin[c + d*x]*((3*
Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2])/(2 + n) + Si
n[c + d*x]*((3*Hypergeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2
])/(3 + n) + (Hypergeometric2F1[-5/2, (4 + n)/2, (6 + n)/2, Sin[c + d*x]^2]
*Sin[c + d*x])/(4 + n)))))/d
```

Maple [F] time = 14.569, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^6 (\sin(dx+c))^n (a+a\sin(dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^3 \sin(dx+c)^n \cos(dx+c)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,a^3\cos(dx+c)^8 - 4\,a^3\cos(dx+c)^6 + \left(a^3\cos(dx+c)^8 - 4\,a^3\cos(dx+c)^6\right)\sin(dx+c)\right)\sin(dx+c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + (a^3*cos(d*x + c)^
8 - 4*a^3*cos(d*x + c)^6)*sin(d*x + c))*sin(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.654 $\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=200

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*Cos[c
+ d*x]*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (a^2*Cos[c + d*x]*Hyperg
eometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 + n
))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.242237, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2873, 2577}

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*Cos[c
+ d*x]*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (a^2*Cos[c + d*x]*Hyperg
eometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 + n
))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2])
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^n(c+dx)+2a^2\cos^6(c+dx)\sin^{1+n}(c+d\right.$$
$$= a^2\int \cos^6(c+dx)\sin^n(c+dx)\,dx + a^2\int \cos^6(c+dx)\sin^{2+n}(c$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}} +$$

Mathematica [A] time = 0.282645, size = 164, normalized size = 0.82

$$\frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{n+1}(c+dx)\left(\left(n^2+5n+6\right){}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+(n+1)\sin(c+dx)\left(2(n\right.\right.}{d(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*((6 + 5*n + n^2
)*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + (1 + n)*S
in[c + d*x]*(2*(3 + n)*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2] + (2 + n)*Hypergeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d
*x]^2]*Sin[c + d*x])))/(d*(1 + n)*(2 + n)*(3 + n))
```

Maple [F] time = 10.043, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^6 (\sin(dx+c))^n (a+a\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^2 \sin(dx+c)^n \cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2\cos(dx+c)^8 - 2a^2\cos(dx+c)^6\sin(dx+c) - 2a^2\cos(dx+c)^6\right)\sin(dx+c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-(a^2*cos(d*x + c)^8 - 2*a^2*cos(d*x + c)^6*sin(d*x + c) - 2*a^2*c
os(d*x + c)^6)*sin(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^2\sin(dx+c)^n\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

3.655 $\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (a*Cos[c + d*x]
*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.136853, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2838, 2577}

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (a*Cos[c + d*x]
*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin^n(c+dx)\,dx + a\int \cos^6(c+dx)\sin^{1+n}(c+d$$

$$= \frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}+\frac{a}{}$$

Mathematica [F] time = 0.297752, size = 0, normalized size = 0.

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x]), x]
```

Maple [F] time = 5.675, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^{6} (\sin (dx+c))^{n} (a+a\sin (dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin (dx+c)+a)\sin (dx+c)^{n}\cos (dx+c)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cos (dx+c)^{6}\sin (dx+c)+a\cos (dx+c)^{6}\right)\sin (dx+c)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^6*sin(d*x + c) + a*cos(d*x + c)^6)*sin(d*x + c)^n,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (dx + c) + a) \sin (dx + c)^n \cos (dx + c)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

3.656 $\int \cos^7(c+dx)\sin^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$-\frac{a\sin^{14}(c+dx)}{14d}-\frac{a\sin^{13}(c+dx)}{13d}+\frac{a\sin^{12}(c+dx)}{4d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{3a\sin^{10}(c+dx)}{10d}-\frac{a\sin^{9}(c+dx)}{3d}+\frac{a\sin^{8}(c+dx)}{8d}$$

```
[Out] (a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^8)/(8*d) - (a*Sin[c + d*x]^9)/(3
*d) - (3*a*Sin[c + d*x]^10)/(10*d) + (3*a*Sin[c + d*x]^11)/(11*d) + (a*Sin[
c + d*x]^12)/(4*d) - (a*Sin[c + d*x]^13)/(13*d) - (a*Sin[c + d*x]^14)/(14*d
)
```

Rubi [A] time = 0.100681, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^{14}(c+dx)}{14d}-\frac{a\sin^{13}(c+dx)}{13d}+\frac{a\sin^{12}(c+dx)}{4d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{3a\sin^{10}(c+dx)}{10d}-\frac{a\sin^{9}(c+dx)}{3d}+\frac{a\sin^{8}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^8)/(8*d) - (a*Sin[c + d*x]^9)/(3
*d) - (3*a*Sin[c + d*x]^10)/(10*d) + (3*a*Sin[c + d*x]^11)/(11*d) + (a*Sin[
c + d*x]^12)/(4*d) - (a*Sin[c + d*x]^13)/(13*d) - (a*Sin[c + d*x]^14)/(14*d
)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^7(c+dx)\sin^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^6(a+x)^4}{a^6}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^6(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^{13}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^7x^6+a^6x^7-3a^5x^8-3a^4x^9+3a^3x^{10}+3a^2x^{11}-ax^{12}\right.\right.}{a^{13}d}$$
$$= \frac{a\sin^7(c+dx)}{7d}+\frac{a\sin^8(c+dx)}{8d}-\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^{10}(c+d}{10d}$$

Mathematica [A] time = 1.03223, size = 117, normalized size = 0.91

$$-\frac{a(-1201200\sin(c+dx)+300300\sin(3(c+dx))+180180\sin(5(c+dx))-51480\sin(7(c+dx))-40040\sin(9(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(525525*Cos[2*(c + d*x)] - 105105*Cos[6*(c + d*x)] + 21021*Cos[10*(c +
d*x)] - 2145*Cos[14*(c + d*x)] - 1201200*Sin[c + d*x] + 300300*Sin[3*(c + d
*x)] + 180180*Sin[5*(c + d*x)] - 51480*Sin[7*(c + d*x)] - 40040*Sin[9*(c +
d*x)] + 5460*Sin[11*(c + d*x)] + 4620*Sin[13*(c + d*x)]))/(246005760*d)
```

Maple [A] time = 0.035, size = 166, normalized size = 1.3

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^6(\cos(dx+c))^8}{14}-\frac{(\sin(dx+c))^4(\cos(dx+c))^8}{28}-\frac{(\sin(dx+c))^2(\cos(dx+c))^8}{70}-\frac{(\cos(dx+c)}{280}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/14*sin(d*x+c)^6*cos(d*x+c)^8-1/28*sin(d*x+c)^4*cos(d*x+c)^8-1/70
*sin(d*x+c)^2*cos(d*x+c)^8-1/280*cos(d*x+c)^8)+a*(-1/13*sin(d*x+c)^5*cos(d*
x+c)^8-5/143*sin(d*x+c)^3*cos(d*x+c)^8-5/429*sin(d*x+c)*cos(d*x+c)^8+5/3003
*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.05377, size = 127, normalized size = 0.98

$$-\frac{8580\,a\sin(dx+c)^{14}+9240\,a\sin(dx+c)^{13}-30030\,a\sin(dx+c)^{12}-32760\,a\sin(dx+c)^{11}+36036\,a\sin(dx+c)^{1}}{120120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/120120*(8580*a*sin(d*x + c)^14 + 9240*a*sin(d*x + c)^13 - 30030*a*sin(d*
x + c)^12 - 32760*a*sin(d*x + c)^11 + 36036*a*sin(d*x + c)^10 + 40040*a*sin
(d*x + c)^9 - 15015*a*sin(d*x + c)^8 - 17160*a*sin(d*x + c)^7)/d
```

Fricas [A] time = 1.68238, size = 369, normalized size = 2.86

$$\frac{8580\,a\cos\left(dx+c\right)^{14}-30030\,a\cos\left(dx+c\right)^{12}+36036\,a\cos\left(dx+c\right)^{10}-15015\,a\cos\left(dx+c\right)^{8}-40\left(231\,a\cos\left(dx+c\right)^{12}\right.}{12012}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120120*(8580*a*cos(d*x + c)^14 - 30030*a*cos(d*x + c)^12 + 36036*a*cos(d*
x + c)^10 - 15015*a*cos(d*x + c)^8 - 40*(231*a*cos(d*x + c)^12 - 567*a*cos(
d*x + c)^10 + 371*a*cos(d*x + c)^8 - 5*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^
4 - 8*a*cos(d*x + c)^2 - 16*a)*sin(d*x + c))/d
```

Sympy [A] time = 175.577, size = 184, normalized size = 1.43

$$\begin{cases} \frac{16a\sin^{13}(c+dx)}{3003d}+\frac{8a\sin^{11}(c+dx)\cos^{2}(c+dx)}{231d}+\frac{2a\sin^{9}(c+dx)\cos^{4}(c+dx)}{21d}+\frac{a\sin^{7}(c+dx)\cos^{6}(c+dx)}{7d}-\frac{a\sin^{6}(c+dx)\cos^{8}(c+dx)}{8d}-\frac{3a\sin^{4}(c+dx)\cos}{40d} \\ x\left(a\sin(c)+a\right)\sin^{6}(c)\cos^{7}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**13/(3003*d) + 8*a*sin(c + d*x)**11*cos(c + d*
x)**2/(231*d) + 2*a*sin(c + d*x)**9*cos(c + d*x)**4/(21*d) + a*sin(c + d*x)
**7*cos(c + d*x)**6/(7*d) - a*sin(c + d*x)**6*cos(c + d*x)**8/(8*d) - 3*a*s
in(c + d*x)**4*cos(c + d*x)**10/(40*d) - a*sin(c + d*x)**2*cos(c + d*x)**12
/(40*d) - a*cos(c + d*x)**14/(280*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**
6*cos(c)**7, True))
```

Giac [A] time = 1.38966, size = 220, normalized size = 1.71

$$\frac{a\cos\left(14\,dx+14\,c\right)}{114688\,d}-\frac{7\,a\cos\left(10\,dx+10\,c\right)}{81920\,d}+\frac{7\,a\cos\left(6\,dx+6\,c\right)}{16384\,d}-\frac{35\,a\cos\left(2\,dx+2\,c\right)}{16384\,d}-\frac{a\sin\left(13\,dx+13\,c\right)}{53248\,d}-\frac{a\sin\left(11\right.}{450}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/114688*a*cos(14*d*x + 14*c)/d - 7/81920*a*cos(10*d*x + 10*c)/d + 7/16384*
a*cos(6*d*x + 6*c)/d - 35/16384*a*cos(2*d*x + 2*c)/d - 1/53248*a*sin(13*d*x
 + 13*c)/d - 1/45056*a*sin(11*d*x + 11*c)/d + 1/6144*a*sin(9*d*x + 9*c)/d +
 3/14336*a*sin(7*d*x + 7*c)/d - 3/4096*a*sin(5*d*x + 5*c)/d - 5/4096*a*sin(
3*d*x + 3*c)/d + 5/1024*a*sin(d*x + c)/d
```

3.657 $\int \cos^7(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\sin^{13}(c+dx)}{13d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{a\sin^9(c+dx)}{3d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^8(c}{8d}$$

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(5*d) - (a*Cos[c + d*x]^12)
/(12*d) + (a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^9)/(3*d) + (3*a*Sin[c
+ d*x]^11)/(11*d) - (a*Sin[c + d*x]^13)/(13*d)
```

Rubi [A] time = 0.139859, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2565, 266, 43, 2564, 270}

$$-\frac{a\sin^{13}(c+dx)}{13d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{a\sin^9(c+dx)}{3d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^8(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```


```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(5*d) - (a*Cos[c + d*x]^12)
/(12*d) + (a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^9)/(3*d) + (3*a*Sin[c
+ d*x]^11)/(11*d) - (a*Sin[c + d*x]^13)/(13*d)
```


Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin^5(c+dx)\,dx + a\int \cos^7(c+dx)\sin^6(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^6\left(1-x^2\right.\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \cos^2(c+dx)\right)}{2d} + \frac{a\,\text{Subst}\left(\int \left(x^6-3x^8+\right.\right.}{}$$
$$= \frac{a\sin^7(c+dx)}{7d} - \frac{a\sin^9(c+dx)}{3d} + \frac{3a\sin^{11}(c+dx)}{11d} - \frac{a\sin^{13}(c+dx)}{13d}$$
$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{5d} - \frac{a\cos^{12}(c+dx)}{12d} + \frac{a\sin^7(c+dx}{7d}$$

Mathematica [A] time = 0.724221, size = 137, normalized size = 1.21

$$-\frac{a(-600600\sin(c+dx)+150150\sin(3(c+dx))+90090\sin(5(c+dx))-25740\sin(7(c+dx))-20020\sin(9(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(600600*Cos[2*(c + d*x)] + 75075*Cos[4*(c + d*x)] - 100100*Cos[6*(c + d
*x)] - 30030*Cos[8*(c + d*x)] + 12012*Cos[10*(c + d*x)] + 5005*Cos[12*(c +
d*x)] - 600600*Sin[c + d*x] + 150150*Sin[3*(c + d*x)] + 90090*Sin[5*(c + d*
x)] - 25740*Sin[7*(c + d*x)] - 20020*Sin[9*(c + d*x)] + 2730*Sin[11*(c + d*
x)] + 2310*Sin[13*(c + d*x)]))/(123002880*d)
```

Maple [A] time = 0.034, size = 148, normalized size = 1.3

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^5(\cos(dx+c))^8}{13} - \frac{5(\sin(dx+c))^3(\cos(dx+c))^8}{143} - \frac{5\sin(dx+c)(\cos(dx+c))^8}{429} + \frac{5\sin(dx+c)}{3003}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/13*sin(d*x+c)^5*cos(d*x+c)^8-5/143*sin(d*x+c)^3*cos(d*x+c)^8-5/4
29*sin(d*x+c)*cos(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*c
os(d*x+c)^2)*sin(d*x+c))+a*(-1/12*sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)
^2*cos(d*x+c)^8-1/120*cos(d*x+c)^8))
```

Maxima [A] time = 1.04414, size = 127, normalized size = 1.12

$$-\frac{9240\,a\sin\left(dx+c\right)^{13}+10010\,a\sin\left(dx+c\right)^{12}-32760\,a\sin\left(dx+c\right)^{11}-36036\,a\sin\left(dx+c\right)^{10}+40040\,a\sin\left(dx+c\right)}{120120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/120120*(9240*a*sin(d*x + c)^13 + 10010*a*sin(d*x + c)^12 - 32760*a*sin(d
*x + c)^11 - 36036*a*sin(d*x + c)^10 + 40040*a*sin(d*x + c)^9 + 45045*a*sin
(d*x + c)^8 - 17160*a*sin(d*x + c)^7 - 20020*a*sin(d*x + c)^6)/d
```

Fricas [A] time = 1.64133, size = 336, normalized size = 2.97

$$-\frac{10010\,a\cos\left(dx+c\right)^{12}-24024\,a\cos\left(dx+c\right)^{10}+15015\,a\cos\left(dx+c\right)^{8}+40\left(231\,a\cos\left(dx+c\right)^{12}-567\,a\cos\left(dx+c\right)\right.}{120120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/120120*(10010*a*cos(d*x + c)^12 - 24024*a*cos(d*x + c)^10 + 15015*a*cos(
d*x + c)^8 + 40*(231*a*cos(d*x + c)^12 - 567*a*cos(d*x + c)^10 + 371*a*cos(
d*x + c)^8 - 5*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^2 -
 16*a)*sin(d*x + c))/d
```

Sympy [A] time = 117.722, size = 182, normalized size = 1.61

$$\begin{cases} \frac{16a\sin^{13}(c+dx)}{3003d}+\frac{a\sin^{12}(c+dx)}{120d}+\frac{8a\sin^{11}(c+dx)\cos^{2}(c+dx)}{231d}+\frac{a\sin^{10}(c+dx)\cos^{2}(c+dx)}{20d}+\frac{2a\sin^{9}(c+dx)\cos^{4}(c+dx)}{21d}+\frac{a\sin^{8}(c+dx)\cos^{4}(c+}{8d} \\ x\left(a\sin\left(c\right)+a\right)\sin^{5}\left(c\right)\cos^{7}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**13/(3003*d) + a*sin(c + d*x)**12/(120*d) + 8*
a*sin(c + d*x)**11*cos(c + d*x)**2/(231*d) + a*sin(c + d*x)**10*cos(c + d*x
)**2/(20*d) + 2*a*sin(c + d*x)**9*cos(c + d*x)**4/(21*d) + a*sin(c + d*x)**
8*cos(c + d*x)**4/(8*d) + a*sin(c + d*x)**7*cos(c + d*x)**6/(7*d) + a*sin(c
 + d*x)**6*cos(c + d*x)**6/(6*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**5*co
s(c)**7, True))
```

Giac [A] time = 1.26667, size = 261, normalized size = 2.31

$$-\frac{a\cos\left(12\,dx+12\,c\right)}{24576\,d}-\frac{a\cos\left(10\,dx+10\,c\right)}{10240\,d}+\frac{a\cos\left(8\,dx+8\,c\right)}{4096\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{6144\,d}-\frac{5\,a\cos\left(4\,dx+4\,c\right)}{8192\,d}-\frac{5\,a\cos(}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24576*a*cos(12*d*x + 12*c)/d - 1/10240*a*cos(10*d*x + 10*c)/d + 1/4096*a
*cos(8*d*x + 8*c)/d + 5/6144*a*cos(6*d*x + 6*c)/d - 5/8192*a*cos(4*d*x + 4*
c)/d - 5/1024*a*cos(2*d*x + 2*c)/d - 1/53248*a*sin(13*d*x + 13*c)/d - 1/450
56*a*sin(11*d*x + 11*c)/d + 1/6144*a*sin(9*d*x + 9*c)/d + 3/14336*a*sin(7*d
*x + 7*c)/d - 3/4096*a*sin(5*d*x + 5*c)/d - 5/4096*a*sin(3*d*x + 3*c)/d + 5
/1024*a*sin(d*x + c)/d
```

3.658 $\int \cos^7(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^8(c+dx)}{8d}$$

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(5*d) - (a*Cos[c + d*x]^12)
/(12*d) + (a*Sin[c + d*x]^5)/(5*d) - (3*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c
+ d*x]^9)/(3*d) - (a*Sin[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.136612, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2564, 270, 2565, 266, 43}

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(5*d) - (a*Cos[c + d*x]^12)
/(12*d) + (a*Sin[c + d*x]^5)/(5*d) - (3*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c
+ d*x]^9)/(3*d) - (a*Sin[c + d*x]^11)/(11*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^7(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin^4(c+dx)\,dx + a\int \cos^7(c+dx)\sin^5(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^4\left(1-x^2\right.\right.}{\ldots}$$
$$= -\frac{a\,\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \cos^2(c+dx)\right)}{2d} + \frac{a\,\text{Subst}\left(\int \left(x^4-3x^6+\ldots\right.\right.}{\ldots}$$
$$= \frac{a\sin^5(c+dx)}{5d} - \frac{3a\sin^7(c+dx)}{7d} + \frac{a\sin^9(c+dx)}{3d} - \frac{a\sin^{11}(c+dx)}{11d} - \ldots$$
$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{5d} - \frac{a\cos^{12}(c+dx)}{12d} + \frac{a\sin^5(c+dx)}{5d}\ldots$$

Mathematica [A] time = 0.558947, size = 127, normalized size = 1.12

$$-\frac{a(-129360\sin(c+dx)+18480\sin(3(c+dx))+20328\sin(5(c+dx))+1320\sin(7(c+dx))-3080\sin(9(c+dx))-840\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(46200*Cos[2*(c + d*x)] + 5775*Cos[4*(c + d*x)] - 7700*Cos[6*(c + d*x)]
 - 2310*Cos[8*(c + d*x)] + 924*Cos[10*(c + d*x)] + 385*Cos[12*(c + d*x)] -
129360*Sin[c + d*x] + 18480*Sin[3*(c + d*x)] + 20328*Sin[5*(c + d*x)] + 132
0*Sin[7*(c + d*x)] - 3080*Sin[9*(c + d*x)] - 840*Sin[11*(c + d*x)]))/(94617
60*d)
```

Maple [A] time = 0.036, size = 130, normalized size = 1.2

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^8}{12} - \frac{(\sin(dx+c))^2(\cos(dx+c))^8}{30} - \frac{(\cos(dx+c))^8}{120}\right) + a\left(-\frac{(\sin(dx+c))^3(\cos(dx+\ldots}{11}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/12*sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)^2*cos(d*x+c)^8-1/12
0*cos(d*x+c)^8)+a*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*cos(d*x+
c)^8+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
))
```

Maxima [A] time = 0.998697, size = 127, normalized size = 1.12

$$-\frac{770\,a\sin\left(dx+c\right)^{12}+840\,a\sin\left(dx+c\right)^{11}-2772\,a\sin\left(dx+c\right)^{10}-3080\,a\sin\left(dx+c\right)^{9}+3465\,a\sin\left(dx+c\right)^{8}+396}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/9240*(770*a*sin(d*x + c)^12 + 840*a*sin(d*x + c)^11 - 2772*a*sin(d*x + c
)^10 - 3080*a*sin(d*x + c)^9 + 3465*a*sin(d*x + c)^8 + 3960*a*sin(d*x + c)^
7 - 1540*a*sin(d*x + c)^6 - 1848*a*sin(d*x + c)^5)/d
```

Fricas [A] time = 1.60779, size = 294, normalized size = 2.6

$$-\frac{770\,a\cos\left(dx+c\right)^{12}-1848\,a\cos\left(dx+c\right)^{10}+1155\,a\cos\left(dx+c\right)^{8}-8\left(105\,a\cos\left(dx+c\right)^{10}-140\,a\cos\left(dx+c\right)^{8}+5\,a}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/9240*(770*a*cos(d*x + c)^12 - 1848*a*cos(d*x + c)^10 + 1155*a*cos(d*x +
c)^8 - 8*(105*a*cos(d*x + c)^10 - 140*a*cos(d*x + c)^8 + 5*a*cos(d*x + c)^6
 + 6*a*cos(d*x + c)^4 + 8*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

Sympy [A] time = 82.1131, size = 182, normalized size = 1.61

$$\begin{cases} \frac{a\sin^{12}(c+dx)}{120d}+\frac{16a\sin^{11}(c+dx)}{1155d}+\frac{a\sin^{10}(c+dx)\cos^{2}(c+dx)}{20d}+\frac{8a\sin^{9}(c+dx)\cos^{2}(c+dx)}{105d}+\frac{a\sin^{8}(c+dx)\cos^{4}(c+dx)}{8d}+\frac{6a\sin^{7}(c+dx)\cos^{4}(c+d}{35d} \\ x\left(a\sin\left(c\right)+a\right)\sin^{4}\left(c\right)\cos^{7}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**12/(120*d) + 16*a*sin(c + d*x)**11/(1155*d) + a*
sin(c + d*x)**10*cos(c + d*x)**2/(20*d) + 8*a*sin(c + d*x)**9*cos(c + d*x)*
*2/(105*d) + a*sin(c + d*x)**8*cos(c + d*x)**4/(8*d) + 6*a*sin(c + d*x)**7*
cos(c + d*x)**4/(35*d) + a*sin(c + d*x)**6*cos(c + d*x)**6/(6*d) + a*sin(c
+ d*x)**5*cos(c + d*x)**6/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**4*cos
(c)**7, True))
```

Giac [A] time = 1.31886, size = 240, normalized size = 2.12

$$-\frac{a\cos\left(12\,dx+12\,c\right)}{24576\,d}-\frac{a\cos\left(10\,dx+10\,c\right)}{10240\,d}+\frac{a\cos\left(8\,dx+8\,c\right)}{4096\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{6144\,d}-\frac{5\,a\cos\left(4\,dx+4\,c\right)}{8192\,d}-\frac{5\,a\cos\left(\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24576*a*cos(12*d*x + 12*c)/d - 1/10240*a*cos(10*d*x + 10*c)/d + 1/4096*a
*cos(8*d*x + 8*c)/d + 5/6144*a*cos(6*d*x + 6*c)/d - 5/8192*a*cos(4*d*x + 4*
c)/d - 5/1024*a*cos(2*d*x + 2*c)/d + 1/11264*a*sin(11*d*x + 11*c)/d + 1/307
2*a*sin(9*d*x + 9*c)/d - 1/7168*a*sin(7*d*x + 7*c)/d - 11/5120*a*sin(5*d*x
+ 5*c)/d - 1/512*a*sin(3*d*x + 3*c)/d + 7/512*a*sin(d*x + c)/d
```

3.659 $\int \cos^7(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(10*d) + (a*Sin[c + d*x]^5)
/(5*d) - (3*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^9)/(3*d) - (a*Sin[c +
 d*x]^11)/(11*d)
```

Rubi [A] time = 0.131623, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2565, 14, 2564, 270}

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(10*d) + (a*Sin[c + d*x]^5)
/(5*d) - (3*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^9)/(3*d) - (a*Sin[c +
 d*x]^11)/(11*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin^3(c+dx)\,dx + a\int \cos^7(c+dx)\sin^4(c+dx)\,dx$$

$$= -\frac{a\,\text{Subst}\left(\int x^7\left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^4\left(1-x^2\right)\right.}{d}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(x^7-x^9\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \left(x^4-3x^6+\right.\right.}{}$$

$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{10d} + \frac{a\sin^5(c+dx)}{5d} - \frac{3a\sin^7(c+dx}{7d}$$

Mathematica [A] time = 0.605329, size = 117, normalized size = 1.21

$$\frac{a(16170\sin(c+dx) - 2310\sin(3(c+dx)) - 2541\sin(5(c+dx)) - 165\sin(7(c+dx)) + 385\sin(9(c+dx)) + 105\sin(11(c}{11827}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-16170*Cos[2*(c + d*x)] - 4620*Cos[4*(c + d*x)] + 1155*Cos[6*(c + d*x)]
 + 1155*Cos[8*(c + d*x)] + 231*Cos[10*(c + d*x)] + 16170*Sin[c + d*x] - 231
0*Sin[3*(c + d*x)] - 2541*Sin[5*(c + d*x)] - 165*Sin[7*(c + d*x)] + 385*Sin
[9*(c + d*x)] + 105*Sin[11*(c + d*x)]))/(1182720*d)
```

Maple [A] time = 0.036, size = 112, normalized size = 1.2

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^8}{11} - \frac{\sin(dx+c)(\cos(dx+c))^8}{33} + \frac{\sin(dx+c)}{231}\left(\frac{16}{5} + (\cos(dx+c))^6 + \frac{6\,(\cos(dx+c)}{5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*cos(d*x+c)^8+1/231*
(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/10*
sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x+c)^8))
```

Maxima [A] time = 1.02365, size = 127, normalized size = 1.31

$$-\frac{840\,a\sin(dx+c)^{11} + 924\,a\sin(dx+c)^{10} - 3080\,a\sin(dx+c)^9 - 3465\,a\sin(dx+c)^8 + 3960\,a\sin(dx+c)^7 + 4620\,a\,\mathrm{s}}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/9240*(840*a*sin(d*x + c)^11 + 924*a*sin(d*x + c)^10 - 3080*a*sin(d*x + c
)^9 - 3465*a*sin(d*x + c)^8 + 3960*a*sin(d*x + c)^7 + 4620*a*sin(d*x + c)^6
 - 1848*a*sin(d*x + c)^5 - 2310*a*sin(d*x + c)^4)/d
```

Fricas [A] time = 1.60166, size = 259, normalized size = 2.67

$$\frac{924\,a\cos\left(dx+c\right)^{10}-1155\,a\cos\left(dx+c\right)^{8}+8\left(105\,a\cos\left(dx+c\right)^{10}-140\,a\cos\left(dx+c\right)^{8}+5\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(\right.\right.}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9240*(924*a*cos(d*x + c)^10 - 1155*a*cos(d*x + c)^8 + 8*(105*a*cos(d*x +
c)^10 - 140*a*cos(d*x + c)^8 + 5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 + 8*
a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

Sympy [A] time = 59.8253, size = 138, normalized size = 1.42

$$\begin{cases}\frac{16a\sin^{11}(c+dx)}{1155d}+\frac{8a\sin^{9}(c+dx)\cos^{2}(c+dx)}{105d}+\frac{6a\sin^{7}(c+dx)\cos^{4}(c+dx)}{35d}+\frac{a\sin^{5}(c+dx)\cos^{6}(c+dx)}{5d}-\frac{a\sin^{2}(c+dx)\cos^{8}(c+dx)}{8d}-\frac{a\cos^{10}(c+dx}{40d} \\ x\left(a\sin(c)+a\right)\sin^{3}(c)\cos^{7}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**11/(1155*d) + 8*a*sin(c + d*x)**9*cos(c + d*x
)**2/(105*d) + 6*a*sin(c + d*x)**7*cos(c + d*x)**4/(35*d) + a*sin(c + d*x)*
*5*cos(c + d*x)**6/(5*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - a*cos(
c + d*x)**10/(40*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**7, True
))
```

Giac [A] time = 1.25992, size = 220, normalized size = 2.27

$$\frac{a\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{a\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{a\cos\left(6\,dx+6\,c\right)}{1024\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{7\,a\cos\left(2\,dx+2\,c\right)}{512\,d}+\frac{a\sin\left(11\,dx+\right.}{11264\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/5120*a*cos(10*d*x + 10*c)/d + 1/1024*a*cos(8*d*x + 8*c)/d + 1/1024*a*cos(
6*d*x + 6*c)/d - 1/256*a*cos(4*d*x + 4*c)/d - 7/512*a*cos(2*d*x + 2*c)/d +
1/11264*a*sin(11*d*x + 11*c)/d + 1/3072*a*sin(9*d*x + 9*c)/d - 1/7168*a*sin
(7*d*x + 7*c)/d - 11/5120*a*sin(5*d*x + 5*c)/d - 1/512*a*sin(3*d*x + 3*c)/d
 + 7/512*a*sin(d*x + c)/d
```

3.660 $\int \cos^7(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(10*d) + (a*Sin[c + d*x]^3)
/(3*d) - (3*a*Sin[c + d*x]^5)/(5*d) + (3*a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c
 + d*x]^9)/(9*d)
```

Rubi [A] time = 0.128225, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2564, 270, 2565, 14}

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Cos[c + d*x]^10)/(10*d) + (a*Sin[c + d*x]^3)
/(3*d) - (3*a*Sin[c + d*x]^5)/(5*d) + (3*a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c
 + d*x]^9)/(9*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin^2(c+dx)\,dx + a\int \cos^7(c+dx)\sin^3(c+dx)$$

$$= -\frac{a\,\text{Subst}\left(\int x^7\left(1-x^2\right)dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\left(1-\right.\right.}{}$$

$$= \frac{a\,\text{Subst}\left(\int\left(x^2-3x^4+3x^6-x^8\right)dx, x, \sin(c+dx)\right)}{d} - a\,\text{Subst}\left(\right.$$

$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{10d} + \frac{a\sin^3(c+dx)}{3d} - \frac{3a\sin^5(c+}{5d}$$

Mathematica [A] time = 0.464554, size = 97, normalized size = 1.

$$-\frac{a(-17640\sin(c+dx)+2016\sin(5(c+dx))+900\sin(7(c+dx))+140\sin(9(c+dx))+4410\cos(2(c+dx))+1260}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(4410*Cos[2*(c + d*x)] + 1260*Cos[4*(c + d*x)] - 315*Cos[6*(c + d*x)] -
 315*Cos[8*(c + d*x)] - 63*Cos[10*(c + d*x)] - 17640*Sin[c + d*x] + 2016*Si
n[5*(c + d*x)] + 900*Sin[7*(c + d*x)] + 140*Sin[9*(c + d*x)]))/(322560*d)
```

Maple [A] time = 0.029, size = 94, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^8}{10}-\frac{(\cos(dx+c))^8}{40}\right)+a\left(-\frac{\sin(dx+c)(\cos(dx+c))^8}{9}+\frac{\sin(dx+c)}{63}\left(\frac{16}{5}+(\cos(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x+c)^8)+a*(-1/9*sin(d*x+
c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*
sin(d*x+c)))
```

Maxima [A] time = 1.05781, size = 127, normalized size = 1.31

$$-\frac{252\,a\sin(dx+c)^{10}+280\,a\sin(dx+c)^9-945\,a\sin(dx+c)^8-1080\,a\sin(dx+c)^7+1260\,a\sin(dx+c)^6+1512\,a\,s}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(252*a*sin(d*x + c)^10 + 280*a*sin(d*x + c)^9 - 945*a*sin(d*x + c)^
8 - 1080*a*sin(d*x + c)^7 + 1260*a*sin(d*x + c)^6 + 1512*a*sin(d*x + c)^5 -
 630*a*sin(d*x + c)^4 - 840*a*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.36665, size = 224, normalized size = 2.31

$$\frac{252\,a\cos\left(dx+c\right)^{10}-315\,a\cos\left(dx+c\right)^{8}-8\left(35\,a\cos\left(dx+c\right)^{8}-5\,a\cos\left(dx+c\right)^{6}-6\,a\cos\left(dx+c\right)^{4}-8\,a\cos\left(dx+c\right)^{2}\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(252*a*cos(d*x + c)^10 - 315*a*cos(d*x + c)^8 - 8*(35*a*cos(d*x + c)
^8 - 5*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^2 - 16*a)*s
in(d*x + c))/d
```

Sympy [A] time = 33.7869, size = 138, normalized size = 1.42

$$\begin{cases}\frac{16a\sin^{9}(c+dx)}{315d}+\frac{8a\sin^{7}(c+dx)\cos^{2}(c+dx)}{35d}+\frac{2a\sin^{5}(c+dx)\cos^{4}(c+dx)}{5d}+\frac{a\sin^{3}(c+dx)\cos^{6}(c+dx)}{3d}-\frac{a\sin^{2}(c+dx)\cos^{8}(c+dx)}{8d}-\frac{a\cos^{10}(c+dx)}{40d} & \\ x\left(a\sin\left(c\right)+a\right)\sin^{2}\left(c\right)\cos^{7}\left(c\right) & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**9/(315*d) + 8*a*sin(c + d*x)**7*cos(c + d*x)*
*2/(35*d) + 2*a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + a*sin(c + d*x)**3*c
os(c + d*x)**6/(3*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - a*cos(c +
d*x)**10/(40*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**7, True))
```

Giac [A] time = 1.15064, size = 180, normalized size = 1.86

$$\frac{a\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{a\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{a\cos\left(6\,dx+6\,c\right)}{1024\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{7\,a\cos\left(2\,dx+2\,c\right)}{512\,d}-\frac{a\sin\left(9\,dx+9\,c\right)}{2304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/5120*a*cos(10*d*x + 10*c)/d + 1/1024*a*cos(8*d*x + 8*c)/d + 1/1024*a*cos(
6*d*x + 6*c)/d - 1/256*a*cos(4*d*x + 4*c)/d - 7/512*a*cos(2*d*x + 2*c)/d -
1/2304*a*sin(9*d*x + 9*c)/d - 5/1792*a*sin(7*d*x + 7*c)/d - 1/160*a*sin(5*d
*x + 5*c)/d + 7/128*a*sin(d*x + c)/d
```

3.661 $\int \cos^7(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^8(c+dx)}{8d}$$

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(3*d) - (3*a*Sin[c + d*x]^5)
/(5*d) + (3*a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.08965, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2834, 2565, 30, 2564, 270}

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(3*d) - (3*a*Sin[c + d*x]^5)
/(5*d) + (3*a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^9)/(9*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin(c+dx)\,dx + a\int \cos^7(c+dx)\sin^2(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^7\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int x^2\left(1-x^2\right)^3\,dx, x, \text{si}\right.}{d}$$
$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\,\text{Subst}\left(\int \left(x^2-3x^4+3x^6-x^8\right)dx, x, \sin(c+dx\right.}{d}$$
$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\sin^3(c+dx)}{3d} - \frac{3a\sin^5(c+dx)}{5d} + \frac{3a\sin^7(c+dx)}{7d}$$

Mathematica [A] time = 0.380543, size = 60, normalized size = 0.74

$$\frac{a\left(\sin^3(c+dx)(1389\cos(2(c+dx))+330\cos(4(c+dx))+35\cos(6(c+dx))+1606)-1260\cos^8(c+dx)\right)}{10080d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-1260*Cos[c + d*x]^8 + (1606 + 1389*Cos[2*(c + d*x)] + 330*Cos[4*(c + d
*x)] + 35*Cos[6*(c + d*x)])*Sin[c + d*x]^3))/(10080*d)
```

Maple [A] time = 0.025, size = 74, normalized size = 0.9

$$\frac{1}{d}\left(a\left(-\frac{\sin(dx+c)(\cos(dx+c))^8}{9} + \frac{\sin(dx+c)}{63}\left(\frac{16}{5} + (\cos(dx+c))^6 + \frac{6(\cos(dx+c))^4}{5} + \frac{8(\cos(dx+c))^2}{5}\right)\right) - \frac{a(\text{co}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)
^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*a*cos(d*x+c)^8)
```

Maxima [A] time = 1.00755, size = 127, normalized size = 1.57

$$-\frac{280\,a\sin(dx+c)^9 + 315\,a\sin(dx+c)^8 - 1080\,a\sin(dx+c)^7 - 1260\,a\sin(dx+c)^6 + 1512\,a\sin(dx+c)^5 + 1890\,a\sin(}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*a*sin(d*x + c)^9 + 315*a*sin(d*x + c)^8 - 1080*a*sin(d*x + c)^
7 - 1260*a*sin(d*x + c)^6 + 1512*a*sin(d*x + c)^5 + 1890*a*sin(d*x + c)^4 -
 840*a*sin(d*x + c)^3 - 1260*a*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.48095, size = 193, normalized size = 2.38

$$-\frac{315\,a\cos\left(dx+c\right)^{8}+8\left(35\,a\cos\left(dx+c\right)^{8}-5\,a\cos\left(dx+c\right)^{6}-6\,a\cos\left(dx+c\right)^{4}-8\,a\cos\left(dx+c\right)^{2}-16\,a\right)\sin\left(dx+c\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2520*(315*a*cos(d*x + c)^8 + 8*(35*a*cos(d*x + c)^8 - 5*a*cos(d*x + c)^6
 - 6*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^2 - 16*a)*sin(d*x + c))/d
```

Sympy [A] time = 21.0276, size = 114, normalized size = 1.41

$$\begin{cases} \frac{16a\sin^{9}(c+dx)}{315d}+\frac{8a\sin^{7}(c+dx)\cos^{2}(c+dx)}{35d}+\frac{2a\sin^{5}(c+dx)\cos^{4}(c+dx)}{5d}+\frac{a\sin^{3}(c+dx)\cos^{6}(c+dx)}{3d}-\frac{a\cos^{8}(c+dx)}{8d} & \text{for } d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)\sin\left(c\right)\cos^{7}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**9/(315*d) + 8*a*sin(c + d*x)**7*cos(c + d*x)*
*2/(35*d) + 2*a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + a*sin(c + d*x)**3*c
os(c + d*x)**6/(3*d) - a*cos(c + d*x)**8/(8*d), Ne(d, 0)), (x*(a*sin(c) + a
)*sin(c)*cos(c)**7, True))
```

Giac [A] time = 1.19917, size = 159, normalized size = 1.96

$$-\frac{a\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a\cos\left(6\,dx+6\,c\right)}{128\,d}-\frac{7\,a\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{7\,a\cos\left(2\,dx+2\,c\right)}{128\,d}-\frac{a\sin\left(9\,dx+9\,c\right)}{2304\,d}-\frac{5\,a\sin\left(7\,dx+7\,c\right)}{1792\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1024*a*cos(8*d*x + 8*c)/d - 1/128*a*cos(6*d*x + 6*c)/d - 7/256*a*cos(4*d
*x + 4*c)/d - 7/128*a*cos(2*d*x + 2*c)/d - 1/2304*a*sin(9*d*x + 9*c)/d - 5/
1792*a*sin(7*d*x + 7*c)/d - 1/160*a*sin(5*d*x + 5*c)/d + 7/128*a*sin(d*x +
c)/d
```

3.662 $\int \cos^6(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{3a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (3*a*Sin[c + d*x]^2)/(2*d) -
 (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^4)/(4*d) + (3*a*Sin[c + d*x]^5)/(
5*d) - (a*Sin[c + d*x]^6)/(6*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0766889, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 88}

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{3a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (3*a*Sin[c + d*x]^2)/(2*d) -
 (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^4)/(4*d) + (3*a*Sin[c + d*x]^5)/(
5*d) - (a*Sin[c + d*x]^6)/(6*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^6(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^3(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6+\frac{a^7}{x}-3a^5x-3a^4x^2+3a^3x^3+3a^2x^4-ax^5-x^6\right) dx\right.}{a^6 d}$$
$$= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}-\frac{a\sin^3(c+}{d}$$

Mathematica [A] time = 0.138611, size = 106, normalized size = 0.9

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\left(-2\sin^6(c+dx)+9\sin^4(c+dx)-18\sin^2(c+\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^5)/(5*d) - (a
*Sin[c + d*x]^7)/(7*d) + (a*(12*Log[Sin[c + d*x]] - 18*Sin[c + d*x]^2 + 9*S
in[c + d*x]^4 - 2*Sin[c + d*x]^6))/(12*d)
```

Maple [A] time = 0.053, size = 128, normalized size = 1.1

$$\frac{16\,a\sin(dx+c)}{35\,d}+\frac{\sin(dx+c)(\cos(dx+c))^6 a}{7\,d}+\frac{6\sin(dx+c)(\cos(dx+c))^4 a}{35\,d}+\frac{8(\cos(dx+c))^2\sin(dx+c)\,a}{35\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 16/35*a*sin(d*x+c)/d+1/7/d*cos(d*x+c)^6*sin(d*x+c)*a+6/35/d*cos(d*x+c)^4*si
n(d*x+c)*a+8/35/d*cos(d*x+c)^2*sin(d*x+c)*a+1/6*a*cos(d*x+c)^6/d+1/4*a*cos(
d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.00251, size = 123, normalized size = 1.04

$$-\frac{60\,a\sin(dx+c)^7+70\,a\sin(dx+c)^6-252\,a\sin(dx+c)^5-315\,a\sin(dx+c)^4+420\,a\sin(dx+c)^3+630\,a\sin(dx}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 252*a*sin(d*x + c)^5 -
315*a*sin(d*x + c)^4 + 420*a*sin(d*x + c)^3 + 630*a*sin(d*x + c)^2 - 420*a*
log(sin(d*x + c)) - 420*a*sin(d*x + c))/d
```

Fricas [A] time = 1.54706, size = 263, normalized size = 2.23

$$\frac{70\,a\cos\left(dx+c\right)^{6}+105\,a\cos\left(dx+c\right)^{4}+210\,a\cos\left(dx+c\right)^{2}+420\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+12\,\left(5\,a\cos\left(dx+c\right)^{6}+6\,a\cos\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*a*cos(d*x + c)^6 + 105*a*cos(d*x + c)^4 + 210*a*cos(d*x + c)^2 +
420*a*log(1/2*sin(d*x + c)) + 12*(5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 +
 8*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17745, size = 124, normalized size = 1.05

$$-\frac{60\,a\sin\left(dx+c\right)^{7}+70\,a\sin\left(dx+c\right)^{6}-252\,a\sin\left(dx+c\right)^{5}-315\,a\sin\left(dx+c\right)^{4}+420\,a\sin\left(dx+c\right)^{3}+630\,a\sin\left(dx+c\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 252*a*sin(d*x + c)^5 -
315*a*sin(d*x + c)^4 + 420*a*sin(d*x + c)^3 + 630*a*sin(d*x + c)^2 - 420*a*
log(abs(sin(d*x + c))) - 420*a*sin(d*x + c))/d
```

3.663 $\int \cos^5(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=114

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (3*a*Sin[c + d*x])/d - (3
*a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^4)/(4*d
) - (a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.0874338, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (3*a*Sin[c + d*x])/d - (3
*a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^4)/(4*d
) - (a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^6)/(6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^5(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^3(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^5 + \frac{a^7}{x^2} + \frac{a^6}{x} - 3a^4x + 3a^3x^2 + 3a^2x^3 - ax^4 - x^5\right) dx, x$$
$$= -\frac{a\csc(c+dx)}{d} + \frac{a\log(\sin(c+dx))}{d} - \frac{3a\sin(c+dx)}{d} - \frac{3a\sin^2(c+$$

Mathematica [A] time = 0.137165, size = 102, normalized size = 0.89

$$-\frac{a\sin^5(c+dx)}{5d} + \frac{a\sin^3(c+dx)}{d} - \frac{3a\sin(c+dx)}{d} - \frac{a\csc(c+dx)}{d} + \frac{a\left(-2\sin^6(c+dx) + 9\sin^4(c+dx) - 18\sin^2(c+dx\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (3*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^3)/d - (a*Si
n[c + d*x]^5)/(5*d) + (a*(12*Log[Sin[c + d*x]] - 18*Sin[c + d*x]^2 + 9*Sin[
c + d*x]^4 - 2*Sin[c + d*x]^6))/(12*d)
```

Maple [A] time = 0.052, size = 150, normalized size = 1.3

$$\frac{a(\cos(dx+c))^6}{6d} + \frac{a(\cos(dx+c))^4}{4d} + \frac{a(\cos(dx+c))^2}{2d} + \frac{a\ln(\sin(dx+c))}{d} - \frac{a(\cos(dx+c))^8}{d\sin(dx+c)} - \frac{16a\sin(dx+c)}{5d} - \text{sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/6*a*cos(d*x+c)^6/d+1/4*a*cos(d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x
+c))/d-1/d*a/sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*s
in(d*x+c)*a-6/5/d*cos(d*x+c)^4*sin(d*x+c)*a-8/5/d*cos(d*x+c)^2*sin(d*x+c)*a
```

Maxima [A] time = 1.04824, size = 123, normalized size = 1.08

$$-\frac{10a\sin(dx+c)^6 + 12a\sin(dx+c)^5 - 45a\sin(dx+c)^4 - 60a\sin(dx+c)^3 + 90a\sin(dx+c)^2 - 60a\log(\sin(dx+c)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 45*a*sin(d*x + c)^4 - 60
*a*sin(d*x + c)^3 + 90*a*sin(d*x + c)^2 - 60*a*log(sin(d*x + c)) + 180*a*si
n(d*x + c) + 60*a/sin(d*x + c))/d
```

Fricas [A] time = 1.63326, size = 312, normalized size = 2.74

$$\frac{48\,a\cos\left(dx+c\right)^{6}+96\,a\cos\left(dx+c\right)^{4}+384\,a\cos\left(dx+c\right)^{2}+240\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+5\left(8\,a\cos\left(dx+\right.\right.}{240\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^6 + 96*a*cos(d*x + c)^4 + 384*a*cos(d*x + c)^2 + 2
40*a*log(1/2*sin(d*x + c))*sin(d*x + c) + 5*(8*a*cos(d*x + c)^6 + 12*a*cos(
d*x + c)^4 + 24*a*cos(d*x + c)^2 - 19*a)*sin(d*x + c) - 768*a)/(d*sin(d*x +
 c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17089, size = 136, normalized size = 1.19

$$-\frac{10\,a\sin\left(dx+c\right)^{6}+12\,a\sin\left(dx+c\right)^{5}-45\,a\sin\left(dx+c\right)^{4}-60\,a\sin\left(dx+c\right)^{3}+90\,a\sin\left(dx+c\right)^{2}-60\,a\log\left(\left|\sin\left(dx\right.\right.\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 45*a*sin(d*x + c)^4 - 60
*a*sin(d*x + c)^3 + 90*a*sin(d*x + c)^2 - 60*a*log(abs(sin(d*x + c))) + 180
*a*sin(d*x + c) + 60*(a*sin(d*x + c) + a)/sin(d*x + c))/d
```

3.664 $\int \cos^4(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-$$

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (3*a*Log[Sin[c + d*x]])/
d - (3*a*Sin[c + d*x])/d + (3*a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/
d - (a*Sin[c + d*x]^4)/(4*d) - (a*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0869306, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (3*a*Log[Sin[c + d*x]])/
d - (3*a*Sin[c + d*x])/d + (3*a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/
d - (a*Sin[c + d*x]^4)/(4*d) - (a*Sin[c + d*x]^5)/(5*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^4(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^3(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^4+\frac{a^7}{x^3}+\frac{a^6}{x^2}-\frac{3a^5}{x}+3a^3x+3a^2x^2-ax^3-x^4\right)dx, x\right.}{a^4d}$$
$$= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{3a\log(\sin(c+dx))}{d}-\frac{3a\sin(c}{d}$$

Mathematica [A] time = 0.125145, size = 100, normalized size = 0.87

$$-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}-\frac{a\left(\sin^4(c+dx)-6\sin^2(c+dx)+2\csc^2(c+dx)\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (3*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^3)/d - (a*Si
n[c + d*x]^5)/(5*d) - (a*(2*Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] - 6*Sin[c
 + d*x]^2 + Sin[c + d*x]^4))/(4*d)
```

Maple [A] time = 0.061, size = 173, normalized size = 1.5

$$-\frac{a(\cos(dx+c))^8}{d\sin(dx+c)}-\frac{16a\sin(dx+c)}{5d}-\frac{\sin(dx+c)(\cos(dx+c))^6a}{d}-\frac{6\sin(dx+c)(\cos(dx+c))^4a}{5d}-\frac{8(\cos(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*sin(d*x
+c)*a-6/5/d*cos(d*x+c)^4*sin(d*x+c)*a-8/5/d*cos(d*x+c)^2*sin(d*x+c)*a-1/2/d
*a/sin(d*x+c)^2*cos(d*x+c)^8-1/2*a*cos(d*x+c)^6/d-3/4*a*cos(d*x+c)^4/d-3/2*
a*cos(d*x+c)^2/d-3*a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.02327, size = 122, normalized size = 1.06

$$-\frac{4a\sin(dx+c)^5+5a\sin(dx+c)^4-20a\sin(dx+c)^3-30a\sin(dx+c)^2+60a\log(\sin(dx+c))+60a\sin(dx+c}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/20*(4*a*sin(d*x + c)^5 + 5*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30*a
*sin(d*x + c)^2 + 60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + 10*(2*a*sin(
d*x + c) + a)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.67183, size = 328, normalized size = 2.85

$$-\frac{40\,a\cos\left(dx+c\right)^{6}+120\,a\cos\left(dx+c\right)^{4}-255\,a\cos\left(dx+c\right)^{2}+480\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+32\left(a\cos\right.}{160\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/160*(40*a*cos(d*x + c)^6 + 120*a*cos(d*x + c)^4 - 255*a*cos(d*x + c)^2 +
 480*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 32*(a*cos(d*x + c)^6 +
2*a*cos(d*x + c)^4 + 8*a*cos(d*x + c)^2 - 16*a)*sin(d*x + c) + 15*a)/(d*cos
(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21013, size = 140, normalized size = 1.22

$$-\frac{4\,a\sin\left(dx+c\right)^{5}+5\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-30\,a\sin\left(dx+c\right)^{2}+60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a\sin\left(dx+c\right)\cdot}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/20*(4*a*sin(d*x + c)^5 + 5*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30*a
*sin(d*x + c)^2 + 60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - 10*(9*a
*sin(d*x + c)^2 - 2*a*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

3.665 $\int \cos^3(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$-\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}+\frac{3a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+d}{d}$$

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (3*a*Log[Sin[c + d*x]])/d + (3*a*Sin[c + d*x])/d + (3*a*Sin[c + d*x]^2)/(
2*d) - (a*Sin[c + d*x]^3)/(3*d) - (a*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0864115, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}+\frac{3a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (3*a*Log[Sin[c + d*x]])/d + (3*a*Sin[c + d*x])/d + (3*a*Sin[c + d*x]^2)/(
2*d) - (a*Sin[c + d*x]^3)/(3*d) - (a*Sin[c + d*x]^4)/(4*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^3(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^3(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^3+\frac{a^7}{x^4}+\frac{a^6}{x^3}-\frac{3a^5}{x^2}-\frac{3a^4}{x}+3a^2x-ax^2-x^3\right)dx, x, a\sin(\right.}{a^3 d}$$
$$= \frac{3a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{3a\log(\sin(c+dx}{d}$$

Mathematica [A] time = 0.219729, size = 103, normalized size = 0.87

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\csc(c+dx)}{d}-\frac{a\left(\sin^4(c+dx)-6\sin^2(c+dx)+2\csc^2(c+dx)+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (3*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^3)/(3*d) - (a*(2*Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] - 6*Si
n[c + d*x]^2 + Sin[c + d*x]^4))/(4*d)
```

Maple [A] time = 0.068, size = 195, normalized size = 1.7

$$-\frac{a(\cos(dx+c))^8}{2d(\sin(dx+c))^2}-\frac{a(\cos(dx+c))^6}{2d}-\frac{3a(\cos(dx+c))^4}{4d}-\frac{3a(\cos(dx+c))^2}{2d}-3\frac{a\ln(\sin(dx+c))}{d}-\frac{a(\cos(dx+c)}{3d(\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^8-1/2*a*cos(d*x+c)^6/d-3/4*a*cos(d*x+c)^4/
d-3/2*a*cos(d*x+c)^2/d-3*a*ln(sin(d*x+c))/d-1/3/d*a/sin(d*x+c)^3*cos(d*x+c)
^8+5/3/d*a/sin(d*x+c)*cos(d*x+c)^8+16/3*a*sin(d*x+c)/d+5/3/d*cos(d*x+c)^6*s
in(d*x+c)*a+2/d*cos(d*x+c)^4*sin(d*x+c)*a+8/3/d*cos(d*x+c)^2*sin(d*x+c)*a
```

Maxima [A] time = 1.02428, size = 124, normalized size = 1.05

$$-\frac{3a\sin(dx+c)^4+4a\sin(dx+c)^3-18a\sin(dx+c)^2+36a\log(\sin(dx+c))-36a\sin(dx+c)-\frac{2\left(18a\sin(dx+c)^2-3a\sin\right.}{\sin(dx+c)^3}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 36*a
*log(sin(d*x + c)) - 36*a*sin(d*x + c) - 2*(18*a*sin(d*x + c)^2 - 3*a*sin(d
```

```
*x + c) - 2*a)/sin(d*x + c)^3)/d
```

Fricas [A] time = 1.74723, size = 369, normalized size = 3.13

$$-\frac{32\,a\cos\left(dx+c\right)^{6}+192\,a\cos\left(dx+c\right)^{4}-768\,a\cos\left(dx+c\right)^{2}+288\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx\right.}{96\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/96*(32*a*cos(d*x + c)^6 + 192*a*cos(d*x + c)^4 - 768*a*cos(d*x + c)^2 +
288*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(8*a*cos(
d*x + c)^6 + 24*a*cos(d*x + c)^4 - 51*a*cos(d*x + c)^2 + 3*a)*sin(d*x + c)
+ 512*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31783, size = 140, normalized size = 1.19

$$-\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-18\,a\sin\left(dx+c\right)^{2}+36\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-36\,a\sin\left(dx+c\right)-\frac{2\,\left(33\,a\sin\left(dx+c\right)^{3}+\right.}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 36*a
*log(abs(sin(d*x + c))) - 36*a*sin(d*x + c) - 2*(33*a*sin(d*x + c)^3 + 18*a
*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 2*a)/sin(d*x + c)^3)/d
```

3.666 $\int \cos^2(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+dx)}{d}$$

```
[Out] (3*a*Csc[c + d*x])/d + (3*a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d
) - (a*Csc[c + d*x]^4)/(4*d) + (3*a*Log[Sin[c + d*x]])/d + (3*a*Sin[c + d*x
])/d - (a*Sin[c + d*x]^2)/(2*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0894501, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d + (3*a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d
) - (a*Csc[c + d*x]^4)/(4*d) + (3*a*Log[Sin[c + d*x]])/d + (3*a*Sin[c + d*x
])/d - (a*Sin[c + d*x]^2)/(2*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^2(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a-x)^3(a+x)^4}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2+\frac{a^7}{x^5}+\frac{a^6}{x^4}-\frac{3a^5}{x^3}-\frac{3a^4}{x^2}+\frac{3a^3}{x}-ax-x^2\right) dx, x, a\sin\right.}{a^2 d}$$
$$= \frac{3a\csc(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+d}{4d}$$

Mathematica [A] time = 0.572343, size = 105, normalized size = 0.89

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\csc(c+dx)}{d}+\frac{a\left(-2\sin^2(c+dx)-\csc^4(c+dx)+6\csc^2(c+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (3*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^3)/(3*d) + (a*(6*Csc[c + d*x]^2 - Csc[c + d*x]^4 + 12*Log[Sin
[c + d*x]] - 2*Sin[c + d*x]^2))/(4*d)
```

Maple [A] time = 0.063, size = 217, normalized size = 1.8

$$-\frac{a(\cos(dx+c))^8}{3d(\sin(dx+c))^3}+\frac{5a(\cos(dx+c))^8}{3d\sin(dx+c)}+\frac{16a\sin(dx+c)}{3d}+\frac{5\sin(dx+c)(\cos(dx+c))^6 a}{3d}+2\frac{\sin(dx+c)(\cos(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^8+5/3/d*a/sin(d*x+c)*cos(d*x+c)^8+16/3*a*s
in(d*x+c)/d+5/3/d*cos(d*x+c)^6*sin(d*x+c)*a+2/d*cos(d*x+c)^4*sin(d*x+c)*a+8
/3/d*cos(d*x+c)^2*sin(d*x+c)*a-1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^8+1/2/d*a/si
n(d*x+c)^2*cos(d*x+c)^8+1/2*a*cos(d*x+c)^6/d+3/4*a*cos(d*x+c)^4/d+3/2*a*cos
(d*x+c)^2/d+3*a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.0378, size = 124, normalized size = 1.05

$$-\frac{4a\sin(dx+c)^3+6a\sin(dx+c)^2-36a\log(\sin(dx+c))-36a\sin(dx+c)-\frac{36a\sin(dx+c)^3+18a\sin(dx+c)^2-4a\sin(dx+c)}{\sin(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*sin(d*x + c)^3 + 6*a*sin(d*x + c)^2 - 36*a*log(sin(d*x + c)) - 3
6*a*sin(d*x + c) - (36*a*sin(d*x + c)^3 + 18*a*sin(d*x + c)^2 - 4*a*sin(d*x
```

```
+ c) - 3*a)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.6996, size = 375, normalized size = 3.18

$$\frac{6\,a\cos\left(dx+c\right)^{6}-15\,a\cos\left(dx+c\right)^{4}-6\,a\cos\left(dx+c\right)^{2}+36\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)}{12\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*cos(d*x + c)^6 - 15*a*cos(d*x + c)^4 - 6*a*cos(d*x + c)^2 + 36*(a
*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c)) + 4*(a*cos(
d*x + c)^6 + 6*a*cos(d*x + c)^4 - 24*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c)
+ 12*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18705, size = 139, normalized size = 1.18

$$-\frac{4\,a\sin\left(dx+c\right)^{3}+6\,a\sin\left(dx+c\right)^{2}-36\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-36\,a\sin\left(dx+c\right)+\frac{75\,a\sin(dx+c)^{4}-36\,a\sin(dx+c)^{3}-18\,a\sin(dx+c)^{2}+}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(4*a*sin(d*x + c)^3 + 6*a*sin(d*x + c)^2 - 36*a*log(abs(sin(d*x + c))
) - 36*a*sin(d*x + c) + (75*a*sin(d*x + c)^4 - 36*a*sin(d*x + c)^3 - 18*a*s
in(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin(d*x + c)^4)/d
```

3.667 $\int \cos(c+dx)\cot^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx}{d}$$

```
[Out] (-3*a*Csc[c + d*x])/d + (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d -
 (a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (3*a*Log[Sin[c + d*x
]])/d - (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0815798, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 88}

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d -
 (a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (3*a*Log[Sin[c + d*x
]])/d - (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos(c+dx)\cot^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^3(a+x)^4}{x^6}dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^6}dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^7}{x^6}+\frac{a^6}{x^5}-\frac{3a^5}{x^4}-\frac{3a^4}{x^3}+\frac{3a^3}{x^2}+\frac{3a^2}{x}-x\right)dx, x, a\sin(c+a\right.}{ad}$$
$$= -\frac{3a\csc(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}$$

Mathematica [A] time = 0.185646, size = 102, normalized size = 0.89

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc(c+dx)}{d}+\frac{a\left(-2\sin^2(c+dx)-\csc^4(c+dx)+6\csc^2(c+dx)+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) - (
a*Sin[c + d*x])/d + (a*(6*Csc[c + d*x]^2 - Csc[c + d*x]^4 + 12*Log[Sin[c +
d*x]] - 2*Sin[c + d*x]^2))/(4*d)
```

Maple [B] time = 0.063, size = 239, normalized size = 2.1

$$-\frac{a(\cos(dx+c))^8}{4d(\sin(dx+c))^4}+\frac{a(\cos(dx+c))^8}{2d(\sin(dx+c))^2}+\frac{a(\cos(dx+c))^6}{2d}+\frac{3a(\cos(dx+c))^4}{4d}+\frac{3a(\cos(dx+c))^2}{2d}+3\frac{a\ln(\sin(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^8+1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^8+1/2*a*
cos(d*x+c)^6/d+3/4*a*cos(d*x+c)^4/d+3/2*a*cos(d*x+c)^2/d+3*a*ln(sin(d*x+c))
/d-1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^8+1/5/d*a/sin(d*x+c)^3*cos(d*x+c)^8-1/d*
a/sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*sin(d*x+c)*a
-6/5/d*cos(d*x+c)^4*sin(d*x+c)*a-8/5/d*cos(d*x+c)^2*sin(d*x+c)*a
```

Maxima [A] time = 1.04542, size = 123, normalized size = 1.07

$$-\frac{10\,a\sin(dx+c)^2-60\,a\log(\sin(dx+c))+20\,a\sin(dx+c)+\frac{60\,a\sin(dx+c)^4-30\,a\sin(dx+c)^3-20\,a\sin(dx+c)^2+5\,a\sin(dx+c)+4\,a}{\sin(dx+c)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/20*(10*a*sin(d*x + c)^2 - 60*a*log(sin(d*x + c)) + 20*a*sin(d*x + c) + (
60*a*sin(d*x + c)^4 - 30*a*sin(d*x + c)^3 - 20*a*sin(d*x + c)^2 + 5*a*sin(d
```

```
*x + c) + 4*a)/sin(d*x + c)^5)/d
```

Fricas [A] time = 1.38546, size = 419, normalized size = 3.64

$$\frac{20\,a\cos\left(dx+c\right)^{6}-120\,a\cos\left(dx+c\right)^{4}+160\,a\cos\left(dx+c\right)^{2}+60\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\sin\left(d\right.\right.}{20\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/20*(20*a*cos(d*x + c)^6 - 120*a*cos(d*x + c)^4 + 160*a*cos(d*x + c)^2 + 6
0*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c))*sin(d*x
 + c) + 5*(2*a*cos(d*x + c)^6 - 5*a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + 4
*a)*sin(d*x + c) - 64*a)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18585, size = 139, normalized size = 1.21

$$-\frac{10\,a\sin\left(dx+c\right)^{2}-60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+20\,a\sin\left(dx+c\right)+\frac{137\,a\sin(dx+c)^{5}+60\,a\sin(dx+c)^{4}-30\,a\sin(dx+c)^{3}-20\,a\sin(dx+c)^{2}+}{\sin(dx+c)^{5}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/20*(10*a*sin(d*x + c)^2 - 60*a*log(abs(sin(d*x + c))) + 20*a*sin(d*x + c
) + (137*a*sin(d*x + c)^5 + 60*a*sin(d*x + c)^4 - 30*a*sin(d*x + c)^3 - 20*
a*sin(d*x + c)^2 + 5*a*sin(d*x + c) + 4*a)/sin(d*x + c)^5)/d
```

3.668 $\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

```
[Out] (-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d +
 (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)
/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0530714, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d +
 (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)
/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^7}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{a^7}{x^7}+\frac{a^6}{x^6}-\frac{3a^5}{x^5}-\frac{3a^4}{x^4}+\frac{3a^3}{x^3}+\frac{3a^2}{x^2}-\frac{a}{x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{3a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^5(c}{5d}\end{aligned}$$

Mathematica [A] time = 0.383324, size = 111, normalized size = 0.97

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc(c+dx)}{d}-\frac{a\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+dx)-\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) - (
a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos[c +
d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) - (a*Sin[c + d*x])/d
```

Maple [A] time = 0.069, size = 195, normalized size = 1.7

$$-\frac{a\left(\cos\left(dx+c\right)\right)^{8}}{5\,d\left(\sin\left(dx+c\right)\right)^{5}}+\frac{a\left(\cos\left(dx+c\right)\right)^{8}}{5\,d\left(\sin\left(dx+c\right)\right)^{3}}-\frac{a\left(\cos\left(dx+c\right)\right)^{8}}{d\sin\left(dx+c\right)}-\frac{16\,a\sin\left(dx+c\right)}{5\,d}-\frac{\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{6}a}{d}-\frac{6\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^8+1/5/d*a/sin(d*x+c)^3*cos(d*x+c)^8-1/d*a/
sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*sin(d*x+c)*a-6
/5/d*cos(d*x+c)^4*sin(d*x+c)*a-8/5/d*cos(d*x+c)^2*sin(d*x+c)*a-1/6*a*cot(d*
x+c)^6/d+1/4/d*a*cot(d*x+c)^4-1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.03066, size = 123, normalized size = 1.07

$$-\frac{60\,a\log\left(\sin\left(dx+c\right)\right)+60\,a\sin\left(dx+c\right)+\frac{180\,a\sin\left(dx+c\right)^{5}+90\,a\sin\left(dx+c\right)^{4}-60\,a\sin\left(dx+c\right)^{3}-45\,a\sin\left(dx+c\right)^{2}+12\,a\sin\left(dx+c\right)+10\,a}{\sin\left(dx+c\right)^{6}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + (180*a*sin(d*x + c)^5 +
 90*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 - 45*a*sin(d*x + c)^2 + 12*a*sin
(d*x + c) + 10*a)/sin(d*x + c)^6)/d
```

Fricas [A] time = 1.16157, size = 412, normalized size = 3.58

$$\frac{90\,a\cos\left(dx+c\right)^{4}-135\,a\cos\left(dx+c\right)^{2}-60\left(a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}+3\,a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\sin\left(dx\right.\right.}{60\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(90*a*cos(d*x + c)^4 - 135*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^6 - 3
*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) - 12*(5*a
*cos(d*x + c)^6 - 30*a*cos(d*x + c)^4 + 40*a*cos(d*x + c)^2 - 16*a)*sin(d*x
 + c) + 55*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30576, size = 140, normalized size = 1.22

$$-\frac{60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a\sin\left(dx+c\right)-\frac{147\,a\sin(dx+c)^{6}-180\,a\sin(dx+c)^{5}-90\,a\sin(dx+c)^{4}+60\,a\sin(dx+c)^{3}+45\,a\sin(dx+c)^{2}-12\,a\sin(dx+}{\sin(dx+c)^{6}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - (147*a*sin(d*x + c
)^6 - 180*a*sin(d*x + c)^5 - 90*a*sin(d*x + c)^4 + 60*a*sin(d*x + c)^3 + 45
*a*sin(d*x + c)^2 - 12*a*sin(d*x + c) - 10*a)/sin(d*x + c)^6)/d
```

3.669 $\int \cot^7(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}+\frac{3a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc(c+}{d}$$

```
[Out] (a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/d + (3
*a*Csc[c + d*x]^4)/(4*d) + (3*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/
(6*d) - (a*Csc[c + d*x]^7)/(7*d) - (a*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0845058, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 88}

$$-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}+\frac{3a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/d + (3
*a*Csc[c + d*x]^4)/(4*d) + (3*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/
(6*d) - (a*Csc[c + d*x]^7)/(7*d) - (a*Log[Sin[c + d*x]])/d
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^7(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^8(a-x)^3(a+x)^4}{x^8}dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^8}dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{a^6}{x^7}-\frac{3a^5}{x^6}-\frac{3a^4}{x^5}+\frac{3a^3}{x^4}+\frac{3a^2}{x^3}-\frac{a}{x^2}-\frac{1}{x}\right)dx, x, a\sin(c+\right.}{d}$$
$$= \frac{a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+dx)}{4d}+$$

Mathematica [A] time = 0.390895, size = 115, normalized size = 0.97

$$-\frac{a\csc^7(c+dx)}{7d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{d}+\frac{a\csc(c+dx)}{d}-\frac{a\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+dx)\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^5)/(5*d) - (a
*Csc[c + d*x]^7)/(7*d) - (a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c
+ d*x]^6 + 12*Log[Cos[c + d*x]] + 12*Log[Tan[c + d*x]]))/(12*d)
```

Maple [A] time = 0.065, size = 217, normalized size = 1.8

$$-\frac{a(\cot(dx+c))^6}{6d}+\frac{a(\cot(dx+c))^4}{4d}-\frac{a(\cot(dx+c))^2}{2d}-\frac{a\ln(\sin(dx+c))}{d}-\frac{a(\cos(dx+c))^8}{7d(\sin(dx+c))^7}+\frac{a(\cos(dx+c))^8}{35d(\sin(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6*a*cot(d*x+c)^6/d+1/4/d*a*cot(d*x+c)^4-1/2*a*cot(d*x+c)^2/d-a*ln(sin(d*
x+c))/d-1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^8+1/35/d*a/sin(d*x+c)^5*cos(d*x+c)^
8-1/35/d*a/sin(d*x+c)^3*cos(d*x+c)^8+1/7/d*a/sin(d*x+c)*cos(d*x+c)^8+16/35*
a*sin(d*x+c)/d+1/7/d*cos(d*x+c)^6*sin(d*x+c)*a+6/35/d*cos(d*x+c)^4*sin(d*x+
c)*a+8/35/d*cos(d*x+c)^2*sin(d*x+c)*a
```

Maxima [A] time = 1.0397, size = 127, normalized size = 1.07

$$-\frac{420\,a\log(\sin(dx+c))-\frac{420\,a\sin(dx+c)^6-630\,a\sin(dx+c)^5-420\,a\sin(dx+c)^4+315\,a\sin(dx+c)^3+252\,a\sin(dx+c)^2-70\,a\sin(dx+c)-60\,a}{\sin(dx+c)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(420*a*log(sin(d*x + c)) - (420*a*sin(d*x + c)^6 - 630*a*sin(d*x + c
)^5 - 420*a*sin(d*x + c)^4 + 315*a*sin(d*x + c)^3 + 252*a*sin(d*x + c)^2 -
```

```
70*a*sin(d*x + c) - 60*a)/sin(d*x + c)^7)/d
```

Fricas [A] time = 1.2071, size = 458, normalized size = 3.85

$$\frac{420\,a\cos\left(dx+c\right)^{6}-840\,a\cos\left(dx+c\right)^{4}+672\,a\cos\left(dx+c\right)^{2}-420\left(a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}+3\,a\cos\left(dx+\right.\right.}{420\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*cos(d*x + c)^6 - 840*a*cos(d*x + c)^4 + 672*a*cos(d*x + c)^2 -
 420*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1
/2*sin(d*x + c))*sin(d*x + c) + 35*(18*a*cos(d*x + c)^4 - 27*a*cos(d*x + c)
^2 + 11*a)*sin(d*x + c) - 192*a)/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 +
3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33715, size = 143, normalized size = 1.2

$$-\frac{420\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{1089\,a\sin(dx+c)^{7}+420\,a\sin(dx+c)^{6}-630\,a\sin(dx+c)^{5}-420\,a\sin(dx+c)^{4}+315\,a\sin(dx+c)^{3}+252\,a\sin(dx+c)^{2}-70\,a\sin(\ldots}{\sin(dx+c)^{7}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(sin(d*x + c))) - (1089*a*sin(d*x + c)^7 + 420*a*sin(d
*x + c)^6 - 630*a*sin(d*x + c)^5 - 420*a*sin(d*x + c)^4 + 315*a*sin(d*x + c
)^3 + 252*a*sin(d*x + c)^2 - 70*a*sin(d*x + c) - 60*a)/sin(d*x + c)^7)/d
```

3.670 $\int \cot^7(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^7(c+dx)}{7d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{d}+\frac{a\csc(c+dx)}{d}$$

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) + (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*
a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.107246, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 30, 2606, 194}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^7(c+dx)}{7d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{d}+\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```


```
[Out] -(a*Cot[c + d*x]^8)/(8*d) + (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*
a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```


Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cot^7(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc(c+dx)\,dx + a\int \cot^7(c+dx)\csc^2(c+dx)\,d$$
$$= -\frac{a\,\mathrm{Subst}\left(\int x^7\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\mathrm{Subst}\left(\int \left(-1+x^2\right)^3\,dx\right.}{d}$$
$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\,\mathrm{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)dx, x, \csc(c\right.}{d}$$
$$= -\frac{a\cot^8(c+dx)}{8d} + \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{d} + \frac{3a\csc^5(c+d}{5d}$$

Mathematica [A] time = 0.0328331, size = 74, normalized size = 1.

$$-\frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^7(c+dx)}{7d} + \frac{3a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{d} + \frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) + (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*
a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Maple [B] time = 0.068, size = 138, normalized size = 1.9

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^8}{7(\sin(dx+c))^7} + \frac{(\cos(dx+c))^8}{35(\sin(dx+c))^5} - \frac{(\cos(dx+c))^8}{35(\sin(dx+c))^3} + \frac{(\cos(dx+c))^8}{7\sin(dx+c)} + \frac{\sin(dx+c)}{7}\left(\frac{16}{5} + (\cos(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/
sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*a/sin(d*x+c)^8*cos(d*x
+c)^8)
```

Maxima [A] time = 1.04174, size = 124, normalized size = 1.68

$$\frac{280\,a\sin(dx+c)^7 + 140\,a\sin(dx+c)^6 - 280\,a\sin(dx+c)^5 - 210\,a\sin(dx+c)^4 + 168\,a\sin(dx+c)^3 + 140\,a\sin(dx}{280\,d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/280*(280*a*sin(d*x + c)^7 + 140*a*sin(d*x + c)^6 - 280*a*sin(d*x + c)^5 -
 210*a*sin(d*x + c)^4 + 168*a*sin(d*x + c)^3 + 140*a*sin(d*x + c)^2 - 40*a*
sin(d*x + c) - 35*a)/(d*sin(d*x + c)^8)
```

Fricas [A] time = 1.13352, size = 351, normalized size = 4.74

$$-\frac{140\,a\cos\left(dx+c\right)^{6}-210\,a\cos\left(dx+c\right)^{4}+140\,a\cos\left(dx+c\right)^{2}+8\left(35\,a\cos\left(dx+c\right)^{6}-70\,a\cos\left(dx+c\right)^{4}+56\,a\cos\left(dx\right.\right.}{280\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/280*(140*a*cos(d*x + c)^6 - 210*a*cos(d*x + c)^4 + 140*a*cos(d*x + c)^2
+ 8*(35*a*cos(d*x + c)^6 - 70*a*cos(d*x + c)^4 + 56*a*cos(d*x + c)^2 - 16*a
)*sin(d*x + c) - 35*a)/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x
 + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35942, size = 124, normalized size = 1.68

$$\frac{280\,a\sin\left(dx+c\right)^{7}+140\,a\sin\left(dx+c\right)^{6}-280\,a\sin\left(dx+c\right)^{5}-210\,a\sin\left(dx+c\right)^{4}+168\,a\sin\left(dx+c\right)^{3}+140\,a\sin\left(dx+c\right.}{280\,d\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/280*(280*a*sin(d*x + c)^7 + 140*a*sin(d*x + c)^6 - 280*a*sin(d*x + c)^5 -
 210*a*sin(d*x + c)^4 + 168*a*sin(d*x + c)^3 + 140*a*sin(d*x + c)^2 - 40*a*
sin(d*x + c) - 35*a)/(d*sin(d*x + c)^8)
```

3.671 $\int \cot^7(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) + (a*Csc[c + d*x]^3)/(3*d) - (3*a*Csc[c + d*x]^5)
/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.11891, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 30}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) + (a*Csc[c + d*x]^3)/(3*d) - (3*a*Csc[c + d*x]^5)
/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^7(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^2(c+dx)\,dx + a\int \cot^7(c+dx)\csc^3(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^7\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,\right.}{d}$$
$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\,\text{Subst}\left(\int\left(-x^2+3x^4-3x^6+x^8\right)dx, x, \csc(c+\right.}{d}$$
$$= -\frac{a\cot^8(c+dx)}{8d} + \frac{a\csc^3(c+dx)}{3d} - \frac{3a\csc^5(c+dx)}{5d} + \frac{3a\csc^7(c+dx)}{7d}$$

Mathematica [A] time = 0.0683213, size = 81, normalized size = 1.

$$-\frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^9(c+dx)}{9d} + \frac{3a\csc^7(c+dx)}{7d} - \frac{3a\csc^5(c+dx)}{5d} + \frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) + (a*Csc[c + d*x]^3)/(3*d) - (3*a*Csc[c + d*x]^5)
/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Maple [B] time = 0.066, size = 156, normalized size = 1.9

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^8}{8(\sin(dx+c))^8} + a\left(-\frac{(\cos(dx+c))^8}{9(\sin(dx+c))^9} - \frac{(\cos(dx+c))^8}{63(\sin(dx+c))^7} + \frac{(\cos(dx+c))^8}{315(\sin(dx+c))^5} - \frac{(\cos(dx+c))^8}{315(\sin(dx+c))^3} + \frac{(\cos(dx}{63\sin(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/8*a/sin(d*x+c)^8*cos(d*x+c)^8+a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^8-1/6
3/sin(d*x+c)^7*cos(d*x+c)^8+1/315/sin(d*x+c)^5*cos(d*x+c)^8-1/315/sin(d*x+c
)^3*cos(d*x+c)^8+1/63/sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.02975, size = 124, normalized size = 1.53

$$\frac{1260\,a\sin(dx+c)^7 + 840\,a\sin(dx+c)^6 - 1890\,a\sin(dx+c)^5 - 1512\,a\sin(dx+c)^4 + 1260\,a\sin(dx+c)^3 + 1080\,a\sin(}{2520\,d\sin(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2520*(1260*a*sin(d*x + c)^7 + 840*a*sin(d*x + c)^6 - 1890*a*sin(d*x + c)^
5 - 1512*a*sin(d*x + c)^4 + 1260*a*sin(d*x + c)^3 + 1080*a*sin(d*x + c)^2 -
 315*a*sin(d*x + c) - 280*a)/(d*sin(d*x + c)^9)
```

Fricas [A] time = 1.12393, size = 370, normalized size = 4.57

$$-\frac{840\,a\cos\left(dx+c\right)^{6}-1008\,a\cos\left(dx+c\right)^{4}+576\,a\cos\left(dx+c\right)^{2}+315\left(4\,a\cos\left(dx+c\right)^{6}-6\,a\cos\left(dx+c\right)^{4}+4\,a\cos(}{2520\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)\sin(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(840*a*cos(d*x + c)^6 - 1008*a*cos(d*x + c)^4 + 576*a*cos(d*x + c)^
2 + 315*(4*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 - a)*
sin(d*x + c) - 128*a)/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x
 + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**10*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33645, size = 124, normalized size = 1.53

$$\frac{1260\,a\sin\left(dx+c\right)^{7}+840\,a\sin\left(dx+c\right)^{6}-1890\,a\sin\left(dx+c\right)^{5}-1512\,a\sin\left(dx+c\right)^{4}+1260\,a\sin\left(dx+c\right)^{3}+1080\,a\,\mathrm{s}}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2520*(1260*a*sin(d*x + c)^7 + 840*a*sin(d*x + c)^6 - 1890*a*sin(d*x + c)^
5 - 1512*a*sin(d*x + c)^4 + 1260*a*sin(d*x + c)^3 + 1080*a*sin(d*x + c)^2 -
 315*a*sin(d*x + c) - 280*a)/(d*sin(d*x + c)^9)
```

3.672 $\int \cot^7(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(10*d) + (a*Csc[c + d*x]^3)
/(3*d) - (3*a*Csc[c + d*x]^5)/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c
 + d*x]^9)/(9*d)
```

Rubi [A] time = 0.125265, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 14, 2606, 270}

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(10*d) + (a*Csc[c + d*x]^3)
/(3*d) - (3*a*Csc[c + d*x]^5)/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c
 + d*x]^9)/(9*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^7(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^3(c+dx)\,dx + a\int \cot^7(c+dx)\csc^4(c+dx)$$

$$= -\frac{a\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x^7\left(1\right.\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(-x^2+3x^4-3x^6+x^8\right)dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subs}}{}$$

$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{10d} + \frac{a\csc^3(c+dx)}{3d} - \frac{3a\csc^5(c+}{5d}$$

Mathematica [A] time = 0.184479, size = 86, normalized size = 0.89

$$-\frac{a\csc^3(c+dx)\left(252\csc^7(c+dx)+280\csc^6(c+dx)-945\csc^5(c+dx)-1080\csc^4(c+dx)+1260\csc^3(c+dx)+15\right.}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^3*(-840 - 630*Csc[c + d*x] + 1512*Csc[c + d*x]^2 + 1260*Cs
c[c + d*x]^3 - 1080*Csc[c + d*x]^4 - 945*Csc[c + d*x]^5 + 280*Csc[c + d*x]^
6 + 252*Csc[c + d*x]^7))/(2520*d)
```

Maple [B] time = 0.062, size = 176, normalized size = 1.8

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^8}{9(\sin(dx+c))^9}-\frac{(\cos(dx+c))^8}{63(\sin(dx+c))^7}+\frac{(\cos(dx+c))^8}{315(\sin(dx+c))^5}-\frac{(\cos(dx+c))^8}{315(\sin(dx+c))^3}+\frac{(\cos(dx+c))^8}{63\sin(dx+c)}+\frac{\sin(dx}{63}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^8-1/63/sin(d*x+c)^7*cos(d*x+c)^8+1/315
/sin(d*x+c)^5*cos(d*x+c)^8-1/315/sin(d*x+c)^3*cos(d*x+c)^8+1/63/sin(d*x+c)*
cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin
(d*x+c))+a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^8-1/40/sin(d*x+c)^8*cos(d*x+c)^8
))
```

Maxima [A] time = 1.03559, size = 124, normalized size = 1.28

$$\frac{840\,a\sin(dx+c)^7+630\,a\sin(dx+c)^6-1512\,a\sin(dx+c)^5-1260\,a\sin(dx+c)^4+1080\,a\sin(dx+c)^3+945\,a\sin}{2520\,d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2520*(840*a*sin(d*x + c)^7 + 630*a*sin(d*x + c)^6 - 1512*a*sin(d*x + c)^5
 - 1260*a*sin(d*x + c)^4 + 1080*a*sin(d*x + c)^3 + 945*a*sin(d*x + c)^2 - 2
80*a*sin(d*x + c) - 252*a)/(d*sin(d*x + c)^10)
```

Fricas [A] time = 1.16464, size = 386, normalized size = 3.98

$$\frac{630\,a\cos\left(dx+c\right)^{6}-630\,a\cos\left(dx+c\right)^{4}+315\,a\cos\left(dx+c\right)^{2}+8\left(105\,a\cos\left(dx+c\right)^{6}-126\,a\cos\left(dx+c\right)^{4}+72\,a\cos\left(dx\right.\right.}{2520\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2520*(630*a*cos(d*x + c)^6 - 630*a*cos(d*x + c)^4 + 315*a*cos(d*x + c)^2
+ 8*(105*a*cos(d*x + c)^6 - 126*a*cos(d*x + c)^4 + 72*a*cos(d*x + c)^2 - 16
*a)*sin(d*x + c) - 63*a)/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos
(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**11*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29762, size = 124, normalized size = 1.28

$$\frac{840\,a\sin\left(dx+c\right)^{7}+630\,a\sin\left(dx+c\right)^{6}-1512\,a\sin\left(dx+c\right)^{5}-1260\,a\sin\left(dx+c\right)^{4}+1080\,a\sin\left(dx+c\right)^{3}+945\,a\sin\left(dx\right.}{2520\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2520*(840*a*sin(d*x + c)^7 + 630*a*sin(d*x + c)^6 - 1512*a*sin(d*x + c)^5
 - 1260*a*sin(d*x + c)^4 + 1080*a*sin(d*x + c)^3 + 945*a*sin(d*x + c)^2 - 2
80*a*sin(d*x + c) - 252*a)/(d*sin(d*x + c)^10)
```

3.673 $\int \cot^7(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^5(c+dx)}{5d}$$

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(10*d) + (a*Csc[c + d*x]^5)
/(5*d) - (3*a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^9)/(3*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rubi [A] time = 0.124318, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 14}

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(10*d) + (a*Csc[c + d*x]^5)
/(5*d) - (3*a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^9)/(3*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^7(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^4(c+dx)\,dx + a\int \cot^7(c+dx)\csc^5(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^4\left(-1+x^2\right)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x^7\left(1+\right.\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int \left(x^7+x^9\right)dx, x, -\cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(-x^4+3x\right.\right.}{}$$
$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{10d} + \frac{a\csc^5(c+dx)}{5d} - \frac{3a\csc^7(c+dx)}{7d}$$

Mathematica [A] time = 0.138561, size = 86, normalized size = 0.89

$$-\frac{a\csc^4(c+dx)\left(840\csc^7(c+dx)+924\csc^6(c+dx)-3080\csc^5(c+dx)-3465\csc^4(c+dx)+3960\csc^3(c+dx)+4620\right.}{9240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^4*(-2310 - 1848*Csc[c + d*x] + 4620*Csc[c + d*x]^2 + 3960*
Csc[c + d*x]^3 - 3465*Csc[c + d*x]^4 - 3080*Csc[c + d*x]^5 + 924*Csc[c + d*
x]^6 + 840*Csc[c + d*x]^7))/(9240*d)
```

Maple [B] time = 0.066, size = 194, normalized size = 2.

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^8}{10(\sin(dx+c))^{10}}-\frac{(\cos(dx+c))^8}{40(\sin(dx+c))^8}\right)+a\left(-\frac{(\cos(dx+c))^8}{11(\sin(dx+c))^{11}}-\frac{(\cos(dx+c))^8}{33(\sin(dx+c))^9}-\frac{(\cos(dx+c))^8}{231(\sin(dx+c))^7}+\frac{}{11}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^8-1/40/sin(d*x+c)^8*cos(d*x+c)^8)+a*
(-1/11/sin(d*x+c)^11*cos(d*x+c)^8-1/33/sin(d*x+c)^9*cos(d*x+c)^8-1/231/sin(
d*x+c)^7*cos(d*x+c)^8+1/1155/sin(d*x+c)^5*cos(d*x+c)^8-1/1155/sin(d*x+c)^3*
cos(d*x+c)^8+1/231/sin(d*x+c)*cos(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+6/5*cos
(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.03288, size = 124, normalized size = 1.28

$$\frac{2310\,a\sin(dx+c)^7+1848\,a\sin(dx+c)^6-4620\,a\sin(dx+c)^5-3960\,a\sin(dx+c)^4+3465\,a\sin(dx+c)^3+3080\,a\sin}{9240\,d\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/9240*(2310*a*sin(d*x + c)^7 + 1848*a*sin(d*x + c)^6 - 4620*a*sin(d*x + c)
^5 - 3960*a*sin(d*x + c)^4 + 3465*a*sin(d*x + c)^3 + 3080*a*sin(d*x + c)^2
- 924*a*sin(d*x + c) - 840*a)/(d*sin(d*x + c)^11)
```

Fricas [A] time = 1.22169, size = 405, normalized size = 4.18

$$\frac{1848\,a\cos\left(dx+c\right)^{6}-1584\,a\cos\left(dx+c\right)^{4}+704\,a\cos\left(dx+c\right)^{2}+231\left(10\,a\cos\left(dx+c\right)^{6}-10\,a\cos\left(dx+c\right)^{4}+5\,a\cos\right.}{9240\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/9240*(1848*a*cos(d*x + c)^6 - 1584*a*cos(d*x + c)^4 + 704*a*cos(d*x + c)^
2 + 231*(10*a*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a
)*sin(d*x + c) - 128*a)/((d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos
(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**12*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33868, size = 124, normalized size = 1.28

$$\frac{2310\,a\sin\left(dx+c\right)^{7}+1848\,a\sin\left(dx+c\right)^{6}-4620\,a\sin\left(dx+c\right)^{5}-3960\,a\sin\left(dx+c\right)^{4}+3465\,a\sin\left(dx+c\right)^{3}+3080\,a}{9240\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/9240*(2310*a*sin(d*x + c)^7 + 1848*a*sin(d*x + c)^6 - 4620*a*sin(d*x + c)
^5 - 3960*a*sin(d*x + c)^4 + 3465*a*sin(d*x + c)^3 + 3080*a*sin(d*x + c)^2
- 924*a*sin(d*x + c) - 840*a)/(d*sin(d*x + c)^11)
```

3.674 $\int \cot^7(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^5(c+dx)}{5d}$$

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(5*d) - (a*Cot[c + d*x]^12)
/(12*d) + (a*Csc[c + d*x]^5)/(5*d) - (3*a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c
+ d*x]^9)/(3*d) - (a*Csc[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.133233, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2607, 266, 43, 2606, 270}

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(5*d) - (a*Cot[c + d*x]^12)
/(12*d) + (a*Csc[c + d*x]^5)/(5*d) - (3*a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c
+ d*x]^9)/(3*d) - (a*Csc[c + d*x]^11)/(11*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^7(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^5(c+dx)\,dx + a\int \cot^7(c+dx)\csc^6(c+dx)$$
$$= -\frac{a\,\text{Subst}\left(\int x^4\left(-1+x^2\right)^3 dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x^7\left(1\right.\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int x^3(1+x)^2\,dx, x, \cot^2(c+dx)\right)}{2d} - \frac{a\,\text{Subst}\left(\int\left(-x^4+\right.\right.}{}$$
$$= \frac{a\csc^5(c+dx)}{5d} - \frac{3a\csc^7(c+dx)}{7d} + \frac{a\csc^9(c+dx)}{3d} - \frac{a\csc^{11}(c+d}{11d}$$
$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{5d} - \frac{a\cot^{12}(c+dx)}{12d} + \frac{a\csc^5(c+}{5d}$$

Mathematica [A] time = 0.231248, size = 86, normalized size = 0.76

$$-\frac{a\csc^{12}(c+dx)(-45\sin(c+dx)+1111\sin(3(c+dx))+363\sin(5(c+dx))+231\sin(7(c+dx))+3003\cos(2(c+dx}{73920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^12*(1617 + 3003*Cos[2*(c + d*x)] + 1155*Cos[4*(c + d*x)] +
 385*Cos[6*(c + d*x)] - 45*Sin[c + d*x] + 1111*Sin[3*(c + d*x)] + 363*Sin[5
*(c + d*x)] + 231*Sin[7*(c + d*x)]))/(73920*d)
```

Maple [B] time = 0.06, size = 212, normalized size = 1.9

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^8}{11(\sin(dx+c))^{11}} - \frac{(\cos(dx+c))^8}{33(\sin(dx+c))^9} - \frac{(\cos(dx+c))^8}{231(\sin(dx+c))^7} + \frac{(\cos(dx+c))^8}{1155(\sin(dx+c))^5} - \frac{(\cos(dx+c))^8}{1155(\sin(dx+c))^3} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^8-1/33/sin(d*x+c)^9*cos(d*x+c)^8-1/2
31/sin(d*x+c)^7*cos(d*x+c)^8+1/1155/sin(d*x+c)^5*cos(d*x+c)^8-1/1155/sin(d*
x+c)^3*cos(d*x+c)^8+1/231/sin(d*x+c)*cos(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+
6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/12/sin(d*x+c)^12*cos(d
*x+c)^8-1/30/sin(d*x+c)^10*cos(d*x+c)^8-1/120/sin(d*x+c)^8*cos(d*x+c)^8))
```

Maxima [A] time = 1.03558, size = 124, normalized size = 1.1

$$\frac{1848\,a\sin(dx+c)^7+1540\,a\sin(dx+c)^6-3960\,a\sin(dx+c)^5-3465\,a\sin(dx+c)^4+3080\,a\sin(dx+c)^3+2772\,a\sin}{9240\,d\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/9240*(1848*a*sin(d*x + c)^7 + 1540*a*sin(d*x + c)^6 - 3960*a*sin(d*x + c)
^5 - 3465*a*sin(d*x + c)^4 + 3080*a*sin(d*x + c)^3 + 2772*a*sin(d*x + c)^2
- 840*a*sin(d*x + c) - 770*a)/(d*sin(d*x + c)^12)
```

Fricas [A] time = 1.25645, size = 421, normalized size = 3.73

$$-\frac{1540\,a\cos(dx+c)^6-1155\,a\cos(dx+c)^4+462\,a\cos(dx+c)^2+8\left(231\,a\cos(dx+c)^6-198\,a\cos(dx+c)^4+88\,a\cos\right.}{9240\left(d\cos(dx+c)^{12}-6\,d\cos(dx+c)^{10}+15\,d\cos(dx+c)^8-20\,d\cos(dx+c)^6+15\,d\cos(dx+c)^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/9240*(1540*a*cos(d*x + c)^6 - 1155*a*cos(d*x + c)^4 + 462*a*cos(d*x + c)
^2 + 8*(231*a*cos(d*x + c)^6 - 198*a*cos(d*x + c)^4 + 88*a*cos(d*x + c)^2 -
 16*a)*sin(d*x + c) - 77*a)/(d*cos(d*x + c)^12 - 6*d*cos(d*x + c)^10 + 15*d
*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*d*cos(d*x +
 c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**13*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36302, size = 124, normalized size = 1.1

$$\frac{1848\,a\sin(dx+c)^7+1540\,a\sin(dx+c)^6-3960\,a\sin(dx+c)^5-3465\,a\sin(dx+c)^4+3080\,a\sin(dx+c)^3+2772\,a\sin}{9240\,d\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/9240*(1848*a*sin(d*x + c)^7 + 1540*a*sin(d*x + c)^6 - 3960*a*sin(d*x + c)
^5 - 3465*a*sin(d*x + c)^4 + 3080*a*sin(d*x + c)^3 + 2772*a*sin(d*x + c)^2
- 840*a*sin(d*x + c) - 770*a)/(d*sin(d*x + c)^12)
```

3.675 $\int \cot^7(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^{8}(c+dx)}{8d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{a\csc^{9}(c+dx)}{3d}+\frac{a\csc^{7}(c+dx)}{7d}$$

```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(5*d) - (a*Cot[c + d*x]^12)
/(12*d) + (a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(3*d) + (3*a*Csc[c
+ d*x]^11)/(11*d) - (a*Csc[c + d*x]^13)/(13*d)
```

Rubi [A] time = 0.134878, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2606, 270, 2607, 266, 43}

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^{8}(c+dx)}{8d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{a\csc^{9}(c+dx)}{3d}+\frac{a\csc^{7}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```


```
[Out] -(a*Cot[c + d*x]^8)/(8*d) - (a*Cot[c + d*x]^10)/(5*d) - (a*Cot[c + d*x]^12)
/(12*d) + (a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(3*d) + (3*a*Csc[c
+ d*x]^11)/(11*d) - (a*Csc[c + d*x]^13)/(13*d)
```


Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^6(c+dx)\,dx + a\int \cot^7(c+dx)\csc^7(c+dx)$$

$$= -\frac{a\,\text{Subst}\left(\int x^6\left(-1+x^2\right)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int x^7\left(1\right.\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int x^3(1+x)^2\,dx, x, \cot^2(c+dx)\right)}{2d} - \frac{a\,\text{Subst}\left(\int\left(-x^6+\right.\right.}{}$$

$$= \frac{a\csc^7(c+dx)}{7d} - \frac{a\csc^9(c+dx)}{3d} + \frac{3a\csc^{11}(c+dx)}{11d} - \frac{a\csc^{13}(c+}{13d}$$

$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{5d} - \frac{a\cot^{12}(c+dx)}{12d} + \frac{a\csc^7(c+}{7d}$$

Mathematica [A] time = 0.215459, size = 86, normalized size = 0.76

$$-\frac{a\csc^{13}(c+dx)(3003\sin(c+dx)+24024\sin(3(c+dx))+10010\sin(5(c+dx))+5005\sin(7(c+dx))+70460\cos(2}{1921920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^13*(40200 + 70460*Cos[2*(c + d*x)] + 28600*Cos[4*(c + d*x)
] + 8580*Cos[6*(c + d*x)] + 3003*Sin[c + d*x] + 24024*Sin[3*(c + d*x)] + 10
010*Sin[5*(c + d*x)] + 5005*Sin[7*(c + d*x)]))/(1921920*d)
```

Maple [B] time = 0.066, size = 230, normalized size = 2.

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^8}{12(\sin(dx+c))^{12}} - \frac{(\cos(dx+c))^8}{30(\sin(dx+c))^{10}} - \frac{(\cos(dx+c))^8}{120(\sin(dx+c))^8}\right) + a\left(-\frac{(\cos(dx+c))^8}{13(\sin(dx+c))^{13}} - \frac{5(\cos(dx+c))^8}{143(\sin(dx+c))^1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/12/sin(d*x+c)^12*cos(d*x+c)^8-1/30/sin(d*x+c)^10*cos(d*x+c)^8-1/
120/sin(d*x+c)^8*cos(d*x+c)^8)+a*(-1/13/sin(d*x+c)^13*cos(d*x+c)^8-5/143/si
n(d*x+c)^11*cos(d*x+c)^8-5/429/sin(d*x+c)^9*cos(d*x+c)^8-5/3003/sin(d*x+c)^
7*cos(d*x+c)^8+1/3003/sin(d*x+c)^5*cos(d*x+c)^8-1/3003/sin(d*x+c)^3*cos(d*x
+c)^8+5/3003/sin(d*x+c)*cos(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+
c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.05411, size = 124, normalized size = 1.1

$$\frac{20020\,a\sin(dx+c)^7+17160\,a\sin(dx+c)^6-45045\,a\sin(dx+c)^5-40040\,a\sin(dx+c)^4+36036\,a\sin(dx+c)^3+327}{120120\,d\sin(dx+c)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/120120*(20020*a*sin(d*x + c)^7 + 17160*a*sin(d*x + c)^6 - 45045*a*sin(d*x
 + c)^5 - 40040*a*sin(d*x + c)^4 + 36036*a*sin(d*x + c)^3 + 32760*a*sin(d*x
 + c)^2 - 10010*a*sin(d*x + c) - 9240*a)/(d*sin(d*x + c)^13)
```

Fricas [A] time = 1.25785, size = 446, normalized size = 3.95

$$-\frac{17160\,a\cos(dx+c)^6-11440\,a\cos(dx+c)^4+4160\,a\cos(dx+c)^2+1001\left(20\,a\cos(dx+c)^6-15\,a\cos(dx+c)^4+6\,a\right.}{120120\left(d\cos(dx+c)^{12}-6\,d\cos(dx+c)^{10}+15\,d\cos(dx+c)^8-20\,d\cos(dx+c)^6+15\,d\cos(dx+c)^4-6\,d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/120120*(17160*a*cos(d*x + c)^6 - 11440*a*cos(d*x + c)^4 + 4160*a*cos(d*x
 + c)^2 + 1001*(20*a*cos(d*x + c)^6 - 15*a*cos(d*x + c)^4 + 6*a*cos(d*x + c
)^2 - a)*sin(d*x + c) - 640*a)/((d*cos(d*x + c)^12 - 6*d*cos(d*x + c)^10 +
15*d*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*d*cos(d
*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**14*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33499, size = 124, normalized size = 1.1

$$\frac{20020\,a\sin(dx+c)^7+17160\,a\sin(dx+c)^6-45045\,a\sin(dx+c)^5-40040\,a\sin(dx+c)^4+36036\,a\sin(dx+c)^3+327}{120120\,d\sin(dx+c)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120120*(20020*a*sin(d*x + c)^7 + 17160*a*sin(d*x + c)^6 - 45045*a*sin(d*x
 + c)^5 - 40040*a*sin(d*x + c)^4 + 36036*a*sin(d*x + c)^3 + 32760*a*sin(d*x
 + c)^2 - 10010*a*sin(d*x + c) - 9240*a)/(d*sin(d*x + c)^13)
```

3.676 $\int \cot^7(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$-\frac{a\csc^{14}(c+dx)}{14d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{a\csc^{12}(c+dx)}{4d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{3a\csc^{10}(c+dx)}{10d}-\frac{a\csc^{9}(c+dx)}{3d}+\frac{a\csc^{8}(c-}{8d}$$

```
[Out] (a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^9)/(3
*d) - (3*a*Csc[c + d*x]^10)/(10*d) + (3*a*Csc[c + d*x]^11)/(11*d) + (a*Csc[
c + d*x]^12)/(4*d) - (a*Csc[c + d*x]^13)/(13*d) - (a*Csc[c + d*x]^14)/(14*d
)
```

Rubi [A] time = 0.098505, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a\csc^{14}(c+dx)}{14d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{a\csc^{12}(c+dx)}{4d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{3a\csc^{10}(c+dx)}{10d}-\frac{a\csc^{9}(c+dx)}{3d}+\frac{a\csc^{8}(c-}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^8*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^9)/(3
*d) - (3*a*Csc[c + d*x]^10)/(10*d) + (3*a*Csc[c + d*x]^11)/(11*d) + (a*Csc[
c + d*x]^12)/(4*d) - (a*Csc[c + d*x]^13)/(13*d) - (a*Csc[c + d*x]^14)/(14*d
)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^7(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^{15}(a-x)^3(a+x)^4}{x^{15}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^{15}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \left(\frac{a^7}{x^{15}}+\frac{a^6}{x^{14}}-\frac{3a^5}{x^{13}}-\frac{3a^4}{x^{12}}+\frac{3a^3}{x^{11}}+\frac{3a^2}{x^{10}}-\frac{a}{x^9}-\frac{1}{x^8}\right)dx, x, a$$
$$= \frac{a\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^{10}(c+a$$

Mathematica [A] time = 0.233332, size = 86, normalized size = 0.67

$$-\frac{a\csc^{14}(c+dx)(9940\sin(c+dx)+41860\sin(3(c+dx))+20020\sin(5(c+dx))+8580\sin(7(c+dx))+129129\cos(}{3843840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^8*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^14*(76362 + 129129*Cos[2*(c + d*x)] + 54054*Cos[4*(c + d*x
)] + 15015*Cos[6*(c + d*x)] + 9940*Sin[c + d*x] + 41860*Sin[3*(c + d*x)] +
20020*Sin[5*(c + d*x)] + 8580*Sin[7*(c + d*x)]))/(3843840*d)
```

Maple [B] time = 0.063, size = 248, normalized size = 1.9

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^8}{13(\sin(dx+c))^{13}}-\frac{5(\cos(dx+c))^8}{143(\sin(dx+c))^{11}}-\frac{5(\cos(dx+c))^8}{429(\sin(dx+c))^9}-\frac{5(\cos(dx+c))^8}{3003(\sin(dx+c))^7}+\frac{(\cos(dx+c))^8}{3003(\sin(dx+c))^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/13/sin(d*x+c)^13*cos(d*x+c)^8-5/143/sin(d*x+c)^11*cos(d*x+c)^8-5
/429/sin(d*x+c)^9*cos(d*x+c)^8-5/3003/sin(d*x+c)^7*cos(d*x+c)^8+1/3003/sin(
d*x+c)^5*cos(d*x+c)^8-1/3003/sin(d*x+c)^3*cos(d*x+c)^8+5/3003/sin(d*x+c)*co
s(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin
(d*x+c))+a*(-1/14/sin(d*x+c)^14*cos(d*x+c)^8-1/28/sin(d*x+c)^12*cos(d*x+c)^
8-1/70/sin(d*x+c)^10*cos(d*x+c)^8-1/280/sin(d*x+c)^8*cos(d*x+c)^8))
```

Maxima [A] time = 1.01359, size = 124, normalized size = 0.96

$$\frac{17160\,a\sin(dx+c)^7+15015\,a\sin(dx+c)^6-40040\,a\sin(dx+c)^5-36036\,a\sin(dx+c)^4+32760\,a\sin(dx+c)^3+}{120120\,d\sin(dx+c)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/120120*(17160*a*sin(d*x + c)^7 + 15015*a*sin(d*x + c)^6 - 40040*a*sin(d*x
 + c)^5 - 36036*a*sin(d*x + c)^4 + 32760*a*sin(d*x + c)^3 + 30030*a*sin(d*x
 + c)^2 - 9240*a*sin(d*x + c) - 8580*a)/(d*sin(d*x + c)^14)
```

Fricas [A] time = 1.27976, size = 460, normalized size = 3.57

$$\frac{15015\,a\cos\left(dx+c\right)^{6}-9009\,a\cos\left(dx+c\right)^{4}+3003\,a\cos\left(dx+c\right)^{2}+40\left(429\,a\cos\left(dx+c\right)^{6}-286\,a\cos\left(dx+c\right)^{4}+104\,a}{120120\left(d\cos\left(dx+c\right)^{14}-7\,d\cos\left(dx+c\right)^{12}+21\,d\cos\left(dx+c\right)^{10}-35\,d\cos\left(dx+c\right)^{8}+35\,d\cos\left(dx+c\right)^{6}-21\,d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/120120*(15015*a*cos(d*x + c)^6 - 9009*a*cos(d*x + c)^4 + 3003*a*cos(d*x +
 c)^2 + 40*(429*a*cos(d*x + c)^6 - 286*a*cos(d*x + c)^4 + 104*a*cos(d*x + c
)^2 - 16*a)*sin(d*x + c) - 429*a)/(d*cos(d*x + c)^14 - 7*d*cos(d*x + c)^12
+ 21*d*cos(d*x + c)^10 - 35*d*cos(d*x + c)^8 + 35*d*cos(d*x + c)^6 - 21*d*c
os(d*x + c)^4 + 7*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**15*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37425, size = 124, normalized size = 0.96

$$\frac{17160\,a\sin\left(dx+c\right)^{7}+15015\,a\sin\left(dx+c\right)^{6}-40040\,a\sin\left(dx+c\right)^{5}-36036\,a\sin\left(dx+c\right)^{4}+32760\,a\sin\left(dx+c\right)^{3}+300}{120120\,d\sin\left(dx+c\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120120*(17160*a*sin(d*x + c)^7 + 15015*a*sin(d*x + c)^6 - 40040*a*sin(d*x
 + c)^5 - 36036*a*sin(d*x + c)^4 + 32760*a*sin(d*x + c)^3 + 30030*a*sin(d*x
 + c)^2 - 9240*a*sin(d*x + c) - 8580*a)/(d*sin(d*x + c)^14)
```

3.677 $\int \frac{\cos^7(c+dx)\sin^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sin^{12}(c+dx)}{12ad}+\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{5ad}-\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}$$

```
[Out] Sin[c + d*x]^7/(7*a*d) - Sin[c + d*x]^8/(8*a*d) - (2*Sin[c + d*x]^9)/(9*a*d
) + Sin[c + d*x]^10/(5*a*d) + Sin[c + d*x]^11/(11*a*d) - Sin[c + d*x]^12/(1
2*a*d)
```

Rubi [A] time = 0.127646, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^{12}(c+dx)}{12ad}+\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{5ad}-\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^7/(7*a*d) - Sin[c + d*x]^8/(8*a*d) - (2*Sin[c + d*x]^9)/(9*a*d
) + Sin[c + d*x]^10/(5*a*d) + Sin[c + d*x]^11/(11*a*d) - Sin[c + d*x]^12/(1
2*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^6(a+x)^2}{a^6}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^6(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^{13}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^6-a^4x^7-2a^3x^8+2a^2x^9+ax^{10}-x^{11}\right)dx, x, a\sin(c+dx)\right)}{a^{13}d}$$
$$= \frac{\sin^7(c+dx)}{7ad}-\frac{\sin^8(c+dx)}{8ad}-\frac{2\sin^9(c+dx)}{9ad}+\frac{\sin^{10}(c+dx)}{5ad}+\frac{\sin^{11}(c+dx)}{11ad}-\frac{\sin^{12}(}{12}$$

Mathematica [A] time = 0.597994, size = 68, normalized size = 0.62

$$\frac{\sin^7(c+dx)\left(-2310\sin^5(c+dx)+2520\sin^4(c+dx)+5544\sin^3(c+dx)-6160\sin^2(c+dx)-3465\sin(c+dx)+3960\right)}{27720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^7*(3960 - 3465*Sin[c + d*x] - 6160*Sin[c + d*x]^2 + 5544*Sin[
c + d*x]^3 + 2520*Sin[c + d*x]^4 - 2310*Sin[c + d*x]^5))/(27720*a*d)
```

Maple [A] time = 0.115, size = 69, normalized size = 0.6

$$\frac{1}{da}\left(-\frac{(\sin(dx+c))^{12}}{12}+\frac{(\sin(dx+c))^{11}}{11}+\frac{(\sin(dx+c))^{10}}{5}-\frac{2(\sin(dx+c))^9}{9}-\frac{(\sin(dx+c))^8}{8}+\frac{(\sin(dx+c))^7}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/12*sin(d*x+c)^12+1/11*sin(d*x+c)^11+1/5*sin(d*x+c)^10-2/9*sin(d*x
+c)^9-1/8*sin(d*x+c)^8+1/7*sin(d*x+c)^7)
```

Maxima [A] time = 1.04859, size = 93, normalized size = 0.85

$$-\frac{2310\sin(dx+c)^{12}-2520\sin(dx+c)^{11}-5544\sin(dx+c)^{10}+6160\sin(dx+c)^9+3465\sin(dx+c)^8-3960\sin(d}{27720\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/27720*(2310*sin(d*x + c)^12 - 2520*sin(d*x + c)^11 - 5544*sin(d*x + c)^1
0 + 6160*sin(d*x + c)^9 + 3465*sin(d*x + c)^8 - 3960*sin(d*x + c)^7)/(a*d)
```

Fricas [A] time = 1.21247, size = 311, normalized size = 2.85

$$-\frac{2310\cos(dx+c)^{12}-8316\cos(dx+c)^{10}+10395\cos(dx+c)^8-4620\cos(dx+c)^6+40\left(63\cos(dx+c)^{10}-161\cos\right.}{27720\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/27720*(2310*cos(d*x + c)^12 - 8316*cos(d*x + c)^10 + 10395*cos(d*x + c)^
8 - 4620*cos(d*x + c)^6 + 40*(63*cos(d*x + c)^10 - 161*cos(d*x + c)^8 + 113
*cos(d*x + c)^6 - 3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3122, size = 93, normalized size = 0.85

$$-\frac{2310 \sin(dx+c)^{12} - 2520 \sin(dx+c)^{11} - 5544 \sin(dx+c)^{10} + 6160 \sin(dx+c)^{9} + 3465 \sin(dx+c)^{8} - 3960 \sin}{27720\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/27720*(2310*sin(d*x + c)^12 - 2520*sin(d*x + c)^11 - 5544*sin(d*x + c)^1
0 + 6160*sin(d*x + c)^9 + 3465*sin(d*x + c)^8 - 3960*sin(d*x + c)^7)/(a*d)
```

3.678 $\int \frac{\cos^7(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{10ad}+\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{4ad}-\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^6(c+dx)}{6ad}$$

```
[Out] Sin[c + d*x]^6/(6*a*d) - Sin[c + d*x]^7/(7*a*d) - Sin[c + d*x]^8/(4*a*d) +
(2*Sin[c + d*x]^9)/(9*a*d) + Sin[c + d*x]^10/(10*a*d) - Sin[c + d*x]^11/(11
*a*d)
```

Rubi [A] time = 0.126867, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{10ad}+\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{4ad}-\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^6/(6*a*d) - Sin[c + d*x]^7/(7*a*d) - Sin[c + d*x]^8/(4*a*d) +
(2*Sin[c + d*x]^9)/(9*a*d) + Sin[c + d*x]^10/(10*a*d) - Sin[c + d*x]^11/(11
*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^5(a+x)^2}{a^5}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^5(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^{12}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^5-a^4x^6-2a^3x^7+2a^2x^8+ax^9-x^{10}\right)dx, x, a\sin(c+dx)\right)}{a^{12}d}$$
$$= \frac{\sin^6(c+dx)}{6ad} - \frac{\sin^7(c+dx)}{7ad} - \frac{\sin^8(c+dx)}{4ad} + \frac{2\sin^9(c+dx)}{9ad} + \frac{\sin^{10}(c+dx)}{10ad} - \sin$$

Mathematica [A] time = 0.908169, size = 68, normalized size = 0.62

$$\frac{\sin^6(c+dx)\left(-1260\sin^5(c+dx)+1386\sin^4(c+dx)+3080\sin^3(c+dx)-3465\sin^2(c+dx)-1980\sin(c+dx)+23\right.}{13860ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^6*(2310 - 1980*Sin[c + d*x] - 3465*Sin[c + d*x]^2 + 3080*Sin[
c + d*x]^3 + 1386*Sin[c + d*x]^4 - 1260*Sin[c + d*x]^5))/(13860*a*d)
```

Maple [A] time = 0.105, size = 69, normalized size = 0.6

$$\frac{1}{da}\left(-\frac{(\sin(dx+c))^{11}}{11}+\frac{(\sin(dx+c))^{10}}{10}+\frac{2(\sin(dx+c))^{9}}{9}-\frac{(\sin(dx+c))^{8}}{4}-\frac{(\sin(dx+c))^{7}}{7}+\frac{(\sin(dx+c))^{6}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/11*sin(d*x+c)^11+1/10*sin(d*x+c)^10+2/9*sin(d*x+c)^9-1/4*sin(d*x+
c)^8-1/7*sin(d*x+c)^7+1/6*sin(d*x+c)^6)
```

Maxima [A] time = 1.03119, size = 93, normalized size = 0.85

$$-\frac{1260\sin(dx+c)^{11}-1386\sin(dx+c)^{10}-3080\sin(dx+c)^{9}+3465\sin(dx+c)^{8}+1980\sin(dx+c)^{7}-2310\sin}{13860\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/13860*(1260*sin(d*x + c)^11 - 1386*sin(d*x + c)^10 - 3080*sin(d*x + c)^9
 + 3465*sin(d*x + c)^8 + 1980*sin(d*x + c)^7 - 2310*sin(d*x + c)^6)/(a*d)
```

Fricas [A] time = 1.15045, size = 278, normalized size = 2.55

$$-\frac{1386\cos(dx+c)^{10}-3465\cos(dx+c)^{8}+2310\cos(dx+c)^{6}-20\left(63\cos(dx+c)^{10}-161\cos(dx+c)^{8}+113\cos\right.}{13860\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/13860*(1386*cos(d*x + c)^10 - 3465*cos(d*x + c)^8 + 2310*cos(d*x + c)^6
- 20*(63*cos(d*x + c)^10 - 161*cos(d*x + c)^8 + 113*cos(d*x + c)^6 - 3*cos(
d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28024, size = 93, normalized size = 0.85

$$-\frac{1260 \sin(dx+c)^{11} - 1386 \sin(dx+c)^{10} - 3080 \sin(dx+c)^{9} + 3465 \sin(dx+c)^{8} + 1980 \sin(dx+c)^{7} - 2310 \sin(dx}{13860\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13860*(1260*sin(d*x + c)^11 - 1386*sin(d*x + c)^10 - 3080*sin(d*x + c)^9
 + 3465*sin(d*x + c)^8 + 1980*sin(d*x + c)^7 - 2310*sin(d*x + c)^6)/(a*d)
```

3.679 $\int \frac{\cos^7(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sin^{10}(c+dx)}{10ad}+\frac{\sin^9(c+dx)}{9ad}+\frac{\sin^8(c+dx)}{4ad}-\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}+\frac{\sin^5(c+dx)}{5ad}$$

```
[Out] Sin[c + d*x]^5/(5*a*d) - Sin[c + d*x]^6/(6*a*d) - (2*Sin[c + d*x]^7)/(7*a*d
) + Sin[c + d*x]^8/(4*a*d) + Sin[c + d*x]^9/(9*a*d) - Sin[c + d*x]^10/(10*a
*d)
```

Rubi [A] time = 0.127047, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^{10}(c+dx)}{10ad}+\frac{\sin^9(c+dx)}{9ad}+\frac{\sin^8(c+dx)}{4ad}-\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}+\frac{\sin^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^5/(5*a*d) - Sin[c + d*x]^6/(6*a*d) - (2*Sin[c + d*x]^7)/(7*a*d
) + Sin[c + d*x]^8/(4*a*d) + Sin[c + d*x]^9/(9*a*d) - Sin[c + d*x]^10/(10*a
*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^4(a+x)^2}{a^4}\,dx,x,a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^4(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^{11}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^4-a^4x^5-2a^3x^6+2a^2x^7+ax^8-x^9\right)dx,x,a\sin(c+dx)\right)}{a^{11}d}$$
$$= \frac{\sin^5(c+dx)}{5ad}-\frac{\sin^6(c+dx)}{6ad}-\frac{2\sin^7(c+dx)}{7ad}+\frac{\sin^8(c+dx)}{4ad}+\frac{\sin^9(c+dx)}{9ad}-\frac{\sin^{10}(c}{10a}$$

Mathematica [A] time = 0.414949, size = 68, normalized size = 0.62

$$\frac{\sin^5(c+dx)\left(-126\sin^5(c+dx)+140\sin^4(c+dx)+315\sin^3(c+dx)-360\sin^2(c+dx)-210\sin(c+dx)+252\right)}{1260ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^5*(252 - 210*Sin[c + d*x] - 360*Sin[c + d*x]^2 + 315*Sin[c +
d*x]^3 + 140*Sin[c + d*x]^4 - 126*Sin[c + d*x]^5))/(1260*a*d)
```

Maple [A] time = 0.1, size = 69, normalized size = 0.6

$$\frac{1}{da}\left(-\frac{(\sin(dx+c))^{10}}{10}+\frac{(\sin(dx+c))^9}{9}+\frac{(\sin(dx+c))^8}{4}-\frac{2(\sin(dx+c))^7}{7}-\frac{(\sin(dx+c))^6}{6}+\frac{(\sin(dx+c))^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/10*sin(d*x+c)^10+1/9*sin(d*x+c)^9+1/4*sin(d*x+c)^8-2/7*sin(d*x+c)
^7-1/6*sin(d*x+c)^6+1/5*sin(d*x+c)^5)
```

Maxima [A] time = 1.12277, size = 93, normalized size = 0.85

$$-\frac{126\sin(dx+c)^{10}-140\sin(dx+c)^9-315\sin(dx+c)^8+360\sin(dx+c)^7+210\sin(dx+c)^6-252\sin(dx+c)^5}{1260\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/1260*(126*sin(d*x + c)^10 - 140*sin(d*x + c)^9 - 315*sin(d*x + c)^8 + 36
0*sin(d*x + c)^7 + 210*sin(d*x + c)^6 - 252*sin(d*x + c)^5)/(a*d)
```

Fricas [A] time = 1.16745, size = 239, normalized size = 2.19

$$\frac{126\cos(dx+c)^{10}-315\cos(dx+c)^8+210\cos(dx+c)^6+4\left(35\cos(dx+c)^8-50\cos(dx+c)^6+3\cos(dx+c)^4+4\right.}{1260\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1260*(126*cos(d*x + c)^10 - 315*cos(d*x + c)^8 + 210*cos(d*x + c)^6 + 4*(
35*cos(d*x + c)^8 - 50*cos(d*x + c)^6 + 3*cos(d*x + c)^4 + 4*cos(d*x + c)^2
 + 8)*sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29074, size = 93, normalized size = 0.85

$$-\frac{126 \sin(dx+c)^{10} - 140 \sin(dx+c)^{9} - 315 \sin(dx+c)^{8} + 360 \sin(dx+c)^{7} + 210 \sin(dx+c)^{6} - 252 \sin(dx+c)^{5}}{1260\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1260*(126*sin(d*x + c)^10 - 140*sin(d*x + c)^9 - 315*sin(d*x + c)^8 + 36
0*sin(d*x + c)^7 + 210*sin(d*x + c)^6 - 252*sin(d*x + c)^5)/(a*d)
```

3.680 $\int \frac{\cos^7(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{\sin^9(c+dx)}{9ad}+\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\cos^8(c+dx)}{8ad}-\frac{\cos^6(c+dx)}{6ad}$$

```
[Out] -Cos[c + d*x]^6/(6*a*d) + Cos[c + d*x]^8/(8*a*d) - Sin[c + d*x]^5/(5*a*d) +
 (2*Sin[c + d*x]^7)/(7*a*d) - Sin[c + d*x]^9/(9*a*d)
```

Rubi [A] time = 0.160386, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2835, 2565, 14, 2564, 270}

$$-\frac{\sin^9(c+dx)}{9ad}+\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\cos^8(c+dx)}{8ad}-\frac{\cos^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cos[c + d*x]^6/(6*a*d) + Cos[c + d*x]^8/(8*a*d) - Sin[c + d*x]^5/(5*a*d) +
 (2*Sin[c + d*x]^7)/(7*a*d) - Sin[c + d*x]^9/(9*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^5(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^5(c+dx)\sin^4(c+dx)\,dx}{a}$$
$$= -\frac{\text{Subst}\left(\int x^5\left(1-x^2\right)dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)^2 dx, x, \sin(c+dx\right.}{ad}$$
$$= -\frac{\text{Subst}\left(\int \left(x^5-x^7\right)dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx, x, \sin(c+\right.}{ad}$$
$$= -\frac{\cos^6(c+dx)}{6ad} + \frac{\cos^8(c+dx)}{8ad} - \frac{\sin^5(c+dx)}{5ad} + \frac{2\sin^7(c+dx)}{7ad} - \frac{\sin^9(c+dx)}{9ad}$$

Mathematica [A] time = 0.565633, size = 68, normalized size = 0.75

$$\frac{\sin^4(c+dx)\left(-280\sin^5(c+dx)+315\sin^4(c+dx)+720\sin^3(c+dx)-840\sin^2(c+dx)-504\sin(c+dx)+630\right)}{2520ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^4*(630 - 504*Sin[c + d*x] - 840*Sin[c + d*x]^2 + 720*Sin[c + 
d*x]^3 + 315*Sin[c + d*x]^4 - 280*Sin[c + d*x]^5))/(2520*a*d)
```

Maple [A] time = 0.088, size = 69, normalized size = 0.8

$$\frac{1}{da}\left(-\frac{(\sin(dx+c))^9}{9}+\frac{(\sin(dx+c))^8}{8}+\frac{2(\sin(dx+c))^7}{7}-\frac{(\sin(dx+c))^6}{3}-\frac{(\sin(dx+c))^5}{5}+\frac{(\sin(dx+c))^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/9*sin(d*x+c)^9+1/8*sin(d*x+c)^8+2/7*sin(d*x+c)^7-1/3*sin(d*x+c)^6
-1/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4)
```

Maxima [A] time = 1.11612, size = 93, normalized size = 1.02

$$-\frac{280\sin(dx+c)^9-315\sin(dx+c)^8-720\sin(dx+c)^7+840\sin(dx+c)^6+504\sin(dx+c)^5-630\sin(dx+c)^4}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*sin(d*x + c)^9 - 315*sin(d*x + c)^8 - 720*sin(d*x + c)^7 + 840
*sin(d*x + c)^6 + 504*sin(d*x + c)^5 - 630*sin(d*x + c)^4)/(a*d)
```

Fricas [A] time = 1.15522, size = 209, normalized size = 2.3

$$\frac{315 \cos(dx+c)^8 - 420 \cos(dx+c)^6 - 8\left(35 \cos(dx+c)^8 - 50 \cos(dx+c)^6 + 3 \cos(dx+c)^4 + 4 \cos(dx+c)^2 + 8\right) \mathrm{si}}{2520\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*cos(d*x + c)^8 - 420*cos(d*x + c)^6 - 8*(35*cos(d*x + c)^8 - 50
*cos(d*x + c)^6 + 3*cos(d*x + c)^4 + 4*cos(d*x + c)^2 + 8)*sin(d*x + c))/(a
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28428, size = 93, normalized size = 1.02

$$-\frac{280 \sin(dx+c)^9 - 315 \sin(dx+c)^8 - 720 \sin(dx+c)^7 + 840 \sin(dx+c)^6 + 504 \sin(dx+c)^5 - 630 \sin(dx+c)^4}{2520\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2520*(280*sin(d*x + c)^9 - 315*sin(d*x + c)^8 - 720*sin(d*x + c)^7 + 840
*sin(d*x + c)^6 + 504*sin(d*x + c)^5 - 630*sin(d*x + c)^4)/(a*d)
```

3.681 $\int \frac{\cos^7(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\sin^7(c+dx)}{7ad}-\frac{2\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^8(c+dx)}{8ad}+\frac{\cos^6(c+dx)}{6ad}$$

```
[Out] Cos[c + d*x]^6/(6*a*d) - Cos[c + d*x]^8/(8*a*d) + Sin[c + d*x]^3/(3*a*d) -
(2*Sin[c + d*x]^5)/(5*a*d) + Sin[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.161681, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2835, 2564, 270, 2565, 14}

$$\frac{\sin^7(c+dx)}{7ad}-\frac{2\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^8(c+dx)}{8ad}+\frac{\cos^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^6/(6*a*d) - Cos[c + d*x]^8/(8*a*d) + Sin[c + d*x]^3/(3*a*d) -
(2*Sin[c + d*x]^5)/(5*a*d) + Sin[c + d*x]^7/(7*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^5(c+dx)\sin^2(c+dx)\,dx}{a}-\frac{\int \cos^5(c+dx)\sin^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sin(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(x^5-x^7\right)\,dx,x,\cos(c+dx)\right)}{ad}\\ &= \frac{\cos^6(c+dx)}{6ad}-\frac{\cos^8(c+dx)}{8ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{2\sin^5(c+dx)}{5ad}+\frac{\sin^7(c+dx)}{7ad}\end{aligned}$$

Mathematica [A] time = 0.291162, size = 68, normalized size = 0.75

$$\frac{\sin^3(c+dx)\left(-105\sin^5(c+dx)+120\sin^4(c+dx)+280\sin^3(c+dx)-336\sin^2(c+dx)-210\sin(c+dx)+280\right)}{840ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^3*(280 - 210*Sin[c + d*x] - 336*Sin[c + d*x]^2 + 280*Sin[c + 
d*x]^3 + 120*Sin[c + d*x]^4 - 105*Sin[c + d*x]^5))/(840*a*d)
```

Maple [A] time = 0.082, size = 69, normalized size = 0.8

$$\frac{1}{da}\left(-\frac{(\sin(dx+c))^8}{8}+\frac{(\sin(dx+c))^7}{7}+\frac{(\sin(dx+c))^6}{3}-\frac{2\,(\sin(dx+c))^5}{5}-\frac{(\sin(dx+c))^4}{4}+\frac{(\sin(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/8*sin(d*x+c)^8+1/7*sin(d*x+c)^7+1/3*sin(d*x+c)^6-2/5*sin(d*x+c)^5
-1/4*sin(d*x+c)^4+1/3*sin(d*x+c)^3)
```

Maxima [A] time = 1.01142, size = 93, normalized size = 1.02

$$-\frac{105\sin(dx+c)^8-120\sin(dx+c)^7-280\sin(dx+c)^6+336\sin(dx+c)^5+210\sin(dx+c)^4-280\sin(dx+c)^3}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*(105*sin(d*x + c)^8 - 120*sin(d*x + c)^7 - 280*sin(d*x + c)^6 + 336*
sin(d*x + c)^5 + 210*sin(d*x + c)^4 - 280*sin(d*x + c)^3)/(a*d)
```

Fricas [A] time = 1.10554, size = 182, normalized size = 2.

$$-\frac{105 \cos(dx+c)^8 - 140 \cos(dx+c)^6 + 8\left(15 \cos(dx+c)^6 - 3 \cos(dx+c)^4 - 4 \cos(dx+c)^2 - 8\right)\sin(dx+c)}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(105*cos(d*x + c)^8 - 140*cos(d*x + c)^6 + 8*(15*cos(d*x + c)^6 - 3*
cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22116, size = 93, normalized size = 1.02

$$-\frac{105 \sin(dx+c)^8 - 120 \sin(dx+c)^7 - 280 \sin(dx+c)^6 + 336 \sin(dx+c)^5 + 210 \sin(dx+c)^4 - 280 \sin(dx+c)^3}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/840*(105*sin(d*x + c)^8 - 120*sin(d*x + c)^7 - 280*sin(d*x + c)^6 + 336*
sin(d*x + c)^5 + 210*sin(d*x + c)^4 - 280*sin(d*x + c)^3)/(a*d)
```

3.682 $\int \frac{\cos^7(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\sin^7(c+dx)}{7ad}+\frac{2\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^6(c+dx)}{6ad}$$

```
[Out] -Cos[c + d*x]^6/(6*a*d) - Sin[c + d*x]^3/(3*a*d) + (2*Sin[c + d*x]^5)/(5*a*
d) - Sin[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.114699, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2835, 2565, 30, 2564, 270}

$$-\frac{\sin^7(c+dx)}{7ad}+\frac{2\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cos[c + d*x]^6/(6*a*d) - Sin[c + d*x]^3/(3*a*d) + (2*Sin[c + d*x]^5)/(5*a*
d) - Sin[c + d*x]^7/(7*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^7(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^5(c+dx)\sin(c+dx)\,dx}{a}-\frac{\int \cos^5(c+dx)\sin^2(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int x^5\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(c+dx)\right)}{ad}\\
&= -\frac{\cos^6(c+dx)}{6ad}-\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sin(c+dx)\right)}{ad}\\
&= -\frac{\cos^6(c+dx)}{6ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{2\sin^5(c+dx)}{5ad}-\frac{\sin^7(c+dx)}{7ad}
\end{aligned}
$$

Mathematica [A] time = 0.26847, size = 68, normalized size = 0.93

$$\frac{\sin^2(c+dx)\left(-30\sin^5(c+dx)+35\sin^4(c+dx)+84\sin^3(c+dx)-105\sin^2(c+dx)-70\sin(c+dx)+105\right)}{210ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^2*(105 - 70*Sin[c + d*x] - 105*Sin[c + d*x]^2 + 84*Sin[c + d*
x]^3 + 35*Sin[c + d*x]^4 - 30*Sin[c + d*x]^5))/(210*a*d)
```

Maple [A] time = 0.065, size = 69, normalized size = 1.

$$\frac{1}{da}\left(-\frac{(\sin(dx+c))^7}{7}+\frac{(\sin(dx+c))^6}{6}+\frac{2(\sin(dx+c))^5}{5}-\frac{(\sin(dx+c))^4}{2}-\frac{(\sin(dx+c))^3}{3}+\frac{(\sin(dx+c))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/7*sin(d*x+c)^7+1/6*sin(d*x+c)^6+2/5*sin(d*x+c)^5-1/2*sin(d*x+c)^4
-1/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2)
```

Maxima [A] time = 1.02475, size = 93, normalized size = 1.27

$$-\frac{30\sin(dx+c)^7-35\sin(dx+c)^6-84\sin(dx+c)^5+105\sin(dx+c)^4+70\sin(dx+c)^3-105\sin(dx+c)^2}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(30*sin(d*x + c)^7 - 35*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin
(d*x + c)^4 + 70*sin(d*x + c)^3 - 105*sin(d*x + c)^2)/(a*d)
```

Fricas [A] time = 1.15168, size = 153, normalized size = 2.1

$$-\frac{35 \cos (dx+c)^6 - 2\left(15 \cos (dx+c)^6 - 3 \cos (dx+c)^4 - 4 \cos (dx+c)^2 - 8\right) \sin (dx+c)}{210\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/210*(35*cos(d*x + c)^6 - 2*(15*cos(d*x + c)^6 - 3*cos(d*x + c)^4 - 4*cos
(d*x + c)^2 - 8)*sin(d*x + c))/(a*d)
```

Sympy [A] time = 120.397, size = 1794, normalized size = 24.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-5*tan(c/2 + d*x/2)**14/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*
tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d
*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 73
5*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 175*tan(c/2 + d*x/2)**12/(105*a*d*ta
n(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/
2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205
*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 280*tan
(c/2 + d*x/2)**11/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)*
*12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a
*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d
*x/2)**2 + 105*a*d) + 105*tan(c/2 + d*x/2)**10/(105*a*d*tan(c/2 + d*x/2)**1
4 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d
*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*
x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 224*tan(c/2 + d*x/2)**9/
(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan
(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2
)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d
) + 525*tan(c/2 + d*x/2)**8/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2
 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**
8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*t
an(c/2 + d*x/2)**2 + 105*a*d) - 912*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2 +
d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 +
 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan
(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 525*tan(c/2 + d
*x/2)**6/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 220
5*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/
2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2
+ 105*a*d) + 224*tan(c/2 + d*x/2)**5/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*
d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 +
 d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 +
735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 105*tan(c/2 + d*x/2)**4/(105*a*d*t
an(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x
/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 220
5*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 280*ta
n(c/2 + d*x/2)**3/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)*
```

```
*12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a
*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d
*x/2)**2 + 105*a*d) + 175*tan(c/2 + d*x/2)**2/(105*a*d*tan(c/2 + d*x/2)**14
 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*
tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x
/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 5/(105*a*d*tan(c/2 + d*x/
2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 367
5*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2
 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d), Ne(d, 0)), (x*sin(c)
*cos(c)**7/(a*sin(c) + a), True))
```

Giac [A] time = 1.29933, size = 93, normalized size = 1.27

$$-\frac{30\sin(dx+c)^7-35\sin(dx+c)^6-84\sin(dx+c)^5+105\sin(dx+c)^4+70\sin(dx+c)^3-105\sin(dx+c)^2}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/210*(30*sin(d*x + c)^7 - 35*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin
(d*x + c)^4 + 70*sin(d*x + c)^3 - 105*sin(d*x + c)^2)/(a*d)
```

3.683 $\int \frac{\cos^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(a-a\sin(c+dx))^5}{5a^6d}-\frac{(a-a\sin(c+dx))^4}{a^5d}$$

```
[Out] -((a - a*Sin[c + d*x])^4/(a^5*d)) + (4*(a - a*Sin[c + d*x])^5)/(5*a^6*d) -
(a - a*Sin[c + d*x])^6/(6*a^7*d)
```

Rubi [A] time = 0.0628442, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$-\frac{(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(a-a\sin(c+dx))^5}{5a^6d}-\frac{(a-a\sin(c+dx))^4}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a - a*Sin[c + d*x])^4/(a^5*d)) + (4*(a - a*Sin[c + d*x])^5)/(5*a^6*d) -
(a - a*Sin[c + d*x])^6/(6*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int (a-x)^3(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\operatorname{Subst}\left(\int \left(4a^2(a-x)^3-4a(a-x)^4+(a-x)^5\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= -\frac{(a-a\sin(c+dx))^4}{a^5d}+\frac{4(a-a\sin(c+dx))^5}{5a^6d}-\frac{(a-a\sin(c+dx))^6}{6a^7d}
\end{aligned}$$

Mathematica [A] time = 0.187619, size = 66, normalized size = 0.97

$$-\frac{\sin(c+dx)\left(5\sin^5(c+dx)-6\sin^4(c+dx)-15\sin^3(c+dx)+20\sin^2(c+dx)+15\sin(c+dx)-30\right)}{30ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sin[c + d*x]*(-30 + 15*Sin[c + d*x] + 20*Sin[c + d*x]^2 - 15*Sin[c + d*x]
^3 - 6*Sin[c + d*x]^4 + 5*Sin[c + d*x]^5))/(30*a*d)
```

Maple [A] time = 0.072, size = 65, normalized size = 1.

$$\frac{1}{da}\left(-\frac{(\sin(dx+c))^6}{6}+\frac{(\sin(dx+c))^5}{5}+\frac{(\sin(dx+c))^4}{2}-\frac{2(\sin(dx+c))^3}{3}-\frac{(\sin(dx+c))^2}{2}+\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/6*sin(d*x+c)^6+1/5*sin(d*x+c)^5+1/2*sin(d*x+c)^4-2/3*sin(d*x+c)^3
-1/2*sin(d*x+c)^2+sin(d*x+c))
```

Maxima [A] time = 1.03408, size = 90, normalized size = 1.32

$$-\frac{5\sin(dx+c)^6-6\sin(dx+c)^5-15\sin(dx+c)^4+20\sin(dx+c)^3+15\sin(dx+c)^2-30\sin(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(5*sin(d*x + c)^6 - 6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*x
 + c)^3 + 15*sin(d*x + c)^2 - 30*sin(d*x + c))/(a*d)
```

Fricas [A] time = 1.09109, size = 122, normalized size = 1.79

$$\frac{5\cos(dx+c)^6+2\left(3\cos(dx+c)^4+4\cos(dx+c)^2+8\right)\sin(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(5*cos(d*x + c)^6 + 2*(3*cos(d*x + c)^4 + 4*cos(d*x + c)^2 + 8)*sin(d*
x + c))/(a*d)
```

Sympy [A] time = 74.0354, size = 1096, normalized size = 16.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((30*tan(c/2 + d*x/2)**11/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*ta
n(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)
**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) -
30*tan(c/2 + d*x/2)**10/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x
/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a
*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 70*tan(c/2
+ d*x/2)**9/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 22
5*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 +
 d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 156*tan(c/2 + d*x/2)**7
/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c
/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4
+ 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 100*tan(c/2 + d*x/2)**6/(15*a*d*ta
n(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)
**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*ta
n(c/2 + d*x/2)**2 + 15*a*d) + 156*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*x
/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a
*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x
/2)**2 + 15*a*d) + 70*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**12 + 90
*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 +
 d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*
a*d) - 30*tan(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2
 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 +
 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*ta
n(c/2 + d*x/2)/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 +
 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/
2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d), Ne(d, 0)), (x*cos(c)*
*7/(a*sin(c) + a), True))
```

Giac [A] time = 1.28969, size = 90, normalized size = 1.32

$$-\frac{5\,\sin\left(dx+c\right)^{6}-6\,\sin\left(dx+c\right)^{5}-15\,\sin\left(dx+c\right)^{4}+20\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)^{2}-30\,\sin\left(dx+c\right)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(5*sin(d*x + c)^6 - 6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*x
 + c)^3 + 15*sin(d*x + c)^2 - 30*sin(d*x + c))/(a*d)
```

3.684 $\int \frac{\cos^6(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=99

$$-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}+\frac{2\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(a*d) + (2*Si
n[c + d*x]^3)/(3*a*d) + Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.10026, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}+\frac{2\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(a*d) + (2*Si
n[c + d*x]^3)/(3*a*d) + Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^3(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4+\frac{a^5}{x}-2a^3x+2a^2x^2+ax^3-x^4\right)dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{ad}+\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^5(c+dx)}{5ad}$$

Mathematica [A] time = 0.0722493, size = 68, normalized size = 0.69

$$\frac{-12\sin^5(c+dx)+15\sin^4(c+dx)+40\sin^3(c+dx)-60\sin^2(c+dx)-60\sin(c+dx)+60\log(\sin(c+dx))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (60*Log[Sin[c + d*x]] - 60*Sin[c + d*x] - 60*Sin[c + d*x]^2 + 40*Sin[c + d*
x]^3 + 15*Sin[c + d*x]^4 - 12*Sin[c + d*x]^5)/(60*a*d)
```

Maple [A] time = 0.101, size = 94, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{da}-\frac{\sin(dx+c)}{da}-\frac{(\sin(dx+c))^2}{da}+\frac{2(\sin(dx+c))^3}{3da}+\frac{(\sin(dx+c))^4}{4da}-\frac{(\sin(dx+c))^5}{5da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/d/a-sin(d*x+c)^2/d/a+2/3*sin(d*x+c)^3/d/a+1/4
*sin(d*x+c)^4/d/a-1/5*sin(d*x+c)^5/d/a
```

Maxima [A] time = 1.03304, size = 96, normalized size = 0.97

$$-\frac{\frac{12\sin(dx+c)^5-15\sin(dx+c)^4-40\sin(dx+c)^3+60\sin(dx+c)^2+60\sin(dx+c)}{a}-\frac{60\log(\sin(dx+c))}{a}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 40*sin(d*x + c)^3 + 60*sin(
d*x + c)^2 + 60*sin(d*x + c))/a - 60*log(sin(d*x + c))/a)/d
```

Fricas [A] time = 1.16531, size = 186, normalized size = 1.88

$$\frac{15 \cos(dx+c)^4 + 30 \cos(dx+c)^2 - 4\left(3 \cos(dx+c)^4 + 4 \cos(dx+c)^2 + 8\right)\sin(dx+c) + 60 \log\left(\frac{1}{2} \sin(dx+c)\right)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^4 + 4*cos(d
*x + c)^2 + 8)*sin(d*x + c) + 60*log(1/2*sin(d*x + c)))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25604, size = 119, normalized size = 1.2

$$\frac{\frac{60 \log(|\sin(dx+c)|)}{a} - \frac{12 a^4 \sin(dx+c)^5 - 15 a^4 \sin(dx+c)^4 - 40 a^4 \sin(dx+c)^3 + 60 a^4 \sin(dx+c)^2 + 60 a^4 \sin(dx+c)}{a^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(sin(d*x + c)))/a - (12*a^4*sin(d*x + c)^5 - 15*a^4*sin(d*x
 + c)^4 - 40*a^4*sin(d*x + c)^3 + 60*a^4*sin(d*x + c)^2 + 60*a^4*sin(d*x +
c))/a^5)/d
```

3.685 $\int \frac{\cos^5(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{ad}-\frac{2\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) - (2*Sin[c + d*x])/(a*d) +
Sin[c + d*x]^2/(a*d) + Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d)
```

Rubi [A] time = 0.119503, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{ad}-\frac{2\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) - (2*Sin[c + d*x])/(a*d) +
Sin[c + d*x]^2/(a*d) + Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^3(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^3+\frac{a^5}{x^2}-\frac{a^4}{x}+2a^2x+ax^2-x^3\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}-\frac{2\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{ad}+\frac{\sin^3(c+dx)}{3ad}-$$

Mathematica [A] time = 0.128917, size = 66, normalized size = 0.69

$$-\frac{3\sin^4(c+dx)-4\sin^3(c+dx)-12\sin^2(c+dx)+24\sin(c+dx)+12\csc(c+dx)+12\log(\sin(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(12*Csc[c + d*x] + 12*Log[Sin[c + d*x]] + 24*Sin[c + d*x] - 12*Sin[c + d*x
]^2 - 4*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4)/(12*a*d)
```

Maple [A] time = 0.118, size = 94, normalized size = 1.

$$-\frac{(\sin(dx+c))^4}{4da}+\frac{(\sin(dx+c))^3}{3da}+\frac{(\sin(dx+c))^2}{da}-2\frac{\sin(dx+c)}{da}-\frac{1}{da\sin(dx+c)}-\frac{\ln(\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4*sin(d*x+c)^4/d/a+1/3*sin(d*x+c)^3/d/a+sin(d*x+c)^2/d/a-2*sin(d*x+c)/d/
a-1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.04327, size = 100, normalized size = 1.05

$$-\frac{\frac{3\sin(dx+c)^4-4\sin(dx+c)^3-12\sin(dx+c)^2+24\sin(dx+c)}{a}+\frac{12\log(\sin(dx+c))}{a}+\frac{12}{a\sin(dx+c)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 - 12*sin(d*x + c)^2 + 24*sin(d*
x + c))/a + 12*log(sin(d*x + c))/a + 12/(a*sin(d*x + c)))/d
```

Fricas [A] time = 1.15926, size = 234, normalized size = 2.46

$$\frac{32\cos(dx+c)^4+128\cos(dx+c)^2-3\left(8\cos(dx+c)^4+16\cos(dx+c)^2-11\right)\sin(dx+c)-96\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-256}{96\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(32*cos(d*x + c)^4 + 128*cos(d*x + c)^2 - 3*(8*cos(d*x + c)^4 + 16*cos
(d*x + c)^2 - 11)*sin(d*x + c) - 96*log(1/2*sin(d*x + c))*sin(d*x + c) - 25
6)/(a*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30527, size = 128, normalized size = 1.35

$$-\frac{\frac{12\log(|\sin(dx+c)|)}{a}-\frac{12(\sin(dx+c)-1)}{a\sin(dx+c)}+\frac{3a^3\sin(dx+c)^4-4a^3\sin(dx+c)^3-12a^3\sin(dx+c)^2+24a^3\sin(dx+c)}{a^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*log(abs(sin(d*x + c)))/a - 12*(sin(d*x + c) - 1)/(a*sin(d*x + c))
 + (3*a^3*sin(d*x + c)^4 - 4*a^3*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 2
4*a^3*sin(d*x + c))/a^4)/d
```

3.686 $\int \frac{\cos^4(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}+\frac{2\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{2\log(\sin(c+dx))}{ad}$$

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (2*Log[Sin[c + d*x]])/(a*d) +
 (2*Sin[c + d*x])/(a*d) + Sin[c + d*x]^2/(2*a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.118378, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}+\frac{2\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{2\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (2*Log[Sin[c + d*x]])/(a*d) +
 (2*Sin[c + d*x])/(a*d) + Sin[c + d*x]^2/(2*a*d) - Sin[c + d*x]^3/(3*a*d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^3(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a^2+\frac{a^5}{x^3}-\frac{a^4}{x^2}-\frac{2a^3}{x}+ax-x^2\right)dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}-\frac{2\log(\sin(c+dx))}{ad}+\frac{2\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}-\sin$$

Mathematica [A] time = 0.108717, size = 66, normalized size = 0.68

$$\frac{-2\sin^3(c+dx)+3\sin^2(c+dx)+12\sin(c+dx)-3\csc^2(c+dx)+6\csc(c+dx)-12\log(\sin(c+dx))}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*Csc[c + d*x] - 3*Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] + 12*Sin[c + d*x]
 + 3*Sin[c + d*x]^2 - 2*Sin[c + d*x]^3)/(6*a*d)
```

Maple [A] time = 0.128, size = 94, normalized size = 1.

$$-\frac{(\sin(dx+c))^3}{3da}+\frac{(\sin(dx+c))^2}{2da}+2\frac{\sin(dx+c)}{da}+\frac{1}{da\sin(dx+c)}-2\frac{\ln(\sin(dx+c))}{da}-\frac{1}{2da(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3*sin(d*x+c)^3/d/a+1/2*sin(d*x+c)^2/d/a+2*sin(d*x+c)/d/a+1/d/a/sin(d*x+c
)-2*ln(sin(d*x+c))/a/d-1/2/d/a/sin(d*x+c)^2
```

Maxima [A] time = 1.02273, size = 100, normalized size = 1.03

$$-\frac{\frac{2\sin(dx+c)^3-3\sin(dx+c)^2-12\sin(dx+c)}{a}+\frac{12\log(\sin(dx+c))}{a}-\frac{3(2\sin(dx+c)-1)}{a\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((2*sin(d*x + c)^3 - 3*sin(d*x + c)^2 - 12*sin(d*x + c))/a + 12*log(si
n(d*x + c))/a - 3*(2*sin(d*x + c) - 1)/(a*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.15647, size = 244, normalized size = 2.52

$$-\frac{6\,\cos\left(dx+c\right)^4-9\,\cos\left(dx+c\right)^2+24\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\left(\cos\left(dx+c\right)^4+4\,\cos\left(dx+c\right)^2-8\right.}{12\left(ad\cos\left(dx+c\right)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^4 - 9*cos(d*x + c)^2 + 24*(cos(d*x + c)^2 - 1)*log(1/
2*sin(d*x + c)) - 4*(cos(d*x + c)^4 + 4*cos(d*x + c)^2 - 8)*sin(d*x + c) -
3)/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33759, size = 127, normalized size = 1.31

$$-\frac{\frac{12\,\log\left(|\sin\left(dx+c\right)|\right)}{a}+\frac{2\,a^2\sin\left(dx+c\right)^3-3\,a^2\sin\left(dx+c\right)^2-12\,a^2\sin\left(dx+c\right)}{a^3}-\frac{3\left(6\,\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)-1\right)}{a\sin\left(dx+c\right)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*log(abs(sin(d*x + c)))/a + (2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x +
 c)^2 - 12*a^2*sin(d*x + c))/a^3 - 3*(6*sin(d*x + c)^2 + 2*sin(d*x + c) - 1
)/(a*sin(d*x + c)^2))/d
```

3.687 $\int \frac{\cos^3(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{2\csc(c+dx)}{ad}+\frac{2\log(\sin(c+dx))}{ad}$$

```
[Out] (2*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) +
(2*Log[Sin[c + d*x]])/(a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.120062, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{2\csc(c+dx)}{ad}+\frac{2\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) +
(2*Log[Sin[c + d*x]])/(a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^3(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a+\frac{a^5}{x^4}-\frac{a^4}{x^3}-\frac{2a^3}{x^2}+\frac{2a^2}{x}-x\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{2\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{2\log(\sin(c+dx))}{ad}+\frac{\sin(c+dx)}{ad}-$$

Mathematica [A] time = 0.17654, size = 66, normalized size = 0.68

$$\frac{-3\sin^2(c+dx)+6\sin(c+dx)-2\csc^3(c+dx)+3\csc^2(c+dx)+12\csc(c+dx)+12\log(\sin(c+dx))}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (12*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 2*Csc[c + d*x]^3 + 12*Log[Sin[c + d*x
]] + 6*Sin[c + d*x] - 3*Sin[c + d*x]^2)/(6*a*d)
```

Maple [A] time = 0.129, size = 94, normalized size = 1.

$$-\frac{(\sin(dx+c))^2}{2da}+\frac{\sin(dx+c)}{da}+2\frac{1}{da\sin(dx+c)}+2\frac{\ln(\sin(dx+c))}{da}-\frac{1}{3da(\sin(dx+c))^3}+\frac{1}{2da(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2*sin(d*x+c)^2/d/a+sin(d*x+c)/d/a+2/d/a/sin(d*x+c)+2*ln(sin(d*x+c))/a/d-
1/3/d/a/sin(d*x+c)^3+1/2/d/a/sin(d*x+c)^2
```

Maxima [A] time = 1.02851, size = 99, normalized size = 1.02

$$-\frac{\frac{3\left(\sin(dx+c)^2-2\sin(dx+c)\right)}{a}-\frac{12\log(\sin(dx+c))}{a}-\frac{12\sin(dx+c)^2+3\sin(dx+c)-2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(sin(d*x + c)^2 - 2*sin(d*x + c))/a - 12*log(sin(d*x + c))/a - (12*
sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.16546, size = 289, normalized size = 2.98

$$-\frac{12\cos(dx+c)^4-24\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-48\cos(dx+c)^2-3\left(2\cos(dx+c)^4-3\cos\right.}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*cos(d*x + c)^4 - 24*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*si
n(d*x + c) - 48*cos(d*x + c)^2 - 3*(2*cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 1
)*sin(d*x + c) + 32)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29019, size = 117, normalized size = 1.21

$$\frac{\frac{12\log(|\sin(dx+c)|)}{a}-\frac{3\left(a\sin(dx+c)^2-2a\sin(dx+c)\right)}{a^2}-\frac{22\sin(dx+c)^3-12\sin(dx+c)^2-3\sin(dx+c)+2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*log(abs(sin(d*x + c)))/a - 3*(a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/
a^2 - (22*sin(d*x + c)^3 - 12*sin(d*x + c)^2 - 3*sin(d*x + c) + 2)/(a*sin(d
*x + c)^3))/d
```

3.688 $\int \frac{\cos^2(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{\sin(c+dx)}{ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{ad}-\frac{2\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] (-2*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(a*d) + Csc[c + d*x]^3/(3*a*d) - C
sc[c + d*x]^4/(4*a*d) + Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d)
```

Rubi [A] time = 0.119448, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\sin(c+dx)}{ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{ad}-\frac{2\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-2*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(a*d) + Csc[c + d*x]^3/(3*a*d) - C
sc[c + d*x]^4/(4*a*d) + Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d)
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a-x)^3(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^5}{x^5}-\frac{a^4}{x^4}-\frac{2a^3}{x^3}+\frac{2a^2}{x^2}+\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= -\frac{2\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\log(\sin(c+dx))}{ad}-\text{si}$$

Mathematica [A] time = 0.315159, size = 66, normalized size = 0.7

$$-\frac{12\sin(c+dx)+3\csc^4(c+dx)-4\csc^3(c+dx)-12\csc^2(c+dx)+24\csc(c+dx)-12\log(\sin(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(24*Csc[c + d*x] - 12*Csc[c + d*x]^2 - 4*Csc[c + d*x]^3 + 3*Csc[c + d*x]^4
 - 12*Log[Sin[c + d*x]] + 12*Sin[c + d*x])/(12*a*d)
```

Maple [A] time = 0.133, size = 93, normalized size = 1.

$$-\frac{\sin(dx+c)}{da}-2\frac{1}{da\sin(dx+c)}-\frac{1}{4da(\sin(dx+c))^4}+\frac{\ln(\sin(dx+c))}{da}+\frac{1}{3da(\sin(dx+c))^3}+\frac{1}{da(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] -sin(d*x+c)/d/a-2/d/a/sin(d*x+c)-1/4/d/a/sin(d*x+c)^4+ln(sin(d*x+c))/a/d+1/
3/d/a/sin(d*x+c)^3+1/d/a/sin(d*x+c)^2
```

Maxima [A] time = 1.00428, size = 97, normalized size = 1.03

$$\frac{\frac{12\log(\sin(dx+c))}{a}-\frac{12\sin(dx+c)}{a}-\frac{24\sin(dx+c)^3-12\sin(dx+c)^2-4\sin(dx+c)+3}{a\sin(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*log(sin(d*x + c))/a - 12*sin(d*x + c)/a - (24*sin(d*x + c)^3 - 12*
sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*sin(d*x + c)^4))/d
```

Fricas [A] time = 1.15473, size = 281, normalized size = 2.99

$$-\frac{12\cos\left(dx+c\right)^{2}-12\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+4\left(3\cos\left(dx+c\right)^{4}-12\cos\left(dx+c\right)\right.}{12\left(ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx+c\right)^{2}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*sin(d*x + c)) + 4*(3*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 8)*sin(d*x + c
) - 9)/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32078, size = 112, normalized size = 1.19

$$\frac{\frac{12\log\left(|\sin\left(dx+c\right)|\right)}{a}-\frac{12\sin\left(dx+c\right)}{a}-\frac{25\sin\left(dx+c\right)^{4}+24\sin\left(dx+c\right)^{3}-12\sin\left(dx+c\right)^{2}-4\sin\left(dx+c\right)+3}{a\sin\left(dx+c\right)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*log(abs(sin(d*x + c)))/a - 12*sin(d*x + c)/a - (25*sin(d*x + c)^4
+ 24*sin(d*x + c)^3 - 12*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*sin(d*x +
c)^4))/d
```

3.689 $\int \frac{\cos(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}+\frac{2\csc^3(c+dx)}{3ad}-\frac{\csc^2(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - Csc[c + d*x]^2/(a*d) + (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^4/(4*a*d) - Csc[c + d*x]^5/(5*a*d) - Log[Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.104356, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}+\frac{2\csc^3(c+dx)}{3ad}-\frac{\csc^2(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Csc[c + d*x]^2/(a*d) + (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^4/(4*a*d) - Csc[c + d*x]^5/(5*a*d) - Log[Sin[c + d*x]]/(a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^3(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}-\frac{a^4}{x^5}-\frac{2a^3}{x^4}+\frac{2a^2}{x^3}+\frac{a}{x^2}-\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{ad}+\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^4(c+dx)}{4ad}-\frac{\csc^5(c+dx)}{5ad}-\text{log(}$$

Mathematica [A] time = 0.102729, size = 68, normalized size = 0.68

$$-\frac{12\csc^5(c+dx)-15\csc^4(c+dx)-40\csc^3(c+dx)+60\csc^2(c+dx)+60\csc(c+dx)+60\log(\sin(c+dx))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(60*Csc[c + d*x] + 60*Csc[c + d*x]^2 - 40*Csc[c + d*x]^3 - 15*Csc[c + d*x]
^4 + 12*Csc[c + d*x]^5 + 60*Log[Sin[c + d*x]])/(60*a*d)
```

Maple [A] time = 0.144, size = 97, normalized size = 1.

$$-\frac{1}{da\sin(dx+c)}-\frac{1}{5da(\sin(dx+c))^5}+\frac{1}{4da(\sin(dx+c))^4}-\frac{\ln(\sin(dx+c))}{da}+\frac{2}{3da(\sin(dx+c))^3}-\frac{1}{da(\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a/sin(d*x+c)-1/5/d/a/sin(d*x+c)^5+1/4/d/a/sin(d*x+c)^4-ln(sin(d*x+c))/
a/d+2/3/d/a/sin(d*x+c)^3-1/d/a/sin(d*x+c)^2
```

Maxima [A] time = 1.04601, size = 95, normalized size = 0.95

$$-\frac{\frac{60\log(\sin(dx+c))}{a}+\frac{60\sin(dx+c)^4+60\sin(dx+c)^3-40\sin(dx+c)^2-15\sin(dx+c)+12}{a\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(sin(d*x + c))/a + (60*sin(d*x + c)^4 + 60*sin(d*x + c)^3 - 40
*sin(d*x + c)^2 - 15*sin(d*x + c) + 12)/(a*sin(d*x + c)^5))/d
```

Fricas [A] time = 1.15792, size = 321, normalized size = 3.21

$$-\frac{60 \cos(dx+c)^4 + 60\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c) - 80\cos(dx+c)^2 - 15\left(4 \cdots\right.}{60\left(ad\cos(dx+c)^4 - 2\,ad\cos(dx+c)^2 + ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^4 + 60*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*sin(d*x + c))*sin(d*x + c) - 80*cos(d*x + c)^2 - 15*(4*cos(d*x + c)^2 -
3)*sin(d*x + c) + 32)/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*si
n(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.295, size = 111, normalized size = 1.11

$$-\frac{\frac{60\log(|\sin(dx+c)|)}{a} - \frac{137\sin(dx+c)^5 - 60\sin(dx+c)^4 - 60\sin(dx+c)^3 + 40\sin(dx+c)^2 + 15\sin(dx+c) - 12}{a\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*log(abs(sin(d*x + c)))/a - (137*sin(d*x + c)^5 - 60*sin(d*x + c)^
4 - 60*sin(d*x + c)^3 + 40*sin(d*x + c)^2 + 15*sin(d*x + c) - 12)/(a*sin(d*
x + c)^5))/d
```

3.690 $\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

```
[Out] -Cot[c + d*x]^6/(6*a*d) + Csc[c + d*x]/(a*d) - (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.0895555, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^6/(6*a*d) + Csc[c + d*x]/(a*d) - (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^5/(5*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^2(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^6(c+dx)}{6ad}+\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^6(c+dx)}{6ad}+\frac{\csc(c+dx)}{ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^5(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [A] time = 0.134506, size = 61, normalized size = 0.9

$$
\frac{\csc^6(c+dx)(78\sin(c+dx)-5(7\sin(3(c+dx))-3\sin(5(c+dx))+5)-15\cos(4(c+dx)))}{240ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^6*(-15*Cos[4*(c + d*x)] + 78*Sin[c + d*x] - 5*(5 + 7*Sin[3*(c
 + d*x)] - 3*Sin[5*(c + d*x)])))/(240*a*d)
```

Maple [A] time = 0.149, size = 67, normalized size = 1.

$$
\frac{1}{da}\left((\sin(dx+c))^{-1}+\frac{1}{5\,(\sin(dx+c))^5}+\frac{1}{2\,(\sin(dx+c))^4}-\frac{1}{6\,(\sin(dx+c))^6}-\frac{2}{3\,(\sin(dx+c))^3}-\frac{1}{2\,(\sin(dx+c))^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/sin(d*x+c)+1/5/sin(d*x+c)^5+1/2/sin(d*x+c)^4-1/6/sin(d*x+c)^6-2/3/
sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A] time = 0.99108, size = 89, normalized size = 1.31

$$
\frac{30\,\sin(dx+c)^5-15\,\sin(dx+c)^4-20\,\sin(dx+c)^3+15\,\sin(dx+c)^2+6\,\sin(dx+c)-5}{30\,ad\sin(dx+c)^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*
x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

Fricas [A] time = 1.10049, size = 248, normalized size = 3.65

$$
\frac{15\,\cos(dx+c)^4-15\,\cos(dx+c)^2-2\left(15\,\cos(dx+c)^4-20\,\cos(dx+c)^2+8\right)\sin(dx+c)+5}{30\left(ad\cos(dx+c)^6-3\,ad\cos(dx+c)^4+3\,ad\cos(dx+c)^2-ad\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*cos(d*x + c)^4 - 15*cos(d*x + c)^2 - 2*(15*cos(d*x + c)^4 - 20*cos
(d*x + c)^2 + 8)*sin(d*x + c) + 5)/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)
^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33578, size = 89, normalized size = 1.31

$$\frac{30 \sin\left(dx+c\right)^{5} - 15 \sin\left(dx+c\right)^{4} - 20 \sin\left(dx+c\right)^{3} + 15 \sin\left(dx+c\right)^{2} + 6 \sin\left(dx+c\right) - 5}{30 \, ad \sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*
x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

3.691 $\int \frac{\cot^7(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{2\csc^5(c+dx)}{5ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] Cot[c + d*x]^6/(6*a*d) - Csc[c + d*x]^3/(3*a*d) + (2*Csc[c + d*x]^5)/(5*a*d
) - Csc[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.138536, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2835, 2606, 270, 2607, 30}

$$\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{2\csc^5(c+dx)}{5ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^6/(6*a*d) - Csc[c + d*x]^3/(3*a*d) + (2*Csc[c + d*x]^5)/(5*a*d
) - Csc[c + d*x]^7/(7*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^7(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{ad}\\
&= \frac{\cot^6(c+dx)}{6ad}-\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \csc(c+dx)\right)}{ad}\\
&= \frac{\cot^6(c+dx)}{6ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{2\csc^5(c+dx)}{5ad}-\frac{\csc^7(c+dx)}{7ad}
\end{aligned}
$$

Mathematica [A] time = 0.151838, size = 68, normalized size = 0.93

$$\frac{\csc^2(c+dx)\left(-30\csc^5(c+dx)+35\csc^4(c+dx)+84\csc^3(c+dx)-105\csc^2(c+dx)-70\csc(c+dx)+105\right)}{210ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^2*(105 - 70*Csc[c + d*x] - 105*Csc[c + d*x]^2 + 84*Csc[c + d*
x]^3 + 35*Csc[c + d*x]^4 - 30*Csc[c + d*x]^5))/(210*a*d)
```

Maple [A] time = 0.158, size = 69, normalized size = 1.

$$\frac{1}{da}\left(-\frac{1}{7\left(\sin(dx+c)\right)^7}+\frac{2}{5\left(\sin(dx+c)\right)^5}-\frac{1}{2\left(\sin(dx+c)\right)^4}+\frac{1}{6\left(\sin(dx+c)\right)^6}-\frac{1}{3\left(\sin(dx+c)\right)^3}+\frac{1}{2\left(\sin(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/7/sin(d*x+c)^7+2/5/sin(d*x+c)^5-1/2/sin(d*x+c)^4+1/6/sin(d*x+c)^6
-1/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.01652, size = 89, normalized size = 1.22

$$\frac{105\sin(dx+c)^5-70\sin(dx+c)^4-105\sin(dx+c)^3+84\sin(dx+c)^2+35\sin(dx+c)-30}{210\,ad\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(105*sin(d*x + c)^5 - 70*sin(d*x + c)^4 - 105*sin(d*x + c)^3 + 84*sin
(d*x + c)^2 + 35*sin(d*x + c) - 30)/(a*d*sin(d*x + c)^7)
```

Fricas [A] time = 1.13911, size = 270, normalized size = 3.7

$$\frac{70 \cos(dx+c)^4 - 56 \cos(dx+c)^2 - 35 \left(3 \cos(dx+c)^4 - 3 \cos(dx+c)^2 + 1\right) \sin(dx+c) + 16}{210 \left(ad \cos(dx+c)^6 - 3\, ad \cos(dx+c)^4 + 3\, ad \cos(dx+c)^2 - ad\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(70*cos(d*x + c)^4 - 56*cos(d*x + c)^2 - 35*(3*cos(d*x + c)^4 - 3*cos
(d*x + c)^2 + 1)*sin(d*x + c) + 16)/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x +
c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32596, size = 89, normalized size = 1.22

$$\frac{105 \sin(dx+c)^5 - 70 \sin(dx+c)^4 - 105 \sin(dx+c)^3 + 84 \sin(dx+c)^2 + 35 \sin(dx+c) - 30}{210\, ad \sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/210*(105*sin(d*x + c)^5 - 70*sin(d*x + c)^4 - 105*sin(d*x + c)^3 + 84*sin
(d*x + c)^2 + 35*sin(d*x + c) - 30)/(a*d*sin(d*x + c)^7)
```

3.692 $\int \frac{\cot^7(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{\cot^8(c+dx)}{8ad}-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{2\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] -Cot[c + d*x]^6/(6*a*d) - Cot[c + d*x]^8/(8*a*d) + Csc[c + d*x]^3/(3*a*d) -
 (2*Csc[c + d*x]^5)/(5*a*d) + Csc[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.160092, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2835, 2607, 14, 2606, 270}

$$-\frac{\cot^8(c+dx)}{8ad}-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{2\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^6/(6*a*d) - Cot[c + d*x]^8/(8*a*d) + Csc[c + d*x]^3/(3*a*d) -
 (2*Csc[c + d*x]^5)/(5*a*d) + Csc[c + d*x]^7/(7*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^7(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^4(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^5\left(1+x^2\right)\,dx,x,-\cot(c+dx)\right.}{ad}\\
&= \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(x^5+x^7\right)\,dx,x,-\cot(c+dx\right.}{ad}\\
&= -\frac{\cot^6(c+dx)}{6ad}-\frac{\cot^8(c+dx)}{8ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{2\csc^5(c+dx)}{5ad}+\frac{\csc^7(c+dx)}{7ad}
\end{aligned}
$$

Mathematica [A] time = 0.140122, size = 68, normalized size = 0.75

$$
\frac{\csc^3(c+dx)\left(-105\csc^5(c+dx)+120\csc^4(c+dx)+280\csc^3(c+dx)-336\csc^2(c+dx)-210\csc(c+dx)+280\right)}{840ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^3*(280 - 210*Csc[c + d*x] - 336*Csc[c + d*x]^2 + 280*Csc[c +
d*x]^3 + 120*Csc[c + d*x]^4 - 105*Csc[c + d*x]^5))/(840*a*d)
```

Maple [A] time = 0.167, size = 69, normalized size = 0.8

$$
\frac{1}{da}\left(\frac{1}{7\left(\sin(dx+c)\right)^7}-\frac{1}{8\left(\sin(dx+c)\right)^8}-\frac{2}{5\left(\sin(dx+c)\right)^5}-\frac{1}{4\left(\sin(dx+c)\right)^4}+\frac{1}{3\left(\sin(dx+c)\right)^6}+\frac{1}{3\left(\sin(dx+c)\right)^3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/7/sin(d*x+c)^7-1/8/sin(d*x+c)^8-2/5/sin(d*x+c)^5-1/4/sin(d*x+c)^4+
1/3/sin(d*x+c)^6+1/3/sin(d*x+c)^3)
```

Maxima [A] time = 1.03196, size = 89, normalized size = 0.98

$$
\frac{280\sin(dx+c)^5-210\sin(dx+c)^4-336\sin(dx+c)^3+280\sin(dx+c)^2+120\sin(dx+c)-105}{840\,ad\sin(dx+c)^8}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(280*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 336*sin(d*x + c)^3 + 280*s
in(d*x + c)^2 + 120*sin(d*x + c) - 105)/(a*d*sin(d*x + c)^8)
```

Fricas [A] time = 1.20344, size = 286, normalized size = 3.14

$$-\frac{210\cos(dx+c)^4-140\cos(dx+c)^2-8\left(35\cos(dx+c)^4-28\cos(dx+c)^2+8\right)\sin(dx+c)+35}{840\left(ad\cos(dx+c)^8-4\,ad\cos(dx+c)^6+6\,ad\cos(dx+c)^4-4\,ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(210*cos(d*x + c)^4 - 140*cos(d*x + c)^2 - 8*(35*cos(d*x + c)^4 - 28
*cos(d*x + c)^2 + 8)*sin(d*x + c) + 35)/(a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x
 + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32088, size = 89, normalized size = 0.98

$$\frac{280\sin(dx+c)^5-210\sin(dx+c)^4-336\sin(dx+c)^3+280\sin(dx+c)^2+120\sin(dx+c)-105}{840\,ad\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/840*(280*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 336*sin(d*x + c)^3 + 280*s
in(d*x + c)^2 + 120*sin(d*x + c) - 105)/(a*d*sin(d*x + c)^8)
```

3.693 $\int \frac{\cot^7(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\cot^8(c+dx)}{8ad}+\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^9(c+dx)}{9ad}+\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] Cot[c + d*x]^6/(6*a*d) + Cot[c + d*x]^8/(8*a*d) - Csc[c + d*x]^5/(5*a*d) +
(2*Csc[c + d*x]^7)/(7*a*d) - Csc[c + d*x]^9/(9*a*d)
```

Rubi [A] time = 0.160435, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2835, 2606, 270, 2607, 14}

$$\frac{\cot^8(c+dx)}{8ad}+\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^9(c+dx)}{9ad}+\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^6/(6*a*d) + Cot[c + d*x]^8/(8*a*d) - Csc[c + d*x]^5/(5*a*d) +
(2*Csc[c + d*x]^7)/(7*a*d) - Csc[c + d*x]^9/(9*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^7(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc^4(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^5(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^5\left(1+x^2\right)\,dx,x,-\cot(c\right.}{ad}\\
&= \frac{\text{Subst}\left(\int \left(x^5+x^7\right)\,dx,x,-\cot(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\csc(c+\right.}{ad}\\
&= \frac{\cot^6(c+dx)}{6ad}+\frac{\cot^8(c+dx)}{8ad}-\frac{\csc^5(c+dx)}{5ad}+\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^9(c+dx)}{9ad}
\end{aligned}
$$

Mathematica [A] time = 0.155548, size = 68, normalized size = 0.75

$$\frac{\csc^4(c+dx)\left(-280\csc^5(c+dx)+315\csc^4(c+dx)+720\csc^3(c+dx)-840\csc^2(c+dx)-504\csc(c+dx)+630\right)}{2520ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^4*(630 - 504*Csc[c + d*x] - 840*Csc[c + d*x]^2 + 720*Csc[c +
d*x]^3 + 315*Csc[c + d*x]^4 - 280*Csc[c + d*x]^5))/(2520*a*d)
```

Maple [A] time = 0.177, size = 69, normalized size = 0.8

$$\frac{1}{da}\left(\frac{2}{7\left(\sin(dx+c)\right)^7}+\frac{1}{8\left(\sin(dx+c)\right)^8}-\frac{1}{5\left(\sin(dx+c)\right)^5}+\frac{1}{4\left(\sin(dx+c)\right)^4}-\frac{1}{9\left(\sin(dx+c)\right)^9}-\frac{1}{3\left(\sin(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(2/7/sin(d*x+c)^7+1/8/sin(d*x+c)^8-1/5/sin(d*x+c)^5+1/4/sin(d*x+c)^4-
1/9/sin(d*x+c)^9-1/3/sin(d*x+c)^6)
```

Maxima [A] time = 1.01303, size = 89, normalized size = 0.98

$$\frac{630\sin(dx+c)^5-504\sin(dx+c)^4-840\sin(dx+c)^3+720\sin(dx+c)^2+315\sin(dx+c)-280}{2520\,ad\sin(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2520*(630*sin(d*x + c)^5 - 504*sin(d*x + c)^4 - 840*sin(d*x + c)^3 + 720*
sin(d*x + c)^2 + 315*sin(d*x + c) - 280)/(a*d*sin(d*x + c)^9)
```

Fricas [A] time = 1.13092, size = 308, normalized size = 3.38

$$-\frac{504 \cos\left(dx+c\right)^{4}-288 \cos\left(dx+c\right)^{2}-105 \left(6 \cos\left(dx+c\right)^{4}-4 \cos\left(dx+c\right)^{2}+1\right) \sin\left(dx+c\right)+64}{2520 \left(ad \cos\left(dx+c\right)^{8}-4 \,ad \cos\left(dx+c\right)^{6}+6 \,ad \cos\left(dx+c\right)^{4}-4 \,ad \cos\left(dx+c\right)^{2}+ad\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(504*cos(d*x + c)^4 - 288*cos(d*x + c)^2 - 105*(6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1)*sin(d*x + c) + 64)/((a*d*cos(d*x + c)^8 - 4*a*d*cos(d
*x + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20144, size = 89, normalized size = 0.98

$$\frac{630 \sin\left(dx+c\right)^{5}-504 \sin\left(dx+c\right)^{4}-840 \sin\left(dx+c\right)^{3}+720 \sin\left(dx+c\right)^{2}+315 \sin\left(dx+c\right)-280}{2520 \,ad \sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2520*(630*sin(d*x + c)^5 - 504*sin(d*x + c)^4 - 840*sin(d*x + c)^3 + 720*
sin(d*x + c)^2 + 315*sin(d*x + c) - 280)/(a*d*sin(d*x + c)^9)
```

3.694 $\int \frac{\cot^7(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc^{10}(c+dx)}{10ad}+\frac{\csc^9(c+dx)}{9ad}+\frac{\csc^8(c+dx)}{4ad}-\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] Csc[c + d*x]^5/(5*a*d) - Csc[c + d*x]^6/(6*a*d) - (2*Csc[c + d*x]^7)/(7*a*d
) + Csc[c + d*x]^8/(4*a*d) + Csc[c + d*x]^9/(9*a*d) - Csc[c + d*x]^10/(10*a
*d)
```

Rubi [A] time = 0.121184, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc^{10}(c+dx)}{10ad}+\frac{\csc^9(c+dx)}{9ad}+\frac{\csc^8(c+dx)}{4ad}-\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^5/(5*a*d) - Csc[c + d*x]^6/(6*a*d) - (2*Csc[c + d*x]^7)/(7*a*d
) + Csc[c + d*x]^8/(4*a*d) + Csc[c + d*x]^9/(9*a*d) - Csc[c + d*x]^10/(10*a
*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^{11}(a-x)^3(a+x)^2}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \left(\frac{a^5}{x^{11}} - \frac{a^4}{x^{10}} - \frac{2a^3}{x^9} + \frac{2a^2}{x^8} + \frac{a}{x^7} - \frac{1}{x^6}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^5(c+dx)}{5ad} - \frac{\csc^6(c+dx)}{6ad} - \frac{2\csc^7(c+dx)}{7ad} + \frac{\csc^8(c+dx)}{4ad} + \frac{\csc^9(c+dx)}{9ad} - \frac{\csc^{10}(c}{10}$$

Mathematica [A] time = 0.106386, size = 68, normalized size = 0.62

$$\frac{\csc^5(c+dx)\left(-126\csc^5(c+dx)+140\csc^4(c+dx)+315\csc^3(c+dx)-360\csc^2(c+dx)-210\csc(c+dx)+252\right)}{1260ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^5*(252 - 210*Csc[c + d*x] - 360*Csc[c + d*x]^2 + 315*Csc[c +
d*x]^3 + 140*Csc[c + d*x]^4 - 126*Csc[c + d*x]^5))/(1260*a*d)
```

Maple [A] time = 0.19, size = 69, normalized size = 0.6

$$\frac{1}{da}\left(-\frac{1}{10\,(\sin(dx+c))^{10}} - \frac{2}{7\,(\sin(dx+c))^{7}} + \frac{1}{4\,(\sin(dx+c))^{8}} + \frac{1}{5\,(\sin(dx+c))^{5}} + \frac{1}{9\,(\sin(dx+c))^{9}} - \frac{1}{6\,(\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/10/sin(d*x+c)^10-2/7/sin(d*x+c)^7+1/4/sin(d*x+c)^8+1/5/sin(d*x+c)
^5+1/9/sin(d*x+c)^9-1/6/sin(d*x+c)^6)
```

Maxima [A] time = 0.995203, size = 89, normalized size = 0.82

$$\frac{252\sin(dx+c)^5 - 210\sin(dx+c)^4 - 360\sin(dx+c)^3 + 315\sin(dx+c)^2 + 140\sin(dx+c) - 126}{1260\,ad\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/1260*(252*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 360*sin(d*x + c)^3 + 315*
sin(d*x + c)^2 + 140*sin(d*x + c) - 126)/(a*d*sin(d*x + c)^10)
```

Fricas [A] time = 1.13609, size = 321, normalized size = 2.94

$$\frac{210\cos(dx+c)^4-105\cos(dx+c)^2-4\left(63\cos(dx+c)^4-36\cos(dx+c)^2+8\right)\sin(dx+c)+21}{1260\left(ad\cos(dx+c)^{10}-5\,ad\cos(dx+c)^8+10\,ad\cos(dx+c)^6-10\,ad\cos(dx+c)^4+5\,ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1260*(210*cos(d*x + c)^4 - 105*cos(d*x + c)^2 - 4*(63*cos(d*x + c)^4 - 36
*cos(d*x + c)^2 + 8)*sin(d*x + c) + 21)/(a*d*cos(d*x + c)^10 - 5*a*d*cos(d*
x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*cos(d*x + c)^4 + 5*a*d*cos(d*x +
c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41387, size = 89, normalized size = 0.82

$$\frac{252\sin(dx+c)^5-210\sin(dx+c)^4-360\sin(dx+c)^3+315\sin(dx+c)^2+140\sin(dx+c)-126}{1260\,ad\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1260*(252*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 360*sin(d*x + c)^3 + 315*
sin(d*x + c)^2 + 140*sin(d*x + c) - 126)/(a*d*sin(d*x + c)^10)
```

3.695 $\int \frac{\cot^7(c+dx)\csc^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{10ad}+\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{4ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}$$

```
[Out] Csc[c + d*x]^6/(6*a*d) - Csc[c + d*x]^7/(7*a*d) - Csc[c + d*x]^8/(4*a*d) +
(2*Csc[c + d*x]^9)/(9*a*d) + Csc[c + d*x]^10/(10*a*d) - Csc[c + d*x]^11/(11
*a*d)
```

Rubi [A] time = 0.122985, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{10ad}+\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{4ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^6/(6*a*d) - Csc[c + d*x]^7/(7*a*d) - Csc[c + d*x]^8/(4*a*d) +
(2*Csc[c + d*x]^9)/(9*a*d) + Csc[c + d*x]^10/(10*a*d) - Csc[c + d*x]^11/(11
*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^{12}(a-x)^3(a+x)^2}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{a^5}{x^{12}} - \frac{a^4}{x^{11}} - \frac{2a^3}{x^{10}} + \frac{2a^2}{x^9} + \frac{a}{x^8} - \frac{1}{x^7}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^6(c+dx)}{6ad} - \frac{\csc^7(c+dx)}{7ad} - \frac{\csc^8(c+dx)}{4ad} + \frac{2\csc^9(c+dx)}{9ad} + \frac{\csc^{10}(c+dx)}{10ad} - \text{cs}$$

Mathematica [A] time = 0.10756, size = 68, normalized size = 0.62

$$\frac{\csc^6(c+dx)\left(-1260\csc^5(c+dx) + 1386\csc^4(c+dx) + 3080\csc^3(c+dx) - 3465\csc^2(c+dx) - 1980\csc(c+dx) + 2\right.}{13860ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^6*(2310 - 1980*Csc[c + d*x] - 3465*Csc[c + d*x]^2 + 3080*Csc[
c + d*x]^3 + 1386*Csc[c + d*x]^4 - 1260*Csc[c + d*x]^5))/(13860*a*d)
```

Maple [A] time = 0.201, size = 69, normalized size = 0.6

$$\frac{1}{da}\left(\frac{1}{10\,(\sin(dx+c))^{10}} - \frac{1}{7\,(\sin(dx+c))^{7}} - \frac{1}{11\,(\sin(dx+c))^{11}} - \frac{1}{4\,(\sin(dx+c))^{8}} + \frac{2}{9\,(\sin(dx+c))^{9}} + \frac{1}{6\,(\sin(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/10/sin(d*x+c)^10-1/7/sin(d*x+c)^7-1/11/sin(d*x+c)^11-1/4/sin(d*x+c
)^8+2/9/sin(d*x+c)^9+1/6/sin(d*x+c)^6)
```

Maxima [A] time = 1.04613, size = 89, normalized size = 0.82

$$\frac{2310\sin(dx+c)^5 - 1980\sin(dx+c)^4 - 3465\sin(dx+c)^3 + 3080\sin(dx+c)^2 + 1386\sin(dx+c) - 1260}{13860\,ad\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/13860*(2310*sin(d*x + c)^5 - 1980*sin(d*x + c)^4 - 3465*sin(d*x + c)^3 +
3080*sin(d*x + c)^2 + 1386*sin(d*x + c) - 1260)/(a*d*sin(d*x + c)^11)
```

Fricas [A] time = 1.10201, size = 347, normalized size = 3.18

$$\frac{1980\,\cos\left(dx+c\right)^{4}-880\,\cos\left(dx+c\right)^{2}-231\,\left(10\,\cos\left(dx+c\right)^{4}-5\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)+160}{13860\,\left(ad\cos\left(dx+c\right)^{10}-5\,ad\cos\left(dx+c\right)^{8}+10\,ad\cos\left(dx+c\right)^{6}-10\,ad\cos\left(dx+c\right)^{4}+5\,ad\cos\left(dx+c\right)^{2}-ad\right)\sin\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/13860*(1980*cos(d*x + c)^4 - 880*cos(d*x + c)^2 - 231*(10*cos(d*x + c)^4
- 5*cos(d*x + c)^2 + 1)*sin(d*x + c) + 160)/((a*d*cos(d*x + c)^10 - 5*a*d*c
os(d*x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*cos(d*x + c)^4 + 5*a*d*cos(d
*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**12/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28163, size = 89, normalized size = 0.82

$$\frac{2310\,\sin\left(dx+c\right)^{5}-1980\,\sin\left(dx+c\right)^{4}-3465\,\sin\left(dx+c\right)^{3}+3080\,\sin\left(dx+c\right)^{2}+1386\,\sin\left(dx+c\right)-1260}{13860\,ad\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13860*(2310*sin(d*x + c)^5 - 1980*sin(d*x + c)^4 - 3465*sin(d*x + c)^3 +
3080*sin(d*x + c)^2 + 1386*sin(d*x + c) - 1260)/(a*d*sin(d*x + c)^11)
```

3.696 $\int \frac{\cot^7(c+dx)\csc^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc^{12}(c+dx)}{12ad}+\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{5ad}-\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] Csc[c + d*x]^7/(7*a*d) - Csc[c + d*x]^8/(8*a*d) - (2*Csc[c + d*x]^9)/(9*a*d
) + Csc[c + d*x]^10/(5*a*d) + Csc[c + d*x]^11/(11*a*d) - Csc[c + d*x]^12/(1
2*a*d)
```

Rubi [A] time = 0.121952, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{\csc^{12}(c+dx)}{12ad}+\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{5ad}-\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^7/(7*a*d) - Csc[c + d*x]^8/(8*a*d) - (2*Csc[c + d*x]^9)/(9*a*d
) + Csc[c + d*x]^10/(5*a*d) + Csc[c + d*x]^11/(11*a*d) - Csc[c + d*x]^12/(1
2*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^{13}(a-x)^3(a+x)^2}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{a^5}{x^{13}} - \frac{a^4}{x^{12}} - \frac{2a^3}{x^{11}} + \frac{2a^2}{x^{10}} + \frac{a}{x^9} - \frac{1}{x^8}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^7(c+dx)}{7ad} - \frac{\csc^8(c+dx)}{8ad} - \frac{2\csc^9(c+dx)}{9ad} + \frac{\csc^{10}(c+dx)}{5ad} + \frac{\csc^{11}(c+dx)}{11ad} - \frac{\csc^{12}}{1}$$

Mathematica [A] time = 0.108163, size = 68, normalized size = 0.62

$$\frac{\csc^7(c+dx)\left(-2310\csc^5(c+dx) + 2520\csc^4(c+dx) + 5544\csc^3(c+dx) - 6160\csc^2(c+dx) - 3465\csc(c+dx) + 3960\right.}{27720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^7*(3960 - 3465*Csc[c + d*x] - 6160*Csc[c + d*x]^2 + 5544*Csc[
c + d*x]^3 + 2520*Csc[c + d*x]^4 - 2310*Csc[c + d*x]^5))/(27720*a*d)
```

Maple [A] time = 0.215, size = 69, normalized size = 0.6

$$\frac{1}{da}\left(\frac{1}{5\,(\sin(dx+c))^{10}} + \frac{1}{7\,(\sin(dx+c))^7} + \frac{1}{11\,(\sin(dx+c))^{11}} - \frac{1}{8\,(\sin(dx+c))^8} - \frac{2}{9\,(\sin(dx+c))^9} - \frac{1}{12\,(\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/5/sin(d*x+c)^10+1/7/sin(d*x+c)^7+1/11/sin(d*x+c)^11-1/8/sin(d*x+c)
^8-2/9/sin(d*x+c)^9-1/12/sin(d*x+c)^12)
```

Maxima [A] time = 1.0265, size = 89, normalized size = 0.82

$$\frac{3960\sin(dx+c)^5 - 3465\sin(dx+c)^4 - 6160\sin(dx+c)^3 + 5544\sin(dx+c)^2 + 2520\sin(dx+c) - 2310}{27720\,ad\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/27720*(3960*sin(d*x + c)^5 - 3465*sin(d*x + c)^4 - 6160*sin(d*x + c)^3 +
5544*sin(d*x + c)^2 + 2520*sin(d*x + c) - 2310)/(a*d*sin(d*x + c)^12)
```

Fricas [A] time = 1.24107, size = 363, normalized size = 3.33

$$-\frac{3465 \cos(dx+c)^4 - 1386 \cos(dx+c)^2 - 40\left(99 \cos(dx+c)^4 - 44 \cos(dx+c)^2 + 8\right)\sin(dx+c) +}{27720\left(ad\cos(dx+c)^{12} - 6\,ad\cos(dx+c)^{10} + 15\,ad\cos(dx+c)^8 - 20\,ad\cos(dx+c)^6 + 15\,ad\cos(dx+c)^4 - 6\,ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/27720*(3465*cos(d*x + c)^4 - 1386*cos(d*x + c)^2 - 40*(99*cos(d*x + c)^4
 - 44*cos(d*x + c)^2 + 8)*sin(d*x + c) + 231)/(a*d*cos(d*x + c)^12 - 6*a*d*
cos(d*x + c)^10 + 15*a*d*cos(d*x + c)^8 - 20*a*d*cos(d*x + c)^6 + 15*a*d*co
s(d*x + c)^4 - 6*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**13/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32478, size = 89, normalized size = 0.82

$$\frac{3960 \sin(dx+c)^5 - 3465 \sin(dx+c)^4 - 6160 \sin(dx+c)^3 + 5544 \sin(dx+c)^2 + 2520 \sin(dx+c) - 2310}{27720\,ad\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/27720*(3960*sin(d*x + c)^5 - 3465*sin(d*x + c)^4 - 6160*sin(d*x + c)^3 +
5544*sin(d*x + c)^2 + 2520*sin(d*x + c) - 2310)/(a*d*sin(d*x + c)^12)
```

3.697 $\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=184

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}-\frac{8a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{6a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{6a^3\sin^{n+6}(c+dx)}{d(n+6)}+\frac{8a^3\sin^{n+7}(c+}{d(n+7)}$$

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (8*a^3*Sin[c + d*x]^(4 + n))/(d*(4 + n)) - (6*a^3*Sin[c + d*x]^(5
+ n))/(d*(5 + n)) + (6*a^3*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (8*a^3*Sin[c
 + d*x]^(7 + n))/(d*(7 + n)) - (3*a^3*Sin[c + d*x]^(9 + n))/(d*(9 + n)) - (
a^3*Sin[c + d*x]^(10 + n))/(d*(10 + n))
```

Rubi [A] time = 0.177537, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}-\frac{8a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{6a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{6a^3\sin^{n+6}(c+dx)}{d(n+6)}+\frac{8a^3\sin^{n+7}(c+}{d(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (8*a^3*Sin[c + d*x]^(4 + n))/(d*(4 + n)) - (6*a^3*Sin[c + d*x]^(5
+ n))/(d*(5 + n)) + (6*a^3*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (8*a^3*Sin[c
 + d*x]^(7 + n))/(d*(7 + n)) - (3*a^3*Sin[c + d*x]^(9 + n))/(d*(9 + n)) - (
a^3*Sin[c + d*x]^(10 + n))/(d*(10 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx=\frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)^6\,dx,x,a\sin(c+dx)\right)}{a^7d}$$

$$=\frac{\text{Subst}\left(\int \left(a^9\left(\frac{x}{a}\right)^n+3a^9\left(\frac{x}{a}\right)^{1+n}-8a^9\left(\frac{x}{a}\right)^{3+n}-6a^9\left(\frac{x}{a}\right)^{4+n}+6a^9\left(\frac{x}{a}\right.\right.\right.}{a^7d}$$

$$=\frac{a^3\sin^{1+n}(c+dx)}{d(1+n)}+\frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)}-\frac{8a^3\sin^{4+n}(c+dx)}{d(4+n)}-\frac{6a^3}{}$$

Mathematica [A] time = 0.904192, size = 126, normalized size = 0.68

$$\frac{a^3 \sin^{n+1}(c+dx)\left(-\frac{\sin^9(c+dx)}{n+10}-\frac{3\sin^8(c+dx)}{n+9}+\frac{8\sin^6(c+dx)}{n+7}+\frac{6\sin^5(c+dx)}{n+6}-\frac{6\sin^4(c+dx)}{n+5}-\frac{8\sin^3(c+dx)}{n+4}+\frac{3\sin(c+dx)}{n+2}+\frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (3*Sin[c + d*x])/(2 + n) - (8*Sin
[c + d*x]^3)/(4 + n) - (6*Sin[c + d*x]^4)/(5 + n) + (6*Sin[c + d*x]^5)/(6 +
 n) + (8*Sin[c + d*x]^6)/(7 + n) - (3*Sin[c + d*x]^8)/(9 + n) - Sin[c + d*x
]^9/(10 + n)))/d
```

Maple [F] time = 20.262, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^7 (\sin (dx+c))^n (a+a\sin (dx+c))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71032, size = 1751, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] ((a^3*n^7 + 34*a^3*n^6 + 472*a^3*n^5 + 3442*a^3*n^4 + 14083*a^3*n^3 + 31804
*a^3*n^2 + 35844*a^3*n + 15120*a^3)*cos(d*x + c)^10 - 5*(a^3*n^7 + 34*a^3*n
^6 + 472*a^3*n^5 + 3442*a^3*n^4 + 14083*a^3*n^3 + 31804*a^3*n^2 + 35844*a^3
*n + 15120*a^3)*cos(d*x + c)^8 + 192*a^3*n^4 + 4*(a^3*n^7 + 28*a^3*n^6 + 30
4*a^3*n^5 + 1618*a^3*n^4 + 4375*a^3*n^3 + 5554*a^3*n^2 + 2520*a^3*n)*cos(d*
x + c)^6 + 4224*a^3*n^3 + 31488*a^3*n^2 + 24*(a^3*n^6 + 24*a^3*n^5 + 208*a^
```

```
3*n^4 + 786*a^3*n^3 + 1231*a^3*n^2 + 630*a^3*n)*cos(d*x + c)^4 + 87936*a^3*
n + 60480*a^3 + 96*(a^3*n^5 + 22*a^3*n^4 + 164*a^3*n^3 + 458*a^3*n^2 + 315*
a^3*n)*cos(d*x + c)^2 - (3*(a^3*n^7 + 35*a^3*n^6 + 497*a^3*n^5 + 3689*a^3*n
^4 + 15302*a^3*n^3 + 34916*a^3*n^2 + 39640*a^3*n + 16800*a^3)*cos(d*x + c)^
8 - 192*a^3*n^4 - 4*(a^3*n^7 + 31*a^3*n^6 + 385*a^3*n^5 + 2485*a^3*n^4 + 89
74*a^3*n^3 + 18004*a^3*n^2 + 18360*a^3*n + 7200*a^3)*cos(d*x + c)^6 - 4224*
a^3*n^3 - 31488*a^3*n^2 - 24*(a^3*n^6 + 26*a^3*n^5 + 255*a^3*n^4 + 1210*a^3
*n^3 + 2924*a^3*n^2 + 3384*a^3*n + 1440*a^3)*cos(d*x + c)^4 - 93696*a^3*n -
 92160*a^3 - 96*(a^3*n^5 + 23*a^3*n^4 + 186*a^3*n^3 + 652*a^3*n^2 + 968*a^3
*n + 480*a^3)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x + c)^n/(d*n^8 + 44*d*n^
7 + 812*d*n^6 + 8162*d*n^5 + 48503*d*n^4 + 172634*d*n^3 + 353884*d*n^2 + 37
3560*d*n + 151200*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36572, size = 1836, normalized size = 9.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((n^3*sin(d*x + c)^n*sin(d*x + c)^10 + 18*n^2*sin(d*x + c)^n*sin(d*x + c)^
10 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^8 + 104*n*sin(d*x + c)^n*sin(d*x + c
)^10 - 60*n^2*sin(d*x + c)^n*sin(d*x + c)^8 + 192*sin(d*x + c)^n*sin(d*x +
c)^10 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)^6 - 372*n*sin(d*x + c)^n*sin(d*x
+ c)^8 + 66*n^2*sin(d*x + c)^n*sin(d*x + c)^6 - 720*sin(d*x + c)^n*sin(d*x
+ c)^8 - n^3*sin(d*x + c)^n*sin(d*x + c)^4 + 456*n*sin(d*x + c)^n*sin(d*x +
 c)^6 - 24*n^2*sin(d*x + c)^n*sin(d*x + c)^4 + 960*sin(d*x + c)^n*sin(d*x +
 c)^6 - 188*n*sin(d*x + c)^n*sin(d*x + c)^4 - 480*sin(d*x + c)^n*sin(d*x +
c)^4)*a^3/(n^4 + 28*n^3 + 284*n^2 + 1232*n + 1920) + 3*(n^3*sin(d*x + c)^n*
sin(d*x + c)^9 + 15*n^2*sin(d*x + c)^n*sin(d*x + c)^9 - 3*n^3*sin(d*x + c)^
n*sin(d*x + c)^7 + 71*n*sin(d*x + c)^n*sin(d*x + c)^9 - 51*n^2*sin(d*x + c)
^n*sin(d*x + c)^7 + 105*sin(d*x + c)^n*sin(d*x + c)^9 + 3*n^3*sin(d*x + c)^
n*sin(d*x + c)^5 - 261*n*sin(d*x + c)^n*sin(d*x + c)^7 + 57*n^2*sin(d*x + c
)^n*sin(d*x + c)^5 - 405*sin(d*x + c)^n*sin(d*x + c)^7 - n^3*sin(d*x + c)^n
*sin(d*x + c)^3 + 333*n*sin(d*x + c)^n*sin(d*x + c)^5 - 21*n^2*sin(d*x + c)
^n*sin(d*x + c)^3 + 567*sin(d*x + c)^n*sin(d*x + c)^5 - 143*n*sin(d*x + c)^
n*sin(d*x + c)^3 - 315*sin(d*x + c)^n*sin(d*x + c)^3)*a^3/(n^4 + 24*n^3 + 2
06*n^2 + 744*n + 945) + 3*(n^3*sin(d*x + c)^n*sin(d*x + c)^8 + 12*n^2*sin(d
*x + c)^n*sin(d*x + c)^8 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^6 + 44*n*sin(d
*x + c)^n*sin(d*x + c)^8 - 42*n^2*sin(d*x + c)^n*sin(d*x + c)^6 + 48*sin(d*
x + c)^n*sin(d*x + c)^8 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)^4 - 168*n*sin(d
*x + c)^n*sin(d*x + c)^6 + 48*n^2*sin(d*x + c)^n*sin(d*x + c)^4 - 192*sin(d
*x + c)^n*sin(d*x + c)^6 - n^3*sin(d*x + c)^n*sin(d*x + c)^2 + 228*n*sin(d*
```

```
x + c)^n*sin(d*x + c)^4 - 18*n^2*sin(d*x + c)^n*sin(d*x + c)^2 + 288*sin(d*
x + c)^n*sin(d*x + c)^4 - 104*n*sin(d*x + c)^n*sin(d*x + c)^2 - 192*sin(d*x
 + c)^n*sin(d*x + c)^2)*a^3/(n^4 + 20*n^3 + 140*n^2 + 400*n + 384) + (n^3*s
in(d*x + c)^n*sin(d*x + c)^7 + 9*n^2*sin(d*x + c)^n*sin(d*x + c)^7 - 3*n^3*
sin(d*x + c)^n*sin(d*x + c)^5 + 23*n*sin(d*x + c)^n*sin(d*x + c)^7 - 33*n^2
*sin(d*x + c)^n*sin(d*x + c)^5 + 15*sin(d*x + c)^n*sin(d*x + c)^7 + 3*n^3*s
in(d*x + c)^n*sin(d*x + c)^3 - 93*n*sin(d*x + c)^n*sin(d*x + c)^5 + 39*n^2*
sin(d*x + c)^n*sin(d*x + c)^3 - 63*sin(d*x + c)^n*sin(d*x + c)^5 - n^3*sin(
d*x + c)^n*sin(d*x + c) + 141*n*sin(d*x + c)^n*sin(d*x + c)^3 - 15*n^2*sin(
d*x + c)^n*sin(d*x + c) + 105*sin(d*x + c)^n*sin(d*x + c)^3 - 71*n*sin(d*x
+ c)^n*sin(d*x + c) - 105*sin(d*x + c)^n*sin(d*x + c))*a^3/(n^4 + 16*n^3 +
86*n^2 + 176*n + 105))/d
```

3.698 $\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=184

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a^2\sin^{n+3}(c+dx)}{d(n+3)}-\frac{6a^2\sin^{n+4}(c+dx)}{d(n+4)}+\frac{6a^2\sin^{n+6}(c+dx)}{d(n+6)}+\frac{2a^2\sin^{n+7}(c+dx)}{d(n+7)}$$

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (2*a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (6*a^2*Sin[c + d*x]^(4
+ n))/(d*(4 + n)) + (6*a^2*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (2*a^2*Sin[c
 + d*x]^(7 + n))/(d*(7 + n)) - (2*a^2*Sin[c + d*x]^(8 + n))/(d*(8 + n)) - (
a^2*Sin[c + d*x]^(9 + n))/(d*(9 + n))
```

Rubi [A] time = 0.175899, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a^2\sin^{n+3}(c+dx)}{d(n+3)}-\frac{6a^2\sin^{n+4}(c+dx)}{d(n+4)}+\frac{6a^2\sin^{n+6}(c+dx)}{d(n+6)}+\frac{2a^2\sin^{n+7}(c+dx)}{d(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (2*a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (6*a^2*Sin[c + d*x]^(4
+ n))/(d*(4 + n)) + (6*a^2*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (2*a^2*Sin[c
 + d*x]^(7 + n))/(d*(7 + n)) - (2*a^2*Sin[c + d*x]^(8 + n))/(d*(8 + n)) - (
a^2*Sin[c + d*x]^(9 + n))/(d*(9 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)^5\,dx,x,a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\text{Subst}\left(\int \left(a^8\left(\frac{x}{a}\right)^n+2a^8\left(\frac{x}{a}\right)^{1+n}-2a^8\left(\frac{x}{a}\right)^{2+n}-6a^8\left(\frac{x}{a}\right)^{3+n}+6a^8\left(\frac{x}{a}\right.\right.\right.}{a^7d}\\ &= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)}+\frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)}-\frac{2a^2\sin^{3+n}(c+dx)}{d(3+n)}-\frac{6a^2}{}\end{aligned}$$

Mathematica [A] time = 0.73897, size = 126, normalized size = 0.68

$$\frac{a^2\sin^{n+1}(c+dx)\left(-\frac{\sin^8(c+dx)}{n+9}-\frac{2\sin^7(c+dx)}{n+8}+\frac{2\sin^6(c+dx)}{n+7}+\frac{6\sin^5(c+dx)}{n+6}-\frac{6\sin^3(c+dx)}{n+4}-\frac{2\sin^2(c+dx)}{n+3}+\frac{2\sin(c+dx)}{n+2}+\frac{1}{n+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (2*Sin[c + d*x])/(2 + n) - (2*Sin
[c + d*x]^2)/(3 + n) - (6*Sin[c + d*x]^3)/(4 + n) + (6*Sin[c + d*x]^5)/(6 +
 n) + (2*Sin[c + d*x]^6)/(7 + n) - (2*Sin[c + d*x]^7)/(8 + n) - Sin[c + d*x
]^8/(9 + n)))/d
```

Maple [F] time = 13.977, size = 0, normalized size = 0.

$$\int(\cos(dx+c))^7(\sin(dx+c))^n(a+a\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64247, size = 1567, normalized size = 8.52

$$-\frac{\left(2\left(a^2n^7+32\,a^2n^6+414\,a^2n^5+2788\,a^2n^4+10469\,a^2n^3+21708\,a^2n^2+22716\,a^2n+9072\,a^2\right)\cos(dx+c)^8-2\left(a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(2*(a^2*n^7 + 32*a^2*n^6 + 414*a^2*n^5 + 2788*a^2*n^4 + 10469*a^2*n^3 + 21
708*a^2*n^2 + 22716*a^2*n + 9072*a^2)*cos(d*x + c)^8 - 2*(a^2*n^7 + 26*a^2*
n^6 + 258*a^2*n^5 + 1240*a^2*n^4 + 3029*a^2*n^3 + 3534*a^2*n^2 + 1512*a^2*n
)*cos(d*x + c)^6 - 96*a^2*n^4 - 1920*a^2*n^3 - 12*(a^2*n^6 + 22*a^2*n^5 + 1
70*a^2*n^4 + 560*a^2*n^3 + 789*a^2*n^2 + 378*a^2*n)*cos(d*x + c)^4 - 12480*
```

```
a^2*n^2 - 28800*a^2*n - 48*(a^2*n^5 + 20*a^2*n^4 + 130*a^2*n^3 + 300*a^2*n^
2 + 189*a^2*n)*cos(d*x + c)^2 - 18144*a^2 + ((a^2*n^7 + 31*a^2*n^6 + 391*a^
2*n^5 + 2581*a^2*n^4 + 9544*a^2*n^3 + 19564*a^2*n^2 + 20304*a^2*n + 8064*a^
2)*cos(d*x + c)^8 - 2*(a^2*n^7 + 29*a^2*n^6 + 343*a^2*n^5 + 2135*a^2*n^4 +
7504*a^2*n^3 + 14756*a^2*n^2 + 14832*a^2*n + 5760*a^2)*cos(d*x + c)^6 - 96*
a^2*n^4 - 1920*a^2*n^3 - 12*(a^2*n^6 + 24*a^2*n^5 + 223*a^2*n^4 + 1020*a^2*
n^3 + 2404*a^2*n^2 + 2736*a^2*n + 1152*a^2)*cos(d*x + c)^4 - 13440*a^2*n^2
- 38400*a^2*n - 48*(a^2*n^5 + 21*a^2*n^4 + 160*a^2*n^3 + 540*a^2*n^2 + 784*
a^2*n + 384*a^2)*cos(d*x + c)^2 - 36864*a^2)*sin(d*x + c))*sin(d*x + c)^n/(
d*n^8 + 40*d*n^7 + 670*d*n^6 + 6100*d*n^5 + 32773*d*n^4 + 105460*d*n^3 + 19
6380*d*n^2 + 190800*d*n + 72576*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32205, size = 1376, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((n^3*sin(d*x + c)^n*sin(d*x + c)^9 + 15*n^2*sin(d*x + c)^n*sin(d*x + c)^9
 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^7 + 71*n*sin(d*x + c)^n*sin(d*x + c)^9
 - 51*n^2*sin(d*x + c)^n*sin(d*x + c)^7 + 105*sin(d*x + c)^n*sin(d*x + c)^9
 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)^5 - 261*n*sin(d*x + c)^n*sin(d*x + c)^
7 + 57*n^2*sin(d*x + c)^n*sin(d*x + c)^5 - 405*sin(d*x + c)^n*sin(d*x + c)^
7 - n^3*sin(d*x + c)^n*sin(d*x + c)^3 + 333*n*sin(d*x + c)^n*sin(d*x + c)^5
 - 21*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 567*sin(d*x + c)^n*sin(d*x + c)^5
 - 143*n*sin(d*x + c)^n*sin(d*x + c)^3 - 315*sin(d*x + c)^n*sin(d*x + c)^3)
*a^2/(n^4 + 24*n^3 + 206*n^2 + 744*n + 945) + 2*(n^3*sin(d*x + c)^n*sin(d*x
 + c)^8 + 12*n^2*sin(d*x + c)^n*sin(d*x + c)^8 - 3*n^3*sin(d*x + c)^n*sin(d
*x + c)^6 + 44*n*sin(d*x + c)^n*sin(d*x + c)^8 - 42*n^2*sin(d*x + c)^n*sin(
d*x + c)^6 + 48*sin(d*x + c)^n*sin(d*x + c)^8 + 3*n^3*sin(d*x + c)^n*sin(d*
x + c)^4 - 168*n*sin(d*x + c)^n*sin(d*x + c)^6 + 48*n^2*sin(d*x + c)^n*sin(
d*x + c)^4 - 192*sin(d*x + c)^n*sin(d*x + c)^6 - n^3*sin(d*x + c)^n*sin(d*x
 + c)^2 + 228*n*sin(d*x + c)^n*sin(d*x + c)^4 - 18*n^2*sin(d*x + c)^n*sin(d
*x + c)^2 + 288*sin(d*x + c)^n*sin(d*x + c)^4 - 104*n*sin(d*x + c)^n*sin(d*
x + c)^2 - 192*sin(d*x + c)^n*sin(d*x + c)^2)*a^2/(n^4 + 20*n^3 + 140*n^2 +
 400*n + 384) + (n^3*sin(d*x + c)^n*sin(d*x + c)^7 + 9*n^2*sin(d*x + c)^n*s
in(d*x + c)^7 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^5 + 23*n*sin(d*x + c)^n*s
in(d*x + c)^7 - 33*n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 15*sin(d*x + c)^n*si
n(d*x + c)^7 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)^3 - 93*n*sin(d*x + c)^n*si
n(d*x + c)^5 + 39*n^2*sin(d*x + c)^n*sin(d*x + c)^3 - 63*sin(d*x + c)^n*sin
(d*x + c)^5 - n^3*sin(d*x + c)^n*sin(d*x + c) + 141*n*sin(d*x + c)^n*sin(d*
x + c)^3 - 15*n^2*sin(d*x + c)^n*sin(d*x + c) + 105*sin(d*x + c)^n*sin(d*x
```

```
+ c)^3 - 71*n*sin(d*x + c)^n*sin(d*x + c) - 105*sin(d*x + c)^n*sin(d*x + c)
)*a^2/(n^4 + 16*n^3 + 86*n^2 + 176*n + 105))/d
```

3.699 $\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=167

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{3a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{3a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{3a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{3a\sin^{n+6}(c+dx)}{d(n+6)}-$$

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (3*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (3*a*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (3*a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (3*a*Sin[c + d*x]^(6 + n
))/(d*(6 + n)) - (a*Sin[c + d*x]^(7 + n))/(d*(7 + n)) - (a*Sin[c + d*x]^(8
+ n))/(d*(8 + n))
```

Rubi [A] time = 0.137666, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2836, 88}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{3a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{3a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{3a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{3a\sin^{n+6}(c+dx)}{d(n+6)}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (3*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (3*a*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (3*a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (3*a*Sin[c + d*x]^(6 + n
))/(d*(6 + n)) - (a*Sin[c + d*x]^(7 + n))/(d*(7 + n)) - (a*Sin[c + d*x]^(8
+ n))/(d*(8 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^7 d}\\
&= \frac{\text{Subst}\left(\int \left(a^7\left(\frac{x}{a}\right)^n+a^7\left(\frac{x}{a}\right)^{1+n}-3a^7\left(\frac{x}{a}\right)^{2+n}-3a^7\left(\frac{x}{a}\right)^{3+n}+3a^7\left(\frac{x}{a}\right)^{4+}\right.\right.}{a^7 d}\\
&= \frac{a\sin^{1+n}(c+dx)}{d(1+n)}+\frac{a\sin^{2+n}(c+dx)}{d(2+n)}-\frac{3a\sin^{3+n}(c+dx)}{d(3+n)}-\frac{3a\sin^{4+n}}{d(4+}
\end{aligned}$$

Mathematica [B] time = 3.15482, size = 659, normalized size = 3.95

$$a\sin^{n+1}(c+dx)\left(5n^7\sin(c+dx)+9n^7\sin(3(c+dx))+5n^7\sin(5(c+dx))+n^7\sin(7(c+dx))+188n^6\sin(c+dx)+\dots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n)*(1755648 + 2521536*n + 1486096*n^2 + 481280*n^3 + 9
4012*n^4 + 11084*n^5 + 724*n^6 + 20*n^7 + 6*(114816 + 262064*n + 219828*n^2
 + 90640*n^3 + 20499*n^4 + 2611*n^5 + 177*n^6 + 5*n^7)*Cos[2*(c + d*x)] + 1
2*(10368 + 25776*n + 24372*n^2 + 11584*n^3 + 3027*n^4 + 439*n^5 + 33*n^6 +
n^7)*Cos[4*(c + d*x)] + 11520*Cos[6*(c + d*x)] + 29664*n*Cos[6*(c + d*x)] +
 29512*n^2*Cos[6*(c + d*x)] + 15008*n^3*Cos[6*(c + d*x)] + 4270*n^4*Cos[6*(
c + d*x)] + 686*n^5*Cos[6*(c + d*x)] + 58*n^6*Cos[6*(c + d*x)] + 2*n^7*Cos[
6*(c + d*x)] + 468720*Sin[c + d*x] + 879324*n*Sin[c + d*x] + 552236*n^2*Sin
[c + d*x] + 167669*n^3*Sin[c + d*x] + 28904*n^4*Sin[c + d*x] + 3050*n^5*Sin
[c + d*x] + 188*n^6*Sin[c + d*x] + 5*n^7*Sin[c + d*x] + 186480*Sin[3*(c + d
*x)] + 439836*n*Sin[3*(c + d*x)] + 384948*n^2*Sin[3*(c + d*x)] + 165273*n^3
*Sin[3*(c + d*x)] + 38232*n^4*Sin[3*(c + d*x)] + 4866*n^5*Sin[3*(c + d*x)]
+ 324*n^6*Sin[3*(c + d*x)] + 9*n^7*Sin[3*(c + d*x)] + 45360*Sin[5*(c + d*x)
] + 114252*n*Sin[5*(c + d*x)] + 110036*n^2*Sin[5*(c + d*x)] + 53525*n^3*Sin
[5*(c + d*x)] + 14360*n^4*Sin[5*(c + d*x)] + 2138*n^5*Sin[5*(c + d*x)] + 16
4*n^6*Sin[5*(c + d*x)] + 5*n^7*Sin[5*(c + d*x)] + 5040*Sin[7*(c + d*x)] + 1
3068*n*Sin[7*(c + d*x)] + 13132*n^2*Sin[7*(c + d*x)] + 6769*n^3*Sin[7*(c +
d*x)] + 1960*n^4*Sin[7*(c + d*x)] + 322*n^5*Sin[7*(c + d*x)] + 28*n^6*Sin[7
*(c + d*x)] + n^7*Sin[7*(c + d*x)]))/(64*d*(1 + n)*(2 + n)*(3 + n)*(4 + n)*
(5 + n)*(6 + n)*(7 + n)*(8 + n))
```

Maple [F] time = 8.074, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^7 (\sin(dx+c))^n (a+a\sin(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57113, size = 1219, normalized size = 7.3

$$-\frac{\left(\left(an^{7}+28\,an^{6}+322\,an^{5}+1960\,an^{4}+6769\,an^{3}+13132\,an^{2}+13068\,an+5040\,a\right)\cos\left(dx+c\right)^{8}-\left(an^{7}+22\,an^{6}+19\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a*n^7 + 28*a*n^6 + 322*a*n^5 + 1960*a*n^4 + 6769*a*n^3 + 13132*a*n^2 + 1
3068*a*n + 5040*a)*cos(d*x + c)^8 - (a*n^7 + 22*a*n^6 + 190*a*n^5 + 820*a*n
^4 + 1849*a*n^3 + 2038*a*n^2 + 840*a*n)*cos(d*x + c)^6 - 48*a*n^4 - 6*(a*n^
6 + 18*a*n^5 + 118*a*n^4 + 348*a*n^3 + 457*a*n^2 + 210*a*n)*cos(d*x + c)^4
- 768*a*n^3 - 4128*a*n^2 - 24*(a*n^5 + 16*a*n^4 + 86*a*n^3 + 176*a*n^2 + 10
5*a*n)*cos(d*x + c)^2 - 8448*a*n - ((a*n^7 + 29*a*n^6 + 343*a*n^5 + 2135*a*
n^4 + 7504*a*n^3 + 14756*a*n^2 + 14832*a*n + 5760*a)*cos(d*x + c)^6 + 48*a*
n^4 + 6*(a*n^6 + 24*a*n^5 + 223*a*n^4 + 1020*a*n^3 + 2404*a*n^2 + 2736*a*n
+ 1152*a)*cos(d*x + c)^4 + 960*a*n^3 + 6720*a*n^2 + 24*(a*n^5 + 21*a*n^4 +
160*a*n^3 + 540*a*n^2 + 784*a*n + 384*a)*cos(d*x + c)^2 + 19200*a*n + 18432
*a)*sin(d*x + c) - 5040*a)*sin(d*x + c)^n/(d*n^8 + 36*d*n^7 + 546*d*n^6 + 4
536*d*n^5 + 22449*d*n^4 + 67284*d*n^3 + 118124*d*n^2 + 109584*d*n + 40320*d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20704, size = 910, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((n^3*sin(d*x + c)^n*sin(d*x + c)^8 + 12*n^2*sin(d*x + c)^n*sin(d*x + c)^8
 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^6 + 44*n*sin(d*x + c)^n*sin(d*x + c)^8
 - 42*n^2*sin(d*x + c)^n*sin(d*x + c)^6 + 48*sin(d*x + c)^n*sin(d*x + c)^8
+ 3*n^3*sin(d*x + c)^n*sin(d*x + c)^4 - 168*n*sin(d*x + c)^n*sin(d*x + c)^6
 + 48*n^2*sin(d*x + c)^n*sin(d*x + c)^4 - 192*sin(d*x + c)^n*sin(d*x + c)^6
 - n^3*sin(d*x + c)^n*sin(d*x + c)^2 + 228*n*sin(d*x + c)^n*sin(d*x + c)^4
- 18*n^2*sin(d*x + c)^n*sin(d*x + c)^2 + 288*sin(d*x + c)^n*sin(d*x + c)^4
- 104*n*sin(d*x + c)^n*sin(d*x + c)^2 - 192*sin(d*x + c)^n*sin(d*x + c)^2)*
a/(n^4 + 20*n^3 + 140*n^2 + 400*n + 384) + (n^3*sin(d*x + c)^n*sin(d*x + c)
^7 + 9*n^2*sin(d*x + c)^n*sin(d*x + c)^7 - 3*n^3*sin(d*x + c)^n*sin(d*x + c
)^5 + 23*n*sin(d*x + c)^n*sin(d*x + c)^7 - 33*n^2*sin(d*x + c)^n*sin(d*x +
c)^5 + 15*sin(d*x + c)^n*sin(d*x + c)^7 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)
^3 - 93*n*sin(d*x + c)^n*sin(d*x + c)^5 + 39*n^2*sin(d*x + c)^n*sin(d*x + c
```

```
)^3 - 63*sin(d*x + c)^n*sin(d*x + c)^5 - n^3*sin(d*x + c)^n*sin(d*x + c) +
141*n*sin(d*x + c)^n*sin(d*x + c)^3 - 15*n^2*sin(d*x + c)^n*sin(d*x + c) +
105*sin(d*x + c)^n*sin(d*x + c)^3 - 71*n*sin(d*x + c)^n*sin(d*x + c) - 105*
sin(d*x + c)^n*sin(d*x + c))*a/(n^4 + 16*n^3 + 86*n^2 + 176*n + 105))/d
```

3.700 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)}-\frac{\sin^{n+2}(c+dx)}{ad(n+2)}-\frac{2\sin^{n+3}(c+dx)}{ad(n+3)}+\frac{2\sin^{n+4}(c+dx)}{ad(n+4)}+\frac{\sin^{n+5}(c+dx)}{ad(n+5)}-\frac{\sin^{n+6}(c+dx)}{ad(n+6)}$$

```
[Out] Sin[c + d*x]^(1 + n)/(a*d*(1 + n)) - Sin[c + d*x]^(2 + n)/(a*d*(2 + n)) - (
2*Sin[c + d*x]^(3 + n))/(a*d*(3 + n)) + (2*Sin[c + d*x]^(4 + n))/(a*d*(4 +
n)) + Sin[c + d*x]^(5 + n)/(a*d*(5 + n)) - Sin[c + d*x]^(6 + n)/(a*d*(6 + n
))
```

Rubi [A] time = 0.164255, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 88}

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)}-\frac{\sin^{n+2}(c+dx)}{ad(n+2)}-\frac{2\sin^{n+3}(c+dx)}{ad(n+3)}+\frac{2\sin^{n+4}(c+dx)}{ad(n+4)}+\frac{\sin^{n+5}(c+dx)}{ad(n+5)}-\frac{\sin^{n+6}(c+dx)}{ad(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a*d*(1 + n)) - Sin[c + d*x]^(2 + n)/(a*d*(2 + n)) - (
2*Sin[c + d*x]^(3 + n))/(a*d*(3 + n)) + (2*Sin[c + d*x]^(4 + n))/(a*d*(4 +
n)) + Sin[c + d*x]^(5 + n)/(a*d*(5 + n)) - Sin[c + d*x]^(6 + n)/(a*d*(6 + n
))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\text{Subst}\left(\int \left(a^5\left(\frac{x}{a}\right)^n-a^5\left(\frac{x}{a}\right)^{1+n}-2a^5\left(\frac{x}{a}\right)^{2+n}+2a^5\left(\frac{x}{a}\right)^{3+n}+a^5\left(\frac{x}{a}\right)^{4+n}-a^5\left(\frac{x}{a}\right)^{5+n}\right)dx,\right.}{a^7d}\\ &= \frac{\sin^{1+n}(c+dx)}{ad(1+n)}-\frac{\sin^{2+n}(c+dx)}{ad(2+n)}-\frac{2\sin^{3+n}(c+dx)}{ad(3+n)}+\frac{2\sin^{4+n}(c+dx)}{ad(4+n)}+\frac{\sin^{5+n}(c+d}{ad(5+n)}\end{aligned}$$

Mathematica [A] time = 0.307137, size = 95, normalized size = 0.69

$$\frac{\sin^{n+1}(c+dx)\left(-\frac{\sin^5(c+dx)}{n+6}+\frac{\sin^4(c+dx)}{n+5}+\frac{2\sin^3(c+dx)}{n+4}-\frac{2\sin^2(c+dx)}{n+3}-\frac{\sin(c+dx)}{n+2}+\frac{1}{n+1}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((1 + n)^(-1) - Sin[c + d*x]/(2 + n) - (2*Sin[c + d*x
]^2)/(3 + n) + (2*Sin[c + d*x]^3)/(4 + n) + Sin[c + d*x]^4/(5 + n) - Sin[c
+ d*x]^5/(6 + n)))/(a*d)
```

Maple [F] time = 3.677, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^7(\sin(dx+c))^n}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

Maxima [A] time = 1.20259, size = 325, normalized size = 2.37

$$-\frac{\left((n^5+15n^4+85n^3+225n^2+274n+120)\sin(dx+c)^6-(n^5+16n^4+95n^3+260n^2+324n+144)\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*sin(d*x + c)^6 - (n^5 + 1
6*n^4 + 95*n^3 + 260*n^2 + 324*n + 144)*sin(d*x + c)^5 - 2*(n^5 + 17*n^4 +
107*n^3 + 307*n^2 + 396*n + 180)*sin(d*x + c)^4 + 2*(n^5 + 18*n^4 + 121*n^3
 + 372*n^2 + 508*n + 240)*sin(d*x + c)^3 + (n^5 + 19*n^4 + 137*n^3 + 461*n^
2 + 702*n + 360)*sin(d*x + c)^2 - (n^5 + 20*n^4 + 155*n^3 + 580*n^2 + 1044*
n + 720)*sin(d*x + c))*sin(d*x + c)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 +
1624*n^2 + 1764*n + 720)*a*d)
```

Fricas [A] time = 1.29196, size = 639, normalized size = 4.66

$$\frac{\left((n^5+15n^4+85n^3+225n^2+274n+120)\cos(dx+c)^6-(n^5+11n^4+41n^3+61n^2+30n)\cos(dx+c)^4-8n^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*cos(d*x + c)^6 - (n^5 + 11
*n^4 + 41*n^3 + 61*n^2 + 30*n)*cos(d*x + c)^4 - 8*n^3 - 4*(n^4 + 9*n^3 + 23
*n^2 + 15*n)*cos(d*x + c)^2 - 72*n^2 + ((n^5 + 16*n^4 + 95*n^3 + 260*n^2 +
324*n + 144)*cos(d*x + c)^4 + 8*n^3 + 4*(n^4 + 13*n^3 + 56*n^2 + 92*n + 48)
*cos(d*x + c)^2 + 96*n^2 + 352*n + 384)*sin(d*x + c) - 184*n - 120)*sin(d*x
 + c)^n/(a*d*n^6 + 21*a*d*n^5 + 175*a*d*n^4 + 735*a*d*n^3 + 1624*a*d*n^2 +
1764*a*d*n + 720*a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.52594, size = 188, normalized size = 1.37

$$-\frac{\frac{\sin(dx+c)^n \sin(dx+c)^6}{n+6} - \frac{\sin(dx+c)^n \sin(dx+c)^5}{n+5} - \frac{2 \sin(dx+c)^n \sin(dx+c)^4}{n+4} + \frac{2 \sin(dx+c)^n \sin(dx+c)^3}{n+3} + \frac{\sin(dx+c)^n \sin(dx+c)^2}{n+2} - \frac{\sin(dx+c)^{n+1}}{n+1}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(sin(d*x + c)^n*sin(d*x + c)^6/(n + 6) - sin(d*x + c)^n*sin(d*x + c)^5/(n
+ 5) - 2*sin(d*x + c)^n*sin(d*x + c)^4/(n + 4) + 2*sin(d*x + c)^n*sin(d*x +
 c)^3/(n + 3) + sin(d*x + c)^n*sin(d*x + c)^2/(n + 2) - sin(d*x + c)^(n + 1
)/(n + 1))/(a*d)
```

3.701 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)}-\frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)}+\frac{2\sin^{n+4}(c+dx)}{a^2d(n+4)}-\frac{\sin^{n+5}(c+dx)}{a^2d(n+5)}$$

```
[Out] Sin[c + d*x]^(1 + n)/(a^2*d*(1 + n)) - (2*Sin[c + d*x]^(2 + n))/(a^2*d*(2 +
 n)) + (2*Sin[c + d*x]^(4 + n))/(a^2*d*(4 + n)) - Sin[c + d*x]^(5 + n)/(a^2
*d*(5 + n))
```

Rubi [A] time = 0.142898, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 75}

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)}-\frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)}+\frac{2\sin^{n+4}(c+dx)}{a^2d(n+4)}-\frac{\sin^{n+5}(c+dx)}{a^2d(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^2*d*(1 + n)) - (2*Sin[c + d*x]^(2 + n))/(a^2*d*(2 +
 n)) + (2*Sin[c + d*x]^(4 + n))/(a^2*d*(4 + n)) - Sin[c + d*x]^(5 + n)/(a^2
*d*(5 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\operatorname{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)\,dx,x,a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^4\left(\frac{x}{a}\right)^n-2a^4\left(\frac{x}{a}\right)^{1+n}+2a^4\left(\frac{x}{a}\right)^{3+n}-a^4\left(\frac{x}{a}\right)^{4+n}\right)dx,x,a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\sin^{1+n}(c+dx)}{a^2d(1+n)}-\frac{2\sin^{2+n}(c+dx)}{a^2d(2+n)}+\frac{2\sin^{4+n}(c+dx)}{a^2d(4+n)}-\frac{\sin^{5+n}(c+dx)}{a^2d(5+n)}\end{aligned}$$

Mathematica [A] time = 0.340678, size = 117, normalized size = 1.27

$$\frac{\sin^{n+1}(c+dx)\left(-\left(n^3+7n^2+14n+8\right)\sin^4(c+dx)+2\left(n^3+8n^2+17n+10\right)\sin^3(c+dx)-2\left(n^3+10n^2+29n+20\right)\mathrm{s}\right.}{a^2d(n+1)(n+2)(n+4)(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(40 + 38*n + 11*n^2 + n^3 - 2*(20 + 29*n + 10*n^2 + n
^3)*Sin[c + d*x] + 2*(10 + 17*n + 8*n^2 + n^3)*Sin[c + d*x]^3 - (8 + 14*n +
 7*n^2 + n^3)*Sin[c + d*x]^4))/(a^2*d*(1 + n)*(2 + n)*(4 + n)*(5 + n))
```

Maple [F] time = 2.848, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^7(\sin(dx+c))^n}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

Maxima [A] time = 1.20429, size = 170, normalized size = 1.85

$$-\frac{\left(\left(n^3+7\,n^2+14\,n+8\right)\sin(dx+c)^5-2\left(n^3+8\,n^2+17\,n+10\right)\sin(dx+c)^4+2\left(n^3+10\,n^2+29\,n+20\right)\sin(dx+c)^2\right.}{\left(n^4+12\,n^3+49\,n^2+78\,n+40\right)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -((n^3 + 7*n^2 + 14*n + 8)*sin(d*x + c)^5 - 2*(n^3 + 8*n^2 + 17*n + 10)*sin
(d*x + c)^4 + 2*(n^3 + 10*n^2 + 29*n + 20)*sin(d*x + c)^2 - (n^3 + 11*n^2 +
 38*n + 40)*sin(d*x + c))*sin(d*x + c)^n/((n^4 + 12*n^3 + 49*n^2 + 78*n + 4
0)*a^2*d)
```

Fricas [A] time = 1.22766, size = 414, normalized size = 4.5

$$\frac{\left(2\left(n^3+8\,n^2+17\,n+10\right)\cos(dx+c)^4-2\left(n^3+6\,n^2+5\,n\right)\cos(dx+c)^2-4\,n^2-\left(\left(n^3+7\,n^2+14\,n+8\right)\cos(dx+c)^4\right.\right.}{a^2dn^4+12\,a^2dn^3+49\,a^2dn^2+78\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (2*(n^3 + 8*n^2 + 17*n + 10)*cos(d*x + c)^4 - 2*(n^3 + 6*n^2 + 5*n)*cos(d*x
 + c)^2 - 4*n^2 - ((n^3 + 7*n^2 + 14*n + 8)*cos(d*x + c)^4 - 2*(n^3 + 7*n^2
```

```
 + 14*n + 8)*cos(d*x + c)^2 - 4*n^2 - 24*n - 32)*sin(d*x + c) - 24*n - 20)*
sin(d*x + c)^n/(a^2*d*n^4 + 12*a^2*d*n^3 + 49*a^2*d*n^2 + 78*a^2*d*n + 40*a
^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.58285, size = 127, normalized size = 1.38

$$-\frac{\frac{\sin(dx+c)^n\sin(dx+c)^5}{n+5}-\frac{2\sin(dx+c)^n\sin(dx+c)^4}{n+4}+\frac{2\sin(dx+c)^n\sin(dx+c)^2}{n+2}-\frac{\sin(dx+c)^{n+1}}{n+1}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(sin(d*x + c)^n*sin(d*x + c)^5/(n + 5) - 2*sin(d*x + c)^n*sin(d*x + c)^4/(
n + 4) + 2*sin(d*x + c)^n*sin(d*x + c)^2/(n + 2) - sin(d*x + c)^(n + 1)/(n
+ 1))/(a^2*d)
```

3.702 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$\frac{\sin^{n+1}(c+dx)}{a^3d(n+1)}-\frac{3\sin^{n+2}(c+dx)}{a^3d(n+2)}+\frac{3\sin^{n+3}(c+dx)}{a^3d(n+3)}-\frac{\sin^{n+4}(c+dx)}{a^3d(n+4)}$$

```
[Out] Sin[c + d*x]^(1 + n)/(a^3*d*(1 + n)) - (3*Sin[c + d*x]^(2 + n))/(a^3*d*(2 +
 n)) + (3*Sin[c + d*x]^(3 + n))/(a^3*d*(3 + n)) - Sin[c + d*x]^(4 + n)/(a^3
*d*(4 + n))
```

Rubi [A] time = 0.137075, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 43}

$$\frac{\sin^{n+1}(c+dx)}{a^3d(n+1)}-\frac{3\sin^{n+2}(c+dx)}{a^3d(n+2)}+\frac{3\sin^{n+3}(c+dx)}{a^3d(n+3)}-\frac{\sin^{n+4}(c+dx)}{a^3d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^3*d*(1 + n)) - (3*Sin[c + d*x]^(2 + n))/(a^3*d*(2 +
 n)) + (3*Sin[c + d*x]^(3 + n))/(a^3*d*(3 + n)) - Sin[c + d*x]^(4 + n)/(a^3
*d*(4 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n\,dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \left(a^3\left(\frac{x}{a}\right)^n-3a^3\left(\frac{x}{a}\right)^{1+n}+3a^3\left(\frac{x}{a}\right)^{2+n}-a^3\left(\frac{x}{a}\right)^{3+n}\right)dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\sin^{1+n}(c+dx)}{a^3d(1+n)}-\frac{3\sin^{2+n}(c+dx)}{a^3d(2+n)}+\frac{3\sin^{3+n}(c+dx)}{a^3d(3+n)}-\frac{\sin^{4+n}(c+dx)}{a^3d(4+n)}
\end{aligned}$$

Mathematica [A] time = 0.188185, size = 66, normalized size = 0.72

$$\frac{\sin^{n+1}(c+dx)\left(-\frac{\sin^3(c+dx)}{n+4}+\frac{3\sin^2(c+dx)}{n+3}-\frac{3\sin(c+dx)}{n+2}+\frac{1}{n+1}\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((1 + n)^(-1) - (3*Sin[c + d*x])/(2 + n) + (3*Sin[c +
 d*x]^2)/(3 + n) - Sin[c + d*x]^3/(4 + n)))/(a^3*d)
```

Maple [F] time = 2.116, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^7(\sin(dx+c))^n}{(a+a\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

Maxima [A] time = 1.21105, size = 170, normalized size = 1.85

$$-\frac{\left(\left(n^3+6n^2+11n+6\right)\sin(dx+c)^4-3\left(n^3+7n^2+14n+8\right)\sin(dx+c)^3+3\left(n^3+8n^2+19n+12\right)\sin(dx+c)^2\right.}{\left(n^4+10n^3+35n^2+50n+24\right)a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -((n^3 + 6*n^2 + 11*n + 6)*sin(d*x + c)^4 - 3*(n^3 + 7*n^2 + 14*n + 8)*sin(
d*x + c)^3 + 3*(n^3 + 8*n^2 + 19*n + 12)*sin(d*x + c)^2 - (n^3 + 9*n^2 + 26
*n + 24)*sin(d*x + c))*sin(d*x + c)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*
a^3*d)
```

Fricas [A] time = 1.18212, size = 389, normalized size = 4.23

$$-\frac{\left(\left(n^3+6n^2+11n+6\right)\cos(dx+c)^4+4n^3-\left(5n^3+36n^2+79n+48\right)\cos(dx+c)^2+30n^2-\left(4n^3-3\left(n^3+7n^2\right.\right.\right.}{a^3dn^4+10a^3dn^3+35a^3dn^2+50a^3dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -((n^3 + 6*n^2 + 11*n + 6)*cos(d*x + c)^4 + 4*n^3 - (5*n^3 + 36*n^2 + 79*n
+ 48)*cos(d*x + c)^2 + 30*n^2 - (4*n^3 - 3*(n^3 + 7*n^2 + 14*n + 8)*cos(d*x
```

```
 + c)^2 + 30*n^2 + 68*n + 48)*sin(d*x + c) + 68*n + 42)*sin(d*x + c)^n/(a^3
*d*n^4 + 10*a^3*d*n^3 + 35*a^3*d*n^2 + 50*a^3*d*n + 24*a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.64291, size = 127, normalized size = 1.38

$$-\frac{\frac{\sin(dx+c)^n \sin(dx+c)^4}{n+4} - \frac{3 \sin(dx+c)^n \sin(dx+c)^3}{n+3} + \frac{3 \sin(dx+c)^n \sin(dx+c)^2}{n+2} - \frac{\sin(dx+c)^{n+1}}{n+1}}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(sin(d*x + c)^n*sin(d*x + c)^4/(n + 4) - 3*sin(d*x + c)^n*sin(d*x + c)^3/(
n + 3) + 3*sin(d*x + c)^n*sin(d*x + c)^2/(n + 2) - sin(d*x + c)^(n + 1)/(n
+ 1))/(a^3*d)
```

3.703 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=109

$$\frac{8\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}-\frac{7\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+2}(c+dx)}{a^4d(n+2)}-\frac{\sin^{n+3}(c+dx)}{a^4d(n+3)}$$

```
[Out] (-7*Sin[c + d*x]^(1 + n))/(a^4*d*(1 + n)) + (8*Hypergeometric2F1[1, 1 + n,
2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^4*d*(1 + n)) + (4*Sin[c + d*
x]^(2 + n))/(a^4*d*(2 + n)) - Sin[c + d*x]^(3 + n)/(a^4*d*(3 + n))
```

Rubi [A] time = 0.174651, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2836, 88, 43, 64}

$$\frac{8\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}-\frac{7\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+2}(c+dx)}{a^4d(n+2)}-\frac{\sin^{n+3}(c+dx)}{a^4d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-7*Sin[c + d*x]^(1 + n))/(a^4*d*(1 + n)) + (8*Hypergeometric2F1[1, 1 + n,
2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^4*d*(1 + n)) + (4*Sin[c + d*
x]^(2 + n))/(a^4*d*(2 + n)) - Sin[c + d*x]^(3 + n)/(a^4*d*(3 + n))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$

$$= \frac{\text{Subst}\left(\int \left(-4a^2\left(\frac{x}{a}\right)^n - 2a(a-x)\left(\frac{x}{a}\right)^n - (a-x)^2\left(\frac{x}{a}\right)^n + \frac{8a^3\left(\frac{x}{a}\right)^n}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^7d}$$

$$= -\frac{4\sin^{1+n}(c+dx)}{a^4d(1+n)} - \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n\,dx, x, a\sin(c+dx)\right)}{a^7d} - \frac{2\,\text{Subst}\left(\int (a-x)\left(\right.\right.}{}$$

$$= -\frac{4\sin^{1+n}(c+dx)}{a^4d(1+n)} + \frac{8\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d(1+n)} - \frac{\text{Subst}\left(\int \left(a\right.\right.}{}$$

$$= -\frac{7\sin^{1+n}(c+dx)}{a^4d(1+n)} + \frac{8\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d(1+n)} + \frac{4\sin^{2+n}(c+}{a^4d(2+n}$$

Mathematica [A] time = 0.254182, size = 104, normalized size = 0.95

$$\frac{\frac{8a^3\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{n+1} - \frac{7a^3\sin^{n+1}(c+dx)}{n+1} + \frac{4a^3\sin^{n+2}(c+dx)}{n+2} - \frac{a^3\sin^{n+3}(c+dx)}{n+3}}{a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] ((-7*a^3*Sin[c + d*x]^(1 + n))/(1 + n) + (8*a^3*Hypergeometric2F1[1, 1 + n,
 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(1 + n) + (4*a^3*Sin[c + d*x]^
(2 + n))/(2 + n) - (a^3*Sin[c + d*x]^(3 + n))/(3 + n))/(a^7*d)
```

Maple [F] time = 1.497, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^7(\sin(dx+c))^n}{(a+a\sin(dx+c))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^7}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{7}}{a^{4}\cos\left(dx+c\right)^{4}-8\,a^{4}\cos\left(dx+c\right)^{2}+8\,a^{4}-4\left(a^{4}\cos\left(dx+c\right)^{2}-2\,a^{4}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^7/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x
+ c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(dx+c\right)^{n}\cos\left(dx+c\right)^{7}}{\left(a\sin\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^4, x)
```

3.704 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^5}\,dx$

Optimal. Leaf size=160

$$-\frac{4(2n+3)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^5d(n+1)}+\frac{\sin^{n+1}(c+dx)\left(a(2n+7)\sin(c+dx)+a\left(8n^2+30n+27\right.\right.}{d\left(n^2+3n+2\right)\left(a^6\sin(c+dx)+a^6\right)}$$

```
[Out] (-4*(3 + 2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x
]^(1 + n))/(a^5*d*(1 + n)) - (Sin[c + d*x]^(1 + n)*(a - a*Sin[c + d*x])^2)/
(d*(2 + n)*(a^7 + a^7*Sin[c + d*x])) + (Sin[c + d*x]^(1 + n)*(a*(27 + 30*n
+ 8*n^2) + a*(7 + 2*n)*Sin[c + d*x]))/(d*(2 + 3*n + n^2)*(a^6 + a^6*Sin[c +
 d*x]))
```

Rubi [A] time = 0.203876, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2836, 100, 146, 64}

$$-\frac{4(2n+3)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^5d(n+1)}+\frac{\sin^{n+1}(c+dx)\left(a(2n+7)\sin(c+dx)+a\left(8n^2+30n+27\right.\right.}{d\left(n^2+3n+2\right)\left(a^6\sin(c+dx)+a^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^5,x]
```

```
[Out] (-4*(3 + 2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x
]^(1 + n))/(a^5*d*(1 + n)) - (Sin[c + d*x]^(1 + n)*(a - a*Sin[c + d*x])^2)/
(d*(2 + n)*(a^7 + a^7*Sin[c + d*x])) + (Sin[c + d*x]^(1 + n)*(a*(27 + 30*n
+ 8*n^2) + a*(7 + 2*n)*Sin[c + d*x]))/(d*(2 + 3*n + n^2)*(a^6 + a^6*Sin[c +
 d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
```

```
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^5}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3\left(\frac{x}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= -\frac{\sin^{1+n}(c+dx)(a-a\sin(c+dx))^2}{d(2+n)\left(a^7+a^7\sin(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{(a-x)\left(\frac{x}{a}\right)^n(a(3+2n)+(-7-2n)x)}{(a+x)^2}\,dx, x, a\sin\right.}{a^6 d(2+n)}$$

$$= -\frac{\sin^{1+n}(c+dx)(a-a\sin(c+dx))^2}{d(2+n)\left(a^7+a^7\sin(c+dx)\right)} + \frac{\sin^{1+n}(c+dx)\left(a\left(27+30n+8n^2\right)+a(7+\right.}{d(1+n)(2+n)\left(a^6+a^6\sin(c+\right.}$$

$$= -\frac{4(3+2n)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^5 d(1+n)} - \frac{\sin^{1+n}(c+dx)(a-a}{d(2+n)\left(a^7+a^7\,\mathrm{s}\right.}$$

Mathematica [A] time = 0.206406, size = 108, normalized size = 0.68

$$\frac{\sin^{n+1}(c+dx)\left(-4\left(2n^2+7n+6\right)(\sin(c+dx)+1)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))-(n+1)\sin^2(c+dx)+(4n+9\right.}{a^5 d(n+1)(n+2)(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^5,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(26 + 29*n + 8*n^2 + (9 + 4*n)*Sin[c + d*x] - (1 + n)
*Sin[c + d*x]^2 - 4*(6 + 7*n + 2*n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -S
in[c + d*x]]*(1 + Sin[c + d*x])))/(a^5*d*(1 + n)*(2 + n)*(1 + Sin[c + d*x])
)
```

Maple [F] time = 1.239, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^7(\sin(dx+c))^n}{(a+a\sin(dx+c))^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^7}{(a\sin(dx+c)+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)^n \cos(dx+c)^7}{5\,a^5\cos(dx+c)^4 - 20\,a^5\cos(dx+c)^2 + 16\,a^5 + \left(a^5\cos(dx+c)^4 - 12\,a^5\cos(dx+c)^2 + 16\,a^5\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x, algorithm="fricas
")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^7/(5*a^5*cos(d*x + c)^4 - 20*a^5*cos(d
*x + c)^2 + 16*a^5 + (a^5*cos(d*x + c)^4 - 12*a^5*cos(d*x + c)^2 + 16*a^5)*
sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^7}{(a\sin(dx+c)+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^5, x)
```

3.705 $\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=209

$$-\frac{\cos^{11}(c+dx)}{11ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^7(c+dx)}{12ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{24ad}+\frac{\sin(c+d}{}$$

```
[Out] (-5*x)/(1024*a) - Cos[c + d*x]^7/(7*a*d) + (2*Cos[c + d*x]^9)/(9*a*d) - Cos
[c + d*x]^11/(11*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(1024*a*d) - (5*Cos[c
 + d*x]^3*Sin[c + d*x])/(1536*a*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(384*a*d
) + (Cos[c + d*x]^7*Sin[c + d*x])/(64*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^3
)/(24*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^5)/(12*a*d)
```

Rubi [A] time = 0.281858, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2565, 270, 2568, 2635, 8}

$$-\frac{\cos^{11}(c+dx)}{11ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^7(c+dx)}{12ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{24ad}+\frac{\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-5*x)/(1024*a) - Cos[c + d*x]^7/(7*a*d) + (2*Cos[c + d*x]^9)/(9*a*d) - Cos
[c + d*x]^11/(11*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(1024*a*d) - (5*Cos[c
 + d*x]^3*Sin[c + d*x])/(1536*a*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(384*a*d
) + (Cos[c + d*x]^7*Sin[c + d*x])/(64*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^3
)/(24*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^5)/(12*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
```

```
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^6(c+dx)\sin^5(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^6(c+dx)\,dx}{a}$$

$$= \frac{\cos^7(c+dx)\sin^5(c+dx)}{12ad} - \frac{5\int \cos^6(c+dx)\sin^4(c+dx)\,dx}{12a} - \frac{\text{Subst}\left(\int x^6\left(1-x^2\right)^2$$

$$= \frac{\cos^7(c+dx)\sin^3(c+dx)}{24ad} + \frac{\cos^7(c+dx)\sin^5(c+dx)}{12ad} - \frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{8a}$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} + \frac{\cos^7(c+dx)\sin(c+dx)}{64ad} + \frac{\cos^7(c}{}$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{384ad} + \frac{\cos^7(c}{}$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} - \frac{5\cos^3(c+dx)\sin(c+dx)}{1536ad} - \frac{\cos^5(c}{}$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} - \frac{5\cos(c+dx)\sin(c+dx)}{1024ad} - \frac{5\cos^3(}{}$$

$$= -\frac{5x}{1024a} - \frac{\cos^7(c+dx)}{7ad} + \frac{2\cos^9(c+dx)}{9ad} - \frac{\cos^{11}(c+dx)}{11ad} - \frac{5\cos(c+dx)\sin(c+dx)}{1024ad}$$

Mathematica [B] time = 14.4436, size = 518, normalized size = 2.48

$$-\frac{55440dx\sin\left(\frac{c}{2}\right) - 55440\sin\left(\frac{c}{2}+dx\right) + 55440\sin\left(\frac{3c}{2}+dx\right) - 18480\sin\left(\frac{5c}{2}+3dx\right) + 18480\sin\left(\frac{7c}{2}+3dx\right) - 10395\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(55440*d*x*Cos[c/2] + 55440*Cos[c/2 + d*x] + 55440*Cos[(3*c)/2 + d*x] + 18
480*Cos[(5*c)/2 + 3*d*x] + 18480*Cos[(7*c)/2 + 3*d*x] - 10395*Cos[(7*c)/2 +
 4*d*x] + 10395*Cos[(9*c)/2 + 4*d*x] - 5544*Cos[(9*c)/2 + 5*d*x] - 5544*Cos
[(11*c)/2 + 5*d*x] - 3960*Cos[(13*c)/2 + 7*d*x] - 3960*Cos[(15*c)/2 + 7*d*x
] + 2079*Cos[(15*c)/2 + 8*d*x] - 2079*Cos[(17*c)/2 + 8*d*x] + 616*Cos[(17*c
)/2 + 9*d*x] + 616*Cos[(19*c)/2 + 9*d*x] + 504*Cos[(21*c)/2 + 11*d*x] + 504
*Cos[(23*c)/2 + 11*d*x] - 231*Cos[(23*c)/2 + 12*d*x] + 231*Cos[(25*c)/2 + 1
2*d*x] + 99792*Sin[c/2] + 55440*d*x*Sin[c/2] - 55440*Sin[c/2 + d*x] + 55440
*Sin[(3*c)/2 + d*x] - 18480*Sin[(5*c)/2 + 3*d*x] + 18480*Sin[(7*c)/2 + 3*d*
x] - 10395*Sin[(7*c)/2 + 4*d*x] - 10395*Sin[(9*c)/2 + 4*d*x] + 5544*Sin[(9*
c)/2 + 5*d*x] - 5544*Sin[(11*c)/2 + 5*d*x] + 3960*Sin[(13*c)/2 + 7*d*x] - 3
```

```
960*Sin[(15*c)/2 + 7*d*x] + 2079*Sin[(15*c)/2 + 8*d*x] + 2079*Sin[(17*c)/2
+ 8*d*x] - 616*Sin[(17*c)/2 + 9*d*x] + 616*Sin[(19*c)/2 + 9*d*x] - 504*Sin[
(21*c)/2 + 11*d*x] + 504*Sin[(23*c)/2 + 11*d*x] - 231*Sin[(23*c)/2 + 12*d*x
] - 231*Sin[(25*c)/2 + 12*d*x])/(11354112*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.128, size = 755, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] -16/693/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12+5/512/d/a/(1+tan(1/2*d*x+1/2*c)^2)^
12*tan(1/2*d*x+1/2*c)-64/231/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/
2*c)^2+175/1536/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^3-32/21/
d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^4+311/512/d/a/(1+tan(1/2
*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^5+352/63/d/a/(1+tan(1/2*d*x+1/2*c)^2)^
12*tan(1/2*d*x+1/2*c)^6-8361/512/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*
x+1/2*c)^7-192/7/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^8+42259
/768/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^9+96/7/d/a/(1+tan(1
/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^10-25295/256/d/a/(1+tan(1/2*d*x+1/2*
c)^2)^12*tan(1/2*d*x+1/2*c)^11-32/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2
*d*x+1/2*c)^12+25295/256/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)
^13-32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^14-42259/768/d/a/
(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^15+16/d/a/(1+tan(1/2*d*x+1/2
*c)^2)^12*tan(1/2*d*x+1/2*c)^16+8361/512/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*ta
n(1/2*d*x+1/2*c)^17-32/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)
^18-311/512/d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^19-175/1536/
d/a/(1+tan(1/2*d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^21-5/512/d/a/(1+tan(1/2*
d*x+1/2*c)^2)^12*tan(1/2*d*x+1/2*c)^23-5/512/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.63672, size = 952, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/354816*((3465*sin(d*x + c)/(cos(d*x + c) + 1) - 98304*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 40425*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 540672*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 215523*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 + 1982464*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 5794173*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 - 9732096*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 19523658*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 4866048*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 - 35058870*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 3784704*sin(d*x
 + c)^12/(cos(d*x + c) + 1)^12 + 35058870*sin(d*x + c)^13/(cos(d*x + c) + 1
)^13 - 11354112*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 19523658*sin(d*x +
c)^15/(cos(d*x + c) + 1)^15 + 5677056*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
 + 5794173*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 3784704*sin(d*x + c)^18/
(cos(d*x + c) + 1)^18 - 215523*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 4042
5*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 - 3465*sin(d*x + c)^23/(cos(d*x + c
) + 1)^23 - 8192)/(a + 12*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a*sin(
```

```
d*x + c)^4/(cos(d*x + c) + 1)^4 + 220*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + 495*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 792*a*sin(d*x + c)^10/(cos(d
*x + c) + 1)^10 + 924*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 792*a*sin(d
*x + c)^14/(cos(d*x + c) + 1)^14 + 495*a*sin(d*x + c)^16/(cos(d*x + c) + 1)
^16 + 220*a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 66*a*sin(d*x + c)^20/(c
os(d*x + c) + 1)^20 + 12*a*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 + a*sin(d*
x + c)^24/(cos(d*x + c) + 1)^24) - 3465*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a)/d
```

Fricas [A] time = 1.20768, size = 324, normalized size = 1.55

$$-\frac{64512\cos(dx+c)^{11} - 157696\cos(dx+c)^{9} + 101376\cos(dx+c)^{7} + 3465\,dx - 231\left(256\cos(dx+c)^{11} - 640\cos(dx\right.}{709632\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/709632*(64512*cos(d*x + c)^11 - 157696*cos(d*x + c)^9 + 101376*cos(d*x +
 c)^7 + 3465*d*x - 231*(256*cos(d*x + c)^11 - 640*cos(d*x + c)^9 + 432*cos(
d*x + c)^7 - 8*cos(d*x + c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*
x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29103, size = 417, normalized size = 2.

$$-\frac{\frac{3465\,(dx+c)}{a} + \frac{2\left(3465\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{23} + 40425\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{21} + 215523\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19} + 3784704\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{18} - 5794173\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17} - 56770\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/709632*(3465*(d*x + c)/a + 2*(3465*tan(1/2*d*x + 1/2*c)^23 + 40425*tan(1
/2*d*x + 1/2*c)^21 + 215523*tan(1/2*d*x + 1/2*c)^19 + 3784704*tan(1/2*d*x +
 1/2*c)^18 - 5794173*tan(1/2*d*x + 1/2*c)^17 - 5677056*tan(1/2*d*x + 1/2*c)
^16 + 19523658*tan(1/2*d*x + 1/2*c)^15 + 11354112*tan(1/2*d*x + 1/2*c)^14 -
 35058870*tan(1/2*d*x + 1/2*c)^13 + 3784704*tan(1/2*d*x + 1/2*c)^12 + 35058
870*tan(1/2*d*x + 1/2*c)^11 - 4866048*tan(1/2*d*x + 1/2*c)^10 - 19523658*ta
n(1/2*d*x + 1/2*c)^9 + 9732096*tan(1/2*d*x + 1/2*c)^8 + 5794173*tan(1/2*d*x
 + 1/2*c)^7 - 1982464*tan(1/2*d*x + 1/2*c)^6 - 215523*tan(1/2*d*x + 1/2*c)^
```

```
5 + 540672*tan(1/2*d*x + 1/2*c)^4 - 40425*tan(1/2*d*x + 1/2*c)^3 + 98304*ta
n(1/2*d*x + 1/2*c)^2 - 3465*tan(1/2*d*x + 1/2*c) + 8192)/((tan(1/2*d*x + 1/
2*c)^2 + 1)^12*a))/d
```

3.706 $\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=183

$$\frac{\cos^{11}(c+dx)}{11ad}-\frac{2\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}-\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}+\frac{\sin(c+dx)\cos}{160a}$$

```
[Out] (3*x)/(256*a) + Cos[c + d*x]^7/(7*a*d) - (2*Cos[c + d*x]^9)/(9*a*d) + Cos[c
 + d*x]^11/(11*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(256*a*d) + (Cos[c + d*
x]^3*Sin[c + d*x])/(128*a*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(160*a*d) - (3
*Cos[c + d*x]^7*Sin[c + d*x])/(80*a*d) - (Cos[c + d*x]^7*Sin[c + d*x]^3)/(1
0*a*d)
```

Rubi [A] time = 0.240174, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 270}

$$\frac{\cos^{11}(c+dx)}{11ad}-\frac{2\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}-\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}+\frac{\sin(c+dx)\cos}{160a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (3*x)/(256*a) + Cos[c + d*x]^7/(7*a*d) - (2*Cos[c + d*x]^9)/(9*a*d) + Cos[c
 + d*x]^11/(11*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(256*a*d) + (Cos[c + d*
x]^3*Sin[c + d*x])/(128*a*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(160*a*d) - (3
*Cos[c + d*x]^7*Sin[c + d*x])/(80*a*d) - (Cos[c + d*x]^7*Sin[c + d*x]^3)/(1
0*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^6(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^5(c+dx)\,dx}{a}$$

$$= -\frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} + \frac{3\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^6\left(1-\right.\right.}{}$$

$$= -\frac{3\cos^7(c+dx)\sin(c+dx)}{80ad} - \frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} + \frac{3\int \cos^6(c+dx)\,dx}{80a} + \frac{\text{S}}{}$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{\cos^5(c+dx)\sin(c+dx)}{160ad} - \frac{3\cos^7}{}$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{128ad} + \frac{\cos^5(}{}$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{3\cos(c+dx)\sin(c+dx)}{256ad} + \frac{\cos^3(}{}$$

$$= \frac{3x}{256a} + \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{3\cos(c+dx)\sin(c+dx)}{256ad}$$

Mathematica [B] time = 12.2312, size = 573, normalized size = 3.13

$$\frac{97020\sin^2\left(\frac{1}{2}(c+dx)\right)}{d(a\sin(c+dx)+a)} + \frac{103950\sin(c)\sin(dx)}{ad} - \frac{66990\sin(3c)\sin(3dx)}{ad} + \frac{24948\sin(5c)\sin(5dx)}{ad} - \frac{1980\sin(7c)\sin(7dx)}{ad} - \frac{76230\sin(2(c+dx))}{ad} + \frac{27}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((97020*c)/(a*d) + (83160*x)/a - (103950*Cos[c]*Cos[d*x])/(a*d) + (66990*Co
s[3*c]*Cos[3*d*x])/(a*d) - (24948*Cos[5*c]*Cos[5*d*x])/(a*d) + (1980*Cos[7*
c]*Cos[7*d*x])/(a*d) + (173250*Cos[c + d*x])/(a*d) - (43890*Cos[3*(c + d*x)
])/(a*d) + (18018*Cos[5*(c + d*x)])/(a*d) - (6930*Cos[7*(c + d*x)])/(a*d) +
 (770*Cos[9*(c + d*x)])/(a*d) + (630*Cos[11*(c + d*x)])/(a*d) + (90090*Cos[
2*d*x]*Sin[2*c])/(a*d) - (55440*Cos[4*d*x]*Sin[4*c])/(a*d) + (4620*Cos[6*d*
x]*Sin[6*c])/(a*d) + (103950*Sin[c]*Sin[d*x])/(a*d) + (90090*Cos[2*c]*Sin[2
*d*x])/(a*d) - (66990*Sin[3*c]*Sin[3*d*x])/(a*d) - (55440*Cos[4*c]*Sin[4*d*
x])/(a*d) + (24948*Sin[5*c]*Sin[5*d*x])/(a*d) + (4620*Cos[6*c]*Sin[6*d*x])/
(a*d) - (1980*Sin[7*c]*Sin[7*d*x])/(a*d) - (76230*Sin[(d*x)/2])/(a*d*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (20790*Sin[(c + d*
x)/2])/(a*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (48510*Sin[c + d*x])/(
a*d*(1 + Sin[c + d*x])) + (97020*Sin[(c + d*x)/2]^2)/(d*(a + a*Sin[c + d*x]
)) - (76230*Sin[2*(c + d*x)])/(a*d) + (27720*Sin[4*(c + d*x)])/(a*d) - (115
```

```
50*Sin[6*(c + d*x)])/(a*d) + (3465*Sin[8*(c + d*x)])/(a*d) + (1386*Sin[10*(
c + d*x)])/(a*d))/7096320
```

Maple [B] time = 0.119, size = 653, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 16/693/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11-3/128/d/a/(1+tan(1/2*d*x+1/2*c)^2)^1
1*tan(1/2*d*x+1/2*c)+16/63/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*
c)^2-1/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^3+80/63/d/a/(1+
tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^4+3323/640/d/a/(1+tan(1/2*d*x+1
/2*c)^2)^11*tan(1/2*d*x+1/2*c)^5-48/7/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1
/2*d*x+1/2*c)^6-54/5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^7+2
40/7/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^8+841/64/d/a/(1+tan
(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^9-48/d/a/(1+tan(1/2*d*x+1/2*c)^2)^
11*tan(1/2*d*x+1/2*c)^10+176/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+
1/2*c)^12-841/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^13-80/3
/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^14+54/5/d/a/(1+tan(1/2*
d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^15+32/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11
*tan(1/2*d*x+1/2*c)^16-3323/640/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x
+1/2*c)^17+1/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^19+3/128/
d/a/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^21+3/128/a/d*arctan(tan(
1/2*d*x+1/2*c))
```

Maxima [B] time = 1.72599, size = 842, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/443520*((10395*sin(d*x + c)/(cos(d*x + c) + 1) - 112640*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 110880*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 563200*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2302839*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 + 3041280*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4790016*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 15206400*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 582
8130*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 21288960*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 - 26019840*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 5828130*s
in(d*x + c)^13/(cos(d*x + c) + 1)^13 + 11827200*sin(d*x + c)^14/(cos(d*x +
c) + 1)^14 - 4790016*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 4730880*sin(d*
x + c)^16/(cos(d*x + c) + 1)^16 + 2302839*sin(d*x + c)^17/(cos(d*x + c) + 1
)^17 - 110880*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 10395*sin(d*x + c)^21
/(cos(d*x + c) + 1)^21 - 10240)/(a + 11*a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 55*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 165*a*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 330*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 462*a*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10 + 462*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12
 + 330*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 165*a*sin(d*x + c)^16/(cos
(d*x + c) + 1)^16 + 55*a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 11*a*sin(d
*x + c)^20/(cos(d*x + c) + 1)^20 + a*sin(d*x + c)^22/(cos(d*x + c) + 1)^22)
```

```
 - 10395*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.22871, size = 294, normalized size = 1.61

$$\frac{80640\,\cos\left(dx+c\right)^{11}-197120\,\cos\left(dx+c\right)^{9}+126720\,\cos\left(dx+c\right)^{7}+10395\,dx+693\,\left(128\,\cos\left(dx+c\right)^{9}-176\,\cos\left(\right.\right.}{887040\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/887040*(80640*cos(d*x + c)^11 - 197120*cos(d*x + c)^9 + 126720*cos(d*x +
c)^7 + 10395*d*x + 693*(128*cos(d*x + c)^9 - 176*cos(d*x + c)^7 + 8*cos(d*x
 + c)^5 + 10*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32072, size = 365, normalized size = 1.99

$$\frac{\frac{10395\,(dx+c)}{a}+\frac{2\left(10395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{21}+110880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19}-2302839\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+4730880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}+4790016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/887040*(10395*(d*x + c)/a + 2*(10395*tan(1/2*d*x + 1/2*c)^21 + 110880*tan
(1/2*d*x + 1/2*c)^19 - 2302839*tan(1/2*d*x + 1/2*c)^17 + 4730880*tan(1/2*d*
x + 1/2*c)^16 + 4790016*tan(1/2*d*x + 1/2*c)^15 - 11827200*tan(1/2*d*x + 1/
2*c)^14 - 5828130*tan(1/2*d*x + 1/2*c)^13 + 26019840*tan(1/2*d*x + 1/2*c)^1
2 - 21288960*tan(1/2*d*x + 1/2*c)^10 + 5828130*tan(1/2*d*x + 1/2*c)^9 + 152
06400*tan(1/2*d*x + 1/2*c)^8 - 4790016*tan(1/2*d*x + 1/2*c)^7 - 3041280*tan
(1/2*d*x + 1/2*c)^6 + 2302839*tan(1/2*d*x + 1/2*c)^5 + 563200*tan(1/2*d*x +
 1/2*c)^4 - 110880*tan(1/2*d*x + 1/2*c)^3 + 112640*tan(1/2*d*x + 1/2*c)^2 -
 10395*tan(1/2*d*x + 1/2*c) + 10240)/((tan(1/2*d*x + 1/2*c)^2 + 1)^11*a))/d
```

3.707 $\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}+\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{160ad}-\sin(c$$

```
[Out] (-3*x)/(256*a) - Cos[c + d*x]^7/(7*a*d) + Cos[c + d*x]^9/(9*a*d) - (3*Cos[c
 + d*x]*Sin[c + d*x])/(256*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(128*a*d) -
 (Cos[c + d*x]^5*Sin[c + d*x])/(160*a*d) + (3*Cos[c + d*x]^7*Sin[c + d*x])/
(80*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*a*d)
```

Rubi [A] time = 0.237026, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2565, 14, 2568, 2635, 8}

$$\frac{\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}+\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{160ad}-\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(256*a) - Cos[c + d*x]^7/(7*a*d) + Cos[c + d*x]^9/(9*a*d) - (3*Cos[c
 + d*x]*Sin[c + d*x])/(256*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(128*a*d) -
 (Cos[c + d*x]^5*Sin[c + d*x])/(160*a*d) + (3*Cos[c + d*x]^7*Sin[c + d*x])/
(80*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^6(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^4(c+dx)\,dx}{a}$$

$$= \frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} - \frac{3\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{10a} - \frac{\text{Subst}\left(\int x^6\left(1-x^2\right.\right.}{a}$$

$$= \frac{3\cos^7(c+dx)\sin(c+dx)}{80ad} + \frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} - \frac{3\int \cos^6(c+dx)\,dx}{80a} - \frac{\text{Sul}}{}$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{160ad} + \frac{3\cos^7(c+dx)\sin(c+}{80ad}$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{128ad} - \frac{\cos^5(c+dx)\sin(c+dx}{160ad}$$

$$= -\frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{3\cos(c+dx)\sin(c+dx)}{256ad} - \frac{\cos^3(c+dx)\sin(c+d}{128ad}$$

$$= -\frac{3x}{256a} - \frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{3\cos(c+dx)\sin(c+dx)}{256ad} - \frac{\cos^3(c+dx)\text{s}}{128a}$$

Mathematica [B] time = 15.1624, size = 533, normalized size = 3.23

$$-\frac{15120dx\sin\left(\frac{c}{2}\right) - 15120\sin\left(\frac{c}{2}+dx\right) + 15120\sin\left(\frac{3c}{2}+dx\right) + 1260\sin\left(\frac{3c}{2}+2dx\right) + 1260\sin\left(\frac{5c}{2}+2dx\right) - 6720\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(-1260*(25*c - 12*d*x)*Cos[c/2] + 15120*Cos[c/2 + d*x] + 15120*Cos[(3*c)/2
 + d*x] + 1260*Cos[(3*c)/2 + 2*d*x] - 1260*Cos[(5*c)/2 + 2*d*x] + 6720*Cos[
(5*c)/2 + 3*d*x] + 6720*Cos[(7*c)/2 + 3*d*x] - 2520*Cos[(7*c)/2 + 4*d*x] +
2520*Cos[(9*c)/2 + 4*d*x] - 630*Cos[(11*c)/2 + 6*d*x] + 630*Cos[(13*c)/2 +
6*d*x] - 1080*Cos[(13*c)/2 + 7*d*x] - 1080*Cos[(15*c)/2 + 7*d*x] + 315*Cos[
(15*c)/2 + 8*d*x] - 315*Cos[(17*c)/2 + 8*d*x] - 280*Cos[(17*c)/2 + 9*d*x] -
 280*Cos[(19*c)/2 + 9*d*x] + 126*Cos[(19*c)/2 + 10*d*x] - 126*Cos[(21*c)/2
+ 10*d*x] + 37800*Sin[c/2] - 31500*c*Sin[c/2] + 15120*d*x*Sin[c/2] - 15120*
Sin[c/2 + d*x] + 15120*Sin[(3*c)/2 + d*x] + 1260*Sin[(3*c)/2 + 2*d*x] + 126
0*Sin[(5*c)/2 + 2*d*x] - 6720*Sin[(5*c)/2 + 3*d*x] + 6720*Sin[(7*c)/2 + 3*d
*x] - 2520*Sin[(7*c)/2 + 4*d*x] - 2520*Sin[(9*c)/2 + 4*d*x] - 630*Sin[(11*c
)/2 + 6*d*x] - 630*Sin[(13*c)/2 + 6*d*x] + 1080*Sin[(13*c)/2 + 7*d*x] - 108
0*Sin[(15*c)/2 + 7*d*x] + 315*Sin[(15*c)/2 + 8*d*x] + 315*Sin[(17*c)/2 + 8*
d*x] + 280*Sin[(17*c)/2 + 9*d*x] - 280*Sin[(19*c)/2 + 9*d*x] + 126*Sin[(19*
c)/2 + 10*d*x] + 126*Sin[(21*c)/2 + 10*d*x])/(1290240*a*d*(Cos[c/2] + Sin[c
```

```
/2]))
```

Maple [B] time = 0.11, size = 619, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -4/63/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10+3/128/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10
*tan(1/2*d*x+1/2*c)-40/63/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c
)^2+29/128/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^3+8/7/d/a/(1+
tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^4-867/160/d/a/(1+tan(1/2*d*x+1/
2*c)^2)^10*tan(1/2*d*x+1/2*c)^5-72/7/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/
2*d*x+1/2*c)^6+519/32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^7-
1879/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^9-8/d/a/(1+tan(1
/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^10+1879/64/d/a/(1+tan(1/2*d*x+1/2*c)
^2)^10*tan(1/2*d*x+1/2*c)^11-40/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d
*x+1/2*c)^12-519/32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^13+8
/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^14+867/160/d/a/(1+tan
(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^15-4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^
10*tan(1/2*d*x+1/2*c)^16-29/128/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x
+1/2*c)^17-3/128/d/a/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^19-3/12
8/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.75912, size = 787, normalized size = 4.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/40320*((945*sin(d*x + c)/(cos(d*x + c) + 1) - 25600*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 9135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 46080*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 218484*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4
14720*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 653940*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 - 1183770*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 322560*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10 + 1183770*sin(d*x + c)^11/(cos(d*x + c) + 1)^
11 - 537600*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 653940*sin(d*x + c)^13/
(cos(d*x + c) + 1)^13 + 107520*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 2184
84*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 161280*sin(d*x + c)^16/(cos(d*x
+ c) + 1)^16 - 9135*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 945*sin(d*x + c
)^19/(cos(d*x + c) + 1)^19 - 2560)/(a + 10*a*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 45*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 120*a*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6 + 210*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 252*a*sin(d
*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a*sin(d*x + c)^12/(cos(d*x + c) + 1)
^12 + 120*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a*sin(d*x + c)^16/(c
os(d*x + c) + 1)^16 + 10*a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + a*sin(d*
x + c)^20/(cos(d*x + c) + 1)^20) - 945*arctan(sin(d*x + c)/(cos(d*x + c) +
1))/a)/d
```

Fricas [A] time = 1.17474, size = 252, normalized size = 1.53

$$\frac{8960 \cos(dx+c)^9 - 11520 \cos(dx+c)^7 - 945\,dx - 63\left(128 \cos(dx+c)^9 - 176 \cos(dx+c)^7 + 8 \cos(dx+c)^5 + 10\right.}{80640\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80640*(8960*cos(d*x + c)^9 - 11520*cos(d*x + c)^7 - 945*d*x - 63*(128*cos
(d*x + c)^9 - 176*cos(d*x + c)^7 + 8*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 1
5*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29525, size = 347, normalized size = 2.1

$$-\frac{\frac{945\,(dx+c)}{a} + \frac{2\left(945 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19} + 9135 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17} + 161280 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16} - 218484 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} - 107520 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14} + 653940\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(945*(d*x + c)/a + 2*(945*tan(1/2*d*x + 1/2*c)^19 + 9135*tan(1/2*d
*x + 1/2*c)^17 + 161280*tan(1/2*d*x + 1/2*c)^16 - 218484*tan(1/2*d*x + 1/2*
c)^15 - 107520*tan(1/2*d*x + 1/2*c)^14 + 653940*tan(1/2*d*x + 1/2*c)^13 + 5
37600*tan(1/2*d*x + 1/2*c)^12 - 1183770*tan(1/2*d*x + 1/2*c)^11 + 322560*ta
n(1/2*d*x + 1/2*c)^10 + 1183770*tan(1/2*d*x + 1/2*c)^9 - 653940*tan(1/2*d*x
 + 1/2*c)^7 + 414720*tan(1/2*d*x + 1/2*c)^6 + 218484*tan(1/2*d*x + 1/2*c)^5
 - 46080*tan(1/2*d*x + 1/2*c)^4 - 9135*tan(1/2*d*x + 1/2*c)^3 + 25600*tan(1
/2*d*x + 1/2*c)^2 - 945*tan(1/2*d*x + 1/2*c) + 2560)/((tan(1/2*d*x + 1/2*c)
^2 + 1)^10*a))/d
```

3.708 $\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}+\frac{5\sin}{}$$

```
[Out] (5*x)/(128*a) + Cos[c + d*x]^7/(7*a*d) - Cos[c + d*x]^9/(9*a*d) + (5*Cos[c
+ d*x]*Sin[c + d*x])/(128*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(192*a*d)
+ (Cos[c + d*x]^5*Sin[c + d*x])/(48*a*d) - (Cos[c + d*x]^7*Sin[c + d*x])/(8
*a*d)
```

Rubi [A] time = 0.196626, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2568, 2635, 8, 2565, 14}

$$-\frac{\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}+\frac{5\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (5*x)/(128*a) + Cos[c + d*x]^7/(7*a*d) - Cos[c + d*x]^9/(9*a*d) + (5*Cos[c
+ d*x]*Sin[c + d*x])/(128*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(192*a*d)
+ (Cos[c + d*x]^5*Sin[c + d*x])/(48*a*d) - (Cos[c + d*x]^7*Sin[c + d*x])/(8
*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^3(c+dx)\,dx}{a}\\
&= -\frac{\cos^7(c+dx)\sin(c+dx)}{8ad} + \frac{\int \cos^6(c+dx)\,dx}{8a} + \frac{\mathrm{Subst}\left(\int x^6\left(1-x^2\right)dx, x, \cos(c\right.}{ad}\\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{48ad} - \frac{\cos^7(c+dx)\sin(c+dx)}{8ad} + \frac{5\int \cos^4(c+dx)\,dx}{48a} + \mathrm{Subst}\\
&= \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{192ad} + \frac{\cos^5(c+dx)\sin(c+d}{48ad}\\
&= \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+dx)\sin(c+d}{192ad}\\
&= \frac{5x}{128a} + \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+dx)s}{192a}
\end{aligned}$$

Mathematica [B] time = 9.54328, size = 479, normalized size = 3.45

$$-\frac{-5040dx\sin\left(\frac{c}{2}\right)+1512\sin\left(\frac{c}{2}+dx\right)-1512\sin\left(\frac{3c}{2}+dx\right)-1008\sin\left(\frac{3c}{2}+2dx\right)-1008\sin\left(\frac{5c}{2}+2dx\right)+672\sin\left(\frac{5c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(2520*(c - 2*d*x)*Cos[c/2] - 1512*Cos[c/2 + d*x] - 1512*Cos[(3*c)/2 + d*x]
 - 1008*Cos[(3*c)/2 + 2*d*x] + 1008*Cos[(5*c)/2 + 2*d*x] - 672*Cos[(5*c)/2
+ 3*d*x] - 672*Cos[(7*c)/2 + 3*d*x] + 504*Cos[(7*c)/2 + 4*d*x] - 504*Cos[(9
*c)/2 + 4*d*x] + 336*Cos[(11*c)/2 + 6*d*x] - 336*Cos[(13*c)/2 + 6*d*x] + 10
8*Cos[(13*c)/2 + 7*d*x] + 108*Cos[(15*c)/2 + 7*d*x] + 63*Cos[(15*c)/2 + 8*d
*x] - 63*Cos[(17*c)/2 + 8*d*x] + 28*Cos[(17*c)/2 + 9*d*x] + 28*Cos[(19*c)/2
 + 9*d*x] - 7560*Sin[c/2] + 2520*c*Sin[c/2] - 5040*d*x*Sin[c/2] + 1512*Sin[
c/2 + d*x] - 1512*Sin[(3*c)/2 + d*x] - 1008*Sin[(3*c)/2 + 2*d*x] - 1008*Sin
[(5*c)/2 + 2*d*x] + 672*Sin[(5*c)/2 + 3*d*x] - 672*Sin[(7*c)/2 + 3*d*x] + 5
04*Sin[(7*c)/2 + 4*d*x] + 504*Sin[(9*c)/2 + 4*d*x] + 336*Sin[(11*c)/2 + 6*d
*x] + 336*Sin[(13*c)/2 + 6*d*x] - 108*Sin[(13*c)/2 + 7*d*x] + 108*Sin[(15*c
)/2 + 7*d*x] + 63*Sin[(15*c)/2 + 8*d*x] + 63*Sin[(17*c)/2 + 8*d*x] - 28*Sin
[(17*c)/2 + 9*d*x] + 28*Sin[(19*c)/2 + 9*d*x])/(129024*a*d*(Cos[c/2] + Sin[
c/2]))
```

Maple [B] time = 0.106, size = 551, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 4/63/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9-5/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan
(1/2*d*x+1/2*c)+4/7/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^2+191
/96/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^3-12/7/d/a/(1+tan(1/2
*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^4-83/32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*
tan(1/2*d*x+1/2*c)^5+12/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^6
+145/32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^7-12/d/a/(1+tan(1
/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^8+20/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*t
an(1/2*d*x+1/2*c)^10-145/32/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*
c)^11-20/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^12+83/32/d/a/(
1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^13+4/d/a/(1+tan(1/2*d*x+1/2*c)
^2)^9*tan(1/2*d*x+1/2*c)^14-191/96/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d
*x+1/2*c)^15+5/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^17+5/64
/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.61433, size = 705, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4032*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 2304*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 8022*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6912*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 10458*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4838
4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18270*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 + 48384*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 80640*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 + 18270*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 26880
*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 10458*sin(d*x + c)^13/(cos(d*x + c
) + 1)^13 - 16128*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 8022*sin(d*x + c)
^15/(cos(d*x + c) + 1)^15 - 315*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 256
)/(a + 9*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 36*a*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 84*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 126*a*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 + 126*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
84*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 36*a*sin(d*x + c)^14/(cos(d*x
+ c) + 1)^14 + 9*a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 + a*sin(d*x + c)^1
8/(cos(d*x + c) + 1)^18) - 315*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.19651, size = 220, normalized size = 1.58

$$-\frac{896\,\cos\left(dx+c\right)^{9}-1152\,\cos\left(dx+c\right)^{7}-315\,dx+21\,\left(48\,\cos\left(dx+c\right)^{7}-8\,\cos\left(dx+c\right)^{5}-10\,\cos\left(dx+c\right)^{3}-15\,\cos\left(dx\right.\right.}{8064\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8064*(896*cos(d*x + c)^9 - 1152*cos(d*x + c)^7 - 315*d*x + 21*(48*cos(d*
x + c)^7 - 8*cos(d*x + c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x
+ c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17643, size = 312, normalized size = 2.24

$$\frac{315\,(dx+c)}{a} + \frac{2\left(315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17} - 8022\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} + 16128\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14} + 10458\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 26880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} - 18270\,\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8064*(315*(d*x + c)/a + 2*(315*tan(1/2*d*x + 1/2*c)^17 - 8022*tan(1/2*d*x
 + 1/2*c)^15 + 16128*tan(1/2*d*x + 1/2*c)^14 + 10458*tan(1/2*d*x + 1/2*c)^1
3 - 26880*tan(1/2*d*x + 1/2*c)^12 - 18270*tan(1/2*d*x + 1/2*c)^11 + 80640*t
an(1/2*d*x + 1/2*c)^10 - 48384*tan(1/2*d*x + 1/2*c)^8 + 18270*tan(1/2*d*x +
 1/2*c)^7 + 48384*tan(1/2*d*x + 1/2*c)^6 - 10458*tan(1/2*d*x + 1/2*c)^5 - 6
912*tan(1/2*d*x + 1/2*c)^4 + 8022*tan(1/2*d*x + 1/2*c)^3 + 2304*tan(1/2*d*x
 + 1/2*c)^2 - 315*tan(1/2*d*x + 1/2*c) + 256)/((tan(1/2*d*x + 1/2*c)^2 + 1)
^9*a))/d
```

3.709 $\int \frac{\cos^8(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}-\frac{5\sin(c+dx)\cos(c+$$

```
[Out] (-5*x)/(128*a) - Cos[c + d*x]^7/(7*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(12
8*a*d) - (5*Cos[c + d*x]^3*Sin[c + d*x])/(192*a*d) - (Cos[c + d*x]^5*Sin[c
+ d*x])/(48*a*d) + (Cos[c + d*x]^7*Sin[c + d*x])/(8*a*d)
```

Rubi [A] time = 0.145237, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2565, 30, 2568, 2635, 8}

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}-\frac{5\sin(c+dx)\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*x)/(128*a) - Cos[c + d*x]^7/(7*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(12
8*a*d) - (5*Cos[c + d*x]^3*Sin[c + d*x])/(192*a*d) - (Cos[c + d*x]^5*Sin[c
+ d*x])/(48*a*d) + (Cos[c + d*x]^7*Sin[c + d*x])/(8*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^8(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^6(c+dx)\sin(c+dx)\,dx}{a}-\frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a}\\
&= \frac{\cos^7(c+dx)\sin(c+dx)}{8ad}-\frac{\int \cos^6(c+dx)\,dx}{8a}-\frac{\text{Subst}\left(\int x^6\,dx,x,\cos(c+dx)\right)}{ad}\\
&= -\frac{\cos^7(c+dx)}{7ad}-\frac{\cos^5(c+dx)\sin(c+dx)}{48ad}+\frac{\cos^7(c+dx)\sin(c+dx)}{8ad}-\frac{5\int \cos^4(c+\\
&= -\frac{\cos^7(c+dx)}{7ad}-\frac{5\cos^3(c+dx)\sin(c+dx)}{192ad}-\frac{\cos^5(c+dx)\sin(c+dx)}{48ad}+\frac{\cos^7(c+d\\
&= -\frac{\cos^7(c+dx)}{7ad}-\frac{5\cos(c+dx)\sin(c+dx)}{128ad}-\frac{5\cos^3(c+dx)\sin(c+dx)}{192ad}-\frac{\cos^5(c+d\\
&= -\frac{5x}{128a}-\frac{\cos^7(c+dx)}{7ad}-\frac{5\cos(c+dx)\sin(c+dx)}{128ad}-\frac{5\cos^3(c+dx)\sin(c+dx)}{192ad}-\text{co}\end{aligned}$$

Mathematica [B] time = 11.3751, size = 481, normalized size = 3.98

$$-\frac{1680dx\sin\left(\frac{c}{2}\right)-1680\sin\left(\frac{c}{2}+dx\right)+1680\sin\left(\frac{3c}{2}+dx\right)+336\sin\left(\frac{3c}{2}+2dx\right)+336\sin\left(\frac{5c}{2}+2dx\right)-1008\sin\left(\frac{5c}{2}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(-336*(7*c - 5*d*x)*Cos[c/2] + 1680*Cos[c/2 + d*x] + 1680*Cos[(3*c)/2 + d*
x] + 336*Cos[(3*c)/2 + 2*d*x] - 336*Cos[(5*c)/2 + 2*d*x] + 1008*Cos[(5*c)/2
 + 3*d*x] + 1008*Cos[(7*c)/2 + 3*d*x] - 168*Cos[(7*c)/2 + 4*d*x] + 168*Cos[
(9*c)/2 + 4*d*x] + 336*Cos[(9*c)/2 + 5*d*x] + 336*Cos[(11*c)/2 + 5*d*x] - 1
12*Cos[(11*c)/2 + 6*d*x] + 112*Cos[(13*c)/2 + 6*d*x] + 48*Cos[(13*c)/2 + 7*
d*x] + 48*Cos[(15*c)/2 + 7*d*x] - 21*Cos[(15*c)/2 + 8*d*x] + 21*Cos[(17*c)/
2 + 8*d*x] + 4704*Sin[c/2] - 2352*c*Sin[c/2] + 1680*d*x*Sin[c/2] - 1680*Sin
[c/2 + d*x] + 1680*Sin[(3*c)/2 + d*x] + 336*Sin[(3*c)/2 + 2*d*x] + 336*Sin[
(5*c)/2 + 2*d*x] - 1008*Sin[(5*c)/2 + 3*d*x] + 1008*Sin[(7*c)/2 + 3*d*x] -
168*Sin[(7*c)/2 + 4*d*x] - 168*Sin[(9*c)/2 + 4*d*x] - 336*Sin[(9*c)/2 + 5*d
*x] + 336*Sin[(11*c)/2 + 5*d*x] - 112*Sin[(11*c)/2 + 6*d*x] - 112*Sin[(13*c
)/2 + 6*d*x] - 48*Sin[(13*c)/2 + 7*d*x] + 48*Sin[(15*c)/2 + 7*d*x] - 21*Sin
[(15*c)/2 + 8*d*x] - 21*Sin[(17*c)/2 + 8*d*x])/(43008*a*d*(Cos[c/2] + Sin[c
/2]))
```

Maple [B] time = 0.082, size = 551, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/7/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8+5/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan
(1/2*d*x+1/2*c)-2/7/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2-397
/192/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3-6/d/a/(1+tan(1/2*d
*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4+895/192/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*
tan(1/2*d*x+1/2*c)^5-6/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6-
1765/192/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7-10/d/a/(1+tan(
1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8+1765/192/d/a/(1+tan(1/2*d*x+1/2*c)
^2)^8*tan(1/2*d*x+1/2*c)^9-10/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/
2*c)^10-895/192/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11-2/d/a/
(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12+397/192/d/a/(1+tan(1/2*d*x
+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13-2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2
*d*x+1/2*c)^14-5/64/d/a/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-5/
64/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.56373, size = 676, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1344*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 384*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 2779*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 8064*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 6265*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8064*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 12355*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 13440*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 12355*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 13440*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6265*sin(d*
x + c)^11/(cos(d*x + c) + 1)^11 - 2688*sin(d*x + c)^12/(cos(d*x + c) + 1)^1
2 + 2779*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 2688*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 384)/(a + 8*
a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 + 56*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos
(d*x + c) + 1)^8 + 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*
x + c)^12/(cos(d*x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14
 + a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.13154, size = 189, normalized size = 1.56

$$-\frac{384\,\cos\left(dx+c\right)^{7}+105\,dx-7\left(48\,\cos\left(dx+c\right)^{7}-8\,\cos\left(dx+c\right)^{5}-10\,\cos\left(dx+c\right)^{3}-15\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2688\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2688*(384*cos(d*x + c)^7 + 105*d*x - 7*(48*cos(d*x + c)^7 - 8*cos(d*x +
c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18302, size = 312, normalized size = 2.58

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+2688\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}-2779\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2688\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+6265\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+13440\,\tan\left(\frac{1}{2}\,d\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2688*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 2688*tan(1/2*d*
x + 1/2*c)^14 - 2779*tan(1/2*d*x + 1/2*c)^13 + 2688*tan(1/2*d*x + 1/2*c)^12
 + 6265*tan(1/2*d*x + 1/2*c)^11 + 13440*tan(1/2*d*x + 1/2*c)^10 - 12355*tan
(1/2*d*x + 1/2*c)^9 + 13440*tan(1/2*d*x + 1/2*c)^8 + 12355*tan(1/2*d*x + 1/
2*c)^7 + 8064*tan(1/2*d*x + 1/2*c)^6 - 6265*tan(1/2*d*x + 1/2*c)^5 + 8064*t
an(1/2*d*x + 1/2*c)^4 + 2779*tan(1/2*d*x + 1/2*c)^3 + 384*tan(1/2*d*x + 1/2
*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 384)/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a))
/d
```

3.710 $\int \frac{\cos^7(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=143

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{5\sin(c+dx)\cos(c}{16ad}$$

```
[Out] (-5*x)/(16*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) + Cos[c +
d*x]^3/(3*a*d) + Cos[c + d*x]^5/(5*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(16
*a*d) - (5*Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) - (Cos[c + d*x]^5*Sin[c +
d*x])/(6*a*d)
```

Rubi [A] time = 0.145968, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2592, 302, 206, 2635, 8}

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{5\sin(c+dx)\cos(c}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-5*x)/(16*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) + Cos[c +
d*x]^3/(3*a*d) + Cos[c + d*x]^5/(5*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(16
*a*d) - (5*Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) - (Cos[c + d*x]^5*Sin[c +
d*x])/(6*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^6(c+dx)\,dx}{a} + \frac{\int \cos^5(c+dx)\cot(c+dx)\,dx}{a}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \frac{5\int \cos^4(c+dx)\,dx}{6a} - \frac{\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad}$$

$$= -\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \frac{5\int \cos^2(c+dx)\,dx}{8a} - \text{Sub}$$

$$= \frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{5\cos(c+dx)\sin(c+dx)}{16ad} - \frac{5\cos^3(c+}{}$$

$$= -\frac{5x}{16a} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{5\cos}{}$$

Mathematica [A] time = 0.270593, size = 102, normalized size = 0.71

$$-\frac{225\sin(2(c+dx)) + 45\sin(4(c+dx)) + 5\sin(6(c+dx)) - 1320\cos(c+dx) - 140\cos(3(c+dx)) - 12\cos(5(c+dx)}{960ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(300*c + 300*d*x - 1320*Cos[c + d*x] - 140*Cos[3*(c + d*x)] - 12*Cos[5*(c
+ d*x)] + 960*Log[Cos[(c + d*x)/2]] - 960*Log[Sin[(c + d*x)/2]] + 225*Sin[2
*(c + d*x)] + 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)])/(960*a*d)
```

Maple [B] time = 0.124, size = 432, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 11/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+6/d/a/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10-5/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^9+18/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8+
15/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+92/3/d/a/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6-15/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^5+28/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4
+5/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+62/5/d/a/(1+tan(1
```

```
/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2-11/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6
*tan(1/2*d*x+1/2*c)+46/15/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6-5/8/a/d*arctan(tan
(1/2*d*x+1/2*c))+1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.57289, size = 543, normalized size = 3.8

$$-\frac{\frac{\frac{165 \sin(dx+c)}{\cos(dx+c)+1}-\frac{1488 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{25 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3360 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{450 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3680 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{450 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{2160 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{25 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{720 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}}{120\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 1488*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 25*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3360*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 450*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3680*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 2160*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 25*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 - 720*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 165*sin(d*x + c)^11/(
cos(d*x + c) + 1)^11 - 368)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) + 75*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a - 120*log(sin(d*x + c)/(cos(d*x + c) +
1))/a)/d
```

Fricas [A] time = 1.15296, size = 302, normalized size = 2.11

$$\frac{48 \cos(dx+c)^5+80 \cos(dx+c)^3-75\, dx-5\left(8 \cos(dx+c)^5+10 \cos(dx+c)^3+15 \cos(dx+c)\right)\sin(dx+c)+240}{240\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*cos(d*x + c)^5 + 80*cos(d*x + c)^3 - 75*d*x - 5*(8*cos(d*x + c)^5
 + 10*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(d*x + c) + 240*cos(d*x + c) - 1
20*log(1/2*cos(d*x + c) + 1/2) + 120*log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20717, size = 263, normalized size = 1.84

$$-\frac{\frac{75\,(dx+c)}{a}-\frac{240\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\left(165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+720\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2160\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+450\,\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(75*(d*x + c)/a - 240*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(165*tan(
1/2*d*x + 1/2*c)^11 + 720*tan(1/2*d*x + 1/2*c)^10 - 25*tan(1/2*d*x + 1/2*c)
^9 + 2160*tan(1/2*d*x + 1/2*c)^8 + 450*tan(1/2*d*x + 1/2*c)^7 + 3680*tan(1/
2*d*x + 1/2*c)^6 - 450*tan(1/2*d*x + 1/2*c)^5 + 3360*tan(1/2*d*x + 1/2*c)^4
 + 25*tan(1/2*d*x + 1/2*c)^3 + 1488*tan(1/2*d*x + 1/2*c)^2 - 165*tan(1/2*d*
x + 1/2*c) + 368)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a))/d
```

3.711 $\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{15\cot(c+dx)}{8ad}+\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}+\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}+$$

```
[Out] (-15*x)/(8*a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cos[c +
d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) - (15*Cot[c + d*x])/(8*a*d) + (5*Co
s[c + d*x]^2*Cot[c + d*x])/(8*a*d) + (Cos[c + d*x]^4*Cot[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.173902, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2839, 2591, 288, 321, 203, 2592, 302, 206}

$$-\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{15\cot(c+dx)}{8ad}+\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}+\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-15*x)/(8*a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cos[c +
d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) - (15*Cot[c + d*x])/(8*a*d) + (5*Co
s[c + d*x]^2*Cot[c + d*x])/(8*a*d) + (Cos[c + d*x]^4*Cot[c + d*x])/(4*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos^5(c+dx)\cot(c+dx)\,dx}{a}+\frac{\int \cos^4(c+dx)\cot^2(c+dx)\,dx}{a}\\ &= \frac{\operatorname{Subst}\left(\int \frac{x^6}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3}\,dx,x,\cot(c+dx)\right)}{ad}\\ &= \frac{\cos^4(c+dx)\cot(c+dx)}{4ad}+\frac{\operatorname{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)dx,x,\cos(c+dx)\right)}{ad}-\ldots 5\\ &= -\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}+\frac{\cos^4(\ldots}{}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}-\frac{15\cot(c+dx\ldots}{8ad}\\ &= -\frac{15x}{8a}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}-\frac{15\,\mathrm{co}\ldots}{}\end{aligned}$$

Mathematica [A] time = 0.705235, size = 146, normalized size = 1.07

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(1800c\sin(c+dx)+1800dx\sin(c+dx)+590\sin(2(c+dx))+64\sin(4(c+dx))+6\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(1200*Cos[c + d*x] - 225*Cos[3*(c + d*x
)] - 15*Cos[5*(c + d*x)] + 1800*c*Sin[c + d*x] + 1800*d*x*Sin[c + d*x] - 96
0*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 960*Log[Sin[(c + d*x)/2]]*Sin[c + d*
x] + 590*Sin[2*(c + d*x)] + 64*Sin[4*(c + d*x)] + 6*Sin[6*(c + d*x)]))/(192
0*a*d)

Maple [B] time = 0.135, size = 367, normalized size = 2.7

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{9}{4da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-5}-6\frac{(\tan(1/2\,dx+c/2))^{8}}{da\left(1+(\tan(1/2\,dx+c/2))^{2}\right)^{5}}+\frac{5}{2da}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x)

[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)+9/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1
/2*c)^9-6/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8+5/2/d/a/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7-12/d/a/(1+tan(1/2*d*x+1/2*c)^2)^
5*tan(1/2*d*x+1/2*c)^6-56/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*
c)^4-5/2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3-28/3/d/a/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2-9/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)-46/15/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5-15/4/a/d*arctan(
tan(1/2*d*x+1/2*c))-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+1/2*c))

Maxima [B] time = 1.57044, size = 512, normalized size = 3.74

$$-\frac{\frac{\frac{184\sin(dx+c)}{\cos(dx+c)+1}+\frac{285\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{560\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{450\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{1120\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{300\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{720\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{360\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{105\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+30}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{10a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{10a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{5a\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{a\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}+\frac{225\,\mathrm{arct}}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/60*((184*sin(d*x + c)/(cos(d*x + c) + 1) + 285*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 560*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 450*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 1120*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 300*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 720*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3
60*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 105*sin(d*x + c)^10/(cos(d*x + c)
+ 1)^10 + 30)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 10*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 5*a*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + a*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11) + 225*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a + 60*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 30*sin(d*x + c)/(a*
(cos(d*x + c) + 1)))/d

Fricas [A] time = 1.151, size = 354, normalized size = 2.58

$$\frac{30\cos(dx+c)^{5}+75\cos(dx+c)^{3}-\left(24\cos(dx+c)^{5}+40\cos(dx+c)^{3}+225\,dx+120\cos(dx+c)\right)\sin(dx+c)+60}{120\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(30*cos(d*x + c)^5 + 75*cos(d*x + c)^3 - (24*cos(d*x + c)^5 + 40*cos(
d*x + c)^3 + 225*d*x + 120*cos(d*x + c))*sin(d*x + c) + 60*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) - 60*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2
25*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21551, size = 269, normalized size = 1.96

$$-\frac{\frac{225\,(dx+c)}{a}+\frac{120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{60\left(2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+150\,\tan\left(\cdots\right.\right.}{\cdots}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(225*(d*x + c)/a + 120*log(abs(tan(1/2*d*x + 1/2*c)))/a - 60*tan(1/2
*d*x + 1/2*c)/a - 60*(2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c))
- 2*(135*tan(1/2*d*x + 1/2*c)^9 - 360*tan(1/2*d*x + 1/2*c)^8 + 150*tan(1/2*
d*x + 1/2*c)^7 - 720*tan(1/2*d*x + 1/2*c)^6 - 1120*tan(1/2*d*x + 1/2*c)^4 -
 150*tan(1/2*d*x + 1/2*c)^3 - 560*tan(1/2*d*x + 1/2*c)^2 - 135*tan(1/2*d*x
+ 1/2*c) - 184)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a))/d
```

3.712 $\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$-\frac{5\cos^3(c+dx)}{6ad}-\frac{5\cos(c+dx)}{2ad}+\frac{15\cot(c+dx)}{8ad}-\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}-\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}-\frac{5\cos^2(c+d}{8}$$

```
[Out] (15*x)/(8*a) + (5*ArcTanh[Cos[c + d*x]])/(2*a*d) - (5*Cos[c + d*x])/(2*a*d)
 - (5*Cos[c + d*x]^3)/(6*a*d) + (15*Cot[c + d*x])/(8*a*d) - (5*Cos[c + d*x]
^2*Cot[c + d*x])/(8*a*d) - (Cos[c + d*x]^4*Cot[c + d*x])/(4*a*d) - (Cos[c +
 d*x]^3*Cot[c + d*x]^2)/(2*a*d)
```

Rubi [A] time = 0.18867, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2839, 2592, 288, 302, 206, 2591, 321, 203}

$$-\frac{5\cos^3(c+dx)}{6ad}-\frac{5\cos(c+dx)}{2ad}+\frac{15\cot(c+dx)}{8ad}-\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}-\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}-\frac{5\cos^2(c+d}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (15*x)/(8*a) + (5*ArcTanh[Cos[c + d*x]])/(2*a*d) - (5*Cos[c + d*x])/(2*a*d)
 - (5*Cos[c + d*x]^3)/(6*a*d) + (15*Cot[c + d*x])/(8*a*d) - (5*Cos[c + d*x]
^2*Cot[c + d*x])/(8*a*d) - (Cos[c + d*x]^4*Cot[c + d*x])/(4*a*d) - (Cos[c +
 d*x]^3*Cot[c + d*x]^2)/(2*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos^4(c+dx)\cot^2(c+dx)\,dx}{a}+\frac{\int \cos^3(c+dx)\cot^3(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx,x,\cos(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3}\,dx,x,\cot(c+dx)\right)}{ad}\\
&= -\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}-\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx,x,c\right.}{4ad}\\
&= -\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}-\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}-\frac{\cos^3(c+dx)\cot^2(c+dx}{2ad}\\
&= -\frac{5\cos(c+dx)}{2ad}-\frac{5\cos^3(c+dx)}{6ad}+\frac{15\cot(c+dx)}{8ad}-\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}-\frac{c}{}\\
&= \frac{15x}{8a}+\frac{5\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{5\cos(c+dx)}{2ad}-\frac{5\cos^3(c+dx)}{6ad}+\frac{15\cot(c+dx)}{8ad}-
\end{aligned}$$

Mathematica [A] time = 0.523759, size = 179, normalized size = 1.19

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-285\sin(2(c+dx))+42\sin(4(c+dx))+3\sin(6(c+dx))+400\cos(c+dx)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(-360*c - 360*d*x + 400*Cos[c + d
*x] - 200*Cos[3*(c + d*x)] - 8*Cos[5*(c + d*x)] - 480*Log[Cos[(c + d*x)/2]]
 + 120*Cos[2*(c + d*x)]*(3*c + 3*d*x + 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[
(c + d*x)/2]]) + 480*Log[Sin[(c + d*x)/2]] - 285*Sin[2*(c + d*x)] + 42*Sin[
4*(c + d*x)] + 3*Sin[6*(c + d*x)]))/(1536*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.148, size = 371, normalized size = 2.5

$$\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{2\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{9}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}-6\,\frac{(\tan(1/2\,dx+c/2}{da\left(1+(\tan(1/2\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/2/d/a*tan(1/2*d*x+1/2*c)-9/4/d/a/(1+tan(1/2*
d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-6/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)^6-1/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-14/
d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4+1/4/d/a/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-38/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)^2+9/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-14/3/d
/a/(1+tan(1/2*d*x+1/2*c)^2)^4+15/4/a/d*arctan(tan(1/2*d*x+1/2*c))-1/8/d/a/t
an(1/2*d*x+1/2*c)^2+1/2/d/a/tan(1/2*d*x+1/2*c)-5/2/d/a*ln(tan(1/2*d*x+1/2*c
))
```

Maxima [B] time = 1.54507, size = 517, normalized size = 3.45

$$\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{124\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{102\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{322\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{78\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{348\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{42\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{147\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{42\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-3}{\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{3\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\right.}{a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*((12*sin(d*x + c)/(cos(d*x + c) + 1) - 124*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 102*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 322*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 78*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 348*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 42*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 147*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 - 42*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 3)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 + 6*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4*a*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 + a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 3*(4*sin(d
*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a + 90*ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 60*log(sin(d*x + c)/(cos(d*x + c)
 + 1))/a)/d
```

Fricas [A] time = 1.21775, size = 413, normalized size = 2.75

$$-\frac{8\cos(dx+c)^5-45\,dx\cos(dx+c)^2+40\cos(dx+c)^3+45\,dx-30\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3}{24\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*cos(d*x + c)^5 - 45*d*x*cos(d*x + c)^2 + 40*cos(d*x + c)^3 + 45*d*
x - 30*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 30*(cos(d*x + c)^
2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 3*(2*cos(d*x + c)^5 + 5*cos(d*x + c)^
3 - 15*cos(d*x + c))*sin(d*x + c) - 60*cos(d*x + c))/(a*d*cos(d*x + c)^2 -
a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21191, size = 292, normalized size = 1.95

$$\frac{\frac{45\,(dx+c)}{a}-\frac{60\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^2}+\frac{3\left(30\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-\frac{2\left(27\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(45*(d*x + c)/a - 60*log(abs(tan(1/2*d*x + 1/2*c)))/a + 3*(a*tan(1/2*d
*x + 1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c))/a^2 + 3*(30*tan(1/2*d*x + 1/2*c)^
2 + 4*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)^2) - 2*(27*tan(1/2*
d*x + 1/2*c)^7 + 72*tan(1/2*d*x + 1/2*c)^6 + 3*tan(1/2*d*x + 1/2*c)^5 + 168
*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x + 1/2*c)^3 + 152*tan(1/2*d*x + 1/2*
c)^2 - 27*tan(1/2*d*x + 1/2*c) + 56)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.713 $\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=146

$$\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cos(c+dx)}{2ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{5\cot(c+dx)}{2ad}+\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot^3(c+dx}{2ad}$$

```
[Out] (5*x)/(2*a) - (5*ArcTanh[Cos[c + d*x]])/(2*a*d) + (5*Cos[c + d*x])/(2*a*d)
+ (5*Cos[c + d*x]^3)/(6*a*d) + (5*Cot[c + d*x])/(2*a*d) + (Cos[c + d*x]^3*C
ot[c + d*x]^2)/(2*a*d) - (5*Cot[c + d*x]^3)/(6*a*d) + (Cos[c + d*x]^2*Cot[c
 + d*x]^3)/(2*a*d)
```

Rubi [A] time = 0.180188, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2839, 2591, 288, 302, 203, 2592, 206}

$$\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cos(c+dx)}{2ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{5\cot(c+dx)}{2ad}+\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot^3(c+dx}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*x)/(2*a) - (5*ArcTanh[Cos[c + d*x]])/(2*a*d) + (5*Cos[c + d*x])/(2*a*d)
+ (5*Cos[c + d*x]^3)/(6*a*d) + (5*Cot[c + d*x])/(2*a*d) + (Cos[c + d*x]^3*C
ot[c + d*x]^2)/(2*a*d) - (5*Cot[c + d*x]^3)/(6*a*d) + (Cos[c + d*x]^2*Cot[c
 + d*x]^3)/(2*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^3(c+dx)\cot^3(c+dx)\,dx}{a}+\frac{\int \cos^2(c+dx)\cot^4(c+dx)\,dx}{a}$$

$$=\frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx,x,\cot(c+dx)\right)}{ad}$$

$$=\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{5\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\cos\right.}{2ad}$$

$$=\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{5\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-}\right.\right.}{2ad}$$

$$=\frac{5\cos(c+dx)}{2ad}+\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cot(c+dx)}{2ad}+\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}-\frac{5\,\text{co}}{}$$

$$=\frac{5x}{2a}-\frac{5\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{5\cos(c+dx)}{2ad}+\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cot(c+dx)}{2ad}+\frac{\text{c}}{}$$

Mathematica [A] time = 0.789702, size = 197, normalized size = 1.35

$$-\frac{\csc^3(c+dx)\left(-180c\sin(c+dx)-180dx\sin(c+dx)-75\sin(2(c+dx))+60c\sin(3(c+dx))+60dx\sin(3(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^3*(-30*Cos[c + d*x] + 65*Cos[3*(c + d*x)] - 3*Cos[5*(c + d*x
)] - 180*c*Sin[c + d*x] - 180*d*x*Sin[c + d*x] + 180*Log[Cos[(c + d*x)/2]]*
Sin[c + d*x] - 180*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 75*Sin[2*(c + d*x)]
 + 60*c*Sin[3*(c + d*x)] + 60*d*x*Sin[3*(c + d*x)] - 60*Log[Cos[(c + d*x)/2
]]*Sin[3*(c + d*x)] + 60*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 24*Sin[4*
(c + d*x)] + Sin[6*(c + d*x)]))/(96*a*d)
```

Maple [B] time = 0.148, size = 306, normalized size = 2.1

$$\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{9}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a*tan(1/2*d*x+1/2*c)^2-9/8/d/a*tan(1/2*
d*x+1/2*c)-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+6/d/a/(1+t
an(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^
3*tan(1/2*d*x+1/2*c)^2+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+
14/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3+5/a/d*arctan(tan(1/2*d*x+1/2*c))-1/24/d
/a/tan(1/2*d*x+1/2*c)^3+1/8/d/a/tan(1/2*d*x+1/2*c)^2+9/8/d/a/tan(1/2*d*x+1/
2*c)+5/2/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.54203, size = 489, normalized size = 3.35

$$-\frac{\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{121\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{102\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{201\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{80\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{147\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+3}{\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((27*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - (3*sin(d*x + c)/(cos(d*x
 + c) + 1) + 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 121*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 102*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 201*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 80*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 147*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 1)/(a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 3*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9) - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.18203, size = 470, normalized size = 3.22

$$-\frac{6\cos(dx+c)^5-40\cos(dx+c)^3+15\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-15\left(\cos(dx+c)^2-1\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*(cos(d*x + c)^2 - 1)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(cos(d*x + c)^2 - 1)*log(-1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) - 2*(2*cos(d*x + c)^5 + 15*d*x*cos(d*x + c)^2
```

```
+ 10*cos(d*x + c)^3 - 15*d*x - 15*cos(d*x + c))*sin(d*x + c) + 30*cos(d*x +
 c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27305, size = 308, normalized size = 2.11

$$\frac{\frac{180\,(dx+c)}{a} + \frac{180\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} + \frac{3\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 27\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^3} - \frac{110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/72*(180*(d*x + c)/a + 180*log(abs(tan(1/2*d*x + 1/2*c)))/a + 3*(a^2*tan(1
/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 27*a^2*tan(1/2*d*x + 1/2
*c))/a^3 - (110*tan(1/2*d*x + 1/2*c)^9 - 9*tan(1/2*d*x + 1/2*c)^8 - 111*tan
(1/2*d*x + 1/2*c)^7 - 240*tan(1/2*d*x + 1/2*c)^6 - 273*tan(1/2*d*x + 1/2*c)
^5 - 306*tan(1/2*d*x + 1/2*c)^4 - 253*tan(1/2*d*x + 1/2*c)^3 - 72*tan(1/2*d
*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 3)/((tan(1/2*d*x + 1/2*c)^3 + tan(
1/2*d*x + 1/2*c))^3*a))/d
```

3.714 $\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$\frac{15\cos(c+dx)}{8ad}+\frac{5\cot^3(c+dx)}{6ad}-\frac{5\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}+\frac{5\cos(c+dx)}{8a}$$

```
[Out] (-5*x)/(2*a) - (15*ArcTanh[Cos[c + d*x]])/(8*a*d) + (15*Cos[c + d*x])/(8*a*
d) - (5*Cot[c + d*x])/(2*a*d) + (5*Cos[c + d*x]*Cot[c + d*x]^2)/(8*a*d) + (
5*Cot[c + d*x]^3)/(6*a*d) - (Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*a*d) - (Cos[
c + d*x]*Cot[c + d*x]^4)/(4*a*d)
```

Rubi [A] time = 0.176904, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2839, 2592, 288, 321, 206, 2591, 302, 203}

$$\frac{15\cos(c+dx)}{8ad}+\frac{5\cot^3(c+dx)}{6ad}-\frac{5\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}+\frac{5\cos(c+dx)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-5*x)/(2*a) - (15*ArcTanh[Cos[c + d*x]])/(8*a*d) + (15*Cos[c + d*x])/(8*a*
d) - (5*Cot[c + d*x])/(2*a*d) + (5*Cos[c + d*x]*Cot[c + d*x]^2)/(8*a*d) + (
5*Cot[c + d*x]^3)/(6*a*d) - (Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*a*d) - (Cos[
c + d*x]*Cot[c + d*x]^4)/(4*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos^2(c+dx)\cot^4(c+dx)\,dx}{a}+\frac{\int \cos(c+dx)\cot^5(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx,x,\cos(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx,x,\cot(c+dx)\right)}{ad}\\
&= -\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}+\frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx,x,c\right.}{4ad}\\
&= \frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}-\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}\\
&= \frac{15\cos(c+dx)}{8ad}-\frac{5\cot(c+dx)}{2ad}+\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}+\frac{5\cot^3(c+dx)}{6ad}-\text{co}\\
&= -\frac{5x}{2a}-\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{15\cos(c+dx)}{8ad}-\frac{5\cot(c+dx)}{2ad}+\frac{5\cos(c+dx)\,\text{c}}{8ad}
\end{aligned}$$

Mathematica [A] time = 0.733414, size = 252, normalized size = 1.68

$$-\frac{\csc^4(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(95\sin(2(c+dx))-68\sin(4(c+dx))+3\sin(6(c+dx))+60c\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(180*c + 180*d*x -
 30*Cos[c + d*x] + 90*Cos[3*(c + d*x)] + 60*c*Cos[4*(c + d*x)] + 60*d*x*Cos
[4*(c + d*x)] - 12*Cos[5*(c + d*x)] + 135*Log[Cos[(c + d*x)/2]] + 45*Cos[4*
(c + d*x)]*Log[Cos[(c + d*x)/2]] - 60*Cos[2*(c + d*x)]*(4*c + 4*d*x + 3*Log
[Cos[(c + d*x)/2]] - 3*Log[Sin[(c + d*x)/2]]) - 135*Log[Sin[(c + d*x)/2]] -
 45*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 95*Sin[2*(c + d*x)] - 68*Sin[4
*(c + d*x)] + 3*Sin[6*(c + d*x)]))/(192*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.148, size = 310, normalized size = 2.1

$$\frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{9}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/4/d/a*tan(1/2
*d*x+1/2*c)^2+9/8/d/a*tan(1/2*d*x+1/2*c)+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)^3+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1
/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+2/d/a/(1+tan(1/2*d*x+1/2
*c)^2)^2-5/a/d*arctan(tan(1/2*d*x+1/2*c))-1/64/d/a/tan(1/2*d*x+1/2*c)^4+1/2
4/d/a/tan(1/2*d*x+1/2*c)^3+1/4/d/a/tan(1/2*d*x+1/2*c)^2-9/8/d/a/tan(1/2*d*x
+1/2*c)+15/8/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.59504, size = 459, normalized size = 3.06

$$\frac{\frac{\frac{216\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a}+\frac{\frac{8\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{200\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{477\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{616\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{432\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{24\,\mathrm{s}}{(\cos}}{\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/192*((216*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a + (8*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 477*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 - 616*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4
32*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 24*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 - 3)/(a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 960*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a + 360*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)
/d
```

Fricas [A] time = 1.17538, size = 536, normalized size = 3.57

$$-\frac{120\,dx\cos(dx+c)^4-48\,\cos(dx+c)^5-240\,dx\cos(dx+c)^2+150\,\cos(dx+c)^3+120\,dx+45\left(\cos(dx+c)^4-2\,\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(120*d*x*cos(d*x + c)^4 - 48*cos(d*x + c)^5 - 240*d*x*cos(d*x + c)^2
+ 150*cos(d*x + c)^3 + 120*d*x + 45*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)
*log(1/2*cos(d*x + c) + 1/2) - 45*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*l
og(-1/2*cos(d*x + c) + 1/2) + 8*(3*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 15*
cos(d*x + c))*sin(d*x + c) - 90*cos(d*x + c))/(a*d*cos(d*x + c)^4 - 2*a*d*c
os(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.51842, size = 302, normalized size = 2.01

$$-\frac{\frac{480\,(dx+c)}{a} - \frac{360\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} - \frac{192\,\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a} - \frac{3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(480*(d*x + c)/a - 360*log(abs(tan(1/2*d*x + 1/2*c)))/a - 192*(tan(1
/2*d*x + 1/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 2)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^2*a) - (3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 48*a^3*tan(1/2*d*x + 1/2*c)^2 + 216*a^3*tan(1/2*d*x
+ 1/2*c))/a^4 + (750*tan(1/2*d*x + 1/2*c)^4 + 216*tan(1/2*d*x + 1/2*c)^3 -
48*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/
2*c)^4))/d
```

3.715 $\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{15\cos(c+dx)}{8ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}-\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}+$$

```
[Out] -(x/a) + (15*ArcTanh[Cos[c + d*x]])/(8*a*d) - (15*Cos[c + d*x])/(8*a*d) - C
ot[c + d*x]/(a*d) - (5*Cos[c + d*x]*Cot[c + d*x]^2)/(8*a*d) + Cot[c + d*x]^
3/(3*a*d) + (Cos[c + d*x]*Cot[c + d*x]^4)/(4*a*d) - Cot[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.151258, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2839, 3473, 8, 2592, 288, 321, 206}

$$-\frac{15\cos(c+dx)}{8ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}-\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) + (15*ArcTanh[Cos[c + d*x]])/(8*a*d) - (15*Cos[c + d*x])/(8*a*d) - C
ot[c + d*x]/(a*d) - (5*Cos[c + d*x]*Cot[c + d*x]^2)/(8*a*d) + Cot[c + d*x]^
3/(3*a*d) + (Cos[c + d*x]*Cot[c + d*x]^4)/(4*a*d) - Cot[c + d*x]^5/(5*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos(c+dx)\cot^5(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\,dx}{a}$$

$$= -\frac{\cot^5(c+dx)}{5ad}-\frac{\int \cot^4(c+dx)\,dx}{a}+\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx,x,\cos(c+dx)\right)}{ad}$$

$$= \frac{\cot^3(c+dx)}{3ad}+\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\int \cot^2(c+dx)\,dx}{a}-\frac{5\,\text{S}}{}$$

$$= -\frac{\cot(c+dx)}{ad}-\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\cos(c+dx)\cot^4(c+}{4ad}$$

$$= -\frac{x}{a}-\frac{15\cos(c+dx)}{8ad}-\frac{\cot(c+dx)}{ad}-\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}+\frac{\cot^3(c+dx)}{3ad}+$$

$$= -\frac{x}{a}+\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{15\cos(c+dx)}{8ad}-\frac{\cot(c+dx)}{ad}-\frac{5\cos(c+dx)\cot^2}{8ad}$$

Mathematica [A] time = 0.994794, size = 264, normalized size = 1.91

$$-\frac{\csc^5(c+dx)\left(1200c\sin(c+dx)+1200dx\sin(c+dx)+600\sin(2(c+dx))-600c\sin(3(c+dx))-600dx\sin(3(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^5*(400*Cos[c + d*x] - 200*Cos[3*(c + d*x)] + 184*Cos[5*(c +
d*x)] + 1200*c*Sin[c + d*x] + 1200*d*x*Sin[c + d*x] - 2250*Log[Cos[(c + d*x
)/2]]*Sin[c + d*x] + 2250*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 600*Sin[2*(c
 + d*x)] - 600*c*Sin[3*(c + d*x)] - 600*d*x*Sin[3*(c + d*x)] + 1125*Log[Cos
[(c + d*x)/2]]*Sin[3*(c + d*x)] - 1125*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x
)] - 510*Sin[4*(c + d*x)] + 120*c*Sin[5*(c + d*x)] + 120*d*x*Sin[5*(c + d*x
)] - 225*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 225*Log[Sin[(c + d*x)/2]]
*Sin[5*(c + d*x)] + 60*Sin[6*(c + d*x)]))/(1920*a*d)
```

Maple [A] time = 0.154, size = 249, normalized size = 1.8

$$\frac{1}{160\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{7}{96\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{11}{16\,da}\tan\left(\frac{dx}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a*tan(1/2*d*x+1/2*c)^4-7/96/d/a*tan(1
/2*d*x+1/2*c)^3+1/4/d/a*tan(1/2*d*x+1/2*c)^2+11/16/d/a*tan(1/2*d*x+1/2*c)-2
/a/d/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*arctan(tan(1/2*d*x+1/2*c))-1/160/d/a/ta
n(1/2*d*x+1/2*c)^5+1/64/d/a/tan(1/2*d*x+1/2*c)^4+7/96/d/a/tan(1/2*d*x+1/2*c
)^3-1/4/d/a/tan(1/2*d*x+1/2*c)^2-11/16/d/a/tan(1/2*d*x+1/2*c)-15/8/d/a*ln(t
an(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.52293, size = 431, normalized size = 3.12

$$\frac{\frac{\frac{660\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{240\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{70\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{15\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}+\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{64\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{225\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{590\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2160\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{660}{(\mathrm{co}}}{\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*((660*sin(d*x + c)/(cos(d*x + c) + 1) + 240*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 70*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a + (15*sin(d*x
+ c)/(cos(d*x + c) + 1) + 64*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 225*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 - 590*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
 2160*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 660*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 - 6)/(a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7) - 1920*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 1800*
log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.16632, size = 593, normalized size = 4.3

$$-\frac{368\,\cos(dx+c)^5-560\,\cos(dx+c)^3-225\left(\cos(dx+c)^4-2\,\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*cos(d*x + c)^5 - 560*cos(d*x + c)^3 - 225*(cos(d*x + c)^4 - 2*c
os(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 225*(cos(d*x
+ c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
30*(8*d*x*cos(d*x + c)^4 + 8*cos(d*x + c)^5 - 16*d*x*cos(d*x + c)^2 - 25*co
s(d*x + c)^3 + 8*d*x + 15*cos(d*x + c))*sin(d*x + c) + 240*cos(d*x + c))/((
a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38203, size = 293, normalized size = 2.12

$$-\frac{\frac{960\,(dx+c)}{a}+\frac{1800\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{1920}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a}-\frac{6\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+240\,a^{4}\tan(}{a^{5}}}{960}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/960*(960*(d*x + c)/a + 1800*log(abs(tan(1/2*d*x + 1/2*c)))/a + 1920/((ta
n(1/2*d*x + 1/2*c)^2 + 1)*a) - (6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^4*tan(1
/2*d*x + 1/2*c)^4 - 70*a^4*tan(1/2*d*x + 1/2*c)^3 + 240*a^4*tan(1/2*d*x + 1
/2*c)^2 + 660*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (4110*tan(1/2*d*x + 1/2*c)^5
- 660*tan(1/2*d*x + 1/2*c)^4 - 240*tan(1/2*d*x + 1/2*c)^3 + 70*tan(1/2*d*x
+ 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c) - 6)/(a*tan(1/2*d*x + 1/2*c)^5))/d
```

3.716 $\int \frac{\cos(c+dx)\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=142

$$\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}+\frac{5\cot^3(c+dx)\csc(c+$$

```
[Out] x/a + (5*ArcTanh[Cos[c + d*x]])/(16*a*d) + Cot[c + d*x]/(a*d) - Cot[c + d*x
]^3/(3*a*d) + Cot[c + d*x]^5/(5*a*d) - (5*Cot[c + d*x]*Csc[c + d*x])/(16*a*
d) + (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) - (Cot[c + d*x]^5*Csc[c + d*x
])/(6*a*d)
```

Rubi [A] time = 0.178634, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2611, 3770, 3473, 8}

$$\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}+\frac{5\cot^3(c+dx)\csc(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + (5*ArcTanh[Cos[c + d*x]])/(16*a*d) + Cot[c + d*x]/(a*d) - Cot[c + d*x
]^3/(3*a*d) + Cot[c + d*x]^5/(5*a*d) - (5*Cot[c + d*x]*Csc[c + d*x])/(16*a*
d) + (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) - (Cot[c + d*x]^5*Csc[c + d*x
])/(6*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc(c+dx)\,dx}{a}$$
$$= \frac{\cot^5(c+dx)}{5ad} - \frac{\cot^5(c+dx)\csc(c+dx)}{6ad} - \frac{5\int \cot^4(c+dx)\csc(c+dx)\,dx}{6a} + \frac{\int \cot^4}{}$$
$$= -\frac{\cot^3(c+dx)}{3ad} + \frac{\cot^5(c+dx)}{5ad} + \frac{5\cot^3(c+dx)\csc(c+dx)}{24ad} - \frac{\cot^5(c+dx)\csc(c+d}{6ad}$$
$$= \frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\cot^5(c+dx)}{5ad} - \frac{5\cot(c+dx)\csc(c+dx)}{16ad} + \frac{5\cot^3(c+}{}$$
$$= \frac{x}{a} + \frac{5\tanh^{-1}(\cos(c+dx))}{16ad} + \frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\cot^5(c+dx)}{5ad} - \frac{5\cot(c}{}$$

Mathematica [B] time = 0.944289, size = 317, normalized size = 2.23

$$-\frac{\csc^6(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-1200\sin(2(c+dx))+768\sin(4(c+dx))-368\sin(6(c+dx))-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(-2400*c - 2400*d*
x + 900*Cos[c + d*x] + 50*Cos[3*(c + d*x)] - 1440*c*Cos[4*(c + d*x)] - 1440
*d*x*Cos[4*(c + d*x)] + 330*Cos[5*(c + d*x)] + 240*c*Cos[6*(c + d*x)] + 240
*d*x*Cos[6*(c + d*x)] - 750*Log[Cos[(c + d*x)/2]] - 450*Cos[4*(c + d*x)]*Lo
g[Cos[(c + d*x)/2]] + 75*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 225*Cos[2
*(c + d*x)]*(16*(c + d*x) + 5*Log[Cos[(c + d*x)/2]] - 5*Log[Sin[(c + d*x)/2
]]) + 750*Log[Sin[(c + d*x)/2]] + 450*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]
] - 75*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 1200*Sin[2*(c + d*x)] + 768
*Sin[4*(c + d*x)] - 368*Sin[6*(c + d*x)]))/(7680*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.156, size = 264, normalized size = 1.9

$$\frac{1}{384\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6 - \frac{1}{160\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{3}{128\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + \frac{7}{96\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{15}{128\,da}\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/384/d/a*tan(1/2*d*x+1/2*c)^6-1/160/d/a*tan(1/2*d*x+1/2*c)^5-3/128/d/a*tan
(1/2*d*x+1/2*c)^4+7/96/d/a*tan(1/2*d*x+1/2*c)^3+15/128/d/a*tan(1/2*d*x+1/2*
c)^2-11/16/d/a*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/2*d*x+1/2*c))-1/384/d/
a/tan(1/2*d*x+1/2*c)^6+1/160/d/a/tan(1/2*d*x+1/2*c)^5+3/128/d/a/tan(1/2*d*x
+1/2*c)^4-7/96/d/a/tan(1/2*d*x+1/2*c)^3-15/128/d/a/tan(1/2*d*x+1/2*c)^2+11/
16/d/a/tan(1/2*d*x+1/2*c)-5/16/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.53579, size = 402, normalized size = 2.83

$$-\frac{\frac{\frac{1320\sin(dx+c)}{\cos(dx+c)+1}-\frac{225\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{140\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{12\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a}-\frac{3840\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{600\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\ldots}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/1920*((1320*sin(d*x + c)/(cos(d*x + c) + 1) - 225*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 140*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 12*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6)/a - 3840*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a + 600*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (12*sin(d*x + c)/(cos(d
*x + c) + 1) + 45*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 140*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 - 225*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1320*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c) + 1)^6/(a*sin(d*x + c)^6)
)/d
```

Fricas [A] time = 1.2034, size = 660, normalized size = 4.65

$$\frac{480\,dx\cos(dx+c)^6-1440\,dx\cos(dx+c)^4+330\cos(dx+c)^5+1440\,dx\cos(dx+c)^2-400\cos(dx+c)^3-480\,dx+\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(480*d*x*cos(d*x + c)^6 - 1440*d*x*cos(d*x + c)^4 + 330*cos(d*x + c)^
5 + 1440*d*x*cos(d*x + c)^2 - 400*cos(d*x + c)^3 - 480*d*x + 75*(cos(d*x +
c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
 - 75*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*c
os(d*x + c) + 1/2) - 32*(23*cos(d*x + c)^5 - 35*cos(d*x + c)^3 + 15*cos(d*x
 + c))*sin(d*x + c) + 150*cos(d*x + c))/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x
 + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36038, size = 302, normalized size = 2.13

$$\frac{\frac{1920(dx+c)}{a}-\frac{600\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{5a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-12a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-45a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+140a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+225a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}\ldots\right.}{a^6}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(1920*(d*x + c)/a - 600*log(abs(tan(1/2*d*x + 1/2*c)))/a + (5*a^5*ta
n(1/2*d*x + 1/2*c)^6 - 12*a^5*tan(1/2*d*x + 1/2*c)^5 - 45*a^5*tan(1/2*d*x +
 1/2*c)^4 + 140*a^5*tan(1/2*d*x + 1/2*c)^3 + 225*a^5*tan(1/2*d*x + 1/2*c)^2
 - 1320*a^5*tan(1/2*d*x + 1/2*c))/a^6 + (1470*tan(1/2*d*x + 1/2*c)^6 + 1320
*tan(1/2*d*x + 1/2*c)^5 - 225*tan(1/2*d*x + 1/2*c)^4 - 140*tan(1/2*d*x + 1/
2*c)^3 + 45*tan(1/2*d*x + 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*c) - 5)/(a*tan(1/
2*d*x + 1/2*c)^6))/d
```

3.717 $\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}$$

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Co
t[c + d*x]^5*Csc[c + d*x])/(6*a*d)
```

Rubi [A] time = 0.147515, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Co
t[c + d*x]^5*Csc[c + d*x])/(6*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= \frac{\cot^5(c+dx)\csc(c+dx)}{6ad} + \frac{5\int \cot^4(c+dx)\csc(c+dx)\,dx}{6a} + \frac{\text{Subst}\left(\int x^6\,dx, x, -\cot(c+d\right.}{ad}$$
$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{5\cot^3(c+dx)\csc(c+dx)}{24ad} + \frac{\cot^5(c+dx)\csc(c+dx)}{6ad} - \frac{5\int \cot^2(c+dx)\,c}{8a}$$
$$= -\frac{\cot^7(c+dx)}{7ad} + \frac{5\cot(c+dx)\csc(c+dx)}{16ad} - \frac{5\cot^3(c+dx)\csc(c+dx)}{24ad} + \frac{\cot^5(c+dx)\csc(}{6ad}$$
$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{16ad} - \frac{\cot^7(c+dx)}{7ad} + \frac{5\cot(c+dx)\csc(c+dx)}{16ad} - \frac{5\cot^3(c+dx)\csc(c}{24ad}$$

Mathematica [B] time = 0.909543, size = 284, normalized size = 2.68

$$-\frac{\csc^5(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-1190\sin(2(c+dx))+392\sin(4(c+dx))-462\sin(6(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^5*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(1680*Cos[c + d*x]
 + 1008*Cos[3*(c + d*x)] + 336*Cos[5*(c + d*x)] + 48*Cos[7*(c + d*x)] + 367
5*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 3675*Log[Sin[(c + d*x)/2]]*Sin[c + d
*x] - 1190*Sin[2*(c + d*x)] - 2205*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] +
 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 392*Sin[4*(c + d*x)] + 735*L
og[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*Log[Sin[(c + d*x)/2]]*Sin[5*(c
+ d*x)] - 462*Sin[6*(c + d*x)] - 105*Log[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)]
 + 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(86016*a*d*(1 + Sin[c + d*x
]))
```

Maple [B] time = 0.163, size = 284, normalized size = 2.7

$$\frac{1}{896\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{1}{384\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6 - \frac{1}{128\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{3}{128\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + \frac{3}{128\,da}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/896/d/a*tan(1/2*d*x+1/2*c)^7-1/384/d/a*tan(1/2*d*x+1/2*c)^6-1/128/d/a*tan
(1/2*d*x+1/2*c)^5+3/128/d/a*tan(1/2*d*x+1/2*c)^4+3/128/d/a*tan(1/2*d*x+1/2*
c)^3-15/128/d/a*tan(1/2*d*x+1/2*c)^2-5/128/d/a*tan(1/2*d*x+1/2*c)-1/896/d/a
/tan(1/2*d*x+1/2*c)^7+5/128/d/a/tan(1/2*d*x+1/2*c)+1/128/d/a/tan(1/2*d*x+1/
2*c)^5-3/128/d/a/tan(1/2*d*x+1/2*c)^4+5/16/d/a*ln(tan(1/2*d*x+1/2*c))+1/384
/d/a/tan(1/2*d*x+1/2*c)^6-3/128/d/a/tan(1/2*d*x+1/2*c)^3+15/128/d/a/tan(1/2
*d*x+1/2*c)^2
```

Maxima [B] time = 1.03114, size = 425, normalized size = 4.01

$$-\frac{\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1}+\frac{315 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{63 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{7 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}-\frac{840 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{7 \sin(dx+c)}{\cos(dx+c)+1}+\frac{21 \sin}{(\cos(d}\right.}{}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2688*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 63*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a - 840*
log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (7*sin(d*x + c)/(cos(d*x + c) + 1)
 + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 63*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos(d*x + c
) + 1)^7/(a*sin(d*x + c)^7))/d
```

Fricas [B] time = 1.18875, size = 545, normalized size = 5.14

$$\frac{96 \cos(dx+c)^7-105\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+105\,(}{672\left(ad\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/672*(96*cos(d*x + c)^7 - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)
^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 14*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))
*sin(d*x + c))/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x
+ c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36103, size = 329, normalized size = 3.1

$$\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{3a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-7a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-21a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-315a^6\tan\left(\frac{1}{2}dx\right.}{a^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^6*tan(1/2*d*x + 1/2*c)^
7 - 7*a^6*tan(1/2*d*x + 1/2*c)^6 - 21*a^6*tan(1/2*d*x + 1/2*c)^5 + 63*a^6*t
an(1/2*d*x + 1/2*c)^4 + 63*a^6*tan(1/2*d*x + 1/2*c)^3 - 315*a^6*tan(1/2*d*x
 + 1/2*c)^2 - 105*a^6*tan(1/2*d*x + 1/2*c))/a^7 - (2178*tan(1/2*d*x + 1/2*c
)^7 - 105*tan(1/2*d*x + 1/2*c)^6 - 315*tan(1/2*d*x + 1/2*c)^5 + 63*tan(1/2*
d*x + 1/2*c)^4 + 63*tan(1/2*d*x + 1/2*c)^3 - 21*tan(1/2*d*x + 1/2*c)^2 - 7*
tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^7))/d
```

3.718 $\int \frac{\cot^8(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{\cot^7(c+dx)}{7ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}-\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}+\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}-\frac{5\cot(c+dx)\csc^3(c+}{64ad}$$

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(128*a*d) + Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(128*a*d) - (5*Cot[c + d*x]*Csc[c + d*x]^3)/(64*a*d) + (5
*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*a*d) - (Cot[c + d*x]^5*Csc[c + d*x]^3)/
(8*a*d)
```

Rubi [A] time = 0.218482, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2839, 2611, 3768, 3770, 2607, 30}

$$\frac{\cot^7(c+dx)}{7ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}-\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}+\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}-\frac{5\cot(c+dx)\csc^3(c+}{64ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(128*a*d) + Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(128*a*d) - (5*Cot[c + d*x]*Csc[c + d*x]^3)/(64*a*d) + (5
*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*a*d) - (Cot[c + d*x]^5*Csc[c + d*x]^3)/
(8*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^3(c+dx)\,dx}{a}$$

$$= -\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad} - \frac{5\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^6\,dx, x, \cdots\right.}{ad}$$

$$= \frac{\cot^7(c+dx)}{7ad} + \frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad} - \frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad} + \frac{5\int \cot^2(\cdots}{\cdots}$$

$$= \frac{\cot^7(c+dx)}{7ad} - \frac{5\cot(c+dx)\csc^3(c+dx)}{64ad} + \frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad} - \frac{\cot^5(c+\cdots}{\cdots}$$

$$= \frac{\cot^7(c+dx)}{7ad} + \frac{5\cot(c+dx)\csc(c+dx)}{128ad} - \frac{5\cot(c+dx)\csc^3(c+dx)}{64ad} + \frac{5\cot^3(c+\cdots}{\cdots}$$

$$= \frac{5\tanh^{-1}(\cos(c+dx))}{128ad} + \frac{\cot^7(c+dx)}{7ad} + \frac{5\cot(c+dx)\csc(c+dx)}{128ad} - \frac{5\cot(c+dx)\,\mathrm{c}\cdots}{64a\cdots}$$

Mathematica [B] time = 0.997881, size = 291, normalized size = 2.17

$$\frac{\csc^8(c+dx)\left(5376\sin(2(c+dx)) + 5376\sin(4(c+dx)) + 2304\sin(6(c+dx)) + 384\sin(8(c+dx)) - 24710\cos(c+d\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^8*(-24710*Cos[c + d*x] - 12530*Cos[3*(c + d*x)] - 5558*Cos[5*
(c + d*x)] - 210*Cos[7*(c + d*x)] + 3675*Log[Cos[(c + d*x)/2]] - 5880*Cos[2
*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 2940*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)
/2]] - 840*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 105*Cos[8*(c + d*x)]*Lo
g[Cos[(c + d*x)/2]] - 3675*Log[Sin[(c + d*x)/2]] + 5880*Cos[2*(c + d*x)]*Lo
g[Sin[(c + d*x)/2]] - 2940*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 840*Cos
[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 105*Cos[8*(c + d*x)]*Log[Sin[(c + d*x
)/2]] + 5376*Sin[2*(c + d*x)] + 5376*Sin[4*(c + d*x)] + 2304*Sin[6*(c + d*x
)] + 384*Sin[8*(c + d*x)]))/(344064*a*d)
```

Maple [B] time = 0.174, size = 322, normalized size = 2.4

$$\frac{1}{2048\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8 - \frac{1}{896\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{1}{384\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6 + \frac{1}{128\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{1}{256\,da}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2048/d/a*tan(1/2*d*x+1/2*c)^8-1/896/d/a*tan(1/2*d*x+1/2*c)^7-1/384/d/a*ta
n(1/2*d*x+1/2*c)^6+1/128/d/a*tan(1/2*d*x+1/2*c)^5+1/256/d/a*tan(1/2*d*x+1/2
*c)^4-3/128/d/a*tan(1/2*d*x+1/2*c)^3+1/128/d/a*tan(1/2*d*x+1/2*c)^2+5/128/d
/a*tan(1/2*d*x+1/2*c)+1/896/d/a/tan(1/2*d*x+1/2*c)^7-5/128/d/a/tan(1/2*d*x+
1/2*c)-1/2048/d/a/tan(1/2*d*x+1/2*c)^8-1/128/d/a/tan(1/2*d*x+1/2*c)^5-1/256
/d/a/tan(1/2*d*x+1/2*c)^4-5/128/d/a*ln(tan(1/2*d*x+1/2*c))+1/384/d/a/tan(1/
2*d*x+1/2*c)^6+3/128/d/a/tan(1/2*d*x+1/2*c)^3-1/128/d/a/tan(1/2*d*x+1/2*c)^
2
```

Maxima [B] time = 1.00927, size = 478, normalized size = 3.57

$$\frac{\frac{\frac{1680\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{336\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1008\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{168\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{336\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{112\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{48\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{21\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}{a}-\frac{1680\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{48}{\cos\cdots}\right.}{}}{43008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/43008*((1680*sin(d*x + c)/(cos(d*x + c) + 1) + 336*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 1008*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 168*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 336*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 112*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 48*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 +
 21*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/a - 1680*log(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + (48*sin(d*x + c)/(cos(d*x + c) + 1) + 112*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 336*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 168*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 33
6*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1680*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 21)*(cos(d*x + c) + 1)^8/(a*sin(d*x + c)^8))/d
```

Fricas [A] time = 1.16482, size = 602, normalized size = 4.49

$$\frac{768\,\cos(dx+c)^7\sin(dx+c)-210\,\cos(dx+c)^7-1022\,\cos(dx+c)^5+770\,\cos(dx+c)^3+105\left(\cos(dx+c)^8-4\,\cos(\cdots\right.}{5376\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/5376*(768*cos(d*x + c)^7*sin(d*x + c) - 210*cos(d*x + c)^7 - 1022*cos(d*x
 + c)^5 + 770*cos(d*x + c)^3 + 105*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*c
os(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 105*(co
s(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*
log(-1/2*cos(d*x + c) + 1/2) - 210*cos(d*x + c))/(a*d*cos(d*x + c)^8 - 4*a*
d*cos(d*x + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43603, size = 370, normalized size = 2.76

$$-\frac{\frac{1680 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a}-\frac{21 a^7 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8-48 a^7 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7-112 a^7 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6+336 a^7 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5+168 a^7 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4-10}{a^8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/43008*(1680*log(abs(tan(1/2*d*x + 1/2*c)))/a - (21*a^7*tan(1/2*d*x + 1/2
*c)^8 - 48*a^7*tan(1/2*d*x + 1/2*c)^7 - 112*a^7*tan(1/2*d*x + 1/2*c)^6 + 33
6*a^7*tan(1/2*d*x + 1/2*c)^5 + 168*a^7*tan(1/2*d*x + 1/2*c)^4 - 1008*a^7*ta
n(1/2*d*x + 1/2*c)^3 + 336*a^7*tan(1/2*d*x + 1/2*c)^2 + 1680*a^7*tan(1/2*d*
x + 1/2*c))/a^8 - (4566*tan(1/2*d*x + 1/2*c)^8 - 1680*tan(1/2*d*x + 1/2*c)^
7 - 336*tan(1/2*d*x + 1/2*c)^6 + 1008*tan(1/2*d*x + 1/2*c)^5 - 168*tan(1/2*
d*x + 1/2*c)^4 - 336*tan(1/2*d*x + 1/2*c)^3 + 112*tan(1/2*d*x + 1/2*c)^2 +
48*tan(1/2*d*x + 1/2*c) - 21)/(a*tan(1/2*d*x + 1/2*c)^8))/d
```

3.719 $\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=152

$$-\frac{\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}+\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}-\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}+\frac{5\,\mathrm{co}}{}$$

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(128*a*d) - Cot[c + d*x]^7/(7*a*d) - Cot[c + d*x
]^9/(9*a*d) - (5*Cot[c + d*x]*Csc[c + d*x])/(128*a*d) + (5*Cot[c + d*x]*Csc
[c + d*x]^3)/(64*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*a*d) + (Cot[c
 + d*x]^5*Csc[c + d*x]^3)/(8*a*d)
```

Rubi [A] time = 0.235424, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2607, 14, 2611, 3768, 3770}

$$-\frac{\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}+\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}-\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}+\frac{5\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(128*a*d) - Cot[c + d*x]^7/(7*a*d) - Cot[c + d*x
]^9/(9*a*d) - (5*Cot[c + d*x]*Csc[c + d*x])/(128*a*d) + (5*Cot[c + d*x]*Csc
[c + d*x]^3)/(64*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*a*d) + (Cot[c
 + d*x]^5*Csc[c + d*x]^3)/(8*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc^3(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^4(c+dx)\,dx}{a}$$
$$= \frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad} + \frac{5\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^6\left(1+x^{2}\right.\right.}{}$$
$$= -\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad} + \frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad} - \frac{5\int \cot^2(c+dx)\csc^3(c}{16a}$$
$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{\cot^9(c+dx)}{9ad} + \frac{5\cot(c+dx)\csc^3(c+dx)}{64ad} - \frac{5\cot^3(c+dx)\csc^3(c}{48ad}$$
$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{\cot^9(c+dx)}{9ad} - \frac{5\cot(c+dx)\csc(c+dx)}{128ad} + \frac{5\cot(c+dx)\csc^3(c+}{64ad}$$
$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{128ad} - \frac{\cot^7(c+dx)}{7ad} - \frac{\cot^9(c+dx)}{9ad} - \frac{5\cot(c+dx)\csc(c+dx)}{128ad}$$

Mathematica [B] time = 1.35241, size = 313, normalized size = 2.06

$$-\frac{\csc^9(c+dx)\left(-36540\sin(2(c+dx)) - 20916\sin(4(c+dx)) - 16044\sin(6(c+dx)) - 630\sin(8(c+dx)) + 129024\,\text{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^9*(129024*Cos[c + d*x] + 75264*Cos[3*(c + d*x)] + 23040*Cos[
5*(c + d*x)] + 2304*Cos[7*(c + d*x)] - 256*Cos[9*(c + d*x)] + 39690*Log[Cos
[(c + d*x)/2]]*Sin[c + d*x] - 39690*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 36
540*Sin[2*(c + d*x)] - 26460*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] + 26460
*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 20916*Sin[4*(c + d*x)] + 11340*Lo
g[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 11340*Log[Sin[(c + d*x)/2]]*Sin[5*(c
 + d*x)] - 16044*Sin[6*(c + d*x)] - 2835*Log[Cos[(c + d*x)/2]]*Sin[7*(c + d
*x)] + 2835*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 630*Sin[8*(c + d*x)] +
 315*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 315*Log[Sin[(c + d*x)/2]]*Sin
[9*(c + d*x)]))/(2064384*a*d)
```

Maple [B] time = 0.179, size = 322, normalized size = 2.1

$$\frac{1}{4608\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9 - \frac{1}{2048\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8 - \frac{3}{3584\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 + \frac{1}{384\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6 - \frac{1}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4608/d/a*tan(1/2*d*x+1/2*c)^9-1/2048/d/a*tan(1/2*d*x+1/2*c)^8-3/3584/d/a*
tan(1/2*d*x+1/2*c)^7+1/384/d/a*tan(1/2*d*x+1/2*c)^6-1/256/d/a*tan(1/2*d*x+1
/2*c)^4+1/192/d/a*tan(1/2*d*x+1/2*c)^3-1/128/d/a*tan(1/2*d*x+1/2*c)^2-3/256
/d/a*tan(1/2*d*x+1/2*c)+3/3584/d/a/tan(1/2*d*x+1/2*c)^7+3/256/d/a/tan(1/2*d
*x+1/2*c)+1/2048/d/a/tan(1/2*d*x+1/2*c)^8+1/256/d/a/tan(1/2*d*x+1/2*c)^4-1/
4608/d/a/tan(1/2*d*x+1/2*c)^9+5/128/d/a*ln(tan(1/2*d*x+1/2*c))-1/384/d/a/ta
n(1/2*d*x+1/2*c)^6-1/192/d/a/tan(1/2*d*x+1/2*c)^3+1/128/d/a/tan(1/2*d*x+1/2
*c)^2
```

Maxima [B] time = 1.02169, size = 479, normalized size = 3.15

$$-\frac{\frac{\frac{1512\sin(dx+c)}{\cos(dx+c)+1}+\frac{1008\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{672\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{504\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{336\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{108\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{63\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{28\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\left(\cdots\right.}{129024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/129024*((1512*sin(d*x + c)/(cos(d*x + c) + 1) + 1008*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 672*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 504*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 - 336*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 108*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7 + 63*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 28*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a - 5040*log(sin(d*x + c)/(cos(d
*x + c) + 1))/a - (63*sin(d*x + c)/(cos(d*x + c) + 1) + 108*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 336*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 504*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 672*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1
008*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1512*sin(d*x + c)^8/(cos(d*x + c)
 + 1)^8 - 28)*(cos(d*x + c) + 1)^9/(a*sin(d*x + c)^9))/d
```

Fricas [A] time = 1.23934, size = 689, normalized size = 4.53

$$\frac{512\cos(dx+c)^9-2304\cos(dx+c)^7-315\left(\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1\right)\log\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/16128*(512*cos(d*x + c)^9 - 2304*cos(d*x + c)^7 - 315*(cos(d*x + c)^8 - 4
*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + 315*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d
*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 + 42*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d
*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^6 + 6*a*d*
cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3315, size = 369, normalized size = 2.43

$$\frac{\frac{5040 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{28\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-63\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-108\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+336\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-504\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+672}{a^{9}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/129024*(5040*log(abs(tan(1/2*d*x + 1/2*c)))/a + (28*a^8*tan(1/2*d*x + 1/2
*c)^9 - 63*a^8*tan(1/2*d*x + 1/2*c)^8 - 108*a^8*tan(1/2*d*x + 1/2*c)^7 + 33
6*a^8*tan(1/2*d*x + 1/2*c)^6 - 504*a^8*tan(1/2*d*x + 1/2*c)^4 + 672*a^8*tan
(1/2*d*x + 1/2*c)^3 - 1008*a^8*tan(1/2*d*x + 1/2*c)^2 - 1512*a^8*tan(1/2*d*
x + 1/2*c))/a^9 - (14258*tan(1/2*d*x + 1/2*c)^9 - 1512*tan(1/2*d*x + 1/2*c)
^8 - 1008*tan(1/2*d*x + 1/2*c)^7 + 672*tan(1/2*d*x + 1/2*c)^6 - 504*tan(1/2
*d*x + 1/2*c)^5 + 336*tan(1/2*d*x + 1/2*c)^3 - 108*tan(1/2*d*x + 1/2*c)^2 -
 63*tan(1/2*d*x + 1/2*c) + 28)/(a*tan(1/2*d*x + 1/2*c)^9))/d
```

3.720 $\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{\cot^9(c+dx)}{9ad}+\frac{\cot^7(c+dx)}{7ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}-\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad}-\frac{\cot(c+$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(256*a*d) + Cot[c + d*x]^7/(7*a*d) + Cot[c + d*x]
^9/(9*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(256*a*d) + (Cot[c + d*x]*Csc[c
+ d*x]^3)/(128*a*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(32*a*d) + (Cot[c + d*x
]^3*Csc[c + d*x]^5)/(16*a*d) - (Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*a*d)
```

Rubi [A] time = 0.247625, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2611, 3768, 3770, 2607, 14}

$$\frac{\cot^9(c+dx)}{9ad}+\frac{\cot^7(c+dx)}{7ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}-\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad}-\frac{\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (3*ArcTanh[Cos[c + d*x]])/(256*a*d) + Cot[c + d*x]^7/(7*a*d) + Cot[c + d*x]
^9/(9*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(256*a*d) + (Cot[c + d*x]*Csc[c
+ d*x]^3)/(128*a*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(32*a*d) + (Cot[c + d*x
]^3*Csc[c + d*x]^5)/(16*a*d) - (Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*a*d)
```


Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc^4(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^5(c+dx)\,dx}{a}$$
$$= -\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} - \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{2a} - \frac{\text{Subst}\left(\int x^6\left(1+x^2\right.\right.}{}$$
$$= \frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad} - \frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} + \frac{3\int \cot^2(c+dx)\csc^5(c+}{16a}$$
$$= \frac{\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} - \frac{\cot(c+dx)\csc^5(c+dx)}{32ad} + \frac{\cot^3(c+dx)\csc^5(c+dx}{16ad}$$
$$= \frac{\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} + \frac{\cot(c+dx)\csc^3(c+dx)}{128ad} - \frac{\cot(c+dx)\csc^5(c+dx)}{32ad}$$
$$= \frac{\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} + \frac{3\cot(c+dx)\csc(c+dx)}{256ad} + \frac{\cot(c+dx)\csc^3(c+dx}{128ad}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{256ad} + \frac{\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} + \frac{3\cot(c+dx)\csc(c+dx)}{256ad}$$

Mathematica [B] time = 1.50859, size = 386, normalized size = 2.19

$$-\frac{\csc^9(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-537600\sin(2(c+dx))-522240\sin(4(c+dx))-207360\sin(6(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^9*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(2367540*Cos[c + d
*x] + 1307880*Cos[3*(c + d*x)] + 436968*Cos[5*(c + d*x)] + 18270*Cos[7*(c +
 d*x)] - 1890*Cos[9*(c + d*x)] - 119070*Log[Cos[(c + d*x)/2]] + 198450*Cos[
2*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 113400*Cos[4*(c + d*x)]*Log[Cos[(c + d
*x)/2]] + 42525*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 9450*Cos[8*(c + d*
x)]*Log[Cos[(c + d*x)/2]] + 945*Cos[10*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 1
19070*Log[Sin[(c + d*x)/2]] - 198450*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]]
 + 113400*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 42525*Cos[6*(c + d*x)]*L
og[Sin[(c + d*x)/2]] + 9450*Cos[8*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 945*Co
s[10*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 537600*Sin[2*(c + d*x)] - 522240*Si
n[4*(c + d*x)] - 207360*Sin[6*(c + d*x)] - 25600*Sin[8*(c + d*x)] + 2560*Si
n[10*(c + d*x)]))/(165150720*a*d*(1 + Csc[c + d*x]))
```

Maple [B] time = 0.188, size = 360, normalized size = 2.1

$$\frac{1}{10240\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}-\frac{1}{4608\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}-\frac{1}{4096\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}+\frac{3}{3584\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}-\frac{1}{204}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/10240/d/a*tan(1/2*d*x+1/2*c)^10-1/4608/d/a*tan(1/2*d*x+1/2*c)^9-1/4096/d/
a*tan(1/2*d*x+1/2*c)^8+3/3584/d/a*tan(1/2*d*x+1/2*c)^7-1/2048/d/a*tan(1/2*d
*x+1/2*c)^6+1/512/d/a*tan(1/2*d*x+1/2*c)^4-1/192/d/a*tan(1/2*d*x+1/2*c)^3+1
/1024/d/a*tan(1/2*d*x+1/2*c)^2+3/256/d/a*tan(1/2*d*x+1/2*c)-1/10240/d/a/tan
(1/2*d*x+1/2*c)^10-3/3584/d/a/tan(1/2*d*x+1/2*c)^7-3/256/d/a/tan(1/2*d*x+1/
2*c)+1/4096/d/a/tan(1/2*d*x+1/2*c)^8-1/512/d/a/tan(1/2*d*x+1/2*c)^4+1/4608/
d/a/tan(1/2*d*x+1/2*c)^9-3/256/d/a*ln(tan(1/2*d*x+1/2*c))+1/2048/d/a/tan(1/
2*d*x+1/2*c)^6+1/192/d/a/tan(1/2*d*x+1/2*c)^3-1/1024/d/a/tan(1/2*d*x+1/2*c)
^2
```

Maxima [B] time = 1.05124, size = 532, normalized size = 3.02

$$\frac{\frac{\frac{15120\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1260\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6720\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{630\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1080\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{280\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{126\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}{a}-15120\log(}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/1290240*((15120*sin(d*x + c)/(cos(d*x + c) + 1) + 1260*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 6720*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2520*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 630*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 10
80*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 315*sin(d*x + c)^8/(cos(d*x + c) +
 1)^8 - 280*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 126*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10)/a - 15120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (280*s
in(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
1080*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 630*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 2520*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 6720*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 - 1260*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15120*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 - 126)*(cos(d*x + c) + 1)^10/(a*sin(d*x + c)
^10))/d
```

Fricas [A] time = 1.24031, size = 768, normalized size = 4.36

$$-\frac{1890\,\cos(dx+c)^9-8820\,\cos(dx+c)^7-16128\,\cos(dx+c)^5+8820\,\cos(dx+c)^3-945\left(\cos(dx+c)^{10}-5\,\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/161280*(1890*cos(d*x + c)^9 - 8820*cos(d*x + c)^7 - 16128*cos(d*x + c)^5
 + 8820*cos(d*x + c)^3 - 945*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d
*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c)
+ 1/2) + 945*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*c
os(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2560*(
2*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*sin(d*x + c) - 1890*cos(d*x + c))/(a*d
*cos(d*x + c)^10 - 5*a*d*cos(d*x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*co
s(d*x + c)^4 + 5*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39131, size = 409, normalized size = 2.32

$$-\frac{\frac{15120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{126a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-280a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-315a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+1080a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-630a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{a^{10}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1290240*(15120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (126*a^9*tan(1/2*d*x +
 1/2*c)^10 - 280*a^9*tan(1/2*d*x + 1/2*c)^9 - 315*a^9*tan(1/2*d*x + 1/2*c)^
8 + 1080*a^9*tan(1/2*d*x + 1/2*c)^7 - 630*a^9*tan(1/2*d*x + 1/2*c)^6 + 2520
*a^9*tan(1/2*d*x + 1/2*c)^4 - 6720*a^9*tan(1/2*d*x + 1/2*c)^3 + 1260*a^9*ta
n(1/2*d*x + 1/2*c)^2 + 15120*a^9*tan(1/2*d*x + 1/2*c))/a^10 - (44286*tan(1/
2*d*x + 1/2*c)^10 - 15120*tan(1/2*d*x + 1/2*c)^9 - 1260*tan(1/2*d*x + 1/2*c
)^8 + 6720*tan(1/2*d*x + 1/2*c)^7 - 2520*tan(1/2*d*x + 1/2*c)^6 + 630*tan(1
/2*d*x + 1/2*c)^4 - 1080*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c)^
2 + 280*tan(1/2*d*x + 1/2*c) - 126)/(a*tan(1/2*d*x + 1/2*c)^10))/d
```

3.721 $\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{\cot^{11}(c+dx)}{11ad}-\frac{2\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}+\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}-\frac{\cot^3(c+dx)\csc}{16ad}$$

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(256*a*d) - Cot[c + d*x]^7/(7*a*d) - (2*Cot[c +
d*x]^9)/(9*a*d) - Cot[c + d*x]^11/(11*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/
(256*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(128*a*d) + (Cot[c + d*x]*Csc[c +
 d*x]^5)/(32*a*d) - (Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*a*d) + (Cot[c + d*x
]^5*Csc[c + d*x]^5)/(10*a*d)
```

Rubi [A] time = 0.252739, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2607, 270, 2611, 3768, 3770}

$$-\frac{\cot^{11}(c+dx)}{11ad}-\frac{2\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}+\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}-\frac{\cot^3(c+dx)\csc}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(256*a*d) - Cot[c + d*x]^7/(7*a*d) - (2*Cot[c +
d*x]^9)/(9*a*d) - Cot[c + d*x]^11/(11*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/
(256*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(128*a*d) + (Cot[c + d*x]*Csc[c +
 d*x]^5)/(32*a*d) - (Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*a*d) + (Cot[c + d*x
]^5*Csc[c + d*x]^5)/(10*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc^5(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^6(c+dx)\,dx}{a}$$

$$= \frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} + \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{2a} + \frac{\mathrm{Subst}\left(\int x^6\left(1+x^2\right)\right.}{}$$

$$= -\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad} + \frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} - \frac{3\int \cot^2(c+dx)\csc^5(c+}{16a}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{32ad} - \cot^3$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{128ad} + \cot($$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} - \frac{3\cot(c+dx)\csc(c+dx)}{256ad} - \cot($$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{256ad} - \frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} - 3\cot($$

Mathematica [A] time = 2.93717, size = 187, normalized size = 0.96

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-2661120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{10}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(-2661120*(Log[Cos[(c + d*x)/2]] -
 Log[Sin[(c + d*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^10*(6840320 + 9973760*C
os[2*(c + d*x)] + 3543040*Cos[4*(c + d*x)] + 343040*Cos[6*(c + d*x)] - 6144
0*Cos[8*(c + d*x)] + 5120*Cos[10*(c + d*x)] - 3219678*Sin[c + d*x] - 260845
2*Sin[3*(c + d*x)] - 2181564*Sin[5*(c + d*x)] - 121275*Sin[7*(c + d*x)] + 1
0395*Sin[9*(c + d*x)])))/(227082240*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.209, size = 436, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/22528/d/a*tan(1/2*d*x+1/2*c)^11-1/10240/d/a*tan(1/2*d*x+1/2*c)^10-1/18432
/d/a*tan(1/2*d*x+1/2*c)^9+1/4096/d/a*tan(1/2*d*x+1/2*c)^8-5/14336/d/a*tan(1
/2*d*x+1/2*c)^7+1/2048/d/a*tan(1/2*d*x+1/2*c)^6+1/2048/d/a*tan(1/2*d*x+1/2*
c)^5-1/512/d/a*tan(1/2*d*x+1/2*c)^4+5/3072/d/a*tan(1/2*d*x+1/2*c)^3-1/1024/
d/a*tan(1/2*d*x+1/2*c)^2-5/1024/d/a*tan(1/2*d*x+1/2*c)+1/10240/d/a/tan(1/2*
d*x+1/2*c)^10+5/14336/d/a/tan(1/2*d*x+1/2*c)^7+5/1024/d/a/tan(1/2*d*x+1/2*c
)-1/22528/d/a/tan(1/2*d*x+1/2*c)^11-1/4096/d/a/tan(1/2*d*x+1/2*c)^8-1/2048/
d/a/tan(1/2*d*x+1/2*c)^5+1/512/d/a/tan(1/2*d*x+1/2*c)^4+1/18432/d/a/tan(1/2
*d*x+1/2*c)^9+3/256/d/a*ln(tan(1/2*d*x+1/2*c))-1/2048/d/a/tan(1/2*d*x+1/2*c
)^6-5/3072/d/a/tan(1/2*d*x+1/2*c)^3+1/1024/d/a/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.13837, size = 641, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/14192640*((69300*sin(d*x + c)/(cos(d*x + c) + 1) + 13860*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 23100*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 27720*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6930*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 - 6930*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4950*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 3465*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 770*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 + 1386*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 630*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11)/a - 166320*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a - (1386*sin(d*x + c)/(cos(d*x + c) + 1) + 770*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - 3465*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4950*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 6930*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
 6930*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 27720*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 - 23100*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 13860*sin(d*x + c)
^9/(cos(d*x + c) + 1)^9 + 69300*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 630
)*(cos(d*x + c) + 1)^11/(a*sin(d*x + c)^11))/d
```

Fricas [A] time = 1.23436, size = 861, normalized size = 4.44

$$\frac{20480\cos(dx+c)^{11} - 112640\cos(dx+c)^9 + 253440\cos(dx+c)^7 - 10395\left(\cos(dx+c)^{10} - 5\cos(dx+c)^8 + 10\cos(\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1774080*(20480*cos(d*x + c)^11 - 112640*cos(d*x + c)^9 + 253440*cos(d*x +
 c)^7 - 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*
cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) + 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*c
```

```
os(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) + 1386*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x + c)^5 + 7
0*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^10 - 5
*a*d*cos(d*x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*cos(d*x + c)^4 + 5*a*d
*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**12/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27398, size = 486, normalized size = 2.51

$$\frac{\frac{166320 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a} + \frac{630 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{11} - 1386 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{10} - 770 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{9} + 3465 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{8} - 4950 a^{10} \tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/14192640*(166320*log(abs(tan(1/2*d*x + 1/2*c)))/a + (630*a^10*tan(1/2*d*x
 + 1/2*c)^11 - 1386*a^10*tan(1/2*d*x + 1/2*c)^10 - 770*a^10*tan(1/2*d*x + 1
/2*c)^9 + 3465*a^10*tan(1/2*d*x + 1/2*c)^8 - 4950*a^10*tan(1/2*d*x + 1/2*c)
^7 + 6930*a^10*tan(1/2*d*x + 1/2*c)^6 + 6930*a^10*tan(1/2*d*x + 1/2*c)^5 -
27720*a^10*tan(1/2*d*x + 1/2*c)^4 + 23100*a^10*tan(1/2*d*x + 1/2*c)^3 - 138
60*a^10*tan(1/2*d*x + 1/2*c)^2 - 69300*a^10*tan(1/2*d*x + 1/2*c))/a^11 - (5
02266*tan(1/2*d*x + 1/2*c)^11 - 69300*tan(1/2*d*x + 1/2*c)^10 - 13860*tan(1
/2*d*x + 1/2*c)^9 + 23100*tan(1/2*d*x + 1/2*c)^8 - 27720*tan(1/2*d*x + 1/2*
c)^7 + 6930*tan(1/2*d*x + 1/2*c)^6 + 6930*tan(1/2*d*x + 1/2*c)^5 - 4950*tan
(1/2*d*x + 1/2*c)^4 + 3465*tan(1/2*d*x + 1/2*c)^3 - 770*tan(1/2*d*x + 1/2*c
)^2 - 1386*tan(1/2*d*x + 1/2*c) + 630)/(a*tan(1/2*d*x + 1/2*c)^11))/d
```

3.722 $\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$\frac{\cos^{11}(c+dx)}{11a^2d}-\frac{4\cos^9(c+dx)}{9a^2d}+\frac{5\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^5(c+dx)\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}$$

```
[Out] (-3*x)/(128*a^2) - (2*Cos[c + d*x]^5)/(5*a^2*d) + (5*Cos[c + d*x]^7)/(7*a^2
*d) - (4*Cos[c + d*x]^9)/(9*a^2*d) + Cos[c + d*x]^11/(11*a^2*d) - (3*Cos[c
+ d*x]*Sin[c + d*x])/(128*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(64*a^2*d)
 + (Cos[c + d*x]^5*Sin[c + d*x])/(16*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^
3)/(8*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.412395, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2565, 270, 2568, 2635, 8}

$$\frac{\cos^{11}(c+dx)}{11a^2d}-\frac{4\cos^9(c+dx)}{9a^2d}+\frac{5\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^5(c+dx)\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-3*x)/(128*a^2) - (2*Cos[c + d*x]^5)/(5*a^2*d) + (5*Cos[c + d*x]^7)/(7*a^2
*d) - (4*Cos[c + d*x]^9)/(9*a^2*d) + Cos[c + d*x]^11/(11*a^2*d) - (3*Cos[c
+ d*x]*Sin[c + d*x])/(128*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(64*a^2*d)
 + (Cos[c + d*x]^5*Sin[c + d*x])/(16*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^
3)/(8*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^5)/(5*a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^5(c+dx) - 2a^2\cos^4(c+dx)\sin^6(c+dx) + a^2\cos^4(c+dx)\sin\ldots\right.}{a^4}$$

$$= \frac{\int \cos^4(c+dx)\sin^5(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^7(c+dx)\,dx}{a^2} - \frac{2\int \cos^4(c+dx)\ldots}{a^2}$$

$$= \frac{\cos^5(c+dx)\sin^5(c+dx)}{5a^2d} - \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)^2\ldots\right.}{a^2d}$$

$$= \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} + \frac{\cos^5(c+dx)\sin^5(c+dx)}{5a^2d} - \frac{3\int \cos^4(c+dx)\sin^2(c+\ldots}{8a^2}$$

$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} + \frac{\cos^5(c+dx)}{16a\ldots}$$

$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} - \frac{\cos^3(c+dx)}{64a\ldots}$$

$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} - \frac{3\cos(c+dx)\ldots}{128\ldots}$$

$$= -\frac{3x}{128a^2} - \frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} - \frac{3\cos\ldots}{}$$

Mathematica [B] time = 10.3357, size = 1453, normalized size = 7.16

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*Cos[c + d*x]*(1 + 2*Sin[c + d*x]))/(3072*a^2*d*(1 + Sin[c + d*x])^2) +
(27720*(c + d*x) + 41580*Cos[c + d*x] - 7056*Cos[3*(c + d*x)] + 1764*Cos[5*
(c + d*x)] - 360*Cos[7*(c + d*x)] + 28*Cos[9*(c + d*x)] + (42*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 21/(Cos[(c + d*x)/2] + Sin[(
```

```
c + d*x)/2])^2 - (15204*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]) - 15120*Sin[2*(c + d*x)] + 3528*Sin[4*(c + d*x)] - 840*Sin[6*(c + d*x)
] + 126*Sin[8*(c + d*x)])/(86016*a^2*d) + (-360360*(c + d*x) - 566280*Cos[c
 + d*x] + 108900*Cos[3*(c + d*x)] - 33264*Cos[5*(c + d*x)] + 9900*Cos[7*(c
+ d*x)] - 2200*Cos[9*(c + d*x)] + 180*Cos[11*(c + d*x)] - (330*Sin[(c + d*x
)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 165/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2 + (166980*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]) + 217800*Sin[2*(c + d*x)] - 59400*Sin[4*(c + d*x)] + 18480*Sin[6*(c
 + d*x)] - 4950*Sin[8*(c + d*x)] + 792*Sin[10*(c + d*x)])/(2027520*a^2*d) +
 (25*(36*d*x*Cos[(d*x)/2] - 21*Cos[c + (d*x)/2] + 35*Cos[c + (3*d*x)/2] - 1
2*d*x*Cos[2*c + (3*d*x)/2] - 3*Cos[3*c + (5*d*x)/2] - 57*Sin[(d*x)/2] + 36*
d*x*Sin[c + (d*x)/2] + 12*d*x*Sin[c + (3*d*x)/2] + 9*Sin[2*c + (3*d*x)/2] +
 3*Sin[2*c + (5*d*x)/2]))/(12288*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^3) + (5*(180*d*x*Cos[(d*x)/2] - 21*Cos[c + (d*x)/2]
 + 147*Cos[c + (3*d*x)/2] - 60*d*x*Cos[2*c + (3*d*x)/2] - 15*Cos[3*c + (5*d
*x)/2] + 3*Cos[3*c + (7*d*x)/2] + Cos[5*c + (9*d*x)/2] - 201*Sin[(d*x)/2] +
 180*d*x*Sin[c + (d*x)/2] + 60*d*x*Sin[c + (3*d*x)/2] + 73*Sin[2*c + (3*d*x
)/2] + 15*Sin[2*c + (5*d*x)/2] + 3*Sin[4*c + (7*d*x)/2] - Sin[4*c + (9*d*x)
/2]))/(12288*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3) - (7*(2520*d*x*Cos[(d*x)/2] + 165*Cos[c + (d*x)/2] + 1905*Cos[c + (3
*d*x)/2] - 840*d*x*Cos[2*c + (3*d*x)/2] - 210*Cos[3*c + (5*d*x)/2] + 42*Cos
[3*c + (7*d*x)/2] + 14*Cos[5*c + (9*d*x)/2] - 6*Cos[5*c + (11*d*x)/2] - 3*C
os[7*c + (13*d*x)/2] - 2355*Sin[(d*x)/2] + 2520*d*x*Sin[c + (d*x)/2] + 840*
d*x*Sin[c + (3*d*x)/2] + 1175*Sin[2*c + (3*d*x)/2] + 210*Sin[2*c + (5*d*x)/
2] + 42*Sin[4*c + (7*d*x)/2] - 14*Sin[4*c + (9*d*x)/2] - 6*Sin[6*c + (11*d*
x)/2] + 3*Sin[6*c + (13*d*x)/2]))/(30720*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^3) + (7560*d*x*Cos[(d*x)/2] + 1239*Cos[c +
(d*x)/2] + 5467*Cos[c + (3*d*x)/2] - 2520*d*x*Cos[2*c + (3*d*x)/2] - 630*Co
s[3*c + (5*d*x)/2] + 126*Cos[3*c + (7*d*x)/2] + 42*Cos[5*c + (9*d*x)/2] - 1
8*Cos[5*c + (11*d*x)/2] - 9*Cos[7*c + (13*d*x)/2] + 5*Cos[7*c + (15*d*x)/2]
 + 3*Cos[9*c + (17*d*x)/2] - 6321*Sin[(d*x)/2] + 7560*d*x*Sin[c + (d*x)/2]
+ 2520*d*x*Sin[c + (3*d*x)/2] + 3773*Sin[2*c + (3*d*x)/2] + 630*Sin[2*c + (
5*d*x)/2] + 126*Sin[4*c + (7*d*x)/2] - 42*Sin[4*c + (9*d*x)/2] - 18*Sin[6*c
 + (11*d*x)/2] + 9*Sin[6*c + (13*d*x)/2] + 5*Sin[8*c + (15*d*x)/2] - 3*Sin[
8*c + (17*d*x)/2])/(43008*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^3)
```

Maple [B] time = 0.137, size = 653, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -272/3465/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11+3/64/d/a^2/(1+tan(1/2*d*x+1/2*c
)^2)^11*tan(1/2*d*x+1/2*c)-272/315/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/
2*d*x+1/2*c)^2+1/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^3-2
72/63/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^4+773/320/d/a^2/
(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^5-16/7/d/a^2/(1+tan(1/2*d*x+
1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^6-148/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*t
an(1/2*d*x+1/2*c)^7+80/7/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*
c)^8+1207/32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^9-464/5/d
/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^10+848/15/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^12-1207/32/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^11*tan(1/2*d*x+1/2*c)^13-112/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan
(1/2*d*x+1/2*c)^14+148/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*
```

```
c)^15-32/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^16-773/320/
d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^17-1/2/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^11*tan(1/2*d*x+1/2*c)^19-3/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^11*tan(1/2*d*x+1/2*c)^21-3/64/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.55959, size = 875, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/221760*((10395*sin(d*x + c)/(cos(d*x + c) + 1) - 191488*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 110880*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 957440*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 535689*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 - 506880*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6564096*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 2534400*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 8364510
*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 20579328*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 + 12536832*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8364510*sin(d
*x + c)^13/(cos(d*x + c) + 1)^13 - 8279040*sin(d*x + c)^14/(cos(d*x + c) +
1)^14 + 6564096*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 2365440*sin(d*x + c
)^16/(cos(d*x + c) + 1)^16 - 535689*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 -
 110880*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 10395*sin(d*x + c)^21/(cos(
d*x + c) + 1)^21 - 17408)/(a^2 + 11*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 55*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 165*a^2*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 330*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 462*a^2*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 + 462*a^2*sin(d*x + c)^12/(cos(d*x + c)
 + 1)^12 + 330*a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 165*a^2*sin(d*x
+ c)^16/(cos(d*x + c) + 1)^16 + 55*a^2*sin(d*x + c)^18/(cos(d*x + c) + 1)^1
8 + 11*a^2*sin(d*x + c)^20/(cos(d*x + c) + 1)^20 + a^2*sin(d*x + c)^22/(cos
(d*x + c) + 1)^22) - 10395*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.19052, size = 332, normalized size = 1.64

$$\frac{40320\,\cos\left(dx+c\right)^{11}-197120\,\cos\left(dx+c\right)^{9}+316800\,\cos\left(dx+c\right)^{7}-177408\,\cos\left(dx+c\right)^{5}-10395\,dx+693\,\left(128\,\cos\ldots\right.}{443520\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/443520*(40320*cos(d*x + c)^11 - 197120*cos(d*x + c)^9 + 316800*cos(d*x +
c)^7 - 177408*cos(d*x + c)^5 - 10395*d*x + 693*(128*cos(d*x + c)^9 - 336*co
s(d*x + c)^7 + 248*cos(d*x + c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*si
n(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31961, size = 365, normalized size = 1.8

$$-\frac{\frac{10395\,(dx+c)}{a^2}+\frac{2\left(10395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{21}+110880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19}+535689\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+2365440\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}-6564096\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+82\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/443520*(10395*(d*x + c)/a^2 + 2*(10395*tan(1/2*d*x + 1/2*c)^21 + 110880*
tan(1/2*d*x + 1/2*c)^19 + 535689*tan(1/2*d*x + 1/2*c)^17 + 2365440*tan(1/2*
d*x + 1/2*c)^16 - 6564096*tan(1/2*d*x + 1/2*c)^15 + 8279040*tan(1/2*d*x + 1
/2*c)^14 + 8364510*tan(1/2*d*x + 1/2*c)^13 - 12536832*tan(1/2*d*x + 1/2*c)^
12 + 20579328*tan(1/2*d*x + 1/2*c)^10 - 8364510*tan(1/2*d*x + 1/2*c)^9 - 25
34400*tan(1/2*d*x + 1/2*c)^8 + 6564096*tan(1/2*d*x + 1/2*c)^7 + 506880*tan(
1/2*d*x + 1/2*c)^6 - 535689*tan(1/2*d*x + 1/2*c)^5 + 957440*tan(1/2*d*x + 1
/2*c)^4 - 110880*tan(1/2*d*x + 1/2*c)^3 + 191488*tan(1/2*d*x + 1/2*c)^2 - 1
0395*tan(1/2*d*x + 1/2*c) + 17408)/((tan(1/2*d*x + 1/2*c)^2 + 1)^11*a^2))/d
```

3.723 $\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=185

$$\frac{2\cos^9(c+dx)}{9a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^5(c+dx)\cos^5(c+dx)}{10a^2d}-\frac{3\sin^3(c+dx)\cos^5(c+dx)}{16a^2d}-\frac{3\sin(c$$

```
[Out] (9*x)/(256*a^2) + (2*Cos[c + d*x]^5)/(5*a^2*d) - (4*Cos[c + d*x]^7)/(7*a^2*
d) + (2*Cos[c + d*x]^9)/(9*a^2*d) + (9*Cos[c + d*x]*Sin[c + d*x])/(256*a^2*
d) + (3*Cos[c + d*x]^3*Sin[c + d*x])/(128*a^2*d) - (3*Cos[c + d*x]^5*Sin[c
+ d*x])/(32*a^2*d) - (3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*a^2*d) - (Cos[c
+ d*x]^5*Sin[c + d*x]^5)/(10*a^2*d)
```

Rubi [A] time = 0.457852, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2568, 2635, 8, 2565, 270}

$$\frac{2\cos^9(c+dx)}{9a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^5(c+dx)\cos^5(c+dx)}{10a^2d}-\frac{3\sin^3(c+dx)\cos^5(c+dx)}{16a^2d}-\frac{3\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (9*x)/(256*a^2) + (2*Cos[c + d*x]^5)/(5*a^2*d) - (4*Cos[c + d*x]^7)/(7*a^2*
d) + (2*Cos[c + d*x]^9)/(9*a^2*d) + (9*Cos[c + d*x]*Sin[c + d*x])/(256*a^2*
d) + (3*Cos[c + d*x]^3*Sin[c + d*x])/(128*a^2*d) - (3*Cos[c + d*x]^5*Sin[c
+ d*x])/(32*a^2*d) - (3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*a^2*d) - (Cos[c
+ d*x]^5*Sin[c + d*x]^5)/(10*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^4(c+dx)-2a^2\cos^4(c+dx)\sin^5(c+dx)+a^2\cos^4(c+dx)\sin^6(c\right.}{a^4}$$
$$= \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2}+\frac{\int \cos^4(c+dx)\sin^6(c+dx)\,dx}{a^2}-\frac{2\int \cos^4(c+dx)\sin}{a^2}$$
$$= -\frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d}-\frac{\cos^5(c+dx)\sin^5(c+dx)}{10a^2d}+\frac{3\int \cos^4(c+dx)\sin^2(c+dx}{8a^2}$$
$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d}-\frac{3\cos^5(c+dx)\sin^3(c+dx)}{16a^2d}-\frac{\cos^5(c+dx)\sin^5(c+dx)}{10a^2d}+$$
$$= \frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^9(c+dx)}{9a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{64a^2d}-\frac{3\cos^5(}{}$$
$$= \frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^9(c+dx)}{9a^2d}+\frac{3\cos(c+dx)\sin(c+dx)}{128a^2d}+\frac{3\cos^3}{}$$
$$= \frac{3x}{128a^2}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^9(c+dx)}{9a^2d}+\frac{9\cos(c+dx)\sin(c+dx)}{256a^2d}$$
$$= \frac{9x}{256a^2}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^9(c+dx)}{9a^2d}+\frac{9\cos(c+dx)\sin(c+dx)}{256a^2d}$$

Mathematica [B] time = 8.43935, size = 585, normalized size = 3.16

$$\frac{45360dx\sin\left(\frac{c}{2}\right)-30240\sin\left(\frac{c}{2}+dx\right)+30240\sin\left(\frac{3c}{2}+dx\right)-1260\sin\left(\frac{3c}{2}+2dx\right)-1260\sin\left(\frac{5c}{2}+2dx\right)-6720\sin\left(\frac{5c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2520*(187*c - 18*d*x)*Cos[c/2] + 30240*Cos[c/2 + d*x] + 30240*Cos[(3*c)/2
 + d*x] - 1260*Cos[(3*c)/2 + 2*d*x] + 1260*Cos[(5*c)/2 + 2*d*x] + 6720*Cos[
(5*c)/2 + 3*d*x] + 6720*Cos[(7*c)/2 + 3*d*x] - 7560*Cos[(7*c)/2 + 4*d*x] +
```

```
7560*Cos[(9*c)/2 + 4*d*x] - 4032*Cos[(9*c)/2 + 5*d*x] - 4032*Cos[(11*c)/2 +
 5*d*x] + 630*Cos[(11*c)/2 + 6*d*x] - 630*Cos[(13*c)/2 + 6*d*x] - 720*Cos[(
13*c)/2 + 7*d*x] - 720*Cos[(15*c)/2 + 7*d*x] + 945*Cos[(15*c)/2 + 8*d*x] -
945*Cos[(17*c)/2 + 8*d*x] + 560*Cos[(17*c)/2 + 9*d*x] + 560*Cos[(19*c)/2 +
9*d*x] - 126*Cos[(19*c)/2 + 10*d*x] + 126*Cos[(21*c)/2 + 10*d*x] + 327180*S
in[c/2] - 471240*c*Sin[c/2] + 45360*d*x*Sin[c/2] - 30240*Sin[c/2 + d*x] + 3
0240*Sin[(3*c)/2 + d*x] - 1260*Sin[(3*c)/2 + 2*d*x] - 1260*Sin[(5*c)/2 + 2*
d*x] - 6720*Sin[(5*c)/2 + 3*d*x] + 6720*Sin[(7*c)/2 + 3*d*x] - 7560*Sin[(7*
c)/2 + 4*d*x] - 7560*Sin[(9*c)/2 + 4*d*x] + 4032*Sin[(9*c)/2 + 5*d*x] - 403
2*Sin[(11*c)/2 + 5*d*x] + 630*Sin[(11*c)/2 + 6*d*x] + 630*Sin[(13*c)/2 + 6*
d*x] + 720*Sin[(13*c)/2 + 7*d*x] - 720*Sin[(15*c)/2 + 7*d*x] + 945*Sin[(15*
c)/2 + 8*d*x] + 945*Sin[(17*c)/2 + 8*d*x] - 560*Sin[(17*c)/2 + 9*d*x] + 560
*Sin[(19*c)/2 + 9*d*x] - 126*Sin[(19*c)/2 + 10*d*x] - 126*Sin[(21*c)/2 + 10
*d*x])/(1290240*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.125, size = 619, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 32/315/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10-9/128/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^10*tan(1/2*d*x+1/2*c)+64/63/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*
x+1/2*c)^2-87/128/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^3+32
/7/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^4+553/160/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^5-64/7/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^10*tan(1/2*d*x+1/2*c)^6+491/32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan
(1/2*d*x+1/2*c)^7+32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^8
-2555/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^9+64/5/d/a^2/
(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^10+2555/64/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^11-32/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
10*tan(1/2*d*x+1/2*c)^12-491/32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d
*x+1/2*c)^13+64/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^14-5
53/160/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^15+87/128/d/a^2
/(1+tan(1/2*d*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^17+9/128/d/a^2/(1+tan(1/2*d
*x+1/2*c)^2)^10*tan(1/2*d*x+1/2*c)^19+9/128/d/a^2*arctan(tan(1/2*d*x+1/2*c)
)
```

Maxima [B] time = 1.58117, size = 817, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/40320*((2835*sin(d*x + c)/(cos(d*x + c) + 1) - 40960*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 27405*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 184320*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 139356*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 + 368640*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 618660*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7 - 1290240*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1609650*sin
```

```
(d*x + c)^9/(cos(d*x + c) + 1)^9 - 516096*sin(d*x + c)^10/(cos(d*x + c) + 1
)^10 - 1609650*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 430080*sin(d*x + c)^
12/(cos(d*x + c) + 1)^12 + 618660*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 8
60160*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 139356*sin(d*x + c)^15/(cos(d
*x + c) + 1)^15 - 27405*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 2835*sin(d*
x + c)^19/(cos(d*x + c) + 1)^19 - 4096)/(a^2 + 10*a^2*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 45*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 120*a^2*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 + 252*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a^2*sin(d*x + c)^12
/(cos(d*x + c) + 1)^12 + 120*a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45
*a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 + 10*a^2*sin(d*x + c)^18/(cos(d*
x + c) + 1)^18 + a^2*sin(d*x + c)^20/(cos(d*x + c) + 1)^20) - 2835*arctan(s
in(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.18664, size = 292, normalized size = 1.58

$$\frac{17920\cos(dx+c)^9-46080\cos(dx+c)^7+32256\cos(dx+c)^5+2835dx-63\left(128\cos(dx+c)^9-496\cos(dx+c)^7\right.}{80640a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/80640*(17920*cos(d*x + c)^9 - 46080*cos(d*x + c)^7 + 32256*cos(d*x + c)^5
 + 2835*d*x - 63*(128*cos(d*x + c)^9 - 496*cos(d*x + c)^7 + 488*cos(d*x + c
)^5 - 30*cos(d*x + c)^3 - 45*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28953, size = 347, normalized size = 1.88

$$\frac{\frac{2835(dx+c)}{a^2}+\frac{2\left(2835\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{19}+27405\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17}-139356\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+860160\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14}-618660\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-430080\,\mathrm{ta}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/80640*(2835*(d*x + c)/a^2 + 2*(2835*tan(1/2*d*x + 1/2*c)^19 + 27405*tan(1
/2*d*x + 1/2*c)^17 - 139356*tan(1/2*d*x + 1/2*c)^15 + 860160*tan(1/2*d*x +
1/2*c)^14 - 618660*tan(1/2*d*x + 1/2*c)^13 - 430080*tan(1/2*d*x + 1/2*c)^12
```

```
 + 1609650*tan(1/2*d*x + 1/2*c)^11 + 516096*tan(1/2*d*x + 1/2*c)^10 - 16096
50*tan(1/2*d*x + 1/2*c)^9 + 1290240*tan(1/2*d*x + 1/2*c)^8 + 618660*tan(1/2
*d*x + 1/2*c)^7 - 368640*tan(1/2*d*x + 1/2*c)^6 + 139356*tan(1/2*d*x + 1/2*
c)^5 + 184320*tan(1/2*d*x + 1/2*c)^4 - 27405*tan(1/2*d*x + 1/2*c)^3 + 40960
*tan(1/2*d*x + 1/2*c)^2 - 2835*tan(1/2*d*x + 1/2*c) + 4096)/((tan(1/2*d*x +
 1/2*c)^2 + 1)^10*a^2))/d
```

3.724 $\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=159

$$-\frac{\cos^9(c+dx)}{9a^2d}+\frac{3\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin(c+dx)\cos}{32a^2}$$

```
[Out] (-3*x)/(64*a^2) - (2*Cos[c + d*x]^5)/(5*a^2*d) + (3*Cos[c + d*x]^7)/(7*a^2*
d) - Cos[c + d*x]^9/(9*a^2*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(64*a^2*d) -
(Cos[c + d*x]^3*Sin[c + d*x])/(32*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(8
*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^3)/(4*a^2*d)
```

Rubi [A] time = 0.363248, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{\cos^9(c+dx)}{9a^2d}+\frac{3\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin(c+dx)\cos}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-3*x)/(64*a^2) - (2*Cos[c + d*x]^5)/(5*a^2*d) + (3*Cos[c + d*x]^7)/(7*a^2*
d) - Cos[c + d*x]^9/(9*a^2*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(64*a^2*d) -
(Cos[c + d*x]^3*Sin[c + d*x])/(32*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(8
*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^3)/(4*a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cos^4(c+dx)\sin^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\cos^4(c+dx)\sin^3(c+dx)-2a^2\cos^4(c+dx)\sin^4(c+dx)+a^2\cos^4(c+dx)\sin\right.}{a^4}\\
&= \frac{\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a^2}+\frac{\int \cos^4(c+dx)\sin^5(c+dx)\,dx}{a^2}-\frac{2\int \cos^4(c+dx)}{a^2}\\
&= \frac{\cos^5(c+dx)\sin^3(c+dx)}{4a^2d}-\frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{4a^2}-\frac{\mathrm{Subst}\left(\int x^4\left(1-x^2\right.\right.}{a^2}\\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^5(c+dx)\sin^3(c+dx)}{4a^2d}-\frac{\int \cos^4(c+dx)\,dx}{8a^2}-\mathrm{Subst}\\
&= -\frac{2\cos^5(c+dx)}{5a^2d}+\frac{3\cos^7(c+dx)}{7a^2d}-\frac{\cos^9(c+dx)}{9a^2d}-\frac{\cos^3(c+dx)\sin(c+dx)}{32a^2d}+\cos\\
&= -\frac{2\cos^5(c+dx)}{5a^2d}+\frac{3\cos^7(c+dx)}{7a^2d}-\frac{\cos^9(c+dx)}{9a^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{64a^2d}-\cos\\
&= -\frac{3x}{64a^2}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{3\cos^7(c+dx)}{7a^2d}-\frac{\cos^9(c+dx)}{9a^2d}-\frac{3\cos(c+dx)\sin(c+d}{64a^2d}
\end{aligned}$$

Mathematica [B] time = 6.46633, size = 430, normalized size = 2.7

$$-\frac{15120dx\sin\left(\frac{c}{2}\right)-11340\sin\left(\frac{c}{2}+dx\right)+11340\sin\left(\frac{3c}{2}+dx\right)-3360\sin\left(\frac{5c}{2}+3dx\right)+3360\sin\left(\frac{7c}{2}+3dx\right)-2520\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(420*(7 + 330*c + 36*d*x)*Cos[c/2] + 11340*Cos[c/2 + d*x] + 11340*Cos[(3*c
)/2 + d*x] + 3360*Cos[(5*c)/2 + 3*d*x] + 3360*Cos[(7*c)/2 + 3*d*x] - 2520*C
os[(7*c)/2 + 4*d*x] + 2520*Cos[(9*c)/2 + 4*d*x] - 1008*Cos[(9*c)/2 + 5*d*x]
 - 1008*Cos[(11*c)/2 + 5*d*x] - 450*Cos[(13*c)/2 + 7*d*x] - 450*Cos[(15*c)/
2 + 7*d*x] + 315*Cos[(15*c)/2 + 8*d*x] - 315*Cos[(17*c)/2 + 8*d*x] + 70*Cos
[(17*c)/2 + 9*d*x] + 70*Cos[(19*c)/2 + 9*d*x] - 78960*Sin[c/2] + 138600*c*S
in[c/2] + 15120*d*x*Sin[c/2] - 11340*Sin[c/2 + d*x] + 11340*Sin[(3*c)/2 + d
*x] - 3360*Sin[(5*c)/2 + 3*d*x] + 3360*Sin[(7*c)/2 + 3*d*x] - 2520*Sin[(7*c
)/2 + 4*d*x] - 2520*Sin[(9*c)/2 + 4*d*x] + 1008*Sin[(9*c)/2 + 5*d*x] - 1008
*Sin[(11*c)/2 + 5*d*x] + 450*Sin[(13*c)/2 + 7*d*x] - 450*Sin[(15*c)/2 + 7*d
*x] + 315*Sin[(15*c)/2 + 8*d*x] + 315*Sin[(17*c)/2 + 8*d*x] - 70*Sin[(17*c)
/2 + 9*d*x] + 70*Sin[(19*c)/2 + 9*d*x])/(322560*a^2*d*(Cos[c/2] + Sin[c/2])
)
```

Maple [B] time = 0.121, size = 551, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -52/315/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9+3/32/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^9*tan(1/2*d*x+1/2*c)-52/35/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1
/2*c)^2+13/16/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^3-68/35/d
/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^4-155/16/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^5+4/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9
*tan(1/2*d*x+1/2*c)^6+169/16/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1
/2*c)^7-164/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^8+12/d/a^
2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^10-169/16/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^11-44/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9
*tan(1/2*d*x+1/2*c)^12+155/16/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+
1/2*c)^13-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^14-13/16/d/
a^2/(1+tan(1/2*d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^15-3/32/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^9*tan(1/2*d*x+1/2*c)^17-3/32/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.61691, size = 732, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/10080*((945*sin(d*x + c)/(cos(d*x + c) + 1) - 14976*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 8190*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 19584*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 97650*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 80
64*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 106470*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 - 330624*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120960*sin(d*x + c)
^10/(cos(d*x + c) + 1)^10 - 106470*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 -
147840*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 97650*sin(d*x + c)^13/(cos(d
*x + c) + 1)^13 - 40320*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 8190*sin(d*
x + c)^15/(cos(d*x + c) + 1)^15 - 945*sin(d*x + c)^17/(cos(d*x + c) + 1)^17
```

```
- 1664)/(a^2 + 9*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 36*a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 84*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
126*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 126*a^2*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10 + 84*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 36*a^2*si
n(d*x + c)^14/(cos(d*x + c) + 1)^14 + 9*a^2*sin(d*x + c)^16/(cos(d*x + c) +
 1)^16 + a^2*sin(d*x + c)^18/(cos(d*x + c) + 1)^18) - 945*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.21003, size = 255, normalized size = 1.6

$$-\frac{2240 \cos(dx+c)^9 - 8640 \cos(dx+c)^7 + 8064 \cos(dx+c)^5 + 945\,dx + 315\left(16 \cos(dx+c)^7 - 24 \cos(dx+c)^5 + \cdots\right.}{20160\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20160*(2240*cos(d*x + c)^9 - 8640*cos(d*x + c)^7 + 8064*cos(d*x + c)^5 +
 945*d*x + 315*(16*cos(d*x + c)^7 - 24*cos(d*x + c)^5 + 2*cos(d*x + c)^3 +
3*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2719, size = 312, normalized size = 1.96

$$-\frac{\frac{945\,(dx+c)}{a^2} + \frac{2\left(945 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17} + 8190 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} + 40320 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14} - 97650 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} + 147840 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} + 106470 \tan \cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20160*(945*(d*x + c)/a^2 + 2*(945*tan(1/2*d*x + 1/2*c)^17 + 8190*tan(1/2
*d*x + 1/2*c)^15 + 40320*tan(1/2*d*x + 1/2*c)^14 - 97650*tan(1/2*d*x + 1/2*
c)^13 + 147840*tan(1/2*d*x + 1/2*c)^12 + 106470*tan(1/2*d*x + 1/2*c)^11 - 1
20960*tan(1/2*d*x + 1/2*c)^10 + 330624*tan(1/2*d*x + 1/2*c)^8 - 106470*tan(
1/2*d*x + 1/2*c)^7 - 8064*tan(1/2*d*x + 1/2*c)^6 + 97650*tan(1/2*d*x + 1/2*
c)^5 + 19584*tan(1/2*d*x + 1/2*c)^4 - 8190*tan(1/2*d*x + 1/2*c)^3 + 14976*t
an(1/2*d*x + 1/2*c)^2 - 945*tan(1/2*d*x + 1/2*c) + 1664)/((tan(1/2*d*x + 1/
2*c)^2 + 1)^9*a^2))/d
```

3.725 $\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=141

$$-\frac{2\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{11\sin(c+dx)\cos^5(c+dx)}{48a^2d}+\frac{11\sin(c+dx)\cos^3(c+dx)}{192a^2d}$$

```
[Out] (11*x)/(128*a^2) + (2*Cos[c + d*x]^5)/(5*a^2*d) - (2*Cos[c + d*x]^7)/(7*a^2
*d) + (11*Cos[c + d*x]*Sin[c + d*x])/(128*a^2*d) + (11*Cos[c + d*x]^3*Sin[c
 + d*x])/(192*a^2*d) - (11*Cos[c + d*x]^5*Sin[c + d*x])/(48*a^2*d) - (Cos[c
 + d*x]^5*Sin[c + d*x]^3)/(8*a^2*d)
```

Rubi [A] time = 0.374303, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2568, 2635, 8, 2565, 14}

$$-\frac{2\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{11\sin(c+dx)\cos^5(c+dx)}{48a^2d}+\frac{11\sin(c+dx)\cos^3(c+dx)}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (11*x)/(128*a^2) + (2*Cos[c + d*x]^5)/(5*a^2*d) - (2*Cos[c + d*x]^7)/(7*a^2
*d) + (11*Cos[c + d*x]*Sin[c + d*x])/(128*a^2*d) + (11*Cos[c + d*x]^3*Sin[c
 + d*x])/(192*a^2*d) - (11*Cos[c + d*x]^5*Sin[c + d*x])/(48*a^2*d) - (Cos[c
 + d*x]^5*Sin[c + d*x]^3)/(8*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^2(c+dx)-2a^2\cos^4(c+dx)\sin^3(c+dx)+a^2\cos^4(c+dx)\sin\right.}{a^4}$$
$$= \frac{\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a^2}+\frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2}-\frac{2\int \cos^4(c+dx)}{a}$$
$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{6a^2d}-\frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d}+\frac{\int \cos^4(c+dx)\,dx}{6a^2}+\frac{3\int c}{}$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{24a^2d}-\frac{11\cos^5(c+dx)\sin(c+dx)}{48a^2d}-\frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d}$$
$$= \frac{2\cos^5(c+dx)}{5a^2d}-\frac{2\cos^7(c+dx)}{7a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{16a^2d}+\frac{11\cos^3(c+dx)\sin(c+}{192a^2d}$$
$$= \frac{x}{16a^2}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{2\cos^7(c+dx)}{7a^2d}+\frac{11\cos(c+dx)\sin(c+dx)}{128a^2d}+\frac{11\cos^3(c+}{19}$$
$$= \frac{11x}{128a^2}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{2\cos^7(c+dx)}{7a^2d}+\frac{11\cos(c+dx)\sin(c+dx)}{128a^2d}+\frac{11\cos^3(c}{}$$

Mathematica [B] time = 4.1311, size = 481, normalized size = 3.41

$$\frac{18480dx\sin\left(\frac{c}{2}\right)-10080\sin\left(\frac{c}{2}+dx\right)+10080\sin\left(\frac{3c}{2}+dx\right)+1680\sin\left(\frac{3c}{2}+2dx\right)+1680\sin\left(\frac{5c}{2}+2dx\right)-3360\sin\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (9240*(15*c + 2*d*x)*Cos[c/2] + 10080*Cos[c/2 + d*x] + 10080*Cos[(3*c)/2 +
d*x] + 1680*Cos[(3*c)/2 + 2*d*x] - 1680*Cos[(5*c)/2 + 2*d*x] + 3360*Cos[(5*
c)/2 + 3*d*x] + 3360*Cos[(7*c)/2 + 3*d*x] - 2520*Cos[(7*c)/2 + 4*d*x] + 252
0*Cos[(9*c)/2 + 4*d*x] - 672*Cos[(9*c)/2 + 5*d*x] - 672*Cos[(11*c)/2 + 5*d*
x] - 560*Cos[(11*c)/2 + 6*d*x] + 560*Cos[(13*c)/2 + 6*d*x] - 480*Cos[(13*c)
/2 + 7*d*x] - 480*Cos[(15*c)/2 + 7*d*x] + 105*Cos[(15*c)/2 + 8*d*x] - 105*C
```

```
os[(17*c)/2 + 8*d*x] - 79800*Sin[c/2] + 138600*c*Sin[c/2] + 18480*d*x*Sin[c
/2] - 10080*Sin[c/2 + d*x] + 10080*Sin[(3*c)/2 + d*x] + 1680*Sin[(3*c)/2 +
2*d*x] + 1680*Sin[(5*c)/2 + 2*d*x] - 3360*Sin[(5*c)/2 + 3*d*x] + 3360*Sin[(
7*c)/2 + 3*d*x] - 2520*Sin[(7*c)/2 + 4*d*x] - 2520*Sin[(9*c)/2 + 4*d*x] + 6
72*Sin[(9*c)/2 + 5*d*x] - 672*Sin[(11*c)/2 + 5*d*x] - 560*Sin[(11*c)/2 + 6*
d*x] - 560*Sin[(13*c)/2 + 6*d*x] + 480*Sin[(13*c)/2 + 7*d*x] - 480*Sin[(15*
c)/2 + 7*d*x] + 105*Sin[(15*c)/2 + 8*d*x] + 105*Sin[(17*c)/2 + 8*d*x])/(215
040*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.097, size = 483, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 8/35/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8-11/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
8*tan(1/2*d*x+1/2*c)+64/35/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2
*c)^2+259/192/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3-8/5/d/a
^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4+1103/192/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5+64/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
8*tan(1/2*d*x+1/2*c)^6-2261/192/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^7+8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^8+2261/192
/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9-1103/192/d/a^2/(1+ta
n(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11+8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^12-259/192/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*
d*x+1/2*c)^13+11/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15+
11/64/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.64853, size = 647, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6720*((1155*sin(d*x + c)/(cos(d*x + c) + 1) - 12288*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 9065*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10752*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 38605*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8
6016*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 79135*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 - 53760*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 79135*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 + 38605*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 5376
0*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 9065*sin(d*x + c)^13/(cos(d*x + c
) + 1)^13 - 1155*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 1536)/(a^2 + 8*a^2
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^2*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 1155*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.12257, size = 230, normalized size = 1.63

$$-\frac{3840\cos(dx+c)^7-5376\cos(dx+c)^5-1155\,dx-35\left(48\cos(dx+c)^7-136\cos(dx+c)^5+22\cos(dx+c)^3+3\right.}{13440\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/13440*(3840*cos(d*x + c)^7 - 5376*cos(d*x + c)^5 - 1155*d*x - 35*(48*cos
(d*x + c)^7 - 136*cos(d*x + c)^5 + 22*cos(d*x + c)^3 + 33*cos(d*x + c))*sin
(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26968, size = 277, normalized size = 1.96

$$\frac{1155\,(dx+c)}{a^2}+\frac{2\left(1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}-9065\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+53760\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-38605\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+79135\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+53760\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(1155*(d*x + c)/a^2 + 2*(1155*tan(1/2*d*x + 1/2*c)^15 - 9065*tan(1/
2*d*x + 1/2*c)^13 + 53760*tan(1/2*d*x + 1/2*c)^12 - 38605*tan(1/2*d*x + 1/2
*c)^11 + 79135*tan(1/2*d*x + 1/2*c)^9 + 53760*tan(1/2*d*x + 1/2*c)^8 - 7913
5*tan(1/2*d*x + 1/2*c)^7 + 86016*tan(1/2*d*x + 1/2*c)^6 + 38605*tan(1/2*d*x
 + 1/2*c)^5 - 10752*tan(1/2*d*x + 1/2*c)^4 + 9065*tan(1/2*d*x + 1/2*c)^3 +
12288*tan(1/2*d*x + 1/2*c)^2 - 1155*tan(1/2*d*x + 1/2*c) + 1536)/((tan(1/2*
d*x + 1/2*c)^2 + 1)^8*a^2))/d
```

3.726 $\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{2\cos^7(c+dx)}{35a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{12a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{x}{8a^2}-\frac{\cos^9(c+d}{5d(a\sin(c+dx}$$

```
[Out] -x/(8*a^2) - (2*Cos[c + d*x]^7)/(35*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(8
*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(12*a^2*d) - (Cos[c + d*x]^5*Sin[c
+ d*x])/(15*a^2*d) - Cos[c + d*x]^9/(5*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.137529, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2859, 2682, 2635, 8}

$$-\frac{2\cos^7(c+dx)}{35a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{12a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{x}{8a^2}-\frac{\cos^9(c+d}{5d(a\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(8*a^2) - (2*Cos[c + d*x]^7)/(35*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(8
*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(12*a^2*d) - (Cos[c + d*x]^5*Sin[c
+ d*x])/(15*a^2*d) - Cos[c + d*x]^9/(5*d*(a + a*Sin[c + d*x])^2)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}dx = -\frac{\cos^9(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\int \frac{\cos^8(c+dx)}{a+a\sin(c+dx)}dx}{5a}$$
$$= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos^9(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\int \cos^6(c+dx)\,dx}{5a^2}$$
$$= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{15a^2d} - \frac{\cos^9(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{\int \cos^4(c+d}{3a^2}$$
$$= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{12a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{15a^2d} - \frac{\cos^9(c}{5d(a+a\sin}$$
$$= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{12a^2d} - \frac{\cos^5(c+dx}{15}$$
$$= -\frac{x}{8a^2} - \frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{12a^2d} - \cos^5($$

Mathematica [B] time = 4.86483, size = 418, normalized size = 3.37

$$-\frac{1680dx\sin\left(\frac{c}{2}\right) - 1155\sin\left(\frac{c}{2}+dx\right) + 1155\sin\left(\frac{3c}{2}+dx\right) + 210\sin\left(\frac{3c}{2}+2dx\right) + 210\sin\left(\frac{5c}{2}+2dx\right) - 525\sin\left(\frac{5c}{2}+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(70*(7 + 24*d*x)*Cos[c/2] + 1155*Cos[c/2 + d*x] + 1155*Cos[(3*c)/2 + d*x]
+ 210*Cos[(3*c)/2 + 2*d*x] - 210*Cos[(5*c)/2 + 2*d*x] + 525*Cos[(5*c)/2 + 3
*d*x] + 525*Cos[(7*c)/2 + 3*d*x] - 210*Cos[(7*c)/2 + 4*d*x] + 210*Cos[(9*c)
/2 + 4*d*x] + 63*Cos[(9*c)/2 + 5*d*x] + 63*Cos[(11*c)/2 + 5*d*x] - 70*Cos[(
11*c)/2 + 6*d*x] + 70*Cos[(13*c)/2 + 6*d*x] - 15*Cos[(13*c)/2 + 7*d*x] - 15
*Cos[(15*c)/2 + 7*d*x] - 490*Sin[c/2] + 1680*d*x*Sin[c/2] - 1155*Sin[c/2 +
d*x] + 1155*Sin[(3*c)/2 + d*x] + 210*Sin[(3*c)/2 + 2*d*x] + 210*Sin[(5*c)/2
 + 2*d*x] - 525*Sin[(5*c)/2 + 3*d*x] + 525*Sin[(7*c)/2 + 3*d*x] - 210*Sin[(
7*c)/2 + 4*d*x] - 210*Sin[(9*c)/2 + 4*d*x] - 63*Sin[(9*c)/2 + 5*d*x] + 63*S
in[(11*c)/2 + 5*d*x] - 70*Sin[(11*c)/2 + 6*d*x] - 70*Sin[(13*c)/2 + 6*d*x]
+ 15*Sin[(13*c)/2 + 7*d*x] - 15*Sin[(15*c)/2 + 7*d*x])/(13440*a^2*d*(Cos[c/
2] + Sin[c/2]))
```

Maple [B] time = 0.089, size = 449, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13-2/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12+11/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^7*tan(1/2*d*x+1/2*c)^11-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1
/2*c)^10-31/12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^9-2/d/a^
2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8-16/d/a^2/(1+tan(1/2*d*x+1
/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6+31/12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(
1/2*d*x+1/2*c)^5-14/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4
```

```
-11/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3-8/5/d/a^2/(1+ta
n(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2+1/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^7*tan(1/2*d*x+1/2*c)-18/35/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^7-1/4/d/a^2*ar
ctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.67944, size = 589, normalized size = 4.75

$$\frac{\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{672 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{1540 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{1176 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{1085 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{6720 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{840 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{1085 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{3360 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{1540 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2 + \frac{7 a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{21 a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{35 a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{35 a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{21 a^2 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{7 a^2 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{a^2 \sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}}{420\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/420*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 672*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1176*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 1085*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6720*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 840*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
- 1085*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3360*sin(d*x + c)^10/(cos(d*x
+ c) + 1)^10 + 1540*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 840*sin(d*x + c
)^12/(cos(d*x + c) + 1)^12 - 105*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 21
6)/(a^2 + 7*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 21*a^2*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 35*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 35*a^2
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a^2*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 + 7*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + a^2*sin(d*x + c)^1
4/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)
/d
```

Fricas [A] time = 1.15135, size = 189, normalized size = 1.52

$$\frac{120 \cos(dx+c)^7 - 336 \cos(dx+c)^5 - 105\, dx + 35 \left(8 \cos(dx+c)^5 - 2 \cos(dx+c)^3 - 3 \cos(dx+c)\right) \sin(dx+c)}{840\, a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*cos(d*x + c)^7 - 336*cos(d*x + c)^5 - 105*d*x + 35*(8*cos(d*x +
c)^5 - 2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28466, size = 259, normalized size = 2.09

$$-\frac{\frac{105\,(dx+c)}{a^2}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+840\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-1540\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1085\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+840\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+\cdots\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7 a^2}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(105*(d*x + c)/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 840*tan(1/2*d*
x + 1/2*c)^12 - 1540*tan(1/2*d*x + 1/2*c)^11 + 3360*tan(1/2*d*x + 1/2*c)^10
 + 1085*tan(1/2*d*x + 1/2*c)^9 + 840*tan(1/2*d*x + 1/2*c)^8 + 6720*tan(1/2*
d*x + 1/2*c)^6 - 1085*tan(1/2*d*x + 1/2*c)^5 + 1176*tan(1/2*d*x + 1/2*c)^4
+ 1540*tan(1/2*d*x + 1/2*c)^3 + 672*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*
x + 1/2*c) + 216)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*a^2))/d
```

3.727 $\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{2a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{\tanh^{-1}(\cos(c+d$$

```
[Out] (-3*x)/(4*a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a^2*d) + Cos
[c + d*x]^3/(3*a^2*d) - Cos[c + d*x]^5/(5*a^2*d) - (3*Cos[c + d*x]*Sin[c +
d*x])/(4*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(2*a^2*d)
```

Rubi [A] time = 0.236758, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2875, 2873, 2635, 8, 2592, 302, 206, 2565, 30}

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{2a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{\tanh^{-1}(\cos(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(4*a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a^2*d) + Cos
[c + d*x]^3/(3*a^2*d) - Cos[c + d*x]^5/(5*a^2*d) - (3*Cos[c + d*x]*Sin[c +
d*x])/(4*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(2*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cos^3(c+dx)\cot(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(-2a^2\cos^4(c+dx)+a^2\cos^3(c+dx)\cot(c+dx)+a^2\cos^4(c+dx)\sin(c+dx)\right) d\text{:}}{a^4}\\ &= \frac{\int \cos^3(c+dx)\cot(c+dx)\,dx}{a^2}+\frac{\int \cos^4(c+dx)\sin(c+dx)\,dx}{a^2}-\frac{2\int \cos^4(c+dx)\,dx}{a^2}\\ &= -\frac{\cos^3(c+dx)\sin(c+dx)}{2a^2d}-\frac{3\int \cos^2(c+dx)\,dx}{2a^2}-\frac{\text{Subst}\left(\int x^4\,dx,x,\cos(c+dx)\right)}{a^2d}-\\ &= -\frac{\cos^5(c+dx)}{5a^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{4a^2d}-\frac{\cos^3(c+dx)\sin(c+dx)}{2a^2d}-\frac{3\int 1\,dx}{4a^2}-\\ &= -\frac{3x}{4a^2}+\frac{\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos^5(c+dx)}{5a^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{4a^2d}-\\ &= -\frac{3x}{4a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos^5(c+dx)}{5a^2d}-\frac{3\cos(}{}\end{aligned}$$

Mathematica [A] time = 0.701307, size = 93, normalized size = 0.78

$$\frac{270\cos(c+dx)+5\cos(3(c+dx))-3\left(40\sin(2(c+dx))+5\sin(4(c+dx))+\cos(5(c+dx))-80\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{240a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (270*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - 3*(60*c + 60*d*x + Cos[5*(c + d*x)
] + 80*Log[Cos[(c + d*x)/2]] - 80*Log[Sin[(c + d*x)/2]] + 40*Sin[2*(c + d*x
)] + 5*Sin[4*(c + d*x)]))/(240*a^2*d)
```

Maple [B] time = 0.132, size = 329, normalized size = 2.8

$$\frac{5}{2\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5}+2\,\frac{(\tan(1/2\,dx+c/2))^8}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^5}+\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 5/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+2/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*t
an(1/2*d*x+1/2*c)^7+12/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^
6+32/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4-1/d/a^2/(1+tan
(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+28/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^5*tan(1/2*d*x+1/2*c)^2-5/2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+
1/2*c)+34/15/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^5-3/2/d/a^2*arctan(tan(1/2*d*x+
1/2*c))+1/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.56378, size = 450, normalized size = 3.78

$$-\frac{\frac{\frac{75\sin(dx+c)}{\cos(dx+c)+1}-\frac{280\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{320\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{360\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{30\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{60\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{75\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-68}{a^2+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*((75*sin(d*x + c)/(cos(d*x + c) + 1) - 280*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 320*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 360*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 30*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 - 60*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 75*sin
(d*x + c)^9/(cos(d*x + c) + 1)^9 - 68)/(a^2 + 5*a^2*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^2*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a
^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^2 - 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.15343, size = 274, normalized size = 2.3

$$-\frac{12\cos(dx+c)^5-20\cos(dx+c)^3+45\,dx+15\left(2\cos(dx+c)^3+3\cos(dx+c)\right)\sin(dx+c)-60\cos(dx+c)+30\,\mathrm{l}}{60\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 45*d*x + 15*(2*cos(d*x + c)^
3 + 3*cos(d*x + c))*sin(d*x + c) - 60*cos(d*x + c) + 30*log(1/2*cos(d*x + c
) + 1/2) - 30*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.312, size = 211, normalized size = 1.77

$$-\frac{\frac{45\,(dx+c)}{a^2}-\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{2\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-\cdots\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(45*(d*x + c)/a^2 - 60*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 2*(75*tan
(1/2*d*x + 1/2*c)^9 + 60*tan(1/2*d*x + 1/2*c)^8 + 30*tan(1/2*d*x + 1/2*c)^7
 + 360*tan(1/2*d*x + 1/2*c)^6 + 320*tan(1/2*d*x + 1/2*c)^4 - 30*tan(1/2*d*x
 + 1/2*c)^3 + 280*tan(1/2*d*x + 1/2*c)^2 - 75*tan(1/2*d*x + 1/2*c) + 68)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^2))/d
```

3.728 $\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{2\tanh^{-1}(\cos(c+$$

```
[Out] (-9*x)/(8*a^2) + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - (2*Cos[c + d*x])/(a^2*
d) - (2*Cos[c + d*x]^3)/(3*a^2*d) - Cot[c + d*x]/(a^2*d) + (Cos[c + d*x]*Si
n[c + d*x])/(8*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^2*d)
```

Rubi [A] time = 0.301899, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2872, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{2\tanh^{-1}(\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-9*x)/(8*a^2) + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - (2*Cos[c + d*x])/(a^2*
d) - (2*Cos[c + d*x]^3)/(3*a^2*d) - Cot[c + d*x]/(a^2*d) + (Cos[c + d*x]*Si
n[c + d*x])/(8*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^2(c+dx)\cot^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(-a^6-2a^6\csc(c+dx)+a^6\csc^2(c+dx)+4a^6\sin(c+dx)-a^6\sin^2(c+dx)-2a\right.}{a^8}$$
$$= -\frac{x}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{\int \sin^2(c+dx)\,dx}{a^2}+\frac{\int \sin^4(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx}{a^2}$$
$$= -\frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{4\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{\cos(c+}{}$$
$$= -\frac{3x}{2a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+$$
$$= -\frac{9x}{8a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+$$

Mathematica [A] time = 1.50911, size = 128, normalized size = 1.1

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-108(c+dx)+3\sin(4(c+dx))-240\cos(c+dx)-16\cos(3(c+dx))+48\tan\right.}{96d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-108*(c + d*x) - 240*Cos[c + d*x]
 - 16*Cos[3*(c + d*x)] - 48*Cot[(c + d*x)/2] + 192*Log[Cos[(c + d*x)/2]] -
192*Log[Sin[(c + d*x)/2]] + 3*Sin[4*(c + d*x)] + 48*Tan[(c + d*x)/2]))/(96*
d*(a + a*Sin[c + d*x])^2)
```

Maple [B] time = 0.148, size = 333, normalized size = 2.9

$$\frac{1}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}-8\frac{(\tan(1/2\,dx+c/2))^6}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^4}+\frac{7}{4da^2}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d
*x+1/2*c)^7-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6+7/4/d/a
^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-16/d/a^2/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4-7/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)^3-40/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+
1/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-16/3/d/a^2/(1+tan(1
/2*d*x+1/2*c)^2)^4-9/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/2/d/a^2/tan(1/2*d
*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.61814, size = 470, normalized size = 4.05

$$-\frac{\frac{\frac{64\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{160\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{57\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{192\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{96\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+6}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4\,a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{27\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*((64*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 160*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 57*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 192*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 96*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 + 6)/(a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 4
*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a^2*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 + 4*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^2*sin(d*x + c)^9/
(cos(d*x + c) + 1)^9) + 27*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 24
*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 6*sin(d*x + c)/(a^2*(cos(d*x +
c) + 1)))/d
```

Fricas [A] time = 1.2009, size = 323, normalized size = 2.78

$$-\frac{6\,\cos(dx+c)^5-9\,\cos(dx+c)^3+\left(16\,\cos(dx+c)^3+27\,dx+48\,\cos(dx+c)\right)\sin(dx+c)-24\,\log\left(\frac{1}{2}\,\cos(dx+c)+\right.}{24\,a^2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*cos(d*x + c)^5 - 9*cos(d*x + c)^3 + (16*cos(d*x + c)^3 + 27*d*x +
48*cos(d*x + c))*sin(d*x + c) - 24*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 + 24*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 27*cos(d*x + c))/(a^2*d*s
in(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31685, size = 251, normalized size = 2.16

$$-\frac{\frac{27\,(dx+c)}{a^2}+\frac{48\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{12\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)/a^2 + 48*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 12*tan(1/
2*d*x + 1/2*c)/a^2 - 12*(4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2
*c)) + 2*(3*tan(1/2*d*x + 1/2*c)^7 + 96*tan(1/2*d*x + 1/2*c)^6 - 21*tan(1/2
*d*x + 1/2*c)^5 + 192*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*x + 1/2*c)^3 +
160*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 64)/((tan(1/2*d*x + 1
/2*c)^2 + 1)^4*a^2))/d
```

3.729 $\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{\cos^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{3x}{a^2}$$

```
[Out] (3*x)/a^2 + ArcTanh[Cos[c + d*x]]/(2*a^2*d) + Cos[c + d*x]^3/(3*a^2*d) + (2
*Cot[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + (Cos[c + d
*x]*Sin[c + d*x])/(a^2*d)
```

Rubi [A] time = 0.252508, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{\cos^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{3x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```



```
[Out] (3*x)/a^2 + ArcTanh[Cos[c + d*x]]/(2*a^2*d) + Cos[c + d*x]^3/(3*a^2*d) + (2
*Cot[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + (Cos[c + d
*x]*Sin[c + d*x])/(a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos(c+dx)\cot^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(4a^6 - a^6\csc(c+dx) - 2a^6\csc^2(c+dx) + a^6\csc^3(c+dx) - a^6\sin(c+dx) - 2a^6 \ldots\right.}{a^8}$$
$$= \frac{4x}{a^2} - \frac{\int \csc(c+dx)\,dx}{a^2} + \frac{\int \csc^3(c+dx)\,dx}{a^2} - \frac{\int \sin(c+dx)\,dx}{a^2} + \frac{\int \sin^3(c+dx)\,dx}{a^2} \ldots$$
$$= \frac{4x}{a^2} + \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cos(c+dx)\ldots}{a^2 \ldots}$$
$$= \frac{3x}{a^2} + \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{\cos^3(c+dx)}{3a^2d} + \frac{2\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+d \ldots}{2a^2d}$$

Mathematica [A] time = 2.04388, size = 158, normalized size = 1.63

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(6\cos(c+dx)+2\cos(3(c+dx))+3\left(4\sin(2(c+dx))-8\tan\left(\frac{1}{2}(c+dx)\right)+8\,\mathrm{co}\ldots\right.\right.}{24a^2d(\sin(\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(6*Cos[c + d*x] + 2*Cos[3*(c + d*x
)] + 3*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 + 4*Log[Cos
[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 + 4*Sin[2*(c
+ d*x)] - 8*Tan[(c + d*x)/2])))/(24*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [B] time = 0.152, size = 234, normalized size = 2.4

$$\frac{1}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{(\tan(1/2\,dx+c/2))^5}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+2\,\frac{(\tan(1/2\,dx+c/2))^4}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)-2/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*t
an(1/2*d*x+1/2*c)^4+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+2
/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3+6/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/8/
d/a^2/tan(1/2*d*x+1/2*c)^2+1/d/a^2/tan(1/2*d*x+1/2*c)-1/2/d/a^2*ln(tan(1/2*
d*x+1/2*c))
```

Maxima [B] time = 1.62956, size = 446, normalized size = 4.6

$$\frac{\frac{\frac{24\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{72\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{45\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{24\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-3}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{3\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2}+\frac{144\arctan}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((24*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 72*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 45*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 24*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3)/(a^2*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 3*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8) - 3*(8*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)/a^2 + 144*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 12
*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.22781, size = 383, normalized size = 3.95

$$\frac{4\cos(dx+c)^5+36\,dx\cos(dx+c)^2-4\cos(dx+c)^3-36\,dx+3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(\right.}{12\left(a^2d\cos(dx+c)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(4*cos(d*x + c)^5 + 36*d*x*cos(d*x + c)^2 - 4*cos(d*x + c)^3 - 36*d*x
+ 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 -
1)*log(-1/2*cos(d*x + c) + 1/2) + 12*(cos(d*x + c)^3 - 3*cos(d*x + c))*sin(
```

```
d*x + c) + 6*cos(d*x + c))/(a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33612, size = 227, normalized size = 2.34

$$\frac{\frac{72\,(dx+c)}{a^2} - \frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} + \frac{3\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4} + \frac{3\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2} - \frac{16\left(3\,\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(72*(d*x + c)/a^2 - 12*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 3*(a^2*tan
(1/2*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 + 3*(6*tan(1/2*d*x +
1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)^2) - 16*(3
*tan(1/2*d*x + 1/2*c)^5 - 3*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x + 1/2*c)
 - 1)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2))/d
```

3.730 $\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{2\cos(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{2a^2}$$

```
[Out] -x/(2*a^2) - (3*ArcTanh[Cos[c + d*x]])/(a^2*d) + (2*Cos[c + d*x])/(a^2*d) -
 Cot[c + d*x]^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d) - (Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*d)
```

Rubi [A] time = 0.243023, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2709, 3770, 3767, 8, 3768, 2638, 2635}

$$\frac{2\cos(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -x/(2*a^2) - (3*ArcTanh[Cos[c + d*x]])/(a^2*d) + (2*Cos[c + d*x])/(a^2*d) -
 Cot[c + d*x]^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d) - (Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(-a^6+4a^6\csc(c+dx)-a^6\csc^2(c+dx)-2a^6\csc^3(c+dx)+a^6\csc^4(c+dx)-2\right.}{a^8}$$
$$= -\frac{x}{a^2}-\frac{\int \csc^2(c+dx)\,dx}{a^2}+\frac{\int \csc^4(c+dx)\,dx}{a^2}+\frac{\int \sin^2(c+dx)\,dx}{a^2}-\frac{2\int \csc^3(c+d}{a^2}$$
$$= -\frac{x}{a^2}-\frac{4\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{\cos(c+}{}$$
$$= -\frac{x}{2a^2}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c}{a^2d}$$

Mathematica [A] time = 2.36625, size = 184, normalized size = 1.9

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)^4\sec^2\left(\frac{1}{2}(c+dx)\right)\left(30\cos(c+dx)-\cos(3(c+dx))+3\left(\cos(5(c+dx))+8\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((1 + Cot[(c + d*x)/2])^4*Sec[(c + d*x)/2]^2*(30*Cos[c + d*x] - Cos[3*(c +
 d*x)] + 3*(Cos[5*(c + d*x)] + 8*(c + d*x - 6*Cos[c + d*x] + 2*Cos[3*(c + d
*x)] + 6*Log[Cos[(c + d*x)/2]] - Cos[2*(c + d*x)]*(c + d*x + 6*Log[Cos[(c +
 d*x)/2]] - 6*Log[Sin[(c + d*x)/2]]) - 6*Log[Sin[(c + d*x)/2]])*Sin[c + d*x
]))*Tan[(c + d*x)/2])/(768*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [B] time = 0.158, size = 272, normalized size = 2.8

$$\frac{1}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{8\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^2*tan(1/2*d*x+1/2*c)^2-1/8/d/a^2*ta
n(1/2*d*x+1/2*c)+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+4/
d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2-1/d/a^2
*arctan(tan(1/2*d*x+1/2*c))-1/24/d/a^2/tan(1/2*d*x+1/2*c)^3+1/4/d/a^2/tan(1
/2*d*x+1/2*c)^2+1/8/d/a^2/tan(1/2*d*x+1/2*c)+3/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.53807, size = 413, normalized size = 4.26

$$\frac{\frac{\frac{6 \sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{108 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{19 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{102 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{27 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}-1}{\frac{a^2 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2 a^2 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^2 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}-\frac{\frac{3 \sin(dx+c)}{\cos(dx+c)+1}+\frac{6 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24 \arctan\left(\frac{\sin}{\cos}\right.}{a^2}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((6*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 108*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 19*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 102*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 27*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 1)/(a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*a^2*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7) - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/a^2 + 72*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.30188, size = 431, normalized size = 4.44

$$\frac{3 \cos(dx+c)^5 - 4 \cos(dx+c)^3 - 9\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + 9\left(\cos(dx+c)^2-1\right)\log\left(\right.}{6\left(a^2 d \cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*cos(d*x + c)^5 - 4*cos(d*x + c)^3 - 9*(cos(d*x + c)^2 - 1)*log(1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) + 9*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 3*(d*x*cos(d*x + c)^2 - 4*cos(d*x + c)^3 - d*x +
6*cos(d*x + c))*sin(d*x + c) + 3*cos(d*x + c))/((a^2*d*cos(d*x + c)^2 - a^2
*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32628, size = 262, normalized size = 2.7

$$-\frac{\frac{12\,(dx+c)}{a^2}-\frac{72\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{24\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{132\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,\mathrm{ta}}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)/a^2 - 72*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 24*(tan(1
/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 4)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (132*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/
2*d*x + 1/2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/2*c)^3)
 - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^4*tan(1
/2*d*x + 1/2*c))/a^6)/d
```

3.731 $\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{\cos(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{9\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc(c+}{8a^2d}$$

```
[Out] (-2*x)/a^2 + (9*ArcTanh[Cos[c + d*x]])/(8*a^2*d) - Cos[c + d*x]/(a^2*d) - (
2*Cot[c + d*x])/(a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) + (Cot[c + d*x]*Csc[
c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rubi [A] time = 0.281237, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2638}

$$-\frac{\cos(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{9\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc(c+}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-2*x)/a^2 + (9*ArcTanh[Cos[c + d*x]])/(8*a^2*d) - Cos[c + d*x]/(a^2*d) - (
2*Cot[c + d*x])/(a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) + (Cot[c + d*x]*Csc[
c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(-2a^6 - a^6\csc(c+dx) + 4a^6\csc^2(c+dx) - a^6\csc^3(c+dx) - 2a^6\csc^4(c+dx) + \right.}{a^8}$$

$$= -\frac{2x}{a^2} - \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{\int \csc^3(c+dx)\,dx}{a^2} + \frac{\int \csc^5(c+dx)\,dx}{a^2} + \frac{\int \sin(c+dx)\,d}{a^2}$$

$$= -\frac{2x}{a^2} + \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx}{4}$$

$$= -\frac{2x}{a^2} + \frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{\cos(c+dx)}{a^2d} - \frac{2\cot(c+dx)}{a^2d} + \frac{2\cot^3(c+dx)}{3a^2d} + \frac{\mathrm{cc}}{}$$

$$= -\frac{2x}{a^2} + \frac{9\tanh^{-1}(\cos(c+dx))}{8a^2d} - \frac{\cos(c+dx)}{a^2d} - \frac{2\cot(c+dx)}{a^2d} + \frac{2\cot^3(c+dx)}{3a^2d} + \frac{\mathrm{cc}}{}$$

Mathematica [A] time = 1.83727, size = 219, normalized size = 1.89

$$-\frac{\sin^5(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^4\left(192\cot(c+dx)+(3\csc(c+dx)-8)\csc^4\left(\frac{1}{2}(c+dx)\right)+(128-6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^4*(192*Cot[c + d*x] + Csc[(c + d*x)
/2]^2*(128 - 6*Csc[c + d*x]) + Csc[(c + d*x)/2]^4*(-8 + 3*Csc[c + d*x]) + 8
*(3*Csc[c + d*x]*(16*(c + d*x) - 9*Log[Cos[(c + d*x)/2]] + 9*Log[Sin[(c + d
*x)/2]]) - (7 + 8*Cos[c + d*x])*Sec[(c + d*x)/2]^4 + 3*Csc[c + d*x]^3*Sin[(
c + d*x)/2]^2 - 6*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4))*Sin[c + d*x]^5)/(3072
*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.167, size = 173, normalized size = 1.5

$$\frac{1}{64\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 - \frac{1}{12\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{5}{4\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 2\,\frac{1}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)} - 4\,\frac{\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-1/12/d/a^2*tan(1/2*d*x+1/2*c)^3+5/4/d/a^2*t
an(1/2*d*x+1/2*c)-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*arctan(tan(1/2*d
*x+1/2*c))-1/64/d/a^2/tan(1/2*d*x+1/2*c)^4+1/12/d/a^2/tan(1/2*d*x+1/2*c)^3-
5/4/d/a^2/tan(1/2*d*x+1/2*c)-9/8/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.54064, size = 355, normalized size = 3.06

$$\frac{\frac{\frac{16 \sin(dx+c)}{\cos(dx+c)+1} - \frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{224 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{384 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{240 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - 3}{\frac{a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{\frac{240 \sin(dx+c)}{\cos(dx+c)+1} - \frac{16 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^2} - \frac{768 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} - \cdots}{192\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/192*((16*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 224*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 384*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 240*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3)/(a^2*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) +
(240*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/a^2 - 768*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/a^2 - 216*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.14847, size = 513, normalized size = 4.42

$$-\frac{96\, dx \cos(dx+c)^4 + 48 \cos(dx+c)^5 - 192\, dx \cos(dx+c)^2 - 90 \cos(dx+c)^3 + 96\, dx - 27 \left(\cos(dx+c)^4 - 2 \cos(dx \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(96*d*x*cos(d*x + c)^4 + 48*cos(d*x + c)^5 - 192*d*x*cos(d*x + c)^2 -
 90*cos(d*x + c)^3 + 96*d*x - 27*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*lo
g(1/2*cos(d*x + c) + 1/2) + 27*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(
-1/2*cos(d*x + c) + 1/2) - 32*(4*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x +
 c) + 54*cos(d*x + c))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29527, size = 215, normalized size = 1.85

$$-\frac{\frac{384\,(dx+c)}{a^2}+\frac{216\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{384}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}-\frac{450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}-\frac{3\,a^6\tan\left(\frac{1}{2}\right.}{}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/192*(384*(d*x + c)/a^2 + 216*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 384/((
tan(1/2*d*x + 1/2*c)^2 + 1)*a^2) - (450*tan(1/2*d*x + 1/2*c)^4 - 240*tan(1/
2*d*x + 1/2*c)^3 + 16*tan(1/2*d*x + 1/2*c) - 3)/(a^2*tan(1/2*d*x + 1/2*c)^4
) - (3*a^6*tan(1/2*d*x + 1/2*c)^4 - 16*a^6*tan(1/2*d*x + 1/2*c)^3 + 240*a^6
*tan(1/2*d*x + 1/2*c))/a^8)/d
```

3.732 $\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=118

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot^3(c+dx)\csc(c+dx)}{2a^2d}-\frac{3\cot(c+dx)\csc(c+}{4a^2d}$$

```
[Out] x/a^2 + (3*ArcTanh[Cos[c + d*x]])/(4*a^2*d) + Cot[c + d*x]/(a^2*d) - Cot[c
+ d*x]^3/(3*a^2*d) - Cot[c + d*x]^5/(5*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x
])/(4*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(2*a^2*d)
```

Rubi [A] time = 0.321298, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 3473, 8, 2611, 3770, 2607, 30}

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot^3(c+dx)\csc(c+dx)}{2a^2d}-\frac{3\cot(c+dx)\csc(c+}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] x/a^2 + (3*ArcTanh[Cos[c + d*x]])/(4*a^2*d) + Cot[c + d*x]/(a^2*d) - Cot[c
+ d*x]^3/(3*a^2*d) - Cot[c + d*x]^5/(5*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x
])/(4*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(2*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^4(c+dx)-2a^2\cot^4(c+dx)\csc(c+dx)+a^2\cot^4(c+dx)\csc^2(c+dx)\right)\,d\ldots}{a^4}$$

$$= \frac{\int \cot^4(c+dx)\,dx}{a^2}+\frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^4(c+dx)\csc(c+dx)\ldots}{a^2}$$

$$= -\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^3(c+dx)\csc(c+dx)}{2a^2d}-\frac{\int \cot^2(c+dx)\,dx}{a^2}+\frac{3\int \cot^2(c+dx)\ldots}{2a^2\ldots}$$

$$= \frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot^3(c+\ldots}{\ldots}$$

$$= \frac{x}{a^2}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{3\cot(\ldots}{\ldots}$$

Mathematica [B] time = 1.0697, size = 254, normalized size = 2.15

$$\frac{\csc^5(c+dx)\left(600c\sin(c+dx)+600dx\sin(c+dx)-60\sin(2(c+dx))-300c\sin(3(c+dx))-300dx\sin(3(c+dx))+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-40*Cos[c + d*x] - 220*Cos[3*(c + d*x)] + 68*Cos[5*(c + d*
x)] + 600*c*Sin[c + d*x] + 600*d*x*Sin[c + d*x] + 450*Log[Cos[(c + d*x)/2]]
*Sin[c + d*x] - 450*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 60*Sin[2*(c + d*x)
] - 300*c*Sin[3*(c + d*x)] - 300*d*x*Sin[3*(c + d*x)] - 225*Log[Cos[(c + d*
x)/2]]*Sin[3*(c + d*x)] + 225*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 150*
Sin[4*(c + d*x)] + 60*c*Sin[5*(c + d*x)] + 60*d*x*Sin[5*(c + d*x)] + 45*Log
[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 45*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d
*x)]))/(960*a^2*d)
```

Maple [B] time = 0.163, size = 226, normalized size = 1.9

$$\frac{1}{160\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{32\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+\frac{1}{96\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{9}{16\,da^2}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/160/d/a^2*tan(1/2*d*x+1/2*c)^5-1/32/d/a^2*tan(1/2*d*x+1/2*c)^4+1/96/d/a^2
*tan(1/2*d*x+1/2*c)^3+1/4/d/a^2*tan(1/2*d*x+1/2*c)^2-9/16/d/a^2*tan(1/2*d*x
+1/2*c)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-1/160/d/a^2/tan(1/2*d*x+1/2*c)^5
-1/96/d/a^2/tan(1/2*d*x+1/2*c)^3+1/32/d/a^2/tan(1/2*d*x+1/2*c)^4-1/4/d/a^2/
tan(1/2*d*x+1/2*c)^2+9/16/d/a^2/tan(1/2*d*x+1/2*c)-3/4/d/a^2*ln(tan(1/2*d*x
+1/2*c))
```

Maxima [B] time = 1.53676, size = 348, normalized size = 2.95

$$-\frac{\frac{\frac{270\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{120\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{960\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\,\sin(d}{(\cos(dx+}\right.}{}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*((270*sin(d*x + c)/(cos(d*x + c) + 1) - 120*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 960*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 360*log(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^2 - (15*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 270*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

Fricas [A] time = 1.1508, size = 567, normalized size = 4.81

$$\frac{136\,\cos(dx+c)^5-280\,\cos(dx+c)^3+45\left(\cos(dx+c)^4-2\,\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-45}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(136*cos(d*x + c)^5 - 280*cos(d*x + c)^3 + 45*(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 45*(cos(d*x + c
)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*
(4*d*x*cos(d*x + c)^4 - 8*d*x*cos(d*x + c)^2 + 5*cos(d*x + c)^3 + 4*d*x - 3
*cos(d*x + c))*sin(d*x + c) + 120*cos(d*x + c))/((a^2*d*cos(d*x + c)^4 - 2*
a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3502, size = 263, normalized size = 2.23

$$\frac{\frac{480\,(dx+c)}{a^2}-\frac{360\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{822\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+270\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}+}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(480*(d*x + c)/a^2 - 360*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (822*ta
n(1/2*d*x + 1/2*c)^5 + 270*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c
)^3 - 5*tan(1/2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c) - 3)/(a^2*tan(1/2*
d*x + 1/2*c)^5) + (3*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^8*tan(1/2*d*x + 1/2*
c)^4 + 5*a^8*tan(1/2*d*x + 1/2*c)^3 + 120*a^8*tan(1/2*d*x + 1/2*c)^2 - 270*
a^8*tan(1/2*d*x + 1/2*c))/a^10)/d
```

3.733 $\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=132

$$\frac{2\cot^5(c+dx)}{5a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}-\frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+d}{8a^2d}$$

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (5*C
ot[c + d*x]*Csc[c + d*x])/(16*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x])/(4*a^2
*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x
]^3)/(6*a^2*d)
```

Rubi [A] time = 0.34245, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2611, 3770, 2607, 30, 3768}

$$\frac{2\cot^5(c+dx)}{5a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}-\frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+d}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (5*C
ot[c + d*x]*Csc[c + d*x])/(16*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x])/(4*a^2
*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x
]^3)/(6*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc(c+dx) - 2a^2\cot^4(c+dx)\csc^2(c+dx) + a^2\cot^4(c+dx)\csc^3\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{a^2} - \frac{2\int \cot^4(c+dx)\,c}{a^2}$$
$$= -\frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d} - \frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d} - \frac{\int \cot^2(c+dx)\csc^3(c+dx}{2a^2}$$
$$= \frac{2\cot^5(c+dx)}{5a^2d} + \frac{3\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d} + \frac{\cot(c+dx}{8}$$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{5\cot(c+dx)\csc(c+dx)}{16a^2d} - \frac{\cot^3(c+dx}{4a}$$
$$= -\frac{7\tanh^{-1}(\cos(c+dx))}{16a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{5\cot(c+dx)\csc(c+dx)}{16a^2d} - \frac{\cot^3(c+dx}{4a}$$

Mathematica [A] time = 1.83826, size = 145, normalized size = 1.1

$$\frac{\csc^6(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(60(32\sin(c+dx)-11)\cos(c+dx)+6(32\sin(c+dx)+45)\cos(5(\right.}{7680a^2d(\sin(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(3360*(-Log[Cos[(c
+ d*x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^6 + 60*Cos[c + d*x]*(-11 +
 32*Sin[c + d*x]) + 6*Cos[5*(c + d*x)]*(45 + 32*Sin[c + d*x]) + 10*Cos[3*(c
 + d*x)]*(-89 + 96*Sin[c + d*x])))/(7680*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [B] time = 0.169, size = 246, normalized size = 1.9

$$\frac{1}{384\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6 - \frac{1}{80\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{1}{128\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + \frac{1}{16\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{17}{128\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/384/d/a^2*tan(1/2*d*x+1/2*c)^6-1/80/d/a^2*tan(1/2*d*x+1/2*c)^5+1/128/d/a^
2*tan(1/2*d*x+1/2*c)^4+1/16/d/a^2*tan(1/2*d*x+1/2*c)^3-17/128/d/a^2*tan(1/2
*d*x+1/2*c)^2-1/8/d/a^2*tan(1/2*d*x+1/2*c)+1/8/d/a^2/tan(1/2*d*x+1/2*c)+1/8
0/d/a^2/tan(1/2*d*x+1/2*c)^5-1/128/d/a^2/tan(1/2*d*x+1/2*c)^4+7/16/d/a^2*ln
(tan(1/2*d*x+1/2*c))-1/384/d/a^2/tan(1/2*d*x+1/2*c)^6-1/16/d/a^2/tan(1/2*d*
x+1/2*c)^3+17/128/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.06664, size = 371, normalized size = 2.81

$$-\frac{\frac{\frac{240 \sin(dx+c)}{\cos(dx+c)+1}+\frac{255 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{120 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{15 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^2}-\frac{840 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{24 \sin(dx+c)}{\cos(dx+c)+1}-\frac{15 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{120 \sin}{(\cos(dx}\right.}{}}{1920\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1920*((240*sin(d*x + c)/(cos(d*x + c) + 1) + 255*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6)/a^2 - 840*log(sin(d*x + c)/(cos(d*x + c) + 1))/
a^2 - (24*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 255*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 240*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x +
c) + 1)^6/(a^2*sin(d*x + c)^6))/d
```

Fricas [A] time = 1.14848, size = 501, normalized size = 3.8

$$-\frac{192 \cos(dx+c)^5 \sin(dx+c) + 270 \cos(dx+c)^5 - 560 \cos(dx+c)^3 + 105 \left(\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(d\right.}{480 \left(a^2 d \cos(dx+c)^6 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(192*cos(d*x + c)^5*sin(d*x + c) + 270*cos(d*x + c)^5 - 560*cos(d*x
+ c)^3 + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log
(1/2*cos(d*x + c) + 1/2) - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) + 210*cos(d*x + c))/(a^2*d*cos(
d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35636, size = 290, normalized size = 2.2

$$\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{2058 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 240 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 255 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 120 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 24 \tan\left(\frac{1}{2}\,dx+ \right.}{a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (2058*tan(1/2*d*x + 1/2*c)
^6 - 240*tan(1/2*d*x + 1/2*c)^5 - 255*tan(1/2*d*x + 1/2*c)^4 + 120*tan(1/2*
d*x + 1/2*c)^3 + 15*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 5)/(
a^2*tan(1/2*d*x + 1/2*c)^6) + (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^10*tan(
1/2*d*x + 1/2*c)^5 + 15*a^10*tan(1/2*d*x + 1/2*c)^4 + 120*a^10*tan(1/2*d*x
+ 1/2*c)^3 - 255*a^10*tan(1/2*d*x + 1/2*c)^2 - 240*a^10*tan(1/2*d*x + 1/2*c
))/a^12)/d
```

3.734 $\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{\cot(c+dx)\csc^5(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc^3(c+dx)}{12a^2d}+\frac{\cot(c$$

```
[Out] ArcTanh[Cos[c + d*x]]/(8*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*
x]^7/(7*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (7*Cot[c + d*x]*Cs
c[c + d*x]^3)/(12*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(3*a^2*d)
```

Rubi [A] time = 0.25998, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 3767, 8, 3768, 3770}

$$-\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{\cot(c+dx)\csc^5(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc^3(c+dx)}{12a^2d}+\frac{\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(8*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*
x]^7/(7*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (7*Cot[c + d*x]*Cs
c[c + d*x]^3)/(12*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(3*a^2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \left(a^6\csc^2(c+dx) - 2a^6\csc^3(c+dx) - a^6\csc^4(c+dx) + 4a^6\csc^5(c+dx) - a^6\csc^6(c+d\right.}{a^8}$$

$$= \frac{\int \csc^2(c+dx)\,dx}{a^2} - \frac{\int \csc^4(c+dx)\,dx}{a^2} - \frac{\int \csc^6(c+dx)\,dx}{a^2} + \frac{\int \csc^8(c+dx)\,dx}{a^2} - \frac{2\int \mathrm{cs}}{}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{a^2d} + \frac{\cot(c+dx)\csc^5(c+dx)}{3a^2d} - \frac{\int \csc(c}{a}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d} - 7\cot($$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{8a^2d} - 7\,\mathrm{co}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{8a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{8a^2d} - 7\cot($$

Mathematica [B] time = 1.08262, size = 251, normalized size = 2.02

$$-\frac{\csc^7(c+dx)\left(-2170\sin(2(c+dx)) - 3080\sin(4(c+dx)) - 210\sin(6(c+dx)) + 5880\cos(c+dx) + 2184\cos(3(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^7*(5880*Cos[c + d*x] + 2184*Cos[3*(c + d*x)] - 168*Cos[5*(c
+ d*x)] - 216*Cos[7*(c + d*x)] - 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] +
3675*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 2170*Sin[2*(c + d*x)] + 2205*Log[
Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c +
d*x)] - 3080*Sin[4*(c + d*x)] - 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)]
+ 735*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 210*Sin[6*(c + d*x)] + 105*L
og[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)] - 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c
+ d*x)]))/(53760*a^2*d)
```

Maple [B] time = 0.183, size = 284, normalized size = 2.3

$$\frac{1}{896\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{1}{192\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6 + \frac{3}{640\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{1}{64\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 - \frac{5}{128\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/896/d/a^2*tan(1/2*d*x+1/2*c)^7-1/192/d/a^2*tan(1/2*d*x+1/2*c)^6+3/640/d/a
^2*tan(1/2*d*x+1/2*c)^5+1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-5/128/d/a^2*tan(1/2
*d*x+1/2*c)^3+1/64/d/a^2*tan(1/2*d*x+1/2*c)^2+11/128/d/a^2*tan(1/2*d*x+1/2*
c)-1/896/d/a^2/tan(1/2*d*x+1/2*c)^7-11/128/d/a^2/tan(1/2*d*x+1/2*c)-3/640/d
/a^2/tan(1/2*d*x+1/2*c)^5-1/64/d/a^2/tan(1/2*d*x+1/2*c)^4-1/8/d/a^2*ln(tan(
1/2*d*x+1/2*c))+1/192/d/a^2/tan(1/2*d*x+1/2*c)^6+5/128/d/a^2/tan(1/2*d*x+1/
2*c)^3-1/64/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.04902, size = 424, normalized size = 3.42

$$\frac{\frac{\frac{1155\sin(dx+c)}{\cos(dx+c)+1}+\frac{210\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{525\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{210\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{70\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}-\frac{1680\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{70\sin(dx+c)}{\cos(dx+c)+1}-\frac{63\sin\ldots}{(\cos(d\ldots}\right.}{}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/13440*((1155*sin(d*x + c)/(cos(d*x + c) + 1) + 210*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 525*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 210*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 70*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^2
 - 1680*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (70*sin(d*x + c)/(cos(d*
x + c) + 1) - 63*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 210*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 525*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 210*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1155*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1
5)*(cos(d*x + c) + 1)^7/(a^2*sin(d*x + c)^7))/d
```

Fricas [A] time = 1.13392, size = 585, normalized size = 4.72

$$-\frac{432\cos(dx+c)^7-672\cos(dx+c)^5-105\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\ldots\right.}{1680\left(a^2d\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1680*(432*cos(d*x + c)^7 - 672*cos(d*x + c)^5 - 105*(cos(d*x + c)^6 - 3*
cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 70*(3*cos(d*x + c)^5 + 8*cos(d*x + c
)^3 - 3*cos(d*x + c))*sin(d*x + c))/((a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*
x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**8/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32531, size = 331, normalized size = 2.67

$$-\frac{\frac{1680\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{4356\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+525\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-63\tan\left(\frac{1}{2}dx\ldots\right.}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/13440*(1680*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (4356*tan(1/2*d*x + 1/2
*c)^7 - 1155*tan(1/2*d*x + 1/2*c)^6 - 210*tan(1/2*d*x + 1/2*c)^5 + 525*tan(
1/2*d*x + 1/2*c)^4 - 210*tan(1/2*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c)^2
 + 70*tan(1/2*d*x + 1/2*c) - 15)/(a^2*tan(1/2*d*x + 1/2*c)^7) - (15*a^12*ta
n(1/2*d*x + 1/2*c)^7 - 70*a^12*tan(1/2*d*x + 1/2*c)^6 + 63*a^12*tan(1/2*d*x
 + 1/2*c)^5 + 210*a^12*tan(1/2*d*x + 1/2*c)^4 - 525*a^12*tan(1/2*d*x + 1/2*
c)^3 + 210*a^12*tan(1/2*d*x + 1/2*c)^2 + 1155*a^12*tan(1/2*d*x + 1/2*c))/a^
14)/d
```

3.735 $\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$\frac{2\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{128a^2d}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}+\frac{\cot}{}$$

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(128*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (2
*Cot[c + d*x]^7)/(7*a^2*d) - (11*Cot[c + d*x]*Csc[c + d*x])/(128*a^2*d) + (
7*Cot[c + d*x]*Csc[c + d*x]^3)/(64*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x]^3)
/(6*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(16*a^2*d) - (Cot[c + d*x]^3*Csc
[c + d*x]^5)/(8*a^2*d)
```

Rubi [A] time = 0.407361, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2611, 3768, 3770, 2607, 14}

$$\frac{2\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{128a^2d}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}+\frac{\cot}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(128*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (2
*Cot[c + d*x]^7)/(7*a^2*d) - (11*Cot[c + d*x]*Csc[c + d*x])/(128*a^2*d) + (
7*Cot[c + d*x]*Csc[c + d*x]^3)/(64*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x]^3)
/(6*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(16*a^2*d) - (Cot[c + d*x]^3*Csc
[c + d*x]^5)/(8*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}dx = \frac{\int \cot^4(c+dx)\csc^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^3(c+dx) - 2a^2\cot^4(c+dx)\csc^4(c+dx) + a^2\cot^4(c+dx)\csc\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{a^2} - \frac{2\int \cot^4(c+dx)}{a^2}$$
$$= -\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d} - \frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d} - \frac{3\int \cot^2(c+dx)\csc^5(c+}{8a^2}$$
$$= \frac{\cot(c+dx)\csc^3(c+dx)}{8a^2d} - \frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d} + \frac{\cot(c+dx)\csc^5(c+dx)}{16a^2d} -$$
$$= \frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\cot^7(c+dx)}{7a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{16a^2d} + \frac{7\cot(c+dx)\csc^3(c+}{64a^2d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{16a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\cot^7(c+dx)}{7a^2d} - \frac{11\cot(c+dx)\csc(c+d}{128a^2d}$$
$$= -\frac{11\tanh^{-1}(\cos(c+dx))}{128a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\cot^7(c+dx)}{7a^2d} - \frac{11\cot(c+dx)\csc(c+}{128a^2d}$$

Mathematica [A] time = 0.894743, size = 291, normalized size = 1.65

$$-\frac{\csc^8(c+dx)\left(-86016\sin(2(c+dx)) - 64512\sin(4(c+dx)) - 12288\sin(6(c+dx)) + 1536\sin(8(c+dx)) + 158270\,c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^8*(158270*Cos[c + d*x] + 77210*Cos[3*(c + d*x)] - 18130*Cos[
5*(c + d*x)] - 2310*Cos[7*(c + d*x)] + 40425*Log[Cos[(c + d*x)/2]] - 64680*
Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 32340*Cos[4*(c + d*x)]*Log[Cos[(c
+ d*x)/2]] - 9240*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 1155*Cos[8*(c +
d*x)]*Log[Cos[(c + d*x)/2]] - 40425*Log[Sin[(c + d*x)/2]] + 64680*Cos[2*(c
```

```
+ d*x)]*Log[Sin[(c + d*x)/2]] - 32340*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]
] + 9240*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 1155*Cos[8*(c + d*x)]*Log
[Sin[(c + d*x)/2]] - 86016*Sin[2*(c + d*x)] - 64512*Sin[4*(c + d*x)] - 1228
8*Sin[6*(c + d*x)] + 1536*Sin[8*(c + d*x)]))/(1720320*a^2*d)
```

Maple [B] time = 0.194, size = 322, normalized size = 1.8

$$\frac{1}{2048\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8-\frac{1}{448\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{1}{384\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6+\frac{1}{320\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{3}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2048/d/a^2*tan(1/2*d*x+1/2*c)^8-1/448/d/a^2*tan(1/2*d*x+1/2*c)^7+1/384/d/
a^2*tan(1/2*d*x+1/2*c)^6+1/320/d/a^2*tan(1/2*d*x+1/2*c)^5-3/256/d/a^2*tan(1
/2*d*x+1/2*c)^4+1/64/d/a^2*tan(1/2*d*x+1/2*c)^3-1/128/d/a^2*tan(1/2*d*x+1/2
*c)^2-3/64/d/a^2*tan(1/2*d*x+1/2*c)+1/448/d/a^2/tan(1/2*d*x+1/2*c)^7+3/64/d
/a^2/tan(1/2*d*x+1/2*c)-1/2048/d/a^2/tan(1/2*d*x+1/2*c)^8-1/320/d/a^2/tan(1
/2*d*x+1/2*c)^5+3/256/d/a^2/tan(1/2*d*x+1/2*c)^4+11/128/d/a^2*ln(tan(1/2*d*
x+1/2*c))-1/384/d/a^2/tan(1/2*d*x+1/2*c)^6-1/64/d/a^2/tan(1/2*d*x+1/2*c)^3+
1/128/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.09685, size = 479, normalized size = 2.72

$$-\frac{\frac{\frac{10080\sin(dx+c)}{\cos(dx+c)+1}+\frac{1680\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3360\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{672\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{560\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{480\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{105\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}{a^2}-\frac{18480\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-}{215040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/215040*((10080*sin(d*x + c)/(cos(d*x + c) + 1) + 1680*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 3360*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2520*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 672*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 56
0*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 480*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 - 105*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/a^2 - 18480*log(sin(d*x + c
)/(cos(d*x + c) + 1))/a^2 - (480*sin(d*x + c)/(cos(d*x + c) + 1) - 560*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 - 672*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 2520*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3360*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 + 1680*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 10080*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 105)*(cos(d*x + c) + 1)^8/(a^2*sin(d*x + c)^8))/d
```

Fricas [A] time = 1.19628, size = 655, normalized size = 3.72

$$\frac{2310\cos(dx+c)^7+490\cos(dx+c)^5-8470\cos(dx+c)^3-1155\left(\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/26880*(2310*cos(d*x + c)^7 + 490*cos(d*x + c)^5 - 8470*cos(d*x + c)^3 - 1
155*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^
2 + 1)*log(1/2*cos(d*x + c) + 1/2) + 1155*(cos(d*x + c)^8 - 4*cos(d*x + c)^
6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) -
 1536*(2*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*sin(d*x + c) + 2310*cos(d*x + c
))/(a^2*d*cos(d*x + c)^8 - 4*a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4
- 4*a^2*d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**9/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37301, size = 369, normalized size = 2.1

$$\frac{\frac{18480 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^2} - \frac{50226 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8 - 10080 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7 - 1680 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6 + 3360 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 - 2520 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 +672}{a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/215040*(18480*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (50226*tan(1/2*d*x + 1
/2*c)^8 - 10080*tan(1/2*d*x + 1/2*c)^7 - 1680*tan(1/2*d*x + 1/2*c)^6 + 3360
*tan(1/2*d*x + 1/2*c)^5 - 2520*tan(1/2*d*x + 1/2*c)^4 + 672*tan(1/2*d*x + 1
/2*c)^3 + 560*tan(1/2*d*x + 1/2*c)^2 - 480*tan(1/2*d*x + 1/2*c) + 105)/(a^2
*tan(1/2*d*x + 1/2*c)^8) + (105*a^14*tan(1/2*d*x + 1/2*c)^8 - 480*a^14*tan(
1/2*d*x + 1/2*c)^7 + 560*a^14*tan(1/2*d*x + 1/2*c)^6 + 672*a^14*tan(1/2*d*x
 + 1/2*c)^5 - 2520*a^14*tan(1/2*d*x + 1/2*c)^4 + 3360*a^14*tan(1/2*d*x + 1/
2*c)^3 - 1680*a^14*tan(1/2*d*x + 1/2*c)^2 - 10080*a^14*tan(1/2*d*x + 1/2*c)
)/a^16)/d
```

3.736 $\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$-\frac{\cot^9(c+dx)}{9a^2d}-\frac{3\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{64a^2d}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d}-\frac{\cot(c+dx)\csc}{8a^2d}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(64*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (3*Co
t[c + d*x]^7)/(7*a^2*d) - Cot[c + d*x]^9/(9*a^2*d) + (3*Cot[c + d*x]*Csc[c
+ d*x])/(64*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(32*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^5)/(8*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x]^5)/(4*a^2*d)
```

Rubi [A] time = 0.386414, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 14, 2611, 3768, 3770, 270}

$$-\frac{\cot^9(c+dx)}{9a^2d}-\frac{3\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{64a^2d}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d}-\frac{\cot(c+dx)\csc}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(64*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (3*Co
t[c + d*x]^7)/(7*a^2*d) - Cot[c + d*x]^9/(9*a^2*d) + (3*Cot[c + d*x]*Csc[c
+ d*x])/(64*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(32*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^5)/(8*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x]^5)/(4*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^4(c+dx)-2a^2\cot^4(c+dx)\csc^5(c+dx)+a^2\cot^4(c+dx)\mathrm{cs}\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^4(c+dx)\,dx}{a^2}+\frac{\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2}-\frac{2\int \cot^4(c+dx)}{a}$$
$$= \frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d}+\frac{3\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{4a^2}+\frac{\mathrm{Subst}\left(\int x^4\left(1+x^{\cdot}\right.\right.}{}$$
$$= -\frac{\cot(c+dx)\csc^5(c+dx)}{8a^2d}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d}-\frac{\int \csc^5(c+dx)\,dx}{8a^2}+\frac{\mathrm{Subs}}{}$$
$$= -\frac{2\cot^5(c+dx)}{5a^2d}-\frac{3\cot^7(c+dx)}{7a^2d}-\frac{\cot^9(c+dx)}{9a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{32a^2d}-\mathrm{cot}$$
$$= -\frac{2\cot^5(c+dx)}{5a^2d}-\frac{3\cot^7(c+dx)}{7a^2d}-\frac{\cot^9(c+dx)}{9a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{64a^2d}+\mathrm{cot}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{64a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{3\cot^7(c+dx)}{7a^2d}-\frac{\cot^9(c+dx)}{9a^2d}+\frac{3\cot(c}{}$$

Mathematica [A] time = 1.79092, size = 313, normalized size = 1.86

$$\frac{\csc^9(c+dx)\left(212940\sin(2(c+dx))+195300\sin(4(c+dx))+16380\sin(6(c+dx))-1890\sin(8(c+dx))-451584\,\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^9*(-451584*Cos[c + d*x] - 155904*Cos[3*(c + d*x)] + 20736*Cos
[5*(c + d*x)] + 14976*Cos[7*(c + d*x)] - 1664*Cos[9*(c + d*x)] + 119070*Log
[Cos[(c + d*x)/2]]*Sin[c + d*x] - 119070*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]
 + 212940*Sin[2*(c + d*x)] - 79380*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] +
 79380*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 195300*Sin[4*(c + d*x)] + 3
4020*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 34020*Log[Sin[(c + d*x)/2]]*S
in[5*(c + d*x)] + 16380*Sin[6*(c + d*x)] - 8505*Log[Cos[(c + d*x)/2]]*Sin[7
*(c + d*x)] + 8505*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 1890*Sin[8*(c +
 d*x)] + 945*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 945*Log[Sin[(c + d*x)
/2]]*Sin[9*(c + d*x)]))/(5160960*a^2*d)
```

Maple [A] time = 0.203, size = 284, normalized size = 1.7

$$\frac{1}{4608\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{1}{1024\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8+\frac{5}{3584\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{1}{320\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4608/d/a^2*tan(1/2*d*x+1/2*c)^9-1/1024/d/a^2*tan(1/2*d*x+1/2*c)^8+5/3584/
d/a^2*tan(1/2*d*x+1/2*c)^7-1/320/d/a^2*tan(1/2*d*x+1/2*c)^5+1/128/d/a^2*tan
(1/2*d*x+1/2*c)^4-1/96/d/a^2*tan(1/2*d*x+1/2*c)^3+9/256/d/a^2*tan(1/2*d*x+1
/2*c)-5/3584/d/a^2/tan(1/2*d*x+1/2*c)^7-9/256/d/a^2/tan(1/2*d*x+1/2*c)+1/10
24/d/a^2/tan(1/2*d*x+1/2*c)^8+1/320/d/a^2/tan(1/2*d*x+1/2*c)^5-1/128/d/a^2/
tan(1/2*d*x+1/2*c)^4-1/4608/d/a^2/tan(1/2*d*x+1/2*c)^9-3/64/d/a^2*ln(tan(1/
2*d*x+1/2*c))+1/96/d/a^2/tan(1/2*d*x+1/2*c)^3
```

Maxima [B] time = 1.0409, size = 424, normalized size = 2.52

$$\frac{\frac{\frac{11340\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{3360\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1008\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{450\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{70\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2}-\frac{15120\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\right.}{}}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/322560*((11340*sin(d*x + c)/(cos(d*x + c) + 1) - 3360*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 2520*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1008*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 315
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 70*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9)/a^2 - 15120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (315*sin(d*x + c
)/(cos(d*x + c) + 1) - 450*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1008*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 2520*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 3360*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 11340*sin(d*x + c)^8/(cos(d*x +
 c) + 1)^8 - 70)*(cos(d*x + c) + 1)^9/(a^2*sin(d*x + c)^9))/d
```

Fricas [A] time = 1.20781, size = 736, normalized size = 4.38

$$-\frac{3328\,\cos(dx+c)^9-14976\,\cos(dx+c)^7+16128\,\cos(dx+c)^5-945\left(\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/40320*(3328*cos(d*x + c)^9 - 14976*cos(d*x + c)^7 + 16128*cos(d*x + c)^5
 - 945*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x +
c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 945*(cos(d*x + c)^8 -
4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 630*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11
*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x + c))/((a^2*d*cos(d*x + c)^8 - 4*
a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^2 + a^
2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**10/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3495, size = 331, normalized size = 1.97

$$-\frac{\frac{15120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{42774\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-11340\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+3360\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-2520\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1008\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-4}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] -1/322560*(15120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (42774*tan(1/2*d*x +
1/2*c)^9 - 11340*tan(1/2*d*x + 1/2*c)^8 + 3360*tan(1/2*d*x + 1/2*c)^6 - 252
0*tan(1/2*d*x + 1/2*c)^5 + 1008*tan(1/2*d*x + 1/2*c)^4 - 450*tan(1/2*d*x +
1/2*c)^2 + 315*tan(1/2*d*x + 1/2*c) - 70)/(a^2*tan(1/2*d*x + 1/2*c)^9) - (7
0*a^16*tan(1/2*d*x + 1/2*c)^9 - 315*a^16*tan(1/2*d*x + 1/2*c)^8 + 450*a^16*
tan(1/2*d*x + 1/2*c)^7 - 1008*a^16*tan(1/2*d*x + 1/2*c)^5 + 2520*a^16*tan(1
/2*d*x + 1/2*c)^4 - 3360*a^16*tan(1/2*d*x + 1/2*c)^3 + 11340*a^16*tan(1/2*d
*x + 1/2*c))/a^18)/d
```

3.737 $\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=218

$$\frac{2\cot^9(c+dx)}{9a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{9\tanh^{-1}(\cos(c+dx))}{256a^2d}-\frac{\cot^3(c+dx)\csc^7(c+dx)}{10a^2d}-\frac{\cot^3(c+dx)\,\mathrm{cs}}{8a^2d}$$

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(256*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (4*
Cot[c + d*x]^7)/(7*a^2*d) + (2*Cot[c + d*x]^9)/(9*a^2*d) - (9*Cot[c + d*x]*
Csc[c + d*x])/(256*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*a^2*d) + (
9*Cot[c + d*x]*Csc[c + d*x]^5)/(160*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x]^5
)/(8*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x]^7)/(80*a^2*d) - (Cot[c + d*x]^3*
Csc[c + d*x]^7)/(10*a^2*d)
```

Rubi [A] time = 0.487213, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2611, 3768, 3770, 2607, 270}

$$\frac{2\cot^9(c+dx)}{9a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{9\tanh^{-1}(\cos(c+dx))}{256a^2d}-\frac{\cot^3(c+dx)\csc^7(c+dx)}{10a^2d}-\frac{\cot^3(c+dx)\,\mathrm{cs}}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(256*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (4*
Cot[c + d*x]^7)/(7*a^2*d) + (2*Cot[c + d*x]^9)/(9*a^2*d) - (9*Cot[c + d*x]*
Csc[c + d*x])/(256*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*a^2*d) + (
9*Cot[c + d*x]*Csc[c + d*x]^5)/(160*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x]^5
)/(8*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x]^7)/(80*a^2*d) - (Cot[c + d*x]^3*
Csc[c + d*x]^7)/(10*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^7(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^5(c+dx) - 2a^2\cot^4(c+dx)\csc^6(c+dx) + a^2\cot^4(c+dx)\text{cs}\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^7(c+dx)\,dx}{a^2} - \frac{2\int \cot^4(c+dx}{a}$$
$$= -\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d} - \frac{\cot^3(c+dx)\csc^7(c+dx)}{10a^2d} - \frac{3\int \cot^2(c+dx)\csc^7(c+}{10a^2}$$
$$= \frac{\cot(c+dx)\csc^5(c+dx)}{16a^2d} - \frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d} + \frac{3\cot(c+dx)\csc^7(c+dx)}{80a^2d}$$
$$= \frac{2\cot^5(c+dx)}{5a^2d} + \frac{4\cot^7(c+dx)}{7a^2d} + \frac{2\cot^9(c+dx)}{9a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{64a^2d} + \frac{9\,\text{c}}{}$$
$$= \frac{2\cot^5(c+dx)}{5a^2d} + \frac{4\cot^7(c+dx)}{7a^2d} + \frac{2\cot^9(c+dx)}{9a^2d} - \frac{3\cot(c+dx)\csc(c+dx)}{128a^2d} - \frac{3\,\text{c}}{}$$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{4\cot^7(c+dx)}{7a^2d} + \frac{2\cot^9(c+dx)}{9a^2d} - \frac{9\,\text{co}}{}$$
$$= -\frac{9\tanh^{-1}(\cos(c+dx))}{256a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{4\cot^7(c+dx)}{7a^2d} + \frac{2\cot^9(c+dx)}{9a^2d} - \frac{9\,\text{co}}{}$$

Mathematica [A] time = 1.60567, size = 353, normalized size = 1.62

$$\frac{\csc^{10}(c+dx)\left(1720320\sin(2(c+dx)) + 1228800\sin(4(c+dx)) + 184320\sin(6(c+dx)) - 40960\sin(8(c+dx)) + 40\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^10*(-3219300*Cos[c + d*x] - 1237320*Cos[3*(c + d*x)] + 278712
*Cos[5*(c + d*x)] + 54810*Cos[7*(c + d*x)] - 5670*Cos[9*(c + d*x)] - 357210
*Log[Cos[(c + d*x)/2]] + 595350*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 34
0200*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 127575*Cos[6*(c + d*x)]*Log[C
os[(c + d*x)/2]] - 28350*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 2835*Cos[
10*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 357210*Log[Sin[(c + d*x)/2]] - 595350
*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 340200*Cos[4*(c + d*x)]*Log[Sin[(
c + d*x)/2]] - 127575*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 28350*Cos[8*
(c + d*x)]*Log[Sin[(c + d*x)/2]] - 2835*Cos[10*(c + d*x)]*Log[Sin[(c + d*x)
/2]] + 1720320*Sin[2*(c + d*x)] + 1228800*Sin[4*(c + d*x)] + 184320*Sin[6*(
c + d*x)] - 40960*Sin[8*(c + d*x)] + 4096*Sin[10*(c + d*x)]))/(41287680*a^2
*d)
```

Maple [B] time = 0.214, size = 398, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/10240/d/a^2*tan(1/2*d*x+1/2*c)^10-1/2304/d/a^2*tan(1/2*d*x+1/2*c)^9+3/409
6/d/a^2*tan(1/2*d*x+1/2*c)^8-1/1792/d/a^2*tan(1/2*d*x+1/2*c)^7-1/2048/d/a^2
*tan(1/2*d*x+1/2*c)^6+1/320/d/a^2*tan(1/2*d*x+1/2*c)^5-3/512/d/a^2*tan(1/2*
d*x+1/2*c)^4+1/192/d/a^2*tan(1/2*d*x+1/2*c)^3+1/1024/d/a^2*tan(1/2*d*x+1/2*
c)^2-3/128/d/a^2*tan(1/2*d*x+1/2*c)-1/10240/d/a^2/tan(1/2*d*x+1/2*c)^10+1/1
792/d/a^2/tan(1/2*d*x+1/2*c)^7+3/128/d/a^2/tan(1/2*d*x+1/2*c)-3/4096/d/a^2/
tan(1/2*d*x+1/2*c)^8-1/320/d/a^2/tan(1/2*d*x+1/2*c)^5+3/512/d/a^2/tan(1/2*d
*x+1/2*c)^4+1/2304/d/a^2/tan(1/2*d*x+1/2*c)^9+9/256/d/a^2*ln(tan(1/2*d*x+1/
2*c))+1/2048/d/a^2/tan(1/2*d*x+1/2*c)^6-1/192/d/a^2/tan(1/2*d*x+1/2*c)^3-1/
1024/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.04572, size = 587, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/1290240*((30240*sin(d*x + c)/(cos(d*x + c) + 1) - 1260*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 6720*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7560*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 - 4032*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
630*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 720*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 - 945*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 560*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 126*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)/a^2 - 45360*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (560*sin(d*x + c)/(cos(d*x + c) +
1) - 945*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 720*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 630*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4032*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 + 7560*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6720*sin
(d*x + c)^7/(cos(d*x + c) + 1)^7 - 1260*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 30240*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 126)*(cos(d*x + c) + 1)^10/(
a^2*sin(d*x + c)^10))/d
```

Fricas [A] time = 1.20639, size = 817, normalized size = 3.75

$$\frac{5670\cos(dx+c)^9 - 26460\cos(dx+c)^7 + 16128\cos(dx+c)^5 + 26460\cos(dx+c)^3 - 2835\left(\cos(dx+c)^{10} - 5\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/161280*(5670*cos(d*x + c)^9 - 26460*cos(d*x + c)^7 + 16128*cos(d*x + c)^5
 + 26460*cos(d*x + c)^3 - 2835*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos
(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c
) + 1/2) + 2835*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 1
0*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 102
4*(8*cos(d*x + c)^9 - 36*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*sin(d*x + c) -
 5670*cos(d*x + c))/(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x + c)^8 + 10*a^
2*d*cos(d*x + c)^6 - 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*x + c)^2 - a^2
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**11/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40215, size = 447, normalized size = 2.05

$$\frac{\frac{45360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} - \frac{132858\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10} - 30240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 + 1260\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8 + 6720\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 7560\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 + 4}{a^2\tan\left(\frac{1}{2}dx+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(45360*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (132858*tan(1/2*d*x +
 1/2*c)^10 - 30240*tan(1/2*d*x + 1/2*c)^9 + 1260*tan(1/2*d*x + 1/2*c)^8 + 6
720*tan(1/2*d*x + 1/2*c)^7 - 7560*tan(1/2*d*x + 1/2*c)^6 + 4032*tan(1/2*d*x
 + 1/2*c)^5 - 630*tan(1/2*d*x + 1/2*c)^4 - 720*tan(1/2*d*x + 1/2*c)^3 + 945
*tan(1/2*d*x + 1/2*c)^2 - 560*tan(1/2*d*x + 1/2*c) + 126)/(a^2*tan(1/2*d*x
+ 1/2*c)^10) + (126*a^18*tan(1/2*d*x + 1/2*c)^10 - 560*a^18*tan(1/2*d*x + 1
/2*c)^9 + 945*a^18*tan(1/2*d*x + 1/2*c)^8 - 720*a^18*tan(1/2*d*x + 1/2*c)^7
 - 630*a^18*tan(1/2*d*x + 1/2*c)^6 + 4032*a^18*tan(1/2*d*x + 1/2*c)^5 - 756
0*a^18*tan(1/2*d*x + 1/2*c)^4 + 6720*a^18*tan(1/2*d*x + 1/2*c)^3 + 1260*a^1
8*tan(1/2*d*x + 1/2*c)^2 - 30240*a^18*tan(1/2*d*x + 1/2*c))/a^20)/d
```

3.738 $\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=210

$$-\frac{\cot^{11}(c+dx)}{11a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d}+\frac{\cot^3(c+dx)\csc^7(c+dx)}{5a^2d}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(128*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (5*C
ot[c + d*x]^7)/(7*a^2*d) - (4*Cot[c + d*x]^9)/(9*a^2*d) - Cot[c + d*x]^11/(
11*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(128*a^2*d) + (Cot[c + d*x]*Csc[c
 + d*x]^3)/(64*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(80*a^2*d) - (3*Cot[c
 + d*x]*Csc[c + d*x]^7)/(40*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x]^7)/(5*a^2
*d)
```

Rubi [A] time = 0.432113, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2873, 2607, 270, 2611, 3768, 3770}

$$-\frac{\cot^{11}(c+dx)}{11a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d}+\frac{\cot^3(c+dx)\csc^7(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(128*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (5*C
ot[c + d*x]^7)/(7*a^2*d) - (4*Cot[c + d*x]^9)/(9*a^2*d) - Cot[c + d*x]^11/(
11*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(128*a^2*d) + (Cot[c + d*x]*Csc[c
 + d*x]^3)/(64*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(80*a^2*d) - (3*Cot[c
 + d*x]*Csc[c + d*x]^7)/(40*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x]^7)/(5*a^2
*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^4(c+dx)\csc^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\cot^4(c+dx)\csc^6(c+dx)-2a^2\cot^4(c+dx)\csc^7(c+dx)+a^2\cot^4(c+dx)\mathrm{cs}\right.}{a^4}\\
&= \frac{\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2}+\frac{\int \cot^4(c+dx)\csc^8(c+dx)\,dx}{a^2}-\frac{2\int \cot^4(c+dx)}{a}\\
&= \frac{\cot^3(c+dx)\csc^7(c+dx)}{5a^2d}+\frac{3\int \cot^2(c+dx)\csc^7(c+dx)\,dx}{5a^2}+\frac{\mathrm{Subst}\left(\int x^4\left(1+x^{2}\right.\right.}{}\\
&= -\frac{3\cot(c+dx)\csc^7(c+dx)}{40a^2d}+\frac{\cot^3(c+dx)\csc^7(c+dx)}{5a^2d}-\frac{3\int \csc^7(c+dx)\,dx}{40a^2}+\frac{\mathrm{S}}{}\\
&= -\frac{2\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{\cot^{11}(c+dx)}{11a^2d}+\frac{\cot(c+dx)\,\mathrm{c}}{80a}\\
&= -\frac{2\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{\cot^{11}(c+dx)}{11a^2d}+\frac{\cot(c+dx)\,\mathrm{c}}{64a}\\
&= -\frac{2\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{\cot^{11}(c+dx)}{11a^2d}+\frac{3\cot(c+dx)}{128a}\\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{\cot^{11}(}{11}
\end{aligned}
$$

Mathematica [A] time = 4.03829, size = 186, normalized size = 0.89

$$
\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4\left(2661120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cot(c+dx)\csc^{10}(c+\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(2661120*(Log[Cos[(c + d*x)/2]] -
Log[Sin[(c + d*x)/2]]) + Cot[c + d*x]*Csc[c + d*x]^10*(-5402624 - 5752832*C
os[2*(c + d*x)] + 346112*Cos[4*(c + d*x)] + 583168*Cos[6*(c + d*x)] - 10444
8*Cos[8*(c + d*x)] + 8704*Cos[10*(c + d*x)] + 2457378*Sin[c + d*x] + 590713
2*Sin[3*(c + d*x)] + 656964*Sin[5*(c + d*x)] - 121275*Sin[7*(c + d*x)] + 10
395*Sin[9*(c + d*x)])))/(113541120*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [B] time = 0.239, size = 436, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/22528/d/a^2*tan(1/2*d*x+1/2*c)^11-1/5120/d/a^2*tan(1/2*d*x+1/2*c)^10+7/18
432/d/a^2*tan(1/2*d*x+1/2*c)^9-1/2048/d/a^2*tan(1/2*d*x+1/2*c)^8+3/14336/d/
a^2*tan(1/2*d*x+1/2*c)^7+1/1024/d/a^2*tan(1/2*d*x+1/2*c)^6-27/10240/d/a^2*t
an(1/2*d*x+1/2*c)^5+1/256/d/a^2*tan(1/2*d*x+1/2*c)^4-11/3072/d/a^2*tan(1/2*
d*x+1/2*c)^3-1/512/d/a^2*tan(1/2*d*x+1/2*c)^2+19/1024/d/a^2*tan(1/2*d*x+1/2
*c)+1/5120/d/a^2/tan(1/2*d*x+1/2*c)^10-3/14336/d/a^2/tan(1/2*d*x+1/2*c)^7-1
9/1024/d/a^2/tan(1/2*d*x+1/2*c)-1/22528/d/a^2/tan(1/2*d*x+1/2*c)^11+1/2048/
d/a^2/tan(1/2*d*x+1/2*c)^8+27/10240/d/a^2/tan(1/2*d*x+1/2*c)^5-1/256/d/a^2/
tan(1/2*d*x+1/2*c)^4-7/18432/d/a^2/tan(1/2*d*x+1/2*c)^9-3/128/d/a^2*ln(tan(
1/2*d*x+1/2*c))-1/1024/d/a^2/tan(1/2*d*x+1/2*c)^6+11/3072/d/a^2/tan(1/2*d*x
+1/2*c)^3+1/512/d/a^2/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.04086, size = 640, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/7096320*((131670*sin(d*x + c)/(cos(d*x + c) + 1) - 13860*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - 25410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 27720*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 18711*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 + 6930*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1485*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 3465*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 2695*sin(d*x + c)
^9/(cos(d*x + c) + 1)^9 - 1386*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 315*
sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^2 - 166320*log(sin(d*x + c)/(cos(d
*x + c) + 1))/a^2 + (1386*sin(d*x + c)/(cos(d*x + c) + 1) - 2695*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 3465*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1485
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6930*sin(d*x + c)^5/(cos(d*x + c) +
1)^5 + 18711*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 27720*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 + 25410*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 13860*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 - 131670*sin(d*x + c)^10/(cos(d*x + c) + 1)^
10 - 315)*(cos(d*x + c) + 1)^11/(a^2*sin(d*x + c)^11))/d
```

Fricas [A] time = 1.27652, size = 910, normalized size = 4.33

$$-\frac{34816 \cos(dx+c)^{11} - 191488 \cos(dx+c)^{9} + 430848 \cos(dx+c)^{7} - 354816 \cos(dx+c)^{5} - 10395 \left(\cos(dx+c)^{10}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/887040*(34816*cos(d*x + c)^11 - 191488*cos(d*x + c)^9 + 430848*cos(d*x +
 c)^7 - 354816*cos(d*x + c)^5 - 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 +
 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) + 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 +
10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) + 1386*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7
+ 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x + c))/(
(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x + c)^8 + 10*a^2*d*cos(d*x + c)^6 -
 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**12/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36872, size = 487, normalized size = 2.32

$$-\frac{\frac{166320 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} - \frac{502266 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11} - 131670 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10} + 13860 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9} + 25410 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8} - 27720 \tan\left(\frac{1}{2}dx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] -1/7096320*(166320*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (502266*tan(1/2*d*x
 + 1/2*c)^11 - 131670*tan(1/2*d*x + 1/2*c)^10 + 13860*tan(1/2*d*x + 1/2*c)^
9 + 25410*tan(1/2*d*x + 1/2*c)^8 - 27720*tan(1/2*d*x + 1/2*c)^7 + 18711*tan
(1/2*d*x + 1/2*c)^6 - 6930*tan(1/2*d*x + 1/2*c)^5 - 1485*tan(1/2*d*x + 1/2*
c)^4 + 3465*tan(1/2*d*x + 1/2*c)^3 - 2695*tan(1/2*d*x + 1/2*c)^2 + 1386*tan
(1/2*d*x + 1/2*c) - 315)/(a^2*tan(1/2*d*x + 1/2*c)^11) - (315*a^20*tan(1/2*
d*x + 1/2*c)^11 - 1386*a^20*tan(1/2*d*x + 1/2*c)^10 + 2695*a^20*tan(1/2*d*x
 + 1/2*c)^9 - 3465*a^20*tan(1/2*d*x + 1/2*c)^8 + 1485*a^20*tan(1/2*d*x + 1/
2*c)^7 + 6930*a^20*tan(1/2*d*x + 1/2*c)^6 - 18711*a^20*tan(1/2*d*x + 1/2*c)
^5 + 27720*a^20*tan(1/2*d*x + 1/2*c)^4 - 25410*a^20*tan(1/2*d*x + 1/2*c)^3
- 13860*a^20*tan(1/2*d*x + 1/2*c)^2 + 131670*a^20*tan(1/2*d*x + 1/2*c))/a^2
2)/d
```

3.739 $\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=161

$$-\frac{3\cos^7(c+dx)}{7a^3d}+\frac{7\cos^5(c+dx)}{5a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{29\sin^3(c+dx)\cos^3(c+dx)}{48a^3d}+\frac{29\sin(\cdots}{\cdots}$$

```
[Out] (-29*x)/(128*a^3) - (4*Cos[c + d*x]^3)/(3*a^3*d) + (7*Cos[c + d*x]^5)/(5*a^
3*d) - (3*Cos[c + d*x]^7)/(7*a^3*d) - (29*Cos[c + d*x]*Sin[c + d*x])/(128*a
^3*d) + (29*Cos[c + d*x]^3*Sin[c + d*x])/(64*a^3*d) + (29*Cos[c + d*x]^3*Si
n[c + d*x]^3)/(48*a^3*d) + (Cos[c + d*x]^3*Sin[c + d*x]^5)/(8*a^3*d)
```

Rubi [A] time = 0.477072, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2565, 14, 2568, 2635, 8, 270}

$$-\frac{3\cos^7(c+dx)}{7a^3d}+\frac{7\cos^5(c+dx)}{5a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{29\sin^3(c+dx)\cos^3(c+dx)}{48a^3d}+\frac{29\sin(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-29*x)/(128*a^3) - (4*Cos[c + d*x]^3)/(3*a^3*d) + (7*Cos[c + d*x]^5)/(5*a^
3*d) - (3*Cos[c + d*x]^7)/(7*a^3*d) - (29*Cos[c + d*x]*Sin[c + d*x])/(128*a
^3*d) + (29*Cos[c + d*x]^3*Sin[c + d*x])/(64*a^3*d) + (29*Cos[c + d*x]^3*Si
n[c + d*x]^3)/(48*a^3*d) + (Cos[c + d*x]^3*Sin[c + d*x]^5)/(8*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cos^2(c+dx)\sin^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(a^3\cos^2(c+dx)\sin^3(c+dx) - 3a^3\cos^2(c+dx)\sin^4(c+dx) + 3a^3\cos^2(c+dx)\,\mathrm{s}\right.}{a^6}$$

$$= \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a^3} - \frac{\int \cos^2(c+dx)\sin^6(c+dx)\,dx}{a^3} - \frac{3\int \cos^2(c+dx)}{a^3}$$

$$= \frac{\cos^3(c+dx)\sin^3(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)\sin^5(c+dx)}{8a^3d} - \frac{5\int \cos^2(c+dx)\sin^4(c+d}{8a^3}$$

$$= \frac{3\cos^3(c+dx)\sin(c+dx)}{8a^3d} + \frac{29\cos^3(c+dx)\sin^3(c+dx)}{48a^3d} + \frac{\cos^3(c+dx)\sin^5(c+d}{8a^3d}$$

$$= -\frac{4\cos^3(c+dx)}{3a^3d} + \frac{7\cos^5(c+dx)}{5a^3d} - \frac{3\cos^7(c+dx)}{7a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{16a^3d} + 2$$

$$= -\frac{3x}{16a^3} - \frac{4\cos^3(c+dx)}{3a^3d} + \frac{7\cos^5(c+dx)}{5a^3d} - \frac{3\cos^7(c+dx)}{7a^3d} - \frac{29\cos(c+dx)\sin(c+}{128a^3d}$$

$$= -\frac{29x}{128a^3} - \frac{4\cos^3(c+dx)}{3a^3d} + \frac{7\cos^5(c+dx)}{5a^3d} - \frac{3\cos^7(c+dx)}{7a^3d} - \frac{29\cos(c+dx)\sin(c}{128a^3d}$$

Mathematica [B] time = 4.18506, size = 482, normalized size = 2.99

$$\frac{-48720dx\sin\left(\frac{c}{2}\right) + 38640\sin\left(\frac{c}{2}+dx\right) - 38640\sin\left(\frac{3c}{2}+dx\right) + 6720\sin\left(\frac{3c}{2}+2dx\right) + 6720\sin\left(\frac{5c}{2}+2dx\right) + 3920\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (84*(-7 + 12870*c - 580*d*x)*Cos[c/2] - 38640*Cos[c/2 + d*x] - 38640*Cos[(3
*c)/2 + d*x] + 6720*Cos[(3*c)/2 + 2*d*x] - 6720*Cos[(5*c)/2 + 2*d*x] - 3920
*Cos[(5*c)/2 + 3*d*x] - 3920*Cos[(7*c)/2 + 3*d*x] + 5880*Cos[(7*c)/2 + 4*d*
x] - 5880*Cos[(9*c)/2 + 4*d*x] + 4368*Cos[(9*c)/2 + 5*d*x] + 4368*Cos[(11*c
)/2 + 5*d*x] - 2240*Cos[(11*c)/2 + 6*d*x] + 2240*Cos[(13*c)/2 + 6*d*x] - 72
0*Cos[(13*c)/2 + 7*d*x] - 720*Cos[(15*c)/2 + 7*d*x] + 105*Cos[(15*c)/2 + 8*
d*x] - 105*Cos[(17*c)/2 + 8*d*x] - 998928*Sin[c/2] + 1081080*c*Sin[c/2] - 4
8720*d*x*Sin[c/2] + 38640*Sin[c/2 + d*x] - 38640*Sin[(3*c)/2 + d*x] + 6720*
Sin[(3*c)/2 + 2*d*x] + 6720*Sin[(5*c)/2 + 2*d*x] + 3920*Sin[(5*c)/2 + 3*d*x
] - 3920*Sin[(7*c)/2 + 3*d*x] + 5880*Sin[(7*c)/2 + 4*d*x] + 5880*Sin[(9*c)/
2 + 4*d*x] - 4368*Sin[(9*c)/2 + 5*d*x] + 4368*Sin[(11*c)/2 + 5*d*x] - 2240*
Sin[(11*c)/2 + 6*d*x] - 2240*Sin[(13*c)/2 + 6*d*x] + 720*Sin[(13*c)/2 + 7*d
*x] - 720*Sin[(15*c)/2 + 7*d*x] + 105*Sin[(15*c)/2 + 8*d*x] + 105*Sin[(17*c
)/2 + 8*d*x])/(215040*a^3*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.106, size = 517, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -76/105/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8+29/64/d/a^3/(1+tan(1/2*d*x+1/2*c)^
2)^8*tan(1/2*d*x+1/2*c)-608/105/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^2+667/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3-24
4/15/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4-1465/192/d/a^3/(
1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5+32/15/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6-5117/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*t
an(1/2*d*x+1/2*c)^7-76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c
)^8+5117/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9-128/3/d/
a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^10+1465/192/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^
8*tan(1/2*d*x+1/2*c)^12-667/192/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^13-29/64/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-29
/64/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.59037, size = 674, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6720*((3045*sin(d*x + c)/(cos(d*x + c) + 1) - 38912*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 23345*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 109312*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 51275*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
14336*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 179095*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 - 170240*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 179095*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 - 286720*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 51275*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 26880*sin(d*x + c)^12/(cos(d
*x + c) + 1)^12 - 23345*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 3045*sin(d*
```

```
x + c)^15/(cos(d*x + c) + 1)^15 - 4864)/(a^3 + 8*a^3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a^3*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 +
 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^3*sin(d*x + c)^12/(cos
(d*x + c) + 1)^12 + 8*a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a^3*sin(d
*x + c)^16/(cos(d*x + c) + 1)^16) - 3045*arctan(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^3)/d
```

Fricas [A] time = 1.12619, size = 263, normalized size = 1.63

$$-\frac{5760 \cos(dx+c)^7 - 18816 \cos(dx+c)^5 + 17920 \cos(dx+c)^3 + 3045\,dx - 35\left(48 \cos(dx+c)^7 - 328 \cos(dx+c)\right.}{13440\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/13440*(5760*cos(d*x + c)^7 - 18816*cos(d*x + c)^5 + 17920*cos(d*x + c)^3
 + 3045*d*x - 35*(48*cos(d*x + c)^7 - 328*cos(d*x + c)^5 + 454*cos(d*x + c)
^3 - 87*cos(d*x + c))*sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31098, size = 294, normalized size = 1.83

$$-\frac{\frac{3045\,(dx+c)}{a^3} + \frac{2\left(3045 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} + 23345 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} + 26880 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} - 51275 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 286720 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} - 17909\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/13440*(3045*(d*x + c)/a^3 + 2*(3045*tan(1/2*d*x + 1/2*c)^15 + 23345*tan(
1/2*d*x + 1/2*c)^13 + 26880*tan(1/2*d*x + 1/2*c)^12 - 51275*tan(1/2*d*x + 1
/2*c)^11 + 286720*tan(1/2*d*x + 1/2*c)^10 - 179095*tan(1/2*d*x + 1/2*c)^9 +
 170240*tan(1/2*d*x + 1/2*c)^8 + 179095*tan(1/2*d*x + 1/2*c)^7 - 14336*tan(
1/2*d*x + 1/2*c)^6 + 51275*tan(1/2*d*x + 1/2*c)^5 + 109312*tan(1/2*d*x + 1/
2*c)^4 - 23345*tan(1/2*d*x + 1/2*c)^3 + 38912*tan(1/2*d*x + 1/2*c)^2 - 3045
*tan(1/2*d*x + 1/2*c) + 4864)/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a^3))/d
```

3.740 $\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{\cos^7(c+dx)}{7a^3d}-\frac{\cos^5(c+dx)}{a^3d}+\frac{4\cos^3(c+dx)}{3a^3d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{2a^3d}-\frac{5\sin(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{5\sin(c+dx)\,c\cdots}{16a^3\cdots}$$

```
[Out] (5*x)/(16*a^3) + (4*Cos[c + d*x]^3)/(3*a^3*d) - Cos[c + d*x]^5/(a^3*d) + Co
s[c + d*x]^7/(7*a^3*d) + (5*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d) - (5*Cos[
c + d*x]^3*Sin[c + d*x])/(8*a^3*d) - (Cos[c + d*x]^3*Sin[c + d*x]^3)/(2*a^3
*d)
```

Rubi [A] time = 0.401399, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2568, 2635, 8, 2565, 14, 270}

$$\frac{\cos^7(c+dx)}{7a^3d}-\frac{\cos^5(c+dx)}{a^3d}+\frac{4\cos^3(c+dx)}{3a^3d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{2a^3d}-\frac{5\sin(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{5\sin(c+dx)\,c\cdots}{16a^3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (5*x)/(16*a^3) + (4*Cos[c + d*x]^3)/(3*a^3*d) - Cos[c + d*x]^5/(a^3*d) + Co
s[c + d*x]^7/(7*a^3*d) + (5*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d) - (5*Cos[
c + d*x]^3*Sin[c + d*x])/(8*a^3*d) - (Cos[c + d*x]^3*Sin[c + d*x]^3)/(2*a^3
*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cos^2(c+dx)\sin^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(a^3\cos^2(c+dx)\sin^2(c+dx) - 3a^3\cos^2(c+dx)\sin^3(c+dx) + 3a^3\cos^2(c+dx)\,\mathrm{s}\right.}{a^6}$$

$$= \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a^3} - \frac{3\int \cos^2(c+dx)}{a^3}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{2a^3d} + \frac{\int \cos^2(c+dx)\,dx}{4a^3} + \frac{3\int \mathrm{c}}{}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{5\cos^3(c+dx)\sin(c+dx)}{8a^3d} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{2a^3d} +$$

$$= \frac{x}{8a^3} + \frac{4\cos^3(c+dx)}{3a^3d} - \frac{\cos^5(c+dx)}{a^3d} + \frac{\cos^7(c+dx)}{7a^3d} + \frac{5\cos(c+dx)\sin(c+dx)}{16a^3d} -$$

$$= \frac{5x}{16a^3} + \frac{4\cos^3(c+dx)}{3a^3d} - \frac{\cos^5(c+dx)}{a^3d} + \frac{\cos^7(c+dx)}{7a^3d} + \frac{5\cos(c+dx)\sin(c+dx)}{16a^3d} -$$

Mathematica [B] time = 9.81987, size = 429, normalized size = 3.23

$$\frac{840dx\sin\left(\frac{c}{2}\right) - 609\sin\left(\frac{c}{2}+dx\right) + 609\sin\left(\frac{3c}{2}+dx\right) - 63\sin\left(\frac{3c}{2}+2dx\right) - 63\sin\left(\frac{5c}{2}+2dx\right) - 91\sin\left(\frac{5c}{2}+3dx\right) + 91}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-168*(99*c - 5*d*x)*Cos[c/2] + 609*Cos[c/2 + d*x] + 609*Cos[(3*c)/2 + d*x]
 - 63*Cos[(3*c)/2 + 2*d*x] + 63*Cos[(5*c)/2 + 2*d*x] + 91*Cos[(5*c)/2 + 3*d
```

```
*x] + 91*Cos[(7*c)/2 + 3*d*x] - 105*Cos[(7*c)/2 + 4*d*x] + 105*Cos[(9*c)/2
+ 4*d*x] - 63*Cos[(9*c)/2 + 5*d*x] - 63*Cos[(11*c)/2 + 5*d*x] + 21*Cos[(11*
c)/2 + 6*d*x] - 21*Cos[(13*c)/2 + 6*d*x] + 3*Cos[(13*c)/2 + 7*d*x] + 3*Cos[
(15*c)/2 + 7*d*x] + 16996*Sin[c/2] - 16632*c*Sin[c/2] + 840*d*x*Sin[c/2] -
609*Sin[c/2 + d*x] + 609*Sin[(3*c)/2 + d*x] - 63*Sin[(3*c)/2 + 2*d*x] - 63*
Sin[(5*c)/2 + 2*d*x] - 91*Sin[(5*c)/2 + 3*d*x] + 91*Sin[(7*c)/2 + 3*d*x] -
105*Sin[(7*c)/2 + 4*d*x] - 105*Sin[(9*c)/2 + 4*d*x] + 63*Sin[(9*c)/2 + 5*d*
x] - 63*Sin[(11*c)/2 + 5*d*x] + 21*Sin[(11*c)/2 + 6*d*x] + 21*Sin[(13*c)/2
+ 6*d*x] - 3*Sin[(13*c)/2 + 7*d*x] + 3*Sin[(15*c)/2 + 7*d*x])/(2688*a^3*d*(
Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.102, size = 415, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 5/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13+3/2/d/a^3/(1+tan
(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11+12/d/a^3/(1+tan(1/2*d*x+1/2*c)^2
)^7*tan(1/2*d*x+1/2*c)^10-119/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*
x+1/2*c)^9+92/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8+8/3/d
/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6+119/8/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5+8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)^4-3/2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^
3+20/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2-5/8/d/a^3/(1+t
an(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)+20/21/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^7+5/8/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.56745, size = 562, normalized size = 4.23

$$-\frac{\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1120 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{252 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{1344 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{2499 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{448 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{5152 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{2499 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{2016 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} - \frac{252 \sin(dx+c)}{(\cos(dx+c)+1)} \cdots}{a^3 + \frac{7 a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{21 a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{35 a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{35 a^3 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{21 a^3 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{7 a^3 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{a^3 \sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}} \cdots}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/168*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 1120*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 252*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1344*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 2499*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 448*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 5152*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 2499*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2016*sin(d*x + c)^10/(cos(d*x
 + c) + 1)^10 - 252*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105*sin(d*x + c
)^13/(cos(d*x + c) + 1)^13 - 160)/(a^3 + 7*a^3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 21*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*a^3*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 35*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a^
3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 7*a^3*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12 + a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.11327, size = 217, normalized size = 1.63

$$\frac{48 \cos(dx+c)^7 - 336 \cos(dx+c)^5 + 448 \cos(dx+c)^3 + 105\,dx + 21\left(8 \cos(dx+c)^5 - 18 \cos(dx+c)^3 + 5 \cos(dx\right.}{336\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/336*(48*cos(d*x + c)^7 - 336*cos(d*x + c)^5 + 448*cos(d*x + c)^3 + 105*d*
x + 21*(8*cos(d*x + c)^5 - 18*cos(d*x + c)^3 + 5*cos(d*x + c))*sin(d*x + c)
)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28112, size = 242, normalized size = 1.82

$$\frac{\frac{105\,(dx+c)}{a^3} + \frac{2\left(105 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} + 252 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 2016 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} - 2499 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} + 5152 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8} + 448 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/336*(105*(d*x + c)/a^3 + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 252*tan(1/2*d*x
 + 1/2*c)^11 + 2016*tan(1/2*d*x + 1/2*c)^10 - 2499*tan(1/2*d*x + 1/2*c)^9 +
 5152*tan(1/2*d*x + 1/2*c)^8 + 448*tan(1/2*d*x + 1/2*c)^6 + 2499*tan(1/2*d*
x + 1/2*c)^5 + 1344*tan(1/2*d*x + 1/2*c)^4 - 252*tan(1/2*d*x + 1/2*c)^3 + 1
120*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 160)/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^7*a^3))/d
```

3.741 $\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$-\frac{7\cos^5(c+dx)}{30a^3d}-\frac{\cos^7(c+dx)}{6d\left(a^3\sin(c+dx)+a^3\right)}-\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^3d}-\frac{7\sin(c+dx)\cos(c+dx)}{16a^3d}-\frac{7x}{16a^3}-\frac{\cos^9(c}{3d(a\sin(c}$$

```
[Out] (-7*x)/(16*a^3) - (7*Cos[c + d*x]^5)/(30*a^3*d) - (7*Cos[c + d*x]*Sin[c + d
*x])/(16*a^3*d) - (7*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^3*d) - Cos[c + d*x]
^9/(3*d*(a + a*Sin[c + d*x])^3) - Cos[c + d*x]^7/(6*d*(a^3 + a^3*Sin[c + d*
x]))
```

Rubi [A] time = 0.16982, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2859, 2679, 2682, 2635, 8}

$$-\frac{7\cos^5(c+dx)}{30a^3d}-\frac{\cos^7(c+dx)}{6d\left(a^3\sin(c+dx)+a^3\right)}-\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^3d}-\frac{7\sin(c+dx)\cos(c+dx)}{16a^3d}-\frac{7x}{16a^3}-\frac{\cos^9(c}{3d(a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-7*x)/(16*a^3) - (7*Cos[c + d*x]^5)/(30*a^3*d) - (7*Cos[c + d*x]*Sin[c + d
*x])/(16*a^3*d) - (7*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^3*d) - Cos[c + d*x]
^9/(3*d*(a + a*Sin[c + d*x])^3) - Cos[c + d*x]^7/(6*d*(a^3 + a^3*Sin[c + d*
x]))
```


Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3} - \frac{\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{a}$$

$$= -\frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3} - \frac{\cos^7(c+dx)}{6d\left(a^3+a^3\sin(c+dx)\right)} - \frac{7\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{6a^2}$$

$$= -\frac{7\cos^5(c+dx)}{30a^3d} - \frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3} - \frac{\cos^7(c+dx)}{6d\left(a^3+a^3\sin(c+dx)\right)} - \frac{7\int \cos^4(c+ }{6a^3}$$

$$= -\frac{7\cos^5(c+dx)}{30a^3d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3} - \frac{\cos^7(}{6d\left(a^3+a^3\right.}$$

$$= -\frac{7\cos^5(c+dx)}{30a^3d} - \frac{7\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \frac{\cos^9}{3d(a+a}$$

$$= -\frac{7x}{16a^3} - \frac{7\cos^5(c+dx)}{30a^3d} - \frac{7\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^3d} - $$

Mathematica [B] time = 2.0293, size = 366, normalized size = 2.79

$$\frac{-840dx\sin\left(\frac{c}{2}\right)+600\sin\left(\frac{c}{2}+dx\right)-600\sin\left(\frac{3c}{2}+dx\right)+15\sin\left(\frac{3c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+2dx\right)+140\sin\left(\frac{5c}{2}+3dx\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-21*(1 + 40*d*x)*Cos[c/2] - 600*Cos[c/2 + d*x] - 600*Cos[(3*c)/2 + d*x] +
15*Cos[(3*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 2*d*x] - 140*Cos[(5*c)/2 + 3*d*x
] - 140*Cos[(7*c)/2 + 3*d*x] + 105*Cos[(7*c)/2 + 4*d*x] - 105*Cos[(9*c)/2 +
 4*d*x] + 36*Cos[(9*c)/2 + 5*d*x] + 36*Cos[(11*c)/2 + 5*d*x] - 5*Cos[(11*c)
/2 + 6*d*x] + 5*Cos[(13*c)/2 + 6*d*x] + 21*Sin[c/2] - 840*d*x*Sin[c/2] + 60
0*Sin[c/2 + d*x] - 600*Sin[(3*c)/2 + d*x] + 15*Sin[(3*c)/2 + 2*d*x] + 15*Si
n[(5*c)/2 + 2*d*x] + 140*Sin[(5*c)/2 + 3*d*x] - 140*Sin[(7*c)/2 + 3*d*x] +
105*Sin[(7*c)/2 + 4*d*x] + 105*Sin[(9*c)/2 + 4*d*x] - 36*Sin[(9*c)/2 + 5*d*
x] + 36*Sin[(11*c)/2 + 5*d*x] - 5*Sin[(11*c)/2 + 6*d*x] - 5*Sin[(13*c)/2 +
6*d*x])/(1920*a^3*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] time = 0.089, size = 415, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -7/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-2/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10+73/24/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)^9-18/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)^8+37/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7-44/3/d/
a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6-37/4/d/a^3/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(
1/2*d*x+1/2*c)^4-73/24/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
3-34/5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2+7/8/d/a^3/(1+t
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-22/15/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^6-7/8/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.55939, size = 531, normalized size = 4.05

$$\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{816\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{365\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{480\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1110\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1760\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1110\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{2160\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{365\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{240\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\cdots}{a^3+\frac{6\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\,a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6\,a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}\cdots}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/120*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 816*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 365*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 480*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 1110*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1760*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 1110*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 2160*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 365*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 240*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 105*sin(d*x + c)^11/
(cos(d*x + c) + 1)^11 - 176)/(a^3 + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^3*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^3*sin(
d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^
12) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.14432, size = 190, normalized size = 1.45

$$\frac{144\,\cos\left(dx+c\right)^5-320\,\cos\left(dx+c\right)^3-105\,dx-5\left(8\,\cos\left(dx+c\right)^5-50\,\cos\left(dx+c\right)^3+21\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(144*cos(d*x + c)^5 - 320*cos(d*x + c)^3 - 105*d*x - 5*(8*cos(d*x + c
)^5 - 50*cos(d*x + c)^3 + 21*cos(d*x + c))*sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27153, size = 242, normalized size = 1.85

$$-\frac{\frac{105\,(dx+c)}{a^3}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-365\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2160\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1760\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(105*(d*x + c)/a^3 + 2*(105*tan(1/2*d*x + 1/2*c)^11 + 240*tan(1/2*d*
x + 1/2*c)^10 - 365*tan(1/2*d*x + 1/2*c)^9 + 2160*tan(1/2*d*x + 1/2*c)^8 -
1110*tan(1/2*d*x + 1/2*c)^7 + 1760*tan(1/2*d*x + 1/2*c)^6 + 1110*tan(1/2*d*
x + 1/2*c)^5 + 480*tan(1/2*d*x + 1/2*c)^4 + 365*tan(1/2*d*x + 1/2*c)^3 + 81
6*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 176)/((tan(1/2*d*x +
1/2*c)^2 + 1)^6*a^3))/d
```

3.742 $\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13x}{8a^3}$$

```
[Out] (-13*x)/(8*a^3) - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(a^3*d) - Co
s[c + d*x]^3/(a^3*d) - (13*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) + (Cos[c +
d*x]^3*Sin[c + d*x])/(4*a^3*d)
```

Rubi [A] time = 0.242914, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2875, 2873, 2635, 8, 2592, 321, 206, 2565, 30, 2568}

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-13*x)/(8*a^3) - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(a^3*d) - Co
s[c + d*x]^3/(a^3*d) - (13*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) + (Cos[c +
d*x]^3*Sin[c + d*x])/(4*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cos(c+dx)\cot(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-3a^3\cos^2(c+dx)+a^3\cos(c+dx)\cot(c+dx)+3a^3\cos^2(c+dx)\sin(c+dx)-a\right.}{a^6}$$

$$= \frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a^3} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{3\int \cos^2(c+dx)\,dx}{a^3}$$

$$= -\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{\int \cos^2(c+dx)\,dx}{4a^3} - \frac{3\int 1\,d}{2a^3}$$

$$= -\frac{3x}{2a^3} + \frac{\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{a^3d} - \frac{13\cos(c+dx)\sin(c+dx)}{8a^3d} + \frac{\cos^3(c+dx)\sin}{4a^3d}$$

$$= -\frac{13x}{8a^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{a^3d} - \frac{13\cos(c+dx)\sin(c+}{8a^3d}$$

Mathematica [A] time = 0.390727, size = 80, normalized size = 0.81

$$\frac{-24\sin(2(c+dx))+\sin(4(c+dx))+8\cos(c+dx)-8\cos(3(c+dx))+32\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-32\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-52*c - 52*d*x + 8*Cos[c + d*x] - 8*Cos[3*(c + d*x)] - 32*Log[Cos[(c + d*x
)/2]] + 32*Log[Sin[(c + d*x)/2]] - 24*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])/
(32*a^3*d)
```

Maple [B] time = 0.135, size = 239, normalized size = 2.4

$$\frac{11}{4da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}-4\frac{(\tan(1/2\,dx+c/2))^6}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^4}+\frac{19}{4da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 11/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-4/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6+19/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)
^4*tan(1/2*d*x+1/2*c)^5-19/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^3+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2-11/4/d/a^3/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-13/4/d/a^3*arctan(tan(1/2*d*x
+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.58602, size = 363, normalized size = 3.67

$$-\frac{\frac{\frac{11\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{19\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{19\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{16\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{11\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{13\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((11*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 19*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 19*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 + 16*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 11*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/(a^3 + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 13*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 - 4*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.17286, size = 238, normalized size = 2.4

$$-\frac{8\cos(dx+c)^3+13\,dx-\left(2\cos(dx+c)^3-13\cos(dx+c)\right)\sin(dx+c)-8\cos(dx+c)+4\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*cos(d*x + c)^3 + 13*d*x - (2*cos(d*x + c)^3 - 13*cos(d*x + c))*sin(
d*x + c) - 8*cos(d*x + c) + 4*log(1/2*cos(d*x + c) + 1/2) - 4*log(-1/2*cos(
d*x + c) + 1/2))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30986, size = 174, normalized size = 1.76

$$-\frac{\frac{13\,(dx+c)}{a^3}-\frac{8\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{2\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-11\,\mathrm{t}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(13*(d*x + c)/a^3 - 8*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 2*(11*tan(1
/2*d*x + 1/2*c)^7 - 16*tan(1/2*d*x + 1/2*c)^6 + 19*tan(1/2*d*x + 1/2*c)^5 -
 19*tan(1/2*d*x + 1/2*c)^3 + 16*tan(1/2*d*x + 1/2*c)^2 - 11*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

3.743 $\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{2a^3}$$

```
[Out] x/(2*a^3) + (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - (3*Cos[c + d*x])/(a^3*d) +
Cos[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]/(a^3*d) + (3*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d)
```

Rubi [A] time = 0.222455, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2709, 3770, 3767, 8, 2638, 2635, 2633}

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/(2*a^3) + (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - (3*Cos[c + d*x])/(a^3*d) +
Cos[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]/(a^3*d) + (3*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(2a^5 - 3a^5\csc(c+dx) + a^5\csc^2(c+dx) + 2a^5\sin(c+dx) - 3a^5\sin^2(c+dx) + a\right.}{a^8}$$
$$= \frac{2x}{a^3} + \frac{\int \csc^2(c+dx)\,dx}{a^3} + \frac{\int \sin^3(c+dx)\,dx}{a^3} + \frac{2\int \sin(c+dx)\,dx}{a^3} - \frac{3\int \csc(c+dx}{a^3}$$
$$= \frac{2x}{a^3} + \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cos(c+dx)}{a^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{3\int 1\,dx}{2a^3}$$
$$= \frac{x}{2a^3} + \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{3\cos(c+dx)}{a^3d} + \frac{\cos^3(c+dx)}{3a^3d} - \frac{\cot(c+dx)}{a^3d} + \frac{3\,\mathrm{co}}{}$$

Mathematica [A] time = 0.958189, size = 126, normalized size = 1.37

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(6(c+dx)+9\sin(2(c+dx))-33\cos(c+dx)+\cos(3(c+dx))+6\tan\left(\frac{1}{2}(c+dx\right.\right.}{12d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(6*(c + d*x) - 33*Cos[c + d*x] + C
os[3*(c + d*x)] - 6*Cot[(c + d*x)/2] + 36*Log[Cos[(c + d*x)/2]] - 36*Log[Si
n[(c + d*x)/2]] + 9*Sin[2*(c + d*x)] + 6*Tan[(c + d*x)/2]))/(12*d*(a + a*Si
n[c + d*x])^3)
```

Maple [B] time = 0.155, size = 230, normalized size = 2.5

$$\frac{1}{2da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 3\frac{(\tan(1/2\,dx+c/2))^5}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3} - 4\frac{(\tan(1/2\,dx+c/2))^4}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3} - 12\frac{(\tan(1/2\,dx+c}{da^3\left(1+(\tan(1/2\,dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x
+1/2*c)^5-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-12/d/a^3/
(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2+3/d/a^3/(1+tan(1/2*d*x+1/2*
c)^2)^3*tan(1/2*d*x+1/2*c)-16/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3+1/d/a^3*ar
ctan(tan(1/2*d*x+1/2*c))-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/2*d*
x+1/2*c))
```

Maxima [B] time = 1.56722, size = 385, normalized size = 4.18

$$-\frac{\frac{\frac{32 \sin(dx+c)}{\cos(dx+c)+1}-\frac{9 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{72 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+3}{\frac{a^3 \sin(dx+c)}{\cos(dx+c)+1}+\frac{3 a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3 a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}} - \frac{6 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{18 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} - \frac{3 \sin(\ldots}{a^3(\cos(\ldots}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*((32*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 72*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 3)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^3*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
+ a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 6*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^3 + 18*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 3*sin(d*x +
 c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.12946, size = 289, normalized size = 3.14

$$-\frac{9 \cos(dx+c)^3 - \left(2 \cos(dx+c)^3 + 3 dx - 18 \cos(dx+c)\right)\sin(dx+c) - 9 \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 9 \log\ldots}{6 a^3 d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(9*cos(d*x + c)^3 - (2*cos(d*x + c)^3 + 3*d*x - 18*cos(d*x + c))*sin(d
*x + c) - 9*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 9*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 3*cos(d*x + c))/(a^3*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

[Out] Timed out

Giac [A] time = 1.2936, size = 198, normalized size = 2.15

$$\frac{\frac{3\,(dx+c)}{a^3}-\frac{18\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{3\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)/a^3 - 18*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 3*tan(1/2*d*
x + 1/2*c)/a^3 + 3*(6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c))
- 2*(9*tan(1/2*d*x + 1/2*c)^5 + 12*tan(1/2*d*x + 1/2*c)^4 + 36*tan(1/2*d*x
+ 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 16)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a
^3))/d
```

3.744 $\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{5x}{2a^3}$$

```
[Out] (5*x)/(2*a^3) - (5*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cos[c + d*x])/(a^3
*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (C
os[c + d*x]*Sin[c + d*x])/(2*a^3*d)
```

Rubi [A] time = 0.246939, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2638, 2635}

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{5x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(2*a^3) - (5*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cos[c + d*x])/(a^3
*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (C
os[c + d*x]*Sin[c + d*x])/(2*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(2a^5+2a^5\csc(c+dx)-3a^5\csc^2(c+dx)+a^5\csc^3(c+dx)-3a^5\sin(c+dx)+a\right.}{a^8}$$
$$= \frac{2x}{a^3}+\frac{\int \csc^3(c+dx)\,dx}{a^3}+\frac{\int \sin^2(c+dx)\,dx}{a^3}+\frac{2\int \csc(c+dx)\,dx}{a^3}-\frac{3\int \csc^2(c+d}{a^3}$$
$$= \frac{2x}{a^3}-\frac{2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{\cos(c+d}{}$$
$$= \frac{5x}{2a^3}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c}{2a^3d}$$

Mathematica [A] time = 0.906118, size = 144, normalized size = 1.47

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(20(c+dx)-2\sin(2(c+dx))+24\cos(c+dx)-12\tan\left(\frac{1}{2}(c+dx)\right)+12\cot\left(\frac{1}{2}(\right.\right.}{8d(a\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(20*(c + d*x) + 24*Cos[c + d*x] +
12*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 20*Log[Cos[(c + d*x)/2]] + 20*Lo
g[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 - 2*Sin[2*(c + d*x)] - 12*Tan[(c +
 d*x)/2]))/(8*d*(a + a*Sin[c + d*x])^3)
```

Maple [B] time = 0.157, size = 234, normalized size = 2.4

$$\frac{1}{8da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{3}{2da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}+6\frac{(\tan(1/2\,dx}{da^3\left(1+(\tan(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^3*tan(1/2*d*x+1/2*c)+1/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2
*tan(1/2*d*x+1/2*c)^2-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)
+6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2+5/d/a^3*arctan(tan(1/2*d*x+1/2*c))-1/8/
d/a^3/tan(1/2*d*x+1/2*c)^2+3/2/d/a^3/tan(1/2*d*x+1/2*c)+5/2/d/a^3*ln(tan(1/
2*d*x+1/2*c))
```

Maxima [B] time = 1.56469, size = 360, normalized size = 3.67

$$\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{46\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{47\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3}+\frac{40\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{20\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8*((12*sin(d*x + c)/(cos(d*x + c) + 1) + 46*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 47*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 20*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^3*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - (12*sin(d*x + c)/(cos(d*x + c) + 1)
 - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 40*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^3 + 20*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.19672, size = 358, normalized size = 3.65

$$\frac{10\,dx\cos(dx+c)^2+12\,\cos(dx+c)^3-10\,dx-5\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+5\left(\cos(dx+c)^2-1\right)\log\left(\right.}{4\left(a^3d\cos(dx+c)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(10*d*x*cos(d*x + c)^2 + 12*cos(d*x + c)^3 - 10*d*x - 5*(cos(d*x + c)^2
 - 1)*log(1/2*cos(d*x + c) + 1/2) + 5*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x
 + c) + 1/2) - 2*(cos(d*x + c)^3 + 5*cos(d*x + c))*sin(d*x + c) - 10*cos(d*
x + c))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.33558, size = 232, normalized size = 2.37

$$\frac{\frac{20\,(dx+c)}{a^3}+\frac{20\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,\tan(}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2 a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(20*(d*x + c)/a^3 + 20*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (10*tan(1/2
*d*x + 1/2*c)^6 - 20*tan(1/2*d*x + 1/2*c)^5 - 27*tan(1/2*d*x + 1/2*c)^4 - 1
6*tan(1/2*d*x + 1/2*c)^3 - 36*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2
*c) + 1)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2*a^3) + (a^3*tan
(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.745 $\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$-\frac{\cos(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{3\cot(c+dx)}{a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3x}{a^3}$$

```
[Out] (-3*x)/a^3 - ArcTanh[Cos[c + d*x]]/(2*a^3*d) - Cos[c + d*x]/(a^3*d) - (3*Co
t[c + d*x])/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*
x])/(2*a^3*d)
```

Rubi [A] time = 0.274305, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2875, 2872, 3770, 3767, 8, 3768, 2638}

$$-\frac{\cos(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{3\cot(c+dx)}{a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 - ArcTanh[Cos[c + d*x]]/(2*a^3*d) - Cos[c + d*x]/(a^3*d) - (3*Co
t[c + d*x])/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*
x])/(2*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-3a^5+2a^5\csc(c+dx)+2a^5\csc^2(c+dx)-3a^5\csc^3(c+dx)+a^5\csc^4(c+dx) \cdots\right.}{a^8}$$
$$= -\frac{3x}{a^3}+\frac{\int \csc^4(c+dx)\,dx}{a^3}+\frac{\int \sin(c+dx)\,dx}{a^3}+\frac{2\int \csc(c+dx)\,dx}{a^3}+\frac{2\int \csc^2(c+d \cdots}{a^3}$$
$$= -\frac{3x}{a^3}-\frac{2\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3\int \csc \cdots}{\cdots}$$
$$= -\frac{3x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cos(c+dx)}{a^3d}-\frac{3\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot \cdots}{\cdots}$$

Mathematica [A] time = 2.52061, size = 132, normalized size = 1.43

$$\frac{\csc^3(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(2(3\sin(c+dx)+8)\cos(3(c+dx))+6(5\sin(c+dx)-4)\cos(c+d \cdots\right.}{24a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-12*(6*(c + d*x) +
 Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3 + 2*Cos[3*(c
 + d*x)]*(8 + 3*Sin[c + d*x]) + 6*Cos[c + d*x]*(-4 + 5*Sin[c + d*x])))/(24*
a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.164, size = 173, normalized size = 1.9

$$\frac{1}{24\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{11}{8\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{1}{da^3\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}-6\,\frac{\mathrm{arcta}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^3*tan(1/2*d*x+1/2*c)^2+11/8/d/a^3*t
an(1/2*d*x+1/2*c)-2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^3*arctan(tan(1/2*d
```

```
*x+1/2*c))-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3+3/8/d/a^3/tan(1/2*d*x+1/2*c)^2-1
1/8/d/a^3/tan(1/2*d*x+1/2*c)+1/2/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.5688, size = 327, normalized size = 3.55

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{34\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{39\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{33\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1}{\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{\frac{33\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{144\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right.}{a^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 34*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 39*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 33*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 1)/(a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^3*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5) + (33*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^3 -
 144*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 12*log(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.16471, size = 413, normalized size = 4.49

$$-\frac{32\cos(dx+c)^3+3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c\right.}{12\left(a^3d\cos(dx+c)^2-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(32*cos(d*x + c)^3 + 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) + 6*(6*d*x*cos(d*x + c)^2 + 2*cos(d*x + c)^3 - 6*d*x + cos(d*x + c)
)*sin(d*x + c) - 36*cos(d*x + c))/((a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x +
 c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28637, size = 212, normalized size = 2.3

$$-\frac{\frac{72\,(dx+c)}{a^3}-\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{48}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}+\frac{22\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\cdots}{\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/24*(72*(d*x + c)/a^3 - 12*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 48/((tan(
1/2*d*x + 1/2*c)^2 + 1)*a^3) + (22*tan(1/2*d*x + 1/2*c)^3 + 33*tan(1/2*d*x
+ 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (a^
6*tan(1/2*d*x + 1/2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 + 33*a^6*tan(1/2*d*
x + 1/2*c))/a^9)/d
```

3.746 $\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{11\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{x}{a^3}$$

```
[Out] x/a^3 + (13*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + Cot[c + d*x]/(a^3*d) + Cot[c
 + d*x]^3/(a^3*d) - (11*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x
]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rubi [A] time = 0.318165, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2875, 2873, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{11\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/a^3 + (13*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + Cot[c + d*x]/(a^3*d) + Cot[c
 + d*x]^3/(a^3*d) - (11*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x
]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \cot^2(c+dx)\csc^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(-a^3\cot^2(c+dx)+3a^3\cot^2(c+dx)\csc(c+dx)-3a^3\cot^2(c+dx)\csc^2(c+dx)\right.}{a^6}\\
&= -\frac{\int \cot^2(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^2(c+dx)\csc(c+dx}{a^3}\\
&= \frac{\cot(c+dx)}{a^3d}-\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{\int \csc^3(c+d}{4a^3}\\
&= \frac{x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{11\cot(c+dx)\csc(c+}{8a^3d}\\
&= \frac{x}{a^3}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{11\cot(c+dx)\csc(c}{8a^3d}
\end{aligned}$$

Mathematica [A] time = 2.37377, size = 165, normalized size = 1.7

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-22\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+22\sec^2\left(\frac{1}{2}(c+dx)\right)+(4\sin(c+dx)-\right.}{64a^3d(\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-22*Csc[(c + d*x)/2]^2 + 22*Sec[(
c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 + 8*(8*c + 8*d*x + 13*Log[Cos[(c + d*x)/
2]] - 13*Log[Sin[(c + d*x)/2]] - 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4) + Csc
```

```
[(c + d*x)/2]^4*(-1 + 4*Sin[c + d*x])))/(64*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [B] time = 0.171, size = 188, normalized size = 1.9

$$\frac{1}{64\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{1}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{3}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{8\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan(\tan(1/2}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/64/d/a^3*tan(1/2*d*x+1/2*c)^4-1/8/d/a^3*tan(1/2*d*x+1/2*c)^3+3/8/d/a^3*ta
n(1/2*d*x+1/2*c)^2-1/8/d/a^3*tan(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+
1/2*c))-1/64/d/a^3/tan(1/2*d*x+1/2*c)^4+1/8/d/a^3/tan(1/2*d*x+1/2*c)^3-3/8/
d/a^3/tan(1/2*d*x+1/2*c)^2+1/8/d/a^3/tan(1/2*d*x+1/2*c)-13/8/d/a^3*ln(tan(1
/2*d*x+1/2*c))
```

Maxima [B] time = 1.55326, size = 294, normalized size = 3.03

$$-\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^3}-\frac{128\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{104\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx}{(\cos(dx+c}\right.}{a^3\sin(dx+c)^4}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/64*((8*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4)/a^3 - 128*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 104*log
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (8*sin(d*x + c)/(cos(d*x + c) + 1)
- 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 - 1)*(cos(d*x + c) + 1)^4/(a^3*sin(d*x + c)^4))/d
```

Fricas [A] time = 1.17891, size = 452, normalized size = 4.66

$$\frac{16\,dx\cos(dx+c)^4-32\,dx\cos(dx+c)^2+22\,\cos(dx+c)^3+16\,dx+13\left(\cos(dx+c)^4-2\,\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\,\cos\right.}{16\left(a^3d\cos(dx+c)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(16*d*x*cos(d*x + c)^4 - 32*d*x*cos(d*x + c)^2 + 22*cos(d*x + c)^3 + 1
6*d*x + 13*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1
/2) - 13*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/
2) + 16*cos(d*x + c)*sin(d*x + c) - 26*cos(d*x + c))/(a^3*d*cos(d*x + c)^4
- 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3419, size = 224, normalized size = 2.31

$$\frac{\frac{192\,(dx+c)}{a^3} - \frac{312\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} + \frac{650\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4} + \frac{3\left(a^9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots\right.}{\cdots}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*(192*(d*x + c)/a^3 - 312*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + (650*ta
n(1/2*d*x + 1/2*c)^4 + 24*tan(1/2*d*x + 1/2*c)^3 - 72*tan(1/2*d*x + 1/2*c)^
2 + 24*tan(1/2*d*x + 1/2*c) - 3)/(a^3*tan(1/2*d*x + 1/2*c)^4) + 3*(a^9*tan(
1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*c)^3 + 24*a^9*tan(1/2*d*x + 1/
2*c)^2 - 8*a^9*tan(1/2*d*x + 1/2*c))/a^12)/d
```

3.747 $\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=100

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^3d}$$

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(8*a^3*d) - (4*Cot[c + d*x]^3)/(3*a^3*d) - Cot[c
 + d*x]^5/(5*a^3*d) + (Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) + (3*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rubi [A] time = 0.337484, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(8*a^3*d) - (4*Cot[c + d*x]^3)/(3*a^3*d) - Cot[c
 + d*x]^5/(5*a^3*d) + (Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) + (3*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^4(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^2(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\csc^2(c+dx)-3a^3\cot^2(c+dx)\right.}{a^6}$$
$$= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^3}+\frac{3\int \cot^2(c+dx}{}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{\int \csc(c+dx)\,dx}{2a^3}+\frac{3\int \csc}{}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot^3(c+dx)}{a^3d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{3\cot(c+dx)\mathrm{cs}}{4a^3d}$$
$$= -\frac{7\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Mathematica [A] time = 1.69672, size = 189, normalized size = 1.89

$$-\frac{\csc^5(c+dx)\left(-780\sin(2(c+dx))+30\sin(4(c+dx))+560\cos(c+dx)-40\cos(3(c+dx))-136\cos(5(c+dx))-1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]^5*(560*Cos[c + d*x] - 40*Cos[3*(c + d*x)] - 136*Cos[5*(c + d
*x)] + 1050*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 1050*Log[Sin[(c + d*x)/2]]
*Sin[c + d*x] - 780*Sin[2*(c + d*x)] - 525*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
 d*x)] + 525*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 30*Sin[4*(c + d*x)] +
 105*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 105*Log[Sin[(c + d*x)/2]]*Sin
[5*(c + d*x)]))/(1920*a^3*d)
```

Maple [B] time = 0.171, size = 208, normalized size = 2.1

$$\frac{1}{160\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{3}{64\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+\frac{13}{96\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{8\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{7}{16\,d a^3}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/160/d/a^3*tan(1/2*d*x+1/2*c)^5-3/64/d/a^3*tan(1/2*d*x+1/2*c)^4+13/96/d/a^
3*tan(1/2*d*x+1/2*c)^3-1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-7/16/d/a^3*tan(1/2*d*
x+1/2*c)+7/16/d/a^3/tan(1/2*d*x+1/2*c)-1/160/d/a^3/tan(1/2*d*x+1/2*c)^5+3/6
4/d/a^3/tan(1/2*d*x+1/2*c)^4+7/8/d/a^3*ln(tan(1/2*d*x+1/2*c))-13/96/d/a^3/t
an(1/2*d*x+1/2*c)^3+1/8/d/a^3/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.02244, size = 317, normalized size = 3.17

$$-\frac{\frac{\frac{420\sin(dx+c)}{\cos(dx+c)+1}+\frac{120\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{130\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{130\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{420\sin\ldots}{(\cos(d\ldots}\right.}{a^3\sin(dx+c)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/960*((420*sin(d*x + c)/(cos(d*x + c) + 1) + 120*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 130*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 840*log(
sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (45*sin(d*x + c)/(cos(d*x + c) + 1)
 - 130*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 120*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 420*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1)^
5/(a^3*sin(d*x + c)^5))/d
```

Fricas [A] time = 1.15458, size = 468, normalized size = 4.68

$$\frac{272\cos(dx+c)^5-320\cos(dx+c)^3-105\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+1\ldots}{240\left(a^3d\cos(dx+c)^4-2a^3\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(272*cos(d*x + c)^5 - 320*cos(d*x + c)^3 - 105*(cos(d*x + c)^4 - 2*co
s(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x +
 c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3
0*(cos(d*x + c)^3 - 7*cos(d*x + c))*sin(d*x + c))/((a^3*d*cos(d*x + c)^4 -
2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31928, size = 251, normalized size = 2.51

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{1918\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 420\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 130\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 6}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5} + \frac{6a^{12}\tan(}{}}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (1918*tan(1/2*d*x + 1/2*c)^
5 - 420*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c)^3 + 130*tan(1/2*d
*x + 1/2*c)^2 - 45*tan(1/2*d*x + 1/2*c) + 6)/(a^3*tan(1/2*d*x + 1/2*c)^5) +
 (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^12*tan(1/2*d*x + 1/2*c)^4 + 130*a^12
*tan(1/2*d*x + 1/2*c)^3 - 120*a^12*tan(1/2*d*x + 1/2*c)^2 - 420*a^12*tan(1/
2*d*x + 1/2*c))/a^15)/d
```

3.748 $\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=124

$$\frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\cos(c+dx))}{16a^3d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^3d}-\frac{17\cot(c+dx)\csc^3(c+dx)}{24a^3d}+\frac{7\,\mathrm{c}}{}$$

```
[Out] (7*ArcTanh[Cos[c + d*x]])/(16*a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (3*Co
t[c + d*x]^5)/(5*a^3*d) + (7*Cot[c + d*x]*Csc[c + d*x])/(16*a^3*d) - (17*Co
t[c + d*x]*Csc[c + d*x]^3)/(24*a^3*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^
3*d)
```

Rubi [A] time = 0.363856, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2875, 2873, 2607, 30, 2611, 3768, 3770, 14}

$$\frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\cos(c+dx))}{16a^3d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^3d}-\frac{17\cot(c+dx)\csc^3(c+dx)}{24a^3d}+\frac{7\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*ArcTanh[Cos[c + d*x]])/(16*a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (3*Co
t[c + d*x]^5)/(5*a^3*d) + (7*Cot[c + d*x]*Csc[c + d*x])/(16*a^3*d) - (17*Co
t[c + d*x]*Csc[c + d*x]^3)/(24*a^3*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^
3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^5(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^2(c+dx)\csc^2(c+dx)+3a^3\cot^2(c+dx)\csc^3(c+dx)-3a^3\cot^2(c+dx)\right.}{a^6}$$
$$= -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a^3}+\frac{3\int \cot^2(c+d}{}$$
$$= -\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^3d}-\frac{\int \csc^5(c+dx)\,dx}{6a^3}-\frac{3\int \mathrm{c}}{}$$
$$= \frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{8a^3d}-\frac{17\cot(c+dx)\csc^3(c+dx)}{24a^3d}-\frac{\cot(c+d}{}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}+\frac{7\cot(c+dx)\csc(c+d}{16a^3d}$$
$$= \frac{7\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}+\frac{7\cot(c+dx)\csc(c+d}{16a^3d}$$

Mathematica [A] time = 0.971315, size = 242, normalized size = 1.95

$$\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)^6\left(704\tan\left(\tfrac{1}{2}(c+dx)\right)-704\cot\left(\tfrac{1}{2}(c+dx)\right)+210\csc^2\left(\tfrac{1}{2}(c+dx)\right)+5\sec^6\left(\tfrac{1}{2}(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-704*Cot[(c + d*x)/2] + 210*Csc[(
c + d*x)/2]^2 + 840*Log[Cos[(c + d*x)/2]] - 840*Log[Sin[(c + d*x)/2]] - 210
*Sec[(c + d*x)/2]^2 + 90*Sec[(c + d*x)/2]^4 + 5*Sec[(c + d*x)/2]^6 - 544*Cs
c[c + d*x]^3*Sin[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6*(-5 + 18*Sin[c + d*x])
```

```
 + Csc[(c + d*x)/2]^4*(-90 + 34*Sin[c + d*x]) + 704*Tan[(c + d*x)/2] - 36*S
ec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(1920*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [B] time = 0.187, size = 246, normalized size = 2.

$$\frac{1}{384\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6-\frac{3}{160\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7}{128\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4-\frac{7}{96\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{128\,da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/384/d/a^3*tan(1/2*d*x+1/2*c)^6-3/160/d/a^3*tan(1/2*d*x+1/2*c)^5+7/128/d/a
^3*tan(1/2*d*x+1/2*c)^4-7/96/d/a^3*tan(1/2*d*x+1/2*c)^3-1/128/d/a^3*tan(1/2
*d*x+1/2*c)^2+5/16/d/a^3*tan(1/2*d*x+1/2*c)-5/16/d/a^3/tan(1/2*d*x+1/2*c)+3
/160/d/a^3/tan(1/2*d*x+1/2*c)^5-7/128/d/a^3/tan(1/2*d*x+1/2*c)^4-7/16/d/a^3
*ln(tan(1/2*d*x+1/2*c))-1/384/d/a^3/tan(1/2*d*x+1/2*c)^6+7/96/d/a^3/tan(1/2
*d*x+1/2*c)^3+1/128/d/a^3/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.06775, size = 370, normalized size = 2.98

$$\frac{\frac{\frac{600\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{140\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{105\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^3}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\left(\frac{36\sin(dx+c)}{\cos(dx+c)+1}-\frac{105\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{140\sin(\ldots}{(\cos(dx+\ldots}\right.}{}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/1920*((600*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 140*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 105*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6)/a^3 - 840*log(sin(d*x + c)/(cos(d*x + c) + 1))/a
^3 + (36*sin(d*x + c)/(cos(d*x + c) + 1) - 105*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 140*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 600*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c
) + 1)^6/(a^3*sin(d*x + c)^6))/d
```

Fricas [A] time = 1.13023, size = 532, normalized size = 4.29

$$-\frac{210\cos(dx+c)^5-80\cos(dx+c)^3-105\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\ldots\right.}{480\left(a^3d\cos(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/480*(210*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 105*(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 105*(cos
```

```
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c
) + 1/2) - 32*(11*cos(d*x + c)^5 - 20*cos(d*x + c)^3)*sin(d*x + c) - 210*co
s(d*x + c))/(a^3*d*cos(d*x + c)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*
x + c)^2 - a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34516, size = 292, normalized size = 2.35

$$-\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{2058\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-600\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-5}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6}-\frac{5a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-36a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+105a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-140a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+600a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{18}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/1920*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (2058*tan(1/2*d*x + 1/2*c
)^6 - 600*tan(1/2*d*x + 1/2*c)^5 + 15*tan(1/2*d*x + 1/2*c)^4 + 140*tan(1/2*
d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) - 5)/
(a^3*tan(1/2*d*x + 1/2*c)^6) - (5*a^15*tan(1/2*d*x + 1/2*c)^6 - 36*a^15*tan
(1/2*d*x + 1/2*c)^5 + 105*a^15*tan(1/2*d*x + 1/2*c)^4 - 140*a^15*tan(1/2*d*
x + 1/2*c)^3 - 15*a^15*tan(1/2*d*x + 1/2*c)^2 + 600*a^15*tan(1/2*d*x + 1/2*
c))/a^18)/d
```

3.749 $\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d}+\frac{\cot(c+dx)\csc^3(c}{8a^3d}$$

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a^3*d) - (4*Cot[c + d*x]^3)/(3*a^3*d) - Cot[
c + d*x]^5/(a^3*d) - Cot[c + d*x]^7/(7*a^3*d) - (5*Cot[c + d*x]*Csc[c + d*x
])/(16*a^3*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^3*d) + (Cot[c + d*x]*Csc
[c + d*x]^5)/(2*a^3*d)
```

Rubi [A] time = 0.231049, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2709, 3768, 3770, 3767}

$$-\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d}+\frac{\cot(c+dx)\csc^3(c}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a^3*d) - (4*Cot[c + d*x]^3)/(3*a^3*d) - Cot[
c + d*x]^5/(a^3*d) - Cot[c + d*x]^7/(7*a^3*d) - (5*Cot[c + d*x]*Csc[c + d*x
])/(16*a^3*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^3*d) + (Cot[c + d*x]*Csc
[c + d*x]^5)/(2*a^3*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(a^5\csc^3(c+dx) - 3a^5\csc^4(c+dx) + 2a^5\csc^5(c+dx) + 2a^5\csc^6(c+dx) - 3a^5\csc^7(c\right.}{a^8}$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^3} + \frac{\int \csc^8(c+dx)\,dx}{a^3} + \frac{2\int \csc^5(c+dx)\,dx}{a^3} + \frac{2\int \csc^6(c+dx)\,dx}{a^3} - \frac{3\int}{}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2a^3d} - \frac{\cot(c+dx)\csc^3(c+dx)}{2a^3d} + \frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d} + \frac{\int \csc}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{4\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{a^3d} - \frac{\cot^7(c+dx)}{7a^3d} - \frac{5\cot(c+dx)\,\mathrm{c}}{4a^3d}$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{4a^3d} - \frac{4\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{a^3d} - \frac{\cot^7(c+dx)}{7a^3d} - \frac{5\cot(c+dx)}{16a}$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{16a^3d} - \frac{4\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{a^3d} - \frac{\cot^7(c+dx)}{7a^3d} - \frac{5\cot(c+dx)}{16a}$$

Mathematica [A] time = 0.962836, size = 251, normalized size = 1.79

$$\frac{\csc^7(c+dx)\left(4998\sin(2(c+dx)) + 504\sin(4(c+dx)) - 210\sin(6(c+dx)) - 4704\cos(c+dx) + 672\cos(3(c+dx)) +\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^7*(-4704*Cos[c + d*x] + 672*Cos[3*(c + d*x)] + 1120*Cos[5*(c
+ d*x)] - 160*Cos[7*(c + d*x)] - 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] +
3675*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 4998*Sin[2*(c + d*x)] + 2205*Log[
Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c +
d*x)] + 504*Sin[4*(c + d*x)] - 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] +
 735*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 210*Sin[6*(c + d*x)] + 105*Lo
g[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)] - 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c +
 d*x)]))/(21504*a^3*d)
```

Maple [B] time = 0.198, size = 284, normalized size = 2.

$$\frac{1}{896\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{1}{128\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6 + \frac{3}{128\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{5}{128\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 + \frac{1}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/896/d/a^3*tan(1/2*d*x+1/2*c)^7-1/128/d/a^3*tan(1/2*d*x+1/2*c)^6+3/128/d/a
^3*tan(1/2*d*x+1/2*c)^5-5/128/d/a^3*tan(1/2*d*x+1/2*c)^4+13/384/d/a^3*tan(1
/2*d*x+1/2*c)^3+3/128/d/a^3*tan(1/2*d*x+1/2*c)^2-29/128/d/a^3*tan(1/2*d*x+1
/2*c)-1/896/d/a^3/tan(1/2*d*x+1/2*c)^7+29/128/d/a^3/tan(1/2*d*x+1/2*c)-3/12
8/d/a^3/tan(1/2*d*x+1/2*c)^5+5/128/d/a^3/tan(1/2*d*x+1/2*c)^4+5/16/d/a^3*ln
(tan(1/2*d*x+1/2*c))+1/128/d/a^3/tan(1/2*d*x+1/2*c)^6-13/384/d/a^3/tan(1/2*
d*x+1/2*c)^3-3/128/d/a^3/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.01462, size = 425, normalized size = 3.04

$$-\frac{\frac{\frac{609 \sin(dx+c)}{\cos(dx+c)+1}-\frac{63 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{91 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{105 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{63 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}-\frac{840 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\left(\frac{21 \sin(dx+c)}{\cos(dx+c)+1}-\frac{63 \sin\cdots}{(\cos(dx\cdots}\right.}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2688*((609*sin(d*x + c)/(cos(d*x + c) + 1) - 63*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 91*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 105*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^3 - 8
40*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (21*sin(d*x + c)/(cos(d*x + c
) + 1) - 63*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 105*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 91*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 63*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 + 609*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos(d
*x + c) + 1)^7/(a^3*sin(d*x + c)^7))/d
```

Fricas [A] time = 1.17624, size = 612, normalized size = 4.37

$$\frac{320 \cos(dx+c)^7-1120 \cos(dx+c)^5+896 \cos(dx+c)^3-105\left(\cos(dx+c)^6-3 \cos(dx+c)^4+3 \cos(dx+c)^2-1\right)\text{l}\cdots}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/672*(320*cos(d*x + c)^7 - 1120*cos(d*x + c)^5 + 896*cos(d*x + c)^3 - 105*
(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 42*(5*cos(d*x +
c)^5 - 8*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(d*x + c))/((a^3*d*cos(d*x + c
)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**8/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37335, size = 329, normalized size = 2.35

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{2178\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-609\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+91\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\dots}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (2178*tan(1/2*d*x + 1/2*c)
^7 - 609*tan(1/2*d*x + 1/2*c)^6 + 63*tan(1/2*d*x + 1/2*c)^5 + 91*tan(1/2*d*
x + 1/2*c)^4 - 105*tan(1/2*d*x + 1/2*c)^3 + 63*tan(1/2*d*x + 1/2*c)^2 - 21*
tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2*d*x + 1/2*c)^7) + (3*a^18*tan(1/2*d*
x + 1/2*c)^7 - 21*a^18*tan(1/2*d*x + 1/2*c)^6 + 63*a^18*tan(1/2*d*x + 1/2*c
)^5 - 105*a^18*tan(1/2*d*x + 1/2*c)^4 + 91*a^18*tan(1/2*d*x + 1/2*c)^3 + 63
*a^18*tan(1/2*d*x + 1/2*c)^2 - 609*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

3.750 $\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=166

$$\frac{3\cot^7(c+dx)}{7a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{29\tanh^{-1}(\cos(c+dx))}{128a^3d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8a^3d}-\frac{23\cot(c+dx)}{48a}$$

```
[Out] (29*ArcTanh[Cos[c + d*x]])/(128*a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (7*
Cot[c + d*x]^5)/(5*a^3*d) + (3*Cot[c + d*x]^7)/(7*a^3*d) + (29*Cot[c + d*x]
*Csc[c + d*x])/(128*a^3*d) + (29*Cot[c + d*x]*Csc[c + d*x]^3)/(192*a^3*d) -
 (23*Cot[c + d*x]*Csc[c + d*x]^5)/(48*a^3*d) - (Cot[c + d*x]*Csc[c + d*x]^7
)/(8*a^3*d)
```

Rubi [A] time = 0.405674, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2875, 2873, 2607, 14, 2611, 3768, 3770, 270}

$$\frac{3\cot^7(c+dx)}{7a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{29\tanh^{-1}(\cos(c+dx))}{128a^3d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8a^3d}-\frac{23\cot(c+dx)}{48a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (29*ArcTanh[Cos[c + d*x]])/(128*a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (7*
Cot[c + d*x]^5)/(5*a^3*d) + (3*Cot[c + d*x]^7)/(7*a^3*d) + (29*Cot[c + d*x]
*Csc[c + d*x])/(128*a^3*d) + (29*Cot[c + d*x]*Csc[c + d*x]^3)/(192*a^3*d) -
 (23*Cot[c + d*x]*Csc[c + d*x]^5)/(48*a^3*d) - (Cot[c + d*x]*Csc[c + d*x]^7
)/(8*a^3*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \cot^2(c+dx)\csc^7(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(-a^3\cot^2(c+dx)\csc^4(c+dx)+3a^3\cot^2(c+dx)\csc^5(c+dx)-3a^3\cot^2(c+dx)\cdots\right.}{a^6}\\
&= -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^7(c+dx)\,dx}{a^3}+\frac{3\int \cot^2(c+dx\cdots}{\cdots}\\
&= -\frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8a^3d}-\frac{\int \csc^7(c+dx)\,dx}{8a^3}-\frac{\int \csc^5(\cdots}{\cdots}\\
&= \frac{\cot(c+dx)\csc^3(c+dx)}{8a^3d}-\frac{23\cot(c+dx)\csc^5(c+dx)}{48a^3d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8a^3d}-\cdots\\
&= \frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+dx)}{7a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{16a^3d}+\frac{29\,\cdots}{\cdots}\\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+dx)}{7a^3d}+\frac{29\cot\cdots}{\cdots}\\
&= \frac{29\tanh^{-1}(\cos(c+dx))}{128a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+dx)}{7a^3d}+\frac{29\,\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 4.96387, size = 317, normalized size = 1.91

$$
-\frac{\sin^7(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^6\left(15(7\csc(c+dx)-24)\csc^8\left(\frac{1}{2}(c+dx)\right)+4(455\csc(c+dx)-276\cdots\right.}{\cdots}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^6*(Csc[(c + d*x)/2]^4*(1328 - 210*C
sc[c + d*x]) + 15*Csc[(c + d*x)/2]^8*(-24 + 7*Csc[c + d*x]) + 4*Csc[(c + d*
x)/2]^6*(-276 + 455*Csc[c + d*x]) - 4*Csc[(c + d*x)/2]^2*(-4864 + 3045*Csc[
c + d*x]) - 8*(6090*Csc[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)
/2]]) + ((2833 + 4616*Cos[c + d*x] + 1907*Cos[2*(c + d*x)] + 304*Cos[3*(c +
 d*x)])*Sec[(c + d*x)/2]^8)/4 - 6090*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 - 42
0*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4 + 14560*Csc[c + d*x]^7*Sin[(c + d*x)/2]
^6 + 3360*Csc[c + d*x]^9*Sin[(c + d*x)/2]^8))*Sin[c + d*x]^7)/(13762560*a^3
*d*(1 + Sin[c + d*x])^3)
```

Maple [B] time = 0.206, size = 322, normalized size = 1.9

$$\frac{1}{2048\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}-\frac{3}{896\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}+\frac{1}{96\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}-\frac{13}{640\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}+\frac{7}{256\,da^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2048/d/a^3*tan(1/2*d*x+1/2*c)^8-3/896/d/a^3*tan(1/2*d*x+1/2*c)^7+1/96/d/a
^3*tan(1/2*d*x+1/2*c)^6-13/640/d/a^3*tan(1/2*d*x+1/2*c)^5+7/256/d/a^3*tan(1
/2*d*x+1/2*c)^4-7/384/d/a^3*tan(1/2*d*x+1/2*c)^3-1/32/d/a^3*tan(1/2*d*x+1/2
*c)^2+23/128/d/a^3*tan(1/2*d*x+1/2*c)+3/896/d/a^3/tan(1/2*d*x+1/2*c)^7-23/1
28/d/a^3/tan(1/2*d*x+1/2*c)-1/2048/d/a^3/tan(1/2*d*x+1/2*c)^8+13/640/d/a^3/
tan(1/2*d*x+1/2*c)^5-7/256/d/a^3/tan(1/2*d*x+1/2*c)^4-29/128/d/a^3*ln(tan(1
/2*d*x+1/2*c))-1/96/d/a^3/tan(1/2*d*x+1/2*c)^6+7/384/d/a^3/tan(1/2*d*x+1/2*
c)^3+1/32/d/a^3/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.07134, size = 478, normalized size = 2.88

$$\frac{\frac{\frac{38640\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{6720\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{3920\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5880\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{4368\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2240\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{720\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{105\,\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}{a^{3}}-\frac{48720\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}+}{215040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/215040*((38640*sin(d*x + c)/(cos(d*x + c) + 1) - 6720*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 3920*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5880*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 4368*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 22
40*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 720*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 105*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/a^3 - 48720*log(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 + (720*sin(d*x + c)/(cos(d*x + c) + 1) - 2240*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4368*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 5880*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3920*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 + 6720*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 38640*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 - 105)*(cos(d*x + c) + 1)^8/(a^3*sin(d*x + c)^8))/
d
```

Fricas [A] time = 1.17516, size = 691, normalized size = 4.16

$$-\frac{6090 \cos(dx+c)^7 - 22330 \cos(dx+c)^5 + 13510 \cos(dx+c)^3 - 3045 \left(\cos(dx+c)^8 - 4 \cos(dx+c)^6 + 6 \cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/26880*(6090*cos(d*x + c)^7 - 22330*cos(d*x + c)^5 + 13510*cos(d*x + c)^3
 - 3045*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x +
 c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) + 3045*(cos(d*x + c)^8 - 4*cos(d*x +
 c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/
2) - 256*(38*cos(d*x + c)^7 - 133*cos(d*x + c)^5 + 140*cos(d*x + c)^3)*sin(
d*x + c) + 6090*cos(d*x + c))/(a^3*d*cos(d*x + c)^8 - 4*a^3*d*cos(d*x + c)^
6 + 6*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**9/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28704, size = 370, normalized size = 2.23

$$-\frac{\frac{48720 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{132414 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 - 38640 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 6720 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 + 3920 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 5880 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 +}{a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/215040*(48720*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (132414*tan(1/2*d*x +
 1/2*c)^8 - 38640*tan(1/2*d*x + 1/2*c)^7 + 6720*tan(1/2*d*x + 1/2*c)^6 + 39
20*tan(1/2*d*x + 1/2*c)^5 - 5880*tan(1/2*d*x + 1/2*c)^4 + 4368*tan(1/2*d*x
+ 1/2*c)^3 - 2240*tan(1/2*d*x + 1/2*c)^2 + 720*tan(1/2*d*x + 1/2*c) - 105)/
(a^3*tan(1/2*d*x + 1/2*c)^8) - (105*a^21*tan(1/2*d*x + 1/2*c)^8 - 720*a^21*
tan(1/2*d*x + 1/2*c)^7 + 2240*a^21*tan(1/2*d*x + 1/2*c)^6 - 4368*a^21*tan(1
/2*d*x + 1/2*c)^5 + 5880*a^21*tan(1/2*d*x + 1/2*c)^4 - 3920*a^21*tan(1/2*d*
x + 1/2*c)^3 - 6720*a^21*tan(1/2*d*x + 1/2*c)^2 + 38640*a^21*tan(1/2*d*x +
1/2*c))/a^24)/d
```

3.751 $\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=82

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{2a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

```
[Out] (-3*a*x)/2 + (2*a*Cos[c + d*x])/d - (a*Cos[c + d*x]^3)/(3*d) + (a*Sec[c + d
*x])/d + (3*a*Tan[c + d*x])/(2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.130793, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2591, 288, 321, 203, 2590, 270}

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{2a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*x)/2 + (2*a*Cos[c + d*x])/d - (a*Cos[c + d*x]^3)/(3*d) + (a*Sec[c + d
*x])/d + (3*a*Tan[c + d*x])/(2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin^2(c+dx)\tan^2(c+dx)\,dx + a\int \sin^3(c+dx)\tan^2(c+dx)$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x\right.}{d}$$

$$= -\frac{a\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx, x, c\right.}{d}$$

$$= \frac{2a\cos(c+dx)}{d} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} + \frac{3a\tan(c+dx}{2d}$$

$$= -\frac{3ax}{2} + \frac{2a\cos(c+dx)}{d} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} + \frac{3a\,\text{ta}}{}$$

Mathematica [A] time = 0.300721, size = 82, normalized size = 1.

$$-\frac{3a(c+dx)}{2d} + \frac{a\sin(2(c+dx))}{4d} + \frac{7a\cos(c+dx)}{4d} - \frac{a\cos(3(c+dx))}{12d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (7*a*Cos[c + d*x])/(4*d) - (a*Cos[3*(c + d*x)])/(1
2*d) + (a*Sec[c + d*x])/d + (a*Sin[2*(c + d*x)])/(4*d) + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.05, size = 104, normalized size = 1.3

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^6}{\cos(dx+c)} + \left(\frac{8}{3} + (\sin(dx+c))^4 + \frac{4(\sin(dx+c))^2}{3}\right)\cos(dx+c)\right) + a\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)} + \left((\sin(dx+c))^3 + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x
+c))+a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/
```

```
2*d*x-3/2*c))
```

Maxima [A] time = 1.53837, size = 101, normalized size = 1.23

$$-\frac{2\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a+3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\,\tan\left(dx+c\right)\right)a}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a + 3*(3*d*x + 3
*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a)/d
```

Fricas [A] time = 1.11707, size = 333, normalized size = 4.06

$$-\frac{2\,a\cos\left(dx+c\right)^{4}-a\cos\left(dx+c\right)^{3}+9\,adx-12\,a\cos\left(dx+c\right)^{2}+3\left(3\,adx-5\,a\right)\cos\left(dx+c\right)-\left(2\,a\cos\left(dx+c\right)^{3}+9\,adx\right.}{6\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*cos(d*x + c)^4 - a*cos(d*x + c)^3 + 9*a*d*x - 12*a*cos(d*x + c)^2
 + 3*(3*a*d*x - 5*a)*cos(d*x + c) - (2*a*cos(d*x + c)^3 + 9*a*d*x + 3*a*cos
(d*x + c)^2 - 9*a*cos(d*x + c) + 6*a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c) -
 d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13676, size = 142, normalized size = 1.73

$$-\frac{9\left(dx+c\right)a+\frac{12\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-10\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a + 12*a/(tan(1/2*d*x + 1/2*c) - 1) + 2*(3*a*tan(1/2*d*x
+ 1/2*c)^5 - 6*a*tan(1/2*d*x + 1/2*c)^4 - 24*a*tan(1/2*d*x + 1/2*c)^2 - 3*a
*tan(1/2*d*x + 1/2*c) - 10*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.752 $\int \sin(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

```
[Out] (-3*a*x)/2 + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (3*a*Tan[c + d*x])/(
2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.102326, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2838, 2590, 14, 2591, 288, 321, 203}

$$\frac{a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*x)/2 + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (3*a*Tan[c + d*x])/(
2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin(c+dx)\tan^2(c+dx)\,dx + a\int \sin^2(c+dx)\tan^2(c+dx)\,d$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \text{ta}\right.}{d}$$

$$= -\frac{a\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(c+\right.}{d}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3a\tan(c+dx)}{2d} - \frac{a\sin^2(c+dx)\,\text{t}}{2d}$$

$$= -\frac{3ax}{2} + \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3a\tan(c+dx)}{2d} - \frac{a\sin^2(}{}$$

Mathematica [A] time = 0.102209, size = 63, normalized size = 0.97

$$-\frac{3a(c+dx)}{2d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\cos(c+dx)}{d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (a*Sin[2
*(c + d*x)])/(4*d) + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.05, size = 94, normalized size = 1.5

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)} + \left((\sin(dx+c))^3 + \frac{3\sin(dx+c)}{2}\right)\cos(dx+c) - \frac{3dx}{2} - \frac{3c}{2}\right) + a\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)} + \left(2 + (\sin(d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

[Out] 1/d*(a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))

Maxima [A] time = 1.55021, size = 84, normalized size = 1.29

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\,\tan\left(dx+c\right)\right)a-2\,a\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a - 2*a*(1/cos(d*x + c) + cos(d*x + c)))/d

---

Fricas [A] time = 1.0835, size = 263, normalized size = 4.05

$$\frac{a\cos\left(dx+c\right)^{3}-3\,adx+2\,a\cos\left(dx+c\right)^{2}-3\left(adx-a\right)\cos\left(dx+c\right)+\left(3\,adx+a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+2\,a\right)\sin\left(\right.}{2\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(a*cos(d*x + c)^3 - 3*a*d*x + 2*a*cos(d*x + c)^2 - 3*(a*d*x - a)*cos(d*x + c) + (3*a*d*x + a*cos(d*x + c)^2 - a*cos(d*x + c) + 2*a)*sin(d*x + c) + 2*a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [A] time = 1.28156, size = 122, normalized size = 1.88

$$-\frac{3\left(dx+c\right)a+\frac{4\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")

```
[Out] -1/2*(3*(d*x + c)*a + 4*a/(tan(1/2*d*x + 1/2*c) - 1) + 2*(a*tan(1/2*d*x + 1
/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(1
/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.753 $\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=39

$$\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}-ax$$

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (a*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rubi [A] time = 0.10432, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2708, 2746, 12, 2735, 2648}

$$\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (a*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx &= a^2\int \frac{\sin^2(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= \frac{a\cos(c+dx)}{d}+a\int \frac{a\sin(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= \frac{a\cos(c+dx)}{d}+a^2\int \frac{\sin(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= -ax+\frac{a\cos(c+dx)}{d}+a^2\int \frac{1}{a-a\sin(c+dx)}\,dx\\ &= -ax+\frac{a\cos(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0333655, size = 47, normalized size = 1.21

$$\frac{a\cos(c+dx)}{d}-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

Maple [A] time = 0.046, size = 59, normalized size = 1.5

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+a(\tan(dx+c)-dx-c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a*(tan(d*x+c)-
d*x-c))
```

Maxima [A] time = 1.57888, size = 53, normalized size = 1.36

$$-\frac{(dx+c-\tan(dx+c))a-a\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [B] time = 1.04934, size = 194, normalized size = 4.97

$$-\frac{adx - a\cos(dx + c)^2 + (adx - 2a)\cos(dx + c) - (adx - a\cos(dx + c) + a)\sin(dx + c) - a}{d\cos(dx + c) - d\sin(dx + c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x - a*cos(d*x + c)^2 + (a*d*x - 2*a)*cos(d*x + c) - (a*d*x - a*cos(d*
x + c) + a)*sin(d*x + c) - a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25164, size = 109, normalized size = 2.79

$$-\frac{(dx + c)a + \frac{2\left(a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2a\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) + 2*a)
/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) -
1))/d
```

3.754 $\int \sec(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=27

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-ax$$

```
[Out] -(a*x) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0465878, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2838, 2606, 8, 3473}

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(a*x) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx &= a\int \sec(c+dx)\tan(c+dx)\,dx+a\int \tan^2(c+dx)\,dx\\ &= \frac{a\tan(c+dx)}{d}-a\int 1\,dx+\frac{a\,\text{Subst}\left(\int 1\,dx,x,\sec(c+dx)\right)}{d}\\ &= -ax+\frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0197155, size = 36, normalized size = 1.33

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.037, size = 32, normalized size = 1.2

$$\frac{1}{d}\left(a\left(\tan\left(dx+c\right)-dx-c\right)+\frac{a}{\cos\left(dx+c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+a/cos(d*x+c))
```

Maxima [A] time = 1.65113, size = 43, normalized size = 1.59

$$-\frac{(dx+c-\tan\left(dx+c\right))a-\frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - a/cos(d*x + c))/d
```

Fricas [B] time = 1.06085, size = 143, normalized size = 5.3

$$-\frac{adx+(adx-a)\cos\left(dx+c\right)-(adx+a)\sin\left(dx+c\right)-a}{d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x + (a*d*x - a)*cos(d*x + c) - (a*d*x + a)*sin(d*x + c) - a)/(d*cos(d
*x + c) - d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int\sin^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2*sec
(c + d*x)**2, x))
```

Giac [A] time = 1.28639, size = 39, normalized size = 1.44

$$-\frac{(dx+c)a+\frac{2a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + 2*a/(tan(1/2*d*x + 1/2*c) - 1))/d
```

3.755 $\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=36

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0742867, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2622, 321, 207, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]

[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \sec^2(c+dx)\,dx + a\int \csc(c+dx)\sec^2(c+dx)\,dx$$
$$= -\frac{a\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c$$
$$= \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}$$

Mathematica [A] time = 0.0322275, size = 56, normalized size = 1.56

$$\frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[(c + d*x)/2]])/d) + (a*Log[Sin[(c + d*x)/2]])/d + (a*Sec[c + d
*x])/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.066, size = 47, normalized size = 1.3

$$\frac{a\tan(dx+c)}{d} + \frac{a}{d\cos(dx+c)} + \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*tan(d*x+c)/d+1/d*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.06426, size = 65, normalized size = 1.81

$$\frac{a\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) + 2\,a\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 2
*a*tan(d*x + c))/d
```

Fricas [B] time = 1.08253, size = 302, normalized size = 8.39

$$\frac{2\,a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)-a\sin\left(dx+c\right)+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a\cos\left(dx+c\right)-a\sin\left(dx+c\right)+a\right)\log\left(\cdot\right.}{2\,\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cos(d*x + c) - (a*cos(d*x + c) - a*sin(d*x + c) + a)*log(1/2*cos(d
*x + c) + 1/2) + (a*cos(d*x + c) - a*sin(d*x + c) + a)*log(-1/2*cos(d*x + c
) + 1/2) + 2*a*sin(d*x + c) + 2*a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26279, size = 46, normalized size = 1.28

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*a/(tan(1/2*d*x + 1/2*c) - 1))/d
```

3.756 $\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=48

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (a*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

Rubi [A] time = 0.108878, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2620, 14, 2622, 321, 207}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (a*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx &= a\int \csc(c+dx)\sec^2(c+dx)\,dx + a\int \csc^2(c+dx)\sec^2(c+dx)\,dx\\ &= \frac{a\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c\right.}{d}\\ &= \frac{a\sec(c+dx)}{d} + \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\right.}{}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+}{d}\end{aligned}$$

Mathematica [A] time = 0.0752202, size = 68, normalized size = 1.42

$$\frac{a\tan(c+dx)}{d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/
2]])/d + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.073, size = 69, normalized size = 1.4

$$\frac{a}{d\cos(dx+c)} + \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d} + \frac{a}{d\sin(dx+c)\cos(dx+c)} - 2\frac{a\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/d*a/sin(d*x+c)/cos(d*x+c
)-2*a*cot(d*x+c)/d
```

Maxima [A] time = 1.09502, size = 80, normalized size = 1.67

$$\frac{a\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 2a\left(\frac{1}{\tan(dx+c)} - \tan(dx+c)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 2
*a*(1/tan(d*x + c) - tan(d*x + c)))/d
```

Fricas [B] time = 1.11007, size = 446, normalized size = 9.29

$$-\frac{4\,a\cos\left(dx+c\right)^{2}+2\,a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^{2}+\left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(d\cos\left(dx+c\right)^{2}+\left(d\,c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) + (a*cos(d*x + c)^2 + (a*cos(d*
x + c) + a)*sin(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) - (a*cos(d*x + c)
^2 + (a*cos(d*x + c) + a)*sin(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/2) -
2*(2*a*cos(d*x + c) + a)*sin(d*x + c) - 2*a)/(d*cos(d*x + c)^2 + (d*cos(d*x
 + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2505, size = 117, normalized size = 2.44

$$\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*log(abs(tan(1/2*d*x + 1/2*c))) + a*tan(1/2*d*x + 1/2*c) - (a*tan(1
/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2*d*x + 1/2*c)^2 -
 tan(1/2*d*x + 1/2*c)))/d
```

3.757 $\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=75

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d + (3*a*Sec[c + d*x]
)/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.13013, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2622, 288, 321, 207, 2620, 14}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d + (3*a*Sec[c + d*x]
)/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (a*Tan[c + d*x])/d
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^2(c+dx)\,dx + a\int \csc^3(c+dx)\sec^2(c+dx)$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, t\right.}{d}$$

$$= -\frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, \tan(c+\right.}{d}$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d} - \frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + $$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d} - $$

Mathematica [B] time = 1.48626, size = 172, normalized size = 2.29

$$-\frac{2a\cot(2(c+dx))}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} +$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[2*(c + d*x)])/d - (a*Csc[(c + d*x)/2]^2)/(8*d) - (3*a*Log[Cos[(c
+ d*x)/2]])/(2*d) + (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]
^2)/(8*d) + (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
- (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.086, size = 93, normalized size = 1.2

$$\frac{a}{d\sin(dx+c)\cos(dx+c)} - 2\frac{a\cot(dx+c)}{d} - \frac{a}{2d(\sin(dx+c))^2\cos(dx+c)} + \frac{3a}{2d\cos(dx+c)} + \frac{3a\ln(\csc(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*a/sin(d*x+c)/cos(d*x+c)-2*a*cot(d*x+c)/d-1/2/d*a/sin(d*x+c)^2/cos(d*x+c
)+3/2/d*a/cos(d*x+c)+3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.13041, size = 113, normalized size = 1.51

$$\frac{a\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-4a\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*a*(1/tan(d*x + c) - tan(d*x
+ c)))/d
```

Fricas [B] time = 1.13302, size = 660, normalized size = 8.8

$$\frac{8\,a\cos\left(dx+c\right)^3+6\,a\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)-\left(a\cos\left(dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*cos(d*x + c)^3 + 6*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 3*(a*cos(
d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*sin
(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x + c)^3 + a*cos(d*
x + c)^2 - a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*sin(d*x + c) - a)*log(-1
/2*cos(d*x + c) + 1/2) + 2*(4*a*cos(d*x + c)^2 + a*cos(d*x + c) - 2*a)*sin(
d*x + c) - 4*a)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - (d*
cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29286, size = 138, normalized size = 1.84

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{16\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{18\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 12*a*log(abs(tan(1/2*d*x + 1/2*c))) + 4*a*t
an(1/2*d*x + 1/2*c) - 16*a/(tan(1/2*d*x + 1/2*c) - 1) - (18*a*tan(1/2*d*x +
 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.758 $\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=91

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}-\frac{2a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a*Cot[c + d*x])/d - (a*Cot[c + d*x]
^3)/(3*d) + (3*a*Sec[c + d*x])/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d
) + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.131479, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2620, 270, 2622, 288, 321, 207}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}-\frac{2a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a*Cot[c + d*x])/d - (a*Cot[c + d*x]
^3)/(3*d) + (3*a*Sec[c + d*x])/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d
) + (a*Tan[c + d*x])/d
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc^3(c+dx)\sec^2(c+dx)\,dx + a\int \csc^4(c+dx)\sec^2(c+dx)$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x,\right.}{d}$$

$$= -\frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx, x, \tan\right.}{d}$$

$$= -\frac{2a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} + \frac{3a\sec(c+dx)}{2d} - \frac{a\csc^2(c+d}{}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} + \frac{3a}{}$$

Mathematica [B] time = 4.8328, size = 205, normalized size = 2.25

$$\frac{a\tan(c+dx)}{d} - \frac{5a\cot(c+dx)}{3d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{3a\log\left(\cos\left(\frac{1}{2}\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*a*Cot[c + d*x])/(3*d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(3*d) - (3*a*Log[Cos[(c + d*x)/2]])/(2*d) + (3*a*Log[Sin[(c
 + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d) + (a*Sin[(c + d*x)/2])/(d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (a*Sin[(c + d*x)/2])/(d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])) + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.084, size = 116, normalized size = 1.3

$$-\frac{a}{2d(\sin(dx+c))^2\cos(dx+c)} + \frac{3a}{2d\cos(dx+c)} + \frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d} - \frac{a}{3d(\sin(dx+c))^3\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2/cos(d*x+c)+3/2/d*a/cos(d*x+c)+3/2/d*a*ln(csc(d*x+c)-c
ot(d*x+c))-1/3/d*a/sin(d*x+c)^3/cos(d*x+c)+4/3/d*a/sin(d*x+c)/cos(d*x+c)-8/
3*a*cot(d*x+c)/d
```

Maxima [A] time = 1.08284, size = 132, normalized size = 1.45

$$\frac{3\,a\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)} - 3\,\log\left(\cos\left(dx+c\right)+1\right) + 3\,\log\left(\cos\left(dx+c\right)-1\right)\right) - 4\,a\left(\frac{6\,\tan(dx+c)^2+1}{\tan(dx+c)^3} - 3\,\tan\left(dx+c\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log
(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*a*((6*tan(d*x + c)^2 + 1)
/tan(d*x + c)^3 - 3*tan(d*x + c)))/d
```

Fricas [B] time = 1.12838, size = 817, normalized size = 8.98

$$-\frac{32\,a\cos\left(dx+c\right)^4 + 14\,a\cos\left(dx+c\right)^3 - 48\,a\cos\left(dx+c\right)^2 - 18\,a\cos\left(dx+c\right) + 9\left(a\cos\left(dx+c\right)^4 - 2\,a\cos\left(dx+c\right)^2 + \left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(32*a*cos(d*x + c)^4 + 14*a*cos(d*x + c)^3 - 48*a*cos(d*x + c)^2 - 18
*a*cos(d*x + c) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + (a*cos(d*x + c
)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(1/2*cos(
d*x + c) + 1/2) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + (a*cos(d*x + c
)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(-1/2*cos
(d*x + c) + 1/2) - 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 - 15*a*cos(d
*x + c) - 6*a)*sin(d*x + c) + 12*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2
+ (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) +
 d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27197, size = 176, normalized size = 1.93

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{48\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{6\cdots}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 36*a*log(abs(
tan(1/2*d*x + 1/2*c))) + 21*a*tan(1/2*d*x + 1/2*c) - 48*a/(tan(1/2*d*x + 1/
2*c) - 1) - (66*a*tan(1/2*d*x + 1/2*c)^3 + 21*a*tan(1/2*d*x + 1/2*c)^2 + 3*
a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.759 $\int \sin(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=89

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{3a^2\cos(c+dx)}{d}+\frac{3a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sin^2(c+dx)\tan(c+dx)}{d}-3a^2x$$

```
[Out] -3*a^2*x + (3*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]^3)/(3*d) + (2*a^2*Sec
[c + d*x])/d + (3*a^2*Tan[c + d*x])/d - (a^2*Sin[c + d*x]^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.208373, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2873, 2590, 14, 2591, 288, 321, 203, 270}

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{3a^2\cos(c+dx)}{d}+\frac{3a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sin^2(c+dx)\tan(c+dx)}{d}-3a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -3*a^2*x + (3*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]^3)/(3*d) + (2*a^2*Sec
[c + d*x])/d + (3*a^2*Tan[c + d*x])/d - (a^2*Sin[c + d*x]^2*Tan[c + d*x])/d
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\sin(c+dx)\tan^2(c+dx)+2a^2\sin^2(c+dx)\tan^2(c+dx)+\right.$$

$$= a^2\int \sin(c+dx)\tan^2(c+dx)\,dx + a^2\int \sin^3(c+dx)\tan^2(c+dx$$

$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a^2\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx, x\right.}{d}$$

$$= -\frac{a^2\sin^2(c+dx)\tan(c+dx)}{d} - \frac{a^2\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cos(\right.}{d}$$

$$= \frac{3a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c+dx)}{3d} + \frac{2a^2\sec(c+dx)}{d} + \frac{3a^2\tan(c}{d}$$

$$= -3a^2x + \frac{3a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c+dx)}{3d} + \frac{2a^2\sec(c+dx)}{d} + $$

Mathematica [A] time = 0.514425, size = 161, normalized size = 1.81

$$-\frac{a^2(\sin(c+dx)+1)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)(-6\sin(2(c+dx))-33\cos(c+dx)+\cos(3(c+dx))+36c+36dx)-\sin\left(\frac{1}{2}(c\right.\right.}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(36*c + 36*d*x - 33*Cos[c + d*
x] + Cos[3*(c + d*x)] - 6*Sin[2*(c + d*x)]) - Sin[(c + d*x)/2]*(48 + 36*c +
 36*d*x - 33*Cos[c + d*x] + Cos[3*(c + d*x)] - 6*Sin[2*(c + d*x)])))/(12*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^4)
```

Maple [A] time = 0.062, size = 148, normalized size = 1.7

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^6}{\cos(dx+c)}+\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4(\sin(dx+c))^2}{3}\right)\cos(dx+c)\right)+2a^2\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\sin(dx+c))^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+2*a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x
+c)-3/2*d*x-3/2*c)+a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)
))
```

Maxima [A] time = 1.59813, size = 132, normalized size = 1.48

$$-\frac{\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)a^2+3\left(3dx+3c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan(dx+c)\right)a^2-3a^2\left(\frac{1}{\cos(dx+c)}+\cos(dx\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*((cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^2 + 3*(3*d*x + 3
*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^2 - 3*a^2*(1/cos
(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.08702, size = 362, normalized size = 4.07

$$-\frac{a^2\cos(dx+c)^4-2a^2\cos(dx+c)^3+9a^2dx-9a^2\cos(dx+c)^2-6a^2+3\left(3a^2dx-4a^2\right)\cos(dx+c)-\left(a^2\cos(dx+c\right.}{3\left(d\cos(dx+c)-d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3 + 9*a^2*d*x - 9*a^2*cos(d*x
 + c)^2 - 6*a^2 + 3*(3*a^2*d*x - 4*a^2)*cos(d*x + c) - (a^2*cos(d*x + c)^3
+ 9*a^2*d*x + 3*a^2*cos(d*x + c)^2 - 6*a^2*cos(d*x + c) + 6*a^2)*sin(d*x +
c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28154, size = 161, normalized size = 1.81

$$-\frac{9\,(dx+c)a^2+\frac{12\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(9*(d*x + c)*a^2 + 12*a^2/(tan(1/2*d*x + 1/2*c) - 1) + 2*(3*a^2*tan(1/
2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^4 - 18*a^2*tan(1/2*d*x + 1/2*
c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c) - 8*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/
d
```

3.760 $\int (a + a\sin(c + dx))^2 \tan^2(c + dx)\, dx$

Optimal. Leaf size=71

$$\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{5a^2x}{2}$$

```
[Out] (-5*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c
+ d*x])) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0857938, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 2648, 2638, 2635, 8}

$$\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{5a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-5*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c
+ d*x])) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = a^2\int\left(-2-\frac{2}{-1+\sin(c+dx)}-2\sin(c+dx)-\sin^2(c+dx)\right)dx$$
$$= -2a^2x - a^2\int\sin^2(c+dx)\,dx-\left(2a^2\right)\int\frac{1}{-1+\sin(c+dx)}\,dx-\left(2a^2\right)\int\sin($$
$$= -2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}-$$
$$= -\frac{5a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [B] time = 0.390829, size = 145, normalized size = 2.04

$$-\frac{a^2(\sin(c+dx)+1)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)(10(c+dx)-\sin(2(c+dx))-8\cos(c+dx))+\sin\left(\frac{1}{2}(c+dx)\right)(-2(5c+5dx+\right.}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(10*(c + d*x) - 8*Cos[c + d*x]
 - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(8 + 5*c + 5*d*x) + 8*Cos[c + d
*x] + Sin[2*(c + d*x)])))/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.056, size = 117, normalized size = 1.7

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3\,dx}{2}-\frac{3\,c}{2}\right)+2\,a^2\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\mathrm{si}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+2*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+
a^2*(tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.63799, size = 113, normalized size = 1.59

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)a^2+2\,(dx+c-\tan(dx+c))a^2-4\,a^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^
2 + 2*(d*x + c - tan(d*x + c))*a^2 - 4*a^2*(1/cos(d*x + c) + cos(d*x + c)))
```

```
/d
```

Fricas [A] time = 1.08748, size = 296, normalized size = 4.17

$$\frac{a^2 \cos(dx+c)^3 - 5a^2dx + 4a^2 \cos(dx+c)^2 + 4a^2 - \left(5a^2dx - 7a^2\right)\cos(dx+c) + \left(5a^2dx + a^2\cos(dx+c)^2 - 3a^2\cos(\right.}{2\left(d\cos(dx+c) - d\sin(dx+c) + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*cos(d*x + c)^3 - 5*a^2*d*x + 4*a^2*cos(d*x + c)^2 + 4*a^2 - (5*a^2
*d*x - 7*a^2)*cos(d*x + c) + (5*a^2*d*x + a^2*cos(d*x + c)^2 - 3*a^2*cos(d*
x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20161, size = 138, normalized size = 1.94

$$-\frac{5(dx+c)a^2 + \frac{8a^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1} + \frac{2\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 4a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 4a^2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)*a^2 + 8*a^2/(tan(1/2*d*x + 1/2*c) - 1) + 2*(a^2*tan(1/2*d
*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c) - 4
*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.761 $\int \sec(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{2a^2\cos(c+dx)}{d}-2a^2x+\frac{\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

```
[Out] -2*a^2*x + (2*a^2*Cos[c + d*x])/d + (Sec[c + d*x]*(a + a*Sin[c + d*x])^2)/d
```

Rubi [A] time = 0.0568371, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2855, 2638}

$$\frac{2a^2\cos(c+dx)}{d}-2a^2x+\frac{\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -2*a^2*x + (2*a^2*Cos[c + d*x])/d + (Sec[c + d*x]*(a + a*Sin[c + d*x])^2)/d
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx&=\frac{\sec(c+dx)(a+a\sin(c+dx))^2}{d}-(2a)\int(a+a\sin(c+dx))\,dx\\&=-2a^2x+\frac{\sec(c+dx)(a+a\sin(c+dx))^2}{d}-\left(2a^2\right)\int\sin(c+dx)\,dx\\&=-2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+a\sin(c+dx))^2}{d}\end{aligned}$$

Mathematica [B] time = 0.369367, size = 90, normalized size = 2.09

$$\frac{(a\sin(c+dx)+a)^2\left(-2(c+dx)+\cos(c+dx)+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] ((-2*(c + d*x) + Cos[c + d*x] + (4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]))*(a + a*Sin[c + d*x])^2)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^4)
```

Maple [A] time = 0.049, size = 76, normalized size = 1.8

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+2a^2(\tan(dx+c)-dx-c)+\frac{a^2}{\cos(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(tan(d
*x+c)-d*x-c)+a^2/cos(d*x+c))
```

Maxima [A] time = 1.70752, size = 77, normalized size = 1.79

$$-\frac{2(dx+c-\tan(dx+c))a^2-a^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*x + c - tan(d*x + c))*a^2 - a^2*(1/cos(d*x + c) + cos(d*x + c)) - a^
2/cos(d*x + c))/d
```

Fricas [B] time = 1.08957, size = 230, normalized size = 5.35

$$-\frac{2a^2dx-a^2\cos(dx+c)^2-2a^2+\left(2a^2dx-3a^2\right)\cos(dx+c)-\left(2a^2dx-a^2\cos(dx+c)+2a^2\right)\sin(dx+c)}{d\cos(dx+c)-d\sin(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*d*x - a^2*cos(d*x + c)^2 - 2*a^2 + (2*a^2*d*x - 3*a^2)*cos(d*x + c)
 - (2*a^2*d*x - a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d
*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16254, size = 120, normalized size = 2.79

$$-\frac{2\left((dx+c)a^2+\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((d*x + c)*a^2 + (2*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c
) + 3*a^2)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x +
 1/2*c) - 1))/d
```

3.762 $\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=44

$$\frac{2a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d + (2*a^2*Tan[c +
d*x])/d
```

Rubi [A] time = 0.126434, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2873, 3767, 8, 2622, 321, 207, 2606}

$$\frac{2a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d + (2*a^2*Tan[c +
d*x])/d
```


Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(2a^2\sec^2(c+dx)+a^2\csc(c+dx)\sec^2(c+dx)+a^2\sec(c+dx\right.\\ &= a^2\int \csc(c+dx)\sec^2(c+dx)\,dx+a^2\int \sec(c+dx)\tan(c+dx)\,c\\ &= \frac{a^2\,\mathrm{Subst}(\int 1\,dx,x,\sec(c+dx))}{d}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\sec(c\right.}{d}\\ &= \frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sec\right.}{d}\\ &= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.114016, size = 69, normalized size = 1.57

$$\frac{a^2\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]] + (4*Sin[(c + d*x)/2])
/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])))/d
```

Maple [A] time = 0.089, size = 55, normalized size = 1.3

$$2\,\frac{a^2}{d\cos(dx+c)}+2\,\frac{a^2\tan(dx+c)}{d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d*a^2/cos(d*x+c)+2*a^2*tan(d*x+c)/d+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.08082, size = 88, normalized size = 2.

$$\frac{a^2\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)+4\,a^2\tan(dx+c)+\frac{2\,a^2}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) +
 4*a^2*tan(d*x + c) + 2*a^2/cos(d*x + c))/d
```

Fricas [B] time = 1.12321, size = 327, normalized size = 7.43

$$\frac{4\,a^2\cos\left(dx+c\right)+4\,a^2\sin\left(dx+c\right)+4\,a^2-\left(a^2\cos\left(dx+c\right)-a^2\sin\left(dx+c\right)+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^2\cos\left(dx\right.\right.}{2\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*cos(d*x + c) + 4*a^2*sin(d*x + c) + 4*a^2 - (a^2*cos(d*x + c) -
a^2*sin(d*x + c) + a^2)*log(1/2*cos(d*x + c) + 1/2) + (a^2*cos(d*x + c) - a
^2*sin(d*x + c) + a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c) - d*si
n(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26659, size = 51, normalized size = 1.16

$$\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{4\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 4*a^2/(tan(1/2*d*x + 1/2*c) - 1))/d
```

3.763 $\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=58

$$\frac{2a^2\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] (-2*a^2*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d + (2*a^2*Sec[c + d*
x])/d + (2*a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.222525, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2873, 3767, 8, 2622, 321, 207, 2620, 14}

$$\frac{2a^2\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d + (2*a^2*Sec[c + d*
x])/d + (2*a^2*Tan[c + d*x])/d
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\sec^2(c+dx)+2a^2\csc(c+dx)\sec^2(c+dx)+a^2\csc^2(c+dx)\right.$$
$$= a^2\int \sec^2(c+dx)\,dx + a^2\int \csc^2(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)\int$$
$$= -\frac{a^2\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} + \frac{a^2\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c\right.}{d}$$
$$= \frac{2a^2\sec(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan\right.}{d}$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cot(c+dx)}{d} + \frac{2a^2\sec(c+dx)}{d} + \frac{2a}{}$$

Mathematica [A] time = 0.37997, size = 96, normalized size = 1.66

$$\frac{a^2\left(\tan\left(\frac{1}{2}(c+dx)\right)-\cot\left(\frac{1}{2}(c+dx)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-Cot[(c + d*x)/2] - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]]
 + (8*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + Tan[(c + d*
x)/2]))/(2*d)
```

Maple [A] time = 0.094, size = 92, normalized size = 1.6

$$\frac{a^2\tan(dx+c)}{d}+2\frac{a^2}{d\cos(dx+c)}+2\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{a^2}{d\sin(dx+c)\cos(dx+c)}-2\frac{a^2\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] a^2*tan(d*x+c)/d+2/d*a^2/cos(d*x+c)+2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+1/d*a
^2/sin(d*x+c)/cos(d*x+c)-2*a^2*cot(d*x+c)/d
```

Maxima [A] time = 1.16119, size = 97, normalized size = 1.67

$$\frac{a^2\left(\frac{2}{\cos(dx+c)} - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right)\right) - a^2\left(\frac{1}{\tan(dx+c)} - \tan\left(dx+c\right)\right) + a^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (a^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - a^2
*(1/tan(d*x + c) - tan(d*x + c)) + a^2*tan(d*x + c))/d
```

Fricas [B] time = 1.12086, size = 473, normalized size = 8.16

$$-\frac{3\,a^2\cos\left(dx+c\right)^2 + a^2\cos\left(dx+c\right) - 2\,a^2 + \left(a^2\cos\left(dx+c\right)^2 - a^2 + \left(a^2\cos\left(dx+c\right) + a^2\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(d\right.\right.}{d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(3*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - 2*a^2 + (a^2*cos(d*x + c)^2 - a
^2 + (a^2*cos(d*x + c) + a^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (
a^2*cos(d*x + c)^2 - a^2 + (a^2*cos(d*x + c) + a^2)*sin(d*x + c))*log(-1/2*
cos(d*x + c) + 1/2) - (3*a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))/(d*cos(d*x
 + c)^2 + (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27156, size = 132, normalized size = 2.28

$$\frac{4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 7\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a
^2*tan(1/2*d*x + 1/2*c)^2 + 7*a^2*tan(1/2*d*x + 1/2*c) - a^2)/(tan(1/2*d*x
+ 1/2*c)^2 - tan(1/2*d*x + 1/2*c)))/d
```

3.764 $\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=86

$$\frac{2a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{5a^2\sec(c+dx)}{2d}-\frac{5a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (-5*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a^2*Cot[c + d*x])/d + (5*a^2*Sec[
c + d*x])/(2*d) - (a^2*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (2*a^2*Tan[c +
d*x])/d
```

Rubi [A] time = 0.210062, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2873, 2622, 321, 207, 2620, 14, 288}

$$\frac{2a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{5a^2\sec(c+dx)}{2d}-\frac{5a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a^2*Cot[c + d*x])/d + (5*a^2*Sec[
c + d*x])/(2*d) - (a^2*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (2*a^2*Tan[c +
d*x])/d
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\csc(c+dx)\sec^2(c+dx)+2a^2\csc^2(c+dx)\sec^2(c+dx)+a^2\right.$$

$$= a^2\int \csc(c+dx)\sec^2(c+dx)\,dx + a^2\int \csc^3(c+dx)\sec^2(c+dx)\,d$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, s\right.}{d}$$

$$= \frac{a^2\sec(c+dx)}{d} - \frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,d\right.}{d}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^2\cot(c+dx)}{d} + \frac{5a^2\sec(c+dx)}{2d} - \frac{a^2}{}$$

$$= -\frac{5a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^2\cot(c+dx)}{d} + \frac{5a^2\sec(c+dx)}{2d} - \frac{a}{}$$

Mathematica [A] time = 1.13372, size = 124, normalized size = 1.44

$$\frac{a^2\left(8\tan\left(\frac{1}{2}(c+dx)\right)-8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+20\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-20\log\left(\cos\left(\right.\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-8*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 20*Log[Cos[(c + d*x)/2]] +
 20*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 + (32*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]) + 8*Tan[(c + d*x)/2]))/(8*d)
```

Maple [A] time = 0.111, size = 104, normalized size = 1.2

$$\frac{5\,a^2}{2\,d\cos(dx+c)} + \frac{5\,a^2\ln(\csc(dx+c)-\cot(dx+c))}{2\,d} + 2\,\frac{a^2}{d\sin(dx+c)\cos(dx+c)} - 4\,\frac{a^2\cot(dx+c)}{d} - \frac{a}{2\,d\,(\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 5/2/d*a^2/cos(d*x+c)+5/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+2/d*a^2/sin(d*x+c)
/cos(d*x+c)-4*a^2*cot(d*x+c)/d-1/2/d*a^2/sin(d*x+c)^2/cos(d*x+c)
```

Maxima [A] time = 1.18263, size = 167, normalized size = 1.94

$$\frac{a^2\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)} - 3\log\left(\cos\left(dx+c\right)+1\right) + 3\log\left(\cos\left(dx+c\right)-1\right)\right) + 2a^2\left(\frac{2}{\cos(dx+c)} - \log\left(\cos\left(dx+c\right)+1\right) + l\cdots\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*a^2*(2/cos(d*x + c) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 8*a^2*(1/tan(d*x + c) - tan(d*
x + c)))/d
```

Fricas [B] time = 1.13002, size = 718, normalized size = 8.35

$$\frac{16\,a^2\cos\left(dx+c\right)^3 + 10\,a^2\cos\left(dx+c\right)^2 - 14\,a^2\cos\left(dx+c\right) - 8\,a^2 - 5\left(a^2\cos\left(dx+c\right)^3 + a^2\cos\left(dx+c\right)^2 - a^2\cos\left(dx\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(16*a^2*cos(d*x + c)^3 + 10*a^2*cos(d*x + c)^2 - 14*a^2*cos(d*x + c) -
8*a^2 - 5*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2
 - (a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 5
*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2 - (a^2*c
os(d*x + c)^2 - a^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(8*a^2*
cos(d*x + c)^2 + 3*a^2*cos(d*x + c) - 4*a^2)*sin(d*x + c))/(d*cos(d*x + c)^
3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - (d*cos(d*x + c)^2 - d)*sin(d*x + c)
 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

[Out] Timed out

Giac [A] time = 1.32934, size = 157, normalized size = 1.83

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{32\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,a^2\tan\left(\frac{1}{2}\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 20*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 8
*a^2*tan(1/2*d*x + 1/2*c) - 32*a^2/(tan(1/2*d*x + 1/2*c) - 1) - (30*a^2*tan
(1/2*d*x + 1/2*c)^2 + 8*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c
)^2)/d
```

3.765 $\int \sin(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=111

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{7a^3\cos(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{19a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] (-51*a^3*x)/8 + (7*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/d + (4*a^3*Co
s[c + d*x])/(d*(1 - Sin[c + d*x])) + (19*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*
d) + (a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.170108, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 2648, 2638, 2635, 8, 2633}

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{7a^3\cos(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{19a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-51*a^3*x)/8 + (7*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/d + (4*a^3*Co
s[c + d*x])/(d*(1 - Sin[c + d*x])) + (19*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*
d) + (a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = a^2\int\left(-4a-\frac{4a}{-1+\sin(c+dx)}-4a\sin(c+dx)-4a\sin^2(c+dx)-3\right.$$
$$=-4a^3x-a^3\int\sin^4(c+dx)\,dx-\left(3a^3\right)\int\sin^3(c+dx)\,dx-\left(4a^3\right)\int$$
$$=-4a^3x+\frac{4a^3\cos(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)\sin}{d}$$
$$=-6a^3x+\frac{7a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+$$
$$=-\frac{51a^3x}{8}+\frac{7a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+$$

Mathematica [A] time = 0.802426, size = 125, normalized size = 1.13

$$\frac{(a\sin(c+dx)+a)^3\left(-204(c+dx)+40\sin(2(c+dx))-\sin(4(c+dx))+200\cos(c+dx)-8\cos(3(c+dx))+\frac{256\sin(}{\cos\left(\frac{1}{2}(c+dx)\right)}\right.}{32d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-204*(c + d*x) + 200*Cos[c + d*x] - 8*Cos[3*(c + d
*x)] + (256*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 40*Si
n[2*(c + d*x)] - Sin[4*(c + d*x)]))/(32*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^6)
```

Maple [B] time = 0.069, size = 212, normalized size = 1.9

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^7}{\cos(dx+c)}+\left((\sin(dx+c))^5+\frac{5(\sin(dx+c))^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+3a^3\left(\frac{(\sin(}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d
*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+3*a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(
d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(
d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+a^3*(sin(d*x+c)^4/cos(d*
x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.70465, size = 219, normalized size = 1.97

$$-\frac{8\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^{3}+\left(15\,dx+15\,c-\frac{9\,\tan(dx+c)^{3}+7\,\tan(dx+c)}{\tan(dx+c)^{4}+2\,\tan(dx+c)^{2}+1}-8\,\tan\left(dx+c\right)\right)a^{3}+12\left(3\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/8*(8*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + (15*d*x +
15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)
^2 + 1) - 8*tan(d*x + c))*a^3 + 12*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c
)^2 + 1) - 2*tan(d*x + c))*a^3 - 8*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.14145, size = 440, normalized size = 3.96

$$-\frac{2\,a^{3}\cos\left(dx+c\right)^{5}+8\,a^{3}\cos\left(dx+c\right)^{4}-15\,a^{3}\cos\left(dx+c\right)^{3}+51\,a^{3}dx-56\,a^{3}\cos\left(dx+c\right)^{2}-32\,a^{3}+\left(51\,a^{3}dx-67\,a^{3}\right.}{8\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/8*(2*a^3*cos(d*x + c)^5 + 8*a^3*cos(d*x + c)^4 - 15*a^3*cos(d*x + c)^3 +
 51*a^3*d*x - 56*a^3*cos(d*x + c)^2 - 32*a^3 + (51*a^3*d*x - 67*a^3)*cos(d*
x + c) + (2*a^3*cos(d*x + c)^4 - 6*a^3*cos(d*x + c)^3 - 51*a^3*d*x - 21*a^3
*cos(d*x + c)^2 + 35*a^3*cos(d*x + c) - 32*a^3)*sin(d*x + c))/(d*cos(d*x +
c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28266, size = 225, normalized size = 2.03

$$-\frac{51\left(dx+c\right)a^{3}+\frac{64\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(19\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-32\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+27\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-144\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-27\,a^{3}\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(51*(d*x + c)*a^3 + 64*a^3/(tan(1/2*d*x + 1/2*c) - 1) + 2*(19*a^3*tan(
1/2*d*x + 1/2*c)^7 - 32*a^3*tan(1/2*d*x + 1/2*c)^6 + 27*a^3*tan(1/2*d*x + 1
/2*c)^5 - 144*a^3*tan(1/2*d*x + 1/2*c)^4 - 27*a^3*tan(1/2*d*x + 1/2*c)^3 -
160*a^3*tan(1/2*d*x + 1/2*c)^2 - 19*a^3*tan(1/2*d*x + 1/2*c) - 48*a^3)/(tan
(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.766 $\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=89

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

```
[Out] (-11*a^3*x)/2 + (5*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) + (4*a^
3*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/
(2*d)
```

Rubi [A] time = 0.12408, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 2648, 2638, 2635, 8, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-11*a^3*x)/2 + (5*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) + (4*a^
3*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/
(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a + a\sin(c + dx))^3 \tan^2(c + dx)\,dx = a^2 \int \left(-4a - \frac{4a}{-1 + \sin(c + dx)} - 4a\sin(c + dx) - 3a\sin^2(c + dx) - a\sin^3(c + dx\right.$$

$$= -4a^3x - a^3 \int \sin^3(c + dx)\,dx - \left(3a^3\right)\int \sin^2(c + dx)\,dx - \left(4a^3\right)\int \frac{1}{-1 + \sin(c +}$$

$$= -4a^3x + \frac{4a^3\cos(c + dx)}{d} + \frac{4a^3\cos(c + dx)}{d(1 - \sin(c + dx))} + \frac{3a^3\cos(c + dx)\sin(c + dx)}{2d} - \frac{1}{2}$$

$$= -\frac{11a^3x}{2} + \frac{5a^3\cos(c + dx)}{d} - \frac{a^3\cos^3(c + dx)}{3d} + \frac{4a^3\cos(c + dx)}{d(1 - \sin(c + dx))} + \frac{3a^3\cos(c +}{}$$

Mathematica [A] time = 0.473272, size = 115, normalized size = 1.29

$$\frac{(a\sin(c + dx) + a)^3\left(-66(c + dx) + 9\sin(2(c + dx)) + 57\cos(c + dx) - \cos(3(c + dx)) + \frac{96\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{12d\left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-66*(c + d*x) + 57*Cos[c + d*x] - Cos[3*(c + d*x)]
 + (96*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 9*Sin[2*(c
 + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.061, size = 167, normalized size = 1.9

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx + c))^6}{\cos(dx + c)} + \left(\frac{8}{3} + (\sin(dx + c))^4 + \frac{4(\sin(dx + c))^2}{3}\right)\cos(dx + c)\right) + 3a^3\left(\frac{(\sin(dx + c))^5}{\cos(dx + c)} + \left((\sin(dx + c))^3 +\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x
+c)-3/2*d*x-3/2*c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+
c))+a^3*(tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.65322, size = 158, normalized size = 1.78

$$-\frac{2\left(\cos(dx + c)^3 - \frac{3}{\cos(dx+c)} - 6\cos(dx + c)\right)a^3 + 9\left(3dx + 3c - \frac{\tan(dx+c)}{\tan(dx+c)^2+1} - 2\tan(dx + c)\right)a^3 + 6(dx + c - \tan(dx +}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + 9*(3*d*x +
 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^3 + 6*(d*x + c
 - tan(d*x + c))*a^3 - 18*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.11567, size = 378, normalized size = 4.25

$$-\frac{2\,a^3\cos\left(dx+c\right)^4 - 7\,a^3\cos\left(dx+c\right)^3 + 33\,a^3dx - 30\,a^3\cos\left(dx+c\right)^2 - 24\,a^3 + 3\left(11\,a^3dx - 15\,a^3\right)\cos\left(dx+c\right) - \left(2\,a\right.}{6\left(d\cos\left(dx+c\right) - d\sin\left(dx+c\right) + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*a^3*cos(d*x + c)^4 - 7*a^3*cos(d*x + c)^3 + 33*a^3*d*x - 30*a^3*cos
(d*x + c)^2 - 24*a^3 + 3*(11*a^3*d*x - 15*a^3)*cos(d*x + c) - (2*a^3*cos(d*
x + c)^3 + 33*a^3*d*x + 9*a^3*cos(d*x + c)^2 - 21*a^3*cos(d*x + c) + 24*a^3
)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3063, size = 161, normalized size = 1.81

$$-\frac{33\,(dx+c)a^3 + \frac{48\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1} + \frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 28\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)*a^3 + 48*a^3/(tan(1/2*d*x + 1/2*c) - 1) + 2*(9*a^3*tan(1
/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4 - 60*a^3*tan(1/2*d*x + 1/
2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 28*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
3)/d
```

3.767 $\int \sec(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=67

$$\frac{6a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{9a^3x}{2}+\frac{\sec(c+dx)(a\sin(c+dx)+a)^3}{d}$$

```
[Out] (-9*a^3*x)/2 + (6*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(
2*d) + (Sec[c + d*x]*(a + a*Sin[c + d*x])^3)/d
```

Rubi [A] time = 0.063891, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2855, 2644}

$$\frac{6a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{9a^3x}{2}+\frac{\sec(c+dx)(a\sin(c+dx)+a)^3}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-9*a^3*x)/2 + (6*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(
2*d) + (Sec[c + d*x]*(a + a*Sin[c + d*x])^3)/d
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx &= \frac{\sec(c+dx)(a+a\sin(c+dx))^3}{d}-(3a)\int (a+a\sin(c+dx))^2\,dx\\ &= -\frac{9a^3x}{2}+\frac{6a^3\cos(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{\sec(c+dx
\end{aligned}$$

Mathematica [B] time = 0.500428, size = 145, normalized size = 2.16

$$-\frac{a^3(\sin(c+dx)+1)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)(18(c+dx)-\sin(2(c+dx))-12\cos(c+dx))+\sin\left(\frac{1}{2}(c+dx)\right)(-2(9c+9dx+16}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -(a^3*(1 + Sin[c + d*x])^3*(Cos[(c + d*x)/2]*(18*(c + d*x) - 12*Cos[c + d*x
] - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(16 + 9*c + 9*d*x) + 12*Cos[c
+ d*x] + Sin[2*(c + d*x)])))/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [B] time = 0.053, size = 130, normalized size = 1.9

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+
3*a^3*(tan(d*x+c)-d*x-c)+a^3/cos(d*x+c))
```

Maxima [A] time = 1.5146, size = 131, normalized size = 1.96

$$-\frac{\left(3dx+3c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan(dx+c)\right)a^3+6(dx+c-\tan(dx+c))a^3-6a^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-\frac{2a^3}{\cos(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^
3 + 6*(d*x + c - tan(d*x + c))*a^3 - 6*a^3*(1/cos(d*x + c) + cos(d*x + c))
- 2*a^3/cos(d*x + c))/d
```

Fricas [A] time = 1.08699, size = 297, normalized size = 4.43

$$\frac{a^3\cos(dx+c)^3-9a^3dx+6a^3\cos(dx+c)^2+8a^3-\left(9a^3dx-13a^3\right)\cos(dx+c)+\left(9a^3dx+a^3\cos(dx+c)^2-5a^3\right.}{2(d\cos(dx+c)-d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*cos(d*x + c)^3 - 9*a^3*d*x + 6*a^3*cos(d*x + c)^2 + 8*a^3 - (9*a^3
*d*x - 13*a^3)*cos(d*x + c) + (9*a^3*d*x + a^3*cos(d*x + c)^2 - 5*a^3*cos(d
*x + c) + 8*a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26537, size = 138, normalized size = 2.06

$$-\frac{9\,(dx+c)a^3 + \frac{16\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1} + \frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 6\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(9*(d*x + c)*a^3 + 16*a^3/(tan(1/2*d*x + 1/2*c) - 1) + 2*(a^3*tan(1/2*
d*x + 1/2*c)^3 - 6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) -
6*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.768 $\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=48

$$\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+a^3(-x)$$

```
[Out] -(a^3*x) - (a^3*ArcTanh[Cos[c + d*x]])/d + (4*a^3*Cos[c + d*x])/(d*(1 - Sin
[c + d*x]))
```

Rubi [A] time = 0.104201, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2872, 3770, 2648}

$$\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+a^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) - (a^3*ArcTanh[Cos[c + d*x]])/d + (4*a^3*Cos[c + d*x])/(d*(1 - Sin
[c + d*x]))
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx &= a^2\int\left(-a+a\csc(c+dx)-\frac{4a}{-1+\sin(c+dx)}\right)dx\\ &= -a^3x+a^3\int\csc(c+dx)\,dx-\left(4a^3\right)\int\frac{1}{-1+\sin(c+dx)}\,dx\\ &= -a^3x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.133245, size = 74, normalized size = 1.54

$$-\frac{a^3\left(-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+c+dx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*(c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]] - (8*Sin[(c
 + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])))/d)
```

Maple [A] time = 0.096, size = 70, normalized size = 1.5

$$-a^3x+4\,\frac{a^3\tan(dx+c)}{d}-\frac{a^3c}{d}+4\,\frac{a^3}{d\cos(dx+c)}+\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -a^3*x+4*a^3*tan(d*x+c)/d-1/d*a^3*c+4/d*a^3/cos(d*x+c)+1/d*a^3*ln(csc(d*x+c
)-cot(d*x+c))
```

Maxima [A] time = 1.47459, size = 113, normalized size = 2.35

$$-\frac{2(dx+c-\tan(dx+c))a^3-a^3\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-6\,a^3\tan(dx+c)-\frac{6\,a^3}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^3 - a^3*(2/cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)) - 6*a^3*tan(d*x + c) - 6*a^3/cos(d*x + c))
/d
```

Fricas [B] time = 1.12461, size = 382, normalized size = 7.96

$$-\frac{2\,a^3dx-8\,a^3+2\left(a^3dx-4\,a^3\right)\cos(dx+c)+\left(a^3\cos(dx+c)-a^3\sin(dx+c)+a^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(a^3\cos\right.}{2\left(d\cos(dx+c)-d\sin(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*d*x - 8*a^3 + 2*(a^3*d*x - 4*a^3)*cos(d*x + c) + (a^3*cos(d*x +
 c) - a^3*sin(d*x + c) + a^3)*log(1/2*cos(d*x + c) + 1/2) - (a^3*cos(d*x +
c) - a^3*sin(d*x + c) + a^3)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*d*x + 4*
```

```
a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27733, size = 66, normalized size = 1.38

$$-\frac{(dx+c)a^3 - a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + \frac{8\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 8*a^3/(tan(1/2*d*x +
 1/2*c) - 1))/d
```

3.769 $\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=56

$$-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (4*a^3*Cos[c + d*
x])/(d*(1 - Sin[c + d*x]))
```

Rubi [A] time = 0.135202, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2872, 3770, 3767, 8, 2648}

$$-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (4*a^3*Cos[c + d*
x])/(d*(1 - Sin[c + d*x]))
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^2\int\left(3a\csc(c+dx)+a\csc^2(c+dx)-\frac{4a}{-1+\sin(c+dx)}\right)dx$$
$$= a^3\int\csc^2(c+dx)\,dx+\left(3a^3\right)\int\csc(c+dx)\,dx-\left(4a^3\right)\int\frac{}{-1+\mathrm{s}}$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{a^3\,\mathrm{Subst}(\int 1}{}$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

Mathematica [A] time = 0.357578, size = 96, normalized size = 1.71

$$\frac{a^3\left(\tan\left(\frac{1}{2}(c+dx)\right)-\cot\left(\frac{1}{2}(c+dx)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{16\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-Cot[(c + d*x)/2] - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]]
 + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + Tan[(c + d
*x)/2]))/(2*d)
```

Maple [A] time = 0.106, size = 93, normalized size = 1.7

$$4\frac{a^3}{d\cos(dx+c)}+3\frac{a^3\tan(dx+c)}{d}+3\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{a^3}{d\sin(dx+c)\cos(dx+c)}-2\frac{a^3\cot(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/d*a^3/cos(d*x+c)+3*a^3*tan(d*x+c)/d+3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+1/d
*a^3/sin(d*x+c)/cos(d*x+c)-2*a^3*cot(d*x+c)/d
```

Maxima [A] time = 0.986882, size = 119, normalized size = 2.12

$$\frac{3a^3\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-2a^3\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)+6a^3\tan(dx+c)+\frac{}{\mathrm{co}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(3*a^3*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1))
 - 2*a^3*(1/tan(d*x + c) - tan(d*x + c)) + 6*a^3*tan(d*x + c) + 2*a^3/cos(d
*x + c))/d
```

Fricas [B] time = 1.1081, size = 490, normalized size = 8.75

$$-\frac{10\,a^3\cos\left(dx+c\right)^2+2\,a^3\cos\left(dx+c\right)-8\,a^3+3\,\left(a^3\cos\left(dx+c\right)^2-a^3+\left(a^3\cos\left(dx+c\right)+a^3\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(\right.\right.}{2\,\left(d\cos\left(dx+c\right)^{\cdot}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(10*a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) - 8*a^3 + 3*(a^3*cos(d*x +
 c)^2 - a^3 + (a^3*cos(d*x + c) + a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) +
 1/2) - 3*(a^3*cos(d*x + c)^2 - a^3 + (a^3*cos(d*x + c) + a^3)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2) - 2*(5*a^3*cos(d*x + c) + 4*a^3)*sin(d*x + c
))/(d*cos(d*x + c)^2 + (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27519, size = 132, normalized size = 2.36

$$\frac{6\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+14\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + a^3*tan(1/2*d*x + 1/2*c) - (3*a
^3*tan(1/2*d*x + 1/2*c)^2 + 14*a^3*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d*x
 + 1/2*c)^2 - tan(1/2*d*x + 1/2*c)))/d
```

3.770 $\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] (-9*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c
+ d*x]*Csc[c + d*x])/(2*d) + (4*a^3*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rubi [A] time = 0.156333, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 3768, 2648}

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-9*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c
+ d*x]*Csc[c + d*x])/(2*d) + (4*a^3*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx &= a^2\int\left(4a\csc(c+dx)+3a\csc^2(c+dx)+a\csc^3(c+dx)-\frac{4a}{-1+\sin(\right.\\ &= a^3\int\csc^3(c+dx)\,dx+\left(3a^3\right)\int\csc^2(c+dx)\,dx+\left(4a^3\right)\int\csc(c+\\ &= -\frac{4a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{4a^3\cos(}{d(1-\sin}\\ &= -\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc(c}{2d}\end{aligned}$$

Mathematica [A] time = 1.13107, size = 124, normalized size = 1.55

$$\frac{a^3\left(12\tan\left(\frac{1}{2}(c+dx)\right)-12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+36\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-36\log\left(\mathrm{co}\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-12*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 36*Log[Cos[(c + d*x)/2]]
+ 36*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 + (64*Sin[(c + d*x)/2])/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]) + 12*Tan[(c + d*x)/2]))/(8*d)
```

Maple [A] time = 0.119, size = 117, normalized size = 1.5

$$\frac{a^3\tan(dx+c)}{d}+\frac{9a^3}{2d\cos(dx+c)}+\frac{9a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}+3\frac{a^3}{d\sin(dx+c)\cos(dx+c)}-6\frac{a^3\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] a^3*tan(d*x+c)/d+9/2/d*a^3/cos(d*x+c)+9/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3
/d*a^3/sin(d*x+c)/cos(d*x+c)-6*a^3*cot(d*x+c)/d-1/2/d*a^3/sin(d*x+c)^2/cos(
d*x+c)
```

Maxima [A] time = 1.04362, size = 182, normalized size = 2.28

$$\frac{a^3\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+6a^3\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^3*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a^3*(2/cos(d*x + c) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*a^3*(1/tan(d*x + c) - tan(d
*x + c)) + 4*a^3*tan(d*x + c))/d
```

Fricas [B] time = 1.15538, size = 721, normalized size = 9.01

$$28\,a^3\cos(dx+c)^3+18\,a^3\cos(dx+c)^2-26\,a^3\cos(dx+c)-16\,a^3-9\left(a^3\cos(dx+c)^3+a^3\cos(dx+c)^2-a^3\cos(d\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(28*a^3*cos(d*x + c)^3 + 18*a^3*cos(d*x + c)^2 - 26*a^3*cos(d*x + c) -
16*a^3 - 9*(a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - a^
3 - (a^3*cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
9*(a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - a^3 - (a^3*
cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(14*a^
3*cos(d*x + c)^2 + 5*a^3*cos(d*x + c) - 8*a^3)*sin(d*x + c))/(d*cos(d*x + c
)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - (d*cos(d*x + c)^2 - d)*sin(d*x +
c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30554, size = 157, normalized size = 1.96

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{64\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{54\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^3\ldots}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\ldots\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 1
2*a^3*tan(1/2*d*x + 1/2*c) - 64*a^3/(tan(1/2*d*x + 1/2*c) - 1) - (54*a^3*ta
n(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2
*c)^2)/d
```

3.771 $\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=98

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{5a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] (-11*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (4*a^3*Cos[c +
 d*x])/(d*(1 - Sin[c + d*x]))
```

Rubi [A] time = 0.175406, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 3768, 2648}

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{5a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (4*a^3*Cos[c +
 d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^2\int \Big(4a\csc(c+dx)+4a\csc^2(c+dx)+3a\csc^3(c+dx)+a\csc$$

$$= a^3\int \csc^4(c+dx)\,dx+\left(3a^3\right)\int \csc^3(c+dx)\,dx+\left(4a^3\right)\int \csc(c$$

$$= -\frac{4a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{4a^3}{d(1-}$$

$$= -\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}$$

Mathematica [B] time = 6.14217, size = 211, normalized size = 2.15

$$a^3\left(\frac{7\tan\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{7\cot\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{11\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{11\,l}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-7*Cot[(c + d*x)/2])/(3*d) - (3*Csc[(c + d*x)/2]^2)/(8*d) - (Cot[(c +
 d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) - (11*Log[Cos[(c + d*x)/2]])/(2*d) + (1
1*Log[Sin[(c + d*x)/2]])/(2*d) + (3*Sec[(c + d*x)/2]^2)/(8*d) + (8*Sin[(c +
 d*x)/2])/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (7*Tan[(c + d*x)/2])/
(3*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d))
```

Maple [A] time = 0.128, size = 128, normalized size = 1.3

$$\frac{11\,a^3}{2\,d\cos(dx+c)}+\frac{11\,a^3\ln(\csc(dx+c)-\cot(dx+c))}{2\,d}+\frac{13\,a^3}{3\,d\sin(dx+c)\cos(dx+c)}-\frac{26\,a^3\cot(dx+c)}{3\,d}-\frac{}{2\,d(\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 11/2/d*a^3/cos(d*x+c)+11/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+13/3/d*a^3/sin(d
*x+c)/cos(d*x+c)-26/3*a^3*cot(d*x+c)/d-3/2/d*a^3/sin(d*x+c)^2/cos(d*x+c)-1/
3/d*a^3/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [A] time = 1.01845, size = 216, normalized size = 2.2

$$\frac{9\,a^3\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+6\,a^3\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(9*a^3*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a^3*(2/cos(d*x + c) - l
og(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 36*a^3*(1/tan(d*x + c) - ta
n(d*x + c)) - 4*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3*tan(d*x + c)
))/d
```

Fricas [B] time = 1.18183, size = 887, normalized size = 9.05

$$-\frac{104\,a^3\cos(dx+c)^4+38\,a^3\cos(dx+c)^3-156\,a^3\cos(dx+c)^2-42\,a^3\cos(dx+c)+48\,a^3+33\left(a^3\cos(dx+c)^4-2\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 + 38*a^3*cos(d*x + c)^3 - 156*a^3*cos(d*x + c
)^2 - 42*a^3*cos(d*x + c) + 48*a^3 + 33*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x
 + c)^2 + a^3 + (a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a^3*cos(d*x + c)
 - a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 33*(a^3*cos(d*x + c)^4
- 2*a^3*cos(d*x + c)^2 + a^3 + (a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a
^3*cos(d*x + c) - a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(52*a
^3*cos(d*x + c)^3 + 33*a^3*cos(d*x + c)^2 - 45*a^3*cos(d*x + c) - 24*a^3)*s
in(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + (d*cos(d*x + c)^3 + d
*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2463, size = 200, normalized size = 2.04

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+132\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+57\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{192\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x + 1/2*c)^2 + 132*a^3*l
og(abs(tan(1/2*d*x + 1/2*c))) + 57*a^3*tan(1/2*d*x + 1/2*c) - 192*a^3/(tan(
1/2*d*x + 1/2*c) - 1) - (242*a^3*tan(1/2*d*x + 1/2*c)^3 + 57*a^3*tan(1/2*d*
x + 1/2*c)^2 + 9*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.772 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{\cos(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\sec(c+dx)}{ad}+\frac{x}{a}$$

```
[Out] x/a + Cos[c + d*x]/(a*d) + (2*Sec[c + d*x])/(a*d) - Sec[c + d*x]^3/(3*a*d)
- Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.142375, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2839, 3473, 8, 2590, 270}

$$\frac{\cos(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\sec(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(a*d) + (2*Sec[c + d*x])/(a*d) - Sec[c + d*x]^3/(3*a*d)
- Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \tan^4(c+dx)\,dx}{a} - \frac{\int \sin(c+dx)\tan^4(c+dx)\,dx}{a}$$
$$= \frac{\tan^3(c+dx)}{3ad} - \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= -\frac{\tan(c+dx)}{ad} + \frac{\tan^3(c+dx)}{3ad} + \frac{\int 1\,dx}{a} + \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, \cos(c+dx\right.}{ad}$$
$$= \frac{x}{a} + \frac{\cos(c+dx)}{ad} + \frac{2\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} - \frac{\tan(c+dx)}{ad} + \frac{\tan^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.377016, size = 148, normalized size = 1.78

$$\frac{11\sin(c+dx)+6c\sin(2(c+dx))+6dx\sin(2(c+dx))-11\sin(2(c+dx))+3\sin(3(c+dx))+2(6c+6dx-11)\cos(c}{12ad(\sin(c+dx)+1)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (18 + 2*(-11 + 6*c + 6*d*x)*Cos[c + d*x] + 14*Cos[2*(c + d*x)] + 11*Sin[c +
 d*x] - 11*Sin[2*(c + d*x)] + 6*c*Sin[2*(c + d*x)] + 6*d*x*Sin[2*(c + d*x)]
 + 3*Sin[3*(c + d*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.081, size = 126, normalized size = 1.5

$$-\frac{1}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\,\frac{1}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)}+2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da}-\frac{2}{3\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d/a/(tan(1/2*d*x+1/2*c)-1)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)+2/a/d*arctan
(tan(1/2*d*x+1/2*c))-2/3/d/a/(tan(1/2*d*x+1/2*c)+1)^3+1/d/a/(tan(1/2*d*x+1/
2*c)+1)^2+5/2/a/d/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.49787, size = 319, normalized size = 3.84

$$\frac{2\left(\frac{\frac{13\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+8}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*((13*sin(d*x + c)/(cos(d*x + c) + 1) + 2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 6*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 8)/(a + 2*a*sin(d*x +
c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - a*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 2*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6) + 3*arctan(sin(d*x + c)/(cos(d*x + c) +
1))/a)/d
```

Fricas [A] time = 1.05527, size = 215, normalized size = 2.59

$$\frac{3\,dx\cos\left(dx+c\right)+7\,\cos\left(dx+c\right)^{2}+\left(3\,dx\cos\left(dx+c\right)+3\,\cos\left(dx+c\right)^{2}+2\right)\sin\left(dx+c\right)+1}{3\,\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c) + 7*cos(d*x + c)^2 + (3*d*x*cos(d*x + c) + 3*cos(d*
x + c)^2 + 2)*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16784, size = 169, normalized size = 2.04

$$\frac{\frac{6\,(dx+c)}{a}-\frac{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)a}+\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/a - 3*(tan(1/2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*c) + 5
)/((tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c)
- 1)*a) + (15*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) + 17)/(a*(ta
n(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.773 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}-\frac{x}{a}$$

```
[Out] -(x/a) - Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]/(a*d) -
 Tan[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.110768, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2839, 2606, 3473, 8}

$$-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]/(a*d) -
 Tan[c + d*x]^3/(3*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \tan^4(c+dx)\,dx}{a}$$
$$= -\frac{\tan^3(c+dx)}{3ad} + \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= -\frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{\tan(c+dx)}{ad} - \frac{\tan^3(c+dx)}{3ad} - \frac{\int 1\,dx}{a}$$
$$= -\frac{x}{a} - \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{\tan(c+dx)}{ad} - \frac{\tan^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.351386, size = 111, normalized size = 1.59

$$\frac{-2\sin(c+dx)+4\cos(2(c+dx))+(6c+6dx-5)(\sin(c+dx)+1)\cos(c+dx)}{6ad(\sin(c+dx)+1)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (4*Cos[2*(c + d*x)] - 2*Sin[c + d*x] + (-5 + 6*c + 6*d*x)*Cos[c + d*x]*(1 +
 Sin[c + d*x]))/(6*a*d*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.075, size = 104, normalized size = 1.5

$$-\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}+\frac{2}{3da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d/a/(tan(1/2*d*x+1/2*c)-1)-2/a/d*arctan(tan(1/2*d*x+1/2*c))+2/3/d/a/(t
an(1/2*d*x+1/2*c)+1)^3-1/d/a/(tan(1/2*d*x+1/2*c)+1)^2-3/2/a/d/(tan(1/2*d*x+
1/2*c)+1)
```

Maxima [B] time = 1.69134, size = 208, normalized size = 2.97

$$-\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+2}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/3*((sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 - 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2)/(a + 2*a*sin(d*x + c)/(cos
(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4
```

```
/(cos(d*x + c) + 1)^4) + 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.13662, size = 190, normalized size = 2.71

$$-\frac{3\,dx\cos(dx+c)+4\,\cos(dx+c)^2+(3\,dx\cos(dx+c)-1)\sin(dx+c)-2}{3\,(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*d*x*cos(d*x + c) + 4*cos(d*x + c)^2 + (3*d*x*cos(d*x + c) - 1)*sin(
d*x + c) - 2)/(a*d*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15634, size = 104, normalized size = 1.49

$$-\frac{\frac{6\,(dx+c)}{a}+\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)/a + 3/(a*(tan(1/2*d*x + 1/2*c) - 1)) + (9*tan(1/2*d*x + 1
/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.774 $\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0897598, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2706, 2607, 30, 2606}

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{\tan^3(c+dx)}{3ad}$$

Mathematica [B] time = 0.138543, size = 106, normalized size = 2.12

$$\frac{8\sin(c+dx)-5\sin(2(c+dx))-10\cos(c+dx)+2\cos(2(c+dx))+6}{12ad(\sin(c+dx)+1)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6 - 10*Cos[c + d*x] + 2*Cos[2*(c + d*x)] + 8*Sin[c + d*x] - 5*Sin[2*(c + d
*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.068, size = 70, normalized size = 1.4

$$8\,\frac{1}{da}\left(-1/16\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-1}-1/12\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-3}+1/8\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-2}+1/16\,\left(\tan(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 8/d/a*(-1/16/(tan(1/2*d*x+1/2*c)-1)-1/12/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan(
1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A] time = 1.01635, size = 122, normalized size = 2.44

$$\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x
 + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4)*d)
```

Fricas [A] time = 1.02146, size = 127, normalized size = 2.54

$$\frac{\cos\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1}{3\,\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^2 + 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) +
a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sin^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.28596, size = 92, normalized size = 1.84

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 5)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.775 $\int \frac{\sec(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\tan^3(c+dx)}{3ad}$$

```
[Out] Sec[c + d*x]^3/(3*a*d) - Tan[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.093388, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2839, 2606, 30, 2607}

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\tan^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^3/(3*a*d) - Tan[c + d*x]^3/(3*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^3(c+dx)\tan(c+dx)\,dx}{a} - \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int x^2\,dx, x, \sec(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{\sec^3(c+dx)}{3ad} - \frac{\tan^3(c+dx)}{3ad}$$

Mathematica [B] time = 0.131107, size = 104, normalized size = 2.81

$$\frac{-2\sin(c+dx)+\frac{1}{2}\sin(2(c+dx))+\cos(c+dx)+\cos(2(c+dx))-3}{6ad(\sin(c+dx)+1)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3 + Cos[c + d*x] + Cos[2*(c + d*x)] - 2*Sin[c + d*x] + Sin[2*(c + d*x)]/2
)/(6*a*d*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.056, size = 70, normalized size = 1.9

$$4\,\frac{1}{da}\left(-1/8\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-1}+1/6\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-3}-1/4\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-2}+1/8\,\left(\tan\left(1/2\,dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 4/d/a*(-1/8/(tan(1/2*d*x+1/2*c)-1)+1/6/(tan(1/2*d*x+1/2*c)+1)^3-1/4/(tan(1/
2*d*x+1/2*c)+1)^2+1/8/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.02291, size = 149, normalized size = 4.03

$$\frac{2\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d)
```

Fricas [A] time = 1.0251, size = 126, normalized size = 3.41

$$-\frac{\cos(dx+c)^2-\sin(dx+c)-2}{3\,(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 - sin(d*x + c) - 2)/(a*d*cos(d*x + c)*sin(d*x + c) + a
*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sin(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.23009, size = 77, normalized size = 2.08

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*
(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.776 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d
) - Tan[c + d*x]/(a*d) - Tan[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.119843, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2622, 302, 207, 3767}

$$-\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d
) - Tan[c + d*x]/(a*d) - Tan[c + d*x]^3/(3*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \sec^4(c+dx)\,dx}{a}+\frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(1+x^2\right)dx, x, -\tan(c+dx)\right)}{ad}$$
$$= -\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{s}\right.}{ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.606308, size = 149, normalized size = 1.89

$$\frac{6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\left(-\frac{11}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}-\frac{2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.}\right.}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]] + (Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^(-2) + Sin[(c + d*x)/2]*(3/(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]) - 2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 11/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])))/(6*a*d)
```

Maple [A] time = 0.08, size = 103, normalized size = 1.3

$$-\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{2}{3da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{5}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d/a/(tan(1/2*d*x+1/2*c)-1)+2/3/d/a/(tan(1/2*d*x+1/2*c)+1)^3-1/d/a/(tan
(1/2*d*x+1/2*c)+1)^2+5/2/a/d/(tan(1/2*d*x+1/2*c)+1)+1/d/a*ln(tan(1/2*d*x+1/
2*c))
```

Maxima [A] time = 1.01971, size = 184, normalized size = 2.33

$$\frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4\right)}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 4)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + 3*log(sin(d*
x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.12055, size = 333, normalized size = 4.22

$$\frac{4\cos(dx+c)^2 - 3(\cos(dx+c)\sin(dx+c) + \cos(dx+c))\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + 3(\cos(dx+c)\sin(dx+c) + \cos(}{6(ad\cos(dx+c)\sin(dx+c) + ad\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(4*cos(d*x + c)^2 - 3*(cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*log(1/
2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2) + 2*sin(d*x + c) + 4)/(a*d*cos(d*x + c)*sin(d*x + c)
 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.29881, size = 112, normalized size = 1.42

$$\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)} + \frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3/(a*(tan(1/2*d*x + 1/2*c) - 1))
+ (15*tan(1/2*d*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 13)/(a*(tan(1/2*d*
x + 1/2*c) + 1)^3))/d
```

3.777 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{\tan^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d) - Sec[c + d*x]/(a*d) - Sec
[c + d*x]^3/(3*a*d) + (2*Tan[c + d*x])/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.194132, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2620, 270, 2622, 302, 207}

$$\frac{\tan^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d) - Sec[c + d*x]/(a*d) - Sec
[c + d*x]^3/(3*a*d) + (2*Tan[c + d*x])/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a}+\frac{\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,dx,x,\tan(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx,x,\tan(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx,x,\sec(c+\right.}{ad}\\ &= -\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\text{Subst}\left(\right.}{}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}+\frac{\text{t}}{}\end{aligned}$$

Mathematica [B] time = 0.586477, size = 245, normalized size = 2.63

$$-\frac{\csc^3(c+dx)\left(4\sin(c+dx)-16\sin(2(c+dx))+8\sin(3(c+dx))+10\cos(2(c+dx))+8\cos(3(c+dx))+6\sin(2(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^3*(2 + 10*Cos[2*(c + d*x)] + 8*Cos[3*(c + d*x)] + 3*Cos[3*(c
 + d*x)]*Log[Cos[(c + d*x)/2]] - 3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] +
 Cos[c + d*x]*(-8 - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]]) + 4*
Sin[c + d*x] - 16*Sin[2*(c + d*x)] - 6*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x
)] + 6*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]))/(3*a*d
*(Csc[(c + d*x)/2] - Sec[(c + d*x)/2])*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2]
)*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.092, size = 139, normalized size = 1.5

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{2}{3da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{7}{2da}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-1/2/d/a/(tan(1/2*d*x+1/2*c)-1)-2/3/d/a/(tan(1/2*
d*x+1/2*c)+1)^3+1/d/a/(tan(1/2*d*x+1/2*c)+1)^2-7/2/a/d/(tan(1/2*d*x+1/2*c)+
```

```
1)-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.0261, size = 290, normalized size = 3.12

$$-\frac{\frac{\frac{22\sin(dx+c)}{\cos(dx+c)+1}+\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{30\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((22*sin(d*x + c)/(cos(d*x + c) + 1) + 8*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 3)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 2*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5) + 6*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3
*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.15789, size = 440, normalized size = 4.73

$$\frac{10\cos(dx+c)^2+3\left(\cos(dx+c)^3-\cos(dx+c)\sin(dx+c)-\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+c\right.\cdots}{6\left(ad\cos(dx+c)^3-ad\cos(dx+c)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^3 - cos(d*x + c)*sin(d*x + c) - co
s(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^3 - cos(d*x + c)*
sin(d*x + c) - cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(8*cos(d*x +
c)^2 - 1)*sin(d*x + c) - 4)/(a*d*cos(d*x + c)^3 - a*d*cos(d*x + c)*sin(d*x
+ c) - a*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28112, size = 180, normalized size = 1.94

$$-\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a}+\frac{21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+19}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*tan(1/2*d*x + 1/2*c)/a - 3*(ta
n(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/((tan(1/2*d*x + 1/2*c)^2
 - tan(1/2*d*x + 1/2*c))*a) + (21*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x +
 1/2*c) + 19)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.778 $\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{9\tan^5(c+dx)}{10a^2d}-\frac{3\tan^3(c+dx)}{2a^2d}+\frac{9\tan(c+dx)}{2a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}$$

```
[Out] (-9*x)/(2*a^2) - (2*Cos[c + d*x])/(a^2*d) - (6*Sec[c + d*x])/(a^2*d) + (2*S
ec[c + d*x]^3)/(a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + (9*Tan[c + d*x])/(2
*a^2*d) - (3*Tan[c + d*x]^3)/(2*a^2*d) + (9*Tan[c + d*x]^5)/(10*a^2*d) - (S
in[c + d*x]^2*Tan[c + d*x]^5)/(2*a^2*d)
```

Rubi [A] time = 0.306488, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2875, 2710, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{9\tan^5(c+dx)}{10a^2d}-\frac{3\tan^3(c+dx)}{2a^2d}+\frac{9\tan(c+dx)}{2a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*x)/(2*a^2) - (2*Cos[c + d*x])/(a^2*d) - (6*Sec[c + d*x])/(a^2*d) + (2*S
ec[c + d*x]^3)/(a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + (9*Tan[c + d*x])/(2
*a^2*d) - (3*Tan[c + d*x]^3)/(2*a^2*d) + (9*Tan[c + d*x]^5)/(10*a^2*d) - (S
in[c + d*x]^2*Tan[c + d*x]^5)/(2*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int (a-a\sin(c+dx))^2\tan^6(c+dx)\,dx}{a^4}\\
&= \frac{\int \left(a^2\tan^6(c+dx)-2a^2\sin(c+dx)\tan^6(c+dx)+a^2\sin^2(c+dx)\tan^6(c+dx)\right)dx}{a^4}\\
&= \frac{\int \tan^6(c+dx)\,dx}{a^2}+\frac{\int \sin^2(c+dx)\tan^6(c+dx)\,dx}{a^2}-\frac{2\int \sin(c+dx)\tan^6(c+dx)\,dx}{a^2}\\
&= \frac{\tan^5(c+dx)}{5a^2d}-\frac{\int \tan^4(c+dx)\,dx}{a^2}+\frac{\operatorname{Subst}\left(\int \frac{x^8}{\left(1+x^2\right)^2}\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{2\operatorname{Subst}\left(\right.}{}\\
&= -\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan^5(c+dx)}{5a^2d}-\frac{\sin^2(c+dx)\tan^5(c+dx)}{2a^2d}+\frac{\int \tan^2(c+dx)\,dx}{a^2}+\frac{2\,\mathrm{S}}{}\\
&= -\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(c+dx)}{a^2d}-\mathrm{ta}\\
&= -\frac{x}{a^2}-\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{9\tan(c+dx}{2a^2d}\\
&= -\frac{9x}{2a^2}-\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{9\tan(c+a}{2a^2d}
\end{aligned}
$$

Mathematica [A] time = 0.573373, size = 191, normalized size = 1.28

$$-\frac{250\sin(c+dx)+720c\sin(2(c+dx))+720dx\sin(2(c+dx))-824\sin(2(c+dx))+351\sin(3(c+dx))+5\sin(5(c+}{160a^2d\left(\cos\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(500 + 10*(-103 + 90*c + 90*d*x)*Cos[c + d*x] + 544*Cos[2*(c + d*x)] + 206
*Cos[3*(c + d*x)] - 180*c*Cos[3*(c + d*x)] - 180*d*x*Cos[3*(c + d*x)] - 20*
Cos[4*(c + d*x)] + 250*Sin[c + d*x] - 824*Sin[2*(c + d*x)] + 720*c*Sin[2*(c
 + d*x)] + 720*d*x*Sin[2*(c + d*x)] + 351*Sin[3*(c + d*x)] + 5*Sin[5*(c + d
*x)])/(160*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.105, size = 267, normalized size = 1.8

$$-\frac{1}{4da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-4\frac{(\tan(1/2\,dx+c/2))^2}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(tan(1/2*d*x+1/2*c)-1)-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/
2*d*x+1/2*c)^3-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d/
a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-4/d/a^2/(1+tan(1/2*d*x+1/
2*c)^2)^2-9/d/a^2*arctan(tan(1/2*d*x+1/2*c))-4/5/d/a^2/(tan(1/2*d*x+1/2*c)+
1)^5+2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^4+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3-7/2/
d/a^2/(tan(1/2*d*x+1/2*c)+1)^2-31/4/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.5556, size = 568, normalized size = 3.81

$$-\frac{\frac{\frac{211\sin(dx+c)}{\cos(dx+c)+1}+\frac{268\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{212\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{174\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{300\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{300\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{180\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{45\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+64}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{7a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{7a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{45\,\mathrm{arc}}{}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/5*((211*sin(d*x + c)/(cos(d*x + c) + 1) + 268*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 212*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 174*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 300*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 300*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 180*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 45*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 + 64)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 7*a^2*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a^2*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
```

```
)^6 - 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*a^2*sin(d*x + c)^8/(cos
(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10) + 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a^2)/d
```

Fricas [A] time = 1.15781, size = 350, normalized size = 2.35

$$-\frac{45\,dx\cos\left(dx+c\right)^{3}+10\,\cos\left(dx+c\right)^{4}-90\,dx\cos\left(dx+c\right)-78\,\cos\left(dx+c\right)^{2}-\left(5\,\cos\left(dx+c\right)^{4}+90\,dx\cos\left(dx+c\right)+8\cdots\right.}{10\left(a^{2}d\cos\left(dx+c\right)^{3}-2\,a^{2}d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/10*(45*d*x*cos(d*x + c)^3 + 10*cos(d*x + c)^4 - 90*d*x*cos(d*x + c) - 78
*cos(d*x + c)^2 - (5*cos(d*x + c)^4 + 90*d*x*cos(d*x + c) + 84*cos(d*x + c)
^2 - 6)*sin(d*x + c) + 4)/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(
d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29314, size = 216, normalized size = 1.45

$$-\frac{\frac{90\,(dx+c)}{a^{2}}+\frac{20\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{2}}+\frac{5}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+690\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1120\,\mathrm{ta}\cdots}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\cdots\right.\right.}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(90*(d*x + c)/a^2 + 20*(tan(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*
c)^2 - tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + 5/(
a^2*(tan(1/2*d*x + 1/2*c) - 1)) + (155*tan(1/2*d*x + 1/2*c)^4 + 690*tan(1/2
*d*x + 1/2*c)^3 + 1120*tan(1/2*d*x + 1/2*c)^2 + 750*tan(1/2*d*x + 1/2*c) +
181)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.779 $\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{5\sec^3(c+dx)}{3a^2d}+\frac{4\sec(c+dx)}{a^2d}+$$

```
[Out] (2*x)/a^2 + Cos[c + d*x]/(a^2*d) + (4*Sec[c + d*x])/(a^2*d) - (5*Sec[c + d*
x]^3)/(3*a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x])/(a^2*d) +
 (2*Tan[c + d*x]^3)/(3*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.277804, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2606, 194, 3473, 8, 2590, 270}

$$\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{5\sec^3(c+dx)}{3a^2d}+\frac{4\sec(c+dx)}{a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + Cos[c + d*x]/(a^2*d) + (4*Sec[c + d*x])/(a^2*d) - (5*Sec[c + d*
x]^3)/(3*a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x])/(a^2*d) +
 (2*Tan[c + d*x]^3)/(3*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec(c+dx)(a-a\sin(c+dx))^2\tan^5(c+dx)\,dx}{a^4}$$

$$= \frac{\int \left(a^2\sec(c+dx)\tan^5(c+dx)-2a^2\tan^6(c+dx)+a^2\sin(c+dx)\tan^6(c+dx)\right)dx}{a^4}$$

$$= \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a^2}+\frac{\int \sin(c+dx)\tan^6(c+dx)\,dx}{a^2}-\frac{2\int \tan^6(c+dx)\,dx}{a^2}$$

$$= -\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\int \tan^4(c+dx)\,dx}{a^2}-\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx,x,\cos(c+dx)\right)}{a^2d}+\text{Subs}\ldots$$

$$= \frac{2\tan^3(c+dx)}{3a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\int \tan^2(c+dx)\,dx}{a^2}-\frac{\text{Subst}\left(\int\left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\ldots\right.\right.}{a^2d}$$

$$= \frac{\cos(c+dx)}{a^2d}+\frac{4\sec(c+dx)}{a^2d}-\frac{5\sec^3(c+dx)}{3a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\,\text{ta}\ldots}{}$$

$$= \frac{2x}{a^2}+\frac{\cos(c+dx)}{a^2d}+\frac{4\sec(c+dx)}{a^2d}-\frac{5\sec^3(c+dx)}{3a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\tan(c+dx)}{a^2d}\ldots$$

Mathematica [A] time = 0.563298, size = 148, normalized size = 1.23

$$\frac{\sec(c+dx)(400\sin(c+dx)+480c\sin(2(c+dx))+480dx\sin(2(c+dx))-796\sin(2(c+dx))+304\sin(3(c+dx))+(60\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(550 + (-995 + 600*c + 600*d*x)*Cos[c + d*x] + 376*Cos[2*(c +
 d*x)] + 199*Cos[3*(c + d*x)] - 120*c*Cos[3*(c + d*x)] - 120*d*x*Cos[3*(c +
 d*x)] - 30*Cos[4*(c + d*x)] + 400*Sin[c + d*x] - 796*Sin[2*(c + d*x)] + 48
0*c*Sin[2*(c + d*x)] + 480*d*x*Sin[2*(c + d*x)] + 304*Sin[3*(c + d*x)]))/(2
40*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.109, size = 169, normalized size = 1.4

$$-\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\,\frac{1}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)}+4\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^2}+\frac{4}{5\,da^2}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(tan(1/2*d*x+1/2*c)-1)+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)+4/d/a^2*
arctan(tan(1/2*d*x+1/2*c))+4/5/d/a^2/(tan(1/2*d*x+1/2*c)+1)^5-2/d/a^2/(tan(
1/2*d*x+1/2*c)+1)^4-1/3/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3+5/2/d/a^2/(tan(1/2*d
*x+1/2*c)+1)^2+17/4/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.57332, size = 452, normalized size = 3.77

$$\frac{4\left(\frac{\frac{97\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{108\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{27\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{40\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{85\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{60\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+28}{a^2+\frac{4\,a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\,a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{4\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4\,a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 4/15*((97*sin(d*x + c)/(cos(d*x + c) + 1) + 108*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 27*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 40*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 85*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 60*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 28)/(a^2
+ 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 6*a^2*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 4*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 4*a^2*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 6*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4*a^2*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.1041, size = 325, normalized size = 2.71

$$\frac{30\,dx\cos(dx+c)^3+15\,\cos(dx+c)^4-60\,dx\cos(dx+c)-62\,\cos(dx+c)^2-2\left(30\,dx\cos(dx+c)+38\,\cos(dx+c)\right.}{15\left(a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)\sin(dx+c)-2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/15*(30*d*x*cos(d*x + c)^3 + 15*cos(d*x + c)^4 - 60*d*x*cos(d*x + c) - 62*
cos(d*x + c)^2 - 2*(30*d*x*cos(d*x + c) + 38*cos(d*x + c)^2 + 3)*sin(d*x +
c) - 9)/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d
*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28805, size = 204, normalized size = 1.7

$$\frac{\frac{120\,(dx+c)}{a^2}-\frac{15\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)a^2}+\frac{255\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1170\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1310\,\tan\left(\frac{1}{2}\,d\right.}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(120*(d*x + c)/a^2 - 15*(tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*
c) + 9)/((tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1
/2*c) - 1)*a^2) + (255*tan(1/2*d*x + 1/2*c)^4 + 1170*tan(1/2*d*x + 1/2*c)^3
 + 1960*tan(1/2*d*x + 1/2*c)^2 + 1310*tan(1/2*d*x + 1/2*c) + 313)/(a^2*(tan
(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.780 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=106

$$\frac{2\tan^5(c+dx)}{5a^2d}-\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[Out] -(x/a^2) - (2*Sec[c + d*x])/(a^2*d) + (4*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec
[c + d*x]^5)/(5*a^2*d) + Tan[c + d*x]/(a^2*d) - Tan[c + d*x]^3/(3*a^2*d) +
(2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.282228, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 30, 2606, 194, 3473, 8}

$$\frac{2\tan^5(c+dx)}{5a^2d}-\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Sec[c + d*x])/(a^2*d) + (4*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec
[c + d*x]^5)/(5*a^2*d) + Tan[c + d*x]/(a^2*d) - Tan[c + d*x]^3/(3*a^2*d) +
(2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^2(c+dx)(a-a\sin(c+dx))^2\tan^4(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^2(c+dx)\tan^4(c+dx)-2a^2\sec(c+dx)\tan^5(c+dx)+a^2\tan^6(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2}+\frac{\int \tan^6(c+dx)\,dx}{a^2}-\frac{2\int \sec(c+dx)\tan^5(c+dx)\,dx}{a^2}$$
$$= \frac{\tan^5(c+dx)}{5a^2d}-\frac{\int \tan^4(c+dx)\,dx}{a^2}+\frac{\text{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\,\text{Subst}\left(\int (-\right.}{}$$
$$= -\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\int \tan^2(c+dx)\,dx}{a^2}-\frac{2\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx,\right.}{a^2d}$$
$$= -\frac{2\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\,\text{t}}{}$$
$$= -\frac{x}{a^2}-\frac{2\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\tan^3(c+dx)}{3a^2d}$$

Mathematica [A] time = 0.546513, size = 143, normalized size = 1.35

$$-\frac{\sec(c+dx)\left(-10\sin(c+dx)+60c\sin(2(c+dx))+60dx\sin(2(c+dx))-89\sin(2(c+dx))+26\sin(3(c+dx))+\frac{5}{4}(60c\right.}{60a^2d(\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]*(20 + (5*(-89 + 60*c + 60*d*x)*Cos[c + d*x])/4 + 44*Cos[2*(c
 + d*x)] + (89*Cos[3*(c + d*x)])/4 - 15*c*Cos[3*(c + d*x)] - 15*d*x*Cos[3*(
c + d*x)] - 10*Sin[c + d*x] - 89*Sin[2*(c + d*x)] + 60*c*Sin[2*(c + d*x)] +
 60*d*x*Sin[2*(c + d*x)] + 26*Sin[3*(c + d*x)]))/(60*a^2*d*(1 + Sin[c + d*x
])^2)
```

Maple [A] time = 0.089, size = 146, normalized size = 1.4

$$-\frac{1}{4\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^{2}}-\frac{4}{5\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}+2\,\frac{1}{da^{2}\left(\tan\left(1/2\,dx+c/2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(tan(1/2*d*x+1/2*c)-1)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-4/5/d/
a^2/(tan(1/2*d*x+1/2*c)+1)^5+2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^4-1/3/d/a^2/(ta
n(1/2*d*x+1/2*c)+1)^3-3/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2-7/4/d/a^2/(tan(1/2
*d*x+1/2*c)+1)
```

Maxima [B] time = 1.51984, size = 336, normalized size = 3.17

$$-\frac{2\left(\frac{\frac{49\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{70\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{60\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{15\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+16}{a^{2}+\frac{4\,a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{5\,a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{4\,a^{2}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -2/15*((49*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 70*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 60*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 16)/(a^2 + 4*a^2
*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 15*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.09352, size = 297, normalized size = 2.8

$$-\frac{15\,dx\cos\left(dx+c\right)^{3}-30\,dx\cos\left(dx+c\right)-22\,\cos\left(dx+c\right)^{2}-\left(30\,dx\cos\left(dx+c\right)+26\,\cos\left(dx+c\right)^{2}-9\right)\sin\left(dx+c\right)\cdot}{15\left(a^{2}d\cos\left(dx+c\right)^{3}-2\,a^{2}d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/15*(15*d*x*cos(d*x + c)^3 - 30*d*x*cos(d*x + c) - 22*cos(d*x + c)^2 - (3
0*d*x*cos(d*x + c) + 26*cos(d*x + c)^2 - 9)*sin(d*x + c) + 6)/(a^2*d*cos(d*
x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19629, size = 139, normalized size = 1.31

$$-\frac{\frac{60\,(dx+c)}{a^2}+\frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+510\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+920\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+610\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+143}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)/a^2 + 15/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) + (105*tan(1/
2*d*x + 1/2*c)^4 + 510*tan(1/2*d*x + 1/2*c)^3 + 920*tan(1/2*d*x + 1/2*c)^2
+ 610*tan(1/2*d*x + 1/2*c) + 143)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.781 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=66

$$-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^3(c+dx)}{a^2d}+\frac{\sec(c+dx)}{a^2d}$$

```
[Out] Sec[c + d*x]/(a^2*d) - Sec[c + d*x]^3/(a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d
) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.258677, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2606, 14, 2607, 30, 194}

$$-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^3(c+dx)}{a^2d}+\frac{\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]/(a^2*d) - Sec[c + d*x]^3/(a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d
) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^2\tan^3(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^3(c+dx)\tan^3(c+dx) - 2a^2\sec^2(c+dx)\tan^4(c+dx) + a^2\sec(c+dx)\tan^5(c\right.}{a^4}$$
$$= \frac{\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a^2} + \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a^2} - \frac{2\int \sec^2(c+dx)\tan}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2 dx, x, \sec(c+dx)\right)}{a^2d} -$$
$$= -\frac{2\tan^5(c+dx)}{5a^2d} + \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int \left(-x^2+x^4\right)\right.}{a^2d}$$
$$= \frac{\sec(c+dx)}{a^2d} - \frac{\sec^3(c+dx)}{a^2d} + \frac{2\sec^5(c+dx)}{5a^2d} - \frac{2\tan^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.27068, size = 84, normalized size = 1.27

$$\frac{\sec(c+dx)(40\sin(c+dx) - 52\sin(2(c+dx)) + 8\sin(3(c+dx)) - 65\cos(c+dx) - 8\cos(2(c+dx)) + 13\cos(3(c+dx))}{80a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(40 - 65*Cos[c + d*x] - 8*Cos[2*(c + d*x)] + 13*Cos[3*(c + d*
x)] + 40*Sin[c + d*x] - 52*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]))/(80*a^2*
d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.088, size = 100, normalized size = 1.5

$$16\,\frac{1}{da^2}\left(-\frac{1}{64\,\tan(1/2\,dx+c/2)-64} + 1/20\,(\tan(1/2\,dx+c/2)+1)^{-5} - 1/8\,(\tan(1/2\,dx+c/2)+1)^{-4} + 1/16\,(\tan(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 16/d/a^2*(-1/64/(tan(1/2*d*x+1/2*c)-1)+1/20/(tan(1/2*d*x+1/2*c)+1)^5-1/8/(t
an(1/2*d*x+1/2*c)+1)^4+1/16/(tan(1/2*d*x+1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*
```

```
c)+1)^2+1/64/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.03021, size = 221, normalized size = 3.35

$$\frac{4\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{5\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 4/5*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6)*d)
```

Fricas [A] time = 1.12167, size = 197, normalized size = 2.98

$$\frac{\cos(dx+c)^2-2\left(\cos(dx+c)^2+1\right)\sin(dx+c)-3}{5\left(a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)\sin(dx+c)-2a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/5*(cos(d*x + c)^2 - 2*(cos(d*x + c)^2 + 1)*sin(d*x + c) - 3)/(a^2*d*cos(d
*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19185, size = 127, normalized size = 1.92

$$-\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+80\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+50\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+11}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) - (5*tan(1/2*d*x + 1/2*c)^4 + 30*
tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 + 50*tan(1/2*d*x + 1/2*c
) + 11)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.782 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{2\sec^3(c+dx)}{3a^2d}$$

```
[Out] (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + Tan[c + d*x]^
3/(3*a^2*d) + (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.189761, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2711, 2607, 14, 2606, 30}

$$\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{2\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + Tan[c + d*x]^
3/(3*a^2*d) + (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \left(a^2\sec^4(c+dx)\tan^2(c+dx) - 2a^2\sec^3(c+dx)\tan^3(c+dx) + a^2\sec^2(c+dx)\tan^4(c+d\right.}{a^4}$$
$$= \frac{\int \sec^4(c+dx)\tan^2(c+dx)\,dx}{a^2} + \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2} - \frac{2\int \sec^3(c+dx)\tan^3(c}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^4\,dx, x, \tan(c+dx)\right)}{a^2 d} + \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)dx, x, \tan(c+dx)\right)}{a^2 d} - \frac{2\,\text{Subst}\left(\int x^2\right.}{}$$
$$= \frac{\tan^5(c+dx)}{5a^2 d} + \frac{\text{Subst}\left(\int \left(x^2+x^4\right)dx, x, \tan(c+dx)\right)}{a^2 d} - \frac{2\,\text{Subst}\left(\int \left(-x^2+x^4\right)dx, x, \sec(c\right.}{a^2 d}$$
$$= \frac{2\sec^3(c+dx)}{3a^2 d} - \frac{2\sec^5(c+dx)}{5a^2 d} + \frac{\tan^3(c+dx)}{3a^2 d} + \frac{2\tan^5(c+dx)}{5a^2 d}$$

Mathematica [A] time = 0.236509, size = 86, normalized size = 1.18

$$-\frac{\sec(c+dx)\left(-35\sin(c+dx) + 11\sin(2(c+dx)) + \sin(3(c+dx)) + \frac{55}{4}\cos(c+dx) + 4\cos(2(c+dx)) - \frac{11}{4}\cos(3(c+dx)\right.}{60a^2 d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]*(-20 + (55*Cos[c + d*x])/4 + 4*Cos[2*(c + d*x)] - (11*Cos[3*
(c + d*x)])/4 - 35*Sin[c + d*x] + 11*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/
(60*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.089, size = 100, normalized size = 1.4

$$8\frac{1}{da^2}\left(-1/32\,(\tan(1/2\,dx+c/2)-1)^{-1} - 1/10\,(\tan(1/2\,dx+c/2)+1)^{-5} + 1/4\,(\tan(1/2\,dx+c/2)+1)^{-4} - \frac{5}{24\,(\tan(1/2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 8/d/a^2*(-1/32/(tan(1/2*d*x+1/2*c)-1)-1/10/(tan(1/2*d*x+1/2*c)+1)^5+1/4/(ta
n(1/2*d*x+1/2*c)+1)^4-5/24/(tan(1/2*d*x+1/2*c)+1)^3+1/16/(tan(1/2*d*x+1/2*c
)+1)^2+1/32/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.02728, size = 248, normalized size = 3.4

$$\frac{8\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1} + \frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + 1\right)}{15\left(a^2 + \frac{4a^2\sin(dx+c)}{\cos(dx+c)+1} + \frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 8/15*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a^2 + 4*a^2*sin(d*x + c
)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*a^2*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d)
```

Fricas [A] time = 1.05302, size = 198, normalized size = 2.71

$$\frac{2\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^2-9\right)\sin\left(dx+c\right)-6}{15\left(a^2d\cos\left(dx+c\right)^3-2\,a^2d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^2d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/15*(2*cos(d*x + c)^2 + (cos(d*x + c)^2 - 9)*sin(d*x + c) - 6)/(a^2*d*cos(
d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sin^2(c+dx)\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A] time = 1.21631, size = 127, normalized size = 1.74

$$-\frac{\frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) - (15*tan(1/2*d*x + 1/2*c)^4 + 9
0*tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 + 70*tan(1/2*d*x + 1/2
*c) + 17)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.783 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$\frac{4\tan(c+dx)}{15a^2d}-\frac{2\sec(c+dx)}{15d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

```
[Out] Sec[c + d*x]/(5*d*(a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x])/(15*d*(a^2 + a
^2*Sin[c + d*x])) + (4*Tan[c + d*x])/(15*a^2*d)
```

Rubi [A] time = 0.128079, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2859, 2672, 3767, 8}

$$\frac{4\tan(c+dx)}{15a^2d}-\frac{2\sec(c+dx)}{15d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]/(5*d*(a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x])/(15*d*(a^2 + a
^2*Sin[c + d*x])) + (4*Tan[c + d*x])/(15*a^2*d)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} + \frac{2\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{5a}$$
$$= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\sec(c+dx)}{15d\left(a^2+a^2\sin(c+dx)\right)} + \frac{4\int \sec^2(c+dx)\,dx}{15a^2}$$
$$= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\sec(c+dx)}{15d\left(a^2+a^2\sin(c+dx)\right)} - \frac{4\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx)}{15a^2d}$$
$$= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\sec(c+dx)}{15d\left(a^2+a^2\sin(c+dx)\right)} + \frac{4\tan(c+dx)}{15a^2d}$$

Mathematica [A] time = 0.226141, size = 82, normalized size = 1.15

$$-\frac{\sec(c+dx)(-80\sin(c+dx)-4\sin(2(c+dx))+16\sin(3(c+dx))-5\cos(c+dx)+64\cos(2(c+dx))+\cos(3(c+d}{240a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]*(-80 - 5*Cos[c + d*x] + 64*Cos[2*(c + d*x)] + Cos[3*(c + d*x
)] - 80*Sin[c + d*x] - 4*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)]))/(240*a^2*
d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.082, size = 100, normalized size = 1.4

$$4\,\frac{1}{da^2}\left(-1/16\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-1}+1/5\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-5}-1/2\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-4}+\frac{}{12\,(\tan(1/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 4/d/a^2*(-1/16/(tan(1/2*d*x+1/2*c)-1)+1/5/(tan(1/2*d*x+1/2*c)+1)^5-1/2/(tan
(1/2*d*x+1/2*c)+1)^4+7/12/(tan(1/2*d*x+1/2*c)+1)^3-3/8/(tan(1/2*d*x+1/2*c)+
1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.03107, size = 275, normalized size = 3.87

$$\frac{2\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}{15\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/15*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin
```

```
(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 4*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6)*d)
```

Fricas [A] time = 1.01259, size = 204, normalized size = 2.87

$$\frac{8 \cos(dx+c)^2 + 2\left(2 \cos(dx+c)^2 - 3\right)\sin(dx+c) - 9}{15\left(a^2 d \cos(dx+c)^3 - 2\,a^2 d \cos(dx+c)\sin(dx+c) - 2\,a^2 d \cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(d*x + c)^2 + 2*(2*cos(d*x + c)^2 - 3)*sin(d*x + c) - 9)/(a^2*d*
cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sin(c+dx)\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A] time = 1.25893, size = 127, normalized size = 1.79

$$-\frac{\frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)} - \frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 50\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 7}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) - (15*tan(1/2*d*x + 1/2*c)^4 - 3
0*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2*d*x + 1/2*c)^2 - 50*tan(1/2*d*x + 1/2
*c) - 7)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.784 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+\ldots}{a^2d}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x])/(a^2*d) - (4*Tan[
c + d*x]^3)/(3*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.266254, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2606, 30}

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+\ldots}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x])/(a^2*d) - (4*Tan[
c + d*x]^3)/(3*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc(c+dx)\sec^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(-2a^2\sec^6(c+dx)+a^2\csc(c+dx)\sec^6(c+dx)+a^2\sec^5(c+dx)\tan(c+dx)\right)dx}{a^4}$$

$$= \frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a^2} + \frac{\int \sec^5(c+dx)\tan(c+dx)\,dx}{a^2} - \frac{2\int \sec^6(c+dx)\,dx}{a^2}$$

$$= \frac{\text{Subst}\left(\int x^4\,dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2d} + \frac{2\,\text{Subst}\left(\int \left(1+\right.\right.}{}$$

$$= \frac{\sec^5(c+dx)}{5a^2d} - \frac{2\tan(c+dx)}{a^2d} - \frac{4\tan^3(c+dx)}{3a^2d} - \frac{2\tan^5(c+dx)}{5a^2d} + \frac{\text{Subst}\left(\int \left(1+x^2+\right.\right.}{}$$

$$= \frac{\sec(c+dx)}{a^2d} + \frac{\sec^3(c+dx)}{3a^2d} + \frac{2\sec^5(c+dx)}{5a^2d} - \frac{2\tan(c+dx)}{a^2d} - \frac{4\tan^3(c+dx)}{3a^2d} - \frac{2\tan}{5}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\sec(c+dx)}{a^2d} + \frac{\sec^3(c+dx)}{3a^2d} + \frac{2\sec^5(c+dx)}{5a^2d} - \frac{2\tan(c+dx)}{a^2d}$$

Mathematica [A] time = 0.478946, size = 196, normalized size = 1.7

$$\frac{\sec(c+dx)\left(160\sin(c+dx)-316\sin(2(c+dx))+64\sin(3(c+dx))+136\cos(2(c+dx))+79\cos(3(c+dx))+240\sin(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(280 + 136*Cos[2*(c + d*x)] + 79*Cos[3*(c + d*x)] + 60*Cos[3*
(c + d*x)]*Log[Cos[(c + d*x)/2]] - 5*Cos[c + d*x]*(79 + 60*Log[Cos[(c + d*x
)/2]] - 60*Log[Sin[(c + d*x)/2]]) - 60*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2
]] + 160*Sin[c + d*x] - 316*Sin[2*(c + d*x)] - 240*Log[Cos[(c + d*x)/2]]*Si
n[2*(c + d*x)] + 240*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] + 64*Sin[3*(c +
```

```
d*x)]))/(240*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.105, size = 145, normalized size = 1.3

$$-\frac{1}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{4}{5\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}-2\,\frac{1}{da^2\left(\tan\left(1/2\,dx+c/2\right)+1\right)^4}+\frac{11}{3\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(tan(1/2*d*x+1/2*c)-1)+4/5/d/a^2/(tan(1/2*d*x+1/2*c)+1)^5-2/d/a^
2/(tan(1/2*d*x+1/2*c)+1)^4+11/3/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3-7/2/d/a^2/(t
an(1/2*d*x+1/2*c)+1)^2+17/4/d/a^2/(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*ln(tan(1/2
*d*x+1/2*c))
```

Maxima [B] time = 1.02998, size = 338, normalized size = 2.94

$$\frac{\frac{4\left(\frac{37\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{30\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+13\right)}{a^2+\frac{4\,a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{15\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/15*(4*(37*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 30*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 13)/(a^2 + 4*a^
2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 15*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.16387, size = 471, normalized size = 4.1

$$-\frac{34\,\cos\left(dx+c\right)^2+15\left(\cos\left(dx+c\right)^3-2\,\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-15\left(\cos\right.}{30\left(a^2d\cos\left(dx+c\right)^3-2\,a^2d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(34*cos(d*x + c)^2 + 15*(cos(d*x + c)^3 - 2*cos(d*x + c)*sin(d*x + c)
 - 2*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(cos(d*x + c)^3 - 2*cos
(d*x + c)*sin(d*x + c) - 2*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(
8*cos(d*x + c)^2 + 3)*sin(d*x + c) + 18)/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*co
s(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28113, size = 147, normalized size = 1.28

$$\frac{\frac{60 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)} + \frac{255 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+810 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1120 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+710 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+193}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 15/(a^2*(tan(1/2*d*x + 1/2*c)
 - 1)) + (255*tan(1/2*d*x + 1/2*c)^4 + 810*tan(1/2*d*x + 1/2*c)^3 + 1120*ta
n(1/2*d*x + 1/2*c)^2 + 710*tan(1/2*d*x + 1/2*c) + 193)/(a^2*(tan(1/2*d*x +
1/2*c) + 1)^5))/d
```

3.785 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=130

$$\frac{2\tan^5(c+dx)}{5a^2d}+\frac{5\tan^3(c+dx)}{3a^2d}+\frac{4\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}+$$

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Sec[c + d*x])
/(a^2*d) - (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + (4
*Tan[c + d*x])/(a^2*d) + (5*Tan[c + d*x]^3)/(3*a^2*d) + (2*Tan[c + d*x]^5)/
(5*a^2*d)
```

Rubi [A] time = 0.310337, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2620, 270}

$$\frac{2\tan^5(c+dx)}{5a^2d}+\frac{5\tan^3(c+dx)}{3a^2d}+\frac{4\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Sec[c + d*x])
/(a^2*d) - (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + (4
*Tan[c + d*x])/(a^2*d) + (5*Tan[c + d*x]^3)/(3*a^2*d) + (2*Tan[c + d*x]^5)/
(5*a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^2(c+dx)\sec^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^6(c+dx)-2a^2\csc(c+dx)\sec^6(c+dx)+a^2\csc^2(c+dx)\sec^6(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \sec^6(c+dx)\,dx}{a^2}+\frac{\int \csc^2(c+dx)\sec^6(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\sec^6(c+dx)\,dx}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^2}\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx,x,-\tan(c+dx)\right)}{a^2d}-$$
$$= \frac{\tan(c+dx)}{a^2d}+\frac{2\tan^3(c+dx)}{3a^2d}+\frac{\tan^5(c+dx)}{5a^2d}+\frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+3x^2+x^4\right)dx,x,\text{t}\right.}{a^2d}$$
$$= -\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{4\tan(c+dx)}{a^2d}+\frac{5\,\text{t}}{}$$
$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx}{5a^2d}$$

Mathematica [B] time = 0.731926, size = 289, normalized size = 2.22

$$-\frac{\csc^3(c+dx)\left(58\sin(c+dx)-168\sin(2(c+dx))+82\sin(3(c+dx))+28\sin(4(c+dx))+48\cos(2(c+dx))+112\cos(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^3*(40 + 48*Cos[2*(c + d*x)] + 112*Cos[3*(c + d*x)] - 28*Cos[
4*(c + d*x)] + 60*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 4*Cos[c + d*x]*(
```

```
28 + 15*Log[Cos[(c + d*x)/2]] - 15*Log[Sin[(c + d*x)/2]]) - 60*Cos[3*(c + d
*x)]*Log[Sin[(c + d*x)/2]] + 58*Sin[c + d*x] - 168*Sin[2*(c + d*x)] - 90*Lo
g[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)] + 90*Log[Sin[(c + d*x)/2]]*Sin[2*(c +
d*x)] + 82*Sin[3*(c + d*x)] + 28*Sin[4*(c + d*x)] + 15*Log[Cos[(c + d*x)/2]
]*Sin[4*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]))/(15*a^2*d*
(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.13, size = 182, normalized size = 1.4

$$\frac{1}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{4}{5da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}+2\frac{1}{da^2\left(\tan\left(1/2\,dx+c/2\right)+1\right)^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/4/d/a^2/(tan(1/2*d*x+1/2*c)-1)-4/5/d/a^2/(ta
n(1/2*d*x+1/2*c)+1)^5+2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^4-13/3/d/a^2/(tan(1/2*
d*x+1/2*c)+1)^3+9/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2-31/4/d/a^2/(tan(1/2*d*x+
1/2*c)+1)-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.07389, size = 417, normalized size = 3.21

$$-\frac{\frac{\frac{244\sin(dx+c)}{\cos(dx+c)+1}+\frac{571\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{320\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{475\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{660\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{255\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+15}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{15\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*((244*sin(d*x + c)/(cos(d*x + c) + 1) + 571*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 320*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 475*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 660*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 255*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 15)/(a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 4
*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^2*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 5*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4*a^2*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 - a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 60*log(
sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 15*sin(d*x + c)/(a^2*(cos(d*x + c) +
 1)))/d
```

Fricas [A] time = 1.14054, size = 591, normalized size = 4.55

$$-\frac{56\cos(dx+c)^4-80\cos(dx+c)^2-15\left(2\cos(dx+c)^3+\left(\cos(dx+c)^3-2\cos(dx+c)\right)\sin(dx+c)-2\cos(dx+\right.}{15\left(2a^2d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

[Out] -1/15*(56*cos(d*x + c)^4 - 80*cos(d*x + c)^2 - 15*(2*cos(d*x + c)^3 + (cos(
d*x + c)^3 - 2*cos(d*x + c))*sin(d*x + c) - 2*cos(d*x + c))*log(1/2*cos(d*x
 + c) + 1/2) + 15*(2*cos(d*x + c)^3 + (cos(d*x + c)^3 - 2*cos(d*x + c))*sin
(d*x + c) - 2*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(41*cos(d*x +
c)^2 - 3)*sin(d*x + c) + 9)/(2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)
+ (a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c))*sin(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 1.27618, size = 217, normalized size = 1.67

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}-\frac{15\left(4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a^2}+\frac{465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1590\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2240\tan\left(\cdots\right.}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\cdots\right.}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/60*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 30*tan(1/2*d*x + 1/2*c)/a^2
 - 15*(4*tan(1/2*d*x + 1/2*c)^2 - 7*tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*x
 + 1/2*c)^2 - tan(1/2*d*x + 1/2*c))*a^2) + (465*tan(1/2*d*x + 1/2*c)^4 + 15
90*tan(1/2*d*x + 1/2*c)^3 + 2240*tan(1/2*d*x + 1/2*c)^2 + 1450*tan(1/2*d*x
+ 1/2*c) + 383)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d

3.786 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=158

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{6\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{9\sec^5(c+dx)}{10a^2d}+\frac{3\sec^3(c+dx)}{2a^2d}+\frac{9\sec(c+dx)}{2a^2d}$$

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (9*Sec[c
+ d*x])/(2*a^2*d) + (3*Sec[c + d*x]^3)/(2*a^2*d) + (9*Sec[c + d*x]^5)/(10*a
^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^5)/(2*a^2*d) - (6*Tan[c + d*x])/(a^2*d
) - (2*Tan[c + d*x]^3)/(a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.346225, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2622, 302, 207, 2620, 270, 288}

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{6\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{9\sec^5(c+dx)}{10a^2d}+\frac{3\sec^3(c+dx)}{2a^2d}+\frac{9\sec(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (9*Sec[c
+ d*x])/(2*a^2*d) + (3*Sec[c + d*x]^3)/(2*a^2*d) + (9*Sec[c + d*x]^5)/(10*a
^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^5)/(2*a^2*d) - (6*Tan[c + d*x])/(a^2*d
) - (2*Tan[c + d*x]^3)/(a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^3(c+dx)\sec^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\csc(c+dx)\sec^6(c+dx) - 2a^2\csc^2(c+dx)\sec^6(c+dx) + a^2\csc^3(c+dx)\sec^6(c\right.}{a^4}$$

$$= \frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a^2} + \frac{\int \csc^3(c+dx)\sec^6(c+dx)\,dx}{a^2} - \frac{2\int \csc^2(c+dx)\sec^6}{a^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2d} - \frac{2\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\csc^2(c+dx)\sec^5(c+dx)}{2a^2d} + \frac{\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)dx, x, \sec(c+dx)\right)}{a^2d} - \frac{2\,\text{S}}{}$$

$$= \frac{2\cot(c+dx)}{a^2d} + \frac{\sec(c+dx)}{a^2d} + \frac{\sec^3(c+dx)}{3a^2d} + \frac{\sec^5(c+dx)}{5a^2d} - \frac{\csc^2(c+dx)\sec^5(c+dx}{2a^2d}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{2\cot(c+dx)}{a^2d} + \frac{9\sec(c+dx)}{2a^2d} + \frac{3\sec^3(c+dx)}{2a^2d} + \frac{9\sec^5(c+}{10a^2d}$$

$$= -\frac{9\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{2\cot(c+dx)}{a^2d} + \frac{9\sec(c+dx)}{2a^2d} + \frac{3\sec^3(c+dx)}{2a^2d} + \frac{9\sec^5(c+}{10a^2d}$$

Mathematica [B] time = 0.699453, size = 328, normalized size = 2.08

$$-\frac{\csc^2(c+dx)\sec(c+dx)\left(-432\sin(c+dx) + 744\sin(2(c+dx)) - 176\sin(3(c+dx)) - 372\sin(4(c+dx)) + 128\sin(5(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^2*Sec[c + d*x]*(-348 + 176*Cos[2*(c + d*x)] - 651*Cos[3*(c +
 d*x)] + 332*Cos[4*(c + d*x)] + 93*Cos[5*(c + d*x)] - 630*Cos[3*(c + d*x)]*
Log[Cos[(c + d*x)/2]] + 90*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 18*Cos[
c + d*x]*(31 + 30*Log[Cos[(c + d*x)/2]] - 30*Log[Sin[(c + d*x)/2]]) + 630*C
os[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 90*Cos[5*(c + d*x)]*Log[Sin[(c + d*
x)/2]] - 432*Sin[c + d*x] + 744*Sin[2*(c + d*x)] + 720*Log[Cos[(c + d*x)/2]
]*Sin[2*(c + d*x)] - 720*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 176*Sin[3
*(c + d*x)] - 372*Sin[4*(c + d*x)] - 360*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d
*x)] + 360*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] + 128*Sin[5*(c + d*x)]))/
(320*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.141, size = 219, normalized size = 1.4

$$\frac{1}{8da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{4}{5da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}-2\frac{}{da^2(\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)-1/4/d/a^2/(tan(1/
2*d*x+1/2*c)-1)+4/5/d/a^2/(tan(1/2*d*x+1/2*c)+1)^5-2/d/a^2/(tan(1/2*d*x+1/2
*c)+1)^4+5/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3-11/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)
^2+49/4/d/a^2/(tan(1/2*d*x+1/2*c)+1)-1/8/d/a^2/tan(1/2*d*x+1/2*c)^2+1/d/a^2
/tan(1/2*d*x+1/2*c)+9/2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.05704, size = 478, normalized size = 3.03

$$\frac{\frac{\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{567\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1448\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{985\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{820\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1355\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{520\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-5}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{5a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{5\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2}+\frac{180\,\mathrm{lo}}{}}{40d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/40*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 567*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 1448*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 985*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 820*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1355*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 - 520*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5)
/(a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^2*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 5*a^2*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 4*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^2*s
in(d*x + c)^8/(cos(d*x + c) + 1)^8) - 5*(8*sin(d*x + c)/(cos(d*x + c) + 1)
- sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^2 + 180*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^2)/d
```

Fricas [A] time = 1.14918, size = 699, normalized size = 4.42

$$-\frac{166\cos(dx+c)^4 - 144\cos(dx+c)^2 + 45\left(\cos(dx+c)^5 - 3\cos(dx+c)^3 - 2\left(\cos(dx+c)^3 - \cos(dx+c)\right)\sin(dx+c\ldots}{2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20*(166*cos(d*x + c)^4 - 144*cos(d*x + c)^2 + 45*(cos(d*x + c)^5 - 3*cos
(d*x + c)^3 - 2*(cos(d*x + c)^3 - cos(d*x + c))*sin(d*x + c) + 2*cos(d*x +
c))*log(1/2*cos(d*x + c) + 1/2) - 45*(cos(d*x + c)^5 - 3*cos(d*x + c)^3 - 2
*(cos(d*x + c)^3 - cos(d*x + c))*sin(d*x + c) + 2*cos(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2) + 4*(32*cos(d*x + c)^4 - 35*cos(d*x + c)^2 - 2)*sin(d*x +
 c) - 12)/(a^2*d*cos(d*x + c)^5 - 3*a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x
+ c) - 2*(a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33743, size = 252, normalized size = 1.59

$$\frac{\frac{180\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} + \frac{5\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^4} - \frac{10}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)} - \frac{5\left(54\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} + \frac{2\left(245\ldots\right.}{\ldots}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40*(180*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 5*(a^2*tan(1/2*d*x + 1/2*c)^
2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 - 10/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) -
 5*(54*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*
x + 1/2*c)^2) + 2*(245*tan(1/2*d*x + 1/2*c)^4 + 870*tan(1/2*d*x + 1/2*c)^3
+ 1240*tan(1/2*d*x + 1/2*c)^2 + 810*tan(1/2*d*x + 1/2*c) + 211)/(a^2*(tan(1
/2*d*x + 1/2*c) + 1)^5))/d
```

3.787 $\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=151

$$\frac{\cos(c+dx)}{a^3d}+\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{a^3d}-\frac{3\tan(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{13\sec^5(c+dx)}{5a^3d}-$$

```
[Out] (3*x)/a^3 + Cos[c + d*x]/(a^3*d) + (7*Sec[c + d*x])/(a^3*d) - (5*Sec[c + d*
x]^3)/(a^3*d) + (13*Sec[c + d*x]^5)/(5*a^3*d) - (4*Sec[c + d*x]^7)/(7*a^3*d
) - (3*Tan[c + d*x])/(a^3*d) + Tan[c + d*x]^3/(a^3*d) - (3*Tan[c + d*x]^5)/
(5*a^3*d) + (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rubi [A] time = 0.34146, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2875, 2873, 2607, 30, 2606, 194, 3473, 8, 2590, 270}

$$\frac{\cos(c+dx)}{a^3d}+\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{a^3d}-\frac{3\tan(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{13\sec^5(c+dx)}{5a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (3*x)/a^3 + Cos[c + d*x]/(a^3*d) + (7*Sec[c + d*x])/(a^3*d) - (5*Sec[c + d*
x]^3)/(a^3*d) + (13*Sec[c + d*x]^5)/(5*a^3*d) - (4*Sec[c + d*x]^7)/(7*a^3*d
) - (3*Tan[c + d*x])/(a^3*d) + Tan[c + d*x]^3/(a^3*d) - (3*Tan[c + d*x]^5)/
(5*a^3*d) + (4*Tan[c + d*x]^7)/(7*a^3*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^2(c+dx)(a-a\sin(c+dx))^3\tan^6(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^2(c+dx)\tan^6(c+dx) - 3a^3\sec(c+dx)\tan^7(c+dx) + 3a^3\tan^8(c+dx) - a^3\right.}{a^6}$$

$$= \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^3} - \frac{\int \sin(c+dx)\tan^8(c+dx)\,dx}{a^3} - \frac{3\int \sec(c+dx)\tan^7}{a^3}$$

$$= \frac{3\tan^7(c+dx)}{7a^3d} - \frac{3\int \tan^6(c+dx)\,dx}{a^3} + \frac{\operatorname{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{a^3d} + \frac{\operatorname{Subst}\left(\int \right.}{}$$

$$= -\frac{3\tan^5(c+dx)}{5a^3d} + \frac{4\tan^7(c+dx)}{7a^3d} + \frac{3\int \tan^4(c+dx)\,dx}{a^3} + \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^8}-\frac{4}{x^6}+\right.\right.}{a^3}$$

$$= \frac{\cos(c+dx)}{a^3d} + \frac{7\sec(c+dx)}{a^3d} - \frac{5\sec^3(c+dx)}{a^3d} + \frac{13\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7(c+dx)}{7a^3d} + \text{ta}$$

$$= \frac{\cos(c+dx)}{a^3d} + \frac{7\sec(c+dx)}{a^3d} - \frac{5\sec^3(c+dx)}{a^3d} + \frac{13\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7(c+dx)}{7a^3d} - 3\text{t}$$

$$= \frac{3x}{a^3} + \frac{\cos(c+dx)}{a^3d} + \frac{7\sec(c+dx)}{a^3d} - \frac{5\sec^3(c+dx)}{a^3d} + \frac{13\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7(c+dx)}{7a^3d}$$

Mathematica [A] time = 0.650508, size = 224, normalized size = 1.48

$$\frac{8008\sin(c+dx)+11760c\sin(2(c+dx))+11760dx\sin(2(c+dx))-20762\sin(2(c+dx))+6588\sin(3(c+dx))-84}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (8400 + 14*(-1483 + 840*c + 840*d*x)*Cos[c + d*x] + 5152*Cos[2*(c + d*x)] +
 8898*Cos[3*(c + d*x)] - 5040*c*Cos[3*(c + d*x)] - 5040*d*x*Cos[3*(c + d*x)
] - 2288*Cos[4*(c + d*x)] + 8008*Sin[c + d*x] - 20762*Sin[2*(c + d*x)] + 11
760*c*Sin[2*(c + d*x)] + 11760*d*x*Sin[2*(c + d*x)] + 6588*Sin[3*(c + d*x)]
 + 1483*Sin[4*(c + d*x)] - 840*c*Sin[4*(c + d*x)] - 840*d*x*Sin[4*(c + d*x)
] - 140*Sin[5*(c + d*x)])/(2240*a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A] time = 0.12, size = 211, normalized size = 1.4

$$-\frac{1}{8da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\frac{1}{da^3\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}+6\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^3}-\frac{8}{7da^3}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/d/a^3/(tan(1/2*d*x+1/2*c)-1)+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)+6/d/a^3*
arctan(tan(1/2*d*x+1/2*c))-8/7/d/a^3/(tan(1/2*d*x+1/2*c)+1)^7+4/d/a^3/(tan(
1/2*d*x+1/2*c)+1)^6-14/5/d/a^3/(tan(1/2*d*x+1/2*c)+1)^5-3/d/a^3/(tan(1/2*d*
x+1/2*c)+1)^4+1/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3+17/4/d/a^3/(tan(1/2*d*x+1/
2*c)+1)^2+49/8/d/a^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.56908, size = 568, normalized size = 3.76

$$\frac{2\left(\frac{\frac{951\sin(dx+c)}{\cos(dx+c)+1}+\frac{2010\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1980\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{574\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{966\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1890\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{1540\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{630\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{105\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+176}{a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{15a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{20a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{14a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{20a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\right.}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2/35*((951*sin(d*x + c)/(cos(d*x + c) + 1) + 2010*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1980*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 574*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - 966*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1890*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 1540*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 630*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 105*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 + 176)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 15*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 20*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 14*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 14*a^3*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 - 20*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 15*a^3*sin(d*
```

```
x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.14304, size = 431, normalized size = 2.85

$$\frac{315\,dx\cos(dx+c)^3+286\,\cos(dx+c)^4-420\,dx\cos(dx+c)-447\,\cos(dx+c)^2+\left(105\,dx\cos(dx+c)^3+35\,\cos(dx+\right.}{35\left(3\,a^3d\cos(dx+c)^3-4\,a^3d\cos(dx+c)+\left(a^3d\cos(dx+c)^3-4\,a^3d\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(315*d*x*cos(d*x + c)^3 + 286*cos(d*x + c)^4 - 420*d*x*cos(d*x + c) -
447*cos(d*x + c)^2 + (105*d*x*cos(d*x + c)^3 + 35*cos(d*x + c)^4 - 420*d*x*
cos(d*x + c) - 438*cos(d*x + c)^2 - 20)*sin(d*x + c) - 15)/(3*a^3*d*cos(d*x
 + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x +
c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1958, size = 239, normalized size = 1.58

$$\frac{\frac{840\,(dx+c)}{a^3}-\frac{35\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)a^3}+\frac{1715\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+11480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31815\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+45920\,\tan}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/280*(840*(d*x + c)/a^3 - 35*(tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x + 1/
2*c) + 17)/((tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
+ 1/2*c) - 1)*a^3) + (1715*tan(1/2*d*x + 1/2*c)^6 + 11480*tan(1/2*d*x + 1/2
*c)^5 + 31815*tan(1/2*d*x + 1/2*c)^4 + 45920*tan(1/2*d*x + 1/2*c)^3 + 35161
*tan(1/2*d*x + 1/2*c)^2 + 13832*tan(1/2*d*x + 1/2*c) + 2221)/(a^3*(tan(1/2*
d*x + 1/2*c) + 1)^7))/d
```

3.788 $\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=142

$$-\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}$$

```
[Out] -(x/a^3) - (3*Sec[c + d*x])/(a^3*d) + (10*Sec[c + d*x]^3)/(3*a^3*d) - (11*S
ec[c + d*x]^5)/(5*a^3*d) + (4*Sec[c + d*x]^7)/(7*a^3*d) + Tan[c + d*x]/(a^3
*d) - Tan[c + d*x]^3/(3*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) - (4*Tan[c + d*x]
^7)/(7*a^3*d)
```

Rubi [A] time = 0.339515, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2875, 2873, 2606, 270, 2607, 30, 194, 3473, 8}

$$-\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (3*Sec[c + d*x])/(a^3*d) + (10*Sec[c + d*x]^3)/(3*a^3*d) - (11*S
ec[c + d*x]^5)/(5*a^3*d) + (4*Sec[c + d*x]^7)/(7*a^3*d) + Tan[c + d*x]/(a^3
*d) - Tan[c + d*x]^3/(3*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) - (4*Tan[c + d*x]
^7)/(7*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^3\tan^5(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^3(c+dx)\tan^5(c+dx)-3a^3\sec^2(c+dx)\tan^6(c+dx)+3a^3\sec(c+dx)\tan^7\right.}{a^6}$$
$$= \frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^3}-\frac{\int \tan^8(c+dx)\,dx}{a^3}-\frac{3\int \sec^2(c+dx)\tan^6(c+dx)\,d}{a^3}$$
$$= -\frac{\tan^7(c+dx)}{7a^3d}+\frac{\int \tan^6(c+dx)\,dx}{a^3}+\frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\sec(c+dx)\right)}{a^3d}-\frac{3}{}$$
$$= \frac{\tan^5(c+dx)}{5a^3d}-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{\int \tan^4(c+dx)\,dx}{a^3}+\frac{\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\right.}{a^3d}$$
$$= -\frac{3\sec(c+dx)}{a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+$$
$$= -\frac{3\sec(c+dx)}{a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\tan(c+dx)}{a^3d}-$$
$$= -\frac{x}{a^3}-\frac{3\sec(c+dx)}{a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\tan(c+}{a^3d}$$

Mathematica [A] time = 0.822684, size = 214, normalized size = 1.51

$$-\frac{2688\sin(c+dx)+11760c\sin(2(c+dx))+11760dx\sin(2(c+dx))-23282\sin(2(c+dx))+5568\sin(3(c+dx))-840c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(4200 + 14*(-1663 + 840*c + 840*d*x)*Cos[c + d*x] + 6272*Cos[2*(c + d*x)]
+ 9978*Cos[3*(c + d*x)] - 5040*c*Cos[3*(c + d*x)] - 5040*d*x*Cos[3*(c + d*x
)] - 1768*Cos[4*(c + d*x)] + 2688*Sin[c + d*x] - 23282*Sin[2*(c + d*x)] + 1
1760*c*Sin[2*(c + d*x)] + 11760*d*x*Sin[2*(c + d*x)] + 5568*Sin[3*(c + d*x)
] + 1663*Sin[4*(c + d*x)] - 840*c*Sin[4*(c + d*x)] - 840*d*x*Sin[4*(c + d*x
)])/(6720*a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^7)
```

Maple [A] time = 0.117, size = 187, normalized size = 1.3

$$-\frac{1}{8\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{d a^3}+\frac{8}{7\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-7}-4\,\frac{1}{d a^3\left(\tan\left(1/2\,dx+c/2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/d/a^3/(tan(1/2*d*x+1/2*c)-1)-2/d/a^3*arctan(tan(1/2*d*x+1/2*c))+8/7/d/
a^3/(tan(1/2*d*x+1/2*c)+1)^7-4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^6+18/5/d/a^3/(t
an(1/2*d*x+1/2*c)+1)^5+1/d/a^3/(tan(1/2*d*x+1/2*c)+1)^4-5/6/d/a^3/(tan(1/2*
d*x+1/2*c)+1)^3-7/4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2-15/8/d/a^3/(tan(1/2*d*x+
1/2*c)+1)
```

Maxima [B] time = 1.62881, size = 452, normalized size = 3.18

$$-\frac{2\left(\frac{\frac{711\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1274\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{469\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1260\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1435\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{630\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{105\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+136}{a^3+\frac{6\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6\,a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{105\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -2/105*((711*sin(d*x + c)/(cos(d*x + c) + 1) + 1274*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 469*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1260*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 1435*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 630*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 105*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
+ 136)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(co
s(d*x + c) + 1)^8) + 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.15031, size = 408, normalized size = 2.87

$$-\frac{315\,dx\cos\left(dx+c\right)^3+221\,\cos\left(dx+c\right)^4-420\,dx\cos\left(dx+c\right)-417\,\cos\left(dx+c\right)^2+3\left(35\,dx\cos\left(dx+c\right)^3-140\,dx\,c\right.}{105\left(3\,a^3d\cos\left(dx+c\right)^3-4\,a^3d\cos\left(dx+c\right)+\left(a^3d\cos\left(dx+c\right)^3-4\,a^3d\cos\left(dx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/105*(315*d*x*cos(d*x + c)^3 + 221*cos(d*x + c)^4 - 420*d*x*cos(d*x + c)
- 417*cos(d*x + c)^2 + 3*(35*d*x*cos(d*x + c)^3 - 140*d*x*cos(d*x + c) - 11
6*cos(d*x + c)^2 + 15)*sin(d*x + c) + 60)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d
*cos(d*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28277, size = 174, normalized size = 1.23

$$-\frac{\frac{840\,(dx+c)}{a^3}+\frac{105}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{1575\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+10920\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31675\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48160\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36981\,\tan\left(\frac{1}{2}\,dx+\cdots\right.}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)/a^3 + 105/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) + (1575*ta
n(1/2*d*x + 1/2*c)^6 + 10920*tan(1/2*d*x + 1/2*c)^5 + 31675*tan(1/2*d*x + 1
/2*c)^4 + 48160*tan(1/2*d*x + 1/2*c)^3 + 36981*tan(1/2*d*x + 1/2*c)^2 + 143
92*tan(1/2*d*x + 1/2*c) + 2281)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.789 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{9\sec^5(c+dx)}{5a^3d}-\frac{2\sec^3(c+dx)}{a^3d}+\frac{\sec(c+dx)}{a^3d}$$

```
[Out] Sec[c + d*x]/(a^3*d) - (2*Sec[c + d*x]^3)/(a^3*d) + (9*Sec[c + d*x]^5)/(5*a
^3*d) - (4*Sec[c + d*x]^7)/(7*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) + (4*Tan[c
+ d*x]^7)/(7*a^3*d)
```

Rubi [A] time = 0.328677, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 14, 2606, 270, 30, 194}

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{9\sec^5(c+dx)}{5a^3d}-\frac{2\sec^3(c+dx)}{a^3d}+\frac{\sec(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]/(a^3*d) - (2*Sec[c + d*x]^3)/(a^3*d) + (9*Sec[c + d*x]^5)/(5*a
^3*d) - (4*Sec[c + d*x]^7)/(7*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) + (4*Tan[c
+ d*x]^7)/(7*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^4(c+dx)(a-a\sin(c+dx))^3\tan^4(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^4(c+dx)\tan^4(c+dx)-3a^3\sec^3(c+dx)\tan^5(c+dx)+3a^3\sec^2(c+dx)\tan\right.}{a^6}$$
$$= \frac{\int \sec^4(c+dx)\tan^4(c+dx)\,dx}{a^3}-\frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a^3}-\frac{3\int \sec^3(c+dx)\tan}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\text{Subst}\left(\int x^4\left(1+x^2\right)\,dx,x,\tan(c+dx)\right)}{a^3d}$$
$$= \frac{3\tan^7(c+dx)}{7a^3d}-\frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\text{Subst}\left(\int \left(x^4\right.\right.}{}$$
$$= \frac{\sec(c+dx)}{a^3d}-\frac{2\sec^3(c+dx)}{a^3d}+\frac{9\sec^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}+\frac{4\tan}{}$$

Mathematica [A] time = 0.477402, size = 104, normalized size = 1.02

$$\frac{\sec(c+dx)(1344\sin(c+dx)-1946\sin(2(c+dx))+64\sin(3(c+dx))+139\sin(4(c+dx))-1946\cos(c+dx)-224\cos}{2240a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(840 - 1946*Cos[c + d*x] - 224*Cos[2*(c + d*x)] + 834*Cos[3*(
c + d*x)] - 104*Cos[4*(c + d*x)] + 1344*Sin[c + d*x] - 1946*Sin[2*(c + d*x)
] + 64*Sin[3*(c + d*x)] + 139*Sin[4*(c + d*x)]))/(2240*a^3*d*(1 + Sin[c + d
*x])^3)
```

Maple [A] time = 0.107, size = 130, normalized size = 1.3

$$32\frac{1}{da^3}\left(-\frac{1}{256\tan(1/2dx+c/2)-256}-1/28\left(\tan(1/2dx+c/2)+1\right)^{-7}+1/8\left(\tan(1/2dx+c/2)+1\right)^{-6}-\frac{1}{80\left(\tan(1/2d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 32/d/a^3*(-1/256/(tan(1/2*d*x+1/2*c)-1)-1/28/(tan(1/2*d*x+1/2*c)+1)^7+1/8/(
tan(1/2*d*x+1/2*c)+1)^6-11/80/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/
2*c)+1)^4+1/64/(tan(1/2*d*x+1/2*c)+1)^3+1/128/(tan(1/2*d*x+1/2*c)+1)^2+1/25
6/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.05044, size = 311, normalized size = 3.05

$$\frac{16\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+1\right)}{35\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 16/35*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 14*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 14*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a^3 + 6*a^3*sin(d*x
+ c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a
^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d*x + c
) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A] time = 1.12336, size = 266, normalized size = 2.61

$$\frac{13\cos(dx+c)^4-6\cos(dx+c)^2-4\left(\cos(dx+c)^2+5\right)\sin(dx+c)-15}{35\left(3a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)+\left(a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(13*cos(d*x + c)^4 - 6*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + 5)*sin(d*x
 + c) - 15)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x
 + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25186, size = 162, normalized size = 1.59

$$-\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1015\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1673\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+616\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (35*tan(1/2*d*x + 1/2*c)^6 +
280*tan(1/2*d*x + 1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 2240*tan(1/2*d*x
 + 1/2*c)^3 + 1673*tan(1/2*d*x + 1/2*c)^2 + 616*tan(1/2*d*x + 1/2*c) + 93)/
(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.790 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{7\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{a^3d}$$

```
[Out] Sec[c + d*x]^3/(a^3*d) - (7*Sec[c + d*x]^5)/(5*a^3*d) + (4*Sec[c + d*x]^7)/
(7*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3*d) - (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rubi [A] time = 0.314914, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2606, 14, 2607, 270, 30}

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{7\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]^3/(a^3*d) - (7*Sec[c + d*x]^5)/(5*a^3*d) + (4*Sec[c + d*x]^7)/
(7*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3*d) - (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^5(c+dx)(a-a\sin(c+dx))^3\tan^3(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^5(c+dx)\tan^3(c+dx)-3a^3\sec^4(c+dx)\tan^4(c+dx)+3a^3\sec^3(c+dx)\tan^5\right.}{a^6}$$
$$= \frac{\int \sec^5(c+dx)\tan^3(c+dx)\,dx}{a^3}-\frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^3}-\frac{3\int \sec^4(c+dx)\tan}{a^3}$$
$$= -\frac{\text{Subst}\left(\int x^6\,dx,x,\tan(c+dx)\right)}{a^3d}+\frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)dx,x,\sec(c+dx)\right)}{a^3d}+\frac{3\,\text{Subs}}{}$$
$$= -\frac{\tan^7(c+dx)}{7a^3d}+\frac{\text{Subst}\left(\int\left(-x^4+x^6\right)dx,x,\sec(c+dx)\right)}{a^3d}+\frac{3\,\text{Subst}\left(\int\left(x^2-2x^4+x^6\right.\right.}{a^3d}$$
$$= \frac{\sec^3(c+dx)}{a^3d}-\frac{7\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{4\tan^7(c+dx)}{7a^3d}$$

Mathematica [A] time = 0.376032, size = 104, normalized size = 1.18

$$\frac{\sec(c+dx)(1008\sin(c+dx)-602\sin(2(c+dx))+48\sin(3(c+dx))+43\sin(4(c+dx))-602\cos(c+dx)-448\cos(2(}{2240a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(840 - 602*Cos[c + d*x] - 448*Cos[2*(c + d*x)] + 258*Cos[3*(c
 + d*x)] - 8*Cos[4*(c + d*x)] + 1008*Sin[c + d*x] - 602*Sin[2*(c + d*x)] +
48*Sin[3*(c + d*x)] + 43*Sin[4*(c + d*x)]))/(2240*a^3*d*(1 + Sin[c + d*x])^
3)
```

Maple [A] time = 0.102, size = 130, normalized size = 1.5

$$16\,\frac{1}{da^3}\left(-\frac{1}{128\,\tan(1/2\,dx+c/2)-128}+1/14\,(\tan(1/2\,dx+c/2)+1)^{-7}-1/4\,(\tan(1/2\,dx+c/2)+1)^{-6}+\frac{13}{40\,(\tan(1/2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 16/d/a^3*(-1/128/(tan(1/2*d*x+1/2*c)-1)+1/14/(tan(1/2*d*x+1/2*c)+1)^7-1/4/(
tan(1/2*d*x+1/2*c)+1)^6+13/40/(tan(1/2*d*x+1/2*c)+1)^5-3/16/(tan(1/2*d*x+1/
2*c)+1)^4+1/32/(tan(1/2*d*x+1/2*c)+1)^3+1/64/(tan(1/2*d*x+1/2*c)+1)^2+1/128
/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.07027, size = 338, normalized size = 3.84

$$\frac{4\left(\frac{18\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{42\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{35\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)}{35\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/35*(18*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 42*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 35*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 3)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A] time = 1.10806, size = 263, normalized size = 2.99

$$\frac{\cos(dx+c)^4+13\cos(dx+c)^2-3\left(\cos(dx+c)^2+5\right)\sin(dx+c)-20}{35\left(3a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)+\left(a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(cos(d*x + c)^4 + 13*cos(d*x + c)^2 - 3*(cos(d*x + c)^2 + 5)*sin(d*x +
 c) - 20)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x +
 c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2061, size = 162, normalized size = 1.84

$$-\frac{\frac{35}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1015\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1001\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+392\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{7}}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (35*tan(1/2*d*x + 1/2*c)^6 +
280*tan(1/2*d*x + 1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 1120*tan(1/2*d*x
 + 1/2*c)^3 + 1001*tan(1/2*d*x + 1/2*c)^2 + 392*tan(1/2*d*x + 1/2*c) + 61)/
(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.791 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{3a^3d}$$

```
[Out] -Sec[c + d*x]^3/(3*a^3*d) + Sec[c + d*x]^5/(a^3*d) - (4*Sec[c + d*x]^7)/(7*
a^3*d) + Tan[c + d*x]^3/(3*a^3*d) + Tan[c + d*x]^5/(a^3*d) + (4*Tan[c + d*x
]^7)/(7*a^3*d)
```

Rubi [A] time = 0.243329, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2711, 2607, 270, 2606, 14}

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Sec[c + d*x]^3/(3*a^3*d) + Sec[c + d*x]^5/(a^3*d) - (4*Sec[c + d*x]^7)/(7*
a^3*d) + Tan[c + d*x]^3/(3*a^3*d) + Tan[c + d*x]^5/(a^3*d) + (4*Tan[c + d*x
]^7)/(7*a^3*d)
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(a^3\sec^6(c+dx)\tan^2(c+dx) - 3a^3\sec^5(c+dx)\tan^3(c+dx) + 3a^3\sec^4(c+dx)\tan^4(c+\right.}{a^6}$$
$$= \frac{\int \sec^6(c+dx)\tan^2(c+dx)\,dx}{a^3} - \frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^3} - \frac{3\int \sec^5(c+dx)\tan^3(c-}{a^3}$$
$$= -\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2 dx, x, \sec(c+dx)\right)}{a^3 d} + \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^2 dx, x, \tan(c+dx)\right)}{a^3 d} -$$
$$= -\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right) dx, x, \sec(c+dx)\right)}{a^3 d} + \frac{\text{Subst}\left(\int \left(x^2+2x^4+x^6\right) dx, x, \tan(c+dx\right.}{a^3 d}$$
$$= -\frac{\sec^3(c+dx)}{3a^3 d} + \frac{\sec^5(c+dx)}{a^3 d} - \frac{4\sec^7(c+dx)}{7a^3 d} + \frac{\tan^3(c+dx)}{3a^3 d} + \frac{\tan^5(c+dx)}{a^3 d} + \frac{4\tan^7(c+}{7a^3 d}$$

Mathematica [A] time = 0.338808, size = 104, normalized size = 1.01

$$\frac{\sec(c+dx)(672\sin(c+dx) - 70\sin(2(c+dx)) - 96\sin(3(c+dx)) + 5\sin(4(c+dx)) - 70\cos(c+dx) - 224\cos(2(c+d}{1344a^3 d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(336 - 70*Cos[c + d*x] - 224*Cos[2*(c + d*x)] + 30*Cos[3*(c +
 d*x)] + 16*Cos[4*(c + d*x)] + 672*Sin[c + d*x] - 70*Sin[2*(c + d*x)] - 96*
Sin[3*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(1344*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.104, size = 130, normalized size = 1.3

$$8\frac{1}{da^3}\left(-\frac{1}{64\tan(1/2\,dx+c/2)-64} - 1/7\,(\tan(1/2\,dx+c/2)+1)^{-7} + 1/2\,(\tan(1/2\,dx+c/2)+1)^{-6} - 3/4\,(\tan(1/2\,dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 8/d/a^3*(-1/64/(tan(1/2*d*x+1/2*c)-1)-1/7/(tan(1/2*d*x+1/2*c)+1)^7+1/2/(tan
(1/2*d*x+1/2*c)+1)^6-3/4/(tan(1/2*d*x+1/2*c)+1)^5+5/8/(tan(1/2*d*x+1/2*c)+1
)^4-13/48/(tan(1/2*d*x+1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*c)+1)^2+1/64/(tan(
1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.04846, size = 365, normalized size = 3.54

$$\frac{4\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1} + \frac{14\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{28\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{14\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + 1\right)}{21\left(a^3 + \frac{6a^3\sin(dx+c)}{\cos(dx+c)+1} + \frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/21*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 14*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 28*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 14*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a^3 + 6*a^3*si
n(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A] time = 1.14163, size = 265, normalized size = 2.57

$$-\frac{2\cos\left(dx+c\right)^4-9\cos\left(dx+c\right)^2-6\left(\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)+9}{21\left(3a^3d\cos\left(dx+c\right)^3-4a^3d\cos\left(dx+c\right)+\left(a^3d\cos\left(dx+c\right)^3-4a^3d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/21*(2*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 - 2)*sin(d*x
 + c) + 9)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x
+ c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24318, size = 162, normalized size = 1.57

$$-\frac{\frac{21}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+168\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+161\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+224\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+56\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/168*(21/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (21*tan(1/2*d*x + 1/2*c)^6 +
168*tan(1/2*d*x + 1/2*c)^5 + 161*tan(1/2*d*x + 1/2*c)^4 + 224*tan(1/2*d*x +
 1/2*c)^3 + 63*tan(1/2*d*x + 1/2*c)^2 + 56*tan(1/2*d*x + 1/2*c) + 11)/(a^3*
(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.792 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$\frac{6\tan(c+dx)}{35a^3d}-\frac{3\sec(c+dx)}{35d\left(a^3\sin(c+dx)+a^3\right)}-\frac{3\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}+\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

```
[Out] Sec[c + d*x]/(7*d*(a + a*Sin[c + d*x])^3) - (3*Sec[c + d*x])/(35*a*d*(a + a
*Sin[c + d*x])^2) - (3*Sec[c + d*x])/(35*d*(a^3 + a^3*Sin[c + d*x])) + (6*T
an[c + d*x])/(35*a^3*d)
```

Rubi [A] time = 0.141934, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2859, 2672, 3767, 8}

$$\frac{6\tan(c+dx)}{35a^3d}-\frac{3\sec(c+dx)}{35d\left(a^3\sin(c+dx)+a^3\right)}-\frac{3\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}+\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]/(7*d*(a + a*Sin[c + d*x])^3) - (3*Sec[c + d*x])/(35*a*d*(a + a
*Sin[c + d*x])^2) - (3*Sec[c + d*x])/(35*d*(a^3 + a^3*Sin[c + d*x])) + (6*T
an[c + d*x])/(35*a^3*d)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} + \frac{3\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{7a}$$
$$= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} + \frac{9\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{35a^2}$$
$$= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)} + \frac{6\int}{}$$
$$= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)} - \frac{6\,\mathrm{Su}}{}$$
$$= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)} + \frac{6\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.329024, size = 104, normalized size = 1.05

$$\frac{\sec(c+dx)(672\sin(c+dx)+182\sin(2(c+dx))-288\sin(3(c+dx))-13\sin(4(c+dx))+182\cos(c+dx)-672\cos}{2240a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(560 + 182*Cos[c + d*x] - 672*Cos[2*(c + d*x)] - 78*Cos[3*(c
+ d*x)] + 48*Cos[4*(c + d*x)] + 672*Sin[c + d*x] + 182*Sin[2*(c + d*x)] - 2
88*Sin[3*(c + d*x)] - 13*Sin[4*(c + d*x)]))/(2240*a^3*d*(1 + Sin[c + d*x])^
3)
```

Maple [A] time = 0.096, size = 130, normalized size = 1.3

$$4\,\frac{1}{da^3}\left(-1/32\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-1}+2/7\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-7}-\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-6}+\frac{17}{10\,(\tan(1/2\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/d/a^3*(-1/32/(tan(1/2*d*x+1/2*c)-1)+2/7/(tan(1/2*d*x+1/2*c)+1)^7-1/(tan(1
/2*d*x+1/2*c)+1)^6+17/10/(tan(1/2*d*x+1/2*c)+1)^5-7/4/(tan(1/2*d*x+1/2*c)+1
)^4+9/8/(tan(1/2*d*x+1/2*c)+1)^3-7/16/(tan(1/2*d*x+1/2*c)+1)^2+1/32/(tan(1/
2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.03065, size = 392, normalized size = 3.96

$$-\frac{2\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{56\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{105\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{70\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{35\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}{35\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/35*(6*sin(d*x + c)/(cos(d*x + c) + 1) - 21*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 56*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 105*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 70*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 35*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) +
14*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a
^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A] time = 1.08124, size = 270, normalized size = 2.73

$$-\frac{6\cos(dx+c)^4-27\cos(dx+c)^2-3\left(6\cos(dx+c)^2-5\right)\sin(dx+c)+20}{35\left(3a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)+\left(a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(6*cos(d*x + c)^4 - 27*cos(d*x + c)^2 - 3*(6*cos(d*x + c)^2 - 5)*sin(
d*x + c) + 20)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(
d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sin(c+dx)\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A] time = 1.31122, size = 162, normalized size = 1.64

$$-\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-280\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-665\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-791\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-392\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (35*tan(1/2*d*x + 1/2*c)^6 -
280*tan(1/2*d*x + 1/2*c)^5 - 665*tan(1/2*d*x + 1/2*c)^4 - 1120*tan(1/2*d*x
+ 1/2*c)^3 - 791*tan(1/2*d*x + 1/2*c)^2 - 392*tan(1/2*d*x + 1/2*c) - 51)/(a
^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.793 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=151

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{11\tan^5(c+dx)}{5a^3d}-\frac{10\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx}{3a^3d}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + Sec[c + d*x]/(a^3*d) + Sec[c + d*x]^3/(3
*a^3*d) + Sec[c + d*x]^5/(5*a^3*d) + (4*Sec[c + d*x]^7)/(7*a^3*d) - (3*Tan[
c + d*x])/(a^3*d) - (10*Tan[c + d*x]^3)/(3*a^3*d) - (11*Tan[c + d*x]^5)/(5*
a^3*d) - (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rubi [A] time = 0.293391, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2606, 30, 2607, 270}

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{11\tan^5(c+dx)}{5a^3d}-\frac{10\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + Sec[c + d*x]/(a^3*d) + Sec[c + d*x]^3/(3
*a^3*d) + Sec[c + d*x]^5/(5*a^3*d) + (4*Sec[c + d*x]^7)/(7*a^3*d) - (3*Tan[
c + d*x])/(a^3*d) - (10*Tan[c + d*x]^3)/(3*a^3*d) - (11*Tan[c + d*x]^5)/(5*
a^3*d) - (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(-3a^3\sec^8(c+dx)+a^3\csc(c+dx)\sec^8(c+dx)+3a^3\sec^7(c+dx)\tan(c+dx)-a^3 \text{s}\right.}{a^6}\\
&= \frac{\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^3}-\frac{\int \sec^6(c+dx)\tan^2(c+dx)\,dx}{a^3}-\frac{3\int \sec^8(c+dx)\,dx}{a^3}+\\
&= \frac{\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx,x,\tan(c+dx)\right)}{a^3d}+\frac{3\,\text{Sul}}{}\\
&= \frac{3\sec^7(c+dx)}{7a^3d}-\frac{3\tan(c+dx)}{a^3d}-\frac{3\tan^3(c+dx)}{a^3d}-\frac{9\tan^5(c+dx)}{5a^3d}-\frac{3\tan^7(c+dx)}{7a^3d}-\\
&= \frac{\sec(c+dx)}{a^3d}+\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\tan(c+dx)}{a^3d}-\frac{10\tan^3}{3a}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\sec(c+dx)}{a^3d}+\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-
\end{aligned}
$$

Mathematica [B] time = 0.407884, size = 341, normalized size = 2.26

$$\frac{105\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-2281\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5+353\left(\sin\left(\frac{1}{2}(c+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (60 - (120*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 324*Si
n[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 162*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2 - 706*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^3 + 353*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 2281*Sin[(c + d
*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 840*Log[Cos[(c + d*x)/2]]*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6 + 840*Log[Sin[(c + d*x)/2]]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^6 + (105*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^6)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(840*d*(a + a
*Sin[c + d*x])^3)
```

Maple [A] time = 0.133, size = 187, normalized size = 1.2

$$-\frac{1}{8da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{8}{7da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-7}-4\frac{1}{da^3\left(\tan\left(1/2\,dx+c/2\right)+1\right)^6}+\frac{42}{5da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/8/d/a^3/(tan(1/2*d*x+1/2*c)-1)+8/7/d/a^3/(tan(1/2*d*x+1/2*c)+1)^7-4/d/a^
3/(tan(1/2*d*x+1/2*c)+1)^6+42/5/d/a^3/(tan(1/2*d*x+1/2*c)+1)^5-11/d/a^3/(ta
n(1/2*d*x+1/2*c)+1)^4+67/6/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3-31/4/d/a^3/(tan(1
/2*d*x+1/2*c)+1)^2+49/8/d/a^3/(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x
+1/2*c))
```

Maxima [B] time = 1.06973, size = 454, normalized size = 3.01

$$\frac{\frac{2\left(\frac{1011\sin(dx+c)}{\cos(dx+c)+1}+\frac{1939\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1379\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{525\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1715\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1155\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{315\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+221\right)}{a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{105\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/105*(2*(1011*sin(d*x + c)/(cos(d*x + c) + 1) + 1939*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 1379*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 525*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 1715*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1155*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 315*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 221)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*
a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x +
```

```
c) + 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8) + 105*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.15971, size = 602, normalized size = 3.99

$$\frac{272 \cos(dx+c)^4 - 594 \cos(dx+c)^2 - 105\left(3 \cos(dx+c)^3 + \left(\cos(dx+c)^3 - 4 \cos(dx+c)\right)\sin(dx+c) - 4 \cos(dx+\right.}{210\left(3 a^3 d \cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/210*(272*cos(d*x + c)^4 - 594*cos(d*x + c)^2 - 105*(3*cos(d*x + c)^3 + (c
os(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c) - 4*cos(d*x + c))*log(1/2*cos(
d*x + c) + 1/2) + 105*(3*cos(d*x + c)^3 + (cos(d*x + c)^3 - 4*cos(d*x + c))
*sin(d*x + c) - 4*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 6*(101*cos(d
*x + c)^2 + 15)*sin(d*x + c) - 120)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d
*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24654, size = 182, normalized size = 1.21

$$\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^3} - \frac{105}{a^3\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)} + \frac{5145 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6 + 24360 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 + 54005 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 + 66080 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 + 4769}{a^3\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^7}}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 105/(a^3*(tan(1/2*d*x + 1/2
*c) - 1)) + (5145*tan(1/2*d*x + 1/2*c)^6 + 24360*tan(1/2*d*x + 1/2*c)^5 + 5
4005*tan(1/2*d*x + 1/2*c)^4 + 66080*tan(1/2*d*x + 1/2*c)^3 + 47691*tan(1/2*
d*x + 1/2*c)^2 + 18872*tan(1/2*d*x + 1/2*c) + 3431)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^7))/d
```

3.794 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=162

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{13\tan^5(c+dx)}{5a^3d}+\frac{5\tan^3(c+dx)}{a^3d}+\frac{7\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (3*Sec[c + d*x])
/(a^3*d) - Sec[c + d*x]^3/(a^3*d) - (3*Sec[c + d*x]^5)/(5*a^3*d) - (4*Sec[c
 + d*x]^7)/(7*a^3*d) + (7*Tan[c + d*x])/(a^3*d) + (5*Tan[c + d*x]^3)/(a^3*d
) + (13*Tan[c + d*x]^5)/(5*a^3*d) + (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rubi [A] time = 0.347003, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2620, 270, 2606, 30}

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{13\tan^5(c+dx)}{5a^3d}+\frac{5\tan^3(c+dx)}{a^3d}+\frac{7\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (3*Sec[c + d*x])
/(a^3*d) - Sec[c + d*x]^3/(a^3*d) - (3*Sec[c + d*x]^5)/(5*a^3*d) - (4*Sec[c
 + d*x]^7)/(7*a^3*d) + (7*Tan[c + d*x])/(a^3*d) + (5*Tan[c + d*x]^3)/(a^3*d
) + (13*Tan[c + d*x]^5)/(5*a^3*d) + (4*Tan[c + d*x]^7)/(7*a^3*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^2(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(3a^3\sec^8(c+dx)-3a^3\csc(c+dx)\sec^8(c+dx)+a^3\csc^2(c+dx)\sec^8(c+dx)-a^3 s\right.}{a^6}\\
&= \frac{\int \csc^2(c+dx)\sec^8(c+dx)\,dx}{a^3}-\frac{\int \sec^7(c+dx)\tan(c+dx)\,dx}{a^3}+\frac{3\int \sec^8(c+dx)\,dx}{a^3}\\
&= -\frac{\text{Subst}\left(\int x^6\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^2}\,dx,x,\tan(c+dx)\right)}{a^3d}-\frac{3\,\text{Subst}\left(\int\right.}{}\\
&= -\frac{\sec^7(c+dx)}{7a^3d}+\frac{3\tan(c+dx)}{a^3d}+\frac{3\tan^3(c+dx)}{a^3d}+\frac{9\tan^5(c+dx)}{5a^3d}+\frac{3\tan^7(c+dx)}{7a^3d}+\\
&= -\frac{\cot(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{7\,\text{ta}}{}\\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}
\end{aligned}$$

Mathematica [B] time = 0.848432, size = 351, normalized size = 2.17

$$\frac{\csc^3(c+dx)\left(-1316\sin(c+dx)+3520\sin(2(c+dx))-1380\sin(3(c+dx))-1056\sin(4(c+dx))+176\sin(5(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(-966 - 440*Cos[2*(c + d*x)] - 2640*Cos[3*(c + d*x)] + 846*
Cos[4*(c + d*x)] + 176*Cos[5*(c + d*x)] - 1575*Cos[3*(c + d*x)]*Log[Cos[(c
+ d*x)/2]] + 105*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 14*Cos[c + d*x]*(
176 + 105*Log[Cos[(c + d*x)/2]] - 105*Log[Sin[(c + d*x)/2]]) + 1575*Cos[3*(
c + d*x)]*Log[Sin[(c + d*x)/2]] - 105*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]
] - 1316*Sin[c + d*x] + 3520*Sin[2*(c + d*x)] + 2100*Log[Cos[(c + d*x)/2]]*
Sin[2*(c + d*x)] - 2100*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 1380*Sin[3
*(c + d*x)] - 1056*Sin[4*(c + d*x)] - 630*Log[Cos[(c + d*x)/2]]*Sin[4*(c +
d*x)] + 630*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] + 176*Sin[5*(c + d*x)]))
/(140*a^3*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.15, size = 224, normalized size = 1.4

$$\frac{1}{2da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{8da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{8}{7da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-7}+4\frac{1}{da^3\left(\tan(1/2\,dx+c/2)+1\right)^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/8/d/a^3/(tan(1/2*d*x+1/2*c)-1)-8/7/d/a^3/(ta
n(1/2*d*x+1/2*c)+1)^7+4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^6-46/5/d/a^3/(tan(1/2*
d*x+1/2*c)+1)^5+13/d/a^3/(tan(1/2*d*x+1/2*c)+1)^4-31/2/d/a^3/(tan(1/2*d*x+1
/2*c)+1)^3+49/4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2-111/8/d/a^3/(tan(1/2*d*x+1/2
*c)+1)-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.06049, size = 533, normalized size = 3.29

$$-\frac{\frac{\frac{934\sin(dx+c)}{\cos(dx+c)+1}+\frac{3854\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6566\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3556\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3710\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7070\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{1015\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+35}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{14a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{14a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{6a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{210\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a^3}}{70d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/70*((934*sin(d*x + c)/(cos(d*x + c) + 1) + 3854*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 6566*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3556*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 3710*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7070*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4270*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 - 1015*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 35)/(a^3*sin(d*x + c)/(cos(d
```

```
*x + c) + 1) + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 14*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1
4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 14*a^3*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 - 6*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - a^3*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9) + 210*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 35
*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.5394, size = 713, normalized size = 4.4

$$\frac{846 \cos(dx+c)^4 - 956 \cos(dx+c)^2 + 105\left(\cos(dx+c)^5 - 5\cos(dx+c)^3 - \left(3\cos(dx+c)^3 - 4\cos(dx+c)\right)\sin(dx+\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/70*(846*cos(d*x + c)^4 - 956*cos(d*x + c)^2 + 105*(cos(d*x + c)^5 - 5*cos
(d*x + c)^3 - (3*cos(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c) + 4*cos(d*x
+ c))*log(1/2*cos(d*x + c) + 1/2) - 105*(cos(d*x + c)^5 - 5*cos(d*x + c)^3
- (3*cos(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c) + 4*cos(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2) + 2*(176*cos(d*x + c)^4 - 477*cos(d*x + c)^2 + 15)*s
in(d*x + c) + 40)/(a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^3 + 4*a^3*d*
cos(d*x + c) - (3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30927, size = 252, normalized size = 1.56

$$-\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3} - \frac{35\left(12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 17\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a^3} + \frac{3885\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 + 19880\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 454\ldots}{}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 140*tan(1/2*d*x + 1/2*c)/a
^3 - 35*(12*tan(1/2*d*x + 1/2*c)^2 - 17*tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2
*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c))*a^3) + (3885*tan(1/2*d*x + 1/2*c)^6
 + 19880*tan(1/2*d*x + 1/2*c)^5 + 45465*tan(1/2*d*x + 1/2*c)^4 + 57120*tan(
```

```
1/2*d*x + 1/2*c)^3 + 41671*tan(1/2*d*x + 1/2*c)^2 + 16632*tan(1/2*d*x + 1/2
*c) + 2931)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.795 $\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{a\cos^3(c+dx)}{3d}-\frac{3a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{3a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d}$$

```
[Out] (5*a*x)/2 - (3*a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (3*a*Sec[c +
d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (5*a*Tan[c + d*x])/(2*d) + (5*a*Tan[c
+ d*x]^3)/(6*d) - (a*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rubi [A] time = 0.134516, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2591, 288, 302, 203, 2590, 270}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{3a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{3a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```


```
[Out] (5*a*x)/2 - (3*a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (3*a*Sec[c +
d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (5*a*Tan[c + d*x])/(2*d) + (5*a*Tan[c
+ d*x]^3)/(6*d) - (a*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx &= a\int \sin^2(c+dx)\tan^4(c+dx)\,dx + a\int \sin^3(c+dx)\tan^4(c+dx)\\
&= -\frac{a\,\mathrm{Subst}\left(\int \frac{(1-x^2)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x\right.}{d}\\
&= -\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d} - \frac{a\,\mathrm{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)dx\right.}{d}\\
&= -\frac{3a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\sec(c+dx)}{d} + \frac{a\sec^3(c+}{3d}\\
&= -\frac{3a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\sec(c+dx)}{d} + \frac{a\sec^3(c+}{3d}\\
&= \frac{5ax}{2} - \frac{3a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\sec(c+dx)}{d} + \frac{a\sec^3}{}\end{aligned}$$

Mathematica [A] time = 0.411456, size = 84, normalized size = 0.72

$$\frac{a\left(-3\sin(2(c+dx)) - 33\cos(c+dx) + \cos(3(c+dx)) - 28\tan(c+dx) + 4\sec^3(c+dx) - 36\sec(c+dx) + 4\tan(c+\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*(30*c + 30*d*x - 33*Cos[c + d*x] + Cos[3*(c + d*x)] - 36*Sec[c + d*x] +
4*Sec[c + d*x]^3 - 3*Sin[2*(c + d*x)] - 28*Tan[c + d*x] + 4*Sec[c + d*x]^2*
Tan[c + d*x]))/(12*d)
```

Maple [A] time = 0.074, size = 164, normalized size = 1.4

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^8}{3(\cos(dx+c))^3} - \frac{5(\sin(dx+c))^8}{3\cos(dx+c)} - \frac{5\cos(dx+c)}{3}\left(\frac{16}{5} + (\sin(dx+c))^6 + \frac{6(\sin(dx+c))^4}{5} + \frac{8(\sin(dx+c)}{5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5
+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*sin(d*x
+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*sin(d*
x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A] time = 1.51775, size = 130, normalized size = 1.11

$$\frac{2\left(\cos\left(dx+c\right)^{3}-\frac{9\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}-9\cos\left(dx+c\right)\right)a+\left(2\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\tan\left(dx+c\right)\right)a}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
+ c))*a + (2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^
2 + 1) - 12*tan(d*x + c))*a)/d
```

Fricas [A] time = 1.4633, size = 285, normalized size = 2.44

$$\frac{a\cos\left(dx+c\right)^{4}-15\,adx\cos\left(dx+c\right)+29\,a\cos\left(dx+c\right)^{2}+\left(2\,a\cos\left(dx+c\right)^{4}+15\,adx\cos\left(dx+c\right)-15\,a\cos\left(dx+c\right)^{2}-4\right.}{6\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*cos(d*x + c)^4 - 15*a*d*x*cos(d*x + c) + 29*a*cos(d*x + c)^2 + (2*a*
cos(d*x + c)^4 + 15*a*d*x*cos(d*x + c) - 15*a*cos(d*x + c)^2 - 4*a)*sin(d*x
 + c) + 2*a)/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28295, size = 246, normalized size = 2.1

$$\frac{15\left(dx+c\right)a-\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{33\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-102\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+200\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-330\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+402\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-41}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)*a - 3*a/(tan(1/2*d*x + 1/2*c) + 1) + (33*a*tan(1/2*d*x +
1/2*c)^8 - 102*a*tan(1/2*d*x + 1/2*c)^7 + 200*a*tan(1/2*d*x + 1/2*c)^6 - 33
0*a*tan(1/2*d*x + 1/2*c)^5 + 402*a*tan(1/2*d*x + 1/2*c)^4 - 410*a*tan(1/2*d
*x + 1/2*c)^3 + 264*a*tan(1/2*d*x + 1/2*c)^2 - 150*a*tan(1/2*d*x + 1/2*c) +
 61*a)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2
*c) - 1)^3)/d
```

3.796 $\int \sin(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d}+\frac{5a}{2}$$

```
[Out] (5*a*x)/2 - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/
(3*d) - (5*a*Tan[c + d*x])/(2*d) + (5*a*Tan[c + d*x]^3)/(6*d) - (a*Sin[c +
d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rubi [A] time = 0.130055, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2838, 2590, 270, 2591, 288, 302, 203}

$$-\frac{a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d}+\frac{5a}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (5*a*x)/2 - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/
(3*d) - (5*a*Tan[c + d*x])/(2*d) + (5*a*Tan[c + d*x]^3)/(6*d) - (a*Sin[c +
d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx = a\int \sin(c+dx)\tan^4(c+dx)\,dx + a\int \sin^2(c+dx)\tan^4(c+dx)\,d$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x,\right.}{d}$$

$$= -\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d} - \frac{a\,\mathrm{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, \cos\right.}{d}$$

$$= -\frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\sin^2(c+dx)}{2}$$

$$= -\frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{5a\tan(c+dx}{2d}$$

$$= \frac{5ax}{2} - \frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{5a\tan(c}{2d}$$

Mathematica [A] time = 0.245321, size = 76, normalized size = 0.75

$$\frac{a\left(-3\sin(2(c+dx)) - 12\cos(c+dx) - 28\tan(c+dx) + 4\sec^3(c+dx) - 24\sec(c+dx) + 4\tan(c+dx)\sec^2(c+dx)\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*(30*c + 30*d*x - 12*Cos[c + d*x] - 24*Sec[c + d*x] + 4*Sec[c + d*x]^3 -
3*Sin[2*(c + d*x)] - 28*Tan[c + d*x] + 4*Sec[c + d*x]^2*Tan[c + d*x]))/(12*
d)
```

Maple [A] time = 0.069, size = 154, normalized size = 1.5

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^7}{3(\cos(dx+c))^3} - \frac{4(\sin(dx+c))^7}{3\cos(dx+c)} - \frac{4\cos(dx+c)}{3}\left((\sin(dx+c))^5 + \frac{5(\sin(dx+c))^3}{4} + \frac{15\sin(dx+c)}{8}\right) + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(
d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+a*(1/3
*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*si
n(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.69724, size = 117, normalized size = 1.16

$$\frac{\left(2\,\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\,\tan\left(dx+c\right)}{\tan\left(dx+c\right)^{2}+1}-12\,\tan\left(dx+c\right)\right)a-2\,a\left(\frac{6\,\cos\left(dx+c\right)^{2}-1}{\cos\left(dx+c\right)^{3}}+3\,\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a - 2*a*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos
(d*x + c)))/d
```

Fricas [A] time = 1.40778, size = 258, normalized size = 2.55

$$\frac{3\,a\cos\left(dx+c\right)^{4}-15\,adx\cos\left(dx+c\right)+17\,a\cos\left(dx+c\right)^{2}+\left(15\,adx\cos\left(dx+c\right)-3\,a\cos\left(dx+c\right)^{2}+2\,a\right)\sin\left(dx+c\right)-}{6\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^4 - 15*a*d*x*cos(d*x + c) + 17*a*cos(d*x + c)^2 + (15
*a*d*x*cos(d*x + c) - 3*a*cos(d*x + c)^2 + 2*a)*sin(d*x + c) - 4*a)/(d*cos(
d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26608, size = 181, normalized size = 1.79

$$\frac{15\,(dx+c)a+\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{6\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}+\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+23\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)*a + 3*a/(tan(1/2*d*x + 1/2*c) + 1) + 6*(a*tan(1/2*d*x + 1
/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(1
/2*d*x + 1/2*c)^2 + 1)^2 + (21*a*tan(1/2*d*x + 1/2*c)^2 - 48*a*tan(1/2*d*x
+ 1/2*c) + 23*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.797 $\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=72

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

```
[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0960119, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2710, 3473, 8, 2590, 270}

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx = \int \left(a\tan^4(c+dx)+a\sin(c+dx)\tan^4(c+dx)\right)dx$$
$$= a\int \tan^4(c+dx)\,dx + a\int \sin(c+dx)\tan^4(c+dx)\,dx$$
$$= \frac{a\tan^3(c+dx)}{3d} - a\int \tan^2(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} + a\int 1\,dx - \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x$$
$$= ax - \frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a\tan$$

Mathematica [A] time = 0.0408189, size = 81, normalized size = 1.12

$$-\frac{a\cos(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} + \frac{a\tan^{-1}(\tan(c+dx))}{d} - \frac{a\tan(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{2a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a
*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.063, size = 98, normalized size = 1.4

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^6}{3(\cos(dx+c))^3} - \frac{(\sin(dx+c))^6}{\cos(dx+c)} - \left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4(\sin(dx+c))^2}{3}\right)\cos(dx+c)\right)+a\left(\frac{(\tan(dx+c))^3}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+
c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A] time = 1.53912, size = 88, normalized size = 1.22

$$\frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a - a\left(\frac{6\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\cos(dx+c)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - a*((6*cos(d*x + c)
^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A] time = 1.36546, size = 224, normalized size = 3.11

$$-\frac{3\,adx\cos\left(dx+c\right)-7\,a\cos\left(dx+c\right)^{2}-\left(3\,adx\cos\left(dx+c\right)-3\,a\cos\left(dx+c\right)^{2}-2\,a\right)\sin\left(dx+c\right)-a}{3\,\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*d*x*cos(d*x + c) - 7*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) - 3
*a*cos(d*x + c)^2 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x + c) - d
*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26258, size = 167, normalized size = 2.32

$$\frac{6\,(dx+c)a-\frac{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-36\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a - 3*(a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)
 + 5*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/
2*c) + 1) + (15*a*tan(1/2*d*x + 1/2*c)^2 - 36*a*tan(1/2*d*x + 1/2*c) + 17*a
)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.798 $\int \sec(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}+ax$$

```
[Out] a*x - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d +
(a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0978578, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2838, 2606, 3473, 8}

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] a*x - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d +
(a*Tan[c + d*x]^3)/(3*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx = a\int \sec(c+dx)\tan^3(c+dx)\,dx + a\int \tan^4(c+dx)\,dx$$
$$= \frac{a\tan^3(c+dx)}{3d} - a\int \tan^2(c+dx)\,dx + \frac{a\,\text{Subst}\left(\int \left(-1+x^2\right)dx, x, s\right.}{d}$$
$$= -\frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} +$$
$$= ax - \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d}$$

Mathematica [A] time = 0.041551, size = 69, normalized size = 1.15

$$\frac{a\tan^3(c+dx)}{3d} + \frac{a\tan^{-1}(\tan(c+dx))}{d} - \frac{a\tan(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
 - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.064, size = 88, normalized size = 1.5

$$\frac{1}{d}\left(a\left(\frac{(\tan(dx+c))^3}{3} - \tan(dx+c) + dx + c\right) + a\left(\frac{(\sin(dx+c))^4}{3(\cos(dx+c))^3} - \frac{(\sin(dx+c))^4}{3\cos(dx+c)} - \frac{\left(2+(\sin(dx+c))^2\right)\cos(dx+c}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+a*(1/3*sin(d*x+c)^4/cos(d*x+c)^3
-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.53314, size = 74, normalized size = 1.23

$$\frac{\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a - \frac{\left(3\cos(dx+c)^2-1\right)a}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - (3*cos(d*x + c)^2
- 1)*a/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.42534, size = 196, normalized size = 3.27

$$-\frac{3\,adx\cos(dx+c) - 4\,a\cos(dx+c)^2 - (3\,adx\cos(dx+c) + a)\sin(dx+c) + 2\,a}{3\,(d\cos(dx+c)\sin(dx+c) - d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*d*x*cos(d*x + c) - 4*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) + a
)*sin(d*x + c) + 2*a)/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23333, size = 100, normalized size = 1.67

$$\frac{6\,(dx+c)a+\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a + 3*a/(tan(1/2*d*x + 1/2*c) + 1) + (9*a*tan(1/2*d*x + 1/
2*c)^2 - 24*a*tan(1/2*d*x + 1/2*c) + 11*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.799 $\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=45

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}$$

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.103911, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2838, 2607, 30, 2606}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx &= a\int \sec^2(c+dx)\tan^2(c+dx)\,dx+a\int \sec(c+dx)\tan^3(c+dx)\,dx\\ &= \frac{a\,\text{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{d}+\frac{a\,\text{Subst}\left(\int\left(-1+x^2\right)dx,x,\sec(\right.}{d}\\ &= -\frac{a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0373222, size = 45, normalized size = 1.

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.055, size = 82, normalized size = 1.8

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^4}{3(\cos(dx+c))^3}-\frac{(\sin(dx+c))^4}{3\cos(dx+c)}-\frac{\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}\right)+\frac{a(\sin(dx+c))^3}{3(\cos(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+si
n(d*x+c)^2)*cos(d*x+c))+1/3*a*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A] time = 1.01503, size = 53, normalized size = 1.18

$$\frac{a\tan(dx+c)^3-\frac{\left(3\cos(dx+c)^2-1\right)a}{\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(a*tan(d*x + c)^3 - (3*cos(d*x + c)^2 - 1)*a/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.53109, size = 127, normalized size = 2.82

$$\frac{a\cos(dx+c)^2-2a\sin(dx+c)+a}{3(d\cos(dx+c)\sin(dx+c)-d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(d*x + c)^2 - 2*a*sin(d*x + c) + a)/(d*cos(d*x + c)*sin(d*x + c)
- d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26336, size = 90, normalized size = 2.

$$-\frac{\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*a/(tan(1/2*d*x + 1/2*c) + 1) - (3*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*t
an(1/2*d*x + 1/2*c) + 5*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.800 $\int \sec^3(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=33

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}$$

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0838112, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2838, 2606, 30, 2607}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx &= a\int \sec^3(c+dx)\tan(c+dx)\,dx+a\int \sec^2(c+dx)\tan^2(c+dx)\,d\\ &=\frac{a\,\text{Subst}\left(\int x^2\,dx,x,\sec(c+dx)\right)}{d}+\frac{a\,\text{Subst}\left(\int x^2\,dx,x,\tan(c+d\right.}{d}\\ &=\frac{a\sec^3(c+dx)}{3d}+\frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0212681, size = 33, normalized size = 1.

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.047, size = 36, normalized size = 1.1

$$\frac{1}{d}\left(\frac{a(\sin(dx+c))^3}{3(\cos(dx+c))^3}+\frac{a}{3(\cos(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*sin(d*x+c)^3/cos(d*x+c)^3+1/3*a/cos(d*x+c)^3)
```

Maxima [A] time = 1.04008, size = 35, normalized size = 1.06

$$\frac{a\tan(dx+c)^3+\frac{a}{\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(a*tan(d*x + c)^3 + a/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.56659, size = 127, normalized size = 3.85

$$\frac{a\cos(dx+c)^2+a\sin(dx+c)-2a}{3(d\cos(dx+c)\sin(dx+c)-d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(d*x + c)^2 + a*sin(d*x + c) - 2*a)/(d*cos(d*x + c)*sin(d*x + c)
- d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24004, size = 72, normalized size = 2.18

$$\frac{\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a/(tan(1/2*d*x + 1/2*c) + 1) - (3*a*tan(1/2*d*x + 1/2*c)^2 + a)/(tan
(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.801 $\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=68

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3
*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0874933, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2838, 2622, 302, 207, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3
*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \sec^4(c+dx)\,dx + a\int \csc(c+dx)\sec^4(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\right.}{d}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} + \frac{a\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx\right.}{d}$$
$$= \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} +$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan(}{}$$

Mathematica [A] time = 0.116491, size = 85, normalized size = 1.25

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[(c + d*x)/2]])/d) + (a*Log[Sin[(c + d*x)/2]])/d + (a*Sec[c + d
*x])/d + (a*Sec[c + d*x]^3)/(3*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.082, size = 82, normalized size = 1.2

$$\frac{2\,a\tan(dx+c)}{3\,d} + \frac{a\tan(dx+c)\left(\sec(dx+c)\right)^2}{3\,d} + \frac{a}{3\,d\left(\cos(dx+c)\right)^3} + \frac{a}{d\cos(dx+c)} + \frac{a\ln\left(\csc(dx+c)-\cot(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/3/d*a/cos(d*x+c)^3+1/d
*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.17048, size = 99, normalized size = 1.46

$$\frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)a + a\left(\frac{2\left(3\cos(dx+c)^2+1\right)}{\cos(dx+c)^3} - 3\log\left(\cos(dx+c)+1\right) + 3\log\left(\cos(dx+c)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a + a*(2*(3*cos(d*x + c)^2 + 1)/co
s(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.67432, size = 348, normalized size = 5.12

$$-\frac{4\,a\cos\left(dx+c\right)^{2}+3\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)-a\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)\right.}{6\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^2 + 3*(a*cos(d*x + c)*sin(d*x + c) - a*cos(d*x + c))
*log(1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c)*sin(d*x + c) - a*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) - 2*a*sin(d*x + c) + 4*a)/(d*cos(d*x + c)
*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28876, size = 109, normalized size = 1.6

$$\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*a*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a/(tan(1/2*d*x + 1/2*c) + 1) -
(15*a*tan(1/2*d*x + 1/2*c)^2 - 24*a*tan(1/2*d*x + 1/2*c) + 13*a)/(tan(1/2*d
*x + 1/2*c) - 1)^3)/d
```

3.802 $\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (a*Sec[c + d*x])/d +
(a*Sec[c + d*x]^3)/(3*d) + (2*a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.123752, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2620, 270, 2622, 302, 207}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (a*Sec[c + d*x])/d +
(a*Sec[c + d*x]^3)/(3*d) + (2*a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc(c+dx)\sec^4(c+dx)\,dx + a\int \csc^2(c+dx)\sec^4(c+dx)\,dx$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x, \tan(\right.}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, \tan(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \left(1+x^2+\right.\right.}{}$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{2a\tan(c+dx)}{d} +$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c-}{3d}$$

Mathematica [A] time = 0.0566675, size = 109, normalized size = 1.35

$$\frac{5a\tan(c+dx)}{3d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/
2]])/d + (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) + (5*a*Tan[c + d*x])
/(3*d) + (a*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Maple [A] time = 0.098, size = 106, normalized size = 1.3

$$\frac{a}{3d(\cos(dx+c))^3} + \frac{a}{d\cos(dx+c)} + \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d} + \frac{a}{3d\sin(dx+c)(\cos(dx+c))^3} + \frac{4}{3d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3/d*a/cos(d*x+c)^3+1/d*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/3/d
*a/sin(d*x+c)/cos(d*x+c)^3+4/3/d*a/sin(d*x+c)/cos(d*x+c)-8/3*a*cot(d*x+c)/d
```

Maxima [A] time = 1.01255, size = 112, normalized size = 1.38

$$\frac{2\left(\tan(dx+c)^3 - \frac{3}{\tan(dx+c)} + 6\tan(dx+c)\right)a + a\left(\frac{2\left(3\cos(dx+c)^2+1\right)}{\cos(dx+c)^3} - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a + a*(2*(3*cos(d
*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c
) - 1)))/d
```

Fricas [B] time = 1.61541, size = 458, normalized size = 5.65

$$-\frac{10\,a\cos\left(dx+c\right)^{2}+3\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)\sin\left(dx+c\right)-a\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a\cos\right.}{6\left(d\cos\left(dx+c\right)^{3}+d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(10*a*cos(d*x + c)^2 + 3*(a*cos(d*x + c)^3 + a*cos(d*x + c)*sin(d*x +
c) - a*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c)^3 + a*
cos(d*x + c)*sin(d*x + c) - a*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) -
2*(8*a*cos(d*x + c)^2 - a)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)^3 + d*cos(d*
x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28096, size = 174, normalized size = 2.15

$$\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-36\,a\tan\left(\frac{1}{2}\,dx+\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*a*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a*tan(1/2*d*x + 1/2*c) - 3*(a*t
an(1/2*d*x + 1/2*c)^2 + 3*a*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)
^2 + tan(1/2*d*x + 1/2*c)) - (21*a*tan(1/2*d*x + 1/2*c)^2 - 36*a*tan(1/2*d*
x + 1/2*c) + 19*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.803 $\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=110

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{5a\sec^3(c+dx)}{6d}+\frac{5a\sec(c+dx)}{2d}-\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(}{}$$

```
[Out] (-5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d + (5*a*Sec[c + d*x]
)/(2*d) + (5*a*Sec[c + d*x]^3)/(6*d) - (a*Csc[c + d*x]^2*Sec[c + d*x]^3)/(2
*d) + (2*a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.136468, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2622, 288, 302, 207, 2620, 270}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{5a\sec^3(c+dx)}{6d}+\frac{5a\sec(c+dx)}{2d}-\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d + (5*a*Sec[c + d*x]
)/(2*d) + (5*a*Sec[c + d*x]^3)/(6*d) - (a*Csc[c + d*x]^2*Sec[c + d*x]^3)/(2
*d) + (2*a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^4(c+dx)\,dx + a\int \csc^3(c+dx)\sec^4(c+dx)$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\,\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x,\right.}{d}$$

$$= -\frac{a\csc^2(c+dx)\sec^3(c+dx)}{2d} + \frac{a\,\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, \text{ta}\right.}{d}$$

$$= -\frac{a\cot(c+dx)}{d} - \frac{a\csc^2(c+dx)\sec^3(c+dx)}{2d} + \frac{2a\tan(c+dx)}{d} +$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{5a\sec(c+dx)}{2d} + \frac{5a\sec^3(c+dx)}{6d} - \frac{a\csc^2(c+a}{}$$

$$= -\frac{5a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)}{d} + \frac{5a\sec(c+dx)}{2d} + \frac{5a\,\text{s}}{}$$

Mathematica [B] time = 6.08636, size = 359, normalized size = 3.26

$$\frac{5a\tan(c+dx)}{3d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{5a\log\left(\cos\left(\frac{1}{2}\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (5*a*Log[Cos[(c + d*
x)/2]])/(2*d) + (5*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/
(8*d) + a/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (a*Sin[(c + d*x)
/2])/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (13*a*Sin[(c + d*x)/2]
)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (a*Sin[(c + d*x)/2])/(6*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + a/(12*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2) - (13*a*Sin[(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])) + (5*a*Tan[c + d*x])/(3*d) + (a*Sec[c + d*x]^2*Tan[c + d*x])/(3*
d)
```

Maple [A] time = 0.106, size = 138, normalized size = 1.3

$$\frac{a}{3d\sin(dx+c)(\cos(dx+c))^3}+\frac{4a}{3d\sin(dx+c)\cos(dx+c)}-\frac{8a\cot(dx+c)}{3d}+\frac{a}{3d(\sin(dx+c))^2(\cos(dx+c))^3}-\frac{\quad}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3/d*a/sin(d*x+c)/cos(d*x+c)^3+4/3/d*a/sin(d*x+c)/cos(d*x+c)-8/3*a*cot(d*x
+c)/d+1/3/d*a/sin(d*x+c)^2/cos(d*x+c)^3-5/6/d*a/sin(d*x+c)^2/cos(d*x+c)+5/2
/d*a/cos(d*x+c)+5/2/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.1672, size = 143, normalized size = 1.3

$$\frac{4\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\tan(dx+c)\right)a+a\left(\frac{2\left(15\cos(dx+c)^4-10\cos(dx+c)^2-2\right)}{\cos(dx+c)^5-\cos(dx+c)^3}-15\log(\cos(dx+c)+1)+15\log(\cos\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a + a*(2*(15*cos
(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15
*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.46833, size = 575, normalized size = 5.23

$$\frac{32a\cos(dx+c)^4-18a\cos(dx+c)^2-15\left(a\cos(dx+c)^3-a\cos(dx+c)-\left(a\cos(dx+c)^3-a\cos(dx+c)\right)\sin(dx+c\right.}{12\left(d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(32*a*cos(d*x + c)^4 - 18*a*cos(d*x + c)^2 - 15*(a*cos(d*x + c)^3 - a*
cos(d*x + c) - (a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) + 15*(a*cos(d*x + c)^3 - a*cos(d*x + c) - (a*cos(d*x + c)
^3 - a*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(a*cos(
d*x + c)^2 + 2*a)*sin(d*x + c) - 8*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c) -
(d*cos(d*x + c)^3 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21085, size = 200, normalized size = 1.82

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+60\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{3\left(30\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,a\,\mathrm{ta}\cdots\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*a*tan(1/2*d*x + 1/2*c)^2 + 60*a*log(abs(tan(1/2*d*x + 1/2*c))) + 12
*a*tan(1/2*d*x + 1/2*c) + 12*a/(tan(1/2*d*x + 1/2*c) + 1) - 3*(30*a*tan(1/2
*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2 - 4*
(27*a*tan(1/2*d*x + 1/2*c)^2 - 48*a*tan(1/2*d*x + 1/2*c) + 25*a)/(tan(1/2*d
*x + 1/2*c) - 1)^3)/d
```

3.804 $\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{11a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{7a^2x}{2}$$

```
[Out] (7*a^2*x)/2 - (2*a^2*Cos[c + d*x])/d + (a^2*Cos[c + d*x])/(3*d*(1 - Sin[c +
 d*x])^2) - (11*a^2*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (a^2*Cos[c + d
*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.205375, antiderivative size = 120, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2708, 2765, 2977, 2734}

$$-\frac{16a^2\cos(c+dx)}{3d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{8a^2\sin^2(c+dx)\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{7a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (7*a^2*x)/2 - (16*a^2*Cos[c + d*x])/(3*d) - (7*a^2*Cos[c + d*x]*Sin[c + d*x
])/(2*d) - (8*a^2*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(1 - Sin[c + d*x])) + (
a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx = a^4\int \frac{\sin^4(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\sin^2(c+dx)(-3a-5a\sin(c+dx))}{a-a\sin(c+dx)}\,dx$$
$$= -\frac{8a^2\cos(c+dx)\sin^2(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{1}{3}\int \sin(c+d$$
$$= \frac{7a^2x}{2} - \frac{16a^2\cos(c+dx)}{3d} - \frac{7a^2\cos(c+dx)\sin(c+dx)}{2d} - \frac{8a^2\cos(c+dx)\sin}{3d(1-\sin(c+}$$

Mathematica [A] time = 1.23498, size = 159, normalized size = 1.57

$$-\frac{a^2\left(-21(12c+12dx+7)\cos\left(\frac{1}{2}(c+dx)\right)+(84c+84dx+239)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-5\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\frac{7}{2}(c+\right.\right.\right.}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -(a^2*(-21*(7 + 12*c + 12*d*x)*Cos[(c + d*x)/2] + (239 + 84*c + 84*d*x)*Cos
[(3*(c + d*x))/2] + 3*(-5*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2] + 2*(
50 + 56*c + 56*d*x + (-27 + 28*c + 28*d*x)*Cos[c + d*x] - 6*Cos[2*(c + d*x)
] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(48*d*(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])^3)
```

Maple [A] time = 0.083, size = 186, normalized size = 1.8

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^7}{3(\cos(dx+c))^3} - \frac{4(\sin(dx+c))^7}{3\cos(dx+c)} - \frac{4\cos(dx+c)}{3}\left((\sin(dx+c))^5 + \frac{5(\sin(dx+c))^3}{4} + \frac{15\sin(dx+c)}{8}\right) + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(si
n(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+2*a^
2*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+
4/3*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A] time = 1.68, size = 162, normalized size = 1.6

$$\frac{\left(2\tan(dx+c)^3+15dx+15c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan(dx+c)\right)a^2+2\left(\tan(dx+c)^3+3dx+3c-3\tan(dx+c)\right)a^2}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a^2 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c)
)*a^2 - 4*a^2*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [B] time = 1.40218, size = 468, normalized size = 4.63

$$\frac{3\,a^2\cos\left(dx+c\right)^4-6\,a^2\cos\left(dx+c\right)^3-42\,a^2dx+\left(21\,a^2dx+31\,a^2\right)\cos\left(dx+c\right)^2-2\,a^2-\left(21\,a^2dx-38\,a^2\right)\cos\left(dx+c\right)-}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^3 - 42*a^2*d*x + (21*a^2*d*x
 + 31*a^2)*cos(d*x + c)^2 - 2*a^2 - (21*a^2*d*x - 38*a^2)*cos(d*x + c) - (3
*a^2*cos(d*x + c)^3 - 42*a^2*d*x + 9*a^2*cos(d*x + c)^2 + 2*a^2 - (21*a^2*d
*x - 40*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c)
 + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22969, size = 182, normalized size = 1.8

$$\frac{21\,(dx+c)a^2+\frac{6\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{4\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*(d*x + c)*a^2 + 6*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x +
 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
2 + 4*(9*a^2*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*tan(1/2*d*x + 1/2*c) + 10*a^2)
/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.805 $\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=86

$$-\frac{4a^2\cos(c+dx)}{3d}+\frac{a^4\sin^2(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+2a^2x$$

```
[Out] 2*a^2*x - (4*a^2*Cos[c + d*x])/(3*d) - (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c +
 d*x])) + (a^4*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.245919, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2869, 2765, 2968, 3023, 12, 2735, 2648}

$$-\frac{4a^2\cos(c+dx)}{3d}+\frac{a^4\sin^2(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```


```
[Out] 2*a^2*x - (4*a^2*Cos[c + d*x])/(3*d) - (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c +
 d*x])) + (a^4*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(a - a*Sin[c + d*x])^2)
```


Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = a^4\int \frac{\sin^3(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\sin(c+dx)(-2a-4a\sin(c+d}{a-a\sin(c+dx)}$$
$$= \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{-2a\sin(c+dx)-4a\sin^2(c+a}{a-a\sin(c+dx)}$$
$$= -\frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{1}{3}a\int \frac{6a^2\sin(c+}{a-a\sin(c}$$
$$= -\frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - \left(2a^3\right)\int \frac{\sin(c+}{a-a\sin(}$$
$$= 2a^2x - \frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - \left(2a^3\right)\int \frac{}{a-}$$
$$= 2a^2x - \frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{2a^3\cos(c}{d(a-a\sin(c}$$

Mathematica [A] time = 1.0341, size = 131, normalized size = 1.52

$$\frac{a^2(\sin(c+dx)+1)^2\left(-3\cos(c+dx)+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(8\sin(c+dx)-7)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+6c+6dx\right)}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(6*c + 6*d*x - 3*Cos[c + d*x] + (Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^(-2) + (2*Sin[(c + d*x)/2]*(-7 + 8*Sin[c + d*x]))/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^4)
```

Maple [A] time = 0.073, size = 162, normalized size = 1.9

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^6}{3(\cos(dx+c))^3}-\frac{(\sin(dx+c))^6}{\cos(dx+c)}-\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4(\sin(dx+c))^2}{3}\right)\cos(dx+c)\right)+2a^2\left(1/3\ (\tan(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*
x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x
+c)+a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+s
in(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.64283, size = 128, normalized size = 1.49

$$\frac{2\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^2-a^2\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos(dx+c)\right)-\frac{\left(3\,\cos(dx+c)^2-1\right)a^2}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2 - a^2*((6*cos(d*
x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - (3*cos(d*x + c)^2 - 1)*a^2
/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.38732, size = 400, normalized size = 4.65

$$-\frac{3\,a^2\cos(dx+c)^3+12\,a^2dx-\left(6\,a^2dx+11\,a^2\right)\cos(dx+c)^2+a^2+\left(6\,a^2dx-13\,a^2\right)\cos(dx+c)-\left(12\,a^2dx-3\,a^2\cos\right.}{3\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2\,d)\sin(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(3*a^2*cos(d*x + c)^3 + 12*a^2*d*x - (6*a^2*d*x + 11*a^2)*cos(d*x + c)
^2 + a^2 + (6*a^2*d*x - 13*a^2)*cos(d*x + c) - (12*a^2*d*x - 3*a^2*cos(d*x
+ c)^2 - a^2 + 2*(3*a^2*d*x - 7*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2729, size = 116, normalized size = 1.35

$$\frac{2\left(3\,(dx+c)a^2 - \frac{3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + \frac{6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 7\,a^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(d*x + c)*a^2 - 3*a^2/(tan(1/2*d*x + 1/2*c)^2 + 1) + (6*a^2*tan(1/2*
d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 7*a^2)/(tan(1/2*d*x + 1/2*c)
 - 1)^3)/d
```

3.806 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=63

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{5a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+a^2x$$

```
[Out] a^2*x - (5*a^2*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) + (a^4*Cos[c + d*x])/
(3*d*(a - a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.206785, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2869, 2758, 2735, 2648}

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{5a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] a^2*x - (5*a^2*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) + (a^4*Cos[c + d*x])/
(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = a^4\int \frac{\sin^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{-2a-3a\sin(c+dx)}{a-a\sin(c+dx)}\,dx$$
$$= a^2x + \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{1}{3}\left(5a^3\right)\int \frac{1}{a-a\sin(c+dx)}\,dx$$
$$= a^2x + \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{5a^3\cos(c+dx)}{3d(a-a\sin(c+dx))}$$

Mathematica [A] time = 0.0514261, size = 79, normalized size = 1.25

$$\frac{2a^2\tan^3(c+dx)}{3d} + \frac{a^2\tan^{-1}(\tan(c+dx))}{d} - \frac{a^2\tan(c+dx)}{d} + \frac{2a^2\sec^3(c+dx)}{3d} - \frac{2a^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (a^2*ArcTan[Tan[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (2*a^2*Sec[c + d*x]
^3)/(3*d) - (a^2*Tan[c + d*x])/d + (2*a^2*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.074, size = 114, normalized size = 1.8

$$\frac{1}{d}\left(a^2\left(\frac{(\tan(dx+c))^3}{3} - \tan(dx+c) + dx + c\right) + 2a^2\left(1/3\,\frac{(\sin(dx+c))^4}{(\cos(dx+c))^3} - 1/3\,\frac{(\sin(dx+c))^4}{\cos(dx+c)} - 1/3\,\left(2+(\sin(dx+c))^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a^2*(1/3*sin(d*x+c)^4/cos(d*
x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+1/3*a^2
*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A] time = 1.65301, size = 96, normalized size = 1.52

$$\frac{a^2\tan(dx+c)^3 + \left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^2 - \frac{2\left(3\,\cos(dx+c)^2-1\right)a^2}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a
^2 - 2*(3*cos(d*x + c)^2 - 1)*a^2/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.41047, size = 329, normalized size = 5.22

$$-\frac{6\,a^2dx - \left(3\,a^2dx + 5\,a^2\right)\cos\left(dx+c\right)^2 + a^2 + \left(3\,a^2dx - 4\,a^2\right)\cos\left(dx+c\right) - \left(6\,a^2dx - a^2 + \left(3\,a^2dx - 5\,a^2\right)\cos\left(dx+c\right.\right.}{3\left(d\cos\left(dx+c\right)^2 - d\cos\left(dx+c\right) + \left(d\cos\left(dx+c\right) + 2\,d\right)\sin\left(dx+c\right) - 2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(6*a^2*d*x - (3*a^2*d*x + 5*a^2)*cos(d*x + c)^2 + a^2 + (3*a^2*d*x - 4
*a^2)*cos(d*x + c) - (6*a^2*d*x - a^2 + (3*a^2*d*x - 5*a^2)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*si
n(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21985, size = 90, normalized size = 1.43

$$\frac{3\,(dx+c)a^2 + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*tan(1/2*d*x
+ 1/2*c) + 4*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.807 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=60

$$-\frac{2a^2\tan(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

```
[Out] (-2*a^2*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(3*d)
 - (2*a^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0864827, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2855, 2669, 3767, 8}

$$-\frac{2a^2\tan(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (-2*a^2*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(3*d)
 - (2*a^2*Tan[c + d*x])/(3*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} - \frac{1}{3}(2a)\int \sec^2(c+dx)(a+a\sin$$
$$= -\frac{2a^2\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} - \frac{1}{3}\left(2a^2\right)\int$$
$$= -\frac{2a^2\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} + \frac{\left(2a^2\right)\mathrm{Sub}}{}$$
$$= -\frac{2a^2\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} - \frac{2a^2\tan(c}{3d}$$

Mathematica [A] time = 0.282257, size = 72, normalized size = 1.2

$$\frac{a^2\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)-2\cos\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (a^2*(3*Cos[(c + d*x)/2] - 2*Cos[(3*(c + d*x))/2] - 3*Sin[(c + d*x)/2]))/(3
*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
```

Maple [A] time = 0.059, size = 99, normalized size = 1.7

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^4}{3(\cos(dx+c))^3} - \frac{(\sin(dx+c))^4}{3\cos(dx+c)} - \frac{\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}\right) + \frac{2a^2(\sin(dx+c))^3}{3(\cos(dx+c))^3} + \frac{a^2}{3(\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+2/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+1/3*a^2/cos(d*x
+c)^3)
```

Maxima [A] time = 1.12491, size = 76, normalized size = 1.27

$$\frac{2a^2\tan(dx+c)^3 - \frac{\left(3\cos(dx+c)^2-1\right)a^2}{\cos(dx+c)^3} + \frac{a^2}{\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2*tan(d*x + c)^3 - (3*cos(d*x + c)^2 - 1)*a^2/cos(d*x + c)^3 + a^2
/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.40958, size = 236, normalized size = 3.93

$$\frac{2\,a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)-a^{2}-\left(2\,a^{2}\cos\left(dx+c\right)+a^{2}\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - a^2 - (2*a^2*cos(d*x + c) +
a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2
*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27201, size = 51, normalized size = 0.85

$$-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{2}\right)}{3\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*d*x + 1/2*c) - a^2)/(d*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

3.808 $\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=73

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (4*a^2*Cos[c + d*x])/(3*d*(1 - Sin[c + d
*x])) + (a^4*Cos[c + d*x])/(3*d*(a - a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.187461, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2869, 2766, 2978, 12, 3770}

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (4*a^2*Cos[c + d*x])/(3*d*(1 - Sin[c + d
*x])) + (a^4*Cos[c + d*x])/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx = a^4\int \frac{\csc(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\csc(c+dx)(3a+a\sin(c+dx))}{a-a\sin(c+dx)}\,dx$$
$$= \frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}\int 3a^2\csc(c+dx)\,d$$
$$= \frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} + a^2\int \csc(c+dx)\,dx$$
$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+d}$$

Mathematica [A] time = 0.501146, size = 142, normalized size = 1.95

$$\frac{a^2(\sin(c+dx)+1)^2\left(3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(4\sin(c+dx)-5)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\right.\right.}\right.}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2
]] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) + (2*Sin[(c + d*x)/2]*(-5 +
 4*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.103, size = 92, normalized size = 1.3

$$\frac{2a^2}{3d(\cos(dx+c))^3} + \frac{4a^2\tan(dx+c)}{3d} + \frac{2a^2\tan(dx+c)(\sec(dx+c))^2}{3d} + \frac{a^2}{d\cos(dx+c)} + \frac{a^2\ln(\csc(dx+c)-\cot(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/3/d*a^2/cos(d*x+c)^3+4/3*a^2*tan(d*x+c)/d+2/3/d*a^2*tan(d*x+c)*sec(d*x+c)
^2+1/d*a^2/cos(d*x+c)+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.02628, size = 122, normalized size = 1.67

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{2}+a^{2}\left(\frac{2\left(3\,\cos\left(dx+c\right)^{2}+1\right)}{\cos\left(dx+c\right)^{3}}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{2}{\cos(d}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + a^2*(2*(3*cos(d*x + c)^2 + 1
)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*a
^2/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.41095, size = 578, normalized size = 7.92

$$-\frac{8\,a^{2}\cos\left(dx+c\right)^{2}+10\,a^{2}\cos\left(dx+c\right)+2\,a^{2}+3\left(a^{2}\cos\left(dx+c\right)^{2}-a^{2}\cos\left(dx+c\right)-2\,a^{2}+\left(a^{2}\cos\left(dx+c\right)+2\,a^{2}\right)\mathrm{s}}{6\,(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(8*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c) + 2*a^2 + 3*(a^2*cos(d*x +
 c)^2 - a^2*cos(d*x + c) - 2*a^2 + (a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))
*log(1/2*cos(d*x + c) + 1/2) - 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2
*a^2 + (a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2
) - 2*(4*a^2*cos(d*x + c) - a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*
x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26965, size = 99, normalized size = 1.36

$$\frac{3\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(6\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(6*a^2*tan(1/2*d*x + 1/2*c)^2
 - 9*a^2*tan(1/2*d*x + 1/2*c) + 5*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.809 $\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=87

$$-\frac{10a^2\cot(c+dx)}{3d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] (-2*a^2*ArcTanh[Cos[c + d*x]])/d - (10*a^2*Cot[c + d*x])/(3*d) + (2*a^2*Cot
[c + d*x])/(d*(1 - Sin[c + d*x])) + (a^4*Cot[c + d*x])/(3*d*(a - a*Sin[c +
d*x])^2)
```

Rubi [A] time = 0.27391, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2869, 2766, 2978, 2748, 3767, 8, 3770}

$$-\frac{10a^2\cot(c+dx)}{3d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*ArcTanh[Cos[c + d*x]])/d - (10*a^2*Cot[c + d*x])/(3*d) + (2*a^2*Cot
[c + d*x])/(d*(1 - Sin[c + d*x])) + (a^4*Cot[c + d*x])/(3*d*(a - a*Sin[c +
d*x])^2)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx = a^4\int \frac{\csc^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\csc^2(c+dx)(4a+2a\sin(c+dx}{a-a\sin(c+dx)}$$
$$= \frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}\int \csc^2(c+dx)$$
$$= \frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2} + \left(2a^2\right)\int \csc(c+d$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))} + \frac{a^4\cot(c+}{3d(a-a\sin(c}$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{10a^2\cot(c+dx)}{3d} + \frac{2a^2\cot(c+dx}{d(1-\sin(c+d}$$

Mathematica [A] time = 0.932694, size = 135, normalized size = 1.55

$$\frac{a^2\left(3\tan\left(\frac{1}{2}(c+dx)\right)-3\cot\left(\frac{1}{2}(c+dx)\right)+12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)(7\sin(c+dx}{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+d\right.\right.}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-3*Cot[(c + d*x)/2] - 12*Log[Cos[(c + d*x)/2]] + 12*Log[Sin[(c + d*x)
/2]] + 2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*Sin[(c + d*x)/2]*(-8
+ 7*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 3*Tan[(c + d*
x)/2]))/(6*d)
```

Maple [A] time = 0.122, size = 156, normalized size = 1.8

$$\frac{2a^2\tan(dx+c)}{3d}+\frac{a^2\tan(dx+c)(\sec(dx+c))^2}{3d}+\frac{2a^2}{3d(\cos(dx+c))^3}+2\frac{a^2}{d\cos(dx+c)}+2\frac{a^2\ln(\csc(dx+c)-\cot(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/3*a^2*tan(d*x+c)/d+1/3/d*a^2*tan(d*x+c)*sec(d*x+c)^2+2/3/d*a^2/cos(d*x+c)
^3+2/d*a^2/cos(d*x+c)+2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+1/3/d*a^2/sin(d*x+c
)/cos(d*x+c)^3+4/3/d*a^2/sin(d*x+c)/cos(d*x+c)-8/3*a^2*cot(d*x+c)/d
```

Maxima [A] time = 1.15638, size = 144, normalized size = 1.66

$$\frac{\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\tan(dx+c)\right)a^2+\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2+a^2\left(\frac{2\left(3\cos(dx+c)^2+1\right)}{\cos(dx+c)^3}-3\log(\cos(dx\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*((tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^2 + (tan(d*x + c)
^3 + 3*tan(d*x + c))*a^2 + a^2*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.42352, size = 810, normalized size = 9.31

$$\frac{10a^2\cos(dx+c)^3-4a^2\cos(dx+c)^2-13a^2\cos(dx+c)+a^2-3\left(a^2\cos(dx+c)^3+2a^2\cos(dx+c)^2-a^2\cos(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/3*(10*a^2*cos(d*x + c)^3 - 4*a^2*cos(d*x + c)^2 - 13*a^2*cos(d*x + c) + a
^2 - 3*(a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^
2 - (a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x + c))*log(1/2*c
os(d*x + c) + 1/2) + 3*(a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 - a^2*cos
(d*x + c) - 2*a^2 - (a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x
 + c))*log(-1/2*cos(d*x + c) + 1/2) + (10*a^2*cos(d*x + c)^2 + 14*a^2*cos(d
*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - d*cos
(d*x + c) - (d*cos(d*x + c)^2 - d*cos(d*x + c) - 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32088, size = 159, normalized size = 1.83

$$\frac{12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{4\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^2*tan(1/2*d*x + 1/2*c) - 3
*(4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) - 4*(9*a^2*tan(1/2
*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 8*a^2)/(tan(1/2*d*x + 1/2*c
) - 1)^3)/d
```

3.810 $\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=125

$$-\frac{16a^2\cot(c+dx)}{3d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{7a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{8a^2\cot(c+ \ldots}{3d(1-\mathrm{si}\ldots}$$

```
[Out] (-7*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (16*a^2*Cot[c + d*x])/(3*d) - (7*a^2
*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (8*a^2*Cot[c + d*x]*Csc[c + d*x])/(3*d*
(1 - Sin[c + d*x])) + (a^4*Cot[c + d*x]*Csc[c + d*x])/(3*d*(a - a*Sin[c + d
*x])^2)
```

Rubi [A] time = 0.30795, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2869, 2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{16a^2\cot(c+dx)}{3d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{7a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{8a^2\cot(c+ \ldots}{3d(1-\mathrm{si}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (16*a^2*Cot[c + d*x])/(3*d) - (7*a^2
*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (8*a^2*Cot[c + d*x]*Csc[c + d*x])/(3*d*
(1 - Sin[c + d*x])) + (a^4*Cot[c + d*x]*Csc[c + d*x])/(3*d*(a - a*Sin[c + d
*x])^2)
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx = a^4\int \frac{\csc^3(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\csc^3(c+dx)(5a+3a\sin(c+$$
$$= \frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}\int$$
$$= \frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}(16$$
$$= -\frac{7a^2\cot(c+dx)\csc(c+dx)}{2d} + \frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4}{}$$
$$= -\frac{7a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{16a^2\cot(c+dx)}{3d} - \frac{7a^2\cot(c+dx)}{2d}$$

Mathematica [A] time = 2.11198, size = 190, normalized size = 1.52

$$\frac{a^2\left(24\tan\left(\frac{1}{2}(c+dx)\right) - 24\cot\left(\frac{1}{2}(c+dx)\right) - 3\csc^2\left(\frac{1}{2}(c+dx)\right) + 3\sec^2\left(\frac{1}{2}(c+dx)\right) + 84\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 84\,l\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-24*Cot[(c + d*x)/2] - 3*Csc[(c + d*x)/2]^2 - 84*Log[Cos[(c + d*x)/2]
] + 84*Log[Sin[(c + d*x)/2]] + 3*Sec[(c + d*x)/2]^2 + 8/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^3 + (160*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) +
24*Tan[(c + d*x)/2]))/(24*d)
```

Maple [A] time = 0.135, size = 168, normalized size = 1.3

$$\frac{a^2}{3\,d\,(\cos(dx+c))^3}+\frac{7\,a^2}{2\,d\cos(dx+c)}+\frac{7\,a^2\ln(\csc(dx+c)-\cot(dx+c))}{2\,d}+\frac{2\,a^2}{3\,d\sin(dx+c)\,(\cos(dx+c))^3}+\frac{}{3\,d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/3/d*a^2/cos(d*x+c)^3+7/2/d*a^2/cos(d*x+c)+7/2/d*a^2*ln(csc(d*x+c)-cot(d*x
+c))+2/3/d*a^2/sin(d*x+c)/cos(d*x+c)^3+8/3/d*a^2/sin(d*x+c)/cos(d*x+c)-16/3
*a^2*cot(d*x+c)/d+1/3/d*a^2/sin(d*x+c)^2/cos(d*x+c)^3-5/6/d*a^2/sin(d*x+c)^
2/cos(d*x+c)
```

Maxima [A] time = 1.19111, size = 216, normalized size = 1.73

$$\frac{8\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\,\tan(dx+c)\right)a^2+a^2\left(\frac{2\left(15\,\cos(dx+c)^4-10\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^5-\cos(dx+c)^3}-15\,\log(\cos(dx+c)+1)+15\,\log(\mathrm{co}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(8*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^2 + a^2*(2*(15
*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3)
- 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2*a^2*(2*(3*cos(d*
x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c)
 - 1)))/d
```

Fricas [B] time = 1.46823, size = 1062, normalized size = 8.5

$$-\frac{64\,a^2\cos(dx+c)^4+86\,a^2\cos(dx+c)^3-54\,a^2\cos(dx+c)^2-80\,a^2\cos(dx+c)-4\,a^2+21\left(a^2\cos(dx+c)^4-a^2\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/12*(64*a^2*cos(d*x + c)^4 + 86*a^2*cos(d*x + c)^3 - 54*a^2*cos(d*x + c)^
2 - 80*a^2*cos(d*x + c) - 4*a^2 + 21*(a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)
^3 - 3*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) + 2*a^2 + (a^2*cos(d*x + c)^3
+ 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x + c))*log(1/2*co
```

```
s(d*x + c) + 1/2) - 21*(a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 3*a^2*cos
(d*x + c)^2 + a^2*cos(d*x + c) + 2*a^2 + (a^2*cos(d*x + c)^3 + 2*a^2*cos(d*
x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) +
1/2) - 2*(32*a^2*cos(d*x + c)^3 - 11*a^2*cos(d*x + c)^2 - 38*a^2*cos(d*x +
c) + 2*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 3*d*cos(d*
x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - d*cos(
d*x + c) - 2*d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34292, size = 203, normalized size = 1.62

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 84\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + 24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{3\left(42\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*tan(1/2*d*x + 1/2*c)^2 + 84*a^2*log(abs(tan(1/2*d*x + 1/2*c)))
+ 24*a^2*tan(1/2*d*x + 1/2*c) - 3*(42*a^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*ta
n(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^2 - 16*(12*a^2*tan(1/2*d*x +
 1/2*c)^2 - 21*a^2*tan(1/2*d*x + 1/2*c) + 11*a^2)/(tan(1/2*d*x + 1/2*c) - 1
)^3)/d
```

3.811 $\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{6a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^3x}{2}$$

```
[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3
*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 -
 Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.193788, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{6a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3
*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 -
 Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx = a^4\int\left(\frac{7}{a}+\frac{2}{a(-1+\sin(c+dx))^2}+\frac{9}{a(-1+\sin(c+dx))}+\frac{5\sin(c+dx)}{a}+\frac{3\,\mathrm{s}\ldots}{}\right.$$

$$= 7a^3x + a^3\int \sin^3(c+dx)\,dx + \left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx + \left(3a^3\right)\int \sin\ldots$$

$$= 7a^3x - \frac{5a^3\cos(c+dx)}{d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{9a^3\cos(c+dx)}{d(1-\sin(c+dx))} - \frac{3a^3\,\mathrm{co}\ldots}{}$$

$$= \frac{17a^3x}{2} - \frac{6a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{25a^3\,\mathrm{c}\ldots}{3d(1-\ldots}$$

Mathematica [A] time = 2.2261, size = 177, normalized size = 1.49

$$\frac{(a\sin(c+dx)+a)^3\left(102(c+dx)-9\sin(2(c+dx))-69\cos(c+dx)+\cos(3(c+dx))-\frac{200\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{\ldots}{(\cos(\ldots}\right.}{12d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(102*(c + d*x) - 69*Cos[c + d*x] + Cos[3*(c + d*x)]
 + 8/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])^3 - (200*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]) - 9*Sin[2*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^6)
```

Maple [B] time = 0.098, size = 266, normalized size = 2.2

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^8}{3(\cos(dx+c))^3}-\frac{5(\sin(dx+c))^8}{3\cos(dx+c)}-\frac{5\cos(dx+c)}{3}\left(\frac{16}{5}+(\sin(dx+c))^6+\frac{6(\sin(dx+c))^4}{5}+\frac{8(\sin(dx+\ldots}{5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16
/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*s
```

```
in(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*
sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+3*a^3*(1/3*sin(d*x+
c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^
2)*cos(d*x+c))+a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A] time = 1.56687, size = 223, normalized size = 1.87

$$\frac{2\left(\cos\left(dx+c\right)^{3}-\frac{9\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}-9\cos\left(dx+c\right)\right)a^{3}+3\left(2\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\tan\left(dx+c\right)\right)a^{3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
+ c))*a^3 + 3*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x +
 c)^2 + 1) - 12*tan(d*x + c))*a^3 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan
(d*x + c))*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
 c)))/d
```

Fricas [B] time = 1.46836, size = 539, normalized size = 4.53

$$\frac{2\,a^{3}\cos\left(dx+c\right)^{5}+7\,a^{3}\cos\left(dx+c\right)^{4}-22\,a^{3}\cos\left(dx+c\right)^{3}-102\,a^{3}dx-4\,a^{3}+\left(51\,a^{3}dx+77\,a^{3}\right)\cos\left(dx+c\right)^{2}-\left(51\,a^{3}dx\right.}{6\left(d\cos\left(dx+c\right)^{2}-d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^5 + 7*a^3*cos(d*x + c)^4 - 22*a^3*cos(d*x + c)^3 -
102*a^3*d*x - 4*a^3 + (51*a^3*d*x + 77*a^3)*cos(d*x + c)^2 - (51*a^3*d*x -
100*a^3)*cos(d*x + c) + (2*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + 102*
a^3*d*x - 27*a^3*cos(d*x + c)^2 - 4*a^3 + (51*a^3*d*x - 104*a^3)*cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*
d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32719, size = 252, normalized size = 2.12

$$\frac{51\,(dx+c)a^3+\frac{2\left(51\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-153\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+289\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-459\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+501\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-511\,a^3\tan\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(51*(d*x + c)*a^3 + 2*(51*a^3*tan(1/2*d*x + 1/2*c)^8 - 153*a^3*tan(1/2*
d*x + 1/2*c)^7 + 289*a^3*tan(1/2*d*x + 1/2*c)^6 - 459*a^3*tan(1/2*d*x + 1/2
*c)^5 + 501*a^3*tan(1/2*d*x + 1/2*c)^4 - 511*a^3*tan(1/2*d*x + 1/2*c)^3 + 3
27*a^3*tan(1/2*d*x + 1/2*c)^2 - 189*a^3*tan(1/2*d*x + 1/2*c) + 80*a^3)/(tan
(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) - 1)^3)
/d
```

3.812 $\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{19a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{11a^3x}{2}$$

```
[Out] (11*a^3*x)/2 - (3*a^3*Cos[c + d*x])/d + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[
c + d*x])^2) - (19*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (a^3*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.162259, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2872, 2650, 2648, 2638, 2635, 8}

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{19a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{11a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (11*a^3*x)/2 - (3*a^3*Cos[c + d*x])/d + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[
c + d*x])^2) - (19*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (a^3*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx = a^4\int\left(\frac{5}{a}+\frac{2}{a(-1+\sin(c+dx))^2}+\frac{7}{a(-1+\sin(c+dx))}+\frac{3\sin(c\right.$$

$$= 5a^3x + a^3\int\sin^2(c+dx)\,dx + \left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx +$$

$$= 5a^3x - \frac{3a^3\cos(c+dx)}{d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{7a^3\cos(c+dx}{d(1-\sin(c+dx}$$

$$= \frac{11a^3x}{2} - \frac{3a^3\cos(c+dx)}{d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{19a^3\cos(c+}{3d(1-\sin(c+}$$

Mathematica [A] time = 1.45991, size = 159, normalized size = 1.57

$$-\frac{a^3\left(-3(132c+132dx+89)\cos\left(\frac{1}{2}(c+dx)\right)+(132c+132dx+403)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-9\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\frac{7}{2}\right.\right.\right.}{48d\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -(a^3*(-3*(89 + 132*c + 132*d*x)*Cos[(c + d*x)/2] + (403 + 132*c + 132*d*x)
*Cos[(3*(c + d*x))/2] + 3*(-9*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2] +
 2*(86 + 88*c + 88*d*x + (-43 + 44*c + 44*d*x)*Cos[c + d*x] - 10*Cos[2*(c +
 d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(48*d*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^3)
```

Maple [B] time = 0.088, size = 246, normalized size = 2.4

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^7}{3(\cos(dx+c))^3}-\frac{4(\sin(dx+c))^7}{3\cos(dx+c)}-\frac{4\cos(dx+c)}{3}\left((\sin(dx+c))^5+\frac{5(\sin(dx+c))^3}{4}+\frac{15\sin(dx+c)}{8}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(si
n(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+3*a^
3*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+
4/3*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+a^3
*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+
c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.70007, size = 196, normalized size = 1.94

$$\frac{\left(2 \tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3 \tan(dx+c)}{\tan(dx+c)^{2}+1}-12 \tan\left(dx+c\right)\right)a^{3}+6\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3 \tan\left(dx+c\right)\right)a^{3}-6}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a^3 + 6*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c)
)*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - 2*
(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.4259, size = 471, normalized size = 4.66

$$\frac{3\,a^{3}\cos\left(dx+c\right)^{4}-12\,a^{3}\cos\left(dx+c\right)^{3}-66\,a^{3}dx-4\,a^{3}+\left(33\,a^{3}dx+53\,a^{3}\right)\cos\left(dx+c\right)^{2}-\left(33\,a^{3}dx-64\,a^{3}\right)\cos\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)+\left(d\cos\left(dx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^3*cos(d*x + c)^4 - 12*a^3*cos(d*x + c)^3 - 66*a^3*d*x - 4*a^3 + (3
3*a^3*d*x + 53*a^3)*cos(d*x + c)^2 - (33*a^3*d*x - 64*a^3)*cos(d*x + c) - (
3*a^3*cos(d*x + c)^3 - 66*a^3*d*x + 15*a^3*cos(d*x + c)^2 + 4*a^3 - (33*a^3
*d*x - 68*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x +
c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21789, size = 182, normalized size = 1.8

$$\frac{33\,(dx+c)a^{3}+\frac{6\left(a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^{3}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}+\frac{4\left(15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-36\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\,a^{3}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(33*(d*x + c)*a^3 + 6*(a^3*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*tan(1/2*d*x +
 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
2 + 4*(15*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a^3*tan(1/2*d*x + 1/2*c) + 17*a^3
)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.813 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=77

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}+3a^3x+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

```
[Out] 3*a^3*x - (3*a^3*Cos[c + d*x])/d - (2*a^5*Cos[c + d*x]^3)/(d*(a - a*Sin[c +
 d*x])^2) + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d)
```

Rubi [A] time = 0.218969, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2871, 2670, 2680, 2682, 8}

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}+3a^3x+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] 3*a^3*x - (3*a^3*Cos[c + d*x])/d - (2*a^5*Cos[c + d*x]^3)/(d*(a - a*Sin[c +
 d*x])^2) + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d)
```

Rule 2871

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] - Dist[1/g^2, Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && EqQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \int \sec^2(c+dx)(a+a\sin(c+$$
$$= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - a^6\int \frac{\cos^4(c+dx)}{(a-a\sin(c+dx))^3}\,dx$$
$$= -\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} + \big(3a$$
$$= -\frac{3a^3\cos(c+dx)}{d} - \frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2} + \frac{\sec^3(c+dx)(a+a\,\mathrm{s}}{3d}$$
$$= 3a^3x - \frac{3a^3\cos(c+dx)}{d} - \frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2} + \frac{\sec^3(c+dx)(a}{}$$

Mathematica [A] time = 1.25296, size = 133, normalized size = 1.73

$$\frac{a^3(\sin(c+dx)+1)^3\left(-3\cos(c+dx)+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(13\sin(c+dx)-11)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+9c+9dx\right)}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(9*c + 9*d*x - 3*Cos[c + d*x] + 2/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-11 + 13*Sin[c + d*x]))/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^6)
```

Maple [B] time = 0.08, size = 184, normalized size = 2.4

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^6}{3(\cos(dx+c))^3}-\frac{(\sin(dx+c))^6}{\cos(dx+c)}-\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4(\sin(dx+c))^2}{3}\right)\cos(dx+c)\right)+3a^3\left(1/3(\tan(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*
x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x
+c)+3*a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2
+sin(d*x+c)^2)*cos(d*x+c))+1/3*a^3*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A] time = 1.68552, size = 144, normalized size = 1.87

$$\frac{a^3\tan(dx+c)^3+3\left(\tan(dx+c)^3+3dx+3c-3\tan(dx+c)\right)a^3-a^3\left(\frac{6\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\cos(dx+c)\right)-\frac{3\left(3\cos(dx+c)^2\right.}{\cos(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/3*(a^3*tan(d*x + c)^3 + 3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))
*a^3 - a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - 3*(3*
cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.37299, size = 404, normalized size = 5.25

$$-\frac{3\,a^3\cos\left(dx+c\right)^3+18\,a^3dx+2\,a^3-\left(9\,a^3dx+16\,a^3\right)\cos\left(dx+c\right)^2+\left(9\,a^3dx-17\,a^3\right)\cos\left(dx+c\right)-\left(18\,a^3dx-3\,a^3\cos\right.}{3\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/3*(3*a^3*cos(d*x + c)^3 + 18*a^3*d*x + 2*a^3 - (9*a^3*d*x + 16*a^3)*cos(
d*x + c)^2 + (9*a^3*d*x - 17*a^3)*cos(d*x + c) - (18*a^3*d*x - 3*a^3*cos(d*
x + c)^2 - 2*a^3 + (9*a^3*d*x - 19*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29811, size = 117, normalized size = 1.52

$$\frac{9\,(dx+c)a^3-\frac{6\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(9*(d*x + c)*a^3 - 6*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(9*a^3*tan(1/
2*d*x + 1/2*c)^2 - 24*a^3*tan(1/2*d*x + 1/2*c) + 11*a^3)/(tan(1/2*d*x + 1/2
*c) - 1)^3)/d
```

3.814 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=64

$$-\frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+a^3x+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

```
[Out] a^3*x + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d) - (2*a^5*Cos[c + d*x]
)/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.138811, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2855, 2670, 2680, 8}

$$-\frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+a^3x+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] a^3*x + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d) - (2*a^5*Cos[c + d*x]
)/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - a\int \sec^2(c+dx)(a+a\sin(c+dx$$
$$= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - a^5\int \frac{\cos^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)} + a^3\int 1\,d$$
$$= a^3x + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.719077, size = 107, normalized size = 1.67

$$-\frac{a^3\left(-9(c+dx+2)\cos\left(\frac{1}{2}(c+dx)\right)+(3c+3dx+14)\cos\left(\frac{3}{2}(c+dx)\right)+6\sin\left(\frac{1}{2}(c+dx)\right)(2(c+dx+2)+(c+dx)\cos(c\right.}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -(a^3*(-9*(2 + c + d*x)*Cos[(c + d*x)/2] + (14 + 3*c + 3*d*x)*Cos[(3*(c + d
*x))/2] + 6*(2*(2 + c + d*x) + (c + d*x)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(
6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
```

Maple [B] time = 0.071, size = 126, normalized size = 2.

$$\frac{1}{d}\left(a^3\left(\frac{(\tan(dx+c))^3}{3}-\tan(dx+c)+dx+c\right)+3a^3\left(1/3\,\frac{(\sin(dx+c))^4}{(\cos(dx+c))^3}-1/3\,\frac{(\sin(dx+c))^4}{\cos(dx+c)}-1/3\,\left(2+(\sin(dx+c))^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+3*a^3*(1/3*sin(d*x+c)^4/cos(d*
x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+a^3*sin
(d*x+c)^3/cos(d*x+c)^3+1/3*a^3/cos(d*x+c)^3)
```

Maxima [A] time = 1.69052, size = 113, normalized size = 1.77

$$\frac{3\,a^3\tan(dx+c)^3+\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^3-\frac{3\left(3\,\cos(dx+c)^2-1\right)a^3}{\cos(dx+c)^3}+\frac{a^3}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))
*a^3 - 3*(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3 + a^3/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.35476, size = 335, normalized size = 5.23

$$-\frac{6a^3dx + 2a^3 - \left(3a^3dx + 7a^3\right)\cos\left(dx+c\right)^2 + \left(3a^3dx - 5a^3\right)\cos\left(dx+c\right) - \left(6a^3dx - 2a^3 + \left(3a^3dx - 7a^3\right)\cos\left(dx\right.\right.}{3\left(d\cos\left(dx+c\right)^2 - d\cos\left(dx+c\right) + \left(d\cos\left(dx+c\right) + 2d\right)\sin\left(dx+c\right) - 2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*d*x + 2*a^3 - (3*a^3*d*x + 7*a^3)*cos(d*x + c)^2 + (3*a^3*d*x -
 5*a^3)*cos(d*x + c) - (6*a^3*d*x - 2*a^3 + (3*a^3*d*x - 7*a^3)*cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d
)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25368, size = 90, normalized size = 1.41

$$\frac{3\left(dx+c\right)a^3 + \frac{2\left(3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 12a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 5a^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^3 + 2*(3*a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x
 + 1/2*c) + 5*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.815 $\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=72

$$\frac{5a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d
*x])^2) + (5*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x]))
```

Rubi [A] time = 0.140726, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2872, 3770, 2650, 2648}

$$\frac{5a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d
*x])^2) + (5*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x]))
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx = a^4\int\left(\frac{\csc(c+dx)}{a}+\frac{2}{a(-1+\sin(c+dx))^2}-\frac{1}{a(-1+\sin(c+dx))}\right.$$
$$= a^3\int\csc(c+dx)\,dx - a^3\int\frac{1}{-1+\sin(c+dx)}\,dx + \left(2a^3\right)\int\frac{}{(-1+}$$
$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{a^3\cos(c+d}{d(1-\sin(c+}$$
$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{5a^3\cos(c+}{3d(1-\sin(c+}$$

Mathematica [A] time = 0.625892, size = 144, normalized size = 2.

$$\frac{a^3(\sin(c+dx)+1)^3\left(3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(5\sin(c+dx)-7)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\right.\right.}\right.}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2
]] + 2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-7 +
5*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.121, size = 115, normalized size = 1.6

$$\frac{a^3(\sin(dx+c))^3}{3d(\cos(dx+c))^3}+\frac{4a^3}{3d(\cos(dx+c))^3}+2\frac{a^3\tan(dx+c)}{d}+\frac{a^3\tan(dx+c)(\sec(dx+c))^2}{d}+\frac{a^3}{d\cos(dx+c)}+\frac{a^3\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/3/d*a^3*sin(d*x+c)^3/cos(d*x+c)^3+4/3/d*a^3/cos(d*x+c)^3+2*a^3*tan(d*x+c)
/d+1/d*a^3*tan(d*x+c)*sec(d*x+c)^2+1/d*a^3/cos(d*x+c)+1/d*a^3*ln(csc(d*x+c)
-cot(d*x+c))
```

Maxima [A] time = 1.15318, size = 139, normalized size = 1.93

$$\frac{2a^3\tan(dx+c)^3+6\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3+a^3\left(\frac{2\left(3\cos(dx+c)^2+1\right)}{\cos(dx+c)^3}-3\log(\cos(dx+c)+1)+3\log(\cos(\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*tan(d*x + c)^3 + 6*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 + a^3*(
2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(c
```

```
os(d*x + c) - 1)) + 6*a^3/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.47947, size = 582, normalized size = 8.08

$$-\frac{10\,a^3\cos\left(dx+c\right)^2+14\,a^3\cos\left(dx+c\right)+4\,a^3+3\left(a^3\cos\left(dx+c\right)^2-a^3\cos\left(dx+c\right)-2\,a^3+\left(a^3\cos\left(dx+c\right)+2\,a^3\right)\sin}{6\left(d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(10*a^3*cos(d*x + c)^2 + 14*a^3*cos(d*x + c) + 4*a^3 + 3*(a^3*cos(d*x
+ c)^2 - a^3*cos(d*x + c) - 2*a^3 + (a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c)
)*log(1/2*cos(d*x + c) + 1/2) - 3*(a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) -
2*a^3 + (a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/
2) - 2*(5*a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos
(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24262, size = 99, normalized size = 1.38

$$\frac{3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(9*a^3*tan(1/2*d*x + 1/2*c)^2
 - 12*a^3*tan(1/2*d*x + 1/2*c) + 7*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.816 $\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{a^3\cot(c+dx)}{d}+\frac{11a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (2*a^3*Cos[c + d*
x])/(3*d*(1 - Sin[c + d*x])^2) + (11*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*
x]))
```

Rubi [A] time = 0.169914, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 3770, 3767, 8, 2650, 2648}

$$-\frac{a^3\cot(c+dx)}{d}+\frac{11a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (2*a^3*Cos[c + d*
x])/(3*d*(1 - Sin[c + d*x])^2) + (11*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*
x]))
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx = a^4\int\left(\frac{3\csc(c+dx)}{a}+\frac{\csc^2(c+dx)}{a}+\frac{2}{a(-1+\sin(c+dx))^2}-\frac{}{a(-1}\right.$$
$$= a^3\int\csc^2(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\left(3a^3\right)\int c$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{3a^3\cos(c+d}{d(1-\sin(c+d}$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}$$

Mathematica [A] time = 0.954951, size = 135, normalized size = 1.57

$$\frac{a^3\left(3\tan\left(\frac{1}{2}(c+dx)\right)-3\cot\left(\frac{1}{2}(c+dx)\right)+18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)(11\sin(c+dx)-1}{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right.}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-3*Cot[(c + d*x)/2] - 18*Log[Cos[(c + d*x)/2]] + 18*Log[Sin[(c + d*x)
/2]] + 4/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*Sin[(c + d*x)/2]*(-13
 + 11*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 3*Tan[(c +
d*x)/2]))/(6*d)
```

Maple [A] time = 0.136, size = 155, normalized size = 1.8

$$\frac{4\,a^3}{3\,d\,(\cos(dx+c))^3}+2\,\frac{a^3\tan(dx+c)}{d}+\frac{a^3\tan(dx+c)\,(\sec(dx+c))^2}{d}+3\,\frac{a^3}{d\cos(dx+c)}+3\,\frac{a^3\ln(\csc(dx+c)-\cot(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/3/d*a^3/cos(d*x+c)^3+2*a^3*tan(d*x+c)/d+1/d*a^3*tan(d*x+c)*sec(d*x+c)^2+3
/d*a^3/cos(d*x+c)+3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+1/3/d*a^3/sin(d*x+c)/co
s(d*x+c)^3+4/3/d*a^3/sin(d*x+c)/cos(d*x+c)-8/3*a^3*cot(d*x+c)/d
```

Maxima [A] time = 1.10084, size = 166, normalized size = 1.93

$$\frac{2\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\,\tan(dx+c)\right)a^3+6\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^3+3\,a^3\left(\frac{2\left(3\,\cos(dx+c)^2+1\right)}{\cos(dx+c)^3}-3\,\log(\mathrm{co}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^3 + 6*(tan(d*x
+ c)^3 + 3*tan(d*x + c))*a^3 + 3*a^3*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)
^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*a^3/cos(d*x + c
)^3)/d
```

Fricas [B] time = 1.56735, size = 819, normalized size = 9.52

$$\frac{28\,a^{3}\cos\left(dx+c\right)^{3}-10\,a^{3}\cos\left(dx+c\right)^{2}-34\,a^{3}\cos\left(dx+c\right)+4\,a^{3}-9\left(a^{3}\cos\left(dx+c\right)^{3}+2\,a^{3}\cos\left(dx+c\right)^{2}-a^{3}\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(28*a^3*cos(d*x + c)^3 - 10*a^3*cos(d*x + c)^2 - 34*a^3*cos(d*x + c) +
4*a^3 - 9*(a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2
*a^3 - (a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))*log(1/
2*cos(d*x + c) + 1/2) + 9*(a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - a^3*
cos(d*x + c) - 2*a^3 - (a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(14*a^3*cos(d*x + c)^2 + 19*a^3*
cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2
- d*cos(d*x + c) - (d*cos(d*x + c)^2 - d*cos(d*x + c) - 2*d)*sin(d*x + c) -
 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.323, size = 159, normalized size = 1.85

$$\frac{18\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{4\left(15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c) - 3
*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c) - 4*(15*a^3*tan(1/
2*d*x + 1/2*c)^2 - 24*a^3*tan(1/2*d*x + 1/2*c) + 13*a^3)/(tan(1/2*d*x + 1/2
*c) - 1)^3)/d
```

3.817 $\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=110

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{17a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] (-11*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]*Csc[c + d*x])/(2*d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^
2) + (17*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x]))
```

Rubi [A] time = 0.190362, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 3768, 2650, 2648}

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{17a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]*Csc[c + d*x])/(2*d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^
2) + (17*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x]))
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx = a^4\int\left(\frac{5\csc(c+dx)}{a}+\frac{3\csc^2(c+dx)}{a}+\frac{\csc^3(c+dx)}{a}+\frac{2}{a(-1+\sin(\right.$$

$$= a^3\int\csc^3(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\left(3a^3\right)\int c$$

$$= -\frac{5a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{2a^3\cos}{3d(1-\sin}$$

$$= -\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc(}{2d}$$

Mathematica [A] time = 2.11083, size = 190, normalized size = 1.73

$$\frac{a^3\left(36\tan\left(\frac{1}{2}(c+dx)\right)-36\cot\left(\frac{1}{2}(c+dx)\right)-3\csc^2\left(\frac{1}{2}(c+dx)\right)+3\sec^2\left(\frac{1}{2}(c+dx)\right)+132\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-132\,\mathrm{lc}\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-36*Cot[(c + d*x)/2] - 3*Csc[(c + d*x)/2]^2 - 132*Log[Cos[(c + d*x)/2
]] + 132*Log[Sin[(c + d*x)/2]] + 3*Sec[(c + d*x)/2]^2 + 16/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2 + (32*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])^3 + (272*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
 + 36*Tan[(c + d*x)/2]))/(24*d)
```

Maple [A] time = 0.244, size = 202, normalized size = 1.8

$$\frac{2\,a^3\tan(dx+c)}{3\,d}+\frac{a^3\tan(dx+c)(\sec(dx+c))^2}{3\,d}+\frac{a^3}{d(\cos(dx+c))^3}+\frac{11\,a^3}{2\,d\cos(dx+c)}+\frac{11\,a^3\ln(\csc(dx+c)-\cot(dx}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/3*a^3*tan(d*x+c)/d+1/3/d*a^3*tan(d*x+c)*sec(d*x+c)^2+1/d*a^3/cos(d*x+c)^3
+11/2/d*a^3/cos(d*x+c)+11/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+1/d*a^3/sin(d*x
+c)/cos(d*x+c)^3+4/d*a^3/sin(d*x+c)/cos(d*x+c)-8*a^3*cot(d*x+c)/d+1/3/d*a^3
```

```
/sin(d*x+c)^2/cos(d*x+c)^3-5/6/d*a^3/sin(d*x+c)^2/cos(d*x+c)
```

Maxima [A] time = 1.18138, size = 246, normalized size = 2.24

$$\frac{12\left(\tan(dx+c)^3 - \frac{3}{\tan(dx+c)} + 6\tan(dx+c)\right)a^3 + 4\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^3 + a^3\left(\frac{2\left(15\cos(dx+c)^4 - 10\cos(dx+c)\right.}{\cos(dx+c)^5 - \cos(dx+c)^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(12*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^3 + 4*(tan(d*
x + c)^3 + 3*tan(d*x + c))*a^3 + a^3*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c
)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15*log(cos(d*x + c) + 1) + 15*
log(cos(d*x + c) - 1)) + 6*a^3*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.4775, size = 1067, normalized size = 9.7

$$-\frac{104\,a^3\cos(dx+c)^4 + 142\,a^3\cos(dx+c)^3 - 90\,a^3\cos(dx+c)^2 - 136\,a^3\cos(dx+c) - 8\,a^3 + 33\left(a^3\cos(dx+c)^4 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 + 142*a^3*cos(d*x + c)^3 - 90*a^3*cos(d*x + c
)^2 - 136*a^3*cos(d*x + c) - 8*a^3 + 33*(a^3*cos(d*x + c)^4 - a^3*cos(d*x +
 c)^3 - 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3 + (a^3*cos(d*x + c)
^3 + 2*a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))*log(1/2
*cos(d*x + c) + 1/2) - 33*(a^3*cos(d*x + c)^4 - a^3*cos(d*x + c)^3 - 3*a^3*
cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3 + (a^3*cos(d*x + c)^3 + 2*a^3*cos
(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2) - 2*(52*a^3*cos(d*x + c)^3 - 19*a^3*cos(d*x + c)^2 - 64*a^3*cos(d*x
 + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 3*d*cos
(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - d*c
os(d*x + c) - 2*d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20275, size = 203, normalized size = 1.85

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+132\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(66\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^3*tan(1/2*d*x + 1/2*c)^2 + 132*a^3*log(abs(tan(1/2*d*x + 1/2*c)))
 + 36*a^3*tan(1/2*d*x + 1/2*c) - 3*(66*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*
tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2 - 16*(21*a^3*tan(1/2*d*x
 + 1/2*c)^2 - 36*a^3*tan(1/2*d*x + 1/2*c) + 19*a^3)/(tan(1/2*d*x + 1/2*c) -
 1)^3)/d
```

3.818 $\int \csc^4(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=128

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{6a^3\cot(c+dx)}{d}+\frac{23a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{2d}-3a^3\,\mathrm{c}$$

```
[Out] (-17*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (6*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (2*a^3*Cos[c +
 d*x])/(3*d*(1 - Sin[c + d*x])^2) + (23*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c +
 d*x]))
```

Rubi [A] time = 0.206043, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2872, 3770, 3767, 8, 3768, 2650, 2648}

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{6a^3\cot(c+dx)}{d}+\frac{23a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{2d}-3a^3\,\mathrm{c}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-17*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (6*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (2*a^3*Cos[c +
 d*x])/(3*d*(1 - Sin[c + d*x])^2) + (23*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c +
 d*x]))
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx = a^4\int\left(\frac{7\csc(c+dx)}{a}+\frac{5\csc^2(c+dx)}{a}+\frac{3\csc^3(c+dx)}{a}+\frac{\csc^4(c+a}{a}\right.$$

$$= a^3\int\csc^4(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\left(3a^3\right)\int c$$

$$= -\frac{7a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{2a^3 c}{3d(1-s}$$

$$= -\frac{17a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{6a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-$$

Mathematica [B] time = 6.18308, size = 287, normalized size = 2.24

$$a^3\left(\frac{17\tan\left(\frac{1}{2}(c+dx)\right)}{6d}-\frac{17\cot\left(\frac{1}{2}(c+dx)\right)}{6d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{17\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{17\log}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-17*Cot[(c + d*x)/2])/(6*d) - (3*Csc[(c + d*x)/2]^2)/(8*d) - (Cot[(c
+ d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) - (17*Log[Cos[(c + d*x)/2]])/(2*d) + (
17*Log[Sin[(c + d*x)/2]])/(2*d) + (3*Sec[(c + d*x)/2]^2)/(8*d) + 2/(3*d*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*Sin[(c + d*x)/2])/(3*d*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^3) + (46*Sin[(c + d*x)/2])/(3*d*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])) + (17*Tan[(c + d*x)/2])/(6*d) + (Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(24*d))
```

Maple [A] time = 0.164, size = 214, normalized size = 1.7

$$\frac{a^3}{3d(\cos(dx+c))^3}+\frac{17a^3}{2d\cos(dx+c)}+\frac{17a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}+\frac{a^3}{d\sin(dx+c)(\cos(dx+c))^3}+\frac{}{3d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/3/d*a^3/cos(d*x+c)^3+17/2/d*a^3/cos(d*x+c)+17/2/d*a^3*ln(csc(d*x+c)-cot(d
*x+c))+1/d*a^3/sin(d*x+c)/cos(d*x+c)^3+20/3/d*a^3/sin(d*x+c)/cos(d*x+c)-40/
3*a^3*cot(d*x+c)/d+1/d*a^3/sin(d*x+c)^2/cos(d*x+c)^3-5/2/d*a^3/sin(d*x+c)^2
/cos(d*x+c)+1/3/d*a^3/sin(d*x+c)^3/cos(d*x+c)^3-2/3/d*a^3/sin(d*x+c)^3/cos(
d*x+c)
```

Maxima [A] time = 1.20841, size = 277, normalized size = 2.16

$$12\left(\tan(dx+c)^3 - \frac{3}{\tan(dx+c)} + 6\tan(dx+c)\right)a^3 + 4\left(\tan(dx+c)^3 - \frac{9\tan(dx+c)^2+1}{\tan(dx+c)^3} + 9\tan(dx+c)\right)a^3 + 3a^3\left(\frac{2(15\cos\ldots}{\cos\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(12*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^3 + 4*(tan(d*
x + c)^3 - (9*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 + 9*tan(d*x + c))*a^3 + 3*
a^3*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*
x + c)^3) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2*a^3*(2
*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(co
s(d*x + c) - 1)))/d
```

Fricas [B] time = 1.52212, size = 1314, normalized size = 10.27

$$\frac{160\,a^3\cos(dx+c)^5 - 58\,a^3\cos(dx+c)^4 - 356\,a^3\cos(dx+c)^3 + 70\,a^3\cos(dx+c)^2 + 200\,a^3\cos(dx+c) - 8\,a^3 - 51\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(160*a^3*cos(d*x + c)^5 - 58*a^3*cos(d*x + c)^4 - 356*a^3*cos(d*x + c)
^3 + 70*a^3*cos(d*x + c)^2 + 200*a^3*cos(d*x + c) - 8*a^3 - 51*(a^3*cos(d*x
 + c)^5 + 2*a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 - 4*a^3*cos(d*x + c)^
2 + a^3*cos(d*x + c) + 2*a^3 - (a^3*cos(d*x + c)^4 - a^3*cos(d*x + c)^3 - 3
*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) + 51*(a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 2*a^3*cos(
d*x + c)^3 - 4*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3 - (a^3*cos(d*x
 + c)^4 - a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*
a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(80*a^3*cos(d*x + c)^4
+ 109*a^3*cos(d*x + c)^3 - 69*a^3*cos(d*x + c)^2 - 104*a^3*cos(d*x + c) - 4
*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 2*d*cos(d*x +
c)^3 - 4*d*cos(d*x + c)^2 + d*cos(d*x + c) - (d*cos(d*x + c)^4 - d*cos(d*x
+ c)^3 - 3*d*cos(d*x + c)^2 + d*cos(d*x + c) + 2*d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21421, size = 262, normalized size = 2.05

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+204\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+69\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{187\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x + 1/2*c)^2 + 204*a^3*l
og(abs(tan(1/2*d*x + 1/2*c))) + 69*a^3*tan(1/2*d*x + 1/2*c) - (187*a^3*tan(
1/2*d*x + 1/2*c)^6 - 60*a^3*tan(1/2*d*x + 1/2*c)^5 - 405*a^3*tan(1/2*d*x +
1/2*c)^4 + 394*a^3*tan(1/2*d*x + 1/2*c)^3 - 45*a^3*tan(1/2*d*x + 1/2*c)^2 -
 6*a^3*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x +
1/2*c))^3)/d
```

3.819 $\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx$

Optimal. Leaf size=143

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{16a^4\cos(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{35a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{56a^4\cos(c+dx)}{3d(1-\sin(c+dx))}$$

```
[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4
*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*
(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c
 + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.203745, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{16a^4\cos(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{35a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{56a^4\cos(c+dx)}{3d(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4
*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*
(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c
 + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx = a^4\int \left(16+\frac{4}{(-1+\sin(c+dx))^2}+\frac{20}{-1+\sin(c+dx)}+12\sin(c+dx)+8\sin^2(c\right.$$

$$= 16a^4x + a^4\int \sin^4(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{(-1+\sin(c+dx))^2}\,dx + \left(4a^4\right)\int \sin^3($$

$$= 16a^4x - \frac{12a^4\cos(c+dx)}{d} + \frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{20a^4\cos(c+dx)}{d(1-\sin(c+dx))} - \frac{4a^4\cos}{}$$

$$= 20a^4x - \frac{16a^4\cos(c+dx)}{d} + \frac{4a^4\cos^3(c+dx)}{3d} + \frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{56a^4\cos}{3d(1-\sin}$$

$$= \frac{163a^4x}{8} - \frac{16a^4\cos(c+dx)}{d} + \frac{4a^4\cos^3(c+dx)}{3d} + \frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{56a^4}{3d(1-}$$

Mathematica [A] time = 1.57654, size = 252, normalized size = 1.76

$$\frac{a^4\left(-11736c\sin\left(\frac{1}{2}(c+dx)\right)-11736dx\sin\left(\frac{1}{2}(c+dx)\right)-16488\sin\left(\frac{1}{2}(c+dx)\right)-3912c\sin\left(\frac{3}{2}(c+dx)\right)-3912dx\sin\left(\frac{3}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (a^4*(24*(209 + 489*c + 489*d*x)*Cos[(c + d*x)/2] - 24*(453 + 163*c + 163*d
*x)*Cos[(3*(c + d*x))/2] + 885*Cos[(5*(c + d*x))/2] - 129*Cos[(7*(c + d*x))
/2] - 23*Cos[(9*(c + d*x))/2] + 3*Cos[(11*(c + d*x))/2] - 16488*Sin[(c + d*
x)/2] - 11736*c*Sin[(c + d*x)/2] - 11736*d*x*Sin[(c + d*x)/2] + 3704*Sin[(3
*(c + d*x))/2] - 3912*c*Sin[(3*(c + d*x))/2] - 3912*d*x*Sin[(3*(c + d*x))/2
] + 885*Sin[(5*(c + d*x))/2] + 129*Sin[(7*(c + d*x))/2] - 23*Sin[(9*(c + d*
x))/2] - 3*Sin[(11*(c + d*x))/2]))/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^3)
```

Maple [B] time = 0.101, size = 360, normalized size = 2.5

$$\frac{1}{d}\left(a^4\left(\frac{(\sin(dx+c))^9}{3(\cos(dx+c))^3}-2\frac{(\sin(dx+c))^9}{\cos(dx+c)}-2\left((\sin(dx+c))^7+7/6(\sin(dx+c))^5+\frac{35(\sin(dx+c))^3}{24}+\frac{35\sin(dx+}{16}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/3*sin(d*x+c)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*
x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/
8*d*x+35/8*c)+4*a^4*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x
+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+6
*a^4*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*
x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+4*a^4*(1
/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*
sin(d*x+c)^2)*cos(d*x+c))+a^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A] time = 1.6844, size = 321, normalized size = 2.24

$$32\left(\cos(dx+c)^3 - \frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3} - 9\cos(dx+c)\right)a^4 + \left(8\tan(dx+c)^3 + 105\,dx + 105\,c - \frac{3\left(13\tan(dx+c)^3+11\tan(dx+c)\right)}{\tan(dx+c)^4+2\tan(dx+c)^2+1} \cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/24*(32*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*
x + c))*a^4 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*tan(d*x + c)^3 +
11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 72*tan(d*x + c))
*a^4 + 24*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^
2 + 1) - 12*tan(d*x + c))*a^4 + 8*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^4 - 32*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)
))/d
```

Fricas [A] time = 1.51897, size = 620, normalized size = 4.34

$$-\frac{6\,a^4\cos(dx+c)^6 - 20\,a^4\cos(dx+c)^5 - 85\,a^4\cos(dx+c)^4 + 214\,a^4\cos(dx+c)^3 + 978\,a^4dx + 32\,a^4 - \left(489\,a^4dx + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^6 - 20*a^4*cos(d*x + c)^5 - 85*a^4*cos(d*x + c)^4
 + 214*a^4*cos(d*x + c)^3 + 978*a^4*d*x + 32*a^4 - (489*a^4*d*x + 721*a^4)*
cos(d*x + c)^2 + (489*a^4*d*x - 962*a^4)*cos(d*x + c) - (6*a^4*cos(d*x + c)
^5 + 26*a^4*cos(d*x + c)^4 - 59*a^4*cos(d*x + c)^3 + 978*a^4*d*x - 273*a^4*
cos(d*x + c)^2 - 32*a^4 + (489*a^4*d*x - 994*a^4)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c)
 - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30111, size = 270, normalized size = 1.89

$$\frac{489\,(dx+c)a^4+\frac{64\left(12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-288\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+129\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1\cdots\right.}{\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(489*(d*x + c)*a^4 + 64*(12*a^4*tan(1/2*d*x + 1/2*c)^2 - 27*a^4*tan(1/
2*d*x + 1/2*c) + 13*a^4)/(tan(1/2*d*x + 1/2*c) - 1)^3 + 2*(105*a^4*tan(1/2*
d*x + 1/2*c)^7 - 288*a^4*tan(1/2*d*x + 1/2*c)^6 + 129*a^4*tan(1/2*d*x + 1/2
*c)^5 - 1056*a^4*tan(1/2*d*x + 1/2*c)^4 - 129*a^4*tan(1/2*d*x + 1/2*c)^3 -
1120*a^4*tan(1/2*d*x + 1/2*c)^2 - 105*a^4*tan(1/2*d*x + 1/2*c) - 352*a^4)/(
tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.820 $\int \sec^2(c+dx)(a+a\sin(c+dx))^4\tan^2(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{32a^4\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^4x}{2}$$

```
[Out] (17*a^4*x)/2 - (4*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x])/(3*d*(1 - Sin[
c + d*x])^2) - (32*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (a^4*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.16332, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2872, 2650, 2648, 2638, 2635, 8}

$$-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{32a^4\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```


```
[Out] (17*a^4*x)/2 - (4*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x])/(3*d*(1 - Sin[
c + d*x])^2) - (32*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (a^4*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```


Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^4\tan^2(c+dx)\,dx = a^4\int\left(8+\frac{4}{(-1+\sin(c+dx))^2}+\frac{12}{-1+\sin(c+dx)}+4\sin(c+dx)\right.$$
$$=8a^4x+a^4\int\sin^2(c+dx)\,dx+\left(4a^4\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\left(4\right.$$
$$=8a^4x-\frac{4a^4\cos(c+dx)}{d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{12a^4\cos(c+dx)}{d(1-\sin(c+dx))}$$
$$=\frac{17a^4x}{2}-\frac{4a^4\cos(c+dx)}{d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{32a^4\cos(c+d}{3d(1-\sin(c+d}$$

Mathematica [A] time = 1.9119, size = 158, normalized size = 1.56

$$-\frac{a^4\left(-3(204c+204dx+161)\cos\left(\frac{1}{2}(c+dx)\right)+(204c+204dx+647)\cos\left(\frac{3}{2}(c+dx)\right)-39\cos\left(\frac{5}{2}(c+dx)\right)+3\cos\left(\frac{7}{2}(c\right.\right.}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] -(a^4*(-3*(161 + 204*c + 204*d*x)*Cos[(c + d*x)/2] + (647 + 204*c + 204*d*x
)*Cos[(3*(c + d*x))/2] - 39*Cos[(5*(c + d*x))/2] + 3*Cos[(7*(c + d*x))/2] +
 6*(146 + 136*c + 136*d*x + (-59 + 68*c + 68*d*x)*Cos[c + d*x] - 14*Cos[2*(
c + d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^3)
```

Maple [B] time = 0.101, size = 268, normalized size = 2.7

$$\frac{1}{d}\left(a^4\left(\frac{(\sin(dx+c))^7}{3(\cos(dx+c))^3}-\frac{4(\sin(dx+c))^7}{3\cos(dx+c)}-\frac{4\cos(dx+c)}{3}\left((\sin(dx+c))^5+\frac{5(\sin(dx+c))^3}{4}+\frac{15\sin(dx+c)}{8}\right)+\frac{5d}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(si
n(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+4*a^
4*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+
4/3*sin(d*x+c)^2)*cos(d*x+c))+6*a^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+4*a
^4*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*
x+c)^2)*cos(d*x+c))+1/3*a^4*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A] time = 1.62033, size = 213, normalized size = 2.11

$$\frac{2\,a^4\tan\left(dx+c\right)^3+\left(2\,\tan\left(dx+c\right)^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\,\tan\left(dx+c\right)\right)a^4+12\left(\tan\left(dx+c\right)^3+3\,dx+3\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*a^4*tan(d*x + c)^3 + (2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x +
 c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c))*a^4 + 12*(tan(d*x + c)^3 + 3*d*
x + 3*c - 3*tan(d*x + c))*a^4 - 8*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^
3 + 3*cos(d*x + c)) - 8*(3*cos(d*x + c)^2 - 1)*a^4/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.45128, size = 477, normalized size = 4.72

$$\frac{3\,a^4\cos\left(dx+c\right)^4-18\,a^4\cos\left(dx+c\right)^3-102\,a^4dx-8\,a^4+17\left(3\,a^4dx+5\,a^4\right)\cos\left(dx+c\right)^2-\left(51\,a^4dx-98\,a^4\right)\cos\left(dx\right.}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^4*cos(d*x + c)^4 - 18*a^4*cos(d*x + c)^3 - 102*a^4*d*x - 8*a^4 + 1
7*(3*a^4*d*x + 5*a^4)*cos(d*x + c)^2 - (51*a^4*d*x - 98*a^4)*cos(d*x + c) -
 (3*a^4*cos(d*x + c)^3 - 102*a^4*d*x + 21*a^4*cos(d*x + c)^2 + 8*a^4 - (51*
a^4*d*x - 106*a^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*
x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31767, size = 182, normalized size = 1.8

$$\frac{51\left(dx+c\right)a^4+\frac{6\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{16\left(6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(51*(d*x + c)*a^4 + 6*(a^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^4*tan(1/2*d*x +
 1/2*c)^2 - a^4*tan(1/2*d*x + 1/2*c) - 8*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
2 + 16*(6*a^4*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/2*c) + 7*a^4)
/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.821 $\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{\cos(c+dx)}{ad}+\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec(c+dx)}{ad}-\frac{x}{a}$$

```
[Out] -(x/a) - Cos[c + d*x]/(a*d) - (3*Sec[c + d*x])/(a*d) + Sec[c + d*x]^3/(a*d)
 - Sec[c + d*x]^5/(5*a*d) + Tan[c + d*x]/(a*d) - Tan[c + d*x]^3/(3*a*d) + T
an[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.158352, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2839, 3473, 8, 2590, 270}

$$-\frac{\cos(c+dx)}{ad}+\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec(c+dx)}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - Cos[c + d*x]/(a*d) - (3*Sec[c + d*x])/(a*d) + Sec[c + d*x]^3/(a*d)
 - Sec[c + d*x]^5/(5*a*d) + Tan[c + d*x]/(a*d) - Tan[c + d*x]^3/(3*a*d) + T
an[c + d*x]^5/(5*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \tan^6(c+dx)\,dx}{a} - \frac{\int \sin(c+dx)\tan^6(c+dx)\,dx}{a}$$

$$= \frac{\tan^5(c+dx)}{5ad} - \frac{\int \tan^4(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, \cos(c+dx)\right)}{ad}$$

$$= -\frac{\tan^3(c+dx)}{3ad} + \frac{\tan^5(c+dx)}{5ad} + \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right) d\right.}{ad}$$

$$= -\frac{\cos(c+dx)}{ad} - \frac{3\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan(c+dx)}{ad} - \frac{\tan^3(c}{3ad}$$

$$= -\frac{x}{a} - \frac{\cos(c+dx)}{ad} - \frac{3\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan(c+dx)}{ad} - \frac{\tan}{}$$

Mathematica [A] time = 0.674661, size = 224, normalized size = 1.91

$$-\frac{216\sin(c+dx)+240c\sin(2(c+dx))+240dx\sin(2(c+dx))-618\sin(2(c+dx))+532\sin(3(c+dx))+120c\sin(4(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(1200 + 18*(-103 + 40*c + 40*d*x)*Cos[c + d*x] + 1568*Cos[2*(c + d*x)] - 6
18*Cos[3*(c + d*x)] + 240*c*Cos[3*(c + d*x)] + 240*d*x*Cos[3*(c + d*x)] + 3
04*Cos[4*(c + d*x)] + 216*Sin[c + d*x] - 618*Sin[2*(c + d*x)] + 240*c*Sin[2
*(c + d*x)] + 240*d*x*Sin[2*(c + d*x)] + 532*Sin[3*(c + d*x)] - 309*Sin[4*(
c + d*x)] + 120*c*Sin[4*(c + d*x)] + 120*d*x*Sin[4*(c + d*x)] + 60*Sin[5*(c
 + d*x)])/(960*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.096, size = 210, normalized size = 1.8

$$-\frac{1}{6da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - \frac{1}{4da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2} + \frac{7}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} - 2\frac{1}{da\left(1+(\tan(1/2\,dx+c/2))^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/d/a/(tan(1/2*d*x+1/2*c)-1)^3-1/4/d/a/(tan(1/2*d*x+1/2*c)-1)^2+7/8/d/a/
(tan(1/2*d*x+1/2*c)-1)-2/a/d/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*arctan(tan(1/2*
d*x+1/2*c))-2/5/d/a/(tan(1/2*d*x+1/2*c)+1)^5+1/d/a/(tan(1/2*d*x+1/2*c)+1)^4
+1/3/d/a/(tan(1/2*d*x+1/2*c)+1)^3-3/2/d/a/(tan(1/2*d*x+1/2*c)+1)^2-23/8/a/d
/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.58587, size = 540, normalized size = 4.62

$$-\frac{2\left(\frac{\frac{81\sin(dx+c)}{\cos(dx+c)+1}-\frac{78\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{172\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{26\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{22\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{70\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{20\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{30\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+48}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{15\,\text{arcta}}{}\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/15*((81*sin(d*x + c)/(cos(d*x + c) + 1) - 78*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 172*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 26*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 22*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 70*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 20*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 30*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8 - 15*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 4
8)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 4*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2*a*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4*a*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a*si
n(d*x + c)^9/(cos(d*x + c) + 1)^9 - a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10
) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.5073, size = 288, normalized size = 2.46

$$-\frac{15\,dx\cos\left(dx+c\right)^{3}+38\,\cos\left(dx+c\right)^{4}+11\,\cos\left(dx+c\right)^{2}+\left(15\,dx\cos\left(dx+c\right)^{3}+15\,\cos\left(dx+c\right)^{4}+22\,\cos\left(dx+c\right)^{2}\cdots}{15\left(ad\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*d*x*cos(d*x + c)^3 + 38*cos(d*x + c)^4 + 11*cos(d*x + c)^2 + (15*
d*x*cos(d*x + c)^3 + 15*cos(d*x + c)^4 + 22*cos(d*x + c)^2 - 4)*sin(d*x + c
) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29895, size = 201, normalized size = 1.72

$$-\frac{\frac{120\,(dx+c)}{a}+\frac{240}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a}-\frac{5\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+23\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}+\frac{345\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1560\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2570\,\tan\left(\frac{1}{2}\,dx\cdots\right.}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)/a + 240/((tan(1/2*d*x + 1/2*c)^2 + 1)*a) - 5*(21*tan(
1/2*d*x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 23)/(a*(tan(1/2*d*x + 1/2*c)
 - 1)^3) + (345*tan(1/2*d*x + 1/2*c)^4 + 1560*tan(1/2*d*x + 1/2*c)^3 + 2570
*tan(1/2*d*x + 1/2*c)^2 + 1720*tan(1/2*d*x + 1/2*c) + 413)/(a*(tan(1/2*d*x
+ 1/2*c) + 1)^5))/d
```

3.822 $\int \frac{\sin(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}+\frac{x}{a}$$

```
[Out] x/a + Sec[c + d*x]/(a*d) - (2*Sec[c + d*x]^3)/(3*a*d) + Sec[c + d*x]^5/(5*a
*d) - Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.129149, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2606, 194, 3473, 8}

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Sec[c + d*x]/(a*d) - (2*Sec[c + d*x]^3)/(3*a*d) + Sec[c + d*x]^5/(5*a
*d) - Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \tan^6(c+dx)\,dx}{a}$$
$$= -\frac{\tan^5(c+dx)}{5ad} + \frac{\int \tan^4(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2 dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{\tan^3(c+dx)}{3ad} - \frac{\tan^5(c+dx)}{5ad} - \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right) dx, x, \text{sec}\right.}{ad}$$
$$= \frac{\sec(c+dx)}{ad} - \frac{2\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan(c+dx)}{ad} + \frac{\tan^3(c+dx)}{3ad} - \frac{\tan^5(c+}{5ad}$$
$$= \frac{x}{a} + \frac{\sec(c+dx)}{ad} - \frac{2\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan(c+dx)}{ad} + \frac{\tan^3(c+dx)}{3ad} - \frac{\tan^5}{}$$

Mathematica [A] time = 0.643285, size = 191, normalized size = 1.82

$$-\frac{\sec^3(c+dx)\left(8\sin(c+dx) - 30c\sin(2(c+dx)) - 30dx\sin(2(c+dx)) + \frac{89}{4}\sin(2(c+dx)) + 16\sin(3(c+dx)) - 15c\sin(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^3*(-25 + (267/4 - 90*c - 90*d*x)*Cos[c + d*x] - 16*Cos[2*(c
+ d*x)] + (89*Cos[3*(c + d*x)])/4 - 30*c*Cos[3*(c + d*x)] - 30*d*x*Cos[3*(c
 + d*x)] - 23*Cos[4*(c + d*x)] + 8*Sin[c + d*x] + (89*Sin[2*(c + d*x)])/4 -
 30*c*Sin[2*(c + d*x)] - 30*d*x*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)] + (8
9*Sin[4*(c + d*x)])/8 - 15*c*Sin[4*(c + d*x)] - 15*d*x*Sin[4*(c + d*x)]))/(
120*a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.09, size = 166, normalized size = 1.6

$$-\frac{1}{6da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - \frac{1}{4da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2} + \frac{5}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + 2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/d/a/(tan(1/2*d*x+1/2*c)-1)^3-1/4/d/a/(tan(1/2*d*x+1/2*c)-1)^2+5/8/d/a/
(tan(1/2*d*x+1/2*c)-1)+2/a/d*arctan(tan(1/2*d*x+1/2*c))+2/5/d/a/(tan(1/2*d*
x+1/2*c)+1)^5-1/d/a/(tan(1/2*d*x+1/2*c)+1)^4+1/d/a/(tan(1/2*d*x+1/2*c)+1)^2
+11/8/a/d/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.69175, size = 429, normalized size = 4.09

$$\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1} - \frac{46\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{13\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{100\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{35\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{30\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + 8}{a + \frac{2a\sin(dx+c)}{\cos(dx+c)+1} - \frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} + \frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 2/15*((sin(d*x + c)/(cos(d*x + c) + 1) - 46*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 13*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 100*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 30*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d

---

Fricas [A] time = 1.39791, size = 258, normalized size = 2.46

$$\frac{15\,dx\cos(dx+c)^3 + 23\cos(dx+c)^4 - 19\cos(dx+c)^2 + \left(15\,dx\cos(dx+c)^3 - 8\cos(dx+c)^2 + 1\right)\sin(dx+c) + 4}{15\left(ad\cos(dx+c)^3\sin(dx+c) + ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/15*(15*d*x*cos(d*x + c)^3 + 23*cos(d*x + c)^4 - 19*cos(d*x + c)^2 + (15*d*x*cos(d*x + c)^3 - 8*cos(d*x + c)^2 + 1)*sin(d*x + c) + 4)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [A] time = 1.3074, size = 176, normalized size = 1.68

$$\frac{\frac{120\,(dx+c)}{a} + \frac{5\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 36\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3} + \frac{3\left(55\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 260\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 450\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 300\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/120*(120*(d*x + c)/a + 5*(15*tan(1/2*d*x + 1/2*c)^2 - 36*tan(1/2*d*x + 1/2*c) + 17)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) + 3*(55*tan(1/2*d*x + 1/2*c)^4 + 260*tan(1/2*d*x + 1/2*c)^3 + 450*tan(1/2*d*x + 1/2*c)^2 + 300*tan(1/2*d*x + 1/2*c) + 71)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d

3.823 $\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

```
[Out] -(Sec[c + d*x]/(a*d)) + (2*Sec[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^5/(5*a*d)
 + Tan[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.0956588, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]/(a*d)) + (2*Sec[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^5/(5*a*d)
 + Tan[c + d*x]^5/(5*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a} \\
&= \frac{\text{Subst}\left(\int x^4\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \sec(c+dx)\right)}{ad} \\
&= \frac{\tan^5(c+dx)}{5ad} - \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \sec(c+dx)\right)}{ad} \\
&= -\frac{\sec(c+dx)}{ad} + \frac{2\sec^3(c+dx)}{3ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan^5(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [A] time = 0.269237, size = 106, normalized size = 1.54

$$
-\frac{\sec^3(c+dx)(-64\sin(c+dx)-178\sin(2(c+dx))+192\sin(3(c+dx))-89\sin(4(c+dx))-534\cos(c+dx)+288\,\text{c}}{960ad(\sin(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^3*(200 - 534*Cos[c + d*x] + 288*Cos[2*(c + d*x)] - 178*Cos[3
*(c + d*x)] + 24*Cos[4*(c + d*x)] - 64*Sin[c + d*x] - 178*Sin[2*(c + d*x)]
+ 192*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)]))/(960*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.086, size = 130, normalized size = 1.9

$$
32\,\frac{1}{da}\left(-\frac{1}{192\,(\tan(1/2\,dx+c/2)-1)^3} - \frac{1}{128\,(\tan(1/2\,dx+c/2)-1)^2} + \frac{3}{256\,\tan(1/2\,dx+c/2)-256} - \frac{1}{80\,(\tan(1/2\,d}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 32/d/a*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+3/25
6/(tan(1/2*d*x+1/2*c)-1)-1/80/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/
2*c)+1)^4-1/96/(tan(1/2*d*x+1/2*c)+1)^3-1/64/(tan(1/2*d*x+1/2*c)+1)^2-3/256
/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.03692, size = 289, normalized size = 4.19

$$
-\frac{16\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1} - \frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{6\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + 1\right)}{15\left(a + \frac{2a\sin(dx+c)}{\cos(dx+c)+1} - \frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -16/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a + 2*a*sin(d*x + c)/
(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x
```

```
+ c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A] time = 1.37555, size = 194, normalized size = 2.81

$$-\frac{3\cos(dx+c)^4+6\cos(dx+c)^2+4\left(3\cos(dx+c)^2-1\right)\sin(dx+c)-1}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 4*(3*cos(d*x + c)^2 - 1)*sin(d
*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28909, size = 162, normalized size = 2.35

$$\frac{\frac{5\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+11\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+490\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+320\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+73}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(
1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 240*tan(1/2*d*x + 1
/2*c)^3 + 490*tan(1/2*d*x + 1/2*c)^2 + 320*tan(1/2*d*x + 1/2*c) + 73)/(a*(t
an(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.824 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}$$

```
[Out] -Sec[c + d*x]^3/(3*a*d) + Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.138481, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2606, 14, 2607, 30}

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Sec[c + d*x]^3/(3*a*d) + Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a}$$
$$= -\frac{\text{Subst}\left(\int x^4\,dx, x, \tan(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \sec(c+dx)\right)}{ad}$$
$$= -\frac{\tan^5(c+dx)}{5ad} + \frac{\text{Subst}\left(\int \left(-x^2+x^4\right)dx, x, \sec(c+dx)\right)}{ad}$$
$$= -\frac{\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan^5(c+dx)}{5ad}$$

Mathematica [A] time = 0.288426, size = 106, normalized size = 1.93

$$\frac{\sec^3(c+dx)(16\sin(c+dx)+22\sin(2(c+dx))-48\sin(3(c+dx))+11\sin(4(c+dx))+66\cos(c+dx)-192\cos(2(c+dx)) \ldots}{960ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^3*(40 + 66*Cos[c + d*x] - 192*Cos[2*(c + d*x)] + 22*Cos[3*(c
+ d*x)] + 24*Cos[4*(c + d*x)] + 16*Sin[c + d*x] + 22*Sin[2*(c + d*x)] - 48*
Sin[3*(c + d*x)] + 11*Sin[4*(c + d*x)]))/(960*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.081, size = 115, normalized size = 2.1

$$16\,\frac{1}{da}\left(-\frac{1}{96\,(\tan(1/2\,dx+c/2)-1)^3}-\frac{1}{64\,(\tan(1/2\,dx+c/2)-1)^2}+\frac{1}{128\,\tan(1/2\,dx+c/2)-128}+1/40\,(\tan(1/2\,dx+ \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 16/d/a*(-1/96/(tan(1/2*d*x+1/2*c)-1)^3-1/64/(tan(1/2*d*x+1/2*c)-1)^2+1/128/
(tan(1/2*d*x+1/2*c)-1)+1/40/(tan(1/2*d*x+1/2*c)+1)^5-1/16/(tan(1/2*d*x+1/2*
c)+1)^4+1/24/(tan(1/2*d*x+1/2*c)+1)^3-1/128/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.06878, size = 316, normalized size = 5.75

$$-\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*s
```

```
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8)*d)
```

Fricas [A] time = 1.34623, size = 190, normalized size = 3.45

$$\frac{3 \cos(dx+c)^4 - 9 \cos(dx+c)^2 - \left(3 \cos(dx+c)^2 - 1\right) \sin(dx+c) + 4}{15 \left(ad \cos(dx+c)^3 \sin(dx+c) + ad \cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 1)*sin(d*x
+ c) + 4)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24477, size = 162, normalized size = 2.95

$$\frac{\frac{5\left(3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 - 12 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+5\right)}{a\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)^3} - \frac{15 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 + 60 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 + 10 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 20 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+7}{a\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^5}}{120 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(3*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 5)/(a*(tan(1
/2*d*x + 1/2*c) - 1)^3) - (15*tan(1/2*d*x + 1/2*c)^4 + 60*tan(1/2*d*x + 1/2
*c)^3 + 10*tan(1/2*d*x + 1/2*c)^2 + 20*tan(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/
2*d*x + 1/2*c) + 1)^5))/d
```

3.825 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}$$

```
[Out] Sec[c + d*x]^3/(3*a*d) - Sec[c + d*x]^5/(5*a*d) + Tan[c + d*x]^3/(3*a*d) +
Tan[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.159939, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2839, 2607, 14, 2606}

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^3/(3*a*d) - Sec[c + d*x]^5/(5*a*d) + Tan[c + d*x]^3/(3*a*d) +
Tan[c + d*x]^5/(5*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^4(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a}$$
$$= -\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \sec(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{\text{Subst}\left(\int \left(-x^2+x^4\right)dx, x, \sec(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \left(x^2+x^4\right)dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{\sec^3(c+dx)}{3ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan^3(c+dx)}{3ad} + \frac{\tan^5(c+dx)}{5ad}$$

Mathematica [A] time = 0.332771, size = 106, normalized size = 1.45

$$-\frac{\sec^3(c+dx)(-224\sin(c+dx)+22\sin(2(c+dx))+32\sin(3(c+dx))+11\sin(4(c+dx))+66\cos(c+dx)-32\cos(}{960ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^3*(-80 + 66*Cos[c + d*x] - 32*Cos[2*(c + d*x)] + 22*Cos[3*(c
 + d*x)] - 16*Cos[4*(c + d*x)] - 224*Sin[c + d*x] + 22*Sin[2*(c + d*x)] + 3
2*Sin[3*(c + d*x)] + 11*Sin[4*(c + d*x)]))/(960*a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.074, size = 130, normalized size = 1.8

$$8\frac{1}{da}\left(-1/48\,(\tan(1/2\,dx+c/2)-1)^{-3}-1/32\,(\tan(1/2\,dx+c/2)-1)^{-2}-\frac{1}{64\tan(1/2\,dx+c/2)-64}-1/20\,(\tan(1/2\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 8/d/a*(-1/48/(tan(1/2*d*x+1/2*c)-1)^3-1/32/(tan(1/2*d*x+1/2*c)-1)^2-1/64/(t
an(1/2*d*x+1/2*c)-1)-1/20/(tan(1/2*d*x+1/2*c)+1)^5+1/8/(tan(1/2*d*x+1/2*c)+
1)^4-1/8/(tan(1/2*d*x+1/2*c)+1)^3+1/16/(tan(1/2*d*x+1/2*c)+1)^2+1/64/(tan(1
/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.0749, size = 343, normalized size = 4.7

$$\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+1\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 10*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a + 2*a*sin(d*x +
 c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(
```

```
d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A] time = 1.37406, size = 188, normalized size = 2.58

$$\frac{2\cos(dx+c)^4-\cos(dx+c)^2-2\left(\cos(dx+c)^2-2\right)\sin(dx+c)+1}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(2*cos(d*x + c)^4 - cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 2)*sin(d*x +
c) + 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25863, size = 147, normalized size = 2.01

$$-\frac{\frac{5\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{3\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+50\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+9\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(5*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) -
 3*(5*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 + 50*tan(1/2*d*x +
 1/2*c)^2 + 40*tan(1/2*d*x + 1/2*c) + 9)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/
d
```

3.826 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}$$

```
[Out] Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]^3/(3*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.107935, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2606, 30, 2607, 14}

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]^3/(3*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^5(c+dx)\tan(c+dx)\,dx}{a} - \frac{\int \sec^4(c+dx)\tan^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int x^4\,dx, x, \sec(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{\sec^5(c+dx)}{5ad} - \frac{\text{Subst}\left(\int \left(x^2+x^4\right)dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{\sec^5(c+dx)}{5ad} - \frac{\tan^3(c+dx)}{3ad} - \frac{\tan^5(c+dx)}{5ad}$$

Mathematica [A] time = 0.259062, size = 106, normalized size = 1.93

$$-\frac{\sec^3(c+dx)(-96\sin(c+dx)+18\sin(2(c+dx))-32\sin(3(c+dx))+9\sin(4(c+dx))+54\cos(c+dx)+32\cos(2(c+}{960ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^3*(-240 + 54*Cos[c + d*x] + 32*Cos[2*(c + d*x)] + 18*Cos[3*(
c + d*x)] + 16*Cos[4*(c + d*x)] - 96*Sin[c + d*x] + 18*Sin[2*(c + d*x)] - 3
2*Sin[3*(c + d*x)] + 9*Sin[4*(c + d*x)]))/(960*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.067, size = 130, normalized size = 2.4

$$4\,\frac{1}{da}\left(-1/24\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-3}-1/16\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-2}-\frac{3}{32\,\tan\left(1/2\,dx+c/2\right)-32}-1/4\,(\tan(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 4/d/a*(-1/24/(tan(1/2*d*x+1/2*c)-1)^3-1/16/(tan(1/2*d*x+1/2*c)-1)^2-3/32/(t
an(1/2*d*x+1/2*c)-1)-1/4/(tan(1/2*d*x+1/2*c)+1)^4+1/10/(tan(1/2*d*x+1/2*c)+
1)^5+1/3/(tan(1/2*d*x+1/2*c)+1)^3-1/4/(tan(1/2*d*x+1/2*c)+1)^2+3/32/(tan(1/
2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.18115, size = 370, normalized size = 6.73

$$\frac{2\left(\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+3\right)}{15\left(a+\frac{2\,a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6\,a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6\,a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2\,a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 10*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 3)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d
```

```
*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*
x + c) + 1)^8)*d)
```

Fricas [A] time = 1.31488, size = 189, normalized size = 3.44

$$-\frac{2\cos(dx+c)^4-\cos(dx+c)^2-\left(2\cos(dx+c)^2+1\right)\sin(dx+c)-4}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(d*x + c)^4 - cos(d*x + c)^2 - (2*cos(d*x + c)^2 + 1)*sin(d*x +
 c) - 4)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sin(c+dx)\sec^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [B] time = 1.21636, size = 162, normalized size = 2.95

$$-\frac{\frac{5\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+60\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+70\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 7)/(a*(tan(
1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 60*tan(1/2*d*x + 1/
2*c)^3 + 70*tan(1/2*d*x + 1/2*c)^2 + 20*tan(1/2*d*x + 1/2*c) + 13)/(a*(tan(
1/2*d*x + 1/2*c) + 1)^5))/d
```

3.827 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=115

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{2\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d
) + Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]/(a*d) - (2*Tan[c + d*x]^3)/(3*a*d
) - Tan[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.124173, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2839, 2622, 302, 207, 3767}

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{2\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d
) + Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]/(a*d) - (2*Tan[c + d*x]^3)/(3*a*d
) - Tan[c + d*x]^5/(5*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \sec^6(c+dx)\,dx}{a} + \frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -\tan(c+dx)\right)}{ad}$$
$$= -\frac{\tan(c+dx)}{ad} - \frac{2\tan^3(c+dx)}{3ad} - \frac{\tan^5(c+dx)}{5ad} + \frac{\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right) d\right.}{ad}$$
$$= \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan(c+dx)}{ad} - \frac{2\tan^3(c+dx)}{3ad} - \frac{\tan^5(}{5}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan(c+dx)}{ad}$$

Mathematica [B] time = 0.639078, size = 267, normalized size = 2.32

$$-\frac{\sec^3(c+dx)\left(-22\sin(c+dx)+\frac{149}{4}\sin(2(c+dx))-14\sin(3(c+dx))+\frac{149}{8}\sin(4(c+dx))-76\cos(2(c+dx))+\frac{149}{4}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^3*(-100 - 76*Cos[2*(c + d*x)] + (149*Cos[3*(c + d*x)])/4 - 8
*Cos[4*(c + d*x)] + 30*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + Cos[c + d*x
]*(447/4 + 90*Log[Cos[(c + d*x)/2]] - 90*Log[Sin[(c + d*x)/2]]) - 30*Cos[3*
(c + d*x)]*Log[Sin[(c + d*x)/2]] - 22*Sin[c + d*x] + (149*Sin[2*(c + d*x)])
/4 + 30*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)] - 30*Log[Sin[(c + d*x)/2]]*S
in[2*(c + d*x)] - 14*Sin[3*(c + d*x)] + (149*Sin[4*(c + d*x)])/8 + 15*Log[C
os[(c + d*x)/2]]*Sin[4*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x
)]))/(120*a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.098, size = 187, normalized size = 1.6

$$-\frac{1}{6\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - \frac{1}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2} - \frac{7}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{2}{5\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/6/d/a/(tan(1/2*d*x+1/2*c)-1)^3-1/4/d/a/(tan(1/2*d*x+1/2*c)-1)^2-7/8/d/a/
(tan(1/2*d*x+1/2*c)-1)+2/5/d/a/(tan(1/2*d*x+1/2*c)+1)^5-1/d/a/(tan(1/2*d*x+
1/2*c)+1)^4+2/d/a/(tan(1/2*d*x+1/2*c)+1)^3-2/d/a/(tan(1/2*d*x+1/2*c)+1)^2+2
3/8/a/d/(tan(1/2*d*x+1/2*c)+1)+1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.03177, size = 432, normalized size = 3.76

$$\frac{\frac{2\left(\frac{31\sin(dx+c)}{\cos(dx+c)+1}-\frac{31\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{73\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{25\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{65\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+23\right)}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/15*(2*(31*sin(d*x + c)/(cos(d*x + c) + 1) - 31*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 73*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 25*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 65*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23)/(a +
 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin
(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.39562, size = 413, normalized size = 3.59

$$\frac{16\cos(dx+c)^4+22\cos(dx+c)^2-15\left(\cos(dx+c)^3\sin(dx+c)+\cos(dx+c)^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15\left(\cos(d\ldots\right.}{30\left(ad\cos(dx+c)^3\sin(dx+c)+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(16*cos(d*x + c)^4 + 22*cos(d*x + c)^2 - 15*(cos(d*x + c)^3*sin(d*x +
c) + cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^3*sin(d
*x + c) + cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(7*cos(d*x + c)^
2 + 1)*sin(d*x + c) + 8)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c
)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20221, size = 184, normalized size = 1.6

$$\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{5\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+19\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}+\frac{3\left(115\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+380\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+530\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+340\tan\ldots\right.}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - 5*(21*tan(1/2*d*x + 1/2*c)^2
- 36*tan(1/2*d*x + 1/2*c) + 19)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) + 3*(115*t
an(1/2*d*x + 1/2*c)^4 + 380*tan(1/2*d*x + 1/2*c)^3 + 530*tan(1/2*d*x + 1/2*
c)^2 + 340*tan(1/2*d*x + 1/2*c) + 91)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.828 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{ad}+\frac{3\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos}{ad}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d) - Sec[c + d*x]/(a*d) - Sec
[c + d*x]^3/(3*a*d) - Sec[c + d*x]^5/(5*a*d) + (3*Tan[c + d*x])/(a*d) + Tan
[c + d*x]^3/(a*d) + Tan[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.166008, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2839, 2620, 270, 2622, 302, 207}

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{ad}+\frac{3\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d) - Sec[c + d*x]/(a*d) - Sec
[c + d*x]^3/(3*a*d) - Sec[c + d*x]^5/(5*a*d) + (3*Tan[c + d*x])/(a*d) + Tan
[c + d*x]^3/(a*d) + Tan[c + d*x]^5/(5*a*d)
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a}+\frac{\int \csc^2(c+dx)\sec^6(c+dx)\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^2}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+3x^2+x^4\right)dx, x, \tan(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right.\right.}{ad}$$

$$= -\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{3\tan(c+dx)}{ad}+\frac{\tan^3}{}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+$$

Mathematica [B] time = 0.596397, size = 341, normalized size = 2.71

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(352\sin(c+dx)-596\sin(2(c+dx))+864\sin(3(c+dx))-298\sin(4(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(176 + 1216*Cos[2*(c + d
*x)] + 149*Cos[3*(c + d*x)] + 528*Cos[4*(c + d*x)] + 149*Cos[5*(c + d*x)] +
 120*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 120*Cos[5*(c + d*x)]*Log[Cos[
(c + d*x)/2]] - 120*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 120*Cos[5*(c +
 d*x)]*Log[Sin[(c + d*x)/2]] + Cos[c + d*x]*(-298 - 240*Log[Cos[(c + d*x)/2
]] + 240*Log[Sin[(c + d*x)/2]]) + 352*Sin[c + d*x] - 596*Sin[2*(c + d*x)] -
 480*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)] + 480*Log[Sin[(c + d*x)/2]]*Sin
[2*(c + d*x)] + 864*Sin[3*(c + d*x)] - 298*Sin[4*(c + d*x)] - 240*Log[Cos[(
c + d*x)/2]]*Sin[4*(c + d*x)] + 240*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]
+ 384*Sin[5*(c + d*x)]))/(3840*a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.102, size = 223, normalized size = 1.8

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{6da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{4da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{9}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{2}{5da}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-1/6/d/a/(tan(1/2*d*x+1/2*c)-1)^3-1/4/d/a/(tan(1/
2*d*x+1/2*c)-1)^2-9/8/d/a/(tan(1/2*d*x+1/2*c)-1)-2/5/d/a/(tan(1/2*d*x+1/2*c
)+1)^5+1/d/a/(tan(1/2*d*x+1/2*c)+1)^4-7/3/d/a/(tan(1/2*d*x+1/2*c)+1)^3+5/2/
d/a/(tan(1/2*d*x+1/2*c)+1)^2-39/8/a/d/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a/tan(1/
2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.10039, size = 512, normalized size = 4.06

$$-\frac{\frac{\frac{122 \sin(dx+c)}{\cos(dx+c)+1}-\frac{26 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{454 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{252 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{510 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{330 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{210 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{195 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+15}{\frac{a \sin(dx+c)}{\cos(dx+c)+1}+\frac{2 a \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2 a \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{6 a \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6 a \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{2 a \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{2 a \sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{a \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{30 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{1}{a(\cdots}}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*((122*sin(d*x + c)/(cos(d*x + c) + 1) - 26*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 454*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 252*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 510*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 330*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 - 210*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 195
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 15)/(a*sin(d*x + c)/(cos(d*x + c) +
1) + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*a*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 6*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*a*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 2*a*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - a*sin(d*x + c)^9/(cos(d*x + c) + 1)^9) + 30
*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 15*sin(d*x + c)/(a*(cos(d*x + c)
+ 1)))/d
```

Fricas [A] time = 1.57092, size = 514, normalized size = 4.08

$$\frac{66 \cos(dx+c)^4 - 28 \cos(dx+c)^2 + 15\left(\cos(dx+c)^5 - \cos(dx+c)^3 \sin(dx+c) - \cos(dx+c)^3\right)\log\left(\frac{1}{2}\cos(dx+c) + \cdots\right.}{30\left(ad\cos(dx+c)^5 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(66*cos(d*x + c)^4 - 28*cos(d*x + c)^2 + 15*(cos(d*x + c)^5 - cos(d*x
+ c)^3*sin(d*x + c) - cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) - 15*(cos
(d*x + c)^5 - cos(d*x + c)^3*sin(d*x + c) - cos(d*x + c)^3)*log(-1/2*cos(d*
x + c) + 1/2) + 2*(48*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 1)*sin(d*x + c) -
 8)/(a*d*cos(d*x + c)^5 - a*d*cos(d*x + c)^3*sin(d*x + c) - a*d*cos(d*x + c
)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28615, size = 240, normalized size = 1.9

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{60 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{60\left(2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{5\left(27 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}+\frac{585 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - 60*tan(1/2*d*x + 1/2*c)/a -
60*(2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)) + 5*(27*tan(1/2*d*
x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 25)/(a*(tan(1/2*d*x + 1/2*c) - 1)^
3) + (585*tan(1/2*d*x + 1/2*c)^4 + 2040*tan(1/2*d*x + 1/2*c)^3 + 2890*tan(1
/2*d*x + 1/2*c)^2 + 1880*tan(1/2*d*x + 1/2*c) + 493)/(a*(tan(1/2*d*x + 1/2*
c) + 1)^5))/d
```

3.829 $\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{7\sec^5(c+dx)}{5a^2d}+$$

```
[Out] (-2*x)/a^2 - Cos[c + d*x]/(a^2*d) - (5*Sec[c + d*x])/(a^2*d) + (3*Sec[c + d
*x]^3)/(a^2*d) - (7*Sec[c + d*x]^5)/(5*a^2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d
) + (2*Tan[c + d*x])/(a^2*d) - (2*Tan[c + d*x]^3)/(3*a^2*d) + (2*Tan[c + d*
x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.289752, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2606, 194, 3473, 8, 2590, 270}

$$-\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{7\sec^5(c+dx)}{5a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-2*x)/a^2 - Cos[c + d*x]/(a^2*d) - (5*Sec[c + d*x])/(a^2*d) + (3*Sec[c + d
*x]^3)/(a^2*d) - (7*Sec[c + d*x]^5)/(5*a^2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d
) + (2*Tan[c + d*x])/(a^2*d) - (2*Tan[c + d*x]^3)/(3*a^2*d) + (2*Tan[c + d*
x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec(c+dx)(a-a\sin(c+dx))^2\tan^7(c+dx)\,dx}{a^4}$$

$$= \frac{\int \left(a^2\sec(c+dx)\tan^7(c+dx) - 2a^2\tan^8(c+dx) + a^2\sin(c+dx)\tan^8(c+dx)\right) d}{a^4}$$

$$= \frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a^2} + \frac{\int \sin(c+dx)\tan^8(c+dx)\,dx}{a^2} - \frac{2\int \tan^8(c+dx)}{a^2}$$

$$= -\frac{2\tan^7(c+dx)}{7a^2d} + \frac{2\int \tan^6(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^8}\,dx, x, \cos(c+dx)\right)}{a^2d} + \text{Su}$$

$$= \frac{2\tan^5(c+dx)}{5a^2d} - \frac{2\tan^7(c+dx)}{7a^2d} - \frac{2\int \tan^4(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^8}-\frac{4}{x^6}+\right.\right.}{}$$

$$= -\frac{\cos(c+dx)}{a^2d} - \frac{5\sec(c+dx)}{a^2d} + \frac{3\sec^3(c+dx)}{a^2d} - \frac{7\sec^5(c+dx)}{5a^2d} + \frac{2\sec^7(c+dx)}{7a^2d} -$$

$$= -\frac{\cos(c+dx)}{a^2d} - \frac{5\sec(c+dx)}{a^2d} + \frac{3\sec^3(c+dx)}{a^2d} - \frac{7\sec^5(c+dx)}{5a^2d} + \frac{2\sec^7(c+dx)}{7a^2d} +$$

$$= -\frac{2x}{a^2} - \frac{\cos(c+dx)}{a^2d} - \frac{5\sec(c+dx)}{a^2d} + \frac{3\sec^3(c+dx)}{a^2d} - \frac{7\sec^5(c+dx)}{5a^2d} + \frac{2\sec^7(c+}{7a^2d}$$

Mathematica [A] time = 0.779097, size = 267, normalized size = 1.72

$$-\frac{5488\sin(c+dx) + 6720c\sin(2(c+dx)) + 6720dx\sin(2(c+dx)) - 13224\sin(2(c+dx)) + 8376\sin(3(c+dx)) + 33}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(11172 + 42*(-551 + 280*c + 280*d*x)*Cos[c + d*x] + 14834*Cos[2*(c + d*x)]
 - 4959*Cos[3*(c + d*x)] + 2520*c*Cos[3*(c + d*x)] + 2520*d*x*Cos[3*(c + d*
x)] + 1852*Cos[4*(c + d*x)] + 1653*Cos[5*(c + d*x)] - 840*c*Cos[5*(c + d*x)
] - 840*d*x*Cos[5*(c + d*x)] - 210*Cos[6*(c + d*x)] + 5488*Sin[c + d*x] - 1
```

```
3224*Sin[2*(c + d*x)] + 6720*c*Sin[2*(c + d*x)] + 6720*d*x*Sin[2*(c + d*x)]
 + 8376*Sin[3*(c + d*x)] - 6612*Sin[4*(c + d*x)] + 3360*c*Sin[4*(c + d*x)]
+ 3360*d*x*Sin[4*(c + d*x)] + 2248*Sin[5*(c + d*x)])/(6720*a^2*d*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A] time = 0.144, size = 253, normalized size = 1.6

$$-\frac{1}{12\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{8\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{1}{2\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-2\,\frac{1}{da^{2}\left(1+(\tan(1/2\,dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2+1/2
/d/a^2/(tan(1/2*d*x+1/2*c)-1)-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*arct
an(tan(1/2*d*x+1/2*c))+4/7/d/a^2/(tan(1/2*d*x+1/2*c)+1)^7-2/d/a^2/(tan(1/2*
d*x+1/2*c)+1)^6+6/5/d/a^2/(tan(1/2*d*x+1/2*c)+1)^5+2/d/a^2/(tan(1/2*d*x+1/2
*c)+1)^4-1/12/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3-23/8/d/a^2/(tan(1/2*d*x+1/2*c)
+1)^2-9/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.70179, size = 684, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -4/105*((759*sin(d*x + c)/(cos(d*x + c) + 1) + 444*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 1249*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1816*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 454*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 616*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 1274*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
+ 560*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 385*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 420*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 105*sin(d*x + c)^11/
(cos(d*x + c) + 1)^11 + 216)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) +
 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*a^2*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 11*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 8*a^2*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 11*
a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 4*a^2*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 4*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 4*a^2*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 - a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) + 10
5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.71709, size = 404, normalized size = 2.61

$$-\frac{210\,dx\cos\left(dx+c\right)^{5}+105\,\cos\left(dx+c\right)^{6}-420\,dx\cos\left(dx+c\right)^{3}-389\,\cos\left(dx+c\right)^{4}-173\,\cos\left(dx+c\right)^{2}-2\left(210\,dx\cos\right.}{105\left(a^{2}d\cos\left(dx+c\right)^{5}-2\,a^{2}d\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2\,a^{2}d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(210*d*x*cos(d*x + c)^5 + 105*cos(d*x + c)^6 - 420*d*x*cos(d*x + c)^
3 - 389*cos(d*x + c)^4 - 173*cos(d*x + c)^2 - 2*(210*d*x*cos(d*x + c)^3 + 2
81*cos(d*x + c)^4 + 51*cos(d*x + c)^2 - 5)*sin(d*x + c) + 25)/(a^2*d*cos(d*
x + c)^5 - 2*a^2*d*cos(d*x + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3241, size = 236, normalized size = 1.52

$$-\frac{\frac{1680\,(dx+c)}{a^2}+\frac{1680}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}-\frac{35\left(12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{3780\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+25095\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+68845\,\mathrm{ta}}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(1680*(d*x + c)/a^2 + 1680/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^2) - 35*(
12*tan(1/2*d*x + 1/2*c)^2 - 27*tan(1/2*d*x + 1/2*c) + 13)/(a^2*(tan(1/2*d*x
 + 1/2*c) - 1)^3) + (3780*tan(1/2*d*x + 1/2*c)^6 + 25095*tan(1/2*d*x + 1/2*
c)^5 + 68845*tan(1/2*d*x + 1/2*c)^4 + 98350*tan(1/2*d*x + 1/2*c)^3 + 75222*
tan(1/2*d*x + 1/2*c)^2 + 29659*tan(1/2*d*x + 1/2*c) + 4777)/(a^2*(tan(1/2*d
*x + 1/2*c) + 1)^7))/d
```

3.830 $\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=140

$$\frac{2\tan^7(c+dx)}{7a^2d}-\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{6\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{a^2d}+\frac{2\sec}{}$$

```
[Out] x/a^2 + (2*Sec[c + d*x])/(a^2*d) - (2*Sec[c + d*x]^3)/(a^2*d) + (6*Sec[c +
d*x]^5)/(5*a^2*d) - (2*Sec[c + d*x]^7)/(7*a^2*d) - Tan[c + d*x]/(a^2*d) + T
an[c + d*x]^3/(3*a^2*d) - Tan[c + d*x]^5/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*
a^2*d)
```

Rubi [A] time = 0.300188, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 30, 2606, 194, 3473, 8}

$$\frac{2\tan^7(c+dx)}{7a^2d}-\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{6\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{a^2d}+\frac{2\sec}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] x/a^2 + (2*Sec[c + d*x])/(a^2*d) - (2*Sec[c + d*x]^3)/(a^2*d) + (6*Sec[c +
d*x]^5)/(5*a^2*d) - (2*Sec[c + d*x]^7)/(7*a^2*d) - Tan[c + d*x]/(a^2*d) + T
an[c + d*x]^3/(3*a^2*d) - Tan[c + d*x]^5/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*
a^2*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^2(c+dx)(a-a\sin(c+dx))^2\tan^6(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^2(c+dx)\tan^6(c+dx)-2a^2\sec(c+dx)\tan^7(c+dx)+a^2\tan^8(c+dx)\right) \ldots}{a^4}$$
$$= \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^2}+\frac{\int \tan^8(c+dx)\,dx}{a^2}-\frac{2\int \sec(c+dx)\tan^7(c+dx) \ldots}{a^2}$$
$$= \frac{\tan^7(c+dx)}{7a^2d}-\frac{\int \tan^6(c+dx)\,dx}{a^2}+\frac{\text{Subst}\left(\int x^6\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$
$$= -\frac{\tan^5(c+dx)}{5a^2d}+\frac{2\tan^7(c+dx)}{7a^2d}+\frac{\int \tan^4(c+dx)\,dx}{a^2}-\frac{2\,\text{Subst}\left(\int \left(-1+3x^2-3x \ldots\right.\right.}{a^2 \ldots}$$
$$= \frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{a^2d}+\frac{6\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\ldots$$
$$= \frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{a^2d}+\frac{6\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}-\frac{\tan(c+dx)}{a^2d}+\ldots$$
$$= \frac{x}{a^2}+\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{a^2d}+\frac{6\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}-\frac{\tan(c+d \ldots}{a^2d}$$

Mathematica [A] time = 0.558876, size = 257, normalized size = 1.84

$$\frac{2128\sin(c+dx)+6720c\sin(2(c+dx))+6720dx\sin(2(c+dx))-9144\sin(2(c+dx))+456\sin(3(c+dx))+3360c\,\text{s}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4032 + 42*(-381 + 280*c + 280*d*x)*Cos[c + d*x] + 5504*Cos[2*(c + d*x)] -
3429*Cos[3*(c + d*x)] + 2520*c*Cos[3*(c + d*x)] + 2520*d*x*Cos[3*(c + d*x)]
 + 2752*Cos[4*(c + d*x)] + 1143*Cos[5*(c + d*x)] - 840*c*Cos[5*(c + d*x)] -
 840*d*x*Cos[5*(c + d*x)] + 2128*Sin[c + d*x] - 9144*Sin[2*(c + d*x)] + 672
0*c*Sin[2*(c + d*x)] + 6720*d*x*Sin[2*(c + d*x)] + 456*Sin[3*(c + d*x)] - 4
```

```
572*Sin[4*(c + d*x)] + 3360*c*Sin[4*(c + d*x)] + 3360*d*x*Sin[4*(c + d*x)]
+ 1528*Sin[5*(c + d*x)])/(13440*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A] time = 0.119, size = 230, normalized size = 1.6

$$-\frac{1}{12\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{3}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\,\frac{\arctan(\tan(1/2\,dx+c/}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2+3/8
/d/a^2/(tan(1/2*d*x+1/2*c)-1)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-4/7/d/a^2/
(tan(1/2*d*x+1/2*c)+1)^7+2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^6-8/5/d/a^2/(tan(1/
2*d*x+1/2*c)+1)^5-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)^4+5/12/d/a^2/(tan(1/2*d*x+
1/2*c)+1)^3+11/8/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+13/8/d/a^2/(tan(1/2*d*x+1/2
*c)+1)
```

Maxima [B] time = 1.76889, size = 568, normalized size = 4.06

$$\frac{2\left(\frac{\frac{279\sin(dx+c)}{\cos(dx+c)+1}-\frac{132\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1048\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{364\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1554\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{980\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{280\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{420\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{105\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+96}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{105\,\mathrm{arc}}{\;}\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 2/105*((279*sin(d*x + c)/(cos(d*x + c) + 1) - 132*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 1048*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 364*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 1554*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 980*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 280*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
420*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 105*sin(d*x + c)^9/(cos(d*x + c)
+ 1)^9 + 96)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14
*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^2)/d
```

Fricas [A] time = 1.68706, size = 373, normalized size = 2.66

$$\frac{105\,dx\cos(dx+c)^5-210\,dx\cos(dx+c)^3-172\cos(dx+c)^4+86\cos(dx+c)^2-\left(210\,dx\cos(dx+c)^3+191\cos(dx+\right.}{105\left(a^2d\cos(dx+c)^5-2\,a^2d\cos(dx+c)^3\sin(dx+c)-2\,a^2d\cos(dx+c)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/105*(105*d*x*cos(d*x + c)^5 - 210*d*x*cos(d*x + c)^3 - 172*cos(d*x + c)^4
 + 86*cos(d*x + c)^2 - (210*d*x*cos(d*x + c)^3 + 191*cos(d*x + c)^4 - 129*c
os(d*x + c)^2 + 25)*sin(d*x + c) - 10)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(
d*x + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23087, size = 209, normalized size = 1.49

$$\frac{\frac{840\,(dx+c)}{a^2}+\frac{35\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{1365\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+9345\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+26600\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+39410\,\tan\left(\frac{1}{2}\,dx+\right.}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(840*(d*x + c)/a^2 + 35*(9*tan(1/2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x +
1/2*c) + 10)/(a^2*(tan(1/2*d*x + 1/2*c) - 1)^3) + (1365*tan(1/2*d*x + 1/2*c
)^6 + 9345*tan(1/2*d*x + 1/2*c)^5 + 26600*tan(1/2*d*x + 1/2*c)^4 + 39410*ta
n(1/2*d*x + 1/2*c)^3 + 30261*tan(1/2*d*x + 1/2*c)^2 + 11837*tan(1/2*d*x + 1
/2*c) + 1886)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.831 $\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{\sec^5(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)}{a^2d}$$

```
[Out] -(Sec[c + d*x]/(a^2*d)) + (4*Sec[c + d*x]^3)/(3*a^2*d) - Sec[c + d*x]^5/(a^
2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.271521, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2606, 270, 2607, 30, 194}

$$-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{\sec^5(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]/(a^2*d)) + (4*Sec[c + d*x]^3)/(3*a^2*d) - Sec[c + d*x]^5/(a^
2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^2\tan^5(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^3(c+dx)\tan^5(c+dx) - 2a^2\sec^2(c+dx)\tan^6(c+dx) + a^2\sec(c+dx)\tan\right.}{a^4}$$
$$= \frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^2} + \frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a^2} - \frac{2\int \sec^2(c+dx)\,t}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx, x, \sec(c+dx\right.}{a^2d}$$
$$= -\frac{2\tan^7(c+dx)}{7a^2d} + \frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\sec(c+dx)}{a^2d} + \frac{4\sec^3(c+dx)}{3a^2d} - \frac{\sec^5(c+dx)}{a^2d} + \frac{2\sec^7(c+dx)}{7a^2d} - \frac{2\tan^7(c+dx)}{7a^2d}$$

Mathematica [A] time = 0.263848, size = 126, normalized size = 1.48

$$-\frac{\sec^3(c+dx)(28\sin(c+dx) - 104\sin(2(c+dx)) + 66\sin(3(c+dx)) - 52\sin(4(c+dx)) + 6\sin(5(c+dx)) - 182\cos}{336a^2d(\sin(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^3*(42 - 182*Cos[c + d*x] + 104*Cos[2*(c + d*x)] - 39*Cos[3*(
c + d*x)] - 18*Cos[4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 28*Sin[c + d*x] - 1
04*Sin[2*(c + d*x)] + 66*Sin[3*(c + d*x)] - 52*Sin[4*(c + d*x)] + 6*Sin[5*(
c + d*x)]))/(336*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.114, size = 145, normalized size = 1.7

$$64\frac{1}{da^2}\left(-\frac{1}{768\left(\tan(1/2\,dx+c/2)-1\right)^3} - \frac{1}{512\left(\tan(1/2\,dx+c/2)-1\right)^2} + \frac{1}{256\tan(1/2\,dx+c/2)-256} + \frac{}{112\left(\tan(1/\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 64/d/a^2*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2+1/
256/(tan(1/2*d*x+1/2*c)-1)+1/112/(tan(1/2*d*x+1/2*c)+1)^7-1/32/(tan(1/2*d*x
+1/2*c)+1)^6+1/32/(tan(1/2*d*x+1/2*c)+1)^5-5/768/(tan(1/2*d*x+1/2*c)+1)^3-3
/512/(tan(1/2*d*x+1/2*c)+1)^2-1/256/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.11376, size = 400, normalized size = 4.71

$$-\frac{16\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}{21\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -16/21*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

Fricas [A] time = 1.6397, size = 263, normalized size = 3.09

$$-\frac{9\cos(dx+c)^4-22\cos(dx+c)^2-2\left(3\cos(dx+c)^4+6\cos(dx+c)^2-1\right)\sin(dx+c)+5}{21\left(a^2d\cos(dx+c)^5-2a^2d\cos(dx+c)^3\sin(dx+c)-2a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/21*(9*cos(d*x + c)^4 - 22*cos(d*x + c)^2 - 2*(3*cos(d*x + c)^4 + 6*cos(d
*x + c)^2 - 1)*sin(d*x + c) + 5)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x +
c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2793, size = 197, normalized size = 2.32

$$\frac{\frac{7\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+1015\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+1750\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+1344\tan\left(\frac{1}{2}dx+\right.}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{7}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/168*(7*(6*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 7)/(a^2*(tan
(1/2*d*x + 1/2*c) - 1)^3) - (42*tan(1/2*d*x + 1/2*c)^6 + 315*tan(1/2*d*x +
1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 1750*tan(1/2*d*x + 1/2*c)^3 + 1344
*tan(1/2*d*x + 1/2*c)^2 + 511*tan(1/2*d*x + 1/2*c) + 79)/(a^2*(tan(1/2*d*x
+ 1/2*c) + 1)^7))/d
```

3.832 $\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{\tan^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{4\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}$$

```
[Out] (-2*Sec[c + d*x]^3)/(3*a^2*d) + (4*Sec[c + d*x]^5)/(5*a^2*d) - (2*Sec[c + d
*x]^7)/(7*a^2*d) + Tan[c + d*x]^5/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.156614, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2711, 2607, 14, 2606, 270, 30}

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{\tan^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{4\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Sec[c + d*x]^3)/(3*a^2*d) + (4*Sec[c + d*x]^5)/(5*a^2*d) - (2*Sec[c + d
*x]^7)/(7*a^2*d) + Tan[c + d*x]^5/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \left(a^2\sec^4(c+dx)\tan^4(c+dx)-2a^2\sec^3(c+dx)\tan^5(c+dx)+a^2\sec^2(c+dx)\tan^6(c$$
$$= \frac{\int \sec^4(c+dx)\tan^4(c+dx)\,dx}{a^2}+\frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^2}-\frac{2\int \sec^3(c+dx)\tan^5}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^6\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int x^4\left(1+x^2\right)dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{\tan^7(c+dx)}{7a^2d}+\frac{\text{Subst}\left(\int \left(x^4+x^6\right)dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx,\right.}{a^2d}$$
$$= -\frac{2\sec^3(c+dx)}{3a^2d}+\frac{4\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\tan^5(c+dx)}{5a^2d}+\frac{2\tan^7(c+dx)}{7a^2d}$$

Mathematica [A] time = 0.247231, size = 126, normalized size = 1.38

$$-\frac{\sec^3(c+dx)(-1232\sin(c+dx)-824\sin(2(c+dx))+1896\sin(3(c+dx))-412\sin(4(c+dx))-72\sin(5(c+dx))-}{13440a^2d(\sin(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^3*(672 - 1442*Cos[c + d*x] + 1664*Cos[2*(c + d*x)] - 309*Cos
[3*(c + d*x)] - 288*Cos[4*(c + d*x)] + 103*Cos[5*(c + d*x)] - 1232*Sin[c +
d*x] - 824*Sin[2*(c + d*x)] + 1896*Sin[3*(c + d*x)] - 412*Sin[4*(c + d*x)]
- 72*Sin[5*(c + d*x)]))/(13440*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.122, size = 160, normalized size = 1.8

$$32\,\frac{1}{da^2}\left(-\frac{1}{384\,(\tan(1/2\,dx+c/2)-1)^3}-\frac{1}{256\,(\tan(1/2\,dx+c/2)-1)^2}+\frac{1}{256\,\tan(1/2\,dx+c/2)-256}-\frac{}{56\,(\tan(1/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 32/d/a^2*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2+1/
256/(tan(1/2*d*x+1/2*c)-1)-1/56/(tan(1/2*d*x+1/2*c)+1)^7+1/16/(tan(1/2*d*x+
1/2*c)+1)^6-3/40/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/2*c)+1)^4+1/3
84/(tan(1/2*d*x+1/2*c)+1)^3-1/256/(tan(1/2*d*x+1/2*c)+1)^2-1/256/(tan(1/2*d
*x+1/2*c)+1))
```

Maxima [B] time = 1.08625, size = 427, normalized size = 4.69

$$-\frac{32\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+1\right)}{105\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -32/105*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a^2 + 4*a^2*s
in(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*a^2*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4
*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x + c)^10/(cos(d*x + c
) + 1)^10)*d)
```

Fricas [A] time = 1.63311, size = 267, normalized size = 2.93

$$-\frac{18\,\cos\left(dx+c\right)^{4}-44\,\cos\left(dx+c\right)^{2}+\left(9\,\cos\left(dx+c\right)^{4}-66\,\cos\left(dx+c\right)^{2}+25\right)\sin\left(dx+c\right)+10}{105\left(a^{2}d\cos\left(dx+c\right)^{5}-2\,a^{2}d\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(18*cos(d*x + c)^4 - 44*cos(d*x + c)^2 + (9*cos(d*x + c)^4 - 66*cos(
d*x + c)^2 + 25)*sin(d*x + c) + 10)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x
 + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1964, size = 197, normalized size = 2.16

$$\frac{\frac{35\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+735\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+2030\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2030\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1701\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{7}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(35*(3*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 4)/(a^2*(tan
(1/2*d*x + 1/2*c) - 1)^3) - (105*tan(1/2*d*x + 1/2*c)^6 + 735*tan(1/2*d*x +
```

```
 1/2*c)^5 + 2030*tan(1/2*d*x + 1/2*c)^4 + 2030*tan(1/2*d*x + 1/2*c)^3 + 170
1*tan(1/2*d*x + 1/2*c)^2 + 707*tan(1/2*d*x + 1/2*c) + 116)/(a^2*(tan(1/2*d*
x + 1/2*c) + 1)^7))/d
```

3.833 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{3\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

```
[Out] Sec[c + d*x]^3/(3*a^2*d) - (3*Sec[c + d*x]^5)/(5*a^2*d) + (2*Sec[c + d*x]^7
)/(7*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.288258, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2875, 2873, 2606, 14, 2607, 270}

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{3\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^2*d) - (3*Sec[c + d*x]^5)/(5*a^2*d) + (2*Sec[c + d*x]^7
)/(7*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sec^5(c+dx)(a-a\sin(c+dx))^2\tan^3(c+dx)\,dx}{a^4}\\ &= \frac{\int \left(a^2\sec^5(c+dx)\tan^3(c+dx)-2a^2\sec^4(c+dx)\tan^4(c+dx)+a^2\sec^3(c+dx)\,\mathrm{ta}\right.}{a^4}\\ &= \frac{\int \sec^5(c+dx)\tan^3(c+dx)\,dx}{a^2}+\frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^2}-\frac{2\int \sec^4(c+dx)}{a}\\ &= \frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2dx,x,\sec(c+d\right.}{a^2d}\\ &= \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int \left(-x^4+x^6\right)dx,x,\sec(c+\right.}{a^2d}\\ &= \frac{\sec^3(c+dx)}{3a^2d}-\frac{3\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}\end{aligned}$$

Mathematica [A] time = 0.324578, size = 126, normalized size = 1.38

$$\frac{\sec^3(c+dx)(448\sin(c+dx)-104\sin(2(c+dx))-144\sin(3(c+dx))-52\sin(4(c+dx))+48\sin(5(c+dx))-182\,\mathrm{co}}{6720a^2d(\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*(672 - 182*Cos[c + d*x] - 736*Cos[2*(c + d*x)] - 39*Cos[3*(
c + d*x)] + 192*Cos[4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 448*Sin[c + d*x] -
 104*Sin[2*(c + d*x)] - 144*Sin[3*(c + d*x)] - 52*Sin[4*(c + d*x)] + 48*Sin
[5*(c + d*x)]))/(6720*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.107, size = 130, normalized size = 1.4

$$16\,\frac{1}{da^2}\left(-\frac{1}{192\,(\tan(1/2\,dx+c/2)-1)^3}-\frac{1}{128\,(\tan(1/2\,dx+c/2)-1)^2}+1/28\,(\tan(1/2\,dx+c/2)+1)^{-7}-1/8\,(\tan(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 16/d/a^2*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+1/
28/(tan(1/2*d*x+1/2*c)+1)^7-1/8/(tan(1/2*d*x+1/2*c)+1)^6+7/40/(tan(1/2*d*x+
1/2*c)+1)^5-1/8/(tan(1/2*d*x+1/2*c)+1)^4+7/192/(tan(1/2*d*x+1/2*c)+1)^3+1/1
28/(tan(1/2*d*x+1/2*c)+1)^2)
```

Maxima [B] time = 1.23808, size = 454, normalized size = 4.99

$$\frac{4\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{91\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{105\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}{105\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin}{(\cos(d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 4/105*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 91*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 84*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*
a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) +
 1)^9 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

Fricas [A] time = 1.61421, size = 267, normalized size = 2.93

$$-\frac{24\cos(dx+c)^4-47\cos(dx+c)^2+2\left(6\cos(dx+c)^4-9\cos(dx+c)^2+5\right)\sin(dx+c)+25}{105\left(a^2d\cos(dx+c)^5-2a^2d\cos(dx+c)^3\sin(dx+c)-2a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(24*cos(d*x + c)^4 - 47*cos(d*x + c)^2 + 2*(6*cos(d*x + c)^4 - 9*cos
(d*x + c)^2 + 5)*sin(d*x + c) + 25)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x
 + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28072, size = 162, normalized size = 1.78

$$-\frac{\frac{35\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1015\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1330\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1302\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+469\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+67}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(3*tan(1/2*d*x + 1/2*c) - 1)/(a^2*(tan(1/2*d*x + 1/2*c) - 1)^3)
- (105*tan(1/2*d*x + 1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 1330*tan(1/2*
d*x + 1/2*c)^3 + 1302*tan(1/2*d*x + 1/2*c)^2 + 469*tan(1/2*d*x + 1/2*c) + 6
7)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.834 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}$$

```
[Out] (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Sec[c + d*x]^7)/(7*a^2*d) + Tan[c + d*x]^
3/(3*a^2*d) + (3*Tan[c + d*x]^5)/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.305774, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2875, 2873, 2607, 270, 2606, 14}

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Sec[c + d*x]^7)/(7*a^2*d) + Tan[c + d*x]^
3/(3*a^2*d) + (3*Tan[c + d*x]^5)/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^6(c+dx)(a-a\sin(c+dx))^2\tan^2(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^6(c+dx)\tan^2(c+dx) - 2a^2\sec^5(c+dx)\tan^3(c+dx) + a^2\sec^4(c+dx)\,\mathrm{t}\right.}{a^4}$$
$$= \frac{\int \sec^6(c+dx)\tan^2(c+dx)\,dx}{a^2} + \frac{\int \sec^4(c+dx)\tan^4(c+dx)\,dx}{a^2} - \frac{2\int \sec^5(c+dx}{}$$
$$= \frac{\mathrm{Subst}\left(\int x^4\left(1+x^2\right)dx, x, \tan(c+dx)\right)}{a^2d} + \frac{\mathrm{Subst}\left(\int x^2\left(1+x^2\right)^2 dx, x, \tan(c+dx\right.}{a^2d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(x^4+x^6\right)dx, x, \tan(c+dx)\right)}{a^2d} + \frac{\mathrm{Subst}\left(\int \left(x^2+2x^4+x^6\right)dx, x, \tan(c+\right.}{a^2d}$$
$$= \frac{2\sec^5(c+dx)}{5a^2d} - \frac{2\sec^7(c+dx)}{7a^2d} + \frac{\tan^3(c+dx)}{3a^2d} + \frac{3\tan^5(c+dx)}{5a^2d} + \frac{2\tan^7(c+dx)}{7a^2d}$$

Mathematica [A] time = 0.374074, size = 126, normalized size = 1.38

$$\frac{\sec^3(c+dx)(3136\sin(c+dx) - 408\sin(2(c+dx)) - 48\sin(3(c+dx)) - 204\sin(4(c+dx)) + 16\sin(5(c+dx)) - 714}{13440a^2d(\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*(1344 - 714*Cos[c + d*x] + 128*Cos[2*(c + d*x)] - 153*Cos[3
*(c + d*x)] + 64*Cos[4*(c + d*x)] + 51*Cos[5*(c + d*x)] + 3136*Sin[c + d*x]
 - 408*Sin[2*(c + d*x)] - 48*Sin[3*(c + d*x)] - 204*Sin[4*(c + d*x)] + 16*S
in[5*(c + d*x)]))/(13440*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.111, size = 160, normalized size = 1.8

$$8\,\frac{1}{da^2}\left(-\frac{1}{96\,(\tan(1/2\,dx+c/2)-1)^3} - \frac{1}{64\,(\tan(1/2\,dx+c/2)-1)^2} - \frac{1}{64\,\tan(1/2\,dx+c/2)-64} - 1/14\,(\tan(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 8/d/a^2*(-1/96/(tan(1/2*d*x+1/2*c)-1)^3-1/64/(tan(1/2*d*x+1/2*c)-1)^2-1/64/
(tan(1/2*d*x+1/2*c)-1)-1/14/(tan(1/2*d*x+1/2*c)+1)^7+1/4/(tan(1/2*d*x+1/2*c
)+1)^6-2/5/(tan(1/2*d*x+1/2*c)+1)^5+3/8/(tan(1/2*d*x+1/2*c)+1)^4-19/96/(tan
(1/2*d*x+1/2*c)+1)^3+3/64/(tan(1/2*d*x+1/2*c)+1)^2+1/64/(tan(1/2*d*x+1/2*c)
```

```
+1))
```

Maxima [B] time = 1.15331, size = 481, normalized size = 5.29

$$\frac{8\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{7\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{42\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{35\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35\sin(dx+c)}{(\cos(dx+c)+1}\right.}{105\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin}{(\cos(d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 8/105*(12*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 11*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 7*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 42*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 35*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 35*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3)/((a^2 + 4
*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*a^2*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)
^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x + c)^10/(cos(d
*x + c) + 1)^10)*d)
```

Fricas [A] time = 1.61468, size = 263, normalized size = 2.89

$$-\frac{4\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2+\left(2\cos\left(dx+c\right)^4-3\cos\left(dx+c\right)^2+25\right)\sin\left(dx+c\right)+10}{105\left(a^2d\cos\left(dx+c\right)^5-2a^2d\cos\left(dx+c\right)^3\sin\left(dx+c\right)-2a^2d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(4*cos(d*x + c)^4 - 2*cos(d*x + c)^2 + (2*cos(d*x + c)^4 - 3*cos(d*x
 + c)^2 + 25)*sin(d*x + c) + 10)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x +
c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25787, size = 197, normalized size = 2.16

$$-\frac{\frac{35\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+945\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1820\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1617\tan\left(\frac{1}{2}\right.}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 2)/(a^2*(ta
n(1/2*d*x + 1/2*c) - 1)^3) - (105*tan(1/2*d*x + 1/2*c)^6 + 945*tan(1/2*d*x
+ 1/2*c)^5 + 1820*tan(1/2*d*x + 1/2*c)^4 + 2450*tan(1/2*d*x + 1/2*c)^3 + 16
17*tan(1/2*d*x + 1/2*c)^2 + 749*tan(1/2*d*x + 1/2*c) + 122)/(a^2*(tan(1/2*d
*x + 1/2*c) + 1)^7))/d
```

3.835 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=93

$$\frac{8\tan^3(c+dx)}{105a^2d}+\frac{8\tan(c+dx)}{35a^2d}-\frac{2\sec^3(c+dx)}{35d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

```
[Out] Sec[c + d*x]^3/(7*d*(a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x]^3)/(35*d*(a^2
 + a^2*Sin[c + d*x])) + (8*Tan[c + d*x])/(35*a^2*d) + (8*Tan[c + d*x]^3)/(1
05*a^2*d)
```

Rubi [A] time = 0.11545, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2859, 2672, 3767}

$$\frac{8\tan^3(c+dx)}{105a^2d}+\frac{8\tan(c+dx)}{35a^2d}-\frac{2\sec^3(c+dx)}{35d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]^3/(7*d*(a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x]^3)/(35*d*(a^2
 + a^2*Sin[c + d*x])) + (8*Tan[c + d*x])/(35*a^2*d) + (8*Tan[c + d*x]^3)/(1
05*a^2*d)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} + \frac{2\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{7a}$$
$$= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} - \frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)} + \frac{8\int \sec^4(c+dx)\,dx}{35a^2}$$
$$= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} - \frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)} - \frac{8\,\mathrm{Subst}\left(\int \left(1+x^2\right)dx, x, -\mathrm{ta}\right.}{35a^2d}$$
$$= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} - \frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)} + \frac{8\tan(c+dx)}{35a^2d} + \frac{8\tan^3(c+dx}{105a^2d}$$

Mathematica [A] time = 0.301531, size = 134, normalized size = 1.44

$$-\frac{\sec^3(c+dx)\left(-56\sin(c+dx)+3\sin(2(c+dx))-12\sin(3(c+dx))+\frac{3}{2}\sin(4(c+dx))+4\sin(5(c+dx))+\frac{21}{4}\cos(c\right.}{420a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^3*(-84 + (21*Cos[c + d*x])/4 + 32*Cos[2*(c + d*x)] + (9*Cos[
3*(c + d*x)])/8 + 16*Cos[4*(c + d*x)] - (3*Cos[5*(c + d*x)])/8 - 56*Sin[c +
 d*x] + 3*Sin[2*(c + d*x)] - 12*Sin[3*(c + d*x)] + (3*Sin[4*(c + d*x)])/2 +
 4*Sin[5*(c + d*x)]))/(420*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.105, size = 160, normalized size = 1.7

$$4\frac{1}{da^2}\left(-1/48\,(\tan(1/2\,dx+c/2)-1)^{-3}-1/32\,(\tan(1/2\,dx+c/2)-1)^{-2}-1/16\,(\tan(1/2\,dx+c/2)-1)^{-1}+1/7\,(\tan(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 4/d/a^2*(-1/48/(tan(1/2*d*x+1/2*c)-1)^3-1/32/(tan(1/2*d*x+1/2*c)-1)^2-1/16/
(tan(1/2*d*x+1/2*c)-1)+1/7/(tan(1/2*d*x+1/2*c)+1)^7-1/2/(tan(1/2*d*x+1/2*c)
+1)^6+9/10/(tan(1/2*d*x+1/2*c)+1)^5-1/(tan(1/2*d*x+1/2*c)+1)^4+35/48/(tan(1
/2*d*x+1/2*c)+1)^3-11/32/(tan(1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+
1))
```

Maxima [B] time = 1.16389, size = 508, normalized size = 5.46

$$\frac{2\left(\frac{36\sin(dx+c)}{\cos(dx+c)+1}+\frac{132\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{68\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{14\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{84\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{140\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{140\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\right.}{105\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a}{(c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/105*(36*sin(d*x + c)/(cos(d*x + c) + 1) + 132*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 68*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 14*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 84*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 140*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 140*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 105*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 9)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x +
 c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

Fricas [A] time = 1.61269, size = 267, normalized size = 2.87

$$\frac{32 \cos\left(dx+c\right)^{4} - 16 \cos\left(dx+c\right)^{2} + 2 \left(8 \cos\left(dx+c\right)^{4} - 12 \cos\left(dx+c\right)^{2} - 5\right) \sin\left(dx+c\right) - 25}{105 \left(a^{2} d \cos\left(dx+c\right)^{5} - 2 a^{2} d \cos\left(dx+c\right)^{3} \sin\left(dx+c\right) - 2 a^{2} d \cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(32*cos(d*x + c)^4 - 16*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^4 - 12*cos
(d*x + c)^2 - 5)*sin(d*x + c) - 25)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x
 + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19218, size = 197, normalized size = 2.12

$$-\frac{\frac{35 \left(6 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2} - 9 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+5\right)}{a^{2} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)^{3}} - \frac{210 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6} + 105 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5} - 175 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4} - 910 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3} - 756 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}}{a^{2} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^{7}}}{840 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(6*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 5)/(a^2*(ta
n(1/2*d*x + 1/2*c) - 1)^3) - (210*tan(1/2*d*x + 1/2*c)^6 + 105*tan(1/2*d*x
+ 1/2*c)^5 - 175*tan(1/2*d*x + 1/2*c)^4 - 910*tan(1/2*d*x + 1/2*c)^3 - 756*
tan(1/2*d*x + 1/2*c)^2 - 427*tan(1/2*d*x + 1/2*c) - 31)/(a^2*(tan(1/2*d*x +
 1/2*c) + 1)^7))/d
```

3.836 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{6\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d) + Sec[c + d*x]^5/(5*a^2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d) - (2*Tan[
c + d*x])/(a^2*d) - (2*Tan[c + d*x]^3)/(a^2*d) - (6*Tan[c + d*x]^5)/(5*a^2*
d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.252569, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2606, 30}

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{6\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d) + Sec[c + d*x]^5/(5*a^2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d) - (2*Tan[
c + d*x])/(a^2*d) - (2*Tan[c + d*x]^3)/(a^2*d) - (6*Tan[c + d*x]^5)/(5*a^2*
d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(-2a^2\sec^8(c+dx)+a^2\csc(c+dx)\sec^8(c+dx)+a^2\sec^7(c+dx)\tan(c+dx)\right)dx}{a^4}$$

$$= \frac{\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^2}+\frac{\int \sec^7(c+dx)\tan(c+dx)\,dx}{a^2}-\frac{2\int \sec^8(c+dx)\,dx}{a^2}$$

$$= \frac{\mathrm{Subst}\left(\int x^6\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{2\,\mathrm{Subst}\left(\int (1+\right.}{}$$

$$= \frac{\sec^7(c+dx)}{7a^2d}-\frac{2\tan(c+dx)}{a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{6\tan^5(c+dx)}{5a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{\mathrm{Su}}{}$$

$$= \frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan(c+dx)}{a^2d}-\frac{2\tan^3(}{a^2}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-$$

Mathematica [B] time = 0.600631, size = 352, normalized size = 2.36

$$\frac{2464\sin(c+dx)-4472\sin(2(c+dx))+2208\sin(3(c+dx))-2236\sin(4(c+dx))+384\sin(5(c+dx))+5312\cos(2(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (6216 + 5312*Cos[2*(c + d*x)] - 1677*Cos[3*(c + d*x)] + 696*Cos[4*(c + d*x)
] + 559*Cos[5*(c + d*x)] - 1260*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 42
```

```
0*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 14*Cos[c + d*x]*(559 + 420*Log[C
os[(c + d*x)/2]] - 420*Log[Sin[(c + d*x)/2]]) + 1260*Cos[3*(c + d*x)]*Log[S
in[(c + d*x)/2]] - 420*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 2464*Sin[c
+ d*x] - 4472*Sin[2*(c + d*x)] - 3360*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)
] + 3360*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] + 2208*Sin[3*(c + d*x)] - 2
236*Sin[4*(c + d*x)] - 1680*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] + 1680*L
og[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] + 384*Sin[5*(c + d*x)])/(6720*a^2*d*(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^7)
```

Maple [A] time = 0.131, size = 229, normalized size = 1.5

$$-\frac{1}{12\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{1}{2\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{4}{7\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2
/d/a^2/(tan(1/2*d*x+1/2*c)-1)+4/7/d/a^2/(tan(1/2*d*x+1/2*c)+1)^7-2/d/a^2/(t
an(1/2*d*x+1/2*c)+1)^6+22/5/d/a^2/(tan(1/2*d*x+1/2*c)+1)^5-6/d/a^2/(tan(1/2
*d*x+1/2*c)+1)^4+79/12/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3-39/8/d/a^2/(tan(1/2*d
*x+1/2*c)+1)^2+9/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*ln(tan(1/2*d*x+1/2*
c))
```

Maxima [B] time = 1.0648, size = 570, normalized size = 3.83

$$\frac{\frac{2\left(\frac{554\sin(dx+c)}{\cos(dx+c)+1}+\frac{258\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1108\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1204\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{504\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1470\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{420\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{210\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+191\right)}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{10}{}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(2*(554*sin(d*x + c)/(cos(d*x + c) + 1) + 258*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 1108*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1204*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 504*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1470*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 420*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 - 315*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 210*sin(d*x + c)^9/(cos(d*x +
c) + 1)^9 + 191)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x
+ c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a
^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*log(sin(d*x + c)/(cos(d*x +
 c) + 1))/a^2)/d
```

Fricas [A] time = 1.8116, size = 549, normalized size = 3.68

$$-\frac{174\cos(dx+c)^4+158\cos(dx+c)^2+105\left(\cos(dx+c)^5-2\cos(dx+c)^3\sin(dx+c)-2\cos(dx+c)^3\right)\log\left(\frac{1}{2}\cos(d\right.}{210\left(a^2d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/210*(174*cos(d*x + c)^4 + 158*cos(d*x + c)^2 + 105*(cos(d*x + c)^5 - 2*c
os(d*x + c)^3*sin(d*x + c) - 2*cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2)
- 105*(cos(d*x + c)^5 - 2*cos(d*x + c)^3*sin(d*x + c) - 2*cos(d*x + c)^3)*l
og(-1/2*cos(d*x + c) + 1/2) + 4*(48*cos(d*x + c)^4 + 33*cos(d*x + c)^2 + 5)
*sin(d*x + c) + 50)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)^3*sin(d*x
+ c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28044, size = 217, normalized size = 1.46

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{35\left(12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+11\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}+\frac{3780\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+18585\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+41755\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+517}{a^2\left(\tan\left(\frac{1}{2}dx\right.\right.}}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 35*(12*tan(1/2*d*x + 1/2*c)
^2 - 21*tan(1/2*d*x + 1/2*c) + 11)/(a^2*(tan(1/2*d*x + 1/2*c) - 1)^3) + (37
80*tan(1/2*d*x + 1/2*c)^6 + 18585*tan(1/2*d*x + 1/2*c)^5 + 41755*tan(1/2*d*
x + 1/2*c)^4 + 51730*tan(1/2*d*x + 1/2*c)^3 + 37506*tan(1/2*d*x + 1/2*c)^2
+ 14917*tan(1/2*d*x + 1/2*c) + 2671)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.837 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{7\tan^5(c+dx)}{5a^2d}+\frac{3\tan^3(c+dx)}{a^2d}+\frac{5\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-$$

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Sec[c + d*x])
/(a^2*d) - (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) - (2
*Sec[c + d*x]^7)/(7*a^2*d) + (5*Tan[c + d*x])/(a^2*d) + (3*Tan[c + d*x]^3)/
(a^2*d) + (7*Tan[c + d*x]^5)/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.329989, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2620, 270}

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{7\tan^5(c+dx)}{5a^2d}+\frac{3\tan^3(c+dx)}{a^2d}+\frac{5\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Sec[c + d*x])
/(a^2*d) - (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) - (2
*Sec[c + d*x]^7)/(7*a^2*d) + (5*Tan[c + d*x])/(a^2*d) + (3*Tan[c + d*x]^3)/
(a^2*d) + (7*Tan[c + d*x]^5)/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^2(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^8(c+dx)-2a^2\csc(c+dx)\sec^8(c+dx)+a^2\csc^2(c+dx)\sec^8(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \sec^8(c+dx)\,dx}{a^2}+\frac{\int \csc^2(c+dx)\sec^8(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^2}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^4}{x^2}\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{\operatorname{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)dx,x,-\tan(c+\right.}{a^2d}$$
$$= \frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{\tan^7(c+dx)}{7a^2d}+\frac{\operatorname{Subst}\left(\int \left(4+\frac{1}{x^2}+6\right.\right.}{}$$
$$= -\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{5\,\mathrm{t}}{}$$
$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx}{5a^2d}$$

Mathematica [B] time = 6.09012, size = 442, normalized size = 2.7

$$\frac{16\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{13\sin\left(\frac{1}{2}(c+dx)\right)}{384d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{384d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (16*(-Cot[(c + d*x)/2]/(32*d) + Log[Cos[(c + d*x)/2]]/(8*d) - Log[Sin[(c +
d*x)/2]]/(8*d) + 1/(768*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + Sin[(c
```

```
 + d*x)/2]/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (13*Sin[(c + d
*x)/2])/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(2
24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7) - 1/(448*d*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^6) + (3*Sin[(c + d*x)/2])/(140*d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^5) - 3/(280*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + (9
97*Sin[(c + d*x)/2])/(13440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - 99
7/(26880*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4777*Sin[(c + d*x)/2
])/(13440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + Tan[(c + d*x)/2]/(32*d
)))/a^2
```

Maple [A] time = 0.135, size = 266, normalized size = 1.6

$$\frac{1}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{12\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{5}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/12/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/
(tan(1/2*d*x+1/2*c)-1)^2-5/8/d/a^2/(tan(1/2*d*x+1/2*c)-1)-4/7/d/a^2/(tan(1/
2*d*x+1/2*c)+1)^7+2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^6-24/5/d/a^2/(tan(1/2*d*x+
1/2*c)+1)^5+7/d/a^2/(tan(1/2*d*x+1/2*c)+1)^4-107/12/d/a^2/(tan(1/2*d*x+1/2*
c)+1)^3+59/8/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2-75/8/d/a^2/(tan(1/2*d*x+1/2*c)+
1)-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.06658, size = 649, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/210*((1828*sin(d*x + c)/(cos(d*x + c) + 1) + 3847*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 1656*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 12734*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 - 7952*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 9702
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 12600*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 315*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 5460*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 2205*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 105)/(a^2*si
n(d*x + c)/(cos(d*x + c) + 1) + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 3*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 8*a^2*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 14*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 14*a^2*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 8*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 3*
a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 4*a^2*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 - a^2*sin(d*x + c)^11/(cos(d*x + c) + 1)^11) + 420*log(sin(d*x +
 c)/(cos(d*x + c) + 1))/a^2 - 105*sin(d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.8143, size = 671, normalized size = 4.09

$$-\frac{432\,\cos\left(dx+c\right)^6-660\,\cos\left(dx+c\right)^4+98\,\cos\left(dx+c\right)^2-105\left(2\,\cos\left(dx+c\right)^5-2\,\cos\left(dx+c\right)^3+\left(\cos\left(dx+c\right)^5-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(432*cos(d*x + c)^6 - 660*cos(d*x + c)^4 + 98*cos(d*x + c)^2 - 105*(
2*cos(d*x + c)^5 - 2*cos(d*x + c)^3 + (cos(d*x + c)^5 - 2*cos(d*x + c)^3)*s
in(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 105*(2*cos(d*x + c)^5 - 2*cos(d*
x + c)^3 + (cos(d*x + c)^5 - 2*cos(d*x + c)^3)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) - 2*(327*cos(d*x + c)^4 - 41*cos(d*x + c)^2 - 5)*sin(d*x + c
) + 25)/(2*a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)^3 + (a^2*d*cos(d*x +
 c)^5 - 2*a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30984, size = 275, normalized size = 1.68

$$-\frac{\frac{1680 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} - \frac{420 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2} - \frac{420\left(4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)} + \frac{35\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-27\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+14\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3} + \frac{7875\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right. \cdots}{\cdots}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(1680*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 420*tan(1/2*d*x + 1/2*c)/
a^2 - 420*(4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)) + 35*(15*
tan(1/2*d*x + 1/2*c)^2 - 27*tan(1/2*d*x + 1/2*c) + 14)/(a^2*(tan(1/2*d*x +
1/2*c) - 1)^3) + (7875*tan(1/2*d*x + 1/2*c)^6 + 41055*tan(1/2*d*x + 1/2*c)^
5 + 94640*tan(1/2*d*x + 1/2*c)^4 + 119630*tan(1/2*d*x + 1/2*c)^3 + 87507*ta
n(1/2*d*x + 1/2*c)^2 + 34979*tan(1/2*d*x + 1/2*c) + 6122)/(a^2*(tan(1/2*d*x
 + 1/2*c) + 1)^7))/d
```

3.838 $\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=194

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{8\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{a^2d}-\frac{8\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{11\sec^7(c+dx)}{14a^2d}+\frac{11\sec^5(c+dx)}{10a^2d}$$

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (11*Sec[
c + d*x])/(2*a^2*d) + (11*Sec[c + d*x]^3)/(6*a^2*d) + (11*Sec[c + d*x]^5)/(
10*a^2*d) + (11*Sec[c + d*x]^7)/(14*a^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^7
)/(2*a^2*d) - (8*Tan[c + d*x])/(a^2*d) - (4*Tan[c + d*x]^3)/(a^2*d) - (8*Ta
n[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.365604, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2622, 302, 207, 2620, 270, 288}

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{8\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{a^2d}-\frac{8\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{11\sec^7(c+dx)}{14a^2d}+\frac{11\sec^5(c+dx)}{10a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (11*Sec[
c + d*x])/(2*a^2*d) + (11*Sec[c + d*x]^3)/(6*a^2*d) + (11*Sec[c + d*x]^5)/(
10*a^2*d) + (11*Sec[c + d*x]^7)/(14*a^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^7
)/(2*a^2*d) - (8*Tan[c + d*x])/(a^2*d) - (4*Tan[c + d*x]^3)/(a^2*d) - (8*Ta
n[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^3(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\csc(c+dx)\sec^8(c+dx) - 2a^2\csc^2(c+dx)\sec^8(c+dx) + a^2\csc^3(c+dx)\sec^8(c\right.}{a^4}$$

$$= \frac{\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^2} + \frac{\int \csc^3(c+dx)\sec^8(c+dx)\,dx}{a^2} - \frac{2\int \csc^2(c+dx)\sec^8}{a^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2d} - \frac{2\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\csc^2(c+dx)\sec^7(c+dx)}{2a^2d} + \frac{\text{Subst}\left(\int \left(1+x^2+x^4+x^6+\frac{1}{-1+x^2}\right)dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2\cot(c+dx)}{a^2d} + \frac{\sec(c+dx)}{a^2d} + \frac{\sec^3(c+dx)}{3a^2d} + \frac{\sec^5(c+dx)}{5a^2d} + \frac{\sec^7(c+dx)}{7a^2d} - \frac{\csc^2(c+}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{2\cot(c+dx)}{a^2d} + \frac{11\sec(c+dx)}{2a^2d} + \frac{11\sec^3(c+dx)}{6a^2d} + \frac{11\sec^5(c}{10a^2}$$

$$= -\frac{11\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{2\cot(c+dx)}{a^2d} + \frac{11\sec(c+dx)}{2a^2d} + \frac{11\sec^3(c+dx)}{6a^2d} + \frac{11\sec^5}{10}$$

Mathematica [A] time = 0.582447, size = 277, normalized size = 1.43

$$-36960\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\tfrac{1}{2}(c+dx)\right)\right)^7 + 36960\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-36960*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7 + 369
60*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7 + (Csc[c +
 d*x]^2*(4510 - 6908*Cos[c + d*x] - 563*Cos[2*(c + d*x)] + 4396*Cos[3*(c +
d*x)] - 5390*Cos[4*(c + d*x)] + 3140*Cos[5*(c + d*x)] - 1917*Cos[6*(c + d*x
)] - 628*Cos[7*(c + d*x)] + 4488*Sin[c + d*x] - 7536*Sin[2*(c + d*x)] + 383
6*Sin[3*(c + d*x)] - 780*Sin[5*(c + d*x)] + 2512*Sin[6*(c + d*x)] - 768*Sin
[7*(c + d*x)]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)/(6720*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*Sin[c + d*x])^2)
```

Maple [A] time = 0.162, size = 303, normalized size = 1.6

$$\frac{1}{8da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{12da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{8da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{3}{4da^2}\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2*tan(1/2*d*x+1/2*c)-1/12/d/a^2/(tan(1
/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2-3/4/d/a^2/(tan(1/2*d*
x+1/2*c)-1)+4/7/d/a^2/(tan(1/2*d*x+1/2*c)+1)^7-2/d/a^2/(tan(1/2*d*x+1/2*c)+
1)^6+26/5/d/a^2/(tan(1/2*d*x+1/2*c)+1)^5-8/d/a^2/(tan(1/2*d*x+1/2*c)+1)^4+1
39/12/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3-83/8/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+67
/4/d/a^2/(tan(1/2*d*x+1/2*c)+1)-1/8/d/a^2/tan(1/2*d*x+1/2*c)^2+1/d/a^2/tan(
1/2*d*x+1/2*c)+11/2/d/a^2*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.15984, size = 710, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/840*((420*sin(d*x + c)/(cos(d*x + c) + 1) + 15173*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 38432*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 894*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 - 95344*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 77182
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 61992*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 101115*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 11340*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9 - 33495*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 14280*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105)/(a^2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 4*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 8*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^2
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 14*a^2*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 8*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3*a^2*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 4*a^2*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^2*s
in(d*x + c)^12/(cos(d*x + c) + 1)^12) - 105*(8*sin(d*x + c)/(cos(d*x + c) +
 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^2 + 4620*log(sin(d*x + c)/(cos
```

```
(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.90948, size = 786, normalized size = 4.05

$$-\frac{3834 \cos(dx+c)^6 - 3056 \cos(dx+c)^4 - 468 \cos(dx+c)^2 + 1155 \left(\cos(dx+c)^7 - 3 \cos(dx+c)^5 + 2 \cos(dx+c)^3 - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/420*(3834*cos(d*x + c)^6 - 3056*cos(d*x + c)^4 - 468*cos(d*x + c)^2 + 11
55*(cos(d*x + c)^7 - 3*cos(d*x + c)^5 + 2*cos(d*x + c)^3 - 2*(cos(d*x + c)^
5 - cos(d*x + c)^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 1155*(cos(d
*x + c)^7 - 3*cos(d*x + c)^5 + 2*cos(d*x + c)^3 - 2*(cos(d*x + c)^5 - cos(d
*x + c)^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(768*cos(d*x + c)
^6 - 765*cos(d*x + c)^4 - 98*cos(d*x + c)^2 - 10)*sin(d*x + c) - 100)/(a^2*
d*cos(d*x + c)^7 - 3*a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^3 - 2*(a^2
*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33592, size = 321, normalized size = 1.65

$$\frac{\frac{4620 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^2} + \frac{105\left(a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 8 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{a^4} - \frac{105\left(66 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 8 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)}{a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2} - \frac{35\left(18 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 3\cdots\right.}{a^2\left(\tan\left(\frac{1}{2} dx + \cdots\right.\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(4620*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 105*(a^2*tan(1/2*d*x + 1/2
*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 - 105*(66*tan(1/2*d*x + 1/2*c)^2 -
8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/2*c)^2) - 35*(18*tan(1/2*d
*x + 1/2*c)^2 - 33*tan(1/2*d*x + 1/2*c) + 17)/(a^2*(tan(1/2*d*x + 1/2*c) -
1)^3) + (14070*tan(1/2*d*x + 1/2*c)^6 + 75705*tan(1/2*d*x + 1/2*c)^5 + 1772
05*tan(1/2*d*x + 1/2*c)^4 + 226450*tan(1/2*d*x + 1/2*c)^3 + 166488*tan(1/2*
d*x + 1/2*c)^2 + 66661*tan(1/2*d*x + 1/2*c) + 11533)/(a^2*(tan(1/2*d*x + 1/
2*c) + 1)^7))/d
```

3.839 $\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$-\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+$$

```
[Out] x/a^3 + (3*Sec[c + d*x])/(a^3*d) - (13*Sec[c + d*x]^3)/(3*a^3*d) + (21*Sec[
c + d*x]^5)/(5*a^3*d) - (15*Sec[c + d*x]^7)/(7*a^3*d) + (4*Sec[c + d*x]^9)/
(9*a^3*d) - Tan[c + d*x]/(a^3*d) + Tan[c + d*x]^3/(3*a^3*d) - Tan[c + d*x]^
5/(5*a^3*d) + Tan[c + d*x]^7/(7*a^3*d) - (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.367597, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2875, 2873, 2606, 270, 2607, 30, 194, 3473, 8}

$$-\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] x/a^3 + (3*Sec[c + d*x])/(a^3*d) - (13*Sec[c + d*x]^3)/(3*a^3*d) + (21*Sec[
c + d*x]^5)/(5*a^3*d) - (15*Sec[c + d*x]^7)/(7*a^3*d) + (4*Sec[c + d*x]^9)/
(9*a^3*d) - Tan[c + d*x]/(a^3*d) + Tan[c + d*x]^3/(3*a^3*d) - Tan[c + d*x]^
5/(5*a^3*d) + Tan[c + d*x]^7/(7*a^3*d) - (4*Tan[c + d*x]^9)/(9*a^3*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^3\tan^7(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^3(c+dx)\tan^7(c+dx)-3a^3\sec^2(c+dx)\tan^8(c+dx)+3a^3\sec(c+dx)\tan^9\right.}{a^6}$$
$$= \frac{\int \sec^3(c+dx)\tan^7(c+dx)\,dx}{a^3}-\frac{\int \tan^{10}(c+dx)\,dx}{a^3}-\frac{3\int \sec^2(c+dx)\tan^8(c+dx)\,d}{a^3}$$
$$= -\frac{\tan^9(c+dx)}{9a^3d}+\frac{\int \tan^8(c+dx)\,dx}{a^3}+\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,\sec(c+dx)\right)}{a^3d}-\frac{3}{}$$
$$= \frac{\tan^7(c+dx)}{7a^3d}-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{\int \tan^6(c+dx)\,dx}{a^3}+\frac{\text{Subst}\left(\int\left(-x^2+3x^4-3x^6+x\right.\right.}{a^3d}$$
$$= \frac{3\sec(c+dx)}{a^3d}-\frac{13\sec^3(c+dx)}{3a^3d}+\frac{21\sec^5(c+dx)}{5a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}$$
$$= \frac{3\sec(c+dx)}{a^3d}-\frac{13\sec^3(c+dx)}{3a^3d}+\frac{21\sec^5(c+dx)}{5a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}$$
$$= \frac{3\sec(c+dx)}{a^3d}-\frac{13\sec^3(c+dx)}{3a^3d}+\frac{21\sec^5(c+dx)}{5a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}$$
$$= \frac{x}{a^3}+\frac{3\sec(c+dx)}{a^3d}-\frac{13\sec^3(c+dx)}{3a^3d}+\frac{21\sec^5(c+dx)}{5a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+}{9a^3d}$$

Mathematica [A] time = 0.51058, size = 273, normalized size = 1.53

$$\frac{93312\sin(c+dx)+272160(c+dx)\sin(2(c+dx))-506277\sin(2(c+dx))+125248\sin(3(c+dx))+120960(c+dx)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (169344 - 675036*Cos[c + d*x] + 362880*(c + d*x)*Cos[c + d*x] + 173952*Cos[
2*(c + d*x)] - 37502*Cos[3*(c + d*x)] + 20160*(c + d*x)*Cos[3*(c + d*x)] +
54912*Cos[4*(c + d*x)] + 112506*Cos[5*(c + d*x)] - 60480*(c + d*x)*Cos[5*(c
 + d*x)] - 21376*Cos[6*(c + d*x)] + 93312*Sin[c + d*x] - 506277*Sin[2*(c +
d*x)] + 272160*(c + d*x)*Sin[2*(c + d*x)] + 125248*Sin[3*(c + d*x)] - 22501
2*Sin[4*(c + d*x)] + 120960*(c + d*x)*Sin[4*(c + d*x)] + 67776*Sin[5*(c + d
*x)] + 18751*Sin[6*(c + d*x)] - 10080*(c + d*x)*Sin[6*(c + d*x)])/(322560*d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3*(a + a*Sin[c + d*x])^3)
```

Maple [A] time = 0.16, size = 272, normalized size = 1.5

$$-\frac{1}{24\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{16\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{7}{32\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\,\frac{\arctan(\tan(1/2\,d}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/24/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-1/16/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2+7/
32/d/a^3/(tan(1/2*d*x+1/2*c)-1)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))+8/9/d/a^
3/(tan(1/2*d*x+1/2*c)+1)^9-4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^8+40/7/d/a^3/(tan
(1/2*d*x+1/2*c)+1)^7-4/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^6-21/10/d/a^3/(tan(1/
2*d*x+1/2*c)+1)^5-3/4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^4+3/4/d/a^3/(tan(1/2*d*x
+1/2*c)+1)^3+13/8/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+57/32/d/a^3/(tan(1/2*d*x+1
/2*c)+1)
```

Maxima [B] time = 1.7886, size = 657, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2/315*((1893*sin(d*x + c)/(cos(d*x + c) + 1) + 2526*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 2939*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9936*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 - 3546*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 11172*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 9702*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 - 3675*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1890*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10 - 315*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 368)/(a^3 +
6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10
/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*
sin(d*x + c)^12/(cos(d*x + c) + 1)^12) + 315*arctan(sin(d*x + c)/(cos(d*x +
 c) + 1))/a^3)/d
```

Fricas [A] time = 1.8831, size = 478, normalized size = 2.69

$$\frac{945\,dx\cos(dx+c)^5+668\,\cos(dx+c)^6-1260\,dx\cos(dx+c)^3-1431\,\cos(dx+c)^4+465\,\cos(dx+c)^2+\left(315\,dx\cos(\right.}{315\left(3\,a^3d\cos(dx+c)^5-4\,a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/315*(945*d*x*cos(d*x + c)^5 + 668*cos(d*x + c)^6 - 1260*d*x*cos(d*x + c)^
3 - 1431*cos(d*x + c)^4 + 465*cos(d*x + c)^2 + (315*d*x*cos(d*x + c)^5 - 12
60*d*x*cos(d*x + c)^3 - 1059*cos(d*x + c)^4 + 305*cos(d*x + c)^2 - 35)*sin(
d*x + c) - 70)/(3*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*co
s(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29692, size = 244, normalized size = 1.37

$$\frac{\frac{10080\,(dx+c)}{a^3}+\frac{105\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+23\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{17955\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+160020\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+624960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1387260\,\mathrm{ta}}{}}{10080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/10080*(10080*(d*x + c)/a^3 + 105*(21*tan(1/2*d*x + 1/2*c)^2 - 48*tan(1/2*
d*x + 1/2*c) + 23)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^3) + (17955*tan(1/2*d*x
+ 1/2*c)^8 + 160020*tan(1/2*d*x + 1/2*c)^7 + 624960*tan(1/2*d*x + 1/2*c)^6
+ 1387260*tan(1/2*d*x + 1/2*c)^5 + 1884582*tan(1/2*d*x + 1/2*c)^4 + 1556268
*tan(1/2*d*x + 1/2*c)^3 + 774792*tan(1/2*d*x + 1/2*c)^2 + 215748*tan(1/2*d*
x + 1/2*c) + 25967)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^9))/d
```

3.840 $\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=121

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{13\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{a^3d}+\frac{7\sec^3(c+dx)}{3a^3d}-\frac{\sec(c+dx)}{a^3d}$$

```
[Out] -(Sec[c + d*x]/(a^3*d)) + (7*Sec[c + d*x]^3)/(3*a^3*d) - (3*Sec[c + d*x]^5)
/(a^3*d) + (13*Sec[c + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + T
an[c + d*x]^7/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.346745, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2875, 2873, 2607, 14, 2606, 270, 30, 194}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{13\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{a^3d}+\frac{7\sec^3(c+dx)}{3a^3d}-\frac{\sec(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]/(a^3*d)) + (7*Sec[c + d*x]^3)/(3*a^3*d) - (3*Sec[c + d*x]^5)
/(a^3*d) + (13*Sec[c + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + T
an[c + d*x]^7/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^4(c+dx)(a-a\sin(c+dx))^3\tan^6(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^4(c+dx)\tan^6(c+dx)-3a^3\sec^3(c+dx)\tan^7(c+dx)+3a^3\sec^2(c+dx)\tan\right.}{a^6}$$
$$= \frac{\int \sec^4(c+dx)\tan^6(c+dx)\,dx}{a^3} - \frac{\int \sec(c+dx)\tan^9(c+dx)\,dx}{a^3} - \frac{3\int \sec^3(c+dx)\tan}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)^4 dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int x^6\left(1+x^2\right) dx, x, \tan(c+dx)\right)}{a^3d}$$
$$= \frac{\tan^9(c+dx)}{3a^3d} - \frac{\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right) dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int \right.}{}$$
$$= -\frac{\sec(c+dx)}{a^3d} + \frac{7\sec^3(c+dx)}{3a^3d} - \frac{3\sec^5(c+dx)}{a^3d} + \frac{13\sec^7(c+dx)}{7a^3d} - \frac{4\sec^9(c+dx)}{9a^3d} +$$

Mathematica [A] time = 0.376476, size = 185, normalized size = 1.53

$$\frac{-2304\sin(c+dx)+27189\sin(2(c+dx))-16256\sin(3(c+dx))+12084\sin(4(c+dx))+384\sin(5(c+dx))-1007\sin(}{64512d(a\sin(c+dx)+a)^3\left(\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9408 + 36252*Cos[c + d*x] - 12384*Cos[2*(c + d*x)] + 2014*Cos[3*(c + d*x)
] + 4800*Cos[4*(c + d*x)] - 6042*Cos[5*(c + d*x)] + 608*Cos[6*(c + d*x)] -
2304*Sin[c + d*x] + 27189*Sin[2*(c + d*x)] - 16256*Sin[3*(c + d*x)] + 12084
*Sin[4*(c + d*x)] + 384*Sin[5*(c + d*x)] - 1007*Sin[6*(c + d*x)])/(64512*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3*(a + a*Sin[c + d*x])^3)
```

Maple [A] time = 0.142, size = 190, normalized size = 1.6

$$128\,\frac{1}{d{a}^{3}}\left(-\frac{1}{3072\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{3}}-\frac{1}{2048\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{2}}+\frac{5}{4096\,\tan\left(1/2\,dx+c/2\right)-4096}-\frac{1}{144\,(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 128/d/a^3*(-1/3072/(tan(1/2*d*x+1/2*c)-1)^3-1/2048/(tan(1/2*d*x+1/2*c)-1)^2
+5/4096/(tan(1/2*d*x+1/2*c)-1)-1/144/(tan(1/2*d*x+1/2*c)+1)^9+1/32/(tan(1/2
*d*x+1/2*c)+1)^8-11/224/(tan(1/2*d*x+1/2*c)+1)^7+5/192/(tan(1/2*d*x+1/2*c)+
1)^6+1/256/(tan(1/2*d*x+1/2*c)+1)^5-1/512/(tan(1/2*d*x+1/2*c)+1)^4-1/384/(t
an(1/2*d*x+1/2*c)+1)^3-1/512/(tan(1/2*d*x+1/2*c)+1)^2-5/4096/(tan(1/2*d*x+1
/2*c)+1))
```

Maxima [B] time = 1.20072, size = 489, normalized size = 4.04

$$-\frac{32\left(\frac{6\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36\,\mathrm{s}}{(\cos(}\right.}{63\left(a^{3}+\frac{6\,a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2\,a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36\,a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{36\,a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{27\,a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -32/63*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a^3 + 6*a^3*s
in(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*
x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
- 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d
*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x
 + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A] time = 1.69633, size = 328, normalized size = 2.71

$$-\frac{19\,\cos\left(dx+c\right)^{6}+9\,\cos\left(dx+c\right)^{4}-51\,\cos\left(dx+c\right)^{2}+2\left(3\,\cos\left(dx+c\right)^{4}-34\,\cos\left(dx+c\right)^{2}+7\right)\sin\left(dx+c\right)+7}{63\left(3\,a^{3}d\cos\left(dx+c\right)^{5}-4\,a^{3}d\cos\left(dx+c\right)^{3}+\left(a^{3}d\cos\left(dx+c\right)^{5}-4\,a^{3}d\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/63*(19*cos(d*x + c)^6 + 9*cos(d*x + c)^4 - 51*cos(d*x + c)^2 + 2*(3*cos(
d*x + c)^4 - 34*cos(d*x + c)^2 + 7)*sin(d*x + c) + 7)/(3*a^3*d*cos(d*x + c)
^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^
3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19524, size = 232, normalized size = 1.92

$$\frac{\frac{21\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+17\right)}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+3024\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+13020\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+32760\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+51282\tan\left(\right.}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right.}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2016*(21*(15*tan(1/2*d*x + 1/2*c)^2 - 36*tan(1/2*d*x + 1/2*c) + 17)/(a^3*
(tan(1/2*d*x + 1/2*c) - 1)^3) - (315*tan(1/2*d*x + 1/2*c)^8 + 3024*tan(1/2*
d*x + 1/2*c)^7 + 13020*tan(1/2*d*x + 1/2*c)^6 + 32760*tan(1/2*d*x + 1/2*c)^
5 + 51282*tan(1/2*d*x + 1/2*c)^4 + 43008*tan(1/2*d*x + 1/2*c)^3 + 20988*tan
(1/2*d*x + 1/2*c)^2 + 5688*tan(1/2*d*x + 1/2*c) + 667)/(a^3*(tan(1/2*d*x +
1/2*c) + 1)^9))/d
```

3.841 $\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{3\tan^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{11\sec^7(c+dx)}{7a^3d}+\frac{2\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}$$

```
[Out] -(Sec[c + d*x]^3/(a^3*d)) + (2*Sec[c + d*x]^5)/(a^3*d) - (11*Sec[c + d*x]^7
)/(7*a^3*d) + (4*Sec[c + d*x]^9)/(9*a^3*d) - (3*Tan[c + d*x]^7)/(7*a^3*d) -
 (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.338446, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2875, 2873, 2606, 270, 2607, 14, 30}

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{3\tan^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{11\sec^7(c+dx)}{7a^3d}+\frac{2\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]^3/(a^3*d)) + (2*Sec[c + d*x]^5)/(a^3*d) - (11*Sec[c + d*x]^7
)/(7*a^3*d) + (4*Sec[c + d*x]^9)/(9*a^3*d) - (3*Tan[c + d*x]^7)/(7*a^3*d) -
 (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^5(c+dx)(a-a\sin(c+dx))^3\tan^5(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^5(c+dx)\tan^5(c+dx) - 3a^3\sec^4(c+dx)\tan^6(c+dx) + 3a^3\sec^3(c+dx)\tan^7 \ldots}{a^6}$$
$$= \frac{\int \sec^5(c+dx)\tan^5(c+dx)\,dx}{a^3} - \frac{\int \sec^2(c+dx)\tan^8(c+dx)\,dx}{a^3} - \frac{3\int \sec^4(c+dx)\tan \ldots}{a^3}$$
$$= -\frac{\text{Subst}\left(\int x^8\,dx, x, \tan(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{3\,\text{Sub}\ldots}{\ldots}$$
$$= -\frac{\tan^9(c+dx)}{9a^3d} + \frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx, x, \sec(c+dx)\right)}{a^3d} + \frac{3\,\text{Subst}\left(\int \left(-x^2+3x \ldots\right.\right.}{\ldots}$$
$$= -\frac{\sec^3(c+dx)}{a^3d} + \frac{2\sec^5(c+dx)}{a^3d} - \frac{11\sec^7(c+dx)}{7a^3d} + \frac{4\sec^9(c+dx)}{9a^3d} - \frac{3\tan^7(c+dx)}{7a^3d} - \ldots$$

Mathematica [A] time = 0.219221, size = 185, normalized size = 1.76

$$\frac{-1152\sin(c+dx) + 6507\sin(2(c+dx)) - 8128\sin(3(c+dx)) + 2892\sin(4(c+dx)) + 192\sin(5(c+dx)) - 241\sin(6(c+ \ldots}{64512d(a\sin(c+dx)+a)^3\left(\cos\left(\frac{1}{2}(c+ \ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-1344 + 8676*Cos[c + d*x] - 11232*Cos[2*(c + d*x)] + 482*Cos[3*(c + d*x)]
+ 4416*Cos[4*(c + d*x)] - 1446*Cos[5*(c + d*x)] - 32*Cos[6*(c + d*x)] - 115
2*Sin[c + d*x] + 6507*Sin[2*(c + d*x)] - 8128*Sin[3*(c + d*x)] + 2892*Sin[4
*(c + d*x)] + 192*Sin[5*(c + d*x)] - 241*Sin[6*(c + d*x)])/(64512*d*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a
 + a*Sin[c + d*x])^3)
```

Maple [A] time = 0.139, size = 190, normalized size = 1.8

$$64\frac{1}{da^3}\left(-\frac{1}{1536\left(\tan(1/2\,dx+c/2)-1\right)^3} - \frac{1}{1024\left(\tan(1/2\,dx+c/2)-1\right)^2} + \frac{3}{2048\tan(1/2\,dx+c/2)-2048} + \frac{\ldots}{72\left(\tan(1 \ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 64/d/a^3*(-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(1/2*d*x+1/2*c)-1)^2+
3/2048/(tan(1/2*d*x+1/2*c)-1)+1/72/(tan(1/2*d*x+1/2*c)+1)^9-1/16/(tan(1/2*d
*x+1/2*c)+1)^8+3/28/(tan(1/2*d*x+1/2*c)+1)^7-1/12/(tan(1/2*d*x+1/2*c)+1)^6+
3/128/(tan(1/2*d*x+1/2*c)+1)^5+1/256/(tan(1/2*d*x+1/2*c)+1)^4-1/768/(tan(1/
2*d*x+1/2*c)+1)^3-1/512/(tan(1/2*d*x+1/2*c)+1)^2-3/2048/(tan(1/2*d*x+1/2*c)
+1))
```

Maxima [B] time = 1.15885, size = 516, normalized size = 4.91

$$-\frac{16\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right.}{63\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -16/63*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 42*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 1
2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27
*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x +
c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A] time = 1.76602, size = 323, normalized size = 3.08

$$\frac{\cos(dx+c)^6-36\cos(dx+c)^4+57\cos(dx+c)^2-\left(3\cos(dx+c)^4-34\cos(dx+c)^2+7\right)\sin(dx+c)-14}{63\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/63*(cos(d*x + c)^6 - 36*cos(d*x + c)^4 + 57*cos(d*x + c)^2 - (3*cos(d*x +
 c)^4 - 34*cos(d*x + c)^2 + 7)*sin(d*x + c) - 14)/(3*a^3*d*cos(d*x + c)^5 -
 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*s
in(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30418, size = 232, normalized size = 2.21

$$\frac{\frac{21\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+11\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{189\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+1764\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+7224\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+16380\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+19026\tan\left(\frac{1}{2}\right.}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^9}}{2016d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2016*(21*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a^3*(
tan(1/2*d*x + 1/2*c) - 1)^3) - (189*tan(1/2*d*x + 1/2*c)^8 + 1764*tan(1/2*d
*x + 1/2*c)^7 + 7224*tan(1/2*d*x + 1/2*c)^6 + 16380*tan(1/2*d*x + 1/2*c)^5
+ 19026*tan(1/2*d*x + 1/2*c)^4 + 16380*tan(1/2*d*x + 1/2*c)^3 + 8352*tan(1/
2*d*x + 1/2*c)^2 + 2340*tan(1/2*d*x + 1/2*c) + 281)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^9))/d
```

3.842 $\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{5\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{9\sec^7(c+dx)}{7a^3d}-\frac{6\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{3a^3d}$$

```
[Out] Sec[c + d*x]^3/(3*a^3*d) - (6*Sec[c + d*x]^5)/(5*a^3*d) + (9*Sec[c + d*x]^7
)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) + (5*
Tan[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.223219, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2711, 2607, 270, 2606, 14}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{5\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{9\sec^7(c+dx)}{7a^3d}-\frac{6\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^3*d) - (6*Sec[c + d*x]^5)/(5*a^3*d) + (9*Sec[c + d*x]^7
)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) + (5*
Tan[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(a^3\sec^6(c+dx)\tan^4(c+dx) - 3a^3\sec^5(c+dx)\tan^5(c+dx) + 3a^3\sec^4(c+dx)\tan^6(c+ \right.}{a^6}$$

$$= \frac{\int \sec^6(c+dx)\tan^4(c+dx)\,dx}{a^3} - \frac{\int \sec^3(c+dx)\tan^7(c+dx)\,dx}{a^3} - \frac{3\int \sec^5(c+dx)\tan^5(c-}{a^3}$$

$$= -\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3 dx, x, \sec(c+dx)\right)}{a^3 d} + \frac{\text{Subst}\left(\int x^4\left(1+x^2\right)^2 dx, x, \tan(c+dx)\right)}{a^3 d} -$$

$$= -\frac{\text{Subst}\left(\int \left(-x^2+3x^4-3x^6+x^8\right) dx, x, \sec(c+dx)\right)}{a^3 d} + \frac{\text{Subst}\left(\int \left(x^4+2x^6+x^8\right) dx, x, \text{ta}\right.}{a^3 d}$$

$$= \frac{\sec^3(c+dx)}{3a^3 d} - \frac{6\sec^5(c+dx)}{5a^3 d} + \frac{9\sec^7(c+dx)}{7a^3 d} - \frac{4\sec^9(c+dx)}{9a^3 d} + \frac{\tan^5(c+dx)}{5a^3 d} + \frac{5\tan^7(c}{7a^3}$$

Mathematica [A] time = 0.279077, size = 185, normalized size = 1.46

$$\frac{39168\sin(c+dx) + 837\sin(2(c+dx)) - 28288\sin(3(c+dx)) + 372\sin(4(c+dx)) + 4224\sin(5(c+dx)) - 31\sin(6(c+}{322560d(a\sin(c+dx)+a)^3\left(\cos\left(\frac{1}{2}(c+d\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5376 + 1116*Cos[c + d*x] - 21312*Cos[2*(c + d*x)] + 62*Cos[3*(c + d*x)] +
8448*Cos[4*(c + d*x)] - 186*Cos[5*(c + d*x)] - 704*Cos[6*(c + d*x)] + 39168
*Sin[c + d*x] + 837*Sin[2*(c + d*x)] - 28288*Sin[3*(c + d*x)] + 372*Sin[4*(
c + d*x)] + 4224*Sin[5*(c + d*x)] - 31*Sin[6*(c + d*x)])/(322560*d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a
+ a*Sin[c + d*x])^3)
```

Maple [A] time = 0.141, size = 175, normalized size = 1.4

$$32\,\frac{1}{da^3}\left(-\frac{1}{768\,(\tan(1/2\,dx+c/2)-1)^3} - \frac{1}{512\,(\tan(1/2\,dx+c/2)-1)^2} + \frac{1}{1024\,\tan(1/2\,dx+c/2)-1024} - 1/36\,(\tan(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 32/d/a^3*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2+1/
1024/(tan(1/2*d*x+1/2*c)-1)-1/36/(tan(1/2*d*x+1/2*c)+1)^9+1/8/(tan(1/2*d*x+
1/2*c)+1)^8-13/56/(tan(1/2*d*x+1/2*c)+1)^7+11/48/(tan(1/2*d*x+1/2*c)+1)^6-3
9/320/(tan(1/2*d*x+1/2*c)+1)^5+3/128/(tan(1/2*d*x+1/2*c)+1)^4+1/192/(tan(1/
2*d*x+1/2*c)+1)^3-1/1024/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.06783, size = 543, normalized size = 4.28

$$-\frac{16\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1} + \frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{2\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{27\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{162\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{126\sin(}{(\cos(dx+}\right.}{315\left(a^3 + \frac{6a^3\sin(dx+c)}{\cos(dx+c)+1} + \frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{2}{(c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -16/315*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 162*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 126*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 126*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1)/((a^
3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36
*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x +
 c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c
)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 -
a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A] time = 1.7035, size = 335, normalized size = 2.64

$$\frac{22\cos(dx+c)^6 - 99\cos(dx+c)^4 + 120\cos(dx+c)^2 - 2\left(33\cos(dx+c)^4 - 80\cos(dx+c)^2 + 35\right)\sin(dx+c) - 3}{315\left(3a^3d\cos(dx+c)^5 - 4a^3d\cos(dx+c)^3 + \left(a^3d\cos(dx+c)^5 - 4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/315*(22*cos(d*x + c)^6 - 99*cos(d*x + c)^4 + 120*cos(d*x + c)^2 - 2*(33*c
os(d*x + c)^4 - 80*cos(d*x + c)^2 + 35)*sin(d*x + c) - 35)/(3*a^3*d*cos(d*x
 + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x
+ c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25984, size = 215, normalized size = 1.69

$$\frac{\frac{105\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3} - \frac{315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+2520\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+7140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1638\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8232\tan\left(\frac{1}{2}\right.}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^9}}{10080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/10080*(105*(3*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 5)/(a^3*
(tan(1/2*d*x + 1/2*c) - 1)^3) - (315*tan(1/2*d*x + 1/2*c)^8 + 2520*tan(1/2*
d*x + 1/2*c)^7 + 7140*tan(1/2*d*x + 1/2*c)^6 - 1638*tan(1/2*d*x + 1/2*c)^4
- 8232*tan(1/2*d*x + 1/2*c)^3 - 2988*tan(1/2*d*x + 1/2*c)^2 - 432*tan(1/2*d
*x + 1/2*c) - 13)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^9))/d
```

3.843 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{\tan^7(c+dx)}{a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{\sec^7(c+dx)}{a^3d}+\frac{3\sec^5(c+dx)}{5a^3d}$$

```
[Out] (3*Sec[c + d*x]^5)/(5*a^3*d) - Sec[c + d*x]^7/(a^3*d) + (4*Sec[c + d*x]^9)/
(9*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3*d) - Tan[c + d*x]^7/(a^3*d) - (4*Tan[
c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.33538, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2875, 2873, 2606, 14, 2607, 270}

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{\tan^7(c+dx)}{a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{\sec^7(c+dx)}{a^3d}+\frac{3\sec^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Sec[c + d*x]^5)/(5*a^3*d) - Sec[c + d*x]^7/(a^3*d) + (4*Sec[c + d*x]^9)/
(9*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3*d) - Tan[c + d*x]^7/(a^3*d) - (4*Tan[
c + d*x]^9)/(9*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \sec^7(c+dx)(a-a\sin(c+dx))^3\tan^3(c+dx)\,dx}{a^6}\\ &= \frac{\int \left(a^3\sec^7(c+dx)\tan^3(c+dx)-3a^3\sec^6(c+dx)\tan^4(c+dx)+3a^3\sec^5(c+dx)\tan^5\right.}{a^6}\\ &= \frac{\int \sec^7(c+dx)\tan^3(c+dx)\,dx}{a^3}-\frac{\int \sec^4(c+dx)\tan^6(c+dx)\,dx}{a^3}-\frac{3\int \sec^6(c+dx)\tan}{a^3}\\ &= \frac{\text{Subst}\left(\int x^6\left(-1+x^2\right)dx,x,\sec(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^6\left(1+x^2\right)dx,x,\tan(c+dx)\right)}{a^3d}+\\ &= \frac{\text{Subst}\left(\int \left(-x^6+x^8\right)dx,x,\sec(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int \left(x^6+x^8\right)dx,x,\tan(c+dx)\right)}{a^3d}+3\\ &= \frac{3\sec^5(c+dx)}{5a^3d}-\frac{\sec^7(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{\tan^7(c+dx)}{a^3d}-4\tan\end{aligned}$$

Mathematica [A] time = 0.315734, size = 185, normalized size = 1.76

$$\frac{4608\sin(c+dx)-1323\sin(2(c+dx))-128\sin(3(c+dx))-588\sin(4(c+dx))+384\sin(5(c+dx))+49\sin(6(c+dx))}{46080d(a\sin(c+dx)+a)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5376 - 1764*Cos[c + d*x] - 4032*Cos[2*(c + d*x)] - 98*Cos[3*(c + d*x)] + 7
68*Cos[4*(c + d*x)] + 294*Cos[5*(c + d*x)] - 64*Cos[6*(c + d*x)] + 4608*Sin
[c + d*x] - 1323*Sin[2*(c + d*x)] - 128*Sin[3*(c + d*x)] - 588*Sin[4*(c + d
*x)] + 384*Sin[5*(c + d*x)] + 49*Sin[6*(c + d*x)])/(46080*d*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*Sin
[c + d*x])^3)
```

Maple [A] time = 0.131, size = 190, normalized size = 1.8

$$16\frac{1}{da^3}\left(-\frac{1}{384\,(\tan(1/2\,dx+c/2)-1)^3}-\frac{1}{256\,(\tan(1/2\,dx+c/2)-1)^2}-\frac{1}{512\,\tan(1/2\,dx+c/2)-512}+1/18\,(\tan(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 16/d/a^3*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2-1/
512/(tan(1/2*d*x+1/2*c)-1)+1/18/(tan(1/2*d*x+1/2*c)+1)^9-1/4/(tan(1/2*d*x+1
/2*c)+1)^8+1/2/(tan(1/2*d*x+1/2*c)+1)^7-7/12/(tan(1/2*d*x+1/2*c)+1)^6+67/16
0/(tan(1/2*d*x+1/2*c)+1)^5-11/64/(tan(1/2*d*x+1/2*c)+1)^4+5/192/(tan(1/2*d*
x+1/2*c)+1)^3+1/128/(tan(1/2*d*x+1/2*c)+1)^2+1/512/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.09962, size = 570, normalized size = 5.43

$$\frac{4\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{18\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{84\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\cdots\right.}{45\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/45*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 18*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 18*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 84*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 54*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 45*sin(d*x +
c)^8/(cos(d*x + c) + 1)^8 + 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1
) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^1
2)*d)
```

Fricas [A] time = 1.68134, size = 323, normalized size = 3.08

$$\frac{2\cos(dx+c)^6-9\cos(dx+c)^4+15\cos(dx+c)^2-\left(6\cos(dx+c)^4-5\cos(dx+c)^2+5\right)\sin(dx+c)-10}{45\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/45*(2*cos(d*x + c)^6 - 9*cos(d*x + c)^4 + 15*cos(d*x + c)^2 - (6*cos(d*x
+ c)^4 - 5*cos(d*x + c)^2 + 5)*sin(d*x + c) - 10)/(3*a^3*d*cos(d*x + c)^5 -
 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*s
in(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)

[Out] Timed out

Giac [A] time = 1.27307, size = 217, normalized size = 2.07

$$-\frac{\frac{15\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+540\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+3120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+5940\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+8298\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+6372\tan\left(\frac{1}{2}d\right.}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{9}}}{1440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/1440*(15*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^8 + 540*tan(1/2*d*x + 1/2*c)^7 + 3120*tan(1/2*d*x + 1/2*c)^6 + 5940*tan(1/2*d*x + 1/2*c)^5 + 8298*tan(1/2*d*x + 1/2*c)^4 + 6372*tan(1/2*d*x + 1/2*c)^3 + 3528*tan(1/2*d*x + 1/2*c)^2 + 972*tan(1/2*d*x + 1/2*c) + 113)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^9))/d

3.844 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{9\tan^7(c+dx)}{7a^3d}+\frac{6\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{5\sec^7(c+dx)}{7a^3d}-\frac{\sec^5(c+dx)}{5a^3d}$$

```
[Out] -Sec[c + d*x]^5/(5*a^3*d) + (5*Sec[c + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^
9)/(9*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) + (6*Tan[c + d*x]^5)/(5*a^3*d) + (9
*Tan[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.362324, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2875, 2873, 2607, 270, 2606, 14}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{9\tan^7(c+dx)}{7a^3d}+\frac{6\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{5\sec^7(c+dx)}{7a^3d}-\frac{\sec^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Sec[c + d*x]^5/(5*a^3*d) + (5*Sec[c + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^
9)/(9*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) + (6*Tan[c + d*x]^5)/(5*a^3*d) + (9
*Tan[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^8(c+dx)(a-a\sin(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^8(c+dx)\tan^2(c+dx)-3a^3\sec^7(c+dx)\tan^3(c+dx)+3a^3\sec^6(c+dx)\tan\right.}{a^6}$$
$$= \frac{\int \sec^8(c+dx)\tan^2(c+dx)\,dx}{a^3}-\frac{\int \sec^5(c+dx)\tan^5(c+dx)\,dx}{a^3}-\frac{3\int \sec^7(c+dx)\,\mathrm{ta}}{a^3}$$
$$= -\frac{\mathrm{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\mathrm{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx,x,\tan(c+d\right.}{a^3d}$$
$$= -\frac{\mathrm{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\mathrm{Subst}\left(\int \left(x^2+3x^4+3x^6+x^8\right)dx,\right.}{a^3d}$$
$$= -\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\tan^3(c+dx)}{3a^3d}+\frac{6\tan^5(c+dx)}{5a^3d}+$$

Mathematica [A] time = 0.328035, size = 185, normalized size = 1.46

$$\frac{73728\sin(c+dx)-7263\sin(2(c+dx))+512\sin(3(c+dx))-3228\sin(4(c+dx))-1536\sin(5(c+dx))+269\sin(6(c+}{322560d(a\sin(c+dx)+a)^3\left(\cos\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (32256 - 9684*Cos[c + d*x] - 6912*Cos[2*(c + d*x)] - 538*Cos[3*(c + d*x)] -
 3072*Cos[4*(c + d*x)] + 1614*Cos[5*(c + d*x)] + 256*Cos[6*(c + d*x)] + 737
28*Sin[c + d*x] - 7263*Sin[2*(c + d*x)] + 512*Sin[3*(c + d*x)] - 3228*Sin[4
*(c + d*x)] - 1536*Sin[5*(c + d*x)] + 269*Sin[6*(c + d*x)])/(322560*d*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*
(a + a*Sin[c + d*x])^3)
```

Maple [A] time = 0.125, size = 190, normalized size = 1.5

$$8\,\frac{1}{da^3}\left(-\frac{1}{192\,(\tan(1/2\,dx+c/2)-1)^3}-\frac{1}{128\,(\tan(1/2\,dx+c/2)-1)^2}-\frac{3}{256\,\tan(1/2\,dx+c/2)-256}-1/9\,(\tan(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 8/d/a^3*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2-3/2
56/(tan(1/2*d*x+1/2*c)-1)-1/9/(tan(1/2*d*x+1/2*c)+1)^9+1/2/(tan(1/2*d*x+1/2
*c)+1)^8-15/14/(tan(1/2*d*x+1/2*c)+1)^7+17/12/(tan(1/2*d*x+1/2*c)+1)^6-99/8
0/(tan(1/2*d*x+1/2*c)+1)^5+23/32/(tan(1/2*d*x+1/2*c)+1)^4-1/4/(tan(1/2*d*x+
1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*c)+1)^2+3/256/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.16055, size = 597, normalized size = 4.7

$$\frac{4\left(\frac{66\sin(dx+c)}{\cos(dx+c)+1} + \frac{132\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{232\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{108\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{84\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{504\sin}{(\cos(dx}\right.}{315\left(a^3 + \frac{6a^3\sin(dx+c)}{\cos(dx+c)+1} + \frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/315*(66*sin(d*x + c)/(cos(d*x + c) + 1) + 132*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 232*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 18*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 108*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 84*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 504*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 315*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 210*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
 + 11)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*
sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c)
 + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A] time = 1.6539, size = 333, normalized size = 2.62

$$-\frac{8\cos(dx+c)^6 - 36\cos(dx+c)^4 + 15\cos(dx+c)^2 - 2\left(12\cos(dx+c)^4 - 10\cos(dx+c)^2 - 35\right)\sin(dx+c) + 35}{315\left(3a^3d\cos(dx+c)^5 - 4a^3d\cos(dx+c)^3 + \left(a^3d\cos(dx+c)^5 - 4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/315*(8*cos(d*x + c)^6 - 36*cos(d*x + c)^4 + 15*cos(d*x + c)^2 - 2*(12*co
s(d*x + c)^4 - 10*cos(d*x + c)^2 - 35)*sin(d*x + c) + 35)/(3*a^3*d*cos(d*x
+ c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x +
 c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29245, size = 232, normalized size = 1.83

$$-\frac{\frac{105\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{945\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+10080\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+23940\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+42840\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+41958\tan\cdots}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\cdots\right.}}{10080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/10080*(105*(9*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 7)/(a^3
*(tan(1/2*d*x + 1/2*c) - 1)^3) - (945*tan(1/2*d*x + 1/2*c)^8 + 10080*tan(1/
2*d*x + 1/2*c)^7 + 23940*tan(1/2*d*x + 1/2*c)^6 + 42840*tan(1/2*d*x + 1/2*c
)^5 + 41958*tan(1/2*d*x + 1/2*c)^4 + 32592*tan(1/2*d*x + 1/2*c)^3 + 14148*t
an(1/2*d*x + 1/2*c)^2 + 5112*tan(1/2*d*x + 1/2*c) + 673)/(a^3*(tan(1/2*d*x
+ 1/2*c) + 1)^9))/d
```

3.845 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=123

$$\frac{4\tan^3(c+dx)}{63a^3d}+\frac{4\tan(c+dx)}{21a^3d}-\frac{\sec^3(c+dx)}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}+\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

```
[Out] Sec[c + d*x]^3/(9*d*(a + a*Sin[c + d*x])^3) - Sec[c + d*x]^3/(21*a*d*(a + a
*Sin[c + d*x])^2) - Sec[c + d*x]^3/(21*d*(a^3 + a^3*Sin[c + d*x])) + (4*Tan
[c + d*x])/(21*a^3*d) + (4*Tan[c + d*x]^3)/(63*a^3*d)
```

Rubi [A] time = 0.160907, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2859, 2672, 3767}

$$\frac{4\tan^3(c+dx)}{63a^3d}+\frac{4\tan(c+dx)}{21a^3d}-\frac{\sec^3(c+dx)}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}+\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]^3/(9*d*(a + a*Sin[c + d*x])^3) - Sec[c + d*x]^3/(21*a*d*(a + a
*Sin[c + d*x])^2) - Sec[c + d*x]^3/(21*d*(a^3 + a^3*Sin[c + d*x])) + (4*Tan
[c + d*x])/(21*a^3*d) + (4*Tan[c + d*x]^3)/(63*a^3*d)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} + \frac{\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{3a}$$

$$= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} + \frac{5\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{21a^2}$$

$$= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} - \frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)} + \frac{4\int \sec \ldots}{\ldots}$$

$$= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} - \frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)} - \frac{4\,\mathrm{Subs}\ldots}{\ldots}$$

$$= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} - \frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)} + \frac{4\tan(\ldots}{21\ldots}$$

Mathematica [A] time = 0.276557, size = 185, normalized size = 1.5

$$\frac{9216\sin(c+dx)+675\sin(2(c+dx))+512\sin(3(c+dx))+300\sin(4(c+dx))-1536\sin(5(c+dx))-25\sin(6(c+dx))\ldots}{64512d(a\sin(c+dx)+a)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (10752 + 900*Cos[c + d*x] - 6912*Cos[2*(c + d*x)] + 50*Cos[3*(c + d*x)] - 3
072*Cos[4*(c + d*x)] - 150*Cos[5*(c + d*x)] + 256*Cos[6*(c + d*x)] + 9216*S
in[c + d*x] + 675*Sin[2*(c + d*x)] + 512*Sin[3*(c + d*x)] + 300*Sin[4*(c +
d*x)] - 1536*Sin[5*(c + d*x)] - 25*Sin[6*(c + d*x)])/(64512*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*S
in[c + d*x])^3)
```

Maple [A] time = 0.113, size = 190, normalized size = 1.5

$$4\,\frac{1}{da^3}\left(-\frac{1}{96\,(\tan(1/2\,dx+c/2)-1)^3}-\frac{1}{64\,(\tan(1/2\,dx+c/2)-1)^2}-\frac{5}{128\,\tan(1/2\,dx+c/2)-128}+2/9\,(\tan(1/2\,dx+c\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/d/a^3*(-1/96/(tan(1/2*d*x+1/2*c)-1)^3-1/64/(tan(1/2*d*x+1/2*c)-1)^2-5/128
/(tan(1/2*d*x+1/2*c)-1)+2/9/(tan(1/2*d*x+1/2*c)+1)^9-1/(tan(1/2*d*x+1/2*c)+
1)^8+16/7/(tan(1/2*d*x+1/2*c)+1)^7-10/3/(tan(1/2*d*x+1/2*c)+1)^6+27/8/(tan(
1/2*d*x+1/2*c)+1)^5-39/16/(tan(1/2*d*x+1/2*c)+1)^4+59/48/(tan(1/2*d*x+1/2*c
)+1)^3-13/32/(tan(1/2*d*x+1/2*c)+1)^2+5/128/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.18718, size = 597, normalized size = 4.85

$$\frac{2\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{75\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{128\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{162\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{42\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{189\sin(dx+\ldots}{(\cos(dx+c)+\ldots}\right.}{63\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a^3\,\mathrm{s}\ldots}{(\cos(\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/63*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 75*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 128*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 162*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 42*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 189*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 126*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 + 63*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin
(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) +
1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A] time = 1.68669, size = 328, normalized size = 2.67

$$-\frac{8\cos(dx+c)^6-36\cos(dx+c)^4+15\cos(dx+c)^2-\left(24\cos(dx+c)^4-20\cos(dx+c)^2-7\right)\sin(dx+c)+14}{63\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/63*(8*cos(d*x + c)^6 - 36*cos(d*x + c)^4 + 15*cos(d*x + c)^2 - (24*cos(d
*x + c)^4 - 20*cos(d*x + c)^2 - 7)*sin(d*x + c) + 14)/(3*a^3*d*cos(d*x + c)
^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^
3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31876, size = 232, normalized size = 1.89

$$-\frac{\frac{21\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-756\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-4200\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-11340\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-14994\,\text{ta}}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}}{2016d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2016*(21*(15*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 13)/(a^3
*(tan(1/2*d*x + 1/2*c) - 1)^3) - (315*tan(1/2*d*x + 1/2*c)^8 - 756*tan(1/2*
d*x + 1/2*c)^7 - 4200*tan(1/2*d*x + 1/2*c)^6 - 11340*tan(1/2*d*x + 1/2*c)^5
 - 14994*tan(1/2*d*x + 1/2*c)^4 - 13356*tan(1/2*d*x + 1/2*c)^3 - 6768*tan(1
/2*d*x + 1/2*c)^2 - 2196*tan(1/2*d*x + 1/2*c) - 209)/(a^3*(tan(1/2*d*x + 1/
2*c) + 1)^9))/d
```

3.846 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{15\tan^7(c+dx)}{7a^3d}-\frac{21\tan^5(c+dx)}{5a^3d}-\frac{13\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\sec^7(c+}{7a^3d}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + Sec[c + d*x]/(a^3*d) + Sec[c + d*x]^3/(3
*a^3*d) + Sec[c + d*x]^5/(5*a^3*d) + Sec[c + d*x]^7/(7*a^3*d) + (4*Sec[c +
d*x]^9)/(9*a^3*d) - (3*Tan[c + d*x])/(a^3*d) - (13*Tan[c + d*x]^3)/(3*a^3*d
) - (21*Tan[c + d*x]^5)/(5*a^3*d) - (15*Tan[c + d*x]^7)/(7*a^3*d) - (4*Tan[
c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.360108, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2606, 30, 2607, 270}

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{15\tan^7(c+dx)}{7a^3d}-\frac{21\tan^5(c+dx)}{5a^3d}-\frac{13\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\sec^7(c+}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + Sec[c + d*x]/(a^3*d) + Sec[c + d*x]^3/(3
*a^3*d) + Sec[c + d*x]^5/(5*a^3*d) + Sec[c + d*x]^7/(7*a^3*d) + (4*Sec[c +
d*x]^9)/(9*a^3*d) - (3*Tan[c + d*x])/(a^3*d) - (13*Tan[c + d*x]^3)/(3*a^3*d
) - (21*Tan[c + d*x]^5)/(5*a^3*d) - (15*Tan[c + d*x]^7)/(7*a^3*d) - (4*Tan[
c + d*x]^9)/(9*a^3*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc(c+dx)\sec^{10}(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-3a^3\sec^{10}(c+dx)+a^3\csc(c+dx)\sec^{10}(c+dx)+3a^3\sec^9(c+dx)\tan(c+dx)\right.}{a^6}$$
$$= \frac{\int \csc(c+dx)\sec^{10}(c+dx)\,dx}{a^3} - \frac{\int \sec^8(c+dx)\tan^2(c+dx)\,dx}{a^3} - \frac{3\int \sec^{10}(c+dx)}{a^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^3d} - \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx, x, \tan(c+dx)\right)}{a^3d} + $$
$$= \frac{\sec^9(c+dx)}{3a^3d} - \frac{3\tan(c+dx)}{a^3d} - \frac{4\tan^3(c+dx)}{a^3d} - \frac{18\tan^5(c+dx)}{5a^3d} - \frac{12\tan^7(c+dx)}{7a^3d}$$
$$= \frac{\sec(c+dx)}{a^3d} + \frac{\sec^3(c+dx)}{3a^3d} + \frac{\sec^5(c+dx)}{5a^3d} + \frac{\sec^7(c+dx)}{7a^3d} + \frac{4\sec^9(c+dx)}{9a^3d} - $$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\sec(c+dx)}{a^3d} + \frac{\sec^3(c+dx)}{3a^3d} + \frac{\sec^5(c+dx)}{5a^3d} + \frac{\sec^7(c+dx)}{7a^3d}$$

Mathematica [A] time = 1.39842, size = 204, normalized size = 1.09

$$\frac{322560\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 322560\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \frac{196992\sin(c+dx)-383157\sin(2(c+dx))+211648\sin(3(c+dx))-170292}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-322560*Log[Cos[(c + d*x)/2]] + 322560*Log[Sin[(c + d*x)/2]] + (357504 - 5
10876*Cos[c + d*x] + 317952*Cos[2*(c + d*x)] - 28382*Cos[3*(c + d*x)] + 203
52*Cos[4*(c + d*x)] + 85146*Cos[5*(c + d*x)] - 11776*Cos[6*(c + d*x)] + 196
992*Sin[c + d*x] - 383157*Sin[2*(c + d*x)] + 211648*Sin[3*(c + d*x)] - 1702
92*Sin[4*(c + d*x)] + 50496*Sin[5*(c + d*x)] + 14191*Sin[6*(c + d*x)])/((Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
9))/(322560*a^3*d)
```

Maple [A] time = 0.158, size = 271, normalized size = 1.5

$$-\frac{1}{24\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - \frac{1}{16\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2} - \frac{9}{32\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{8}{9\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/24/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-1/16/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2-9/
32/d/a^3/(tan(1/2*d*x+1/2*c)-1)+8/9/d/a^3/(tan(1/2*d*x+1/2*c)+1)^9-4/d/a^3/
(tan(1/2*d*x+1/2*c)+1)^8+72/7/d/a^3/(tan(1/2*d*x+1/2*c)+1)^7-52/3/d/a^3/(ta
n(1/2*d*x+1/2*c)+1)^6+219/10/d/a^3/(tan(1/2*d*x+1/2*c)+1)^5-83/4/d/a^3/(tan
(1/2*d*x+1/2*c)+1)^4+193/12/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3-75/8/d/a^3/(tan(
1/2*d*x+1/2*c)+1)^2+201/32/d/a^3/(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*
d*x+1/2*c))
```

Maxima [B] time = 1.10388, size = 686, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/315*(2*(3063*sin(d*x + c)/(cos(d*x + c) + 1) + 4866*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 1289*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 11736*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 10566*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 52
92*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 13482*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 6300*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2625*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 - 3150*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 945*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 + 668)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x
+ c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a
^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a
^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c
) + 1)^12) + 315*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 2.0486, size = 676, normalized size = 3.61

$$\frac{736 \cos(dx+c)^6 - 1422 \cos(dx+c)^4 - 510 \cos(dx+c)^2 - 315 \left(3 \cos(dx+c)^5 - 4 \cos(dx+c)^3 + \left(\cos(dx+c)^5 - 4 \cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/630*(736*cos(d*x + c)^6 - 1422*cos(d*x + c)^4 - 510*cos(d*x + c)^2 - 315*
(3*cos(d*x + c)^5 - 4*cos(d*x + c)^3 + (cos(d*x + c)^5 - 4*cos(d*x + c)^3)*
sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 315*(3*cos(d*x + c)^5 - 4*cos(d
*x + c)^3 + (cos(d*x + c)^5 - 4*cos(d*x + c)^3)*sin(d*x + c))*log(-1/2*cos(
d*x + c) + 1/2) - 2*(789*cos(d*x + c)^4 + 235*cos(d*x + c)^2 + 35)*sin(d*x
+ c) - 140)/(3*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d
*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24968, size = 252, normalized size = 1.35

$$\frac{\frac{10080 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^3} - \frac{105\left(27 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-48 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+25\right)}{a^3\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)^3} + \frac{63315 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8+412020 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7+1273440 \tan\left(\frac{1}{2} dx\right.}{}}{10080}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/10080*(10080*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 105*(27*tan(1/2*d*x + 1
/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 25)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^3)
+ (63315*tan(1/2*d*x + 1/2*c)^8 + 412020*tan(1/2*d*x + 1/2*c)^7 + 1273440*t
an(1/2*d*x + 1/2*c)^6 + 2324700*tan(1/2*d*x + 1/2*c)^5 + 2731302*tan(1/2*d*
x + 1/2*c)^4 + 2097228*tan(1/2*d*x + 1/2*c)^3 + 1032552*tan(1/2*d*x + 1/2*c
)^2 + 297828*tan(1/2*d*x + 1/2*c) + 40127)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^
9))/d
```

3.847 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=200

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{17\tan^7(c+dx)}{7a^3d}+\frac{28\tan^5(c+dx)}{5a^3d}+\frac{22\tan^3(c+dx)}{3a^3d}+\frac{8\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (3*Sec[c + d*x])
/(a^3*d) - Sec[c + d*x]^3/(a^3*d) - (3*Sec[c + d*x]^5)/(5*a^3*d) - (3*Sec[c
 + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + (8*Tan[c + d*x])/(a^3
*d) + (22*Tan[c + d*x]^3)/(3*a^3*d) + (28*Tan[c + d*x]^5)/(5*a^3*d) + (17*T
an[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.393419, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2875, 2873, 3767, 2622, 302, 207, 2620, 270, 2606, 30}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{17\tan^7(c+dx)}{7a^3d}+\frac{28\tan^5(c+dx)}{5a^3d}+\frac{22\tan^3(c+dx)}{3a^3d}+\frac{8\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (3*Sec[c + d*x])
/(a^3*d) - Sec[c + d*x]^3/(a^3*d) - (3*Sec[c + d*x]^5)/(5*a^3*d) - (3*Sec[c
 + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + (8*Tan[c + d*x])/(a^3
*d) + (22*Tan[c + d*x]^3)/(3*a^3*d) + (28*Tan[c + d*x]^5)/(5*a^3*d) + (17*T
an[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^2(c+dx)\sec^{10}(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(3a^3\sec^{10}(c+dx) - 3a^3\csc(c+dx)\sec^{10}(c+dx) + a^3\csc^2(c+dx)\sec^{10}(c+dx)\right.}{a^6}$$

$$= \frac{\int \csc^2(c+dx)\sec^{10}(c+dx)\,dx}{a^3} - \frac{\int \sec^9(c+dx)\tan(c+dx)\,dx}{a^3} + \frac{3\int \sec^{10}(c+dx}{a^3}$$

$$= -\frac{\text{Subst}\left(\int x^8\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int \frac{(1+x^2)^5}{x^2}\,dx, x, \tan(c+dx)\right)}{a^3d} - \frac{3\,\text{Subst}}{}$$

$$= -\frac{\sec^9(c+dx)}{9a^3d} + \frac{3\tan(c+dx)}{a^3d} + \frac{4\tan^3(c+dx)}{a^3d} + \frac{18\tan^5(c+dx)}{5a^3d} + \frac{12\tan^7(c+}{7a^3d}$$

$$= -\frac{\cot(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\sec^3(c+dx)}{a^3d} - \frac{3\sec^5(c+dx)}{5a^3d} - \frac{3\sec^7(c+dx)}{7a^3d} - 4$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{\cot(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\sec^3(c+dx)}{a^3d} - \frac{3\sec^5(c+}{5a^3d}$$

Mathematica [A] time = 0.653642, size = 230, normalized size = 1.15

$$-1935360\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+1935360\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\csc(c+dx)(-707328\sin(c+dx)+1364182\sin(2(c+dx))-1161600\sin(3(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (1935360*Log[Cos[(c + d*x)/2]] - 1935360*Log[Sin[(c + d*x)/2]] + (Csc[c + d
*x]*(-590976 + 1083321*Cos[c + d*x] - 653248*Cos[2*(c + d*x)] - 601845*Cos[
3*(c + d*x)] + 340096*Cos[4*(c + d*x)] - 521599*Cos[5*(c + d*x)] + 259008*C
os[6*(c + d*x)] + 40123*Cos[7*(c + d*x)] - 707328*Sin[c + d*x] + 1364182*Si
n[2*(c + d*x)] - 1161600*Sin[3*(c + d*x)] + 320984*Sin[4*(c + d*x)] - 32934
4*Sin[5*(c + d*x)] - 240738*Sin[6*(c + d*x)] + 53248*Sin[7*(c + d*x)]))/((C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^9))/(645120*a^3*d)
```

Maple [A] time = 0.171, size = 308, normalized size = 1.5

$$\frac{1}{2da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{24da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{16da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{11}{32da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{8}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/24/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-1/16/d/a^3
/(tan(1/2*d*x+1/2*c)-1)^2-11/32/d/a^3/(tan(1/2*d*x+1/2*c)-1)-8/9/d/a^3/(tan
(1/2*d*x+1/2*c)+1)^9+4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^8-76/7/d/a^3/(tan(1/2*d
*x+1/2*c)+1)^7+58/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^6-267/10/d/a^3/(tan(1/2*d*
x+1/2*c)+1)^5+111/4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^4-25/d/a^3/(tan(1/2*d*x+1/
2*c)+1)^3+67/4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2-501/32/d/a^3/(tan(1/2*d*x+1/2
*c)+1)-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.15626, size = 765, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/630*((8786*sin(d*x + c)/(cos(d*x + c) + 1) + 35076*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 43062*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 41753*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 152172*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
99072*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 93324*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 157689*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 44730*sin(d*x + c
)^9/(cos(d*x + c) + 1)^9 - 50820*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 42
210*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 10395*sin(d*x + c)^12/(cos(d*x
+ c) + 1)^12 + 315)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 6*a^3*sin(d*x +
```

```
c)^2/(cos(d*x + c) + 1)^2 + 12*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*
a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 27*a^3*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 - 36*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 36*a^3*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + 27*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2*a
^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 12*a^3*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 6*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - a^3*sin(d*x +
c)^13/(cos(d*x + c) + 1)^13) + 1890*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 315*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.87536, size = 794, normalized size = 3.97

$$\frac{8094\cos(dx+c)^6 - 9484\cos(dx+c)^4 + 620\cos(dx+c)^2 + 945\left(\cos(dx+c)^7 - 5\cos(dx+c)^5 + 4\cos(dx+c)^3 - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/630*(8094*cos(d*x + c)^6 - 9484*cos(d*x + c)^4 + 620*cos(d*x + c)^2 + 945
*(cos(d*x + c)^7 - 5*cos(d*x + c)^5 + 4*cos(d*x + c)^3 - (3*cos(d*x + c)^5
- 4*cos(d*x + c)^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 945*(cos(d*
x + c)^7 - 5*cos(d*x + c)^5 + 4*cos(d*x + c)^3 - (3*cos(d*x + c)^5 - 4*cos(
d*x + c)^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(1664*cos(d*x +
c)^6 - 4653*cos(d*x + c)^4 + 285*cos(d*x + c)^2 + 35)*sin(d*x + c) + 140)/(
a^3*d*cos(d*x + c)^7 - 5*a^3*d*cos(d*x + c)^5 + 4*a^3*d*cos(d*x + c)^3 - (3
*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3084, size = 311, normalized size = 1.56

$$-\frac{\frac{30240\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{5040\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3} - \frac{5040\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)} + \frac{105\left(33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-60\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+31\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3} + \frac{157815\tan\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/10080*(30240*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5040*tan(1/2*d*x + 1/2
*c)/a^3 - 5040*(6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)) + 10
```

```
5*(33*tan(1/2*d*x + 1/2*c)^2 - 60*tan(1/2*d*x + 1/2*c) + 31)/(a^3*(tan(1/2*
d*x + 1/2*c) - 1)^3) + (157815*tan(1/2*d*x + 1/2*c)^8 + 1093680*tan(1/2*d*x
 + 1/2*c)^7 + 3488940*tan(1/2*d*x + 1/2*c)^6 + 6524280*tan(1/2*d*x + 1/2*c)
^5 + 7788186*tan(1/2*d*x + 1/2*c)^4 + 6052704*tan(1/2*d*x + 1/2*c)^3 + 2995
596*tan(1/2*d*x + 1/2*c)^2 + 864504*tan(1/2*d*x + 1/2*c) + 113591)/(a^3*(ta
n(1/2*d*x + 1/2*c) + 1)^9))/d
```

3.848 $\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$\frac{8\tan^{11}(c+dx)}{11a^4d}+\frac{16\tan^9(c+dx)}{9a^4d}+\frac{9\tan^7(c+dx)}{7a^4d}+\frac{\tan^5(c+dx)}{5a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{20\sec^9(c+dx)}{9a^4d}-\frac{16\sec^7(c}{7a^4d}$$

```
[Out] (4*Sec[c + d*x]^5)/(5*a^4*d) - (16*Sec[c + d*x]^7)/(7*a^4*d) + (20*Sec[c +
d*x]^9)/(9*a^4*d) - (8*Sec[c + d*x]^11)/(11*a^4*d) + Tan[c + d*x]^5/(5*a^4*
d) + (9*Tan[c + d*x]^7)/(7*a^4*d) + (16*Tan[c + d*x]^9)/(9*a^4*d) + (8*Tan[
c + d*x]^11)/(11*a^4*d)
```

Rubi [A] time = 0.31329, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2711, 2607, 270, 2606, 14}

$$\frac{8\tan^{11}(c+dx)}{11a^4d}+\frac{16\tan^9(c+dx)}{9a^4d}+\frac{9\tan^7(c+dx)}{7a^4d}+\frac{\tan^5(c+dx)}{5a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{20\sec^9(c+dx)}{9a^4d}-\frac{16\sec^7(c}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (4*Sec[c + d*x]^5)/(5*a^4*d) - (16*Sec[c + d*x]^7)/(7*a^4*d) + (20*Sec[c +
d*x]^9)/(9*a^4*d) - (8*Sec[c + d*x]^11)/(11*a^4*d) + Tan[c + d*x]^5/(5*a^4*
d) + (9*Tan[c + d*x]^7)/(7*a^4*d) + (16*Tan[c + d*x]^9)/(9*a^4*d) + (8*Tan[
c + d*x]^11)/(11*a^4*d)
```


Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(a^4\sec^8(c+dx)\tan^4(c+dx) - 4a^4\sec^7(c+dx)\tan^5(c+dx) + 6a^4\sec^6(c+dx)\tan^6(c+\cdots}{a^8}$$

$$= \frac{\int \sec^8(c+dx)\tan^4(c+dx)\,dx}{a^4} + \frac{\int \sec^4(c+dx)\tan^8(c+dx)\,dx}{a^4} - \frac{4\int \sec^7(c+dx)\tan^5(c\cdots}{a^4}$$

$$= \frac{\text{Subst}\left(\int x^8\left(1+x^2\right)dx, x, \tan(c+dx)\right)}{a^4 d} + \frac{\text{Subst}\left(\int x^4\left(1+x^2\right)^3 dx, x, \tan(c+dx)\right)}{a^4 d} - \frac{4\,\text{Su}\cdots}{\cdots}$$

$$= \frac{\text{Subst}\left(\int \left(x^8+x^{10}\right)dx, x, \tan(c+dx)\right)}{a^4 d} + \frac{\text{Subst}\left(\int \left(x^4+3x^6+3x^8+x^{10}\right)dx, x, \tan(c+d\cdots}{a^4 d}$$

$$= \frac{4\sec^5(c+dx)}{5a^4d} - \frac{16\sec^7(c+dx)}{7a^4d} + \frac{20\sec^9(c+dx)}{9a^4d} - \frac{8\sec^{11}(c+dx)}{11a^4d} + \frac{\tan^5(c+dx)}{5a^4d} + \frac{9\,\text{ta}\cdots}{\cdots}$$

Mathematica [A] time = 0.461469, size = 166, normalized size = 1.14

$$\frac{\sec^3(c+dx)(501600\sin(c+dx) - 70136\sin(2(c+dx)) - 200288\sin(3(c+dx)) - 25504\sin(4(c+dx)) + 48800\sin(5(c\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(168960 - 78903*Cos[c + d*x] - 183040*Cos[2*(c + d*x)] + 87
67*Cos[3*(c + d*x)] + 62464*Cos[4*(c + d*x)] + 19925*Cos[5*(c + d*x)] - 156
16*Cos[6*(c + d*x)] - 797*Cos[7*(c + d*x)] + 501600*Sin[c + d*x] - 70136*Si
n[2*(c + d*x)] - 200288*Sin[3*(c + d*x)] - 25504*Sin[4*(c + d*x)] + 48800*S
in[5*(c + d*x)] + 6376*Sin[6*(c + d*x)] - 1952*Sin[7*(c + d*x)]))/(3548160*
a^4*d*(1 + Sin[c + d*x])^4)
```

Maple [A] time = 0.135, size = 190, normalized size = 1.3

$$32\,\frac{1}{da^4}\left(-\frac{1}{1536\,(\tan(1/2\,dx+c/2)-1)^3} - \frac{1}{1024\,(\tan(1/2\,dx+c/2)-1)^2} - 1/22\,(\tan(1/2\,dx+c/2)+1)^{-11} + 1/4\,(\tan(1/\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 32/d/a^4*(-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(1/2*d*x+1/2*c)-1)^2-
1/22/(tan(1/2*d*x+1/2*c)+1)^11+1/4/(tan(1/2*d*x+1/2*c)+1)^10-11/18/(tan(1/2
*d*x+1/2*c)+1)^9+7/8/(tan(1/2*d*x+1/2*c)+1)^8-179/224/(tan(1/2*d*x+1/2*c)+1
)^7+89/192/(tan(1/2*d*x+1/2*c)+1)^6-49/320/(tan(1/2*d*x+1/2*c)+1)^5+1/64/(t
an(1/2*d*x+1/2*c)+1)^4+7/1536/(tan(1/2*d*x+1/2*c)+1)^3+1/1024/(tan(1/2*d*x+
1/2*c)+1)^2)
```

Maxima [B] time = 1.21688, size = 659, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 32/3465*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 50*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 64*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 22*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 517*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 726*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 1650*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 924*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 693*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 + 2)/((a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 25*a^4*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 32*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 11*a
^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 88*a^4*sin(d*x + c)^5/(cos(d*x + c
) + 1)^5 - 99*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 99*a^4*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 + 88*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 11*a
^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 32*a^4*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 25*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^4*sin(d*x
 + c)^13/(cos(d*x + c) + 1)^13 - a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14)
*d)
```

Fricas [A] time = 1.71831, size = 409, normalized size = 2.82

$$-\frac{488 \cos(dx+c)^6 - 1220 \cos(dx+c)^4 + 1120 \cos(dx+c)^2 + \left(122 \cos(dx+c)^6 - 915 \cos(dx+c)^4 + 1400 \cos(d\cdots\right.}{3465 \left(a^4 d \cos(dx+c)^7 - 8 a^4 d \cos(dx+c)^5 + 8 a^4 d \cos(dx+c)^3 - 4 \left(a^4 d \cos(dx+c)^5 - 2 a^4 d \cos(dx \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/3465*(488*cos(d*x + c)^6 - 1220*cos(d*x + c)^4 + 1120*cos(d*x + c)^2 + (
122*cos(d*x + c)^6 - 915*cos(d*x + c)^4 + 1400*cos(d*x + c)^2 - 735)*sin(d*
x + c) - 420)/(a^4*d*cos(d*x + c)^7 - 8*a^4*d*cos(d*x + c)^5 + 8*a^4*d*cos(
d*x + c)^3 - 4*(a^4*d*cos(d*x + c)^5 - 2*a^4*d*cos(d*x + c)^3)*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34311, size = 232, normalized size = 1.6

$$-\frac{\frac{1155\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{3465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+47355\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+309540\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+588588\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+891198\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+7}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{110880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/110880*(1155*(3*tan(1/2*d*x + 1/2*c) - 1)/(a^4*(tan(1/2*d*x + 1/2*c) - 1
)^3) - (3465*tan(1/2*d*x + 1/2*c)^9 + 47355*tan(1/2*d*x + 1/2*c)^8 + 309540
*tan(1/2*d*x + 1/2*c)^7 + 588588*tan(1/2*d*x + 1/2*c)^6 + 891198*tan(1/2*d*
x + 1/2*c)^5 + 747450*tan(1/2*d*x + 1/2*c)^4 + 481140*tan(1/2*d*x + 1/2*c)^
3 + 172700*tan(1/2*d*x + 1/2*c)^2 + 35233*tan(1/2*d*x + 1/2*c) + 3203)/(a^4
*(tan(1/2*d*x + 1/2*c) + 1)^11))/d
```

3.849 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$-\frac{8\tan^{11}(c+dx)}{11a^4d}-\frac{20\tan^9(c+dx)}{9a^4d}-\frac{16\tan^7(c+dx)}{7a^4d}-\frac{4\tan^5(c+dx)}{5a^4d}+\frac{8\sec^{11}(c+dx)}{11a^4d}-\frac{16\sec^9(c+dx)}{9a^4d}+\frac{9\sec^7}{7}$$

```
[Out] -Sec[c + d*x]^5/(5*a^4*d) + (9*Sec[c + d*x]^7)/(7*a^4*d) - (16*Sec[c + d*x]
^9)/(9*a^4*d) + (8*Sec[c + d*x]^11)/(11*a^4*d) - (4*Tan[c + d*x]^5)/(5*a^4*
d) - (16*Tan[c + d*x]^7)/(7*a^4*d) - (20*Tan[c + d*x]^9)/(9*a^4*d) - (8*Tan
[c + d*x]^11)/(11*a^4*d)
```

Rubi [A] time = 0.40922, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2875, 2873, 2606, 14, 2607, 270}

$$-\frac{8\tan^{11}(c+dx)}{11a^4d}-\frac{20\tan^9(c+dx)}{9a^4d}-\frac{16\tan^7(c+dx)}{7a^4d}-\frac{4\tan^5(c+dx)}{5a^4d}+\frac{8\sec^{11}(c+dx)}{11a^4d}-\frac{16\sec^9(c+dx)}{9a^4d}+\frac{9\sec^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```


```
[Out] -Sec[c + d*x]^5/(5*a^4*d) + (9*Sec[c + d*x]^7)/(7*a^4*d) - (16*Sec[c + d*x]
^9)/(9*a^4*d) + (8*Sec[c + d*x]^11)/(11*a^4*d) - (4*Tan[c + d*x]^5)/(5*a^4*
d) - (16*Tan[c + d*x]^7)/(7*a^4*d) - (20*Tan[c + d*x]^9)/(9*a^4*d) - (8*Tan
[c + d*x]^11)/(11*a^4*d)
```


Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \sec^9(c+dx)(a-a\sin(c+dx))^4\tan^3(c+dx)\,dx}{a^8}$$

$$= \frac{\int \left(a^4\sec^9(c+dx)\tan^3(c+dx) - 4a^4\sec^8(c+dx)\tan^4(c+dx) + 6a^4\sec^7(c+dx)\tan^5\right.}{a^8}$$

$$= \frac{\int \sec^9(c+dx)\tan^3(c+dx)\,dx}{a^4} + \frac{\int \sec^5(c+dx)\tan^7(c+dx)\,dx}{a^4} - \frac{4\int \sec^8(c+dx)\tan}{a^4}$$

$$= \frac{\text{Subst}\left(\int x^8\left(-1+x^2\right)dx, x, \sec(c+dx)\right)}{a^4d} + \frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)^3 dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int\left(-x^4+3x^6-3x^8+x^{10}\right)dx, x, \sec(c+dx)\right)}{a^4d} + \frac{\text{Subst}\left(\int\left(-x^8+x^{10}\right)dx, x, \text{se}\right.}{a^4d}$$

$$= -\frac{\sec^5(c+dx)}{5a^4d} + \frac{9\sec^7(c+dx)}{7a^4d} - \frac{16\sec^9(c+dx)}{9a^4d} + \frac{8\sec^{11}(c+dx)}{11a^4d} - \frac{4\tan^5(c+dx)}{5a^4d} -$$

Mathematica [A] time = 0.457422, size = 166, normalized size = 1.14

$$\frac{\sec^3(c+dx)(844800\sin(c+dx) - 191752\sin(2(c+dx)) + 11264\sin(3(c+dx)) - 69728\sin(4(c+dx)) + 25600\sin(5(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(844800 - 215721*Cos[c + d*x] - 619520*Cos[2*(c + d*x)] + 2
3969*Cos[3*(c + d*x)] + 32768*Cos[4*(c + d*x)] + 54475*Cos[5*(c + d*x)] - 8
192*Cos[6*(c + d*x)] - 2179*Cos[7*(c + d*x)] + 844800*Sin[c + d*x] - 191752
*Sin[2*(c + d*x)] + 11264*Sin[3*(c + d*x)] - 69728*Sin[4*(c + d*x)] + 25600
*Sin[5*(c + d*x)] + 17432*Sin[6*(c + d*x)] - 1024*Sin[7*(c + d*x)]))/(70963
20*a^4*d*(1 + Sin[c + d*x])^4)
```

Maple [A] time = 0.131, size = 220, normalized size = 1.5

$$16\,\frac{1}{da^4}\left(-\frac{1}{768\,(\tan(1/2\,dx+c/2)-1)^3} - \frac{1}{512\,(\tan(1/2\,dx+c/2)-1)^2} - \frac{1}{512\,\tan(1/2\,dx+c/2)-512} + 1/11\,(\tan(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 16/d/a^4*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2-1/
512/(tan(1/2*d*x+1/2*c)-1)+1/11/(tan(1/2*d*x+1/2*c)+1)^11-1/2/(tan(1/2*d*x+
1/2*c)+1)^10+23/18/(tan(1/2*d*x+1/2*c)+1)^9-2/(tan(1/2*d*x+1/2*c)+1)^8+235/
112/(tan(1/2*d*x+1/2*c)+1)^7-145/96/(tan(1/2*d*x+1/2*c)+1)^6+29/40/(tan(1/2
*d*x+1/2*c)+1)^5-13/64/(tan(1/2*d*x+1/2*c)+1)^4+13/768/(tan(1/2*d*x+1/2*c)+
1)^3+3/512/(tan(1/2*d*x+1/2*c)+1)^2+1/512/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.15319, size = 686, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 4/3465*(488*sin(d*x + c)/(cos(d*x + c) + 1) + 1525*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1952*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2794*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 176*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4818*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5280*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 + 10857*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 5544*sin(d*x + c)^9/(cos(d*x
 + c) + 1)^9 + 3465*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 61)/((a^4 + 8*a
^4*sin(d*x + c)/(cos(d*x + c) + 1) + 25*a^4*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 32*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 11*a^4*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 88*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*a^4*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 99*a^4*sin(d*x + c)^8/(cos(d*x + c) +
1)^8 + 88*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 11*a^4*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 32*a^4*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 25*a
^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^4*sin(d*x + c)^13/(cos(d*x +
 c) + 1)^13 - a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14)*d)
```

Fricas [A] time = 1.74186, size = 402, normalized size = 2.77

$$-\frac{128 \cos\left(dx+c\right)^{6} - 320 \cos\left(dx+c\right)^{4} + 805 \cos\left(dx+c\right)^{2} + 4 \left(8 \cos\left(dx+c\right)^{6} - 60 \cos\left(dx+c\right)^{4} + 35 \cos\left(dx+c\right)^{2}}{3465 \left(a^{4} d \cos\left(dx+c\right)^{7} - 8 a^{4} d \cos\left(dx+c\right)^{5} + 8 a^{4} d \cos\left(dx+c\right)^{3} - 4 \left(a^{4} d \cos\left(dx+c\right)^{5} - 2 a^{4} d \cos\left(dx+c\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/3465*(128*cos(d*x + c)^6 - 320*cos(d*x + c)^4 + 805*cos(d*x + c)^2 + 4*(
8*cos(d*x + c)^6 - 60*cos(d*x + c)^4 + 35*cos(d*x + c)^2 - 105)*sin(d*x + c
) - 735)/(a^4*d*cos(d*x + c)^7 - 8*a^4*d*cos(d*x + c)^5 + 8*a^4*d*cos(d*x +
 c)^3 - 4*(a^4*d*cos(d*x + c)^5 - 2*a^4*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33721, size = 267, normalized size = 1.84

$$-\frac{\frac{1155\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)}{a^{4}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{3465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+45045\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+279510\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+669900\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+12053}{}}{11088}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/110880*(1155*(3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 2)/(a^
4*(tan(1/2*d*x + 1/2*c) - 1)^3) - (3465*tan(1/2*d*x + 1/2*c)^10 + 45045*tan
(1/2*d*x + 1/2*c)^9 + 279510*tan(1/2*d*x + 1/2*c)^8 + 669900*tan(1/2*d*x +
1/2*c)^7 + 1205358*tan(1/2*d*x + 1/2*c)^6 + 1334718*tan(1/2*d*x + 1/2*c)^5
+ 1144440*tan(1/2*d*x + 1/2*c)^4 + 627660*tan(1/2*d*x + 1/2*c)^3 + 257345*t
an(1/2*d*x + 1/2*c)^2 + 57013*tan(1/2*d*x + 1/2*c) + 5498)/(a^4*(tan(1/2*d*
x + 1/2*c) + 1)^11))/d
```

3.850 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=143

$$\frac{8\tan^{11}(c+dx)}{11a^4d}+\frac{8\tan^9(c+dx)}{3a^4d}+\frac{25\tan^7(c+dx)}{7a^4d}+\frac{2\tan^5(c+dx)}{a^4d}+\frac{\tan^3(c+dx)}{3a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{4\sec^9(c+}{3a^4d}$$

```
[Out] (-4*Sec[c + d*x]^7)/(7*a^4*d) + (4*Sec[c + d*x]^9)/(3*a^4*d) - (8*Sec[c + d
*x]^11)/(11*a^4*d) + Tan[c + d*x]^3/(3*a^4*d) + (2*Tan[c + d*x]^5)/(a^4*d)
+ (25*Tan[c + d*x]^7)/(7*a^4*d) + (8*Tan[c + d*x]^9)/(3*a^4*d) + (8*Tan[c +
 d*x]^11)/(11*a^4*d)
```

Rubi [A] time = 0.35645, antiderivative size = 184, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2870, 2672, 3767, 8}

$$\frac{8\tan(c+dx)}{231a^4d}-\frac{4\sec(c+dx)}{231d\left(a^4\sin(c+dx)+a^4\right)}-\frac{4\sec(c+dx)}{231d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\sec^3(c+dx)}{6ad(a\sin(c+dx)+a)^3}-\frac{5\sec(c+}{231ad(a\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(a*Sec[c + d*x])/(22*d*(a + a*Sin[c + d*x])^5) - Sec[c + d*x]/(33*d*(a + a
*Sin[c + d*x])^4) - (5*Sec[c + d*x])/(231*a*d*(a + a*Sin[c + d*x])^3) + Sec
[c + d*x]^3/(6*a*d*(a + a*Sin[c + d*x])^3) - (4*Sec[c + d*x])/(231*d*(a^2 +
 a^2*Sin[c + d*x])^2) - (4*Sec[c + d*x])/(231*d*(a^4 + a^4*Sin[c + d*x])) +
 (8*Tan[c + d*x])/(231*a^4*d)
```

Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\sec^3(c+dx)}{6ad(a+a\sin(c+dx))^3} + \frac{1}{2}a\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^5}\,dx$$

$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} + \frac{\sec^3(c+dx)}{6ad(a+a\sin(c+dx))^3} + \frac{3}{11}\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$

$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} + \frac{\sec^3(c+dx)}{6ad(a+a\sin(c+dx))^3} + \frac{5\int}{}$$

$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx))^3} + \frac{}{6ad}$$

$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx))^3} + \frac{}{6ad}$$

$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx))^3} + \frac{}{6ad}$$

$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx))^3} + \frac{}{6ad}$$

$$= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+dx)}{231ad(a+a\sin(c+dx))^3} + \frac{}{6ad}$$

Mathematica [A] time = 0.492419, size = 166, normalized size = 1.16

$$\frac{\sec^3(c+dx)(26048\sin(c+dx) - 1144\sin(2(c+dx)) - 704\sin(3(c+dx)) - 416\sin(4(c+dx)) - 1600\sin(5(c+dx)) + 1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(11264 - 1287*Cos[c + d*x] - 5632*Cos[2*(c + d*x)] + 143*Co
s[3*(c + d*x)] - 2048*Cos[4*(c + d*x)] + 325*Cos[5*(c + d*x)] + 512*Cos[6*(
c + d*x)] - 13*Cos[7*(c + d*x)] + 26048*Sin[c + d*x] - 1144*Sin[2*(c + d*x)
] - 704*Sin[3*(c + d*x)] - 416*Sin[4*(c + d*x)] - 1600*Sin[5*(c + d*x)] + 1
04*Sin[6*(c + d*x)] + 64*Sin[7*(c + d*x)]))/(118272*a^4*d*(1 + Sin[c + d*x]
)^4)
```

Maple [A] time = 0.144, size = 218, normalized size = 1.5

$$8\frac{1}{da^4}\left(-\frac{1}{384\,(\tan(1/2\,dx+c/2)-1)^3} - \frac{1}{256\,(\tan(1/2\,dx+c/2)-1)^2} - \frac{1}{128\,\tan(1/2\,dx+c/2)-128} - 2/11\,(\tan(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 8/d/a^4*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2-1/1
28/(tan(1/2*d*x+1/2*c)-1)-2/11/(tan(1/2*d*x+1/2*c)+1)^11+1/(tan(1/2*d*x+1/2
*c)+1)^10-8/3/(tan(1/2*d*x+1/2*c)+1)^9+9/2/(tan(1/2*d*x+1/2*c)+1)^8-295/56/
(tan(1/2*d*x+1/2*c)+1)^7+71/16/(tan(1/2*d*x+1/2*c)+1)^6-43/16/(tan(1/2*d*x+
```

```
1/2*c)+1)^5+9/8/(tan(1/2*d*x+1/2*c)+1)^4-109/384/(tan(1/2*d*x+1/2*c)+1)^3+5
/256/(tan(1/2*d*x+1/2*c)+1)^2+1/128/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.25567, size = 713, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 8/231*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 50*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 141*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 132*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 132*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 44*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 110*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 154*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 308*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
 + 154*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 77*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 + 2)/((a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 25*a^4*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 32*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 - 11*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 88*a^4*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 - 99*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 99*a^4*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 88*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1
)^9 + 11*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 32*a^4*sin(d*x + c)^11
/(cos(d*x + c) + 1)^11 - 25*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a
^4*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - a^4*sin(d*x + c)^14/(cos(d*x + c
) + 1)^14)*d)
```

Fricas [A] time = 1.62218, size = 390, normalized size = 2.73

$$\frac{32 \cos(dx+c)^6 - 80 \cos(dx+c)^4 + 28 \cos(dx+c)^2 + \left(8 \cos(dx+c)^6 - 60 \cos(dx+c)^4 + 35 \cos(dx+c)^2 + 49\right) s}{231 \left(a^4 d \cos(dx+c)^7 - 8 a^4 d \cos(dx+c)^5 + 8 a^4 d \cos(dx+c)^3 - 4 \left(a^4 d \cos(dx+c)^5 - 2 a^4 d \cos(dx+c)^3\right) \mathrm{si}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/231*(32*cos(d*x + c)^6 - 80*cos(d*x + c)^4 + 28*cos(d*x + c)^2 + (8*cos(d
*x + c)^6 - 60*cos(d*x + c)^4 + 35*cos(d*x + c)^2 + 49)*sin(d*x + c) + 28)/
(a^4*d*cos(d*x + c)^7 - 8*a^4*d*cos(d*x + c)^5 + 8*a^4*d*cos(d*x + c)^3 - 4
*(a^4*d*cos(d*x + c)^5 - 2*a^4*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35932, size = 267, normalized size = 1.87

$$-\frac{\frac{77\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\right)}{a^{4}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{462\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+5775\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+14399\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+29260\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+30800\tan\left(\frac{1}{2}\right.}{}}{7392\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/7392*(77*(6*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 5)/(a^4*(t
an(1/2*d*x + 1/2*c) - 1)^3) - (462*tan(1/2*d*x + 1/2*c)^10 + 5775*tan(1/2*d
*x + 1/2*c)^9 + 14399*tan(1/2*d*x + 1/2*c)^8 + 29260*tan(1/2*d*x + 1/2*c)^7
 + 30800*tan(1/2*d*x + 1/2*c)^6 + 27874*tan(1/2*d*x + 1/2*c)^5 + 12650*tan(
1/2*d*x + 1/2*c)^4 + 6556*tan(1/2*d*x + 1/2*c)^3 + 1210*tan(1/2*d*x + 1/2*c
)^2 + 935*tan(1/2*d*x + 1/2*c) + 127)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^11))/
d
```

3.851 $\int \sin(c+dx)(a+a\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=133

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{39a\log(1-$$

```
[Out] (-39*a*Log[1 - Sin[c + d*x]])/(16*d) - (9*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) + a^3/(8*d*(a - a*Sin[c + d*
x])^2) - (5*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x])
)
```

Rubi [A] time = 0.111147, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{39a\log(1-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (-39*a*Log[1 - Sin[c + d*x]])/(16*d) - (9*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) + a^3/(8*d*(a - a*Sin[c + d*
x])^2) - (5*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x])
)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))\tan^5(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^6}{a^6(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^4}{4(a-x)^3}-\frac{5a^3}{4(a-x)^2}+\frac{39a^2}{16(a-x)}-x+\frac{a^3}{8(a+x)^2}-\frac{9a^2}{16(a+x)}\right)dx, \ldots}{ad}$$

$$= -\frac{39a\log(1-\sin(c+dx))}{16d}-\frac{9a\log(1+\sin(c+dx))}{16d}-\frac{a\sin(c+dx)}{d} - \ldots$$

Mathematica [A] time = 0.439807, size = 133, normalized size = 1.

$$-\frac{a\sin(c+dx)\tan^4(c+dx)}{d}-\frac{a\left(2\sin^2(c+dx)-\sec^4(c+dx)+6\sec^2(c+dx)+12\log(\cos(c+dx))\right)}{4d}-\frac{5a\left(6\tan(c+dx\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -(a*(12*Log[Cos[c + d*x]] + 6*Sec[c + d*x]^2 - Sec[c + d*x]^4 + 2*Sin[c + d
*x]^2))/(4*d) - (a*Sin[c + d*x]*Tan[c + d*x]^4)/d - (5*a*(6*Sec[c + d*x]^3*
Tan[c + d*x] - 8*Sec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x])))/(8*d)
```

Maple [A] time = 0.072, size = 205, normalized size = 1.5

$$\frac{a(\sin(dx+c))^8}{4d(\cos(dx+c))^4}-\frac{a(\sin(dx+c))^8}{2d(\cos(dx+c))^2}-\frac{a(\sin(dx+c))^6}{2d}-\frac{3a(\sin(dx+c))^4}{4d}-\frac{3(\sin(dx+c))^2a}{2d}-3\frac{a\ln(\cos(dx+c\ldots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a*sin(d*x+c)^8/cos(d*x+c)^2-1/2*a*s
in(d*x+c)^6/d-3/4*a*sin(d*x+c)^4/d-3/2*a*sin(d*x+c)^2/d-3*a*ln(cos(d*x+c))/
d+1/4/d*a*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a*sin(d*x+c)^7/cos(d*x+c)^2-3/8*a
*sin(d*x+c)^5/d-5/8*a*sin(d*x+c)^3/d-15/8*a*sin(d*x+c)/d+15/8/d*a*ln(sec(d*
x+c)+tan(d*x+c))
```

Maxima [A] time = 1.11454, size = 143, normalized size = 1.08

$$-\frac{8a\sin(dx+c)^2+9a\log(\sin(dx+c)+1)+39a\log(\sin(dx+c)-1)+16a\sin(dx+c)-\frac{2\left(9a\sin(dx+c)^2+3a\sin(dx+c)-10\ldots\right.}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)\ldots}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(8*a*sin(d*x + c)^2 + 9*a*log(sin(d*x + c) + 1) + 39*a*log(sin(d*x +
c) - 1) + 16*a*sin(d*x + c) - 2*(9*a*sin(d*x + c)^2 + 3*a*sin(d*x + c) - 10
*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.54359, size = 440, normalized size = 3.31

$$\frac{8\,a\cos\left(dx+c\right)^{4}+6\,a\cos\left(dx+c\right)^{2}-9\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-39\left(a\cos\right.}{16\left(d\cos\left(dx+c\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*a*cos(d*x + c)^4 + 6*a*cos(d*x + c)^2 - 9*(a*cos(d*x + c)^2*sin(d*x
 + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 39*(a*cos(d*x + c)^2*sin(
d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^4
 + 6*a*cos(d*x + c)^2 - 3*a)*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2*sin(d*x
+ c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39309, size = 153, normalized size = 1.15

$$-\frac{16\,a\sin\left(dx+c\right)^{2}+18\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+78\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+32\,a\sin\left(dx+c\right)-\frac{2\,(9\,a\sin(dx+c)+7\,a)}{\sin(dx+c)+1}-}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(16*a*sin(d*x + c)^2 + 18*a*log(abs(sin(d*x + c) + 1)) + 78*a*log(abs
(sin(d*x + c) - 1)) + 32*a*sin(d*x + c) - 2*(9*a*sin(d*x + c) + 7*a)/(sin(d
*x + c) + 1) - (117*a*sin(d*x + c)^2 - 194*a*sin(d*x + c) + 81*a)/(sin(d*x
+ c) - 1)^2)/d
```

3.852 $\int (a+a\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log}{}$$

```
[Out] (-23*a*Log[1 - Sin[c + d*x]])/(16*d) + (7*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(d*(a - a*Sin[
c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0680902, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (-23*a*Log[1 - Sin[c + d*x]])/(16*d) + (7*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(d*(a - a*Sin[
c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))\tan^5(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{4(a-x)^3}-\frac{a^2}{(a-x)^2}+\frac{23a}{16(a-x)}-\frac{a^2}{8(a+x)^2}+\frac{7a}{16(a+x)}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log(1+\sin(c+dx))}{16d}-\frac{a\sin(c+dx)}{d}+\frac{}{8d(a-a\sin}\end{aligned}$$

Mathematica [A] time = 0.266773, size = 123, normalized size = 1.07

$$-\frac{a\sin(c+dx)\tan^4(c+dx)}{d}-\frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d}-\frac{5a\left(6\tan(c+dx)\sec^3(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*S
ec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(8*d)
```

Maple [A] time = 0.071, size = 147, normalized size = 1.3

$$\frac{a(\sin(dx+c))^7}{4d(\cos(dx+c))^4}-\frac{3a(\sin(dx+c))^7}{8d(\cos(dx+c))^2}-\frac{3a(\sin(dx+c))^5}{8d}-\frac{5a(\sin(dx+c))^3}{8d}-\frac{15a\sin(dx+c)}{8d}+\frac{15a\ln(\sec(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a*sin(d*x+c)^7/cos(d*x+c)^2-3/8*a*s
in(d*x+c)^5/d-5/8*a*sin(d*x+c)^3/d-15/8*a*sin(d*x+c)/d+15/8/d*a*ln(sec(d*x+
c)+tan(d*x+c))+1/4*a*tan(d*x+c)^4/d-1/2*a*tan(d*x+c)^2/d-a*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.1957, size = 128, normalized size = 1.11

$$\frac{7a\log(\sin(dx+c)+1)-23a\log(\sin(dx+c)-1)-16a\sin(dx+c)+\frac{2\left(9a\sin(dx+c)^2-a\sin(dx+c)-6a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) - 16*a*sin(d*x
 + c) + 2*(9*a*sin(d*x + c)^2 - a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin
(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.57403, size = 410, normalized size = 3.57

$$\frac{16a\cos(dx+c)^4+2a\cos(dx+c)^2+7\left(a\cos(dx+c)^2\sin(dx+c)-a\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-23\left(a\cos\right.}{16\left(d\cos(dx+c)^2\sin(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*cos(d*x + c)^4 + 2*a*cos(d*x + c)^2 + 7*(a*cos(d*x + c)^2*sin(d*
x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 23*(a*cos(d*x + c)^2*sin
(d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(8*a*cos(d*x + c)^
2 + a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42373, size = 136, normalized size = 1.18

$$\frac{14\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-46\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-32\,a\sin\left(dx+c\right)-\frac{2\,(7\,a\sin(dx+c)+5\,a)}{\sin(dx+c)+1}+\frac{69\,a\sin(dx+c)^{2}-106\,a\sin(dx+c)+}{(\sin(dx+c)-1)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(14*a*log(abs(sin(d*x + c) + 1)) - 46*a*log(abs(sin(d*x + c) - 1)) - 3
2*a*sin(d*x + c) - 2*(7*a*sin(d*x + c) + 5*a)/(sin(d*x + c) + 1) + (69*a*si
n(d*x + c)^2 - 106*a*sin(d*x + c) + 41*a)/(sin(d*x + c) - 1)^2)/d
```

3.853 $\int \sec(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=105

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}-\frac{5a\log(\sin(c+dx}{16d}$$

```
[Out] (-11*a*Log[1 - Sin[c + d*x]])/(16*d) - (5*a*Log[1 + Sin[c + d*x]])/(16*d) +
 a^3/(8*d*(a - a*Sin[c + d*x])^2) - (3*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^
2/(8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.094418, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}-\frac{5a\log(\sin(c+dx}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (-11*a*Log[1 - Sin[c + d*x]])/(16*d) - (5*a*Log[1 + Sin[c + d*x]])/(16*d) +
 a^3/(8*d*(a - a*Sin[c + d*x])^2) - (3*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^
2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx = \frac{a^5 \operatorname{Subst}\left(\int \frac{x^4}{a^4(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a \operatorname{Subst}\left(\int \frac{x^4}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a \operatorname{Subst}\left(\int \left(\frac{a^2}{4(a-x)^3} - \frac{3a}{4(a-x)^2} + \frac{11}{16(a-x)} + \frac{a}{8(a+x)^2} - \frac{5}{16(a+x)}\right) dx, x, a\sin(\right.}{d}$$
$$= -\frac{11a\log(1-\sin(c+dx))}{16d} - \frac{5a\log(1+\sin(c+dx))}{16d} + \frac{a^3}{8d(a-a\sin(c}$$

Mathematica [A] time = 0.230451, size = 106, normalized size = 1.01

$$\frac{a\tan^3(c+dx)\sec(c+dx)}{d} - \frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d} - \frac{a\left(6\tan(c+dx)\sec^3(c+dx)-3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*Sec[c + d*x]*Tan[c + d*x]^3)/d - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x
]^2 - Tan[c + d*x]^4))/(4*d) - (a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 3*(ArcTa
nh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])))/(8*d)
```

Maple [A] time = 0.069, size = 133, normalized size = 1.3

$$\frac{a(\tan(dx+c))^4}{4d} - \frac{a(\tan(dx+c))^2}{2d} - \frac{a\ln(\cos(dx+c))}{d} + \frac{a(\sin(dx+c))^5}{4d(\cos(dx+c))^4} - \frac{a(\sin(dx+c))^5}{8d(\cos(dx+c))^2} - \frac{a(\sin(dx+c))^3}{8d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4*a*tan(d*x+c)^4/d-1/2*a*tan(d*x+c)^2/d-a*ln(cos(d*x+c))/d+1/4/d*a*sin(d*
x+c)^5/cos(d*x+c)^4-1/8/d*a*sin(d*x+c)^5/cos(d*x+c)^2-1/8*a*sin(d*x+c)^3/d-
3/8*a*sin(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01102, size = 116, normalized size = 1.1

$$-\frac{5\,a\log(\sin(dx+c)+1) + 11\,a\log(\sin(dx+c)-1) - \frac{2\left(5\,a\sin(dx+c)^2+3\,a\sin(dx+c)-6\,a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(5*a*log(sin(d*x + c) + 1) + 11*a*log(sin(d*x + c) - 1) - 2*(5*a*sin(
d*x + c)^2 + 3*a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin
(d*x + c) + 1))/d
```

Fricas [A] time = 1.53321, size = 351, normalized size = 3.34

$$\frac{10\,a\cos\left(dx+c\right)^2-5\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-11\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)\right.}{16\left(d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(10*a*cos(d*x + c)^2 - 5*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) - 11*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1) - 6*a*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2
*sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49534, size = 126, normalized size = 1.2

$$-\frac{10\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+22\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,(5\,a\sin(dx+c)+3\,a)}{\sin(dx+c)+1}-\frac{33\,a\sin(dx+c)^2-42\,a\sin(dx+c)+13\,a}{(\sin(dx+c)-1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(10*a*log(abs(sin(d*x + c) + 1)) + 22*a*log(abs(sin(d*x + c) - 1)) -
2*(5*a*sin(d*x + c) + 3*a)/(sin(d*x + c) + 1) - (33*a*sin(d*x + c)^2 - 42*a
*sin(d*x + c) + 13*a)/(sin(d*x + c) - 1)^2)/d
```

3.854 $\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=84

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/
(2*d*(a - a*Sin[c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.101589, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2836, 12, 88, 206}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/
(2*d*(a - a*Sin[c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a}{4(a-x)^3} - \frac{1}{2(a-x)^2} - \frac{1}{8(a+x)^2} + \frac{3}{8(a^2-x^2)}\right) dx, x, a\sin(c+\right.}{d}$$

$$= \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{a^2}{2d(a-a\sin(c+dx))} + \frac{a^2}{8d(a+a\sin(c+}$$

$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{a^2}{2d(a-a\sin(c}$$

Mathematica [A] time = 0.201194, size = 84, normalized size = 1.

$$\frac{a\tan^4(c+dx)}{4d} + \frac{a\tan^3(c+dx)\sec(c+dx)}{d} - \frac{a\left(6\tan(c+dx)\sec^3(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\,\text{s}\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a*Tan[c + d*x]^4)/(4*d) - (a*(6*Sec[c
+ d*x]^3*Tan[c + d*x] - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x
])))/(8*d)
```

Maple [A] time = 0.067, size = 114, normalized size = 1.4

$$\frac{a(\sin(dx+c))^5}{4d(\cos(dx+c))^4} - \frac{a(\sin(dx+c))^5}{8d(\cos(dx+c))^2} - \frac{a(\sin(dx+c))^3}{8d} - \frac{3a\sin(dx+c)}{8d} + \frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{8d} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a*sin(d*x+c)^5/cos(d*x+c)^2-1/8*a*s
in(d*x+c)^3/d-3/8*a*sin(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a*
sin(d*x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 1.02721, size = 116, normalized size = 1.38

$$\frac{3a\log(\sin(dx+c)+1) - 3a\log(\sin(dx+c)-1) + \frac{2\left(5a\sin(dx+c)^2 - a\sin(dx+c) - 2a\right)}{\sin(dx+c)^3 - \sin(dx+c)^2 - \sin(dx+c) + 1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a*log(sin(d*x + c) + 1) - 3*a*log(sin(d*x + c) - 1) + 2*(5*a*sin(d*
x + c)^2 - a*sin(d*x + c) - 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x
 + c) + 1))/d
```

Fricas [A] time = 1.35258, size = 350, normalized size = 4.17

$$\frac{10\,a\cos\left(dx+c\right)^{2}+3\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\right.}{16\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(10*a*cos(d*x + c)^2 + 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) - 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 2*a*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*
sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33009, size = 122, normalized size = 1.45

$$\frac{6\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(3\,a\sin\left(dx+c\right)+a\right)}{\sin\left(dx+c\right)+1}+\frac{9\,a\sin\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)-3\,a}{\left(\sin\left(dx+c\right)-1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*a*log(abs(sin(d*x + c) + 1)) - 6*a*log(abs(sin(d*x + c) - 1)) - 2*(
3*a*sin(d*x + c) + a)/(sin(d*x + c) + 1) + (9*a*sin(d*x + c)^2 - 2*a*sin(d*
x + c) - 3*a)/(sin(d*x + c) - 1)^2)/d
```

3.855 $\int \sec^3(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=84

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(
4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0978452, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2836, 12, 88, 206}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(
4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = \frac{a^5 \operatorname{Subst}\left(\int \frac{x^2}{a^2(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3 \operatorname{Subst}\left(\int \frac{x^2}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3 \operatorname{Subst}\left(\int \left(\frac{1}{4(a-x)^3} - \frac{1}{4a(a-x)^2} + \frac{1}{8a(a+x)^2} - \frac{1}{8a\left(a^2-x^2\right)}\right) dx, x, a\sin(c+\right.}{d}$$
$$= \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{a^2}{4d(a-a\sin(c+dx))} - \frac{a^2}{8d(a+a\sin(c+dx}$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{a^2}{4d(a-a\sin(c+}$$

Mathematica [A] time = 0.0307131, size = 74, normalized size = 0.88

$$\frac{a\tan^4(c+dx)}{4d} - \frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.059, size = 100, normalized size = 1.2

$$\frac{a(\sin(dx+c))^4}{4d(\cos(dx+c))^4} + \frac{a(\sin(dx+c))^3}{4d(\cos(dx+c))^4} + \frac{a(\sin(dx+c))^3}{8d(\cos(dx+c))^2} + \frac{a\sin(dx+c)}{8d} - \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*a*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a
*sin(d*x+c)^3/cos(d*x+c)^2+1/8*a*sin(d*x+c)/d-1/8/d*a*ln(sec(d*x+c)+tan(d*x
+c))
```

Maxima [A] time = 1.11409, size = 113, normalized size = 1.35

$$-\frac{a\log(\sin(dx+c)+1) - a\log(\sin(dx+c)-1) - \frac{2\left(a\sin(dx+c)^2+3a\sin(dx+c)-2a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c
)^2 + 3*a*sin(d*x + c) - 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x +
```

```
c) + 1))/d
```

Fricas [A] time = 1.44748, size = 343, normalized size = 4.08

$$\frac{2\,a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-\right.}{16\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(sin(d*x + c) + 1) + (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(-sin(d*x + c) + 1) - 6*a*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2*sin(d
*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32023, size = 123, normalized size = 1.46

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(a\sin\left(dx+c\right)-a\right)}{\sin\left(dx+c\right)+1}+\frac{3\,a\sin\left(dx+c\right)^{2}-14\,a\sin\left(dx+c\right)+7\,a}{\left(\sin\left(dx+c\right)-1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*a*log(abs(sin(d*x + c) + 1)) - 2*a*log(abs(sin(d*x + c) - 1)) - 2*
(a*sin(d*x + c) - a)/(sin(d*x + c) + 1) + (3*a*sin(d*x + c)^2 - 14*a*sin(d*
x + c) + 7*a)/(sin(d*x + c) - 1)^2)/d
```

3.856 $\int \sec^4(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=61

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(
8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0667725, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2836, 12, 77, 206}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(
8*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x}{a(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{x}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \left(\frac{1}{4a(a-x)^3} - \frac{1}{8a^2(a+x)^2} - \frac{1}{8a^2(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3}{8d(a-a\sin(c+dx))^2} + \frac{a^2}{8d(a+a\sin(c+dx))} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{a^2-x}\right.}{}$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3}{8d(a-a\sin(c+dx))^2} + \frac{a^2}{8d(a+a\sin(c}$$

Mathematica [A] time = 0.0291179, size = 74, normalized size = 1.21

$$\frac{a\sec^4(c+dx)}{4d} - \frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Sec[c + d*x]^4)/(4*d) - (a*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.053, size = 92, normalized size = 1.5

$$\frac{a(\sin(dx+c))^3}{4d(\cos(dx+c))^4} + \frac{a(\sin(dx+c))^3}{8d(\cos(dx+c))^2} + \frac{a\sin(dx+c)}{8d} - \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{a}{4d(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a*sin(d*x+c)^3/cos(d*x+c)^2+1/8*a*s
in(d*x+c)/d-1/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a/cos(d*x+c)^4
```

Maxima [A] time = 1.05883, size = 113, normalized size = 1.85

$$-\frac{a\log(\sin(dx+c)+1) - a\log(\sin(dx+c)-1) - \frac{2\left(a\sin(dx+c)^2 - a\sin(dx+c) + 2a\right)}{\sin(dx+c)^3 - \sin(dx+c)^2 - \sin(dx+c) + 1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c
)^2 - a*sin(d*x + c) + 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c)
```

```
+ 1))/d
```

Fricas [B] time = 1.49932, size = 343, normalized size = 5.62

$$\frac{2\,a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\ldots}{16\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(sin(d*x + c) + 1) + (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(-sin(d*x + c) + 1) + 2*a*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d
*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31221, size = 123, normalized size = 2.02

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(a\sin\left(dx+c\right)+3\,a\right)}{\sin\left(dx+c\right)+1}+\frac{3\,a\sin\left(dx+c\right)^{2}-6\,a\sin\left(dx+c\right)-a}{\left(\sin\left(dx+c\right)-1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*a*log(abs(sin(d*x + c) + 1)) - 2*a*log(abs(sin(d*x + c) - 1)) - 2*
(a*sin(d*x + c) + 3*a)/(sin(d*x + c) + 1) + (3*a*sin(d*x + c)^2 - 6*a*sin(d
*x + c) - a)/(sin(d*x + c) - 1)^2)/d
```

3.857 $\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=117

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] (-11*a*Log[1 - Sin[c + d*x]])/(16*d) + (a*Log[Sin[c + d*x]])/d - (5*a*Log[1
 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(2*d*(a -
 a*Sin[c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.107274, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-11*a*Log[1 - Sin[c + d*x]])/(16*d) + (a*Log[Sin[c + d*x]])/d - (5*a*Log[1
 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(2*d*(a -
 a*Sin[c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a}{(a-x)^3x(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{1}{(a-x)^3x(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{1}{4a^3(a-x)^3} + \frac{1}{2a^4(a-x)^2} + \frac{11}{16a^5(a-x)} + \frac{1}{a^5x} - \frac{1}{8a^4(a+x)^2} - \frac{5}{16a^5(a+}\right.\right.}{d}$$
$$= -\frac{11a\log(1-\sin(c+dx))}{16d} + \frac{a\log(\sin(c+dx))}{d} - \frac{5a\log(1+\sin(c+d}{16d}$$

Mathematica [A] time = 0.209148, size = 99, normalized size = 0.85

$$\frac{a\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{a\left(-\sec^4(c+dx) - 2\sec^2(c+dx) - 4\log(\sin(c+dx)) + 4\log(\cos(c+dx))\right)}{4d} + \frac{3a\left(\tanh^{-1}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec[c +
 d*x]^4))/(4*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin
[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A] time = 0.085, size = 100, normalized size = 0.9

$$\frac{a\tan(dx+c)(\sec(dx+c))^3}{4d} + \frac{3a\sec(dx+c)\tan(dx+c)}{8d} + \frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{a}{4d(\cos(dx+c))^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*tan(d*x+c)*sec(d*x+c)^3+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+1/4/d*a/cos(d*x+c)^4+1/2/d*a/cos(d*x+c)^2+a*ln(tan(d*x
+c))/d
```

Maxima [A] time = 1.01337, size = 128, normalized size = 1.09

$$-\frac{5a\log(\sin(dx+c)+1) + 11a\log(\sin(dx+c)-1) - 16a\log(\sin(dx+c)) + \frac{2\left(3a\sin(dx+c)^2 + a\sin(dx+c) - 6a\right)}{\sin(dx+c)^3 - \sin(dx+c)^2 - \sin(dx+c) + 1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(5*a*log(sin(d*x + c) + 1) + 11*a*log(sin(d*x + c) - 1) - 16*a*log(si
n(d*x + c)) + 2*(3*a*sin(d*x + c)^2 + a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3
 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.44537, size = 456, normalized size = 3.9

$$-\frac{6\,a\cos\left(dx+c\right)^2-16\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+5\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right.\right.}{16\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*cos(d*x + c)^2 - 16*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
 c)^2)*log(1/2*sin(d*x + c)) + 5*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
 + c)^2)*log(sin(d*x + c) + 1) + 11*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(
d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*a*sin(d*x + c) + 6*a)/(d*cos(d*x + c
)^2*sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32802, size = 140, normalized size = 1.2

$$-\frac{10\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+22\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-32\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{2\,(5\,a\sin(dx+c)+7\,a)}{\sin(dx+c)+1}-\frac{33\,a\sin(dx+c)^2-8}{(\sin(dx}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(10*a*log(abs(sin(d*x + c) + 1)) + 22*a*log(abs(sin(d*x + c) - 1)) -
32*a*log(abs(sin(d*x + c))) - 2*(5*a*sin(d*x + c) + 7*a)/(sin(d*x + c) + 1)
 - (33*a*sin(d*x + c)^2 - 82*a*sin(d*x + c) + 53*a)/(sin(d*x + c) - 1)^2)/d
```

3.858 $\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{a\log}{}$$

```
[Out] -((a*Csc[c + d*x])/d) - (23*a*Log[1 - Sin[c + d*x]])/(16*d) + (a*Log[Sin[c
+ d*x]])/d + (7*a*Log[1 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d
*x])^2) + (3*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]
))
```

Rubi [A] time = 0.12108, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{a\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (23*a*Log[1 - Sin[c + d*x]])/(16*d) + (a*Log[Sin[c
+ d*x]])/d + (7*a*Log[1 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d
*x])^2) + (3*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]
))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^2}{(a-x)^3x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{1}{4a^4(a-x)^3} + \frac{3}{4a^5(a-x)^2} + \frac{23}{16a^6(a-x)} + \frac{1}{a^5x^2} + \frac{1}{a^6x} + \frac{1}{8a^5(a+x}\right.\right.}{d}$$
$$= -\frac{a\csc(c+dx)}{d} - \frac{23a\log(1-\sin(c+dx))}{16d} + \frac{a\log(\sin(c+dx))}{d} +$$

Mathematica [C] time = 0.189154, size = 76, normalized size = 0.59

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d} - \frac{a\left(-\sec^4(c+dx) - 2\sec^2(c+dx) - 4\log(\sin(c+dx)) + 4\log(\cos(c+d\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (a*
(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec[c + d*x
]^4))/(4*d)
```

Maple [A] time = 0.109, size = 120, normalized size = 0.9

$$\frac{a}{4d(\cos(dx+c))^4} + \frac{a}{2d(\cos(dx+c))^2} + \frac{a\ln(\tan(dx+c))}{d} + \frac{a}{4d\sin(dx+c)(\cos(dx+c))^4} + \frac{5a}{8d\sin(dx+c)(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/cos(d*x+c)^4+1/2/d*a/cos(d*x+c)^2+a*ln(tan(d*x+c))/d+1/4/d*a/sin(d*
x+c)/cos(d*x+c)^4+5/8/d*a/sin(d*x+c)/cos(d*x+c)^2-15/8/d*a/sin(d*x+c)+15/8/
d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.08075, size = 154, normalized size = 1.19

$$\frac{7a\log(\sin(dx+c)+1) - 23a\log(\sin(dx+c)-1) + 16a\log(\sin(dx+c)) - \frac{2\left(15a\sin(dx+c)^3 - 11a\sin(dx+c)^2 - 14a\sin(dx+c)\right.}{\sin(dx+c)^4 - \sin(dx+c)^3 - \sin(dx+c)^2 + \sin(dx+c}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) + 16*a*log(sin
(d*x + c)) - 2*(15*a*sin(d*x + c)^3 - 11*a*sin(d*x + c)^2 - 14*a*sin(d*x +
c) + 8*a)/(sin(d*x + c)^4 - sin(d*x + c)^3 - sin(d*x + c)^2 + sin(d*x + c))
```

```
)/d
```

Fricas [A] time = 1.53628, size = 593, normalized size = 4.6

$$-\frac{22\,a\cos\left(dx+c\right)^{2}-16\left(a\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-7\left(a\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(22*a*cos(d*x + c)^2 - 16*(a*cos(d*x + c)^4 + a*cos(d*x + c)^2*sin(d*
x + c) - a*cos(d*x + c)^2)*log(1/2*sin(d*x + c)) - 7*(a*cos(d*x + c)^4 + a*
cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 23*
(a*cos(d*x + c)^4 + a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) - 2*(15*a*cos(d*x + c)^2 - a)*sin(d*x + c) - 6*a)/(d*cos(
d*x + c)^4 + d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34861, size = 163, normalized size = 1.26

$$\frac{14\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-46\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+32\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{23\,a\sin\left(dx+c\right)^{2}+59\,a\sin\left(dx+c\right)+32\,a}{\sin\left(dx+c\right)^{2}+\sin\left(dx+c\right)}+\frac{69\,a\sin\left(\right.}{}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(14*a*log(abs(sin(d*x + c) + 1)) - 46*a*log(abs(sin(d*x + c) - 1)) + 3
2*a*log(abs(sin(d*x + c))) - (23*a*sin(d*x + c)^2 + 59*a*sin(d*x + c) + 32*
a)/(sin(d*x + c)^2 + sin(d*x + c)) + (69*a*sin(d*x + c)^2 - 162*a*sin(d*x +
 c) + 97*a)/(sin(d*x + c) - 1)^2)/d
```

3.859 $\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{39a\log(1-s}{16}$$

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (39*a*Log[1 - Sin[c + d*
x]])/(16*d) + (3*a*Log[Sin[c + d*x]])/d - (9*a*Log[1 + Sin[c + d*x]])/(16*d
) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(d*(a - a*Sin[c + d*x])) + a^2/(
8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.132132, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{39a\log(1-s}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (39*a*Log[1 - Sin[c + d*
x]])/(16*d) + (3*a*Log[Sin[c + d*x]])/d - (9*a*Log[1 + Sin[c + d*x]])/(16*d
) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(d*(a - a*Sin[c + d*x])) + a^2/(
8*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\,\mathrm{Subst}\left(\int \frac{a^3}{(a-x)^3x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^8\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^3x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^8\,\mathrm{Subst}\left(\int \left(\frac{1}{4a^5(a-x)^3} + \frac{1}{a^6(a-x)^2} + \frac{39}{16a^7(a-x)} + \frac{1}{a^5x^3} + \frac{1}{a^6x^2} + \frac{3}{a^7x} - \frac{}{8a^6(}\right.\right.}{d}$$

$$= -\frac{a\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{39a\log(1-\sin(c+dx))}{16d} + \frac{3a\log}{}$$

Mathematica [C] time = 0.753603, size = 86, normalized size = 0.6

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d} - \frac{a\left(2\csc^2(c+dx) - \sec^4(c+dx) - 4\sec^2(c+dx) - 12\log(\sin(c+dx)) + 12\,\mathrm{l}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (a*
(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Sec[c +
 d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

Maple [A] time = 0.112, size = 151, normalized size = 1.1

$$\frac{a}{4\,d\sin(dx+c)(\cos(dx+c))^4} + \frac{5\,a}{8\,d\sin(dx+c)(\cos(dx+c))^2} - \frac{15\,a}{8\,d\sin(dx+c)} + \frac{15\,a\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a/sin(d*x+c)/cos(d*x+c)^2-15/8/d*a/si
n(d*x+c)+15/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a/sin(d*x+c)^2/cos(d*x+c)
^4+3/4/d*a/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a/sin(d*x+c)^2+3*a*ln(tan(d*x+c)
)/d
```

Maxima [A] time = 1.13342, size = 171, normalized size = 1.2

$$-\frac{9\,a\log(\sin(dx+c)+1) + 39\,a\log(\sin(dx+c)-1) - 48\,a\log(\sin(dx+c)) + \frac{2\left(15\,a\sin(dx+c)^4 - 3\,a\sin(dx+c)^3 - 22\,a\sin(dx+c)^2 + 4\right.}{\sin(dx+c)^5 - \sin(dx+c)^4 - \sin(dx+c)^3 + \sin(}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(9*a*log(sin(d*x + c) + 1) + 39*a*log(sin(d*x + c) - 1) - 48*a*log(si
n(d*x + c)) + 2*(15*a*sin(d*x + c)^4 - 3*a*sin(d*x + c)^3 - 22*a*sin(d*x +
c)^2 + 4*a*sin(d*x + c) + 4*a)/(sin(d*x + c)^5 - sin(d*x + c)^4 - sin(d*x +
```

```
c)^3 + sin(d*x + c)^2))/d
```

Fricas [B] time = 1.58582, size = 733, normalized size = 5.13

$$\frac{30\,a\cos\left(dx+c\right)^{4}-16\,a\cos\left(dx+c\right)^{2}+48\left(a\cos\left(dx+c\right)^{4}-a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)^{4}-a\cos\left(dx+c\right)^{2}\right)\sin\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(30*a*cos(d*x + c)^4 - 16*a*cos(d*x + c)^2 + 48*(a*cos(d*x + c)^4 - a*
cos(d*x + c)^2 - (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*x + c))*log(1/
2*sin(d*x + c)) - 9*(a*cos(d*x + c)^4 - a*cos(d*x + c)^2 - (a*cos(d*x + c)^
4 - a*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 39*(a*cos(d*x +
 c)^4 - a*cos(d*x + c)^2 - (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*x +
c))*log(-sin(d*x + c) + 1) + 2*(3*a*cos(d*x + c)^2 + a)*sin(d*x + c) - 6*a)
/(d*cos(d*x + c)^4 - d*cos(d*x + c)^2 - (d*cos(d*x + c)^4 - d*cos(d*x + c)^
2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34488, size = 169, normalized size = 1.18

$$-\frac{36\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+156\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-192\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{4\,(9\,a\sin(dx+c)+11\,a)}{\sin(dx+c)+1}+\frac{27\,a\sin(dx+c}{}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/64*(36*a*log(abs(sin(d*x + c) + 1)) + 156*a*log(abs(sin(d*x + c) - 1)) -
 192*a*log(abs(sin(d*x + c))) - 4*(9*a*sin(d*x + c) + 11*a)/(sin(d*x + c) +
 1) + (27*a*sin(d*x + c)^4 + 74*a*sin(d*x + c)^3 - 141*a*sin(d*x + c)^2 + 3
2*a)/(sin(d*x + c)^2 - sin(d*x + c))^2)/d
```

3.860 $\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=162

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

```
[Out] (-3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
 - (59*a*Log[1 - Sin[c + d*x]])/(16*d) + (3*a*Log[Sin[c + d*x]])/d + (11*a*
Log[1 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + (5*a^2)/
(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.140866, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
 - (59*a*Log[1 - Sin[c + d*x]])/(16*d) + (3*a*Log[Sin[c + d*x]])/d + (11*a*
Log[1 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + (5*a^2)/
(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^4}{(a-x)^3x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \left(\frac{1}{4a^6(a-x)^3}+\frac{5}{4a^7(a-x)^2}+\frac{59}{16a^8(a-x)}+\frac{1}{a^5x^4}+\frac{1}{a^6x^3}+\frac{3}{a^7x^2}+\right.\right.}{d}$$
$$= -\frac{3a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{59a\log(1-}{16}$$

Mathematica [C] time = 1.21069, size = 90, normalized size = 0.56

$$-\frac{a\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(c+dx)\right)}{3d}-\frac{a\left(2\csc^2(c+dx)-\sec^4(c+dx)-4\sec^2(c+dx)-12\log(\sin(c+dx))\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[c + d*x]^2])/(3*d)
- (a*(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Se
c[c + d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

Maple [A] time = 0.123, size = 173, normalized size = 1.1

$$\frac{a}{4\,d\left(\sin(dx+c)\right)^2\left(\cos(dx+c)\right)^4}+\frac{3\,a}{4\,d\left(\sin(dx+c)\right)^2\left(\cos(dx+c)\right)^2}-\frac{3\,a}{2\,d\left(\sin(dx+c)\right)^2}+3\,\frac{a\ln(\tan(dx+c))}{d}+\frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*a/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a
/sin(d*x+c)^2+3*a*ln(tan(d*x+c))/d+1/4/d*a/sin(d*x+c)^3/cos(d*x+c)^4-7/12/d
*a/sin(d*x+c)^3/cos(d*x+c)^2+35/24/d*a/sin(d*x+c)/cos(d*x+c)^2-35/8/d*a/sin
(d*x+c)+35/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.03477, size = 186, normalized size = 1.15

$$\frac{33\,a\log(\sin(dx+c)+1)-177\,a\log(\sin(dx+c)-1)+144\,a\log(\sin(dx+c))-\frac{2\left(105\,a\sin(dx+c)^5-69\,a\sin(dx+c)^4-106\,a\sin\right.}{\sin(dx+c)^6-\sin(dx+c)^5-}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(33*a*log(sin(d*x + c) + 1) - 177*a*log(sin(d*x + c) - 1) + 144*a*log(
sin(d*x + c)) - 2*(105*a*sin(d*x + c)^5 - 69*a*sin(d*x + c)^4 - 106*a*sin(d
*x + c)^3 + 52*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 8*a)/(sin(d*x + c)^6 -
```

```
sin(d*x + c)^5 - sin(d*x + c)^4 + sin(d*x + c)^3))/d
```

Fricas [B] time = 1.60815, size = 892, normalized size = 5.51

$$-\frac{138\,a\cos\left(dx+c\right)^{4}-172\,a\cos\left(dx+c\right)^{2}-144\,\left(a\cos\left(dx+c\right)^{6}-2\,a\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{2}+\left(a\cos\left(dx+c\right)^{4}-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(138*a*cos(d*x + c)^4 - 172*a*cos(d*x + c)^2 - 144*(a*cos(d*x + c)^6
- 2*a*cos(d*x + c)^4 + a*cos(d*x + c)^2 + (a*cos(d*x + c)^4 - a*cos(d*x + c
)^2)*sin(d*x + c))*log(1/2*sin(d*x + c)) - 33*(a*cos(d*x + c)^6 - 2*a*cos(d
*x + c)^4 + a*cos(d*x + c)^2 + (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*
x + c))*log(sin(d*x + c) + 1) + 177*(a*cos(d*x + c)^6 - 2*a*cos(d*x + c)^4
+ a*cos(d*x + c)^2 + (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*x + c))*lo
g(-sin(d*x + c) + 1) - 2*(105*a*cos(d*x + c)^4 - 104*a*cos(d*x + c)^2 + 3*a
)*sin(d*x + c) + 18*a)/(d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4 + d*cos(d*x +
 c)^2 + (d*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37193, size = 201, normalized size = 1.24

$$\frac{66\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-354\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+288\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{6\,(11\,a\sin(dx+c)+13\,a)}{\sin(dx+c)+1}+\frac{3\left(177\,a\sin(dx+c)\right.}{(\sin}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/96*(66*a*log(abs(sin(d*x + c) + 1)) - 354*a*log(abs(sin(d*x + c) - 1)) +
288*a*log(abs(sin(d*x + c))) - 6*(11*a*sin(d*x + c) + 13*a)/(sin(d*x + c) +
 1) + 3*(177*a*sin(d*x + c)^2 - 394*a*sin(d*x + c) + 221*a)/(sin(d*x + c) -
 1)^2 - 16*(33*a*sin(d*x + c)^3 + 18*a*sin(d*x + c)^2 + 3*a*sin(d*x + c) +
2*a)/sin(d*x + c)^3)/d
```

3.861 $\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=119

$$-\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-a^2\cdots$$

```
[Out] (-31*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) -
 (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*Sin[
c + d*x])^2) - (9*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0878471, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-a^2\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-31*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) -
 (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*Sin[
c + d*x])^2) - (9*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^4}{2(a-x)^3}-\frac{9a^3}{4(a-x)^2}+\frac{31a^2}{8(a-x)}-x-\frac{a^2}{8(a+x)}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{2a^2\sin(c+dx)}{d}-a^2\cdots\end{aligned}$$

Mathematica [A] time = 0.246645, size = 75, normalized size = 0.63

$$-\frac{a^2\left(4\sin^2(c+dx)+16\sin(c+dx)-\frac{18}{\sin(c+dx)-1}-\frac{2}{(\sin(c+dx)-1)^2}+31\log(1-\sin(c+dx))+\log(\sin(c+dx)+1)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -(a^2*(31*Log[1 - Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c + d
*x])^2 - 18/(-1 + Sin[c + d*x]) + 16*Sin[c + d*x] + 4*Sin[c + d*x]^2))/(8*d
)
```

Maple [B] time = 0.091, size = 261, normalized size = 2.2

$$\frac{a^2 (\sin (dx+c))^8}{4 d (\cos (dx+c))^4} - \frac{a^2 (\sin (dx+c))^8}{2 d (\cos (dx+c))^2} - \frac{a^2 (\sin (dx+c))^6}{2 d} - \frac{3 (\sin (dx+c))^4 a^2}{4 d} - \frac{3 a^2 (\sin (dx+c))^2}{2 d} - 4 \frac{a^2 \ln (\cos (dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a^2*sin(d*x+c)^8/cos(d*x+c)^2-1/2
*a^2*sin(d*x+c)^6/d-3/4*a^2*sin(d*x+c)^4/d-3/2*a^2*sin(d*x+c)^2/d-4/d*a^2*l
n(cos(d*x+c))+1/2/d*a^2*sin(d*x+c)^7/cos(d*x+c)^4-3/4/d*a^2*sin(d*x+c)^7/co
s(d*x+c)^2-3/4*a^2*sin(d*x+c)^5/d-5/4*a^2*sin(d*x+c)^3/d-15/4*a^2*sin(d*x+c
)/d+15/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*t
an(d*x+c)^2
```

Maxima [A] time = 1.11912, size = 130, normalized size = 1.09

$$-\frac{4 a^2 \sin (dx+c)^2 + a^2 \log (\sin (dx+c)+1) + 31 a^2 \log (\sin (dx+c)-1) + 16 a^2 \sin (dx+c) - \frac{2 \left(9 a^2 \sin(dx+c) - 8 a^2\right)}{\sin(dx+c)^2 - 2 \sin(dx+c) + 1}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(4*a^2*sin(d*x + c)^2 + a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x
 + c) - 1) + 16*a^2*sin(d*x + c) - 2*(9*a^2*sin(d*x + c) - 8*a^2)/(sin(d*x
+ c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.5842, size = 412, normalized size = 3.46

$$\frac{4 a^2 \cos (dx+c)^4 + 22 a^2 \cos (dx+c)^2 - 12 a^2 - \left(a^2 \cos (dx+c)^2 + 2 a^2 \sin (dx+c) - 2 a^2\right) \log (\sin (dx+c)+1) - 31 \left(a^2\right.}{8 \left(d \cos (dx+c)^2 + 2 d \sin (d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(4*a^2*cos(d*x + c)^4 + 22*a^2*cos(d*x + c)^2 - 12*a^2 - (a^2*cos(d*x +
 c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(sin(d*x + c) + 1) - 31*(a^2*cos(d*x
```

```
 + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos
(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27293, size = 138, normalized size = 1.16

$$-\frac{8\,a^2\sin\left(dx+c\right)^2+2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+62\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+32\,a^2\sin\left(dx+c\right)-\frac{93\,a^2\sin(dx+c)^2-150\,a}{(\sin(dx+c}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(8*a^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c) + 1)) + 62*a^2*log
(abs(sin(d*x + c) - 1)) + 32*a^2*sin(d*x + c) - (93*a^2*sin(d*x + c)^2 - 15
0*a^2*sin(d*x + c) + 61*a^2)/(sin(d*x + c) - 1)^2)/d
```

3.862 $\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\sin(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

```
[Out] (-17*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[1 + Sin[c + d*x]])/(8*d) -
 (a^2*Sin[c + d*x])/d + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (7*a^3)/(4*d*(a
- a*Sin[c + d*x]))
```

Rubi [A] time = 0.130506, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\sin(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (-17*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[1 + Sin[c + d*x]])/(8*d) -
 (a^2*Sin[c + d*x])/d + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (7*a^3)/(4*d*(a
- a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^4(c+dx)\,dx = \frac{a^5\,\mathrm{Subst}\left(\int \frac{x^4}{a^4(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\mathrm{Subst}\left(\int \frac{x^4}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\mathrm{Subst}\left(\int \left(-1+\frac{a^3}{2(a-x)^3}-\frac{7a^2}{4(a-x)^2}+\frac{17a}{8(a-x)}+\frac{a}{8(a+x)}\right)dx, x, a\sin(c\right.}{d}$$
$$= -\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{a^2\sin(c+}{d}$$

Mathematica [A] time = 0.119892, size = 67, normalized size = 0.66

$$-\frac{a^2\left(8\sin(c+dx)-\frac{14}{\sin(c+dx)-1}-\frac{2}{(\sin(c+dx)-1)^2}+17\log(1-\sin(c+dx))-\log(\sin(c+dx)+1)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -(a^2*(17*Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c + d
*x])^2 - 14/(-1 + Sin[c + d*x]) + 8*Sin[c + d*x]))/(8*d)
```

Maple [B] time = 0.088, size = 213, normalized size = 2.1

$$\frac{a^2(\sin(dx+c))^7}{4d(\cos(dx+c))^4}-\frac{3a^2(\sin(dx+c))^7}{8d(\cos(dx+c))^2}-\frac{3(\sin(dx+c))^5a^2}{8d}-\frac{3a^2(\sin(dx+c))^3}{4d}-\frac{9a^2\sin(dx+c)}{4d}+\frac{9a^2\ln(\sec(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a^2*sin(d*x+c)^7/cos(d*x+c)^2-3/8
*a^2*sin(d*x+c)^5/d-3/4*a^2*sin(d*x+c)^3/d-9/4*a^2*sin(d*x+c)/d+9/4/d*a^2*l
n(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2*tan(d*x+c)^4-1/d*a^2*tan(d*x+c)^2-2/d*a^
2*ln(cos(d*x+c))+1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^2*sin(d*x+c)^5
/cos(d*x+c)^2
```

Maxima [A] time = 0.98556, size = 112, normalized size = 1.11

$$\frac{a^2\log(\sin(dx+c)+1)-17a^2\log(\sin(dx+c)-1)-8a^2\sin(dx+c)+\frac{2(7a^2\sin(dx+c)-6a^2)}{\sin(dx+c)^2-2\sin(dx+c)+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - 17*a^2*log(sin(d*x + c) - 1) - 8*a^2*sin(d
*x + c) + 2*(7*a^2*sin(d*x + c) - 6*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) +
```

```
1))/d
```

Fricas [A] time = 1.46687, size = 377, normalized size = 3.73

$$\frac{16\,a^2\cos\left(dx+c\right)^2-4\,a^2+\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-17\left(a^2\cos\left(dx+c\right)^2+2\,a^2\right.}{8\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(16*a^2*cos(d*x + c)^2 - 4*a^2 + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x +
c) - 2*a^2)*log(sin(d*x + c) + 1) - 17*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x
+ c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos(d*x + c)^2 - a^2)*sin(d
*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29202, size = 119, normalized size = 1.18

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-34\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-16\,a^2\sin\left(dx+c\right)+\frac{51\,a^2\sin(dx+c)^2-74\,a^2\sin(dx+c)+27\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 34*a^2*log(abs(sin(d*x + c) - 1))
- 16*a^2*sin(d*x + c) + (51*a^2*sin(d*x + c)^2 - 74*a^2*sin(d*x + c) + 27*a
^2)/(sin(d*x + c) - 1)^2)/d
```

3.863 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=87

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

```
[Out] (-7*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) +
a^4/(4*d*(a - a*Sin[c + d*x])^2) - (5*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.138701, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (-7*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) +
a^4/(4*d*(a - a*Sin[c + d*x])^2) - (5*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx &= \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(a-x)^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(a-x)^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^2\,\text{Subst}\left(\int \left(\frac{a^2}{2(a-x)^3}-\frac{5a}{4(a-x)^2}+\frac{7}{8(a-x)}-\frac{1}{8(a+x)}\right)dx,x,a\sin(c+d\right.}{d}\\ &= -\frac{7a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}+\frac{}{4d(a-a\text{ s}}\end{aligned}$$

Mathematica [A] time = 0.381434, size = 91, normalized size = 1.05

$$\frac{a^2\left(3\tanh^{-1}(\sin(c+dx))-6\tan(c+dx)\sec^3(c+dx)+\tan(c+dx)\left(8\tan^2(c+dx)+3\right)\sec(c+dx)-2\left(-\tan^4(c+dx\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(3*ArcTanh[Sin[c + d*x]] - 6*Sec[c + d*x]^3*Tan[c + d*x] + Sec[c + d*x
]*Tan[c + d*x]*(3 + 8*Tan[c + d*x]^2) - 2*(2*Log[Cos[c + d*x]] + Tan[c + d*
x]^2 - Tan[c + d*x]^4)))/(4*d)
```

Maple [B] time = 0.079, size = 173, normalized size = 2.

$$\frac{a^2(\tan(dx+c))^4}{4d}-\frac{a^2(\tan(dx+c))^2}{2d}-\frac{a^2\ln(\cos(dx+c))}{d}+\frac{(\sin(dx+c))^5a^2}{2d(\cos(dx+c))^4}-\frac{(\sin(dx+c))^5a^2}{4d(\cos(dx+c))^2}-\frac{a^2(\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*tan(d*x+c)^2-1/d*a^2*ln(cos(d*x+c))+1/2/d*
a^2*sin(d*x+c)^5/cos(d*x+c)^4-1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2-1/4*a^2*s
in(d*x+c)^3/d-3/4*a^2*sin(d*x+c)/d+3/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/
d*a^2*sin(d*x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 1.08351, size = 97, normalized size = 1.11

$$-\frac{a^2\log(\sin(dx+c)+1)+7a^2\log(\sin(dx+c)-1)-\frac{2\left(5a^2\sin(dx+c)-4a^2\right)}{\sin(dx+c)^2-2\sin(dx+c)+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) + 7*a^2*log(sin(d*x + c) - 1) - 2*(5*a^2*si
n(d*x + c) - 4*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.52263, size = 312, normalized size = 3.59

$$-\frac{10a^2\sin(dx+c)-8a^2+\left(a^2\cos(dx+c)^2+2a^2\sin(dx+c)-2a^2\right)\log(\sin(dx+c)+1)+7\left(a^2\cos(dx+c)^2+2a^2\text{s}\right.}{8\left(d\cos(dx+c)^2+2d\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(10*a^2*sin(d*x + c) - 8*a^2 + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c
) - 2*a^2)*log(sin(d*x + c) + 1) + 7*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x +
c) - 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) -
2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28445, size = 105, normalized size = 1.21

$$-\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+14\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{21\,a^2\sin(dx+c)^2-22\,a^2\sin(dx+c)+5\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) + 14*a^2*log(abs(sin(d*x + c) - 1))
 - (21*a^2*sin(d*x + c)^2 - 22*a^2*sin(d*x + c) + 5*a^2)/(sin(d*x + c) - 1)
^2)/d
```

3.864 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=64

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{3a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (3*a
^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.11848, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2836, 12, 88, 206}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{3a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (3*a
^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^2}{a^2(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{a}{2(a-x)^3} - \frac{3}{4(a-x)^2} + \frac{1}{4(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4}{4d(a-a\sin(c+dx))^2} - \frac{3a^3}{4d(a-a\sin(c+dx))} + \frac{a^3\,\text{Subst}\left(\int \frac{}{a^2}\right.}{}$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{a^4}{4d(a-a\sin(c+dx))^2} - \frac{3a^3}{4d(a-a\sin(}$$

Mathematica [A] time = 0.10705, size = 39, normalized size = 0.61

$$\frac{a^2\left(\frac{3\sin(c+dx)-2}{(\sin(c+dx)-1)^2} + \tanh^{-1}(\sin(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (a^2*(ArcTanh[Sin[c + d*x]] + (-2 + 3*Sin[c + d*x])/(-1 + Sin[c + d*x])^2))
/(4*d)
```

Maple [B] time = 0.075, size = 174, normalized size = 2.7

$$\frac{(\sin(dx+c))^5 a^2}{4d(\cos(dx+c))^4} - \frac{(\sin(dx+c))^5 a^2}{8d(\cos(dx+c))^2} - \frac{a^2(\sin(dx+c))^3}{8d} - \frac{a^2\sin(dx+c)}{4d} + \frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{4d} + \frac{(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2-1/8
*a^2*sin(d*x+c)^3/d-1/4*a^2*sin(d*x+c)/d+1/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/2/d*a^2*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1
/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2
```

Maxima [A] time = 1.08203, size = 97, normalized size = 1.52

$$\frac{a^2\log(\sin(dx+c)+1) - a^2\log(\sin(dx+c)-1) + \frac{2(3a^2\sin(dx+c)-2a^2)}{\sin(dx+c)^2-2\sin(dx+c)+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - a^2*log(sin(d*x + c) - 1) + 2*(3*a^2*sin(d
*x + c) - 2*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [B] time = 1.38907, size = 308, normalized size = 4.81

$$-\frac{6\,a^2\sin\left(dx+c\right)-4\,a^2-\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\right.}{8\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(6*a^2*sin(d*x + c) - 4*a^2 - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
 - 2*a^2)*log(sin(d*x + c) + 1) + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
- 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26176, size = 104, normalized size = 1.62

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{3\,a^2\sin\left(dx+c\right)^2+6\,a^2\sin\left(dx+c\right)-5\,a^2}{\left(\sin\left(dx+c\right)-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 2*a^2*log(abs(sin(d*x + c) - 1)) +
 (3*a^2*sin(d*x + c)^2 + 6*a^2*sin(d*x + c) - 5*a^2)/(sin(d*x + c) - 1)^2)/
d
```

3.865 $\int \sec^4(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=64

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

```
[Out] -(a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) - a^3
/(4*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0869577, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2836, 12, 77, 206}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -(a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) - a^3
/(4*d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{a^5\operatorname{Subst}\left(\int \frac{x}{a(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\operatorname{Subst}\left(\int \frac{x}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\operatorname{Subst}\left(\int \left(\frac{1}{2(a-x)^3}-\frac{1}{4a(a-x)^2}-\frac{1}{4a(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{a^3}{4d(a-a\sin(c+dx))}-\frac{a^3\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,d\right.}{4}$$
$$= -\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{a^3}{4d(a-a\sin(c+}$$

Mathematica [A] time = 0.0800397, size = 36, normalized size = 0.56

$$-\frac{a^2\left(\tanh^{-1}(\sin(c+dx))-\frac{\sin(c+dx)}{(\sin(c+dx)-1)^2}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -(a^2*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x]/(-1 + Sin[c + d*x])^2))/(4*d)
```

Maple [B] time = 0.067, size = 126, normalized size = 2.

$$\frac{(\sin(dx+c))^4a^2}{4d(\cos(dx+c))^4}+\frac{a^2(\sin(dx+c))^3}{2d(\cos(dx+c))^4}+\frac{a^2(\sin(dx+c))^3}{4d(\cos(dx+c))^2}+\frac{a^2\sin(dx+c)}{4d}-\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{4d}+\frac{}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^4/cos(d*x+c)^4+1/2/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/4
/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/4*a^2*sin(d*x+c)/d-1/4/d*a^2*ln(sec(d*x+
c)+tan(d*x+c))+1/4/d*a^2/cos(d*x+c)^4
```

Maxima [A] time = 1.15226, size = 86, normalized size = 1.34

$$-\frac{a^2\log(\sin(dx+c)+1)-a^2\log(\sin(dx+c)-1)-\frac{2a^2\sin(dx+c)}{\sin(dx+c)^2-2\sin(dx+c)+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) - a^2*log(sin(d*x + c) - 1) - 2*a^2*sin(d*x
 + c)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.51917, size = 297, normalized size = 4.64

$$-\frac{2\,a^2\sin\left(dx+c\right)+\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx\right.\right.}{8\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a^2*sin(d*x + c) + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2
)*log(sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)
*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22033, size = 128, normalized size = 2.

$$-\frac{a^2\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)+2\right|\right)-a^2\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)-2\right|\right)+\frac{a^2\left(\frac{1}{\sin(dx+c)}+\sin(dx+c)\right)-6\,a^2}{\frac{1}{\sin(dx+c)}+\sin(dx+c)-2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(a^2*log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - a^2*log(abs(1/sin(
d*x + c) + sin(d*x + c) - 2)) + (a^2*(1/sin(d*x + c) + sin(d*x + c)) - 6*a^
2)/(1/sin(d*x + c) + sin(d*x + c) - 2))/d
```

3.866 $\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{3a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

```
[Out] (-7*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[Sin[c + d*x]])/d - (a^2*Log
[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (3*a^3)/(4*d
*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.114926, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 72}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{3a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[Sin[c + d*x]])/d - (a^2*Log
[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (3*a^3)/(4*d
*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a}{(a-x)^3x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{1}{(a-x)^3x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(\frac{1}{2a^2(a-x)^3}+\frac{3}{4a^3(a-x)^2}+\frac{7}{8a^4(a-x)}+\frac{1}{a^4x}-\frac{1}{8a^4(a+x)}\right) dx, x\right.}{d}$$
$$= -\frac{7a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(1+\sin(}{8d}$$

Mathematica [A] time = 0.292746, size = 66, normalized size = 0.65

$$-\frac{a^2\left(\frac{6}{\sin(c+dx)-1}-\frac{2}{(\sin(c+dx)-1)^2}+7\log(1-\sin(c+dx))-8\log(\sin(c+dx))+\log(\sin(c+dx)+1)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(7*Log[1 - Sin[c + d*x]] - 8*Log[Sin[c + d*x]] + Log[1 + Sin[c + d*x]
] - 2/(-1 + Sin[c + d*x])^2 + 6/(-1 + Sin[c + d*x])))/(8*d)
```

Maple [A] time = 0.109, size = 112, normalized size = 1.1

$$\frac{a^2}{2\,d\,(\cos(dx+c))^4}+\frac{a^2\tan(dx+c)\,(\sec(dx+c))^3}{2\,d}+\frac{3\,a^2\sec(dx+c)\tan(dx+c)}{4\,d}+\frac{3\,a^2\ln(\sec(dx+c)+\tan(dx+}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d*a^2/cos(d*x+c)^4+1/2/d*a^2*tan(d*x+c)*sec(d*x+c)^3+3/4/d*a^2*sec(d*x+
c)*tan(d*x+c)+3/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2/cos(d*x+c)^2+1/
d*a^2*ln(tan(d*x+c))
```

Maxima [A] time = 1.04557, size = 113, normalized size = 1.12

$$-\frac{a^2\log(\sin(dx+c)+1)+7\,a^2\log(\sin(dx+c)-1)-8\,a^2\log(\sin(dx+c))+\frac{2\left(3\,a^2\sin(dx+c)-4\,a^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) + 7*a^2*log(sin(d*x + c) - 1) - 8*a^2*log(s
in(d*x + c)) + 2*(3*a^2*sin(d*x + c) - 4*a^2)/(sin(d*x + c)^2 - 2*sin(d*x +
 c) + 1))/d
```

Fricas [A] time = 1.58213, size = 412, normalized size = 4.08

$$\frac{6\,a^2\sin(dx+c)-8\,a^2+8\left(a^2\cos(dx+c)^2+2\,a^2\sin(dx+c)-2\,a^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-\left(a^2\cos(dx+c)^2+2\,a^2\sin(\ldots\right.}{8\left(d\cos(dx+c)^2+2\,d\sin(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^2*sin(d*x + c) - 8*a^2 + 8*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c
) - 2*a^2)*log(1/2*sin(d*x + c)) - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
 - 2*a^2)*log(sin(d*x + c) + 1) - 7*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c
) - 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23969, size = 123, normalized size = 1.22

$$-\frac{2\,a^2\log\left(|\sin(dx+c)+1|\right)+14\,a^2\log\left(|\sin(dx+c)-1|\right)-16\,a^2\log\left(|\sin(dx+c)|\right)-\frac{21\,a^2\sin(dx+c)^2-54\,a^2\sin(dx+c)+37\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) + 14*a^2*log(abs(sin(d*x + c) - 1))
 - 16*a^2*log(abs(sin(d*x + c))) - (21*a^2*sin(d*x + c)^2 - 54*a^2*sin(d*x
+ c) + 37*a^2)/(sin(d*x + c) - 1)^2)/d
```

3.867 $\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{2a^2\log(\sin(c+dx))}{d}+$$

```
[Out] -((a^2*Csc[c + d*x])/d) - (17*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (2*a^2*Log
[Sin[c + d*x]])/d + (a^2*Log[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin
[c + d*x])^2) + (5*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.141646, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{2a^2\log(\sin(c+dx))}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) - (17*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (2*a^2*Log
[Sin[c + d*x]])/d + (a^2*Log[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin
[c + d*x])^2) + (5*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\operatorname{Subst}\left(\int \frac{a^2}{(a-x)^3x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\operatorname{Subst}\left(\int \frac{1}{(a-x)^3x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\operatorname{Subst}\left(\int \left(\frac{1}{2a^3(a-x)^3}+\frac{5}{4a^4(a-x)^2}+\frac{17}{8a^5(a-x)}+\frac{1}{a^4x^2}+\frac{2}{a^5x}+\frac{1}{8a^5(a+x)}\right)\right.}{d}$$
$$= -\frac{a^2\csc(c+dx)}{d} - \frac{17a^2\log(1-\sin(c+dx))}{8d} + \frac{2a^2\log(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.262242, size = 74, normalized size = 0.64

$$\frac{a^2\left(-\frac{10}{\sin(c+dx)-1}+\frac{2}{(\sin(c+dx)-1)^2}-8\csc(c+dx)-17\log(1-\sin(c+dx))+16\log(\sin(c+dx))+\log(\sin(c+dx)+1)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-8*Csc[c + d*x] - 17*Log[1 - Sin[c + d*x]] + 16*Log[Sin[c + d*x]] + L
og[1 + Sin[c + d*x]] + 2/(-1 + Sin[c + d*x])^2 - 10/(-1 + Sin[c + d*x])))/(
8*d)
```

Maple [A] time = 0.128, size = 176, normalized size = 1.5

$$\frac{a^2\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{3a^2\sec(dx+c)\tan(dx+c)}{8d}+\frac{9a^2\ln(\sec(dx+c)+\tan(dx+c))}{4d}+\frac{a^2}{2d(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^2*sec(d*x+c)*tan(d*x+c)+9/4/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2/cos(d*x+c)^4+1/d*a^2/cos(d*x+c)^2+2/d*
a^2*ln(tan(d*x+c))+1/4/d*a^2/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a^2/sin(d*x+c)/c
os(d*x+c)^2-15/8/d*a^2/sin(d*x+c)
```

Maxima [A] time = 1.05377, size = 140, normalized size = 1.21

$$\frac{a^2\log(\sin(dx+c)+1)-17a^2\log(\sin(dx+c)-1)+16a^2\log(\sin(dx+c))-\frac{2\left(9a^2\sin(dx+c)^2-14a^2\sin(dx+c)+4a^2\right)}{\sin(dx+c)^3-2\sin(dx+c)^2+\sin(dx+c)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - 17*a^2*log(sin(d*x + c) - 1) + 16*a^2*log(
sin(d*x + c)) - 2*(9*a^2*sin(d*x + c)^2 - 14*a^2*sin(d*x + c) + 4*a^2)/(sin
```

```
(d*x + c)^3 - 2*sin(d*x + c)^2 + sin(d*x + c)))/d
```

Fricas [B] time = 1.48133, size = 582, normalized size = 5.02

$$\frac{18\,a^2\cos\left(dx+c\right)^2 + 28\,a^2\sin\left(dx+c\right) - 26\,a^2 + 16\left(2\,a^2\cos\left(dx+c\right)^2 - 2\,a^2 - \left(a^2\cos\left(dx+c\right)^2 - 2\,a^2\right)\sin\left(dx+c\right)\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(18*a^2*cos(d*x + c)^2 + 28*a^2*sin(d*x + c) - 26*a^2 + 16*(2*a^2*cos(d
*x + c)^2 - 2*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2)*sin(d*x + c))*log(1/2*sin(
d*x + c)) + (2*a^2*cos(d*x + c)^2 - 2*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2)*si
n(d*x + c))*log(sin(d*x + c) + 1) - 17*(2*a^2*cos(d*x + c)^2 - 2*a^2 - (a^2
*cos(d*x + c)^2 - 2*a^2)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(2*d*cos(d*x
 + c)^2 - (d*cos(d*x + c)^2 - 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24168, size = 155, normalized size = 1.34

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right) - 34\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right) + 32\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right) - \frac{16\left(2\,a^2\sin(dx+c)+a^2\right)}{\sin(dx+c)} + \frac{51\,a^2\sin(dx+c)}{(\mathrm{si}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 34*a^2*log(abs(sin(d*x + c) - 1))
+ 32*a^2*log(abs(sin(d*x + c))) - 16*(2*a^2*sin(d*x + c) + a^2)/sin(d*x + c
) + (51*a^2*sin(d*x + c)^2 - 122*a^2*sin(d*x + c) + 75*a^2)/(sin(d*x + c) -
 1)^2)/d
```

3.868 $\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=134

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}+\frac{4a^2\log}{}$$

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - (31*a^2*Log[1 - Sin[
c + d*x]])/(8*d) + (4*a^2*Log[Sin[c + d*x]])/d - (a^2*Log[1 + Sin[c + d*x]]
)/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (7*a^3)/(4*d*(a - a*Sin[c + d*
x]))
```

Rubi [A] time = 0.148136, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}+\frac{4a^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - (31*a^2*Log[1 - Sin[
c + d*x]])/(8*d) + (4*a^2*Log[Sin[c + d*x]])/d - (a^2*Log[1 + Sin[c + d*x]]
)/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (7*a^3)/(4*d*(a - a*Sin[c + d*
x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^3}{(a-x)^3x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \left(\frac{1}{2a^4(a-x)^3} + \frac{7}{4a^5(a-x)^2} + \frac{31}{8a^6(a-x)} + \frac{1}{a^4x^3} + \frac{2}{a^5x^2} + \frac{4}{a^6x} - \right.\right.}{d}$$
$$= -\frac{2a^2\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{31a^2\log(1-\sin(c+dx))}{8d}$$

Mathematica [A] time = 1.18508, size = 84, normalized size = 0.63

$$-\frac{a^2\left(\frac{14}{\sin(c+dx)-1} - \frac{2}{(\sin(c+dx)-1)^2} + 4\csc^2(c+dx) + 16\csc(c+dx) + 31\log(1-\sin(c+dx)) - 32\log(\sin(c+dx)) + \log(\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(16*Csc[c + d*x] + 4*Csc[c + d*x]^2 + 31*Log[1 - Sin[c + d*x]] - 32*L
og[Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c + d*x])^2 + 14/(-1
 + Sin[c + d*x])))/(8*d)
```

Maple [A] time = 0.141, size = 199, normalized size = 1.5

$$\frac{a^2}{4\,d\,(\cos(dx+c))^4} + \frac{a^2}{2\,d\,(\cos(dx+c))^2} + 4\,\frac{a^2\ln(\tan(dx+c))}{d} + \frac{a^2}{2\,d\sin(dx+c)\,(\cos(dx+c))^4} + \frac{5\,a^2}{4\,d\sin(dx+c)\,(\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/cos(d*x+c)^4+1/2/d*a^2/cos(d*x+c)^2+4/d*a^2*ln(tan(d*x+c))+1/2/d*
a^2/sin(d*x+c)/cos(d*x+c)^4+5/4/d*a^2/sin(d*x+c)/cos(d*x+c)^2-15/4/d*a^2/si
n(d*x+c)+15/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^2/sin(d*x+c)^2/cos(d*
x+c)^4+3/4/d*a^2/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a^2/sin(d*x+c)^2
```

Maxima [A] time = 0.992041, size = 161, normalized size = 1.2

$$-\frac{a^2\log(\sin(dx+c)+1) + 31\,a^2\log(\sin(dx+c)-1) - 32\,a^2\log(\sin(dx+c)) + \frac{2\left(15\,a^2\sin(dx+c)^3 - 22\,a^2\sin(dx+c)^2 + 4\,a^2\sin(\right.}{\sin(dx+c)^4 - 2\,\sin(dx+c)^3 + \sin(dx+c}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x + c) - 1) - 32*a^2*log
(sin(d*x + c)) + 2*(15*a^2*sin(d*x + c)^3 - 22*a^2*sin(d*x + c)^2 + 4*a^2*s
```

```
in(d*x + c) + 2*a^2)/(sin(d*x + c)^4 - 2*sin(d*x + c)^3 + sin(d*x + c)^2))/
d
```

Fricas [B] time = 1.5414, size = 732, normalized size = 5.46

$$-\frac{44\,a^2\cos(dx+c)^2 - 40\,a^2 - 32\left(a^2\cos(dx+c)^4 - 3\,a^2\cos(dx+c)^2 + 2\,a^2 + 2\left(a^2\cos(dx+c)^2 - a^2\right)\sin(dx+c)\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(44*a^2*cos(d*x + c)^2 - 40*a^2 - 32*(a^2*cos(d*x + c)^4 - 3*a^2*cos(d
*x + c)^2 + 2*a^2 + 2*(a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))*log(1/2*sin(
d*x + c)) + (a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 + 2*a^2 + 2*(a^2*cos
(d*x + c)^2 - a^2)*sin(d*x + c))*log(sin(d*x + c) + 1) + 31*(a^2*cos(d*x +
c)^4 - 3*a^2*cos(d*x + c)^2 + 2*a^2 + 2*(a^2*cos(d*x + c)^2 - a^2)*sin(d*x
+ c))*log(-sin(d*x + c) + 1) - 2*(15*a^2*cos(d*x + c)^2 - 19*a^2)*sin(d*x +
 c))/(d*cos(d*x + c)^4 - 3*d*cos(d*x + c)^2 + 2*(d*cos(d*x + c)^2 - d)*sin(
d*x + c) + 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27012, size = 169, normalized size = 1.26

$$-\frac{4\,a^2\log\left(|\sin(dx+c)+1|\right) + 124\,a^2\log\left(|\sin(dx+c)-1|\right) - 128\,a^2\log\left(|\sin(dx+c)|\right) + \frac{3\,a^2\sin(dx+c)^4 + 114\,a^2\sin(dx+c)^3 - 173\,a^2}{\left(\sin(dx+c)^2 - \sin\right.}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*a^2*log(abs(sin(d*x + c) + 1)) + 124*a^2*log(abs(sin(d*x + c) - 1)
) - 128*a^2*log(abs(sin(d*x + c))) + (3*a^2*sin(d*x + c)^4 + 114*a^2*sin(d*
x + c)^3 - 173*a^2*sin(d*x + c)^2 + 32*a^2*sin(d*x + c) + 16*a^2)/(sin(d*x
+ c)^2 - sin(d*x + c))^2)/d
```

3.869 $\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=150

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{4a^2\csc(c+dx)}{d}-\frac{49a^2\log(1-s}{8d}$$

```
[Out] (-4*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]^3)/(3*
d) - (49*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (6*a^2*Log[Sin[c + d*x]])/d + (
a^2*Log[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (9*a^
3)/(4*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.164349, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{4a^2\csc(c+dx)}{d}-\frac{49a^2\log(1-s}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]^3)/(3*
d) - (49*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (6*a^2*Log[Sin[c + d*x]])/d + (
a^2*Log[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (9*a^
3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^4}{(a-x)^3x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \left(\frac{1}{2a^5(a-x)^3} + \frac{9}{4a^6(a-x)^2} + \frac{49}{8a^7(a-x)} + \frac{1}{a^4x^4} + \frac{2}{a^5x^3} + \frac{4}{a^6x^2} + \frac{6}{a^7 \cdots}\right.\right.}{d}$$
$$= -\frac{4a^2\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{d} - \frac{a^2\csc^3(c+dx)}{3d} - \frac{49a^2\log(1 \cdots}{\cdots}$$

Mathematica [A] time = 6.05746, size = 133, normalized size = 0.89

$$\frac{a^9\left(\frac{9}{4a^6(a-a\sin(c+dx))} + \frac{1}{4a^5(a-a\sin(c+dx))^2} - \frac{\csc^3(c+dx)}{3a^7} - \frac{\csc^2(c+dx)}{a^7} - \frac{4\csc(c+dx)}{a^7} - \frac{49\log(1-\sin(c+dx))}{8a^7} + \frac{6\log(\sin(c+dx))}{a^7} + \frac{\log(\sin(c+dx) \cdots}{8a^7}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^9*((-4*Csc[c + d*x])/a^7 - Csc[c + d*x]^2/a^7 - Csc[c + d*x]^3/(3*a^7) -
 (49*Log[1 - Sin[c + d*x]])/(8*a^7) + (6*Log[Sin[c + d*x]])/a^7 + Log[1 + S
in[c + d*x]]/(8*a^7) + 1/(4*a^5*(a - a*Sin[c + d*x])^2) + 9/(4*a^6*(a - a*S
in[c + d*x]))))/d
```

Maple [A] time = 0.152, size = 215, normalized size = 1.4

$$\frac{a^2}{4\,d\sin(dx+c)(\cos(dx+c))^4} + \frac{25\,a^2}{12\,d\sin(dx+c)(\cos(dx+c))^2} - \frac{25\,a^2}{4\,d\sin(dx+c)} + \frac{25\,a^2\ln(\sec(dx+c)+\tan(dx+c \cdots}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/sin(d*x+c)/cos(d*x+c)^4+25/12/d*a^2/sin(d*x+c)/cos(d*x+c)^2-25/4/
d*a^2/sin(d*x+c)+25/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2/sin(d*x+c)^
2/cos(d*x+c)^4+3/2/d*a^2/sin(d*x+c)^2/cos(d*x+c)^2-3/d*a^2/sin(d*x+c)^2+6/d
*a^2*ln(tan(d*x+c))+1/4/d*a^2/sin(d*x+c)^3/cos(d*x+c)^4-7/12/d*a^2/sin(d*x+
c)^3/cos(d*x+c)^2
```

Maxima [A] time = 1.13435, size = 180, normalized size = 1.2

$$\frac{3\,a^2\log(\sin(dx+c)+1) - 147\,a^2\log(\sin(dx+c)-1) + 144\,a^2\log(\sin(dx+c)) - \frac{2\left(75\,a^2\sin(dx+c)^4 - 114\,a^2\sin(dx+c)^3 + 28\,a^2\sin \cdots\right.}{\sin(dx+c)^5 - 2\sin(dx+c)^4 + \text{s} \cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*(3*a^2*log(sin(d*x + c) + 1) - 147*a^2*log(sin(d*x + c) - 1) + 144*a^2
*log(sin(d*x + c)) - 2*(75*a^2*sin(d*x + c)^4 - 114*a^2*sin(d*x + c)^3 + 28
*a^2*sin(d*x + c)^2 + 4*a^2*sin(d*x + c) + 4*a^2)/(sin(d*x + c)^5 - 2*sin(d
*x + c)^4 + sin(d*x + c)^3))/d
```

Fricas [B] time = 1.65177, size = 906, normalized size = 6.04

$$150\,a^2\cos\left(dx+c\right)^4 - 356\,a^2\cos\left(dx+c\right)^2 + 214\,a^2 + 144\left(2\,a^2\cos\left(dx+c\right)^4 - 4\,a^2\cos\left(dx+c\right)^2 + 2\,a^2 - \left(a^2\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(150*a^2*cos(d*x + c)^4 - 356*a^2*cos(d*x + c)^2 + 214*a^2 + 144*(2*a^
2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + 2*a^2 - (a^2*cos(d*x + c)^4 - 3*a
^2*cos(d*x + c)^2 + 2*a^2)*sin(d*x + c))*log(1/2*sin(d*x + c)) + 3*(2*a^2*c
os(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + 2*a^2 - (a^2*cos(d*x + c)^4 - 3*a^2*
cos(d*x + c)^2 + 2*a^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 147*(2*a^2*co
s(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + 2*a^2 - (a^2*cos(d*x + c)^4 - 3*a^2*c
os(d*x + c)^2 + 2*a^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 4*(57*a^2*cos
(d*x + c)^2 - 55*a^2)*sin(d*x + c))/(2*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^
2 - (d*cos(d*x + c)^4 - 3*d*cos(d*x + c)^2 + 2*d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25799, size = 192, normalized size = 1.28

$$\frac{6\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right) - 294\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right) + 288\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right) + \frac{3\left(147\,a^2\sin(dx+c)^2 - 330\,a^2\sin(dx+c)\cdots\right.}{(\sin(dx+c)-1)^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(6*a^2*log(abs(sin(d*x + c) + 1)) - 294*a^2*log(abs(sin(d*x + c) - 1))
 + 288*a^2*log(abs(sin(d*x + c))) + 3*(147*a^2*sin(d*x + c)^2 - 330*a^2*sin
(d*x + c) + 187*a^2)/(sin(d*x + c) - 1)^2 - 16*(33*a^2*sin(d*x + c)^3 + 12*
a^2*sin(d*x + c)^2 + 3*a^2*sin(d*x + c) + a^2)/sin(d*x + c)^3)/d
```

3.870 $\int (a+a\sin(c+dx))^3\tan^5(c+dx)\,dx$

Optimal. Leaf size=114

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{5a^4}{d(a-a\sin(c+dx))}-\frac{6a^3\sin(c+dx)}{d}-\frac{10a^3\log(1-\sin}{d}$$

```
[Out] (-10*a^3*Log[1 - Sin[c + d*x]])/d - (6*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c +
 d*x]^2)/(2*d) - (a^3*Sin[c + d*x]^3)/(3*d) + a^5/(2*d*(a - a*Sin[c + d*x])
^2) - (5*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0840697, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{5a^4}{d(a-a\sin(c+dx))}-\frac{6a^3\sin(c+dx)}{d}-\frac{10a^3\log(1-\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] (-10*a^3*Log[1 - Sin[c + d*x]])/d - (6*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c +
 d*x]^2)/(2*d) - (a^3*Sin[c + d*x]^3)/(3*d) + a^5/(2*d*(a - a*Sin[c + d*x])
^2) - (5*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^3\tan^5(c+dx)\,dx&=\frac{\text{Subst}\left(\int\frac{x^5}{(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(-6a^2+\frac{a^5}{(a-x)^3}-\frac{5a^4}{(a-x)^2}+\frac{10a^3}{a-x}-3ax-x^2\right)dx,x,a\sin(c+dx)\right)}{d}\\&=-\frac{10a^3\log(1-\sin(c+dx))}{d}-\frac{6a^3\sin(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{a^3\sin^3(c+d}{3d}\end{aligned}$$

Mathematica [A] time = 0.306308, size = 73, normalized size = 0.64

$$-\frac{a^3\left(2\sin^3(c+dx)+9\sin^2(c+dx)+36\sin(c+dx)+\frac{27-30\sin(c+dx)}{(\sin(c+dx)-1)^2}+60\log(1-\sin(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -(a^3*(60*Log[1 - Sin[c + d*x]] + (27 - 30*Sin[c + d*x])/(-1 + Sin[c + d*x]
)^2 + 36*Sin[c + d*x] + 9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/(6*d)
```

Maple [B] time = 0.1, size = 325, normalized size = 2.9

$$\frac{a^3 (\sin (dx+c))^9}{4 d (\cos (dx+c))^4}-\frac{5 a^3 (\sin (dx+c))^9}{8 d (\cos (dx+c))^2}-\frac{5 a^3 (\sin (dx+c))^7}{8 d}-2 \frac{a^3 (\sin (dx+c))^5}{d}-\frac{10 a^3 (\sin (dx+c))^3}{3 d}-10 \frac{a^3 \sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^9/cos(d*x+c)^4-5/8/d*a^3*sin(d*x+c)^9/cos(d*x+c)^2-5/8
*a^3*sin(d*x+c)^7/d-2*a^3*sin(d*x+c)^5/d-10/3*a^3*sin(d*x+c)^3/d-10*a^3*sin
(d*x+c)/d+10/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^3*sin(d*x+c)^8/cos(d*x
+c)^4-3/2/d*a^3*sin(d*x+c)^8/cos(d*x+c)^2-3/2*a^3*sin(d*x+c)^6/d-9/4*a^3*si
n(d*x+c)^4/d-9/2*a^3*sin(d*x+c)^2/d-10/d*a^3*ln(cos(d*x+c))+3/4/d*a^3*sin(d
*x+c)^7/cos(d*x+c)^4-9/8/d*a^3*sin(d*x+c)^7/cos(d*x+c)^2+1/4/d*a^3*tan(d*x+
c)^4-1/2/d*a^3*tan(d*x+c)^2
```

Maxima [A] time = 1.07241, size = 130, normalized size = 1.14

$$-\frac{2 a^3 \sin (dx+c)^3+9 a^3 \sin (dx+c)^2+60 a^3 \log (\sin (dx+c)-1)+36 a^3 \sin (dx+c)-\frac{3 \left(10 a^3 \sin (dx+c)-9 a^3\right)}{\sin (dx+c)^2-2 \sin (dx+c)+1}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 60*a^3*log(sin(d*x + c)
 - 1) + 36*a^3*sin(d*x + c) - 3*(10*a^3*sin(d*x + c) - 9*a^3)/(sin(d*x + c)
^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.44393, size = 351, normalized size = 3.08

$$\frac{10 a^3 \cos (dx+c)^4+115 a^3 \cos (dx+c)^2-80 a^3-120 \left(a^3 \cos (dx+c)^2+2 a^3 \sin (dx+c)-2 a^3\right) \log (-\sin (dx+c)+}{12 \left(d \cos (dx+c)^2+2 d \sin (dx+c)-2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(10*a^3*cos(d*x + c)^4 + 115*a^3*cos(d*x + c)^2 - 80*a^3 - 120*(a^3*co
s(d*x + c)^2 + 2*a^3*sin(d*x + c) - 2*a^3)*log(-sin(d*x + c) + 1) + 2*(2*a^
```

```
3*cos(d*x + c)^4 - 24*a^3*cos(d*x + c)^2 + 37*a^3)*sin(d*x + c))/(d*cos(d*x
 + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25699, size = 327, normalized size = 2.87

$$\frac{30\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-60\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+183\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(30*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 60*a^3*log(abs(tan(1/2*d*x +
1/2*c) - 1)) - (55*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)
^5 + 183*a^3*tan(1/2*d*x + 1/2*c)^4 + 80*a^3*tan(1/2*d*x + 1/2*c)^3 + 183*a
^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*tan(1/2*d*x + 1/2*c) + 55*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^3 + (125*a^3*tan(1/2*d*x + 1/2*c)^4 - 524*a^3*tan(1/2*d
*x + 1/2*c)^3 + 804*a^3*tan(1/2*d*x + 1/2*c)^2 - 524*a^3*tan(1/2*d*x + 1/2*
c) + 125*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

3.871 $\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=96

$$-\frac{a^3\sin^2(c+dx)}{2d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{4a^4}{d(a-a\sin(c+dx))}-\frac{3a^3\sin(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] (-6*a^3*Log[1 - Sin[c + d*x]])/d - (3*a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*
x]^2)/(2*d) + a^5/(2*d*(a - a*Sin[c + d*x])^2) - (4*a^4)/(d*(a - a*Sin[c +
d*x]))
```

Rubi [A] time = 0.110984, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 43}

$$-\frac{a^3\sin^2(c+dx)}{2d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{4a^4}{d(a-a\sin(c+dx))}-\frac{3a^3\sin(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (-6*a^3*Log[1 - Sin[c + d*x]])/d - (3*a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*
x]^2)/(2*d) + a^5/(2*d*(a - a*Sin[c + d*x])^2) - (4*a^4)/(d*(a - a*Sin[c +
d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^4(c+dx)\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{x^4}{a^4(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(-3a+\frac{a^4}{(a-x)^3}-\frac{4a^3}{(a-x)^2}+\frac{6a^2}{a-x}-x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{6a^3\log(1-\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\sin^2(c+dx)}{2d}+\frac{}{2d(}$$

Mathematica [A] time = 0.236321, size = 61, normalized size = 0.64

$$-\frac{a^3\left(\sin^2(c+dx)+6\sin(c+dx)+\frac{7-8\sin(c+dx)}{(\sin(c+dx)-1)^2}+12\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] -(a^3*(12*Log[1 - Sin[c + d*x]] + (7 - 8*Sin[c + d*x])/(-1 + Sin[c + d*x])^
2 + 6*Sin[c + d*x] + Sin[c + d*x]^2))/(2*d)
```

Maple [B] time = 0.095, size = 309, normalized size = 3.2

$$\frac{a^3(\sin(dx+c))^8}{4d(\cos(dx+c))^4}-\frac{a^3(\sin(dx+c))^8}{2d(\cos(dx+c))^2}-\frac{a^3(\sin(dx+c))^6}{2d}-\frac{3a^3(\sin(dx+c))^4}{4d}-\frac{3a^3(\sin(dx+c))^2}{2d}-6\frac{a^3\ln(\cos(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a^3*sin(d*x+c)^8/cos(d*x+c)^2-1/2
*a^3*sin(d*x+c)^6/d-3/4*a^3*sin(d*x+c)^4/d-3/2*a^3*sin(d*x+c)^2/d-6/d*a^3*l
n(cos(d*x+c))+3/4/d*a^3*sin(d*x+c)^7/cos(d*x+c)^4-9/8/d*a^3*sin(d*x+c)^7/co
s(d*x+c)^2-9/8*a^3*sin(d*x+c)^5/d-2*a^3*sin(d*x+c)^3/d-6*a^3*sin(d*x+c)/d+6
/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^3*tan(d*x+c)^4-3/2/d*a^3*tan(d*x+c
)^2+1/4/d*a^3*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2
```

Maxima [A] time = 1.00627, size = 111, normalized size = 1.16

$$-\frac{a^3\sin(dx+c)^2+12\,a^3\log(\sin(dx+c)-1)+6\,a^3\sin(dx+c)-\frac{8\,a^3\sin(dx+c)-7\,a^3}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(a^3*sin(d*x + c)^2 + 12*a^3*log(sin(d*x + c) - 1) + 6*a^3*sin(d*x + c
) - (8*a^3*sin(d*x + c) - 7*a^3)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.49283, size = 311, normalized size = 3.24

$$\frac{2\,a^3\cos(dx+c)^4+19\,a^3\cos(dx+c)^2-8\,a^3-24\left(a^3\cos(dx+c)^2+2\,a^3\sin(dx+c)-2\,a^3\right)\log(-\sin(dx+c)+1)-\cdots}{4\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(2*a^3*cos(d*x + c)^4 + 19*a^3*cos(d*x + c)^2 - 8*a^3 - 24*(a^3*cos(d*x
 + c)^2 + 2*a^3*sin(d*x + c) - 2*a^3)*log(-sin(d*x + c) + 1) - 2*(4*a^3*cos
(d*x + c)^2 - 3*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20934, size = 282, normalized size = 2.94

$$\frac{6\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (6*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - (9*a^3*tan(1/2*d*x + 1/2*c)^4 + 6*a^3*tan(1/2*d*x + 1/2*c)^3 + 20
*a^3*tan(1/2*d*x + 1/2*c)^2 + 6*a^3*tan(1/2*d*x + 1/2*c) + 9*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^2 + (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 106*a^3*tan(1/2*d*
x + 1/2*c)^3 + 164*a^3*tan(1/2*d*x + 1/2*c)^2 - 106*a^3*tan(1/2*d*x + 1/2*c
) + 25*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

3.872 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=78

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\sin(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] (-3*a^3*Log[1 - Sin[c + d*x]])/d - (a^3*Sin[c + d*x])/d + a^5/(2*d*(a - a*S
in[c + d*x])^2) - (3*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.118999, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\sin(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (-3*a^3*Log[1 - Sin[c + d*x]])/d - (a^3*Sin[c + d*x])/d + a^5/(2*d*(a - a*S
in[c + d*x])^2) - (3*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx &= \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^2\,\text{Subst}\left(\int \left(-1+\frac{a^3}{(a-x)^3}-\frac{3a^2}{(a-x)^2}+\frac{3a}{a-x}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{3a^3\log(1-\sin(c+dx))}{d}-\frac{a^3\sin(c+dx)}{d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 0.219748, size = 53, normalized size = 0.68

$$-\frac{a^3\left(2\sin(c+dx)+\frac{5-6\sin(c+dx)}{(\sin(c+dx)-1)^2}+6\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -(a^3*(6*Log[1 - Sin[c + d*x]] + (5 - 6*Sin[c + d*x])/(-1 + Sin[c + d*x])^2
 + 2*Sin[c + d*x]))/(2*d)
```

Maple [B] time = 0.091, size = 237, normalized size = 3.

$$\frac{a^3(\sin(dx+c))^7}{4d(\cos(dx+c))^4}-\frac{3a^3(\sin(dx+c))^7}{8d(\cos(dx+c))^2}-\frac{3a^3(\sin(dx+c))^5}{8d}-\frac{a^3(\sin(dx+c))^3}{d}-3\frac{a^3\sin(dx+c)}{d}+3\frac{a^3\ln(\sec(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a^3*sin(d*x+c)^7/cos(d*x+c)^2-3/8
*a^3*sin(d*x+c)^5/d-a^3*sin(d*x+c)^3/d-3*a^3*sin(d*x+c)/d+3/d*a^3*ln(sec(d*
x+c)+tan(d*x+c))+3/4/d*a^3*tan(d*x+c)^4-3/2/d*a^3*tan(d*x+c)^2-3/d*a^3*ln(c
os(d*x+c))+3/4/d*a^3*sin(d*x+c)^5/cos(d*x+c)^4-3/8/d*a^3*sin(d*x+c)^5/cos(d
*x+c)^2+1/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 0.991603, size = 95, normalized size = 1.22

$$-\frac{6a^3\log(\sin(dx+c)-1)+2a^3\sin(dx+c)-\frac{6a^3\sin(dx+c)-5a^3}{\sin(dx+c)^2-2\sin(dx+c)+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(6*a^3*log(sin(d*x + c) - 1) + 2*a^3*sin(d*x + c) - (6*a^3*sin(d*x + c
) - 5*a^3)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.44588, size = 269, normalized size = 3.45

$$\frac{4a^3\cos(dx+c)^2+a^3-6\left(a^3\cos(dx+c)^2+2a^3\sin(dx+c)-2a^3\right)\log(-\sin(dx+c)+1)-2\left(a^3\cos(dx+c)^2+a^3\right.}{2\left(d\cos(dx+c)^2+2d\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(4*a^3*cos(d*x + c)^2 + a^3 - 6*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x + c
) - 2*a^3)*log(-sin(d*x + c) + 1) - 2*(a^3*cos(d*x + c)^2 + a^3)*sin(d*x +
c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21339, size = 240, normalized size = 3.08

$$\frac{6\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{25\,a^3\tan\left(\frac{1}{2}\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*a^3*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^2 + 2*a^3*tan(1/2*d*x + 1/2*c)
+ 3*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1) + (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 10
8*a^3*tan(1/2*d*x + 1/2*c)^3 + 170*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a^3*tan
(1/2*d*x + 1/2*c) + 25*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

3.873 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=64

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] -((a^3*Log[1 - Sin[c + d*x]])/d) + a^5/(2*d*(a - a*Sin[c + d*x])^2) - (2*a^
4)/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.110029, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 43}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -((a^3*Log[1 - Sin[c + d*x]])/d) + a^5/(2*d*(a - a*Sin[c + d*x])^2) - (2*a^
4)/(d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx &= \frac{a^5\operatorname{Subst}\left(\int \frac{x^2}{a^2(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^3\operatorname{Subst}\left(\int \frac{x^2}{(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^3\operatorname{Subst}\left(\int \left(\frac{a^2}{(a-x)^3}-\frac{2a}{(a-x)^2}+\frac{1}{a-x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{2a^4}{d(a-a\sin(}\end{aligned}$$

Mathematica [A] time = 0.144288, size = 45, normalized size = 0.7

$$-\frac{a^3\left(\frac{3-4\sin(c+dx)}{(\sin(c+dx)-1)^2}+2\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -(a^3*(2*Log[1 - Sin[c + d*x]] + (3 - 4*Sin[c + d*x])/(-1 + Sin[c + d*x])^2
))/(2*d)
```

Maple [B] time = 0.083, size = 220, normalized size = 3.4

$$\frac{a^3(\tan(dx+c))^4}{4d}-\frac{a^3(\tan(dx+c))^2}{2d}-\frac{a^3\ln(\cos(dx+c))}{d}+\frac{3a^3(\sin(dx+c))^5}{4d(\cos(dx+c))^4}-\frac{3a^3(\sin(dx+c))^5}{8d(\cos(dx+c))^2}-\frac{3a^3(\sin(dx+}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)^4-1/2/d*a^3*tan(d*x+c)^2-1/d*a^3*ln(cos(d*x+c))+3/4/d*
a^3*sin(d*x+c)^5/cos(d*x+c)^4-3/8/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2-3/8*a^3*s
in(d*x+c)^3/d-a^3*sin(d*x+c)/d+1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^3*
sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^3*sin
(d*x+c)^3/cos(d*x+c)^2
```

Maxima [A] time = 1.08402, size = 80, normalized size = 1.25

$$-\frac{2a^3\log(\sin(dx+c)-1)-\frac{4a^3\sin(dx+c)-3a^3}{\sin(dx+c)^2-2\sin(dx+c)+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*a^3*log(sin(d*x + c) - 1) - (4*a^3*sin(d*x + c) - 3*a^3)/(sin(d*x +
 c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.44517, size = 211, normalized size = 3.3

$$-\frac{4a^3\sin(dx+c)-3a^3+2\left(a^3\cos(dx+c)^2+2a^3\sin(dx+c)-2a^3\right)\log(-\sin(dx+c)+1)}{2\left(d\cos(dx+c)^2+2d\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(4*a^3*sin(d*x + c) - 3*a^3 + 2*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x +
c) - 2*a^3)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) -
2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27287, size = 169, normalized size = 2.64

$$\frac{6\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-112\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+186\,a^3\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*a^3*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) + (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 112*a^3*tan(1/2*d*x + 1/2*c)
^3 + 186*a^3*tan(1/2*d*x + 1/2*c)^2 - 112*a^3*tan(1/2*d*x + 1/2*c) + 25*a^3
)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

3.874 $\int \sec^4(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=31

$$\frac{a^5\sin^2(c+dx)}{2d(a-a\sin(c+dx))^2}$$

```
[Out] (a^5*Sin[c + d*x]^2)/(2*d*(a - a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.059388, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 37}

$$\frac{a^5\sin^2(c+dx)}{2d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (a^5*Sin[c + d*x]^2)/(2*d*(a - a*Sin[c + d*x])^2)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx &= \frac{a^5\,\text{Subst}\left(\int \frac{x}{a(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^4\,\text{Subst}\left(\int \frac{x}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^5\sin^2(c+dx)}{2d(a-a\sin(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 0.0304367, size = 30, normalized size = 0.97

$$\frac{a^3\sin^2(c+dx)}{2d(1-\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (a^3*Sin[c + d*x]^2)/(2*d*(1 - Sin[c + d*x])^2)
```

Maple [B] time = 0.073, size = 154, normalized size = 5.

$$\frac{a^3(\sin(dx+c))^5}{4d(\cos(dx+c))^4}-\frac{a^3(\sin(dx+c))^5}{8d(\cos(dx+c))^2}-\frac{a^3(\sin(dx+c))^3}{8d}+\frac{3a^3(\sin(dx+c))^4}{4d(\cos(dx+c))^4}+\frac{3a^3(\sin(dx+c))^3}{4d(\cos(dx+c))^4}+\frac{3a^3(\sin(dx}{8d(\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2-1/8
*a^3*sin(d*x+c)^3/d+3/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3/4/d*a^3*sin(d*x+c
)^3/cos(d*x+c)^4+3/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+1/4/d*a^3/cos(d*x+c)^4
```

Maxima [A] time = 1.10199, size = 57, normalized size = 1.84

$$\frac{2a^3\sin(dx+c)-a^3}{2\left(\sin(dx+c)^2-2\sin(dx+c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*sin(d*x + c) - a^3)/((sin(d*x + c)^2 - 2*sin(d*x + c) + 1)*d)
```

Fricas [A] time = 1.32256, size = 104, normalized size = 3.35

$$-\frac{2a^3\sin(dx+c)-a^3}{2\left(d\cos(dx+c)^2+2d\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*sin(d*x + c) - a^3)/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19195, size = 43, normalized size = 1.39

$$\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*a^3*tan(1/2*d*x + 1/2*c)^2/(d*(tan(1/2*d*x + 1/2*c) - 1)^4)
```

3.875 $\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=77

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

```
[Out] -((a^3*Log[1 - Sin[c + d*x]])/d) + (a^3*Log[Sin[c + d*x]])/d + a^5/(2*d*(a
- a*Sin[c + d*x])^2) + a^4/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.105017, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 44}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Log[1 - Sin[c + d*x]])/d) + (a^3*Log[Sin[c + d*x]])/d + a^5/(2*d*(a
- a*Sin[c + d*x])^2) + a^4/(d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{a^5\,\text{Subst}\left(\int \frac{a}{(a-x)^3x}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^6\,\text{Subst}\left(\int \frac{1}{(a-x)^3x}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^6\,\text{Subst}\left(\int \left(\frac{1}{a(a-x)^3}+\frac{1}{a^2(a-x)^2}+\frac{1}{a^3(a-x)}+\frac{1}{a^3x}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{a^5}{2d(a-a\sin(c+
\end{aligned}$$

Mathematica [A] time = 0.220527, size = 54, normalized size = 0.7

$$\frac{a^3\left(\frac{3-2\sin(c+dx)}{(\sin(c+dx)-1)^2}-2\log(1-\sin(c+dx))+2\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-2*Log[1 - Sin[c + d*x]] + 2*Log[Sin[c + d*x]] + (3 - 2*Sin[c + d*x])
/(-1 + Sin[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.121, size = 172, normalized size = 2.2

$$\frac{a^3 (\sin(dx+c))^3}{4d(\cos(dx+c))^4}+\frac{a^3 (\sin(dx+c))^3}{8d(\cos(dx+c))^2}+\frac{a^3\sin(dx+c)}{8d}+\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^3}{d(\cos(dx+c))^4}+\frac{3a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+1/8
*a^3*sin(d*x+c)/d+1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3/cos(d*x+c)^4+3/
4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+9/8/d*a^3*sec(d*x+c)*tan(d*x+c)+1/2/d*a^3/c
os(d*x+c)^2+1/d*a^3*ln(tan(d*x+c))
```

Maxima [A] time = 1.08578, size = 95, normalized size = 1.23

$$-\frac{2a^3\log(\sin(dx+c)-1)-2a^3\log(\sin(dx+c))+\frac{2a^3\sin(dx+c)-3a^3}{\sin(dx+c)^2-2\sin(dx+c)+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*log(sin(d*x + c) - 1) - 2*a^3*log(sin(d*x + c)) + (2*a^3*sin(d*
x + c) - 3*a^3)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.42778, size = 312, normalized size = 4.05

$$\frac{2a^3\sin(dx+c)-3a^3+2\left(a^3\cos(dx+c)^2+2a^3\sin(dx+c)-2a^3\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-2\left(a^3\cos(dx+c)^2+2a^3\sin\right.}{2\left(d\cos(dx+c)^2+2d\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*sin(d*x + c) - 3*a^3 + 2*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x + c
) - 2*a^3)*log(1/2*sin(d*x + c)) - 2*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x +
```

```
c) - 2*a^3)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) -
2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31277, size = 166, normalized size = 2.16

$$-\frac{12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-6\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-76\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+114\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(12*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*a^3*log(abs(tan(1/2*d*x
 + 1/2*c))) - (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 76*a^3*tan(1/2*d*x + 1/2*c)^
3 + 114*a^3*tan(1/2*d*x + 1/2*c)^2 - 76*a^3*tan(1/2*d*x + 1/2*c) + 25*a^3)/
(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

3.876 $\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=93

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a^3*Log[1 - Sin[c + d*x]])/d + (3*a^3*Log[Sin[
c + d*x]])/d + a^5/(2*d*(a - a*Sin[c + d*x])^2) + (2*a^4)/(d*(a - a*Sin[c +
 d*x]))
```

Rubi [A] time = 0.157522, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 44}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a^3*Log[1 - Sin[c + d*x]])/d + (3*a^3*Log[Sin[
c + d*x]])/d + a^5/(2*d*(a - a*Sin[c + d*x])^2) + (2*a^4)/(d*(a - a*Sin[c +
 d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{a^2}{(a-x)^3x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{1}{(a-x)^3x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{1}{a^2(a-x)^3}+\frac{2}{a^3(a-x)^2}+\frac{3}{a^4(a-x)}+\frac{1}{a^3x^2}+\frac{3}{a^4x}\right)dx, x, a\sin\right.}{d}$$
$$= -\frac{a^3\csc(c+dx)}{d} - \frac{3a^3\log(1-\sin(c+dx))}{d} + \frac{3a^3\log(\sin(c+dx}{d}$$

Mathematica [A] time = 0.221222, size = 63, normalized size = 0.68

$$\frac{a^3\left(-\frac{4}{\sin(c+dx)-1}+\frac{1}{(\sin(c+dx)-1)^2}-2\csc(c+dx)-6\log(1-\sin(c+dx))+6\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-2*Csc[c + d*x] - 6*Log[1 - Sin[c + d*x]] + 6*Log[Sin[c + d*x]] + (-1
 + Sin[c + d*x])^(-2) - 4/(-1 + Sin[c + d*x])))/(2*d)
```

Maple [A] time = 0.136, size = 176, normalized size = 1.9

$$\frac{a^3}{d(\cos(dx+c))^4}+\frac{3a^3\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{9a^3\sec(dx+c)\tan(dx+c)}{8d}+3\frac{a^3\ln(\sec(dx+c)+\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3/cos(d*x+c)^4+3/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+9/8/d*a^3*sec(d*x+c)
*tan(d*x+c)+3/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3/cos(d*x+c)^2+3/d*a^
3*ln(tan(d*x+c))+1/4/d*a^3/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a^3/sin(d*x+c)/cos
(d*x+c)^2-15/8/d*a^3/sin(d*x+c)
```

Maxima [A] time = 1.12938, size = 122, normalized size = 1.31

$$-\frac{6a^3\log(\sin(dx+c)-1)-6a^3\log(\sin(dx+c))+\frac{6a^3\sin(dx+c)^2-9a^3\sin(dx+c)+2a^3}{\sin(dx+c)^3-2\sin(dx+c)^2+\sin(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(6*a^3*log(sin(d*x + c) - 1) - 6*a^3*log(sin(d*x + c)) + (6*a^3*sin(d*
x + c)^2 - 9*a^3*sin(d*x + c) + 2*a^3)/(sin(d*x + c)^3 - 2*sin(d*x + c)^2 +
```

```
sin(d*x + c)))/d
```

Fricas [A] time = 1.4574, size = 441, normalized size = 4.74

$$\frac{6\,a^3\cos\left(dx+c\right)^2+9\,a^3\sin\left(dx+c\right)-8\,a^3+6\left(2\,a^3\cos\left(dx+c\right)^2-2\,a^3-\left(a^3\cos\left(dx+c\right)^2-2\,a^3\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\sin\ldots\right.}{2\left(2\,d\cos\left(dx+c\right)^2-\left(d\cos\left(dx+c\right)^2-2\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(6*a^3*cos(d*x + c)^2 + 9*a^3*sin(d*x + c) - 8*a^3 + 6*(2*a^3*cos(d*x +
 c)^2 - 2*a^3 - (a^3*cos(d*x + c)^2 - 2*a^3)*sin(d*x + c))*log(1/2*sin(d*x
+ c)) - 6*(2*a^3*cos(d*x + c)^2 - 2*a^3 - (a^3*cos(d*x + c)^2 - 2*a^3)*sin(
d*x + c))*log(-sin(d*x + c) + 1))/(2*d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 -
 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20734, size = 224, normalized size = 2.41

$$-\frac{12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-6\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{25\,a^3\tan\left(\frac{1}{2}\,d\ldots\right.}{\ldots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(12*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*a^3*log(abs(tan(1/2*d*x
 + 1/2*c))) + a^3*tan(1/2*d*x + 1/2*c) + (6*a^3*tan(1/2*d*x + 1/2*c) + a^3)
/tan(1/2*d*x + 1/2*c) - (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 88*a^3*tan(1/2*d*x
 + 1/2*c)^3 + 130*a^3*tan(1/2*d*x + 1/2*c)^2 - 88*a^3*tan(1/2*d*x + 1/2*c)
+ 25*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

3.877 $\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}+\frac{6a^3\log}{}$$

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (6*a^3*Log[1 - Sin[c
 + d*x]])/d + (6*a^3*Log[Sin[c + d*x]])/d + a^5/(2*d*(a - a*Sin[c + d*x])^2
) + (3*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.147573, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 44}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}+\frac{6a^3\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (6*a^3*Log[1 - Sin[c
 + d*x]])/d + (6*a^3*Log[Sin[c + d*x]])/d + a^5/(2*d*(a - a*Sin[c + d*x])^2
) + (3*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^5 \operatorname{Subst}\left(\int \frac{a^3}{(a-x)^3x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^8 \operatorname{Subst}\left(\int \frac{1}{(a-x)^3x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^8 \operatorname{Subst}\left(\int \left(\frac{1}{a^3(a-x)^3} + \frac{3}{a^4(a-x)^2} + \frac{6}{a^5(a-x)} + \frac{1}{a^3x^3} + \frac{3}{a^4x^2} + \frac{6}{a^5x}\right) dx, x, \ldots}{d}$$

$$= -\frac{3a^3\csc(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d} - \frac{6a^3\log(1-\sin(c+dx))}{d} + \frac{6a \ldots}{\ldots}$$

Mathematica [A] time = 0.77874, size = 73, normalized size = 0.66

$$-\frac{a^3\left(\frac{6}{\sin(c+dx)-1} - \frac{1}{(\sin(c+dx)-1)^2} + \csc^2(c+dx) + 6\csc(c+dx) + 12\log(1-\sin(c+dx)) - 12\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(6*Csc[c + d*x] + Csc[c + d*x]^2 + 12*Log[1 - Sin[c + d*x]] - 12*Log[
Sin[c + d*x]] - (-1 + Sin[c + d*x])^(-2) + 6/(-1 + Sin[c + d*x])))/(2*d)
```

Maple [B] time = 0.149, size = 241, normalized size = 2.2

$$\frac{a^3\tan(dx+c)(\sec(dx+c))^3}{4d} + \frac{3a^3\sec(dx+c)\tan(dx+c)}{8d} + 6\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{3a^3}{4d(\cos(dx+c))^4} \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*sec(d*x+c)*tan(d*x+c)+6/d*a^3*l
n(sec(d*x+c)+tan(d*x+c))+3/4/d*a^3/cos(d*x+c)^4+3/2/d*a^3/cos(d*x+c)^2+6/d*
a^3*ln(tan(d*x+c))+3/4/d*a^3/sin(d*x+c)/cos(d*x+c)^4+15/8/d*a^3/sin(d*x+c)/
cos(d*x+c)^2-45/8/d*a^3/sin(d*x+c)+1/4/d*a^3/sin(d*x+c)^2/cos(d*x+c)^4+3/4/
d*a^3/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a^3/sin(d*x+c)^2
```

Maxima [A] time = 1.14212, size = 139, normalized size = 1.25

$$-\frac{12a^3\log(\sin(dx+c)-1) - 12a^3\log(\sin(dx+c)) + \frac{12a^3\sin(dx+c)^3 - 18a^3\sin(dx+c)^2 + 4a^3\sin(dx+c) + a^3}{\sin(dx+c)^4 - 2\sin(dx+c)^3 + \sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(12*a^3*log(sin(d*x + c) - 1) - 12*a^3*log(sin(d*x + c)) + (12*a^3*sin
(d*x + c)^3 - 18*a^3*sin(d*x + c)^2 + 4*a^3*sin(d*x + c) + a^3)/(sin(d*x +
```

```
c)^4 - 2*sin(d*x + c)^3 + sin(d*x + c)^2))/d
```

Fricas [B] time = 1.53028, size = 567, normalized size = 5.11

$$-\frac{18\,a^3\cos\left(dx+c\right)^2-17\,a^3-12\left(a^3\cos\left(dx+c\right)^4-3\,a^3\cos\left(dx+c\right)^2+2\,a^3+2\left(a^3\cos\left(dx+c\right)^2-a^3\right)\sin\left(dx+c\right)\right)\mathrm{lc}}{2\left(d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(18*a^3*cos(d*x + c)^2 - 17*a^3 - 12*(a^3*cos(d*x + c)^4 - 3*a^3*cos(d
*x + c)^2 + 2*a^3 + 2*(a^3*cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(1/2*sin(
d*x + c)) + 12*(a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^2 + 2*a^3 + 2*(a^3*
cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 4*(3*a^3*cos(d
*x + c)^2 - 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4 - 3*d*cos(d*x + c)^2 + 2
*(d*cos(d*x + c)^2 - d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27533, size = 267, normalized size = 2.41

$$-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+96\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-48\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{7}{-}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 96*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 48*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^3*tan(1/2*d*x + 1/2*c) + (
72*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*
d*x + 1/2*c)^2 - 8*(25*a^3*tan(1/2*d*x + 1/2*c)^4 - 92*a^3*tan(1/2*d*x + 1/
2*c)^3 + 136*a^3*tan(1/2*d*x + 1/2*c)^2 - 92*a^3*tan(1/2*d*x + 1/2*c) + 25*
a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

3.878 $\int \frac{\sin^4(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=236

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{3a^2}{16d(a\sin(c+dx)+a)^3}+\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}-\frac{1}{128d(a-a}$$

```
[Out] (515*Log[1 - Sin[c + d*x]])/(256*a*d) - (1795*Log[1 + Sin[c + d*x]])/(256*a
*d) + (5*Sin[c + d*x])/(a*d) - Sin[c + d*x]^2/(2*a*d) + Sin[c + d*x]^3/(3*a
*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (17*a)/(128*d*(a - a*Sin[c + d*x]
)^2) + 125/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a + a*Sin[c + d*x])^4)
 - (3*a^2)/(16*d*(a + a*Sin[c + d*x])^3) + (71*a)/(64*d*(a + a*Sin[c + d*x]
)^2) - 5/(d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.252007, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{3a^2}{16d(a\sin(c+dx)+a)^3}+\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}-\frac{1}{128d(a-a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (515*Log[1 - Sin[c + d*x]])/(256*a*d) - (1795*Log[1 + Sin[c + d*x]])/(256*a
*d) + (5*Sin[c + d*x])/(a*d) - Sin[c + d*x]^2/(2*a*d) + Sin[c + d*x]^3/(3*a
*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (17*a)/(128*d*(a - a*Sin[c + d*x]
)^2) + 125/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a + a*Sin[c + d*x])^4)
 - (3*a^2)/(16*d*(a + a*Sin[c + d*x])^3) + (71*a)/(64*d*(a + a*Sin[c + d*x]
)^2) - 5/(d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^4(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7 \operatorname{Subst}\left(\int \frac{x^{11}}{a^{11}(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^{11}}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(5a^2 + \frac{a^6}{32(a-x)^4} - \frac{17a^5}{64(a-x)^3} + \frac{125a^4}{128(a-x)^2} - \frac{515a^3}{256(a-x)} - ax + x^2 - \frac{a^7}{16(a+x)^5} + \frac{9a}{16(a+}\right.\right.}{a^4 d}$$

$$= \frac{515\log(1-\sin(c+dx))}{256ad} - \frac{1795\log(1+\sin(c+dx))}{256ad} + \frac{5\sin(c+dx)}{ad} - \frac{\sin^2(c+dx}{2ad}$$

Mathematica [A] time = 6.12358, size = 153, normalized size = 0.65

$$\frac{256\sin^3(c+dx) - 384\sin^2(c+dx) + 3840\sin(c+dx) + \frac{750}{1-\sin(c+dx)} - \frac{3840}{\sin(c+dx)+1} - \frac{102}{(1-\sin(c+dx))^2} + \frac{852}{(\sin(c+dx)+1)^2} + \frac{}{(1-\sin}}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (1545*Log[1 - Sin[c + d*x]] - 5385*Log[1 + Sin[c + d*x]] + 8/(1 - Sin[c + d
*x])^3 - 102/(1 - Sin[c + d*x])^2 + 750/(1 - Sin[c + d*x]) + 3840*Sin[c + d
*x] - 384*Sin[c + d*x]^2 + 256*Sin[c + d*x]^3 + 12/(1 + Sin[c + d*x])^4 - 1
44/(1 + Sin[c + d*x])^3 + 852/(1 + Sin[c + d*x])^2 - 3840/(1 + Sin[c + d*x]
))/(768*a*d)
```

Maple [A] time = 0.113, size = 208, normalized size = 0.9

$$\frac{(\sin(dx+c))^3}{3da} - \frac{(\sin(dx+c))^2}{2da} + 5\frac{\sin(dx+c)}{da} - \frac{1}{96da(\sin(dx+c)-1)^3} - \frac{17}{128da(\sin(dx+c)-1)^2} - \frac{1}{128da(\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/3*sin(d*x+c)^3/d/a-1/2*sin(d*x+c)^2/d/a+5*sin(d*x+c)/d/a-1/96/d/a/(sin(d*
x+c)-1)^3-17/128/d/a/(sin(d*x+c)-1)^2-125/128/a/d/(sin(d*x+c)-1)+515/256/a/
d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4-3/16/d/a/(1+sin(d*x+c))^3+71/6
4/a/d/(1+sin(d*x+c))^2-5/a/d/(1+sin(d*x+c))-1795/256*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.02129, size = 282, normalized size = 1.19

$$-\frac{\frac{2\left(2295\sin(dx+c)^6+375\sin(dx+c)^5-5480\sin(dx+c)^4-680\sin(dx+c)^3+4473\sin(dx+c)^2+313\sin(dx+c)-1232\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a} - \frac{128\left(2\sin(dx+c)^3-3\sin(dx+c)^2+30\right.}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/768*(2*(2295*sin(d*x + c)^6 + 375*sin(d*x + c)^5 - 5480*sin(d*x + c)^4 -
 680*sin(d*x + c)^3 + 4473*sin(d*x + c)^2 + 313*sin(d*x + c) - 1232)/(a*sin
(d*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 +
3*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 128*(2*sin(
d*x + c)^3 - 3*sin(d*x + c)^2 + 30*sin(d*x + c))/a + 5385*log(sin(d*x + c)
+ 1)/a - 1545*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.8645, size = 585, normalized size = 2.48

$$\frac{256\,\cos(dx+c)^{10} - 3968\,\cos(dx+c)^{8} - 686\,\cos(dx+c)^{6} + 2810\,\cos(dx+c)^{4} - 796\,\cos(dx+c)^{2} - 5385\left(\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/768*(256*cos(d*x + c)^10 - 3968*cos(d*x + c)^8 - 686*cos(d*x + c)^6 + 281
0*cos(d*x + c)^4 - 796*cos(d*x + c)^2 - 5385*(cos(d*x + c)^6*sin(d*x + c) +
 cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 1545*(cos(d*x + c)^6*sin(d*x + c)
+ cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(64*cos(d*x + c)^8 + 1952*cos(
d*x + c)^6 + 375*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 8)*sin(d*x + c) + 112
)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36217, size = 242, normalized size = 1.03

$$-\frac{\frac{21540\,\log(|\sin(dx+c)+1|)}{a} - \frac{6180\,\log(|\sin(dx+c)-1|)}{a} - \frac{512\left(2\,a^{2}\sin(dx+c)^{3} - 3\,a^{2}\sin(dx+c)^{2} + 30\,a^{2}\sin(dx+c)\right)}{a^{3}} + \frac{2\left(5665\,\sin(dx+c)^{3} - 15495\,\sin(dx+c)^{2} + \right.}{a(\sin(dx+c)-1)^{3}}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(21540*log(abs(sin(d*x + c) + 1))/a - 6180*log(abs(sin(d*x + c) - 1
))/a - 512*(2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c)^2 + 30*a^2*sin(d*x +
c))/a^3 + 2*(5665*sin(d*x + c)^3 - 15495*sin(d*x + c)^2 + 14199*sin(d*x + c
) - 4353)/(a*(sin(d*x + c) - 1)^3) - (44875*sin(d*x + c)^4 + 164140*sin(d*x
 + c)^3 + 226578*sin(d*x + c)^2 + 139660*sin(d*x + c) + 32395)/(a*(sin(d*x
+ c) + 1)^4))/d
```

3.879 $\int \frac{\sin^3(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=220

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{6d(a\sin(c+dx)+a)^3}+\frac{\sin^2(c+dx)}{2ad}-\frac{15a}{128d(a-a\sin(c+dx))}$$

```
[Out] (325*Log[1 - Sin[c + d*x]])/(256*a*d) + (955*Log[1 + Sin[c + d*x]])/(256*a*
d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d) + a^2/(96*d*(a - a*Sin[c +
 d*x])^3) - (15*a)/(128*d*(a - a*Sin[c + d*x])^2) + 95/(128*d*(a - a*Sin[c
+ d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) + a^2/(6*d*(a + a*Sin[c + d*x]
)^3) - (55*a)/(64*d*(a + a*Sin[c + d*x])^2) + 105/(32*d*(a + a*Sin[c + d*x]
))
```

Rubi [A] time = 0.228588, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{6d(a\sin(c+dx)+a)^3}+\frac{\sin^2(c+dx)}{2ad}-\frac{15a}{128d(a-a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (325*Log[1 - Sin[c + d*x]])/(256*a*d) + (955*Log[1 + Sin[c + d*x]])/(256*a*
d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d) + a^2/(96*d*(a - a*Sin[c +
 d*x])^3) - (15*a)/(128*d*(a - a*Sin[c + d*x])^2) + 95/(128*d*(a - a*Sin[c
+ d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) + a^2/(6*d*(a + a*Sin[c + d*x]
)^3) - (55*a)/(64*d*(a + a*Sin[c + d*x])^2) + 105/(32*d*(a + a*Sin[c + d*x]
))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{x^{10}}{a^{10}(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^5}{32(a-x)^4}-\frac{15a^4}{64(a-x)^3}+\frac{95a^3}{128(a-x)^2}-\frac{325a^2}{256(a-x)}+x+\frac{a^6}{16(a+x)^5}-\frac{a^5}{2(a+x)^4}+\frac{55a^4}{32(a+}\right.\right.}{a^3 d}$$
$$= \frac{325\log(1-\sin(c+dx))}{256ad}+\frac{955\log(1+\sin(c+dx))}{256ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}+\frac{}{9}$$

Mathematica [A] time = 6.14101, size = 143, normalized size = 0.65

$$\frac{384\sin^2(c+dx)-768\sin(c+dx)+\frac{570}{1-\sin(c+dx)}+\frac{2520}{\sin(c+dx)+1}-\frac{90}{(1-\sin(c+dx))^2}-\frac{660}{(\sin(c+dx)+1)^2}+\frac{8}{(1-\sin(c+dx))^3}+\frac{128}{(\sin(c+dx)+1)^3}-}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (975*Log[1 - Sin[c + d*x]] + 2865*Log[1 + Sin[c + d*x]] + 8/(1 - Sin[c + d*
x])^3 - 90/(1 - Sin[c + d*x])^2 + 570/(1 - Sin[c + d*x]) - 768*Sin[c + d*x]
 + 384*Sin[c + d*x]^2 - 12/(1 + Sin[c + d*x])^4 + 128/(1 + Sin[c + d*x])^3
- 660/(1 + Sin[c + d*x])^2 + 2520/(1 + Sin[c + d*x]))/(768*a*d)
```

Maple [A] time = 0.112, size = 192, normalized size = 0.9

$$\frac{(\sin(dx+c))^2}{2\,da}-\frac{\sin(dx+c)}{da}-\frac{1}{96\,da\,(\sin(dx+c)-1)^3}-\frac{15}{128\,da\,(\sin(dx+c)-1)^2}-\frac{95}{128\,da\,(\sin(dx+c)-1)}+\frac{325\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2*sin(d*x+c)^2/d/a-sin(d*x+c)/d/a-1/96/d/a/(sin(d*x+c)-1)^3-15/128/d/a/(s
in(d*x+c)-1)^2-95/128/a/d/(sin(d*x+c)-1)+325/256/a/d*ln(sin(d*x+c)-1)-1/64/
d/a/(1+sin(d*x+c))^4+1/6/d/a/(1+sin(d*x+c))^3-55/64/a/d/(1+sin(d*x+c))^2+10
5/32/a/d/(1+sin(d*x+c))+955/256*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.03663, size = 266, normalized size = 1.21

$$\frac{\frac{2\left(975\sin(dx+c)^6-945\sin(dx+c)^5-3240\sin(dx+c)^4+1560\sin(dx+c)^3+3489\sin(dx+c)^2-671\sin(dx+c)-1232\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{384\left(\sin(dx+c)^2-2\sin(dx+c)\right)}{a}+\frac{2865\log}{}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/768*(2*(975*sin(d*x + c)^6 - 945*sin(d*x + c)^5 - 3240*sin(d*x + c)^4 + 1
560*sin(d*x + c)^3 + 3489*sin(d*x + c)^2 - 671*sin(d*x + c) - 1232)/(a*sin(
d*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3
*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 384*(sin(d*x
 + c)^2 - 2*sin(d*x + c))/a + 2865*log(sin(d*x + c) + 1)/a + 975*log(sin(d*
x + c) - 1)/a)/d
```

Fricas [A] time = 1.89573, size = 554, normalized size = 2.52

$$\frac{384 \cos(dx+c)^8 + 1374 \cos(dx+c)^6 + 630 \cos(dx+c)^4 - 132 \cos(dx+c)^2 + 2865 \left(\cos(dx+c)^6 \sin(dx+c) + \cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/768*(384*cos(d*x + c)^8 + 1374*cos(d*x + c)^6 + 630*cos(d*x + c)^4 - 132*
cos(d*x + c)^2 + 2865*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(si
n(d*x + c) + 1) + 975*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-s
in(d*x + c) + 1) - 2*(192*cos(d*x + c)^8 + 288*cos(d*x + c)^6 - 945*cos(d*x
 + c)^4 + 330*cos(d*x + c)^2 - 56)*sin(d*x + c) + 16)/(a*d*cos(d*x + c)^6*s
in(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39147, size = 217, normalized size = 0.99

$$\frac{\frac{11460 \log(|\sin(dx+c)+1|)}{a} + \frac{3900 \log(|\sin(dx+c)-1|)}{a} + \frac{1536\left(a\sin(dx+c)^2 - 2a\sin(dx+c)\right)}{a^2} - \frac{2\left(3575 \sin(dx+c)^3 - 9585 \sin(dx+c)^2 + 8625 \sin(dx+c) - 25\right.}{a(\sin(dx+c)-1)^3}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(11460*log(abs(sin(d*x + c) + 1))/a + 3900*log(abs(sin(d*x + c) - 1)
)/a + 1536*(a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/a^2 - 2*(3575*sin(d*x + c)
^3 - 9585*sin(d*x + c)^2 + 8625*sin(d*x + c) - 2599)/(a*(sin(d*x + c) - 1)^
3) - (23875*sin(d*x + c)^4 + 85420*sin(d*x + c)^3 + 115650*sin(d*x + c)^2 +
 70028*sin(d*x + c) + 15971)/(a*(sin(d*x + c) + 1)^4))/d
```

3.880 $\int \frac{\sin^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{7a^2}{48d(a\sin(c+dx)+a)^3}-\frac{13a}{128d(a-a\sin(c+dx))^2}+\frac{41a}{64d(a\sin(c+dx)+a)^2}$$

```
[Out] (187*Log[1 - Sin[c + d*x]])/(256*a*d) - (443*Log[1 + Sin[c + d*x]])/(256*a*
d) + Sin[c + d*x]/(a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (13*a)/(128*d
*(a - a*Sin[c + d*x])^2) + 69/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a +
 a*Sin[c + d*x])^4) - (7*a^2)/(48*d*(a + a*Sin[c + d*x])^3) + (41*a)/(64*d*
(a + a*Sin[c + d*x])^2) - 2/(d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.212357, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{7a^2}{48d(a\sin(c+dx)+a)^3}-\frac{13a}{128d(a-a\sin(c+dx))^2}+\frac{41a}{64d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (187*Log[1 - Sin[c + d*x]])/(256*a*d) - (443*Log[1 + Sin[c + d*x]])/(256*a*
d) + Sin[c + d*x]/(a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (13*a)/(128*d
*(a - a*Sin[c + d*x])^2) + 69/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a +
 a*Sin[c + d*x])^4) - (7*a^2)/(48*d*(a + a*Sin[c + d*x])^3) + (41*a)/(64*d*
(a + a*Sin[c + d*x])^2) - 2/(d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{x^9}{a^9(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^9}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^4}{32(a-x)^4}-\frac{13a^3}{64(a-x)^3}+\frac{69a^2}{128(a-x)^2}-\frac{187a}{256(a-x)}-\frac{a^5}{16(a+x)^5}+\frac{7a^4}{16(a+x)^4}-\frac{41a^3}{32(a+x)^3}\right.\right.}{a^2d}$$
$$= \frac{187\log(1-\sin(c+dx))}{256ad}-\frac{443\log(1+\sin(c+dx))}{256ad}+\frac{\sin(c+dx)}{ad}+\frac{a^2}{96d(a-a\sin(}$$

Mathematica [A] time = 6.13077, size = 133, normalized size = 0.67

$$\frac{768\sin(c+dx)+\frac{414}{1-\sin(c+dx)}-\frac{1536}{\sin(c+dx)+1}-\frac{78}{(1-\sin(c+dx))^2}+\frac{492}{(\sin(c+dx)+1)^2}+\frac{8}{(1-\sin(c+dx))^3}-\frac{112}{(\sin(c+dx)+1)^3}+\frac{12}{(\sin(c+dx)+1)^4}+}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (561*Log[1 - Sin[c + d*x]] - 1329*Log[1 + Sin[c + d*x]] + 8/(1 - Sin[c + d*
x])^3 - 78/(1 - Sin[c + d*x])^2 + 414/(1 - Sin[c + d*x]) + 768*Sin[c + d*x]
 + 12/(1 + Sin[c + d*x])^4 - 112/(1 + Sin[c + d*x])^3 + 492/(1 + Sin[c + d*
x])^2 - 1536/(1 + Sin[c + d*x]))/(768*a*d)
```

Maple [A] time = 0.105, size = 175, normalized size = 0.9

$$\frac{\sin(dx+c)}{da}-\frac{1}{96\,da\,(\sin(dx+c)-1)^3}-\frac{13}{128\,da\,(\sin(dx+c)-1)^2}-\frac{69}{128\,da\,(\sin(dx+c)-1)}+\frac{187\ln(\sin(dx+c)-}{256\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] sin(d*x+c)/d/a-1/96/d/a/(sin(d*x+c)-1)^3-13/128/d/a/(sin(d*x+c)-1)^2-69/128
/a/d/(sin(d*x+c)-1)+187/256/a/d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4-
7/48/d/a/(1+sin(d*x+c))^3+41/64/a/d/(1+sin(d*x+c))^2-2/a/d/(1+sin(d*x+c))-4
43/256*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.05821, size = 251, normalized size = 1.26

$$-\frac{\frac{2\left(975\sin(dx+c)^6+207\sin(dx+c)^5-2088\sin(dx+c)^4-360\sin(dx+c)^3+1569\sin(dx+c)^2+161\sin(dx+c)-400\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{1329\log(\sin(dx+c)+1)}{a}-\frac{561\log(\sin}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(975*sin(d*x + c)^6 + 207*sin(d*x + c)^5 - 2088*sin(d*x + c)^4 -
360*sin(d*x + c)^3 + 1569*sin(d*x + c)^2 + 161*sin(d*x + c) - 400)/(a*sin(d
```

```
*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*
a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 1329*log(sin(
d*x + c) + 1)/a - 561*log(sin(d*x + c) - 1)/a - 768*sin(d*x + c)/a)/d
```

Fricas [A] time = 1.64101, size = 527, normalized size = 2.65

$$-\frac{768 \cos(dx+c)^8 + 1182 \cos(dx+c)^6 - 1674 \cos(dx+c)^4 + 636 \cos(dx+c)^2 + 1329 \left(\cos(dx+c)^6 \sin(dx+c) + \text{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(768*cos(d*x + c)^8 + 1182*cos(d*x + c)^6 - 1674*cos(d*x + c)^4 + 63
6*cos(d*x + c)^2 + 1329*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(
sin(d*x + c) + 1) - 561*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(
-sin(d*x + c) + 1) - 2*(384*cos(d*x + c)^6 + 207*cos(d*x + c)^4 - 54*cos(d*
x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*co
s(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34441, size = 198, normalized size = 0.99

$$-\frac{\frac{5316 \log(|\sin(dx+c)+1|)}{a} - \frac{2244 \log(|\sin(dx+c)-1|)}{a} - \frac{3072 \sin(dx+c)}{a} + \frac{2\left(2057 \sin(dx+c)^3 - 5343 \sin(dx+c)^2 + 4671 \sin(dx+c) - 1369\right)}{a(\sin(dx+c)-1)^3} - \frac{11075 \sin(dx+}{}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(5316*log(abs(sin(d*x + c) + 1))/a - 2244*log(abs(sin(d*x + c) - 1)
)/a - 3072*sin(d*x + c)/a + 2*(2057*sin(d*x + c)^3 - 5343*sin(d*x + c)^2 +
4671*sin(d*x + c) - 1369)/(a*(sin(d*x + c) - 1)^3) - (11075*sin(d*x + c)^4
+ 38156*sin(d*x + c)^3 + 49986*sin(d*x + c)^2 + 29356*sin(d*x + c) + 6499)/
(a*(sin(d*x + c) + 1)^4))/d
```

3.881 $\int \frac{\sin(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=188

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{8d(a\sin(c+dx)+a)^3}-\frac{11a}{128d(a-a\sin(c+dx))^2}-\frac{2}{64d(a\sin(c}$$

```
[Out] (93*Log[1 - Sin[c + d*x]])/(256*a*d) + (163*Log[1 + Sin[c + d*x]])/(256*a*d
) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (11*a)/(128*d*(a - a*Sin[c + d*x])^
2) + 47/(128*d*(a - a*Sin[c + d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) +
a^2/(8*d*(a + a*Sin[c + d*x])^3) - (29*a)/(64*d*(a + a*Sin[c + d*x])^2) + 3
5/(32*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.183577, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{8d(a\sin(c+dx)+a)^3}-\frac{11a}{128d(a-a\sin(c+dx))^2}-\frac{2}{64d(a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (93*Log[1 - Sin[c + d*x]])/(256*a*d) + (163*Log[1 + Sin[c + d*x]])/(256*a*d
) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (11*a)/(128*d*(a - a*Sin[c + d*x])^
2) + 47/(128*d*(a - a*Sin[c + d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) +
a^2/(8*d*(a + a*Sin[c + d*x])^3) - (29*a)/(64*d*(a + a*Sin[c + d*x])^2) + 3
5/(32*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\operatorname{Subst}\left(\int \frac{x^8}{a^8(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{x^8}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{32(a-x)^4} - \frac{11a^2}{64(a-x)^3} + \frac{47a}{128(a-x)^2} - \frac{93}{256(a-x)} + \frac{a^4}{16(a+x)^5} - \frac{3a^3}{8(a+x)^4} + \frac{29a^2}{32(a+x)^3} - \frac{35a}{32(a+x}\right.\right.}{ad}$$
$$= \frac{93\log(1-\sin(c+dx))}{256ad} + \frac{163\log(1+\sin(c+dx))}{256ad} + \frac{a^2}{96d(a-a\sin(c+dx))^3} - \frac{}{128d(a}$$

Mathematica [A] time = 3.95042, size = 117, normalized size = 0.62

$$\frac{\frac{2\left(279\sin^6(c+dx)-489\sin^5(c+dx)-1000\sin^4(c+dx)+728\sin^3(c+dx)+1113\sin^2(c+dx)-295\sin(c+dx)-400\right)}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4} + 279\log(1-\sin(c+dx)) + 489\log(si}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (279*Log[1 - Sin[c + d*x]] + 489*Log[1 + Sin[c + d*x]] + (2*(-400 - 295*Sin
[c + d*x] + 1113*Sin[c + d*x]^2 + 728*Sin[c + d*x]^3 - 1000*Sin[c + d*x]^4
- 489*Sin[c + d*x]^5 + 279*Sin[c + d*x]^6))/((-1 + Sin[c + d*x])^3*(1 + Sin
[c + d*x])^4))/(768*a*d)
```

Maple [A] time = 0.104, size = 162, normalized size = 0.9

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3} - \frac{11}{128\,da\,(\sin(dx+c)-1)^2} - \frac{47}{128\,da\,(\sin(dx+c)-1)} + \frac{93\,\ln(\sin(dx+c)-1)}{256\,da} - \frac{}{64\,da\,(1+s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3-11/128/d/a/(sin(d*x+c)-1)^2-47/128/a/d/(sin(d*x+
c)-1)+93/256/a/d*ln(sin(d*x+c)-1)-1/64/d/a/(1+sin(d*x+c))^4+1/8/d/a/(1+sin(
d*x+c))^3-29/64/a/d/(1+sin(d*x+c))^2+35/32/a/d/(1+sin(d*x+c))+163/256*ln(1+
sin(d*x+c))/a/d
```

Maxima [A] time = 1.13474, size = 236, normalized size = 1.26

$$\frac{\frac{2\left(279\sin(dx+c)^6-489\sin(dx+c)^5-1000\sin(dx+c)^4+728\sin(dx+c)^3+1113\sin(dx+c)^2-295\sin(dx+c)-400\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a} + \frac{489\log(\sin(dx+c)+1)}{a} + \frac{279\log(\sin(dx+c)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(279*sin(d*x + c)^6 - 489*sin(d*x + c)^5 - 1000*sin(d*x + c)^4 + 7
28*sin(d*x + c)^3 + 1113*sin(d*x + c)^2 - 295*sin(d*x + c) - 400)/(a*sin(d*
```

```
x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a
*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 489*log(sin(d*
x + c) + 1)/a + 279*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.59043, size = 466, normalized size = 2.48

$$\frac{558 \cos(dx+c)^6 + 326 \cos(dx+c)^4 - 100 \cos(dx+c)^2 + 489 \left(\cos(dx+c)^6 \sin(dx+c) + \cos(dx+c)^6\right) \log(\sin(d}{768 \left(ad \cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(558*cos(d*x + c)^6 + 326*cos(d*x + c)^4 - 100*cos(d*x + c)^2 + 489*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 279*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(4
89*cos(d*x + c)^4 - 250*cos(d*x + c)^2 + 56)*sin(d*x + c) + 16)/(a*d*cos(d*
x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36621, size = 184, normalized size = 0.98

$$\frac{\frac{1956 \log(|\sin(dx+c)+1|)}{a} + \frac{1116 \log(|\sin(dx+c)-1|)}{a} - \frac{2\left(1023 \sin(dx+c)^3 - 2505 \sin(dx+c)^2 + 2073 \sin(dx+c) - 575\right)}{a(\sin(dx+c)-1)^3} - \frac{4075 \sin(dx+c)^4 + 12940 \sin(dx+c}{a(s}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(1956*log(abs(sin(d*x + c) + 1))/a + 1116*log(abs(sin(d*x + c) - 1))
/a - 2*(1023*sin(d*x + c)^3 - 2505*sin(d*x + c)^2 + 2073*sin(d*x + c) - 575
)/(a*(sin(d*x + c) - 1)^3) - (4075*sin(d*x + c)^4 + 12940*sin(d*x + c)^3 +
15762*sin(d*x + c)^2 + 8620*sin(d*x + c) + 1771)/(a*(sin(d*x + c) + 1)^4))/
d
```

3.882 $\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{\tan^8(c+dx)}{8ad} - \frac{35\tanh^{-1}(\sin(c+dx))}{128ad} - \frac{\tan^7(c+dx)\sec(c+dx)}{8ad} + \frac{7\tan^5(c+dx)\sec(c+dx)}{48ad} - \frac{35\tan^3(c+dx)\sec(c}{192ad}$$

```
[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128
*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c
+ d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8
/(8*a*d)
```

Rubi [A] time = 0.171751, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^8(c+dx)}{8ad} - \frac{35\tanh^{-1}(\sin(c+dx))}{128ad} - \frac{\tan^7(c+dx)\sec(c+dx)}{8ad} + \frac{7\tan^5(c+dx)\sec(c+dx)}{48ad} - \frac{35\tan^3(c+dx)\sec(c}{192ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128
*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c
+ d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8
/(8*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^2(c+dx)\tan^7(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^8(c+dx)\,dx}{a}$$

$$= -\frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{7\int \sec(c+dx)\tan^6(c+dx)\,dx}{8a} + \frac{\operatorname{Subst}\left(\int x^7\,dx, x, \tan(c+ \ldots\right.}{ad}$$

$$= \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{\tan^8(c+dx)}{8ad} - \frac{35\int \sec(c+dx)\,\mathrm{t}\ldots}{48a}$$

$$= -\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \ldots$$

$$= \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \ldots$$

$$= -\frac{35\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \ldots$$

Mathematica [A] time = 0.923232, size = 101, normalized size = 0.78

$$-\frac{\frac{279\sin^6(c+dx)+87\sin^5(c+dx)-424\sin^4(c+dx)-136\sin^3(c+dx)+249\sin^2(c+dx)+57\sin(c+dx)-48}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4}+105\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(105*ArcTanh[Sin[c + d*x]] + (-48 + 57*Sin[c + d*x] + 249*Sin[c + d*x]^2 -
 136*Sin[c + d*x]^3 - 424*Sin[c + d*x]^4 + 87*Sin[c + d*x]^5 + 279*Sin[c +
d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

Maple [A] time = 0.095, size = 162, normalized size = 1.3

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3} - \frac{9}{128\,da\,(\sin(dx+c)-1)^2} - \frac{29}{128\,da\,(\sin(dx+c)-1)} + \frac{35\ln(\sin(dx+c)-1)}{256\,da} + \frac{\ldots}{64\,da\,(1\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3-9/128/d/a/(sin(d*x+c)-1)^2-29/128/a/d/(sin(d*x+c
)-1)+35/256/a/d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4-5/48/d/a/(1+sin(
d*x+c))^3+19/64/a/d/(1+sin(d*x+c))^2-1/2/a/d/(1+sin(d*x+c))-35/256*ln(1+sin
(d*x+c))/a/d
```

Maxima [A] time = 1.06466, size = 236, normalized size = 1.82

$$-\frac{\frac{2\left(279\sin(dx+c)^6+87\sin(dx+c)^5-424\sin(dx+c)^4-136\sin(dx+c)^3+249\sin(dx+c)^2+57\sin(dx+c)-48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{105\log(\sin(dx+c)+1)}{a}-\frac{105\log(\sin(dx+c\ldots}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(279*sin(d*x + c)^6 + 87*sin(d*x + c)^5 - 424*sin(d*x + c)^4 - 13
6*sin(d*x + c)^3 + 249*sin(d*x + c)^2 + 57*sin(d*x + c) - 48)/(a*sin(d*x +
c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin
(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 105*log(sin(d*x +
c) + 1)/a - 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.68086, size = 464, normalized size = 3.57

$$-\frac{558\cos(dx+c)^6 - 826\cos(dx+c)^4 + 476\cos(dx+c)^2 + 105\left(\cos(dx+c)^6\sin(dx+c) + \cos(dx+c)^6\right)\log(\sin(dx}{768\left(ad\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(558*cos(d*x + c)^6 - 826*cos(d*x + c)^4 + 476*cos(d*x + c)^2 + 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(
87*cos(d*x + c)^4 - 38*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x
 + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32385, size = 184, normalized size = 1.42

$$-\frac{\frac{420\log(|\sin(dx+c)+1|)}{a} - \frac{420\log(|\sin(dx+c)-1|)}{a} + \frac{2\left(385\sin(dx+c)^3-807\sin(dx+c)^2+567\sin(dx+c)-129\right)}{a(\sin(dx+c)-1)^3} - \frac{875\sin(dx+c)^4+1964\sin(dx+c)^3+1554\,\mathrm{s}}{a(\sin(dx+c)+1}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 2*(385*sin(d*x + c)^3 - 807*sin(d*x + c)^2 + 567*sin(d*x + c) - 129)/(a
*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 1964*sin(d*x + c)^3 + 1554*s
in(d*x + c)^2 + 396*sin(d*x + c) - 21)/(a*(sin(d*x + c) + 1)^4))/d
```

3.883 $\int \frac{\sec(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}-\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}+\frac{5\tan(c+dx)\mathrm{se}}{64ad}$$

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(128*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(128*a
*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) - (5*Sec[c + d*x]^3*Tan[c +
d*x]^3)/(48*a*d) + (Sec[c + d*x]^3*Tan[c + d*x]^5)/(8*a*d) - Tan[c + d*x]^8
/(8*a*d)
```

Rubi [A] time = 0.236483, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2835, 2611, 3768, 3770, 2607, 30}

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}-\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}+\frac{5\tan(c+dx)\mathrm{se}}{64ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(128*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(128*a
*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) - (5*Sec[c + d*x]^3*Tan[c +
d*x]^3)/(48*a*d) + (Sec[c + d*x]^3*Tan[c + d*x]^5)/(8*a*d) - Tan[c + d*x]^8
/(8*a*d)
```


Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{a} - \frac{\int \sec^2(c+dx)\tan^7(c+dx)\,dx}{a}$$
$$= \frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} - \frac{5\int \sec^3(c+dx)\tan^4(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^7\,dx, x, \tan(\right.}{ad}$$
$$= -\frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} - \frac{\tan^8(c+dx)}{8ad} + \frac{5\int \sec^3(c}{}$$
$$= \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad}$$
$$= -\frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad}$$
$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{128ad} - \frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{5\,\text{s}}{}$$

Mathematica [A] time = 0.915934, size = 101, normalized size = 0.75

$$-\frac{\frac{-15\sin^6(c+dx)+177\sin^5(c+dx)+104\sin^4(c+dx)-184\sin^3(c+dx)-129\sin^2(c+dx)+63\sin(c+dx)+48}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4}+15\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(15*ArcTanh[Sin[c + d*x]] + (48 + 63*Sin[c + d*x] - 129*Sin[c + d*x]^2 - 1
84*Sin[c + d*x]^3 + 104*Sin[c + d*x]^4 + 177*Sin[c + d*x]^5 - 15*Sin[c + d*
x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

Maple [A] time = 0.091, size = 162, normalized size = 1.2

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3} - \frac{7}{128\,da\,(\sin(dx+c)-1)^2} - \frac{15}{128\,da\,(\sin(dx+c)-1)} + \frac{5\ln(\sin(dx+c)-1)}{256\,da} - \frac{1}{64\,da\,(1+\text{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3-7/128/d/a/(sin(d*x+c)-1)^2-15/128/a/d/(sin(d*x+c
)-1)+5/256/a/d*ln(sin(d*x+c)-1)-1/64/d/a/(1+sin(d*x+c))^4+1/12/d/a/(1+sin(d
*x+c))^3-11/64/a/d/(1+sin(d*x+c))^2+5/32/a/d/(1+sin(d*x+c))-5/256*ln(1+sin(
```

d*x+c))/a/d

Maxima [A] time = 1.06677, size = 236, normalized size = 1.76

$$\frac{\frac{2\left(15\sin(dx+c)^6-177\sin(dx+c)^5-104\sin(dx+c)^4+184\sin(dx+c)^3+129\sin(dx+c)^2-63\sin(dx+c)-48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-1)}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(15*sin(d*x + c)^6 - 177*sin(d*x + c)^5 - 104*sin(d*x + c)^4 + 184
*sin(d*x + c)^3 + 129*sin(d*x + c)^2 - 63*sin(d*x + c) - 48)/(a*sin(d*x + c
)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(
d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c)
 + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.46796, size = 460, normalized size = 3.43

$$\frac{30\cos(dx+c)^6+118\cos(dx+c)^4-68\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+}{768\left(ad\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(30*cos(d*x + c)^6 + 118*cos(d*x + c)^4 - 68*cos(d*x + c)^2 - 15*(cos
(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(cos(
d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(177*c
os(d*x + c)^4 - 170*cos(d*x + c)^2 + 56)*sin(d*x + c) + 16)/(a*d*cos(d*x +
c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35526, size = 184, normalized size = 1.37

$$-\frac{\frac{60\log(|\sin(dx+c)+1|)}{a}-\frac{60\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(55\sin(dx+c)^3+15\sin(dx+c)^2-111\sin(dx+c)+57\right)}{a(\sin(dx+c)-1)^3}-\frac{125\sin(dx+c)^4+980\sin(dx+c)^3+1662\sin(}{a(\sin(dx+c)+1)}}{3072d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
+ 2*(55*sin(d*x + c)^3 + 15*sin(d*x + c)^2 - 111*sin(d*x + c) + 57)/(a*(sin
(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 980*sin(d*x + c)^3 + 1662*sin(d*x
 + c)^2 + 1140*sin(d*x + c) + 285)/(a*(sin(d*x + c) + 1)^4))/d
```

3.884 $\int \frac{\sec^2(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{\tan^8(c+dx)}{8ad}+\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}+\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}-5$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(128*a*d) + (5*Sec[c + d*x]*Tan[c + d*x])/(128*a*
d) - (5*Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) + (5*Sec[c + d*x]^3*Tan[c + d
*x]^3)/(48*a*d) - (Sec[c + d*x]^3*Tan[c + d*x]^5)/(8*a*d) + Tan[c + d*x]^6/
(6*a*d) + Tan[c + d*x]^8/(8*a*d)
```

Rubi [A] time = 0.244631, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2607, 14, 2611, 3768, 3770}

$$\frac{\tan^8(c+dx)}{8ad}+\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}+\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}-5$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(128*a*d) + (5*Sec[c + d*x]*Tan[c + d*x])/(128*a*
d) - (5*Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) + (5*Sec[c + d*x]^3*Tan[c + d
*x]^3)/(48*a*d) - (Sec[c + d*x]^3*Tan[c + d*x]^5)/(8*a*d) + Tan[c + d*x]^6/
(6*a*d) + Tan[c + d*x]^8/(8*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^4(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{a}$$
$$= -\frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} + \frac{5\int \sec^3(c+dx)\tan^4(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^5\left(1+x^2\right.\right.}{a}$$
$$= \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} - \frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} - \frac{5\int \sec^3(c+dx)\tan^2(c+}{16a}$$
$$= -\frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} + \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} - \frac{\sec^3(c+dx)\tan^5(c+dx}{8ad}$$
$$= \frac{5\sec(c+dx)\tan(c+dx)}{128ad} - \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} + \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad}$$
$$= \frac{5\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{5\sec(c+dx)\tan(c+dx)}{128ad} - \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} + 5\text{s}$$

Mathematica [A] time = 1.1749, size = 92, normalized size = 0.61

$$\frac{-\frac{15}{\sin(c+dx)-1} - \frac{15}{(\sin(c+dx)-1)^2} + \frac{30}{(\sin(c+dx)+1)^2} - \frac{4}{(\sin(c+dx)-1)^3} - \frac{24}{(\sin(c+dx)+1)^3} + \frac{6}{(\sin(c+dx)+1)^4} + 15\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] - 4/(-1 + Sin[c + d*x])^3 - 15/(-1 + Sin[c + d*x]
)^2 - 15/(-1 + Sin[c + d*x]) + 6/(1 + Sin[c + d*x])^4 - 24/(1 + Sin[c + d*x
])^3 + 30/(1 + Sin[c + d*x])^2)/(384*a*d)
```

Maple [A] time = 0.089, size = 144, normalized size = 1.

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3} - \frac{5}{128\,da\,(\sin(dx+c)-1)^2} - \frac{5}{128\,da\,(\sin(dx+c)-1)} - \frac{5\ln(\sin(dx+c)-1)}{256\,da} + \frac{1}{64\,da\,(1+\text{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3-5/128/d/a/(sin(d*x+c)-1)^2-5/128/a/d/(sin(d*x+c)
-1)-5/256/a/d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4-1/16/d/a/(1+sin(d*
```

```
x+c))^3+5/64/a/d/(1+sin(d*x+c))^2+5/256*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.02561, size = 234, normalized size = 1.54

$$-\frac{\frac{2\left(15\sin(dx+c)^6+15\sin(dx+c)^5+88\sin(dx+c)^4-8\sin(dx+c)^3-63\sin(dx+c)^2+\sin(dx+c)+16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(15*sin(d*x + c)^6 + 15*sin(d*x + c)^5 + 88*sin(d*x + c)^4 - 8*si
n(d*x + c)^3 - 63*sin(d*x + c)^2 + sin(d*x + c) + 16)/(a*sin(d*x + c)^7 + a
*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c
)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c) + 1)/a
 + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.63004, size = 460, normalized size = 3.03

$$-\frac{30\cos(dx+c)^6-266\cos(dx+c)^4+316\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx}{768\left(ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(30*cos(d*x + c)^6 - 266*cos(d*x + c)^4 + 316*cos(d*x + c)^2 - 15*(c
os(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(co
s(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*
cos(d*x + c)^4 - 22*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x +
c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36729, size = 184, normalized size = 1.21

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a}-\frac{60\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(55\sin(dx+c)^3-225\sin(dx+c)^2+225\sin(dx+c)-71\right)}{a(\sin(dx+c)-1)^3}-\frac{125\sin(dx+c)^4+500\sin(dx+c)^3+510\sin(dx}{a(\sin(dx+c)+1)^4}}{3072d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a +
 2*(55*sin(d*x + c)^3 - 225*sin(d*x + c)^2 + 225*sin(d*x + c) - 71)/(a*(sin
(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 500*sin(d*x + c)^3 + 510*sin(d*x
+ c)^2 + 212*sin(d*x + c) + 29)/(a*(sin(d*x + c) + 1)^4))/d
```

3.885 $\int \frac{\sec^3(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{\tan^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}+\text{ta}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(128*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(128*a*
d) + (Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])
/(16*a*d) + (Sec[c + d*x]^5*Tan[c + d*x]^3)/(8*a*d) - Tan[c + d*x]^6/(6*a*d
) - Tan[c + d*x]^8/(8*a*d)
```

Rubi [A] time = 0.225859, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2611, 3768, 3770, 2607, 14}

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{\tan^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}+\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(128*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(128*a*
d) + (Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])
/(16*a*d) + (Sec[c + d*x]^5*Tan[c + d*x]^3)/(8*a*d) - Tan[c + d*x]^6/(6*a*d
) - Tan[c + d*x]^8/(8*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{a} - \frac{\int \sec^4(c+dx)\tan^5(c+dx)\,dx}{a}$$
$$= \frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad} - \frac{3\int \sec^5(c+dx)\tan^2(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^5\left(1+x^2\right)\right.}{ad}$$
$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{16ad} + \frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad} + \frac{\int \sec^5(c+dx)\,dx}{16a} - \text{Subst}$$
$$= \frac{\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{\sec^5(c+dx)\tan(c+dx)}{16ad} + \frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad} - \text{ta}$$
$$= \frac{3\sec(c+dx)\tan(c+dx)}{128ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{\sec^5(c+dx)\tan(c+dx)}{16ad} + \text{se}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{3\sec(c+dx)\tan(c+dx)}{128ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{64ad} - \sec^5$$

Mathematica [A] time = 0.632404, size = 101, normalized size = 0.67

$$\frac{\frac{-9\sin^6(c+dx)-9\sin^5(c+dx)+24\sin^4(c+dx)-72\sin^3(c+dx)-39\sin^2(c+dx)+25\sin(c+dx)+16}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4}+9\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (9*ArcTanh[Sin[c + d*x]] + (16 + 25*Sin[c + d*x] - 39*Sin[c + d*x]^2 - 72*S
in[c + d*x]^3 + 24*Sin[c + d*x]^4 - 9*Sin[c + d*x]^5 - 9*Sin[c + d*x]^6)/((
-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

Maple [A] time = 0.086, size = 162, normalized size = 1.1

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3} - \frac{3}{128\,da\,(\sin(dx+c)-1)^2} + \frac{1}{128\,da\,(\sin(dx+c)-1)} - \frac{3\ln(\sin(dx+c)-1)}{256\,da} - \frac{1}{64\,da\,(1+\text{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3-3/128/d/a/(sin(d*x+c)-1)^2+1/128/a/d/(sin(d*x+c)
-1)-3/256/a/d*ln(sin(d*x+c)-1)-1/64/d/a/(1+sin(d*x+c))^4+1/24/d/a/(1+sin(d*
```

```
x+c))^3-1/64/a/d/(1+sin(d*x+c))^2-1/32/a/d/(1+sin(d*x+c))+3/256*ln(1+sin(d*
x+c))/a/d
```

Maxima [A] time = 1.11721, size = 236, normalized size = 1.57

$$-\frac{\frac{2\left(9\sin(dx+c)^6+9\sin(dx+c)^5-24\sin(dx+c)^4+72\sin(dx+c)^3+39\sin(dx+c)^2-25\sin(dx+c)-16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{9\log(\sin(dx+c)+1)}{a}+\frac{9\log(\sin(dx+c)-1)}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(9*sin(d*x + c)^6 + 9*sin(d*x + c)^5 - 24*sin(d*x + c)^4 + 72*sin
(d*x + c)^3 + 39*sin(d*x + c)^2 - 25*sin(d*x + c) - 16)/(a*sin(d*x + c)^7 +
 a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x +
 c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 9*log(sin(d*x + c) + 1)/
a + 9*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.46116, size = 452, normalized size = 3.01

$$-\frac{18\cos(dx+c)^6-6\cos(dx+c)^4+36\cos(dx+c)^2-9\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)}{768\left(ad\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(18*cos(d*x + c)^6 - 6*cos(d*x + c)^4 + 36*cos(d*x + c)^2 - 9*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 9*(cos(d*x
 + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*
x + c)^4 - 90*cos(d*x + c)^2 + 56)*sin(d*x + c) - 16)/(a*d*cos(d*x + c)^6*s
in(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33109, size = 184, normalized size = 1.23

$$\frac{\frac{36\log(|\sin(dx+c)+1|)}{a}-\frac{36\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(33\sin(dx+c)^3-87\sin(dx+c)^2+39\sin(dx+c)-1\right)}{a(\sin(dx+c)-1)^3}-\frac{75\sin(dx+c)^4+396\sin(dx+c)^3+786\sin(dx+c)^2}{a(\sin(dx+c)+1)^4}}{3072d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(36*log(abs(sin(d*x + c) + 1))/a - 36*log(abs(sin(d*x + c) - 1))/a +
 2*(33*sin(d*x + c)^3 - 87*sin(d*x + c)^2 + 39*sin(d*x + c) - 1)/(a*(sin(d*
x + c) - 1)^3) - (75*sin(d*x + c)^4 + 396*sin(d*x + c)^3 + 786*sin(d*x + c)
^2 + 556*sin(d*x + c) + 139)/(a*(sin(d*x + c) + 1)^4))/d
```

3.886 $\int \frac{\sec^4(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$\frac{\sec^8(c+dx)}{8ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}-\tan(c$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(128*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x
]^8/(8*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (Sec[c + d*x]^3*Tan
[c + d*x])/(64*a*d) + (Sec[c + d*x]^5*Tan[c + d*x])/(16*a*d) - (Sec[c + d*x
]^5*Tan[c + d*x]^3)/(8*a*d)
```

Rubi [A] time = 0.224721, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2606, 14, 2611, 3768, 3770}

$$\frac{\sec^8(c+dx)}{8ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}-\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(128*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x
]^8/(8*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (Sec[c + d*x]^3*Tan
[c + d*x])/(64*a*d) + (Sec[c + d*x]^5*Tan[c + d*x])/(16*a*d) - (Sec[c + d*x
]^5*Tan[c + d*x]^3)/(8*a*d)
```


Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}dx = \frac{\int \sec^6(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{a}$$

$$= -\frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad} + \frac{3\int \sec^5(c+dx)\tan^2(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^5\left(-1+x\right.\right.}{a}$$

$$= \frac{\sec^5(c+dx)\tan(c+dx)}{16ad} - \frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad} - \frac{\int \sec^5(c+dx)\,dx}{16a} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{64ad} + \frac{\sec^5(c+dx)\tan(c+dx)}{16ad}$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{8ad} - \frac{3\sec(c+dx)\tan(c+dx)}{128ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{64ad}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{128ad} - \frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{8ad} - \frac{3\sec(c+dx)\tan(c+dx)}{128ad} -$$

Mathematica [A] time = 0.878819, size = 92, normalized size = 0.61

$$-\frac{-\frac{9}{\sin(c+dx)-1} + \frac{3}{(\sin(c+dx)-1)^2} + \frac{6}{(\sin(c+dx)+1)^2} + \frac{4}{(\sin(c+dx)-1)^3} + \frac{8}{(\sin(c+dx)+1)^3} - \frac{6}{(\sin(c+dx)+1)^4} + 9\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(9*ArcTanh[Sin[c + d*x]] + 4/(-1 + Sin[c + d*x])^3 + 3/(-1 + Sin[c + d*x])
^2 - 9/(-1 + Sin[c + d*x]) - 6/(1 + Sin[c + d*x])^4 + 8/(1 + Sin[c + d*x])^
3 + 6/(1 + Sin[c + d*x])^2)/(384*a*d)
```

Maple [A] time = 0.082, size = 144, normalized size = 1.

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3} - \frac{1}{128\,da\,(\sin(dx+c)-1)^2} + \frac{3}{128\,da\,(\sin(dx+c)-1)} + \frac{3\ln(\sin(dx+c)-1)}{256\,da} + \frac{1}{64\,da\,(1+\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3-1/128/d/a/(sin(d*x+c)-1)^2+3/128/a/d/(sin(d*x+c)
-1)+3/256/a/d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4-1/48/d/a/(1+sin(d*
```

```
x+c))^3-1/64/a/d/(1+sin(d*x+c))^2-3/256*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.26131, size = 236, normalized size = 1.57

$$\frac{\frac{2\left(9\sin(dx+c)^6+9\sin(dx+c)^5-24\sin(dx+c)^4-24\sin(dx+c)^3-57\sin(dx+c)^2+7\sin(dx+c)+16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{9\log(\sin(dx+c)+1)}{a}+\frac{9\log(\sin(dx+c)-1)}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(9*sin(d*x + c)^6 + 9*sin(d*x + c)^5 - 24*sin(d*x + c)^4 - 24*sin(
d*x + c)^3 - 57*sin(d*x + c)^2 + 7*sin(d*x + c) + 16)/(a*sin(d*x + c)^7 + a
*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c
)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 9*log(sin(d*x + c) + 1)/a
+ 9*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.5533, size = 451, normalized size = 3.01

$$\frac{18\cos(dx+c)^6-6\cos(dx+c)^4-156\cos(dx+c)^2-9\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)}{768\left(ad\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(18*cos(d*x + c)^6 - 6*cos(d*x + c)^4 - 156*cos(d*x + c)^2 - 9*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 9*(cos(d*x
 + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*
x + c)^4 + 6*cos(d*x + c)^2 - 8)*sin(d*x + c) + 112)/(a*d*cos(d*x + c)^6*si
n(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40873, size = 184, normalized size = 1.23

$$-\frac{\frac{36\log(|\sin(dx+c)+1|)}{a}-\frac{36\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(33\sin(dx+c)^3-135\sin(dx+c)^2+183\sin(dx+c)-65\right)}{a(\sin(dx+c)-1)^3}-\frac{75\sin(dx+c)^4+300\sin(dx+c)^3+402\sin(d}{a(\sin(dx+c)+1)^4}}{3072d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(36*log(abs(sin(d*x + c) + 1))/a - 36*log(abs(sin(d*x + c) - 1))/a
+ 2*(33*sin(d*x + c)^3 - 135*sin(d*x + c)^2 + 183*sin(d*x + c) - 65)/(a*(si
n(d*x + c) - 1)^3) - (75*sin(d*x + c)^4 + 300*sin(d*x + c)^3 + 402*sin(d*x
+ c)^2 + 140*sin(d*x + c) + 11)/(a*(sin(d*x + c) + 1)^4))/d
```

3.887 $\int \frac{\sec^5(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{\sec^8(c+dx)}{8ad}+\frac{\sec^6(c+dx)}{6ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}-\frac{5\tan}{}$$

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(128*a*d) + Sec[c + d*x]^6/(6*a*d) - Sec[c + d*x
]^8/(8*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (5*Sec[c + d*x]^3*T
an[c + d*x])/(192*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])/(48*a*d) + (Sec[c +
d*x]^7*Tan[c + d*x])/(8*a*d)
```

Rubi [A] time = 0.199473, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2611, 3768, 3770, 2606, 14}

$$-\frac{\sec^8(c+dx)}{8ad}+\frac{\sec^6(c+dx)}{6ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}-\frac{5\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(128*a*d) + Sec[c + d*x]^6/(6*a*d) - Sec[c + d*x
]^8/(8*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (5*Sec[c + d*x]^3*T
an[c + d*x])/(192*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])/(48*a*d) + (Sec[c +
d*x]^7*Tan[c + d*x])/(8*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{a}-\frac{\int \sec^6(c+dx)\tan^3(c+dx)\,dx}{a}\\ &= \frac{\sec^7(c+dx)\tan(c+dx)}{8ad}-\frac{\int \sec^7(c+dx)\,dx}{8a}-\frac{\text{Subst}\left(\int x^5\left(-1+x^2\right)\,dx,x,\sec(c+d\right.}{ad}\\ &= -\frac{\sec^5(c+dx)\tan(c+dx)}{48ad}+\frac{\sec^7(c+dx)\tan(c+dx)}{8ad}-\frac{5\int \sec^5(c+dx)\,dx}{48a}-\frac{\text{Subst}}{}\\ &= \frac{\sec^6(c+dx)}{6ad}-\frac{\sec^8(c+dx)}{8ad}-\frac{5\sec^3(c+dx)\tan(c+dx)}{192ad}-\frac{\sec^5(c+dx)\tan(c+dx)}{48ad}\\ &= \frac{\sec^6(c+dx)}{6ad}-\frac{\sec^8(c+dx)}{8ad}-\frac{5\sec(c+dx)\tan(c+dx)}{128ad}-\frac{5\sec^3(c+dx)\tan(c+dx)}{192ad}\\ &= -\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\sec^6(c+dx)}{6ad}-\frac{\sec^8(c+dx)}{8ad}-\frac{5\sec(c+dx)\tan(c+dx)}{128ad}-\end{aligned}$$

Mathematica [A] time = 0.523227, size = 92, normalized size = 0.62

$$-\frac{-\frac{3}{\sin(c+dx)-1}-\frac{12}{\sin(c+dx)+1}-\frac{3}{(\sin(c+dx)-1)^2}-\frac{6}{(\sin(c+dx)+1)^2}+\frac{4}{(\sin(c+dx)-1)^3}+\frac{6}{(\sin(c+dx)+1)^4}+15\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(15*ArcTanh[Sin[c + d*x]] + 4/(-1 + Sin[c + d*x])^3 - 3/(-1 + Sin[c + d*x]
)^2 - 3/(-1 + Sin[c + d*x]) + 6/(1 + Sin[c + d*x])^4 - 6/(1 + Sin[c + d*x])
^2 - 12/(1 + Sin[c + d*x]))/(384*a*d)
```

Maple [A] time = 0.081, size = 144, normalized size = 1.

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3}+\frac{1}{128\,da\,(\sin(dx+c)-1)^2}+\frac{1}{128\,da\,(\sin(dx+c)-1)}+\frac{5\,\ln(\sin(dx+c)-1)}{256\,da}-\frac{1}{64\,da\,(1+\text{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3+1/128/d/a/(sin(d*x+c)-1)^2+1/128/a/d/(sin(d*x+c)
-1)+5/256/a/d*ln(sin(d*x+c)-1)-1/64/d/a/(1+sin(d*x+c))^4+1/64/a/d/(1+sin(d*
```

```
x+c))^2+1/32/a/d/(1+sin(d*x+c))-5/256*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.26687, size = 236, normalized size = 1.59

$$\frac{\frac{2\left(15\sin(dx+c)^6+15\sin(dx+c)^5-40\sin(dx+c)^4-40\sin(dx+c)^3+33\sin(dx+c)^2-31\sin(dx+c)-16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-1)}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(15*sin(d*x + c)^6 + 15*sin(d*x + c)^5 - 40*sin(d*x + c)^4 - 40*si
n(d*x + c)^3 + 33*sin(d*x + c)^2 - 31*sin(d*x + c) - 16)/(a*sin(d*x + c)^7
+ a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x
+ c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c) + 1
)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.58372, size = 455, normalized size = 3.07

$$\frac{30\cos(dx+c)^6-10\cos(dx+c)^4-4\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)}{768\left(ad\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(30*cos(d*x + c)^6 - 10*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 15*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(cos(d*
x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*cos(
d*x + c)^4 + 10*cos(d*x + c)^2 - 56)*sin(d*x + c) + 16)/(a*d*cos(d*x + c)^6
*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34551, size = 184, normalized size = 1.24

$$-\frac{\frac{60\log(|\sin(dx+c)+1|)}{a}-\frac{60\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(55\sin(dx+c)^3-177\sin(dx+c)^2+177\sin(dx+c)-39\right)}{a(\sin(dx+c)-1)^3}-\frac{125\sin(dx+c)^4+596\sin(dx+c)^3+1086\sin}{a(\sin(dx+c)+1}}{3072d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
+ 2*(55*sin(d*x + c)^3 - 177*sin(d*x + c)^2 + 177*sin(d*x + c) - 39)/(a*(si
n(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 596*sin(d*x + c)^3 + 1086*sin(d*
x + c)^2 + 884*sin(d*x + c) + 221)/(a*(sin(d*x + c) + 1)^4))/d
```

3.888 $\int \frac{\sec^6(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{\sec^8(c+dx)}{8ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}+\frac{5\tan(c+dx)\sec^3(c-}{192ad}$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(128*a*d) + Sec[c + d*x]^8/(8*a*d) + (5*Sec[c + d
*x]*Tan[c + d*x])/(128*a*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(192*a*d) + (
Sec[c + d*x]^5*Tan[c + d*x])/(48*a*d) - (Sec[c + d*x]^7*Tan[c + d*x])/(8*a*
d)
```

Rubi [A] time = 0.147518, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2835, 2606, 30, 2611, 3768, 3770}

$$\frac{\sec^8(c+dx)}{8ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}+\frac{5\tan(c+dx)\sec^3(c-}{192ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^6*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(128*a*d) + Sec[c + d*x]^8/(8*a*d) + (5*Sec[c + d
*x]*Tan[c + d*x])/(128*a*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(192*a*d) + (
Sec[c + d*x]^5*Tan[c + d*x])/(48*a*d) - (Sec[c + d*x]^7*Tan[c + d*x])/(8*a*
d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^8(c+dx)\tan(c+dx)\,dx}{a}-\frac{\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= -\frac{\sec^7(c+dx)\tan(c+dx)}{8ad}+\frac{\int \sec^7(c+dx)\,dx}{8a}+\frac{\text{Subst}\left(\int x^7\,dx,x,\sec(c+dx)\right)}{ad}\\ &= \frac{\sec^8(c+dx)}{8ad}+\frac{\sec^5(c+dx)\tan(c+dx)}{48ad}-\frac{\sec^7(c+dx)\tan(c+dx)}{8ad}+\frac{5\int \sec^5(c+dx)}{48a}\\ &= \frac{\sec^8(c+dx)}{8ad}+\frac{5\sec^3(c+dx)\tan(c+dx)}{192ad}+\frac{\sec^5(c+dx)\tan(c+dx)}{48ad}-\frac{\sec^7(c+dx)\tan}{8ad}\\ &= \frac{\sec^8(c+dx)}{8ad}+\frac{5\sec(c+dx)\tan(c+dx)}{128ad}+\frac{5\sec^3(c+dx)\tan(c+dx)}{192ad}+\frac{\sec^5(c+dx)}{48a}\\ &= \frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\sec^8(c+dx)}{8ad}+\frac{5\sec(c+dx)\tan(c+dx)}{128ad}+\frac{5\sec^3(c+dx)\tan}{192ad}\end{aligned}$$

Mathematica [A] time = 0.870656, size = 92, normalized size = 0.71

$$\frac{-\frac{15}{\sin(c+dx)-1}+\frac{9}{(\sin(c+dx)-1)^2}+\frac{6}{(\sin(c+dx)+1)^2}-\frac{4}{(\sin(c+dx)-1)^3}+\frac{8}{(\sin(c+dx)+1)^3}+\frac{6}{(\sin(c+dx)+1)^4}+15\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^6*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] - 4/(-1 + Sin[c + d*x])^3 + 9/(-1 + Sin[c + d*x])
^2 - 15/(-1 + Sin[c + d*x]) + 6/(1 + Sin[c + d*x])^4 + 8/(1 + Sin[c + d*x])
^3 + 6/(1 + Sin[c + d*x])^2)/(384*a*d)
```

Maple [A] time = 0.068, size = 144, normalized size = 1.1

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3}+\frac{3}{128\,da\,(\sin(dx+c)-1)^2}-\frac{5}{128\,da\,(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{256\,da}+\frac{1}{64\,da\,(1+\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3+3/128/d/a/(sin(d*x+c)-1)^2-5/128/a/d/(sin(d*x+c)
-1)-5/256/a/d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4+1/48/d/a/(1+sin(d*
```

```
x+c))^3+1/64/a/d/(1+sin(d*x+c))^2+5/256*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.27393, size = 236, normalized size = 1.82

$$-\frac{\frac{2\left(15\sin(dx+c)^6+15\sin(dx+c)^5-40\sin(dx+c)^4-40\sin(dx+c)^3+33\sin(dx+c)^2+33\sin(dx+c)+48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(15*sin(d*x + c)^6 + 15*sin(d*x + c)^5 - 40*sin(d*x + c)^4 - 40*s
in(d*x + c)^3 + 33*sin(d*x + c)^2 + 33*sin(d*x + c) + 48)/(a*sin(d*x + c)^7
 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x
 + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c) +
1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.6565, size = 456, normalized size = 3.51

$$-\frac{30\cos(dx+c)^6-10\cos(dx+c)^4-4\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c}{768\left(ad\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(30*cos(d*x + c)^6 - 10*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 15*(cos(
d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*cos
(d*x + c)^4 + 10*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x + c)^
6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28742, size = 184, normalized size = 1.42

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a}-\frac{60\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(55\sin(dx+c)^3-225\sin(dx+c)^2+321\sin(dx+c)-167\right)}{a(\sin(dx+c)-1)^3}-\frac{125\sin(dx+c)^4+500\sin(dx+c)^3+702\sin(}{a(\sin(dx+c)+1)^4}}{3072d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a +
 2*(55*sin(d*x + c)^3 - 225*sin(d*x + c)^2 + 321*sin(d*x + c) - 167)/(a*(si
n(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 500*sin(d*x + c)^3 + 702*sin(d*x
 + c)^2 + 340*sin(d*x + c) - 35)/(a*(sin(d*x + c) + 1)^4))/d
```

3.889 $\int \frac{\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=165

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{5a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

```
[Out] (35*ArcTanh[Sin[c + d*x]])/(128*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) +
(5*a)/(128*d*(a - a*Sin[c + d*x])^2) + 15/(128*d*(a - a*Sin[c + d*x])) - a^
3/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(24*d*(a + a*Sin[c + d*x])^3) - (5*a)
/(64*d*(a + a*Sin[c + d*x])^2) - 5/(32*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.125898, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{5a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```


```
[Out] (35*ArcTanh[Sin[c + d*x]])/(128*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) +
(5*a)/(128*d*(a - a*Sin[c + d*x])^2) + 15/(128*d*(a - a*Sin[c + d*x])) - a^
3/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(24*d*(a + a*Sin[c + d*x])^3) - (5*a)
/(64*d*(a + a*Sin[c + d*x])^2) - 5/(32*d*(a + a*Sin[c + d*x]))
```


Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^4}+\frac{5}{64a^6(a-x)^3}+\frac{15}{128a^7(a-x)^2}+\frac{1}{16a^4(a+x)^5}+\frac{1}{8a^5(a+x)^4}+\frac{5}{32a^6(a+x)^3}+\frac{5}{32a^7(a+x)^2}\right.\right.}{d}$$

$$= \frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{5a}{128d(a-a\sin(c+dx))^2}+\frac{15}{128d(a-a\sin(c+dx))}-\frac{a^3}{64d(a+a\sin(}$$

$$= \frac{35\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{5a}{128d(a-a\sin(c+dx))^2}+\frac{15}{128d(a-a\sin}$$

Mathematica [A] time = 0.527043, size = 145, normalized size = 0.88

$$-\frac{\sec^6(c+dx)\left(-105\sin^6(c+dx)-105\sin^5(c+dx)+280\sin^4(c+dx)+280\sin^3(c+dx)-231\sin^2(c+dx)-231\sin(c\right.}{384ad(\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^6*(48 - 105*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8 - 231*Sin[c + d*x] -
231*Sin[c + d*x]^2 + 280*Sin[c + d*x]^3 + 280*Sin[c + d*x]^4 - 105*Sin[c +
d*x]^5 - 105*Sin[c + d*x]^6))/(384*a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.073, size = 162, normalized size = 1.

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3}+\frac{5}{128\,da\,(\sin(dx+c)-1)^2}-\frac{15}{128\,da\,(\sin(dx+c)-1)}-\frac{35\,\ln(\sin(dx+c)-1)}{256\,da}-\frac{}{64\,da\,(1+s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3+5/128/d/a/(sin(d*x+c)-1)^2-15/128/a/d/(sin(d*x+c
)-1)-35/256/a/d*ln(sin(d*x+c)-1)-1/64/d/a/(1+sin(d*x+c))^4-1/24/d/a/(1+sin(
d*x+c))^3-5/64/a/d/(1+sin(d*x+c))^2-5/32/a/d/(1+sin(d*x+c))+35/256*ln(1+sin
(d*x+c))/a/d
```

Maxima [A] time = 1.06108, size = 236, normalized size = 1.43

$$-\frac{\frac{2\left(105\sin(dx+c)^6+105\sin(dx+c)^5-280\sin(dx+c)^4-280\sin(dx+c)^3+231\sin(dx+c)^2+231\sin(dx+c)-48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{105\log(\sin(dx+c)+1)}{a}+\frac{105\log(\sin(dx+c)-}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(105*sin(d*x + c)^6 + 105*sin(d*x + c)^5 - 280*sin(d*x + c)^4 - 2
80*sin(d*x + c)^3 + 231*sin(d*x + c)^2 + 231*sin(d*x + c) - 48)/(a*sin(d*x
+ c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*s
in(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 105*log(sin(d*x
+ c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.51134, size = 462, normalized size = 2.8

$$-\frac{210\cos(dx+c)^6 - 70\cos(dx+c)^4 - 28\cos(dx+c)^2 - 105\left(\cos(dx+c)^6\sin(dx+c) + \cos(dx+c)^6\right)\log(\sin(dx}{768\left(ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(210*cos(d*x + c)^6 - 70*cos(d*x + c)^4 - 28*cos(d*x + c)^2 - 105*(c
os(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 105*(c
os(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 14*(1
5*cos(d*x + c)^4 + 10*cos(d*x + c)^2 + 8)*sin(d*x + c) - 16)/(a*d*cos(d*x +
 c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32448, size = 184, normalized size = 1.12

$$\frac{\frac{420\log(|\sin(dx+c)+1|)}{a} - \frac{420\log(|\sin(dx+c)-1|)}{a} + \frac{2\left(385\sin(dx+c)^3-1335\sin(dx+c)^2+1575\sin(dx+c)-641\right)}{a(\sin(dx+c)-1)^3} - \frac{875\sin(dx+c)^4+3980\sin(dx+c)^3+69}{a(\sin(d}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/a
 + 2*(385*sin(d*x + c)^3 - 1335*sin(d*x + c)^2 + 1575*sin(d*x + c) - 641)/(
a*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 3980*sin(d*x + c)^3 + 6930*
sin(d*x + c)^2 + 5548*sin(d*x + c) + 1771)/(a*(sin(d*x + c) + 1)^4))/d
```

3.890 $\int \frac{\csc(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=202

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{16d(a\sin(c+dx)+a)^3}+\frac{7a}{128d(a-a\sin(c+dx))^2}+\frac{11a}{64d(a\sin(c+$$

```
[Out] (-93*Log[1 - Sin[c + d*x]])/(256*a*d) + Log[Sin[c + d*x]]/(a*d) - (163*Log[
1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (7*a)/(1
28*d*(a - a*Sin[c + d*x])^2) + 29/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*
(a + a*Sin[c + d*x])^4) + a^2/(16*d*(a + a*Sin[c + d*x])^3) + (11*a)/(64*d*
(a + a*Sin[c + d*x])^2) + 1/(2*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.200673, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{16d(a\sin(c+dx)+a)^3}+\frac{7a}{128d(a-a\sin(c+dx))^2}+\frac{11a}{64d(a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-93*Log[1 - Sin[c + d*x]])/(256*a*d) + Log[Sin[c + d*x]]/(a*d) - (163*Log[
1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (7*a)/(1
28*d*(a - a*Sin[c + d*x])^2) + 29/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*
(a + a*Sin[c + d*x])^4) + a^2/(16*d*(a + a*Sin[c + d*x])^3) + (11*a)/(64*d*
(a + a*Sin[c + d*x])^2) + 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7 \operatorname{Subst}\left(\int \frac{a}{(a-x)^4x(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^8 \operatorname{Subst}\left(\int \frac{1}{(a-x)^4x(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^8 \operatorname{Subst}\left(\int \left(\frac{1}{32a^6(a-x)^4} + \frac{7}{64a^7(a-x)^3} + \frac{29}{128a^8(a-x)^2} + \frac{93}{256a^9(a-x)} + \frac{1}{a^9x} - \frac{1}{16a^5(a+x)^5} - \frac{3}{16a^6(\cdots}\right.\right.}{d}$$

$$= -\frac{93\log(1-\sin(c+dx))}{256ad} + \frac{\log(\sin(c+dx))}{ad} - \frac{163\log(1+\sin(c+dx))}{256ad} + \frac{}{96d(a-a\cdots}$$

Mathematica [A] time = 6.12995, size = 189, normalized size = 0.94

$$\frac{a^8\left(\frac{29}{128a^8(a-a\sin(c+dx))} + \frac{1}{2a^8(a\sin(c+dx)+a)} + \frac{7}{128a^7(a-a\sin(c+dx))^2} + \frac{11}{64a^7(a\sin(c+dx)+a)^2} + \frac{1}{96a^6(a-a\sin(c+dx))^3} + \frac{1}{16a^6(a\sin(c+dx)+a)^3}\cdots\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^8*((-93*Log[1 - Sin[c + d*x]])/(256*a^9) + Log[Sin[c + d*x]]/a^9 - (163*
Log[1 + Sin[c + d*x]])/(256*a^9) + 1/(96*a^6*(a - a*Sin[c + d*x])^3) + 7/(1
28*a^7*(a - a*Sin[c + d*x])^2) + 29/(128*a^8*(a - a*Sin[c + d*x])) + 1/(64*
a^5*(a + a*Sin[c + d*x])^4) + 1/(16*a^6*(a + a*Sin[c + d*x])^3) + 11/(64*a^
7*(a + a*Sin[c + d*x])^2) + 1/(2*a^8*(a + a*Sin[c + d*x]))))/d
```

Maple [A] time = 0.089, size = 176, normalized size = 0.9

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3} + \frac{7}{128\,da\,(\sin(dx+c)-1)^2} - \frac{29}{128\,da\,(\sin(dx+c)-1)} - \frac{93\,\ln(\sin(dx+c)-1)}{256\,da} + \frac{}{64\,da\,(1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3+7/128/d/a/(sin(d*x+c)-1)^2-29/128/a/d/(sin(d*x+c
)-1)-93/256/a/d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4+1/16/d/a/(1+sin(
d*x+c))^3+11/64/a/d/(1+sin(d*x+c))^2+1/2/a/d/(1+sin(d*x+c))-163/256*ln(1+si
n(d*x+c))/a/d+ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.02196, size = 252, normalized size = 1.25

$$\frac{\frac{2\left(105\sin(dx+c)^6-87\sin(dx+c)^5-472\sin(dx+c)^4+200\sin(dx+c)^3+711\sin(dx+c)^2-121\sin(dx+c)-400\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a} - \frac{489\log(\sin(dx+c)+1)}{a} - \frac{279\log(\sin(dx+c)\cdots}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(105*sin(d*x + c)^6 - 87*sin(d*x + c)^5 - 472*sin(d*x + c)^4 + 200
*sin(d*x + c)^3 + 711*sin(d*x + c)^2 - 121*sin(d*x + c) - 400)/(a*sin(d*x +
 c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*si
n(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 489*log(sin(d*x +
 c) + 1)/a - 279*log(sin(d*x + c) - 1)/a + 768*log(sin(d*x + c))/a)/d
```

Fricas [A] time = 1.64071, size = 564, normalized size = 2.79

$$\frac{210 \cos(dx+c)^6 + 314 \cos(dx+c)^4 + 164 \cos(dx+c)^2 + 768 \left(\cos(dx+c)^6 \sin(dx+c) + \cos(dx+c)^6\right) \log\left(\frac{1}{2} \sin(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(210*cos(d*x + c)^6 + 314*cos(d*x + c)^4 + 164*cos(d*x + c)^2 + 768*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(1/2*sin(d*x + c)) - 489*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 279*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(8
7*cos(d*x + c)^4 + 26*cos(d*x + c)^2 + 8)*sin(d*x + c) + 112)/(a*d*cos(d*x
+ c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34404, size = 201, normalized size = 1.

$$-\frac{\frac{1956 \log(|\sin(dx+c)+1|)}{a} + \frac{1116 \log(|\sin(dx+c)-1|)}{a} - \frac{3072 \log(|\sin(dx+c)|)}{a} - \frac{2\left(1023 \sin(dx+c)^3 - 3417 \sin(dx+c)^2 + 3849 \sin(dx+c) - 1471\right)}{a(\sin(dx+c)-1)^3} - \frac{4075 \sin}{}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(1956*log(abs(sin(d*x + c) + 1))/a + 1116*log(abs(sin(d*x + c) - 1)
)/a - 3072*log(abs(sin(d*x + c)))/a - 2*(1023*sin(d*x + c)^3 - 3417*sin(d*x
 + c)^2 + 3849*sin(d*x + c) - 1471)/(a*(sin(d*x + c) - 1)^3) - (4075*sin(d*
x + c)^4 + 17836*sin(d*x + c)^3 + 29586*sin(d*x + c)^2 + 22156*sin(d*x + c)
 + 6379)/(a*(sin(d*x + c) + 1)^4))/d
```

3.891 $\int \frac{\csc^2(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=217

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{12d(a\sin(c+dx)+a)^3}+\frac{9a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - (187*Log[1 - Sin[c + d*x]])/(256*a*d) - Log[Sin[c +
 d*x]]/(a*d) + (443*Log[1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin
[c + d*x])^3) + (9*a)/(128*d*(a - a*Sin[c + d*x])^2) + 47/(128*d*(a - a*Sin
[c + d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(12*d*(a + a*Sin[c +
d*x])^3) - (19*a)/(64*d*(a + a*Sin[c + d*x])^2) - 35/(32*d*(a + a*Sin[c + d
*x]))
```

Rubi [A] time = 0.239394, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{12d(a\sin(c+dx)+a)^3}+\frac{9a}{128d(a-a\sin(c+dx))^2}-\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (187*Log[1 - Sin[c + d*x]])/(256*a*d) - Log[Sin[c +
 d*x]]/(a*d) + (443*Log[1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin
[c + d*x])^3) + (9*a)/(128*d*(a - a*Sin[c + d*x])^2) + 47/(128*d*(a - a*Sin
[c + d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(12*d*(a + a*Sin[c +
d*x])^3) - (19*a)/(64*d*(a + a*Sin[c + d*x])^2) - 35/(32*d*(a + a*Sin[c + d
*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\operatorname{Subst}\left(\int \frac{a^2}{(a-x)^4x^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\operatorname{Subst}\left(\int \frac{1}{(a-x)^4x^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\operatorname{Subst}\left(\int \left(\frac{1}{32a^7(a-x)^4}+\frac{9}{64a^8(a-x)^3}+\frac{47}{128a^9(a-x)^2}+\frac{187}{256a^{10}(a-x)}+\frac{1}{a^9x^2}-\frac{1}{a^{10}x}+\frac{1}{16a^6(a+x)^5}+\right.\right.}{d}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{187\log(1-\sin(c+dx))}{256ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{443\log(1+\sin(c+dx))}{256ad}$$

Mathematica [A] time = 6.13852, size = 201, normalized size = 0.93

$$\frac{a^9\left(\frac{47}{128a^9(a-a\sin(c+dx))}-\frac{35}{32a^9(a\sin(c+dx)+a)}+\frac{9}{128a^8(a-a\sin(c+dx))^2}-\frac{19}{64a^8(a\sin(c+dx)+a)^2}+\frac{1}{96a^7(a-a\sin(c+dx))^3}-\frac{1}{12a^7(a\sin(c+dx)+a)^3}-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^9*(-(Csc[c + d*x]/a^10) - (187*Log[1 - Sin[c + d*x]])/(256*a^10) - Log[S
in[c + d*x]]/a^10 + (443*Log[1 + Sin[c + d*x]])/(256*a^10) + 1/(96*a^7*(a -
 a*Sin[c + d*x])^3) + 9/(128*a^8*(a - a*Sin[c + d*x])^2) + 47/(128*a^9*(a -
 a*Sin[c + d*x])) - 1/(64*a^6*(a + a*Sin[c + d*x])^4) - 1/(12*a^7*(a + a*Si
n[c + d*x])^3) - 19/(64*a^8*(a + a*Sin[c + d*x])^2) - 35/(32*a^9*(a + a*Sin
[c + d*x]))))/d
```

Maple [A] time = 0.103, size = 193, normalized size = 0.9

$$-\frac{1}{96\,da\left(\sin(dx+c)-1\right)^3}+\frac{9}{128\,da\left(\sin(dx+c)-1\right)^2}-\frac{47}{128\,da\left(\sin(dx+c)-1\right)}-\frac{187\,\ln(\sin(dx+c)-1)}{256\,da}-\frac{}{64\,da\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3+9/128/d/a/(sin(d*x+c)-1)^2-47/128/a/d/(sin(d*x+c
)-1)-187/256/a/d*ln(sin(d*x+c)-1)-1/64/d/a/(1+sin(d*x+c))^4-1/12/d/a/(1+sin
(d*x+c))^3-19/64/a/d/(1+sin(d*x+c))^2-35/32/a/d/(1+sin(d*x+c))+443/256*ln(1
+sin(d*x+c))/a/d-1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.02794, size = 277, normalized size = 1.28

$$-\frac{\frac{2\left(945\sin(dx+c)^7+753\sin(dx+c)^6-2712\sin(dx+c)^5-2040\sin(dx+c)^4+2559\sin(dx+c)^3+1727\sin(dx+c)^2-784\sin(dx+c)-384\right)}{a\sin(dx+c)^8+a\sin(dx+c)^7-3a\sin(dx+c)^6-3a\sin(dx+c)^5+3a\sin(dx+c)^4+3a\sin(dx+c)^3-a\sin(dx+c)^2-a\sin(dx+c)}-\frac{1329\log(\sin(dx+c)+1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(945*sin(d*x + c)^7 + 753*sin(d*x + c)^6 - 2712*sin(d*x + c)^5 -
2040*sin(d*x + c)^4 + 2559*sin(d*x + c)^3 + 1727*sin(d*x + c)^2 - 784*sin(d
*x + c) - 384)/(a*sin(d*x + c)^8 + a*sin(d*x + c)^7 - 3*a*sin(d*x + c)^6 -
3*a*sin(d*x + c)^5 + 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c)^3 - a*sin(d*x +
c)^2 - a*sin(d*x + c)) - 1329*log(sin(d*x + c) + 1)/a + 561*log(sin(d*x + c
) - 1)/a + 768*log(sin(d*x + c))/a)/d
```

Fricas [A] time = 1.67313, size = 694, normalized size = 3.2

$$\frac{1506\cos(dx+c)^6 - 438\cos(dx+c)^4 - 188\cos(dx+c)^2 - 768\left(\cos(dx+c)^8 - \cos(dx+c)^6\sin(dx+c) - \cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(1506*cos(d*x + c)^6 - 438*cos(d*x + c)^4 - 188*cos(d*x + c)^2 - 768*
(cos(d*x + c)^8 - cos(d*x + c)^6*sin(d*x + c) - cos(d*x + c)^6)*log(1/2*sin
(d*x + c)) + 1329*(cos(d*x + c)^8 - cos(d*x + c)^6*sin(d*x + c) - cos(d*x +
 c)^6)*log(sin(d*x + c) + 1) - 561*(cos(d*x + c)^8 - cos(d*x + c)^6*sin(d*x
 + c) - cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(945*cos(d*x + c)^6 - 12
3*cos(d*x + c)^4 - 30*cos(d*x + c)^2 - 8)*sin(d*x + c) - 112)/(a*d*cos(d*x
+ c)^8 - a*d*cos(d*x + c)^6*sin(d*x + c) - a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36998, size = 230, normalized size = 1.06

$$\frac{\frac{5316\log(|\sin(dx+c)+1|)}{a} - \frac{2244\log(|\sin(dx+c)-1|)}{a} - \frac{3072\log(|\sin(dx+c)|)}{a} + \frac{3072(\sin(dx+c)-1)}{a\sin(dx+c)} + \frac{2\left(2057\sin(dx+c)^3 - 6735\sin(dx+c)^2 + 7407\sin(\right.}{a(\sin(dx+c)-1)^3}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(5316*log(abs(sin(d*x + c) + 1))/a - 2244*log(abs(sin(d*x + c) - 1))
/a - 3072*log(abs(sin(d*x + c)))/a + 3072*(sin(d*x + c) - 1)/(a*sin(d*x + c
)) + 2*(2057*sin(d*x + c)^3 - 6735*sin(d*x + c)^2 + 7407*sin(d*x + c) - 274
5)/(a*(sin(d*x + c) - 1)^3) - (11075*sin(d*x + c)^4 + 47660*sin(d*x + c)^3
+ 77442*sin(d*x + c)^2 + 56460*sin(d*x + c) + 15651)/(a*(sin(d*x + c) + 1)^
4))/d
```

3.892 $\int \frac{\csc^3(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{5a^2}{48d(a\sin(c+dx)+a)^3}+\frac{11a}{128d(a-a\sin(c+dx))^2}+\frac{29a}{64d(a\sin(c+$$

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (325*Log[1 - Sin[c + d*x]])/(
256*a*d) + (5*Log[Sin[c + d*x]])/(a*d) - (955*Log[1 + Sin[c + d*x]])/(256*a
*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (11*a)/(128*d*(a - a*Sin[c + d*x]
)^2) + 69/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a + a*Sin[c + d*x])^4)
+ (5*a^2)/(48*d*(a + a*Sin[c + d*x])^3) + (29*a)/(64*d*(a + a*Sin[c + d*x])
^2) + 2/(d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.246226, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{5a^2}{48d(a\sin(c+dx)+a)^3}+\frac{11a}{128d(a-a\sin(c+dx))^2}+\frac{29a}{64d(a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```


```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (325*Log[1 - Sin[c + d*x]])/(
256*a*d) + (5*Log[Sin[c + d*x]])/(a*d) - (955*Log[1 + Sin[c + d*x]])/(256*a
*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (11*a)/(128*d*(a - a*Sin[c + d*x]
)^2) + 69/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a + a*Sin[c + d*x])^4)
+ (5*a^2)/(48*d*(a + a*Sin[c + d*x])^3) + (29*a)/(64*d*(a + a*Sin[c + d*x])
^2) + 2/(d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\mathrm{Subst}\left(\int \frac{a^3}{(a-x)^4x^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^{10}\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^4x^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^{10}\,\mathrm{Subst}\left(\int \left(\frac{1}{32a^8(a-x)^4} + \frac{11}{64a^9(a-x)^3} + \frac{69}{128a^{10}(a-x)^2} + \frac{325}{256a^{11}(a-x)} + \frac{1}{a^9x^3} - \frac{1}{a^{10}x^2} + \frac{5}{a^{11}x}\right.\right.}{d}$$
$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad} - \frac{325\log(1-\sin(c+dx))}{256ad} + \frac{5\log(\sin(c+dx))}{ad} - \frac{955\,\mathrm{l}}{}$$

Mathematica [A] time = 6.16423, size = 213, normalized size = 0.92

$$\frac{a^{10}\left(\frac{69}{128a^{10}(a-a\sin(c+dx))} + \frac{2}{a^{10}(a\sin(c+dx)+a)} + \frac{11}{128a^9(a-a\sin(c+dx))^2} + \frac{29}{64a^9(a\sin(c+dx)+a)^2} + \frac{1}{96a^8(a-a\sin(c+dx))^3} + \frac{5}{48a^8(a\sin(c+dx)+a)}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^10*(Csc[c + d*x]/a^11 - Csc[c + d*x]^2/(2*a^11) - (325*Log[1 - Sin[c + d
*x]])/(256*a^11) + (5*Log[Sin[c + d*x]])/a^11 - (955*Log[1 + Sin[c + d*x]])
/(256*a^11) + 1/(96*a^8*(a - a*Sin[c + d*x])^3) + 11/(128*a^9*(a - a*Sin[c
+ d*x])^2) + 69/(128*a^10*(a - a*Sin[c + d*x])) + 1/(64*a^7*(a + a*Sin[c +
d*x])^4) + 5/(48*a^8*(a + a*Sin[c + d*x])^3) + 29/(64*a^9*(a + a*Sin[c + d*
x])^2) + 2/(a^10*(a + a*Sin[c + d*x]))))/d
```

Maple [A] time = 0.108, size = 208, normalized size = 0.9

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3} + \frac{11}{128\,da\,(\sin(dx+c)-1)^2} - \frac{69}{128\,da\,(\sin(dx+c)-1)} - \frac{325\,\ln(\sin(dx+c)-1)}{256\,da} + \frac{}{64\,da\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3+11/128/d/a/(sin(d*x+c)-1)^2-69/128/a/d/(sin(d*x+
c)-1)-325/256/a/d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4+5/48/d/a/(1+si
n(d*x+c))^3+29/64/a/d/(1+sin(d*x+c))^2+2/a/d/(1+sin(d*x+c))-955/256*ln(1+si
n(d*x+c))/a/d-1/2/d/a/sin(d*x+c)^2+1/d/a/sin(d*x+c)+5*ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.08348, size = 293, normalized size = 1.26

$$\frac{\frac{2\left(945\sin(dx+c)^8-15\sin(dx+c)^7-3480\sin(dx+c)^6-120\sin(dx+c)^5+4479\sin(dx+c)^4+319\sin(dx+c)^3-2192\sin(dx+c)^2-192\sin(dx+c)+192\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-3a\sin(dx+c)^7-3a\sin(dx+c)^6+3a\sin(dx+c)^5+3a\sin(dx+c)^4-a\sin(dx+c)^3-a\sin(dx+c)^2} - \frac{2865\,\mathrm{lo}}{}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(945*sin(d*x + c)^8 - 15*sin(d*x + c)^7 - 3480*sin(d*x + c)^6 - 12
0*sin(d*x + c)^5 + 4479*sin(d*x + c)^4 + 319*sin(d*x + c)^3 - 2192*sin(d*x
+ c)^2 - 192*sin(d*x + c) + 192)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 3*a
*sin(d*x + c)^7 - 3*a*sin(d*x + c)^6 + 3*a*sin(d*x + c)^5 + 3*a*sin(d*x + c
)^4 - a*sin(d*x + c)^3 - a*sin(d*x + c)^2) - 2865*log(sin(d*x + c) + 1)/a -
 975*log(sin(d*x + c) - 1)/a + 3840*log(sin(d*x + c))/a)/d
```

Fricas [A] time = 1.72846, size = 830, normalized size = 3.58

$$\frac{1890 \cos(dx+c)^8 - 600 \cos(dx+c)^6 - 582 \cos(dx+c)^4 - 212 \cos(dx+c)^2 + 3840 \left(\cos(dx+c)^8 - \cos(dx+c)^6 + (\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(1890*cos(d*x + c)^8 - 600*cos(d*x + c)^6 - 582*cos(d*x + c)^4 - 212*
cos(d*x + c)^2 + 3840*(cos(d*x + c)^8 - cos(d*x + c)^6 + (cos(d*x + c)^8 -
cos(d*x + c)^6)*sin(d*x + c))*log(1/2*sin(d*x + c)) - 2865*(cos(d*x + c)^8
- cos(d*x + c)^6 + (cos(d*x + c)^8 - cos(d*x + c)^6)*sin(d*x + c))*log(sin(
d*x + c) + 1) - 975*(cos(d*x + c)^8 - cos(d*x + c)^6 + (cos(d*x + c)^8 - co
s(d*x + c)^6)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(15*cos(d*x + c)^6 -
 165*cos(d*x + c)^4 - 34*cos(d*x + c)^2 - 8)*sin(d*x + c) - 112)/(a*d*cos(d
*x + c)^8 - a*d*cos(d*x + c)^6 + (a*d*cos(d*x + c)^8 - a*d*cos(d*x + c)^6)*
sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39307, size = 246, normalized size = 1.06

$$-\frac{\frac{11460 \log(|\sin(dx+c)+1|)}{a} + \frac{3900 \log(|\sin(dx+c)-1|)}{a} - \frac{15360 \log(|\sin(dx+c)|)}{a} + \frac{1536 \left(15 \sin(dx+c)^2 - 2 \sin(dx+c) + 1\right)}{a \sin(dx+c)^2} - \frac{2 \left(3575 \sin(dx+c)^3 - 11553 \sin\cdots\right.}{a(\sin(\cdots}}{3072 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(11460*log(abs(sin(d*x + c) + 1))/a + 3900*log(abs(sin(d*x + c) - 1
))/a - 15360*log(abs(sin(d*x + c)))/a + 1536*(15*sin(d*x + c)^2 - 2*sin(d*x
 + c) + 1)/(a*sin(d*x + c)^2) - 2*(3575*sin(d*x + c)^3 - 11553*sin(d*x + c)
^2 + 12513*sin(d*x + c) - 4551)/(a*(sin(d*x + c) - 1)^3) - (23875*sin(d*x +
 c)^4 + 101644*sin(d*x + c)^3 + 163074*sin(d*x + c)^2 + 117036*sin(d*x + c)
 + 31779)/(a*(sin(d*x + c) + 1)^4))/d
```

3.893 $\int \frac{\csc^4(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=253

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{8d(a\sin(c+dx)+a)^3}+\frac{13a}{128d(a-a\sin(c+dx))^2}-\frac{4}{64d(a\sin(c}$$

```
[Out] (-5*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) -
 (515*Log[1 - Sin[c + d*x]])/(256*a*d) - (5*Log[Sin[c + d*x]])/(a*d) + (179
5*Log[1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (1
3*a)/(128*d*(a - a*Sin[c + d*x])^2) + 95/(128*d*(a - a*Sin[c + d*x])) - a^3
/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(8*d*(a + a*Sin[c + d*x])^3) - (41*a)/
(64*d*(a + a*Sin[c + d*x])^2) - 105/(32*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.264146, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{8d(a\sin(c+dx)+a)^3}+\frac{13a}{128d(a-a\sin(c+dx))^2}-\frac{4}{64d(a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^4*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```



```
[Out] (-5*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) -
 (515*Log[1 - Sin[c + d*x]])/(256*a*d) - (5*Log[Sin[c + d*x]])/(a*d) + (179
5*Log[1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (1
3*a)/(128*d*(a - a*Sin[c + d*x])^2) + 95/(128*d*(a - a*Sin[c + d*x])) - a^3
/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(8*d*(a + a*Sin[c + d*x])^3) - (41*a)/
(64*d*(a + a*Sin[c + d*x])^2) - 105/(32*d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\csc^4(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{a^4}{(a-x)^4x^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\,\text{Subst}\left(\int \frac{1}{(a-x)^4x^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\,\text{Subst}\left(\int \left(\frac{1}{32a^9(a-x)^4} + \frac{13}{64a^{10}(a-x)^3} + \frac{95}{128a^{11}(a-x)^2} + \frac{515}{256a^{12}(a-x)} + \frac{1}{a^9x^4} - \frac{1}{a^{10}x^3} + \frac{5}{a^{11}x^2} - \right.\right.}{}$$

$$= -\frac{5\csc(c+dx)}{ad} + \frac{\csc^2(c+dx)}{2ad} - \frac{\csc^3(c+dx)}{3ad} - \frac{515\log(1-\sin(c+dx))}{256ad} - \frac{5\log(\sin(}{ad}$$

Mathematica [A] time = 6.13887, size = 231, normalized size = 0.91

$$\frac{a^{11}\left(\frac{95}{128a^{11}(a-a\sin(c+dx))} - \frac{105}{32a^{11}(a\sin(c+dx)+a)} + \frac{13}{128a^{10}(a-a\sin(c+dx))^2} - \frac{41}{64a^{10}(a\sin(c+dx)+a)^2} + \frac{1}{96a^9(a-a\sin(c+dx))^3} - \frac{1}{8a^9(a\sin(c+dx)+a)^3}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^4*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^11*((-5*Csc[c + d*x])/a^12 + Csc[c + d*x]^2/(2*a^12) - Csc[c + d*x]^3/(3
*a^12) - (515*Log[1 - Sin[c + d*x]])/(256*a^12) - (5*Log[Sin[c + d*x]])/a^1
2 + (1795*Log[1 + Sin[c + d*x]])/(256*a^12) + 1/(96*a^9*(a - a*Sin[c + d*x]
)^3) + 13/(128*a^10*(a - a*Sin[c + d*x])^2) + 95/(128*a^11*(a - a*Sin[c + d
*x])) - 1/(64*a^8*(a + a*Sin[c + d*x])^4) - 1/(8*a^9*(a + a*Sin[c + d*x])^3
) - 41/(64*a^10*(a + a*Sin[c + d*x])^2) - 105/(32*a^11*(a + a*Sin[c + d*x])
)))/d
```

Maple [A] time = 0.114, size = 225, normalized size = 0.9

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^3} + \frac{13}{128\,da\,(\sin(dx+c)-1)^2} - \frac{95}{128\,da\,(\sin(dx+c)-1)} - \frac{515\ln(\sin(dx+c)-1)}{256\,da} - \frac{}{64\,da\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3+13/128/d/a/(sin(d*x+c)-1)^2-95/128/a/d/(sin(d*x+
c)-1)-515/256/a/d*ln(sin(d*x+c)-1)-1/64/d/a/(1+sin(d*x+c))^4-1/8/d/a/(1+sin
(d*x+c))^3-41/64/a/d/(1+sin(d*x+c))^2-105/32/a/d/(1+sin(d*x+c))+1795/256*ln
(1+sin(d*x+c))/a/d-1/3/d/a/sin(d*x+c)^3+1/2/d/a/sin(d*x+c)^2-5/d/a/sin(d*x+
c)-5*ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.06219, size = 306, normalized size = 1.21

$$-\frac{\frac{2\left(3465\sin(dx+c)^9+2505\sin(dx+c)^8-10200\sin(dx+c)^7-6840\sin(dx+c)^6+10023\sin(dx+c)^5+5863\sin(dx+c)^4-3344\sin(dx+c)^3-1344\sin(dx+c)^2+64\sin(\right.}{a\sin(dx+c)^{10}+a\sin(dx+c)^9-3a\sin(dx+c)^8-3a\sin(dx+c)^7+3a\sin(dx+c)^6+3a\sin(dx+c)^5-a\sin(dx+c)^4-a\sin(dx+c)^3}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(3465*sin(d*x + c)^9 + 2505*sin(d*x + c)^8 - 10200*sin(d*x + c)^7
 - 6840*sin(d*x + c)^6 + 10023*sin(d*x + c)^5 + 5863*sin(d*x + c)^4 - 3344*
sin(d*x + c)^3 - 1344*sin(d*x + c)^2 + 64*sin(d*x + c) - 128)/(a*sin(d*x +
c)^10 + a*sin(d*x + c)^9 - 3*a*sin(d*x + c)^8 - 3*a*sin(d*x + c)^7 + 3*a*si
n(d*x + c)^6 + 3*a*sin(d*x + c)^5 - a*sin(d*x + c)^4 - a*sin(d*x + c)^3) -
5385*log(sin(d*x + c) + 1)/a + 1545*log(sin(d*x + c) - 1)/a + 3840*log(sin(
d*x + c))/a)/d
```

Fricas [A] time = 1.70826, size = 979, normalized size = 3.87

$$5010 \cos(dx+c)^8 - 6360 \cos(dx+c)^6 + 746 \cos(dx+c)^4 + 236 \cos(dx+c)^2 - 3840 \left(\cos(dx+c)^{10} - 2 \cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(5010*cos(d*x + c)^8 - 6360*cos(d*x + c)^6 + 746*cos(d*x + c)^4 + 236
*cos(d*x + c)^2 - 3840*(cos(d*x + c)^10 - 2*cos(d*x + c)^8 + cos(d*x + c)^6
 - (cos(d*x + c)^8 - cos(d*x + c)^6)*sin(d*x + c))*log(1/2*sin(d*x + c)) +
5385*(cos(d*x + c)^10 - 2*cos(d*x + c)^8 + cos(d*x + c)^6 - (cos(d*x + c)^8
 - cos(d*x + c)^6)*sin(d*x + c))*log(sin(d*x + c) + 1) - 1545*(cos(d*x + c)
^10 - 2*cos(d*x + c)^8 + cos(d*x + c)^6 - (cos(d*x + c)^8 - cos(d*x + c)^6)
*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(3465*cos(d*x + c)^8 - 3660*cos(d
*x + c)^6 + 213*cos(d*x + c)^4 + 38*cos(d*x + c)^2 + 8)*sin(d*x + c) + 112)
/(a*d*cos(d*x + c)^10 - 2*a*d*cos(d*x + c)^8 + a*d*cos(d*x + c)^6 - (a*d*co
s(d*x + c)^8 - a*d*cos(d*x + c)^6)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39332, size = 252, normalized size = 1.

$$\frac{\frac{21540 \log(|\sin(dx+c)+1|)}{a} - \frac{6180 \log(|\sin(dx+c)-1|)}{a} - \frac{15360 \log(|\sin(dx+c)|)}{a} + \frac{19745 \sin(dx+c)^6 - 76875 \sin(dx+c)^5 + 111723 \sin(dx+c)^4 - 74081 \sin(}{\left(\sin(dx+c)^2 - \sin(dx+c)\right.}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(21540*log(abs(sin(d*x + c) + 1))/a - 6180*log(abs(sin(d*x + c) - 1)
)/a - 15360*log(abs(sin(d*x + c)))/a + (19745*sin(d*x + c)^6 - 76875*sin(d*
x + c)^5 + 111723*sin(d*x + c)^4 - 74081*sin(d*x + c)^3 + 23040*sin(d*x + c
)^2 - 4608*sin(d*x + c) + 1024)/((sin(d*x + c)^2 - sin(d*x + c))^3*a) - (44
875*sin(d*x + c)^4 + 189580*sin(d*x + c)^3 + 301458*sin(d*x + c)^2 + 214060
*sin(d*x + c) + 57355)/(a*(sin(d*x + c) + 1)^4))/d
```

3.894 $\int \sec^5(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{2a^2\tan^7(c+dx)}{7d}+\frac{2a^2\tan^5(c+dx)}{5d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{3a^2\sec^5(c+dx)}{5d}+\frac{a^2\sec^3(c+dx)}{3d}$$

```
[Out] (a^2*Sec[c + d*x]^3)/(3*d) - (3*a^2*Sec[c + d*x]^5)/(5*d) + (2*a^2*Sec[c +
d*x]^7)/(7*d) + (2*a^2*Tan[c + d*x]^5)/(5*d) + (2*a^2*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.21374, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2873, 2606, 14, 2607, 270}

$$\frac{2a^2\tan^7(c+dx)}{7d}+\frac{2a^2\tan^5(c+dx)}{5d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{3a^2\sec^5(c+dx)}{5d}+\frac{a^2\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Sec[c + d*x]^3)/(3*d) - (3*a^2*Sec[c + d*x]^5)/(5*d) + (2*a^2*Sec[c +
d*x]^7)/(7*d) + (2*a^2*Tan[c + d*x]^5)/(5*d) + (2*a^2*Tan[c + d*x]^7)/(7*d)
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = \int \left(a^2\sec^5(c+dx)\tan^3(c+dx)+2a^2\sec^4(c+dx)\tan^4(c+dx)+\right.$$
$$= a^2\int \sec^5(c+dx)\tan^3(c+dx)\,dx + a^2\int \sec^3(c+dx)\tan^5(c+dx$$
$$= \frac{a^2\,\text{Subst}\left(\int x^4\left(-1+x^2\right)dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\,\text{Subst}\left(\int x^2\left(-1\right.\right.}{}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\,\text{Subst}\left(\int \left(-x\right.\right.}{}$$
$$= \frac{a^2\sec^3(c+dx)}{3d} - \frac{3a^2\sec^5(c+dx)}{5d} + \frac{2a^2\sec^7(c+dx)}{7d} + \frac{2a^2\tan^5(}{5d}$$

Mathematica [A] time = 0.984646, size = 139, normalized size = 1.53

$$-\frac{a^2\sec^7(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4(448\sin(c+dx)-104\sin(2(c+dx))-144\sin(3(c+dx))-52\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] -(a^2*Sec[c + d*x]^7*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-672 + 182*Co
s[c + d*x] + 736*Cos[2*(c + d*x)] + 39*Cos[3*(c + d*x)] - 192*Cos[4*(c + d*
x)] - 13*Cos[5*(c + d*x)] + 448*Sin[c + d*x] - 104*Sin[2*(c + d*x)] - 144*S
in[3*(c + d*x)] - 52*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)]))/(6720*d)
```

Maple [B] time = 0.089, size = 248, normalized size = 2.7

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^6}{7(\cos(dx+c))^7}+\frac{(\sin(dx+c))^6}{35(\cos(dx+c))^5}-\frac{(\sin(dx+c))^6}{105(\cos(dx+c))^3}+\frac{(\sin(dx+c))^6}{35\cos(dx+c)}+\frac{\cos(dx+c)}{35}\left(\frac{8}{3}+(\sin(dx+c))^4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/7*sin(d*x+c)^6/cos(d*x+c)^7+1/35*sin(d*x+c)^6/cos(d*x+c)^5-1/10
5*sin(d*x+c)^6/cos(d*x+c)^3+1/35*sin(d*x+c)^6/cos(d*x+c)+1/35*(8/3+sin(d*x+
c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/3
5*sin(d*x+c)^5/cos(d*x+c)^5)+a^2*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*
x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*
x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.03654, size = 123, normalized size = 1.35

$$\frac{6\left(5\tan(dx+c)^7+7\tan(dx+c)^5\right)a^2+\frac{\left(35\cos(dx+c)^4-42\cos(dx+c)^2+15\right)a^2}{\cos(dx+c)^7}-\frac{3\left(7\cos(dx+c)^2-5\right)a^2}{\cos(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/105*(6*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*a^2 + (35*cos(d*x + c)^4 - 4
2*cos(d*x + c)^2 + 15)*a^2/cos(d*x + c)^7 - 3*(7*cos(d*x + c)^2 - 5)*a^2/co
s(d*x + c)^7)/d
```

Fricas [A] time = 1.3343, size = 284, normalized size = 3.12

$$-\frac{24\,a^2\cos\left(dx+c\right)^4-47\,a^2\cos\left(dx+c\right)^2+25\,a^2-2\left(6\,a^2\cos\left(dx+c\right)^4-9\,a^2\cos\left(dx+c\right)^2+5\,a^2\right)\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^5+2\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(24*a^2*cos(d*x + c)^4 - 47*a^2*cos(d*x + c)^2 + 25*a^2 - 2*(6*a^2*c
os(d*x + c)^4 - 9*a^2*cos(d*x + c)^2 + 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)
^5 + 2*d*cos(d*x + c)^3*sin(d*x + c) - 2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28235, size = 186, normalized size = 2.04

$$-\frac{\frac{35\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}-\frac{105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1015\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1302\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+469\,a^2\tan\bigl(\frac{1}{2}\,dx+\frac{1}{2}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(3*a^2*tan(1/2*d*x + 1/2*c) + a^2)/(tan(1/2*d*x + 1/2*c) + 1)^3
- (105*a^2*tan(1/2*d*x + 1/2*c)^5 - 1015*a^2*tan(1/2*d*x + 1/2*c)^4 + 1330*
a^2*tan(1/2*d*x + 1/2*c)^3 - 1302*a^2*tan(1/2*d*x + 1/2*c)^2 + 469*a^2*tan(
1/2*d*x + 1/2*c) - 67*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

3.895 $\int \frac{\sin^3(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{19a^3}{256d(a\sin(c+dx)+a)^4}-\frac{3a^2}{64d(a-a\sin(c+dx))^3}-\frac{5}{128d(a\sin}$$

```
[Out] (-843*Log[1 - Sin[c + d*x]])/(512*a*d) - (2229*Log[1 + Sin[c + d*x]])/(512*
a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d) + a^3/(256*d*(a - a*Sin[
c + d*x])^4) - (3*a^2)/(64*d*(a - a*Sin[c + d*x])^3) + (141*a)/(512*d*(a -
a*Sin[c + d*x])^2) - 39/(32*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*Sin
[c + d*x])^5) + (19*a^3)/(256*d*(a + a*Sin[c + d*x])^4) - (53*a^2)/(128*d*(
a + a*Sin[c + d*x])^3) + (765*a)/(512*d*(a + a*Sin[c + d*x])^2) - 1155/(256
*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.282715, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{19a^3}{256d(a\sin(c+dx)+a)^4}-\frac{3a^2}{64d(a-a\sin(c+dx))^3}-\frac{5}{128d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-843*Log[1 - Sin[c + d*x]])/(512*a*d) - (2229*Log[1 + Sin[c + d*x]])/(512*
a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d) + a^3/(256*d*(a - a*Sin[
c + d*x])^4) - (3*a^2)/(64*d*(a - a*Sin[c + d*x])^3) + (141*a)/(512*d*(a -
a*Sin[c + d*x])^2) - 39/(32*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*Sin
[c + d*x])^5) + (19*a^3)/(256*d*(a + a*Sin[c + d*x])^4) - (53*a^2)/(128*d*(
a + a*Sin[c + d*x])^3) + (765*a)/(512*d*(a + a*Sin[c + d*x])^2) - 1155/(256
*d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{x^{12}}{a^{12}(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^{12}}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(a+\frac{a^6}{64(a-x)^5}-\frac{9a^5}{64(a-x)^4}+\frac{141a^4}{256(a-x)^3}-\frac{39a^3}{32(a-x)^2}+\frac{843a^2}{512(a-x)}-x+\frac{a^7}{32(a+x)^6}-\frac{1}{64(}\right.\right.}{a^3 d}$$
$$= -\frac{843\log(1-\sin(c+dx))}{512ad}-\frac{2229\log(1+\sin(c+dx))}{512ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

Mathematica [A] time = 6.1625, size = 169, normalized size = 0.64

$$-\frac{1280\sin^2(c+dx)-2560\sin(c+dx)+\frac{3120}{1-\sin(c+dx)}+\frac{11550}{\sin(c+dx)+1}-\frac{705}{(1-\sin(c+dx))^2}-\frac{3825}{(\sin(c+dx)+1)^2}+\frac{120}{(1-\sin(c+dx))^3}+\frac{106}{(\sin(c+d}}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(4215*Log[1 - Sin[c + d*x]] + 11145*Log[1 + Sin[c + d*x]] - 10/(1 - Sin[c
+ d*x])^4 + 120/(1 - Sin[c + d*x])^3 - 705/(1 - Sin[c + d*x])^2 + 3120/(1 -
 Sin[c + d*x]) - 2560*Sin[c + d*x] + 1280*Sin[c + d*x]^2 + 16/(1 + Sin[c +
d*x])^5 - 190/(1 + Sin[c + d*x])^4 + 1060/(1 + Sin[c + d*x])^3 - 3825/(1 +
Sin[c + d*x])^2 + 11550/(1 + Sin[c + d*x]))/(2560*a*d)
```

Maple [A] time = 0.112, size = 227, normalized size = 0.9

$$-\frac{(\sin(dx+c))^2}{2da}+\frac{\sin(dx+c)}{da}+\frac{1}{256\,da(\sin(dx+c)-1)^4}+\frac{3}{64\,da(\sin(dx+c)-1)^3}+\frac{141}{512\,da(\sin(dx+c)-1)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2*sin(d*x+c)^2/d/a+sin(d*x+c)/d/a+1/256/d/a/(sin(d*x+c)-1)^4+3/64/d/a/(s
in(d*x+c)-1)^3+141/512/d/a/(sin(d*x+c)-1)^2+39/32/a/d/(sin(d*x+c)-1)-843/51
2/a/d*ln(sin(d*x+c)-1)-1/160/d/a/(1+sin(d*x+c))^5+19/256/d/a/(1+sin(d*x+c))
^4-53/128/d/a/(1+sin(d*x+c))^3+765/512/a/d/(1+sin(d*x+c))^2-1155/256/a/d/(1
+sin(d*x+c))-2229/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.03996, size = 319, normalized size = 1.21

$$-\frac{\frac{2\left(4215\sin(dx+c)^8-5385\sin(dx+c)^7-18655\sin(dx+c)^6+13345\sin(dx+c)^5+30113\sin(dx+c)^4-11487\sin(dx+c)^3-21257\sin(dx+c)^2+3383\sin(dx+c)+5\right.}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2560*(2*(4215*sin(d*x + c)^8 - 5385*sin(d*x + c)^7 - 18655*sin(d*x + c)^
6 + 13345*sin(d*x + c)^5 + 30113*sin(d*x + c)^4 - 11487*sin(d*x + c)^3 - 21
257*sin(d*x + c)^2 + 3383*sin(d*x + c) + 5568)/(a*sin(d*x + c)^9 + a*sin(d*
x + c)^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6
*a*sin(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c
) + a) + 1280*(sin(d*x + c)^2 - 2*sin(d*x + c))/a + 11145*log(sin(d*x + c)
+ 1)/a + 4215*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.97715, size = 626, normalized size = 2.37

$$-\frac{1280 \cos(dx+c)^{10} + 6510 \cos(dx+c)^{8} + 3590 \cos(dx+c)^{6} - 1124 \cos(dx+c)^{4} + 272 \cos(dx+c)^{2} + 11145 \left(\cos(\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2560*(1280*cos(d*x + c)^10 + 6510*cos(d*x + c)^8 + 3590*cos(d*x + c)^6 -
 1124*cos(d*x + c)^4 + 272*cos(d*x + c)^2 + 11145*(cos(d*x + c)^8*sin(d*x +
 c) + cos(d*x + c)^8)*log(sin(d*x + c) + 1) + 4215*(cos(d*x + c)^8*sin(d*x
+ c) + cos(d*x + c)^8)*log(-sin(d*x + c) + 1) - 2*(640*cos(d*x + c)^10 + 96
0*cos(d*x + c)^8 - 5385*cos(d*x + c)^6 + 2810*cos(d*x + c)^4 - 952*cos(d*x
+ c)^2 + 144)*sin(d*x + c) - 32)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos
(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**12/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35359, size = 244, normalized size = 0.92

$$-\frac{\frac{44580 \log(|\sin(dx+c)+1|)}{a} + \frac{16860 \log(|\sin(dx+c)-1|)}{a} + \frac{5120\left(a\sin(dx+c)^{2} - 2a\sin(dx+c)\right)}{a^{2}} - \frac{5\left(7025 \sin(dx+c)^{4} - 25604 \sin(dx+c)^{3} + 35226 \sin(dx+c)^{2} - \ldots\right.}{a(\sin(dx+c)-1)^{4}}}{10240\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(44580*log(abs(sin(d*x + c) + 1))/a + 16860*log(abs(sin(d*x + c) -
 1))/a + 5120*(a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/a^2 - 5*(7025*sin(d*x +
 c)^4 - 25604*sin(d*x + c)^3 + 35226*sin(d*x + c)^2 - 21644*sin(d*x + c) +
5005)/(a*(sin(d*x + c) - 1)^4) - (101791*sin(d*x + c)^5 + 462755*sin(d*x +
c)^4 + 848410*sin(d*x + c)^3 + 782370*sin(d*x + c)^2 + 362335*sin(d*x + c)
+ 67347)/(a*(sin(d*x + c) + 1)^5))/d
```

3.896 $\int \frac{\sin^2(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{17a^3}{256d(a\sin(c+dx)+a)^4}-\frac{a^2}{24d(a-a\sin(c+dx))^3}+\frac{}{384d(a\,\mathrm{s}}$$

```
[Out] (-437*Log[1 - Sin[c + d*x]])/(512*a*d) + (949*Log[1 + Sin[c + d*x]])/(512*a
*d) - Sin[c + d*x]/(a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) - a^2/(24*d*(
a - a*Sin[c + d*x])^3) + (109*a)/(512*d*(a - a*Sin[c + d*x])^2) - 203/(256*
d*(a - a*Sin[c + d*x])) + a^4/(160*d*(a + a*Sin[c + d*x])^5) - (17*a^3)/(25
6*d*(a + a*Sin[c + d*x])^4) + (125*a^2)/(384*d*(a + a*Sin[c + d*x])^3) - (5
15*a)/(512*d*(a + a*Sin[c + d*x])^2) + 5/(2*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.26132, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{17a^3}{256d(a\sin(c+dx)+a)^4}-\frac{a^2}{24d(a-a\sin(c+dx))^3}+\frac{}{384d(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-437*Log[1 - Sin[c + d*x]])/(512*a*d) + (949*Log[1 + Sin[c + d*x]])/(512*a
*d) - Sin[c + d*x]/(a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) - a^2/(24*d*(
a - a*Sin[c + d*x])^3) + (109*a)/(512*d*(a - a*Sin[c + d*x])^2) - 203/(256*
d*(a - a*Sin[c + d*x])) + a^4/(160*d*(a + a*Sin[c + d*x])^5) - (17*a^3)/(25
6*d*(a + a*Sin[c + d*x])^4) + (125*a^2)/(384*d*(a + a*Sin[c + d*x])^3) - (5
15*a)/(512*d*(a + a*Sin[c + d*x])^2) + 5/(2*d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{x^{11}}{a^{11}(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{11}}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$

$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^5}{64(a-x)^5}-\frac{a^4}{8(a-x)^4}+\frac{109a^3}{256(a-x)^3}-\frac{203a^2}{256(a-x)^2}+\frac{437a}{512(a-x)}-\frac{a^6}{32(a+x)^6}+\frac{17a^5}{64(a+x)^5}\right.\right.}{a^2d}$$

$$= -\frac{437\log(1-\sin(c+dx))}{512ad}+\frac{949\log(1+\sin(c+dx))}{512ad}-\frac{\sin(c+dx)}{ad}+\frac{a^3}{256d(a-a\sin(}$$

Mathematica [A] time = 6.17697, size = 159, normalized size = 0.64

$$-\frac{7680\sin(c+dx)+\frac{6090}{1-\sin(c+dx)}-\frac{19200}{\sin(c+dx)+1}-\frac{1635}{(1-\sin(c+dx))^2}+\frac{7725}{(\sin(c+dx)+1)^2}+\frac{320}{(1-\sin(c+dx))^3}-\frac{2500}{(\sin(c+dx)+1)^3}-\frac{30}{(1-\sin(c+dx))^4}+}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(6555*Log[1 - Sin[c + d*x]] - 14235*Log[1 + Sin[c + d*x]] - 30/(1 - Sin[c
+ d*x])^4 + 320/(1 - Sin[c + d*x])^3 - 1635/(1 - Sin[c + d*x])^2 + 6090/(1
- Sin[c + d*x]) + 7680*Sin[c + d*x] - 48/(1 + Sin[c + d*x])^5 + 510/(1 + Si
n[c + d*x])^4 - 2500/(1 + Sin[c + d*x])^3 + 7725/(1 + Sin[c + d*x])^2 - 192
00/(1 + Sin[c + d*x]))/(7680*a*d)
```

Maple [A] time = 0.113, size = 212, normalized size = 0.9

$$-\frac{\sin(dx+c)}{da}+\frac{1}{256\,da\,(\sin(dx+c)-1)^4}+\frac{1}{24\,da\,(\sin(dx+c)-1)^3}+\frac{109}{512\,da\,(\sin(dx+c)-1)^2}+\frac{203}{256\,da\,(\sin(dx+c)-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x)
```

```
[Out] -sin(d*x+c)/d/a+1/256/d/a/(sin(d*x+c)-1)^4+1/24/d/a/(sin(d*x+c)-1)^3+109/51
2/d/a/(sin(d*x+c)-1)^2+203/256/a/d/(sin(d*x+c)-1)-437/512/a/d*ln(sin(d*x+c)
-1)+1/160/d/a/(1+sin(d*x+c))^5-17/256/d/a/(1+sin(d*x+c))^4+125/384/d/a/(1+s
in(d*x+c))^3-515/512/a/d/(1+sin(d*x+c))^2+5/2/a/d/(1+sin(d*x+c))+949/512*ln
(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.05169, size = 304, normalized size = 1.23

$$\frac{\frac{2\left(12645\sin(dx+c)^8+3045\sin(dx+c)^7-36765\sin(dx+c)^6-7965\sin(dx+c)^5+42339\sin(dx+c)^4+7139\sin(dx+c)^3-22171\sin(dx+c)^2-2171\sin(dx+c)+4384\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}+}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/7680*(2*(12645*sin(d*x + c)^8 + 3045*sin(d*x + c)^7 - 36765*sin(d*x + c)^
6 - 7965*sin(d*x + c)^5 + 42339*sin(d*x + c)^4 + 7139*sin(d*x + c)^3 - 2217
1*sin(d*x + c)^2 - 2171*sin(d*x + c) + 4384)/(a*sin(d*x + c)^9 + a*sin(d*x
+ c)^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a
*sin(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c)
+ a) + 14235*log(sin(d*x + c) + 1)/a - 6555*log(sin(d*x + c) - 1)/a - 7680*
sin(d*x + c)/a)/d
```

Fricas [A] time = 1.89572, size = 602, normalized size = 2.44

$$\frac{7680 \cos(dx+c)^{10} + 17610 \cos(dx+c)^{8} - 27630 \cos(dx+c)^{6} + 15828 \cos(dx+c)^{4} - 5584 \cos(dx+c)^{2} + 14235}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/7680*(7680*cos(d*x + c)^10 + 17610*cos(d*x + c)^8 - 27630*cos(d*x + c)^6
+ 15828*cos(d*x + c)^4 - 5584*cos(d*x + c)^2 + 14235*(cos(d*x + c)^8*sin(d*
x + c) + cos(d*x + c)^8)*log(sin(d*x + c) + 1) - 6555*(cos(d*x + c)^8*sin(d
*x + c) + cos(d*x + c)^8)*log(-sin(d*x + c) + 1) - 2*(3840*cos(d*x + c)^8 +
 3045*cos(d*x + c)^6 - 1170*cos(d*x + c)^4 + 344*cos(d*x + c)^2 - 48)*sin(d
*x + c) + 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32017, size = 225, normalized size = 0.91

$$\frac{\frac{56940 \log(|\sin(dx+c)+1|)}{a} - \frac{26220 \log(|\sin(dx+c)-1|)}{a} - \frac{30720 \sin(dx+c)}{a} + \frac{5\left(10925 \sin(dx+c)^{4} - 38828 \sin(dx+c)^{3} + 52242 \sin(dx+c)^{2} - 31444 \sin(dx+c)}{a(\sin(dx+c)-1)^{4}}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(56940*log(abs(sin(d*x + c) + 1))/a - 26220*log(abs(sin(d*x + c) -
1))/a - 30720*sin(d*x + c)/a + 5*(10925*sin(d*x + c)^4 - 38828*sin(d*x + c)
^3 + 52242*sin(d*x + c)^2 - 31444*sin(d*x + c) + 7129)/(a*(sin(d*x + c) - 1
)^4) - (130013*sin(d*x + c)^5 + 573265*sin(d*x + c)^4 + 1023830*sin(d*x + c
)^3 + 922030*sin(d*x + c)^2 + 417605*sin(d*x + c) + 75961)/(a*(sin(d*x + c)
 + 1)^5))/d
```

3.897 $\int \frac{\sin(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=233

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{15a^3}{256d(a\sin(c+dx)+a)^4}-\frac{7a^2}{192d(a-a\sin(c+dx))^3}-\frac{}{384d(a\sin}$$

```
[Out] (-193*Log[1 - Sin[c + d*x]])/(512*a*d) - (319*Log[1 + Sin[c + d*x]])/(512*a
*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) - (7*a^2)/(192*d*(a - a*Sin[c + d*
x])^3) + (81*a)/(512*d*(a - a*Sin[c + d*x])^2) - 61/(128*d*(a - a*Sin[c + d
*x])) - a^4/(160*d*(a + a*Sin[c + d*x])^5) + (15*a^3)/(256*d*(a + a*Sin[c +
 d*x])^4) - (95*a^2)/(384*d*(a + a*Sin[c + d*x])^3) + (325*a)/(512*d*(a + a
*Sin[c + d*x])^2) - 315/(256*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.234589, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{15a^3}{256d(a\sin(c+dx)+a)^4}-\frac{7a^2}{192d(a-a\sin(c+dx))^3}-\frac{}{384d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-193*Log[1 - Sin[c + d*x]])/(512*a*d) - (319*Log[1 + Sin[c + d*x]])/(512*a
*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) - (7*a^2)/(192*d*(a - a*Sin[c + d*
x])^3) + (81*a)/(512*d*(a - a*Sin[c + d*x])^2) - 61/(128*d*(a - a*Sin[c + d
*x])) - a^4/(160*d*(a + a*Sin[c + d*x])^5) + (15*a^3)/(256*d*(a + a*Sin[c +
 d*x])^4) - (95*a^2)/(384*d*(a + a*Sin[c + d*x])^3) + (325*a)/(512*d*(a + a
*Sin[c + d*x])^2) - 315/(256*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{x^{10}}{a^{10}(a-x)^5(a+x)^6}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{(a-x)^5(a+x)^6}\,dx,x,a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4}{64(a-x)^5}-\frac{7a^3}{64(a-x)^4}+\frac{81a^2}{256(a-x)^3}-\frac{61a}{128(a-x)^2}+\frac{193}{512(a-x)}+\frac{a^5}{32(a+x)^6}-\frac{15a^4}{64(a+x)^5}+\right.\right.}{ad}$$
$$= -\frac{193\log(1-\sin(c+dx))}{512ad}-\frac{319\log(1+\sin(c+dx))}{512ad}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-$$

Mathematica [A] time = 4.91451, size = 137, normalized size = 0.59

$$-\frac{\frac{2\left(2895\sin^8(c+dx)-6705\sin^7(c+dx)-13815\sin^6(c+dx)+14985\sin^5(c+dx)+23049\sin^4(c+dx)-12151\sin^3(c+dx)-16561\sin^2(c+dx)+3439\sin(c+dx)+4384\right.}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(2895*Log[1 - Sin[c + d*x]] + 4785*Log[1 + Sin[c + d*x]] + (2*(4384 + 3439
*Sin[c + d*x] - 16561*Sin[c + d*x]^2 - 12151*Sin[c + d*x]^3 + 23049*Sin[c +
 d*x]^4 + 14985*Sin[c + d*x]^5 - 13815*Sin[c + d*x]^6 - 6705*Sin[c + d*x]^7
 + 2895*Sin[c + d*x]^8))/((-1 + Sin[c + d*x])^4*(1 + Sin[c + d*x])^5))/(768
0*a*d)
```

Maple [A] time = 0.109, size = 198, normalized size = 0.9

$$\frac{1}{256\,da\left(\sin\left(dx+c\right)-1\right)^4}+\frac{7}{192\,da\left(\sin\left(dx+c\right)-1\right)^3}+\frac{81}{512\,da\left(\sin\left(dx+c\right)-1\right)^2}+\frac{61}{128\,da\left(\sin\left(dx+c\right)-1\right)}-\frac{193\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4+7/192/d/a/(sin(d*x+c)-1)^3+81/512/d/a/(sin(d*x+c
)-1)^2+61/128/a/d/(sin(d*x+c)-1)-193/512/a/d*ln(sin(d*x+c)-1)-1/160/d/a/(1+
sin(d*x+c))^5+15/256/d/a/(1+sin(d*x+c))^4-95/384/d/a/(1+sin(d*x+c))^3+325/5
12/a/d/(1+sin(d*x+c))^2-315/256/a/d/(1+sin(d*x+c))-319/512*ln(1+sin(d*x+c))
/a/d
```

Maxima [A] time = 1.05919, size = 289, normalized size = 1.24

$$-\frac{\frac{2\left(2895\,\sin\left(dx+c\right)^8-6705\,\sin\left(dx+c\right)^7-13815\,\sin\left(dx+c\right)^6+14985\,\sin\left(dx+c\right)^5+23049\,\sin\left(dx+c\right)^4-12151\,\sin\left(dx+c\right)^3-16561\,\sin\left(dx+c\right)^2+3439\,\sin\left(dx+c\right)+4\right.}{a\sin\left(dx+c\right)^9+a\sin\left(dx+c\right)^8-4\,a\sin\left(dx+c\right)^7-4\,a\sin\left(dx+c\right)^6+6\,a\sin\left(dx+c\right)^5+6\,a\sin\left(dx+c\right)^4-4\,a\sin\left(dx+c\right)^3-4\,a\sin\left(dx+c\right)^2+a\sin\left(dx+c\right)+a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/7680*(2*(2895*sin(d*x + c)^8 - 6705*sin(d*x + c)^7 - 13815*sin(d*x + c)^
6 + 14985*sin(d*x + c)^5 + 23049*sin(d*x + c)^4 - 12151*sin(d*x + c)^3 - 16
561*sin(d*x + c)^2 + 3439*sin(d*x + c) + 4384)/(a*sin(d*x + c)^9 + a*sin(d*
x + c)^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6
*a*sin(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c
) + a) + 4785*log(sin(d*x + c) + 1)/a + 2895*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.1346, size = 537, normalized size = 2.3

$$-\frac{5790\cos(dx+c)^8 + 4470\cos(dx+c)^6 - 2052\cos(dx+c)^4 + 656\cos(dx+c)^2 + 4785\left(\cos(dx+c)^8\sin(dx+c) + c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/7680*(5790*cos(d*x + c)^8 + 4470*cos(d*x + c)^6 - 2052*cos(d*x + c)^4 +
656*cos(d*x + c)^2 + 4785*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*lo
g(sin(d*x + c) + 1) + 2895*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*l
og(-sin(d*x + c) + 1) + 2*(6705*cos(d*x + c)^6 - 5130*cos(d*x + c)^4 + 2296
*cos(d*x + c)^2 - 432)*sin(d*x + c) - 96)/(a*d*cos(d*x + c)^8*sin(d*x + c)
+ a*d*cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35615, size = 211, normalized size = 0.91

$$-\frac{\frac{19140\log(|\sin(dx+c)+1|)}{a} + \frac{11580\log(|\sin(dx+c)-1|)}{a} - \frac{5\left(4825\sin(dx+c)^4 - 16372\sin(dx+c)^3 + 21138\sin(dx+c)^2 - 12236\sin(dx+c) + 2669\right)}{a(\sin(dx+c)-1)^4} - \frac{43703\sin\cdots}{\cdots}}{30720d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(19140*log(abs(sin(d*x + c) + 1))/a + 11580*log(abs(sin(d*x + c) -
 1))/a - 5*(4825*sin(d*x + c)^4 - 16372*sin(d*x + c)^3 + 21138*sin(d*x + c)
^2 - 12236*sin(d*x + c) + 2669)/(a*(sin(d*x + c) - 1)^4) - (43703*sin(d*x +
 c)^5 + 180715*sin(d*x + c)^4 + 305330*sin(d*x + c)^3 + 261130*sin(d*x + c)
^2 + 112415*sin(d*x + c) + 19411)/(a*(sin(d*x + c) + 1)^5))/d
```

3.898 $\int \frac{\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{63\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^9(c+dx)\sec(c+dx)}{10ad}+\frac{9\tan^7(c+dx)\sec(c+dx)}{80ad}-\frac{21\tan^5(c+dx)\sec(c+dx)}{160ad}$$

```
[Out] (63*ArcTanh[Sin[c + d*x]])/(256*a*d) - (63*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (21*Sec[c + d*x]*Tan[c + d*x]^3)/(128*a*d) - (21*Sec[c + d*x]*Tan[c
+ d*x]^5)/(160*a*d) + (9*Sec[c + d*x]*Tan[c + d*x]^7)/(80*a*d) - (Sec[c + d
*x]*Tan[c + d*x]^9)/(10*a*d) + Tan[c + d*x]^10/(10*a*d)
```

Rubi [A] time = 0.192562, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{63\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^9(c+dx)\sec(c+dx)}{10ad}+\frac{9\tan^7(c+dx)\sec(c+dx)}{80ad}-\frac{21\tan^5(c+dx)\sec(c+dx)}{160ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```


```
[Out] (63*ArcTanh[Sin[c + d*x]])/(256*a*d) - (63*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (21*Sec[c + d*x]*Tan[c + d*x]^3)/(128*a*d) - (21*Sec[c + d*x]*Tan[c
+ d*x]^5)/(160*a*d) + (9*Sec[c + d*x]*Tan[c + d*x]^7)/(80*a*d) - (Sec[c + d
*x]*Tan[c + d*x]^9)/(10*a*d) + Tan[c + d*x]^10/(10*a*d)
```


Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^2(c+dx)\tan^9(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^{10}(c+dx)\,dx}{a}$$
$$= -\frac{\sec(c+dx)\tan^9(c+dx)}{10ad} + \frac{9\int \sec(c+dx)\tan^8(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^9\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{9\sec(c+dx)\tan^7(c+dx)}{80ad} - \frac{\sec(c+dx)\tan^9(c+dx)}{10ad} + \frac{\tan^{10}(c+dx)}{10ad} - \frac{63\int \sec(c+dx)\tan$$
$$= -\frac{21\sec(c+dx)\tan^5(c+dx)}{160ad} + \frac{9\sec(c+dx)\tan^7(c+dx)}{80ad} - \frac{\sec(c+dx)\tan^9(c+dx)}{10ad} + \frac{\tan^{10}}{1}$$
$$= \frac{21\sec(c+dx)\tan^3(c+dx)}{128ad} - \frac{21\sec(c+dx)\tan^5(c+dx)}{160ad} + \frac{9\sec(c+dx)\tan^7(c+dx)}{80ad} - \sec($$
$$= -\frac{63\sec(c+dx)\tan(c+dx)}{256ad} + \frac{21\sec(c+dx)\tan^3(c+dx)}{128ad} - \frac{21\sec(c+dx)\tan^5(c+dx)}{160ad} + 9\,\text{s}$$
$$= \frac{63\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{63\sec(c+dx)\tan(c+dx)}{256ad} + \frac{21\sec(c+dx)\tan^3(c+dx)}{128ad} - 21\sec(c$$

Mathematica [A] time = 2.52125, size = 122, normalized size = 0.79

$$\frac{\frac{2\left(965\sin^8(c+dx)+325\sin^7(c+dx)-2045\sin^6(c+dx)-765\sin^5(c+dx)+1923\sin^4(c+dx)+643\sin^3(c+dx)-827\sin^2(c+dx)-187\sin(c+dx)+128\right)}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5}+630\tanh^{-1}}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (630*ArcTanh[Sin[c + d*x]] + (2*(128 - 187*Sin[c + d*x] - 827*Sin[c + d*x]^
2 + 643*Sin[c + d*x]^3 + 1923*Sin[c + d*x]^4 - 765*Sin[c + d*x]^5 - 2045*Si
n[c + d*x]^6 + 325*Sin[c + d*x]^7 + 965*Sin[c + d*x]^8))/((-1 + Sin[c + d*x
])^4*(1 + Sin[c + d*x])^5))/(2560*a*d)
```

Maple [A] time = 0.105, size = 198, normalized size = 1.3

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} + \frac{1}{32\,da\,(\sin(dx+c)-1)^3} + \frac{57}{512\,da\,(\sin(dx+c)-1)^2} + \frac{65}{256\,da\,(\sin(dx+c)-1)} - \frac{63\ln(\sin}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4+1/32/d/a/(sin(d*x+c)-1)^3+57/512/d/a/(sin(d*x+c)
-1)^2+65/256/a/d/(sin(d*x+c)-1)-63/512/a/d*ln(sin(d*x+c)-1)+1/160/d/a/(1+si
n(d*x+c))^5-13/256/d/a/(1+sin(d*x+c))^4+23/128/d/a/(1+sin(d*x+c))^3-187/512
/a/d/(1+sin(d*x+c))^2+1/2/a/d/(1+sin(d*x+c))+63/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.05284, size = 289, normalized size = 1.88

$$\frac{\frac{2\left(965 \sin(dx+c)^8+325 \sin(dx+c)^7-2045 \sin(dx+c)^6-765 \sin(dx+c)^5+1923 \sin(dx+c)^4+643 \sin(dx+c)^3-827 \sin(dx+c)^2-187 \sin(dx+c)+128\right)}{a \sin(dx+c)^9+a \sin(dx+c)^8-4 a \sin(dx+c)^7-4 a \sin(dx+c)^6+6 a \sin(dx+c)^5+6 a \sin(dx+c)^4-4 a \sin(dx+c)^3-4 a \sin(dx+c)^2+a \sin(dx+c)+a}+\frac{315 \log}{}}{2560 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(965*sin(d*x + c)^8 + 325*sin(d*x + c)^7 - 2045*sin(d*x + c)^6 -
765*sin(d*x + c)^5 + 1923*sin(d*x + c)^4 + 643*sin(d*x + c)^3 - 827*sin(d*x
 + c)^2 - 187*sin(d*x + c) + 128)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*
a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x +
c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) + 315*
log(sin(d*x + c) + 1)/a - 315*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.04701, size = 529, normalized size = 3.44

$$\frac{1930 \cos(dx+c)^8 - 3630 \cos(dx+c)^6 + 3156 \cos(dx+c)^4 - 1488 \cos(dx+c)^2 + 315\left(\cos(dx+c)^8 \sin(dx+c) + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(1930*cos(d*x + c)^8 - 3630*cos(d*x + c)^6 + 3156*cos(d*x + c)^4 - 1
488*cos(d*x + c)^2 + 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log
(sin(d*x + c) + 1) - 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log
(-sin(d*x + c) + 1) - 2*(325*cos(d*x + c)^6 - 210*cos(d*x + c)^4 + 88*cos(d
*x + c)^2 - 16)*sin(d*x + c) + 288)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*
cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33374, size = 211, normalized size = 1.37

$$\frac{\frac{1260 \log(|\sin(dx+c)+1|)}{a} - \frac{1260 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(525 \sin(dx+c)^4-1580 \sin(dx+c)^3+1818 \sin(dx+c)^2-932 \sin(dx+c)+177\right)}{a(\sin(dx+c)-1)^4} - \frac{2877 \sin(dx+c)^5+}{}}{10240 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10240*(1260*log(abs(sin(d*x + c) + 1))/a - 1260*log(abs(sin(d*x + c) - 1)
)/a + 5*(525*sin(d*x + c)^4 - 1580*sin(d*x + c)^3 + 1818*sin(d*x + c)^2 - 9
32*sin(d*x + c) + 177)/(a*(sin(d*x + c) - 1)^4) - (2877*sin(d*x + c)^5 + 92
65*sin(d*x + c)^4 + 12030*sin(d*x + c)^3 + 7430*sin(d*x + c)^2 + 1965*sin(d
*x + c) + 113)/(a*(sin(d*x + c) + 1)^5))/d
```

3.899 $\int \frac{\sec(c+dx)\tan^8(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=160

$$-\frac{\tan^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}-\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}+\frac{7\tan^3(c+dx)}{96a}$$

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(256*a*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(256*a*
d) - (7*Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (7*Sec[c + d*x]^3*Tan[c +
d*x]^3)/(96*a*d) - (7*Sec[c + d*x]^3*Tan[c + d*x]^5)/(80*a*d) + (Sec[c + d*
x]^3*Tan[c + d*x]^7)/(10*a*d) - Tan[c + d*x]^10/(10*a*d)
```

Rubi [A] time = 0.270722, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2835, 2611, 3768, 3770, 2607, 30}

$$-\frac{\tan^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}-\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}+\frac{7\tan^3(c+dx)}{96a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^8)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (7*ArcTanh[Sin[c + d*x]])/(256*a*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(256*a*
d) - (7*Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (7*Sec[c + d*x]^3*Tan[c +
d*x]^3)/(96*a*d) - (7*Sec[c + d*x]^3*Tan[c + d*x]^5)/(80*a*d) + (Sec[c + d*
x]^3*Tan[c + d*x]^7)/(10*a*d) - Tan[c + d*x]^10/(10*a*d)
```


Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^8(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^3(c+dx)\tan^8(c+dx)\,dx}{a} - \frac{\int \sec^2(c+dx)\tan^9(c+dx)\,dx}{a}$$
$$= \frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} - \frac{7\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{10a} - \frac{\text{Subst}\left(\int x^9\,dx, x, \tan(\right.}{ad}$$
$$= -\frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} + \frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} - \frac{\tan^{10}(c+dx)}{10ad} + \frac{7\int \sec^3(}{}$$
$$= \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} - \frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} + \frac{\sec(c+dx)\tan^7(c+dx)}{10ad}$$
$$= -\frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} - \frac{7\sec^3(c+dx)\tan^5(c+d}{80ad}$$
$$= \frac{7\sec(c+dx)\tan(c+dx)}{256ad} - \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} -$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{7\sec(c+dx)\tan(c+dx)}{256ad} - \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{7\,\text{se}}{}$$

Mathematica [A] time = 2.44905, size = 121, normalized size = 0.76

$$\frac{\frac{-210\sin^8(c+dx)+3630\sin^7(c+dx)+2050\sin^6(c+dx)-5630\sin^5(c+dx)-3838\sin^4(c+dx)+3842\sin^3(c+dx)+2862\sin^2(c+dx)-978\sin(c+dx)-768}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5} + 210\tanh}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^8)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (210*ArcTanh[Sin[c + d*x]] + (-768 - 978*Sin[c + d*x] + 2862*Sin[c + d*x]^2
 + 3842*Sin[c + d*x]^3 - 3838*Sin[c + d*x]^4 - 5630*Sin[c + d*x]^5 + 2050*S
in[c + d*x]^6 + 3630*Sin[c + d*x]^7 - 210*Sin[c + d*x]^8)/((-1 + Sin[c + d*
x])^4*(1 + Sin[c + d*x])^5))/(7680*a*d)
```

Maple [A] time = 0.107, size = 198, normalized size = 1.2

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} + \frac{5}{192\,da\,(\sin(dx+c)-1)^3} + \frac{37}{512\,da\,(\sin(dx+c)-1)^2} + \frac{7}{64\,da\,(\sin(dx+c)-1)} - \frac{7\,\ln(\sin(}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4+5/192/d/a/(sin(d*x+c)-1)^3+37/512/d/a/(sin(d*x+c
)-1)^2+7/64/a/d/(sin(d*x+c)-1)-7/512/a/d*ln(sin(d*x+c)-1)-1/160/d/a/(1+sin(
d*x+c))^5+11/256/d/a/(1+sin(d*x+c))^4-47/384/d/a/(1+sin(d*x+c))^3+93/512/a/
d/(1+sin(d*x+c))^2-35/256/a/d/(1+sin(d*x+c))+7/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.02855, size = 289, normalized size = 1.81

$$-\frac{\frac{2\left(105 \sin(dx+c)^{8}-1815 \sin(dx+c)^{7}-1025 \sin(dx+c)^{6}+2815 \sin(dx+c)^{5}+1919 \sin(dx+c)^{4}-1921 \sin(dx+c)^{3}-1431 \sin(dx+c)^{2}+489 \sin(dx+c)+384\right)}{a \sin(dx+c)^{9}+a \sin(dx+c)^{8}-4 a \sin(dx+c)^{7}-4 a \sin(dx+c)^{6}+6 a \sin(dx+c)^{5}+6 a \sin(dx+c)^{4}-4 a \sin(dx+c)^{3}-4 a \sin(dx+c)^{2}+a \sin(dx+c)+a} - \cdots}{7680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(105*sin(d*x + c)^8 - 1815*sin(d*x + c)^7 - 1025*sin(d*x + c)^6
+ 2815*sin(d*x + c)^5 + 1919*sin(d*x + c)^4 - 1921*sin(d*x + c)^3 - 1431*si
n(d*x + c)^2 + 489*sin(d*x + c) + 384)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8
 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d
*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) -
 105*log(sin(d*x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.02497, size = 533, normalized size = 3.33

$$-\frac{210 \cos(dx+c)^{8} + 1210 \cos(dx+c)^{6} - 1052 \cos(dx+c)^{4} + 496 \cos(dx+c)^{2} - 105 \left(\cos(dx+c)^{8} \sin(dx+c) + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(210*cos(d*x + c)^8 + 1210*cos(d*x + c)^6 - 1052*cos(d*x + c)^4 + 4
96*cos(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
sin(d*x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
-sin(d*x + c) + 1) + 2*(1815*cos(d*x + c)^6 - 2630*cos(d*x + c)^4 + 1736*co
s(d*x + c)^2 - 432)*sin(d*x + c) - 96)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a
*d*cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4009, size = 211, normalized size = 1.32

$$\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a} - \frac{420 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(175 \sin(dx+c)^4 - 28 \sin(dx+c)^3 - 522 \sin(dx+c)^2 + 588 \sin(dx+c) - 189\right)}{a(\sin(dx+c)-1)^4} - \frac{959 \sin(dx+c)^5 + 8995 \sin(d}{}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 5*(175*sin(d*x + c)^4 - 28*sin(d*x + c)^3 - 522*sin(d*x + c)^2 + 588*si
n(d*x + c) - 189)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 8995*sin
(d*x + c)^4 + 20810*sin(d*x + c)^3 + 21810*sin(d*x + c)^2 + 11055*sin(d*x +
 c) + 2211)/(a*(sin(d*x + c) + 1)^5))/d
```

3.900 $\int \frac{\sec^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{\tan^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}+\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}-$$

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(256*a*d) - (7*Sec[c + d*x]*Tan[c + d*x])/(256*a
*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) - (7*Sec[c + d*x]^3*Tan[c +
 d*x]^3)/(96*a*d) + (7*Sec[c + d*x]^3*Tan[c + d*x]^5)/(80*a*d) - (Sec[c + d
*x]^3*Tan[c + d*x]^7)/(10*a*d) + Tan[c + d*x]^8/(8*a*d) + Tan[c + d*x]^10/(
10*a*d)
```

Rubi [A] time = 0.280451, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2607, 14, 2611, 3768, 3770}

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{\tan^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}+\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(256*a*d) - (7*Sec[c + d*x]*Tan[c + d*x])/(256*a
*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) - (7*Sec[c + d*x]^3*Tan[c +
 d*x]^3)/(96*a*d) + (7*Sec[c + d*x]^3*Tan[c + d*x]^5)/(80*a*d) - (Sec[c + d
*x]^3*Tan[c + d*x]^7)/(10*a*d) + Tan[c + d*x]^8/(8*a*d) + Tan[c + d*x]^10/(
10*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^4(c+dx)\tan^7(c+dx)\,dx}{a} - \frac{\int \sec^3(c+dx)\tan^8(c+dx)\,dx}{a}$$

$$= -\frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} + \frac{7\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^7\left(1+x^2\right.\right.}{a}$$

$$= \frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} - \frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} - \frac{7\int \sec^3(c+dx)\tan^4(c+}{16a}$$

$$= -\frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} + \frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} - \frac{\sec^3(c+dx)\tan^7(c+}{10ad}$$

$$= \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} + \frac{7\sec^3(c+dx)\tan^5(c+dx}{80ad}$$

$$= -\frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{7\sec^3(c+dx)\tan^3(c+dx}{96ad}$$

$$= -\frac{7\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} - 7$$

Mathematica [A] time = 1.66669, size = 124, normalized size = 0.7

$$-\frac{\frac{210}{1-\sin(c+dx)} - \frac{315}{(1-\sin(c+dx))^2} + \frac{525}{(\sin(c+dx)+1)^2} + \frac{160}{(1-\sin(c+dx))^3} - \frac{580}{(\sin(c+dx)+1)^3} - \frac{30}{(1-\sin(c+dx))^4} + \frac{270}{(\sin(c+dx)+1)^4} - \frac{48}{(\sin(c+dx)+1)^5} + 21}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(210*ArcTanh[Sin[c + d*x]] - 30/(1 - Sin[c + d*x])^4 + 160/(1 - Sin[c + d*
x])^3 - 315/(1 - Sin[c + d*x])^2 + 210/(1 - Sin[c + d*x]) - 48/(1 + Sin[c +
 d*x])^5 + 270/(1 + Sin[c + d*x])^4 - 580/(1 + Sin[c + d*x])^3 + 525/(1 + S
in[c + d*x])^2)/(7680*a*d)
```

Maple [A] time = 0.096, size = 180, normalized size = 1.

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} + \frac{1}{48\,da\,(\sin(dx+c)-1)^3} + \frac{21}{512\,da\,(\sin(dx+c)-1)^2} + \frac{7}{256\,da\,(\sin(dx+c)-1)} + \frac{7\ln(\sin(}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4+1/48/d/a/(sin(d*x+c)-1)^3+21/512/d/a/(sin(d*x+c)
-1)^2+7/256/a/d/(sin(d*x+c)-1)+7/512/a/d*ln(sin(d*x+c)-1)+1/160/d/a/(1+sin(
d*x+c))^5-9/256/d/a/(1+sin(d*x+c))^4+29/384/d/a/(1+sin(d*x+c))^3-35/512/a/d
/(1+sin(d*x+c))^2-7/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.04846, size = 289, normalized size = 1.62

$$\frac{\frac{2\left(105\sin(dx+c)^8+105\sin(dx+c)^7+895\sin(dx+c)^6-65\sin(dx+c)^5-961\sin(dx+c)^4-\sin(dx+c)^3+489\sin(dx+c)^2+9\sin(dx+c)-96\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{105\log(\sin(dx}{a}}{7680d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/7680*(2*(105*sin(d*x + c)^8 + 105*sin(d*x + c)^7 + 895*sin(d*x + c)^6 - 6
5*sin(d*x + c)^5 - 961*sin(d*x + c)^4 - sin(d*x + c)^3 + 489*sin(d*x + c)^2
 + 9*sin(d*x + c) - 96)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(d*x
+ c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 - 4*a
*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 105*log(sin(d*
x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.33712, size = 529, normalized size = 2.97

$$\frac{210\cos(dx+c)^8-2630\cos(dx+c)^6+4708\cos(dx+c)^4-3344\cos(dx+c)^2-105\left(\cos(dx+c)^8\sin(dx+c)+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(210*cos(d*x + c)^8 - 2630*cos(d*x + c)^6 + 4708*cos(d*x + c)^4 - 33
44*cos(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
sin(d*x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
-sin(d*x + c) + 1) - 2*(105*cos(d*x + c)^6 - 250*cos(d*x + c)^4 + 184*cos(d
*x + c)^2 - 48)*sin(d*x + c) + 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*
cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36264, size = 211, normalized size = 1.19

$$-\frac{\frac{420 \log\left(|\sin(dx+c)+1|\right)}{a} - \frac{420 \log\left(|\sin(dx+c)-1|\right)}{a} + \frac{5\left(175 \sin(dx+c)^4-868 \sin(dx+c)^3+1302 \sin(dx+c)^2-828 \sin(dx+c)+195\right)}{a(\sin(dx+c)-1)^4} - \frac{959 \sin(dx+c)^5+4795 \mathrm{s}}{}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))
/a + 5*(175*sin(d*x + c)^4 - 868*sin(d*x + c)^3 + 1302*sin(d*x + c)^2 - 828
*sin(d*x + c) + 195)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 4795*
sin(d*x + c)^4 + 7490*sin(d*x + c)^3 + 5610*sin(d*x + c)^2 + 2055*sin(d*x +
 c) + 291)/(a*(sin(d*x + c) + 1)^5))/d
```

3.901 $\int \frac{\sec^3(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=176

$$-\frac{\tan^{10}(c+dx)}{10ad}-\frac{\tan^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{16ad}+\cdots$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(256*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(256*a
*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (Sec[c + d*x]^5*Tan[c + d*x
])/(32*a*d) - (Sec[c + d*x]^5*Tan[c + d*x]^3)/(16*a*d) + (Sec[c + d*x]^5*Ta
n[c + d*x]^5)/(10*a*d) - Tan[c + d*x]^8/(8*a*d) - Tan[c + d*x]^10/(10*a*d)
```

Rubi [A] time = 0.266218, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2611, 3768, 3770, 2607, 14}

$$-\frac{\tan^{10}(c+dx)}{10ad}-\frac{\tan^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{16ad}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(256*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(256*a
*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (Sec[c + d*x]^5*Tan[c + d*x
])/(32*a*d) - (Sec[c + d*x]^5*Tan[c + d*x]^3)/(16*a*d) + (Sec[c + d*x]^5*Ta
n[c + d*x]^5)/(10*a*d) - Tan[c + d*x]^8/(8*a*d) - Tan[c + d*x]^10/(10*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^5(c+dx)\tan^6(c+dx)\,dx}{a} - \frac{\int \sec^4(c+dx)\tan^7(c+dx)\,dx}{a}$$
$$= \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} - \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{2a} - \frac{\text{Subst}\left(\int x^7\left(1+x^2\right)d\right.}{ad}$$
$$= -\frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} + \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} + \frac{3\int \sec^5(c+dx)\tan^2(c+}{16a}$$
$$= \frac{\sec^5(c+dx)\tan(c+dx)}{32ad} - \frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} + \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} - $$
$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{\sec^5(c+dx)\tan(c+dx)}{32ad} - \frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} + $$
$$= -\frac{3\sec(c+dx)\tan(c+dx)}{256ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{\sec^5(c+dx)\tan(c+dx)}{32ad} - $$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{3\sec(c+dx)\tan(c+dx)}{256ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} + $$

Mathematica [A] time = 2.76586, size = 122, normalized size = 0.69

$$-\frac{30\tanh^{-1}(\sin(c+dx)) - \frac{2\left(15\sin^8(c+dx)+15\sin^7(c+dx)-55\sin^6(c+dx)+265\sin^5(c+dx)+137\sin^4(c+dx)-183\sin^3(c+dx)-113\sin^2(c+dx)+47\sin(c+\right.}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5}}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(30*ArcTanh[Sin[c + d*x]] - (2*(32 + 47*Sin[c + d*x] - 113*Sin[c + d*x]^2
- 183*Sin[c + d*x]^3 + 137*Sin[c + d*x]^4 + 265*Sin[c + d*x]^5 - 55*Sin[c +
 d*x]^6 + 15*Sin[c + d*x]^7 + 15*Sin[c + d*x]^8))/((-1 + Sin[c + d*x])^4*(1
 + Sin[c + d*x])^5))/(2560*a*d)
```

Maple [A] time = 0.095, size = 198, normalized size = 1.1

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} + \frac{1}{64\,da\,(\sin(dx+c)-1)^3} + \frac{9}{512\,da\,(\sin(dx+c)-1)^2} - \frac{1}{128\,da\,(\sin(dx+c)-1)} + \frac{3\ln(\sin(}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4+1/64/d/a/(sin(d*x+c)-1)^3+9/512/d/a/(sin(d*x+c)-
1)^2-1/128/a/d/(sin(d*x+c)-1)+3/512/a/d*ln(sin(d*x+c)-1)-1/160/d/a/(1+sin(d
*x+c))^5+7/256/d/a/(1+sin(d*x+c))^4-5/128/d/a/(1+sin(d*x+c))^3+5/512/a/d/(1
+sin(d*x+c))^2+5/256/a/d/(1+sin(d*x+c))-3/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.03646, size = 289, normalized size = 1.64

$$\frac{\frac{2\left(15\sin(dx+c)^8+15\sin(dx+c)^7-55\sin(dx+c)^6+265\sin(dx+c)^5+137\sin(dx+c)^4-183\sin(dx+c)^3-113\sin(dx+c)^2+47\sin(dx+c)+32\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{15\log(\sin(dx+}{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(15*sin(d*x + c)^8 + 15*sin(d*x + c)^7 - 55*sin(d*x + c)^6 + 265*
sin(d*x + c)^5 + 137*sin(d*x + c)^4 - 183*sin(d*x + c)^3 - 113*sin(d*x + c)
^2 + 47*sin(d*x + c) + 32)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(d
*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 -
4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 15*log(sin(
d*x + c) + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.56189, size = 518, normalized size = 2.94

$$\frac{30\cos(dx+c)^8-10\cos(dx+c)^6+124\cos(dx+c)^4-112\cos(dx+c)^2-15\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(30*cos(d*x + c)^8 - 10*cos(d*x + c)^6 + 124*cos(d*x + c)^4 - 112*co
s(d*x + c)^2 - 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*
x + c) + 1) + 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d*
x + c) + 1) - 2*(15*cos(d*x + c)^6 - 310*cos(d*x + c)^4 + 392*cos(d*x + c)^
2 - 144)*sin(d*x + c) + 32)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
+ c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33182, size = 211, normalized size = 1.2

$$-\frac{\frac{60\,\log\left(|\sin(dx+c)+1|\right)}{a}-\frac{60\,\log\left(|\sin(dx+c)-1|\right)}{a}+\frac{5\left(25\,\sin(dx+c)^4-84\,\sin(dx+c)^3+66\,\sin(dx+c)^2-12\,\sin(dx+c)-3\right)}{a(\sin(dx+c)-1)^4}-\frac{137\,\sin(dx+c)^5+885\,\sin(dx+c)^4+2}{}}{10240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
 + 5*(25*sin(d*x + c)^4 - 84*sin(d*x + c)^3 + 66*sin(d*x + c)^2 - 12*sin(d*
x + c) - 3)/(a*(sin(d*x + c) - 1)^4) - (137*sin(d*x + c)^5 + 885*sin(d*x +
c)^4 + 2270*sin(d*x + c)^3 + 2470*sin(d*x + c)^2 + 1265*sin(d*x + c) + 253)
/(a*(sin(d*x + c) + 1)^5))/d
```

3.902 $\int \frac{\sec^4(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{4ad}+\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}+\frac{\tan^3(c+dx)\sec}{16ad}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^6/(6*a*d) - Sec[c + d*x]
^8/(4*a*d) + Sec[c + d*x]^10/(10*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) - (Sec[c + d*x]^5*Tan[c + d*
x])/(32*a*d) + (Sec[c + d*x]^5*Tan[c + d*x]^3)/(16*a*d) - (Sec[c + d*x]^5*T
an[c + d*x]^5)/(10*a*d)
```

Rubi [A] time = 0.273114, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2835, 2606, 266, 43, 2611, 3768, 3770}

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{4ad}+\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}+\frac{\tan^3(c+dx)\sec}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^6/(6*a*d) - Sec[c + d*x]
^8/(4*a*d) + Sec[c + d*x]^10/(10*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) - (Sec[c + d*x]^5*Tan[c + d*
x])/(32*a*d) + (Sec[c + d*x]^5*Tan[c + d*x]^3)/(16*a*d) - (Sec[c + d*x]^5*T
an[c + d*x]^5)/(10*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^6(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \sec^5(c+dx)\tan^6(c+dx)\,dx}{a}$$

$$= -\frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} + \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x^5\left(-1+x^2\right.\right.}{a\ldots}$$

$$= \frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} - \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} - \frac{3\int \sec^5(c+dx)\tan^2(c+dx\ldots}{16a}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{32ad} + \frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} - \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} + \ldots$$

$$= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{4ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{\sec^5(c+d\ldots}{3\ldots}$$

$$= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{4ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\sec(c+dx)\tan(c+dx)}{256ad} + \frac{\sec^3(c+\ldots}{1\ldots}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{4ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\sec(c+dx)\ldots}{256\ldots}$$

Mathematica [A] time = 5.85349, size = 116, normalized size = 0.6

$$\frac{-\frac{90}{\sin(c+dx)-1} + \frac{15}{(\sin(c+dx)-1)^2} + \frac{75}{(\sin(c+dx)+1)^2} + \frac{80}{(\sin(c+dx)-1)^3} + \frac{100}{(\sin(c+dx)+1)^3} + \frac{30}{(\sin(c+dx)-1)^4} - \frac{150}{(\sin(c+dx)+1)^4} + \frac{48}{(\sin(c+dx)+1)^5} + 90\ldots}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (90*ArcTanh[Sin[c + d*x]] + 30/(-1 + Sin[c + d*x])^4 + 80/(-1 + Sin[c + d*x
])^3 + 15/(-1 + Sin[c + d*x])^2 - 90/(-1 + Sin[c + d*x]) + 48/(1 + Sin[c +
d*x])^5 - 150/(1 + Sin[c + d*x])^4 + 100/(1 + Sin[c + d*x])^3 + 75/(1 + Sin
[c + d*x])^2)/(7680*a*d)
```

Maple [A] time = 0.089, size = 180, normalized size = 0.9

$$\frac{1}{256da(\sin(dx+c)-1)^4}+\frac{1}{96da(\sin(dx+c)-1)^3}+\frac{1}{512da(\sin(dx+c)-1)^2}-\frac{3}{256da(\sin(dx+c)-1)}-\frac{3\ln(\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4+1/96/d/a/(sin(d*x+c)-1)^3+1/512/d/a/(sin(d*x+c)-
1)^2-3/256/a/d/(sin(d*x+c)-1)-3/512/a/d*ln(sin(d*x+c)-1)+1/160/d/a/(1+sin(d
*x+c))^5-5/256/d/a/(1+sin(d*x+c))^4+5/384/d/a/(1+sin(d*x+c))^3+5/512/a/d/(1
+sin(d*x+c))^2+3/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.01659, size = 289, normalized size = 1.49

$$-\frac{\frac{2\left(45\sin(dx+c)^8+45\sin(dx+c)^7-165\sin(dx+c)^6-165\sin(dx+c)^5-549\sin(dx+c)^4+91\sin(dx+c)^3+301\sin(dx+c)^2-19\sin(dx+c)-64\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{45\log(\sin(d}{a}}{7680d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(45*sin(d*x + c)^8 + 45*sin(d*x + c)^7 - 165*sin(d*x + c)^6 - 16
5*sin(d*x + c)^5 - 549*sin(d*x + c)^4 + 91*sin(d*x + c)^3 + 301*sin(d*x + c
)^2 - 19*sin(d*x + c) - 64)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(
d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 -
 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 45*log(sin
(d*x + c) + 1)/a + 45*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.33805, size = 521, normalized size = 2.69

$$-\frac{90\cos(dx+c)^8-30\cos(dx+c)^6-1548\cos(dx+c)^4+2224\cos(dx+c)^2-45\left(\cos(dx+c)^8\sin(dx+c)+\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(90*cos(d*x + c)^8 - 30*cos(d*x + c)^6 - 1548*cos(d*x + c)^4 + 2224
*cos(d*x + c)^2 - 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin
(d*x + c) + 1) + 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin
(d*x + c) + 1) - 2*(45*cos(d*x + c)^6 + 30*cos(d*x + c)^4 - 104*cos(d*x + c
)^2 + 48)*sin(d*x + c) - 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*
x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28089, size = 211, normalized size = 1.09

$$\frac{\frac{180 \log(|\sin(dx+c)+1|)}{a} - \frac{180 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(75 \sin(dx+c)^4 - 372 \sin(dx+c)^3 + 678 \sin(dx+c)^2 - 476 \sin(dx+c) + 119\right)}{a(\sin(dx+c)-1)^4} - \frac{411 \sin(dx+c)^5 + 2055 \sin(d}{}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(180*log(abs(sin(d*x + c) + 1))/a - 180*log(abs(sin(d*x + c) - 1))/
a + 5*(75*sin(d*x + c)^4 - 372*sin(d*x + c)^3 + 678*sin(d*x + c)^2 - 476*si
n(d*x + c) + 119)/(a*(sin(d*x + c) - 1)^4) - (411*sin(d*x + c)^5 + 2055*sin
(d*x + c)^4 + 3810*sin(d*x + c)^3 + 2810*sin(d*x + c)^2 + 955*sin(d*x + c)
+ 119)/(a*(sin(d*x + c) + 1)^5))/d
```

3.903 $\int \frac{\sec^5(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=192

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{4ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}-\frac{3\tan(c+dx)\mathrm{s}}{80a}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(256*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x]
^8/(4*a*d) - Sec[c + d*x]^10/(10*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (Sec[c + d*x]^5*Tan[c + d*
x])/(160*a*d) - (3*Sec[c + d*x]^7*Tan[c + d*x])/(80*a*d) + (Sec[c + d*x]^7*
Tan[c + d*x]^3)/(10*a*d)
```

Rubi [A] time = 0.25289, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2835, 2611, 3768, 3770, 2606, 266, 43}

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{4ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}-\frac{3\tan(c+dx)\mathrm{s}}{80a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(256*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x]
^8/(4*a*d) - Sec[c + d*x]^10/(10*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (Sec[c + d*x]^5*Tan[c + d*
x])/(160*a*d) - (3*Sec[c + d*x]^7*Tan[c + d*x])/(80*a*d) + (Sec[c + d*x]^7*
Tan[c + d*x]^3)/(10*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^7(c+dx)\tan^4(c+dx)\,dx}{a} - \frac{\int \sec^6(c+dx)\tan^5(c+dx)\,dx}{a}$$

$$= \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} - \frac{3\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{10a} - \frac{\text{Subst}\left(\int x^5\left(-1+x^2\right.\right.}{ad}$$

$$= -\frac{3\sec^7(c+dx)\tan(c+dx)}{80ad} + \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} + \frac{3\int \sec^7(c+dx)\,dx}{80a} - \text{Sub}$$

$$= \frac{\sec^5(c+dx)\tan(c+dx)}{160ad} - \frac{3\sec^7(c+dx)\tan(c+dx)}{80ad} + \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} +$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{4ad} - \frac{\sec^{10}(c+dx)}{10ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{\sec^5(c+}{}$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{4ad} - \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\sec(c+dx)\tan(c+dx)}{256ad} + \frac{\sec^3(c+}{}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{4ad} - \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\sec(c+dx)}{256}$$

Mathematica [A] time = 5.72551, size = 116, normalized size = 0.6

$$\frac{-\frac{90}{\sin(c+dx)+1} - \frac{45}{(\sin(c+dx)-1)^2} - \frac{45}{(\sin(c+dx)+1)^2} + \frac{40}{(\sin(c+dx)-1)^3} + \frac{20}{(\sin(c+dx)+1)^3} + \frac{30}{(\sin(c+dx)-1)^4} + \frac{90}{(\sin(c+dx)+1)^4} - \frac{48}{(\sin(c+dx)+1)^5} + 90}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (90*ArcTanh[Sin[c + d*x]] + 30/(-1 + Sin[c + d*x])^4 + 40/(-1 + Sin[c + d*x
])^3 - 45/(-1 + Sin[c + d*x])^2 - 48/(1 + Sin[c + d*x])^5 + 90/(1 + Sin[c +
 d*x])^4 + 20/(1 + Sin[c + d*x])^3 - 45/(1 + Sin[c + d*x])^2 - 90/(1 + Sin[
c + d*x]))/(7680*a*d)
```

Maple [A] time = 0.087, size = 180, normalized size = 0.9

$$\frac{1}{256da(\sin(dx+c)-1)^4}+\frac{1}{192da(\sin(dx+c)-1)^3}-\frac{3}{512da(\sin(dx+c)-1)^2}-\frac{3\ln(\sin(dx+c)-1)}{512da}-\frac{}{160da(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4+1/192/d/a/(sin(d*x+c)-1)^3-3/512/d/a/(sin(d*x+c)
-1)^2-3/512/a/d*ln(sin(d*x+c)-1)-1/160/d/a/(1+sin(d*x+c))^5+3/256/d/a/(1+si
n(d*x+c))^4+1/384/d/a/(1+sin(d*x+c))^3-3/512/a/d/(1+sin(d*x+c))^2-3/256/a/d
/(1+sin(d*x+c))+3/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.0931, size = 289, normalized size = 1.51

$$-\frac{\frac{2\left(45\sin(dx+c)^8+45\sin(dx+c)^7-165\sin(dx+c)^6-165\sin(dx+c)^5+219\sin(dx+c)^4-421\sin(dx+c)^3-211\sin(dx+c)^2+109\sin(dx+c)+64\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{45\log(\sin(}{a}}{7680d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(45*sin(d*x + c)^8 + 45*sin(d*x + c)^7 - 165*sin(d*x + c)^6 - 16
5*sin(d*x + c)^5 + 219*sin(d*x + c)^4 - 421*sin(d*x + c)^3 - 211*sin(d*x +
c)^2 + 109*sin(d*x + c) + 64)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*si
n(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4
 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 45*log(s
in(d*x + c) + 1)/a + 45*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.07818, size = 517, normalized size = 2.69

$$-\frac{90\cos(dx+c)^8-30\cos(dx+c)^6-12\cos(dx+c)^4+176\cos(dx+c)^2-45\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(90*cos(d*x + c)^8 - 30*cos(d*x + c)^6 - 12*cos(d*x + c)^4 + 176*co
s(d*x + c)^2 - 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*
x + c) + 1) + 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d*
x + c) + 1) - 2*(45*cos(d*x + c)^6 + 30*cos(d*x + c)^4 - 616*cos(d*x + c)^2
 + 432)*sin(d*x + c) - 96)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x +
 c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4095, size = 211, normalized size = 1.1

$$\frac{\frac{180\log(|\sin(dx+c)+1|)}{a} - \frac{180\log(|\sin(dx+c)-1|)}{a} + \frac{5\left(75\sin(dx+c)^4-300\sin(dx+c)^3+414\sin(dx+c)^2-196\sin(dx+c)+31\right)}{a(\sin(dx+c)-1)^4} - \frac{411\sin(dx+c)^5+2415\sin(dx}{}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(180*log(abs(sin(d*x + c) + 1))/a - 180*log(abs(sin(d*x + c) - 1))/
a + 5*(75*sin(d*x + c)^4 - 300*sin(d*x + c)^3 + 414*sin(d*x + c)^2 - 196*si
n(d*x + c) + 31)/(a*(sin(d*x + c) - 1)^4) - (411*sin(d*x + c)^5 + 2415*sin(
d*x + c)^4 + 5730*sin(d*x + c)^3 + 6730*sin(d*x + c)^2 + 3515*sin(d*x + c)
+ 703)/(a*(sin(d*x + c) + 1)^5))/d
```

3.904 $\int \frac{\sec^6(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}+\frac{3\tan(c+dx)\sec^7(c+dx)}{80ad}-\text{ta}$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(256*a*d) - Sec[c + d*x]^8/(8*a*d) + Sec[c + d*x
]^10/(10*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(256*a*d) - (Sec[c + d*x]^3*T
an[c + d*x])/(128*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])/(160*a*d) + (3*Sec[c
 + d*x]^7*Tan[c + d*x])/(80*a*d) - (Sec[c + d*x]^7*Tan[c + d*x]^3)/(10*a*d)
```

Rubi [A] time = 0.241546, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2606, 14, 2611, 3768, 3770}

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}+\frac{3\tan(c+dx)\sec^7(c+dx)}{80ad}-\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^6*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(256*a*d) - Sec[c + d*x]^8/(8*a*d) + Sec[c + d*x
]^10/(10*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(256*a*d) - (Sec[c + d*x]^3*T
an[c + d*x])/(128*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])/(160*a*d) + (3*Sec[c
 + d*x]^7*Tan[c + d*x])/(80*a*d) - (Sec[c + d*x]^7*Tan[c + d*x]^3)/(10*a*d)
```


Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^8(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \sec^7(c+dx)\tan^4(c+dx)\,dx}{a}$$
$$= -\frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} + \frac{3\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^7\left(-1+x\right.\right.}{}$$
$$= \frac{3\sec^7(c+dx)\tan(c+dx)}{80ad} - \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} - \frac{3\int \sec^7(c+dx)\,dx}{80a} + \frac{\text{Subs}}{}$$
$$= -\frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{\sec^5(c+dx)\tan(c+dx)}{160ad} + \frac{3\sec^7(c+dx)\tan(c+d}{80ad}$$
$$= -\frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{\sec^5(c+dx)\tan(c+dx)}{160ad}$$
$$= -\frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{3\sec(c+dx)\tan(c+dx)}{256ad} - \frac{\sec^3(c+dx)\tan(c+dx}{128ad}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{3\sec(c+dx)\tan(c+dx)}{256ad} -$$

Mathematica [A] time = 2.90546, size = 104, normalized size = 0.6

$$-\frac{-\frac{30}{\sin(c+dx)-1} + \frac{15}{(\sin(c+dx)-1)^2} + \frac{15}{(\sin(c+dx)+1)^2} + \frac{20}{(\sin(c+dx)+1)^3} - \frac{10}{(\sin(c+dx)-1)^4} + \frac{10}{(\sin(c+dx)+1)^4} - \frac{16}{(\sin(c+dx)+1)^5} + 30\tanh^{-1}(\sin(c}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^6*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(30*ArcTanh[Sin[c + d*x]] - 10/(-1 + Sin[c + d*x])^4 + 15/(-1 + Sin[c + d*
x])^2 - 30/(-1 + Sin[c + d*x]) - 16/(1 + Sin[c + d*x])^5 + 10/(1 + Sin[c +
d*x])^4 + 20/(1 + Sin[c + d*x])^3 + 15/(1 + Sin[c + d*x])^2)/(2560*a*d)
```

Maple [A] time = 0.084, size = 162, normalized size = 0.9

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} - \frac{3}{512\,da\,(\sin(dx+c)-1)^2} + \frac{3}{256\,da\,(\sin(dx+c)-1)} + \frac{3\ln(\sin(dx+c)-1)}{512\,da} + \frac{}{160\,da\,(1+\text{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4-3/512/d/a/(sin(d*x+c)-1)^2+3/256/a/d/(sin(d*x+c)
-1)+3/512/a/d*ln(sin(d*x+c)-1)+1/160/d/a/(1+sin(d*x+c))^5-1/256/d/a/(1+sin(
d*x+c))^4-1/128/d/a/(1+sin(d*x+c))^3-3/512/a/d/(1+sin(d*x+c))^2-3/512*ln(1+
sin(d*x+c))/a/d
```

Maxima [A] time = 1.05806, size = 289, normalized size = 1.66

$$\frac{\frac{2\left(15\sin(dx+c)^8+15\sin(dx+c)^7-55\sin(dx+c)^6-55\sin(dx+c)^5+73\sin(dx+c)^4+73\sin(dx+c)^3+143\sin(dx+c)^2-17\sin(dx+c)-32\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{15\log(\sin(dx+}{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(15*sin(d*x + c)^8 + 15*sin(d*x + c)^7 - 55*sin(d*x + c)^6 - 55*s
in(d*x + c)^5 + 73*sin(d*x + c)^4 + 73*sin(d*x + c)^3 + 143*sin(d*x + c)^2
- 17*sin(d*x + c) - 32)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(d*x
+ c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 - 4*a
*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 15*log(sin(d*x
 + c) + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.15705, size = 512, normalized size = 2.94

$$\frac{30\cos(dx+c)^8-10\cos(dx+c)^6-4\cos(dx+c)^4-368\cos(dx+c)^2-15\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(30*cos(d*x + c)^8 - 10*cos(d*x + c)^6 - 4*cos(d*x + c)^4 - 368*cos(
d*x + c)^2 - 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*x
+ c) + 1) + 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d*x
+ c) + 1) - 2*(15*cos(d*x + c)^6 + 10*cos(d*x + c)^4 + 8*cos(d*x + c)^2 - 1
6)*sin(d*x + c) + 288)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x + c)^
8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35749, size = 211, normalized size = 1.21

$$-\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a} - \frac{60 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(25 \sin(dx+c)^4-124 \sin(dx+c)^3+234 \sin(dx+c)^2-196 \sin(dx+c)+53\right)}{a(\sin(dx+c)-1)^4} - \frac{137 \sin(dx+c)^5+685 \sin(dx+c)^4+\cdots}{\cdots}}{10240\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
 + 5*(25*sin(d*x + c)^4 - 124*sin(d*x + c)^3 + 234*sin(d*x + c)^2 - 196*sin
(d*x + c) + 53)/(a*(sin(d*x + c) - 1)^4) - (137*sin(d*x + c)^5 + 685*sin(d*
x + c)^4 + 1310*sin(d*x + c)^3 + 1110*sin(d*x + c)^2 + 305*sin(d*x + c) + 2
1)/(a*(sin(d*x + c) + 1)^5))/d
```

3.905 $\int \frac{\sec^7(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=172

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}-\frac{7\,\mathrm{ta}}{}$$

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^8/(8*a*d) - Sec[c + d*x
]^10/(10*a*d) - (7*Sec[c + d*x]*Tan[c + d*x])/(256*a*d) - (7*Sec[c + d*x]^3
*Tan[c + d*x])/(384*a*d) - (7*Sec[c + d*x]^5*Tan[c + d*x])/(480*a*d) - (Sec
[c + d*x]^7*Tan[c + d*x])/(80*a*d) + (Sec[c + d*x]^9*Tan[c + d*x])/(10*a*d)
```

Rubi [A] time = 0.219143, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2835, 2611, 3768, 3770, 2606, 14}

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}-\frac{7\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^8/(8*a*d) - Sec[c + d*x
]^10/(10*a*d) - (7*Sec[c + d*x]*Tan[c + d*x])/(256*a*d) - (7*Sec[c + d*x]^3
*Tan[c + d*x])/(384*a*d) - (7*Sec[c + d*x]^5*Tan[c + d*x])/(480*a*d) - (Sec
[c + d*x]^7*Tan[c + d*x])/(80*a*d) + (Sec[c + d*x]^9*Tan[c + d*x])/(10*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^7(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^9(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec^8(c+dx)\tan^3(c+dx)\,dx}{a}\\
&= \frac{\sec^9(c+dx)\tan(c+dx)}{10ad} - \frac{\int \sec^9(c+dx)\,dx}{10a} - \frac{\text{Subst}\left(\int x^7\left(-1+x^2\right)\,dx, x, \sec(c+
\end{aligned}$$

$$= -\frac{\sec^7(c+dx)\tan(c+dx)}{80ad} + \frac{\sec^9(c+dx)\tan(c+dx)}{10ad} - \frac{7\int \sec^7(c+dx)\,dx}{80a} - \text{Subst}$$

$$= \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec^5(c+dx)\tan(c+dx)}{480ad} - \frac{\sec^7(c+dx)\tan(c+dx)}{80ad}$$

$$= \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec^3(c+dx)\tan(c+dx)}{384ad} - \frac{7\sec^5(c+dx)\tan(c+d}{480ad}$$

$$= \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec(c+dx)\tan(c+dx)}{256ad} - \frac{7\sec^3(c+dx)\tan(c+dx}{384ad}$$

$$= -\frac{7\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec(c+dx)\tan(c+dx)}{256ad} -$$

Mathematica [A] time = 2.59189, size = 122, normalized size = 0.71

$$-\frac{210\tanh^{-1}(\sin(c+dx)) - \frac{2\left(105\sin^8(c+dx)+105\sin^7(c+dx)-385\sin^6(c+dx)-385\sin^5(c+dx)+511\sin^4(c+dx)+511\sin^3(c+dx)-279\sin^2(c+dx)+201}{(\sin(c+dx)-1)^4(\sin(c+dx)+1)^5}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(210*ArcTanh[Sin[c + d*x]] - (2*(96 + 201*Sin[c + d*x] - 279*Sin[c + d*x]^
2 + 511*Sin[c + d*x]^3 + 511*Sin[c + d*x]^4 - 385*Sin[c + d*x]^5 - 385*Sin[
c + d*x]^6 + 105*Sin[c + d*x]^7 + 105*Sin[c + d*x]^8))/((-1 + Sin[c + d*x])
^4*(1 + Sin[c + d*x])^5))/(7680*a*d)
```

Maple [A] time = 0.079, size = 198, normalized size = 1.2

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} - \frac{1}{192\,da\,(\sin(dx+c)-1)^3} + \frac{1}{512\,da\,(\sin(dx+c)-1)^2} + \frac{1}{128\,da\,(\sin(dx+c)-1)} + \frac{7\ln(\sin}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4-1/192/d/a/(sin(d*x+c)-1)^3+1/512/d/a/(sin(d*x+c)
-1)^2+1/128/a/d/(sin(d*x+c)-1)+7/512/a/d*ln(sin(d*x+c)-1)-1/160/d/a/(1+sin(
d*x+c))^5-1/256/d/a/(1+sin(d*x+c))^4+1/384/d/a/(1+sin(d*x+c))^3+5/512/a/d/(
1+sin(d*x+c))^2+5/256/a/d/(1+sin(d*x+c))-7/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.03582, size = 289, normalized size = 1.68

$$\frac{\frac{2\left(105 \sin(dx+c)^8+105 \sin(dx+c)^7-385 \sin(dx+c)^6-385 \sin(dx+c)^5+511 \sin(dx+c)^4+511 \sin(dx+c)^3-279 \sin(dx+c)^2+201 \sin(dx+c)+96\right)}{a \sin(dx+c)^9+a \sin(dx+c)^8-4 a \sin(dx+c)^7-4 a \sin(dx+c)^6+6 a \sin(dx+c)^5+6 a \sin(dx+c)^4-4 a \sin(dx+c)^3-4 a \sin(dx+c)^2+a \sin(dx+c)+a} - \frac{105 \log(\mathrm{si}}{}}{7680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/7680*(2*(105*sin(d*x + c)^8 + 105*sin(d*x + c)^7 - 385*sin(d*x + c)^6 - 3
85*sin(d*x + c)^5 + 511*sin(d*x + c)^4 + 511*sin(d*x + c)^3 - 279*sin(d*x +
 c)^2 + 201*sin(d*x + c) + 96)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*s
in(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^
4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 105*log
(sin(d*x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.13217, size = 518, normalized size = 3.01

$$\frac{210 \cos(dx+c)^8 - 70 \cos(dx+c)^6 - 28 \cos(dx+c)^4 - 16 \cos(dx+c)^2 - 105\left(\cos(dx+c)^8 \sin(dx+c) + \cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(210*cos(d*x + c)^8 - 70*cos(d*x + c)^6 - 28*cos(d*x + c)^4 - 16*cos
(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*
x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d
*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 70*cos(d*x + c)^4 + 56*cos(d*x + c)^
2 - 432)*sin(d*x + c) + 96)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
+ c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26182, size = 211, normalized size = 1.23

$$-\frac{\frac{420\log(|\sin(dx+c)+1|)}{a}-\frac{420\log(|\sin(dx+c)-1|)}{a}+\frac{5\left(175\sin(dx+c)^4-748\sin(dx+c)^3+1182\sin(dx+c)^2-788\sin(dx+c)+155\right)}{a(\sin(dx+c)-1)^4}-\frac{959\sin(dx+c)^5+5395\,\mathrm{s}}{}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))
/a + 5*(175*sin(d*x + c)^4 - 748*sin(d*x + c)^3 + 1182*sin(d*x + c)^2 - 788
*sin(d*x + c) + 155)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 5395*
sin(d*x + c)^4 + 12290*sin(d*x + c)^3 + 14170*sin(d*x + c)^2 + 8135*sin(d*x
 + c) + 1627)/(a*(sin(d*x + c) + 1)^5))/d
```

3.906 $\int \frac{\sec^8(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{\sec^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}+\frac{7\tan(c+dx)\sec^5(c}{480ad}$$

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^10/(10*a*d) + (7*Sec[c +
 d*x]*Tan[c + d*x])/(256*a*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(384*a*d) +
 (7*Sec[c + d*x]^5*Tan[c + d*x])/(480*a*d) + (Sec[c + d*x]^7*Tan[c + d*x])/
(80*a*d) - (Sec[c + d*x]^9*Tan[c + d*x])/(10*a*d)
```

Rubi [A] time = 0.165216, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2835, 2606, 30, 2611, 3768, 3770}

$$\frac{\sec^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}+\frac{7\tan(c+dx)\sec^5(c}{480ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^8*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^10/(10*a*d) + (7*Sec[c +
 d*x]*Tan[c + d*x])/(256*a*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(384*a*d) +
 (7*Sec[c + d*x]^5*Tan[c + d*x])/(480*a*d) + (Sec[c + d*x]^7*Tan[c + d*x])/
(80*a*d) - (Sec[c + d*x]^9*Tan[c + d*x])/(10*a*d)
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^{10}(c+dx)\tan(c+dx)\,dx}{a} - \frac{\int \sec^9(c+dx)\tan^2(c+dx)\,dx}{a}$$
$$= -\frac{\sec^9(c+dx)\tan(c+dx)}{10ad} + \frac{\int \sec^9(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^9\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{\sec^{10}(c+dx)}{10ad} + \frac{\sec^7(c+dx)\tan(c+dx)}{80ad} - \frac{\sec^9(c+dx)\tan(c+dx)}{10ad} + \frac{7\int \sec^7(c+dx)}{80a}$$
$$= \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec^5(c+dx)\tan(c+dx)}{480ad} + \frac{\sec^7(c+dx)\tan(c+dx)}{80ad} - \frac{\sec^9(c+dx)}{10a}$$
$$= \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{384ad} + \frac{7\sec^5(c+dx)\tan(c+dx)}{480ad} + \frac{\sec^7(c+dx)}{8}$$
$$= \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{384ad} + \frac{7\sec^5(c+d}{48}$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\,\text{ta}}{384ad}$$

Mathematica [A] time = 5.86991, size = 116, normalized size = 0.75

$$\frac{-\frac{210}{\sin(c+dx)-1} + \frac{135}{(\sin(c+dx)-1)^2} + \frac{75}{(\sin(c+dx)+1)^2} - \frac{80}{(\sin(c+dx)-1)^3} + \frac{100}{(\sin(c+dx)+1)^3} + \frac{30}{(\sin(c+dx)-1)^4} + \frac{90}{(\sin(c+dx)+1)^4} + \frac{48}{(\sin(c+dx)+1)^5} + 21}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^8*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (210*ArcTanh[Sin[c + d*x]] + 30/(-1 + Sin[c + d*x])^4 - 80/(-1 + Sin[c + d*
x])^3 + 135/(-1 + Sin[c + d*x])^2 - 210/(-1 + Sin[c + d*x]) + 48/(1 + Sin[c
 + d*x])^5 + 90/(1 + Sin[c + d*x])^4 + 100/(1 + Sin[c + d*x])^3 + 75/(1 + S
in[c + d*x])^2)/(7680*a*d)
```

Maple [A] time = 0.071, size = 180, normalized size = 1.2

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} - \frac{1}{96\,da\,(\sin(dx+c)-1)^3} + \frac{9}{512\,da\,(\sin(dx+c)-1)^2} - \frac{7}{256\,da\,(\sin(dx+c)-1)} - \frac{7\ln(\sin(}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4-1/96/d/a/(sin(d*x+c)-1)^3+9/512/d/a/(sin(d*x+c)-
1)^2-7/256/a/d/(sin(d*x+c)-1)-7/512/a/d*ln(sin(d*x+c)-1)+1/160/d/a/(1+sin(d
*x+c))^5+3/256/d/a/(1+sin(d*x+c))^4+5/384/d/a/(1+sin(d*x+c))^3+5/512/a/d/(1
+sin(d*x+c))^2+7/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.07135, size = 289, normalized size = 1.88

$$-\frac{\frac{2\left(105\sin(dx+c)^8+105\sin(dx+c)^7-385\sin(dx+c)^6-385\sin(dx+c)^5+511\sin(dx+c)^4+511\sin(dx+c)^3-279\sin(dx+c)^2-279\sin(dx+c)-384\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{105\log}{}}{7680d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(105*sin(d*x + c)^8 + 105*sin(d*x + c)^7 - 385*sin(d*x + c)^6 -
385*sin(d*x + c)^5 + 511*sin(d*x + c)^4 + 511*sin(d*x + c)^3 - 279*sin(d*x
+ c)^2 - 279*sin(d*x + c) - 384)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a
*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c
)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 105*l
og(sin(d*x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.18468, size = 520, normalized size = 3.38

$$-\frac{210\cos(dx+c)^8-70\cos(dx+c)^6-28\cos(dx+c)^4-16\cos(dx+c)^2-105\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(210*cos(d*x + c)^8 - 70*cos(d*x + c)^6 - 28*cos(d*x + c)^4 - 16*co
s(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d
*x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(
d*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 70*cos(d*x + c)^4 + 56*cos(d*x + c)
^2 + 48)*sin(d*x + c) - 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
 + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31615, size = 211, normalized size = 1.37

$$\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a} - \frac{420 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(175 \sin(dx+c)^4-868 \sin(dx+c)^3+1662 \sin(dx+c)^2-1484 \sin(dx+c)+539\right)}{a(\sin(dx+c)-1)^4} - \frac{959 \sin(dx+c)^5+4795 \mathrm{si}}{}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 5*(175*sin(d*x + c)^4 - 868*sin(d*x + c)^3 + 1662*sin(d*x + c)^2 - 1484
*sin(d*x + c) + 539)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 4795*
sin(d*x + c)^4 + 9290*sin(d*x + c)^3 + 8290*sin(d*x + c)^2 + 2735*sin(d*x +
 c) - 293)/(a*(sin(d*x + c) + 1)^5))/d
```

3.907 $\int \frac{\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=210

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{5a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{64d(a-a\sin(c+dx))^3}-\frac{}{128d(a}$$

```
[Out] (63*ArcTanh[Sin[c + d*x]])/(256*a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) +
 a^2/(64*d*(a - a*Sin[c + d*x])^3) + (21*a)/(512*d*(a - a*Sin[c + d*x])^2)
+ 7/(64*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*Sin[c + d*x])^5) - (5*a
^3)/(256*d*(a + a*Sin[c + d*x])^4) - (5*a^2)/(128*d*(a + a*Sin[c + d*x])^3)
 - (35*a)/(512*d*(a + a*Sin[c + d*x])^2) - 35/(256*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.167037, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{5a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{64d(a-a\sin(c+dx))^3}-\frac{}{128d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```


```
[Out] (63*ArcTanh[Sin[c + d*x]])/(256*a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) +
 a^2/(64*d*(a - a*Sin[c + d*x])^3) + (21*a)/(512*d*(a - a*Sin[c + d*x])^2)
+ 7/(64*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*Sin[c + d*x])^5) - (5*a
^3)/(256*d*(a + a*Sin[c + d*x])^4) - (5*a^2)/(128*d*(a + a*Sin[c + d*x])^3)
 - (35*a)/(512*d*(a + a*Sin[c + d*x])^2) - 35/(256*d*(a + a*Sin[c + d*x]))
```


Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9 \operatorname{Subst}\left(\int \frac{1}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^9 \operatorname{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^5} + \frac{3}{64a^7(a-x)^4} + \frac{21}{256a^8(a-x)^3} + \frac{7}{64a^9(a-x)^2} + \frac{1}{32a^5(a+x)^6} + \frac{5}{64a^6(a+x)^5} + \frac{15}{128a^7(a+x)}\right.\right.}{d}$$

$$= \frac{a^3}{256d(a-a\sin(c+dx))^4} + \frac{a^2}{64d(a-a\sin(c+dx))^3} + \frac{21a}{512d(a-a\sin(c+dx))^2} + \frac{7}{64d(a-a\sin}$$

$$= \frac{63\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{a^3}{256d(a-a\sin(c+dx))^4} + \frac{a^2}{64d(a-a\sin(c+dx))^3} + \frac{21a}{512d(a-a\sin}$$

Mathematica [A] time = 1.4314, size = 165, normalized size = 0.79

$$\frac{\sec^8(c+dx)\left(-315\sin^8(c+dx) - 315\sin^7(c+dx) + 1155\sin^6(c+dx) + 1155\sin^5(c+dx) - 1533\sin^4(c+dx) - 1533\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^8*(-128 + 315*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^8*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^10 + 837*Sin[c + d*x]
+ 837*Sin[c + d*x]^2 - 1533*Sin[c + d*x]^3 - 1533*Sin[c + d*x]^4 + 1155*Sin
[c + d*x]^5 + 1155*Sin[c + d*x]^6 - 315*Sin[c + d*x]^7 - 315*Sin[c + d*x]^8
))/(1280*a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.073, size = 198, normalized size = 0.9

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} - \frac{1}{64\,da\,(\sin(dx+c)-1)^3} + \frac{21}{512\,da\,(\sin(dx+c)-1)^2} - \frac{7}{64\,da\,(\sin(dx+c)-1)} - \frac{63\ln(\sin(}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4-1/64/d/a/(sin(d*x+c)-1)^3+21/512/d/a/(sin(d*x+c)
-1)^2-7/64/a/d/(sin(d*x+c)-1)-63/512/a/d*ln(sin(d*x+c)-1)-1/160/d/a/(1+sin(
d*x+c))^5-5/256/d/a/(1+sin(d*x+c))^4-5/128/d/a/(1+sin(d*x+c))^3-35/512/a/d/
(1+sin(d*x+c))^2-35/256/a/d/(1+sin(d*x+c))+63/512*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.04454, size = 289, normalized size = 1.38

$$-\frac{\frac{2\left(315\sin(dx+c)^8+315\sin(dx+c)^7-1155\sin(dx+c)^6-1155\sin(dx+c)^5+1533\sin(dx+c)^4+1533\sin(dx+c)^3-837\sin(dx+c)^2-837\sin(dx+c)+128\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a} - \frac{315\log}{}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2560*(2*(315*sin(d*x + c)^8 + 315*sin(d*x + c)^7 - 1155*sin(d*x + c)^6 -
 1155*sin(d*x + c)^5 + 1533*sin(d*x + c)^4 + 1533*sin(d*x + c)^3 - 837*sin(
d*x + c)^2 - 837*sin(d*x + c) + 128)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 -
 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x
 + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 3
15*log(sin(d*x + c) + 1)/a + 315*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.10542, size = 520, normalized size = 2.48

$$-\frac{630\cos(dx+c)^8 - 210\cos(dx+c)^6 - 84\cos(dx+c)^4 - 48\cos(dx+c)^2 - 315\left(\cos(dx+c)^8\sin(dx+c) + \cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2560*(630*cos(d*x + c)^8 - 210*cos(d*x + c)^6 - 84*cos(d*x + c)^4 - 48*c
os(d*x + c)^2 - 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(
d*x + c) + 1) + 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin
(d*x + c) + 1) - 6*(105*cos(d*x + c)^6 + 70*cos(d*x + c)^4 + 56*cos(d*x + c
)^2 + 48)*sin(d*x + c) - 32)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
 + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33553, size = 211, normalized size = 1.

$$\frac{\frac{1260\log(|\sin(dx+c)+1|)}{a} - \frac{1260\log(|\sin(dx+c)-1|)}{a} + \frac{5\left(525\sin(dx+c)^4 - 2324\sin(dx+c)^3 + 3906\sin(dx+c)^2 - 2972\sin(dx+c) + 873\right)}{a(\sin(dx+c)-1)^4} - \frac{2877\sin(dx+c)^5}{}}{10240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10240*(1260*log(abs(sin(d*x + c) + 1))/a - 1260*log(abs(sin(d*x + c) - 1)
)/a + 5*(525*sin(d*x + c)^4 - 2324*sin(d*x + c)^3 + 3906*sin(d*x + c)^2 - 2
972*sin(d*x + c) + 873)/(a*(sin(d*x + c) - 1)^4) - (2877*sin(d*x + c)^5 + 1
5785*sin(d*x + c)^4 + 35070*sin(d*x + c)^3 + 39670*sin(d*x + c)^2 + 23085*s
in(d*x + c) + 5641)/(a*(sin(d*x + c) + 1)^5))/d
```

3.908 $\int \frac{\csc(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{7a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{48d(a-a\sin(c+dx))^3}+\frac{29}{384d(a\sin(}$$

```
[Out] (-193*Log[1 - Sin[c + d*x]])/(512*a*d) + Log[Sin[c + d*x]]/(a*d) - (319*Log
[1 + Sin[c + d*x]])/(512*a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) + a^2/(4
8*d*(a - a*Sin[c + d*x])^3) + (37*a)/(512*d*(a - a*Sin[c + d*x])^2) + 65/(2
56*d*(a - a*Sin[c + d*x])) + a^4/(160*d*(a + a*Sin[c + d*x])^5) + (7*a^3)/(
256*d*(a + a*Sin[c + d*x])^4) + (29*a^2)/(384*d*(a + a*Sin[c + d*x])^3) + (
93*a)/(512*d*(a + a*Sin[c + d*x])^2) + 1/(2*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.244415, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2836, 12, 88}

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{7a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{48d(a-a\sin(c+dx))^3}+\frac{29}{384d(a\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-193*Log[1 - Sin[c + d*x]])/(512*a*d) + Log[Sin[c + d*x]]/(a*d) - (319*Log
[1 + Sin[c + d*x]])/(512*a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) + a^2/(4
8*d*(a - a*Sin[c + d*x])^3) + (37*a)/(512*d*(a - a*Sin[c + d*x])^2) + 65/(2
56*d*(a - a*Sin[c + d*x])) + a^4/(160*d*(a + a*Sin[c + d*x])^5) + (7*a^3)/(
256*d*(a + a*Sin[c + d*x])^4) + (29*a^2)/(384*d*(a + a*Sin[c + d*x])^3) + (
93*a)/(512*d*(a + a*Sin[c + d*x])^2) + 1/(2*d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{a}{(a-x)^5x(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^{10}\,\text{Subst}\left(\int \frac{1}{(a-x)^5x(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^{10}\,\text{Subst}\left(\int \left(\frac{1}{64a^7(a-x)^5} + \frac{1}{16a^8(a-x)^4} + \frac{37}{256a^9(a-x)^3} + \frac{65}{256a^{10}(a-x)^2} + \frac{193}{512a^{11}(a-x)} + \frac{1}{a^{11}x} - \right.\right.}{}$$
$$= -\frac{193\log(1-\sin(c+dx))}{512ad} + \frac{\log(\sin(c+dx))}{ad} - \frac{319\log(1+\sin(c+dx))}{512ad} + \frac{}{256d(a-}$$

Mathematica [A] time = 6.19715, size = 228, normalized size = 0.92

$$a^{10}\left(\frac{65}{256a^{10}(a-a\sin(c+dx))} + \frac{1}{2a^{10}(a\sin(c+dx)+a)} + \frac{37}{512a^9(a-a\sin(c+dx))^2} + \frac{93}{512a^9(a\sin(c+dx)+a)^2} + \frac{1}{48a^8(a-a\sin(c+dx))^3} + \frac{29}{384a^8(a\sin(c+dx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^10*((-193*Log[1 - Sin[c + d*x]])/(512*a^11) + Log[Sin[c + d*x]]/a^11 - (
319*Log[1 + Sin[c + d*x]])/(512*a^11) + 1/(256*a^7*(a - a*Sin[c + d*x])^4)
+ 1/(48*a^8*(a - a*Sin[c + d*x])^3) + 37/(512*a^9*(a - a*Sin[c + d*x])^2) +
 65/(256*a^10*(a - a*Sin[c + d*x])) + 1/(160*a^6*(a + a*Sin[c + d*x])^5) +
7/(256*a^7*(a + a*Sin[c + d*x])^4) + 29/(384*a^8*(a + a*Sin[c + d*x])^3) +
93/(512*a^9*(a + a*Sin[c + d*x])^2) + 1/(2*a^10*(a + a*Sin[c + d*x]))))/d
```

Maple [A] time = 0.1, size = 212, normalized size = 0.9

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} - \frac{1}{48\,da\,(\sin(dx+c)-1)^3} + \frac{37}{512\,da\,(\sin(dx+c)-1)^2} - \frac{65}{256\,da\,(\sin(dx+c)-1)} - \frac{193\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4-1/48/d/a/(sin(d*x+c)-1)^3+37/512/d/a/(sin(d*x+c)
-1)^2-65/256/a/d/(sin(d*x+c)-1)-193/512/a/d*ln(sin(d*x+c)-1)+1/160/d/a/(1+s
in(d*x+c))^5+7/256/d/a/(1+sin(d*x+c))^4+29/384/d/a/(1+sin(d*x+c))^3+93/512/
a/d/(1+sin(d*x+c))^2+1/2/a/d/(1+sin(d*x+c))-319/512*ln(1+sin(d*x+c))/a/d+ln
(sin(d*x+c))/a/d
```

Maxima [A] time = 1.02366, size = 305, normalized size = 1.23

$$\frac{\frac{2\left(945\sin(dx+c)^8-975\sin(dx+c)^7-5385\sin(dx+c)^6+3255\sin(dx+c)^5+11319\sin(dx+c)^4-3721\sin(dx+c)^3-10831\sin(dx+c)^2+1489\sin(dx+c)+4384\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a} -}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/7680*(2*(945*sin(d*x + c)^8 - 975*sin(d*x + c)^7 - 5385*sin(d*x + c)^6 +
3255*sin(d*x + c)^5 + 11319*sin(d*x + c)^4 - 3721*sin(d*x + c)^3 - 10831*si
n(d*x + c)^2 + 1489*sin(d*x + c) + 4384)/(a*sin(d*x + c)^9 + a*sin(d*x + c)
^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin
(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a)
 - 4785*log(sin(d*x + c) + 1)/a - 2895*log(sin(d*x + c) - 1)/a + 7680*log(s
in(d*x + c))/a)/d
```

Fricas [A] time = 2.22443, size = 636, normalized size = 2.57

$$\frac{1890 \cos(dx+c)^8 + 3210 \cos(dx+c)^6 + 1668 \cos(dx+c)^4 + 1136 \cos(dx+c)^2 + 7680 \left(\cos(dx+c)^8 \sin(dx+c) + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(1890*cos(d*x + c)^8 + 3210*cos(d*x + c)^6 + 1668*cos(d*x + c)^4 + 1
136*cos(d*x + c)^2 + 7680*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*lo
g(1/2*sin(d*x + c)) - 4785*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*l
og(sin(d*x + c) + 1) - 2895*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*
log(-sin(d*x + c) + 1) + 2*(975*cos(d*x + c)^6 + 330*cos(d*x + c)^4 + 136*c
os(d*x + c)^2 + 48)*sin(d*x + c) + 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) +
a*d*cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33199, size = 228, normalized size = 0.92

$$-\frac{\frac{19140 \log(|\sin(dx+c)+1|)}{a} + \frac{11580 \log(|\sin(dx+c)-1|)}{a} - \frac{30720 \log(|\sin(dx+c)|)}{a} - \frac{5\left(4825 \sin(dx+c)^4 - 20860 \sin(dx+c)^3 + 34074 \sin(dx+c)^2 - 24996 \sin(\cdots\right.}{a(\sin(dx+c)-1)^4}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(19140*log(abs(sin(d*x + c) + 1))/a + 11580*log(abs(sin(d*x + c) -
 1))/a - 30720*log(abs(sin(d*x + c)))/a - 5*(4825*sin(d*x + c)^4 - 20860*si
n(d*x + c)^3 + 34074*sin(d*x + c)^2 - 24996*sin(d*x + c) + 6981)/(a*(sin(d*
x + c) - 1)^4) - (43703*sin(d*x + c)^5 + 233875*sin(d*x + c)^4 + 504050*sin
(d*x + c)^3 + 548250*sin(d*x + c)^2 + 302175*sin(d*x + c) + 67995)/(a*(sin(
d*x + c) + 1)^5))/d
```

3.909 $\int \frac{\csc^2(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=262

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{9a^3}{256d(a\sin(c+dx)+a)^4}+\frac{5a^2}{192d(a-a\sin(c+dx))^3}-\frac{}{384d(}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - (437*Log[1 - Sin[c + d*x]])/(512*a*d) - Log[Sin[c +
 d*x]]/(a*d) + (949*Log[1 + Sin[c + d*x]])/(512*a*d) + a^3/(256*d*(a - a*Si
n[c + d*x])^4) + (5*a^2)/(192*d*(a - a*Sin[c + d*x])^3) + (57*a)/(512*d*(a
- a*Sin[c + d*x])^2) + 61/(128*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*
Sin[c + d*x])^5) - (9*a^3)/(256*d*(a + a*Sin[c + d*x])^4) - (47*a^2)/(384*d
*(a + a*Sin[c + d*x])^3) - (187*a)/(512*d*(a + a*Sin[c + d*x])^2) - 315/(25
6*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.290825, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{9a^3}{256d(a\sin(c+dx)+a)^4}+\frac{5a^2}{192d(a-a\sin(c+dx))^3}-\frac{}{384d(}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```


```
[Out] -(Csc[c + d*x]/(a*d)) - (437*Log[1 - Sin[c + d*x]])/(512*a*d) - Log[Sin[c +
 d*x]]/(a*d) + (949*Log[1 + Sin[c + d*x]])/(512*a*d) + a^3/(256*d*(a - a*Si
n[c + d*x])^4) + (5*a^2)/(192*d*(a - a*Sin[c + d*x])^3) + (57*a)/(512*d*(a
- a*Sin[c + d*x])^2) + 61/(128*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*
Sin[c + d*x])^5) - (9*a^3)/(256*d*(a + a*Sin[c + d*x])^4) - (47*a^2)/(384*d
*(a + a*Sin[c + d*x])^3) - (187*a)/(512*d*(a + a*Sin[c + d*x])^2) - 315/(25
6*d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\operatorname{Subst}\left(\int \frac{a^2}{(a-x)^5x^2(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\operatorname{Subst}\left(\int \frac{1}{(a-x)^5x^2(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\operatorname{Subst}\left(\int \left(\frac{1}{64a^8(a-x)^5} + \frac{5}{64a^9(a-x)^4} + \frac{57}{256a^{10}(a-x)^3} + \frac{61}{128a^{11}(a-x)^2} + \frac{437}{512a^{12}(a-x)} + \frac{1}{a^{11}x^2} - \ldots\right.\right.}{}$$

$$= -\frac{\csc(c+dx)}{ad} - \frac{437\log(1-\sin(c+dx))}{512ad} - \frac{\log(\sin(c+dx))}{ad} + \frac{949\log(1+\sin(c+dx))}{512ad}\ldots$$

Mathematica [A] time = 6.20331, size = 240, normalized size = 0.92

$$\frac{a^{11}\left(\frac{61}{128a^{11}(a-a\sin(c+dx))} - \frac{315}{256a^{11}(a\sin(c+dx)+a)} + \frac{57}{512a^{10}(a-a\sin(c+dx))^2} - \frac{187}{512a^{10}(a\sin(c+dx)+a)^2} + \frac{5}{192a^9(a-a\sin(c+dx))^3} - \frac{47}{384a^9(a\sin(c+dx\ldots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^11*(-(Csc[c + d*x]/a^12) - (437*Log[1 - Sin[c + d*x]])/(512*a^12) - Log[
Sin[c + d*x]]/a^12 + (949*Log[1 + Sin[c + d*x]])/(512*a^12) + 1/(256*a^8*(a
 - a*Sin[c + d*x])^4) + 5/(192*a^9*(a - a*Sin[c + d*x])^3) + 57/(512*a^10*(
a - a*Sin[c + d*x])^2) + 61/(128*a^11*(a - a*Sin[c + d*x])) - 1/(160*a^7*(a
 + a*Sin[c + d*x])^5) - 9/(256*a^8*(a + a*Sin[c + d*x])^4) - 47/(384*a^9*(a
 + a*Sin[c + d*x])^3) - 187/(512*a^10*(a + a*Sin[c + d*x])^2) - 315/(256*a^
11*(a + a*Sin[c + d*x]))))/d
```

Maple [A] time = 0.104, size = 229, normalized size = 0.9

$$\frac{1}{256\,da\left(\sin(dx+c)-1\right)^4} - \frac{5}{192\,da\left(\sin(dx+c)-1\right)^3} + \frac{57}{512\,da\left(\sin(dx+c)-1\right)^2} - \frac{61}{128\,da\left(\sin(dx+c)-1\right)} - \frac{437\ln(\sin\ldots}{5\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4-5/192/d/a/(sin(d*x+c)-1)^3+57/512/d/a/(sin(d*x+c
)-1)^2-61/128/a/d/(sin(d*x+c)-1)-437/512/a/d*ln(sin(d*x+c)-1)-1/160/d/a/(1+
sin(d*x+c))^5-9/256/d/a/(1+sin(d*x+c))^4-47/384/d/a/(1+sin(d*x+c))^3-187/51
2/a/d/(1+sin(d*x+c))^2-315/256/a/d/(1+sin(d*x+c))+949/512*ln(1+sin(d*x+c))/
a/d-1/d/a/sin(d*x+c)-ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.02458, size = 331, normalized size = 1.26

$$-\frac{\frac{2\left(10395\sin(dx+c)^9+8475\sin(dx+c)^8-40035\sin(dx+c)^7-31395\sin(dx+c)^6+57309\sin(dx+c)^5+42269\sin(dx+c)^4-35941\sin(dx+c)^3-23621\sin(dx+c)^2+8\ldots\right.}{a\sin(dx+c)^{10}+a\sin(dx+c)^9-4a\sin(dx+c)^8-4a\sin(dx+c)^7+6a\sin(dx+c)^6+6a\sin(dx+c)^5-4a\sin(dx+c)^4-4a\sin(dx+c)^3+a\sin(dx+c)^2+a\sin\ldots}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(10395*sin(d*x + c)^9 + 8475*sin(d*x + c)^8 - 40035*sin(d*x + c)
^7 - 31395*sin(d*x + c)^6 + 57309*sin(d*x + c)^5 + 42269*sin(d*x + c)^4 - 3
5941*sin(d*x + c)^3 - 23621*sin(d*x + c)^2 + 8224*sin(d*x + c) + 3840)/(a*s
in(d*x + c)^10 + a*sin(d*x + c)^9 - 4*a*sin(d*x + c)^8 - 4*a*sin(d*x + c)^7
 + 6*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 - 4*a*sin(d*x + c)^4 - 4*a*sin(d
*x + c)^3 + a*sin(d*x + c)^2 + a*sin(d*x + c)) - 14235*log(sin(d*x + c) + 1
)/a + 6555*log(sin(d*x + c) - 1)/a + 7680*log(sin(d*x + c))/a)/d
```

Fricas [A] time = 2.19109, size = 774, normalized size = 2.95

$$\frac{16950 \cos(dx+c)^8 - 5010 \cos(dx+c)^6 - 2132 \cos(dx+c)^4 - 1264 \cos(dx+c)^2 - 7680 \left(\cos(dx+c)^{10} - \cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(16950*cos(d*x + c)^8 - 5010*cos(d*x + c)^6 - 2132*cos(d*x + c)^4 -
1264*cos(d*x + c)^2 - 7680*(cos(d*x + c)^10 - cos(d*x + c)^8*sin(d*x + c) -
 cos(d*x + c)^8)*log(1/2*sin(d*x + c)) + 14235*(cos(d*x + c)^10 - cos(d*x +
 c)^8*sin(d*x + c) - cos(d*x + c)^8)*log(sin(d*x + c) + 1) - 6555*(cos(d*x
+ c)^10 - cos(d*x + c)^8*sin(d*x + c) - cos(d*x + c)^8)*log(-sin(d*x + c) +
 1) + 2*(10395*cos(d*x + c)^8 - 1545*cos(d*x + c)^6 - 426*cos(d*x + c)^4 -
152*cos(d*x + c)^2 - 48)*sin(d*x + c) - 864)/(a*d*cos(d*x + c)^10 - a*d*cos
(d*x + c)^8*sin(d*x + c) - a*d*cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23339, size = 257, normalized size = 0.98

$$\frac{\frac{56940 \log(|\sin(dx+c)+1|)}{a} - \frac{26220 \log(|\sin(dx+c)-1|)}{a} - \frac{30720 \log(|\sin(dx+c)|)}{a} + \frac{30720 (\sin(dx+c)-1)}{a \sin(dx+c)} + \frac{5\left(10925 \sin(dx+c)^4 - 46628 \sin(dx+c)^3 + 75\right.}{a(\sin(dx}}{307}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(56940*log(abs(sin(d*x + c) + 1))/a - 26220*log(abs(sin(d*x + c) -
1))/a - 30720*log(abs(sin(d*x + c)))/a + 30720*(sin(d*x + c) - 1)/(a*sin(d*
x + c)) + 5*(10925*sin(d*x + c)^4 - 46628*sin(d*x + c)^3 + 75018*sin(d*x +
```

```
c)^2 - 54012*sin(d*x + c) + 14721)/(a*(sin(d*x + c) - 1)^4) - (130013*sin(d
*x + c)^5 + 687865*sin(d*x + c)^4 + 1462550*sin(d*x + c)^3 + 1564350*sin(d*
x + c)^2 + 843525*sin(d*x + c) + 184065)/(a*(sin(d*x + c) + 1)^5))/d
```

3.910 $\int \frac{\csc^3(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=279

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{11a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{32d(a-a\sin(c+dx))^3}+\frac{}{128d(a\,\mathrm{s}}$$

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (843*Log[1 - Sin[c + d*x]])/(
512*a*d) + (6*Log[Sin[c + d*x]])/(a*d) - (2229*Log[1 + Sin[c + d*x]])/(512*
a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) + a^2/(32*d*(a - a*Sin[c + d*x])^
3) + (81*a)/(512*d*(a - a*Sin[c + d*x])^2) + 203/(256*d*(a - a*Sin[c + d*x]
)) + a^4/(160*d*(a + a*Sin[c + d*x])^5) + (11*a^3)/(256*d*(a + a*Sin[c + d*
x])^4) + (23*a^2)/(128*d*(a + a*Sin[c + d*x])^3) + (325*a)/(512*d*(a + a*Si
n[c + d*x])^2) + 5/(2*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.302406, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 12, 88}

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{11a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{32d(a-a\sin(c+dx))^3}+\frac{}{128d(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (843*Log[1 - Sin[c + d*x]])/(
512*a*d) + (6*Log[Sin[c + d*x]])/(a*d) - (2229*Log[1 + Sin[c + d*x]])/(512*
a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) + a^2/(32*d*(a - a*Sin[c + d*x])^
3) + (81*a)/(512*d*(a - a*Sin[c + d*x])^2) + 203/(256*d*(a - a*Sin[c + d*x]
)) + a^4/(160*d*(a + a*Sin[c + d*x])^5) + (11*a^3)/(256*d*(a + a*Sin[c + d*
x])^4) + (23*a^2)/(128*d*(a + a*Sin[c + d*x])^3) + (325*a)/(512*d*(a + a*Si
n[c + d*x])^2) + 5/(2*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\,\text{Subst}\left(\int \frac{a^3}{(a-x)^5x^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^{12}\,\text{Subst}\left(\int \frac{1}{(a-x)^5x^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^{12}\,\text{Subst}\left(\int \left(\frac{1}{64a^9(a-x)^5} + \frac{3}{32a^{10}(a-x)^4} + \frac{81}{256a^{11}(a-x)^3} + \frac{203}{256a^{12}(a-x)^2} + \frac{843}{512a^{13}(a-x)} + \frac{1}{a^{11}x^3} - \right.\right.}{}$$
$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad} - \frac{843\log(1-\sin(c+dx))}{512ad} + \frac{6\log(\sin(c+dx))}{ad} - \frac{2229\log}{}$$

Mathematica [A] time = 6.22719, size = 254, normalized size = 0.91

$$a^{12}\left(\frac{203}{256a^{12}(a-a\sin(c+dx))} + \frac{5}{2a^{12}(a\sin(c+dx)+a)} + \frac{81}{512a^{11}(a-a\sin(c+dx))^2} + \frac{325}{512a^{11}(a\sin(c+dx)+a)^2} + \frac{1}{32a^{10}(a-a\sin(c+dx))^3} + \frac{23}{128a^{10}(a\sin(c+dx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^12*(Csc[c + d*x]/a^13 - Csc[c + d*x]^2/(2*a^13) - (843*Log[1 - Sin[c + d
*x]])/(512*a^13) + (6*Log[Sin[c + d*x]])/a^13 - (2229*Log[1 + Sin[c + d*x]]
)/(512*a^13) + 1/(256*a^9*(a - a*Sin[c + d*x])^4) + 1/(32*a^10*(a - a*Sin[c
 + d*x])^3) + 81/(512*a^11*(a - a*Sin[c + d*x])^2) + 203/(256*a^12*(a - a*S
in[c + d*x])) + 1/(160*a^8*(a + a*Sin[c + d*x])^5) + 11/(256*a^9*(a + a*Sin
[c + d*x])^4) + 23/(128*a^10*(a + a*Sin[c + d*x])^3) + 325/(512*a^11*(a + a
*Sin[c + d*x])^2) + 5/(2*a^12*(a + a*Sin[c + d*x]))))/d
```

Maple [A] time = 0.112, size = 244, normalized size = 0.9

$$\frac{1}{256\,da\,(\sin(dx+c)-1)^4} - \frac{1}{32\,da\,(\sin(dx+c)-1)^3} + \frac{81}{512\,da\,(\sin(dx+c)-1)^2} - \frac{203}{256\,da\,(\sin(dx+c)-1)} - \frac{843\ln(\sin}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/256/d/a/(sin(d*x+c)-1)^4-1/32/d/a/(sin(d*x+c)-1)^3+81/512/d/a/(sin(d*x+c)
-1)^2-203/256/a/d/(sin(d*x+c)-1)-843/512/a/d*ln(sin(d*x+c)-1)+1/160/d/a/(1+
sin(d*x+c))^5+11/256/d/a/(1+sin(d*x+c))^4+23/128/d/a/(1+sin(d*x+c))^3+325/5
12/a/d/(1+sin(d*x+c))^2+5/2/a/d/(1+sin(d*x+c))-2229/512*ln(1+sin(d*x+c))/a/
d-1/2/d/a/sin(d*x+c)^2+1/d/a/sin(d*x+c)+6*ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.03702, size = 347, normalized size = 1.24

$$\frac{\frac{2\left(3465\sin(dx+c)^{10} - 375\sin(dx+c)^9 - 16545\sin(dx+c)^8 + 735\sin(dx+c)^7 + 30303\sin(dx+c)^6 + 223\sin(dx+c)^5 - 25847\sin(dx+c)^4 - 1207\sin(dx+c)^3 + 9408\sin(d\right.}{a\sin(dx+c)^{11} + a\sin(dx+c)^{10} - 4a\sin(dx+c)^9 - 4a\sin(dx+c)^8 + 6a\sin(dx+c)^7 + 6a\sin(dx+c)^6 - 4a\sin(dx+c)^5 - 4a\sin(dx+c)^4 + a\sin(dx+c)^3 + a\sin}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(3465*sin(d*x + c)^10 - 375*sin(d*x + c)^9 - 16545*sin(d*x + c)^8
 + 735*sin(d*x + c)^7 + 30303*sin(d*x + c)^6 + 223*sin(d*x + c)^5 - 25847*s
in(d*x + c)^4 - 1207*sin(d*x + c)^3 + 9408*sin(d*x + c)^2 + 640*sin(d*x + c
) - 640)/(a*sin(d*x + c)^11 + a*sin(d*x + c)^10 - 4*a*sin(d*x + c)^9 - 4*a*
sin(d*x + c)^8 + 6*a*sin(d*x + c)^7 + 6*a*sin(d*x + c)^6 - 4*a*sin(d*x + c)
^5 - 4*a*sin(d*x + c)^4 + a*sin(d*x + c)^3 + a*sin(d*x + c)^2) - 11145*log(
sin(d*x + c) + 1)/a - 4215*log(sin(d*x + c) - 1)/a + 15360*log(sin(d*x + c)
)/a)/d
```

Fricas [A] time = 2.33834, size = 910, normalized size = 3.26

$$\frac{6930 \cos(dx+c)^{10} - 1560 \cos(dx+c)^{8} - 2454 \cos(dx+c)^{6} - 884 \cos(dx+c)^{4} - 464 \cos(dx+c)^{2} + 15360 \left(\cos(\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(6930*cos(d*x + c)^10 - 1560*cos(d*x + c)^8 - 2454*cos(d*x + c)^6 -
884*cos(d*x + c)^4 - 464*cos(d*x + c)^2 + 15360*(cos(d*x + c)^10 - cos(d*x
+ c)^8 + (cos(d*x + c)^10 - cos(d*x + c)^8)*sin(d*x + c))*log(1/2*sin(d*x +
 c)) - 11145*(cos(d*x + c)^10 - cos(d*x + c)^8 + (cos(d*x + c)^10 - cos(d*x
 + c)^8)*sin(d*x + c))*log(sin(d*x + c) + 1) - 4215*(cos(d*x + c)^10 - cos(
d*x + c)^8 + (cos(d*x + c)^10 - cos(d*x + c)^8)*sin(d*x + c))*log(-sin(d*x
+ c) + 1) + 2*(375*cos(d*x + c)^8 - 765*cos(d*x + c)^6 - 178*cos(d*x + c)^4
 - 56*cos(d*x + c)^2 - 16)*sin(d*x + c) - 288)/(a*d*cos(d*x + c)^10 - a*d*c
os(d*x + c)^8 + (a*d*cos(d*x + c)^10 - a*d*cos(d*x + c)^8)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28225, size = 273, normalized size = 0.98

$$-\frac{\frac{44580 \log(|\sin(dx+c)+1|)}{a} + \frac{16860 \log(|\sin(dx+c)-1|)}{a} - \frac{61440 \log(|\sin(dx+c)|)}{a} + \frac{5120\left(18 \sin(dx+c)^{2} - 2 \sin(dx+c) + 1\right)}{a \sin(dx+c)^{2}} - \frac{5\left(7025 \sin(dx+c)^{4} - 2972\ldots\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(44580*log(abs(sin(d*x + c) + 1))/a + 16860*log(abs(sin(d*x + c) -
 1))/a - 61440*log(abs(sin(d*x + c)))/a + 5120*(18*sin(d*x + c)^2 - 2*sin(d
*x + c) + 1)/(a*sin(d*x + c)^2) - 5*(7025*sin(d*x + c)^4 - 29724*sin(d*x +
c)^3 + 47346*sin(d*x + c)^2 - 33684*sin(d*x + c) + 9045)/(a*(sin(d*x + c) -
 1)^4) - (101791*sin(d*x + c)^5 + 534555*sin(d*x + c)^4 + 1126810*sin(d*x +
 c)^3 + 1192850*sin(d*x + c)^2 + 634975*sin(d*x + c) + 136235)/(a*(sin(d*x
+ c) + 1)^5))/d
```

3.911 $\int (g\sec(e+fx))^p(d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=127

$$\frac{\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}(g\sec(e+fx))^p(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F_1\Big(n}{df(n+1)}$$

```
[Out] (AppellF1[1 + n, (1 + p)/2, (1 - 2*m + p)/2, 2 + n, Sin[e + f*x], -Sin[e +
f*x]]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(1 - Sin[e + f*x])^((1 + p)/2)*(d*Sin
[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(a + a*Sin[e + f*x]
)^m)/(d*f*(1 + n))
```

Rubi [A] time = 0.349434, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2926, 2886, 135, 133}

$$\frac{\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}(g\sec(e+fx))^p(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F_1\Big(n}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```


```
[Out] (AppellF1[1 + n, (1 + p)/2, (1 - 2*m + p)/2, 2 + n, Sin[e + f*x], -Sin[e +
f*x]]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(1 - Sin[e + f*x])^((1 + p)/2)*(d*Sin
[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(a + a*Sin[e + f*x]
)^m)/(d*f*(1 + n))
```


Rule 2926

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[((a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(g*Cos[e + f*x])^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && !IntegerQ[p]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (g\sec(e+fx))^p(d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx = \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int (g\cos(e+fx))^{-p}(d\sin(e$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+fx))^{\frac{1+p}{2}}(a+a\sin(\right.}{}$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(1-\sin(e+fx))^{\frac{1}{2}+\frac{p}{2}}(a-a\sin(e\right.}{}$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(1-\sin(e+fx))^{\frac{1}{2}+\frac{p}{2}}(1+\sin(e+\right.}{}$$

$$= \frac{F_1\left(1+n;\frac{1+p}{2},\frac{1}{2}(1-2m+p);2+n;\sin(e+fx),-\sin(e+fx\right.}{}$$

Mathematica [B] time = 3.26272, size = 347, normalized size = 2.73

$$\frac{g(p-3)(a(\sin(e+fx)+1))^m(d\sin}{f(p-1)\left(2\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(nF_1\left(\frac{3-p}{2};1-n,m+n-p+1;\frac{5-p}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+2fx-\pi\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (g*(-3 + p)*AppellF1[(1 - p)/2, -n, 1 + m + n - p, (3 - p)/2, Cot[(2*e + Pi
 + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(g*Sec[e + f*x])^(-1 + p)*(d*
Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^m)/(f*(-1 + p)*((-3 + p)*AppellF1[(1
 - p)/2, -n, 1 + m + n - p, (3 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2
*e - Pi + 2*f*x)/4]^2] + 2*(n*AppellF1[(3 - p)/2, 1 - n, 1 + m + n - p, (5
- p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m
 + n - p)*AppellF1[(3 - p)/2, -n, 2 + m + n - p, (5 - p)/2, Cot[(2*e + Pi +
 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F] time = 7.592, size = 0, normalized size = 0.

$$\int \left(g\sec\left(fx+e\right)\right)^p\left(d\sin\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

3.912 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=88

$$\frac{(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)}$$

```
[Out] (Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a
+ a*Sin[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(1 + m)*((c + d*Sin[
e + f*x])/(c - d))^n)
```

Rubi [A] time = 0.13977, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2833, 70, 69}

$$\frac{(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a
+ a*Sin[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(1 + m)*((c + d*Sin[
e + f*x])/(c - d))^n)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\operatorname{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^n dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\left((c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\right)\operatorname{Subst}\left(\int (a+x)^m \left(\right.\right.}{af}$$

$$= \frac{{}_2F_1\left(1+m,-n;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))}{af(1+m)}$$

Mathematica [A] time = 0.140462, size = 88, normalized size = 1.

$$\frac{(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a
+ a*Sin[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(1 + m)*((c + d*Sin[
e + f*x])/(c - d))^n)
```

Maple [F] time = 0.33, size = 0, normalized size = 0.

$$\int \cos(fx+e)\left(a+a\sin(fx+e)\right)^m\left(c+d\sin(fx+e)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(fx+e)+a\right)^m\left(d\sin(fx+e)+c\right)^n\cos(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a\sin(fx+e)+a\right)^m\left(d\sin(fx+e)+c\right)^n\cos(fx+e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e), x)
```

3.913 $\int \cos(e+fx)(a+a\sin(e+fx))^4(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=175

$$\frac{a^4(c-d)^4(c+d\sin(e+fx))^{n+1}}{d^5f(n+1)}-\frac{4a^4(c-d)^3(c+d\sin(e+fx))^{n+2}}{d^5f(n+2)}+\frac{6a^4(c-d)^2(c+d\sin(e+fx))^{n+3}}{d^5f(n+3)}-\frac{4a^4(c-d)(c+d\sin(e+fx))^{n+4}}{d^5f(n+4)}$$

```
[Out] (a^4*(c - d)^4*(c + d*Sin[e + f*x])^(1 + n))/(d^5*f*(1 + n)) - (4*a^4*(c -
d)^3*(c + d*Sin[e + f*x])^(2 + n))/(d^5*f*(2 + n)) + (6*a^4*(c - d)^2*(c +
d*Sin[e + f*x])^(3 + n))/(d^5*f*(3 + n)) - (4*a^4*(c - d)*(c + d*Sin[e + f*
x])^(4 + n))/(d^5*f*(4 + n)) + (a^4*(c + d*Sin[e + f*x])^(5 + n))/(d^5*f*(5
 + n))
```

Rubi [A] time = 0.205881, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{a^4(c-d)^4(c+d\sin(e+fx))^{n+1}}{d^5f(n+1)}-\frac{4a^4(c-d)^3(c+d\sin(e+fx))^{n+2}}{d^5f(n+2)}+\frac{6a^4(c-d)^2(c+d\sin(e+fx))^{n+3}}{d^5f(n+3)}-\frac{4a^4(c-d)(c+d\sin(e+fx))^{n+4}}{d^5f(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^4*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^4*(c - d)^4*(c + d*Sin[e + f*x])^(1 + n))/(d^5*f*(1 + n)) - (4*a^4*(c -
d)^3*(c + d*Sin[e + f*x])^(2 + n))/(d^5*f*(2 + n)) + (6*a^4*(c - d)^2*(c +
d*Sin[e + f*x])^(3 + n))/(d^5*f*(3 + n)) - (4*a^4*(c - d)*(c + d*Sin[e + f*
x])^(4 + n))/(d^5*f*(4 + n)) + (a^4*(c + d*Sin[e + f*x])^(5 + n))/(d^5*f*(5
 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^4(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)^4\left(c+\frac{dx}{a}\right)^n dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^4(c-d)^4\left(c+\frac{dx}{a}\right)^n}{d^4} - \frac{4a^4(c-d)^3\left(c+\frac{dx}{a}\right)^{1+n}}{d^4} + \frac{6a^4(c-d)^2\left(c+\frac{dx}{a}\right)^{2+}}{d^4}\right.\right.}{af}$$

$$= \frac{a^4(c-d)^4(c+d\sin(e+fx))^{1+n}}{d^5f(1+n)} - \frac{4a^4(c-d)^3(c+d\sin(e+f}{d^5f(2+n)}$$

Mathematica [A] time = 0.595756, size = 130, normalized size = 0.74

$$\frac{a^4(c+d\sin(e+fx))^{n+1}\left(-\frac{4(c-d)^3(c+d\sin(e+fx))}{n+2}+\frac{6(c-d)^2(c+d\sin(e+fx))^2}{n+3}-\frac{4(c-d)(c+d\sin(e+fx))^3}{n+4}+\frac{(c+d\sin(e+fx))^4}{n+5}+\frac{(c-d)^4}{n+1}\right)}{d^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^4*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^4*(c + d*Sin[e + f*x])^(1 + n)*((c - d)^4/(1 + n) - (4*(c - d)^3*(c + d*
Sin[e + f*x]))/(2 + n) + (6*(c - d)^2*(c + d*Sin[e + f*x])^2)/(3 + n) - (4*
(c - d)*(c + d*Sin[e + f*x])^3)/(4 + n) + (c + d*Sin[e + f*x])^4/(5 + n)))/
(d^5*f)
```

Maple [F] time = 0.565, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^4\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.7658, size = 1897, normalized size = 10.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] (24*a^4*c^5 - 120*a^4*c^4*d + 240*a^4*c^3*d^2 - 240*a^4*c^2*d^3 + 120*a^4*c
*d^4 + 360*a^4*d^5 + 8*(a^4*c*d^4 + a^4*d^5)*n^4 + (120*a^4*d^5 + (a^4*c*d^
4 + 4*a^4*d^5)*n^4 + 2*(3*a^4*c*d^4 + 22*a^4*d^5)*n^3 + (11*a^4*c*d^4 + 164
*a^4*d^5)*n^2 + 2*(3*a^4*c*d^4 + 122*a^4*d^5)*n)*cos(f*x + e)^4 - 16*(a^4*c
^2*d^3 - 5*a^4*c*d^4 - 6*a^4*d^5)*n^3 + 8*(3*a^4*c^3*d^2 - 15*a^4*c^2*d^3 +
 32*a^4*c*d^4 + 50*a^4*d^5)*n^2 - 4*(120*a^4*d^5 + (2*a^4*c*d^4 + 3*a^4*d^5
)*n^4 - (3*a^4*c^2*d^3 - 18*a^4*c*d^4 - 35*a^4*d^5)*n^3 + (3*a^4*c^3*d^2 -
18*a^4*c^2*d^3 + 49*a^4*c*d^4 + 141*a^4*d^5)*n^2 + (3*a^4*c^3*d^2 - 15*a^4*
c^2*d^3 + 33*a^4*c*d^4 + 229*a^4*d^5)*n)*cos(f*x + e)^2 - 8*(3*a^4*c^4*d -
15*a^4*c^3*d^2 + 31*a^4*c^2*d^3 - 35*a^4*c*d^4 - 84*a^4*d^5)*n + (384*a^4*d
^5 + 8*(a^4*c*d^4 + a^4*d^5)*n^4 + (a^4*d^5*n^4 + 10*a^4*d^5*n^3 + 35*a^4*d
^5*n^2 + 50*a^4*d^5*n + 24*a^4*d^5)*cos(f*x + e)^4 - 16*(a^4*c^2*d^3 - 5*a^
4*c*d^4 - 6*a^4*d^5)*n^3 + 8*(3*a^4*c^3*d^2 - 15*a^4*c^2*d^3 + 32*a^4*c*d^4
 + 50*a^4*d^5)*n^2 - 4*(72*a^4*d^5 + (a^4*c*d^4 + 2*a^4*d^5)*n^4 - (a^4*c^2
*d^3 - 8*a^4*c*d^4 - 23*a^4*d^5)*n^3 - (3*a^4*c^2*d^3 - 17*a^4*c*d^4 - 91*a
^4*d^5)*n^2 - 2*(a^4*c^2*d^3 - 5*a^4*c*d^4 - 71*a^4*d^5)*n)*cos(f*x + e)^2
- 8*(3*a^4*c^4*d - 15*a^4*c^3*d^2 + 31*a^4*c^2*d^3 - 35*a^4*c*d^4 - 84*a^4*
d^5)*n)*sin(f*x + e))*(d*sin(f*x + e) + c)^n/(d^5*f*n^5 + 15*d^5*f*n^4 + 85
*d^5*f*n^3 + 225*d^5*f*n^2 + 274*d^5*f*n + 120*d^5*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**4*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21482, size = 2484, normalized size = 14.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] (((d*sin(f*x + e) + c)^5*(d*sin(f*x + e) + c)^n*n^4 - 4*(d*sin(f*x + e) + c
)^4*(d*sin(f*x + e) + c)^n*c*n^4 + 6*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e)
 + c)^n*c^2*n^4 - 4*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^3*n^4 +
 (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^4*n^4 + 10*(d*sin(f*x + e) +
 c)^5*(d*sin(f*x + e) + c)^n*n^3 - 44*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e
) + c)^n*c*n^3 + 72*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*c^2*n^3 -
 52*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^3*n^3 + 14*(d*sin(f*x +
 e) + c)*(d*sin(f*x + e) + c)^n*c^4*n^3 + 35*(d*sin(f*x + e) + c)^5*(d*sin(
f*x + e) + c)^n*n^2 - 164*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*c*n
^2 + 294*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*c^2*n^2 - 236*(d*sin
```

(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^3*n^2 + 71*(d*sin(f*x + e) + c)*(
d*sin(f*x + e) + c)^n*c^4*n^2 + 50*(d*sin(f*x + e) + c)^5*(d*sin(f*x + e) +
 c)^n*n - 244*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*c*n + 468*(d*si
n(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*c^2*n - 428*(d*sin(f*x + e) + c)^2
*(d*sin(f*x + e) + c)^n*c^3*n + 154*(d*sin(f*x + e) + c)*(d*sin(f*x + e) +
c)^n*c^4*n + 24*(d*sin(f*x + e) + c)^5*(d*sin(f*x + e) + c)^n - 120*(d*sin(
f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*c + 240*(d*sin(f*x + e) + c)^3*(d*si
n(f*x + e) + c)^n*c^2 - 240*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c
^3 + 120*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^4)*a^4/(d^4*n^5 + 15
*d^4*n^4 + 85*d^4*n^3 + 225*d^4*n^2 + 274*d^4*n + 120*d^4) + 4*((d*sin(f*x
+ e) + c)^4*(d*sin(f*x + e) + c)^n*n^3 - 3*(d*sin(f*x + e) + c)^3*(d*sin(f*
x + e) + c)^n*c*n^3 + 3*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2*n
^3 - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^3 + 6*(d*sin(f*x + e
) + c)^4*(d*sin(f*x + e) + c)^n*n^2 - 21*(d*sin(f*x + e) + c)^3*(d*sin(f*x
+ e) + c)^n*c*n^2 + 24*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2*n^
2 - 9*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^2 + 11*(d*sin(f*x +
 e) + c)^4*(d*sin(f*x + e) + c)^n*n - 42*(d*sin(f*x + e) + c)^3*(d*sin(f*x
+ e) + c)^n*c*n + 57*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2*n -
26*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n + 6*(d*sin(f*x + e) +
c)^4*(d*sin(f*x + e) + c)^n - 24*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c
)^n*c + 36*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2 - 24*(d*sin(f*
x + e) + c)*(d*sin(f*x + e) + c)^n*c^3)*a^4/(d^3*n^4 + 10*d^3*n^3 + 35*d^3*
n^2 + 50*d^3*n + 24*d^3) + 6*((d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n
n^2 - 2(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n^2 + (d*sin(f*x +
 e) + c)*(d*sin(f*x + e) + c)^n*c^2*n^2 + 3*(d*sin(f*x + e) + c)^3*(d*sin(f
*x + e) + c)^n*n - 8*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n + 5*
(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2*n + 2*(d*sin(f*x + e) + c)^
3*(d*sin(f*x + e) + c)^n - 6*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*
c + 6*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2)*a^4/(d^2*n^3 + 6*d^2
*n^2 + 11*d^2*n + 6*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^4/(n + 1) + 4*((d
*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*n - (d*sin(f*x + e) + c)*(d*sin
(f*x + e) + c)^n*c*n + (d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n - 2*(d
*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c)*a^4/((n^2 + 3*n + 2)*d))/(d*f)

3.914 $\int \cos(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=139

$$-\frac{a^3(c-d)^3(c+d\sin(e+fx))^{n+1}}{d^4f(n+1)}+\frac{3a^3(c-d)^2(c+d\sin(e+fx))^{n+2}}{d^4f(n+2)}-\frac{3a^3(c-d)(c+d\sin(e+fx))^{n+3}}{d^4f(n+3)}+\frac{a^3(c+d\sin(e+fx))^{n+4}}{d^4f(n+4)}$$

```
[Out] -((a^3*(c - d)^3*(c + d*Sin[e + f*x])^(1 + n))/(d^4*f*(1 + n))) + (3*a^3*(c
 - d)^2*(c + d*Sin[e + f*x])^(2 + n))/(d^4*f*(2 + n)) - (3*a^3*(c - d)*(c +
 d*Sin[e + f*x])^(3 + n))/(d^4*f*(3 + n)) + (a^3*(c + d*Sin[e + f*x])^(4 +
n))/(d^4*f*(4 + n))
```

Rubi [A] time = 0.165452, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$-\frac{a^3(c-d)^3(c+d\sin(e+fx))^{n+1}}{d^4f(n+1)}+\frac{3a^3(c-d)^2(c+d\sin(e+fx))^{n+2}}{d^4f(n+2)}-\frac{3a^3(c-d)(c+d\sin(e+fx))^{n+3}}{d^4f(n+3)}+\frac{a^3(c+d\sin(e+fx))^{n+4}}{d^4f(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((a^3*(c - d)^3*(c + d*Sin[e + f*x])^(1 + n))/(d^4*f*(1 + n))) + (3*a^3*(c
 - d)^2*(c + d*Sin[e + f*x])^(2 + n))/(d^4*f*(2 + n)) - (3*a^3*(c - d)*(c +
 d*Sin[e + f*x])^(3 + n))/(d^4*f*(3 + n)) + (a^3*(c + d*Sin[e + f*x])^(4 +
n))/(d^4*f*(4 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\left(c+\frac{dx}{a}\right)^n dx, x, a\sin(e+fx)\right)}{af}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a^3(c-d)^3\left(c+\frac{dx}{a}\right)^n}{d^3}+\frac{3a^3(c-d)^2\left(c+\frac{dx}{a}\right)^{1+n}}{d^3}-\frac{3a^3(c-d)\left(c+\frac{dx}{a}\right)^{2+n}}{d^3}+\dots\right)\right)}{af}\\ &= -\frac{a^3(c-d)^3(c+d\sin(e+fx))^{1+n}}{d^4f(1+n)}+\frac{3a^3(c-d)^2(c+d\sin(e+fx))^{2+n}}{d^4f(2+n)}\dots\end{aligned}$$

Mathematica [A] time = 0.319519, size = 105, normalized size = 0.76

$$\frac{a^3(c+d\sin(e+fx))^{n+1}\left(\frac{3(c-d)^2(c+d\sin(e+fx))}{n+2}-\frac{3(c-d)(c+d\sin(e+fx))^2}{n+3}+\frac{(c+d\sin(e+fx))^3}{n+4}-\frac{(c-d)^3}{n+1}\right)}{d^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^3*(c + d*Sin[e + f*x])^(1 + n)*(-((c - d)^3/(1 + n)) + (3*(c - d)^2*(c +
 d*Sin[e + f*x]))/(2 + n) - (3*(c - d)*(c + d*Sin[e + f*x])^2)/(3 + n) + (c
 + d*Sin[e + f*x])^3/(4 + n)))/(d^4*f)
```

Maple [F] time = 0.458, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^3\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.36936, size = 1125, normalized size = 8.09

$$-\frac{\left(6\,a^3c^4-24\,a^3c^3d+36\,a^3c^2d^2-24\,a^3cd^3-42\,a^3d^4-\left(a^3d^4n^3+6\,a^3d^4n^2+11\,a^3d^4n+6\,a^3d^4\right)\cos\left(fx+e\right)^4-4\left(a^3cd^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] -(6*a^3*c^4 - 24*a^3*c^3*d + 36*a^3*c^2*d^2 - 24*a^3*c*d^3 - 42*a^3*d^4 - (
a^3*d^4*n^3 + 6*a^3*d^4*n^2 + 11*a^3*d^4*n + 6*a^3*d^4)*cos(f*x + e)^4 - 4*
(a^3*c*d^3 + a^3*d^4)*n^3 + 6*(a^3*c^2*d^2 - 4*a^3*c*d^3 - 5*a^3*d^4)*n^2 +
 (48*a^3*d^4 + (3*a^3*c*d^3 + 5*a^3*d^4)*n^3 - 3*(a^3*c^2*d^2 - 5*a^3*c*d^3
 - 12*a^3*d^4)*n^2 - (3*a^3*c^2*d^2 - 12*a^3*c*d^3 - 79*a^3*d^4)*n)*cos(f*x
 + e)^2 - 2*(3*a^3*c^3*d - 12*a^3*c^2*d^2 + 19*a^3*c*d^3 + 34*a^3*d^4)*n -
```

```
(48*a^3*d^4 + 4*(a^3*c*d^3 + a^3*d^4)*n^3 - 6*(a^3*c^2*d^2 - 4*a^3*c*d^3 -
5*a^3*d^4)*n^2 - (24*a^3*d^4 + (a^3*c*d^3 + 3*a^3*d^4)*n^3 + 3*(a^3*c*d^3 +
 7*a^3*d^4)*n^2 + 2*(a^3*c*d^3 + 21*a^3*d^4)*n)*cos(f*x + e)^2 + 2*(3*a^3*c
^3*d - 12*a^3*c^2*d^2 + 19*a^3*c*d^3 + 34*a^3*d^4)*n)*sin(f*x + e))*(d*sin(
f*x + e) + c)^n/(d^4*f*n^4 + 10*d^4*f*n^3 + 35*d^4*f*n^2 + 50*d^4*f*n + 24*
d^4*f)
```

Sympy [A] time = 173.645, size = 5722, normalized size = 41.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Piecewise((c**n*(a**3*sin(e + f*x)**3/f - a**3*sin(e + f*x)**2*cos(e + f*x)
**2/(2*f) + a**3*sin(e + f*x)/f - a**3*cos(e + f*x)**4/(4*f) - 3*a**3*cos(e
 + f*x)**2/(2*f)), Eq(d, 0)), (x*(c + d*sin(e))**n*(a*sin(e) + a)**3*cos(e)
, Eq(f, 0)), (6*a**3*c**4*log(c/d + sin(e + f*x))/(6*c**4*d**4*f + 18*c**3*
d**5*f*sin(e + f*x) + 18*c**2*d**6*f*sin(e + f*x)**2 + 6*c*d**7*f*sin(e + f
*x)**3) + 5*a**3*c**4/(6*c**4*d**4*f + 18*c**3*d**5*f*sin(e + f*x) + 18*c**
2*d**6*f*sin(e + f*x)**2 + 6*c*d**7*f*sin(e + f*x)**3) + 18*a**3*c**3*d*log
(c/d + sin(e + f*x))*sin(e + f*x)/(6*c**4*d**4*f + 18*c**3*d**5*f*sin(e + f
*x) + 18*c**2*d**6*f*sin(e + f*x)**2 + 6*c*d**7*f*sin(e + f*x)**3) + 9*a**3
*c**3*d*sin(e + f*x)/(6*c**4*d**4*f + 18*c**3*d**5*f*sin(e + f*x) + 18*c**2
*d**6*f*sin(e + f*x)**2 + 6*c*d**7*f*sin(e + f*x)**3) + 18*a**3*c**2*d**2*l
og(c/d + sin(e + f*x))*sin(e + f*x)**2/(6*c**4*d**4*f + 18*c**3*d**5*f*sin(
e + f*x) + 18*c**2*d**6*f*sin(e + f*x)**2 + 6*c*d**7*f*sin(e + f*x)**3) - 3
*a**3*c**2*d**2/(6*c**4*d**4*f + 18*c**3*d**5*f*sin(e + f*x) + 18*c**2*d**6
*f*sin(e + f*x)**2 + 6*c*d**7*f*sin(e + f*x)**3) + 6*a**3*c*d**3*log(c/d +
sin(e + f*x))*sin(e + f*x)**3/(6*c**4*d**4*f + 18*c**3*d**5*f*sin(e + f*x)
+ 18*c**2*d**6*f*sin(e + f*x)**2 + 6*c*d**7*f*sin(e + f*x)**3) - 6*a**3*c*d
**3*sin(e + f*x)**3/(6*c**4*d**4*f + 18*c**3*d**5*f*sin(e + f*x) + 18*c**2*
d**6*f*sin(e + f*x)**2 + 6*c*d**7*f*sin(e + f*x)**3) - 9*a**3*c*d**3*sin(e
+ f*x)/(6*c**4*d**4*f + 18*c**3*d**5*f*sin(e + f*x) + 18*c**2*d**6*f*sin(e
+ f*x)**2 + 6*c*d**7*f*sin(e + f*x)**3) - 2*a**3*c*d**3/(6*c**4*d**4*f + 18
*c**3*d**5*f*sin(e + f*x) + 18*c**2*d**6*f*sin(e + f*x)**2 + 6*c*d**7*f*sin
(e + f*x)**3) + 6*a**3*d**4*sin(e + f*x)**3/(6*c**4*d**4*f + 18*c**3*d**5*f
*sin(e + f*x) + 18*c**2*d**6*f*sin(e + f*x)**2 + 6*c*d**7*f*sin(e + f*x)**3
), Eq(n, -4)), (-6*a**3*c**3*log(c/d + sin(e + f*x))/(2*c**2*d**4*f + 4*c*d
**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 9*a**3*c**3/(2*c**2*d**4*f
 + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 12*a**3*c**2*d*log
(c/d + sin(e + f*x))*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x)
+ 2*d**6*f*sin(e + f*x)**2) + 6*a**3*c**2*d*log(c/d + sin(e + f*x))/(2*c**2
*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 12*a**3*c**
2*d*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e
+ f*x)**2) + 9*a**3*c**2*d/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**
6*f*sin(e + f*x)**2) - 6*a**3*c*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)**
2/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 12
*a**3*c*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5
*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 12*a**3*c*d**2*sin(e + f*x)/(
2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 3*a**
3*c*d**2/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**
2) + 6*a**3*d**3*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(2*c**2*d**4*f + 4
*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 2*a**3*d**3*sin(e + f*
x)**3/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2)
- 6*a**3*d**3*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**
```

```
6*f*sin(e + f*x)**2) - a**3*d**3/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) +
 2*d**6*f*sin(e + f*x)**2), Eq(n, -3)), (6*a**3*c**4*log(c/d + sin(e + f*x)
)/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x)) + 6*a**3*c**4/(2*c**2*d**4*f +
2*c*d**5*f*sin(e + f*x)) + 6*a**3*c**3*d*log(c/d + sin(e + f*x))*sin(e + f*
x)/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x)) - 12*a**3*c**3*d*log(c/d + sin
(e + f*x))/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x)) - 12*a**3*c**3*d/(2*c*
*2*d**4*f + 2*c*d**5*f*sin(e + f*x)) - 12*a**3*c**2*d**2*log(c/d + sin(e +
f*x))*sin(e + f*x)/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x)) + 6*a**3*c**2*
d**2*log(c/d + sin(e + f*x))/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x)) + 3*
a**3*c**2*d**2*cos(e + f*x)**2/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x)) +
6*a**3*c**2*d**2/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x)) + 6*a**3*c*d**3*
log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*
x)) + 4*a**3*c*d**3*sin(e + f*x)**3/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x
)) + 3*a**3*c*d**3*sin(e + f*x)*cos(e + f*x)**2/(2*c**2*d**4*f + 2*c*d**5*f
*sin(e + f*x)) - 6*a**3*c*d**3*cos(e + f*x)**2/(2*c**2*d**4*f + 2*c*d**5*f*
sin(e + f*x)) - 2*a**3*c*d**3/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x)) - 6
*a**3*d**4*sin(e + f*x)**3/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*x)) - 6*a*
*3*d**4*sin(e + f*x)*cos(e + f*x)**2/(2*c**2*d**4*f + 2*c*d**5*f*sin(e + f*
x)), Eq(n, -2)), (-a**3*c**3*log(c/d + sin(e + f*x))/(d**4*f) + 3*a**3*c**2
*log(c/d + sin(e + f*x))/(d**3*f) + a**3*c**2*sin(e + f*x)/(d**3*f) - 3*a**
3*c*log(c/d + sin(e + f*x))/(d**2*f) - 3*a**3*c*sin(e + f*x)/(d**2*f) + a**
3*c*cos(e + f*x)**2/(2*d**2*f) + a**3*log(c/d + sin(e + f*x))/(d*f) + a**3*
sin(e + f*x)**3/(3*d*f) + 3*a**3*sin(e + f*x)/(d*f) - 3*a**3*cos(e + f*x)**
2/(2*d*f), Eq(n, -1)), (-6*a**3*c**4*(c + d*sin(e + f*x))**n/(d**4*f*n**4 +
 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 6*a**3*c**3*d
*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*
d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 6*a**3*c**3*d*n*(c + d*sin(e + f*x
))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**
4*f) + 24*a**3*c**3*d*(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3
 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) - 3*a**3*c**2*d**2*n**2*(c + d
*sin(e + f*x))**n*sin(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f
*n**2 + 50*d**4*f*n + 24*d**4*f) - 6*a**3*c**2*d**2*n**2*(c + d*sin(e + f*x
))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4
*f*n + 24*d**4*f) - 3*a**3*c**2*d**2*n**2*(c + d*sin(e + f*x))**n/(d**4*f*n
**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) - 3*a**3*c
**2*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f
*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) - 24*a**3*c**2*d**2*n*(c
+ d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f
*n**2 + 50*d**4*f*n + 24*d**4*f) - 21*a**3*c**2*d**2*n*(c + d*sin(e + f*x))
**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*
f) - 36*a**3*c**2*d**2*(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**
3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + a**3*c*d**3*n**3*(c + d*sin
(e + f*x))**n*sin(e + f*x)**3/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**
2 + 50*d**4*f*n + 24*d**4*f) + 3*a**3*c*d**3*n**3*(c + d*sin(e + f*x))**n*s
in(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n
 + 24*d**4*f) + 3*a**3*c*d**3*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d*
*4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + a*
*3*c*d**3*n**3*(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d
**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 3*a**3*c*d**3*n**2*(c + d*sin(e + f
*x))**n*sin(e + f*x)**3/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50
*d**4*f*n + 24*d**4*f) + 15*a**3*c*d**3*n**2*(c + d*sin(e + f*x))**n*sin(e
+ f*x)**2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24
*d**4*f) + 21*a**3*c*d**3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f
*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 9*a**3
*c*d**3*n**2*(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**
4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 2*a**3*c*d**3*n*(c + d*sin(e + f*x))*
*n*sin(e + f*x)**3/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4
*f*n + 24*d**4*f) + 12*a**3*c*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**
2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f)
```

```
 + 36*a**3*c*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*
d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 26*a**3*c*d**3*n*
(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50
*d**4*f*n + 24*d**4*f) + 24*a**3*c*d**3*(c + d*sin(e + f*x))**n/(d**4*f*n**
4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + a**3*d**4*
n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**4/(d**4*f*n**4 + 10*d**4*f*n**3
+ 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 3*a**3*d**4*n**3*(c + d*sin(e
 + f*x))**n*sin(e + f*x)**3/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2
+ 50*d**4*f*n + 24*d**4*f) + 3*a**3*d**4*n**3*(c + d*sin(e + f*x))**n*sin(e
 + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 2
4*d**4*f) + a**3*d**4*n**3*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n**
4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 6*a**3*d**
4*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**4/(d**4*f*n**4 + 10*d**4*f*n**
3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 21*a**3*d**4*n**2*(c + d*si
n(e + f*x))**n*sin(e + f*x)**3/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n*
*2 + 50*d**4*f*n + 24*d**4*f) + 24*a**3*d**4*n**2*(c + d*sin(e + f*x))**n*s
in(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n
 + 24*d**4*f) + 9*a**3*d**4*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**4
*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 11*a
**3*d**4*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**4/(d**4*f*n**4 + 10*d**4*f
*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 42*a**3*d**4*n*(c + d*s
in(e + f*x))**n*sin(e + f*x)**3/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n
**2 + 50*d**4*f*n + 24*d**4*f) + 57*a**3*d**4*n*(c + d*sin(e + f*x))**n*sin
(e + f*x)**2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n +
 24*d**4*f) + 26*a**3*d**4*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n
**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 6*a**3*d
**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**4/(d**4*f*n**4 + 10*d**4*f*n**3 +
 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f) + 24*a**3*d**4*(c + d*sin(e + f*
x))**n*sin(e + f*x)**3/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*
d**4*f*n + 24*d**4*f) + 36*a**3*d**4*(c + d*sin(e + f*x))**n*sin(e + f*x)**
2/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f)
 + 24*a**3*d**4*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**4*f*n**4 + 10*d**4
*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f), True))
```

Giac [B] time = 1.22337, size = 1351, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] (((d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*n^3 - 3*(d*sin(f*x + e) + c
)^3*(d*sin(f*x + e) + c)^n*c*n^3 + 3*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e)
 + c)^n*c^2*n^3 - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^3 + 6*(
d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*n^2 - 21*(d*sin(f*x + e) + c)^
3*(d*sin(f*x + e) + c)^n*c*n^2 + 24*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e)
+ c)^n*c^2*n^2 - 9*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^2 + 11
*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*n - 42*(d*sin(f*x + e) + c)^
3*(d*sin(f*x + e) + c)^n*c*n + 57*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) +
c)^n*c^2*n - 26*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n + 6*(d*si
n(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n - 24*(d*sin(f*x + e) + c)^3*(d*sin
(f*x + e) + c)^n*c + 36*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2 -
 24*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3)*a^3/(d^3*n^4 + 10*d^3*
n^3 + 35*d^3*n^2 + 50*d^3*n + 24*d^3) + 3*((d*sin(f*x + e) + c)^3*(d*sin(f*
x + e) + c)^n*n^2 - 2*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n^2 +
```

```
 (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2*n^2 + 3*(d*sin(f*x + e) +
c)^3*(d*sin(f*x + e) + c)^n*n - 8*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) +
c)^n*c*n + 5*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2*n + 2*(d*sin(f
*x + e) + c)^3*(d*sin(f*x + e) + c)^n - 6*(d*sin(f*x + e) + c)^2*(d*sin(f*x
 + e) + c)^n*c + 6*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2)*a^3/(d^
2*n^3 + 6*d^2*n^2 + 11*d^2*n + 6*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^3/(n
 + 1) + 3*((d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*n - (d*sin(f*x + e
) + c)*(d*sin(f*x + e) + c)^n*c*n + (d*sin(f*x + e) + c)^2*(d*sin(f*x + e)
+ c)^n - 2*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c)*a^3/((n^2 + 3*n +
 2)*d))/(d*f)
```

3.915 $\int \cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=101

$$\frac{a^2(c-d)^2(c+d\sin(e+fx))^{n+1}}{d^3f(n+1)}-\frac{2a^2(c-d)(c+d\sin(e+fx))^{n+2}}{d^3f(n+2)}+\frac{a^2(c+d\sin(e+fx))^{n+3}}{d^3f(n+3)}$$

```
[Out] (a^2*(c - d)^2*(c + d*Sin[e + f*x])^(1 + n))/(d^3*f*(1 + n)) - (2*a^2*(c -
d)*(c + d*Sin[e + f*x])^(2 + n))/(d^3*f*(2 + n)) + (a^2*(c + d*Sin[e + f*x]
)^(3 + n))/(d^3*f*(3 + n))
```

Rubi [A] time = 0.149943, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{a^2(c-d)^2(c+d\sin(e+fx))^{n+1}}{d^3f(n+1)}-\frac{2a^2(c-d)(c+d\sin(e+fx))^{n+2}}{d^3f(n+2)}+\frac{a^2(c+d\sin(e+fx))^{n+3}}{d^3f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^2*(c - d)^2*(c + d*Sin[e + f*x])^(1 + n))/(d^3*f*(1 + n)) - (2*a^2*(c -
d)*(c + d*Sin[e + f*x])^(2 + n))/(d^3*f*(2 + n)) + (a^2*(c + d*Sin[e + f*x]
)^(3 + n))/(d^3*f*(3 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx=\frac{\text{Subst}\left(\int(a+x)^2\left(c+\frac{dx}{a}\right)^n\,dx,x,a\sin(e+fx)\right)}{af}$$

$$=\frac{\text{Subst}\left(\int\left(\frac{a^2(c-d)^2\left(c+\frac{dx}{a}\right)^n}{d^2}-\frac{2a^2(c-d)\left(c+\frac{dx}{a}\right)^{1+n}}{d^2}+\frac{a^2\left(c+\frac{dx}{a}\right)^{2+n}}{d^2}\right)\right.}{af}$$

$$=\frac{a^2(c-d)^2(c+d\sin(e+fx))^{1+n}}{d^3f(1+n)}-\frac{2a^2(c-d)(c+d\sin(e+}{d^3f(2+n)}$$

Mathematica [A] time = 0.359379, size = 78, normalized size = 0.77

$$\frac{a^2(c+d\sin(e+fx))^{n+1}\left(-\frac{2(c-d)(c+d\sin(e+fx))}{n+2}+\frac{(c+d\sin(e+fx))^2}{n+3}+\frac{(c-d)^2}{n+1}\right)}{d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^2*(c + d*Sin[e + f*x])^(1 + n)*((c - d)^2/(1 + n) - (2*(c - d)*(c + d*Si
n[e + f*x]))/(2 + n) + (c + d*Sin[e + f*x])^2/(3 + n)))/(d^3*f)
```

Maple [F] time = 0.466, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01878, size = 599, normalized size = 5.93

$$\frac{\left(2\,a^2c^3-6\,a^2c^2d+6\,a^2cd^2+6\,a^2d^3+2\left(a^2cd^2+a^2d^3\right)n^2-\left(6\,a^2d^3+\left(a^2cd^2+2\,a^2d^3\right)n^2+\left(a^2cd^2+8\,a^2d^3\right)n\right)\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] (2*a^2*c^3 - 6*a^2*c^2*d + 6*a^2*c*d^2 + 6*a^2*d^3 + 2*(a^2*c*d^2 + a^2*d^3
)*n^2 - (6*a^2*d^3 + (a^2*c*d^2 + 2*a^2*d^3)*n^2 + (a^2*c*d^2 + 8*a^2*d^3)*
n)*cos(f*x + e)^2 - 2*(a^2*c^2*d - 3*a^2*c*d^2 - 4*a^2*d^3)*n + (8*a^2*d^3
+ 2*(a^2*c*d^2 + a^2*d^3)*n^2 - (a^2*d^3*n^2 + 3*a^2*d^3*n + 2*a^2*d^3)*cos
(f*x + e)^2 - 2*(a^2*c^2*d - 3*a^2*c*d^2 - 4*a^2*d^3)*n)*sin(f*x + e))*(d*s
in(f*x + e) + c)^n/(d^3*f*n^3 + 6*d^3*f*n^2 + 11*d^3*f*n + 6*d^3*f)
```

Sympy [A] time = 29.7327, size = 2278, normalized size = 22.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Piecewise((c**n*(a**2*sin(e + f*x)**3/(3*f) + a**2*sin(e + f*x)/f - a**2*co
s(e + f*x)**2/f), Eq(d, 0)), (x*(c + d*sin(e))**n*(a*sin(e) + a)**2*cos(e),
 Eq(f, 0)), (2*a**2*c**2*log(c/d + sin(e + f*x))/(2*c**2*d**3*f + 4*c*d**4*
f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 3*a**2*c**2/(2*c**2*d**3*f + 4
*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 4*a**2*c*d*log(c/d + s
in(e + f*x))*sin(e + f*x)/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5
*f*sin(e + f*x)**2) + 4*a**2*c*d*sin(e + f*x)/(2*c**2*d**3*f + 4*c*d**4*f*s
in(e + f*x) + 2*d**5*f*sin(e + f*x)**2) - 2*a**2*c*d/(2*c**2*d**3*f + 4*c*d
**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 2*a**2*d**2*log(c/d + sin(
e + f*x))*sin(e + f*x)**2/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5
*f*sin(e + f*x)**2) - 4*a**2*d**2*sin(e + f*x)/(2*c**2*d**3*f + 4*c*d**4*f*
sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) - a**2*d**2/(2*c**2*d**3*f + 4*c*d
**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2), Eq(n, -3)), (-2*a**2*c**3*l
og(c/d + sin(e + f*x))/(c**2*d**3*f + c*d**4*f*sin(e + f*x)) - 2*a**2*c**3/
(c**2*d**3*f + c*d**4*f*sin(e + f*x)) - 2*a**2*c**2*d*log(c/d + sin(e + f*x
))*sin(e + f*x)/(c**2*d**3*f + c*d**4*f*sin(e + f*x)) + 2*a**2*c**2*d*log(c
/d + sin(e + f*x))/(c**2*d**3*f + c*d**4*f*sin(e + f*x)) + 2*a**2*c**2*d/(c
**2*d**3*f + c*d**4*f*sin(e + f*x)) + 2*a**2*c*d**2*log(c/d + sin(e + f*x))
*sin(e + f*x)/(c**2*d**3*f + c*d**4*f*sin(e + f*x)) - a**2*c*d**2*cos(e + f
*x)**2/(c**2*d**3*f + c*d**4*f*sin(e + f*x)) - a**2*c*d**2/(c**2*d**3*f + c
*d**4*f*sin(e + f*x)) - a**2*d**3*sin(e + f*x)**3/(c**2*d**3*f + c*d**4*f*s
in(e + f*x)) - a**2*d**3*sin(e + f*x)*cos(e + f*x)**2/(c**2*d**3*f + c*d**4
*f*sin(e + f*x)), Eq(n, -2)), (a**2*c**2*log(c/d + sin(e + f*x))/(d**3*f) -
 2*a**2*c*log(c/d + sin(e + f*x))/(d**2*f) - a**2*c*sin(e + f*x)/(d**2*f) +
 a**2*log(c/d + sin(e + f*x))/(d*f) + 2*a**2*sin(e + f*x)/(d*f) - a**2*cos(
e + f*x)**2/(2*d*f), Eq(n, -1)), (2*a**2*c**3*(c + d*sin(e + f*x))**n/(d**3
*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) - 2*a**2*c**2*d*n*(c + d*
sin(e + f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n +
6*d**3*f) - 2*a**2*c**2*d*n*(c + d*sin(e + f*x))**n/(d**3*f*n**3 + 6*d**3*f
*n**2 + 11*d**3*f*n + 6*d**3*f) - 6*a**2*c**2*d*(c + d*sin(e + f*x))**n/(d*
*3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + a**2*c*d**2*n**2*(c +
 d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*
f*n + 6*d**3*f) + 2*a**2*c*d**2*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)/(
d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + a**2*c*d**2*n**2*(c
 + d*sin(e + f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f
) + a**2*c*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f*n**3 + 6*
d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 6*a**2*c*d**2*n*(c + d*sin(e + f*x)
)**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) +
5*a**2*c*d**2*n*(c + d*sin(e + f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d
**3*f*n + 6*d**3*f) + 6*a**2*c*d**2*(c + d*sin(e + f*x))**n/(d**3*f*n**3 +
6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + a**2*d**3*n**2*(c + d*sin(e + f*x
))**n*sin(e + f*x)**3/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f
) + 2*a**2*d**3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f*n**3 +
 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + a**2*d**3*n**2*(c + d*sin(e + f*
x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f)
+ 3*a**2*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**3*f*n**3 + 6*d*
*3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 8*a**2*d**3*n*(c + d*sin(e + f*x))**n
*sin(e + f*x)**2/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 5
*a**2*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n
**2 + 11*d**3*f*n + 6*d**3*f) + 2*a**2*d**3*(c + d*sin(e + f*x))**n*sin(e +
```

```
 f*x)**3/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 6*a**2*d*
*3*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f*n**3 + 6*d**3*f*n**2 + 1
1*d**3*f*n + 6*d**3*f) + 6*a**2*d**3*(c + d*sin(e + f*x))**n*sin(e + f*x)/(
d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f), True))
```

Giac [B] time = 1.13679, size = 625, normalized size = 6.19

$$\left(\left(d\sin\left(fx+e\right)+c\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{n}n^{2}-2\left(d\sin\left(fx+e\right)+c\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{n}cn^{2}+\left(d\sin\left(fx+e\right)+c\right)\left(d\sin\left(fx+e\right)+c\right)^{n}c^{2}n^{2}+3\left(d\sin\left(fx+e\right)+c\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] (((d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*n^2 - 2*(d*sin(f*x + e) + c
)^2*(d*sin(f*x + e) + c)^n*c*n^2 + (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c
)^n*c^2*n^2 + 3*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*n - 8*(d*sin(
f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n + 5*(d*sin(f*x + e) + c)*(d*sin(
f*x + e) + c)^n*c^2*n + 2*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n - 6
*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c + 6*(d*sin(f*x + e) + c)*(
d*sin(f*x + e) + c)^n*c^2)*a^2/(d^2*n^3 + 6*d^2*n^2 + 11*d^2*n + 6*d^2) + (
d*sin(f*x + e) + c)^(n + 1)*a^2/(n + 1) + 2*((d*sin(f*x + e) + c)^2*(d*sin(
f*x + e) + c)^n*n - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c*n + (d*si
n(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n - 2*(d*sin(f*x + e) + c)*(d*sin(f*
x + e) + c)^n*c)*a^2/((n^2 + 3*n + 2)*d))/(d*f)
```

3.916 $\int \cos(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=61

$$\frac{a(c+d\sin(e+fx))^{n+2}}{d^2f(n+2)}-\frac{a(c-d)(c+d\sin(e+fx))^{n+1}}{d^2f(n+1)}$$

```
[Out] -((a*(c - d)*(c + d*Sin[e + f*x])^(1 + n))/(d^2*f*(1 + n))) + (a*(c + d*Sin
[e + f*x])^(2 + n))/(d^2*f*(2 + n))
```

Rubi [A] time = 0.09559, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{a(c+d\sin(e+fx))^{n+2}}{d^2f(n+2)}-\frac{a(c-d)(c+d\sin(e+fx))^{n+1}}{d^2f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((a*(c - d)*(c + d*Sin[e + f*x])^(1 + n))/(d^2*f*(1 + n))) + (a*(c + d*Sin
[e + f*x])^(2 + n))/(d^2*f*(2 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(c+\frac{dx}{a}\right)^n\,dx,x,a\sin(e+fx)\right)}{af}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a(c-d)\left(c+\frac{dx}{a}\right)^n}{d}+\frac{a\left(c+\frac{dx}{a}\right)^{1+n}}{d}\right)dx,x,a\sin(e+fx)\right)}{af}\\ &= -\frac{a(c-d)(c+d\sin(e+fx))^{1+n}}{d^2f(1+n)}+\frac{a(c+d\sin(e+fx))^{2+n}}{d^2f(2+n)}\end{aligned}$$

Mathematica [A] time = 0.486797, size = 52, normalized size = 0.85

$$\frac{a(c+d\sin(e+fx))^{n+1}(-c+d(n+1)\sin(e+fx)+d(n+2))}{d^2f(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a*(c + d*Sin[e + f*x])^(1 + n)*(-c + d*(2 + n) + d*(1 + n)*Sin[e + f*x]))/
(d^2*f*(1 + n)*(2 + n))
```

Maple [F] time = 0.269, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94721, size = 250, normalized size = 4.1

$$-\frac{\left(ac^{2}-2\,acd-ad^{2}+\left(ad^{2}n+ad^{2}\right)\cos\left(fx+e\right)^{2}-\left(acd+ad^{2}\right)n-\left(2\,ad^{2}+\left(acd+ad^{2}\right)n\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)+\right.}{d^{2}fn^{2}+3\,d^{2}fn+2\,d^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fric
as")
```

```
[Out] -(a*c^2 - 2*a*c*d - a*d^2 + (a*d^2*n + a*d^2)*cos(f*x + e)^2 - (a*c*d + a*d
^2)*n - (2*a*d^2 + (a*c*d + a*d^2)*n)*sin(f*x + e))*(d*sin(f*x + e) + c)^n/
(d^2*f*n^2 + 3*d^2*f*n + 2*d^2*f)
```

Sympy [A] time = 10.7143, size = 586, normalized size = 9.61

$$\begin{cases} c^n\left(\frac{a\sin(e+fx)}{f}-\frac{a\cos^2(e+fx)}{2f}\right) \\ x(c+d\sin(e))^n(a\sin(e)+a)\cos(e) \\ \frac{ac\log\left(\frac{c}{d}+\sin(e+fx)\right)}{cd^2f+d^3f\sin(e+fx)}+\frac{ac}{cd^2f+d^3f\sin(e+fx)}+\frac{ad\log\left(\frac{c}{d}+\sin(e+fx)\right)\sin(e+fx)}{cd^2f+d^3f\sin(e+fx)}-\frac{ad}{cd^2f+d^3f\sin(e+fx)} \\ -\frac{ac\log\left(\frac{c}{d}+\sin(e+fx)\right)}{d^2f}+\frac{a\log\left(\frac{c}{d}+\sin(e+fx)\right)}{df}+\frac{a\sin(e+fx)}{df} \\ -\frac{ac^2(c+d\sin(e+fx))^n}{d^2fn^2+3d^2fn+2d^2f}+\frac{acdn(c+d\sin(e+fx))^n\sin(e+fx)}{d^2fn^2+3d^2fn+2d^2f}+\frac{acdn(c+d\sin(e+fx))^n}{d^2fn^2+3d^2fn+2d^2f}+\frac{2acd(c+d\sin(e+fx))^n}{d^2fn^2+3d^2fn+2d^2f}+\frac{ad^2n(c+d\sin(e+fx))^n\sin^2(e+fx)}{d^2fn^2+3d^2fn+2d^2f} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Piecewise((c**n*(a*sin(e + f*x)/f - a*cos(e + f*x)**2/(2*f)), Eq(d, 0)), (x
*(c + d*sin(e))**n*(a*sin(e) + a)*cos(e), Eq(f, 0)), (a*c*log(c/d + sin(e +
 f*x))/(c*d**2*f + d**3*f*sin(e + f*x)) + a*c/(c*d**2*f + d**3*f*sin(e + f*
x)) + a*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(c*d**2*f + d**3*f*sin(e + f
*x)) - a*d/(c*d**2*f + d**3*f*sin(e + f*x)), Eq(n, -2)), (-a*c*log(c/d + si
n(e + f*x))/(d**2*f) + a*log(c/d + sin(e + f*x))/(d*f) + a*sin(e + f*x)/(d*
f), Eq(n, -1)), (-a*c**2*(c + d*sin(e + f*x))**n/(d**2*f*n**2 + 3*d**2*f*n
+ 2*d**2*f) + a*c*d*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**2*f*n**2 + 3
*d**2*f*n + 2*d**2*f) + a*c*d*n*(c + d*sin(e + f*x))**n/(d**2*f*n**2 + 3*d*
*2*f*n + 2*d**2*f) + 2*a*c*d*(c + d*sin(e + f*x))**n/(d**2*f*n**2 + 3*d**2*
f*n + 2*d**2*f) + a*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**2*f*
n**2 + 3*d**2*f*n + 2*d**2*f) + a*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*
x)/(d**2*f*n**2 + 3*d**2*f*n + 2*d**2*f) + a*d**2*(c + d*sin(e + f*x))**n*s
in(e + f*x)**2/(d**2*f*n**2 + 3*d**2*f*n + 2*d**2*f) + 2*a*d**2*(c + d*sin(
e + f*x))**n*sin(e + f*x)/(d**2*f*n**2 + 3*d**2*f*n + 2*d**2*f), True))
```

Giac [B] time = 1.26225, size = 211, normalized size = 3.46

$$\frac{\frac{(d\sin(fx+e)+c)^{n+1}a}{n+1}+\frac{\left((d\sin(fx+e)+c)^2(d\sin(fx+e)+c)^n n-(d\sin(fx+e)+c)(d\sin(fx+e)+c)^n cn+(d\sin(fx+e)+c)^2(d\sin(fx+e)+c)^n-2(d\sin(fx+e)+\cdots\right.}{(n^2+3n+2)d}}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac
")
```

```
[Out] ((d*sin(f*x + e) + c)^(n + 1)*a/(n + 1) + ((d*sin(f*x + e) + c)^2*(d*sin(f*
x + e) + c)^n*n - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c*n + (d*sin(
f*x + e) + c)^2*(d*sin(f*x + e) + c)^n - 2*(d*sin(f*x + e) + c)*(d*sin(f*x
+ e) + c)^n*c)*a/((n^2 + 3*n + 2)*d))/(d*f)
```

3.917 $\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{af(n+1)(c-d)}$$

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)))
```

Rubi [A] time = 0.124484, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$-\frac{(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{af(n+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(c+\frac{dx}{a}\right)^n}{a+x}\,dx,x,a\sin(e+fx)\right)}{af}\\ &= -\frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a(c-d)f(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0694949, size = 60, normalized size = 1.

$$-\frac{(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{af(n+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)))
```

Maple [F] time = 1.207, size = 0, normalized size = 0.

$$\int \frac{\cos \left(fx+e\right) \left(c+d \sin \left(fx+e\right)\right)^{n}}{a+a \sin \left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin \left(fx+e\right)+c\right)^{n} \cos \left(fx+e\right)}{a \sin \left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d \sin \left(fx+e\right)+c\right)^{n} \cos \left(fx+e\right)}{a \sin \left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a), x)
```

3.918 $\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=60

$$\frac{d(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^2f(n+1)(c-d)^2}$$

```
[Out] (d*Hypergeometric2F1[2, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a^2*(c - d)^2*f*(1 + n))
```

Rubi [A] time = 0.109314, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$\frac{d(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^2f(n+1)(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*Hypergeometric2F1[2, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a^2*(c - d)^2*f*(1 + n))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(c+\frac{dx}{a}\right)^n}{(a+x)^2}\,dx,x,a\sin(e+fx)\right)}{af}\\ &= \frac{d\,{}_2F_1\left(2,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a^2(c-d)^2f(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0641619, size = 61, normalized size = 1.02

$$\frac{d(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;-\frac{c+d\sin(e+fx)}{d-c}\right)}{a^2f(n+1)(d-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*Hypergeometric2F1[2, 1 + n, 2 + n, -((c + d*Sin[e + f*x])/(-c + d))]*(c
+ d*Sin[e + f*x])^(1 + n))/(a^2*(-c + d)^2*f*(1 + n))
```

Maple [F] time = 0.884, size = 0, normalized size = 0.

$$\int \frac{\cos \left(fx+e\right) \left(c+d \sin \left(fx+e\right)\right)^{n}}{\left(a+a \sin \left(fx+e\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin \left(fx+e\right)+c\right)^{n} \cos \left(fx+e\right)}{\left(a \sin \left(fx+e\right)+a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d \sin \left(fx+e\right)+c\right)^{n} \cos \left(fx+e\right)}{a^{2} \cos \left(fx+e\right)^{2}-2 \, a^{2} \sin \left(fx+e\right)-2 \, a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)/(a^2*cos(f*x + e)^2 - 2*a^2*s
in(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^2, x)
```

3.919 $\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=63

$$-\frac{d^2(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^3f(n+1)(c-d)^3}$$

```
[Out] -((d^2*Hypergeometric2F1[3, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c
+ d*Sin[e + f*x])^(1 + n))/(a^3*(c - d)^3*f*(1 + n)))
```

Rubi [A] time = 0.117261, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$-\frac{d^2(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^3f(n+1)(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((d^2*Hypergeometric2F1[3, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c
+ d*Sin[e + f*x])^(1 + n))/(a^3*(c - d)^3*f*(1 + n)))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(c+\frac{dx}{a}\right)^n}{(a+x)^3}\,dx, x, a\sin(e+fx)\right)}{af}\\ &= -\frac{d^2\,{}_2F_1\left(3,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a^3(c-d)^3f(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0622529, size = 63, normalized size = 1.

$$\frac{d^2(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(3,n+1;n+2;-\frac{c+d\sin(e+fx)}{d-c}\right)}{a^3f(n+1)(d-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^2*Hypergeometric2F1[3, 1 + n, 2 + n, -((c + d*Sin[e + f*x])/(-c + d))]*(
c + d*Sin[e + f*x])^(1 + n))/(a^3*(-c + d)^3*f*(1 + n))
```

Maple [F] time = 1.088, size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)/(3*a^3*cos(f*x + e)^2 - 4*a^3
 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^3, x)
```

3.920 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^4\,dx$

Optimal. Leaf size=170

$$\frac{6d^2(c-d)^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{4d^3(c-d)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{4d(c-d)^3(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{d^4(a\sin(e$$

```
[Out] ((c - d)^4*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (4*(c - d)^3*d*(a
+ a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (6*(c - d)^2*d^2*(a + a*Sin[e
+ f*x])^(3 + m))/(a^3*f*(3 + m)) + (4*(c - d)*d^3*(a + a*Sin[e + f*x])^(4 +
 m))/(a^4*f*(4 + m)) + (d^4*(a + a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

Rubi [A] time = 0.181892, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{6d^2(c-d)^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{4d^3(c-d)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{4d(c-d)^3(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{d^4(a\sin(e$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((c - d)^4*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (4*(c - d)^3*d*(a
+ a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (6*(c - d)^2*d^2*(a + a*Sin[e
+ f*x])^(3 + m))/(a^3*f*(3 + m)) + (4*(c - d)*d^3*(a + a*Sin[e + f*x])^(4 +
 m))/(a^4*f*(4 + m)) + (d^4*(a + a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^4\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^4\,dx,x,a\sin(e+fx)\right)}{af}\\ &= \frac{\text{Subst}\left(\int \left((c-d)^4(a+x)^m+\frac{4(c-d)^3d(a+x)^{1+m}}{a}+\frac{6(c-d)^2d^2(a+}{a^2}\right.\right.}{af}\\ &= \frac{(c-d)^4(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{4(c-d)^3d(a+a\sin(e+}{a^2f(2+m)}\end{aligned}$$

Mathematica [A] time = 0.65945, size = 143, normalized size = 0.84

$$\frac{(a(\sin(e+fx)+1))^{m+1}\left(\frac{4a^4d^3(c-d)(\sin(e+fx)+1)^3}{m+4}+\frac{6a^4d^2(c-d)^2(\sin(e+fx)+1)^2}{m+3}+\frac{4a^4d(c-d)^3(\sin(e+fx)+1)}{m+2}+\frac{a^4(c-d)^4}{m+1}+\frac{d^4(a\sin(e+fx)+a)^4}{m+5}\right)}{a^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*((a^4*(c - d)^4)/(1 + m) + (4*a^4*(c - d)^3
*d*(1 + Sin[e + f*x]))/(2 + m) + (6*a^4*(c - d)^2*d^2*(1 + Sin[e + f*x])^2)
/(3 + m) + (4*a^4*(c - d)*d^3*(1 + Sin[e + f*x])^3)/(4 + m) + (d^4*(a + a*S
in[e + f*x])^4)/(5 + m)))/(a^5*f)
```

Maple [F] time = 7.776, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.47408, size = 1706, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] ((c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*m^4 + ((4*c*d^3 + d^4)*m^4 + 1
20*c*d^3 + 2*(22*c*d^3 + 3*d^4)*m^3 + (164*c*d^3 + 11*d^4)*m^2 + 2*(122*c*d
^3 + 3*d^4)*m)*cos(f*x + e)^4 + 120*c^4 + 240*c^2*d^2 + 24*d^4 + 2*(7*c^4 +
 24*c^3*d + 30*c^2*d^2 + 16*c*d^3 + 3*d^4)*m^3 + (71*c^4 + 188*c^3*d + 186*
c^2*d^2 + 92*c*d^3 + 23*d^4)*m^2 - 2*((2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)
*m^4 + 120*c^3*d + 120*c*d^3 + 2*(13*c^3*d + 15*c^2*d^2 + 19*c*d^3 + 3*d^4)
```

```
*m^3 + (118*c^3*d + 87*c^2*d^2 + 128*c*d^3 + 17*d^4)*m^2 + 2*(107*c^3*d + 3
0*c^2*d^2 + 107*c*d^3 + 6*d^4)*m)*cos(f*x + e)^2 + 2*(77*c^4 + 120*c^3*d +
114*c^2*d^2 + 80*c*d^3 + 9*d^4)*m + ((c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 +
 d^4)*m^4 + (d^4*m^4 + 10*d^4*m^3 + 35*d^4*m^2 + 50*d^4*m + 24*d^4)*cos(f*x
 + e)^4 + 120*c^4 + 240*c^2*d^2 + 24*d^4 + 2*(7*c^4 + 24*c^3*d + 30*c^2*d^2
 + 16*c*d^3 + 3*d^4)*m^3 + (71*c^4 + 188*c^3*d + 186*c^2*d^2 + 92*c*d^3 + 2
3*d^4)*m^2 - 2*((3*c^2*d^2 + 2*c*d^3 + d^4)*m^4 + 120*c^2*d^2 + 24*d^4 + 4*
(9*c^2*d^2 + 4*c*d^3 + 2*d^4)*m^3 + (147*c^2*d^2 + 34*c*d^3 + 29*d^4)*m^2 +
 2*(117*c^2*d^2 + 10*c*d^3 + 23*d^4)*m)*cos(f*x + e)^2 + 2*(77*c^4 + 120*c^
3*d + 114*c^2*d^2 + 80*c*d^3 + 9*d^4)*m)*sin(f*x + e))*(a*sin(f*x + e) + a)
^m/(f*m^5 + 15*f*m^4 + 85*f*m^3 + 225*f*m^2 + 274*f*m + 120*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32179, size = 2491, normalized size = 14.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] (6*((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m^2 - 2*(a*sin(f*x + e) +
 a)^2*(a*sin(f*x + e) + a)^m*a*m^2 + (a*sin(f*x + e) + a)*(a*sin(f*x + e) +
 a)^m*a^2*m^2 + 3*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 8*(a*si
n(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a*m + 5*(a*sin(f*x + e) + a)*(a*si
n(f*x + e) + a)^m*a^2*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m -
 6*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a + 6*(a*sin(f*x + e) + a)
*(a*sin(f*x + e) + a)^m*a^2)*c^2*d^2/(a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^
2) + 4*((a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^3 - 3*(a*sin(f*x +
e) + a)^3*(a*sin(f*x + e) + a)^m*a*m^3 + 3*(a*sin(f*x + e) + a)^2*(a*sin(f*
x + e) + a)^m*a^2*m^3 - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^3
 + 6*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^2 - 21*(a*sin(f*x + e)
 + a)^3*(a*sin(f*x + e) + a)^m*a*m^2 + 24*(a*sin(f*x + e) + a)^2*(a*sin(f*x
 + e) + a)^m*a^2*m^2 - 9*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^
2 + 11*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m - 42*(a*sin(f*x + e)
 + a)^3*(a*sin(f*x + e) + a)^m*a*m + 57*(a*sin(f*x + e) + a)^2*(a*sin(f*x +
 e) + a)^m*a^2*m - 26*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m + 6
*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m - 24*(a*sin(f*x + e) + a)^3*
(a*sin(f*x + e) + a)^m*a + 36*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m
*a^2 - 24*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3)*c*d^3/(a^3*m^4 +
 10*a^3*m^3 + 35*a^3*m^2 + 50*a^3*m + 24*a^3) + ((a*sin(f*x + e) + a)^5*(a*
sin(f*x + e) + a)^m*m^4 - 4*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a
*m^4 + 6*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a^2*m^4 - 4*(a*sin(f
*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^3*m^4 + (a*sin(f*x + e) + a)*(a*sin
```

```
(f*x + e) + a)^m*a^4*m^4 + 10*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m
*m^3 - 44*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a*m^3 + 72*(a*sin(f
*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a^2*m^3 - 52*(a*sin(f*x + e) + a)^2*(
a*sin(f*x + e) + a)^m*a^3*m^3 + 14*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a
)^m*a^4*m^3 + 35*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m^2 - 164*(a
*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a*m^2 + 294*(a*sin(f*x + e) + a
)^3*(a*sin(f*x + e) + a)^m*a^2*m^2 - 236*(a*sin(f*x + e) + a)^2*(a*sin(f*x
+ e) + a)^m*a^3*m^2 + 71*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^4*m^
2 + 50*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m - 244*(a*sin(f*x + e
) + a)^4*(a*sin(f*x + e) + a)^m*a*m + 468*(a*sin(f*x + e) + a)^3*(a*sin(f*x
 + e) + a)^m*a^2*m - 428*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^3*
m + 154*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^4*m + 24*(a*sin(f*x +
 e) + a)^5*(a*sin(f*x + e) + a)^m - 120*(a*sin(f*x + e) + a)^4*(a*sin(f*x +
 e) + a)^m*a + 240*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a^2 - 240*
(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^3 + 120*(a*sin(f*x + e) + a
)*(a*sin(f*x + e) + a)^m*a^4)*d^4/(a^4*m^5 + 15*a^4*m^4 + 85*a^4*m^3 + 225*
a^4*m^2 + 274*a^4*m + 120*a^4) + (a*sin(f*x + e) + a)^(m + 1)*c^4/(m + 1) +
 4*((a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*m - (a*sin(f*x + e) + a)*
(a*sin(f*x + e) + a)^m*a*m + (a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m
- 2*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a)*c^3*d/((m^2 + 3*m + 2)*a
))/(a*f)
```

3.921 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=133

$$\frac{3d^2(c-d)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{3d(c-d)^2(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{d^3(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{(c-d)^3(a\sin(e+f}{af(m+1}$$

```
[Out] ((c - d)^3*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (3*(c - d)^2*d*(a
+ a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (3*(c - d)*d^2*(a + a*Sin[e +
f*x])^(3 + m))/(a^3*f*(3 + m)) + (d^3*(a + a*Sin[e + f*x])^(4 + m))/(a^4*f*
(4 + m))
```

Rubi [A] time = 0.140071, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{3d^2(c-d)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{3d(c-d)^2(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{d^3(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{(c-d)^3(a\sin(e+f}{af(m+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((c - d)^3*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (3*(c - d)^2*d*(a
+ a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (3*(c - d)*d^2*(a + a*Sin[e +
f*x])^(3 + m))/(a^3*f*(3 + m)) + (d^3*(a + a*Sin[e + f*x])^(4 + m))/(a^4*f*
(4 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^3\,dx,x,a\sin(e+fx)\right)}{af}\\
&= \frac{\text{Subst}\left(\int \left((c-d)^3(a+x)^m+\frac{3(c-d)^2d(a+x)^{1+m}}{a}+\frac{3(c-d)d^2(a+x}{a^2}\right.\right.}{af}\\
&= \frac{(c-d)^3(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{3(c-d)^2d(a+a\sin(e+}{a^2f(2+m)}
\end{aligned}$$

Mathematica [A] time = 0.381506, size = 113, normalized size = 0.85

$$\frac{(a(\sin(e+fx)+1))^{m+1}\left(\frac{3a^3d^2(c-d)(\sin(e+fx)+1)^2}{m+3}+\frac{3a^3d(c-d)^2(\sin(e+fx)+1)}{m+2}+\frac{a^3(c-d)^3}{m+1}+\frac{d^3(a\sin(e+fx)+a)^3}{m+4}\right)}{a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*((a^3*(c - d)^3)/(1 + m) + (3*a^3*(c - d)^2
*d*(1 + Sin[e + f*x]))/(2 + m) + (3*a^3*(c - d)*d^2*(1 + Sin[e + f*x])^2)/(
3 + m) + (d^3*(a + a*Sin[e + f*x])^3)/(4 + m)))/(a^4*f)
```

Maple [F] time = 3.055, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42002, size = 905, normalized size = 6.8

$$\frac{\left(\left(d^3m^3+6\,d^3m^2+11\,d^3m+6\,d^3\right)\cos\left(fx+e\right)^4+\left(c^3+3\,c^2d+3\,cd^2+d^3\right)m^3+24\,c^3+24\,cd^2+3\left(3\,c^3+7\,c^2d+5\,cd^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] ((d^3*m^3 + 6*d^3*m^2 + 11*d^3*m + 6*d^3)*cos(f*x + e)^4 + (c^3 + 3*c^2*d +
 3*c*d^2 + d^3)*m^3 + 24*c^3 + 24*c*d^2 + 3*(3*c^3 + 7*c^2*d + 5*c*d^2 + d^
3)*m^2 - ((3*c^2*d + 3*c*d^2 + 2*d^3)*m^3 + 36*c^2*d + 12*d^3 + 3*(8*c^2*d
+ 5*c*d^2 + 3*d^3)*m^2 + (57*c^2*d + 12*c*d^2 + 19*d^3)*m)*cos(f*x + e)^2 +
 2*(13*c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*m + ((c^3 + 3*c^2*d + 3*c*d^2 + d^
3)*m^3 + 24*c^3 + 24*c*d^2 + 3*(3*c^3 + 7*c^2*d + 5*c*d^2 + d^3)*m^2 - ((3*
```

```
c*d^2 + d^3)*m^3 + 24*c*d^2 + 3*(7*c*d^2 + d^3)*m^2 + 2*(21*c*d^2 + d^3)*m)
*cos(f*x + e)^2 + 2*(13*c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*m)*sin(f*x + e))*
(a*sin(f*x + e) + a)^m/(f*m^4 + 10*f*m^3 + 35*f*m^2 + 50*f*m + 24*f)
```

Sympy [A] time = 109.548, size = 4308, normalized size = 32.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((x*(c + d*sin(e))**3*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-2*c**
3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e +
 f*x) + 6*a**4*f) - 9*c**2*d*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**3 + 18*a*
*4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 3*c**2*d/(6*a**
4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) +
6*a**4*f) + 6*c*d**2*sin(e + f*x)**3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*
sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 6*d**3*log(sin(e + f
*x) + 1)*sin(e + f*x)**3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)
**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 18*d**3*log(sin(e + f*x) + 1)*si
n(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a*
*4*f*sin(e + f*x) + 6*a**4*f) + 18*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/
(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f
*x) + 6*a**4*f) + 6*d**3*log(sin(e + f*x) + 1)/(6*a**4*f*sin(e + f*x)**3 +
18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 6*d**3*sin
(e + f*x)**3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**
4*f*sin(e + f*x) + 6*a**4*f) + 9*d**3*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**
3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 5*d**3
/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e +
f*x) + 6*a**4*f), Eq(m, -4)), (-c**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*s
in(e + f*x) + 2*a**3*f) - 6*c**2*d*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 +
 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 3*c**2*d/(2*a**3*f*sin(e + f*x)**2 + 4
*a**3*f*sin(e + f*x) + 2*a**3*f) + 6*c*d**2*log(sin(e + f*x) + 1)*sin(e + f
*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*c
*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3
*f*sin(e + f*x) + 2*a**3*f) + 6*c*d**2*log(sin(e + f*x) + 1)/(2*a**3*f*sin(
e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*c*d**2*sin(e + f*x)/(2
*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 9*c*d**2/(2*a
**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 6*d**3*log(sin(
e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e +
f*x) + 2*a**3*f) - 12*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin
(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 6*d**3*log(sin(e + f*x)
+ 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 2*d**3
*sin(e + f*x)**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3
*f) - 12*d**3*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x
) + 2*a**3*f) - 9*d**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) +
2*a**3*f), Eq(m, -3)), (-2*c**3/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*c**2
*d*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) +
6*c**2*d*log(sin(e + f*x) + 1)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*c**2*
d/(2*a**2*f*sin(e + f*x) + 2*a**2*f) - 12*c*d**2*log(sin(e + f*x) + 1)*sin(
e + f*x)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) - 12*c*d**2*log(sin(e + f*x) +
1)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) - 6*c*d**2*sin(e + f*x)**3/(2*a**2*f*
sin(e + f*x) + 2*a**2*f) - 6*c*d**2*sin(e + f*x)*cos(e + f*x)**2/(2*a**2*f*
sin(e + f*x) + 2*a**2*f) - 6*c*d**2*cos(e + f*x)**2/(2*a**2*f*sin(e + f*x)
+ 2*a**2*f) - 12*c*d**2/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*d**3*log(sin
(e + f*x) + 1)*sin(e + f*x)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*d**3*log
```

```
(sin(e + f*x) + 1)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 4*d**3*sin(e + f*x)
**3/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 3*d**3*sin(e + f*x)*cos(e + f*x)**
2/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 3*d**3*cos(e + f*x)**2/(2*a**2*f*sin
(e + f*x) + 2*a**2*f) + 6*d**3/(2*a**2*f*sin(e + f*x) + 2*a**2*f), Eq(m, -2
)), (c**3*log(sin(e + f*x) + 1)/(a*f) - 3*c**2*d*log(sin(e + f*x) + 1)/(a*f
) + 3*c**2*d*sin(e + f*x)/(a*f) + 3*c*d**2*log(sin(e + f*x) + 1)/(a*f) - 3*
c*d**2*sin(e + f*x)/(a*f) - 3*c*d**2*cos(e + f*x)**2/(2*a*f) - d**3*log(sin
(e + f*x) + 1)/(a*f) + d**3*sin(e + f*x)**3/(3*a*f) + d**3*sin(e + f*x)/(a*
f) + d**3*cos(e + f*x)**2/(2*a*f), Eq(m, -1)), (c**3*m**3*(a*sin(e + f*x) +
 a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + c**3
*m**3*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24
*f) + 9*c**3*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3
+ 35*f*m**2 + 50*f*m + 24*f) + 9*c**3*m**2*(a*sin(e + f*x) + a)**m/(f*m**4
+ 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 26*c**3*m*(a*sin(e + f*x) + a)**
m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 26*c**3*m
*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) +
 24*c**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m*
*2 + 50*f*m + 24*f) + 24*c**3*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 +
 35*f*m**2 + 50*f*m + 24*f) + 3*c**2*d*m**3*(a*sin(e + f*x) + a)**m*sin(e +
 f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 3*c**2*d*m**3*(
a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*
m + 24*f) + 24*c**2*d*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4
+ 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 21*c**2*d*m**2*(a*sin(e + f*x) +
 a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 3*c*
*2*d*m**2*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m
+ 24*f) + 57*c**2*d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*
f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 36*c**2*d*m*(a*sin(e + f*x) + a)**m*s
in(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 21*c**2*d*m*
(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) +
36*c**2*d*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*
f*m**2 + 50*f*m + 24*f) - 36*c**2*d*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*
m**3 + 35*f*m**2 + 50*f*m + 24*f) + 3*c*d**2*m**3*(a*sin(e + f*x) + a)**m*s
in(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 3*c*d**2*
m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**
2 + 50*f*m + 24*f) + 21*c*d**2*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3
/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 15*c*d**2*m**2*(a*sin(e
 + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m +
24*f) - 6*c*d**2*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m
**3 + 35*f*m**2 + 50*f*m + 24*f) + 42*c*d**2*m*(a*sin(e + f*x) + a)**m*sin(
e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 12*c*d**2*m*
(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 5
0*f*m + 24*f) - 24*c*d**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 +
10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 6*c*d**2*m*(a*sin(e + f*x) + a)**m
/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 24*c*d**2*(a*sin(e + f*
x) + a)**m*sin(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f)
 + 24*c*d**2*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f
*m + 24*f) + d**3*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**4 + 10
*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + d**3*m**3*(a*sin(e + f*x) + a)**m*si
n(e + f*x)**3/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 6*d**3*m**
2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**4 + 10*f*m**3 + 35*f*m**2 +
 50*f*m + 24*f) + 3*d**3*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m*
*4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 3*d**3*m**2*(a*sin(e + f*x) +
 a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 1
1*d**3*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**4 + 10*f*m**3 + 35*f
*m**2 + 50*f*m + 24*f) + 2*d**3*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(
f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 3*d**3*m*(a*sin(e + f*x)
+ a)**m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) +
6*d**3*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m*
*2 + 50*f*m + 24*f) + 6*d**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4/(f*m**
```

```
4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) - 6*d**3*(a*sin(e + f*x) + a)**m
/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f), True))
```

Giac [B] time = 1.20513, size = 1354, normalized size = 10.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] (3*((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m^2 - 2*(a*sin(f*x + e) +
 a)^2*(a*sin(f*x + e) + a)^m*a*m^2 + (a*sin(f*x + e) + a)*(a*sin(f*x + e) +
 a)^m*a^2*m^2 + 3*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 8*(a*si
n(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a*m + 5*(a*sin(f*x + e) + a)*(a*si
n(f*x + e) + a)^m*a^2*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m -
 6*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a + 6*(a*sin(f*x + e) + a)
*(a*sin(f*x + e) + a)^m*a^2)*c*d^2/(a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^2)
 + ((a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^3 - 3*(a*sin(f*x + e) +
 a)^3*(a*sin(f*x + e) + a)^m*a*m^3 + 3*(a*sin(f*x + e) + a)^2*(a*sin(f*x +
e) + a)^m*a^2*m^3 - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^3 + 6
*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^2 - 21*(a*sin(f*x + e) + a
)^3*(a*sin(f*x + e) + a)^m*a*m^2 + 24*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e
) + a)^m*a^2*m^2 - 9*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^2 +
11*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m - 42*(a*sin(f*x + e) + a
)^3*(a*sin(f*x + e) + a)^m*a*m + 57*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e)
+ a)^m*a^2*m - 26*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m + 6*(a*
sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m - 24*(a*sin(f*x + e) + a)^3*(a*s
in(f*x + e) + a)^m*a + 36*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^2
 - 24*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3)*d^3/(a^3*m^4 + 10*a^
3*m^3 + 35*a^3*m^2 + 50*a^3*m + 24*a^3) + (a*sin(f*x + e) + a)^(m + 1)*c^3/
(m + 1) + 3*((a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*m - (a*sin(f*x +
 e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*sin(f*x + e) + a)^2*(a*sin(f*x + e
) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a)*c^2*d/((m^2 + 3
*m + 2)*a))/(a*f)
```

3.922 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=96

$$\frac{2d(c-d)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{d^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{(c-d)^2(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

```
[Out] ((c - d)^2*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (2*(c - d)*d*(a +
a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (d^2*(a + a*Sin[e + f*x])^(3 + m
))/(a^3*f*(3 + m))
```

Rubi [A] time = 0.111723, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 43}

$$\frac{2d(c-d)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{d^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{(c-d)^2(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((c - d)^2*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (2*(c - d)*d*(a +
a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (d^2*(a + a*Sin[e + f*x])^(3 + m
))/(a^3*f*(3 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx=\frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^2\,dx,x,a\sin(e+fx)\right)}{af}$$
$$=\frac{\text{Subst}\left(\int\left((c-d)^2(a+x)^m+\frac{2(c-d)d(a+x)^{1+m}}{a}+\frac{d^2(a+x)^{2+m}}{a^2}\right)dx,\right.}{af}$$
$$=\frac{(c-d)^2(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{2(c-d)d(a+a\sin(e+fx))}{a^2f(2+m)}$$

Mathematica [A] time = 0.393096, size = 83, normalized size = 0.86

$$\frac{(a(\sin(e+fx)+1))^{m+1}\left(\frac{2a^2d(c-d)(\sin(e+fx)+1)}{m+2}+\frac{a^2(c-d)^2}{m+1}+\frac{d^2(a\sin(e+fx)+a)^2}{m+3}\right)}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*((a^2*(c - d)^2)/(1 + m) + (2*a^2*(c - d)*d
*(1 + Sin[e + f*x]))/(2 + m) + (d^2*(a + a*Sin[e + f*x])^2)/(3 + m)))/(a^3*
f)
```

Maple [F] time = 2.322, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27003, size = 427, normalized size = 4.45

$$\frac{\left(\left(c^2+2\,cd+d^2\right)m^2-\left(\left(2\,cd+d^2\right)m^2+6\,cd+\left(8\,cd+d^2\right)m\right)\cos\left(fx+e\right)^2+6\,c^2+2\,d^2+\left(5\,c^2+6\,cd+d^2\right)m+\left(\left(c^2\right.\right.\right.}{fm^3+6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] ((c^2 + 2*c*d + d^2)*m^2 - ((2*c*d + d^2)*m^2 + 6*c*d + (8*c*d + d^2)*m)*co
s(f*x + e)^2 + 6*c^2 + 2*d^2 + (5*c^2 + 6*c*d + d^2)*m + ((c^2 + 2*c*d + d^
2)*m^2 - (d^2*m^2 + 3*d^2*m + 2*d^2)*cos(f*x + e)^2 + 6*c^2 + 2*d^2 + (5*c^
2 + 6*c*d + d^2)*m)*sin(f*x + e))*(a*sin(f*x + e) + a)^m/(f*m^3 + 6*f*m^2 +
 11*f*m + 6*f)
```

Sympy [A] time = 28.4177, size = 1686, normalized size = 17.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((x*(c + d*sin(e))**2*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-c**2/
(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 4*c*d*sin(e
 + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 2*c
*d/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 2*d**2*l
og(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*s
in(e + f*x) + 2*a**3*f) + 4*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3
*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 2*d**2*log(sin(e +
 f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) +
4*d**2*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a
**3*f) + 3*d**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*
f), Eq(m, -3)), (-c**2/(a**2*f*sin(e + f*x) + a**2*f) + 2*c*d*log(sin(e + f
*x) + 1)*sin(e + f*x)/(a**2*f*sin(e + f*x) + a**2*f) + 2*c*d*log(sin(e + f*
x) + 1)/(a**2*f*sin(e + f*x) + a**2*f) + 2*c*d/(a**2*f*sin(e + f*x) + a**2*
f) - 2*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(a**2*f*sin(e + f*x) + a**2*
f) - 2*d**2*log(sin(e + f*x) + 1)/(a**2*f*sin(e + f*x) + a**2*f) - d**2*sin
(e + f*x)**3/(a**2*f*sin(e + f*x) + a**2*f) - d**2*sin(e + f*x)*cos(e + f*x
)**2/(a**2*f*sin(e + f*x) + a**2*f) - d**2*cos(e + f*x)**2/(a**2*f*sin(e +
f*x) + a**2*f) - 2*d**2/(a**2*f*sin(e + f*x) + a**2*f), Eq(m, -2)), (c**2*l
og(sin(e + f*x) + 1)/(a*f) - 2*c*d*log(sin(e + f*x) + 1)/(a*f) + 2*c*d*sin(
e + f*x)/(a*f) + d**2*log(sin(e + f*x) + 1)/(a*f) - d**2*sin(e + f*x)/(a*f)
 - d**2*cos(e + f*x)**2/(2*a*f), Eq(m, -1)), (c**2*m**2*(a*sin(e + f*x) + a
)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + c**2*m**2*(a*sin(e +
 f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 5*c**2*m*(a*sin(e + f*x)
 + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 5*c**2*m*(a*sin(
e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 6*c**2*(a*sin(e + f*x
) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 6*c**2*(a*sin(e
 + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 2*c*d*m**2*(a*sin(e +
f*x) + a)**m*sin(e + f*x)**2/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 2*c*d*m**
2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) +
 8*c*d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**3 + 6*f*m**2 + 11*f*
m + 6*f) + 6*c*d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2
+ 11*f*m + 6*f) - 2*c*d*m*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f
*m + 6*f) + 6*c*d*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**3 + 6*f*m**
2 + 11*f*m + 6*f) - 6*c*d*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f
*m + 6*f) + d**2*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**3 + 6*f
*m**2 + 11*f*m + 6*f) + d**2*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(
f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 3*d**2*m*(a*sin(e + f*x) + a)**m*sin(e
+ f*x)**3/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + d**2*m*(a*sin(e + f*x) + a)*
*m*sin(e + f*x)**2/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) - 2*d**2*m*(a*sin(e +
 f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 2*d**2*(a*s
in(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 2*
d**2*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f), True))
```

Giac [B] time = 1.23389, size = 624, normalized size = 6.5

$$\left(\left(a\sin\left(fx+e\right)+a\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}m^{2}-2\left(a\sin\left(fx+e\right)+a\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}am^{2}+\left(a\sin\left(fx+e\right)+a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}a^{2}m^{2}+3\left(a\sin\left(fx+e\right)+a\right)^{3}\left(a\sin\left(fx+e\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m^2 - 2*(a*sin(f*x + e) + a
)^2*(a*sin(f*x + e) + a)^m*a*m^2 + (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a
)^m*a^2*m^2 + 3*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 8*(a*sin(
f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a*m + 5*(a*sin(f*x + e) + a)*(a*sin(
f*x + e) + a)^m*a^2*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m - 6
*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a + 6*(a*sin(f*x + e) + a)*(
a*sin(f*x + e) + a)^m*a^2)*d^2/(a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^2) + (
a*sin(f*x + e) + a)^(m + 1)*c^2/(m + 1) + 2*((a*sin(f*x + e) + a)^2*(a*sin(
f*x + e) + a)^m*m - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*si
n(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*
x + e) + a)^m*a)*c*d/((m^2 + 3*m + 2)*a))/(a*f)
```

3.923 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=59

$$\frac{d(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

```
[Out] ((c - d)*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (d*(a + a*Sin[e + f*
x])^(2 + m))/(a^2*f*(2 + m))
```

Rubi [A] time = 0.0662621, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{d(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((c - d)*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (d*(a + a*Sin[e + f*
x])^(2 + m))/(a^2*f*(2 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)dx,x,a\sin(e+fx)\right)}{af}\\ &= \frac{\text{Subst}\left(\int \left((c-d)(a+x)^m+\frac{d(a+x)^{1+m}}{a}\right)dx,x,a\sin(e+fx)\right)}{af}\\ &= \frac{(c-d)(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{d(a+a\sin(e+fx))^{2+m}}{a^2f(2+m)}\end{aligned}$$

Mathematica [A] time = 0.120484, size = 51, normalized size = 0.86

$$\frac{(a(\sin(e+fx)+1))^{m+1}(c(m+2)+d(m+1)\sin(e+fx)-d)}{af(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*(-d + c*(2 + m) + d*(1 + m)*Sin[e + f*x]))/
(a*f*(1 + m)*(2 + m))
```

Maple [F] time = 1.645, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04999, size = 169, normalized size = 2.86

$$-\frac{\left((dm+d)\cos\left(fx+e\right)^{2}-(c+d)m-((c+d)m+2\,c)\sin\left(fx+e\right)-2\,c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{fm^{2}+3\,fm+2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -((d*m + d)*cos(f*x + e)^2 - (c + d)*m - ((c + d)*m + 2*c)*sin(f*x + e) - 2
*c)*(a*sin(f*x + e) + a)^m/(f*m^2 + 3*f*m + 2*f)
```

Sympy [A] time = 9.60676, size = 428, normalized size = 7.25

$$\begin{cases} x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^{m}\cos\left(e\right) \\ -\frac{c}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{d\log\left(\sin\left(e+fx\right)+1\right)\sin\left(e+fx\right)}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{d\log\left(\sin\left(e+fx\right)+1\right)}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{d}{a^{2}f\sin\left(e+fx\right)+a^{2}f} \\ \frac{c\log\left(\sin\left(e+fx\right)+1\right)}{af}-\frac{d\log\left(\sin\left(e+fx\right)+1\right)}{af}+\frac{d\sin\left(e+fx\right)}{af} \\ \frac{cm\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{cm\left(a\sin\left(e+fx\right)+a\right)^{m}}{fm^{2}+3fm+2f}+\frac{2c\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{2c\left(a\sin\left(e+fx\right)+a\right)^{m}}{fm^{2}+3fm+2f}+\frac{dm\left(a\sin\left(e+fx\right)+a\right)^{m}\sin^{2}\left(e\right.}{fm^{2}+3fm+2f} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((x*(c + d*sin(e))*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-c/(a**2*
f*sin(e + f*x) + a**2*f) + d*log(sin(e + f*x) + 1)*sin(e + f*x)/(a**2*f*sin
(e + f*x) + a**2*f) + d*log(sin(e + f*x) + 1)/(a**2*f*sin(e + f*x) + a**2*f
) + d/(a**2*f*sin(e + f*x) + a**2*f), Eq(m, -2)), (c*log(sin(e + f*x) + 1)/
(a*f) - d*log(sin(e + f*x) + 1)/(a*f) + d*sin(e + f*x)/(a*f), Eq(m, -1)), (
c*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + c*m*(a*si
n(e + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f) + 2*c*(a*sin(e + f*x) + a)**m*sin
(e + f*x)/(f*m**2 + 3*f*m + 2*f) + 2*c*(a*sin(e + f*x) + a)**m/(f*m**2 + 3*
f*m + 2*f) + d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m +
2*f) + d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + d*
(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m + 2*f) - d*(a*sin(e
 + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f), True))
```

Giac [B] time = 1.21849, size = 211, normalized size = 3.58

$$\frac{\frac{\left(a\sin\left(fx+e\right)+a\right)^{m+1}c}{m+1}+\frac{\left(\left(a\sin\left(fx+e\right)+a\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}m-\left(a\sin\left(fx+e\right)+a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}am+\left(a\sin\left(fx+e\right)+a\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}-2\left(a\sin\left(fx+e\right)+a\right. \cdots}{\left(m^{2}+3m+2\right)a}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] ((a*sin(f*x + e) + a)^(m + 1)*c/(m + 1) + ((a*sin(f*x + e) + a)^2*(a*sin(f*
x + e) + a)^m*m - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*sin(
f*x + e) + a)^2*(a*sin(f*x + e) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*x
+ e) + a)^m*a)*d/((m^2 + 3*m + 2)*a))/(a*f)
```

3.924 $\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)}$$

```
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)*f*(1 + m))
```

Rubi [A] time = 0.102836, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)*f*(1 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{c+\frac{dx}{a}}\,dx,x,a\sin(e+fx)\right)}{af}\\ &= \frac{{}_2F_1\left(1,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)f(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0970181, size = 59, normalized size = 1.

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)*f*(1 + m))
```

Maple [F] time = 1.079, size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c), x)
```

3.925 $\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=59

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^2}$$

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^2*f*(1 + m))
```

Rubi [A] time = 0.0971774, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^2*f*(1 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{\left(c+\frac{dx}{a}\right)^2}\,dx, x, a\sin(e+fx)\right)}{af}\\ &= \frac{{}_2F_1\left(2,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^2f(1+m)}\end{aligned}$$

Mathematica [A] time = 0.102513, size = 59, normalized size = 1.

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^2*f*(1 + m))
```

Maple [F] time = 1.174, size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c+d\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)/(d^2*cos(f*x + e)^2 - 2*c*d*s
in(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^2, x)
```

3.926 $\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=59

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^3}$$

```
[Out] (Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^3*f*(1 + m))
```

Rubi [A] time = 0.103192, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2833, 68}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^3*f*(1 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{\left(c+\frac{dx}{a}\right)^3}\,dx, x, a\sin(e+fx)\right)}{af}\\ &= \frac{{}_2F_1\left(3,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^3f(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0904704, size = 59, normalized size = 1.

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^3*f*(1 + m))
```

Maple [F] time = 1.373, size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c+d\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)/(3*c*d^2*cos(f*x + e)^2 - c^3
 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^3, x)
```

3.927 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=54

$$-\frac{\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(1,m+n+2;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

```
[Out] -((Hypergeometric2F1[1, 2 + m + n, 2 + m, 1 + Sin[c + d*x]]*Sin[c + d*x]^(1
 + n)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*(1 + m)))
```

Rubi [A] time = 0.0778548, antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 66, 64}

$$\frac{(\sin(c+dx)+1)^{-m}\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^m\,{}_2F_1(-m,n+1;n+2;-\sin(c+dx))}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[-m, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a
 + a*Sin[c + d*x])^m)/(d*(1 + n)*(1 + Sin[c + d*x])^m)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^m\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\left((1+\sin(c+dx))^{-m}(a+a\sin(c+dx))^m\right)\text{Subst}\left(\int \left(\frac{x}{a}\right)^n \left(1+\frac{x}{a}\right)^m d\right.}{ad}$$

$$= \frac{{}_2F_1(-m,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)(1+\sin(c+dx))^{-}}{d(1+n)}$$

Mathematica [A] time = 0.0799043, size = 61, normalized size = 1.13

$$\frac{(\sin(c+dx)+1)^{-m}\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^m\,{}_2F_1(-m,n+1;n+2;-\sin(c+dx))}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[-m, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a
 + a*Sin[c + d*x])^m)/(d*(1 + n)*(1 + Sin[c + d*x])^m)
```

Maple [F] time = 3.917, size = 0, normalized size = 0.

$$\int \cos(dx+c)(\sin(dx+c))^n(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m\sin(dx+c)^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sin(d*x + c)^n*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin(dx+c)+a)^m\sin(dx+c)^n\cos(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sin(d*x + c)^n*cos(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m \sin(dx+c)^n \cos(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sin(d*x + c)^n*cos(d*x + c), x)
```

3.928 $\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$-\frac{4(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{6(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}+\frac{(a\sin(c+\cdots}{ad(m\cdots}$$

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m)) - (4*(a + a*Sin[c + d*x])^(2 + m
))/(a^2*d*(2 + m)) + (6*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) - (4*
(a + a*Sin[c + d*x])^(4 + m))/(a^4*d*(4 + m)) + (a + a*Sin[c + d*x])^(5 + m
)/(a^5*d*(5 + m))
```

Rubi [A] time = 0.119252, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{4(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{6(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}+\frac{(a\sin(c+\cdots}{ad(m\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m)) - (4*(a + a*Sin[c + d*x])^(2 + m
))/(a^2*d*(2 + m)) + (6*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) - (4*
(a + a*Sin[c + d*x])^(4 + m))/(a^4*d*(4 + m)) + (a + a*Sin[c + d*x])^(5 + m
)/(a^5*d*(5 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+x)^m}{a^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^4(a+x)^m\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4(a+x)^m - 4a^3(a+x)^{1+m} + 6a^2(a+x)^{2+m} - 4a(a+x)^3\right.\right.}{a^5 d}$$
$$= \frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)} - \frac{4(a+a\sin(c+dx))^{2+m}}{a^2 d(2+m)} + \frac{6(a+a\sin(c+}{a^3 d(3+m}$$

Mathematica [A] time = 1.40365, size = 150, normalized size = 1.12

$$\frac{(a(\sin(c+dx)+1))^{m+1}\left(\frac{3\left(-2\left(m^2+3m+2\right)\cos(2(c+dx))-8(m+1)\sin(c+dx)+m^2+m+6\right)}{(m+1)(m+2)(m+3)} + \frac{16(\sin(c+dx)+1)^4}{m+5} - \frac{64(\sin(c+dx)+1)^3}{m+4} + \frac{84(\sin(c+dx)+1)}{m+3}\right.}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(1 + m)*(7/(1 + m) - (40*(1 + Sin[c + d*x]))/(2 + m
) + (84*(1 + Sin[c + d*x])^2)/(3 + m) - (64*(1 + Sin[c + d*x])^3)/(4 + m) +
 (16*(1 + Sin[c + d*x])^4)/(5 + m) + (3*(6 + m + m^2 - 2*(2 + 3*m + m^2)*Co
s[2*(c + d*x)] - 8*(1 + m)*Sin[c + d*x]))/((1 + m)*(2 + m)*(3 + m))))/(16*a
*d)
```

Maple [F] time = 2.483, size = 0, normalized size = 0.

$$\int \cos(dx+c)(\sin(dx+c))^4(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

Maxima [A] time = 1.07683, size = 215, normalized size = 1.6

$$\frac{\left(\left(m^4+10m^3+35m^2+50m+24\right)a^m\sin(dx+c)^5 + \left(m^4+6m^3+11m^2+6m\right)a^m\sin(dx+c)^4 - 4\left(m^3+3m^2+2m\right)a\right.}{\left(m^5+15m^4+85m^3+225m^2+27\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*a^m*sin(d*x + c)^5 + (m^4 + 6*m^3 + 11
*m^2 + 6*m)*a^m*sin(d*x + c)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^m*sin(d*x + c)^3 +
 12*(m^2 + m)*a^m*sin(d*x + c)^2 - 24*a^m*m*sin(d*x + c) + 24*a^m)*(sin(d*x
 + c) + 1)^m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*d)
```

Fricas [A] time = 1.85389, size = 501, normalized size = 3.74

$$\frac{\left(\left(m^4+6\,m^3+11\,m^2+6\,m\right)\cos\left(dx+c\right)^4+m^4+6\,m^3-2\left(m^4+6\,m^3+17\,m^2+12\,m\right)\cos\left(dx+c\right)^2+23\,m^2+\left(\left(m^4\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^4 + 6*m^3 + 11*m^2 + 6*m)*cos(d*x + c)^4 + m^4 + 6*m^3 - 2*(m^4 + 6*m^3
 + 17*m^2 + 12*m)*cos(d*x + c)^2 + 23*m^2 + ((m^4 + 10*m^3 + 35*m^2 + 50*m
+ 24)*cos(d*x + c)^4 + m^4 + 6*m^3 - 2*(m^4 + 8*m^3 + 29*m^2 + 46*m + 24)*c
os(d*x + c)^2 + 23*m^2 + 18*m + 24)*sin(d*x + c) + 18*m + 24)*(a*sin(d*x +
c) + a)^m/(d*m^5 + 15*d*m^4 + 85*d*m^3 + 225*d*m^2 + 274*d*m + 120*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27006, size = 1068, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^5*(a*sin(d*x + c) + a)^m*m^4 - 4*(a*sin(d*x + c) + a)
^4*(a*sin(d*x + c) + a)^m*a*m^4 + 6*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c)
+ a)^m*a^2*m^4 - 4*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^3*m^4 +
(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^4*m^4 + 10*(a*sin(d*x + c) +
a)^5*(a*sin(d*x + c) + a)^m*m^3 - 44*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c)
 + a)^m*a*m^3 + 72*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*a^2*m^3 -
52*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^3*m^3 + 14*(a*sin(d*x +
c) + a)*(a*sin(d*x + c) + a)^m*a^4*m^3 + 35*(a*sin(d*x + c) + a)^5*(a*sin(d
*x + c) + a)^m*m^2 - 164*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a*m^
2 + 294*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*a^2*m^2 - 236*(a*sin(
d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^3*m^2 + 71*(a*sin(d*x + c) + a)*(a
*sin(d*x + c) + a)^m*a^4*m^2 + 50*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c) +
a)^m*m - 244*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a*m + 468*(a*sin
(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*a^2*m - 428*(a*sin(d*x + c) + a)^2*
(a*sin(d*x + c) + a)^m*a^3*m + 154*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a
)^m*a^4*m + 24*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c) + a)^m - 120*(a*sin(d
*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a + 240*(a*sin(d*x + c) + a)^3*(a*sin
(d*x + c) + a)^m*a^2 - 240*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^
3 + 120*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^4)/((a^4*m^5 + 15*a^4
*m^4 + 85*a^4*m^3 + 225*a^4*m^2 + 274*a^4*m + 120*a^4)*a*d)
```

3.929 $\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=108

$$\frac{3(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{3(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}+\frac{(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

```
[Out] -((a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))) + (3*(a + a*Sin[c + d*x])^(2
+ m))/(a^2*d*(2 + m)) - (3*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) +
(a + a*Sin[c + d*x])^(4 + m)/(a^4*d*(4 + m))
```

Rubi [A] time = 0.0969453, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{3(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{3(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}+\frac{(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))) + (3*(a + a*Sin[c + d*x])^(2
+ m))/(a^2*d*(2 + m)) - (3*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) +
(a + a*Sin[c + d*x])^(4 + m)/(a^4*d*(4 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+x)^m}{a^3}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int x^3(a+x)^m\,dx,x,a\sin(c+dx)\right)}{a^4d}\\ &= \frac{\text{Subst}\left(\int \left(-a^3(a+x)^m+3a^2(a+x)^{1+m}-3a(a+x)^{2+m}+(a+x)^{3+m}\right.\right.}{a^4d}\\ &= -\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}+\frac{3(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}-\frac{3(a+a\sin(c+}{a^3d(3+}\end{aligned}$$

Mathematica [A] time = 0.571176, size = 94, normalized size = 0.87

$$\frac{\left(\left(m^3+6m^2+11m+6\right)\sin^3(c+dx)-3\left(m^2+3m+2\right)\sin^2(c+dx)+6(m+1)\sin(c+dx)-6\right)(a(\sin(c+dx)+1))^m}{ad(m+1)(m+2)(m+3)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(1 + m)*(-6 + 6*(1 + m)*Sin[c + d*x] - 3*(2 + 3*m +
 m^2)*Sin[c + d*x]^2 + (6 + 11*m + 6*m^2 + m^3)*Sin[c + d*x]^3))/(a*d*(1 +
m)*(2 + m)*(3 + m)*(4 + m))
```

Maple [F] time = 1.652, size = 0, normalized size = 0.

$$\int \cos(dx+c)(\sin(dx+c))^3(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

Maxima [A] time = 1.03799, size = 161, normalized size = 1.49

$$\frac{\left(\left(m^3+6m^2+11m+6\right)a^m\sin(dx+c)^4+\left(m^3+3m^2+2m\right)a^m\sin(dx+c)^3-3\left(m^2+m\right)a^m\sin(dx+c)^2+6a^m m\sin\ldots}{\left(m^4+10m^3+35m^2+50m+24\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(d*x + c)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(
d*x + c)^3 - 3*(m^2 + m)*a^m*sin(d*x + c)^2 + 6*a^m*m*sin(d*x + c) - 6*a^m)
*(sin(d*x + c) + 1)^m/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*d)
```

Fricas [A] time = 1.80017, size = 336, normalized size = 3.11

$$\frac{\left(\left(m^3+6m^2+11m+6\right)\cos(dx+c)^4+m^3-\left(2m^3+9m^2+19m+12\right)\cos(dx+c)^2+3m^2+\left(m^3-\left(m^3+3m^2+\ldots\right.\right.\right.}{dm^4+10dm^3+35dm^2+50dm+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^3 + 6*m^2 + 11*m + 6)*cos(d*x + c)^4 + m^3 - (2*m^3 + 9*m^2 + 19*m + 12
)*cos(d*x + c)^2 + 3*m^2 + (m^3 - (m^3 + 3*m^2 + 2*m)*cos(d*x + c)^2 + 3*m^
2 + 8*m)*sin(d*x + c) + 8*m)*(a*sin(d*x + c) + a)^m/(d*m^4 + 10*d*m^3 + 35*
d*m^2 + 50*d*m + 24*d)
```

Sympy [A] time = 70.7406, size = 1547, normalized size = 14.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)**3*cos(c), Eq(d, 0)), (6*log(sin(c +
d*x) + 1)*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x
)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*log(sin(c + d*x) + 1)*sin(c
+ d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d
*sin(c + d*x) + 6*a**4*d) + 18*log(sin(c + d*x) + 1)*sin(c + d*x)/(6*a**4*d
*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a
**4*d) + 6*log(sin(c + d*x) + 1)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(
c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) - 6*sin(c + d*x)**3/(6*a**
4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) +
6*a**4*d) + 9*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*
x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 5/(6*a**4*d*sin(c + d*x)**3 +
18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d), Eq(m, -4)),
 (-6*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a*
*3*d*sin(c + d*x) + 2*a**3*d) - 12*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a*
*3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 6*log(sin(c + d*
x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*s
in(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d
) - 12*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a
**3*d) - 9/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), E
q(m, -3)), (6*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**2*d*sin(c + d*x) + 2
*a**2*d) + 6*log(sin(c + d*x) + 1)/(2*a**2*d*sin(c + d*x) + 2*a**2*d) + 4*s
in(c + d*x)**3/(2*a**2*d*sin(c + d*x) + 2*a**2*d) + 3*sin(c + d*x)*cos(c +
d*x)**2/(2*a**2*d*sin(c + d*x) + 2*a**2*d) + 3*cos(c + d*x)**2/(2*a**2*d*si
n(c + d*x) + 2*a**2*d) + 6/(2*a**2*d*sin(c + d*x) + 2*a**2*d), Eq(m, -2)),
(-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**3/(3*a*d) + sin(c + d*x)/(a*d
) + cos(c + d*x)**2/(2*a*d), Eq(m, -1)), (m**3*(a*sin(c + d*x) + a)**m*sin(
c + d*x)**4/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) + m**3*(a*sin(
c + d*x) + a)**m*sin(c + d*x)**3/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m +
 24*d) + 6*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**4 + 10*d*m**3
 + 35*d*m**2 + 50*d*m + 24*d) + 3*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)
**3/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) - 3*m**2*(a*sin(c + d*
x) + a)**m*sin(c + d*x)**2/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d)
 + 11*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**4 + 10*d*m**3 + 35*d*
m**2 + 50*d*m + 24*d) + 2*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**4
 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) - 3*m*(a*sin(c + d*x) + a)**m*sin
(c + d*x)**2/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) + 6*m*(a*sin(
c + d*x) + a)**m*sin(c + d*x)/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24
*d) + 6*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**4 + 10*d*m**3 + 35*d*
m**2 + 50*d*m + 24*d) - 6*(a*sin(c + d*x) + a)**m/(d*m**4 + 10*d*m**3 + 35*
d*m**2 + 50*d*m + 24*d), True))
```

Giac [B] time = 1.24764, size = 686, normalized size = 6.35

$$\frac{(a\sin(dx+c)+a)^4(a\sin(dx+c)+a)^m m^3 - 3(a\sin(dx+c)+a)^3(a\sin(dx+c)+a)^m a m^3 + 3(a\sin(dx+c)+a)^2(a\sin\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*m^3 - 3*(a*sin(d*x + c) + a)
^3*(a*sin(d*x + c) + a)^m*a*m^3 + 3*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c)
+ a)^m*a^2*m^3 - (a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^3*m^3 + 6*(a
*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*m^2 - 21*(a*sin(d*x + c) + a)^3
*(a*sin(d*x + c) + a)^m*a*m^2 + 24*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) +
 a)^m*a^2*m^2 - 9*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^3*m^2 + 11*
(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*m - 42*(a*sin(d*x + c) + a)^3
*(a*sin(d*x + c) + a)^m*a*m + 57*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a
)^m*a^2*m - 26*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^3*m + 6*(a*sin
(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m - 24*(a*sin(d*x + c) + a)^3*(a*sin(
d*x + c) + a)^m*a + 36*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^2 -
24*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^3)/((a^3*m^4 + 10*a^3*m^3
+ 35*a^3*m^2 + 50*a^3*m + 24*a^3)*a*d)
```

3.930 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=80

$$-\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}+\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m)) - (2*(a + a*Sin[c + d*x])^(2 + m
))/(a^2*d*(2 + m)) + (a + a*Sin[c + d*x])^(3 + m)/(a^3*d*(3 + m))
```

Rubi [A] time = 0.0842618, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$-\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}+\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m)) - (2*(a + a*Sin[c + d*x])^(2 + m
))/(a^2*d*(2 + m)) + (a + a*Sin[c + d*x])^(3 + m)/(a^3*d*(3 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+x)^m}{a^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int x^2(a+x)^m\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(a^2(a+x)^m-2a(a+x)^{1+m}+(a+x)^{2+m}\right)dx,x,a\sin(c+\right.}{a^3d}\\ &= \frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}-\frac{2(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}+\frac{(a+a\sin(c+d}{a^3d(3+m}\end{aligned}$$

Mathematica [A] time = 0.187009, size = 77, normalized size = 0.96

$$-\frac{(a(\sin(c+dx)+1))^{m+1}\left(\left(m^2+3m+2\right)\cos(2(c+dx))+4(m+1)\sin(c+dx)-m^2-3m-6\right)}{2ad(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a*(1 + Sin[c + d*x]))^(1 + m)*(-6 - 3*m - m^2 + (2 + 3*m + m^2)*Cos[2*(c
 + d*x)] + 4*(1 + m)*Sin[c + d*x]))/(2*a*d*(1 + m)*(2 + m)*(3 + m))
```

Maple [F] time = 1.47, size = 0, normalized size = 0.

$$\int \cos(dx+c)\left(\sin(dx+c)\right)^2\left(a+a\sin(dx+c)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

Maxima [A] time = 1.04014, size = 113, normalized size = 1.41

$$\frac{\left(\left(m^2+3\,m+2\right)a^m\sin(dx+c)^3+\left(m^2+m\right)a^m\sin(dx+c)^2-2\,a^m m\sin(dx+c)+2\,a^m\right)\left(\sin(dx+c)+1\right)^m}{\left(m^3+6\,m^2+11\,m+6\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^2 + 3*m + 2)*a^m*sin(d*x + c)^3 + (m^2 + m)*a^m*sin(d*x + c)^2 - 2*a^m*
m*sin(d*x + c) + 2*a^m)*(sin(d*x + c) + 1)^m/((m^3 + 6*m^2 + 11*m + 6)*d)
```

Fricas [A] time = 1.8318, size = 217, normalized size = 2.71

$$-\frac{\left(\left(m^2+m\right)\cos(dx+c)^2-m^2+\left(\left(m^2+3\,m+2\right)\cos(dx+c)^2-m^2-m-2\right)\sin(dx+c)-m-2\right)\left(a\sin(dx+c)+a\right.}{dm^3+6\,dm^2+11\,dm+6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -((m^2 + m)*cos(d*x + c)^2 - m^2 + ((m^2 + 3*m + 2)*cos(d*x + c)^2 - m^2 -
m - 2)*sin(d*x + c) - m - 2)*(a*sin(d*x + c) + a)^m/(d*m^3 + 6*d*m^2 + 11*d
*m + 6*d)
```

Sympy [A] time = 21.4815, size = 756, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)**2*cos(c), Eq(d, 0)), (2*log(sin(c +
d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x)
 + 2*a**3*d) + 4*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*log(sin(c + d*x) + 1)/(2*a**3*d*
sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 4*sin(c + d*x)/(2*a**
3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 3/(2*a**3*d*sin(c
 + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Eq(m, -3)), (-2*log(sin(c +
 d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) - 2*log(sin(c + d*x)
 + 1)/(a**2*d*sin(c + d*x) + a**2*d) - sin(c + d*x)**3/(a**2*d*sin(c + d*x)
 + a**2*d) - sin(c + d*x)*cos(c + d*x)**2/(a**2*d*sin(c + d*x) + a**2*d) -
cos(c + d*x)**2/(a**2*d*sin(c + d*x) + a**2*d) - 2/(a**2*d*sin(c + d*x) + a
**2*d), Eq(m, -2)), (log(sin(c + d*x) + 1)/(a*d) - sin(c + d*x)/(a*d) - cos
(c + d*x)**2/(2*a*d), Eq(m, -1)), (m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x
)**3/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + m**2*(a*sin(c + d*x) + a)**m*sin(
c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 3*m*(a*sin(c + d*x) + a)**
m*sin(c + d*x)**3/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + m*(a*sin(c + d*x) +
a)**m*sin(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) - 2*m*(a*sin(c + d
*x) + a)**m*sin(c + d*x)/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 2*(a*sin(c +
d*x) + a)**m*sin(c + d*x)**3/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 2*(a*sin(
c + d*x) + a)**m/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d), True))
```

Giac [B] time = 1.25146, size = 387, normalized size = 4.84

$$(a\sin(dx+c)+a)^3(a\sin(dx+c)+a)^m m^2 - 2(a\sin(dx+c)+a)^2(a\sin(dx+c)+a)^m am^2 + (a\sin(dx+c)+a)(a\sin(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*m^2 - 2*(a*sin(d*x + c) + a)
^2*(a*sin(d*x + c) + a)^m*a*m^2 + (a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)
^m*a^2*m^2 + 3*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*m - 8*(a*sin(d
*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a*m + 5*(a*sin(d*x + c) + a)*(a*sin(d
*x + c) + a)^m*a^2*m + 2*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m - 6*
(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a + 6*(a*sin(d*x + c) + a)*(a
*sin(d*x + c) + a)^m*a^2)/((a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^2)*a*d)
```

3.931 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=54

$$\frac{(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

```
[Out] -((a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))) + (a + a*Sin[c + d*x])^(2 + m
)/(a^2*d*(2 + m))
```

Rubi [A] time = 0.0526407, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))) + (a + a*Sin[c + d*x])^(2 + m
)/(a^2*d*(2 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^m\,dx&=\frac{\text{Subst}\left(\int \frac{x(a+x)^m}{a}\,dx,x,a\sin(c+dx)\right)}{ad}\\&=\frac{\text{Subst}\left(\int x(a+x)^m\,dx,x,a\sin(c+dx)\right)}{a^2d}\\&=\frac{\text{Subst}\left(\int \left(-a(a+x)^m+(a+x)^{1+m}\right)dx,x,a\sin(c+dx)\right)}{a^2d}\\&=-\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}+\frac{(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}\end{aligned}$$

Mathematica [A] time = 0.0266722, size = 43, normalized size = 0.8

$$\frac{((m+1)\sin(c+dx)-1)(a(\sin(c+dx)+1))^{m+1}}{ad(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(1 + m)*(-1 + (1 + m)*Sin[c + d*x]))/(a*d*(1 + m)*(
2 + m))
```

Maple [F] time = 0.974, size = 0, normalized size = 0.

$$\int \cos(dx+c)\sin(dx+c)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

Maxima [A] time = 1.03093, size = 76, normalized size = 1.41

$$\frac{\left(a^m(m+1)\sin(dx+c)^2+a^m m\sin(dx+c)-a^m\right)(\sin(dx+c)+1)^m}{\left(m^2+3m+2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (a^m*(m + 1)*sin(d*x + c)^2 + a^m*m*sin(d*x + c) - a^m)*(sin(d*x + c) + 1)^
m/((m^2 + 3*m + 2)*d)
```

Fricas [A] time = 1.90926, size = 126, normalized size = 2.33

$$-\frac{\left((m+1)\cos(dx+c)^2-m\sin(dx+c)-m\right)(a\sin(dx+c)+a)^m}{dm^2+3\,dm+2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -((m + 1)*cos(d*x + c)^2 - m*sin(d*x + c) - m)*(a*sin(d*x + c) + a)^m/(d*m^
2 + 3*d*m + 2*d)
```

Sympy [A] time = 7.61462, size = 248, normalized size = 4.59

$$\begin{cases} x\left(a\sin\left(c\right)+a\right)^{m}\sin\left(c\right)\cos\left(c\right) & \text{for } d=0\\ \frac{\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}+\frac{\log\left(\sin\left(c+dx\right)+1\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}+\frac{1}{a^{2}d\sin\left(c+dx\right)+a^{2}d} & \text{for } m=-2\\ -\frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin\left(c+dx\right)}{ad} & \text{for } m=-1\\ \frac{m\left(a\sin\left(c+dx\right)+a\right)^{m}\sin^{2}\left(c+dx\right)}{dm^{2}+3dm+2d}+\frac{m\left(a\sin\left(c+dx\right)+a\right)^{m}\sin\left(c+dx\right)}{dm^{2}+3dm+2d}+\frac{\left(a\sin\left(c+dx\right)+a\right)^{m}\sin^{2}\left(c+dx\right)}{dm^{2}+3dm+2d}-\frac{\left(a\sin\left(c+dx\right)+a\right)^{m}}{dm^{2}+3dm+2d} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)*cos(c), Eq(d, 0)), (log(sin(c + d*x)
+ 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) + log(sin(c + d*x) + 1)/(a
**2*d*sin(c + d*x) + a**2*d) + 1/(a**2*d*sin(c + d*x) + a**2*d), Eq(m, -2))
, (-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)/(a*d), Eq(m, -1)), (m*(a*sin
(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**2 + 3*d*m + 2*d) + m*(a*sin(c + d*x
) + a)**m*sin(c + d*x)/(d*m**2 + 3*d*m + 2*d) + (a*sin(c + d*x) + a)**m*sin
(c + d*x)**2/(d*m**2 + 3*d*m + 2*d) - (a*sin(c + d*x) + a)**m/(d*m**2 + 3*d
*m + 2*d), True))
```

Giac [B] time = 1.22055, size = 162, normalized size = 3.

$$\frac{\left(a\sin\left(dx+c\right)+a\right)^{2}\left(a\sin\left(dx+c\right)+a\right)^{m}m-\left(a\sin\left(dx+c\right)+a\right)\left(a\sin\left(dx+c\right)+a\right)^{m}am+\left(a\sin\left(dx+c\right)+a\right)^{2}\left(a\sin\left(d\right.\right.}{\left(m^{2}+3m+2\right)a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*m - (a*sin(d*x + c) + a)*(a*
sin(d*x + c) + a)^m*a*m + (a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m - 2
*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a)/((m^2 + 3*m + 2)*a^2*d)
```

3.932 $\int \cot(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=43

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Rubi [A] time = 0.0414507, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 65}

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{{}_2F_1(1,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0547175, size = 43, normalized size = 1.

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Maple [F] time = 0.971, size = 0, normalized size = 0.

$$\int \cos(dx+c)\csc(dx+c)\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c), x)
```

3.933 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=42

$$\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(2,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x])^
(1 + m))/(a*d*(1 + m))
```

Rubi [A] time = 0.0579094, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 65}

$$\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(2,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x])^
(1 + m))/(a*d*(1 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{a^2(a+x)^m}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^m}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{{}_2F_1(2,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0553152, size = 42, normalized size = 1.

$$\frac{(a\sin(c+dx)+a)^{m+1}\,_2F_1(2,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x])^
(1 + m))/(a*d*(1 + m))
```

Maple [F] time = 0.668, size = 0, normalized size = 0.

$$\int \cos(dx+c)\left(\csc(dx+c)\right)^2\left(a+a\sin(dx+c)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^2, x)
```

3.934 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=43

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(3,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

```
[Out] -((Hypergeometric2F1[3, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Rubi [A] time = 0.0811075, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 65}

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(3,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[3, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{a^3(a+x)^m}{x^3}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{a^2\,\text{Subst}\left(\int \frac{(a+x)^m}{x^3}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{{}_2F_1(3,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.0603483, size = 43, normalized size = 1.

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(3,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[3, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Maple [F] time = 0.714, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(\csc(dx+c))^3\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(dx+c)+a)^m\cos(dx+c)\csc(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m \cos(dx+c)\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^3, x)
```

3.935 $\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=79

$$-\frac{a(c+d)\cos^3(e+fx)}{3f}+\frac{a(4c+d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ax(4c+d)-\frac{ad\sin(e+fx)\cos^3(e+fx)}{4f}$$

```
[Out] (a*(4*c + d)*x)/8 - (a*(c + d)*Cos[e + f*x]^3)/(3*f) + (a*(4*c + d)*Cos[e +
 f*x]*Sin[e + f*x])/(8*f) - (a*d*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rubi [A] time = 0.0918696, antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2669, 2635, 8}

$$-\frac{a(4c+d)\cos^3(e+fx)}{12f}+\frac{a(4c+d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ax(4c+d)-\frac{d\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(4*c + d)*x)/8 - (a*(4*c + d)*Cos[e + f*x]^3)/(12*f) + (a*(4*c + d)*Cos[
e + f*x]*Sin[e + f*x])/(8*f) - (d*Cos[e + f*x]^3*(a + a*Sin[e + f*x]))/(4*f
)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{d\cos^3(e+fx)(a+a\sin(e+fx))}{4f}+\frac{1}{4}(4c+d)\int \cos^2(e+fx$$
$$= -\frac{a(4c+d)\cos^3(e+fx)}{12f}-\frac{d\cos^3(e+fx)(a+a\sin(e+fx))}{4f}+$$
$$= -\frac{a(4c+d)\cos^3(e+fx)}{12f}+\frac{a(4c+d)\cos(e+fx)\sin(e+fx)}{8f}-$$
$$= \frac{1}{8}a(4c+d)x-\frac{a(4c+d)\cos^3(e+fx)}{12f}+\frac{a(4c+d)\cos(e+fx)s}{8f}$$

Mathematica [A] time = 0.608047, size = 64, normalized size = 0.81

$$-\frac{a(24(c+d)\cos(e+fx)+8(c+d)\cos(3(e+fx))-12fx(4c+d)-24c\sin(2(e+fx))+3d\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] -(a*(-12*(4*c + d)*f*x + 24*(c + d)*Cos[e + f*x] + 8*(c + d)*Cos[3*(e + f*x
)] - 24*c*Sin[2*(e + f*x)] + 3*d*Sin[4*(e + f*x)]))/(96*f)
```

Maple [A] time = 0.053, size = 96, normalized size = 1.2

$$\frac{1}{f}\left(da\left(-\frac{\sin(fx+e)(\cos(fx+e))^3}{4}+\frac{\sin(fx+e)\cos(fx+e)}{8}+\frac{fx}{8}+\frac{e}{8}\right)-\frac{(\cos(fx+e))^3ac}{3}-\frac{da(\cos(fx+e))^3}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(d*a*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/
8*e)-1/3*cos(f*x+e)^3*a*c-1/3*d*a*cos(f*x+e)^3+c*a*(1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 1.0376, size = 100, normalized size = 1.27

$$-\frac{32\,ac\cos(fx+e)^3+32\,ad\cos(fx+e)^3-24\,(2\,fx+2\,e+\sin(2\,fx+2\,e))ac-3\,(4\,fx+4\,e-\sin(4\,fx+4\,e))ad}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*a*c*cos(f*x + e)^3 + 32*a*d*cos(f*x + e)^3 - 24*(2*f*x + 2*e + si
n(2*f*x + 2*e))*a*c - 3*(4*f*x + 4*e - sin(4*f*x + 4*e))*a*d)/f
```

Fricas [A] time = 1.7148, size = 177, normalized size = 2.24

$$-\frac{8\,(ac+ad)\cos\left(fx+e\right)^{3}-3\,(4\,ac+ad)fx+3\left(2\,ad\cos\left(fx+e\right)^{3}-(4\,ac+ad)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/24*(8*(a*c + a*d)*cos(f*x + e)^3 - 3*(4*a*c + a*d)*f*x + 3*(2*a*d*cos(f*
x + e)^3 - (4*a*c + a*d)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 1.31873, size = 199, normalized size = 2.52

$$\begin{cases} \frac{acx\sin^2(e+fx)}{2}+\frac{acx\cos^2(e+fx)}{2}+\frac{ac\sin(e+fx)\cos(e+fx)}{2f}-\frac{ac\cos^3(e+fx)}{3f}+\frac{adx\sin^4(e+fx)}{8}+\frac{adx\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{adx\cos^4(e+ \\ x\,(c+d\sin(e))\,(a\sin(e)+a)\cos^2(e) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x*sin(e + f*x)**2/2 + a*c*x*cos(e + f*x)**2/2 + a*c*sin(e +
f*x)*cos(e + f*x)/(2*f) - a*c*cos(e + f*x)**3/(3*f) + a*d*x*sin(e + f*x)**4
/8 + a*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + a*d*x*cos(e + f*x)**4/8 + a*
d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - a*d*sin(e + f*x)*cos(e + f*x)**3/(8*
f) - a*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) + a)
*cos(e)**2, True))
```

Giac [A] time = 1.27094, size = 117, normalized size = 1.48

$$\frac{1}{8}\,(4\,ac+ad)x-\frac{ad\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{ac\sin\left(2\,fx+2\,e\right)}{4\,f}-\frac{(ac+ad)\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{(ac+ad)\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*a*c + a*d)*x - 1/32*a*d*sin(4*f*x + 4*e)/f + 1/4*a*c*sin(2*f*x + 2*e
)/f - 1/12*(a*c + a*d)*cos(3*f*x + 3*e)/f - 1/4*(a*c + a*d)*cos(f*x + e)/f
```

3.936 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=123

$$\frac{2\sqrt{c+d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}\sqrt{d}f(c-d)}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)}$$

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(a^(3/2)*(c - d)*f) + (2*Sqrt[c + d]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e +
f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)*Sqrt[d]*f)
```

Rubi [A] time = 0.551369, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2916, 2985, 2649, 206, 2773, 208}

$$\frac{2\sqrt{c+d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}\sqrt{d}f(c-d)}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(a^(3/2)*(c - d)*f) + (2*Sqrt[c + d]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e +
f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)*Sqrt[d]*f)
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/b^2, Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ
[2*m, 2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \frac{\int \frac{a-a\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{a^2}$$

$$= \frac{2\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{a(c-d)} - \frac{(c+d)\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a^2(c-d)}$$

$$= -\frac{4\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a(c-d)f} + \frac{(2(c+d))\,\mathrm{Subst}\left(\int \frac{1}{ac+ad-d}\right.}{a(c-d}$$

$$= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)f} + \frac{2\sqrt{c+d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right.}{a^{3/2}(c-d)\sqrt{d}f}$$

Mathematica [C] time = 2.72914, size = 220, normalized size = 1.79

$$\frac{(-1)^{3/4}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sqrt[4]{-1}\sqrt{c+d}\left(\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\,\mathrm{c}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] ((-1)^(3/4)*((-4 - 4*I)*Sqrt[d]*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
 + f*x)/4])] + (-1)^(1/4)*Sqrt[c + d]*(Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d]
+ Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])] - Log[Sec[(e + f*x)
/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(Sqrt[d]*(-c + d)*f*(a*(1 + Sin[
e + f*x]))^(3/2))
```

Maple [A] time = 1.283, size = 160, normalized size = 1.3

$$-2\,\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}}{a^{3/2}\left(c-d\right)\sqrt{a\left(c+d\right)d}\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}f}\left(\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{a}}\right)\sqrt{a\left(c+d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/a^(3/2)*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(2^(1/2)*arctanh(1/2*(
-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)-arctanh((-a*(-
1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*c-arctanh((-a*(-1+sin(f*x
+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*d)/(c-d)/(a*(c+d)*d)^(1/2)/cos(f*x
+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.73173, size = 1661, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt((c + d)/(a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*
x + e)^2 - c^2 - 2*c*d - d^2 - 4*(d^2*cos(f*x + e)^2 - c*d - 3*d^2 - (c*d +
 2*d^2)*cos(f*x + e) + (d^2*cos(f*x + e) + c*d + 3*d^2)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt((c + d)/(a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e
) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e
))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 -
 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f
*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 2*sqrt(2)*log(-(cos(f*x + e)^
2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(c
os(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)
^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c -
a*d)*f), (sqrt(-(c + d)/(a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f
*x + e) - c - 2*d)*sqrt(-(c + d)/(a*d))/((c + d)*cos(f*x + e))) - sqrt(2)*l
og(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*si
n(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e)
+ 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))
/sqrt(a))/((a*c - a*d)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.937 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{d}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f\sqrt{c-d}}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]])])/(a^(3/2)*Sqrt[d]*f) - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[
c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]])])/(a^(3/2)*Sqrt[c - d]*f)
```

Rubi [A] time = 0.671545, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2916, 2982, 2782, 208, 2775, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{d}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```


```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]])])/(a^(3/2)*Sqrt[d]*f) - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[
c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]])])/(a^(3/2)*Sqrt[c - d]*f)
```


Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/b^2, Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ
[2*m, 2*n]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\int \frac{a-a\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2} + \frac{2\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} + 2\,\text{Subst}\ldots$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{d}f} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(\ldots}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{\ldots}}\right.}{a^{3/2}\sqrt{c-d}f}$$

Mathematica [C] time = 33.2878, size = 208404, normalized size = 1478.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.319, size = 4463, normalized size = 31.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f/d^2/(-(d^2/c^2)^(1/2)*c)^(1/2)/(c^2-2*c*d+d^2)/(2*c-2*d)^(1/2)*((d^2/
c^2)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1
```

/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e)
)/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*
c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(
d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*co
s(f*x+e)-d)/(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d
^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d))*(2*c-2*d)^(1/2)*c*cos(f*x+e)-4*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*
ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(-1+cos(f*x
+e)-sin(f*x+e)))*c^2*d^2*sin(f*x+e)-(d^2/c^2)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6
*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d
^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)
)^(1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^
(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)
^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/(d*(c+d*sin(f*x+e))/((
d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f
x+e)+d))(2*c-2*d)^(1/2)*c+8*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)+c*sin(f*x+e)
-d*sin(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f*x+e)))*c*d^3*sin(f*x+e)+(d^2/c^2)^(
1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(
-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)
+d))^(1/2))*(2*c-2*d)^(1/2)*c^3*d*sin(f*x+e)-2*(d^2/c^2)^(1/2)*(d*(c+d*sin(
f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*
c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-
2*d)^(1/2)*c^2*d^2*sin(f*x+e)+(d^2/c^2)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2
)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*
sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-2*d)^(1/2)*c*d^3*
sin(f*x+e)+(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1
/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e)
)/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*
c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(
d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*co
s(f*x+e)-d)/(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d
^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d))*(2*c-2*d)^(1/2)*d*cos(f*x+e)-((
(d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2
-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^
(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^
(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2
)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/(
d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/2)
*c*sin(f*x+e)-d*cos(f*x+e)+d))*(2*c-2*d)^(1/2)*d*sin(f*x+e)+(d^2/c^2)^(1/2)
(d(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^
2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))
^(1/2))*(2*c-2*d)^(1/2)*c^3*d*cos(f*x+e)-2*(d^2/c^2)^(1/2)*(d*(c+d*sin(f*x+
e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(
1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-2*d)
^(1/2)*c^2*d^2*cos(f*x+e)+(d^2/c^2)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1
/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(
f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-2*d)^(1/2)*c*d^3*cos(
f*x+e)+2*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan
(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*
x+e)+d))^(1/2))*(2*c-2*d)^(1/2)*c*d^3*sin(f*x+e)+2*(d^2/c^2)^(1/2)*(d*(c+d*
sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1
/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(
2*c-2*d)^(1/2)*c^2*d^2-(d^2/c^2)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1
/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)
c)^(1/2)(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arcta
n(((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(

```
d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/
c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2
)*c*sin(f*x+e)+d))^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d))*(2*
c-2*d)^(1/2)*c*sin(f*x+e)+(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)
+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^
2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-2*d)^(1/2)*d^4+(d*(c+d*sin(f*x+e))/((
d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(
d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-2*d)^(1/2)
*c^2*d^2-2*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arct
an(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(
f*x+e)+d))^(1/2))*(2*c-2*d)^(1/2)*c*d^3-(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2
)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*
x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-2*d)^(1/2)*d^4*cos(f*x+
e)-(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(
d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d
))^(1/2))*(2*c-2*d)^(1/2)*d^4*sin(f*x+e)-(d^2/c^2)^(1/2)*(d*(c+d*sin(f*x+e)
)/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/
2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-2*d)^(
1/2)*c*d^3-(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1
/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e)
)/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(((d^2/c^2)^(1/2)*c^2-d^2)*
c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(
d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*co
s(f*x+e)-d)/(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)/((d
^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d))*(2*c-2*d)^(1/2)*d-(d*(c+d*sin(f
*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c
)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-2
*d)^(1/2)*c^2*d^2*cos(f*x+e)+2*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f
*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d
^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2))*(2*c-2*d)^(1/2)*c*d^3*cos(f*x+e)-(d*(
c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*arctan(1/(-(d^2/c^2
)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2
))*(2*c-2*d)^(1/2)*c^2*d^2*sin(f*x+e)-4*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+
e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)+c*
sin(f*x+e)-d*sin(f*x+e)-c+d)/(-1+cos(f*x+e)-sin(f*x+e)))*d^4*sin(f*x+e)-(d^
2/c^2)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d))^(1/2)*ar
ctan(1/(-(d^2/c^2)^(1/2)*c)^(1/2)*(d*(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*si
n(f*x+e)+d))^(1/2))*(2*c-2*d)^(1/2)*c^3*d)/(c+d*sin(f*x+e))^(1/2)*(sin(f*x+
e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1
+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) +
c)), x)
```

Fricas [B] time = 6.68613, size = 5009, normalized size = 35.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*a*d*log(-((c - 3*d)*cos(f*x + e)^2 - 2*sqrt(2)*((c - d)*cos
(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*s
in(f*x + e) + c)/sqrt(a*c - a*d) + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(
f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x
+ e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a*c - a*d) - sqrt(-a*d)*lo
g((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
 a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*
d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*
a*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x
 + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d
^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16
*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d
^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^4
 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (12
8*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^
4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*
a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*c
os(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/(a^2*d*f), 1
/2*(2*sqrt(2)*a*d*log(-((c - 3*d)*cos(f*x + e)^2 - 2*sqrt(2)*((c - d)*cos(f
*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)/sqrt(a*c - a*d) + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*
x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a*c - a*d) - sqrt(a*d)*arcta
n(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x +
 e))*sqrt(a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*d^3*c
os(f*x + e)^3 - (3*a*c*d^2 - a*d^3)*cos(f*x + e)*sin(f*x + e) - (a*c^2*d -
a*c*d^2 + 2*a*d^3)*cos(f*x + e))))/(a^2*d*f), 1/4*(8*sqrt(2)*a*d*sqrt(-1/(a
*c - a*d))*arctan(sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)
*sqrt(-1/(a*c - a*d))/cos(f*x + e)) - sqrt(-a*d)*log((128*a*d^4*cos(f*x + e
)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3
- a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x
+ e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 -
8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d
 - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^
3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^
3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(
5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*d)*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^4 - 28*a*c^3*d + 230*a*c^
2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 +
 a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4
)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 +
32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e
))/(cos(f*x + e) + sin(f*x + e) + 1)))/(a^2*d*f), 1/2*(4*sqrt(2)*a*d*sqrt(-
1/(a*c - a*d))*arctan(sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)*sqrt(-1/(a*c - a*d))/cos(f*x + e)) - sqrt(a*d)*arctan(1/4*(8*d^2*cos(f
*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*d)*sqr
t(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*d^3*cos(f*x + e)^3 - (3
*a*c*d^2 - a*d^3)*cos(f*x + e)*sin(f*x + e) - (a*c^2*d - a*c*d^2 + 2*a*d^3)
*cos(f*x + e))))/(a^2*d*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(c + d*sin(e +
f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(fx+e)^2}{(a\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{d\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) +
c)), x)
```

3.938 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=135

$$\frac{2\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{3}{2};-\frac{1}{2},-n;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),-\right.}{af(2m+3)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, -n, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c
+ d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e +
f*x])/(c - d))^n)
```

Rubi [A] time = 0.244927, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2918, 140, 139, 138}

$$\frac{2\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{3}{2};-\frac{1}{2},-n;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),-\right.}{af(2m+3)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, -n, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c
+ d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e +
f*x])/(c - d))^n)
```

Rule 2918

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[e + f*
x]/(a^(p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[I
nt[(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&
 IntegerQ[p/2] && !IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \sqrt{a-ax}(a+ax)^{\frac{1}{2}+m}(c+dx)^n\,dx, x\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(\sqrt{2}\cos(e+fx)\right)\text{Subst}\left(\int \sqrt{\frac{1}{2}-\frac{x}{2}}(a+ax)^{\frac{1}{2}+m}(c+dx)^n\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(\sqrt{2}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{a(c+d\sin(e+fx))}{ac-ad}\right)^{-n}\right)\text{S}}{f\sqrt{\frac{a-a\sin(e+f}{a}}}$$

$$= \frac{2\sqrt{2}F_1\left(\frac{3}{2}+m;-\frac{1}{2},-n;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\mathrm{si}}{}\right.}{af(}$$

Mathematica [A] time = 0.676294, size = 158, normalized size = 1.17

$$-\frac{4\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos(e+fx)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m-\frac{1}{2}}(a(\sin(e+fx)+1))^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+}{c+d}\right.}{3f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*AppellF1[3/2, -1/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c
+ d*Sin[e + f*x])^n*Sin[(2*e - Pi + 2*f*x)/4]^2*(Sin[(2*e + Pi + 2*f*x)/4]^
2)^(-1/2 - m))/(3*f*((c + d*Sin[e + f*x])/(c + d))^n)
```

Maple [F] time = 0.437, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\cos(fx+e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

3.939 $\int \cos^2(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$-\frac{16\sqrt{2}a^3(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{7}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e}{c+d}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-16*Sqrt[2]*a^3*AppellF1[3/2, -7/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x
])^n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.180305, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{16\sqrt{2}a^3(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{7}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e}{c+d}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-16*Sqrt[2]*a^3*AppellF1[3/2, -7/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x
])^n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx = \frac{\left(a^3\cos(e+fx)\right)\text{Subst}\left(\int \sqrt{1-x}(1+x)^{7/2}(c+dx)^n\,dx,x,\text{si}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}}{f\sqrt{1-\sin(e+fx)}\sqrt{}}$$

$$= -\frac{16\sqrt{2}a^3F_1\left(\frac{3}{2};-\frac{7}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\text{c}}{3f\sqrt{1+}}$$

Mathematica [F] time = 113.736, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.467, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^3\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3a^3\cos\left(fx+e\right)^4-4a^3\cos\left(fx+e\right)^2+\left(a^3\cos\left(fx+e\right)^4-4a^3\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^
4 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

3.940 $\int \cos^2(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$-\frac{8\sqrt{2}a^2(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-8*Sqrt[2]*a^2*AppellF1[3/2, -5/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.178233, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{8\sqrt{2}a^2(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^2*AppellF1[3/2, -5/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \sqrt{1-x}(1+x)^{5/2}(c+dx)^n\,dx, x\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Su}}{f\sqrt{1-\sin(e+f}}$$

$$= -\frac{8\sqrt{2}a^2F_1\left(\frac{3}{2};-\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3f\sqrt{1}}$$

Mathematica [F] time = 74.3619, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2\sin\left(fx+e\right)-2\,a^2\cos\left(fx+e\right)^2\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*c
os(f*x + e)^2)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^2(d\sin(fx+e)+c)^n\cos(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

3.941 $\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=117

$$-\frac{4\sqrt{2}a(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+f}{c+d}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-4*Sqrt[2]*a*AppellF1[3/2, -3/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^
n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.130826, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2868, 139, 138}

$$-\frac{4\sqrt{2}a(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+f}{c+d}\right.}{3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*Sqrt[2]*a*AppellF1[3/2, -3/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^
n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2868

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(c*g*(g
*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(1 + (d*x)/c)^((p + 1)/2)*(1 - (d*x)/c)^((p - 1)/
2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \sqrt{1-x}(1+x)^{3/2}(c+dx)^n\,dx,x,\sin(\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1}}$$

$$= -\frac{4\sqrt{2}aF_1\left(\frac{3}{2};-\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(}{3f\sqrt{1+\text{si}}}$$

Mathematica [F] time = 16.0657, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]

[Out] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")

[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\cos\left(fx+e\right)^2\sin\left(fx+e\right)+a\cos\left(fx+e\right)^2\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

3.942 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{2}(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{1}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3af\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(Sqrt[2]*AppellF1[3/2, 1/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e +
 f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(3
*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.228802, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2914, 2755, 139, 138}

$$-\frac{\sqrt{2}(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{1}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(Sqrt[2]*AppellF1[3/2, 1/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e +
 f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(3
*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2914

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^(2*m), I
nt[(c + d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\int (a-a\sin(e+fx))(c+d\sin(e+fx))^n\,dx}{a^2}$$

$$= \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{\sqrt{1-x}(c+dx)^n}{\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\sqrt{1-x}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right.}{\sqrt{1+x}}\right.}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}F_1\left(\frac{3}{2};\frac{1}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+}{3af\sqrt{1+\sin(e+fx)}}$$

Mathematica [A] time = 0.994821, size = 229, normalized size = 1.92

$$-\frac{\sec(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}(c+d\sin(e+fx))^{n+1}\left((n+1)(c+d\sin(e+fx))F_1\right.}{adf(n+1)(n+2)(d-c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Sec[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[-((d*(-1 + Sin
[e + f*x]))/(c + d))]*(c + d*Sin[e + f*x])^(1 + n)*(-((c + d)*(2 + n)*Appel
lF1[1 + n, 1/2, 1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[e + f*
x])/(c + d)]) + (1 + n)*AppellF1[2 + n, 1/2, 1/2, 3 + n, (c + d*Sin[e + f*x
])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])))/(a*d*(-c +
 d)*f*(1 + n)*(2 + n)*Sqrt[(d*(1 + Sin[e + f*x]))/(-c + d)]))
```

Maple [F] time = 0.414, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a), x)
```

3.943 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(AppellF1[3/2, 3/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(3*Sqrt[2]
*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.216091, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(AppellF1[3/2, 3/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(3*Sqrt[2]
*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{\sqrt{1-x}(c+dx)^n}{(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\sqrt{1-x}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{(1+x)^{3/2}}\right.}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{3}{2};\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))(}{3\sqrt{2}a^2 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 8.58602, size = 0, normalized size = 0.

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2, x
]
```

Maple [F] time = 0.5, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a^2*cos(f*x + e)^2 - 2*a^2
*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^2, x)
```

3.944 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=119

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{6\sqrt{2}a^3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(AppellF1[3/2, 5/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(6*Sqrt[2]
*a^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.213685, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{6\sqrt{2}a^3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] -(AppellF1[3/2, 5/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(6*Sqrt[2]
*a^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```


Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{\sqrt{1-x}(c+dx)^n}{(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^3 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\sqrt{1-x}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right.}{(1+x)^{5/2}}\right.}{a^3 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{3}{2};\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx}{6\sqrt{2}a^3 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 16.3984, size = 0, normalized size = 0.

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3, x
]
```

Maple [F] time = 0.72, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(3*a^3*cos(f*x + e)^2 - 4*a
^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^3, x)
```

3.945 $\int \cos^4(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=135

$$\frac{4\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+2}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{5}{2};-\frac{3}{2},-n;m+\frac{7}{2};\frac{1}{2}(\sin(e+fx)+1\right.}{a^2f(2m+5)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, -n, 7/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(2 + m)*(c
+ d*Sin[e + f*x])^n)/(a^2*f*(5 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e
+ f*x])/(c - d))^n)
```

Rubi [A] time = 0.22617, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2918, 140, 139, 138}

$$\frac{4\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+2}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{5}{2};-\frac{3}{2},-n;m+\frac{7}{2};\frac{1}{2}(\sin(e+fx)+1\right.}{a^2f(2m+5)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```


```
[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, -n, 7/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(2 + m)*(c
+ d*Sin[e + f*x])^n)/(a^2*f*(5 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e
+ f*x])/(c - d))^n)
```


Rule 2918

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[e + f*
x]/(a^(p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[I
nt[(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&
 IntegerQ[p/2] &&  !IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{\frac{3}{2}+m}(c+dx)^n\,dx, x, \ldots\right.}{a^2 f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(2\sqrt{2}\cos(e+fx)\right)\text{Subst}\left(\int\left(\frac{1}{2}-\frac{x}{2}\right)^{3/2}(a+ax)^{\frac{3}{2}+m}(c+dx)\ldots\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(2\sqrt{2}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{a(c+d\sin(e+fx))}{ac-ad}\right)^{-n}\right)\text{Su}\ldots}{af\sqrt{\frac{a-a\sin(e+f\ldots}{a}}\ldots}$$

$$= \frac{4\sqrt{2}F_1\left(\frac{5}{2}+m;-\frac{3}{2},-n;\frac{7}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e\ldots}{c-d}\ldots\right.}{a^2 f(5-\ldots}$$

Mathematica [A] time = 1.28988, size = 160, normalized size = 1.19

$$-\frac{4\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^3(e+fx)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m-\frac{3}{2}}(a(\sin(e+fx)+1))^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx\ldots}{c+d}\right.\ldots}{5f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*AppellF1[5/2, -3/2 - m, -n, 7/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[e + f*x]^3*(a*(1 + Sin[e + f*x]))^m*(
c + d*Sin[e + f*x])^n*Sin[(2*e - Pi + 2*f*x)/4]^2*(Sin[(2*e + Pi + 2*f*x)/4
]^2)^(-3/2 - m))/(5*f*((c + d*Sin[e + f*x])/(c + d))^n)
```

Maple [F] time = 0.495, size = 0, normalized size = 0.

$$\int\left(\cos\left(fx+e\right)\right)^4\left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] Timed out
```

3.946 $\int \cos^4(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=121

$$-\frac{16\sqrt{2}a^2(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+f}{c+d}\right.}{5f\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-16*Sqrt[2]*a^2*AppellF1[5/2, -7/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f
*x])^n)/(5*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.177085, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{16\sqrt{2}a^2(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+f}{c+d}\right.}{5f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-16*Sqrt[2]*a^2*AppellF1[5/2, -7/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f
*x])^n)/(5*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int(1-x)^{3/2}(1+x)^{7/2}(c+dx)^n\,dx,\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\operatorname{Su}}{f\sqrt{1-\sin(e+f}}$$

$$= -\frac{16\sqrt{2}a^2F_1\left(\frac{5}{2};-\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right.}{5f\sqrt{}}$$

Mathematica [F] time = 1.59877, size = 0, normalized size = 0.

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.539, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^4\left(a+a\sin\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^6-2\,a^2\cos\left(fx+e\right)^4\sin\left(fx+e\right)-2\,a^2\cos\left(fx+e\right)^4\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^6 - 2*a^2*cos(f*x + e)^4*sin(f*x + e) - 2*a^2*c
os(f*x + e)^4)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^2 (d\sin(fx+e)+c)^n \cos(fx+e)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

3.947 $\int \cos^4(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$-\frac{8\sqrt{2}a(1-\sin(e+fx))\cos^3(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{5f(\sin(e+fx)+1)^{3/2}}$$

```
[Out] (-8*Sqrt[2]*a*AppellF1[5/2, -5/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]^3*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n)/(5*f*(1 + Sin[e + f*x])^(3/2)*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.141013, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2868, 139, 138}

$$-\frac{8\sqrt{2}a(1-\sin(e+fx))\cos^3(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{5f(\sin(e+fx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```


```
[Out] (-8*Sqrt[2]*a*AppellF1[5/2, -5/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]^3*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n)/(5*f*(1 + Sin[e + f*x])^(3/2)*((c + d*Sin[e + f*x])/(c + d))^n)
```


Rule 2868

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(c*g*(g
*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(1 + (d*x)/c)^((p + 1)/2)*(1 - (d*x)/c)^((p - 1)/
2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{\left(a\cos^3(e+fx)\right)\text{Subst}\left(\int(1-x)^{3/2}(1+x)^{5/2}(c+dx)^n\,dx,x,\text{si}\right.}{f(1-\sin(e+fx))^{3/2}(1+\sin(e+fx))^{3/2}}$$

$$= \frac{\left(a\cos^3(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\right.}{f(1-\sin(e+fx))^{3/2}(1}$$

$$= -\frac{8\sqrt{2}aF_1\left(\frac{5}{2};-\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos^3}{5f(1+\sin}$$

Mathematica [F] time = 0.737515, size = 0, normalized size = 0.

$$\int \cos^4(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.432, size = 0, normalized size = 0.

$$\int (\cos(fx+e))^4(a+a\sin(fx+e))(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\cos(fx+e)^4\sin(fx+e)+a\cos(fx+e)^4\right)(d\sin(fx+e)+c)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*cos(f*x + e)^4*sin(f*x + e) + a*cos(f*x + e)^4)*(d*sin(f*x + e)
 + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)(d\sin(fx+e)+c)^n\cos(fx+e)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

3.948 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=121

$$-\frac{2\sqrt{2}(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5af\sqrt{\sin(e+fx)+1}}$$

```
[Out] (-2*Sqrt[2]*AppellF1[5/2, -1/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[
e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^
n)/(5*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.181231, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{2\sqrt{2}(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```


```
[Out] (-2*Sqrt[2]*AppellF1[5/2, -1/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[
e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^
n)/(5*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```


Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int (1-x)^{3/2}\sqrt{1+x}(c+dx)^n\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int (1-x)^{3/2}\sqrt{1+\right.}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{2\sqrt{2}F_1\left(\frac{5}{2};-\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(}{5af\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 29.2282, size = 0, normalized size = 0.

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]), x]
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int \frac{(\cos(fx+e))^4(c+d\sin(fx+e))^n}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\sin(fx+e)+c)^n\cos(fx+e)^4}{a\sin(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a), x)
```

3.949 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{2}(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5a^2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(Sqrt[2]*AppellF1[5/2, 1/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e +
 f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/
(5*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.244807, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2914, 2784, 139, 138}

$$-\frac{\sqrt{2}(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] -(Sqrt[2]*AppellF1[5/2, 1/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e +
 f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/
(5*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```


Rule 2914

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^(2*m), I
nt[(c + d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\int (a-a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx}{a^4}$$

$$= \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\mathrm{Subst}\left(\int \frac{(1-x)^{3/2}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)}{\sqrt{1+x}} \ldots}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}F_1\left(\frac{5}{2};\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx \ldots}{5a^2f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 16.9785, size = 0, normalized size = 0.

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2, x
]
```

Maple [F] time = 0.57, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^4\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a^2*cos(f*x + e)^2 - 2*a^2
*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^2, x)
```

3.950 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=121

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{3}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5\sqrt{2}a^3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(AppellF1[5/2, 3/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(5*Sqrt[
2]*a^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.18151, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{3}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5\sqrt{2}a^3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] -(AppellF1[5/2, 3/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(5*Sqrt[
2]*a^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```


Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{(1+x)^{3/2}}\,dx,x,\sin(e+fx)\right)}{a^3f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1-x)^{3/2}\left(-\frac{c}{-c-d}-\ldots\right.}{(1+x)^{3/2}}\right.}{a^3f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{F_1\left(\frac{5}{2};\frac{3}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+f\ldots}{5\sqrt{2}a^3f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 18.076, size = 0, normalized size = 0.

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3, x
]
```

Maple [F] time = 0.658, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^4\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4}{\left(a\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(3*a^3*cos(f*x + e)^2 - 4*a
^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^3, x)
```

3.951 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$

Optimal. Leaf size=121

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{10\sqrt{2}a^4f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(AppellF1[5/2, 5/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(10*Sqrt
[2]*a^4*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.185634, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{10\sqrt{2}a^4f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4,x]
```

```
[Out] -(AppellF1[5/2, 5/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(10*Sqrt
[2]*a^4*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^4f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1-x)^{3/2}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)}{(1+x)^{5/2}}\right.}{a^4f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{5}{2};\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))^2}{10\sqrt{2}a^4f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 17.7048, size = 0, normalized size = 0.

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4, x
]
```

Maple [F] time = 0.927, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^4\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4}{\left(a\sin\left(fx+e\right)+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{a^{4}\cos\left(fx+e\right)^{4}-8\,a^{4}\cos\left(fx+e\right)^{2}+8\,a^{4}-4\left(a^{4}\cos\left(fx+e\right)^{2}-2\,a^{4}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorithm="
fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a^4*cos(f*x + e)^4 - 8*a^4*
cos(f*x + e)^2 + 8*a^4 - 4*(a^4*cos(f*x + e)^2 - 2*a^4)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^4, x)
```

3.952 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^5}\,dx$

Optimal. Leaf size=121

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{20\sqrt{2}a^5f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(AppellF1[5/2, 7/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(20*Sqrt
[2]*a^5*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.178409, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2917, 139, 138}

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{20\sqrt{2}a^5f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^5,x]
```


```
[Out] -(AppellF1[5/2, 7/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(20*Sqrt
[2]*a^5*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```


Rule 2917

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e
 + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(1 +
(b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^5}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{(1+x)^{7/2}}\,dx, x, \sin(e+fx)\right)}{a^5 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1-x)^{3/2}\left(-\frac{c}{-c-d}-\frac{}{-}\right.}{(1+x)^{7/2}}\right.}{a^5 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{5}{2};\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx}{20\sqrt{2}a^5 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 2.24102, size = 0, normalized size = 0.

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^5,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^5, x
]
```

Maple [F] time = 1.125, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^4\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^4}{\left(a\sin\left(fx+e\right)+a\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{5\,a^{5}\cos\left(fx+e\right)^{4}-20\,a^{5}\cos\left(fx+e\right)^{2}+16\,a^{5}+\left(a^{5}\cos\left(fx+e\right)^{4}-12\,a^{5}\cos\left(fx+e\right)^{2}+16\,a^{5}\right)\sin\left(fx+e\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x, algorithm="
fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(5*a^5*cos(f*x + e)^4 - 20*a
^5*cos(f*x + e)^2 + 16*a^5 + (a^5*cos(f*x + e)^4 - 12*a^5*cos(f*x + e)^2 +
16*a^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^5, x)
```

3.953 $\int \cos^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{(A-7B)(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(A-3B)(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(3A-5B)(a\sin(c+dx)+a)^6}{3a^5d}+\frac{8(A-B)(a\sin(c\cdot}{5a^4d}$$

```
[Out] (8*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^4*d) - (2*(3*A - 5*B)*(a + a*Sin[c
+ d*x])^6)/(3*a^5*d) + (6*(A - 3*B)*(a + a*Sin[c + d*x])^7)/(7*a^6*d) - ((A
 - 7*B)*(a + a*Sin[c + d*x])^8)/(8*a^7*d) - (B*(a + a*Sin[c + d*x])^9)/(9*a
^8*d)
```

Rubi [A] time = 0.142071, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$-\frac{(A-7B)(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(A-3B)(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(3A-5B)(a\sin(c+dx)+a)^6}{3a^5d}+\frac{8(A-B)(a\sin(c\cdot}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (8*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^4*d) - (2*(3*A - 5*B)*(a + a*Sin[c
+ d*x])^6)/(3*a^5*d) + (6*(A - 3*B)*(a + a*Sin[c + d*x])^7)/(7*a^6*d) - ((A
 - 7*B)*(a + a*Sin[c + d*x])^8)/(8*a^7*d) - (B*(a + a*Sin[c + d*x])^9)/(9*a
^8*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^4\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^4-4a^2(3A-5B)(a+x)^5+6\right.\right.}{}\\ &= \frac{8(A-B)(a+a\sin(c+dx))^5}{5a^4d}-\frac{2(3A-5B)(a+a\sin(c+d}{3a^5d}\end{aligned}$$

Mathematica [A] time = 0.792265, size = 194, normalized size = 1.45

$$\frac{a(\sin(c+dx)+1)(-17640(A+B)\cos(2(c+dx))-8820(A+B)\cos(4(c+dx))+176400A\sin(c+dx)+35280A\sin(3(c+dx))\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(1 + Sin[c + d*x])*(-17640*(A + B)*Cos[2*(c + d*x)] - 8820*(A + B)*Cos[4
*(c + d*x)] - 2520*A*Cos[6*(c + d*x)] - 2520*B*Cos[6*(c + d*x)] - 315*A*Cos
[8*(c + d*x)] - 315*B*Cos[8*(c + d*x)] + 176400*A*Sin[c + d*x] + 17640*B*Si
n[c + d*x] + 35280*A*Sin[3*(c + d*x)] + 7056*A*Sin[5*(c + d*x)] - 2016*B*Si
n[5*(c + d*x)] + 720*A*Sin[7*(c + d*x)] - 900*B*Sin[7*(c + d*x)] - 140*B*Si
n[9*(c + d*x)]))/(322560*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)
```

Maple [A] time = 0.058, size = 128, normalized size = 1.

$$\frac{1}{d}\left(aB\left(-\frac{(\cos(dx+c))^8\sin(dx+c)}{9}+\frac{\sin(dx+c)}{63}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6\,(\cos(dx+c))^4}{5}+\frac{8\,(\cos(dx+c))^2}{5}\right)\right)-\frac{aA}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+
c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*a*A*cos(d*x+c)^8-1/8*a*B*cos(d*x+c)^
8+1/7*a*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.02365, size = 181, normalized size = 1.35

$$-\frac{280\,Ba\sin(dx+c)^9+315\,(A+B)a\sin(dx+c)^8+360\,(A-3\,B)a\sin(dx+c)^7-1260\,(A+B)a\sin(dx+c)^6-1512\,(A\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2520*(280*B*a*sin(d*x + c)^9 + 315*(A + B)*a*sin(d*x + c)^8 + 360*(A - 3
*B)*a*sin(d*x + c)^7 - 1260*(A + B)*a*sin(d*x + c)^6 - 1512*(A - B)*a*sin(d
*x + c)^5 + 1890*(A + B)*a*sin(d*x + c)^4 + 840*(3*A - B)*a*sin(d*x + c)^3
- 1260*(A + B)*a*sin(d*x + c)^2 - 2520*A*a*sin(d*x + c))/d
```

Fricas [A] time = 1.87929, size = 261, normalized size = 1.95

$$-\frac{315\,(A+B)a\cos(dx+c)^8+8\left(35\,Ba\cos(dx+c)^8-5\,(9\,A+B)a\cos(dx+c)^6-6\,(9\,A+B)a\cos(dx+c)^4-8\,(9\,A+\ldots\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2520*(315*(A + B)*a*cos(d*x + c)^8 + 8*(35*B*a*cos(d*x + c)^8 - 5*(9*A +
 B)*a*cos(d*x + c)^6 - 6*(9*A + B)*a*cos(d*x + c)^4 - 8*(9*A + B)*a*cos(d*x
 + c)^2 - 16*(9*A + B)*a)*sin(d*x + c))/d
```

Sympy [A] time = 22.8752, size = 228, normalized size = 1.7

$$\begin{cases} \frac{16Aa\sin^7(c+dx)}{35d} + \frac{8Aa\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2Aa\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{Aa\sin(c+dx)\cos^6(c+dx)}{d} - \frac{Aa\cos^8(c+dx)}{8d} + \frac{16Ba\sin^9(c+dx)}{315d} \\ x(A+B\sin(c))(a\sin(c)+a)\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a*sin(c + d*x)**7/(35*d) + 8*A*a*sin(c + d*x)**5*cos(c + d*
x)**2/(5*d) + 2*A*a*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a*sin(c + d*x)*co
s(c + d*x)**6/d - A*a*cos(c + d*x)**8/(8*d) + 16*B*a*sin(c + d*x)**9/(315*d
) + 8*B*a*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 2*B*a*sin(c + d*x)**5*co
s(c + d*x)**4/(5*d) + B*a*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) - B*a*cos(c
 + d*x)**8/(8*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)*cos(c)**7, Tr
ue))
```

Giac [A] time = 1.32345, size = 246, normalized size = 1.84

$$-\frac{Ba\sin(9\,dx+9\,c)}{2304\,d} + \frac{7\,Aa\sin(3\,dx+3\,c)}{64\,d} - \frac{(Aa+Ba)\cos(8\,dx+8\,c)}{1024\,d} - \frac{(Aa+Ba)\cos(6\,dx+6\,c)}{128\,d} - \frac{7\,(Aa+Ba)\,c}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2304*B*a*sin(9*d*x + 9*c)/d + 7/64*A*a*sin(3*d*x + 3*c)/d - 1/1024*(A*a
+ B*a)*cos(8*d*x + 8*c)/d - 1/128*(A*a + B*a)*cos(6*d*x + 6*c)/d - 7/256*(A
*a + B*a)*cos(4*d*x + 4*c)/d - 7/128*(A*a + B*a)*cos(2*d*x + 2*c)/d + 1/179
2*(4*A*a - 5*B*a)*sin(7*d*x + 7*c)/d + 1/320*(7*A*a - 2*B*a)*sin(5*d*x + 5*
c)/d + 7/128*(10*A*a + B*a)*sin(d*x + c)/d
```

3.954 $\int \cos^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{(A-5B)(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{a^3d}+\frac{B(a\sin(c+dx)+a)^7}{7a^6d}$$

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(a^3*d) - (4*(A - 2*B)*(a + a*Sin[c + d*x]
)^5)/(5*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^6)/(6*a^5*d) + (B*(a + a*S
in[c + d*x])^7)/(7*a^6*d)
```

Rubi [A] time = 0.10771, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$\frac{(A-5B)(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{a^3d}+\frac{B(a\sin(c+dx)+a)^7}{7a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(a^3*d) - (4*(A - 2*B)*(a + a*Sin[c + d*x]
)^5)/(5*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^6)/(6*a^5*d) + (B*(a + a*S
in[c + d*x])^7)/(7*a^6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^3\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^3-4a(A-2B)(a+x)^4+(A-5\right.\right.}{a^5d}\\ &= \frac{(A-B)(a+a\sin(c+dx))^4}{a^3d}-\frac{4(A-2B)(a+a\sin(c+dx))^5}{5a^4d}+\end{aligned}$$

Mathematica [A] time = 0.67241, size = 130, normalized size = 1.27

$$-\frac{a(525(A+B)\cos(2(c+dx))+210(A+B)\cos(4(c+dx))-4200A\sin(c+dx)-700A\sin(3(c+dx))-84A\sin(5(c$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a*(525*(A + B)*Cos[2*(c + d*x)] + 210*(A + B)*Cos[4*(c + d*x)] + 35*A*Cos
[6*(c + d*x)] + 35*B*Cos[6*(c + d*x)] - 4200*A*Sin[c + d*x] - 525*B*Sin[c +
 d*x] - 700*A*Sin[3*(c + d*x)] + 35*B*Sin[3*(c + d*x)] - 84*A*Sin[5*(c + d*
x)] + 63*B*Sin[5*(c + d*x)] + 15*B*Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.059, size = 108, normalized size = 1.1

$$\frac{1}{d}\left(aB\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)\right)-\frac{aA(\cos(dx+c))^6}{6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/6*a*A*cos(d*x+c)^6-1/6*a*B*cos(d*x+c)^6+1/5*a*A*(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.05107, size = 140, normalized size = 1.37

$$\frac{30\,Ba\sin(dx+c)^7+35\,(A+B)a\sin(dx+c)^6+42\,(A-2\,B)a\sin(dx+c)^5-105\,(A+B)a\sin(dx+c)^4-70\,(2\,A-}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/210*(30*B*a*sin(d*x + c)^7 + 35*(A + B)*a*sin(d*x + c)^6 + 42*(A - 2*B)*a
*sin(d*x + c)^5 - 105*(A + B)*a*sin(d*x + c)^4 - 70*(2*A - B)*a*sin(d*x + c
)^3 + 105*(A + B)*a*sin(d*x + c)^2 + 210*A*a*sin(d*x + c))/d
```

Fricas [A] time = 1.79138, size = 215, normalized size = 2.11

$$-\frac{35\,(A+B)a\cos(dx+c)^6+2\left(15\,Ba\cos(dx+c)^6-3\,(7\,A+B)a\cos(dx+c)^4-4\,(7\,A+B)a\cos(dx+c)^2-8\,(7\,A-}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/210*(35*(A + B)*a*cos(d*x + c)^6 + 2*(15*B*a*cos(d*x + c)^6 - 3*(7*A + B
)*a*cos(d*x + c)^4 - 4*(7*A + B)*a*cos(d*x + c)^2 - 8*(7*A + B)*a)*sin(d*x
+ c))/d
```

Sympy [A] time = 7.57911, size = 178, normalized size = 1.75

$$\begin{cases} \frac{8Aa\sin^5(c+dx)}{15d} + \frac{4Aa\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Aa\sin(c+dx)\cos^4(c+dx)}{d} - \frac{Aa\cos^6(c+dx)}{6d} + \frac{8Ba\sin^7(c+dx)}{105d} + \frac{4Ba\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{Ba\,s \\ x\,(A + B\sin(c))\,(a\sin(c) + a)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a*sin(c + d*x)**5/(15*d) + 4*A*a*sin(c + d*x)**3*cos(c + d*x
)**2/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**4/d - A*a*cos(c + d*x)**6/(6*d)
 + 8*B*a*sin(c + d*x)**7/(105*d) + 4*B*a*sin(c + d*x)**5*cos(c + d*x)**2/(1
5*d) + B*a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - B*a*cos(c + d*x)**6/(6*d
), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)*cos(c)**5, True))
```

Giac [A] time = 1.27205, size = 196, normalized size = 1.92

$$-\frac{Ba\sin(7\,dx + 7\,c)}{448\,d} - \frac{(Aa + Ba)\cos(6\,dx + 6\,c)}{192\,d} - \frac{(Aa + Ba)\cos(4\,dx + 4\,c)}{32\,d} - \frac{5\,(Aa + Ba)\cos(2\,dx + 2\,c)}{64\,d} + \frac{(4\,Aa - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/448*B*a*sin(7*d*x + 7*c)/d - 1/192*(A*a + B*a)*cos(6*d*x + 6*c)/d - 1/32
*(A*a + B*a)*cos(4*d*x + 4*c)/d - 5/64*(A*a + B*a)*cos(2*d*x + 2*c)/d + 1/3
20*(4*A*a - 3*B*a)*sin(5*d*x + 5*c)/d + 1/192*(20*A*a - B*a)*sin(3*d*x + 3*
c)/d + 5/64*(8*A*a + B*a)*sin(d*x + c)/d
```

3.955 $\int \cos^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-\frac{(A-3B)(a\sin(c+dx)+a)^4}{4a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^3}{3a^2d}-\frac{B(a\sin(c+dx)+a)^5}{5a^4d}$$

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*
x])^4)/(4*a^3*d) - (B*(a + a*Sin[c + d*x])^5)/(5*a^4*d)
```

Rubi [A] time = 0.0940424, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$-\frac{(A-3B)(a\sin(c+dx)+a)^4}{4a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^3}{3a^2d}-\frac{B(a\sin(c+dx)+a)^5}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*
x])^4)/(4*a^3*d) - (B*(a + a*Sin[c + d*x])^5)/(5*a^4*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx=\frac{\text{Subst}\left(\int (a-x)(a+x)^2\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^3d}$$
$$=\frac{\text{Subst}\left(\int \left(2a(A-B)(a+x)^2+(-A+3B)(a+x)^3-\frac{B(a+x)}{a}\right.\right.}{a^3d}$$
$$=\frac{2(A-B)(a+a\sin(c+dx))^3}{3a^2d}-\frac{(A-3B)(a+a\sin(c+dx)}{4a^3d}$$

Mathematica [A] time = 0.804491, size = 78, normalized size = 1.

$$-\frac{a(-4(100A+11B)\sin(c+dx)+3\cos(4(c+dx))(5(A+B)+4B\sin(c+dx))+\cos(2(c+dx))((32B-80A)\sin(c+}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a*(-4*(100*A + 11*B)*Sin[c + d*x] + 3*Cos[4*(c + d*x)]*(5*(A + B) + 4*B*S
in[c + d*x]) + Cos[2*(c + d*x)]*(60*(A + B) + (-80*A + 32*B)*Sin[c + d*x]))
)/(480*d)
```

Maple [A] time = 0.054, size = 88, normalized size = 1.1

$$\frac{1}{d}\left(aB\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)-\frac{aA(\cos(dx+c))^4}{4}-\frac{aB(\cos(dx+c))^4}{4}+\frac{aA}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
4*a*A*cos(d*x+c)^4-1/4*a*B*cos(d*x+c)^4+1/3*a*A*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

Maxima [A] time = 0.972714, size = 97, normalized size = 1.24

$$-\frac{12\,Ba\sin(dx+c)^5+15\,(A+B)a\sin(dx+c)^4+20\,(A-B)a\sin(dx+c)^3-30\,(A+B)a\sin(dx+c)^2-60\,Aa\sin(dx+}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/60*(12*B*a*sin(d*x + c)^5 + 15*(A + B)*a*sin(d*x + c)^4 + 20*(A - B)*a*s
in(d*x + c)^3 - 30*(A + B)*a*sin(d*x + c)^2 - 60*A*a*sin(d*x + c))/d
```

Fricas [A] time = 1.66021, size = 167, normalized size = 2.14

$$-\frac{15\,(A+B)a\cos(dx+c)^4+4\left(3\,Ba\cos(dx+c)^4-(5\,A+B)a\cos(dx+c)^2-2\,(5\,A+B)a\right)\sin(dx+c)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/60*(15*(A + B)*a*cos(d*x + c)^4 + 4*(3*B*a*cos(d*x + c)^4 - (5*A + B)*a*
cos(d*x + c)^2 - 2*(5*A + B)*a)*sin(d*x + c))/d
```

Sympy [A] time = 2.44826, size = 128, normalized size = 1.64

$$\begin{cases} \frac{2Aa\sin^3(c+dx)}{3d} + \frac{Aa\sin(c+dx)\cos^2(c+dx)}{d} - \frac{Aa\cos^4(c+dx)}{4d} + \frac{2Ba\sin^5(c+dx)}{15d} + \frac{Ba\sin^3(c+dx)\cos^2(c+dx)}{3d} - \frac{Ba\cos^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a*sin(c + d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d
 - A*a*cos(c + d*x)**4/(4*d) + 2*B*a*sin(c + d*x)**5/(15*d) + B*a*sin(c + d
*x)**3*cos(c + d*x)**2/(3*d) - B*a*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(A
+ B*sin(c))*(a*sin(c) + a)*cos(c)**3, True))
```

Giac [A] time = 1.27429, size = 135, normalized size = 1.73

$$-\frac{12\,Ba\sin(dx+c)^5 + 15\,Aa\sin(dx+c)^4 + 15\,Ba\sin(dx+c)^4 + 20\,Aa\sin(dx+c)^3 - 20\,Ba\sin(dx+c)^3 - 30\,Aa\sin}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/60*(12*B*a*sin(d*x + c)^5 + 15*A*a*sin(d*x + c)^4 + 15*B*a*sin(d*x + c)^
4 + 20*A*a*sin(d*x + c)^3 - 20*B*a*sin(d*x + c)^3 - 30*A*a*sin(d*x + c)^2 -
 30*B*a*sin(d*x + c)^2 - 60*A*a*sin(d*x + c))/d
```

3.956 $\int \cos(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=49

$$\frac{a(A+B)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{aB\sin^3(c+dx)}{3d}$$

```
[Out] (a*A*Sin[c + d*x])/d + (a*(A + B)*Sin[c + d*x]^2)/(2*d) + (a*B*Sin[c + d*x]
^3)/(3*d)
```

Rubi [A] time = 0.0631775, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{a(A+B)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{aB\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + (a*(A + B)*Sin[c + d*x]^2)/(2*d) + (a*B*Sin[c + d*x]
^3)/(3*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(aA+(A+B)x+\frac{Bx^2}{a}\right)dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{aA\sin(c+dx)}{d}+\frac{a(A+B)\sin^2(c+dx)}{2d}+\frac{aB\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.394483, size = 46, normalized size = 0.94

$$-\frac{a(\cos(2(c+dx))(3(A+B)+2B\sin(c+dx))-2(6A+B)\sin(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a*(-2*(6*A + B)*Sin[c + d*x] + Cos[2*(c + d*x)]*(3*(A + B) + 2*B*Sin[c +
d*x])))/(12*d)
```

Maple [A] time = 0.026, size = 44, normalized size = 0.9

$$\frac{1}{d}\left(\frac{aB\left(\sin\left(dx+c\right)\right)^{3}}{3}+\frac{\left(aA+aB\right)\left(\sin\left(dx+c\right)\right)^{2}}{2}+A\sin\left(dx+c\right)a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*B*sin(d*x+c)^3+1/2*(A*a+B*a)*sin(d*x+c)^2+A*sin(d*x+c)*a)
```

Maxima [A] time = 1.00871, size = 57, normalized size = 1.16

$$\frac{2\,Ba\sin\left(dx+c\right)^{3}+3\,(A+B)a\sin\left(dx+c\right)^{2}+6\,Aa\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/6*(2*B*a*sin(d*x + c)^3 + 3*(A + B)*a*sin(d*x + c)^2 + 6*A*a*sin(d*x + c)
)/d
```

Fricas [A] time = 1.64018, size = 120, normalized size = 2.45

$$-\frac{3\,(A+B)a\cos\left(dx+c\right)^{2}+2\left(Ba\cos\left(dx+c\right)^{2}-(3\,A+B)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -1/6*(3*(A + B)*a*cos(d*x + c)^2 + 2*(B*a*cos(d*x + c)^2 - (3*A + B)*a)*sin
(d*x + c))/d
```

Sympy [A] time = 0.630478, size = 75, normalized size = 1.53

$$\begin{cases}\frac{Aa\sin\left(c+dx\right)}{d}-\frac{Aa\cos^{2}\left(c+dx\right)}{2d}+\frac{Ba\sin^{3}\left(c+dx\right)}{3d}-\frac{Ba\cos^{2}\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x\left(A+B\sin\left(c\right)\right)\left(a\sin\left(c\right)+a\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise(((A*a*sin(c + d*x)/d - A*a*cos(c + d*x)**2/(2*d) + B*a*sin(c + d*x
)**3/(3*d) - B*a*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin
(c) + a)*cos(c), True))
```

Giac [A] time = 1.26491, size = 70, normalized size = 1.43

$$\frac{2\,Ba\sin\left(dx+c\right)^{3}+3\,Aa\sin\left(dx+c\right)^{2}+3\,Ba\sin\left(dx+c\right)^{2}+6\,Aa\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*B*a*sin(d*x + c)^3 + 3*A*a*sin(d*x + c)^2 + 3*B*a*sin(d*x + c)^2 + 6
*A*a*sin(d*x + c))/d
```

3.957 $\int \sec(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=34

$$-\frac{a(A+B)\log(1-\sin(c+dx))}{d}-\frac{aB\sin(c+dx)}{d}$$

```
[Out] -((a*(A + B)*Log[1 - Sin[c + d*x]])/d) - (a*B*Sin[c + d*x])/d
```

Rubi [A] time = 0.0695984, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2836, 43}

$$-\frac{a(A+B)\log(1-\sin(c+dx))}{d}-\frac{aB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a*(A + B)*Log[1 - Sin[c + d*x]])/d) - (a*B*Sin[c + d*x])/d
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{a\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{a-x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a\,\text{Subst}\left(\int \left(-\frac{B}{a}+\frac{A+B}{a-x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a(A+B)\log(1-\sin(c+dx))}{d}-\frac{aB\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0337236, size = 68, normalized size = 2.

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}-\frac{aA\log(\cos(c+dx))}{d}-\frac{aB\sin(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}-\frac{aB\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d - (a*A*Log[Co
s[c + d*x]])/d - (a*B*Log[Cos[c + d*x]])/d - (a*B*Sin[c + d*x])/d
```

Maple [A] time = 0.065, size = 47, normalized size = 1.4

$$-\frac{a\ln(\sin(dx+c)-1)A}{d}-\frac{aB\sin(dx+c)}{d}-\frac{a\ln(\sin(dx+c)-1)B}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] -1/d*a*ln(sin(d*x+c)-1)*A-a*B*sin(d*x+c)/d-1/d*a*ln(sin(d*x+c)-1)*B
```

Maxima [A] time = 1.01055, size = 39, normalized size = 1.15

$$-\frac{(A+B)a\log(\sin(dx+c)-1)+Ba\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -((A + B)*a*log(sin(d*x + c) - 1) + B*a*sin(d*x + c))/d
```

Fricas [A] time = 1.77425, size = 78, normalized size = 2.29

$$-\frac{(A+B)a\log(-\sin(dx+c)+1)+Ba\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -((A + B)*a*log(-sin(d*x + c) + 1) + B*a*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec(c+dx)\,dx+\int A\sin(c+dx)\sec(c+dx)\,dx+\int B\sin(c+dx)\sec(c+dx)\,dx+\int B\sin^2(c+dx)\sec(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*sin(c + d*x)*sec(c + d*x), x) +
 Integral(B*sin(c + d*x)*sec(c + d*x), x) + Integral(B*sin(c + d*x)**2*sec(
c + d*x), x))
```

Giac [B] time = 1.37615, size = 154, normalized size = 4.53

$$\frac{(Aa+Ba)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-2\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,Ba}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((A*a + B*a)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*(A*a + B*a)*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - (A*a*tan(1/2*d*x + 1/2*c)^2 + B*a*tan(1/2*d*x + 1/2*
c)^2 + 2*B*a*tan(1/2*d*x + 1/2*c) + A*a + B*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)
)/d
```

3.958 $\int \sec^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{a^2(A+B)}{2d(a-a\sin(c+dx))}+\frac{a(A-B)\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] (a*(A - B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(A + B))/(2*d*(a - a*Sin[c +
 d*x]))
```

Rubi [A] time = 0.0864544, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$\frac{a^2(A+B)}{2d(a-a\sin(c+dx))}+\frac{a(A-B)\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```


```
[Out] (a*(A - B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(A + B))/(2*d*(a - a*Sin[c +
 d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{a^3\operatorname{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\operatorname{Subst}\left(\int \left(\frac{A+B}{2a(a-x)^2}+\frac{A-B}{2a(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^2(A+B)}{2d(a-a\sin(c+dx))} + \frac{\left(a^2(A-B)\right)\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\right.}{2d}$$

$$= \frac{a(A-B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2(A+B)}{2d(a-a\sin(c+dx))}$$

Mathematica [C] time = 0.613797, size = 260, normalized size = 5.53

$$\frac{a\left(2i(A-B)(\sin(c+dx)-1)\tan^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)+(A-B)\sin(c+dx)\left(2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(2*A + 2*B + I*A*d*x - I*B*d*x - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 2*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2] - B*Log[(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2]
 + (2*I)*(A - B)*ArcTan[Tan[(c + d*x)/2]]*(-1 + Sin[c + d*x]) + (A - B)*((-
I)*d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^2])*Sin[c + d*x]))/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2)
```

Maple [B] time = 0.089, size = 129, normalized size = 2.7

$$\frac{aA}{2d(\cos(dx+c))^2} + \frac{aB(\sin(dx+c))^3}{2d(\cos(dx+c))^2} + \frac{aB\sin(dx+c)}{2d} - \frac{aB\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{aA\sec(dx+c)\tan}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a*A/cos(d*x+c)^2+1/2/d*a*B*sin(d*x+c)^3/cos(d*x+c)^2+1/2*a*B*sin(d*x+
c)/d-1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*A*sec(d*x+c)*tan(d*x+c)+1/
2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*B/cos(d*x+c)^2
```

Maxima [A] time = 0.993578, size = 74, normalized size = 1.57

$$\frac{(A-B)a\log(\sin(dx+c)+1)-(A-B)a\log(\sin(dx+c)-1)-\frac{2(A+B)a}{\sin(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((A - B)*a*log(sin(d*x + c) + 1) - (A - B)*a*log(sin(d*x + c) - 1) - 2*
(A + B)*a/(sin(d*x + c) - 1))/d
```

Fricas [B] time = 1.83575, size = 221, normalized size = 4.7

$$-\frac{2(A+B)a-((A-B)a\sin(dx+c)-(A-B)a)\log(\sin(dx+c)+1)+((A-B)a\sin(dx+c)-(A-B)a)\log(-\sin(dx}{4(d\sin(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/4*(2*(A + B)*a - ((A - B)*a*sin(d*x + c) - (A - B)*a)*log(sin(d*x + c) +
 1) + ((A - B)*a*sin(d*x + c) - (A - B)*a)*log(-sin(d*x + c) + 1))/(d*sin(d
*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36947, size = 113, normalized size = 2.4

$$\frac{(Aa-Ba)\log(|\sin(dx+c)+1|)-(Aa-Ba)\log(|\sin(dx+c)-1|)+\frac{Aa\sin(dx+c)-Ba\sin(dx+c)-3\,Aa-Ba}{\sin(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*((A*a - B*a)*log(abs(sin(d*x + c) + 1)) - (A*a - B*a)*log(abs(sin(d*x +
 c) - 1)) + (A*a*sin(d*x + c) - B*a*sin(d*x + c) - 3*A*a - B*a)/(sin(d*x +
c) - 1))/d
```

3.959 $\int \sec^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=100

$$\frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2}-\frac{a^2(A-B)}{8d(a\sin(c+dx)+a)}+\frac{a^2A}{4d(a-a\sin(c+dx))}+\frac{a(3A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] (a*(3*A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(A + B))/(8*d*(a - a*Sin[c
 + d*x])^2) + (a^2*A)/(4*d*(a - a*Sin[c + d*x])) - (a^2*(A - B))/(8*d*(a +
a*Sin[c + d*x]))
```

Rubi [A] time = 0.122077, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$\frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2}-\frac{a^2(A-B)}{8d(a\sin(c+dx)+a)}+\frac{a^2A}{4d(a-a\sin(c+dx))}+\frac{a(3A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```


```
[Out] (a*(3*A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(A + B))/(8*d*(a - a*Sin[c
 + d*x])^2) + (a^2*A)/(4*d*(a - a*Sin[c + d*x])) - (a^2*(A - B))/(8*d*(a +
a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{a^5 \operatorname{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5 \operatorname{Subst}\left(\int \left(\frac{A+B}{4a^2(a-x)^3} + \frac{A}{4a^3(a-x)^2} + \frac{A-B}{8a^3(a+x)^2} + \frac{3A-B}{8a^3(a^2-x^2)}\right) dx, x\right.}{d}$$

$$= \frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2} + \frac{a^2A}{4d(a-a\sin(c+dx))} - \frac{a^2(A-}{8d(a+a\sin(}$$

$$= \frac{a(3A-B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2} + \frac{}{4d}$$

Mathematica [C] time = 1.52678, size = 357, normalized size = 3.57

$$\frac{a(\sin(c+dx)+1)\left(ix(3A-B)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2 + \frac{2(A+B)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} - \frac{2(3A-B)\left(\sin\left(\frac{1}{2}(c+dx)\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*((2*(-A + B))/d + I*(3*A - B)*x*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
- ((2*I)*(3*A - B)*ArcTan[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^2)/d - (2*(3*A - B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2)/d + ((3*A - B)*Log[(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/d + (2*(A + B)*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^4) + (4*A*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(d*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])^2))*(1 + Sin[c + d*x]))/(16*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^4)
```

Maple [A] time = 0.096, size = 173, normalized size = 1.7

$$\frac{aA}{4d(\cos(dx+c))^4} + \frac{aB(\sin(dx+c))^3}{4d(\cos(dx+c))^4} + \frac{aB(\sin(dx+c))^3}{8d(\cos(dx+c))^2} + \frac{aB\sin(dx+c)}{8d} - \frac{aB\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{aA}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*A/cos(d*x+c)^4+1/4/d*a*B*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a*B*sin(d*
x+c)^3/cos(d*x+c)^2+1/8*a*B*sin(d*x+c)/d-1/8/d*a*B*ln(sec(d*x+c)+tan(d*x+c)
)+1/4/d*a*A*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a*A*sec(d*x+c)*tan(d*x+c)+3/8/d*a
*A*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a*B/cos(d*x+c)^4
```

Maxima [A] time = 1.02367, size = 155, normalized size = 1.55

$$\frac{(3A-B)a\log(\sin(dx+c)+1) - (3A-B)a\log(\sin(dx+c)-1) - \frac{2\left((3A-B)a\sin(dx+c)^2-(3A-B)a\sin(dx+c)-2(A+B)a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*((3*A - B)*a*log(sin(d*x + c) + 1) - (3*A - B)*a*log(sin(d*x + c) - 1)
 - 2*((3*A - B)*a*sin(d*x + c)^2 - (3*A - B)*a*sin(d*x + c) - 2*(A + B)*a)/
(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.81289, size = 439, normalized size = 4.39

$$-\frac{2\,(3\,A-B)a\cos\left(dx+c\right)^{2}+2\,(3\,A-B)a\sin\left(dx+c\right)-2\,(A-3\,B)a-\left((3\,A-B)a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-(3\,A-}{16\left(d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/16*(2*(3*A - B)*a*cos(d*x + c)^2 + 2*(3*A - B)*a*sin(d*x + c) - 2*(A - 3
*B)*a - ((3*A - B)*a*cos(d*x + c)^2*sin(d*x + c) - (3*A - B)*a*cos(d*x + c)
^2)*log(sin(d*x + c) + 1) + ((3*A - B)*a*cos(d*x + c)^2*sin(d*x + c) - (3*A
 - B)*a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2*sin(d*x +
 c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34597, size = 205, normalized size = 2.05

$$\frac{2\,(3\,Aa-Ba)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,(3\,Aa-Ba)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,(3\,Aa\sin(dx+c)-Ba\sin(dx+c)+5\,Aa-3\,Ba)}{\sin(dx+c)+1}+\frac{9\,Aa}{}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*(2*(3*A*a - B*a)*log(abs(sin(d*x + c) + 1)) - 2*(3*A*a - B*a)*log(abs(
sin(d*x + c) - 1)) - 2*(3*A*a*sin(d*x + c) - B*a*sin(d*x + c) + 5*A*a - 3*B
*a)/(sin(d*x + c) + 1) + (9*A*a*sin(d*x + c)^2 - 3*B*a*sin(d*x + c)^2 - 26*
A*a*sin(d*x + c) + 6*B*a*sin(d*x + c) + 21*A*a + B*a)/(sin(d*x + c) - 1)^2)
/d
```

3.960 $\int \sec^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=157

$$\frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3}+\frac{a^3(3A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a^3(A-B)}{32d(a\sin(c+dx)+a)^2}-\frac{a^2(2A-B)}{16d(a\sin(c+dx)+a)}+\frac{3a^2A}{16d(a-a\sin(c+$$

```
[Out] (a*(5*A - B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(A + B))/(24*d*(a - a*Sin
[c + d*x])^3) + (a^3*(3*A + B))/(32*d*(a - a*Sin[c + d*x])^2) + (3*a^2*A)/(
16*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(32*d*(a + a*Sin[c + d*x])^2) -
(a^2*(2*A - B))/(16*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.167853, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$\frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3}+\frac{a^3(3A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a^3(A-B)}{32d(a\sin(c+dx)+a)^2}-\frac{a^2(2A-B)}{16d(a\sin(c+dx)+a)}+\frac{3a^2A}{16d(a-a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```


```
[Out] (a*(5*A - B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(A + B))/(24*d*(a - a*Sin
[c + d*x])^3) + (a^3*(3*A + B))/(32*d*(a - a*Sin[c + d*x])^2) + (3*a^2*A)/(
16*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(32*d*(a + a*Sin[c + d*x])^2) -
(a^2*(2*A - B))/(16*d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{a^7\operatorname{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\operatorname{Subst}\left(\int \left(\frac{A+B}{8a^3(a-x)^4} + \frac{3A+B}{16a^4(a-x)^3} + \frac{3A}{16a^5(a-x)^2} + \frac{A-B}{16a^4(a+x)^3}\right.\right.}{d}$$

$$= \frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3} + \frac{a^3(3A+B)}{32d(a-a\sin(c+dx))^2} + \frac{}{16d(a}$$

$$= \frac{a(5A-B)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3}$$

Mathematica [C] time = 1.68966, size = 451, normalized size = 2.87

$$\frac{a(\sin(c+dx)+1)\left(3ix(5A-B)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4 + \frac{3(3A+B)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} + \frac{4(A+B)\left(\sin\left(\frac{1}{2}(\right.\right.}{d\left(\cos\left(\frac{1}{2}(c+\right.\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*((3*(-A + B))/d - (6*(2*A - B)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/
d + (3*I)*(5*A - B)*x*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - ((6*I)*(5*A
 - B)*ArcTan[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/d -
 (6*(5*A - B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^4)/d + (3*(5*A - B)*Log[(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/d + (4*(A + B)*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6) +
 (3*(3*A + B)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^4) + (18*A*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2))*(1 + Sin[c + d*x]))/(96*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.104, size = 217, normalized size = 1.4

$$\frac{aA}{6d(\cos(dx+c))^6} + \frac{aB(\sin(dx+c))^3}{6d(\cos(dx+c))^6} + \frac{aB(\sin(dx+c))^3}{8d(\cos(dx+c))^4} + \frac{aB(\sin(dx+c))^3}{16d(\cos(dx+c))^2} + \frac{aB\sin(dx+c)}{16d} - \frac{aB\ln(\sec(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/6/d*a*A/cos(d*x+c)^6+1/6/d*a*B*sin(d*x+c)^3/cos(d*x+c)^6+1/8/d*a*B*sin(d*
x+c)^3/cos(d*x+c)^4+1/16/d*a*B*sin(d*x+c)^3/cos(d*x+c)^2+1/16*a*B*sin(d*x+c
)/d-1/16/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*a*A*tan(d*x+c)*sec(d*x+c)^5+
5/24/d*a*A*tan(d*x+c)*sec(d*x+c)^3+5/16/d*a*A*sec(d*x+c)*tan(d*x+c)+5/16/d*
a*A*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*a*B/cos(d*x+c)^6
```

Maxima [A] time = 1.08341, size = 231, normalized size = 1.47

$$\frac{3\,(5\,A-B)a\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A-B)a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(5\,A-B)a\sin(dx+c)^4-3\,(5\,A-B)a\sin(dx+c)^3-5\,(5\,A-B)a\sin}{\sin(dx+c)^5-\sin(dx+c)^4-2\,\sin(dx+c)^3+2\,\sin}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(5*A - B)*a*log(sin(d*x + c) + 1) - 3*(5*A - B)*a*log(sin(d*x + c)
- 1) - 2*(3*(5*A - B)*a*sin(d*x + c)^4 - 3*(5*A - B)*a*sin(d*x + c)^3 - 5*(
5*A - B)*a*sin(d*x + c)^2 + 5*(5*A - B)*a*sin(d*x + c) + 8*(A + B)*a)/(sin(
d*x + c)^5 - sin(d*x + c)^4 - 2*sin(d*x + c)^3 + 2*sin(d*x + c)^2 + sin(d*x
 + c) - 1))/d
```

Fricas [A] time = 1.85864, size = 533, normalized size = 3.39

$$-\frac{6\,(5\,A-B)a\cos\left(dx+c\right)^4-2\,(5\,A-B)a\cos\left(dx+c\right)^2-4\,(A-5\,B)a-3\left((5\,A-B)a\cos\left(dx+c\right)^4\sin\left(dx+c\right)-(5\,A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/96*(6*(5*A - B)*a*cos(d*x + c)^4 - 2*(5*A - B)*a*cos(d*x + c)^2 - 4*(A -
 5*B)*a - 3*((5*A - B)*a*cos(d*x + c)^4*sin(d*x + c) - (5*A - B)*a*cos(d*x
+ c)^4)*log(sin(d*x + c) + 1) + 3*((5*A - B)*a*cos(d*x + c)^4*sin(d*x + c)
- (5*A - B)*a*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(3*(5*A - B)*a*cos
(d*x + c)^2 + 2*(5*A - B)*a)*sin(d*x + c))/(d*cos(d*x + c)^4*sin(d*x + c) -
 d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42576, size = 271, normalized size = 1.73

$$\frac{6\,(5\,Aa-Ba)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\,(5\,Aa-Ba)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{3\left(15\,Aa\sin(dx+c)^2-3\,Ba\sin(dx+c)^2+38\,Aa\sin(dx+c)-10}{(\sin(dx+c)+1)^2}}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*(6*(5*A*a - B*a)*log(abs(sin(d*x + c) + 1)) - 6*(5*A*a - B*a)*log(abs
(sin(d*x + c) - 1)) - 3*(15*A*a*sin(d*x + c)^2 - 3*B*a*sin(d*x + c)^2 + 38*
A*a*sin(d*x + c) - 10*B*a*sin(d*x + c) + 25*A*a - 9*B*a)/(sin(d*x + c) + 1)
^2 + (55*A*a*sin(d*x + c)^3 - 11*B*a*sin(d*x + c)^3 - 201*A*a*sin(d*x + c)^
2 + 33*B*a*sin(d*x + c)^2 + 255*A*a*sin(d*x + c) - 27*B*a*sin(d*x + c) - 11
7*A*a - 3*B*a)/(sin(d*x + c) - 1)^3)/d
```

3.961 $\int \cos^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=138

$$-\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{a(8A+B)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a(8A+B)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a(8A+B)\sin}{}$$

```
[Out] (5*a*(8*A + B)*x)/128 - (a*(8*A + B)*Cos[c + d*x]^7)/(56*d) + (5*a*(8*A + B
)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*a*(8*A + B)*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + (a*(8*A + B)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (B*Cos
[c + d*x]^7*(a + a*Sin[c + d*x]))/(8*d)
```

Rubi [A] time = 0.140802, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2669, 2635, 8}

$$-\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{a(8A+B)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a(8A+B)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a(8A+B)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (5*a*(8*A + B)*x)/128 - (a*(8*A + B)*Cos[c + d*x]^7)/(56*d) + (5*a*(8*A + B
)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*a*(8*A + B)*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + (a*(8*A + B)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (B*Cos
[c + d*x]^7*(a + a*Sin[c + d*x]))/(8*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))}{8d} + \frac{1}{8}(8A+B)\int \cos^6(c$$

$$= -\frac{a(8A+B)\cos^7(c+dx)}{56d} - \frac{B\cos^7(c+dx)(a+a\sin(c+d$$

$$= -\frac{a(8A+B)\cos^7(c+dx)}{56d} + \frac{a(8A+B)\cos^5(c+dx)\sin(c+}{48d}$$

$$= -\frac{a(8A+B)\cos^7(c+dx)}{56d} + \frac{5a(8A+B)\cos^3(c+dx)\sin(c}{192d}$$

$$= -\frac{a(8A+B)\cos^7(c+dx)}{56d} + \frac{5a(8A+B)\cos(c+dx)\sin(c+}{128d}$$

$$= \frac{5}{128}a(8A+B)x - \frac{a(8A+B)\cos^7(c+dx)}{56d} + \frac{5a(8A+B)\mathrm{c}}{}$$

Mathematica [A] time = 0.880102, size = 164, normalized size = 1.19

$$-\frac{a(1680(A+B)\cos(c+dx)+1008(A+B)\cos(3(c+dx))-5040A\sin(2(c+dx))-1008A\sin(4(c+dx))-112A\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a*(-6720*A*d*x - 840*B*d*x + 1680*(A + B)*Cos[c + d*x] + 1008*(A + B)*Cos
[3*(c + d*x)] + 336*A*Cos[5*(c + d*x)] + 336*B*Cos[5*(c + d*x)] + 48*A*Cos[
7*(c + d*x)] + 48*B*Cos[7*(c + d*x)] - 5040*A*Sin[2*(c + d*x)] - 336*B*Sin[
2*(c + d*x)] - 1008*A*Sin[4*(c + d*x)] + 168*B*Sin[4*(c + d*x)] - 112*A*Sin
[6*(c + d*x)] + 112*B*Sin[6*(c + d*x)] + 21*B*Sin[8*(c + d*x)]))/(21504*d)
```

Maple [A] time = 0.056, size = 138, normalized size = 1.

$$\frac{1}{d}\left(aB\left(-\frac{\sin(dx+c)(\cos(dx+c))^7}{8}+\frac{\sin(dx+c)}{48}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{128}+\frac{5}{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-1/7*a*A*cos(d*x+c)^7-1/7*a*B
*cos(d*x+c)^7+a*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(
d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.03704, size = 167, normalized size = 1.21

$$-\frac{3072\,Aa\cos(dx+c)^7+3072\,Ba\cos(dx+c)^7+112\left(4\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\sin(4\,dx+4\,c)-48\sin(\right.}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/21504*(3072*A*a*cos(d*x + c)^7 + 3072*B*a*cos(d*x + c)^7 + 112*(4*sin(2*
d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*
a - 7*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*si
n(4*d*x + 4*c))*B*a)/d
```

Fricas [A] time = 1.91591, size = 267, normalized size = 1.93

$$-\frac{384\,(A+B)a\cos\left(dx+c\right)^{7}-105\,(8\,A+B)adx+7\left(48\,Ba\cos\left(dx+c\right)^{7}-8\,(8\,A+B)a\cos\left(dx+c\right)^{5}-10\,(8\,A+B)a\cos\right.}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2688*(384*(A + B)*a*cos(d*x + c)^7 - 105*(8*A + B)*a*d*x + 7*(48*B*a*cos
(d*x + c)^7 - 8*(8*A + B)*a*cos(d*x + c)^5 - 10*(8*A + B)*a*cos(d*x + c)^3
- 15*(8*A + B)*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 13.8277, size = 416, normalized size = 3.01

$$\begin{cases} \frac{5Aax\sin^{6}(c+dx)}{16}+\frac{15Aax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{15Aax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{5Aax\cos^{6}(c+dx)}{16}+\frac{5Aa\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{5Aa\sin^{3}(c+\cdots}{\cdots} \\ x\left(A+B\sin\left(c\right)\right)\left(a\sin\left(c\right)+a\right)\cos^{6}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*A*a*x*sin(c + d*x)**6/16 + 15*A*a*x*sin(c + d*x)**4*cos(c + d*
x)**2/16 + 15*A*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*A*a*x*cos(c + d*
x)**6/16 + 5*A*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*A*a*sin(c + d*x)**
3*cos(c + d*x)**3/(6*d) + 11*A*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - A*a*
cos(c + d*x)**7/(7*d) + 5*B*a*x*sin(c + d*x)**8/128 + 5*B*a*x*sin(c + d*x)*
*6*cos(c + d*x)**2/32 + 15*B*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*B*a
*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 5*B*a*x*cos(c + d*x)**8/128 + 5*B*a
*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*B*a*sin(c + d*x)**5*cos(c + d*x)
**3/(384*d) + 73*B*a*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) - 5*B*a*sin(c
+ d*x)*cos(c + d*x)**7/(128*d) - B*a*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(
A + B*sin(c))*(a*sin(c) + a)*cos(c)**6, True))
```

Giac [A] time = 1.327, size = 238, normalized size = 1.72

$$\frac{5}{128}\,(8\,Aa+Ba)x-\frac{Ba\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{(Aa+Ba)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{(Aa+Ba)\cos\left(5\,dx+5\,c\right)}{64\,d}-\frac{3\,(Aa+Ba)\cos\left(3\,dx\cdots\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 5/128*(8*A*a + B*a)*x - 1/1024*B*a*sin(8*d*x + 8*c)/d - 1/448*(A*a + B*a)*c
os(7*d*x + 7*c)/d - 1/64*(A*a + B*a)*cos(5*d*x + 5*c)/d - 3/64*(A*a + B*a)*
cos(3*d*x + 3*c)/d - 5/64*(A*a + B*a)*cos(d*x + c)/d + 1/192*(A*a - B*a)*si
n(6*d*x + 6*c)/d + 1/128*(6*A*a - B*a)*sin(4*d*x + 4*c)/d + 1/64*(15*A*a +
B*a)*sin(2*d*x + 2*c)/d
```

3.962 $\int \cos^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=111

$$-\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a(6A+B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}ax(6A+B)-$$

```
[Out] (a*(6*A + B)*x)/16 - (a*(6*A + B)*Cos[c + d*x]^5)/(30*d) + (a*(6*A + B)*Cos
[c + d*x]*Sin[c + d*x])/(16*d) + (a*(6*A + B)*Cos[c + d*x]^3*Sin[c + d*x])/
(24*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x]))/(6*d)
```

Rubi [A] time = 0.112641, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2669, 2635, 8}

$$-\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a(6A+B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}ax(6A+B)-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(6*A + B)*x)/16 - (a*(6*A + B)*Cos[c + d*x]^5)/(30*d) + (a*(6*A + B)*Cos
[c + d*x]*Sin[c + d*x])/(16*d) + (a*(6*A + B)*Cos[c + d*x]^3*Sin[c + d*x])/
(24*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x]))/(6*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))}{6d} + \frac{1}{6}(6A+B)\int \cos^4(c$$

$$= -\frac{a(6A+B)\cos^5(c+dx)}{30d} - \frac{B\cos^5(c+dx)(a+a\sin(c+dx}{6d}$$

$$= -\frac{a(6A+B)\cos^5(c+dx)}{30d} + \frac{a(6A+B)\cos^3(c+dx)\sin(c+}{24d}$$

$$= -\frac{a(6A+B)\cos^5(c+dx)}{30d} + \frac{a(6A+B)\cos(c+dx)\sin(c+}{16d}$$

$$= \frac{1}{16}a(6A+B)x - \frac{a(6A+B)\cos^5(c+dx)}{30d} + \frac{a(6A+B)\cos(}{}$$

Mathematica [A] time = 0.619302, size = 120, normalized size = 1.08

$$-\frac{a(120(A+B)\cos(c+dx)+60(A+B)\cos(3(c+dx))-240A\sin(2(c+dx))-30A\sin(4(c+dx))+12A\cos(5(c+d}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a*(-360*A*d*x - 60*B*d*x + 120*(A + B)*Cos[c + d*x] + 60*(A + B)*Cos[3*(c
 + d*x)] + 12*A*Cos[5*(c + d*x)] + 12*B*Cos[5*(c + d*x)] - 240*A*Sin[2*(c +
 d*x)] - 15*B*Sin[2*(c + d*x)] - 30*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*
x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.055, size = 118, normalized size = 1.1

$$\frac{1}{d}\left(aB\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6}+\frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{dx}{16}+\frac{c}{16}\right)-\frac{aA(\cos(dx+c))}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-1/5*a*A*cos(d*x+c)^5-1/5*a*B*cos(d*x+c)^5+a*A*(1
/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.01507, size = 132, normalized size = 1.19

$$-\frac{192\,Aa\cos(dx+c)^5+192\,Ba\cos(dx+c)^5-30\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa-5\left(4\,\sin(2\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/960*(192*A*a*cos(d*x + c)^5 + 192*B*a*cos(d*x + c)^5 - 30*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 5*(4*sin(2*d*x + 2*c)^3 + 1
```

```
2*d*x + 12*c - 3*sin(4*d*x + 4*c))*B*a)/d
```

Fricas [A] time = 1.77524, size = 217, normalized size = 1.95

$$-\frac{48\,(A+B)a\cos{(dx+c)}^5-15\,(6\,A+B)adx+5\left(8\,Ba\cos{(dx+c)}^5-2\,(6\,A+B)a\cos{(dx+c)}^3-3\,(6\,A+B)a\cos{(dx+\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/240*(48*(A + B)*a*cos(d*x + c)^5 - 15*(6*A + B)*a*d*x + 5*(8*B*a*cos(d*x
 + c)^5 - 2*(6*A + B)*a*cos(d*x + c)^3 - 3*(6*A + B)*a*cos(d*x + c))*sin(d*
x + c))/d
```

Sympy [A] time = 4.8359, size = 306, normalized size = 2.76

$$\begin{cases} \frac{3Aax\sin^4{(c+dx)}}{8}+\frac{3Aax\sin^2{(c+dx)}\cos^2{(c+dx)}}{4}+\frac{3Aax\cos^4{(c+dx)}}{8}+\frac{3Aa\sin^3{(c+dx)}\cos{(c+dx)}}{8d}+\frac{5Aa\sin{(c+dx)}\cos^3{(c+dx)}}{8d}-\frac{Aa\cos^5{(c+dx)}}{5d}+ \\ x\,(A+B\sin{(c)})\,(a\sin{(c)}+a)\cos^4{(c)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a*x*sin(c + d*x)**4/8 + 3*A*a*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 3*A*a*x*cos(c + d*x)**4/8 + 3*A*a*sin(c + d*x)**3*cos(c + d*x)/(8*d
) + 5*A*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) - A*a*cos(c + d*x)**5/(5*d) +
B*a*x*sin(c + d*x)**6/16 + 3*B*a*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B
*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + B*a*x*cos(c + d*x)**6/16 + B*a*si
n(c + d*x)**5*cos(c + d*x)/(16*d) + B*a*sin(c + d*x)**3*cos(c + d*x)**3/(6*
d) - B*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - B*a*cos(c + d*x)**5/(5*d), N
e(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)*cos(c)**4, True))
```

Giac [A] time = 1.29168, size = 180, normalized size = 1.62

$$\frac{1}{16}\,(6\,Aa+Ba)x-\frac{Ba\sin{(6\,dx+6\,c)}}{192\,d}-\frac{(Aa+Ba)\cos{(5\,dx+5\,c)}}{80\,d}-\frac{(Aa+Ba)\cos{(3\,dx+3\,c)}}{16\,d}-\frac{(Aa+Ba)\cos{(dx+c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/16*(6*A*a + B*a)*x - 1/192*B*a*sin(6*d*x + 6*c)/d - 1/80*(A*a + B*a)*cos(
5*d*x + 5*c)/d - 1/16*(A*a + B*a)*cos(3*d*x + 3*c)/d - 1/8*(A*a + B*a)*cos(
d*x + c)/d + 1/64*(2*A*a - B*a)*sin(4*d*x + 4*c)/d + 1/64*(16*A*a + B*a)*si
n(2*d*x + 2*c)/d
```

3.963 $\int \cos^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=84

$$-\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+B)-\frac{B\cos^3(c+dx)(a\sin(c+dx)+a)}{4d}$$

```
[Out] (a*(4*A + B)*x)/8 - (a*(4*A + B)*Cos[c + d*x]^3)/(12*d) + (a*(4*A + B)*Cos[
c + d*x]*Sin[c + d*x])/(8*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c + d*x]))/(4*d
)
```

Rubi [A] time = 0.0953011, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2860, 2669, 2635, 8}

$$-\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+B)-\frac{B\cos^3(c+dx)(a\sin(c+dx)+a)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(4*A + B)*x)/8 - (a*(4*A + B)*Cos[c + d*x]^3)/(12*d) + (a*(4*A + B)*Cos[
c + d*x]*Sin[c + d*x])/(8*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c + d*x]))/(4*d
)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))}{4d}+\frac{1}{4}(4A+B)\int \cos^2(c+a$$
$$= -\frac{a(4A+B)\cos^3(c+dx)}{12d}-\frac{B\cos^3(c+dx)(a+a\sin(c+dx))}{4d}$$
$$= -\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\cos(c+dx)\sin(c+dx)}{8d}$$
$$= \frac{1}{8}a(4A+B)x-\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\cos(c+}{8d}$$

Mathematica [A] time = 0.661985, size = 64, normalized size = 0.76

$$-\frac{a(24(A+B)\cos(c+dx)+8(A+B)\cos(3(c+dx))-12dx(4A+B)-24A\sin(2(c+dx))+3B\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a*(-12*(4*A + B)*d*x + 24*(A + B)*Cos[c + d*x] + 8*(A + B)*Cos[3*(c + d*x
)] - 24*A*Sin[2*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.052, size = 96, normalized size = 1.1

$$\frac{1}{d}\left(aB\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{aA(\cos(dx+c))^3}{3}-\frac{aB(\cos(dx+c))^3}{3}+a.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-1/3*a*A*cos(d*x+c)^3-1/3*a*B*cos(d*x+c)^3+a*A*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.01098, size = 100, normalized size = 1.19

$$-\frac{32\,Aa\cos(dx+c)^3+32\,Ba\cos(dx+c)^3-24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa-3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))Ba}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*A*a*cos(d*x + c)^3 + 32*B*a*cos(d*x + c)^3 - 24*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*a - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*B*a)/d
```

Fricas [A] time = 1.76367, size = 169, normalized size = 2.01

$$-\frac{8\,(A+B)a\cos(dx+c)^3-3\,(4\,A+B)adx+3\left(2\,Ba\cos(dx+c)^3-(4\,A+B)a\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/24*(8*(A + B)*a*cos(d*x + c)^3 - 3*(4*A + B)*a*d*x + 3*(2*B*a*cos(d*x +
c)^3 - (4*A + B)*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.34164, size = 199, normalized size = 2.37

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} - \frac{Aa\cos^3(c+dx)}{3d} + \frac{Bax\sin^4(c+dx)}{8} + \frac{Bax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Bax\cos^4(c+dx)}{8} \\ x(A + B\sin(c))(a\sin(c) + a)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) - A*a*cos(c + d*x)**3/(3*d) + B*a*x*sin(c + d*x)**4
/8 + B*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a*x*cos(c + d*x)**4/8 + B*
a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - B*a*sin(c + d*x)*cos(c + d*x)**3/(8*
d) - B*a*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)
*cos(c)**2, True))
```

Giac [A] time = 1.32078, size = 112, normalized size = 1.33

$$\frac{1}{8}(4\,Aa + Ba)x - \frac{Ba\sin(4\,dx + 4\,c)}{32\,d} + \frac{Aa\sin(2\,dx + 2\,c)}{4\,d} - \frac{(Aa + Ba)\cos(3\,dx + 3\,c)}{12\,d} - \frac{(Aa + Ba)\cos(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + B*a)*x - 1/32*B*a*sin(4*d*x + 4*c)/d + 1/4*A*a*sin(2*d*x + 2*c
)/d - 1/12*(A*a + B*a)*cos(3*d*x + 3*c)/d - 1/4*(A*a + B*a)*cos(d*x + c)/d
```

3.964 $\int \sec^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=29

$$\frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)}{d}-aBx$$

```
[Out] -(a*B*x) + ((A + B)*Sec[c + d*x]*(a + a*Sin[c + d*x]))/d
```

Rubi [A] time = 0.0488888, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2855, 8}

$$\frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)}{d}-aBx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a*B*x) + ((A + B)*Sec[c + d*x]*(a + a*Sin[c + d*x]))/d
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx&=\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))}{d}-(aB)\int 1\,dx\\&=-aBx+\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))}{d}\end{aligned}$$

Mathematica [B] time = 0.339527, size = 85, normalized size = 2.93

$$\frac{a\left(2(A+B)\sin\left(\frac{dx}{2}\right)+Bdx\sin\left(c+\frac{dx}{2}\right)-Bdx\cos\left(\frac{dx}{2}\right)\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(-(B*d*x*Cos[(d*x)/2]) + 2*(A + B)*Sin[(d*x)/2] + B*d*x*Sin[c + (d*x)/2]
))/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
```

Maple [A] time = 0.064, size = 54, normalized size = 1.9

$$\frac{1}{d}\left(\frac{aA}{\cos(dx+c)}+aB(\tan(dx+c)-dx-c)+aA\tan(dx+c)+\frac{aB}{\cos(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*A/cos(d*x+c)+a*B*(tan(d*x+c)-d*x-c)+a*A*tan(d*x+c)+a*B/cos(d*x+c))
```

Maxima [A] time = 1.5162, size = 76, normalized size = 2.62

$$-\frac{(dx+c-\tan(dx+c))Ba-Aa\tan(dx+c)-\frac{Aa}{\cos(dx+c)}-\frac{Ba}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -((d*x + c - tan(d*x + c))*B*a - A*a*tan(d*x + c) - A*a/cos(d*x + c) - B*a/
cos(d*x + c))/d
```

Fricas [B] time = 1.82678, size = 184, normalized size = 6.34

$$-\frac{Badx-(A+B)a+(Badx-(A+B)a)\cos(dx+c)-(Badx+(A+B)a)\sin(dx+c)}{d\cos(dx+c)-d\sin(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -(B*a*d*x - (A + B)*a + (B*a*d*x - (A + B)*a)*cos(d*x + c) - (B*a*d*x + (A
+ B)*a)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec^2(c+dx)\,dx+\int A\sin(c+dx)\sec^2(c+dx)\,dx+\int B\sin(c+dx)\sec^2(c+dx)\,dx+\int B\sin^2(c+dx)\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*sin(c + d*x)*sec(c + d*x)**2
, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**2, x) + Integral(B*sin(c + d*x
)**2*sec(c + d*x)**2, x))
```

Giac [A] time = 1.3602, size = 49, normalized size = 1.69

$$-\frac{(dx+c)Ba+\frac{2\,(Aa+Ba)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((d*x + c)*B*a + 2*(A*a + B*a)/(tan(1/2*d*x + 1/2*c) - 1))/d
```

3.965 $\int \sec^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=50

$$\frac{a(2A-B)\tan(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)}{3d}$$

```
[Out] ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c + d*x]))/(3*d) + (a*(2*A - B)*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0678648, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2855, 3767, 8}

$$\frac{a(2A-B)\tan(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c + d*x]))/(3*d) + (a*(2*A - B)*Tan[c + d*x])/(3*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d}+\frac{1}{3}(a(2A-B))\int\\ &= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d}-\frac{(a(2A-B))\,\mathrm{Sub}}{}\\ &= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d}+\frac{a(2A-B)\tan(c}{3d}\end{aligned}$$

Mathematica [A] time = 0.587961, size = 97, normalized size = 1.94

$$\frac{a\sec(c)(\sin(c+dx)+1)\sec^3(c+dx)(-2(A+B)\cos(c+dx)+A\sin(2(c+dx))+4A\cos(c+2dx)+8A\sin(dx)+B\sin(}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*Sec[c + d*x]^3*(1 + Sin[c + d*x])*(6*B*Cos[c] - 2*(A + B)*Cos[c +
 d*x] + 4*A*Cos[c + 2*d*x] - 2*B*Cos[c + 2*d*x] + 8*A*Sin[d*x] - 4*B*Sin[d*
x] + A*Sin[2*(c + d*x)] + B*Sin[2*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.084, size = 72, normalized size = 1.4

$$\frac{1}{d}\left(\frac{aA}{3\left(\cos\left(dx+c\right)\right)^3}+\frac{aB\left(\sin\left(dx+c\right)\right)^3}{3\left(\cos\left(dx+c\right)\right)^3}-aA\left(-\frac{2}{3}-\frac{\left(\sec\left(dx+c\right)\right)^2}{3}\right)\tan\left(dx+c\right)+\frac{aB}{3\left(\cos\left(dx+c\right)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*A/cos(d*x+c)^3+1/3*a*B*sin(d*x+c)^3/cos(d*x+c)^3-a*A*(-2/3-1/3*s
ec(d*x+c)^2)*tan(d*x+c)+1/3*a*B/cos(d*x+c)^3)
```

Maxima [A] time = 1.02611, size = 80, normalized size = 1.6

$$\frac{Ba\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aa+\frac{Aa}{\cos(dx+c)^3}+\frac{Ba}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*(B*a*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + A*a/cos(d
*x + c)^3 + B*a/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.62345, size = 166, normalized size = 3.32

$$-\frac{(2\,A-B)a\cos\left(dx+c\right)^2+(2\,A-B)a\sin\left(dx+c\right)-(A-2\,B)a}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/3*((2*A - B)*a*cos(d*x + c)^2 + (2*A - B)*a*sin(d*x + c) - (A - 2*B)*a)/
(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32178, size = 127, normalized size = 2.54

$$-\frac{\frac{3(Aa-Ba)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}+\frac{9Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7Aa+Ba}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(3*(A*a - B*a)/(tan(1/2*d*x + 1/2*c) + 1) + (9*A*a*tan(1/2*d*x + 1/2*c
)^2 + 3*B*a*tan(1/2*d*x + 1/2*c)^2 - 12*A*a*tan(1/2*d*x + 1/2*c) + 7*A*a +
B*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.966 $\int \sec^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=73

$$\frac{a(4A-B)\tan^3(c+dx)}{15d}+\frac{a(4A-B)\tan(c+dx)}{5d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)}{5d}$$

```
[Out] ((A + B)*Sec[c + d*x]^5*(a + a*Sin[c + d*x]))/(5*d) + (a*(4*A - B)*Tan[c +
d*x])/(5*d) + (a*(4*A - B)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.0729733, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2855, 3767}

$$\frac{a(4A-B)\tan^3(c+dx)}{15d}+\frac{a(4A-B)\tan(c+dx)}{5d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^5*(a + a*Sin[c + d*x]))/(5*d) + (a*(4*A - B)*Tan[c +
d*x])/(5*d) + (a*(4*A - B)*Tan[c + d*x]^3)/(15*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx=\frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d}+\frac{1}{5}(a(4A-B))\int \text{sec}$$
$$=\frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d}-\frac{(a(4A-B))\,\text{Subst}\,(}{}$$
$$=\frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d}+\frac{a(4A-B)\tan(c+}{5d}$$

Mathematica [B] time = 1.2376, size = 223, normalized size = 3.05

$$\frac{a\sec(c)(-54(A+B)\cos(c+dx)+18A\sin(2(c+dx))+9A\sin(4(c+dx))+128A\sin(2c+3dx)-18A\cos(3(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*(240*B*Cos[c] - 54*(A + B)*Cos[c + d*x] - 18*A*Cos[3*(c + d*x)] -
 18*B*Cos[3*(c + d*x)] + 128*A*Cos[c + 2*d*x] - 32*B*Cos[c + 2*d*x] + 64*A*
Cos[3*c + 4*d*x] - 16*B*Cos[3*c + 4*d*x] + 384*A*Sin[d*x] - 96*B*Sin[d*x] +
 18*A*Sin[2*(c + d*x)] + 18*B*Sin[2*(c + d*x)] + 9*A*Sin[4*(c + d*x)] + 9*B
*Sin[4*(c + d*x)] + 128*A*Sin[2*c + 3*d*x] - 32*B*Sin[2*c + 3*d*x]))/(960*d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3)
```

Maple [A] time = 0.099, size = 102, normalized size = 1.4

$$\frac{1}{d}\left(\frac{aA}{5\,(\cos(dx+c))^5}+aB\left(\frac{(\sin(dx+c))^3}{5\,(\cos(dx+c))^5}+\frac{2\,(\sin(dx+c))^3}{15\,(\cos(dx+c))^3}\right)-aA\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4\,(\sec(dx+c))^2}{15}\right)\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a*A/cos(d*x+c)^5+a*B*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c
)^3/cos(d*x+c)^3)-a*A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)
+1/5*a*B/cos(d*x+c)^5)
```

Maxima [A] time = 1.04565, size = 116, normalized size = 1.59

$$\frac{\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa+\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)Ba+\frac{3\,Aa}{\cos(dx+c)^5}+\frac{3\,l}{\cos(d}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a + (3*tan
(d*x + c)^5 + 5*tan(d*x + c)^3)*B*a + 3*A*a/cos(d*x + c)^5 + 3*B*a/cos(d*x
+ c)^5)/d
```

Fricas [A] time = 1.73992, size = 259, normalized size = 3.55

$$-\frac{2\,(4\,A-B)a\cos(dx+c)^4-(4\,A-B)a\cos(dx+c)^2-(A-4\,B)a+\left(2\,(4\,A-B)a\cos(dx+c)^2+(4\,A-B)a\right)\sin(dx}{15\left(d\cos(dx+c)^3\sin(dx+c)-d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/15*(2*(4*A - B)*a*cos(d*x + c)^4 - (4*A - B)*a*cos(d*x + c)^2 - (A - 4*B
)*a + (2*(4*A - B)*a*cos(d*x + c)^2 + (4*A - B)*a)*sin(d*x + c))/(d*cos(d*x
```

```
+ c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)

[Out] Timed out
```

Giac [B] time = 1.28708, size = 304, normalized size = 4.16

$$-\frac{\frac{5\left(15\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,Aa-7\,Ba\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}+\frac{165\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+45\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")

[Out] -1/120*(5*(15*A*a*tan(1/2*d*x + 1/2*c)^2 - 9*B*a*tan(1/2*d*x + 1/2*c)^2 + 2
4*A*a*tan(1/2*d*x + 1/2*c) - 12*B*a*tan(1/2*d*x + 1/2*c) + 13*A*a - 7*B*a)/
(tan(1/2*d*x + 1/2*c) + 1)^3 + (165*A*a*tan(1/2*d*x + 1/2*c)^4 + 45*B*a*tan
(1/2*d*x + 1/2*c)^4 - 480*A*a*tan(1/2*d*x + 1/2*c)^3 - 60*B*a*tan(1/2*d*x +
 1/2*c)^3 + 650*A*a*tan(1/2*d*x + 1/2*c)^2 + 70*B*a*tan(1/2*d*x + 1/2*c)^2
- 400*A*a*tan(1/2*d*x + 1/2*c) - 20*B*a*tan(1/2*d*x + 1/2*c) + 113*A*a + 13
*B*a)/(tan(1/2*d*x + 1/2*c) - 1)^5)/d
```

3.967 $\int \sec^8(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=96

$$\frac{a(6A-B)\tan^5(c+dx)}{35d}+\frac{2a(6A-B)\tan^3(c+dx)}{21d}+\frac{a(6A-B)\tan(c+dx)}{7d}+\frac{(A+B)\sec^7(c+dx)(a\sin(c+dx)+a}{7d}$$

```
[Out] ((A + B)*Sec[c + d*x]^7*(a + a*Sin[c + d*x]))/(7*d) + (a*(6*A - B)*Tan[c + d*x])/(7*d) + (2*a*(6*A - B)*Tan[c + d*x]^3)/(21*d) + (a*(6*A - B)*Tan[c + d*x]^5)/(35*d)
```

Rubi [A] time = 0.0797963, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2855, 3767}

$$\frac{a(6A-B)\tan^5(c+dx)}{35d}+\frac{2a(6A-B)\tan^3(c+dx)}{21d}+\frac{a(6A-B)\tan(c+dx)}{7d}+\frac{(A+B)\sec^7(c+dx)(a\sin(c+dx)+a}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^7*(a + a*Sin[c + d*x]))/(7*d) + (a*(6*A - B)*Tan[c + d*x])/(7*d) + (2*a*(6*A - B)*Tan[c + d*x]^3)/(21*d) + (a*(6*A - B)*Tan[c + d*x]^5)/(35*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d}+\frac{1}{7}(a(6A-B))\int\\ &= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d}-\frac{(a(6A-B))\,\text{Sub}}{}\\ &= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d}+\frac{a(6A-B)\tan(c}{7d}\end{aligned}$$

Mathematica [B] time = 2.02256, size = 315, normalized size = 3.28

$$a\sec(c)(-1500(A+B)\cos(c+dx)+375A\sin(2(c+dx))+300A\sin(4(c+dx))+75A\sin(6(c+dx))+7680A\sin(2$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*(8960*B*Cos[c] - 1500*(A + B)*Cos[c + d*x] - 750*A*Cos[3*(c + d*x
)] - 750*B*Cos[3*(c + d*x)] - 150*A*Cos[5*(c + d*x)] - 150*B*Cos[5*(c + d*x
)] + 3840*A*Cos[c + 2*d*x] - 640*B*Cos[c + 2*d*x] + 3072*A*Cos[3*c + 4*d*x]
 - 512*B*Cos[3*c + 4*d*x] + 768*A*Cos[5*c + 6*d*x] - 128*B*Cos[5*c + 6*d*x]
 + 15360*A*Sin[d*x] - 2560*B*Sin[d*x] + 375*A*Sin[2*(c + d*x)] + 375*B*Sin[
2*(c + d*x)] + 300*A*Sin[4*(c + d*x)] + 300*B*Sin[4*(c + d*x)] + 75*A*Sin[6
*(c + d*x)] + 75*B*Sin[6*(c + d*x)] + 7680*A*Sin[2*c + 3*d*x] - 1280*B*Sin[
2*c + 3*d*x] + 1536*A*Sin[4*c + 5*d*x] - 256*B*Sin[4*c + 5*d*x]))/(53760*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^5)
```

Maple [A] time = 0.105, size = 130, normalized size = 1.4

$$\frac{1}{d}\left(\frac{aA}{7\left(\cos\left(dx+c\right)\right)^{7}}+aB\left(\frac{\left(\sin\left(dx+c\right)\right)^{3}}{7\left(\cos\left(dx+c\right)\right)^{7}}+\frac{4\left(\sin\left(dx+c\right)\right)^{3}}{35\left(\cos\left(dx+c\right)\right)^{5}}+\frac{8\left(\sin\left(dx+c\right)\right)^{3}}{105\left(\cos\left(dx+c\right)\right)^{3}}\right)-aA\left(-\frac{16}{35}-\frac{\left(\sec\left(dx+c\right)\right)^{6}}{7}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/7*a*A/cos(d*x+c)^7+a*B*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c
)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)-a*A*(-16/35-1/7*sec(d*x+c
)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c)+1/7*a*B/cos(d*x+c)^7)
```

Maxima [A] time = 1.01267, size = 144, normalized size = 1.5

$$\frac{3\left(5\tan\left(dx+c\right)^{7}+21\tan\left(dx+c\right)^{5}+35\tan\left(dx+c\right)^{3}+35\tan\left(dx+c\right)\right)Aa+\left(15\tan\left(dx+c\right)^{7}+42\tan\left(dx+c\right)^{5}+\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/105*(3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan
(d*x + c))*A*a + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3
)*B*a + 15*A*a/cos(d*x + c)^7 + 15*B*a/cos(d*x + c)^7)/d
```

Fricas [A] time = 1.78687, size = 350, normalized size = 3.65

$$-\frac{8\left(6A-B\right)a\cos\left(dx+c\right)^{6}-4\left(6A-B\right)a\cos\left(dx+c\right)^{4}-\left(6A-B\right)a\cos\left(dx+c\right)^{2}-3\left(A-6B\right)a+\left(8\left(6A-B\right)a\cos\left(dx\right.\right.}{105\left(d\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

[Out] -1/105*(8*(6*A - B)*a*cos(d*x + c)^6 - 4*(6*A - B)*a*cos(d*x + c)^4 - (6*A - B)*a*cos(d*x + c)^2 - 3*(A - 6*B)*a + (8*(6*A - B)*a*cos(d*x + c)^4 + 4*(6*A - B)*a*cos(d*x + c)^2 + 3*(6*A - B)*a)*sin(d*x + c))/(d*cos(d*x + c)^5*sin(d*x + c) - d*cos(d*x + c)^5)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)

[Out] Timed out

Giac [B] time = 1.26275, size = 466, normalized size = 4.85

$$-\frac{\frac{7\left(165\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-75\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+540\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-210\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+750\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-280\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+48}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")

[Out] -1/1680*(7*(165*A*a*tan(1/2*d*x + 1/2*c)^4 - 75*B*a*tan(1/2*d*x + 1/2*c)^4 + 540*A*a*tan(1/2*d*x + 1/2*c)^3 - 210*B*a*tan(1/2*d*x + 1/2*c)^3 + 750*A*a*tan(1/2*d*x + 1/2*c)^2 - 280*B*a*tan(1/2*d*x + 1/2*c)^2 + 480*A*a*tan(1/2*d*x + 1/2*c) - 170*B*a*tan(1/2*d*x + 1/2*c) + 129*A*a - 49*B*a)/(tan(1/2*d*x + 1/2*c) + 1)^5 + (2205*A*a*tan(1/2*d*x + 1/2*c)^6 + 525*B*a*tan(1/2*d*x + 1/2*c)^6 - 10080*A*a*tan(1/2*d*x + 1/2*c)^5 - 1470*B*a*tan(1/2*d*x + 1/2*c)^5 + 21945*A*a*tan(1/2*d*x + 1/2*c)^4 + 2555*B*a*tan(1/2*d*x + 1/2*c)^4 - 26460*A*a*tan(1/2*d*x + 1/2*c)^3 - 2240*B*a*tan(1/2*d*x + 1/2*c)^3 + 18963*A*a*tan(1/2*d*x + 1/2*c)^2 + 1407*B*a*tan(1/2*d*x + 1/2*c)^2 - 7476*A*a*tan(1/2*d*x + 1/2*c) - 434*B*a*tan(1/2*d*x + 1/2*c) + 1383*A*a + 137*B*a)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d

3.968 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{a(8A-B)\tan^7(c+dx)}{63d}+\frac{a(8A-B)\tan^5(c+dx)}{15d}+\frac{a(8A-B)\tan^3(c+dx)}{9d}+\frac{a(8A-B)\tan(c+dx)}{9d}+\frac{(A+B)\sec^9(c\cdots}{}$$

```
[Out] ((A + B)*Sec[c + d*x]^9*(a + a*Sin[c + d*x]))/(9*d) + (a*(8*A - B)*Tan[c +
d*x])/(9*d) + (a*(8*A - B)*Tan[c + d*x]^3)/(9*d) + (a*(8*A - B)*Tan[c + d*x
]^5)/(15*d) + (a*(8*A - B)*Tan[c + d*x]^7)/(63*d)
```

Rubi [A] time = 0.0865134, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2855, 3767}

$$\frac{a(8A-B)\tan^7(c+dx)}{63d}+\frac{a(8A-B)\tan^5(c+dx)}{15d}+\frac{a(8A-B)\tan^3(c+dx)}{9d}+\frac{a(8A-B)\tan(c+dx)}{9d}+\frac{(A+B)\sec^9(c\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^9*(a + a*Sin[c + d*x]))/(9*d) + (a*(8*A - B)*Tan[c +
d*x])/(9*d) + (a*(8*A - B)*Tan[c + d*x]^3)/(9*d) + (a*(8*A - B)*Tan[c + d*x
]^5)/(15*d) + (a*(8*A - B)*Tan[c + d*x]^7)/(63*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{10}(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d}+\frac{1}{9}(a(8A-B))\int \mathrm{se}\cdots\\ &= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d}-\frac{(a(8A-B))\,\mathrm{Subst}\cdots}{}\\ &= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d}+\frac{a(8A-B)\tan(c+\cdots}{9d}\end{aligned}$$

Mathematica [B] time = 4.27944, size = 407, normalized size = 3.42

$$\frac{a\sec(c)(-85750(A+B)\cos(c+dx)+17150A\sin(2(c+dx))+17150A\sin(4(c+dx))+7350A\sin(6(c+dx))+1225A\,\mathrm{s}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*(645120*B*Cos[c] - 85750*(A + B)*Cos[c + d*x] - 51450*A*Cos[3*(c
+ d*x)] - 51450*B*Cos[3*(c + d*x)] - 17150*A*Cos[5*(c + d*x)] - 17150*B*Cos
[5*(c + d*x)] - 2450*A*Cos[7*(c + d*x)] - 2450*B*Cos[7*(c + d*x)] + 229376*
A*Cos[c + 2*d*x] - 28672*B*Cos[c + 2*d*x] + 229376*A*Cos[3*c + 4*d*x] - 286
72*B*Cos[3*c + 4*d*x] + 98304*A*Cos[5*c + 6*d*x] - 12288*B*Cos[5*c + 6*d*x]
 + 16384*A*Cos[7*c + 8*d*x] - 2048*B*Cos[7*c + 8*d*x] + 1146880*A*Sin[d*x]
- 143360*B*Sin[d*x] + 17150*A*Sin[2*(c + d*x)] + 17150*B*Sin[2*(c + d*x)] +
 17150*A*Sin[4*(c + d*x)] + 17150*B*Sin[4*(c + d*x)] + 7350*A*Sin[6*(c + d*
x)] + 7350*B*Sin[6*(c + d*x)] + 1225*A*Sin[8*(c + d*x)] + 1225*B*Sin[8*(c +
 d*x)] + 688128*A*Sin[2*c + 3*d*x] - 86016*B*Sin[2*c + 3*d*x] + 229376*A*Si
n[4*c + 5*d*x] - 28672*B*Sin[4*c + 5*d*x] + 32768*A*Sin[6*c + 7*d*x] - 4096
*B*Sin[6*c + 7*d*x]))/(5160960*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^9*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A] time = 0.119, size = 158, normalized size = 1.3

$$\frac{1}{d}\left(\frac{aA}{9\,(\cos(dx+c))^9}+aB\left(\frac{(\sin(dx+c))^3}{9\,(\cos(dx+c))^9}+\frac{2\,(\sin(dx+c))^3}{21\,(\cos(dx+c))^7}+\frac{8\,(\sin(dx+c))^3}{105\,(\cos(dx+c))^5}+\frac{16\,(\sin(dx+c))^3}{315\,(\cos(dx+c))^3}\right)-aA\Big(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/9*a*A/cos(d*x+c)^9+a*B*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c
)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*
x+c)^3)-a*A*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^
4-64/315*sec(d*x+c)^2)*tan(d*x+c)+1/9*a*B/cos(d*x+c)^9)
```

Maxima [A] time = 1.02318, size = 170, normalized size = 1.43

$$\frac{\left(35\,\tan(dx+c)^9+180\,\tan(dx+c)^7+378\,\tan(dx+c)^5+420\,\tan(dx+c)^3+315\,\tan(dx+c)\right)Aa+\left(35\,\tan(dx\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="max
ima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*A*a + (35*tan(d*x + c)^9 + 135*tan(d*x +
c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*B*a + 35*A*a/cos(d*x + c)^9
 + 35*B*a/cos(d*x + c)^9)/d
```

Fricas [A] time = 1.91598, size = 436, normalized size = 3.66

$$-\frac{16\,(8\,A-B)a\cos(dx+c)^8-8\,(8\,A-B)a\cos(dx+c)^6-2\,(8\,A-B)a\cos(dx+c)^4-(8\,A-B)a\cos(dx+c)^2-5\,(A}{315\,\left(d\cos(dx+c)^7\,\mathrm{si}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fri
cas")
```

```
[Out] -1/315*(16*(8*A - B)*a*cos(d*x + c)^8 - 8*(8*A - B)*a*cos(d*x + c)^6 - 2*(8
*A - B)*a*cos(d*x + c)^4 - (8*A - B)*a*cos(d*x + c)^2 - 5*(A - 8*B)*a + (16
*(8*A - B)*a*cos(d*x + c)^6 + 8*(8*A - B)*a*cos(d*x + c)^4 + 6*(8*A - B)*a*
cos(d*x + c)^2 + 5*(8*A - B)*a)*sin(d*x + c))/(d*cos(d*x + c)^7*sin(d*x + c
) - d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36597, size = 628, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="gia
c")
```

```
[Out] -1/40320*(3*(9765*A*a*tan(1/2*d*x + 1/2*c)^6 - 3675*B*a*tan(1/2*d*x + 1/2*c
)^6 + 48720*A*a*tan(1/2*d*x + 1/2*c)^5 - 15960*B*a*tan(1/2*d*x + 1/2*c)^5 +
 109865*A*a*tan(1/2*d*x + 1/2*c)^4 - 33775*B*a*tan(1/2*d*x + 1/2*c)^4 + 136
640*A*a*tan(1/2*d*x + 1/2*c)^3 - 39760*B*a*tan(1/2*d*x + 1/2*c)^3 + 99183*A
*a*tan(1/2*d*x + 1/2*c)^2 - 28161*B*a*tan(1/2*d*x + 1/2*c)^2 + 39536*A*a*ta
n(1/2*d*x + 1/2*c) - 11032*B*a*tan(1/2*d*x + 1/2*c) + 7043*A*a - 2101*B*a)/
(tan(1/2*d*x + 1/2*c) + 1)^7 + (51345*A*a*tan(1/2*d*x + 1/2*c)^8 + 11025*B*
a*tan(1/2*d*x + 1/2*c)^8 - 322560*A*a*tan(1/2*d*x + 1/2*c)^7 - 47880*B*a*ta
n(1/2*d*x + 1/2*c)^7 + 976500*A*a*tan(1/2*d*x + 1/2*c)^6 + 117180*B*a*tan(1
/2*d*x + 1/2*c)^6 - 1753920*A*a*tan(1/2*d*x + 1/2*c)^5 - 168840*B*a*tan(1/2
*d*x + 1/2*c)^5 + 2037294*A*a*tan(1/2*d*x + 1/2*c)^4 + 165942*B*a*tan(1/2*d
*x + 1/2*c)^4 - 1550976*A*a*tan(1/2*d*x + 1/2*c)^3 - 106008*B*a*tan(1/2*d*x
 + 1/2*c)^3 + 760644*A*a*tan(1/2*d*x + 1/2*c)^2 + 47772*B*a*tan(1/2*d*x + 1
/2*c)^2 - 219456*A*a*tan(1/2*d*x + 1/2*c) - 12888*B*a*tan(1/2*d*x + 1/2*c)
+ 30089*A*a + 2657*B*a)/(tan(1/2*d*x + 1/2*c) - 1)^9)/d
```

3.969 $\int \cos^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{(A-7B)(a\sin(c+dx)+a)^9}{9a^7d}+\frac{3(A-3B)(a\sin(c+dx)+a)^8}{4a^6d}-\frac{4(3A-5B)(a\sin(c+dx)+a)^7}{7a^5d}+\frac{4(A-B)(a\sin(c\cdots}{3a^4d}$$

```
[Out] (4*(A - B)*(a + a*Sin[c + d*x])^6)/(3*a^4*d) - (4*(3*A - 5*B)*(a + a*Sin[c
+ d*x])^7)/(7*a^5*d) + (3*(A - 3*B)*(a + a*Sin[c + d*x])^8)/(4*a^6*d) - ((A
 - 7*B)*(a + a*Sin[c + d*x])^9)/(9*a^7*d) - (B*(a + a*Sin[c + d*x])^10)/(10
*a^8*d)
```

Rubi [A] time = 0.177766, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-7B)(a\sin(c+dx)+a)^9}{9a^7d}+\frac{3(A-3B)(a\sin(c+dx)+a)^8}{4a^6d}-\frac{4(3A-5B)(a\sin(c+dx)+a)^7}{7a^5d}+\frac{4(A-B)(a\sin(c\cdots}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (4*(A - B)*(a + a*Sin[c + d*x])^6)/(3*a^4*d) - (4*(3*A - 5*B)*(a + a*Sin[c
+ d*x])^7)/(7*a^5*d) + (3*(A - 3*B)*(a + a*Sin[c + d*x])^8)/(4*a^6*d) - ((A
 - 7*B)*(a + a*Sin[c + d*x])^9)/(9*a^7*d) - (B*(a + a*Sin[c + d*x])^10)/(10
*a^8*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx=\frac{\text{Subst}\left(\int (a-x)^3(a+x)^5\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^7d}$$

$$=\frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^5-4a^2(3A-5B)(a+x)^6+\cdots\right.\right.}{}$$

$$=\frac{4(A-B)(a+a\sin(c+dx))^6}{3a^4d}-\frac{4(3A-5B)(a+a\sin(c+\cdots}{7a^5d}$$

Mathematica [A] time = 1.1852, size = 86, normalized size = 0.64

$$-\frac{a^2(\sin(c+dx)+1)^6\left(28(5A-17B)\sin^3(c+dx)+(651B-525A)\sin^2(c+dx)+6(115A-61B)\sin(c+dx)-325A+1\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^2*(1 + Sin[c + d*x])^6*(-325*A + 61*B + 6*(115*A - 61*B)*Sin[c + d*x] +
 (-525*A + 651*B)*Sin[c + d*x]^2 + 28*(5*A - 17*B)*Sin[c + d*x]^3 + 126*B*S
in[c + d*x]^4))/(1260*d)
```

Maple [A] time = 0.069, size = 231, normalized size = 1.7

$$\frac{1}{d}\left(a^2A\left(-\frac{(\cos(dx+c))^8\sin(dx+c)}{9}+\frac{\sin(dx+c)}{63}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))^4}{5}+\frac{8(\cos(dx+c))^2}{5}\right)\right)+Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+B*a^2*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8
-1/40*cos(d*x+c)^8)-1/4*a^2*A*cos(d*x+c)^8+2*B*a^2*(-1/9*cos(d*x+c)^8*sin(d
*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
)+1/7*a^2*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c
)-1/8*B*a^2*cos(d*x+c)^8)
```

Maxima [A] time = 1.04507, size = 227, normalized size = 1.69

$$-\frac{126\,Ba^2\sin(dx+c)^{10}+140\,(A+2\,B)a^2\sin(dx+c)^9+315\,(A-B)a^2\sin(dx+c)^8-360\,(A+3\,B)a^2\sin(dx+c)^7-12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/1260*(126*B*a^2*sin(d*x + c)^10 + 140*(A + 2*B)*a^2*sin(d*x + c)^9 + 315
*(A - B)*a^2*sin(d*x + c)^8 - 360*(A + 3*B)*a^2*sin(d*x + c)^7 - 1260*A*a^2
*sin(d*x + c)^6 + 1512*B*a^2*sin(d*x + c)^5 + 630*(3*A + B)*a^2*sin(d*x + c
)^4 + 840*(A - B)*a^2*sin(d*x + c)^3 - 630*(2*A + B)*a^2*sin(d*x + c)^2 - 1
260*A*a^2*sin(d*x + c))/d
```

Fricas [A] time = 2.08347, size = 328, normalized size = 2.45

$$\frac{126\,Ba^2\cos(dx+c)^{10}-315\,(A+B)a^2\cos(dx+c)^8-4\left(35\,(A+2\,B)a^2\cos(dx+c)^8-10\,(5\,A+B)a^2\cos(dx+c)^6-12\right.}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1260*(126*B*a^2*cos(d*x + c)^10 - 315*(A + B)*a^2*cos(d*x + c)^8 - 4*(35*
(A + 2*B)*a^2*cos(d*x + c)^8 - 10*(5*A + B)*a^2*cos(d*x + c)^6 - 12*(5*A +
B)*a^2*cos(d*x + c)^4 - 16*(5*A + B)*a^2*cos(d*x + c)^2 - 32*(5*A + B)*a^2)
*sin(d*x + c))/d
```

Sympy [A] time = 37.7904, size = 389, normalized size = 2.9

$$\begin{cases} \frac{16Aa^2\sin^9(c+dx)}{315d} + \frac{8Aa^2\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{16Aa^2\sin^7(c+dx)}{35d} + \frac{2Aa^2\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{8Aa^2\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{Aa^2\sin \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a**2*sin(c + d*x)**9/(315*d) + 8*A*a**2*sin(c + d*x)**7*cos
(c + d*x)**2/(35*d) + 16*A*a**2*sin(c + d*x)**7/(35*d) + 2*A*a**2*sin(c + d
*x)**5*cos(c + d*x)**4/(5*d) + 8*A*a**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*
d) + A*a**2*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + 2*A*a**2*sin(c + d*x)**
3*cos(c + d*x)**4/d + A*a**2*sin(c + d*x)*cos(c + d*x)**6/d - A*a**2*cos(c
+ d*x)**8/(4*d) + 32*B*a**2*sin(c + d*x)**9/(315*d) + 16*B*a**2*sin(c + d*x
)**7*cos(c + d*x)**2/(35*d) + 4*B*a**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*d
) + 2*B*a**2*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) - B*a**2*sin(c + d*x)**2
*cos(c + d*x)**8/(8*d) - B*a**2*cos(c + d*x)**10/(40*d) - B*a**2*cos(c + d*
x)**8/(8*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**7, True
))
```

Giac [A] time = 1.29702, size = 323, normalized size = 2.41

$$\frac{Ba^2\cos(10\,dx+10\,c)}{5120\,d} - \frac{Aa^2\cos(8\,dx+8\,c)}{512\,d} + \frac{7\,Aa^2\sin(3\,dx+3\,c)}{64\,d} - \frac{\left(16\,Aa^2+7\,Ba^2\right)\cos(6\,dx+6\,c)}{1024\,d} - \frac{(7\,Aa^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/5120*B*a^2*cos(10*d*x + 10*c)/d - 1/512*A*a^2*cos(8*d*x + 8*c)/d + 7/64*A
*a^2*sin(3*d*x + 3*c)/d - 1/1024*(16*A*a^2 + 7*B*a^2)*cos(6*d*x + 6*c)/d -
1/128*(7*A*a^2 + 4*B*a^2)*cos(4*d*x + 4*c)/d - 7/512*(8*A*a^2 + 5*B*a^2)*co
s(2*d*x + 2*c)/d - 1/2304*(A*a^2 + 2*B*a^2)*sin(9*d*x + 9*c)/d - 1/1792*(A*
a^2 + 10*B*a^2)*sin(7*d*x + 7*c)/d + 1/320*(5*A*a^2 - 4*B*a^2)*sin(5*d*x +
5*c)/d + 7/128*(11*A*a^2 + 2*B*a^2)*sin(d*x + c)/d
```

3.970 $\int \cos^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{(A-5B)(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(A-2B)(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(A-B)(a\sin(c+dx)+a)^5}{5a^3d}+\frac{B(a\sin(c+dx)+a)^8}{8a^6d}$$

```
[Out] (4*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(A - 2*B)*(a + a*Sin[c +
d*x])^6)/(3*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^7)/(7*a^5*d) + (B*(a +
 a*Sin[c + d*x])^8)/(8*a^6*d)
```

Rubi [A] time = 0.148639, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{(A-5B)(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(A-2B)(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(A-B)(a\sin(c+dx)+a)^5}{5a^3d}+\frac{B(a\sin(c+dx)+a)^8}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (4*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(A - 2*B)*(a + a*Sin[c +
d*x])^6)/(3*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^7)/(7*a^5*d) + (B*(a +
 a*Sin[c + d*x])^8)/(8*a^6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx=\frac{\text{Subst}\left(\int (a-x)^2(a+x)^4\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^5d}$$
$$=\frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^4-4a(A-2B)(a+x)^5+(A-\right.\right.}{a^5d}$$
$$=\frac{4(A-B)(a+a\sin(c+dx))^5}{5a^3d}-\frac{2(A-2B)(a+a\sin(c+dx))^6}{3a^4d}$$

Mathematica [A] time = 0.343973, size = 70, normalized size = 0.67

$$\frac{a^2(\sin(c+dx)+1)^5\left(15(8A-19B)\sin^2(c+dx)-5(64A-47B)\sin(c+dx)+232A+105B\sin^3(c+dx)-47B\right)}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^5*(232*A - 47*B - 5*(64*A - 47*B)*Sin[c + d*x] + 15
*(8*A - 19*B)*Sin[c + d*x]^2 + 105*B*Sin[c + d*x]^3))/(840*d)
```

Maple [B] time = 0.067, size = 201, normalized size = 1.9

$$\frac{1}{d}\left(a^2A\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)\right)+Ba^2\left(-\frac{(\sin(dx+c))}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))+B*a^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)
-1/3*a^2*A*cos(d*x+c)^6+2*B*a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c)-1/6*B*a^2*cos(d*x+c)^6)
```

Maxima [A] time = 1.01849, size = 192, normalized size = 1.83

$$\frac{105\,Ba^2\sin(dx+c)^8+120\,(A+2\,B)a^2\sin(dx+c)^7+140\,(2\,A-B)a^2\sin(dx+c)^6-168\,(A+4\,B)a^2\sin(dx+c)^5-}{840}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/840*(105*B*a^2*sin(d*x + c)^8 + 120*(A + 2*B)*a^2*sin(d*x + c)^7 + 140*(2
*A - B)*a^2*sin(d*x + c)^6 - 168*(A + 4*B)*a^2*sin(d*x + c)^5 - 210*(4*A +
B)*a^2*sin(d*x + c)^4 - 280*(A - 2*B)*a^2*sin(d*x + c)^3 + 420*(2*A + B)*a^
2*sin(d*x + c)^2 + 840*A*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.9253, size = 277, normalized size = 2.64

$$\frac{105\,Ba^2\cos(dx+c)^8-280\,(A+B)a^2\cos(dx+c)^6-8\left(15\,(A+2\,B)a^2\cos(dx+c)^6-6\,(4\,A+B)a^2\cos(dx+c)^4-8\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/840*(105*B*a^2*cos(d*x + c)^8 - 280*(A + B)*a^2*cos(d*x + c)^6 - 8*(15*(A
 + 2*B)*a^2*cos(d*x + c)^6 - 6*(4*A + B)*a^2*cos(d*x + c)^4 - 8*(4*A + B)*a
^2*cos(d*x + c)^2 - 16*(4*A + B)*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 14.0238, size = 335, normalized size = 3.19

$$\begin{cases} \frac{8Aa^2\sin^7(c+dx)}{105d} + \frac{4Aa^2\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{8Aa^2\sin^5(c+dx)}{15d} + \frac{Aa^2\sin^3(c+dx)\cos^4(c+dx)}{3d} + \frac{4Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)\cos}{d} \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a**2*sin(c + d*x)**7/(105*d) + 4*A*a**2*sin(c + d*x)**5*cos(
c + d*x)**2/(15*d) + 8*A*a**2*sin(c + d*x)**5/(15*d) + A*a**2*sin(c + d*x)*
*3*cos(c + d*x)**4/(3*d) + 4*A*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) +
 A*a**2*sin(c + d*x)*cos(c + d*x)**4/d - A*a**2*cos(c + d*x)**6/(3*d) + B*a
**2*sin(c + d*x)**8/(24*d) + 16*B*a**2*sin(c + d*x)**7/(105*d) + B*a**2*sin
(c + d*x)**6*cos(c + d*x)**2/(6*d) + 8*B*a**2*sin(c + d*x)**5*cos(c + d*x)*
*2/(15*d) + B*a**2*sin(c + d*x)**4*cos(c + d*x)**4/(4*d) + 2*B*a**2*sin(c +
 d*x)**3*cos(c + d*x)**4/(3*d) - B*a**2*cos(c + d*x)**6/(6*d), Ne(d, 0)), (
x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**5, True))
```

Giac [B] time = 1.36602, size = 273, normalized size = 2.6

$$\frac{Ba^2\cos(8\,dx+8\,c)}{1024\,d} - \frac{\left(4\,Aa^2+Ba^2\right)\cos(6\,dx+6\,c)}{384\,d} - \frac{\left(16\,Aa^2+9\,Ba^2\right)\cos(4\,dx+4\,c)}{256\,d} - \frac{\left(20\,Aa^2+13\,Ba^2\right)\cos(2\,dx+}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1024*B*a^2*cos(8*d*x + 8*c)/d - 1/384*(4*A*a^2 + B*a^2)*cos(6*d*x + 6*c)/
d - 1/256*(16*A*a^2 + 9*B*a^2)*cos(4*d*x + 4*c)/d - 1/128*(20*A*a^2 + 13*B*
a^2)*cos(2*d*x + 2*c)/d - 1/448*(A*a^2 + 2*B*a^2)*sin(7*d*x + 7*c)/d + 1/32
0*(A*a^2 - 6*B*a^2)*sin(5*d*x + 5*c)/d + 1/192*(19*A*a^2 - 2*B*a^2)*sin(3*d
*x + 3*c)/d + 5/64*(9*A*a^2 + 2*B*a^2)*sin(d*x + c)/d
```

3.971 $\int \cos^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-\frac{(A-3B)(a\sin(c+dx)+a)^5}{5a^3d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{2a^2d}-\frac{B(a\sin(c+dx)+a)^6}{6a^4d}$$

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(2*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*x]
)^5)/(5*a^3*d) - (B*(a + a*Sin[c + d*x])^6)/(6*a^4*d)
```

Rubi [A] time = 0.109171, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-3B)(a\sin(c+dx)+a)^5}{5a^3d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{2a^2d}-\frac{B(a\sin(c+dx)+a)^6}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(2*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*x]
)^5)/(5*a^3*d) - (B*(a + a*Sin[c + d*x])^6)/(6*a^4*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^3\left(A+\frac{Bx}{a}\right)dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(A-B)(a+x)^3+(-A+3B)(a+x)^4-\frac{B(a+}{a}\right.\right.}{a^3d}\\ &= \frac{(A-B)(a+a\sin(c+dx))^4}{2a^2d}-\frac{(A-3B)(a+a\sin(c+dx))}{5a^3d}\end{aligned}$$

Mathematica [A] time = 0.405951, size = 66, normalized size = 0.85

$$\frac{a^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^8(-4(3A-4B)\sin(c+dx)+18A+5B\cos(2(c+dx))-9B)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*(18*A - 9*B + 5*B*Cos[2*(c + d
*x)] - 4*(3*A - 4*B)*Sin[c + d*x]))/(60*d)
```

Maple [B] time = 0.068, size = 171, normalized size = 2.2

$$\frac{1}{d}\left(a^2A\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)+Ba^2\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{6}-\frac{(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))+
B*a^2*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)-1/2*a^2*A*cos(d*x+
c)^4+2*B*a^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c)
)+1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)-1/4*B*a^2*cos(d*x+c)^4)
```

Maxima [A] time = 0.978895, size = 130, normalized size = 1.67

$$-\frac{5\,Ba^2\sin(dx+c)^6+6\,(A+2\,B)a^2\sin(dx+c)^5+15\,Aa^2\sin(dx+c)^4-20\,Ba^2\sin(dx+c)^3-15\,(2\,A+B)a^2\sin(dx+}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/30*(5*B*a^2*sin(d*x + c)^6 + 6*(A + 2*B)*a^2*sin(d*x + c)^5 + 15*A*a^2*s
in(d*x + c)^4 - 20*B*a^2*sin(d*x + c)^3 - 15*(2*A + B)*a^2*sin(d*x + c)^2 -
 30*A*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.79339, size = 224, normalized size = 2.87

$$\frac{5\,Ba^2\cos(dx+c)^6-15\,(A+B)a^2\cos(dx+c)^4-2\left(3\,(A+2\,B)a^2\cos(dx+c)^4-2\,(3\,A+B)a^2\cos(dx+c)^2-4\,(3\,A+}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/30*(5*B*a^2*cos(d*x + c)^6 - 15*(A + B)*a^2*cos(d*x + c)^4 - 2*(3*(A + 2*
B)*a^2*cos(d*x + c)^4 - 2*(3*A + B)*a^2*cos(d*x + c)^2 - 4*(3*A + B)*a^2)*s
in(d*x + c))/d
```

Sympy [A] time = 4.96456, size = 228, normalized size = 2.92

$$\begin{cases} \frac{2Aa^2\sin^5(c+dx)}{15d} + \frac{Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{2Aa^2\sin^3(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)\cos^2(c+dx)}{d} - \frac{Aa^2\cos^4(c+dx)}{2d} + \frac{Ba^2\sin^6(c+dx)}{12d} + \frac{4Ba^2\sin \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a**2*sin(c + d*x)**5/(15*d) + A*a**2*sin(c + d*x)**3*cos(c +
 d*x)**2/(3*d) + 2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c
 + d*x)**2/d - A*a**2*cos(c + d*x)**4/(2*d) + B*a**2*sin(c + d*x)**6/(12*d)
 + 4*B*a**2*sin(c + d*x)**5/(15*d) + B*a**2*sin(c + d*x)**4*cos(c + d*x)**2
/(4*d) + 2*B*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - B*a**2*cos(c + d*
x)**4/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**3, True
))
```

Giac [A] time = 1.28829, size = 157, normalized size = 2.01

$$-\frac{5Ba^2\sin(dx+c)^6 + 6Aa^2\sin(dx+c)^5 + 12Ba^2\sin(dx+c)^5 + 15Aa^2\sin(dx+c)^4 - 20Ba^2\sin(dx+c)^3 - 30Aa^2}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/30*(5*B*a^2*sin(d*x + c)^6 + 6*A*a^2*sin(d*x + c)^5 + 12*B*a^2*sin(d*x +
 c)^5 + 15*A*a^2*sin(d*x + c)^4 - 20*B*a^2*sin(d*x + c)^3 - 30*A*a^2*sin(d*
x + c)^2 - 15*B*a^2*sin(d*x + c)^2 - 30*A*a^2*sin(d*x + c))/d
```

3.972 $\int \cos(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{B(a\sin(c+dx)+a)^4}{4a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^3}{3ad}$$

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^3)/(3*a*d) + (B*(a + a*Sin[c + d*x])^4)/(4*a^
2*d)
```

Rubi [A] time = 0.069862, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B(a\sin(c+dx)+a)^4}{4a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^3)/(3*a*d) + (B*(a + a*Sin[c + d*x])^4)/(4*a^
2*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left((A-B)(a+x)^2+\frac{B(a+x)^3}{a}\right)dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{(A-B)(a+a\sin(c+dx))^3}{3ad}+\frac{B(a+a\sin(c+dx))^4}{4a^2d}\end{aligned}$$

Mathematica [A] time = 0.0932027, size = 49, normalized size = 0.96

$$\frac{\frac{1}{3}(A-B)(a\sin(c+dx)+a)^3+\frac{B(a\sin(c+dx)+a)^4}{4a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (((A - B)*(a + a*Sin[c + d*x])^3)/3 + (B*(a + a*Sin[c + d*x])^4)/(4*a))/(a*
d)
```

Maple [A] time = 0.032, size = 75, normalized size = 1.5

$$\frac{1}{d}\left(\frac{Ba^2(\sin(dx+c))^4}{4}+\frac{\left(a^2A+2\,Ba^2\right)(\sin(dx+c))^3}{3}+\frac{\left(2\,a^2A+Ba^2\right)(\sin(dx+c))^2}{2}+a^2A\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/4*B*a^2*sin(d*x+c)^4+1/3*(A*a^2+2*B*a^2)*sin(d*x+c)^3+1/2*(2*A*a^2+B
*a^2)*sin(d*x+c)^2+a^2*A*sin(d*x+c))
```

Maxima [A] time = 1.06434, size = 92, normalized size = 1.8

$$\frac{3\,Ba^2\sin(dx+c)^4+4\,(A+2\,B)a^2\sin(dx+c)^3+6\,(2\,A+B)a^2\sin(dx+c)^2+12\,Aa^2\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*B*a^2*sin(d*x + c)^4 + 4*(A + 2*B)*a^2*sin(d*x + c)^3 + 6*(2*A + B)
*a^2*sin(d*x + c)^2 + 12*A*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.83226, size = 177, normalized size = 3.47

$$\frac{3\,Ba^2\cos(dx+c)^4-12\,(A+B)a^2\cos(dx+c)^2-4\left((A+2\,B)a^2\cos(dx+c)^2-2\,(2\,A+B)a^2\right)\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*B*a^2*cos(d*x + c)^4 - 12*(A + B)*a^2*cos(d*x + c)^2 - 4*((A + 2*B)
*a^2*cos(d*x + c)^2 - 2*(2*A + B)*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 1.36024, size = 143, normalized size = 2.8

$$\begin{cases}\frac{Aa^2\sin^3(c+dx)}{3d}+\frac{Aa^2\sin(c+dx)}{d}-\frac{Aa^2\cos^2(c+dx)}{d}+\frac{2Ba^2\sin^3(c+dx)}{3d}-\frac{Ba^2\sin^2(c+dx)\cos^2(c+dx)}{2d}-\frac{Ba^2\cos^4(c+dx)}{4d}-\frac{Ba^2\cos^2(c+dx)}{2d} & \text{fo} \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos(c) & \text{o}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)/d - A*a**2*co
s(c + d*x)**2/d + 2*B*a**2*sin(c + d*x)**3/(3*d) - B*a**2*sin(c + d*x)**2*c
os(c + d*x)**2/(2*d) - B*a**2*cos(c + d*x)**4/(4*d) - B*a**2*cos(c + d*x)**
2/(2*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c), True))
```

Giac [A] time = 1.36954, size = 119, normalized size = 2.33

$$\frac{3 Ba^2 \sin(dx+c)^4 + 4 Aa^2 \sin(dx+c)^3 + 8 Ba^2 \sin(dx+c)^3 + 12 Aa^2 \sin(dx+c)^2 + 6 Ba^2 \sin(dx+c)^2 + 12 Aa^2 \sin(d}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/12*(3*B*a^2*sin(d*x + c)^4 + 4*A*a^2*sin(d*x + c)^3 + 8*B*a^2*sin(d*x + c
)^3 + 12*A*a^2*sin(d*x + c)^2 + 6*B*a^2*sin(d*x + c)^2 + 12*A*a^2*sin(d*x +
 c))/d
```

3.973 $\int \sec(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$-\frac{a^2(A+B)\sin(c+dx)}{d}-\frac{2a^2(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^2}{2d}$$

```
[Out] (-2*a^2*(A + B)*Log[1 - Sin[c + d*x]])/d - (a^2*(A + B)*Sin[c + d*x])/d - (
B*(a + a*Sin[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.0962114, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$-\frac{a^2(A+B)\sin(c+dx)}{d}-\frac{2a^2(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*(A + B)*Log[1 - Sin[c + d*x]])/d - (a^2*(A + B)*Sin[c + d*x])/d - (
B*(a + a*Sin[c + d*x])^2)/(2*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= \frac{a\,\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{a}\right)}{a-x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a\,\text{Subst}\left(\int \left(-A-B+\frac{2a(A+B)}{a-x}-\frac{B(a+x)}{a}\right)dx,x,a\sin(c+dx\right.}{d}\\ &= -\frac{2a^2(A+B)\log(1-\sin(c+dx))}{d}-\frac{a^2(A+B)\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0786707, size = 51, normalized size = 0.85

$$\frac{a\left(-a(A+2B)\sin(c+dx)-2a(A+B)\log(1-\sin(c+dx))-\frac{1}{2}aB\sin^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(-2*a*(A + B)*Log[1 - Sin[c + d*x]] - a*(A + 2*B)*Sin[c + d*x] - (a*B*Si
n[c + d*x]^2)/2))/d
```

Maple [B] time = 0.075, size = 127, normalized size = 2.1

$$-\frac{a^2A\sin(dx+c)}{d}+2\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{Ba^2(\sin(dx+c))^2}{2d}-2\frac{Ba^2\ln(\cos(dx+c))}{d}-2\frac{a^2A\ln(\cos}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] -1/d*a^2*A*sin(d*x+c)+2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))-1/2/d*B*a^2*sin(d
*x+c)^2-2/d*B*a^2*ln(cos(d*x+c))-2/d*a^2*A*ln(cos(d*x+c))-2/d*B*a^2*sin(d*x
+c)+2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01459, size = 70, normalized size = 1.17

$$-\frac{Ba^2\sin(dx+c)^2+4(A+B)a^2\log(\sin(dx+c)-1)+2(A+2B)a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(B*a^2*sin(d*x + c)^2 + 4*(A + B)*a^2*log(sin(d*x + c) - 1) + 2*(A + 2
*B)*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.84076, size = 135, normalized size = 2.25

$$\frac{Ba^2\cos(dx+c)^2-4(A+B)a^2\log(-\sin(dx+c)+1)-2(A+2B)a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(B*a^2*cos(d*x + c)^2 - 4*(A + B)*a^2*log(-sin(d*x + c) + 1) - 2*(A + 2
*B)*a^2*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int A \sec(c+dx)\,dx + \int 2A \sin(c+dx) \sec(c+dx)\,dx + \int A \sin^2(c+dx) \sec(c+dx)\,dx + \int B \sin(c+dx) \sec \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*sin(c + d*x)*sec(c + d*x),
 x) + Integral(A*sin(c + d*x)**2*sec(c + d*x), x) + Integral(B*sin(c + d*x)
*sec(c + d*x), x) + Integral(2*B*sin(c + d*x)**2*sec(c + d*x), x) + Integra
l(B*sin(c + d*x)**3*sec(c + d*x), x))
```

Giac [B] time = 1.34599, size = 297, normalized size = 4.95

$$2\left(Aa^2 + Ba^2\right)\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right) - 4\left(Aa^2 + Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{3Aa^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 3Ba^2\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] (2*(A*a^2 + B*a^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 4*(A*a^2 + B*a^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) - (3*A*a^2*tan(1/2*d*x + 1/2*c)^4 + 3*B*a^2*
tan(1/2*d*x + 1/2*c)^4 + 2*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^2*tan(1/2*d
*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c)^2 + 8*B*a^2*tan(1/2*d*x + 1/2*
c)^2 + 2*A*a^2*tan(1/2*d*x + 1/2*c) + 4*B*a^2*tan(1/2*d*x + 1/2*c) + 3*A*a^
2 + 3*B*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.974 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a^3(A+B)}{d(a-a\sin(c+dx))}+\frac{a^2B\log(1-\sin(c+dx))}{d}$$

```
[Out] (a^2*B*Log[1 - Sin[c + d*x]])/d + (a^3*(A + B))/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0903995, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 43}

$$\frac{a^3(A+B)}{d(a-a\sin(c+dx))}+\frac{a^2B\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*B*Log[1 - Sin[c + d*x]])/d + (a^3*(A + B))/(d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= \frac{a^3\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\,\text{Subst}\left(\int \left(\frac{A+B}{(a-x)^2}-\frac{B}{a(a-x)}\right)dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^2B\log(1-\sin(c+dx))}{d}+\frac{a^3(A+B)}{d(a-a\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.0742413, size = 41, normalized size = 0.95

$$\frac{a^3\left(\frac{A+B}{a-a\sin(c+dx)}+\frac{B\log(1-\sin(c+dx))}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*((B*Log[1 - Sin[c + d*x]])/a + (A + B)/(a - a*Sin[c + d*x])))/d
```

Maple [B] time = 0.103, size = 189, normalized size = 4.4

$$\frac{a^2 A (\sin(dx+c))^3}{2d(\cos(dx+c))^2} + \frac{a^2 A \sin(dx+c)}{2d} + \frac{Ba^2 (\tan(dx+c))^2}{2d} + \frac{Ba^2 \ln(\cos(dx+c))}{d} + \frac{a^2 A}{d(\cos(dx+c))^2} + \frac{Ba^2 (\sin(d}{d(\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*a^2*A*sin(d*x+c)+1/2/d*B*a^2*ta
n(d*x+c)^2+1/d*B*a^2*ln(cos(d*x+c))+1/d*a^2*A/cos(d*x+c)^2+1/d*B*a^2*sin(d*
x+c)^3/cos(d*x+c)^2+1/d*B*a^2*sin(d*x+c)-1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/2/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^2/cos(d*x+c)^2
```

Maxima [A] time = 1.08152, size = 50, normalized size = 1.16

$$\frac{Ba^2 \log(\sin(dx+c)-1) - \frac{(A+B)a^2}{\sin(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] (B*a^2*log(sin(d*x + c) - 1) - (A + B)*a^2/(sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.73785, size = 123, normalized size = 2.86

$$-\frac{(A+B)a^2 - \left(Ba^2 \sin(dx+c) - Ba^2\right) \log(-\sin(dx+c)+1)}{d \sin(dx+c) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -((A + B)*a^2 - (B*a^2*sin(d*x + c) - B*a^2)*log(-sin(d*x + c) + 1))/(d*sin
(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33581, size = 151, normalized size = 3.51

$$-\frac{Ba^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-2\,Ba^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -(B*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*B*a^2*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + (3*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 2*A*a^2*tan(1/2*d*x + 1/2*c)
- 8*B*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^2)/d
```

3.975 $\int \sec^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{a^3(A-B)}{4d(a-a\sin(c+dx))}+\frac{a^2(A-B)\tanh^{-1}(\sin(c+dx))}{4d}$$

```
[Out] (a^2*(A - B)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^4*(A + B))/(4*d*(a - a*Sin[c
 + d*x])^2) + (a^3*(A - B))/(4*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.118901, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{a^3(A-B)}{4d(a-a\sin(c+dx))}+\frac{a^2(A-B)\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(A - B)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^4*(A + B))/(4*d*(a - a*Sin[c
 + d*x])^2) + (a^3*(A - B))/(4*d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{A+B}{2a(a-x)^3}+\frac{A-B}{4a^2(a-x)^2}+\frac{A-B}{4a^2(a^2-x^2)}\right)dx,x,a\sin(c+\right.}{d}$$
$$= \frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{a^3(A-B)}{4d(a-a\sin(c+dx))}+\frac{\left(a^3(A-B)\right)}{}$$
$$= \frac{a^2(A-B)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{}{4a}$$

Mathematica [A] time = 0.136888, size = 75, normalized size = 0.97

$$\frac{a^5\left(\frac{A-B}{4a^2(a-a\sin(c+dx))}+\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{4a^3}+\frac{A+B}{4a(a-a\sin(c+dx))^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^5*(((A - B)*ArcTanh[Sin[c + d*x]])/(4*a^3) + (A + B)/(4*a*(a - a*Sin[c +
 d*x])^2) + (A - B)/(4*a^2*(a - a*Sin[c + d*x]))))/d
```

Maple [B] time = 0.112, size = 281, normalized size = 3.7

$$\frac{a^2A(\sin(dx+c))^3}{4d(\cos(dx+c))^4}+\frac{a^2A(\sin(dx+c))^3}{8d(\cos(dx+c))^2}+\frac{a^2A\sin(dx+c)}{8d}+\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{4d}+\frac{Ba^2(\sin(dx+c))^4}{4d(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^2
+1/8/d*a^2*A*sin(d*x+c)+1/4/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*B*a^2*s
in(d*x+c)^4/cos(d*x+c)^4+1/2/d*a^2*A/cos(d*x+c)^4+1/2/d*B*a^2*sin(d*x+c)^3/
cos(d*x+c)^4+1/4/d*B*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/4/d*B*a^2*sin(d*x+c)-1
/4/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^2*A*tan(d*x+c)*sec(d*x+c)^3+3/
8/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/4/d*B*a^2/cos(d*x+c)^4
```

Maxima [A] time = 1.03936, size = 117, normalized size = 1.52

$$\frac{(A-B)a^2\log(\sin(dx+c)+1)-(A-B)a^2\log(\sin(dx+c)-1)-\frac{2\left((A-B)a^2\sin(dx+c)-2\,Aa^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/8*((A - B)*a^2*log(sin(d*x + c) + 1) - (A - B)*a^2*log(sin(d*x + c) - 1)
- 2*((A - B)*a^2*sin(d*x + c) - 2*A*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) +
 1))/d
```

Fricas [B] time = 1.81465, size = 385, normalized size = 5.

$$\frac{2\,(A-B)a^{2}\sin\left(dx+c\right)-4\,Aa^{2}+\left((A-B)a^{2}\cos\left(dx+c\right)^{2}+2\,(A-B)a^{2}\sin\left(dx+c\right)-2\,(A-B)a^{2}\right)\log\left(\sin\left(dx+c\right)\right.}{8\left(d\cos\left(dx+c\right)^{2}+2\,d\sin\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/8*(2*(A - B)*a^2*sin(d*x + c) - 4*A*a^2 + ((A - B)*a^2*cos(d*x + c)^2 + 2
*(A - B)*a^2*sin(d*x + c) - 2*(A - B)*a^2)*log(sin(d*x + c) + 1) - ((A - B)
*a^2*cos(d*x + c)^2 + 2*(A - B)*a^2*sin(d*x + c) - 2*(A - B)*a^2)*log(-sin(
d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36705, size = 176, normalized size = 2.29

$$\frac{2\left(Aa^{2}-Ba^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\left(Aa^{2}-Ba^{2}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{3\,Aa^{2}\sin(dx+c)^{2}-3\,Ba^{2}\sin(dx+c)^{2}-10\,Aa^{2}\sin(dx+}{(\sin(dx+c)-1)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/16*(2*(A*a^2 - B*a^2)*log(abs(sin(d*x + c) + 1)) - 2*(A*a^2 - B*a^2)*log(
abs(sin(d*x + c) - 1)) + (3*A*a^2*sin(d*x + c)^2 - 3*B*a^2*sin(d*x + c)^2 -
 10*A*a^2*sin(d*x + c) + 10*B*a^2*sin(d*x + c) + 11*A*a^2 - 3*B*a^2)/(sin(d
*x + c) - 1)^2)/d
```

3.976 $\int \sec^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3}+\frac{a^3(3A-B)}{16d(a-a\sin(c+dx))}-\frac{a^3(A-B)}{16d(a\sin(c+dx)+a)}+\frac{a^2(2A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a}{8d(a-a\,s}$$

```
[Out] (a^2*(2*A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^5*(A + B))/(12*d*(a - a*Si
n[c + d*x])^3) + (a^4*A)/(8*d*(a - a*Sin[c + d*x])^2) + (a^3*(3*A - B))/(16
*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(16*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.156748, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3}+\frac{a^3(3A-B)}{16d(a-a\sin(c+dx))}-\frac{a^3(A-B)}{16d(a\sin(c+dx)+a)}+\frac{a^2(2A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a}{8d(a-a\,s}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(2*A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^5*(A + B))/(12*d*(a - a*Si
n[c + d*x])^3) + (a^4*A)/(8*d*(a - a*Sin[c + d*x])^2) + (a^3*(3*A - B))/(16
*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(16*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{A+B}{4a^2(a-x)^4} + \frac{A}{4a^3(a-x)^3} + \frac{3A-B}{16a^4(a-x)^2} + \frac{A-B}{16a^4(a+x)^2}\right.\right.}{d}$$

$$= \frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3} + \frac{a^4A}{8d(a-a\sin(c+dx))^2} + \frac{a}{16d(a}$$

$$= \frac{a^2(2A-B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3}$$

Mathematica [A] time = 0.656443, size = 90, normalized size = 0.68

$$\frac{a^2\left(\frac{3B-9A}{\sin(c+dx)-1} - \frac{3(A-B)}{\sin(c+dx)+1} - \frac{4(A+B)}{(\sin(c+dx)-1)^3} + 6(2A-B)\tanh^{-1}(\sin(c+dx)) + \frac{6A}{(\sin(c+dx)-1)^2}\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(6*(2*A - B)*ArcTanh[Sin[c + d*x]] - (4*(A + B))/(-1 + Sin[c + d*x])^3
 + (6*A)/(-1 + Sin[c + d*x])^2 + (-9*A + 3*B)/(-1 + Sin[c + d*x]) - (3*(A -
 B))/(1 + Sin[c + d*x])))/(48*d)
```

Maple [B] time = 0.115, size = 379, normalized size = 2.9

$$\frac{a^2A(\sin(dx+c))^3}{6d(\cos(dx+c))^6} + \frac{a^2A(\sin(dx+c))^3}{8d(\cos(dx+c))^4} + \frac{a^2A(\sin(dx+c))^3}{16d(\cos(dx+c))^2} + \frac{a^2A\sin(dx+c)}{16d} + \frac{a^2A\ln(\sec(dx+c)+\tan(dx+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/6/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^6+1/8/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^4
+1/16/d*a^2*A*sin(d*x+c)^3/cos(d*x+c)^2+1/16/d*a^2*A*sin(d*x+c)+1/4/d*a^2*A
*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*B*a^2*sin(d*x+c)^4/cos(d*x+c)^6+1/12/d*B*a
^2*sin(d*x+c)^4/cos(d*x+c)^4+1/3/d*a^2*A/cos(d*x+c)^6+1/3/d*B*a^2*sin(d*x+c
)^3/cos(d*x+c)^6+1/4/d*B*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*B*a^2*sin(d*x+
c)^3/cos(d*x+c)^2+1/8/d*B*a^2*sin(d*x+c)-1/8/d*B*a^2*ln(sec(d*x+c)+tan(d*x+
c))+1/6/d*a^2*A*tan(d*x+c)*sec(d*x+c)^5+5/24/d*a^2*A*tan(d*x+c)*sec(d*x+c)^
3+5/16/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/6/d*B*a^2/cos(d*x+c)^6
```

Maxima [A] time = 1.02922, size = 200, normalized size = 1.52

$$\frac{3(2A-B)a^2\log(\sin(dx+c)+1) - 3(2A-B)a^2\log(\sin(dx+c)-1) - \frac{2\left(3(2A-B)a^2\sin(dx+c)^3 - 6(2A-B)a^2\sin(dx+c)^2 + (2A-\right.}{\sin(dx+c)^4 - 2\sin(dx+c)^3 + 2\sin(d}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(3*(2*A - B)*a^2*log(sin(d*x + c) + 1) - 3*(2*A - B)*a^2*log(sin(d*x +
 c) - 1) - 2*(3*(2*A - B)*a^2*sin(d*x + c)^3 - 6*(2*A - B)*a^2*sin(d*x + c)
^2 + (2*A - B)*a^2*sin(d*x + c) + 2*(4*A + B)*a^2)/(sin(d*x + c)^4 - 2*sin(
d*x + c)^3 + 2*sin(d*x + c) - 1))/d
```

Fricas [B] time = 1.77586, size = 640, normalized size = 4.85

$$-\frac{12\,(2\,A-B)a^2\cos\left(dx+c\right)^2-8\,(A-2\,B)a^2-3\left((2\,A-B)a^2\cos\left(dx+c\right)^4+2\,(2\,A-B)a^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/48*(12*(2*A - B)*a^2*cos(d*x + c)^2 - 8*(A - 2*B)*a^2 - 3*((2*A - B)*a^2
*cos(d*x + c)^4 + 2*(2*A - B)*a^2*cos(d*x + c)^2*sin(d*x + c) - 2*(2*A - B)
*a^2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*((2*A - B)*a^2*cos(d*x + c)^
4 + 2*(2*A - B)*a^2*cos(d*x + c)^2*sin(d*x + c) - 2*(2*A - B)*a^2*cos(d*x +
 c)^2)*log(-sin(d*x + c) + 1) - 2*(3*(2*A - B)*a^2*cos(d*x + c)^2 - 4*(2*A
- B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2*sin(d*x + c)
 - 2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36955, size = 282, normalized size = 2.14

$$\frac{6\left(2\,Aa^2-Ba^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\left(2\,Aa^2-Ba^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{6\left(2\,Aa^2\sin(dx+c)-Ba^2\sin(dx+c)+3\,Aa^2-2\,Ba^2\right)}{\sin(dx+c)+1}+}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/96*(6*(2*A*a^2 - B*a^2)*log(abs(sin(d*x + c) + 1)) - 6*(2*A*a^2 - B*a^2)*
log(abs(sin(d*x + c) - 1)) - 6*(2*A*a^2*sin(d*x + c) - B*a^2*sin(d*x + c) +
 3*A*a^2 - 2*B*a^2)/(sin(d*x + c) + 1) + (22*A*a^2*sin(d*x + c)^3 - 11*B*a^
2*sin(d*x + c)^3 - 84*A*a^2*sin(d*x + c)^2 + 39*B*a^2*sin(d*x + c)^2 + 114*
A*a^2*sin(d*x + c) - 45*B*a^2*sin(d*x + c) - 60*A*a^2 + 9*B*a^2)/(sin(d*x +
 c) - 1)^3)/d
```

3.977 $\int \cos^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=196

$$-\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}-\frac{(9A+2B)\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{72d}+\frac{a^2(9A+2B)\sin(c+dx)\cos^5(c+dx)}{48d}+$$

```
[Out] (5*a^2*(9*A + 2*B)*x)/128 - (a^2*(9*A + 2*B)*Cos[c + d*x]^7)/(56*d) + (5*a^
2*(9*A + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*a^2*(9*A + 2*B)*Cos[c
 + d*x]^3*Sin[c + d*x])/(192*d) + (a^2*(9*A + 2*B)*Cos[c + d*x]^5*Sin[c + d
*x])/(48*d) - (B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2)/(9*d) - ((9*A + 2*B
)*Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(72*d)
```

Rubi [A] time = 0.211018, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}-\frac{(9A+2B)\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{72d}+\frac{a^2(9A+2B)\sin(c+dx)\cos^5(c+dx)}{48d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (5*a^2*(9*A + 2*B)*x)/128 - (a^2*(9*A + 2*B)*Cos[c + d*x]^7)/(56*d) + (5*a^
2*(9*A + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*a^2*(9*A + 2*B)*Cos[c
 + d*x]^3*Sin[c + d*x])/(192*d) + (a^2*(9*A + 2*B)*Cos[c + d*x]^5*Sin[c + d
*x])/(48*d) - (B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2)/(9*d) - ((9*A + 2*B
)*Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(72*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d} + \frac{1}{9}(9A+2B)\int \cos^6(c$$
$$= -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d} - \frac{(9A+2B)\cos^7(c+dx}{72}$$
$$= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d} - \frac{B\cos^7(c+dx)(a+a\sin(c+dx}{9d}$$
$$= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d} + \frac{a^2(9A+2B)\cos^5(c+dx)\sin(}{48d}$$
$$= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d} + \frac{5a^2(9A+2B)\cos^3(c+dx)\sin}{192d}$$
$$= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d} + \frac{5a^2(9A+2B)\cos(c+dx)\sin(}{128d}$$
$$= \frac{5}{128}a^2(9A+2B)x - \frac{a^2(9A+2B)\cos^7(c+dx)}{56d} + \frac{5a^2(9A+2}{}$$

Mathematica [A] time = 5.05663, size = 216, normalized size = 1.1

$$-\frac{a^2\cos(c+dx)\left(32(135A+86B)\cos(2(c+dx))+16(108A+59B)\cos(4(c+dx))+\frac{2520(9A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}-13671A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^2*Cos[c + d*x]*(2880*A + 1900*B + (2520*(9*A + 2*B)*ArcSin[Sqrt[1 - Sin
[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 32*(135*A + 86*B)*Cos[2*(c + d*
x)] + 16*(108*A + 59*B)*Cos[4*(c + d*x)] + 288*A*Cos[6*(c + d*x)] + 64*B*Co
s[6*(c + d*x)] - 28*B*Cos[8*(c + d*x)] - 13671*A*Sin[c + d*x] - 2478*B*Sin[
c + d*x] - 2457*A*Sin[3*(c + d*x)] + 462*B*Sin[3*(c + d*x)] - 63*A*Sin[5*(c
 + d*x)] + 546*B*Sin[5*(c + d*x)] + 63*A*Sin[7*(c + d*x)] + 126*B*Sin[7*(c
+ d*x)]))/(32256*d)
```

Maple [A] time = 0.069, size = 245, normalized size = 1.3

$$\frac{1}{d}\left(a^2A\left(-\frac{\sin(dx+c)(\cos(dx+c))^7}{8}+\frac{\sin(dx+c)}{48}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{128}+\frac{5c}{128}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^
3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+B*a^2*(-1/9*sin(d*x+c)^2*c
os(d*x+c)^7-2/63*cos(d*x+c)^7)-2/7*a^2*A*cos(d*x+c)^7+2*B*a^2*(-1/8*sin(d*x
+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d
*x+c)+5/128*d*x+5/128*c)+a^2*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos
(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)-1/7*B*a^2*cos(d*x+c)^7)
```

Maxima [A] time = 1.03578, size = 281, normalized size = 1.43

$$-\frac{18432\,Aa^2\cos\left(dx+c\right)^7+9216\,Ba^2\cos\left(dx+c\right)^7-21\left(64\,\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/64512*(18432*A*a^2*cos(d*x + c)^7 + 9216*B*a^2*cos(d*x + c)^7 - 21*(64*s
in(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4
*c))*A*a^2 + 336*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c)
 - 48*sin(2*d*x + 2*c))*A*a^2 - 1024*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*
B*a^2 - 42*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) -
24*sin(4*d*x + 4*c))*B*a^2)/d
```

Fricas [A] time = 2.02562, size = 343, normalized size = 1.75

$$\frac{896\,Ba^2\cos\left(dx+c\right)^9-2304\,(A+B)a^2\cos\left(dx+c\right)^7+315\,(9\,A+2\,B)a^2dx-21\left(48\,(A+2\,B)a^2\cos\left(dx+c\right)^7-8\,(9\,A\right.}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/8064*(896*B*a^2*cos(d*x + c)^9 - 2304*(A + B)*a^2*cos(d*x + c)^7 + 315*(9
*A + 2*B)*a^2*d*x - 21*(48*(A + 2*B)*a^2*cos(d*x + c)^7 - 8*(9*A + 2*B)*a^2
*cos(d*x + c)^5 - 10*(9*A + 2*B)*a^2*cos(d*x + c)^3 - 15*(9*A + 2*B)*a^2*co
s(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 26.0963, size = 719, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*A*a**2*x*sin(c + d*x)**8/128 + 5*A*a**2*x*sin(c + d*x)**6*cos(
c + d*x)**2/32 + 5*A*a**2*x*sin(c + d*x)**6/16 + 15*A*a**2*x*sin(c + d*x)**
```

```
4*cos(c + d*x)**4/64 + 15*A*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 5*A
*a**2*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 15*A*a**2*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 5*A*a**2*x*cos(c + d*x)**8/128 + 5*A*a**2*x*cos(c + d*x)
**6/16 + 5*A*a**2*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*A*a**2*sin(c +
d*x)**5*cos(c + d*x)**3/(384*d) + 5*A*a**2*sin(c + d*x)**5*cos(c + d*x)/(16
*d) + 73*A*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 5*A*a**2*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) - 5*A*a**2*sin(c + d*x)*cos(c + d*x)**7/(128*
d) + 11*A*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 2*A*a**2*cos(c + d*x)*
*7/(7*d) + 5*B*a**2*x*sin(c + d*x)**8/64 + 5*B*a**2*x*sin(c + d*x)**6*cos(c
 + d*x)**2/16 + 15*B*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 5*B*a**2*x
*sin(c + d*x)**2*cos(c + d*x)**6/16 + 5*B*a**2*x*cos(c + d*x)**8/64 + 5*B*a
**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*B*a**2*sin(c + d*x)**5*cos(c +
 d*x)**3/(192*d) + 73*B*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(192*d) - B*a*
*2*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 5*B*a**2*sin(c + d*x)*cos(c + d*
x)**7/(64*d) - 2*B*a**2*cos(c + d*x)**9/(63*d) - B*a**2*cos(c + d*x)**7/(7*
d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**6, True))
```

Giac [A] time = 1.35269, size = 317, normalized size = 1.62

$$\frac{Ba^2\cos(9\,dx+9\,c)}{2304\,d} - \frac{Ba^2\sin(6\,dx+6\,c)}{96\,d} + \frac{5}{128}\left(9\,Aa^2+2\,Ba^2\right)x - \frac{\left(8\,Aa^2+Ba^2\right)\cos(7\,dx+7\,c)}{1792\,d} - \frac{\left(2\,Aa^2+Ba^2\right)\cos}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2304*B*a^2*cos(9*d*x + 9*c)/d - 1/96*B*a^2*sin(6*d*x + 6*c)/d + 5/128*(9*
A*a^2 + 2*B*a^2)*x - 1/1792*(8*A*a^2 + B*a^2)*cos(7*d*x + 7*c)/d - 1/64*(2*
A*a^2 + B*a^2)*cos(5*d*x + 5*c)/d - 1/192*(18*A*a^2 + 11*B*a^2)*cos(3*d*x +
 3*c)/d - 1/128*(20*A*a^2 + 13*B*a^2)*cos(d*x + c)/d - 1/1024*(A*a^2 + 2*B*
a^2)*sin(8*d*x + 8*c)/d + 1/128*(5*A*a^2 - 2*B*a^2)*sin(4*d*x + 4*c)/d + 1/
32*(8*A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d
```

3.978 $\int \cos^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=165

$$-\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}-\frac{(7A+2B)\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{42d}+\frac{a^2(7A+2B)\sin(c+dx)\cos^3(c+dx)}{24d}+$$

```
[Out] (a^2*(7*A + 2*B)*x)/16 - (a^2*(7*A + 2*B)*Cos[c + d*x]^5)/(30*d) + (a^2*(7*
A + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(7*A + 2*B)*Cos[c + d*x]^
3*Sin[c + d*x])/(24*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2)/(7*d) -
((7*A + 2*B)*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(42*d)
```

Rubi [A] time = 0.189213, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}-\frac{(7A+2B)\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{42d}+\frac{a^2(7A+2B)\sin(c+dx)\cos^3(c+dx)}{24d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(7*A + 2*B)*x)/16 - (a^2*(7*A + 2*B)*Cos[c + d*x]^5)/(30*d) + (a^2*(7*
A + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(7*A + 2*B)*Cos[c + d*x]^
3*Sin[c + d*x])/(24*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2)/(7*d) -
((7*A + 2*B)*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(42*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d} + \frac{1}{7}(7A+2B)\int \cos^4(c$$

$$= -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d} - \frac{(7A+2B)\cos^5(c+dx}{42}$$

$$= -\frac{a^2(7A+2B)\cos^5(c+dx)}{30d} - \frac{B\cos^5(c+dx)(a+a\sin(c+d}{7d}$$

$$= -\frac{a^2(7A+2B)\cos^5(c+dx)}{30d} + \frac{a^2(7A+2B)\cos^3(c+dx)\sin(c}{24d}$$

$$= -\frac{a^2(7A+2B)\cos^5(c+dx)}{30d} + \frac{a^2(7A+2B)\cos(c+dx)\sin(c}{16d}$$

$$= \frac{1}{16}a^2(7A+2B)x - \frac{a^2(7A+2B)\cos^5(c+dx)}{30d} + \frac{a^2(7A+2B)}{}$$

Mathematica [A] time = 1.70164, size = 171, normalized size = 1.04

$$-\frac{a^2\cos(c+dx)\left((672A+447B)\cos(2(c+dx))+6(28A+13B)\cos(4(c+dx))+\frac{420(7A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}-1645A\sin(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^2*Cos[c + d*x]*(504*A + 354*B + (420*(7*A + 2*B)*ArcSin[Sqrt[1 - Sin[c
+ d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + (672*A + 447*B)*Cos[2*(c + d*x)] +
 6*(28*A + 13*B)*Cos[4*(c + d*x)] - 15*B*Cos[6*(c + d*x)] - 1645*A*Sin[c +
d*x] - 350*B*Sin[c + d*x] - 140*A*Sin[3*(c + d*x)] + 140*B*Sin[3*(c + d*x)]
 + 35*A*Sin[5*(c + d*x)] + 70*B*Sin[5*(c + d*x)]))/(3360*d)
```

Maple [A] time = 0.073, size = 215, normalized size = 1.3

$$\frac{1}{d}\left(a^2A\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6}+\frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{dx}{16}+\frac{c}{16}\right)+Ba^2\left(-\frac{(\sin(dx+c)}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))
*sin(d*x+c)+1/16*d*x+1/16*c)+B*a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos
```

```
(d*x+c)^5)-2/5*a^2*A*cos(d*x+c)^5+2*B*a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/2
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*A*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/5*B*a^2*cos(d*x+c)^5)
```

Maxima [A] time = 1.05968, size = 231, normalized size = 1.4

$$-\frac{2688\,Aa^2\cos(dx+c)^5 + 1344\,Ba^2\cos(dx+c)^5 - 35\left(4\,\sin(2\,dx+2\,c)^3 + 12\,dx + 12\,c - 3\,\sin(4\,dx+4\,c)\right)Aa^2 - 21}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/6720*(2688*A*a^2*cos(d*x + c)^5 + 1344*B*a^2*cos(d*x + c)^5 - 35*(4*sin(
2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*A*a^2 - 210*(12*d*x +
12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 192*(5*cos(d*x + c)^7
 - 7*cos(d*x + c)^5)*B*a^2 - 70*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*s
in(4*d*x + 4*c))*B*a^2)/d
```

Fricas [A] time = 2.03827, size = 290, normalized size = 1.76

$$\frac{240\,Ba^2\cos(dx+c)^7 - 672\,(A+B)a^2\cos(dx+c)^5 + 105\,(7\,A+2\,B)a^2dx - 35\left(8\,(A+2\,B)a^2\cos(dx+c)^5 - 2\,(7\,A+\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(240*B*a^2*cos(d*x + c)^7 - 672*(A + B)*a^2*cos(d*x + c)^5 + 105*(7*
A + 2*B)*a^2*d*x - 35*(8*(A + 2*B)*a^2*cos(d*x + c)^5 - 2*(7*A + 2*B)*a^2*c
os(d*x + c)^3 - 3*(7*A + 2*B)*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 9.31177, size = 539, normalized size = 3.27

$$\begin{cases} \frac{Aa^2x\sin^6(c+dx)}{16} + \frac{3Aa^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3Aa^2x\sin^4(c+dx)}{8} + \frac{3Aa^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Aa^2x}{} \\ x\left(A+B\sin(c)\right)\left(a\sin(c)+a\right)^2\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**6/16 + 3*A*a**2*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 3*A*a**2*x*sin(c + d*x)**4/8 + 3*A*a**2*x*sin(c + d*x)**2*cos
(c + d*x)**4/16 + 3*A*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + A*a**2*x*c
os(c + d*x)**6/16 + 3*A*a**2*x*cos(c + d*x)**4/8 + A*a**2*sin(c + d*x)**5*c
os(c + d*x)/(16*d) + A*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 3*A*a**
2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A*a**2*sin(c + d*x)*cos(c + d*x)**5/
(16*d) + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*A*a**2*cos(c + d*x
)**5/(5*d) + B*a**2*x*sin(c + d*x)**6/8 + 3*B*a**2*x*sin(c + d*x)**4*cos(c
```

```
+ d*x)**2/8 + 3*B*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + B*a**2*x*cos(c
 + d*x)**6/8 + B*a**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) + B*a**2*sin(c + d
*x)**3*cos(c + d*x)**3/(3*d) - B*a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d)
 - B*a**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 2*B*a**2*cos(c + d*x)**7/(35
*d) - B*a**2*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c)
+ a)**2*cos(c)**4, True))
```

Giac [A] time = 1.30595, size = 259, normalized size = 1.57

$$\frac{Ba^2\cos(7\,dx+7\,c)}{448\,d}+\frac{1}{16}\left(7\,Aa^2+2\,Ba^2\right)x-\frac{\left(8\,Aa^2+3\,Ba^2\right)\cos(5\,dx+5\,c)}{320\,d}-\frac{\left(8\,Aa^2+5\,Ba^2\right)\cos(3\,dx+3\,c)}{64\,d}-\frac{(16\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/448*B*a^2*cos(7*d*x + 7*c)/d + 1/16*(7*A*a^2 + 2*B*a^2)*x - 1/320*(8*A*a^
2 + 3*B*a^2)*cos(5*d*x + 5*c)/d - 1/64*(8*A*a^2 + 5*B*a^2)*cos(3*d*x + 3*c)
/d - 1/64*(16*A*a^2 + 11*B*a^2)*cos(d*x + c)/d - 1/192*(A*a^2 + 2*B*a^2)*si
n(6*d*x + 6*c)/d + 1/64*(A*a^2 - 2*B*a^2)*sin(4*d*x + 4*c)/d + 1/64*(17*A*a
^2 + 2*B*a^2)*sin(2*d*x + 2*c)/d
```

3.979 $\int \cos^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}-\frac{(5A+2B)\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{20d}+\frac{a^2(5A+2B)\sin(c+dx)\cos(c+dx)}{8d}+$$

```
[Out] (a^2*(5*A + 2*B)*x)/8 - (a^2*(5*A + 2*B)*Cos[c + d*x]^3)/(12*d) + (a^2*(5*A
 + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c +
 d*x])^2)/(5*d) - ((5*A + 2*B)*Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/(20
*d)
```

Rubi [A] time = 0.16846, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}-\frac{(5A+2B)\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{20d}+\frac{a^2(5A+2B)\sin(c+dx)\cos(c+dx)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(5*A + 2*B)*x)/8 - (a^2*(5*A + 2*B)*Cos[c + d*x]^3)/(12*d) + (a^2*(5*A
 + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c +
 d*x])^2)/(5*d) - ((5*A + 2*B)*Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/(20
*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d} + \frac{1}{5}(5A+2B)\int \cos^2(c$$

$$= -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d} - \frac{(5A+2B)\cos^3(c+dx}{2(}$$

$$= -\frac{a^2(5A+2B)\cos^3(c+dx)}{12d} - \frac{B\cos^3(c+dx)(a+a\sin(c+d}{5d}$$

$$= -\frac{a^2(5A+2B)\cos^3(c+dx)}{12d} + \frac{a^2(5A+2B)\cos(c+dx)\sin(c}{8d}$$

$$= \frac{1}{8}a^2(5A+2B)x - \frac{a^2(5A+2B)\cos^3(c+dx)}{12d} + \frac{a^2(5A+2B)\,c}{}$$

Mathematica [A] time = 0.954653, size = 133, normalized size = 0.99

$$-\frac{a^2\cos(c+dx)\left(8(10A+7B)\cos(2(c+dx)) + \frac{60(5A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}} - 135A\sin(c+dx) + 15A\sin(3(c+dx)) + 80A\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^2*Cos[c + d*x]*(80*A + 62*B + (60*(5*A + 2*B)*ArcSin[Sqrt[1 - Sin[c + d
*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 8*(10*A + 7*B)*Cos[2*(c + d*x)] - 6*B
*Cos[4*(c + d*x)] - 135*A*Sin[c + d*x] - 30*B*Sin[c + d*x] + 15*A*Sin[3*(c
+ d*x)] + 30*B*Sin[3*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.064, size = 182, normalized size = 1.4

$$\frac{1}{d}\left(a^2A\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4} + \frac{\cos(dx+c)\sin(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) + Ba^2\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^3}{5} - \frac{2\,(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+
1/8*c)+B*a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)-2/3*a^2*A*c
os(d*x+c)^3+2*B*a^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)
+1/8*d*x+1/8*c)+a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/3*B*a^2*c
os(d*x+c)^3)
```

Maxima [A] time = 1.02022, size = 181, normalized size = 1.35

$$-\frac{320\,Aa^2\cos\left(dx+c\right)^3+160\,Ba^2\cos\left(dx+c\right)^3-15\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)Aa^2-120\left(2\,dx+2\,c+\sin\left(2\,dx+2\right.\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/480*(320*A*a^2*cos(d*x + c)^3 + 160*B*a^2*cos(d*x + c)^3 - 15*(4*d*x + 4
*c - sin(4*d*x + 4*c))*A*a^2 - 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 -
 32*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*B*a^2 - 30*(4*d*x + 4*c - sin(4*d
*x + 4*c))*B*a^2)/d
```

Fricas [A] time = 1.99813, size = 235, normalized size = 1.75

$$\frac{24\,Ba^2\cos\left(dx+c\right)^5-80\left(A+B\right)a^2\cos\left(dx+c\right)^3+15\left(5\,A+2\,B\right)a^2dx-15\left(2\left(A+2\,B\right)a^2\cos\left(dx+c\right)^3-\left(5\,A+2\,B\right)a\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(24*B*a^2*cos(d*x + c)^5 - 80*(A + B)*a^2*cos(d*x + c)^3 + 15*(5*A +
2*B)*a^2*d*x - 15*(2*(A + 2*B)*a^2*cos(d*x + c)^3 - (5*A + 2*B)*a^2*cos(d*x
 + c))*sin(d*x + c))/d
```

Sympy [A] time = 3.05206, size = 371, normalized size = 2.77

$$\begin{cases} \frac{Aa^2x\sin^4(c+dx)}{8}+\frac{Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{Aa^2x\sin^2(c+dx)}{2}+\frac{Aa^2x\cos^4(c+dx)}{8}+\frac{Aa^2x\cos^2(c+dx)}{2}+\frac{Aa^2\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{Aa}{} \\ x\left(A+B\sin\left(c\right)\right)\left(a\sin\left(c\right)+a\right)^2\cos^2\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**4/8 + A*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**4/8 + A*a**2*
x*cos(c + d*x)**2/2 + A*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A*a**2*si
n(c + d*x)*cos(c + d*x)**3/(8*d) + A*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) -
 2*A*a**2*cos(c + d*x)**3/(3*d) + B*a**2*x*sin(c + d*x)**4/4 + B*a**2*x*sin
(c + d*x)**2*cos(c + d*x)**2/2 + B*a**2*x*cos(c + d*x)**4/4 + B*a**2*sin(c
+ d*x)**3*cos(c + d*x)/(4*d) - B*a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d)
 - B*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) - 2*B*a**2*cos(c + d*x)**5/(15
*d) - B*a**2*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c)
+ a)**2*cos(c)**2, True))
```

Giac [A] time = 1.32936, size = 176, normalized size = 1.31

$$\frac{Ba^2\cos(5\,dx+5\,c)}{80\,d}+\frac{Aa^2\sin(2\,dx+2\,c)}{4\,d}+\frac{1}{8}\left(5\,Aa^2+2\,Ba^2\right)x-\frac{\left(8\,Aa^2+5\,Ba^2\right)\cos(3\,dx+3\,c)}{48\,d}-\frac{\left(4\,Aa^2+3\,Ba^2\right)\mathrm{co}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/80*B*a^2*cos(5*d*x + 5*c)/d + 1/4*A*a^2*sin(2*d*x + 2*c)/d + 1/8*(5*A*a^2
 + 2*B*a^2)*x - 1/48*(8*A*a^2 + 5*B*a^2)*cos(3*d*x + 3*c)/d - 1/8*(4*A*a^2
+ 3*B*a^2)*cos(d*x + c)/d - 1/32*(A*a^2 + 2*B*a^2)*sin(4*d*x + 4*c)/d
```

3.980 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=55

$$\frac{a^2(A+2B)\cos(c+dx)}{d}+a^2x(-(A+2B))+\frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

```
[Out] -(a^2*(A + 2*B)*x) + (a^2*(A + 2*B)*Cos[c + d*x])/d + ((A + B)*Sec[c + d*x]
*(a + a*Sin[c + d*x])^2)/d
```

Rubi [A] time = 0.0933537, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2855, 2638}

$$\frac{a^2(A+2B)\cos(c+dx)}{d}+a^2x(-(A+2B))+\frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^2*(A + 2*B)*x) + (a^2*(A + 2*B)*Cos[c + d*x])/d + ((A + B)*Sec[c + d*x]
*(a + a*Sin[c + d*x])^2)/d
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))^2}{d}-(a(A+2B))\int (\\ &= -a^2(A+2B)x+\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))^2}{d}-\\ &= -a^2(A+2B)x+\frac{a^2(A+2B)\cos(c+dx)}{d}+\frac{(A+B)\sec(c\end{aligned}$$

Mathematica [A] time = 0.241612, size = 91, normalized size = 1.65

$$\frac{a^2\sec(c+dx)\left(4(A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{\cos^2(c+dx)}+4A\sin(c+dx)+4A+4B\sin(c+dx)+B\cos(2(c+d\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*Sec[c + d*x]*(4*A + 5*B + 4*(A + 2*B)*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sq
rt[2]]*Sqrt[Cos[c + d*x]^2] + B*Cos[2*(c + d*x)] + 4*A*Sin[c + d*x] + 4*B*S
in[c + d*x]))/(2*d)
```

Maple [B] time = 0.079, size = 123, normalized size = 2.2

$$\frac{1}{d}\left(a^2A(\tan(dx+c)-dx-c)+Ba^2\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+2\frac{a^2A}{\cos(dx+c)}+2Ba^2(\tan($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(tan(d*x+c)-d*x-c)+B*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^
2)*cos(d*x+c))+2*a^2*A/cos(d*x+c)+2*B*a^2*(tan(d*x+c)-d*x-c)+a^2*A*tan(d*x+
c)+B*a^2/cos(d*x+c))
```

Maxima [A] time = 1.52915, size = 140, normalized size = 2.55

$$-\frac{(dx+c-\tan(dx+c))Aa^2+2(dx+c-\tan(dx+c))Ba^2-Ba^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-Aa^2\tan(dx+c)-\frac{2Aa^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -((d*x + c - tan(d*x + c))*A*a^2 + 2*(d*x + c - tan(d*x + c))*B*a^2 - B*a^2
*(1/cos(d*x + c) + cos(d*x + c)) - A*a^2*tan(d*x + c) - 2*A*a^2/cos(d*x + c
) - B*a^2/cos(d*x + c))/d
```

Fricas [B] time = 2.09241, size = 302, normalized size = 5.49

$$-\frac{(A+2B)a^2dx-Ba^2\cos(dx+c)^2-2(A+B)a^2+\left((A+2B)a^2dx-(2A+3B)a^2\right)\cos(dx+c)-\left((A+2B)a^2dx-Ba^2\right.}{d\cos(dx+c)-d\sin(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -((A + 2*B)*a^2*d*x - B*a^2*cos(d*x + c)^2 - 2*(A + B)*a^2 + ((A + 2*B)*a^2
*d*x - (2*A + 3*B)*a^2)*cos(d*x + c) - ((A + 2*B)*a^2*d*x - B*a^2*cos(d*x +
 c) + 2*(A + B)*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3381, size = 169, normalized size = 3.07

$$-\frac{\left(Aa^2+2\,Ba^2\right)(dx+c)+\frac{2\left(2\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Aa^2+3\,Ba^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -((A*a^2 + 2*B*a^2)*(d*x + c) + 2*(2*A*a^2*tan(1/2*d*x + 1/2*c)^2 + 2*B*a^2
*tan(1/2*d*x + 1/2*c)^2 - B*a^2*tan(1/2*d*x + 1/2*c) + 2*A*a^2 + 3*B*a^2)/(
tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) - 1)
)/d
```

3.981 $\int \sec^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=73

$$\frac{a^2(A-2B)\tan(c+dx)}{3d}+\frac{a^2(A-2B)\sec(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

```
[Out] (a^2*(A - 2*B)*Sec[c + d*x])/(3*d) + ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c +
 d*x])^2)/(3*d) + (a^2*(A - 2*B)*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.11585, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2855, 2669, 3767, 8}

$$\frac{a^2(A-2B)\tan(c+dx)}{3d}+\frac{a^2(A-2B)\sec(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(A - 2*B)*Sec[c + d*x])/(3*d) + ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c +
 d*x])^2)/(3*d) + (a^2*(A - 2*B)*Tan[c + d*x])/(3*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} + \frac{1}{3}(a(A-2B))$$
$$= \frac{a^2(A-2B)\sec(c+dx)}{3d} + \frac{(A+B)\sec^3(c+dx)(a+a\sin($$
$$= \frac{a^2(A-2B)\sec(c+dx)}{3d} + \frac{(A+B)\sec^3(c+dx)(a+a\sin($$
$$= \frac{a^2(A-2B)\sec(c+dx)}{3d} + \frac{(A+B)\sec^3(c+dx)(a+a\sin($$

Mathematica [A] time = 0.0183405, size = 121, normalized size = 1.66

$$-\frac{a^2A\tan^3(c+dx)}{3d} + \frac{2a^2A\sec^3(c+dx)}{3d} + \frac{a^2A\tan(c+dx)\sec^2(c+dx)}{d} + \frac{2a^2B\tan^3(c+dx)}{3d} - \frac{a^2B\sec^3(c+dx)}{3d} + $$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*A*Sec[c + d*x]^3)/(3*d) - (a^2*B*Sec[c + d*x]^3)/(3*d) + (a^2*A*Sec[
c + d*x]^2*Tan[c + d*x])/d + (a^2*B*Sec[c + d*x]*Tan[c + d*x]^2)/d - (a^2*A
*Tan[c + d*x]^3)/(3*d) + (2*a^2*B*Tan[c + d*x]^3)/(3*d)
```

Maple [B] time = 0.095, size = 162, normalized size = 2.2

$$\frac{1}{d}\left(\frac{a^2A(\sin(dx+c))^3}{3(\cos(dx+c))^3} + Ba^2\left(\frac{(\sin(dx+c))^4}{3(\cos(dx+c))^3} - \frac{(\sin(dx+c))^4}{3\cos(dx+c)} - \frac{\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}\right) + \frac{2a^2A}{3(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a^2*A*sin(d*x+c)^3/cos(d*x+c)^3+B*a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)
^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+2/3*a^2*A/c
os(d*x+c)^3+2/3*B*a^2*sin(d*x+c)^3/cos(d*x+c)^3-a^2*A*(-2/3-1/3*sec(d*x+c)^
2)*tan(d*x+c)+1/3*B*a^2/cos(d*x+c)^3)
```

Maxima [A] time = 1.01528, size = 146, normalized size = 2.

$$\frac{Aa^2\tan(dx+c)^3 + 2Ba^2\tan(dx+c)^3 + \left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Aa^2 - \frac{\left(3\cos(dx+c)^2-1\right)Ba^2}{\cos(dx+c)^3} + \frac{2Aa^2}{\cos(dx+c)^3} + \frac{Ba}{\cos(dx}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/3*(A*a^2*tan(d*x + c)^3 + 2*B*a^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*ta
n(d*x + c))*A*a^2 - (3*cos(d*x + c)^2 - 1)*B*a^2/cos(d*x + c)^3 + 2*A*a^2/c
os(d*x + c)^3 + B*a^2/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.92604, size = 294, normalized size = 4.03

$$-\frac{(A-2B)a^2\cos(dx+c)^2+(2A-B)a^2\cos(dx+c)+(A+B)a^2-\left((A-2B)a^2\cos(dx+c)-(A+B)a^2\right)\sin(dx+c)}{3\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2d)\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/3*((A - 2*B)*a^2*cos(d*x + c)^2 + (2*A - B)*a^2*cos(d*x + c) + (A + B)*a
^2 - ((A - 2*B)*a^2*cos(d*x + c) - (A + B)*a^2)*sin(d*x + c))/(d*cos(d*x +
c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33111, size = 105, normalized size = 1.44

$$-\frac{2\left(3Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2Aa^2-Ba^2\right)}{3d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -2/3*(3*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a
^2*tan(1/2*d*x + 1/2*c) + 2*A*a^2 - B*a^2)/(d*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

3.982 $\int \sec^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=104

$$\frac{a^2(3A-2B)\tan^3(c+dx)}{15d}+\frac{a^2(3A-2B)\tan(c+dx)}{5d}+\frac{a^2(3A-2B)\sec^3(c+dx)}{15d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+d$$

```
[Out] (a^2*(3*A - 2*B)*Sec[c + d*x]^3)/(15*d) + ((A + B)*Sec[c + d*x]^5*(a + a*Si
n[c + d*x])^2)/(5*d) + (a^2*(3*A - 2*B)*Tan[c + d*x])/(5*d) + (a^2*(3*A - 2
*B)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.124831, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2669, 3767}

$$\frac{a^2(3A-2B)\tan^3(c+dx)}{15d}+\frac{a^2(3A-2B)\tan(c+dx)}{5d}+\frac{a^2(3A-2B)\sec^3(c+dx)}{15d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(3*A - 2*B)*Sec[c + d*x]^3)/(15*d) + ((A + B)*Sec[c + d*x]^5*(a + a*Si
n[c + d*x])^2)/(5*d) + (a^2*(3*A - 2*B)*Tan[c + d*x])/(5*d) + (a^2*(3*A - 2
*B)*Tan[c + d*x]^3)/(15*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^2}{5d} + \frac{1}{5}(a(3A-2B)) \int$$
$$= \frac{a^2(3A-2B)\sec^3(c+dx)}{15d} + \frac{(A+B)\sec^5(c+dx)(a+a\sin(c}{5d}$$
$$= \frac{a^2(3A-2B)\sec^3(c+dx)}{15d} + \frac{(A+B)\sec^5(c+dx)(a+a\sin(c}{5d}$$
$$= \frac{a^2(3A-2B)\sec^3(c+dx)}{15d} + \frac{(A+B)\sec^5(c+dx)(a+a\sin(c}{5d}$$

Mathematica [A] time = 0.0200992, size = 178, normalized size = 1.71

$$\frac{2a^2A\tan^5(c+dx)}{5d} + \frac{2a^2A\sec^5(c+dx)}{5d} - \frac{a^2A\tan^3(c+dx)\sec^2(c+dx)}{d} + \frac{a^2A\tan(c+dx)\sec^4(c+dx)}{d} - \frac{4a^2B\tan^5(c}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*A*Sec[c + d*x]^5)/(5*d) + (a^2*B*Sec[c + d*x]^5)/(15*d) + (a^2*A*Sec
[c + d*x]^4*Tan[c + d*x])/d + (a^2*B*Sec[c + d*x]^3*Tan[c + d*x]^2)/(3*d) -
 (a^2*A*Sec[c + d*x]^2*Tan[c + d*x]^3)/d + (2*a^2*B*Sec[c + d*x]^2*Tan[c +
d*x]^3)/(3*d) + (2*a^2*A*Tan[c + d*x]^5)/(5*d) - (4*a^2*B*Tan[c + d*x]^5)/(
15*d)
```

Maple [B] time = 0.106, size = 231, normalized size = 2.2

$$\frac{1}{d}\left(a^2A\left(\frac{(\sin(dx+c))^3}{5(\cos(dx+c))^5} + \frac{2(\sin(dx+c))^3}{15(\cos(dx+c))^3}\right) + Ba^2\left(\frac{(\sin(dx+c))^4}{5(\cos(dx+c))^5} + \frac{(\sin(dx+c))^4}{15(\cos(dx+c))^3} - \frac{(\sin(dx+c))^4}{15\cos(dx+c)} - \frac{(2+(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+B
*a^2*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15*sin
(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+2/5*a^2*A/cos(d*x+c)
^5+2*B*a^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)-a
^2*A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/5*B*a^2/cos(d*
x+c)^5)
```

Maxima [A] time = 1.02389, size = 198, normalized size = 1.9

$$\frac{\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)Aa^2 + \left(3\tan(dx+c)^5 + 5\tan(dx+c)^3\right)Aa^2 + 2\left(3\tan(dx+c)^5\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^2 + (3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*A*a^2 + 2*(3*tan(d*x + c)^5 + 5*tan(d*x +
 c)^3)*B*a^2 - (5*cos(d*x + c)^2 - 3)*B*a^2/cos(d*x + c)^5 + 6*A*a^2/cos(d*
x + c)^5 + 3*B*a^2/cos(d*x + c)^5)/d
```

Fricas [A] time = 1.83644, size = 273, normalized size = 2.62

$$-\frac{4\,(3\,A-2\,B)a^2\cos{(dx+c)}^2-3\,(2\,A-3\,B)a^2-\left(2\,(3\,A-2\,B)a^2\cos{(dx+c)}^2-3\,(3\,A-2\,B)a^2\right)\sin{(dx+c)}}{15\left(d\cos{(dx+c)}^3+2\,d\cos{(dx+c)}\sin{(dx+c)}-2\,d\cos{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/15*(4*(3*A - 2*B)*a^2*cos(d*x + c)^2 - 3*(2*A - 3*B)*a^2 - (2*(3*A - 2*B
)*a^2*cos(d*x + c)^2 - 3*(3*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^3 +
 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24262, size = 259, normalized size = 2.49

$$-\frac{\frac{15\left(Aa^2-Ba^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{105\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-270\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+360\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-40}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/60*(15*(A*a^2 - B*a^2)/(tan(1/2*d*x + 1/2*c) + 1) + (105*A*a^2*tan(1/2*d
*x + 1/2*c)^4 + 15*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 270*A*a^2*tan(1/2*d*x + 1
/2*c)^3 + 30*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 360*A*a^2*tan(1/2*d*x + 1/2*c)^
2 - 40*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 210*A*a^2*tan(1/2*d*x + 1/2*c) + 50*B
*a^2*tan(1/2*d*x + 1/2*c) + 63*A*a^2 - 7*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^
5)/d
```

3.983 $\int \sec^8(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a^2(5A-2B)\tan^5(c+dx)}{35d}+\frac{2a^2(5A-2B)\tan^3(c+dx)}{21d}+\frac{a^2(5A-2B)\tan(c+dx)}{7d}+\frac{a^2(5A-2B)\sec^5(c+dx)}{35d}+\frac{(A+$$

```
[Out] (a^2*(5*A - 2*B)*Sec[c + d*x]^5)/(35*d) + ((A + B)*Sec[c + d*x]^7*(a + a*Si
n[c + d*x])^2)/(7*d) + (a^2*(5*A - 2*B)*Tan[c + d*x])/(7*d) + (2*a^2*(5*A -
 2*B)*Tan[c + d*x]^3)/(21*d) + (a^2*(5*A - 2*B)*Tan[c + d*x]^5)/(35*d)
```

Rubi [A] time = 0.133749, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2669, 3767}

$$\frac{a^2(5A-2B)\tan^5(c+dx)}{35d}+\frac{2a^2(5A-2B)\tan^3(c+dx)}{21d}+\frac{a^2(5A-2B)\tan(c+dx)}{7d}+\frac{a^2(5A-2B)\sec^5(c+dx)}{35d}+\frac{(A+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(5*A - 2*B)*Sec[c + d*x]^5)/(35*d) + ((A + B)*Sec[c + d*x]^7*(a + a*Si
n[c + d*x])^2)/(7*d) + (a^2*(5*A - 2*B)*Tan[c + d*x])/(7*d) + (2*a^2*(5*A -
 2*B)*Tan[c + d*x]^3)/(21*d) + (a^2*(5*A - 2*B)*Tan[c + d*x]^5)/(35*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d} + \frac{1}{7}(a(5A-2B)$$
$$= \frac{a^2(5A-2B)\sec^5(c+dx)}{35d} + \frac{(A+B)\sec^7(c+dx)(a+a\sin$$
$$= \frac{a^2(5A-2B)\sec^5(c+dx)}{35d} + \frac{(A+B)\sec^7(c+dx)(a+a\sin$$
$$= \frac{a^2(5A-2B)\sec^5(c+dx)}{35d} + \frac{(A+B)\sec^7(c+dx)(a+a\sin$$

Mathematica [A] time = 0.345351, size = 130, normalized size = 1.01

$$\frac{a^2\left(8(2B-5A)\tan^7(c+dx)+(30A+9B)\sec^7(c+dx)-35(5A-2B)\tan^3(c+dx)\sec^4(c+dx)+28(5A-2B)\tan^5\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*((30*A + 9*B)*Sec[c + d*x]^7 + 105*A*Sec[c + d*x]^6*Tan[c + d*x] + 21*
B*Sec[c + d*x]^5*Tan[c + d*x]^2 - 35*(5*A - 2*B)*Sec[c + d*x]^4*Tan[c + d*x
]^3 + 28*(5*A - 2*B)*Sec[c + d*x]^2*Tan[c + d*x]^5 + 8*(-5*A + 2*B)*Tan[c +
 d*x]^7))/(105*d)
```

Maple [B] time = 0.119, size = 295, normalized size = 2.3

$$\frac{1}{d}\left(a^2A\left(\frac{(\sin(dx+c))^3}{7(\cos(dx+c))^7}+\frac{4(\sin(dx+c))^3}{35(\cos(dx+c))^5}+\frac{8(\sin(dx+c))^3}{105(\cos(dx+c))^3}\right)+Ba^2\left(\frac{(\sin(dx+c))^4}{7(\cos(dx+c))^7}+\frac{3(\sin(dx+c))^4}{35(\cos(dx+c))^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/
105*sin(d*x+c)^3/cos(d*x+c)^3)+B*a^2*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*si
n(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/co
s(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c))+2/7*a^2*A/cos(d*x+c)^7+2*B*a^2*(
1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+
c)^3/cos(d*x+c)^3)-a^2*A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*se
c(d*x+c)^2)*tan(d*x+c)+1/7*B*a^2/cos(d*x+c)^7)
```

Maxima [A] time = 1.03039, size = 240, normalized size = 1.86

$$\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan(dx+c)^3\right)Aa^2+3\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/105*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*A*a^2 +
3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x +
c))*A*a^2 + 2*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*B
*a^2 - 3*(7*cos(d*x + c)^2 - 5)*B*a^2/cos(d*x + c)^7 + 30*A*a^2/cos(d*x + c
)^7 + 15*B*a^2/cos(d*x + c)^7)/d
```

Fricas [A] time = 1.61766, size = 377, normalized size = 2.92

$$-\frac{16(5A-2B)a^2\cos(dx+c)^4-8(5A-2B)a^2\cos(dx+c)^2-5(2A-5B)a^2-\left(8(5A-2B)a^2\cos(dx+c)^4-12(5A\right.}{105\left(d\cos(dx+c)^5+2d\cos(dx+c)^3\sin(dx+c)-2d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/105*(16*(5*A - 2*B)*a^2*cos(d*x + c)^4 - 8*(5*A - 2*B)*a^2*cos(d*x + c)^
2 - 5*(2*A - 5*B)*a^2 - (8*(5*A - 2*B)*a^2*cos(d*x + c)^4 - 12*(5*A - 2*B)*
a^2*cos(d*x + c)^2 - 5*(5*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5 + 2
*d*cos(d*x + c)^3*sin(d*x + c) - 2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29088, size = 439, normalized size = 3.4

$$-\frac{\frac{35\left(9Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-9Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8Aa^2-5Ba^2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}+\frac{1365Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+210Ba^2\tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/840*(35*(9*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*B*a^2*tan(1/2*d*x + 1/2*c)^2
 + 15*A*a^2*tan(1/2*d*x + 1/2*c) - 9*B*a^2*tan(1/2*d*x + 1/2*c) + 8*A*a^2 -
 5*B*a^2)/(tan(1/2*d*x + 1/2*c) + 1)^3 + (1365*A*a^2*tan(1/2*d*x + 1/2*c)^6
 + 210*B*a^2*tan(1/2*d*x + 1/2*c)^6 - 5775*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 1
05*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 12250*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 175*
B*a^2*tan(1/2*d*x + 1/2*c)^4 - 14350*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 910*B*a
^2*tan(1/2*d*x + 1/2*c)^3 + 10185*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 756*B*a^2*
tan(1/2*d*x + 1/2*c)^2 - 3955*A*a^2*tan(1/2*d*x + 1/2*c) + 427*B*a^2*tan(1/
2*d*x + 1/2*c) + 760*A*a^2 - 31*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

3.984 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=154

$$\frac{a^2(7A-2B)\tan^7(c+dx)}{63d}+\frac{a^2(7A-2B)\tan^5(c+dx)}{15d}+\frac{a^2(7A-2B)\tan^3(c+dx)}{9d}+\frac{a^2(7A-2B)\tan(c+dx)}{9d}+a^2(\ldots$$

```
[Out] (a^2*(7*A - 2*B)*Sec[c + d*x]^7)/(63*d) + ((A + B)*Sec[c + d*x]^9*(a + a*Si
n[c + d*x])^2)/(9*d) + (a^2*(7*A - 2*B)*Tan[c + d*x])/(9*d) + (a^2*(7*A - 2
*B)*Tan[c + d*x]^3)/(9*d) + (a^2*(7*A - 2*B)*Tan[c + d*x]^5)/(15*d) + (a^2*
(7*A - 2*B)*Tan[c + d*x]^7)/(63*d)
```

Rubi [A] time = 0.141179, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2669, 3767}

$$\frac{a^2(7A-2B)\tan^7(c+dx)}{63d}+\frac{a^2(7A-2B)\tan^5(c+dx)}{15d}+\frac{a^2(7A-2B)\tan^3(c+dx)}{9d}+\frac{a^2(7A-2B)\tan(c+dx)}{9d}+a^2(\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(7*A - 2*B)*Sec[c + d*x]^7)/(63*d) + ((A + B)*Sec[c + d*x]^9*(a + a*Si
n[c + d*x])^2)/(9*d) + (a^2*(7*A - 2*B)*Tan[c + d*x])/(9*d) + (a^2*(7*A - 2
*B)*Tan[c + d*x]^3)/(9*d) + (a^2*(7*A - 2*B)*Tan[c + d*x]^5)/(15*d) + (a^2*
(7*A - 2*B)*Tan[c + d*x]^7)/(63*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^2}{9d} + \frac{1}{9}(a(7A-2B))\int$$

$$= \frac{a^2(7A-2B)\sec^7(c+dx)}{63d} + \frac{(A+B)\sec^9(c+dx)(a+a\sin(}{9d}$$

$$= \frac{a^2(7A-2B)\sec^7(c+dx)}{63d} + \frac{(A+B)\sec^9(c+dx)(a+a\sin(}{9d}$$

$$= \frac{a^2(7A-2B)\sec^7(c+dx)}{63d} + \frac{(A+B)\sec^9(c+dx)(a+a\sin(}{9d}$$

Mathematica [A] time = 0.448487, size = 156, normalized size = 1.01

$$\frac{a^2\left(16(7A-2B)\tan^9(c+dx)+5(14A+5B)\sec^9(c+dx)-105(7A-2B)\tan^3(c+dx)\sec^6(c+dx)+126(7A-2B)\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(5*(14*A + 5*B)*Sec[c + d*x]^9 + 315*A*Sec[c + d*x]^8*Tan[c + d*x] + 4
5*B*Sec[c + d*x]^7*Tan[c + d*x]^2 - 105*(7*A - 2*B)*Sec[c + d*x]^6*Tan[c +
d*x]^3 + 126*(7*A - 2*B)*Sec[c + d*x]^4*Tan[c + d*x]^5 - 72*(7*A - 2*B)*Sec
[c + d*x]^2*Tan[c + d*x]^7 + 16*(7*A - 2*B)*Tan[c + d*x]^9))/(315*d)
```

Maple [B] time = 0.131, size = 359, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/
105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+B*a^2*(1/9*
sin(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/
cos(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/
63*(2+sin(d*x+c)^2)*cos(d*x+c))+2/9*a^2*A/cos(d*x+c)^9+2*B*a^2*(1/9*sin(d*x
+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*
x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)-a^2*A*(-128/315-1/9*sec(d*x+c)^8-8
/63*sec(d*x+c)^6-16/105*sec(d*x+c)^4-64/315*sec(d*x+c)^2)*tan(d*x+c)+1/9*B*
a^2/cos(d*x+c)^9)
```

Maxima [A] time = 1.04463, size = 279, normalized size = 1.81

$$\frac{\left(35\tan(dx+c)^9+180\tan(dx+c)^7+378\tan(dx+c)^5+420\tan(dx+c)^3+315\tan(dx+c)\right)Aa^2+\left(35\tan(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*A*a^2 + (35*tan(d*x + c)^9 + 135*tan(d*x
+ c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*A*a^2 + 2*(35*tan(d*x + c
)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*B*a^2 -
 5*(9*cos(d*x + c)^2 - 7)*B*a^2/cos(d*x + c)^9 + 70*A*a^2/cos(d*x + c)^9 +
35*B*a^2/cos(d*x + c)^9)/d
```

Fricas [A] time = 1.84669, size = 475, normalized size = 3.08

$$-\frac{32\,(7\,A-2\,B)a^2\cos\left(dx+c\right)^6-16\,(7\,A-2\,B)a^2\cos\left(dx+c\right)^4-4\,(7\,A-2\,B)a^2\cos\left(dx+c\right)^2-7\,(2\,A-7\,B)a^2-\left(16\right.}{315\,\left(d\cos\left(dx+c\right)^7+2\,d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/315*(32*(7*A - 2*B)*a^2*cos(d*x + c)^6 - 16*(7*A - 2*B)*a^2*cos(d*x + c)
^4 - 4*(7*A - 2*B)*a^2*cos(d*x + c)^2 - 7*(2*A - 7*B)*a^2 - (16*(7*A - 2*B)
*a^2*cos(d*x + c)^6 - 24*(7*A - 2*B)*a^2*cos(d*x + c)^4 - 10*(7*A - 2*B)*a^
2*cos(d*x + c)^2 - 7*(7*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^7 + 2*d
*cos(d*x + c)^5*sin(d*x + c) - 2*d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31404, size = 622, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="g
iac")
```

```
[Out] -1/20160*(21*(435*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 225*B*a^2*tan(1/2*d*x + 1/
2*c)^4 + 1470*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 690*B*a^2*tan(1/2*d*x + 1/2*c)
^3 + 2060*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 940*B*a^2*tan(1/2*d*x + 1/2*c)^2 +
 1330*A*a^2*tan(1/2*d*x + 1/2*c) - 590*B*a^2*tan(1/2*d*x + 1/2*c) + 353*A*a
^2 - 163*B*a^2)/(tan(1/2*d*x + 1/2*c) + 1)^5 + (31185*A*a^2*tan(1/2*d*x + 1
/2*c)^8 + 4725*B*a^2*tan(1/2*d*x + 1/2*c)^8 - 185220*A*a^2*tan(1/2*d*x + 1/
2*c)^7 - 11340*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 546840*A*a^2*tan(1/2*d*x + 1/
2*c)^6 + 15120*B*a^2*tan(1/2*d*x + 1/2*c)^6 - 961380*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 3780*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 1101618*A*a^2*tan(1/2*d*x + 1/
```

```
2*c)^4 - 24318*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 828492*A*a^2*tan(1/2*d*x + 1/
2*c)^3 + 33852*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 404208*A*a^2*tan(1/2*d*x + 1/
2*c)^2 - 19368*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 116172*A*a^2*tan(1/2*d*x + 1/
2*c) + 6732*B*a^2*tan(1/2*d*x + 1/2*c) + 16373*A*a^2 - 223*B*a^2)/(tan(1/2*
d*x + 1/2*c) - 1)^9)/d
```

3.985 $\int \sec^{12}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=179

$$\frac{a^2(9A-2B)\tan^9(c+dx)}{99d}+\frac{4a^2(9A-2B)\tan^7(c+dx)}{77d}+\frac{6a^2(9A-2B)\tan^5(c+dx)}{55d}+\frac{4a^2(9A-2B)\tan^3(c+dx)}{33d}+$$

```
[Out] (a^2*(9*A - 2*B)*Sec[c + d*x]^9)/(99*d) + ((A + B)*Sec[c + d*x]^11*(a + a*S
in[c + d*x])^2)/(11*d) + (a^2*(9*A - 2*B)*Tan[c + d*x])/(11*d) + (4*a^2*(9*
A - 2*B)*Tan[c + d*x]^3)/(33*d) + (6*a^2*(9*A - 2*B)*Tan[c + d*x]^5)/(55*d)
 + (4*a^2*(9*A - 2*B)*Tan[c + d*x]^7)/(77*d) + (a^2*(9*A - 2*B)*Tan[c + d*x
]^9)/(99*d)
```

Rubi [A] time = 0.150762, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2669, 3767}

$$\frac{a^2(9A-2B)\tan^9(c+dx)}{99d}+\frac{4a^2(9A-2B)\tan^7(c+dx)}{77d}+\frac{6a^2(9A-2B)\tan^5(c+dx)}{55d}+\frac{4a^2(9A-2B)\tan^3(c+dx)}{33d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(9*A - 2*B)*Sec[c + d*x]^9)/(99*d) + ((A + B)*Sec[c + d*x]^11*(a + a*S
in[c + d*x])^2)/(11*d) + (a^2*(9*A - 2*B)*Tan[c + d*x])/(11*d) + (4*a^2*(9*
A - 2*B)*Tan[c + d*x]^3)/(33*d) + (6*a^2*(9*A - 2*B)*Tan[c + d*x]^5)/(55*d)
 + (4*a^2*(9*A - 2*B)*Tan[c + d*x]^7)/(77*d) + (a^2*(9*A - 2*B)*Tan[c + d*x
]^9)/(99*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{12}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^{11}(c+dx)(a+a\sin(c+dx))^2}{11d} + \frac{1}{11}(a(9A-2B))$$
$$= \frac{a^2(9A-2B)\sec^9(c+dx)}{99d} + \frac{(A+B)\sec^{11}(c+dx)(a+a\sin}{11d}$$
$$= \frac{a^2(9A-2B)\sec^9(c+dx)}{99d} + \frac{(A+B)\sec^{11}(c+dx)(a+a\sin}{11d}$$
$$= \frac{a^2(9A-2B)\sec^9(c+dx)}{99d} + \frac{(A+B)\sec^{11}(c+dx)(a+a\sin}{11d}$$

Mathematica [A] time = 0.909396, size = 181, normalized size = 1.01

$$\frac{a^2\left(128(2B-9A)\tan^{11}(c+dx)+35(18A+7B)\sec^{11}(c+dx)-1155(9A-2B)\tan^3(c+dx)\sec^8(c+dx)+1848(9A-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(35*(18*A + 7*B)*Sec[c + d*x]^11 + 3465*A*Sec[c + d*x]^10*Tan[c + d*x]
 + 385*B*Sec[c + d*x]^9*Tan[c + d*x]^2 - 1155*(9*A - 2*B)*Sec[c + d*x]^8*Ta
n[c + d*x]^3 + 1848*(9*A - 2*B)*Sec[c + d*x]^6*Tan[c + d*x]^5 - 1584*(9*A -
 2*B)*Sec[c + d*x]^4*Tan[c + d*x]^7 + 704*(9*A - 2*B)*Sec[c + d*x]^2*Tan[c
+ d*x]^9 + 128*(-9*A + 2*B)*Tan[c + d*x]^11))/(3465*d)
```

Maple [B] time = 0.236, size = 423, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/11*sin(d*x+c)^3/cos(d*x+c)^11+8/99*sin(d*x+c)^3/cos(d*x+c)^9+
16/231*sin(d*x+c)^3/cos(d*x+c)^7+64/1155*sin(d*x+c)^3/cos(d*x+c)^5+128/3465
*sin(d*x+c)^3/cos(d*x+c)^3)+B*a^2*(1/11*sin(d*x+c)^4/cos(d*x+c)^11+7/99*sin
(d*x+c)^4/cos(d*x+c)^9+5/99*sin(d*x+c)^4/cos(d*x+c)^7+1/33*sin(d*x+c)^4/cos
(d*x+c)^5+1/99*sin(d*x+c)^4/cos(d*x+c)^3-1/99*sin(d*x+c)^4/cos(d*x+c)-1/99*
(2+sin(d*x+c)^2)*cos(d*x+c))+2/11*a^2*A/cos(d*x+c)^11+2*B*a^2*(1/11*sin(d*x
+c)^3/cos(d*x+c)^11+8/99*sin(d*x+c)^3/cos(d*x+c)^9+16/231*sin(d*x+c)^3/cos(
d*x+c)^7+64/1155*sin(d*x+c)^3/cos(d*x+c)^5+128/3465*sin(d*x+c)^3/cos(d*x+c)
^3)-a^2*A*(-256/693-1/11*sec(d*x+c)^10-10/99*sec(d*x+c)^8-80/693*sec(d*x+c)
^6-32/231*sec(d*x+c)^4-128/693*sec(d*x+c)^2)*tan(d*x+c)+1/11*B*a^2/cos(d*x+
c)^11)
```

Maxima [A] time = 1.04184, size = 321, normalized size = 1.79

$$\frac{\left(315\,\tan\left(dx+c\right)^{11}+1540\,\tan\left(dx+c\right)^{9}+2970\,\tan\left(dx+c\right)^{7}+2772\,\tan\left(dx+c\right)^{5}+1155\,\tan\left(dx+c\right)^{3}\right)Aa^2+5\left(63\,\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/3465*((315*tan(d*x + c)^11 + 1540*tan(d*x + c)^9 + 2970*tan(d*x + c)^7 +
2772*tan(d*x + c)^5 + 1155*tan(d*x + c)^3)*A*a^2 + 5*(63*tan(d*x + c)^11 +
385*tan(d*x + c)^9 + 990*tan(d*x + c)^7 + 1386*tan(d*x + c)^5 + 1155*tan(d*
x + c)^3 + 693*tan(d*x + c))*A*a^2 + 2*(315*tan(d*x + c)^11 + 1540*tan(d*x
+ c)^9 + 2970*tan(d*x + c)^7 + 2772*tan(d*x + c)^5 + 1155*tan(d*x + c)^3)*B
*a^2 - 35*(11*cos(d*x + c)^2 - 9)*B*a^2/cos(d*x + c)^11 + 630*A*a^2/cos(d*x
 + c)^11 + 315*B*a^2/cos(d*x + c)^11)/d
```

Fricas [A] time = 1.89079, size = 583, normalized size = 3.26

$$-\frac{256\,(9\,A-2\,B)a^2\cos{(dx+c)}^8-128\,(9\,A-2\,B)a^2\cos{(dx+c)}^6-32\,(9\,A-2\,B)a^2\cos{(dx+c)}^4-16\,(9\,A-2\,B)a^2\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/3465*(256*(9*A - 2*B)*a^2*cos(d*x + c)^8 - 128*(9*A - 2*B)*a^2*cos(d*x +
 c)^6 - 32*(9*A - 2*B)*a^2*cos(d*x + c)^4 - 16*(9*A - 2*B)*a^2*cos(d*x + c)
^2 - 45*(2*A - 9*B)*a^2 - (128*(9*A - 2*B)*a^2*cos(d*x + c)^8 - 192*(9*A -
2*B)*a^2*cos(d*x + c)^6 - 80*(9*A - 2*B)*a^2*cos(d*x + c)^4 - 56*(9*A - 2*B
)*a^2*cos(d*x + c)^2 - 45*(9*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^9
+ 2*d*cos(d*x + c)^7*sin(d*x + c) - 2*d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44411, size = 806, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="g
iac")
```

```
[Out] -1/443520*(33*(6825*A*a^2*tan(1/2*d*x + 1/2*c)^6 - 2940*B*a^2*tan(1/2*d*x +
 1/2*c)^6 + 34965*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 13755*B*a^2*tan(1/2*d*x +
1/2*c)^5 + 79800*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 30065*B*a^2*tan(1/2*d*x + 1
/2*c)^4 + 100170*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 36470*B*a^2*tan(1/2*d*x + 1
```

```
/2*c)^3 + 73017*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 26166*B*a^2*tan(1/2*d*x + 1/
2*c)^2 + 29169*A*a^2*tan(1/2*d*x + 1/2*c) - 10367*B*a^2*tan(1/2*d*x + 1/2*c
) + 5142*A*a^2 - 1901*B*a^2)/(tan(1/2*d*x + 1/2*c) + 1)^7 + (661815*A*a^2*t
an(1/2*d*x + 1/2*c)^10 + 97020*B*a^2*tan(1/2*d*x + 1/2*c)^10 - 5083155*A*a^
2*tan(1/2*d*x + 1/2*c)^9 - 405405*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 19355490*A
*a^2*tan(1/2*d*x + 1/2*c)^8 + 952875*B*a^2*tan(1/2*d*x + 1/2*c)^8 - 4544694
0*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 1122660*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 722
95146*A*a^2*tan(1/2*d*x + 1/2*c)^6 + 557172*B*a^2*tan(1/2*d*x + 1/2*c)^6 -
80611146*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 563178*B*a^2*tan(1/2*d*x + 1/2*c)^5
 + 63771840*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 1126950*B*a^2*tan(1/2*d*x + 1/2*
c)^4 - 35253900*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 955020*B*a^2*tan(1/2*d*x + 1
/2*c)^3 + 13119975*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 406120*B*a^2*tan(1/2*d*x
+ 1/2*c)^2 - 2978811*A*a^2*tan(1/2*d*x + 1/2*c) + 97163*B*a^2*tan(1/2*d*x +
 1/2*c) + 330966*A*a^2 - 13*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^11)/d
```

3.986 $\int \cos^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{(A-7B)(a\sin(c+dx)+a)^{10}}{10a^7d}+\frac{2(A-3B)(a\sin(c+dx)+a)^9}{3a^6d}-\frac{(3A-5B)(a\sin(c+dx)+a)^8}{2a^5d}+\frac{8(A-B)(a\sin(c-}{7a^4d}$$

```
[Out] (8*(A - B)*(a + a*Sin[c + d*x])^7)/(7*a^4*d) - ((3*A - 5*B)*(a + a*Sin[c +
d*x])^8)/(2*a^5*d) + (2*(A - 3*B)*(a + a*Sin[c + d*x])^9)/(3*a^6*d) - ((A -
 7*B)*(a + a*Sin[c + d*x])^10)/(10*a^7*d) - (B*(a + a*Sin[c + d*x])^11)/(11
*a^8*d)
```

Rubi [A] time = 0.182323, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-7B)(a\sin(c+dx)+a)^{10}}{10a^7d}+\frac{2(A-3B)(a\sin(c+dx)+a)^9}{3a^6d}-\frac{(3A-5B)(a\sin(c+dx)+a)^8}{2a^5d}+\frac{8(A-B)(a\sin(c-}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (8*(A - B)*(a + a*Sin[c + d*x])^7)/(7*a^4*d) - ((3*A - 5*B)*(a + a*Sin[c +
d*x])^8)/(2*a^5*d) + (2*(A - 3*B)*(a + a*Sin[c + d*x])^9)/(3*a^6*d) - ((A -
 7*B)*(a + a*Sin[c + d*x])^10)/(10*a^7*d) - (B*(a + a*Sin[c + d*x])^11)/(11
*a^8*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx=\frac{\text{Subst}\left(\int (a-x)^3(a+x)^6\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^7d}$$

$$=\frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^6-4a^2(3A-5B)(a+x)^7+\right.\right.}{}$$

$$=\frac{8(A-B)(a+a\sin(c+dx))^7}{7a^4d}-\frac{(3A-5B)(a+a\sin(c+d}{2a^5d}$$

Mathematica [A] time = 1.54663, size = 86, normalized size = 0.64

$$-\frac{a^3(\sin(c+dx)+1)^7\left(21(11A-37B)\sin^3(c+dx)+(1029B-847A)\sin^2(c+dx)+14(77A-39B)\sin(c+dx)-484A\right.}{2310d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^3*(1 + Sin[c + d*x])^7*(-484*A + 78*B + 14*(77*A - 39*B)*Sin[c + d*x] +
 (-847*A + 1029*B)*Sin[c + d*x]^2 + 21*(11*A - 37*B)*Sin[c + d*x]^3 + 210*B
*Sin[c + d*x]^4))/(2310*d)
```

Maple [B] time = 0.07, size = 345, normalized size = 2.6

$$\frac{1}{d}\left(a^3A\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^8}{10}-\frac{(\cos(dx+c))^8}{40}\right)+Ba^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^8}{11}-\frac{(\cos(dx+c))^8\sin(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x+c)^8)+B*a^3*(-1/11
*sin(d*x+c)^3*cos(d*x+c)^8-1/33*cos(d*x+c)^8*sin(d*x+c)+1/231*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*A*(-1/9*cos(d*x+
c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*
sin(d*x+c))+3*B*a^3*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x+c)^8)-3/8
*a^3*A*cos(d*x+c)^8+3*B*a^3*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*
x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+1/7*a^3*A*(16/5+cos(d
*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)-1/8*B*a^3*cos(d*x+c)^
8)
```

Maxima [A] time = 1.01753, size = 246, normalized size = 1.84

$$-\frac{210\,Ba^3\sin(dx+c)^{11}+231\,(A+3\,B)a^3\sin(dx+c)^{10}+770\,Aa^3\sin(dx+c)^9-2310\,Ba^3\sin(dx+c)^8-660\,(4\,A+3\,B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2310*(210*B*a^3*sin(d*x + c)^11 + 231*(A + 3*B)*a^3*sin(d*x + c)^10 + 77
0*A*a^3*sin(d*x + c)^9 - 2310*B*a^3*sin(d*x + c)^8 - 660*(4*A + 3*B)*a^3*si
n(d*x + c)^7 - 2310*(A - B)*a^3*sin(d*x + c)^6 + 924*(3*A + 4*B)*a^3*sin(d*
x + c)^5 + 4620*A*a^3*sin(d*x + c)^4 - 2310*B*a^3*sin(d*x + c)^3 - 1155*(3*
A + B)*a^3*sin(d*x + c)^2 - 2310*A*a^3*sin(d*x + c))/d
```

Fricas [A] time = 2.02686, size = 400, normalized size = 2.99

$$\frac{231\,(A+3\,B)a^3\cos(dx+c)^{10}-1155\,(A+B)a^3\cos(dx+c)^8+2\left(105\,Ba^3\cos(dx+c)^{10}-35\,(11\,A+15\,B)a^3\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2310*(231*(A + 3*B)*a^3*cos(d*x + c)^10 - 1155*(A + B)*a^3*cos(d*x + c)^8
 + 2*(105*B*a^3*cos(d*x + c)^10 - 35*(11*A + 15*B)*a^3*cos(d*x + c)^8 + 20*
(11*A + 3*B)*a^3*cos(d*x + c)^6 + 24*(11*A + 3*B)*a^3*cos(d*x + c)^4 + 32*(
11*A + 3*B)*a^3*cos(d*x + c)^2 + 64*(11*A + 3*B)*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 65.6018, size = 530, normalized size = 3.96

$$\begin{cases} \frac{16Aa^3\sin^9(c+dx)}{105d} + \frac{24Aa^3\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{16Aa^3\sin^7(c+dx)}{35d} + \frac{6Aa^3\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{8Aa^3\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{Aa^3\sin \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a**3*sin(c + d*x)**9/(105*d) + 24*A*a**3*sin(c + d*x)**7*co
s(c + d*x)**2/(35*d) + 16*A*a**3*sin(c + d*x)**7/(35*d) + 6*A*a**3*sin(c +
d*x)**5*cos(c + d*x)**4/(5*d) + 8*A*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(5
*d) + A*a**3*sin(c + d*x)**3*cos(c + d*x)**6/d + 2*A*a**3*sin(c + d*x)**3*c
os(c + d*x)**4/d - A*a**3*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) + A*a**3*si
n(c + d*x)*cos(c + d*x)**6/d - A*a**3*cos(c + d*x)**10/(40*d) - 3*A*a**3*co
s(c + d*x)**8/(8*d) + 16*B*a**3*sin(c + d*x)**11/(1155*d) + 8*B*a**3*sin(c
+ d*x)**9*cos(c + d*x)**2/(105*d) + 16*B*a**3*sin(c + d*x)**9/(105*d) + 6*B
*a**3*sin(c + d*x)**7*cos(c + d*x)**4/(35*d) + 24*B*a**3*sin(c + d*x)**7*co
s(c + d*x)**2/(35*d) + B*a**3*sin(c + d*x)**5*cos(c + d*x)**6/(5*d) + 6*B*a
**3*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + B*a**3*sin(c + d*x)**3*cos(c +
d*x)**6/d - 3*B*a**3*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - 3*B*a**3*cos(c
 + d*x)**10/(40*d) - B*a**3*cos(c + d*x)**8/(8*d), Ne(d, 0)), (x*(A + B*sin
(c))*(a*sin(c) + a)**3*cos(c)**7, True))
```

Giac [B] time = 1.38131, size = 382, normalized size = 2.85

$$\frac{Ba^3\sin(11\,dx+11\,c)}{11264\,d} + \frac{\left(Aa^3+3\,Ba^3\right)\cos(10\,dx+10\,c)}{5120\,d} - \frac{\left(Aa^3-Ba^3\right)\cos(8\,dx+8\,c)}{512\,d} - \frac{\left(23\,Aa^3+5\,Ba^3\right)\cos(6\,dx}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/11264*B*a^3*sin(11*d*x + 11*c)/d + 1/5120*(A*a^3 + 3*B*a^3)*cos(10*d*x +
10*c)/d - 1/512*(A*a^3 - B*a^3)*cos(8*d*x + 8*c)/d - 1/1024*(23*A*a^3 + 5*B
*a^3)*cos(6*d*x + 6*c)/d - 1/128*(11*A*a^3 + 5*B*a^3)*cos(4*d*x + 4*c)/d -
7/512*(13*A*a^3 + 7*B*a^3)*cos(2*d*x + 2*c)/d - 1/3072*(4*A*a^3 + 3*B*a^3)*
sin(9*d*x + 9*c)/d - 1/7168*(44*A*a^3 + 61*B*a^3)*sin(7*d*x + 7*c)/d + 1/51
20*(16*A*a^3 - 107*B*a^3)*sin(5*d*x + 5*c)/d + 1/512*(56*A*a^3 - B*a^3)*sin
(3*d*x + 3*c)/d + 91/512*(4*A*a^3 + B*a^3)*sin(d*x + c)/d
```

3.987 $\int \cos^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{(A-5B)(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(A-B)(a\sin(c+dx)+a)^6}{3a^3d}+\frac{B(a\sin(c+dx)+a)^9}{9a^6d}$$

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (4*(A - 2*B)*(a + a*Sin[c +
d*x])^7)/(7*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^8)/(8*a^5*d) + (B*(a +
 a*Sin[c + d*x])^9)/(9*a^6*d)
```

Rubi [A] time = 0.152259, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{(A-5B)(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(A-B)(a\sin(c+dx)+a)^6}{3a^3d}+\frac{B(a\sin(c+dx)+a)^9}{9a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (4*(A - 2*B)*(a + a*Sin[c +
d*x])^7)/(7*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^8)/(8*a^5*d) + (B*(a +
 a*Sin[c + d*x])^9)/(9*a^6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx=\frac{\text{Subst}\left(\int (a-x)^2(a+x)^5\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^5d}$$

$$=\frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^5-4a(A-2B)(a+x)^6+(A-\right.\right.}{a^5d}$$

$$=\frac{2(A-B)(a+a\sin(c+dx))^6}{3a^3d}-\frac{4(A-2B)(a+a\sin(c+dx))^7}{7a^4d}$$

Mathematica [A] time = 0.434802, size = 70, normalized size = 0.67

$$\frac{a^3(\sin(c+dx)+1)^6\left(21(3A-7B)\sin^2(c+dx)-6(27A-19B)\sin(c+dx)+111A+56B\sin^3(c+dx)-19B\right)}{504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^6*(111*A - 19*B - 6*(27*A - 19*B)*Sin[c + d*x] + 21
*(3*A - 7*B)*Sin[c + d*x]^2 + 56*B*Sin[c + d*x]^3))/(504*d)
```

Maple [B] time = 0.072, size = 305, normalized size = 2.9

$$\frac{1}{d}\left(a^3A\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{8}-\frac{(\cos(dx+c))^6}{24}\right)+Ba^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^6}{9}-\frac{\sin(dx+c)(\cos}{21}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+B*a^3*(-1/9*s
in(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)
^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*A*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*B*a^3*(-1/8*sin(d*x+c)^2
*cos(d*x+c)^6-1/24*cos(d*x+c)^6)-1/2*a^3*A*cos(d*x+c)^6+3*B*a^3*(-1/7*sin(d
*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+1/5
*a^3*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*B*a^3*cos(d*x+c)^
6)
```

Maxima [A] time = 1.13474, size = 213, normalized size = 2.03

$$56\,Ba^3\sin(dx+c)^9+63\,(A+3\,B)a^3\sin(dx+c)^8+72\,(3\,A+B)a^3\sin(dx+c)^7+84\,(A-5\,B)a^3\sin(dx+c)^6-504$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/504*(56*B*a^3*sin(d*x + c)^9 + 63*(A + 3*B)*a^3*sin(d*x + c)^8 + 72*(3*A
+ B)*a^3*sin(d*x + c)^7 + 84*(A - 5*B)*a^3*sin(d*x + c)^6 - 504*(A + B)*a^3
*sin(d*x + c)^5 - 126*(5*A - B)*a^3*sin(d*x + c)^4 + 168*(A + 3*B)*a^3*sin(
d*x + c)^3 + 252*(3*A + B)*a^3*sin(d*x + c)^2 + 504*A*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.92373, size = 321, normalized size = 3.06

$$\frac{63\,(A+3\,B)a^3\cos(dx+c)^8-336\,(A+B)a^3\cos(dx+c)^6+8\left(7\,Ba^3\cos(dx+c)^8-(27\,A+37\,B)a^3\cos(dx+c)^6+6\right.}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/504*(63*(A + 3*B)*a^3*cos(d*x + c)^8 - 336*(A + B)*a^3*cos(d*x + c)^6 + 8
*(7*B*a^3*cos(d*x + c)^8 - (27*A + 37*B)*a^3*cos(d*x + c)^6 + 6*(3*A + B)*a
^3*cos(d*x + c)^4 + 8*(3*A + B)*a^3*cos(d*x + c)^2 + 16*(3*A + B)*a^3)*sin(
d*x + c))/d
```

Sympy [A] time = 24.6646, size = 471, normalized size = 4.49

$$\begin{cases} \frac{Aa^3\sin^8(c+dx)}{24d} + \frac{8Aa^3\sin^7(c+dx)}{35d} + \frac{Aa^3\sin^6(c+dx)\cos^2(c+dx)}{6d} + \frac{4Aa^3\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{8Aa^3\sin^5(c+dx)}{15d} + \frac{Aa^3\sin^4(c+dx)\cos^4(c+dx)}{4d} \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)**8/(24*d) + 8*A*a**3*sin(c + d*x)**7/(35*d)
+ A*a**3*sin(c + d*x)**6*cos(c + d*x)**2/(6*d) + 4*A*a**3*sin(c + d*x)**5*c
os(c + d*x)**2/(5*d) + 8*A*a**3*sin(c + d*x)**5/(15*d) + A*a**3*sin(c + d*x
)**4*cos(c + d*x)**4/(4*d) + A*a**3*sin(c + d*x)**3*cos(c + d*x)**4/d + 4*A
*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + A*a**3*sin(c + d*x)*cos(c + d
*x)**4/d - A*a**3*cos(c + d*x)**6/(2*d) + 8*B*a**3*sin(c + d*x)**9/(315*d)
+ B*a**3*sin(c + d*x)**8/(8*d) + 4*B*a**3*sin(c + d*x)**7*cos(c + d*x)**2/(
35*d) + 8*B*a**3*sin(c + d*x)**7/(35*d) + B*a**3*sin(c + d*x)**6*cos(c + d*
x)**2/(2*d) + B*a**3*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 4*B*a**3*sin(c
 + d*x)**5*cos(c + d*x)**2/(5*d) + 3*B*a**3*sin(c + d*x)**4*cos(c + d*x)**4
/(4*d) + B*a**3*sin(c + d*x)**3*cos(c + d*x)**4/d - B*a**3*cos(c + d*x)**6/
(6*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**5, True))
```

Giac [B] time = 1.32641, size = 311, normalized size = 2.96

$$\frac{Ba^3\sin(9\,dx+9\,c)}{2304\,d} + \frac{\left(Aa^3+3\,Ba^3\right)\cos(8\,dx+8\,c)}{1024\,d} - \frac{\left(5\,Aa^3-Ba^3\right)\cos(6\,dx+6\,c)}{384\,d} - \frac{\left(25\,Aa^3+11\,Ba^3\right)\cos(4\,dx+4\,c)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2304*B*a^3*sin(9*d*x + 9*c)/d + 1/1024*(A*a^3 + 3*B*a^3)*cos(8*d*x + 8*c)
/d - 1/384*(5*A*a^3 - B*a^3)*cos(6*d*x + 6*c)/d - 1/256*(25*A*a^3 + 11*B*a^
3)*cos(4*d*x + 4*c)/d - 1/128*(33*A*a^3 + 19*B*a^3)*cos(2*d*x + 2*c)/d - 1/
1792*(12*A*a^3 + 11*B*a^3)*sin(7*d*x + 7*c)/d - 1/64*(A*a^3 + 2*B*a^3)*sin(
5*d*x + 5*c)/d + 1/192*(17*A*a^3 - 4*B*a^3)*sin(3*d*x + 3*c)/d + 11/128*(10
*A*a^3 + 3*B*a^3)*sin(d*x + c)/d
```

3.988 $\int \cos^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-\frac{(A-3B)(a\sin(c+dx)+a)^6}{6a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^5}{5a^2d}-\frac{B(a\sin(c+dx)+a)^7}{7a^4d}$$

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*
x])^6)/(6*a^3*d) - (B*(a + a*Sin[c + d*x])^7)/(7*a^4*d)
```

Rubi [A] time = 0.133522, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-3B)(a\sin(c+dx)+a)^6}{6a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^5}{5a^2d}-\frac{B(a\sin(c+dx)+a)^7}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*
x])^6)/(6*a^3*d) - (B*(a + a*Sin[c + d*x])^7)/(7*a^4*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx=\frac{\text{Subst}\left(\int (a-x)(a+x)^4\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^3d}$$

$$=\frac{\text{Subst}\left(\int \left(2a(A-B)(a+x)^4+(-A+3B)(a+x)^5-\frac{B(a+}{a}\right.\right.}{a^3d}$$

$$=\frac{2(A-B)(a+a\sin(c+dx))^5}{5a^2d}-\frac{(A-3B)(a+a\sin(c+dx}{6a^3d}$$

Mathematica [A] time = 0.240138, size = 53, normalized size = 0.68

$$-\frac{a^3(\sin(c+dx)+1)^5\left(5(7A-9B)\sin(c+dx)-49A+30B\sin^2(c+dx)+9B\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^3*(1 + Sin[c + d*x])^5*(-49*A + 9*B + 5*(7*A - 9*B)*Sin[c + d*x] + 30*B
*Sin[c + d*x]^2))/(210*d)
```

Maple [B] time = 0.069, size = 265, normalized size = 3.4

$$\frac{1}{d}\left(a^3A\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{6}-\frac{(\cos(dx+c))^4}{12}\right)+Ba^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^4}{7}-\frac{3\sin(dx+c)(\cos(}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+B*a^3*(-1/7*s
in(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+cos(d*x+c)^2)
*sin(d*x+c))+3*a^3*A*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*si
n(d*x+c))+3*B*a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)-3/4*a^
3*A*cos(d*x+c)^4+3*B*a^3*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2
)*sin(d*x+c))+1/3*a^3*A*(2+cos(d*x+c)^2)*sin(d*x+c)-1/4*B*a^3*cos(d*x+c)^4)
```

Maxima [A] time = 1.02209, size = 170, normalized size = 2.18

$$-\frac{30\,Ba^3\sin(dx+c)^7+35\,(A+3\,B)a^3\sin(dx+c)^6+42\,(3\,A+2\,B)a^3\sin(dx+c)^5+105\,(A-B)a^3\sin(dx+c)^4-70\,(2}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/210*(30*B*a^3*sin(d*x + c)^7 + 35*(A + 3*B)*a^3*sin(d*x + c)^6 + 42*(3*A
 + 2*B)*a^3*sin(d*x + c)^5 + 105*(A - B)*a^3*sin(d*x + c)^4 - 70*(2*A + 3*B
)*a^3*sin(d*x + c)^3 - 105*(3*A + B)*a^3*sin(d*x + c)^2 - 210*A*a^3*sin(d*x
 + c))/d
```

Fricas [A] time = 1.79773, size = 286, normalized size = 3.67

$$\frac{35\,(A+3\,B)a^3\cos(dx+c)^6-210\,(A+B)a^3\cos(dx+c)^4+2\left(15\,Ba^3\cos(dx+c)^6-3\,(21\,A+29\,B)a^3\cos(dx+c)^4+8\right.}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/210*(35*(A + 3*B)*a^3*cos(d*x + c)^6 - 210*(A + B)*a^3*cos(d*x + c)^4 + 2
*(15*B*a^3*cos(d*x + c)^6 - 3*(21*A + 29*B)*a^3*cos(d*x + c)^4 + 8*(7*A + 3
*B)*a^3*cos(d*x + c)^2 + 16*(7*A + 3*B)*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 8.32572, size = 313, normalized size = 4.01

$$\begin{cases} \frac{Aa^3\sin^6(c+dx)}{12d} + \frac{2Aa^3\sin^5(c+dx)}{5d} + \frac{Aa^3\sin^4(c+dx)\cos^2(c+dx)}{4d} + \frac{Aa^3\sin^3(c+dx)\cos^2(c+dx)}{d} + \frac{2Aa^3\sin^3(c+dx)}{3d} + \frac{Aa^3\sin(c+dx)\cos^2(c+dx)}{d} \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)**6/(12*d) + 2*A*a**3*sin(c + d*x)**5/(5*d) +
 A*a**3*sin(c + d*x)**4*cos(c + d*x)**2/(4*d) + A*a**3*sin(c + d*x)**3*cos(
c + d*x)**2/d + 2*A*a**3*sin(c + d*x)**3/(3*d) + A*a**3*sin(c + d*x)*cos(c
+ d*x)**2/d - 3*A*a**3*cos(c + d*x)**4/(4*d) + 2*B*a**3*sin(c + d*x)**7/(35
*d) + B*a**3*sin(c + d*x)**6/(4*d) + B*a**3*sin(c + d*x)**5*cos(c + d*x)**2
/(5*d) + 2*B*a**3*sin(c + d*x)**5/(5*d) + 3*B*a**3*sin(c + d*x)**4*cos(c +
d*x)**2/(4*d) + B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d - B*a**3*cos(c + d
*x)**4/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**3, Tru
e))
```

Giac [B] time = 1.34238, size = 232, normalized size = 2.97

$$-\frac{30\,Ba^3\sin(dx+c)^7 + 35\,Aa^3\sin(dx+c)^6 + 105\,Ba^3\sin(dx+c)^6 + 126\,Aa^3\sin(dx+c)^5 + 84\,Ba^3\sin(dx+c)^5 + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/210*(30*B*a^3*sin(d*x + c)^7 + 35*A*a^3*sin(d*x + c)^6 + 105*B*a^3*sin(d
*x + c)^6 + 126*A*a^3*sin(d*x + c)^5 + 84*B*a^3*sin(d*x + c)^5 + 105*A*a^3*
sin(d*x + c)^4 - 105*B*a^3*sin(d*x + c)^4 - 140*A*a^3*sin(d*x + c)^3 - 210*
B*a^3*sin(d*x + c)^3 - 315*A*a^3*sin(d*x + c)^2 - 105*B*a^3*sin(d*x + c)^2
- 210*A*a^3*sin(d*x + c))/d
```

3.989 $\int \cos(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{B(a\sin(c+dx)+a)^5}{5a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{4ad}$$

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(4*a*d) + (B*(a + a*Sin[c + d*x])^5)/(5*a^
2*d)
```

Rubi [A] time = 0.062236, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B(a\sin(c+dx)+a)^5}{5a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(4*a*d) + (B*(a + a*Sin[c + d*x])^5)/(5*a^
2*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left((A-B)(a+x)^3+\frac{B(a+x)^4}{a}\right)dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{(A-B)(a+a\sin(c+dx))^4}{4ad}+\frac{B(a+a\sin(c+dx))^5}{5a^2d}\end{aligned}$$

Mathematica [A] time = 0.0882942, size = 36, normalized size = 0.71

$$\frac{a^3(\sin(c+dx)+1)^4(5A+4B\sin(c+dx)-B)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^4*(5*A - B + 4*B*Sin[c + d*x]))/(20*d)
```

Maple [B] time = 0.033, size = 98, normalized size = 1.9

$$\frac{1}{d}\left(\frac{Ba^3(\sin(dx+c))^5}{5}+\frac{\left(a^3A+3Ba^3\right)(\sin(dx+c))^4}{4}+\frac{\left(3a^3A+3Ba^3\right)(\sin(dx+c))^3}{3}+\frac{\left(3a^3A+Ba^3\right)(\sin(dx+c\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*B*a^3*sin(d*x+c)^5+1/4*(A*a^3+3*B*a^3)*sin(d*x+c)^4+1/3*(3*A*a^3+3
*B*a^3)*sin(d*x+c)^3+1/2*(3*A*a^3+B*a^3)*sin(d*x+c)^2+a^3*A*sin(d*x+c))
```

Maxima [A] time = 1.05471, size = 113, normalized size = 2.22

$$\frac{4Ba^3\sin(dx+c)^5+5(A+3B)a^3\sin(dx+c)^4+20(A+B)a^3\sin(dx+c)^3+10(3A+B)a^3\sin(dx+c)^2+20Aa^3\,\mathrm{s}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/20*(4*B*a^3*sin(d*x + c)^5 + 5*(A + 3*B)*a^3*sin(d*x + c)^4 + 20*(A + B)*
a^3*sin(d*x + c)^3 + 10*(3*A + B)*a^3*sin(d*x + c)^2 + 20*A*a^3*sin(d*x + c
))/d
```

Fricas [B] time = 1.68586, size = 224, normalized size = 4.39

$$\frac{5(A+3B)a^3\cos(dx+c)^4-40(A+B)a^3\cos(dx+c)^2+4\left(Ba^3\cos(dx+c)^4-(5A+7B)a^3\cos(dx+c)^2+2(5A+\right.}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/20*(5*(A + 3*B)*a^3*cos(d*x + c)^4 - 40*(A + B)*a^3*cos(d*x + c)^2 + 4*(B
*a^3*cos(d*x + c)^4 - (5*A + 7*B)*a^3*cos(d*x + c)^2 + 2*(5*A + 3*B)*a^3)*s
in(d*x + c))/d
```

Sympy [A] time = 2.70239, size = 204, normalized size = 4.

$$\begin{cases}\frac{Aa^3\sin^3(c+dx)}{d}-\frac{Aa^3\sin^2(c+dx)\cos^2(c+dx)}{2d}+\frac{Aa^3\sin(c+dx)}{d}-\frac{Aa^3\cos^4(c+dx)}{4d}-\frac{3Aa^3\cos^2(c+dx)}{2d}+\frac{Ba^3\sin^5(c+dx)}{5d}+\frac{Ba^3\sin^3(c+dx)}{d}-3\\ x(A+B\sin(c))(a\sin(c)+a)^3\cos(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)**3/d - A*a**3*sin(c + d*x)**2*cos(c + d*x)**
2/(2*d) + A*a**3*sin(c + d*x)/d - A*a**3*cos(c + d*x)**4/(4*d) - 3*A*a**3*c
os(c + d*x)**2/(2*d) + B*a**3*sin(c + d*x)**5/(5*d) + B*a**3*sin(c + d*x)**
3/d - 3*B*a**3*sin(c + d*x)**2*cos(c + d*x)**2/(2*d) - 3*B*a**3*cos(c + d*x
)**4/(4*d) - B*a**3*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*
sin(c) + a)**3*cos(c), True))
```

Giac [B] time = 1.32333, size = 157, normalized size = 3.08

$$\frac{4\,Ba^3\sin\left(dx+c\right)^5+5\,Aa^3\sin\left(dx+c\right)^4+15\,Ba^3\sin\left(dx+c\right)^4+20\,Aa^3\sin\left(dx+c\right)^3+20\,Ba^3\sin\left(dx+c\right)^3+30\,Aa^3\sin}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/20*(4*B*a^3*sin(d*x + c)^5 + 5*A*a^3*sin(d*x + c)^4 + 15*B*a^3*sin(d*x +
c)^4 + 20*A*a^3*sin(d*x + c)^3 + 20*B*a^3*sin(d*x + c)^3 + 30*A*a^3*sin(d*x
 + c)^2 + 10*B*a^3*sin(d*x + c)^2 + 20*A*a^3*sin(d*x + c))/d
```

3.990 $\int \sec(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a^3(A+B)\sin^2(c+dx)}{2d}-\frac{3a^3(A+B)\sin(c+dx)}{d}-\frac{4a^3(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^3}{3d}$$

```
[Out] (-4*a^3*(A + B)*Log[1 - Sin[c + d*x]])/d - (3*a^3*(A + B)*Sin[c + d*x])/d -
 (a^3*(A + B)*Sin[c + d*x]^2)/(2*d) - (B*(a + a*Sin[c + d*x])^3)/(3*d)
```

Rubi [A] time = 0.0947624, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2836, 77}

$$-\frac{a^3(A+B)\sin^2(c+dx)}{2d}-\frac{3a^3(A+B)\sin(c+dx)}{d}-\frac{4a^3(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-4*a^3*(A + B)*Log[1 - Sin[c + d*x]])/d - (3*a^3*(A + B)*Sin[c + d*x])/d -
 (a^3*(A + B)*Sin[c + d*x]^2)/(2*d) - (B*(a + a*Sin[c + d*x])^3)/(3*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx=\frac{a\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{a}\right)}{a-x}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$=\frac{a\,\text{Subst}\left(\int \left(-3a(A+B)+\frac{4a^2(A+B)}{a-x}-(A+B)x-\frac{B(a+x)^2}{a}\right)\right.}{d}$$
$$=-\frac{4a^3(A+B)\log(1-\sin(c+dx))}{d}-\frac{3a^3(A+B)\sin(c+dx}{d}$$

Mathematica [A] time = 0.1262, size = 68, normalized size = 0.84

$$-\frac{a^3\left(3(A+3B)\sin^2(c+dx)+6(3A+4B)\sin(c+dx)+24(A+B)\log(1-\sin(c+dx))+2B\sin^3(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^3*(24*(A + B)*Log[1 - Sin[c + d*x]] + 6*(3*A + 4*B)*Sin[c + d*x] + 3*(A
 + 3*B)*Sin[c + d*x]^2 + 2*B*Sin[c + d*x]^3))/(6*d)
```

Maple [B] time = 0.087, size = 161, normalized size = 2.

$$-\frac{a^3A(\sin(dx+c))^2}{2d}-4\frac{a^3A\ln(\cos(dx+c))}{d}-\frac{Ba^3(\sin(dx+c))^3}{3d}-4\frac{Ba^3\sin(dx+c)}{d}+4\frac{Ba^3\ln(\sec(dx+c)+\tan(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a^3*A*sin(d*x+c)^2-4/d*a^3*A*ln(cos(d*x+c))-1/3/d*B*a^3*sin(d*x+c)^3
-4*a^3*B*sin(d*x+c)/d+4/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))-3/d*a^3*A*sin(d*x
+c)+4/d*a^3*A*ln(sec(d*x+c)+tan(d*x+c))-3/2/d*B*a^3*sin(d*x+c)^2-4/d*B*a^3*
ln(cos(d*x+c))
```

Maxima [A] time = 1.03075, size = 99, normalized size = 1.22

$$-\frac{2\,Ba^3\sin(dx+c)^3+3\,(A+3\,B)a^3\sin(dx+c)^2+24\,(A+B)a^3\log(\sin(dx+c)-1)+6\,(3\,A+4\,B)a^3\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/6*(2*B*a^3*sin(d*x + c)^3 + 3*(A + 3*B)*a^3*sin(d*x + c)^2 + 24*(A + B)*
a^3*log(sin(d*x + c) - 1) + 6*(3*A + 4*B)*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.78676, size = 188, normalized size = 2.32

$$\frac{3\,(A+3\,B)a^3\cos(dx+c)^2-24\,(A+B)a^3\log(-\sin(dx+c)+1)+2\left(Ba^3\cos(dx+c)^2-(9\,A+13\,B)a^3\right)\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(A + 3*B)*a^3*cos(d*x + c)^2 - 24*(A + B)*a^3*log(-sin(d*x + c) + 1)
 + 2*(B*a^3*cos(d*x + c)^2 - (9*A + 13*B)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38332, size = 390, normalized size = 4.81

$$2\left(6\left(Aa^3+Ba^3\right)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\left(Aa^3+Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{11\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+11\,Ba^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 2/3*(6*(A*a^3 + B*a^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*(A*a^3 + B*a^3)
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - (11*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 11
*B*a^3*tan(1/2*d*x + 1/2*c)^6 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^3*t
an(1/2*d*x + 1/2*c)^5 + 36*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 42*B*a^3*tan(1/2*
d*x + 1/2*c)^4 + 18*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 28*B*a^3*tan(1/2*d*x + 1
/2*c)^3 + 36*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 42*B*a^3*tan(1/2*d*x + 1/2*c)^2
 + 9*A*a^3*tan(1/2*d*x + 1/2*c) + 12*B*a^3*tan(1/2*d*x + 1/2*c) + 11*A*a^3
+ 11*B*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.991 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{2a^4(A+B)}{d(a-a\sin(c+dx))}+\frac{a^3(A+3B)\log(1-\sin(c+dx))}{d}+\frac{a^3B\sin(c+dx)}{d}$$

```
[Out] (a^3*(A + 3*B)*Log[1 - Sin[c + d*x]])/d + (a^3*B*Sin[c + d*x])/d + (2*a^4*(
A + B))/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.108573, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{2a^4(A+B)}{d(a-a\sin(c+dx))}+\frac{a^3(A+3B)\log(1-\sin(c+dx))}{d}+\frac{a^3B\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(A + 3*B)*Log[1 - Sin[c + d*x]])/d + (a^3*B*Sin[c + d*x])/d + (2*a^4*(
A + B))/(d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= \frac{a^3\,\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{a}\right)}{(a-x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^3\,\text{Subst}\left(\int \left(\frac{B}{a}+\frac{2a(A+B)}{(a-x)^2}+\frac{-A-3B}{a-x}\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^3(A+3B)\log(1-\sin(c+dx))}{d}+\frac{a^3B\sin(c+dx)}{d}+\frac{2a^4}{d(a-a}\end{aligned}$$

Mathematica [A] time = 0.114809, size = 48, normalized size = 0.77

$$\frac{a^3\left(-\frac{2(A+B)}{\sin(c+dx)-1}+(A+3B)\log(1-\sin(c+dx))+B\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*((A + 3*B)*Log[1 - Sin[c + d*x]] - (2*(A + B))/(-1 + Sin[c + d*x]) + B
*Sin[c + d*x]))/d
```

Maple [B] time = 0.116, size = 290, normalized size = 4.7

$$\frac{a^3A(\tan(dx+c))^2}{2d}+\frac{a^3A\ln(\cos(dx+c))}{d}+\frac{Ba^3(\sin(dx+c))^5}{2d(\cos(dx+c))^2}+\frac{Ba^3(\sin(dx+c))^3}{2d}+3\frac{Ba^3\sin(dx+c)}{d}-3\frac{Ba^3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a^3*A*tan(d*x+c)^2+1/d*a^3*A*ln(cos(d*x+c))+1/2/d*B*a^3*sin(d*x+c)^5/
cos(d*x+c)^2+1/2/d*B*a^3*sin(d*x+c)^3+3*a^3*B*sin(d*x+c)/d-3/d*B*a^3*ln(sec
(d*x+c)+tan(d*x+c))+3/2/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^2+3/2/d*a^3*A*sin(d
*x+c)-1/d*a^3*A*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*B*a^3*tan(d*x+c)^2+3/d*B*a^
3*ln(cos(d*x+c))+3/2/d*a^3*A/cos(d*x+c)^2+3/2/d*B*a^3*sin(d*x+c)^3/cos(d*x+
c)^2+1/2/d*a^3*A*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^3/cos(d*x+c)^2
```

Maxima [A] time = 1.02333, size = 70, normalized size = 1.13

$$\frac{(A+3B)a^3\log(\sin(dx+c)-1)+Ba^3\sin(dx+c)-\frac{2(A+B)a^3}{\sin(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] ((A + 3*B)*a^3*log(sin(d*x + c) - 1) + B*a^3*sin(d*x + c) - 2*(A + B)*a^3/(
sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.74517, size = 207, normalized size = 3.34

$$-\frac{Ba^3\cos(dx+c)^2+Ba^3\sin(dx+c)+(2A+B)a^3-\left((A+3B)a^3\sin(dx+c)-(A+3B)a^3\right)\log(-\sin(dx+c)+1)}{d\sin(dx+c)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -(B*a^3*cos(d*x + c)^2 + B*a^3*sin(d*x + c) + (2*A + B)*a^3 - ((A + 3*B)*a^
3*sin(d*x + c) - (A + 3*B)*a^3)*log(-sin(d*x + c) + 1))/(d*sin(d*x + c) - d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4351, size = 308, normalized size = 4.97

$$-\frac{\left(Aa^3+3\,Ba^3\right)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-2\left(Aa^3+3\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,Ba^3\tan\left(\frac{1}{2}\,dx\cdots\right.}{\tan\left(\frac{1}{2}\cdots\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -((A*a^3 + 3*B*a^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*(A*a^3 + 3*B*a^3)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - (A*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*B*a^3
*tan(1/2*d*x + 1/2*c)^2 + 2*B*a^3*tan(1/2*d*x + 1/2*c) + A*a^3 + 3*B*a^3)/(
tan(1/2*d*x + 1/2*c)^2 + 1) + (3*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 9*B*a^3*tan
(1/2*d*x + 1/2*c)^2 - 10*A*a^3*tan(1/2*d*x + 1/2*c) - 22*B*a^3*tan(1/2*d*x
+ 1/2*c) + 3*A*a^3 + 9*B*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^2)/d
```

3.992 $\int \sec^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a^3(aA+aB\sin(c+dx))^2}{2d(A+B)(a-a\sin(c+dx))^2}$$

```
[Out] (a^3*(a*A + a*B*Sin[c + d*x])^2)/(2*(A + B)*d*(a - a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0817873, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 37}

$$\frac{a^3(aA+aB\sin(c+dx))^2}{2d(A+B)(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(a*A + a*B*Sin[c + d*x])^2)/(2*(A + B)*d*(a - a*Sin[c + d*x])^2)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= \frac{a^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^3(aA+aB\sin(c+dx))^2}{2(A+B)d(a-a\sin(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.0440062, size = 37, normalized size = 0.86

$$\frac{a^3(A+B\sin(c+dx))^2}{2d(A+B)(\sin(c+dx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(A + B*Sin[c + d*x])^2)/(2*(A + B)*d*(-1 + Sin[c + d*x])^2)
```

Maple [B] time = 0.123, size = 312, normalized size = 7.3

$$\frac{a^3A(\sin(dx+c))^4}{4d(\cos(dx+c))^4}+\frac{Ba^3(\sin(dx+c))^5}{4d(\cos(dx+c))^4}-\frac{Ba^3(\sin(dx+c))^5}{8d(\cos(dx+c))^2}-\frac{Ba^3(\sin(dx+c))^3}{8d}+\frac{3a^3A(\sin(dx+c))^3}{4d(\cos(dx+c))^4}+\frac{3a^3A(\sin}{8d(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a^3*A*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^4
-1/8/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^2-1/8/d*B*a^3*sin(d*x+c)^3+3/4/d*a^3*A
*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^2+3/8/d*a^3*
A*sin(d*x+c)+3/4/d*B*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3/4/d*a^3*A/cos(d*x+c)^4
+3/4/d*B*a^3*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*B*a^3*sin(d*x+c)^3/cos(d*x+c)^
2+1/4/d*a^3*A*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*A*sec(d*x+c)*tan(d*x+c)+1/4
/d*B*a^3/cos(d*x+c)^4
```

Maxima [A] time = 1.03375, size = 63, normalized size = 1.47

$$\frac{2\,Ba^3\sin(dx+c)+(A-B)a^3}{2\left(\sin(dx+c)^2-2\,\sin(dx+c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*B*a^3*sin(d*x + c) + (A - B)*a^3)/((sin(d*x + c)^2 - 2*sin(d*x + c)
+ 1)*d)
```

Fricas [A] time = 1.57837, size = 117, normalized size = 2.72

$$-\frac{2\,Ba^3\sin(dx+c)+(A-B)a^3}{2\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*B*a^3*sin(d*x + c) + (A - B)*a^3)/(d*cos(d*x + c)^2 + 2*d*sin(d*x +
 c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40641, size = 111, normalized size = 2.58

$$\frac{2\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^3*tan(1/2*d*x + 1/2*c)^2 + B*a^3*tan(
1/2*d*x + 1/2*c)^2 + A*a^3*tan(1/2*d*x + 1/2*c))/(d*(tan(1/2*d*x + 1/2*c) -
 1)^4)
```

3.993 $\int \sec^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3}+\frac{a^5(A-B)}{8d(a-a\sin(c+dx))^2}+\frac{a^4(A-B)}{8d(a-a\sin(c+dx))}+\frac{a^3(A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] (a^3*(A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^6*(A + B))/(6*d*(a - a*Sin[c
 + d*x])^3) + (a^5*(A - B))/(8*d*(a - a*Sin[c + d*x])^2) + (a^4*(A - B))/(8
*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.135148, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3}+\frac{a^5(A-B)}{8d(a-a\sin(c+dx))^2}+\frac{a^4(A-B)}{8d(a-a\sin(c+dx))}+\frac{a^3(A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^6*(A + B))/(6*d*(a - a*Sin[c
 + d*x])^3) + (a^5*(A - B))/(8*d*(a - a*Sin[c + d*x])^2) + (a^4*(A - B))/(8
*d*(a - a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\,\text{Subst}\left(\int \left(\frac{A+B}{2a(a-x)^4}+\frac{A-B}{4a^2(a-x)^3}+\frac{A-B}{8a^3(a-x)^2}+\frac{A-B}{8a^3(a^2-x^2)}\right) d\right.}{d}$$
$$= \frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3}+\frac{a^5(A-B)}{8d(a-a\sin(c+dx))^2}+\frac{a^4}{8d(a-}$$
$$= \frac{a^3(A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3}+$$

Mathematica [A] time = 0.253383, size = 95, normalized size = 0.9

$$\frac{a^3\left(-3(A-B)\sin^2(c+dx)+9(A-B)\sin(c+dx)-3(A-B)\tanh^{-1}(\sin(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{24d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(2*(-5*A + B) - 3*(A - B)*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^6 + 9*(A - B)*Sin[c + d*x] - 3*(A - B)*Sin[c + d*x]^2))/(24
*d*(-1 + Sin[c + d*x])^3)
```

Maple [B] time = 0.129, size = 521, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 3/16/d*a^3*A*sin(d*x+c)-1/48/d*B*a^3*sin(d*x+c)^3+1/8*a^3*B*sin(d*x+c)/d-1/
8/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^3*A/cos(d*x+c)^6+1/6/d*B*a^3/co
s(d*x+c)^6+1/8/d*a^3*A*ln(sec(d*x+c)+tan(d*x+c))+1/12/d*a^3*A*sin(d*x+c)^4/
cos(d*x+c)^4+1/4/d*B*a^3*sin(d*x+c)^4/cos(d*x+c)^4+1/2/d*B*a^3*sin(d*x+c)^3
/cos(d*x+c)^6+3/8/d*B*a^3*sin(d*x+c)^3/cos(d*x+c)^4+3/16/d*B*a^3*sin(d*x+c)
^3/cos(d*x+c)^2+1/6/d*a^3*A*tan(d*x+c)*sec(d*x+c)^5+5/24/d*a^3*A*tan(d*x+c)
*sec(d*x+c)^3+5/16/d*a^3*A*sec(d*x+c)*tan(d*x+c)+1/6/d*B*a^3*sin(d*x+c)^5/c
os(d*x+c)^6+1/24/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^4-1/48/d*B*a^3*sin(d*x+c)^
5/cos(d*x+c)^2+1/2/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^6+3/8/d*a^3*A*sin(d*x+c)
^3/cos(d*x+c)^4+3/16/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*B*a^3*sin(d*x+
c)^4/cos(d*x+c)^6+1/6/d*a^3*A*sin(d*x+c)^4/cos(d*x+c)^6
```

Maxima [A] time = 1.06411, size = 166, normalized size = 1.58

$$\frac{3(A-B)a^3\log(\sin(dx+c)+1)-3(A-B)a^3\log(\sin(dx+c)-1)-\frac{2\left(3(A-B)a^3\sin(dx+c)^2-9(A-B)a^3\sin(dx+c)+2(5A-B)a^3\right)}{\sin(dx+c)^3-3\sin(dx+c)^2+3\sin(dx+c)-1}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(3*(A - B)*a^3*log(sin(d*x + c) + 1) - 3*(A - B)*a^3*log(sin(d*x + c)
- 1) - 2*(3*(A - B)*a^3*sin(d*x + c)^2 - 9*(A - B)*a^3*sin(d*x + c) + 2*(5*
A - B)*a^3)/(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c) - 1))/d
```

Fricas [B] time = 1.77478, size = 570, normalized size = 5.43

$$\frac{6\,(A-B)a^3\cos{(dx+c)}^2+18\,(A-B)a^3\sin{(dx+c)}-2\,(13\,A-5\,B)a^3+3\left(3\,(A-B)a^3\cos{(dx+c)}^2-4\,(A-B)a^3-\left((A\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(6*(A - B)*a^3*cos(d*x + c)^2 + 18*(A - B)*a^3*sin(d*x + c) - 2*(13*A
- 5*B)*a^3 + 3*(3*(A - B)*a^3*cos(d*x + c)^2 - 4*(A - B)*a^3 - ((A - B)*a^3
*cos(d*x + c)^2 - 4*(A - B)*a^3)*sin(d*x + c))*log(sin(d*x + c) + 1) - 3*(3
*(A - B)*a^3*cos(d*x + c)^2 - 4*(A - B)*a^3 - ((A - B)*a^3*cos(d*x + c)^2 -
 4*(A - B)*a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(3*d*cos(d*x + c)^2 -
 (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40851, size = 213, normalized size = 2.03

$$\frac{6\left(Aa^3-Ba^3\right)\log\left(|\sin{(dx+c)}+1|\right)-6\left(Aa^3-Ba^3\right)\log\left(|\sin{(dx+c)}-1|\right)+\frac{11\,Aa^3\sin(dx+c)^3-11\,Ba^3\sin(dx+c)^3-45\,Aa^3\sin(dx+c)}{}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/96*(6*(A*a^3 - B*a^3)*log(abs(sin(d*x + c) + 1)) - 6*(A*a^3 - B*a^3)*log(
abs(sin(d*x + c) - 1)) + (11*A*a^3*sin(d*x + c)^3 - 11*B*a^3*sin(d*x + c)^3
 - 45*A*a^3*sin(d*x + c)^2 + 45*B*a^3*sin(d*x + c)^2 + 69*A*a^3*sin(d*x + c
) - 69*B*a^3*sin(d*x + c) - 51*A*a^3 + 19*B*a^3)/(sin(d*x + c) - 1)^3)/d
```

3.994 $\int \sec^9(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=162

$$\frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4}+\frac{a^5(3A-B)}{32d(a-a\sin(c+dx))^2}+\frac{a^4(2A-B)}{16d(a-a\sin(c+dx))}-\frac{a^4(A-B)}{32d(a\sin(c+dx)+a)}+\frac{a^3(5A-3B)\,\mathrm{ta}}{}$$

```
[Out] (a^3*(5*A - 3*B)*ArcTanh[Sin[c + d*x]])/(32*d) + (a^7*(A + B))/(16*d*(a - a
*Sin[c + d*x])^4) + (a^6*A)/(12*d*(a - a*Sin[c + d*x])^3) + (a^5*(3*A - B))
/(32*d*(a - a*Sin[c + d*x])^2) + (a^4*(2*A - B))/(16*d*(a - a*Sin[c + d*x])
) - (a^4*(A - B))/(32*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.186258, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4}+\frac{a^5(3A-B)}{32d(a-a\sin(c+dx))^2}+\frac{a^4(2A-B)}{16d(a-a\sin(c+dx))}-\frac{a^4(A-B)}{32d(a\sin(c+dx)+a)}+\frac{a^3(5A-3B)\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(5*A - 3*B)*ArcTanh[Sin[c + d*x]])/(32*d) + (a^7*(A + B))/(16*d*(a - a
*Sin[c + d*x])^4) + (a^6*A)/(12*d*(a - a*Sin[c + d*x])^3) + (a^5*(3*A - B))
/(32*d*(a - a*Sin[c + d*x])^2) + (a^4*(2*A - B))/(16*d*(a - a*Sin[c + d*x])
) - (a^4*(A - B))/(32*d*(a + a*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^9(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a^9 \operatorname{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^5(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^9 \operatorname{Subst}\left(\int \left(\frac{A+B}{4a^2(a-x)^5} + \frac{A}{4a^3(a-x)^4} + \frac{3A-B}{16a^4(a-x)^3} + \frac{2A-B}{16a^5(a-x)^2} + \cdots\right.\right.}{d}$$

$$= \frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4} + \frac{a^6A}{12d(a-a\sin(c+dx))^3} + \frac{a^5\cdots}{32d(a-\cdots}$$

$$= \frac{a^3(5A-3B)\tanh^{-1}(\sin(c+dx))}{32d} + \frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4} \cdots$$

Mathematica [A] time = 0.631038, size = 151, normalized size = 0.93

$$\frac{a^9\left(\frac{2A-B}{16a^5(a-a\sin(c+dx))} - \frac{A-B}{32a^5(a\sin(c+dx)+a)} + \frac{3A-B}{32a^4(a-a\sin(c+dx))^2} + \frac{A+B}{16a^2(a-a\sin(c+dx))^4} + \frac{(5A-3B)\tanh^{-1}(\sin(c+dx))}{32a^6} + \frac{A}{12a^3(a-a\sin(c+dx))^3}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^9*(((5*A - 3*B)*ArcTanh[Sin[c + d*x]])/(32*a^6) + (A + B)/(16*a^2*(a - a
*Sin[c + d*x])^4) + A/(12*a^3*(a - a*Sin[c + d*x])^3) + (3*A - B)/(32*a^4*(
a - a*Sin[c + d*x])^2) + (2*A - B)/(16*a^5*(a - a*Sin[c + d*x])) - (A - B)/
(32*a^5*(a + a*Sin[c + d*x]))))/d
```

Maple [B] time = 0.223, size = 669, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 3/8/d*a^3*A/cos(d*x+c)^8+1/8/d*B*a^3/cos(d*x+c)^8+1/16/d*B*a^3*sin(d*x+c)^5
/cos(d*x+c)^6+5/16/d*a^3*A*sin(d*x+c)^3/cos(d*x+c)^6+1/4/d*B*a^3*sin(d*x+c)
^4/cos(d*x+c)^6+1/12/d*a^3*A*sin(d*x+c)^4/cos(d*x+c)^6+15/128/d*a^3*A*sin(d
*x+c)^3/cos(d*x+c)^2+15/128/d*B*a^3*sin(d*x+c)^3/cos(d*x+c)^2+35/128/d*a^3*
A*sec(d*x+c)*tan(d*x+c)+3/8/d*B*a^3*sin(d*x+c)^4/cos(d*x+c)^8+3/8/d*B*a^3*s
in(d*x+c)^3/cos(d*x+c)^8+1/8/d*a^3*A*tan(d*x+c)*sec(d*x+c)^7+1/8/d*a^3*A*si
n(d*x+c)^4/cos(d*x+c)^8+1/8/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^8+3/8/d*a^3*A*s
in(d*x+c)^3/cos(d*x+c)^8+5/16/d*B*a^3*sin(d*x+c)^3/cos(d*x+c)^6+7/48/d*a^3*
A*tan(d*x+c)*sec(d*x+c)^5-1/128/d*B*a^3*sin(d*x+c)^3-3/32/d*B*a^3*ln(sec(d*
x+c)+tan(d*x+c))+15/128/d*a^3*A*sin(d*x+c)+5/32/d*a^3*A*ln(sec(d*x+c)+tan(d
*x+c))+1/24/d*a^3*A*sin(d*x+c)^4/cos(d*x+c)^4-1/128/d*B*a^3*sin(d*x+c)^5/co
s(d*x+c)^2+1/64/d*B*a^3*sin(d*x+c)^5/cos(d*x+c)^4+15/64/d*a^3*A*sin(d*x+c)^
3/cos(d*x+c)^4+1/8/d*B*a^3*sin(d*x+c)^4/cos(d*x+c)^4+15/64/d*B*a^3*sin(d*x+
c)^3/cos(d*x+c)^4+35/192/d*a^3*A*tan(d*x+c)*sec(d*x+c)^3+3/32*a^3*B*sin(d*x
+c)/d
```

Maxima [A] time = 1.06497, size = 250, normalized size = 1.54

$$\frac{3\,(5\,A-3\,B)a^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A-3\,B)a^3\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(5\,A-3\,B)a^3\sin(dx+c)^4-9\,(5\,A-3\,B)a^3\sin(dx+c)\right.}{\sin(dx+c)^5-3\,\sin(dx+c)^4+2\,\mathrm{si}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/192*(3*(5*A - 3*B)*a^3*log(sin(d*x + c) + 1) - 3*(5*A - 3*B)*a^3*log(sin(
d*x + c) - 1) - 2*(3*(5*A - 3*B)*a^3*sin(d*x + c)^4 - 9*(5*A - 3*B)*a^3*sin
(d*x + c)^3 + 7*(5*A - 3*B)*a^3*sin(d*x + c)^2 + 3*(5*A - 3*B)*a^3*sin(d*x
+ c) - 32*A*a^3)/(sin(d*x + c)^5 - 3*sin(d*x + c)^4 + 2*sin(d*x + c)^3 + 2*
sin(d*x + c)^2 - 3*sin(d*x + c) + 1))/d
```

Fricas [B] time = 1.86546, size = 846, normalized size = 5.22

$$\frac{6\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^4-26\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^2+12\,(3\,A-5\,B)a^3+3\left(3\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^4-4\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/192*(6*(5*A - 3*B)*a^3*cos(d*x + c)^4 - 26*(5*A - 3*B)*a^3*cos(d*x + c)^2
 + 12*(3*A - 5*B)*a^3 + 3*(3*(5*A - 3*B)*a^3*cos(d*x + c)^4 - 4*(5*A - 3*B)
*a^3*cos(d*x + c)^2 - ((5*A - 3*B)*a^3*cos(d*x + c)^4 - 4*(5*A - 3*B)*a^3*c
os(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 3*(3*(5*A - 3*B)*a^3*c
os(d*x + c)^4 - 4*(5*A - 3*B)*a^3*cos(d*x + c)^2 - ((5*A - 3*B)*a^3*cos(d*x
 + c)^4 - 4*(5*A - 3*B)*a^3*cos(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c)
 + 1) + 6*(3*(5*A - 3*B)*a^3*cos(d*x + c)^2 - 2*(5*A - 3*B)*a^3)*sin(d*x +
c))/(3*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 - (d*cos(d*x + c)^4 - 4*d*cos(
d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.50438, size = 320, normalized size = 1.98

$$\frac{12\left(5\,Aa^3-3\,Ba^3\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-12\left(5\,Aa^3-3\,Ba^3\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{12\left(5\,Aa^3\sin(dx+c)-3\,Ba^3\sin(dx+c)+7\right.}{\sin(dx+c)+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/768*(12*(5*A*a^3 - 3*B*a^3)*log(abs(sin(d*x + c) + 1)) - 12*(5*A*a^3 - 3*
B*a^3)*log(abs(sin(d*x + c) - 1)) - 12*(5*A*a^3*sin(d*x + c) - 3*B*a^3*sin(
d*x + c) + 7*A*a^3 - 5*B*a^3)/(sin(d*x + c) + 1) + (125*A*a^3*sin(d*x + c)^
4 - 75*B*a^3*sin(d*x + c)^4 - 596*A*a^3*sin(d*x + c)^3 + 348*B*a^3*sin(d*x
+ c)^3 + 1110*A*a^3*sin(d*x + c)^2 - 618*B*a^3*sin(d*x + c)^2 - 996*A*a^3*s
in(d*x + c) + 492*B*a^3*sin(d*x + c) + 405*A*a^3 - 99*B*a^3)/(sin(d*x + c)
- 1)^4)/d
```

3.995 $\int \cos^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=231

$$-\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}-\frac{11(10A+3B)\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{720d}+\frac{11a^3(10A+3B)\sin(c+dx)\cos^5(}{480d}$$

```
[Out] (11*a^3*(10*A + 3*B)*x)/256 - (11*a^3*(10*A + 3*B)*Cos[c + d*x]^7)/(560*d)
+ (11*a^3*(10*A + 3*B)*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (11*a^3*(10*A +
 3*B)*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (11*a^3*(10*A + 3*B)*Cos[c + d
*x]^5*Sin[c + d*x])/(480*d) - (a*(10*A + 3*B)*Cos[c + d*x]^7*(a + a*Sin[c +
 d*x])^2)/(90*d) - (B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(10*d) - (11*(
10*A + 3*B)*Cos[c + d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(720*d)
```

Rubi [A] time = 0.268184, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}-\frac{11(10A+3B)\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{720d}+\frac{11a^3(10A+3B)\sin(c+dx)\cos^5(}{480d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (11*a^3*(10*A + 3*B)*x)/256 - (11*a^3*(10*A + 3*B)*Cos[c + d*x]^7)/(560*d)
+ (11*a^3*(10*A + 3*B)*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (11*a^3*(10*A +
 3*B)*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (11*a^3*(10*A + 3*B)*Cos[c + d
*x]^5*Sin[c + d*x])/(480*d) - (a*(10*A + 3*B)*Cos[c + d*x]^7*(a + a*Sin[c +
 d*x])^2)/(90*d) - (B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(10*d) - (11*(
10*A + 3*B)*Cos[c + d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(720*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))^3}{10d}+\frac{1}{10}(10A+3B)\int \cos^6$$
$$= -\frac{a(10A+3B)\cos^7(c+dx)(a+a\sin(c+dx))^2}{90d}-\frac{B\cos^7(c+}{}$$
$$= -\frac{a(10A+3B)\cos^7(c+dx)(a+a\sin(c+dx))^2}{90d}-\frac{B\cos^7(c+}{}$$
$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}-\frac{a(10A+3B)\cos^7(c+dx)(a}{90d}$$
$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}+\frac{11a^3(10A+3B)\cos^5(c+d}{480d}$$
$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}+\frac{11a^3(10A+3B)\cos^3(c+d}{384d}$$
$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}+\frac{11a^3(10A+3B)\cos(c+dx}{256d}$$
$$= \frac{11}{256}a^3(10A+3B)x-\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}+\frac{11a^3(1}{}$$

Mathematica [A] time = 6.05706, size = 344, normalized size = 1.49

$$-\frac{32\sqrt{2}a^2(10aA+3aB)\left(\frac{1}{2}(\sin(c+dx)-1)+1\right)^{13/2}\left(\frac{385\left(\frac{\sqrt{2}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}}{\sqrt{\frac{1}{2}(\sin(c+dx)-1)+1}}-\frac{2}{15}(1-\sin(c+dx))^3-\frac{1}{3}(1-\sin(c+dx))^2+\sin(c+dx)-}{8192\left(\frac{1}{2}(\sin(c+dx)-1)+1\right)^6(1-\sin(c+dx))^4}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(10*d) - (32*Sqrt[2]*a^2*(10*a*A
 + 3*a*B)*Cos[c + d*x]^7*(1 + (-1 + Sin[c + d*x])/2)^(13/2)*((7*(99/(2048*(
1 + (-1 + Sin[c + d*x])/2)^6) + 33/(256*(1 + (-1 + Sin[c + d*x])/2)^5) + 33
/(128*(1 + (-1 + Sin[c + d*x])/2)^4) + 99/(224*(1 + (-1 + Sin[c + d*x])/2)^
3) + 11/(16*(1 + (-1 + Sin[c + d*x])/2)^2) + (1 + (-1 + Sin[c + d*x])/2)^(-
1)))/18 + (385*(-1 + (Sqrt[2]*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1
 - Sin[c + d*x]])/Sqrt[1 + (-1 + Sin[c + d*x])/2] - (1 - Sin[c + d*x])^2/3
- (2*(1 - Sin[c + d*x])^3)/15 + Sin[c + d*x]))/(8192*(1 + (-1 + Sin[c + d*x
])/2)^6*(1 - Sin[c + d*x])^4)))/(35*d*(1 + Sin[c + d*x])^(7/2))
```

Maple [A] time = 0.072, size = 363, normalized size = 1.6

$$\frac{1}{d}\left(a^3A\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^7}{9}-\frac{2(\cos(dx+c))^7}{63}\right)+Ba^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^7}{10}-\frac{3\sin(dx+c)(}{8(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+B*a^3*(-1/10*
sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+
5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+3*a^3*A*(-1
/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x
+c))*sin(d*x+c)+5/128*d*x+5/128*c)+3*B*a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-
2/63*cos(d*x+c)^7)-3/7*a^3*A*cos(d*x+c)^7+3*B*a^3*(-1/8*sin(d*x+c)*cos(d*x+
c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*
d*x+5/128*c)+a^3*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin
(d*x+c)+5/16*d*x+5/16*c)-1/7*B*a^3*cos(d*x+c)^7)
```

Maxima [A] time = 1.08704, size = 383, normalized size = 1.66

$$-\frac{276480\,Aa^3\cos(dx+c)^7+92160\,Ba^3\cos(dx+c)^7-10240\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)Aa^3-630\left(64\sin(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/645120*(276480*A*a^3*cos(d*x + c)^7 + 92160*B*a^3*cos(d*x + c)^7 - 10240
*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*A*a^3 - 630*(64*sin(2*d*x + 2*c)^3 +
 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*A*a^3 + 3360*(
4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
2*c))*A*a^3 - 30720*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*B*a^3 - 63*(32*si
n(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*
c))*B*a^3 - 630*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*
c) - 24*sin(4*d*x + 4*c))*B*a^3)/d
```

Fricas [A] time = 2.1073, size = 410, normalized size = 1.77

$$\frac{8960\,(A+3\,B)a^3\cos(dx+c)^9-46080\,(A+B)a^3\cos(dx+c)^7+3465\,(10\,A+3\,B)a^3dx+21\left(384\,Ba^3\cos(dx+c)^9-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/80640*(8960*(A + 3*B)*a^3*cos(d*x + c)^9 - 46080*(A + B)*a^3*cos(d*x + c)
^7 + 3465*(10*A + 3*B)*a^3*d*x + 21*(384*B*a^3*cos(d*x + c)^9 - 48*(30*A +
41*B)*a^3*cos(d*x + c)^7 + 88*(10*A + 3*B)*a^3*cos(d*x + c)^5 + 110*(10*A +
 3*B)*a^3*cos(d*x + c)^3 + 165*(10*A + 3*B)*a^3*cos(d*x + c))*sin(d*x + c))
```

/d

Sympy [A] time = 46.5322, size = 1042, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((15*A*a**3*x*sin(c + d*x)**8/128 + 15*A*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**2/32 + 5*A*a**3*x*sin(c + d*x)**6/16 + 45*A*a**3*x*sin(c + d*x)
**4*cos(c + d*x)**4/64 + 15*A*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 1
5*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 15*A*a**3*x*sin(c + d*x)**2
*cos(c + d*x)**4/16 + 15*A*a**3*x*cos(c + d*x)**8/128 + 5*A*a**3*x*cos(c +
d*x)**6/16 + 15*A*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*A*a**3*sin
(c + d*x)**5*cos(c + d*x)**3/(128*d) + 5*A*a**3*sin(c + d*x)**5*cos(c + d*x
)/(16*d) + 73*A*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + 5*A*a**3*sin
(c + d*x)**3*cos(c + d*x)**3/(6*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)**7
/(7*d) - 15*A*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) + 11*A*a**3*sin(c +
 d*x)*cos(c + d*x)**5/(16*d) - 2*A*a**3*cos(c + d*x)**9/(63*d) - 3*A*a**3*c
os(c + d*x)**7/(7*d) + 3*B*a**3*x*sin(c + d*x)**10/256 + 15*B*a**3*x*sin(c
+ d*x)**8*cos(c + d*x)**2/256 + 15*B*a**3*x*sin(c + d*x)**8/128 + 15*B*a**3
*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*B*a**3*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 15*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 45*B*a**3*
x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*B*a**3*x*sin(c + d*x)**2*cos(c +
d*x)**8/256 + 15*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*B*a**3*x*c
os(c + d*x)**10/256 + 15*B*a**3*x*cos(c + d*x)**8/128 + 3*B*a**3*sin(c + d*
x)**9*cos(c + d*x)/(256*d) + 7*B*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*
d) + 15*B*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + B*a**3*sin(c + d*x)**
5*cos(c + d*x)**5/(10*d) + 55*B*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d
) - 7*B*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) + 73*B*a**3*sin(c + d*
x)**3*cos(c + d*x)**5/(128*d) - 3*B*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7
*d) - 3*B*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 15*B*a**3*sin(c + d*x
)*cos(c + d*x)**7/(128*d) - 2*B*a**3*cos(c + d*x)**9/(21*d) - B*a**3*cos(c
+ d*x)**7/(7*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**6,
True))
```

Giac [A] time = 1.4006, size = 369, normalized size = 1.6

$$\frac{Ba^3 \sin(10\,dx + 10\,c)}{5120\,d} + \frac{11}{256}\left(10\,Aa^3 + 3\,Ba^3\right)x + \frac{\left(Aa^3 + 3\,Ba^3\right)\cos(9\,dx + 9\,c)}{2304\,d} - \frac{\left(9\,Aa^3 - 5\,Ba^3\right)\cos(7\,dx + 7\,c)}{1792\,d} - \frac{(3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/5120*B*a^3*sin(10*d*x + 10*c)/d + 11/256*(10*A*a^3 + 3*B*a^3)*x + 1/2304*
(A*a^3 + 3*B*a^3)*cos(9*d*x + 9*c)/d - 1/1792*(9*A*a^3 - 5*B*a^3)*cos(7*d*x
 + 7*c)/d - 1/64*(3*A*a^3 + B*a^3)*cos(5*d*x + 5*c)/d - 1/192*(29*A*a^3 + 1
5*B*a^3)*cos(3*d*x + 3*c)/d - 1/128*(33*A*a^3 + 19*B*a^3)*cos(d*x + c)/d -
1/2048*(6*A*a^3 + 5*B*a^3)*sin(8*d*x + 8*c)/d - 1/3072*(32*A*a^3 + 51*B*a^3
)*sin(6*d*x + 6*c)/d + 1/256*(6*A*a^3 - 7*B*a^3)*sin(4*d*x + 4*c)/d + 1/512
*(144*A*a^3 + 25*B*a^3)*sin(2*d*x + 2*c)/d
```

3.996 $\int \cos^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=200

$$-\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}-\frac{3(8A+3B)\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{3a^3(8A+3B)\sin(c+dx)\cos^3(c+dx)}{64d}\ldots$$

```
[Out] (9*a^3*(8*A + 3*B)*x)/128 - (3*a^3*(8*A + 3*B)*Cos[c + d*x]^5)/(80*d) + (9*
a^3*(8*A + 3*B)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (3*a^3*(8*A + 3*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(64*d) - (a*(8*A + 3*B)*Cos[c + d*x]^5*(a + a*Sin
[c + d*x])^2)/(56*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3)/(8*d) - (3
*(8*A + 3*B)*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(112*d)
```

Rubi [A] time = 0.238976, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}-\frac{3(8A+3B)\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{3a^3(8A+3B)\sin(c+dx)\cos^3(c+dx)}{64d}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (9*a^3*(8*A + 3*B)*x)/128 - (3*a^3*(8*A + 3*B)*Cos[c + d*x]^5)/(80*d) + (9*
a^3*(8*A + 3*B)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (3*a^3*(8*A + 3*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(64*d) - (a*(8*A + 3*B)*Cos[c + d*x]^5*(a + a*Sin
[c + d*x])^2)/(56*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3)/(8*d) - (3
*(8*A + 3*B)*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(112*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))^3}{8d}+\frac{1}{8}(8A+3B)\int \cos^4(c$$
$$= -\frac{a(8A+3B)\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{B\cos^5(c+d}{}$$
$$= -\frac{a(8A+3B)\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{B\cos^5(c+d}{}$$
$$= -\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}-\frac{a(8A+3B)\cos^5(c+dx)(a+}{56d}$$
$$= -\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}+\frac{3a^3(8A+3B)\cos^3(c+dx)\,\mathrm{si}}{64d}$$
$$= -\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}+\frac{9a^3(8A+3B)\cos(c+dx)\sin}{128d}$$
$$= \frac{9}{128}a^3(8A+3B)x-\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}+\frac{9a^3(8A+}{}$$

Mathematica [A] time = 2.18421, size = 183, normalized size = 0.92

$$-\frac{a^3\cos(c+dx)\left(16(373A+223B)\cos(2(c+dx))+32(41A+11B)\cos(4(c+dx))+\frac{2520(8A+3B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}-10640A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^3*Cos[c + d*x]*(4576*A + 2976*B + (2520*(8*A + 3*B)*ArcSin[Sqrt[1 - Sin
[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 16*(373*A + 223*B)*Cos[2*(c + d
*x)] + 32*(41*A + 11*B)*Cos[4*(c + d*x)] - 80*A*Cos[6*(c + d*x)] - 240*B*Co
s[6*(c + d*x)] - 10640*A*Sin[c + d*x] - 3045*B*Sin[c + d*x] + 1365*B*Sin[3*
(c + d*x)] + 560*A*Sin[5*(c + d*x)] + 595*B*Sin[5*(c + d*x)] - 35*B*Sin[7*(
c + d*x)]))/(17920*d)
```

Maple [A] time = 0.07, size = 323, normalized size = 1.6

$$\frac{1}{d}\left(a^3A\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7}-\frac{2(\cos(dx+c))^5}{35}\right)+Ba^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^5}{8}-\frac{\sin(dx+c)(\cos(}{16}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+B*a^3*(-1/8*s
in(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+3*a^3*A*(-1/6*sin(d*x+c)*cos(d*
x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+3*B*a
^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)-3/5*a^3*A*cos(d*x+c)^
5+3*B*a^3*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*
sin(d*x+c)+1/16*d*x+1/16*c)+a^3*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*
x+c)+3/8*d*x+3/8*c)-1/5*B*a^3*cos(d*x+c)^5)
```

Maxima [A] time = 1.10521, size = 313, normalized size = 1.56

$$-\frac{21504\,Aa^3\cos(dx+c)^5 + 7168\,Ba^3\cos(dx+c)^5 - 1024\left(5\cos(dx+c)^7 - 7\cos(dx+c)^5\right)Aa^3 - 560\left(4\sin(2\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/35840*(21504*A*a^3*cos(d*x + c)^5 + 7168*B*a^3*cos(d*x + c)^5 - 1024*(5*
cos(d*x + c)^7 - 7*cos(d*x + c)^5)*A*a^3 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d
*x + 12*c - 3*sin(4*d*x + 4*c))*A*a^3 - 1120*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 3072*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5
)*B*a^3 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*B
*a^3 - 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*B*a^3)/d
```

Fricas [A] time = 1.91957, size = 342, normalized size = 1.71

$$\frac{640\,(A+3\,B)a^3\cos(dx+c)^7 - 3584\,(A+B)a^3\cos(dx+c)^5 + 315\,(8\,A+3\,B)a^3dx + 35\left(16\,Ba^3\cos(dx+c)^7 - 8\,(8\,A\right.}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4480*(640*(A + 3*B)*a^3*cos(d*x + c)^7 - 3584*(A + B)*a^3*cos(d*x + c)^5
+ 315*(8*A + 3*B)*a^3*d*x + 35*(16*B*a^3*cos(d*x + c)^7 - 8*(8*A + 11*B)*a^
3*cos(d*x + c)^5 + 6*(8*A + 3*B)*a^3*cos(d*x + c)^3 + 9*(8*A + 3*B)*a^3*cos
(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 16.3624, size = 823, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**6/16 + 9*A*a**3*x*sin(c + d*x)**4*cos(c
 + d*x)**2/16 + 3*A*a**3*x*sin(c + d*x)**4/8 + 9*A*a**3*x*sin(c + d*x)**2*c
os(c + d*x)**4/16 + 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*A*a**3
*x*cos(c + d*x)**6/16 + 3*A*a**3*x*cos(c + d*x)**4/8 + 3*A*a**3*sin(c + d*x
)**5*cos(c + d*x)/(16*d) + A*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) + 3
*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A*a**3*sin(c + d*x)**2*cos(c +
 d*x)**5/(5*d) - 3*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*A*a**3*si
n(c + d*x)*cos(c + d*x)**3/(8*d) - 2*A*a**3*cos(c + d*x)**7/(35*d) - 3*A*a*
*3*cos(c + d*x)**5/(5*d) + 3*B*a**3*x*sin(c + d*x)**8/128 + 3*B*a**3*x*sin(
c + d*x)**6*cos(c + d*x)**2/32 + 3*B*a**3*x*sin(c + d*x)**6/16 + 9*B*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**4/64 + 9*B*a**3*x*sin(c + d*x)**4*cos(c + d*
x)**2/16 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 9*B*a**3*x*sin(c
 + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**3*x*cos(c + d*x)**8/128 + 3*B*a**3*x
*cos(c + d*x)**6/16 + 3*B*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 11*B*
a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) + 3*B*a**3*sin(c + d*x)**5*cos
(c + d*x)/(16*d) - 11*B*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + B*a*
*3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*B*a**3*sin(c + d*x)**2*cos(c +
 d*x)**5/(5*d) - 3*B*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 3*B*a**3*s
in(c + d*x)*cos(c + d*x)**5/(16*d) - 6*B*a**3*cos(c + d*x)**7/(35*d) - B*a*
*3*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*co
s(c)**4, True))
```

Giac [A] time = 1.33106, size = 293, normalized size = 1.46

$$\frac{Ba^3\sin\left(8\,dx+8\,c\right)}{1024\,d}+\frac{9}{128}\left(8\,Aa^3+3\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{\left(11\,Aa^3+Ba^3\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{\left(13\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1024*B*a^3*sin(8*d*x + 8*c)/d + 9/128*(8*A*a^3 + 3*B*a^3)*x + 1/448*(A*a^
3 + 3*B*a^3)*cos(7*d*x + 7*c)/d - 1/320*(11*A*a^3 + B*a^3)*cos(5*d*x + 5*c)
/d - 1/64*(13*A*a^3 + 7*B*a^3)*cos(3*d*x + 3*c)/d - 1/64*(27*A*a^3 + 17*B*a
^3)*cos(d*x + c)/d - 1/64*(A*a^3 + B*a^3)*sin(6*d*x + 6*c)/d - 1/128*(2*A*a
^3 + 7*B*a^3)*sin(4*d*x + 4*c)/d + 1/64*(19*A*a^3 + 3*B*a^3)*sin(2*d*x + 2*
c)/d
```

3.997 $\int \cos^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=159

$$-\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}-\frac{7(2A+B)\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{40d}+\frac{7a^3(2A+B)\sin(c+dx)\cos(c+dx)}{16d}+$$

```
[Out] (7*a^3*(2*A + B)*x)/16 - (7*a^3*(2*A + B)*Cos[c + d*x]^3)/(24*d) + (7*a^3*(
2*A + B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a*(2*A + B)*Cos[c + d*x]^3*(a
 + a*Sin[c + d*x])^2)/(10*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(6
*d) - (7*(2*A + B)*Cos[c + d*x]^3*(a^3 + a^3*Sin[c + d*x]))/(40*d)
```

Rubi [A] time = 0.21743, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2860, 2678, 2669, 2635, 8}

$$-\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}-\frac{7(2A+B)\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{40d}+\frac{7a^3(2A+B)\sin(c+dx)\cos(c+dx)}{16d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (7*a^3*(2*A + B)*x)/16 - (7*a^3*(2*A + B)*Cos[c + d*x]^3)/(24*d) + (7*a^3*(
2*A + B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a*(2*A + B)*Cos[c + d*x]^3*(a
 + a*Sin[c + d*x])^2)/(10*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(6
*d) - (7*(2*A + B)*Cos[c + d*x]^3*(a^3 + a^3*Sin[c + d*x]))/(40*d)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d}+\frac{1}{2}(2A+B)\int \cos^2(c+$$
$$= -\frac{a(2A+B)\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\frac{B\cos^3(c+dx}{}$$
$$= -\frac{a(2A+B)\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\frac{B\cos^3(c+dx}{}$$
$$= -\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}-\frac{a(2A+B)\cos^3(c+dx)(a+a\,\mathrm{s}}{10d}$$
$$= -\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}+\frac{7a^3(2A+B)\cos(c+dx)\sin(c}{16d}$$
$$= \frac{7}{16}a^3(2A+B)x-\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}+\frac{7a^3(2A+B)\,\mathrm{c}}{}$$

Mathematica [A] time = 1.36457, size = 146, normalized size = 0.92

$$-\frac{a^3\cos(c+dx)\left(16(17A+11B)\cos(2(c+dx))-12(A+3B)\cos(4(c+dx))+\frac{420(2A+B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}-330A\sin(c+d\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^3*Cos[c + d*x]*(284*A + 212*B + (420*(2*A + B)*ArcSin[Sqrt[1 - Sin[c +
d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 16*(17*A + 11*B)*Cos[2*(c + d*x)] -
12*(A + 3*B)*Cos[4*(c + d*x)] - 330*A*Sin[c + d*x] - 95*B*Sin[c + d*x] + 90
*A*Sin[3*(c + d*x)] + 110*B*Sin[3*(c + d*x)] - 5*B*Sin[5*(c + d*x)]))/(480*
d)
```

Maple [A] time = 0.069, size = 279, normalized size = 1.8

$$\frac{1}{d}\left(a^3A\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^3}{5}-\frac{2\,(\cos(dx+c))^3}{15}\right)+Ba^3\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^3}{6}-\frac{(\cos(dx+c))^3\sin}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+B*a^3*(-1/6*s
in(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*
x+c)+1/16*d*x+1/16*c)+3*a^3*A*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*
```

```
sin(d*x+c)+1/8*d*x+1/8*c)+3*B*a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(
d*x+c)^3)-a^3*A*cos(d*x+c)^3+3*B*a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(
d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+a^3*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1
/2*c)-1/3*B*a^3*cos(d*x+c)^3)
```

Maxima [A] time = 1.13797, size = 269, normalized size = 1.69

$$-\frac{960\,Aa^3\cos(dx+c)^3+320\,Ba^3\cos(dx+c)^3-64\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)Aa^3-90\,(4\,dx+4\,c-\sin(4\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/960*(960*A*a^3*cos(d*x + c)^3 + 320*B*a^3*cos(d*x + c)^3 - 64*(3*cos(d*x
 + c)^5 - 5*cos(d*x + c)^3)*A*a^3 - 90*(4*d*x + 4*c - sin(4*d*x + 4*c))*A*a
^3 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 - 192*(3*cos(d*x + c)^5 - 5
*cos(d*x + c)^3)*B*a^3 + 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*
d*x + 4*c))*B*a^3 - 90*(4*d*x + 4*c - sin(4*d*x + 4*c))*B*a^3)/d
```

Fricas [A] time = 1.83732, size = 285, normalized size = 1.79

$$\frac{48\,(A+3\,B)a^3\cos(dx+c)^5-320\,(A+B)a^3\cos(dx+c)^3+105\,(2\,A+B)a^3dx+5\left(8\,Ba^3\cos(dx+c)^5-2\,(18\,A+25\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(48*(A + 3*B)*a^3*cos(d*x + c)^5 - 320*(A + B)*a^3*cos(d*x + c)^3 + 1
05*(2*A + B)*a^3*d*x + 5*(8*B*a^3*cos(d*x + c)^5 - 2*(18*A + 25*B)*a^3*cos(
d*x + c)^3 + 21*(2*A + B)*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 5.75516, size = 588, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**4/8 + A
*a**3*x*cos(c + d*x)**2/2 + 3*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A
*a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 3*A*a**3*sin(c + d*x)*cos(c +
 d*x)**3/(8*d) + A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*A*a**3*cos(c +
d*x)**5/(15*d) - A*a**3*cos(c + d*x)**3/d + B*a**3*x*sin(c + d*x)**6/16 + 3
*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B*a**3*x*sin(c + d*x)**4/8
 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**3*x*sin(c + d*x)*
*2*cos(c + d*x)**2/4 + B*a**3*x*cos(c + d*x)**6/16 + 3*B*a**3*x*cos(c + d*x
```

```
)**4/8 + B*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - B*a**3*sin(c + d*x)**
3*cos(c + d*x)**3/(6*d) + 3*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - B*a
**3*sin(c + d*x)**2*cos(c + d*x)**3/d - B*a**3*sin(c + d*x)*cos(c + d*x)**5
/(16*d) - 3*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*B*a**3*cos(c + d*
x)**5/(5*d) - B*a**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a
*sin(c) + a)**3*cos(c)**2, True))
```

Giac [A] time = 1.3148, size = 223, normalized size = 1.4

$$\frac{Ba^3 \sin(6\,dx + 6\,c)}{192\,d} + \frac{7}{16}\left(2\,Aa^3 + Ba^3\right)x + \frac{\left(Aa^3 + 3\,Ba^3\right)\cos(5\,dx + 5\,c)}{80\,d} - \frac{\left(13\,Aa^3 + 7\,Ba^3\right)\cos(3\,dx + 3\,c)}{48\,d} - \frac{\left(7\,Aa^3 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*a^3*sin(6*d*x + 6*c)/d + 7/16*(2*A*a^3 + B*a^3)*x + 1/80*(A*a^3 + 3
*B*a^3)*cos(5*d*x + 5*c)/d - 1/48*(13*A*a^3 + 7*B*a^3)*cos(3*d*x + 3*c)/d -
 1/8*(7*A*a^3 + 5*B*a^3)*cos(d*x + c)/d - 1/64*(6*A*a^3 + 7*B*a^3)*sin(4*d*
x + 4*c)/d + 1/64*(16*A*a^3 - B*a^3)*sin(2*d*x + 2*c)/d
```

3.998 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=91

$$\frac{2a^3(2A+3B)\cos(c+dx)}{d}+\frac{a^3(2A+3B)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3}{2}a^3x(2A+3B)+\frac{(A+B)\sec(c+dx)(a\sin(c+d}{d}$$

```
[Out] (-3*a^3*(2*A + 3*B)*x)/2 + (2*a^3*(2*A + 3*B)*Cos[c + d*x])/d + (a^3*(2*A +
 3*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + ((A + B)*Sec[c + d*x]*(a + a*Sin[c
 + d*x])^3)/d
```

Rubi [A] time = 0.104856, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2855, 2644}

$$\frac{2a^3(2A+3B)\cos(c+dx)}{d}+\frac{a^3(2A+3B)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3}{2}a^3x(2A+3B)+\frac{(A+B)\sec(c+dx)(a\sin(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-3*a^3*(2*A + 3*B)*x)/2 + (2*a^3*(2*A + 3*B)*Cos[c + d*x])/d + (a^3*(2*A +
 3*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + ((A + B)*Sec[c + d*x]*(a + a*Sin[c
 + d*x])^3)/d
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))^3}{d}-(a(2A+3B))\int \\ &= -\frac{3}{2}a^3(2A+3B)x+\frac{2a^3(2A+3B)\cos(c+dx)}{d}+\frac{a^3(2A+}{}\end{aligned}$$

Mathematica [C] time = 0.243196, size = 82, normalized size = 0.9

$$\frac{\sec(c+dx)\left(4\sqrt{2}a^3(2A+3B)\sqrt{\sin(c+dx)+1}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)-B(a\sin(c+dx)+a)^3\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(4*Sqrt[2]*a^3*(2*A + 3*B)*Hypergeometric2F1[-3/2, -1/2, 1/2,
 (1 - Sin[c + d*x])/2]*Sqrt[1 + Sin[c + d*x]] - B*(a + a*Sin[c + d*x])^3))/
(2*d)
```

Maple [B] time = 0.087, size = 219, normalized size = 2.4

$$\frac{1}{d}\left(a^3A\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+Ba^3\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)\mathrm{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+B*a^3*(sin
(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c
)+3*a^3*A*(tan(d*x+c)-d*x-c)+3*B*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)
^2)*cos(d*x+c))+3*a^3*A/cos(d*x+c)+3*B*a^3*(tan(d*x+c)-d*x-c)+a^3*A*tan(d*x
+c)+B*a^3/cos(d*x+c))
```

Maxima [A] time = 1.61656, size = 225, normalized size = 2.47

$$-\frac{6\,(dx+c-\tan(dx+c))Aa^3+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)Ba^3+6\,(dx+c-\tan(dx+c))Ba^3-2\,Aa^3\left(\cdots\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2*(6*(d*x + c - tan(d*x + c))*A*a^3 + (3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*B*a^3 + 6*(d*x + c - tan(d*x + c))*B*a^3 -
 2*A*a^3*(1/cos(d*x + c) + cos(d*x + c)) - 6*B*a^3*(1/cos(d*x + c) + cos(d*
x + c)) - 2*A*a^3*tan(d*x + c) - 6*A*a^3/cos(d*x + c) - 2*B*a^3/cos(d*x + c
))/d
```

Fricas [A] time = 1.68607, size = 414, normalized size = 4.55

$$\frac{Ba^3\cos(dx+c)^3-3\,(2\,A+3\,B)a^3dx+2\,(A+3\,B)a^3\cos(dx+c)^2+8\,(A+B)a^3-\left(3\,(2\,A+3\,B)a^3dx-(10\,A+13\,B)a^3\cdots\right.}{2\,(d\cos(dx+c)-d\sin\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*a^3*cos(d*x + c)^3 - 3*(2*A + 3*B)*a^3*d*x + 2*(A + 3*B)*a^3*cos(d*x
 + c)^2 + 8*(A + B)*a^3 - (3*(2*A + 3*B)*a^3*d*x - (10*A + 13*B)*a^3)*cos(d
```

```
*x + c) + (3*(2*A + 3*B)*a^3*d*x + B*a^3*cos(d*x + c)^2 - (2*A + 5*B)*a^3*c
os(d*x + c) + 8*(A + B)*a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c)
 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30437, size = 198, normalized size = 2.18

$$-\frac{3\left(2\,Aa^{3}+3\,Ba^{3}\right)(dx+c)+\frac{16\left(Aa^{3}+Ba^{3}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-6\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*(3*(2*A*a^3 + 3*B*a^3)*(d*x + c) + 16*(A*a^3 + B*a^3)/(tan(1/2*d*x + 1
/2*c) - 1) + 2*(B*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*tan(1/2*d*x + 1/2*c)
^2 - 6*B*a^3*tan(1/2*d*x + 1/2*c)^2 - B*a^3*tan(1/2*d*x + 1/2*c) - 2*A*a^3
- 6*B*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.999 $\int \sec^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=69

$$-\frac{2a^5B\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+a^3Bx+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

```
[Out] a^3*B*x + ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d) - (2*a^5*B*
Cos[c + d*x])/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.154012, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2855, 2670, 2680, 8}

$$-\frac{2a^5B\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+a^3Bx+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] a^3*B*x + ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d) - (2*a^5*B*
Cos[c + d*x])/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - (aB)\int \sec^2(c$$
$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \left(a^5B\right)\int \frac{\mathrm{co}}{(a-a}$$
$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{2a^5B\cos(c}{d\left(a^2-a^2\sin(\right.}$$
$$= a^3Bx + \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{2a^5}{d\left(a^2-\right.}$$

Mathematica [A] time = 1.05802, size = 121, normalized size = 1.75

$$-\frac{a^3\left(-3\cos\left(\frac{1}{2}(c+dx)\right)(2A+3B(c+dx+2))+\cos\left(\frac{3}{2}(c+dx)\right)(2A+B(3c+3dx+14))+6B\sin\left(\frac{1}{2}(c+dx)\right)(2(c+\right.}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^3*(-3*(2*A + 3*B*(2 + c + d*x))*Cos[(c + d*x)/2] + (2*A + B*(14 + 3*c +
 3*d*x))*Cos[(3*(c + d*x))/2] + 6*B*(2*(2 + c + d*x) + (c + d*x)*Cos[c + d*
x])*Sin[(c + d*x)/2]))/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
```

Maple [B] time = 0.126, size = 248, normalized size = 3.6

$$\frac{1}{d}\left(a^3A\left(\frac{(\sin(dx+c))^4}{3(\cos(dx+c))^3} - \frac{(\sin(dx+c))^4}{3\cos(dx+c)} - \frac{\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}\right) + Ba^3\left(\frac{(\tan(dx+c))^3}{3} - \tan(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(
2+sin(d*x+c)^2)*cos(d*x+c))+B*a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+a^3*A
*sin(d*x+c)^3/cos(d*x+c)^3+3*B*a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d
*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+a^3*A/cos(d*x+c)^3+B*a^
3*sin(d*x+c)^3/cos(d*x+c)^3-a^3*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*B*
a^3/cos(d*x+c)^3)
```

Maxima [B] time = 1.65021, size = 221, normalized size = 3.2

$$\frac{3\,Aa^3\tan(dx+c)^3 + 3\,Ba^3\tan(dx+c)^3 + \left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)Aa^3 + \left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/3*(3*A*a^3*tan(d*x + c)^3 + 3*B*a^3*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*
tan(d*x + c))*A*a^3 + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*B*a^3
 - (3*cos(d*x + c)^2 - 1)*A*a^3/cos(d*x + c)^3 - 3*(3*cos(d*x + c)^2 - 1)*B
*a^3/cos(d*x + c)^3 + 3*A*a^3/cos(d*x + c)^3 + B*a^3/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.60412, size = 402, normalized size = 5.83

$$-\frac{6\,Ba^3dx + 2\,(A+B)a^3 - \left(3\,Ba^3dx + (A+7\,B)a^3\right)\cos\left(dx+c\right)^2 + \left(3\,Ba^3dx + (A-5\,B)a^3\right)\cos\left(dx+c\right) - \left(6\,Ba^3dx - 2\,(A\right.}{3\left(d\cos\left(dx+c\right)^2 - d\cos\left(dx+c\right) + \left(d\cos\left(dx+c\right) + 2\,d\right)\sin\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/3*(6*B*a^3*d*x + 2*(A + B)*a^3 - (3*B*a^3*d*x + (A + 7*B)*a^3)*cos(d*x +
 c)^2 + (3*B*a^3*d*x + (A - 5*B)*a^3)*cos(d*x + c) - (6*B*a^3*d*x - 2*(A +
B)*a^3 + (3*B*a^3*d*x - (A + 7*B)*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3302, size = 126, normalized size = 1.83

$$\frac{3\,(dx+c)Ba^3 - \frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 3\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 12\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + Aa^3 - 5\,Ba^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(d*x + c)*B*a^3 - 2*(3*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*B*a^3*tan(1/
2*d*x + 1/2*c)^2 + 12*B*a^3*tan(1/2*d*x + 1/2*c) + A*a^3 - 5*B*a^3)/(tan(1/
2*d*x + 1/2*c) - 1)^3)/d
```

3.1000 $\int \sec^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=107

$$\frac{a^5(2A-3B)\cos(c+dx)}{15d\left(a^2-a^2\sin(c+dx)\right)}+\frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)^3}{5d}$$

```
[Out] (a^5*(2*A - 3*B)*Cos[c + d*x])/(15*d*(a - a*Sin[c + d*x])^2) + ((A + B)*Sec
[c + d*x]^5*(a + a*Sin[c + d*x])^3)/(5*d) + (a^5*(2*A - 3*B)*Cos[c + d*x])/
(15*d*(a^2 - a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.153776, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2855, 2670, 2650, 2648}

$$\frac{a^5(2A-3B)\cos(c+dx)}{15d\left(a^2-a^2\sin(c+dx)\right)}+\frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)^3}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^5*(2*A - 3*B)*Cos[c + d*x])/(15*d*(a - a*Sin[c + d*x])^2) + ((A + B)*Sec
[c + d*x]^5*(a + a*Sin[c + d*x])^3)/(5*d) + (a^5*(2*A - 3*B)*Cos[c + d*x])/
(15*d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^3}{5d} + \frac{1}{5}(a(2A-3B)) \int$$
$$= \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^3}{5d} + \frac{1}{5}\left(a^5(2A-3B)\right)$$
$$= \frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2} + \frac{(A+B)\sec^5(c+dx)(a+a\sin(c}{5d}$$
$$= \frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2} + \frac{a^4(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))} + \frac{(A+}{}$$

Mathematica [A] time = 0.164378, size = 94, normalized size = 0.88

$$-\frac{a^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(6(2A-3B)\sin(c+dx)+(2A-3B)\cos(2(c+dx))-16A+9B)}{30d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-16*A + 9*B + (2*A - 3*B)*Cos[
2*(c + d*x)] + 6*(2*A - 3*B)*Sin[c + d*x]))/(30*d*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^5)
```

Maple [B] time = 0.119, size = 333, normalized size = 3.1

$$\frac{1}{d}\left(a^3A\left(\frac{(\sin(dx+c))^4}{5(\cos(dx+c))^5}+\frac{(\sin(dx+c))^4}{15(\cos(dx+c))^3}-\frac{(\sin(dx+c))^4}{15\cos(dx+c)}-\frac{\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{15}\right)+\frac{Ba^3(\sin(dx+c}{5(\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/
15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+1/5*B*a^3*sin(
d*x+c)^5/cos(d*x+c)^5+3*a^3*A*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c
)^3/cos(d*x+c)^3)+3*B*a^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/
cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))
+3/5*a^3*A/cos(d*x+c)^5+3*B*a^3*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x
+c)^3/cos(d*x+c)^3)-a^3*A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*
x+c)+1/5*B*a^3/cos(d*x+c)^5)
```

Maxima [A] time = 1.06842, size = 254, normalized size = 2.37

$$\frac{3\,Ba^3\tan(dx+c)^5+\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^3+3\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)A}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/15*(3*B*a^3*tan(d*x + c)^5 + (3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*t
an(d*x + c))*A*a^3 + 3*(3*tan(d*x + c)^5 + 5*tan(d*x + c)^3)*A*a^3 + 3*(3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*B*a^3 - (5*cos(d*x + c)^2 - 3)*A*a^3/cos(
d*x + c)^5 - 3*(5*cos(d*x + c)^2 - 3)*B*a^3/cos(d*x + c)^5 + 9*A*a^3/cos(d*
x + c)^5 + 3*B*a^3/cos(d*x + c)^5)/d
```

Fricas [A] time = 1.65749, size = 464, normalized size = 4.34

$$\frac{(2A-3B)a^3\cos(dx+c)^3-2(2A-3B)a^3\cos(dx+c)^2-3(3A-2B)a^3\cos(dx+c)-3(A+B)a^3+\left((2A-3B)a\right.}{15\left(d\cos(dx+c)^3+3d\cos(dx+c)^2-2d\cos(dx+c)-\left(d\cos(dx+c)^2-2d\,\mathrm{co}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/15*((2*A - 3*B)*a^3*cos(d*x + c)^3 - 2*(2*A - 3*B)*a^3*cos(d*x + c)^2 - 3
*(3*A - 2*B)*a^3*cos(d*x + c) - 3*(A + B)*a^3 + ((2*A - 3*B)*a^3*cos(d*x +
c)^2 + 3*(2*A - 3*B)*a^3*cos(d*x + c) - 3*(A + B)*a^3)*sin(d*x + c))/(d*cos
(d*x + c)^3 + 3*d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2 - 2
*d*cos(d*x + c) - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37364, size = 197, normalized size = 1.84

$$-\frac{2\left(15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,B\right.}{15\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -2/15*(15*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 30*A*a^3*tan(1/2*d*x + 1/2*c)^3 +
15*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^
3*tan(1/2*d*x + 1/2*c)^2 - 20*A*a^3*tan(1/2*d*x + 1/2*c) + 15*B*a^3*tan(1/2
*d*x + 1/2*c) + 7*A*a^3 - 3*B*a^3)/(d*(tan(1/2*d*x + 1/2*c) - 1)^5)
```

3.1001 $\int \sec^8(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$\frac{a^3(4A-3B)\tan^3(c+dx)}{35d}+\frac{3a^3(4A-3B)\tan(c+dx)}{35d}+\frac{2(4A-3B)\sec^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{35d}+\frac{(A+B)\sec^7(c$$

```
[Out] ((A + B)*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(7*d) + (2*(4*A - 3*B)*Sec[
c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(35*d) + (3*a^3*(4*A - 3*B)*Tan[c + d*
x])/(35*d) + (a^3*(4*A - 3*B)*Tan[c + d*x]^3)/(35*d)
```

Rubi [A] time = 0.145609, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2676, 3767}

$$\frac{a^3(4A-3B)\tan^3(c+dx)}{35d}+\frac{3a^3(4A-3B)\tan(c+dx)}{35d}+\frac{2(4A-3B)\sec^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{35d}+\frac{(A+B)\sec^7(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```


```
[Out] ((A + B)*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(7*d) + (2*(4*A - 3*B)*Sec[
c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(35*d) + (3*a^3*(4*A - 3*B)*Tan[c + d*
x])/(35*d) + (a^3*(4*A - 3*B)*Tan[c + d*x]^3)/(35*d)
```


Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d} + \frac{1}{7}(a(4A-3B)$$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d} + \frac{2(4A-3B)\text{se}}{}$$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d} + \frac{2(4A-3B)\text{se}}{}$$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d} + \frac{2(4A-3B)\text{se}}{}$$

Mathematica [A] time = 0.456818, size = 135, normalized size = 1.17

$$\frac{a^3(14(4A-3B)\cos(2(c+dx))+(3B-4A)\cos(4(c+dx))+56A\sin(c+dx)-24A\sin(3(c+dx))-42B\sin(c+dx)}{140d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(35*B + 14*(4*A - 3*B)*Cos[2*(c + d*x)] + (-4*A + 3*B)*Cos[4*(c + d*x)
] + 56*A*Sin[c + d*x] - 42*B*Sin[c + d*x] - 24*A*Sin[3*(c + d*x)] + 18*B*Si
n[3*(c + d*x)]))/(140*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]))
```

Maple [B] time = 0.123, size = 435, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/
35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c
)^2)*cos(d*x+c))+B*a^3*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos
(d*x+c)^5)+3*a^3*A*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x
+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+3*B*a^3*(1/7*sin(d*x+c)^4/cos(d*x+c)
^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d
*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c))+3/7*a^3*A/cos(d*x+c)^7
+3*B*a^3*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/10
5*sin(d*x+c)^3/cos(d*x+c)^3)-a^3*A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)
^4-8/35*sec(d*x+c)^2)*tan(d*x+c)+1/7*B*a^3/cos(d*x+c)^7)
```

Maxima [B] time = 1.05954, size = 308, normalized size = 2.68

$$\frac{\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan(dx+c)^3\right)Aa^3+\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/35*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*A*a^3 + (
5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))
*A*a^3 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*B*a^3
+ (5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*B*a^3 - (7*cos(d*x + c)^2 - 5)*A*a^
3/cos(d*x + c)^7 - 3*(7*cos(d*x + c)^2 - 5)*B*a^3/cos(d*x + c)^7 + 15*A*a^3
/cos(d*x + c)^7 + 5*B*a^3/cos(d*x + c)^7)/d
```

Fricas [A] time = 1.66649, size = 350, normalized size = 3.04

$$\frac{2\,(4\,A-3\,B)a^3\cos{(dx+c)}^4-9\,(4\,A-3\,B)a^3\cos{(dx+c)}^2+5\,(3\,A-4\,B)a^3+\left(6\,(4\,A-3\,B)a^3\cos{(dx+c)}^2-5\,(4\,A-3\right.}{35\left(3\,d\cos{(dx+c)}^3-4\,d\cos{(dx+c)}-\left(d\cos{(dx+c)}^3-4\,d\cos{(dx+c)}\right)\sin{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/35*(2*(4*A - 3*B)*a^3*cos(d*x + c)^4 - 9*(4*A - 3*B)*a^3*cos(d*x + c)^2 +
 5*(3*A - 4*B)*a^3 + (6*(4*A - 3*B)*a^3*cos(d*x + c)^2 - 5*(4*A - 3*B)*a^3)
*sin(d*x + c))/(3*d*cos(d*x + c)^3 - 4*d*cos(d*x + c) - (d*cos(d*x + c)^3 -
 4*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36736, size = 351, normalized size = 3.05

$$-\frac{\frac{35\left(Aa^3-Ba^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{525\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+35\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+280\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4025\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-665}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/280*(35*(A*a^3 - B*a^3)/(tan(1/2*d*x + 1/2*c) + 1) + (525*A*a^3*tan(1/2*
d*x + 1/2*c)^6 + 35*B*a^3*tan(1/2*d*x + 1/2*c)^6 - 1960*A*a^3*tan(1/2*d*x +
 1/2*c)^5 + 280*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 4025*A*a^3*tan(1/2*d*x + 1/2
*c)^4 - 665*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 4480*A*a^3*tan(1/2*d*x + 1/2*c)^
```

```
3 + 1120*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 3143*A*a^3*tan(1/2*d*x + 1/2*c)^2 -
 791*B*a^3*tan(1/2*d*x + 1/2*c)^2 - 1176*A*a^3*tan(1/2*d*x + 1/2*c) + 392*B
*a^3*tan(1/2*d*x + 1/2*c) + 243*A*a^3 - 51*B*a^3)/(tan(1/2*d*x + 1/2*c) - 1
)^7)/d
```

3.1002 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{a^3(2A-B)\tan^5(c+dx)}{21d}+\frac{10a^3(2A-B)\tan^3(c+dx)}{63d}+\frac{5a^3(2A-B)\tan(c+dx)}{21d}+\frac{2(2A-B)\sec^7(c+dx)\left(a^3\sin(c+\right.}{21d}$$

```
[Out] ((A + B)*Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3)/(9*d) + (2*(2*A - B)*Sec[c
+ d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(21*d) + (5*a^3*(2*A - B)*Tan[c + d*x])/
(21*d) + (10*a^3*(2*A - B)*Tan[c + d*x]^3)/(63*d) + (a^3*(2*A - B)*Tan[c +
d*x]^5)/(21*d)
```

Rubi [A] time = 0.151401, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2855, 2676, 3767}

$$\frac{a^3(2A-B)\tan^5(c+dx)}{21d}+\frac{10a^3(2A-B)\tan^3(c+dx)}{63d}+\frac{5a^3(2A-B)\tan(c+dx)}{21d}+\frac{2(2A-B)\sec^7(c+dx)\left(a^3\sin(c+\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3)/(9*d) + (2*(2*A - B)*Sec[c
+ d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(21*d) + (5*a^3*(2*A - B)*Tan[c + d*x])/
(21*d) + (10*a^3*(2*A - B)*Tan[c + d*x]^3)/(63*d) + (a^3*(2*A - B)*Tan[c +
d*x]^5)/(21*d)
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d} + \frac{1}{3}(a(2A-B))$$
$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d} + \frac{2(2A-B)\sec}{}$$
$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d} + \frac{2(2A-B)\sec}{}$$
$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d} + \frac{2(2A-B)\sec}{}$$

Mathematica [A] time = 0.58123, size = 176, normalized size = 1.26

$$-\frac{a^3(27(B-2A)\cos(2(c+dx))+12(B-2A)\cos(4(c+dx))-72A\sin(c+dx)-4A\sin(3(c+dx))+12A\sin(5(c+d}{252d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^3*(-42*B + 27*(-2*A + B)*Cos[2*(c + d*x)] + 12*(-2*A + B)*Cos[4*(c + d*
x)] + 2*A*Cos[6*(c + d*x)] - B*Cos[6*(c + d*x)] - 72*A*Sin[c + d*x] + 36*B*
Sin[c + d*x] - 4*A*Sin[3*(c + d*x)] + 2*B*Sin[3*(c + d*x)] + 12*A*Sin[5*(c
+ d*x)] - 6*B*Sin[5*(c + d*x)]))/(252*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [B] time = 0.138, size = 535, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(a^3*A*(1/9*sin(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/
21*sin(d*x+c)^4/cos(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)
^4/cos(d*x+c)-1/63*(2+sin(d*x+c)^2)*cos(d*x+c))+B*a^3*(1/9*sin(d*x+c)^5/cos
(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)+3
*a^3*A*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*
sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3*B*a^3*(1/9*si
n(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/co
s(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63
*(2+sin(d*x+c)^2)*cos(d*x+c))+1/3*a^3*A/cos(d*x+c)^9+3*B*a^3*(1/9*sin(d*x+c
)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+
c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)-a^3*A*(-128/315-1/9*sec(d*x+c)^8-8/6
3*sec(d*x+c)^6-16/105*sec(d*x+c)^4-64/315*sec(d*x+c)^2)*tan(d*x+c)+1/9*B*a^
3/cos(d*x+c)^9)
```

Maxima [B] time = 1.0801, size = 365, normalized size = 2.61

$$\frac{\left(35\tan(dx+c)^9+180\tan(dx+c)^7+378\tan(dx+c)^5+420\tan(dx+c)^3+315\tan(dx+c)\right)Aa^3+3\left(35\tan(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*A*a^3 + 3*(35*tan(d*x + c)^9 + 135*tan(d*
x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*A*a^3 + 3*(35*tan(d*x +
 c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*B*a^3
 + (35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*B*a^3 - 5*(9
*cos(d*x + c)^2 - 7)*A*a^3/cos(d*x + c)^9 - 15*(9*cos(d*x + c)^2 - 7)*B*a^3
/cos(d*x + c)^9 + 105*A*a^3/cos(d*x + c)^9 + 35*B*a^3/cos(d*x + c)^9)/d
```

Fricas [A] time = 1.94768, size = 436, normalized size = 3.11

$$\frac{8\,(2\,A-B)a^3\cos\left(dx+c\right)^6-36\,(2\,A-B)a^3\cos\left(dx+c\right)^4+15\,(2\,A-B)a^3\cos\left(dx+c\right)^2+7\,(A-2\,B)a^3+\Big(24\,(2\,A-B)a}{63\,\Big(3\,d\cos\left(dx+c\right)^5-4\,d\cos\left(dx+c\right)^3-\Big(d\cos\left(dx+c\right)^5-4\,d\cos(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] 1/63*(8*(2*A - B)*a^3*cos(d*x + c)^6 - 36*(2*A - B)*a^3*cos(d*x + c)^4 + 15
*(2*A - B)*a^3*cos(d*x + c)^2 + 7*(A - 2*B)*a^3 + (24*(2*A - B)*a^3*cos(d*x
 + c)^4 - 20*(2*A - B)*a^3*cos(d*x + c)^2 - 7*(2*A - B)*a^3)*sin(d*x + c))/
(3*d*cos(d*x + c)^5 - 4*d*cos(d*x + c)^3 - (d*cos(d*x + c)^5 - 4*d*cos(d*x
+ c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31763, size = 531, normalized size = 3.79

$$-\frac{\frac{21\left(21\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+19\,Aa^3-13\,Ba^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}+\frac{3591\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+315\,Ba^3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="g
iac")
```

```
[Out] -1/2016*(21*(21*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^3*tan(1/2*d*x + 1/2*c
)^2 + 36*A*a^3*tan(1/2*d*x + 1/2*c) - 24*B*a^3*tan(1/2*d*x + 1/2*c) + 19*A*
a^3 - 13*B*a^3)/(tan(1/2*d*x + 1/2*c) + 1)^3 + (3591*A*a^3*tan(1/2*d*x + 1/
2*c)^8 + 315*B*a^3*tan(1/2*d*x + 1/2*c)^8 - 19656*A*a^3*tan(1/2*d*x + 1/2*c
)^7 + 756*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 56196*A*a^3*tan(1/2*d*x + 1/2*c)^6
 - 4200*B*a^3*tan(1/2*d*x + 1/2*c)^6 - 95760*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
 11340*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 107730*A*a^3*tan(1/2*d*x + 1/2*c)^4 -
 14994*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 79464*A*a^3*tan(1/2*d*x + 1/2*c)^3 +
13356*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 38484*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 6
768*B*a^3*tan(1/2*d*x + 1/2*c)^2 - 10944*A*a^3*tan(1/2*d*x + 1/2*c) + 2196*
B*a^3*tan(1/2*d*x + 1/2*c) + 1615*A*a^3 - 209*B*a^3)/(tan(1/2*d*x + 1/2*c)
- 1)^9)/d
```

3.1003 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{(A+5B)(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(A+2B)(a-a\sin(c+dx))^5}{5a^6d}-\frac{(A+B)(a-a\sin(c+dx))^4}{a^5d}+\frac{B(a-a\sin(c+dx))^7}{7a^8d}$$

```
[Out] -(((A + B)*(a - a*Sin[c + d*x])^4)/(a^5*d)) + (4*(A + 2*B)*(a - a*Sin[c + d
*x])^5)/(5*a^6*d) - ((A + 5*B)*(a - a*Sin[c + d*x])^6)/(6*a^7*d) + (B*(a -
a*Sin[c + d*x])^7)/(7*a^8*d)
```

Rubi [A] time = 0.14689, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A+5B)(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(A+2B)(a-a\sin(c+dx))^5}{5a^6d}-\frac{(A+B)(a-a\sin(c+dx))^4}{a^5d}+\frac{B(a-a\sin(c+dx))^7}{7a^8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(((A + B)*(a - a*Sin[c + d*x])^4)/(a^5*d)) + (4*(A + 2*B)*(a - a*Sin[c + d
*x])^5)/(5*a^6*d) - ((A + 5*B)*(a - a*Sin[c + d*x])^6)/(6*a^7*d) + (B*(a -
a*Sin[c + d*x])^7)/(7*a^8*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^2\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(A+B)(a-x)^3-4a(A+2B)(a-x)^4+(A+5B)(a-x)^5-\frac{B(a-x)}{a}\right.\right.}{a^7d}\\ &= -\frac{(A+B)(a-a\sin(c+dx))^4}{a^5d}+\frac{4(A+2B)(a-a\sin(c+dx))^5}{5a^6d}-\frac{(A+5B)(a-a}{6a^7}\end{aligned}$$

Mathematica [A] time = 0.216757, size = 69, normalized size = 0.66

$$-\frac{(\sin(c+dx)-1)^4\left(5(7A+17B)\sin^2(c+dx)+(98A+76B)\sin(c+dx)+77A+30B\sin^3(c+dx)+19B\right)}{210ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((-1 + Sin[c + d*x])^4*(77*A + 19*B + (98*A + 76*B)*Sin[c + d*x] + 5*(7*A
+ 17*B)*Sin[c + d*x]^2 + 30*B*Sin[c + d*x]^3))/(210*a*d)
```

Maple [A] time = 0.108, size = 107, normalized size = 1.

$$\frac{1}{da}\left(-\frac{B(\sin(dx+c))^7}{7}+\frac{(-A+B)(\sin(dx+c))^6}{6}+\frac{(A+2B)(\sin(dx+c))^5}{5}+\frac{(2A-2B)(\sin(dx+c))^4}{4}+\frac{(-2A-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/7*B*sin(d*x+c)^7+1/6*(-A+B)*sin(d*x+c)^6+1/5*(A+2*B)*sin(d*x+c)^5
+1/4*(2*A-2*B)*sin(d*x+c)^4+1/3*(-2*A-B)*sin(d*x+c)^3+1/2*(-A+B)*sin(d*x+c)
^2+A*sin(d*x+c))
```

Maxima [A] time = 1.06914, size = 140, normalized size = 1.33

$$-\frac{30B\sin(dx+c)^7+35(A-B)\sin(dx+c)^6-42(A+2B)\sin(dx+c)^5-105(A-B)\sin(dx+c)^4+70(2A+B)\mathrm{s}}{210ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/210*(30*B*sin(d*x + c)^7 + 35*(A - B)*sin(d*x + c)^6 - 42*(A + 2*B)*sin(
d*x + c)^5 - 105*(A - B)*sin(d*x + c)^4 + 70*(2*A + B)*sin(d*x + c)^3 + 105
*(A - B)*sin(d*x + c)^2 - 210*A*sin(d*x + c))/(a*d)
```

Fricas [A] time = 1.82632, size = 204, normalized size = 1.94

$$\frac{35(A-B)\cos(dx+c)^6+2\left(15B\cos(dx+c)^6+3(7A-B)\cos(dx+c)^4+4(7A-B)\cos(dx+c)^2+56A-8B\right)\mathrm{si}}{210ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/210*(35*(A - B)*cos(d*x + c)^6 + 2*(15*B*cos(d*x + c)^6 + 3*(7*A - B)*cos
(d*x + c)^4 + 4*(7*A - B)*cos(d*x + c)^2 + 56*A - 8*B)*sin(d*x + c))/(a*d)
```

Sympy [A] time = 138.232, size = 3896, normalized size = 37.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-35*A*tan(c/2 + d*x/2)**14/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*
a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2
 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4
+ 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) + 420*A*tan(c/2 + d*x/2)**13/(210
*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/
2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**
6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d)
- 665*A*tan(c/2 + d*x/2)**12/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c
/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)
**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*
d*tan(c/2 + d*x/2)**2 + 210*a*d) + 1400*A*tan(c/2 + d*x/2)**11/(210*a*d*tan
(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/
2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410
*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) - 1155*A
*tan(c/2 + d*x/2)**10/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*
x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7
350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c
/2 + d*x/2)**2 + 210*a*d) + 3164*A*tan(c/2 + d*x/2)**9/(210*a*d*tan(c/2 + d
*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 +
 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan
(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) - 2625*A*tan(c/2
 + d*x/2)**8/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12
+ 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*t
an(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/
2)**2 + 210*a*d) + 4368*A*tan(c/2 + d*x/2)**7/(210*a*d*tan(c/2 + d*x/2)**14
 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d
*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*
x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) - 2625*A*tan(c/2 + d*x/2)
**6/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*
d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 +
d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 2
10*a*d) + 3164*A*tan(c/2 + d*x/2)**5/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a
*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2
+ d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 +
 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) - 1155*A*tan(c/2 + d*x/2)**4/(210*
a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2
 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6
 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) +
 1400*A*tan(c/2 + d*x/2)**3/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/
2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)*
*8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d
*tan(c/2 + d*x/2)**2 + 210*a*d) - 665*A*tan(c/2 + d*x/2)**2/(210*a*d*tan(c/
2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)*
*10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*
d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) + 420*A*tan
(c/2 + d*x/2)/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12
 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*
tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x
/2)**2 + 210*a*d) - 35*A/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 +
 d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8
```

```
+ 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*ta
n(c/2 + d*x/2)**2 + 210*a*d) - 10*B*tan(c/2 + d*x/2)**14/(210*a*d*tan(c/2 +
 d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10
 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*t
an(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) + 350*B*tan(c/
2 + d*x/2)**12/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**1
2 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d
*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*
x/2)**2 + 210*a*d) - 560*B*tan(c/2 + d*x/2)**11/(210*a*d*tan(c/2 + d*x/2)**
14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a
*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 +
d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) + 210*B*tan(c/2 + d*x/2
)**10/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*
a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2
+ d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 +
 210*a*d) + 448*B*tan(c/2 + d*x/2)**9/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*
a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2
 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4
+ 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) + 1050*B*tan(c/2 + d*x/2)**8/(210
*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/
2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**
6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d)
- 1824*B*tan(c/2 + d*x/2)**7/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c
/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)
**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*
d*tan(c/2 + d*x/2)**2 + 210*a*d) + 1050*B*tan(c/2 + d*x/2)**6/(210*a*d*tan(
c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2
)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*
a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) + 448*B*t
an(c/2 + d*x/2)**5/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2
)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350
*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2
+ d*x/2)**2 + 210*a*d) + 210*B*tan(c/2 + d*x/2)**4/(210*a*d*tan(c/2 + d*x/2
)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 735
0*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2
 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) - 560*B*tan(c/2 + d*
x/2)**3/(210*a*d*tan(c/2 + d*x/2)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 441
0*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/
2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2
 + 210*a*d) + 350*B*tan(c/2 + d*x/2)**2/(210*a*d*tan(c/2 + d*x/2)**14 + 147
0*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c
/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2 + d*x/2)**
4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d) - 10*B/(210*a*d*tan(c/2 + d*x/2
)**14 + 1470*a*d*tan(c/2 + d*x/2)**12 + 4410*a*d*tan(c/2 + d*x/2)**10 + 735
0*a*d*tan(c/2 + d*x/2)**8 + 7350*a*d*tan(c/2 + d*x/2)**6 + 4410*a*d*tan(c/2
 + d*x/2)**4 + 1470*a*d*tan(c/2 + d*x/2)**2 + 210*a*d), Ne(d, 0)), (x*(A +
B*sin(c))*cos(c)**7/(a*sin(c) + a), True))
```

Giac [A] time = 1.29216, size = 188, normalized size = 1.79

$$-\frac{30\,B\sin\left(dx+c\right)^{7}+35\,A\sin\left(dx+c\right)^{6}-35\,B\sin\left(dx+c\right)^{6}-42\,A\sin\left(dx+c\right)^{5}-84\,B\sin\left(dx+c\right)^{5}-105\,A\sin\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/210*(30*B*sin(d*x + c)^7 + 35*A*sin(d*x + c)^6 - 35*B*sin(d*x + c)^6 - 4
2*A*sin(d*x + c)^5 - 84*B*sin(d*x + c)^5 - 105*A*sin(d*x + c)^4 + 105*B*sin
(d*x + c)^4 + 140*A*sin(d*x + c)^3 + 70*B*sin(d*x + c)^3 + 105*A*sin(d*x +
c)^2 - 105*B*sin(d*x + c)^2 - 210*A*sin(d*x + c))/(a*d)
```

3.1004 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{(A+3B)(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(A+B)(a-a\sin(c+dx))^3}{3a^4d}-\frac{B(a-a\sin(c+dx))^5}{5a^6d}$$

```
[Out] (-2*(A + B)*(a - a*Sin[c + d*x])^3)/(3*a^4*d) + ((A + 3*B)*(a - a*Sin[c + d
*x])^4)/(4*a^5*d) - (B*(a - a*Sin[c + d*x])^5)/(5*a^6*d)
```

Rubi [A] time = 0.116178, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{(A+3B)(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(A+B)(a-a\sin(c+dx))^3}{3a^4d}-\frac{B(a-a\sin(c+dx))^5}{5a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*(A + B)*(a - a*Sin[c + d*x])^3)/(3*a^4*d) + ((A + 3*B)*(a - a*Sin[c + d
*x])^4)/(4*a^5*d) - (B*(a - a*Sin[c + d*x])^5)/(5*a^6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx=\frac{\text{Subst}\left(\int (a-x)^2(a+x)\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^5d}$$

$$=\frac{\text{Subst}\left(\int \left(2a(A+B)(a-x)^2+(-A-3B)(a-x)^3+\frac{B(a-x)^4}{a}\right)dx,x,a\sin(c+\right.}{a^5d}$$

$$=-\frac{2(A+B)(a-a\sin(c+dx))^3}{3a^4d}+\frac{(A+3B)(a-a\sin(c+dx))^4}{4a^5d}-\frac{B(a-a\sin(}{5a^6}$$

Mathematica [A] time = 0.149055, size = 72, normalized size = 0.91

$$\frac{\sin(c+dx)\left(15(A-B)\sin^3(c+dx)-20(A+B)\sin^2(c+dx)-30(A-B)\sin(c+dx)+60A+12B\sin^4(c+dx)\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(60*A - 30*(A - B)*Sin[c + d*x] - 20*(A + B)*Sin[c + d*x]^2 +
 15*(A - B)*Sin[c + d*x]^3 + 12*B*Sin[c + d*x]^4))/(60*a*d)
```

Maple [A] time = 0.095, size = 75, normalized size = 1.

$$\frac{1}{da}\left(\frac{B(\sin(dx+c))^5}{5}+\frac{(A-B)(\sin(dx+c))^4}{4}+\frac{(-A-B)(\sin(dx+c))^3}{3}+\frac{(-A+B)(\sin(dx+c))^2}{2}+A\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/5*B*sin(d*x+c)^5+1/4*(A-B)*sin(d*x+c)^4+1/3*(-A-B)*sin(d*x+c)^3+1/
2*(-A+B)*sin(d*x+c)^2+A*sin(d*x+c))
```

Maxima [A] time = 1.10156, size = 97, normalized size = 1.23

$$\frac{12\,B\sin(dx+c)^5+15\,(A-B)\sin(dx+c)^4-20\,(A+B)\sin(dx+c)^3-30\,(A-B)\sin(dx+c)^2+60\,A\sin(dx+c)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/60*(12*B*sin(d*x + c)^5 + 15*(A - B)*sin(d*x + c)^4 - 20*(A + B)*sin(d*x
+ c)^3 - 30*(A - B)*sin(d*x + c)^2 + 60*A*sin(d*x + c))/(a*d)
```

Fricas [A] time = 1.66173, size = 159, normalized size = 2.01

$$\frac{15\,(A-B)\cos(dx+c)^4+4\left(3\,B\cos(dx+c)^4+(5\,A-B)\cos(dx+c)^2+10\,A-2\,B\right)\sin(dx+c)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/60*(15*(A - B)*cos(d*x + c)^4 + 4*(3*B*cos(d*x + c)^4 + (5*A - B)*cos(d*x
 + c)^2 + 10*A - 2*B)*sin(d*x + c))/(a*d)
```

Sympy [A] time = 46.5843, size = 1703, normalized size = 21.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((30*A*tan(c/2 + d*x/2)**9/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*t
an(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)
**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**8/(15*a
*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*
x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d
) + 80*A*tan(c/2 + d*x/2)**7/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2
+ d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 7
5*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**6/(15*a*d*tan(
c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6
 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 100
*A*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/
2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*
tan(c/2 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 +
d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150
*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 80*A*tan(
c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 +
 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2
 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)*
*10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*ta
n(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*A*tan(c/2 + d
*x/2)/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*t
an(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)*
*2 + 15*a*d) + 30*B*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a
*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*
x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 40*B*tan(c/2 + d*x/2)**7/(
15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2
+ d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15
*a*d) + 30*B*tan(c/2 + d*x/2)**6/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(
c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4
 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 16*B*tan(c/2 + d*x/2)**5/(15*a*d*
tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2
)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) +
 30*B*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d
*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a
*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 40*B*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2
 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 +
150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*B*t
an(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**
8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(
c/2 + d*x/2)**2 + 15*a*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)**5/(a*sin(c)
 + a), True))
```

Giac [A] time = 1.34271, size = 128, normalized size = 1.62

$$\frac{12\,B\sin\left(dx+c\right)^{5}+15\,A\sin\left(dx+c\right)^{4}-15\,B\sin\left(dx+c\right)^{4}-20\,A\sin\left(dx+c\right)^{3}-20\,B\sin\left(dx+c\right)^{3}-30\,A\sin\left(dx+c\right)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/60*(12*B*sin(d*x + c)^5 + 15*A*sin(d*x + c)^4 - 15*B*sin(d*x + c)^4 - 20*
```

```
A*sin(d*x + c)^3 - 20*B*sin(d*x + c)^3 - 30*A*sin(d*x + c)^2 + 30*B*sin(d*x
 + c)^2 + 60*A*sin(d*x + c))/(a*d)
```

3.1005 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{(A-B)\sin^2(c+dx)}{2ad}+\frac{A\sin(c+dx)}{ad}-\frac{B\sin^3(c+dx)}{3ad}$$

```
[Out] (A*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x]^2)/(2*a*d) - (B*Sin[c + d*x]
^3)/(3*a*d)
```

Rubi [A] time = 0.0954406, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 43}

$$-\frac{(A-B)\sin^2(c+dx)}{2ad}+\frac{A\sin(c+dx)}{ad}-\frac{B\sin^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (A*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x]^2)/(2*a*d) - (B*Sin[c + d*x]
^3)/(3*a*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(aA-(A-B)x-\frac{Bx^2}{a}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{A\sin(c+dx)}{ad}-\frac{(A-B)\sin^2(c+dx)}{2ad}-\frac{B\sin^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.0941487, size = 44, normalized size = 0.77

$$\frac{\sin(c+dx)\left(-3(A-B)\sin(c+dx)+6A-2B\sin^2(c+dx)\right)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(6*A - 3*(A - B)*Sin[c + d*x] - 2*B*Sin[c + d*x]^2))/(6*a*d)
```

Maple [A] time = 0.085, size = 43, normalized size = 0.8

$$\frac{1}{da}\left(-\frac{B\left(\sin\left(dx+c\right)\right)^{3}}{3}+\frac{\left(-A+B\right)\left(\sin\left(dx+c\right)\right)^{2}}{2}+A\sin\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/3*B*sin(d*x+c)^3+1/2*(-A+B)*sin(d*x+c)^2+A*sin(d*x+c))
```

Maxima [A] time = 1.04723, size = 59, normalized size = 1.04

$$-\frac{2\,B\sin\left(dx+c\right)^{3}+3\,\left(A-B\right)\sin\left(dx+c\right)^{2}-6\,A\sin\left(dx+c\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/6*(2*B*sin(d*x + c)^3 + 3*(A - B)*sin(d*x + c)^2 - 6*A*sin(d*x + c))/(a*
d)
```

Fricas [A] time = 1.72857, size = 113, normalized size = 1.98

$$\frac{3\,\left(A-B\right)\cos\left(dx+c\right)^{2}+2\,\left(B\cos\left(dx+c\right)^{2}+3\,A-B\right)\sin\left(dx+c\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(A - B)*cos(d*x + c)^2 + 2*(B*cos(d*x + c)^2 + 3*A - B)*sin(d*x + c)
)/(a*d)
```

Sympy [A] time = 13.6641, size = 588, normalized size = 10.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((6*A*tan(c/2 + d*x/2)**5/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c
/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 6*A*tan(c/2 + d*x/2)*
*4/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 +
 d*x/2)**2 + 3*a*d) + 12*A*tan(c/2 + d*x/2)**3/(3*a*d*tan(c/2 + d*x/2)**6 +
 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 6*A*tan(c
/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a
*d*tan(c/2 + d*x/2)**2 + 3*a*d) + 6*A*tan(c/2 + d*x/2)/(3*a*d*tan(c/2 + d*x
/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) + 6
*B*tan(c/2 + d*x/2)**4/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)*
*4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 8*B*tan(c/2 + d*x/2)**3/(3*a*d*ta
n(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 +
 3*a*d) + 6*B*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/
2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d), Ne(d, 0)), (x*(A + B*si
n(c))*cos(c)**3/(a*sin(c) + a), True))
```

Giac [A] time = 1.30967, size = 69, normalized size = 1.21

$$-\frac{2 B \sin(dx+c)^3 + 3 A \sin(dx+c)^2 - 3 B \sin(dx+c)^2 - 6 A \sin(dx+c)}{6 ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(2*B*sin(d*x + c)^3 + 3*A*sin(d*x + c)^2 - 3*B*sin(d*x + c)^2 - 6*A*si
n(d*x + c))/(a*d)
```

3.1006 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{(A-B)\log(\sin(c+dx)+1)}{ad}+\frac{B\sin(c+dx)}{ad}$$

```
[Out] ((A - B)*Log[1 + Sin[c + d*x]])/(a*d) + (B*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.0817913, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{(A-B)\log(\sin(c+dx)+1)}{ad}+\frac{B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*Log[1 + Sin[c + d*x]])/(a*d) + (B*Sin[c + d*x])/(a*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{a+x}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(\frac{B}{a}+\frac{A-B}{a+x}\right)dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{(A-B)\log(1+\sin(c+dx))}{ad}+\frac{B\sin(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0316451, size = 31, normalized size = 0.86

$$\frac{(A-B)\log(\sin(c+dx)+1)+B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*Log[1 + Sin[c + d*x]] + B*Sin[c + d*x])/(a*d)
```

Maple [A] time = 0.032, size = 51, normalized size = 1.4

$$\frac{\ln \left(1+\sin \left(dx+c\right)\right) A}{da}-\frac{\ln \left(1+\sin \left(dx+c\right)\right) B}{da}+\frac{B \sin \left(dx+c\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*ln(1+sin(d*x+c))*A-1/d/a*ln(1+sin(d*x+c))*B+B*sin(d*x+c)/d/a
```

Maxima [A] time = 1.06111, size = 46, normalized size = 1.28

$$\frac{\frac{(A-B)\log(\sin(dx+c)+1)}{a}+\frac{B\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] ((A - B)*log(sin(d*x + c) + 1)/a + B*sin(d*x + c)/a)/d
```

Fricas [A] time = 1.79853, size = 76, normalized size = 2.11

$$\frac{(A-B)\log \left(\sin \left(dx+c\right)+1\right)+B\sin \left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] ((A - B)*log(sin(d*x + c) + 1) + B*sin(d*x + c))/(a*d)
```

Sympy [A] time = 0.677483, size = 60, normalized size = 1.67

$$\begin{cases} \frac{A\log\left(\sin\left(c+dx\right)+1\right)}{ad}-\frac{B\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{B\sin\left(c+dx\right)}{ad} & \text{for}\ d\neq 0\\ \frac{x(A+B\sin\left(c\right))\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*log(sin(c + d*x) + 1)/(a*d) - B*log(sin(c + d*x) + 1)/(a*d) +
B*sin(c + d*x)/(a*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)/(a*sin(c) + a), T
rue))
```

Giac [A] time = 1.35858, size = 47, normalized size = 1.31

$$\frac{\frac{(A-B)\log(|\sin(dx+c)+1|)}{a}+\frac{B\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((A - B)*log(abs(sin(d*x + c) + 1))/a + B*sin(d*x + c)/a)/d
```

3.1007 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{A-B}{2d(a\sin(c+dx)+a)}$$

```
[Out] ((A + B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (A - B)/(2*d*(a + a*Sin[c + d*x])
)
```

Rubi [A] time = 0.0941355, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{A-B}{2d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (A - B)/(2*d*(a + a*Sin[c + d*x])
)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{A-B}{2a(a+x)^2}+\frac{A+B}{2a(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{A-B}{2d(a+a\sin(c+dx))}+\frac{(A+B)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{2d}$$
$$= \frac{(A+B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{A-B}{2d(a+a\sin(c+dx))}$$

Mathematica [A] time = 0.060957, size = 44, normalized size = 0.98

$$\frac{(A+B)(\sin(c+dx)+1)\tanh^{-1}(\sin(c+dx))-A+B}{2ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-A + B + (A + B)*ArcTanh[Sin[c + d*x]]*(1 + Sin[c + d*x]))/(2*a*d*(1 + Sin
[c + d*x]))
```

Maple [B] time = 0.092, size = 112, normalized size = 2.5

$$-\frac{\ln(\sin(dx+c)-1)\,A}{4\,da}-\frac{\ln(\sin(dx+c)-1)\,B}{4\,da}-\frac{A}{2\,da\,(1+\sin(dx+c))}+\frac{B}{2\,da\,(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d/a*ln(sin(d*x+c)-1)*A-1/4/d/a*ln(sin(d*x+c)-1)*B-1/2/d/a/(1+sin(d*x+c
))*A+1/2/d/a/(1+sin(d*x+c))*B+1/4/d/a*ln(1+sin(d*x+c))*B+1/4/d/a*ln(1+sin(d
*x+c))*A
```

Maxima [A] time = 1.017, size = 78, normalized size = 1.73

$$\frac{\frac{(A+B)\log(\sin(dx+c)+1)}{a}-\frac{(A+B)\log(\sin(dx+c)-1)}{a}-\frac{2\,(A-B)}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/4*((A + B)*log(sin(d*x + c) + 1)/a - (A + B)*log(sin(d*x + c) - 1)/a - 2*
(A - B)/(a*sin(d*x + c) + a))/d
```

Fricas [A] time = 1.71091, size = 207, normalized size = 4.6

$$\frac{((A+B)\sin(dx+c)+A+B)\log(\sin(dx+c)+1)-((A+B)\sin(dx+c)+A+B)\log(-\sin(dx+c)+1)-2A+2}{4(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/4*(((A + B)*sin(d*x + c) + A + B)*log(sin(d*x + c) + 1) - ((A + B)*sin(d*
x + c) + A + B)*log(-sin(d*x + c) + 1) - 2*A + 2*B)/(a*d*sin(d*x + c) + a*d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec(c+dx)}{\sin(c+dx)+1}\,dx+\int \frac{B\sin(c+dx)\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(B*sin(c + d*x)*s
ec(c + d*x)/(sin(c + d*x) + 1), x))/a
```

Giac [A] time = 1.3048, size = 107, normalized size = 2.38

$$\frac{\frac{(A+B)\log(|\sin(dx+c)+1|)}{a}-\frac{(A+B)\log(|\sin(dx+c)-1|)}{a}-\frac{A\sin(dx+c)+B\sin(dx+c)+3A-B}{a(\sin(dx+c)+1)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*((A + B)*log(abs(sin(d*x + c) + 1))/a - (A + B)*log(abs(sin(d*x + c) -
1))/a - (A*sin(d*x + c) + B*sin(d*x + c) + 3*A - B)/(a*(sin(d*x + c) + 1)))
/d
```

3.1008 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a(A-B)}{8d(a\sin(c+dx)+a)^2}+\frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{A}{4d(a\sin(c+dx)+a)}$$

```
[Out] ((3*A + B)*ArcTanh[Sin[c + d*x]])/(8*a*d) + (A + B)/(8*d*(a - a*Sin[c + d*x
])) - (a*(A - B))/(8*d*(a + a*Sin[c + d*x])^2) - A/(4*d*(a + a*Sin[c + d*x]
))
```

Rubi [A] time = 0.138204, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a(A-B)}{8d(a\sin(c+dx)+a)^2}+\frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{A}{4d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((3*A + B)*ArcTanh[Sin[c + d*x]])/(8*a*d) + (A + B)/(8*d*(a - a*Sin[c + d*x
])) - (a*(A - B))/(8*d*(a + a*Sin[c + d*x])^2) - A/(4*d*(a + a*Sin[c + d*x]
))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a^3\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{A+B}{8a^3(a-x)^2} + \frac{A-B}{4a^2(a+x)^3} + \frac{A}{4a^3(a+x)^2} + \frac{3A+B}{8a^3(a^2-x^2)}\right) dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{A+B}{8d(a-a\sin(c+dx))} - \frac{a(A-B)}{8d(a+a\sin(c+dx))^2} - \frac{A}{4d(a+a\sin(c+dx))} + \frac{(3A}{}$$

$$= \frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{8ad} + \frac{A+B}{8d(a-a\sin(c+dx))} - \frac{a(A-B)}{8d(a+a\sin(c+dx}$$

Mathematica [A] time = 0.25448, size = 75, normalized size = 0.82

$$\frac{\frac{A+B}{a-a\sin(c+dx)} + \frac{B-A}{a(\sin(c+dx)+1)^2} + \frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{a} - \frac{2A}{a\sin(c+dx)+a}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (((3*A + B)*ArcTanh[Sin[c + d*x]])/a + (-A + B)/(a*(1 + Sin[c + d*x])^2) +
(A + B)/(a - a*Sin[c + d*x]) - (2*A)/(a + a*Sin[c + d*x]))/(8*d)
```

Maple [B] time = 0.134, size = 169, normalized size = 1.9

$$-\frac{3\ln(\sin(dx+c)-1)A}{16da} - \frac{\ln(\sin(dx+c)-1)B}{16da} - \frac{A}{8da(\sin(dx+c)-1)} - \frac{B}{8da(\sin(dx+c)-1)} - \frac{A}{4da(1+\sin(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] -3/16/d/a*ln(sin(d*x+c)-1)*A-1/16/d/a*ln(sin(d*x+c)-1)*B-1/8/d/a/(sin(d*x+c
)-1)*A-1/8/d/a/(sin(d*x+c)-1)*B-1/4/d/a/(1+sin(d*x+c))*A-1/8/d/a/(1+sin(d*x
+c))^2*A+1/8/d/a/(1+sin(d*x+c))^2*B+3/16/d/a*ln(1+sin(d*x+c))*A+1/16/d/a*ln
(1+sin(d*x+c))*B
```

Maxima [A] time = 1.01793, size = 153, normalized size = 1.68

$$\frac{\frac{(3A+B)\log(\sin(dx+c)+1)}{a} - \frac{(3A+B)\log(\sin(dx+c)-1)}{a} - \frac{2\left((3A+B)\sin(dx+c)^2+(3A+B)\sin(dx+c)-2A+2B\right)}{a\sin(dx+c)^3+a\sin(dx+c)^2-a\sin(dx+c)-a}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*((3*A + B)*log(sin(d*x + c) + 1)/a - (3*A + B)*log(sin(d*x + c) - 1)/a
 - 2*((3*A + B)*sin(d*x + c)^2 + (3*A + B)*sin(d*x + c) - 2*A + 2*B)/(a*sin
```

```
(d*x + c)^3 + a*sin(d*x + c)^2 - a*sin(d*x + c) - a))/d
```

Fricas [A] time = 1.50304, size = 423, normalized size = 4.65

$$-\frac{2\,(3\,A+B)\cos\left(dx+c\right)^{2}-\left((3\,A+B)\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+(3\,A+B)\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+\left((3\,A\right.}{16\left(ad\cos\left(dx+c\right)^{2}\sin\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/16*(2*(3*A + B)*cos(d*x + c)^2 - ((3*A + B)*cos(d*x + c)^2*sin(d*x + c)
+ (3*A + B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((3*A + B)*cos(d*x + c)
^2*sin(d*x + c) + (3*A + B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(3*A
 + B)*sin(d*x + c) - 2*A - 6*B)/(a*d*cos(d*x + c)^2*sin(d*x + c) + a*d*cos(
d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^{3}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx+\int \frac{B\sin\left(c+dx\right)\sec^{3}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(B*sin(c + d*x
)*sec(c + d*x)**3/(sin(c + d*x) + 1), x))/a
```

Giac [A] time = 1.33967, size = 198, normalized size = 2.18

$$\frac{\frac{2\,(3\,A+B)\log(|\sin(dx+c)+1|)}{a}-\frac{2\,(3\,A+B)\log(|\sin(dx+c)-1|)}{a}+\frac{2\,(3\,A\sin(dx+c)+B\sin(dx+c)-5\,A-3\,B)}{a(\sin(dx+c)-1)}-\frac{9\,A\sin(dx+c)^{2}+3\,B\sin(dx+c)^{2}+26\,A\sin(dx+c)+6}{a(\sin(dx+c)+1)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*(2*(3*A + B)*log(abs(sin(d*x + c) + 1))/a - 2*(3*A + B)*log(abs(sin(d*
x + c) - 1))/a + 2*(3*A*sin(d*x + c) + B*sin(d*x + c) - 5*A - 3*B)/(a*(sin(
d*x + c) - 1)) - (9*A*sin(d*x + c)^2 + 3*B*sin(d*x + c)^2 + 26*A*sin(d*x +
c) + 6*B*sin(d*x + c) + 21*A - B)/(a*(sin(d*x + c) + 1)^2))/d
```

3.1009 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2(A-B)}{24d(a\sin(c+dx)+a)^3}+\frac{a(A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a(3A-B)}{32d(a\sin(c+dx)+a)^2}+\frac{2A+B}{16d(a-a\sin(c+dx))}+\frac{(5A+B)\,\mathrm{ta}}{}$$

```
[Out] ((5*A + B)*ArcTanh[Sin[c + d*x]])/(16*a*d) + (a*(A + B))/(32*d*(a - a*Sin[c
 + d*x])^2) + (2*A + B)/(16*d*(a - a*Sin[c + d*x])) - (a^2*(A - B))/(24*d*(
a + a*Sin[c + d*x])^3) - (a*(3*A - B))/(32*d*(a + a*Sin[c + d*x])^2) - (3*A
)/(16*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.18878, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$-\frac{a^2(A-B)}{24d(a\sin(c+dx)+a)^3}+\frac{a(A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a(3A-B)}{32d(a\sin(c+dx)+a)^2}+\frac{2A+B}{16d(a-a\sin(c+dx))}+\frac{(5A+B)\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```


```
[Out] ((5*A + B)*ArcTanh[Sin[c + d*x]])/(16*a*d) + (a*(A + B))/(32*d*(a - a*Sin[c
 + d*x])^2) + (2*A + B)/(16*d*(a - a*Sin[c + d*x])) - (a^2*(A - B))/(24*d*(
a + a*Sin[c + d*x])^3) - (a*(3*A - B))/(32*d*(a + a*Sin[c + d*x])^2) - (3*A
)/(16*d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a^5 \operatorname{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5 \operatorname{Subst}\left(\int \left(\frac{A+B}{16a^4(a-x)^3} + \frac{2A+B}{16a^5(a-x)^2} + \frac{A-B}{8a^3(a+x)^4} + \frac{3A-B}{16a^4(a+x)^3} + \frac{3A}{16a^5(a+x)^2} + \frac{5A+B}{16a^5(a^2-\right.\right.}{d}$$

$$= \frac{a(A+B)}{32d(a-a\sin(c+dx))^2} + \frac{2A+B}{16d(a-a\sin(c+dx))} - \frac{a^2(A-B)}{24d(a+a\sin(c+dx))^3} - \frac{}{32}$$

$$= \frac{(5A+B)\tanh^{-1}(\sin(c+dx))}{16ad} + \frac{a(A+B)}{32d(a-a\sin(c+dx))^2} + \frac{2A+B}{16d(a-a\sin(c+dx}$$

Mathematica [A] time = 0.523395, size = 105, normalized size = 0.72

$$\frac{-\frac{6(2A+B)}{\sin(c+dx)-1} + \frac{3(A+B)}{(\sin(c+dx)-1)^2} + \frac{3B-9A}{(\sin(c+dx)+1)^2} - \frac{4(A-B)}{(\sin(c+dx)+1)^3} + 6(5A+B)\tanh^{-1}(\sin(c+dx)) - \frac{18A}{\sin(c+dx)+1}}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*(5*A + B)*ArcTanh[Sin[c + d*x]] + (3*(A + B))/(-1 + Sin[c + d*x])^2 - (6
*(2*A + B))/(-1 + Sin[c + d*x]) - (4*(A - B))/(1 + Sin[c + d*x])^3 + (-9*A
+ 3*B)/(1 + Sin[c + d*x])^2 - (18*A)/(1 + Sin[c + d*x]))/(96*a*d)
```

Maple [A] time = 0.099, size = 245, normalized size = 1.7

$$-\frac{5\ln(\sin(dx+c)-1)A}{32da} - \frac{\ln(\sin(dx+c)-1)B}{32da} + \frac{A}{32da(\sin(dx+c)-1)^2} + \frac{B}{32da(\sin(dx+c)-1)^2} - \frac{A}{8da(\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] -5/32/d/a*ln(sin(d*x+c)-1)*A-1/32/d/a*ln(sin(d*x+c)-1)*B+1/32/d/a/(sin(d*x+
c)-1)^2*A+1/32/d/a/(sin(d*x+c)-1)^2*B-1/8/d/a/(sin(d*x+c)-1)*A-1/16/d/a/(si
n(d*x+c)-1)*B-3/16/d/a/(1+sin(d*x+c))*A-1/24/d/a/(1+sin(d*x+c))^3*A+1/24/d/
a/(1+sin(d*x+c))^3*B-3/32/d/a/(1+sin(d*x+c))^2*A+1/32/d/a/(1+sin(d*x+c))^2*
B+5/32/d/a*ln(1+sin(d*x+c))*A+1/32/d/a*ln(1+sin(d*x+c))*B
```

Maxima [A] time = 1.09045, size = 223, normalized size = 1.53

$$\frac{\frac{3(5A+B)\log(\sin(dx+c)+1)}{a} - \frac{3(5A+B)\log(\sin(dx+c)-1)}{a} - \frac{2\left(3(5A+B)\sin(dx+c)^4 + 3(5A+B)\sin(dx+c)^3 - 5(5A+B)\sin(dx+c)^2 - 5(5A+B)\sin(dx+c) + 8A\right.}{a\sin(dx+c)^5 + a\sin(dx+c)^4 - 2a\sin(dx+c)^3 - 2a\sin(dx+c)^2 + a\sin(dx+c) + a}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(5*A + B)*log(sin(d*x + c) + 1)/a - 3*(5*A + B)*log(sin(d*x + c) -
1)/a - 2*(3*(5*A + B)*sin(d*x + c)^4 + 3*(5*A + B)*sin(d*x + c)^3 - 5*(5*A
+ B)*sin(d*x + c)^2 - 5*(5*A + B)*sin(d*x + c) + 8*A - 8*B)/(a*sin(d*x + c)
^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 - 2*a*sin(d*x + c)^2 + a*sin(d*x
 + c) + a))/d
```

Fricas [A] time = 1.64501, size = 512, normalized size = 3.51

$$-\frac{6\,(5\,A+B)\cos\left(dx+c\right)^4-2\,(5\,A+B)\cos\left(dx+c\right)^2-3\,\left((5\,A+B)\cos\left(dx+c\right)^4\sin\left(dx+c\right)+(5\,A+B)\cos\left(dx+c\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/96*(6*(5*A + B)*cos(d*x + c)^4 - 2*(5*A + B)*cos(d*x + c)^2 - 3*((5*A +
B)*cos(d*x + c)^4*sin(d*x + c) + (5*A + B)*cos(d*x + c)^4)*log(sin(d*x + c)
 + 1) + 3*((5*A + B)*cos(d*x + c)^4*sin(d*x + c) + (5*A + B)*cos(d*x + c)^4
)*log(-sin(d*x + c) + 1) - 2*(3*(5*A + B)*cos(d*x + c)^2 + 10*A + 2*B)*sin(
d*x + c) - 4*A - 20*B)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^
4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39077, size = 259, normalized size = 1.77

$$\frac{\frac{6\,(5\,A+B)\log\left(|\sin(dx+c)+1|\right)}{a}-\frac{6\,(5\,A+B)\log\left(|\sin(dx+c)-1|\right)}{a}+\frac{3\left(15\,A\sin(dx+c)^2+3\,B\sin(dx+c)^2-38\,A\sin(dx+c)-10\,B\sin(dx+c)+25\,A+9\,B\right)}{a(\sin(dx+c)-1)^2}-\frac{55\,A\,\mathrm{s}\cdots}{\cdots}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*(6*(5*A + B)*log(abs(sin(d*x + c) + 1))/a - 6*(5*A + B)*log(abs(sin(d
*x + c) - 1))/a + 3*(15*A*sin(d*x + c)^2 + 3*B*sin(d*x + c)^2 - 38*A*sin(d*
x + c) - 10*B*sin(d*x + c) + 25*A + 9*B)/(a*(sin(d*x + c) - 1)^2) - (55*A*s
in(d*x + c)^3 + 11*B*sin(d*x + c)^3 + 201*A*sin(d*x + c)^2 + 33*B*sin(d*x +
 c)^2 + 255*A*sin(d*x + c) + 27*B*sin(d*x + c) + 117*A - 3*B)/(a*(sin(d*x +
 c) + 1)^3))/d
```

3.1010 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=205

$$-\frac{a^3(A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3}-\frac{a^2(2A-B)}{48d(a\sin(c+dx)+a)^3}+\frac{a(5A+3B)}{128d(a-a\sin(c+dx))^2}-\frac{a(5A-}{64d(a\sin(c+}$$

```
[Out] (5*(7*A + B)*ArcTanh[Sin[c + d*x]])/(128*a*d) + (a^2*(A + B))/(96*d*(a - a*
Sin[c + d*x])^3) + (a*(5*A + 3*B))/(128*d*(a - a*Sin[c + d*x])^2) + (5*(3*A
 + B))/(128*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(64*d*(a + a*Sin[c + d*
x])^4) - (a^2*(2*A - B))/(48*d*(a + a*Sin[c + d*x])^3) - (a*(5*A - B))/(64*
d*(a + a*Sin[c + d*x])^2) - (5*A)/(32*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.248532, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$-\frac{a^3(A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3}-\frac{a^2(2A-B)}{48d(a\sin(c+dx)+a)^3}+\frac{a(5A+3B)}{128d(a-a\sin(c+dx))^2}-\frac{a(5A-}{64d(a\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```


```
[Out] (5*(7*A + B)*ArcTanh[Sin[c + d*x]])/(128*a*d) + (a^2*(A + B))/(96*d*(a - a*
Sin[c + d*x])^3) + (a*(5*A + 3*B))/(128*d*(a - a*Sin[c + d*x])^2) + (5*(3*A
 + B))/(128*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(64*d*(a + a*Sin[c + d*
x])^4) - (a^2*(2*A - B))/(48*d*(a + a*Sin[c + d*x])^3) - (a*(5*A - B))/(64*
d*(a + a*Sin[c + d*x])^2) - (5*A)/(32*d*(a + a*Sin[c + d*x]))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\text{Subst}\left(\int\left(\frac{A+B}{32a^5(a-x)^4} + \frac{5A+3B}{64a^6(a-x)^3} + \frac{5(3A+B)}{128a^7(a-x)^2} + \frac{A-B}{16a^4(a+x)^5} + \frac{2A-B}{16a^5(a+x)^4} + \ldots\right.\right.}{d}$$

$$= \frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3} + \frac{a(5A+3B)}{128d(a-a\sin(c+dx))^2} + \frac{5(3A+B)}{128d(a-a\sin(c+dx))} \ldots$$

$$= \frac{5(7A+B)\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3} + \frac{a(5A+3\ldots}{128d(a-a\sin(\ldots}$$

Mathematica [A] time = 0.836141, size = 142, normalized size = 0.69

$$\frac{\frac{-15(7A+B)\sin^6(c+dx)-15(7A+B)\sin^5(c+dx)+40(7A+B)\sin^4(c+dx)+40(7A+B)\sin^3(c+dx)-33(7A+B)\sin^2(c+dx)-33(7A+B)\sin(c+dx)+48(A-B)}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4} + 15(\ldots}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*(7*A + B)*ArcTanh[Sin[c + d*x]] + (48*(A - B) - 33*(7*A + B)*Sin[c + d*
x] - 33*(7*A + B)*Sin[c + d*x]^2 + 40*(7*A + B)*Sin[c + d*x]^3 + 40*(7*A +
B)*Sin[c + d*x]^4 - 15*(7*A + B)*Sin[c + d*x]^5 - 15*(7*A + B)*Sin[c + d*x]
^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

Maple [A] time = 0.109, size = 321, normalized size = 1.6

$$-\frac{35\ln(\sin(dx+c)-1)A}{256\,da} - \frac{5\ln(\sin(dx+c)-1)B}{256\,da} + \frac{5A}{128\,da(\sin(dx+c)-1)^2} + \frac{3B}{128\,da(\sin(dx+c)-1)^2} - \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] -35/256/d/a*ln(sin(d*x+c)-1)*A-5/256/d/a*ln(sin(d*x+c)-1)*B+5/128/d/a/(sin(
d*x+c)-1)^2*A+3/128/d/a/(sin(d*x+c)-1)^2*B-1/96/d/a/(sin(d*x+c)-1)^3*A-1/96
/d/a/(sin(d*x+c)-1)^3*B-15/128/d/a/(sin(d*x+c)-1)*A-5/128/d/a/(sin(d*x+c)-1
)*B-5/32/d/a/(1+sin(d*x+c))*A-1/64/d/a/(1+sin(d*x+c))^4*A+1/64/d/a/(1+sin(d
*x+c))^4*B-1/24/d/a/(1+sin(d*x+c))^3*A+1/48/d/a/(1+sin(d*x+c))^3*B-5/64/d/a
/(1+sin(d*x+c))^2*A+1/64/d/a/(1+sin(d*x+c))^2*B+35/256/d/a*ln(1+sin(d*x+c))
*A+5/256/d/a*ln(1+sin(d*x+c))*B
```

Maxima [A] time = 1.05065, size = 297, normalized size = 1.45

$$\frac{\frac{15(7A+B)\log(\sin(dx+c)+1)}{a} - \frac{15(7A+B)\log(\sin(dx+c)-1)}{a} - \frac{2\left(15(7A+B)\sin(dx+c)^6+15(7A+B)\sin(dx+c)^5-40(7A+B)\sin(dx+c)^4-40(7A+B)\sin\ldots\right.}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\,\text{s}\ldots}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/768*(15*(7*A + B)*log(sin(d*x + c) + 1)/a - 15*(7*A + B)*log(sin(d*x + c)
 - 1)/a - 2*(15*(7*A + B)*sin(d*x + c)^6 + 15*(7*A + B)*sin(d*x + c)^5 - 40
*(7*A + B)*sin(d*x + c)^4 - 40*(7*A + B)*sin(d*x + c)^3 + 33*(7*A + B)*sin(
d*x + c)^2 + 33*(7*A + B)*sin(d*x + c) - 48*A + 48*B)/(a*sin(d*x + c)^7 + a
*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c
)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a))/d
```

Fricas [A] time = 1.60582, size = 602, normalized size = 2.94

$$-\frac{30\,(7\,A+B)\cos\left(dx+c\right)^{6}-10\,(7\,A+B)\cos\left(dx+c\right)^{4}-4\,(7\,A+B)\cos\left(dx+c\right)^{2}-15\left(\left(7\,A+B\right)\cos\left(dx+c\right)^{6}\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/768*(30*(7*A + B)*cos(d*x + c)^6 - 10*(7*A + B)*cos(d*x + c)^4 - 4*(7*A
+ B)*cos(d*x + c)^2 - 15*((7*A + B)*cos(d*x + c)^6*sin(d*x + c) + (7*A + B)
*cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*((7*A + B)*cos(d*x + c)^6*sin(d
*x + c) + (7*A + B)*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*(7*A + B
)*cos(d*x + c)^4 + 10*(7*A + B)*cos(d*x + c)^2 + 56*A + 8*B)*sin(d*x + c) -
 16*A - 112*B)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43809, size = 319, normalized size = 1.56

$$\frac{\frac{60\,(7\,A+B)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}-\frac{60\,(7\,A+B)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a}+\frac{2\left(385\,A\sin\left(dx+c\right)^{3}+55\,B\sin\left(dx+c\right)^{3}-1335\,A\sin\left(dx+c\right)^{2}-225\,B\sin\left(dx+c\right)^{2}+1575\,A\sin\left(dx\right.\right.}{a\left(\sin\left(dx+c\right)-1\right)^{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/3072*(60*(7*A + B)*log(abs(sin(d*x + c) + 1))/a - 60*(7*A + B)*log(abs(si
n(d*x + c) - 1))/a + 2*(385*A*sin(d*x + c)^3 + 55*B*sin(d*x + c)^3 - 1335*A
*sin(d*x + c)^2 - 225*B*sin(d*x + c)^2 + 1575*A*sin(d*x + c) + 321*B*sin(d*
```

```
x + c) - 641*A - 167*B)/(a*(sin(d*x + c) - 1)^3) - (875*A*sin(d*x + c)^4 +
125*B*sin(d*x + c)^4 + 3980*A*sin(d*x + c)^3 + 500*B*sin(d*x + c)^3 + 6930*
A*sin(d*x + c)^2 + 702*B*sin(d*x + c)^2 + 5548*A*sin(d*x + c) + 340*B*sin(d
*x + c) + 1771*A - 35*B)/(a*(sin(d*x + c) + 1)^4))/d
```

3.1011 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{(A+3B)(a-a\sin(c+dx))^5}{5a^7d}-\frac{(A+B)(a-a\sin(c+dx))^4}{2a^6d}-\frac{B(a-a\sin(c+dx))^6}{6a^8d}$$

```
[Out] -((A + B)*(a - a*Sin[c + d*x])^4)/(2*a^6*d) + ((A + 3*B)*(a - a*Sin[c + d*x
])^5)/(5*a^7*d) - (B*(a - a*Sin[c + d*x])^6)/(6*a^8*d)
```

Rubi [A] time = 0.124974, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{(A+3B)(a-a\sin(c+dx))^5}{5a^7d}-\frac{(A+B)(a-a\sin(c+dx))^4}{2a^6d}-\frac{B(a-a\sin(c+dx))^6}{6a^8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((A + B)*(a - a*Sin[c + d*x])^4)/(2*a^6*d) + ((A + 3*B)*(a - a*Sin[c + d*x
])^5)/(5*a^7*d) - (B*(a - a*Sin[c + d*x])^6)/(6*a^8*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx&=\frac{\text{Subst}\left(\int (a-x)^3(a+x)\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin(c+dx)\right)}{a^7d}\\&=\frac{\text{Subst}\left(\int \left(2a(A+B)(a-x)^3+(-A-3B)(a-x)^4+\frac{B(a-x)^5}{a}\right)\,dx,x,a\sin(c+dx)\right)}{a^7d}\\&=-\frac{(A+B)(a-a\sin(c+dx))^4}{2a^6d}+\frac{(A+3B)(a-a\sin(c+dx))^5}{5a^7d}-\frac{B(a-a\sin(c+dx))^6}{6a^8d}\end{aligned}$$

Mathematica [A] time = 0.16659, size = 52, normalized size = 0.66

$$-\frac{(\sin(c+dx)-1)^4\left((6A+8B)\sin(c+dx)+9A+5B\sin^2(c+dx)+2B\right)}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((-1 + Sin[c + d*x])^4*(9*A + 2*B + (6*A + 8*B)*Sin[c + d*x] + 5*B*Sin[c +
 d*x]^2))/(30*a^2*d)
```

Maple [A] time = 0.162, size = 82, normalized size = 1.

$$\frac{1}{da^2}\left(-\frac{B(\sin(dx+c))^6}{6}+\frac{(-A+2B)(\sin(dx+c))^5}{5}+\frac{A(\sin(dx+c))^4}{2}-\frac{2B(\sin(dx+c))^3}{3}+\frac{(-2A+B)(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/6*B*sin(d*x+c)^6+1/5*(-A+2*B)*sin(d*x+c)^5+1/2*A*sin(d*x+c)^4-2
/3*B*sin(d*x+c)^3+1/2*(-2*A+B)*sin(d*x+c)^2+A*sin(d*x+c))
```

Maxima [A] time = 1.02463, size = 112, normalized size = 1.42

$$-\frac{5B\sin(dx+c)^6+6(A-2B)\sin(dx+c)^5-15A\sin(dx+c)^4+20B\sin(dx+c)^3+15(2A-B)\sin(dx+c)^2-30}{30a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/30*(5*B*sin(d*x + c)^6 + 6*(A - 2*B)*sin(d*x + c)^5 - 15*A*sin(d*x + c)^
4 + 20*B*sin(d*x + c)^3 + 15*(2*A - B)*sin(d*x + c)^2 - 30*A*sin(d*x + c))/
(a^2*d)
```

Fricas [A] time = 1.39974, size = 204, normalized size = 2.58

$$\frac{5B\cos(dx+c)^6+15(A-B)\cos(dx+c)^4-2\left(3(A-2B)\cos(dx+c)^4-2(3A-B)\cos(dx+c)^2-12A+4B\right)\sin}{30a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/30*(5*B*cos(d*x + c)^6 + 15*(A - B)*cos(d*x + c)^4 - 2*(3*(A - 2*B)*cos(d
*x + c)^4 - 2*(3*A - B)*cos(d*x + c)^2 - 12*A + 4*B)*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.29591, size = 128, normalized size = 1.62

$$-\frac{5\,B\sin\left(dx+c\right)^{6}+6\,A\sin\left(dx+c\right)^{5}-12\,B\sin\left(dx+c\right)^{5}-15\,A\sin\left(dx+c\right)^{4}+20\,B\sin\left(dx+c\right)^{3}+30\,A\sin\left(dx+c\right)^{2}-}{30\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/30*(5*B*sin(d*x + c)^6 + 6*A*sin(d*x + c)^5 - 12*B*sin(d*x + c)^5 - 15*A
*sin(d*x + c)^4 + 20*B*sin(d*x + c)^3 + 30*A*sin(d*x + c)^2 - 15*B*sin(d*x
+ c)^2 - 30*A*sin(d*x + c))/(a^2*d)
```

3.1012 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=51

$$\frac{B(a-a\sin(c+dx))^4}{4a^6d}-\frac{(A+B)(a-a\sin(c+dx))^3}{3a^5d}$$

```
[Out] -((A + B)*(a - a*Sin[c + d*x])^3)/(3*a^5*d) + (B*(a - a*Sin[c + d*x])^4)/(4
*a^6*d)
```

Rubi [A] time = 0.102889, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 43}

$$\frac{B(a-a\sin(c+dx))^4}{4a^6d}-\frac{(A+B)(a-a\sin(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((A + B)*(a - a*Sin[c + d*x])^3)/(3*a^5*d) + (B*(a - a*Sin[c + d*x])^4)/(4
*a^6*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2\left(A+\frac{Bx}{a}\right)dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left((A+B)(a-x)^2-\frac{B(a-x)^3}{a}\right)dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= -\frac{(A+B)(a-a\sin(c+dx))^3}{3a^5d}+\frac{B(a-a\sin(c+dx))^4}{4a^6d}
\end{aligned}$$

Mathematica [A] time = 0.0631584, size = 34, normalized size = 0.67

$$\frac{(\sin(c+dx)-1)^3(4A+3B\sin(c+dx)+B)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((-1 + Sin[c + d*x])^3*(4*A + B + 3*B*Sin[c + d*x]))/(12*a^2*d)
```

Maple [A] time = 0.109, size = 58, normalized size = 1.1

$$\frac{1}{da^2}\left(\frac{B(\sin(dx+c))^4}{4}+\frac{(A-2B)(\sin(dx+c))^3}{3}+\frac{(-2A+B)(\sin(dx+c))^2}{2}+A\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(1/4*B*sin(d*x+c)^4+1/3*(A-2*B)*sin(d*x+c)^3+1/2*(-2*A+B)*sin(d*x+c
)^2+A*sin(d*x+c))
```

Maxima [A] time = 1.03872, size = 82, normalized size = 1.61

$$\frac{3B\sin(dx+c)^4+4(A-2B)\sin(dx+c)^3-6(2A-B)\sin(dx+c)^2+12A\sin(dx+c)}{12a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(3*B*sin(d*x + c)^4 + 4*(A - 2*B)*sin(d*x + c)^3 - 6*(2*A - B)*sin(d*x
 + c)^2 + 12*A*sin(d*x + c))/(a^2*d)
```

Fricas [A] time = 1.45174, size = 161, normalized size = 3.16

$$\frac{3B\cos(dx+c)^4+12(A-B)\cos(dx+c)^2-4\left((A-2B)\cos(dx+c)^2-4A+2B\right)\sin(dx+c)}{12a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*B*cos(d*x + c)^4 + 12*(A - B)*cos(d*x + c)^2 - 4*((A - 2*B)*cos(d*x
 + c)^2 - 4*A + 2*B)*sin(d*x + c))/(a^2*d)
```

Sympy [A] time = 85.0489, size = 1545, normalized size = 30.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-2*A*tan(c/2 + d*x/2)**8/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2
*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2
+ d*x/2)**2 + 9*a**2*d) + 18*A*tan(c/2 + d*x/2)**7/(9*a**2*d*tan(c/2 + d*x/
2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*
a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) - 44*A*tan(c/2 + d*x/2)**6/(9*a**2*d
*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 +
d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) + 78*A*tan(c/2 + d*x/
2)**5/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a*
*2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) - 84*A
*tan(c/2 + d*x/2)**4/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*
x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9
*a**2*d) + 78*A*tan(c/2 + d*x/2)**3/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2
*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2
+ d*x/2)**2 + 9*a**2*d) - 44*A*tan(c/2 + d*x/2)**2/(9*a**2*d*tan(c/2 + d*x/
2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*
a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) + 18*A*tan(c/2 + d*x/2)/(9*a**2*d*ta
n(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x
/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) - 2*A/(9*a**2*d*tan(c/2
+ d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4
 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) - 2*B*tan(c/2 + d*x/2)**8/(9*a
**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c
/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) + 10*B*tan(c/2 +
 d*x/2)**6/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 +
54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) -
 48*B*tan(c/2 + d*x/2)**5/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2
 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**
2 + 9*a**2*d) + 60*B*tan(c/2 + d*x/2)**4/(9*a**2*d*tan(c/2 + d*x/2)**8 + 36
*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4 + 36*a**2*d*tan
(c/2 + d*x/2)**2 + 9*a**2*d) - 48*B*tan(c/2 + d*x/2)**3/(9*a**2*d*tan(c/2 +
 d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d*x/2)**4
+ 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) + 10*B*tan(c/2 + d*x/2)**2/(9*a
**2*d*tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c
/2 + d*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d) - 2*B/(9*a**2*d*
tan(c/2 + d*x/2)**8 + 36*a**2*d*tan(c/2 + d*x/2)**6 + 54*a**2*d*tan(c/2 + d
*x/2)**4 + 36*a**2*d*tan(c/2 + d*x/2)**2 + 9*a**2*d), Ne(d, 0)), (x*(A + B*
sin(c))*cos(c)**5/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.33772, size = 99, normalized size = 1.94

$$\frac{3\,B\sin\left(dx+c\right)^{4}+4\,A\sin\left(dx+c\right)^{3}-8\,B\sin\left(dx+c\right)^{3}-12\,A\sin\left(dx+c\right)^{2}+6\,B\sin\left(dx+c\right)^{2}+12\,A\sin\left(dx+c\right)}{12\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/12*(3*B*sin(d*x + c)^4 + 4*A*sin(d*x + c)^3 - 8*B*sin(d*x + c)^3 - 12*A*s
in(d*x + c)^2 + 6*B*sin(d*x + c)^2 + 12*A*sin(d*x + c))/(a^2*d)
```

3.1013 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{(A-B)\sin(c+dx)}{a^2d}+\frac{2(A-B)\log(\sin(c+dx)+1)}{a^2d}-\frac{B(a-a\sin(c+dx))^2}{2a^4d}$$

```
[Out] (2*(A - B)*Log[1 + Sin[c + d*x]])/(a^2*d) - ((A - B)*Sin[c + d*x])/(a^2*d)
- (B*(a - a*Sin[c + d*x])^2)/(2*a^4*d)
```

Rubi [A] time = 0.107344, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$-\frac{(A-B)\sin(c+dx)}{a^2d}+\frac{2(A-B)\log(\sin(c+dx)+1)}{a^2d}-\frac{B(a-a\sin(c+dx))^2}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*(A - B)*Log[1 + Sin[c + d*x]])/(a^2*d) - ((A - B)*Sin[c + d*x])/(a^2*d)
- (B*(a - a*Sin[c + d*x])^2)/(2*a^4*d)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)\left(A+\frac{Bx}{a}\right)}{a+x}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(-A+B+\frac{B(a-x)}{a}+\frac{2a(A-B)}{a+x}\right)dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{2(A-B)\log(1+\sin(c+dx))}{a^2d}-\frac{(A-B)\sin(c+dx)}{a^2d}-\frac{B(a-a\sin(c+dx))^2}{2a^4d}\end{aligned}$$

Mathematica [A] time = 0.0931061, size = 51, normalized size = 0.77

$$-\frac{2(A-2B)\sin(c+dx)-4(A-B)\log(\sin(c+dx)+1)+B\sin^2(c+dx)+B}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(B - 4*(A - B)*Log[1 + Sin[c + d*x]] + 2*(A - 2*B)*Sin[c + d*x] + B*Sin[c
+ d*x]^2)/(2*a^2*d)
```

Maple [A] time = 0.109, size = 85, normalized size = 1.3

$$-\frac{B(\sin(dx+c))^2}{2da^2}-\frac{A\sin(dx+c)}{da^2}+2\frac{B\sin(dx+c)}{da^2}+2\frac{\ln(1+\sin(dx+c))A}{da^2}-2\frac{B\ln(1+\sin(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d/a^2*B*sin(d*x+c)^2-1/d/a^2*sin(d*x+c)*A+2/d/a^2*B*sin(d*x+c)+2/d/a^2
*ln(1+sin(d*x+c))*A-2*B*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.02946, size = 73, normalized size = 1.11

$$\frac{\frac{4(A-B)\log(\sin(dx+c)+1)}{a^2}-\frac{B\sin(dx+c)^2+2(A-2B)\sin(dx+c)}{a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(4*(A - B)*log(sin(d*x + c) + 1)/a^2 - (B*sin(d*x + c)^2 + 2*(A - 2*B)*
sin(d*x + c))/a^2)/d
```

Fricas [A] time = 1.47416, size = 126, normalized size = 1.91

$$\frac{B\cos(dx+c)^2+4(A-B)\log(\sin(dx+c)+1)-2(A-2B)\sin(dx+c)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*cos(d*x + c)^2 + 4*(A - B)*log(sin(d*x + c) + 1) - 2*(A - 2*B)*sin(d
*x + c))/(a^2*d)
```

Sympy [A] time = 27.5817, size = 1428, normalized size = 21.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((28*A*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(7*a**2*d*tan
(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) + 56*A*log(tan
(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a
**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) + 28*A*log(tan(c/2 + d*x/2) + 1)/(7*a
**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) - 14*
A*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(7*a**2*d*tan(c/2 + d*x/
2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) - 28*A*log(tan(c/2 + d*x/
2)**2 + 1)*tan(c/2 + d*x/2)**2/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*ta
n(c/2 + d*x/2)**2 + 7*a**2*d) - 14*A*log(tan(c/2 + d*x/2)**2 + 1)/(7*a**2*d
*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) + 6*A*tan(
c/2 + d*x/2)**4/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)*
*2 + 7*a**2*d) - 14*A*tan(c/2 + d*x/2)**3/(7*a**2*d*tan(c/2 + d*x/2)**4 + 1
4*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) + 12*A*tan(c/2 + d*x/2)**2/(7*a**2
*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) - 14*A*t
an(c/2 + d*x/2)/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)*
*2 + 7*a**2*d) + 6*A/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*
x/2)**2 + 7*a**2*d) - 28*B*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(7
*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) - 5
6*B*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(7*a**2*d*tan(c/2 + d*x/2
)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) - 28*B*log(tan(c/2 + d*x/2
) + 1)/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a*
*2*d) + 14*B*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(7*a**2*d*tan
(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) + 28*B*log(tan
(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(7*a**2*d*tan(c/2 + d*x/2)**4 + 1
4*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) + 14*B*log(tan(c/2 + d*x/2)**2 + 1
)/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d)
 - 10*B*tan(c/2 + d*x/2)**4/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c
/2 + d*x/2)**2 + 7*a**2*d) + 28*B*tan(c/2 + d*x/2)**3/(7*a**2*d*tan(c/2 + d
*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2*d) - 34*B*tan(c/2 + d*x/2
)**2/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c/2 + d*x/2)**2 + 7*a**2
*d) + 28*B*tan(c/2 + d*x/2)/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d*tan(c
/2 + d*x/2)**2 + 7*a**2*d) - 10*B/(7*a**2*d*tan(c/2 + d*x/2)**4 + 14*a**2*d
*tan(c/2 + d*x/2)**2 + 7*a**2*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)**3/(a
*sin(c) + a)**2, True))
```

Giac [A] time = 1.42122, size = 124, normalized size = 1.88

$$-\frac{\frac{4\,(A-B)\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}+\frac{(a\sin(dx+c)+a)^2\left(B+\frac{2\,(Aa^2-3\,Ba^2)}{(a\sin(dx+c)+a)a}\right)}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(A - B)*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a))
)/a^2 + (a*sin(d*x + c) + a)^2*(B + 2*(A*a^2 - 3*B*a^2)/((a*sin(d*x + c) +
a)*a))/a^4)/d
```

3.1014 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=44

$$\frac{B\log(\sin(c+dx)+1)}{a^2d}-\frac{A-B}{d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] (B*Log[1 + Sin[c + d*x]])/(a^2*d) - (A - B)/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0678509, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B\log(\sin(c+dx)+1)}{a^2d}-\frac{A-B}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (B*Log[1 + Sin[c + d*x]])/(a^2*d) - (A - B)/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(\frac{A-B}{(a+x)^2}+\frac{B}{a(a+x)}\right)dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{B\log(1+\sin(c+dx))}{a^2d}-\frac{A-B}{d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.0733169, size = 41, normalized size = 0.93

$$\frac{\frac{B\log(\sin(c+dx)+1)}{a}-\frac{A-B}{a\sin(c+dx)+a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((B*Log[1 + Sin[c + d*x]])/a - (A - B)/(a + a*Sin[c + d*x]))/(a*d)
```

Maple [A] time = 0.048, size = 56, normalized size = 1.3

$$-\frac{A}{da^2\left(1+\sin\left(dx+c\right)\right)}+\frac{B}{da^2\left(1+\sin\left(dx+c\right)\right)}+\frac{B\ln\left(1+\sin\left(dx+c\right)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+sin(d*x+c))*A+1/d/a^2/(1+sin(d*x+c))*B+B*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.02342, size = 58, normalized size = 1.32

$$-\frac{\frac{A-B}{a^2\sin(dx+c)+a^2}-\frac{B\log(\sin(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -((A - B)/(a^2*sin(d*x + c) + a^2) - B*log(sin(d*x + c) + 1)/a^2)/d
```

Fricas [A] time = 1.4452, size = 112, normalized size = 2.55

$$\frac{\left(B\sin\left(dx+c\right)+B\right)\log\left(\sin\left(dx+c\right)+1\right)-A+B}{a^2d\sin\left(dx+c\right)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] ((B*sin(d*x + c) + B)*log(sin(d*x + c) + 1) - A + B)/(a^2*d*sin(d*x + c) +
a^2*d)
```

Sympy [A] time = 1.0167, size = 121, normalized size = 2.75

$$\begin{cases}-\frac{A}{a^2d\sin\left(c+dx\right)+a^2d}+\frac{B\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{a^2d\sin\left(c+dx\right)+a^2d}+\frac{B\log\left(\sin\left(c+dx\right)+1\right)}{a^2d\sin\left(c+dx\right)+a^2d}+\frac{B}{a^2d\sin\left(c+dx\right)+a^2d} & \text{for}\: d\neq 0\\\frac{x\left(A+B\sin\left(c\right)\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-A/(a**2*d*sin(c + d*x) + a**2*d) + B*log(sin(c + d*x) + 1)*sin(
c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) + B*log(sin(c + d*x) + 1)/(a**2*d*s
in(c + d*x) + a**2*d) + B/(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*(A
+ B*sin(c))*cos(c)/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.33329, size = 103, normalized size = 2.34

$$-\frac{\frac{B\left(\frac{\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a}-\frac{1}{a\sin(dx+c)+a}\right)}{a}+\frac{A}{(a\sin(dx+c)+a)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -(B*(log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a - 1/(a*
sin(d*x + c) + a))/a + A/((a*sin(d*x + c) + a)*a))/d
```

3.1015 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{A+B}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{A-B}{4d(a\sin(c+dx)+a)^2}$$

```
[Out] ((A + B)*ArcTanh[Sin[c + d*x]])/(4*a^2*d) - (A - B)/(4*d*(a + a*Sin[c + d*x
])^2) - (A + B)/(4*d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.1075, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 77, 206}

$$-\frac{A+B}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{A-B}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((A + B)*ArcTanh[Sin[c + d*x]])/(4*a^2*d) - (A - B)/(4*d*(a + a*Sin[c + d*x
])^2) - (A + B)/(4*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{A-B}{2a(a+x)^3}+\frac{A+B}{4a^2(a+x)^2}+\frac{A+B}{4a^2(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{A-B}{4d(a+a\sin(c+dx))^2}-\frac{A+B}{4d\left(a^2+a^2\sin(c+dx)\right)}+\frac{(A+B)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}$$
$$= \frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{A-B}{4d(a+a\sin(c+dx))^2}-\frac{A+B}{4d\left(a^2+a^2\sin(c+d\right.}$$

Mathematica [A] time = 0.119526, size = 69, normalized size = 0.97

$$\frac{a\left(-\frac{A+B}{4a^2(a\sin(c+dx)+a)}+\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^3}-\frac{A-B}{4a(a\sin(c+dx)+a)^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a*(((A + B)*ArcTanh[Sin[c + d*x]])/(4*a^3) - (A - B)/(4*a*(a + a*Sin[c + d
*x])^2) - (A + B)/(4*a^2*(a + a*Sin[c + d*x]))))/d
```

Maple [B] time = 0.115, size = 150, normalized size = 2.1

$$-\frac{\ln(\sin(dx+c)-1)A}{8da^2}-\frac{\ln(\sin(dx+c)-1)B}{8da^2}-\frac{A}{4da^2(1+\sin(dx+c))^2}+\frac{B}{4da^2(1+\sin(dx+c))^2}-\frac{A}{4da^2(1+\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d/a^2*ln(sin(d*x+c)-1)*A-1/8/d/a^2*ln(sin(d*x+c)-1)*B-1/4/d/a^2/(1+sin
(d*x+c))^2*A+1/4/d/a^2/(1+sin(d*x+c))^2*B-1/4/d/a^2/(1+sin(d*x+c))*A-1/4/d/
a^2/(1+sin(d*x+c))*B+1/8/d/a^2*ln(1+sin(d*x+c))*A+1/8*B*ln(1+sin(d*x+c))/a^
2/d
```

Maxima [A] time = 1.04265, size = 113, normalized size = 1.59

$$-\frac{\frac{2((A+B)\sin(dx+c)+2A)}{a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2}-\frac{(A+B)\log(\sin(dx+c)+1)}{a^2}+\frac{(A+B)\log(\sin(dx+c)-1)}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/8*(2*((A + B)*sin(d*x + c) + 2*A)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x +
c) + a^2) - (A + B)*log(sin(d*x + c) + 1)/a^2 + (A + B)*log(sin(d*x + c) -
```

```
1)/a^2)/d
```

Fricas [B] time = 1.53666, size = 358, normalized size = 5.04

$$\frac{\left((A+B)\cos\left(dx+c\right)^2-2\,(A+B)\sin\left(dx+c\right)-2\,A-2\,B\right)\log\left(\sin\left(dx+c\right)+1\right)-\left((A+B)\cos\left(dx+c\right)^2-2\,(A+B)\sin\right.}{8\left(a^2d\cos\left(dx+c\right)^2-2\,a^2d\sin\left(dx+c\right)-2\,a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/8*(((A + B)*cos(d*x + c)^2 - 2*(A + B)*sin(d*x + c) - 2*A - 2*B)*log(sin(
d*x + c) + 1) - ((A + B)*cos(d*x + c)^2 - 2*(A + B)*sin(d*x + c) - 2*A - 2*
B)*log(-sin(d*x + c) + 1) + 2*(A + B)*sin(d*x + c) + 4*A)/(a^2*d*cos(d*x +
c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx+\int\frac{B\sin\left(c+dx\right)\sec\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x) + Integ
ral(B*sin(c + d*x)*sec(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x))
/a**2
```

Giac [A] time = 1.38443, size = 140, normalized size = 1.97

$$\frac{\frac{2\,(A+B)\log\left(\left|\sin(dx+c)+1\right|\right)}{a^2}-\frac{2\,(A+B)\log\left(\left|\sin(dx+c)-1\right|\right)}{a^2}-\frac{3\,A\sin(dx+c)^2+3\,B\sin(dx+c)^2+10\,A\sin(dx+c)+10\,B\sin(dx+c)+11\,A+3\,B}{a^2(\sin(dx+c)+1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/16*(2*(A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 2*(A + B)*log(abs(sin(d*x
+ c) - 1))/a^2 - (3*A*sin(d*x + c)^2 + 3*B*sin(d*x + c)^2 + 10*A*sin(d*x +
c) + 10*B*sin(d*x + c) + 11*A + 3*B)/(a^2*(sin(d*x + c) + 1)^2))/d
```

3.1016 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$\frac{A+B}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{3A+B}{16d\left(a^2\sin(c+dx)+a^2\right)}+\frac{(2A+B)\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{a(A-B)}{12d(a\sin(c+dx)+a)^3}-\frac{}{8d}$$

```
[Out] ((2*A + B)*ArcTanh[Sin[c + d*x]])/(8*a^2*d) - (a*(A - B))/(12*d*(a + a*Sin[
c + d*x])^3) - A/(8*d*(a + a*Sin[c + d*x])^2) + (A + B)/(16*d*(a^2 - a^2*Si
n[c + d*x])) - (3*A + B)/(16*d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.154616, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$\frac{A+B}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{3A+B}{16d\left(a^2\sin(c+dx)+a^2\right)}+\frac{(2A+B)\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{a(A-B)}{12d(a\sin(c+dx)+a)^3}-\frac{}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((2*A + B)*ArcTanh[Sin[c + d*x]])/(8*a^2*d) - (a*(A - B))/(12*d*(a + a*Sin[
c + d*x])^3) - A/(8*d*(a + a*Sin[c + d*x])^2) + (A + B)/(16*d*(a^2 - a^2*Si
n[c + d*x])) - (3*A + B)/(16*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a^3\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{A+B}{16a^4(a-x)^2}+\frac{A-B}{4a^2(a+x)^4}+\frac{A}{4a^3(a+x)^3}+\frac{3A+B}{16a^4(a+x)^2}+\frac{2A+B}{8a^4(a^2-x^2)}\right) dx, x, a\,\text{si}\right.}{d}$$

$$= -\frac{a(A-B)}{12d(a+a\sin(c+dx))^3}-\frac{A}{8d(a+a\sin(c+dx))^2}+\frac{A+B}{16d\left(a^2-a^2\sin(c+dx)\right)} -$$

$$= \frac{(2A+B)\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{a(A-B)}{12d(a+a\sin(c+dx))^3}-\frac{A}{8d(a+a\sin(c+dx))}$$

Mathematica [A] time = 0.678867, size = 87, normalized size = 0.71

$$-\frac{\frac{3(A+B)}{\sin(c+dx)-1}+\frac{3(3A+B)}{\sin(c+dx)+1}+\frac{4(A-B)}{(\sin(c+dx)+1)^3}-6(2A+B)\tanh^{-1}(\sin(c+dx))+\frac{6A}{(\sin(c+dx)+1)^2}}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(-6*(2*A + B)*ArcTanh[Sin[c + d*x]] + (3*(A + B))/(-1 + Sin[c + d*x]) + (4
*(A - B))/(1 + Sin[c + d*x])^3 + (6*A)/(1 + Sin[c + d*x])^2 + (3*(3*A + B))
/(1 + Sin[c + d*x]))/(48*a^2*d)
```

Maple [A] time = 0.138, size = 207, normalized size = 1.7

$$-\frac{\ln(\sin(dx+c)-1)A}{8\,da^2}-\frac{\ln(\sin(dx+c)-1)B}{16\,da^2}-\frac{A}{16\,da^2(\sin(dx+c)-1)}-\frac{B}{16\,da^2(\sin(dx+c)-1)}-\frac{A}{8\,da^2(1+\sin(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d/a^2*ln(sin(d*x+c)-1)*A-1/16/d/a^2*ln(sin(d*x+c)-1)*B-1/16/d/a^2/(sin
(d*x+c)-1)*A-1/16/d/a^2/(sin(d*x+c)-1)*B-1/8/d/a^2/(1+sin(d*x+c))^2*A-1/12/
d/a^2/(1+sin(d*x+c))^3*A+1/12/d/a^2/(1+sin(d*x+c))^3*B+1/8/d/a^2*ln(1+sin(d
*x+c))*A+1/16*B*ln(1+sin(d*x+c))/a^2/d-3/16/d/a^2/(1+sin(d*x+c))*A-1/16/d/a
^2/(1+sin(d*x+c))*B
```

Maxima [A] time = 1.03721, size = 188, normalized size = 1.53

$$-\frac{\frac{2\left(3\,(2\,A+B)\sin(dx+c)^3+6\,(2\,A+B)\sin(dx+c)^2+(2\,A+B)\sin(dx+c)-8\,A+2\,B\right)}{a^2\sin(dx+c)^4+2\,a^2\sin(dx+c)^3-2\,a^2\sin(dx+c)-a^2}-\frac{3\,(2\,A+B)\log(\sin(dx+c)+1)}{a^2}+\frac{3\,(2\,A+B)\log(\sin(dx+c)-1)}{a^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/48*(2*(3*(2*A + B)*sin(d*x + c)^3 + 6*(2*A + B)*sin(d*x + c)^2 + (2*A +
B)*sin(d*x + c) - 8*A + 2*B)/(a^2*sin(d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2
*a^2*sin(d*x + c) - a^2) - 3*(2*A + B)*log(sin(d*x + c) + 1)/a^2 + 3*(2*A +
 B)*log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [B] time = 1.52442, size = 597, normalized size = 4.85

$$\frac{12\,(2\,A+B)\cos\left(dx+c\right)^{2}+3\left(\left(2\,A+B\right)\cos\left(dx+c\right)^{4}-2\,(2\,A+B)\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-2\,(2\,A+B)\cos\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/48*(12*(2*A + B)*cos(d*x + c)^2 + 3*((2*A + B)*cos(d*x + c)^4 - 2*(2*A +
B)*cos(d*x + c)^2*sin(d*x + c) - 2*(2*A + B)*cos(d*x + c)^2)*log(sin(d*x +
c) + 1) - 3*((2*A + B)*cos(d*x + c)^4 - 2*(2*A + B)*cos(d*x + c)^2*sin(d*x
+ c) - 2*(2*A + B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(3*(2*A + B)*
cos(d*x + c)^2 - 8*A - 4*B)*sin(d*x + c) - 8*A - 16*B)/(a^2*d*cos(d*x + c)^
4 - 2*a^2*d*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36114, size = 228, normalized size = 1.85

$$\frac{\frac{6\,(2\,A+B)\log\left(|\sin(dx+c)+1|\right)}{a^{2}}-\frac{6\,(2\,A+B)\log\left(|\sin(dx+c)-1|\right)}{a^{2}}+\frac{6\,(2\,A\sin(dx+c)+B\sin(dx+c)-3\,A-2\,B)}{a^{2}(\sin(dx+c)-1)}-\frac{22\,A\sin(dx+c)^{3}+11\,B\sin(dx+c)^{3}+84\,A\sin(d}{}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/96*(6*(2*A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 6*(2*A + B)*log(abs(sin(
d*x + c) - 1))/a^2 + 6*(2*A*sin(d*x + c) + B*sin(d*x + c) - 3*A - 2*B)/(a^2
*(sin(d*x + c) - 1)) - (22*A*sin(d*x + c)^3 + 11*B*sin(d*x + c)^3 + 84*A*si
n(d*x + c)^2 + 39*B*sin(d*x + c)^2 + 114*A*sin(d*x + c) + 45*B*sin(d*x + c)
 + 60*A + 9*B)/(a^2*(sin(d*x + c) + 1)^3))/d
```

3.1017 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$-\frac{a^2(A-B)}{32d(a\sin(c+dx)+a)^4}+\frac{5A+3B}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5A+B}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5(3A+B)\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{}{48}$$

```
[Out] (5*(3*A + B)*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + (A + B)/(64*d*(a - a*Sin[c
 + d*x])^2) - (a^2*(A - B))/(32*d*(a + a*Sin[c + d*x])^4) - (a*(3*A - B))/(
48*d*(a + a*Sin[c + d*x])^3) - (3*A)/(32*d*(a + a*Sin[c + d*x])^2) + (5*A +
 3*B)/(64*d*(a^2 - a^2*Sin[c + d*x])) - (5*A + B)/(32*d*(a^2 + a^2*Sin[c +
d*x]))
```

Rubi [A] time = 0.205736, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$-\frac{a^2(A-B)}{32d(a\sin(c+dx)+a)^4}+\frac{5A+3B}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5A+B}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5(3A+B)\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{}{48}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*(3*A + B)*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + (A + B)/(64*d*(a - a*Sin[c
 + d*x])^2) - (a^2*(A - B))/(32*d*(a + a*Sin[c + d*x])^4) - (a*(3*A - B))/(
48*d*(a + a*Sin[c + d*x])^3) - (3*A)/(32*d*(a + a*Sin[c + d*x])^2) + (5*A +
 3*B)/(64*d*(a^2 - a^2*Sin[c + d*x])) - (5*A + B)/(32*d*(a^2 + a^2*Sin[c +
d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int\left(\frac{A+B}{32a^5(a-x)^3} + \frac{5A+3B}{64a^6(a-x)^2} + \frac{A-B}{8a^3(a+x)^5} + \frac{3A-B}{16a^4(a+x)^4} + \frac{3A}{16a^5(a+x)^3} + \frac{5A}{32a^6(}\right.\right.}{d}$$

$$= \frac{A+B}{64d(a-a\sin(c+dx))^2} - \frac{a^2(A-B)}{32d(a+a\sin(c+dx))^4} - \frac{a(3A-B)}{48d(a+a\sin(c+dx))^3}$$

$$= \frac{5(3A+B)\tanh^{-1}(\sin(c+dx))}{64a^2d} + \frac{A+B}{64d(a-a\sin(c+dx))^2} - \frac{a^2(A-B}{32d(a+a\sin(c}$$

Mathematica [A] time = 0.664118, size = 123, normalized size = 0.69

$$\frac{-\frac{3(5A+3B)}{\sin(c+dx)-1} - \frac{6(5A+B)}{\sin(c+dx)+1} + \frac{3(A+B)}{(\sin(c+dx)-1)^2} + \frac{4(B-3A)}{(\sin(c+dx)+1)^3} - \frac{6(A-B)}{(\sin(c+dx)+1)^4} + 15(3A+B)\tanh^{-1}(\sin(c+dx)) - \frac{18A}{(\sin(c+dx)+1)^2}}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (15*(3*A + B)*ArcTanh[Sin[c + d*x]] + (3*(A + B))/(-1 + Sin[c + d*x])^2 - (
3*(5*A + 3*B))/(-1 + Sin[c + d*x]) - (6*(A - B))/(1 + Sin[c + d*x])^4 + (4*
(-3*A + B))/(1 + Sin[c + d*x])^3 - (18*A)/(1 + Sin[c + d*x])^2 - (6*(5*A +
B))/(1 + Sin[c + d*x]))/(192*a^2*d)
```

Maple [A] time = 0.135, size = 283, normalized size = 1.6

$$-\frac{15\ln(\sin(dx+c)-1)A}{128\,da^2} - \frac{5\ln(\sin(dx+c)-1)B}{128\,da^2} + \frac{A}{64\,da^2(\sin(dx+c)-1)^2} + \frac{B}{64\,da^2(\sin(dx+c)-1)^2} - \frac{}{64\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -15/128/d/a^2*ln(sin(d*x+c)-1)*A-5/128/d/a^2*ln(sin(d*x+c)-1)*B+1/64/d/a^2/
(sin(d*x+c)-1)^2*A+1/64/d/a^2/(sin(d*x+c)-1)^2*B-5/64/d/a^2/(sin(d*x+c)-1)*
A-3/64/d/a^2/(sin(d*x+c)-1)*B-3/32/d/a^2/(1+sin(d*x+c))^2*A-1/32/d/a^2/(1+s
in(d*x+c))^4*A+1/32/d/a^2/(1+sin(d*x+c))^4*B-1/16/d/a^2/(1+sin(d*x+c))^3*A+
1/48/d/a^2/(1+sin(d*x+c))^3*B-5/32/d/a^2/(1+sin(d*x+c))*A-1/32/d/a^2/(1+sin
(d*x+c))*B+15/128/d/a^2*ln(1+sin(d*x+c))*A+5/128*B*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.05767, size = 279, normalized size = 1.56

$$-\frac{\frac{2\left(15(3A+B)\sin(dx+c)^5+30(3A+B)\sin(dx+c)^4-10(3A+B)\sin(dx+c)^3-50(3A+B)\sin(dx+c)^2-17(3A+B)\sin(dx+c)+48A-16B\right)}{a^2\sin(dx+c)^6+2a^2\sin(dx+c)^5-a^2\sin(dx+c)^4-4a^2\sin(dx+c)^3-a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2} - \frac{15(3A+B)\log(\text{si}}{a^2}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/384*(2*(15*(3*A + B)*sin(d*x + c)^5 + 30*(3*A + B)*sin(d*x + c)^4 - 10*(
3*A + B)*sin(d*x + c)^3 - 50*(3*A + B)*sin(d*x + c)^2 - 17*(3*A + B)*sin(d*
x + c) + 48*A - 16*B)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x + c)^5 - a^2*sin(
d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c)
 + a^2) - 15*(3*A + B)*log(sin(d*x + c) + 1)/a^2 + 15*(3*A + B)*log(sin(d*x
 + c) - 1)/a^2)/d
```

Fricas [A] time = 1.65888, size = 687, normalized size = 3.84

$$\frac{60\,(3\,A+B)\cos\left(dx+c\right)^4-20\,(3\,A+B)\cos\left(dx+c\right)^2+15\left((3\,A+B)\cos\left(dx+c\right)^6-2\,(3\,A+B)\cos\left(dx+c\right)^4\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/384*(60*(3*A + B)*cos(d*x + c)^4 - 20*(3*A + B)*cos(d*x + c)^2 + 15*((3*A
 + B)*cos(d*x + c)^6 - 2*(3*A + B)*cos(d*x + c)^4*sin(d*x + c) - 2*(3*A + B
)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*((3*A + B)*cos(d*x + c)^6 - 2*
(3*A + B)*cos(d*x + c)^4*sin(d*x + c) - 2*(3*A + B)*cos(d*x + c)^4)*log(-si
n(d*x + c) + 1) + 2*(15*(3*A + B)*cos(d*x + c)^4 - 20*(3*A + B)*cos(d*x + c
)^2 - 36*A - 12*B)*sin(d*x + c) - 24*A - 72*B)/(a^2*d*cos(d*x + c)^6 - 2*a^
2*d*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44561, size = 289, normalized size = 1.61

$$\frac{\frac{60\,(3\,A+B)\log(|\sin(dx+c)+1|)}{a^2}-\frac{60\,(3\,A+B)\log(|\sin(dx+c)-1|)}{a^2}+\frac{6\left(45\,A\sin(dx+c)^2+15\,B\sin(dx+c)^2-110\,A\sin(dx+c)-42\,B\sin(dx+c)+69\,A+31\,B\right)}{a^2(\sin(dx+c)-1)^2}-\frac{375}{}}{153}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/1536*(60*(3*A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 60*(3*A + B)*log(abs(
sin(d*x + c) - 1))/a^2 + 6*(45*A*sin(d*x + c)^2 + 15*B*sin(d*x + c)^2 - 110
```

```
*A*sin(d*x + c) - 42*B*sin(d*x + c) + 69*A + 31*B)/(a^2*(sin(d*x + c) - 1)^
2) - (375*A*sin(d*x + c)^4 + 125*B*sin(d*x + c)^4 + 1740*A*sin(d*x + c)^3 +
 548*B*sin(d*x + c)^3 + 3114*A*sin(d*x + c)^2 + 894*B*sin(d*x + c)^2 + 2604
*A*sin(d*x + c) + 612*B*sin(d*x + c) + 903*A + 93*B)/(a^2*(sin(d*x + c) + 1
)^4))/d
```

3.1018 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=236

$$-\frac{a^3(A-B)}{80d(a\sin(c+dx)+a)^5}-\frac{a^2(2A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{3(7A+3B)}{256d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5(7A+B)}{256d\left(a^2\sin(c+dx)+a^2\right)}+\frac{7(4A+$$

```
[Out] (7*(4*A + B)*ArcTanh[Sin[c + d*x]])/(128*a^2*d) + (a*(A + B))/(192*d*(a - a
*Sin[c + d*x])^3) + (3*A + 2*B)/(128*d*(a - a*Sin[c + d*x])^2) - (a^3*(A -
B))/(80*d*(a + a*Sin[c + d*x])^5) - (a^2*(2*A - B))/(64*d*(a + a*Sin[c + d*
x])^4) - (a*(5*A - B))/(96*d*(a + a*Sin[c + d*x])^3) - (5*A)/(64*d*(a + a*S
in[c + d*x])^2) + (3*(7*A + 3*B))/(256*d*(a^2 - a^2*Sin[c + d*x])) - (5*(7*
A + B))/(256*d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.278553, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 77, 206}

$$-\frac{a^3(A-B)}{80d(a\sin(c+dx)+a)^5}-\frac{a^2(2A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{3(7A+3B)}{256d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5(7A+B)}{256d\left(a^2\sin(c+dx)+a^2\right)}+\frac{7(4A+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*(4*A + B)*ArcTanh[Sin[c + d*x]])/(128*a^2*d) + (a*(A + B))/(192*d*(a - a
*Sin[c + d*x])^3) + (3*A + 2*B)/(128*d*(a - a*Sin[c + d*x])^2) - (a^3*(A -
B))/(80*d*(a + a*Sin[c + d*x])^5) - (a^2*(2*A - B))/(64*d*(a + a*Sin[c + d*
x])^4) - (a*(5*A - B))/(96*d*(a + a*Sin[c + d*x])^3) - (5*A)/(64*d*(a + a*S
in[c + d*x])^2) + (3*(7*A + 3*B))/(256*d*(a^2 - a^2*Sin[c + d*x])) - (5*(7*
A + B))/(256*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a^7\,\mathrm{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\mathrm{Subst}\left(\int\left(\frac{A+B}{64a^6(a-x)^4}+\frac{3A+2B}{64a^7(a-x)^3}+\frac{3(7A+3B)}{256a^8(a-x)^2}+\frac{A-B}{16a^4(a+x)^6}+\frac{2A-B}{16a^5(a+x)^5}+\ldots\right.\right.}{d}$$

$$= \frac{a(A+B)}{192d(a-a\sin(c+dx))^3}+\frac{3A+2B}{128d(a-a\sin(c+dx))^2}-\frac{a^3(A-B)}{80d(a+a\sin(c+dx))\ldots}$$

$$= \frac{7(4A+B)\tanh^{-1}(\sin(c+dx))}{128a^2d}+\frac{a(A+B)}{192d(a-a\sin(c+dx))^3}+\frac{3A+2\ldots}{128d(a-a\sin\ldots}$$

Mathematica [A] time = 1.37362, size = 160, normalized size = 0.68

$$\frac{210(4A+B)\tanh^{-1}(\sin(c+dx))-\frac{2\left(105(4A+B)\sin^7(c+dx)+210(4A+B)\sin^6(c+dx)-175(4A+B)\sin^5(c+dx)-560(4A+B)\sin^4(c+dx)-49(4A+B\ldots\right.}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^5}}{3840a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (210*(4*A + B)*ArcTanh[Sin[c + d*x]] - (2*(48*(-8*A + 3*B) + 183*(4*A + B)*
Sin[c + d*x] + 462*(4*A + B)*Sin[c + d*x]^2 - 49*(4*A + B)*Sin[c + d*x]^3 -
 560*(4*A + B)*Sin[c + d*x]^4 - 175*(4*A + B)*Sin[c + d*x]^5 + 210*(4*A + B
)*Sin[c + d*x]^6 + 105*(4*A + B)*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^3*(1
 + Sin[c + d*x])^5))/(3840*a^2*d)
```

Maple [A] time = 0.141, size = 359, normalized size = 1.5

$$-\frac{7\ln(\sin(dx+c)-1)A}{64da^2}-\frac{7\ln(\sin(dx+c)-1)B}{256da^2}+\frac{3A}{128da^2(\sin(dx+c)-1)^2}+\frac{B}{64da^2(\sin(dx+c)-1)^2}-\frac{\ldots}{192\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -7/64/d/a^2*ln(sin(d*x+c)-1)*A-7/256/d/a^2*ln(sin(d*x+c)-1)*B+3/128/d/a^2/(
sin(d*x+c)-1)^2*A+1/64/d/a^2/(sin(d*x+c)-1)^2*B-1/192/d/a^2/(sin(d*x+c)-1)^
3*A-1/192/d/a^2/(sin(d*x+c)-1)^3*B-21/256/d/a^2/(sin(d*x+c)-1)*A-9/256/d/a^
2/(sin(d*x+c)-1)*B-5/64/d/a^2/(1+sin(d*x+c))^2*A-1/80/d/a^2/(1+sin(d*x+c))^
5*A+1/80/d/a^2/(1+sin(d*x+c))^5*B-1/32/d/a^2/(1+sin(d*x+c))^4*A+1/64/d/a^2/
(1+sin(d*x+c))^4*B-5/96/d/a^2/(1+sin(d*x+c))^3*A+1/96/d/a^2/(1+sin(d*x+c))^
3*B+7/64/d/a^2*ln(1+sin(d*x+c))*A+7/256*B*ln(1+sin(d*x+c))/a^2/d-35/256/d/a
^2/(1+sin(d*x+c))*A-5/256/d/a^2/(1+sin(d*x+c))*B
```

Maxima [A] time = 1.05955, size = 340, normalized size = 1.44

$$-\frac{\frac{2\left(105(4A+B)\sin(dx+c)^7+210(4A+B)\sin(dx+c)^6-175(4A+B)\sin(dx+c)^5-560(4A+B)\sin(dx+c)^4-49(4A+B)\sin(dx+c)^3+462(4A+B)\sin(dx+c)^2+1\ldots\right.}{a^2\sin(dx+c)^8+2a^2\sin(dx+c)^7-2a^2\sin(dx+c)^6-6a^2\sin(dx+c)^5+6a^2\sin(dx+c)^3+2a^2\sin(dx+c)^2-2a^2\sin(dx+c)-a^2\ldots}}{3840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/3840*(2*(105*(4*A + B)*sin(d*x + c)^7 + 210*(4*A + B)*sin(d*x + c)^6 - 1
75*(4*A + B)*sin(d*x + c)^5 - 560*(4*A + B)*sin(d*x + c)^4 - 49*(4*A + B)*s
in(d*x + c)^3 + 462*(4*A + B)*sin(d*x + c)^2 + 183*(4*A + B)*sin(d*x + c) -
 384*A + 144*B)/(a^2*sin(d*x + c)^8 + 2*a^2*sin(d*x + c)^7 - 2*a^2*sin(d*x
+ c)^6 - 6*a^2*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^3 + 2*a^2*sin(d*x + c)^2
 - 2*a^2*sin(d*x + c) - a^2) - 105*(4*A + B)*log(sin(d*x + c) + 1)/a^2 + 10
5*(4*A + B)*log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A] time = 1.78837, size = 782, normalized size = 3.31

$$\frac{420\,(4\,A+B)\cos{(dx+c)}^{6}-140\,(4\,A+B)\cos{(dx+c)}^{4}-56\,(4\,A+B)\cos{(dx+c)}^{2}+105\,\Big((4\,A+B)\cos{(dx+c)}^{8}-2\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3840*(420*(4*A + B)*cos(d*x + c)^6 - 140*(4*A + B)*cos(d*x + c)^4 - 56*(4
*A + B)*cos(d*x + c)^2 + 105*((4*A + B)*cos(d*x + c)^8 - 2*(4*A + B)*cos(d*
x + c)^6*sin(d*x + c) - 2*(4*A + B)*cos(d*x + c)^6)*log(sin(d*x + c) + 1) -
 105*((4*A + B)*cos(d*x + c)^8 - 2*(4*A + B)*cos(d*x + c)^6*sin(d*x + c) -
2*(4*A + B)*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(105*(4*A + B)*cos(d
*x + c)^6 - 140*(4*A + B)*cos(d*x + c)^4 - 84*(4*A + B)*cos(d*x + c)^2 - 25
6*A - 64*B)*sin(d*x + c) - 128*A - 512*B)/(a^2*d*cos(d*x + c)^8 - 2*a^2*d*c
os(d*x + c)^6*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6953, size = 348, normalized size = 1.47

$$\frac{\frac{420\,(4\,A+B)\log(|\sin(dx+c)+1|)}{a^{2}}-\frac{420\,(4\,A+B)\log(|\sin(dx+c)-1|)}{a^{2}}+\frac{10\left(308\,A\sin(dx+c)^{3}+77\,B\sin(dx+c)^{3}-1050\,A\sin(dx+c)^{2}-285\,B\sin(dx+c)^{2}+1212\,A\sin\cdots\right.}{a^{2}(\sin(dx+c)-1)^{3}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/15360*(420*(4*A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 420*(4*A + B)*log(a
bs(sin(d*x + c) - 1))/a^2 + 10*(308*A*sin(d*x + c)^3 + 77*B*sin(d*x + c)^3
- 1050*A*sin(d*x + c)^2 - 285*B*sin(d*x + c)^2 + 1212*A*sin(d*x + c) + 363*
B*sin(d*x + c) - 478*A - 163*B)/(a^2*(sin(d*x + c) - 1)^3) - (3836*A*sin(d*
x + c)^5 + 959*B*sin(d*x + c)^5 + 21280*A*sin(d*x + c)^4 + 5095*B*sin(d*x +
 c)^4 + 47960*A*sin(d*x + c)^3 + 10790*B*sin(d*x + c)^3 + 55360*A*sin(d*x +
 c)^2 + 11230*B*sin(d*x + c)^2 + 33260*A*sin(d*x + c) + 5435*B*sin(d*x + c)
 + 8608*A + 667*B)/(a^2*(sin(d*x + c) + 1)^5))/d
```

3.1019 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=170

$$-\frac{a2^{\frac{1}{2}(2m+p+1)}(A(m+p+1)+Bm)(a\sin(e+fx)+a)^{m-1}(g\cos(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+\right.}{fg(p+1)(m+p+1)}$$

```
[Out] -((2^((1 + 2*m + p)/2)*a*(B*m + A*(1 + m + p))*(g*Cos[e + f*x])^(1 + p)*Hyp
ergeometric2F1[(1 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[e + f*x])/2]
*(1 + Sin[e + f*x])^((1 - 2*m - p)/2)*(a + a*Sin[e + f*x])^(-1 + m))/(f*g*(
1 + p)*(1 + m + p))) - (B*(g*Cos[e + f*x])^(1 + p)*(a + a*Sin[e + f*x])^m)/
(f*g*(1 + m + p))
```

Rubi [A] time = 0.269419, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2860, 2689, 70, 69}

$$-\frac{a2^{\frac{1}{2}(2m+p+1)}(A(m+p+1)+Bm)(a\sin(e+fx)+a)^{m-1}(g\cos(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+\right.}{fg(p+1)(m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^((1 + 2*m + p)/2)*a*(B*m + A*(1 + m + p))*(g*Cos[e + f*x])^(1 + p)*Hyp
ergeometric2F1[(1 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[e + f*x])/2]
*(1 + Sin[e + f*x])^((1 - 2*m - p)/2)*(a + a*Sin[e + f*x])^(-1 + m))/(f*g*(
1 + p)*(1 + m + p))) - (B*(g*Cos[e + f*x])^(1 + p)*(a + a*Sin[e + f*x])^m)/
(f*g*(1 + m + p))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m}{fg(1+m+p)} + \left(A+\frac{}{1+}\right.$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m}{fg(1+m+p)} + \frac{\left(a^2\left(A+\right.\right.}{}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m}{fg(1+m+p)} + \frac{\left(2^{-\frac{1}{2}+m+}\right.}{}$$

$$= -\frac{2^{\frac{1}{2}(1+2m+p)}a\left(A+\frac{Bm}{1+m+p}\right)(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1\right.}{}$$

Mathematica [A] time = 0.447754, size = 154, normalized size = 0.91

$$-\frac{\cos(e+fx)(a(\sin(e+fx)+1))^m(g\cos(e+fx))^p(\sin(e+fx)+1)^{\frac{1}{2}(-2m-p-1)}\left(2^{\frac{1}{2}(2m+p+1)}(A(m+p+1)+Bm)\,{}_2F_1\left(\frac{1}{2}\right.\right.}{f(p+1)(m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(g*Cos[e + f*x])^p*(1 + Sin[e + f*x])^((-1 - 2*m - p)/2)*(a
*(1 + Sin[e + f*x]))^m*(2^((1 + 2*m + p)/2)*(B*m + A*(1 + m + p))*Hypergeom
etric2F1[(1 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[e + f*x])/2] + B*(
1 + p)*(1 + Sin[e + f*x])^((1 + 2*m + p)/2)))/(f*(1 + p)*(1 + m + p)))
```

Maple [F] time = 4.148, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^p\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] Exception raised: AttributeError
```

3.1020 $\int \cos^7(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=159

$$\frac{8(A-B)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}-\frac{4(3A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}+\frac{6(A-3B)(a\sin(e+fx)+a)^{m+6}}{a^6f(m+6)}-\frac{(A-7B)(a$$

```
[Out] (8*(A - B)*(a + a*Sin[e + f*x])^(4 + m))/(a^4*f*(4 + m)) - (4*(3*A - 5*B)*(
a + a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m)) + (6*(A - 3*B)*(a + a*Sin[e +
f*x])^(6 + m))/(a^6*f*(6 + m)) - ((A - 7*B)*(a + a*Sin[e + f*x])^(7 + m))/(
a^7*f*(7 + m)) - (B*(a + a*Sin[e + f*x])^(8 + m))/(a^8*f*(8 + m))
```

Rubi [A] time = 0.166194, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{8(A-B)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}-\frac{4(3A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}+\frac{6(A-3B)(a\sin(e+fx)+a)^{m+6}}{a^6f(m+6)}-\frac{(A-7B)(a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^7*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```


```
[Out] (8*(A - B)*(a + a*Sin[e + f*x])^(4 + m))/(a^4*f*(4 + m)) - (4*(3*A - 5*B)*(
a + a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m)) + (6*(A - 3*B)*(a + a*Sin[e +
f*x])^(6 + m))/(a^6*f*(6 + m)) - ((A - 7*B)*(a + a*Sin[e + f*x])^(7 + m))/(
a^7*f*(7 + m)) - (B*(a + a*Sin[e + f*x])^(8 + m))/(a^8*f*(8 + m))
```


Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^7(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{3+m}\left(A+\frac{Bx}{a}\right)dx, x, a\sin(e+fx)\right)}{a^7 f}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^{3+m} - 4a^2(3A-5B)(a+x)^{4+m}\right.\right.}{}$$
$$= \frac{8(A-B)(a+a\sin(e+fx))^{4+m}}{a^4 f(4+m)} - \frac{4(3A-5B)(a+a\sin(e+}{a^5 f(5+m)}$$

Mathematica [A] time = 0.729528, size = 132, normalized size = 0.83

$$\frac{(a(\sin(e+fx)+1))^{m+4}\left(-\frac{a^4(A-7B)(\sin(e+fx)+1)^3}{m+7}+\frac{6a^4(A-3B)(\sin(e+fx)+1)^2}{m+6}-\frac{4a^4(3A-5B)(\sin(e+fx)+1)}{m+5}+\frac{8a^4(A-B)}{m+4}-\frac{B(a\sin(e+fx)+a)^4}{m+8}\right.}{a^8 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^7*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(4 + m)*((8*a^4*(A - B))/(4 + m) - (4*a^4*(3*A - 5*
B)*(1 + Sin[e + f*x]))/(5 + m) + (6*a^4*(A - 3*B)*(1 + Sin[e + f*x])^2)/(6
+ m) - (a^4*(A - 7*B)*(1 + Sin[e + f*x])^3)/(7 + m) - (B*(a + a*Sin[e + f*x
])^4)/(8 + m)))/(a^8*f)
```

Maple [F] time = 13.852, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^7 \left(a+a\sin\left(fx+e\right)\right)^m \left(A+B\sin\left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.00434, size = 867, normalized size = 5.45

$$-\frac{\left(\left(Bm^4+22\,Bm^3+179\,Bm^2+638\,Bm+840\,B\right)\cos\left(fx+e\right)^8-\left((A+B)m^4+(17\,A+9\,B)m^3+4\,(23\,A+5\,B)m^2+16\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -((B*m^4 + 22*B*m^3 + 179*B*m^2 + 638*B*m + 840*B)*cos(f*x + e)^8 - ((A + B
)*m^4 + (17*A + 9*B)*m^3 + 4*(23*A + 5*B)*m^2 + 160*A*m)*cos(f*x + e)^6 - 1
2*((A + B)*m^3 + (11*A + 3*B)*m^2 + 24*A*m)*cos(f*x + e)^4 - 96*((A + B)*m^
2 + 8*A*m)*cos(f*x + e)^2 - 384*(A + B)*m - (((A + B)*m^4 + (23*A + 15*B)*m
^3 + 2*(97*A + 37*B)*m^2 + 8*(89*A + 15*B)*m + 960*A)*cos(f*x + e)^6 + 12*(
(A + B)*m^3 + (15*A + 7*B)*m^2 + 4*(17*A + 3*B)*m + 96*A)*cos(f*x + e)^4 +
96*((A + B)*m^2 + 2*(5*A + B)*m + 16*A)*cos(f*x + e)^2 + 384*(A + B)*m + 30
72*A)*sin(f*x + e) - 3072*A)*(a*sin(f*x + e) + a)^m/(f*m^5 + 30*f*m^4 + 355
*f*m^3 + 2070*f*m^2 + 5944*f*m + 6720*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**7*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36109, size = 1893, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] -(((a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*m^3 - 6*(a*sin(f*x + e) +
a)^6*(a*sin(f*x + e) + a)^m*a*m^3 + 12*(a*sin(f*x + e) + a)^5*(a*sin(f*x +
e) + a)^m*a^2*m^3 - 8*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a^3*m^3
 + 15*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*m^2 - 96*(a*sin(f*x + e
) + a)^6*(a*sin(f*x + e) + a)^m*a*m^2 + 204*(a*sin(f*x + e) + a)^5*(a*sin(f
*x + e) + a)^m*a^2*m^2 - 144*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*
a^3*m^2 + 74*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*m - 498*(a*sin(f
*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a*m + 1128*(a*sin(f*x + e) + a)^5*(a*
sin(f*x + e) + a)^m*a^2*m - 856*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)
^m*a^3*m + 120*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m - 840*(a*sin(f
*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a + 2016*(a*sin(f*x + e) + a)^5*(a*si
n(f*x + e) + a)^m*a^2 - 1680*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*
a^3)*A/(a^6*m^4 + 22*a^6*m^3 + 179*a^6*m^2 + 638*a^6*m + 840*a^6) + ((a*sin
(f*x + e) + a)^8*(a*sin(f*x + e) + a)^m*m^4 - 7*(a*sin(f*x + e) + a)^7*(a*s
in(f*x + e) + a)^m*a*m^4 + 18*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m
*a^2*m^4 - 20*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m^4 + 8*(a*
sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a^4*m^4 + 22*(a*sin(f*x + e) + a
)^8*(a*sin(f*x + e) + a)^m*m^3 - 161*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e)
 + a)^m*a*m^3 + 432*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2*m^3 -
 500*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m^3 + 208*(a*sin(f*x
```

```
 + e) + a)^4*(a*sin(f*x + e) + a)^m*a^4*m^3 + 179*(a*sin(f*x + e) + a)^8*(a
*sin(f*x + e) + a)^m*m^2 - 1358*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)
^m*a*m^2 + 3798*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2*m^2 - 460
0*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m^2 + 2008*(a*sin(f*x +
 e) + a)^4*(a*sin(f*x + e) + a)^m*a^4*m^2 + 638*(a*sin(f*x + e) + a)^8*(a*s
in(f*x + e) + a)^m*m - 4984*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*a
*m + 14472*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2*m - 18400*(a*s
in(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m + 8528*(a*sin(f*x + e) + a)
^4*(a*sin(f*x + e) + a)^m*a^4*m + 840*(a*sin(f*x + e) + a)^8*(a*sin(f*x + e
) + a)^m - 6720*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*a + 20160*(a*
sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2 - 26880*(a*sin(f*x + e) + a)
^5*(a*sin(f*x + e) + a)^m*a^3 + 13440*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e
) + a)^m*a^4)*B/((a^6*m^5 + 30*a^6*m^4 + 355*a^6*m^3 + 2070*a^6*m^2 + 5944*
a^6*m + 6720*a^6)*a))/(a*f)
```

3.1021 $\int \cos^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=123

$$\frac{4(A-B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}-\frac{4(A-2B)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{(A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}+\frac{B(a\sin(e+f}{a^6f(m}$$

```
[Out] (4*(A - B)*(a + a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (4*(A - 2*B)*(a
+ a*Sin[e + f*x])^(4 + m))/(a^4*f*(4 + m)) + ((A - 5*B)*(a + a*Sin[e + f*x]
)^(5 + m))/(a^5*f*(5 + m)) + (B*(a + a*Sin[e + f*x])^(6 + m))/(a^6*f*(6 + m
))
```

Rubi [A] time = 0.135305, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{4(A-B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}-\frac{4(A-2B)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{(A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}+\frac{B(a\sin(e+f}{a^6f(m}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (4*(A - B)*(a + a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (4*(A - 2*B)*(a
+ a*Sin[e + f*x])^(4 + m))/(a^4*f*(4 + m)) + ((A - 5*B)*(a + a*Sin[e + f*x]
)^(5 + m))/(a^5*f*(5 + m)) + (B*(a + a*Sin[e + f*x])^(6 + m))/(a^6*f*(6 + m
))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{2+m}\left(A+\frac{Bx}{a}\right)dx, x, a\sin(e+fx)\right)}{a^5 f}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^{2+m} - 4a(A-2B)(a+x)^{3+m} + \right.\right.}{a^5 f}$$
$$= \frac{4(A-B)(a+a\sin(e+fx))^{3+m}}{a^3 f(3+m)} - \frac{4(A-2B)(a+a\sin(e+f}{a^4 f(4+m)}$$

Mathematica [A] time = 0.382035, size = 103, normalized size = 0.84

$$\frac{(a(\sin(e+fx)+1))^{m+3}\left(\frac{a^3(A-5B)(\sin(e+fx)+1)^2}{m+5} - \frac{4a^3(A-2B)(\sin(e+fx)+1)}{m+4} + \frac{4a^3(A-B)}{m+3} + \frac{B(a\sin(e+fx)+a)^3}{m+6}\right)}{a^6 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3 + m)*((4*a^3*(A - B))/(3 + m) - (4*a^3*(A - 2*B)
*(1 + Sin[e + f*x]))/(4 + m) + (a^3*(A - 5*B)*(1 + Sin[e + f*x])^2)/(5 + m)
 + (B*(a + a*Sin[e + f*x])^3)/(6 + m)))/(a^6*f)
```

Maple [F] time = 5.923, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^5 \left(a+a\sin\left(fx+e\right)\right)^m \left(A+B\sin\left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71917, size = 563, normalized size = 4.58

$$-\frac{\left(\left(Bm^3+12\,Bm^2+47\,Bm+60\,B\right)\cos\left(fx+e\right)^6 - \left((A+B)m^3+3\,(3\,A+B)m^2+18\,Am\right)\cos\left(fx+e\right)^4 - 8\left((A+B)m^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -((B*m^3 + 12*B*m^2 + 47*B*m + 60*B)*cos(f*x + e)^6 - ((A + B)*m^3 + 3*(3*A
 + B)*m^2 + 18*A*m)*cos(f*x + e)^4 - 8*((A + B)*m^2 + 6*A*m)*cos(f*x + e)^2
 - 32*(A + B)*m - (((A + B)*m^3 + (13*A + 7*B)*m^2 + 6*(9*A + 2*B)*m + 72*A
)*cos(f*x + e)^4 + 8*((A + B)*m^2 + 2*(4*A + B)*m + 12*A)*cos(f*x + e)^2 +
32*(A + B)*m + 192*A)*sin(f*x + e) - 192*A)*(a*sin(f*x + e) + a)^m/(f*m^4 +
 18*f*m^3 + 119*f*m^2 + 342*f*m + 360*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2915, size = 1162, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] (((a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m^2 - 4*(a*sin(f*x + e) + a
)^4*(a*sin(f*x + e) + a)^m*a*m^2 + 4*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e)
 + a)^m*a^2*m^2 + 7*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m - 32*(a
*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a*m + 36*(a*sin(f*x + e) + a)^3
*(a*sin(f*x + e) + a)^m*a^2*m + 12*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) +
 a)^m - 60*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a + 80*(a*sin(f*x
+ e) + a)^3*(a*sin(f*x + e) + a)^m*a^2)*A/(a^4*m^3 + 12*a^4*m^2 + 47*a^4*m
+ 60*a^4) + ((a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*m^3 - 5*(a*sin(f
*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a*m^3 + 8*(a*sin(f*x + e) + a)^4*(a*s
in(f*x + e) + a)^m*a^2*m^3 - 4*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^
m*a^3*m^3 + 12*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*m^2 - 65*(a*si
n(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a*m^2 + 112*(a*sin(f*x + e) + a)^4
*(a*sin(f*x + e) + a)^m*a^2*m^2 - 60*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e)
 + a)^m*a^3*m^2 + 47*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*m - 270*
(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a*m + 504*(a*sin(f*x + e) + a
)^4*(a*sin(f*x + e) + a)^m*a^2*m - 296*(a*sin(f*x + e) + a)^3*(a*sin(f*x +
e) + a)^m*a^3*m + 60*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m - 360*(a
*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a + 720*(a*sin(f*x + e) + a)^4*
(a*sin(f*x + e) + a)^m*a^2 - 480*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a
)^m*a^3)*B/((a^4*m^4 + 18*a^4*m^3 + 119*a^4*m^2 + 342*a^4*m + 360*a^4)*a))/
(a*f)
```

3.1022 $\int \cos^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=93

$$\frac{2(A-B)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}-\frac{(A-3B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}-\frac{B(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}$$

```
[Out] (2*(A - B)*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) - ((A - 3*B)*(a +
a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (B*(a + a*Sin[e + f*x])^(4 + m))
/(a^4*f*(4 + m))
```

Rubi [A] time = 0.117641, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2836, 77}

$$\frac{2(A-B)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}-\frac{(A-3B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}-\frac{B(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) - ((A - 3*B)*(a +
a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (B*(a + a*Sin[e + f*x])^(4 + m))
/(a^4*f*(4 + m))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^{1+m}\left(A+\frac{Bx}{a}\right)dx,x,a\sin(e+fx)\right)}{a^3f}$$

$$= \frac{\text{Subst}\left(\int \left(2a(A-B)(a+x)^{1+m}+(-A+3B)(a+x)^{2+m}-\frac{B}{}\right.\right.}{a^3f}$$

$$= \frac{2(A-B)(a+a\sin(e+fx))^{2+m}}{a^2f(2+m)}-\frac{(A-3B)(a+a\sin(e+fx}{a^3f(3+m)}$$

Mathematica [A] time = 0.297884, size = 93, normalized size = 1.

$$\frac{2(A-B)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}-\frac{(A-3B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}-\frac{B(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) - ((A - 3*B)*(a +
a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (B*(a + a*Sin[e + f*x])^(4 + m))
/(a^4*f*(4 + m))
```

Maple [F] time = 2.612, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{3}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5578, size = 333, normalized size = 3.58

$$-\frac{\left(\left(Bm^2+5\,Bm+6\,B\right)\cos\left(fx+e\right)^4-\left((A+B)m^2+4\,Am\right)\cos\left(fx+e\right)^2-4\,(A+B)m-\left(\left(\left((A+B)m^2+2\,(3\,A+B)\right.\right.\right.}{fm^3+9\,fm^2+26\,fm+24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -((B*m^2 + 5*B*m + 6*B)*cos(f*x + e)^4 - ((A + B)*m^2 + 4*A*m)*cos(f*x + e)
^2 - 4*(A + B)*m - (((A + B)*m^2 + 2*(3*A + B)*m + 8*A)*cos(f*x + e)^2 + 4*
(A + B)*m + 16*A)*sin(f*x + e) - 16*A)*(a*sin(f*x + e) + a)^m/(f*m^3 + 9*f*
m^2 + 26*f*m + 24*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27483, size = 618, normalized size = 6.65

$$-\frac{\frac{\left(\left(a\sin\left(fx+e\right)+a\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}m-2\left(a\sin\left(fx+e\right)+a\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}am+2\left(a\sin\left(fx+e\right)+a\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}-6\left(a\sin\left(fx+e\right)+a\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{\cdots}}{a^{2}m^{2}+5a^{2}m+6a^{2}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] -(((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 2*(a*sin(f*x + e) + a)
^2*(a*sin(f*x + e) + a)^m*a*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) +
a)^m - 6*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a)*A/(a^2*m^2 + 5*a^
2*m + 6*a^2) + ((a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^2 - 3*(a*si
n(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a*m^2 + 2*(a*sin(f*x + e) + a)^2*(
a*sin(f*x + e) + a)^m*a^2*m^2 + 5*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) +
a)^m*m - 18*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a*m + 14*(a*sin(f
*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^2*m + 6*(a*sin(f*x + e) + a)^4*(a*s
in(f*x + e) + a)^m - 24*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a + 2
4*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^2)*B/((a^2*m^3 + 9*a^2*m^
2 + 26*a^2*m + 24*a^2)*a))/(a*f)
```

3.1023 $\int \cos(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=59

$$\frac{B(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(A-B)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

```
[Out] ((A - B)*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (B*(a + a*Sin[e + f*
x])^(2 + m))/(a^2*f*(2 + m))
```

Rubi [A] time = 0.0705771, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(A-B)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (B*(a + a*Sin[e + f*
x])^(2 + m))/(a^2*f*(2 + m))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\left(A+\frac{Bx}{a}\right)dx,x,a\sin(e+fx)\right)}{af}\\ &= \frac{\text{Subst}\left(\int \left((A-B)(a+x)^m+\frac{B(a+x)^{1+m}}{a}\right)dx,x,a\sin(e+f\right.}{af}\\ &= \frac{(A-B)(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{B(a+a\sin(e+fx))^{2+m}}{a^2f(2+m)}\end{aligned}$$

Mathematica [A] time = 0.122547, size = 51, normalized size = 0.86

$$\frac{(a(\sin(e+fx)+1))^{m+1}(A(m+2)+B(m+1)\sin(e+fx)-B)}{af(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*(-B + A*(2 + m) + B*(1 + m)*Sin[e + f*x]))/
(a*f*(1 + m)*(2 + m))
```

Maple [F] time = 1.661, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49354, size = 169, normalized size = 2.86

$$-\frac{\left((Bm+B)\cos\left(fx+e\right)^{2}-(A+B)m-((A+B)m+2A)\sin\left(fx+e\right)-2A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{fm^{2}+3fm+2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -((B*m + B)*cos(f*x + e)^2 - (A + B)*m - ((A + B)*m + 2*A)*sin(f*x + e) - 2
*A)*(a*sin(f*x + e) + a)^m/(f*m^2 + 3*f*m + 2*f)
```

Sympy [A] time = 8.30572, size = 428, normalized size = 7.25

$$\begin{cases} x\left(A+B\sin(e)\right)\left(a\sin(e)+a\right)^{m}\cos(e) \\ -\frac{A}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{B\log\left(\sin\left(e+fx\right)+1\right)\sin\left(e+fx\right)}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{B\log\left(\sin\left(e+fx\right)+1\right)}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{B}{a^{2}f\sin\left(e+fx\right)+a^{2}f} \\ \frac{A\log\left(\sin\left(e+fx\right)+1\right)}{af}-\frac{B\log\left(\sin\left(e+fx\right)+1\right)}{af}+\frac{B\sin\left(e+fx\right)}{af} \\ \frac{Am\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{Am\left(a\sin\left(e+fx\right)+a\right)^{m}}{fm^{2}+3fm+2f}+\frac{2A\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{2A\left(a\sin\left(e+fx\right)+a\right)^{m}}{fm^{2}+3fm+2f}+\frac{Bm\left(a\sin\left(e+fx\right)+a\right)^{m}\sin^{2}\left(e+\right.}{fm^{2}+3fm+2f} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((x*(A + B*sin(e))*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-A/(a**2*
f*sin(e + f*x) + a**2*f) + B*log(sin(e + f*x) + 1)*sin(e + f*x)/(a**2*f*sin
(e + f*x) + a**2*f) + B*log(sin(e + f*x) + 1)/(a**2*f*sin(e + f*x) + a**2*f
) + B/(a**2*f*sin(e + f*x) + a**2*f), Eq(m, -2)), (A*log(sin(e + f*x) + 1)/
(a*f) - B*log(sin(e + f*x) + 1)/(a*f) + B*sin(e + f*x)/(a*f), Eq(m, -1)), (
A*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + A*m*(a*si
n(e + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f) + 2*A*(a*sin(e + f*x) + a)**m*sin
(e + f*x)/(f*m**2 + 3*f*m + 2*f) + 2*A*(a*sin(e + f*x) + a)**m/(f*m**2 + 3*
f*m + 2*f) + B*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m +
2*f) + B*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + B*
(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m + 2*f) - B*(a*sin(e
 + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f), True))
```

Giac [B] time = 1.32114, size = 211, normalized size = 3.58

$$\frac{\frac{(a\sin(fx+e)+a)^{m+1}A}{m+1}+\frac{\left((a\sin(fx+e)+a)^{2}(a\sin(fx+e)+a)^{m}m-(a\sin(fx+e)+a)(a\sin(fx+e)+a)^{m}am+(a\sin(fx+e)+a)^{2}(a\sin(fx+e)+a)^{m}-2\,(a\sin(fx\cdots\right)}{(m^{2}+3\,m+2)a}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] ((a*sin(f*x + e) + a)^(m + 1)*A/(m + 1) + ((a*sin(f*x + e) + a)^2*(a*sin(f*
x + e) + a)^m*m - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*sin(
f*x + e) + a)^2*(a*sin(f*x + e) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*x
+ e) + a)^m*a)*B/((m^2 + 3*m + 2)*a))/(a*f)
```

3.1024 $\int \sec(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=80

$$\frac{(A+B)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}+\frac{(A-B)(a\sin(e+fx)+a)^m}{2fm}$$

```
[Out] ((A - B)*(a + a*Sin[e + f*x])^m)/(2*f*m) + ((A + B)*Hypergeometric2F1[1, 1
+ m, 2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(1 + m))/(4*a*f*(1 +
 m))
```

Rubi [A] time = 0.105848, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2836, 79, 68}

$$\frac{(A+B)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}+\frac{(A-B)(a\sin(e+fx)+a)^m}{2fm}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[e + f*x])^m)/(2*f*m) + ((A + B)*Hypergeometric2F1[1, 1
+ m, 2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(1 + m))/(4*a*f*(1 +
 m))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{a\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}\left(A+\frac{Bx}{a}\right)}{a-x}dx, x, a\sin(e+fx)\right)}{f}$$
$$= \frac{(A-B)(a+a\sin(e+fx))^m}{2fm} + \frac{(A+B)\,\text{Subst}\left(\int \frac{(a+x)^m}{a-x}d\right.}{2f}$$
$$= \frac{(A-B)(a+a\sin(e+fx))^m}{2fm} + \frac{(A+B)\,{}_2F_1\left(1,1+m;2+\right.}{}$$

Mathematica [A] time = 0.111482, size = 71, normalized size = 0.89

$$\frac{(a(\sin(e+fx)+1))^m\left(m(A+B)(\sin(e+fx)+1)\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)+2(m+1)(A-B)\right)}{4fm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(2*(A - B)*(1 + m) + (A + B)*m*Hypergeometric2F1[
1, 1 + m, 2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))/(4*f*m*(1 + m))
```

Maple [F] time = 1., size = 0, normalized size = 0.

$$\int \sec(fx+e)\,(a+a\sin(fx+e))^m\,(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\sec(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec(fx+e)\sin(fx+e)+A\sec(fx+e))(a\sin(fx+e)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral((B*sec(f*x + e)*sin(f*x + e) + A*sec(f*x + e))*(a*sin(f*x + e) + a
)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e), x)
```

3.1025 $\int \sec^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=100

$$\frac{a^2(A+B)(a\sin(e+fx)+a)^{m-1}}{2f(a-a\sin(e+fx))}-\frac{a(A(2-m)-Bm)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}$$

```
[Out] -(a*(A*(2 - m) - B*m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]
*(a + a*Sin[e + f*x])^(-1 + m))/(4*f*(1 - m)) + (a^2*(A + B)*(a + a*Sin[e +
 f*x])^(-1 + m))/(2*f*(a - a*Sin[e + f*x]))
```

Rubi [A] time = 0.132801, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 78, 68}

$$\frac{a^2(A+B)(a\sin(e+fx)+a)^{m-1}}{2f(a-a\sin(e+fx))}-\frac{a(A(2-m)-Bm)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(a*(A*(2 - m) - B*m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]
*(a + a*Sin[e + f*x])^(-1 + m))/(4*f*(1 - m)) + (a^2*(A + B)*(a + a*Sin[e +
 f*x])^(-1 + m))/(2*f*(a - a*Sin[e + f*x]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sec^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{a^3 \,\text{Subst}\left(\int \frac{(a+x)^{-2+m}\left(A+\frac{Bx}{a}\right)}{(a-x)^2}dx, x, a\sin(e+fx)\right)}{f}$$

$$= \frac{a^2(A+B)(a+a\sin(e+fx))^{-1+m}}{2f(a-a\sin(e+fx))} + \frac{\left(a^2(A(2-m)-Bm)\right) \text{S}}{}$$

$$= -\frac{a(A(2-m)-Bm)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\sin(e+fx))\right)(a}{4f(1-m)}$$

Mathematica [A] time = 0.163781, size = 82, normalized size = 0.82

$$-\frac{a(a(\sin(e+fx)+1))^{m-1}\left((A(m-2)+Bm)(\sin(e+fx)-1)\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)+2(m-1)(A+B)\right)}{4f(m-1)(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(a*(2*(A + B)*(-1 + m) + (A*(-2 + m) + B*m)*Hypergeometric2F1[1, -1 + m, m
, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]))*(a*(1 + Sin[e + f*x]))^(-1 + m
))/(4*f*(-1 + m)*(-1 + Sin[e + f*x]))
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int (\sec(fx+e))^3 (a+a\sin(fx+e))^m (A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m \sec(fx+e)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec(fx+e)^3\sin(fx+e)+A\sec(fx+e)^3\right)(a\sin(fx+e)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^3*sin(f*x + e) + A*sec(f*x + e)^3)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\sec(fx+e)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

3.1026 $\int \sec^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=104

$$\frac{a^4(A+B)(a\sin(e+fx)+a)^{m-2}}{4f(a-a\sin(e+fx))^2}-\frac{a^2(A(4-m)-Bm)(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(2,m-2;m-1;\frac{1}{2}(\sin(e+fx)+1)\right)}{16f(2-m)}$$

```
[Out] -(a^2*(A*(4 - m) - B*m)*Hypergeometric2F1[2, -2 + m, -1 + m, (1 + Sin[e + f
*x])/2]*(a + a*Sin[e + f*x])^(-2 + m))/(16*f*(2 - m)) + (a^4*(A + B)*(a + a
*Sin[e + f*x])^(-2 + m))/(4*f*(a - a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.139399, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2836, 78, 68}

$$\frac{a^4(A+B)(a\sin(e+fx)+a)^{m-2}}{4f(a-a\sin(e+fx))^2}-\frac{a^2(A(4-m)-Bm)(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(2,m-2;m-1;\frac{1}{2}(\sin(e+fx)+1)\right)}{16f(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(a^2*(A*(4 - m) - B*m)*Hypergeometric2F1[2, -2 + m, -1 + m, (1 + Sin[e + f
*x])/2]*(a + a*Sin[e + f*x])^(-2 + m))/(16*f*(2 - m)) + (a^4*(A + B)*(a + a
*Sin[e + f*x])^(-2 + m))/(4*f*(a - a*Sin[e + f*x])^2)
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sec^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{(a+x)^{-3+m}\left(A+\frac{Bx}{a}\right)}{(a-x)^3}dx, x, a\sin(e+fx)\right)}{f}$$

$$= \frac{a^4(A+B)(a+a\sin(e+fx))^{-2+m}}{4f(a-a\sin(e+fx))^2} + \frac{\left(a^4(A(4-m)-Bm\right.}{}$$

$$= -\frac{a^2(A(4-m)-Bm)\,{}_2F_1\left(2,-2+m;-1+m;\frac{1}{2}(1+\sin(e\right.}{16f(2-m)}$$

Mathematica [A] time = 0.16677, size = 76, normalized size = 0.73

$$\frac{a^2(a(\sin(e+fx)+1))^{m-2}\left(\frac{4(A+B)}{(\sin(e+fx)-1)^2}-\frac{(A(m-4)+Bm)\,{}_2F_1\left(2,m-2;m-1;\frac{1}{2}(\sin(e+fx)+1)\right)}{m-2}\right)}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*(-(((A*(-4 + m) + B*m)*Hypergeometric2F1[2, -2 + m, -1 + m, (1 + Sin[e
 + f*x])/2])/(-2 + m)) + (4*(A + B))/(-1 + Sin[e + f*x])^2)*(a*(1 + Sin[e +
 f*x]))^(-2 + m))/(16*f)
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int (\sec(fx+e))^5(a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\sec(fx+e)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec(fx+e)^5\sin(fx+e)+A\sec(fx+e)^5\right)(a\sin(fx+e)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^5*sin(f*x + e) + A*sec(f*x + e)^5)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m \sec(fx+e)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^5, x)
```

3.1027 $\int \cos^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=129

$$-\frac{a^3 2^{m+\frac{7}{2}}(A(m+7)+Bm)\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e+f\right.}{7f(m+7)}$$

```
[Out] -(2^(7/2 + m)*a^3*(B*m + A*(7 + m))*Cos[e + f*x]^7*Hypergeometric2F1[7/2, -
5/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Si
n[e + f*x])^(-3 + m))/(7*f*(7 + m)) - (B*Cos[e + f*x]^7*(a + a*Sin[e + f*x]
)^m)/(f*(7 + m))
```

Rubi [A] time = 0.198795, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$-\frac{a^3 2^{m+\frac{7}{2}}(A(m+7)+Bm)\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e+f\right.}{7f(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(2^(7/2 + m)*a^3*(B*m + A*(7 + m))*Cos[e + f*x]^7*Hypergeometric2F1[7/2, -
5/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Si
n[e + f*x])^(-3 + m))/(7*f*(7 + m)) - (B*Cos[e + f*x]^7*(a + a*Sin[e + f*x]
)^m)/(f*(7 + m))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos^7(e+fx)(a+a\sin(e+fx))^m}{f(7+m)} + \left(A+\frac{Bm}{7+m}\right)\int \cos$$

$$= -\frac{B\cos^7(e+fx)(a+a\sin(e+fx))^m}{f(7+m)} + \frac{\left(a^2\left(A+\frac{Bm}{7+m}\right)\cos^7(\right.}{f(}$$

$$= -\frac{B\cos^7(e+fx)(a+a\sin(e+fx))^m}{f(7+m)} + \frac{\left(2^{\frac{5}{2}+m}a^4\left(A+\frac{Bm}{7+m}\right)\right.}{}$$

$$= -\frac{2^{\frac{7}{2}+m}a^3\left(A+\frac{Bm}{7+m}\right)\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},-\frac{5}{2}-m;\frac{9}{2};\frac{1}{2}(1-\mathrm{si}\right.}{7}$$

Mathematica [A] time = 0.865395, size = 111, normalized size = 0.86

$$-\frac{\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{7}{2}}(a(\sin(e+fx)+1))^m\left(2^{m+\frac{7}{2}}(A(m+7)+Bm)\,{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)\right.}{7f(m+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(Cos[e + f*x]^7*(1 + Sin[e + f*x])^(-7/2 - m)*(a*(1 + Sin[e + f*x]))^m*(2^
(7/2 + m)*(B*m + A*(7 + m))*Hypergeometric2F1[7/2, -5/2 - m, 9/2, (1 - Sin[
e + f*x])/2] + 7*B*(1 + Sin[e + f*x])^(7/2 + m)))/(7*f*(7 + m))
```

Maple [F] time = 10.54, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^6\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(fx+e\right)^{6}\sin\left(fx+e\right)+A\cos\left(fx+e\right)^{6}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(f*x + e)^6*sin(f*x + e) + A*cos(f*x + e)^6)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^6, x)
```

3.1028 $\int \cos^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=129

$$-\frac{a^2 2^{m+\frac{5}{2}}(A(m+5)+Bm)\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f(m+5)}$$

```
[Out] -(2^(5/2 + m)*a^2*(B*m + A*(5 + m))*Cos[e + f*x]^5*Hypergeometric2F1[5/2, -
3/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Si
n[e + f*x])^(-2 + m))/(5*f*(5 + m)) - (B*Cos[e + f*x]^5*(a + a*Sin[e + f*x]
)^m)/(f*(5 + m))
```

Rubi [A] time = 0.197929, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$-\frac{a^2 2^{m+\frac{5}{2}}(A(m+5)+Bm)\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```



```
[Out] -(2^(5/2 + m)*a^2*(B*m + A*(5 + m))*Cos[e + f*x]^5*Hypergeometric2F1[5/2, -
3/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Si
n[e + f*x])^(-2 + m))/(5*f*(5 + m)) - (B*Cos[e + f*x]^5*(a + a*Sin[e + f*x]
)^m)/(f*(5 + m))
```


Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos^5(e+fx)(a+a\sin(e+fx))^m}{f(5+m)} + \left(A+\frac{Bm}{5+m}\right)\int c$$

$$= -\frac{B\cos^5(e+fx)(a+a\sin(e+fx))^m}{f(5+m)} + \frac{\left(a^2\left(A+\frac{Bm}{5+m}\right)co\right.}{}$$

$$= -\frac{B\cos^5(e+fx)(a+a\sin(e+fx))^m}{f(5+m)} + \frac{\left(2^{\frac{3}{2}+m}a^3\left(A+\frac{B}{5+}\right.\right.}{}$$

$$= -\frac{2^{\frac{5}{2}+m}a^2\left(A+\frac{Bm}{5+m}\right)\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\frac{7}{2};\frac{1}{2}(1\right.}{}$$

Mathematica [A] time = 0.463207, size = 111, normalized size = 0.86

$$-\frac{\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{5}{2}}(a(\sin(e+fx)+1))^m\left(2^{m+\frac{5}{2}}(A(m+5)+Bm)\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)\right.\right.}{5f(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(Cos[e + f*x]^5*(1 + Sin[e + f*x])^(-5/2 - m)*(a*(1 + Sin[e + f*x]))^m*(2^
(5/2 + m)*(B*m + A*(5 + m))*Hypergeometric2F1[5/2, -3/2 - m, 7/2, (1 - Sin[
e + f*x])/2] + 5*B*(1 + Sin[e + f*x])^(5/2 + m)))/(5*f*(5 + m))
```

Maple [F] time = 3.684, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^4\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(fx+e\right)^{4}\sin\left(fx+e\right)+A\cos\left(fx+e\right)^{4}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(f*x + e)^4*sin(f*x + e) + A*cos(f*x + e)^4)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^4, x)
```

3.1029 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=127

$$-\frac{a2^{m+\frac{3}{2}}(A(m+3)+Bm)\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx\right.}{3f(m+3)}$$

```
[Out] -(2^(3/2 + m)*a*(B*m + A*(3 + m))*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/
2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[
e + f*x])^(-1 + m))/(3*f*(3 + m)) - (B*Cos[e + f*x]^3*(a + a*Sin[e + f*x])^
m)/(f*(3 + m))
```

Rubi [A] time = 0.185491, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$-\frac{a2^{m+\frac{3}{2}}(A(m+3)+Bm)\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx\right.}{3f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```


```
[Out] -(2^(3/2 + m)*a*(B*m + A*(3 + m))*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/
2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[
e + f*x])^(-1 + m))/(3*f*(3 + m)) - (B*Cos[e + f*x]^3*(a + a*Sin[e + f*x])^
m)/(f*(3 + m))
```


Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos^3(e+fx)(a+a\sin(e+fx))^m}{f(3+m)}+\left(A+\frac{Bm}{3+m}\right)\int \cos$$

$$= -\frac{B\cos^3(e+fx)(a+a\sin(e+fx))^m}{f(3+m)}+\frac{\left(a^2\left(A+\frac{Bm}{3+m}\right)\cos^3(\right.}{f(}$$

$$= -\frac{B\cos^3(e+fx)(a+a\sin(e+fx))^m}{f(3+m)}+\frac{\left(2^{\frac{1}{2}+m}a^2\left(A+\frac{Bm}{3+m}\right)\right.}{}$$

$$= -\frac{2^{\frac{3}{2}+m}a\left(A+\frac{Bm}{3+m}\right)\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin\right.}{3f}$$

Mathematica [A] time = 0.318708, size = 111, normalized size = 0.87

$$-\frac{\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{3}{2}}(a(\sin(e+fx)+1))^m\left(2^{m+\frac{3}{2}}(A(m+3)+Bm)\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)\right.}{3f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(Cos[e + f*x]^3*(1 + Sin[e + f*x])^(-3/2 - m)*(a*(1 + Sin[e + f*x]))^m*(2^
(3/2 + m)*(B*m + A*(3 + m))*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 - Sin[
e + f*x])/2] + 3*B*(1 + Sin[e + f*x])^(3/2 + m)))/(3*f*(3 + m))
```

Maple [F] time = 1.891, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(fx+e\right)^2\sin\left(fx+e\right)+A\cos\left(fx+e\right)^2\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(f*x + e)^2*sin(f*x + e) + A*cos(f*x + e)^2)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

3.1030 $\int \sec^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=123

$$\frac{2^{m-\frac{1}{2}}(A(1-m)-Bm)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)}+\frac{B\sec\ldots}{\ldots}$$

```
[Out] (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)) + (2^(-1/2 + m)*(A*(1 -
 m) - B*m)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[
e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m))
```

Rubi [A] time = 0.187409, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$\frac{2^{m-\frac{1}{2}}(A(1-m)-Bm)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)}+\frac{B\sec\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)) + (2^(-1/2 + m)*(A*(1 -
 m) - B*m)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[
e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \left(A-\frac{Bm}{1-m}\right)\int \sec^2$$

$$= \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \frac{\left(a^2\left(A-\frac{Bm}{1-m}\right)\sec(e\right.}{}$$

$$= \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \frac{\left(2^{-\frac{3}{2}+m}a\left(A-\frac{Bm}{1-m}\right)\right.}{}$$

$$= \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \frac{2^{-\frac{1}{2}+m}\left(A-\frac{Bm}{1-m}\right)2^{}}{}$$

Mathematica [C] time = 24.9891, size = 6104, normalized size = 49.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.272, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(fx+e\right)^2\sin\left(fx+e\right)+A\sec\left(fx+e\right)^2\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^2*sin(f*x + e) + A*sec(f*x + e)^2)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

3.1031 $\int \sec^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=135

$$\frac{2^{m-\frac{3}{2}}(A(3-m)-Bm)\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3af(3-m)}$$

```
[Out] (B*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m)/(f*(3 - m)) + (2^(-3/2 + m)*(A*(3
 - m) - B*m)*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3
*a*f*(3 - m))
```

Rubi [A] time = 0.196221, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$\frac{2^{m-\frac{3}{2}}(A(3-m)-Bm)\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3af(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (B*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m)/(f*(3 - m)) + (2^(-3/2 + m)*(A*(3
 - m) - B*m)*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3
*a*f*(3 - m))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \left(A-\frac{Bm}{3-m}\right)\int \sec^4(e$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \frac{\left(a^2\left(A-\frac{Bm}{3-m}\right)\sec^3(e\right.}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \frac{\left(2^{-\frac{5}{2}+m}\left(A-\frac{Bm}{3-m}\right)\sec\right.}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \frac{2^{-\frac{3}{2}+m}\left(A-\frac{Bm}{3-m}\right){}_2F_1}{}$$

Mathematica [F] time = 1.55136, size = 0, normalized size = 0.

$$\int \sec^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] Integrate[Sec[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]), x]
```

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^4\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(fx+e\right)^{4}\sin\left(fx+e\right)+A\sec\left(fx+e\right)^{4}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^4*sin(f*x + e) + A*sec(f*x + e)^4)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

3.1032 $\int \sec^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=135

$$\frac{2^{m-\frac{5}{2}}(A(5-m)-Bm)\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2f(5-m)}+$$

```
[Out] (B*Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m)/(f*(5 - m)) + (2^(-5/2 + m)*(A*(5
 - m) - B*m)*Hypergeometric2F1[-5/2, 7/2 - m, -3/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/(5
*a^2*f*(5 - m))
```

Rubi [A] time = 0.193075, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2860, 2689, 70, 69}

$$\frac{2^{m-\frac{5}{2}}(A(5-m)-Bm)\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2f(5-m)}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```


```
[Out] (B*Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m)/(f*(5 - m)) + (2^(-5/2 + m)*(A*(5
 - m) - B*m)*Hypergeometric2F1[-5/2, 7/2 - m, -3/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/(5
*a^2*f*(5 - m))
```


Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \left(A-\frac{Bm}{5-m}\right)\int \sec$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \frac{\left(a^2\left(A-\frac{Bm}{5-m}\right)\sec^5\right.}{}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \frac{\left(2^{-\frac{7}{2}+m}\left(A-\frac{Bm}{5-m}\right)\right.}{}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \frac{2^{-\frac{5}{2}+m}\left(A-\frac{Bm}{5-m}\right)2}{}$$

Mathematica [F] time = 3.50869, size = 0, normalized size = 0.

$$\int \sec^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] Integrate[Sec[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]), x]
```

Maple [F] time = 0.404, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{6}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(fx+e\right)^{6}\sin\left(fx+e\right)+A\sec\left(fx+e\right)^{6}\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^6*sin(f*x + e) + A*sec(f*x + e)^6)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^6, x)
```

3.1033 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-p}\,dx$

Optimal. Leaf size=239

$$\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{c^2fg(p+3)(p+5)(p+7)}+\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{c^3fg(p+1)(p+3)(p+5)(p+7)}$$

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-4 - p))/(f*g*(7 +
p)) + ((3*A - B*(4 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-3
- p))/(c*f*g*(5 + p)*(7 + p)) + (2*(3*A - B*(4 + p))*(g*Cos[e + f*x])^(1 +
p)*(c - c*Sin[e + f*x])^(-2 - p))/(c^2*f*g*(3 + p)*(5 + p)*(7 + p)) + (2*(3
*A - B*(4 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 - p))/(c^
3*f*g*(1 + p)*(3 + p)*(5 + p)*(7 + p))
```

Rubi [A] time = 0.440974, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2859, 2672, 2671}

$$\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{c^2fg(p+3)(p+5)(p+7)}+\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{c^3fg(p+1)(p+3)(p+5)(p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-4 - p))/(f*g*(7 +
p)) + ((3*A - B*(4 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-3
- p))/(c*f*g*(5 + p)*(7 + p)) + (2*(3*A - B*(4 + p))*(g*Cos[e + f*x])^(1 +
p)*(c - c*Sin[e + f*x])^(-2 - p))/(c^2*f*g*(3 + p)*(5 + p)*(7 + p)) + (2*(3
*A - B*(4 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 - p))/(c^
3*f*g*(1 + p)*(3 + p)*(5 + p)*(7 + p))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-p}\,dx = \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-4-p}}{fg(7+p)} + \frac{(3A}{}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-4-p}}{fg(7+p)} + \frac{(3A}{}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-4-p}}{fg(7+p)} + \frac{(3A}{}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-4-p}}{fg(7+p)} + \frac{(3A}{}$$

Mathematica [A] time = 0.526017, size = 160, normalized size = 0.67

$$\frac{\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\left(\left(p^2+8p+13\right)(B(p+4)-3A)\sin(e+fx)+(2B(p+4)-6A)\sin^3(e+\right.}{c^4f(p+1)(p+3)(p+5)(p+7)(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4
- p),x]
```

```
[Out] (Cos[e + f*x]*(g*Cos[e + f*x])^p*(-(B*(13 + 8*p + p^2)) + A*(36 + 41*p + 12
*p^2 + p^3) + (13 + 8*p + p^2)*(-3*A + B*(4 + p))*Sin[e + f*x] - 2*(4 + p)*
(-3*A + B*(4 + p))*Sin[e + f*x]^2 + (-6*A + 2*B*(4 + p))*Sin[e + f*x]^3))/(
c^4*f*(1 + p)*(3 + p)*(5 + p)*(7 + p)*(-1 + Sin[e + f*x])^4*(c - c*Sin[e +
f*x])^p)
```

Maple [F] time = 0.726, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^p\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-4-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x, algo
rithm="maxima")
```

[Out] Timed out

Fricas [A] time = 1.65411, size = 478, normalized size = 2.

$$\frac{\left(2\left(Bp^2-(3A-8B)p-12A+16B\right)\cos\left(fx+e\right)^3+\left(Ap^3+3(4A-B)p^2+(47A-24B)p+60A-45B\right)\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x, algo
rithm="fricas")
```

```
[Out] (2*(B*p^2 - (3*A - 8*B)*p - 12*A + 16*B)*cos(f*x + e)^3 + (A*p^3 + 3*(4*A -
 B)*p^2 + (47*A - 24*B)*p + 60*A - 45*B)*cos(f*x + e) - (2*(B*p - 3*A + 4*B
)*cos(f*x + e)^3 - (B*p^3 - 3*(A - 4*B)*p^2 - (24*A - 47*B)*p - 45*A + 60*B
)*cos(f*x + e))*sin(f*x + e))*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
- 4)/(f*p^4 + 16*f*p^3 + 86*f*p^2 + 176*f*p + 105*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-4-p),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x, algo
rithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.1034 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-p}\,dx$

Optimal. Leaf size=168

$$\frac{(2A-B(p+3))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{c^2fg(p+1)(p+3)(p+5)}+\frac{(A+B)(c-c\sin(e+fx))^{-p-3}(g\cos(e+fx))^{p+1}}{fg(p+5)}+\frac{(2A-B(p$$

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-3 - p))/(f*g*(5 +
p)) + ((2*A - B*(3 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-2
- p))/(c*f*g*(3 + p)*(5 + p)) + ((2*A - B*(3 + p))*(g*Cos[e + f*x])^(1 + p)
*(c - c*Sin[e + f*x])^(-1 - p))/(c^2*f*g*(1 + p)*(3 + p)*(5 + p))
```

Rubi [A] time = 0.306062, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2859, 2672, 2671}

$$\frac{(2A-B(p+3))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{c^2fg(p+1)(p+3)(p+5)}+\frac{(A+B)(c-c\sin(e+fx))^{-p-3}(g\cos(e+fx))^{p+1}}{fg(p+5)}+\frac{(2A-B(p$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-3 - p))/(f*g*(5 +
p)) + ((2*A - B*(3 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-2
- p))/(c*f*g*(3 + p)*(5 + p)) + ((2*A - B*(3 + p))*(g*Cos[e + f*x])^(1 + p)
*(c - c*Sin[e + f*x])^(-1 - p))/(c^2*f*g*(1 + p)*(3 + p)*(5 + p))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-p}\,dx = \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-3-p}}{fg(5+p)} +$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-3-p}}{fg(5+p)} +$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-3-p}}{fg(5+p)} +$$

Mathematica [A] time = 0.261668, size = 119, normalized size = 0.71

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\left((2A-B(p+3))\sin^2(e+fx)+(p+3)(B(p+3)-2A)\sin(e+fx)\right.}{c^3f(p+1)(p+3)(p+5)(\sin(e+fx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3
- p),x]
```

```
[Out] -((Cos[e + f*x]*(g*Cos[e + f*x])^p*(-(B*(3 + p)) + A*(7 + 6*p + p^2) + (3 +
 p)*(-2*A + B*(3 + p))*Sin[e + f*x] + (2*A - B*(3 + p))*Sin[e + f*x]^2))/(c
^3*f*(1 + p)*(3 + p)*(5 + p)*(-1 + Sin[e + f*x])^3*(c - c*Sin[e + f*x])^p))
```

Maple [F] time = 0.632, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^p\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-3-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x, algo
rithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.43571, size = 317, normalized size = 1.89

$$\frac{\left(\left(Bp-2A+3B\right)\cos\left(fx+e\right)^3+\left(Bp^2-2(A-3B)p-6A+9B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(Ap^2+2(3A-B)p+\right.\right.}{fp^3+9fp^2+23fp+15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x, algo
rithm="fricas")
```

```
[Out] ((B*p - 2*A + 3*B)*cos(f*x + e)^3 + (B*p^2 - 2*(A - 3*B)*p - 6*A + 9*B)*cos
(f*x + e)*sin(f*x + e) + (A*p^2 + 2*(3*A - B)*p + 9*A - 6*B)*cos(f*x + e))*
(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p - 3)/(f*p^3 + 9*f*p^2 + 23*f*p
 + 15*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-3-p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x, algo
rithm="giac")
```

```
[Out] sage2
```

3.1035 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-p}\,dx$

Optimal. Leaf size=102

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{fg(p+3)}+\frac{(A-B(p+2))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{cfg(p+1)(p+3)}$$

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-2 - p))/(f*g*(3 +
p)) + ((A - B*(2 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 -
p))/(c*f*g*(1 + p)*(3 + p))
```

Rubi [A] time = 0.2122, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2859, 2671}

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{fg(p+3)}+\frac{(A-B(p+2))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{cfg(p+1)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-2 - p))/(f*g*(3 +
p)) + ((A - B*(2 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 -
p))/(c*f*g*(1 + p)*(3 + p))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-p}\,dx &= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-2-p}}{fg(3+p)}+\\ &= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-2-p}}{fg(3+p)}+\end{aligned}$$

Mathematica [A] time = 0.147402, size = 83, normalized size = 0.81

$$\frac{\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p((B(p+2)-A)\sin(e+fx)+A(p+2)-B)}{c^2f(p+1)(p+3)(\sin(e+fx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2
- p),x]
```

```
[Out] (Cos[e + f*x]*(g*Cos[e + f*x])^p*(-B + A*(2 + p) + (-A + B*(2 + p))*Sin[e +
 f*x]))/(c^2*f*(1 + p)*(3 + p)*(-1 + Sin[e + f*x])^2*(c - c*Sin[e + f*x])^p
)
```

Maple [F] time = 0.595, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-2-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 2), x)
```

Fricas [A] time = 1.49969, size = 200, normalized size = 1.96

$$\frac{\left(\left(Bp-A+2\,B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(Ap+2\,A-B\right)\cos\left(fx+e\right)\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-2}}{fp^{2}+4\,fp+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x, algo
rithm="fricas")
```

```
[Out] ((B*p - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (A*p + 2*A - B)*cos(f*x + e))*
(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p - 2)/(f*p^2 + 4*f*p + 3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-2-p),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x, algo
rithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.1036 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-p}\,dx$

Optimal. Leaf size=151

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{fg(p+1)}-\frac{B2^{\frac{1}{2}-\frac{p}{2}}(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F}{fg(p+1)}$$

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 +
p)) - (2^(1/2 - p/2)*B*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 + p)/2
, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((1 + p)/2
)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p))
```

Rubi [A] time = 0.264259, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2859, 2689, 70, 69}

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{fg(p+1)}-\frac{B2^{\frac{1}{2}-\frac{p}{2}}(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 +
p)) - (2^(1/2 - p/2)*B*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 + p)/2
, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((1 + p)/2
)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-p}\,dx = \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1-p}}{fg(1+p)} - \cdots$$

$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1-p}}{fg(1+p)} - \cdots$$

$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1-p}}{fg(1+p)} - \cdots$$

$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1-p}}{fg(1+p)} - \cdots$$

Mathematica [C] time = 3.44466, size = 300, normalized size = 1.99

$$-\frac{2^{-p}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^p\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-p-1)-2p}\left(\frac{1-\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{1}{\cos(e+fx)+1}}}\right)^{2p}\left(\frac{1-\tan\left(\frac{1}{2}(e\cdots\right.}{\sqrt{\sec^2\left(\frac{1}{2}(e\cdots\right.}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1
- p),x]
```

```
[Out] -(((g*Cos[e + f*x])^p*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2*(-1 - p) -
2*p)*(c - c*Sin[e + f*x])^(-1 - p)*((1 - Tan[(e + f*x)/2])/Sqrt[(1 + Cos[e
+ f*x])^(-1)])^(2*p)*((-I)*B*(1 + p)*Hypergeometric2F1[1, -p, 1 - p, ((-I)*
(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])]*(-1 + Tan[(e + f*x)/2]) +
I*B*(1 + p)*Hypergeometric2F1[1, -p, 1 - p, (I*(-1 + Tan[(e + f*x)/2]))/(1
+ Tan[(e + f*x)/2])]*(-1 + Tan[(e + f*x)/2]) + (A + B)*p*(1 + Tan[(e + f*x)
/2])))/(2^p*f*p*(1 + p)*((1 - Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(
2*p)*(-1 + Tan[(e + f*x)/2])))
```

Maple [F] time = 0.432, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-1-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x, algo
rithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
- 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-1-p),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x, algo
rithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.1037 $\int (g\cos(e+fx))^p (A+B\sin(e+fx))(c-c\sin(e+fx))^{-p}\,dx$

Optimal. Leaf size=147

$$\frac{c2^{\frac{1}{2}-\frac{p}{2}}(A+Bp)(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(p+1)}-$$

```
[Out] (2^(1/2 - p/2)*c*(A + B*p)*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 +
p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((1 +
p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p)) - (B*(g*Cos[e + f*x])^(1
 + p))/(f*g*(c - c*Sin[e + f*x])^p)
```

Rubi [A] time = 0.209082, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2860, 2689, 70, 69}

$$\frac{c2^{\frac{1}{2}-\frac{p}{2}}(A+Bp)(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(p+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^p,x]
```

```
[Out] (2^(1/2 - p/2)*c*(A + B*p)*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 +
p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((1 +
p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p)) - (B*(g*Cos[e + f*x])^(1
 + p))/(f*g*(c - c*Sin[e + f*x])^p)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-p}\,dx = -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-p}}{fg} + (A+Bp)\int($$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-p}}{fg} + \frac{\left(c^2(A+Bp)\right.}{}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-p}}{fg} + \frac{\left(2^{-\frac{1}{2}-\frac{p}{2}}c^2(A\right.}{}$$

$$= \frac{2^{\frac{1}{2}-\frac{p}{2}}c(A+Bp)(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1+p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1\right.}{fg(1}$$

Mathematica [A] time = 0.540496, size = 144, normalized size = 0.98

$$-\frac{2^{\frac{1}{2}(-p-1)}\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\left(2(A+Bp)(1-\sin(e+fx))^{\frac{p+1}{2}}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx\right.\right.}{f(p+1)(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^p*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^p,
x]
```

```
[Out] -((2^((-1 - p)/2)*Cos[e + f*x]*(g*Cos[e + f*x])^p*(2*(A + B*p)*Hypergeometr
ic2F1[(1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2) + 2^((1 + p)/2)*B*(1 + p)*(-1 + Sin[e + f*x])))/(f*(1 + p)
*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^p))
```

Maple [F] time = 1.438, size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p\left(A+B\sin\left(fx+e\right)\right)}{\left(c-c\sin\left(fx+e\right)\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}}{\left(-c\sin\left(fx+e\right)+c\right)^{p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p/(-c*sin(f*x + e) + c)^p,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}}{\left(-c\sin\left(fx+e\right)+c\right)^{p}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p/(-c*sin(f*x + e) + c)^p, x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**p),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.1038 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-p}\,dx$

Optimal. Leaf size=160

$$\frac{c^2 2^{\frac{1}{2}-\frac{p}{2}}(2A-B(1-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(p+1)}$$

```
[Out] (2^(1/2 - p/2)*c^2*(2*A - B*(1 - p))*(g*Cos[e + f*x])^(1 + p)*Hypergeometri
c2F1[(-1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p)) - (B*(g*Cos[e
 + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(1 - p))/(2*f*g)
```

Rubi [A] time = 0.272417, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2860, 2689, 70, 69}

$$\frac{c^2 2^{\frac{1}{2}-\frac{p}{2}}(2A-B(1-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 - p),x]
```

```
[Out] (2^(1/2 - p/2)*c^2*(2*A - B*(1 - p))*(g*Cos[e + f*x])^(1 + p)*Hypergeometri
c2F1[(-1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p)) - (B*(g*Cos[e
 + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(1 - p))/(2*f*g)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-p}\,dx = -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{1-p}}{2fg} - \frac{(-2Ac}{}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{1-p}}{2fg} - \frac{\left(c(-2A\right.}{}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{1-p}}{2fg} - \frac{\left(2^{-\frac{1}{2}-\frac{p}{2}}\right.}{}$$

$$= \frac{2^{\frac{1}{2}-\frac{p}{2}}c^2(2A-B(1-p))(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-1+\right.}{}$$

Mathematica [A] time = 0.638409, size = 150, normalized size = 0.94

$$\frac{c2^{\frac{1}{2}(-p-3)}\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\left(B2^{\frac{p+1}{2}}(p+1)(\sin(e+fx)-1)^2-4(2A+B(p-1))(1-\sin(\right.}{f(p+1)(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 -
 p),x]
```

```
[Out] (2^((-3 - p)/2)*c*Cos[e + f*x]*(g*Cos[e + f*x])^p*(-4*(2*A + B*(-1 + p))*Hy
pergeometric2F1[(-1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1
- Sin[e + f*x])^((1 + p)/2) + 2^((1 + p)/2)*B*(1 + p)*(-1 + Sin[e + f*x])^2
))/(f*(1 + p)*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^p)
```

Maple [F] time = 0.468, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^p\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{1-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^p\left(-c\sin\left(fx+e\right)+c\right)^{-p+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
+ 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-p),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.1039 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-p}\,dx$

Optimal. Leaf size=163

$$\frac{c^3 2^{\frac{5}{2}-\frac{p}{2}}(3A-B(2-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-3}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+\right.}{3fg(p+1)}$$

```
[Out] (2^(5/2 - p/2)*c^3*(3*A - B*(2 - p))*(g*Cos[e + f*x])^(1 + p)*Hypergeometri
c2F1[(-3 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(3*f*g*(1 + p)) - (B*(g*Cos
[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(2 - p))/(3*f*g)
```

Rubi [A] time = 0.275605, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2860, 2689, 70, 69}

$$\frac{c^3 2^{\frac{5}{2}-\frac{p}{2}}(3A-B(2-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-3}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+\right.}{3fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 - p),x]
```

```
[Out] (2^(5/2 - p/2)*c^3*(3*A - B*(2 - p))*(g*Cos[e + f*x])^(1 + p)*Hypergeometri
c2F1[(-3 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(3*f*g*(1 + p)) - (B*(g*Cos
[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(2 - p))/(3*f*g)
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-p}\,dx = -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{2-p}}{3fg} - \frac{1}{3}(-3A+$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{2-p}}{3fg} - \frac{\left(c^2(-3A+\right.}{}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{2-p}}{3fg} - \frac{\left(2^{\frac{3}{2}-\frac{p}{2}}c^4(-\right.}{}$$

$$= \frac{2^{\frac{5}{2}-\frac{p}{2}}c^3(3A-B(2-p))(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-3+p)\right.}{}$$

Mathematica [A] time = 0.870102, size = 155, normalized size = 0.95

$$-\frac{c^2 2^{\frac{1}{2}(-p-1)}\cos(e+fx)(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^p\left(8(3A+B(p-2))(1-\sin(e+fx))^{\frac{p+1}{2}}\,{}_2F_1\left(\frac{p-3}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\mathrm{si}\right.\right.}{3f(p+1)(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 -
 p),x]
```

```
[Out] -(2^((-1 - p)/2)*c^2*Cos[e + f*x]*(g*Cos[e + f*x])^p*(8*(3*A + B*(-2 + p))*
Hypergeometric2F1[(-3 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(
1 - Sin[e + f*x])^((1 + p)/2) + 2^((1 + p)/2)*B*(1 + p)*(-1 + Sin[e + f*x])
^3))/(3*f*(1 + p)*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^p)
```

Maple [F] time = 0.515, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^p\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{2-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^p\left(-c\sin\left(fx+e\right)+c\right)^{-p+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
+ 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-p),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.1040 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(Am-A(1+m+p)\sin(e+fx))\,dx$

Optimal. Leaf size=32

$$\frac{A(a\sin(e+fx)+a)^m(g\cos(e+fx))^{p+1}}{fg}$$

```
[Out] (A*(g*Cos[e + f*x])^(1 + p)*(a + a*Sin[e + f*x])^m)/(f*g)
```

Rubi [A] time = 0.117022, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2854}

$$\frac{A(a\sin(e+fx)+a)^m(g\cos(e+fx))^{p+1}}{fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A*m - A*(1 + m + p)*Sin[e + 
f*x]),x]
```

```
[Out] (A*(g*Cos[e + f*x])^(1 + p)*(a + a*Sin[e + f*x])^m)/(f*g)
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; Fr
eeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c
*(m + p + 1), 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(Am-A(1+m+p)\sin(e+fx))\,dx=\frac{A(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m}{fg}$$

Mathematica [A] time = 0.172178, size = 33, normalized size = 1.03

$$\frac{A\cos(e+fx)(a(\sin(e+fx)+1))^m(g\cos(e+fx))^p}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A*m - A*(1 + m + p)*Si
n[e + f*x]),x]
```

```
[Out] (A*Cos[e + f*x]*(g*Cos[e + f*x])^p*(a*(1 + Sin[e + f*x]))^m)/f
```

Maple [F] time = 4.252, size = 0, normalized size = 0.

$$\int \left(g \cos \left(f x+e\right)\right)^{p} \left(a+a \sin \left(f x+e\right)\right)^{m} \left(A m-A \left(1+m+p\right) \sin \left(f x+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(A\left(m+p+1\right) \sin \left(fx+e\right)-Am\right) \left(g \cos \left(fx+e\right)\right)^{p} \left(a \sin \left(fx+e\right)+a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x,
 algorithm="maxima")
```

```
[Out] -integrate((A*(m + p + 1)*sin(f*x + e) - A*m)*(g*cos(f*x + e))^p*(a*sin(f*x
 + e) + a)^m, x)
```

Fricas [A] time = 1.55306, size = 81, normalized size = 2.53

$$\frac{\left(g \cos \left(fx+e\right)\right)^{p} \left(a \sin \left(fx+e\right)+a\right)^{m} A \cos \left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x,
 algorithm="fricas")
```

```
[Out] (g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*A*cos(f*x + e)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(A*m-A*(1+m+p)*sin(f*x+e)),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x,
 algorithm="giac")
```

```
[Out] sage2
```

3.1041 $\int (g\cos(e+fx))^p(a-a\sin(e+fx))^m(Am+A(1+m+p)\sin(e+fx))\,dx$

Optimal. Leaf size=34

$$-\frac{A(a-a\sin(e+fx))^m(g\cos(e+fx))^{p+1}}{fg}$$

```
[Out] -((A*(g*Cos[e + f*x])^(1 + p)*(a - a*Sin[e + f*x])^m)/(f*g))
```

Rubi [A] time = 0.115491, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2854}

$$-\frac{A(a-a\sin(e+fx))^m(g\cos(e+fx))^{p+1}}{fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a - a*Sin[e + f*x])^m*(A*m + A*(1 + m + p)*Sin[e + f*x]),x]
```

```
[Out] -((A*(g*Cos[e + f*x])^(1 + p)*(a - a*Sin[e + f*x])^m)/(f*g))
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + p + 1), 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a-a\sin(e+fx))^m(Am+A(1+m+p)\sin(e+fx))\,dx = -\frac{A(g\cos(e+fx))^{1+p}(a-a\sin(e+fx))}{fg}$$

Mathematica [A] time = 0.0613699, size = 35, normalized size = 1.03

$$-\frac{A\cos(e+fx)(a-a\sin(e+fx))^m(g\cos(e+fx))^p}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a - a*Sin[e + f*x])^m*(A*m + A*(1 + m + p)*Sin[e + f*x]),x]
```

```
[Out] -((A*Cos[e + f*x]*(g*Cos[e + f*x])^p*(a - a*Sin[e + f*x])^m)/f)
```

Maple [F] time = 4.184, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{p} \left(a-a\sin\left(fx+e\right)\right)^{m} \left(Am+A\left(1+m+p\right)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A\left(m+p+1\right)\sin\left(fx+e\right)+Am\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x,
 algorithm="maxima")
```

```
[Out] integrate((A*(m + p + 1)*sin(f*x + e) + A*m)*(g*cos(f*x + e))^p*(-a*sin(f*x
 + e) + a)^m, x)
```

Fricas [A] time = 1.42943, size = 84, normalized size = 2.47

$$-\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(-a\sin\left(fx+e\right)+a\right)^{m}A\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x,
 algorithm="fricas")
```

```
[Out] -(g*cos(f*x + e))^p*(-a*sin(f*x + e) + a)^m*A*cos(f*x + e)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a-a*sin(f*x+e))**m*(A*m+A*(1+m+p)*sin(f*x+e)),
x)
```

```
[Out] Timed out
```

Giac [B] time = 32.0174, size = 2515, normalized size = 73.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x,
 algorithm="giac")
```

```
[Out] -(A*e^(-m*log(2) - p*log(2) + p*log(2*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2
- 1)*abs(g)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + 2*m*log(4*abs(tan(-1/
8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + p*log(4*
abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))
 + m*log(abs(a)))*tan(-1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1/2*
f*x + 1/2*e) + g)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(g) + 1/2*pi*p*floor(1
/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 3/4) + pi*m*flo
or(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1/2*
pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4
) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*p*floor(1/2*f*x/pi + 1/2*e
/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1) - 1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2
*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1) + 1/4*pi*p*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi
*m + 1/4*pi*p)^2*tan(1/2*f*x + 1/2*e)^2 - A*e^(-m*log(2) - p*log(2) + p*log
(2*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 1)*abs(g)/(tan(-1/8*pi + 1/4*f*x
+ 1/4*e)^2 + 1)) + 2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*
pi + 1/4*f*x + 1/4*e)^2 + 1)) + p*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))
/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a)))*tan(-1/4*pi*p*sgn
(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g)*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1)*sgn(g) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*
x/pi + 1/2*e/pi + 1/2) + 3/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/
2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi -
floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/p
i + 1/2) + pi*p*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a)
 + 1) - 1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) +
g) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*p*sgn(tan(1/2*f*x +
1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m + 1/4*pi*p)^2 - A*e^(-m*log(2) -
 p*log(2) + p*log(2*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 1)*abs(g)/(tan(-
1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + 2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1
/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + p*log(4*abs(tan(-1/8*pi +
1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a)))*
tan(1/2*f*x + 1/2*e)^2 + A*e^(-m*log(2) - p*log(2) + p*log(2*abs(tan(-1/8*p
i + 1/4*f*x + 1/4*e)^2 - 1)*abs(g)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))
+ 2*m*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1/4*f*x + 1/
4*e)^2 + 1)) + p*log(4*abs(tan(-1/8*pi + 1/4*f*x + 1/4*e))/(tan(-1/8*pi + 1
/4*f*x + 1/4*e)^2 + 1)) + m*log(abs(a))))/(f*tan(-1/4*pi*p*sgn(g*tan(1/2*f*
x + 1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1
)*sgn(g) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/
pi + 1/2) + 3/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/
2*e/pi + 1/2) + 1/4) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x
/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi
*p*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1) - 1/4*p
i*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g) + 1/2*pi*m
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*p*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
 + 1/4*pi*m*sgn(a) + 1/4*pi*m + 1/4*pi*p)^2*tan(1/2*f*x + 1/2*e)^2 + f*tan(
-1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g)*sgn(
tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(g) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi -
 floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 3/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/
pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1/2*pi*p*floor(1/2*f*x/pi
+ 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x
/pi + 1/2*e/pi + 1/2) + pi*p*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floo
r(-1/4*sgn(a) + 1) - 1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1/2*f*
x + 1/2*e) + g) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*p*sgn(t
an(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m + 1/4*pi*p)^2 + f*t
```

```
an(1/2*f*x + 1/2*e)^2 + f)
```

3.1042 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=168

$$\frac{g2^{\frac{p+1}{2}}(1-\sin(e+fx))^{\frac{1-p}{2}}(a\sin(e+fx)+a)^{m+1}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(\frac{1}{2}(2m+p+\right.}{af(2m+p+1)}$$

```
[Out] (2^((1 + p)/2)*g*AppellF1[(1 + 2*m + p)/2, (1 - p)/2, -n, (3 + 2*m + p)/2,
(1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*(g*Cos[e + f*x])^(
-1 + p)*(1 - Sin[e + f*x])^((1 - p)/2)*(a + a*Sin[e + f*x])^(1 + m)*(c + d*
Sin[e + f*x])^n)/(a*f*(1 + 2*m + p)*((c + d*Sin[e + f*x])/(c - d))^n)
```

Rubi [A] time = 0.28029, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2921, 140, 139, 138}

$$\frac{g2^{\frac{p+1}{2}}(1-\sin(e+fx))^{\frac{1-p}{2}}(a\sin(e+fx)+a)^{m+1}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(\frac{1}{2}(2m+p+\right.}{af(2m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```


```
[Out] (2^((1 + p)/2)*g*AppellF1[(1 + 2*m + p)/2, (1 - p)/2, -n, (3 + 2*m + p)/2,
(1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*(g*Cos[e + f*x])^(
-1 + p)*(1 - Sin[e + f*x])^((1 - p)/2)*(a + a*Sin[e + f*x])^(1 + m)*(c + d*
Sin[e + f*x])^n)/(a*f*(1 + 2*m + p)*((c + d*Sin[e + f*x])/(c - d))^n)
```


Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(g
*(g*Cos[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(a-a\sin(e+fx))^{\frac{1-p}{2}}(a+a\sin(e+\cdots\right.}{\cdots}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{p}{2}}g(g\cos(e+fx))^{-1+p}(a-a\sin(e+fx))^{-\frac{1}{2}+\frac{1-p}{2}+\frac{p}{2}}\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{p}{2}}g(g\cos(e+fx))^{-1+p}(a-a\sin(e+fx))^{-\frac{1}{2}+\frac{1-p}{2}+\frac{p}{2}}\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{2^{\frac{1+p}{2}}gF_1\left(\frac{1}{2}(1+2m+p);\frac{1-p}{2},-n;\frac{1}{2}(3+2m+p);\frac{1}{2}(1+\mathrm{si}\cdots\right.}{\cdots}$$

Mathematica [B] time = 8.34151, size = 798, normalized size = 4.75

$$-\frac{\cdots}{f\left(-\frac{dnF_1\left(\frac{p+1}{2};m+n+p+1,-n;\frac{p+3}{2};-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)\cos^2(e+fx)}{c+d\sin(e+fx)}+\frac{2(p+1)\left((c-d)nF_1\left(\frac{p+3}{2};m+n+p+1,1-n;\frac{p+5}{2};-\tan^2\left(\frac{1}{4}(2e+\cdots\right.\right.\right.}{\cdots}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (-2*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*
x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*(g*Cos[e + f*x])
^p*Cos[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^
n*Sin[(2*e + Pi + 2*f*x)/4])/(f*(AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3
+ p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^
2)/(c + d))] + (2*(1 + p)*((c - d)*n*AppellF1[(3 + p)/2, 1 + m + n + p, 1 -
 n, (5 + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f
*x)/4]^2)/(c + d))] - (c + d)*(1 + m + n + p)*AppellF1[(3 + p)/2, 2 + m + n
 + p, -n, (5 + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi
 + 2*f*x)/4]^2)/(c + d))])*Cot[(2*e + Pi + 2*f*x)/4]^2)/((c + d)*(3 + p)) +
 p*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*x
```

```
)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Sin[e + f*x] - (d
*n*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*x
)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Cos[e + f*x]^2)/(
c + d*Sin[e + f*x]) + 2*(n + p)*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 +
 p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2
)/(c + d))]*Sin[(2*e - Pi + 2*f*x)/4]^2 - (2*(c - d)*n*AppellF1[(1 + p)/2,
1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[
(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[
e + f*x])))
```

Maple [F] time = 2.763, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

3.1043 $\int (g\cos(e+fx))^p (a+a\sin(e+fx))^2 (c+d\sin(e+fx))^n \, dx$

Optimal. Leaf size=149

$$-\frac{a^2 2^{\frac{p}{2}+\frac{5}{2}} (\sin(e+fx)+1)^{\frac{1}{2}(-p-5)+2} (g\cos(e+fx))^{p+1} (c+d\sin(e+fx))^n \left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{p+1}{2}; \frac{1}{2}(-p-3), -n; \frac{p+3}{2}; \frac{1}{2} \ldots\right)}{fg(p+1)}$$

```
[Out] -((2^(5/2 + p/2)*a^2*AppellF1[(1 + p)/2, (-3 - p)/2, -n, (3 + p)/2, (1 - Si
n[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(1 + p)*(1
+ Sin[e + f*x])^(2 + (-5 - p)/2)*(c + d*Sin[e + f*x])^n)/(f*g*(1 + p)*((c +
 d*Sin[e + f*x])/(c + d))^n))
```

Rubi [A] time = 0.222132, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{a^2 g 2^{\frac{p+5}{2}} (1-\sin(e+fx)) (\sin(e+fx)+1)^{\frac{1-p}{2}} (g\cos(e+fx))^{p-1} (c+d\sin(e+fx))^n \left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{p+1}{2}; \frac{1}{2}(-p-3 \ldots\right)}{f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((2^((5 + p)/2)*a^2*g*AppellF1[(1 + p)/2, (-3 - p)/2, -n, (3 + p)/2, (1 -
Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*
(1 - Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
```

```
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+f\right.}{}$$

$$= \frac{\left(a^2g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+f\right.}{}$$

$$= -\frac{2^{\frac{5+p}{2}}a^2gF_1\left(\frac{1+p}{2};\frac{1}{2}(-3-p),-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\right.}{}$$

Mathematica [F] time = 19.8071, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,
 x]
```

Maple [F] time = 0.616, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a+a\sin\left(fx+e\right)\right)^{2}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\left(g\cos\left(fx+e\right)\right)^p\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(g*cos(f*x + e)
)^p*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.1044 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=145

$$-\frac{a2^{\frac{p}{2}+\frac{3}{2}}(\sin(e+fx)+1)^{\frac{1}{2}(-p-1)}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{1}{2}(-p-1),-n;\frac{p+3}{2};\frac{1}{2}(1-\sin$$ $$\overline{\qquad fg(p+1) \qquad}$$

```
[Out] -((2^(3/2 + p/2)*a*AppellF1[(1 + p)/2, (-1 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(1 + p)*(1 +
Sin[e + f*x])^((-1 - p)/2)*(c + d*Sin[e + f*x])^n)/(f*g*(1 + p)*((c + d*Sin
[e + f*x])/(c + d))^n))
```

Rubi [A] time = 0.165132, antiderivative size = 151, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2868, 139, 138}

$$-\frac{ag2^{\frac{p+3}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{1}{2}(-p-1),-n$$ $$\overline{\qquad f(p+1) \qquad}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((2^((3 + p)/2)*a*g*AppellF1[(1 + p)/2, (-1 - p)/2, -n, (3 + p)/2, (1 - Si
n[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1
 - Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(f*
(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 2868

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(c*g*(g
*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(1 + (d*x)/c)^((p + 1)/2)*(1 - (d*x)/c)^((p - 1)/
2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
```

```
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{\left(ag(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+\ldots\right.}{\ldots}$$

$$= \frac{\left(ag(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+\ldots\right.}{\ldots}$$

$$= -\frac{2^{\frac{3+p}{2}}agF_1\left(\frac{1+p}{2};\frac{1}{2}(-1-p),-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\ldots\right.}{\ldots}$$

Mathematica [F] time = 3.82345, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x
]
```

Maple [F] time = 0.394, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a+a\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n, x
)
```

3.1045 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{\frac{p}{2}-\frac{1}{2}}(\sin(e+fx)+1)^{\frac{1-p}{2}-1}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{3-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+\right.}{afg(p+1)}$$

```
[Out] -((2^(-1/2 + p/2)*AppellF1[(1 + p)/2, (3 - p)/2, -n, (3 + p)/2, (1 - Sin[e
+ f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(1 + p)*(1 + Si
n[e + f*x])^(-1 + (1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a*f*g*(1 + p)*((c + d
*Sin[e + f*x])/(c + d))^n))
```

Rubi [A] time = 0.247344, antiderivative size = 155, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{g2^{\frac{p}{2}-\frac{1}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{3-p}{2},-n;\frac{p+}{2}\right.}{af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((2^(-1/2 + p/2)*g*AppellF1[(1 + p)/2, (3 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a*f*
(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}\right)\operatorname{Subst}\left(\int(1-\cdots\right.}{af}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}(c+d\sin(e+\cdots\right.}{\cdots}$$

$$= -\frac{2^{-\frac{1}{2}+\frac{p}{2}}gF_1\left(\frac{1+p}{2};\frac{3-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e+f\cdots}{af\cdots}$$

Mathematica [F] time = 6.91244, size = 0, normalized size = 0.

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),
x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),
 x]
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p\left(c+d\sin\left(fx+e\right)\right)^n}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p\left(d\sin\left(fx+e\right)+c\right)^n}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\left(g\cos\left(e+fx\right)\right)^{p}\left(c+d\sin\left(e+fx\right)\right)^{n}}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral((g*cos(e + f*x))**p*(c + d*sin(e + f*x))**n/(sin(e + f*x) + 1), x)
/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x
)
```

3.1046 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{\frac{p}{2}-\frac{3}{2}}(\sin(e+fx)+1)^{\frac{3-p}{2}-2}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{5-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+fx\right.}{a^2fg(p+1)}$$

```
[Out] -((2^(-3/2 + p/2)*AppellF1[(1 + p)/2, (5 - p)/2, -n, (3 + p)/2, (1 - Sin[e
+ f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(1 + p)*(1 + Si
n[e + f*x])^(-2 + (3 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^2*f*g*(1 + p)*((c +
 d*Sin[e + f*x])/(c + d))^n))
```

Rubi [A] time = 0.229998, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{g2^{\frac{p-3}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{5-p}{2},-n;\frac{p+3}{2};\frac{1}{2}\right.}{a^2f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((2^((-3 + p)/2)*g*AppellF1[(1 + p)/2, (5 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^2*
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}\right)\text{Subst}\left(\int\right.}{a^2 f}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}(c+d\sin(e\right.}{}$$

$$= -\frac{2^{\frac{1}{2}(-3+p)}gF_1\left(\frac{1+p}{2};\frac{5-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e}{}$$

Mathematica [F] time = 10.3247, size = 0, normalized size = 0.

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
2, x]
```

Maple [F] time = 0.53, size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p\left(d\sin\left(fx+e\right)+c\right)^n}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 - 2
*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2,
 x)
```

3.1047 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{\frac{p}{2}-\frac{5}{2}}(\sin(e+fx)+1)^{\frac{5-p}{2}-3}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{7-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+\right.}{a^3fg(p+1)}$$

```
[Out] -((2^(-5/2 + p/2)*AppellF1[(1 + p)/2, (7 - p)/2, -n, (3 + p)/2, (1 - Sin[e
+ f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(1 + p)*(1 + Si
n[e + f*x])^(-3 + (5 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^3*f*g*(1 + p)*((c +
 d*Sin[e + f*x])/(c + d))^n))
```

Rubi [A] time = 0.232628, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{g2^{\frac{p-5}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{7-p}{2},-n;\frac{p+3}{2}\right.}{a^3f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((2^((-5 + p)/2)*g*AppellF1[(1 + p)/2, (7 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^3*
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}\right)\text{Subst}\left(\int(1-\right.}{a^3f}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}(c+d\sin(e+\right.}{}$$

$$= -\frac{2^{\frac{1}{2}(-5+p)}gF_1\left(\frac{1+p}{2};\frac{7-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e+}{a^3}$$

Mathematica [F] time = 14.1332, size = 0, normalized size = 0.

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
3, x]
```

Maple [F] time = 0.628, size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(3*a^3*cos(f*x + e)^2 -
 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3,
 x)
```

3.1048 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{\frac{p}{2}-\frac{7}{2}}(\sin(e+fx)+1)^{\frac{7-p}{2}-4}(g\cos(e+fx))^{p+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{9-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+fx\right.}{a^4fg(p+1)}$$

```
[Out] -((2^(-7/2 + p/2)*AppellF1[(1 + p)/2, (9 - p)/2, -n, (3 + p)/2, (1 - Sin[e
+ f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(1 + p)*(1 + Si
n[e + f*x])^(-4 + (7 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^4*f*g*(1 + p)*((c +
 d*Sin[e + f*x])/(c + d))^n))
```

Rubi [A] time = 0.233877, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2920, 139, 138}

$$-\frac{g2^{\frac{p-7}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{9-p}{2},-n;\frac{p+3}{2};\frac{1}{2}\right.}{a^4f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4,x]
```

```
[Out] -((2^((-7 + p)/2)*g*AppellF1[(1 + p)/2, (9 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^4*
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}\right)\text{Subst}\left(\int\right.}{a^4f}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1-p}{2}}(c+d\sin(e\right.}{}$$

$$= -\frac{2^{\frac{1}{2}(-7+p)}gF_1\left(\frac{1+p}{2};\frac{9-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e}{}$$

Mathematica [F] time = 18.6434, size = 0, normalized size = 0.

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
4,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
4, x]
```

Maple [F] time = 0.76, size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^{4}\cos\left(fx+e\right)^{4}-8\,a^{4}\cos\left(fx+e\right)^{2}+8\,a^{4}-4\left(a^{4}\cos\left(fx+e\right)^{2}-2\,a^{4}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a^4*cos(f*x + e)^4 - 8*
a^4*cos(f*x + e)^2 + 8*a^4 - 4*(a^4*cos(f*x + e)^2 - 2*a^4)*sin(f*x + e)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^4,
 x)
```

3.1049 $\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=175

$$\frac{2^{\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^{m+1}(g\sec(e+fx))^p(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(\frac{1}{2}(2\right.}{af(2m-p+1)}$$

```
[Out] (2^(1/2 - p/2)*AppellF1[(1 + 2*m - p)/2, (1 + p)/2, -n, (3 + 2*m - p)/2, (1
 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*(g*Sec[
e + f*x])^p*(1 - Sin[e + f*x])^((1 + p)/2)*(a + a*Sin[e + f*x])^(1 + m)*(c
+ d*Sin[e + f*x])^n)/(a*f*(1 + 2*m - p)*((c + d*Sin[e + f*x])/(c - d))^n)
```

Rubi [A] time = 0.429306, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2926, 2921, 140, 139, 138}

$$\frac{2^{\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^{m+1}(g\sec(e+fx))^p(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(\frac{1}{2}(2\right.}{af(2m-p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 - p/2)*AppellF1[(1 + 2*m - p)/2, (1 + p)/2, -n, (3 + 2*m - p)/2, (1
 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*(g*Sec[
e + f*x])^p*(1 - Sin[e + f*x])^((1 + p)/2)*(a + a*Sin[e + f*x])^(1 + m)*(c
+ d*Sin[e + f*x])^n)/(a*f*(1 + 2*m - p)*((c + d*Sin[e + f*x])/(c - d))^n)
```

Rule 2926

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[((a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(g*Cos[e + f*x])^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] &&  !IntegerQ[p]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(g
*(g*Cos[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
```

```
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int (g\cos(e+fx))^{-p}(a+$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+fx))^{\frac{1+p}{2}}(a+a\,\text{s}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+fx))^{-\frac{1}{2}-}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+fx))^{-\frac{1}{2}-}\right.}{}$$

$$= \frac{2^{\frac{1}{2}-\frac{p}{2}}F_1\left(\frac{1}{2}(1+2m-p);\frac{1+p}{2},-n;\frac{1}{2}(3+2m-p);\frac{1}{2}(1+\sin\right.}{}$$

Mathematica [B] time = 9.43978, size = 852, normalized size = 4.87

$$\frac{}{f\left(\frac{dnF_1\left(\frac{1-p}{2};m+n-p+1,-n;\frac{3-p}{2};-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)\cos^2(e+fx)}{c+d\sin(e+fx)}+\frac{2(1-p)\left((m+n-p+1)F_1\left(\frac{3-p}{2};m+n-p+2,-n;\frac{5-p}{2};-\tan^2\left(\frac{1}{4}(2e+2\right.\right.\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (2*AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[(2*e - Pi + 2*f*x
)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Cos[(2*e + Pi + 2
*f*x)/4]*(g*Sec[e + f*x])^p*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^n
*Sin[(2*e + Pi + 2*f*x)/4])/(f*(-AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3
- p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^
2)/(c + d))] + (2*(1 - p)*(-(((c - d)*n*AppellF1[(3 - p)/2, 1 + m + n - p,
1 - n, (5 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi +
2*f*x)/4]^2)/(c + d))])/(c + d)) + (1 + m + n - p)*AppellF1[(3 - p)/2, 2 +
m + n - p, -n, (5 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e
 - Pi + 2*f*x)/4]^2)/(c + d))])*Cot[(2*e + Pi + 2*f*x)/4]^2)/(3 - p) + p*Ap
pellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^
2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Sin[e + f*x] + (d*n*Ap
pellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^
2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Cos[e + f*x]^2)/(c + d
*Sin[e + f*x]) - 2*(n - p)*AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2
, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c
+ d))]*Sin[(2*e - Pi + 2*f*x)/4]^2 + (2*(c - d)*n*AppellF1[(1 - p)/2, 1 + m
 + n - p, -n, (3 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e
- Pi + 2*f*x)/4]^2)/(c + d))]*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f
*x])))
```

Maple [F] time = 2.746, size = 0, normalized size = 0.

$$\int \left(g \sec \left(f x+e\right)\right)^{p} \left(a+a \sin \left(f x+e\right)\right)^{m} \left(c+d \sin \left(f x+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g \sec \left(f x+e\right)\right)^{p} \left(a \sin \left(f x+e\right)+a\right)^{m} \left(d \sin \left(f x+e\right)+c\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g \sec \left(f x+e\right)\right)^{p} \left(a \sin \left(f x+e\right)+a\right)^{m} \left(d \sin \left(f x+e\right)+c\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\sec\left(fx+e\right)\right)^{p}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

3.1050 $\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{b\sin(c+dx)\cos(c+dx)}{16d}$$

```
[Out] (b*x)/16 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) - (b*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (b*Cos
[c + d*x]^3*Sin[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.163955, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{b\sin(c+dx)\cos(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/16 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) - (b*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (b*Cos
[c + d*x]^3*Sin[c + d*x]^3)/(6*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin^3(c+dx)\,dx + b\int \cos^2(c+dx)\sin^4(c+dx)\,dx$$
$$= -\frac{b\cos^3(c+dx)\sin^3(c+dx)}{6d} + \frac{1}{2}b\int \cos^2(c+dx)\sin^2(c+dx)\,dx -$$
$$= -\frac{b\cos^3(c+dx)\sin(c+dx)}{8d} - \frac{b\cos^3(c+dx)\sin^3(c+dx)}{6d} + \frac{1}{8}b\int \mathrm{co}$$
$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{b\cos(c+dx)\sin(c+dx)}{16d} - \frac{b\cos}{}$$
$$= \frac{bx}{16} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{b\cos(c+dx)\sin(c+dx)}{16d} -$$

Mathematica [A] time = 0.186917, size = 77, normalized size = 0.73

$$\frac{-120a\cos(c+dx) - 20a\cos(3(c+dx)) + 12a\cos(5(c+dx)) - 15b\sin(2(c+dx)) - 15b\sin(4(c+dx)) + 5b\sin(6(c+dx)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*b*d*x - 120*a*Cos[c + d*x] - 20*a*Cos[3*(c + d*x)] + 12*a*Cos[5*(c + d*
x)] - 15*b*Sin[2*(c + d*x)] - 15*b*Sin[4*(c + d*x)] + 5*b*Sin[6*(c + d*x)])
/(960*d)
```

Maple [A] time = 0.03, size = 95, normalized size = 0.9

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^3}{5} - \frac{2(\cos(dx+c))^3}{15}\right) + b\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^3}{6} - \frac{(\cos(dx+c))^3\sin(dx+}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+b*(-1/6*sin(d*x+c
)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/1
6*d*x+1/16*c))
```

Maxima [A] time = 1.10122, size = 88, normalized size = 0.84

$$\frac{64\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a - 5\left(4\sin(2dx+2c)^3 - 12dx - 12c + 3\sin(4dx+4c)\right)b}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a - 5*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*b)/d
```

Fricas [A] time = 1.52557, size = 193, normalized size = 1.84

$$\frac{48\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+15\,bdx+5\,\left(8\,b\cos\left(dx+c\right)^{5}-14\,b\cos\left(dx+c\right)^{3}+3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*b*d*x + 5*(8*b*cos(d*
x + c)^5 - 14*b*cos(d*x + c)^3 + 3*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 4.25816, size = 192, normalized size = 1.83

$$\begin{cases} -\frac{a\sin^{2}(c+dx)\cos^{3}(c+dx)}{3d}-\frac{2a\cos^{5}(c+dx)}{15d}+\frac{bx\sin^{6}(c+dx)}{16}+\frac{3bx\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3bx\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{bx\cos^{6}(c+dx)}{16}+\frac{b\sin \ldots}{} \\ x\left(a+b\sin\left(c\right)\right)\sin^{3}\left(c\right)\cos^{2}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*a*cos(c + d*x)**5/(
15*d) + b*x*sin(c + d*x)**6/16 + 3*b*x*sin(c + d*x)**4*cos(c + d*x)**2/16 +
 3*b*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b*x*cos(c + d*x)**6/16 + b*sin(
c + d*x)**5*cos(c + d*x)/(16*d) - b*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) -
 b*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)
**3*cos(c)**2, True))
```

Giac [A] time = 1.30691, size = 124, normalized size = 1.18

$$\frac{1}{16}\,bx+\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}+\frac{b\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{b\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*b*x + 1/80*a*cos(5*d*x + 5*c)/d - 1/48*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d + 1/192*b*sin(6*d*x + 6*c)/d - 1/64*b*sin(4*d*x + 4*c)/d - 1/6
4*b*sin(2*d*x + 2*c)/d
```

3.1051 $\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}+\frac{b\cos^5(c+dx)}{5d}-\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] (a*x)/8 - (b*Cos[c + d*x]^3)/(3*d) + (b*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
d*x]*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.130606, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}+\frac{b\cos^5(c+dx)}{5d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/8 - (b*Cos[c + d*x]^3)/(3*d) + (b*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
d*x]*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin^2(c+dx)\,dx + b\int \cos^2(c+dx)\sin^3(c+dx)$$

$$= -\frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^2(c+dx)\,dx - \frac{b\,\mathrm{Subst}\,(\int}{}$$

$$= \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}a\int 1$$

$$= \frac{ax}{8} - \frac{b\cos^3(c+dx)}{3d} + \frac{b\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{8d}$$

Mathematica [A] time = 0.0964036, size = 59, normalized size = 0.73

$$\frac{-15a\sin(4(c+dx))+60ac+60adx-60b\cos(c+dx)-10b\cos(3(c+dx))+6b\cos(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*a*c + 60*a*d*x - 60*b*Cos[c + d*x] - 10*b*Cos[3*(c + d*x)] + 6*b*Cos[5*
(c + d*x)] - 15*a*Sin[4*(c + d*x)])/(480*d)
```

Maple [A] time = 0.029, size = 77, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^3}{5}-\frac{2\,(\cos}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*
c)+b*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3))
```

Maxima [A] time = 1.04165, size = 70, normalized size = 0.86

$$\frac{15\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a+32\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)b}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a + 32*(3*cos(d*x + c)^5 - 5*cos
(d*x + c)^3)*b)/d
```

Fricas [A] time = 1.43723, size = 162, normalized size = 2.

$$\frac{24\,b\cos\left(dx+c\right)^{5}-40\,b\cos\left(dx+c\right)^{3}+15\,adx-15\left(2\,a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*b*cos(d*x + c)^5 - 40*b*cos(d*x + c)^3 + 15*a*d*x - 15*(2*a*cos(d
*x + c)^3 - a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 2.20325, size = 144, normalized size = 1.78

$$\begin{cases}\frac{ax\sin^{4}(c+dx)}{8}+\frac{ax\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{ax\cos^{4}(c+dx)}{8}+\frac{a\sin^{3}(c+dx)\cos(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^{3}(c+dx)}{8d}-\frac{b\sin^{2}(c+dx)\cos^{3}(c+dx)}{3d}-\frac{2b\cos\ldots}{} \\ x\left(a+b\sin(c)\right)\sin^{2}(c)\cos^{2}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**4/8 + a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
a*x*cos(c + d*x)**4/8 + a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a*sin(c + d*
x)*cos(c + d*x)**3/(8*d) - b*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b*co
s(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**2*cos(c)**2, Tru
e))
```

Giac [A] time = 1.25384, size = 84, normalized size = 1.04

$$\frac{1}{8}\,ax+\frac{b\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{b\cos\left(dx+c\right)}{8\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*a*x + 1/80*b*cos(5*d*x + 5*c)/d - 1/48*b*cos(3*d*x + 3*c)/d - 1/8*b*cos
(d*x + c)/d - 1/32*a*sin(4*d*x + 4*c)/d
```

3.1052 $\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b\sin(c+dx)\cos(c+dx)}{8d}+\frac{bx}{8}$$

```
[Out] (b*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(8*d) -
(b*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0942328, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b\sin(c+dx)\cos(c+dx)}{8d}+\frac{bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(8*d) -
(b*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin(c+dx)\,dx + b\int \cos^2(c+dx)\sin^2(c+dx)\,dx$$
$$= -\frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}b\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int x^2 \right.}{}$$
$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{b\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\cos^3(c+dx)\sin(c+}{4d}$$
$$= \frac{bx}{8} - \frac{a\cos^3(c+dx)}{3d} + \frac{b\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\cos^3(c+dx)\sin(}{4d}$$

Mathematica [A] time = 0.110726, size = 61, normalized size = 0.94

$$-\frac{a\cos^3(c+dx)}{3d} + \frac{1}{8}b\left(-\frac{\sin(4c)\cos(4dx)}{4d} - \frac{\cos(4c)\sin(4dx)}{4d}\right) + \frac{bx}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (b*(-(Cos[4*d*x]*Sin[4*c])/(4*d) - (Co
s[4*c]*Sin[4*d*x])/(4*d)))/8
```

Maple [A] time = 0.026, size = 57, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^3}{3} + b\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4} + \frac{\cos(dx+c)\sin(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/3*a*cos(d*x+c)^3+b*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin
(d*x+c)+1/8*d*x+1/8*c))
```

Maxima [A] time = 1.12417, size = 53, normalized size = 0.82

$$-\frac{32\,a\cos(dx+c)^3 - 3\,(4\,dx + 4\,c - \sin(4\,dx + 4\,c))b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*a*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*b)/d
```

Fricas [A] time = 1.46703, size = 128, normalized size = 1.97

$$-\frac{8\,a\cos\left(dx+c\right)^{3}-3\,bdx+3\left(2\,b\cos\left(dx+c\right)^{3}-b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*a*cos(d*x + c)^3 - 3*b*d*x + 3*(2*b*cos(d*x + c)^3 - b*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [A] time = 1.13681, size = 119, normalized size = 1.83

$$\begin{cases} -\frac{a\cos^{3}(c+dx)}{3d}+\frac{bx\sin^{4}(c+dx)}{8}+\frac{bx\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{bx\cos^{4}(c+dx)}{8}+\frac{b\sin^{3}(c+dx)\cos(c+dx)}{8d}-\frac{b\sin(c+dx)\cos^{3}(c+dx)}{8d} & \text{for}\ d \neq \\ x\left(a+b\sin\left(c\right)\right)\sin\left(c\right)\cos^{2}\left(c\right) & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)**3/(3*d) + b*x*sin(c + d*x)**4/8 + b*x*sin(c + d
*x)**2*cos(c + d*x)**2/4 + b*x*cos(c + d*x)**4/8 + b*sin(c + d*x)**3*cos(c
+ d*x)/(8*d) - b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*s
in(c))*sin(c)*cos(c)**2, True))
```

Giac [A] time = 1.26649, size = 63, normalized size = 0.97

$$\frac{1}{8}\,bx-\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{a\cos\left(dx+c\right)}{4\,d}-\frac{b\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*b*x - 1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/32*b*sin(4*d
*x + 4*c)/d
```

3.1053 $\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

```
[Out] (b*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (b*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0694966, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2838, 2592, 321, 206, 2635, 8}

$$\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (b*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos(c+dx)\cot(c+dx)\,dx + b\int \cos^2(c+dx)\,dx$$
$$= \frac{b\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}b\int 1\,dx - \frac{a\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos\right.}{d}$$
$$= \frac{bx}{2} + \frac{a\cos(c+dx)}{d} + \frac{b\cos(c+dx)\sin(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{}$$
$$= \frac{bx}{2} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{b\cos(c+dx)\sin(c}{2d}$$

Mathematica [A] time = 0.0541021, size = 74, normalized size = 1.45

$$\frac{a\cos(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{b(c+dx)}{2d} + \frac{b\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a
*Log[Sin[(c + d*x)/2]])/d + (b*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.048, size = 63, normalized size = 1.2

$$\frac{\cos(dx+c)\,a}{d} + \frac{a\ln(\csc(dx+c) - \cot(dx+c))}{d} + \frac{b\cos(dx+c)\sin(dx+c)}{2\,d} + \frac{bx}{2} + \frac{cb}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/2*b*cos(d*x+c)*sin(d*x+c)/
d+1/2*b*x+1/2*b*c/d
```

Maxima [A] time = 1.03717, size = 77, normalized size = 1.51

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))b + 2\,a(2\,\cos(dx+c) - \log(\cos(dx+c) + 1) + \log(\cos(dx+c) - 1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*b + 2*a*(2*cos(d*x + c) - log(cos(d*x
 + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.47206, size = 174, normalized size = 3.41

$$\frac{bdx + b\cos(dx+c)\sin(dx+c) + 2\,a\cos(dx+c) - a\log\left(\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right) + a\log\left(-\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + b*cos(d*x + c)*sin(d*x + c) + 2*a*cos(d*x + c) - a*log(1/2*cos
(d*x + c) + 1/2) + a*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sin(c+dx))\cos^2(c+dx)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**2*csc(c + d*x), x)
```

Giac [A] time = 1.32054, size = 117, normalized size = 2.29

$$\frac{(dx+c)b + 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*b + 2*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(b*tan(1/2*d*x +
1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.1054 $\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -(a*x) - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rubi [A] time = 0.0561516, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2722, 2592, 321, 206, 3473, 8}

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)dx\\ &= a\int \cot^2(c+dx)\,dx+b\int \cos(c+dx)\cot(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)}{d}-a\int 1\,dx-\frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -ax+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -ax-\frac{b\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}\end{aligned}$$

Mathematica [C] time = 0.0312949, size = 75, normalized size = 1.83

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/2]])/d
```

Maple [A] time = 0.046, size = 57, normalized size = 1.4

$$-ax-\frac{a\cot(dx+c)}{d}+\frac{b\cos(dx+c)}{d}+\frac{b\ln(\csc(dx+c)-\cot(dx+c))}{d}-\frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -a*x-a*cot(d*x+c)/d+b*cos(d*x+c)/d+1/d*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a
```

Maxima [A] time = 1.56275, size = 73, normalized size = 1.78

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-b(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - b*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.50713, size = 236, normalized size = 5.76

$$-\frac{b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+2\,a\cos(dx+c)+2\,(adx-b\cos(}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - b*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37987, size = 146, normalized size = 3.56

$$-\frac{6\,(dx+c)a-6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*b*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

3.1055 $\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot(c+dx)}{d}-bx$$

```
[Out] -(b*x) + (a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x])/d - (a*Cot[c +
d*x]*Csc[c + d*x])/(2*d)
```

Rubi [A] time = 0.079695, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2838, 2611, 3770, 3473, 8}

$$\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*x) + (a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x])/d - (a*Cot[c +
d*x]*Csc[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc(c+dx)\,dx + b\int \cot^2(c+dx)\,dx$$
$$= -\frac{b\cot(c+dx)}{d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}a\int \csc(c+dx)\,dx$$
$$= -bx + \frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b\cot(c+dx)}{d} - \frac{a\cot(c+dx)\csc(}{2d}$$

Mathematica [C] time = 0.0438881, size = 109, normalized size = 2.1

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^2)/(8*d) - (b*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, -Tan[c + d*x]^2])/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Sin[(c
+ d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.052, size = 81, normalized size = 1.6

$$-\frac{a(\cos(dx+c))^3}{2d(\sin(dx+c))^2} - \frac{\cos(dx+c)a}{2d} - \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d} - bx - \frac{b\cot(dx+c)}{d} - \frac{cb}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a*cos(d*x+c)/d-1/2/d*a*ln(csc(d*x+c)
-cot(d*x+c))-b*x-b*cot(d*x+c)/d-b*c/d
```

Maxima [A] time = 1.73384, size = 89, normalized size = 1.71

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)b - a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} + \log(\cos(dx+c)+1) - \log(\cos(dx+c)-1)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*b - a*(2*cos(d*x + c)/(cos(d*x + c)^2 -
1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.48262, size = 300, normalized size = 5.77

$$-\frac{4bdx\cos(dx+c)^2 - 4bdx - 4b\cos(dx+c)\sin(dx+c) - 2a\cos(dx+c) - \left(a\cos(dx+c)^2 - a\right)\log\left(\frac{1}{2}\cos(dx+c\right.}{4\left(d\cos(dx+c)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*d*x*cos(d*x + c)^2 - 4*b*d*x - 4*b*cos(d*x + c)*sin(d*x + c) - 2*
a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + (a*co
s(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30955, size = 128, normalized size = 2.46

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,(dx+c)b-4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 8*(d*x + c)*b - 4*a*log(abs(tan(1/2*d*x + 1
/2*c))) + 4*b*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(
1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.1056 $\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] (b*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]^3)/(3*d) - (b*Cot[c + d*x
]*Csc[c + d*x])/(2*d)
```

Rubi [A] time = 0.109089, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]^3)/(3*d) - (b*Cot[c + d*x
]*Csc[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^2(c+dx)\,dx + b\int \cot^2(c+dx)\csc(c+dx)\,dx$$
$$= -\frac{b\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}b\int \csc(c+dx)\,dx + \frac{a\,\text{Subst}\left(\int x^2 d\right.}{}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\cot(c+dx)\csc(c+dx)}{2d}$$

Mathematica [A] time = 0.0361305, size = 95, normalized size = 1.83

$$-\frac{a\cot^3(c+dx)}{3d} - \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^3)/(3*d) - (b*Csc[(c + d*x)/2]^2)/(8*d) + (b*Log[Cos[(c +
d*x)/2]])/(2*d) - (b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2)/
(8*d)
```

Maple [A] time = 0.054, size = 80, normalized size = 1.5

$$-\frac{a(\cos(dx+c))^3}{3d(\sin(dx+c))^3} - \frac{b(\cos(dx+c))^3}{2d(\sin(dx+c))^2} - \frac{b\cos(dx+c)}{2d} - \frac{b\ln(\csc(dx+c)-\cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^3-1/2/d*b/sin(d*x+c)^2*cos(d*x+c)^3-1/2*b*
cos(d*x+c)/d-1/2/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.1028, size = 82, normalized size = 1.58

$$\frac{3b\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} + \log(\cos(dx+c)+1) - \log(\cos(dx+c)-1)\right) - \frac{4a}{\tan(dx+c)^3}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - lo
g(cos(d*x + c) - 1)) - 4*a/tan(d*x + c)^3)/d
```

Fricas [B] time = 1.35262, size = 316, normalized size = 6.08

$$\frac{4a\cos(dx+c)^3 + 6b\cos(dx+c)\sin(dx+c) + 3\left(b\cos(dx+c)^2 - b\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 3\left(b\cos(d\right.}{12\left(d\cos(dx+c)^2 - d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*cos(d*x + c)^3 + 6*b*cos(d*x + c)*sin(d*x + c) + 3*(b*cos(d*x + c
)^2 - b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(b*cos(d*x + c)^2 - b
)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26198, size = 155, normalized size = 2.98

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{22\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a}{\mathrm{ta}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 - 12*b*log(abs(
tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x + 1/2*c) + (22*b*tan(1/2*d*x + 1/2
*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*
d*x + 1/2*c)^3)/d
```

3.1057 $\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}$$

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (b*Cot[c + d*x]^3)/(3*d) + (a*Cot[c + d*x
]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.127812, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (b*Cot[c + d*x]^3)/(3*d) + (a*Cot[c + d*x
]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^3(c+dx)\,dx + b\int \cot^2(c+dx)\csc^2(c+dx)$$
$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a\int \csc^3(c+dx)\,dx + \frac{b\,\mathrm{Subst}\big(\int$$
$$= -\frac{b\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(}{4d}$$
$$= \frac{a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc(c+d}{8d}$$

Mathematica [A] time = 0.0406093, size = 135, normalized size = 1.82

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{a\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cot[c + d*x]^3)/(3*d) + (a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Log[Cos[(c + d*x)/2]])/(8*d) - (a*Log[Sin[(c + d*x)/2]]
)/(8*d) - (a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.061, size = 102, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^3}{4d(\sin(dx+c))^4} - \frac{a(\cos(dx+c))^3}{8d(\sin(dx+c))^2} - \frac{\cos(dx+c)\,a}{8d} - \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{8d} - \frac{b(\cos(dx+c))^3}{3d(\sin(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^3-1/8*a*
cos(d*x+c)/d-1/8/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*b/sin(d*x+c)^3*cos(d*x
+c)^3
```

Maxima [A] time = 1.08119, size = 108, normalized size = 1.46

$$-\frac{3a\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) + \frac{16b}{\tan(dx+c)^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/48*(3*a*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x +
 c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*b/tan(d*x
+ c)^3)/d
```

Fricas [B] time = 1.4037, size = 377, normalized size = 5.09

$$-\frac{16\,b\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+6\,a\cos\left(dx+c\right)^{3}+6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\cos\cdots\right.}{48\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(16*b*cos(d*x + c)^3*sin(d*x + c) + 6*a*cos(d*x + c)^3 + 6*a*cos(d*x
+ c) - 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) +
 1/2) + 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c)
 + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3505, size = 157, normalized size = 2.12

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{50\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+24\,b\cdots}{\cdots}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*log(a
bs(tan(1/2*d*x + 1/2*c))) - 24*b*tan(1/2*d*x + 1/2*c) + (50*a*tan(1/2*d*x +
 1/2*c)^4 + 24*b*tan(1/2*d*x + 1/2*c)^3 - 8*b*tan(1/2*d*x + 1/2*c) - 3*a)/t
an(1/2*d*x + 1/2*c)^4)/d
```

3.1058 $\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{b\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] (b*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]^5)/(5*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Cot[c + d*x]*Csc[c +
d*x]^3)/(4*d)
```

Rubi [A] time = 0.133439, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{b\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]^5)/(5*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Cot[c + d*x]*Csc[c +
d*x]^3)/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^4(c+dx)\,dx + b\int \cot^2(c+dx)\csc^3(c+dx)\,dx$$
$$= -\frac{b\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}b\int \csc^3(c+dx)\,dx + \frac{a\,\mathrm{Subst}\left(\int x^2\right.}{}$$
$$= \frac{b\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{8}b\int \csc(c$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} + \frac{b\cot(c+}{}$$

Mathematica [A] time = 0.0688998, size = 177, normalized size = 1.97

$$\frac{2a\cot(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} + \frac{a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{b\,s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(15*d) + (b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*Cot[c + d*x]*C
sc[c + d*x]^4)/(5*d) + (b*Log[Cos[(c + d*x)/2]])/(8*d) - (b*Log[Sin[(c + d*
x)/2]])/(8*d) - (b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*
d)
```

Maple [A] time = 0.059, size = 124, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^3}{5d(\sin(dx+c))^5} - \frac{2a(\cos(dx+c))^3}{15d(\sin(dx+c))^3} - \frac{b(\cos(dx+c))^3}{4d(\sin(dx+c))^4} - \frac{b(\cos(dx+c))^3}{8d(\sin(dx+c))^2} - \frac{b\cos(dx+c)}{8d} - \frac{b\ln(\csc(dx+c)-}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^3-2/15/d*a/sin(d*x+c)^3*cos(d*x+c)^3-1/4/d
*b/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*b/sin(d*x+c)^2*cos(d*x+c)^3-1/8*b*cos(d*
x+c)/d-1/8/d*b*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.04767, size = 124, normalized size = 1.38

$$-\frac{15\,b\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\left(5\,\tan(dx+c)^2+3\right)a}{\tan(dx+c)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(15*b*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*(5*tan(d
*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Fricas [B] time = 1.51351, size = 466, normalized size = 5.18

$$\frac{32\,a\cos\left(dx+c\right)^5-80\,a\cos\left(dx+c\right)^3+15\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right.}{240\left(d\cos\left(dx+c\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(b*cos
(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 30*(b*cos(d*x + c)^3 + b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28988, size = 194, normalized size = 2.16

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-60\,a\tan\left(\frac{1}{2}\,dx\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(
1/2*d*x + 1/2*c)^3 - 120*b*log(abs(tan(1/2*d*x + 1/2*c))) - 60*a*tan(1/2*d*
x + 1/2*c) + (274*b*tan(1/2*d*x + 1/2*c)^5 + 60*a*tan(1/2*d*x + 1/2*c)^4 -
10*a*tan(1/2*d*x + 1/2*c)^2 - 15*b*tan(1/2*d*x + 1/2*c) - 6*a)/tan(1/2*d*x
+ 1/2*c)^5)/d
```

3.1059 $\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=190

$$\frac{(7a^2+4b^2)\cos^3(c+dx)}{105d}-\frac{(7a^2+4b^2)\cos(c+dx)}{35d}+\frac{(2a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{ab\sin^5(c+dx)\cos(c+d$$

```
[Out] (a*b*x)/8 - ((7*a^2 + 4*b^2)*Cos[c + d*x])/(35*d) + ((7*a^2 + 4*b^2)*Cos[c
+ d*x]^3)/(105*d) - (a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a*b*Cos[c + d*
x]*Sin[c + d*x]^3)/(12*d) + ((2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(35
*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]^5)/(21*d) + (Cos[c + d*x]*Sin[c + d*x]
^4*(a + b*Sin[c + d*x])^2)/(7*d)
```

Rubi [A] time = 0.382177, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3033, 3023, 2748, 2633, 2635, 8}

$$\frac{(7a^2+4b^2)\cos^3(c+dx)}{105d}-\frac{(7a^2+4b^2)\cos(c+dx)}{35d}+\frac{(2a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{ab\sin^5(c+dx)\cos(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*x)/8 - ((7*a^2 + 4*b^2)*Cos[c + d*x])/(35*d) + ((7*a^2 + 4*b^2)*Cos[c
+ d*x]^3)/(105*d) - (a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a*b*Cos[c + d*
x]*Sin[c + d*x]^3)/(12*d) + ((2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(35
*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]^5)/(21*d) + (Cos[c + d*x]*Sin[c + d*x]
^4*(a + b*Sin[c + d*x])^2)/(7*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
```

```
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx = \int \sin^3(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2}{7d} + \frac{1}{7}\int \sin^3(c+dx)(a+$$

$$= \frac{ab\cos(c+dx)\sin^5(c+dx)}{21d} + \frac{\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+$$

$$= \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{35d} + \frac{ab\cos(c+dx)\sin^5(c+dx)}{21d} +$$

$$= \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{35d} + \frac{ab\cos(c+dx)\sin^5(c+dx)}{21d} +$$

$$= -\frac{ab\cos(c+dx)\sin^3(c+dx)}{12d} + \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{35d}$$

$$= -\frac{\left(7a^2+4b^2\right)\cos(c+dx)}{35d} + \frac{\left(7a^2+4b^2\right)\cos^3(c+dx)}{105d} - \frac{ab\cos(c+}{}$$

$$= \frac{abx}{8} - \frac{\left(7a^2+4b^2\right)\cos(c+dx)}{35d} + \frac{\left(7a^2+4b^2\right)\cos^3(c+dx)}{105d} - \frac{ab\cos}{}$$

Mathematica [A] time = 0.551489, size = 132, normalized size = 0.69

$$\frac{-105\left(8a^2+5b^2\right)\cos(c+dx)-35\left(4a^2+b^2\right)\cos(3(c+dx))+84a^2\cos(5(c+dx))-210ab\sin(2(c+dx))-210ab\sin}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (840*a*b*c + 840*a*b*d*x - 105*(8*a^2 + 5*b^2)*Cos[c + d*x] - 35*(4*a^2 + b
^2)*Cos[3*(c + d*x)] + 84*a^2*Cos[5*(c + d*x)] + 63*b^2*Cos[5*(c + d*x)] -
15*b^2*Cos[7*(c + d*x)] - 210*a*b*Sin[2*(c + d*x)] - 210*a*b*Sin[4*(c + d*x
)] + 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.043, size = 150, normalized size = 0.8

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^3}{5}-\frac{2(\cos(dx+c))^3}{15}\right)+2ab\left(-1/6(\sin(dx+c))^3(\cos(dx+c))^3-1/8(\cos(dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+2*a*b*(-1/6*sin
(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*x+
c)+1/16*d*x+1/16*c)+b^2*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*c
os(d*x+c)^3-8/105*cos(d*x+c)^3))
```

Maxima [A] time = 1.12241, size = 140, normalized size = 0.74

$$\frac{224\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)a^2-35\left(4\sin(2dx+2c)^3-12dx-12c+3\sin(4dx+4c)\right)ab-32\left(15\cos(\right.}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3360*(224*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2 - 35*(4*sin(2*d*x + 2
*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a*b - 32*(15*cos(d*x + c)^7 - 4
2*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*b^2)/d
```

Fricas [A] time = 1.49346, size = 274, normalized size = 1.44

$$-\frac{120b^2\cos(dx+c)^7-168\left(a^2+2b^2\right)\cos(dx+c)^5-105abdx+280\left(a^2+b^2\right)\cos(dx+c)^3-35\left(8ab\cos(dx+c)^5-\right.}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + 2*b^2)*cos(d*x + c)^5 - 105*a*b
*d*x + 280*(a^2 + b^2)*cos(d*x + c)^3 - 35*(8*a*b*cos(d*x + c)^5 - 14*a*b*c
os(d*x + c)^3 + 3*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 7.76245, size = 275, normalized size = 1.45

$$\begin{cases} -\frac{a^2\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{2a^2\cos^5(c+dx)}{15d} + \frac{abx\sin^6(c+dx)}{8} + \frac{3abx\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3abx\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{abx\cos^6(c+dx)}{8} + \cdots \\ x\left(a+b\sin(c)\right)^2\sin^3(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*a**2*cos(c + d*x
)**5/(15*d) + a*b*x*sin(c + d*x)**6/8 + 3*a*b*x*sin(c + d*x)**4*cos(c + d*x
)**2/8 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + a*b*x*cos(c + d*x)**6/
8 + a*b*sin(c + d*x)**5*cos(c + d*x)/(8*d) - a*b*sin(c + d*x)**3*cos(c + d*
x)**3/(3*d) - a*b*sin(c + d*x)*cos(c + d*x)**5/(8*d) - b**2*sin(c + d*x)**4
*cos(c + d*x)**3/(3*d) - 4*b**2*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 8*
b**2*cos(c + d*x)**7/(105*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**3*cos
(c)**2, True))
```

Giac [A] time = 1.19229, size = 190, normalized size = 1.

$$\frac{1}{8}abx - \frac{b^2\cos(7dx+7c)}{448d} + \frac{ab\sin(6dx+6c)}{96d} - \frac{ab\sin(4dx+4c)}{32d} - \frac{ab\sin(2dx+2c)}{32d} + \frac{\left(4a^2+3b^2\right)\cos(5dx+5c)}{320d} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a*b*x - 1/448*b^2*cos(7*d*x + 7*c)/d + 1/96*a*b*sin(6*d*x + 6*c)/d - 1/
32*a*b*sin(4*d*x + 4*c)/d - 1/32*a*b*sin(2*d*x + 2*c)/d + 1/320*(4*a^2 + 3*
b^2)*cos(5*d*x + 5*c)/d - 1/192*(4*a^2 + b^2)*cos(3*d*x + 3*c)/d - 1/64*(8*
a^2 + 5*b^2)*cos(d*x + c)/d
```

3.1060 $\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=163

$$\frac{(2a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{(2a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(2a^2+b^2)+\frac{2ab\cos^3(c+dx)}{15d}-\frac{2ab\,c}{}$$

```
[Out] ((2*a^2 + b^2)*x)/16 - (2*a*b*Cos[c + d*x])/(5*d) + (2*a*b*Cos[c + d*x]^3)/
(15*d) - ((2*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((2*a^2 - b^2)*
Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]^4)/(15
*d) + (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(6*d)
```

Rubi [A] time = 0.381583, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3033, 3023, 2748, 2635, 8, 2633}

$$\frac{(2a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{(2a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(2a^2+b^2)+\frac{2ab\cos^3(c+dx)}{15d}-\frac{2ab\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] ((2*a^2 + b^2)*x)/16 - (2*a*b*Cos[c + d*x])/(5*d) + (2*a*b*Cos[c + d*x]^3)/
(15*d) - ((2*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((2*a^2 - b^2)*
Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]^4)/(15
*d) + (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(6*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
```

```
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = \int \sin^2(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2}{6d} + \frac{1}{6}\int \sin^2(c+dx)(a+$$

$$= \frac{ab\cos(c+dx)\sin^4(c+dx)}{15d} + \frac{\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+$$

$$= \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{ab\cos(c+dx)\sin^4(c+dx)}{15d} +$$

$$= \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{ab\cos(c+dx)\sin^4(c+dx)}{15d} +$$

$$= -\frac{\left(2a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{\left(2a^2-b^2\right)\cos(c+dx)\sin^3(c}{24d}$$

$$= \frac{1}{16}\left(2a^2+b^2\right)x - \frac{2ab\cos(c+dx)}{5d} + \frac{2ab\cos^3(c+dx)}{15d} - \frac{\left(2a^2+b^2\right)}{}$$

Mathematica [A] time = 0.213002, size = 120, normalized size = 0.74

$$\frac{-30a^2\sin(4(c+dx)) + 120a^2c + 120a^2dx - 240ab\cos(c+dx) - 40ab\cos(3(c+dx)) + 24ab\cos(5(c+dx)) - 15b^2\sin(2(}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (120*a^2*c + 60*b^2*c + 120*a^2*d*x + 60*b^2*d*x - 240*a*b*Cos[c + d*x] - 4
0*a*b*Cos[3*(c + d*x)] + 24*a*b*Cos[5*(c + d*x)] - 15*b^2*Sin[2*(c + d*x)]
- 30*a^2*Sin[4*(c + d*x)] - 15*b^2*Sin[4*(c + d*x)] + 5*b^2*Sin[6*(c + d*x)
])/(960*d)
```

Maple [A] time = 0.042, size = 141, normalized size = 0.9

$$\frac{1}{d}\left(a^2\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+2ab\left(-1/5\,(\sin(dx+c))^2(\cos(dx+c))^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)+2*a*b*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+b^2*(-1/6*sin
(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*x+
c)+1/16*d*x+1/16*c))
```

Maxima [A] time = 1.16233, size = 124, normalized size = 0.76

$$\frac{30\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2+128\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)ab-5\left(4\,\sin(2\,dx+2\,c)^3-12\,dx-12\,c+\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/960*(30*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 + 128*(3*cos(d*x + c)^5 - 5*
cos(d*x + c)^3)*a*b - 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x
 + 4*c))*b^2)/d
```

Fricas [A] time = 1.48997, size = 252, normalized size = 1.55

$$\frac{96\,ab\cos(dx+c)^5-160\,ab\cos(dx+c)^3+15\left(2\,a^2+b^2\right)dx+5\left(8\,b^2\cos(dx+c)^5-2\left(6\,a^2+7\,b^2\right)\cos(dx+c)^3+3\left(\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/240*(96*a*b*cos(d*x + c)^5 - 160*a*b*cos(d*x + c)^3 + 15*(2*a^2 + b^2)*d*
x + 5*(8*b^2*cos(d*x + c)^5 - 2*(6*a^2 + 7*b^2)*cos(d*x + c)^3 + 3*(2*a^2 +
 b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 4.69886, size = 309, normalized size = 1.9

$$\begin{cases} \frac{a^2x\sin^4(c+dx)}{8} + \frac{a^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^2x\cos^4(c+dx)}{8} + \frac{a^2\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{a^2\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{2ab\sin^2(c+dx)\cos^3(c+dx)}{3d} \\ x(a+b\sin(c))^2\sin^2(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**4/8 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**
2/4 + a**2*x*cos(c + d*x)**4/8 + a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) -
a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a*b*sin(c + d*x)**2*cos(c + d*x
)**3/(3*d) - 4*a*b*cos(c + d*x)**5/(15*d) + b**2*x*sin(c + d*x)**6/16 + 3*b
**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**4/16 + b**2*x*cos(c + d*x)**6/16 + b**2*sin(c + d*x)**5*cos(c + d*x)
/(16*d) - b**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**2*sin(c + d*x)*co
s(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**2,
True))
```

Giac [A] time = 1.21461, size = 155, normalized size = 0.95

$$\frac{1}{16}\left(2a^2+b^2\right)x + \frac{ab\cos(5dx+5c)}{40d} - \frac{ab\cos(3dx+3c)}{24d} - \frac{ab\cos(dx+c)}{4d} + \frac{b^2\sin(6dx+6c)}{192d} - \frac{b^2\sin(2dx+2c)}{64d} - \left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2 + b^2)*x + 1/40*a*b*cos(5*d*x + 5*c)/d - 1/24*a*b*cos(3*d*x + 3
*c)/d - 1/4*a*b*cos(d*x + c)/d + 1/192*b^2*sin(6*d*x + 6*c)/d - 1/64*b^2*si
n(2*d*x + 2*c)/d - 1/64*(2*a^2 + b^2)*sin(4*d*x + 4*c)/d
```

3.1061 $\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=106

$$-\frac{(a^2+4b^2)\cos^3(c+dx)}{30d}-\frac{\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}-\frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d}+\frac{ab\sin(c+dx)\cos(}{4d}$$

```
[Out] (a*b*x)/4 - ((a^2 + 4*b^2)*Cos[c + d*x]^3)/(30*d) + (a*b*Cos[c + d*x]*Sin[c
 + d*x])/(4*d) - (a*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(10*d) - (Cos[c +
d*x]^3*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rubi [A] time = 0.158925, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{(a^2+4b^2)\cos^3(c+dx)}{30d}-\frac{\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}-\frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d}+\frac{ab\sin(c+dx)\cos(}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*x)/4 - ((a^2 + 4*b^2)*Cos[c + d*x]^3)/(30*d) + (a*b*Cos[c + d*x]*Sin[c
 + d*x])/(4*d) - (a*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(10*d) - (Cos[c +
d*x]^3*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d} + \frac{1}{5}\int \cos^2(c+dx)(2b+2a\sin(c$$
$$= -\frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d} - \frac{\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}$$
$$= -\frac{\left(a^2+4b^2\right)\cos^3(c+dx)}{30d} - \frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d} - \cos^3($$
$$= -\frac{\left(a^2+4b^2\right)\cos^3(c+dx)}{30d} + \frac{ab\cos(c+dx)\sin(c+dx)}{4d} - \frac{a\cos^3(c+}{}$$
$$= \frac{abx}{4} - \frac{\left(a^2+4b^2\right)\cos^3(c+dx)}{30d} + \frac{ab\cos(c+dx)\sin(c+dx)}{4d} - a\cos^3$$

Mathematica [A] time = 0.365439, size = 77, normalized size = 0.73

$$\frac{-30\left(2a^2+b^2\right)\cos(c+dx)-5\left(4a^2+b^2\right)\cos(3(c+dx))+3b(20a(c+dx)-5a\sin(4(c+dx))+b\cos(5(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*(2*a^2 + b^2)*Cos[c + d*x] - 5*(4*a^2 + b^2)*Cos[3*(c + d*x)] + 3*b*(2
0*a*(c + d*x) + b*Cos[5*(c + d*x)] - 5*a*Sin[4*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.037, size = 94, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{a^2(\cos(dx+c))^3}{3}+2ab\left(-1/4(\cos(dx+c))^3\sin(dx+c)+1/8\cos(dx+c)\sin(dx+c)+1/8dx+c/8\right)+b^2\left(-\frac{(\sin}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/3*a^2*cos(d*x+c)^3+2*a*b*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+
c)*sin(d*x+c)+1/8*d*x+1/8*c)+b^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d
*x+c)^3))
```

Maxima [A] time = 1.15251, size = 92, normalized size = 0.87

$$-\frac{80a^2\cos(dx+c)^3-15(4dx+4c-\sin(4dx+4c))ab-16\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)b^2}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(80*a^2*cos(d*x + c)^3 - 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b - 1
6*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*b^2)/d
```

Fricas [A] time = 1.38892, size = 185, normalized size = 1.75

$$\frac{12\,b^2\cos\left(dx+c\right)^5+15\,abdx-20\,\left(a^2+b^2\right)\cos\left(dx+c\right)^3-15\,\left(2\,ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^2*cos(d*x + c)^5 + 15*a*b*d*x - 20*(a^2 + b^2)*cos(d*x + c)^3 -
15*(2*a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 2.36347, size = 172, normalized size = 1.62

$$\begin{cases} -\frac{a^2\cos^3\left(c+dx\right)}{3d}+\frac{abx\sin^4\left(c+dx\right)}{4}+\frac{abx\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{2}+\frac{abx\cos^4\left(c+dx\right)}{4}+\frac{ab\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{ab\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{4d}-b^2\,\mathrm{s} \\ x\left(a+b\sin\left(c\right)\right)^2\sin\left(c\right)\cos^2\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**3/(3*d) + a*b*x*sin(c + d*x)**4/4 + a*b*x*si
n(c + d*x)**2*cos(c + d*x)**2/2 + a*b*x*cos(c + d*x)**4/4 + a*b*sin(c + d*x
)**3*cos(c + d*x)/(4*d) - a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) - b**2*sin
(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b**2*cos(c + d*x)**5/(15*d), Ne(d, 0
)), (x*(a + b*sin(c))**2*sin(c)*cos(c)**2, True))
```

Giac [A] time = 1.20691, size = 111, normalized size = 1.05

$$\frac{1}{4}\,abx+\frac{b^2\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{ab\sin\left(4\,dx+4\,c\right)}{16\,d}-\frac{\left(4\,a^2+b^2\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{\left(2\,a^2+b^2\right)\cos\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*a*b*x + 1/80*b^2*cos(5*d*x + 5*c)/d - 1/16*a*b*sin(4*d*x + 4*c)/d - 1/4
8*(4*a^2 + b^2)*cos(3*d*x + 3*c)/d - 1/8*(2*a^2 + b^2)*cos(d*x + c)/d
```

3.1062 $\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=90

$$\frac{(2a^2-b^2)\cos(c+dx)}{3d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos(c+dx)}{3d}+\frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}+abx$$

```
[Out] a*b*x - (a^2*ArcTanh[Cos[c + d*x]])/d + ((2*a^2 - b^2)*Cos[c + d*x])/(3*d)
+ (a*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (Cos[c + d*x]*(a + b*Sin[c + d*x]
)^2)/(3*d)
```

Rubi [A] time = 0.253013, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2889, 3050, 3033, 3023, 2735, 3770}

$$\frac{(2a^2-b^2)\cos(c+dx)}{3d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos(c+dx)}{3d}+\frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] a*b*x - (a^2*ArcTanh[Cos[c + d*x]])/d + ((2*a^2 - b^2)*Cos[c + d*x])/(3*d)
+ (a*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (Cos[c + d*x]*(a + b*Sin[c + d*x]
)^2)/(3*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \csc(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx \\
&= \frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}+\frac{1}{3}\int \csc(c+dx)(a+b\sin(c+d \\
&= \frac{ab\cos(c+dx)\sin(c+dx)}{3d}+\frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}+\frac{1}{6} \\
&= \frac{\left(2a^2-b^2\right)\cos(c+dx)}{3d}+\frac{ab\cos(c+dx)\sin(c+dx)}{3d}+\frac{\cos(c+dx)}{} \\
&= abx+\frac{\left(2a^2-b^2\right)\cos(c+dx)}{3d}+\frac{ab\cos(c+dx)\sin(c+dx)}{3d}+\frac{\cos(c}{} \\
&= abx-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{\left(2a^2-b^2\right)\cos(c+dx)}{3d}+\frac{ab\cos(c}{}
\end{aligned}
$$

Mathematica [A] time = 0.209251, size = 91, normalized size = 1.01

$$
\frac{3\left(4a^2-b^2\right)\cos(c+dx)+6a\left(2a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+b\sin(2(c+dx))+2bc+2bdx\right)+}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*(4*a^2 - b^2)*Cos[c + d*x] - b^2*Cos[3*(c + d*x)] + 6*a*(2*b*c + 2*b*d*x
 - 2*a*Log[Cos[(c + d*x)/2]] + 2*a*Log[Sin[(c + d*x)/2]] + b*Sin[2*(c + d*x
)]))/(12*d)
```

Maple [A] time = 0.072, size = 83, normalized size = 0.9

$$
\frac{a^2\cos(dx+c)}{d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{ab\cos(dx+c)\sin(dx+c)}{d}+abx+\frac{abc}{d}-\frac{b^2(\cos(dx+c))^3}{3d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] a^2*cos(d*x+c)/d+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+a*b*cos(d*x+c)*sin(d*x+c
)/d+a*b*x+1/d*a*b*c-1/3*b^2*cos(d*x+c)^3/d
```

Maxima [A] time = 1.12878, size = 100, normalized size = 1.11

$$-\frac{2\,b^2\cos\left(dx+c\right)^3-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)ab-3\,a^2\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^2*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b - 3*a^2
*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.43112, size = 231, normalized size = 2.57

$$-\frac{2\,b^2\cos\left(dx+c\right)^3-6\,abdx-6\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,a^2\cos\left(dx+c\right)+3\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,a^2\log\left(\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^2*cos(d*x + c)^3 - 6*a*b*d*x - 6*a*b*cos(d*x + c)*sin(d*x + c) -
6*a^2*cos(d*x + c) + 3*a^2*log(1/2*cos(d*x + c) + 1/2) - 3*a^2*log(-1/2*cos
(d*x + c) + 1/2))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3309, size = 180, normalized size = 2.

$$\frac{3\left(dx+c\right)ab+3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a*b + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*a*b*tan(
1/2*d*x + 1/2*c)^5 - 3*a^2*tan(1/2*d*x + 1/2*c)^4 + 3*b^2*tan(1/2*d*x + 1/2
*c)^4 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2 +
 b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.1063 $\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=78

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

```
[Out] -(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x
])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0976003, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473}

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x
])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)+2ab\cos(c+dx)\cot(c+dx)+a^2\cot^2(c+dx)\right)dx$$
$$= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot(c+dx)\,dx+b^2\int \cos^2(c+d$$
$$= -\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}-a^2\int 1\,dx+\frac{1}{2}b^2\int 1\,dx-\frac{(2a}{}$$
$$= -a^2x+\frac{b^2x}{2}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}$$
$$= -a^2x+\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+$$

Mathematica [A] time = 0.389094, size = 116, normalized size = 1.49

$$\frac{2a^2\tan\left(\frac{1}{2}(c+dx)\right)-2a^2\cot\left(\frac{1}{2}(c+dx)\right)-4a^2c-4a^2dx+8ab\cos(c+dx)+8ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8ab\log\left(\cos\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*b*Cos[c + d*x] - 2*a^2*Co
t[(c + d*x)/2] - 8*a*b*Log[Cos[(c + d*x)/2]] + 8*a*b*Log[Sin[(c + d*x)/2]]
+ b^2*Sin[2*(c + d*x)] + 2*a^2*Tan[(c + d*x)/2])/(4*d)
```

Maple [A] time = 0.066, size = 102, normalized size = 1.3

$$-a^2x-\frac{a^2\cot(dx+c)}{d}-\frac{a^2c}{d}+2\frac{ab\cos(dx+c)}{d}+2\frac{ab\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{b^2\cos(dx+c)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -a^2*x-a^2*cot(d*x+c)/d-1/d*c*a^2+2*a*b*cos(d*x+c)/d+2/d*a*b*ln(csc(d*x+c)-
cot(d*x+c))+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d+1/2*b^2*x+1/2/d*b^2*c
```

Maxima [A] time = 1.72419, size = 107, normalized size = 1.37

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-(2\,dx+2\,c+\sin(2\,dx+2\,c))b^2-4\,ab(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*b
^2 - 4*a*b*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1))
)/d
```

Fricas [A] time = 1.37342, size = 308, normalized size = 3.95

$$-\frac{b^2\cos(dx+c)^3+2\,ab\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,ab\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+\left(2\,a^2-b^2\right)\mathrm{co}}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^3 + 2*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c) + ((2*a^2 - b^2)*d*x - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36085, size = 200, normalized size = 2.56

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\left(2\,a^2-b^2\right)(dx+c)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a
^2 - b^2)*(d*x + c) - (4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*
c) - 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.1064 $\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=89

$$\frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{ab\cot(c+dx)}{d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d}-2abx+\frac{3b^2\cos(c+dx)}{2d}$$

```
[Out] -2*a*b*x + ((a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (3*b^2*Cos[c + d*x
])/(2*d) - (a*b*Cot[c + d*x])/d - (Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c +
 d*x])^2)/(2*d)
```

Rubi [A] time = 0.305784, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2889, 3048, 3031, 3023, 2735, 3770}

$$\frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{ab\cot(c+dx)}{d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d}-2abx+\frac{3b^2\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -2*a*b*x + ((a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (3*b^2*Cos[c + d*x
])/(2*d) - (a*b*Cot[c + d*x])/d - (Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c +
 d*x])^2)/(2*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^3(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)dx$$
$$= -\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d} + \frac{1}{2}\int \csc^2(c+dx)($$
$$= -\frac{ab\cot(c+dx)}{d} - \frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d} -$$
$$= \frac{3b^2\cos(c+dx)}{2d} - \frac{ab\cot(c+dx)}{d} - \frac{\cot(c+dx)\csc(c+dx)(a+b}{2d}$$
$$= -2abx + \frac{3b^2\cos(c+dx)}{2d} - \frac{ab\cot(c+dx)}{d} - \frac{\cot(c+dx)\csc(c+}{}$$
$$= -2abx + \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} + \frac{3b^2\cos(c+dx)}{2d} - \frac{ab}{}$$

Mathematica [A] time = 0.900336, size = 155, normalized size = 1.74

$$\frac{a^2\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right) + a^2\sec^2\left(\frac{1}{2}(c+dx)\right) - 4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 8ab\tan\left(\frac{1}{2}(c+\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-16*a*b*c - 16*a*b*d*x + 8*b^2*Cos[c + d*x] - 8*a*b*Cot[(c + d*x)/2] - a^2
*Csc[(c + d*x)/2]^2 + 4*a^2*Log[Cos[(c + d*x)/2]] - 8*b^2*Log[Cos[(c + d*x)
/2]] - 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log[Sin[(c + d*x)/2]] + a^2*Sec[
(c + d*x)/2]^2 + 8*a*b*Tan[(c + d*x)/2])/(8*d)
```

Maple [A] time = 0.079, size = 126, normalized size = 1.4

$$-\frac{a^2(\cos(dx+c))^3}{2d(\sin(dx+c))^2} - \frac{a^2\cos(dx+c)}{2d} - \frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d} - 2abx - 2\frac{ab\cot(dx+c)}{d} - 2\frac{abc}{d} + \frac{b^2\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a^2*cos(d*x+c)/d-1/2/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))-2*a*b*x-2*a*b*cot(d*x+c)/d-2/d*a*b*c+b^2*cos(d*x+c)/d+1/
d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.68321, size = 139, normalized size = 1.56

$$-\frac{8\left(dx+c+\frac{1}{\tan(dx+c)}\right)ab-a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)-2b^2(2\cos(dx+c)-\log(c}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(8*(d*x + c + 1/tan(d*x + c))*a*b - a^2*(2*cos(d*x + c)/(cos(d*x + c)^
2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 2*b^2*(2*cos(d*x
+ c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.44467, size = 414, normalized size = 4.65

$$-\frac{8\,abdx\cos(dx+c)^2-4\,b^2\cos(dx+c)^3-8\,abdx-8\,ab\cos(dx+c)\sin(dx+c)-2\left(a^2-2\,b^2\right)\cos(dx+c)-\left(\left(a^2-2\,b\right.\right.}{4\left(d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/4*(8*a*b*d*x*cos(d*x + c)^2 - 4*b^2*cos(d*x + c)^3 - 8*a*b*d*x - 8*a*b*c
os(d*x + c)*sin(d*x + c) - 2*(a^2 - 2*b^2)*cos(d*x + c) - ((a^2 - 2*b^2)*co
s(d*x + c)^2 - a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2 - 2*b^2)*co
s(d*x + c)^2 - a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2
 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37059, size = 200, normalized size = 2.25

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,(dx+c)ab+8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,\left(a^2-2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{16\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*a*b + 8*a*b*tan(1/2*d*x + 1/
2*c) - 4*(a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 16*b^2/(tan(1/2*d*x
 + 1/2*c)^2 + 1) + (6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2
*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.1065 $\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=96

$$\frac{(a^2-2b^2)\cot(c+dx)}{3d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{ab\cot(c+dx)\csc(c+dx)}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+d$$

```
[Out] -(b^2*x) + (a*b*ArcTanh[Cos[c + d*x]])/d + ((a^2 - 2*b^2)*Cot[c + d*x])/(3*
d) - (a*b*Cot[c + d*x]*Csc[c + d*x])/(3*d) - (Cot[c + d*x]*Csc[c + d*x]^2*(
a + b*Sin[c + d*x])^2)/(3*d)
```

Rubi [A] time = 0.389709, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2889, 3048, 3031, 3021, 2735, 3770}

$$\frac{(a^2-2b^2)\cot(c+dx)}{3d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{ab\cot(c+dx)\csc(c+dx)}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -(b^2*x) + (a*b*ArcTanh[Cos[c + d*x]])/d + ((a^2 - 2*b^2)*Cot[c + d*x])/(3*
d) - (a*b*Cot[c + d*x]*Csc[c + d*x])/(3*d) - (Cot[c + d*x]*Csc[c + d*x]^2*(
a + b*Sin[c + d*x])^2)/(3*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^4(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)dx$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{3d} + \frac{1}{3}\int \csc^3(c+d$$

$$= -\frac{ab\cot(c+dx)\csc(c+dx)}{3d} - \frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin}{3d}$$

$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)}{3d} - \frac{ab\cot(c+dx)\csc(c+dx)}{3d} - \cot(c+d$$

$$= -b^2x + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{3d} - \frac{ab\cot(c+dx)\csc(c+dx)}{3d} - \text{co}$$

$$= -b^2x + \frac{ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{3d} - ab\,\text{co}$$

Mathematica [B] time = 6.16953, size = 538, normalized size = 5.6

$$\frac{\sin^2(c+dx)\csc\left(\frac{1}{2}(c+dx)\right)\left(a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)(a\csc(c+dx)+b)^2}{6d(a+b\sin(c+dx))^2} + \frac{\sin^2(c+dx)\sec\left(\frac{1}{2}(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b^2*(c + d*x)*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(d*(a + b*Sin[c + d
*x])^2)) + ((a^2*Cos[(c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2
]*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(6*d*(a + b*Sin[c + d*x])^2) - (a*
b*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(4*d*(a + b*Sin
[c + d*x])^2) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x
])^2*Sin[c + d*x]^2)/(24*d*(a + b*Sin[c + d*x])^2) + (a*b*(b + a*Csc[c + d*
x])^2*Log[Cos[(c + d*x)/2]]*Sin[c + d*x]^2)/(d*(a + b*Sin[c + d*x])^2) - (a
*b*(b + a*Csc[c + d*x])^2*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^2)/(d*(a + b*S
```

```
in[c + d*x])^2) + (a*b*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^2*Sin[c + d*
x]^2)/(4*d*(a + b*Sin[c + d*x])^2) + ((b + a*Csc[c + d*x])^2*Sec[(c + d*x)/
2]*(-(a^2*Sin[(c + d*x)/2]) + 3*b^2*Sin[(c + d*x)/2])*Sin[c + d*x]^2)/(6*d*
(a + b*Sin[c + d*x])^2) + (a^2*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^2*Si
n[c + d*x]^2*Tan[(c + d*x)/2])/(24*d*(a + b*Sin[c + d*x])^2)
```

Maple [A] time = 0.077, size = 114, normalized size = 1.2

$$-\frac{a^2(\cos(dx+c))^3}{3d(\sin(dx+c))^3}-\frac{ab(\cos(dx+c))^3}{d(\sin(dx+c))^2}-\frac{ab\cos(dx+c)}{d}-\frac{ab\ln(\csc(dx+c)-\cot(dx+c))}{d}-b^2x-\frac{b^2\cot(dx+c)}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1/d*a*b/sin(d*x+c)^2*cos(d*x+c)^3-a*b*
cos(d*x+c)/d-1/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-b^2*x-b^2*cot(d*x+c)/d-1/d*b
^2*c
```

Maxima [A] time = 1.56373, size = 111, normalized size = 1.16

$$-\frac{6\left(dx+c+\frac{1}{\tan(dx+c)}\right)b^2-3ab\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)+\frac{2a^2}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6*(6*(d*x + c + 1/tan(d*x + c))*b^2 - 3*a*b*(2*cos(d*x + c)/(cos(d*x + c
)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 2*a^2/tan(d*x +
 c)^3)/d
```

Fricas [A] time = 1.51407, size = 423, normalized size = 4.41

$$\frac{2\left(a^2-3b^2\right)\cos(dx+c)^3+6b^2\cos(dx+c)+3\left(ab\cos(dx+c)^2-ab\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3\left(ab\cos(\right.}{6\left(d\cos(dx+c)^2-d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*(a^2 - 3*b^2)*cos(d*x + c)^3 + 6*b^2*cos(d*x + c) + 3*(a*b*cos(d*x +
 c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a*b*cos(d*x + c)
^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(b^2*d*x*cos(d*x +
c)^2 - b^2*d*x - a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26827, size = 225, normalized size = 2.34

$$\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24(dx+c)b^2-24ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 24*(d*x +
 c)*b^2 - 24*a*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 12*b^2*tan(1/2*d*x + 1/2*c) + (44*a*b*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x +
1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.1066 $\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=123

$$\frac{(a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}+\frac{2ab\cot(c+dx)}{3d}-\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d}-$$

```
[Out] ((a^2 + 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (2*a*b*Cot[c + d*x])/(3*d) +
((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*b*Cot[c + d*x]*Csc[c +
 d*x]^2)/(6*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(4*d)
```

Rubi [A] time = 0.364544, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{(a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}+\frac{2ab\cot(c+dx)}{3d}-\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (2*a*b*Cot[c + d*x])/(3*d) +
((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*b*Cot[c + d*x]*Csc[c +
 d*x]^2)/(6*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(4*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \csc^5(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)dx \\
&= -\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{4d}+\frac{1}{4}\int \csc^4(c+dx \\
&= -\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d}-\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin}{4d} \\
&= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d}-\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d} \\
&= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d}-\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d} \\
&= \frac{\left(a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+}{8d} \\
&= \frac{\left(a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{2ab\cot(c+dx)}{3d}+\frac{\left(a^2-2b^2\right)}{}
\end{aligned}$$

Mathematica [B] time = 6.17054, size = 579, normalized size = 4.71

$$\frac{\left(a^2-4b^2\right)\sin^2(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a\csc(c+dx)+b)^2}{32d(a+b\sin(c+dx))^2}+\frac{\left(-a^2-4b^2\right)\sin^2(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\csc(c}{8d(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Cot[(c + d*x)/2]*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(3*d*(a + b*Si
n[c + d*x])^2) + ((a^2 - 4*b^2)*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^2*S
in[c + d*x]^2)/(32*d*(a + b*Sin[c + d*x])^2) - (a*b*Cot[(c + d*x)/2]*Csc[(c
 + d*x)/2]^2*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(12*d*(a + b*Sin[c + d*
x])^2) - (a^2*Csc[(c + d*x)/2]^4*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(64
*d*(a + b*Sin[c + d*x])^2) + ((a^2 + 4*b^2)*(b + a*Csc[c + d*x])^2*Log[Cos[
(c + d*x)/2]]*Sin[c + d*x]^2)/(8*d*(a + b*Sin[c + d*x])^2) + ((-a^2 - 4*b^2
)*(b + a*Csc[c + d*x])^2*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^2)/(8*d*(a + b*
Sin[c + d*x])^2) + ((-a^2 + 4*b^2)*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^
2*Sin[c + d*x]^2)/(32*d*(a + b*Sin[c + d*x])^2) + (a^2*(b + a*Csc[c + d*x])
^2*Sec[(c + d*x)/2]^4*Sin[c + d*x]^2)/(64*d*(a + b*Sin[c + d*x])^2) - (a*b*
(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2*Tan[(c + d*x)/2])/(3*d*(a + b*Sin[c +
 d*x])^2) + (a*b*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^2*Sin[c + d*x]^2*T
an[(c + d*x)/2])/(12*d*(a + b*Sin[c + d*x])^2)
```

Maple [A] time = 0.082, size = 173, normalized size = 1.4

$$-\frac{a^2(\cos(dx+c))^3}{4d(\sin(dx+c))^4}-\frac{a^2(\cos(dx+c))^3}{8d(\sin(dx+c))^2}-\frac{a^2\cos(dx+c)}{8d}-\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{2ab(\cos(dx+c))^3}{3d(\sin(dx+c))^3}-\frac{b^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-1/
8*a^2*cos(d*x+c)/d-1/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/3/d*a*b/sin(d*x+c)
^3*cos(d*x+c)^3-1/2/d*b^2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*b^2*cos(d*x+c)/d-1/
2/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.06572, size = 174, normalized size = 1.41

$$-\frac{3a^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12b^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(3*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*b^2*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c)
 - 1)) + 32*a*b/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.4891, size = 504, normalized size = 4.1

$$-\frac{32ab\cos(dx+c)^3\sin(dx+c)+6\left(a^2-4b^2\right)\cos(dx+c)^3+6\left(a^2+4b^2\right)\cos(dx+c)-3\left(\left(a^2+4b^2\right)\cos(dx+c)^4-2\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(32*a*b*cos(d*x + c)^3*sin(d*x + c) + 6*(a^2 - 4*b^2)*cos(d*x + c)^3
+ 6*(a^2 + 4*b^2)*cos(d*x + c) - 3*((a^2 + 4*b^2)*cos(d*x + c)^4 - 2*(a^2 +
 4*b^2)*cos(d*x + c)^2 + a^2 + 4*b^2)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^2
 + 4*b^2)*cos(d*x + c)^4 - 2*(a^2 + 4*b^2)*cos(d*x + c)^2 + a^2 + 4*b^2)*lo
g(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22435, size = 246, normalized size = 2.

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\left(a^2+4\,b^2\right)\log}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*b^
2*tan(1/2*d*x + 1/2*c)^2 - 48*a*b*tan(1/2*d*x + 1/2*c) - 24*(a^2 + 4*b^2)*l
og(abs(tan(1/2*d*x + 1/2*c))) + (50*a^2*tan(1/2*d*x + 1/2*c)^4 + 200*b^2*ta
n(1/2*d*x + 1/2*c)^4 + 48*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*b^2*tan(1/2*d*x +
 1/2*c)^2 - 16*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.1067 $\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=148

$$\frac{(2a^2+5b^2)\cot(c+dx)}{15d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{15d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{4d}-\frac{ab\cot(c+dx)\csc^3(c+dx)}{10d}+$$

```
[Out] (a*b*ArcTanh[Cos[c + d*x]])/(4*d) + ((2*a^2 + 5*b^2)*Cot[c + d*x])/(15*d) +
 (a*b*Cot[c + d*x]*Csc[c + d*x])/(4*d) + ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c
+ d*x]^2)/(15*d) - (a*b*Cot[c + d*x]*Csc[c + d*x]^3)/(10*d) - (Cot[c + d*x]
*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rubi [A] time = 0.386162, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2889, 3048, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2a^2+5b^2)\cot(c+dx)}{15d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{15d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{4d}-\frac{ab\cot(c+dx)\csc^3(c+dx)}{10d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (a*b*ArcTanh[Cos[c + d*x]])/(4*d) + ((2*a^2 + 5*b^2)*Cot[c + d*x])/(15*d) +
 (a*b*Cot[c + d*x]*Csc[c + d*x])/(4*d) + ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c
+ d*x]^2)/(15*d) - (a*b*Cot[c + d*x]*Csc[c + d*x]^3)/(10*d) - (Cot[c + d*x]
*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(5*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
```

```
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^6(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx$$
$$= -\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{5d} + \frac{1}{5}\int \csc^5(c+dx)(a$$
$$= -\frac{ab\cot(c+dx)\csc^3(c+dx)}{10d} - \frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c$$
$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{ab\cot(c+dx)\csc^3(c+dx)}{10d} -$$
$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{ab\cot(c+dx)\csc^3(c+dx)}{10d} -$$
$$= \frac{ab\cot(c+dx)\csc(c+dx)}{4d} + \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)}{15d} -$$
$$= \frac{ab\tanh^{-1}(\cos(c+dx))}{4d} + \frac{\left(2a^2+5b^2\right)\cot(c+dx)}{15d} + \frac{ab\cot(c+dx)}{4d}$$

Mathematica [A] time = 0.819497, size = 236, normalized size = 1.59

$$\frac{\csc^5(c+dx)\left(-40\left(4a^2+b^2\right)\cos(c+dx)+20\left(b^2-2a^2\right)\cos(3(c+dx))+8a^2\cos(5(c+dx))-180ab\sin(2(c+dx))-30a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-40*(4*a^2 + b^2)*Cos[c + d*x] + 20*(-2*a^2 + b^2)*Cos[3*(
c + d*x)] + 8*a^2*Cos[5*(c + d*x)] + 20*b^2*Cos[5*(c + d*x)] + 150*a*b*Log[
Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*a*b*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]
 - 180*a*b*Sin[2*(c + d*x)] - 75*a*b*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)]
 + 75*a*b*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*a*b*Sin[4*(c + d*x)]
+ 15*a*b*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*a*b*Log[Sin[(c + d*x)/
2]]*Sin[5*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.082, size = 156, normalized size = 1.1

$$-\frac{a^2(\cos(dx+c))^3}{5d(\sin(dx+c))^5} - \frac{2a^2(\cos(dx+c))^3}{15d(\sin(dx+c))^3} - \frac{ab(\cos(dx+c))^3}{2d(\sin(dx+c))^4} - \frac{ab(\cos(dx+c))^3}{4d(\sin(dx+c))^2} - \frac{ab\cos(dx+c)}{4d} - \frac{ab\ln(\csc(dx+c}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^3-2/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^3-1
/2/d*a*b/sin(d*x+c)^4*cos(d*x+c)^3-1/4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^3-1/4*
a*b*cos(d*x+c)/d-1/4/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*b^2/sin(d*x+c)^3
*cos(d*x+c)^3
```

Maxima [A] time = 1.08261, size = 146, normalized size = 0.99

$$-\frac{15\,ab\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) + \frac{40\,b^2}{\tan(dx+c)^3} + \frac{8\left(5\tan(dx+c)^2+3\right)a^2}{\tan(dx+c)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*(15*a*b*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 40*b^2/ta
n(d*x + c)^3 + 8*(5*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.39364, size = 518, normalized size = 3.5

$$\frac{8\left(2a^2+5b^2\right)\cos\left(dx+c\right)^5-40\left(a^2+b^2\right)\cos\left(dx+c\right)^3+15\left(ab\cos\left(dx+c\right)^4-2ab\cos\left(dx+c\right)^2+ab\right)\log\left(\frac{1}{2}\cos\left(d\right.\right.}{120\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(8*(2*a^2 + 5*b^2)*cos(d*x + c)^5 - 40*(a^2 + b^2)*cos(d*x + c)^3 + 1
5*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*log(1/2*cos(d*x + c) +
1/2)*sin(d*x + c) - 15*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*lo
g(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(a*b*cos(d*x + c)^3 + a*b*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28206, size = 300, normalized size = 2.03

$$\frac{3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+15ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+5a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+20b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-120ab\log\left(\left|\tan\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 + 5*a^2
*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b*log(abs(t
an(1/2*d*x + 1/2*c))) - 30*a^2*tan(1/2*d*x + 1/2*c) - 60*b^2*tan(1/2*d*x +
1/2*c) + (274*a*b*tan(1/2*d*x + 1/2*c)^5 + 30*a^2*tan(1/2*d*x + 1/2*c)^4 +
```

```
60*b^2*tan(1/2*d*x + 1/2*c)^4 - 5*a^2*tan(1/2*d*x + 1/2*c)^2 - 20*b^2*tan(1
/2*d*x + 1/2*c)^2 - 15*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*
c)^5)/d
```

3.1068 $\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=170

$$\frac{(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{(a^2+2b^2)\cot(c+dx)\csc(c+dx)}{16d}+\frac{2ab\cot}{1}$$

```
[Out] ((a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (2*a*b*Cot[c + d*x])/(5*d) +
 (2*a*b*Cot[c + d*x]^3)/(15*d) + ((a^2 + 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(16*d) + ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a*b*Cot[c +
d*x]*Csc[c + d*x]^4)/(15*d) - (Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d
*x])^2)/(6*d)
```

Rubi [A] time = 0.39605, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3031, 3021, 2748, 3767, 3768, 3770}

$$\frac{(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{(a^2+2b^2)\cot(c+dx)\csc(c+dx)}{16d}+\frac{2ab\cot}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] ((a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (2*a*b*Cot[c + d*x])/(5*d) +
 (2*a*b*Cot[c + d*x]^3)/(15*d) + ((a^2 + 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(16*d) + ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a*b*Cot[c +
d*x]*Csc[c + d*x]^4)/(15*d) - (Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d
*x])^2)/(6*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
```

```
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^7(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)dx$$
$$= -\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2}{6d} + \frac{1}{6}\int \csc^6(c+dx)(a$$
$$= -\frac{ab\cot(c+dx)\csc^4(c+dx)}{15d} - \frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c}{6d}$$
$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{ab\cot(c+dx)\csc^4(c+dx)}{15d} -$$
$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{ab\cot(c+dx)\csc^4(c+dx)}{15d} -$$
$$= \frac{\left(a^2+2b^2\right)\cot(c+dx)\csc(c+dx)}{16d} + \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc^3(c}{24d}$$
$$= \frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} + \frac{2ab\cot(c+dx)}{5d} + \frac{2ab\cot^3(c+}{15d}$$

Mathematica [A] time = 0.776761, size = 296, normalized size = 1.74

$$\frac{30\left(a^2+2b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+5a^2\sec^6\left(\frac{1}{2}(c+dx)\right)-30a^2\sec^2\left(\frac{1}{2}(c+dx)\right)-120a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+120a^2\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (256*a*b*Cot[(c + d*x)/2] + 30*(a^2 + 2*b^2)*Csc[(c + d*x)/2]^2 + 120*a^2*L
og[Cos[(c + d*x)/2]] + 240*b^2*Log[Cos[(c + d*x)/2]] - 120*a^2*Log[Sin[(c +
 d*x)/2]] - 240*b^2*Log[Sin[(c + d*x)/2]] - 30*a^2*Sec[(c + d*x)/2]^2 - 60*
b^2*Sec[(c + d*x)/2]^2 + 30*b^2*Sec[(c + d*x)/2]^4 + 5*a^2*Sec[(c + d*x)/2]
^6 - 64*a*b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 768*a*b*Csc[c + d*x]^5*Sin[
(c + d*x)/2]^6 - a*Csc[(c + d*x)/2]^6*(5*a + 12*b*Sin[c + d*x]) + Csc[(c +
d*x)/2]^4*(-30*b^2 + 4*a*b*Sin[c + d*x]) - 256*a*b*Tan[(c + d*x)/2])/(1920*
d)
```

Maple [A] time = 0.083, size = 244, normalized size = 1.4

$$-\frac{a^2(\cos(dx+c))^3}{6d(\sin(dx+c))^6}-\frac{a^2(\cos(dx+c))^3}{8d(\sin(dx+c))^4}-\frac{a^2(\cos(dx+c))^3}{16d(\sin(dx+c))^2}-\frac{a^2\cos(dx+c)}{16d}-\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{16d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)^3-1/8/d*a^2/sin(d*x+c)^4*cos(d*x+c)^3-1/
16/d*a^2/sin(d*x+c)^2*cos(d*x+c)^3-1/16*a^2*cos(d*x+c)/d-1/16/d*a^2*ln(csc(
d*x+c)-cot(d*x+c))-2/5/d*a*b/sin(d*x+c)^5*cos(d*x+c)^3-4/15/d*a*b/sin(d*x+c
)^3*cos(d*x+c)^3-1/4/d*b^2/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*b^2/sin(d*x+c)^2
*cos(d*x+c)^3-1/8*b^2*cos(d*x+c)/d-1/8/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.11435, size = 251, normalized size = 1.48

$$-\frac{5a^2\left(\frac{2\left(3\cos(dx+c)^5-8\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+30b^2\left(\frac{2\left(\cos(dx+c)^3+\text{co}\right.}{\cos(dx+c)^4-2\cos(}\right.}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^2*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 30*b^2*(2*(cos(d*x + c)^3 + cos(d*x + c
))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(co
s(d*x + c) - 1)) + 64*(5*tan(d*x + c)^2 + 3)*a*b/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.5846, size = 697, normalized size = 4.1

$$-\frac{30\left(a^2+2b^2\right)\cos(dx+c)^5-80a^2\cos(dx+c)^3-30\left(a^2+2b^2\right)\cos(dx+c)-15\left(\left(a^2+2b^2\right)\cos(dx+c)^6-3\left(a^2+2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(30*(a^2 + 2*b^2)*cos(d*x + c)^5 - 80*a^2*cos(d*x + c)^3 - 30*(a^2 +
 2*b^2)*cos(d*x + c) - 15*((a^2 + 2*b^2)*cos(d*x + c)^6 - 3*(a^2 + 2*b^2)*c
os(d*x + c)^4 + 3*(a^2 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(1/2*cos(d
*x + c) + 1/2) + 15*((a^2 + 2*b^2)*cos(d*x + c)^6 - 3*(a^2 + 2*b^2)*cos(d*x
 + c)^4 + 3*(a^2 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(-1/2*cos(d*x +
c) + 1/2) + 64*(2*a*b*cos(d*x + c)^5 - 5*a*b*cos(d*x + c)^3)*sin(d*x + c))/
(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25234, size = 373, normalized size = 2.19

$$\frac{5a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+24ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+30b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+40ab\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a*b*tan(1/2*d*x + 1/2*c)^5 + 15*a
^2*tan(1/2*d*x + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 + 40*a*b*tan(1/2*
d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 240*a*b*tan(1/2*d*x + 1/2*
c) - 120*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (294*a^2*tan(1/2*d*
x + 1/2*c)^6 + 588*b^2*tan(1/2*d*x + 1/2*c)^6 + 240*a*b*tan(1/2*d*x + 1/2*c
)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 - 40*a*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^
2*tan(1/2*d*x + 1/2*c)^2 - 30*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a*b*tan(1/2*d
*x + 1/2*c) - 5*a^2)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.1069 $\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=232

$$\frac{b\left(21a^2+4b^2\right)\cos^3(c+dx)}{105d}-\frac{b\left(21a^2+4b^2\right)\cos(c+dx)}{35d}+\frac{b\left(a^2-b^2\right)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{a\left(2a^2-7b^2\right)\sin^3(c}{56}$$

```
[Out] (a*(2*a^2 + 3*b^2)*x)/16 - (b*(21*a^2 + 4*b^2)*Cos[c + d*x])/(35*d) + (b*(2
1*a^2 + 4*b^2)*Cos[c + d*x]^3)/(105*d) - (a*(2*a^2 + 3*b^2)*Cos[c + d*x]*Si
n[c + d*x])/(16*d) + (a*(2*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(56*d)
 + (b*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(35*d) + (a*Cos[c + d*x]*Sin
[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(14*d) + (Cos[c + d*x]*Sin[c + d*x]^3*(
a + b*Sin[c + d*x])^3)/(7*d)
```

Rubi [A] time = 0.571816, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2889, 3050, 3049, 3033, 3023, 2748, 2635, 8, 2633}

$$\frac{b\left(21a^2+4b^2\right)\cos^3(c+dx)}{105d}-\frac{b\left(21a^2+4b^2\right)\cos(c+dx)}{35d}+\frac{b\left(a^2-b^2\right)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{a\left(2a^2-7b^2\right)\sin^3(c}{56}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (a*(2*a^2 + 3*b^2)*x)/16 - (b*(21*a^2 + 4*b^2)*Cos[c + d*x])/(35*d) + (b*(2
1*a^2 + 4*b^2)*Cos[c + d*x]^3)/(105*d) - (a*(2*a^2 + 3*b^2)*Cos[c + d*x]*Si
n[c + d*x])/(16*d) + (a*(2*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(56*d)
 + (b*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(35*d) + (a*Cos[c + d*x]*Sin
[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(14*d) + (Cos[c + d*x]*Sin[c + d*x]^3*(
a + b*Sin[c + d*x])^3)/(7*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \sin^2(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)dx$$
$$= \frac{\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^3}{7d} + \frac{1}{7}\int \sin^2(c+dx)($$
$$= \frac{a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2}{14d} + \frac{\cos(c+dx)\sin^3}{}$$
$$= \frac{b\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{35d} + \frac{a\cos(c+dx)\sin^3(c+dx)}{14d}$$
$$= \frac{a\left(2a^2-7b^2\right)\cos(c+dx)\sin^3(c+dx)}{56d} + \frac{b\left(a^2-b^2\right)\cos(c+dx)}{35d}$$
$$= \frac{a\left(2a^2-7b^2\right)\cos(c+dx)\sin^3(c+dx)}{56d} + \frac{b\left(a^2-b^2\right)\cos(c+dx)}{35d}$$
$$= -\frac{a\left(2a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\left(2a^2-7b^2\right)\cos(c+}{56d}$$
$$= \frac{1}{16}a\left(2a^2+3b^2\right)x - \frac{b\left(21a^2+4b^2\right)\cos(c+dx)}{35d} + \frac{b\left(21a^2+4b^2\right)}{105}$$

Mathematica [A] time = 0.808312, size = 157, normalized size = 0.68

$$\frac{105a\left(-\left(2a^2+3b^2\right)\sin(4(c+dx))+8a^2c+8a^2dx-3b^2\sin(2(c+dx))+b^2\sin(6(c+dx))+12b^2c+12b^2dx\right)-105b}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-105*b*(24*a^2 + 5*b^2)*Cos[c + d*x] - 35*(12*a^2*b + b^3)*Cos[3*(c + d*x)
] + 63*(4*a^2*b + b^3)*Cos[5*(c + d*x)] - 15*b^3*Cos[7*(c + d*x)] + 105*a*(
8*a^2*c + 12*b^2*c + 8*a^2*d*x + 12*b^2*d*x - 3*b^2*Sin[2*(c + d*x)] - (2*a
^2 + 3*b^2)*Sin[4*(c + d*x)] + b^2*Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.049, size = 196, normalized size = 0.8

$$\frac{1}{d}\left(a^3\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+3a^2b\left(-1/5\,(\sin(dx+c))^2(\cos(dx+c))^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)+3*a^2*b*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a*b^2*(-1
/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*si
n(d*x+c)+1/16*d*x+1/16*c)+b^3*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+
c)^2*cos(d*x+c)^3-8/105*cos(d*x+c)^3))
```

Maxima [A] time = 1.1982, size = 177, normalized size = 0.76

$$\frac{210\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^3+1344\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)a^2b-105\left(4\,\sin(2\,dx+2\,c)^3-12\,dx-\right.}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6720*(210*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^3 + 1344*(3*cos(d*x + c)^5 -
 5*cos(d*x + c)^3)*a^2*b - 105*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*si
n(4*d*x + 4*c))*a*b^2 - 64*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(
d*x + c)^3)*b^3)/d
```

Fricas [A] time = 1.5761, size = 343, normalized size = 1.48

$$-\frac{240\,b^3\cos(dx+c)^7-336\left(3\,a^2b+2\,b^3\right)\cos(dx+c)^5+560\left(3\,a^2b+b^3\right)\cos(dx+c)^3-105\left(2\,a^3+3\,ab^2\right)dx-105\left(8\,a\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1680*(240*b^3*cos(d*x + c)^7 - 336*(3*a^2*b + 2*b^3)*cos(d*x + c)^5 + 56
0*(3*a^2*b + b^3)*cos(d*x + c)^3 - 105*(2*a^3 + 3*a*b^2)*d*x - 105*(8*a*b^2
*cos(d*x + c)^5 - 2*(2*a^3 + 7*a*b^2)*cos(d*x + c)^3 + (2*a^3 + 3*a*b^2)*co
s(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 8.26901, size = 394, normalized size = 1.7

$$\begin{cases}\frac{a^3x\sin^4(c+dx)}{8}+\frac{a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{a^3x\cos^4(c+dx)}{8}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{a^3\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{a^2b\sin^2(c+dx)\cos^3(c+dx)}{d} \\ x\left(a+b\sin(c)\right)^3\sin^2(c)\cos^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**4/8 + a**3*x*sin(c + d*x)**2*cos(c + d*x)**
2/4 + a**3*x*cos(c + d*x)**4/8 + a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) -
a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**2*b*sin(c + d*x)**2*cos(c + d*
x)**3/d - 2*a**2*b*cos(c + d*x)**5/(5*d) + 3*a*b**2*x*sin(c + d*x)**6/16 +
9*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a*b**2*x*sin(c + d*x)**2*
cos(c + d*x)**4/16 + 3*a*b**2*x*cos(c + d*x)**6/16 + 3*a*b**2*sin(c + d*x)*
*5*cos(c + d*x)/(16*d) - a*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a
*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b**3*sin(c + d*x)**4*cos(c + d*
x)**3/(3*d) - 4*b**3*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 8*b**3*cos(c
+ d*x)**7/(105*d), Ne(d, 0)), (x*(a + b*sin(c))**3*sin(c)**2*cos(c)**2, Tru
e))
```

Giac [A] time = 1.18634, size = 224, normalized size = 0.97

$$-\frac{b^3\cos(7\,dx+7\,c)}{448\,d}+\frac{ab^2\sin(6\,dx+6\,c)}{64\,d}-\frac{3\,ab^2\sin(2\,dx+2\,c)}{64\,d}+\frac{1}{16}\left(2\,a^3+3\,ab^2\right)x+\frac{3\left(4\,a^2b+b^3\right)\cos(5\,dx+5\,c)}{320\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/448*b^3*cos(7*d*x + 7*c)/d + 1/64*a*b^2*sin(6*d*x + 6*c)/d - 3/64*a*b^2*
sin(2*d*x + 2*c)/d + 1/16*(2*a^3 + 3*a*b^2)*x + 3/320*(4*a^2*b + b^3)*cos(5
*d*x + 5*c)/d - 1/192*(12*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 1/64*(24*a^2*b
+ 5*b^3)*cos(d*x + c)/d - 1/64*(2*a^3 + 3*a*b^2)*sin(4*d*x + 4*c)/d
```

3.1070 $\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=163

$$-\frac{a\left(2a^2+33b^2\right)\cos^3(c+dx)}{120d}-\frac{\left(2a^2+5b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))}{40d}+\frac{b\left(6a^2+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}b$$

```
[Out] (b*(6*a^2 + b^2)*x)/16 - (a*(2*a^2 + 33*b^2)*Cos[c + d*x]^3)/(120*d) + (b*(
6*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - ((2*a^2 + 5*b^2)*Cos[c + d
*x]^3*(a + b*Sin[c + d*x]))/(40*d) - (a*Cos[c + d*x]^3*(a + b*Sin[c + d*x])
^2)/(10*d) - (Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(6*d)
```

Rubi [A] time = 0.294542, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{a\left(2a^2+33b^2\right)\cos^3(c+dx)}{120d}-\frac{\left(2a^2+5b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))}{40d}+\frac{b\left(6a^2+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}b$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (b*(6*a^2 + b^2)*x)/16 - (a*(2*a^2 + 33*b^2)*Cos[c + d*x]^3)/(120*d) + (b*(
6*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - ((2*a^2 + 5*b^2)*Cos[c + d
*x]^3*(a + b*Sin[c + d*x]))/(40*d) - (a*Cos[c + d*x]^3*(a + b*Sin[c + d*x])
^2)/(10*d) - (Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(6*d)
```


Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{\cos^3(c+dx)(a+b\sin(c+dx))^3}{6d}+\frac{1}{6}\int \cos^2(c+dx)(3b+3a\sin$$
$$= -\frac{a\cos^3(c+dx)(a+b\sin(c+dx))^2}{10d}-\frac{\cos^3(c+dx)(a+b\sin(c+$$
$$= -\frac{\left(2a^2+5b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))}{40d}-\frac{a\cos^3(c+dx)(a+}{10}$$
$$= -\frac{a\left(2a^2+33b^2\right)\cos^3(c+dx)}{120d}-\frac{\left(2a^2+5b^2\right)\cos^3(c+dx)(a+b\sin}{40d}$$
$$= -\frac{a\left(2a^2+33b^2\right)\cos^3(c+dx)}{120d}+\frac{b\left(6a^2+b^2\right)\cos(c+dx)\sin(c+}{16d}$$
$$= \frac{1}{16}b\left(6a^2+b^2\right)x-\frac{a\left(2a^2+33b^2\right)\cos^3(c+dx)}{120d}+\frac{b\left(6a^2+b^2\right)\cos}{}$$

Mathematica [A] time = 0.76512, size = 138, normalized size = 0.85

$$\frac{-120a\left(2a^2+3b^2\right)\cos(c+dx)-20\left(4a^3+3ab^2\right)\cos(3(c+dx))+b\left(5\left(-3\left(6a^2+b^2\right)\sin(4(c+dx))+72a^2c+72a^2dx\right.\right.}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-120*a*(2*a^2 + 3*b^2)*Cos[c + d*x] - 20*(4*a^3 + 3*a*b^2)*Cos[3*(c + d*x)
] + b*(36*a*b*Cos[5*(c + d*x)] + 5*(72*a^2*c + 18*b^2*c + 72*a^2*d*x + 12*b
^2*d*x - 3*b^2*Sin[2*(c + d*x)] - 3*(6*a^2 + b^2)*Sin[4*(c + d*x)] + b^2*Si
n[6*(c + d*x)])))/(960*d)
```

Maple [A] time = 0.046, size = 158, normalized size = 1.

$$\frac{1}{d}\left(-\frac{a^3(\cos(dx+c))^3}{3}+3a^2b\left(-1/4(\cos(dx+c))^3\sin(dx+c)+1/8\cos(dx+c)\sin(dx+c)+1/8dx+c/8\right)+3ab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/3*a^3*cos(d*x+c)^3+3*a^2*b*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*
x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+3*a*b^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15
*cos(d*x+c)^3)+b^3*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x
+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A] time = 1.13537, size = 146, normalized size = 0.9

$$-\frac{320a^3\cos(dx+c)^3-90(4dx+4c-\sin(4dx+4c))a^2b-192\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)ab^2+5\left(4\sin(2\right.}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(320*a^3*cos(d*x + c)^3 - 90*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2*b
- 192*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a*b^2 + 5*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*b^3)/d
```

Fricas [A] time = 1.47875, size = 281, normalized size = 1.72

$$\frac{144\,ab^2\cos(dx+c)^5 - 80\left(a^3+3\,ab^2\right)\cos(dx+c)^3 + 15\left(6\,a^2b+b^3\right)dx + 5\left(8\,b^3\cos(dx+c)^5 - 2\left(18\,a^2b+7\,b^3\right)\cos(dx\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(144*a*b^2*cos(d*x + c)^5 - 80*(a^3 + 3*a*b^2)*cos(d*x + c)^3 + 15*(6
*a^2*b + b^3)*d*x + 5*(8*b^3*cos(d*x + c)^5 - 2*(18*a^2*b + 7*b^3)*cos(d*x
+ c)^3 + 3*(6*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 4.75514, size = 340, normalized size = 2.09

$$\begin{cases} -\frac{a^3\cos^3(c+dx)}{3d} + \frac{3a^2bx\sin^4(c+dx)}{8} + \frac{3a^2bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^2bx\cos^4(c+dx)}{8} + \frac{3a^2b\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{3a^2b\sin(c+dx)\cos^3(c+dx)}{8d} \\ x\left(a+b\sin(c)\right)^3\sin(c)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-a**3*cos(c + d*x)**3/(3*d) + 3*a**2*b*x*sin(c + d*x)**4/8 + 3*a
**2*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**2*b*x*cos(c + d*x)**4/8 +
3*a**2*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a**2*b*sin(c + d*x)*cos(c +
 d*x)**3/(8*d) - a*b**2*sin(c + d*x)**2*cos(c + d*x)**3/d - 2*a*b**2*cos(c
+ d*x)**5/(5*d) + b**3*x*sin(c + d*x)**6/16 + 3*b**3*x*sin(c + d*x)**4*cos(
c + d*x)**2/16 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b**3*x*cos(c
 + d*x)**6/16 + b**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - b**3*sin(c + d*x
)**3*cos(c + d*x)**3/(6*d) - b**3*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d
, 0)), (x*(a + b*sin(c))**3*sin(c)*cos(c)**2, True))
```

Giac [A] time = 1.19758, size = 188, normalized size = 1.15

$$\frac{3\,ab^2\cos(5\,dx+5\,c)}{80\,d} + \frac{b^3\sin(6\,dx+6\,c)}{192\,d} - \frac{b^3\sin(2\,dx+2\,c)}{64\,d} + \frac{1}{16}\left(6\,a^2b+b^3\right)x - \frac{\left(4\,a^3+3\,ab^2\right)\cos(3\,dx+3\,c)}{48\,d} - \frac{(2\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 3/80*a*b^2*cos(5*d*x + 5*c)/d + 1/192*b^3*sin(6*d*x + 6*c)/d - 1/64*b^3*sin
(2*d*x + 2*c)/d + 1/16*(6*a^2*b + b^3)*x - 1/48*(4*a^3 + 3*a*b^2)*cos(3*d*x
 + 3*c)/d - 1/8*(2*a^3 + 3*a*b^2)*cos(d*x + c)/d - 1/64*(6*a^2*b + b^3)*sin
(4*d*x + 4*c)/d
```

3.1071 $\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=136

$$\frac{a\left(a^2-2b^2\right)\cos(c+dx)}{2d}+\frac{b\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(12a^2+b^2\right)-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c$$

```
[Out] (b*(12*a^2 + b^2)*x)/8 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a*(a^2 - 2*b^2)*C
os[c + d*x])/(2*d) + (b*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(4*d) + (Cos[c + d*x]*(a + b*Sin[c +
d*x])^3)/(4*d)
```

Rubi [A] time = 0.413054, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2889, 3050, 3049, 3033, 3023, 2735, 3770}

$$\frac{a\left(a^2-2b^2\right)\cos(c+dx)}{2d}+\frac{b\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(12a^2+b^2\right)-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (b*(12*a^2 + b^2)*x)/8 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a*(a^2 - 2*b^2)*C
os[c + d*x])/(2*d) + (b*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(4*d) + (Cos[c + d*x]*(a + b*Sin[c +
d*x])^3)/(4*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$

$$= \frac{\cos(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{1}{4}\int \csc(c+dx)(a+b\sin(c+dx))^2$$

$$= \frac{a\cos(c+dx)(a+b\sin(c+dx))^2}{4d} + \frac{\cos(c+dx)(a+b\sin(c+dx))^3}{4d} +$$

$$= \frac{b\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos(c+dx)(a+b\sin(c+dx))}{4d}$$

$$= \frac{a\left(a^2-2b^2\right)\cos(c+dx)}{2d} + \frac{b\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\,\mathrm{co}}{}$$

$$= \frac{1}{8}b\left(12a^2+b^2\right)x + \frac{a\left(a^2-2b^2\right)\cos(c+dx)}{2d} + \frac{b\left(2a^2-b^2\right)\cos(c+dx}{8d}$$

$$= \frac{1}{8}b\left(12a^2+b^2\right)x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\left(a^2-2b^2\right)\cos(c+dx)}{2d}$$

Mathematica [A] time = 0.289677, size = 129, normalized size = 0.95

$$\frac{8a\left(4a^2-3b^2\right)\cos(c+dx)+24a^2b\sin(2(c+dx))+48a^2bc+48a^2bdx+32a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-32a^3\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (48*a^2*b*c + 4*b^3*c + 48*a^2*b*d*x + 4*b^3*d*x + 8*a*(4*a^2 - 3*b^2)*Cos[
c + d*x] - 8*a*b^2*Cos[3*(c + d*x)] - 32*a^3*Log[Cos[(c + d*x)/2]] + 32*a^3
*Log[Sin[(c + d*x)/2]] + 24*a^2*b*Sin[2*(c + d*x)] - b^3*Sin[4*(c + d*x)])/
(32*d)
```

Maple [A] time = 0.091, size = 150, normalized size = 1.1

$$\frac{a^3\cos(dx+c)}{d}+\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{3a^2b\cos(dx+c)\sin(dx+c)}{2d}+\frac{3a^2bx}{2}+\frac{3a^2bc}{2d}-\frac{ab^2(\cos(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3/2/d*a^2*b*cos(d*x+c)*s
in(d*x+c)+3/2*a^2*b*x+3/2/d*a^2*b*c-a*b^2*cos(d*x+c)^3/d-1/4/d*b^3*cos(d*x+
c)^3*sin(d*x+c)+1/8*b^3*cos(d*x+c)*sin(d*x+c)/d+1/8*b^3*x+1/8/d*b^3*c
```

Maxima [A] time = 1.15422, size = 136, normalized size = 1.

$$-\frac{32ab^2\cos(dx+c)^3-24(2dx+2c+\sin(2dx+2c))a^2b-(4dx+4c-\sin(4dx+4c))b^3-16a^3(2\cos(dx+c)-}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*a*b^2*cos(d*x + c)^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b
- (4*d*x + 4*c - sin(4*d*x + 4*c))*b^3 - 16*a^3*(2*cos(d*x + c) - log(cos(d
*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.63186, size = 297, normalized size = 2.18

$$-\frac{8ab^2\cos(dx+c)^3-8a^3\cos(dx+c)+4a^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-4a^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(12a^2b+b^3\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a*b^2*cos(d*x + c)^3 - 8*a^3*cos(d*x + c) + 4*a^3*log(1/2*cos(d*x +
 c) + 1/2) - 4*a^3*log(-1/2*cos(d*x + c) + 1/2) - (12*a^2*b + b^3)*d*x + (2
*b^3*cos(d*x + c)^3 - (12*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24492, size = 396, normalized size = 2.91

$$8\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\left(12\,a^{2}b+b^{3}\right)(dx+c)-\frac{2\left(12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+24\,ab^{2}\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + (12*a^2*b + b^3)*(d*x + c) - 2*
(12*a^2*b*tan(1/2*d*x + 1/2*c)^7 - b^3*tan(1/2*d*x + 1/2*c)^7 - 8*a^3*tan(1
/2*d*x + 1/2*c)^6 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 + 7*b^3*tan(1/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4 +
 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 7*b^3*
tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*a*b^2*tan(1/2*d*
x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) - 8
*a^3 + 8*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.1072 $\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=102

$$\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3}{}$$

```
[Out] -(a^3*x) + (3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + (3*a^2*b*Cos
[c + d*x])/d - (b^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.142007, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473, 2565, 30}

$$\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + (3*a^2*b*Cos
[c + d*x])/d - (b^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx &= \int \left(3ab^2\cos^2(c+dx)+3a^2b\cos(c+dx)\cot(c+dx)+a^3\cot^2(c+dx)+b^3\cos^2\right.\\ &= a^3\int \cot^2(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot(c+dx)\,dx+\left(3ab^2\right)\int \cos^2(c+\\ &= -\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos(c+dx)\sin(c+dx)}{2d}-a^3\int 1\,dx+\frac{1}{2}\left(3ab^2\right)\int 1\,dx-\\ &= -a^3x+\frac{3}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos}{}\\ &= -a^3x+\frac{3}{2}ab^2x-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 1.2903, size = 143, normalized size = 1.4

$$\frac{\left(36a^2b-3b^3\right)\cos(c+dx)+6a\left(a^2\tan\left(\frac{1}{2}(c+dx)\right)-2a^2c-2a^2dx+6ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((36*a^2*b - 3*b^3)*Cos[c + d*x] - b^3*Cos[3*(c + d*x)] - 6*a^3*Cot[(c + d*
x)/2] + 9*a*b^2*Sin[2*(c + d*x)] + 6*a*(-2*a^2*c + 3*b^2*c - 2*a^2*d*x + 3*
b^2*d*x - 6*a*b*Log[Cos[(c + d*x)/2]] + 6*a*b*Log[Sin[(c + d*x)/2]] + a^2*T
an[(c + d*x)/2]))/(12*d)
```

Maple [A] time = 0.084, size = 125, normalized size = 1.2

$$-a^3x-\frac{a^3\cot(dx+c)}{d}-\frac{a^3c}{d}+3\frac{a^2b\cos(dx+c)}{d}+3\frac{a^2b\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{3ab^2\cos(dx+c)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -a^3*x-a^3*cot(d*x+c)/d-1/d*a^3*c+3*a^2*b*cos(d*x+c)/d+3/d*a^2*b*ln(csc(d*x
+c)-cot(d*x+c))+3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d+3/2*a*b^2*x+3/2/d*a*b^2*c
-1/3*b^3*cos(d*x+c)^3/d
```

Maxima [A] time = 1.62115, size = 128, normalized size = 1.25

$$-\frac{4b^3\cos(dx+c)^3+12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3-9(2dx+2c+\sin(2dx+2c))ab^2-18a^2b(2\cos(dx+c)-\log(\cos$$

$$12d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(4*b^3*cos(d*x + c)^3 + 12*(d*x + c + 1/tan(d*x + c))*a^3 - 9*(2*d*x
+ 2*c + sin(2*d*x + 2*c))*a*b^2 - 18*a^2*b*(2*cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.5575, size = 370, normalized size = 3.63

$$-\frac{9ab^2\cos(dx+c)^3+9a^2b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9a^2b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3(2}{6d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(9*a*b^2*cos(d*x + c)^3 + 9*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 9*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(2*a^3 - 3*a*b
^2)*cos(d*x + c) + (2*b^3*cos(d*x + c)^3 - 18*a^2*b*cos(d*x + c) + 3*(2*a^3
 - 3*a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18499, size = 269, normalized size = 2.64

$$\frac{18\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\left(2\,a^3-3\,ab^2\right)(dx+c)-\frac{3\left(6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(9\,ab^2\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c) -
 3*(2*a^3 - 3*a*b^2)*(d*x + c) - 3*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan
(1/2*d*x + 1/2*c) - 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*
x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)
^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 2*b^3)/(tan(1/2*d*x + 1/2*c)
^2 + 1)^3)/d
```

3.1073 $\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=138

$$\frac{a(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{1}{2}bx(6a^2-b^2)+\frac{15ab^2\cos(c+dx)}{2d}-\frac{3b\cot(c+dx)(a+b\sin(c+dx))^2}{2d}-\frac{\cot(c}{}$$

```
[Out] -(b*(6*a^2 - b^2)*x)/2 + (a*(a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (1
5*a*b^2*Cos[c + d*x])/(2*d) + (5*b^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (3*
b*Cot[c + d*x]*(a + b*Sin[c + d*x])^2)/(2*d) - (Cot[c + d*x]*Csc[c + d*x]*(
a + b*Sin[c + d*x])^3)/(2*d)
```

Rubi [A] time = 0.465199, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2889, 3048, 3047, 3033, 3023, 2735, 3770}

$$\frac{a(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{1}{2}bx(6a^2-b^2)+\frac{15ab^2\cos(c+dx)}{2d}-\frac{3b\cot(c+dx)(a+b\sin(c+dx))^2}{2d}-\frac{\cot(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -(b*(6*a^2 - b^2)*x)/2 + (a*(a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (1
5*a*b^2*Cos[c + d*x])/(2*d) + (5*b^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (3*
b*Cot[c + d*x]*(a + b*Sin[c + d*x])^2)/(2*d) - (Cot[c + d*x]*Csc[c + d*x]*(
a + b*Sin[c + d*x])^3)/(2*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^3(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)dx$$
$$= -\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{2d} + \frac{1}{2}\int \csc^2(c+dx)(a+$$
$$= -\frac{3b\cot(c+dx)(a+b\sin(c+dx))^2}{2d} - \frac{\cot(c+dx)\csc(c+dx)(a+b\,s}{2d}$$
$$= \frac{5b^3\cos(c+dx)\sin(c+dx)}{2d} - \frac{3b\cot(c+dx)(a+b\sin(c+dx))^2}{2d} - c$$
$$= \frac{15ab^2\cos(c+dx)}{2d} + \frac{5b^3\cos(c+dx)\sin(c+dx)}{2d} - \frac{3b\cot(c+dx)(a-}{2d}$$
$$= -\frac{1}{2}b\left(6a^2-b^2\right)x + \frac{15ab^2\cos(c+dx)}{2d} + \frac{5b^3\cos(c+dx)\sin(c+dx)}{2d}$$
$$= -\frac{1}{2}b\left(6a^2-b^2\right)x + \frac{a\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} + \frac{15ab^2\cos(c-}{2d}$$

Mathematica [A] time = 1.33709, size = 192, normalized size = 1.39

$$12a^2b\tan\left(\tfrac{1}{2}(c+dx)\right) - 12a^2b\cot\left(\tfrac{1}{2}(c+dx)\right) - 24a^2bc - 24a^2bdx + a^3\left(-\csc^2\left(\tfrac{1}{2}(c+dx)\right)\right) + a^3\sec^2\left(\tfrac{1}{2}(c+dx)\right) - 4a^3$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-24*a^2*b*c + 4*b^3*c - 24*a^2*b*d*x + 4*b^3*d*x + 24*a*b^2*Cos[c + d*x] -
 12*a^2*b*Cot[(c + d*x)/2] - a^3*Csc[(c + d*x)/2]^2 + 4*a^3*Log[Cos[(c + d*
x)/2]] - 24*a*b^2*Log[Cos[(c + d*x)/2]] - 4*a^3*Log[Sin[(c + d*x)/2]] + 24*
a*b^2*Log[Sin[(c + d*x)/2]] + a^3*Sec[(c + d*x)/2]^2 + 2*b^3*Sin[2*(c + d*x
)] + 12*a^2*b*Tan[(c + d*x)/2])/(8*d)
```

Maple [A] time = 0.098, size = 171, normalized size = 1.2

$$-\frac{a^3(\cos(dx+c))^3}{2d(\sin(dx+c))^2}-\frac{a^3\cos(dx+c)}{2d}-\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}-3a^2bx-3\frac{a^2b\cot(dx+c)}{d}-3\frac{a^2bc}{d}+3\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-1/2*a^3*cos(d*x+c)/d-1/2/d*a^3*ln(csc(
d*x+c)-cot(d*x+c))-3*a^2*b*x-3*a^2*b*cot(d*x+c)/d-3/d*a^2*b*c+3*a*b^2*cos(d
*x+c)/d+3/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))+1/2*b^3*cos(d*x+c)*sin(d*x+c)/d
+1/2*b^3*x+1/2/d*b^3*c
```

Maxima [A] time = 1.63022, size = 173, normalized size = 1.25

$$-\frac{12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2b-(2dx+2c+\sin(2dx+2c))b^3-a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(12*(d*x + c + 1/tan(d*x + c))*a^2*b - (2*d*x + 2*c + sin(2*d*x + 2*c)
)*b^3 - a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) -
log(cos(d*x + c) - 1)) - 6*a*b^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.69171, size = 510, normalized size = 3.7

$$\frac{12ab^2\cos(dx+c)^3-2\left(6a^2b-b^3\right)dx\cos(dx+c)^2+2\left(6a^2b-b^3\right)dx+2\left(a^3-6ab^2\right)\cos(dx+c)-\left(a^3-6ab^2-\left(a^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(12*a*b^2*cos(d*x + c)^3 - 2*(6*a^2*b - b^3)*d*x*cos(d*x + c)^2 + 2*(6*
a^2*b - b^3)*d*x + 2*(a^3 - 6*a*b^2)*cos(d*x + c) - (a^3 - 6*a*b^2 - (a^3 -
 6*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - 6*a*b^2 - (a
^3 - 6*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b^3*cos(d*x
 + c)^3 + (6*a^2*b - b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27662, size = 367, normalized size = 2.66

$$\frac{a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 12\,a^2 b \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 4\left(6\,a^2 b - b^3\right)(dx + c) - 4\left(a^3 - 6\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + \frac{2\,a^3 \tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) - 4*(6*a^2*
b - b^3)*(d*x + c) - 4*(a^3 - 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (2*
a^3*tan(1/2*d*x + 1/2*c)^6 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*b*tan
(1/2*d*x + 1/2*c)^5 - 8*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/
2*c)^4 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^3
+ 8*b^3*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b
*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)
)^2)/d
```

3.1074 $\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=138

$$\frac{a\left(a^2-3b^2\right)\cot(c+dx)}{3d}+\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d}-3ab^2x-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{3d}$$

```
[Out] -3*a*b^2*x + (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (11*b^3*Cos[
c + d*x])/(6*d) + (a*(a^2 - 3*b^2)*Cot[c + d*x])/(3*d) - (b*Cot[c + d*x]*Cs
c[c + d*x]*(a + b*Sin[c + d*x])^2)/(2*d) - (Cot[c + d*x]*Csc[c + d*x]^2*(a
+ b*Sin[c + d*x])^3)/(3*d)
```

Rubi [A] time = 0.482681, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2889, 3048, 3047, 3031, 3023, 2735, 3770}

$$\frac{a\left(a^2-3b^2\right)\cot(c+dx)}{3d}+\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d}-3ab^2x-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -3*a*b^2*x + (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (11*b^3*Cos[
c + d*x])/(6*d) + (a*(a^2 - 3*b^2)*Cot[c + d*x])/(3*d) - (b*Cot[c + d*x]*Cs
c[c + d*x]*(a + b*Sin[c + d*x])^2)/(2*d) - (Cot[c + d*x]*Csc[c + d*x]^2*(a
+ b*Sin[c + d*x])^3)/(3*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^4(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{3d} + \frac{1}{3}\int \csc^3(c+dx)(a$$

$$= -\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d} - \frac{\cot(c+dx)\csc^2(c$$

$$= \frac{a\left(a^2-3b^2\right)\cot(c+dx)}{3d} - \frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+d$$

$$= \frac{11b^3\cos(c+dx)}{6d} + \frac{a\left(a^2-3b^2\right)\cot(c+dx)}{3d} - \frac{b\cot(c+dx)\csc(c+$$

$$= -3ab^2x + \frac{11b^3\cos(c+dx)}{6d} + \frac{a\left(a^2-3b^2\right)\cot(c+dx)}{3d} - \frac{b\cot(c+d$$

$$= -3ab^2x + \frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} + \frac{11b^3\cos(c+dx)}{6d} + $$

Mathematica [B] time = 6.19941, size = 615, normalized size = 4.46

$$\frac{\sin^3(c+dx)\csc\left(\frac{1}{2}(c+dx)\right)\left(a^3\cos\left(\frac{1}{2}(c+dx)\right)-9ab^2\cos\left(\frac{1}{2}(c+dx)\right)\right)(a\csc(c+dx)+b)^3}{6d(a+b\sin(c+dx))^3}+\frac{\left(2b^3-3a^2b\right)\sin^3(c+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*b^2*(c + d*x)*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(d*(a + b*Sin[c
+ d*x])^3) + (b^3*Cos[c + d*x]*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(d*(a
 + b*Sin[c + d*x])^3) + ((a^3*Cos[(c + d*x)/2] - 9*a*b^2*Cos[(c + d*x)/2])*
Csc[(c + d*x)/2]*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(6*d*(a + b*Sin[c +
 d*x])^3) - (3*a^2*b*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^3*Sin[c + d*x]
^3)/(8*d*(a + b*Sin[c + d*x])^3) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2
*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(24*d*(a + b*Sin[c + d*x])^3) + ((3
*a^2*b - 2*b^3)*(b + a*Csc[c + d*x])^3*Log[Cos[(c + d*x)/2]]*Sin[c + d*x]^3
)/(2*d*(a + b*Sin[c + d*x])^3) + ((-3*a^2*b + 2*b^3)*(b + a*Csc[c + d*x])^3
*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^3)/(2*d*(a + b*Sin[c + d*x])^3) + (3*a^
2*b*(b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]^2*Sin[c + d*x]^3)/(8*d*(a + b*S
in[c + d*x])^3) + ((b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]*(-(a^3*Sin[(c +
d*x)/2]) + 9*a*b^2*Sin[(c + d*x)/2])*Sin[c + d*x]^3)/(6*d*(a + b*Sin[c + d*
x])^3) + (a^3*(b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]^2*Sin[c + d*x]^3*Tan[
(c + d*x)/2])/(24*d*(a + b*Sin[c + d*x])^3)
```

Maple [A] time = 0.101, size = 159, normalized size = 1.2

$$-\frac{a^3(\cos(dx+c))^3}{3d(\sin(dx+c))^3}-\frac{3a^2b(\cos(dx+c))^3}{2d(\sin(dx+c))^2}-\frac{3a^2b\cos(dx+c)}{2d}-\frac{3a^2b\ln(\csc(dx+c)-\cot(dx+c))}{2d}-3ab^2x-3\frac{ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d*a^3/sin(d*x+c)^3*cos(d*x+c)^3-3/2/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^3-
3/2*a^2*b*cos(d*x+c)/d-3/2/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-3*a*b^2*x-3*a*
b^2*cot(d*x+c)/d-3/d*a*b^2*c+b^3*cos(d*x+c)/d+1/d*b^3*ln(csc(d*x+c)-cot(d*x
+c))
```

Maxima [A] time = 1.64517, size = 161, normalized size = 1.17

$$-\frac{36\left(dx+c+\frac{1}{\tan(dx+c)}\right)ab^2-9a^2b\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)-6b^3(2\cos(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(36*(d*x + c + 1/tan(d*x + c))*a*b^2 - 9*a^2*b*(2*cos(d*x + c)/(cos(d
*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 6*b^3*(2*
cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 4*a^3/tan(d
```

```
*x + c)^3)/d
```

Fricas [A] time = 1.60852, size = 562, normalized size = 4.07

$$\frac{36\,ab^2\cos\left(dx+c\right)+4\left(a^3-9\,ab^2\right)\cos\left(dx+c\right)^3-3\left(3\,a^2b-2\,b^3-\left(3\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(36*a*b^2*cos(d*x + c) + 4*(a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(3*a^2*b
 - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*si
n(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/
2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(6*a*b^2*d*x*cos(d*x + c)^2 - 2*b^3*
cos(d*x + c)^3 - 6*a*b^2*d*x - (3*a^2*b - 2*b^3)*cos(d*x + c))*sin(d*x + c)
)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30907, size = 300, normalized size = 2.17

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)ab^2-3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4\cdots}{\tan\left(\frac{1}{2}\,d\cdots\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x
 + c)*a*b^2 - 3*a^3*tan(1/2*d*x + 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) +
48*b^3/(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*(3*a^2*b - 2*b^3)*log(abs(tan(1/2*
d*x + 1/2*c))) + (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44*b^3*tan(1/2*d*x + 1/
2*c)^3 + 3*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9
*a^2*b*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.1075 $\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=152

$$\frac{b\left(2a^2-b^2\right)\cot(c+dx)}{2d}+\frac{a\left(a^2+12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot(c+dx)\,\mathrm{c}}{}$$

```
[Out] -(b^3*x) + (a*(a^2 + 12*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (b*(2*a^2 - b^2
)*Cot[c + d*x])/(2*d) + (a*(a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) -
 (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(4*d) - (Cot[c + d*
x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(4*d)
```

Rubi [A] time = 0.510627, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2889, 3048, 3047, 3031, 3021, 2735, 3770}

$$\frac{b\left(2a^2-b^2\right)\cot(c+dx)}{2d}+\frac{a\left(a^2+12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot(c+dx)\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -(b^3*x) + (a*(a^2 + 12*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (b*(2*a^2 - b^2
)*Cot[c + d*x])/(2*d) + (a*(a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) -
 (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(4*d) - (Cot[c + d*
x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(4*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^5(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{1}{4}\int \csc^4(c+dx)(a$$

$$= -\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\cot(c+dx)\csc^3(}{}$$

$$= \frac{a\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot(c+dx)\csc^2(c+dx)(a}{4d}$$

$$= \frac{b\left(2a^2-b^2\right)\cot(c+dx)}{2d} + \frac{a\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{8d} - \frac{b}{}$$

$$= -b^3x + \frac{b\left(2a^2-b^2\right)\cot(c+dx)}{2d} + \frac{a\left(a^2-2b^2\right)\cot(c+dx)\csc(c+}{8d}$$

$$= -b^3x + \frac{a\left(a^2+12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{b\left(2a^2-b^2\right)\cot(c+dx}{2d}$$

Mathematica [B] time = 6.21487, size = 690, normalized size = 4.54

$$\frac{\left(a^3-12ab^2\right)\sin^3(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a\csc(c+dx)+b)^3}{32d(a+b\sin(c+dx))^3}+\frac{\sin^3(c+dx)\csc\left(\frac{1}{2}(c+dx)\right)\left(a^2b\cos\left(\frac{1}{2}(c+dx)\right)-b\right.}{2d(a+b\sin(c+d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((b^3*(c + d*x)*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(d*(a + b*Sin[c + d
*x])^3)) + ((a^2*b*Cos[(c + d*x)/2] - b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2
]*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(2*d*(a + b*Sin[c + d*x])^3) + ((a
^3 - 12*a*b^2)*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(3
2*d*(a + b*Sin[c + d*x])^3) - (a^2*b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(b
 + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(8*d*(a + b*Sin[c + d*x])^3) - (a^3*Cs
c[(c + d*x)/2]^4*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(64*d*(a + b*Sin[c
+ d*x])^3) + ((a^3 + 12*a*b^2)*(b + a*Csc[c + d*x])^3*Log[Cos[(c + d*x)/2]]
*Sin[c + d*x]^3)/(8*d*(a + b*Sin[c + d*x])^3) + ((-a^3 - 12*a*b^2)*(b + a*C
sc[c + d*x])^3*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^3)/(8*d*(a + b*Sin[c + d*
x])^3) + ((-a^3 + 12*a*b^2)*(b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]^2*Sin[c
 + d*x]^3)/(32*d*(a + b*Sin[c + d*x])^3) + (a^3*(b + a*Csc[c + d*x])^3*Sec[
(c + d*x)/2]^4*Sin[c + d*x]^3)/(64*d*(a + b*Sin[c + d*x])^3) + ((b + a*Csc[
c + d*x])^3*Sec[(c + d*x)/2]*(-(a^2*b*Sin[(c + d*x)/2]) + b^3*Sin[(c + d*x)
/2])*Sin[c + d*x]^3)/(2*d*(a + b*Sin[c + d*x])^3) + (a^2*b*(b + a*Csc[c + d
*x])^3*Sec[(c + d*x)/2]^2*Sin[c + d*x]^3*Tan[(c + d*x)/2])/(8*d*(a + b*Sin[
c + d*x])^3)
```

Maple [A] time = 0.098, size = 207, normalized size = 1.4

$$-\frac{a^3(\cos(dx+c))^3}{4d(\sin(dx+c))^4}-\frac{a^3(\cos(dx+c))^3}{8d(\sin(dx+c))^2}-\frac{a^3\cos(dx+c)}{8d}-\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{a^2b(\cos(dx+c))^3}{d(\sin(dx+c))^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^3-1/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-1/
8*a^3*cos(d*x+c)/d-1/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-1/d*a^2*b/sin(d*x+c)
^3*cos(d*x+c)^3-3/2/d*a*b^2/sin(d*x+c)^2*cos(d*x+c)^3-3/2*a*b^2*cos(d*x+c)/
d-3/2/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-b^3*x-1/d*cot(d*x+c)*b^3-1/d*b^3*c
```

Maxima [A] time = 1.63156, size = 201, normalized size = 1.32

$$-\frac{16\left(dx+c+\frac{1}{\tan(dx+c)}\right)b^3+a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12ab^2\left(\frac{2\cos}{\cos(d}\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*(d*x + c + 1/tan(d*x + c))*b^3 + a^3*(2*(cos(d*x + c)^3 + cos(d*x
 + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + lo
```

```
g(cos(d*x + c) - 1)) - 12*a*b^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(
cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 16*a^2*b/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.63152, size = 666, normalized size = 4.38

$$-\frac{16\,b^3dx\cos\left(dx+c\right)^4-32\,b^3dx\cos\left(dx+c\right)^2+16\,b^3dx+2\left(a^3-12\,ab^2\right)\cos\left(dx+c\right)^3+2\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)-((}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(16*b^3*d*x*cos(d*x + c)^4 - 32*b^3*d*x*cos(d*x + c)^2 + 16*b^3*d*x +
 2*(a^3 - 12*a*b^2)*cos(d*x + c)^3 + 2*(a^3 + 12*a*b^2)*cos(d*x + c) - ((a^
3 + 12*a*b^2)*cos(d*x + c)^4 + a^3 + 12*a*b^2 - 2*(a^3 + 12*a*b^2)*cos(d*x
+ c)^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^3 + 12*a*b^2)*cos(d*x + c)^4 + a^
3 + 12*a*b^2 - 2*(a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2) + 16*(b^3*cos(d*x + c) + (a^2*b - b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35072, size = 316, normalized size = 2.08

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+72\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-192\,(dx+c)b^3-72\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*tan(1/2*d*x + 1/2*c)^4 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 72*
a*b^2*tan(1/2*d*x + 1/2*c)^2 - 192*(d*x + c)*b^3 - 72*a^2*b*tan(1/2*d*x + 1
/2*c) + 96*b^3*tan(1/2*d*x + 1/2*c) - 24*(a^3 + 12*a*b^2)*log(abs(tan(1/2*d
*x + 1/2*c))) + (50*a^3*tan(1/2*d*x + 1/2*c)^4 + 600*a*b^2*tan(1/2*d*x + 1/
2*c)^4 + 72*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 96*b^3*tan(1/2*d*x + 1/2*c)^3 -
72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/ta
n(1/2*d*x + 1/2*c)^4)/d
```

3.1076 $\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=183

$$\frac{a\left(2a^2+15b^2\right)\cot(c+dx)}{15d}+\frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a\left(2a^2-3b^2\right)\cot(c+dx)\csc^2(c+dx)}{30d}+\frac{3b\left(5a^2-\right.}{}$$

```
[Out] (b*(3*a^2 + 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (a*(2*a^2 + 15*b^2)*Cot[c
 + d*x])/(15*d) + (3*b*(5*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(40*d) +
(a*(2*a^2 - 3*b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*d) - (3*b*Cot[c + d*x]*
Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(20*d) - (Cot[c + d*x]*Csc[c + d*x]^
4*(a + b*Sin[c + d*x])^3)/(5*d)
```

Rubi [A] time = 0.568896, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2889, 3048, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{a\left(2a^2+15b^2\right)\cot(c+dx)}{15d}+\frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a\left(2a^2-3b^2\right)\cot(c+dx)\csc^2(c+dx)}{30d}+\frac{3b\left(5a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(3*a^2 + 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (a*(2*a^2 + 15*b^2)*Cot[c
 + d*x])/(15*d) + (3*b*(5*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(40*d) +
(a*(2*a^2 - 3*b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*d) - (3*b*Cot[c + d*x]*
Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(20*d) - (Cot[c + d*x]*Csc[c + d*x]^
4*(a + b*Sin[c + d*x])^3)/(5*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^6(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)dx$$
$$= -\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{5d} + \frac{1}{5}\int \csc^5(c+d$$
$$= -\frac{3b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{20d} - \frac{\cot(c+dx)\,c}{}$$
$$= \frac{a\left(2a^2-3b^2\right)\cot(c+dx)\csc^2(c+dx)}{30d} - \frac{3b\cot(c+dx)\csc^3(c+}{2}$$
$$= \frac{3b\left(5a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{40d} + \frac{a\left(2a^2-3b^2\right)\cot(c+d}{30d}$$
$$= \frac{3b\left(5a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{40d} + \frac{a\left(2a^2-3b^2\right)\cot(c+d}{30d}$$
$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{3b\left(5a^2-2b^2\right)\cot(c+dx)}{40d}$$
$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{a\left(2a^2+15b^2\right)\cot(c+dx)}{15d}$$

Mathematica [A] time = 1.24525, size = 344, normalized size = 1.88

$$\frac{32\left(2a^3+15ab^2\right)\cot\left(\frac{1}{2}(c+dx)\right)+30\left(3a^2b-4b^3\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+a\csc^4\left(\frac{1}{2}(c+dx)\right)\left(\left(a^2-60b^2\right)\sin(c+dx)-45\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (32*(2*a^3 + 15*a*b^2)*Cot[(c + d*x)/2] + 30*(3*a^2*b - 4*b^3)*Csc[(c + d*x
)/2]^2 + 360*a^2*b*Log[Cos[(c + d*x)/2]] + 480*b^3*Log[Cos[(c + d*x)/2]] -
360*a^2*b*Log[Sin[(c + d*x)/2]] - 480*b^3*Log[Sin[(c + d*x)/2]] - 90*a^2*b*
Sec[(c + d*x)/2]^2 + 120*b^3*Sec[(c + d*x)/2]^2 + 45*a^2*b*Sec[(c + d*x)/2]
^4 - 16*a^3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 960*a*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 - 3*a^3*Csc[(c + d*x)/2]^6*Sin[c + d*x] + a*Csc[(c + d*x)/
2]^4*(-45*a*b + (a^2 - 60*b^2)*Sin[c + d*x]) - 64*a^3*Tan[(c + d*x)/2] - 48
0*a*b^2*Tan[(c + d*x)/2] + 6*a^3*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*
d)
```

Maple [A] time = 0.105, size = 227, normalized size = 1.2

$$-\frac{a^3(\cos(dx+c))^3}{5d(\sin(dx+c))^5} - \frac{2a^3(\cos(dx+c))^3}{15d(\sin(dx+c))^3} - \frac{3a^2b(\cos(dx+c))^3}{4d(\sin(dx+c))^4} - \frac{3a^2b(\cos(dx+c))^3}{8d(\sin(dx+c))^2} - \frac{3a^2b\cos(dx+c)}{8d} - \frac{3a^2b\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^3-2/15/d*a^3/sin(d*x+c)^3*cos(d*x+c)^3-3
/4/d*a^2*b/sin(d*x+c)^4*cos(d*x+c)^3-3/8/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^3-
3/8*a^2*b*cos(d*x+c)/d-3/8/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*a*b^2/sin(
d*x+c)^3*cos(d*x+c)^3-1/2/d*b^3/sin(d*x+c)^2*cos(d*x+c)^3-1/2*b^3*cos(d*x+c
)/d-1/2/d*b^3*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.16253, size = 212, normalized size = 1.16

$$-\frac{45\,a^2b\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-60\,b^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/240*(45*a^2*b*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 60*b^3*
(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x
+ c) - 1)) + 240*a*b^2/tan(d*x + c)^3 + 16*(5*tan(d*x + c)^2 + 3)*a^3/tan(d
*x + c)^5)/d
```

Fricas [A] time = 1.53561, size = 675, normalized size = 3.69

$$\frac{16\left(2\,a^3+15\,ab^2\right)\cos\left(dx+c\right)^5-80\left(a^3+3\,ab^2\right)\cos\left(dx+c\right)^3+15\left(\left(3\,a^2b+4\,b^3\right)\cos\left(dx+c\right)^4+3\,a^2b+4\,b^3-2\left(3\,a^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(16*(2*a^3 + 15*a*b^2)*cos(d*x + c)^5 - 80*(a^3 + 3*a*b^2)*cos(d*x +
c)^3 + 15*((3*a^2*b + 4*b^3)*cos(d*x + c)^4 + 3*a^2*b + 4*b^3 - 2*(3*a^2*b
+ 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*((3*
a^2*b + 4*b^3)*cos(d*x + c)^4 + 3*a^2*b + 4*b^3 - 2*(3*a^2*b + 4*b^3)*cos(d
*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*((3*a^2*b - 4*b^3
)*cos(d*x + c)^3 + (3*a^2*b + 4*b^3)*cos(d*x + c))*sin(d*x + c))/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27483, size = 392, normalized size = 2.14

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,b^3\tan\left(\frac{1}{2}\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 10*
a^3*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan
(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c) - 360*a*b^2*tan(1/2*d*x +
 1/2*c) - 120*(3*a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (822*a^2*b
*tan(1/2*d*x + 1/2*c)^5 + 1096*b^3*tan(1/2*d*x + 1/2*c)^5 + 60*a^3*tan(1/2*
d*x + 1/2*c)^4 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*b^3*tan(1/2*d*x + 1
/2*c)^3 - 10*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^2*tan(1/2*d*x + 1/2*c)^2
- 45*a^2*b*tan(1/2*d*x + 1/2*c) - 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.1077 $\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=212

$$\frac{b(6a^2+5b^2)\cot(c+dx)}{15d}+\frac{a(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a(5a^2-6b^2)\cot(c+dx)\csc^3(c+dx)}{120d}+\frac{b(3a^2-b^2)\cot}{}$$

```
[Out] (a*(a^2 + 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (b*(6*a^2 + 5*b^2)*Cot[c +
 d*x])/(15*d) + (a*(a^2 + 6*b^2)*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (b*(3*
a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) + (a*(5*a^2 - 6*b^2)*Cot[c +
 d*x]*Csc[c + d*x]^3)/(120*d) - (b*Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c
 + d*x])^2)/(10*d) - (Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(
6*d)
```

Rubi [A] time = 0.598271, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2889, 3048, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{b(6a^2+5b^2)\cot(c+dx)}{15d}+\frac{a(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a(5a^2-6b^2)\cot(c+dx)\csc^3(c+dx)}{120d}+\frac{b(3a^2-b^2)\cot}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (a*(a^2 + 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (b*(6*a^2 + 5*b^2)*Cot[c +
 d*x])/(15*d) + (a*(a^2 + 6*b^2)*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (b*(3*
a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) + (a*(5*a^2 - 6*b^2)*Cot[c +
 d*x]*Csc[c + d*x]^3)/(120*d) - (b*Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c
 + d*x])^2)/(10*d) - (Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(
6*d)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^7(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$

$$= -\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{6d} + \frac{1}{6}\int \csc^6(c+dx)(a$$

$$= -\frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{10d} - \frac{\cot(c+dx)\csc^5(c$$

$$= \frac{a\left(5a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)}{120d} - \frac{b\cot(c+dx)\csc^4(c+dx)(}{10d}$$

$$= \frac{b\left(3a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)}{15d} + \frac{a\left(5a^2-6b^2\right)\cot(c+dx)\,cs}{120d}$$

$$= \frac{b\left(3a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)}{15d} + \frac{a\left(5a^2-6b^2\right)\cot(c+dx)\,cs}{120d}$$

$$= \frac{a\left(a^2+6b^2\right)\cot(c+dx)\csc(c+dx)}{16d} + \frac{b\left(3a^2-b^2\right)\cot(c+dx)\csc^2}{15d}$$

$$= \frac{a\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} + \frac{b\left(6a^2+5b^2\right)\cot(c+dx)}{15d} + \frac{a\,(}{}$$

Mathematica [A] time = 2.02905, size = 369, normalized size = 1.74

$$-\frac{-64\left(6a^2b+5b^3\right)\cot\left(\frac{1}{2}(c+dx)\right)-30\left(a^3+6ab^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+2b\csc^4\left(\frac{1}{2}(c+dx)\right)\left(\left(20b^2-3a^2\right)\sin(c+dx)+45\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(-64*(6*a^2*b + 5*b^3)*Cot[(c + d*x)/2] - 30*(a^3 + 6*a*b^2)*Csc[(c + d*x)
/2]^2 - 120*a^3*Log[Cos[(c + d*x)/2]] - 720*a*b^2*Log[Cos[(c + d*x)/2]] + 1
20*a^3*Log[Sin[(c + d*x)/2]] + 720*a*b^2*Log[Sin[(c + d*x)/2]] + 30*a^3*Sec
[(c + d*x)/2]^2 + 180*a*b^2*Sec[(c + d*x)/2]^2 - 90*a*b^2*Sec[(c + d*x)/2]^
4 - 5*a^3*Sec[(c + d*x)/2]^6 + 96*a^2*b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 -
 640*b^3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + a^2*Csc[(c + d*x)/2]^6*(5*a +
18*b*Sin[c + d*x]) + 2*b*Csc[(c + d*x)/2]^4*(45*a*b + (-3*a^2 + 20*b^2)*Sin
[c + d*x]) + 384*a^2*b*Tan[(c + d*x)/2] + 320*b^3*Tan[(c + d*x)/2] - 36*a^2
*b*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(1920*d)
```

Maple [A] time = 0.106, size = 276, normalized size = 1.3

$$-\frac{a^3(\cos(dx+c))^3}{6d(\sin(dx+c))^6} - \frac{a^3(\cos(dx+c))^3}{8d(\sin(dx+c))^4} - \frac{a^3(\cos(dx+c))^3}{16d(\sin(dx+c))^2} - \frac{a^3\cos(dx+c)}{16d} - \frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{16d} - \frac{3\,}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/6/d*a^3/sin(d*x+c)^6*cos(d*x+c)^3-1/8/d*a^3/sin(d*x+c)^4*cos(d*x+c)^3-1/
16/d*a^3/sin(d*x+c)^2*cos(d*x+c)^3-1/16*a^3*cos(d*x+c)/d-1/16/d*a^3*ln(csc(
d*x+c)-cot(d*x+c))-3/5/d*a^2*b/sin(d*x+c)^5*cos(d*x+c)^3-2/5/d*a^2*b/sin(d*
```

```
x+c)^3*cos(d*x+c)^3-3/4/d*a*b^2/sin(d*x+c)^4*cos(d*x+c)^3-3/8/d*a*b^2/sin(d
*x+c)^2*cos(d*x+c)^3-3/8*a*b^2*cos(d*x+c)/d-3/8/d*a*b^2*ln(csc(d*x+c)-cot(d
*x+c))-1/3/d*b^3/sin(d*x+c)^3*cos(d*x+c)^3
```

Maxima [A] time = 1.12276, size = 273, normalized size = 1.29

$$-\frac{5\,a^{3}\left(\frac{2\left(3\,\cos(dx+c)^{5}-8\,\cos(dx+c)^{3}-3\,\cos(dx+c)\right)}{\cos(dx+c)^{6}-3\,\cos(dx+c)^{4}+3\,\cos(dx+c)^{2}-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+90\,ab^{2}\left(\frac{2\left(\cos(dx+c)^{3}+c\right.}{\cos(dx+c)^{4}-2\,\mathrm{co}}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^3*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 90*a*b^2*(2*(cos(d*x + c)^3 + cos(d*x +
 c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(
cos(d*x + c) - 1)) + 160*b^3/tan(d*x + c)^3 + 96*(5*tan(d*x + c)^2 + 3)*a^2
*b/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.50196, size = 755, normalized size = 3.56

$$\frac{80\,a^{3}\cos\left(dx+c\right)^{3}-30\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{5}+30\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)+15\left(\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{6}-3\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(80*a^3*cos(d*x + c)^3 - 30*(a^3 + 6*a*b^2)*cos(d*x + c)^5 + 30*(a^3
+ 6*a*b^2)*cos(d*x + c) + 15*((a^3 + 6*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 6*a
*b^2)*cos(d*x + c)^4 - a^3 - 6*a*b^2 + 3*(a^3 + 6*a*b^2)*cos(d*x + c)^2)*lo
g(1/2*cos(d*x + c) + 1/2) - 15*((a^3 + 6*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 6
*a*b^2)*cos(d*x + c)^4 - a^3 - 6*a*b^2 + 3*(a^3 + 6*a*b^2)*cos(d*x + c)^2)*
log(-1/2*cos(d*x + c) + 1/2) - 32*((6*a^2*b + 5*b^3)*cos(d*x + c)^5 - 5*(3*
a^2*b + b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x
+ c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32878, size = 478, normalized size = 2.25

$$5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+60\,a^2b\tan\left(\frac{1}{2}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 15
*a^3*tan(1/2*d*x + 1/2*c)^4 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 60*a^2*b*ta
n(1/2*d*x + 1/2*c)^3 + 80*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x +
 1/2*c)^2 - 360*a^2*b*tan(1/2*d*x + 1/2*c) - 240*b^3*tan(1/2*d*x + 1/2*c) -
 120*(a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (294*a^3*tan(1/2*d*x
+ 1/2*c)^6 + 1764*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 360*a^2*b*tan(1/2*d*x + 1/
2*c)^5 + 240*b^3*tan(1/2*d*x + 1/2*c)^5 + 15*a^3*tan(1/2*d*x + 1/2*c)^4 - 6
0*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 80*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan
(1/2*d*x + 1/2*c)^2 - 90*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 36*a^2*b*tan(1/2*d*
x + 1/2*c) - 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.1078 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$\frac{(12a^2-b^2)\cos(c+dx)}{3b^4d}-\frac{2a^2(4a^2-3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{ax(4a^2-b^2)}{b^5}-\frac{2a\sin(c+dx)\cos(c+dx)}{b^3d}-\frac{\sin^3}{bd}$$

```
[Out] (a*(4*a^2 - b^2)*x)/b^5 - (2*a^2*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + ((12*a^2 - b^2)*Cos[c + d
*x])/(3*b^4*d) - (2*a*Cos[c + d*x]*Sin[c + d*x])/(b^3*d) + (4*Cos[c + d*x]*
Sin[c + d*x]^2)/(3*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(b*d*(a + b*Sin[c
 + d*x]))
```

Rubi [A] time = 0.741585, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2889, 3048, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{(12a^2-b^2)\cos(c+dx)}{3b^4d}-\frac{2a^2(4a^2-3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{ax(4a^2-b^2)}{b^5}-\frac{2a\sin(c+dx)\cos(c+dx)}{b^3d}-\frac{\sin^3}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*(4*a^2 - b^2)*x)/b^5 - (2*a^2*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + ((12*a^2 - b^2)*Cos[c + d
*x])/(3*b^4*d) - (2*a*Cos[c + d*x]*Sin[c + d*x])/(b^3*d) + (4*Cos[c + d*x]*
Sin[c + d*x]^2)/(3*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(b*d*(a + b*Sin[c
 + d*x]))
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\sin^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin^2(c+dx)\left(-3\left(a^2-b^2\right)+4\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{b\left(a^2-b^2\right)}$$

$$= \frac{4\cos(c+dx)\sin^2(c+dx)}{3b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin(c+dx)\left(8a\left(a^2-b^2\right)-b\left(a^2-b^2\right)s\right.}{a+b\,\mathrm{si}}}{3b^2\left(\right.}$$

$$= -\frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(c+dx)\sin^2(c+dx)}{3b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{bd(a+b\sin(c+dx))}$$

$$= \frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(c+dx)\sin^2(c+dx)}{3b^2d} -$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} + \frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(c+a}{}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} + \frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(c+a}{}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} + \frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(c+a}{}$$

$$= \frac{a\left(4a^2-b^2\right)x}{b^5} - \frac{2a^2\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}d} + \frac{\left(12a^2-b^2\right)\cos(c+dx)}{3b^4d}$$

Mathematica [A] time = 2.47963, size = 246, normalized size = 1.31

$$\frac{\frac{24a^2b^2\sin(2(c+dx))+12ab\left(8a^2-b^2\right)\cos(c+dx)-24a^2b^2c-24a^2b^2dx+96a^3bc\sin(c+dx)+96a^3bdx\sin(c+dx)+96a^4c+96a^4dx-24ab^3c\sin(c+dx)-24ab^3dx\sin(c+}{a+b\sin(c+dx)}}{24b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-48*a^2*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] + (96*a^4*c - 24*a^2*b^2*c + 96*a^4*d*x - 24*a^2*b^2*d*x +
 12*a*b*(8*a^2 - b^2)*Cos[c + d*x] + 4*a*b^3*Cos[3*(c + d*x)] + 96*a^3*b*c*
Sin[c + d*x] - 24*a*b^3*c*Sin[c + d*x] + 96*a^3*b*d*x*Sin[c + d*x] - 24*a*b
^3*d*x*Sin[c + d*x] + 24*a^2*b^2*Sin[2*(c + d*x)] - 2*b^4*Sin[2*(c + d*x)]
- b^4*Sin[4*(c + d*x)])/(a + b*Sin[c + d*x]))/(24*b^5*d)
```

Maple [B] time = 0.125, size = 460, normalized size = 2.5

$$2\,\frac{a\left(\tan(1/2\,dx+c/2)\right)^5}{db^3\left(1+\left(\tan(1/2\,dx+c/2)\right)^2\right)^3} + 6\,\frac{\left(\tan(1/2\,dx+c/2)\right)^4a^2}{db^4\left(1+\left(\tan(1/2\,dx+c/2)\right)^2\right)^3} - 2\,\frac{\left(\tan(1/2\,dx+c/2)\right)^4}{db^2\left(1+\left(\tan(1/2\,dx+c/2)\right)^2\right)^3} + 12\,\frac{a^2\,(}{db^4\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5+6/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)
^3*tan(1/2*d*x+1/2*c)^4+12/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*a^2*tan(1/2*d*x
+1/2*c)^2-2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)+6/d/b^4/(
1+tan(1/2*d*x+1/2*c)^2)^3*a^2-2/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3+8/d/b^5*
arctan(tan(1/2*d*x+1/2*c))*a^3-2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a+2/d*a^2
/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2
/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-8/d*a^4/b^5/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d*
a^2/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86673, size = 1381, normalized size = 7.35

$$\left[\frac{4\left(a^3b^3 - ab^5\right)\cos\left(dx+c\right)^3 + 6\left(4\,a^6 - 5\,a^4b^2 + a^2b^4\right)dx + 3\left(4\,a^5 - 3\,a^3b^2 + \left(4\,a^4b - 3\,a^2b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/6*(4*(a^3*b^3 - a*b^5)*cos(d*x + c)^3 + 6*(4*a^6 - 5*a^4*b^2 + a^2*b^4)*
d*x + 3*(4*a^5 - 3*a^3*b^2 + (4*a^4*b - 3*a^2*b^3)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(4*a^5*b - 5*a^3*b^3 +
a*b^5)*cos(d*x + c) - 2*((a^2*b^4 - b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 5*a^
3*b^3 + a*b^5)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^
2*b^6 - b^8)*d*sin(d*x + c) + (a^3*b^5 - a*b^7)*d), 1/3*(2*(a^3*b^3 - a*b^5
)*cos(d*x + c)^3 + 3*(4*a^6 - 5*a^4*b^2 + a^2*b^4)*d*x + 3*(4*a^5 - 3*a^3*b
^2 + (4*a^4*b - 3*a^2*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 3*(4*a^5*b - 5*a^3*b^3 + a*b^5
)*cos(d*x + c) - ((a^2*b^4 - b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 5*a^3*b^3 +
 a*b^5)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^6 -
 b^8)*d*sin(d*x + c) + (a^3*b^5 - a*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31987, size = 352, normalized size = 1.87

$$\frac{\frac{3\left(4a^3-ab^2\right)(dx+c)}{b^5}-\frac{6\left(4a^4-3a^2b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{6\left(a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^3\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)b^4}+\frac{2\left(3ab\tan\left(\frac{1}{2}d\right.\right.}{}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(4*a^3 - a*b^2)*(d*x + c)/b^5 - 6*(4*a^4 - 3*a^2*b^2)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
 b^2)))/(sqrt(a^2 - b^2)*b^5) + 6*(a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/((a*ta
n(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^4) + 2*(3*a*b*tan(1/
2*d*x + 1/2*c)^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^4 - 3*b^2*tan(1/2*d*x + 1/2*c
)^4 + 18*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*a^2 -
b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4))/d
```

3.1079 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=153

$$\frac{2a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{x\left(6a^2-b^2\right)}{2b^4}-\frac{3a\cos(c+dx)}{b^3d}-\frac{\sin^2(c+dx)\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{3\sin(c+dx)\cos(c+dx)}{2b^2d}$$

```
[Out] -((6*a^2 - b^2)*x)/(2*b^4) + (2*a*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2]*d) - (3*a*Cos[c + d*x])/(b^3*
d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^2
)/(b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.491838, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3050, 3023, 2735, 2660, 618, 204}

$$\frac{2a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{x\left(6a^2-b^2\right)}{2b^4}-\frac{3a\cos(c+dx)}{b^3d}-\frac{\sin^2(c+dx)\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{3\sin(c+dx)\cos(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((6*a^2 - b^2)*x)/(2*b^4) + (2*a*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2]*d) - (3*a*Cos[c + d*x])/(b^3*
d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^2
)/(b*d*(a + b*Sin[c + d*x]))
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
```

```
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\sin^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin(c+dx)\left(-2\left(a^2-b^2\right)+3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{b\left(a^2-b^2\right)}$$

$$= \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{3a\left(a^2-b^2\right)-b\left(a^2-b^2\right)\sin(c+dx)-6a\left(a^2\right.}{a+b\sin(c+dx)}}{2b^2\left(a^2-b^2\right)}$$

$$= -\frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{3ab\left(a^2-b^2\right)+}{}}{2}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))}$$

$$= -\frac{\left(6a^2-b^2\right)x}{2b^4} + \frac{2a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\,d} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+d}{2}$$

Mathematica [A] time = 0.376863, size = 129, normalized size = 0.84

$$\frac{2\left(b^2-6a^2\right)(c+dx) + \frac{8a\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{4a^2b\cos(c+dx)}{a+b\sin(c+dx)} - 8ab\cos(c+dx) + b^2\sin(2(c+dx))}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(-6*a^2 + b^2)*(c + d*x) + (8*a*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 8*a*b*Cos[c + d*x] - (4*a^2*b*C
os[c + d*x])/(a + b*Sin[c + d*x]) + b^2*Sin[2*(c + d*x)])/(4*b^4*d)
```

Maple [B] time = 0.117, size = 353, normalized size = 2.3

$$-\frac{1}{db^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2} - 4\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^2 a}{db^3\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)^2} + \frac{1}{db^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-4/d/b^3/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*a-6/d/b^4*arctan(tan(
```

```
1/2*d*x+1/2*c))*a^2+1/d/b^2*arctan(tan(1/2*d*x+1/2*c))-2/d/b^2/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+1/2*c)-2/d/b^3/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^2+6/d/b^4*a^3/(a^2-b^2)^(1/2)*a
rctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d/b^2*a/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66269, size = 1237, normalized size = 8.08

$$\left[-\frac{\left(a^{2} b^{3}-b^{5}\right) \cos \left(d x+c\right)^{3}+\left(6 \, a^{5}-7 \, a^{3} b^{2}+a b^{4}\right) d x-\left(3 \, a^{4}-2 \, a^{2} b^{2}+\left(3 \, a^{3} b-2 \, a b^{3}\right) \sin \left(d x+c\right)\right) \sqrt{-a^{2}+b^{2}} \log \left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*((a^2*b^3 - b^5)*cos(d*x + c)^3 + (6*a^5 - 7*a^3*b^2 + a*b^4)*d*x - (
3*a^4 - 2*a^2*b^2 + (3*a^3*b - 2*a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + (6*a^4*b - 7*a^2*b^3 + b^5)*cos(d*x
 + c) + ((6*a^4*b - 7*a^2*b^3 + b^5)*d*x + 3*(a^3*b^2 - a*b^4)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^5 - b^7)*d*sin(d*x + c) + (a^3*b^4 - a*b^6)*d), -1/
2*((a^2*b^3 - b^5)*cos(d*x + c)^3 + (6*a^5 - 7*a^3*b^2 + a*b^4)*d*x + 2*(3*
a^4 - 2*a^2*b^2 + (3*a^3*b - 2*a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(
-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (6*a^4*b - 7*a^2*b^
3 + b^5)*cos(d*x + c) + ((6*a^4*b - 7*a^2*b^3 + b^5)*d*x + 3*(a^3*b^2 - a*b
^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d*sin(d*x + c) + (a^3*b^4
- a*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30521, size = 285, normalized size = 1.86

$$-\frac{\frac{\left(6\,a^{2}-b^{2}\right)(dx+c)}{b^{4}}-\frac{4\left(3\,a^{3}-2\,ab^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,b^{4}}+\frac{4\left(ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{2}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)b^{3}}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}b^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((6*a^2 - b^2)*(d*x + c)/b^4 - 4*(3*a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*b^4) + 4*(a*b*tan(1/2*d*x + 1/2*c) + a^2)/((a*tan(1/2*d
*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^3) + 2*(b*tan(1/2*d*x + 1/2
*c)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 4*a)/((tan(1/
2*d*x + 1/2*c)^2 + 1)^2*b^3))/d
```

3.1080 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=106

$$-\frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}+\frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}+\frac{2ax}{b^3}$$

```
[Out] (2*a*x)/b^3 - (2*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^3*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]*(2*a + b*Sin[c + d*x]))/(b^2*
d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.152587, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2863, 2735, 2660, 618, 204}

$$-\frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}+\frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}+\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*x)/b^3 - (2*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^3*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]*(2*a + b*Sin[c + d*x]))/(b^2*
d*(a + b*Sin[c + d*x]))
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))} - \frac{\int \frac{-b-2a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{2ax}{b^3} + \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))} - \frac{\left(2a^2-b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^3}$$

$$= \frac{2ax}{b^3} + \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))} - \frac{\left(2\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}\right.\right.}{b^3d}$$

$$= \frac{2ax}{b^3} + \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))} + \frac{\left(4\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+\right.}{b^3d}$$

$$= \frac{2ax}{b^3} - \frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d} + \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.853078, size = 130, normalized size = 1.23

$$\frac{\frac{4a^2c+4a^2dx+4ab(c+dx)\sin(c+dx)+4ab\cos(c+dx)+b^2\sin(2(c+dx))}{a+b\sin(c+dx)} - \frac{4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] + (4*a^2*c + 4*a^2*d*x + 4*a*b*Cos[c + d*x] + 4*a*b*(c + d*x)*Sin
[c + d*x] + b^2*Sin[2*(c + d*x)])/(a + b*Sin[c + d*x]))/(2*b^3*d)
```

Maple [B] time = 0.107, size = 229, normalized size = 2.2

$$2\,\frac{1}{db^2\left(1+(\tan(1/2\,dx+c/2))^2\right)} + 4\,\frac{\arctan(\tan(1/2\,dx+c/2))\,a}{db^3} + 2\,\frac{\tan(1/2\,dx+c/2)}{bd\left((\tan(1/2\,dx+c/2))^2\,a+2\,\tan(1/2\,dx+c/2)\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)+4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a+2/d/b
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/b
```

```
^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a-4/d*a^2/b^3/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/b/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63915, size = 1025, normalized size = 9.67

$$\left[\frac{4\left(a^4-a^2b^2\right)dx+\left(2a^3-ab^2+\left(2a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+\right.}{b^2\cos(dx+c)^2-2ab\sin(dx+c)}\right.}{2\left(\left(a^2b^4-b^6\right)d\sin\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*(a^4 - a^2*b^2)*d*x + (2*a^3 - a*b^2 + (2*a^2*b - b^3)*sin(d*x + c)
)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b
^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 4*(a^3*b - a*
b^3)*cos(d*x + c) + 2*(2*(a^3*b - a*b^3)*d*x + (a^2*b^2 - b^4)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^4 - b^6)*d*sin(d*x + c) + (a^3*b^3 - a*b^5)*d), (2*
(a^4 - a^2*b^2)*d*x + (2*a^3 - a*b^2 + (2*a^2*b - b^3)*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*
(a^3*b - a*b^3)*cos(d*x + c) + (2*(a^3*b - a*b^3)*d*x + (a^2*b^2 - b^4)*cos
(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*sin(d*x + c) + (a^3*b^3 - a*b^5
)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21664, size = 258, normalized size = 2.43

$$\frac{2\left(\frac{(dx+c)a}{b^3}-\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)(2a^2-b^2)}{\sqrt{a^2-b^2}b^3}+\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((d*x + c)*a/b^3 - (pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*(2*a^2 - b^2)/(sqrt(a^2 - b^2)*b
^3) + (b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 3*b*tan(1/2*
d*x + 1/2*c) + 2*a)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3
 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^2))/d
```

3.1081 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{ad(a+b\sin(c+dx))}$$

```
[Out] (-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^2
]*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a*d*(a + b*Sin[c + d*x
]))
```

Rubi [A] time = 0.242228, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2889, 3056, 12, 2747, 3770, 2660, 618, 204}

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^2
]*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a*d*(a + b*Sin[c + d*x
]))
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\left(a^2-b^2\right)\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{b\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c\right.\right.}{a^2d}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2\right.}{a^2d}$$

$$= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.211391, size = 97, normalized size = 1.05

$$\frac{-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{a\cos(c+dx)}{a+b\sin(c+dx)}+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]] + (a*Cos[c + d*x])/(a + b*Sin
[c + d*x]))/(a^2*d)
```

Maple [A] time = 0.141, size = 153, normalized size = 1.7

$$2\,\frac{\tan(1/2\,dx+c/2)\,b}{da^2\left((\tan(1/2\,dx+c/2))^2\,a+2\,\tan(1/2\,dx+c/2)\,b+a\right)}+2\,\frac{1}{da\left((\tan(1/2\,dx+c/2))^2\,a+2\,\tan(1/2\,dx+c/2)\,b+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c
)*b+2/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-2/d/a^2*b/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a^2
*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02589, size = 1111, normalized size = 12.08

$$\left[-\frac{\left(b^2\sin(dx+c)+ab\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2}{2\left(\left(a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*sin(d*x + c) + a*b)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c)
```

```
+ b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2)) - 2*(a^3 - a*b^2)*cos(d*x + c) + (a^3 - a*b^2 + (a^2*b - b^
3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^3 - a*b^2 + (a^2*b - b^3)
*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^4*b - a^2*b^3)*d*sin(d*x +
 c) + (a^5 - a^3*b^2)*d), 1/2*(2*(b^2*sin(d*x + c) + a*b)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*(a^3 - a*b^
2)*cos(d*x + c) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c))*log(1/2*cos(d*
x + c) + 1/2) + (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c))*log(-1/2*cos(d*x
 + c) + 1/2))/((a^4*b - a^2*b^3)*d*sin(d*x + c) + (a^5 - a^3*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.22913, size = 176, normalized size = 1.91

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}a^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a^2) - log(abs(tan(1/2*d*x + 1
/2*c)))/a^2 - 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 + 2
*b*tan(1/2*d*x + 1/2*c) + a)*a^2))/d
```

3.1082 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*Sq
rt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(
a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.435172, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*Sq
rt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(
a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(-2b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int \csc(c+dx)\,dx}{a^3} - \frac{\left(a^2-2b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,d\ldots}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{\left(2\left(a^2-2b^2\right)\right)\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\left(4\left(a^2-2b^2\right)\right)\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(\ldots}{ad(a+b\ldots}$$

Mathematica [A] time = 0.726565, size = 139, normalized size = 1.21

$$-\frac{\frac{4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2ab\cos(c+dx)}{a+b\sin(c+dx)} - a\tan\left(\frac{1}{2}(c+dx)\right) + a\cot\left(\frac{1}{2}(c+dx)\right) + 4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 4b\log\left(\text{c}\ldots\right.}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] + a*Cot[(c + d*x)/2] - 4*b*Log[Cos[(c + d*x)/2]] + 4*b*Log[Sin[(c
 + d*x)/2]] + (2*a*b*Cos[c + d*x])/(a + b*Sin[c + d*x]) - a*Tan[(c + d*x)/2
])/(2*a^3*d)
```

Maple [B] time = 0.166, size = 245, normalized size = 2.1

$$\frac{1}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{b^2\tan(1/2\,dx+c/2)}{da^3\left((\tan(1/2\,dx+c/2))^2\,a+2\,\tan(1/2\,dx+c/2)\,b+a\right)}-2\,\frac{b}{da^2\left((\tan(1/2\,dx+c/2))^2\,a+2\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)*b-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x
+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^3*b*ln
(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29363, size = 1681, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(4*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*b^3 - (a^2*b
 - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*l
og(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^4 - a^2*b^2)*cos(d*x + c) -
2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x
```

```
 + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos
(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(
(a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^2)*d*sin(d*x + c) - (a^5*
b - a^3*b^3)*d), (2*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*
b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (a
^4 - a^2*b^2)*cos(d*x + c) - (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^
2 + (a^3*b - a*b^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^2*b^2 -
b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2))/((a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^
2)*d*sin(d*x + c) - (a^5*b - a^3*b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.24506, size = 294, normalized size = 2.56

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)(a^2-2\,b^2)}{\sqrt{a^2-b^2}\,a^3}-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\cdots\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\,\mathrm{ta}\cdots\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
+ 12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^3) - (4*a*b*tan
(1/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*b^2*tan(1/2*d*x + 1/
2*c)^2 - 14*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/((a*tan(1/2*d*x + 1/2*c)^3 +
2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3))/d
```

3.1083 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$\frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}+\frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{3b\cot(c+dx)}{a^3d}-\frac{3\cot(c+dx)\csc(c+dx)}{2a^2d}$$

```
[Out] (2*b*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*Sqrt[a^2 - b^2]*d) + ((a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + (3*
b*Cot[c + d*x])/(a^3*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + (Cot[c
+ d*x]*Csc[c + d*x])/(a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.767619, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}+\frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{3b\cot(c+dx)}{a^3d}-\frac{3\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (2*b*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*Sqrt[a^2 - b^2]*d) + ((a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + (3*
b*Cot[c + d*x])/(a^3*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + (Cot[c
+ d*x]*Csc[c + d*x])/(a*d*(a + b*Sin[c + d*x]))
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)\left(3(a^2-b^2)-2(a^2-b^2)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(-6b(a^2-b^2)-a(a^2-b^2\ldots}{a+b\sin\ldots}}{2a^2\left(\ldots\right.}$$

$$= \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)(-\ldots}{}}{}$$

$$= \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} - \frac{\left(a^2-6b^2\right)\int\ldots}{\ldots}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot\ldots}{ad\ldots}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot\ldots}{ad\ldots}$$

$$= \frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}\,d} + \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{3b\cot(c\ldots}{a^3d\ldots}$$

Mathematica [A] time = 3.09739, size = 196, normalized size = 1.25

$$\frac{-\frac{16b\left(3b^2-2a^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 4\left(a^2-6b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4\left(a^2-6b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - a^2\csc^2\left(\frac{1}{2}(\ldots\right.}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-16*b*(-2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] + 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 4*(a^2
- 6*b^2)*Log[Cos[(c + d*x)/2]] - 4*(a^2 - 6*b^2)*Log[Sin[(c + d*x)/2]] + a^
2*Sec[(c + d*x)/2]^2 + (8*a*b^2*Cos[c + d*x])/(a + b*Sin[c + d*x]) - 8*a*b*
Tan[(c + d*x)/2])/(8*a^4*d)
```

Maple [B] time = 0.19, size = 307, normalized size = 2.

$$\frac{1}{8da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{b}{da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\frac{b^3\tan(1/2\,dx+c/2)}{da^4\left(\left(\tan(1/2\,dx+c/2)\right)^2a+2\tan(1/2\,dx+c/2)\,b+a\right)} + 2\frac{}{da^3\left((\mathrm{ta}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3*tan(1/2*d*x+1/2*c)*b+2/d*b^3/a^4/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*b^2/a
```

```
^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+4/d/a^2*b/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d*b^3/a^4/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/8/
d/a^2/tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*ln(tan(1/2*d*x+1/2*c))+3/d/a^4*ln(tan(
1/2*d*x+1/2*c))*b^2+1/d*b/a^3/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.37467, size = 2500, normalized size = 15.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/4*(12*(a^3*b^2 - a*b^4)*cos(d*x + c)^3 - 6*(a^4*b - a^2*b^3)*cos(d*x + c
)*sin(d*x + c) - 2*(2*a^3*b - 3*a*b^3 - (2*a^3*b - 3*a*b^3)*cos(d*x + c)^2
+ (2*a^2*b^2 - 3*b^4 - (2*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sq
rt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2)
)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^5 - 7*a^3*b
^2 + 6*a*b^4)*cos(d*x + c) - (a^5 - 7*a^3*b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2
+ 6*a*b^4)*cos(d*x + c)^2 + (a^4*b - 7*a^2*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3
 + 6*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^5
- 7*a^3*b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2 + (a^4*b
 - 7*a^2*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - a^5*b^2)*d*cos(d*x + c)^2 - (a^
7 - a^5*b^2)*d + ((a^6*b - a^4*b^3)*d*cos(d*x + c)^2 - (a^6*b - a^4*b^3)*d)
*sin(d*x + c)), 1/4*(12*(a^3*b^2 - a*b^4)*cos(d*x + c)^3 - 6*(a^4*b - a^2*b
^3)*cos(d*x + c)*sin(d*x + c) + 4*(2*a^3*b - 3*a*b^3 - (2*a^3*b - 3*a*b^3)*
cos(d*x + c)^2 + (2*a^2*b^2 - 3*b^4 - (2*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*s
in(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*
cos(d*x + c))) + 2*(a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c) - (a^5 - 7*a^3*
b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2 + (a^4*b - 7*a^2
*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2)*sin(d*x + c))*lo
g(1/2*cos(d*x + c) + 1/2) + (a^5 - 7*a^3*b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2 +
 6*a*b^4)*cos(d*x + c)^2 + (a^4*b - 7*a^2*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3
+ 6*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7
- a^5*b^2)*d*cos(d*x + c)^2 - (a^7 - a^5*b^2)*d + ((a^6*b - a^4*b^3)*d*cos(
d*x + c)^2 - (a^6*b - a^4*b^3)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.33662, size = 347, normalized size = 2.21

$$-\frac{\frac{4\left(a^2-6b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}-\frac{16\left(2a^2b-3b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}-\cdots}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*(a^2 - 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 16*(2*a^2*b - 3*
b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (a^2*tan(1/2*d*x + 1/2*c
)^2 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 - 16*(b^3*tan(1/2*d*x + 1/2*c) + a*b^
2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^4) - (6*a^2
*tan(1/2*d*x + 1/2*c)^2 - 36*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x
 + 1/2*c) - a^2)/(a^4*tan(1/2*d*x + 1/2*c)^2))/d
```

3.1084 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=193

$$-\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d}-\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{2b\cot(c+dx)}{a^3d}$$

```
[Out] (-2*b^2*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^5*Sqrt[a^2 - b^2]*d) - (b*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) +
((a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) + (2*b*Cot[c + d*x]*Csc[c + d*x])/(
a^3*d) - (4*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) + (Cot[c + d*x]*Csc[c +
d*x]^2)/(a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.03841, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d}-\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{2b\cot(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^5*Sqrt[a^2 - b^2]*d) - (b*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) +
((a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) + (2*b*Cot[c + d*x]*Csc[c + d*x])/(
a^3*d) - (4*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) + (Cot[c + d*x]*Csc[c +
d*x]^2)/(a*d*(a + b*Sin[c + d*x]))
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^4(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)\csc^2(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^4(c+dx)\left(4\left(a^2-b^2\right)-3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d} + \frac{\cot(c+dx)\csc^2(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)\left(-12b\left(a^2-b^2\right)-a\left(a^2-b^2\right.\right.}{a+b\,\mathrm{si}}}{3a^2\left(\right.}$$

$$= \frac{2b\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d} + \frac{\cot(c+dx)\csc^2(c+dx)}{ad(a+b\sin(c+dx))} +$$

$$= \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d} +$$

$$= \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d} +$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)\csc(c+}{a^3d}$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)\csc(c+}{a^3d}$$

$$= -\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}d} - \frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(a^2-1\right.}{}$$

Mathematica [A] time = 6.34173, size = 385, normalized size = 1.99

$$\frac{\left(a^2b-4b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{\left(4b^3-a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} - \frac{b^3\cos(c+dx)}{a^4d(a+b\sin(c+dx))} + \frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(a^2\cos\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(3*a^2 - 4*b^2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Si
n[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((a^2*Cos[(c +
 d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*Csc[(c
+ d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^2*d) +
 ((-(a^2*b) + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((a^2*b - 4*b^3)*Log[
Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (Sec[(c + d
*x)/2]*(-(a^2*Sin[(c + d*x)/2]) + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*d) - (b^3
*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(24*a^2*d)
```

Maple [B] time = 0.176, size = 390, normalized size = 2.

$$\frac{1}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{b}{4\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{1}{8\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{3\,b^2}{2\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 2\,\frac{b^4}{da^5\left((\tan(1/2\,dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*tan(1/2*d*x+1/2*c)^2*b-1/8/d/a^2*
tan(1/2*d*x+1/2*c)+3/2/d/a^4*b^2*tan(1/2*d*x+1/2*c)-2/d*b^4/a^5/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d*b^3/a^4/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-6/d/a^3/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+8/d*b^4/a^5/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/24/d/a^2
/tan(1/2*d*x+1/2*c)^3+1/8/d/a^2/tan(1/2*d*x+1/2*c)-3/2/d/a^4/tan(1/2*d*x+1/
2*c)*b^2+1/4/d/a^3*b/tan(1/2*d*x+1/2*c)^2+1/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-
4/d/a^5*b^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.83606, size = 3228, normalized size = 16.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/6*(2*(a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 - 3*(3*a^2*b^3 - 4*b^
5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - 2*(3*a^2*b^3 - 4*b^5)*cos(d*x + c)
^2 + (3*a^3*b^2 - 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2
+ b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 12*(a^4*b^
2 - a^2*b^4)*cos(d*x + c) + 3*(a^4*b^2 - 5*a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a
^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x +
c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x
 + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^4*b^2 - 5*a^2*b^4
 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5*a^
2*b^4 + 4*b^6)*cos(d*x + c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a
^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2
) + 2*((a^5*b - 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3*b^
3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - a^5*b^3)*d*cos(d*x + c)^
4 - 2*(a^7*b - a^5*b^3)*d*cos(d*x + c)^2 + (a^7*b - a^5*b^3)*d - ((a^8 - a^
6*b^2)*d*cos(d*x + c)^2 - (a^8 - a^6*b^2)*d)*sin(d*x + c)), -1/6*(2*(a^6 -
7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 - 6*(3*a^2*b^3 - 4*b^5 + (3*a^2*b^3 -
 4*b^5)*cos(d*x + c)^4 - 2*(3*a^2*b^3 - 4*b^5)*cos(d*x + c)^2 + (3*a^3*b^2
- 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 -
```

```
b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 12*(a^4
*b^2 - a^2*b^4)*cos(d*x + c) + 3*(a^4*b^2 - 5*a^2*b^4 + 4*b^6 + (a^4*b^2 -
5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x
 + c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(
d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^4*b^2 - 5*a^2*
b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5
*a^2*b^4 + 4*b^6)*cos(d*x + c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b -
5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) +
1/2) + 2*((a^5*b - 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3
*b^3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - a^5*b^3)*d*cos(d*x +
c)^4 - 2*(a^7*b - a^5*b^3)*d*cos(d*x + c)^2 + (a^7*b - a^5*b^3)*d - ((a^8 -
 a^6*b^2)*d*cos(d*x + c)^2 - (a^8 - a^6*b^2)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38767, size = 444, normalized size = 2.3

$$\frac{\frac{24\left(a^2b-4b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5}-\frac{48\left(3a^2b^2-4b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^5}+\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^4\tan\cdots}{a^6}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 48*(3*a^2*b^2
 - 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/2*d*x +
1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 3*a^4*tan(1/2*d*x + 1/2*c) + 36
*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 - 48*(b^4*tan(1/2*d*x + 1/2*c) + a*b^3)/
((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (44*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 176*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*
x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*
c) + a^3)/(a^5*tan(1/2*d*x + 1/2*c)^3))/d
```

3.1085 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=266

$$-\frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4d\left(a^2-b^2\right)}+\frac{a\left(-19a^2b^2+12a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\left(a^2-b^2\right)^{3/2}}-\frac{\left(4a^2-3b^2\right)\sin^2(c+dx)\cos(c+d}{2b^2d\left(a^2-b^2\right)(a+b\sin(c+dx)}$$

```
[Out] -((12*a^2 - b^2)*x)/(2*b^5) + (a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*(a^2 - b^2)^(3/2)*d) - (a*(12*a^
2 - 11*b^2)*Cos[c + d*x])/(2*b^4*(a^2 - b^2)*d) + ((6*a^2 - 5*b^2)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(2
*b*d*(a + b*Sin[c + d*x])^2) - ((4*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^2
)/(2*b^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.877846, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4d\left(a^2-b^2\right)}+\frac{a\left(-19a^2b^2+12a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\left(a^2-b^2\right)^{3/2}}-\frac{\left(4a^2-3b^2\right)\sin^2(c+dx)\cos(c+d}{2b^2d\left(a^2-b^2\right)(a+b\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((12*a^2 - b^2)*x)/(2*b^5) + (a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*(a^2 - b^2)^(3/2)*d) - (a*(12*a^
2 - 11*b^2)*Cos[c + d*x])/(2*b^4*(a^2 - b^2)*d) + ((6*a^2 - 5*b^2)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(2
*b*d*(a + b*Sin[c + d*x])^2) - ((4*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^2
)/(2*b^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\sin^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\int \frac{\sin^2(c+dx)\left(-3\left(a^2-b^2\right)+4\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\left(4a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\sin(c+dx)\left(2\left(4a^4 \ldots\right.\right.}{\ldots}}{\ldots}$$

$$= \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\cos(c+dx)\sin^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\left(4a^2-3b^2\right)\cos(c \ldots}{2b^2\left(a^2-b^2\right)d(a \ldots}$$

$$= -\frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\cos(c+dx)\,\mathrm{s}\ldots}{2bd(a+b\sin \ldots}$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} - \frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} \ldots$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} - \frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} \ldots$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} - \frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} \ldots$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} + \frac{a\left(12a^4-19a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\left(a^2-b^2\right)^{3/2}d} - \frac{a\left(12a^2-11b^2\right) \ldots}{2b^4\left(a^2 - \ldots\right.}$$

Mathematica [A] time = 5.70776, size = 288, normalized size = 1.08

$$\frac{\frac{4a\left(-19a^2b^2+12a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{-2b^2\left(-13a^2b^2+12a^4+b^4\right)(c+dx)\sin^2(c+dx)+a^2\left(-\left(2\left(-13a^2b^2+12a^4+b^4\right)(c+dx)+\left(18a^2b^2-17b^4\right)\sin(2(\ldots\right.\right.}{\ldots}}{4b^5d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((4*a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] + (-4*a*b*(12*a^4 - 13*a^2*b^2 + b^4)*(c + d*x)*
Sin[c + d*x] - 2*b^2*(12*a^4 - 13*a^2*b^2 + b^4)*(c + d*x)*Sin[c + d*x]^2 +
 Cos[c + d*x]*(-24*a^5*b + 22*a^3*b^3 - 8*a*b^3*(a^2 - b^2)*Sin[c + d*x]^2
+ 2*b^4*(a^2 - b^2)*Sin[c + d*x]^3) - a^2*(2*(12*a^4 - 13*a^2*b^2 + b^4)*(c
 + d*x) + (18*a^2*b^2 - 17*b^4)*Sin[2*(c + d*x)]))/(a + b*Sin[c + d*x])^2)/
(4*(a - b)*b^5*(a + b)*d)
```

Maple [B] time = 0.149, size = 845, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*a-12/d/b^5*arctan(tan
(1/2*d*x+1/2*c))*a^2+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))-5/d*a^4/b^3/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3
+4/d*a^2/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*ta
n(1/2*d*x+1/2*c)^3-6/d*a^5/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-7/d*a^3/b^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2+10/d*a/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-1
9/d*a^4/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*t
an(1/2*d*x+1/2*c)+16/d*a^2/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)-6/d*a^5/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)+5/d*a^3/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)+12/d*a^5/b^5/(a^2-b^2)^(3/2)*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-19/d*a^3/b^3/(a^2-b^2)^(3/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d*a/b/(a^2-b^2)^(3
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1118, size = 2309, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(12*a^6*b^2 - 25*a^4*b^4 + 14*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 +
8*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^3 - 2*(12*a^8 - 13*a^6*b^2 - 1
1*a^4*b^4 + 13*a^2*b^6 - b^8)*d*x + (12*a^7 - 7*a^5*b^2 - 13*a^3*b^4 + 6*a*
b^6 - (12*a^5*b^2 - 19*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(12*a^6*b - 19
*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x +
 c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2)) - 2*(12*a^7*b - 19*a^5*b^3 + 3*a^3*b^5 + 4*a*b^7)*cos(d
*x + c) - 2*((a^4*b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^3 + 2*(12*a^7*b - 25*
a^5*b^3 + 14*a^3*b^5 - a*b^7)*d*x + (18*a^6*b^2 - 36*a^4*b^4 + 19*a^2*b^6 -
 b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^7 - 2*a^2*b^9 + b^11)*d*cos(d*x +
```

```
c)^2 - 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d*sin(d*x + c) - (a^6*b^5 - a^4*b^
7 - a^2*b^9 + b^11)*d), -1/2*((12*a^6*b^2 - 25*a^4*b^4 + 14*a^2*b^6 - b^8)*
d*x*cos(d*x + c)^2 + 4*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^3 - (12*a
^8 - 13*a^6*b^2 - 11*a^4*b^4 + 13*a^2*b^6 - b^8)*d*x - (12*a^7 - 7*a^5*b^2
- 13*a^3*b^4 + 6*a*b^6 - (12*a^5*b^2 - 19*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2
 + 2*(12*a^6*b - 19*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arct
an(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (12*a^7*b - 19*a
^5*b^3 + 3*a^3*b^5 + 4*a*b^7)*cos(d*x + c) - ((a^4*b^4 - 2*a^2*b^6 + b^8)*c
os(d*x + c)^3 + 2*(12*a^7*b - 25*a^5*b^3 + 14*a^3*b^5 - a*b^7)*d*x + (18*a^
6*b^2 - 36*a^4*b^4 + 19*a^2*b^6 - b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^
7 - 2*a^2*b^9 + b^11)*d*cos(d*x + c)^2 - 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d
*sin(d*x + c) - (a^6*b^5 - a^4*b^7 - a^2*b^9 + b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36636, size = 722, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(12*a^5 - 19*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^2*b^5 - b^
7)*sqrt(a^2 - b^2)) - 2*(6*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 5*a^2*b^3*tan(1/2
*d*x + 1/2*c)^7 + 12*a^5*tan(1/2*d*x + 1/2*c)^6 + 5*a^3*b^2*tan(1/2*d*x + 1
/2*c)^6 - 14*a*b^4*tan(1/2*d*x + 1/2*c)^6 + 54*a^4*b*tan(1/2*d*x + 1/2*c)^5
 - 45*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 4*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^
5*tan(1/2*d*x + 1/2*c)^4 + 15*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 44*a*b^4*tan
(1/2*d*x + 1/2*c)^4 + 90*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 87*a^2*b^3*tan(1/2*
d*x + 1/2*c)^3 + 4*b^5*tan(1/2*d*x + 1/2*c)^3 + 36*a^5*tan(1/2*d*x + 1/2*c)
^2 - a^3*b^2*tan(1/2*d*x + 1/2*c)^2 - 30*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 42*
a^4*b*tan(1/2*d*x + 1/2*c) - 39*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a^5 - 11*
a^3*b^2)/((a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2
*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - (12
*a^2 - b^2)*(d*x + c)/b^5)/d
```

3.1086 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(-9a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\left(a^2-b^2\right)^{3/2}}+\frac{a\left(3a^2-2b^2\right)\cos(c+dx)}{2b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3ax}{b^4}-\frac{\sin^2(c+dx)\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+$$

```
[Out] (3*a*x)/b^4 - ((6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(3/2)*d) + (3*Cos[c + d*x])/(2*b^3*d) -
(Cos[c + d*x]*Sin[c + d*x]^2)/(2*b*d*(a + b*Sin[c + d*x])^2) + (a*(3*a^2 -
2*b^2)*Cos[c + d*x])/(2*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.562332, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3048, 3032, 3023, 2735, 2660, 618, 204}

$$-\frac{\left(-9a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\left(a^2-b^2\right)^{3/2}}+\frac{a\left(3a^2-2b^2\right)\cos(c+dx)}{2b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3ax}{b^4}-\frac{\sin^2(c+dx)\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*x)/b^4 - ((6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(3/2)*d) + (3*Cos[c + d*x])/(2*b^3*d) -
(Cos[c + d*x]*Sin[c + d*x]^2)/(2*b*d*(a + b*Sin[c + d*x])^2) + (a*(3*a^2 -
2*b^2)*Cos[c + d*x])/(2*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
```

```
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\sin^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\int \frac{\sin(c+dx)\left(-2\left(a^2-b^2\right)+3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{b\left(3a^4-5a^2b^2+2b^4\right)+3a(a}{}}{2}$$

$$= \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{b^2(3a}{}}{}$$

$$= \frac{3ax}{b^4} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} -$$

$$= \frac{3ax}{b^4} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} -$$

$$= \frac{3ax}{b^4} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\left(3a^2-2b^2\right)\cos(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} +$$

$$= \frac{3ax}{b^4} - \frac{\left(6a^4-9a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}d} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(}{2bd(a+b\sin(c}$$

Mathematica [A] time = 1.13289, size = 159, normalized size = 0.88

$$\frac{-\frac{2\left(-9a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{ab\left(5a^2-4b^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2} + 6a(c+dx) + 2b\cos(c+dx)}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (6*a*(c + d*x) - (2*(6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 2*b*Cos[c + d*x] - (a^2*b*Cos[c
+ d*x])/(a + b*Sin[c + d*x])^2 + (a*b*(5*a^2 - 4*b^2)*Cos[c + d*x])/((a - b
)*(a + b)*(a + b*Sin[c + d*x]))))/(2*b^4*d)
```

Maple [B] time = 0.139, size = 711, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)+6/d/b^4*a*arctan(tan(1/2*d*x+1/2*c))+3/d/b
^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^3/(a^2-b^2)*tan(1/
```

```
2*d*x+1/2*c)^3-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a/(a
^2-b^2)*tan(1/2*d*x+1/2*c)^3+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2*a^4+5/d/b/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2*a^2-6/d*b/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2
*c)^2+13/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^3/(a^2
-b^2)*tan(1/2*d*x+1/2*c)-10/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2*a^4/(a^2-b^2)-3/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2*a^2/(a^2-b^2)-6/d/b^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^4+9/d/b^2/(a^2-b^2)^(3/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2-2/d/(a^2-b^2)^(3/2)*a
rctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90576, size = 1975, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*x*cos(d*x + c)^2 + 4*(a^4*b^3 - 2*
a^2*b^5 + b^7)*cos(d*x + c)^3 - 12*(a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*x -
(6*a^6 - 3*a^4*b^2 - 7*a^2*b^4 + 2*b^6 - (6*a^4*b^2 - 9*a^2*b^4 + 2*b^6)*co
s(d*x + c)^2 + 2*(6*a^5*b - 9*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 +
b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2
*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(6*a^6*b - 9*a^4*b^3 + a^
2*b^5 + 2*b^7)*cos(d*x + c) - 2*(12*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*x + (9*
a^5*b^2 - 17*a^3*b^4 + 8*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a
^2*b^8 + b^10)*d*cos(d*x + c)^2 - 2*(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d*sin(d*x
 + c) - (a^6*b^4 - a^4*b^6 - a^2*b^8 + b^10)*d), 1/2*(6*(a^5*b^2 - 2*a^3*b^
4 + a*b^6)*d*x*cos(d*x + c)^2 + 2*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^
3 - 6*(a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*x - (6*a^6 - 3*a^4*b^2 - 7*a^2*b^
4 + 2*b^6 - (6*a^4*b^2 - 9*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(6*a^5*b - 9
*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
 b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (6*a^6*b - 9*a^4*b^3 + a^2*b^5 + 2*b^
7)*cos(d*x + c) - (12*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*x + (9*a^5*b^2 - 17*a
^3*b^4 + 8*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)
*d*cos(d*x + c)^2 - 2*(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d*sin(d*x + c) - (a^6*b
^4 - a^4*b^6 - a^2*b^8 + b^10)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38279, size = 408, normalized size = 2.27

$$-\frac{\frac{\left(6a^4-9a^2b^2+2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{3a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+5a^2b^2\tan\left(\frac{1}{2}dx+\cdots\right.}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\cdots\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((6*a^4 - 9*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a
^2 - b^2)) - (3*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)
^3 + 4*a^4*tan(1/2*d*x + 1/2*c)^2 + 5*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*b^
4*tan(1/2*d*x + 1/2*c)^2 + 13*a^3*b*tan(1/2*d*x + 1/2*c) - 10*a*b^3*tan(1/2
*d*x + 1/2*c) + 4*a^4 - 3*a^2*b^2)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)
^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - 3*(d*x + c)*a/b^4 - 2/((tan(1/2*d*x
 + 1/2*c)^2 + 1)*b^3))/d
```

3.1087 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=167

$$\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\left(a^2-b^2\right)^{3/2}}-\frac{a\cos^3(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+dx)\right)}{2b^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-$$

```
[Out] -(x/b^3) + (a*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(b^3*(a^2 - b^2)^(3/2)*d) - (a*Cos[c + d*x]^3)/(2*(a^2 - b^2)*d*(a + b
*Sin[c + d*x])^2) - (Cos[c + d*x]*(2*(a^2 - b^2) + a*b*Sin[c + d*x]))/(2*b^
2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.27907, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2864, 2863, 2735, 2660, 618, 204}

$$\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\left(a^2-b^2\right)^{3/2}}-\frac{a\cos^3(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+dx)\right)}{2b^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(x/b^3) + (a*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(b^3*(a^2 - b^2)^(3/2)*d) - (a*Cos[c + d*x]^3)/(2*(a^2 - b^2)*d*(a + b
*Sin[c + d*x])^2) - (Cos[c + d*x]*(2*(a^2 - b^2) + a*b*Sin[c + d*x]))/(2*b^
2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= -\frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}-\frac{\int \frac{\cos^2(c+dx)(2b+a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}\\
&= -\frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+dx)\right)}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx))}+\frac{\int \frac{-ab-\cdots}{\cdots}}{2\cdots}\\
&= -\frac{x}{b^3}-\frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+dx)\right)}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx))}+\cdots\\
&= -\frac{x}{b^3}-\frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+dx)\right)}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx))}+\cdots\\
&= -\frac{x}{b^3}-\frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)+ab\sin(c+dx)\right)}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx))}-\cdots\\
&= -\frac{x}{b^3}+\frac{a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}d}-\frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}-\frac{\cos(c\cdots}{2\cdots}
\end{aligned}
$$

Mathematica [A] time = 1.89661, size = 289, normalized size = 1.73

$$
\frac{\frac{\frac{2a\left(-20a^2b^2+8a^4+15b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{3b\left(-7a^2b^2+4a^4+2b^4\right)\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{ab\left(4a^2-3b^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}-8(c+dx)}{b^3}-\frac{\frac{6ab\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\cos(c+dx)\left(b\left(a^2+2\cdots\right.\right.}{(a+b\cdots}}{(a-b)^2(a+b)^2}\cdots}{8d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*Cos[c +
 d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2*b^2 +
 2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))))/b^3 - ((6
*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (C
os[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x]))/(a + b*Sin[c
+ d*x])^2)/((a - b)^2*(a + b)^2))/(8*d)
```

Maple [B] time = 0.132, size = 576, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -2/d/b^3*arctan(tan(1/2*d*x+1/2*c))-1/d*a^2/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-2/d*a^3/b^2/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-3
/d*a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*
d*x+1/2*c)^2+2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/
(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-7/d*a^2/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)+4/d*b/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)-2/d*a^3/b^2/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)+1/d/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a/(a^2-b^2)+2/d*a^3/b^3/(a^2-b^2)^(3
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/d*a/b/(a^2-b
^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.68554, size = 1702, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 - 4*(a^6 - a^4*b^2
- a^2*b^4 + b^6)*d*x - (2*a^5 - a^3*b^2 - 3*a*b^4 - (2*a^3*b^2 - 3*a*b^4)*c
os(d*x + c)^2 + 2*(2*a^4*b - 3*a^2*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
```

```
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(
d*x + c) - 2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x + (3*a^4*b^2 - 5*a^2*b^4 +
2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x +
 c)^2 - 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*sin(d*x + c) - (a^6*b^3 - a^4*b^5
 - a^2*b^7 + b^9)*d), -1/2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x*cos(d*x + c)^
2 - 2*(a^6 - a^4*b^2 - a^2*b^4 + b^6)*d*x - (2*a^5 - a^3*b^2 - 3*a*b^4 - (2
*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(2*a^4*b - 3*a^2*b^3)*sin(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))
) - (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c) - (4*(a^5*b - 2*a^3*b^3 + a*
b^5)*d*x + (3*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^
4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2 - 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)
*d*sin(d*x + c) - (a^6*b^3 - a^4*b^5 - a^2*b^7 + b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28848, size = 346, normalized size = 2.07

$$\frac{\frac{\left(2a^3-3ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}-\frac{a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+7a^3b}{\left(a^3b^2-ab^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^3 - 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2))
- (a^3*b*tan(1/2*d*x + 1/2*c)^3 + 2*a^4*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^2*
tan(1/2*d*x + 1/2*c)^2 - 2*b^4*tan(1/2*d*x + 1/2*c)^2 + 7*a^3*b*tan(1/2*d*x
 + 1/2*c) - 4*a*b^3*tan(1/2*d*x + 1/2*c) + 2*a^4 - a^2*b^2)/((a^3*b^2 - a*b
^4)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - (d*x + c
)/b^3)/d
```

3.1088 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=154

$$-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}+\frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

```
[Out] -((b*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3
*(a^2 - b^2)^(3/2)*d)) - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(2*a*
d*(a + b*Sin[c + d*x])^2) + ((a^2 - 2*b^2)*Cos[c + d*x])/(2*a^2*(a^2 - b^2)
*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.477451, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2889, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}+\frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((b*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3
*(a^2 - b^2)^(3/2)*d)) - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(2*a*
d*(a + b*Sin[c + d*x])^2) + ((a^2 - 2*b^2)*Cos[c + d*x])/(2*a^2*(a^2 - b^2)
*d*(a + b*Sin[c + d*x]))
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc(c+dx)\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(2\left(a^2-b^2\right)^2-ab\left(a^2-b \right.\right.}{a+b\sin(c+dx)}}{2a^2\left(a^2-b^2\right)}$$

$$= \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int\csc(c+dx)\,dx}{a^3} - \left(b\left(3a\right.\right.$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} -$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} +$$

$$= -\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

Mathematica [A] time = 1.08966, size = 154, normalized size = 1.

$$\frac{\frac{2b(2b^2-3a^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\cos(c+dx)\left(b(a^2-2b^2)\sin(c+dx)+2a^3-3ab^2\right)}{(a-b)(a+b)(a+b\sin(c+dx))^2}+2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*b*(-3*a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(3/2) - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + (a*Co
s[c + d*x]*(2*a^3 - 3*a*b^2 + b*(a^2 - 2*b^2)*Sin[c + d*x]))/((a - b)*(a +
b)*(a + b*Sin[c + d*x])^2))/(2*a^3*d)
```

Maple [B] time = 0.18, size = 632, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 3/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b/(a^2-b^2)*tan(1/2
*d*x+1/2*c)^3-4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b
^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+2/d*a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2+1/d*b^2/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-6/d/a^3/(
tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/
2*c)^2*b^4+5/d*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b
^2)*tan(1/2*d*x+1/2*c)-8/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2*b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)+2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2*a/(a^2-b^2)-3/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2*b^2/(a^2-b^2)-3/d/a*b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^3*b^3/(a^2-b^2)^(3/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a^3*ln(tan(1/2*d*x+1/2*c
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.69597, size = 2190, normalized size = 14.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) - (3*a^4*b
 + a^2*b^3 - 2*b^5 - (3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 2*
a*b^4)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) + 2*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c) - 2*(a^6 - a^4*b^2 -
a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a
^3*b^3 + a*b^5)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^6 - a^4*b^
2 - a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b -
 2*a^3*b^3 + a*b^5)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7*b^2 -
 2*a^5*b^4 + a^3*b^6)*d*cos(d*x + c)^2 - 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*
sin(d*x + c) - (a^9 - a^7*b^2 - a^5*b^4 + a^3*b^6)*d), -1/2*((a^5*b - 3*a^3
*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) + (3*a^4*b + a^2*b^3 - 2*b^5 - (3
*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 2*a*b^4)*sin(d*x + c))*sq
rt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))
+ (2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c) - (a^6 - a^4*b^2 - a^2*b^4 +
 b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 +
a*b^5)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^6 - a^4*b^2 - a^2*b^4
 + b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3
+ a*b^5)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7*b^2 - 2*a^5*b^4
+ a^3*b^6)*d*cos(d*x + c)^2 - 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*sin(d*x + c
) - (a^9 - a^7*b^2 - a^5*b^4 + a^3*b^6)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.37014, size = 374, normalized size = 2.43

$$-\frac{\frac{\left(3a^2b-2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}-\frac{3a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\,\mathrm{t}\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((3*a^2*b - 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2))
 - (3*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^4
*tan(1/2*d*x + 1/2*c)^2 + a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*b^4*tan(1/2*d*
```

```
x + 1/2*c)^2 + 5*a^3*b*tan(1/2*d*x + 1/2*c) - 8*a*b^3*tan(1/2*d*x + 1/2*c)
+ 2*a^4 - 3*a^2*b^2)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1
/2*d*x + 1/2*c) + a)^2) - log(abs(tan(1/2*d*x + 1/2*c)))/a^3)/d
```

3.1089 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$-\frac{\left(-9a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{3/2}}-\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3d\left(a^2-b^2\right)}+\frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3b\tanh}{}$$

```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((
5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a
 + b*Sin[c + d*x])^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.794895, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2723, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{\left(-9a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{3/2}}-\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3d\left(a^2-b^2\right)}+\frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3b\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((
5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a
 + b*Sin[c + d*x])^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(5a^4-11a^2b^2+6b^4-ab\left(a^2 \ldots\right.\right.}{a \ldots}}{2a \ldots}$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \ldots$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \ldots$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3 \ldots\right.}{2a^2\left(a^2-b^2\right) \ldots}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3 \ldots\right.}{2a^2\left(a^2-b^2\right) \ldots}$$

$$= -\frac{\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c \ldots}{2a^3\left(a^2-b^2\right) \ldots}$$

Mathematica [A] time = 5.47132, size = 195, normalized size = 0.97

$$\frac{-\frac{2\left(-9a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{ab\left(4b^2-3a^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2} + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) - 6b \ldots}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^2 - b^2)^(3/2) - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]]
- 6*b*Log[Sin[(c + d*x)/2]] - (a^2*b*Cos[c + d*x])/(a + b*Sin[c + d*x])^2 +
 (a*b*(-3*a^2 + 4*b^2)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x]))
 + a*Tan[(c + d*x)/2])/(2*a^4*d)
```

Maple [B] time = 0.197, size = 729, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-5/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+6/d/a^3/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-4/d*b/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2
*c)^2-3/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-
b^2)*tan(1/2*d*x+1/2*c)^2+10/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*b^5/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-11/d/a/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)+14/d/a^3/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x
+1/2*c)-4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b/(a^2-b^2)
+5/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)-
2/d/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)+9/d/a^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))*b^2-6/d/a^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))*b^4-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(1/2*d*x
+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.55402, size = 3044, normalized size = 15.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - 2*(8*a^6*b - 1
7*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 +
12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^
4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)
*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*s
in(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*
sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
2*(2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 6*(2*a^5*b^2 - 4*a
^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b
- a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^5*b^2 - 4*a^3*b^4 + 2*a*b^6
 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3 - a^2*
b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^2
- 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d*co
s(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), -1/2*
((5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - (8*a^6*b - 17*a^4*b^3
+ 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 + 12*a*b^5 -
```

```
2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^4*b^2 - 3*
a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
+ c))) - (2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 3*(2*a^5*b^
2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)
^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(2*a^5*b^2 - 4*a^3*b^4 +
2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3
 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x
+ c)^2 - 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^
6)*d*cos(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.29914, size = 458, normalized size = 2.27

$$-\frac{\frac{2\left(2a^4-9a^2b^2+6b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}+\frac{2\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b^3\tan\left(\right.\right.}{\left(a^6-a^4b^2\right)\left(a\tan\left(\frac{1}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*
sqrt(a^2 - b^2)) + 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^3*tan(1/2*d*x + 1/2
*c)^2 - 10*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1
4*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b - 5*a^2*b^3)/((a^6 - a^4*b^2)*(a*tan
(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/
2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 - (6*b*tan(1/2*d*x + 1/2*c)
 - a)/(a^4*tan(1/2*d*x + 1/2*c)))/d
```

3.1090 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=269

$$\frac{b\left(-19a^2b^2+6a^4+12b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\left(a^2-b^2\right)^{3/2}}+\frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4d\left(a^2-b^2\right)}+\frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d}-$$

```
[Out] (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^5*(a^2 - b^2)^(3/2)*d) + ((a^2 - 12*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^5*d) + (b*(11*a^2 - 12*b^2)*Cot[c + d*x])/(2*a^4*(a^2 - b^2)*d) - ((5*
a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*(a^2 - b^2)*d) + (Cot[c + d*
x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 4*b^2)*Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.15081, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{b\left(-19a^2b^2+6a^4+12b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\left(a^2-b^2\right)^{3/2}}+\frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4d\left(a^2-b^2\right)}+\frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^5*(a^2 - b^2)^(3/2)*d) + ((a^2 - 12*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^5*d) + (b*(11*a^2 - 12*b^2)*Cot[c + d*x])/(2*a^4*(a^2 - b^2)*d) - ((5*
a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*(a^2 - b^2)*d) + (Cot[c + d*
x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 4*b^2)*Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
```

```
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\csc^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^3(c+dx)\left(4\left(a^2-b^2\right)-3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)\left(2\left(5a^4-11\right.\right.}{}}{}$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\cot(c}{2a^2\left(a^2-b^2\right)d(a}$$

$$= \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)\csc}{2ad(a+b\sin(c}$$

$$= \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)\csc}{2ad(a+b\sin(c}$$

$$= \frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} + \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cot(c}{2a^3\left(a^2\right.}$$

$$= \frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} + \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cot(c}{2a^3\left(a^2\right.}$$

$$= \frac{b\left(6a^4-19a^2b^2+12b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{3/2}d} + \frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d}$$

Mathematica [A] time = 6.31739, size = 330, normalized size = 1.23

$$\frac{\left(12b^2-a^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{\left(a^2-12b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{b^2\cos(c+dx)}{2a^3d(a+b\sin(c+dx))^2} + \frac{5a^2b^2\cos(c+dx)}{2a^4d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/
2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2)^(3/2)*d) + (3*
b*Cot[(c + d*x)/2])/(2*a^4*d) - Csc[(c + d*x)/2]^2/(8*a^3*d) + ((a^2 - 12*b
^2)*Log[Cos[(c + d*x)/2]])/(2*a^5*d) + ((-a^2 + 12*b^2)*Log[Sin[(c + d*x)/2
]])/(2*a^5*d) + Sec[(c + d*x)/2]^2/(8*a^3*d) + (b^2*Cos[c + d*x])/(2*a^3*d*
(a + b*Sin[c + d*x])^2) + (5*a^2*b^2*Cos[c + d*x] - 6*b^4*Cos[c + d*x])/(2*
a^4*(a - b)*(a + b)*d*(a + b*Sin[c + d*x])) - (3*b*Tan[(c + d*x)/2])/(2*a^4
*d)
```

Maple [B] time = 0.207, size = 803, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^4*tan(1/2*d*x+1/2*c)*b+7/d/a^2/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)*tan(1/2*d*x+1/
2*c)^3-8/d*b^5/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2
-b^2)*tan(1/2*d*x+1/2*c)^3+6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2+5/d/a^3/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2*b^4-14/d*b^6/a
^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*
x+1/2*c)^2+17/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3
/(a^2-b^2)*tan(1/2*d*x+1/2*c)-20/d*b^5/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)+6/d/a/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^2/(a^2-b^2)-7/d*b^4/a^3/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)+6/d/a*b/(a^2-b^2)^(3/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-19/d/a^3*b^3/(a^2-b^2)^(
3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+12/d*b^5/a^5/
(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/
8/d/a^3/tan(1/2*d*x+1/2*c)^2-1/2/d/a^3*ln(tan(1/2*d*x+1/2*c))+6/d/a^5*ln(ta
n(1/2*d*x+1/2*c))*b^2+3/2/d*b/a^4/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.44917, size = 4301, normalized size = 15.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(17*a^6*b^2 - 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c)^3 - (6*a^6*b -
 13*a^4*b^3 - 7*a^2*b^5 + 12*b^7 + (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7)*cos(d*
x + c)^4 - (6*a^6*b - 7*a^4*b^3 - 26*a^2*b^5 + 24*b^7)*cos(d*x + c)^2 + 2*(
6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6 - (6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*(a^8 - 19*a^6*b^2 + 36*a^4*b^4 - 18*a^2*b^6)*cos(d*x + c) -
(a^8 - 13*a^6*b^2 + 11*a^4*b^4 + 13*a^2*b^6 - 12*b^8 + (a^6*b^2 - 14*a^4*b^
4 + 25*a^2*b^6 - 12*b^8)*cos(d*x + c)^4 - (a^8 - 12*a^6*b^2 - 3*a^4*b^4 + 3
8*a^2*b^6 - 24*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 1
2*a*b^7 - (a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*cos(d*x + c)^2)*sin(
```

```
d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^8 - 13*a^6*b^2 + 11*a^4*b^4 + 13
*a^2*b^6 - 12*b^8 + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*cos(d*x +
c)^4 - (a^8 - 12*a^6*b^2 - 3*a^4*b^4 + 38*a^2*b^6 - 24*b^8)*cos(d*x + c)^2
+ 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7 - (a^7*b - 14*a^5*b^3 + 25*
a^3*b^5 - 12*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1
/2) + 2*((11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7)*cos(d*x + c)^3 - (4*a^7*b + 3
*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 2
*a^7*b^4 + a^5*b^6)*d*cos(d*x + c)^4 - (a^11 - 3*a^7*b^4 + 2*a^5*b^6)*d*cos
(d*x + c)^2 + (a^11 - a^9*b^2 - a^7*b^4 + a^5*b^6)*d - 2*((a^10*b - 2*a^8*b
^3 + a^6*b^5)*d*cos(d*x + c)^2 - (a^10*b - 2*a^8*b^3 + a^6*b^5)*d)*sin(d*x
+ c)), -1/4*(2*(17*a^6*b^2 - 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c)^3 + 2*(6
*a^6*b - 13*a^4*b^3 - 7*a^2*b^5 + 12*b^7 + (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7
)*cos(d*x + c)^4 - (6*a^6*b - 7*a^4*b^3 - 26*a^2*b^5 + 24*b^7)*cos(d*x + c)
^2 + 2*(6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6 - (6*a^5*b^2 - 19*a^3*b^4 + 12*a*
b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c)
+ b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*(a^8 - 19*a^6*b^2 + 36*a^4*b^4 - 1
8*a^2*b^6)*cos(d*x + c) - (a^8 - 13*a^6*b^2 + 11*a^4*b^4 + 13*a^2*b^6 - 12*
b^8 + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*cos(d*x + c)^4 - (a^8 -
12*a^6*b^2 - 3*a^4*b^4 + 38*a^2*b^6 - 24*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 1
4*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7 - (a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a
*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^8 - 13
*a^6*b^2 + 11*a^4*b^4 + 13*a^2*b^6 - 12*b^8 + (a^6*b^2 - 14*a^4*b^4 + 25*a^
2*b^6 - 12*b^8)*cos(d*x + c)^4 - (a^8 - 12*a^6*b^2 - 3*a^4*b^4 + 38*a^2*b^6
 - 24*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7 -
 (a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))
*log(-1/2*cos(d*x + c) + 1/2) + 2*((11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7)*cos
(d*x + c)^3 - (4*a^7*b + 3*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c))*s
in(d*x + c))/((a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d*cos(d*x + c)^4 - (a^11 - 3*
a^7*b^4 + 2*a^5*b^6)*d*cos(d*x + c)^2 + (a^11 - a^9*b^2 - a^7*b^4 + a^5*b^6
)*d - 2*((a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c)^2 - (a^10*b - 2*a^8*
b^3 + a^6*b^5)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+b*sin(d*x+c))**3,x)

[Out] Timed out
```

Giac [B] time = 1.33237, size = 710, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/8*(8*(6*a^4*b - 19*a^2*b^3 + 12*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^7 - a^5*b^
2)*sqrt(a^2 - b^2)) + (2*a^6*tan(1/2*d*x + 1/2*c)^6 - 26*a^4*b^2*tan(1/2*d*
x + 1/2*c)^6 + 24*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 + 20*a^5*b*tan(1/2*d*x + 1
```

/2*c)^5 - 60*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 + 32*a*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*a^6*tan(1/2*d*x + 1/2*c)^4 + 53*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 - 64*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 16*b^6*tan(1/2*d*x + 1/2*c)^4 + 28*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 60*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 112*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 68*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 76*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 8*a^5*b*tan(1/2*d*x + 1/2*c) - 8*a^3*b^3*tan(1/2*d*x + 1/2*c) - a^6 + a^4*b^2)/((a^7 - a^5*b^2)*(a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2) - 4*(a^2 - 12*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6)/d

3.1091 $\int \frac{\cos^2(e+fx)}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=347

$$\frac{4b\cos(e+fx)}{3af\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}-\frac{4\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{d\sin(e+fx)}}\right.\right.}{3a^2\sqrt{d}f\sqrt{a+b}}$$

```
[Out] (2*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(3*a*d*f*(a + b*Sin[e + f*x])^(3/2))
+ (4*b*Cos[e + f*x])/(3*a*(a^2 - b^2)*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin
[e + f*x]]) - (4*b*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e
+ f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt
[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(3*a^3*Sq
rt[a + b]*Sqrt[d]*f) - (4*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 +
 Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(3
*a^2*Sqrt[a + b]*Sqrt[d]*f)
```

Rubi [A] time = 0.774143, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2887, 2800, 2998, 2816, 2994}

$$\frac{4b\cos(e+fx)}{3af\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}-\frac{4\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{d\sin(e+fx)}}\right.\right.}{3a^2\sqrt{d}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (2*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(3*a*d*f*(a + b*Sin[e + f*x])^(3/2))
+ (4*b*Cos[e + f*x])/(3*a*(a^2 - b^2)*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin
[e + f*x]]) - (4*b*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e
+ f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt
[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(3*a^3*Sq
rt[a + b]*Sqrt[d]*f) - (4*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 +
 Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(3
*a^2*Sqrt[a + b]*Sqrt[d]*f)
```

Rule 2887

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -Simp[(g*(g*C
os[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(m + 1))/(a*
d*f*(m + 1)), x] + Dist[(g^2*(2*m + 3))/(2*a*(m + 1)), Int[((g*Cos[e + f*x]
)^(p - 2)*(a + b*Sin[e + f*x])^(m + 1))/Sqrt[d*Sin[e + f*x]], x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p +
 1/2, 0]
```

Rule 2800

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(3/2)), x_Symbol] :> Simp[(2*b*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(b + a
```

```
*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}}+\frac{2\int \frac{1}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{3/2}}\,dx}{3a}$$

$$= \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}}+\frac{4b\cos(e+fx)}{3a\left(a^2-b^2\right)f\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+}}$$

$$= \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}}+\frac{4b\cos(e+fx)}{3a\left(a^2-b^2\right)f\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+}}$$

$$= \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}}+\frac{4b\cos(e+fx)}{3a\left(a^2-b^2\right)f\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+}}$$

Mathematica [B] time = 21.5127, size = 3348, normalized size = 9.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((2*Cos[e + f*x])/(3*a*(a + b*Sin[e
+ f*x])^2) - (4*b^2*Cos[e + f*x])/(3*a^2*(a^2 - b^2)*(a + b*Sin[e + f*x])))
)/(f*Sqrt[d*Sin[e + f*x]]) + (4*Sqrt[a + b*Sin[e + f*x]]*((2*Sqrt[a + b*Sin
[e + f*x]])/(3*a*(a^2 - b^2)*Sqrt[Sin[e + f*x]]) - (4*b*Sqrt[Sin[e + f*x]]*
Sqrt[a + b*Sin[e + f*x]])/(3*a^2*(a^2 - b^2)))*(-2*b*Sin[(e + f*x)/2]^2 - (
2*a*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])
/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*
Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(
e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^
2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan
[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e
+ f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-
b + Sqrt[-a^2 + b^2])])))/(3*a^2*(a^2 - b^2)*f*Sqrt[d*Sin[e + f*x]]*((2*b*C
os[e + f*x]*(-2*b*Sin[(e + f*x)/2]^2 - (2*a*(-(b*EllipticE[ArcSin[Sqrt[(-b
+ Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqr
t[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt
[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2]
)/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2
]))])))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(
a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(3*a^2*(a
^2 - b^2)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (2*Cos[e + f*x]*Sq
rt[a + b*Sin[e + f*x]]*(-2*b*Sin[(e + f*x)/2]^2 - (2*a*(-(b*EllipticE[ArcSi
n[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[
2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*El
lipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 +
 b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(
e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt
[-a^2 + b^2]))])))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e
 + f*x]))/(a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]))
)/(3*a^2*(a^2 - b^2)*Sin[e + f*x]^(3/2)) + (4*Sqrt[a + b*Sin[e + f*x]]*(-2*
b*Cos[(e + f*x)/2]*Sin[(e + f*x)/2] + (a^2*Sec[(e + f*x)/2]^2*(-(b*Elliptic
E[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]
]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2])
 + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt
[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(
a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b
 + Sqrt[-a^2 + b^2]))])))/(2*Sqrt[-a^2 + b^2]*(-b + Sqrt[-a^2 + b^2])*Sqrt[(
a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*((a*Tan[(e + f*x)/2
])/(-b + Sqrt[-a^2 + b^2]))^(3/2)) + (a*((a*b*Cos[e + f*x]*Sec[(e + f*x)/2]
^2)/(a^2 - b^2) + (a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x])*Tan[(e + f*x)/
2])/(a^2 - b^2))*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[
(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-
a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b
^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/
(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*
Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])))/(Sqrt[-a^2 + b^2]*((
a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2))^(3/2)*Sqrt[(a*Tan[(
e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) - (2*a*(-(b*EllipticE[ArcSin[Sqrt[(-
b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*S
qrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]^2)/2 - (a^2*Elli
pticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b
^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2
]^2*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])/(4*(b + Sqrt[-a^2 +
 b^2])*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]) + (a^2*Ellipti
cF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]
]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]^2
*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])/(4*(-b + Sqrt[-a^2 +
 b^2])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) + (a*b*Sec[(e +
```

```
f*x)/2]^2*Tan[(e + f*x)/2]*Sqrt[1 - (-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x
)/2])/(-b + Sqrt[-a^2 + b^2])])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[(-b + Sqrt
[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (-b + Sqrt[-a
^2 + b^2] - a*Tan[(e + f*x)/2])/(2*Sqrt[-a^2 + b^2])]) + (a^2*Sec[(e + f*x)
/2]^2*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e +
 f*x)/2])/(b + Sqrt[-a^2 + b^2]))]])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[(b + S
qrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (b + Sqrt[
-a^2 + b^2] + a*Tan[(e + f*x)/2])/(2*Sqrt[-a^2 + b^2])]*Sqrt[1 - (b + Sqrt[
-a^2 + b^2] + a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2])])))/(Sqrt[-a^2 + b
^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*Sqrt[(a*T
an[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(3*a^2*(a^2 - b^2)*Sqrt[Sin[e
+ f*x]])))
```

Maple [B] time = 0.409, size = 4546, normalized size = 13.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*2^(1/2)/(a^2-b^2)/a^3*(4*(-a^2+b^2)^(1/2)*sin(f*x+e)*cos(f*x+e)*(((-
a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))
/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-
a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2
))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-c
os(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2
+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^3-2*(-a^2+b^2)^(1/2)*sin(f*x+e)*cos
(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2
+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+c
os(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a
^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*
sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/
2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*b-4*sin(f*x+e)*cos(f*
x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^
2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(
f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+
b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin
(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*
((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*b^2+4*sin(f*x+e)*cos(f*x
+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2
)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f
*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b
^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(
f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*(
(b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^4+4*(-a^2+b^2)^(1/2)*sin(f*
x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^
2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(
f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+
b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin
(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*
((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^3-2*(-a^2+b^2)^(1/2)*sin(f
*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b
^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos
(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2
+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*si
n(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)
```

```
*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*b+4*(-a^2+b^2)^(1/2)*co
s(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^
2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+
cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-
a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b
*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1
/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b^2-2*(-a^2+b^2)^(1/2)
*cos(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(
-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+
e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b
+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-a^2+b^2)^(1/2)*sin(f*x+e
)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2
^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^3-4*sin(f*x+e)*(((-
a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))
/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-
a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2
))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-c
os(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2
+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*b^2+4*sin(f*x+e)*(((-a^2+b^2)^(1/
2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))
^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2
)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e
))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+
a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))
/(-a^2+b^2)^(1/2))^(1/2))*b^4-4*cos(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*
sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b
^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*Ellipti
cE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)
^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2
))^(1/2))*a^3*b+4*cos(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos
(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(
f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2
)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*
x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b^
3+4*(-a^2+b^2)^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*
a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b
*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*
sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1
/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b^2-2*(-a^
2+b^2)^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+
(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x
+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(
b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-a^2+b^2)^(1/2)*sin(f*x+
e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*
2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^3-4*(((-a^2+b^2)^(
1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e
))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b
^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x
+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*
a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2
))/(-a^2+b^2)^(1/2))^(1/2))*a^3*b+4*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x
+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/
2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-
a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))
/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2
```

```
))*a*b^3-sin(f*x+e)*cos(f*x+e)*2^(1/2)*a^4-sin(f*x+e)*cos(f*x+e)*2^(1/2)*a^
2*b^2-2*cos(f*x+e)^2*2^(1/2)*a*b^3+4*sin(f*x+e)*2^(1/2)*a^2*b^2-2*cos(f*x+e
)*2^(1/2)*a^3*b+2*2^(1/2)*a^3*b+2*2^(1/2)*a*b^3)*(a+b*sin(f*x+e))^(1/2)/(-b
^2*cos(f*x+e)^2+2*sin(f*x+e)*a*b+a^2+b^2)/(d*sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)}\cos\left(fx+e\right)^2}{b^3d\cos\left(fx+e\right)^4-\left(3a^2b+2b^3\right)d\cos\left(fx+e\right)^2+\left(3a^2b+b^3\right)d-\left(3ab^2d\cos\left(fx+e\right)^2-\left(a^3+3ab^2\right)d\right)\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*cos(f*x + e)^2/(b^3*
d*cos(f*x + e)^4 - (3*a^2*b + 2*b^3)*d*cos(f*x + e)^2 + (3*a^2*b + b^3)*d -
 (3*a*b^2*d*cos(f*x + e)^2 - (a^3 + 3*a*b^2)*d)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e))),
 x)
```

3.1092 $\int \cos^4(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3a\sin(c+dx)\cos(c+dx)}{128d}+\frac{3a}{12}$$

```
[Out] (3*a*x)/128 - (b*Cos[c + d*x]^5)/(5*d) + (2*b*Cos[c + d*x]^7)/(7*d) - (b*Co
s[c + d*x]^9)/(9*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c +
d*x]^3*Sin[c + d*x])/(64*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*C
os[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rubi [A] time = 0.190996, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 270}

$$-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3a\sin(c+dx)\cos(c+dx)}{128d}+\frac{3a}{12}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/128 - (b*Cos[c + d*x]^5)/(5*d) + (2*b*Cos[c + d*x]^7)/(7*d) - (b*Co
s[c + d*x]^9)/(9*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c +
d*x]^3*Sin[c + d*x])/(64*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*C
os[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^4(c+dx)\,dx + b\int \cos^4(c+dx)\sin^5(c+dx)$$

$$= -\frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3a)\int \cos^4(c+dx)\sin^2(c+dx)$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{16}a$$

$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{2b\cos^7(c+dx)}{7d} - \frac{b\cos^9(c+dx)}{9d} + \frac{a\cos^3(c+}{}$$

$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{2b\cos^7(c+dx)}{7d} - \frac{b\cos^9(c+dx)}{9d} + \frac{3a\cos(c+}{}$$

$$= \frac{3ax}{128} - \frac{b\cos^5(c+dx)}{5d} + \frac{2b\cos^7(c+dx)}{7d} - \frac{b\cos^9(c+dx)}{9d} + \frac{3a\,\mathrm{co}}{}$$

Mathematica [A] time = 0.249909, size = 92, normalized size = 0.64

$$\frac{-2520a\sin(4(c+dx)) + 315a\sin(8(c+dx)) + 7560ac + 7560adx - 7560b\cos(c+dx) - 1680b\cos(3(c+dx)) + 1008}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (7560*a*c + 7560*a*d*x - 7560*b*Cos[c + d*x] - 1680*b*Cos[3*(c + d*x)] + 10
08*b*Cos[5*(c + d*x)] + 180*b*Cos[7*(c + d*x)] - 140*b*Cos[9*(c + d*x)] - 2
520*a*Sin[4*(c + d*x)] + 315*a*Sin[8*(c + d*x)])/(322560*d)
```

Maple [A] time = 0.035, size = 124, normalized size = 0.9

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^5}{8} - \frac{\sin(dx+c)(\cos(dx+c))^5}{16} + \frac{\sin(dx+c)}{64}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+b*(-1/9*sin(d*x+c
)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5))
```

Maxima [A] time = 1.00608, size = 96, normalized size = 0.67

$$\frac{315\,(24\,dx + 24\,c + \sin(8\,dx + 8\,c) - 8\,\sin(4\,dx + 4\,c))a - 1024\,\left(35\,\cos(dx + c)^{9} - 90\,\cos(dx + c)^{7} + 63\,\cos(dx + c)^{5}\right)b}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/322560*(315*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a - 1
024*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b)/d
```

Fricas [A] time = 1.51164, size = 270, normalized size = 1.89

$$-\frac{4480\,b\cos(dx + c)^{9} - 11520\,b\cos(dx + c)^{7} + 8064\,b\cos(dx + c)^{5} - 945\,adx - 315\,\left(16\,a\cos(dx + c)^{7} - 24\,a\cos(dx + c)^{5} + 2\,a\cos(dx + c)^{3} + 3\,a\cos(dx + c)\right)\sin(dx + c)}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40320*(4480*b*cos(d*x + c)^9 - 11520*b*cos(d*x + c)^7 + 8064*b*cos(d*x +
 c)^5 - 945*a*d*x - 315*(16*a*cos(d*x + c)^7 - 24*a*cos(d*x + c)^5 + 2*a*co
s(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 24.7718, size = 272, normalized size = 1.9

$$\begin{cases} \frac{3ax\sin^{8}(c+dx)}{128} + \frac{3ax\sin^{6}(c+dx)\cos^{2}(c+dx)}{32} + \frac{9ax\sin^{4}(c+dx)\cos^{4}(c+dx)}{64} + \frac{3ax\sin^{2}(c+dx)\cos^{6}(c+dx)}{32} + \frac{3ax\cos^{8}(c+dx)}{128} + \frac{3a\sin^{7}(c+dx)\cos(c+dx)}{128d} + \dots \\ x\,(a + b\sin(c))\sin^{4}(c)\cos^{4}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**8/128 + 3*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 9*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a*x*sin(c + d*x)**2*cos
(c + d*x)**6/32 + 3*a*x*cos(c + d*x)**8/128 + 3*a*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + 11*a*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*a*sin(c + d
*x)**3*cos(c + d*x)**5/(128*d) - 3*a*sin(c + d*x)*cos(c + d*x)**7/(128*d) -
 b*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b*sin(c + d*x)**2*cos(c + d*x)
**7/(35*d) - 8*b*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(
c)**4*cos(c)**4, True))
```

Giac [A] time = 1.28232, size = 144, normalized size = 1.01

$$\frac{3}{128}\,ax - \frac{b\cos(9\,dx + 9\,c)}{2304\,d} + \frac{b\cos(7\,dx + 7\,c)}{1792\,d} + \frac{b\cos(5\,dx + 5\,c)}{320\,d} - \frac{b\cos(3\,dx + 3\,c)}{192\,d} - \frac{3\,b\cos(dx + c)}{128\,d} + \frac{a\sin(8\,dx + \dots}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*a*x - 1/2304*b*cos(9*d*x + 9*c)/d + 1/1792*b*cos(7*d*x + 7*c)/d + 1/3
20*b*cos(5*d*x + 5*c)/d - 1/192*b*cos(3*d*x + 3*c)/d - 3/128*b*cos(d*x + c)
/d + 1/1024*a*sin(8*d*x + 8*c)/d - 1/128*a*sin(4*d*x + 4*c)/d
```

3.1093 $\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=127

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{64d}+$$

```
[Out] (3*b*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (3*b*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (b*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) -
 (b*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (b*Cos[c + d*x]^5*Sin[c + d*x]^3)
/(8*d)
```

Rubi [A] time = 0.186913, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2565, 14, 2568, 2635, 8}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{64d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (3*b*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (b*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) -
 (b*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (b*Cos[c + d*x]^5*Sin[c + d*x]^3)
/(8*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^3(c+dx)\,dx + b\int \cos^4(c+dx)\sin^4(c+dx)$$

$$= -\frac{b\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3b)\int \cos^4(c+dx)\sin^2(c+dx)$$

$$= -\frac{b\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{b\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{16}b$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{b\cos^3(c+dx)\sin(c+dx)}{64d} -$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3b\cos(c+dx)\sin(c+dx)}{128d} +$$

$$= \frac{3bx}{128} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3b\cos(c+dx)\sin(c+d}{128d}$$

Mathematica [A] time = 0.187602, size = 77, normalized size = 0.61

$$\frac{-1680a\cos(c+dx) - 560a\cos(3(c+dx)) + 112a\cos(5(c+dx)) + 80a\cos(7(c+dx)) - 280b\sin(4(c+dx)) + 35b\sin}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (840*b*d*x - 1680*a*Cos[c + d*x] - 560*a*Cos[3*(c + d*x)] + 112*a*Cos[5*(c
+ d*x)] + 80*a*Cos[7*(c + d*x)] - 280*b*Sin[4*(c + d*x)] + 35*b*Sin[8*(c +
d*x)])/(35840*d)
```

Maple [A] time = 0.032, size = 106, normalized size = 0.8

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7} - \frac{2(\cos(dx+c))^5}{35}\right) + b\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^5}{8} - \frac{\sin(dx+c)(\cos(dx}{16}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+b*(-1/8*sin(d*x+c
)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*
x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

Maxima [A] time = 1.0884, size = 82, normalized size = 0.65

$$\frac{1024 \left(5 \cos (dx+c)^7 - 7 \cos (dx+c)^5\right)a + 35 \,(24\,dx + 24\,c + \sin (8\,dx+8\,c) - 8 \sin (4\,dx+4\,c))b}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/35840*(1024*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a + 35*(24*d*x + 24*c +
 sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*b)/d
```

Fricas [A] time = 1.47906, size = 230, normalized size = 1.81

$$\frac{640\,a \cos (dx+c)^7 - 896\,a \cos (dx+c)^5 + 105\,bdx + 35\left(16\,b \cos (dx+c)^7 - 24\,b \cos (dx+c)^5 + 2\,b \cos (dx+c)^3 + 3\,b\,c\right.}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4480*(640*a*cos(d*x + c)^7 - 896*a*cos(d*x + c)^5 + 105*b*d*x + 35*(16*b*
cos(d*x + c)^7 - 24*b*cos(d*x + c)^5 + 2*b*cos(d*x + c)^3 + 3*b*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [A] time = 15.0204, size = 248, normalized size = 1.95

$$\begin{cases} -\frac{a \sin^2 (c+dx) \cos^5 (c+dx)}{5d} - \frac{2a \cos^7 (c+dx)}{35d} + \frac{3bx \sin^8 (c+dx)}{128} + \frac{3bx \sin^6 (c+dx) \cos^2 (c+dx)}{32} + \frac{9bx \sin^4 (c+dx) \cos^4 (c+dx)}{64} + \frac{3bx \sin^2 (c+dx) \cos^6 (c+}{32} \\ x\,(a + b \sin (c)) \sin^3 (c) \cos^4 (c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*a*cos(c + d*x)**7/(
35*d) + 3*b*x*sin(c + d*x)**8/128 + 3*b*x*sin(c + d*x)**6*cos(c + d*x)**2/3
2 + 9*b*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*b*x*sin(c + d*x)**2*cos(c
+ d*x)**6/32 + 3*b*x*cos(c + d*x)**8/128 + 3*b*sin(c + d*x)**7*cos(c + d*x)
/(128*d) + 11*b*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b*sin(c + d*x)
**3*cos(c + d*x)**5/(128*d) - 3*b*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(
d, 0)), (x*(a + b*sin(c))*sin(c)**3*cos(c)**4, True))
```

Giac [A] time = 1.40348, size = 124, normalized size = 0.98

$$\frac{3}{128}\,bx + \frac{a \cos (7\,dx+7\,c)}{448\,d} + \frac{a \cos (5\,dx+5\,c)}{320\,d} - \frac{a \cos (3\,dx+3\,c)}{64\,d} - \frac{3\,a \cos (dx+c)}{64\,d} + \frac{b \sin (8\,dx+8\,c)}{1024\,d} - \frac{b \sin (4\,dx+}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*b*x + 1/448*a*cos(7*d*x + 7*c)/d + 1/320*a*cos(5*d*x + 5*c)/d - 1/64*
a*cos(3*d*x + 3*c)/d - 3/64*a*cos(d*x + c)/d + 1/1024*b*sin(8*d*x + 8*c)/d
- 1/128*b*sin(4*d*x + 4*c)/d
```

3.1094 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=103

$$-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}+\frac{b\cos^7(c+dx)}{7d}-\frac{b\cos^5(c+}{5d}$$

```
[Out] (a*x)/16 - (b*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]^7)/(7*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Co
s[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.148456, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2568, 2635, 8, 2565, 14}

$$-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}+\frac{b\cos^7(c+dx)}{7d}-\frac{b\cos^5(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (b*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]^7)/(7*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Co
s[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^2(c+dx)\,dx + b\int \cos^4(c+dx)\sin^3(c+dx)$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}a\int \cos^4(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{8}a\int c$$

$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{b\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} + \frac{a}{}$$

$$= \frac{ax}{16} - \frac{b\cos^5(c+dx)}{5d} + \frac{b\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d}$$

Mathematica [A] time = 0.186332, size = 88, normalized size = 0.85

$$\frac{105a\sin(2(c+dx)) - 105a\sin(4(c+dx)) - 35a\sin(6(c+dx)) + 420adx - 315b\cos(c+dx) - 105b\cos(3(c+dx)) + 2}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (420*a*d*x - 315*b*Cos[c + d*x] - 105*b*Cos[3*(c + d*x)] + 21*b*Cos[5*(c +
d*x)] + 15*b*Cos[7*(c + d*x)] + 105*a*Sin[2*(c + d*x)] - 105*a*Sin[4*(c + d
*x)] - 35*a*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.03, size = 88, normalized size = 0.9

$$\frac{1}{d}\left(a\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6} + \frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{dx}{16} + \frac{c}{16}\right) + b\left(-\frac{(\sin(dx+c))^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin
(d*x+c)+1/16*d*x+1/16*c)+b*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^
5))
```

Maxima [A] time = 1.15628, size = 88, normalized size = 0.85

$$\frac{35\left(4\sin(2dx+2c)^3 + 12dx + 12c - 3\sin(4dx+4c)\right)a + 192\left(5\cos(dx+c)^7 - 7\cos(dx+c)^5\right)b}{6720d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6720*(35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a +
192*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b)/d
```

Fricas [A] time = 1.46273, size = 198, normalized size = 1.92

$$\frac{240\,b\cos\left(dx+c\right)^{7}-336\,b\cos\left(dx+c\right)^{5}+105\,adx-35\left(8\,a\cos\left(dx+c\right)^{5}-2\,a\cos\left(dx+c\right)^{3}-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*b*cos(d*x + c)^7 - 336*b*cos(d*x + c)^5 + 105*a*d*x - 35*(8*a*c
os(d*x + c)^5 - 2*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 8.06608, size = 192, normalized size = 1.86

$$\begin{cases}\frac{ax\sin^{6}(c+dx)}{16}+\frac{3ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{ax\cos^{6}(c+dx)}{16}+\frac{a\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{a\sin^{3}(c+dx)\cos^{3}(c+dx)}{6d}-\\x\left(a+b\sin\left(c\right)\right)\sin^{2}\left(c\right)\cos^{4}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b*sin(c + d*x)**2*cos(c + d*x)*
*5/(5*d) - 2*b*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)*
*2*cos(c)**4, True))
```

Giac [A] time = 1.36333, size = 144, normalized size = 1.4

$$\frac{1}{16}\,ax+\frac{b\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{b\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\,b\cos\left(dx+c\right)}{64\,d}-\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+4\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x + 1/448*b*cos(7*d*x + 7*c)/d + 1/320*b*cos(5*d*x + 5*c)/d - 1/64*b
*cos(3*d*x + 3*c)/d - 3/64*b*cos(d*x + c)/d - 1/192*a*sin(6*d*x + 6*c)/d -
1/64*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2*d*x + 2*c)/d
```

3.1095 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b\sin(c+dx)\cos(c+dx)}{16d}+\frac{bx}{16}$$

```
[Out] (b*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(16*d)
+ (b*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (b*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d)
```

Rubi [A] time = 0.110956, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2565, 30, 2568, 2635, 8}

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b\sin(c+dx)\cos(c+dx)}{16d}+\frac{bx}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(16*d)
+ (b*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (b*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos^4(c+dx)\sin(c+dx)\,dx+b\int \cos^4(c+dx)\sin^2(c+dx)\,dx\\ &= -\frac{b\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}b\int \cos^4(c+dx)\,dx-\frac{a\,\mathrm{Subst}\left(\int x^4\right.}{}\\ &= -\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{b\cos^5(c+dx)\sin(c+}{6d}\\ &= -\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin(c+dx)}{16d}+\frac{b\cos^3(c+dx)\sin(c+}{24d}\\ &= \frac{bx}{16}-\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin(c+dx)}{16d}+\frac{b\cos^3(c+dx)\sin(}{24d}\end{aligned}$$

Mathematica [A] time = 0.161803, size = 77, normalized size = 0.89

$$-\frac{120a\cos(c+dx)+60a\cos(3(c+dx))+12a\cos(5(c+dx))-15b\sin(2(c+dx))+15b\sin(4(c+dx))+5b\sin(6(c+dx)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-60*b*d*x + 120*a*Cos[c + d*x] + 60*a*Cos[3*(c + d*x)] + 12*a*Cos[5*(c +
d*x)] - 15*b*Sin[2*(c + d*x)] + 15*b*Sin[4*(c + d*x)] + 5*b*Sin[6*(c + d*x)
])/(960*d)
```

Maple [A] time = 0.027, size = 68, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^5}{5}+b\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6}+\frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{dx}{16}+\frac{c}{16}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/5*a*cos(d*x+c)^5+b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3
+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A] time = 1.10714, size = 70, normalized size = 0.8

$$-\frac{192\,a\cos(dx+c)^5-5\left(4\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\sin(4\,dx+4\,c)\right)b}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/960*(192*a*cos(d*x + c)^5 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*
sin(4*d*x + 4*c))*b)/d
```

Fricas [A] time = 1.37966, size = 163, normalized size = 1.87

$$-\frac{48\,a\cos\left(dx+c\right)^{5}-15\,bdx+5\,\left(8\,b\cos\left(dx+c\right)^{5}-2\,b\cos\left(dx+c\right)^{3}-3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(48*a*cos(d*x + c)^5 - 15*b*d*x + 5*(8*b*cos(d*x + c)^5 - 2*b*cos(d*
x + c)^3 - 3*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 4.68937, size = 167, normalized size = 1.92

$$\begin{cases} -\frac{a\cos^{5}(c+dx)}{5d}+\frac{bx\sin^{6}(c+dx)}{16}+\frac{3bx\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3bx\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{bx\cos^{6}(c+dx)}{16}+\frac{b\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{b\sin^{3} \\ x\left(a+b\sin(c)\right)\sin(c)\cos^{4}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)**5/(5*d) + b*x*sin(c + d*x)**6/16 + 3*b*x*sin(c
+ d*x)**4*cos(c + d*x)**2/16 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b
*x*cos(c + d*x)**6/16 + b*sin(c + d*x)**5*cos(c + d*x)/(16*d) + b*sin(c + d
*x)**3*cos(c + d*x)**3/(6*d) - b*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d,
 0)), (x*(a + b*sin(c))*sin(c)*cos(c)**4, True))
```

Giac [A] time = 1.31486, size = 124, normalized size = 1.43

$$\frac{1}{16}\,bx-\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}-\frac{b\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{b\sin\left(2\,dx+\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*b*x - 1/80*a*cos(5*d*x + 5*c)/d - 1/16*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d - 1/192*b*sin(6*d*x + 6*c)/d - 1/64*b*sin(4*d*x + 4*c)/d + 1/6
4*b*sin(2*d*x + 2*c)/d
```

3.1096 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] (3*b*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d
*x]^3)/(3*d) + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b*Cos[c + d*x]^3*Si
n[c + d*x])/(4*d)
```

Rubi [A] time = 0.0998257, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2592, 302, 206, 2635, 8}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```


```
[Out] (3*b*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d
*x]^3)/(3*d) + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b*Cos[c + d*x]^3*Si
n[c + d*x])/(4*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^3(c+dx)\cot(c+dx)\,dx + b\int \cos^4(c+dx)\,dx$$
$$= \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3b)\int \cos^2(c+dx)\,dx - \frac{a\,\text{Subst}\Big(\ldots}{\ldots}$$
$$= \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}(3b)\int \ldots$$
$$= \frac{3bx}{8} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{3b\cos(c+dx)\sin(c+dx)}{8d} \ldots$$
$$= \frac{3bx}{8} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \ldots$$

Mathematica [A] time = 0.122046, size = 109, normalized size = 1.22

$$\frac{5a\cos(c+dx)}{4d} + \frac{a\cos(3(c+dx))}{12d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{3b(c+dx)}{8d} + \frac{b\sin(2(c+dx))}{4d} \ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) + (5*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12
*d) - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d + (b*Sin[2*
(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.055, size = 97, normalized size = 1.1

$$\frac{a(\cos(dx+c))^3}{3d} + \frac{\cos(dx+c)\,a}{d} + \frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d} + \frac{b(\cos(dx+c))^3\sin(dx+c)}{4d} + \frac{3b\cos(dx+c\ldots}{8\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/3*a*cos(d*x+c)^3/d+a*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/4*b*c
os(d*x+c)^3*sin(d*x+c)/d+3/8*b*cos(d*x+c)*sin(d*x+c)/d+3/8*b*x+3/8*b*c/d
```

Maxima [A] time = 1.11138, size = 109, normalized size = 1.22

$$\frac{16\left(2\cos(dx+c)^3 + 6\cos(dx+c) - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1)\right)a + 3(12dx + 12c + \sin(4dx \ldots}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*b)/d
```

Fricas [A] time = 1.68077, size = 252, normalized size = 2.83

$$\frac{8\,a\cos\left(dx+c\right)^{3}+9\,bdx+24\,a\cos\left(dx+c\right)-12\,a\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+12\,a\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(2\,b\cos\left(\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*b*d*x + 24*a*cos(d*x + c) - 12*a*log(1/2*cos(d
*x + c) + 1/2) + 12*a*log(-1/2*cos(d*x + c) + 1/2) + 3*(2*b*cos(d*x + c)^3
+ 3*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35741, size = 196, normalized size = 2.2

$$\frac{9\,(dx+c)b+24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-96\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+9\,b\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*b + 24*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(15*b*tan(1/2
*d*x + 1/2*c)^7 - 48*a*tan(1/2*d*x + 1/2*c)^6 - 9*b*tan(1/2*d*x + 1/2*c)^5
- 96*a*tan(1/2*d*x + 1/2*c)^4 + 9*b*tan(1/2*d*x + 1/2*c)^3 - 80*a*tan(1/2*d
*x + 1/2*c)^2 - 15*b*tan(1/2*d*x + 1/2*c) - 32*a)/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

3.1097 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3ax}{2}+\frac{b\cos^3(c+dx)}{3d}+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] (-3*a*x)/2 - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d + (b*Cos[c +
d*x]^3)/(3*d) - (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/
(2*d)
```

Rubi [A] time = 0.133641, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2838, 2591, 288, 321, 203, 2592, 302, 206}

$$-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3ax}{2}+\frac{b\cos^3(c+dx)}{3d}+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*x)/2 - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d + (b*Cos[c +
d*x]^3)/(3*d) - (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/
(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^2(c+dx)\cot^2(c+dx)\,dx + b\int \cos^3(c+dx)\cot(c+dx)\,dx$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(\right.}{d}$$

$$= \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \cot(c+dx)\right)}{2d}$$

$$= \frac{b\cos(c+dx)}{d} + \frac{b\cos^3(c+dx)}{3d} - \frac{3a\cot(c+dx)}{2d} + \frac{a\cos^2(c+dx)\,\text{co}}{2d}$$

$$= -\frac{3ax}{2} - \frac{b\tanh^{-1}(\cos(c+dx))}{d} + \frac{b\cos(c+dx)}{d} + \frac{b\cos^3(c+dx)}{3d} - 3$$

Mathematica [A] time = 0.328251, size = 105, normalized size = 1.27

$$-\frac{3a(c+dx)}{2d} - \frac{a\sin(2(c+dx))}{4d} - \frac{a\cot(c+dx)}{d} + \frac{5b\cos(c+dx)}{4d} + \frac{b\cos(3(c+dx))}{12d} + \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{b\log\left(\text{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (5*b*Cos[c + d*x])/(4*d) + (b*Cos[3*(c + d*x)])/(1
2*d) - (a*Cot[c + d*x])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d
*x)/2]])/d - (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.053, size = 119, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^5}{d\sin(dx+c)} - \frac{a(\cos(dx+c))^3\sin(dx+c)}{d} - \frac{3\cos(dx+c)\,a\sin(dx+c)}{2d} - \frac{3ax}{2} - \frac{3ca}{2d} + \frac{b(\cos(dx+c))^3}{3d} + \frac{b\,\text{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^5-a*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a*cos(d*x+c)
*sin(d*x+c)/d-3/2*a*x-3/2/d*c*a+1/3*b*cos(d*x+c)^3/d+b*cos(d*x+c)/d+1/d*b*l
n(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.69459, size = 123, normalized size = 1.48

$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a-\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a - (2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*lo
g(cos(d*x + c) - 1))*b)/d
```

Fricas [A] time = 1.79935, size = 300, normalized size = 3.61

$$\frac{3\,a\cos(dx+c)^3-3\,b\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\,b\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9\,a\cos(dx+}{6\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^3 - 3*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*
b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*a*cos(d*x + c) + (2*b*cos(d
*x + c)^3 - 9*a*d*x + 6*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38756, size = 192, normalized size = 2.31

$$-\frac{9\,(dx+c)a-6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,b\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + 3*(2*b*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c) - 2*(3*a*t
an(1/2*d*x + 1/2*c)^5 + 12*b*tan(1/2*d*x + 1/2*c)^4 + 12*b*tan(1/2*d*x + 1/
2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) + 8*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.1098 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cot(c+dx)}{2d}+\frac{b\cos^2(c+dx)\cot(c+dx)}{2d}$$

```
[Out] (-3*b*x)/2 + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) -
 (3*b*Cot[c + d*x])/(2*d) + (b*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (a*Cos[
c + d*x]*Cot[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.121987, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2838, 2592, 288, 321, 206, 2591, 203}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cot(c+dx)}{2d}+\frac{b\cos^2(c+dx)\cot(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*b*x)/2 + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) -
 (3*b*Cot[c + d*x])/(2*d) + (b*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (a*Cos[
c + d*x]*Cot[c + d*x]^2)/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos(c+dx)\cot^3(c+dx)\,dx + b\int \cos^2(c+dx)\cot^2(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot\right.}{d}$$
$$= \frac{b\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} + \frac{(3a)\,\text{Subst}}{}$$
$$= -\frac{3a\cos(c+dx)}{2d} - \frac{3b\cot(c+dx)}{2d} + \frac{b\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\,\text{co}}{}$$
$$= -\frac{3bx}{2} + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} - \frac{3b\cot(c+dx)}{2d} +$$

Mathematica [A] time = 1.53349, size = 132, normalized size = 1.4

$$-\frac{a\cos(c+dx)}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{3b(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*b*(c + d*x))/(2*d) - (a*Cos[c + d*x])/d - (b*Cot[c + d*x])/d - (a*Csc[(
c + d*x)/2]^2)/(8*d) + (3*a*Log[Cos[(c + d*x)/2]])/(2*d) - (3*a*Log[Sin[(c
+ d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d) - (b*Sin[2*(c + d*x)])/(4*
d)
```

Maple [A] time = 0.056, size = 143, normalized size = 1.5

$$-\frac{a(\cos(dx+c))^5}{2d(\sin(dx+c))^2} - \frac{a(\cos(dx+c))^3}{2d} - \frac{3\cos(dx+c)a}{2d} - \frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d} - \frac{b(\cos(dx+c))^5}{d\sin(dx+c)} - \frac{b(\text{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a*cos(d*x+c)^3/d-3/2*a*cos(d*x+c)/d-
3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*b/sin(d*x+c)*cos(d*x+c)^5-b*cos(d*x+c
)^3*sin(d*x+c)/d-3/2*b*cos(d*x+c)*sin(d*x+c)/d-3/2*b*x-3/2*b*c/d
```

Maxima [A] time = 1.71044, size = 136, normalized size = 1.45

$$-\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b-a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*b - a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos
(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.83968, size = 365, normalized size = 3.88

$$-\frac{6\,bdx\cos\left(dx+c\right)^2+4\,a\cos\left(dx+c\right)^3-6\,bdx-6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3}{4\left(d\cos\left(dx+c\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*b*d*x*cos(d*x + c)^2 + 4*a*cos(d*x + c)^3 - 6*b*d*x - 6*a*cos(d*x +
 c) - 3*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x +
 c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b*cos(d*x + c)^3 - 3*b*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44834, size = 220, normalized size = 2.34

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\left(dx+c\right)b-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x + c)*b - 12*a*log(abs(tan(1/2*d*x +
 1/2*c))) + 4*b*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^6 + 4*b*ta
n(1/2*d*x + 1/2*c)^5 - 5*a*tan(1/2*d*x + 1/2*c)^4 - 16*b*tan(1/2*d*x + 1/2*
c)^3 - 12*a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2
*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2)/d
```

3.1099 $\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

```
[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rubi [A] time = 0.0847967, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2722, 2592, 288, 321, 206, 3473, 8}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx\\
&= a\int \cot^4(c+dx)\,dx+b\int \cos(c+dx)\cot^3(c+dx)\,dx\\
&= -\frac{a\cot^3(c+dx)}{3d}-a\int \cot^2(c+dx)\,dx-\frac{b\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\cos(c+dx)\right)}{d}\\
&= \frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+a\int 1\,dx+\frac{(3b)\,\text{Su}}{}\\
&= ax-\frac{3b\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx}{3d}\\
&= ax+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\,\text{co}}{2d}
\end{aligned}$$

Mathematica [C] time = 0.0438929, size = 125, normalized size = 1.52

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}-\frac{b\cos(c+dx)}{d}-\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{3b\log\left(\sin\right(}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (b*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*b*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2
)/(8*d)
```

Maple [A] time = 0.062, size = 106, normalized size = 1.3

$$-\frac{a(\cot(dx+c))^3}{3d}+\frac{a\cot(dx+c)}{d}+ax+\frac{ca}{d}-\frac{b(\cos(dx+c))^5}{2d(\sin(dx+c))^2}-\frac{b(\cos(dx+c))^3}{2d}-\frac{3b\cos(dx+c)}{2d}-\frac{3b\ln(\csc(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1/d*c*a-1/2/d*b/sin(d*x+c)^2*cos(d
*x+c)^5-1/2*b*cos(d*x+c)^3/d-3/2*b*cos(d*x+c)/d-3/2/d*b*ln(csc(d*x+c)-cot(d
*x+c))
```

Maxima [A] time = 1.64372, size = 124, normalized size = 1.51

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)-1)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*b*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.8177, size = 425, normalized size = 5.18

$$\frac{16\,a\cos(dx+c)^3+9\left(b\cos(dx+c)^2-b\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9\left(b\cos(dx+c)^2-b\right)\log\left(-\frac{1}{2}\,\cos(\ldots\right.}{12\left(d\cos(dx+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(b*cos(d*x + c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*b*cos(d*x
+ c)^3 - 2*a*d*x + 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41656, size = 190, normalized size = 2.32

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,(dx+c)a-36\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\ldots}{\tan(\ldots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*b*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*b/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.1100 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=88

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}+\frac{b\cot(c+dx)}{d}$$

```
[Out] b*x - (3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*Cot[c + d*x])/d - (b*Cot[c + d
*x]^3)/(3*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Cs
c[c + d*x])/(4*d)
```

Rubi [A] time = 0.111163, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2838, 2611, 3770, 3473, 8}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}+\frac{b\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] b*x - (3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*Cot[c + d*x])/d - (b*Cot[c + d
*x]^3)/(3*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Cs
c[c + d*x])/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc(c+dx)\,dx + b\int \cot^4(c+dx)\,dx$$
$$= -\frac{b\cot^3(c+dx)}{3d} - \frac{a\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3a)\int \cot^2(c+dx)$$
$$= \frac{b\cot(c+dx)}{d} - \frac{b\cot^3(c+dx)}{3d} + \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot^3(}{}$$
$$= bx - \frac{3a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{b\cot(c+dx)}{d} - \frac{b\cot^3(c+dx)}{3d} + \frac{3a\,c}{}$$

Mathematica [C] time = 0.0507189, size = 153, normalized size = 1.74

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{3a\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (5*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Cot[c
+ d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) - (3*a*Lo
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (5*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.063, size = 128, normalized size = 1.5

$$-\frac{a(\cos(dx+c))^5}{4d(\sin(dx+c))^4} + \frac{a(\cos(dx+c))^5}{8d(\sin(dx+c))^2} + \frac{a(\cos(dx+c))^3}{8d} + \frac{3\cos(dx+c)a}{8d} + \frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{8d} - \frac{b(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/8*a*
cos(d*x+c)^3/d+3/8*a*cos(d*x+c)/d+3/8/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3*b*c
ot(d*x+c)^3/d+b*cot(d*x+c)/d+b*x+b*c/d
```

Maxima [A] time = 1.63938, size = 144, normalized size = 1.64

$$\frac{16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b-3\,a\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos(dx+c)+1\right)-3\,\log\left(\cos(dx+c)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*b - 3*a*(2*(
5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)
+ 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.81543, size = 494, normalized size = 5.61

$$\frac{48\,bdx\cos\left(dx+c\right)^{4}-96\,bdx\cos\left(dx+c\right)^{2}-30\,a\cos\left(dx+c\right)^{3}+48\,bdx+18\,a\cos\left(dx+c\right)-9\left(a\cos\left(dx+c\right)^{4}-2\,a\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*b*d*x*cos(d*x + c)^4 - 96*b*d*x*cos(d*x + c)^2 - 30*a*cos(d*x + c)
^3 + 48*b*d*x + 18*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^
2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) - 16*(4*b*cos(d*x + c)^3 - 3*b*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43739, size = 207, normalized size = 2.35

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+192\,(dx+c)b+72\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*(d*x + c)*b + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 120*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 - 120*b*tan(1/2
*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) +
3*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.1101 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3b\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] (-3*b*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^5)/(5*d) + (3*b*Cot[c
+ d*x]*Csc[c + d*x])/(8*d) - (b*Cot[c + d*x]^3*Csc[c + d*x])/(4*d)
```

Rubi [A] time = 0.132982, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2838, 2607, 30, 2611, 3770}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3b\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*b*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^5)/(5*d) + (3*b*Cot[c
+ d*x]*Csc[c + d*x])/(8*d) - (b*Cot[c + d*x]^3*Csc[c + d*x])/(4*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^2(c+dx)\,dx + b\int \cot^4(c+dx)\csc(c+dx)\,d$$
$$= -\frac{b\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3b)\int \cot^2(c+dx)\csc(c+dx)\,d$$
$$= -\frac{a\cot^5(c+dx)}{5d} + \frac{3b\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot^3(c+dx)\csc}{4d}$$
$$= -\frac{3b\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^5(c+dx)}{5d} + \frac{3b\cot(c+dx)\csc(c}{8d}$$

Mathematica [A] time = 0.0354158, size = 135, normalized size = 1.82

$$-\frac{a\cot^5(c+dx)}{5d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5b\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3b\log\left(\sin\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^5)/(5*d) + (5*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d
*x)/2]^4)/(64*d) - (3*b*Log[Cos[(c + d*x)/2]])/(8*d) + (3*b*Log[Sin[(c + d*
x)/2]])/(8*d) - (5*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(6
4*d)
```

Maple [A] time = 0.063, size = 116, normalized size = 1.6

$$-\frac{a(\cos(dx+c))^5}{5d(\sin(dx+c))^5} - \frac{b(\cos(dx+c))^5}{4d(\sin(dx+c))^4} + \frac{b(\cos(dx+c))^5}{8d(\sin(dx+c))^2} + \frac{b(\cos(dx+c))^3}{8d} + \frac{3b\cos(dx+c)}{8d} + \frac{3b\ln(\csc(dx+}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^5-1/4/d*b/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*
b/sin(d*x+c)^2*cos(d*x+c)^5+1/8*b*cos(d*x+c)^3/d+3/8*b*cos(d*x+c)/d+3/8/d*b
*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.12415, size = 116, normalized size = 1.57

$$-\frac{5b\left(\frac{2\left(5\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1} + 3\log(\cos(dx+c)+1) - 3\log(\cos(dx+c)-1)\right) + \frac{16a}{\tan(dx+c)^5}}{80d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*b*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 16*a/
tan(d*x + c)^5)/d
```

Fricas [B] time = 1.72965, size = 441, normalized size = 5.96

$$-\frac{16\,a\cos\left(dx+c\right)^{5}+15\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\left(b\cos\left(dx+c\right)^{4}\right.}{80\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*a*cos(d*x + c)^5 + 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(b*cos(d*x + c)^4 - 2*b*cos(
d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(5*b*cos(d*x
 + c)^3 - 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x
+ c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39985, size = 234, normalized size = 3.16

$$\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+5\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+120\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/320*(2*a*tan(1/2*d*x + 1/2*c)^5 + 5*b*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1
/2*d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^2 + 120*b*log(abs(tan(1/2*d*x
 + 1/2*c))) + 20*a*tan(1/2*d*x + 1/2*c) - (274*b*tan(1/2*d*x + 1/2*c)^5 + 2
0*a*tan(1/2*d*x + 1/2*c)^4 - 40*b*tan(1/2*d*x + 1/2*c)^3 - 10*a*tan(1/2*d*x
 + 1/2*c)^2 + 5*b*tan(1/2*d*x + 1/2*c) + 2*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.1102 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=98

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}-\frac{b\,\mathrm{cc}}{}$$

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(16*d) - (b*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.165285, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}-\frac{b\,\mathrm{cc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(16*d) - (b*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^3(c+dx)\,dx + b\int \cot^4(c+dx)\csc^2(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}a\int \cot^2(c+dx)\csc^3(c+dx)\,dx +$$
$$= -\frac{b\cot^5(c+dx)}{5d} + \frac{a\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a\cot^3(c+dx)\csc^3(c}{6d}$$
$$= -\frac{b\cot^5(c+dx)}{5d} - \frac{a\cot(c+dx)\csc(c+dx)}{16d} + \frac{a\cot(c+dx)\csc^3(c+}{8d}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{b\cot^5(c+dx)}{5d} - \frac{a\cot(c+dx)\csc(c+dx}{16d}$$

Mathematica [A] time = 0.0426912, size = 175, normalized size = 1.79

$$-\frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^2\left(\frac{1}{2}(c+\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cot[c + d*x]^5)/(5*d) - (a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*x
)/2]^4)/(64*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) - (a*Log[Cos[(c + d*x)/2]])
/(16*d) + (a*Log[Sin[(c + d*x)/2]])/(16*d) + (a*Sec[(c + d*x)/2]^2)/(64*d)
- (a*Sec[(c + d*x)/2]^4)/(64*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A] time = 0.066, size = 138, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^5}{6d(\sin(dx+c))^6} - \frac{a(\cos(dx+c))^5}{24d(\sin(dx+c))^4} + \frac{a(\cos(dx+c))^5}{48d(\sin(dx+c))^2} + \frac{a(\cos(dx+c))^3}{48d} + \frac{\cos(dx+c)a}{16d} + \frac{a\ln(\csc(dx+c)-}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/6/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/48/
d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a*cos(d*x+c)^3/d+1/16*a*cos(d*x+c)/d+1/1
6/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*b/sin(d*x+c)^5*cos(d*x+c)^5
```

Maxima [A] time = 1.14976, size = 143, normalized size = 1.46

$$\frac{5a\left(\frac{2\left(3\cos(dx+c)^5+8\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1} - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1)\right) - \frac{96b}{\tan(dx+c)^5}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(5*a*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d
*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) +
 1) + 3*log(cos(d*x + c) - 1)) - 96*b/tan(d*x + c)^5)/d
```

Fricas [B] time = 1.81841, size = 497, normalized size = 5.07

$$\frac{96\,b\cos(dx+c)^5\sin(dx+c)+30\,a\cos(dx+c)^5+80\,a\cos(dx+c)^3-30\,a\cos(dx+c)-15\left(a\cos(dx+c)^6-3\,a\,\mathrm{cc}\right.}{480\left(d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(96*b*cos(d*x + c)^5*sin(d*x + c) + 30*a*cos(d*x + c)^5 + 80*a*cos(d*
x + c)^3 - 30*a*cos(d*x + c) - 15*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 +
3*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 15*(a*cos(d*x + c)^6
- 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)
)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42804, size = 271, normalized size = 2.77

$$\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+12\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-60\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*a*tan(1/2*d*x + 1/2*c)^6 + 12*b*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan
(1/2*d*x + 1/2*c)^4 - 60*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*
c)^2 + 120*a*log(abs(tan(1/2*d*x + 1/2*c))) + 120*b*tan(1/2*d*x + 1/2*c) -
(294*a*tan(1/2*d*x + 1/2*c)^6 + 120*b*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan(1/2
*d*x + 1/2*c)^4 - 60*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*c)^2
 + 12*b*tan(1/2*d*x + 1/2*c) + 5*a)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.1103 $\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=114

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{b\tanh^{-1}(\cos(c+dx))}{16d}-\frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{b\cot(c+dx)\csc^3(c+dx)}{8d}-$$

```
[Out] -(b*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]^7)/(7*d) - (b*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (b*Cot[c + d*x]*Csc[c
 + d*x]^3)/(8*d) - (b*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.173139, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2607, 14, 2611, 3768, 3770}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{b\tanh^{-1}(\cos(c+dx))}{16d}-\frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{b\cot(c+dx)\csc^3(c+dx)}{8d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```


```
[Out] -(b*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]^7)/(7*d) - (b*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (b*Cot[c + d*x]*Csc[c
 + d*x]^3)/(8*d) - (b*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d)
```


Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^4(c+dx)\,dx + b\int \cot^4(c+dx)\csc^3(c+dx)$$
$$= -\frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}b\int \cot^2(c+dx)\csc^3(c+dx)\,dx$$
$$= \frac{b\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d} + \frac{1}{8}b\int$$
$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{b\cot(c+dx)\csc(c+dx)}{16d} + \frac{b}{}$$
$$= -\frac{b\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{b\,\mathrm{co}}{}$$

Mathematica [B] time = 0.0831098, size = 239, normalized size = 2.1

$$-\frac{2a\cot(c+dx)}{35d} - \frac{a\cot(c+dx)\csc^6(c+dx)}{7d} + \frac{8a\cot(c+dx)\csc^4(c+dx)}{35d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{35d} - \frac{b\csc^6\left(\frac{1}{2}(c\right.}{384}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(35*d) - (b*Csc[(c + d*x)/2]^2)/(64*d) + (b*Csc[(c + d*
x)/2]^4)/(64*d) - (b*Csc[(c + d*x)/2]^6)/(384*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^2)/(35*d) + (8*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^6)/(7*d) - (b*Log[Cos[(c + d*x)/2]])/(16*d) + (b*Log[Sin[(c +
 d*x)/2]])/(16*d) + (b*Sec[(c + d*x)/2]^2)/(64*d) - (b*Sec[(c + d*x)/2]^4)/
(64*d) + (b*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A] time = 0.066, size = 160, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^5}{7d(\sin(dx+c))^7} - \frac{2a(\cos(dx+c))^5}{35d(\sin(dx+c))^5} - \frac{b(\cos(dx+c))^5}{6d(\sin(dx+c))^6} - \frac{b(\cos(dx+c))^5}{24d(\sin(dx+c))^4} + \frac{b(\cos(dx+c))^5}{48d(\sin(dx+c))^2} + \frac{b(\cos(d}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/7/d*a/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a/sin(d*x+c)^5*cos(d*x+c)^5-1/6/d
*b/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*b/sin(d*x+c)^4*cos(d*x+c)^5+1/48/d*b/si
n(d*x+c)^2*cos(d*x+c)^5+1/48*b*cos(d*x+c)^3/d+1/16*b*cos(d*x+c)/d+1/16/d*b*
ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.15978, size = 159, normalized size = 1.39

$$\frac{35\,b\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\left(7\,\tan(dx+c)^2+5\right)a}{\tan(dx+c)^7}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3360*(35*b*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 96*(7*tan(d*x + c)^2 + 5)*a/tan(d*x + c)
^7)/d
```

Fricas [B] time = 1.7855, size = 593, normalized size = 5.2

$$-\frac{192\,a\cos\left(dx+c\right)^7-672\,a\cos\left(dx+c\right)^5+105\left(b\cos\left(dx+c\right)^6-3\,b\cos\left(dx+c\right)^4+3\,b\cos\left(dx+c\right)^2-b\right)\log\left(\frac{1}{2}\,\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3360*(192*a*cos(d*x + c)^7 - 672*a*cos(d*x + c)^5 + 105*(b*cos(d*x + c)^
6 - 3*b*cos(d*x + c)^4 + 3*b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 105*(b*cos(d*x + c)^6 - 3*b*cos(d*x + c)^4 + 3*b*cos(d*x +
 c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*(3*b*cos(d*x + c)
^5 + 8*b*cos(d*x + c)^3 - 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27675, size = 309, normalized size = 2.71

$$\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+35\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 + 35*b*tan(1/2*d*x + 1/2*c)^6 - 21*a*t
an(1/2*d*x + 1/2*c)^5 - 105*b*tan(1/2*d*x + 1/2*c)^4 - 105*a*tan(1/2*d*x +
1/2*c)^3 - 105*b*tan(1/2*d*x + 1/2*c)^2 + 840*b*log(abs(tan(1/2*d*x + 1/2*c
))) + 315*a*tan(1/2*d*x + 1/2*c) - (2178*b*tan(1/2*d*x + 1/2*c)^7 + 315*a*t
an(1/2*d*x + 1/2*c)^6 - 105*b*tan(1/2*d*x + 1/2*c)^5 - 105*a*tan(1/2*d*x +
1/2*c)^4 - 105*b*tan(1/2*d*x + 1/2*c)^3 - 21*a*tan(1/2*d*x + 1/2*c)^2 + 35*
b*tan(1/2*d*x + 1/2*c) + 15*a)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.1104 $\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=136

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{3a\,\mathrm{cc}}{}$$

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(128*d) - (b*Cot[c + d*x]^5)/(5*d) - (b*Cot[c
+ d*x]^7)/(7*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^3)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rubi [A] time = 0.18414, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2611, 3768, 3770, 2607, 14}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{3a\,\mathrm{cc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(128*d) - (b*Cot[c + d*x]^5)/(5*d) - (b*Cot[c
+ d*x]^7)/(7*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^3)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^5(c+dx)\,dx + b\int \cot^4(c+dx)\csc^4(c+dx)$$
$$= -\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{1}{8}(3a)\int \cot^2(c+dx)\csc^5(c+dx$$
$$= \frac{a\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d} + \frac{1}{16}a\int$$
$$= -\frac{b\cot^5(c+dx)}{5d} - \frac{b\cot^7(c+dx)}{7d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{64d} +$$
$$= -\frac{b\cot^5(c+dx)}{5d} - \frac{b\cot^7(c+dx)}{7d} - \frac{3a\cot(c+dx)\csc(c+dx)}{128d} -$$
$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{b\cot^5(c+dx)}{5d} - \frac{b\cot^7(c+dx)}{7d} - \frac{3a}{}$$

Mathematica [B] time = 0.0794382, size = 279, normalized size = 2.05

$$-\frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} - \frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{a\sec^8\left(\frac{1}{2}(c+dx)\right)}{2048d} - \frac{a\sec^6\left(\frac{1}{2}\right.}{51}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*Cot[c + d*x])/(35*d) - (3*a*Csc[(c + d*x)/2]^2)/(512*d) + (a*Csc[(c +
 d*x)/2]^4)/(1024*d) + (a*Csc[(c + d*x)/2]^6)/(512*d) - (a*Csc[(c + d*x)/2]
^8)/(2048*d) - (b*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) + (8*b*Cot[c + d*x]*C
sc[c + d*x]^4)/(35*d) - (b*Cot[c + d*x]*Csc[c + d*x]^6)/(7*d) - (3*a*Log[Co
s[(c + d*x)/2]])/(128*d) + (3*a*Log[Sin[(c + d*x)/2]])/(128*d) + (3*a*Sec[(
c + d*x)/2]^2)/(512*d) - (a*Sec[(c + d*x)/2]^4)/(1024*d) - (a*Sec[(c + d*x)
/2]^6)/(512*d) + (a*Sec[(c + d*x)/2]^8)/(2048*d)
```

Maple [A] time = 0.066, size = 182, normalized size = 1.3

$$-\frac{a(\cos(dx+c))^5}{8d(\sin(dx+c))^8} - \frac{a(\cos(dx+c))^5}{16d(\sin(dx+c))^6} - \frac{a(\cos(dx+c))^5}{64d(\sin(dx+c))^4} + \frac{a(\cos(dx+c))^5}{128d(\sin(dx+c))^2} + \frac{a(\cos(dx+c))^3}{128d} + \frac{3\cos(d}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/8/d*a/sin(d*x+c)^8*cos(d*x+c)^5-1/16/d*a/sin(d*x+c)^6*cos(d*x+c)^5-1/64/
d*a/sin(d*x+c)^4*cos(d*x+c)^5+1/128/d*a/sin(d*x+c)^2*cos(d*x+c)^5+1/128*a*c
os(d*x+c)^3/d+3/128*a*cos(d*x+c)/d+3/128/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/7/
d*b/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*b/sin(d*x+c)^5*cos(d*x+c)^5
```

Maxima [A] time = 1.00146, size = 186, normalized size = 1.37

$$\frac{35\,a\left(\frac{2\left(3\,\cos(dx+c)^{7}-11\,\cos(dx+c)^{5}-11\,\cos(dx+c)^{3}+3\,\cos(dx+c)\right)}{\cos(dx+c)^{8}-4\,\cos(dx+c)^{6}+6\,\cos(dx+c)^{4}-4\,\cos(dx+c)^{2}+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{256\left(7\,\tan(dx+\cdots\right.}{\tan(dx+c\cdots}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8960*(35*a*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 +
 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
256*(7*tan(d*x + c)^2 + 5)*b/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.85859, size = 656, normalized size = 4.82

$$\frac{210\,a\cos\left(dx+c\right)^{7}-770\,a\cos\left(dx+c\right)^{5}-770\,a\cos\left(dx+c\right)^{3}+210\,a\cos\left(dx+c\right)-105\left(a\cos\left(dx+c\right)^{8}-4\,a\cos\left(dx+c\right.\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8960*(210*a*cos(d*x + c)^7 - 770*a*cos(d*x + c)^5 - 770*a*cos(d*x + c)^3
+ 210*a*cos(d*x + c) - 105*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos
(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 105*(a*
cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 256*(2*b*cos(d*x + c)^7 - 7*b*cos(d*
x + c)^5)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*
x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38671, size = 271, normalized size = 1.99

$$35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+80\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-112\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-560\,b\tan\left(\frac{1}{2}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/71680*(35*a*tan(1/2*d*x + 1/2*c)^8 + 80*b*tan(1/2*d*x + 1/2*c)^7 - 112*b*
tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 560*b*tan(1/2*d*x +
 1/2*c)^3 + 1680*a*log(abs(tan(1/2*d*x + 1/2*c))) + 1680*b*tan(1/2*d*x + 1/
2*c) - (4566*a*tan(1/2*d*x + 1/2*c)^8 + 1680*b*tan(1/2*d*x + 1/2*c)^7 - 560
*b*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 112*b*tan(1/2*d*
x + 1/2*c)^3 + 80*b*tan(1/2*d*x + 1/2*c) + 35*a)/tan(1/2*d*x + 1/2*c)^8)/d
```

3.1105 $\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=301

$$\frac{(9a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{(9a^2+4b^2)\cos(c+dx)}{105d}-\frac{a(10a^2-29b^2)\sin^5(c+dx)\cos(c+dx)}{504bd}-\frac{5(3a^2-8b^2)\sin^4(c+dx)}{}$$

```
[Out] (3*a*b*x)/64 - ((9*a^2 + 4*b^2)*Cos[c + d*x])/(105*d) + ((9*a^2 + 4*b^2)*Co
s[c + d*x]^3)/(315*d) - (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(64*d) - (a*b*Cos
[c + d*x]*Sin[c + d*x]^3)/(32*d) - ((15*a^4 - 44*a^2*b^2 + 6*b^4)*Cos[c + d
*x]*Sin[c + d*x]^4)/(630*b^2*d) - (a*(10*a^2 - 29*b^2)*Cos[c + d*x]*Sin[c +
 d*x]^5)/(504*b*d) - (5*(3*a^2 - 8*b^2)*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*
Sin[c + d*x])^2)/(252*b^2*d) + (a*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c
+ d*x])^3)/(12*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^3
)/(9*b*d)
```

Rubi [A] time = 0.653838, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2895, 3049, 3033, 3023, 2748, 2633, 2635, 8}

$$\frac{(9a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{(9a^2+4b^2)\cos(c+dx)}{105d}-\frac{a(10a^2-29b^2)\sin^5(c+dx)\cos(c+dx)}{504bd}-\frac{5(3a^2-8b^2)\sin^4(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```



```
[Out] (3*a*b*x)/64 - ((9*a^2 + 4*b^2)*Cos[c + d*x])/(105*d) + ((9*a^2 + 4*b^2)*Co
s[c + d*x]^3)/(315*d) - (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(64*d) - (a*b*Cos
[c + d*x]*Sin[c + d*x]^3)/(32*d) - ((15*a^4 - 44*a^2*b^2 + 6*b^4)*Cos[c + d
*x]*Sin[c + d*x]^4)/(630*b^2*d) - (a*(10*a^2 - 29*b^2)*Cos[c + d*x]*Sin[c +
 d*x]^5)/(504*b*d) - (5*(3*a^2 - 8*b^2)*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*
Sin[c + d*x])^2)/(252*b^2*d) + (a*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c
+ d*x])^3)/(12*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^3
)/(9*b*d)
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
```

```
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{a\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^3}{12b^2d} - \frac{\cos(c+dx)\sin^5(c+\cdots}{}$$
$$= -\frac{5\left(3a^2-8b^2\right)\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2}{252b^2d} + \frac{a\cos(\cdots}{}$$
$$= -\frac{a\left(10a^2-29b^2\right)\cos(c+dx)\sin^5(c+dx)}{504bd} - \frac{5\left(3a^2-8b^2\right)\cos(c+d\cdots}{}$$
$$= -\frac{\left(15a^4-44a^2b^2+6b^4\right)\cos(c+dx)\sin^4(c+dx)}{630b^2d} - \frac{a\left(10a^2-29b^2\right)\cdots}{}$$
$$= -\frac{\left(15a^4-44a^2b^2+6b^4\right)\cos(c+dx)\sin^4(c+dx)}{630b^2d} - \frac{a\left(10a^2-29b^2\right)\cdots}{}$$
$$= -\frac{ab\cos(c+dx)\sin^3(c+dx)}{32d} - \frac{\left(15a^4-44a^2b^2+6b^4\right)\cos(c+dx)\,\mathrm{si}\cdots}{630b^2d}$$
$$= -\frac{\left(9a^2+4b^2\right)\cos(c+dx)}{105d} + \frac{\left(9a^2+4b^2\right)\cos^3(c+dx)}{315d} - \frac{3ab\cos(c+\cdots}{}$$
$$= \frac{3abx}{64} - \frac{\left(9a^2+4b^2\right)\cos(c+dx)}{105d} + \frac{\left(9a^2+4b^2\right)\cos^3(c+dx)}{315d} - \frac{3ab\,\mathrm{c}\cdots}{}$$

Mathematica [A] time = 0.956657, size = 144, normalized size = 0.48

$$\frac{-3780\left(2a^2+b^2\right)\cos(c+dx)-840\left(3a^2+b^2\right)\cos(3(c+dx))+504a^2\cos(5(c+dx))+360a^2\cos(7(c+dx))-2520ab\,\mathrm{si}\cdots}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (7560*a*b*c + 7560*a*b*d*x - 3780*(2*a^2 + b^2)*Cos[c + d*x] - 840*(3*a^2 +
 b^2)*Cos[3*(c + d*x)] + 504*a^2*Cos[5*(c + d*x)] + 504*b^2*Cos[5*(c + d*x)
] + 360*a^2*Cos[7*(c + d*x)] + 90*b^2*Cos[7*(c + d*x)] - 70*b^2*Cos[9*(c +
d*x)] - 2520*a*b*Sin[4*(c + d*x)] + 315*a*b*Sin[8*(c + d*x)])/(161280*d)
```

Maple [A] time = 0.043, size = 161, normalized size = 0.5

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7}-\frac{2(\cos(dx+c))^5}{35}\right)+2ab\left(-1/8(\sin(dx+c))^3(\cos(dx+c))^5-1/16\sin(dx+c)\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+2*a*b*(-1/8*sin
(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+b^2*(-1/9*sin(d*x+c)^4*cos(d*x+c)
^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5))
```

Maxima [A] time = 0.998958, size = 135, normalized size = 0.45

$$\frac{4608\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^2+315(24dx+24c+\sin(8dx+8c)-8\sin(4dx+4c))ab-512\left(35\cos(dx\cdots\right.}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/161280*(4608*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 + 315*(24*d*x + 24
*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a*b - 512*(35*cos(d*x + c)^9 -
90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^2)/d
```

Fricas [A] time = 1.86421, size = 315, normalized size = 1.05

$$-\frac{2240\,b^2\cos(dx+c)^9 - 2880\left(a^2+2\,b^2\right)\cos(dx+c)^7 + 4032\left(a^2+b^2\right)\cos(dx+c)^5 - 945\,abdx - 315\left(16\,ab\cos(dx\right.}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20160*(2240*b^2*cos(d*x + c)^9 - 2880*(a^2 + 2*b^2)*cos(d*x + c)^7 + 403
2*(a^2 + b^2)*cos(d*x + c)^5 - 945*a*b*d*x - 315*(16*a*b*cos(d*x + c)^7 - 2
4*a*b*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^3 + 3*a*b*cos(d*x + c))*sin(d*x +
 c))/d
```

Sympy [A] time = 23.89, size = 335, normalized size = 1.11

$$\begin{cases} -\frac{a^2\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{2a^2\cos^7(c+dx)}{35d} + \frac{3abx\sin^8(c+dx)}{64} + \frac{3abx\sin^6(c+dx)\cos^2(c+dx)}{16} + \frac{9abx\sin^4(c+dx)\cos^4(c+dx)}{32} + \frac{3abx\sin^2(c+dx)}{16} \\ x\left(a+b\sin(c)\right)^2\sin^3(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*a**2*cos(c + d*x
)**7/(35*d) + 3*a*b*x*sin(c + d*x)**8/64 + 3*a*b*x*sin(c + d*x)**6*cos(c +
d*x)**2/16 + 9*a*b*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 3*a*b*x*sin(c + d
*x)**2*cos(c + d*x)**6/16 + 3*a*b*x*cos(c + d*x)**8/64 + 3*a*b*sin(c + d*x)
**7*cos(c + d*x)/(64*d) + 11*a*b*sin(c + d*x)**5*cos(c + d*x)**3/(64*d) - 1
1*a*b*sin(c + d*x)**3*cos(c + d*x)**5/(64*d) - 3*a*b*sin(c + d*x)*cos(c + d
*x)**7/(64*d) - b**2*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b**2*sin(c +
 d*x)**2*cos(c + d*x)**7/(35*d) - 8*b**2*cos(c + d*x)**9/(315*d), Ne(d, 0))
, (x*(a + b*sin(c))**2*sin(c)**3*cos(c)**4, True))
```

Giac [A] time = 1.35921, size = 192, normalized size = 0.64

$$\frac{3}{64}\,abx - \frac{b^2\cos(9\,dx+9\,c)}{2304\,d} + \frac{ab\sin(8\,dx+8\,c)}{512\,d} - \frac{ab\sin(4\,dx+4\,c)}{64\,d} + \frac{\left(4\,a^2+b^2\right)\cos(7\,dx+7\,c)}{1792\,d} + \frac{\left(a^2+b^2\right)\cos(5}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/64*a*b*x - 1/2304*b^2*cos(9*d*x + 9*c)/d + 1/512*a*b*sin(8*d*x + 8*c)/d -
 1/64*a*b*sin(4*d*x + 4*c)/d + 1/1792*(4*a^2 + b^2)*cos(7*d*x + 7*c)/d + 1/
320*(a^2 + b^2)*cos(5*d*x + 5*c)/d - 1/192*(3*a^2 + b^2)*cos(3*d*x + 3*c)/d
 - 3/128*(2*a^2 + b^2)*cos(d*x + c)/d
```

3.1106 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=278

$$-\frac{a\left(20a^2-69b^2\right)\sin^4(c+dx)\cos(c+dx)}{840bd}-\frac{\left(20a^2-63b^2\right)\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{336b^2d}-\frac{\left(-140a^2b^2\right.}{}$$

```
[Out] ((8*a^2 + 3*b^2)*x)/128 - (6*a*b*Cos[c + d*x])/(35*d) + (2*a*b*Cos[c + d*x]
^3)/(35*d) - ((8*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(128*d) - ((40*a^4
 - 140*a^2*b^2 + 21*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(1344*b^2*d) - (a*(20
*a^2 - 69*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(840*b*d) - ((20*a^2 - 63*b^2)*
Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(336*b^2*d) + (5*a*Cos[
c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(56*b^2*d) - (Cos[c + d*x]*
Sin[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(8*b*d)
```

Rubi [A] time = 0.626873, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2895, 3049, 3033, 3023, 2748, 2635, 8, 2633}

$$-\frac{a\left(20a^2-69b^2\right)\sin^4(c+dx)\cos(c+dx)}{840bd}-\frac{\left(20a^2-63b^2\right)\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{336b^2d}-\frac{\left(-140a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```



```
[Out] ((8*a^2 + 3*b^2)*x)/128 - (6*a*b*Cos[c + d*x])/(35*d) + (2*a*b*Cos[c + d*x]
^3)/(35*d) - ((8*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(128*d) - ((40*a^4
 - 140*a^2*b^2 + 21*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(1344*b^2*d) - (a*(20
*a^2 - 69*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(840*b*d) - ((20*a^2 - 63*b^2)*
Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(336*b^2*d) + (5*a*Cos[
c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(56*b^2*d) - (Cos[c + d*x]*
Sin[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(8*b*d)
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{5a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^3}{56b^2d} - \frac{\cos(c+dx)\sin}{}$$
$$= -\frac{\left(20a^2-63b^2\right)\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2}{336b^2d} + \frac{5}{}$$
$$= -\frac{a\left(20a^2-69b^2\right)\cos(c+dx)\sin^4(c+dx)}{840bd} - \frac{\left(20a^2-63b^2\right)\cos(c}{}$$
$$= -\frac{\left(40a^4-140a^2b^2+21b^4\right)\cos(c+dx)\sin^3(c+dx)}{1344b^2d} - \frac{a\left(20a^2-6\right.}{}$$
$$= -\frac{\left(40a^4-140a^2b^2+21b^4\right)\cos(c+dx)\sin^3(c+dx)}{1344b^2d} - \frac{a\left(20a^2-6\right.}{}$$
$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{128d} - \frac{\left(40a^4-140a^2b^2+21b^4\right)}{134}$$
$$= \frac{1}{128}\left(8a^2+3b^2\right)x - \frac{6ab\cos(c+dx)}{35d} + \frac{2ab\cos^3(c+dx)}{35d} - \frac{\left(8a^2\right.}{}$$

Mathematica [A] time = 0.581002, size = 141, normalized size = 0.51

$$\frac{840a^2\sin(2(c+dx)) - 840a^2\sin(4(c+dx)) - 280a^2\sin(6(c+dx)) + 3360a^2dx - 5040ab\cos(c+dx) - 1680ab\cos(3(}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (1680*b^2*c + 3360*a^2*d*x + 1260*b^2*d*x - 5040*a*b*Cos[c + d*x] - 1680*a*
b*Cos[3*(c + d*x)] + 336*a*b*Cos[5*(c + d*x)] + 240*a*b*Cos[7*(c + d*x)] +
840*a^2*Sin[2*(c + d*x)] - 840*a^2*Sin[4*(c + d*x)] - 420*b^2*Sin[4*(c + d*
x)] - 280*a^2*Sin[6*(c + d*x)] + (105*b^2*Sin[8*(c + d*x)])/2)/(53760*d)
```

Maple [A] time = 0.044, size = 163, normalized size = 0.6

$$\frac{1}{d}\left(a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6} + \frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{dx}{16} + \frac{c}{16}\right) + 2ab\left(-1/7\,(\sin(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d
*x+c)^5)+b^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1
/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

Maxima [A] time = 1.00985, size = 136, normalized size = 0.49

$$\frac{560\left(4\sin(2dx+2c)^3 + 12dx + 12c - 3\sin(4dx+4c)\right)a^2 + 6144\left(5\cos(dx+c)^7 - 7\cos(dx+c)^5\right)ab + 105\,(24d}{107520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/107520*(560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a
^2 + 6144*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a*b + 105*(24*d*x + 24*c +
sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A] time = 1.85615, size = 316, normalized size = 1.14

$$\frac{3840\,ab\cos\left(dx+c\right)^{7}-5376\,ab\cos\left(dx+c\right)^{5}+105\left(8\,a^{2}+3\,b^{2}\right)dx+35\left(48\,b^{2}\cos\left(dx+c\right)^{7}-8\left(8\,a^{2}+9\,b^{2}\right)\cos\left(dx+c\right)\right.}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/13440*(3840*a*b*cos(d*x + c)^7 - 5376*a*b*cos(d*x + c)^5 + 105*(8*a^2 + 3
*b^2)*d*x + 35*(48*b^2*cos(d*x + c)^7 - 8*(8*a^2 + 9*b^2)*cos(d*x + c)^5 +
2*(8*a^2 + 3*b^2)*cos(d*x + c)^3 + 3*(8*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [A] time = 13.5831, size = 420, normalized size = 1.51

$$\begin{cases} \frac{a^{2}x\sin^{6}\left(c+dx\right)}{16}+\frac{3a^{2}x\sin^{4}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{16}+\frac{3a^{2}x\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{16}+\frac{a^{2}x\cos^{6}\left(c+dx\right)}{16}+\frac{a^{2}\sin^{5}\left(c+dx\right)\cos\left(c+dx\right)}{16d}+\frac{a^{2}\sin^{3}\left(c+dx\right)\cos^{3}\left(c+\right.}{6d} \\ x\left(a+b\sin\left(c\right)\right)^{2}\sin^{2}\left(c\right)\cos^{4}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/16 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a**2*x*cos(c + d*x)
**6/16 + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a**2*sin(c + d*x)**3*co
s(c + d*x)**3/(6*d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 2*a*b*sin(
c + d*x)**2*cos(c + d*x)**5/(5*d) - 4*a*b*cos(c + d*x)**7/(35*d) + 3*b**2*x
*sin(c + d*x)**8/128 + 3*b**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 9*b**2
*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*
x)**6/32 + 3*b**2*x*cos(c + d*x)**8/128 + 3*b**2*sin(c + d*x)**7*cos(c + d*
x)/(128*d) + 11*b**2*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b**2*sin(
c + d*x)**3*cos(c + d*x)**5/(128*d) - 3*b**2*sin(c + d*x)*cos(c + d*x)**7/(
128*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**4, True))
```

Giac [A] time = 1.22418, size = 203, normalized size = 0.73

$$\frac{1}{128}\left(8\,a^{2}+3\,b^{2}\right)x+\frac{ab\cos\left(7\,dx+7\,c\right)}{224\,d}+\frac{ab\cos\left(5\,dx+5\,c\right)}{160\,d}-\frac{ab\cos\left(3\,dx+3\,c\right)}{32\,d}-\frac{3\,ab\cos\left(dx+c\right)}{32\,d}+\frac{b^{2}\sin\left(8\,dx+8\,c\right.}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/128*(8*a^2 + 3*b^2)*x + 1/224*a*b*cos(7*d*x + 7*c)/d + 1/160*a*b*cos(5*d*
x + 5*c)/d - 1/32*a*b*cos(3*d*x + 3*c)/d - 3/32*a*b*cos(d*x + c)/d + 1/1024
*b^2*sin(8*d*x + 8*c)/d - 1/192*a^2*sin(6*d*x + 6*c)/d + 1/64*a^2*sin(2*d*x
 + 2*c)/d - 1/128*(2*a^2 + b^2)*sin(4*d*x + 4*c)/d
```

3.1107 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$-\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}-\frac{\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}-\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}+\frac{ab\sin(c+dx)\cos^3(c-}{12d}$$

```
[Out] (a*b*x)/8 - ((a^2 + 6*b^2)*Cos[c + d*x]^5)/(105*d) + (a*b*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(12*d) - (a*Cos[c + d*x
]^5*(a + b*Sin[c + d*x]))/(21*d) - (Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/
(7*d)
```

Rubi [A] time = 0.180754, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}-\frac{\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}-\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}+\frac{ab\sin(c+dx)\cos^3(c-}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*x)/8 - ((a^2 + 6*b^2)*Cos[c + d*x]^5)/(105*d) + (a*b*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(12*d) - (a*Cos[c + d*x
]^5*(a + b*Sin[c + d*x]))/(21*d) - (Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/
(7*d)
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}+\frac{1}{7}\int \cos^4(c+dx)(2b+2a\sin$$
$$= -\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}-\frac{\cos^5(c+dx)(a+b\sin(c+d$$
$$= -\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}-\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}-$$
$$= -\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}+\frac{ab\cos^3(c+dx)\sin(c+dx)}{12d}-\frac{a\cos^5(}{}$$
$$= -\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}+\frac{ab\cos(c+dx)\sin(c+dx)}{8d}+\frac{ab\cos^3(}{}$$
$$= \frac{abx}{8}-\frac{\left(a^2+6b^2\right)\cos^5(c+dx)}{105d}+\frac{ab\cos(c+dx)\sin(c+dx)}{8d}+\frac{ab}{}$$

Mathematica [A] time = 0.42195, size = 132, normalized size = 1.02

$$\frac{-105\left(8a^2+3b^2\right)\cos(c+dx)-105\left(4a^2+b^2\right)\cos(3(c+dx))-84a^2\cos(5(c+dx))+210ab\sin(2(c+dx))-210ab\sin}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (840*a*b*c + 840*a*b*d*x - 105*(8*a^2 + 3*b^2)*Cos[c + d*x] - 105*(4*a^2 +
b^2)*Cos[3*(c + d*x)] - 84*a^2*Cos[5*(c + d*x)] + 21*b^2*Cos[5*(c + d*x)] +
 15*b^2*Cos[7*(c + d*x)] + 210*a*b*Sin[2*(c + d*x)] - 210*a*b*Sin[4*(c + d*
x)] - 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.037, size = 105, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{a^2(\cos(dx+c))^5}{5}+2ab\left(-1/6\sin(dx+c)(\cos(dx+c))^5+1/24\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/5*a^2*cos(d*x+c)^5+2*a*b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+b^2*(-1/7*sin(d*x+c)^2*c
os(d*x+c)^5-2/35*cos(d*x+c)^5))
```

Maxima [A] time = 0.999241, size = 109, normalized size = 0.84

$$-\frac{672a^2\cos(dx+c)^5-35\left(4\sin(2dx+2c)^3+12dx+12c-3\sin(4dx+4c)\right)ab-96\left(5\cos(dx+c)^7-7\cos(dx\right.}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(672*a^2*cos(d*x + c)^5 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c
- 3*sin(4*d*x + 4*c))*a*b - 96*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^2)/d
```

Fricas [A] time = 1.79268, size = 224, normalized size = 1.74

$$\frac{120\,b^2\cos\left(dx+c\right)^7-168\left(a^2+b^2\right)\cos\left(dx+c\right)^5+105\,abdx-35\left(8\,ab\cos\left(dx+c\right)^5-2\,ab\cos\left(dx+c\right)^3-3\,ab\cos\left(dx+\right.\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + b^2)*cos(d*x + c)^5 + 105*a*b*d*
x - 35*(8*a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*s
in(d*x + c))/d
```

Sympy [A] time = 8.16441, size = 223, normalized size = 1.73

$$\begin{cases} -\frac{a^2\cos^5\left(c+dx\right)}{5d}+\frac{abx\sin^6\left(c+dx\right)}{8}+\frac{3abx\sin^4\left(c+dx\right)\cos^2\left(c+dx\right)}{8}+\frac{3abx\sin^2\left(c+dx\right)\cos^4\left(c+dx\right)}{8}+\frac{abx\cos^6\left(c+dx\right)}{8}+\frac{ab\sin^5\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{ab}{} \\ x\left(a+b\sin\left(c\right)\right)^2\sin\left(c\right)\cos^4\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**5/(5*d) + a*b*x*sin(c + d*x)**6/8 + 3*a*b*x*
sin(c + d*x)**4*cos(c + d*x)**2/8 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**4
/8 + a*b*x*cos(c + d*x)**6/8 + a*b*sin(c + d*x)**5*cos(c + d*x)/(8*d) + a*b
*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - a*b*sin(c + d*x)*cos(c + d*x)**5/(
8*d) - b**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*b**2*cos(c + d*x)**7/
(35*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)*cos(c)**4, True))
```

Giac [A] time = 1.22672, size = 190, normalized size = 1.47

$$\frac{1}{8}\,abx+\frac{b^2\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{ab\sin\left(6\,dx+6\,c\right)}{96\,d}-\frac{ab\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{ab\sin\left(2\,dx+2\,c\right)}{32\,d}-\frac{\left(4\,a^2-b^2\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a*b*x + 1/448*b^2*cos(7*d*x + 7*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d - 1/
32*a*b*sin(4*d*x + 4*c)/d + 1/32*a*b*sin(2*d*x + 2*c)/d - 1/320*(4*a^2 - b^
2)*cos(5*d*x + 5*c)/d - 1/64*(4*a^2 + b^2)*cos(3*d*x + 3*c)/d - 1/64*(8*a^2
 + 3*b^2)*cos(d*x + c)/d
```

3.1108 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\cos(c+d}{4d}$$

```
[Out] (3*a*b*x)/4 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) - (b^2*Cos[c + d*x]^5)/(5*d) + (3*a*b*Cos[c + d*x]*Sin
[c + d*x])/(4*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.447365, antiderivative size = 190, normalized size of antiderivative = 1.64, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2895, 3049, 3033, 3023, 2735, 3770}

$$-\frac{\left(-14a^2b^2+a^4+3b^4\right)\cos(c+dx)}{15b^2d}-\frac{\left(a^2-12b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{30b^2d}-\frac{a\left(2a^2-27b^2\right)\sin(c+dx)\cos(c+}{60bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/4 - (a^2*ArcTanh[Cos[c + d*x]])/d - ((a^4 - 14*a^2*b^2 + 3*b^4)*C
os[c + d*x])/(15*b^2*d) - (a*(2*a^2 - 27*b^2)*Cos[c + d*x]*Sin[c + d*x])/(6
0*b*d) - ((a^2 - 12*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(30*b^2*d) +
(a*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(10*b^2*d) - (Cos[c + d*x]*Sin[c +
d*x]*(a + b*Sin[c + d*x])^3)/(5*b*d)
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{a\cos(c+dx)(a+b\sin(c+dx))^3}{10b^2d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\sin\\ &= -\frac{\left(a^2-12b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{30b^2d}+\frac{a\cos(c+dx)(a+b\sin\\ &= -\frac{a\left(2a^2-27b^2\right)\cos(c+dx)\sin(c+dx)}{60bd}-\frac{\left(a^2-12b^2\right)\cos(c+dx)(a\\ &= -\frac{\left(a^4-14a^2b^2+3b^4\right)\cos(c+dx)}{15b^2d}-\frac{a\left(2a^2-27b^2\right)\cos(c+dx)\sin(c\\ &= \frac{3abx}{4}-\frac{\left(a^4-14a^2b^2+3b^4\right)\cos(c+dx)}{15b^2d}-\frac{a\left(2a^2-27b^2\right)\cos(c+dx\\ &= \frac{3abx}{4}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{\left(a^4-14a^2b^2+3b^4\right)\cos(c+dx)}{15b^2d}-\end{aligned}$$

Mathematica [A] time = 0.516802, size = 125, normalized size = 1.08

$$\frac{30\left(10a^2-b^2\right)\cos(c+dx)+5\left(4a^2-3b^2\right)\cos(3(c+dx))+15a\left(4\left(4a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\right.\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (30*(10*a^2 - b^2)*Cos[c + d*x] + 5*(4*a^2 - 3*b^2)*Cos[3*(c + d*x)] - 3*b^
2*Cos[5*(c + d*x)] + 15*a*(4*(3*b*(c + d*x) - 4*a*Log[Cos[(c + d*x)/2]] + 4
```

```
*a*Log[Sin[(c + d*x)/2]]) + 8*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)]))/(24
0*d)
```

Maple [A] time = 0.083, size = 123, normalized size = 1.1

$$\frac{a^2 (\cos (dx+c))^3}{3d} + \frac{a^2 \cos (dx+c)}{d} + \frac{a^2 \ln (\csc (dx+c) - \cot (dx+c))}{d} + \frac{ab (\cos (dx+c))^3 \sin (dx+c)}{2d} + \frac{3\, ab \cos (d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)/d+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+1
/2*a*b*cos(d*x+c)^3*sin(d*x+c)/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+3/4*a*b*x+
3/4/d*a*b*c-1/5*b^2*cos(d*x+c)^5/d
```

Maxima [A] time = 1.00125, size = 131, normalized size = 1.13

$$-\frac{48\, b^2 \cos (dx+c)^5 - 40 \left(2 \cos (dx+c)^3 + 6 \cos (dx+c) - 3 \log (\cos (dx+c)+1) + 3 \log (\cos (dx+c)-1)\right) a^2 - 15}{240\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(48*b^2*cos(d*x + c)^5 - 40*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^2 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b)/d
```

Fricas [A] time = 1.94262, size = 309, normalized size = 2.66

$$-\frac{12\, b^2 \cos (dx+c)^5 - 20\, a^2 \cos (dx+c)^3 - 45\, abdx - 60\, a^2 \cos (dx+c) + 30\, a^2 \log \left(\frac{1}{2} \cos (dx+c) + \frac{1}{2}\right) - 30\, a^2 \log \left(-\right.}{60\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*b^2*cos(d*x + c)^5 - 20*a^2*cos(d*x + c)^3 - 45*a*b*d*x - 60*a^2*
cos(d*x + c) + 30*a^2*log(1/2*cos(d*x + c) + 1/2) - 30*a^2*log(-1/2*cos(d*x
 + c) + 1/2) - 15*(2*a*b*cos(d*x + c)^3 + 3*a*b*cos(d*x + c))*sin(d*x + c))
/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3056, size = 288, normalized size = 2.48

$$\frac{45\,(dx+c)ab + 60\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - \frac{2\left(75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 + 60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 + 30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - \cdots\right.}{\cdots}}{60\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*(d*x + c)*a*b + 60*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(75*a*b*
tan(1/2*d*x + 1/2*c)^9 - 120*a^2*tan(1/2*d*x + 1/2*c)^8 + 60*b^2*tan(1/2*d*
x + 1/2*c)^8 + 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 360*a^2*tan(1/2*d*x + 1/2*c)
^6 - 440*a^2*tan(1/2*d*x + 1/2*c)^4 + 120*b^2*tan(1/2*d*x + 1/2*c)^4 - 30*a
*b*tan(1/2*d*x + 1/2*c)^3 - 280*a^2*tan(1/2*d*x + 1/2*c)^2 - 75*a*b*tan(1/2
*d*x + 1/2*c) - 80*a^2 + 12*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.1109 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=181

$$\frac{a(a^2+28b^2)\cos(c+dx)}{6bd}+\frac{(a^2+12b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{12abd}+\frac{(2a^2+39b^2)\sin(c+dx)\cos(c+dx)}{24d}-\frac{3}{8}x$$

```
[Out] (-3*(4*a^2 - b^2)*x)/8 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (a*(a^2 + 28*b^2
)*Cos[c + d*x])/(6*b*d) + ((2*a^2 + 39*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*
d) + ((a^2 + 12*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(12*a*b*d) - (Cos
[c + d*x]*(a + b*Sin[c + d*x])^3)/(4*b*d) - (Cot[c + d*x]*(a + b*Sin[c + d*
x])^3)/(a*d)
```

Rubi [A] time = 0.521179, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2894, 3049, 3033, 3023, 2735, 3770}

$$\frac{a(a^2+28b^2)\cos(c+dx)}{6bd}+\frac{(a^2+12b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{12abd}+\frac{(2a^2+39b^2)\sin(c+dx)\cos(c+dx)}{24d}-\frac{3}{8}x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```



```
[Out] (-3*(4*a^2 - b^2)*x)/8 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (a*(a^2 + 28*b^2
)*Cos[c + d*x])/(6*b*d) + ((2*a^2 + 39*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*
d) + ((a^2 + 12*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(12*a*b*d) - (Cos
[c + d*x]*(a + b*Sin[c + d*x])^3)/(4*b*d) - (Cot[c + d*x]*(a + b*Sin[c + d*
x])^3)/(a*d)
```


Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx &= -\frac{\cos(c+dx)(a+b\sin(c+dx))^3}{4bd}-\frac{\cot(c+dx)(a+b\sin(c+dx))^3}{ad}\\
&= \frac{\left(a^2+12b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{12abd}-\frac{\cos(c+dx)(a+b\sin}{4bd}\\
&= \frac{\left(2a^2+39b^2\right)\cos(c+dx)\sin(c+dx)}{24d}+\frac{\left(a^2+12b^2\right)\cos(c+dx)(a+}{12abd}\\
&= \frac{a\left(a^2+28b^2\right)\cos(c+dx)}{6bd}+\frac{\left(2a^2+39b^2\right)\cos(c+dx)\sin(c+dx)}{24d}+\\
&= -\frac{3}{8}\left(4a^2-b^2\right)x+\frac{a\left(a^2+28b^2\right)\cos(c+dx)}{6bd}+\frac{\left(2a^2+39b^2\right)\cos(c+}{24d}\\
&= -\frac{3}{8}\left(4a^2-b^2\right)x-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\left(a^2+28b^2\right)\cos(c+}{6bd}
\end{aligned}
$$

Mathematica [A] time = 0.706621, size = 167, normalized size = 0.92

$$
-\frac{3a^2(c+dx)}{2d}-\frac{a^2\sin(2(c+dx))}{4d}-\frac{a^2\cot(c+dx)}{d}+\frac{5ab\cos(c+dx)}{2d}+\frac{ab\cos(3(c+dx))}{6d}+\frac{2ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a^2*(c + d*x))/(2*d) + (3*b^2*(c + d*x))/(8*d) + (5*a*b*Cos[c + d*x])/(
2*d) + (a*b*Cos[3*(c + d*x)])/(6*d) - (a^2*Cot[c + d*x])/d - (2*a*b*Log[Cos
[(c + d*x)/2]])/d + (2*a*b*Log[Sin[(c + d*x)/2]])/d - (a^2*Sin[2*(c + d*x)]
)/(4*d) + (b^2*Sin[2*(c + d*x)])/(4*d) + (b^2*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.073, size = 191, normalized size = 1.1

$$-\frac{a^2 (\cos(dx+c))^5}{d\sin(dx+c)} - \frac{a^2 (\cos(dx+c))^3 \sin(dx+c)}{d} - \frac{3a^2\cos(dx+c)\sin(dx+c)}{2d} - \frac{3a^2x}{2} - \frac{3a^2c}{2d} + \frac{2ab(\cos(dx+ \ldots}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^5-a^2*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a^2*cos(
d*x+c)*sin(d*x+c)/d-3/2*a^2*x-3/2/d*c*a^2+2/3*a*b*cos(d*x+c)^3/d+2*a*b*cos(
d*x+c)/d+2/d*a*b*ln(csc(d*x+c)-cot(d*x+c))+1/4*b^2*cos(d*x+c)^3*sin(d*x+c)/
d+3/8*b^2*cos(d*x+c)*sin(d*x+c)/d+3/8*b^2*x+3/8/d*b^2*c
```

Maxima [A] time = 1.51355, size = 171, normalized size = 0.94

$$-\frac{48\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2 - 32\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(\cos(dx+c)+1)+3\,\log(\cos \ldots}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/96*(48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
 c)))*a^2 - 32*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1)
 + 3*log(cos(d*x + c) - 1))*a*b - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*b^2)/d
```

Fricas [A] time = 1.88502, size = 398, normalized size = 2.2

$$-\frac{6\,b^2\cos(dx+c)^5 - 3\left(4\,a^2-b^2\right)\cos(dx+c)^3 + 24\,ab\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 24\,ab\log\left(-\frac{1}{2}\,\cos(dx+ \ldots}{24\,d\,\mathrm{s}\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*b^2*cos(d*x + c)^5 - 3*(4*a^2 - b^2)*cos(d*x + c)^3 + 24*a*b*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 24*a*b*log(-1/2*cos(d*x + c) + 1/2)*s
in(d*x + c) + 9*(4*a^2 - b^2)*cos(d*x + c) - (16*a*b*cos(d*x + c)^3 - 9*(4*
a^2 - b^2)*d*x + 48*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31508, size = 370, normalized size = 2.04

$$48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\left(4\,a^2-b^2\right)(dx+c)-\frac{12\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(48*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^2*tan(1/2*d*x + 1/2*c) -
 9*(4*a^2 - b^2)*(d*x + c) - 12*(4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*
d*x + 1/2*c) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 - 15*b^2*tan(1/2*d*x + 1/2*
c)^7 + 96*a*b*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*tan(1/2*d*x + 1/2*c)^5 + 9*b^
2*tan(1/2*d*x + 1/2*c)^5 + 192*a*b*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*tan(1/2*
d*x + 1/2*c)^3 - 9*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*a*b*tan(1/2*d*x + 1/2*c
)^2 - 12*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c) + 64*a*b)/(
tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.1110 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=189

$$-\frac{(4a^2-23b^2)\cos(c+dx)}{6d}-\frac{(2a^2-3b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{6a^2d}-\frac{b(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{3ad}+\frac{(3a^2}{}$$

```
[Out] -3*a*b*x + ((3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - ((4*a^2 - 23*b^2
)*Cos[c + d*x])/(6*d) - (b*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(3*a*d)
 - ((2*a^2 - 3*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(6*a^2*d) - (b*Cot
[c + d*x]*(a + b*Sin[c + d*x])^3)/(2*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a
 + b*Sin[c + d*x])^3)/(2*a*d)
```

Rubi [A] time = 0.483869, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3049, 3033, 3023, 2735, 3770}

$$-\frac{(4a^2-23b^2)\cos(c+dx)}{6d}-\frac{(2a^2-3b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{6a^2d}-\frac{b(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{3ad}+\frac{(3a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -3*a*b*x + ((3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - ((4*a^2 - 23*b^2
)*Cos[c + d*x])/(6*d) - (b*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(3*a*d)
 - ((2*a^2 - 3*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(6*a^2*d) - (b*Cot
[c + d*x]*(a + b*Sin[c + d*x])^3)/(2*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a
 + b*Sin[c + d*x])^3)/(2*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{b\cot(c+dx)(a+b\sin(c+dx))^3}{2a^2d} - \frac{\cot(c+dx)\csc(c+dx)(a+b\sin}{2ad}$$

$$= -\frac{\left(2a^2-3b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{6a^2d} - \frac{b\cot(c+dx)(a+b\sin}{2a^2d}$$

$$= -\frac{b\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{3ad} - \frac{\left(2a^2-3b^2\right)\cos(c+dx)(a+}{6a^2d}$$

$$= -\frac{\left(4a^2-23b^2\right)\cos(c+dx)}{6d} - \frac{b\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{3ad} -$$

$$= -3abx - \frac{\left(4a^2-23b^2\right)\cos(c+dx)}{6d} - \frac{b\left(a^2-3b^2\right)\cos(c+dx)\sin(c+}{3ad}$$

$$= -3abx + \frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} - \frac{\left(4a^2-23b^2\right)\cos(c+dx}{6d}$$

Mathematica [A] time = 3.56679, size = 191, normalized size = 1.01

$$\frac{-6\left(4a^2-5b^2\right)\cos(c+dx)+3\left(a^2\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)-12a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^2\log\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-6*(4*a^2 - 5*b^2)*Cos[c + d*x] + 2*b^2*Cos[3*(c + d*x)] + 3*(-24*a*b*c -
24*a*b*d*x - 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 12*a^2*Log[C
os[(c + d*x)/2]] - 8*b^2*Log[Cos[(c + d*x)/2]] - 12*a^2*Log[Sin[(c + d*x)/2
]] + 8*b^2*Log[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 - 4*a*b*Sin[2*(c
+ d*x)] + 8*a*b*Tan[(c + d*x)/2]))/(24*d)
```

Maple [A] time = 0.089, size = 208, normalized size = 1.1

$$-\frac{a^2(\cos(dx+c))^5}{2d(\sin(dx+c))^2}-\frac{a^2(\cos(dx+c))^3}{2d}-\frac{3a^2\cos(dx+c)}{2d}-\frac{3a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-2\frac{ab(\cos(dx+c))}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a^2*cos(d*x+c)^3/d-3/2*a^2*cos(d*x
+c)/d-3/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/d*a*b/sin(d*x+c)*cos(d*x+c)^5-2
*a*b*cos(d*x+c)^3*sin(d*x+c)/d-3*a*b*cos(d*x+c)*sin(d*x+c)/d-3*a*b*x-3/d*a*
b*c+1/3*b^2*cos(d*x+c)^3/d+b^2*cos(d*x+c)/d+1/d*b^2*ln(csc(d*x+c)-cot(d*x+c
))
```

Maxima [A] time = 1.49058, size = 203, normalized size = 1.07

$$-\frac{12\left(3dx+3c+\frac{3\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab-2\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(d\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*(12*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
 c)))*a*b - 2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1)
+ 3*log(cos(d*x + c) - 1))*b^2 - 3*a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1)
 - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.85399, size = 518, normalized size = 2.74

$$\frac{4b^2\cos(dx+c)^5-36abdx\cos(dx+c)^2+36abdx-4\left(3a^2-2b^2\right)\cos(dx+c)^3+6\left(3a^2-2b^2\right)\cos(dx+c)+3\left(\left(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(4*b^2*cos(d*x + c)^5 - 36*a*b*d*x*cos(d*x + c)^2 + 36*a*b*d*x - 4*(3*
a^2 - 2*b^2)*cos(d*x + c)^3 + 6*(3*a^2 - 2*b^2)*cos(d*x + c) + 3*((3*a^2 -
2*b^2)*cos(d*x + c)^2 - 3*a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) - 3*((3*
a^2 - 2*b^2)*cos(d*x + c)^2 - 3*a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2) -
```

```
12*(a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33913, size = 340, normalized size = 1.8

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)ab+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\left(3\,a^2-2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{3\left(18\,a^2\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x + c)*a*b + 24*a*b*tan(1/2*d*x
+ 1/2*c) - 12*(3*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 3*(18*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x +
1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 16*(3*a*b*tan(1/2*d*x + 1/2*c)^5 - 3
*a^2*tan(1/2*d*x + 1/2*c)^4 + 6*b^2*tan(1/2*d*x + 1/2*c)^4 - 6*a^2*tan(1/2*
d*x + 1/2*c)^2 + 6*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c)
- 3*a^2 + 4*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.1111 $\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=133

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
 d*x]^3)/(3*d)
```

Rubi [A] time = 0.162409, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 2591, 288, 321, 203, 2592, 206, 3473, 8}

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
 d*x]^3)/(3*d)
```


Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^2(c+dx)+2ab\cos(c+dx)\cot^3(c+dx)+a^2\cot^4(c+dx\right.$$

$$= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^3(c+dx)\,dx+b^2\int \cos^2(c+dx)$$

$$= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\cos(c+\right.}{d}$$

$$= \frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}-\frac{a^2}{}$$

$$= a^2x-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot(}{2d}$$

$$= a^2x-\frac{3b^2x}{2}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b}{}$$

Mathematica [B] time = 6.18259, size = 293, normalized size = 2.2

$$\frac{\left(2a^2-3b^2\right)(c+dx)}{2d}+\frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{6d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right.}{6d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*(c + d*x))/(2*d) - (2*a*b*Cos[c + d*x])/d + ((4*a^2*Cos[(c
 + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c
+ d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (3*
a*b*Log[Cos[(c + d*x)/2]])/d - (3*a*b*Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(
c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 3*b^2*S
in[(c + d*x)/2]))/(6*d) - (b^2*Sin[2*(c + d*x)])/(4*d) + (a^2*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A] time = 0.087, size = 199, normalized size = 1.5

$$-\frac{a^2(\cot(dx+c))^3}{3d}+\frac{a^2\cot(dx+c)}{d}+a^2x+\frac{a^2c}{d}-\frac{ab(\cos(dx+c))^5}{d(\sin(dx+c))^2}-\frac{ab(\cos(dx+c))^3}{d}-3\frac{ab\cos(dx+c)}{d}-3\frac{ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d+a^2*x+1/d*c*a^2-1/d*a*b/sin(d*x+c)
^2*cos(d*x+c)^5-a*b*cos(d*x+c)^3/d-3*a*b*cos(d*x+c)/d-3/d*a*b*ln(csc(d*x+c)
-cot(d*x+c))-1/d*b^2/sin(d*x+c)*cos(d*x+c)^5-b^2*cos(d*x+c)^3*sin(d*x+c)/d-
3/2*b^2*cos(d*x+c)*sin(d*x+c)/d-3/2*b^2*x-3/2/d*b^2*c
```

Maxima [A] time = 1.51813, size = 186, normalized size = 1.4

$$\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b^2+3\,ab\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\cos\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*(3*d*x
 + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^2 + 3*a*
b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c
) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.8642, size = 539, normalized size = 4.05

$$\frac{3\,b^2\cos(dx+c)^5+4\left(2\,a^2-3\,b^2\right)\cos(dx+c)^3+9\left(ab\cos(dx+c)^2-ab\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*b^2*cos(d*x + c)^5 + 4*(2*a^2 - 3*b^2)*cos(d*x + c)^3 + 9*(a*b*cos(d
*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a*b*cos(d*x
+ c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(2*a^2 - 3*b^2)
```

```
*cos(d*x + c) + 3*((2*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)^
3 - (2*a^2 - 3*b^2)*d*x + 6*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36112, size = 325, normalized size = 2.44

$$a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 6\,ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 72\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 15\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 12\,b^2\tan\left(\frac{1}{2}\,dx + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 72*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2
*d*x + 1/2*c) + 12*(2*a^2 - 3*b^2)*(d*x + c) + 24*(b^2*tan(1/2*d*x + 1/2*c)
^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x + 1/2*
c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.1112 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=178

$$-\frac{b^2\left(39a^2+2b^2\right)\cos(c+dx)}{24a^2d}-\frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{12a^2d}+1$$

```
[Out] 2*a*b*x - (3*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) - (b^2*(39*a^2 + 2*
b^2)*Cos[c + d*x])/(24*a^2*d) + (17*a*b*Cot[c + d*x])/(12*d) + (5*Cot[c + d
*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^2)/(8*d) + (b*Cot[c + d*x]*Csc[c + d*
x]^2*(a + b*Sin[c + d*x])^3)/(12*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a +
 b*Sin[c + d*x])^3)/(4*a*d)
```

Rubi [A] time = 0.462778, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3047, 3031, 3023, 2735, 3770}

$$-\frac{b^2\left(39a^2+2b^2\right)\cos(c+dx)}{24a^2d}-\frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{12a^2d}+1$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] 2*a*b*x - (3*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) - (b^2*(39*a^2 + 2*
b^2)*Cos[c + d*x])/(24*a^2*d) + (17*a*b*Cot[c + d*x])/(12*d) + (5*Cot[c + d
*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^2)/(8*d) + (b*Cot[c + d*x]*Csc[c + d*
x]^2*(a + b*Sin[c + d*x])^3)/(12*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a +
 b*Sin[c + d*x])^3)/(4*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{12a^2d} - \frac{\cot(c+dx)\csc^3(c+ \ldots}{\ldots}$$

$$= \frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{8d} + \frac{b\cot(c+dx)\csc^2(c \ldots}{1 \ldots}$$

$$= \frac{17ab\cot(c+dx)}{12d} + \frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{8d} + \ldots$$

$$= -\frac{b^2\left(39a^2+2b^2\right)\cos(c+dx)}{24a^2d} + \frac{17ab\cot(c+dx)}{12d} + \frac{5\cot(c+dx)\csc \ldots}{\ldots}$$

$$= 2abx - \frac{b^2\left(39a^2+2b^2\right)\cos(c+dx)}{24a^2d} + \frac{17ab\cot(c+dx)}{12d} + \frac{5\cot(c+ \ldots}{\ldots}$$

$$= 2abx - \frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^2\left(39a^2+2b^2\right)\cos(c+ \ldots}{24a^2d}$$

Mathematica [A] time = 2.71773, size = 270, normalized size = 1.52

$$\frac{-3a^2\csc^4\left(\frac{1}{2}(c+dx)\right)+30a^2\csc^2\left(\frac{1}{2}(c+dx)\right)+3a^2\sec^4\left(\frac{1}{2}(c+dx)\right)-30a^2\sec^2\left(\frac{1}{2}(c+dx)\right)+72a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right) \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (384*a*b*c + 384*a*b*d*x - 192*b^2*Cos[c + d*x] + 256*a*b*Cot[(c + d*x)/2]
+ 30*a^2*Csc[(c + d*x)/2]^2 - 24*b^2*Csc[(c + d*x)/2]^2 - 3*a^2*Csc[(c + d*
x)/2]^4 - 72*a^2*Log[Cos[(c + d*x)/2]] + 288*b^2*Log[Cos[(c + d*x)/2]] + 72
*a^2*Log[Sin[(c + d*x)/2]] - 288*b^2*Log[Sin[(c + d*x)/2]] - 30*a^2*Sec[(c
+ d*x)/2]^2 + 24*b^2*Sec[(c + d*x)/2]^2 + 3*a^2*Sec[(c + d*x)/2]^4 + 128*a*
b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 8*a*b*Csc[(c + d*x)/2]^4*Sin[c + d*x]
 - 256*a*b*Tan[(c + d*x)/2])/(192*d)
```

Maple [A] time = 0.088, size = 223, normalized size = 1.3

$$-\frac{a^2(\cos(dx+c))^5}{4d(\sin(dx+c))^4}+\frac{a^2(\cos(dx+c))^5}{8d(\sin(dx+c))^2}+\frac{a^2(\cos(dx+c))^3}{8d}+\frac{3a^2\cos(dx+c)}{8d}+\frac{3a^2\ln(\csc(dx+c)-\cot(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+1/
8*a^2*cos(d*x+c)^3/d+3/8*a^2*cos(d*x+c)/d+3/8/d*a^2*ln(csc(d*x+c)-cot(d*x+c
))-2/3*a*b*cot(d*x+c)^3/d+2*a*b*cot(d*x+c)/d+2*a*b*x+2/d*a*b*c-1/2/d*b^2/si
n(d*x+c)^2*cos(d*x+c)^5-1/2*b^2*cos(d*x+c)^3/d-3/2*b^2*cos(d*x+c)/d-3/2/d*b
^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.61168, size = 224, normalized size = 1.26

$$\frac{32\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)ab-3\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)-1)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/48*(32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a*b - 3*a^2*
(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 +
 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 12*b^2*(2*cos(d*
x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*
log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.83193, size = 666, normalized size = 3.74

$$\frac{96\,abdx\cos(dx+c)^4-48\,b^2\cos(dx+c)^5-192\,abdx\cos(dx+c)^2+96\,abdx-30\left(a^2-4\,b^2\right)\cos(dx+c)^3+18\left(a^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(96*a*b*d*x*cos(d*x + c)^4 - 48*b^2*cos(d*x + c)^5 - 192*a*b*d*x*cos(d
*x + c)^2 + 96*a*b*d*x - 30*(a^2 - 4*b^2)*cos(d*x + c)^3 + 18*(a^2 - 4*b^2)
*cos(d*x + c) - 9*((a^2 - 4*b^2)*cos(d*x + c)^4 - 2*(a^2 - 4*b^2)*cos(d*x +
 c)^2 + a^2 - 4*b^2)*log(1/2*cos(d*x + c) + 1/2) + 9*((a^2 - 4*b^2)*cos(d*x
 + c)^4 - 2*(a^2 - 4*b^2)*cos(d*x + c)^2 + a^2 - 4*b^2)*log(-1/2*cos(d*x +
c) + 1/2) - 32*(4*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37084, size = 329, normalized size = 1.85

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+384\,(dx+c)ab-24}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 384*(d*x + c)*a*
b - 240*a*b*tan(1/2*d*x + 1/2*c) + 72*(a^2 - 4*b^2)*log(abs(tan(1/2*d*x + 1
/2*c))) - 384*b^2/(tan(1/2*d*x + 1/2*c)^2 + 1) - (150*a^2*tan(1/2*d*x + 1/2
*c)^4 - 600*b^2*tan(1/2*d*x + 1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 2
4*a^2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1
/2*d*x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.1113 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=209

$$-\frac{\left(-14a^2b^2+3a^4+b^4\right)\cot(c+dx)}{15a^2d}+\frac{b\left(27a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{60ad}+\frac{\left(12a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)(a}{30a^2d}$$

```
[Out] b^2*x - (3*a*b*ArcTanh[Cos[c + d*x]])/(4*d) - ((3*a^4 - 14*a^2*b^2 + b^4)*C
ot[c + d*x])/(15*a^2*d) + (b*(27*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(6
0*a*d) + ((12*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2
)/(30*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(10*a
^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(5*a*d)
```

Rubi [A] time = 0.515992, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2893, 3047, 3031, 3021, 2735, 3770}

$$-\frac{\left(-14a^2b^2+3a^4+b^4\right)\cot(c+dx)}{15a^2d}+\frac{b\left(27a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{60ad}+\frac{\left(12a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)(a}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] b^2*x - (3*a*b*ArcTanh[Cos[c + d*x]])/(4*d) - ((3*a^4 - 14*a^2*b^2 + b^4)*C
ot[c + d*x])/(15*a^2*d) + (b*(27*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(6
0*a*d) + ((12*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2
)/(30*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(10*a
^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(5*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{10a^2d} - \frac{\cot(c+dx)\csc^4(c\ldots}{}$$

$$= \frac{\left(12a^2-b^2\right)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{30a^2d} + \frac{b\cot(c+\ldots}{}$$

$$= \frac{b\left(27a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{60ad} + \frac{\left(12a^2-b^2\right)\cot(c+dx)\csc\ldots}{3\ldots}$$

$$= -\frac{\left(3a^4-14a^2b^2+b^4\right)\cot(c+dx)}{15a^2d} + \frac{b\left(27a^2-2b^2\right)\cot(c+dx)\csc(\ldots}{60ad}$$

$$= b^2x - \frac{\left(3a^4-14a^2b^2+b^4\right)\cot(c+dx)}{15a^2d} + \frac{b\left(27a^2-2b^2\right)\cot(c+dx)\ldots}{60ad}$$

$$= b^2x - \frac{3ab\tanh^{-1}(\cos(c+dx))}{4d} - \frac{\left(3a^4-14a^2b^2+b^4\right)\cot(c+dx)}{15a^2d} + \ldots$$

Mathematica [A] time = 1.45704, size = 285, normalized size = 1.36

$$\frac{\left(640b^2-96a^2\right)\cot\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)\left(\left(21a^2-20b^2\right)\sin(c+dx)-30ab\right)+96a^2\tan\left(\frac{1}{2}(c+dx)\right)+192a^2\sin\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (960*b^2*c + 960*b^2*d*x + (-96*a^2 + 640*b^2)*Cot[(c + d*x)/2] + 300*a*b*C
sc[(c + d*x)/2]^2 - 720*a*b*Log[Cos[(c + d*x)/2]] + 720*a*b*Log[Sin[(c + d*
x)/2]] - 300*a*b*Sec[(c + d*x)/2]^2 + 30*a*b*Sec[(c + d*x)/2]^4 - 336*a^2*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*b^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^
4 + 192*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - 3*a^2*Csc[(c + d*x)/2]^6*Si
n[c + d*x] + Csc[(c + d*x)/2]^4*(-30*a*b + (21*a^2 - 20*b^2)*Sin[c + d*x])
+ 96*a^2*Tan[(c + d*x)/2] - 640*b^2*Tan[(c + d*x)/2])/(960*d)
```

Maple [A] time = 0.089, size = 165, normalized size = 0.8

$$-\frac{a^2(\cos(dx+c))^5}{5d(\sin(dx+c))^5}-\frac{ab(\cos(dx+c))^5}{2d(\sin(dx+c))^4}+\frac{ab(\cos(dx+c))^5}{4d(\sin(dx+c))^2}+\frac{ab(\cos(dx+c))^3}{4d}+\frac{3ab\cos(dx+c)}{4d}+\frac{3ab\ln(\csc(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1/2/d*a*b/sin(d*x+c)^4*cos(d*x+c)^5+1/
4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^5+1/4*a*b*cos(d*x+c)^3/d+3/4*a*b*cos(d*x+c)
/d+3/4/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/3*b^2*cot(d*x+c)^3/d+b^2*cot(d*x+c
)/d+b^2*x+1/d*b^2*c
```

Maxima [A] time = 1.51504, size = 166, normalized size = 0.79

$$\frac{40\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b^2-15\,ab\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)-1)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/120*(40*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*b^2 - 15*a*
b*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) - 24*a^2/tan(d*x
 + c)^5)/d
```

Fricas [A] time = 1.82271, size = 637, normalized size = 3.05

$$-\frac{8\left(3\,a^2-20\,b^2\right)\cos(dx+c)^5+280\,b^2\cos(dx+c)^3-120\,b^2\cos(dx+c)+45\left(ab\cos(dx+c)^4-2\,ab\cos(dx+c)^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/120*(8*(3*a^2 - 20*b^2)*cos(d*x + c)^5 + 280*b^2*cos(d*x + c)^3 - 120*b^
2*cos(d*x + c) + 45*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 45*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*
x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(4*b^2*d*x*c
os(d*x + c)^4 - 8*b^2*d*x*cos(d*x + c)^2 - 5*a*b*cos(d*x + c)^3 + 4*b^2*d*x
 + 3*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35887, size = 355, normalized size = 1.7

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 15*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b*tan(1/2*
d*x + 1/2*c)^2 + 480*(d*x + c)*b^2 + 360*a*b*log(abs(tan(1/2*d*x + 1/2*c)))
 + 30*a^2*tan(1/2*d*x + 1/2*c) - 300*b^2*tan(1/2*d*x + 1/2*c) - (822*a*b*ta
n(1/2*d*x + 1/2*c)^5 + 30*a^2*tan(1/2*d*x + 1/2*c)^4 - 300*b^2*tan(1/2*d*x
+ 1/2*c)^4 - 120*a*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c)^2
 + 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan
(1/2*d*x + 1/2*c)^5)/d
```

3.1114 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=236

$$-\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{b(13a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{60ad}-\frac{(-80a^2b^2+15a^4+12b^4)\cot(c+dx)\,\mathrm{cs}}{240a^2d}$$

```
[Out] -((a^2 + 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a*b*Cot[c + d*x])/(5*d)
- ((15*a^4 - 80*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(240*a^2*d) +
(b*(13*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a*d) + ((35*a^2 - 6*b^
2)*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(120*a^2*d) + (b*Cot
[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(10*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(6*a*d)
```

Rubi [A] time = 0.603552, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2893, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$-\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{b(13a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{60ad}-\frac{(-80a^2b^2+15a^4+12b^4)\cot(c+dx)\,\mathrm{cs}}{240a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2 + 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a*b*Cot[c + d*x])/(5*d)
- ((15*a^4 - 80*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(240*a^2*d) +
(b*(13*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a*d) + ((35*a^2 - 6*b^
2)*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(120*a^2*d) + (b*Cot
[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(10*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(6*a*d)
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{10a^2d} - \frac{\cot(c+dx)\csc^5}{}$$
$$= \frac{(35a^2-6b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{120a^2d} + \frac{b\,\text{co}}{}$$
$$= \frac{b(13a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{60ad} + \frac{(35a^2-6b^2)\cot(c+}{}$$
$$= -\frac{(15a^4-80a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{240a^2d} + \frac{b(13a^2-2b}{}$$
$$= -\frac{(15a^4-80a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{240a^2d} + \frac{b(13a^2-2b}{}$$
$$= -\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{(15a^4-80a^2b^2+12b^4)\cot(}{240a^2d}$$
$$= -\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2ab\cot(c+dx)}{5d} - \frac{(15a^4-8}{}$$

Mathematica [A] time = 0.825759, size = 319, normalized size = 1.35

$$\frac{-30(a^2-10b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)+6\csc^4\left(\frac{1}{2}(c+dx)\right)(5a^2+14ab\sin(c+dx)-5b^2)+5a^2\sec^6\left(\frac{1}{2}(c+dx)\right)-30a^2\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-384*a*b*Cot[(c + d*x)/2] - 30*(a^2 - 10*b^2)*Csc[(c + d*x)/2]^2 - 120*a^2
*Log[Cos[(c + d*x)/2]] - 720*b^2*Log[Cos[(c + d*x)/2]] + 120*a^2*Log[Sin[(c
 + d*x)/2]] + 720*b^2*Log[Sin[(c + d*x)/2]] + 30*a^2*Sec[(c + d*x)/2]^2 - 3
00*b^2*Sec[(c + d*x)/2]^2 - 30*a^2*Sec[(c + d*x)/2]^4 + 30*b^2*Sec[(c + d*x
)/2]^4 + 5*a^2*Sec[(c + d*x)/2]^6 - 1344*a*b*Csc[c + d*x]^3*Sin[(c + d*x)/2
]^4 + 768*a*b*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - a*Csc[(c + d*x)/2]^6*(5*a
 + 12*b*Sin[c + d*x]) + 6*Csc[(c + d*x)/2]^4*(5*a^2 - 5*b^2 + 14*a*b*Sin[c
+ d*x]) + 384*a*b*Tan[(c + d*x)/2])/(1920*d)
```

Maple [A] time = 0.089, size = 253, normalized size = 1.1

$$-\frac{a^2(\cos(dx+c))^5}{6d(\sin(dx+c))^6} - \frac{a^2(\cos(dx+c))^5}{24d(\sin(dx+c))^4} + \frac{a^2(\cos(dx+c))^5}{48d(\sin(dx+c))^2} + \frac{a^2(\cos(dx+c))^3}{48d} + \frac{a^2\cos(dx+c)}{16d} + \frac{a^2\ln(\csc(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*a^2/sin(d*x+c)^4*cos(d*x+c)^5+1
/48/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a^2*cos(d*x+c)^3/d+1/16*a^2*cos(d*
x+c)/d+1/16/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/5/d*a*b/sin(d*x+c)^5*cos(d*x+
c)^5-1/4/d*b^2/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*b^2/sin(d*x+c)^2*cos(d*x+c)^
5+1/8*b^2*cos(d*x+c)^3/d+3/8*b^2*cos(d*x+c)/d+3/8/d*b^2*ln(csc(d*x+c)-cot(d
*x+c))
```

Maxima [A] time = 1.15141, size = 243, normalized size = 1.03

$$\frac{5\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-30\,b^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(d\right.}{\cos(dx+c)^4-2\,\cos(dx+c}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/480*(5*a^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 30*b^2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x
+ c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3
*log(cos(d*x + c) - 1)) - 192*a*b/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.90136, size = 675, normalized size = 2.86

$$\frac{192\,ab\cos\left(dx+c\right)^5\sin\left(dx+c\right)+30\left(a^2-10\,b^2\right)\cos\left(dx+c\right)^5+80\left(a^2+6\,b^2\right)\cos\left(dx+c\right)^3-30\left(a^2+6\,b^2\right)\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(192*a*b*cos(d*x + c)^5*sin(d*x + c) + 30*(a^2 - 10*b^2)*cos(d*x + c)
^5 + 80*(a^2 + 6*b^2)*cos(d*x + c)^3 - 30*(a^2 + 6*b^2)*cos(d*x + c) - 15*(
(a^2 + 6*b^2)*cos(d*x + c)^6 - 3*(a^2 + 6*b^2)*cos(d*x + c)^4 + 3*(a^2 + 6*
b^2)*cos(d*x + c)^2 - a^2 - 6*b^2)*log(1/2*cos(d*x + c) + 1/2) + 15*((a^2 +
 6*b^2)*cos(d*x + c)^6 - 3*(a^2 + 6*b^2)*cos(d*x + c)^4 + 3*(a^2 + 6*b^2)*c
os(d*x + c)^2 - a^2 - 6*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^
6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37018, size = 417, normalized size = 1.77

$$\frac{5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,ab\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a*b*tan(1/2*d*x + 1/2*c)^5 - 15*a
^2*tan(1/2*d*x + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*a*b*tan(1/2
*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 240*b^2*tan(1/2*d*x + 1/2
*c)^2 + 240*a*b*tan(1/2*d*x + 1/2*c) + 120*(a^2 + 6*b^2)*log(abs(tan(1/2*d*
x + 1/2*c))) - (294*a^2*tan(1/2*d*x + 1/2*c)^6 + 1764*b^2*tan(1/2*d*x + 1/2
*c)^6 + 240*a*b*tan(1/2*d*x + 1/2*c)^5 - 15*a^2*tan(1/2*d*x + 1/2*c)^4 - 24
0*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*a*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(
1/2*d*x + 1/2*c)^2 + 30*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a*b*tan(1/2*d*x + 1
/2*c) + 5*a^2)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.1115 $\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=261

$$-\frac{(2a^2+7b^2)\cot(c+dx)}{35d}+\frac{b(53a^2-12b^2)\cot(c+dx)\csc^3(c+dx)}{420ad}-\frac{(-18a^2b^2+3a^4+4b^4)\cot(c+dx)\csc^2(c+dx)}{105a^2d}+$$

```
[Out] -(a*b*ArcTanh[Cos[c + d*x]])/(8*d) - ((2*a^2 + 7*b^2)*Cot[c + d*x])/(35*d)
- (a*b*Cot[c + d*x]*Csc[c + d*x])/(8*d) - ((3*a^4 - 18*a^2*b^2 + 4*b^4)*Cot
[c + d*x]*Csc[c + d*x]^2)/(105*a^2*d) + (b*(53*a^2 - 12*b^2)*Cot[c + d*x]*C
sc[c + d*x]^3)/(420*a*d) + (2*(4*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a
+ b*Sin[c + d*x])^2)/(35*a^2*d) + (2*b*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*S
in[c + d*x])^3)/(21*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*
x])^3)/(7*a*d)
```

Rubi [A] time = 0.628624, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2893, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$-\frac{(2a^2+7b^2)\cot(c+dx)}{35d}+\frac{b(53a^2-12b^2)\cot(c+dx)\csc^3(c+dx)}{420ad}-\frac{(-18a^2b^2+3a^4+4b^4)\cot(c+dx)\csc^2(c+dx)}{105a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -(a*b*ArcTanh[Cos[c + d*x]])/(8*d) - ((2*a^2 + 7*b^2)*Cot[c + d*x])/(35*d)
- (a*b*Cot[c + d*x]*Csc[c + d*x])/(8*d) - ((3*a^4 - 18*a^2*b^2 + 4*b^4)*Cot
[c + d*x]*Csc[c + d*x]^2)/(105*a^2*d) + (b*(53*a^2 - 12*b^2)*Cot[c + d*x]*C
sc[c + d*x]^3)/(420*a*d) + (2*(4*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a
+ b*Sin[c + d*x])^2)/(35*a^2*d) + (2*b*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*S
in[c + d*x])^3)/(21*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*
x])^3)/(7*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
```

```
b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{2b\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{21a^2d} - \frac{\cot(c+dx)\csc^6(c}{}$$
$$= \frac{2\left(4a^2-b^2\right)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{35a^2d} + \frac{2b\cot(c}{}$$
$$= \frac{b\left(53a^2-12b^2\right)\cot(c+dx)\csc^3(c+dx)}{420ad} + \frac{2\left(4a^2-b^2\right)\cot(c+dx)}{}$$
$$= -\frac{\left(3a^4-18a^2b^2+4b^4\right)\cot(c+dx)\csc^2(c+dx)}{105a^2d} + \frac{b\left(53a^2-12b^2\right)c}{}$$
$$= -\frac{\left(3a^4-18a^2b^2+4b^4\right)\cot(c+dx)\csc^2(c+dx)}{105a^2d} + \frac{b\left(53a^2-12b^2\right)c}{}$$
$$= -\frac{ab\cot(c+dx)\csc(c+dx)}{8d} - \frac{\left(3a^4-18a^2b^2+4b^4\right)\cot(c+dx)\csc}{105a^2d}$$
$$= -\frac{ab\tanh^{-1}(\cos(c+dx))}{8d} - \frac{\left(2a^2+7b^2\right)\cot(c+dx)}{35d} - \frac{ab\cot(c+dx}{8}$$

Mathematica [A] time = 1.29396, size = 322, normalized size = 1.23

$$-\frac{\csc^7(c+dx)\left(840\left(6a^2+b^2\right)\cos(c+dx)+168\left(14a^2-b^2\right)\cos(3(c+dx))+336a^2\cos(5(c+dx))-48a^2\cos(7(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^7*(840*(6*a^2 + b^2)*Cos[c + d*x] + 168*(14*a^2 - b^2)*Cos[3
*(c + d*x)] + 336*a^2*Cos[5*(c + d*x)] - 504*b^2*Cos[5*(c + d*x)] - 48*a^2*
Cos[7*(c + d*x)] - 168*b^2*Cos[7*(c + d*x)] + 3675*a*b*Log[Cos[(c + d*x)/2]
]*Sin[c + d*x] - 3675*a*b*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 2170*a*b*Sin
[2*(c + d*x)] - 2205*a*b*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] + 2205*a*b*
Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 3080*a*b*Sin[4*(c + d*x)] + 735*a*
b*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*a*b*Log[Sin[(c + d*x)/2]]*Si
n[5*(c + d*x)] + 210*a*b*Sin[6*(c + d*x)] - 105*a*b*Log[Cos[(c + d*x)/2]]*S
in[7*(c + d*x)] + 105*a*b*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(53760*d
)
```

Maple [A] time = 0.106, size = 194, normalized size = 0.7

$$-\frac{a^2(\cos(dx+c))^5}{7d(\sin(dx+c))^7} - \frac{2a^2(\cos(dx+c))^5}{35d(\sin(dx+c))^5} - \frac{ab(\cos(dx+c))^5}{3d(\sin(dx+c))^6} - \frac{ab(\cos(dx+c))^5}{12d(\sin(dx+c))^4} + \frac{ab(\cos(dx+c))^5}{24d(\sin(dx+c))^2} + \frac{ab(\cos(dx}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/7/d*a^2/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a^2/sin(d*x+c)^5*cos(d*x+c)^5-1
/3/d*a*b/sin(d*x+c)^6*cos(d*x+c)^5-1/12/d*a*b/sin(d*x+c)^4*cos(d*x+c)^5+1/2
4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^5+1/24*a*b*cos(d*x+c)^3/d+1/8*a*b*cos(d*x+c
)/d+1/8/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*b^2/sin(d*x+c)^5*cos(d*x+c)^5
```

Maxima [A] time = 1.112, size = 181, normalized size = 0.69

$$\frac{35\,ab\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{336\,b^2}{\tan(dx+c)^5}-\frac{48\left(7\,\tan(d\right.}{\tan(d}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1680*(35*a*b*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(c
os(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x +
c) + 1) + 3*log(cos(d*x + c) - 1)) - 336*b^2/tan(d*x + c)^5 - 48*(7*tan(d*x
 + c)^2 + 5)*a^2/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.85664, size = 653, normalized size = 2.5

$$-\frac{48\left(2\,a^2+7\,b^2\right)\cos\left(dx+c\right)^7-336\left(a^2+b^2\right)\cos\left(dx+c\right)^5+105\left(ab\cos\left(dx+c\right)^6-3\,ab\cos\left(dx+c\right)^4+3\,ab\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1680*(48*(2*a^2 + 7*b^2)*cos(d*x + c)^7 - 336*(a^2 + b^2)*cos(d*x + c)^5
 + 105*(a*b*cos(d*x + c)^6 - 3*a*b*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)^2 -
a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(a*b*cos(d*x + c)^6 - 3
*a*b*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) - 70*(3*a*b*cos(d*x + c)^5 + 8*a*b*cos(d*x + c)^3 - 3*a*b*
cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*c
os(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32978, size = 468, normalized size = 1.79

$$\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+84\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-210\,ab\tan\left(\frac{1}{2}\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 + 70*a*b*tan(1/2*d*x + 1/2*c)^6 - 21
*a^2*tan(1/2*d*x + 1/2*c)^5 + 84*b^2*tan(1/2*d*x + 1/2*c)^5 - 210*a*b*tan(1
/2*d*x + 1/2*c)^4 - 105*a^2*tan(1/2*d*x + 1/2*c)^3 - 420*b^2*tan(1/2*d*x +
1/2*c)^3 - 210*a*b*tan(1/2*d*x + 1/2*c)^2 + 1680*a*b*log(abs(tan(1/2*d*x +
1/2*c))) + 315*a^2*tan(1/2*d*x + 1/2*c) + 840*b^2*tan(1/2*d*x + 1/2*c) - (4
356*a*b*tan(1/2*d*x + 1/2*c)^7 + 315*a^2*tan(1/2*d*x + 1/2*c)^6 + 840*b^2*t
an(1/2*d*x + 1/2*c)^6 - 210*a*b*tan(1/2*d*x + 1/2*c)^5 - 105*a^2*tan(1/2*d*
x + 1/2*c)^4 - 420*b^2*tan(1/2*d*x + 1/2*c)^4 - 210*a*b*tan(1/2*d*x + 1/2*c
)^3 - 21*a^2*tan(1/2*d*x + 1/2*c)^2 + 84*b^2*tan(1/2*d*x + 1/2*c)^2 + 70*a*
b*tan(1/2*d*x + 1/2*c) + 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.1116 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=354

$$\frac{b\left(27a^2+4b^2\right)\cos^3(c+dx)}{315d}-\frac{b\left(27a^2+4b^2\right)\cos(c+dx)}{105d}-\frac{\left(-93a^2b^2+20a^4+24b^4\right)\sin^4(c+dx)\cos(c+dx)}{2520bd}-\frac{5\left(a^2\right.}{}$$

```
[Out] (a*(8*a^2 + 9*b^2)*x)/128 - (b*(27*a^2 + 4*b^2)*Cos[c + d*x])/(105*d) + (b*
(27*a^2 + 4*b^2)*Cos[c + d*x]^3)/(315*d) - (a*(8*a^2 + 9*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(128*d) - (a*(40*a^4 - 188*a^2*b^2 + 189*b^4)*Cos[c + d*x]*Si
n[c + d*x]^3)/(4032*b^2*d) - ((20*a^4 - 93*a^2*b^2 + 24*b^4)*Cos[c + d*x]*S
in[c + d*x]^4)/(2520*b*d) - (a*(20*a^2 - 87*b^2)*Cos[c + d*x]*Sin[c + d*x]^
3*(a + b*Sin[c + d*x])^2)/(1008*b^2*d) - (5*(a^2 - 4*b^2)*Cos[c + d*x]*Sin[
c + d*x]^3*(a + b*Sin[c + d*x])^3)/(126*b^2*d) + (5*a*Cos[c + d*x]*Sin[c +
d*x]^3*(a + b*Sin[c + d*x])^4)/(72*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^4*(a
 + b*Sin[c + d*x])^4)/(9*b*d)
```

Rubi [A] time = 0.927725, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2895, 3049, 3033, 3023, 2748, 2635, 8, 2633}

$$\frac{b\left(27a^2+4b^2\right)\cos^3(c+dx)}{315d}-\frac{b\left(27a^2+4b^2\right)\cos(c+dx)}{105d}-\frac{\left(-93a^2b^2+20a^4+24b^4\right)\sin^4(c+dx)\cos(c+dx)}{2520bd}-\frac{5\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (a*(8*a^2 + 9*b^2)*x)/128 - (b*(27*a^2 + 4*b^2)*Cos[c + d*x])/(105*d) + (b*
(27*a^2 + 4*b^2)*Cos[c + d*x]^3)/(315*d) - (a*(8*a^2 + 9*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(128*d) - (a*(40*a^4 - 188*a^2*b^2 + 189*b^4)*Cos[c + d*x]*Si
n[c + d*x]^3)/(4032*b^2*d) - ((20*a^4 - 93*a^2*b^2 + 24*b^4)*Cos[c + d*x]*S
in[c + d*x]^4)/(2520*b*d) - (a*(20*a^2 - 87*b^2)*Cos[c + d*x]*Sin[c + d*x]^
3*(a + b*Sin[c + d*x])^2)/(1008*b^2*d) - (5*(a^2 - 4*b^2)*Cos[c + d*x]*Sin[
c + d*x]^3*(a + b*Sin[c + d*x])^3)/(126*b^2*d) + (5*a*Cos[c + d*x]*Sin[c +
d*x]^3*(a + b*Sin[c + d*x])^4)/(72*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^4*(a
 + b*Sin[c + d*x])^4)/(9*b*d)
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{5a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^4}{72b^2d} - \frac{\cos(c+dx)\sin}{}$$
$$= -\frac{5\left(a^2-4b^2\right)\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^3}{126b^2d} + \frac{5a\,c}{}$$
$$= -\frac{a\left(20a^2-87b^2\right)\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2}{1008b^2d} -$$
$$= -\frac{\left(20a^4-93a^2b^2+24b^4\right)\cos(c+dx)\sin^4(c+dx)}{2520bd} - \frac{a\left(20a^2-8\right.}{}$$
$$= -\frac{a\left(40a^4-188a^2b^2+189b^4\right)\cos(c+dx)\sin^3(c+dx)}{4032b^2d} - \frac{\left(20a^4-\right.}{}$$
$$= -\frac{a\left(40a^4-188a^2b^2+189b^4\right)\cos(c+dx)\sin^3(c+dx)}{4032b^2d} - \frac{\left(20a^4-\right.}{}$$
$$= -\frac{a\left(8a^2+9b^2\right)\cos(c+dx)\sin(c+dx)}{128d} - \frac{a\left(40a^4-188a^2b^2+18\right.}{4}$$
$$= \frac{1}{128}a\left(8a^2+9b^2\right)x - \frac{b\left(27a^2+4b^2\right)\cos(c+dx)}{105d} + \frac{b\left(27a^2+4b^2\right.}{3}$$

Mathematica [A] time = 1.21064, size = 204, normalized size = 0.58

$$\frac{-3780b\left(6a^2+b^2\right)\cos(c+dx) - 840\left(9a^2b+b^3\right)\cos(3(c+dx)) + 1512a^2b\cos(5(c+dx)) + 1080a^2b\cos(7(c+dx)) +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (15120*a*b^2*c + 10080*a^3*d*x + 11340*a*b^2*d*x - 3780*b*(6*a^2 + b^2)*Cos
[c + d*x] - 840*(9*a^2*b + b^3)*Cos[3*(c + d*x)] + 1512*a^2*b*Cos[5*(c + d*
x)] + 504*b^3*Cos[5*(c + d*x)] + 1080*a^2*b*Cos[7*(c + d*x)] + 90*b^3*Cos[7
*(c + d*x)] - 70*b^3*Cos[9*(c + d*x)] + 2520*a^3*Sin[2*(c + d*x)] - 2520*a^
3*Sin[4*(c + d*x)] - 3780*a*b^2*Sin[4*(c + d*x)] - 840*a^3*Sin[6*(c + d*x)]
 + (945*a*b^2*Sin[8*(c + d*x)])/2)/(161280*d)
```

Maple [A] time = 0.052, size = 218, normalized size = 0.6

$$\frac{1}{d}\left(a^3\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6} + \frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{dx}{16} + \frac{c}{16}\right) + 3a^2b\left(-1/7\,(\sin(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)+3*a^2*b*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos
(d*x+c)^5)+3*a*b^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+
c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+b^3*(
-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x
+c)^5))
```

Maxima [A] time = 1.12258, size = 189, normalized size = 0.53

$$\frac{1680\left(4\sin(2dx+2c)^3+12dx+12c-3\sin(4dx+4c)\right)a^3+27648\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^2b+945\,(24d}{322560}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/322560*(1680*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*
a^3 + 27648*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2*b + 945*(24*d*x + 24*
c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a*b^2 - 1024*(35*cos(d*x + c)^9
- 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^3)/d
```

Fricas [A] time = 1.94399, size = 405, normalized size = 1.14

$$-\frac{4480\,b^3\cos(dx+c)^9-5760\left(3\,a^2b+2\,b^3\right)\cos(dx+c)^7+8064\left(3\,a^2b+b^3\right)\cos(dx+c)^5-315\left(8\,a^3+9\,ab^2\right)dx-105\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40320*(4480*b^3*cos(d*x + c)^9 - 5760*(3*a^2*b + 2*b^3)*cos(d*x + c)^7 +
 8064*(3*a^2*b + b^3)*cos(d*x + c)^5 - 315*(8*a^3 + 9*a*b^2)*d*x - 105*(144
*a*b^2*cos(d*x + c)^7 - 8*(8*a^3 + 27*a*b^2)*cos(d*x + c)^5 + 2*(8*a^3 + 9*
a*b^2)*cos(d*x + c)^3 + 3*(8*a^3 + 9*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 27.0008, size = 505, normalized size = 1.43

$$\begin{cases} \frac{a^3x\sin^6(c+dx)}{16}+\frac{3a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3a^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{a^3x\cos^6(c+dx)}{16}+\frac{a^3\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{a^3\sin^3(c+dx)\cos^3(c+}{6d} \\ x\,(a+b\sin(c))^3\sin^2(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**6/16 + 3*a**3*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a**3*x*cos(c + d*x)
**6/16 + a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a**3*sin(c + d*x)**3*co
s(c + d*x)**3/(6*d) - a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 3*a**2*b*s
in(c + d*x)**2*cos(c + d*x)**5/(5*d) - 6*a**2*b*cos(c + d*x)**7/(35*d) + 9*
a*b**2*x*sin(c + d*x)**8/128 + 9*a*b**2*x*sin(c + d*x)**6*cos(c + d*x)**2/3
2 + 27*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 9*a*b**2*x*sin(c + d*x
)**2*cos(c + d*x)**6/32 + 9*a*b**2*x*cos(c + d*x)**8/128 + 9*a*b**2*sin(c +
 d*x)**7*cos(c + d*x)/(128*d) + 33*a*b**2*sin(c + d*x)**5*cos(c + d*x)**3/(
128*d) - 33*a*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 9*a*b**2*sin(c
 + d*x)*cos(c + d*x)**7/(128*d) - b**3*sin(c + d*x)**4*cos(c + d*x)**5/(5*d
) - 4*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 8*b**3*cos(c + d*x)**9/
(315*d), Ne(d, 0)), (x*(a + b*sin(c))**3*sin(c)**2*cos(c)**4, True))
```

Giac [A] time = 1.38893, size = 275, normalized size = 0.78

$$-\frac{b^3\cos(9\,dx+9\,c)}{2304\,d}+\frac{3\,ab^2\sin(8\,dx+8\,c)}{1024\,d}-\frac{a^3\sin(6\,dx+6\,c)}{192\,d}+\frac{a^3\sin(2\,dx+2\,c)}{64\,d}+\frac{1}{128}\left(8\,a^3+9\,ab^2\right)x+\frac{\left(12\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2304*b^3*cos(9*d*x + 9*c)/d + 3/1024*a*b^2*sin(8*d*x + 8*c)/d - 1/192*a^
3*sin(6*d*x + 6*c)/d + 1/64*a^3*sin(2*d*x + 2*c)/d + 1/128*(8*a^3 + 9*a*b^2
)*x + 1/1792*(12*a^2*b + b^3)*cos(7*d*x + 7*c)/d + 1/320*(3*a^2*b + b^3)*co
s(5*d*x + 5*c)/d - 1/192*(9*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 3/128*(6*a^2*
b + b^3)*cos(d*x + c)/d - 1/128*(2*a^3 + 3*a*b^2)*sin(4*d*x + 4*c)/d
```

3.1117 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{a(2a^2+61b^2)\cos^5(c+dx)}{560d}-\frac{(2a^2+7b^2)\cos^5(c+dx)(a+b\sin(c+dx))}{112d}+\frac{b(8a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3b}{}$$

```
[Out] (3*b*(8*a^2 + b^2)*x)/128 - (a*(2*a^2 + 61*b^2)*Cos[c + d*x]^5)/(560*d) + (
3*b*(8*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (b*(8*a^2 + b^2)*Cos
[c + d*x]^3*Sin[c + d*x])/(64*d) - ((2*a^2 + 7*b^2)*Cos[c + d*x]^5*(a + b*S
in[c + d*x]))/(112*d) - (3*a*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/(56*d)
- (Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(8*d)
```

Rubi [A] time = 0.327028, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{a(2a^2+61b^2)\cos^5(c+dx)}{560d}-\frac{(2a^2+7b^2)\cos^5(c+dx)(a+b\sin(c+dx))}{112d}+\frac{b(8a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(8*a^2 + b^2)*x)/128 - (a*(2*a^2 + 61*b^2)*Cos[c + d*x]^5)/(560*d) + (
3*b*(8*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (b*(8*a^2 + b^2)*Cos
[c + d*x]^3*Sin[c + d*x])/(64*d) - ((2*a^2 + 7*b^2)*Cos[c + d*x]^5*(a + b*S
in[c + d*x]))/(112*d) - (3*a*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/(56*d)
- (Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(8*d)
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{\cos^5(c+dx)(a+b\sin(c+dx))^3}{8d}+\frac{1}{8}\int \cos^4(c+dx)(3b+3a\sin$$
$$= -\frac{3a\cos^5(c+dx)(a+b\sin(c+dx))^2}{56d}-\frac{\cos^5(c+dx)(a+b\sin(c+$$
$$= -\frac{\left(2a^2+7b^2\right)\cos^5(c+dx)(a+b\sin(c+dx))}{112d}-\frac{3a\cos^5(c+dx)(a}{56}$$
$$= -\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}-\frac{\left(2a^2+7b^2\right)\cos^5(c+dx)(a+b\sin}{112d}$$
$$= -\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}+\frac{b\left(8a^2+b^2\right)\cos^3(c+dx)\sin(c+}{64d}$$
$$= -\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}+\frac{3b\left(8a^2+b^2\right)\cos(c+dx)\sin(c+}{128d}$$
$$= \frac{3}{128}b\left(8a^2+b^2\right)x-\frac{a\left(2a^2+61b^2\right)\cos^5(c+dx)}{560d}+\frac{3b\left(8a^2+b^2\right)c}{}$$

Mathematica [A] time = 0.839165, size = 189, normalized size = 0.97

$$\frac{-280a\left(8a^2+9b^2\right)\cos(c+dx)-280\left(4a^3+3ab^2\right)\cos(3(c+dx))+840a^2b\sin(2(c+dx))-840a^2b\sin(4(c+dx))-2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3360*a^2*b*c + 840*b^3*c + 3360*a^2*b*d*x + 420*b^3*d*x - 280*a*(8*a^2 + 9
*b^2)*Cos[c + d*x] - 280*(4*a^3 + 3*a*b^2)*Cos[3*(c + d*x)] - 224*a^3*Cos[5
*(c + d*x)] + 168*a*b^2*Cos[5*(c + d*x)] + 120*a*b^2*Cos[7*(c + d*x)] + 840
*a^2*b*Sin[2*(c + d*x)] - 840*a^2*b*Sin[4*(c + d*x)] - 140*b^3*Sin[4*(c + d
*x)] - 280*a^2*b*Sin[6*(c + d*x)] + (35*b^3*Sin[8*(c + d*x)])/2)/(17920*d)
```

Maple [A] time = 0.047, size = 180, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{a^3(\cos(dx+c))^5}{5}+3a^2b\left(-1/6\sin(dx+c)(\cos(dx+c))^5+1/24\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/5*a^3*cos(d*x+c)^5+3*a^2*b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(
d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+3*a*b^2*(-1/7*sin(d*x+
c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+b^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/
16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
128*d*x+3/128*c))
```

Maxima [A] time = 1.16054, size = 158, normalized size = 0.81

$$-\frac{7168\,a^3\cos\left(dx+c\right)^5 - 560\left(4\,\sin\left(2\,dx+2\,c\right)^3 + 12\,dx + 12\,c - 3\,\sin\left(4\,dx+4\,c\right)\right)a^2b - 3072\left(5\,\cos\left(dx+c\right)^7 - 7\,\cos\left(\right.\right.}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35840*(7168*a^3*cos(d*x + c)^5 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12
*c - 3*sin(4*d*x + 4*c))*a^2*b - 3072*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)
*a*b^2 - 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*b^3)/d
```

Fricas [A] time = 1.85865, size = 335, normalized size = 1.73

$$\frac{1920\,ab^2\cos\left(dx+c\right)^7 - 896\left(a^3 + 3\,ab^2\right)\cos\left(dx+c\right)^5 + 105\left(8\,a^2b + b^3\right)dx + 35\left(16\,b^3\cos\left(dx+c\right)^7 - 8\left(8\,a^2b + 3\,b^3\right)\mathrm{co}\right.}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4480*(1920*a*b^2*cos(d*x + c)^7 - 896*(a^3 + 3*a*b^2)*cos(d*x + c)^5 + 10
5*(8*a^2*b + b^3)*d*x + 35*(16*b^3*cos(d*x + c)^7 - 8*(8*a^2*b + 3*b^3)*cos
(d*x + c)^5 + 2*(8*a^2*b + b^3)*cos(d*x + c)^3 + 3*(8*a^2*b + b^3)*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [A] time = 14.9345, size = 456, normalized size = 2.35

$$\begin{cases} -\frac{a^3\cos^5(c+dx)}{5d} + \frac{3a^2bx\sin^6(c+dx)}{16} + \frac{9a^2bx\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9a^2bx\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^2bx\cos^6(c+dx)}{16} + \frac{3a^2b\sin^5(c+dx)\cos(c+d}{16d} \\ x\left(a+b\sin(c)\right)^3\sin(c)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-a**3*cos(c + d*x)**5/(5*d) + 3*a**2*b*x*sin(c + d*x)**6/16 + 9*
a**2*b*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a**2*b*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 3*a**2*b*x*cos(c + d*x)**6/16 + 3*a**2*b*sin(c + d*x)**5
*cos(c + d*x)/(16*d) + a**2*b*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a**
2*b*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 3*a*b**2*sin(c + d*x)**2*cos(c +
d*x)**5/(5*d) - 6*a*b**2*cos(c + d*x)**7/(35*d) + 3*b**3*x*sin(c + d*x)**8/
128 + 3*b**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 9*b**3*x*sin(c + d*x)**
4*cos(c + d*x)**4/64 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*b**3
*x*cos(c + d*x)**8/128 + 3*b**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 11*b
**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b**3*sin(c + d*x)**3*cos(c
 + d*x)**5/(128*d) - 3*b**3*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0))
, (x*(a + b*sin(c))**3*sin(c)*cos(c)**4, True))
```

Giac [A] time = 1.32209, size = 248, normalized size = 1.28

$$\frac{3\,ab^2\cos\left(7\,dx+7\,c\right)}{448\,d} + \frac{b^3\sin\left(8\,dx+8\,c\right)}{1024\,d} - \frac{a^2b\sin\left(6\,dx+6\,c\right)}{64\,d} + \frac{3\,a^2b\sin\left(2\,dx+2\,c\right)}{64\,d} + \frac{3}{128}\left(8\,a^2b+b^3\right)x - \frac{\left(4\,a^3 - 3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 3/448*a*b^2*cos(7*d*x + 7*c)/d + 1/1024*b^3*sin(8*d*x + 8*c)/d - 1/64*a^2*b
*sin(6*d*x + 6*c)/d + 3/64*a^2*b*sin(2*d*x + 2*c)/d + 3/128*(8*a^2*b + b^3)
*x - 1/320*(4*a^3 - 3*a*b^2)*cos(5*d*x + 5*c)/d - 1/64*(4*a^3 + 3*a*b^2)*co
s(3*d*x + 3*c)/d - 1/64*(8*a^3 + 9*a*b^2)*cos(d*x + c)/d - 1/128*(6*a^2*b +
 b^3)*sin(4*d*x + 4*c)/d
```

3.1118 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=250

$$-\frac{a\left(-43a^2b^2+2a^4+36b^4\right)\cos(c+dx)}{60b^2d}-\frac{\left(2a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{120b^2d}-\frac{a\left(2a^2-39b^2\right)\cos(c+dx)(a+b}{120b^2d}$$

```
[Out] (b*(18*a^2 + b^2)*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/d - (a*(2*a^4 - 43*a^
2*b^2 + 36*b^4)*Cos[c + d*x])/(60*b^2*d) - ((4*a^4 - 84*a^2*b^2 + 15*b^4)*C
os[c + d*x]*Sin[c + d*x])/(240*b*d) - (a*(2*a^2 - 39*b^2)*Cos[c + d*x]*(a +
 b*Sin[c + d*x])^2)/(120*b^2*d) - ((2*a^2 - 35*b^2)*Cos[c + d*x]*(a + b*Sin
[c + d*x])^3)/(120*b^2*d) + (a*Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(15*b^2
*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^4)/(6*b*d)
```

Rubi [A] time = 0.659357, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2895, 3049, 3033, 3023, 2735, 3770}

$$-\frac{a\left(-43a^2b^2+2a^4+36b^4\right)\cos(c+dx)}{60b^2d}-\frac{\left(2a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{120b^2d}-\frac{a\left(2a^2-39b^2\right)\cos(c+dx)(a+b}{120b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (b*(18*a^2 + b^2)*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/d - (a*(2*a^4 - 43*a^
2*b^2 + 36*b^4)*Cos[c + d*x])/(60*b^2*d) - ((4*a^4 - 84*a^2*b^2 + 15*b^4)*C
os[c + d*x]*Sin[c + d*x])/(240*b*d) - (a*(2*a^2 - 39*b^2)*Cos[c + d*x]*(a +
 b*Sin[c + d*x])^2)/(120*b^2*d) - ((2*a^2 - 35*b^2)*Cos[c + d*x]*(a + b*Sin
[c + d*x])^3)/(120*b^2*d) + (a*Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(15*b^2
*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^4)/(6*b*d)
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{a\cos(c+dx)(a+b\sin(c+dx))^4}{15b^2d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\\
&= -\frac{\left(2a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{120b^2d}+\frac{a\cos(c+dx)(a\\
&= -\frac{a\left(2a^2-39b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{120b^2d}-\frac{\left(2a^2-35b^2\right)c\\
&= -\frac{\left(4a^4-84a^2b^2+15b^4\right)\cos(c+dx)\sin(c+dx)}{240bd}-\frac{a\left(2a^2-39b^2\right)\\
&= -\frac{a\left(2a^4-43a^2b^2+36b^4\right)\cos(c+dx)}{60b^2d}-\frac{\left(4a^4-84a^2b^2+15b^4\right)c}{240b\\
&= \frac{1}{16}b\left(18a^2+b^2\right)x-\frac{a\left(2a^4-43a^2b^2+36b^4\right)\cos(c+dx)}{60b^2d}-\frac{\left(4a^4\\
&= \frac{1}{16}b\left(18a^2+b^2\right)x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a\left(2a^4-43a^2b^2+}{60b\end{aligned}$$

Mathematica [A] time = 0.51171, size = 191, normalized size = 0.76

$$120a\left(10a^2-3b^2\right)\cos(c+dx)+20\left(4a^3-9ab^2\right)\cos(3(c+dx))+720a^2b\sin(2(c+dx))+90a^2b\sin(4(c+dx))+1080$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (1080*a^2*b*c + 60*b^3*c + 1080*a^2*b*d*x + 60*b^3*d*x + 120*a*(10*a^2 - 3*
b^2)*Cos[c + d*x] + 20*(4*a^3 - 9*a*b^2)*Cos[3*(c + d*x)] - 36*a*b^2*Cos[5*
(c + d*x)] - 960*a^3*Log[Cos[(c + d*x)/2]] + 960*a^3*Log[Sin[(c + d*x)/2]]
+ 720*a^2*b*Sin[2*(c + d*x)] + 15*b^3*Sin[2*(c + d*x)] + 90*a^2*b*Sin[4*(c
+ d*x)] - 15*b^3*Sin[4*(c + d*x)] - 5*b^3*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.098, size = 211, normalized size = 0.8

$$\frac{a^3 (\cos(dx+c))^3}{3d} + \frac{a^3 \cos(dx+c)}{d} + \frac{a^3 \ln(\csc(dx+c) - \cot(dx+c))}{d} + \frac{3a^2 b \sin(dx+c)(\cos(dx+c))^3}{4d} + \frac{9a^2 b \cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/3*a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)/d+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3
/4/d*a^2*b*sin(d*x+c)*cos(d*x+c)^3+9/8/d*a^2*b*cos(d*x+c)*sin(d*x+c)+9/8*a^
2*b*x+9/8/d*a^2*b*c-3/5/d*cos(d*x+c)^5*a*b^2-1/6/d*b^3*sin(d*x+c)*cos(d*x+c
)^5+1/24/d*b^3*cos(d*x+c)^3*sin(d*x+c)+1/16*b^3*cos(d*x+c)*sin(d*x+c)/d+1/1
6*b^3*x+1/16/d*b^3*c
```

Maxima [A] time = 1.159, size = 185, normalized size = 0.74

$$-\frac{576\,ab^2 \cos(dx+c)^5 - 160\left(2\cos(dx+c)^3 + 6\cos(dx+c) - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1)\right)a^3 - 9}{960}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(576*a*b^2*cos(d*x + c)^5 - 160*(2*cos(d*x + c)^3 + 6*cos(d*x + c) -
 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^3 - 90*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 5*(4*sin(2*d*x + 2*c)^3 +
 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*b^3)/d
```

Fricas [A] time = 1.96491, size = 401, normalized size = 1.6

$$-\frac{144\,ab^2 \cos(dx+c)^5 - 80\,a^3 \cos(dx+c)^3 - 240\,a^3 \cos(dx+c) + 120\,a^3 \log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - 120\,a^3 \log\left(-\frac{1}{2}\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a*b^2*cos(d*x + c)^5 - 80*a^3*cos(d*x + c)^3 - 240*a^3*cos(d*x
+ c) + 120*a^3*log(1/2*cos(d*x + c) + 1/2) - 120*a^3*log(-1/2*cos(d*x + c)
+ 1/2) - 15*(18*a^2*b + b^3)*d*x + 5*(8*b^3*cos(d*x + c)^5 - 2*(18*a^2*b +
```

```
b^3)*cos(d*x + c)^3 - 3*(18*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30763, size = 576, normalized size = 2.3

$$240\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+15\left(18\,a^2b+b^3\right)(dx+c)-\frac{2\left(450\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(240*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 15*(18*a^2*b + b^3)*(d*x +
c) - 2*(450*a^2*b*tan(1/2*d*x + 1/2*c)^11 - 15*b^3*tan(1/2*d*x + 1/2*c)^11
- 480*a^3*tan(1/2*d*x + 1/2*c)^10 + 720*a*b^2*tan(1/2*d*x + 1/2*c)^10 + 630
*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 235*b^3*tan(1/2*d*x + 1/2*c)^9 - 1920*a^3*t
an(1/2*d*x + 1/2*c)^8 + 720*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 180*a^2*b*tan(1/
2*d*x + 1/2*c)^7 - 390*b^3*tan(1/2*d*x + 1/2*c)^7 - 3200*a^3*tan(1/2*d*x +
1/2*c)^6 + 1440*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 180*a^2*b*tan(1/2*d*x + 1/2*
c)^5 + 390*b^3*tan(1/2*d*x + 1/2*c)^5 - 2880*a^3*tan(1/2*d*x + 1/2*c)^4 + 1
440*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 630*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 235*b
^3*tan(1/2*d*x + 1/2*c)^3 - 1440*a^3*tan(1/2*d*x + 1/2*c)^2 + 144*a*b^2*tan
(1/2*d*x + 1/2*c)^2 - 450*a^2*b*tan(1/2*d*x + 1/2*c) + 15*b^3*tan(1/2*d*x +
 1/2*c) - 320*a^3 + 144*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.1119 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=229

$$\frac{(56a^2b^2+a^4-2b^4)\cos(c+dx)}{10bd}+\frac{(a^2+20b^2)\cos(c+dx)(a+b\sin(c+dx))^3}{20abd}+\frac{(a^2+28b^2)\cos(c+dx)(a+b\sin(c+dx)}{20bd}$$

```
[Out] (-3*a*(4*a^2 - 3*b^2)*x)/8 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + ((a^4 + 56
*a^2*b^2 - 2*b^4)*Cos[c + d*x])/(10*b*d) + (a*(2*a^2 + 83*b^2)*Cos[c + d*x]
*Sin[c + d*x])/(40*d) + ((a^2 + 28*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2
)/(20*b*d) + ((a^2 + 20*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(20*a*b*d
) - (Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(5*b*d) - (Cot[c + d*x]*(a + b*Si
n[c + d*x])^4)/(a*d)
```

Rubi [A] time = 0.677261, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2894, 3049, 3033, 3023, 2735, 3770}

$$\frac{(56a^2b^2+a^4-2b^4)\cos(c+dx)}{10bd}+\frac{(a^2+20b^2)\cos(c+dx)(a+b\sin(c+dx))^3}{20abd}+\frac{(a^2+28b^2)\cos(c+dx)(a+b\sin(c+dx)}{20bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*(4*a^2 - 3*b^2)*x)/8 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + ((a^4 + 56
*a^2*b^2 - 2*b^4)*Cos[c + d*x])/(10*b*d) + (a*(2*a^2 + 83*b^2)*Cos[c + d*x]
*Sin[c + d*x])/(40*d) + ((a^2 + 28*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2
)/(20*b*d) + ((a^2 + 20*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(20*a*b*d
) - (Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(5*b*d) - (Cot[c + d*x]*(a + b*Si
n[c + d*x])^4)/(a*d)
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^3\,dx &= -\frac{\cos(c+dx)(a+b\sin(c+dx))^4}{5bd}-\frac{\cot(c+dx)(a+b\sin(c+dx)}{ad}\\
&= \frac{\left(a^2+20b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{20abd}-\frac{\cos(c+dx)(a+b}{5bd}\\
&= \frac{\left(a^2+28b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{20bd}+\frac{\left(a^2+20b^2\right)\cos(c}{}\\
&= \frac{a\left(2a^2+83b^2\right)\cos(c+dx)\sin(c+dx)}{40d}+\frac{\left(a^2+28b^2\right)\cos(c+dx}{20b}\\
&= \frac{\left(a^4+56a^2b^2-2b^4\right)\cos(c+dx)}{10bd}+\frac{a\left(2a^2+83b^2\right)\cos(c+dx)\sin}{40d}\\
&= -\frac{3}{8}a\left(4a^2-3b^2\right)x+\frac{\left(a^4+56a^2b^2-2b^4\right)\cos(c+dx)}{10bd}+\frac{a\left(2a^2+\right.}{}\\
&= -\frac{3}{8}a\left(4a^2-3b^2\right)x-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{\left(a^4+56a^2b^2-\right.}{1}
\end{aligned}$$

Mathematica [A] time = 2.96152, size = 194, normalized size = 0.85

$$\frac{-20b\left(b^2-30a^2\right)\cos(c+dx)+10\left(4a^2b-b^3\right)\cos(3(c+dx))+480a^2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-480a^2b\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-240*a^3*c + 180*a*b^2*c - 240*a^3*d*x + 180*a*b^2*d*x - 20*b*(-30*a^2 + b
^2)*Cos[c + d*x] + 10*(4*a^2*b - b^3)*Cos[3*(c + d*x)] - 2*b^3*Cos[5*(c + d
*x)] - 80*a^3*Cot[(c + d*x)/2] - 480*a^2*b*Log[Cos[(c + d*x)/2]] + 480*a^2*
b*Log[Sin[(c + d*x)/2]] - 40*a^3*Sin[2*(c + d*x)] + 120*a*b^2*Sin[2*(c + d*
x)] + 15*a*b^2*Sin[4*(c + d*x)] + 80*a^3*Tan[(c + d*x)/2])/(160*d)
```

Maple [A] time = 0.099, size = 216, normalized size = 0.9

$$-\frac{a^3(\cos(dx+c))^5}{d\sin(dx+c)}-\frac{a^3(\cos(dx+c))^3\sin(dx+c)}{d}-\frac{3a^3\cos(dx+c)\sin(dx+c)}{2d}-\frac{3a^3x}{2}-\frac{3a^3c}{2d}+\frac{a^2b(\cos(dx+c))^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d*a^3/sin(d*x+c)*cos(d*x+c)^5-a^3*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a^3*cos(
d*x+c)*sin(d*x+c)/d-3/2*a^3*x-3/2/d*a^3*c+1/d*a^2*b*cos(d*x+c)^3+3*a^2*b*co
s(d*x+c)/d+3/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))+3/4/d*a*b^2*sin(d*x+c)*cos(d
*x+c)^3+9/8*a*b^2*cos(d*x+c)*sin(d*x+c)/d+9/8*a*b^2*x+9/8/d*a*b^2*c-1/5/d*c
os(d*x+c)^5*b^3
```

Maxima [A] time = 1.54116, size = 193, normalized size = 0.84

$$-\frac{32b^3\cos(dx+c)^5+80\left(3dx+3c+\frac{3\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^3-80\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)\right.}{160d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/160*(32*b^3*cos(d*x + c)^5 + 80*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(t
an(d*x + c)^3 + tan(d*x + c)))*a^3 - 80*(2*cos(d*x + c)^3 + 6*cos(d*x + c)
- 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^2*b - 15*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b^2)/d
```

Fricas [A] time = 1.92587, size = 464, normalized size = 2.03

$$-\frac{30ab^2\cos(dx+c)^5+60a^2b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-60a^2b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-5\left(4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40*(30*a*b^2*cos(d*x + c)^5 + 60*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 60*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 5*(4*a^3 - 3
*a*b^2)*cos(d*x + c)^3 + 15*(4*a^3 - 3*a*b^2)*cos(d*x + c) + (8*b^3*cos(d*x
 + c)^5 - 40*a^2*b*cos(d*x + c)^3 - 120*a^2*b*cos(d*x + c) + 15*(4*a^3 - 3*
a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31719, size = 466, normalized size = 2.03

$$120\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\left(4\,a^3-3\,ab^2\right)(dx+c)-\frac{20\left(6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(20\,a^3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/40*(120*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 20*a^3*tan(1/2*d*x + 1/2*c
) - 15*(4*a^3 - 3*a*b^2)*(d*x + c) - 20*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3
)/tan(1/2*d*x + 1/2*c) + 2*(20*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*a*b^2*tan(1/
2*d*x + 1/2*c)^9 + 240*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 40*b^3*tan(1/2*d*x +
1/2*c)^8 + 40*a^3*tan(1/2*d*x + 1/2*c)^7 - 30*a*b^2*tan(1/2*d*x + 1/2*c)^7
+ 720*a^2*b*tan(1/2*d*x + 1/2*c)^6 + 880*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 80*
b^3*tan(1/2*d*x + 1/2*c)^4 - 40*a^3*tan(1/2*d*x + 1/2*c)^3 + 30*a*b^2*tan(1
/2*d*x + 1/2*c)^3 + 560*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 20*a^3*tan(1/2*d*x +
 1/2*c) + 75*a*b^2*tan(1/2*d*x + 1/2*c) + 160*a^2*b - 8*b^3)/(tan(1/2*d*x +
 1/2*c)^2 + 1)^5)/d
```

3.1120 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=231

$$-\frac{a\left(a^2-17b^2\right)\cos(c+dx)}{2d}-\frac{\left(a^2-4b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{4a^2d}-\frac{\left(a^2-6b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{4ad}-\frac{b(}{}$$

```
[Out] (-3*b*(12*a^2 - b^2)*x)/8 + (3*a*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d)
 - (a*(a^2 - 17*b^2)*Cos[c + d*x])/(2*d) - (b*(2*a^2 - 21*b^2)*Cos[c + d*x]
*Sin[c + d*x])/(8*d) - ((a^2 - 6*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/
(4*a*d) - ((a^2 - 4*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(4*a^2*d) - (
b*Cot[c + d*x]*(a + b*Sin[c + d*x])^4)/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x]
*(a + b*Sin[c + d*x])^4)/(2*a*d)
```

Rubi [A] time = 0.691722, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3049, 3033, 3023, 2735, 3770}

$$-\frac{a\left(a^2-17b^2\right)\cos(c+dx)}{2d}-\frac{\left(a^2-4b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{4a^2d}-\frac{\left(a^2-6b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{4ad}-\frac{b(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-3*b*(12*a^2 - b^2)*x)/8 + (3*a*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d)
 - (a*(a^2 - 17*b^2)*Cos[c + d*x])/(2*d) - (b*(2*a^2 - 21*b^2)*Cos[c + d*x]
*Sin[c + d*x])/(8*d) - ((a^2 - 6*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/
(4*a*d) - ((a^2 - 4*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(4*a^2*d) - (
b*Cot[c + d*x]*(a + b*Sin[c + d*x])^4)/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x]
*(a + b*Sin[c + d*x])^4)/(2*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{b\cot(c+dx)(a+b\sin(c+dx))^4}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)(a+}{2ad}$$

$$= -\frac{\left(a^2-4b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{4a^2d} - \frac{b\cot(c+dx)(a+b}{a^2d}$$

$$= -\frac{\left(a^2-6b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{4ad} - \frac{\left(a^2-4b^2\right)\cos(c+}{}$$

$$= -\frac{b\left(2a^2-21b^2\right)\cos(c+dx)\sin(c+dx)}{8d} - \frac{\left(a^2-6b^2\right)\cos(c+dx)(}{4ad}$$

$$= -\frac{a\left(a^2-17b^2\right)\cos(c+dx)}{2d} - \frac{b\left(2a^2-21b^2\right)\cos(c+dx)\sin(c+d}{8d}$$

$$= -\frac{3}{8}b\left(12a^2-b^2\right)x - \frac{a\left(a^2-17b^2\right)\cos(c+dx)}{2d} - \frac{b\left(2a^2-21b^2\right)\mathrm{co}}{}$$

$$= -\frac{3}{8}b\left(12a^2-b^2\right)x + \frac{3a\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\left(a^2-\right.}{}$$

Mathematica [A] time = 6.16045, size = 252, normalized size = 1.09

$$\frac{3b\left(b^2-12a^2\right)(c+dx)}{8d} + \frac{b\left(b^2-3a^2\right)\sin(2(c+dx))}{4d} - \frac{a\left(4a^2-15b^2\right)\cos(c+dx)}{4d} - \frac{3\left(a^3-2ab^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(-12*a^2 + b^2)*(c + d*x))/(8*d) - (a*(4*a^2 - 15*b^2)*Cos[c + d*x])/(
4*d) + (a*b^2*Cos[3*(c + d*x)])/(4*d) - (3*a^2*b*Cot[(c + d*x)/2])/(2*d) -
(a^3*Csc[(c + d*x)/2]^2)/(8*d) + (3*(a^3 - 2*a*b^2)*Log[Cos[(c + d*x)/2]])/
(2*d) - (3*(a^3 - 2*a*b^2)*Log[Sin[(c + d*x)/2]])/(2*d) + (a^3*Sec[(c + d*x
)/2]^2)/(8*d) + (b*(-3*a^2 + b^2)*Sin[2*(c + d*x)])/(4*d) + (b^3*Sin[4*(c +
 d*x)])/(32*d) + (3*a^2*b*Tan[(c + d*x)/2])/(2*d)
```

Maple [A] time = 0.103, size = 279, normalized size = 1.2

$$-\frac{a^3(\cos(dx+c))^5}{2d(\sin(dx+c))^2}-\frac{a^3(\cos(dx+c))^3}{2d}-\frac{3a^3\cos(dx+c)}{2d}-\frac{3a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}-3\frac{a^2b(\cos(dx+c))^5}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a^3*cos(d*x+c)^3/d-3/2*a^3*cos(d*x
+c)/d-3/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/d*a^2*b/sin(d*x+c)*cos(d*x+c)^5
-3/d*a^2*b*sin(d*x+c)*cos(d*x+c)^3-9/2/d*a^2*b*cos(d*x+c)*sin(d*x+c)-9/2*a^
2*b*x-9/2/d*a^2*b*c+a*b^2*cos(d*x+c)^3/d+3*a*b^2*cos(d*x+c)/d+3/d*a*b^2*ln(
csc(d*x+c)-cot(d*x+c))+1/4/d*b^3*cos(d*x+c)^3*sin(d*x+c)+3/8*b^3*cos(d*x+c)
*sin(d*x+c)/d+3/8*b^3*x+3/8/d*b^3*c
```

Maxima [A] time = 1.54, size = 251, normalized size = 1.09

$$-\frac{48\left(3dx+3c+\frac{3\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2b-16\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/32*(48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
 c)))*a^2*b - 16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) +
1) + 3*log(cos(d*x + c) - 1))*a*b^2 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8
*sin(2*d*x + 2*c))*b^3 - 8*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos
(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.99461, size = 616, normalized size = 2.67

$$\frac{8ab^2\cos(dx+c)^5-3\left(12a^2b-b^3\right)dx\cos(dx+c)^2-8\left(a^3-2ab^2\right)\cos(dx+c)^3+3\left(12a^2b-b^3\right)dx+12\left(a^3-2ab^2\right)\text{co}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8*(8*a*b^2*cos(d*x + c)^5 - 3*(12*a^2*b - b^3)*d*x*cos(d*x + c)^2 - 8*(a^
3 - 2*a*b^2)*cos(d*x + c)^3 + 3*(12*a^2*b - b^3)*d*x + 12*(a^3 - 2*a*b^2)*c
os(d*x + c) - 6*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2)*log(1/2*co
s(d*x + c) + 1/2) + 6*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2)*log(
-1/2*cos(d*x + c) + 1/2) + (2*b^3*cos(d*x + c)^5 - (12*a^2*b - b^3)*cos(d*x
 + c)^3 + 3*(12*a^2*b - b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2
- d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32357, size = 540, normalized size = 2.34

$$a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 12\,a^2 b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 3\left(12\,a^2 b - b^3\right)(dx + c) - 12\left(a^3 - 2\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + \frac{18}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) - 3*(12*a^2
*b - b^3)*(d*x + c) - 12*(a^3 - 2*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (
18*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*
tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x + 1/2*c)^2 + 2*(12*a^2*b*tan(1/2*d*
x + 1/2*c)^7 - 5*b^3*tan(1/2*d*x + 1/2*c)^7 - 8*a^3*tan(1/2*d*x + 1/2*c)^6
+ 48*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^3
*tan(1/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4 + 96*a*b^2*tan(1/2*
d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*
c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 + 80*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12
*a^2*b*tan(1/2*d*x + 1/2*c) + 5*b^3*tan(1/2*d*x + 1/2*c) - 8*a^3 + 32*a*b^2
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.1121 $\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+a^3x$$

```
[Out] a^3*x - (9*a*b^2*x)/2 + (9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTan
h[Cos[c + d*x]])/d - (9*a^2*b*Cos[c + d*x])/(2*d) + (b^3*Cos[c + d*x])/d +
(b^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/
(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (3*a^2*b*Cos[c + d*x]
*Cot[c + d*x]^2)/(2*d) - (a^3*Cot[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.22253, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 2592, 302, 206, 2591, 288, 321, 203, 3473, 8}

$$-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] a^3*x - (9*a*b^2*x)/2 + (9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTan
h[Cos[c + d*x]])/d - (9*a^2*b*Cos[c + d*x])/(2*d) + (b^3*Cos[c + d*x])/d +
(b^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/
(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (3*a^2*b*Cos[c + d*x]
*Cot[c + d*x]^2)/(2*d) - (a^3*Cot[c + d*x]^3)/(3*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\cos^3(c+dx)\cot(c+dx)+3ab^2\cos^2(c+dx)\cot^2(c+dx)+3a^2b\cos(c$$

$$= a^3\int \cot^4(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot^3(c+dx)\,dx+\left(3ab^2\right)\int \cos$$

$$= -\frac{a^3\cot^3(c+dx)}{3d}-a^3\int \cot^2(c+dx)\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx,x,\cos\right.}{d}$$

$$= \frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+d}{2d}$$

$$= a^3x-\frac{9a^2b\cos(c+dx)}{2d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}$$

$$= a^3x-\frac{9}{2}ab^2x+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{9a^2b\,c}{}$$

Mathematica [A] time = 6.21857, size = 355, normalized size = 1.83

$$\frac{a(2a^2-9b^2)(c+dx)}{2d}+\frac{b(5b^2-12a^2)\cos(c+dx)}{4d}+\frac{(2b^3-9a^2b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{(9a^2b-2b^3)\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 9*b^2)*(c + d*x))/(2*d) + (b*(-12*a^2 + 5*b^2)*Cos[c + d*x])/(4
*d) + (b^3*Cos[3*(c + d*x)])/(12*d) + ((4*a^3*Cos[(c + d*x)/2] - 9*a*b^2*Co
s[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (3*a^2*b*Csc[(c + d*x)/2]^2)/(8*d
) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + ((9*a^2*b - 2*b^3)*L
og[Cos[(c + d*x)/2]])/(2*d) + ((-9*a^2*b + 2*b^3)*Log[Sin[(c + d*x)/2]])/(2
*d) + (3*a^2*b*Sec[(c + d*x)/2]^2)/(8*d) + (Sec[(c + d*x)/2]*(-4*a^3*Sin[(c
 + d*x)/2] + 9*a*b^2*Sin[(c + d*x)/2]))/(6*d) - (3*a*b^2*Sin[2*(c + d*x)])/
(4*d) + (a^3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A] time = 0.106, size = 264, normalized size = 1.4

$$-\frac{a^3(\cot(dx+c))^3}{3d}+\frac{a^3\cot(dx+c)}{d}+a^3x+\frac{a^3c}{d}-\frac{3a^2b(\cos(dx+c))^5}{2d(\sin(dx+c))^2}-\frac{3a^2b(\cos(dx+c))^3}{2d}-\frac{9a^2b\cos(dx+c)}{2d}-\frac{9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cot(d*x+c)^3/d+a^3*cot(d*x+c)/d+a^3*x+1/d*a^3*c-3/2/d*a^2*b/sin(d*
x+c)^2*cos(d*x+c)^5-3/2/d*a^2*b*cos(d*x+c)^3-9/2*a^2*b*cos(d*x+c)/d-9/2/d*a
^2*b*ln(csc(d*x+c)-cot(d*x+c))-3/d*a*b^2/sin(d*x+c)*cos(d*x+c)^5-3/d*a*b^2*
sin(d*x+c)*cos(d*x+c)^3-9/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d-9/2*a*b^2*x-9/2/d
*a*b^2*c+1/3*b^3*cos(d*x+c)^3/d+b^3*cos(d*x+c)/d+1/d*b^3*ln(csc(d*x+c)-cot(
d*x+c))
```

Maxima [A] time = 1.66792, size = 252, normalized size = 1.3

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3-18\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab^2+2\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 18*(3*d
*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a*b^2 +
2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(cos(
d*x + c) - 1))*b^3 + 9*a^2*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d
*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.94293, size = 699, normalized size = 3.6

$$18\,ab^2\cos\left(dx+c\right)^5+8\left(2\,a^3-9\,ab^2\right)\cos\left(dx+c\right)^3-3\left(9\,a^2b-2\,b^3-\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(18*a*b^2*cos(d*x + c)^5 + 8*(2*a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(9*a
^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(9*a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log
(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(2*a^3 - 9*a*b^2)*cos(d*x + c) +
 2*(2*b^3*cos(d*x + c)^5 + 3*(2*a^3 - 9*a*b^2)*d*x*cos(d*x + c)^2 - 2*(9*a^
2*b - 2*b^3)*cos(d*x + c)^3 - 3*(2*a^3 - 9*a*b^2)*d*x + 3*(9*a^2*b - 2*b^3)
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3599, size = 568, normalized size = 2.93

$$3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+108\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\left(2\,a^3-9\,ab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 45*a
^3*tan(1/2*d*x + 1/2*c) + 108*a*b^2*tan(1/2*d*x + 1/2*c) + 36*(2*a^3 - 9*a*
b^2)*(d*x + c) - 36*(9*a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (198
*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 44*b^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^3*tan(
1/2*d*x + 1/2*c)^8 + 108*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 135*a^2*b*tan(1/2*d
*x + 1/2*c)^7 + 156*b^3*tan(1/2*d*x + 1/2*c)^7 + 132*a^3*tan(1/2*d*x + 1/2*
c)^6 - 324*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 351*a^2*b*tan(1/2*d*x + 1/2*c)^5
+ 156*b^3*tan(1/2*d*x + 1/2*c)^5 + 126*a^3*tan(1/2*d*x + 1/2*c)^4 - 540*a*b
^2*tan(1/2*d*x + 1/2*c)^4 - 315*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 148*b^3*tan(
1/2*d*x + 1/2*c)^3 + 36*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 - 27*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))^3)/d
```

3.1122 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=187

$$-\frac{b^2\left(73a^2-2b^2\right)\cos(c+dx)}{8ad}-\frac{3a\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{3}{2}bx\left(2a^2-b^2\right)+\frac{17b\cot(c+dx)(a+b\sin(c+dx)}{8d}$$

```
[Out] (3*b*(2*a^2 - b^2)*x)/2 - (3*a*(a^2 - 12*b^2)*ArcTanh[Cos[c + d*x]])/(8*d)
- (b^2*(73*a^2 - 2*b^2)*Cos[c + d*x])/(8*a*d) - (13*b^3*Cos[c + d*x]*Sin[c
+ d*x])/(4*d) + (17*b*Cot[c + d*x]*(a + b*Sin[c + d*x])^2)/(8*d) + (5*Cot[c
 + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^3)/(8*d) - (Cot[c + d*x]*Csc[c +
d*x]^3*(a + b*Sin[c + d*x])^4)/(4*a*d)
```

Rubi [A] time = 0.65629, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3047, 3033, 3023, 2735, 3770}

$$-\frac{b^2\left(73a^2-2b^2\right)\cos(c+dx)}{8ad}-\frac{3a\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{3}{2}bx\left(2a^2-b^2\right)+\frac{17b\cot(c+dx)(a+b\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (3*b*(2*a^2 - b^2)*x)/2 - (3*a*(a^2 - 12*b^2)*ArcTanh[Cos[c + d*x]])/(8*d)
- (b^2*(73*a^2 - 2*b^2)*Cos[c + d*x])/(8*a*d) - (13*b^3*Cos[c + d*x]*Sin[c
+ d*x])/(4*d) + (17*b*Cot[c + d*x]*(a + b*Sin[c + d*x])^2)/(8*d) + (5*Cot[c
 + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^3)/(8*d) - (Cot[c + d*x]*Csc[c +
d*x]^3*(a + b*Sin[c + d*x])^4)/(4*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^4}{4ad} - \frac{\int \csc^3(c+dx)(a$$

$$= \frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{8d} - \frac{\cot(c+dx)\csc^3(c$$

$$= \frac{17b\cot(c+dx)(a+b\sin(c+dx))^2}{8d} + \frac{5\cot(c+dx)\csc(c+dx)(a}{8d}$$

$$= -\frac{13b^3\cos(c+dx)\sin(c+dx)}{4d} + \frac{17b\cot(c+dx)(a+b\sin(c+dx)}{8d}$$

$$= -\frac{b^2\left(73a^2-2b^2\right)\cos(c+dx)}{8ad} - \frac{13b^3\cos(c+dx)\sin(c+dx)}{4d} + \frac{17}{}$$

$$= \frac{3}{2}b\left(2a^2-b^2\right)x - \frac{b^2\left(73a^2-2b^2\right)\cos(c+dx)}{8ad} - \frac{13b^3\cos(c+dx)}{4d}$$

$$= \frac{3}{2}b\left(2a^2-b^2\right)x - \frac{3a\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^2\left(73a^2\right.}{}$$

Mathematica [B] time = 6.25432, size = 381, normalized size = 2.04

$$-\frac{3b\left(b^2-2a^2\right)(c+dx)}{2d} + \frac{\left(5a^3-12ab^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{\left(12ab^2-5a^3\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3\left(a^3-12ab^2\right)\log\left(\sin\right.}{8d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(-2*a^2 + b^2)*(c + d*x))/(2*d) - (3*a*b^2*Cos[c + d*x])/d + ((4*a^2*
b*Cos[(c + d*x)/2] - b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(2*d) + ((5*a^
3 - 12*a*b^2)*Csc[(c + d*x)/2]^2)/(32*d) - (a^2*b*Cot[(c + d*x)/2]*Csc[(c +
 d*x)/2]^2)/(8*d) - (a^3*Csc[(c + d*x)/2]^4)/(64*d) - (3*(a^3 - 12*a*b^2)*L
og[Cos[(c + d*x)/2]])/(8*d) + (3*(a^3 - 12*a*b^2)*Log[Sin[(c + d*x)/2]])/(8
*d) + ((-5*a^3 + 12*a*b^2)*Sec[(c + d*x)/2]^2)/(32*d) + (a^3*Sec[(c + d*x)/
2]^4)/(64*d) + (Sec[(c + d*x)/2]*(-4*a^2*b*Sin[(c + d*x)/2] + b^3*Sin[(c +
d*x)/2]))/(2*d) - (b^3*Sin[2*(c + d*x)])/(4*d) + (a^2*b*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/(8*d)
```

Maple [A] time = 0.112, size = 316, normalized size = 1.7

$$-\frac{a^3(\cos(dx+c))^5}{4d(\sin(dx+c))^4}+\frac{a^3(\cos(dx+c))^5}{8d(\sin(dx+c))^2}+\frac{a^3(\cos(dx+c))^3}{8d}+\frac{3a^3\cos(dx+c)}{8d}+\frac{3a^3\ln(\csc(dx+c)-\cot(dx+c))}{8d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+1/
8*a^3*cos(d*x+c)^3/d+3/8*a^3*cos(d*x+c)/d+3/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c
))-1/d*a^2*b*cot(d*x+c)^3+3*a^2*b*x+3*a^2*b*cot(d*x+c)/d+3/d*a^2*b*c-3/2/d*
a*b^2/sin(d*x+c)^2*cos(d*x+c)^5-3/2*a*b^2*cos(d*x+c)^3/d-9/2*a*b^2*cos(d*x+
c)/d-9/2/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-1/d*b^3/sin(d*x+c)*cos(d*x+c)^5-
1/d*b^3*cos(d*x+c)^3*sin(d*x+c)-3/2*b^3*cos(d*x+c)*sin(d*x+c)/d-3/2*b^3*x-3
/2/d*b^3*c
```

Maxima [A] time = 1.67864, size = 286, normalized size = 1.53

$$\frac{16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2b-8\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b^3-a^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/16*(16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2*b - 8*(3
*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^3 -
a^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)
^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 12*a*b^2*(2*
cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1
) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.99008, size = 807, normalized size = 4.32

$$-\frac{48\,ab^2\cos(dx+c)^5-24\left(2\,a^2b-b^3\right)dx\cos(dx+c)^4+48\left(2\,a^2b-b^3\right)dx\cos(dx+c)^2+10\left(a^3-12\,ab^2\right)\cos(dx+c)^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(48*a*b^2*cos(d*x + c)^5 - 24*(2*a^2*b - b^3)*d*x*cos(d*x + c)^4 + 48
*(2*a^2*b - b^3)*d*x*cos(d*x + c)^2 + 10*(a^3 - 12*a*b^2)*cos(d*x + c)^3 -
24*(2*a^2*b - b^3)*d*x - 6*(a^3 - 12*a*b^2)*cos(d*x + c) + 3*((a^3 - 12*a*b
^2)*cos(d*x + c)^4 + a^3 - 12*a*b^2 - 2*(a^3 - 12*a*b^2)*cos(d*x + c)^2)*lo
g(1/2*cos(d*x + c) + 1/2) - 3*((a^3 - 12*a*b^2)*cos(d*x + c)^4 + a^3 - 12*a
*b^2 - 2*(a^3 - 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) + 8*
(b^3*cos(d*x + c)^5 + 4*(2*a^2*b - b^3)*cos(d*x + c)^3 - 3*(2*a^2*b - b^3)*
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29662, size = 463, normalized size = 2.48

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,a^2b\tan\left(\frac{1}{2}\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*t
an(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*b*tan(1/2
*d*x + 1/2*c) + 32*b^3*tan(1/2*d*x + 1/2*c) + 96*(2*a^2*b - b^3)*(d*x + c)
+ 24*(a^3 - 12*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 64*(b^3*tan(1/2*d*x
+ 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c) - 6*
a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (50*a^3*tan(1/2*d*x + 1/2*c)^4 - 60
0*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 32*b^3*
tan(1/2*d*x + 1/2*c)^3 - 8*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*
x + 1/2*c)^2 + 8*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^4)/
d
```

3.1123 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=227

$$-\frac{b^3\left(83a^2+2b^2\right)\cos(c+dx)}{40a^2d}-\frac{a\left(4a^2-29b^2\right)\cot(c+dx)}{20d}-\frac{3b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\cot(c+dx)\csc^3(c\cdots}{2\cdots}$$

```
[Out] 3*a*b^2*x - (3*b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) - (b^3*(83*a^
2 + 2*b^2)*Cos[c + d*x])/(40*a^2*d) - (a*(4*a^2 - 29*b^2)*Cot[c + d*x])/(20
*d) + (27*b*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^2)/(40*d) + (2*C
ot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3)/(5*d) + (b*Cot[c + d*x]*
Csc[c + d*x]^3*(a + b*Sin[c + d*x])^4)/(20*a^2*d) - (Cot[c + d*x]*Csc[c + d
*x]^4*(a + b*Sin[c + d*x])^4)/(5*a*d)
```

Rubi [A] time = 0.711053, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2893, 3047, 3031, 3023, 2735, 3770}

$$-\frac{b^3\left(83a^2+2b^2\right)\cos(c+dx)}{40a^2d}-\frac{a\left(4a^2-29b^2\right)\cot(c+dx)}{20d}-\frac{3b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\cot(c+dx)\csc^3(c\cdots}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] 3*a*b^2*x - (3*b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) - (b^3*(83*a^
2 + 2*b^2)*Cos[c + d*x])/(40*a^2*d) - (a*(4*a^2 - 29*b^2)*Cot[c + d*x])/(20
*d) + (27*b*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^2)/(40*d) + (2*C
ot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3)/(5*d) + (b*Cot[c + d*x]*
Csc[c + d*x]^3*(a + b*Sin[c + d*x])^4)/(20*a^2*d) - (Cot[c + d*x]*Csc[c + d
*x]^4*(a + b*Sin[c + d*x])^4)/(5*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^4}{20a^2d} - \frac{\cot(c+dx)\csc^4 \cdots}{\cdots}\\ &= \frac{2\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{5d} + \frac{b\cot(c+dx)\csc \cdots}{\cdots}\\ &= \frac{27b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{40d} + \frac{2\cot(c+dx) \cdots}{\cdots}\\ &= -\frac{a\left(4a^2-29b^2\right)\cot(c+dx)}{20d} + \frac{27b\cot(c+dx)\csc(c+dx)(a+b \cdots}{40d}\\ &= -\frac{b^3\left(83a^2+2b^2\right)\cos(c+dx)}{40a^2d} - \frac{a\left(4a^2-29b^2\right)\cot(c+dx)}{20d} + \frac{27 \cdots}{\cdots}\\ &= 3ab^2x - \frac{b^3\left(83a^2+2b^2\right)\cos(c+dx)}{40a^2d} - \frac{a\left(4a^2-29b^2\right)\cot(c+dx \cdots}{20d}\\ &= 3ab^2x - \frac{3b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^3\left(83a^2+2b^2\right)\cos \cdots}{40a^2d}\end{aligned}$$

Mathematica [A] time = 1.24748, size = 405, normalized size = 1.78

$$\frac{-32\left(a^3-20ab^2\right)\cot\left(\frac{1}{2}(c+dx)\right) - 15a^2b\csc^4\left(\frac{1}{2}(c+dx)\right) + 150a^2b\csc^2\left(\frac{1}{2}(c+dx)\right) + 15a^2b\sec^4\left(\frac{1}{2}(c+dx)\right) - 150a^2 \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (960*a*b^2*c + 960*a*b^2*d*x - 320*b^3*Cos[c + d*x] - 32*(a^3 - 20*a*b^2)*C
ot[(c + d*x)/2] + 150*a^2*b*Csc[(c + d*x)/2]^2 - 40*b^3*Csc[(c + d*x)/2]^2
- 15*a^2*b*Csc[(c + d*x)/2]^4 - 360*a^2*b*Log[Cos[(c + d*x)/2]] + 480*b^3*L
og[Cos[(c + d*x)/2]] + 360*a^2*b*Log[Sin[(c + d*x)/2]] - 480*b^3*Log[Sin[(c
 + d*x)/2]] - 150*a^2*b*Sec[(c + d*x)/2]^2 + 40*b^3*Sec[(c + d*x)/2]^2 + 15
*a^2*b*Sec[(c + d*x)/2]^4 - 112*a^3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320
*a*b^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 64*a^3*Csc[c + d*x]^5*Sin[(c + d
*x)/2]^6 + 7*a^3*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a*b^2*Csc[(c + d*x)/2
]^4*Sin[c + d*x] - a^3*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 32*a^3*Tan[(c + d*
x)/2] - 640*a*b^2*Tan[(c + d*x)/2])/(320*d)
```

Maple [A] time = 0.109, size = 260, normalized size = 1.2

$$-\frac{a^3 (\cos (dx+c))^5}{5 d (\sin (dx+c))^5} - \frac{3 a^2 b (\cos (dx+c))^5}{4 d (\sin (dx+c))^4} + \frac{3 a^2 b (\cos (dx+c))^5}{8 d (\sin (dx+c))^2} + \frac{3 a^2 b (\cos (dx+c))^3}{8 d} + \frac{9 a^2 b \cos (dx+c)}{8 d} + \frac{9 a^2 b \ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/5/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-3/4/d*a^2*b/sin(d*x+c)^4*cos(d*x+c)^5+
3/8/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^5+3/8/d*a^2*b*cos(d*x+c)^3+9/8*a^2*b*co
s(d*x+c)/d+9/8/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*a*b^2*cot(d*x+c)^3+3*a
*b^2*x+3*a*b^2*cot(d*x+c)/d+3/d*a*b^2*c-1/2/d*b^3/sin(d*x+c)^2*cos(d*x+c)^5
-1/2*b^3*cos(d*x+c)^3/d-3/2*b^3*cos(d*x+c)/d-3/2/d*b^3*ln(csc(d*x+c)-cot(d*
x+c))
```

Maple [A] time = 1.68249, size = 246, normalized size = 1.08

$$\frac{80 \left(3 dx + 3 c + \frac{3 \tan(dx+c)^2 - 1}{\tan(dx+c)^3}\right) a b^2 - 15 a^2 b \left(\frac{2 \left(5 \cos(dx+c)^3 - 3 \cos(dx+c)\right)}{\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1} + 3 \log(\cos(dx+c)+1) - 3 \log(\cos(dx+c)-1)\right)}{80 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/80*(80*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a*b^2 - 15*a
^2*b*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c
)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 20*b^3*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
 - 3*log(cos(d*x + c) - 1)) - 16*a^3/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.92508, size = 832, normalized size = 3.67

$$-\frac{560 a b^2 \cos(dx+c)^3 + 16 \left(a^3 - 20 a b^2\right) \cos(dx+c)^5 - 240 a b^2 \cos(dx+c) + 15 \left(\left(3 a^2 b - 4 b^3\right) \cos(dx+c)^4 + 3 a^2 b - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/80*(560*a*b^2*cos(d*x + c)^3 + 16*(a^3 - 20*a*b^2)*cos(d*x + c)^5 - 240*
a*b^2*cos(d*x + c) + 15*((3*a^2*b - 4*b^3)*cos(d*x + c)^4 + 3*a^2*b - 4*b^3
 - 2*(3*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 15*((3*a^2*b - 4*b^3)*cos(d*x + c)^4 + 3*a^2*b - 4*b^3 - 2*(3*a^2*b
- 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(24
*a*b^2*d*x*cos(d*x + c)^4 - 8*b^3*cos(d*x + c)^5 - 48*a*b^2*d*x*cos(d*x + c
)^2 + 24*a*b^2*d*x - 5*(3*a^2*b - 4*b^3)*cos(d*x + c)^3 + 3*(3*a^2*b - 4*b^
3)*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43498, size = 481, normalized size = 2.12

$$\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a^2b\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/320*(2*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*
a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a^2*b*ta
n(1/2*d*x + 1/2*c)^2 + 40*b^3*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a*b^2
+ 20*a^3*tan(1/2*d*x + 1/2*c) - 600*a*b^2*tan(1/2*d*x + 1/2*c) - 640*b^3/(t
an(1/2*d*x + 1/2*c)^2 + 1) + 120*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/
2*c))) - (822*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 1096*b^3*tan(1/2*d*x + 1/2*c)^
5 + 20*a^3*tan(1/2*d*x + 1/2*c)^4 - 600*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 40*b^3*tan(1/2*d*x + 1/2*c)^3 - 10*a^3*tan(1
/2*d*x + 1/2*c)^2 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*b*tan(1/2*d*x
+ 1/2*c) + 2*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.1124 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=275

$$-\frac{b\left(-43a^2b^2+36a^4+2b^4\right)\cot(c+dx)}{60a^2d}-\frac{a\left(a^2+18b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}-\frac{\left(-84a^2b^2+15a^4+4b^4\right)\cot(c+dx)\csc(c}{240ad}$$

```
[Out] b^3*x - (a*(a^2 + 18*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (b*(36*a^4 - 43*a
^2*b^2 + 2*b^4)*Cot[c + d*x])/(60*a^2*d) - ((15*a^4 - 84*a^2*b^2 + 4*b^4)*C
ot[c + d*x]*Csc[c + d*x])/(240*a*d) + (b*(39*a^2 - 2*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2*(a + b*Sin[c + d*x])^2)/(120*a^2*d) + ((35*a^2 - 2*b^2)*Cot[c +
d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(120*a^2*d) + (b*Cot[c + d*x]*C
sc[c + d*x]^4*(a + b*Sin[c + d*x])^4)/(15*a^2*d) - (Cot[c + d*x]*Csc[c + d*
x]^5*(a + b*Sin[c + d*x])^4)/(6*a*d)
```

Rubi [A] time = 0.755893, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2893, 3047, 3031, 3021, 2735, 3770}

$$-\frac{b\left(-43a^2b^2+36a^4+2b^4\right)\cot(c+dx)}{60a^2d}-\frac{a\left(a^2+18b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}-\frac{\left(-84a^2b^2+15a^4+4b^4\right)\cot(c+dx)\csc(c}{240ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] b^3*x - (a*(a^2 + 18*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (b*(36*a^4 - 43*a
^2*b^2 + 2*b^4)*Cot[c + d*x])/(60*a^2*d) - ((15*a^4 - 84*a^2*b^2 + 4*b^4)*C
ot[c + d*x]*Csc[c + d*x])/(240*a*d) + (b*(39*a^2 - 2*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2*(a + b*Sin[c + d*x])^2)/(120*a^2*d) + ((35*a^2 - 2*b^2)*Cot[c +
d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(120*a^2*d) + (b*Cot[c + d*x]*C
sc[c + d*x]^4*(a + b*Sin[c + d*x])^4)/(15*a^2*d) - (Cot[c + d*x]*Csc[c + d*
x]^5*(a + b*Sin[c + d*x])^4)/(6*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
```

```
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^4}{15a^2d} - \frac{\cot(c+dx)\csc^5}{}$$

$$= \frac{\left(35a^2-2b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{120a^2d} + \frac{b\,\mathrm{co}}{}$$

$$= \frac{b\left(39a^2-2b^2\right)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{120a^2d} + \frac{(3}{}$$

$$= -\frac{\left(15a^4-84a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{240ad} + \frac{b\left(39a^2-2b^2\right.}{}$$

$$= -\frac{b\left(36a^4-43a^2b^2+2b^4\right)\cot(c+dx)}{60a^2d} - \frac{\left(15a^4-84a^2b^2+4b^4\right)\mathrm{c}}{240}$$

$$= b^3x - \frac{b\left(36a^4-43a^2b^2+2b^4\right)\cot(c+dx)}{60a^2d} - \frac{\left(15a^4-84a^2b^2+4\right.}{}$$

$$= b^3x - \frac{a\left(a^2+18b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{b\left(36a^4-43a^2b^2+2\right.}{60a^2d}$$

Mathematica [A] time = 1.77393, size = 408, normalized size = 1.48

$$-64\left(9a^2b-20b^3\right)\cot\left(\frac{1}{2}(c+dx)\right)-30\left(a^3-30ab^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+2\csc^4\left(\frac{1}{2}(c+dx)\right)\left(b\left(63a^2-20b^2\right)\sin(c+dx)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (1920*b^3*c + 1920*b^3*d*x - 64*(9*a^2*b - 20*b^3)*Cot[(c + d*x)/2] - 30*(a
^3 - 30*a*b^2)*Csc[(c + d*x)/2]^2 - 120*a^3*Log[Cos[(c + d*x)/2]] - 2160*a*
b^2*Log[Cos[(c + d*x)/2]] + 120*a^3*Log[Sin[(c + d*x)/2]] + 2160*a*b^2*Log[
Sin[(c + d*x)/2]] + 30*a^3*Sec[(c + d*x)/2]^2 - 900*a*b^2*Sec[(c + d*x)/2]^
2 - 30*a^3*Sec[(c + d*x)/2]^4 + 90*a*b^2*Sec[(c + d*x)/2]^4 + 5*a^3*Sec[(c
+ d*x)/2]^6 - 2016*a^2*b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 640*b^3*Csc[c
+ d*x]^3*Sin[(c + d*x)/2]^4 - a^2*Csc[(c + d*x)/2]^6*(5*a + 18*b*Sin[c + d*
x]) + 2*Csc[(c + d*x)/2]^4*(15*(a^3 - 3*a*b^2) + b*(63*a^2 - 20*b^2)*Sin[c
+ d*x]) + 576*a^2*b*Tan[(c + d*x)/2] - 1280*b^3*Tan[(c + d*x)/2] + 36*a^2*b
*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(1920*d)
```

Maple [A] time = 0.112, size = 302, normalized size = 1.1

$$-\frac{a^3(\cos(dx+c))^5}{6d(\sin(dx+c))^6}-\frac{a^3(\cos(dx+c))^5}{24d(\sin(dx+c))^4}+\frac{a^3(\cos(dx+c))^5}{48d(\sin(dx+c))^2}+\frac{a^3(\cos(dx+c))^3}{48d}+\frac{a^3\cos(dx+c)}{16d}+\frac{a^3\ln(\csc(dx+}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/6/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5-1/24/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+1
/48/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+1/48*a^3*cos(d*x+c)^3/d+1/16*a^3*cos(d*
x+c)/d+1/16/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/5/d*a^2*b/sin(d*x+c)^5*cos(d*
x+c)^5-3/4/d*a*b^2/sin(d*x+c)^4*cos(d*x+c)^5+3/8/d*a*b^2/sin(d*x+c)^2*cos(d
*x+c)^5+3/8*a*b^2*cos(d*x+c)^3/d+9/8*a*b^2*cos(d*x+c)/d+9/8/d*a*b^2*ln(csc(
d*x+c)-cot(d*x+c))-1/3/d*b^3*cot(d*x+c)^3+1/d*cot(d*x+c)*b^3+b^3*x+1/d*b^3*
c
```

Maxima [A] time = 1.64595, size = 293, normalized size = 1.07

$$\frac{160\left(3dx+3c+\frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b^3+5a^3\left(\frac{2\left(3\cos(dx+c)^5+8\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+\right.}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/480*(160*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*b^3 + 5*a^
3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^6
 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1)) - 90*a*b^2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(
```

```
d*x + c) - 1)) - 288*a^2*b/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.90801, size = 941, normalized size = 3.42

$$\frac{480\, b^3 dx \cos(dx+c)^6 - 1440\, b^3 dx \cos(dx+c)^4 + 1440\, b^3 dx \cos(dx+c)^2 + 30\left(a^3 - 30\, ab^2\right)\cos(dx+c)^5 - 480\, b^3 dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(480*b^3*d*x*cos(d*x + c)^6 - 1440*b^3*d*x*cos(d*x + c)^4 + 1440*b^3*
d*x*cos(d*x + c)^2 + 30*(a^3 - 30*a*b^2)*cos(d*x + c)^5 - 480*b^3*d*x + 80*
(a^3 + 18*a*b^2)*cos(d*x + c)^3 - 30*(a^3 + 18*a*b^2)*cos(d*x + c) - 15*((a
^3 + 18*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 18*a*b^2)*cos(d*x + c)^4 - a^3 - 1
8*a*b^2 + 3*(a^3 + 18*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) +
15*((a^3 + 18*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 18*a*b^2)*cos(d*x + c)^4 - a
^3 - 18*a*b^2 + 3*(a^3 + 18*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) +
1/2) + 32*(35*b^3*cos(d*x + c)^3 + (9*a^2*b - 20*b^3)*cos(d*x + c)^5 - 15*b
^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d
*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45618, size = 539, normalized size = 1.96

$$\frac{5\, a^3 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^6 + 36\, a^2 b \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^5 - 15\, a^3 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^4 + 90\, ab^2 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^4 - 180\, a^2 b \tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 15
*a^3*tan(1/2*d*x + 1/2*c)^4 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 180*a^2*b*t
an(1/2*d*x + 1/2*c)^3 + 80*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x
+ 1/2*c)^2 - 720*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 1920*(d*x + c)*b^3 + 360*a^
2*b*tan(1/2*d*x + 1/2*c) - 1200*b^3*tan(1/2*d*x + 1/2*c) + 120*(a^3 + 18*a*
b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - (294*a^3*tan(1/2*d*x + 1/2*c)^6 + 529
2*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 360*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 1200*b^
```

```
3*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*tan(1/2*d*x + 1/2*c)^4 - 720*a*b^2*tan(1/
2*d*x + 1/2*c)^4 - 180*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 80*b^3*tan(1/2*d*x +
1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^2
+ 36*a^2*b*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.1125 $\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=303

$$-\frac{a\left(2a^2+21b^2\right)\cot(c+dx)}{35d}-\frac{3b\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}-\frac{\left(-19a^2b^2+4a^4+2b^4\right)\cot(c+dx)\csc^2(c+dx)}{140ad}-$$

```
[Out] (-3*b*(a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (a*(2*a^2 + 21*b^2)*Cot
[c + d*x])/(35*d) - (b*(105*a^4 - 116*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c
+ d*x])/(560*a^2*d) - ((4*a^4 - 19*a^2*b^2 + 2*b^4)*Cot[c + d*x]*Csc[c + d*
x]^2)/(140*a*d) + (b*(53*a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Si
n[c + d*x])^2)/(280*a^2*d) + ((8*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a
+ b*Sin[c + d*x])^3)/(35*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin
[c + d*x])^4)/(14*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*x]
)^4)/(7*a*d)
```

Rubi [A] time = 0.860202, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2893, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$-\frac{a\left(2a^2+21b^2\right)\cot(c+dx)}{35d}-\frac{3b\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}-\frac{\left(-19a^2b^2+4a^4+2b^4\right)\cot(c+dx)\csc^2(c+dx)}{140ad}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-3*b*(a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (a*(2*a^2 + 21*b^2)*Cot
[c + d*x])/(35*d) - (b*(105*a^4 - 116*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c
+ d*x])/(560*a^2*d) - ((4*a^4 - 19*a^2*b^2 + 2*b^4)*Cot[c + d*x]*Csc[c + d*
x]^2)/(140*a*d) + (b*(53*a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Si
n[c + d*x])^2)/(280*a^2*d) + ((8*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a
+ b*Sin[c + d*x])^3)/(35*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin
[c + d*x])^4)/(14*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*x]
)^4)/(7*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
```

```
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^4}{14a^2d} - \frac{\cot(c+dx)\csc^6}{}$$

$$= \frac{\left(8a^2-b^2\right)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{35a^2d} + \frac{b\cot(}{}$$

$$= \frac{b\left(53a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{280a^2d} + \frac{(8}{}$$

$$= -\frac{\left(4a^4-19a^2b^2+2b^4\right)\cot(c+dx)\csc^2(c+dx)}{140ad} + \frac{b\left(53a^2-6b^2\right.}{}$$

$$= -\frac{b\left(105a^4-116a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{560a^2d} - \frac{\left(4a^4-1\right.}{}$$

$$= -\frac{b\left(105a^4-116a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{560a^2d} - \frac{\left(4a^4-1\right.}{}$$

$$= -\frac{3b\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{b\left(105a^4-116a^2b^2+12b\right.}{560}$$

$$= -\frac{3b\left(a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\left(2a^2+21b^2\right)\cot(c+dx}{35d}$$

Mathematica [A] time = 0.934619, size = 324, normalized size = 1.07

$$-\frac{56a\left(14a^2-3b^2\right)\cos(3(c+dx))\csc^7(c+dx)+70\cot(c+dx)\csc^6(c+dx)\left(b\left(31a^2-18b^2\right)\sin(c+dx)+12a\left(2a^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(56*a*(14*a^2 - 3*b^2)*Cos[3*(c + d*x)]*Csc[c + d*x]^7 + 112*a^3*Cos[5*(c
+ d*x)]*Csc[c + d*x]^7 - 504*a*b^2*Cos[5*(c + d*x)]*Csc[c + d*x]^7 - 16*a^3
*Cos[7*(c + d*x)]*Csc[c + d*x]^7 - 168*a*b^2*Cos[7*(c + d*x)]*Csc[c + d*x]^
7 + 3360*a^2*b*Log[Cos[(c + d*x)/2]] + 6720*b^3*Log[Cos[(c + d*x)/2]] - 336
0*a^2*b*Log[Sin[(c + d*x)/2]] - 6720*b^3*Log[Sin[(c + d*x)/2]] + 70*Cot[c +
 d*x]*Csc[c + d*x]^6*(12*a*(2*a^2 + b^2) + b*(31*a^2 - 18*b^2)*Sin[c + d*x]
) + 1540*a^2*b*Csc[c + d*x]^7*Sin[4*(c + d*x)] + 840*b^3*Csc[c + d*x]^7*Sin
[4*(c + d*x)] + 105*a^2*b*Csc[c + d*x]^7*Sin[6*(c + d*x)] - 350*b^3*Csc[c +
 d*x]^7*Sin[6*(c + d*x)])/(17920*d)
```

Maple [A] time = 0.116, size = 309, normalized size = 1.

$$-\frac{a^3(\cos(dx+c))^5}{7d(\sin(dx+c))^7} - \frac{2a^3(\cos(dx+c))^5}{35d(\sin(dx+c))^5} - \frac{a^2b(\cos(dx+c))^5}{2d(\sin(dx+c))^6} - \frac{a^2b(\cos(dx+c))^5}{8d(\sin(dx+c))^4} + \frac{a^2b(\cos(dx+c))^5}{16d(\sin(dx+c))^2} + \frac{a^2b(\cos}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/7/d*a^3/sin(d*x+c)^7*cos(d*x+c)^5-2/35/d*a^3/sin(d*x+c)^5*cos(d*x+c)^5-1
/2/d*a^2*b/sin(d*x+c)^6*cos(d*x+c)^5-1/8/d*a^2*b/sin(d*x+c)^4*cos(d*x+c)^5+
1/16/d*a^2*b/sin(d*x+c)^2*cos(d*x+c)^5+1/16/d*a^2*b*cos(d*x+c)^3+3/16*a^2*b
*cos(d*x+c)/d+3/16/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-3/5/d*a*b^2/sin(d*x+c)
^5*cos(d*x+c)^5-1/4/d*b^3/sin(d*x+c)^4*cos(d*x+c)^5+1/8/d*b^3/sin(d*x+c)^2*
cos(d*x+c)^5+1/8*b^3*cos(d*x+c)^3/d+3/8*b^3*cos(d*x+c)/d+3/8/d*b^3*ln(csc(d
```

```
*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.05677, size = 281, normalized size = 0.93

$$\frac{35\,a^2b\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-70\,b^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos\right.}{\cos(dx+c)^4-2\,\cos(dx}\right.}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/1120*(35*a^2*b*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/
(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x
+ c) + 1) + 3*log(cos(d*x + c) - 1)) - 70*b^3*(2*(5*cos(d*x + c)^3 - 3*cos(
d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1)
 - 3*log(cos(d*x + c) - 1)) - 672*a*b^2/tan(d*x + c)^5 - 32*(7*tan(d*x + c)
^2 + 5)*a^3/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.88397, size = 837, normalized size = 2.76

$$-\frac{32\left(2\,a^3+21\,ab^2\right)\cos\left(dx+c\right)^7-224\left(a^3+3\,ab^2\right)\cos\left(dx+c\right)^5+105\left(\left(a^2b+2\,b^3\right)\cos\left(dx+c\right)^6-3\left(a^2b+2\,b^3\right)\cos\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1120*(32*(2*a^3 + 21*a*b^2)*cos(d*x + c)^7 - 224*(a^3 + 3*a*b^2)*cos(d*x
 + c)^5 + 105*((a^2*b + 2*b^3)*cos(d*x + c)^6 - 3*(a^2*b + 2*b^3)*cos(d*x +
 c)^4 - a^2*b - 2*b^3 + 3*(a^2*b + 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 105*((a^2*b + 2*b^3)*cos(d*x + c)^6 - 3*(a^2*b +
2*b^3)*cos(d*x + c)^4 - a^2*b - 2*b^3 + 3*(a^2*b + 2*b^3)*cos(d*x + c)^2)*l
og(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*((3*a^2*b - 10*b^3)*cos(d*x +
 c)^5 + 8*(a^2*b + 2*b^3)*cos(d*x + c)^3 - 3*(a^2*b + 2*b^3)*cos(d*x + c))*
sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2
- d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29045, size = 616, normalized size = 2.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*tan(1/2*d*x + 1/2*c)^7 + 35*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 7*
a^3*tan(1/2*d*x + 1/2*c)^5 + 84*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 105*a^2*b*ta
n(1/2*d*x + 1/2*c)^4 + 70*b^3*tan(1/2*d*x + 1/2*c)^4 - 35*a^3*tan(1/2*d*x +
 1/2*c)^3 - 420*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 105*a^2*b*tan(1/2*d*x + 1/2*
c)^2 - 560*b^3*tan(1/2*d*x + 1/2*c)^2 + 105*a^3*tan(1/2*d*x + 1/2*c) + 840*
a*b^2*tan(1/2*d*x + 1/2*c) + 840*(a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*
c))) - (2178*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 4356*b^3*tan(1/2*d*x + 1/2*c)^7
 + 105*a^3*tan(1/2*d*x + 1/2*c)^6 + 840*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 105*
a^2*b*tan(1/2*d*x + 1/2*c)^5 - 560*b^3*tan(1/2*d*x + 1/2*c)^5 - 35*a^3*tan(
1/2*d*x + 1/2*c)^4 - 420*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 105*a^2*b*tan(1/2*d
*x + 1/2*c)^3 + 70*b^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c)^
2 + 84*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2*b*tan(1/2*d*x + 1/2*c) + 5*a^3
)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.1126 $\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=334

$$-\frac{b\left(6a^2+7b^2\right)\cot(c+dx)}{35d}-\frac{3a\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d}-\frac{\left(-148a^2b^2+35a^4+24b^4\right)\cot(c+dx)\csc^3(c+dx)}{2240ad}-$$

```
[Out] (-3*a*(a^2 + 8*b^2)*ArcTanh[Cos[c + d*x]])/(128*d) - (b*(6*a^2 + 7*b^2)*Cot
[c + d*x])/(35*d) - (3*a*(a^2 + 8*b^2)*Cot[c + d*x]*Csc[c + d*x])/(128*d) -
 (b*(24*a^4 - 25*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(280*a^2*d)
- ((35*a^4 - 148*a^2*b^2 + 24*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(2240*a*d)
+ (3*b*(23*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2)
/(560*a^2*d) + ((21*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c +
 d*x])^3)/(112*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*x])
^4)/(14*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^7*(a + b*Sin[c + d*x])^4)/(8*a*
d)
```

Rubi [A] time = 0.907272, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2893, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$-\frac{b\left(6a^2+7b^2\right)\cot(c+dx)}{35d}-\frac{3a\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d}-\frac{\left(-148a^2b^2+35a^4+24b^4\right)\cot(c+dx)\csc^3(c+dx)}{2240ad}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-3*a*(a^2 + 8*b^2)*ArcTanh[Cos[c + d*x]])/(128*d) - (b*(6*a^2 + 7*b^2)*Cot
[c + d*x])/(35*d) - (3*a*(a^2 + 8*b^2)*Cot[c + d*x]*Csc[c + d*x])/(128*d) -
 (b*(24*a^4 - 25*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(280*a^2*d)
- ((35*a^4 - 148*a^2*b^2 + 24*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(2240*a*d)
+ (3*b*(23*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2)
/(560*a^2*d) + ((21*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c +
 d*x])^3)/(112*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*x])
^4)/(14*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^7*(a + b*Sin[c + d*x])^4)/(8*a*
d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^4}{14a^2d} - \frac{\cot(c+dx)\csc^7(c+}{}$$
$$= \frac{\left(21a^2-4b^2\right)\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{112a^2d} + \frac{b\cot(c}{}$$
$$= \frac{3b\left(23a^2-4b^2\right)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{560a^2d} + \frac{(21a}{}$$
$$= -\frac{\left(35a^4-148a^2b^2+24b^4\right)\cot(c+dx)\csc^3(c+dx)}{2240ad} + \frac{3b\left(23a^2-4b\right.}{}$$
$$= -\frac{b\left(24a^4-25a^2b^2+4b^4\right)\cot(c+dx)\csc^2(c+dx)}{280a^2d} - \frac{\left(35a^4-148a^2\right.}{}$$
$$= -\frac{b\left(24a^4-25a^2b^2+4b^4\right)\cot(c+dx)\csc^2(c+dx)}{280a^2d} - \frac{\left(35a^4-148a^2\right.}{}$$
$$= -\frac{3a\left(a^2+8b^2\right)\cot(c+dx)\csc(c+dx)}{128d} - \frac{b\left(24a^4-25a^2b^2+4b^4\right)c}{280a}$$
$$= -\frac{3a\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d} - \frac{b\left(6a^2+7b^2\right)\cot(c+dx)}{35d} -$$

Mathematica [A] time = 1.53012, size = 268, normalized size = 0.8

$$-\frac{-6720a\left(a^2+8b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6720a\left(a^2+8b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\csc^8(c+dx)\left(35a\left(671a^2+248b^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(6720*a*(a^2 + 8*b^2)*Log[Cos[(c + d*x)/2]] - 6720*a*(a^2 + 8*b^2)*Log[Sin
[(c + d*x)/2]] + Csc[c + d*x]^8*(35*a*(671*a^2 + 248*b^2)*Cos[c + d*x] + 35
*(333*a^3 + 104*a*b^2)*Cos[3*(c + d*x)] + 805*a^3*Cos[5*(c + d*x)] - 11480*
a*b^2*Cos[5*(c + d*x)] - 105*a^3*Cos[7*(c + d*x)] - 840*a*b^2*Cos[7*(c + d*
x)] + 21504*a^2*b*Sin[2*(c + d*x)] + 2688*b^3*Sin[2*(c + d*x)] + 16128*a^2*
b*Sin[4*(c + d*x)] + 896*b^3*Sin[4*(c + d*x)] + 3072*a^2*b*Sin[6*(c + d*x)]
 - 896*b^3*Sin[6*(c + d*x)] - 384*a^2*b*Sin[8*(c + d*x)] - 448*b^3*Sin[8*(c
 + d*x)]))/(286720*d)
```

Maple [A] time = 0.117, size = 358, normalized size = 1.1

$$-\frac{a^3(\cos(dx+c))^5}{8d(\sin(dx+c))^8} - \frac{a^3(\cos(dx+c))^5}{16d(\sin(dx+c))^6} - \frac{a^3(\cos(dx+c))^5}{64d(\sin(dx+c))^4} + \frac{a^3(\cos(dx+c))^5}{128d(\sin(dx+c))^2} + \frac{a^3(\cos(dx+c))^3}{128d} + \frac{3a^3\cos(d}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/8/d*a^3/sin(d*x+c)^8*cos(d*x+c)^5-1/16/d*a^3/sin(d*x+c)^6*cos(d*x+c)^5-1
/64/d*a^3/sin(d*x+c)^4*cos(d*x+c)^5+1/128/d*a^3/sin(d*x+c)^2*cos(d*x+c)^5+1
/128*a^3*cos(d*x+c)^3/d+3/128*a^3*cos(d*x+c)/d+3/128/d*a^3*ln(csc(d*x+c)-co
t(d*x+c))-3/7/d*a^2*b/sin(d*x+c)^7*cos(d*x+c)^5-6/35/d*a^2*b/sin(d*x+c)^5*c
os(d*x+c)^5-1/2/d*a*b^2/sin(d*x+c)^6*cos(d*x+c)^5-1/8/d*a*b^2/sin(d*x+c)^4*
```

```
cos(d*x+c)^5+1/16/d*a*b^2/sin(d*x+c)^2*cos(d*x+c)^5+1/16*a*b^2*cos(d*x+c)^3
/d+3/16*a*b^2*cos(d*x+c)/d+3/16/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-1/5/d*b^3
/sin(d*x+c)^5*cos(d*x+c)^5
```

Maxima [A] time = 1.05045, size = 335, normalized size = 1.

$$\frac{35\,a^3\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+280\,ab^2\left(\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8960*(35*a^3*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3
 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))
+ 280*a*b^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
+ 1) + 3*log(cos(d*x + c) - 1)) - 1792*b^3/tan(d*x + c)^5 - 768*(7*tan(d*x
+ c)^2 + 5)*a^2*b/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.91738, size = 957, normalized size = 2.87

$$\frac{210\left(a^3+8\,ab^2\right)\cos\left(dx+c\right)^7-70\left(11\,a^3-40\,ab^2\right)\cos\left(dx+c\right)^5-770\left(a^3+8\,ab^2\right)\cos\left(dx+c\right)^3+210\left(a^3+8\,ab^2\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8960*(210*(a^3 + 8*a*b^2)*cos(d*x + c)^7 - 70*(11*a^3 - 40*a*b^2)*cos(d*x
 + c)^5 - 770*(a^3 + 8*a*b^2)*cos(d*x + c)^3 + 210*(a^3 + 8*a*b^2)*cos(d*x
+ c) - 105*((a^3 + 8*a*b^2)*cos(d*x + c)^8 - 4*(a^3 + 8*a*b^2)*cos(d*x + c)
^6 + 6*(a^3 + 8*a*b^2)*cos(d*x + c)^4 + a^3 + 8*a*b^2 - 4*(a^3 + 8*a*b^2)*c
os(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 105*((a^3 + 8*a*b^2)*cos(d*x +
 c)^8 - 4*(a^3 + 8*a*b^2)*cos(d*x + c)^6 + 6*(a^3 + 8*a*b^2)*cos(d*x + c)^4
 + a^3 + 8*a*b^2 - 4*(a^3 + 8*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c)
+ 1/2) + 256*((6*a^2*b + 7*b^3)*cos(d*x + c)^7 - 7*(3*a^2*b + b^3)*cos(d*x
+ c)^5)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x
+ c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37819, size = 617, normalized size = 1.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/71680*(35*a^3*tan(1/2*d*x + 1/2*c)^8 + 240*a^2*b*tan(1/2*d*x + 1/2*c)^7 +
 560*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 336*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 448*
b^3*tan(1/2*d*x + 1/2*c)^5 - 280*a^3*tan(1/2*d*x + 1/2*c)^4 - 1680*a*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 2240*b^3*tan(1/2
*d*x + 1/2*c)^3 - 1680*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 5040*a^2*b*tan(1/2*d*
x + 1/2*c) + 4480*b^3*tan(1/2*d*x + 1/2*c) + 1680*(a^3 + 8*a*b^2)*log(abs(t
an(1/2*d*x + 1/2*c))) - (4566*a^3*tan(1/2*d*x + 1/2*c)^8 + 36528*a*b^2*tan(
1/2*d*x + 1/2*c)^8 + 5040*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 4480*b^3*tan(1/2*d
*x + 1/2*c)^7 - 1680*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 1680*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 - 2240*b^3*tan(1/2*d*x + 1/2*c)^5 - 280*a^3*tan(1/2*d*x + 1/2*c)
^4 - 1680*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 336*a^2*b*tan(1/2*d*x + 1/2*c)^3 +
 448*b^3*tan(1/2*d*x + 1/2*c)^3 + 560*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*a^
2*b*tan(1/2*d*x + 1/2*c) + 35*a^3)/tan(1/2*d*x + 1/2*c)^8)/d
```

3.1127 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=307

$$-\frac{(-25a^2b^2+30a^4+b^4)\cos(c+dx)}{5b^6d}+\frac{6a^2(-3a^2b^2+2a^4+b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}-\frac{(a^2-b^2)\sin^4(c+dx)\cos(c}{ab^2d(a+b\sin(c+dx)}$$

```
[Out] (-3*a*(8*a^4 - 8*a^2*b^2 + b^4)*x)/(4*b^7) + (6*a^2*(2*a^4 - 3*a^2*b^2 + b^
4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*d
) - ((30*a^4 - 25*a^2*b^2 + b^4)*Cos[c + d*x])/(5*b^6*d) + (3*a*(4*a^2 - 3*
b^2)*Cos[c + d*x]*Sin[c + d*x])/(4*b^5*d) - ((10*a^2 - 7*b^2)*Cos[c + d*x]*
Sin[c + d*x]^2)/(5*b^4*d) + ((3*a^2 - 2*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(
2*a*b^3*d) - (Cos[c + d*x]*Sin[c + d*x]^4)/(5*b^2*d) - ((a^2 - b^2)*Cos[c +
 d*x]*Sin[c + d*x]^4)/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.01584, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2892, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{(-25a^2b^2+30a^4+b^4)\cos(c+dx)}{5b^6d}+\frac{6a^2(-3a^2b^2+2a^4+b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}-\frac{(a^2-b^2)\sin^4(c+dx)\cos(c}{ab^2d(a+b\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-3*a*(8*a^4 - 8*a^2*b^2 + b^4)*x)/(4*b^7) + (6*a^2*(2*a^4 - 3*a^2*b^2 + b^
4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*d
) - ((30*a^4 - 25*a^2*b^2 + b^4)*Cos[c + d*x])/(5*b^6*d) + (3*a*(4*a^2 - 3*
b^2)*Cos[c + d*x]*Sin[c + d*x])/(4*b^5*d) - ((10*a^2 - 7*b^2)*Cos[c + d*x]*
Sin[c + d*x]^2)/(5*b^4*d) + ((3*a^2 - 2*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(
2*a*b^3*d) - (Cos[c + d*x]*Sin[c + d*x]^4)/(5*b^2*d) - ((a^2 - b^2)*Cos[c +
 d*x]*Sin[c + d*x]^4)/(a*b^2*d*(a + b*Sin[c + d*x]))
```


Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d(a+b\sin(c+dx))} + \int \frac{\sin^3(c+dx)\left(3\left(8a^2-5\right.\right.}{}$$

$$= \frac{\left(3a^2-2b^2\right)\cos(c+dx)\sin^3(c+dx)}{2ab^3d} - \frac{\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\left(a^2-b^2\right)\cos(c+}{ab^2d(a+b\,\mathrm{s}}$$

$$= -\frac{\left(10a^2-7b^2\right)\cos(c+dx)\sin^2(c+dx)}{5b^4d} + \frac{\left(3a^2-2b^2\right)\cos(c+dx)\sin^3(c+dx)}{2ab^3d} - \mathrm{co}$$

$$= \frac{3a\left(4a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{4b^5d} - \frac{\left(10a^2-7b^2\right)\cos(c+dx)\sin^2(c+dx)}{5b^4d} + \frac{(3}{}$$

$$= -\frac{\left(30a^4-25a^2b^2+b^4\right)\cos(c+dx)}{5b^6d} + \frac{3a\left(4a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{4b^5d} - \frac{(10a}{}$$

$$= -\frac{3a\left(8a^4-8a^2b^2+b^4\right)x}{4b^7} - \frac{\left(30a^4-25a^2b^2+b^4\right)\cos(c+dx)}{5b^6d} + \frac{3a\left(4a^2-3b^2\right)\cos(}{4b}$$

$$= -\frac{3a\left(8a^4-8a^2b^2+b^4\right)x}{4b^7} - \frac{\left(30a^4-25a^2b^2+b^4\right)\cos(c+dx)}{5b^6d} + \frac{3a\left(4a^2-3b^2\right)\cos(}{4b}$$

$$= -\frac{3a\left(8a^4-8a^2b^2+b^4\right)x}{4b^7} - \frac{\left(30a^4-25a^2b^2+b^4\right)\cos(c+dx)}{5b^6d} + \frac{3a\left(4a^2-3b^2\right)\cos(}{4b}$$

$$= -\frac{3a\left(8a^4-8a^2b^2+b^4\right)x}{4b^7} + \frac{6a^2\left(2a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d} - \frac{(30a^4}{}$$

Mathematica [A] time = 4.11241, size = 378, normalized size = 1.23

$$\frac{\frac{960a^2\left(-3a^2b^2+2a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{240a^4b^2\sin(2(c+dx))-960a^3b^3c\sin(c+dx)-960a^3b^3dx\sin(c+dx)-200a^2b^4\sin(2(c+dx))-10a^2b^4\sin(4(c}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((960*a^2*(2*a^4 - 3*a^2*b^2 + b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] - (960*a^6*c - 960*a^4*b^2*c + 120*a^2*b^4*c + 9
60*a^6*d*x - 960*a^4*b^2*d*x + 120*a^2*b^4*d*x + 60*a*b*(16*a^4 - 14*a^2*b^
2 + b^4)*Cos[c + d*x] + 5*(8*a^3*b^3 - 5*a*b^5)*Cos[3*(c + d*x)] - 3*a*b^5*
Cos[5*(c + d*x)] + 960*a^5*b*c*Sin[c + d*x] - 960*a^3*b^3*c*Sin[c + d*x] +
120*a*b^5*c*Sin[c + d*x] + 960*a^5*b*d*x*Sin[c + d*x] - 960*a^3*b^3*d*x*Sin
[c + d*x] + 120*a*b^5*d*x*Sin[c + d*x] + 240*a^4*b^2*Sin[2*(c + d*x)] - 200
*a^2*b^4*Sin[2*(c + d*x)] + 5*b^6*Sin[2*(c + d*x)] - 10*a^2*b^4*Sin[4*(c +
d*x)] + 4*b^6*Sin[4*(c + d*x)] + b^6*Sin[6*(c + d*x)])/(a + b*Sin[c + d*x])
)/(160*b^7*d)
```

Maple [B] time = 0.139, size = 1119, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 12/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^3-3/2/d/b^3*arctan(tan(1/2*d*x+1/2*c)
)*a-2/5/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5-2/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8-4/d/
b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4-10/d/b^6/(1+tan(1/2*d*x
+1/2*c)^2)^5*a^4+8/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*a^2-12/d/b^7*arctan(tan
(1/2*d*x+1/2*c))*a^5+12/d*a^6/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-18/d*a^4/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+5/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^
2)^5*tan(1/2*d*x+1/2*c)^9*a-10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x
+1/2*c)^8*a^4+12/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^2-
8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3+6/d*a^2/b^3/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/b
^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a-40/d/b^6/(1+tan(1/2*d*
x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^4+36/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^6*a^2-60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/
2*c)^4*a^4+44/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^2+8/d
/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a^3-1/d/b^3/(1+tan(1/2
*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a-40/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5
*tan(1/2*d*x+1/2*c)^2*a^4-4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/
2*c)^9*a^3+28/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^2+4/d
/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a^3-5/2/d/b^3/(1+tan(1/2
*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a-2/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*a^2/b^3/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.34225, size = 1451, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/20*(6*a*b^5*cos(d*x + c)^5 - 5*(4*a^3*b^3 - a*b^5)*cos(d*x + c)^3 - 15*(
8*a^6 - 8*a^4*b^2 + a^2*b^4)*d*x - 30*(2*a^5 - a^3*b^2 + (2*a^4*b - a^2*b^3
)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*
sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))
```

```
*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
 15*(8*a^5*b - 8*a^3*b^3 + a*b^5)*cos(d*x + c) - (4*b^6*cos(d*x + c)^5 - 10
*a^2*b^4*cos(d*x + c)^3 + 15*(8*a^5*b - 8*a^3*b^3 + a*b^5)*d*x + 15*(4*a^4*
b^2 - 3*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)
, 1/20*(6*a*b^5*cos(d*x + c)^5 - 5*(4*a^3*b^3 - a*b^5)*cos(d*x + c)^3 - 15*
(8*a^6 - 8*a^4*b^2 + a^2*b^4)*d*x - 60*(2*a^5 - a^3*b^2 + (2*a^4*b - a^2*b^
3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b
^2)*cos(d*x + c))) - 15*(8*a^5*b - 8*a^3*b^3 + a*b^5)*cos(d*x + c) - (4*b^6
*cos(d*x + c)^5 - 10*a^2*b^4*cos(d*x + c)^3 + 15*(8*a^5*b - 8*a^3*b^3 + a*b
^5)*d*x + 15*(4*a^4*b^2 - 3*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin
(d*x + c) + a*b^7*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23183, size = 724, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(15*(8*a^5 - 8*a^3*b^2 + a*b^4)*(d*x + c)/b^7 - 120*(2*a^6 - 3*a^4*b^
2 + a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d
*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 40*(a^4*b*tan(1/
2*d*x + 1/2*c) - a^2*b^3*tan(1/2*d*x + 1/2*c) + a^5 - a^3*b^2)/((a*tan(1/2*
d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^6) + 2*(40*a^3*b*tan(1/2*d
*x + 1/2*c)^9 - 25*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 100*a^4*tan(1/2*d*x + 1/2
*c)^8 - 120*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 20*b^4*tan(1/2*d*x + 1/2*c)^8
+ 80*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 10*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 400*a
^4*tan(1/2*d*x + 1/2*c)^6 - 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 600*a^4*ta
n(1/2*d*x + 1/2*c)^4 - 440*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 40*b^4*tan(1/2*
d*x + 1/2*c)^4 - 80*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 10*a*b^3*tan(1/2*d*x + 1
/2*c)^3 + 400*a^4*tan(1/2*d*x + 1/2*c)^2 - 280*a^2*b^2*tan(1/2*d*x + 1/2*c)
^2 - 40*a^3*b*tan(1/2*d*x + 1/2*c) + 25*a*b^3*tan(1/2*d*x + 1/2*c) + 100*a^
4 - 80*a^2*b^2 + 4*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*b^6))/d
```

3.1128 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=267

$$\frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d}-\frac{2a\left(-7a^2b^2+5a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}-\frac{\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}+\frac{(5a}{}$$

```
[Out] ((40*a^4 - 36*a^2*b^2 + 3*b^4)*x)/(8*b^6) - (2*a*(5*a^4 - 7*a^2*b^2 + 2*b^4
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d)
 + (a*(15*a^2 - 11*b^2)*Cos[c + d*x])/(3*b^5*d) - ((20*a^2 - 13*b^2)*Cos[c
+ d*x]*Sin[c + d*x])/(8*b^4*d) + ((5*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]
^2)/(3*a*b^3*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) - ((a^2 - b^2)*Co
s[c + d*x]*Sin[c + d*x]^3)/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.758161, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2892, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d}-\frac{2a\left(-7a^2b^2+5a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}-\frac{\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}+\frac{(5a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((40*a^4 - 36*a^2*b^2 + 3*b^4)*x)/(8*b^6) - (2*a*(5*a^4 - 7*a^2*b^2 + 2*b^4
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d)
 + (a*(15*a^2 - 11*b^2)*Cos[c + d*x])/(3*b^5*d) - ((20*a^2 - 13*b^2)*Cos[c
+ d*x]*Sin[c + d*x])/(8*b^4*d) + ((5*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]
^2)/(3*a*b^3*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) - ((a^2 - b^2)*Co
s[c + d*x]*Sin[c + d*x]^3)/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
```

```
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d(a+b\sin(c+dx))} + \int \frac{\sin^2(c+dx)\left(15a^2-8b^2-a\right.}{a}$$

$$= \frac{\left(5a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{3ab^3d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx}{ab^2d(a+b\sin(}$$

$$= -\frac{\left(20a^2-13b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{\left(5a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{3ab^3d} - \cos(c$$

$$= \frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d} - \frac{\left(20a^2-13b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{\left(5a^2-3b^2\right)\cos}{3}$$

$$= \frac{\left(40a^4-36a^2b^2+3b^4\right)x}{8b^6} + \frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d} - \frac{\left(20a^2-13b^2\right)\cos(c+dx)\sin}{8b^4d}$$

$$= \frac{\left(40a^4-36a^2b^2+3b^4\right)x}{8b^6} + \frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d} - \frac{\left(20a^2-13b^2\right)\cos(c+dx)\sin}{8b^4d}$$

$$= \frac{\left(40a^4-36a^2b^2+3b^4\right)x}{8b^6} + \frac{a\left(15a^2-11b^2\right)\cos(c+dx)}{3b^5d} - \frac{\left(20a^2-13b^2\right)\cos(c+dx)\sin}{8b^4d}$$

$$= \frac{\left(40a^4-36a^2b^2+3b^4\right)x}{8b^6} - \frac{2a\left(5a^4-7a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d} + \frac{a\left(15a^2-\right.}{}$$

Mathematica [A] time = 3.56595, size = 325, normalized size = 1.22

$$\frac{240a^3b^2\sin(2(c+dx))-864a^2b^3c\sin(c+dx)-864a^2b^3dx\sin(c+dx)+24b\left(-31a^2b^2+40a^4+b^4\right)\cos(c+dx)+\left(40a^2b^3-21b^5\right)\cos(3(c+dx))-864a^3b^2c-864a^3b^2dx+960a^4}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-384*a*(5*a^4 - 7*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/Sqrt[a^2 - b^2] + (960*a^5*c - 864*a^3*b^2*c + 72*a*b^4*c + 960
*a^5*d*x - 864*a^3*b^2*d*x + 72*a*b^4*d*x + 24*b*(40*a^4 - 31*a^2*b^2 + b^4
)*Cos[c + d*x] + (40*a^2*b^3 - 21*b^5)*Cos[3*(c + d*x)] - 3*b^5*Cos[5*(c +
d*x)] + 960*a^4*b*c*Sin[c + d*x] - 864*a^2*b^3*c*Sin[c + d*x] + 72*b^5*c*Si
n[c + d*x] + 960*a^4*b*d*x*Sin[c + d*x] - 864*a^2*b^3*d*x*Sin[c + d*x] + 72
*b^5*d*x*Sin[c + d*x] + 240*a^3*b^2*Sin[2*(c + d*x)] - 176*a*b^4*Sin[2*(c +
 d*x)] - 10*a*b^4*Sin[4*(c + d*x)])/(a + b*Sin[c + d*x]))/(192*b^6*d)
```

Maple [B] time = 0.128, size = 938, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -10/d/b^6*a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))+14/d/b^4*a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))-4/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d
*x+1/2*c)^7*a^2+8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a^3
-5/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+3/4/d/b^2/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-3/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)^3+5/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)+8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*a^3-16/3/d/b^3/(1+tan(1/2*d*x+1/2*
c)^2)^4*a+10/d/b^6*arctan(tan(1/2*d*x+1/2*c))*a^4-9/d/b^4*arctan(tan(1/2*d*
x+1/2*c))*a^2+2/d/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^4
-2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^2-2/d/b^2/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+1/2*c)+3/4/d/b^
2*arctan(tan(1/2*d*x+1/2*c))-16/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*
x+1/2*c)^4*a-3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a^2+24
/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3-40/3/d/b^3/(1+ta
n(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a-3/d/b^4/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)*a^2+2/d/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)*a^3*tan(1/2*d*x+1/2*c)-8/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)^6*a+3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^
2+24/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20357, size = 1391, normalized size = 5.21

$$\left[-\frac{6\,b^5\cos\left(dx+c\right)^5-\left(20\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^3-3\left(40\,a^5-36\,a^3b^2+3\,ab^4\right)dx+12\left(5\,a^4-2\,a^2b^2+\left(5\,a^3b-2\,ab\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/24*(6*b^5*cos(d*x + c)^5 - (20*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 3*(40*
a^5 - 36*a^3*b^2 + 3*a*b^4)*d*x + 12*(5*a^4 - 2*a^2*b^2 + (5*a^3*b - 2*a*b^
3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c
))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))
 - 3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*cos(d*x + c) + (10*a*b^4*cos(d*x + c)^
3 - 3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*d*x - 3*(20*a^3*b^2 - 13*a*b^4)*cos(d
*x + c))*sin(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d), -1/24*(6*b^5*cos(d*x
 + c)^5 - (20*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 3*(40*a^5 - 36*a^3*b^2 + 3*
a*b^4)*d*x - 24*(5*a^4 - 2*a^2*b^2 + (5*a^3*b - 2*a*b^3)*sin(d*x + c))*sqrt
```

```
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) -
3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*cos(d*x + c) + (10*a*b^4*cos(d*x + c)^3 -
 3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*d*x - 3*(20*a^3*b^2 - 13*a*b^4)*cos(d*x
+ c))*sin(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31052, size = 606, normalized size = 2.27

$$\frac{3\left(40a^4-36a^2b^2+3b^4\right)(dx+c)}{b^6} - \frac{48\left(5a^5-7a^3b^2+2ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6} + \frac{48\left(a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^4-a^2\cdots\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)b^5}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(40*a^4 - 36*a^2*b^2 + 3*b^4)*(d*x + c)/b^6 - 48*(5*a^5 - 7*a^3*b^2
 + 2*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*
x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 48*(a^3*b*tan(1/2
*d*x + 1/2*c) - a*b^3*tan(1/2*d*x + 1/2*c) + a^4 - a^2*b^2)/((a*tan(1/2*d*x
 + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^5) + 2*(36*a^2*b*tan(1/2*d*x
+ 1/2*c)^7 - 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*a^3*tan(1/2*d*x + 1/2*c)^6
- 96*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^3
*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*tan(1/2*d*x + 1/2*c)^4 - 192*a*b^2*tan(1/
2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 9*b^3*tan(1/2*d*x + 1/
2*c)^3 + 288*a^3*tan(1/2*d*x + 1/2*c)^2 - 160*a*b^2*tan(1/2*d*x + 1/2*c)^2
- 36*a^2*b*tan(1/2*d*x + 1/2*c) + 15*b^3*tan(1/2*d*x + 1/2*c) + 96*a^3 - 64
*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^5))/d
```

3.1129 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=163

$$\frac{2\left(-5a^2b^2+4a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}-\frac{\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{b^4d}-\frac{ax\left(4a^2-3b^2\right)}{b^5}+\frac{\cos^3(c+}{3b^2}$$

```
[Out] -((a*(4*a^2 - 3*b^2)*x)/b^5) + (2*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b + a*T
an[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]^
3*(4*a + b*Sin[c + d*x]))/(3*b^2*d*(a + b*Sin[c + d*x])) - (Cos[c + d*x]*(4
*a^2 - b^2 - 2*a*b*Sin[c + d*x]))/(b^4*d)
```

Rubi [A] time = 0.30615, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2863, 2865, 2735, 2660, 618, 204}

$$\frac{2\left(-5a^2b^2+4a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}-\frac{\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{b^4d}-\frac{ax\left(4a^2-3b^2\right)}{b^5}+\frac{\cos^3(c+}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -((a*(4*a^2 - 3*b^2)*x)/b^5) + (2*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b + a*T
an[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]^
3*(4*a + b*Sin[c + d*x]))/(3*b^2*d*(a + b*Sin[c + d*x])) - (Cos[c + d*x]*(4
*a^2 - b^2 - 2*a*b*Sin[c + d*x]))/(b^4*d)
```


Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\int \frac{\cos^2(c+dx)(-b-4a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\left(4a^2-b^2-2ab\sin(c+dx)\right)}{b^4d} - \int \frac{2b(2a^2-}{}$$

$$= -\frac{a\left(4a^2-3b^2\right)x}{b^5} + \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\left(4a^2-b^2-2ab\sin(c\right.}{b^4d}$$

$$= -\frac{a\left(4a^2-3b^2\right)x}{b^5} + \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\left(4a^2-b^2-2ab\sin(c\right.}{b^4d}$$

$$= -\frac{a\left(4a^2-3b^2\right)x}{b^5} + \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\left(4a^2-b^2-2ab\sin(c\right.}{b^4d}$$

$$= -\frac{a\left(4a^2-3b^2\right)x}{b^5} + \frac{2\left(4a^4-5a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d} + \frac{\cos^3(c+dx)(4a+b\,\mathrm{s}}{3b^2d(a+b\sin(c}$$

Mathematica [A] time = 2.1696, size = 247, normalized size = 1.52

$$\frac{\frac{48\left(-5a^2b^2+4a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{-24a^2b^2\sin(2(c+dx))+\left(60ab^3-96a^3b\right)\cos(c+dx)+72a^2b^2c+72a^2b^2dx-96a^3bc\sin(c+dx)-96a^3bdx\sin(c+dx)-9}{a+b\sin(}}{24b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((48*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/Sqrt[a^2 - b^2] + (-96*a^4*c + 72*a^2*b^2*c - 96*a^4*d*x + 72*a^2*b^2
```

```
*d*x + (-96*a^3*b + 60*a*b^3)*Cos[c + d*x] - 4*a*b^3*Cos[3*(c + d*x)] - 96*
a^3*b*c*Sin[c + d*x] + 72*a*b^3*c*Sin[c + d*x] - 96*a^3*b*d*x*Sin[c + d*x]
+ 72*a*b^3*d*x*Sin[c + d*x] - 24*a^2*b^2*Sin[2*(c + d*x)] + 14*b^4*Sin[2*(c
 + d*x)] + b^4*Sin[4*(c + d*x)])/(a + b*Sin[c + d*x]))/(24*b^5*d)
```

Maple [B] time = 0.119, size = 627, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5-6/d/b^4/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2+4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2
)^3*tan(1/2*d*x+1/2*c)^4-12/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/
2*c)^2*a^2+4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2+2/d/b^3/
(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)-6/d/b^4/(1+tan(1/2*d*x+1/2*
c)^2)^3*a^2+8/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3-8/d/b^5*arctan(tan(1/2*d*x
+1/2*c))*a^3+6/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a-2/d*a^2/b^3/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/b/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d*a^3/b^4/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)*a+8/d*a^4/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-10/d*a^2/b^3/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/b/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07301, size = 1141, normalized size = 7.

$$\left[-\frac{4\,ab^3\cos\left(dx+c\right)^3+6\left(4\,a^4-3\,a^2b^2\right)dx+3\left(4\,a^3-ab^2+\left(4\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(4*a*b^3*cos(d*x + c)^3 + 6*(4*a^4 - 3*a^2*b^2)*d*x + 3*(4*a^3 - a*b^
2 + (4*a^2*b - b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d
*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c)
```

```
 + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
c) - a^2 - b^2)) + 6*(4*a^3*b - 3*a*b^3)*cos(d*x + c) - 2*(b^4*cos(d*x + c)
^3 - 3*(4*a^3*b - 3*a*b^3)*d*x - 3*(2*a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x
+ c))/(b^6*d*sin(d*x + c) + a*b^5*d), -1/3*(2*a*b^3*cos(d*x + c)^3 + 3*(4*a
^4 - 3*a^2*b^2)*d*x + 3*(4*a^3 - a*b^2 + (4*a^2*b - b^3)*sin(d*x + c))*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) +
3*(4*a^3*b - 3*a*b^3)*cos(d*x + c) - (b^4*cos(d*x + c)^3 - 3*(4*a^3*b - 3*a
*b^3)*d*x - 3*(2*a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d*sin(d*x
+ c) + a*b^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20063, size = 405, normalized size = 2.48

$$-\frac{\frac{3\left(4a^3-3ab^2\right)(dx+c)}{b^5}-\frac{6\left(4a^4-5a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{6\left(a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^3-ab^2\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)b^4}+\frac{2\left(3ab\cdots\right.}{\cdots}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(4*a^3 - 3*a*b^2)*(d*x + c)/b^5 - 6*(4*a^4 - 5*a^2*b^2 + b^4)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/s
qrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) + 6*(a^2*b*tan(1/2*d*x + 1/2*c) - b^
3*tan(1/2*d*x + 1/2*c) + a^3 - a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(
1/2*d*x + 1/2*c) + a)*b^4) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*tan(1/
2*d*x + 1/2*c)^4 - 6*b^2*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*tan(1/2*d*x + 1/2*
c)^2 - 6*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*a^2 -
4*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4))/d
```

3.1130 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$\frac{2\sqrt{a^2-b^2}\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2ax}{b^3}-\frac{\cos(c+dx)}{b^2d}$$

```
[Out] (-2*a*x)/b^3 + (2*Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2*b^3*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cos[c
+ d*x]/(b^2*d) - ((a^2 - b^2)*Cos[c + d*x])/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.273874, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2892, 3057, 2660, 618, 204, 3770}

$$\frac{2\sqrt{a^2-b^2}\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2ax}{b^3}-\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-2*a*x)/b^3 + (2*Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2*b^3*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cos[c
+ d*x]/(b^2*d) - ((a^2 - b^2)*Cos[c + d*x])/(a*b^2*d*(a + b*Sin[c + d*x]))
```


Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)}{b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(b^2-ab\sin(c+dx)-2a^2\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{ab^2}$$

$$= -\frac{2ax}{b^3} - \frac{\cos(c+dx)}{b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{\left(-2a^4+a^2b^2+b^4 \right.}{a^2\ldots}$$

$$= -\frac{2ax}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} - \frac{\left(2\left(-2a^4+a^2b\ldots\right.\right.}{\ldots}$$

$$= -\frac{2ax}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{b^2d} - \frac{\left(a^2-b^2\right)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\left(4\left(-2a^4+a^2b\ldots\right.\right.}{\ldots}$$

$$= -\frac{2ax}{b^3} + \frac{2\left(2a^4-a^2b^2-b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+d\ldots}{b^2d}$$

Mathematica [A] time = 0.70204, size = 161, normalized size = 1.18

$$\frac{\frac{2\left(-a^2b^2+2a^4-b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3\sqrt{a^2-b^2}} + \frac{\left(b^2-a^2\right)\cos(c+dx)}{ab^2(a+b\sin(c+dx))} + \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} - \frac{2a(c+dx)}{b^3} - \frac{\cos(c+dx)}{b^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(c + d*x))/b^3 + (2*(2*a^4 - a^2*b^2 - b^4)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^3*Sqrt[a^2 - b^2]) - Cos[c + d*x]/b^2 - Lo
g[Cos[(c + d*x)/2]]/a^2 + Log[Sin[(c + d*x)/2]]/a^2 + ((-a^2 + b^2)*Cos[c +
 d*x])/(a*b^2*(a + b*Sin[c + d*x])))/d
```

Maple [B] time = 0.151, size = 380, normalized size = 2.8

$$-2\frac{1}{db^2\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}-4\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)a}{db^3}-2\,\frac{\tan\left(1/2\,dx+c/2\right)}{bd\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a+2\,\tan\left(1/2\,dx+c/\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)-4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a-2/d/
b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/
a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b-
2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a+2/d/a/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+4/d*a^2/b^3/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/b/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a^2*b/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a^2*ln(t
an(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.02764, size = 1226, normalized size = 8.95

$$\left[-\frac{4\,a^4dx-\left(2\,a^3+ab^2+\left(2\,a^2b+b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(d}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a^4*d*x - (2*a^3 + a*b^2 + (2*a^2*b + b^3)*sin(d*x + c))*sqrt(-a^2
 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(2*a^3*b - a*b^3)*cos
(d*x + c) + (b^4*sin(d*x + c) + a*b^3)*log(1/2*cos(d*x + c) + 1/2) - (b^4*s
in(d*x + c) + a*b^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*a^3*b*d*x + a^2*b^
2*cos(d*x + c))*sin(d*x + c))/(a^2*b^4*d*sin(d*x + c) + a^3*b^3*d), -1/2*(4
*a^4*d*x + 2*(2*a^3 + a*b^2 + (2*a^2*b + b^3)*sin(d*x + c))*sqrt(a^2 - b^2)
*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*(2*a^3*b
- a*b^3)*cos(d*x + c) + (b^4*sin(d*x + c) + a*b^3)*log(1/2*cos(d*x + c) + 1
/2) - (b^4*sin(d*x + c) + a*b^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*a^3*b*
d*x + a^2*b^2*cos(d*x + c))*sin(d*x + c))/(a^2*b^4*d*sin(d*x + c) + a^3*b^3
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [B] time = 1.38625, size = 386, normalized size = 2.82

$$-\frac{\frac{2\,(dx+c)a}{b^3}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{2\left(2\,a^4-a^2b^2-b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^2b^3}+\frac{2\left(a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a^{\cdots}\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\cdots\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(d*x + c)*a/b^3 - log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 2*(2*a^4 - a^2*b
^2 - b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^2*b^3) + 2*(a^2*b*tan(1/
2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^3*tan(1/2*d*x + 1/2*c)^
2 - a*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - b^3*tan(1
/2*d*x + 1/2*c) + 2*a^3 - a*b^2)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*
a^2*b^2))/d
```

3.1131 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$-\frac{2\left(a^2b^2+a^4-2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2d\sqrt{a^2-b^2}}+\frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{ad(a+b\sin(c+a}$$

```
[Out] x/b^2 - (2*(a^4 + a^2*b^2 - 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^3*b^2*Sqrt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d)
 + ((a^2 - 2*b^2)*Cos[c + d*x])/(a^2*b*d*(a + b*Sin[c + d*x])) - Cot[c + d*
x]/(a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.303716, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2890, 3057, 2660, 618, 204, 3770}

$$-\frac{2\left(a^2b^2+a^4-2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2d\sqrt{a^2-b^2}}+\frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{ad(a+b\sin(c+a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] x/b^2 - (2*(a^4 + a^2*b^2 - 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^3*b^2*Sqrt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d)
 + ((a^2 - 2*b^2)*Cos[c + d*x])/(a^2*b*d*(a + b*Sin[c + d*x])) - Cot[c + d*
x]/(a*d*(a + b*Sin[c + d*x]))
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(-2b^2-ab\sin(c+dx)+a^2\sin^2(c+dx\right.}{a+b\sin(c+dx)}}{a^2b}$$

$$= \frac{x}{b^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int\csc(c+dx)\,dx}{a^3} - \frac{\left(a^4+\right.}{}$$

$$= \frac{x}{b^2} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{\left(2\right(}{}$$

$$= \frac{x}{b^2} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\left(4\right(}{}$$

$$= \frac{x}{b^2} - \frac{2\left(a^4+a^2b^2-2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^2\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\left(a^2-2b\right.}{a^2bd(a+}$$

Mathematica [A] time = 1.76709, size = 182, normalized size = 1.18

$$\frac{-\frac{4\left(a^2b^2+a^4-2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2\sqrt{a^2-b^2}} + \frac{2\left(a^2-b^2\right)\cos(c+dx)}{a^2b(a+b\sin(c+dx))} - \frac{4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{a^2} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{a^2} +}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*(c + d*x))/b^2 - (4*(a^4 + a^2*b^2 - 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^3*b^2*Sqrt[a^2 - b^2]) - Cot[(c + d*x)/2]/a^2 + (
4*b*Log[Cos[(c + d*x)/2]])/a^3 - (4*b*Log[Sin[(c + d*x)/2]])/a^3 + (2*(a^2
- b^2)*Cos[c + d*x])/(a^2*b*(a + b*Sin[c + d*x])) + Tan[(c + d*x)/2]/a^2)/(
```

2*d)

Maple [B] time = 0.161, size = 396, normalized size = 2.6

$$\frac{1}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{\arctan(\tan(1/2dx+c/2))}{db^2}+2\frac{\tan(1/2dx+c/2)}{d\left((\tan(1/2dx+c/2))^2a+2\tan(1/2dx+c/2)b+a\right)a}-2\frac{}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)+2/d/b^2*arctan(tan(1/2*d*x+1/2*c))+2/d/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)-2/d/a^3/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)+2/d/b
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-2/d/a^2/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b-2/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/2/d/a^2/tan(1/2
*d*x+1/2*c)-2/d/a^3*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.93085, size = 1557, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(2*a^3*b*d*x*cos(d*x + c)^2 - 2*a^3*b*d*x + 2*a^2*b^2*cos(d*x + c) - (
a^2*b + 2*b^3 - (a^2*b + 2*b^3)*cos(d*x + c)^2 + (a^3 + 2*a*b^2)*sin(d*x +
c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
 b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(b^4*cos(
d*x + c)^2 - a*b^3*sin(d*x + c) - b^4)*log(1/2*cos(d*x + c) + 1/2) - 2*(b^4
*cos(d*x + c)^2 - a*b^3*sin(d*x + c) - b^4)*log(-1/2*cos(d*x + c) + 1/2) -
2*(a^4*d*x + (a^3*b - 2*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos(d
*x + c)^2 - a^4*b^2*d*sin(d*x + c) - a^3*b^3*d), (a^3*b*d*x*cos(d*x + c)^2
- a^3*b*d*x + a^2*b^2*cos(d*x + c) - (a^2*b + 2*b^3 - (a^2*b + 2*b^3)*cos(d
```

```
*x + c)^2 + (a^3 + 2*a*b^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*
x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (b^4*cos(d*x + c)^2 - a*b^3*s
in(d*x + c) - b^4)*log(1/2*cos(d*x + c) + 1/2) - (b^4*cos(d*x + c)^2 - a*b^
3*sin(d*x + c) - b^4)*log(-1/2*cos(d*x + c) + 1/2) - (a^4*d*x + (a^3*b - 2*
a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos(d*x + c)^2 - a^4*b^2*d*si
n(d*x + c) - a^3*b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37536, size = 351, normalized size = 2.28

$$\frac{\frac{6\,(dx+c)}{b^2} - \frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} + \frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2} - \frac{12\,\left(a^4+a^2b^2-2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3b^2} + \frac{4\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\cdots}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/b^2 - 12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 3*tan(1/2*
d*x + 1/2*c)/a^2 - 12*(a^4 + a^2*b^2 - 2*b^4)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a
^2 - b^2)*a^3*b^2) + (4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*b*tan(1/2*d*x
+ 1/2*c)^2 - 4*b^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) - 1
4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*c)^3 + 2*b*ta
n(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3*b))/d
```

3.1132 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=158

$$\frac{6b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{a^3d(a+b\sin(c+dx))}+\frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{2b\cot(c+dx)}{a^3d}-\ldots$$

```
[Out] (6*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) + (3*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - Cos[c + d*x]/(2*a
^2*d*(1 - Cos[c + d*x]^2)) + (2*b*Cot[c + d*x])/(a^3*d) - ((a^2 - b^2)*Cos[
c + d*x])/(a^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.449638, antiderivative size = 180, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{6b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd}+\frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\left(2a^2-3b^2\right)\cot(c\ldots}{2a^2bd(a+b\sin(c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) + (3*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - ((a^2 - 3*b^2)*Co
t[c + d*x])/(a^3*b*d) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*b*d*(a + b*Si
n[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x]))
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))}+\frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-3b^2\right)-ab\sin(c+dx)+3b^2\right.}{a+b\sin(c+dx)}}{2a^2b}\\
&=-\frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd}+\frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))}+\int \frac{\csc(c}{}\\
&=-\frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd}+\frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))}-\frac{\left(3\left(a^2\right.\right.}{}\\
&=\frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}-\frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd}+\frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))}-\\
&=\frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}-\frac{\left(a^2-3b^2\right)\cot(c+dx)}{a^3bd}+\frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))}-\\
&=\frac{6b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}-\frac{\left(a^2-3b^2\right)\text{co}}{a^3bd}\end{aligned}$$

Mathematica [A] time = 4.16124, size = 191, normalized size = 1.21

$$\frac{48b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)-12\left(a^2-2b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\left(a^2-2b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{8a(b^2}{a-}}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (48*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 8*
a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 12*(a^2 - 2*b^2)*Log[Cos[(c
 + d*x)/2]] - 12*(a^2 - 2*b^2)*Log[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]
^2 + (8*a*(-a^2 + b^2)*Cos[c + d*x])/(a + b*Sin[c + d*x]) - 8*a*b*Tan[(c +
d*x)/2])/(8*a^4*d)
```

Maple [B] time = 0.171, size = 339, normalized size = 2.2

$$\frac{1}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{b}{da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\tan(1/2\,dx+c/2)\,b}{da^2\left((\tan(1/2\,dx+c/2))^2\,a+2\,\tan(1/2\,dx+c/2)\,b+a\right)}+2\,\frac{}{da^4\left((\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3*tan(1/2*d*x+1/2*c)*b-2/d/a^2/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b+2/d*b^3/a^4
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d/a
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d/a^3/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2+6/d/a^4*b*(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/8/d/a^2/tan(1/2*d*x+1/2*c)^
2-3/2/d/a^2*ln(tan(1/2*d*x+1/2*c))+3/d/a^4*ln(tan(1/2*d*x+1/2*c))*b^2+1/d*b
/a^3/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.61294, size = 1891, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/4*(6*a^2*b*cos(d*x + c)*sin(d*x + c) + 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3
 - 6*(a*b*cos(d*x + c)^2 - a*b + (b^2*cos(d*x + c)^2 - b^2)*sin(d*x + c))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a^3 - 2*a*b^
2)*cos(d*x + c) + 3*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2 + (a^2*
b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) - 3*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2 + (a^2*b - 2
*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c)
+ 1/2))/(a^5*d*cos(d*x + c)^2 - a^5*d + (a^4*b*d*cos(d*x + c)^2 - a^4*b*d)*
sin(d*x + c)), -1/4*(6*a^2*b*cos(d*x + c)*sin(d*x + c) + 4*(a^3 - 3*a*b^2)*
cos(d*x + c)^3 + 12*(a*b*cos(d*x + c)^2 - a*b + (b^2*cos(d*x + c)^2 - b^2)*
sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)
*cos(d*x + c))) - 6*(a^3 - 2*a*b^2)*cos(d*x + c) + 3*(a^3 - 2*a*b^2 - (a^3
- 2*a*b^2)*cos(d*x + c)^2 + (a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2
)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^3 - 2*a*b^2 - (a^3 - 2*a
*b^2)*cos(d*x + c)^2 + (a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*sin
(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(a^5*d*cos(d*x + c)^2 - a^5*d + (a
^4*b*d*cos(d*x + c)^2 - a^4*b*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28195, size = 371, normalized size = 2.35

$$-\frac{\frac{12\,(a^2-2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{48\,(a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}+\frac{16\,(a^2b\tan}{(a\tan}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(12*(a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 48*(a^2*b - b^3
)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
 + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (a^2*tan(1/2*d*x + 1/2*c)^2
 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 + 16*(a^2*b*tan(1/2*d*x + 1/2*c) - b^3*t
an(1/2*d*x + 1/2*c) + a^3 - a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2
*d*x + 1/2*c) + a)*a^4) - (18*a^2*tan(1/2*d*x + 1/2*c)^2 - 36*b^2*tan(1/2*d
*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) - a^2)/(a^4*tan(1/2*d*x + 1/2*c)
^2))/d
```

3.1133 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=238

$$\frac{2\left(-5a^2b^2+a^4+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d}-\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{(a^2}{}$$

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^5*Sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^
5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d
*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(
3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a +
 b*Sin[c + d*x]))
```

Rubi [A] time = 0.703252, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2\left(-5a^2b^2+a^4+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d}-\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{(a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^5*Sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^
5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d
*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(
3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a +
 b*Sin[c + d*x]))
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)\left(6\left(a^2-2b^2\right)-ab\,s\right.}{a+b}}{}$$

$$= -\frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{3ad(a+b\,s}$$

$$= \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)}{3a^2bd(a+b\sin(c}$$

$$= \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)}{3a^2bd(a+b\sin(c}$$

$$= -\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)}{a^3bd}$$

$$= -\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)}{a^3bd}$$

$$= \frac{2\left(a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} - \frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2}{}$$

Mathematica [A] time = 6.36736, size = 403, normalized size = 1.69

$$\frac{\left(3a^2b-4b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{\left(4b^3-3a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{a^2b\cos(c+dx)-b^3\cos(c+dx)}{a^4d(a+b\sin(c+dx))} + \frac{\csc\left(\frac{1}{2}(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] +
 a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((4*a^2*C
os[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*
Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a
^2*d) + ((-3*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((3*a^2*b - 4*
b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (S
ec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*
d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) +
 (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [B] time = 0.178, size = 527, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*tan(1/2*d*x+1/2*c)^2*b-5/8/d/a^2*
tan(1/2*d*x+1/2*c)+3/2/d/a^4*b^2*tan(1/2*d*x+1/2*c)+2/d/a^3/(tan(1/2*d*x+1/
```

```
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)-2/d*b^4/a^5/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a^2/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b-2/d*b^3/a^4/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-10/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+8/d*b^4/a^5/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/24/d/a^2/tan
(1/2*d*x+1/2*c)^3+5/8/d/a^2/tan(1/2*d*x+1/2*c)-3/2/d/a^4/tan(1/2*d*x+1/2*c)
*b^2+1/4/d/a^3*b/tan(1/2*d*x+1/2*c)^2+3/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-4/d/
a^5*b^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.98326, size = 2639, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/6*(4*(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 3*((a^2*b - 4*b^3)*cos(d*x +
c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 -
(a^3 - 4*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
- b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*
sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2)) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) + 3*((3*a^2
*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*co
s(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c
)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 + (3*a^3*b -
 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^3*b - 4*a*
b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*
x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c)), -1/6*(4*
(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 6*((a^2*b - 4*b^3)*cos(d*x + c)^4 + a^
2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 - (a^3 - 4*
a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c
) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) +
 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 -
 4*b^4)*cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*c
os(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 +
(3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(
-1/2*cos(d*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^
```

```
3*b - 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*
b*d*cos(d*x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c))
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31562, size = 481, normalized size = 2.02

$$\frac{\frac{24\left(3a^2b-4b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5}+\frac{48\left(a^4-5a^2b^2+4b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^5}+\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + 48*(a^4 - 5
*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(
1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/
2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/
2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 + 48*(a^2*b^2*tan(1/2*d*x + 1/2
*c) - b^4*tan(1/2*d*x + 1/2*c) + a^3*b - a*b^3)/((a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (132*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
176*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*t
an(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/2*d*
x + 1/2*c)^3))/d
```

3.1134 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=292

$$-\frac{2b\left(-7a^2b^2+2a^4+5b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}-\frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d}-\frac{\left(-36a^2b^2+3a^4+40b^4\right)\tanh^{-1}(\cos(c$$

```
[Out] (-2*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^6*Sqrt[a^2 - b^2]*d) - ((3*a^4 - 36*a^2*b^2 + 40*b^4)*ArcTanh[C
os[c + d*x]])/(8*a^6*d) - (b*(11*a^2 - 15*b^2)*Cot[c + d*x])/(3*a^5*d) + ((
13*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) - ((3*a^2 - 5*b^2)*Co
t[c + d*x]*Csc[c + d*x]^2)/(3*a^3*b*d) + ((4*a^2 - 5*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2)/(4*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^3
)/(4*a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.04726, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b\left(-7a^2b^2+2a^4+5b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}-\frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d}-\frac{\left(-36a^2b^2+3a^4+40b^4\right)\tanh^{-1}(\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^6*Sqrt[a^2 - b^2]*d) - ((3*a^4 - 36*a^2*b^2 + 40*b^4)*ArcTanh[C
os[c + d*x]])/(8*a^6*d) - (b*(11*a^2 - 15*b^2)*Cot[c + d*x])/(3*a^5*d) + ((
13*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) - ((3*a^2 - 5*b^2)*Co
t[c + d*x]*Csc[c + d*x]^2)/(3*a^3*b*d) + ((4*a^2 - 5*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2)/(4*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^3
)/(4*a*d*(a + b*Sin[c + d*x]))
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\left(4a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^4(c+dx)\left(4\left(3a^2-5b^2\right)\right.}{}}{}$$

$$= -\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{3a^3bd} + \frac{\left(4a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+}{4ad(a}$$

$$= \frac{\left(13a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{3a^3bd} + \frac{\left(4a^2-\right.}{4}$$

$$= -\frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d} + \frac{\left(13a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\left(3a^2-5b^2\right)\cot}{}$$

$$= -\frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d} + \frac{\left(13a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\left(3a^2-5b^2\right)\cot}{}$$

$$= -\frac{\left(3a^4-36a^2b^2+40b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d} + \frac{\left(13a^2-2\right.}{}$$

$$= -\frac{\left(3a^4-36a^2b^2+40b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{b\left(11a^2-15b^2\right)\cot(c+dx)}{3a^5d} + \frac{\left(13a^2-2\right.}{}$$

$$= -\frac{2b\left(2a^4-7a^2b^2+5b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{\left(3a^4-36a^2b^2+40b^4\right)\tanh^{-1}(\cos(c}{8a^6d}$$

Mathematica [A] time = 6.26046, size = 496, normalized size = 1.7

$$\frac{\left(5a^2-12b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^4d} + \frac{\left(12b^2-5a^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^4d} + \frac{\left(-36a^2b^2+3a^4+40b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^6d} + \frac{\left(36a^2b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)
/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (2*(
2*a^2*b*Cos[(c + d*x)/2] - 3*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5
*d) + ((5*a^2 - 12*b^2)*Csc[(c + d*x)/2]^2)/(32*a^4*d) + (b*Cot[(c + d*x)/2
]*Csc[(c + d*x)/2]^2)/(12*a^3*d) - Csc[(c + d*x)/2]^4/(64*a^2*d) + ((-3*a^4
 + 36*a^2*b^2 - 40*b^4)*Log[Cos[(c + d*x)/2]])/(8*a^6*d) + ((3*a^4 - 36*a^2
*b^2 + 40*b^4)*Log[Sin[(c + d*x)/2]])/(8*a^6*d) + ((-5*a^2 + 12*b^2)*Sec[(c
 + d*x)/2]^2)/(32*a^4*d) + Sec[(c + d*x)/2]^4/(64*a^2*d) + (2*Sec[(c + d*x)
/2]*(2*a^2*b*Sin[(c + d*x)/2] - 3*b^3*Sin[(c + d*x)/2]))/(3*a^5*d) + (-(a^2
*b^2*Cos[c + d*x]) + b^4*Cos[c + d*x])/(a^5*d*(a + b*Sin[c + d*x])) - (b*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(12*a^3*d)
```

Maple [B] time = 0.189, size = 634, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-1/12/d/a^3*tan(1/2*d*x+1/2*c)^3*b-1/8/d/a^2
*tan(1/2*d*x+1/2*c)^2+3/8/d/a^4*b^2*tan(1/2*d*x+1/2*c)^2+5/4/d/a^3*tan(1/2*
d*x+1/2*c)*b-2/d/a^5*b^3*tan(1/2*d*x+1/2*c)-2/d*b^3/a^4/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*b^5/a^6/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-2/d/a^3/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2+2/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)-4/d/a^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+14/d*b^3/a^4/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-10/d*b^5/a^6/(a^2-b^2)^(1/
2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/64/d/a^2/tan(
1/2*d*x+1/2*c)^4+1/8/d/a^2/tan(1/2*d*x+1/2*c)^2-3/8/d/a^4/tan(1/2*d*x+1/2*c
)^2*b^2+3/8/d/a^2*ln(tan(1/2*d*x+1/2*c))-9/2/d/a^4*ln(tan(1/2*d*x+1/2*c))*b
^2+5/d/a^6*ln(tan(1/2*d*x+1/2*c))*b^4+1/12/d/a^3*b/tan(1/2*d*x+1/2*c)^3-5/4
/d*b/a^3/tan(1/2*d*x+1/2*c)+2/d*b^3/a^5/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.6618, size = 3688, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/48*(16*(11*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^5 + 2*(15*a^5 - 196*a^3*b^2
 + 240*a*b^4)*cos(d*x + c)^3 + 24*((2*a^3*b - 5*a*b^3)*cos(d*x + c)^4 + 2*a
^3*b - 5*a*b^3 - 2*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + ((2*a^2*b^2 - 5*b^4
)*cos(d*x + c)^4 + 2*a^2*b^2 - 5*b^4 - 2*(2*a^2*b^2 - 5*b^4)*cos(d*x + c)^2
)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c)
)*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))
 - 6*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c) + 3*(3*a^5 - 36*a^3*b^2 +
40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 36*a
^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^2*b^3 + 40*b^5 + (3*a^4
*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 36*a^2*b^3 + 40*b^5
)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(3*a^5 - 36
*a^3*b^2 + 40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^4 - 2*(3
*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^2*b^3 + 40*b
^5 + (3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 36*a^2*b
^3 + 40*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2
*((49*a^4*b - 60*a^2*b^3)*cos(d*x + c)^3 - 3*(13*a^4*b - 20*a^2*b^3)*cos(d*
```

```
x + c))*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*
d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x +
 c)), -1/48*(16*(11*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^5 + 2*(15*a^5 - 196*a^
3*b^2 + 240*a*b^4)*cos(d*x + c)^3 - 48*((2*a^3*b - 5*a*b^3)*cos(d*x + c)^4
+ 2*a^3*b - 5*a*b^3 - 2*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + ((2*a^2*b^2 -
5*b^4)*cos(d*x + c)^4 + 2*a^2*b^2 - 5*b^4 - 2*(2*a^2*b^2 - 5*b^4)*cos(d*x +
 c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
 - b^2)*cos(d*x + c))) - 6*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c) + 3
*(3*a^5 - 36*a^3*b^2 + 40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x +
 c)^4 - 2*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^
2*b^3 + 40*b^5 + (3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*
b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c)
 + 1/2) - 3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4
)*cos(d*x + c)^4 - 2*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^
4*b - 36*a^2*b^3 + 40*b^5 + (3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4
- 2*(3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*
cos(d*x + c) + 1/2) - 2*((49*a^4*b - 60*a^2*b^3)*cos(d*x + c)^3 - 3*(13*a^4
*b - 20*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*
d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)
^2 + a^6*b*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+b*sin(d*x+c))**2,x)

[Out] Timed out
```

Giac [A] time = 1.39535, size = 622, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/192*(24*(3*a^4 - 36*a^2*b^2 + 40*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6
- 384*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*
a^6) - 384*(a^2*b^3*tan(1/2*d*x + 1/2*c) - b^5*tan(1/2*d*x + 1/2*c) + a^3*b
^2 - a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^6)
 + (3*a^6*tan(1/2*d*x + 1/2*c)^4 - 16*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 24*a^6
*tan(1/2*d*x + 1/2*c)^2 + 72*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*a^5*b*tan
(1/2*d*x + 1/2*c) - 384*a^3*b^3*tan(1/2*d*x + 1/2*c))/a^8 - (150*a^4*tan(1/
2*d*x + 1/2*c)^4 - 1800*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 2000*b^4*tan(1/2*d
*x + 1/2*c)^4 + 240*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 384*a*b^3*tan(1/2*d*x +
1/2*c)^3 - 24*a^4*tan(1/2*d*x + 1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^
2 - 16*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/(a^6*tan(1/2*d*x + 1/2*c)^4))/d
```

3.1135 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=331

$$\frac{a\left(30a^2-13b^2\right)\cos(c+dx)}{2b^6d}-\frac{3a\left(-11a^2b^2+10a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}+\frac{\left(7a^2-2b^2\right)\sin^4(c+dx)\cos(c+d}{2a^2b^2d(a+b\sin(c+dx))}$$

```
[Out] (3*(40*a^4 - 24*a^2*b^2 + b^4)*x)/(8*b^7) - (3*a*(10*a^4 - 11*a^2*b^2 + 2*b
^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*
d) + (a*(30*a^2 - 13*b^2)*Cos[c + d*x])/(2*b^6*d) - (3*(20*a^2 - 7*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(8*b^5*d) + ((10*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c +
d*x]^2)/(2*a*b^4*d) - ((15*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^2
*b^3*d) - ((a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(2*a*b^2*d*(a + b*Sin[c
 + d*x])^2) + ((7*a^2 - 2*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(2*a^2*b^2*d*(a
 + b*Sin[c + d*x]))
```

Rubi [A] time = 1.00846, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2891, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{a\left(30a^2-13b^2\right)\cos(c+dx)}{2b^6d}-\frac{3a\left(-11a^2b^2+10a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}+\frac{\left(7a^2-2b^2\right)\sin^4(c+dx)\cos(c+d}{2a^2b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (3*(40*a^4 - 24*a^2*b^2 + b^4)*x)/(8*b^7) - (3*a*(10*a^4 - 11*a^2*b^2 + 2*b
^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*
d) + (a*(30*a^2 - 13*b^2)*Cos[c + d*x])/(2*b^6*d) - (3*(20*a^2 - 7*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(8*b^5*d) + ((10*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c +
d*x]^2)/(2*a*b^4*d) - ((15*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^2
*b^3*d) - ((a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(2*a*b^2*d*(a + b*Sin[c
 + d*x])^2) + ((7*a^2 - 2*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(2*a^2*b^2*d*(a
 + b*Sin[c + d*x]))
```


Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] && !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{(7a^2-2b^2)\cos(c+dx)\sin^4(c+dx)}{2a^2b^2d(a+b\sin(c+dx))} - \int \frac{\sin^3 \cdots}{\cdots}$$

$$= -\frac{(15a^2-4b^2)\cos(c+dx)\sin^3(c+dx)}{4a^2b^3d} - \frac{(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{(7a^2 \cdots}{\cdots}$$

$$= \frac{(10a^2-3b^2)\cos(c+dx)\sin^2(c+dx)}{2ab^4d} - \frac{(15a^2-4b^2)\cos(c+dx)\sin^3(c+dx)}{4a^2b^3d} - \frac{(a^2 \cdots}{\cdots}$$

$$= -\frac{3(20a^2-7b^2)\cos(c+dx)\sin(c+dx)}{8b^5d} + \frac{(10a^2-3b^2)\cos(c+dx)\sin^2(c+dx)}{2ab^4d} - \cdots$$

$$= \frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d} - \frac{3(20a^2-7b^2)\cos(c+dx)\sin(c+dx)}{8b^5d} + \frac{(10a^2-3b^2 \cdots}{\cdots}$$

$$= \frac{3(40a^4-24a^2b^2+b^4)x}{8b^7} + \frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d} - \frac{3(20a^2-7b^2)\cos(c+d \cdots}{8b^5d}$$

$$= \frac{3(40a^4-24a^2b^2+b^4)x}{8b^7} + \frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d} - \frac{3(20a^2-7b^2)\cos(c+d \cdots}{8b^5d}$$

$$= \frac{3(40a^4-24a^2b^2+b^4)x}{8b^7} + \frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d} - \frac{3(20a^2-7b^2)\cos(c+d \cdots}{8b^5d}$$

$$= \frac{3(40a^4-24a^2b^2+b^4)x}{8b^7} - \frac{3a(10a^4-11a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d} + \frac{a(3 \cdots}{\cdots}$$

Mathematica [B] time = 10.2277, size = 1250, normalized size = 3.78

$$-\frac{-\frac{6\left(-8(c+dx)+\frac{2a(8a^4-20b^2a^2+15b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{3b(4a^4-7b^2a^2+2b^4)\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{ab(4a^2-3b^2)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}\right)}{b^3}+\frac{6\left(\frac{6ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\cos \cdots}{\cdots}\right)}{(a-b)^{\cdots}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((-6*(-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*Co
s[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2*
b^2 + 2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))))/b^3
 + (6*((6*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (Cos[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x]))/(a +
 b*Sin[c + d*x])^2))/((a - b)^2*(a + b)^2) + (2*(-24*(-8*a^2 + b^2)*(c + d*
x) - (6*a*(64*a^6 - 168*a^4*b^2 + 140*a^2*b^4 - 35*b^6)*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 96*a*b*Cos[c + d*x] + (a
*b*(-16*a^4 + 20*a^2*b^2 - 5*b^4)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin
[c + d*x])^2) + (b*(112*a^6 - 220*a^4*b^2 + 115*a^2*b^4 - 10*b^6)*Cos[c + d
*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])) - 8*b^2*Sin[2*(c + d*x)]))/
b^5 + ((12*a*(640*a^8 - 1920*a^6*b^2 + 2016*a^4*b^4 - 840*a^2*b^6 + 105*b^8
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (-3
840*a^10*(c + d*x) + 7680*a^8*b^2*(c + d*x) - 2976*a^6*b^4*(c + d*x) - 1776
```

```
*a^4*b^6*(c + d*x) + 960*a^2*b^8*(c + d*x) - 48*b^10*(c + d*x) - 3840*a^9*b
*Cos[c + d*x] + 8640*a^7*b^3*Cos[c + d*x] - 5696*a^5*b^5*Cos[c + d*x] + 788
*a^3*b^7*Cos[c + d*x] + 114*a*b^9*Cos[c + d*x] + 1920*a^8*b^2*(c + d*x)*Cos
[2*(c + d*x)] - 4800*a^6*b^4*(c + d*x)*Cos[2*(c + d*x)] + 3888*a^4*b^6*(c +
 d*x)*Cos[2*(c + d*x)] - 1056*a^2*b^8*(c + d*x)*Cos[2*(c + d*x)] + 48*b^10*
(c + d*x)*Cos[2*(c + d*x)] + 320*a^7*b^3*Cos[3*(c + d*x)] - 760*a^5*b^5*Cos
[3*(c + d*x)] + 560*a^3*b^7*Cos[3*(c + d*x)] - 120*a*b^9*Cos[3*(c + d*x)] -
 8*a^5*b^5*Cos[5*(c + d*x)] + 16*a^3*b^7*Cos[5*(c + d*x)] - 8*a*b^9*Cos[5*(
c + d*x)] - 7680*a^9*b*(c + d*x)*Sin[c + d*x] + 19200*a^7*b^3*(c + d*x)*Sin
[c + d*x] - 15552*a^5*b^5*(c + d*x)*Sin[c + d*x] + 4224*a^3*b^7*(c + d*x)*S
in[c + d*x] - 192*a*b^9*(c + d*x)*Sin[c + d*x] - 2880*a^8*b^2*Sin[2*(c + d*
x)] + 6880*a^6*b^4*Sin[2*(c + d*x)] - 5182*a^4*b^6*Sin[2*(c + d*x)] + 1221*
a^2*b^8*Sin[2*(c + d*x)] - 36*b^10*Sin[2*(c + d*x)] - 40*a^6*b^4*Sin[4*(c +
 d*x)] + 88*a^4*b^6*Sin[4*(c + d*x)] - 56*a^2*b^8*Sin[4*(c + d*x)] + 8*b^10
*Sin[4*(c + d*x)] + 2*a^4*b^6*Sin[6*(c + d*x)] - 4*a^2*b^8*Sin[6*(c + d*x)]
 + 2*b^10*Sin[6*(c + d*x)])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2))/b^7)/(2
56*d)
```

Maple [B] time = 0.16, size = 1193, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a^2+9/d*a^4/b^5/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-4/d*a^
2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c
)^3+10/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/
2*d*x+1/2*c)^2+15/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)^2-10/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-24/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(
1/2*d*x+1/2*c)^4*a-6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*
a^2+60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3+60/d/b^6/(
1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3+31/d*a^4/b^5/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-16/d*a^2/b^3/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+3/4/d/
b^3*arctan(tan(1/2*d*x+1/2*c))-20/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*
d*x+1/2*c)^2*a+6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2+
20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a^3-12/d/b^4/(1+ta
n(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a+6/d/b^5/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)^5*a^2-18/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2+10/d*
a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-5/d*a^3/b^4/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-5/4/d/b^3/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+3/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)^5-3/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+5/
4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+20/d/b^6/(1+tan(1/2*d
*x+1/2*c)^2)^4*a^3-8/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*a+30/d/b^7*arctan(tan
(1/2*d*x+1/2*c))*a^4-30/d*a^5/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+33/d*a^3/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d*a/b^3/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.88746, size = 2415, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/8*(4*(a^3*b^5 - a*b^7)*cos(d*x + c)^5 - 3*(40*a^6*b^2 - 64*a^4*b^4 + 25
*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 - 2*(20*a^5*b^3 - 27*a^3*b^5 + 7*a*b^7)*
cos(d*x + c)^3 + 3*(40*a^8 - 24*a^6*b^2 - 39*a^4*b^4 + 24*a^2*b^6 - b^8)*d*
x - 6*(10*a^7 - a^5*b^2 - 9*a^3*b^4 + 2*a*b^6 - (10*a^5*b^2 - 11*a^3*b^4 +
2*a*b^6)*cos(d*x + c)^2 + 2*(10*a^6*b - 11*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c
))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
 b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(20*a^7*b
 - 22*a^5*b^3 - a^3*b^5 + 3*a*b^7)*cos(d*x + c) - (2*(a^2*b^6 - b^8)*cos(d*
x + c)^5 - (10*a^4*b^4 - 11*a^2*b^6 + b^8)*cos(d*x + c)^3 - 6*(40*a^7*b - 6
4*a^5*b^3 + 25*a^3*b^5 - a*b^7)*d*x - 3*(60*a^6*b^2 - 91*a^4*b^4 + 32*a^2*b
^6 - b^8)*cos(d*x + c))*sin(d*x + c))/((a^2*b^9 - b^11)*d*cos(d*x + c)^2 -
2*(a^3*b^8 - a*b^10)*d*sin(d*x + c) - (a^4*b^7 - b^11)*d), -1/8*(4*(a^3*b^5
 - a*b^7)*cos(d*x + c)^5 - 3*(40*a^6*b^2 - 64*a^4*b^4 + 25*a^2*b^6 - b^8)*d
*x*cos(d*x + c)^2 - 2*(20*a^5*b^3 - 27*a^3*b^5 + 7*a*b^7)*cos(d*x + c)^3 +
3*(40*a^8 - 24*a^6*b^2 - 39*a^4*b^4 + 24*a^2*b^6 - b^8)*d*x + 12*(10*a^7 -
a^5*b^2 - 9*a^3*b^4 + 2*a*b^6 - (10*a^5*b^2 - 11*a^3*b^4 + 2*a*b^6)*cos(d*x
 + c)^2 + 2*(10*a^6*b - 11*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^
2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(20*a^7
*b - 22*a^5*b^3 - a^3*b^5 + 3*a*b^7)*cos(d*x + c) - (2*(a^2*b^6 - b^8)*cos(
d*x + c)^5 - (10*a^4*b^4 - 11*a^2*b^6 + b^8)*cos(d*x + c)^3 - 6*(40*a^7*b -
 64*a^5*b^3 + 25*a^3*b^5 - a*b^7)*d*x - 3*(60*a^6*b^2 - 91*a^4*b^4 + 32*a^2
*b^6 - b^8)*cos(d*x + c))*sin(d*x + c))/((a^2*b^9 - b^11)*d*cos(d*x + c)^2
- 2*(a^3*b^8 - a*b^10)*d*sin(d*x + c) - (a^4*b^7 - b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36459, size = 729, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(40*a^4 - 24*a^2*b^2 + b^4)*(d*x + c)/b^7 - 24*(10*a^5 - 11*a^3*b^2
+ 2*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
 + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 8*(9*a^4*b*tan(1/2
*d*x + 1/2*c)^3 - 4*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 10*a^5*tan(1/2*d*x + 1
/2*c)^2 + 15*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 - 10*a*b^4*tan(1/2*d*x + 1/2*c)
^2 + 31*a^4*b*tan(1/2*d*x + 1/2*c) - 16*a^2*b^3*tan(1/2*d*x + 1/2*c) + 10*a
^5 - 5*a^3*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^
2*b^6) + 2*(24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 5*b^3*tan(1/2*d*x + 1/2*c)^7
+ 80*a^3*tan(1/2*d*x + 1/2*c)^6 - 48*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*
b*tan(1/2*d*x + 1/2*c)^5 + 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*a^3*tan(1/2*d
*x + 1/2*c)^4 - 96*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/
2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*tan(1/2*d*x + 1/2*c)^2 - 80
*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b*tan(1/2*d*x + 1/2*c) + 5*b^3*tan(1
/2*d*x + 1/2*c) + 80*a^3 - 32*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^6))/
d
```

3.1136 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=284

$$-\frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d}+\frac{(-19a^2b^2+20a^4+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}+\frac{(6a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-$$

```
[Out] (a*(9 - (20*a^2)/b^2)*x)/(2*b^4) + ((20*a^4 - 19*a^2*b^2 + 2*b^4)*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d) - ((60*a^2
 - 17*b^2)*Cos[c + d*x])/(6*b^5*d) + ((5*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*
x])/(a*b^4*d) - ((20*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(6*a^2*b^3*d
) - ((a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(2*a*b^2*d*(a + b*Sin[c + d*x
])^2) + ((6*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(2*a^2*b^2*d*(a + b*Sin
[c + d*x]))
```

Rubi [A] time = 0.738213, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2891, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d}+\frac{(-19a^2b^2+20a^4+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}+\frac{(6a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (a*(9 - (20*a^2)/b^2)*x)/(2*b^4) + ((20*a^4 - 19*a^2*b^2 + 2*b^4)*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d) - ((60*a^2
 - 17*b^2)*Cos[c + d*x])/(6*b^5*d) + ((5*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*
x])/(a*b^4*d) - ((20*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(6*a^2*b^3*d
) - ((a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(2*a*b^2*d*(a + b*Sin[c + d*x
])^2) + ((6*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(2*a^2*b^2*d*(a + b*Sin
[c + d*x]))
```


Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{(6a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{2a^2b^2d(a+b\sin(c+dx))} - \int \frac{\sin^2(c}{}$$

$$= -\frac{(20a^2-3b^2)\cos(c+dx)\sin^2(c+dx)}{6a^2b^3d} - \frac{(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + (6a^2$$

$$= \frac{(5a^2-b^2)\cos(c+dx)\sin(c+dx)}{ab^4d} - \frac{(20a^2-3b^2)\cos(c+dx)\sin^2(c+dx)}{6a^2b^3d} - \frac{(a^2-}{2a}$$

$$= -\frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d} + \frac{(5a^2-b^2)\cos(c+dx)\sin(c+dx)}{ab^4d} - \frac{(20a^2-3b^2)\text{co}}{6}$$

$$= -\frac{a(20a^2-9b^2)x}{2b^6} - \frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d} + \frac{(5a^2-b^2)\cos(c+dx)\sin(c+dx)}{ab^4d}$$

$$= -\frac{a(20a^2-9b^2)x}{2b^6} - \frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d} + \frac{(5a^2-b^2)\cos(c+dx)\sin(c+dx)}{ab^4d}$$

$$= -\frac{a(20a^2-9b^2)x}{2b^6} - \frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d} + \frac{(5a^2-b^2)\cos(c+dx)\sin(c+dx)}{ab^4d}$$

$$= -\frac{a(20a^2-9b^2)x}{2b^6} + \frac{(20a^4-19a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d} - \frac{(60a^2-17b^2)}{6b^5}$$

Mathematica [B] time = 6.25442, size = 1030, normalized size = 3.63

$$-\frac{12\left(-48a(c+dx)+\frac{6(16a^6-40b^2a^4+30b^4a^2-5b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-16b\cos(c+dx)+\frac{ab(-40a^4+72b^2a^2-29b^4)\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{b(8a^4-8b^2a^2+b^4)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}\right)}{b^4}+12\left(2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-12*(-48*a*(c + d*x) + (6*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*ArcT
an[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 16*b*Cos[
c + d*x] + (b*(8*a^4 - 8*a^2*b^2 + b^4)*Cos[c + d*x])/((a - b)*(a + b)*(a +
 b*Sin[c + d*x])^2) + (a*b*(-40*a^4 + 72*a^2*b^2 - 29*b^4)*Cos[c + d*x])/((
a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))))/b^4 + 12*((2*(2*a^2 + b^2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (b*Cos[c +
 d*x]*(4*a^2 - b^2 + 3*a*b*Sin[c + d*x]))/((a - b)^2*(a + b)^2*(a + b*Sin[c
 + d*x])^2)) + (6*((-6*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] + (Cos[c + d*x]*(-(b*(2*a^2 + b^2)) + a*(2*a^2 - 5*b^2)*S
in[c + d*x]))/(a + b*Sin[c + d*x])^2))/((a - b)^2*(a + b)^2) - ((-12*(640*a
^8 - 1792*a^6*b^2 + 1680*a^4*b^4 - 560*a^2*b^6 + 35*b^8)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (3840*a^9*(c + d*x) - 6
912*a^7*b^2*(c + d*x) + 1728*a^5*b^4*(c + d*x) + 1920*a^3*b^6*(c + d*x) - 5
76*a*b^8*(c + d*x) + 3840*a^8*b*Cos[c + d*x] - 7872*a^6*b^3*Cos[c + d*x] +
4256*a^4*b^5*Cos[c + d*x] - 172*a^2*b^7*Cos[c + d*x] - 70*b^9*Cos[c + d*x]
- 1920*a^7*b^2*(c + d*x)*Cos[2*(c + d*x)] + 4416*a^5*b^4*(c + d*x)*Cos[2*(c
 + d*x)] - 3072*a^3*b^6*(c + d*x)*Cos[2*(c + d*x)] + 576*a*b^8*(c + d*x)*Co
```

```
s[2*(c + d*x)] - 320*a^6*b^3*Cos[3*(c + d*x)] + 696*a^4*b^5*Cos[3*(c + d*x)
] - 432*a^2*b^7*Cos[3*(c + d*x)] + 56*b^9*Cos[3*(c + d*x)] + 8*a^4*b^5*Cos[
5*(c + d*x)] - 16*a^2*b^7*Cos[5*(c + d*x)] + 8*b^9*Cos[5*(c + d*x)] + 7680*
a^8*b*(c + d*x)*Sin[c + d*x] - 17664*a^6*b^3*(c + d*x)*Sin[c + d*x] + 12288
*a^4*b^5*(c + d*x)*Sin[c + d*x] - 2304*a^2*b^7*(c + d*x)*Sin[c + d*x] + 288
0*a^7*b^2*Sin[2*(c + d*x)] - 6304*a^5*b^4*Sin[2*(c + d*x)] + 4022*a^3*b^6*S
in[2*(c + d*x)] - 607*a*b^8*Sin[2*(c + d*x)] + 40*a^5*b^4*Sin[4*(c + d*x)]
- 80*a^3*b^6*Sin[4*(c + d*x)] + 40*a*b^8*Sin[4*(c + d*x)])/((a^2 - b^2)^2*(
a + b*Sin[c + d*x])^2))/b^6)/(384*d)
```

Maple [B] time = 0.152, size = 880, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5-12/d/b^5/(1+tan(
1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^
2)^3*tan(1/2*d*x+1/2*c)^4-24/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1
/2*c)^2*a^2+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2+3/d/b^4
/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)-12/d/b^5/(1+tan(1/2*d*x+1/
2*c)^2)^3*a^2+8/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3-20/d/b^6*arctan(tan(1/2*
d*x+1/2*c))*a^3+9/d/b^4*a*arctan(tan(1/2*d*x+1/2*c))-7/d/b^4/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*a^3+2/d/b^2/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*a-8/d/
b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^
2*a^4-13/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*
d*x+1/2*c)^2*a^2+6/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*
tan(1/2*d*x+1/2*c)^2-25/d/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*tan(1/2*d*x+1/2*c)*a^3+10/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*a-8/d/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2*a^4+3/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*a^2+20/d/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2))*a^4-19/d/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.55714, size = 2152, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")

[Out] [1/12*(4*(a^2*b^5 - b^7)*cos(d*x + c)^5 - 6*(20*a^5*b^2 - 29*a^3*b^4 + 9*a*
b^6)*d*x*cos(d*x + c)^2 - 8*(5*a^4*b^3 - 6*a^2*b^5 + b^7)*cos(d*x + c)^3 +
6*(20*a^7 - 9*a^5*b^2 - 20*a^3*b^4 + 9*a*b^6)*d*x + 3*(20*a^6 + a^4*b^2 - 1
7*a^2*b^4 + 2*b^6 - (20*a^4*b^2 - 19*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(2
0*a^5*b - 19*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
- b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*
sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2)) + 6*(20*a^6*b - 19*a^4*b^3 - 3*a^2*b^5 + 2*b^7
)*cos(d*x + c) + 2*(5*(a^3*b^4 - a*b^6)*cos(d*x + c)^3 + 6*(20*a^6*b - 29*a
^4*b^3 + 9*a^2*b^5)*d*x + 3*(30*a^5*b^2 - 41*a^3*b^4 + 11*a*b^6)*cos(d*x +
c))*sin(d*x + c))/((a^2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^3*b^7 - a*b^9)*
d*sin(d*x + c) - (a^4*b^6 - b^10)*d), 1/6*(2*(a^2*b^5 - b^7)*cos(d*x + c)^5
- 3*(20*a^5*b^2 - 29*a^3*b^4 + 9*a*b^6)*d*x*cos(d*x + c)^2 - 4*(5*a^4*b^3
- 6*a^2*b^5 + b^7)*cos(d*x + c)^3 + 3*(20*a^7 - 9*a^5*b^2 - 20*a^3*b^4 + 9*
a*b^6)*d*x + 3*(20*a^6 + a^4*b^2 - 17*a^2*b^4 + 2*b^6 - (20*a^4*b^2 - 19*a^
2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(20*a^5*b - 19*a^3*b^3 + 2*a*b^5)*sin(d*x
+ c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*
x + c))) + 3*(20*a^6*b - 19*a^4*b^3 - 3*a^2*b^5 + 2*b^7)*cos(d*x + c) + (5*
(a^3*b^4 - a*b^6)*cos(d*x + c)^3 + 6*(20*a^6*b - 29*a^4*b^3 + 9*a^2*b^5)*d*
x + 3*(30*a^5*b^2 - 41*a^3*b^4 + 11*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^3*b^7 - a*b^9)*d*sin(d*x + c) - (a^4*
b^6 - b^10)*d)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)

[Out] Timed out

Giac [A] time = 1.25778, size = 531, normalized size = 1.87

$$-\frac{\frac{3\left(20a^3-9ab^2\right)(dx+c)}{b^6}-\frac{6\left(20a^4-19a^2b^2+2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}b^6}+\frac{6\left(7a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8a\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/6*(3*(20*a^3 - 9*a*b^2)*(d*x + c)/b^6 - 6*(20*a^4 - 19*a^2*b^2 + 2*b^4)*
(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 6*(7*a^3*b*tan(1/2*d*x + 1/2*
c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*tan(1/2*d*x + 1/2*c)^2 + 13*a

```
^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*b^4*tan(1/2*d*x + 1/2*c)^2 + 25*a^3*b*tan
(1/2*d*x + 1/2*c) - 10*a*b^3*tan(1/2*d*x + 1/2*c) + 8*a^4 - 3*a^2*b^2)/((a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*b^5) + 2*(9*a*b*ta
n(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*b^2*tan(1/2*d*x +
 1/2*c)^4 + 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 -
 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 - 8*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)
^3*b^5))/d
```

3.1137 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=173

$$-\frac{3a\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{3\cos(c+dx)\left(4a^2+2ab\sin(c+dx)-b^2\right)}{2b^4d(a+b\sin(c+dx))}+\frac{3x\left(4a^2-b^2\right)}{2b^5}+\frac{\cos^3(c+dx)(\cdots}{2b^2d(a+\cdots}$$

```
[Out] (3*(4*a^2 - b^2)*x)/(2*b^5) - (3*a*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]^3*(2*a +
b*Sin[c + d*x]))/(2*b^2*d*(a + b*Sin[c + d*x])^2) + (3*Cos[c + d*x]*(4*a^2
- b^2 + 2*a*b*Sin[c + d*x]))/(2*b^4*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.274943, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2863, 2735, 2660, 618, 204}

$$-\frac{3a\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{3\cos(c+dx)\left(4a^2+2ab\sin(c+dx)-b^2\right)}{2b^4d(a+b\sin(c+dx))}+\frac{3x\left(4a^2-b^2\right)}{2b^5}+\frac{\cos^3(c+dx)(\cdots}{2b^2d(a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*(4*a^2 - b^2)*x)/(2*b^5) - (3*a*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]^3*(2*a +
b*Sin[c + d*x]))/(2*b^2*d*(a + b*Sin[c + d*x])^2) + (3*Cos[c + d*x]*(4*a^2
- b^2 + 2*a*b*Sin[c + d*x]))/(2*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} - \frac{3\int \frac{\cos^2(c+dx)(-2b-4a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{4b^2}$$

$$= \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2+2ab\sin(c+dx)\right)}{2b^4d(a+b\sin(c+dx))} + \frac{3\int \frac{4ab}{}}{}$$

$$= \frac{3\left(4a^2-b^2\right)x}{2b^5} + \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2+2ab\sin(c\right.}{2b^4d(a+b\sin(c+dx))}$$

$$= \frac{3\left(4a^2-b^2\right)x}{2b^5} + \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2+2ab\sin(c\right.}{2b^4d(a+b\sin(c+dx))}$$

$$= \frac{3\left(4a^2-b^2\right)x}{2b^5} + \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2+2ab\sin(c\right.}{2b^4d(a+b\sin(c+dx))}$$

$$= \frac{3\left(4a^2-b^2\right)x}{2b^5} - \frac{3a\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d} + \frac{\cos^3(c+dx)(2a+b\sin(c+d}{2b^2d(a+b\sin(c+dx))^2}$$

Mathematica [A] time = 3.47278, size = 274, normalized size = 1.58

$$\frac{\frac{72a^2b^2\sin(2(c+dx))+12b^2\left(b^2-4a^2\right)(c+dx)\cos(2(c+dx))+24a^2b^2c+24a^2b^2dx+192a^3bc\sin(c+dx)+192a^3bdx\sin(c+dx)+96a^3b\cos(c+dx)+96a^4c+96a^4dx-48ab^3c}{(a+b\sin(c+dx))^2}}{16b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-48*a*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + (96*a^4*c + 24*a^2*b^2*c - 12*b^4*c + 96*a^4*d*x + 24*a^2*
b^2*d*x - 12*b^4*d*x + 96*a^3*b*Cos[c + d*x] + 12*b^2*(-4*a^2 + b^2)*(c + d
*x)*Cos[2*(c + d*x)] - 8*a*b^3*Cos[3*(c + d*x)] + 192*a^3*b*c*Sin[c + d*x]
- 48*a*b^3*c*Sin[c + d*x] + 192*a^3*b*d*x*Sin[c + d*x] - 48*a*b^3*d*x*Sin[c
 + d*x] + 72*a^2*b^2*Sin[2*(c + d*x)] - 10*b^4*Sin[2*(c + d*x)] + b^4*Sin[4
*(c + d*x)])/(a + b*Sin[c + d*x])^2)/(16*b^5*d)
```

Maple [B] time = 0.139, size = 639, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+6/d/b^4/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)+6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*a+12/d/b^5*arctan(tan(
1/2*d*x+1/2*c))*a^2-3/d/b^3*arctan(tan(1/2*d*x+1/2*c))+5/d*a^2/b^3/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+6/d*a^3/b^
4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+
11/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+
1/2*c)^2-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*
d*x+1/2*c)^2+19/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)
^2*tan(1/2*d*x+1/2*c)-4/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)+6/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2-1/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a-
12/d*a^3/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+9/d*a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b
)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35628, size = 1800, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(4*a^4*b^2 - 5*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 + 8*(a^3*b^3 - a*b
^5)*cos(d*x + c)^3 - 6*(4*a^6 - a^4*b^2 - 4*a^2*b^4 + b^6)*d*x - 3*(4*a^5 +
 a^3*b^2 - 3*a*b^4 - (4*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(4*a^4*b - 3*
a^2*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 6*(4*a^5*b - 3*a^3*b^3 - a*b^5)*cos(d*x + c) - 2*((a^2*b^4 - b^6)*c
os(d*x + c)^3 + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5)*d*x + 3*(6*a^4*b^2 - 7*a^2*
b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^7 - b^9)*d*cos(d*x + c)^2 -
2*(a^3*b^6 - a*b^8)*d*sin(d*x + c) - (a^4*b^5 - b^9)*d), 1/2*(3*(4*a^4*b^2
- 5*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 + 4*(a^3*b^3 - a*b^5)*cos(d*x + c)^3
- 3*(4*a^6 - a^4*b^2 - 4*a^2*b^4 + b^6)*d*x - 3*(4*a^5 + a^3*b^2 - 3*a*b^4
- (4*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(4*a^4*b - 3*a^2*b^3)*sin(d*x +
c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x +
 c))) - 3*(4*a^5*b - 3*a^3*b^3 - a*b^5)*cos(d*x + c) - ((a^2*b^4 - b^6)*cos
(d*x + c)^3 + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5)*d*x + 3*(6*a^4*b^2 - 7*a^2*b^
```

```
4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^7 - b^9)*d*cos(d*x + c)^2 - 2*
(a^3*b^6 - a*b^8)*d*sin(d*x + c) - (a^4*b^5 - b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24212, size = 579, normalized size = 3.35

$$\frac{\frac{3\left(4a^2-b^2\right)(dx+c)}{b^5} - \frac{6\left(4a^3-3ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5} + \frac{2\left(6a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+15a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*a^2 - b^2)*(d*x + c)/b^5 - 6*(4*a^3 - 3*a*b^2)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2
)))/(sqrt(a^2 - b^2)*b^5) + 2*(6*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 12*a^4*tan(
1/2*d*x + 1/2*c)^6 + 15*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 2*b^4*tan(1/2*d*x
+ 1/2*c)^6 + 54*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^4*tan(1/2*d*x + 1/2*c)^
4 + 45*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 4*b^4*tan(1/2*d*x + 1/2*c)^4 + 90*a
^3*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 36*a^4*tan(
1/2*d*x + 1/2*c)^2 + 29*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 2*b^4*tan(1/2*d*x
+ 1/2*c)^2 + 42*a^3*b*tan(1/2*d*x + 1/2*c) - 4*a*b^3*tan(1/2*d*x + 1/2*c) +
 12*a^4 - a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3
+ 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a*b^4))/d
```

3.1138 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=175

$$-\frac{\left(-a^2b^2+2a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d\sqrt{a^2-b^2}}+\frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}-\frac{\tanh^{-1}(\cos(}{a^3d}$$

```
[Out] x/b^3 - ((2*a^4 - a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^3*b^3*Sqrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) - ((a
^2 - b^2)*Cos[c + d*x])/(2*a*b^2*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 + 2*b^
2)*Cos[c + d*x])/(2*a^2*b^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.287894, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2891, 3057, 2660, 618, 204, 3770}

$$-\frac{\left(-a^2b^2+2a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d\sqrt{a^2-b^2}}+\frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}-\frac{\tanh^{-1}(\cos(}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] x/b^3 - ((2*a^4 - a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^3*b^3*Sqrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) - ((a
^2 - b^2)*Cos[c + d*x])/(2*a*b^2*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 + 2*b^
2)*Cos[c + d*x])/(2*a^2*b^2*d*(a + b*Sin[c + d*x]))
```


Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= -\frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}+\frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{\int \frac{\csc(c+dx)\left(-2b^2-ab\sin(c+dx)-2a^2\right.}{a+b\sin(c+dx)}}{2a^2b^2} \\
&= \frac{x}{b^3}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}+\frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}+\frac{\int \csc(c+dx)\,dx}{a^3}+\frac{(-2a}{} \\
&= \frac{x}{b^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}+\frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{(}{} \\
&= \frac{x}{b^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}+\frac{\left(3a^2+2b^2\right)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}+\frac{(}{} \\
&= \frac{x}{b^3}-\frac{\left(2a^4-a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3\sqrt{a^2-b^2}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\left(a^2-b^2\right)c}{2ab^2d(a+b\,s}
\end{aligned}$$

Mathematica [A] time = 1.74973, size = 176, normalized size = 1.01

$$\frac{-\frac{2\left(-a^2b^2+2a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3\sqrt{a^2-b^2}}+\frac{\cos(c+dx)\left(b\left(3a^2+2b^2\right)\sin(c+dx)+2a^3+3ab^2\right)}{a^2b^2(a+b\sin(c+dx))^2}+2\left(\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{c+dx}{b^3}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 - a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^3*b^3*Sqrt[a^2 - b^2]) + 2*((c + d*x)/b^3 - Log[Cos[(c + d*x)/2]]/
a^3 + Log[Sin[(c + d*x)/2]]/a^3) + (Cos[c + d*x]*(2*a^3 + 3*a*b^2 + b*(3*a^
2 + 2*b^2)*Sin[c + d*x]))/(a^2*b^2*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.184, size = 600, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 2/d/b^3*arctan(tan(1/2*d*x+1/2*c))+1/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+4/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+2/d*a/b^2/(tan(1/2*d*x+1/2*c
)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+7/d/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2+6/d/a^3*b^2/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+7/d/b/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+8/d/
a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c
)+2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a+3/d/a/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-2/d*a/b^3/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a/b/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a^3*b/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a^
3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.54033, size = 2211, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^5*b^2 - a^3*b^4)*d*x*cos(d*x + c)^2 - 4*(a^7 - a^3*b^4)*d*x + (2
*a^6 + a^4*b^2 + a^2*b^4 + 2*b^6 - (2*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x +
c)^2 + 2*(2*a^5*b - a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*co
s(d*x + c) + 2*(a^4*b^3 - b^7 - (a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^4
 - a*b^6)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 2*(a^4*b^3 - b^7 - (a
^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^4 - a*b^6)*sin(d*x + c))*log(-1/2*c
os(d*x + c) + 1/2) - 2*(4*(a^6*b - a^4*b^3)*d*x + (3*a^5*b^2 - a^3*b^4 - 2*
a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^5 - a^3*b^7)*d*cos(d*x + c)^2 -
```

```
2*(a^6*b^4 - a^4*b^6)*d*sin(d*x + c) - (a^7*b^3 - a^3*b^7)*d), 1/2*(2*(a^5*
b^2 - a^3*b^4)*d*x*cos(d*x + c)^2 - 2*(a^7 - a^3*b^4)*d*x - (2*a^6 + a^4*b^
2 + a^2*b^4 + 2*b^6 - (2*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(2*a
^5*b - a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
+ c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^6*b + a^4*b^3 - 3*a^2*b^5)
*cos(d*x + c) + (a^4*b^3 - b^7 - (a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^
4 - a*b^6)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^4*b^3 - b^7 - (a^
2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^4 - a*b^6)*sin(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2) - (4*(a^6*b - a^4*b^3)*d*x + (3*a^5*b^2 - a^3*b^4 - 2*a*b
^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^5 - a^3*b^7)*d*cos(d*x + c)^2 - 2*(
a^6*b^4 - a^4*b^6)*d*sin(d*x + c) - (a^7*b^3 - a^3*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32965, size = 371, normalized size = 2.12

$$\frac{\frac{dx+c}{b^3} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{\left(2\,a^4-a^2b^2+2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3b^3} + \frac{a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a^4\tan\left(\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/b^3 + log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (2*a^4 - a^2*b^2 + 2*
b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^3*b^3) + (a^3*b*tan(1/2*d*x +
 1/2*c)^3 + 4*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^4*tan(1/2*d*x + 1/2*c)^2 +
 7*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 6*b^4*tan(1/2*d*x + 1/2*c)^2 + 7*a^3*b*
tan(1/2*d*x + 1/2*c) + 8*a*b^3*tan(1/2*d*x + 1/2*c) + 2*a^4 + 3*a^2*b^2)/((
a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^3*b^2))/d
```

3.1139 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$-\frac{3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}-\frac{\left(a^2+6b^2\right)\cos(c+dx)}{2a^3bd(a+b\sin(c+dx))}+\frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2}+\frac{3b\tanh^{-1}(\cos(c+dx}{a^4d}$$

```
[Out] (-3*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*Sq
rt[a^2 - b^2]*d) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Cos
[c + d*x])/(2*a^2*b*d*(a + b*Sin[c + d*x])^2) - Cot[c + d*x]/(a*d*(a + b*Si
n[c + d*x])^2) - ((a^2 + 6*b^2)*Cos[c + d*x])/(2*a^3*b*d*(a + b*Sin[c + d*x
]))
```

Rubi [A] time = 0.471654, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}-\frac{\left(a^2+6b^2\right)\cos(c+dx)}{2a^3bd(a+b\sin(c+dx))}+\frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2}+\frac{3b\tanh^{-1}(\cos(c+dx}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*Sq
rt[a^2 - b^2]*d) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Cos
[c + d*x])/(2*a^2*b*d*(a + b*Sin[c + d*x])^2) - Cot[c + d*x]/(a*d*(a + b*Si
n[c + d*x])^2) - ((a^2 + 6*b^2)*Cos[c + d*x])/(2*a^3*b*d*(a + b*Sin[c + d*x
]))
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc(c+dx)\left(-6b^2-2ab\sin(c+dx)+\left(a^2\right.\right.}{(a+b\sin(c+dx))^2}}{2a^2b}$$

$$= \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \frac{\left(a^2+6b^2\right)\cos(c+dx)}{2a^3bd(a+b\sin(c+dx))} + \frac{\int}{}$$

$$= \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \frac{\left(a^2+6b^2\right)\cos(c+dx)}{2a^3bd(a+b\sin(c+dx))} - (3$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \frac{(a}{2a^3}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \frac{(a}{2a^3}$$

$$= -\frac{3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}\,d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\left(a^2-3b^2\right)\cos}{2a^2bd(a+b\sin(}$$

Mathematica [A] time = 2.48228, size = 184, normalized size = 1.01

$$\frac{-\frac{6\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{a^2\left(a^2-b^2\right)\cos(c+dx)}{b(a+b\sin(c+dx))^2} - \frac{a\left(a^2+4b^2\right)\cos(c+dx)}{b(a+b\sin(c+dx))} + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) - 6b\log\left(\mathrm{s}\right.}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]] - 6*b*Log[Sin[(c
 + d*x)/2]] + (a^2*(a^2 - b^2)*Cos[c + d*x])/(b*(a + b*Sin[c + d*x])^2) - (
a*(a^2 + 4*b^2)*Cos[c + d*x])/(b*(a + b*Sin[c + d*x])) + a*Tan[(c + d*x)/2]
)/(2*a^4*d)
```

Maple [B] time = 0.197, size = 489, normalized size = 2.7

$$\frac{1}{2\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a + 2\,\tan(1/2\,dx+c/2)\,b + a\right)^{-2} - 6\,\frac{(\mathrm{t}}{da^3\left((\tan(1/2\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)+1/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-5/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-10/d/a^4/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3-1/d/a/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-14/d/a^3/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2-5/
```

```
d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b-3/d/a^2/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/a^4/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2
-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.92254, size = 2341, normalized size = 12.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(a^5 + 5*a^3*b^2 - 6*a*b^4)*cos(d*x + c)^3 - 18*(a^4*b - a^2*b^3)*
cos(d*x + c)*sin(d*x + c) + 3*(2*a^3*b - 4*a*b^3 - 2*(a^3*b - 2*a*b^3)*cos(
d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4 - (a^2*b^2 - 2*b^4)*cos(d*x + c)^2)*sin
(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt
(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a
^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c) + 6*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b^2
- a*b^4)*cos(d*x + c)^2 + (a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)*si
n(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b
^2 - a*b^4)*cos(d*x + c)^2 + (a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)
*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^7*b - a^5*b^3)*d*cos(d*x
 + c)^2 - 2*(a^7*b - a^5*b^3)*d + ((a^6*b^2 - a^4*b^4)*d*cos(d*x + c)^2 - (
a^8 - a^4*b^4)*d)*sin(d*x + c)), -1/2*((a^5 + 5*a^3*b^2 - 6*a*b^4)*cos(d*x
+ c)^3 - 9*(a^4*b - a^2*b^3)*cos(d*x + c)*sin(d*x + c) + 3*(2*a^3*b - 4*a*b
^3 - 2*(a^3*b - 2*a*b^3)*cos(d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4 - (a^2*b^2
 - 2*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
+ c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(a^5 + a^3*b^2 - 2*a*b^4)*cos
(d*x + c) + 3*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 + (
a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x
 + c) + 1/2) - 3*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b^2 - a*b^4)*cos(d*x + c)^2
+ (a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos
(d*x + c) + 1/2))/(2*(a^7*b - a^5*b^3)*d*cos(d*x + c)^2 - 2*(a^7*b - a^5*b^
3)*d + ((a^6*b^2 - a^4*b^4)*d*cos(d*x + c)^2 - (a^8 - a^4*b^4)*d)*sin(d*x +
 c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37205, size = 369, normalized size = 2.03

$$-\frac{\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{6\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(a^2-2\,b^2\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 + 6
*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^4) - (6*b*tan(1/2*d
*x + 1/2*c) - a)/(a^4*tan(1/2*d*x + 1/2*c)) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3
 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 10*b^3
*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 14*a*b^2*tan(1/2*d*x +
 1/2*c) - 5*a^2*b)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) +
a)^2*a^4))/d
```

3.1140 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=218

$$\frac{3b\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}-\frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd}+\frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d}+\frac{\left(a^2-2b^2\right)\cot(c}{2a^2bd(a+b\sin(c}$$

```
[Out] (3*b*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5
*Sqrt[a^2 - b^2]*d) + (3*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^5*d) - (
(a^2 - 12*b^2)*Cot[c + d*x])/(2*a^4*b*d) + ((a^2 - 2*b^2)*Cot[c + d*x])/(2*
a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b
*Sin[c + d*x])^2) - (3*b*Cot[c + d*x])/(a^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.771971, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{3b\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}-\frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd}+\frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d}+\frac{\left(a^2-2b^2\right)\cot(c}{2a^2bd(a+b\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5
*Sqrt[a^2 - b^2]*d) + (3*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^5*d) - (
(a^2 - 12*b^2)*Cot[c + d*x])/(2*a^4*b*d) + ((a^2 - 2*b^2)*Cot[c + d*x])/(2*
a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b
*Sin[c + d*x])^2) - (3*b*Cot[c + d*x])/(a^3*d*(a + b*Sin[c + d*x]))
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-6b^2\right)-2ab\sin(c+dx)+\ldots}{(a+b\sin(c+dx))^2}}{4a^2b}$$

$$= \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} - \frac{3b\cot(c+dx)}{a^3d(a+b\sin(c+dx))} + \int \frac{\csc^2(c\ldots}{\ldots}$$

$$= -\frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd} + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} - \frac{3\ldots}{a^3d(\ldots}$$

$$= -\frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd} + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} - \frac{3\ldots}{a^3d(\ldots}$$

$$= \frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} - \frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd} + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^{\ldots}}$$

$$= \frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} - \frac{\left(a^2-12b^2\right)\cot(c+dx)}{2a^4bd} + \frac{\left(a^2-2b^2\right)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^{\ldots}}$$

$$= \frac{3b\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} + \frac{3\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} - \frac{\left(a^2-12b^{\ldots}}{2\ldots}$$

Mathematica [A] time = 6.18532, size = 319, normalized size = 1.46

$$-\frac{3\left(a^2-4b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{3\left(a^2-4b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{6b^2\cos(c+dx)-a^2\cos(c+dx)}{2a^4d(a+b\sin(c+dx))} + \frac{b^2\cos(c+\ldots}{2a^3d(a\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(3*a^2 - 4*b^2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + (3*b*Cot[(c + d*x
)/2])/(2*a^4*d) - Csc[(c + d*x)/2]^2/(8*a^3*d) + (3*(a^2 - 4*b^2)*Log[Cos[(
c + d*x)/2]])/(2*a^5*d) - (3*(a^2 - 4*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^5*d)
 + Sec[(c + d*x)/2]^2/(8*a^3*d) + (-(a^2*Cos[c + d*x]) + b^2*Cos[c + d*x])/
(2*a^3*d*(a + b*Sin[c + d*x])^2) + (-(a^2*Cos[c + d*x]) + 6*b^2*Cos[c + d*x
])/(2*a^4*d*(a + b*Sin[c + d*x])) - (3*b*Tan[(c + d*x)/2])/(2*a^4*d)
```

Maple [B] time = 0.213, size = 642, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^4*tan(1/2*d*x+1/2*c)*b-3/d/a^2*b/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+8/d/a
^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3
```

```
*b^3-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+
1/2*c)^2+3/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*ta
n(1/2*d*x+1/2*c)^2+14/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)^2*b^4-5/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+20/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^3-2/d/a/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2+7/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2*b^2+9/d/a^3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))-12/d/a^5*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/8/d/a^3/tan(1/2*d*x+1/2*c)^2-3/2/d/a^
3*ln(tan(1/2*d*x+1/2*c))+6/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^2+3/2/d*b/a^4/tan
(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.7479, size = 3416, normalized size = 15.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*(a^6 - 10*a^4*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 + 3*(3*a^4*b - a^2*b^
3 - 4*b^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b
^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) - 6*(a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c) + 3*(a^6 - 4*a^4*b^2
- a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3
*a^4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*x + c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b
^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) - 3*(a^6 - 4*a^4*b^2 - a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2
*b^4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3*a^4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*
x + c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*c
os(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*((a^5*b - 13*
a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 + 3*(a^5*b + 3*a^3*b^3 - 4*a*b^5)*cos(d*
x + c))*sin(d*x + c))/((a^7*b^2 - a^5*b^4)*d*cos(d*x + c)^4 - (a^9 + a^7*b^
2 - 2*a^5*b^4)*d*cos(d*x + c)^2 + (a^9 - a^5*b^4)*d - 2*((a^8*b - a^6*b^3)*
d*cos(d*x + c)^2 - (a^8*b - a^6*b^3)*d)*sin(d*x + c)), 1/4*(4*(a^6 - 10*a^4
*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 - 6*(3*a^4*b - a^2*b^3 - 4*b^5 + (3*a^2*b^
3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos(d*x + c)^2 +
2*(3*a^3*b^2 - 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c
```

```
))) - 6*(a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c) + 3*(a^6 - 4*a^4*b^2 - a
^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3*a^
4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*x + c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5
- (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) - 3*(a^6 - 4*a^4*b^2 - a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^
4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3*a^4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*x +
 c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(
d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*((a^5*b - 13*a^3
*b^3 + 12*a*b^5)*cos(d*x + c)^3 + 3*(a^5*b + 3*a^3*b^3 - 4*a*b^5)*cos(d*x +
 c))*sin(d*x + c))/((a^7*b^2 - a^5*b^4)*d*cos(d*x + c)^4 - (a^9 + a^7*b^2 -
 2*a^5*b^4)*d*cos(d*x + c)^2 + (a^9 - a^5*b^4)*d - 2*((a^8*b - a^6*b^3)*d*c
os(d*x + c)^2 - (a^8*b - a^6*b^3)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42662, size = 533, normalized size = 2.44

$$-\frac{\frac{12\left(a^2-4b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5}-\frac{24\left(3a^2b-4b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^5}-\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}-\frac{6a^4\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(12*(a^2 - 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 24*(3*a^2*b - 4
*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/
2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) - (a^3*tan(1/2*d*x + 1/2*
c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6 - (6*a^4*tan(1/2*d*x + 1/2*c)^6 -
 24*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 32*a
*b^3*tan(1/2*d*x + 1/2*c)^5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 + 48*a^2*b^2*tan
(1/2*d*x + 1/2*c)^4 + 16*b^4*tan(1/2*d*x + 1/2*c)^4 + 4*a^3*b*tan(1/2*d*x +
 1/2*c)^3 + 112*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*a^4*tan(1/2*d*x + 1/2*c)^
2 + 76*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^3*b*tan(1/2*d*x + 1/2*c) - a^4)
/((a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x +
1/2*c))^2*a^5))/d
```

3.1141 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=289

$$\frac{\left(-19a^2b^2+2a^4+20b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}+\frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d}-\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d}-$$

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*
a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(6*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.09865, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{\left(-19a^2b^2+2a^4+20b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}+\frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d}-\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*
a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(6*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
```

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^3(c+dx)\left(2\left(3a^2-10b^2\right)-2\right.}{(a+}}{}$$

$$= \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\left(3a^2-20b^2\right)\cot(c+d}{6a^3bd(a+b\sin(c}$$

$$= -\frac{\left(a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{3ad(a+b\,\mathrm{s}}$$

$$= \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{\left(3a^2-5b^2\right)\cot(c+dx}{6a^2bd(a+b\sin(c}$$

$$= \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{\left(3a^2-5b^2\right)\cot(c+dx}{6a^2bd(a+b\sin(c}$$

$$= -\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+d}{a^4bd}$$

$$= -\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+d}{a^4bd}$$

$$= \frac{\left(2a^4-19a^2b^2+20b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(1}{}$$

Mathematica [A] time = 6.20106, size = 459, normalized size = 1.59

$$\frac{\left(9a^2b-20b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} + \frac{\left(20b^3-9a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} + \frac{3a^2b\cos(c+dx)-8b^3\cos(c+dx)}{2a^5d(a+b\sin(c+dx))} + \frac{a^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2]
 + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((2*a^2
*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (
3*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(
24*a^3*d) + ((-9*a^2*b + 20*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + ((9*a^2
*b - 20*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8
*a^4*d) + (Sec[(c + d*x)/2]*(-2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/
2]))/(3*a^5*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(2*a^4*d*(a + b*Si
n[c + d*x])^2) + (3*a^2*b*Cos[c + d*x] - 8*b^3*Cos[c + d*x])/(2*a^5*d*(a +
b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)
```

Maple [B] time = 0.221, size = 780, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b-5/8/d/a^3*
tan(1/2*d*x+1/2*c)+3/d/a^5*b^2*tan(1/2*d*x+1/2*c)+5/d/a^3/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-10/d/a^5/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4+4/d/
a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^
2*b-1/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x
+1/2*c)^2*b^3-18/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*
tan(1/2*d*x+1/2*c)^2*b^5+11/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2-26/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4+4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*b-9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2*b^3+2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))-19/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+20/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-1/24/d/a^3/tan(1/2*d*x+1/2*c
)^3+5/8/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^5/tan(1/2*d*x+1/2*c)*b^2+3/8/d/a^4*b
/tan(1/2*d*x+1/2*c)^2+9/2/d/a^4*b*ln(tan(1/2*d*x+1/2*c))-10/d/a^6*b^3*ln(ta
n(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.47259, size = 4578, normalized size = 15.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3
*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 38*a^3*b^3
+ 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*
b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 +
 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*cos(d*x + c)^4 - (2*a^6 - 15*a^
4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b
^4 + 20*a*b^6)*cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*
a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4
 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 +
(9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 -
```

```
38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 2
0*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c
)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5
+ 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b -
59*a^4*b^3 + 45*a^2*b^5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4
*(a^9*b - a^7*b^3)*d*cos(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a
^6*b^4)*d*cos(d*x + c)^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 +
(a^10 - a^6*b^4)*d)*sin(d*x + c)), 1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*
b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x
 + c)^3 - 6*(4*a^5*b - 38*a^3*b^3 + 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20
*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2
 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 + 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^
6)*cos(d*x + c)^4 - (2*a^6 - 15*a^4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)
^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 -
b^2)*cos(d*x + c))) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b^4 + 20*a*b^6)*cos(d*x
 + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 +
 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x +
 c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^
5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*c
os(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(18*a^5*b^2 -
58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^
4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^
4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)
^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b - 59*a^4*b^3 + 45*a^2*b^
5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c))*si
n(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4*(a^9*b - a^7*b^3)*d*c
os(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a^6*b^4)*d*cos(d*x + c)
^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 + (a^10 - a^6*b^4)*d)*si
n(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+b*sin(d*x+c))**3,x)

[Out] Timed out
```

Giac [A] time = 1.29902, size = 609, normalized size = 2.11

$$\frac{12\left(9a^2b-20b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^6}+\frac{24\left(2a^4-19a^2b^2+20b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^6}+\frac{24\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-10ab^4\,\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(12*(9*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 24*(2*a^4
- 19*a^2*b^2 + 20*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) + 24*(5*
a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*
tan(1/2*d*x + 1/2*c)^2 - a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*b^5*tan(1/2*d*
x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 26*a*b^4*tan(1/2*d*x + 1/2
*c) + 4*a^4*b - 9*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1
/2*c) + a)^2*a^6) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2
*c)^2 - 15*a^6*tan(1/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9
- (198*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a
^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1
/2*d*x + 1/2*c) + a^3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

3.1142 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=340

$$-\frac{3b\left(-11a^2b^2+2a^4+10b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d\sqrt{a^2-b^2}}-\frac{b\left(13a^2-30b^2\right)\cot(c+dx)}{2a^6d}-\frac{3\left(-24a^2b^2+a^4+40b^4\right)\tanh^{-1}(}{8a^7d}$$

```
[Out] (-3*b*(2*a^4 - 11*a^2*b^2 + 10*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*Sqrt[a^2 - b^2]*d) - (3*(a^4 - 24*a^2*b^2 + 40*b^4)*ArcTanh
[Cos[c + d*x]])/(8*a^7*d) - (b*(13*a^2 - 30*b^2)*Cot[c + d*x])/(2*a^6*d) +
(3*(7*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^5*d) - ((3*a^2 - 10*b^2
)*Cot[c + d*x]*Csc[c + d*x]^2)/(2*a^4*b*d) + ((2*a^2 - 3*b^2)*Cot[c + d*x]*
Csc[c + d*x]^2)/(4*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c +
d*x]^3)/(4*a*d*(a + b*Sin[c + d*x])^2) + ((4*a^2 - 15*b^2)*Cot[c + d*x]*Csc
[c + d*x]^2)/(4*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.45867, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2890, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{3b\left(-11a^2b^2+2a^4+10b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d\sqrt{a^2-b^2}}-\frac{b\left(13a^2-30b^2\right)\cot(c+dx)}{2a^6d}-\frac{3\left(-24a^2b^2+a^4+40b^4\right)\tanh^{-1}(}{8a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(2*a^4 - 11*a^2*b^2 + 10*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*Sqrt[a^2 - b^2]*d) - (3*(a^4 - 24*a^2*b^2 + 40*b^4)*ArcTanh
[Cos[c + d*x]])/(8*a^7*d) - (b*(13*a^2 - 30*b^2)*Cot[c + d*x])/(2*a^6*d) +
(3*(7*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^5*d) - ((3*a^2 - 10*b^2
)*Cot[c + d*x]*Csc[c + d*x]^2)/(2*a^4*b*d) + ((2*a^2 - 3*b^2)*Cot[c + d*x]*
Csc[c + d*x]^2)/(4*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c +
d*x]^3)/(4*a*d*(a + b*Sin[c + d*x])^2) + ((4*a^2 - 15*b^2)*Cot[c + d*x]*Csc
[c + d*x]^2)/(4*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{(2a^2-3b^2)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^4(c+dx)(6(2a^2-\ldots}{\ldots}}{\ldots}$$

$$= \frac{(2a^2-3b^2)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))^2} + \frac{(4a^2-15b^2)\cot(\ldots}{4a^3bd(a+\ldots}$$

$$= -\frac{(3a^2-10b^2)\cot(c+dx)\csc^2(c+dx)}{2a^4bd} + \frac{(2a^2-3b^2)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2} - \frac{\cot\ldots}{4a\ldots}$$

$$= \frac{3(7a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{(3a^2-10b^2)\cot(c+dx)\csc^2(c+dx)}{2a^4bd} + \frac{(2\ldots}{\ldots}$$

$$= -\frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d} + \frac{3(7a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{(3a^2-10b\ldots}{\ldots}$$

$$= -\frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d} + \frac{3(7a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{(3a^2-10b\ldots}{\ldots}$$

$$= -\frac{3(a^4-24a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d} - \frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d} + \frac{3(7a\ldots}{\ldots}$$

$$= -\frac{3(a^4-24a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d} - \frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d} + \frac{3(7a\ldots}{\ldots}$$

$$= -\frac{3b(2a^4-11a^2b^2+10b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7\sqrt{a^2-b^2}\,d} - \frac{3(a^4-24a^2b^2+40b^4)\tanh^{-1}(\ldots}{8a^7d}$$

Mathematica [A] time = 4.50849, size = 347, normalized size = 1.02

$$-\frac{\frac{384b(-11a^2b^2+2a^4+10b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 48(-24a^2b^2+a^4+40b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 48(-24a^2b^2+a^4+40b\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((384*b*(2*a^4 - 11*a^2*b^2 + 10*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]])/Sqrt[a^2 - b^2] + 48*(a^4 - 24*a^2*b^2 + 40*b^4)*Log[Cos[(c +
 d*x)/2]] - 48*(a^4 - 24*a^2*b^2 + 40*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Cot
[c + d*x]*Csc[c + d*x]^5*(-4*a^5 + 289*a^3*b^2 - 540*a*b^4 + 4*(5*a^5 - 93*
a^3*b^2 + 180*a*b^4)*Cos[2*(c + d*x)] + (83*a^3*b^2 - 180*a*b^4)*Cos[4*(c +
 d*x)] + 100*a^4*b*Sin[c + d*x] + 20*a^2*b^3*Sin[c + d*x] - 600*b^5*Sin[c +
 d*x] - 44*a^4*b*Sin[3*(c + d*x)] - 50*a^2*b^3*Sin[3*(c + d*x)] + 300*b^5*S
in[3*(c + d*x)] + 26*a^2*b^3*Sin[5*(c + d*x)] - 60*b^5*Sin[5*(c + d*x)]))/(
b + a*Csc[c + d*x])^2)/(128*a^7*d)
```

Maple [B] time = 0.223, size = 889, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)^2*b^4-17/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)*b^3+33/d/a^5*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-5/d/a^6*b^3*tan(1/2*d*x+1/2*c)+15/d/a^7*ln(t
an(1/2*d*x+1/2*c))*b^4+1/8/d/a^4*b/tan(1/2*d*x+1/2*c)^3+11/d*b^4/a^5/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-3/4/d/a^5/tan(1/2*d*x+1/2*c)^
2*b^2+3/8/d/a^3*ln(tan(1/2*d*x+1/2*c))+5/d*b^3/a^6/tan(1/2*d*x+1/2*c)-1/8/d
/a^4*tan(1/2*d*x+1/2*c)^3*b+3/4/d/a^5*b^2*tan(1/2*d*x+1/2*c)^2-6/d/a^3*b^2/
(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-6/
d/a^3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))-1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-7/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^3-9/d/a^5*ln(tan(1/2*d*x+1/2*c
))*b^2-15/8/d*b/a^4/tan(1/2*d*x+1/2*c)+32/d*b^5/a^6/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-30/d*b^5/a^7/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+12/d*b^5/a^6/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+22/d*
b^6/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2
*c)^2-1/64/d/a^3/tan(1/2*d*x+1/2*c)^4+1/64/d/a^3*tan(1/2*d*x+1/2*c)^4+15/8/
d/a^4*tan(1/2*d*x+1/2*c)*b-6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*b^2+1/8/d/a^3/tan(1/2*d*x+1/2*c)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.43483, size = 5862, normalized size = 17.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/16*(2*(83*a^6*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c)^5 + 2*(5*a^8
 - 181*a^6*b^2 + 536*a^4*b^4 - 360*a^2*b^6)*cos(d*x + c)^3 + 12*(2*a^6*b -
9*a^4*b^3 - a^2*b^5 + 10*b^7 - (2*a^4*b^3 - 11*a^2*b^5 + 10*b^7)*cos(d*x +
c)^6 + (2*a^6*b - 5*a^4*b^3 - 23*a^2*b^5 + 30*b^7)*cos(d*x + c)^4 - (4*a^6*
b - 16*a^4*b^3 - 13*a^2*b^5 + 30*b^7)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11*a^
3*b^4 + 10*a*b^6 + (2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^4 - 2*(
2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
- 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*c
```

```
os(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a^8 - 32*a^6*b^2 + 91
*a^4*b^4 - 60*a^2*b^6)*cos(d*x + c) + 3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24
*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x +
c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4 + 152*a^2*b^6 - 120*b^8)*cos(d*x + c)
^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4 + 112*a^2*b^6 - 120*b^8)*cos(d*x + c)
^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3 +
64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4 - 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5
- 40*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(
a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*b^4
 + 64*a^2*b^6 - 40*b^8)*cos(d*x + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4 + 1
52*a^2*b^6 - 120*b^8)*cos(d*x + c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4 + 1
12*a^2*b^6 - 120*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 -
 40*a*b^7 + (a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4 - 2
*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))
*log(-1/2*cos(d*x + c) + 1/2) + 4*(2*(13*a^5*b^3 - 43*a^3*b^5 + 30*a*b^7)*c
os(d*x + c)^5 - (11*a^7*b + 21*a^5*b^3 - 152*a^3*b^5 + 120*a*b^7)*cos(d*x +
 c)^3 + 3*(3*a^7*b - a^5*b^3 - 22*a^3*b^5 + 20*a*b^7)*cos(d*x + c))*sin(d*x
 + c))/((a^9*b^2 - a^7*b^4)*d*cos(d*x + c)^6 - (a^11 + 2*a^9*b^2 - 3*a^7*b^
4)*d*cos(d*x + c)^4 + (2*a^11 + a^9*b^2 - 3*a^7*b^4)*d*cos(d*x + c)^2 - (a^
11 - a^7*b^4)*d - 2*((a^10*b - a^8*b^3)*d*cos(d*x + c)^4 - 2*(a^10*b - a^8*
b^3)*d*cos(d*x + c)^2 + (a^10*b - a^8*b^3)*d)*sin(d*x + c)), 1/16*(2*(83*a^
6*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c)^5 + 2*(5*a^8 - 181*a^6*b^2
+ 536*a^4*b^4 - 360*a^2*b^6)*cos(d*x + c)^3 - 24*(2*a^6*b - 9*a^4*b^3 - a^2
*b^5 + 10*b^7 - (2*a^4*b^3 - 11*a^2*b^5 + 10*b^7)*cos(d*x + c)^6 + (2*a^6*b
 - 5*a^4*b^3 - 23*a^2*b^5 + 30*b^7)*cos(d*x + c)^4 - (4*a^6*b - 16*a^4*b^3
- 13*a^2*b^5 + 30*b^7)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^
6 + (2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^4 - 2*(2*a^5*b^2 - 11*
a^3*b^4 + 10*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(
a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^8 - 32*a^6*b^2 +
 91*a^4*b^4 - 60*a^2*b^6)*cos(d*x + c) + 3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 +
 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x
 + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4 + 152*a^2*b^6 - 120*b^8)*cos(d*x +
 c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4 + 112*a^2*b^6 - 120*b^8)*cos(d*x +
 c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3
 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4 - 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b
^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) -
3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*
b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4
+ 152*a^2*b^6 - 120*b^8)*cos(d*x + c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4
+ 112*a^2*b^6 - 120*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^
5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4
- 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x +
c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(2*(13*a^5*b^3 - 43*a^3*b^5 + 30*a*b^7
)*cos(d*x + c)^5 - (11*a^7*b + 21*a^5*b^3 - 152*a^3*b^5 + 120*a*b^7)*cos(d*
x + c)^3 + 3*(3*a^7*b - a^5*b^3 - 22*a^3*b^5 + 20*a*b^7)*cos(d*x + c))*sin(
d*x + c))/((a^9*b^2 - a^7*b^4)*d*cos(d*x + c)^6 - (a^11 + 2*a^9*b^2 - 3*a^7
*b^4)*d*cos(d*x + c)^4 + (2*a^11 + a^9*b^2 - 3*a^7*b^4)*d*cos(d*x + c)^2 -
(a^11 - a^7*b^4)*d - 2*((a^10*b - a^8*b^3)*d*cos(d*x + c)^4 - 2*(a^10*b - a
^8*b^3)*d*cos(d*x + c)^2 + (a^10*b - a^8*b^3)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38712, size = 743, normalized size = 2.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^4 - 24*a^2*b^2 + 40*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 - 1
92*(2*a^4*b - 11*a^2*b^3 + 10*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a
^7) - 64*(7*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^5*tan(1/2*d*x + 1/2*c)^
3 + 6*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 22*
b^6*tan(1/2*d*x + 1/2*c)^2 + 17*a^3*b^3*tan(1/2*d*x + 1/2*c) - 32*a*b^5*tan
(1/2*d*x + 1/2*c) + 6*a^4*b^2 - 11*a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*
b*tan(1/2*d*x + 1/2*c) + a)^2*a^7) - (50*a^4*tan(1/2*d*x + 1/2*c)^4 - 1200*
a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 2000*b^4*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*a^4*tan(1/2
*d*x + 1/2*c)^2 + 48*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*b*tan(1/2*d*x +
 1/2*c) + a^4)/(a^7*tan(1/2*d*x + 1/2*c)^4) + (a^9*tan(1/2*d*x + 1/2*c)^4 -
 8*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 8*a^9*tan(1/2*d*x + 1/2*c)^2 + 48*a^7*b^2
*tan(1/2*d*x + 1/2*c)^2 + 120*a^8*b*tan(1/2*d*x + 1/2*c) - 320*a^6*b^3*tan(
1/2*d*x + 1/2*c))/a^12)/d
```

3.1143 $\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=463

$$-\frac{10\left(16a^2-33b^2\right)\sin^2(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{1287b^3d}+\frac{8a\left(40a^2-81b^2\right)\sin(c+dx)\cos(c+dx)(a+b\sin(}{3003b^4d}$$

```
[Out] (16*a*(160*a^4 - 279*a^2*b^2 + 27*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]
])/(45045*b^5*d) - (8*(480*a^4 - 937*a^2*b^2 + 231*b^4)*Cos[c + d*x]*(a + b
*Sin[c + d*x])^(3/2))/(45045*b^5*d) + (8*a*(40*a^2 - 81*b^2)*Cos[c + d*x]*S
in[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(3003*b^4*d) - (10*(16*a^2 - 33*b^2
)*Cos[c + d*x]*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(1287*b^3*d) + (2
0*a*Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(143*b^2*d) - (
2*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2))/(13*b*d) - (8*(32
0*a^6 - 798*a^4*b^2 + 435*a^2*b^4 - 693*b^6)*EllipticE[(c - Pi/2 + d*x)/2,
(2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(45045*b^6*d*Sqrt[(a + b*Sin[c + d
*x])/(a + b)]) + (16*a*(160*a^6 - 439*a^4*b^2 + 306*a^2*b^4 - 27*b^6)*Ellip
ticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
/(45045*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.04769, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2895, 3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{10\left(16a^2-33b^2\right)\sin^2(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{1287b^3d}+\frac{8a\left(40a^2-81b^2\right)\sin(c+dx)\cos(c+dx)(a+b\sin(}{3003b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (16*a*(160*a^4 - 279*a^2*b^2 + 27*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]
])/(45045*b^5*d) - (8*(480*a^4 - 937*a^2*b^2 + 231*b^4)*Cos[c + d*x]*(a + b
*Sin[c + d*x])^(3/2))/(45045*b^5*d) + (8*a*(40*a^2 - 81*b^2)*Cos[c + d*x]*S
in[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(3003*b^4*d) - (10*(16*a^2 - 33*b^2
)*Cos[c + d*x]*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(1287*b^3*d) + (2
0*a*Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(143*b^2*d) - (
2*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2))/(13*b*d) - (8*(32
0*a^6 - 798*a^4*b^2 + 435*a^2*b^4 - 693*b^6)*EllipticE[(c - Pi/2 + d*x)/2,
(2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(45045*b^6*d*Sqrt[(a + b*Sin[c + d
*x])/(a + b)]) + (16*a*(160*a^6 - 439*a^4*b^2 + 306*a^2*b^4 - 27*b^6)*Ellip
ticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
/(45045*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
```

```
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{20a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^{3/2}}{143b^2d} - \frac{2\cos(c+dx)}{}$$

$$= -\frac{10\left(16a^2-33b^2\right)\cos(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^{3/2}}{1287b^3d}$$

$$= \frac{8a\left(40a^2-81b^2\right)\cos(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{3/2}}{3003b^4d} - $$

$$= -\frac{8\left(480a^4-937a^2b^2+231b^4\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{45045b^5d} + $$

$$= \frac{16a\left(160a^4-279a^2b^2+27b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{45045b^5d} - $$

$$= \frac{16a\left(160a^4-279a^2b^2+27b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{45045b^5d} - $$

$$= \frac{16a\left(160a^4-279a^2b^2+27b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{45045b^5d} - $$

$$= \frac{16a\left(160a^4-279a^2b^2+27b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{45045b^5d} - $$

Mathematica [A] time = 4.84953, size = 327, normalized size = 0.71

$$\sqrt{a+b\sin(c+dx)}\left(-256a\left(-279a^3b^2+279a^2b^3-160a^4b+160a^5+27ab^4-27b^5\right)F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+128\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(128*(320*a^6 - 798*a^4*b^2 + 435*a^2*b^4 - 693*b
^6)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - 256*a*(160*a^5 - 160*
a^4*b - 279*a^3*b^2 + 279*a^2*b^3 + 27*a*b^4 - 27*b^5)*EllipticF[(-2*c + Pi
 - 2*d*x)/4, (2*b)/(a + b)] - 2*b*Cos[c + d*x]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)]*(10240*a^5 - 21056*a^3*b^2 + 5898*a*b^4 - 1600*(2*a^3*b^2 - 3*a*b^4)
*Cos[2*(c + d*x)] + 630*a*b^4*Cos[4*(c + d*x)] - 7680*a^4*b*Sin[c + d*x] +
13592*a^2*b^3*Sin[c + d*x] - 19866*b^5*Sin[c + d*x] + 1400*a^2*b^3*Sin[3*(c
 + d*x)] + 5775*b^5*Sin[3*(c + d*x)] + 3465*b^5*Sin[5*(c + d*x)])))/(720720
*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
```

Maple [B] time = 1.677, size = 1619, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/45045*(-50*a^3*b^5*sin(d*x+c)^5+10470*a*b^7*sin(d*x+c)^5+80*a^4*b^4*sin(
d*x+c)^4-232*a^2*b^6*sin(d*x+c)^4-160*a^5*b^3*sin(d*x+c)^3+454*a^3*b^5*sin(
d*x+c)^3-8322*a*b^7*sin(d*x+c)^3-640*a^6*b^2*sin(d*x+c)^2+1436*a^4*b^4*sin(
d*x+c)^2-511*a^2*b^6*sin(d*x+c)^2+160*a^5*b^3*sin(d*x+c)-404*a^3*b^5*sin(d*
x+c)+1632*a*b^7*sin(d*x+c)-3780*a*b^7*sin(d*x+c)^7-1516*a^4*b^4+708*a^2*b^6
+640*a^6*b^2-4296*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^2*b^6+35*a^2*b^6*sin(d*x+c)^6-3465*b^8*sin(d*x+c
)^8+9240*b^8*sin(d*x+c)^6-6699*b^8*sin(d*x+c)^4+924*b^8*sin(d*x+c)^2-216*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a*b^7-4932*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^4*b^4-960*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-3512*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3+4472*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a^6*b^2+1280*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b)
)^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^
(1/2),((a-b)/(a+b))^(1/2))*a^7*b+4512*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6+2484*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+24
48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^3*b^5-1280*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^8-2772*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*
sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8+2772*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8)/b^7/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\cos(dx+c)^4\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^{6}-\cos\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

3.1144 $\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=332

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2-7ab\sin(c+dx)-9b^2\right)}{693b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(-24ab\left(a^2-2b^2\right)\sin\right.}{3465b^4d}$$

```
[Out] (-2*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(11*d) + (8*a*(32*a^4 - 93*a^2
*b^2 + 93*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(3465*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*(32*a^6 - 1
01*a^4*b^2 + 114*a^2*b^4 - 45*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
 b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3465*b^5*d*Sqrt[a + b*Sin[c + d*x
]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a^2 - 9*b^2 - 7*a*b*Sin
[c + d*x]))/(693*b^2*d) + (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^4
- 69*a^2*b^2 + 45*b^4 - 24*a*b*(a^2 - 2*b^2)*Sin[c + d*x]))/(3465*b^4*d)
```

Rubi [A] time = 0.633782, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2-7ab\sin(c+dx)-9b^2\right)}{693b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(-24ab\left(a^2-2b^2\right)\sin\right.}{3465b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(11*d) + (8*a*(32*a^4 - 93*a^2
*b^2 + 93*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(3465*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*(32*a^6 - 1
01*a^4*b^2 + 114*a^2*b^4 - 45*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
 b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3465*b^5*d*Sqrt[a + b*Sin[c + d*x
]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a^2 - 9*b^2 - 7*a*b*Sin
[c + d*x]))/(693*b^2*d) + (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^4
- 69*a^2*b^2 + 45*b^4 - 24*a*b*(a^2 - 2*b^2)*Sin[c + d*x]))/(3465*b^4*d)
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
```

```
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2}{11}\int \frac{\cos^4(c+dx)\left(\frac{b}{2}+\frac{1}{2}a\sin(\right.}{\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx}}{693}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx}}{693}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx}}{693}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx}}{693}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{8a\left(32a^4-93a^2b^2+93b^4\right)E\left(\frac{1}{2}\right.}{3465b^5}$$

Mathematica [A] time = 3.90255, size = 326, normalized size = 0.98

$$\frac{b\cos(c+dx)\left(-692a^2b^3\sin(c+dx)-20a^2b^3\sin(3(c+dx))+16\left(4a^3b^2-183ab^4\right)\cos(2(c+dx))-2912a^3b^2+256a^4b\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-64*a*(32*a^5 + 32*a^4*b - 93*a^3*b^2 - 93*a^2*b^3 + 93*a*b^4 + 93*b^5)*El
lipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)] + 64*(32*a^6 - 101*a^4*b^2 + 114*a^2*b^4 - 45*b^6)*EllipticF[(-2*c +
Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*Cos[c
+ d*x]*(1024*a^5 - 2912*a^3*b^2 + 748*a*b^4 + 16*(4*a^3*b^2 - 183*a*b^4)*Co
s[2*(c + d*x)] - 700*a*b^4*Cos[4*(c + d*x)] + 256*a^4*b*Sin[c + d*x] - 692*
a^2*b^3*Sin[c + d*x] + 990*b^5*Sin[c + d*x] - 20*a^2*b^3*Sin[3*(c + d*x)] -
 765*b^5*Sin[3*(c + d*x)] - 315*b^5*Sin[5*(c + d*x)]))/(27720*b^5*d*Sqrt[a
+ b*Sin[c + d*x]])
```

Maple [B] time = 1.645, size = 1356, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3465*(372*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a*b^6-180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*
```

```
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7+350*a*b^6*sin(d*x+c)
^6-5*a^2*b^5*sin(d*x+c)^5+8*a^3*b^4*sin(d*x+c)^4-1066*a*b^6*sin(d*x+c)^4-16
*a^4*b^3*sin(d*x+c)^3+52*a^2*b^5*sin(d*x+c)^3-64*a^5*b^2*sin(d*x+c)^2+170*a
^3*b^4*sin(d*x+c)^2+896*a*b^6*sin(d*x+c)^2+16*a^4*b^3*sin(d*x+c)-47*a^2*b^5
*sin(d*x+c)-178*a^3*b^4-180*a*b^6+64*a^5*b^2-744*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+128*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^6*b-96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a^5*b^2-404*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+288*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+456*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
2*b^5-192*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a*b^6+500*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2+315*b^7*sin(d*x+c)^7-900*b^7*sin(
d*x+c)^5+765*b^7*sin(d*x+c)^3-180*b^7*sin(d*x+c))/b^6/cos(d*x+c)/(a+b*sin(d
*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\cos(dx+c)^4\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sin(dx+c)+a}\cos(dx+c)^4\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\cos(dx+c)^4\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

3.1145 $\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=338

$$-\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d}-\frac{2\left(-53a^2b^2+8a^4-60b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\sin(c+dx)}}+\frac{2a}{}$$

```
[Out] (-2*(8*a^2 - 45*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(105*b^2*d) + (
8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(35*b^2*d) - (2*Cos[c + d*x]*S
in[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(7*b*d) + (2*a*(8*a^2 - 51*b^2)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(105*b^
3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*(8*a^4 - 53*a^2*b^2 - 60*b^4)*
EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)])/(105*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*a*EllipticPi[2, (c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*S
in[c + d*x]])
```

Rubi [A] time = 0.892257, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2895, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d}-\frac{2\left(-53a^2b^2+8a^4-60b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\sin(c+dx)}}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*(8*a^2 - 45*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(105*b^2*d) + (
8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(35*b^2*d) - (2*Cos[c + d*x]*S
in[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(7*b*d) + (2*a*(8*a^2 - 51*b^2)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(105*b^
3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*(8*a^4 - 53*a^2*b^2 - 60*b^4)*
EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)])/(105*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*a*EllipticPi[2, (c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*S
in[c + d*x]])
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{8a\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35b^2d} - \frac{2\cos(c+dx)\sin(c+dx)(a}{7bd}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c+dx)(a}{3}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c+dx)(a}{3}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c+dx)(a}{3}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c+dx)(a}{3}$$

$$= -\frac{2\left(8a^2-45b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\cos(c+dx)(a}{3}$$

Mathematica [C] time = 3.31863, size = 435, normalized size = 1.29

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2-6ab\sin(c+dx)+15b^2\cos(2(c+dx))+75b^2\right) - \frac{8b\left(a^2+30b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2d\right.}{\sqrt{a+b\sin(c+dx)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(-8*a^2 + 51*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^(-1)]) - (8*b*(a^2
 + 30*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*a*(8*a^2 + 159*b^2)*Ellip
ticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/Sqrt[a + b*Sin[c + d*x]] + 2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*
```

```
(8*a^2 + 75*b^2 + 15*b^2*Cos[2*(c + d*x)] - 6*a*b*Sin[c + d*x]))/(210*b^2*d
)
```

Maple [B] time = 1.514, size = 1155, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/105*(15*b^5*sin(d*x+c)^5+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-53*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^
3+111*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a*b^4-60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*b^5-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+59*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-51*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
*b^4-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),
(a-b)/a,((a-b)/(a+b))^(1/2))*a+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b
*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+18*a*b^4*sin(d*x+c)^
4-a^2*b^3*sin(d*x+c)^3-60*b^5*sin(d*x+c)^3-4*a^3*b^2*sin(d*x+c)^2-63*a*b^4*
sin(d*x+c)^2+a^2*b^3*sin(d*x+c)+45*b^5*sin(d*x+c)+4*a^3*b^2+45*a*b^4)/b^4/c
os(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\cos(dx+c)^3\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^3*cot(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.1146 $\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=323

$$\frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd}+\frac{a\left(4a^2+11b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2+57b^2\right)\sqrt{a+b}}{15}$$

```
[Out] ((4*a^2 + 15*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*a*b*d) - (2*Co
s[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(5*b*d) - (Cot[c + d*x]*(a + b*Sin[c
 + d*x])^(3/2))/(a*d) - ((4*a^2 + 57*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(
a + b)]) + (a*(4*a^2 + 11*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^2*d*Sqrt[a + b*Sin[c + d*x]]) +
(b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.875501, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2894, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd}+\frac{a\left(4a^2+11b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2+57b^2\right)\sqrt{a+b}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] ((4*a^2 + 15*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*a*b*d) - (2*Co
s[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(5*b*d) - (Cot[c + d*x]*(a + b*Sin[c
 + d*x])^(3/2))/(a*d) - ((4*a^2 + 57*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(
a + b)]) + (a*(4*a^2 + 11*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^2*d*Sqrt[a + b*Sin[c + d*x]]) +
(b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = -\frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} - \frac{\cot(c+dx)(a+b\sin(c+dx))}{ad}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+dx)(a+b\sin}{5bd}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+dx)(a+b\sin}{5bd}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+dx)(a+b\sin}{5bd}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+dx)(a+b\sin}{5bd}$$

$$= \frac{\left(4a^2+15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(c+dx)(a+b\sin}{5bd}$$

Mathematica [C] time = 3.49874, size = 422, normalized size = 1.31

$$\frac{\frac{2\left(4a^2+27b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{b\sqrt{a+b\sin(c+dx)}} + \frac{2i\left(4a^2+57b^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx}\right.\right.\right.\right.}{ab^3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(4*a^2 + 57*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elli
pticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
(a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^3*Sqrt[-(a + b)^(-1)]) + (184*a
*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(4*a^2 + 27*b^2)*EllipticPi[2, (-2*
c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(b*Sq
rt[a + b*Sin[c + d*x]]) - (4*Sqrt[a + b*Sin[c + d*x]]*(2*a*Cos[c + d*x] + 3
*b*(5*Cot[c + d*x] + Sin[2*(c + d*x)])))/b)/(60*d)
```

Maple [A] time = 1.615, size = 657, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/15*(-6*a*b^4*sin(d*x+c)*cos(d*x+c)^4+(2*a^3*b^2+21*a*b^4)*cos(d*x+c)^2*s
in(d*x+c)-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(4*EllipticE((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5+53*EllipticE((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-57*EllipticE((b/(a-b)*sin(d
*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-4*EllipticF((b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-42*EllipticF((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-11*EllipticF((b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+57*EllipticF((b/
(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-15*EllipticPi(
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4+15*
EllipticPi((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2)
)*b^5)*sin(d*x+c)-8*a^2*b^3*cos(d*x+c)^4+23*a^2*b^3*cos(d*x+c)^2)/a/b^3/sin
(d*x+c)/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\cot\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin\left(c+dx\right)}\cos^{2}\left(c+dx\right)\cot^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)**2*cot(c + d*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.1147 $\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=345

$$-\frac{(8a^2+3b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}-\frac{(8a^2+31b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{12bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-3b^2)\sqrt{a+\ldots}}{1\ldots}$$

```
[Out] -((8*a^2 + 3*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^2*d) + (b*Co
t[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*
x]*(a + b*Sin[c + d*x])^(3/2))/(2*a*d) + ((8*a^2 - 3*b^2)*EllipticE[(c - Pi
/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(12*a*b*d*Sqrt[(a + b
*Sin[c + d*x])/(a + b)]) - ((8*a^2 + 31*b^2)*EllipticF[(c - Pi/2 + d*x)/2,
(2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(12*b*d*Sqrt[a + b*Sin[c
 + d*x]]) - ((12*a^2 + b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.898298, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(8a^2+3b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}-\frac{(8a^2+31b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{12bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-3b^2)\sqrt{a+\ldots}}{1\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((8*a^2 + 3*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^2*d) + (b*Co
t[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*
x]*(a + b*Sin[c + d*x])^(3/2))/(2*a*d) + ((8*a^2 - 3*b^2)*EllipticE[(c - Pi
/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(12*a*b*d*Sqrt[(a + b
*Sin[c + d*x])/(a + b)]) - ((8*a^2 + 31*b^2)*EllipticF[(c - Pi/2 + d*x)/2,
(2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(12*b*d*Sqrt[a + b*Sin[c
 + d*x]]) - ((12*a^2 + b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 3059

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2655

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3002

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2807

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)(a+b}{2ad}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx)(a+b}{4a^2d}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx)(a+b}{4a^2d}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx)(a+b}{4a^2d}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx)(a+b}{4a^2d}$$

$$= -\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c+dx)(a+b}{4a^2d}$$

Mathematica [C] time = 3.3667, size = 450, normalized size = 1.3

$$\frac{\frac{2\left(64a^2+9b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{a\sqrt{a+b\sin(c+dx)}} + \frac{2i\left(8a^2-3b^2\right)\cos(2(c+dx))\csc^2(c+dx)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\left(2a(a-b)E\left(i\sinh\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(8*a^2 - 3*b^2)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*Ellipt
icE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b
)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)
]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1
+ Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^2*b^
2*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (4*(8*a*Cos[c + d*x] + 3*Cot
[c + d*x]*(b + 2*a*Csc[c + d*x]))*Sqrt[a + b*Sin[c + d*x]])/a + (136*b*Elli
pticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(64*a^2 + 9*b^2)*EllipticPi[2, (-2*c + P
i - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*Sqrt[a
```

```
+ b*Sin[c + d*x]]))/(48*d)
```

Maple [B] time = 1.819, size = 1364, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/12*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-42*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2+31*b^3*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^2*sin(d*x+c)^2+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^2+
11*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^3*b^2*sin(d*x+c)^2-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2+36*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*b^2*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^
(1/2))*a^3*sin(d*x+c)^2-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(d*x
+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2+3*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^
(1/2))*a*b^4*sin(d*x+c)^2-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c)
)/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^2+8*a^2*b^3*sin(
d*x+c)^5+8*a^3*b^2*sin(d*x+c)^4+3*a*b^4*sin(d*x+c)^4+a^2*b^3*sin(d*x+c)^3-2
*a^3*b^2*sin(d*x+c)^2-3*a*b^4*sin(d*x+c)^2-9*a^2*b^3*sin(d*x+c)-6*a^3*b^2)/
a^2/b^2/sin(d*x+c)^2/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin(dx+c) + a} \cos(dx+c) \cot(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)*cot(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sin{\left(c + dx \right)}} \cos{\left(c + dx \right)} \cot^{3}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)*cot(c + d*x)**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.1148 $\int \cot^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=351

$$\frac{(32a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d}-\frac{(32a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\sin(c+dx)}}+\frac{(80a^2+3b^2)\sqrt{a+b\sin\cdots}}{24a^2\cdots}$$

```
[Out] ((32*a^2 - 3*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^2*d) + (b*Co
t[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(4*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(3*a*d) + ((80*a^2 + 3*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(24*a
^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((32*a^2 + b^2)*EllipticF[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(24*a*d*Sq
rt[a + b*Sin[c + d*x]]) - (b*(12*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a^2*d*Sqrt[a + b*S
in[c + d*x]])
```

Rubi [A] time = 0.861873, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2725, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(32a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d}-\frac{(32a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\sin(c+dx)}}+\frac{(80a^2+3b^2)\sqrt{a+b\sin\cdots}}{24a^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((32*a^2 - 3*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^2*d) + (b*Co
t[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(4*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(3*a*d) + ((80*a^2 + 3*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(24*a
^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((32*a^2 + b^2)*EllipticF[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(24*a*d*Sq
rt[a + b*Sin[c + d*x]]) - (b*(12*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a^2*d*Sqrt[a + b*S
in[c + d*x]])
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)(a+b\,\mathrm{si}}{3ad}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a+b\,\mathrm{sin}}{4a^2d}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a+b\,\mathrm{sin}}{4a^2d}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a+b\,\mathrm{sin}}{4a^2d}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a+b\,\mathrm{sin}}{4a^2d}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a+b\,\mathrm{sin}}{4a^2d}$$

Mathematica [C] time = 5.50867, size = 473, normalized size = 1.35

$$\frac{-\frac{4\cot(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2\csc^2(c+dx)-32a^2+2ab\csc(c+dx)-3b^2\right)}{a^2} + \frac{-\frac{8a\left(24a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2b\left(8a^2+9b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-\right.}{\sqrt{a+b\sin(c+dx)}}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((-4*Cot[c + d*x]*(-32*a^2 - 3*b^2 + 2*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d*x
]^2)*Sqrt[a + b*Sin[c + d*x]])/a^2 + (((2*I)*(80*a^2 + 3*b^2)*Cos[2*(c + d*
x)]*Csc[c + d*x]^2*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqr
t[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[
-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(
a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/
(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*
(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^
2)) - (8*a*(24*a^2 + b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*b*(8*a^2 +
 9*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin
```

```
[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^2)/(96*d)
```

Maple [B] time = 1.838, size = 1495, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/24*(80*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^5*sin(d*x+c)^3-77*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-3*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
*b^4*sin(d*x+c)^3-48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*sin(d*x+c)^3-32*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3+7
8*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^3*sin(d*x+c)^3-b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+3*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*si
n(d*x+c)^3-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2)
,(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3+36*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b
^3*sin(d*x+c)^3+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-3*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*
sin(d*x+c)^3+32*a^3*b^2*sin(d*x+c)^5+3*a*b^4*sin(d*x+c)^5+32*a^4*b*sin(d*x+
c)^4+a^2*b^3*sin(d*x+c)^4-42*a^3*b^2*sin(d*x+c)^3-3*a*b^4*sin(d*x+c)^3-40*a
^4*b*sin(d*x+c)^2-a^2*b^3*sin(d*x+c)^2+10*a^3*b^2*sin(d*x+c)+8*a^4*b)/a^3/b
/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin(dx+c)+a} \cot(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sin(c + dx)}\cot^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cot(c + d*x)**4, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1149 $\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=412

$$\frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d}+\frac{b\left(196a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^2d\sqrt{a+b\sin(c+dx)}}+\frac{b\left(68a^2-15b\right.}{}$$

```
[Out] (b*(68*a^2 - 15*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(192*a^3*d) + (
5*(4*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(32*a^2
*d) + (5*b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(24*a^2*
d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(4*a*d) + (b*
(68*a^2 - 15*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(192*a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(196*a^2
+ 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(192*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 + 24*a^2*b^2
 - 5*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)])/(64*a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.23542, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d}+\frac{b\left(196a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^2d\sqrt{a+b\sin(c+dx)}}+\frac{b\left(68a^2-15b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (b*(68*a^2 - 15*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(192*a^3*d) + (
5*(4*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(32*a^2
*d) + (5*b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(24*a^2*
d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(4*a*d) + (b*
(68*a^2 - 15*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(192*a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(196*a^2
+ 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(192*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 + 24*a^2*b^2
 - 5*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)])/(64*a^3*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{5b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d} - \frac{\cot(c+dx)\,\mathrm{csc}}{}$$

$$= \frac{5\left(4a^2-b^2\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{32a^2d} + \frac{5b\cot(c}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b^2\right)\cot(}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b^2\right)\cot(}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b^2\right)\cot(}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b^2\right)\cot(}{}$$

$$= \frac{b\left(68a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5\left(4a^2-b^2\right)\cot(}{}$$

Mathematica [C] time = 6.59123, size = 643, normalized size = 1.56

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{5\csc^2(c+dx)\left(12a^2\cos(c+dx)+b^2\cos(c+dx)\right)}{96a^2}+\frac{\csc(c+dx)\left(68a^2b\cos(c+dx)-15b^3\cos(c+dx)\right)}{192a^3}-\frac{b\cot(c+dx)\csc^2(c+dx)}{24a}-\frac{1}{4}\cot(\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((((68*a^2*b*Cos[c + d*x] - 15*b^3*Cos[c + d*x])*Csc[c + d*x])/(192*a^3) +
(5*(12*a^2*Cos[c + d*x] + b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(96*a^2) - (b*C
ot[c + d*x]*Csc[c + d*x]^2)/(24*a) - (Cot[c + d*x]*Csc[c + d*x]^3)/4)*Sqrt[
a + b*Sin[c + d*x]])/d + ((-2*(528*a^3*b - 20*a*b^3)*EllipticF[(-c + Pi/2 -
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[
c + d*x]] - (2*(288*a^4 + 212*a^2*b^2 - 45*b^4)*EllipticPi[2, (-c + Pi/2 -
d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c
 + d*x]] - ((2*I)*(-68*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a
*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*
Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a +
 b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x])
 - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) +
 (a + b*Sin[c + d*x])^2)/b^2)]))/(768*a^3*d)
```

Maple [B] time = 1.859, size = 1761, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/192*(5*a^2*b^3*sin(d*x+c)^5+66*a^3*b^2*sin(d*x+c)^4-56*a^4*b*sin(d*x+c)+2
44*a^4*b*sin(d*x+c)^3-68*a^3*b^2*sin(d*x+c)^6+15*a*b^4*sin(d*x+c)^6-188*a^4
*b*sin(d*x+c)^5-144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b)
)^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))
^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-15*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5
*sin(d*x+c)^4-68*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+2*a^3*b^2*sin(d*x+c)^2-15*a*b^4*si
n(d*x+c)^4-5*a^2*b^3*sin(d*x+c)^3+264*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-48*a^5-120*a^
5*sin(d*x+c)^4+168*a^5*sin(d*x+c)^2+144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+
b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-
15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a*b^4*sin(d*x+c)^4+72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*
```

```
x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(
a+b))^(1/2))*a*b^4*sin(d*x+c)^4+83*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-5*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2
*b^3*sin(d*x+c)^4+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-72*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*s
in(d*x+c)^4-78*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-196*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4)/a
^4/sin(d*x+c)^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin(dx+c)+a} \cot(dx+c)^4 \csc(dx+c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4*csc(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.1150 $\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=484

$$-\frac{\left(332a^2b^2+384a^4-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d}+\frac{\left(116a^2b^2+384a^4-35b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\right.\right.}{1920a^3d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -((384*a^4 + 332*a^2*b^2 - 105*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/
(1920*a^4*d) + (b*(108*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*S
in[c + d*x]])/(960*a^3*d) + ((96*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]^2*
Sqrt[a + b*Sin[c + d*x]])/(240*a^2*d) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3*(a
 + b*Sin[c + d*x])^(3/2))/(40*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*
Sin[c + d*x])^(3/2))/(5*a*d) - ((384*a^4 + 332*a^2*b^2 - 105*b^4)*EllipticE
[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(1920*a^4*d*S
qrt[(a + b*Sin[c + d*x])/(a + b)]) + ((384*a^4 + 116*a^2*b^2 - 35*b^4)*Elli
pticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
)/(1920*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(48*a^4 - 24*a^2*b^2 + 7*b^4)*
EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/(128*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.62211, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(332a^2b^2+384a^4-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d}+\frac{\left(116a^2b^2+384a^4-35b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\right.\right.}{1920a^3d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] -((384*a^4 + 332*a^2*b^2 - 105*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/
(1920*a^4*d) + (b*(108*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*S
in[c + d*x]])/(960*a^3*d) + ((96*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]^2*
Sqrt[a + b*Sin[c + d*x]])/(240*a^2*d) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3*(a
 + b*Sin[c + d*x])^(3/2))/(40*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*
Sin[c + d*x])^(3/2))/(5*a*d) - ((384*a^4 + 332*a^2*b^2 - 105*b^4)*EllipticE
[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(1920*a^4*d*S
qrt[(a + b*Sin[c + d*x])/(a + b)]) + ((384*a^4 + 116*a^2*b^2 - 35*b^4)*Elli
pticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
)/(1920*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(48*a^4 - 24*a^2*b^2 + 7*b^4)*
EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/(128*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
```

```
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{7b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{40a^2d} - \frac{\cot(c+dx)\csc^4(\ldots}{\ldots}$$

$$= \frac{\left(96a^2-35b^2\right)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{240a^2d} + \frac{7b\cot(\ldots}{\ldots}$$

$$= \frac{b\left(108a^2-35b^2\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{960a^3d} + \frac{\left(96a^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d} + \frac{b\left(108\ldots\right.}{\ldots}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d} + \frac{b\left(108\ldots\right.}{\ldots}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d} + \frac{b\left(108\ldots\right.}{\ldots}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d} + \frac{b\left(108\ldots\right.}{\ldots}$$

$$= -\frac{\left(384a^4+332a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d} + \frac{b\left(108\ldots\right.}{\ldots}$$

Mathematica [C] time = 6.55042, size = 702, normalized size = 1.45

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc^3(c+dx)\left(96a^2\cos(c+dx)+7b^2\cos(c+dx)\right)}{240a^2} + \frac{\csc^2(c+dx)\left(108a^2b\cos(c+dx)-35b^3\cos(c+dx)\right)}{960a^3} + \frac{\csc(c+dx)\left(-332a^2b^2\cos(c+dx)-\ldots\right.}{19\ldots}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((((-384*a^4*Cos[c + d*x] - 332*a^2*b^2*Cos[c + d*x] + 105*b^4*Cos[c + d*x]
)*Csc[c + d*x])/(1920*a^4) + ((108*a^2*b*Cos[c + d*x] - 35*b^3*Cos[c + d*x]
)*Csc[c + d*x]^2)/(960*a^3) + ((96*a^2*Cos[c + d*x] + 7*b^2*Cos[c + d*x])*C
sc[c + d*x]^3)/(240*a^2) - (b*Cot[c + d*x]*Csc[c + d*x]^3)/(40*a) - (Cot[c
+ d*x]*Csc[c + d*x]^4)/5)*Sqrt[a + b*Sin[c + d*x]])/d + (b*((-2*(-432*a^3*b
 + 140*a*b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin
[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(1056*a^4 - 1052*a^2*b^2
 + 315*b^4)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(384*a^4 + 332*a^2
*b^2 - 105*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*
a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b
)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sq
rt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c
+ d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])
^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/
```

```
b^2)]))))/(7680*a^4*d)
```

Maple [B] time = 2.067, size = 2075, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/1920*(-402*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-35*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5+10
5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a*b^6*sin(d*x+c)^5+720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*
x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-105*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/
(a+b))^(1/2))*b^7*sin(d*x+c)^5+384*a^6*b-384*a^5*b^2*sin(d*x+c)^7-332*a^3*b
^4*sin(d*x+c)^7+105*a*b^6*sin(d*x+c)^7-384*a^6*b*sin(d*x+c)^6-116*a^4*b^3*s
in(d*x+c)^6+35*a^2*b^5*sin(d*x+c)^6+1152*a^6*b*sin(d*x+c)^4+124*a^4*b^3*sin
(d*x+c)^4-35*a^2*b^5*sin(d*x+c)^4-1416*a^5*b^2*sin(d*x+c)^3+14*a^3*b^4*sin(
d*x+c)^3-1152*a^6*b*sin(d*x+c)^2-8*a^4*b^3*sin(d*x+c)^2+432*a^5*b^2*sin(d*x
+c)+1368*a^5*b^2*sin(d*x+c)^5+318*a^3*b^4*sin(d*x+c)^5-105*a*b^6*sin(d*x+c)
^5+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/
a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+52*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+43
7*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^3*b^4*sin(d*x+c)^5-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+384*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*
b*sin(d*x+c)^5-168*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+116*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c
)^5-720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)
/a,((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^5-360*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*si
n(d*x+c)^5+360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2
),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5-384*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+
c)^5)/a^5/b/sin(d*x+c)^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\cot(dx+c)^4\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4*csc(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.1151 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=528

$$-\frac{2\left(80a^2-221b^2\right)\sin^2(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{2145b^3d}+\frac{8a\left(8a^2-21b^2\right)\sin(c+dx)\cos(c+dx)(a+b\sin(c}{1287b^4d}$$

```
[Out] (8*(64*a^6 - 174*a^4*b^2 + 81*a^2*b^4 - 195*b^6)*Cos[c + d*x]*Sqrt[a + b*Si
n[c + d*x]])/(45045*b^5*d) + (16*a*(32*a^4 - 47*a^2*b^2 - 27*b^4)*Cos[c + d
*x]*(a + b*Sin[c + d*x])^(3/2))/(45045*b^5*d) - (8*(160*a^4 - 375*a^2*b^2 +
 117*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(45045*b^5*d) + (8*a*(8*
a^2 - 21*b^2)*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(1287*b
^4*d) - (2*(80*a^2 - 221*b^2)*Cos[c + d*x]*Sin[c + d*x]^2*(a + b*Sin[c + d*
x])^(5/2))/(2145*b^3*d) + (4*a*Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d
*x])^(5/2))/(39*b^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]
)^(5/2))/(15*b*d) - (16*a*(32*a^6 - 111*a^4*b^2 + 102*a^2*b^4 - 471*b^6)*El
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(45045
*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(64*a^8 - 238*a^6*b^2 + 255
*a^4*b^4 - 276*a^2*b^6 + 195*b^8)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(45045*b^6*d*Sqrt[a + b*Sin[c + d*x
]])
```

Rubi [A] time = 1.20816, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2895, 3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(80a^2-221b^2\right)\sin^2(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{2145b^3d}+\frac{8a\left(8a^2-21b^2\right)\sin(c+dx)\cos(c+dx)(a+b\sin(c}{1287b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (8*(64*a^6 - 174*a^4*b^2 + 81*a^2*b^4 - 195*b^6)*Cos[c + d*x]*Sqrt[a + b*Si
n[c + d*x]])/(45045*b^5*d) + (16*a*(32*a^4 - 47*a^2*b^2 - 27*b^4)*Cos[c + d
*x]*(a + b*Sin[c + d*x])^(3/2))/(45045*b^5*d) - (8*(160*a^4 - 375*a^2*b^2 +
 117*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(45045*b^5*d) + (8*a*(8*
a^2 - 21*b^2)*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(1287*b
^4*d) - (2*(80*a^2 - 221*b^2)*Cos[c + d*x]*Sin[c + d*x]^2*(a + b*Sin[c + d*
x])^(5/2))/(2145*b^3*d) + (4*a*Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d
*x])^(5/2))/(39*b^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]
)^(5/2))/(15*b*d) - (16*a*(32*a^6 - 111*a^4*b^2 + 102*a^2*b^4 - 471*b^6)*El
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(45045
*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(64*a^8 - 238*a^6*b^2 + 255
*a^4*b^4 - 276*a^2*b^6 + 195*b^8)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(45045*b^6*d*Sqrt[a + b*Sin[c + d*x
]])
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
```

```
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{4a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^{5/2}}{39b^2d} - \frac{2\cos(c+dx)}{}$$

$$= -\frac{2\left(80a^2-221b^2\right)\cos(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^{5/2}}{2145b^3d}$$

$$= \frac{8a\left(8a^2-21b^2\right)\cos(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{5/2}}{1287b^4d} -$$

$$= -\frac{8\left(160a^4-375a^2b^2+117b^4\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{45045b^5d}$$

$$= \frac{16a\left(32a^4-47a^2b^2-27b^4\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{45045b^5d} -$$

$$= \frac{8\left(64a^6-174a^4b^2+81a^2b^4-195b^6\right)\cos(c+dx)\sqrt{a+b\sin(c+}}{45045b^5d}$$

$$= \frac{8\left(64a^6-174a^4b^2+81a^2b^4-195b^6\right)\cos(c+dx)\sqrt{a+b\sin(c+}}{45045b^5d}$$

$$= \frac{8\left(64a^6-174a^4b^2+81a^2b^4-195b^6\right)\cos(c+dx)\sqrt{a+b\sin(c+}}{45045b^5d}$$

$$= \frac{8\left(64a^6-174a^4b^2+81a^2b^4-195b^6\right)\cos(c+dx)\sqrt{a+b\sin(c+}}{45045b^5d}$$

Mathematica [A] time = 14.7226, size = 382, normalized size = 0.72

$$\frac{\sqrt{a+b\sin(c+dx)}\left(-256\left(-174a^5b^2+174a^4b^3+81a^3b^4-81a^2b^5-64a^6b+64a^7-195ab^6+195b^7\right)F\left(\frac{1}{4}(-2c-2dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(512*(32*a^7 - 111*a^5*b^2 + 102*a^3*b^4 - 471*a*
b^6)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - 256*(64*a^7 - 64*a^6
*b - 174*a^5*b^2 + 174*a^4*b^3 + 81*a^3*b^4 - 81*a^2*b^5 - 195*a*b^6 + 195*
```

```
b^7)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - 2*b*Cos[c + d*x]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)]*(4096*a^6 - 12416*a^4*b^2 + 8100*a^2*b^4 +
6786*b^6 + (-1280*a^4*b^2 + 3168*a^2*b^4 + 21723*b^6)*Cos[2*(c + d*x)] + 42
*(6*a^2*b^4 - 13*b^6)*Cos[4*(c + d*x)] - 3003*b^6*Cos[6*(c + d*x)] - 3072*a
^5*b*Sin[c + d*x] + 8432*a^3*b^3*Sin[c + d*x] - 41424*a*b^5*Sin[c + d*x] +
560*a^3*b^3*Sin[3*(c + d*x)] + 13776*a*b^5*Sin[3*(c + d*x)] + 7392*a*b^5*Si
n[5*(c + d*x)])))/(1441440*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
```

Maple [B] time = 1.694, size = 1801, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/45045*(-10*a^4*b^5*sin(d*x+c)^5+10362*a^2*b^7*sin(d*x+c)^5+16*a^5*b^4*si
n(d*x+c)^4-62*a^3*b^6*sin(d*x+c)^4-12603*a*b^8*sin(d*x+c)^4-32*a^6*b^3*sin(
d*x+c)^3+122*a^4*b^5*sin(d*x+c)^3-8115*a^2*b^7*sin(d*x+c)^3-128*a^7*b^2*sin
(d*x+c)^2+412*a^5*b^4*sin(d*x+c)^2-305*a^3*b^6*sin(d*x+c)^2+840*a*b^8*sin(d
*x+c)^2+2988*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*a*b^8+17682*a*b^8*sin(d*x+c)^6+128*a^7*b^2-428*a^5*b^4+
360*a^3*b^6+780*a*b^8+780*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*b^9+32*a^6*b^3*sin(d*x+c)-112*a^4*b^5*sin(
d*x+c)+1512*a^2*b^7*sin(d*x+c)-6699*a*b^8*sin(d*x+c)^8-3759*a^2*b^7*sin(d*x
+c)^7+7*a^3*b^6*sin(d*x+c)^6-3003*b^9*sin(d*x+c)^9+7644*b^9*sin(d*x+c)^7-51
09*b^9*sin(d*x+c)^5-312*b^9*sin(d*x+c)^3+780*b^9*sin(d*x+c)-256*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^9+
1144*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*a^7*b^2-1704*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^5*b^4+4584*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^3*b^6-3768*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a*b
^8-192*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*a^7*b^2-952*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^6*b^3+684*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^5*b^4+1020*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^4
*b^5-3480*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*a^3*b^6+256*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^8*b-1104*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^2*b^7)/b^7/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a\cos(dx+c)^6-a\cos(dx+c)^4+\left(b\cos(dx+c)^6-b\cos(dx+c)^4\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*cos(d*x + c)^6 - a*cos(d*x + c)^4 + (b*cos(d*x + c)^6 - b*cos(
d*x + c)^4)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

3.1152 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=394

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a\left(2a^2-5b^2\right)-7b\left(a^2+11b^2\right)\sin(c+dx)\right)}{3003b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(a\left(-1\right.\right.}{}$$

```
[Out] (-6*a*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(143*d) - (2*Cos[c + d*x]^5*
(a + b*Sin[c + d*x])^(3/2))/(13*d) + (8*(32*a^6 - 137*a^4*b^2 + 258*a^2*b^4
 + 231*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c +
 d*x]])/(15015*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*a*(32*a^6 - 1
45*a^4*b^2 + 290*a^2*b^4 - 177*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15015*b^5*d*Sqrt[a + b*Sin[c + d
*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(4*a*(2*a^2 - 5*b^2) - 7
*b*(a^2 + 11*b^2)*Sin[c + d*x]))/(3003*b^2*d) + (4*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(a*(32*a^4 - 113*a^2*b^2 + 177*b^4) - 3*b*(8*a^4 - 27*a^2*b^2
 - 77*b^4)*Sin[c + d*x]))/(15015*b^4*d)
```

Rubi [A] time = 0.858637, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a\left(2a^2-5b^2\right)-7b\left(a^2+11b^2\right)\sin(c+dx)\right)}{3003b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(a\left(-1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-6*a*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(143*d) - (2*Cos[c + d*x]^5*
(a + b*Sin[c + d*x])^(3/2))/(13*d) + (8*(32*a^6 - 137*a^4*b^2 + 258*a^2*b^4
 + 231*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c +
 d*x]])/(15015*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*a*(32*a^6 - 1
45*a^4*b^2 + 290*a^2*b^4 - 177*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15015*b^5*d*Sqrt[a + b*Sin[c + d
*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(4*a*(2*a^2 - 5*b^2) - 7
*b*(a^2 + 11*b^2)*Sin[c + d*x]))/(3003*b^2*d) + (4*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(a*(32*a^4 - 113*a^2*b^2 + 177*b^4) - 3*b*(8*a^4 - 27*a^2*b^2
 - 77*b^4)*Sin[c + d*x]))/(15015*b^4*d)
```


Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
```

```
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d} + \frac{2}{13}\int \cos^4(c+dx)\left(\frac{3b}{2}+\frac{3}{2}\right.$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b\sin(c+}{13d}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx)(a+b\sin(c+}{13d}$$

Mathematica [A] time = 11.3914, size = 382, normalized size = 0.97

$$\frac{-3b\cos(c+dx)\left(2088a^3b^3\sin(c+dx)+40a^3b^3\sin(3(c+dx))+\left(24512a^2b^4-128a^4b^2+8547b^6\right)\cos(2(c+dx))+70\left(8\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-384*(32*a^7 + 32*a^6*b - 137*a^5*b^2 - 137*a^4*b^3 + 258*a^3*b^4 + 258*a^
2*b^5 + 231*a*b^6 + 231*b^7)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 384*a*(32*a^6 - 145*a^4*b^2 + 290*a^
2*b^4 - 177*b^6)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)] - 3*b*Cos[c + d*x]*(-2048*a^6 + 8640*a^4*b^2 + 198
0*a^2*b^4 - 6622*b^6 + (-128*a^4*b^2 + 24512*a^2*b^4 + 8547*b^6)*Cos[2*(c +
 d*x)] + 70*(86*a^2*b^4 - 11*b^6)*Cos[4*(c + d*x)] - 1155*b^6*Cos[6*(c + d*
x)] - 512*a^5*b*Sin[c + d*x] + 2088*a^3*b^3*Sin[c + d*x] - 19492*a*b^5*Sin[
c + d*x] + 40*a^3*b^3*Sin[3*(c + d*x)] + 11870*a*b^5*Sin[3*(c + d*x)] + 525
0*a*b^5*Sin[5*(c + d*x)]))/(720720*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 1.836, size = 1619, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/15015*(-5*a^3*b^5*sin(d*x+c)^5-7390*a*b^7*sin(d*x+c)^5+8*a^4*b^4*sin(d*x+
c)^4-4542*a^2*b^6*sin(d*x+c)^4-16*a^5*b^3*sin(d*x+c)^3+74*a^3*b^5*sin(d*x+c
```

```
)^3+6089*a*b^7*sin(d*x+c)^3-64*a^6*b^2*sin(d*x+c)^2+258*a^4*b^4*sin(d*x+c)^
2+4053*a^2*b^6*sin(d*x+c)^2+16*a^5*b^3*sin(d*x+c)-69*a^3*b^5*sin(d*x+c)-132
4*a*b^7*sin(d*x+c)+2625*a*b^7*sin(d*x+c)^7-266*a^4*b^4-1016*a^2*b^6+64*a^6*
b^2+600*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^2*b^6+1505*a^2*b^6*sin(d*x+c)^6+1155*b^8*sin(d*x+c)^8-3080
*b^8*sin(d*x+c)^6+2233*b^8*sin(d*x+c)^4-308*b^8*sin(d*x+c)^2-708*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b
^7-1580*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b^4-96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-580*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3+676*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*
b^2+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^7*b+108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6+420*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+1160*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*
b^5-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^8+924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^8-924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8)/b^6/cos(d*x+c)/(a+b*sin(d
*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b\cos(dx+c)^6 - a\cos(dx+c)^4\sin(dx+c) - b\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(-(b*cos(d*x + c)^6 - a*cos(d*x + c)^4*sin(d*x + c) - b*cos(d*x + c
)^4)*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

3.1153 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=390

$$-\frac{2\left(8a^2-77b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315b^2d}-\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d}-\frac{2a\left(-95a^2b^2+8a\right.}{}$$

```
[Out] (-2*a*(8*a^2 - 87*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^2*d) -
 (2*(8*a^2 - 77*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(315*b^2*d) +
 (8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(63*b^2*d) - (2*Cos[c + d*x]
*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(9*b*d) + (2*(8*a^4 - 93*a^2*b^2
+ 84*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d
*x]])/(315*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*a*(8*a^4 - 95*a^2
*b^2 - 228*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Si
n[c + d*x])/(a + b)])/(315*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*a^2*Ellipti
cPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.15932, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2895, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(8a^2-77b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315b^2d}-\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d}-\frac{2a\left(-95a^2b^2+8a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(8*a^2 - 87*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^2*d) -
 (2*(8*a^2 - 77*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(315*b^2*d) +
 (8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(63*b^2*d) - (2*Cos[c + d*x]
*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(9*b*d) + (2*(8*a^4 - 93*a^2*b^2
+ 84*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d
*x]])/(315*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*a*(8*a^4 - 95*a^2
*b^2 - 228*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Si
n[c + d*x])/(a + b)])/(315*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*a^2*Ellipti
cPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) && !m < -1 && !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{8a\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63b^2d} - \frac{2\cos(c+dx)\sin(c+dx)(}{9bd}$$

$$= -\frac{2\left(8a^2-77b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315b^2d} + \frac{8a\cos(c+}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2-77b\right.}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2-77b\right.}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2-77b\right.}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2-77b\right.}{}$$

$$= -\frac{2a\left(8a^2-87b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \frac{2\left(8a^2-77b\right.}{}$$

Mathematica [C] time = 3.50949, size = 477, normalized size = 1.22

$$\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(203b^3-12a^2b\right)\sin(c+dx)+16a^3+100ab^2\cos(2(c+dx))+556ab^2+35b^3\sin(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)*(8*a^4 - 93*a^2*b^2 + 84*b^4)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ell
ipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a
- b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x])
```

```
)/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(a*b^2*Sqrt[-(a + b)^(-1
)]) - (8*a*b*(a^2 + 156*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(8*a^4
+ 537*a^2*b^2 + 84*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + Cos[c + d*x
]*Sqrt[a + b*Sin[c + d*x]]*(16*a^3 + 556*a*b^2 + 100*a*b^2*Cos[2*(c + d*x)]
 + (-12*a^2*b + 203*b^3)*Sin[c + d*x] + 35*b^3*Sin[3*(c + d*x)]))/(630*b^2*
d)
```

Maple [B] time = 1.644, size = 1405, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/315*(164*a^2*b^4+85*a*b^5*sin(d*x+c)^5+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b+35*b^6*sin(d*x+c)^6
-112*b^6*sin(d*x+c)^4+77*b^6*sin(d*x+c)^2+53*a^2*b^4*sin(d*x+c)^4-a^3*b^3*s
in(d*x+c)^3-326*a*b^5*sin(d*x+c)^3-4*a^4*b^2*sin(d*x+c)^2-217*a^2*b^4*sin(d
*x+c)^2+a^3*b^3*sin(d*x+c)+241*a*b^5*sin(d*x+c)+4*a^4*b^2-84*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6-8*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^6+84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*b^6-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-95*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3-315*a^2*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*b^4*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2))+315*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(a-b
))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+405*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-228*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5+1
01*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^4*b^2-177*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4)/b^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^3*cot(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral((b*cos(d*x + c)^3*cot(d*x + c)*sin(d*x + c) + a*cos(d*x + c)^3*cot
(d*x + c))*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.1154 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=374

$$\frac{(4a^2+35b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35abd}+\frac{(4a^2+65b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd}+\frac{(61a^2b^2+4a^4+40b^4)\sqrt{}}{35b^2d\sqrt{}}$$

```
[Out] ((4*a^2 + 65*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(35*b*d) + ((4*a^2
 + 35*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(35*a*b*d) - (2*Cos[c +
 d*x]*(a + b*Sin[c + d*x])^(5/2))/(7*b*d) - (Cot[c + d*x]*(a + b*Sin[c + d*
x])^(5/2))/(a*d) - (a*(4*a^2 + 167*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(35*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)]) + ((4*a^4 + 61*a^2*b^2 + 40*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(35*b^2*d*Sqrt[a + b*Sin[c +
d*x]]) + (3*a*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.16537, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2894, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(4a^2+35b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35abd}+\frac{(4a^2+65b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd}+\frac{(61a^2b^2+4a^4+40b^4)\sqrt{}}{35b^2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((4*a^2 + 65*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(35*b*d) + ((4*a^2
 + 35*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(35*a*b*d) - (2*Cos[c +
 d*x]*(a + b*Sin[c + d*x])^(5/2))/(7*b*d) - (Cot[c + d*x]*(a + b*Sin[c + d*
x])^(5/2))/(a*d) - (a*(4*a^2 + 167*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(35*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)]) + ((4*a^4 + 61*a^2*b^2 + 40*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(35*b^2*d*Sqrt[a + b*Sin[c +
d*x]]) + (3*a*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7bd} - \frac{\cot(c+dx)(a+b\sin(c+d}{ad}$$

$$= \frac{\left(4a^2+35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35abd} - \frac{2\cos(c+dx)(a+}{7b}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b^2\right)\cos(c}{}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b^2\right)\cos(c}{}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b^2\right)\cos(c}{}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b^2\right)\cos(c}{}$$

$$= \frac{\left(4a^2+65b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{\left(4a^2+35b^2\right)\cos(c}{}$$

Mathematica [C] time = 3.93311, size = 452, normalized size = 1.21

$$\frac{\frac{8\left(53a^2-20b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2a\left(4a^2-43b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{b\sqrt{a+b\sin(c+dx)}} - 2\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-55b^2\right)\cos(c+dx)+b(1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)*(4*a^2 + 167*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^3*Sqrt[-(a + b)^(-1)]) + (8*(53*a^
2 - 20*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin
[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*a*(4*a^2 - 43*b^2)*Ellip
ticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
```

```
 + b)])/(b*Sqrt[a + b*Sin[c + d*x]]) - (2*Sqrt[a + b*Sin[c + d*x]]*((4*a^2
- 55*b^2)*Cos[c + d*x] + b*(-5*b*Cos[3*(c + d*x)] + 70*a*Cot[c + d*x] + 16*
a*Sin[2*(c + d*x)])))/b)/(140*d)
```

Maple [A] time = 1.58, size = 726, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/35*(-26*a*b^4*sin(d*x+c)*cos(d*x+c)^4+(2*a^3*b^2+31*a*b^4)*cos(d*x+c)^2*
sin(d*x+c)+(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(105*EllipticPi((b/(a-b)*sin(d
*x+c)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4-105*EllipticPi((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5-4*Ellip
ticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5-163*Elli
pticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+167
*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+
4*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b
+102*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^
3*b^2+61*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2)
)*a^2*b^3-207*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^
(1/2))*a*b^4+40*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b)
)^(1/2))*b^5)*sin(d*x+c)+10*b^5*cos(d*x+c)^6+(-18*a^2*b^3+10*b^5)*cos(d*x+c
)^4+(53*a^2*b^3-20*b^5)*cos(d*x+c)^2)/sin(d*x+c)/b^3/cos(d*x+c)/(a+b*sin(d*
x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.1155 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=383

$$-\frac{(8a^2-5b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{20a^2d}-\frac{(8a^2-15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad}-\frac{a(8a^2+37b^2)\sqrt{\frac{a+b\sin}{a\cdot}}}{20bd\sqrt{a}}$$

```
[Out] -((8*a^2 - 15*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(20*a*d) - ((8*a^
2 - 5*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(20*a^2*d) - (b*Cot[c +
 d*x]*(a + b*Sin[c + d*x])^(5/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a
 + b*Sin[c + d*x])^(5/2))/(2*a*d) + ((8*a^2 - 81*b^2)*EllipticE[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(20*b*d*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)]) - (a*(8*a^2 + 37*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(20*b*d*Sqrt[a + b*Sin[c + d
*x]]) - (3*(4*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.16471, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(8a^2-5b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{20a^2d}-\frac{(8a^2-15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad}-\frac{a(8a^2+37b^2)\sqrt{\frac{a+b\sin}{a\cdot}}}{20bd\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] -((8*a^2 - 15*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(20*a*d) - ((8*a^
2 - 5*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(20*a^2*d) - (b*Cot[c +
 d*x]*(a + b*Sin[c + d*x])^(5/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a
 + b*Sin[c + d*x])^(5/2))/(2*a*d) + ((8*a^2 - 81*b^2)*EllipticE[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(20*b*d*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)]) - (a*(8*a^2 + 37*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(20*b*d*Sqrt[a + b*Sin[c + d
*x]]) - (3*(4*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(4*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{b\cot(c+dx)(a+b\sin(c+dx))^{5/2}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)(a}{2ad}$$

$$= -\frac{\left(8a^2-5b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{20a^2d} - \frac{b\cot(c+dx)(a}{}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\right)\cos}{}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\right)\cos}{}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\right)\cos}{}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\right)\cos}{}$$

$$= -\frac{\left(8a^2-15b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{\left(8a^2-5b^2\right)\cos}{}$$

Mathematica [C] time = 3.1366, size = 434, normalized size = 1.13

$$\frac{\frac{2\left(112a^2+51b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2i\left(81b^2-8a^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin}\right.\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)*(-8*a^2 + 81*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^2*Sqrt[-(a + b)^(-1)]) + (472*
a*b*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(112*a^2 + 51*b^2)*EllipticPi[2,
 (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
```

```
Sqrt[a + b*Sin[c + d*x]] + 4*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d
*x]]*(-18*a + 8*a*Cos[2*(c + d*x)] - 31*b*Sin[c + d*x] + 2*b*Sin[3*(c + d*x
)]))/(80*d)
```

Maple [B] time = 1.774, size = 1379, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/20*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-126*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2+37*b^3*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^2*sin(d*x+c)^2+81*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^
2+89*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^2*sin(d*x+c)^2-81*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2+60*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*b^2*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b
))^(1/2))*a^3*sin(d*x+c)^2-60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(
d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2-15*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+
b))^(1/2))*a*b^4*sin(d*x+c)^2+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d
*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^2+8*a*b^4*s
in(d*x+c)^6+24*a^2*b^3*sin(d*x+c)^5+16*a^3*b^2*sin(d*x+c)^4+17*a*b^4*sin(d*
x+c)^4+11*a^2*b^3*sin(d*x+c)^3-6*a^3*b^2*sin(d*x+c)^2-25*a*b^4*sin(d*x+c)^2
-35*a^2*b^3*sin(d*x+c)-10*a^3*b^2)/a/b^2/sin(d*x+c)^2/cos(d*x+c)/(a+b*sin(d
*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)*cot(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.1156 $\int \cot^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=386

$$-\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d}+\frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d}-\frac{\left(16a^2+21b^2\right)\sqrt{\frac{a+b\sin(c+}{a+b}}}{8d\sqrt{a+b}}$$

```
[Out] -(b*(16*a^2 + b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(8*a^2*d) + ((32*
a^2 + b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(24*a^2*d) + (b*Cot[c +
 d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(12*a^2*d) - (Cot[c + d*x]*C
sc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2))/(3*a*d) + ((32*a^2 - b^2)*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(8*a*d*Sqrt
[(a + b*Sin[c + d*x])/(a + b)]) - ((16*a^2 + 21*b^2)*EllipticF[(c - Pi/2 +
d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*d*Sqrt[a + b*
Sin[c + d*x]]) - (b*(36*a^2 + b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a*d*Sqrt[a + b*Sin[c + d*x]
])
```

Rubi [A] time = 1.09974, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2725, 3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d}+\frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d}-\frac{\left(16a^2+21b^2\right)\sqrt{\frac{a+b\sin(c+}{a+b}}}{8d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(b*(16*a^2 + b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(8*a^2*d) + ((32*
a^2 + b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(24*a^2*d) + (b*Cot[c +
 d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(12*a^2*d) - (Cot[c + d*x]*C
sc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2))/(3*a*d) + ((32*a^2 - b^2)*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(8*a*d*Sqrt
[(a + b*Sin[c + d*x])/(a + b)]) - ((16*a^2 + 21*b^2)*EllipticF[(c - Pi/2 +
d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*d*Sqrt[a + b*
Sin[c + d*x]]) - (b*(36*a^2 + b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a*d*Sqrt[a + b*Sin[c + d*x]
])
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}}{12a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)(a+b}{3ad}$$

$$= \frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d} + \frac{b\cot(c+dx)\csc(c+dx)(a+b}{12a^2d}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\,s}{24a^2d}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\,s}{24a^2d}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\,s}{24a^2d}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\,s}{24a^2d}$$

$$= -\frac{b\left(16a^2+b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{\left(32a^2+b^2\right)\cot(c+dx)(a+b\,s}{24a^2d}$$

Mathematica [C] time = 6.29709, size = 486, normalized size = 1.26

$$-\frac{8\left(8a^2-11b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2b\left(40a^2+3b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{a\sqrt{a+b\sin(c+dx)}} - \frac{4\sqrt{a+b\sin(c+dx)}\left(\cot(c+dx)\left(8a^2\csc^2(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)*(32*a^2 - b^2)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*Ellipti
cE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)
] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]
], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 +
 Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^2*b*S
qrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (4*(16*a*b*Cos[c + d*x] + Cot[c
 + d*x]*(-32*a^2 + 3*b^2 + 14*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d*x]^2))*Sqr
t[a + b*Sin[c + d*x]])/(3*a) - (8*(8*a^2 - 11*b^2)*EllipticF[(-2*c + Pi - 2
*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[
c + d*x]] + (2*b*(40*a^2 + 3*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*Sqrt[a + b*Sin[c + d*x]])
)/(32*d)
```

Maple [B] time = 1.836, size = 1511, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/24*(48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)^3+48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-162*b^2*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^3*sin(d*x+c)^3+63*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+3*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)
^3-96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^5*sin(d*x+c)^3+99*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-3*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4
*sin(d*x+c)^3+108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-108*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*
a^2*b^3*sin(d*x+c)^3+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-
b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-3*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))
*b^5*sin(d*x+c)^3+16*a^2*b^3*sin(d*x+c)^6-16*a^3*b^2*sin(d*x+c)^5+3*a*b^4*s
in(d*x+c)^5-32*a^4*b*sin(d*x+c)^4+a^2*b^3*sin(d*x+c)^4+38*a^3*b^2*sin(d*x+c
)^3-3*a*b^4*sin(d*x+c)^3+40*a^4*b*sin(d*x+c)^2-17*a^2*b^3*sin(d*x+c)^2-22*a
^3*b^2*sin(d*x+c)-8*a^4*b)/a^2/b/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(
```

```
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1157 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=408

$$\frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}-\frac{b\left(20a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64ad\sqrt{a+b\sin(c+dx)}}+\frac{b\left(236a^2+3b^2\right)}{}$$

```
[Out] (b*(68*a^2 - 3*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(64*a^2*d) + ((2
0*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(32*a^2*
d) + (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2))/(8*a^2*d) -
 (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2))/(4*a*d) + (b*(236
*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(64*a^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (b*(20*a^2 + b^2)
*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)])/(64*a*d*Sqrt[a + b*Sin[c + d*x]]) + (3*(16*a^4 - 24*a^2*b^2 + b^4)*E
llipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(64*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.23718, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}-\frac{b\left(20a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64ad\sqrt{a+b\sin(c+dx)}}+\frac{b\left(236a^2+3b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (b*(68*a^2 - 3*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(64*a^2*d) + ((2
0*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(32*a^2*
d) + (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2))/(8*a^2*d) -
 (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2))/(4*a*d) + (b*(236
*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(64*a^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (b*(20*a^2 + b^2)
*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)])/(64*a*d*Sqrt[a + b*Sin[c + d*x]]) + (3*(16*a^4 - 24*a^2*b^2 + b^4)*E
llipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(64*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}}{8a^2d} - \frac{\cot(c+dx)\,\mathrm{cs}}{}$$

$$= \frac{\left(20a^2-b^2\right)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{32a^2d} + \frac{b\,\mathrm{co}}{}$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)\cot}{}$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)\cot}{}$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)\cot}{}$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)\cot}{}$$

$$= \frac{b\left(68a^2-3b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{\left(20a^2-b^2\right)\cot}{}$$

Mathematica [C] time = 6.57412, size = 641, normalized size = 1.57

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc^2(c+dx)\left(20a^2\cos(c+dx)-b^2\cos(c+dx)\right)}{32a} + \frac{3\csc(c+dx)\left(36a^2b\cos(c+dx)+b^3\cos(c+dx)\right)}{64a^2} - \frac{1}{4}a\cot(c+dx)\csc^3(c+d\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((3*(36*a^2*b*Cos[c + d*x] + b^3*Cos[c + d*x])*Csc[c + d*x])/(64*a^2) + ((
20*a^2*Cos[c + d*x] - b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(32*a) - (3*b*Cot[c
 + d*x]*Csc[c + d*x]^2)/8 - (a*Cot[c + d*x]*Csc[c + d*x]^3)/4)*Sqrt[a + b*S
in[c + d*x]])/d + ((-2*(432*a^3*b + 4*a*b^3)*EllipticF[(-c + Pi/2 - d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]
] - (2*(96*a^4 + 92*a^2*b^2 + 9*b^4)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*
```

```
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] -
((2*I)*(-236*a^2*b^2 - 3*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a
 - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x
])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*S
qrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b
*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin
[c + d*x])^2)/b^2)]))/(256*a^2*d)
```

Maple [B] time = 1.83, size = 1760, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/64*(a^2*b^3*sin(d*x+c)^5-134*a^3*b^2*sin(d*x+c)^4+40*a^4*b*sin(d*x+c)-18
8*a^4*b*sin(d*x+c)^3+108*a^3*b^2*sin(d*x+c)^6+3*a*b^4*sin(d*x+c)^6+148*a^4*
b*sin(d*x+c)^5+48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-3*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*si
n(d*x+c)^4+236*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+26*a^3*b^2*sin(d*x+c)^2-3*a*b^4*sin(
d*x+c)^4-a^2*b^3*sin(d*x+c)^3-216*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+16*a^5+40*a^5*sin
(d*x+c)^4-56*a^5*sin(d*x+c)^2-48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d
*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-3*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a*b^4*sin(d*x+c)^4+72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a
-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+3*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/
2))*a*b^4*sin(d*x+c)^4-233*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d
*x+c)^4+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b
*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4
+234*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-20*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4)/a^3/sin(d*x+
```

```
c)^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4*csc(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.1158 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=484

$$-\frac{\left(-116a^2b^2+128a^4+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}+\frac{\left(692a^2b^2+128a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2}{a}\right.}{640a^2d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -((128*a^4 - 116*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
640*a^3*d) + (3*b*(36*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin
[c + d*x]])/(320*a^2*d) + ((32*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a
+ b*Sin[c + d*x])^(3/2))/(80*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b
*Sin[c + d*x])^(5/2))/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c
 + d*x])^(5/2))/(5*a*d) - ((128*a^4 - 116*a^2*b^2 + 15*b^4)*EllipticE[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(640*a^3*d*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]) + ((128*a^4 + 692*a^2*b^2 + 5*b^4)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(640*a
^2*d*Sqrt[a + b*Sin[c + d*x]]) + (3*b*(48*a^4 + 8*a^2*b^2 - b^4)*EllipticPi
[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
(128*a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.65152, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-116a^2b^2+128a^4+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}+\frac{\left(692a^2b^2+128a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2}{a}\right.}{640a^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] -((128*a^4 - 116*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
640*a^3*d) + (3*b*(36*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin
[c + d*x]])/(320*a^2*d) + ((32*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a
+ b*Sin[c + d*x])^(3/2))/(80*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b
*Sin[c + d*x])^(5/2))/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c
 + d*x])^(5/2))/(5*a*d) - ((128*a^4 - 116*a^2*b^2 + 15*b^4)*EllipticE[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(640*a^3*d*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]) + ((128*a^4 + 692*a^2*b^2 + 5*b^4)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(640*a
^2*d*Sqrt[a + b*Sin[c + d*x]]) + (3*b*(48*a^4 + 8*a^2*b^2 - b^4)*EllipticPi
[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
(128*a^3*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
```

```
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}}{8a^2d} - \frac{\cot(c+dx)\,\mathrm{c}\cdots}{}$$

$$= \frac{(32a^2-5b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{80a^2d} + \cdots$$

$$= \frac{3b(36a^2-5b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{320a^2d} + \cdots$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d} + \cdots$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d} + \cdots$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d} + \cdots$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d} + \cdots$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d} + \cdots$$

Mathematica [C] time = 6.61802, size = 700, normalized size = 1.45

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc^3(c+dx)(32a^2\cos(c+dx)-b^2\cos(c+dx))}{80a} + \frac{\csc^2(c+dx)(236a^2b\cos(c+dx)+5b^3\cos(c+dx))}{320a^2} + \frac{\csc(c+dx)(116a^2b^2\cos(c+dx)-\cdots}{\cdots}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((((-128*a^4*Cos[c + d*x] + 116*a^2*b^2*Cos[c + d*x] - 15*b^4*Cos[c + d*x])
*Csc[c + d*x])/(640*a^3) + ((236*a^2*b*Cos[c + d*x] + 5*b^3*Cos[c + d*x])*C
sc[c + d*x]^2)/(320*a^2) + ((32*a^2*Cos[c + d*x] - b^2*Cos[c + d*x])*Csc[c
+ d*x]^3)/(80*a) - (11*b*Cot[c + d*x]*Csc[c + d*x]^3)/40 - (a*Cot[c + d*x]*
Csc[c + d*x]^4)/5)*Sqrt[a + b*Sin[c + d*x]])/d + (b*((-2*(1616*a^3*b - 20*a
*b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(1312*a^4 + 356*a^2*b^2 - 45*b^4
)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(128*a^4 - 116*a^2*b^2 + 15*
b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF
[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]
- b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c +
d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b
*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(
-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-(
(a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)])))/(2
560*a^3*d)
```

Maple [B] time = 2.04, size = 2075, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/640*(126*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5-15*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a*b^6*sin(d*x+c)^5+720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c)
)/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+15*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b)
)^(1/2))*b^7*sin(d*x+c)^5+128*a^6*b-128*a^5*b^2*sin(d*x+c)^7+116*a^3*b^4*si
n(d*x+c)^7-15*a*b^6*sin(d*x+c)^7-128*a^6*b*sin(d*x+c)^6+588*a^4*b^3*sin(d*x
+c)^6-5*a^2*b^5*sin(d*x+c)^6+384*a^6*b*sin(d*x+c)^4-772*a^4*b^3*sin(d*x+c)^
4+5*a^2*b^5*sin(d*x+c)^4-1032*a^5*b^2*sin(d*x+c)^3-2*a^3*b^4*sin(d*x+c)^3-3
84*a^6*b*sin(d*x+c)^2+184*a^4*b^3*sin(d*x+c)^2+304*a^5*b^2*sin(d*x+c)+856*a
^5*b^2*sin(d*x+c)^5-114*a^3*b^4*sin(d*x+c)^5+15*a*b^6*sin(d*x+c)^5-15*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+
b))^(1/2))*a*b^6*sin(d*x+c)^5+244*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-131*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
3*b^4*sin(d*x+c)^5+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+128*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)
^5-936*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+692*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*
sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^5-720*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2))*a^4*b^3*sin(d*x+c)^5+120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*s
in(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-1
20*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((
a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^5)/a^4/b/
sin(d*x+c)^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4*csc(d*x + c)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.1159 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=551

$$-\frac{b\left(512a^2b^2+2064a^4-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}+\frac{b\left(176a^2b^2+2544a^4-35b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx\right.\right.}{7680a^3d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -(b*(2064*a^4 + 512*a^2*b^2 - 105*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]
])/(7680*a^4*d) - ((240*a^4 - 168*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*
x]*Sqrt[a + b*Sin[c + d*x]])/(3840*a^3*d) + (b*(156*a^2 - 35*b^2)*Cot[c + d
*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(960*a^2*d) + (7*(4*a^2 - b^2)
*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(96*a^2*d) + (7*b*
Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2))/(60*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2))/(6*a*d) - (b*(2064*a^4 +
 512*a^2*b^2 - 105*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a
 + b*Sin[c + d*x]])/(7680*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(2
544*a^4 + 176*a^2*b^2 - 35*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(7680*a^3*d*Sqrt[a + b*Sin[c + d*x]])
 + ((64*a^6 + 144*a^4*b^2 - 36*a^2*b^4 + 7*b^6)*EllipticPi[2, (c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(512*a^4*d*Sqrt[a
 + b*Sin[c + d*x]])
```

Rubi [A] time = 1.98397, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(512a^2b^2+2064a^4-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}+\frac{b\left(176a^2b^2+2544a^4-35b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx\right.\right.}{7680a^3d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] -(b*(2064*a^4 + 512*a^2*b^2 - 105*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]
])/(7680*a^4*d) - ((240*a^4 - 168*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*
x]*Sqrt[a + b*Sin[c + d*x]])/(3840*a^3*d) + (b*(156*a^2 - 35*b^2)*Cot[c + d
*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(960*a^2*d) + (7*(4*a^2 - b^2)
*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(96*a^2*d) + (7*b*
Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2))/(60*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2))/(6*a*d) - (b*(2064*a^4 +
 512*a^2*b^2 - 105*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a
 + b*Sin[c + d*x]])/(7680*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(2
544*a^4 + 176*a^2*b^2 - 35*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(7680*a^3*d*Sqrt[a + b*Sin[c + d*x]])
 + ((64*a^6 + 144*a^4*b^2 - 36*a^2*b^4 + 7*b^6)*EllipticPi[2, (c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(512*a^4*d*Sqrt[a
 + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
```

```
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{7b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{5/2}}{60a^2d} - \frac{\cot(c+dx)}{}$$

$$= \frac{7\left(4a^2-b^2\right)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{96a^2d} + \frac{7b}{}$$

$$= \frac{b\left(156a^2-35b^2\right)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{960a^2d} +$$

$$= -\frac{\left(240a^4-168a^2b^2+35b^4\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(}}{3840a^3d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}$$

$$= -\frac{b\left(2064a^4+512a^2b^2-105b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}$$

Mathematica [C] time = 6.64049, size = 771, normalized size = 1.4

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\csc^4(c+dx)\left(140a^2\cos(c+dx)-3b^2\cos(c+dx)\right)}{480a} + \frac{\csc^3(c+dx)\left(436a^2b\cos(c+dx)+7b^3\cos(c+dx)\right)}{960a^2} + \frac{\csc^2(c+dx)\left(168a^2b^2\cos(c+d\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((((-2064*a^4*b*Cos[c + d*x] - 512*a^2*b^3*Cos[c + d*x] + 105*b^5*Cos[c + d
*x])*Csc[c + d*x])/(7680*a^4) + ((-240*a^4*Cos[c + d*x] + 168*a^2*b^2*Cos[c
 + d*x] - 35*b^4*Cos[c + d*x])*Csc[c + d*x]^2)/(3840*a^3) + ((436*a^2*b*Cos
[c + d*x] + 7*b^3*Cos[c + d*x])*Csc[c + d*x]^3)/(960*a^2) + ((140*a^2*Cos[c
 + d*x] - 3*b^2*Cos[c + d*x])*Csc[c + d*x]^4)/(480*a) - (13*b*Cot[c + d*x]*
Csc[c + d*x]^4)/60 - (a*Cot[c + d*x]*Csc[c + d*x]^5)/6)*Sqrt[a + b*Sin[c +
d*x]])/d + ((-2*(960*a^5*b - 672*a^3*b^3 + 140*a*b^5)*EllipticF[(-c + Pi/2
- d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin
[c + d*x]] - (2*(1920*a^6 + 2256*a^4*b^2 - 1592*a^2*b^4 + 315*b^6)*Elliptic
Pi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(2064*a^4*b^2 + 512*a^2*b^4 - 105*b^6)
*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*A
rcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*
EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
```

```
]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin
[c + d*x])/(a - b))]])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a
^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2
 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(30720*
a^4*d)
```

Maple [B] time = 2.666, size = 2458, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/7680*(184*a^4*b^3*sin(d*x+c)^5+1280*a^7-105*a*b^6*sin(d*x+c)^6-35*a^2*b^
5*sin(d*x+c)^5+14*a^3*b^4*sin(d*x+c)^4-8*a^4*b^3*sin(d*x+c)^3+1712*a^5*b^2*
sin(d*x+c)^2-2064*a^5*b^2*sin(d*x+c)^8-512*a^3*b^4*sin(d*x+c)^8+105*a*b^6*s
in(d*x+c)^8-2544*a^6*b*sin(d*x+c)^7-176*a^4*b^3*sin(d*x+c)^7+35*a^2*b^5*sin
(d*x+c)^7+8272*a^6*b*sin(d*x+c)^5-5536*a^5*b^2*sin(d*x+c)^4-8672*a^6*b*sin(
d*x+c)^3+2944*a^6*b*sin(d*x+c)+498*a^3*b^4*sin(d*x+c)^6+5888*a^5*b^2*sin(d*
x+c)^6-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a
-b)/a,((a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^6-2064*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6-48
0*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^7*sin(d*x+c)^6+960*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+
c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6+2720*a^7*sin
(d*x+c)^4-3520*a^7*sin(d*x+c)^2-480*a^7*sin(d*x+c)^6-2160*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a
^4*b^3*sin(d*x+c)^6-540*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a
-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6+540*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(
1/2))*a^2*b^5*sin(d*x+c)^6+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x
+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6+1552*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^5*b^2*sin(d*x+c)^6+617*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-105*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*si
n(d*x+c)^6+2544*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^6-1728*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6+
176*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^4*b^3*sin(d*x+c)^6-960*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*si
```

```
n(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^6+2160
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-
b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6-582*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-35*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^2*b^5*sin(d*x+c)^6+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6)/a^5/sin(d*x+c)^6/c
os(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4*csc(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.1160 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=451

$$-\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(-7ab\left(a^2+63b^2\right)\sin(c+dx)-3\right.}{9009b^2d}$$

```
[Out] (-2*(3*a^2 + 13*b^2)*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(429*d) - (2*
a*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2))/(39*d) - (2*Cos[c + d*x]^5*(a
+ b*Sin[c + d*x])^(5/2))/(15*d) + (8*a*(32*a^6 - 189*a^4*b^2 + 570*a^2*b^4
+ 1635*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c +
 d*x]])/(45045*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*(32*a^8 - 197
*a^6*b^2 + 615*a^4*b^4 - 255*a^2*b^6 - 195*b^8)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(45045*b^5*d*Sqrt[a +
 b*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a^4 - 33*
a^2*b^2 - 39*b^4 - 7*a*b*(a^2 + 63*b^2)*Sin[c + d*x]))/(9009*b^2*d) + (4*Co
s[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^6 - 165*a^4*b^2 + 450*a^2*b^4 + 1
95*b^6 - 24*a*b*(a^4 - 5*a^2*b^2 - 60*b^4)*Sin[c + d*x]))/(45045*b^4*d)
```

Rubi [A] time = 1.07487, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(-7ab\left(a^2+63b^2\right)\sin(c+dx)-3\right.}{9009b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-2*(3*a^2 + 13*b^2)*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(429*d) - (2*
a*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2))/(39*d) - (2*Cos[c + d*x]^5*(a
+ b*Sin[c + d*x])^(5/2))/(15*d) + (8*a*(32*a^6 - 189*a^4*b^2 + 570*a^2*b^4
+ 1635*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c +
 d*x]])/(45045*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*(32*a^8 - 197
*a^6*b^2 + 615*a^4*b^4 - 255*a^2*b^6 - 195*b^8)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(45045*b^5*d*Sqrt[a +
 b*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a^4 - 33*
a^2*b^2 - 39*b^4 - 7*a*b*(a^2 + 63*b^2)*Sin[c + d*x]))/(9009*b^2*d) + (4*Co
s[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^6 - 165*a^4*b^2 + 450*a^2*b^4 + 1
95*b^6 - 24*a*b*(a^4 - 5*a^2*b^2 - 60*b^4)*Sin[c + d*x]))/(45045*b^4*d)
```


Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2\cos^5(c+dx)(a+b\sin(c+dx))^{5/2}}{15d} + \frac{2}{15}\int \cos^4(c+dx)\left(\frac{5b}{2}\right.$$

$$= -\frac{2a\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{39d} - \frac{2\cos^5(c+dx)(a+b\sin(c+dx)}{15d}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d} - \frac{2a\cos^5(c+d}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d} - \frac{2a\cos^5(c+d}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d} - \frac{2a\cos^5(c+d}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d} - \frac{2a\cos^5(c+d}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d} - \frac{2a\cos^5(c+d}{}$$

$$= -\frac{2\left(3a^2+13b^2\right)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d} - \frac{2a\cos^5(c+d}{}$$

Mathematica [A] time = 21.1943, size = 450, normalized size = 1.

$$\frac{b\cos(c+dx)\left(-5840a^4b^3\sin(c+dx) - 80a^4b^3\sin(3(c+dx)) + 186768a^2b^5\sin(c+dx) - 101688a^2b^5\sin(3(c+dx)) - \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-256*a*(32*a^7 + 32*a^6*b - 189*a^5*b^2 - 189*a^4*b^3 + 570*a^3*b^4 + 570*
a^2*b^5 + 1635*a*b^6 + 1635*b^7)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 256*(32*a^8 - 197*a^6*b^2 + 615*
a^4*b^4 - 255*a^2*b^6 - 195*b^8)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*Cos[c + d*x]*(4096*a^7 - 23936
*a^5*b^2 - 36512*a^3*b^4 + 67584*a*b^6 + 8*(32*a^5*b^2 - 18192*a^3*b^4 - 18
741*a*b^6)*Cos[2*(c + d*x)] - 224*(161*a^3*b^4 - 54*a*b^6)*Cos[4*(c + d*x)]
 + 20328*a*b^6*Cos[6*(c + d*x)] + 1024*a^6*b*Sin[c + d*x] - 5840*a^4*b^3*Si
n[c + d*x] + 186768*a^2*b^5*Sin[c + d*x] + 8151*b^7*Sin[c + d*x] - 80*a^4*b
^3*Sin[3*(c + d*x)] - 101688*a^2*b^5*Sin[3*(c + d*x)] - 22269*b^7*Sin[3*(c
+ d*x)] - 46536*a^2*b^5*Sin[5*(c + d*x)] - 2457*b^7*Sin[5*(c + d*x)] + 3003
*b^7*Sin[7*(c + d*x)]))/(1441440*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 1.499, size = 1801, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/45045*(-5*a^4*b^5*sin(d*x+c)^5-32532*a^2*b^7*sin(d*x+c)^5+8*a^5*b^4*sin(d
*x+c)^4-13564*a^3*b^6*sin(d*x+c)^4+19302*a*b^8*sin(d*x+c)^4-16*a^6*b^3*sin(
d*x+c)^3+100*a^4*b^5*sin(d*x+c)^3+26382*a^2*b^7*sin(d*x+c)^3-64*a^7*b^2*sin
(d*x+c)^2+362*a^5*b^4*sin(d*x+c)^2+12464*a^3*b^6*sin(d*x+c)^2-1764*a*b^8*si
n(d*x+c)^2-5760*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*a*b^8-26922*a*b^8*sin(d*x+c)^6+64*a^7*b^2-370*a^5*b^
4-3408*a^3*b^6-780*a*b^8-780*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*b^9+16*a^6*b^3*sin(d*x+c)-95*a^4*b^5*si
n(d*x+c)-5484*a^2*b^7*sin(d*x+c)+10164*a*b^8*sin(d*x+c)^8+11634*a^2*b^7*sin
(d*x+c)^7+4508*a^3*b^6*sin(d*x+c)^6+3003*b^9*sin(d*x+c)^9-7644*b^9*sin(d*x+
c)^7+5109*b^9*sin(d*x+c)^5+312*b^9*sin(d*x+c)^3-780*b^9*sin(d*x+c)-128*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*a^9+884*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*a^7*b^2-3036*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^5*b^4-4260*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^3*b^6+6540*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*a*b^8-96*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*a^7*b^2-788*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^6*b^3+576*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^5*b^4+2460*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*a^4*b^5+5280*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*a^3*b^6+128*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^8*b-1020*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*a^2*b^7)/b^6/cos
(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin(dx+c)+a)^{\frac{5}{2}} \cos(dx+c)^4 \sin(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(2\,ab\cos\left(dx+c\right)^{6}-2\,ab\cos\left(dx+c\right)^{4}+\left(b^{2}\cos\left(dx+c\right)^{6}-\left(a^{2}+b^{2}\right)\cos\left(dx+c\right)^{4}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] integral(-(2*a*b*cos(d*x + c)^6 - 2*a*b*cos(d*x + c)^4 + (b^2*cos(d*x + c)^
6 - (a^2 + b^2)*cos(d*x + c)^4)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

3.1161 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=447

$$-\frac{2\left(8a^2-117b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{693b^2d}-\frac{2a\left(8a^2-131b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{693b^2d}-\frac{2\left(-141a^2b^2+8\right.}{}$$

```
[Out] (-2*(8*a^4 - 141*a^2*b^2 + 36*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
693*b^2*d) - (2*a*(8*a^2 - 131*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2)
)/(693*b^2*d) - (2*(8*a^2 - 117*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2
))/(693*b^2*d) + (8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(99*b^2*d) -
 (2*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(11*b*d) + (2*a*(
8*a^4 - 147*a^2*b^2 + 444*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[a + b*Sin[c + d*x]])/(693*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) -
 (2*(8*a^6 - 149*a^4*b^2 - 516*a^2*b^4 - 36*b^6)*EllipticF[(c - Pi/2 + d*x)
/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(693*b^3*d*Sqrt[a +
b*Sin[c + d*x]]) + (2*a^3*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.42183, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2895, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(8a^2-117b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{693b^2d}-\frac{2a\left(8a^2-131b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{693b^2d}-\frac{2\left(-141a^2b^2+8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-2*(8*a^4 - 141*a^2*b^2 + 36*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
693*b^2*d) - (2*a*(8*a^2 - 131*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2)
)/(693*b^2*d) - (2*(8*a^2 - 117*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2
))/(693*b^2*d) + (8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(99*b^2*d) -
 (2*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(11*b*d) + (2*a*(
8*a^4 - 147*a^2*b^2 + 444*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[a + b*Sin[c + d*x]])/(693*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) -
 (2*(8*a^6 - 149*a^4*b^2 - 516*a^2*b^4 - 36*b^6)*EllipticF[(c - Pi/2 + d*x)
/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(693*b^3*d*Sqrt[a +
b*Sin[c + d*x]]) + (2*a^3*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
```

```
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{8a\cos(c+dx)(a+b\sin(c+dx))^{7/2}}{99b^2d} - \frac{2\cos(c+dx)\sin(c+dx)(a+}{11bd}$$

$$= -\frac{2\left(8a^2-117b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{693b^2d} + \frac{8a\cos(c+dx)}{}$$

$$= -\frac{2a\left(8a^2-131b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{693b^2d} - \frac{2\left(8a^2-117b\right.}{}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d} - \frac{2a\left(8a\right.}{}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d} - \frac{2a\left(8a\right.}{}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d} - \frac{2a\left(8a\right.}{}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d} - \frac{2a\left(8a\right.}{}$$

$$= -\frac{2\left(8a^4-141a^2b^2+36b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d} - \frac{2a\left(8a\right.}{}$$

Mathematica [C] time = 3.79175, size = 521, normalized size = 1.17

$$\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4\left(113a^2b^2-54b^4\right)\cos(2(c+dx))+2660a^2b^2-24a^3b\sin(c+dx)+32a^4+1954ab^3\sin(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(((2*I)*(8*a^4 - 147*a^2*b^2 + 444*b^4)*(-2*a*(a - b)*EllipticE[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*
a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b
)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c +
d*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^
(-1)]) + (8*b*(a^4 + 480*a^2*b^2 + 18*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]
] + (2*a*(8*a^4 + 1239*a^2*b^2 + 444*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)
/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d
*x]]) + Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^4 + 2660*a^2*b^2 - 9*b^
4 + 4*(113*a^2*b^2 - 54*b^4)*Cos[2*(c + d*x)] - 63*b^4*Cos[4*(c + d*x)] - 2
4*a^3*b*Sin[c + d*x] + 1954*a*b^3*Sin[c + d*x] + 322*a*b^3*Sin[3*(c + d*x)]
))/(2772*b^2*d)
```

Maple [B] time = 1.829, size = 1573, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/693*(444*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a*b^6-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7+224*a*b^6*sin(d*x+c)^6+2
74*a^2*b^5*sin(d*x+c)^5+116*a^3*b^4*sin(d*x+c)^4-706*a*b^6*sin(d*x+c)^4-a^4
*b^3*sin(d*x+c)^3-1028*a^2*b^5*sin(d*x+c)^3-4*a^5*b^2*sin(d*x+c)^2-505*a^3*
b^4*sin(d*x+c)^2+518*a*b^6*sin(d*x+c)^2+a^4*b^3*sin(d*x+c)+754*a^2*b^5*sin(
d*x+c)-693*a^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*EllipticPi(((a+b*sin(d*x+c))/(a-b))^
(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+693*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*Elliptic
Pi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+389*a^3*b^4-
36*a*b^6+4*a^5*b^2-591*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-6*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-149*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^4*b^3+1107*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^3*b^4-516*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5-408*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6+155*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^5*b^2+63*b^7*sin(d*x+c)^7-180*b^7*sin(d*x+c)^5+153*b^7*sin(d*x+c)^3
```

```
-36*b^7*sin(d*x+c))/b^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^3*cot(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.1162 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=426

$$\frac{(4a^2+63b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63abd}+\frac{(20a^2+469b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315bd}+\frac{a(20a^2+759b^2)}{}$$

```
[Out] (a*(20*a^2 + 759*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b*d) + ((
20*a^2 + 469*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(315*b*d) + ((4*
a^2 + 63*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(63*a*b*d) - (2*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(9*b*d) - (Cot[c + d*x]*(a + b*Sin[c +
 d*x])^(7/2))/(a*d) - ((20*a^4 + 1689*a^2*b^2 - 168*b^4)*EllipticE[(c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^2*d*Sqrt[(a + b
*Sin[c + d*x])/(a + b)]) + (a*(20*a^4 + 739*a^2*b^2 + 816*b^4)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b
^2*d*Sqrt[a + b*Sin[c + d*x]]) + (5*a^2*b*EllipticPi[2, (c - Pi/2 + d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d
*x]])
```

Rubi [A] time = 1.4217, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2894, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(4a^2+63b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63abd}+\frac{(20a^2+469b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315bd}+\frac{a(20a^2+759b^2)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (a*(20*a^2 + 759*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b*d) + ((
20*a^2 + 469*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(315*b*d) + ((4*
a^2 + 63*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(63*a*b*d) - (2*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(9*b*d) - (Cot[c + d*x]*(a + b*Sin[c +
 d*x])^(7/2))/(a*d) - ((20*a^4 + 1689*a^2*b^2 - 168*b^4)*EllipticE[(c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^2*d*Sqrt[(a + b
*Sin[c + d*x])/(a + b)]) + (a*(20*a^4 + 739*a^2*b^2 + 816*b^4)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b
^2*d*Sqrt[a + b*Sin[c + d*x]]) + (5*a^2*b*EllipticPi[2, (c - Pi/2 + d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d
*x]])
```


Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
```

```
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2\cos(c+dx)(a+b\sin(c+dx))^{7/2}}{9bd} - \frac{\cot(c+dx)(a+b\sin(c+dx)}{ad}$$

$$= \frac{\left(4a^2+63b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63abd} - \frac{2\cos(c+dx)(}{}$$

$$= \frac{\left(20a^2+469b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315bd} + \frac{\left(4a^2+63b\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2+46\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2+46\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2+46\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2+46\right.}{}$$

$$= \frac{a\left(20a^2+759b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + \frac{\left(20a^2+46\right.}{}$$

Mathematica [C] time = 4.50656, size = 496, normalized size = 1.16

$$\frac{8ab\left(475a^2-492b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2\left(-1461a^2b^2+20a^4-168b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \sqrt{a+b\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)*(-20*a^4 - 1689*a^2*b^2 + 168*b^4)*(-2*a*(a - b)*EllipticE[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*
```

```
a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b
)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c +
d*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(a*b^2*Sqrt[-(a + b
)^(-1)]) + (8*a*b*(475*a^2 - 492*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (
2*(20*a^4 - 1461*a^2*b^2 - 168*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] -
 Sqrt[a + b*Sin[c + d*x]]*((40*a^3 - 2202*a*b^2)*Cos[c + d*x] + 2*b*(-95*a*
b*Cos[3*(c + d*x)] + 630*a^2*Cot[c + d*x] + (150*a^2 - 119*b^2 - 35*b^2*Cos
[2*(c + d*x)])*Sin[2*(c + d*x)]))))/(1260*b*d)
```

Maple [A] time = 1.6, size = 864, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/315*(70*b^6*sin(d*x+c)*cos(d*x+c)^6+(-340*a^2*b^4+14*b^6)*cos(d*x+c)^4*s
in(d*x+c)+(10*a^4*b^2+57*a^2*b^4-84*b^6)*cos(d*x+c)^2*sin(d*x+c)+(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2)*(1575*EllipticPi((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4-1575*EllipticPi((b/(a-b)*sin(d*x+c)
+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5-20*EllipticE((b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^6-1669*EllipticE((b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+1857*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-168*Ellipt
icE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^6+20*Ellipt
icF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b+930*Ell
ipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+73
9*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b
^3-2673*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*a^2*b^4+816*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*a*b^5+168*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b)
)^(1/2))*b^6)*sin(d*x+c)+260*a*b^5*cos(d*x+c)^6+(-160*a^3*b^3+232*a*b^5)*co
s(d*x+c)^4+(475*a^3*b^3-492*a*b^5)*cos(d*x+c)^2)/sin(d*x+c)/b^3/cos(d*x+c)/
(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.1163 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=430

$$-\frac{\left(8a^2-21b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{28a^2d}-\frac{\left(8a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{28ad}-\frac{\left(8a^2-73b^2\right)\cos(c+ \ldots}{28 \ldots}$$

```
[Out] -((8*a^2 - 73*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(28*d) - ((8*a^2
- 35*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(28*a*d) - ((8*a^2 - 21*
b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(28*a^2*d) - (3*b*Cot[c + d*x
]*(a + b*Sin[c + d*x])^(7/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a + b
*Sin[c + d*x])^(7/2))/(2*a*d) + (a*(8*a^2 - 247*b^2)*EllipticE[(c - Pi/2 +
d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(28*b*d*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)]) - ((8*a^4 + 3*a^2*b^2 - 32*b^4)*EllipticF[(c - Pi/2 + d*x
)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(28*b*d*Sqrt[a + b*
Sin[c + d*x]]) - (3*a*(4*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*d*Sqrt[a + b*Sin[c + d*x
]])
```

Rubi [A] time = 1.42314, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(8a^2-21b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{28a^2d}-\frac{\left(8a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{28ad}-\frac{\left(8a^2-73b^2\right)\cos(c+ \ldots}{28 \ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((8*a^2 - 73*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(28*d) - ((8*a^2
- 35*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(28*a*d) - ((8*a^2 - 21*
b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(28*a^2*d) - (3*b*Cot[c + d*x
]*(a + b*Sin[c + d*x])^(7/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a + b
*Sin[c + d*x])^(7/2))/(2*a*d) + (a*(8*a^2 - 247*b^2)*EllipticE[(c - Pi/2 +
d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(28*b*d*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)]) - ((8*a^4 + 3*a^2*b^2 - 32*b^4)*EllipticF[(c - Pi/2 + d*x
)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(28*b*d*Sqrt[a + b*
Sin[c + d*x]]) - (3*a*(4*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*d*Sqrt[a + b*Sin[c + d*x
]])
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

```
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{3b\cot(c+dx)(a+b\sin(c+dx))^{7/2}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)(a+}{2ad}$$

$$= -\frac{\left(8a^2-21b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{28a^2d} - \frac{3b\cot(c+dx)(a}{4}$$

$$= -\frac{\left(8a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{28ad} - \frac{\left(8a^2-21b^2\right)\cos}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b^2\right)\cos(c}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b^2\right)\cos(c}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b^2\right)\cos(c}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b^2\right)\cos(c}{}$$

$$= -\frac{\left(8a^2-73b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{\left(8a^2-35b^2\right)\cos(c}{}$$

Mathematica [C] time = 5.20971, size = 460, normalized size = 1.07

$$\frac{\frac{8b\left(125a^2-16b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2a\left(160a^2+37b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + 4\sqrt{a+b\sin(c+dx)}\left(\left(22b^2-2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)*(-8*a^2 + 247*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*Arc
```

```
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*El
lipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]]
, (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*
Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^(-1)]) + (8*b*(12
5*a^2 - 16*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b
*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*a*(160*a^2 + 37*b^2)
*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*
x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + 4*Sqrt[a + b*Sin[c + d*x]]*((-24*a
^2 + 22*b^2)*Cos[c + d*x] + 2*b^2*Cos[3*(c + d*x)] - 7*a*Cot[c + d*x]*(9*b
+ 2*a*Csc[c + d*x]) - 12*a*b*Sin[2*(c + d*x)]))/(112*d)
```

Maple [B] time = 1.63, size = 1520, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/28*(8*b^5*sin(d*x+c)^7+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-258*b^2*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin
(d*x+c)^2+3*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2+279*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-32*b^
5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^2+255*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+
c)^2-247*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2+84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^2*EllipticPi(((
a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2-
84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/
a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(
((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c
)^2+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)
/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^2+32*a*b^4*sin(d*x+c)^6+48*a^2*b^3*s
in(d*x+c)^5-32*b^5*sin(d*x+c)^5+24*a^3*b^2*sin(d*x+c)^4+7*a*b^4*sin(d*x+c)^
4+29*a^2*b^3*sin(d*x+c)^3+24*b^5*sin(d*x+c)^3-10*a^3*b^2*sin(d*x+c)^2-39*a*
b^4*sin(d*x+c)^2-77*a^2*b^3*sin(d*x+c)-14*a^3*b^2)/b^2/sin(d*x+c)^2/cos(d*x
+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.1164 $\int \cot^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=429

$$-\frac{b\left(208a^2-25b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{120a^2d}-\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad}+\frac{\left(32a^2-3b^2\right)\mathrm{c}}{}$$

```
[Out] -(b*(96*a^2 - 25*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(40*a*d) - (b*
(208*a^2 - 25*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(120*a^2*d) + (
(32*a^2 - 3*b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(24*a^2*d) - (b*C
ot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(12*a^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(7/2))/(3*a*d) + ((176*a^2 - 167*b
^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/
(40*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (a*(96*a^2 + 179*b^2)*EllipticF
[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(40
*d*Sqrt[a + b*Sin[c + d*x]]) - (5*b*(12*a^2 - b^2)*EllipticPi[2, (c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*d*Sqrt[a +
b*Sin[c + d*x]])
```

Rubi [A] time = 1.35532, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2725, 3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(208a^2-25b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{120a^2d}-\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad}+\frac{\left(32a^2-3b^2\right)\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] -(b*(96*a^2 - 25*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(40*a*d) - (b*
(208*a^2 - 25*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(120*a^2*d) + (
(32*a^2 - 3*b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(24*a^2*d) - (b*C
ot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(12*a^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(7/2))/(3*a*d) + ((176*a^2 - 167*b
^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/
(40*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (a*(96*a^2 + 179*b^2)*EllipticF
[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(40
*d*Sqrt[a + b*Sin[c + d*x]]) - (5*b*(12*a^2 - b^2)*EllipticPi[2, (c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*d*Sqrt[a +
b*Sin[c + d*x]])
```


Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
```

```
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{7/2}}{12a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)(}{3ad}$$

$$= \frac{\left(32a^2-3b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{5/2}}{24a^2d} - \frac{b\cot(c+dx)\csc(c+dx)(a}{12a^2d}$$

$$= -\frac{b\left(208a^2-25b^2\right)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{120a^2d} + \frac{\left(32a^2-3b^2\right)\cot(c+}{2}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\cos(c+}{120}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\cos(c+}{120}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\cos(c+}{120}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\cos(c+}{120}$$

$$= -\frac{b\left(96a^2-25b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b\left(208a^2-25b^2\right)\cos(c+}{120}$$

Mathematica [C] time = 3.4295, size = 466, normalized size = 1.09

$$-\frac{8a(40a^2-173b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}}+\frac{2b(424a^2+117b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}}-\frac{4}{3}\sqrt{a+b\sin(c+dx)}\,(5\cot(c$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)*(-176*a^2 + 167*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b
)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*A
rcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*
EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))
]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - (8*a
*(40*a^2 - 173*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*b*(424*a^2 + 117
*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (4*Sqrt[a + b*Sin[c + d*x]]*(
176*a*b*Cos[c + d*x] + 5*Cot[c + d*x]*(-32*a^2 + 33*b^2 + 26*a*b*Csc[c + d*
x] + 8*a^2*Csc[c + d*x]^2) + 24*b^2*Sin[2*(c + d*x)]))/3)/(160*d)
```

Maple [B] time = 1.86, size = 1526, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/120*(240*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)^3+288*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*
sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-1566*b^2*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^3*sin(d*x+c)^3+537*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+501*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin
(d*x+c)^3-528*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^3+1029*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-501*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a*b^4*sin(d*x+c)^3+900*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+
c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-900*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2))*a^2*b^3*sin(d*x+c)^3-75*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
```

```
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*si
n(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3+75*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)
/(a+b))^(1/2))*b^5*sin(d*x+c)^3+48*a*b^4*sin(d*x+c)^7+224*a^2*b^3*sin(d*x+c
)^6+16*a^3*b^2*sin(d*x+c)^5+117*a*b^4*sin(d*x+c)^5-160*a^4*b*sin(d*x+c)^4+7
1*a^2*b^3*sin(d*x+c)^4+154*a^3*b^2*sin(d*x+c)^3-165*a*b^4*sin(d*x+c)^3+200*
a^4*b*sin(d*x+c)^2-295*a^2*b^3*sin(d*x+c)^2-170*a^3*b^2*sin(d*x+c)-40*a^4*b
)/a/b/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**(5/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out
```

3.1165 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=449

$$-\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}+\frac{5b\left(68a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d}-\frac{b\left(148a^2+169b^2\right)}{64}$$

```
[Out] -(b^2*(196*a^2 + 5*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(64*a^2*d) +
 (5*b*(68*a^2 + b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(192*a^2*d) +
 ((60*a^2 + b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(96*
a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(7/2))/(24*a^2
*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(7/2))/(4*a*d) + (b
*(492*a^2 - 5*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*
Sin[c + d*x]])/(64*a*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (b*(148*a^2 +
169*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(64*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 - 360*a^2*b^2 - 5
*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)])/(64*a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.51137, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2893, 3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}+\frac{5b\left(68a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d}-\frac{b\left(148a^2+169b^2\right)}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] -(b^2*(196*a^2 + 5*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(64*a^2*d) +
 (5*b*(68*a^2 + b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(192*a^2*d) +
 ((60*a^2 + b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(96*
a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(7/2))/(24*a^2
*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(7/2))/(4*a*d) + (b
*(492*a^2 - 5*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*
Sin[c + d*x]])/(64*a*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (b*(148*a^2 +
169*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(64*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 - 360*a^2*b^2 - 5
*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)])/(64*a*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

```
&& !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{7/2}}{24a^2d} - \frac{\cot(c+dx)\,\mathrm{cs}}{}$$

$$= \frac{\left(60a^2+b^2\right)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}}{96a^2d} + \frac{b\,\mathrm{co}}{}$$

$$= \frac{5b\left(68a^2+b^2\right)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d} + \frac{\left(60a^2+b^2\right)\mathrm{c}}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a^2+b\right.}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a^2+b\right.}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a^2+b\right.}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a^2+b\right.}{}$$

$$= -\frac{b^2\left(196a^2+5b^2\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{5b\left(68a^2+b\right.}{}$$

Mathematica [C] time = 6.59333, size = 655, normalized size = 1.46

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{1}{96}\csc^2(c+dx)\left(60a^2\cos(c+dx)-59b^2\cos(c+dx)\right)+\frac{5\csc(c+dx)\left(116a^2b\cos(c+dx)-3b^3\cos(c+dx)\right)}{192a}-\frac{1}{4}a^2\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*b^2*Cos[c + d*x])/3 + (5*(116*a^2*b*Cos[c + d*x] - 3*b^3*Cos[c + d*x]
)*Csc[c + d*x])/(192*a) + ((60*a^2*Cos[c + d*x] - 59*b^2*Cos[c + d*x])*Csc[
c + d*x]^2)/96 - (17*a*b*Cot[c + d*x]*Csc[c + d*x]^2)/24 - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/4)*Sqrt[a + b*Sin[c + d*x]])/d + ((-2*(688*a^3*b - 348*a*
b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(96*a^4 - 228*a^2*b^2 - 15*b^4)*E
llipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(-492*a^2*b^2 + 5*b^4)*Cos[c +
d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticP
i[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x]
)/(a - b))]])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2
+ 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 -
2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(256*a*d)
```

Maple [B] time = 1.879, size = 1777, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/192*(128*a^2*b^3*sin(d*x+c)^7+5*a^2*b^3*sin(d*x+c)^5+706*a^3*b^2*sin(d*x+
c)^4-184*a^4*b*sin(d*x+c)+884*a^4*b*sin(d*x+c)^3-452*a^3*b^2*sin(d*x+c)^6+1
5*a*b^4*sin(d*x+c)^6-700*a^4*b*sin(d*x+c)^5-144*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*sin(d*x
+c)^4-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b
)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^4-1476*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
E(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-254*
a^3*b^2*sin(d*x+c)^2-15*a*b^4*sin(d*x+c)^4-133*a^2*b^3*sin(d*x+c)^3+1032*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^5*sin(d*x+c)^4-48*a^5-120*a^5*sin(d*x+c)^4+168*a^5*sin(d*x+c)^2+144*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b
)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-1080*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b
))^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(
d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+1491*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^3*b^2*sin(d*x+c)^4+507*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*
sin(d*x+c)^4+1080*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-1998*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*
sin(d*x+c)^4+444*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4)/a^2/sin(d*x+c)^4/cos(d*x+c)/(a+
b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.1166 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=482

$$-\frac{\left(-580a^2b^2+128a^4+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d}+\frac{\left(492a^2b^2+128a^4-5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2}{a}\right.}{640ad\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -((128*a^4 - 580*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
640*a^2*d) + (b*(36*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x
])^(3/2))/(64*a^2*d) + ((32*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*S
in[c + d*x])^(5/2))/(80*a^2*d) + (3*b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Si
n[c + d*x])^(7/2))/(40*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c +
 d*x])^(7/2))/(5*a*d) - ((128*a^4 - 2476*a^2*b^2 - 15*b^4)*EllipticE[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(640*a^2*d*Sqrt[(a +
 b*Sin[c + d*x])/(a + b)]) + ((128*a^4 + 492*a^2*b^2 - 5*b^4)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(640*a*
d*Sqrt[a + b*Sin[c + d*x]]) + (3*b*(80*a^4 - 40*a^2*b^2 + b^4)*EllipticPi[2
, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1
28*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.62341, antiderivative size = 482, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2893, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-580a^2b^2+128a^4+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d}+\frac{\left(492a^2b^2+128a^4-5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2}{a}\right.}{640ad\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] -((128*a^4 - 580*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
640*a^2*d) + (b*(36*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x
])^(3/2))/(64*a^2*d) + ((32*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*S
in[c + d*x])^(5/2))/(80*a^2*d) + (3*b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Si
n[c + d*x])^(7/2))/(40*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c +
 d*x])^(7/2))/(5*a*d) - ((128*a^4 - 2476*a^2*b^2 - 15*b^4)*EllipticE[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(640*a^2*d*Sqrt[(a +
 b*Sin[c + d*x])/(a + b)]) + ((128*a^4 + 492*a^2*b^2 - 5*b^4)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(640*a*
d*Sqrt[a + b*Sin[c + d*x]]) + (3*b*(80*a^4 - 40*a^2*b^2 + b^4)*EllipticPi[2
, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1
28*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
```

```
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{3b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{7/2}}{40a^2d} - \frac{\cot(c+dx)\csc\cdots}{\cdots}$$

$$= \frac{(32a^2-b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}}{80a^2d} + \frac{3b\cot\cdots}{\cdots}$$

$$= \frac{b(36a^2-b^2)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{64a^2d} + \frac{(32a^2\cdots}{\cdots}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d} + \frac{b(36\cdots}{\cdots}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d} + \frac{b(36\cdots}{\cdots}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d} + \frac{b(36\cdots}{\cdots}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d} + \frac{b(36\cdots}{\cdots}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d} + \frac{b(36\cdots}{\cdots}$$

Mathematica [C] time = 6.73159, size = 700, normalized size = 1.45

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{1}{80}\csc^3(c+dx)\left(32a^2\cos(c+dx)-31b^2\cos(c+dx)\right)+\frac{\csc^2(c+dx)\left(436a^2b\cos(c+dx)-5b^3\cos(c+dx)\right)}{320a}+\frac{\csc(c+d\cdots}{\cdots}\cdots\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((-128*a^4*Cos[c + d*x] + 1196*a^2*b^2*Cos[c + d*x] + 15*b^4*Cos[c + d*x]
)*Csc[c + d*x])/(640*a^2) + ((436*a^2*b*Cos[c + d*x] - 5*b^3*Cos[c + d*x])*
Csc[c + d*x]^2)/(320*a) + ((32*a^2*Cos[c + d*x] - 31*b^2*Cos[c + d*x])*Csc[
c + d*x]^3)/80 - (21*a*b*Cot[c + d*x]*Csc[c + d*x]^3)/40 - (a^2*Cot[c + d*x
]*Csc[c + d*x]^4)/5)*Sqrt[a + b*Sin[c + d*x]])/d + (b*((-2*(5936*a^3*b + 20
*a*b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(2272*a^4 + 1276*a^2*b^2 + 45*
b^4)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(128*a^4 - 2476*a^2*b^2 -
 15*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*Ellip
ticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a -
b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[
c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b
 + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^
2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqr
t[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))
)/(2560*a^2*d)
```

Maple [B] time = 2.108, size = 2075, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/640*(-2466*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5-15*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a*b^6*sin(d*x+c)^5-1200*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+
c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+15*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+
b))^(1/2))*b^7*sin(d*x+c)^5-128*a^6*b+128*a^5*b^2*sin(d*x+c)^7-1196*a^3*b^4
*sin(d*x+c)^7-15*a*b^6*sin(d*x+c)^7+128*a^6*b*sin(d*x+c)^6-2068*a^4*b^3*sin
(d*x+c)^6-5*a^2*b^5*sin(d*x+c)^6-384*a^6*b*sin(d*x+c)^4+2652*a^4*b^3*sin(d*
x+c)^4+5*a^2*b^5*sin(d*x+c)^4+1592*a^5*b^2*sin(d*x+c)^3-258*a^3*b^4*sin(d*x
+c)^3+384*a^6*b*sin(d*x+c)^2-584*a^4*b^3*sin(d*x+c)^2-464*a^5*b^2*sin(d*x+c
)-1256*a^5*b^2*sin(d*x+c)^5+1454*a^3*b^4*sin(d*x+c)^5+15*a*b^6*sin(d*x+c)^5
-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,(
(a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5-2604*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+246
1*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^3*b^4*sin(d*x+c)^5+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5-128*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b
```

```
*sin(d*x+c)^5+3096*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-492*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c
)^5+1200*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b
)/a,((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^5+600*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*s
in(d*x+c)^5-600*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/
2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5+128*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x
+c)^5)/a^3/b/sin(d*x+c)^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.1167 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=551

$$-\frac{b\left(-176a^2b^2+720a^4+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d}+\frac{b\left(1696a^2b^2+816a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\right.\right.}{1536a^2d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -(b*(720*a^4 - 176*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])
/(1536*a^3*d) - ((16*a^4 - 56*a^2*b^2 + 5*b^4)*Cot[c + d*x]*Csc[c + d*x]*Sq
rt[a + b*Sin[c + d*x]])/(256*a^2*d) + (b*(52*a^2 - 5*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(192*a^2*d) + ((28*a^2 - 3*b^2)*Cot[
c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2))/(96*a^2*d) + (b*Cot[c +
 d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(7/2))/(12*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^(7/2))/(6*a*d) - (b*(720*a^4 - 176*a^2
*b^2 + 15*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(1536*a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(816*a^4 +
1696*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)])/(1536*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + ((64*a^
6 + 720*a^4*b^2 + 60*a^2*b^4 - 5*b^6)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(512*a^3*d*Sqrt[a + b*Sin[c
 + d*x]])
```

Rubi [A] time = 1.97142, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2893, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(-176a^2b^2+720a^4+15b^4\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d}+\frac{b\left(1696a^2b^2+816a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\right.\right.}{1536a^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] -(b*(720*a^4 - 176*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])
/(1536*a^3*d) - ((16*a^4 - 56*a^2*b^2 + 5*b^4)*Cot[c + d*x]*Csc[c + d*x]*Sq
rt[a + b*Sin[c + d*x]])/(256*a^2*d) + (b*(52*a^2 - 5*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(192*a^2*d) + ((28*a^2 - 3*b^2)*Cot[
c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2))/(96*a^2*d) + (b*Cot[c +
 d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(7/2))/(12*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^(7/2))/(6*a*d) - (b*(720*a^4 - 176*a^2
*b^2 + 15*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(1536*a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(816*a^4 +
1696*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)])/(1536*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + ((64*a^
6 + 720*a^4*b^2 + 60*a^2*b^4 - 5*b^6)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(512*a^3*d*Sqrt[a + b*Sin[c
 + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
```

```
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{7/2}}{12a^2d} - \frac{\cot(c+dx)\,\mathrm{c}}{}$$

$$= \frac{(28a^2-3b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}}{96a^2d} + \frac{b}{}$$

$$= \frac{b(52a^2-5b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d} +$$

$$= -\frac{(16a^4-56a^2b^2+5b^4)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+}}{256a^2d}$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d} -$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d} -$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d} -$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d} -$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d} -$$

Mathematica [C] time = 6.69572, size = 771, normalized size = 1.4

$$\sqrt{a+b\sin(c+dx)}\left(\frac{1}{96}\csc^4(c+dx)\left(28a^2\cos(c+dx)-27b^2\cos(c+dx)\right)+\frac{\csc^3(c+dx)\left(164a^2b\cos(c+dx)-b^3\cos(c+dx)\right)}{192a}+\frac{\csc^2(}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((-720*a^4*b*Cos[c + d*x] + 176*a^2*b^3*Cos[c + d*x] - 15*b^5*Cos[c + d*x
])*Csc[c + d*x])/(1536*a^3) + ((-48*a^4*Cos[c + d*x] + 600*a^2*b^2*Cos[c +
d*x] + 5*b^4*Cos[c + d*x])*Csc[c + d*x]^2)/(768*a^2) + ((164*a^2*b*Cos[c +
d*x] - b^3*Cos[c + d*x])*Csc[c + d*x]^3)/(192*a) + ((28*a^2*Cos[c + d*x] -
27*b^2*Cos[c + d*x])*Csc[c + d*x]^4)/96 - (5*a*b*Cot[c + d*x]*Csc[c + d*x]^
4)/12 - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/6)*Sqrt[a + b*Sin[c + d*x]])/d +
((-2*(192*a^5*b + 3744*a^3*b^3 - 20*a*b^5)*EllipticF[(-c + Pi/2 - d*x)/2, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]]
- (2*(384*a^6 + 3600*a^4*b^2 + 536*a^2*b^4 - 45*b^6)*EllipticPi[2, (-c + Pi
/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*
Sin[c + d*x]] - ((2*I)*(720*a^4*b^2 - 176*a^2*b^4 + 15*b^6)*Cos[c + d*x]*Co
s[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a +
b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a -
```

```
 b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a -
b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(
a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a
+ b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(6144*a^3*d)
```

Maple [B] time = 2.677, size = 2458, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/1536*(1816*a^4*b^3*sin(d*x+c)^5-256*a^7-15*a*b^6*sin(d*x+c)^6-5*a^2*b^5*s
in(d*x+c)^5+2*a^3*b^4*sin(d*x+c)^4-440*a^4*b^3*sin(d*x+c)^3-1072*a^5*b^2*si
n(d*x+c)^2+720*a^5*b^2*sin(d*x+c)^8-176*a^3*b^4*sin(d*x+c)^8+15*a*b^6*sin(d
*x+c)^8+816*a^6*b*sin(d*x+c)^7-1376*a^4*b^3*sin(d*x+c)^7+5*a^2*b^5*sin(d*x+
c)^7-2576*a^6*b*sin(d*x+c)^5+3584*a^5*b^2*sin(d*x+c)^4+2656*a^6*b*sin(d*x+c
)^3-896*a^6*b*sin(d*x+c)+174*a^3*b^4*sin(d*x+c)^6-3232*a^5*b^2*sin(d*x+c)^6
-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,(
(a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^6+720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6+96*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^7*sin(d*x+c)^6-192*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b)
)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6-544*a^7*sin(d*x+c)^4+
704*a^7*sin(d*x+c)^2+96*a^7*sin(d*x+c)^6+2160*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticP
i(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d
*x+c)^6-180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(
a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6+180*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^
5*sin(d*x+c)^6+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6-896*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin
(d*x+c)^6+191*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-15*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6-816*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a^6*b*sin(d*x+c)^6+2592*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6-1696*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*
b^3*sin(d*x+c)^6+192*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b)
)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^6-2160*((a+b*sin(d*x+
```

```
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2)
)*a^5*b^2*sin(d*x+c)^6-186*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-5*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin
(d*x+c)^6+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6)/a^4/sin(d*x+c)^6/cos(d*x+c)/(a+b*si
n(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.1168 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=471

$$-\frac{10\left(8a^2-11b^2\right)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{429b^3d}+\frac{4a\left(160a^2-223b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin}}{3003b^4d}$$

```
[Out] (64*a*(80*a^4 - 118*a^2*b^2 + 17*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]
)/(15015*b^6*d) - (8*(480*a^4 - 683*a^2*b^2 + 77*b^4)*Cos[c + d*x]*Sin[c +
d*x]*Sqrt[a + b*Sin[c + d*x]])/(15015*b^5*d) + (4*a*(160*a^2 - 223*b^2)*Cos
[c + d*x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(3003*b^4*d) - (10*(8*a^
2 - 11*b^2)*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(429*b^3*
d) + (24*a*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]])/(143*b^2*d
) - (2*Cos[c + d*x]*Sin[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(13*b*d) + (8*
(1280*a^6 - 2048*a^4*b^2 + 453*a^2*b^4 + 231*b^6)*EllipticE[(c - Pi/2 + d*x
)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15015*b^7*d*Sqrt[(a + b*Sin[
c + d*x])/(a + b)]) - (8*a*(1280*a^6 - 2368*a^4*b^2 + 875*a^2*b^4 + 213*b^6
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(15015*b^7*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.18478, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2895, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{10\left(8a^2-11b^2\right)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{429b^3d}+\frac{4a\left(160a^2-223b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin}}{3003b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (64*a*(80*a^4 - 118*a^2*b^2 + 17*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]
)/(15015*b^6*d) - (8*(480*a^4 - 683*a^2*b^2 + 77*b^4)*Cos[c + d*x]*Sin[c +
d*x]*Sqrt[a + b*Sin[c + d*x]])/(15015*b^5*d) + (4*a*(160*a^2 - 223*b^2)*Cos
[c + d*x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(3003*b^4*d) - (10*(8*a^
2 - 11*b^2)*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(429*b^3*
d) + (24*a*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]])/(143*b^2*d
) - (2*Cos[c + d*x]*Sin[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(13*b*d) + (8*
(1280*a^6 - 2048*a^4*b^2 + 453*a^2*b^4 + 231*b^6)*EllipticE[(c - Pi/2 + d*x
)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15015*b^7*d*Sqrt[(a + b*Sin[
c + d*x])/(a + b)]) - (8*a*(1280*a^6 - 2368*a^4*b^2 + 875*a^2*b^4 + 213*b^6
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(15015*b^7*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
```

```
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{24a\cos(c+dx)\sin^4(c+dx)\sqrt{a+b\sin(c+dx)}}{143b^2d} - \frac{2\cos(c+dx)\sin^5(c+dx)\sqrt{a+b}}{13bd}$$

$$= -\frac{10\left(8a^2-11b^2\right)\cos(c+dx)\sin^3(c+dx)\sqrt{a+b\sin(c+dx)}}{429b^3d} + 24a\cos(c+dx)\sin^4$$

$$= \frac{4a\left(160a^2-223b^2\right)\cos(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}}{3003b^4d} - 10\left(8a^2-11b^2\right)\mathrm{cc}$$

$$= -\frac{8\left(480a^4-683a^2b^2+77b^4\right)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^5d} + 4a\left(160a\right.$$

$$= \frac{64a\left(80a^4-118a^2b^2+17b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - 8\left(480a^4-683a^2b^2+\right.$$

$$= \frac{64a\left(80a^4-118a^2b^2+17b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - 8\left(480a^4-683a^2b^2+\right.$$

$$= \frac{64a\left(80a^4-118a^2b^2+17b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - 8\left(480a^4-683a^2b^2+\right.$$

$$= \frac{64a\left(80a^4-118a^2b^2+17b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - 8\left(480a^4-683a^2b^2+\right.$$

Mathematica [A] time = 5.42158, size = 382, normalized size = 0.81

$$3b\cos(c+dx)\left(-28608a^3b^3\sin(c+dx)-1600a^3b^3\sin(3(c+dx))+\left(-5792a^2b^4+5120a^4b^2-8547b^6\right)\cos(2(c+dx)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-384*(1280*a^7 + 1280*a^6*b - 2048*a^5*b^2 - 2048*a^4*b^3 + 453*a^3*b^4 +
453*a^2*b^5 + 231*a*b^6 + 231*b^7)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(
a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 384*a*(1280*a^6 - 2368*a^4*b^2
 + 875*a^2*b^4 + 213*b^6)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)] + 3*b*Cos[c + d*x]*(81920*a^6 - 125952*a^
4*b^2 + 23760*a^2*b^4 + 6622*b^6 + (5120*a^4*b^2 - 5792*a^2*b^4 - 8547*b^6)
*Cos[2*(c + d*x)] - 70*(8*a^2*b^4 - 11*b^6)*Cos[4*(c + d*x)] + 1155*b^6*Cos
[6*(c + d*x)] + 20480*a^5*b*Sin[c + d*x] - 28608*a^3*b^3*Sin[c + d*x] + 233
2*a*b^5*Sin[c + d*x] - 1600*a^3*b^3*Sin[3*(c + d*x)] + 1390*a*b^5*Sin[3*(c
+ d*x)] + 210*a*b^5*Sin[5*(c + d*x)]))/(720720*b^7*d*Sqrt[a + b*Sin[c + d*x
]])
```

Maple [B] time = 1.55, size = 1619, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15015*(-200*a^3*b^5*sin(d*x+c)^5+410*a*b^7*sin(d*x+c)^5+320*a^4*b^4*sin(d
*x+c)^4-642*a^2*b^6*sin(d*x+c)^4-640*a^5*b^3*sin(d*x+c)^3+1244*a^3*b^5*sin(
d*x+c)^3-541*a*b^7*sin(d*x+c)^3-2560*a^6*b^2*sin(d*x+c)^2+3456*a^4*b^4*sin(
d*x+c)^2-42*a^2*b^6*sin(d*x+c)^2+640*a^5*b^3*sin(d*x+c)-1044*a^3*b^5*sin(d*
x+c)+236*a*b^7*sin(d*x+c)-105*a*b^7*sin(d*x+c)^7-3776*a^4*b^4+544*a^2*b^6+2
560*a^6*b^2-1740*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a^2*b^6+140*a^2*b^6*sin(d*x+c)^6+1155*b^8*sin(d*x+c
)^8-3080*b^8*sin(d*x+c)^6+2233*b^8*sin(d*x+c)^4-308*b^8*sin(d*x+c)^2+852*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a*b^7-10004*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a^4*b^4-3840*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-9472*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3+133
12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^6*b^2+5120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b+888*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6+6504*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+
3500*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^5-5120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a^8+924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8-924*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8)/b^8/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^3}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(dx+c\right)^{6}-\cos\left(dx+c\right)^{4}\right)\sin\left(dx+c\right)}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(cos(d*x + c)^6 - cos(d*x + c)^4)*sin(d*x + c)/sqrt(b*sin(d*x + c
) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)^{3}}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

3.1169 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=405

$$-\frac{2\left(80a^2-117b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^3d}+\frac{8a\left(120a^2-179b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^4d}$$

```
[Out] (-8*(160*a^4 - 247*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])
/(3465*b^5*d) + (8*a*(120*a^2 - 179*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]])/(3465*b^4*d) - (2*(80*a^2 - 117*b^2)*Cos[c + d*x]*Sin[c +
 d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(693*b^3*d) + (20*a*Cos[c + d*x]*Sin[c +
d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(99*b^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^
4*Sqrt[a + b*Sin[c + d*x]])/(11*b*d) - (16*a*(160*a^4 - 267*a^2*b^2 + 69*b^
4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(
3465*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(320*a^6 - 614*a^4*b^2
+ 249*a^2*b^4 + 45*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)])/(3465*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.885941, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2895, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(80a^2-117b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^3d}+\frac{8a\left(120a^2-179b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-8*(160*a^4 - 247*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])
/(3465*b^5*d) + (8*a*(120*a^2 - 179*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]])/(3465*b^4*d) - (2*(80*a^2 - 117*b^2)*Cos[c + d*x]*Sin[c +
 d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(693*b^3*d) + (20*a*Cos[c + d*x]*Sin[c +
d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(99*b^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^
4*Sqrt[a + b*Sin[c + d*x]])/(11*b*d) - (16*a*(160*a^4 - 267*a^2*b^2 + 69*b^
4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(
3465*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(320*a^6 - 614*a^4*b^2
+ 249*a^2*b^4 + 45*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)])/(3465*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
```

```
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx = \frac{20a\cos(c+dx)\sin^3(c+dx)\sqrt{a+b\sin(c+dx)}}{99b^2d} - \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+b\sin}}{11bd}$$

$$= -\frac{2\left(80a^2-117b^2\right)\cos(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^3d} + \frac{20a\cos(c+dx)\sin^3(c}{9}$$

$$= \frac{8a\left(120a^2-179b^2\right)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^4d} - \frac{2\left(80a^2-117b^2\right)\cos(c}{}$$

$$= -\frac{8\left(160a^4-247a^2b^2+45b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a\left(120a^2-179b^2\right)\cos}{}$$

$$= -\frac{8\left(160a^4-247a^2b^2+45b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a\left(120a^2-179b^2\right)\cos}{}$$

$$= -\frac{8\left(160a^4-247a^2b^2+45b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a\left(120a^2-179b^2\right)\cos}{}$$

$$= -\frac{8\left(160a^4-247a^2b^2+45b^4\right)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a\left(120a^2-179b^2\right)\cos}{}$$

Mathematica [A] time = 4.11822, size = 326, normalized size = 0.8

$$\frac{b\cos(c+dx)\left(3752a^2b^3\sin(c+dx)+200a^2b^3\sin(3(c+dx))-128\left(5a^3b^2-6ab^4\right)\cos(2(c+dx))+16448a^3b^2-2560a^4b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (128*a*(160*a^5 + 160*a^4*b - 267*a^3*b^2 - 267*a^2*b^3 + 69*a*b^4 + 69*b^5
)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)] - 64*(320*a^6 - 614*a^4*b^2 + 249*a^2*b^4 + 45*b^6)*EllipticF[(-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*C
os[c + d*x]*(-10240*a^5 + 16448*a^3*b^2 - 3718*a*b^4 - 128*(5*a^3*b^2 - 6*a
*b^4)*Cos[2*(c + d*x)] + 70*a*b^4*Cos[4*(c + d*x)] - 2560*a^4*b*Sin[c + d*x
] + 3752*a^2*b^3*Sin[c + d*x] + 990*b^5*Sin[c + d*x] + 200*a^2*b^3*Sin[3*(c
 + d*x)] - 765*b^5*Sin[3*(c + d*x)] - 315*b^5*Sin[5*(c + d*x)]))/(27720*b^6
*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 1.609, size = 1356, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3465*(552*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
```

```
(a-b)/(a+b))^(1/2))*a*b^6+180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-1280*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7+35*a*b^6*sin(d*x+c
)^6-50*a^2*b^5*sin(d*x+c)^5+80*a^3*b^4*sin(d*x+c)^4-166*a*b^6*sin(d*x+c)^4-
160*a^4*b^3*sin(d*x+c)^3+322*a^2*b^5*sin(d*x+c)^3-640*a^5*b^2*sin(d*x+c)^2+
908*a^3*b^4*sin(d*x+c)^2-49*a*b^6*sin(d*x+c)^2+160*a^4*b^3*sin(d*x+c)-272*a
^2*b^5*sin(d*x+c)-988*a^3*b^4+180*a*b^6+640*a^5*b^2-2688*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+128
0*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^6*b-960*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-2456*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*
sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+1692*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+9
96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^2*b^5-732*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*a*b^6+3416*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-315*b^7*sin(d*x+c)^7
+900*b^7*sin(d*x+c)^5-765*b^7*sin(d*x+c)^3+180*b^7*sin(d*x+c))/b^7/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\cos(dx+c)^6-\cos(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(cos(d*x + c)^6 - cos(d*x + c)^4)/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

3.1170 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=283

$$\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(a\left(32a^2-33b^2\right)-3b\left(8a^2-7b^2\right)\sin(c+dx)\right)}{315b^4d}-\frac{8a\left(-65a^2b^2+32a^4+33b^4\right)\sqrt{\frac{a+b\sin}{a\cdot}}}{315b^5d\sqrt{a+b\sin}}$$

```
[Out] (-2*Cos[c + d*x]^3*(8*a - 7*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(63*b
^2*d) + (8*(32*a^4 - 57*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^5*d*Sqrt[(a + b*Sin[c + d*x])/
(a + b)]) - (8*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b^5*d*Sqrt[a + b
*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*(32*a^2 - 33*
b^2) - 3*b*(8*a^2 - 7*b^2)*Sin[c + d*x]))/(315*b^4*d)
```

Rubi [A] time = 0.452864, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(a\left(32a^2-33b^2\right)-3b\left(8a^2-7b^2\right)\sin(c+dx)\right)}{315b^4d}-\frac{8a\left(-65a^2b^2+32a^4+33b^4\right)\sqrt{\frac{a+b\sin}{a\cdot}}}{315b^5d\sqrt{a+b\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^3*(8*a - 7*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(63*b
^2*d) + (8*(32*a^4 - 57*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^5*d*Sqrt[(a + b*Sin[c + d*x])/
(a + b)]) - (8*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b^5*d*Sqrt[a + b
*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*(32*a^2 - 33*
b^2) - 3*b*(8*a^2 - 7*b^2)*Sin[c + d*x]))/(315*b^4*d)
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\int \frac{\cos^2(c+dx)\left(-\frac{ab}{2}-\frac{1}{2}\left(8a^2-7b^2\right)\sin\right.}{\sqrt{a+b\sin(c+dx)}}}{21b^2}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\cos(c+dx)\sqrt{a+b\sin(c+}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\cos(c+dx)\sqrt{a+b\sin(c+}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\cos(c+dx)\sqrt{a+b\sin(c+}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{8\left(32a^4-57a^2b^2+21b^4\right)E\left(\right.}{315b^5}$$

Mathematica [A] time = 3.05023, size = 275, normalized size = 0.97

$$\frac{-b\cos(c+dx)\left(-8\left(4a^2b^2-21b^4\right)\cos(2(c+dx))+880a^2b^2-128a^3b\sin(c+dx)-512a^4+202ab^3\sin(c+dx)+10ab^3\text{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-32*(32*a^5 + 32*a^4*b - 57*a^3*b^2 - 57*a^2*b^3 + 21*a*b^4 + 21*b^5)*Elli
pticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
```

```
b)] + 32*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4,
(2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*Cos[c + d*x]*(-512*a^
4 + 880*a^2*b^2 - 203*b^4 - 8*(4*a^2*b^2 - 21*b^4)*Cos[2*(c + d*x)] + 35*b^
4*Cos[4*(c + d*x)] - 128*a^3*b*Sin[c + d*x] + 202*a*b^3*Sin[c + d*x] + 10*a
*b^3*Sin[3*(c + d*x)]))/(1260*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 1.483, size = 1190, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/315*(35*b^6*sin(d*x+c)^6-5*a*b^5*sin(d*x+c)^5+128*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-96*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^4*b^2-260*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^3*b^3+180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+132*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-84*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^
6-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^6+356*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-312*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+84*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6+8*a^2*
b^4*sin(d*x+c)^4-112*b^6*sin(d*x+c)^4-16*a^3*b^3*sin(d*x+c)^3+34*a*b^5*sin(
d*x+c)^3-64*a^4*b^2*sin(d*x+c)^2+98*a^2*b^4*sin(d*x+c)^2+77*b^6*sin(d*x+c)^
2+16*a^3*b^3*sin(d*x+c)-29*a*b^5*sin(d*x+c)+64*a^4*b^2-106*a^2*b^4)/b^6/cos
(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^4\sin(dx+c)}{\sqrt{b\sin(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(cos(d*x + c)^4*sin(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(dx+c)^4\sin(dx+c)}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

3.1171 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=288

$$-\frac{2a\left(8a^2-23b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2-21b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+$$

```
[Out] (8*a*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d) - (2*Cos[c + d*x]*Si
n[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(5*b*d) + (2*(8*a^2 - 21*b^2)*Elliptic
E[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^3*d*Sq
rt[(a + b*Sin[c + d*x])/(a + b)]) - (2*a*(8*a^2 - 23*b^2)*EllipticF[(c - Pi
/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^3*d*S
qrt[a + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.654878, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2895, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2a\left(8a^2-23b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2-21b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (8*a*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d) - (2*Cos[c + d*x]*Si
n[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(5*b*d) + (2*(8*a^2 - 21*b^2)*Elliptic
E[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^3*d*Sq
rt[(a + b*Sin[c + d*x])/(a + b)]) - (2*a*(8*a^2 - 23*b^2)*EllipticF[(c - Pi
/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^3*d*S
qrt[a + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{5bd} - \frac{4}{\cdots}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{5bd} + \frac{4}{\cdots}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{5bd} - \frac{(\cdots}{\cdots}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{5bd} + \frac{2}{\cdots}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{5bd} + \frac{2}{\cdots}$$

Mathematica [C] time = 3.55528, size = 408, normalized size = 1.42

$$\frac{-\frac{2(8a^2+9b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2i(21b^2-8a^2)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c\cdots}\right.\right.\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(-8*a^2 + 21*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(a*b^2*Sqrt[-(a + b)^(-1)]) + 4*Cos[c
 + d*x]*(4*a - 3*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]] - (8*a*b*Elliptic
F[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
/Sqrt[a + b*Sin[c + d*x]] - (2*(8*a^2 + 9*b^2)*EllipticPi[2, (-2*c + Pi - 2
*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[
c + d*x]])/(30*b^2*d)
```

Maple [B] time = 1.494, size = 1018, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
```

```
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-23*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+21*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-8*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^5+29*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^2-21*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-15*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*Elli
pticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a+15*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-
b)/(a+b))^(1/2))+3*a*b^4*sin(d*x+c)^4-a^2*b^3*sin(d*x+c)^3-4*a^3*b^2*sin(d*
x+c)^2-3*a*b^4*sin(d*x+c)^2+a^2*b^3*sin(d*x+c)+4*a^3*b^2)/a/b^4/cos(d*x+c)/
(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3 \cot(dx+c)}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^3 \cot(dx+c)}{\sqrt{b\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(cos(d*x + c)^3*cot(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3 \cot(dx+c)}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

3.1172 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=285

$$\frac{(4a^2-7b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2+3b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3ab^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx}{}$$

```
[Out] (-2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b*d) - (Cot[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]])/(a*d) - ((4*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a*b^2*d*Sqrt[(a + b*Sin[c + d*x])
/(a + b)]) + ((4*a^2 - 7*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (b
*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/(a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.672747, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2894, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(4a^2-7b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2+3b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3ab^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (-2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b*d) - (Cot[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]])/(a*d) - ((4*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a*b^2*d*Sqrt[(a + b*Sin[c + d*x])
/(a + b)]) + ((4*a^2 - 7*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (b
*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/(a*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
```

```
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} - \frac{2\int \frac{\csc(c+dx)\left(\frac{3b^2}{4}+\right.}{}}{}$$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} + \frac{2\int \frac{\csc(c+dx)\left(-\frac{3b^3}{4}\right.}{\sqrt{a}}}{}$$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} + \frac{1}{6}\left(-7+\frac{4a^2}{b^2}\right)\int$$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} - \frac{(4a^2+3b^2)\,E\left(\frac{1}{2}\right.}{}$$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} - \frac{(4a^2+3b^2)\,E\left(\frac{1}{2}\right.}{}$$

Mathematica [C] time = 3.46644, size = 416, normalized size = 1.46

$$\frac{\frac{2(4a^2+9b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{ab\sqrt{a+b\sin(c+dx)}} + \frac{2i(4a^2+3b^2)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\right|\right.\right.}{a^2b^3\sqrt{}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(4*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1
)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ellip
ticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (
a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqr
t[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^2*b^3*Sqrt[-(a + b)^(-1)]) - (4*Co
t[c + d*x]*(3*b + 2*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(a*b) + (40*E
llipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(4*a^2 + 9*b^2)*EllipticPi[2, (-2*c +
 Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*b*Sqr
t[a + b*Sin[c + d*x]]))/(12*d)
```

Maple [A] time = 2.826, size = 704, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/3*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-2*a^3*b^2-3*a*b^4)*sin(d*x+c
)*cos(d*x+c)^2+(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/
(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(4*EllipticE((b/(a-b)*sin(
```

```
d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5-EllipticE((b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-3*EllipticE((b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-4*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-6*EllipticF((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+7*EllipticF((b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+3*EllipticF((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+3*EllipticPi((b/
(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(a*b-b^2)/a/b,((a-b)/(a+b))^(1/2))*a*b^4-
3*EllipticPi((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(a*b-b^2)/a/b,((a-b)/(a+b
))^(1/2))*b^5)*sin(d*x+c)+2*a^2*b^3*cos(d*x+c)^4-5*a^2*b^3*cos(d*x+c)^2)/b^
3/(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)/a^2/sin(d*x+c)/cos(d*x+c
)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2\cot(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2\cot(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

3.1173 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=307

$$-\frac{(8a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4abd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2+3b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4a^2bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{3(4a^2-$$

```
[Out] (3*b*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(4*a^2*d) - (Cot[c + d*x]*Csc[c
 + d*x]*Sqrt[a + b*Sin[c + d*x]])/(2*a*d) + ((8*a^2 + 3*b^2)*EllipticE[(c -
 Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(4*a^2*b*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) - ((8*a^2 + b^2)*EllipticF[(c - Pi/2 + d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a*b*d*Sqrt[a + b*Sin
[c + d*x]]) - (3*(4*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a +
 b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.665193, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2893, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(8a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4abd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2+3b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4a^2bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{3(4a^2-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (3*b*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(4*a^2*d) - (Cot[c + d*x]*Csc[c
 + d*x]*Sqrt[a + b*Sin[c + d*x]])/(2*a*d) + ((8*a^2 + 3*b^2)*EllipticE[(c -
 Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(4*a^2*b*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) - ((8*a^2 + b^2)*EllipticF[(c - Pi/2 + d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a*b*d*Sqrt[a + b*Sin
[c + d*x]]) - (3*(4*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a +
 b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
```

```
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad} - \frac{\int \cdots}{\cdots}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad} + \frac{\int \cdots}{\cdots}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad} - \frac{1}{8}\Big(\cdots$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad} + \frac{(8\cdots}{\cdots}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad} + \frac{(8\cdots}{\cdots}$$

Mathematica [C] time = 3.36653, size = 443, normalized size = 1.44

$$\frac{\frac{2(16a^2-9b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{a^2\sqrt{a+b\sin(c+dx)}} + \frac{2i(8a^2+3b^2)\cos(2(c+dx))\csc^2(c+dx)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\Big(2a(a-b)E\Big(i\sinh\cdots}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(8*a^2 + 3*b^2)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*Ellipt
icE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b
)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)
]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1
+ Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^3*b^
2*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (4*Cot[c + d*x]*(-3*b + 2*a*
Csc[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/a^2 - (8*b*EllipticF[(-2*c + Pi - 2
*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*Sqrt[a + b*S
in[c + d*x]]) + (2*(16*a^2 - 9*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a^2*Sqrt[a + b*Sin[c + d*
x]]))/(16*d)
```

Maple [B] time = 3.119, size = 913, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] (-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*(2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b
*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a
```

```
-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2)))+4*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+
c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*
x+c)^2)^(1/2)/a*b*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),-(-a/b+1)/a*b,(
(a-b)/(a+b))^(1/2))-1/2/a*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/sin(d*x+c
)^2+3/4/a^2*b*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/sin(d*x+c)+1/2/a*b*(a
/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(
d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+3/4/a^2*b^2*(a/b-1)*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)
*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))-1/4*(4*a^2+3*b^2)/a^3*(a/b-1)*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*
b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*b*EllipticPi(((a+b*s
in(d*x+c))/(a-b))^(1/2),-(-a/b+1)/a*b,((a-b)/(a+b))^(1/2)))/cos(d*x+c)/(a+b
*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(dx+c\right)\cot\left(dx+c\right)^{3}}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(cos(d*x + c)*cot(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(c+dx\right)\cot^{3}\left(c+dx\right)}{\sqrt{a+b\sin\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)**3/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)\cot(dx+c)^3}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)*cot(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

3.1174 $\int \frac{\cot^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=353

$$\frac{(32a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d}+\frac{(16a^2+5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24a^2d\sqrt{a+b\sin(c+dx)}}+\frac{(32a^2-15b^2)\sqrt{a+}}{24}$$

```
[Out] ((32*a^2 - 15*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^3*d) + (5*b
*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(3*a*d) + ((32*a^2 - 15*b^2)*
EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(24*
a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((16*a^2 + 5*b^2)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(24*a^2
*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(12*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a^3*d*Sqrt[
a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.881176, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2725, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(32a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d}+\frac{(16a^2+5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24a^2d\sqrt{a+b\sin(c+dx)}}+\frac{(32a^2-15b^2)\sqrt{a+}}{24}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((32*a^2 - 15*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^3*d) + (5*b
*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(3*a*d) + ((32*a^2 - 15*b^2)*
EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(24*
a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((16*a^2 + 5*b^2)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(24*a^2
*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(12*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a^3*d*Sqrt[
a + b*Sin[c + d*x]])
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{3ad} -$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}$$

$$= \frac{\left(32a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}$$

Mathematica [C] time = 5.46111, size = 475, normalized size = 1.35

$$\frac{-\frac{4\cot(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2\csc^2(c+dx)-32a^2-10ab\csc(c+dx)+15b^2\right)}{a^3} + \frac{-\frac{8a\left(24a^2-5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2b\left(45b^2-104a^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Gamma}{\sqrt{a+b\sin(c+}}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((-4*Cot[c + d*x]*(-32*a^2 + 15*b^2 - 10*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d
*x]^2)*Sqrt[a + b*Sin[c + d*x]])/a^3 + (((2*I)*(32*a^2 - 15*b^2)*Cos[2*(c +
 d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticP
i[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-(
(b*(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*
x]^2)) - (8*a*(24*a^2 - 5*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
```

```
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*b*(-1
04*a^2 + 45*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^3)/(96*d)
```

Maple [B] time = 1.951, size = 1496, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/24*(48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)^3-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-42*b^2*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^3*sin(d*x+c)^3-5*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+15*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^
3-32*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^5*sin(d*x+c)^3+47*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-15*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4
*sin(d*x+c)^3-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1
/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3+36*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^
2*b^3*sin(d*x+c)^3+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b
))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-15*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))
*b^5*sin(d*x+c)^3-32*a^3*b^2*sin(d*x+c)^5+15*a*b^4*sin(d*x+c)^5-32*a^4*b*si
n(d*x+c)^4+5*a^2*b^3*sin(d*x+c)^4+30*a^3*b^2*sin(d*x+c)^3-15*a*b^4*sin(d*x+
c)^3+40*a^4*b*sin(d*x+c)^2-5*a^2*b^3*sin(d*x+c)^2+2*a^3*b^2*sin(d*x+c)-8*a^
4*b)/a^4/sin(d*x+c)^3/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

3.1175 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=412

$$-\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d}+\frac{b\left(68a^2-35b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^3d\sqrt{a+b\sin(c+dx)}}-\frac{b\left(188a^2\right.}{}$$

```
[Out] -(b*(188*a^2 - 105*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(192*a^4*d)
+ (5*(12*a^2 - 7*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
96*a^3*d) + (7*b*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(24*
a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(4*a*d) - (
b*(188*a^2 - 105*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
 b*Sin[c + d*x]])/(192*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(68*a
^2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)])/(192*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 - 72*a^2
*b^2 + 35*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b
*Sin[c + d*x])/(a + b)])/(64*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.24434, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2893, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d}+\frac{b\left(68a^2-35b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^3d\sqrt{a+b\sin(c+dx)}}-\frac{b\left(188a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```



```
[Out] -(b*(188*a^2 - 105*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(192*a^4*d)
+ (5*(12*a^2 - 7*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
96*a^3*d) + (7*b*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(24*
a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(4*a*d) - (
b*(188*a^2 - 105*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
 b*Sin[c + d*x]])/(192*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(68*a
^2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)])/(192*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 - 72*a^2
*b^2 + 35*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b
*Sin[c + d*x])/(a + b)])/(64*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{7b\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a+b\sin(c+dx)}}{4ad}$$

$$= \frac{5\left(12a^2-7b^2\right)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{96a^3d} + \frac{7b\cot(c+dx)\csc^2(c+dx)}{24a^2}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{96a^3d}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{96a^3d}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{96a^3d}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{96a^3d}$$

$$= -\frac{b\left(188a^2-105b^2\right)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5\left(12a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{96a^3d}$$

Mathematica [C] time = 6.55887, size = 647, normalized size = 1.57

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{5\csc^2(c+dx)\left(12a^2\cos(c+dx)-7b^2\cos(c+dx)\right)}{96a^3} + \frac{\csc(c+dx)\left(105b^3\cos(c+dx)-188a^2b\cos(c+dx)\right)}{192a^4} + \frac{7b\cot(c+dx)\csc^2(c+dx)}{24a^2} - \right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((((-188*a^2*b*Cos[c + d*x] + 105*b^3*Cos[c + d*x])*Csc[c + d*x])/(192*a^4)
 + (5*(12*a^2*Cos[c + d*x] - 7*b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(96*a^3) +
```

```
 (7*b*Cot[c + d*x]*Csc[c + d*x]^2)/(24*a^2) - (Cot[c + d*x]*Csc[c + d*x]^3)
/(4*a))*Sqrt[a + b*Sin[c + d*x]])/d + ((-2*(-240*a^3*b + 140*a*b^3)*Ellipti
cF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
Sqrt[a + b*Sin[c + d*x]] - (2*(288*a^4 - 620*a^2*b^2 + 315*b^4)*EllipticPi[
2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(188*a^2*b^2 - 105*b^4)*Cos[c + d*x]*Cos[
2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)
/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b
)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b)
)])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a
+ b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a +
b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(768*a^4*d)
```

Maple [B] time = 1.953, size = 1761, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/192*(35*a^2*b^3*sin(d*x+c)^5+174*a^3*b^2*sin(d*x+c)^4-8*a^4*b*sin(d*x+c)
+76*a^4*b*sin(d*x+c)^3-188*a^3*b^2*sin(d*x+c)^6+105*a*b^4*sin(d*x+c)^6-68*a
^4*b*sin(d*x+c)^5+144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b
))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-105*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*
b^5*sin(d*x+c)^4-188*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+14*a^3*b^2*sin(d*x+c)^2-105*a*
b^4*sin(d*x+c)^4-35*a^2*b^3*sin(d*x+c)^3+120*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+48*a^5
+120*a^5*sin(d*x+c)^4-168*a^5*sin(d*x+c)^2-144*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
Pi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b*sin(d*
x+c)^4-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+216*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a
+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)
^4+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/
a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+293*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-3
5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^2*b^3*sin(d*x+c)^4+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-216*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1
/2))*a^3*b^2*sin(d*x+c)^4-258*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
```

```
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4+68*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*s
in(d*x+c)^4)/a^5/sin(d*x+c)^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^4 \csc(dx+c)}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(cot(d*x + c)^4*csc(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

3.1176 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=466

$$-\frac{2\left(a^2-b^2\right)\sin^4(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(40a^2-33b^2\right)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{33ab^3d}-\frac{20\left(32a^2-27b^2\right)\text{si}}{}$$

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(a*b^2*d*Sqrt[a + b*Sin[c + d*
x]]) - (8*(640*a^4 - 592*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c +
d*x]])/(1155*b^6*d) + (8*a*(480*a^2 - 419*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sq
rt[a + b*Sin[c + d*x]])/(1155*b^5*d) - (20*(32*a^2 - 27*b^2)*Cos[c + d*x]*S
in[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(231*b^4*d) + (2*(40*a^2 - 33*b^2)*
Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(33*a*b^3*d) - (2*Cos
[c + d*x]*Sin[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]])/(11*b^2*d) - (8*a*(1280*
a^4 - 1344*a^2*b^2 + 123*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[a + b*Sin[c + d*x]])/(1155*b^7*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) +
 (8*(1280*a^6 - 1664*a^4*b^2 + 369*a^2*b^4 + 15*b^6)*EllipticF[(c - Pi/2 +
d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1155*b^7*d*Sqrt
[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.21402, antiderivative size = 466, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2892, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sin^4(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(40a^2-33b^2\right)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{33ab^3d}-\frac{20\left(32a^2-27b^2\right)\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(a*b^2*d*Sqrt[a + b*Sin[c + d*
x]]) - (8*(640*a^4 - 592*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c +
d*x]])/(1155*b^6*d) + (8*a*(480*a^2 - 419*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sq
rt[a + b*Sin[c + d*x]])/(1155*b^5*d) - (20*(32*a^2 - 27*b^2)*Cos[c + d*x]*S
in[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(231*b^4*d) + (2*(40*a^2 - 33*b^2)*
Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(33*a*b^3*d) - (2*Cos
[c + d*x]*Sin[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]])/(11*b^2*d) - (8*a*(1280*
a^4 - 1344*a^2*b^2 + 123*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[a + b*Sin[c + d*x]])/(1155*b^7*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) +
 (8*(1280*a^6 - 1664*a^4*b^2 + 369*a^2*b^4 + 15*b^6)*EllipticF[(c - Pi/2 +
d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1155*b^7*d*Sqrt
[a + b*Sin[c + d*x]])
```


Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
```

```
f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+b\sin(c+dx)}}{11b^2d} +$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{2\left(40a^2-33b^2\right)\cos(c+dx)\sin^3(c+dx)\sqrt{a+b}}{33ab^3d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{20\left(32a^2-27b^2\right)\cos(c+dx)\sin^2(c+dx)\sqrt{a+}}{231b^4d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(480a^2-419b^2\right)\cos(c+dx)\sin(c+dx)\sqrt{a}}{1155b^5d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(640a^4-592a^2b^2+15b^4\right)\cos(c+dx)\sqrt{a+b}}{1155b^6d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(640a^4-592a^2b^2+15b^4\right)\cos(c+dx)\sqrt{a+b}}{1155b^6d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(640a^4-592a^2b^2+15b^4\right)\cos(c+dx)\sqrt{a+b}}{1155b^6d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(640a^4-592a^2b^2+15b^4\right)\cos(c+dx)\sqrt{a+b}}{1155b^6d}$$

Mathematica [A] time = 6.57906, size = 326, normalized size = 0.7

$$\frac{b\cos(c+dx)\left(8672a^2b^3\sin(c+dx)+800a^2b^3\sin(3(c+dx))-16\left(160a^3b^2-93ab^4\right)\cos(2(c+dx))+40448a^3b^2-1024\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (64*a*(1280*a^5 + 1280*a^4*b - 1344*a^3*b^2 - 1344*a^2*b^3 + 123*a*b^4 + 12
3*b^5)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)] - 64*(1280*a^6 - 1664*a^4*b^2 + 369*a^2*b^4 + 15*b^6)*Ellipt
icF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
] + b*Cos[c + d*x]*(-40960*a^5 + 40448*a^3*b^2 - 2728*a*b^4 - 16*(160*a^3*b
^2 - 93*a*b^4)*Cos[2*(c + d*x)] + 280*a*b^4*Cos[4*(c + d*x)] - 10240*a^4*b*
Sin[c + d*x] + 8672*a^2*b^3*Sin[c + d*x] + 330*b^5*Sin[c + d*x] + 800*a^2*b
^3*Sin[3*(c + d*x)] - 255*b^5*Sin[3*(c + d*x)] - 105*b^5*Sin[5*(c + d*x)]))
/(9240*b^7*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 1.581, size = 1356, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/1155*(492*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a*b^6+60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-5120*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7+140*a*b^6*sin(d*x+c
)^6-200*a^2*b^5*sin(d*x+c)^5+320*a^3*b^4*sin(d*x+c)^4-466*a*b^6*sin(d*x+c)^
4-640*a^4*b^3*sin(d*x+c)^3+892*a^2*b^5*sin(d*x+c)^3-2560*a^5*b^2*sin(d*x+c)
^2+2048*a^3*b^4*sin(d*x+c)^2+266*a*b^6*sin(d*x+c)^2+640*a^4*b^3*sin(d*x+c)-
692*a^2*b^5*sin(d*x+c)-2368*a^3*b^4+60*a*b^6+2560*a^5*b^2-5868*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b
^4+5120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^6*b-3840*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-6656*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+4392*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
3*b^4+1476*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^2*b^5-552*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6+10496*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-105*b^7*sin
(d*x+c)^7+300*b^7*sin(d*x+c)^5-255*b^7*sin(d*x+c)^3+60*b^7*sin(d*x+c))/b^8/
cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(\cos(dx+c)^6-\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a}\,\sin(dx+c)}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a)*sin(d*x
 + c)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(3/2), x)
```

3.1177 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=401

$$-\frac{2\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(80a^2-63b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{63ab^3d}-\frac{16\left(60a^2-49b^2\right.}{}$$

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(a*b^2*d*Sqrt[a + b*Sin[c + d*
x]]) + (8*a*(160*a^2 - 139*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315
*b^5*d) - (16*(60*a^2 - 49*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c
+ d*x]])/(315*b^4*d) + (2*(80*a^2 - 63*b^2)*Cos[c + d*x]*Sin[c + d*x]^2*Sqr
t[a + b*Sin[c + d*x]])/(63*a*b^3*d) - (2*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a
 + b*Sin[c + d*x]])/(9*b^2*d) + (8*(320*a^4 - 318*a^2*b^2 + 21*b^4)*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^6*d*
Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (16*a*(160*a^4 - 199*a^2*b^2 + 39*b^4
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(315*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.903799, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2892, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(80a^2-63b^2\right)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{63ab^3d}-\frac{16\left(60a^2-49b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(a*b^2*d*Sqrt[a + b*Sin[c + d*
x]]) + (8*a*(160*a^2 - 139*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315
*b^5*d) - (16*(60*a^2 - 49*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c
+ d*x]])/(315*b^4*d) + (2*(80*a^2 - 63*b^2)*Cos[c + d*x]*Sin[c + d*x]^2*Sqr
t[a + b*Sin[c + d*x]])/(63*a*b^3*d) - (2*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a
 + b*Sin[c + d*x]])/(9*b^2*d) + (8*(320*a^4 - 318*a^2*b^2 + 21*b^4)*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^6*d*
Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (16*a*(160*a^4 - 199*a^2*b^2 + 39*b^4
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(315*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)\sqrt{a+b\sin(c+dx}}{9b^2d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{2\left(80a^2-63b^2\right)\cos(c+dx)\sin^2(c+dx)\sqrt{a}}{63ab^3d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{16\left(60a^2-49b^2\right)\cos(c+dx)\sin(c+dx)\sqrt{a}}{315b^4d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(160a^2-139b^2\right)\cos(c+dx)\sqrt{a+b\sin(c}}{315b^5d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(160a^2-139b^2\right)\cos(c+dx)\sqrt{a+b\sin(c}}{315b^5d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(160a^2-139b^2\right)\cos(c+dx)\sqrt{a+b\sin(c}}{315b^5d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a\left(160a^2-139b^2\right)\cos(c+dx)\sqrt{a+b\sin(c}}{315b^5d}$$

Mathematica [A] time = 5.06707, size = 275, normalized size = 0.69

$$\frac{-b\cos(c+dx)\left(-8\left(40a^2b^2-21b^4\right)\cos(2(c+dx))+4768a^2b^2-1280a^3b\sin(c+dx)-5120a^4+1012ab^3\sin(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-32*(320*a^5 + 320*a^4*b - 318*a^3*b^2 - 318*a^2*b^3 + 21*a*b^4 + 21*b^5)*
EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)] + 64*a*(160*a^4 - 199*a^2*b^2 + 39*b^4)*EllipticF[(-2*c + Pi - 2*d*
x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*Cos[c + d*x]*(-
5120*a^4 + 4768*a^2*b^2 - 203*b^4 - 8*(40*a^2*b^2 - 21*b^4)*Cos[2*(c + d*x)
] + 35*b^4*Cos[4*(c + d*x)] - 1280*a^3*b*Sin[c + d*x] + 1012*a*b^3*Sin[c +
d*x] + 100*a*b^3*Sin[3*(c + d*x)]))/(1260*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 1.694, size = 1190, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/315*(35*b^6*sin(d*x+c)^6-50*a*b^5*sin(d*x+c)^5+1280*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-960*((a+
```

```
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^4*b^2-1592*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^3*b^3+1044*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+312*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-84*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*b^6-1280*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^6+2552*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-1356*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+84*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^
6+80*a^2*b^4*sin(d*x+c)^4-112*b^6*sin(d*x+c)^4-160*a^3*b^3*sin(d*x+c)^3+214
*a*b^5*sin(d*x+c)^3-640*a^4*b^2*sin(d*x+c)^2+476*a^2*b^4*sin(d*x+c)^2+77*b^
6*sin(d*x+c)^2+160*a^3*b^3*sin(d*x+c)-164*a*b^5*sin(d*x+c)+640*a^4*b^2-556*
a^2*b^4)/b^7/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos(dx+c)^6 - \cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a}}{b^2\cos(dx+c)^2 - 2\,ab\sin(dx+c) - a^2 - b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a)/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

3.1178 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-24ab\sin(c+dx)-5b^2\right)}{35b^4d}+\frac{8\left(-37a^2b^2+32a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx\right.\right.}{35b^5d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] (-8*a*(32*a^2 - 29*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a
 + b*Sin[c + d*x]])/(35*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(32*
a^4 - 37*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt
[(a + b*Sin[c + d*x])/(a + b)])/(35*b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Co
s[c + d*x]^3*(8*a + b*Sin[c + d*x]))/(7*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (
4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 5*b^2 - 24*a*b*Sin[c + d*
x]))/(35*b^4*d)
```

Rubi [A] time = 0.424217, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2863, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-24ab\sin(c+dx)-5b^2\right)}{35b^4d}+\frac{8\left(-37a^2b^2+32a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx\right.\right.}{35b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-8*a*(32*a^2 - 29*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a
 + b*Sin[c + d*x]])/(35*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(32*
a^4 - 37*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt
[(a + b*Sin[c + d*x])/(a + b)])/(35*b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Co
s[c + d*x]^3*(8*a + b*Sin[c + d*x]))/(7*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (
4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 5*b^2 - 24*a*b*Sin[c + d*
x]))/(35*b^4*d)
```


Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
```

```
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{12\int \frac{\cos^2(c+dx)\left(-\frac{b}{2}-4a\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}\,dx}{7b^2}$$

$$= \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-5b^2-2\right.}{35b^4d}$$

$$= \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-5b^2-2\right.}{35b^4d}$$

$$= \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-5b^2-2\right.}{35b^4d}$$

$$= -\frac{8a\left(32a^2-29b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{35b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{8\left(32a^4-37a^2b^2+5\right.}{35}$$

Mathematica [A] time = 3.91684, size = 222, normalized size = 0.85

$$\frac{b\cos(c+dx)\left(\left(45b^3-64a^2b\right)\sin(c+dx)-256a^3-16ab^2\cos(2(c+dx))+216ab^2+5b^3\sin(3(c+dx))\right)-16\left(-37a^2b^2+\right.}{70b^5d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a*(32*a^3 + 32*a^2*b - 29*a*b^2 - 29*b^3)*EllipticE[(-2*c + Pi - 2*d*x)
/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 16*(32*a^4 - 37*a^2
*b^2 + 5*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)] + b*Cos[c + d*x]*(-256*a^3 + 216*a*b^2 - 16*a*b^2*Cos
[2*(c + d*x)] + (-64*a^2*b + 45*b^3)*Sin[c + d*x] + 5*b^3*Sin[3*(c + d*x)])
)/(70*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 1.546, size = 943, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/35*(-5*b^5*sin(d*x+c)^5+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-96*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-148*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
2*b^3+96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a*b^4+20*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*b^5-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+244*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-116*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a*b^4+8*a*b^4*sin(d*x+c)^4-16*a^2*b^3*sin(d*x+c)^3+20*b^5*sin(d*x+c)^
3-64*a^3*b^2*sin(d*x+c)^2+42*a*b^4*sin(d*x+c)^2+16*a^2*b^3*sin(d*x+c)-15*b^
5*sin(d*x+c)+64*a^3*b^2-50*a*b^4)/b^6/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)/(b^2*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)
```

3.1179 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{3/2}} dx$

Optimal. Leaf size=296

$$-\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2-5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(8a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c\right.\right.}{3ab^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x])/(a*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (2*Cos[c
 + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*d) - (2*(8*a^2 - 3*b^2)*EllipticE[
(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a*b^3*d*Sqr
t[(a + b*Sin[c + d*x])/(a + b)]) + (2*(8*a^2 - 5*b^2)*EllipticF[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^3*d*Sqrt[a
 + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.678251, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2892, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2-5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(8a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c\right.\right.}{3ab^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x])/(a*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (2*Cos[c
 + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*d) - (2*(8*a^2 - 3*b^2)*EllipticE[
(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a*b^3*d*Sqr
t[(a + b*Sin[c + d*x])/(a + b)]) + (2*(8*a^2 - 5*b^2)*EllipticF[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^3*d*Sqrt[a
 + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(a*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
```

```
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} + \frac{4\int \frac{\csc(c+dx)\left(\frac{3b^2}{4}-\frac{1}{2}ab\sin(c+\ldots\right.}{\sqrt{a+b\ldots}}}{3\ldots}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} - \frac{4\int \frac{\csc(c+dx)\left(-\frac{3b^3}{4}-\frac{1}{4}a\left(8a^2-\ldots\right.\right.}{\sqrt{a+b\sin(c+\ldots}}}{3ab^3}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} + \frac{\int \frac{\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx}{a} + \frac{(8a\ldots}{\ldots}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} - \frac{2\left(8a^2-3b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\ldots\right.\right.}{3ab^3d}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} - \frac{2\left(8a^2-3b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\ldots\right.\right.}{3ab^3d}$$

Mathematica [C] time = 3.67926, size = 419, normalized size = 1.42

$$\frac{-\frac{4\cos(c+dx)\left(4a^2+ab\sin(c+dx)-3b^2\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{2\left(8a^2-9b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2i\left(3b^2-8a^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}\left(b\left(b\ldots\right.\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)*(-8*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[-((b*(1 + Sin[c + d*x]))/(a - b))]])/(a*b^2*Sqrt[-(a + b)^(-1)]) + (8*a*
b*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(8*a^2 - 9*b^2)*EllipticPi[2, (-2*
c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[
a + b*Sin[c + d*x]] - (4*Cos[c + d*x]*(4*a^2 - 3*b^2 + a*b*Sin[c + d*x]))/S
qrt[a + b*Sin[c + d*x]])/(6*a*b^2*d)
```

Maple [B] time = 1.5, size = 1010, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/3*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
```

```
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+3*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-8*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^5+11*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^3*b^2-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*Elliptic
Pi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a-3*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2))-a^2*b^3*sin(d*x+c)^3-4*a^3*b^2*sin(d*x+c)^2+3*a*b^4*sin(d*x+c)^
2+a^2*b^3*sin(d*x+c)+4*a^3*b^2-3*a*b^4)/a^2/b^4/cos(d*x+c)/(a+b*sin(d*x+c))
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^3*cot(d*x + c)/(b^2*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3 \cot(dx+c)}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)
```

3.1180 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=294

$$\frac{\left(2a^2-3b^2\right)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+\right.\right.}{a^2b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] ((2*a^2 - 3*b^2)*Cos[c + d*x])/(a^2*b*d*Sqrt[a + b*Sin[c + d*x]]) - Cot[c +
 d*x]/(a*d*Sqrt[a + b*Sin[c + d*x]]) + ((4*a^2 - 3*b^2)*EllipticE[(c - Pi/2
 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(a^2*b^2*d*Sqrt[(a + b*
Sin[c + d*x])/(a + b)]) - ((4*a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*b^2*d*Sqrt[a + b*Sin[c +
d*x]]) - (3*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*
Sin[c + d*x])/(a + b)])/(a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.705431, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {2890, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(2a^2-3b^2\right)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+\right.\right.}{a^2b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((2*a^2 - 3*b^2)*Cos[c + d*x])/(a^2*b*d*Sqrt[a + b*Sin[c + d*x]]) - Cot[c +
 d*x]/(a*d*Sqrt[a + b*Sin[c + d*x]]) + ((4*a^2 - 3*b^2)*EllipticE[(c - Pi/2
 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(a^2*b^2*d*Sqrt[(a + b*
Sin[c + d*x])/(a + b)]) - ((4*a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*b^2*d*Sqrt[a + b*Sin[c +
d*x]]) - (3*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*
Sin[c + d*x])/(a + b)])/(a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
```

```
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} + \frac{2\int \frac{\csc(c+dx)\left(-\frac{3b^2}{4}-\frac{1}{2}ab\sin(c+dx)+\frac{1}{4}(4\cdots\right.}{\sqrt{a+b\sin(c+dx)}}}{a^2b}$$

$$= \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} + \frac{1}{2}\left(-\frac{3}{a^2}+\frac{4}{b^2}\right)\int \sqrt{a+b\sin(c+\cdots}$$

$$= \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} - \frac{(3b)\int \frac{\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx}{2a^2} - \frac{(4a^2-\cdots}{}$$

$$= \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(\frac{3}{a^2}-\frac{4}{b^2}\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\cdots}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

$$= \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(\frac{3}{a^2}-\frac{4}{b^2}\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\cdots}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.42109, size = 433, normalized size = 1.47

$$\frac{\frac{4a(a^2-b^2)\cos(c+dx)}{b\sqrt{a+b\sin(c+dx)}} - \frac{a(4a^2-9b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{b\sqrt{a+b\sin(c+dx)}} + \frac{i(3b^2-4a^2)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((I*(-4*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[S
qrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elliptic
Pi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
 b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Sin[c + d*x]))/(-a + b)])/(b^3*Sqrt[-(a + b)^(-1)]) + (4*a*(a^2 - b^
2)*Cos[c + d*x])/(b*Sqrt[a + b*Sin[c + d*x]]) - 2*a*Cot[c + d*x]*Sqrt[a + b
*Sin[c + d*x]] + (4*a^2*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (a*(4*a^2 - 9*b
^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
 d*x])/(a + b)])/(b*Sqrt[a + b*Sin[c + d*x]]))/(2*a^3*d)
```

Maple [A] time = 1.711, size = 620, normalized size = 2.1

$$-\frac{1}{a^3b^3\sin(dx+c)\cos(dx+c)d}\left(\left(-2a^3b^2+3ab^4\right)\sin(dx+c)(\cos(dx+c))^2-\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\sqrt{-\frac{b\sin(\cdots}{a+\cdots}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -((-2*a^3*b^2+3*a*b^4)*sin(d*x+c)*cos(d*x+c)^2-(-b/(a-b)*sin(d*x+c)-b/(a-b)
)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
```

```
(1/2)*(4*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2)
)*a^4*b-6*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2
))*a^3*b^2-EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/
2))*a^2*b^3+3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^
(1/2))*a*b^4+3*EllipticPi((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(a-b)/a,((a-
b)/(a+b))^(1/2))*a*b^4-3*EllipticPi((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(a
-b)/a,((a-b)/(a+b))^(1/2))*b^5-4*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2),((a-b)/(a+b))^(1/2))*a^5+7*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2),((a-b)/(a+b))^(1/2))*a^3*b^2-3*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)*sin(d*x+c)+a^2*b^3*cos(d*x+c)^2)/b^3/sin
(d*x+c)/a^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2\cot(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

3.1181 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=366

$$-\frac{(8a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^3bd}+\frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2}{a\cdot}\right.}{4a^2bd\sqrt{a+b\sin(c+dx)}}$$

```
[Out] ((4*a^2 - 5*b^2)*Cot[c + d*x])/(2*a^2*b*d*Sqrt[a + b*Sin[c + d*x]]) - (Cot[
c + d*x]*Csc[c + d*x])/(2*a*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 15*b^2)
*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(4*a^3*b*d) - ((8*a^2 - 15*b^2)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(4*a^3*
b*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((8*a^2 - 5*b^2)*EllipticF[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^2*b*d
*Sqrt[a + b*Sin[c + d*x]]) - (3*(4*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^3*d*Sqrt[a +
 b*Sin[c + d*x]])
```

Rubi [A] time = 0.92834, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2890, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(8a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^3bd}+\frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2}{a\cdot}\right.}{4a^2bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((4*a^2 - 5*b^2)*Cot[c + d*x])/(2*a^2*b*d*Sqrt[a + b*Sin[c + d*x]]) - (Cot[
c + d*x]*Csc[c + d*x])/(2*a*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 15*b^2)
*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(4*a^3*b*d) - ((8*a^2 - 15*b^2)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(4*a^3*
b*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((8*a^2 - 5*b^2)*EllipticF[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^2*b*d
*Sqrt[a + b*Sin[c + d*x]]) - (3*(4*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^3*d*Sqrt[a +
 b*Sin[c + d*x]])
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\left(4a^2-5b^2\right)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{1}{4}\left(8a^2-15b^2\right)-\frac{1}{2}ab\sin(c+dx\right.}{\sqrt{a+b\sin(c+dx)}}}{a^2b}$$

$$= \frac{\left(4a^2-5b^2\right)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(8a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\,\mathrm{s}}}{4a^3bd}$$

$$= \frac{\left(4a^2-5b^2\right)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(8a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\,\mathrm{s}}}{4a^3bd}$$

$$= \frac{\left(4a^2-5b^2\right)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(8a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\,\mathrm{s}}}{4a^3bd}$$

$$= \frac{\left(4a^2-5b^2\right)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(8a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\,\mathrm{s}}}{4a^3bd}$$

$$= \frac{\left(4a^2-5b^2\right)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(8a^2-15b^2\right)\cot(c+dx)\sqrt{a+b\,\mathrm{s}}}{4a^3bd}$$

Mathematica [C] time = 4.81423, size = 435, normalized size = 1.19

$$\frac{\frac{\left(60b^2-32a^2\right)\cos(c+dx)+4a\cot(c+dx)(5b-2a\csc(c+dx))}{a^3\sqrt{a+b\sin(c+dx)}} + \frac{\frac{2\left(32a^2-45b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - 2i\left(15b^2-8a^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\sin(c+d}{a-b}}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-32*a^2 + 60*b^2)*Cos[c + d*x] + 4*a*Cot[c + d*x]*(5*b - 2*a*Csc[c + d*x
]))/(a^3*Sqrt[a + b*Sin[c + d*x]]) + (((-2*I)*(-8*a^2 + 15*b^2)*(-2*a*(a -
b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt
```

```
[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))
])/(a*b^2*Sqrt[-(a + b)^(-1)]) - (40*a*b*EllipticF[(-2*c + Pi - 2*d*x)/4, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]]
+ (2*(32*a^2 - 45*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^3)/(16*d)
```

Maple [B] time = 1.832, size = 1349, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-18*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2-5*b^3*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^2*sin(d*x+c)^2+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^2+
23*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^3*b^2*sin(d*x+c)^2-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-12*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*b^2*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))
^(1/2))*a^3*sin(d*x+c)^2+12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(d*
x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2+15*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b)
)^(1/2))*a*b^4*sin(d*x+c)^2-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x
+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^2-8*a^3*b^2*s
in(d*x+c)^4+15*a*b^4*sin(d*x+c)^4+5*a^2*b^3*sin(d*x+c)^3+6*a^3*b^2*sin(d*x+
c)^2-15*a*b^4*sin(d*x+c)^2-5*a^2*b^3*sin(d*x+c)+2*a^3*b^2)/b^2/sin(d*x+c)^2
/a^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)**3/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)\cot(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)*cot(d*x + c)^3/(b*sin(d*x + c) + a)^(3/2), x)
```

3.1182 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=416

$$\frac{5(16a^2-21b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^4d}-\frac{(32a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24a^3d\sqrt{a+b\sin(c+dx)}}+\frac{5(16a^2-21b^2}{}$$

```
[Out] ((6*a^2 - 7*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*Sqrt[a + b*Sin[c + d
*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*Sqrt[a + b*Sin[c + d*x]]) + (5
*(16*a^2 - 21*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^4*d) - ((24
*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^3*
b*d) + (5*(16*a^2 - 21*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sq
rt[a + b*Sin[c + d*x]])/(24*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((3
2*a^2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Si
n[c + d*x])/(a + b)])/(24*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(36*a^2 - 35
*b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.15481, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2724, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{5(16a^2-21b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^4d}-\frac{(32a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24a^3d\sqrt{a+b\sin(c+dx)}}+\frac{5(16a^2-21b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] ((6*a^2 - 7*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*Sqrt[a + b*Sin[c + d
*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*Sqrt[a + b*Sin[c + d*x]]) + (5
*(16*a^2 - 21*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^4*d) - ((24
*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^3*
b*d) + (5*(16*a^2 - 21*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sq
rt[a + b*Sin[c + d*x]])/(24*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((3
2*a^2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Si
n[c + d*x])/(a + b)])/(24*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(36*a^2 - 35
*b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{2\int \frac{\csc^3(c+dx)\left(\frac{1}{4}\left(24a^2-35\right.\right.}{}}{}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(24a^2-35b^2\right)\cot(c+}{}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)\cot(c}{2}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)\cot(c}{2}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)\cot(c}{2}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)\cot(c}{2}$$

$$= \frac{\left(6a^2-7b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5\left(16a^2-21b^2\right)\cot(c}{2}$$

Mathematica [C] time = 5.56461, size = 468, normalized size = 1.12

$$-\frac{4\left(\left(105b^3-80a^2b\right)\cos(c+dx)+a\cot(c+dx)\left(8a^2\csc^2(c+dx)-32a^2-14ab\csc(c+dx)+35b^2\right)\right)}{a^4\sqrt{a+b\sin(c+dx)}} + \frac{-\frac{8a\left(24a^2-35b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}}+\frac{2b\left(315b^2-2\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((-4*((-80*a^2*b + 105*b^3)*Cos[c + d*x] + a*Cot[c + d*x]*(-32*a^2 + 35*b^2
 - 14*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d*x]^2)))/(a^4*Sqrt[a + b*Sin[c + d*
x]]) + (((10*I)*(-16*a^2 + 21*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-
```

```
(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ellipti
cF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)
] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c
+ d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a
 + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)])
- (8*a*(24*a^2 - 35*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sq
rt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*b*(-296*a^2
 + 315*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b
*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^4)/(96*d)
```

Maple [B] time = 1.809, size = 1496, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/24*(80*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^5*sin(d*x+c)^3-185*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3+105*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a*b^4*sin(d*x+c)^3-48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*sin(d*x+c)^3-32*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^
3+150*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^3*sin(d*x+c)^3+35*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3-105*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a*b^4*sin(d*x+c)^3+108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a
-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-108*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(
1/2))*a^2*b^3*sin(d*x+c)^3-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x
+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3+105*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+
b))^(1/2))*b^5*sin(d*x+c)^3+80*a^3*b^2*sin(d*x+c)^5-105*a*b^4*sin(d*x+c)^5+
32*a^4*b*sin(d*x+c)^4-35*a^2*b^3*sin(d*x+c)^4-66*a^3*b^2*sin(d*x+c)^3+105*a
*b^4*sin(d*x+c)^3-40*a^4*b*sin(d*x+c)^2+35*a^2*b^3*sin(d*x+c)^2-14*a^3*b^2*
sin(d*x+c)+8*a^4*b)/b/a^5/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\cot\left(dx+c\right)^{4}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^{4}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(dx+c\right)^{4}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

3.1183 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=469

$$\frac{2(13a^2-5b^2)\sin^4(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{10(8a^2-3b^2)\sin^3(c+dx)\cos(c+dx)\sqrt{}}{9a^2b^3d}$$

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(3*a*b^2*d*(a + b*Sin[c + d*x]
)^(3/2)) + (2*(13*a^2 - 5*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(3*a^2*b^2*d*Sq
rt[a + b*Sin[c + d*x]]) + (128*a*(40*a^2 - 19*b^2)*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]])/(315*b^6*d) - (8*(480*a^2 - 203*b^2)*Cos[c + d*x]*Sin[c + d*
x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^5*d) + (4*(160*a^2 - 63*b^2)*Cos[c + d*
x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(63*a*b^4*d) - (10*(8*a^2 - 3*b
^2)*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(9*a^2*b^3*d) + (
8*(1280*a^4 - 768*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a
+ b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^7*d*Sqrt[(a + b*Sin[c + d*x])/(a + b
)]) - (8*a*(1280*a^4 - 1088*a^2*b^2 + 123*b^4)*EllipticF[(c - Pi/2 + d*x)/2
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b^7*d*Sqrt[a + b*
Sin[c + d*x]])
```

Rubi [A] time = 1.18088, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2891, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2(13a^2-5b^2)\sin^4(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{10(8a^2-3b^2)\sin^3(c+dx)\cos(c+dx)\sqrt{}}{9a^2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(3*a*b^2*d*(a + b*Sin[c + d*x]
)^(3/2)) + (2*(13*a^2 - 5*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(3*a^2*b^2*d*Sq
rt[a + b*Sin[c + d*x]]) + (128*a*(40*a^2 - 19*b^2)*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]])/(315*b^6*d) - (8*(480*a^2 - 203*b^2)*Cos[c + d*x]*Sin[c + d*
x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^5*d) + (4*(160*a^2 - 63*b^2)*Cos[c + d*
x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(63*a*b^4*d) - (10*(8*a^2 - 3*b
^2)*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(9*a^2*b^3*d) + (
8*(1280*a^4 - 768*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a
+ b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^7*d*Sqrt[(a + b*Sin[c + d*x])/(a + b
)]) - (8*a*(1280*a^4 - 1088*a^2*b^2 + 123*b^4)*EllipticF[(c - Pi/2 + d*x)/2
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b^7*d*Sqrt[a + b*
Sin[c + d*x]])
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
```

```
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{4\int}{}$$

$$=-\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{10(}{}$$

$$=-\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4(1}{}$$

$$=-\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{8(4}{}$$

$$=-\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}+\frac{128}{}$$

$$=-\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}+\frac{128}{}$$

$$=-\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}+\frac{128}{}$$

$$=-\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2\left(13a^2-5b^2\right)\cos(c+dx)\sin^4(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}+\frac{128}{}$$

Mathematica [B] time = 9.29774, size = 1044, normalized size = 2.23

$$\frac{315\left(\frac{\left(\left(a^2+3b^2\right)E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+a(b-a)F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}{(a-b)^2b}-\frac{\cos(c+dx)\left(2a\left(a^2+b^2\right)+b\left(a^2+3b^2\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^2}\right)+\frac{315\left(\left(\left(32a^4-57b^2\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (315*((((a^2 + 3*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + a*(
-a + b)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*
x])/(a + b))^(3/2))/((a - b)^2*b) - (Cos[c + d*x]*(2*a*(a^2 + b^2) + b*(a^2
 + 3*b^2)*Sin[c + d*x]))/(a^2 - b^2)^2) + (315*((((32*a^4 - 57*a^2*b^2 + 21
*b^4)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + a*(-32*a^3 + 32*a^2
*b + 33*a*b^2 - 33*b^3)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((
a + b*Sin[c + d*x])/(a + b))^(3/2))/(a - b)^2 - (b*(4*a*(8*a^4 - 13*a^2*b^2
 + 3*b^4)*Cos[c + d*x] + b*(20*a^4 - 33*a^2*b^2 + 9*b^4)*Sin[2*(c + d*x)]))
/(2*(a^2 - b^2)^2)))/b^3 - (21*((((-2048*a^6 + 4192*a^4*b^2 - 2355*a^2*b^4
+ 231*b^6)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + a*(2048*a^5 -
2048*a^4*b - 2656*a^3*b^2 + 2656*a^2*b^3 + 603*a*b^4 - 603*b^5)*EllipticF[(
-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*x])/(a + b))^(3/2))
```

```
/(a - b)^2 + (b*Cos[c + d*x]*(-64*a*b^2*(a^2 - b^2)^2*Cos[2*(c + d*x)] + b*
(1280*a^6 - 2536*a^4*b^2 + 1347*a^2*b^4 - 111*b^6)*Sin[c + d*x] + 2*(512*a^
7 - 952*a^5*b^2 + 423*a^3*b^4 + 7*a*b^6 + 6*b^3*(a^2 - b^2)^2*Sin[3*(c + d*
x)])))/(a^2 - b^2)^2))/b^5 - (5*(a + b*Sin[c + d*x])*(((-4*b*(-4096*a^7*b +
 8960*a^5*b^3 - 5884*a^3*b^5 + 1041*a*b^7)*EllipticF[(-2*c + Pi - 2*d*x)/4,
 (2*b)/(a + b)] + (65536*a^8 - 161792*a^6*b^2 + 129664*a^4*b^4 - 35109*a^2*
b^6 + 1617*b^8)*((a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] -
a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]))*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/((a - b)^2*(a + b)^2) + b*(a + b*Sin[c + d*x])*(-128*a*(88*a^2
 - 27*b^2)*Cos[c + d*x] + 416*a*b^2*Cos[3*(c + d*x)] + (21*a*(64*a^6 - 112*
a^4*b^2 + 56*a^2*b^4 - 7*b^6)*Cos[c + d*x])/((a^2 - b^2)*(a + b*Sin[c + d*x
])^2) - (21*(1088*a^8 - 2576*a^6*b^2 + 1960*a^4*b^4 - 497*a^2*b^6 + 21*b^8)
*Cos[c + d*x])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])) - 8*b*(-276*a^2 + 35*b^
2)*Sin[2*(c + d*x)] - 56*b^3*Sin[4*(c + d*x)])))/b^7)/(10080*d*(a + b*Sin[c
 + d*x])^(3/2))
```

Maple [B] time = 1.77, size = 2033, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315*(-35*b^7*sin(d*x+c)*cos(d*x+c)^6+(120*a^2*b^5-7*b^7)*cos(d*x+c)^4*sin
(d*x+c)+(3200*a^4*b^3-1740*a^2*b^5+42*b^7)*cos(d*x+c)^2*sin(d*x+c)-4*(-b/(a
-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2)*b*(1280*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2),((a-b)/(a+b))^(1/2))*a^6-2048*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+789*EllipticE((b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-21*EllipticE((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^6-1280*EllipticF((b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b+960*EllipticF((b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+1088*EllipticF((b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3-666*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-123*Ellipti
cF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^5+21*Ellip
ticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^6)*sin(d*x
+c)+60*a*b^6*cos(d*x+c)^6+(-320*a^3*b^4+102*a*b^6)*cos(d*x+c)^4+(2560*a^5*b
^2-896*a^3*b^4-162*a*b^6)*cos(d*x+c)^2+5120*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/
2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^6*
b-3840*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(
1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a
-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-4352*(b/(a-b)*sin(d*x+c)+1/(a-b)*
a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^
(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a
^4*b^3+2664*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+
b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)
+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+492*(b/(a-b)*sin(d*x+c)+1/(a
-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2
))*a^2*b^5-84*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(
a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^6-5120*(b/(a-b)*sin(d*x+c)+1/(
a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a
-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/
```

```
2))*a^7+8192*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-3156*(b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(
a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1
/2))*a^3*b^4+84*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b
/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^6)/(a+b*sin(d*x+c))^(3/2)/b^
8/cos(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\cos(dx+c)^6-\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a}\,\sin(dx+c)}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a)*sin(d*x
 + c)/(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2
*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(5/2), x)
```

3.1184 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=411

$$\frac{2\left(11a^2-3b^2\right)\sin^3(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{2\left(80a^2-21b^2\right)\sin^2(c+dx)\cos(c+dx)\cdots}{21a^2b^3d}$$

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(3*a*b^2*d*(a + b*Sin[c + d*x]
)^(3/2)) + (2*(11*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(3*a^2*b^2*d*Sq
rt[a + b*Sin[c + d*x]]) - (8*(32*a^2 - 11*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[
c + d*x]])/(21*b^5*d) + (8*(24*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[
a + b*Sin[c + d*x]])/(21*a*b^4*d) - (2*(80*a^2 - 21*b^2)*Cos[c + d*x]*Sin[c
 + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(21*a^2*b^3*d) - (16*a*(32*a^2 - 15*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(2
1*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(64*a^4 - 46*a^2*b^2 + 3*b
^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/(21*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.91817, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2891, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(11a^2-3b^2\right)\sin^3(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{2\left(80a^2-21b^2\right)\sin^2(c+dx)\cos(c+dx)\cdots}{21a^2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(3*a*b^2*d*(a + b*Sin[c + d*x]
)^(3/2)) + (2*(11*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(3*a^2*b^2*d*Sq
rt[a + b*Sin[c + d*x]]) - (8*(32*a^2 - 11*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[
c + d*x]])/(21*b^5*d) + (8*(24*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[
a + b*Sin[c + d*x]])/(21*a*b^4*d) - (2*(80*a^2 - 21*b^2)*Cos[c + d*x]*Sin[c
 + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(21*a^2*b^3*d) - (16*a*(32*a^2 - 15*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(2
1*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(64*a^4 - 46*a^2*b^2 + 3*b
^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/(21*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\int}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\left(8\right.}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{8\left(2\right.}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(3\right.}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(3\right.}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(3\right.}{}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(11a^2-3b^2\right)\cos(c+dx)\sin^3(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} - \frac{8\left(3\right.}{}$$

Mathematica [A] time = 7.60269, size = 257, normalized size = 0.63

$$\frac{-\frac{1}{2}b\cos(c+dx)\left(-8\left(8a^2b^2-3b^4\right)\cos(2(c+dx))-288a^2b^2+1280a^3b\sin(c+dx)+1024a^4-516ab^3\sin(c+dx)+12ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (32*a*(a + b)^2*(32*a^2 - 15*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) - 16*(a + b)*(64*a^4 - 46*a^2*b
^2 + 3*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c +
 d*x])/(a + b))^(3/2) - (b*Cos[c + d*x]*(1024*a^4 - 288*a^2*b^2 - 27*b^4 -
8*(8*a^2*b^2 - 3*b^4)*Cos[2*(c + d*x)] + 3*b^4*Cos[4*(c + d*x)] + 1280*a^3*
b*Sin[c + d*x] - 516*a*b^3*Sin[c + d*x] + 12*a*b^3*Sin[3*(c + d*x)]))/2)/(4
2*b^6*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] time = 1.618, size = 1642, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/21*(6*a*b^5*sin(d*x+c)*cos(d*x+c)^4+(160*a^3*b^3-66*a*b^5)*cos(d*x+c)^2*
sin(d*x+c)+4*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b*(64*EllipticF((b/(a-b)*sin
```

```
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-48*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-46*EllipticF((b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+27*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+3*EllipticF((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^5-64*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5+94*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-30*Ellipti
cE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)*sin(d*x
+c)+3*b^6*cos(d*x+c)^6+(-16*a^2*b^4+3*b^6)*cos(d*x+c)^4+(128*a^4*b^2-28*a^2
*b^4-6*b^6)*cos(d*x+c)^2+256*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(
(a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^5*b-192*(b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+
c)-b/(a-b))^(1/2)*a^4*b^2-184*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b
)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),
((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^3*b^3+108*(b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*Elliptic
F((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d
*x+c)-b/(a-b))^(1/2)*a^2*b^4+12*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2
),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a*b^5-256*(b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b
)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),
((a-b)/(a+b))^(1/2))*a^6+376*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-120*(b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)
*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(
(a-b)/(a+b))^(1/2))*a^2*b^4)/(a+b*sin(d*x+c))^(3/2)/b^7/cos(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos(dx+c)^6-\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a}}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((cos(d*x + c)^6 - cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a)/(3*a*b^
2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin
(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

3.1185 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{4\cos(c+dx)\left(32a^2+8ab\sin(c+dx)-9b^2\right)}{15b^4d\sqrt{a+b\sin(c+dx)}}-\frac{8a\left(32a^2-17b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^5d\sqrt{a+b\sin(c+dx)}}+\frac{8\left(32a^2-9b^2\right.}{}$$

```
[Out] (8*(32*a^2 - 9*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b
*Sin[c + d*x]])/(15*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*a*(32*a^
2 - 17*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(15*b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]^3*(
8*a + 3*b*Sin[c + d*x]))/(15*b^2*d*(a + b*Sin[c + d*x])^(3/2)) + (4*Cos[c +
 d*x]*(32*a^2 - 9*b^2 + 8*a*b*Sin[c + d*x]))/(15*b^4*d*Sqrt[a + b*Sin[c + d
*x]])
```

Rubi [A] time = 0.440458, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2863, 2752, 2663, 2661, 2655, 2653}

$$\frac{4\cos(c+dx)\left(32a^2+8ab\sin(c+dx)-9b^2\right)}{15b^4d\sqrt{a+b\sin(c+dx)}}-\frac{8a\left(32a^2-17b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^5d\sqrt{a+b\sin(c+dx)}}+\frac{8\left(32a^2-9b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (8*(32*a^2 - 9*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b
*Sin[c + d*x]])/(15*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*a*(32*a^
2 - 17*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(15*b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]^3*(
8*a + 3*b*Sin[c + d*x]))/(15*b^2*d*(a + b*Sin[c + d*x])^(3/2)) + (4*Cos[c +
 d*x]*(32*a^2 - 9*b^2 + 8*a*b*Sin[c + d*x]))/(15*b^4*d*Sqrt[a + b*Sin[c + d
*x]])
```


Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} - \frac{4\int \frac{\cos^2(c+dx)\left(-\frac{3b}{2}-4a\sin(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}}\,dx}{5b^2}$$

$$= \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)\left(32a^2-9b^2+8ab\sin(c+dx)\right)}{15b^4d\sqrt{a+b\sin(c+dx)}} + \frac{1}{}$$

$$= \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)\left(32a^2-9b^2+8ab\sin(c+dx)\right)}{15b^4d\sqrt{a+b\sin(c+dx)}} - \frac{(4}{}$$

$$= \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)\left(32a^2-9b^2+8ab\sin(c+dx)\right)}{15b^4d\sqrt{a+b\sin(c+dx)}} + \frac{(4}{}$$

$$= \frac{8\left(32a^2-9b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{15b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} - \frac{8a\left(32a^2-17b^2\right)F\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{15b^5d\sqrt{a+b}}$$

Mathematica [A] time = 5.8889, size = 211, normalized size = 0.83

$$\frac{2b\cos(c+dx)\left(b\left(320a^2-69b^2\right)\sin(c+dx)+256a^3-16ab^2\cos(2(c+dx))-24ab^2+3b^3\sin(3(c+dx))\right)+32a\left(32a^2-\right.}{60b^5d(a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32*(a + b)^2*(32*a^2 - 9*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
 b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + 32*a*(a + b)*(32*a^2 - 17*b^2)*
```

```
EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a +
b))^(3/2) + 2*b*Cos[c + d*x]*(256*a^3 - 24*a*b^2 - 16*a*b^2*Cos[2*(c + d*x)
] + b*(320*a^2 - 69*b^2)*Sin[c + d*x] + 3*b^3*Sin[3*(c + d*x)]))/(60*b^5*d*
(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] time = 1.565, size = 1430, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15*(3*b^5*sin(d*x+c)*cos(d*x+c)^4+(80*a^2*b^3-18*b^5)*cos(d*x+c)^2*sin(d*
x+c)+4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b*(32*EllipticF((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b-24*EllipticF((b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2-17*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^3+9*EllipticF((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^4-32*EllipticE((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4+41*EllipticE((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2-9*EllipticE((b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^4)*sin(d*x+c)-8*a*b^4*c
os(d*x+c)^4+(64*a^3*b^2-2*a*b^4)*cos(d*x+c)^2+128*(-b/(a-b)*sin(d*x+c)-b/(a
-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/
2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*a^4*b-96*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3*b^2-68*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2
))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)
*a^2*b^3+36*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a*b^4-128*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)
*a^5+164*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*
(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3*b^2-36*(-b/(a-b)*sin(d*x+c)-b/(a-b)
)^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a
*b^4)/(a+b*sin(d*x+c))^(3/2)/b^6/cos(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\cos(dx+c)^4\sin(dx+c)}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)/(3*a*b^2*cos
(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4\sin(dx+c)}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

3.1186 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=313

$$\frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3ab^3d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2\right.}{}$$

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x])/(3*a*b^2*d*(a + b*Sin[c + d*x])^(3/2)) + (2*(
5*a^2 + 3*b^2)*Cos[c + d*x])/(3*a^2*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*(8
*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(3*a^2*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*(8*a^2 + b^
2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(3*a*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a^2*d*Sqrt[a
+ b*Sin[c + d*x]])
```

Rubi [A] time = 0.680606, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2891, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3ab^3d\sqrt{a+b\sin(c+dx)}}+\frac{2\left(8a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x])/(3*a*b^2*d*(a + b*Sin[c + d*x])^(3/2)) + (2*(
5*a^2 + 3*b^2)*Cos[c + d*x])/(3*a^2*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*(8
*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(3*a^2*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*(8*a^2 + b^
2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(3*a*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a^2*d*Sqrt[a
+ b*Sin[c + d*x]])
```


Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\int \frac{\csc(c+dx)\left(-\frac{3b^2}{4}-\frac{1}{2}ab\sin(\ldots\right.}{\sqrt{a+\ldots}}}{3\ldots}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\csc(c+dx)\left(\frac{3b^3}{4}-\frac{1}{4}a(8a^2+\ldots\right.}{\sqrt{a+b\sin(c+d\ldots}}}{3a^2b^3}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}dx}{a^2} - \frac{(8\ldots}{\ldots}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{2\left(8a^2+3b^2\right)E\left(\frac{1}{2}\left(c-\ldots\right.\right.}{3a^2b^3\ldots}$$

$$= -\frac{2\left(a^2-b^2\right)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2\left(5a^2+3b^2\right)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{2\left(8a^2+3b^2\right)E\left(\frac{1}{2}\left(c-\ldots\right.\right.}{3a^2b^3\ldots}$$

Mathematica [C] time = 4.87962, size = 443, normalized size = 1.42

$$-\frac{\frac{2a^2\left(a^2-b^2\right)\cos(c+dx)}{(a+b\sin(c+dx))^{3/2}} - \frac{2a\left(5a^2+3b^2\right)\cos(c+dx)}{\sqrt{a+b\sin(c+dx)}} + \frac{a\left(8a^2+9b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} + \frac{i\left(8a^2+3b^2\right)\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\frac{b(\ldots}{\ldots}}\ldots}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((I*(8*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[S
qrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elliptic
Pi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
 b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^(-1)]) + (2*a^2*(a^2 -
b^2)*Cos[c + d*x])/(a + b*Sin[c + d*x])^(3/2) - (2*a*(5*a^2 + 3*b^2)*Cos[c
+ d*x])/Sqrt[a + b*Sin[c + d*x]] + (4*a^2*b*EllipticF[(-2*c + Pi - 2*d*x)/4
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x
]] + (a*(8*a^2 + 9*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/(3*a^3*b^2*d
)
```

Maple [B] time = 5.919, size = 1375, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] (-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*(1/b^3*(2*b*(a/b-1)*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1
```

```
/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2)))-4*a*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*
(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c
)-a)*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2)))+1/b^3*(3*a^4-2*a^2*b^2-b^4)/a^2*(2*b*cos(d*x+c)^2/(a^2-b^2)/(-
(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)+2*a/(a^2-b^2)*(a/b-1)*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1
/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))+2/(a^2-b^2)*b*(a/b-1)*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(
-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))))+(-a^4+2*a^2*b^2-b^4)/a/b^3*(2/3/(a^2-b^2)/b*(-(-b*si
n(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+8/3*b*cos(d*x+c)^2/(a^2-
b^2)^2*a/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)+2*(3*a^2+b^2)/(3*a^4-6*a^2
*b^2+3*b^4)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))
^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+8/3*a*b/(
a^2-b^2)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^
(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/
2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))-2/a^3*(a/b-
1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x
+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*b*EllipticPi(
((a+b*sin(d*x+c))/(a-b))^(1/2),-(-a/b+1)/a*b,((a-b)/(a+b))^(1/2)))/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{3}\cot\left(dx+c\right)}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

3.1187 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{\left(4a^2+15b^2\right)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}}+\frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{\left(4a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2+15\right.}{}$$

```
[Out] ((2*a^2 - 5*b^2)*Cos[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])^(3/2)) - Cot
[c + d*x]/(a*d*(a + b*Sin[c + d*x])^(3/2)) - ((4*a^2 + 15*b^2)*Cos[c + d*x]
)/(3*a^3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((4*a^2 + 15*b^2)*EllipticE[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a^3*b^2*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) + ((4*a^2 + 5*b^2)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*a^2*b^2*d*Sqrt[a +
 b*Sin[c + d*x]]) - (5*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.987787, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2890, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(4a^2+15b^2\right)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}}+\frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{\left(4a^2+5b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{\left(4a^2+15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((2*a^2 - 5*b^2)*Cos[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])^(3/2)) - Cot
[c + d*x]/(a*d*(a + b*Sin[c + d*x])^(3/2)) - ((4*a^2 + 15*b^2)*Cos[c + d*x]
)/(3*a^3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((4*a^2 + 15*b^2)*EllipticE[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a^3*b^2*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) + ((4*a^2 + 5*b^2)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*a^2*b^2*d*Sqrt[a +
 b*Sin[c + d*x]]) - (5*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} + \frac{2\int \frac{\csc(c+dx)\left(-\frac{15b^2}{4}-\frac{3}{2}ab\sin(c+dx)+\cdots\right.}{(a+b\sin(c+d\cdots}}{3a^2b}$$

$$= \frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(4a^2+15b^2\right)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}} + \frac{4}{\cdots}$$

$$= \frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(4a^2+15b^2\right)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}} - \frac{4}{\cdots}$$

$$= \frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(4a^2+15b^2\right)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}} + \frac{1}{6}\cdots$$

$$= \frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(4a^2+15b^2\right)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}} - \frac{(4\cdots}{\cdots}$$

$$= \frac{\left(2a^2-5b^2\right)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(4a^2+15b^2\right)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}} - \frac{(4\cdots}{\cdots}$$

Mathematica [C] time = 5.26065, size = 445, normalized size = 1.29

$$\frac{\frac{2\left(4a^2+45b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(c+dx)}} - \frac{2\left(b\left(4a^2+15b^2\right)\sin(2(c+dx))+4a\left(a^2+10b^2\right)\cos(c+dx)+6a^2b\cot(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}} + \frac{2i\left(4a^2+15b^2\right)\sec(c+dx)\sqrt{-\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)*(4*a^2 + 15*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elli
pticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
(a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^2*Sqrt[-(a + b)^(-1)]) + (40*a*
b*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
```

```
/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(4*a^2 + 45*b^2)*EllipticPi[2, (-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt
[a + b*Sin[c + d*x]] - (2*(4*a*(a^2 + 10*b^2)*Cos[c + d*x] + 6*a^2*b*Cot[c
+ d*x] + b*(4*a^2 + 15*b^2)*Sin[2*(c + d*x)]))/(a + b*Sin[c + d*x])^(3/2))/
(12*a^3*b*d)
```

Maple [B] time = 1.895, size = 2112, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/3*(-11*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^2+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^2-15*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+
b))^(1/2))*a*b^5*sin(d*x+c)^2+4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)+6*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*si
n(d*x+c)+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)-11*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^4*b^2*sin(d*x+c)+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)-15*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4
*sin(d*x+c)+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2
),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)+4*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)
^2+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^3*sin(d*x+c)^2+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2-15*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*
b^5*sin(d*x+c)^2-4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^2+3*a^3*b^3-4*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*sin(d*x
+c)-4*a^3*b^3*sin(d*x+c)^4-15*a*b^5*sin(d*x+c)^4-2*a^4*b^2*sin(d*x+c)^3-20*
a^2*b^4*sin(d*x+c)^3+a^3*b^3*sin(d*x+c)^2+20*a^2*b^4*sin(d*x+c)+15*a*b^5*si
```

```
n(d*x+c)^2+2*a^4*b^2*sin(d*x+c)+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin
(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^6*sin(d*x+c)^2)/a^4/si
n(d*x+c)/(a+b*sin(d*x+c))^(3/2)/b^3/cos(d*x+c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2 \cot(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

3.1188 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=407

$$-\frac{\left(8a^2-105b^2\right)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}-\frac{\left(8a^2-35b^2\right)\cot(c+dx)}{12a^3bd\sqrt{a+b\sin(c+dx)}}+\frac{\left(4a^2-7b^2\right)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{\left(8a^2-35b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F}{12a^3bd\sqrt{a+b\sin}}$$

```
[Out] ((4*a^2 - 7*b^2)*Cot[c + d*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^(3/2)) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x])^(3/2)) - ((8*a^2 - 105
*b^2)*Cos[c + d*x])/(12*a^4*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 35*b^2)
*Cot[c + d*x])/(12*a^3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 105*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(12*a
^4*b*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((8*a^2 - 35*b^2)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(12*a^
3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((12*a^2 - 35*b^2)*EllipticPi[2, (c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^4*d*Sqr
t[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.26521, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2890, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(8a^2-105b^2\right)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}-\frac{\left(8a^2-35b^2\right)\cot(c+dx)}{12a^3bd\sqrt{a+b\sin(c+dx)}}+\frac{\left(4a^2-7b^2\right)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{\left(8a^2-35b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F}{12a^3bd\sqrt{a+b\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((4*a^2 - 7*b^2)*Cot[c + d*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^(3/2)) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x])^(3/2)) - ((8*a^2 - 105
*b^2)*Cos[c + d*x])/(12*a^4*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 35*b^2)
*Cot[c + d*x])/(12*a^3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 105*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(12*a
^4*b*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((8*a^2 - 35*b^2)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(12*a^
3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((12*a^2 - 35*b^2)*EllipticPi[2, (c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^4*d*Sqr
t[a + b*Sin[c + d*x]])
```

Rule 2890

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[((a^2*(n + 1) - b^2*(m + n + 2))*Cos[e +
f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^2*b*d^2*f*(n
 + 1)*(m + 1)), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{1}{4}(8a^2-35b^2)-\frac{3}{2}ab\ldots\right)}{(a+b\sin(c\ldots}}{3a^2\ldots}$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-35b^2)\cot(c+dx)}{12a^3bd\sqrt{a+b\sin(c+dx)}}\ldots$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}\ldots$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}\ldots$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}\ldots$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}\ldots$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}\ldots$$

Mathematica [C] time = 6.6142, size = 622, normalized size = 1.53

$$\frac{\sqrt{a+b\sin(c+dx)}\left(-\frac{2(a^2\cos(c+dx)-9b^2\cos(c+dx))}{3a^4(a+b\sin(c+dx))}-\frac{2(a^2\cos(c+dx)-b^2\cos(c+dx))}{3a^3(a+b\sin(c+dx))^2}+\frac{11b\cot(c+dx)}{4a^4}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3}\right)}{d}+\frac{-\frac{2(315b^2-\ldots}{\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*((11*b*Cot[c + d*x])/(4*a^4) - (Cot[c + d*x]*Csc[
c + d*x])/(2*a^3) - (2*(a^2*Cos[c + d*x] - b^2*Cos[c + d*x]))/(3*a^3*(a + b
```

```
*Sin[c + d*x])^2) - (2*(a^2*Cos[c + d*x] - 9*b^2*Cos[c + d*x]))/(3*a^4*(a +
 b*Sin[c + d*x]))))/d + ((-280*a*b*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(-8
0*a^2 + 315*b^2)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(8*a^2 - 105*
b^2)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF
[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]
- b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c +
d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b
*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(
-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-(
(a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(48
*a^4*d)
```

Maple [B] time = 2.039, size = 2617, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/12*(105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-
b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^2+78*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^2
+35*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^3*b^3*sin(d*x+c)^2-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2-36*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))
^(1/2))*a^2*b^4*sin(d*x+c)^3-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d
*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^3-113*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^4*b^2*sin(d*x+c)^2+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin
(d*x+c)^2+36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),
(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^2-36*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^
3*sin(d*x+c)^2-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^
(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2+8*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(
d*x+c)^3-105*a*b^5*sin(d*x+c)^5-140*a^2*b^4*sin(d*x+c)^4-29*a^3*b^3*sin(d*x
+c)^3+105*a*b^5*sin(d*x+c)^3-10*a^4*b^2*sin(d*x+c)^2+140*a^2*b^4*sin(d*x+c)
```

```
^2+21*a^3*b^3*sin(d*x+c)-6*a^4*b^2+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b
*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^6*sin(d*x+c)^3-113
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*a^3*b^3*sin(d*x+c)^3+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^3-8*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2
*sin(d*x+c)^3+78*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^3+35*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3
-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a*b^5*sin(d*x+c)^3+36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(
d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^3+8*a^
3*b^3*sin(d*x+c)^5+16*a^4*b^2*sin(d*x+c)^4+8*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)^2)/a^5/s
in(d*x+c)^2/(a+b*sin(d*x+c))^(3/2)/b^2/cos(d*x+c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)\cot(dx+c)^3}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(cos(d*x + c)*cot(d*x + c)^3/(b*sin(d*x + c) + a)^(5/2), x)
```

3.1189 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=458

$$\frac{b\left(32a^2-105b^2\right)\cos(c+dx)}{8a^5d\sqrt{a+b\sin(c+dx)}}+\frac{\left(16a^2-35b^2\right)\cot(c+dx)}{8a^4d\sqrt{a+b\sin(c+dx)}}-\frac{\left(16a^2-35b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{8a^4d\sqrt{a+b\sin(c+dx)}}+\frac{(32}{}$$

```
[Out] ((2*a^2 - 3*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])
^(3/2)) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^(3/2))
+ (b*(32*a^2 - 105*b^2)*Cos[c + d*x])/(8*a^5*d*Sqrt[a + b*Sin[c + d*x]]) +
((16*a^2 - 35*b^2)*Cot[c + d*x])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a
^2 - 21*b^2)*Cot[c + d*x]*Csc[c + d*x])/(12*a^3*b*d*Sqrt[a + b*Sin[c + d*x]
]) + ((32*a^2 - 105*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[
a + b*Sin[c + d*x]])/(8*a^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((16*a^
2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]]) + (15*b*(4*a^2 - 7*b^2
)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)])/(8*a^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 1.56392, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2724, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(32a^2-105b^2\right)\cos(c+dx)}{8a^5d\sqrt{a+b\sin(c+dx)}}+\frac{\left(16a^2-35b^2\right)\cot(c+dx)}{8a^4d\sqrt{a+b\sin(c+dx)}}-\frac{\left(16a^2-35b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{8a^4d\sqrt{a+b\sin(c+dx)}}+\frac{(32}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] ((2*a^2 - 3*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])
^(3/2)) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^(3/2))
+ (b*(32*a^2 - 105*b^2)*Cos[c + d*x])/(8*a^5*d*Sqrt[a + b*Sin[c + d*x]]) +
((16*a^2 - 35*b^2)*Cot[c + d*x])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a
^2 - 21*b^2)*Cot[c + d*x]*Csc[c + d*x])/(12*a^3*b*d*Sqrt[a + b*Sin[c + d*x]
]) + ((32*a^2 - 105*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[
a + b*Sin[c + d*x]])/(8*a^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((16*a^
2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]]) + (15*b*(4*a^2 - 7*b^2
)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)])/(8*a^5*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{2\int \frac{\csc^3(c+dx)\left(\frac{3}{4}\left(8a^2-21b\right.\right.}{}}{}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} - \frac{\left(8a^2-21b^2\right)\cot(c+}{12a^3bd\sqrt{a+b\sin}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{\left(16a^2-35b^2\right)\cot(c+}{8a^4d\sqrt{a+b\sin(c+}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)\cos(}{8a^5d\sqrt{a+b\sin(c+}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)\cos(}{8a^5d\sqrt{a+b\sin(c+}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)\cos(}{8a^5d\sqrt{a+b\sin(c+}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)\cos(}{8a^5d\sqrt{a+b\sin(c+}}$$

$$= \frac{\left(2a^2-3b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b\left(32a^2-105b^2\right)\cos(}{8a^5d\sqrt{a+b\sin(c+}}$$

Mathematica [C] time = 6.72733, size = 680, normalized size = 1.48

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{8\left(a^2b\cos(c+dx)-3b^3\cos(c+dx)\right)}{3a^5(a+b\sin(c+dx))} + \frac{2\left(a^2b\cos(c+dx)-b^3\cos(c+dx)\right)}{3a^4(a+b\sin(c+dx))^2} + \frac{\csc(c+dx)\left(32a^2\cos(c+dx)-123b^2\cos(c+dx)\right)}{24a^5} + \frac{17b\,\mathrm{co}}{}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(((32*a^2*Cos[c + d*x] - 123*b^2*Cos[c + d*x])*Cs
c[c + d*x])/(24*a^5) + (17*b*Cot[c + d*x]*Csc[c + d*x])/(12*a^4) - (Cot[c +
 d*x]*Csc[c + d*x]^2)/(3*a^3) + (2*(a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x]))
/(3*a^4*(a + b*Sin[c + d*x])^2) + (8*(a^2*b*Cos[c + d*x] - 3*b^3*Cos[c + d*
x]))/(3*a^5*(a + b*Sin[c + d*x]))))/d + ((-2*(32*a^3 - 140*a*b^2)*EllipticF
[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sq
rt[a + b*Sin[c + d*x]] - (2*(152*a^2*b - 315*b^3)*EllipticPi[2, (-c + Pi/2
- d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin
[c + d*x]] - ((2*I)*(-32*a^2*b + 105*b^3)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*
a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]]
, (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b
*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a
+ b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]
) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x])
+ (a + b*Sin[c + d*x])^2)/b^2)]))/(32*a^5*d)
```

Maple [B] time = 2.266, size = 2870, normalized size = 6.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/24*(-315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(
a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3+315*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*
sin(d*x+c)^3+315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1
/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^6*sin(d*x+c)^4+96*a^3*b^3*sin(d*x+c)^6+8
*a^5*b-159*a^3*b^3*sin(d*x+c)^4+315*a*b^5*sin(d*x+c)^4-126*a^4*b^2*sin(d*x+
c)^3+420*a^2*b^4*sin(d*x+c)^3+63*a^3*b^3*sin(d*x+c)^2-18*a^4*b^2*sin(d*x+c)
-315*a*b^5*sin(d*x+c)^6+144*a^4*b^2*sin(d*x+c)^5-420*a^2*b^4*sin(d*x+c)^5+3
2*a^5*b*sin(d*x+c)^4-40*a^5*b*sin(d*x+c)^2+306*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^3+
105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^3*b^3*sin(d*x+c)^3-315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3-180*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b)
)^(1/2))*a^3*b^3*sin(d*x+c)^3+96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)^3-48*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*sin(
d*x+c)^3-411*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^4+315*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^4-48*(
```

```
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^5*b*sin(d*x+c)^4-48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^4+306*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3*
sin(d*x+c)^4+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^4-315*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^4+
180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,(
(a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^4-180*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi
(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*
x+c)^4-315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a
-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^4-411*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)
^3+315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3-48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^3+180*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b)
)^(1/2))*a^4*b^2*sin(d*x+c)^3+96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^4)/sin(d*x+c)^3/a^
6/(a+b*sin(d*x+c))^(3/2)/b/cos(d*x+c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^4}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

3.1190 $\int \frac{\cos^4(e+fx)}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=510

$$\frac{32b\left(2a^2-b^2\right)\cos(e+fx)}{35a^3f\left(a^2-b^2\right)^2\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}+\frac{8\left(a^2-2b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^3df\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}-\frac{8\left(5a^2-3ab-4b^2\right)\text{ta}}{}$$

```
[Out] (2*Cos[e + f*x]^3*Sqrt[d*Sin[e + f*x]])/(7*a*d*f*(a + b*Sin[e + f*x])^(7/2)
) + (12*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(35*a^2*d*f*(a + b*Sin[e + f*x])
^(5/2)) + (8*(a^2 - 2*b^2)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(35*a^3*(a^2
- b^2)*d*f*(a + b*Sin[e + f*x])^(3/2)) + (32*b*(2*a^2 - b^2)*Cos[e + f*x])/
(35*a^3*(a^2 - b^2)^2*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (3
2*b*(2*a^2 - b^2)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e +
 f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[
a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(35*a^5*(a
 - b)*(a + b)^(3/2)*Sqrt[d]*f) - (8*(5*a^2 - 3*a*b - 4*b^2)*Sqrt[(a*(1 - Cs
c[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin
[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -(
(a + b)/(a - b))]*Tan[e + f*x])/(35*a^4*(a - b)*(a + b)^(3/2)*Sqrt[d]*f)
```

Rubi [A] time = 1.91044, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2887, 2889, 3056, 2993, 2998, 2816, 2994}

$$\frac{32b\left(2a^2-b^2\right)\cos(e+fx)}{35a^3f\left(a^2-b^2\right)^2\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}+\frac{8\left(a^2-2b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^3df\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}-\frac{8\left(5a^2-3ab-4b^2\right)\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(9/2)),x]
```

```
[Out] (2*Cos[e + f*x]^3*Sqrt[d*Sin[e + f*x]])/(7*a*d*f*(a + b*Sin[e + f*x])^(7/2)
) + (12*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(35*a^2*d*f*(a + b*Sin[e + f*x])
^(5/2)) + (8*(a^2 - 2*b^2)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(35*a^3*(a^2
- b^2)*d*f*(a + b*Sin[e + f*x])^(3/2)) + (32*b*(2*a^2 - b^2)*Cos[e + f*x])/
(35*a^3*(a^2 - b^2)^2*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (3
2*b*(2*a^2 - b^2)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e +
 f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[
a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(35*a^5*(a
 - b)*(a + b)^(3/2)*Sqrt[d]*f) - (8*(5*a^2 - 3*a*b - 4*b^2)*Sqrt[(a*(1 - Cs
c[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin
[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -(
(a + b)/(a - b))]*Tan[e + f*x])/(35*a^4*(a - b)*(a + b)^(3/2)*Sqrt[d]*f)
```

Rule 2887

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -Simp[(g*(g*C
os[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(m + 1))/(a*
d*f*(m + 1)), x] + Dist[(g^2*(2*m + 3))/(2*a*(m + 1)), Int[((g*Cos[e + f*x]
)^(p - 2)*(a + b*Sin[e + f*x])^(m + 1))/Sqrt[d*Sin[e + f*x]], x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p +
```

```
1/2, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{9/2}}\,dx = \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{6\int \frac{\cos^2(e+fx)}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{7/2}}\,dx}{7a}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{6\int \frac{1-\sin^2(e+fx)}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{7/2}}\,dx}{7a}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{12\int \frac{2(\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{8\left(a^2-\cdots\right.}{35a^3\left(\cdots\right.}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{8\left(a^2-\cdots\right.}{35a^3\left(\cdots\right.}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{8\left(a^2-\cdots\right.}{35a^3\left(\cdots\right.}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}} + \frac{8\left(a^2-\cdots\right.}{35a^3\left(\cdots\right.}$$

Mathematica [C] time = 6.5265, size = 1670, normalized size = 3.27

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(9/2)),
x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((-2*(a^2*Cos[e + f*x] - b^2*Cos[e +
 f*x]))/(7*a*b^2*(a + b*Sin[e + f*x])^4) + (4*(5*a^2*Cos[e + f*x] + 3*b^2*C
os[e + f*x]))/(35*a^2*b^2*(a + b*Sin[e + f*x])^3) - (2*(5*a^4*Cos[e + f*x]
- 9*a^2*b^2*Cos[e + f*x] + 8*b^4*Cos[e + f*x]))/(35*a^3*b^2*(a^2 - b^2)*(a
+ b*Sin[e + f*x])^2) - (32*(2*a^2*b^2*Cos[e + f*x] - b^4*Cos[e + f*x]))/(35
*a^4*(a^2 - b^2)^2*(a + b*Sin[e + f*x]))))/(f*Sqrt[d*Sin[e + f*x]]) + (4*Sq
rt[Sin[e + f*x]]*((4*a*(5*a^4 - 9*a^2*b^2 + 4*b^4)*Sqrt[((a + b)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]
^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x]
)/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]))/((a + b)*S
qrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) + 4*a*(-8*a^3*b + 4*a*b^3)*((Sq
rt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Cs
c[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 -
f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[(
```

```
(a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqr
t[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi
/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Si
n[e + f*x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) + 2*(8*a^
2*b^2 - 4*b^4)*((Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Sin[e + f*x
]]) + (I*Cos[(-e + Pi/2 - f*x)/2]*Csc[e + f*x]*EllipticE[I*ArcSinh[Sin[(-e
+ Pi/2 - f*x)/2]/Sqrt[Sin[e + f*x]]], (-2*a)/(-a - b)]*Sqrt[a + b*Sin[e + f
*x]])/(b*Sqrt[Cos[(-e + Pi/2 - f*x)/2]^2*Csc[e + f*x]]*Sqrt[(Csc[e + f*x]*(
a + b*Sin[e + f*x]))/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*
x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b
*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 -
 f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqr
t[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e
 + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2
*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/
a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sqrt[Sin[
e + f*x]]*Sqrt[a + b*Sin[e + f*x]])))/b)))/(35*a^4*(a - b)^2*(a + b)^2*f*Sq
rt[d*Sin[e + f*x]])
```

Maple [B] time = 0.898, size = 24365, normalized size = 47.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{9}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/((b*sin(f*x + e) + a)^(9/2)*sqrt(d*sin(f*x + e))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin}}{b^5d\cos\left(fx+e\right)^6-\left(10\,a^2b^3+3\,b^5\right)d\cos\left(fx+e\right)^4+\left(5\,a^4b+20\,a^2b^3+3\,b^5\right)d\cos\left(fx+e\right)^2-\left(5\,a^4b+10\,a^2l\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*cos(f*x + e)^4/(b^5
*d*cos(f*x + e)^6 - (10*a^2*b^3 + 3*b^5)*d*cos(f*x + e)^4 + (5*a^4*b + 20*a
^2*b^3 + 3*b^5)*d*cos(f*x + e)^2 - (5*a^4*b + 10*a^2*b^3 + b^5)*d - (5*a*b^
4*d*cos(f*x + e)^4 - 10*(a^3*b^2 + a*b^4)*d*cos(f*x + e)^2 + (a^5 + 10*a^3*
b^2 + 5*a*b^4)*d)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e))**(9/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{9}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/((b*sin(f*x + e) + a)^(9/2)*sqrt(d*sin(f*x + e))),
 x)
```

3.1191 $\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=35

$$\text{Unintegrable}\left(\frac{\sqrt[3]{\sin(c+dx)}\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}},x\right)$$

```
[Out] Unintegrable[(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]],
x]
```

Rubi [A] time = 0.148847, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] Defer[Int][(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx = \int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Mathematica [A] time = 23.2607, size = 0, normalized size = 0.

$$\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]], x]
```

Maple [A] time = 0.5, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^4\sqrt[3]{\sin(dx+c)}\frac{1}{\sqrt{a+b\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^{\frac{1}{3}}}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^(1/3)/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^4 \sin(dx+c)^{\frac{1}{3}}}{\sqrt{b\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(cos(d*x + c)^4*sin(d*x + c)^(1/3)/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**(1/3)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4 \sin(dx+c)^{\frac{1}{3}}}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^(1/3)/sqrt(b*sin(d*x + c) + a), x)
```

3.1192 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p,x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

Rubi [A] time = 0.10087, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
```

```
[Out] Defer[Int][Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx=\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Mathematica [A] time = 5.83462, size = 0, normalized size = 0.

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
```

```
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

Maple [A] time = 0.389, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^4(\sin(dx+c))^n(a+b\sin(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^p\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(dx+c)+a)^p\sin(dx+c)^n\cos(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="fricas
")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1193 $\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=35

$$\text{Unintegrable}\left(\cos^4(c+dx)\sin^{-p-3}(c+dx)(a+b\sin(c+dx))^p,x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p, x
]
```

Rubi [A] time = 0.113725, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p,x]
```

```
[Out] Defer[Int][Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx=\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Mathematica [A] time = 4.57067, size = 0, normalized size = 0.

$$\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p,x]
```

```
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p, x]
```

Maple [A] time = 0.411, size = 0, normalized size = 0.

$$\int \left(\cos(dx+c)\right)^4\left(\sin(dx+c)\right)^{-3-p}\left(a+b\sin(dx+c)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^{-p-3}\cos(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x, algorithm="m
axima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 3)*cos(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(dx+c)+a)^p \sin(dx+c)^{-p-3}\cos(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x, algorithm="f
ricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 3)*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**(-3-p)*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^{-p-3}\cos(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x, algorithm="g
iac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 3)*cos(d*x + c)^4, x)
```

3.1194 $\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=35

$$\text{Unintegrable}\left(\cos^4(c+dx)\sin^{-p-4}(c+dx)(a+b\sin(c+dx))^p,x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p, x
]
```

Rubi [A] time = 0.11212, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p,x]
```

```
[Out] Defer[Int][Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx = \int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Mathematica [A] time = 5.14238, size = 0, normalized size = 0.

$$\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p,x]
```

```
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p, x]
```

Maple [A] time = 0.398, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^4 (\sin(dx+c))^{-4-p} (a+b\sin(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^{-p-4} \cos(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x, algorithm="m
axima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 4)*cos(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(dx+c)+a)^p \sin(dx+c)^{-p-4} \cos(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x, algorithm="f
ricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 4)*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**(-4-p)*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^{-p-4} \cos(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x, algorithm="g
iac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 4)*cos(d*x + c)^4, x)
```

3.1195 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=623

$$\frac{3a\left(a^2(n+6)+3b^2(n+1)\right)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)(n+2)(n+4)(n+6)\sqrt{\cos^2(c+dx)}}+\frac{3b\left(3a^2(n+7)+b^2(n+2)\right)\cos(c}{d(n+2)(n+}$$

```
[Out] (-3*a*(2*a^4*(6 + 5*n + n^2) + 3*b^4*(35 + 12*n + n^2) - 2*a^2*b^2*(58 + 16
*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(1 + n))/(b^2*d*(2 + n)*(4 + n)*(5 + n
)*(6 + n)*(7 + n)) + (3*a*(3*b^2*(1 + n) + a^2*(6 + n))*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n)
)/(d*(1 + n)*(2 + n)*(4 + n)*(6 + n)*Sqrt[Cos[c + d*x]^2]) - (3*(2*a^4*(6 +
 5*n + n^2) + b^4*(24 + 10*n + n^2) - 2*a^2*b^2*(57 + 16*n + n^2))*Cos[c +
d*x]*Sin[c + d*x]^(2 + n))/(b*d*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)) +
(3*b*(b^2*(2 + n) + 3*a^2*(7 + n))*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 +
 n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n)*(3 + n)*
(5 + n)*(7 + n)*Sqrt[Cos[c + d*x]^2]) - (3*a*(a^2*(6 + 5*n + n^2) - b^2*(53
 + 15*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^2)/(
b^2*d*(4 + n)*(5 + n)*(6 + n)*(7 + n)) - ((a^2*(2 + n)*(3 + n) - b^2*(6 + n
)*(8 + n))*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^3)/(b^2*d
*(5 + n)*(6 + n)*(7 + n)) + (a*(3 + n)*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a
 + b*Sin[c + d*x])^4)/(b^2*d*(6 + n)*(7 + n)) - (Cos[c + d*x]*Sin[c + d*x]^
(2 + n)*(a + b*Sin[c + d*x])^4)/(b*d*(7 + n))
```

Rubi [A] time = 1.75916, antiderivative size = 623, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2895, 3049, 3033, 3023, 2748, 2643}

$$\frac{3a\left(a^2(n+6)+3b^2(n+1)\right)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)(n+2)(n+4)(n+6)\sqrt{\cos^2(c+dx)}}+\frac{3b\left(3a^2(n+7)+b^2(n+2)\right)\cos(c}{d(n+2)(n+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-3*a*(2*a^4*(6 + 5*n + n^2) + 3*b^4*(35 + 12*n + n^2) - 2*a^2*b^2*(58 + 16
*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(1 + n))/(b^2*d*(2 + n)*(4 + n)*(5 + n
)*(6 + n)*(7 + n)) + (3*a*(3*b^2*(1 + n) + a^2*(6 + n))*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n)
)/(d*(1 + n)*(2 + n)*(4 + n)*(6 + n)*Sqrt[Cos[c + d*x]^2]) - (3*(2*a^4*(6 +
 5*n + n^2) + b^4*(24 + 10*n + n^2) - 2*a^2*b^2*(57 + 16*n + n^2))*Cos[c +
d*x]*Sin[c + d*x]^(2 + n))/(b*d*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)) +
(3*b*(b^2*(2 + n) + 3*a^2*(7 + n))*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 +
 n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n)*(3 + n)*
(5 + n)*(7 + n)*Sqrt[Cos[c + d*x]^2]) - (3*a*(a^2*(6 + 5*n + n^2) - b^2*(53
 + 15*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^2)/(
b^2*d*(4 + n)*(5 + n)*(6 + n)*(7 + n)) - ((a^2*(2 + n)*(3 + n) - b^2*(6 + n
)*(8 + n))*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^3)/(b^2*d
*(5 + n)*(6 + n)*(7 + n)) + (a*(3 + n)*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a
 + b*Sin[c + d*x])^4)/(b^2*d*(6 + n)*(7 + n)) - (Cos[c + d*x]*Sin[c + d*x]^
(2 + n)*(a + b*Sin[c + d*x])^4)/(b*d*(7 + n))
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
```

```
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{a(3+n)\cos(c+dx)\sin^{1+n}(c+dx)(a+b\sin(c+dx))^4}{b^2d(6+n)(7+n)} - \frac{\cos(c+d\ldots}{}$$

$$= -\frac{\left(a^2(2+n)(3+n)-b^2(6+n)(8+n)\right)\cos(c+dx)\sin^{1+n}(c+dx)(a\ldots}{b^2d(5+n)(6+n)(7+n)}$$

$$= -\frac{3a\left(a^2\left(6+5n+n^2\right)-b^2\left(53+15n+n^2\right)\right)\cos(c+dx)\sin^{1+n}(c+\ldots}{b^2d(4+n)(5+n)(6+n)(7+n)}$$

$$= -\frac{3\left(2a^4\left(6+5n+n^2\right)+b^4\left(24+10n+n^2\right)-2a^2b^2\left(57+16n+n^2\ldots\right.\right.}{bd(3+n)(4+n)(5+n)(6+n)(7+n)\ldots}$$

$$= -\frac{3a\left(2a^4\left(6+5n+n^2\right)+3b^4\left(35+12n+n^2\right)-2a^2b^2\left(58+16n+\ldots\right.\right.}{b^2d(2+n)(4+n)(5+n)(6+n)(7+\ldots}$$

$$= -\frac{3a\left(2a^4\left(6+5n+n^2\right)+3b^4\left(35+12n+n^2\right)-2a^2b^2\left(58+16n+\ldots\right.\right.}{b^2d(2+n)(4+n)(5+n)(6+n)(7+\ldots}$$

$$= -\frac{3a\left(2a^4\left(6+5n+n^2\right)+3b^4\left(35+12n+n^2\right)-2a^2b^2\left(58+16n+\ldots\right.\right.}{b^2d(2+n)(4+n)(5+n)(6+n)(7+\ldots}$$

Mathematica [A] time = 0.801946, size = 195, normalized size = 0.31

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{n+1}(c+dx)\left(b\sin(c+dx)\left(\frac{3a^2\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{n+2}+b\sin(c+dx)\left(\frac{3a\,{}_2F_1\left(-\frac{3}{2},\frac{n+3}{2};\frac{n+5}{2};\sin^2(c+\ldots\right.}{n+3}\ldots\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*((a^3*Hypergeometri
c2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2])/(1 + n) + b*Sin[c + d*x]*
((3*a^2*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2])/(2 +
 n) + b*Sin[c + d*x]*((3*a*Hypergeometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Si
n[c + d*x]^2])/(3 + n) + (b*Hypergeometric2F1[-3/2, (4 + n)/2, (6 + n)/2, S
in[c + d*x]^2]*Sin[c + d*x])/(4 + n)))))/d
```

Maple [F] time = 12.201, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^4 (\sin(dx+c))^n (a+b\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^3\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,ab^2\cos\left(dx+c\right)^6-\left(a^3+3\,ab^2\right)\cos\left(dx+c\right)^4+\left(b^3\cos\left(dx+c\right)^6-\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^4\right)\sin\left(dx+c\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^6 - (a^3 + 3*a*b^2)*cos(d*x + c)^4 + (b^3*c
os(d*x + c)^6 - (3*a^2*b + b^3)*cos(d*x + c)^4)*sin(d*x + c))*sin(d*x + c)^
n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)+a\right)^3\sin\left(dx+c\right)^n\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

3.1196 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=487

$$\frac{3\left(a^2(n+6)+b^2(n+1)\right)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)(n+2)(n+4)(n+6)\sqrt{\cos^2(c+dx)}}-\frac{\left(-2a^2b^2\left(n^2+13n+40\right)+2a^4\left(n^2\right.\right.}{b^2d(n+}$$

```
[Out] -(((3*b^4*(5 + n) + 2*a^4*(6 + 5*n + n^2) - 2*a^2*b^2*(40 + 13*n + n^2))*Co
s[c + d*x]*Sin[c + d*x]^(1 + n))/(b^2*d*(2 + n)*(4 + n)*(5 + n)*(6 + n))) +
 (3*(b^2*(1 + n) + a^2*(6 + n))*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)
/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*(2 + n)*(4
+ n)*(6 + n)*Sqrt[Cos[c + d*x]^2]) - (2*a*(a^2*(6 + 5*n + n^2) - b^2*(39 +
13*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(2 + n))/(b*d*(3 + n)*(4 + n)*(5 + n
)*(6 + n)) + (6*a*b*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/
2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n)*(3 + n)*(5 + n)*Sqrt[Co
s[c + d*x]^2]) - ((a^2*(2 + n)*(3 + n) - b^2*(5 + n)*(7 + n))*Cos[c + d*x]*
Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^2)/(b^2*d*(4 + n)*(5 + n)*(6 + n)
) + (a*(3 + n)*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^3)/(b
^2*d*(5 + n)*(6 + n)) - (Cos[c + d*x]*Sin[c + d*x]^(2 + n)*(a + b*Sin[c + d
*x])^3)/(b*d*(6 + n))
```

Rubi [A] time = 1.12029, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2895, 3049, 3033, 3023, 2748, 2643}

$$\frac{3\left(a^2(n+6)+b^2(n+1)\right)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)(n+2)(n+4)(n+6)\sqrt{\cos^2(c+dx)}}-\frac{\left(-2a^2b^2\left(n^2+13n+40\right)+2a^4\left(n^2\right.\right.}{b^2d(n+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((3*b^4*(5 + n) + 2*a^4*(6 + 5*n + n^2) - 2*a^2*b^2*(40 + 13*n + n^2))*Co
s[c + d*x]*Sin[c + d*x]^(1 + n))/(b^2*d*(2 + n)*(4 + n)*(5 + n)*(6 + n))) +
 (3*(b^2*(1 + n) + a^2*(6 + n))*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)
/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*(2 + n)*(4
+ n)*(6 + n)*Sqrt[Cos[c + d*x]^2]) - (2*a*(a^2*(6 + 5*n + n^2) - b^2*(39 +
13*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(2 + n))/(b*d*(3 + n)*(4 + n)*(5 + n
)*(6 + n)) + (6*a*b*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/
2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n)*(3 + n)*(5 + n)*Sqrt[Co
s[c + d*x]^2]) - ((a^2*(2 + n)*(3 + n) - b^2*(5 + n)*(7 + n))*Cos[c + d*x]*
Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^2)/(b^2*d*(4 + n)*(5 + n)*(6 + n)
) + (a*(3 + n)*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^3)/(b
^2*d*(5 + n)*(6 + n)) - (Cos[c + d*x]*Sin[c + d*x]^(2 + n)*(a + b*Sin[c + d
*x])^3)/(b*d*(6 + n))
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
```

```
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{a(3+n)\cos(c+dx)\sin^{1+n}(c+dx)(a+b\sin(c+dx))^3}{b^2d(5+n)(6+n)} - \frac{\cos(c+d\ldots}{}$$

$$= -\frac{\left(a^2(2+n)(3+n)-b^2(5+n)(7+n)\right)\cos(c+dx)\sin^{1+n}(c+dx)(a\ldots}{b^2d(4+n)(5+n)(6+n)}$$

$$= -\frac{2a\left(a^2\left(6+5n+n^2\right)-b^2\left(39+13n+n^2\right)\right)\cos(c+dx)\sin^{2+n}(c+\ldots}{bd(3+n)(4+n)(5+n)(6+n)}$$

$$= -\frac{\left(3b^4(5+n)+2a^4\left(6+5n+n^2\right)-2a^2b^2\left(40+13n+n^2\right)\right)\cos(c+\ldots}{b^2d(2+n)(4+n)(5+n)(6+n)}$$

$$= -\frac{\left(3b^4(5+n)+2a^4\left(6+5n+n^2\right)-2a^2b^2\left(40+13n+n^2\right)\right)\cos(c+\ldots}{b^2d(2+n)(4+n)(5+n)(6+n)}$$

$$= -\frac{\left(3b^4(5+n)+2a^4\left(6+5n+n^2\right)-2a^2b^2\left(40+13n+n^2\right)\right)\cos(c+\ldots}{b^2d(2+n)(4+n)(5+n)(6+n)}$$

Mathematica [A] time = 0.315392, size = 167, normalized size = 0.34

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{n+1}(c+dx)\left(a^2\left(n^2+5n+6\right){}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+b(n+1)\sin(c+dx)\left(2a(n\ldots\right.\right.}{d(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*(a^2*(6 + 5*n + n^2
)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + b*(1 + n)
*Sin[c + d*x]*(2*a*(3 + n)*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Si
n[c + d*x]^2] + b*(2 + n)*Hypergeometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x])))/(d*(1 + n)*(2 + n)*(3 + n))
```

Maple [F] time = 7.588, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^4 (\sin(dx+c))^n (a+b\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^2 \sin(dx+c)^n \cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2\cos\left(dx+c\right)^6-2\,ab\cos\left(dx+c\right)^4\sin\left(dx+c\right)-\left(a^2+b^2\right)\cos\left(dx+c\right)^4\right)\sin\left(dx+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-(b^2*cos(d*x + c)^6 - 2*a*b*cos(d*x + c)^4*sin(d*x + c) - (a^2 +
b^2)*cos(d*x + c)^4)*sin(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(dx+c\right)+a\right)^2\sin\left(dx+c\right)^n\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

3.1197 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{b\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (b*Cos[c + d*x]
*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.147596, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2838, 2577}

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{b\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx\right.}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (b*Cos[c + d*x]
*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos^4(c+dx)\sin^n(c+dx)\,dx+b\int \cos^4(c+dx)\sin^{1+n}(c+dx)\,d\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}+\frac{b\cos}{}\end{aligned}$$

Mathematica [A] time = 0.1569, size = 111, normalized size = 0.86

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{n+1}(c+dx)\left(a(n+2)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+b(n+1)\sin(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2}\right.\right.}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*(a*(2 + n)*Hypergeo
metric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + b*(1 + n)*Hypergeom
etric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]))/(d*(1 +
 n)*(2 + n))
```

Maple [F] time = 3.066, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^4 (\sin(dx+c))^n (a+b\sin(dx+c))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)\sin(dx+c)^n\cos(dx+c)^4\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos(dx+c)^4\sin(dx+c)+a\cos(dx+c)^4\right)\sin(dx+c)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4*sin(d*x + c) + a*cos(d*x + c)^4)*sin(d*x + c)^n,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)\sin(dx+c)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

3.1198 $\int \cos^5(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^{10}(c+dx)}{10d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^6(c+dx)}{6d}+\frac{b\sin^{11}(c+dx)}{11d}-\frac{2b\sin^9(c+dx)}{9d}+\frac{b\sin^7(c+dx)}{7d}$$

```
[Out] (a*Sin[c + d*x]^6)/(6*d) + (b*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^8)/(4
*d) - (2*b*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*x]^10)/(10*d) + (b*Sin[c +
d*x]^11)/(11*d)
```

Rubi [A] time = 0.129753, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$\frac{a\sin^{10}(c+dx)}{10d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^6(c+dx)}{6d}+\frac{b\sin^{11}(c+dx)}{11d}-\frac{2b\sin^9(c+dx)}{9d}+\frac{b\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^6)/(6*d) + (b*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^8)/(4
*d) - (2*b*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*x]^10)/(10*d) + (b*Sin[c +
d*x]^11)/(11*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)\left(b^2-x^2\right)^2}{b^5}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int x^5(a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^{10}d}$$
$$= \frac{\text{Subst}\left(\int \left(ab^4x^5+b^4x^6-2ab^2x^7-2b^2x^8+ax^9+x^{10}\right)dx, x, b\sin(c\right.}{b^{10}d}$$
$$= \frac{a\sin^6(c+dx)}{6d}+\frac{b\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2b\sin^9(c+dx)}{9d}+$$

Mathematica [A] time = 0.4006, size = 105, normalized size = 1.08

$$\frac{-34650a\cos(2(c+dx))+5775a\cos(6(c+dx))-693a\cos(10(c+dx))+34650b\sin(c+dx)-11550b\sin(3(c+dx))-3}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-34650*a*Cos[2*(c + d*x)] + 5775*a*Cos[6*(c + d*x)] - 693*a*Cos[10*(c + d*
x)] + 34650*b*Sin[c + d*x] - 11550*b*Sin[3*(c + d*x)] - 3465*b*Sin[5*(c + d
*x)] + 2475*b*Sin[7*(c + d*x)] + 385*b*Sin[9*(c + d*x)] - 315*b*Sin[11*(c +
 d*x)])/(3548160*d)
```

Maple [A] time = 0.032, size = 138, normalized size = 1.4

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^6}{10}-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{20}-\frac{(\cos(dx+c))^6}{60}\right)+b\left(-\frac{(\sin(dx+c))^5(\cos(dx+\right.}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60
*cos(d*x+c)^6)+b*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x
+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c)))
```

Maxima [A] time = 1.00014, size = 97, normalized size = 1.

$$\frac{1260\,b\sin(dx+c)^{11}+1386\,a\sin(dx+c)^{10}-3080\,b\sin(dx+c)^9-3465\,a\sin(dx+c)^8+1980\,b\sin(dx+c)^7+2310\,a\,\text{s}}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/13860*(1260*b*sin(d*x + c)^11 + 1386*a*sin(d*x + c)^10 - 3080*b*sin(d*x +
 c)^9 - 3465*a*sin(d*x + c)^8 + 1980*b*sin(d*x + c)^7 + 2310*a*sin(d*x + c)
```

```
^6)/d
```

Fricas [A] time = 2.01031, size = 297, normalized size = 3.06

$$-\frac{1386\,a\cos\left(dx+c\right)^{10}-3465\,a\cos\left(dx+c\right)^{8}+2310\,a\cos\left(dx+c\right)^{6}+20\left(63\,b\cos\left(dx+c\right)^{10}-161\,b\cos\left(dx+c\right)^{8}+11}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/13860*(1386*a*cos(d*x + c)^10 - 3465*a*cos(d*x + c)^8 + 2310*a*cos(d*x +
 c)^6 + 20*(63*b*cos(d*x + c)^10 - 161*b*cos(d*x + c)^8 + 113*b*cos(d*x + c
)^6 - 3*b*cos(d*x + c)^4 - 4*b*cos(d*x + c)^2 - 8*b)*sin(d*x + c))/d
```

Sympy [A] time = 58.4052, size = 136, normalized size = 1.4

$$\begin{cases}-\frac{a\sin^{4}(c+dx)\cos^{6}(c+dx)}{6d}-\frac{a\sin^{2}(c+dx)\cos^{8}(c+dx)}{12d}-\frac{a\cos^{10}(c+dx)}{60d}+\frac{8b\sin^{11}(c+dx)}{693d}+\frac{4b\sin^{9}(c+dx)\cos^{2}(c+dx)}{63d}+\frac{b\sin^{7}(c+dx)\cos^{4}(c+dx)}{7d} \\ x\left(a+b\sin\left(c\right)\right)\sin^{5}\left(c\right)\cos^{5}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**4*cos(c + d*x)**6/(6*d) - a*sin(c + d*x)**2*cos
(c + d*x)**8/(12*d) - a*cos(c + d*x)**10/(60*d) + 8*b*sin(c + d*x)**11/(693
*d) + 4*b*sin(c + d*x)**9*cos(c + d*x)**2/(63*d) + b*sin(c + d*x)**7*cos(c
+ d*x)**4/(7*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**5*cos(c)**5, True))
```

Giac [A] time = 1.19925, size = 180, normalized size = 1.86

$$-\frac{a\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{512\,d}-\frac{b\sin\left(11\,dx+11\,c\right)}{11264\,d}+\frac{b\sin\left(9\,dx+9\,c\right)}{9216\,d}+\frac{5\,b\sin\left(7}{716}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/5120*a*cos(10*d*x + 10*c)/d + 5/3072*a*cos(6*d*x + 6*c)/d - 5/512*a*cos(
2*d*x + 2*c)/d - 1/11264*b*sin(11*d*x + 11*c)/d + 1/9216*b*sin(9*d*x + 9*c)
/d + 5/7168*b*sin(7*d*x + 7*c)/d - 1/1024*b*sin(5*d*x + 5*c)/d - 5/1536*b*s
in(3*d*x + 3*c)/d + 5/512*b*sin(d*x + c)/d
```

3.1199 $\int \cos^5(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{b\sin^{10}(c+dx)}{10d}-\frac{b\sin^8(c+dx)}{4d}+\frac{b\sin^6(c+dx)}{6d}$$

```
[Out] (a*Sin[c + d*x]^5)/(5*d) + (b*Sin[c + d*x]^6)/(6*d) - (2*a*Sin[c + d*x]^7)/
(7*d) - (b*Sin[c + d*x]^8)/(4*d) + (a*Sin[c + d*x]^9)/(9*d) + (b*Sin[c + d*
x]^10)/(10*d)
```

Rubi [A] time = 0.120472, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{b\sin^{10}(c+dx)}{10d}-\frac{b\sin^8(c+dx)}{4d}+\frac{b\sin^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^5)/(5*d) + (b*Sin[c + d*x]^6)/(6*d) - (2*a*Sin[c + d*x]^7)/
(7*d) - (b*Sin[c + d*x]^8)/(4*d) + (a*Sin[c + d*x]^9)/(9*d) + (b*Sin[c + d*
x]^10)/(10*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+x)\left(b^2-x^2\right)^2}{b^4}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int x^4(a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^9 d}$$

$$= \frac{\text{Subst}\left(\int \left(ab^4x^4+b^4x^5-2ab^2x^6-2b^2x^7+ax^8+x^9\right)dx, x, b\sin\right.}{b^9 d}$$

$$= \frac{a\sin^5(c+dx)}{5d}+\frac{b\sin^6(c+dx)}{6d}-\frac{2a\sin^7(c+dx)}{7d}-\frac{b\sin^8(c+dx}{4d}$$

Mathematica [A] time = 0.30239, size = 94, normalized size = 0.97

$$\frac{7560a\sin(c+dx)-1680a\sin(3(c+dx))-1008a\sin(5(c+dx))+180a\sin(7(c+dx))+140a\sin(9(c+dx))-3150b}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3150*b*Cos[2*(c + d*x)] + 525*b*Cos[6*(c + d*x)] - 63*b*Cos[10*(c + d*x)]
 + 7560*a*Sin[c + d*x] - 1680*a*Sin[3*(c + d*x)] - 1008*a*Sin[5*(c + d*x)]
+ 180*a*Sin[7*(c + d*x)] + 140*a*Sin[9*(c + d*x)])/(322560*d)
```

Maple [A] time = 0.033, size = 120, normalized size = 1.2

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^6}{9}-\frac{\sin(dx+c)(\cos(dx+c))^6}{21}+\frac{\sin(dx+c)}{105}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+}{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/10*sin(d*x+c)^4*cos(d*
x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6))
```

Maxima [A] time = 0.99289, size = 97, normalized size = 1.

$$\frac{126\,b\sin(dx+c)^{10}+140\,a\sin(dx+c)^9-315\,b\sin(dx+c)^8-360\,a\sin(dx+c)^7+210\,b\sin(dx+c)^6+252\,a\sin(d}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1260*(126*b*sin(d*x + c)^10 + 140*a*sin(d*x + c)^9 - 315*b*sin(d*x + c)^8
 - 360*a*sin(d*x + c)^7 + 210*b*sin(d*x + c)^6 + 252*a*sin(d*x + c)^5)/d
```

Fricas [A] time = 2.02855, size = 257, normalized size = 2.65

$$-\frac{126\,b\cos\left(dx+c\right)^{10}-315\,b\cos\left(dx+c\right)^{8}+210\,b\cos\left(dx+c\right)^{6}-4\left(35\,a\cos\left(dx+c\right)^{8}-50\,a\cos\left(dx+c\right)^{6}+3\,a\cos\left(dx+\right.\right.}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/1260*(126*b*cos(d*x + c)^10 - 315*b*cos(d*x + c)^8 + 210*b*cos(d*x + c)^
6 - 4*(35*a*cos(d*x + c)^8 - 50*a*cos(d*x + c)^6 + 3*a*cos(d*x + c)^4 + 4*a
*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d
```

Sympy [A] time = 33.5514, size = 136, normalized size = 1.4

$$\begin{cases}\frac{8a\sin^{9}(c+dx)}{315d}+\frac{4a\sin^{7}(c+dx)\cos^{2}(c+dx)}{35d}+\frac{a\sin^{5}(c+dx)\cos^{4}(c+dx)}{5d}-\frac{b\sin^{4}(c+dx)\cos^{6}(c+dx)}{6d}-\frac{b\sin^{2}(c+dx)\cos^{8}(c+dx)}{12d}-\frac{b\cos^{10}(c+dx)}{60d} & \text{fo} \\ x\left(a+b\sin\left(c\right)\right)\sin^{4}\left(c\right)\cos^{5}\left(c\right) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**9/(315*d) + 4*a*sin(c + d*x)**7*cos(c + d*x)**
2/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - b*sin(c + d*x)**4*cos(
c + d*x)**6/(6*d) - b*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - b*cos(c + d*
x)**10/(60*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**4*cos(c)**5, True))
```

Giac [A] time = 1.21775, size = 159, normalized size = 1.64

$$-\frac{b\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,b\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\,b\cos\left(2\,dx+2\,c\right)}{512\,d}+\frac{a\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{a\sin\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a\sin\left(5\,dx+5\right.}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/5120*b*cos(10*d*x + 10*c)/d + 5/3072*b*cos(6*d*x + 6*c)/d - 5/512*b*cos(
2*d*x + 2*c)/d + 1/2304*a*sin(9*d*x + 9*c)/d + 1/1792*a*sin(7*d*x + 7*c)/d
- 1/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 3/128*a*sin(d*x
 + c)/d
```

3.1200 $\int \cos^5(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^9(c+dx)}{9d}-\frac{2b\sin^7(c+dx)}{7d}+\frac{b\sin^5(c+dx)}{5d}$$

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Cos[c + d*x]^8)/(8*d) + (b*Sin[c + d*x]^5)/(
5*d) - (2*b*Sin[c + d*x]^7)/(7*d) + (b*Sin[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.135553, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2565, 14, 2564, 270}

$$\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^9(c+dx)}{9d}-\frac{2b\sin^7(c+dx)}{7d}+\frac{b\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Cos[c + d*x]^8)/(8*d) + (b*Sin[c + d*x]^5)/(
5*d) - (2*b*Sin[c + d*x]^7)/(7*d) + (b*Sin[c + d*x]^9)/(9*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

IGtQ[p, 0]

Rubi steps

$$\int \cos^5(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^3(c+dx)\,dx + b\int \cos^5(c+dx)\sin^4(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^5\left(1-x^2\right)dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int x^4\left(1-x^2\right)\right.}{d}$$
$$= -\frac{a\,\text{Subst}\left(\int \left(x^5-x^7\right)dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \left(x^4-2x^6+\right.\right.}{d}$$
$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^8(c+dx)}{8d} + \frac{b\sin^5(c+dx)}{5d} - \frac{2b\sin^7(c+dx)}{7d}$$

Mathematica [A] time = 0.276294, size = 105, normalized size = 1.3

$$\frac{-7560a\cos(2(c+dx)) - 1260a\cos(4(c+dx)) + 840a\cos(6(c+dx)) + 315a\cos(8(c+dx)) + 7560b\sin(c+dx) - 1680b\,\text{s}}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-7560*a*Cos[2*(c + d*x)] - 1260*a*Cos[4*(c + d*x)] + 840*a*Cos[6*(c + d*x)
] + 315*a*Cos[8*(c + d*x)] + 7560*b*Sin[c + d*x] - 1680*b*Sin[3*(c + d*x)]
- 1008*b*Sin[5*(c + d*x)] + 180*b*Sin[7*(c + d*x)] + 140*b*Sin[9*(c + d*x)]
)/(322560*d)
```

Maple [A] time = 0.033, size = 102, normalized size = 1.3

$$\frac{1}{d}\left(a\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{8} - \frac{(\cos(dx+c))^6}{24}\right) + b\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^6}{9} - \frac{\sin(dx+c)(\cos(dx+c))}{21}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+b*(-1/9*sin(d*x+c
)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 0.97573, size = 97, normalized size = 1.2

$$\frac{280\,b\sin(dx+c)^9 + 315\,a\sin(dx+c)^8 - 720\,b\sin(dx+c)^7 - 840\,a\sin(dx+c)^6 + 504\,b\sin(dx+c)^5 + 630\,a\sin(dx+}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2520*(280*b*sin(d*x + c)^9 + 315*a*sin(d*x + c)^8 - 720*b*sin(d*x + c)^7
- 840*a*sin(d*x + c)^6 + 504*b*sin(d*x + c)^5 + 630*a*sin(d*x + c)^4)/d
```

Fricas [A] time = 1.79075, size = 223, normalized size = 2.75

$$\frac{315\,a\cos\left(dx+c\right)^{8}-420\,a\cos\left(dx+c\right)^{6}+8\left(35\,b\cos\left(dx+c\right)^{8}-50\,b\cos\left(dx+c\right)^{6}+3\,b\cos\left(dx+c\right)^{4}+4\,b\cos\left(dx+\right.\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*cos(d*x + c)^8 - 420*a*cos(d*x + c)^6 + 8*(35*b*cos(d*x + c)^
8 - 50*b*cos(d*x + c)^6 + 3*b*cos(d*x + c)^4 + 4*b*cos(d*x + c)^2 + 8*b)*si
n(d*x + c))/d
```

Sympy [A] time = 21.508, size = 136, normalized size = 1.68

$$\begin{cases} \frac{a\sin^{8}(c+dx)}{24d}+\frac{a\sin^{6}(c+dx)\cos^{2}(c+dx)}{6d}+\frac{a\sin^{4}(c+dx)\cos^{4}(c+dx)}{4d}+\frac{8b\sin^{9}(c+dx)}{315d}+\frac{4b\sin^{7}(c+dx)\cos^{2}(c+dx)}{35d}+\frac{b\sin^{5}(c+dx)\cos^{4}(c+dx)}{5d} \\ x\left(a+b\sin\left(c\right)\right)\sin^{3}\left(c\right)\cos^{5}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**8/(24*d) + a*sin(c + d*x)**6*cos(c + d*x)**2/(6*
d) + a*sin(c + d*x)**4*cos(c + d*x)**4/(4*d) + 8*b*sin(c + d*x)**9/(315*d)
+ 4*b*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + b*sin(c + d*x)**5*cos(c + d*
x)**4/(5*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**3*cos(c)**5, True))
```

Giac [A] time = 1.21713, size = 180, normalized size = 2.22

$$\frac{a\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{a\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{3\,a\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{b\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{b\sin\left(7\,dx+7\,c\right.}{1792\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*a*cos(8*d*x + 8*c)/d + 1/384*a*cos(6*d*x + 6*c)/d - 1/256*a*cos(4*d*
x + 4*c)/d - 3/128*a*cos(2*d*x + 2*c)/d + 1/2304*b*sin(9*d*x + 9*c)/d + 1/1
792*b*sin(7*d*x + 7*c)/d - 1/320*b*sin(5*d*x + 5*c)/d - 1/192*b*sin(3*d*x +
 3*c)/d + 3/128*b*sin(d*x + c)/d
```

3.1201 $\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{b\cos^8(c+dx)}{8d}-\frac{b\cos^6(c+dx)}{6d}$$

```
[Out] -(b*Cos[c + d*x]^6)/(6*d) + (b*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(
3*d) - (2*a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.139311, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2564, 270, 2565, 14}

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{b\cos^8(c+dx)}{8d}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^6)/(6*d) + (b*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(
3*d) - (2*a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^2(c+dx)\,dx + b\int \cos^5(c+dx)\sin^3(c+dx)\,
$$
$$= \frac{a\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \sin(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int x^5\left(1-x\right.\right.}{}$$
$$= \frac{a\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sin(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \left(x^5-\right.\right.}{}$$
$$= -\frac{b\cos^6(c+dx)}{6d} + \frac{b\cos^8(c+dx)}{8d} + \frac{a\sin^3(c+dx)}{3d} - \frac{2a\sin^5(c+}{5d}$$

Mathematica [A] time = 0.293995, size = 94, normalized size = 1.16

$$-\frac{-8400a\sin(c+dx)+560a\sin(3(c+dx))+1008a\sin(5(c+dx))+240a\sin(7(c+dx))+2520b\cos(2(c+dx))+42}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(2520*b*Cos[2*(c + d*x)] + 420*b*Cos[4*(c + d*x)] - 280*b*Cos[6*(c + d*x)]
 - 105*b*Cos[8*(c + d*x)] - 8400*a*Sin[c + d*x] + 560*a*Sin[3*(c + d*x)] +
1008*a*Sin[5*(c + d*x)] + 240*a*Sin[7*(c + d*x)])/(107520*d)
```

Maple [A] time = 0.03, size = 84, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)\right)+b\left(-\frac{(\sin(dx+c))^2(\cos}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))+b*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

Maxima [A] time = 0.988326, size = 97, normalized size = 1.2

$$\frac{105\,b\sin(dx+c)^8+120\,a\sin(dx+c)^7-280\,b\sin(dx+c)^6-336\,a\sin(dx+c)^5+210\,b\sin(dx+c)^4+280\,a\sin(dx}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(105*b*sin(d*x + c)^8 + 120*a*sin(d*x + c)^7 - 280*b*sin(d*x + c)^6 -
 336*a*sin(d*x + c)^5 + 210*b*sin(d*x + c)^4 + 280*a*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.7457, size = 192, normalized size = 2.37

$$\frac{105\,b\cos\left(dx+c\right)^{8}-140\,b\cos\left(dx+c\right)^{6}-8\left(15\,a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}-8\,a\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/840*(105*b*cos(d*x + c)^8 - 140*b*cos(d*x + c)^6 - 8*(15*a*cos(d*x + c)^6
 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

Sympy [A] time = 12.5143, size = 136, normalized size = 1.68

$$\begin{cases} \frac{8a\sin^{7}(c+dx)}{105d}+\frac{4a\sin^{5}(c+dx)\cos^{2}(c+dx)}{15d}+\frac{a\sin^{3}(c+dx)\cos^{4}(c+dx)}{3d}+\frac{b\sin^{8}(c+dx)}{24d}+\frac{b\sin^{6}(c+dx)\cos^{2}(c+dx)}{6d}+\frac{b\sin^{4}(c+dx)\cos^{4}(c+dx)}{4d} & \text{for} \\ x\left(a+b\sin\left(c\right)\right)\sin^{2}\left(c\right)\cos^{5}\left(c\right) & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**7/(105*d) + 4*a*sin(c + d*x)**5*cos(c + d*x)**
2/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) + b*sin(c + d*x)**8/(24*
d) + b*sin(c + d*x)**6*cos(c + d*x)**2/(6*d) + b*sin(c + d*x)**4*cos(c + d*
x)**4/(4*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**2*cos(c)**5, True))
```

Giac [A] time = 1.39575, size = 159, normalized size = 1.96

$$\frac{b\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{b\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{b\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{3\,b\cos\left(2\,dx+2\,c\right)}{128\,d}-\frac{a\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{3\,a\sin\left(5\,dx+5\,c\right)}{320\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*b*cos(8*d*x + 8*c)/d + 1/384*b*cos(6*d*x + 6*c)/d - 1/256*b*cos(4*d*
x + 4*c)/d - 3/128*b*cos(2*d*x + 2*c)/d - 1/448*a*sin(7*d*x + 7*c)/d - 3/32
0*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 5/64*a*sin(d*x + c)/d
```

3.1202 $\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^7(c+dx)}{7d}-\frac{2b\sin^5(c+dx)}{5d}+\frac{b\sin^3(c+dx)}{3d}$$

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (b*Sin[c + d*x]^3)/(3*d) - (2*b*Sin[c + d*x]^5)
/(5*d) + (b*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0977145, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2834, 2565, 30, 2564, 270}

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^7(c+dx)}{7d}-\frac{2b\sin^5(c+dx)}{5d}+\frac{b\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (b*Sin[c + d*x]^3)/(3*d) - (2*b*Sin[c + d*x]^5)
/(5*d) + (b*Sin[c + d*x]^7)/(7*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin(c+dx)\,dx + b\int \cos^5(c+dx)\sin^2(c+dx)\,dx$$

$$= -\frac{a\,\text{Subst}\left(\int x^5\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \text{si}\right.}{d}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{b\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{b\sin^3(c+dx)}{3d} - \frac{2b\sin^5(c+dx)}{5d} + \frac{b\sin^7(c+dx)}{7d}$$

Mathematica [A] time = 0.210732, size = 86, normalized size = 1.32

$$-\frac{525a\cos(2(c+dx))+210a\cos(4(c+dx))+35a\cos(6(c+dx))+350a-525b\sin(c+dx)+35b\sin(3(c+dx))+63b\,\text{si}}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(350*a + 525*a*Cos[2*(c + d*x)] + 210*a*Cos[4*(c + d*x)] + 35*a*Cos[6*(c +
 d*x)] - 525*b*Sin[c + d*x] + 35*b*Sin[3*(c + d*x)] + 63*b*Sin[5*(c + d*x)]
 + 15*b*Sin[7*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.027, size = 64, normalized size = 1.

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^6}{6}+b\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*a*cos(d*x+c)^6+b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+
c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 0.955289, size = 97, normalized size = 1.49

$$\frac{30\,b\sin(dx+c)^7+35\,a\sin(dx+c)^6-84\,b\sin(dx+c)^5-105\,a\sin(dx+c)^4+70\,b\sin(dx+c)^3+105\,a\sin(dx+c)^2}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(30*b*sin(d*x + c)^7 + 35*a*sin(d*x + c)^6 - 84*b*sin(d*x + c)^5 - 10
5*a*sin(d*x + c)^4 + 70*b*sin(d*x + c)^3 + 105*a*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.6543, size = 161, normalized size = 2.48

$$-\frac{35\,a\cos\left(dx+c\right)^{6}+2\,\left(15\,b\cos\left(dx+c\right)^{6}-3\,b\cos\left(dx+c\right)^{4}-4\,b\cos\left(dx+c\right)^{2}-8\,b\right)\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/210*(35*a*cos(d*x + c)^6 + 2*(15*b*cos(d*x + c)^6 - 3*b*cos(d*x + c)^4 -
 4*b*cos(d*x + c)^2 - 8*b)*sin(d*x + c))/d
```

Sympy [A] time = 7.31357, size = 90, normalized size = 1.38

$$\begin{cases} -\frac{a\cos^{6}\left(c+dx\right)}{6d}+\frac{8b\sin^{7}\left(c+dx\right)}{105d}+\frac{4b\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{15d}+\frac{b\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{3d} & \text{for } d\neq 0 \\ x\left(a+b\sin\left(c\right)\right)\sin\left(c\right)\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)**6/(6*d) + 8*b*sin(c + d*x)**7/(105*d) + 4*b*sin
(c + d*x)**5*cos(c + d*x)**2/(15*d) + b*sin(c + d*x)**3*cos(c + d*x)**4/(3*
d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)*cos(c)**5, True))
```

Giac [A] time = 1.22036, size = 139, normalized size = 2.14

$$-\frac{a\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{64\,d}-\frac{b\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{3\,b\sin\left(5\,dx+5\,c\right)}{320\,d}-\frac{b\sin\left(3\,dx+3\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*a*cos(6*d*x + 6*c)/d - 1/32*a*cos(4*d*x + 4*c)/d - 5/64*a*cos(2*d*x
+ 2*c)/d - 1/448*b*sin(7*d*x + 7*c)/d - 3/320*b*sin(5*d*x + 5*c)/d - 1/192*
b*sin(3*d*x + 3*c)/d + 5/64*b*sin(d*x + c)/d
```

3.1203 $\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*b*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (b*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0807859, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2837, 12, 766}

$$\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*b*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (b*Sin[c + d*x]^5)/(5*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx&=\frac{\text{Subst}\left(\int \frac{b(a+x)\left(b^2-x^2\right)^2}{x}\,dx,x,b\sin(c+dx)\right)}{b^5d}\\&=\frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x}\,dx,x,b\sin(c+dx)\right)}{b^4d}\\&=\frac{\text{Subst}\left(\int \left(b^4+\frac{ab^4}{x}-2ab^2x-2b^2x^2+ax^3+x^4\right)dx,x,b\sin(c+dx)\right)}{b^4d}\\&=\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-\frac{2b\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0295842, size = 86, normalized size = 1.

$$\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*b*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (b*Sin[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.053, size = 94, normalized size = 1.1

$$\frac{a(\cos(dx+c))^4}{4d}+\frac{a(\cos(dx+c))^2}{2d}+\frac{a\ln(\sin(dx+c))}{d}+\frac{8b\sin(dx+c)}{15d}+\frac{\sin(dx+c)(\cos(dx+c))^4 b}{5d}+\frac{4(\cos(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4*a*cos(d*x+c)^4/d+1/2*a*cos(d*x+c)^2/d+a*ln(sin(d*x+c))/d+8/15*b*sin(d*x
+c)/d+1/5/d*cos(d*x+c)^4*sin(d*x+c)*b+4/15/d*b*sin(d*x+c)*cos(d*x+c)^2
```

Maxima [A] time = 0.981917, size = 93, normalized size = 1.08

$$\frac{12b\sin(dx+c)^5+15a\sin(dx+c)^4-40b\sin(dx+c)^3-60a\sin(dx+c)^2+60a\log(\sin(dx+c))+60b\sin(dx+\ldots}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*b*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(sin(d*x + c)) + 60*b*sin(d*x + c))/d
```

Fricas [A] time = 1.77281, size = 197, normalized size = 2.29

$$\frac{15a\cos(dx+c)^4+30a\cos(dx+c)^2+60a\log\left(\frac{1}{2}\sin(dx+c)\right)+4\left(3b\cos(dx+c)^4+4b\cos(dx+c)^2+8b\right)\sin(d\ldots}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^2 + 60*a*log(1/2*sin(d*x + c)
) + 4*(3*b*cos(d*x + c)^4 + 4*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33094, size = 95, normalized size = 1.1

$$\frac{12\,b\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-40\,b\sin\left(dx+c\right)^{3}-60\,a\sin\left(dx+c\right)^{2}+60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,b\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(12*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*b*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(abs(sin(d*x + c))) + 60*b*sin(d*x + c))/d
```

3.1204 $\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{b\sin^4(c+dx)}{4d}-\frac{b\sin^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) + (b*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (b
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0926954, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{b\sin^4(c+dx)}{4d}-\frac{b\sin^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (b*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (b
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^4)/(4*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{b^2(a+x)\left(b^2-x^2\right)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}\\ &= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^3 d}\\ &= \frac{\text{Subst}\left(\int \left(-2ab^2+\frac{ab^4}{x^2}+\frac{b^4}{x}-2b^2x+ax^2+x^3\right)dx, x, b\sin(c+d\right.}{b^3 d}\\ &= -\frac{a\csc(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}-\frac{b\sin^2(c+}{d}\end{aligned}$$

Mathematica [A] time = 0.0301926, size = 83, normalized size = 1.

$$\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{b\sin^4(c+dx)}{4d}-\frac{b\sin^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (b*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (b
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.055, size = 116, normalized size = 1.4

$$-\frac{a(\cos(dx+c))^6}{d\sin(dx+c)}-\frac{8a\sin(dx+c)}{3d}-\frac{\sin(dx+c)(\cos(dx+c))^4a}{d}-\frac{4a\sin(dx+c)(\cos(dx+c))^2}{3d}+\frac{b(\cos(dx+c))^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^6-8/3*a*sin(d*x+c)/d-1/d*cos(d*x+c)^4*sin(d*x+
c)*a-4/3/d*a*sin(d*x+c)*cos(d*x+c)^2+1/4/d*b*cos(d*x+c)^4+1/2*b*cos(d*x+c)^
2/d+b*ln(sin(d*x+c))/d
```

Maxima [A] time = 0.971597, size = 93, normalized size = 1.12

$$\frac{3b\sin(dx+c)^4+4a\sin(dx+c)^3-12b\sin(dx+c)^2+12b\log(\sin(dx+c))-24a\sin(dx+c)-\frac{12a}{\sin(dx+c)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*b*sin(d*x + c)^2 + 12*b*
log(sin(d*x + c)) - 24*a*sin(d*x + c) - 12*a/sin(d*x + c))/d
```

Fricas [A] time = 1.75939, size = 250, normalized size = 3.01

$$\frac{32a\cos(dx+c)^4+128a\cos(dx+c)^2+96b\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+3\left(8b\cos(dx+c)^4+16b\cos(dx+c)^2-\right.}{96d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(32*a*cos(d*x + c)^4 + 128*a*cos(d*x + c)^2 + 96*b*log(1/2*sin(d*x + c
))*sin(d*x + c) + 3*(8*b*cos(d*x + c)^4 + 16*b*cos(d*x + c)^2 - 11*b)*sin(d
*x + c) - 256*a)/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29756, size = 107, normalized size = 1.29

$$\frac{3\,b\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-12\,b\sin\left(dx+c\right)^{2}+12\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)-24\,a\sin\left(dx+c\right)-\frac{12\,(b\sin(dx+c)+a)}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*b*sin(d*x + c)^2 + 12*b*
log(abs(sin(d*x + c))) - 24*a*sin(d*x + c) - 12*(b*sin(d*x + c) + a)/sin(d*
x + c))/d
```

3.1205 $\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\sin^2(c+dx)}{2d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}+\frac{b\sin^3(c+dx)}{3d}-\frac{2b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

```
[Out] -((b*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (2*a*Log[Sin[c + d*x]])/
d - (2*b*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d) + (b*Sin[c + d*x]^3)/(3
*d)
```

Rubi [A] time = 0.0917852, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$\frac{a\sin^2(c+dx)}{2d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}+\frac{b\sin^3(c+dx)}{3d}-\frac{2b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (2*a*Log[Sin[c + d*x]])/
d - (2*b*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d) + (b*Sin[c + d*x]^3)/(3
*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^3(a+x)\left(b^2-x^2\right)^2}{x^3}dx,x,b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^3}dx,x,b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-2b^2+\frac{ab^4}{x^3}+\frac{b^4}{x^2}-\frac{2ab^2}{x}+ax+x^2\right)dx,x,b\sin(c+dx)\right)}{b^2d}$$
$$= -\frac{b\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}-\frac{2b\sin(c}{d}$$

Mathematica [A] time = 0.195719, size = 77, normalized size = 0.9

$$-\frac{a\left(-\sin^2(c+dx)+\csc^2(c+dx)+4\log(\sin(c+dx))\right)}{2d}+\frac{b\sin^3(c+dx)}{3d}-\frac{2b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (2*b*Sin[c + d*x])/d + (b*Sin[c + d*x]^3)/(3*d) - (
a*(Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.056, size = 139, normalized size = 1.6

$$-\frac{a(\cos(dx+c))^6}{2d(\sin(dx+c))^2}-\frac{a(\cos(dx+c))^4}{2d}-\frac{a(\cos(dx+c))^2}{d}-2\frac{a\ln(\sin(dx+c))}{d}-\frac{b(\cos(dx+c))^6}{d\sin(dx+c)}-\frac{8b\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^6-1/2*a*cos(d*x+c)^4/d-a*cos(d*x+c)^2/d-2*
a*ln(sin(d*x+c))/d-1/d*b/sin(d*x+c)*cos(d*x+c)^6-8/3*b*sin(d*x+c)/d-1/d*cos
(d*x+c)^4*sin(d*x+c)*b-4/3/d*b*sin(d*x+c)*cos(d*x+c)^2
```

Maxima [A] time = 0.978325, size = 92, normalized size = 1.07

$$\frac{2b\sin(dx+c)^3+3a\sin(dx+c)^2-12a\log(\sin(dx+c))-12b\sin(dx+c)-\frac{3(2b\sin(dx+c)+a)}{\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(sin(d*x + c)) - 12*
b*sin(d*x + c) - 3*(2*b*sin(d*x + c) + a)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.72722, size = 258, normalized size = 3.

$$-\frac{6\,a\cos\left(dx+c\right)^{4}-9\,a\cos\left(dx+c\right)^{2}+24\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\left(b\cos\left(dx+c\right)^{4}+4\,b\cos\left(dx+c\right)^{2}\right.}{12\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*cos(d*x + c)^4 - 9*a*cos(d*x + c)^2 + 24*(a*cos(d*x + c)^2 - a)*
log(1/2*sin(d*x + c)) + 4*(b*cos(d*x + c)^4 + 4*b*cos(d*x + c)^2 - 8*b)*sin
(d*x + c) - 3*a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2354, size = 111, normalized size = 1.29

$$\frac{2\,b\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-12\,b\sin\left(dx+c\right)+\frac{3\left(6\,a\sin\left(dx+c\right)^{2}-2\,b\sin\left(dx+c\right)-a\right)}{\sin\left(dx+c\right)^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(abs(sin(d*x + c)))
- 12*b*sin(d*x + c) + 3*(6*a*sin(d*x + c)^2 - 2*b*sin(d*x + c) - a)/sin(d*x
 + c)^2)/d
```

3.1206 $\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}-\frac{b\csc^2(c+dx)}{2d}-\frac{2b\log(\sin(c+dx))}{d}$$

```
[Out] (2*a*Csc[c + d*x])/d - (b*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (2*b*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d + (b*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0850271, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2837, 12, 766}

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}-\frac{b\csc^2(c+dx)}{2d}-\frac{2b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (b*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (2*b*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{b^4(a+x)\left(b^2-x^2\right)^2}{x^4}\,dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^4}\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(a+\frac{ab^4}{x^4}+\frac{b^4}{x^3}-\frac{2ab^2}{x^2}-\frac{2b^2}{x}+x\right)dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{2a\csc(c+dx)}{d}-\frac{b\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{2b\log(\sin(c+\\}{d}\end{aligned}$$

Mathematica [A] time = 0.145554, size = 76, normalized size = 0.89

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}-\frac{b\left(-\sin^2(c+dx)+\csc^2(c+dx)+4\log(\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (b*(
Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.059, size = 159, normalized size = 1.9

$$-\frac{a(\cos(dx+c))^6}{3d(\sin(dx+c))^3}+\frac{a(\cos(dx+c))^6}{d\sin(dx+c)}+\frac{8a\sin(dx+c)}{3d}+\frac{\sin(dx+c)(\cos(dx+c))^4a}{d}+\frac{4a\sin(dx+c)(\cos(dx+c))^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a/sin(d*x+c)*cos(d*x+c)^6+8/3*a*sin(
d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*a+4/3/d*a*sin(d*x+c)*cos(d*x+c)^2-1/2/
d*b/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*b*cos(d*x+c)^4-b*cos(d*x+c)^2/d-2*b*ln(
sin(d*x+c))/d
```

Maxima [A] time = 1.00204, size = 93, normalized size = 1.09

$$\frac{3b\sin(dx+c)^2-12b\log(\sin(dx+c))+6a\sin(dx+c)+\frac{12a\sin(dx+c)^2-3b\sin(dx+c)-2a}{\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*b*sin(d*x + c)^2 - 12*b*log(sin(d*x + c)) + 6*a*sin(d*x + c) + (12*a
*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/sin(d*x + c)^3)/d
```

Fricas [A] time = 1.73743, size = 300, normalized size = 3.53

$$-\frac{12a\cos(dx+c)^4-48a\cos(dx+c)^2+24\left(b\cos(dx+c)^2-b\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+3\left(2b\cos(dx+c)^4-\right.}{12\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^4 - 48*a*cos(d*x + c)^2 + 24*(b*cos(d*x + c)^2 - b
)*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(2*b*cos(d*x + c)^4 - 3*b*cos(d*x
```

```
+ c)^2 - b)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19465, size = 109, normalized size = 1.28

$$\frac{3\,b\sin\left(dx+c\right)^{2}-12\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)+6\,a\sin\left(dx+c\right)+\frac{22\,b\sin(dx+c)^{3}+12\,a\sin(dx+c)^{2}-3\,b\sin(dx+c)-2\,a}{\sin(dx+c)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*b*sin(d*x + c)^2 - 12*b*log(abs(sin(d*x + c))) + 6*a*sin(d*x + c) +
(22*b*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/sin(d*
x + c)^3)/d
```

3.1207 $\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

```
[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rubi [A] time = 0.0529924, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 766}

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{ab^4}{x^5}+\frac{b^4}{x^4}-\frac{2ab^2}{x^3}-\frac{2b^2}{x^2}+\frac{a}{x}\right)dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{2b\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin(c-}{d}\end{aligned}$$

Mathematica [A] time = 0.243567, size = 87, normalized size = 1.07

$$\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(c+dx))\right)}{4d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (b*Sin[c
 + d*x])/d
```

Maple [A] time = 0.063, size = 136, normalized size = 1.7

$$-\frac{a(\cot(dx+c))^4}{4d}+\frac{a(\cot(dx+c))^2}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{b(\cos(dx+c))^6}{3d(\sin(dx+c))^3}+\frac{b(\cos(dx+c))^6}{d\sin(dx+c)}+\frac{8b\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a*cot(d*x+c)^4+1/2*a*cot(d*x+c)^2/d+a*ln(sin(d*x+c))/d-1/3/d*b/sin(d
*x+c)^3*cos(d*x+c)^6+1/d*b/sin(d*x+c)*cos(d*x+c)^6+8/3*b*sin(d*x+c)/d+1/d*c
os(d*x+c)^4*sin(d*x+c)*b+4/3/d*b*sin(d*x+c)*cos(d*x+c)^2
```

Maxima [A] time = 0.995989, size = 93, normalized size = 1.15

$$\frac{12a\log(\sin(dx+c))+12b\sin(dx+c)+\frac{24b\sin(dx+c)^3+12a\sin(dx+c)^2-4b\sin(dx+c)-3a}{\sin(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*b*sin(d*x + c) + (24*b*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*b*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.70965, size = 292, normalized size = 3.6

$$-\frac{12a\cos(dx+c)^2-12\left(a\cos(dx+c)^4-2a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-4\left(3b\cos(dx+c)^4-12b\cos(dx+c)^2+8b\right)\sin(dx+c)-9a}{12\left(d\cos(dx+c)^4-2d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*b*cos(d*x + c)^4 - 12*b*cos(d*x + c)^2 + 8*b)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19303, size = 111, normalized size = 1.37

$$\frac{12\,a\log\left(|\sin\left(dx+c\right)|\right)+12\,b\sin\left(dx+c\right)-\frac{25\,a\sin(dx+c)^4-24\,b\sin(dx+c)^3-12\,a\sin(dx+c)^2+4\,b\sin(dx+c)+3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*b*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*b*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*b*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

3.1208 $\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}-\frac{b\csc^4(c+dx)}{4d}+\frac{b\csc^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) + (b*Csc[c + d*x]^2)/d + (2*a*Csc[c + d*x]^3)/(3*d) -
 (b*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (b*Log[Sin[c + d*x]]
)/d
```

Rubi [A] time = 0.0835282, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2837, 12, 766}

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}-\frac{b\csc^4(c+dx)}{4d}+\frac{b\csc^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```


```
[Out] -((a*Csc[c + d*x])/d) + (b*Csc[c + d*x]^2)/d + (2*a*Csc[c + d*x]^3)/(3*d) -
 (b*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (b*Log[Sin[c + d*x]]
)/d
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^6(a+x)\left(b^2-x^2\right)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b\,\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(\frac{ab^4}{x^6}+\frac{b^4}{x^5}-\frac{2ab^2}{x^4}-\frac{2b^2}{x^3}+\frac{a}{x^2}+\frac{1}{x}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{b\csc^2(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{b\csc^4(c+dx)}{4d}-$$

Mathematica [A] time = 0.173383, size = 92, normalized size = 1.07

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{b\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*
d) + (b*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Ta
n[c + d*x]]))/(4*d)
```

Maple [A] time = 0.065, size = 160, normalized size = 1.9

$$-\frac{a(\cos(dx+c))^6}{5d(\sin(dx+c))^5}+\frac{a(\cos(dx+c))^6}{15d(\sin(dx+c))^3}-\frac{a(\cos(dx+c))^6}{5d\sin(dx+c)}-\frac{8a\sin(dx+c)}{15d}-\frac{\sin(dx+c)(\cos(dx+c))^4a}{5d}-\frac{4a\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^6+1/15/d*a/sin(d*x+c)^3*cos(d*x+c)^6-1/5/d
*a/sin(d*x+c)*cos(d*x+c)^6-8/15*a*sin(d*x+c)/d-1/5/d*cos(d*x+c)^4*sin(d*x+c
)*a-4/15/d*a*sin(d*x+c)*cos(d*x+c)^2-1/4/d*b*cot(d*x+c)^4+1/2*b*cot(d*x+c)^
2/d+b*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.00327, size = 97, normalized size = 1.13

$$\frac{60\,b\log(\sin(dx+c))-\frac{60\,a\sin(dx+c)^4-60\,b\sin(dx+c)^3-40\,a\sin(dx+c)^2+15\,b\sin(dx+c)+12\,a}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*b*log(sin(d*x + c)) - (60*a*sin(d*x + c)^4 - 60*b*sin(d*x + c)^3 -
 40*a*sin(d*x + c)^2 + 15*b*sin(d*x + c) + 12*a)/sin(d*x + c)^5)/d
```

Fricas [A] time = 1.74423, size = 332, normalized size = 3.86

$$-\frac{60\,a\cos\left(dx+c\right)^{4}-80\,a\cos\left(dx+c\right)^{2}-60\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+}{60\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^4 - 80*a*cos(d*x + c)^2 - 60*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(1/2*sin(d*x + c))*sin(d*x + c) + 15*(4*b*cos(d*x
 + c)^2 - 3*b)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2
 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19506, size = 113, normalized size = 1.31

$$\frac{60\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{137\,b\sin\left(dx+c\right)^{5}+60\,a\sin\left(dx+c\right)^{4}-60\,b\sin\left(dx+c\right)^{3}-40\,a\sin\left(dx+c\right)^{2}+15\,b\sin\left(dx+c\right)+12\,a}{\sin\left(dx+c\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(sin(d*x + c))) - (137*b*sin(d*x + c)^5 + 60*a*sin(d*x +
c)^4 - 60*b*sin(d*x + c)^3 - 40*a*sin(d*x + c)^2 + 15*b*sin(d*x + c) + 12*a
)/sin(d*x + c)^5)/d
```

3.1209 $\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=61

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^5(c+dx)}{5d}+\frac{2b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (b*Csc[c + d*x])/d + (2*b*Csc[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.112362, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 30, 2606, 194}

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^5(c+dx)}{5d}+\frac{2b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (b*Csc[c + d*x])/d + (2*b*Csc[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x]^5)/(5*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^2(c+dx)\,dx + b\int \cot^5(c+dx)\csc(c+dx)\,d$$

$$= -\frac{a\,\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,\right.}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{b\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{b\csc(c+dx)}{d} + \frac{2b\csc^3(c+dx)}{3d} - \frac{b\csc^5(c+d}{5d}$$

Mathematica [A] time = 0.0256761, size = 61, normalized size = 1.

$$-\frac{a\cot^6(c+dx)}{6d} - \frac{b\csc^5(c+dx)}{5d} + \frac{2b\csc^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (b*Csc[c + d*x])/d + (2*b*Csc[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.063, size = 110, normalized size = 1.8

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^6}{6(\sin(dx+c))^6} + b\left(-\frac{(\cos(dx+c))^6}{5(\sin(dx+c))^5} + \frac{(\cos(dx+c))^6}{15(\sin(dx+c))^3} - \frac{(\cos(dx+c))^6}{5\sin(dx+c)} - \frac{\sin(dx+c)}{5}\left(\frac{8}{3} + (\cos(dx+c)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*a/sin(d*x+c)^6*cos(d*x+c)^6+b*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/1
5/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)
^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 0.99776, size = 95, normalized size = 1.56

$$-\frac{30\,b\sin(dx+c)^5 + 15\,a\sin(dx+c)^4 - 20\,b\sin(dx+c)^3 - 15\,a\sin(dx+c)^2 + 6\,b\sin(dx+c) + 5\,a}{30\,d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(30*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*b*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*b*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

Fricas [A] time = 1.58444, size = 254, normalized size = 4.16

$$\frac{15\,a\cos\left(dx+c\right)^4-15\,a\cos\left(dx+c\right)^2+2\left(15\,b\cos\left(dx+c\right)^4-20\,b\cos\left(dx+c\right)^2+8\,b\right)\sin\left(dx+c\right)+5\,a}{30\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*cos(d*x + c)^4 - 15*a*cos(d*x + c)^2 + 2*(15*b*cos(d*x + c)^4 -
20*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) + 5*a)/(d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20206, size = 95, normalized size = 1.56

$$-\frac{30\,b\sin\left(dx+c\right)^5+15\,a\sin\left(dx+c\right)^4-20\,b\sin\left(dx+c\right)^3-15\,a\sin\left(dx+c\right)^2+6\,b\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(30*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*b*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*b*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

3.1210 $\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{b\cot^6(c+dx)}{6d}$$

```
[Out] -(b*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.12248, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 30}

$$-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{b\cot^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^3(c+dx)\,dx + b\int \cot^5(c+dx)\csc^2(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int x^5\,dx, x,\right.}{d}$$
$$= -\frac{b\cot^6(c+dx)}{6d} - \frac{a\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \csc(c+dx)\right)}{d}$$
$$= -\frac{b\cot^6(c+dx)}{6d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d}$$

Mathematica [A] time = 0.026354, size = 65, normalized size = 1.

$$-\frac{a\csc^7(c+dx)}{7d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{3d} - \frac{b\cot^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cot[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Maple [B] time = 0.064, size = 128, normalized size = 2.

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{7(\sin(dx+c))^7} - \frac{(\cos(dx+c))^6}{35(\sin(dx+c))^5} + \frac{(\cos(dx+c))^6}{105(\sin(dx+c))^3} - \frac{(\cos(dx+c))^6}{35\sin(dx+c)} - \frac{\sin(dx+c)}{35}\left(\frac{8}{3} + (\cos(dx+c))^4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105
/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c
)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*b/sin(d*x+c)^6*cos(d*x+c)^6)
```

Maxima [A] time = 0.988483, size = 95, normalized size = 1.46

$$-\frac{105\,b\sin(dx+c)^5 + 70\,a\sin(dx+c)^4 - 105\,b\sin(dx+c)^3 - 84\,a\sin(dx+c)^2 + 35\,b\sin(dx+c) + 30\,a}{210\,d\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(105*b*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*b*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*b*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

Fricas [A] time = 1.62298, size = 273, normalized size = 4.2

$$\frac{70\,a\cos\left(dx+c\right)^{4}-56\,a\cos\left(dx+c\right)^{2}+35\,\left(3\,b\cos\left(dx+c\right)^{4}-3\,b\cos\left(dx+c\right)^{2}+b\right)\sin\left(dx+c\right)+16\,a}{210\,\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(70*a*cos(d*x + c)^4 - 56*a*cos(d*x + c)^2 + 35*(3*b*cos(d*x + c)^4 -
 3*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 16*a)/((d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17815, size = 95, normalized size = 1.46

$$-\frac{105\,b\sin\left(dx+c\right)^{5}+70\,a\sin\left(dx+c\right)^{4}-105\,b\sin\left(dx+c\right)^{3}-84\,a\sin\left(dx+c\right)^{2}+35\,b\sin\left(dx+c\right)+30\,a}{210\,d\sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/210*(105*b*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*b*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*b*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

3.1211 $\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^7(c+dx)}{7d}+\frac{2b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}$$

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Cot[c + d*x]^8)/(8*d) - (b*Csc[c + d*x]^3)/(
3*d) + (2*b*Csc[c + d*x]^5)/(5*d) - (b*Csc[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.129311, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 14, 2606, 270}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^7(c+dx)}{7d}+\frac{2b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Cot[c + d*x]^6)/(6*d) - (a*Cot[c + d*x]^8)/(8*d) - (b*Csc[c + d*x]^3)/(
3*d) + (2*b*Csc[c + d*x]^5)/(5*d) - (b*Csc[c + d*x]^7)/(7*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^4(c+dx)\,dx + b\int \cot^5(c+dx)\csc^3(c+dx)$$

$$= -\frac{a\,\text{Subst}\left(\int x^5\left(1+x^2\right)dx, x, -\cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int x^2\left(-\right.\right.}{}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(x^5+x^7\right)dx, x, -\cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \left(x^2-2\right.\right.}{}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\cot^8(c+dx)}{8d} - \frac{b\csc^3(c+dx)}{3d} + \frac{2b\csc^5(c+}{5d}$$

Mathematica [A] time = 0.0997403, size = 88, normalized size = 1.09

$$-\frac{a\left(3\csc^8(c+dx)-8\csc^6(c+dx)+6\csc^4(c+dx)\right)}{24d} - \frac{b\csc^7(c+dx)}{7d} + \frac{2b\csc^5(c+dx)}{5d} - \frac{b\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Csc[c + d*x]^3)/(3*d) + (2*b*Csc[c + d*x]^5)/(5*d) - (b*Csc[c + d*x]^7)
/(7*d) - (a*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

Maple [B] time = 0.067, size = 148, normalized size = 1.8

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{8(\sin(dx+c))^8} - \frac{(\cos(dx+c))^6}{24(\sin(dx+c))^6}\right) + b\left(-\frac{(\cos(dx+c))^6}{7(\sin(dx+c))^7} - \frac{(\cos(dx+c))^6}{35(\sin(dx+c))^5} + \frac{(\cos(dx+c))^6}{105(\sin(dx+c))^3} - \frac{(c}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6)+b*(-
1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105/sin(d*x+
c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 0.972255, size = 95, normalized size = 1.17

$$-\frac{280\,b\sin(dx+c)^5 + 210\,a\sin(dx+c)^4 - 336\,b\sin(dx+c)^3 - 280\,a\sin(dx+c)^2 + 120\,b\sin(dx+c) + 105\,a}{840\,d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*(280*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*b*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*b*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

Fricas [A] time = 1.60943, size = 289, normalized size = 3.57

$$-\frac{210\,a\cos\left(dx+c\right)^{4}-140\,a\cos\left(dx+c\right)^{2}+8\left(35\,b\cos\left(dx+c\right)^{4}-28\,b\cos\left(dx+c\right)^{2}+8\,b\right)\sin\left(dx+c\right)+35\,a}{840\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(210*a*cos(d*x + c)^4 - 140*a*cos(d*x + c)^2 + 8*(35*b*cos(d*x + c)^
4 - 28*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) + 35*a)/(d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**9*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28645, size = 95, normalized size = 1.17

$$-\frac{280\,b\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-336\,b\sin\left(dx+c\right)^{3}-280\,a\sin\left(dx+c\right)^{2}+120\,b\sin\left(dx+c\right)+105\,a}{840\,d\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/840*(280*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*b*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*b*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

3.1212 $\int \cot^5(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{b\cot^8(c+dx)}{8d}-\frac{b\cot^6(c+dx)}{6d}$$

```
[Out] -(b*Cot[c + d*x]^6)/(6*d) - (b*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^5)/(
5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.131018, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2606, 270, 2607, 14}

$$-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{b\cot^8(c+dx)}{8d}-\frac{b\cot^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cot[c + d*x]^6)/(6*d) - (b*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^5)/(
5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^5(c+dx)\,dx + b\int \cot^5(c+dx)\csc^4(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int x^4\left(-1+x^2\right)^2 dx, x, \csc(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int x^5\left(1+\right.\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right) dx, x, \csc(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \left(x^5+\right.\right.}{}$$
$$= -\frac{b\cot^6(c+dx)}{6d} - \frac{b\cot^8(c+dx)}{8d} - \frac{a\csc^5(c+dx)}{5d} + \frac{2a\csc^7(c+dx)}{7d}$$

Mathematica [A] time = 0.111764, size = 88, normalized size = 1.09

$$-\frac{a\csc^9(c+dx)}{9d} + \frac{2a\csc^7(c+dx)}{7d} - \frac{a\csc^5(c+dx)}{5d} - \frac{b\left(3\csc^8(c+dx) - 8\csc^6(c+dx) + 6\csc^4(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^5)/(5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)
/(9*d) - (b*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

Maple [B] time = 0.066, size = 166, normalized size = 2.1

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{9(\sin(dx+c))^9} - \frac{(\cos(dx+c))^6}{21(\sin(dx+c))^7} - \frac{(\cos(dx+c))^6}{105(\sin(dx+c))^5} + \frac{(\cos(dx+c))^6}{315(\sin(dx+c))^3} - \frac{(\cos(dx+c))^6}{105\sin(dx+c)} - \frac{\sin(dx+c}{105}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x+c)^7*cos(d*x+c)^6-1/105
/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d*x+c)^6-1/105/sin(d*x+c)
*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/8
/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6))
```

Maxima [A] time = 0.983895, size = 95, normalized size = 1.17

$$-\frac{630\,b\sin(dx+c)^5 + 504\,a\sin(dx+c)^4 - 840\,b\sin(dx+c)^3 - 720\,a\sin(dx+c)^2 + 315\,b\sin(dx+c) + 280\,a}{2520\,d\sin(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2520*(630*b*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*b*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*b*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

Fricas [A] time = 1.68691, size = 308, normalized size = 3.8

$$-\frac{504\,a\cos\left(dx+c\right)^{4}-288\,a\cos\left(dx+c\right)^{2}+105\left(6\,b\cos\left(dx+c\right)^{4}-4\,b\cos\left(dx+c\right)^{2}+b\right)\sin\left(dx+c\right)+64\,a}{2520\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(504*a*cos(d*x + c)^4 - 288*a*cos(d*x + c)^2 + 105*(6*b*cos(d*x + c
)^4 - 4*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 64*a)/((d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**10*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22367, size = 95, normalized size = 1.17

$$-\frac{630\,b\sin\left(dx+c\right)^{5}+504\,a\sin\left(dx+c\right)^{4}-840\,b\sin\left(dx+c\right)^{3}-720\,a\sin\left(dx+c\right)^{2}+315\,b\sin\left(dx+c\right)+280\,a}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x, algorithm="giac")
```



```
[Out] -1/2520*(630*b*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*b*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*b*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

3.1213 $\int \cot^5(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^{10}(c+dx)}{10d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^6(c+dx)}{6d}-\frac{b\csc^9(c+dx)}{9d}+\frac{2b\csc^7(c+dx)}{7d}-\frac{b\csc^5(c+dx)}{5d}$$

```
[Out] -(b*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/(6*d) + (2*b*Csc[c + d*x]^7)
/(7*d) + (a*Csc[c + d*x]^8)/(4*d) - (b*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d
*x]^10)/(10*d)
```

Rubi [A] time = 0.0949734, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$-\frac{a\csc^{10}(c+dx)}{10d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^6(c+dx)}{6d}-\frac{b\csc^9(c+dx)}{9d}+\frac{2b\csc^7(c+dx)}{7d}-\frac{b\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/(6*d) + (2*b*Csc[c + d*x]^7)
/(7*d) + (a*Csc[c + d*x]^8)/(4*d) - (b*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d
*x]^10)/(10*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^{11}(a+x)\left(b^2-x^2\right)^2}{x^{11}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b^6\,\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^{11}}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^6\,\text{Subst}\left(\int \left(\frac{ab^4}{x^{11}}+\frac{b^4}{x^{10}}-\frac{2ab^2}{x^9}-\frac{2b^2}{x^8}+\frac{a}{x^7}+\frac{1}{x^6}\right)dx, x, b\sin(c+dx\right)}{d}$$
$$= -\frac{b\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}+\frac{2b\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+}{4d}$$

Mathematica [A] time = 0.103278, size = 88, normalized size = 0.91

$$-\frac{a\left(6\csc^{10}(c+dx)-15\csc^8(c+dx)+10\csc^6(c+dx)\right)}{60d}-\frac{b\csc^9(c+dx)}{9d}+\frac{2b\csc^7(c+dx)}{7d}-\frac{b\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Csc[c + d*x]^5)/(5*d) + (2*b*Csc[c + d*x]^7)/(7*d) - (b*Csc[c + d*x]^9)
/(9*d) - (a*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/(6
0*d)
```

Maple [B] time = 0.066, size = 184, normalized size = 1.9

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{10(\sin(dx+c))^{10}}-\frac{(\cos(dx+c))^6}{20(\sin(dx+c))^8}-\frac{(\cos(dx+c))^6}{60(\sin(dx+c))^6}\right)+b\left(-\frac{(\cos(dx+c))^6}{9(\sin(dx+c))^9}-\frac{(\cos(dx+c))^6}{21(\sin(dx+c))^7}-\frac{}{10}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x+c)^8*cos(d*x+c)^6-1/6
0/sin(d*x+c)^6*cos(d*x+c)^6)+b*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x
+c)^7*cos(d*x+c)^6-1/105/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d
*x+c)^6-1/105/sin(d*x+c)*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c)))
```

Maxima [A] time = 0.990077, size = 95, normalized size = 0.98

$$-\frac{252\,b\sin(dx+c)^5+210\,a\sin(dx+c)^4-360\,b\sin(dx+c)^3-315\,a\sin(dx+c)^2+140\,b\sin(dx+c)+126\,a}{1260\,d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/1260*(252*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*b*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*b*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

Fricas [A] time = 1.73697, size = 321, normalized size = 3.31

$$\frac{210\,a\cos\left(dx+c\right)^{4}-105\,a\cos\left(dx+c\right)^{2}+4\left(63\,b\cos\left(dx+c\right)^{4}-36\,b\cos\left(dx+c\right)^{2}+8\,b\right)\sin\left(dx+c\right)+21\,a}{1260\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1260*(210*a*cos(d*x + c)^4 - 105*a*cos(d*x + c)^2 + 4*(63*b*cos(d*x + c)^
4 - 36*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) + 21*a)/(d*cos(d*x + c)^10 - 5*
d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x
+ c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**11*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2479, size = 95, normalized size = 0.98

$$-\frac{252\,b\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-360\,b\sin\left(dx+c\right)^{3}-315\,a\sin\left(dx+c\right)^{2}+140\,b\sin\left(dx+c\right)+126\,a}{1260\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1260*(252*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*b*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*b*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

3.1214 $\int \cot^5(c+dx)\csc^7(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^{11}(c+dx)}{11d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{b\csc^{10}(c+dx)}{10d}+\frac{b\csc^8(c+dx)}{4d}-\frac{b\csc^6(c+dx)}{6d}$$

```
[Out] -(b*Csc[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^7)/(7*d) + (b*Csc[c + d*x]^8)/(
4*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (b*Csc[c + d*x]^10)/(10*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rubi [A] time = 0.0950315, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 766}

$$-\frac{a\csc^{11}(c+dx)}{11d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{b\csc^{10}(c+dx)}{10d}+\frac{b\csc^8(c+dx)}{4d}-\frac{b\csc^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Csc[c + d*x]^6)/(6*d) - (a*Csc[c + d*x]^7)/(7*d) + (b*Csc[c + d*x]^8)/(
4*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (b*Csc[c + d*x]^10)/(10*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^7(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^{12}(a+x)\left(b^2-x^2\right)^2}{x^{12}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^{12}}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^7\,\text{Subst}\left(\int \left(\frac{ab^4}{x^{12}}+\frac{b^4}{x^{11}}-\frac{2ab^2}{x^{10}}-\frac{2b^2}{x^9}+\frac{a}{x^8}+\frac{1}{x^7}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\csc^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{b\csc^8(c+dx)}{4d}+\frac{2a\csc^9(c+dx)}{9d}$$

Mathematica [A] time = 0.0987624, size = 88, normalized size = 0.91

$$-\frac{a\csc^{11}(c+dx)}{11d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{b\left(6\csc^{10}(c+dx)-15\csc^8(c+dx)+10\csc^6(c+dx)\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^7)/(7*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^11
)/(11*d) - (b*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/
(60*d)
```

Maple [B] time = 0.067, size = 202, normalized size = 2.1

$$\frac{1}{d}\left(a\left(-\frac{(\cos(dx+c))^6}{11(\sin(dx+c))^{11}}-\frac{5(\cos(dx+c))^6}{99(\sin(dx+c))^9}-\frac{5(\cos(dx+c))^6}{231(\sin(dx+c))^7}-\frac{(\cos(dx+c))^6}{231(\sin(dx+c))^5}+\frac{(\cos(dx+c))^6}{693(\sin(dx+c))^3}-\frac{(\cos}{231\text{ s}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^6-5/99/sin(d*x+c)^9*cos(d*x+c)^6-5/2
31/sin(d*x+c)^7*cos(d*x+c)^6-1/231/sin(d*x+c)^5*cos(d*x+c)^6+1/693/sin(d*x+
c)^3*cos(d*x+c)^6-1/231/sin(d*x+c)*cos(d*x+c)^6-1/231*(8/3+cos(d*x+c)^4+4/3
*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x
+c)^8*cos(d*x+c)^6-1/60/sin(d*x+c)^6*cos(d*x+c)^6))
```

Maxima [A] time = 0.969745, size = 95, normalized size = 0.98

$$-\frac{2310\,b\sin(dx+c)^5+1980\,a\sin(dx+c)^4-3465\,b\sin(dx+c)^3-3080\,a\sin(dx+c)^2+1386\,b\sin(dx+c)+1260\,a}{13860\,d\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/13860*(2310*b*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*b*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*b*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

Fricas [A] time = 1.71333, size = 344, normalized size = 3.55

$$\frac{1980\,a\cos(dx+c)^4-880\,a\cos(dx+c)^2+231\left(10\,b\cos(dx+c)^4-5\,b\cos(dx+c)^2+b\right)\sin(dx+c)+160\,a}{13860\left(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/13860*(1980*a*cos(d*x + c)^4 - 880*a*cos(d*x + c)^2 + 231*(10*b*cos(d*x +
 c)^4 - 5*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 160*a)/((d*cos(d*x + c)^10 -
 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d
*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**12*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30291, size = 95, normalized size = 0.98

$$-\frac{2310\,b\sin(dx+c)^5+1980\,a\sin(dx+c)^4-3465\,b\sin(dx+c)^3-3080\,a\sin(dx+c)^2+1386\,b\sin(dx+c)+1260\,a}{13860\,d\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13860*(2310*b*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*b*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*b*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

3.1215 $\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=138

$$\frac{(a^2-2b^2)\sin^7(c+dx)}{7d}-\frac{(2a^2-b^2)\sin^5(c+dx)}{5d}+\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin^8(c+dx)}{4d}-\frac{2ab\sin^6(c+dx)}{3d}+\frac{ab\sin^4(c+d}{2d}$$

```
[Out] (a^2*Sin[c + d*x]^3)/(3*d) + (a*b*Sin[c + d*x]^4)/(2*d) - ((2*a^2 - b^2)*Si
n[c + d*x]^5)/(5*d) - (2*a*b*Sin[c + d*x]^6)/(3*d) + ((a^2 - 2*b^2)*Sin[c +
 d*x]^7)/(7*d) + (a*b*Sin[c + d*x]^8)/(4*d) + (b^2*Sin[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.187817, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin^7(c+dx)}{7d}-\frac{(2a^2-b^2)\sin^5(c+dx)}{5d}+\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin^8(c+dx)}{4d}-\frac{2ab\sin^6(c+dx)}{3d}+\frac{ab\sin^4(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^3)/(3*d) + (a*b*Sin[c + d*x]^4)/(2*d) - ((2*a^2 - b^2)*Si
n[c + d*x]^5)/(5*d) - (2*a*b*Sin[c + d*x]^6)/(3*d) + ((a^2 - 2*b^2)*Sin[c +
 d*x]^7)/(7*d) + (a*b*Sin[c + d*x]^8)/(4*d) + (b^2*Sin[c + d*x]^9)/(9*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^2\left(b^2-x^2\right)^2}{b^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^2\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2b^4x^2+2ab^4x^3+b^2\left(-2a^2+b^2\right)x^4-4ab^2x^5+\left(a^2 -\right.\right.\right.}{b^7 d}$$
$$= \frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{\left(2a^2-b^2\right)\sin^5(c+dx)}{5d}-\frac{2a}{}$$

Mathematica [A] time = 0.741205, size = 169, normalized size = 1.22

$$\frac{12600a^2\sin(c+dx)-840a^2\sin(3(c+dx))-1512a^2\sin(5(c+dx))-360a^2\sin(7(c+dx))-7560ab\cos(2(c+dx))-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-7560*a*b*Cos[2*(c + d*x)] - 1260*a*b*Cos[4*(c + d*x)] + 840*a*b*Cos[6*(c
+ d*x)] + 315*a*b*Cos[8*(c + d*x)] + 12600*a^2*Sin[c + d*x] + 3780*b^2*Sin[
c + d*x] - 840*a^2*Sin[3*(c + d*x)] - 840*b^2*Sin[3*(c + d*x)] - 1512*a^2*S
in[5*(c + d*x)] - 504*b^2*Sin[5*(c + d*x)] - 360*a^2*Sin[7*(c + d*x)] + 90*
b^2*Sin[7*(c + d*x)] + 70*b^2*Sin[9*(c + d*x)])/(161280*d)
```

Maple [A] time = 0.046, size = 155, normalized size = 1.1

$$\frac{1}{d}\left(a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)\right)+2ab\left(-1/8(\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))+2*a*b*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+b
^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 1.06631, size = 146, normalized size = 1.06

$$\frac{140\,b^2\sin(dx+c)^9+315\,ab\sin(dx+c)^8-840\,ab\sin(dx+c)^6+180\left(a^2-2\,b^2\right)\sin(dx+c)^7+630\,ab\sin(dx+c)^4-}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1260*(140*b^2*sin(d*x + c)^9 + 315*a*b*sin(d*x + c)^8 - 840*a*b*sin(d*x +
 c)^6 + 180*(a^2 - 2*b^2)*sin(d*x + c)^7 + 630*a*b*sin(d*x + c)^4 - 252*(2*
a^2 - b^2)*sin(d*x + c)^5 + 420*a^2*sin(d*x + c)^3)/d
```

Fricas [A] time = 1.77027, size = 297, normalized size = 2.15

$$\frac{315\,ab\cos\left(dx+c\right)^{8}-420\,ab\cos\left(dx+c\right)^{6}+4\left(35\,b^{2}\cos\left(dx+c\right)^{8}-5\left(9\,a^{2}+10\,b^{2}\right)\cos\left(dx+c\right)^{6}+3\left(3\,a^{2}+b^{2}\right)\cos\left(dx\right.\right.}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/1260*(315*a*b*cos(d*x + c)^8 - 420*a*b*cos(d*x + c)^6 + 4*(35*b^2*cos(d*x
 + c)^8 - 5*(9*a^2 + 10*b^2)*cos(d*x + c)^6 + 3*(3*a^2 + b^2)*cos(d*x + c)^
4 + 4*(3*a^2 + b^2)*cos(d*x + c)^2 + 24*a^2 + 8*b^2)*sin(d*x + c))/d
```

Sympy [A] time = 22.7959, size = 214, normalized size = 1.55

$$\begin{cases} \frac{8a^2\sin^7(c+dx)}{105d}+\frac{4a^2\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{a^2\sin^3(c+dx)\cos^4(c+dx)}{3d}+\frac{ab\sin^8(c+dx)}{12d}+\frac{ab\sin^6(c+dx)\cos^2(c+dx)}{3d}+\frac{ab\sin^4(c+dx)\cos^4(c+dx)}{2d} \\ x\left(a+b\sin\left(c\right)\right)^2\sin^2\left(c\right)\cos^5\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**7/(105*d) + 4*a**2*sin(c + d*x)**5*cos(c +
d*x)**2/(15*d) + a**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) + a*b*sin(c + d
*x)**8/(12*d) + a*b*sin(c + d*x)**6*cos(c + d*x)**2/(3*d) + a*b*sin(c + d*x
)**4*cos(c + d*x)**4/(2*d) + 8*b**2*sin(c + d*x)**9/(315*d) + 4*b**2*sin(c
+ d*x)**7*cos(c + d*x)**2/(35*d) + b**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*
d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**5, True))
```

Giac [A] time = 1.23439, size = 234, normalized size = 1.7

$$\frac{ab\cos\left(8\,dx+8\,c\right)}{512\,d}+\frac{ab\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{ab\cos\left(4\,dx+4\,c\right)}{128\,d}-\frac{3\,ab\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{b^{2}\sin\left(9\,dx+9\,c\right)}{2304\,d}-\frac{\left(4\,a^{2}-b^{2}\right)\sin}{179}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*a*b*cos(8*d*x + 8*c)/d + 1/192*a*b*cos(6*d*x + 6*c)/d - 1/128*a*b*cos
(4*d*x + 4*c)/d - 3/64*a*b*cos(2*d*x + 2*c)/d + 1/2304*b^2*sin(9*d*x + 9*c)
/d - 1/1792*(4*a^2 - b^2)*sin(7*d*x + 7*c)/d - 1/320*(3*a^2 + b^2)*sin(5*d*
x + 5*c)/d - 1/192*(a^2 + b^2)*sin(3*d*x + 3*c)/d + 1/128*(10*a^2 + 3*b^2)*
sin(d*x + c)/d
```

3.1216 $\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=138

$$\frac{(a^2-2b^2)\sin^6(c+dx)}{6d}-\frac{(2a^2-b^2)\sin^4(c+dx)}{4d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{2ab\sin^7(c+dx)}{7d}-\frac{4ab\sin^5(c+dx)}{5d}+\frac{2ab\sin^3}{3}$$

```
[Out] (a^2*Sin[c + d*x]^2)/(2*d) + (2*a*b*Sin[c + d*x]^3)/(3*d) - ((2*a^2 - b^2)*
Sin[c + d*x]^4)/(4*d) - (4*a*b*Sin[c + d*x]^5)/(5*d) + ((a^2 - 2*b^2)*Sin[c
 + d*x]^6)/(6*d) + (2*a*b*Sin[c + d*x]^7)/(7*d) + (b^2*Sin[c + d*x]^8)/(8*d
)
```

Rubi [A] time = 0.12784, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 772}

$$\frac{(a^2-2b^2)\sin^6(c+dx)}{6d}-\frac{(2a^2-b^2)\sin^4(c+dx)}{4d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{2ab\sin^7(c+dx)}{7d}-\frac{4ab\sin^5(c+dx)}{5d}+\frac{2ab\sin^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^2)/(2*d) + (2*a*b*Sin[c + d*x]^3)/(3*d) - ((2*a^2 - b^2)*
Sin[c + d*x]^4)/(4*d) - (4*a*b*Sin[c + d*x]^5)/(5*d) + ((a^2 - 2*b^2)*Sin[c
 + d*x]^6)/(6*d) + (2*a*b*Sin[c + d*x]^7)/(7*d) + (b^2*Sin[c + d*x]^8)/(8*d
)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{x(a+x)^2(b^2-x^2)^2}{b}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\operatorname{Subst}\left(\int x(a+x)^2\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^6 d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(a^2b^4x + 2ab^4x^2 + b^2\left(-2a^2+b^2\right)x^3 - 4ab^2x^4 + \left(a^2-2b^2\right)\right.\right.}{b^6 d}$$
$$= \frac{a^2\sin^2(c+dx)}{2d} + \frac{2ab\sin^3(c+dx)}{3d} - \frac{\left(2a^2-b^2\right)\sin^4(c+dx)}{4d} - \frac{4ab\,\mathrm{s}}{}$$

Mathematica [A] time = 0.654, size = 138, normalized size = 1.

$$-\frac{840\left(10a^2+3b^2\right)\cos(2(c+dx)) + 420\left(8a^2+b^2\right)\cos(4(c+dx)) + 560a^2\cos(6(c+dx)) - 16800ab\sin(c+dx) + 1120ab}{1075}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(-2590*b^2 + 840*(10*a^2 + 3*b^2)*Cos[2*(c + d*x)] + 420*(8*a^2 + b^2)*Cos
[4*(c + d*x)] + 560*a^2*Cos[6*(c + d*x)] - 280*b^2*Cos[6*(c + d*x)] - 105*b
^2*Cos[8*(c + d*x)] - 16800*a*b*Sin[c + d*x] + 1120*a*b*Sin[3*(c + d*x)] +
2016*a*b*Sin[5*(c + d*x)] + 480*a*b*Sin[7*(c + d*x)])/(107520*d)
```

Maple [A] time = 0.04, size = 101, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{a^2(\cos(dx+c))^6}{6} + 2ab\left(-1/7\sin(dx+c)(\cos(dx+c))^6 + 1/35\left(8/3+(\cos(dx+c))^4 + 4/3(\cos(dx+c))^2\right)\sin(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/6*a^2*cos(d*x+c)^6+2*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+co
s(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^
6-1/24*cos(d*x+c)^6))
```

Maxima [A] time = 0.993132, size = 146, normalized size = 1.06

$$\frac{105b^2\sin(dx+c)^8 + 240ab\sin(dx+c)^7 - 672ab\sin(dx+c)^5 + 140\left(a^2-2b^2\right)\sin(dx+c)^6 + 560ab\sin(dx+c)^3 - 21}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/840*(105*b^2*sin(d*x + c)^8 + 240*a*b*sin(d*x + c)^7 - 672*a*b*sin(d*x +
c)^5 + 140*(a^2 - 2*b^2)*sin(d*x + c)^6 + 560*a*b*sin(d*x + c)^3 - 210*(2*a
^2 - b^2)*sin(d*x + c)^4 + 420*a^2*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.74325, size = 220, normalized size = 1.59

$$\frac{105\,b^2\cos\left(dx+c\right)^8-140\left(a^2+b^2\right)\cos\left(dx+c\right)^6-16\left(15\,ab\cos\left(dx+c\right)^6-3\,ab\cos\left(dx+c\right)^4-4\,ab\cos\left(dx+c\right)^2-8\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^2*cos(d*x + c)^8 - 140*(a^2 + b^2)*cos(d*x + c)^6 - 16*(15*a*b
*cos(d*x + c)^6 - 3*a*b*cos(d*x + c)^4 - 4*a*b*cos(d*x + c)^2 - 8*a*b)*sin(
d*x + c))/d
```

Sympy [A] time = 12.8605, size = 163, normalized size = 1.18

$$\begin{cases} -\frac{a^2\cos^6(c+dx)}{6d}+\frac{16ab\sin^7(c+dx)}{105d}+\frac{8ab\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{2ab\sin^3(c+dx)\cos^4(c+dx)}{3d}+\frac{b^2\sin^8(c+dx)}{24d}+\frac{b^2\sin^6(c+dx)\cos^2(c+dx)}{6d}+ \\ x\left(a+b\sin\left(c\right)\right)^2\sin\left(c\right)\cos^5\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**6/(6*d) + 16*a*b*sin(c + d*x)**7/(105*d) + 8
*a*b*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + 2*a*b*sin(c + d*x)**3*cos(c +
 d*x)**4/(3*d) + b**2*sin(c + d*x)**8/(24*d) + b**2*sin(c + d*x)**6*cos(c +
 d*x)**2/(6*d) + b**2*sin(c + d*x)**4*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*
(a + b*sin(c))**2*sin(c)*cos(c)**5, True))
```

Giac [A] time = 1.21535, size = 205, normalized size = 1.49

$$\frac{b^2\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{ab\sin\left(7\,dx+7\,c\right)}{224\,d}-\frac{3\,ab\sin\left(5\,dx+5\,c\right)}{160\,d}-\frac{ab\sin\left(3\,dx+3\,c\right)}{96\,d}+\frac{5\,ab\sin\left(dx+c\right)}{32\,d}-\frac{\left(2\,a^2-b^2\right)\mathrm{co}}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1024*b^2*cos(8*d*x + 8*c)/d - 1/224*a*b*sin(7*d*x + 7*c)/d - 3/160*a*b*si
n(5*d*x + 5*c)/d - 1/96*a*b*sin(3*d*x + 3*c)/d + 5/32*a*b*sin(d*x + c)/d -
1/384*(2*a^2 - b^2)*cos(6*d*x + 6*c)/d - 1/256*(8*a^2 + b^2)*cos(4*d*x + 4*
c)/d - 1/128*(10*a^2 + 3*b^2)*cos(2*d*x + 2*c)/d
```

3.1217 $\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=130

$$\frac{(a^2-2b^2)\sin^4(c+dx)}{4d}-\frac{(2a^2-b^2)\sin^2(c+dx)}{2d}+\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin^5(c+dx)}{5d}-\frac{4ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}$$

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sin[c +
 d*x]^2)/(2*d) - (4*a*b*Sin[c + d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sin[c + d*x]
^4)/(4*d) + (2*a*b*Sin[c + d*x]^5)/(5*d) + (b^2*Sin[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.133519, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin^4(c+dx)}{4d}-\frac{(2a^2-b^2)\sin^2(c+dx)}{2d}+\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin^5(c+dx)}{5d}-\frac{4ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sin[c +
 d*x]^2)/(2*d) - (4*a*b*Sin[c + d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sin[c + d*x]
^4)/(4*d) + (2*a*b*Sin[c + d*x]^5)/(5*d) + (b^2*Sin[c + d*x]^6)/(6*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b(a+x)^2(b^2-x^2)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(2ab^4 + \frac{a^2b^4}{x} - b^2\left(2a^2-b^2\right)x - 4ab^2x^2 + \left(a^2-2b^2\right)x^3\right.\right.}{b^4 d}$$

$$= \frac{a^2\log(\sin(c+dx))}{d} + \frac{2ab\sin(c+dx)}{d} - \frac{\left(2a^2-b^2\right)\sin^2(c+dx)}{2d} -$$

Mathematica [A] time = 0.156253, size = 105, normalized size = 0.81

$$\frac{15\left(a^2-2b^2\right)\sin^4(c+dx) + 30\left(b^2-2a^2\right)\sin^2(c+dx) + 60a^2\log(\sin(c+dx)) + 24ab\sin^5(c+dx) - 80ab\sin^3(c+dx}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (60*a^2*Log[Sin[c + d*x]] + 120*a*b*Sin[c + d*x] + 30*(-2*a^2 + b^2)*Sin[c
+ d*x]^2 - 80*a*b*Sin[c + d*x]^3 + 15*(a^2 - 2*b^2)*Sin[c + d*x]^4 + 24*a*b
*Sin[c + d*x]^5 + 10*b^2*Sin[c + d*x]^6)/(60*d)
```

Maple [A] time = 0.08, size = 119, normalized size = 0.9

$$\frac{a^2(\cos(dx+c))^4}{4d} + \frac{a^2(\cos(dx+c))^2}{2d} + \frac{a^2\ln(\sin(dx+c))}{d} + \frac{16ab\sin(dx+c)}{15d} + \frac{2ab\sin(dx+c)(\cos(dx+c))^4}{5d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*cos(d*x+c)^4+1/2/d*a^2*cos(d*x+c)^2+a^2*ln(sin(d*x+c))/d+16/15*a*
b*sin(d*x+c)/d+2/5/d*sin(d*x+c)*a*b*cos(d*x+c)^4+8/15/d*sin(d*x+c)*a*b*cos(
d*x+c)^2-1/6/d*cos(d*x+c)^6*b^2
```

Maxima [A] time = 0.966875, size = 142, normalized size = 1.09

$$\frac{10b^2\sin(dx+c)^6 + 24ab\sin(dx+c)^5 - 80ab\sin(dx+c)^3 + 15\left(a^2-2b^2\right)\sin(dx+c)^4 + 60a^2\log(\sin(dx+c)) +}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/60*(10*b^2*sin(d*x + c)^6 + 24*a*b*sin(d*x + c)^5 - 80*a*b*sin(d*x + c)^3
 + 15*(a^2 - 2*b^2)*sin(d*x + c)^4 + 60*a^2*log(sin(d*x + c)) + 120*a*b*sin
(d*x + c) - 30*(2*a^2 - b^2)*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.82381, size = 247, normalized size = 1.9

$$-\frac{10\,b^2\cos(dx+c)^6-15\,a^2\cos(dx+c)^4-30\,a^2\cos(dx+c)^2-60\,a^2\log\left(\frac{1}{2}\sin(dx+c)\right)-8\left(3\,ab\cos(dx+c)^4+4\,ab\,c\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^2*cos(d*x + c)^6 - 15*a^2*cos(d*x + c)^4 - 30*a^2*cos(d*x + c)^
2 - 60*a^2*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x + c)^4 + 4*a*b*cos(d*x
+ c)^2 + 8*a*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27565, size = 159, normalized size = 1.22

$$\frac{10\,b^2\sin(dx+c)^6+24\,ab\sin(dx+c)^5+15\,a^2\sin(dx+c)^4-30\,b^2\sin(dx+c)^4-80\,ab\sin(dx+c)^3-60\,a^2\sin(dx+c}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*sin(d*x + c)^6 + 24*a*b*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4
 - 30*b^2*sin(d*x + c)^4 - 80*a*b*sin(d*x + c)^3 - 60*a^2*sin(d*x + c)^2 +
30*b^2*sin(d*x + c)^2 + 60*a^2*log(abs(sin(d*x + c))) + 120*a*b*sin(d*x + c
))/d
```

3.1218 $\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=125

$$\frac{(a^2-2b^2)\sin^3(c+dx)}{3d}-\frac{(2a^2-b^2)\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{2ab\sin^2(c+dx)}{d}+\frac{2ab\log(\sin}{d}$$

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a*b*Log[Sin[c + d*x]])/d - ((2*a^2 - b^2)*Sin[
c + d*x])/d - (2*a*b*Sin[c + d*x]^2)/d + ((a^2 - 2*b^2)*Sin[c + d*x]^3)/(3*
d) + (a*b*Sin[c + d*x]^4)/(2*d) + (b^2*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.159245, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin^3(c+dx)}{3d}-\frac{(2a^2-b^2)\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{2ab\sin^2(c+dx)}{d}+\frac{2ab\log(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a*b*Log[Sin[c + d*x]])/d - ((2*a^2 - b^2)*Sin[
c + d*x])/d - (2*a*b*Sin[c + d*x]^2)/d + ((a^2 - 2*b^2)*Sin[c + d*x]^3)/(3*
d) + (a*b*Sin[c + d*x]^4)/(2*d) + (b^2*Sin[c + d*x]^5)/(5*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^2(a+x)^2(b^2-x^2)^2}{x^2}dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^2}dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2b^2\left(1-\frac{b^2}{2a^2}\right)+\frac{a^2b^4}{x^2}+\frac{2ab^4}{x}-4ab^2x+\left(a^2-2b^2\right)x^2+\right.\right.}{b^3d}$$
$$= -\frac{a^2\csc(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}-\frac{\left(2a^2-b^2\right)\sin(c+dx)}{d}-$$

Mathematica [A] time = 0.0497468, size = 142, normalized size = 1.14

$$\frac{a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{2ab\sin^2(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}+\frac{b^2\sin^5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a*b*Log[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x]
)/d + (b^2*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^2)/d + (a^2*Sin[c + d*x]^3
)/(3*d) - (2*b^2*Sin[c + d*x]^3)/(3*d) + (a*b*Sin[c + d*x]^4)/(2*d) + (b^2*
Sin[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.076, size = 185, normalized size = 1.5

$$-\frac{a^2(\cos(dx+c))^6}{d\sin(dx+c)}-\frac{8a^2\sin(dx+c)}{3d}-\frac{a^2\sin(dx+c)(\cos(dx+c))^4}{d}-\frac{4a^2\sin(dx+c)(\cos(dx+c))^2}{3d}+\frac{ab(\cos(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^6-8/3*a^2*sin(d*x+c)/d-1/d*a^2*sin(d*x+c)*co
s(d*x+c)^4-4/3/d*a^2*sin(d*x+c)*cos(d*x+c)^2+1/2/d*a*b*cos(d*x+c)^4+1/d*a*b
*cos(d*x+c)^2+2*a*b*ln(sin(d*x+c))/d+8/15*b^2*sin(d*x+c)/d+1/5/d*sin(d*x+c)
*b^2*cos(d*x+c)^4+4/15/d*sin(d*x+c)*b^2*cos(d*x+c)^2
```

Maxima [A] time = 0.979506, size = 142, normalized size = 1.14

$$\frac{6b^2\sin(dx+c)^5+15ab\sin(dx+c)^4-60ab\sin(dx+c)^2+10\left(a^2-2b^2\right)\sin(dx+c)^3+60ab\log(\sin(dx+c))-30\left(2\right.}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 - 60*a*b*sin(d*x + c)^2
+ 10*(a^2 - 2*b^2)*sin(d*x + c)^3 + 60*a*b*log(sin(d*x + c)) - 30*(2*a^2 -
b^2)*sin(d*x + c) - 30*a^2/sin(d*x + c))/d
```

Fricas [A] time = 1.78321, size = 346, normalized size = 2.77

$$-\frac{48\,b^2\cos(dx+c)^6-16\left(5\,a^2-b^2\right)\cos(dx+c)^4-480\,ab\log\left(\frac{1}{2}\,\sin(dx+c)\right)\sin(dx+c)-64\left(5\,a^2-b^2\right)\cos(dx+c}{240\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(48*b^2*cos(d*x + c)^6 - 16*(5*a^2 - b^2)*cos(d*x + c)^4 - 480*a*b*l
og(1/2*sin(d*x + c))*sin(d*x + c) - 64*(5*a^2 - b^2)*cos(d*x + c)^2 + 640*a
^2 - 128*b^2 - 15*(8*a*b*cos(d*x + c)^4 + 16*a*b*cos(d*x + c)^2 - 11*a*b)*s
in(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24842, size = 171, normalized size = 1.37

$$\frac{6\,b^2\sin(dx+c)^5+15\,ab\sin(dx+c)^4+10\,a^2\sin(dx+c)^3-20\,b^2\sin(dx+c)^3-60\,ab\sin(dx+c)^2+60\,ab\log(|\sin}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3
- 20*b^2*sin(d*x + c)^3 - 60*a*b*sin(d*x + c)^2 + 60*a*b*log(abs(sin(d*x +
c))) - 60*a^2*sin(d*x + c) + 30*b^2*sin(d*x + c) - 30*(2*a*b*sin(d*x + c) +
 a^2)/sin(d*x + c))/d
```

3.1219 $\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=127

$$\frac{(a^2-2b^2)\sin^2(c+dx)}{2d}-\frac{(2a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}+\frac{2ab\sin^3(c+dx)}{3d}-\frac{4ab\sin(c+dx)}{d}-\frac{2ab\csc(c}{d}$$

```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((2*a^2 - b^2)*Log[S
in[c + d*x]])/d - (4*a*b*Sin[c + d*x])/d + ((a^2 - 2*b^2)*Sin[c + d*x]^2)/(
2*d) + (2*a*b*Sin[c + d*x]^3)/(3*d) + (b^2*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.157214, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin^2(c+dx)}{2d}-\frac{(2a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}+\frac{2ab\sin^3(c+dx)}{3d}-\frac{4ab\sin(c+dx)}{d}-\frac{2ab\csc(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((2*a^2 - b^2)*Log[S
in[c + d*x]])/d - (4*a*b*Sin[c + d*x])/d + ((a^2 - 2*b^2)*Sin[c + d*x]^2)/(
2*d) + (2*a*b*Sin[c + d*x]^3)/(3*d) + (b^2*Sin[c + d*x]^4)/(4*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^3(a+x)^2\left(b^2-x^2\right)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$

$$= \frac{\text{Subst}\left(\int \left(-4ab^2+\frac{a^2b^4}{x^3}+\frac{2ab^4}{x^2}+\frac{-2a^2b^2+b^4}{x}+\left(a^2-2b^2\right)x+2ax^2\right.\right.}{b^2 d}$$

$$= -\frac{2ab\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{\left(2a^2-b^2\right)\log(\sin(c+dx)}{d}$$

Mathematica [A] time = 0.282604, size = 103, normalized size = 0.81

$$\frac{6\left(a^2-2b^2\right)\sin^2(c+dx)+12\left(b^2-2a^2\right)\log(\sin(c+dx))-6a^2\csc^2(c+dx)+8ab\sin^3(c+dx)-48ab\sin(c+dx)-2}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-24*a*b*Csc[c + d*x] - 6*a^2*Csc[c + d*x]^2 + 12*(-2*a^2 + b^2)*Log[Sin[c
+ d*x]] - 48*a*b*Sin[c + d*x] + 6*(a^2 - 2*b^2)*Sin[c + d*x]^2 + 8*a*b*Sin[
c + d*x]^3 + 3*b^2*Sin[c + d*x]^4)/(12*d)
```

Maple [A] time = 0.087, size = 197, normalized size = 1.6

$$-\frac{a^2(\cos(dx+c))^6}{2d(\sin(dx+c))^2}-\frac{a^2(\cos(dx+c))^4}{2d}-\frac{a^2(\cos(dx+c))^2}{d}-2\frac{a^2\ln(\sin(dx+c))}{d}-2\frac{ab(\cos(dx+c))^6}{d\sin(dx+c)}-\frac{16ab\sin}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*a^2*cos(d*x+c)^4-1/d*a^2*cos(d*x
+c)^2-2*a^2*ln(sin(d*x+c))/d-2/d*a*b/sin(d*x+c)*cos(d*x+c)^6-16/3*a*b*sin(d
*x+c)/d-2/d*sin(d*x+c)*a*b*cos(d*x+c)^4-8/3/d*sin(d*x+c)*a*b*cos(d*x+c)^2+1
/4/d*b^2*cos(d*x+c)^4+1/2/d*b^2*cos(d*x+c)^2+b^2*ln(sin(d*x+c))/d
```

Maxima [A] time = 0.995812, size = 140, normalized size = 1.1

$$\frac{3b^2\sin(dx+c)^4+8ab\sin(dx+c)^3-48ab\sin(dx+c)+6\left(a^2-2b^2\right)\sin(dx+c)^2-12\left(2a^2-b^2\right)\log(\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*b^2*sin(d*x + c)^4 + 8*a*b*sin(d*x + c)^3 - 48*a*b*sin(d*x + c) + 6
*(a^2 - 2*b^2)*sin(d*x + c)^2 - 12*(2*a^2 - b^2)*log(sin(d*x + c)) - 6*(4*a
*b*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.79847, size = 381, normalized size = 3.

$$\frac{24\,b^2\cos(dx+c)^6 - 24\left(2\,a^2-b^2\right)\cos(dx+c)^4 + 9\left(8\,a^2-9\,b^2\right)\cos(dx+c)^2 + 24\,a^2 + 33\,b^2 - 96\left(\left(2\,a^2-b^2\right)\cos(dx+\right.}{96\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*b^2*cos(d*x + c)^6 - 24*(2*a^2 - b^2)*cos(d*x + c)^4 + 9*(8*a^2 -
9*b^2)*cos(d*x + c)^2 + 24*a^2 + 33*b^2 - 96*((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a^2 + b^2)*log(1/2*sin(d*x + c)) - 64*(a*b*cos(d*x + c)^4 + 4*a*b*cos(d
*x + c)^2 - 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30277, size = 189, normalized size = 1.49

$$\frac{3\,b^2\sin(dx+c)^4 + 8\,ab\sin(dx+c)^3 + 6\,a^2\sin(dx+c)^2 - 12\,b^2\sin(dx+c)^2 - 48\,ab\sin(dx+c) - 12\left(2\,a^2-b^2\right)\log(|\sin}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*sin(d*x + c)^4 + 8*a*b*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2 -
12*b^2*sin(d*x + c)^2 - 48*a*b*sin(d*x + c) - 12*(2*a^2 - b^2)*log(abs(sin(
d*x + c))) + 6*(6*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x
 + c) - a^2)/sin(d*x + c)^2)/d
```

3.1220 $\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=120

$$\frac{(a^2-2b^2)\sin(c+dx)}{d}+\frac{(2a^2-b^2)\csc(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}+\frac{ab\sin^2(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{4ab\log(\sin(}{d}$$

```
[Out] ((2*a^2 - b^2)*Csc[c + d*x])/d - (a*b*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]
^3)/(3*d) - (4*a*b*Log[Sin[c + d*x]])/d + ((a^2 - 2*b^2)*Sin[c + d*x])/d +
(a*b*Sin[c + d*x]^2)/d + (b^2*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.140823, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin(c+dx)}{d}+\frac{(2a^2-b^2)\csc(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}+\frac{ab\sin^2(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{4ab\log(\sin(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*Csc[c + d*x])/d - (a*b*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]
^3)/(3*d) - (4*a*b*Log[Sin[c + d*x]])/d + ((a^2 - 2*b^2)*Sin[c + d*x])/d +
(a*b*Sin[c + d*x]^2)/d + (b^2*Sin[c + d*x]^3)/(3*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^4(a+x)^2\left(b^2-x^2\right)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{a^2b^4}{x^4}+\frac{2ab^4}{x^3}+\frac{-2a^2b^2+b^4}{x^2}-\frac{4ab^2}{x}+2ax+x^2\right) \right.}{bd}$$
$$= \frac{\left(2a^2-b^2\right)\csc(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{4ab\log}{}$$

Mathematica [A] time = 0.274211, size = 103, normalized size = 0.86

$$\frac{3\left(a^2-2b^2\right)\sin(c+dx)+\left(6a^2-3b^2\right)\csc(c+dx)-a^2\csc^3(c+dx)+3ab\sin^2(c+dx)-3ab\csc^2(c+dx)-12ab\log(\sin(}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((6*a^2 - 3*b^2)*Csc[c + d*x] - 3*a*b*Csc[c + d*x]^2 - a^2*Csc[c + d*x]^3 -
 12*a*b*Log[Sin[c + d*x]] + 3*(a^2 - 2*b^2)*Sin[c + d*x] + 3*a*b*Sin[c + d*
x]^2 + b^2*Sin[c + d*x]^3)/(3*d)
```

Maple [B] time = 0.087, size = 255, normalized size = 2.1

$$-\frac{a^2(\cos(dx+c))^6}{3d(\sin(dx+c))^3}+\frac{a^2(\cos(dx+c))^6}{d\sin(dx+c)}+\frac{8a^2\sin(dx+c)}{3d}+\frac{a^2\sin(dx+c)(\cos(dx+c))^4}{d}+\frac{4a^2\sin(dx+c)(\cos(dx+}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a^2/sin(d*x+c)*cos(d*x+c)^6+8/3*a^
2*sin(d*x+c)/d+1/d*a^2*sin(d*x+c)*cos(d*x+c)^4+4/3/d*a^2*sin(d*x+c)*cos(d*x
+c)^2-1/d*a*b/sin(d*x+c)^2*cos(d*x+c)^6-1/d*a*b*cos(d*x+c)^4-2/d*a*b*cos(d*
x+c)^2-4*a*b*ln(sin(d*x+c))/d-1/d*b^2/sin(d*x+c)*cos(d*x+c)^6-8/3*b^2*sin(d
*x+c)/d-1/d*sin(d*x+c)*b^2*cos(d*x+c)^4-4/3/d*sin(d*x+c)*b^2*cos(d*x+c)^2
```

Maxima [A] time = 0.992184, size = 139, normalized size = 1.16

$$\frac{b^2\sin(dx+c)^3+3ab\sin(dx+c)^2-12ab\log(\sin(dx+c))+3\left(a^2-2b^2\right)\sin(dx+c)-\frac{3ab\sin(dx+c)-3\left(2a^2-b^2\right)\sin(dx+c)^2+a}{\sin(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(b^2*sin(d*x + c)^3 + 3*a*b*sin(d*x + c)^2 - 12*a*b*log(sin(d*x + c)) +
 3*(a^2 - 2*b^2)*sin(d*x + c) - (3*a*b*sin(d*x + c) - 3*(2*a^2 - b^2)*sin(d
*x + c)^2 + a^2)/sin(d*x + c)^3)/d
```

Fricas [A] time = 1.77982, size = 382, normalized size = 3.18

$$\frac{2\,b^2\cos\left(dx+c\right)^6-6\left(a^2-b^2\right)\cos\left(dx+c\right)^4+24\left(a^2-b^2\right)\cos\left(dx+c\right)^2-24\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)}{6\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*b^2*cos(d*x + c)^6 - 6*(a^2 - b^2)*cos(d*x + c)^4 + 24*(a^2 - b^2)*c
os(d*x + c)^2 - 24*(a*b*cos(d*x + c)^2 - a*b)*log(1/2*sin(d*x + c))*sin(d*x
 + c) - 16*a^2 + 16*b^2 - 3*(2*a*b*cos(d*x + c)^4 - 3*a*b*cos(d*x + c)^2 -
a*b)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19856, size = 171, normalized size = 1.42

$$\frac{b^2\sin\left(dx+c\right)^3+3\,ab\sin\left(dx+c\right)^2-12\,ab\log\left(\left|\sin\left(dx+c\right)\right|\right)+3\,a^2\sin\left(dx+c\right)-6\,b^2\sin\left(dx+c\right)+\frac{22\,ab\sin(dx+c)^3+6\,a^2\cdots}{\cdots}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*sin(d*x + c)^3 + 3*a*b*sin(d*x + c)^2 - 12*a*b*log(abs(sin(d*x + c
))) + 3*a^2*sin(d*x + c) - 6*b^2*sin(d*x + c) + (22*a*b*sin(d*x + c)^3 + 6*
a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 3*a*b*sin(d*x + c) - a^2)/sin(d
*x + c)^3)/d
```

3.1221 $\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\csc(}{d}$$

```
[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[
c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c +
 d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0919727, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 948}

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\csc(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[
c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c +
 d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx&=\frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^5}\,dx,x,b\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(2a+\frac{a^2b^4}{x^5}+\frac{2ab^4}{x^4}+\frac{-2a^2b^2+b^4}{x^3}-\frac{4ab^2}{x^2}+\frac{a^2-2b^2}{x}+x\right)dx,x,b\sin(c+dx)\right)}{d}\\&=\frac{4ab\csc(c+dx)}{d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}\end{aligned}$$

Mathematica [A] time = 0.760486, size = 107, normalized size = 0.85

$$\frac{6(2a^2-b^2)\csc^2(c+dx)+6\left(2(a^2-2b^2)\log(\sin(c+dx))+4ab\sin(c+dx)+b^2\sin^2(c+dx)\right)-3a^2\csc^4(c+dx)-8ab}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (48*a*b*Csc[c + d*x] + 6*(2*a^2 - b^2)*Csc[c + d*x]^2 - 8*a*b*Csc[c + d*x]^
3 - 3*a^2*Csc[c + d*x]^4 + 6*(2*(a^2 - 2*b^2)*Log[Sin[c + d*x]] + 4*a*b*Sin
[c + d*x] + b^2*Sin[c + d*x]^2))/(12*d)
```

Maple [A] time = 0.088, size = 220, normalized size = 1.8

$$-\frac{a^2 (\cot (dx+c))^4}{4d}+\frac{a^2 (\cot (dx+c))^2}{2d}+\frac{a^2 \ln (\sin (dx+c))}{d}-\frac{2ab (\cos (dx+c))^6}{3d (\sin (dx+c))^3}+2\frac{ab (\cos (dx+c))^6}{d \sin (dx+c)}+\frac{16 ab \sin}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c)^2+a^2*ln(sin(d*x+c))/d-2/3/d*a
*b/sin(d*x+c)^3*cos(d*x+c)^6+2/d*a*b/sin(d*x+c)*cos(d*x+c)^6+16/3*a*b*sin(d
*x+c)/d+2/d*sin(d*x+c)*a*b*cos(d*x+c)^4+8/3/d*sin(d*x+c)*a*b*cos(d*x+c)^2-1
/2/d*b^2/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*b^2*cos(d*x+c)^4-1/d*b^2*cos(d*x+c
)^2-2*b^2*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.02934, size = 142, normalized size = 1.13

$$\frac{6 b^2 \sin (dx+c)^2+24 ab \sin (dx+c)+12 \left(a^2-2 b^2\right) \log (\sin (dx+c))+\frac{48 ab \sin (dx+c)^3-8 ab \sin (dx+c)+6 \left(2 a^2-b^2\right) \sin (dx+c)^2-}{\sin (dx+c)^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(sin
(d*x + c)) + (48*a*b*sin(d*x + c)^3 - 8*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*
sin(d*x + c)^2 - 3*a^2)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.76401, size = 437, normalized size = 3.47

$$-\frac{6 b^2 \cos (dx+c)^6-15 b^2 \cos (dx+c)^4+6 \left(2 a^2+b^2\right) \cos (dx+c)^2-9 a^2+3 b^2-12 \left(\left(a^2-2 b^2\right) \cos (dx+c)^4-2 \left(a\right.\right.}{12 \left(d \cos (dx+c)^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/12*(6*b^2*cos(d*x + c)^6 - 15*b^2*cos(d*x + c)^4 + 6*(2*a^2 + b^2)*cos(d
*x + c)^2 - 9*a^2 + 3*b^2 - 12*((a^2 - 2*b^2)*cos(d*x + c)^4 - 2*(a^2 - 2*b
^2)*cos(d*x + c)^2 + a^2 - 2*b^2)*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x
```

```
+ c)^4 - 12*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2
*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27178, size = 186, normalized size = 1.48

$$\frac{6\,b^{2}\sin\left(dx+c\right)^{2}+24\,ab\sin\left(dx+c\right)+12\left(a^{2}-2\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{25\,a^{2}\sin(dx+c)^{4}-50\,b^{2}\sin(dx+c)^{4}-48\,ab\sin(dx+c)^{3}-12\,a^{2}}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(abs
(sin(d*x + c))) - (25*a^2*sin(d*x + c)^4 - 50*b^2*sin(d*x + c)^4 - 48*a*b*s
in(d*x + c)^3 - 12*a^2*sin(d*x + c)^2 + 6*b^2*sin(d*x + c)^2 + 8*a*b*sin(d*
x + c) + 3*a^2)/sin(d*x + c)^4)/d
```

3.1222 $\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=124

$$\frac{(2a^2-b^2)\csc^3(c+dx)}{3d}-\frac{(a^2-2b^2)\csc(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{ab\csc^4(c+dx)}{2d}+\frac{2ab\csc^2(c+dx)}{d}+\frac{2ab\log(\text{si}}{d}$$

```
[Out] -(((a^2 - 2*b^2)*Csc[c + d*x])/d) + (2*a*b*Csc[c + d*x]^2)/d + ((2*a^2 - b^
2)*Csc[c + d*x]^3)/(3*d) - (a*b*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x]^5
)/(5*d) + (2*a*b*Log[Sin[c + d*x]])/d + (b^2*Sin[c + d*x])/d
```

Rubi [A] time = 0.140322, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 948}

$$\frac{(2a^2-b^2)\csc^3(c+dx)}{3d}-\frac{(a^2-2b^2)\csc(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{ab\csc^4(c+dx)}{2d}+\frac{2ab\csc^2(c+dx)}{d}+\frac{2ab\log(\text{si}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -(((a^2 - 2*b^2)*Csc[c + d*x])/d) + (2*a*b*Csc[c + d*x]^2)/d + ((2*a^2 - b^
2)*Csc[c + d*x]^3)/(3*d) - (a*b*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x]^5
)/(5*d) + (2*a*b*Log[Sin[c + d*x]])/d + (b^2*Sin[c + d*x])/d
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^6(a+x)^2(b^2-x^2)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(1+\frac{a^2b^4}{x^6}+\frac{2ab^4}{x^5}+\frac{-2a^2b^2+b^4}{x^4}-\frac{4ab^2}{x^3}+\frac{a^2-2b^2}{x^2}+\frac{2a}{x}\right)dx, x, b$$
$$= -\frac{(a^2-2b^2)\csc(c+dx)}{d}+\frac{2ab\csc^2(c+dx)}{d}+\frac{(2a^2-b^2)\csc^3(c+dx}{3d}$$

Mathematica [A] time = 0.161695, size = 105, normalized size = 0.85

$$\frac{10(2a^2-b^2)\csc^3(c+dx)-30(a^2-2b^2)\csc(c+dx)-6a^2\csc^5(c+dx)-15ab\csc^4(c+dx)+60ab\csc^2(c+dx)+30b(2}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*(a^2 - 2*b^2)*Csc[c + d*x] + 60*a*b*Csc[c + d*x]^2 + 10*(2*a^2 - b^2)*
Csc[c + d*x]^3 - 15*a*b*Csc[c + d*x]^4 - 6*a^2*Csc[c + d*x]^5 + 30*b*(2*a*L
og[Sin[c + d*x]] + b*Sin[c + d*x]))/(30*d)
```

Maple [B] time = 0.089, size = 279, normalized size = 2.3

$$-\frac{a^2(\cos(dx+c))^6}{5d(\sin(dx+c))^5}+\frac{a^2(\cos(dx+c))^6}{15d(\sin(dx+c))^3}-\frac{a^2(\cos(dx+c))^6}{5d\sin(dx+c)}-\frac{8a^2\sin(dx+c)}{15d}-\frac{a^2\sin(dx+c)(\cos(dx+c))^4}{5d}-\frac{4a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^6+1/15/d*a^2/sin(d*x+c)^3*cos(d*x+c)^6-1
/5/d*a^2/sin(d*x+c)*cos(d*x+c)^6-8/15*a^2*sin(d*x+c)/d-1/5/d*a^2*sin(d*x+c)
*cos(d*x+c)^4-4/15/d*a^2*sin(d*x+c)*cos(d*x+c)^2-1/2/d*a*b*cot(d*x+c)^4+1/d
*a*b*cot(d*x+c)^2+2*a*b*ln(sin(d*x+c))/d-1/3/d*b^2/sin(d*x+c)^3*cos(d*x+c)^
6+1/d*b^2/sin(d*x+c)*cos(d*x+c)^6+8/3*b^2*sin(d*x+c)/d+1/d*sin(d*x+c)*b^2*c
os(d*x+c)^4+4/3/d*sin(d*x+c)*b^2*cos(d*x+c)^2
```

Maxima [A] time = 0.990565, size = 142, normalized size = 1.15

$$\frac{60\,ab\log(\sin(dx+c))+30\,b^2\sin(dx+c)+\frac{60\,ab\sin(dx+c)^3-30\,(a^2-2\,b^2)\sin(dx+c)^4-15\,ab\sin(dx+c)+10\,(2\,a^2-b^2)\sin(dx+c)^2-6\,a^2}{\sin(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(60*a*b*log(sin(d*x + c)) + 30*b^2*sin(d*x + c) + (60*a*b*sin(d*x + c)
^3 - 30*(a^2 - 2*b^2)*sin(d*x + c)^4 - 15*a*b*sin(d*x + c) + 10*(2*a^2 - b^
2)*sin(d*x + c)^2 - 6*a^2)/sin(d*x + c)^5)/d
```

Fricas [A] time = 1.8117, size = 425, normalized size = 3.43

$$-\frac{30\,b^2\cos(dx+c)^6+30\left(a^2-5\,b^2\right)\cos(dx+c)^4-40\left(a^2-5\,b^2\right)\cos(dx+c)^2-60\left(ab\cos(dx+c)^4-2\,ab\cos(dx+\right.}{30\left(d\cos(dx+c)^4-2\,d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/30*(30*b^2*cos(d*x + c)^6 + 30*(a^2 - 5*b^2)*cos(d*x + c)^4 - 40*(a^2 -
5*b^2)*cos(d*x + c)^2 - 60*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b
)*log(1/2*sin(d*x + c))*sin(d*x + c) + 16*a^2 - 80*b^2 + 15*(4*a*b*cos(d*x
+ c)^2 - 3*a*b)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*
sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23539, size = 177, normalized size = 1.43

$$\frac{60\,ab\log(|\sin(dx+c)|)+30\,b^2\sin(dx+c)-\frac{137\,ab\sin(dx+c)^5+30\,a^2\sin(dx+c)^4-60\,b^2\sin(dx+c)^4-60\,ab\sin(dx+c)^3-20\,a^2\sin(dx+c)^2+10\,}{\sin(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*log(abs(sin(d*x + c))) + 30*b^2*sin(d*x + c) - (137*a*b*sin(d*
x + c)^5 + 30*a^2*sin(d*x + c)^4 - 60*b^2*sin(d*x + c)^4 - 60*a*b*sin(d*x +
 c)^3 - 20*a^2*sin(d*x + c)^2 + 10*b^2*sin(d*x + c)^2 + 15*a*b*sin(d*x + c)
 + 6*a^2)/sin(d*x + c)^5)/d
```

3.1223 $\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=130

$$\frac{(2a^2-b^2)\csc^4(c+dx)}{4d}-\frac{(a^2-2b^2)\csc^2(c+dx)}{2d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+}{d}$$

```
[Out] (-2*a*b*Csc[c + d*x])/d - ((a^2 - 2*b^2)*Csc[c + d*x]^2)/(2*d) + (4*a*b*Csc
[c + d*x]^3)/(3*d) + ((2*a^2 - b^2)*Csc[c + d*x]^4)/(4*d) - (2*a*b*Csc[c +
d*x]^5)/(5*d) - (a^2*Csc[c + d*x]^6)/(6*d) + (b^2*Log[Sin[c + d*x]])/d
```

Rubi [A] time = 0.158191, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(2a^2-b^2)\csc^4(c+dx)}{4d}-\frac{(a^2-2b^2)\csc^2(c+dx)}{2d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-2*a*b*Csc[c + d*x])/d - ((a^2 - 2*b^2)*Csc[c + d*x]^2)/(2*d) + (4*a*b*Csc
[c + d*x]^3)/(3*d) + ((2*a^2 - b^2)*Csc[c + d*x]^4)/(4*d) - (2*a*b*Csc[c +
d*x]^5)/(5*d) - (a^2*Csc[c + d*x]^6)/(6*d) + (b^2*Log[Sin[c + d*x]])/d
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^7(a+x)^2(b^2-x^2)^2}{x^7}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^7}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \left(\frac{a^2b^4}{x^7}+\frac{2ab^4}{x^6}+\frac{-2a^2b^2+b^4}{x^5}-\frac{4ab^2}{x^4}+\frac{a^2-2b^2}{x^3}+\frac{2a}{x^2}+\frac{1}{x}\right) d\right.}{d}$$

$$= -\frac{2ab\csc(c+dx)}{d}-\frac{(a^2-2b^2)\csc^2(c+dx)}{2d}+\frac{4ab\csc^3(c+dx)}{3d}$$

Mathematica [A] time = 0.165615, size = 107, normalized size = 0.82

$$\frac{15(2a^2-b^2)\csc^4(c+dx)-30(a^2-2b^2)\csc^2(c+dx)-10a^2\csc^6(c+dx)-24ab\csc^5(c+dx)+80ab\csc^3(c+dx)-}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-120*a*b*Csc[c + d*x] - 30*(a^2 - 2*b^2)*Csc[c + d*x]^2 + 80*a*b*Csc[c + d
*x]^3 + 15*(2*a^2 - b^2)*Csc[c + d*x]^4 - 24*a*b*Csc[c + d*x]^5 - 10*a^2*Cs
c[c + d*x]^6 + 60*b^2*Log[Sin[c + d*x]])/(60*d)
```

Maple [A] time = 0.095, size = 196, normalized size = 1.5

$$-\frac{a^2(\cos(dx+c))^6}{6d(\sin(dx+c))^6}-\frac{2ab(\cos(dx+c))^6}{5d(\sin(dx+c))^5}+\frac{2ab(\cos(dx+c))^6}{15d(\sin(dx+c))^3}-\frac{2ab(\cos(dx+c))^6}{5d\sin(dx+c)}-\frac{16ab\sin(dx+c)}{15d}-\frac{2ab\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)^6-2/5/d*a*b/sin(d*x+c)^5*cos(d*x+c)^6+2/
15/d*a*b/sin(d*x+c)^3*cos(d*x+c)^6-2/5/d*a*b/sin(d*x+c)*cos(d*x+c)^6-16/15*
a*b*sin(d*x+c)/d-2/5/d*sin(d*x+c)*a*b*cos(d*x+c)^4-8/15/d*sin(d*x+c)*a*b*co
s(d*x+c)^2-1/4/d*b^2*cot(d*x+c)^4+1/2/d*b^2*cot(d*x+c)^2+b^2*ln(sin(d*x+c))
/d
```

Maxima [A] time = 0.988596, size = 146, normalized size = 1.12

$$\frac{60\,b^2\log(\sin(dx+c))-\frac{120\,ab\sin(dx+c)^5-80\,ab\sin(dx+c)^3+30\,(a^2-2\,b^2)\sin(dx+c)^4+24\,ab\sin(dx+c)-15\,(2\,a^2-b^2)\sin(dx+c)^2+10\,a^2}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(60*b^2*log(sin(d*x + c)) - (120*a*b*sin(d*x + c)^5 - 80*a*b*sin(d*x +
 c)^3 + 30*(a^2 - 2*b^2)*sin(d*x + c)^4 + 24*a*b*sin(d*x + c) - 15*(2*a^2 -
 b^2)*sin(d*x + c)^2 + 10*a^2)/sin(d*x + c)^6)/d
```

Fricas [A] time = 1.80379, size = 448, normalized size = 3.45

$$\frac{30\left(a^2-2b^2\right)\cos\left(dx+c\right)^4-15\left(2a^2-7b^2\right)\cos\left(dx+c\right)^2+10a^2-45b^2+60\left(b^2\cos\left(dx+c\right)^6-3b^2\cos\left(dx+c\right)^4+3b\right.}{60\left(d\cos\left(dx+c\right)^6-3d\cos\left(dx+c\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/60*(30*(a^2 - 2*b^2)*cos(d*x + c)^4 - 15*(2*a^2 - 7*b^2)*cos(d*x + c)^2 +
 10*a^2 - 45*b^2 + 60*(b^2*cos(d*x + c)^6 - 3*b^2*cos(d*x + c)^4 + 3*b^2*co
s(d*x + c)^2 - b^2)*log(1/2*sin(d*x + c)) + 8*(15*a*b*cos(d*x + c)^4 - 20*a
*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x +
c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2441, size = 181, normalized size = 1.39

$$\frac{60b^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{147b^2\sin(dx+c)^6+120ab\sin(dx+c)^5+30a^2\sin(dx+c)^4-60b^2\sin(dx+c)^4-80ab\sin(dx+c)^3-30a^2\sin(dx+c)^2+15b^2\sin(dx+c}{\sin(dx+c)^6}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*b^2*log(abs(sin(d*x + c))) - (147*b^2*sin(d*x + c)^6 + 120*a*b*sin
(d*x + c)^5 + 30*a^2*sin(d*x + c)^4 - 60*b^2*sin(d*x + c)^4 - 80*a*b*sin(d*
x + c)^3 - 30*a^2*sin(d*x + c)^2 + 15*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x +
 c) + 10*a^2)/sin(d*x + c)^6)/d
```

3.1224 $\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{(2a^2-b^2)\csc^5(c+dx)}{5d}-\frac{(a^2-2b^2)\csc^3(c+dx)}{3d}-\frac{a^2\csc^7(c+dx)}{7d}-\frac{ab\csc^6(c+dx)}{3d}+\frac{ab\csc^4(c+dx)}{d}-\frac{ab\csc^2(c+$$

```
[Out] -((b^2*Csc[c + d*x])/d) - (a*b*Csc[c + d*x]^2)/d - ((a^2 - 2*b^2)*Csc[c + d
*x]^3)/(3*d) + (a*b*Csc[c + d*x]^4)/d + ((2*a^2 - b^2)*Csc[c + d*x]^5)/(5*d
) - (a*b*Csc[c + d*x]^6)/(3*d) - (a^2*Csc[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.159315, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(2a^2-b^2)\csc^5(c+dx)}{5d}-\frac{(a^2-2b^2)\csc^3(c+dx)}{3d}-\frac{a^2\csc^7(c+dx)}{7d}-\frac{ab\csc^6(c+dx)}{3d}+\frac{ab\csc^4(c+dx)}{d}-\frac{ab\csc^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b^2*Csc[c + d*x])/d) - (a*b*Csc[c + d*x]^2)/d - ((a^2 - 2*b^2)*Csc[c + d
*x]^3)/(3*d) + (a*b*Csc[c + d*x]^4)/d + ((2*a^2 - b^2)*Csc[c + d*x]^5)/(5*d
) - (a*b*Csc[c + d*x]^6)/(3*d) - (a^2*Csc[c + d*x]^7)/(7*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^8(a+x)^2(b^2-x^2)^2}{x^8}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^8}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^3\,\text{Subst}\left(\int \left(\frac{a^2b^4}{x^8}+\frac{2ab^4}{x^7}+\frac{-2a^2b^2+b^4}{x^6}-\frac{4ab^2}{x^5}+\frac{a^2-2b^2}{x^4}+\frac{2a}{x^3}+\frac{1}{x^2}\right)dx, \right.}{d}$$
$$= -\frac{b^2\csc(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{\left(a^2-2b^2\right)\csc^3(c+dx)}{3d}+\frac{ab\,\mathrm{c}}{}$$

Mathematica [A] time = 0.181635, size = 104, normalized size = 0.81

$$-\frac{\csc(c+dx)\left(21\left(b^2-2a^2\right)\csc^4(c+dx)+35\left(a^2-2b^2\right)\csc^2(c+dx)+15a^2\csc^6(c+dx)+35ab\csc^5(c+dx)-105ab\,\mathrm{csc}\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]*(105*b^2 + 105*a*b*Csc[c + d*x] + 35*(a^2 - 2*b^2)*Csc[c + d
*x]^2 - 105*a*b*Csc[c + d*x]^3 + 21*(-2*a^2 + b^2)*Csc[c + d*x]^4 + 35*a*b*
Csc[c + d*x]^5 + 15*a^2*Csc[c + d*x]^6))/(105*d)
```

Maple [A] time = 0.095, size = 218, normalized size = 1.7

$$\frac{1}{d}\left(a^2\left(-\frac{(\cos(dx+c))^6}{7(\sin(dx+c))^7}-\frac{(\cos(dx+c))^6}{35(\sin(dx+c))^5}+\frac{(\cos(dx+c))^6}{105(\sin(dx+c))^3}-\frac{(\cos(dx+c))^6}{35\sin(dx+c)}-\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/1
05/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x
+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/3*a*b/sin(d*x+c)^6*cos(d*x+c)^6+b^2*(
-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c
)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A] time = 0.990311, size = 143, normalized size = 1.11

$$-\frac{105\,b^2\sin(dx+c)^6+105\,ab\sin(dx+c)^5-105\,ab\sin(dx+c)^3+35\left(a^2-2\,b^2\right)\sin(dx+c)^4+35\,ab\sin(dx+c)-21\,(}{105\,d\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/105*(105*b^2*sin(d*x + c)^6 + 105*a*b*sin(d*x + c)^5 - 105*a*b*sin(d*x +
 c)^3 + 35*(a^2 - 2*b^2)*sin(d*x + c)^4 + 35*a*b*sin(d*x + c) - 21*(2*a^2 -
 b^2)*sin(d*x + c)^2 + 15*a^2)/(d*sin(d*x + c)^7)
```

Fricas [A] time = 1.65978, size = 362, normalized size = 2.81

$$-\frac{105\,b^2\cos\left(dx+c\right)^6-35\left(a^2+7\,b^2\right)\cos\left(dx+c\right)^4+28\left(a^2+7\,b^2\right)\cos\left(dx+c\right)^2-8\,a^2-56\,b^2-35\left(3\,ab\cos\left(dx+c\right)\right.}{105\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(105*b^2*cos(d*x + c)^6 - 35*(a^2 + 7*b^2)*cos(d*x + c)^4 + 28*(a^2
+ 7*b^2)*cos(d*x + c)^2 - 8*a^2 - 56*b^2 - 35*(3*a*b*cos(d*x + c)^4 - 3*a*b
*cos(d*x + c)^2 + a*b)*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^
4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21549, size = 159, normalized size = 1.23

$$-\frac{105\,b^2\sin\left(dx+c\right)^6+105\,ab\sin\left(dx+c\right)^5+35\,a^2\sin\left(dx+c\right)^4-70\,b^2\sin\left(dx+c\right)^4-105\,ab\sin\left(dx+c\right)^3-42\,a^2\sin}{105\,d\sin\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/105*(105*b^2*sin(d*x + c)^6 + 105*a*b*sin(d*x + c)^5 + 35*a^2*sin(d*x +
c)^4 - 70*b^2*sin(d*x + c)^4 - 105*a*b*sin(d*x + c)^3 - 42*a^2*sin(d*x + c)
^2 + 21*b^2*sin(d*x + c)^2 + 35*a*b*sin(d*x + c) + 15*a^2)/(d*sin(d*x + c)^
7)
```

3.1225 $\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=138

$$\frac{(2a^2-b^2)\csc^6(c+dx)}{6d}-\frac{(a^2-2b^2)\csc^4(c+dx)}{4d}-\frac{a^2\csc^8(c+dx)}{8d}-\frac{2ab\csc^7(c+dx)}{7d}+\frac{4ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+\cdots}{3d}$$

```
[Out] -(b^2*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - ((a^2 - 2*b^2)
*Csc[c + d*x]^4)/(4*d) + (4*a*b*Csc[c + d*x]^5)/(5*d) + ((2*a^2 - b^2)*Csc[
c + d*x]^6)/(6*d) - (2*a*b*Csc[c + d*x]^7)/(7*d) - (a^2*Csc[c + d*x]^8)/(8*
d)
```

Rubi [A] time = 0.161012, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(2a^2-b^2)\csc^6(c+dx)}{6d}-\frac{(a^2-2b^2)\csc^4(c+dx)}{4d}-\frac{a^2\csc^8(c+dx)}{8d}-\frac{2ab\csc^7(c+dx)}{7d}+\frac{4ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+\cdots}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(b^2*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - ((a^2 - 2*b^2)
*Csc[c + d*x]^4)/(4*d) + (4*a*b*Csc[c + d*x]^5)/(5*d) + ((2*a^2 - b^2)*Csc[
c + d*x]^6)/(6*d) - (2*a*b*Csc[c + d*x]^7)/(7*d) - (a^2*Csc[c + d*x]^8)/(8*
d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^9(a+x)^2(b^2-x^2)^2}{x^9}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b^4\,\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^9}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^4\,\text{Subst}\left(\int \left(\frac{a^2b^4}{x^9}+\frac{2ab^4}{x^8}+\frac{-2a^2b^2+b^4}{x^7}-\frac{4ab^2}{x^6}+\frac{a^2-2b^2}{x^5}+\frac{2a}{x^4}+\frac{1}{x^3}\right) d\right.}{d}$$
$$= -\frac{b^2\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{(a^2-2b^2)\csc^4(c+dx)}{4d}+$$

Mathematica [A] time = 0.218315, size = 108, normalized size = 0.78

$$-\frac{\csc^2(c+dx)\left(-140\left(2a^2-b^2\right)\csc^4(c+dx)+210\left(a^2-2b^2\right)\csc^2(c+dx)+105a^2\csc^6(c+dx)+240ab\csc^5(c+dx)\right.}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^2*(420*b^2 + 560*a*b*Csc[c + d*x] + 210*(a^2 - 2*b^2)*Csc[c
+ d*x]^2 - 672*a*b*Csc[c + d*x]^3 - 140*(2*a^2 - b^2)*Csc[c + d*x]^4 + 240*
a*b*Csc[c + d*x]^5 + 105*a^2*Csc[c + d*x]^6))/(840*d)
```

Maple [A] time = 0.092, size = 173, normalized size = 1.3

$$\frac{1}{d}\left(a^2\left(-\frac{(\cos(dx+c))^6}{8(\sin(dx+c))^8}-\frac{(\cos(dx+c))^6}{24(\sin(dx+c))^6}\right)+2ab\left(-1/7\frac{(\cos(dx+c))^6}{(\sin(dx+c))^7}-1/35\frac{(\cos(dx+c))^6}{(\sin(dx+c))^5}+\frac{(\cos(dx+c))^6}{105(\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6)+2*
a*b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105/si
n(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*b^2/sin(d*x+c)^6*cos(d*x+c)^6)
```

Maxima [A] time = 0.984058, size = 143, normalized size = 1.04

$$-\frac{420b^2\sin(dx+c)^6+560ab\sin(dx+c)^5-672ab\sin(dx+c)^3+210\left(a^2-2b^2\right)\sin(dx+c)^4+240ab\sin(dx+c)}{840d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/840*(420*b^2*sin(d*x + c)^6 + 560*a*b*sin(d*x + c)^5 - 672*a*b*sin(d*x +
 c)^3 + 210*(a^2 - 2*b^2)*sin(d*x + c)^4 + 240*a*b*sin(d*x + c) - 140*(2*a^
```

```
2 - b^2)*sin(d*x + c)^2 + 105*a^2)/(d*sin(d*x + c)^8)
```

Fricas [A] time = 1.71055, size = 379, normalized size = 2.75

$$\frac{420\,b^2\cos\left(dx+c\right)^6-210\left(a^2+4\,b^2\right)\cos\left(dx+c\right)^4+140\left(a^2+4\,b^2\right)\cos\left(dx+c\right)^2-35\,a^2-140\,b^2-16\left(35\,ab\cos\left(dx+c\right.\right.}{840\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/840*(420*b^2*cos(d*x + c)^6 - 210*(a^2 + 4*b^2)*cos(d*x + c)^4 + 140*(a^2
 + 4*b^2)*cos(d*x + c)^2 - 35*a^2 - 140*b^2 - 16*(35*a*b*cos(d*x + c)^4 - 2
8*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x
 + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**9*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24006, size = 159, normalized size = 1.15

$$-\frac{420\,b^2\sin\left(dx+c\right)^6+560\,ab\sin\left(dx+c\right)^5+210\,a^2\sin\left(dx+c\right)^4-420\,b^2\sin\left(dx+c\right)^4-672\,ab\sin\left(dx+c\right)^3-280\,a^2\sin}{840\,d\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(420*b^2*sin(d*x + c)^6 + 560*a*b*sin(d*x + c)^5 + 210*a^2*sin(d*x +
 c)^4 - 420*b^2*sin(d*x + c)^4 - 672*a*b*sin(d*x + c)^3 - 280*a^2*sin(d*x +
 c)^2 + 140*b^2*sin(d*x + c)^2 + 240*a*b*sin(d*x + c) + 105*a^2)/(d*sin(d*x
 + c)^8)
```

3.1226 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=235

$$\frac{(3a^2-2b^2)\sin^4(c+dx)}{4b^4d}-\frac{4a(a^2-b^2)\sin^3(c+dx)}{3b^5d}+\frac{(-6a^2b^2+5a^4+b^4)\sin^2(c+dx)}{2b^6d}-\frac{2a(-4a^2b^2+3a^4+b^4)\sin}{b^7d}$$

```
[Out] (a^2*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])/(b^8*d) - (2*a*(
3*a^4 - 4*a^2*b^2 + b^4)*Sin[c + d*x])/(b^7*d) + ((5*a^4 - 6*a^2*b^2 + b^4)
*Sin[c + d*x]^2)/(2*b^6*d) - (4*a*(a^2 - b^2)*Sin[c + d*x]^3)/(3*b^5*d) + (
(3*a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b^4*d) - (2*a*Sin[c + d*x]^5)/(5*b^3*d)
+ Sin[c + d*x]^6/(6*b^2*d) + (a^3*(a^2 - b^2)^2)/(b^8*d*(a + b*Sin[c + d*x]
))
```

Rubi [A] time = 0.28051, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(3a^2-2b^2)\sin^4(c+dx)}{4b^4d}-\frac{4a(a^2-b^2)\sin^3(c+dx)}{3b^5d}+\frac{(-6a^2b^2+5a^4+b^4)\sin^2(c+dx)}{2b^6d}-\frac{2a(-4a^2b^2+3a^4+b^4)\sin}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])/(b^8*d) - (2*a*(
3*a^4 - 4*a^2*b^2 + b^4)*Sin[c + d*x])/(b^7*d) + ((5*a^4 - 6*a^2*b^2 + b^4)
*Sin[c + d*x]^2)/(2*b^6*d) - (4*a*(a^2 - b^2)*Sin[c + d*x]^3)/(3*b^5*d) + (
(3*a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b^4*d) - (2*a*Sin[c + d*x]^5)/(5*b^3*d)
+ Sin[c + d*x]^6/(6*b^2*d) + (a^3*(a^2 - b^2)^2)/(b^8*d*(a + b*Sin[c + d*x]
))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{b^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^8 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a\left(3a^4-4a^2b^2+b^4\right)+\left(5a^4-6a^2b^2+b^4\right)x-4a\left(a^2-b^2\right)x^2+\left(3a^2-2b\right.\right.\right.}{b^8 d}$$
$$= \frac{a^2\left(7a^4-10a^2b^2+3b^4\right)\log(a+b\sin(c+dx))}{b^8 d} - \frac{2a\left(3a^4-4a^2b^2+b^4\right)\sin(c+dx)}{b^7 d} + \frac{(}{}$$

Mathematica [A] time = 2.12213, size = 264, normalized size = 1.12

$$\frac{3b^5\left(7a^2-10b^2\right)\sin^5(c+dx)+\left(50ab^6-35a^3b^4\right)\sin^4(c+dx)+10b^3\left(-10a^2b^2+7a^4+3b^4\right)\sin^3(c+dx)-30ab^2\left(-10a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (60*a^3*(a^2 - b^2)*(a^2 - b^2 + (7*a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]]) +
 60*a^2*b*(a^2 - b^2)*(-6*a^2 + 2*b^2 + (7*a^2 - 3*b^2)*Log[a + b*Sin[c + d
*x]])*Sin[c + d*x] - 30*a*b^2*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[c + d*x]^2 +
 10*b^3*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[c + d*x]^3 + (-35*a^3*b^4 + 50*a*b
^6)*Sin[c + d*x]^4 + 3*b^5*(7*a^2 - 10*b^2)*Sin[c + d*x]^5 - 14*a*b^6*Sin[c
 + d*x]^6 + 10*b^7*Sin[c + d*x]^7)/(60*b^8*d*(a + b*Sin[c + d*x]))
```

Maple [A] time = 0.095, size = 342, normalized size = 1.5

$$\frac{(\sin(dx+c))^6}{6b^2d} - \frac{2a(\sin(dx+c))^5}{5b^3d} + \frac{3(\sin(dx+c))^4a^2}{4db^4} - \frac{(\sin(dx+c))^4}{2b^2d} - \frac{4(\sin(dx+c))^3a^3}{3db^5} + \frac{4a(\sin(dx+c))^3}{3b^3d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/6*sin(d*x+c)^6/b^2/d-2/5*a*sin(d*x+c)^5/b^3/d+3/4/d/b^4*sin(d*x+c)^4*a^2-
1/2*sin(d*x+c)^4/b^2/d-4/3/d/b^5*sin(d*x+c)^3*a^3+4/3*a*sin(d*x+c)^3/b^3/d+
5/2/d/b^6*sin(d*x+c)^2*a^4-3/d/b^4*sin(d*x+c)^2*a^2+1/2*sin(d*x+c)^2/b^2/d-
6/d/b^7*a^5*sin(d*x+c)+8/d/b^5*a^3*sin(d*x+c)-2*a*sin(d*x+c)/b^3/d+7/d*a^6/
b^8*ln(a+b*sin(d*x+c))-10/d*a^4/b^6*ln(a+b*sin(d*x+c))+3/d*a^2/b^4*ln(a+b*s
in(d*x+c))+1/d*a^7/b^8/(a+b*sin(d*x+c))-2/d*a^5/b^6/(a+b*sin(d*x+c))+1/d*a^
3/b^4/(a+b*sin(d*x+c))
```

Maxima [A] time = 0.996216, size = 294, normalized size = 1.25

$$\frac{\frac{60\left(a^7-2a^5b^2+a^3b^4\right)}{b^9\sin(dx+c)+ab^8} + \frac{10b^5\sin(dx+c)^6-24ab^4\sin(dx+c)^5+15\left(3a^2b^3-2b^5\right)\sin(dx+c)^4-80\left(a^3b^2-ab^4\right)\sin(dx+c)^3+30\left(5a^4b-6a^2b^3+b^5\right)\sin(dx+c)^2-120\left(3a\right.}{b^7}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(60*(a^7 - 2*a^5*b^2 + a^3*b^4)/(b^9*sin(d*x + c) + a*b^8) + (10*b^5*s
in(d*x + c)^6 - 24*a*b^4*sin(d*x + c)^5 + 15*(3*a^2*b^3 - 2*b^5)*sin(d*x +
c)^4 - 80*(a^3*b^2 - a*b^4)*sin(d*x + c)^3 + 30*(5*a^4*b - 6*a^2*b^3 + b^5)
*sin(d*x + c)^2 - 120*(3*a^5 - 4*a^3*b^2 + a*b^4)*sin(d*x + c))/b^7 + 60*(7
*a^6 - 10*a^4*b^2 + 3*a^2*b^4)*log(b*sin(d*x + c) + a)/b^8)/d
```

Fricas [A] time = 2.20311, size = 671, normalized size = 2.86

$$112\,ab^6\cos(dx+c)^6 + 480\,a^7 - 3240\,a^5b^2 + 3185\,a^3b^4 - 487\,ab^6 - 8\left(35\,a^3b^4 - 8\,ab^6\right)\cos(dx+c)^4 + 16\left(105\,a^5b^2 - 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(112*a*b^6*cos(d*x + c)^6 + 480*a^7 - 3240*a^5*b^2 + 3185*a^3*b^4 - 4
87*a*b^6 - 8*(35*a^3*b^4 - 8*a*b^6)*cos(d*x + c)^4 + 16*(105*a^5*b^2 - 115*
a^3*b^4 + 16*a*b^6)*cos(d*x + c)^2 + 480*(7*a^7 - 10*a^5*b^2 + 3*a^3*b^4 +
(7*a^6*b - 10*a^4*b^3 + 3*a^2*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) -
(80*b^7*cos(d*x + c)^6 - 168*a^2*b^5*cos(d*x + c)^4 + 2880*a^6*b - 3800*a^4
*b^3 + 1007*a^2*b^5 - 25*b^7 + 16*(35*a^4*b^3 - 29*a^2*b^5)*cos(d*x + c)^2)
*sin(d*x + c))/(b^9*d*sin(d*x + c) + a*b^8*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18885, size = 405, normalized size = 1.72

$$\frac{60\left(7\,a^6-10\,a^4b^2+3\,a^2b^4\right)\log(|b\sin(dx+c)+a|)}{b^8} - \frac{60\left(7\,a^6b\sin(dx+c)-10\,a^4b^3\sin(dx+c)+3\,a^2b^5\sin(dx+c)+6\,a^7-8\,a^5b^2+2\,a^3b^4\right)}{(b\sin(dx+c)+a)b^8} + \frac{10\,b^{10}\sin(dx+c)^6-24\,ab\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(7*a^6 - 10*a^4*b^2 + 3*a^2*b^4)*log(abs(b*sin(d*x + c) + a))/b^8
- 60*(7*a^6*b*sin(d*x + c) - 10*a^4*b^3*sin(d*x + c) + 3*a^2*b^5*sin(d*x +
```

```
c) + 6*a^7 - 8*a^5*b^2 + 2*a^3*b^4)/((b*sin(d*x + c) + a)*b^8) + (10*b^10*s
in(d*x + c)^6 - 24*a*b^9*sin(d*x + c)^5 + 45*a^2*b^8*sin(d*x + c)^4 - 30*b^
10*sin(d*x + c)^4 - 80*a^3*b^7*sin(d*x + c)^3 + 80*a*b^9*sin(d*x + c)^3 + 1
50*a^4*b^6*sin(d*x + c)^2 - 180*a^2*b^8*sin(d*x + c)^2 + 30*b^10*sin(d*x +
c)^2 - 360*a^5*b^5*sin(d*x + c) + 480*a^3*b^7*sin(d*x + c) - 120*a*b^9*sin(
d*x + c))/b^12)/d
```

3.1227 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(2-\frac{3a^2}{b^2}\right)\sin^3(c+dx)}{3b^2d}-\frac{2a\left(a^2-b^2\right)\sin^2(c+dx)}{b^5d}+\frac{\left(-6a^2b^2+5a^4+b^4\right)\sin(c+dx)}{b^6d}-\frac{a^2\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))}-\frac{2a\,(}{}$$

```
[Out] (-2*a*(3*a^4 - 4*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^7*d) + ((5*a^4
- 6*a^2*b^2 + b^4)*Sin[c + d*x])/(b^6*d) - (2*a*(a^2 - b^2)*Sin[c + d*x]^2)
/(b^5*d) - ((2 - (3*a^2)/b^2)*Sin[c + d*x]^3)/(3*b^2*d) - (a*Sin[c + d*x]^4
)/(2*b^3*d) + Sin[c + d*x]^5/(5*b^2*d) - (a^2*(a^2 - b^2)^2)/(b^7*d*(a + b*
Sin[c + d*x]))
```

Rubi [A] time = 0.239489, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$-\frac{\left(2-\frac{3a^2}{b^2}\right)\sin^3(c+dx)}{3b^2d}-\frac{2a\left(a^2-b^2\right)\sin^2(c+dx)}{b^5d}+\frac{\left(-6a^2b^2+5a^4+b^4\right)\sin(c+dx)}{b^6d}-\frac{a^2\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))}-\frac{2a\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-2*a*(3*a^4 - 4*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^7*d) + ((5*a^4
- 6*a^2*b^2 + b^4)*Sin[c + d*x])/(b^6*d) - (2*a*(a^2 - b^2)*Sin[c + d*x]^2)
/(b^5*d) - ((2 - (3*a^2)/b^2)*Sin[c + d*x]^3)/(3*b^2*d) - (a*Sin[c + d*x]^4
)/(2*b^3*d) + Sin[c + d*x]^5/(5*b^2*d) - (a^2*(a^2 - b^2)^2)/(b^7*d*(a + b*
Sin[c + d*x]))
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2(b^2-x^2)^2}{b^2(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{x^2(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(5a^4\left(1+\frac{-6a^2b^2+b^4}{5a^4}\right) - 4a\left(a^2-b^2\right)x + \left(3a^2-2b^2\right)x^2 - 2ax^3 + x^4 + \frac{(a^3-ab^2)^2}{(a+x)^2}\right.\right.}{b^7 d}$$
$$= -\frac{2a\left(3a^4-4a^2b^2+b^4\right)\log(a+b\sin(c+dx))}{b^7 d} + \frac{\left(5a^4-6a^2b^2+b^4\right)\sin(c+dx)}{b^6 d} - \frac{2a(\ldots}{}$$

Mathematica [A] time = 1.46236, size = 225, normalized size = 1.17

$$\frac{5b^4\left(3a^2-4b^2\right)\sin^4(c+dx) + \left(40ab^5-30a^3b^3\right)\sin^3(c+dx) + 30b^2\left(-4a^2b^2+3a^4+b^4\right)\sin^2(c+dx) - 30ab\left(a^2-b^2\right)\sin\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*a^2*(a^2 - b^2)*(a^2 - b^2 + (6*a^2 - 2*b^2)*Log[a + b*Sin[c + d*x]])
- 30*a*b*(a^2 - b^2)*(-5*a^2 + b^2 + (6*a^2 - 2*b^2)*Log[a + b*Sin[c + d*x]
])*Sin[c + d*x] + 30*b^2*(3*a^4 - 4*a^2*b^2 + b^4)*Sin[c + d*x]^2 + (-30*a^
3*b^3 + 40*a*b^5)*Sin[c + d*x]^3 + 5*b^4*(3*a^2 - 4*b^2)*Sin[c + d*x]^4 - 9
*a*b^5*Sin[c + d*x]^5 + 6*b^6*Sin[c + d*x]^6)/(30*b^7*d*(a + b*Sin[c + d*x]
))
```

Maple [A] time = 0.094, size = 285, normalized size = 1.5

$$\frac{(\sin(dx+c))^5}{5\,b^2 d} - \frac{a\,(\sin(dx+c))^4}{2\,b^3 d} + \frac{a^2\,(\sin(dx+c))^3}{d b^4} - \frac{2\,(\sin(dx+c))^3}{3\,b^2 d} - 2\,\frac{(\sin(dx+c))^2\,a^3}{d b^5} + 2\,\frac{(\sin(dx+c))^2\,a}{b^3 d} + 5\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/5*sin(d*x+c)^5/b^2/d-1/2*a*sin(d*x+c)^4/b^3/d+1/d/b^4*sin(d*x+c)^3*a^2-2/
3*sin(d*x+c)^3/b^2/d-2/d/b^5*sin(d*x+c)^2*a^3+2*a*sin(d*x+c)^2/b^3/d+5/d/b^
6*a^4*sin(d*x+c)-6/d/b^4*a^2*sin(d*x+c)+sin(d*x+c)/b^2/d-6/d*a^5/b^7*ln(a+b
*sin(d*x+c))+8/d*a^3/b^5*ln(a+b*sin(d*x+c))-2*a*ln(a+b*sin(d*x+c))/b^3/d-1/
d/b^7/(a+b*sin(d*x+c))*a^6+2/d/b^5/(a+b*sin(d*x+c))*a^4-1/d/b^3/(a+b*sin(d*
x+c))*a^2
```

Maxima [A] time = 0.993761, size = 248, normalized size = 1.28

$$-\frac{\frac{30\left(a^6-2\,a^4b^2+a^2b^4\right)}{b^8\sin(dx+c)+ab^7} - \frac{6\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+10\left(3\,a^2b^2-2\,b^4\right)\sin(dx+c)^3-60\left(a^3b-ab^3\right)\sin(dx+c)^2+30\left(5\,a^4-6\,a^2b^2+b^4\right)\sin(dx+c)}{b^6} + \frac{60\,(3\,a^5\ldots}{}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*(30*(a^6 - 2*a^4*b^2 + a^2*b^4)/(b^8*sin(d*x + c) + a*b^7) - (6*b^4*s
in(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 10*(3*a^2*b^2 - 2*b^4)*sin(d*x +
c)^3 - 60*(a^3*b - a*b^3)*sin(d*x + c)^2 + 30*(5*a^4 - 6*a^2*b^2 + b^4)*sin
(d*x + c))/b^6 + 60*(3*a^5 - 4*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) + a)/b^7
)/d
```

Fricas [A] time = 2.1461, size = 590, normalized size = 3.06

$$-\frac{48\,b^{6}\cos\left(dx+c\right)^{6}+240\,a^{6}-1440\,a^{4}b^{2}+1275\,a^{2}b^{4}-128\,b^{6}-8\left(15\,a^{2}b^{4}-2\,b^{6}\right)\cos\left(dx+c\right)^{4}+16\left(45\,a^{4}b^{2}-45\,a^{2}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(48*b^6*cos(d*x + c)^6 + 240*a^6 - 1440*a^4*b^2 + 1275*a^2*b^4 - 128
*b^6 - 8*(15*a^2*b^4 - 2*b^6)*cos(d*x + c)^4 + 16*(45*a^4*b^2 - 45*a^2*b^4
+ 4*b^6)*cos(d*x + c)^2 + 480*(3*a^6 - 4*a^4*b^2 + a^2*b^4 + (3*a^5*b - 4*a
^3*b^3 + a*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (72*a*b^5*cos(d*x +
 c)^4 - 1200*a^5*b + 1440*a^3*b^3 - 293*a*b^5 - 16*(15*a^3*b^3 - 11*a*b^5)*
cos(d*x + c)^2)*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23376, size = 336, normalized size = 1.74

$$-\frac{\frac{60\left(3\,a^{5}-4\,a^{3}b^{2}+ab^{4}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{b^{7}}-\frac{30\left(6\,a^{5}b\sin\left(dx+c\right)-8\,a^{3}b^{3}\sin\left(dx+c\right)+2\,ab^{5}\sin\left(dx+c\right)+5\,a^{6}-6\,a^{4}b^{2}+a^{2}b^{4}\right)}{\left(b\sin\left(dx+c\right)+a\right)b^{7}}-\frac{6\,b^{8}\sin\left(dx+c\right)^{5}-15\,ab^{7}\sin(dx\ldots}{\ldots}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(3*a^5 - 4*a^3*b^2 + a*b^4)*log(abs(b*sin(d*x + c) + a))/b^7 - 30
*(6*a^5*b*sin(d*x + c) - 8*a^3*b^3*sin(d*x + c) + 2*a*b^5*sin(d*x + c) + 5*
a^6 - 6*a^4*b^2 + a^2*b^4)/((b*sin(d*x + c) + a)*b^7) - (6*b^8*sin(d*x + c)
^5 - 15*a*b^7*sin(d*x + c)^4 + 30*a^2*b^6*sin(d*x + c)^3 - 20*b^8*sin(d*x +
 c)^3 - 60*a^3*b^5*sin(d*x + c)^2 + 60*a*b^7*sin(d*x + c)^2 + 150*a^4*b^4*s
in(d*x + c) - 180*a^2*b^6*sin(d*x + c) + 30*b^8*sin(d*x + c))/b^10)/d
```

3.1228 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$\frac{(3a^2-2b^2)\sin^2(c+dx)}{2b^4d}-\frac{4a(a^2-b^2)\sin(c+dx)}{b^5d}+\frac{a(a^2-b^2)^2}{b^6d(a+b\sin(c+dx))}+\frac{(-6a^2b^2+5a^4+b^4)\log(a+b\sin(c+dx))}{b^6d}$$

```
[Out] ((5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^6*d) - (4*a*(a^2 - b
^2)*Sin[c + d*x])/(b^5*d) + ((3*a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^4*d) - (2
*a*Sin[c + d*x]^3)/(3*b^3*d) + Sin[c + d*x]^4/(4*b^2*d) + (a*(a^2 - b^2)^2)
/(b^6*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.154681, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 772}

$$\frac{(3a^2-2b^2)\sin^2(c+dx)}{2b^4d}-\frac{4a(a^2-b^2)\sin(c+dx)}{b^5d}+\frac{a(a^2-b^2)^2}{b^6d(a+b\sin(c+dx))}+\frac{(-6a^2b^2+5a^4+b^4)\log(a+b\sin(c+dx))}{b^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^6*d) - (4*a*(a^2 - b
^2)*Sin[c + d*x])/(b^5*d) + ((3*a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^4*d) - (2
*a*Sin[c + d*x]^3)/(3*b^3*d) + Sin[c + d*x]^4/(4*b^2*d) + (a*(a^2 - b^2)^2)
/(b^6*d*(a + b*Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{b(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(-4(a^3-ab^2)+(3a^2-2b^2)x-2ax^2+x^3-\frac{a(a^2-b^2)^2}{(a+x)^2}+\frac{5a^4-6a^2b^2+b^4}{a+x}\right)dx, x\right.}{b^6 d}$$
$$= \frac{(5a^4-6a^2b^2+b^4)\log(a+b\sin(c+dx))}{b^6 d} - \frac{4a(a^2-b^2)\sin(c+dx)}{b^5 d} + \frac{(3a^2-2b^2)\,\mathrm{si}}{2b^4}$$

Mathematica [A] time = 1.00147, size = 188, normalized size = 1.2

$$\frac{2b^3(5a^2-6b^2)\sin^3(c+dx)-6ab^2(5a^2-6b^2)\sin^2(c+dx)+12b(b^2-a^2)\sin(c+dx)\left((b^2-5a^2)\log(a+b\sin(c+\right.}{12b^6 d(a+b\sin(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (12*a*(a^2 - b^2)*(a^2 - b^2 + (5*a^2 - b^2)*Log[a + b*Sin[c + d*x]]) + 12*
b*(-a^2 + b^2)*(4*a^2 + (-5*a^2 + b^2)*Log[a + b*Sin[c + d*x]])*Sin[c + d*x
] - 6*a*b^2*(5*a^2 - 6*b^2)*Sin[c + d*x]^2 + 2*b^3*(5*a^2 - 6*b^2)*Sin[c +
d*x]^3 - 5*a*b^4*Sin[c + d*x]^4 + 3*b^5*Sin[c + d*x]^5)/(12*b^6*d*(a + b*Si
n[c + d*x]))
```

Maple [A] time = 0.078, size = 229, normalized size = 1.5

$$\frac{(\sin(dx+c))^4}{4b^2d} - \frac{2a(\sin(dx+c))^3}{3b^3d} + \frac{3(\sin(dx+c))^2a^2}{2db^4} - \frac{(\sin(dx+c))^2}{b^2d} - 4\frac{a^3\sin(dx+c)}{db^5} + 4\frac{a\sin(dx+c)}{b^3d} + 5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4*sin(d*x+c)^4/b^2/d-2/3*a*sin(d*x+c)^3/b^3/d+3/2/d/b^4*sin(d*x+c)^2*a^2-
sin(d*x+c)^2/b^2/d-4/d/b^5*a^3*sin(d*x+c)+4*a*sin(d*x+c)/b^3/d+5/d*a^4/b^6*
ln(a+b*sin(d*x+c))-6/d*a^2/b^4*ln(a+b*sin(d*x+c))+1/d/b^2*ln(a+b*sin(d*x+c)
)+1/d*a^5/b^6/(a+b*sin(d*x+c))-2/d*a^3/b^4/(a+b*sin(d*x+c))+1/d*a/b^2/(a+b*
sin(d*x+c))
```

Maxima [A] time = 1.02555, size = 200, normalized size = 1.27

$$\frac{\frac{12(a^5-2a^3b^2+ab^4)}{b^7\sin(dx+c)+ab^6} + \frac{3b^3\sin(dx+c)^4-8ab^2\sin(dx+c)^3+6(3a^2b-2b^3)\sin(dx+c)^2-48(a^3-ab^2)\sin(dx+c)}{b^5} + \frac{12(5a^4-6a^2b^2+b^4)\log(b\sin(dx+c)+a)}{b^6}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(a^5 - 2*a^3*b^2 + a*b^4)/(b^7*sin(d*x + c) + a*b^6) + (3*b^3*sin(
d*x + c)^4 - 8*a*b^2*sin(d*x + c)^3 + 6*(3*a^2*b - 2*b^3)*sin(d*x + c)^2 -
48*(a^3 - a*b^2)*sin(d*x + c))/b^5 + 12*(5*a^4 - 6*a^2*b^2 + b^4)*log(b*sin
(d*x + c) + a)/b^6)/d
```

Fricas [A] time = 1.94763, size = 482, normalized size = 3.07

$$-\frac{40\,ab^4\cos\left(dx+c\right)^4-96\,a^5+504\,a^3b^2-383\,ab^4-16\left(15\,a^3b^2-13\,ab^4\right)\cos\left(dx+c\right)^2-96\left(5\,a^5-6\,a^3b^2+ab^4+\left(5\,a^4b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/96*(40*a*b^4*cos(d*x + c)^4 - 96*a^5 + 504*a^3*b^2 - 383*a*b^4 - 16*(15*
a^3*b^2 - 13*a*b^4)*cos(d*x + c)^2 - 96*(5*a^5 - 6*a^3*b^2 + a*b^4 + (5*a^4
*b - 6*a^2*b^3 + b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) - (24*b^5*cos(d
*x + c)^4 - 384*a^4*b + 392*a^2*b^3 - 33*b^5 - 16*(5*a^2*b^3 - 3*b^5)*cos(d
*x + c)^2)*sin(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20231, size = 262, normalized size = 1.67

$$\frac{\frac{12\left(5\,a^4-6\,a^2b^2+b^4\right)\log\left(\left|b\sin(dx+c)+a\right|\right)}{b^6}-\frac{12\left(5\,a^4b\sin(dx+c)-6\,a^2b^3\sin(dx+c)+b^5\sin(dx+c)+4\,a^5-4\,a^3b^2\right)}{(b\sin(dx+c)+a)b^6}+\frac{3\,b^6\sin(dx+c)^4-8\,ab^5\sin(dx+c)^3+18\,a^2b^4\sin}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(5*a^4 - 6*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/b^6 - 12*(5
*a^4*b*sin(d*x + c) - 6*a^2*b^3*sin(d*x + c) + b^5*sin(d*x + c) + 4*a^5 - 4
*a^3*b^2)/((b*sin(d*x + c) + a)*b^6) + (3*b^6*sin(d*x + c)^4 - 8*a*b^5*sin(
d*x + c)^3 + 18*a^2*b^4*sin(d*x + c)^2 - 12*b^6*sin(d*x + c)^2 - 48*a^3*b^3
*sin(d*x + c) + 48*a*b^5*sin(d*x + c))/b^8)/d
```

3.1229 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{\left(a^2-b^2\right)^2}{ab^4d(a+b\sin(c+dx))}+\frac{\left(3a^2+b^2\right)\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^2b^4d}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{2a\sin(c+dx)}{b^3d}+\frac{\sin^2(c+dx)}{2b^2d}$$

```
[Out] Log[Sin[c + d*x]]/(a^2*d) + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sin[c + d*
x]])/(a^2*b^4*d) - (2*a*Sin[c + d*x])/(b^3*d) + Sin[c + d*x]^2/(2*b^2*d) +
(a^2 - b^2)^2/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.158906, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{\left(a^2-b^2\right)^2}{ab^4d(a+b\sin(c+dx))}+\frac{\left(3a^2+b^2\right)\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^2b^4d}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{2a\sin(c+dx)}{b^3d}+\frac{\sin^2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sin[c + d*
x]])/(a^2*b^4*d) - (2*a*Sin[c + d*x])/(b^3*d) + Sin[c + d*x]^2/(2*b^2*d) +
(a^2 - b^2)^2/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b(b^2-x^2)^2}{x(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{b^4}{a^2x}+x-\frac{(a^2-b^2)^2}{a(a+x)^2}+\frac{(a^2-b^2)(3a^2+b^2)}{a^2(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\log(\sin(c+dx))}{a^2 d}+\frac{(a^2-b^2)(3a^2+b^2)\log(a+b\sin(c+dx))}{a^2b^4d}-\frac{2a\sin(c+dx)}{b^3d}+\frac{\sin^2(}{2}$$

Mathematica [A] time = 0.489321, size = 111, normalized size = 0.92

$$\frac{\frac{2(a^2-b^2)^2}{ab^4(a+b\sin(c+dx))}+\frac{2(a-b)(a+b)(3a^2+b^2)\log(a+b\sin(c+dx))}{a^2b^4}+\frac{2\log(\sin(c+dx))}{a^2}-\frac{4a\sin(c+dx)}{b^3}+\frac{\sin^2(c+dx)}{b^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*Log[Sin[c + d*x]])/a^2 + (2*(a - b)*(a + b)*(3*a^2 + b^2)*Log[a + b*Sin
[c + d*x]])/(a^2*b^4) - (4*a*Sin[c + d*x])/b^3 + Sin[c + d*x]^2/b^2 + (2*(a
^2 - b^2)^2)/(a*b^4*(a + b*Sin[c + d*x]))))/(2*d)
```

Maple [A] time = 0.123, size = 169, normalized size = 1.4

$$\frac{(\sin(dx+c))^2}{2b^2d}-2\frac{a\sin(dx+c)}{b^3d}+\frac{a^3}{db^4(a+b\sin(dx+c))}-2\frac{a}{b^2d(a+b\sin(dx+c))}+\frac{1}{da(a+b\sin(dx+c))}+3\frac{a^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2*sin(d*x+c)^2/b^2/d-2*a*sin(d*x+c)/b^3/d+1/d*a^3/b^4/(a+b*sin(d*x+c))-2/
d*a/b^2/(a+b*sin(d*x+c))+1/d/a/(a+b*sin(d*x+c))+3/d*a^2/b^4*ln(a+b*sin(d*x+
c))-2/d/b^2*ln(a+b*sin(d*x+c))-1/d/a^2*ln(a+b*sin(d*x+c))+ln(sin(d*x+c))/a^
2/d
```

Maxima [A] time = 0.993289, size = 159, normalized size = 1.32

$$\frac{\frac{2(a^4-2a^2b^2+b^4)}{ab^5\sin(dx+c)+a^2b^4}+\frac{2\log(\sin(dx+c))}{a^2}+\frac{b\sin(dx+c)^2-4a\sin(dx+c)}{b^3}+\frac{2(3a^4-2a^2b^2-b^4)\log(b\sin(dx+c)+a)}{a^2b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4 - 2*a^2*b^2 + b^4)/(a*b^5*sin(d*x + c) + a^2*b^4) + 2*log(sin(d
*x + c))/a^2 + (b*sin(d*x + c)^2 - 4*a*sin(d*x + c))/b^3 + 2*(3*a^4 - 2*a^2
*b^2 - b^4)*log(b*sin(d*x + c) + a)/(a^2*b^4))/d
```

Fricas [A] time = 2.20138, size = 425, normalized size = 3.54

$$\frac{6\,a^{3}b^{2}\cos\left(dx+c\right)^{2}+4\,a^{5}-15\,a^{3}b^{2}+4\,ab^{4}+4\left(3\,a^{5}-2\,a^{3}b^{2}-ab^{4}+\left(3\,a^{4}b-2\,a^{2}b^{3}-b^{5}\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx\right.\right.}{4\left(a^{2}b^{5}d\sin\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^3*b^2*cos(d*x + c)^2 + 4*a^5 - 15*a^3*b^2 + 4*a*b^4 + 4*(3*a^5 - 2
*a^3*b^2 - a*b^4 + (3*a^4*b - 2*a^2*b^3 - b^5)*sin(d*x + c))*log(b*sin(d*x
+ c) + a) + 4*(b^5*sin(d*x + c) + a*b^4)*log(-1/2*sin(d*x + c)) - (2*a^2*b^
3*cos(d*x + c)^2 + 8*a^4*b - a^2*b^3)*sin(d*x + c))/(a^2*b^5*d*sin(d*x + c)
 + a^3*b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21857, size = 208, normalized size = 1.73

$$\frac{\frac{2\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{2}}+\frac{b^{2}\sin\left(dx+c\right)^{2}-4\,ab\sin\left(dx+c\right)}{b^{4}}+\frac{2\left(3\,a^{4}-2\,a^{2}b^{2}-b^{4}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{2}b^{4}}-\frac{2\left(3\,a^{4}b\sin\left(dx+c\right)-2\,a^{2}b^{3}\sin\left(dx+c\right)-b^{5}\sin\left(dx+c\right)+2\,a^{5}-\right.}{\left(b\sin\left(dx+c\right)+a\right)a^{2}b^{4}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c)))/a^2 + (b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x + c
))/b^4 + 2*(3*a^4 - 2*a^2*b^2 - b^4)*log(abs(b*sin(d*x + c) + a))/(a^2*b^4)
 - 2*(3*a^4*b*sin(d*x + c) - 2*a^2*b^3*sin(d*x + c) - b^5*sin(d*x + c) + 2*
a^5 - 2*a*b^4)/((b*sin(d*x + c) + a)*a^2*b^4))/d
```

3.1230 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{(a^2-b^2)^2}{a^2b^3d(a+b\sin(c+dx))}-\frac{2(a^4-b^4)\log(a+b\sin(c+dx))}{a^3b^3d}-\frac{2b\log(\sin(c+dx))}{a^3d}-\frac{\csc(c+dx)}{a^2d}+\frac{\sin(c+dx)}{b^2d}$$

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*b*Log[Sin[c + d*x]])/(a^3*d) - (2*(a^4 - b^4)*
Log[a + b*Sin[c + d*x]])/(a^3*b^3*d) + Sin[c + d*x]/(b^2*d) - (a^2 - b^2)^2
/(a^2*b^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.170333, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{(a^2-b^2)^2}{a^2b^3d(a+b\sin(c+dx))}-\frac{2(a^4-b^4)\log(a+b\sin(c+dx))}{a^3b^3d}-\frac{2b\log(\sin(c+dx))}{a^3d}-\frac{\csc(c+dx)}{a^2d}+\frac{\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*b*Log[Sin[c + d*x]])/(a^3*d) - (2*(a^4 - b^4)*
Log[a + b*Sin[c + d*x]])/(a^3*b^3*d) + Sin[c + d*x]/(b^2*d) - (a^2 - b^2)^2
/(a^2*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)^2}{x^2(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^2(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^4}{a^2x^2}-\frac{2b^4}{a^3x}+\frac{(a^2-b^2)^2}{a^2(a+x)^2}-\frac{2(a^4-b^4)}{a^3(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= -\frac{\csc(c+dx)}{a^2d}-\frac{2b\log(\sin(c+dx))}{a^3d}-\frac{2\left(a^4-b^4\right)\log(a+b\sin(c+dx))}{a^3b^3d}+\frac{\sin(c+dx)}{b^2d}$$

Mathematica [A] time = 0.592501, size = 95, normalized size = 0.87

$$-\frac{\frac{(a^2-b^2)^2}{a^2b^3(a+b\sin(c+dx))}+2\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\log(a+b\sin(c+dx))+\frac{2b\log(\sin(c+dx))}{a^3}+\frac{\csc(c+dx)}{a^2}-\frac{\sin(c+dx)}{b^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((Csc[c + d*x]/a^2 + (2*b*Log[Sin[c + d*x]])/a^3 + 2*(a/b^3 - b/a^3)*Log[a
 + b*Sin[c + d*x]] - Sin[c + d*x]/b^2 + (a^2 - b^2)^2/(a^2*b^3*(a + b*Sin[c
 + d*x]))))/d)
```

Maple [A] time = 0.131, size = 151, normalized size = 1.4

$$\frac{\sin(dx+c)}{b^2d}-2\frac{a\ln(a+b\sin(dx+c))}{b^3d}+2\frac{b\ln(a+b\sin(dx+c))}{da^3}-\frac{a^2}{b^3d(a+b\sin(dx+c))}+2\frac{1}{bd(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] sin(d*x+c)/b^2/d-2*a*ln(a+b*sin(d*x+c))/b^3/d+2/d/a^3*b*ln(a+b*sin(d*x+c))-
1/d/b^3/(a+b*sin(d*x+c))*a^2+2/b/d/(a+b*sin(d*x+c))-1/d*b/a^2/(a+b*sin(d*x+
c))-1/d/a^2/sin(d*x+c)-2*b*ln(sin(d*x+c))/a^3/d
```

Maxima [A] time = 0.99743, size = 162, normalized size = 1.49

$$-\frac{\frac{ab^3+\left(a^4-2a^2b^2+2b^4\right)\sin(dx+c)}{a^2b^4\sin(dx+c)^2+a^3b^3\sin(dx+c)}+\frac{2b\log(\sin(dx+c))}{a^3}-\frac{\sin(dx+c)}{b^2}+\frac{2\left(a^4-b^4\right)\log(b\sin(dx+c)+a)}{a^3b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -((a*b^3 + (a^4 - 2*a^2*b^2 + 2*b^4)*sin(d*x + c))/(a^2*b^4*sin(d*x + c)^2
+ a^3*b^3*sin(d*x + c)) + 2*b*log(sin(d*x + c))/a^3 - sin(d*x + c)/b^2 + 2*
(a^4 - b^4)*log(b*sin(d*x + c) + a)/(a^3*b^3))/d
```

Fricas [A] time = 2.13863, size = 473, normalized size = 4.34

$$\frac{a^4 b\cos(dx+c)^2 - a^4 b + a^2 b^3 + 2\left(a^4 b - b^5 - \left(a^4 b - b^5\right)\cos(dx+c)^2 + \left(a^5 - ab^4\right)\sin(dx+c)\right)\log(b\sin(dx+c)+a) -}{a^3 b^4 d\cos(dx+c)^2 - a^4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (a^4*b*cos(d*x + c)^2 - a^4*b + a^2*b^3 + 2*(a^4*b - b^5 - (a^4*b - b^5)*co
s(d*x + c)^2 + (a^5 - a*b^4)*sin(d*x + c))*log(b*sin(d*x + c) + a) - 2*(b^5
*cos(d*x + c)^2 - a*b^4*sin(d*x + c) - b^5)*log(1/2*sin(d*x + c)) + (a^3*b^
2*cos(d*x + c)^2 + a^5 - 3*a^3*b^2 + 2*a*b^4)*sin(d*x + c))/(a^3*b^4*d*cos(
d*x + c)^2 - a^4*b^3*d*sin(d*x + c) - a^3*b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21541, size = 177, normalized size = 1.62

$$-\frac{\frac{2\,b\log\left(\left|\sin(dx+c)\right|\right)}{a^3} - \frac{\sin(dx+c)}{b^2} - \frac{a^3\sin(dx+c)^2 + 2\,a^2 b\sin(dx+c) - 2\,b^3\sin(dx+c) - ab^2}{\left(b\sin(dx+c)^2 + a\sin(dx+c)\right)a^2 b^2} + \frac{2\left(a^4 - b^4\right)\log\left(\left|b\sin(dx+c)+a\right|\right)}{a^3 b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*b*log(abs(sin(d*x + c)))/a^3 - sin(d*x + c)/b^2 - (a^3*sin(d*x + c)^2 +
 2*a^2*b*sin(d*x + c) - 2*b^3*sin(d*x + c) - a*b^2)/((b*sin(d*x + c)^2 + a*
sin(d*x + c))*a^2*b^2) + 2*(a^4 - b^4)*log(abs(b*sin(d*x + c) + a))/(a^3*b^
3))/d
```

3.1231 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=131

$$\frac{(a^2-b^2)^2}{a^3b^2d(a+b\sin(c+dx))}-\frac{(2a^2-3b^2)\log(\sin(c+dx))}{a^4d}+\frac{(2a^2b^2+a^4-3b^4)\log(a+b\sin(c+dx))}{a^4b^2d}+\frac{2b\csc(c+dx)}{a^3d}$$

```
[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((2*a^2 - 3*b^2)*Lo
g[Sin[c + d*x]])/(a^4*d) + ((a^4 + 2*a^2*b^2 - 3*b^4)*Log[a + b*Sin[c + d*x
]])/(a^4*b^2*d) + (a^2 - b^2)^2/(a^3*b^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.199803, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{(a^2-b^2)^2}{a^3b^2d(a+b\sin(c+dx))}-\frac{(2a^2-3b^2)\log(\sin(c+dx))}{a^4d}+\frac{(2a^2b^2+a^4-3b^4)\log(a+b\sin(c+dx))}{a^4b^2d}+\frac{2b\csc(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((2*a^2 - 3*b^2)*Lo
g[Sin[c + d*x]])/(a^4*d) + ((a^4 + 2*a^2*b^2 - 3*b^4)*Log[a + b*Sin[c + d*x
]])/(a^4*b^2*d) + (a^2 - b^2)^2/(a^3*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^3(b^2-x^2)^2}{x^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^3} - \frac{2b^4}{a^3x^2} + \frac{-2a^2b^2+3b^4}{a^4x} - \frac{(a^2-b^2)^2}{a^3(a+x)^2} + \frac{a^4+2a^2b^2-3b^4}{a^4(a+x)}\right) dx, x, b\sin(c+dx)\right)}{b^2 d}$$

$$= \frac{2b\csc(c+dx)}{a^3 d} - \frac{\csc^2(c+dx)}{2a^2 d} - \frac{(2a^2-3b^2)\log(\sin(c+dx))}{a^4 d} + \frac{(a^4+2a^2b^2-3b^4)\log}{a^4b^2}$$

Mathematica [A] time = 0.716844, size = 116, normalized size = 0.89

$$\frac{\frac{2a(a^2-b^2)^2}{b^2(a+b\sin(c+dx))} - 2(2a^2-3b^2)\log(\sin(c+dx)) + \frac{2(2a^2b^2+a^4-3b^4)\log(a+b\sin(c+dx))}{b^2} - a^2\csc^2(c+dx) + 4ab\csc(c+dx)}{2a^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*a*b*Csc[c + d*x] - a^2*Csc[c + d*x]^2 - 2*(2*a^2 - 3*b^2)*Log[Sin[c + d*
x]] + (2*(a^4 + 2*a^2*b^2 - 3*b^4)*Log[a + b*Sin[c + d*x]])/b^2 + (2*a*(a^2
 - b^2)^2)/(b^2*(a + b*Sin[c + d*x])))/(2*a^4*d)
```

Maple [A] time = 0.154, size = 189, normalized size = 1.4

$$\frac{a}{db^2(a+b\sin(dx+c))} - 2\frac{1}{da(a+b\sin(dx+c))} + \frac{b^2}{da^3(a+b\sin(dx+c))} + \frac{\ln(a+b\sin(dx+c))}{db^2} + 2\frac{\ln(a+b\sin(dx}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a/b^2/(a+b*sin(d*x+c))-2/d/a/(a+b*sin(d*x+c))+1/d/a^3*b^2/(a+b*sin(d*x+
c))+1/d/b^2*ln(a+b*sin(d*x+c))+2/d/a^2*ln(a+b*sin(d*x+c))-3/d/a^4*b^2*ln(a+
b*sin(d*x+c))-1/2/d/a^2/sin(d*x+c)^2-2*ln(sin(d*x+c))/a^2/d+3/d/a^4*ln(sin(
d*x+c))*b^2+2/d/a^3*b/sin(d*x+c)
```

Maxima [A] time = 1.01919, size = 198, normalized size = 1.51

$$\frac{\frac{3ab^3\sin(dx+c) - a^2b^2 + 2(a^4-2a^2b^2+3b^4)\sin(dx+c)^2}{a^3b^3\sin(dx+c)^3 + a^4b^2\sin(dx+c)^2} - \frac{2(2a^2-3b^2)\log(\sin(dx+c))}{a^4} + \frac{2(a^4+2a^2b^2-3b^4)\log(b\sin(dx+c)+a)}{a^4b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*((3*a*b^3*sin(d*x + c) - a^2*b^2 + 2*(a^4 - 2*a^2*b^2 + 3*b^4)*sin(d*x
+ c)^2)/(a^3*b^3*sin(d*x + c)^3 + a^4*b^2*sin(d*x + c)^2) - 2*(2*a^2 - 3*b^
2)*log(sin(d*x + c))/a^4 + 2*(a^4 + 2*a^2*b^2 - 3*b^4)*log(b*sin(d*x + c) +
 a)/(a^4*b^2))/d
```

Fricas [B] time = 2.19302, size = 743, normalized size = 5.67

$$-\frac{3\,a^2b^3\sin\left(dx+c\right)+2\,a^5-5\,a^3b^2+6\,ab^4-2\left(a^5-2\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^2+2\left(a^5+2\,a^3b^2-3\,ab^4-\left(a^5+2\,a^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(3*a^2*b^3*sin(d*x + c) + 2*a^5 - 5*a^3*b^2 + 6*a*b^4 - 2*(a^5 - 2*a^3
*b^2 + 3*a*b^4)*cos(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4 - (a^5 + 2*a^
3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + (a^4*b + 2*a^2*b^3 - 3*b^5 - (a^4*b + 2*a
^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) - 2*(
2*a^3*b^2 - 3*a*b^4 - (2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + (2*a^2*b^3 - 3
*b^5 - (2*a^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x +
 c)))/(a^5*b^2*d*cos(d*x + c)^2 - a^5*b^2*d + (a^4*b^3*d*cos(d*x + c)^2 - a
^4*b^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22335, size = 257, normalized size = 1.96

$$-\frac{\frac{2\left(2\,a^2-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^4}-\frac{2\left(a^4+2\,a^2b^2-3\,b^4\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^4b^2}+\frac{2\left(a^4\sin\left(dx+c\right)+2\,a^2b^2\sin\left(dx+c\right)-3\,b^4\sin\left(dx+c\right)+4\,a^3b-4\,ab^3\right)}{\left(b\sin\left(dx+c\right)+a\right)a^4b}-\frac{6\,a^2\sin(d}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^2 - 3*b^2)*log(abs(sin(d*x + c)))/a^4 - 2*(a^4 + 2*a^2*b^2 - 3
*b^4)*log(abs(b*sin(d*x + c) + a))/(a^4*b^2) + 2*(a^4*sin(d*x + c) + 2*a^2*
b^2*sin(d*x + c) - 3*b^4*sin(d*x + c) + 4*a^3*b - 4*a*b^3)/((b*sin(d*x + c)
 + a)*a^4*b) - (6*a^2*sin(d*x + c)^2 - 9*b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x
 + c) - a^2)/(a^4*sin(d*x + c)^2))/d
```

3.1232 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$-\frac{\left(a^2-b^2\right)^2}{a^4bd(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc(c+dx)}{a^4d}+\frac{4b\left(a^2-b^2\right)\log(\sin(c+dx))}{a^5d}-\frac{4b\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^5d}$$

```
[Out] ((2*a^2 - 3*b^2)*Csc[c + d*x])/(a^4*d) + (b*Csc[c + d*x]^2)/(a^3*d) - Csc[c
 + d*x]^3/(3*a^2*d) + (4*b*(a^2 - b^2)*Log[Sin[c + d*x]])/(a^5*d) - (4*b*(a
^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) - (a^2 - b^2)^2/(a^4*b*d*(a + b*
Sin[c + d*x]))
```

Rubi [A] time = 0.205735, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{\left(a^2-b^2\right)^2}{a^4bd(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc(c+dx)}{a^4d}+\frac{4b\left(a^2-b^2\right)\log(\sin(c+dx))}{a^5d}-\frac{4b\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*Csc[c + d*x])/(a^4*d) + (b*Csc[c + d*x]^2)/(a^3*d) - Csc[c
 + d*x]^3/(3*a^2*d) + (4*b*(a^2 - b^2)*Log[Sin[c + d*x]])/(a^5*d) - (4*b*(a
^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) - (a^2 - b^2)^2/(a^4*b*d*(a + b*
Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^4(b^2-x^2)^2}{x^4(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^4(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^4} - \frac{2b^4}{a^3x^3} + \frac{-2a^2b^2+3b^4}{a^4x^2} + \frac{4b^2(a^2-b^2)}{a^5x} + \frac{(a^2-b^2)^2}{a^4(a+x)^2} + \frac{4b^2(-a^2+b^2)}{a^5(a+x)}\right) dx, x, b\sin(c}{bd}$$

$$= \frac{(2a^2-3b^2)\csc(c+dx)}{a^4d} + \frac{b\csc^2(c+dx)}{a^3d} - \frac{\csc^3(c+dx)}{3a^2d} + \frac{4b(a^2-b^2)\log(\sin(c+}{a^5d}$$

Mathematica [A] time = 1.94143, size = 127, normalized size = 0.86

$$\frac{-\frac{3a(a^2-b^2)^2}{b(a+b\sin(c+dx))} + 3a(2a^2-3b^2)\csc(c+dx) + 3a^2b\csc^2(c+dx) - a^3\csc^3(c+dx) + 12b(a-b)(a+b)\log(\sin(c+dx))}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*(2*a^2 - 3*b^2)*Csc[c + d*x] + 3*a^2*b*Csc[c + d*x]^2 - a^3*Csc[c + d*
x]^3 + 12*(a - b)*b*(a + b)*Log[Sin[c + d*x]] - 12*(a - b)*b*(a + b)*Log[a
+ b*Sin[c + d*x]] - (3*a*(a^2 - b^2)^2)/(b*(a + b*Sin[c + d*x])))/(3*a^5*d)
```

Maple [A] time = 0.152, size = 209, normalized size = 1.4

$$-\frac{1}{bd(a+b\sin(dx+c))} + 2\frac{b}{da^2(a+b\sin(dx+c))} - \frac{b^3}{da^4(a+b\sin(dx+c))} - 4\frac{b\ln(a+b\sin(dx+c))}{da^3} + 4\frac{b^3\ln(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/b/d/(a+b*sin(d*x+c))+2/d*b/a^2/(a+b*sin(d*x+c))-1/d/a^4*b^3/(a+b*sin(d*x
+c))-4/d/a^3*b*ln(a+b*sin(d*x+c))+4/d*b^3/a^5*ln(a+b*sin(d*x+c))-1/3/d/a^2/
sin(d*x+c)^3+2/d/a^2/sin(d*x+c)-3/d/a^4/sin(d*x+c)*b^2+1/d/a^3*b/sin(d*x+c)
^2+4*b*ln(sin(d*x+c))/a^3/d-4/d*b^3/a^5*ln(sin(d*x+c))
```

Maxima [A] time = 1.02349, size = 213, normalized size = 1.45

$$\frac{\frac{2a^2b^2\sin(dx+c) - a^3b - 3(a^4-4a^2b^2+4b^4)\sin(dx+c)^3 + 6(a^3b-ab^3)\sin(dx+c)^2}{a^4b^2\sin(dx+c)^4 + a^5b\sin(dx+c)^3} - \frac{12(a^2b-b^3)\log(b\sin(dx+c)+a)}{a^5} + \frac{12(a^2b-b^3)\log(\sin(dx+c))}{a^5}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*((2*a^2*b^2*sin(d*x + c) - a^3*b - 3*(a^4 - 4*a^2*b^2 + 4*b^4)*sin(d*x
+ c)^3 + 6*(a^3*b - a*b^3)*sin(d*x + c)^2)/(a^4*b^2*sin(d*x + c)^4 + a^5*b*
sin(d*x + c)^3) - 12*(a^2*b - b^3)*log(b*sin(d*x + c) + a)/a^5 + 12*(a^2*b
- b^3)*log(sin(d*x + c))/a^5)/d
```

Fricas [B] time = 2.00698, size = 859, normalized size = 5.84

$$\frac{5\,a^4b-6\,a^2b^3-6\left(a^4b-a^2b^3\right)\cos\left(dx+c\right)^2-12\left(a^2b^3-b^5+\left(a^2b^3-b^5\right)\cos\left(dx+c\right)^4-2\left(a^2b^3-b^5\right)\cos\left(dx+c\right)^2+\left(a^3\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/3*(5*a^4*b - 6*a^2*b^3 - 6*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 12*(a^2*b^3
 - b^5 + (a^2*b^3 - b^5)*cos(d*x + c)^4 - 2*(a^2*b^3 - b^5)*cos(d*x + c)^2
+ (a^3*b^2 - a*b^4 - (a^3*b^2 - a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*log(b*
sin(d*x + c) + a) + 12*(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cos(d*x + c)^4 - 2*
(a^2*b^3 - b^5)*cos(d*x + c)^2 + (a^3*b^2 - a*b^4 - (a^3*b^2 - a*b^4)*cos(d
*x + c)^2)*sin(d*x + c))*log(1/2*sin(d*x + c)) - (3*a^5 - 14*a^3*b^2 + 12*a
*b^4 - 3*(a^5 - 4*a^3*b^2 + 4*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*b^2
*d*cos(d*x + c)^4 - 2*a^5*b^2*d*cos(d*x + c)^2 + a^5*b^2*d - (a^6*b*d*cos(d
*x + c)^2 - a^6*b*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19374, size = 285, normalized size = 1.94

$$\frac{\frac{12\left(a^2b-b^3\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^5}-\frac{12\left(a^2b^2-b^4\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^5b}+\frac{3\left(4\,a^2b^3\sin\left(dx+c\right)-4\,b^5\sin\left(dx+c\right)-a^5+6\,a^3b^2-5\,ab^4\right)}{\left(b\sin\left(dx+c\right)+a\right)a^5b}-\frac{22\,a^2b\sin\left(dx+c\right)^3-22\,b^3\sin\left(dx+\cdots\right.}{\cdots}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(12*(a^2*b - b^3)*log(abs(sin(d*x + c)))/a^5 - 12*(a^2*b^2 - b^4)*log(a
bs(b*sin(d*x + c) + a))/(a^5*b) + 3*(4*a^2*b^3*sin(d*x + c) - 4*b^5*sin(d*x
 + c) - a^5 + 6*a^3*b^2 - 5*a*b^4)/((b*sin(d*x + c) + a)*a^5*b) - (22*a^2*b
*sin(d*x + c)^3 - 22*b^3*sin(d*x + c)^3 - 6*a^3*sin(d*x + c)^2 + 9*a*b^2*si
n(d*x + c)^2 - 3*a^2*b*sin(d*x + c) + a^3)/(a^5*sin(d*x + c)^3))/d
```

3.1233 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$\frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}-\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d}+\frac{\left(-6a^2b^2+a^4+5b^4\right)\log(\sin(c+dx))}{a^6d}\cdot$$

```
[Out] (-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a
^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*
b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c +
 d*x]))
```

Rubi [A] time = 0.170519, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}-\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d}+\frac{\left(-6a^2b^2+a^4+5b^4\right)\log(\sin(c+dx))}{a^6d}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a
^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*
b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c +
 d*x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^5} - \frac{2b^4}{a^3x^4} + \frac{-2a^2b^2+3b^4}{a^4x^3} + \frac{4b^2(a^2-b^2)}{a^5x^2} + \frac{a^4-6a^2b^2+5b^4}{a^6x} - \frac{(a^2-b^2)^2}{a^5(a+x)^2} + \frac{-a^4+6a^2b^2-5b^4}{a^6(a+x)}\right) dx, x\right.}{d}$$

$$= -\frac{4b(a^2-b^2)\csc(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+dx)}{4a^2d} + \frac{(a}{}$$

Mathematica [A] time = 6.13748, size = 187, normalized size = 0.99

$$\frac{(a^2-b^2)^2}{a^5d(a+b\sin(c+dx))} + \frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d} + \frac{(-6a^2b^2+a^4+5b^4)\log(\sin(c+dx))}{a^6d} - \frac{(-6a^2b^2+a^4+5b^4)\log(a+}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*(a - b)*b*(a + b)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]
^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) +
 ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2
+ 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin
[c + d*x]))
```

Maple [A] time = 0.155, size = 282, normalized size = 1.5

$$-\frac{\ln(a+b\sin(dx+c))}{da^2} + 6\frac{b^2\ln(a+b\sin(dx+c))}{da^4} - 5\frac{\ln(a+b\sin(dx+c))b^4}{da^6} + \frac{1}{da(a+b\sin(dx+c))} - 2\frac{}{da^3(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*ln(a+b*sin(d*x+c))+6/d/a^4*b^2*ln(a+b*sin(d*x+c))-5/d/a^6*ln(a+b*s
in(d*x+c))*b^4+1/d/a/(a+b*sin(d*x+c))-2/d/a^3*b^2/(a+b*sin(d*x+c))+1/d/a^5/
(a+b*sin(d*x+c))*b^4-1/4/d/a^2/sin(d*x+c)^4+1/d/a^2/sin(d*x+c)^2-3/2/d/a^4/
sin(d*x+c)^2*b^2+ln(sin(d*x+c))/a^2/d-6/d/a^4*ln(sin(d*x+c))*b^2+5/d/a^6*ln
(sin(d*x+c))*b^4+2/3/d/a^3*b/sin(d*x+c)^3-4/d/a^3*b/sin(d*x+c)+4/d*b^3/a^5/
sin(d*x+c)
```

Maxima [A] time = 1.01323, size = 255, normalized size = 1.36

$$\frac{\frac{5a^3b\sin(dx+c)+12(a^4-6a^2b^2+5b^4)\sin(dx+c)^4-3a^4-6(6a^3b-5ab^3)\sin(dx+c)^3+2(6a^4-5a^2b^2)\sin(dx+c)^2}{a^5b\sin(dx+c)^5+a^6\sin(dx+c)^4} - \frac{12(a^4-6a^2b^2+5b^4)\log(b\sin(dx+c)+a)}{a^6} + 12}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*((5*a^3*b*sin(d*x + c) + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*sin(d*x + c)^4 -
 3*a^4 - 6*(6*a^3*b - 5*a*b^3)*sin(d*x + c)^3 + 2*(6*a^4 - 5*a^2*b^2)*sin(d
*x + c)^2)/(a^5*b*sin(d*x + c)^5 + a^6*sin(d*x + c)^4) - 12*(a^4 - 6*a^2*b^
2 + 5*b^4)*log(b*sin(d*x + c) + a)/a^6 + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(s
in(d*x + c))/a^6)/d
```

Fricas [B] time = 2.14108, size = 1241, normalized size = 6.6

$$\frac{21\,a^5 - 82\,a^3b^2 + 60\,ab^4 + 12\left(a^5 - 6\,a^3b^2 + 5\,ab^4\right)\cos\left(dx + c\right)^4 - 2\left(18\,a^5 - 77\,a^3b^2 + 60\,ab^4\right)\cos\left(dx + c\right)^2 - 12\left(a^5 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(21*a^5 - 82*a^3*b^2 + 60*a*b^4 + 12*(a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d
*x + c)^4 - 2*(18*a^5 - 77*a^3*b^2 + 60*a*b^4)*cos(d*x + c)^2 - 12*(a^5 - 6
*a^3*b^2 + 5*a*b^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 -
6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b -
 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x
+ c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 12*(a^5 - 6*a^3*b^2 + 5*a*b
^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 6*a^3*b^2 + 5*a*
b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b - 6*a^2*b^3 + 5*b
^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*sin(d*x + c)) - (31*a^4*b - 30*a^2*b^3 - 6*(6*a^4*b - 5*a^2*
b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x
+ c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b
*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23456, size = 375, normalized size = 1.99

$$\frac{\frac{12\left(a^4-6\,a^2b^2+5\,b^4\right)\log\left(|\sin(dx+c)|\right)}{a^6} - \frac{12\left(a^4b-6\,a^2b^3+5\,b^5\right)\log\left(|b\sin(dx+c)+a|\right)}{a^6b} + \frac{12\left(a^4b\sin(dx+c)-6\,a^2b^3\sin(dx+c)+5\,b^5\sin(dx+c)+2\,a^5-8\,a^3b^2+6\,al\right.}{(b\sin(dx+c)+a)a^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(abs(sin(d*x + c)))/a^6 - 12*(a^4*b -
 6*a^2*b^3 + 5*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6*b) + 12*(a^4*b*sin(d*
```

```
x + c) - 6*a^2*b^3*sin(d*x + c) + 5*b^5*sin(d*x + c) + 2*a^5 - 8*a^3*b^2 +
6*a*b^4)/((b*sin(d*x + c) + a)*a^6) - (25*a^4*sin(d*x + c)^4 - 150*a^2*b^2*
sin(d*x + c)^4 + 125*b^4*sin(d*x + c)^4 + 48*a^3*b*sin(d*x + c)^3 - 48*a*b^
3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 18*a^2*b^2*sin(d*x + c)^2 - 8*a^
3*b*sin(d*x + c) + 3*a^4)/(a^6*sin(d*x + c)^4))/d
```

3.1234 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=226

$$-\frac{b\left(a^2-b^2\right)^2}{a^6d(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc^3(c+dx)}{3a^4d}-\frac{2b\left(a^2-b^2\right)\csc^2(c+dx)}{a^5d}-\frac{\left(-6a^2b^2+a^4+5b^4\right)\csc(c+dx)}{a^6d}-$$

```
[Out] -(((a^4 - 6*a^2*b^2 + 5*b^4)*Csc[c + d*x])/(a^6*d)) - (2*b*(a^2 - b^2)*Csc[
c + d*x]^2)/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^3)/(3*a^4*d) + (b*Csc[c
 + d*x]^4)/(2*a^3*d) - Csc[c + d*x]^5/(5*a^2*d) - (2*b*(a^4 - 4*a^2*b^2 + 3
*b^4)*Log[Sin[c + d*x]])/(a^7*d) + (2*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[a + b
*Sin[c + d*x]])/(a^7*d) - (b*(a^2 - b^2)^2)/(a^6*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.261449, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{b\left(a^2-b^2\right)^2}{a^6d(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc^3(c+dx)}{3a^4d}-\frac{2b\left(a^2-b^2\right)\csc^2(c+dx)}{a^5d}-\frac{\left(-6a^2b^2+a^4+5b^4\right)\csc(c+dx)}{a^6d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -(((a^4 - 6*a^2*b^2 + 5*b^4)*Csc[c + d*x])/(a^6*d)) - (2*b*(a^2 - b^2)*Csc[
c + d*x]^2)/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^3)/(3*a^4*d) + (b*Csc[c
 + d*x]^4)/(2*a^3*d) - Csc[c + d*x]^5/(5*a^2*d) - (2*b*(a^4 - 4*a^2*b^2 + 3
*b^4)*Log[Sin[c + d*x]])/(a^7*d) + (2*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[a + b
*Sin[c + d*x]])/(a^7*d) - (b*(a^2 - b^2)^2)/(a^6*d*(a + b*Sin[c + d*x]))
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^6(b^2-x^2)^2}{x^6(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{b\,\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^6(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \left(\frac{b^4}{a^2x^6} - \frac{2b^4}{a^3x^5} + \frac{-2a^2b^2+3b^4}{a^4x^4} + \frac{4b^2(a^2-b^2)}{a^5x^3} + \frac{a^4-6a^2b^2+5b^4}{a^6x^2} - \frac{2(a^4-4a^2b^2+3b^4)}{a^7x} + \frac{(a^2-b^2)}{a^6(a+x)}\right.\right.}{d}$$
$$= -\frac{(a^4-6a^2b^2+5b^4)\csc(c+dx)}{a^6d} - \frac{2b(a^2-b^2)\csc^2(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\csc^3(c+dx)}{3a^4d}$$

Mathematica [A] time = 3.13538, size = 220, normalized size = 0.97

$$5a^4(4a^2-3b^2)\csc^4(c+dx) + (30a^3b^3-40a^5b)\csc^3(c+dx) - 30a^2(-4a^2b^2+a^4+3b^4)\csc^2(c+dx) - 60b^2(-4a^2b^2+\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*a^2*(a^4 - 4*a^2*b^2 + 3*b^4)*Csc[c + d*x]^2 + (-40*a^5*b + 30*a^3*b^3
)*Csc[c + d*x]^3 + 5*a^4*(4*a^2 - 3*b^2)*Csc[c + d*x]^4 + 9*a^5*b*Csc[c + d
*x]^5 - 6*a^6*Csc[c + d*x]^6 - 60*b^2*(a^4 - 4*a^2*b^2 + 3*b^4)*(Log[Sin[c
+ d*x]] - Log[a + b*Sin[c + d*x]]) - 60*a*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Csc[c
 + d*x]*(1 + Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]]))/(30*a^7*d*(b + a
*Csc[c + d*x]))
```

Maple [A] time = 0.159, size = 343, normalized size = 1.5

$$-\frac{b}{da^2(a+b\sin(dx+c))} + 2\frac{b^3}{da^4(a+b\sin(dx+c))} - \frac{b^5}{da^6(a+b\sin(dx+c))} + 2\frac{b\ln(a+b\sin(dx+c))}{da^3} - 8\frac{b^3\ln(a+\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*b/a^2/(a+b*sin(d*x+c))+2/d/a^4*b^3/(a+b*sin(d*x+c))-1/d*b^5/a^6/(a+b*s
in(d*x+c))+2/d/a^3*b*ln(a+b*sin(d*x+c))-8/d*b^3/a^5*ln(a+b*sin(d*x+c))+6/d*
b^5/a^7*ln(a+b*sin(d*x+c))-1/5/d/a^2/sin(d*x+c)^5+2/3/d/a^2/sin(d*x+c)^3-1/
d/a^4/sin(d*x+c)^3*b^2-1/d/a^2/sin(d*x+c)+6/d/a^4/sin(d*x+c)*b^2-5/d/a^6/si
n(d*x+c)*b^4+1/2/d/a^3*b/sin(d*x+c)^4-2/d/a^3*b/sin(d*x+c)^2+2/d*b^3/a^5/si
n(d*x+c)^2-2*b*ln(sin(d*x+c))/a^3/d+8/d*b^3/a^5*ln(sin(d*x+c))-6/d*b^5/a^7*
ln(sin(d*x+c))
```

Maxima [A] time = 0.989809, size = 304, normalized size = 1.35

$$\frac{\frac{9a^4b\sin(dx+c)-60(a^4b-4a^2b^3+3b^5)\sin(dx+c)^5-6a^5-30(a^5-4a^3b^2+3ab^4)\sin(dx+c)^4-10(4a^4b-3a^2b^3)\sin(dx+c)^3+5(4a^5-3a^3b^2)\sin(dx+c)^2}{a^6b\sin(dx+c)^6+a^7\sin(dx+c)^5} + \frac{60(a^4b-\ldots}{\ldots}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*((9*a^4*b*sin(d*x + c) - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*sin(d*x + c)^5
 - 6*a^5 - 30*(a^5 - 4*a^3*b^2 + 3*a*b^4)*sin(d*x + c)^4 - 10*(4*a^4*b - 3*
a^2*b^3)*sin(d*x + c)^3 + 5*(4*a^5 - 3*a^3*b^2)*sin(d*x + c)^2)/(a^6*b*sin(
d*x + c)^6 + a^7*sin(d*x + c)^5) + 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(b*sin
(d*x + c) + a)/a^7 - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(sin(d*x + c))/a^7)/
d
```

Fricas [B]   time = 2.18314, size = 1553, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/30*(16*a^6 - 105*a^4*b^2 + 90*a^2*b^4 + 30*(a^6 - 4*a^4*b^2 + 3*a^2*b^4)*
cos(d*x + c)^4 - 5*(8*a^6 - 45*a^4*b^2 + 36*a^2*b^4)*cos(d*x + c)^2 + 60*((
a^4*b^2 - 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 + 4*a^2*b^4 - 3*b^6 -
 3*(a^4*b^2 - 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 4*a^2*b^4 +
3*b^6)*cos(d*x + c)^2 - (a^5*b - 4*a^3*b^3 + 3*a*b^5 + (a^5*b - 4*a^3*b^3 +
 3*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*
sin(d*x + c))*log(b*sin(d*x + c) + a) - 60*((a^4*b^2 - 4*a^2*b^4 + 3*b^6)*c
os(d*x + c)^6 - a^4*b^2 + 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 - 4*a^2*b^4 + 3*b^
6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - (a^5*b
 - 4*a^3*b^3 + 3*a*b^5 + (a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^4 - 2*(
a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*sin(d*x
+ c)) + (91*a^5*b - 270*a^3*b^3 + 180*a*b^5 + 60*(a^5*b - 4*a^3*b^3 + 3*a*b
^5)*cos(d*x + c)^4 - 10*(16*a^5*b - 51*a^3*b^3 + 36*a*b^5)*cos(d*x + c)^2)*
sin(d*x + c))/(a^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*
d*cos(d*x + c)^2 - a^7*b*d - (a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2
 + a^8*d)*sin(d*x + c))
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]   time = 1.21143, size = 448, normalized size = 1.98

$$-\frac{\frac{60\left(a^4b-4a^2b^3+3b^5\right)\log(|\sin(dx+c)|)}{a^7}-\frac{60\left(a^4b^2-4a^2b^4+3b^6\right)\log(|b\sin(dx+c)+a|)}{a^7b}+\frac{30\left(2a^4b^2\sin(dx+c)-8a^2b^4\sin(dx+c)+6b^6\sin(dx+c)+3a^5b-10\right.}{(b\sin(dx+c)+a)a^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(abs(sin(d*x + c)))/a^7 - 60*(a^4*
b^2 - 4*a^2*b^4 + 3*b^6)*log(abs(b*sin(d*x + c) + a))/(a^7*b) + 30*(2*a^4*b
^2*sin(d*x + c) - 8*a^2*b^4*sin(d*x + c) + 6*b^6*sin(d*x + c) + 3*a^5*b - 1
0*a^3*b^3 + 7*a*b^5)/((b*sin(d*x + c) + a)*a^7) - (137*a^4*b*sin(d*x + c)^5
 - 548*a^2*b^3*sin(d*x + c)^5 + 411*b^5*sin(d*x + c)^5 - 30*a^5*sin(d*x + c
)^4 + 180*a^3*b^2*sin(d*x + c)^4 - 150*a*b^4*sin(d*x + c)^4 - 60*a^4*b*sin(
d*x + c)^3 + 60*a^2*b^3*sin(d*x + c)^3 + 20*a^5*sin(d*x + c)^2 - 30*a^3*b^2
*sin(d*x + c)^2 + 15*a^4*b*sin(d*x + c) - 6*a^5)/(a^7*sin(d*x + c)^5))/d
```

3.1235 $\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=170

$$-\frac{(2a^2-b^2)\sin^{n+3}(c+dx)}{d(n+3)}+\frac{(a^2-2b^2)\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2ab\sin^{n+2}(c+dx)}{d(n+2)}-\frac{4ab\sin^{n+4}(c+dx)}{d(n+4)}$$

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a*b*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - ((2*a^2 - b^2)*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (4*a*b*Sin[c +
d*x]^(4 + n))/(d*(4 + n)) + ((a^2 - 2*b^2)*Sin[c + d*x]^(5 + n))/(d*(5 + n)
) + (2*a*b*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (b^2*Sin[c + d*x]^(7 + n))/(
d*(7 + n))
```

Rubi [A] time = 0.213198, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 948}

$$-\frac{(2a^2-b^2)\sin^{n+3}(c+dx)}{d(n+3)}+\frac{(a^2-2b^2)\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2ab\sin^{n+2}(c+dx)}{d(n+2)}-\frac{4ab\sin^{n+4}(c+dx)}{d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a*b*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - ((2*a^2 - b^2)*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (4*a*b*Sin[c +
d*x]^(4 + n))/(d*(4 + n)) + ((a^2 - 2*b^2)*Sin[c + d*x]^(5 + n))/(d*(5 + n)
) + (2*a*b*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (b^2*Sin[c + d*x]^(7 + n))/(
d*(7 + n))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \left(\frac{x}{b}\right)^n (a+x)^2\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}\\ &= \frac{\text{Subst}\left(\int \left(a^2b^4\left(\frac{x}{b}\right)^n+2ab^5\left(\frac{x}{b}\right)^{1+n}-b^4\left(2a^2-b^2\right)\left(\frac{x}{b}\right)^{2+n}-4ab^5\right.\right.}{}\\ &= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)}+\frac{2ab\sin^{2+n}(c+dx)}{d(2+n)}-\frac{(2a^2-b^2)\sin^{3+n}(c+}{d(3+n)}\end{aligned}$$

Mathematica [A] time = 0.661141, size = 139, normalized size = 0.82

$$\frac{\sin^{n+1}(c+dx)\left(\frac{(a^2-2b^2)\sin^4(c+dx)}{n+5}-\frac{(2a^2-b^2)\sin^2(c+dx)}{n+3}+\frac{a^2}{n+1}+\frac{2ab\sin^5(c+dx)}{n+6}-\frac{4ab\sin^3(c+dx)}{n+4}+\frac{2ab\sin(c+dx)}{n+2}+\frac{b^2\sin^6(c+dx)}{n+7}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(a^2/(1 + n) + (2*a*b*Sin[c + d*x])/(2 + n) - ((2*a^2
 - b^2)*Sin[c + d*x]^2)/(3 + n) - (4*a*b*Sin[c + d*x]^3)/(4 + n) + ((a^2 -
2*b^2)*Sin[c + d*x]^4)/(5 + n) + (2*a*b*Sin[c + d*x]^5)/(6 + n) + (b^2*Sin[
c + d*x]^6)/(7 + n)))/d
```

Maple [F] time = 10.642, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^5 (\sin(dx+c))^n (a+b\sin(dx+c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34373, size = 1408, normalized size = 8.28

$$-\frac{\left(2\left(abn^6+22\,abn^5+190\,abn^4+820\,abn^3+1849\,abn^2+2038\,abn+840\,ab\right)\cos(dx+c)^6-16\,abn^4-256\,abn^3-2\left(ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(2*(a*b*n^6 + 22*a*b*n^5 + 190*a*b*n^4 + 820*a*b*n^3 + 1849*a*b*n^2 + 2038
*a*b*n + 840*a*b)*cos(d*x + c)^6 - 16*a*b*n^4 - 256*a*b*n^3 - 2*(a*b*n^6 +
18*a*b*n^5 + 118*a*b*n^4 + 348*a*b*n^3 + 457*a*b*n^2 + 210*a*b*n)*cos(d*x +
 c)^4 - 1376*a*b*n^2 - 2816*a*b*n - 8*(a*b*n^5 + 16*a*b*n^4 + 86*a*b*n^3 +
176*a*b*n^2 + 105*a*b*n)*cos(d*x + c)^2 - 1680*a*b + ((b^2*n^6 + 21*b^2*n^5
```

```
 + 175*b^2*n^4 + 735*b^2*n^3 + 1624*b^2*n^2 + 1764*b^2*n + 720*b^2)*cos(d*x
 + c)^6 - 8*(a^2 + b^2)*n^4 - ((a^2 + b^2)*n^6 + (23*a^2 + 17*b^2)*n^5 + 3*
(69*a^2 + 37*b^2)*n^4 + 5*(185*a^2 + 71*b^2)*n^3 + 8*(268*a^2 + 73*b^2)*n^2
 + 1008*a^2 + 144*b^2 + 36*(67*a^2 + 13*b^2)*n)*cos(d*x + c)^4 - 8*(19*a^2
+ 13*b^2)*n^3 - 64*(16*a^2 + 7*b^2)*n^2 - 4*((a^2 + b^2)*n^5 + 2*(10*a^2 +
7*b^2)*n^4 + 3*(49*a^2 + 23*b^2)*n^3 + 4*(121*a^2 + 37*b^2)*n^2 + 336*a^2 +
 48*b^2 + 4*(173*a^2 + 35*b^2)*n)*cos(d*x + c)^2 - 2688*a^2 - 384*b^2 - 32*
(89*a^2 + 23*b^2)*n)*sin(d*x + c))*sin(d*x + c)^n/(d*n^7 + 28*d*n^6 + 322*d
*n^5 + 1960*d*n^4 + 6769*d*n^3 + 13132*d*n^2 + 13068*d*n + 5040*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18212, size = 776, normalized size = 4.56

$$\frac{\left(n^2 \sin(dx+c)^n \sin(dx+c)^5+4\,n \sin(dx+c)^n \sin(dx+c)^5-2\,n^2 \sin(dx+c)^n \sin(dx+c)^3+3\,\sin(dx+c)^n \sin(dx+c)^5-12\,n \sin(dx+c)^n \sin(dx+c)^3+n^2 \sin(dx+c)^n \sin\right.}{n^3+9\,n^2+23\,n+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 4*n*sin(d*x + c)^n*sin(d*x + c)^5 - 2
*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 3*sin(d*x + c)^n*sin(d*x + c)^5 - 12*n
*sin(d*x + c)^n*sin(d*x + c)^3 + n^2*sin(d*x + c)^n*sin(d*x + c) - 10*sin(d
*x + c)^n*sin(d*x + c)^3 + 8*n*sin(d*x + c)^n*sin(d*x + c) + 15*sin(d*x + c
)^n*sin(d*x + c))*a^2/(n^3 + 9*n^2 + 23*n + 15) + 2*(n^2*sin(d*x + c)^n*sin
(d*x + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2*n^2*sin(d*x + c)^n*sin(
d*x + c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n*sin(d*x + c)^n*sin(d*x
+ c)^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin(d*x + c)^n*sin(d*x + c)
^4 + 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*x + c)^n*sin(d*x + c)^2)
*a*b/(n^3 + 12*n^2 + 44*n + 48) + (n^2*sin(d*x + c)^n*sin(d*x + c)^7 + 8*n*
sin(d*x + c)^n*sin(d*x + c)^7 - 2*n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 15*si
n(d*x + c)^n*sin(d*x + c)^7 - 20*n*sin(d*x + c)^n*sin(d*x + c)^5 + n^2*sin(
d*x + c)^n*sin(d*x + c)^3 - 42*sin(d*x + c)^n*sin(d*x + c)^5 + 12*n*sin(d*x
 + c)^n*sin(d*x + c)^3 + 35*sin(d*x + c)^n*sin(d*x + c)^3)*b^2/(n^3 + 15*n^
2 + 71*n + 105))/d
```

3.1236 $\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{b\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2b\sin^{n+4}(c+dx)}{d(n+4)}+\frac{b\sin^{n+6}(c+dx)}{d(n+6)}$$

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (b*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (2*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (2*b*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (b*Sin[c + d*x]^(6 + n))/(
d*(6 + n))
```

Rubi [A] time = 0.137182, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2837, 766}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{b\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2b\sin^{n+4}(c+dx)}{d(n+4)}+\frac{b\sin^{n+6}(c+dx)}{d(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (b*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (2*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (2*b*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (b*Sin[c + d*x]^(6 + n))/(
d*(6 + n))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \left(\frac{x}{b}\right)^n (a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}\\ &= \frac{\text{Subst}\left(\int \left(ab^4\left(\frac{x}{b}\right)^n+b^5\left(\frac{x}{b}\right)^{1+n}-2ab^4\left(\frac{x}{b}\right)^{2+n}-2b^5\left(\frac{x}{b}\right)^{3+n}+ab^4\left(\frac{x}{b}\right.\right.\right.}{b^5 d}\\ &= \frac{a\sin^{1+n}(c+dx)}{d(1+n)}+\frac{b\sin^{2+n}(c+dx)}{d(2+n)}-\frac{2a\sin^{3+n}(c+dx)}{d(3+n)}-\frac{2b\sin^{4+n}}{d(4+}\end{aligned}$$

Mathematica [A] time = 0.195645, size = 97, normalized size = 0.79

$$\frac{\sin^{n+1}(c+dx)\left(\frac{a\sin^4(c+dx)}{n+5}-\frac{2a\sin^2(c+dx)}{n+3}+\frac{a}{n+1}+\frac{b\sin^5(c+dx)}{n+6}-\frac{2b\sin^3(c+dx)}{n+4}+\frac{b\sin(c+dx)}{n+2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(a/(1 + n) + (b*Sin[c + d*x])/(2 + n) - (2*a*Sin[c +
d*x]^2)/(3 + n) - (2*b*Sin[c + d*x]^3)/(4 + n) + (a*Sin[c + d*x]^4)/(5 + n)
 + (b*Sin[c + d*x]^5)/(6 + n)))/d
```

Maple [F] time = 4.69, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^5 (\sin(dx+c))^n (a+b\sin(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95234, size = 713, normalized size = 5.8

$$-\frac{\left(\left(bn^5+15\,bn^4+85\,bn^3+225\,bn^2+274\,bn+120\,b\right)\cos(dx+c)^6-\left(bn^5+11\,bn^4+41\,bn^3+61\,bn^2+30\,bn\right)\cos(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((b*n^5 + 15*b*n^4 + 85*b*n^3 + 225*b*n^2 + 274*b*n + 120*b)*cos(d*x + c)^
6 - (b*n^5 + 11*b*n^4 + 41*b*n^3 + 61*b*n^2 + 30*b*n)*cos(d*x + c)^4 - 8*b*
n^3 - 72*b*n^2 - 4*(b*n^4 + 9*b*n^3 + 23*b*n^2 + 15*b*n)*cos(d*x + c)^2 - 1
84*b*n - ((a*n^5 + 16*a*n^4 + 95*a*n^3 + 260*a*n^2 + 324*a*n + 144*a)*cos(d
*x + c)^4 + 8*a*n^3 + 96*a*n^2 + 4*(a*n^4 + 13*a*n^3 + 56*a*n^2 + 92*a*n +
48*a)*cos(d*x + c)^2 + 352*a*n + 384*a)*sin(d*x + c) - 120*b)*sin(d*x + c)^
n/(d*n^6 + 21*d*n^5 + 175*d*n^4 + 735*d*n^3 + 1624*d*n^2 + 1764*d*n + 720*d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17264, size = 512, normalized size = 4.16

$$\frac{\left(n^2 \sin(dx+c)^n \sin(dx+c)^5+4\, n \sin(dx+c)^n \sin(dx+c)^5-2\, n^2 \sin(dx+c)^n \sin(dx+c)^3+3 \sin(dx+c)^n \sin(dx+c)^5-12\, n \sin(dx+c)^n \sin(dx+c)^3+n^2 \sin(dx+c)^n \sin(dx-\right.}{n^3+9\, n^2+23\, n+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 4*n*sin(d*x + c)^n*sin(d*x + c)^5 - 2
*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 3*sin(d*x + c)^n*sin(d*x + c)^5 - 12*n
*sin(d*x + c)^n*sin(d*x + c)^3 + n^2*sin(d*x + c)^n*sin(d*x + c) - 10*sin(d
*x + c)^n*sin(d*x + c)^3 + 8*n*sin(d*x + c)^n*sin(d*x + c) + 15*sin(d*x + c
)^n*sin(d*x + c))*a/(n^3 + 9*n^2 + 23*n + 15) + (n^2*sin(d*x + c)^n*sin(d*x
 + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2*n^2*sin(d*x + c)^n*sin(d*x
+ c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n*sin(d*x + c)^n*sin(d*x + c)
^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin(d*x + c)^n*sin(d*x + c)^4 +
 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*x + c)^n*sin(d*x + c)^2)*b/(
n^3 + 12*n^2 + 44*n + 48))/d
```

3.1237 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=167

$$\frac{(a^2-b^2)^2\sin^{n+1}(c+dx)\,_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{ab^4d(n+1)}-\frac{a\left(a^2-2b^2\right)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{\left(a^2-2b^2\right)\sin^{n+2}(c+dx)}{b^3d(n+2)}-$$

```
[Out] -((a*(a^2 - 2*b^2)*Sin[c + d*x]^(1 + n))/(b^4*d*(1 + n))) + ((a^2 - b^2)^2*
Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)]*Sin[c + d*x]^(1 +
 n))/(a*b^4*d*(1 + n)) + ((a^2 - 2*b^2)*Sin[c + d*x]^(2 + n))/(b^3*d*(2 + n
)) - (a*Sin[c + d*x]^(3 + n))/(b^2*d*(3 + n)) + Sin[c + d*x]^(4 + n)/(b*d*(
4 + n))
```

Rubi [A] time = 0.338371, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 952, 1620, 64}

$$\frac{(a^2-b^2)^2\sin^{n+1}(c+dx)\,_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{ab^4d(n+1)}-\frac{a\left(a^2-2b^2\right)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{\left(a^2-2b^2\right)\sin^{n+2}(c+dx)}{b^3d(n+2)}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2)*Sin[c + d*x]^(1 + n))/(b^4*d*(1 + n))) + ((a^2 - b^2)^2*
Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)]*Sin[c + d*x]^(1 +
 n))/(a*b^4*d*(1 + n)) + ((a^2 - 2*b^2)*Sin[c + d*x]^(2 + n))/(b^3*d*(2 + n
)) - (a*Sin[c + d*x]^(3 + n))/(b^2*d*(3 + n)) + Sin[c + d*x]^(4 + n)/(b*d*(
4 + n))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(b^2-x^2\right)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\sin^{4+n}(c+dx)}{bd(4+n)} + \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(4+n-\frac{2(4+n)x^2}{b^2}-\frac{a(4+n)x^3}{b^4}\right)}{a+x}\,dx, x, b\sin(c+dx)\right)}{bd(4+n)}$$

$$= \frac{\sin^{4+n}(c+dx)}{bd(4+n)} + \frac{\text{Subst}\left(\int\left(-\frac{a\left(a^2-2b^2\right)(4+n)\left(\frac{x}{b}\right)^n}{b^4} - \frac{\left(-a^2+2b^2\right)(4+n)\left(\frac{x}{b}\right)^{1+n}}{b^3} - \frac{a(4+n)\left(\frac{x}{b}\right)^{2+n}}{b^2} + (-\right.\right.}{bd(4+n)}$$

$$= -\frac{a\left(a^2-2b^2\right)\sin^{1+n}(c+dx)}{b^4d(1+n)} + \frac{\left(a^2-2b^2\right)\sin^{2+n}(c+dx)}{b^3d(2+n)} - \frac{a\sin^{3+n}(c+dx)}{b^2d(3+n)} + \frac{\sin^{4+n}(}{bd(4}$$

$$= -\frac{a\left(a^2-2b^2\right)\sin^{1+n}(c+dx)}{b^4d(1+n)} + \frac{\left(a^2-b^2\right)^2 {}_2F_1\left(1,1+n;2+n;-\frac{b\sin(c+dx)}{a}\right)\sin^{1+n}(c+}{ab^4d(1+n)}$$

Mathematica [A] time = 0.526008, size = 133, normalized size = 0.8

$$\frac{\sin^{n+1}(c+dx)\left(\frac{\left(a^2-b^2\right)^2 {}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a(n+1)} + \frac{b\left(a^2-2b^2\right)\sin(c+dx)}{n+2} - \frac{a^3-2ab^2}{n+1} - \frac{ab^2\sin^2(c+dx)}{n+3} + \frac{b^3\sin^3(c+dx)}{n+4}\right)}{b^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(-((a^3 - 2*a*b^2)/(1 + n)) + ((a^2 - b^2)^2*Hypergeo
metric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)])/(a*(1 + n)) + (b*(a^2 -
2*b^2)*Sin[c + d*x])/(2 + n) - (a*b^2*Sin[c + d*x]^2)/(3 + n) + (b^3*Sin[c
+ d*x]^3)/(4 + n)))/(b^4*d)
```

Maple [F] time = 0.802, size = 0, normalized size = 0.

$$\int \frac{\left(\cos(dx+c)\right)^5\left(\sin(dx+c)\right)^n}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^5}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)^n \cos(dx+c)^5}{b\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n \cos(dx+c)^5}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

3.1238 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=191

$$\frac{\left(a^2-b^2\right)\left(b^2n-a^2(n+4)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a^2b^4d(n+1)}+\frac{\left(3a^2-2b^2\right)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{\left(a^2-b^2\right)^2\sin^{n}}{ab^4d(a+b\sin}$$

```
[Out] ((3*a^2 - 2*b^2)*Sin[c + d*x]^(1 + n))/(b^4*d*(1 + n)) + ((a^2 - b^2)*(b^2*
n - a^2*(4 + n))*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)]*
Sin[c + d*x]^(1 + n))/(a^2*b^4*d*(1 + n)) - (2*a*Sin[c + d*x]^(2 + n))/(b^3
*d*(2 + n)) + Sin[c + d*x]^(3 + n)/(b^2*d*(3 + n)) + ((a^2 - b^2)^2*Sin[c +
 d*x]^(1 + n))/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.364907, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 950, 1620, 64}

$$\frac{\left(a^2-b^2\right)\left(b^2n-a^2(n+4)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a^2b^4d(n+1)}+\frac{\left(3a^2-2b^2\right)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{\left(a^2-b^2\right)^2\sin^{n}}{ab^4d(a+b\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - 2*b^2)*Sin[c + d*x]^(1 + n))/(b^4*d*(1 + n)) + ((a^2 - b^2)*(b^2*
n - a^2*(4 + n))*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)]*
Sin[c + d*x]^(1 + n))/(a^2*b^4*d*(1 + n)) - (2*a*Sin[c + d*x]^(2 + n))/(b^3
*d*(2 + n)) + Sin[c + d*x]^(3 + n)/(b^2*d*(3 + n)) + ((a^2 - b^2)^2*Sin[c +
 d*x]^(1 + n))/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 950

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[(R*(d + e*x
)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)), x] + Dist[1/((m + 1)*(e
*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d*
g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(b^2-x^2\right)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\left(a^2-b^2\right)^2\sin^{1+n}(c+dx)}{ab^4 d(a+b\sin(c+dx))} + \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(-b^3 n-\frac{a^4(1+n)}{b}+2a^2 b(1+n)+a\left(\frac{a^2}{b}-2b\right)x-\frac{a^2x^2}{b}+\frac{ax^3}{b}\right)}{a+x}\,c\right.}{ab^4 d}$$

$$= \frac{\left(a^2-b^2\right)^2\sin^{1+n}(c+dx)}{ab^4 d(a+b\sin(c+dx))} + \frac{\text{Subst}\left(\int\left(\frac{\left(3a^3-2ab^2\right)\left(\frac{x}{b}\right)^n}{b}-2a^2\left(\frac{x}{b}\right)^{1+n}+ab\left(\frac{x}{b}\right)^{2+n}+\frac{(a^2\right.\right.}{ab^4 d}$$

$$= \frac{\left(3a^2-2b^2\right)\sin^{1+n}(c+dx)}{b^4 d(1+n)} - \frac{2a\sin^{2+n}(c+dx)}{b^3 d(2+n)} + \frac{\sin^{3+n}(c+dx)}{b^2 d(3+n)} + \frac{\left(a^2-b^2\right)^2\sin^{1+}}{ab^4 d(a+b\sin}$$

$$= \frac{\left(3a^2-2b^2\right)\sin^{1+n}(c+dx)}{b^4 d(1+n)} + \frac{\left(a^2-b^2\right)\left(b^2 n-a^2(4+n)\right){}_2F_1\left(1,1+n;2+n;-\frac{b\sin(}{}\right.}{a^2 b^4 d(1+n)}$$

Mathematica [A] time = 0.398096, size = 143, normalized size = 0.75

$$\frac{\sin^{n+1}(c+dx)\left(\frac{\left(a^2-b^2\right)^2 {}_2F_1\left(2,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a^2(n+1)}-\frac{4\left(a^2-b^2\right){}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{n+1}+\frac{3a^2-2b^2}{n+1}-\frac{2ab\sin(c+dx)}{n+2}+\frac{b^2\sin^2(c+dx)}{n+3}\right)}{b^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((3*a^2 - 2*b^2)/(1 + n) - (4*(a^2 - b^2)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)])/(1 + n) + ((a^2 - b^2)^2*Hy
pergeometric2F1[2, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)])/(a^2*(1 + n)) - (2
*a*b*Sin[c + d*x])/(2 + n) + (b^2*Sin[c + d*x]^2)/(3 + n)))/(b^4*d)
```

Maple [F] time = 1.326, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^5(\sin(dx+c))^n}{(a+b\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^5}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sin(dx+c)^n\cos(dx+c)^5}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)^5/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^n\cos(dx+c)^5}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a)^2, x)
```

3.1239 $\int \cos^6(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=238

$$-\frac{(a^2+3b^2)\cos^{11}(c+dx)}{11d}+\frac{(2a^2+3b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^5(c+dx)\cos^7(c+dx)}{6d}-\frac{ab\,\mathrm{si}}{}$$

```
[Out] (5*a*b*x)/512 - ((a^2 + b^2)*Cos[c + d*x]^7)/(7*d) + ((2*a^2 + 3*b^2)*Cos[c
 + d*x]^9)/(9*d) - ((a^2 + 3*b^2)*Cos[c + d*x]^11)/(11*d) + (b^2*Cos[c + d*
x]^13)/(13*d) + (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(512*d) + (5*a*b*Cos[c +
d*x]^3*Sin[c + d*x])/(768*d) + (a*b*Cos[c + d*x]^5*Sin[c + d*x])/(192*d) -
(a*b*Cos[c + d*x]^7*Sin[c + d*x])/(32*d) - (a*b*Cos[c + d*x]^7*Sin[c + d*x]
^3)/(12*d) - (a*b*Cos[c + d*x]^7*Sin[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.371907, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2911, 2568, 2635, 8, 3201, 446, 77}

$$-\frac{(a^2+3b^2)\cos^{11}(c+dx)}{11d}+\frac{(2a^2+3b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^5(c+dx)\cos^7(c+dx)}{6d}-\frac{ab\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (5*a*b*x)/512 - ((a^2 + b^2)*Cos[c + d*x]^7)/(7*d) + ((2*a^2 + 3*b^2)*Cos[c
 + d*x]^9)/(9*d) - ((a^2 + 3*b^2)*Cos[c + d*x]^11)/(11*d) + (b^2*Cos[c + d*
x]^13)/(13*d) + (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(512*d) + (5*a*b*Cos[c +
d*x]^3*Sin[c + d*x])/(768*d) + (a*b*Cos[c + d*x]^5*Sin[c + d*x])/(192*d) -
(a*b*Cos[c + d*x]^7*Sin[c + d*x])/(32*d) - (a*b*Cos[c + d*x]^7*Sin[c + d*x]
^3)/(12*d) - (a*b*Cos[c + d*x]^7*Sin[c + d*x]^5)/(6*d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3201

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^6(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^6(c+dx)\,dx + \int \cos^6(c+dx)\sin^5(c+dx)$$

$$= -\frac{ab\cos^7(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5ab)\int \cos^6(c+dx)\sin^4(c+dx$$

$$= -\frac{ab\cos^7(c+dx)\sin^3(c+dx)}{12d} - \frac{ab\cos^7(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{4}(a$$

$$= -\frac{ab\cos^7(c+dx)\sin(c+dx)}{32d} - \frac{ab\cos^7(c+dx)\sin^3(c+dx)}{12d} - \frac{ab\,\mathrm{co}}{}$$

$$= -\frac{\left(a^2+b^2\right)\cos^7(c+dx)}{7d} + \frac{\left(2a^2+3b^2\right)\cos^9(c+dx)}{9d} - \frac{\left(a^2+3b^2\right)\mathrm{co}}{11}$$

$$= -\frac{\left(a^2+b^2\right)\cos^7(c+dx)}{7d} + \frac{\left(2a^2+3b^2\right)\cos^9(c+dx)}{9d} - \frac{\left(a^2+3b^2\right)\mathrm{co}}{11}$$

$$= -\frac{\left(a^2+b^2\right)\cos^7(c+dx)}{7d} + \frac{\left(2a^2+3b^2\right)\cos^9(c+dx)}{9d} - \frac{\left(a^2+3b^2\right)\mathrm{co}}{11}$$

$$= \frac{5abx}{512} - \frac{\left(a^2+b^2\right)\cos^7(c+dx)}{7d} + \frac{\left(2a^2+3b^2\right)\cos^9(c+dx)}{9d} - \frac{\left(a^2+3\right.}{}$$

Mathematica [A] time = 2.15656, size = 210, normalized size = 0.88

$$\frac{-180180\left(2a^2+b^2\right)\cos(c+dx)-15015\left(8a^2+3b^2\right)\cos(3(c+dx))+36036a^2\cos(5(c+dx))+25740a^2\cos(7(c+dx))-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (360360*a*b*c + 360360*a*b*d*x - 180180*(2*a^2 + b^2)*Cos[c + d*x] - 15015*
(8*a^2 + 3*b^2)*Cos[3*(c + d*x)] + 36036*a^2*Cos[5*(c + d*x)] + 27027*b^2*C
os[5*(c + d*x)] + 25740*a^2*Cos[7*(c + d*x)] + 7722*b^2*Cos[7*(c + d*x)] -
4004*a^2*Cos[9*(c + d*x)] - 6006*b^2*Cos[9*(c + d*x)] - 3276*a^2*Cos[11*(c
+ d*x)] - 819*b^2*Cos[11*(c + d*x)] + 693*b^2*Cos[13*(c + d*x)] - 135135*a*
b*Sin[4*(c + d*x)] + 27027*a*b*Sin[8*(c + d*x)] - 3003*a*b*Sin[12*(c + d*x)
])/(36900864*d)
```

Maple [A] time = 0.049, size = 225, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^7}{11}-\frac{4(\sin(dx+c))^2(\cos(dx+c))^7}{99}-\frac{8(\cos(dx+c))^7}{693}\right)+2ab\left(-1/12(\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/
693*cos(d*x+c)^7)+2*a*b*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*
cos(d*x+c)^7-1/64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c
)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+b^2*(-1/13*sin(d*x+c)^
6*cos(d*x+c)^7-6/143*sin(d*x+c)^4*cos(d*x+c)^7-8/429*sin(d*x+c)^2*cos(d*x+c
)^7-16/3003*cos(d*x+c)^7))
```

Maxima [A] time = 0.995607, size = 182, normalized size = 0.76

$$-\frac{53248\left(63\cos(dx+c)^{11}-154\cos(dx+c)^9+99\cos(dx+c)^7\right)a^2-3003\left(4\sin(4dx+4c)^3+120dx+120c+9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/36900864*(53248*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x +
c)^7)*a^2 - 3003*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120*c + 9*sin(8*d*x + 8*
c) - 48*sin(4*d*x + 4*c))*a*b - 12288*(231*cos(d*x + c)^13 - 819*cos(d*x +
c)^11 + 1001*cos(d*x + c)^9 - 429*cos(d*x + c)^7)*b^2)/d
```

Fricas [A] time = 1.99853, size = 456, normalized size = 1.92

$$\frac{354816b^2\cos(dx+c)^{13}-419328\left(a^2+3b^2\right)\cos(dx+c)^{11}+512512\left(2a^2+3b^2\right)\cos(dx+c)^9-658944\left(a^2+b^2\right)\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4612608*(354816*b^2*cos(d*x + c)^13 - 419328*(a^2 + 3*b^2)*cos(d*x + c)^1
1 + 512512*(2*a^2 + 3*b^2)*cos(d*x + c)^9 - 658944*(a^2 + b^2)*cos(d*x + c)
^7 + 45045*a*b*d*x - 3003*(256*a*b*cos(d*x + c)^11 - 640*a*b*cos(d*x + c)^9
```

```
 + 432*a*b*cos(d*x + c)^7 - 8*a*b*cos(d*x + c)^5 - 10*a*b*cos(d*x + c)^3 -
15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 130.965, size = 488, normalized size = 2.05

$$\begin{cases} -\frac{a^2\sin^4(c+dx)\cos^7(c+dx)}{7d} - \frac{4a^2\sin^2(c+dx)\cos^9(c+dx)}{63d} - \frac{8a^2\cos^{11}(c+dx)}{693d} + \frac{5abx\sin^{12}(c+dx)}{512} + \frac{15abx\sin^{10}(c+dx)\cos^2(c+dx)}{256} + \frac{75abx\sin^8(c+dx)}{512} \\ x(a+b\sin(c))^2\sin^5(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 4*a**2*sin(c + d*x
)**2*cos(c + d*x)**9/(63*d) - 8*a**2*cos(c + d*x)**11/(693*d) + 5*a*b*x*sin
(c + d*x)**12/512 + 15*a*b*x*sin(c + d*x)**10*cos(c + d*x)**2/256 + 75*a*b*
x*sin(c + d*x)**8*cos(c + d*x)**4/512 + 25*a*b*x*sin(c + d*x)**6*cos(c + d*
x)**6/128 + 75*a*b*x*sin(c + d*x)**4*cos(c + d*x)**8/512 + 15*a*b*x*sin(c +
 d*x)**2*cos(c + d*x)**10/256 + 5*a*b*x*cos(c + d*x)**12/512 + 5*a*b*sin(c
+ d*x)**11*cos(c + d*x)/(512*d) + 85*a*b*sin(c + d*x)**9*cos(c + d*x)**3/(1
536*d) + 33*a*b*sin(c + d*x)**7*cos(c + d*x)**5/(256*d) - 33*a*b*sin(c + d*
x)**5*cos(c + d*x)**7/(256*d) - 85*a*b*sin(c + d*x)**3*cos(c + d*x)**9/(153
6*d) - 5*a*b*sin(c + d*x)*cos(c + d*x)**11/(512*d) - b**2*sin(c + d*x)**6*c
os(c + d*x)**7/(7*d) - 2*b**2*sin(c + d*x)**4*cos(c + d*x)**9/(21*d) - 8*b*
*2*sin(c + d*x)**2*cos(c + d*x)**11/(231*d) - 16*b**2*cos(c + d*x)**13/(300
3*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**5*cos(c)**6, True))
```

Giac [A] time = 1.30154, size = 289, normalized size = 1.21

$$\frac{5}{512}abx + \frac{b^2\cos(13dx+13c)}{53248d} - \frac{ab\sin(12dx+12c)}{12288d} + \frac{3ab\sin(8dx+8c)}{4096d} - \frac{15ab\sin(4dx+4c)}{4096d} - \frac{(4a^2+b^2)\cos(1}{45056}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/512*a*b*x + 1/53248*b^2*cos(13*d*x + 13*c)/d - 1/12288*a*b*sin(12*d*x + 1
2*c)/d + 3/4096*a*b*sin(8*d*x + 8*c)/d - 15/4096*a*b*sin(4*d*x + 4*c)/d - 1
/45056*(4*a^2 + b^2)*cos(11*d*x + 11*c)/d - 1/18432*(2*a^2 + 3*b^2)*cos(9*d
*x + 9*c)/d + 1/14336*(10*a^2 + 3*b^2)*cos(7*d*x + 7*c)/d + 1/4096*(4*a^2 +
 3*b^2)*cos(5*d*x + 5*c)/d - 5/12288*(8*a^2 + 3*b^2)*cos(3*d*x + 3*c)/d - 5
/1024*(2*a^2 + b^2)*cos(d*x + c)/d
```

3.1240 $\int \cos^6(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=250

$$\frac{(12a^2+25b^2)\sin(c+dx)\cos^9(c+dx)}{120d}-\frac{(44a^2+45b^2)\sin(c+dx)\cos^7(c+dx)}{320d}+\frac{(12a^2+5b^2)\sin(c+dx)\cos^5(c+$$

```
[Out] ((12*a^2 + 5*b^2)*x)/1024 - (2*a*b*Cos[c + d*x]^7)/(7*d) + (4*a*b*Cos[c + d
*x]^9)/(9*d) - (2*a*b*Cos[c + d*x]^11)/(11*d) + ((12*a^2 + 5*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(1024*d) + ((12*a^2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])
/(1536*d) + ((12*a^2 + 5*b^2)*Cos[c + d*x]^5*Sin[c + d*x])/(1920*d) - ((44*
a^2 + 45*b^2)*Cos[c + d*x]^7*Sin[c + d*x])/(320*d) + ((12*a^2 + 25*b^2)*Cos
[c + d*x]^9*Sin[c + d*x])/(120*d) - (b^2*Cos[c + d*x]^11*Sin[c + d*x])/(12*
d)
```

Rubi [A] time = 0.354725, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2911, 2565, 270, 3200, 455, 1157, 385, 199, 203}

$$\frac{(12a^2+25b^2)\sin(c+dx)\cos^9(c+dx)}{120d}-\frac{(44a^2+45b^2)\sin(c+dx)\cos^7(c+dx)}{320d}+\frac{(12a^2+5b^2)\sin(c+dx)\cos^5(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] ((12*a^2 + 5*b^2)*x)/1024 - (2*a*b*Cos[c + d*x]^7)/(7*d) + (4*a*b*Cos[c + d
*x]^9)/(9*d) - (2*a*b*Cos[c + d*x]^11)/(11*d) + ((12*a^2 + 5*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(1024*d) + ((12*a^2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])
/(1536*d) + ((12*a^2 + 5*b^2)*Cos[c + d*x]^5*Sin[c + d*x])/(1920*d) - ((44*
a^2 + 45*b^2)*Cos[c + d*x]^7*Sin[c + d*x])/(320*d) + ((12*a^2 + 25*b^2)*Cos
[c + d*x]^9*Sin[c + d*x])/(120*d) - (b^2*Cos[c + d*x]^11*Sin[c + d*x])/(12*
d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3200

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[(x^n*(a + (a + b)*ff^2*x^2)^p)/
(1 + ff^2*x^2)^((m + n)/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^5(c+dx)\,dx + \int \cos^6(c+dx)\sin^4(c+$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\left(a^2+\left(a^2+b^2\right)x^2\right)}{\left(1+x^2\right)^7}dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int x^6\right.}{}$$

$$= -\frac{b^2\cos^{11}(c+dx)\sin(c+dx)}{12d} - \frac{\text{Subst}\left(\int \frac{-b^2+12b^2x^2-12\left(a^2+b^2\right)x^4}{\left(1+x^2\right)^6}d\right.}{12d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d} + \frac{(1}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d} - \frac{(4}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d} + \frac{(1}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d} + \frac{(1}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+dx)}{11d} + \frac{(1}{}$$

$$= \frac{\left(12a^2+5b^2\right)x}{1024} - \frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos}{}$$

Mathematica [A] time = 1.38634, size = 202, normalized size = 0.81

$$\frac{55440a^2\sin(2(c+dx)) - 110880a^2\sin(4(c+dx)) - 27720a^2\sin(6(c+dx)) + 13860a^2\sin(8(c+dx)) + 5544a^2\sin(10}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (166320*b^2*c + 332640*a^2*d*x + 138600*b^2*d*x - 554400*a*b*Cos[c + d*x] -
 184800*a*b*Cos[3*(c + d*x)] + 55440*a*b*Cos[5*(c + d*x)] + 39600*a*b*Cos[7
*(c + d*x)] - 6160*a*b*Cos[9*(c + d*x)] - 5040*a*b*Cos[11*(c + d*x)] + 5544
0*a^2*Sin[2*(c + d*x)] - 110880*a^2*Sin[4*(c + d*x)] - 51975*b^2*Sin[4*(c +
 d*x)] - 27720*a^2*Sin[6*(c + d*x)] + 13860*a^2*Sin[8*(c + d*x)] + 10395*b^
2*Sin[8*(c + d*x)] + 5544*a^2*Sin[10*(c + d*x)] - 1155*b^2*Sin[12*(c + d*x)
])/(28385280*d)
```

Maple [A] time = 0.05, size = 237, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^7}{10} - \frac{3\sin(dx+c)(\cos(dx+c))^7}{80} + \frac{\sin(dx+c)}{160}\left((\cos(dx+c))^5 + \frac{5(\cos(dx+c}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/16
0*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/25
6*c)+2*a*b*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-
8/693*cos(d*x+c)^7)+b^2*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*
cos(d*x+c)^7-1/64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c
)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c))

Maxima [A] time = 1.01978, size = 185, normalized size = 0.74

$$\frac{2772\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\sin\left(8\,dx+8\,c\right)-40\sin\left(4\,dx+4\,c\right)\right)a^{2}-81920\left(63\cos\left(dx+c\right)^{11}-154\right.}{2838}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")

[Out] 1/28385280*(2772*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8
*c) - 40*sin(4*d*x + 4*c))*a^2 - 81920*(63*cos(d*x + c)^11 - 154*cos(d*x +
c)^9 + 99*cos(d*x + c)^7)*a*b + 1155*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120*
c + 9*sin(8*d*x + 8*c) - 48*sin(4*d*x + 4*c))*b^2)/d

Fricas [A] time = 1.99478, size = 486, normalized size = 1.94

$$-\frac{645120\,ab\cos\left(dx+c\right)^{11}-1576960\,ab\cos\left(dx+c\right)^{9}+1013760\,ab\cos\left(dx+c\right)^{7}-3465\left(12\,a^{2}+5\,b^{2}\right)dx+231\left(1280\,b^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")

[Out] -1/3548160*(645120*a*b*cos(d*x + c)^11 - 1576960*a*b*cos(d*x + c)^9 + 10137
60*a*b*cos(d*x + c)^7 - 3465*(12*a^2 + 5*b^2)*d*x + 231*(1280*b^2*cos(d*x +
 c)^11 - 128*(12*a^2 + 25*b^2)*cos(d*x + c)^9 + 48*(44*a^2 + 45*b^2)*cos(d*
x + c)^7 - 8*(12*a^2 + 5*b^2)*cos(d*x + c)^5 - 10*(12*a^2 + 5*b^2)*cos(d*x
+ c)^3 - 15*(12*a^2 + 5*b^2)*cos(d*x + c))*sin(d*x + c))/d

Sympy [A] time = 90.7959, size = 656, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+b*sin(d*x+c))**2,x)

[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/256 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 15*a**2*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a**2*x*s
in(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)
8/256 + 3*a2*x*cos(c + d*x)**10/256 + 3*a**2*sin(c + d*x)**9*cos(c + d*
x)/(256*d) + 7*a**2*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + a**2*sin(c +
d*x)**5*cos(c + d*x)**5/(10*d) - 7*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(12

```
8*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 2*a*b*sin(c + d*x)**4*
cos(c + d*x)**7/(7*d) - 8*a*b*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 16*a
*b*cos(c + d*x)**11/(693*d) + 5*b**2*x*sin(c + d*x)**12/1024 + 15*b**2*x*si
n(c + d*x)**10*cos(c + d*x)**2/512 + 75*b**2*x*sin(c + d*x)**8*cos(c + d*x)
**4/1024 + 25*b**2*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 75*b**2*x*sin(c
+ d*x)**4*cos(c + d*x)**8/1024 + 15*b**2*x*sin(c + d*x)**2*cos(c + d*x)**10
/512 + 5*b**2*x*cos(c + d*x)**12/1024 + 5*b**2*sin(c + d*x)**11*cos(c + d*x
)/(1024*d) + 85*b**2*sin(c + d*x)**9*cos(c + d*x)**3/(3072*d) + 33*b**2*sin
(c + d*x)**7*cos(c + d*x)**5/(512*d) - 33*b**2*sin(c + d*x)**5*cos(c + d*x)
**7/(512*d) - 85*b**2*sin(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 5*b**2*sin
(c + d*x)*cos(c + d*x)**11/(1024*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)
**4*cos(c)**6, True))
```

Giac [A] time = 1.30758, size = 305, normalized size = 1.22

$$\frac{1}{1024}\left(12\,a^2+5\,b^2\right)x-\frac{ab\cos\left(11\,dx+11\,c\right)}{5632\,d}-\frac{ab\cos\left(9\,dx+9\,c\right)}{4608\,d}+\frac{5\,ab\cos\left(7\,dx+7\,c\right)}{3584\,d}+\frac{ab\cos\left(5\,dx+5\,c\right)}{512\,d}-\frac{5\,ab\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1024*(12*a^2 + 5*b^2)*x - 1/5632*a*b*cos(11*d*x + 11*c)/d - 1/4608*a*b*co
s(9*d*x + 9*c)/d + 5/3584*a*b*cos(7*d*x + 7*c)/d + 1/512*a*b*cos(5*d*x + 5*
c)/d - 5/768*a*b*cos(3*d*x + 3*c)/d - 5/256*a*b*cos(d*x + c)/d - 1/24576*b^
2*sin(12*d*x + 12*c)/d + 1/5120*a^2*sin(10*d*x + 10*c)/d - 1/1024*a^2*sin(6
*d*x + 6*c)/d + 1/512*a^2*sin(2*d*x + 2*c)/d + 1/8192*(4*a^2 + 3*b^2)*sin(8
*d*x + 8*c)/d - 1/8192*(32*a^2 + 15*b^2)*sin(4*d*x + 4*c)/d
```

3.1241 $\int \cos^6(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=187

$$\frac{(a^2+2b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3ab\sin(c+dx)\cos^7(c+dx)}{40d}+\frac{ab\sin(}{}$$

```
[Out] (3*a*b*x)/128 - ((a^2 + b^2)*Cos[c + d*x]^7)/(7*d) + ((a^2 + 2*b^2)*Cos[c +
 d*x]^9)/(9*d) - (b^2*Cos[c + d*x]^11)/(11*d) + (3*a*b*Cos[c + d*x]*Sin[c +
 d*x])/(128*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (a*b*Cos[c + d*
x]^5*Sin[c + d*x])/(80*d) - (3*a*b*Cos[c + d*x]^7*Sin[c + d*x])/(40*d) - (a
*b*Cos[c + d*x]^7*Sin[c + d*x]^3)/(5*d)
```

Rubi [A] time = 0.314548, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2911, 2568, 2635, 8, 3201, 446, 77}

$$\frac{(a^2+2b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3ab\sin(c+dx)\cos^7(c+dx)}{40d}+\frac{ab\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (3*a*b*x)/128 - ((a^2 + b^2)*Cos[c + d*x]^7)/(7*d) + ((a^2 + 2*b^2)*Cos[c +
 d*x]^9)/(9*d) - (b^2*Cos[c + d*x]^11)/(11*d) + (3*a*b*Cos[c + d*x]*Sin[c +
 d*x])/(128*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (a*b*Cos[c + d*
x]^5*Sin[c + d*x])/(80*d) - (3*a*b*Cos[c + d*x]^7*Sin[c + d*x])/(40*d) - (a
*b*Cos[c + d*x]^7*Sin[c + d*x]^3)/(5*d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3201

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^4(c+dx)\,dx + \int \cos^6(c+dx)\sin^3(c+$$

$$= -\frac{ab\cos^7(c+dx)\sin^3(c+dx)}{5d} + \frac{1}{5}(3ab)\int \cos^6(c+dx)\sin^2(c+$$

$$= -\frac{3ab\cos^7(c+dx)\sin(c+dx)}{40d} - \frac{ab\cos^7(c+dx)\sin^3(c+dx)}{5d} +$$

$$= \frac{ab\cos^5(c+dx)\sin(c+dx)}{80d} - \frac{3ab\cos^7(c+dx)\sin(c+dx)}{40d} - ab$$

$$= -\frac{\left(a^2+b^2\right)\cos^7(c+dx)}{7d} + \frac{\left(a^2+2b^2\right)\cos^9(c+dx)}{9d} - \frac{b^2\cos^{11}(c}{11d}$$

$$= -\frac{\left(a^2+b^2\right)\cos^7(c+dx)}{7d} + \frac{\left(a^2+2b^2\right)\cos^9(c+dx)}{9d} - \frac{b^2\cos^{11}(c}{11d}$$

$$= \frac{3abx}{128} - \frac{\left(a^2+b^2\right)\cos^7(c+dx)}{7d} + \frac{\left(a^2+2b^2\right)\cos^9(c+dx)}{9d} - b^2 c$$

Mathematica [A] time = 1.08595, size = 197, normalized size = 1.05

$$-6930\left(12a^2+5b^2\right)\cos(c+dx) - 2310\left(16a^2+5b^2\right)\cos(3(c+dx)) + 5940a^2\cos(7(c+dx)) + 1540a^2\cos(9(c+dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (83160*a*b*c + 83160*a*b*d*x - 6930*(12*a^2 + 5*b^2)*Cos[c + d*x] - 2310*(1
6*a^2 + 5*b^2)*Cos[3*(c + d*x)] + 3465*b^2*Cos[5*(c + d*x)] + 5940*a^2*Cos[
7*(c + d*x)] + 2475*b^2*Cos[7*(c + d*x)] + 1540*a^2*Cos[9*(c + d*x)] - 385*
b^2*Cos[9*(c + d*x)] - 315*b^2*Cos[11*(c + d*x)] + 13860*a*b*Sin[2*(c + d*x
)] - 27720*a*b*Sin[4*(c + d*x)] - 6930*a*b*Sin[6*(c + d*x)] + 3465*a*b*Sin[
```

```
8*(c + d*x)] + 1386*a*b*Sin[10*(c + d*x)])/(3548160*d)
```

Maple [A] time = 0.045, size = 171, normalized size = 0.9

$$\frac{1}{d}\left(a^2\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^7}{9}-\frac{2\,(\cos(dx+c))^7}{63}\right)+2\,ab\left(-1/10\,(\sin(dx+c))^3(\cos(dx+c))^7-\frac{3\,\sin(dx+c)\,(\cdots}{80\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+2*a*b*(-1/10*si
n(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+b^2*(-1/11*si
n(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7))
```

Maxima [A] time = 0.997068, size = 155, normalized size = 0.83

$$\frac{56320\left(7\,\cos(dx+c)^9-9\,\cos(dx+c)^7\right)a^2+693\left(32\,\sin(2\,dx+2\,c)^5+120\,dx+120\,c+5\,\sin(8\,dx+8\,c)-40\,\sin(4\,d\cdots\right.}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3548160*(56320*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2 + 693*(32*sin(2*
d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*
a*b - 5120*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)^7)*b^
2)/d
```

Fricas [A] time = 1.9666, size = 360, normalized size = 1.93

$$-\frac{40320\,b^2\cos(dx+c)^{11}-49280\left(a^2+2\,b^2\right)\cos(dx+c)^9+63360\left(a^2+b^2\right)\cos(dx+c)^7-10395\,abdx-693\left(128\,ab\,\mathrm{co}\cdots\right.}{443520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/443520*(40320*b^2*cos(d*x + c)^11 - 49280*(a^2 + 2*b^2)*cos(d*x + c)^9 +
 63360*(a^2 + b^2)*cos(d*x + c)^7 - 10395*a*b*d*x - 693*(128*a*b*cos(d*x +
c)^9 - 176*a*b*cos(d*x + c)^7 + 8*a*b*cos(d*x + c)^5 + 10*a*b*cos(d*x + c)^
3 + 15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 61.7796, size = 384, normalized size = 2.05

$$\begin{cases}-\frac{a^2\sin^2(c+dx)\cos^7(c+dx)}{7d}-\frac{2a^2\cos^9(c+dx)}{63d}+\frac{3abx\sin^{10}(c+dx)}{128}+\frac{15abx\sin^8(c+dx)\cos^2(c+dx)}{128}+\frac{15abx\sin^6(c+dx)\cos^4(c+dx)}{64}+\frac{15abx\sin^4(c+d\cdots}{64\cdots} \\ x\,(a+b\sin(c))^2\sin^3(c)\cos^6(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 2*a**2*cos(c + d*x
)**9/(63*d) + 3*a*b*x*sin(c + d*x)**10/128 + 15*a*b*x*sin(c + d*x)**8*cos(c
 + d*x)**2/128 + 15*a*b*x*sin(c + d*x)**6*cos(c + d*x)**4/64 + 15*a*b*x*sin
(c + d*x)**4*cos(c + d*x)**6/64 + 15*a*b*x*sin(c + d*x)**2*cos(c + d*x)**8/
128 + 3*a*b*x*cos(c + d*x)**10/128 + 3*a*b*sin(c + d*x)**9*cos(c + d*x)/(12
8*d) + 7*a*b*sin(c + d*x)**7*cos(c + d*x)**3/(64*d) + a*b*sin(c + d*x)**5*c
os(c + d*x)**5/(5*d) - 7*a*b*sin(c + d*x)**3*cos(c + d*x)**7/(64*d) - 3*a*b
*sin(c + d*x)*cos(c + d*x)**9/(128*d) - b**2*sin(c + d*x)**4*cos(c + d*x)**
7/(7*d) - 4*b**2*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 8*b**2*cos(c + d*
x)**11/(693*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**3*cos(c)**6, True))
```

Giac [A] time = 1.25825, size = 293, normalized size = 1.57

$$\frac{3}{128}\,abx-\frac{b^{2}\cos\left(11\,dx+11\,c\right)}{11264\,d}+\frac{b^{2}\cos\left(5\,dx+5\,c\right)}{1024\,d}+\frac{ab\sin\left(10\,dx+10\,c\right)}{2560\,d}+\frac{ab\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{ab\sin\left(6\,dx+6\,c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/128*a*b*x - 1/11264*b^2*cos(11*d*x + 11*c)/d + 1/1024*b^2*cos(5*d*x + 5*c
)/d + 1/2560*a*b*sin(10*d*x + 10*c)/d + 1/1024*a*b*sin(8*d*x + 8*c)/d - 1/5
12*a*b*sin(6*d*x + 6*c)/d - 1/128*a*b*sin(4*d*x + 4*c)/d + 1/256*a*b*sin(2*
d*x + 2*c)/d + 1/9216*(4*a^2 - b^2)*cos(9*d*x + 9*c)/d + 1/7168*(12*a^2 + 5
*b^2)*cos(7*d*x + 7*c)/d - 1/1536*(16*a^2 + 5*b^2)*cos(3*d*x + 3*c)/d - 1/5
12*(12*a^2 + 5*b^2)*cos(d*x + c)/d
```

3.1242 $\int \cos^6(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=201

$$-\frac{(10a^2+11b^2)\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{(10a^2+3b^2)\sin(c+dx)\cos^5(c+dx)}{480d}+\frac{(10a^2+3b^2)\sin(c+dx)\cos^3(c+dx)}{384d}$$

```
[Out] ((10*a^2 + 3*b^2)*x)/256 - (2*a*b*Cos[c + d*x]^7)/(7*d) + (2*a*b*Cos[c + d*
x]^9)/(9*d) + ((10*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(256*d) + ((10*a
^2 + 3*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + ((10*a^2 + 3*b^2)*Cos[c
+ d*x]^5*Sin[c + d*x])/(480*d) - ((10*a^2 + 11*b^2)*Cos[c + d*x]^7*Sin[c +
d*x])/(80*d) + (b^2*Cos[c + d*x]^9*Sin[c + d*x])/(10*d)
```

Rubi [A] time = 0.26689, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2911, 2565, 14, 3200, 455, 385, 199, 203}

$$-\frac{(10a^2+11b^2)\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{(10a^2+3b^2)\sin(c+dx)\cos^5(c+dx)}{480d}+\frac{(10a^2+3b^2)\sin(c+dx)\cos^3(c+dx)}{384d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((10*a^2 + 3*b^2)*x)/256 - (2*a*b*Cos[c + d*x]^7)/(7*d) + (2*a*b*Cos[c + d*
x]^9)/(9*d) + ((10*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(256*d) + ((10*a
^2 + 3*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + ((10*a^2 + 3*b^2)*Cos[c
+ d*x]^5*Sin[c + d*x])/(480*d) - ((10*a^2 + 11*b^2)*Cos[c + d*x]^7*Sin[c +
d*x])/(80*d) + (b^2*Cos[c + d*x]^9*Sin[c + d*x])/(10*d)
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3200

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[(x^n*(a + (a + b)*ff^2*x^2)^p)/
(1 + ff^2*x^2)^((m + n)/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
```

```
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^3(c+dx)\,dx + \int \cos^6(c+dx)\sin^2(c+dx)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2+\left(a^2+b^2\right)x^2\right)}{\left(1+x^2\right)^6}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int x^6\left(1\right.\right.}{}$$

$$= \frac{b^2\cos^9(c+dx)\sin(c+dx)}{10d} - \frac{\text{Subst}\left(\int \frac{b^2-10\left(a^2+b^2\right)x^2}{\left(1+x^2\right)^5}\,dx, x, \tan(c+\right.}{10d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} - \frac{\left(10a^2+11b^2\right)\cos^7(c+dx)}{80d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{\left(10a^2+3b^2\right)\cos^5(c+dx)}{480d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{\left(10a^2+3b^2\right)\cos^3(c+dx)}{384d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{\left(10a^2+3b^2\right)\cos(c+dx)}{256d}$$

$$= \frac{1}{256}\left(10a^2+3b^2\right)x - \frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{(10a^2}{}$$

Mathematica [A] time = 0.880684, size = 193, normalized size = 0.96

$$\frac{5040a^2\sin(2(c+dx)) - 2520a^2\sin(4(c+dx)) - 1680a^2\sin(6(c+dx)) - 315a^2\sin(8(c+dx)) + 12600a^2dx - 15120ab\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6300*b^2*c + 12600*a^2*d*x + 3780*b^2*d*x - 15120*a*b*Cos[c + d*x] - 6720*
a*b*Cos[3*(c + d*x)] + 1080*a*b*Cos[7*(c + d*x)] + 280*a*b*Cos[9*(c + d*x)]
 + 5040*a^2*Sin[2*(c + d*x)] + 630*b^2*Sin[2*(c + d*x)] - 2520*a^2*Sin[4*(c
 + d*x)] - 1260*b^2*Sin[4*(c + d*x)] - 1680*a^2*Sin[6*(c + d*x)] - 315*b^2*
Sin[6*(c + d*x)] - 315*a^2*Sin[8*(c + d*x)] + (315*b^2*Sin[8*(c + d*x)])/2
+ 63*b^2*Sin[10*(c + d*x)])/(322560*d)
```

Maple [A] time = 0.044, size = 183, normalized size = 0.9

$$\frac{1}{d}\left(a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^7}{8} + \frac{\sin(dx+c)}{48}\left((\cos(dx+c))^5 + \frac{5(\cos(dx+c))^3}{4} + \frac{15\cos(dx+c)}{8}\right) + \frac{5dx}{128} + \frac{5c}{128}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+2*a*b*(-1/9*sin(d*x+c)^2*cos
(d*x+c)^7-2/63*cos(d*x+c)^7)+b^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(
d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
```

```
in(d*x+c)+3/256*d*x+3/256*c))
```

Maxima [A] time = 1.00384, size = 171, normalized size = 0.85

$$\frac{210\left(64\,\sin\left(2\,dx+2\,c\right)^{3}+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\,\sin\left(4\,dx+4\,c\right)\right)a^{2}+20480\left(7\,\cos\left(dx+c\right)^{9}-9\,\mathrm{co}\right.}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/645120*(210*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
 - 24*sin(4*d*x + 4*c))*a^2 + 20480*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a
*b + 63*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*
sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A] time = 1.86274, size = 379, normalized size = 1.89

$$\frac{17920\,ab\cos\left(dx+c\right)^{9}-23040\,ab\cos\left(dx+c\right)^{7}+315\left(10\,a^{2}+3\,b^{2}\right)dx+21\left(384\,b^{2}\cos\left(dx+c\right)^{9}-48\left(10\,a^{2}+11\,b^{2}\right)\mathrm{c}\right.}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/80640*(17920*a*b*cos(d*x + c)^9 - 23040*a*b*cos(d*x + c)^7 + 315*(10*a^2
+ 3*b^2)*d*x + 21*(384*b^2*cos(d*x + c)^9 - 48*(10*a^2 + 11*b^2)*cos(d*x +
c)^7 + 8*(10*a^2 + 3*b^2)*cos(d*x + c)^5 + 10*(10*a^2 + 3*b^2)*cos(d*x + c)
^3 + 15*(10*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 39.1656, size = 529, normalized size = 2.63

$$\begin{cases}\frac{5a^{2}x\sin^{8}(c+dx)}{128}+\frac{5a^{2}x\sin^{6}(c+dx)\cos^{2}(c+dx)}{32}+\frac{15a^{2}x\sin^{4}(c+dx)\cos^{4}(c+dx)}{64}+\frac{5a^{2}x\sin^{2}(c+dx)\cos^{6}(c+dx)}{32}+\frac{5a^{2}x\cos^{8}(c+dx)}{128}+\frac{5a^{2}\sin^{7}(c+}{1} \\ x\left(a+b\sin\left(c\right)\right)^{2}\sin^{2}\left(c\right)\cos^{6}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**8/128 + 5*a**2*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*a**2*x*sin(c
+ d*x)**2*cos(c + d*x)**6/32 + 5*a**2*x*cos(c + d*x)**8/128 + 5*a**2*sin(c
+ d*x)**7*cos(c + d*x)/(128*d) + 55*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(3
84*d) + 73*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) - 5*a**2*sin(c + d*
x)*cos(c + d*x)**7/(128*d) - 2*a*b*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) -
4*a*b*cos(c + d*x)**9/(63*d) + 3*b**2*x*sin(c + d*x)**10/256 + 15*b**2*x*si
n(c + d*x)**8*cos(c + d*x)**2/256 + 15*b**2*x*sin(c + d*x)**6*cos(c + d*x)*
*4/128 + 15*b**2*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*b**2*x*sin(c +
d*x)**2*cos(c + d*x)**8/256 + 3*b**2*x*cos(c + d*x)**10/256 + 3*b**2*sin(c
```

```
+ d*x)**9*cos(c + d*x)/(256*d) + 7*b**2*sin(c + d*x)**7*cos(c + d*x)**3/(12
8*d) + b**2*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) - 7*b**2*sin(c + d*x)**3
*cos(c + d*x)**7/(128*d) - 3*b**2*sin(c + d*x)*cos(c + d*x)**9/(256*d), Ne(
d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**6, True))
```

Giac [A] time = 1.19813, size = 255, normalized size = 1.27

$$\frac{1}{256}\left(10\,a^2+3\,b^2\right)x+\frac{ab\cos\left(9\,dx+9\,c\right)}{1152\,d}+\frac{3\,ab\cos\left(7\,dx+7\,c\right)}{896\,d}-\frac{ab\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{3\,ab\cos\left(dx+c\right)}{64\,d}+\frac{b^2\sin\left(10\,dx\right.}{5120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/256*(10*a^2 + 3*b^2)*x + 1/1152*a*b*cos(9*d*x + 9*c)/d + 3/896*a*b*cos(7*
d*x + 7*c)/d - 1/48*a*b*cos(3*d*x + 3*c)/d - 3/64*a*b*cos(d*x + c)/d + 1/51
20*b^2*sin(10*d*x + 10*c)/d - 1/2048*(2*a^2 - b^2)*sin(8*d*x + 8*c)/d - 1/3
072*(16*a^2 + 3*b^2)*sin(6*d*x + 6*c)/d - 1/256*(2*a^2 + b^2)*sin(4*d*x + 4
*c)/d + 1/512*(8*a^2 + b^2)*sin(2*d*x + 2*c)/d
```

3.1243 $\int \cos^6(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=152

$$-\frac{(a^2+8b^2)\cos^7(c+dx)}{252d}-\frac{\cos^7(c+dx)(a+b\sin(c+dx))^2}{9d}-\frac{a\cos^7(c+dx)(a+b\sin(c+dx))}{36d}+\frac{ab\sin(c+dx)\cos^5}{24d}$$

```
[Out] (5*a*b*x)/64 - ((a^2 + 8*b^2)*Cos[c + d*x]^7)/(252*d) + (5*a*b*Cos[c + d*x]
*Sin[c + d*x])/(64*d) + (5*a*b*Cos[c + d*x]^3*Sin[c + d*x])/(96*d) + (a*b*C
os[c + d*x]^5*Sin[c + d*x])/(24*d) - (a*Cos[c + d*x]^7*(a + b*Sin[c + d*x])
)/(36*d) - (Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2)/(9*d)
```

Rubi [A] time = 0.212379, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2862, 2669, 2635, 8}

$$-\frac{(a^2+8b^2)\cos^7(c+dx)}{252d}-\frac{\cos^7(c+dx)(a+b\sin(c+dx))^2}{9d}-\frac{a\cos^7(c+dx)(a+b\sin(c+dx))}{36d}+\frac{ab\sin(c+dx)\cos^5}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*x)/64 - ((a^2 + 8*b^2)*Cos[c + d*x]^7)/(252*d) + (5*a*b*Cos[c + d*x]
*Sin[c + d*x])/(64*d) + (5*a*b*Cos[c + d*x]^3*Sin[c + d*x])/(96*d) + (a*b*C
os[c + d*x]^5*Sin[c + d*x])/(24*d) - (a*Cos[c + d*x]^7*(a + b*Sin[c + d*x])
)/(36*d) - (Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2)/(9*d)
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos^7(c+dx)(a+b\sin(c+dx))^2}{9d} + \frac{1}{9}\int \cos^6(c+dx)(2b+2a\sin(c$$
$$= -\frac{a\cos^7(c+dx)(a+b\sin(c+dx))}{36d} - \frac{\cos^7(c+dx)(a+b\sin(c+dx))^2}{9d}$$
$$= -\frac{(a^2+8b^2)\cos^7(c+dx)}{252d} - \frac{a\cos^7(c+dx)(a+b\sin(c+dx))}{36d} - \cos^7$$
$$= -\frac{(a^2+8b^2)\cos^7(c+dx)}{252d} + \frac{ab\cos^5(c+dx)\sin(c+dx)}{24d} - \frac{a\cos^7(c+}{}$$
$$= -\frac{(a^2+8b^2)\cos^7(c+dx)}{252d} + \frac{5ab\cos^3(c+dx)\sin(c+dx)}{96d} + \frac{ab\cos^5(c}{}$$
$$= -\frac{(a^2+8b^2)\cos^7(c+dx)}{252d} + \frac{5ab\cos(c+dx)\sin(c+dx)}{64d} + \frac{5ab\cos^3(c}{}$$
$$= \frac{5abx}{64} - \frac{(a^2+8b^2)\cos^7(c+dx)}{252d} + \frac{5ab\cos(c+dx)\sin(c+dx)}{64d} + \frac{5ab}{}$$

Mathematica [A] time = 0.999239, size = 161, normalized size = 1.06

$$-\frac{126(10a^2+3b^2)\cos(c+dx)+84(9a^2+2b^2)\cos(3(c+dx))+252a^2\cos(5(c+dx))+36a^2\cos(7(c+dx))-504ab\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(-1260*a*b*c - 1260*a*b*d*x + 126*(10*a^2 + 3*b^2)*Cos[c + d*x] + 84*(9*a^
2 + 2*b^2)*Cos[3*(c + d*x)] + 252*a^2*Cos[5*(c + d*x)] + 36*a^2*Cos[7*(c +
d*x)] - 27*b^2*Cos[7*(c + d*x)] - 7*b^2*Cos[9*(c + d*x)] - 504*a*b*Sin[2*(c
 + d*x)] + 252*a*b*Sin[4*(c + d*x)] + 168*a*b*Sin[6*(c + d*x)] + (63*a*b*Si
n[8*(c + d*x)])/2)/(16128*d)
```

Maple [A] time = 0.042, size = 115, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{a^2(\cos(dx+c))^7}{7} + 2ab\left(-1/8\sin(dx+c)(\cos(dx+c))^7 + 1/48\left((\cos(dx+c))^5 + 5/4(\cos(dx+c))^3 + \frac{15\cos(dx}{8}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/7*a^2*cos(d*x+c)^7+2*a*b*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*
x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+b^2*
(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

Maxima [A] time = 0.986414, size = 124, normalized size = 0.82

$$-\frac{4608a^2\cos(dx+c)^7 - 21(64\sin(2dx+2c)^3 + 120dx + 120c - 3\sin(8dx+8c) - 24\sin(4dx+4c))ab - 512(7\cos}{32256d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32256*(4608*a^2*cos(d*x + c)^7 - 21*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 1
20*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a*b - 512*(7*cos(d*x + c)^
9 - 9*cos(d*x + c)^7)*b^2)/d
```

Fricas [A] time = 1.8221, size = 261, normalized size = 1.72

$$\frac{448 b^2 \cos(dx+c)^9 - 576 \left(a^2+b^2\right) \cos(dx+c)^7 + 315 abdx - 21 \left(48 ab \cos(dx+c)^7 - 8 ab \cos(dx+c)^5 - 10 ab \cos\right.}{4032 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(448*b^2*cos(d*x + c)^9 - 576*(a^2 + b^2)*cos(d*x + c)^7 + 315*a*b*d
*x - 21*(48*a*b*cos(d*x + c)^7 - 8*a*b*cos(d*x + c)^5 - 10*a*b*cos(d*x + c)
^3 - 15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 22.4536, size = 282, normalized size = 1.86

$$\begin{cases} -\frac{a^2\cos^7(c+dx)}{7d} + \frac{5abx\sin^8(c+dx)}{64} + \frac{5abx\sin^6(c+dx)\cos^2(c+dx)}{16} + \frac{15abx\sin^4(c+dx)\cos^4(c+dx)}{32} + \frac{5abx\sin^2(c+dx)\cos^6(c+dx)}{16} + \frac{5abx\cos^8(c\cdot}{64} \\ x\left(a+b\sin(c)\right)^2\sin(c)\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**7/(7*d) + 5*a*b*x*sin(c + d*x)**8/64 + 5*a*b
*x*sin(c + d*x)**6*cos(c + d*x)**2/16 + 15*a*b*x*sin(c + d*x)**4*cos(c + d*
x)**4/32 + 5*a*b*x*sin(c + d*x)**2*cos(c + d*x)**6/16 + 5*a*b*x*cos(c + d*x
)**8/64 + 5*a*b*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*a*b*sin(c + d*x)**
5*cos(c + d*x)**3/(192*d) + 73*a*b*sin(c + d*x)**3*cos(c + d*x)**5/(192*d)
- 5*a*b*sin(c + d*x)*cos(c + d*x)**7/(64*d) - b**2*sin(c + d*x)**2*cos(c +
d*x)**7/(7*d) - 2*b**2*cos(c + d*x)**9/(63*d), Ne(d, 0)), (x*(a + b*sin(c))
**2*sin(c)*cos(c)**6, True))
```

Giac [A] time = 1.20556, size = 238, normalized size = 1.57

$$\frac{5}{64} abx + \frac{b^2 \cos(9 dx + 9 c)}{2304 d} - \frac{a^2 \cos(5 dx + 5 c)}{64 d} - \frac{ab \sin(8 dx + 8 c)}{512 d} - \frac{ab \sin(6 dx + 6 c)}{96 d} - \frac{ab \sin(4 dx + 4 c)}{64 d} + \frac{ab\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/64*a*b*x + 1/2304*b^2*cos(9*d*x + 9*c)/d - 1/64*a^2*cos(5*d*x + 5*c)/d -
1/512*a*b*sin(8*d*x + 8*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d - 1/64*a*b*sin(4
*d*x + 4*c)/d + 1/32*a*b*sin(2*d*x + 2*c)/d - 1/1792*(4*a^2 - 3*b^2)*cos(7*
d*x + 7*c)/d - 1/192*(9*a^2 + 2*b^2)*cos(3*d*x + 3*c)/d - 1/128*(10*a^2 + 3
*b^2)*cos(d*x + c)/d
```

3.1244 $\int \cos^5(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=157

$$\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5ab\sin(c\cdots}{}$$

```
[Out] (5*a*b*x)/8 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) + (a^2*Cos[c + d*x]^5)/(5*d) - (b^2*Cos[c + d*x]^7)/(7
*d) + (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a*b*Cos[c + d*x]^3*Sin[c
 + d*x])/(12*d) + (a*b*Cos[c + d*x]^5*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.207032, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2911, 2635, 8, 14, 207}

$$\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5ab\sin(c\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*x)/8 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) + (a^2*Cos[c + d*x]^5)/(5*d) - (b^2*Cos[c + d*x]^7)/(7
*d) + (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a*b*Cos[c + d*x]^3*Sin[c
 + d*x])/(12*d) + (a*b*Cos[c + d*x]^5*Sin[c + d*x])/(3*d)
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\,dx + \int \cos^5(c+dx)\cot(c+dx)\left(a^2+b^2\sin\right.$$

$$= \frac{ab\cos^5(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}(5ab)\int \cos^4(c+dx)\,dx - \frac{\text{Subst}}{}$$

$$= \frac{5ab\cos^3(c+dx)\sin(c+dx)}{12d} + \frac{ab\cos^5(c+dx)\sin(c+dx)}{3d} + \frac{1}{4}(5$$

$$= \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} + \frac{a^2\cos^5(c+dx)}{5d} - \frac{b^2\cos^7(c+}{7d}$$

$$= \frac{5abx}{8} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+d}{3d}$$

Mathematica [A] time = 0.280247, size = 166, normalized size = 1.06

$$\frac{105\left(88a^2-5b^2\right)\cos(c+dx)+35\left(28a^2-9b^2\right)\cos(3(c+dx))+84a^2\cos(5(c+dx))+6720a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4200*a*b*c + 4200*a*b*d*x + 105*(88*a^2 - 5*b^2)*Cos[c + d*x] + 35*(28*a^2
 - 9*b^2)*Cos[3*(c + d*x)] + 84*a^2*Cos[5*(c + d*x)] - 105*b^2*Cos[5*(c + d
*x)] - 15*b^2*Cos[7*(c + d*x)] - 6720*a^2*Log[Cos[(c + d*x)/2]] + 6720*a^2*
Log[Sin[(c + d*x)/2]] + 3150*a*b*Sin[2*(c + d*x)] + 630*a*b*Sin[4*(c + d*x)
] + 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.086, size = 160, normalized size = 1.

$$\frac{a^2(\cos(dx+c))^5}{5d} + \frac{a^2(\cos(dx+c))^3}{3d} + \frac{a^2\cos(dx+c)}{d} + \frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d} + \frac{ab(\cos(dx+c))^5\sin(}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/5*a^2*cos(d*x+c)^5/d+1/3*a^2*cos(d*x+c)^3/d+a^2*cos(d*x+c)/d+1/d*a^2*ln(c
sc(d*x+c)-cot(d*x+c))+1/3*a*b*cos(d*x+c)^5*sin(d*x+c)/d+5/12*a*b*cos(d*x+c)
^3*sin(d*x+c)/d+5/8*a*b*cos(d*x+c)*sin(d*x+c)/d+5/8*a*b*x+5/8/d*a*b*c-1/7*b
^2*cos(d*x+c)^7/d
```

Maxima [A] time = 0.996883, size = 165, normalized size = 1.05

$$-\frac{480\,b^2\cos(dx+c)^7-112\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\right.}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(480*b^2*cos(d*x + c)^7 - 112*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3
 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a
^2 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*a*b)/d
```

Fricas [A] time = 1.83097, size = 386, normalized size = 2.46

$$-\frac{120\,b^2\cos\left(dx+c\right)^7-168\,a^2\cos\left(dx+c\right)^5-280\,a^2\cos\left(dx+c\right)^3-525\,abdx-840\,a^2\cos\left(dx+c\right)+420\,a^2\log\left(\frac{1}{2}\cos\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/840*(120*b^2*cos(d*x + c)^7 - 168*a^2*cos(d*x + c)^5 - 280*a^2*cos(d*x +
 c)^3 - 525*a*b*d*x - 840*a^2*cos(d*x + c) + 420*a^2*log(1/2*cos(d*x + c) +
 1/2) - 420*a^2*log(-1/2*cos(d*x + c) + 1/2) - 35*(8*a*b*cos(d*x + c)^5 + 1
0*a*b*cos(d*x + c)^3 + 15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22764, size = 393, normalized size = 2.5

$$\frac{525\,(dx+c)ab+840\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(1155\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-2520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+840\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+980\,ab\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(525*(d*x + c)*a*b + 840*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(1155
*a*b*tan(1/2*d*x + 1/2*c)^13 - 2520*a^2*tan(1/2*d*x + 1/2*c)^12 + 840*b^2*t
an(1/2*d*x + 1/2*c)^12 + 980*a*b*tan(1/2*d*x + 1/2*c)^11 - 10080*a^2*tan(1/
2*d*x + 1/2*c)^10 + 2975*a*b*tan(1/2*d*x + 1/2*c)^9 - 20440*a^2*tan(1/2*d*x
 + 1/2*c)^8 + 4200*b^2*tan(1/2*d*x + 1/2*c)^8 - 24640*a^2*tan(1/2*d*x + 1/2
*c)^6 - 2975*a*b*tan(1/2*d*x + 1/2*c)^5 - 16968*a^2*tan(1/2*d*x + 1/2*c)^4
+ 2520*b^2*tan(1/2*d*x + 1/2*c)^4 - 980*a*b*tan(1/2*d*x + 1/2*c)^3 - 6496*a
^2*tan(1/2*d*x + 1/2*c)^2 - 1155*a*b*tan(1/2*d*x + 1/2*c) - 1288*a^2 + 120*
b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

3.1245 $\int \cos^4(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=178

$$-\frac{(6a^2-5b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(14a^2-5b^2)\sin(c+dx)\cos(c+dx)}{16d}-\frac{5}{16}x(6a^2-b^2)-\frac{a^2\cot(c+dx)}{d}+\frac{2ab}{}$$

```
[Out] (-5*(6*a^2 - b^2)*x)/16 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c +
d*x]])/d + (2*a*b*Cos[c + d*x]^3)/(3*d) + (2*a*b*Cos[c + d*x]^5)/(5*d) - (a^
2*Cot[c + d*x])/d - ((14*a^2 - 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (
(6*a^2 - 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^5*S
in[c + d*x])/(6*d)
```

Rubi [A] time = 0.463483, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2911, 2592, 302, 206, 434, 456, 453, 203}

$$-\frac{(6a^2-5b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(14a^2-5b^2)\sin(c+dx)\cos(c+dx)}{16d}-\frac{5}{16}x(6a^2-b^2)-\frac{a^2\cot(c+dx)}{d}+\frac{2ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-5*(6*a^2 - b^2)*x)/16 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c +
d*x]])/d + (2*a*b*Cos[c + d*x]^3)/(3*d) + (2*a*b*Cos[c + d*x]^5)/(5*d) - (a^
2*Cot[c + d*x])/d - ((14*a^2 - 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (
(6*a^2 - 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^5*S
in[c + d*x])/(6*d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^5(c+dx)\cot(c+dx)\,dx + \int \cos^4(c+dx)\cot^2(c+dx)$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+b^2+\frac{a^2}{x^2}}{\left(1+x^2\right)^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x,\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+\left(a^2+b^2\right)x^2}{x^2\left(1+x^2\right)^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \left(-1-x^2\right.\right.}{}$$

$$= \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos^3(c+dx)}{3d} + \frac{2ab\cos^5(c+dx)}{5d} + \frac{b^2\cos^5(c}{}$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos^3(c+dx)}{3d} +$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos^3(c+dx)}{3d} +$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos^3(c+dx)}{3d} +$$

$$= -\frac{5}{16}\left(6a^2-b^2\right)x - \frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2}{}$$

Mathematica [A] time = 0.400461, size = 220, normalized size = 1.24

$$-\frac{15a^2(c+dx)}{8d}-\frac{a^2\sin(2(c+dx))}{2d}-\frac{a^2\sin(4(c+dx))}{32d}-\frac{a^2\cot(c+dx)}{d}+\frac{11ab\cos(c+dx)}{4d}+\frac{7ab\cos(3(c+dx))}{24d}+\frac{al}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a^2*(c + d*x))/(8*d) + (5*b^2*(c + d*x))/(16*d) + (11*a*b*Cos[c + d*x]
)/(4*d) + (7*a*b*Cos[3*(c + d*x)])/(24*d) + (a*b*Cos[5*(c + d*x)])/(40*d) -
 (a^2*Cot[c + d*x])/d - (2*a*b*Log[Cos[(c + d*x)/2]])/d + (2*a*b*Log[Sin[(c
 + d*x)/2]])/d - (a^2*Sin[2*(c + d*x)])/(2*d) + (15*b^2*Sin[2*(c + d*x)])/(
64*d) - (a^2*Sin[4*(c + d*x)])/(32*d) + (3*b^2*Sin[4*(c + d*x)])/(64*d) + (
b^2*Sin[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.078, size = 250, normalized size = 1.4

$$-\frac{a^2(\cos(dx+c))^7}{d\sin(dx+c)}-\frac{a^2(\cos(dx+c))^5\sin(dx+c)}{d}-\frac{5a^2(\cos(dx+c))^3\sin(dx+c)}{4d}-\frac{15a^2\cos(dx+c)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^7-a^2*cos(d*x+c)^5*sin(d*x+c)/d-5/4*a^2*cos(
d*x+c)^3*sin(d*x+c)/d-15/8*a^2*cos(d*x+c)*sin(d*x+c)/d-15/8*a^2*x-15/8/d*c*
a^2+2/5*a*b*cos(d*x+c)^5/d+2/3*a*b*cos(d*x+c)^3/d+2*a*b*cos(d*x+c)/d+2/d*a*
b*ln(csc(d*x+c)-cot(d*x+c))+1/6*b^2*cos(d*x+c)^5*sin(d*x+c)/d+5/24*b^2*cos(
d*x+c)^3*sin(d*x+c)/d+5/16*b^2*cos(d*x+c)*sin(d*x+c)/d+5/16*b^2*x+5/16/d*b^
2*c
```

Maxima [A] time = 1.48727, size = 232, normalized size = 1.3

$$-\frac{120\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+25\,\tan(dx+c)^2+8}{\tan(dx+c)^5+2\,\tan(dx+c)^3+\tan(dx+c)}\right)a^2-64\left(6\,\cos(dx+c)^5+10\,\cos(dx+c)^3+30\,\cos(dx+c)-15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/960*(120*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(t
an(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^2 - 64*(6*cos(d*x + c)^
5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log
(cos(d*x + c) - 1))*a*b + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4
*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^2)/d
```

Fricas [A] time = 1.88746, size = 489, normalized size = 2.75

$$-\frac{40\,b^2\cos(dx+c)^7-10\left(6\,a^2-b^2\right)\cos(dx+c)^5-25\left(6\,a^2-b^2\right)\cos(dx+c)^3+240\,ab\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(40*b^2*cos(d*x + c)^7 - 10*(6*a^2 - b^2)*cos(d*x + c)^5 - 25*(6*a^2
 - b^2)*cos(d*x + c)^3 + 240*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 240*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 75*(6*a^2 - b^2)*cos(d
*x + c) - (96*a*b*cos(d*x + c)^5 + 160*a*b*cos(d*x + c)^3 - 75*(6*a^2 - b^2
)*d*x + 480*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25425, size = 497, normalized size = 2.79

$$480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-75\left(6\,a^2-b^2\right)(dx+c)-\frac{120\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(270\,a^2\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(480*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 120*a^2*tan(1/2*d*x + 1/2*c
) - 75*(6*a^2 - b^2)*(d*x + c) - 120*(4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan
(1/2*d*x + 1/2*c) + 2*(270*a^2*tan(1/2*d*x + 1/2*c)^11 - 165*b^2*tan(1/2*d*
x + 1/2*c)^11 + 1440*a*b*tan(1/2*d*x + 1/2*c)^10 + 570*a^2*tan(1/2*d*x + 1/
2*c)^9 + 25*b^2*tan(1/2*d*x + 1/2*c)^9 + 4320*a*b*tan(1/2*d*x + 1/2*c)^8 +
300*a^2*tan(1/2*d*x + 1/2*c)^7 - 450*b^2*tan(1/2*d*x + 1/2*c)^7 + 7360*a*b*
tan(1/2*d*x + 1/2*c)^6 - 300*a^2*tan(1/2*d*x + 1/2*c)^5 + 450*b^2*tan(1/2*d
*x + 1/2*c)^5 + 6720*a*b*tan(1/2*d*x + 1/2*c)^4 - 570*a^2*tan(1/2*d*x + 1/2
*c)^3 - 25*b^2*tan(1/2*d*x + 1/2*c)^3 + 2976*a*b*tan(1/2*d*x + 1/2*c)^2 - 2
70*a^2*tan(1/2*d*x + 1/2*c) + 165*b^2*tan(1/2*d*x + 1/2*c) + 736*a*b)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.1246 $\int \cos^3(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=180

$$-\frac{(a^2-b^2)\cos^3(c+dx)}{3d}-\frac{(2a^2-b^2)\cos(c+dx)}{d}+\frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-1$$

```
[Out] (-15*a*b*x)/4 + ((5*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - ((2*a^2 - b
^2)*Cos[c + d*x])/d - ((a^2 - b^2)*Cos[c + d*x]^3)/(3*d) + (b^2*Cos[c + d*x
]^5)/(5*d) - (15*a*b*Cot[c + d*x])/(4*d) + (5*a*b*Cos[c + d*x]^2*Cot[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^4*Cot[c + d*x])/(2*d) - (a^2*Cot[c + d*x]*Csc
[c + d*x])/(2*d)
```

Rubi [A] time = 0.304969, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2911, 2591, 288, 321, 203, 455, 1810, 206}

$$-\frac{(a^2-b^2)\cos^3(c+dx)}{3d}-\frac{(2a^2-b^2)\cos(c+dx)}{d}+\frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-1$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-15*a*b*x)/4 + ((5*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - ((2*a^2 - b
^2)*Cos[c + d*x])/d - ((a^2 - b^2)*Cos[c + d*x]^3)/(3*d) + (b^2*Cos[c + d*x
]^5)/(5*d) - (15*a*b*Cot[c + d*x])/(4*d) + (5*a*b*Cos[c + d*x]^2*Cot[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^4*Cot[c + d*x])/(2*d) - (a^2*Cot[c + d*x]*Csc
[c + d*x])/(2*d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^4(c+dx)\cot^2(c+dx)\,dx + \int \cos^3(c+dx)\cot^3(c+dx)$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6\left(a^2+b^2-b^2x^2\right)}{\left(1-x^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right.}\right.}{d}$$

$$= \frac{ab\cos^4(c+dx)\cot(c+dx)}{2d} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{5ab\cos^2(c+dx)\cot(c+dx)}{4d} + \frac{ab\cos^4(c+dx)\cot(c+dx)}{2d} - \frac{a^2\text{co}}{}$$

$$= -\frac{\left(2a^2-b^2\right)\cos(c+dx)}{d} - \frac{\left(a^2-b^2\right)\cos^3(c+dx)}{3d} + \frac{b^2\cos^5(c+dx)}{5d}$$

$$= -\frac{15}{4}abx + \frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d} - \frac{\left(2a^2-b^2\right)\cos(c+dx}{d}$$

Mathematica [A] time = 6.1746, size = 250, normalized size = 1.39

$$-\frac{\left(18a^2-11b^2\right)\cos(c+dx)}{8d} - \frac{\left(4a^2-7b^2\right)\cos(3(c+dx))}{48d} + \frac{\left(2b^2-5a^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{\left(5a^2-2b^2\right)\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a*b*(c + d*x))/(4*d) - ((18*a^2 - 11*b^2)*Cos[c + d*x])/(8*d) - ((4*a^
2 - 7*b^2)*Cos[3*(c + d*x)])/(48*d) + (b^2*Cos[5*(c + d*x)])/(80*d) - (a*b*
Cot[(c + d*x)/2])/d - (a^2*Csc[(c + d*x)/2]^2)/(8*d) + ((5*a^2 - 2*b^2)*Log
[Cos[(c + d*x)/2]])/(2*d) + ((-5*a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]])/(2*d)
+ (a^2*Sec[(c + d*x)/2]^2)/(8*d) - (a*b*Sin[2*(c + d*x)])/d - (a*b*Sin[4*(c
 + d*x)])/(16*d) + (a*b*Tan[(c + d*x)/2])/d
```

Maple [A] time = 0.092, size = 261, normalized size = 1.5

$$-\frac{a^2 (\cos(dx+c))^7}{2d(\sin(dx+c))^2} - \frac{a^2(\cos(dx+c))^5}{2d} - \frac{5a^2(\cos(dx+c))^3}{6d} - \frac{5a^2\cos(dx+c)}{2d} - \frac{5a^2\ln(\csc(dx+c)-\cot(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*a^2*cos(d*x+c)^5/d-5/6*a^2*cos(d*x
+c)^3/d-5/2*a^2*cos(d*x+c)/d-5/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/d*a*b/si
n(d*x+c)*cos(d*x+c)^7-2*a*b*cos(d*x+c)^5*sin(d*x+c)/d-5/2*a*b*cos(d*x+c)^3*
sin(d*x+c)/d-15/4*a*b*cos(d*x+c)*sin(d*x+c)/d-15/4*a*b*x-15/4/d*a*b*c+1/5*b
^2*cos(d*x+c)^5/d+1/3*b^2*cos(d*x+c)^3/d+b^2*cos(d*x+c)/d+1/d*b^2*ln(csc(d*
x+c)-cot(d*x+c))
```

Maxima [A] time = 1.51559, size = 257, normalized size = 1.43

$$-\frac{5\left(4\cos(dx+c)^3 - \frac{6\cos(dx+c)}{\cos(dx+c)^2-1} + 24\cos(dx+c) - 15\log(\cos(dx+c)+1) + 15\log(\cos(dx+c)-1)\right)a^2 + 15\left(15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/60*(5*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 + 15*(15
*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
 2*tan(d*x + c)^3 + tan(d*x + c)))*a*b - 2*(6*cos(d*x + c)^5 + 10*cos(d*x +
 c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1))*b^2)/d
```

Fricas [A] time = 1.88128, size = 610, normalized size = 3.39

$$\frac{12b^2\cos(dx+c)^7 - 225abdx\cos(dx+c)^2 - 4\left(5a^2-2b^2\right)\cos(dx+c)^5 + 225abdx - 20\left(5a^2-2b^2\right)\cos(dx+c)^3 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/60*(12*b^2*cos(d*x + c)^7 - 225*a*b*d*x*cos(d*x + c)^2 - 4*(5*a^2 - 2*b^2
)*cos(d*x + c)^5 + 225*a*b*d*x - 20*(5*a^2 - 2*b^2)*cos(d*x + c)^3 + 30*(5*
a^2 - 2*b^2)*cos(d*x + c) + 15*((5*a^2 - 2*b^2)*cos(d*x + c)^2 - 5*a^2 + 2*
b^2)*log(1/2*cos(d*x + c) + 1/2) - 15*((5*a^2 - 2*b^2)*cos(d*x + c)^2 - 5*a
^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2) - 15*(2*a*b*cos(d*x + c)^5 + 5*a*b
*cos(d*x + c)^3 - 15*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27225, size = 467, normalized size = 2.59

$$15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-450\,(dx+c)ab+120\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-60\left(5\,a^2-2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{15\left(30\,a^2\tan\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/120*(15*a^2*tan(1/2*d*x + 1/2*c)^2 - 450*(d*x + c)*a*b + 120*a*b*tan(1/2*
d*x + 1/2*c) - 60*(5*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 15*(30*a
^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d
*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 4*(135*a*b*tan(1/2*d*x + 1/2*c)
^9 - 180*a^2*tan(1/2*d*x + 1/2*c)^8 + 180*b^2*tan(1/2*d*x + 1/2*c)^8 + 150*
a*b*tan(1/2*d*x + 1/2*c)^7 - 600*a^2*tan(1/2*d*x + 1/2*c)^6 + 360*b^2*tan(1
/2*d*x + 1/2*c)^6 - 800*a^2*tan(1/2*d*x + 1/2*c)^4 + 560*b^2*tan(1/2*d*x +
1/2*c)^4 - 150*a*b*tan(1/2*d*x + 1/2*c)^3 - 520*a^2*tan(1/2*d*x + 1/2*c)^2
+ 280*b^2*tan(1/2*d*x + 1/2*c)^2 - 135*a*b*tan(1/2*d*x + 1/2*c) - 140*a^2 +
 92*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.1247 $\int \cos^2(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=177

$$\frac{(2a^2-b^2)\cot(c+dx)}{d}+\frac{(4a^2-7b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}x(4a^2-3b^2)-\frac{a^2\cot^3(c+dx)}{3d}-\frac{5ab\cos^3(c+dx)}{3d}-$$

```
[Out] (5*(4*a^2 - 3*b^2)*x)/8 + (5*a*b*ArcTanh[Cos[c + d*x]])/d - (5*a*b*Cos[c +
d*x])/d - (5*a*b*Cos[c + d*x]^3)/(3*d) + ((2*a^2 - b^2)*Cot[c + d*x])/d - (
a*b*Cos[c + d*x]^3*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x]^3)/(3*d) + ((4*a^2
 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (b^2*Cos[c + d*x]^3*Sin[c + d*
x])/(4*d)
```

Rubi [A] time = 0.439628, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2911, 2592, 288, 302, 206, 456, 1259, 1261, 203}

$$\frac{(2a^2-b^2)\cot(c+dx)}{d}+\frac{(4a^2-7b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}x(4a^2-3b^2)-\frac{a^2\cot^3(c+dx)}{3d}-\frac{5ab\cos^3(c+dx)}{3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(4*a^2 - 3*b^2)*x)/8 + (5*a*b*ArcTanh[Cos[c + d*x]])/d - (5*a*b*Cos[c +
d*x])/d - (5*a*b*Cos[c + d*x]^3)/(3*d) + ((2*a^2 - b^2)*Cot[c + d*x])/d - (
a*b*Cos[c + d*x]^3*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x]^3)/(3*d) + ((4*a^2
 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (b^2*Cos[c + d*x]^3*Sin[c + d*
x])/(4*d)
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^3(c+dx)\cot^3(c+dx)\,dx + \int \cos^2(c+dx)\cot^4(c+$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+\left(a^2+b^2\right)x^2}{x^4\left(1+x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right.}\right.}{}$$

$$= -\frac{ab\cos^3(c+dx)\cot^2(c+dx)}{d} - \frac{b^2\cos^3(c+dx)\sin(c+dx)}{4d} - -$$

$$= -\frac{ab\cos^3(c+dx)\cot^2(c+dx)}{d} + \frac{\left(4a^2-7b^2\right)\cos(c+dx)\sin(c+}{8d}$$

$$= -\frac{5ab\cos(c+dx)}{d} - \frac{5ab\cos^3(c+dx)}{3d} - \frac{ab\cos^3(c+dx)\cot^2(c+}{d}$$

$$= \frac{5ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{5ab\cos(c+dx)}{d} - \frac{5ab\cos^3(c+dx)}{3d}$$

$$= \frac{5}{8}\left(4a^2-3b^2\right)x + \frac{5ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{5ab\cos(c+dx)}{d} -$$

Mathematica [A] time = 6.25002, size = 336, normalized size = 1.9

$$\frac{5\left(4a^2-3b^2\right)(c+dx)}{8d} + \frac{\left(a^2-2b^2\right)\sin(2(c+dx))}{4d} + \frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(7a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{6d} + \text{se}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(4*a^2 - 3*b^2)*(c + d*x))/(8*d) - (9*a*b*Cos[c + d*x])/(2*d) - (a*b*Cos
[3*(c + d*x)])/(6*d) + ((7*a^2*Cos[(c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*C
sc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c + d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (5*a*b*Log[Cos[(c + d*x)/2]])/d - (5*a*b*
Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/
2]*(-7*a^2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*d) + ((a^2 - 2*b^
2)*Sin[2*(c + d*x)])/(4*d) - (b^2*Sin[4*(c + d*x)])/(32*d) + (a^2*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A] time = 0.089, size = 321, normalized size = 1.8

$$-\frac{a^2(\cos(dx+c))^7}{3d(\sin(dx+c))^3} + \frac{4a^2(\cos(dx+c))^7}{3d\sin(dx+c)} + \frac{4a^2(\cos(dx+c))^5\sin(dx+c)}{3d} + \frac{5a^2(\cos(dx+c))^3\sin(dx+c)}{3d} + \frac{5a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*a^2/sin(d*x+c)*cos(d*x+c)^7+4/3*
a^2*cos(d*x+c)^5*sin(d*x+c)/d+5/3*a^2*cos(d*x+c)^3*sin(d*x+c)/d+5/2*a^2*cos
(d*x+c)*sin(d*x+c)/d+5/2*a^2*x+5/2/d*c*a^2-1/d*a*b/sin(d*x+c)^2*cos(d*x+c)^
7-a*b*cos(d*x+c)^5/d-5/3*a*b*cos(d*x+c)^3/d-5*a*b*cos(d*x+c)/d-5/d*a*b*ln(c
sc(d*x+c)-cot(d*x+c))-1/d*b^2/sin(d*x+c)*cos(d*x+c)^7-b^2*cos(d*x+c)^5*sin(
```

```
d*x+c)/d-5/4*b^2*cos(d*x+c)^3*sin(d*x+c)/d-15/8*b^2*cos(d*x+c)*sin(d*x+c)/d
-15/8*b^2*x-15/8/d*b^2*c
```

Maxima [A] time = 1.52359, size = 255, normalized size = 1.44

$$\frac{4\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^2-4\left(4\,\cos\left(dx+c\right)^3-\frac{6\,\cos(dx+c)}{\cos(dx+c)^2-1}+24\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*(4*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d
*x + c)^5 + tan(d*x + c)^3))*a^2 - 4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(co
s(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos
(d*x + c) - 1))*a*b - 3*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x +
c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*b^2)/d
```

Fricas [A] time = 1.90171, size = 635, normalized size = 3.59

$$\frac{6\,b^2\cos\left(dx+c\right)^7-3\left(4\,a^2-3\,b^2\right)\cos\left(dx+c\right)^5+20\left(4\,a^2-3\,b^2\right)\cos\left(dx+c\right)^3+60\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\,\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*b^2*cos(d*x + c)^7 - 3*(4*a^2 - 3*b^2)*cos(d*x + c)^5 + 20*(4*a^2 -
 3*b^2)*cos(d*x + c)^3 + 60*(a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 60*(a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) - 15*(4*a^2 - 3*b^2)*cos(d*x + c) - (16*a*b*cos(d*x + c
)^5 - 15*(4*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 + 80*a*b*cos(d*x + c)^3 + 15*(4
*a^2 - 3*b^2)*d*x - 120*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2
- d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25046, size = 494, normalized size = 2.79

$$a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 6\,ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 120\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 27\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 12\,b^2\tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 120*a*b*l
og(abs(tan(1/2*d*x + 1/2*c))) - 27*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/
2*d*x + 1/2*c) + 15*(4*a^2 - 3*b^2)*(d*x + c) + (220*a*b*tan(1/2*d*x + 1/2*
c)^3 + 27*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*
b*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3 - 2*(12*a^2*tan(1/2*d*
x + 1/2*c)^7 - 27*b^2*tan(1/2*d*x + 1/2*c)^7 + 144*a*b*tan(1/2*d*x + 1/2*c)
^6 + 12*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 336*a*b
*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 3*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 304*a*b*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*tan(1/2*d*x + 1/2*c)
+ 27*b^2*tan(1/2*d*x + 1/2*c) + 112*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.1248 $\int \cos(c+dx)\cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=174

$$\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{5(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(9a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2\cot(c+dx)\csc^3(\cdot}{4d}$$

```
[Out] 5*a*b*x - (5*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + ((a^2 - 2*b^2)*
Cos[c + d*x])/d - (b^2*Cos[c + d*x]^3)/(3*d) + (5*a*b*Cot[c + d*x])/d - (5*
a*b*Cot[c + d*x]^3)/(3*d) + (a*b*Cos[c + d*x]^2*Cot[c + d*x]^3)/d + ((9*a^2
 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]
^3)/(4*d)
```

Rubi [A] time = 0.278132, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2911, 2591, 288, 302, 203, 455, 1814, 1153, 206}

$$\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{5(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(9a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2\cot(c+dx)\csc^3(\cdot}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] 5*a*b*x - (5*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + ((a^2 - 2*b^2)*
Cos[c + d*x])/d - (b^2*Cos[c + d*x]^3)/(3*d) + (5*a*b*Cot[c + d*x])/d - (5*
a*b*Cot[c + d*x]^3)/(3*d) + (a*b*Cos[c + d*x]^2*Cot[c + d*x]^3)/d + ((9*a^2
 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]
^3)/(4*d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^2(c+dx)\cot^4(c+dx)\,dx + \int \cos(c+dx)\cot^5(c+dx)\,($$

$$= -\frac{\text{Subst}\left(\int \frac{x^6(a^2+b^2-b^2x^2)}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\right.}{d}$$

$$= \frac{ab\cos^2(c+dx)\cot^3(c+dx)}{d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{\text{Subst}}{}$$

$$= \frac{ab\cos^2(c+dx)\cot^3(c+dx)}{d} + \frac{(9a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8d}$$

$$= \frac{5ab\cot(c+dx)}{d} - \frac{5ab\cot^3(c+dx)}{3d} + \frac{ab\cos^2(c+dx)\cot^3(c+dx)}{d}$$

$$= 5abx + \frac{(a^2-2b^2)\cos(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{5ab\cot(c+dx)}{d}$$

$$= 5abx - \frac{5(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{(a^2-2b^2)\cos(c+dx)}{d}$$

Mathematica [A] time = 6.18144, size = 337, normalized size = 1.94

$$\frac{(9a^2-4b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{(4b^2-9a^2)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{5(3a^2-4b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{5(3a^2-4b^2)\log\left(c\right.}{8d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*(c + d*x))/d + ((2*a - 3*b)*(2*a + 3*b)*Cos[c + d*x])/(4*d) - (b^2*C
os[3*(c + d*x)])/(12*d) + (7*a*b*Cot[(c + d*x)/2])/(3*d) + ((9*a^2 - 4*b^2)
*Csc[(c + d*x)/2]^2)/(32*d) - (a*b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(12
*d) - (a^2*Csc[(c + d*x)/2]^4)/(64*d) - (5*(3*a^2 - 4*b^2)*Log[Cos[(c + d*x
)/2]])/(8*d) + (5*(3*a^2 - 4*b^2)*Log[Sin[(c + d*x)/2]])/(8*d) + ((-9*a^2 +
 4*b^2)*Sec[(c + d*x)/2]^2)/(32*d) + (a^2*Sec[(c + d*x)/2]^4)/(64*d) + (a*b
*Sin[2*(c + d*x)])/(2*d) - (7*a*b*Tan[(c + d*x)/2])/(3*d) + (a*b*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/(12*d)
```

Maple [B] time = 0.092, size = 334, normalized size = 1.9

$$-\frac{a^2(\cos(dx+c))^7}{4d(\sin(dx+c))^4} + \frac{3a^2(\cos(dx+c))^7}{8d(\sin(dx+c))^2} + \frac{3a^2(\cos(dx+c))^5}{8d} + \frac{5a^2(\cos(dx+c))^3}{8d} + \frac{15a^2\cos(dx+c)}{8d} + \frac{15a^2\ln(\csc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7+3/
8*a^2*cos(d*x+c)^5/d+5/8*a^2*cos(d*x+c)^3/d+15/8*a^2*cos(d*x+c)/d+15/8/d*a^
2*ln(csc(d*x+c)-cot(d*x+c))-2/3/d*a*b/sin(d*x+c)^3*cos(d*x+c)^7+8/3/d*a*b/s
in(d*x+c)*cos(d*x+c)^7+8/3*a*b*cos(d*x+c)^5*sin(d*x+c)/d+10/3*a*b*cos(d*x+c
)^3*sin(d*x+c)/d+5*a*b*cos(d*x+c)*sin(d*x+c)/d+5*a*b*x+5/d*a*b*c-1/2/d*b^2/
```

```
sin(d*x+c)^2*cos(d*x+c)^7-1/2*b^2*cos(d*x+c)^5/d-5/6*b^2*cos(d*x+c)^3/d-5/2
*b^2*cos(d*x+c)/d-5/2/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.52289, size = 277, normalized size = 1.59

$$16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)ab-4\left(4\,\cos\left(dx+c\right)^3-\frac{6\,\cos(dx+c)}{\cos(dx+c)^2-1}+24\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/48*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(
d*x + c)^5 + tan(d*x + c)^3))*a*b - 4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(c
os(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(co
s(d*x + c) - 1))*b^2 - 3*a^2*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*
x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) +
1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.85872, size = 776, normalized size = 4.46

$$-\frac{16\,b^2\cos\left(dx+c\right)^7-240\,abdx\cos\left(dx+c\right)^4+480\,abdx\cos\left(dx+c\right)^2-16\left(3\,a^2-4\,b^2\right)\cos\left(dx+c\right)^5-240\,abdx+50}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(16*b^2*cos(d*x + c)^7 - 240*a*b*d*x*cos(d*x + c)^4 + 480*a*b*d*x*cos
(d*x + c)^2 - 16*(3*a^2 - 4*b^2)*cos(d*x + c)^5 - 240*a*b*d*x + 50*(3*a^2 -
 4*b^2)*cos(d*x + c)^3 - 30*(3*a^2 - 4*b^2)*cos(d*x + c) + 15*((3*a^2 - 4*b
^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*b^2)*log(
1/2*cos(d*x + c) + 1/2) - 15*((3*a^2 - 4*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4
*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*b^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*(3*
a*b*cos(d*x + c)^5 - 20*a*b*cos(d*x + c)^3 + 15*a*b*cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26861, size = 467, normalized size = 2.68

$$3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-48\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+960\,(dx+c)ab-43$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a*
b - 432*a*b*tan(1/2*d*x + 1/2*c) + 120*(3*a^2 - 4*b^2)*log(abs(tan(1/2*d*x
+ 1/2*c))) - 128*(3*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*tan(1/2*d*x + 1/2*c)
^4 + 9*b^2*tan(1/2*d*x + 1/2*c)^4 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*t
an(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2 + 7*b^2)/(tan(1/
2*d*x + 1/2*c)^2 + 1)^3 - (750*a^2*tan(1/2*d*x + 1/2*c)^4 - 1000*b^2*tan(1/
2*d*x + 1/2*c)^4 - 432*a*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^2*tan(1/2*d*x + 1/
2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1/2*d*x + 1/2*c) + 3*a^
2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.1249 $\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=202

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\,\mathrm{co}}{}$$

```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos
[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a
*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2
*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*C
os[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.188103, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 2591, 288, 302, 203, 2592, 321, 206, 3473, 8}

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos
[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a
*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2
*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*C
os[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```


Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^4(c+dx)+2ab\cos(c+dx)\cot^5(c+dx)+a^2\cot^6(c+\right.$$

$$= a^2\int \cot^6(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^5(c+dx)\,dx+b^2\int \cos^2(c+$$

$$= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx-\frac{(2ab)\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx,x,\cos(c\right.}{d}$$

$$= \frac{a^2\cot^3(c+dx)}{3d}+\frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}$$

$$= -\frac{a^2\cot(c+dx)}{d}+\frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d}+\frac{a^2\cot^3(c+dx)}{3d}+\frac{b^2\cos^2(}{}$$

$$= -a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c+dx)}{d}+\frac{5b^2\cot(c+dx)}{2d}+\frac{5ab\cos(c+d}{4}$$

$$= -a^2x+\frac{5b^2x}{2}-\frac{15ab\tanh^{-1}(\cos(c+dx))}{4d}+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c+dx)}{d}$$

Mathematica [A] time = 1.08303, size = 351, normalized size = 1.74

$$\frac{\left(560b^2-368a^2\right)\cot\left(\frac{1}{2}(c+dx)\right)+368a^2\tan\left(\frac{1}{2}(c+dx)\right)+96a^2\sin^6\left(\frac{1}{2}(c+dx)\right)\csc^5(c+dx)-328a^2\sin^4\left(\frac{1}{2}(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-480*a^2*c + 1200*b^2*c - 480*a^2*d*x + 1200*b^2*d*x + 960*a*b*Cos[c + d*x
] + (-368*a^2 + 560*b^2)*Cot[(c + d*x)/2] + 270*a*b*Csc[(c + d*x)/2]^2 - 15
*a*b*Csc[(c + d*x)/2]^4 - 1800*a*b*Log[Cos[(c + d*x)/2]] + 1800*a*b*Log[Sin
[(c + d*x)/2]] - 270*a*b*Sec[(c + d*x)/2]^2 + 15*a*b*Sec[(c + d*x)/2]^4 - 3
28*a^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 160*b^2*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + 96*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + (41*a^2*Csc[(c + d*x
)/2]^4*Sin[c + d*x])/2 - 10*b^2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - (3*a^2*Cs
c[(c + d*x)/2]^6*Sin[c + d*x])/2 + 120*b^2*Sin[2*(c + d*x)] + 368*a^2*Tan[(
c + d*x)/2] - 560*b^2*Tan[(c + d*x)/2])/(480*d)
```

Maple [A] time = 0.095, size = 302, normalized size = 1.5

$$-\frac{a^2(\cot(dx+c))^5}{5d}+\frac{a^2(\cot(dx+c))^3}{3d}-\frac{a^2\cot(dx+c)}{d}-a^2x-\frac{a^2c}{d}-\frac{ab(\cos(dx+c))^7}{2d(\sin(dx+c))^4}+\frac{3ab(\cos(dx+c))^7}{4d(\sin(dx+c))^2}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5*a^2*cot(d*x+c)^5/d+1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)/d-a^2*x-1/d*c
*a^2-1/2/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7+3/4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^
7+3/4*a*b*cos(d*x+c)^5/d+5/4*a*b*cos(d*x+c)^3/d+15/4*a*b*cos(d*x+c)/d+15/4/
d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*b^2/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*b
^2/sin(d*x+c)*cos(d*x+c)^7+4/3*b^2*cos(d*x+c)^5*sin(d*x+c)/d+5/3*b^2*cos(d*
x+c)^3*sin(d*x+c)/d+5/2*b^2*cos(d*x+c)*sin(d*x+c)/d+5/2*b^2*x+5/2/d*b^2*c
```

Maxima [A] time = 1.54019, size = 247, normalized size = 1.22

$$-\frac{8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-20\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)b^2+15\,ab\left(\frac{2\left(9\,\cos(dx+c)^3-7\right.}{\cos(dx+c)^4-2\,\mathrm{co}}\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d
*x + c)^5)*a^2 - 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2
 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*b^2 + 15*a*b*(2*(9*cos(d*x + c)^3
- 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
 + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.8451, size = 786, normalized size = 3.89

$$-\frac{60\,b^2\cos\left(dx+c\right)^7+92\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^5-140\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^3+225\left(ab\cos\left(dx+c\right)^4-2\,ab\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/120*(60*b^2*cos(d*x + c)^7 + 92*(2*a^2 - 5*b^2)*cos(d*x + c)^5 - 140*(2*
a^2 - 5*b^2)*cos(d*x + c)^3 + 225*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^
2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(a*b*cos(d*x + c)^4
 - 2*a*b*cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
60*(2*a^2 - 5*b^2)*cos(d*x + c) + 30*(2*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^4
- 8*a*b*cos(d*x + c)^5 - 4*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^2 + 25*a*b*cos(
d*x + c)^3 + 2*(2*a^2 - 5*b^2)*d*x - 15*a*b*cos(d*x + c))*sin(d*x + c))/((d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21446, size = 455, normalized size = 2.25

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,ab\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*a*b*tan(1/2*
d*x + 1/2*c)^2 + 1800*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 330*a^2*tan(1/2*
d*x + 1/2*c) - 540*b^2*tan(1/2*d*x + 1/2*c) - 240*(2*a^2 - 5*b^2)*(d*x + c)
 - 480*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (4110*a*b*tan(1
/2*d*x + 1/2*c)^5 + 330*a^2*tan(1/2*d*x + 1/2*c)^4 - 540*b^2*tan(1/2*d*x +
1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 +
 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan(1
/2*d*x + 1/2*c)^5)/d
```

3.1250 $\int \cot^6(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=175

$$\frac{5\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{\left(13a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{\left(11a^2-18b^2\right)\cot(c+dx)\csc(c+dx)}{16d}-\frac{a^2\,c}{}$$

```
[Out] -2*a*b*x + (5*(a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (b^2*Cos[c + d*
x])/d - (2*a*b*Cot[c + d*x])/d + (2*a*b*Cot[c + d*x]^3)/(3*d) - (2*a*b*Cot[
c + d*x]^5)/(5*d) - ((11*a^2 - 18*b^2)*Cot[c + d*x]*Csc[c + d*x])/(16*d) +
((13*a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*C
sc[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.26146, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2911, 3473, 8, 4366, 455, 1814, 1157, 388, 206}

$$\frac{5\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{\left(13a^2-6b^2\right)\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{\left(11a^2-18b^2\right)\cot(c+dx)\csc(c+dx)}{16d}-\frac{a^2\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -2*a*b*x + (5*(a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (b^2*Cos[c + d*
x])/d - (2*a*b*Cot[c + d*x])/d + (2*a*b*Cot[c + d*x]^3)/(3*d) - (2*a*b*Cot[
c + d*x]^5)/(5*d) - ((11*a^2 - 18*b^2)*Cot[c + d*x]*Csc[c + d*x])/(16*d) +
((13*a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*C
sc[c + d*x]^5)/(6*d)
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\,dx + \int \cot^6(c+dx)\csc(c+dx)\left(a^2+b^2\sin^2(c\right.$$

$$= -\frac{2ab\cot^5(c+dx)}{5d} - (2ab)\int \cot^4(c+dx)\,dx - \frac{\text{Subst}\left(\int \frac{x^6\left(a^2+b^2-b^2\right.}{\left(1-x^2\right)^4}\right.}{}$$

$$= \frac{2ab\cot^3(c+dx)}{3d} - \frac{2ab\cot^5(c+dx)}{5d} - \frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d} +$$

$$= -\frac{2ab\cot(c+dx)}{d} + \frac{2ab\cot^3(c+dx)}{3d} - \frac{2ab\cot^5(c+dx)}{5d} + \frac{\left(13a^2-\right.}{}$$

$$= -2abx - \frac{2ab\cot(c+dx)}{d} + \frac{2ab\cot^3(c+dx)}{3d} - \frac{2ab\cot^5(c+dx)}{5d} -$$

$$= -2abx + \frac{b^2\cos(c+dx)}{d} - \frac{2ab\cot(c+dx)}{d} + \frac{2ab\cot^3(c+dx)}{3d} - 2ab$$

$$= -2abx + \frac{5\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{16d} + \frac{b^2\cos(c+dx)}{d} - 2ab\,c$$

Mathematica [B] time = 1.02215, size = 384, normalized size = 2.19

$$\frac{-5a^2\csc^6\left(\frac{1}{2}(c+dx)\right)+60a^2\csc^4\left(\frac{1}{2}(c+dx)\right)-330a^2\csc^2\left(\frac{1}{2}(c+dx)\right)+5a^2\sec^6\left(\frac{1}{2}(c+dx)\right)-60a^2\sec^4\left(\frac{1}{2}(c+dx)\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3840*a*b*c - 3840*a*b*d*x + 1920*b^2*Cos[c + d*x] - 2944*a*b*Cot[(c + d*x
)/2] - 330*a^2*Csc[(c + d*x)/2]^2 + 540*b^2*Csc[(c + d*x)/2]^2 + 60*a^2*Csc
[(c + d*x)/2]^4 - 30*b^2*Csc[(c + d*x)/2]^4 - 5*a^2*Csc[(c + d*x)/2]^6 + 60
0*a^2*Log[Cos[(c + d*x)/2]] - 3600*b^2*Log[Cos[(c + d*x)/2]] - 600*a^2*Log[
Sin[(c + d*x)/2]] + 3600*b^2*Log[Sin[(c + d*x)/2]] + 330*a^2*Sec[(c + d*x)/
2]^2 - 540*b^2*Sec[(c + d*x)/2]^2 - 60*a^2*Sec[(c + d*x)/2]^4 + 30*b^2*Sec[
(c + d*x)/2]^4 + 5*a^2*Sec[(c + d*x)/2]^6 - 2624*a*b*Csc[c + d*x]^3*Sin[(c
+ d*x)/2]^4 + 768*a*b*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + 164*a*b*Csc[(c +
d*x)/2]^4*Sin[c + d*x] - 12*a*b*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 2944*a*b*
Tan[(c + d*x)/2])/(1920*d)
```

Maple [A] time = 0.093, size = 318, normalized size = 1.8

$$-\frac{a^2(\cos(dx+c))^7}{6d(\sin(dx+c))^6} + \frac{a^2(\cos(dx+c))^7}{24d(\sin(dx+c))^4} - \frac{a^2(\cos(dx+c))^7}{16d(\sin(dx+c))^2} - \frac{a^2(\cos(dx+c))^5}{16d} - \frac{5a^2(\cos(dx+c))^3}{48d} - \frac{5a^2\cos(dx}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-1
/16/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-1/16*a^2*cos(d*x+c)^5/d-5/48*a^2*cos(d*
x+c)^3/d-5/16*a^2*cos(d*x+c)/d-5/16/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/5*a*b
```

```
*cot(d*x+c)^5/d+2/3*a*b*cot(d*x+c)^3/d-2*a*b*cot(d*x+c)/d-2*a*b*x-2/d*a*b*c
-1/4/d*b^2/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*b^2/sin(d*x+c)^2*cos(d*x+c)^7+3/
8*b^2*cos(d*x+c)^5/d+5/8*b^2*cos(d*x+c)^3/d+15/8*b^2*cos(d*x+c)/d+15/8/d*b^
2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.47612, size = 296, normalized size = 1.69

$$-\frac{64\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)ab-5\,a^2\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(64*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a*b - 5*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(
d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*l
og(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 30*b^2*(2*(9*cos(d*x + c
)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x
+ c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.88429, size = 919, normalized size = 5.25

$$-\frac{960\,abdx\cos\left(dx+c\right)^6-480\,b^2\cos\left(dx+c\right)^7-2880\,abdx\cos\left(dx+c\right)^4+2880\,abdx\cos\left(dx+c\right)^2-330\left(a^2-6\,b^2\right)c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(960*a*b*d*x*cos(d*x + c)^6 - 480*b^2*cos(d*x + c)^7 - 2880*a*b*d*x*
cos(d*x + c)^4 + 2880*a*b*d*x*cos(d*x + c)^2 - 330*(a^2 - 6*b^2)*cos(d*x +
c)^5 - 960*a*b*d*x + 400*(a^2 - 6*b^2)*cos(d*x + c)^3 - 150*(a^2 - 6*b^2)*c
os(d*x + c) - 75*((a^2 - 6*b^2)*cos(d*x + c)^6 - 3*(a^2 - 6*b^2)*cos(d*x +
c)^4 + 3*(a^2 - 6*b^2)*cos(d*x + c)^2 - a^2 + 6*b^2)*log(1/2*cos(d*x + c) +
 1/2) + 75*((a^2 - 6*b^2)*cos(d*x + c)^6 - 3*(a^2 - 6*b^2)*cos(d*x + c)^4 +
 3*(a^2 - 6*b^2)*cos(d*x + c)^2 - a^2 + 6*b^2)*log(-1/2*cos(d*x + c) + 1/2)
 - 64*(23*a*b*cos(d*x + c)^5 - 35*a*b*cos(d*x + c)^3 + 15*a*b*cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2
- d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29188, size = 455, normalized size = 2.6

$$5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-45\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-280\,ab\tan\left(\frac{1}{2}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a*b*tan(1/2*d*x + 1/2*c)^5 - 45*a
^2*tan(1/2*d*x + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 - 280*a*b*tan(1/2
*d*x + 1/2*c)^3 + 225*a^2*tan(1/2*d*x + 1/2*c)^2 - 480*b^2*tan(1/2*d*x + 1/
2*c)^2 - 3840*(d*x + c)*a*b + 2640*a*b*tan(1/2*d*x + 1/2*c) - 600*(a^2 - 6*
b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 3840*b^2/(tan(1/2*d*x + 1/2*c)^2 + 1)
 + (1470*a^2*tan(1/2*d*x + 1/2*c)^6 - 8820*b^2*tan(1/2*d*x + 1/2*c)^6 - 264
0*a*b*tan(1/2*d*x + 1/2*c)^5 - 225*a^2*tan(1/2*d*x + 1/2*c)^4 + 480*b^2*tan
(1/2*d*x + 1/2*c)^4 + 280*a*b*tan(1/2*d*x + 1/2*c)^3 + 45*a^2*tan(1/2*d*x +
 1/2*c)^2 - 30*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a*b*tan(1/2*d*x + 1/2*c) - 5
*a^2)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.1251 $\int \cot^6(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=158

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{8d}-\frac{ab\cot^5(c+dx)\csc(c+dx)}{3d}+\frac{5ab\cot^3(c+dx)\csc(c+dx)}{12d}-\frac{5ab\cot(}{}$$

```
[Out] -(b^2*x) + (5*a*b*ArcTanh[Cos[c + d*x]])/(8*d) - (b^2*Cot[c + d*x])/d + (b^
2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)
/(7*d) - (5*a*b*Cot[c + d*x]*Csc[c + d*x])/(8*d) + (5*a*b*Cot[c + d*x]^3*Cs
c[c + d*x])/(12*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x])/(3*d)
```

Rubi [A] time = 0.41435, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2911, 2611, 3770, 14, 203}

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{8d}-\frac{ab\cot^5(c+dx)\csc(c+dx)}{3d}+\frac{5ab\cot^3(c+dx)\csc(c+dx)}{12d}-\frac{5ab\cot(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -(b^2*x) + (5*a*b*ArcTanh[Cos[c + d*x]])/(8*d) - (b^2*Cot[c + d*x])/d + (b^
2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)
/(7*d) - (5*a*b*Cot[c + d*x]*Csc[c + d*x])/(8*d) + (5*a*b*Cot[c + d*x]^3*Cs
c[c + d*x])/(12*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x])/(3*d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc(c+dx)\,dx + \int \cot^6(c+dx)\csc^2(c+dx)\,($$

$$= -\frac{ab\cot^5(c+dx)\csc(c+dx)}{3d} - \frac{1}{3}(5ab)\int \cot^4(c+dx)\csc(c+dx)\,d$$

$$= \frac{5ab\cot^3(c+dx)\csc(c+dx)}{12d} - \frac{ab\cot^5(c+dx)\csc(c+dx)}{3d} + \frac{1}{4}(5ab$$

$$= -\frac{b^2\cot(c+dx)}{d} + \frac{b^2\cot^3(c+dx)}{3d} - \frac{b^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^7(c+}{7d}$$

$$= -b^2x + \frac{5ab\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^2\cot(c+dx)}{d} + \frac{b^2\cot^3(c+dx)}{3d}$$

Mathematica [A] time = 1.37271, size = 280, normalized size = 1.77

$$\frac{\csc^7(c+dx)\left(-84\left(15a^2-41b^2\right)\cos(3(c+dx))-28\left(15a^2+71b^2\right)\cos(5(c+dx))-60a^2\cos(7(c+dx))+980ab\sin(4(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-14700*b^2*(c + d*x)*Csc[c + d*x]^6 + 16800*a*b*(Log[Cos[(c + d*x)/2]] - L
og[Sin[(c + d*x)/2]]) - 350*Cot[c + d*x]*Csc[c + d*x]^6*(6*(a^2 + b^2) + 17
*a*b*Sin[c + d*x]) + Csc[c + d*x]^7*(-84*(15*a^2 - 41*b^2)*Cos[3*(c + d*x)]
 - 28*(15*a^2 + 71*b^2)*Cos[5*(c + d*x)] - 60*a^2*Cos[7*(c + d*x)] + 644*b^
2*Cos[7*(c + d*x)] + 8820*b^2*c*Sin[3*(c + d*x)] + 8820*b^2*d*x*Sin[3*(c +
d*x)] + 980*a*b*Sin[4*(c + d*x)] - 2940*b^2*c*Sin[5*(c + d*x)] - 2940*b^2*d
*x*Sin[5*(c + d*x)] - 1155*a*b*Sin[6*(c + d*x)] + 420*b^2*c*Sin[7*(c + d*x)
] + 420*b^2*d*x*Sin[7*(c + d*x)]))/(26880*d)
```

Maple [A] time = 0.108, size = 222, normalized size = 1.4

$$-\frac{a^2(\cos(dx+c))^7}{7d(\sin(dx+c))^7} - \frac{ab(\cos(dx+c))^7}{3d(\sin(dx+c))^6} + \frac{ab(\cos(dx+c))^7}{12d(\sin(dx+c))^4} - \frac{ab(\cos(dx+c))^7}{8d(\sin(dx+c))^2} - \frac{ab(\cos(dx+c))^5}{8d} - \frac{5ab(\cos(dx+}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/7/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/3/d*a*b/sin(d*x+c)^6*cos(d*x+c)^7+1/
12/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7-1/8/d*a*b/sin(d*x+c)^2*cos(d*x+c)^7-1/8*
a*b*cos(d*x+c)^5/d-5/24*a*b*cos(d*x+c)^3/d-5/8*a*b*cos(d*x+c)/d-5/8/d*a*b*l
n(csc(d*x+c)-cot(d*x+c))-1/5*b^2*cot(d*x+c)^5/d+1/3*b^2*cot(d*x+c)^3/d-b^2*
cot(d*x+c)/d-b^2*x-1/d*b^2*c
```

Maxima [A] time = 1.46273, size = 207, normalized size = 1.31

$$-\frac{112\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)b^2-35\,ab\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right.\right.\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1680*(112*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/ta
n(d*x + c)^5)*b^2 - 35*a*b*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*c
os(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 1
5*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 240*a^2/tan(d*x + c)^
7)/d
```

Fricas [B] time = 1.86217, size = 849, normalized size = 5.37

$$\frac{16\left(15\,a^2-161\,b^2\right)\cos\left(dx+c\right)^7+6496\,b^2\cos\left(dx+c\right)^5-5600\,b^2\cos\left(dx+c\right)^3+1680\,b^2\cos\left(dx+c\right)+525\left(ab\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/1680*(16*(15*a^2 - 161*b^2)*cos(d*x + c)^7 + 6496*b^2*cos(d*x + c)^5 - 56
00*b^2*cos(d*x + c)^3 + 1680*b^2*cos(d*x + c) + 525*(a*b*cos(d*x + c)^6 - 3
*a*b*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 525*(a*b*cos(d*x + c)^6 - 3*a*b*cos(d*x + c)^4 + 3*a*b*co
s(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*(24*b^2*
d*x*cos(d*x + c)^6 - 72*b^2*d*x*cos(d*x + c)^4 - 33*a*b*cos(d*x + c)^5 + 72
*b^2*d*x*cos(d*x + c)^2 + 40*a*b*cos(d*x + c)^3 - 24*b^2*d*x - 15*a*b*cos(d
*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*
x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24021, size = 481, normalized size = 3.04

$$\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+84\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-630\,ab\tan\left(\frac{1}{2}\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 + 70*a*b*tan(1/2*d*x + 1/2*c)^6 - 10
5*a^2*tan(1/2*d*x + 1/2*c)^5 + 84*b^2*tan(1/2*d*x + 1/2*c)^5 - 630*a*b*tan(
1/2*d*x + 1/2*c)^4 + 315*a^2*tan(1/2*d*x + 1/2*c)^3 - 980*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3150*a*b*tan(1/2*d*x + 1/2*c)^2 - 13440*(d*x + c)*b^2 - 8400*a*
b*log(abs(tan(1/2*d*x + 1/2*c))) - 525*a^2*tan(1/2*d*x + 1/2*c) + 9240*b^2*
tan(1/2*d*x + 1/2*c) + (21780*a*b*tan(1/2*d*x + 1/2*c)^7 + 525*a^2*tan(1/2*
d*x + 1/2*c)^6 - 9240*b^2*tan(1/2*d*x + 1/2*c)^6 - 3150*a*b*tan(1/2*d*x + 1
/2*c)^5 - 315*a^2*tan(1/2*d*x + 1/2*c)^4 + 980*b^2*tan(1/2*d*x + 1/2*c)^4 +
 630*a*b*tan(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c)^2 - 84*b^2*t
an(1/2*d*x + 1/2*c)^2 - 70*a*b*tan(1/2*d*x + 1/2*c) - 15*a^2)/tan(1/2*d*x +
 1/2*c)^7)/d
```

3.1252 $\int \cot^6(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=159

$$\frac{5\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d}+\frac{\left(17a^2-8b^2\right)\cot(c+dx)\csc^5(c+dx)}{48d}-\frac{\left(59a^2-104b^2\right)\cot(c+dx)\csc^3(c+dx)}{192d}$$

```
[Out] (5*(a^2 + 8*b^2)*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a*b*Cot[c + d*x]^7)/(7
*d) + ((5*a^2 - 88*b^2)*Cot[c + d*x]*Csc[c + d*x])/(128*d) - ((59*a^2 - 104
*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(192*d) + ((17*a^2 - 8*b^2)*Cot[c + d*x]
*Csc[c + d*x]^5)/(48*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^7)/(8*d)
```

Rubi [A] time = 0.319247, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2911, 2607, 30, 4366, 455, 1814, 1157, 385, 206}

$$\frac{5\left(a^2+8b^2\right)\tanh^{-1}(\cos(c+dx))}{128d}+\frac{\left(17a^2-8b^2\right)\cot(c+dx)\csc^5(c+dx)}{48d}-\frac{\left(59a^2-104b^2\right)\cot(c+dx)\csc^3(c+dx)}{192d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(a^2 + 8*b^2)*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a*b*Cot[c + d*x]^7)/(7
*d) + ((5*a^2 - 88*b^2)*Cot[c + d*x]*Csc[c + d*x])/(128*d) - ((59*a^2 - 104
*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(192*d) + ((17*a^2 - 8*b^2)*Cot[c + d*x]
*Csc[c + d*x]^5)/(48*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^7)/(8*d)
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^2(c+dx)\,dx + \int \cot^6(c+dx)\csc^3(c+$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6(a^2+b^2-b^2x^2)}{(1-x^2)^5}dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}\left(\int x^6\right.}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{a^2\cot(c+dx)\csc^7(c+dx)}{8d} + \frac{\text{Subst}\left(\int \frac{a^2+}{}\right.}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} + \frac{(17a^2-8b^2)\cot(c+dx)\csc^5(c+dx)}{48d} - \frac{a^2}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{(59a^2-104b^2)\cot(c+dx)\csc^3(c+dx)}{192d} +$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} + \frac{(5a^2-88b^2)\cot(c+dx)\csc(c+dx)}{128d} - \frac{(59}{}$$

$$= \frac{5(a^2+8b^2)\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2ab\cot^7(c+dx)}{7d} + \frac{(5a^2-8}{}$$

Mathematica [A] time = 0.797125, size = 282, normalized size = 1.77

$$-\frac{7(895a^2-904b^2)\cos(3(c+dx))\csc^8(c+dx)+7\cot(c+dx)\csc^7(c+dx)(1765a^2+1536ab\sin(c+dx)+680b^2)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(7*(895*a^2 - 904*b^2)*Cos[3*(c + d*x)]*Csc[c + d*x]^8 + 2779*a^2*Cos[5*(c
 + d*x)]*Csc[c + d*x]^8 + 3416*b^2*Cos[5*(c + d*x)]*Csc[c + d*x]^8 + 105*a^
2*Cos[7*(c + d*x)]*Csc[c + d*x]^8 - 1848*b^2*Cos[7*(c + d*x)]*Csc[c + d*x]^
8 - 6720*a^2*Log[Cos[(c + d*x)/2]] - 53760*b^2*Log[Cos[(c + d*x)/2]] + 6720
*a^2*Log[Sin[(c + d*x)/2]] + 53760*b^2*Log[Sin[(c + d*x)/2]] + 7*Cot[c + d*
x]*Csc[c + d*x]^7*(1765*a^2 + 680*b^2 + 1536*a*b*Sin[c + d*x]) + 5376*a*b*C
sc[c + d*x]^8*Sin[4*(c + d*x)] + 2304*a*b*Csc[c + d*x]^8*Sin[6*(c + d*x)] +
 384*a*b*Csc[c + d*x]^8*Sin[8*(c + d*x)])/(172032*d)
```

Maple [B] time = 0.099, size = 333, normalized size = 2.1

$$-\frac{a^2(\cos(dx+c))^7}{8d(\sin(dx+c))^8} - \frac{a^2(\cos(dx+c))^7}{48d(\sin(dx+c))^6} + \frac{a^2(\cos(dx+c))^7}{192d(\sin(dx+c))^4} - \frac{a^2(\cos(dx+c))^7}{128d(\sin(dx+c))^2} - \frac{a^2(\cos(dx+c))^5}{128d} - \frac{5a^2(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7-1/48/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1
/192/d*a^2/sin(d*x+c)^4*cos(d*x+c)^7-1/128/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-
1/128*a^2*cos(d*x+c)^5/d-5/384*a^2*cos(d*x+c)^3/d-5/128*a^2*cos(d*x+c)/d-5/
128/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-2/7/d*a*b/sin(d*x+c)^7*cos(d*x+c)^7-1/6
/d*b^2/sin(d*x+c)^6*cos(d*x+c)^7+1/24/d*b^2/sin(d*x+c)^4*cos(d*x+c)^7-1/16/
```

```
d*b^2/sin(d*x+c)^2*cos(d*x+c)^7-1/16*b^2*cos(d*x+c)^5/d-5/48*b^2*cos(d*x+c)
^3/d-5/16*b^2*cos(d*x+c)/d-5/16/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.00587, size = 297, normalized size = 1.87

$$-\frac{7\,a^2\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-56\,b^2\left(\cdots\right.}{5376\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/5376*(7*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^
3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1)) - 56*b^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x
+ c) + 1) - 15*log(cos(d*x + c) - 1)) + 1536*a*b/tan(d*x + c)^7)/d
```

Fricas [B] time = 1.80941, size = 853, normalized size = 5.36

$$-\frac{1536\,ab\cos\left(dx+c\right)^7\sin\left(dx+c\right)+42\left(5\,a^2-88\,b^2\right)\cos\left(dx+c\right)^7+1022\left(a^2+8\,b^2\right)\cos\left(dx+c\right)^5-770\left(a^2+8\,b^2\right)\cos\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/5376*(1536*a*b*cos(d*x + c)^7*sin(d*x + c) + 42*(5*a^2 - 88*b^2)*cos(d*x
 + c)^7 + 1022*(a^2 + 8*b^2)*cos(d*x + c)^5 - 770*(a^2 + 8*b^2)*cos(d*x + c
)^3 + 210*(a^2 + 8*b^2)*cos(d*x + c) - 105*((a^2 + 8*b^2)*cos(d*x + c)^8 -
4*(a^2 + 8*b^2)*cos(d*x + c)^6 + 6*(a^2 + 8*b^2)*cos(d*x + c)^4 - 4*(a^2 +
8*b^2)*cos(d*x + c)^2 + a^2 + 8*b^2)*log(1/2*cos(d*x + c) + 1/2) + 105*((a^
2 + 8*b^2)*cos(d*x + c)^8 - 4*(a^2 + 8*b^2)*cos(d*x + c)^6 + 6*(a^2 + 8*b^2
)*cos(d*x + c)^4 - 4*(a^2 + 8*b^2)*cos(d*x + c)^2 + a^2 + 8*b^2)*log(-1/2*c
os(d*x + c) + 1/2))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x +
c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31952, size = 543, normalized size = 3.42

$$21\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+96\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-112\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+112\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-672\,ab\tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/43008*(21*a^2*tan(1/2*d*x + 1/2*c)^8 + 96*a*b*tan(1/2*d*x + 1/2*c)^7 - 11
2*a^2*tan(1/2*d*x + 1/2*c)^6 + 112*b^2*tan(1/2*d*x + 1/2*c)^6 - 672*a*b*tan
(1/2*d*x + 1/2*c)^5 + 168*a^2*tan(1/2*d*x + 1/2*c)^4 - 1008*b^2*tan(1/2*d*x
 + 1/2*c)^4 + 2016*a*b*tan(1/2*d*x + 1/2*c)^3 + 336*a^2*tan(1/2*d*x + 1/2*c
)^2 + 5040*b^2*tan(1/2*d*x + 1/2*c)^2 - 3360*a*b*tan(1/2*d*x + 1/2*c) - 168
0*(a^2 + 8*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (4566*a^2*tan(1/2*d*x + 1/
2*c)^8 + 36528*b^2*tan(1/2*d*x + 1/2*c)^8 + 3360*a*b*tan(1/2*d*x + 1/2*c)^7
 - 336*a^2*tan(1/2*d*x + 1/2*c)^6 - 5040*b^2*tan(1/2*d*x + 1/2*c)^6 - 2016*
a*b*tan(1/2*d*x + 1/2*c)^5 - 168*a^2*tan(1/2*d*x + 1/2*c)^4 + 1008*b^2*tan(
1/2*d*x + 1/2*c)^4 + 672*a*b*tan(1/2*d*x + 1/2*c)^3 + 112*a^2*tan(1/2*d*x +
 1/2*c)^2 - 112*b^2*tan(1/2*d*x + 1/2*c)^2 - 96*a*b*tan(1/2*d*x + 1/2*c) -
21*a^2)/tan(1/2*d*x + 1/2*c)^8)/d
```

3.1253 $\int \cot^6(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=151

$$-\frac{(a^2+b^2)\cot^7(c+dx)}{7d}-\frac{a^2\cot^9(c+dx)}{9d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{64d}-\frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5ab\cot^3(c+dx)}{24d}$$

```
[Out] (5*a*b*ArcTanh[Cos[c + d*x]])/(64*d) - ((a^2 + b^2)*Cot[c + d*x]^7)/(7*d) -
 (a^2*Cot[c + d*x]^9)/(9*d) + (5*a*b*Cot[c + d*x]*Csc[c + d*x])/(64*d) - (5
*a*b*Cot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (5*a*b*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(24*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.405466, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2911, 2611, 3768, 3770, 14}

$$-\frac{(a^2+b^2)\cot^7(c+dx)}{7d}-\frac{a^2\cot^9(c+dx)}{9d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{64d}-\frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5ab\cot^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (5*a*b*ArcTanh[Cos[c + d*x]])/(64*d) - ((a^2 + b^2)*Cot[c + d*x]^7)/(7*d) -
 (a^2*Cot[c + d*x]^9)/(9*d) + (5*a*b*Cot[c + d*x]*Csc[c + d*x])/(64*d) - (5
*a*b*Cot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (5*a*b*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(24*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x]^3)/(4*d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^3(c+dx)\,dx + \int \cot^6(c+dx)\csc^4(c+$$

$$= -\frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}(5ab)\int \cot^4(c+dx)\csc^3(c+$$

$$= \frac{5ab\cot^3(c+dx)\csc^3(c+dx)}{24d} - \frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d} +$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} - \frac{5ab\cot(c+dx)\csc}{32d}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} + \frac{5ab\cot(c+dx)\csc}{64d}$$

$$= \frac{5ab\tanh^{-1}(\cos(c+dx))}{64d} - \frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+}{9d}$$

Mathematica [A] time = 1.1199, size = 204, normalized size = 1.35

$$-\frac{\csc^9(c+dx)\left(4032\left(8a^2+b^2\right)\cos(c+dx)+18816a^2\cos(3(c+dx))+5760a^2\cos(5(c+dx))+576a^2\cos(7(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(-40320*a*b*Log[Cos[(c + d*x)/2]] + 40320*a*b*Log[Sin[(c + d*x)/2]] + Csc[
c + d*x]^9*(4032*(8*a^2 + b^2)*Cos[c + d*x] + 18816*a^2*Cos[3*(c + d*x)] +
5760*a^2*Cos[5*(c + d*x)] - 2304*b^2*Cos[5*(c + d*x)] + 576*a^2*Cos[7*(c +
d*x)] - 1440*b^2*Cos[7*(c + d*x)] - 64*a^2*Cos[9*(c + d*x)] - 288*b^2*Cos[9
*(c + d*x)] + 18270*a*b*Sin[2*(c + d*x)] + 10458*a*b*Sin[4*(c + d*x)] + 802
2*a*b*Sin[6*(c + d*x)] + 315*a*b*Sin[8*(c + d*x)]))/(516096*d)
```

Maple [A] time = 0.096, size = 232, normalized size = 1.5

$$-\frac{a^2(\cos(dx+c))^7}{9d(\sin(dx+c))^9} - \frac{2a^2(\cos(dx+c))^7}{63d(\sin(dx+c))^7} - \frac{ab(\cos(dx+c))^7}{4d(\sin(dx+c))^8} - \frac{ab(\cos(dx+c))^7}{24d(\sin(dx+c))^6} + \frac{ab(\cos(dx+c))^7}{96d(\sin(dx+c))^4} - \frac{ab(\cos}{64d(\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/9/d*a^2/sin(d*x+c)^9*cos(d*x+c)^7-2/63/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1
/4/d*a*b/sin(d*x+c)^8*cos(d*x+c)^7-1/24/d*a*b/sin(d*x+c)^6*cos(d*x+c)^7+1/9
6/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7-1/64/d*a*b/sin(d*x+c)^2*cos(d*x+c)^7-1/64
*a*b*cos(d*x+c)^5/d-5/192*a*b*cos(d*x+c)^3/d-5/64*a*b*cos(d*x+c)/d-5/64/d*a
```

```
*b*ln(csc(d*x+c)-cot(d*x+c))-1/7/d*b^2/sin(d*x+c)^7*cos(d*x+c)^7
```

Maxima [A] time = 0.990969, size = 208, normalized size = 1.38

$$-\frac{21\,ab\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{1152}{\tan(dx}}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/8064*(21*a*b*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)
^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
 1)) + 1152*b^2/tan(d*x + c)^7 + 128*(9*tan(d*x + c)^2 + 7)*a^2/tan(d*x + c
)^9)/d
```

Fricas [B] time = 1.89782, size = 782, normalized size = 5.18

$$\frac{128\left(2\,a^2+9\,b^2\right)\cos\left(dx+c\right)^9-1152\left(a^2+b^2\right)\cos\left(dx+c\right)^7+315\left(ab\cos\left(dx+c\right)^8-4\,ab\cos\left(dx+c\right)^6+6\,ab\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/8064*(128*(2*a^2 + 9*b^2)*cos(d*x + c)^9 - 1152*(a^2 + b^2)*cos(d*x + c)^
7 + 315*(a*b*cos(d*x + c)^8 - 4*a*b*cos(d*x + c)^6 + 6*a*b*cos(d*x + c)^4 -
 4*a*b*cos(d*x + c)^2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 315
*(a*b*cos(d*x + c)^8 - 4*a*b*cos(d*x + c)^6 + 6*a*b*cos(d*x + c)^4 - 4*a*b*
cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 42*(15*a*
b*cos(d*x + c)^7 + 73*a*b*cos(d*x + c)^5 - 55*a*b*cos(d*x + c)^3 + 15*a*b*c
os(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*co
s(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20013, size = 551, normalized size = 3.65

$$14\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+63\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-54\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+72\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-336\,ab\tan\left(\frac{1}{2}\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/64512*(14*a^2*tan(1/2*d*x + 1/2*c)^9 + 63*a*b*tan(1/2*d*x + 1/2*c)^8 - 54
*a^2*tan(1/2*d*x + 1/2*c)^7 + 72*b^2*tan(1/2*d*x + 1/2*c)^7 - 336*a*b*tan(1
/2*d*x + 1/2*c)^6 - 504*b^2*tan(1/2*d*x + 1/2*c)^5 + 504*a*b*tan(1/2*d*x +
1/2*c)^4 + 336*a^2*tan(1/2*d*x + 1/2*c)^3 + 1512*b^2*tan(1/2*d*x + 1/2*c)^3
 + 1008*a*b*tan(1/2*d*x + 1/2*c)^2 - 5040*a*b*log(abs(tan(1/2*d*x + 1/2*c))
) - 756*a^2*tan(1/2*d*x + 1/2*c) - 2520*b^2*tan(1/2*d*x + 1/2*c) + (14258*a
*b*tan(1/2*d*x + 1/2*c)^9 + 756*a^2*tan(1/2*d*x + 1/2*c)^8 + 2520*b^2*tan(1
/2*d*x + 1/2*c)^8 - 1008*a*b*tan(1/2*d*x + 1/2*c)^7 - 336*a^2*tan(1/2*d*x +
 1/2*c)^6 - 1512*b^2*tan(1/2*d*x + 1/2*c)^6 - 504*a*b*tan(1/2*d*x + 1/2*c)^
5 + 504*b^2*tan(1/2*d*x + 1/2*c)^4 + 336*a*b*tan(1/2*d*x + 1/2*c)^3 + 54*a^
2*tan(1/2*d*x + 1/2*c)^2 - 72*b^2*tan(1/2*d*x + 1/2*c)^2 - 63*a*b*tan(1/2*d
*x + 1/2*c) - 14*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.1254 $\int \cot^6(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=210

$$\frac{(3a^2+10b^2)\tanh^{-1}(\cos(c+dx))}{256d}+\frac{(21a^2-10b^2)\cot(c+dx)\csc^7(c+dx)}{80d}-\frac{(93a^2-170b^2)\cot(c+dx)\csc^5(c+dx)}{480d}+$$

```
[Out] ((3*a^2 + 10*b^2)*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a*b*Cot[c + d*x]^7)/(
7*d) - (2*a*b*Cot[c + d*x]^9)/(9*d) + ((3*a^2 + 10*b^2)*Cot[c + d*x]*Csc[c
+ d*x])/(256*d) + ((3*a^2 - 118*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(384*d) -
 ((93*a^2 - 170*b^2)*Cot[c + d*x]*Csc[c + d*x]^5)/(480*d) + ((21*a^2 - 10*b
^2)*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^9)
/(10*d)
```

Rubi [A] time = 0.344888, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2911, 2607, 14, 4366, 455, 1814, 1157, 385, 199, 206}

$$\frac{(3a^2+10b^2)\tanh^{-1}(\cos(c+dx))}{256d}+\frac{(21a^2-10b^2)\cot(c+dx)\csc^7(c+dx)}{80d}-\frac{(93a^2-170b^2)\cot(c+dx)\csc^5(c+dx)}{480d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] ((3*a^2 + 10*b^2)*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a*b*Cot[c + d*x]^7)/(
7*d) - (2*a*b*Cot[c + d*x]^9)/(9*d) + ((3*a^2 + 10*b^2)*Cot[c + d*x]*Csc[c
+ d*x])/(256*d) + ((3*a^2 - 118*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(384*d) -
 ((93*a^2 - 170*b^2)*Cot[c + d*x]*Csc[c + d*x]^5)/(480*d) + ((21*a^2 - 10*b
^2)*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^9)
/(10*d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
```

```
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^4(c+dx)\,dx + \int \cot^6(c+dx)\csc^5(c+dx)$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6(a^2+b^2-b^2x^2)}{(1-x^2)^6}\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}\left(\int x^6\,(1\right.}{}$$

$$= -\frac{a^2\cot(c+dx)\csc^9(c+dx)}{10d} + \frac{\text{Subst}\left(\int \frac{a^2+10a^2x^2+10a^2x^4-10b^2x^6}{(1-x^2)^5}\,dx,\right.}{10d}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} + \frac{(21a^2-10b^2)\cot(c+dx)}{80d}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} - \frac{(93a^2-170b^2)\cot(c+dx)}{480d}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} + \frac{(3a^2-118b^2)\cot(c+dx)}{384d}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} + \frac{(3a^2+10b^2)\cot(c+dx)\,c}{256d}$$

$$= \frac{(3a^2+10b^2)\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c}{9d}$$

Mathematica [A] time = 1.43241, size = 244, normalized size = 1.16

$$-\frac{80640\left(3a^2+10b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 80640\left(3a^2+10b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \csc^{10}(c+dx)\left(630\left(1879a^2+29\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(-80640*(3*a^2 + 10*b^2)*Log[Cos[(c + d*x)/2]] + 80640*(3*a^2 + 10*b^2)*Lo
g[Sin[(c + d*x)/2]] + Csc[c + d*x]^10*(630*(1879*a^2 + 290*b^2)*Cos[c + d*x
] + 1260*(519*a^2 - 62*b^2)*Cos[3*(c + d*x)] + 218484*a^2*Cos[5*(c + d*x)]
- 24360*b^2*Cos[5*(c + d*x)] + 9135*a^2*Cos[7*(c + d*x)] - 77070*b^2*Cos[7*
(c + d*x)] - 945*a^2*Cos[9*(c + d*x)] - 3150*b^2*Cos[9*(c + d*x)] + 537600*
a*b*Sin[2*(c + d*x)] + 522240*a*b*Sin[4*(c + d*x)] + 207360*a*b*Sin[6*(c +
d*x)] + 25600*a*b*Sin[8*(c + d*x)] - 2560*a*b*Sin[10*(c + d*x)]))/(20643840
*d)
```

Maple [B] time = 0.101, size = 404, normalized size = 1.9

$$-\frac{a^2(\cos(dx+c))^7}{10d(\sin(dx+c))^{10}} - \frac{3a^2(\cos(dx+c))^7}{80d(\sin(dx+c))^8} - \frac{a^2(\cos(dx+c))^7}{160d(\sin(dx+c))^6} + \frac{a^2(\cos(dx+c))^7}{640d(\sin(dx+c))^4} - \frac{3a^2(\cos(dx+c))^7}{1280d(\sin(dx+c))^2} - \frac{3a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/10/d*a^2/sin(d*x+c)^10*cos(d*x+c)^7-3/80/d*a^2/sin(d*x+c)^8*cos(d*x+c)^7
-1/160/d*a^2/sin(d*x+c)^6*cos(d*x+c)^7+1/640/d*a^2/sin(d*x+c)^4*cos(d*x+c)^
```

```
7-3/1280/d*a^2/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*a^2*cos(d*x+c)^5/d-1/256*a^
2*cos(d*x+c)^3/d-3/256*a^2*cos(d*x+c)/d-3/256/d*a^2*ln(csc(d*x+c)-cot(d*x+c
))-2/9/d*a*b/sin(d*x+c)^9*cos(d*x+c)^7-4/63/d*a*b/sin(d*x+c)^7*cos(d*x+c)^7
-1/8/d*b^2/sin(d*x+c)^8*cos(d*x+c)^7-1/48/d*b^2/sin(d*x+c)^6*cos(d*x+c)^7+1
/192/d*b^2/sin(d*x+c)^4*cos(d*x+c)^7-1/128/d*b^2/sin(d*x+c)^2*cos(d*x+c)^7-
1/128*b^2*cos(d*x+c)^5/d-5/384*b^2*cos(d*x+c)^3/d-5/128*b^2*cos(d*x+c)/d-5/
128/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.992257, size = 367, normalized size = 1.75

$$-\frac{63\,a^2\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7-128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log\left(\cos(dx+c)+1\right)+15\log\left(\cos(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/161280*(63*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
 c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
 c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 210*b^2*(2*(15*cos(d*x
+ c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x
+ c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 5120*(9*tan(d*x + c)^2 +
 7)*a*b/tan(d*x + c)^9)/d
```

Fricas [B] time = 1.96849, size = 1138, normalized size = 5.42

$$-\frac{630\left(3\,a^2+10\,b^2\right)\cos\left(dx+c\right)^9-420\left(21\,a^2-58\,b^2\right)\cos\left(dx+c\right)^7-5376\left(3\,a^2+10\,b^2\right)\cos\left(dx+c\right)^5+2940\left(3\,a^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/161280*(630*(3*a^2 + 10*b^2)*cos(d*x + c)^9 - 420*(21*a^2 - 58*b^2)*cos(
d*x + c)^7 - 5376*(3*a^2 + 10*b^2)*cos(d*x + c)^5 + 2940*(3*a^2 + 10*b^2)*c
os(d*x + c)^3 - 630*(3*a^2 + 10*b^2)*cos(d*x + c) - 315*((3*a^2 + 10*b^2)*c
os(d*x + c)^10 - 5*(3*a^2 + 10*b^2)*cos(d*x + c)^8 + 10*(3*a^2 + 10*b^2)*co
s(d*x + c)^6 - 10*(3*a^2 + 10*b^2)*cos(d*x + c)^4 + 5*(3*a^2 + 10*b^2)*cos(
d*x + c)^2 - 3*a^2 - 10*b^2)*log(1/2*cos(d*x + c) + 1/2) + 315*((3*a^2 + 10
*b^2)*cos(d*x + c)^10 - 5*(3*a^2 + 10*b^2)*cos(d*x + c)^8 + 10*(3*a^2 + 10*
b^2)*cos(d*x + c)^6 - 10*(3*a^2 + 10*b^2)*cos(d*x + c)^4 + 5*(3*a^2 + 10*b^
2)*cos(d*x + c)^2 - 3*a^2 - 10*b^2)*log(-1/2*cos(d*x + c) + 1/2) + 5120*(2*
a*b*cos(d*x + c)^9 - 9*a*b*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^10
 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos
(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30524, size = 632, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(126*a^2*tan(1/2*d*x + 1/2*c)^10 + 560*a*b*tan(1/2*d*x + 1/2*c)^9
 - 315*a^2*tan(1/2*d*x + 1/2*c)^8 + 630*b^2*tan(1/2*d*x + 1/2*c)^8 - 2160*a
*b*tan(1/2*d*x + 1/2*c)^7 - 630*a^2*tan(1/2*d*x + 1/2*c)^6 - 3360*b^2*tan(1
/2*d*x + 1/2*c)^6 + 2520*a^2*tan(1/2*d*x + 1/2*c)^4 + 5040*b^2*tan(1/2*d*x
+ 1/2*c)^4 + 13440*a*b*tan(1/2*d*x + 1/2*c)^3 + 1260*a^2*tan(1/2*d*x + 1/2*
c)^2 + 10080*b^2*tan(1/2*d*x + 1/2*c)^2 - 30240*a*b*tan(1/2*d*x + 1/2*c) -
5040*(3*a^2 + 10*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (44286*a^2*tan(1/2*d
*x + 1/2*c)^10 + 147620*b^2*tan(1/2*d*x + 1/2*c)^10 + 30240*a*b*tan(1/2*d*x
 + 1/2*c)^9 - 1260*a^2*tan(1/2*d*x + 1/2*c)^8 - 10080*b^2*tan(1/2*d*x + 1/2
*c)^8 - 13440*a*b*tan(1/2*d*x + 1/2*c)^7 - 2520*a^2*tan(1/2*d*x + 1/2*c)^6
- 5040*b^2*tan(1/2*d*x + 1/2*c)^6 + 630*a^2*tan(1/2*d*x + 1/2*c)^4 + 3360*b
^2*tan(1/2*d*x + 1/2*c)^4 + 2160*a*b*tan(1/2*d*x + 1/2*c)^3 + 315*a^2*tan(1
/2*d*x + 1/2*c)^2 - 630*b^2*tan(1/2*d*x + 1/2*c)^2 - 560*a*b*tan(1/2*d*x +
1/2*c) - 126*a^2)/tan(1/2*d*x + 1/2*c)^10)/d
```

3.1255 $\int \cot^6(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=198

$$-\frac{(2a^2+b^2)\cot^9(c+dx)}{9d}-\frac{(a^2+b^2)\cot^7(c+dx)}{7d}-\frac{a^2\cot^{11}(c+dx)}{11d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{128d}-\frac{ab\cot^5(c+dx)\operatorname{cs}}{5d}$$

```
[Out] (3*a*b*ArcTanh[Cos[c + d*x]])/(128*d) - ((a^2 + b^2)*Cot[c + d*x]^7)/(7*d)
- ((2*a^2 + b^2)*Cot[c + d*x]^9)/(9*d) - (a^2*Cot[c + d*x]^11)/(11*d) + (3*
a*b*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (a*b*Cot[c + d*x]*Csc[c + d*x]^3)/
(64*d) - (a*b*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) + (a*b*Cot[c + d*x]^3*Csc
[c + d*x]^5)/(8*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.456852, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2911, 2611, 3768, 3770, 448}

$$-\frac{(2a^2+b^2)\cot^9(c+dx)}{9d}-\frac{(a^2+b^2)\cot^7(c+dx)}{7d}-\frac{a^2\cot^{11}(c+dx)}{11d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{128d}-\frac{ab\cot^5(c+dx)\operatorname{cs}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```



```
[Out] (3*a*b*ArcTanh[Cos[c + d*x]])/(128*d) - ((a^2 + b^2)*Cot[c + d*x]^7)/(7*d)
- ((2*a^2 + b^2)*Cot[c + d*x]^9)/(9*d) - (a^2*Cot[c + d*x]^11)/(11*d) + (3*
a*b*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (a*b*Cot[c + d*x]*Csc[c + d*x]^3)/
(64*d) - (a*b*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) + (a*b*Cot[c + d*x]^3*Csc
[c + d*x]^5)/(8*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x]^5)/(5*d)
```


Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^5(c+dx)\,dx + \int \cot^6(c+dx)\csc^6(c+dx)$$

$$= -\frac{ab\cot^5(c+dx)\csc^5(c+dx)}{5d} - (ab)\int \cot^4(c+dx)\csc^5(c+dx)\,dx$$

$$= \frac{ab\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{ab\cot^5(c+dx)\csc^5(c+dx)}{5d} + \frac{1}{8}(3a$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot^9(c+dx)}{9d} - \frac{a^2\cot^{11}(c+d}{11d}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot^9(c+dx)}{9d} - \frac{a^2\cot^{11}(c+d}{11d}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot^9(c+dx)}{9d} - \frac{a^2\cot^{11}(c+d}{11d}$$

$$= \frac{3ab\tanh^{-1}(\cos(c+dx))}{128d} - \frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot}{9d}$$

Mathematica [A] time = 1.71349, size = 250, normalized size = 1.26

$$-\frac{\csc^{11}(c+dx)\left(1478400\left(8a^2+b^2\right)\cos(c+dx)+42240\left(160a^2-b^2\right)\cos(3(c+dx))+1943040a^2\cos(5(c+dx))+1408}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(-5322240*a*b*Log[Cos[(c + d*x)/2]] + 5322240*a*b*Log[Sin[(c + d*x)/2]] +
Csc[c + d*x]^11*(1478400*(8*a^2 + b^2)*Cos[c + d*x] + 42240*(160*a^2 - b^2)
*Cos[3*(c + d*x)] + 1943040*a^2*Cos[5*(c + d*x)] - 865920*b^2*Cos[5*(c + d*
x)] + 140800*a^2*Cos[7*(c + d*x)] - 499840*b^2*Cos[7*(c + d*x)] - 28160*a^2
*Cos[9*(c + d*x)] - 77440*b^2*Cos[9*(c + d*x)] + 2560*a^2*Cos[11*(c + d*x)]
 + 7040*b^2*Cos[11*(c + d*x)] + 5828130*a*b*Sin[2*(c + d*x)] + 4790016*a*b*
Sin[4*(c + d*x)] + 2302839*a*b*Sin[6*(c + d*x)] + 110880*a*b*Sin[8*(c + d*x
)] - 10395*a*b*Sin[10*(c + d*x)]))/(227082240*d)
```

Maple [A] time = 0.101, size = 303, normalized size = 1.5

$$-\frac{a^2(\cos(dx+c))^7}{11d(\sin(dx+c))^{11}} - \frac{4a^2(\cos(dx+c))^7}{99d(\sin(dx+c))^9} - \frac{8a^2(\cos(dx+c))^7}{693d(\sin(dx+c))^7} - \frac{ab(\cos(dx+c))^7}{5d(\sin(dx+c))^{10}} - \frac{3ab(\cos(dx+c))^7}{40d(\sin(dx+c))^8} - \frac{ab(\cos}{80d(\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/11/d*a^2/sin(d*x+c)^11*cos(d*x+c)^7-4/99/d*a^2/sin(d*x+c)^9*cos(d*x+c)^7
-8/693/d*a^2/sin(d*x+c)^7*cos(d*x+c)^7-1/5/d*a*b/sin(d*x+c)^10*cos(d*x+c)^7
-3/40/d*a*b/sin(d*x+c)^8*cos(d*x+c)^7-1/80/d*a*b/sin(d*x+c)^6*cos(d*x+c)^7+
1/320/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7-3/640/d*a*b/sin(d*x+c)^2*cos(d*x+c)^7
-3/640*a*b*cos(d*x+c)^5/d-1/128*a*b*cos(d*x+c)^3/d-3/128*a*b*cos(d*x+c)/d-3
/128/d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/9/d*b^2/sin(d*x+c)^9*cos(d*x+c)^7-2/
63/d*b^2/sin(d*x+c)^7*cos(d*x+c)^7
```

Maxima [A] time = 1.00085, size = 265, normalized size = 1.34

$$-\frac{693\,ab\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx\right.\right.\right.}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/887040*(693*a*b*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x
+ c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x
+ c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15
*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 14080*(9*tan(d*x + c)^
2 + 7)*b^2/tan(d*x + c)^9 + 1280*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2 +
63)*a^2/tan(d*x + c)^11)/d
```

Fricas [B] time = 2.02084, size = 990, normalized size = 5.

$$\frac{2560\left(4\,a^2+11\,b^2\right)\cos\left(dx+c\right)^{11}-14080\left(4\,a^2+11\,b^2\right)\cos\left(dx+c\right)^9+126720\left(a^2+b^2\right)\cos\left(dx+c\right)^7+10395\left(ab\,\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/887040*(2560*(4*a^2 + 11*b^2)*cos(d*x + c)^11 - 14080*(4*a^2 + 11*b^2)*co
s(d*x + c)^9 + 126720*(a^2 + b^2)*cos(d*x + c)^7 + 10395*(a*b*cos(d*x + c)^
10 - 5*a*b*cos(d*x + c)^8 + 10*a*b*cos(d*x + c)^6 - 10*a*b*cos(d*x + c)^4 +
 5*a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 103
95*(a*b*cos(d*x + c)^10 - 5*a*b*cos(d*x + c)^8 + 10*a*b*cos(d*x + c)^6 - 10
*a*b*cos(d*x + c)^4 + 5*a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) - 1386*(15*a*b*cos(d*x + c)^9 - 70*a*b*cos(d*x + c)^7 - 12
8*a*b*cos(d*x + c)^5 + 70*a*b*cos(d*x + c)^3 - 15*a*b*cos(d*x + c))*sin(d*x
 + c))/((d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*
d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28743, size = 678, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/7096320*(315*a^2*tan(1/2*d*x + 1/2*c)^11 + 1386*a*b*tan(1/2*d*x + 1/2*c)^
10 - 385*a^2*tan(1/2*d*x + 1/2*c)^9 + 1540*b^2*tan(1/2*d*x + 1/2*c)^9 - 346
5*a*b*tan(1/2*d*x + 1/2*c)^8 - 2475*a^2*tan(1/2*d*x + 1/2*c)^7 - 5940*b^2*t
an(1/2*d*x + 1/2*c)^7 - 6930*a*b*tan(1/2*d*x + 1/2*c)^6 + 3465*a^2*tan(1/2*
d*x + 1/2*c)^5 + 27720*a*b*tan(1/2*d*x + 1/2*c)^4 + 11550*a^2*tan(1/2*d*x +
 1/2*c)^3 + 36960*b^2*tan(1/2*d*x + 1/2*c)^3 + 13860*a*b*tan(1/2*d*x + 1/2*
c)^2 - 166320*a*b*log(abs(tan(1/2*d*x + 1/2*c))) - 34650*a^2*tan(1/2*d*x +
1/2*c) - 83160*b^2*tan(1/2*d*x + 1/2*c) + (502266*a*b*tan(1/2*d*x + 1/2*c)^
11 + 34650*a^2*tan(1/2*d*x + 1/2*c)^10 + 83160*b^2*tan(1/2*d*x + 1/2*c)^10
- 13860*a*b*tan(1/2*d*x + 1/2*c)^9 - 11550*a^2*tan(1/2*d*x + 1/2*c)^8 - 369
60*b^2*tan(1/2*d*x + 1/2*c)^8 - 27720*a*b*tan(1/2*d*x + 1/2*c)^7 - 3465*a^2
*tan(1/2*d*x + 1/2*c)^6 + 6930*a*b*tan(1/2*d*x + 1/2*c)^5 + 2475*a^2*tan(1/
2*d*x + 1/2*c)^4 + 5940*b^2*tan(1/2*d*x + 1/2*c)^4 + 3465*a*b*tan(1/2*d*x +
 1/2*c)^3 + 385*a^2*tan(1/2*d*x + 1/2*c)^2 - 1540*b^2*tan(1/2*d*x + 1/2*c)^
2 - 1386*a*b*tan(1/2*d*x + 1/2*c) - 315*a^2)/tan(1/2*d*x + 1/2*c)^11)/d
```

3.1256 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=525

$$\frac{\left(-1435a^4b^2+588a^2b^4+840a^6-15b^6\right)\cos(c+dx)}{105b^8d}-\frac{2a^2\left(8a^2-3b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{(-30a^2b^2}{}$$

```
[Out] (a*(64*a^6 - 120*a^4*b^2 + 60*a^2*b^4 - 5*b^6)*x)/(8*b^9) - (2*a^2*(8*a^2 -
 3*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(b^9*d) + ((840*a^6 - 1435*a^4*b^2 + 588*a^2*b^4 - 15*b^6)*Cos[c + d*x])/(
105*b^8*d) - (a*(32*a^4 - 52*a^2*b^2 + 19*b^4)*Cos[c + d*x]*Sin[c + d*x])/(
8*b^7*d) + ((280*a^4 - 441*a^2*b^2 + 150*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/
(105*b^6*d) - ((24*a^4 - 37*a^2*b^2 + 12*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/
(12*a*b^5*d) + ((224*a^4 - 340*a^2*b^2 + 105*b^4)*Cos[c + d*x]*Sin[c + d*x]
^4)/(140*a^2*b^4*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d*(a + b*Sin[c + d
*x])) - (3*b*Cos[c + d*x]*Sin[c + d*x]^5)/(20*a^2*d*(a + b*Sin[c + d*x])) -
 ((20*a^4 - 30*a^2*b^2 + 9*b^4)*Cos[c + d*x]*Sin[c + d*x]^5)/(15*a^2*b^3*d*
(a + b*Sin[c + d*x])) - (4*a*Cos[c + d*x]*Sin[c + d*x]^6)/(21*b^2*d*(a + b*
Sin[c + d*x])) + (Cos[c + d*x]*Sin[c + d*x]^7)/(7*b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.9091, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2896, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{\left(-1435a^4b^2+588a^2b^4+840a^6-15b^6\right)\cos(c+dx)}{105b^8d}-\frac{2a^2\left(8a^2-3b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{(-30a^2b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (a*(64*a^6 - 120*a^4*b^2 + 60*a^2*b^4 - 5*b^6)*x)/(8*b^9) - (2*a^2*(8*a^2 -
 3*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(b^9*d) + ((840*a^6 - 1435*a^4*b^2 + 588*a^2*b^4 - 15*b^6)*Cos[c + d*x])/(
105*b^8*d) - (a*(32*a^4 - 52*a^2*b^2 + 19*b^4)*Cos[c + d*x]*Sin[c + d*x])/(
8*b^7*d) + ((280*a^4 - 441*a^2*b^2 + 150*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/
(105*b^6*d) - ((24*a^4 - 37*a^2*b^2 + 12*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/
(12*a*b^5*d) + ((224*a^4 - 340*a^2*b^2 + 105*b^4)*Cos[c + d*x]*Sin[c + d*x]
^4)/(140*a^2*b^4*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d*(a + b*Sin[c + d
*x])) - (3*b*Cos[c + d*x]*Sin[c + d*x]^5)/(20*a^2*d*(a + b*Sin[c + d*x])) -
 ((20*a^4 - 30*a^2*b^2 + 9*b^4)*Cos[c + d*x]*Sin[c + d*x]^5)/(15*a^2*b^3*d*
(a + b*Sin[c + d*x])) - (4*a*Cos[c + d*x]*Sin[c + d*x]^6)/(21*b^2*d*(a + b*
Sin[c + d*x])) + (Cos[c + d*x]*Sin[c + d*x]^7)/(7*b*d*(a + b*Sin[c + d*x]))
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
```

```
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))} - \frac{3b\cos(c+dx)\sin^5(c+dx)}{20a^2d(a+b\sin(c+dx))} - \frac{4a\cos(c+dx)\sin^6(c+dx}{21b^2d(a+b\sin(c+dx))}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))} - \frac{3b\cos(c+dx)\sin^5(c+dx)}{20a^2d(a+b\sin(c+dx))} - \frac{\left(20a^4-30a^2b^2+9b^4\right)\cos}{15a^2b^3d(a+b\,\mathrm{s}}$$

$$= \frac{\left(224a^4-340a^2b^2+105b^4\right)\cos(c+dx)\sin^4(c+dx)}{140a^2b^4d} + \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))} - 3$$

$$= -\frac{\left(24a^4-37a^2b^2+12b^4\right)\cos(c+dx)\sin^3(c+dx)}{12ab^5d} + \frac{\left(224a^4-340a^2b^2+105b^4\right)\mathrm{co}}{140a^2b^4d}$$

$$= \frac{\left(280a^4-441a^2b^2+150b^4\right)\cos(c+dx)\sin^2(c+dx)}{105b^6d} - \frac{\left(24a^4-37a^2b^2+12b^4\right)\cos(}{12ab^5d}$$

$$= -\frac{a\left(32a^4-52a^2b^2+19b^4\right)\cos(c+dx)\sin(c+dx)}{8b^7d} + \frac{\left(280a^4-441a^2b^2+150b^4\right)\mathrm{co}}{105b^6d}$$

$$= \frac{\left(840a^6-1435a^4b^2+588a^2b^4-15b^6\right)\cos(c+dx)}{105b^8d} - \frac{a\left(32a^4-52a^2b^2+19b^4\right)\cos(}{8b^7d}$$

$$= \frac{a\left(64a^6-120a^4b^2+60a^2b^4-5b^6\right)x}{8b^9} + \frac{\left(840a^6-1435a^4b^2+588a^2b^4-15b^6\right)\cos(c}{105b^8d}$$

$$= \frac{a\left(64a^6-120a^4b^2+60a^2b^4-5b^6\right)x}{8b^9} + \frac{\left(840a^6-1435a^4b^2+588a^2b^4-15b^6\right)\cos(c}{105b^8d}$$

$$= \frac{a\left(64a^6-120a^4b^2+60a^2b^4-5b^6\right)x}{8b^9} + \frac{\left(840a^6-1435a^4b^2+588a^2b^4-15b^6\right)\cos(c}{105b^8d}$$

$$= \frac{a\left(64a^6-120a^4b^2+60a^2b^4-5b^6\right)x}{8b^9} - \frac{2a^2\left(8a^2-3b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.}{b^9d}$$

Mathematica [A] time = 8.86982, size = 531, normalized size = 1.01

$$26880a^6b^2\sin(2(c+dx))-201600a^5b^3c\sin(c+dx)-201600a^5b^3dx\sin(c+dx)-45920a^4b^4\sin(2(c+dx))-1120a^4b^4\sin(4(c+dx))+100800a^3b^5c\sin(c+dx)+100800$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-26880*a^2*(8*a^2 - 3*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]] + (107520*a^8*c - 201600*a^6*b^2*c + 100800*a^4*b^4*c -
 8400*a^2*b^6*c + 107520*a^8*d*x - 201600*a^6*b^2*d*x + 100800*a^4*b^4*d*x
- 8400*a^2*b^6*d*x + 840*a*b*(128*a^6 - 224*a^4*b^2 + 98*a^2*b^4 - 5*b^6)*C
os[c + d*x] + 70*(64*a^5*b^3 - 96*a^3*b^5 + 27*a*b^7)*Cos[3*(c + d*x)] - 33
6*a^3*b^5*Cos[5*(c + d*x)] + 350*a*b^7*Cos[5*(c + d*x)] + 40*a*b^7*Cos[7*(c
 + d*x)] + 107520*a^7*b*c*Sin[c + d*x] - 201600*a^5*b^3*c*Sin[c + d*x] + 10
0800*a^3*b^5*c*Sin[c + d*x] - 8400*a*b^7*c*Sin[c + d*x] + 107520*a^7*b*d*x*
Sin[c + d*x] - 201600*a^5*b^3*d*x*Sin[c + d*x] + 100800*a^3*b^5*d*x*Sin[c +
 d*x] - 8400*a*b^7*d*x*Sin[c + d*x] + 26880*a^6*b^2*Sin[2*(c + d*x)] - 4592
0*a^4*b^4*Sin[2*(c + d*x)] + 18480*a^2*b^6*Sin[2*(c + d*x)] - 210*b^8*Sin[2
*(c + d*x)] - 1120*a^4*b^4*Sin[4*(c + d*x)] + 1428*a^2*b^6*Sin[4*(c + d*x)]
 - 210*b^8*Sin[4*(c + d*x)] + 112*a^2*b^6*Sin[6*(c + d*x)] - 90*b^8*Sin[6*(
c + d*x)] - 15*b^8*Sin[8*(c + d*x)])/(a + b*Sin[c + d*x]))/(13440*b^9*d)
```

Maple [B] time = 0.141, size = 2076, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 14/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^6+18/d/b^4/(1+t
an(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^2+28/d/b^5/(1+tan(1/2*d*x+1/
2*c)^2)^7*tan(1/2*d*x+1/2*c)^3*a^3-28/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(
1/2*d*x+1/2*c)^11*a^3-30/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c
)^12*a^4-30/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5*a^5+280/d
/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6*a^6+29/d/b^5/(1+tan(1/
2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^5*a^3-85/12/d/b^3/(1+tan(1/2*d*x+1/2*c
)^2)^7*tan(1/2*d*x+1/2*c)^5*a+210/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*
d*x+1/2*c)^4*a^6-330/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4*
a^4+176/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6*a^2-6/d/b^2/(
1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4-70/3/d/b^6/(1+tan(1/2*d*x+1/
2*c)^2)^7*a^4+46/5/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*a^2+2/d*a^7/b^8/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-4/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)+16/d/b^9*arctan(tan(1/2*d*x+1/2*c))*a^7+15/d/b^5*arctan(ta
n(1/2*d*x+1/2*c))*a^3-5/4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a-30/d/b^7*arcta
n(tan(1/2*d*x+1/2*c))*a^5+14/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*a^6-2/d/b^2/(
1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12-10/d/b^2/(1+tan(1/2*d*x+1/2
*c)^2)^7*tan(1/2*d*x+1/2*c)^8+6/d*a^2/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+7/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^
7*tan(1/2*d*x+1/2*c)^11*a-24/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1
/2*c)^3*a^5+146/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a^2+3
0/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^9*a^5+210/d/b^8/(1+ta
n(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a^6+606/5/d/b^4/(1+tan(1/2*d*x+1
/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4*a^2-1360/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7
*tan(1/2*d*x+1/2*c)^6*a^4+84/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1
/2*c)^2*a^6-400/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2*a^4
+232/5/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2*a^2-6/d/b^7/(1
+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)*a^5+9/d/b^5/(1+tan(1/2*d*x+1/2*
c)^2)^7*tan(1/2*d*x+1/2*c)*a^3-11/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/
```

```
2*d*x+1/2*c)*a+84/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^
6-160/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^4+72/d/b^4/(
1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^2+24/d/b^7/(1+tan(1/2*d*x
+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11*a^5+85/12/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)
^7*tan(1/2*d*x+1/2*c)^9*a+6/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/
2*c)^13*a^5-9/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13*a^3-10
90/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a^4-7/3/d/b^3/(1
+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3*a-29/d/b^5/(1+tan(1/2*d*x+1/2
*c)^2)^7*tan(1/2*d*x+1/2*c)^9*a^3+2/d*a^6/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-4/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*a^2/b^3/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-16/d*a^8/b^9/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+38/d*a^6/b^7
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2
8/d*a^4/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))-2/7/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^7+11/4/d/b^3/(1+tan(1/2*d*x+1/
2*c)^2)^7*tan(1/2*d*x+1/2*c)^13*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.637, size = 2032, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/840*(160*a*b^7*cos(d*x + c)^7 - 14*(24*a^3*b^5 - 5*a*b^7)*cos(d*x + c)^5
 + 35*(32*a^5*b^3 - 36*a^3*b^5 + 5*a*b^7)*cos(d*x + c)^3 + 105*(64*a^8 - 12
0*a^6*b^2 + 60*a^4*b^4 - 5*a^2*b^6)*d*x + 420*(8*a^7 - 11*a^5*b^2 + 3*a^3*b
^4 + (8*a^6*b - 11*a^4*b^3 + 3*a^2*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 105*(64*a^7*b - 120*a^5*b^3 + 60*a^3
*b^5 - 5*a*b^7)*cos(d*x + c) - (120*b^8*cos(d*x + c)^7 - 224*a^2*b^6*cos(d*
x + c)^5 + 70*(8*a^4*b^4 - 7*a^2*b^6)*cos(d*x + c)^3 - 105*(64*a^7*b - 120*
a^5*b^3 + 60*a^3*b^5 - 5*a*b^7)*d*x - 105*(32*a^6*b^2 - 52*a^4*b^4 + 19*a^2
*b^6)*cos(d*x + c))*sin(d*x + c))/(b^10*d*sin(d*x + c) + a*b^9*d), 1/840*(1
60*a*b^7*cos(d*x + c)^7 - 14*(24*a^3*b^5 - 5*a*b^7)*cos(d*x + c)^5 + 35*(32
*a^5*b^3 - 36*a^3*b^5 + 5*a*b^7)*cos(d*x + c)^3 + 105*(64*a^8 - 120*a^6*b^2
 + 60*a^4*b^4 - 5*a^2*b^6)*d*x + 840*(8*a^7 - 11*a^5*b^2 + 3*a^3*b^4 + (8*a
^6*b - 11*a^4*b^3 + 3*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin
(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 105*(64*a^7*b - 120*a^5*b^
3 + 60*a^3*b^5 - 5*a*b^7)*cos(d*x + c) - (120*b^8*cos(d*x + c)^7 - 224*a^2*
```

```
b^6*cos(d*x + c)^5 + 70*(8*a^4*b^4 - 7*a^2*b^6)*cos(d*x + c)^3 - 105*(64*a^
7*b - 120*a^5*b^3 + 60*a^3*b^5 - 5*a*b^7)*d*x - 105*(32*a^6*b^2 - 52*a^4*b^
4 + 19*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/(b^10*d*sin(d*x + c) + a*b^9*d)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27625, size = 1303, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(105*(64*a^7 - 120*a^5*b^2 + 60*a^3*b^4 - 5*a*b^6)*(d*x + c)/b^9 - 16
80*(8*a^8 - 19*a^6*b^2 + 14*a^4*b^4 - 3*a^2*b^6)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqr
t(a^2 - b^2)*b^9) + 1680*(a^6*b*tan(1/2*d*x + 1/2*c) - 2*a^4*b^3*tan(1/2*d*
x + 1/2*c) + a^2*b^5*tan(1/2*d*x + 1/2*c) + a^7 - 2*a^5*b^2 + a^3*b^4)/((a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^8) + 2*(2520*a^5*b
*tan(1/2*d*x + 1/2*c)^13 - 3780*a^3*b^3*tan(1/2*d*x + 1/2*c)^13 + 1155*a*b^
5*tan(1/2*d*x + 1/2*c)^13 + 5880*a^6*tan(1/2*d*x + 1/2*c)^12 - 12600*a^4*b^
2*tan(1/2*d*x + 1/2*c)^12 + 7560*a^2*b^4*tan(1/2*d*x + 1/2*c)^12 - 840*b^6*
tan(1/2*d*x + 1/2*c)^12 + 10080*a^5*b*tan(1/2*d*x + 1/2*c)^11 - 11760*a^3*b
^3*tan(1/2*d*x + 1/2*c)^11 + 980*a*b^5*tan(1/2*d*x + 1/2*c)^11 + 35280*a^6*
tan(1/2*d*x + 1/2*c)^10 - 67200*a^4*b^2*tan(1/2*d*x + 1/2*c)^10 + 30240*a^2
*b^4*tan(1/2*d*x + 1/2*c)^10 + 12600*a^5*b*tan(1/2*d*x + 1/2*c)^9 - 12180*a
^3*b^3*tan(1/2*d*x + 1/2*c)^9 + 2975*a*b^5*tan(1/2*d*x + 1/2*c)^9 + 88200*a
^6*tan(1/2*d*x + 1/2*c)^8 - 152600*a^4*b^2*tan(1/2*d*x + 1/2*c)^8 + 61320*a
^2*b^4*tan(1/2*d*x + 1/2*c)^8 - 4200*b^6*tan(1/2*d*x + 1/2*c)^8 + 117600*a^
6*tan(1/2*d*x + 1/2*c)^6 - 190400*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 + 73920*a^
2*b^4*tan(1/2*d*x + 1/2*c)^6 - 12600*a^5*b*tan(1/2*d*x + 1/2*c)^5 + 12180*a
^3*b^3*tan(1/2*d*x + 1/2*c)^5 - 2975*a*b^5*tan(1/2*d*x + 1/2*c)^5 + 88200*a
^6*tan(1/2*d*x + 1/2*c)^4 - 138600*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 + 50904*a
^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 2520*b^6*tan(1/2*d*x + 1/2*c)^4 - 10080*a^5
*b*tan(1/2*d*x + 1/2*c)^3 + 11760*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 980*a*b^
5*tan(1/2*d*x + 1/2*c)^3 + 35280*a^6*tan(1/2*d*x + 1/2*c)^2 - 56000*a^4*b^2
*tan(1/2*d*x + 1/2*c)^2 + 19488*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 2520*a^5*b
*tan(1/2*d*x + 1/2*c) + 3780*a^3*b^3*tan(1/2*d*x + 1/2*c) - 1155*a*b^5*tan(
1/2*d*x + 1/2*c) + 5880*a^6 - 9800*a^4*b^2 + 3864*a^2*b^4 - 120*b^6)/((tan(
1/2*d*x + 1/2*c)^2 + 1)^7*b^8))/d
```

3.1257 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=471

$$-\frac{a\left(-170a^2b^2+105a^4+61b^4\right)\cos(c+dx)}{15b^7d}+\frac{2a\left(7a^2-2b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{\left(-20a^2b^2+14a^4+\right.}{10a^2b^3d}$$

```
[Out] -((112*a^6 - 200*a^4*b^2 + 90*a^2*b^4 - 5*b^6)*x)/(16*b^8) + (2*a*(7*a^2 -
2*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(b^8*d) - (a*(105*a^4 - 170*a^2*b^2 + 61*b^4)*Cos[c + d*x])/(15*b^7*d) + ((
56*a^4 - 86*a^2*b^2 + 27*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^6*d) - ((35*
a^4 - 52*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*a*b^5*d) + ((42
*a^4 - 61*a^2*b^2 + 16*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^2*b^4*d) + (
Cos[c + d*x]*Sin[c + d*x]^3)/(3*a*d*(a + b*Sin[c + d*x])) - (b*Cos[c + d*x]
*Sin[c + d*x]^4)/(6*a^2*d*(a + b*Sin[c + d*x])) - ((14*a^4 - 20*a^2*b^2 + 5
*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(10*a^2*b^3*d*(a + b*Sin[c + d*x])) - (7
*a*Cos[c + d*x]*Sin[c + d*x]^5)/(30*b^2*d*(a + b*Sin[c + d*x])) + (Cos[c +
d*x]*Sin[c + d*x]^6)/(6*b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.54977, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2896, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(-170a^2b^2+105a^4+61b^4\right)\cos(c+dx)}{15b^7d}+\frac{2a\left(7a^2-2b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{\left(-20a^2b^2+14a^4+\right.}{10a^2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -((112*a^6 - 200*a^4*b^2 + 90*a^2*b^4 - 5*b^6)*x)/(16*b^8) + (2*a*(7*a^2 -
2*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(b^8*d) - (a*(105*a^4 - 170*a^2*b^2 + 61*b^4)*Cos[c + d*x])/(15*b^7*d) + ((
56*a^4 - 86*a^2*b^2 + 27*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^6*d) - ((35*
a^4 - 52*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*a*b^5*d) + ((42
*a^4 - 61*a^2*b^2 + 16*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^2*b^4*d) + (
Cos[c + d*x]*Sin[c + d*x]^3)/(3*a*d*(a + b*Sin[c + d*x])) - (b*Cos[c + d*x]
*Sin[c + d*x]^4)/(6*a^2*d*(a + b*Sin[c + d*x])) - ((14*a^4 - 20*a^2*b^2 + 5
*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(10*a^2*b^3*d*(a + b*Sin[c + d*x])) - (7
*a*Cos[c + d*x]*Sin[c + d*x]^5)/(30*b^2*d*(a + b*Sin[c + d*x])) + (Cos[c +
d*x]*Sin[c + d*x]^6)/(6*b*d*(a + b*Sin[c + d*x]))
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
```

```
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))} - \frac{b\cos(c+dx)\sin^4(c+dx)}{6a^2d(a+b\sin(c+dx))} - \frac{7a\cos(c+dx)\sin^5(c+dx)}{30b^2d(a+b\sin(c+dx))}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))} - \frac{b\cos(c+dx)\sin^4(c+dx)}{6a^2d(a+b\sin(c+dx))} - \frac{\left(14a^4-20a^2b^2+5b^4\right)\cos(}{10a^2b^3d(a+b\sin}$$

$$= \frac{\left(42a^4-61a^2b^2+16b^4\right)\cos(c+dx)\sin^3(c+dx)}{24a^2b^4d} + \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))} - \frac{b\cos}{6a^2}$$

$$= -\frac{\left(35a^4-52a^2b^2+15b^4\right)\cos(c+dx)\sin^2(c+dx)}{15ab^5d} + \frac{\left(42a^4-61a^2b^2+16b^4\right)\cos(c}{24a^2b^4d}$$

$$= \frac{\left(56a^4-86a^2b^2+27b^4\right)\cos(c+dx)\sin(c+dx)}{16b^6d} - \frac{\left(35a^4-52a^2b^2+15b^4\right)\cos(c+}{15ab^5d}$$

$$= -\frac{a\left(105a^4-170a^2b^2+61b^4\right)\cos(c+dx)}{15b^7d} + \frac{\left(56a^4-86a^2b^2+27b^4\right)\cos(c+dx)\sin}{16b^6d}$$

$$= -\frac{\left(112a^6-200a^4b^2+90a^2b^4-5b^6\right)x}{16b^8} - \frac{a\left(105a^4-170a^2b^2+61b^4\right)\cos(c+dx)}{15b^7d} +$$

$$= -\frac{\left(112a^6-200a^4b^2+90a^2b^4-5b^6\right)x}{16b^8} - \frac{a\left(105a^4-170a^2b^2+61b^4\right)\cos(c+dx)}{15b^7d} +$$

$$= -\frac{\left(112a^6-200a^4b^2+90a^2b^4-5b^6\right)x}{16b^8} - \frac{a\left(105a^4-170a^2b^2+61b^4\right)\cos(c+dx)}{15b^7d} +$$

$$= -\frac{\left(112a^6-200a^4b^2+90a^2b^4-5b^6\right)x}{16b^8} + \frac{2a\left(7a^2-2b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.}{b^8d}$$

Mathematica [A] time = 8.43162, size = 462, normalized size = 0.98

$$\frac{3840a\left(7a^2-2b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - \frac{3360a^5b^2\sin(2(c+dx))-24000a^4b^3c\sin(c+dx)-24000a^4b^3dx\sin(c+dx)-5440a^3b^4}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3840*a*(7*a^2 - 2*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]] - (13440*a^7*c - 24000*a^5*b^2*c + 10800*a^3*b^4*c - 600*a*
b^6*c + 13440*a^7*d*x - 24000*a^5*b^2*d*x + 10800*a^3*b^4*d*x - 600*a*b^6*d
*x + 15*b*(896*a^6 - 1488*a^4*b^2 + 576*a^2*b^4 - 15*b^6)*Cos[c + d*x] + 10
*(56*a^4*b^3 - 79*a^2*b^5 + 18*b^7)*Cos[3*(c + d*x)] - 42*a^2*b^5*Cos[5*(c
+ d*x)] + 40*b^7*Cos[5*(c + d*x)] + 5*b^7*Cos[7*(c + d*x)] + 13440*a^6*b*c*
Sin[c + d*x] - 24000*a^4*b^3*c*Sin[c + d*x] + 10800*a^2*b^5*c*Sin[c + d*x]
- 600*b^7*c*Sin[c + d*x] + 13440*a^6*b*d*x*Sin[c + d*x] - 24000*a^4*b^3*d*x
*Sin[c + d*x] + 10800*a^2*b^5*d*x*Sin[c + d*x] - 600*b^7*d*x*Sin[c + d*x] +
 3360*a^5*b^2*Sin[2*(c + d*x)] - 5440*a^3*b^4*Sin[2*(c + d*x)] + 1910*a*b^6
*Sin[2*(c + d*x)] - 140*a^3*b^4*Sin[4*(c + d*x)] + 166*a*b^6*Sin[4*(c + d*x
)] + 14*a*b^6*Sin[6*(c + d*x)])/(a + b*Sin[c + d*x]))/(1920*b^8*d)
```

Maple [B] time = 0.151, size = 1817, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -120/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^5-15/d/b^6/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^4-56/d/b^3/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a-5/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2
*d*x+1/2*c)^11*a^4+27/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)
^11*a^2+560/3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^3-184
/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a+10/d/b^6/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^4-15/2/d/b^4/(1+tan(1/2*d*x+1/2*
c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^2+15/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
2*d*x+1/2*c)^3*a^4-57/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)
^3*a^2-36/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a-60/d/b^7/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^5+88/d/b^5/(1+tan(1/2*d*x
+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^3-11/8/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6
*tan(1/2*d*x+1/2*c)^11-2/d*a^6/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)-14/d/b^8*arctan(tan(1/2*d*x+1/2*c))*a^6+25/d/b^6*arctan(tan(1/2*d
*x+1/2*c))*a^4-45/4/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2+4/d/b^5/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^4-2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)*a^2+5/24/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
2*d*x+1/2*c)^9-15/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+1
5/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-5/24/d/b^2/(1+tan
(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+11/8/d/b^2/(1+tan(1/2*d*x+1/2*c)^
2)^6*tan(1/2*d*x+1/2*c)-92/15/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*a+56/3/d/b^5
/(1+tan(1/2*d*x+1/2*c)^2)^6*a^3-12/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*a^5+4/d
/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^3*tan(1/2*d*x+1/2*
c)-2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+
1/2*c)-32/d/b^6*a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))+22/d/b^4*a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+
1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-12/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*ta
n(1/2*d*x+1/2*c)^10*a-60/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^8*a^5+104/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a^3+57/4/
d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^2-10/d/b^6/(1+tan(1
/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*a^4+15/2/d/b^4/(1+tan(1/2*d*x+1/2*c
)^2)^6*tan(1/2*d*x+1/2*c)^7*a^2-120/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
2*d*x+1/2*c)^6*a^5-124/5/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^2*a+5/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*a^4-27/4/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*a^2-12/d/b^7/(1+tan(1/2*d*x+1
```

```
/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^5+5/8/d/b^2*arctan(tan(1/2*d*x+1/2*c))+2
4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^3+176/d/b^5/(1+t
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^3-2/d*a^5/b^6/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+14/d*a^7/b^8/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47191, size = 1898, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/240*(40*b^7*cos(d*x + c)^7 - 2*(42*a^2*b^5 - 5*b^7)*cos(d*x + c)^5 + 5*
(56*a^4*b^3 - 58*a^2*b^5 + 5*b^7)*cos(d*x + c)^3 + 15*(112*a^7 - 200*a^5*b^
2 + 90*a^3*b^4 - 5*a*b^6)*d*x - 120*(7*a^6 - 9*a^4*b^2 + 2*a^2*b^4 + (7*a^5
*b - 9*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2
)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d
*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2)) + 15*(112*a^6*b - 200*a^4*b^3 + 90*a^2*b^5 - 5*b^7)
*cos(d*x + c) + (56*a*b^6*cos(d*x + c)^5 - 10*(14*a^3*b^4 - 11*a*b^6)*cos(d
*x + c)^3 + 15*(112*a^6*b - 200*a^4*b^3 + 90*a^2*b^5 - 5*b^7)*d*x + 15*(56*
a^5*b^2 - 86*a^3*b^4 + 27*a*b^6)*cos(d*x + c))*sin(d*x + c))/(b^9*d*sin(d*x
 + c) + a*b^8*d), -1/240*(40*b^7*cos(d*x + c)^7 - 2*(42*a^2*b^5 - 5*b^7)*co
s(d*x + c)^5 + 5*(56*a^4*b^3 - 58*a^2*b^5 + 5*b^7)*cos(d*x + c)^3 + 15*(112
*a^7 - 200*a^5*b^2 + 90*a^3*b^4 - 5*a*b^6)*d*x + 240*(7*a^6 - 9*a^4*b^2 + 2
*a^2*b^4 + (7*a^5*b - 9*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*ar
ctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 15*(112*a^6*b
- 200*a^4*b^3 + 90*a^2*b^5 - 5*b^7)*cos(d*x + c) + (56*a*b^6*cos(d*x + c)^5
 - 10*(14*a^3*b^4 - 11*a*b^6)*cos(d*x + c)^3 + 15*(112*a^6*b - 200*a^4*b^3
+ 90*a^2*b^5 - 5*b^7)*d*x + 15*(56*a^5*b^2 - 86*a^3*b^4 + 27*a*b^6)*cos(d*x
 + c))*sin(d*x + c))/(b^9*d*sin(d*x + c) + a*b^8*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27245, size = 1127, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/240*(15*(112*a^6 - 200*a^4*b^2 + 90*a^2*b^4 - 5*b^6)*(d*x + c)/b^8 - 480
*(7*a^7 - 16*a^5*b^2 + 11*a^3*b^4 - 2*a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1
/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^
2 - b^2)*b^8) + 480*(a^5*b*tan(1/2*d*x + 1/2*c) - 2*a^3*b^3*tan(1/2*d*x + 1
/2*c) + a*b^5*tan(1/2*d*x + 1/2*c) + a^6 - 2*a^4*b^2 + a^2*b^4)/((a*tan(1/2
*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^7) + 2*(600*a^4*b*tan(1/2
*d*x + 1/2*c)^11 - 810*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 165*b^5*tan(1/2*d*
x + 1/2*c)^11 + 1440*a^5*tan(1/2*d*x + 1/2*c)^10 - 2880*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^10 + 1440*a*b^4*tan(1/2*d*x + 1/2*c)^10 + 1800*a^4*b*tan(1/2*d*x
+ 1/2*c)^9 - 1710*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 - 25*b^5*tan(1/2*d*x + 1/2
*c)^9 + 7200*a^5*tan(1/2*d*x + 1/2*c)^8 - 12480*a^3*b^2*tan(1/2*d*x + 1/2*c
)^8 + 4320*a*b^4*tan(1/2*d*x + 1/2*c)^8 + 1200*a^4*b*tan(1/2*d*x + 1/2*c)^7
 - 900*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^5*tan(1/2*d*x + 1/2*c)^7 + 14
400*a^5*tan(1/2*d*x + 1/2*c)^6 - 22400*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 736
0*a*b^4*tan(1/2*d*x + 1/2*c)^6 - 1200*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 900*a^
2*b^3*tan(1/2*d*x + 1/2*c)^5 - 450*b^5*tan(1/2*d*x + 1/2*c)^5 + 14400*a^5*t
an(1/2*d*x + 1/2*c)^4 - 21120*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 6720*a*b^4*t
an(1/2*d*x + 1/2*c)^4 - 1800*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1710*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 25*b^5*tan(1/2*d*x + 1/2*c)^3 + 7200*a^5*tan(1/2*d*x
 + 1/2*c)^2 - 10560*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 2976*a*b^4*tan(1/2*d*x
 + 1/2*c)^2 - 600*a^4*b*tan(1/2*d*x + 1/2*c) + 810*a^2*b^3*tan(1/2*d*x + 1/
2*c) - 165*b^5*tan(1/2*d*x + 1/2*c) + 1440*a^5 - 2240*a^3*b^2 + 736*a*b^4)/
((tan(1/2*d*x + 1/2*c)^2 + 1)^6*b^7))/d
```

3.1258 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=231

$$-\frac{2\left(a^2-b^2\right)^{3/2}\left(6a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}-\frac{\cos^3(c+dx)\left(2\left(6a^2-b^2\right)-9ab\sin(c+dx)\right)}{6b^4d}+\frac{\cos(c+dx)\left(4\left(-7\right.\right.}{}$$

```
[Out] (a*(24*a^4 - 40*a^2*b^2 + 15*b^4)*x)/(4*b^7) - (2*(a^2 - b^2)^(3/2)*(6*a^2
- b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) + (Cos[c +
 d*x]^5*(6*a + b*Sin[c + d*x]))/(5*b^2*d*(a + b*Sin[c + d*x])) - (Cos[c + d
*x]^3*(2*(6*a^2 - b^2) - 9*a*b*Sin[c + d*x]))/(6*b^4*d) + (Cos[c + d*x]*(4*
(6*a^4 - 7*a^2*b^2 + b^4) - a*b*(12*a^2 - 11*b^2)*Sin[c + d*x]))/(4*b^6*d)
```

Rubi [A] time = 0.512611, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2863, 2865, 2735, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\left(6a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}-\frac{\cos^3(c+dx)\left(2\left(6a^2-b^2\right)-9ab\sin(c+dx)\right)}{6b^4d}+\frac{\cos(c+dx)\left(4\left(-7\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (a*(24*a^4 - 40*a^2*b^2 + 15*b^4)*x)/(4*b^7) - (2*(a^2 - b^2)^(3/2)*(6*a^2
- b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) + (Cos[c +
 d*x]^5*(6*a + b*Sin[c + d*x]))/(5*b^2*d*(a + b*Sin[c + d*x])) - (Cos[c + d
*x]^3*(2*(6*a^2 - b^2) - 9*a*b*Sin[c + d*x]))/(6*b^4*d) + (Cos[c + d*x]*(4*
(6*a^4 - 7*a^2*b^2 + b^4) - a*b*(12*a^2 - 11*b^2)*Sin[c + d*x]))/(4*b^6*d)
```


Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\int \frac{\cos^4(c+dx)(-b-6a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)\left(2\left(6a^2-b^2\right)-9ab\sin(c+dx)\right)}{6b^4d} - \frac{\int \frac{\text{co}}{}}{}$$

$$= \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)\left(2\left(6a^2-b^2\right)-9ab\sin(c+dx)\right)}{6b^4d} + \frac{\cos(}{}$$

$$= \frac{a\left(24a^4-40a^2b^2+15b^4\right)x}{4b^7} + \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)\left(2\left(6a^2-\right.\right.}{6b}$$

$$= \frac{a\left(24a^4-40a^2b^2+15b^4\right)x}{4b^7} + \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)\left(2\left(6a^2-\right.\right.}{6b}$$

$$= \frac{a\left(24a^4-40a^2b^2+15b^4\right)x}{4b^7} + \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)\left(2\left(6a^2-\right.\right.}{6b}$$

$$= \frac{a\left(24a^4-40a^2b^2+15b^4\right)x}{4b^7} - \frac{2\left(a^2-b^2\right)^{3/2}\left(6a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d} + \frac{\cos^5(c}{5b}$$

Mathematica [A] time = 3.99716, size = 371, normalized size = 1.61

$$\frac{720a^4b^2\sin(2(c+dx))-4800a^3b^3c\sin(c+dx)-4800a^3b^3dx\sin(c+dx)-1080a^2b^4\sin(2(c+dx))-30a^2b^4\sin(4(c+dx))+60ab\left(-74a^2b^2+48a^4+23b^4\right)\cos(c+dx)+5\left(24a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-960*(a^2 - b^2)^(3/2)*(6*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2]])/Sqrt[
a^2 - b^2]] + (2880*a^6*c - 4800*a^4*b^2*c + 1800*a^2*b^4*c + 2880*a^6*d*x
- 4800*a^4*b^2*d*x + 1800*a^2*b^4*d*x + 60*a*b*(48*a^4 - 74*a^2*b^2 + 23*b^
4)*Cos[c + d*x] + 5*(24*a^3*b^3 - 31*a*b^5)*Cos[3*(c + d*x)] - 9*a*b^5*Cos[
5*(c + d*x)] + 2880*a^5*b*c*Sin[c + d*x] - 4800*a^3*b^3*c*Sin[c + d*x] + 18
00*a*b^5*c*Sin[c + d*x] + 2880*a^5*b*d*x*Sin[c + d*x] - 4800*a^3*b^3*d*x*Si
n[c + d*x] + 1800*a*b^5*d*x*Sin[c + d*x] + 720*a^4*b^2*Sin[2*(c + d*x)] - 1
080*a^2*b^4*Sin[2*(c + d*x)] + 295*b^6*Sin[2*(c + d*x)] - 30*a^2*b^4*Sin[4*
(c + d*x)] + 32*b^6*Sin[4*(c + d*x)] + 3*b^6*Sin[6*(c + d*x)])/(a + b*Sin[c
 + d*x]))/(480*b^7*d)
```

Maple [B] time = 0.134, size = 1321, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 46/15/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5+2/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)-4/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)-20/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^3+15/2/d/b^3*arctan(tan(
1/2*d*x+1/2*c))*a+12/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^5+10/d/b^6/(1+tan(1
/2*d*x+1/2*c)^2)^5*a^4-14/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*a^2+2/d/b^2/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a-16/d*a^2/b^3/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/b/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/b/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+6/d/b^2/(1+t
an(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8+12/d/b^2/(1+tan(1/2*d*x+1/2*c)^
2)^5*tan(1/2*d*x+1/2*c)^6+56/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x
+1/2*c)^4+28/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2+10/d/b
^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^4-18/d/b^4/(1+tan(1/2*
d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^2+2/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-4/d*a^2/b^3/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-12/d*a^6/b^7/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+26/d*a^4/b^
5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+
8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3-5/d/b^3/(1+tan(
1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a+40/d/b^6/(1+tan(1/2*d*x+1/2*c)^2
)^5*tan(1/2*d*x+1/2*c)^6*a^4-60/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*
x+1/2*c)^6*a^2+60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^4
-80/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^2-8/d/b^5/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a^3+5/d/b^3/(1+tan(1/2*d*x+1/2*c
)^2)^5*tan(1/2*d*x+1/2*c)^3*a+40/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d
*x+1/2*c)^2*a^4-52/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^
2-4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a^3+9/2/d/b^3/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a+4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)^9*a^3-9/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*
x+1/2*c)^9*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.26105, size = 1619, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(18*a*b^5*cos(d*x + c)^5 - 5*(12*a^3*b^3 - 11*a*b^5)*cos(d*x + c)^3
- 15*(24*a^6 - 40*a^4*b^2 + 15*a^2*b^4)*d*x - 30*(6*a^5 - 7*a^3*b^2 + a*b^4
 + (6*a^4*b - 7*a^2*b^3 + b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*s
in(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2)) - 15*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*cos(d*x
 + c) - (12*b^6*cos(d*x + c)^5 - 10*(3*a^2*b^4 - 2*b^6)*cos(d*x + c)^3 + 15
*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*d*x + 15*(12*a^4*b^2 - 17*a^2*b^4 + 4*b
^6)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d), -1/60*(18*a
*b^5*cos(d*x + c)^5 - 5*(12*a^3*b^3 - 11*a*b^5)*cos(d*x + c)^3 - 15*(24*a^6
 - 40*a^4*b^2 + 15*a^2*b^4)*d*x - 60*(6*a^5 - 7*a^3*b^2 + a*b^4 + (6*a^4*b
- 7*a^2*b^3 + b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 15*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*
cos(d*x + c) - (12*b^6*cos(d*x + c)^5 - 10*(3*a^2*b^4 - 2*b^6)*cos(d*x + c)
^3 + 15*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*d*x + 15*(12*a^4*b^2 - 17*a^2*b^
4 + 4*b^6)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24293, size = 801, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(15*(24*a^5 - 40*a^3*b^2 + 15*a*b^4)*(d*x + c)/b^7 - 120*(6*a^6 - 13*a
^4*b^2 + 8*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
```

```
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 120
*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2
*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*t
an(1/2*d*x + 1/2*c) + a)*b^6) + 2*(120*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 135*a
*b^3*tan(1/2*d*x + 1/2*c)^9 + 300*a^4*tan(1/2*d*x + 1/2*c)^8 - 540*a^2*b^2*
tan(1/2*d*x + 1/2*c)^8 + 180*b^4*tan(1/2*d*x + 1/2*c)^8 + 240*a^3*b*tan(1/2
*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 1200*a^4*tan(1/2*d*x +
 1/2*c)^6 - 1800*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 360*b^4*tan(1/2*d*x + 1/2
*c)^6 + 1800*a^4*tan(1/2*d*x + 1/2*c)^4 - 2400*a^2*b^2*tan(1/2*d*x + 1/2*c)
^4 + 560*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 15
0*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 1200*a^4*tan(1/2*d*x + 1/2*c)^2 - 1560*a^2
*b^2*tan(1/2*d*x + 1/2*c)^2 + 280*b^4*tan(1/2*d*x + 1/2*c)^2 - 120*a^3*b*ta
n(1/2*d*x + 1/2*c) + 135*a*b^3*tan(1/2*d*x + 1/2*c) + 300*a^4 - 420*a^2*b^2
 + 92*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*b^6))/d
```

3.1259 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=266

$$\frac{3\left(a^2-b^2\right)\cos(c+dx)}{b^4d}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{2\left(a^2-b^2\right)^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}+\frac{(a^2}{ab^4d}$$

```
[Out] (a*x)/b^3 + (2*a*(2*a^2 - 3*b^2)*x)/b^5 + (2*(a^2 - b^2)^(3/2)*ArcTan[(b +
a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(b^5*d) - (2*(a^2 - b^2)^(3/2)*(5*a^2
 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a^2*b^5*d) - Arc
Tanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(b^2*d) + (3*(a^2 - b^2)*Cos[c +
d*x])/(b^4*d) - Cos[c + d*x]^3/(3*b^2*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(b
^3*d) + ((a^2 - b^2)^2*Cos[c + d*x])/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.350575, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2897, 3770, 2638, 2635, 8, 2633, 2664, 12, 2660, 618, 204}

$$\frac{3\left(a^2-b^2\right)\cos(c+dx)}{b^4d}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{2\left(a^2-b^2\right)^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}+\frac{(a^2}{ab^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (a*x)/b^3 + (2*a*(2*a^2 - 3*b^2)*x)/b^5 + (2*(a^2 - b^2)^(3/2)*ArcTan[(b +
a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(b^5*d) - (2*(a^2 - b^2)^(3/2)*(5*a^2
 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a^2*b^5*d) - Arc
Tanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(b^2*d) + (3*(a^2 - b^2)*Cos[c +
d*x])/(b^4*d) - Cos[c + d*x]^3/(3*b^2*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(b
^3*d) + ((a^2 - b^2)^2*Cos[c + d*x])/(a*b^4*d*(a + b*Sin[c + d*x]))
```


Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{2\left(-2a^3+3ab^2\right)}{b^5}+\frac{\csc(c+dx)}{a^2}+\frac{3\left(-a^2+b^2\right)\sin(c+dx)}{b^4}+\frac{2a\sin^2(c+dx)}{b^3}-\mathrm{si}\right.$$

$$=\frac{2a\left(2a^2-3b^2\right)x}{b^5}+\frac{\int\csc(c+dx)\,dx}{a^2}+\frac{(2a)\int\sin^2(c+dx)\,dx}{b^3}-\frac{\int\sin^3(c+dx)\,dx}{b^2}-\frac{(3}{}$$

$$=\frac{2a\left(2a^2-3b^2\right)x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{3\left(a^2-b^2\right)\cos(c+dx)}{b^4d}-\frac{a\cos(c+dx)\sin(c}{b^3d}$$

$$=\frac{ax}{b^3}+\frac{2a\left(2a^2-3b^2\right)x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{b^2d}+\frac{3\left(a^2-b^2\right)\cos(c+dx)}{b^4d}$$

$$=\frac{ax}{b^3}+\frac{2a\left(2a^2-3b^2\right)x}{b^5}-\frac{2\left(a^2-b^2\right)^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}-\frac{\tanh^{-1}(\cos}{a^2d}$$

$$=\frac{ax}{b^3}+\frac{2a\left(2a^2-3b^2\right)x}{b^5}-\frac{2\left(a^2-b^2\right)^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}-\frac{\tanh^{-1}(\cos}{a^2d}$$

$$=\frac{ax}{b^3}+\frac{2a\left(2a^2-3b^2\right)x}{b^5}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{2\left(a^2-b^2\right)^{3/2}\left(5a^2+b^2\right.}{a^2}$$

Mathematica [A] time = 1.68633, size = 207, normalized size = 0.78

$$\frac{\frac{12a\left(4a^2-5b^2\right)(c+dx)}{b^5}+\frac{9\left(4a^2-3b^2\right)\cos(c+dx)}{b^4}-\frac{24\left(4a^2+b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^5}+\frac{12\left(a^2-b^2\right)^2\cos(c+dx)}{ab^4(a+b\sin(c+dx))}+\frac{12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}-\frac{12\,\mathrm{lo}}{}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((12*a*(4*a^2 - 5*b^2)*(c + d*x))/b^5 - (24*(a^2 - b^2)^(3/2)*(4*a^2 + b^2)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^5) + (9*(4*a^2 -
3*b^2)*Cos[c + d*x])/b^4 - Cos[3*(c + d*x)]/b^2 - (12*Log[Cos[(c + d*x)/2]]
)/a^2 + (12*Log[Sin[(c + d*x)/2]])/a^2 + (12*(a^2 - b^2)^2*Cos[c + d*x])/(a
*b^4*(a + b*Sin[c + d*x])) - (6*a*Sin[2*(c + d*x)])/b^3)/(12*d)
```

Maple [B] time = 0.17, size = 778, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5+6/d/b^4/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2-6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)
^3*tan(1/2*d*x+1/2*c)^4+12/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2
*c)^2*a^2-8/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-2/d/b^3/(
1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)+6/d/b^4/(1+tan(1/2*d*x+1/2*c
```

```
)^2)^3*a^2-14/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3+8/d/b^5*arctan(tan(1/2*d*x
+1/2*c))*a^3-10/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a+2/d*a^2/b^3/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-4/d/b/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a^2*b/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*a^3/b^4/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-4/d/b^2/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a+2/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)-8/d*a^4/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))+14/d*a^2/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d*b/a^2/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a^2*ln(tan(1/2*d*x+1/2*c
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.95416, size = 1598, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(4*a^3*b^3*cos(d*x + c)^3 + 6*(4*a^6 - 5*a^4*b^2)*d*x - 3*(4*a^5 - 3*a
^3*b^2 - a*b^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a
*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5
)*cos(d*x + c) - 3*(b^6*sin(d*x + c) + a*b^5)*log(1/2*cos(d*x + c) + 1/2) +
 3*(b^6*sin(d*x + c) + a*b^5)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^2*b^4*cos
(d*x + c)^3 - 3*(4*a^5*b - 5*a^3*b^3)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x +
 c))*sin(d*x + c))/(a^2*b^6*d*sin(d*x + c) + a^3*b^5*d), 1/6*(4*a^3*b^3*cos
(d*x + c)^3 + 6*(4*a^6 - 5*a^4*b^2)*d*x + 6*(4*a^5 - 3*a^3*b^2 - a*b^4 + (4
*a^4*b - 3*a^2*b^3 - b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
+ c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5)
*cos(d*x + c) - 3*(b^6*sin(d*x + c) + a*b^5)*log(1/2*cos(d*x + c) + 1/2) +
3*(b^6*sin(d*x + c) + a*b^5)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^2*b^4*cos(
d*x + c)^3 - 3*(4*a^5*b - 5*a^3*b^3)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x +
c))*sin(d*x + c))/(a^2*b^6*d*sin(d*x + c) + a^3*b^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22426, size = 477, normalized size = 1.79

$$\frac{\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{3\left(4\,a^3-5\,ab^2\right)(dx+c)}{b^5}-\frac{6\left(4\,a^6-7\,a^4b^2+2\,a^2b^4+b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^2b^5}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 3*(4*a^3 - 5*a*b^2)*(d*x + c)/b
^5 - 6*(4*a^6 - 7*a^4*b^2 + 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi + 1
/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^
2 - b^2)*a^2*b^5) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*tan(1/2*d*x + 1
/2*c)^4 - 9*b^2*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*tan(1/2*d*x + 1/2*c)^2 - 12
*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*a^2 - 7*b^2)/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4) + 6*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2
*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*
b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^2*b^4))/d
```

3.1260 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=254

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^4d}+\frac{4\left(-3a^4b^2+2a^6+b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^4d\sqrt{a^2-b^2}}-\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^2b^3d(a+b\sin(c+dx))}-$$

```
[Out] -x/(2*b^2) - (3*(a^2 - b^2)*x)/b^4 - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^4*d) + (4*(2*a^6 - 3*a^4*b^2 + b^6)*A
rcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^4*Sqrt[a^2 - b^2]*d
) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*a*Cos[c + d*x])/(b^3*d) - Cot[
c + d*x]/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - ((a^2 - b^2)^2*C
os[c + d*x])/(a^2*b^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.341806, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 2638, 2635, 2664, 12, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^4d}+\frac{4\left(-3a^4b^2+2a^6+b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^4d\sqrt{a^2-b^2}}-\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^2b^3d(a+b\sin(c+dx))}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -x/(2*b^2) - (3*(a^2 - b^2)*x)/b^4 - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^4*d) + (4*(2*a^6 - 3*a^4*b^2 + b^6)*A
rcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^4*Sqrt[a^2 - b^2]*d
) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*a*Cos[c + d*x])/(b^3*d) - Cot[
c + d*x]/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - ((a^2 - b^2)^2*C
os[c + d*x])/(a^2*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(\frac{3\left(-a^2+b^2\right)}{b^4} - \frac{2b\csc(c+dx)}{a^3} + \frac{\csc^2(c+dx)}{a^2} + \frac{2a\sin(c+dx)}{b^3} - \frac{\sin^2(c+dx)}{b^2}\right.$$

$$= -\frac{3\left(a^2-b^2\right)x}{b^4} + \frac{\int \csc^2(c+dx)\,dx}{a^2} + \frac{(2a)\int \sin(c+dx)\,dx}{b^3} - \frac{\int \sin^2(c+dx)\,dx}{b^2} - ($$

$$= -\frac{3\left(a^2-b^2\right)x}{b^4} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2a\cos(c+dx)}{b^3d} + \frac{\cos(c+dx)\sin(c+dx}{2b^2d}$$

$$= -\frac{x}{2b^2} - \frac{3\left(a^2-b^2\right)x}{b^4} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2a\cos(c+dx)}{b^3d} - \frac{\cot(c+dx)}{a^2d} +$$

$$= -\frac{x}{2b^2} - \frac{3\left(a^2-b^2\right)x}{b^4} + \frac{4\left(2a^6-3a^4b^2+b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^4\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(c}{a^3}$$

$$= -\frac{x}{2b^2} - \frac{3\left(a^2-b^2\right)x}{b^4} + \frac{4\left(2a^6-3a^4b^2+b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^4\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(c}{a^3}$$

$$= -\frac{x}{2b^2} - \frac{3\left(a^2-b^2\right)x}{b^4} - \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^4d} + \frac{4\left(2a^6-3a^4b^2+b^6\right.}{a^3b^4}$$

Mathematica [A] time = 2.74542, size = 215, normalized size = 0.85

$$\frac{\frac{2\left(5b^2-6a^2\right)(c+dx)}{b^4} + \frac{8\left(3a^2+2b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^4} - \frac{4\left(a^2-b^2\right)^2\cos(c+dx)}{a^2b^3(a+b\sin(c+dx))} - \frac{8b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{8b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{2\,\mathrm{ta}}{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*(-6*a^2 + 5*b^2)*(c + d*x))/b^4 + (8*(a^2 - b^2)^(3/2)*(3*a^2 + 2*b^2)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^4) - (8*a*Cos[c +
d*x])/b^3 - (2*Cot[(c + d*x)/2])/a^2 + (8*b*Log[Cos[(c + d*x)/2]])/a^3 - (8
*b*Log[Sin[(c + d*x)/2]])/a^3 - (4*(a^2 - b^2)^2*Cos[c + d*x])/(a^2*b^3*(a
+ b*Sin[c + d*x])) + Sin[2*(c + d*x)]/b^2 + (2*Tan[(c + d*x)/2])/a^2)/(4*d)
```

Maple [B] time = 0.172, size = 680, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x
+1/2*c)^3-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b^2
/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-4/d/b^3/(1+tan(1/2*d*x+1/2*c
)^2)^2*a-6/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2+5/d/b^2*arctan(tan(1/2*d*x+
```

```
1/2*c))-2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2
*d*x+1/2*c)+4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2
*d*x+1/2*c)-2/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*t
an(1/2*d*x+1/2*c)-2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)
*a^2+4/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-2/d/a^2*b/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+6/d/b^4*a^3/(a^2-b^2)^(1/2)*a
rctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-8/d/b^2*a/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b
^2-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^3*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.67204, size = 2010, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(3*a^4*b^2*cos(d*x + c)^3 + (6*a^5*b - 5*a^3*b^3)*d*x*cos(d*x + c)^2
- (6*a^5*b - 5*a^3*b^3)*d*x - (3*a^4*b - a^2*b^3 - 2*b^5 - (3*a^4*b - a^2*b
^3 - 2*b^5)*cos(d*x + c)^2 + (3*a^5 - a^3*b^2 - 2*a*b^4)*sin(d*x + c))*sqrt
(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(
b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - (3*a^4*b^2 + 2*a^2*
b^4)*cos(d*x + c) - 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^6)*log(1
/2*cos(d*x + c) + 1/2) + 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^6)*
log(-1/2*cos(d*x + c) + 1/2) - (a^3*b^3*cos(d*x + c)^3 + (6*a^6 - 5*a^4*b^2
)*d*x + (6*a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^3*b^
5*d*cos(d*x + c)^2 - a^4*b^4*d*sin(d*x + c) - a^3*b^5*d), -1/2*(3*a^4*b^2*c
os(d*x + c)^3 + (6*a^5*b - 5*a^3*b^3)*d*x*cos(d*x + c)^2 - (6*a^5*b - 5*a^3
*b^3)*d*x - 2*(3*a^4*b - a^2*b^3 - 2*b^5 - (3*a^4*b - a^2*b^3 - 2*b^5)*cos(
d*x + c)^2 + (3*a^5 - a^3*b^2 - 2*a*b^4)*sin(d*x + c))*sqrt(a^2 - b^2)*arct
an(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (3*a^4*b^2 + 2*a
^2*b^4)*cos(d*x + c) - 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^6)*lo
g(1/2*cos(d*x + c) + 1/2) + 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^
6)*log(-1/2*cos(d*x + c) + 1/2) - (a^3*b^3*cos(d*x + c)^3 + (6*a^6 - 5*a^4*
b^2)*d*x + (6*a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^3
*b^5*d*cos(d*x + c)^2 - a^4*b^4*d*sin(d*x + c) - a^3*b^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26405, size = 518, normalized size = 2.04

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{3\left(6\,a^2-5\,b^2\right)(dx+c)}{b^4}+\frac{6\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 b^3}-\frac{12\left(3\,a^6-\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
+ 3*(6*a^2 - 5*b^2)*(d*x + c)/b^4 + 6*(b*tan(1/2*d*x + 1/2*c)^3 + 4*a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 4*a)/((tan(1/2*d*x + 1/2*c)^2
+ 1)^2*b^3) - 12*(3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2))
)/(sqrt(a^2 - b^2)*a^3*b^4) - (4*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 12*a^4*b*ta
n(1/2*d*x + 1/2*c)^2 + 21*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 4*b^5*tan(1/2*d*
x + 1/2*c)^2 - 12*a^5*tan(1/2*d*x + 1/2*c) + 24*a^3*b^2*tan(1/2*d*x + 1/2*c
) - 14*a*b^4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^3 +
 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3*b^3))/d
```

3.1261 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=251

$$-\frac{6\left(a^2+b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^3d}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}+\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^3b^2d(a+b\sin(c+dx))}+\frac{3(\ldots}{\ldots}$$

```
[Out] (2*a*x)/b^3 + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^2*b^3*d) - (6*(a^2 - b^2)^(3/2)*(a^2 + b^2)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*b^3*d) - ArcTanh[Cos[c + d*x]]/(2*a^2*
d) + (3*(a^2 - b^2)*ArcTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b^2*d) +
 (2*b*Cot[c + d*x])/(a^3*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + ((a^2
 - b^2)^2*Cos[c + d*x])/(a^3*b^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.335801, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 3768, 2638, 2664, 12, 2660, 618, 204}

$$-\frac{6\left(a^2+b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^3d}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}+\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^3b^2d(a+b\sin(c+dx))}+\frac{3(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*x)/b^3 + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^2*b^3*d) - (6*(a^2 - b^2)^(3/2)*(a^2 + b^2)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*b^3*d) - ArcTanh[Cos[c + d*x]]/(2*a^2*
d) + (3*(a^2 - b^2)*ArcTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b^2*d) +
 (2*b*Cot[c + d*x])/(a^3*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + ((a^2
 - b^2)^2*Cos[c + d*x])/(a^3*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(\frac{2a}{b^3} - \frac{3\left(a^2-b^2\right)\csc(c+dx)}{a^4} - \frac{2b\csc^2(c+dx)}{a^3} + \frac{\csc^3(c+dx)}{a^2} - \frac{\sin(c+dx)}{b^2} + \frac{}{a^3}\right.$$

$$= \frac{2ax}{b^3} + \frac{\int\csc^3(c+dx)\,dx}{a^2} - \frac{\int\sin(c+dx)\,dx}{b^2} - \frac{(2b)\int\csc^2(c+dx)\,dx}{a^3} - \frac{\left(3\left(a^2-b^2\right)\right)\int}{}$$

$$= \frac{2ax}{b^3} + \frac{3\left(a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{(a}{a^3b}$$

$$= \frac{2ax}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{3\left(a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^2d} + \frac{2b\cot}{}$$

$$= \frac{2ax}{b^3} - \frac{6\left(a^2-b^2\right)^{3/2}\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^3d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{3\left(a^2\right.}{}$$

$$= \frac{2ax}{b^3} - \frac{6\left(a^2-b^2\right)^{3/2}\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^3d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{3\left(a^2\right.}{}$$

$$= \frac{2ax}{b^3} + \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3d} - \frac{6\left(a^2-b^2\right)^{3/2}\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.}{a^4b^3d}$$

Mathematica [A] time = 6.19018, size = 315, normalized size = 1.25

$$\frac{\left(6b^2-5a^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(5a^2-6b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{-2a^2b^2\cos(c+dx)+a^4\cos(c+dx)+b^4\cos(c}{a^3b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*(c + d*x))/(b^3*d) - (2*(a^2 - b^2)^(3/2)*(2*a^2 + 3*b^2)*ArcTan[(Sec[
(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(
a^4*b^3*d) + Cos[c + d*x]/(b^2*d) + (b*Cot[(c + d*x)/2])/(a^3*d) - Csc[(c +
 d*x)/2]^2/(8*a^2*d) + ((5*a^2 - 6*b^2)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) +
((-5*a^2 + 6*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^4*d) + Sec[(c + d*x)/2]^2/(8*
a^2*d) + (a^4*Cos[c + d*x] - 2*a^2*b^2*Cos[c + d*x] + b^4*Cos[c + d*x])/(a^
3*b^2*d*(a + b*Sin[c + d*x])) - (b*Tan[(c + d*x)/2])/(a^3*d)
```

Maple [B] time = 0.19, size = 618, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3*tan(1/2*d*x+1/2*c)*b+2/d/b^2/(1+tan(
1/2*d*x+1/2*c)^2)+4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a+2/d/b/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-4/d/a^2*b/(tan(1/2*d
```

```
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a^4/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^3+2/d/b^2/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a-4/d/a/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)*b^2-4/d*a^2/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1
/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+8/d*b/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d*b^3/a^4/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/8/d/a^2/tan(1/2*d*x+1/2
*c)^2-5/2/d/a^2*ln(tan(1/2*d*x+1/2*c))+3/d/a^4*ln(tan(1/2*d*x+1/2*c))*b^2+1
/d*b/a^3/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.47255, size = 2700, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/4*(8*a^6*d*x*cos(d*x + c)^2 - 8*a^6*d*x + 4*(2*a^5*b - 2*a^3*b^3 + 3*a*b
^5)*cos(d*x + c)^3 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4 - (2*a^5 + a^3*b^2 - 3*a*
b^4)*cos(d*x + c)^2 + (2*a^4*b + a^2*b^3 - 3*b^5 - (2*a^4*b + a^2*b^3 - 3*b
^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)) - 2*(4*a^5*b - 5*a^3*b^3 + 6*a*b^5)*cos(d*x + c) - (5*a^3
*b^3 - 6*a*b^5 - (5*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^2 + (5*a^2*b^4 - 6*b^6
- (5*a^2*b^4 - 6*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) +
1/2) + (5*a^3*b^3 - 6*a*b^5 - (5*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^2 + (5*a^2
*b^4 - 6*b^6 - (5*a^2*b^4 - 6*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*c
os(d*x + c) + 1/2) + 2*(4*a^5*b*d*x*cos(d*x + c)^2 + 2*a^4*b^2*cos(d*x + c)
^3 - 4*a^5*b*d*x - (2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^5
*b^3*d*cos(d*x + c)^2 - a^5*b^3*d + (a^4*b^4*d*cos(d*x + c)^2 - a^4*b^4*d)*
sin(d*x + c)), 1/4*(8*a^6*d*x*cos(d*x + c)^2 - 8*a^6*d*x + 4*(2*a^5*b - 2*a
^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - 4*(2*a^5 + a^3*b^2 - 3*a*b^4 - (2*a^5 +
a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + (2*a^4*b + a^2*b^3 - 3*b^5 - (2*a^4*b +
 a^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 2*(4*a^5*b - 5*a^3*b^3
+ 6*a*b^5)*cos(d*x + c) - (5*a^3*b^3 - 6*a*b^5 - (5*a^3*b^3 - 6*a*b^5)*cos(
d*x + c)^2 + (5*a^2*b^4 - 6*b^6 - (5*a^2*b^4 - 6*b^6)*cos(d*x + c)^2)*sin(d
*x + c))*log(1/2*cos(d*x + c) + 1/2) + (5*a^3*b^3 - 6*a*b^5 - (5*a^3*b^3 -
6*a*b^5)*cos(d*x + c)^2 + (5*a^2*b^4 - 6*b^6 - (5*a^2*b^4 - 6*b^6)*cos(d*x
```

```
+ c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*a^5*b*d*x*cos(d*x
 + c)^2 + 2*a^4*b^2*cos(d*x + c)^3 - 4*a^5*b*d*x - (2*a^4*b^2 + 3*a^2*b^4)*
cos(d*x + c))*sin(d*x + c))/(a^5*b^3*d*cos(d*x + c)^2 - a^5*b^3*d + (a^4*b^
4*d*cos(d*x + c)^2 - a^4*b^4*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27648, size = 625, normalized size = 2.49

$$\frac{16\,(dx+c)a}{b^3} - \frac{4\left(5\,a^2-6\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4} + \frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4} + \frac{30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab\tan\left(\frac{1}{2}\,dx+\right.}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(16*(d*x + c)*a/b^3 - 4*(5*a^2 - 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/
a^4 + (a^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 + (30*a
^2*tan(1/2*d*x + 1/2*c)^2 - 36*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d
*x + 1/2*c) - a^2)/(a^4*tan(1/2*d*x + 1/2*c)^2) - 16*(2*a^6 - a^4*b^2 - 4*a
^2*b^4 + 3*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/
2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4*b^3) + 16*(a^4*b
*tan(1/2*d*x + 1/2*c)^3 - 2*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + b^5*tan(1/2*d*
x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c)^2 - 2*a^3*b^2*tan(1/2*d*x + 1/2*c
)^2 + a*b^4*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b
^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + 2*a^5 - 2*a^3*b^2 + a*
b^4)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*
d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^4*b^2))/d
```

3.1262 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=287

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2d}+\frac{4\left(-3a^2b^4+a^6+2b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5b^2d\sqrt{a^2-b^2}}+\frac{3\left(a^2-b^2\right)\cot(c+dx)}{a^4d}-\frac{(}{a^4}$$

```
[Out] -(x/b^2) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(a^3*b^2*d) + (4*(a^6 - 3*a^2*b^4 + 2*b^6)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^5*b^2*Sqrt[a^2 - b^2]*d) + (b*ArcTanh[Cos[c + d
*x]])/(a^3*d) - (2*b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) - Cot[c
 + d*x]/(a^2*d) + (3*(a^2 - b^2)*Cot[c + d*x])/(a^4*d) - Cot[c + d*x]^3/(3*
a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - ((a^2 - b^2)^2*Cos[c + d*x
])/(a^4*b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.376761, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2897, 3770, 3767, 8, 3768, 2664, 12, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2d}+\frac{4\left(-3a^2b^4+a^6+2b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5b^2d\sqrt{a^2-b^2}}+\frac{3\left(a^2-b^2\right)\cot(c+dx)}{a^4d}-\frac{(}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -(x/b^2) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(a^3*b^2*d) + (4*(a^6 - 3*a^2*b^4 + 2*b^6)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^5*b^2*Sqrt[a^2 - b^2]*d) + (b*ArcTanh[Cos[c + d
*x]])/(a^3*d) - (2*b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) - Cot[c
 + d*x]/(a^2*d) + (3*(a^2 - b^2)*Cot[c + d*x])/(a^4*d) - Cot[c + d*x]^3/(3*
a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - ((a^2 - b^2)^2*Cos[c + d*x
])/(a^4*b*d*(a + b*Sin[c + d*x]))
```


Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{1}{b^2}+\frac{2\left(3a^2b-2b^3\right)\csc(c+dx)}{a^5}-\frac{3\left(a^2-b^2\right)\csc^2(c+dx)}{a^4}-\frac{2b\csc^3(c+dx)}{a^3}\right.$$

$$= -\frac{x}{b^2}+\frac{\int\csc^4(c+dx)\,dx}{a^2}-\frac{(2b)\int\csc^3(c+dx)\,dx}{a^3}+\frac{\left(2b\left(3a^2-2b^2\right)\right)\int\csc(c+dx)}{a^5}$$

$$= -\frac{x}{b^2}-\frac{2b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{b\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{\left(a^2-b^2\right)}{a^4bd(a+}$$

$$= -\frac{x}{b^2}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{\cot(c+dx)}{a^2d}$$

$$= -\frac{x}{b^2}+\frac{4\left(a^6-3a^2b^4+2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^2\sqrt{a^2-b^2}d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2b(}{}$$

$$= -\frac{x}{b^2}+\frac{4\left(a^6-3a^2b^4+2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^2\sqrt{a^2-b^2}d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2b(}{}$$

$$= -\frac{x}{b^2}-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^2d}+\frac{4\left(a^6-3a^2b^4+2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.}{a^5b^2\sqrt{a^2-b^2}d}$$

Mathematica [A] time = 6.27399, size = 428, normalized size = 1.49

$$\frac{\left(5a^2b-4b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d}+\frac{\left(4b^3-5a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d}+\frac{2a^2b^2\cos(c+dx)+a^4(-\cos(c+dx))-b^4}{a^4bd(a+b\sin(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((c + d*x)/(b^2*d)) + (2*(a^2 - b^2)^(3/2)*(a^2 + 4*b^2)*ArcTan[(Sec[(c +
d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*b
^2*d) + ((7*a^2*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2]
)/(6*a^4*d) + (b*Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c +
 d*x)/2]^2)/(24*a^2*d) + ((-5*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d)
 + ((5*a^2*b - 4*b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^
2)/(4*a^3*d) + (Sec[(c + d*x)/2]*(-7*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c +
d*x)/2]))/(6*a^4*d) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*C
os[c + d*x])/(a^4*b*d*(a + b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(24*a^2*d)
```

Maple [B] time = 0.19, size = 678, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*tan(1/2*d*x+1/2*c)^2*b-9/8/d/a^2*
tan(1/2*d*x+1/2*c)+3/2/d/a^4*b^2*tan(1/2*d*x+1/2*c)-2/d/b^2*arctan(tan(1/2*
d*x+1/2*c))-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2
*d*x+1/2*c)+4/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*t
an(1/2*d*x+1/2*c)-2/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)*tan(1/2*d*x+1/2*c)-2/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)+4/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-2/d*b^3/a^
4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+2/d/b^2*a/(a^2-b^2)^(1/
2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-14/d/a^3/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+
8/d*b^4/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))-1/24/d/a^2/tan(1/2*d*x+1/2*c)^3+9/8/d/a^2/tan(1/2*d*x+1/2*c)-3/2/
d/a^4/tan(1/2*d*x+1/2*c)*b^2+1/4/d/a^3*b/tan(1/2*d*x+1/2*c)^2+5/d/a^3*b*ln(
tan(1/2*d*x+1/2*c))-4/d/a^5*b^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.75213, size = 3244, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/6*(6*a^5*b*d*x*cos(d*x + c)^4 - 12*a^5*b*d*x*cos(d*x + c)^2 + 6*a^5*b*d
*x + 2*(7*a^4*b^2 - 6*a^2*b^4)*cos(d*x + c)^3 + 3*(a^4*b + 3*a^2*b^3 - 4*b^
5 + (a^4*b + 3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 3*a^2*b^3 - 4*b
^5)*cos(d*x + c)^2 + (a^5 + 3*a^3*b^2 - 4*a*b^4 - (a^5 + 3*a^3*b^2 - 4*a*b^
4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c)
+ b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2)) - 12*(a^4*b^2 - a^2*b^4)*cos(d*x + c) + 3*(5*a^2*b^4 - 4*b^
6 + (5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)
^2 + (5*a^3*b^3 - 4*a*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x +
 c))*log(1/2*cos(d*x + c) + 1/2) - 3*(5*a^2*b^4 - 4*b^6 + (5*a^2*b^4 - 4*b^
6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)^2 + (5*a^3*b^3 - 4*a
*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x
 + c) + 1/2) - 2*(3*a^6*d*x*cos(d*x + c)^2 - 3*a^6*d*x + (3*a^5*b - 13*a^3*
b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x +
c))*sin(d*x + c))/(a^5*b^3*d*cos(d*x + c)^4 - 2*a^5*b^3*d*cos(d*x + c)^2 +
a^5*b^3*d - (a^6*b^2*d*cos(d*x + c)^2 - a^6*b^2*d)*sin(d*x + c)), -1/6*(6*a
^5*b*d*x*cos(d*x + c)^4 - 12*a^5*b*d*x*cos(d*x + c)^2 + 6*a^5*b*d*x + 2*(7*
a^4*b^2 - 6*a^2*b^4)*cos(d*x + c)^3 + 6*(a^4*b + 3*a^2*b^3 - 4*b^5 + (a^4*b
```

```
 + 3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 3*a^2*b^3 - 4*b^5)*cos(d*
x + c)^2 + (a^5 + 3*a^3*b^2 - 4*a*b^4 - (a^5 + 3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a
^2 - b^2)*cos(d*x + c))) - 12*(a^4*b^2 - a^2*b^4)*cos(d*x + c) + 3*(5*a^2*b
^4 - 4*b^6 + (5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos
(d*x + c)^2 + (5*a^3*b^3 - 4*a*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*
sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(5*a^2*b^4 - 4*b^6 + (5*a^2*b
^4 - 4*b^6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)^2 + (5*a^3*
b^3 - 4*a*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2) - 2*(3*a^6*d*x*cos(d*x + c)^2 - 3*a^6*d*x + (3*a^5*b
- 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*c
os(d*x + c))*sin(d*x + c))/(a^5*b^3*d*cos(d*x + c)^4 - 2*a^5*b^3*d*cos(d*x
+ c)^2 + a^5*b^3*d - (a^6*b^2*d*cos(d*x + c)^2 - a^6*b^2*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27628, size = 539, normalized size = 1.88

$$-\frac{\frac{24\,(dx+c)}{b^2}-\frac{24\left(5\,a^2b-4\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}-\frac{48\,(a^6 \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/b^2 - 24*(5*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^5 - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 27*
a^4*tan(1/2*d*x + 1/2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 - 48*(a^6 +
 2*a^4*b^2 - 7*a^2*b^4 + 4*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5*
b^2) + 48*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^
5*tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^
2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5*b) + (220*a^2*b*tan(1/2*d*x + 1/2*c)^
3 - 176*b^3*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b
^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/
2*d*x + 1/2*c)^3))/d
```

3.1263 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=303

$$-\frac{10b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(-20a^2b^2+3a^4+15b^4\right)\cot(c+dx)}{3a^5bd}-\frac{5\left(-12a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\cos(c}{8a^6d}$$

```
[Out] (-10*b*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^6*d) - (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d)
+ ((3*a^4 - 20*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(3*a^5*b*d) + (5*(5*a^2 - 4*
b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) - Cot[c + d*x]/(b*d*(a + b*Sin[c
+ d*x])) - ((6*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^3*d*(a + b*Sin[
c + d*x])) + (5*b*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^2*d*(a + b*Sin[c + d*x
])) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.21099, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{10b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(-20a^2b^2+3a^4+15b^4\right)\cot(c+dx)}{3a^5bd}-\frac{5\left(-12a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\cos(c}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-10*b*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^6*d) - (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d)
+ ((3*a^4 - 20*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(3*a^5*b*d) + (5*(5*a^2 - 4*
b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) - Cot[c + d*x]/(b*d*(a + b*Sin[c
+ d*x])) - ((6*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^3*d*(a + b*Sin[
c + d*x])) + (5*b*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^2*d*(a + b*Sin[c + d*x
])) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d*(a + b*Sin[c + d*x]))
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
```

```
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)}{bd(a+b\sin(c+dx))} + \frac{5b\cot(c+dx)\csc^2(c+dx)}{12a^2d(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))} + \int \frac{\text{cs}}{-}$$

$$= -\frac{\cot(c+dx)}{bd(a+b\sin(c+dx))} - \frac{(6a^2-5b^2)\cot(c+dx)\csc(c+dx)}{3a^3d(a+b\sin(c+dx))} + \frac{5b\cot(c+dx)\csc^2(c+}{12a^2d(a+b\sin(c+d}$$

$$= \frac{5(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\cot(c+dx)}{bd(a+b\sin(c+dx))} - \frac{(6a^2-5b^2)\cot(c+dx)}{3a^3d(a+b\sin(c}$$

$$= \frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{3a^5bd} + \frac{5(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\cot(c}{bd(a+b\,\mathrm{s}}$$

$$= \frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{3a^5bd} + \frac{5(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\cot(c}{bd(a+b\,\mathrm{s}}$$

$$= -\frac{5(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{3a^5bd} + \frac{5}{-}$$

$$= -\frac{5(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{3a^5bd} + \frac{5}{-}$$

$$= -\frac{10b(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} - \frac{5(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}$$

Mathematica [A] time = 6.20382, size = 487, normalized size = 1.61

$$\frac{3(3a^2-4b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^4d} - \frac{3(3a^2-4b^2)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^4d} + \frac{5(-12a^2b^2+3a^4+8b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^6d} - \frac{5(-1}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-10*b*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*S
in[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*d) + ((-7*a^2*b*Cos[(c + d*x)/2] +
 6*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (3*(3*a^2 - 4*b^2)*C
sc[(c + d*x)/2]^2)/(32*a^4*d) + (b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(12
*a^3*d) - Csc[(c + d*x)/2]^4/(64*a^2*d) - (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*L
og[Cos[(c + d*x)/2]])/(8*a^6*d) + (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*Log[Sin[(
c + d*x)/2]])/(8*a^6*d) - (3*(3*a^2 - 4*b^2)*Sec[(c + d*x)/2]^2)/(32*a^4*d)
 + Sec[(c + d*x)/2]^4/(64*a^2*d) + (Sec[(c + d*x)/2]*(7*a^2*b*Sin[(c + d*x)
/2] - 6*b^3*Sin[(c + d*x)/2]))/(3*a^5*d) + (a^4*Cos[c + d*x] - 2*a^2*b^2*Co
s[c + d*x] + b^4*Cos[c + d*x])/(a^5*d*(a + b*Sin[c + d*x])) - (b*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/(12*a^3*d)
```

Maple [B] time = 0.187, size = 718, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2*tan(1/2*d*x+1/2*c)^4-1/12/d/a^3*tan(1/2*d*x+1/2*c)^3*b-1/4/d/a^2
*tan(1/2*d*x+1/2*c)^2+3/8/d/a^4*b^2*tan(1/2*d*x+1/2*c)^2+9/4/d/a^3*tan(1/2*
d*x+1/2*c)*b-2/d/a^5*b^3*tan(1/2*d*x+1/2*c)+2/d/a^2*b/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-4/d/a^4/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^3+2/d*b^5/a^6/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)-4/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)*b^2+2/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)-10/d*b/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))+20/d*b^3/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-10/d*b^5/a^6/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/64/d/a^2/tan(1/2*d*x+1/2*c)
^4+1/4/d/a^2/tan(1/2*d*x+1/2*c)^2-3/8/d/a^4/tan(1/2*d*x+1/2*c)^2*b^2+15/8/d
/a^2*ln(tan(1/2*d*x+1/2*c))-15/2/d/a^4*ln(tan(1/2*d*x+1/2*c))*b^2+5/d/a^6*l
n(tan(1/2*d*x+1/2*c))*b^4+1/12/d/a^3*b/tan(1/2*d*x+1/2*c)^3-9/4/d*b/a^3/tan
(1/2*d*x+1/2*c)+2/d*b^3/a^5/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.28851, size = 3613, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/48*(16*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 10*(15*a^5 - 68*
a^3*b^2 + 48*a*b^4)*cos(d*x + c)^3 - 120*((a^3*b - a*b^3)*cos(d*x + c)^4 +
a^3*b - a*b^3 - 2*(a^3*b - a*b^3)*cos(d*x + c)^2 + ((a^2*b^2 - b^4)*cos(d*x
 + c)^4 + a^2*b^2 - b^4 - 2*(a^2*b^2 - b^4)*cos(d*x + c)^2)*sin(d*x + c))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(3*a^5 - 12*
a^3*b^2 + 8*a*b^4)*cos(d*x + c) - 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 + (3*a^5
 - 12*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a*b^4)*
cos(d*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*
b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*sin(
d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 +
(3*a^5 - 12*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a
*b^4)*cos(d*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^
3 + 8*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2
```

```
)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 10*((17*a^4*b - 12*a^2*b^3)*
cos(d*x + c)^3 - 3*(5*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d
*cos(d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4
- 2*a^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x + c)), 1/48*(16*(3*a^5 - 20*a
^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 10*(15*a^5 - 68*a^3*b^2 + 48*a*b^4)*cos
(d*x + c)^3 + 240*((a^3*b - a*b^3)*cos(d*x + c)^4 + a^3*b - a*b^3 - 2*(a^3*
b - a*b^3)*cos(d*x + c)^2 + ((a^2*b^2 - b^4)*cos(d*x + c)^4 + a^2*b^2 - b^4
 - 2*(a^2*b^2 - b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(3*a^5 - 12*a^3*b
^2 + 8*a*b^4)*cos(d*x + c) - 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 + (3*a^5 - 12
*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a*b^4)*cos(d
*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*
cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*sin(d*x +
 c))*log(1/2*cos(d*x + c) + 1/2) + 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 + (3*a^
5 - 12*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a*b^4)
*cos(d*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8
*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*sin
(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 10*((17*a^4*b - 12*a^2*b^3)*cos(d
*x + c)^3 - 3*(5*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(
d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a
^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+b*sin(d*x+c))**2,x)

[Out] Timed out
```

Giac [A] time = 1.29556, size = 641, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/192*(120*(3*a^4 - 12*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6
- 1920*(a^4*b - 2*a^2*b^3 + b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
 arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6
) + 384*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*
tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)*a^6) + (3*a^6*tan(1/2*d*x + 1/2*c)^4 - 16*a
^5*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^6*tan(1/2*d*x + 1/2*c)^2 + 72*a^4*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 432*a^5*b*tan(1/2*d*x + 1/2*c) - 384*a^3*b^3*tan(1/2
*d*x + 1/2*c))/a^8 - (750*a^4*tan(1/2*d*x + 1/2*c)^4 - 3000*a^2*b^2*tan(1/2
*d*x + 1/2*c)^4 + 2000*b^4*tan(1/2*d*x + 1/2*c)^4 + 432*a^3*b*tan(1/2*d*x +
 1/2*c)^3 - 384*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 48*a^4*tan(1/2*d*x + 1/2*c)^
2 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 16*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a
^4)/(a^6*tan(1/2*d*x + 1/2*c)^4))/d
```

3.1264 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=424

$$-\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{\left(-135a^2b^2+38a^4+90b^4\right)\cot(c+dx)}{15a^6d}+\frac{b\left(-40a^2b^2+15a^4+24b\right.}{4a}$$

```
[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]
])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) +
((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((1
5*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) -
(Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b
^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*
Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x
]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.52088, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{\left(-135a^2b^2+38a^4+90b^4\right)\cot(c+dx)}{15a^6d}+\frac{b\left(-40a^2b^2+15a^4+24b\right.}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]
])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) +
((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((1
5*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) -
(Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b
^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*
Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x
]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))
```


Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
```

```
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))} - \frac{\text{c}}{5}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{\left(2a^4-12a^2b^2+9b^4\right)\cot(c+dx}{6a^3b^2d(a+b\sin(c+}$$

$$= -\frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+}{6b^2d(a+}$$

$$= \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx)\csc^2}{30a^4b^2d}$$

$$= -\frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{4a^5bd} -$$

$$= -\frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{4a^5bd} -$$

$$= \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} +$$

$$= \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} +$$

$$= -\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos}{4a^7d}$$

Mathematica [A] time = 1.56744, size = 361, normalized size = 0.85

$$-\frac{1920\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 240b\left(-40a^2b^2+15a^4+24b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 240b\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(1920*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]] - 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Cos[(c + d*x)/2]] +
 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Cot[c +
d*x]*Csc[c + d*x]^5*(196*a^5 - 735*a^3*b^2 + 540*a*b^4 - 12*(16*a^5 - 85*a^
3*b^2 + 60*a*b^4)*Cos[2*(c + d*x)] + (92*a^5 - 285*a^3*b^2 + 180*a*b^4)*Cos
[4*(c + d*x)] + 1162*a^4*b*Sin[c + d*x] - 3060*a^2*b^3*Sin[c + d*x] + 1800*
b^5*Sin[c + d*x] - 562*a^4*b*Sin[3*(c + d*x)] + 1470*a^2*b^3*Sin[3*(c + d*x
)] - 900*b^5*Sin[3*(c + d*x)] + 76*a^4*b*Sin[5*(c + d*x)] - 270*a^2*b^3*Sin
[5*(c + d*x)] + 180*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x]))/(960*a^7*d
)
```

Maple [B] time = 0.19, size = 897, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)^2*b-27/8/d/a^4*b^2*tan(1/2*d*x+1/2*c)+4/d*b^3/
a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+27/8/d/a^4/tan(1/2*d*
x+1/2*c)*b^2-1/2/d/a^3*b/tan(1/2*d*x+1/2*c)^2+10/d/a^5*b^3*ln(tan(1/2*d*x+1
/2*c))+11/16/d/a^2*tan(1/2*d*x+1/2*c)-1/32/d/a^3*tan(1/2*d*x+1/2*c)^4*b+1/8
/d/a^4*tan(1/2*d*x+1/2*c)^3*b^2-1/8/d/a^4/tan(1/2*d*x+1/2*c)^3*b^2-5/2/d/a^
6/tan(1/2*d*x+1/2*c)*b^4+1/32/d/a^3*b/tan(1/2*d*x+1/2*c)^4+1/2/d*b^3/a^5/ta
n(1/2*d*x+1/2*c)^2-6/d/a^7*b^5*ln(tan(1/2*d*x+1/2*c))-1/2/d/a^5*tan(1/2*d*x
+1/2*c)^2*b^3+5/2/d/a^6*b^4*tan(1/2*d*x+1/2*c)-2/d/a^6/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^5-7/96/d/a^2*tan(1/2*d*x+1/2*c)^3+7/96/d/a^
2/tan(1/2*d*x+1/2*c)^3-2/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)*tan(1/2*d*x+1/2*c)+1/160/d/a^2*tan(1/2*d*x+1/2*c)^5-1/160/d/a^2/t
an(1/2*d*x+1/2*c)^5-11/16/d/a^2/tan(1/2*d*x+1/2*c)-15/4/d/a^3*b*ln(tan(1/2*
d*x+1/2*c))-2/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1
/2*d*x+1/2*c)*b^6+12/d/a^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))*b^6-2/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))+4/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)*tan(1/2*d*x+1/2*c)+16/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-26/d*b^4/a^5/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.22019, size = 4674, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/120*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 -
 27*a^4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 60*((a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^
3*b^2 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a
^3*b^2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^
2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c
)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2)) + 30*(4*a^6 - 17*a^4*b^2 + 12*a^2*b^4)*cos(d
```

```
*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 - 15*a^4*b^
2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5*b - 40*a
^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^4 - 2*(
15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos
(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 -
15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5
*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c
)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*lo
g(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 90*a*b^5)*cos(d
*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x + c)^3 + 15*(15*
a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b*d*cos(d*x
 + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2 - a^7*b*d - (
a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*x + c)), 1/120
*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 - 27*a^
4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 120*((a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^3*b^2
 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a^3*b^
2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d
*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(4*a^6 - 17*a^4*b^2 + 12*
a^2*b^4)*cos(d*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6
)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (1
5*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*
x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4
 + 24*b^6)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x +
c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^
5)*cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 9
0*a*b^5)*cos(d*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x +
c)^3 + 15*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a
^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2
 - a^7*b*d - (a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*
x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24538, size = 805, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/480*(120*(15*a^4*b - 40*a^2*b^3 + 24*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))
/a^7 + 960*(a^6 - 8*a^4*b^2 + 13*a^2*b^4 - 6*b^6)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sq
rt(a^2 - b^2)*a^7) + 960*(a^4*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b^4*tan(1/2*
d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c) + a^5*b - 2*a^3*b^3 + a*b^5)/((a*ta
n(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^7) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^8*tan(1/2*d*x + 1/2
*c)^3 + 60*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^7*b*tan(1/2*d*x + 1/2*c)^
2 - 240*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 330*a^8*tan(1/2*d*x + 1/2*c) - 162
0*a^6*b^2*tan(1/2*d*x + 1/2*c) + 1200*a^4*b^4*tan(1/2*d*x + 1/2*c))/a^10 -
(4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 10960*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 +
 6576*b^5*tan(1/2*d*x + 1/2*c)^5 - 330*a^5*tan(1/2*d*x + 1/2*c)^4 + 1620*a^
3*b^2*tan(1/2*d*x + 1/2*c)^4 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^4*
b*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 35*a^5*tan(
1/2*d*x + 1/2*c)^2 - 60*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*tan(1/2*d
*x + 1/2*c) - 3*a^5)/(a^7*tan(1/2*d*x + 1/2*c)^5))/d
```

3.1265 $\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=480

$$\frac{2b\left(2a^2-7b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{b\left(-170a^2b^2+61a^4+105b^4\right)\cot(c+dx)}{15a^7d}+\frac{\left(-90a^4b^2+200a^2b^4\right.}{}$$

```
[Out] (2*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]])/(a^8*d) + ((5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*ArcTa
nh[Cos[c + d*x]])/(16*a^8*d) + (b*(61*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c +
d*x])/(15*a^7*d) - ((27*a^4 - 86*a^2*b^2 + 56*b^4)*Cot[c + d*x]*Csc[c + d*x
])/(16*a^6*d) + ((15*a^4 - 52*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(15*a^5*b*d) - ((16*a^4 - 61*a^2*b^2 + 42*b^4)*Cot[c + d*x]*Csc[c + d*x]^
3)/(24*a^4*b^2*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*b*d*(a + b*Sin[c + d*x
])) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((5*
a^4 - 20*a^2*b^2 + 14*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^3*b^2*d*(a +
b*Sin[c + d*x])) + (7*b*Cot[c + d*x]*Csc[c + d*x]^4)/(30*a^2*d*(a + b*Sin[c
 + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.9592, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\left(2a^2-7b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{b\left(-170a^2b^2+61a^4+105b^4\right)\cot(c+dx)}{15a^7d}+\frac{\left(-90a^4b^2+200a^2b^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (2*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]])/(a^8*d) + ((5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*ArcTa
nh[Cos[c + d*x]])/(16*a^8*d) + (b*(61*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c +
d*x])/(15*a^7*d) - ((27*a^4 - 86*a^2*b^2 + 56*b^4)*Cot[c + d*x]*Csc[c + d*x
])/(16*a^6*d) + ((15*a^4 - 52*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(15*a^5*b*d) - ((16*a^4 - 61*a^2*b^2 + 42*b^4)*Cot[c + d*x]*Csc[c + d*x]^
3)/(24*a^4*b^2*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*b*d*(a + b*Sin[c + d*x
])) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((5*
a^4 - 20*a^2*b^2 + 14*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^3*b^2*d*(a +
b*Sin[c + d*x])) + (7*b*Cot[c + d*x]*Csc[c + d*x]^4)/(30*a^2*d*(a + b*Sin[c
 + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d*(a + b*Sin[c + d*x]))
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
```

```
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)\csc^2(c+dx)}{3bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^3(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{7b\cot(c+dx)\csc^4(c+dx}{30a^2d(a+b\sin(c+dx))}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^3(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{\left(5a^4-20a^2b^2+14b^4\right)\cot}{10a^3b^2d(a+b\,\mathrm{s}}$$

$$= -\frac{\left(16a^4-61a^2b^2+42b^4\right)\cot(c+dx)\csc^3(c+dx)}{24a^4b^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3bd(a+b\sin(c+dx))} + \frac{a\,\mathrm{co}}{6b}$$

$$= \frac{\left(15a^4-52a^2b^2+35b^4\right)\cot(c+dx)\csc^2(c+dx)}{15a^5bd} - \frac{\left(16a^4-61a^2b^2+42b^4\right)\cot(c+a}{24a^4b^2d}$$

$$= -\frac{\left(27a^4-86a^2b^2+56b^4\right)\cot(c+dx)\csc(c+dx)}{16a^6d} + \frac{\left(15a^4-52a^2b^2+35b^4\right)\cot(c+}{15a^5bd}$$

$$= \frac{b\left(61a^4-170a^2b^2+105b^4\right)\cot(c+dx)}{15a^7d} - \frac{\left(27a^4-86a^2b^2+56b^4\right)\cot(c+dx)\csc(c}{16a^6d}$$

$$= \frac{b\left(61a^4-170a^2b^2+105b^4\right)\cot(c+dx)}{15a^7d} - \frac{\left(27a^4-86a^2b^2+56b^4\right)\cot(c+dx)\csc(c}{16a^6d}$$

$$= \frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{b\left(61a^4-170a^2b^2+105b^4}{15a^7d}$$

$$= \frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{b\left(61a^4-170a^2b^2+105b^4}{15a^7d}$$

$$= \frac{2b\left(2a^2-7b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{\left(5a^6-90a^4b^2+200a^2b^4-112\right.}{16a^8d}$$

Mathematica [A] time = 1.6191, size = 447, normalized size = 0.93

$$\frac{15360b\left(2a^2-7b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+480\left(90a^4b^2-200a^2b^4-5a^6+112b^6\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (15360*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]] + 480*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*Log[Cos
[(c + d*x)/2]] + 480*(-5*a^6 + 90*a^4*b^2 - 200*a^2*b^4 + 112*b^6)*Log[Sin[
(c + d*x)/2]] - (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(590*a^6 - 6956*a^4*b^2 +
15280*a^2*b^4 - 8400*b^6 - 8*(35*a^6 - 1289*a^4*b^2 + 2830*a^2*b^4 - 1575*b
^6)*Cos[2*(c + d*x)] + (330*a^6 - 3844*a^4*b^2 + 8720*a^2*b^4 - 5040*b^6)*C
os[4*(c + d*x)] + 488*a^4*b^2*Cos[6*(c + d*x)] - 1360*a^2*b^4*Cos[6*(c + d*
x)] + 840*b^6*Cos[6*(c + d*x)] - 3942*a^5*b*Sin[c + d*x] + 12620*a^3*b^3*Si
n[c + d*x] - 8400*a*b^5*Sin[c + d*x] + 1967*a^5*b*Sin[3*(c + d*x)] - 6590*a
^3*b^3*Sin[3*(c + d*x)] + 4200*a*b^5*Sin[3*(c + d*x)] - 571*a^5*b*Sin[5*(c
+ d*x)] + 1430*a^3*b^3*Sin[5*(c + d*x)] - 840*a*b^5*Sin[5*(c + d*x)]))/(b +
 a*Csc[c + d*x]))/(7680*a^8*d)
```

Maple [B] time = 0.198, size = 1048, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 3/128/d/a^2/tan(1/2*d*x+1/2*c)^4-5/16/d/a^2*ln(tan(1/2*d*x+1/2*c))+7/48/d/a
^3*tan(1/2*d*x+1/2*c)^3*b+1/384/d/a^2*tan(1/2*d*x+1/2*c)^6-1/384/d/a^2/tan(
1/2*d*x+1/2*c)^6+15/128/d/a^2*tan(1/2*d*x+1/2*c)^2-15/128/d/a^2/tan(1/2*d*x
+1/2*c)^2+4/d*b/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))+32/d*b^5/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+
1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/128/d/a^2*tan(1/2*d*x+1/2*c)^4+1/6/d/a^5*b^3
/tan(1/2*d*x+1/2*c)^3+3/d*b^5/a^7/tan(1/2*d*x+1/2*c)-1/80/d/a^3*b*tan(1/2*d
*x+1/2*c)^5+3/64/d/a^4*tan(1/2*d*x+1/2*c)^4*b^2+7/d/a^8*ln(tan(1/2*d*x+1/2*
c))*b^6-3/64/d/a^4/tan(1/2*d*x+1/2*c)^4*b^2-5/8/d/a^6/tan(1/2*d*x+1/2*c)^2*
b^4-1/6/d/a^5*tan(1/2*d*x+1/2*c)^3*b^3+5/8/d/a^6*tan(1/2*d*x+1/2*c)^2*b^4-3
/d/a^7*b^5*tan(1/2*d*x+1/2*c)+2/d/a^7*b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)+1/80/d/a^3*b/tan(1/2*d*x+1/2*c)^5-11/8/d/a^3*tan(1/2*d*x+1
/2*c)*b+2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2+45/8/
d/a^4*ln(tan(1/2*d*x+1/2*c))*b^2+11/8/d*b/a^3/tan(1/2*d*x+1/2*c)+2/d/a^8*b^
7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)-14/d
/a^8*b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^
(1/2))-22/d*b^3/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))+2/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)
*tan(1/2*d*x+1/2*c)*b^3-4/d*b^5/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)*tan(1/2*d*x+1/2*c)-3/4/d/a^4*b^2*tan(1/2*d*x+1/2*c)^2+9/2/d/a^5*
b^3*tan(1/2*d*x+1/2*c)-4/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)+3/4/d/a^4/tan(1/2*d*x+1/2*c)^2*b^2-25/2/d/a^6*ln(tan(1/2*d*x+1/2*
c))*b^4-7/48/d/a^3*b/tan(1/2*d*x+1/2*c)^3-9/2/d*b^3/a^5/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.74573, size = 6056, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

[Out] [1/480*(32*(61*a^5*b^2 - 170*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^7 + 2*(165*a
^7 - 3386*a^5*b^2 + 8440*a^3*b^4 - 5040*a*b^6)*cos(d*x + c)^5 - 80*(5*a^7 -
94*a^5*b^2 + 218*a^3*b^4 - 126*a*b^6)*cos(d*x + c)^3 + 240*((2*a^5*b - 9*a
^3*b^3 + 7*a*b^5)*cos(d*x + c)^6 - 2*a^5*b + 9*a^3*b^3 - 7*a*b^5 - 3*(2*a^5
*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^4 + 3*(2*a^5*b - 9*a^3*b^3 + 7*a*b^5
)*cos(d*x + c)^2 + ((2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^6 - 2*a^4*
b^2 + 9*a^2*b^4 - 7*b^6 - 3*(2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^4
+ 3*(2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
- 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(5*a^7 - 90*a^5*b^2
+ 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c) - 15*(5*a^7 - 90*a^5*b^2 + 200*a^3*
b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^4 - 3*
(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5*a^6*b -
90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*cos(d*x +
c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(5*a^7 - 90*a^5*b^2 +
200*a^3*b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*c
os(d*x + c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^4 - 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5
*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a
^2*b^5 - 112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((571*a^6*b
- 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c)^5 - 40*(23*a^6*b - 68*a^4*b^3 +
42*a^2*b^5)*cos(d*x + c)^3 + 15*(27*a^6*b - 86*a^4*b^3 + 56*a^2*b^5)*cos(d*
x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c)^6 - 3*a^9*d*cos(d*x + c)^4 + 3*a^
9*d*cos(d*x + c)^2 - a^9*d + (a^8*b*d*cos(d*x + c)^6 - 3*a^8*b*d*cos(d*x +
c)^4 + 3*a^8*b*d*cos(d*x + c)^2 - a^8*b*d)*sin(d*x + c)), 1/480*(32*(61*a^5
*b^2 - 170*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^7 + 2*(165*a^7 - 3386*a^5*b^2
+ 8440*a^3*b^4 - 5040*a*b^6)*cos(d*x + c)^5 - 80*(5*a^7 - 94*a^5*b^2 + 218*
a^3*b^4 - 126*a*b^6)*cos(d*x + c)^3 - 480*((2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*
cos(d*x + c)^6 - 2*a^5*b + 9*a^3*b^3 - 7*a*b^5 - 3*(2*a^5*b - 9*a^3*b^3 + 7
*a*b^5)*cos(d*x + c)^4 + 3*(2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2 +
((2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^6 - 2*a^4*b^2 + 9*a^2*b^4 -
7*b^6 - 3*(2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^4 + 3*(2*a^4*b^2 - 9
*a^2*b^4 + 7*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(5*a^7 - 90*a^5*b^2
+ 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c) - 15*(5*a^7 - 90*a^5*b^2 + 200*a^3*
b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^4 - 3*
(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5*a^6*b -
90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*cos(d*x +
c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(5*a^7 - 90*a^5*b^2 +
200*a^3*b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*c
os(d*x + c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^4 - 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5
*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a
^2*b^5 - 112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((571*a^6*b
- 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c)^5 - 40*(23*a^6*b - 68*a^4*b^3 +
42*a^2*b^5)*cos(d*x + c)^3 + 15*(27*a^6*b - 86*a^4*b^3 + 56*a^2*b^5)*cos(d*
x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c)^6 - 3*a^9*d*cos(d*x + c)^4 + 3*a^
9*d*cos(d*x + c)^2 - a^9*d + (a^8*b*d*cos(d*x + c)^6 - 3*a^8*b*d*cos(d*x +

```
c)^4 + 3*a^8*b*d*cos(d*x + c)^2 - a^8*b*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34454, size = 994, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1920*(120*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*log(abs(tan(1/2*d
*x + 1/2*c)))/a^8 - 3840*(2*a^6*b - 11*a^4*b^3 + 16*a^2*b^5 - 7*b^7)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sq
rt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^8) - 3840*(a^4*b^3*tan(1/2*d*x + 1/2*c)
- 2*a^2*b^5*tan(1/2*d*x + 1/2*c) + b^7*tan(1/2*d*x + 1/2*c) + a^5*b^2 - 2*a
^3*b^4 + a*b^6)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*
a^8) - (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^9*b*tan(1/2*d*x + 1/2*c)^5 - 4
5*a^10*tan(1/2*d*x + 1/2*c)^4 + 90*a^8*b^2*tan(1/2*d*x + 1/2*c)^4 + 280*a^9
*b*tan(1/2*d*x + 1/2*c)^3 - 320*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^10*t
an(1/2*d*x + 1/2*c)^2 - 1440*a^8*b^2*tan(1/2*d*x + 1/2*c)^2 + 1200*a^6*b^4*
tan(1/2*d*x + 1/2*c)^2 - 2640*a^9*b*tan(1/2*d*x + 1/2*c) + 8640*a^7*b^3*tan
(1/2*d*x + 1/2*c) - 5760*a^5*b^5*tan(1/2*d*x + 1/2*c))/a^12 - (1470*a^6*tan
(1/2*d*x + 1/2*c)^6 - 26460*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 + 58800*a^2*b^4*
tan(1/2*d*x + 1/2*c)^6 - 32928*b^6*tan(1/2*d*x + 1/2*c)^6 + 2640*a^5*b*tan(
1/2*d*x + 1/2*c)^5 - 8640*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 + 5760*a*b^5*tan(1
/2*d*x + 1/2*c)^5 - 225*a^6*tan(1/2*d*x + 1/2*c)^4 + 1440*a^4*b^2*tan(1/2*d
*x + 1/2*c)^4 - 1200*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 280*a^5*b*tan(1/2*d*x
 + 1/2*c)^3 + 320*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^6*tan(1/2*d*x + 1/2
*c)^2 - 90*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^5*b*tan(1/2*d*x + 1/2*c) -
 5*a^6)/(a^8*tan(1/2*d*x + 1/2*c)^6))/d
```

3.1266 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=536

$$-\frac{a\left(-985a^2b^2+840a^4+213b^4\right)\cos(c+dx)}{30b^8d}+\frac{a\sqrt{a^2-b^2}\left(-47a^2b^2+56a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{\left(-60a^2b^2\right.}{}$$

```
[Out] -((448*a^6 - 600*a^4*b^2 + 180*a^2*b^4 - 5*b^6)*x)/(16*b^9) + (a*Sqrt[a^2 -
 b^2]*(56*a^4 - 47*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(b^9*d) - (a*(840*a^4 - 985*a^2*b^2 + 213*b^4)*Cos[c + d*x])/(30
*b^8*d) + ((224*a^4 - 244*a^2*b^2 + 43*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*
b^7*d) - ((280*a^4 - 291*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(30
*a*b^6*d) + ((168*a^4 - 169*a^2*b^2 + 24*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/
(24*a^2*b^5*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d*(a + b*Sin[c + d*x])^
2) - (b*Cos[c + d*x]*Sin[c + d*x]^5)/(10*a^2*d*(a + b*Sin[c + d*x])^2) - ((
56*a^4 - 60*a^2*b^2 + 9*b^4)*Cos[c + d*x]*Sin[c + d*x]^5)/(60*a^2*b^3*d*(a
+ b*Sin[c + d*x])^2) - (4*a*Cos[c + d*x]*Sin[c + d*x]^6)/(15*b^2*d*(a + b*S
in[c + d*x])^2) + (Cos[c + d*x]*Sin[c + d*x]^7)/(6*b*d*(a + b*Sin[c + d*x])
^2) - ((112*a^4 - 110*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(20*a^
2*b^4*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 2.19938, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2896, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(-985a^2b^2+840a^4+213b^4\right)\cos(c+dx)}{30b^8d}+\frac{a\sqrt{a^2-b^2}\left(-47a^2b^2+56a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{\left(-60a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -((448*a^6 - 600*a^4*b^2 + 180*a^2*b^4 - 5*b^6)*x)/(16*b^9) + (a*Sqrt[a^2 -
 b^2]*(56*a^4 - 47*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(b^9*d) - (a*(840*a^4 - 985*a^2*b^2 + 213*b^4)*Cos[c + d*x])/(30
*b^8*d) + ((224*a^4 - 244*a^2*b^2 + 43*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*
b^7*d) - ((280*a^4 - 291*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(30
*a*b^6*d) + ((168*a^4 - 169*a^2*b^2 + 24*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/
(24*a^2*b^5*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d*(a + b*Sin[c + d*x])^
2) - (b*Cos[c + d*x]*Sin[c + d*x]^5)/(10*a^2*d*(a + b*Sin[c + d*x])^2) - ((
56*a^4 - 60*a^2*b^2 + 9*b^4)*Cos[c + d*x]*Sin[c + d*x]^5)/(60*a^2*b^3*d*(a
+ b*Sin[c + d*x])^2) - (4*a*Cos[c + d*x]*Sin[c + d*x]^6)/(15*b^2*d*(a + b*S
in[c + d*x])^2) + (Cos[c + d*x]*Sin[c + d*x]^7)/(6*b*d*(a + b*Sin[c + d*x])
^2) - ((112*a^4 - 110*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(20*a^
2*b^4*d*(a + b*Sin[c + d*x]))
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
```

```
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^5(c+dx)}{10a^2d(a+b\sin(c+dx))^2} - \frac{4a\cos(c+dx)\sin^6(c+dx)}{15b^2d(a+b\sin(c+dx))^2}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^5(c+dx)}{10a^2d(a+b\sin(c+dx))^2} - \frac{\left(56a^4-60a^2b^2+9b^4\right)\cos(}{60a^2b^3d(a+b\sin}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^5(c+dx)}{10a^2d(a+b\sin(c+dx))^2} - \frac{\left(56a^4-60a^2b^2+9b^4\right)\cos(}{60a^2b^3d(a+b\sin}$$

$$= \frac{\left(168a^4-169a^2b^2+24b^4\right)\cos(c+dx)\sin^3(c+dx)}{24a^2b^5d} + \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\,c}{10}$$

$$= -\frac{\left(280a^4-291a^2b^2+45b^4\right)\cos(c+dx)\sin^2(c+dx)}{30ab^6d} + \frac{\left(168a^4-169a^2b^2+24b^4\right)c}{24a^2b^5}$$

$$= \frac{\left(224a^4-244a^2b^2+43b^4\right)\cos(c+dx)\sin(c+dx)}{16b^7d} - \frac{\left(280a^4-291a^2b^2+45b^4\right)\cos(}{30ab^6d}$$

$$= -\frac{a\left(840a^4-985a^2b^2+213b^4\right)\cos(c+dx)}{30b^8d} + \frac{\left(224a^4-244a^2b^2+43b^4\right)\cos(c+dx)}{16b^7d}$$

$$= -\frac{\left(448a^6-600a^4b^2+180a^2b^4-5b^6\right)x}{16b^9} - \frac{a\left(840a^4-985a^2b^2+213b^4\right)\cos(c+dx)}{30b^8d}$$

$$= -\frac{\left(448a^6-600a^4b^2+180a^2b^4-5b^6\right)x}{16b^9} - \frac{a\left(840a^4-985a^2b^2+213b^4\right)\cos(c+dx)}{30b^8d}$$

$$= -\frac{\left(448a^6-600a^4b^2+180a^2b^4-5b^6\right)x}{16b^9} - \frac{a\left(840a^4-985a^2b^2+213b^4\right)\cos(c+dx)}{30b^8d}$$

$$= -\frac{\left(448a^6-600a^4b^2+180a^2b^4-5b^6\right)x}{16b^9} + \frac{a\sqrt{a^2-b^2}\left(56a^4-47a^2b^2+6b^4\right)\tan^{-1}\Big(}{b^9d}$$

Mathematica [B] time = 14.4196, size = 2015, normalized size = 3.76

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*Cos[c +
d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2*b^2 +
2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/(64*b^3*d)
 - (3*((6*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (Cos[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x]))/(a +
 b*Sin[c + d*x])^2))/(256*(a - b)^2*(a + b)^2*d) - (3*((12*a*(640*a^8 - 192
0*a^6*b^2 + 2016*a^4*b^4 - 840*a^2*b^6 + 105*b^8)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (-3840*a^10*(c + d*x) + 7680*a
^8*b^2*(c + d*x) - 2976*a^6*b^4*(c + d*x) - 1776*a^4*b^6*(c + d*x) + 960*a^
2*b^8*(c + d*x) - 48*b^10*(c + d*x) - 3840*a^9*b*Cos[c + d*x] + 8640*a^7*b^
3*Cos[c + d*x] - 5696*a^5*b^5*Cos[c + d*x] + 788*a^3*b^7*Cos[c + d*x] + 114
*a*b^9*Cos[c + d*x] + 1920*a^8*b^2*(c + d*x)*Cos[2*(c + d*x)] - 4800*a^6*b^
4*(c + d*x)*Cos[2*(c + d*x)] + 3888*a^4*b^6*(c + d*x)*Cos[2*(c + d*x)] - 10
56*a^2*b^8*(c + d*x)*Cos[2*(c + d*x)] + 48*b^10*(c + d*x)*Cos[2*(c + d*x)]
+ 320*a^7*b^3*Cos[3*(c + d*x)] - 760*a^5*b^5*Cos[3*(c + d*x)] + 560*a^3*b^7
*Cos[3*(c + d*x)] - 120*a*b^9*Cos[3*(c + d*x)] - 8*a^5*b^5*Cos[5*(c + d*x)]
 + 16*a^3*b^7*Cos[5*(c + d*x)] - 8*a*b^9*Cos[5*(c + d*x)] - 7680*a^9*b*(c +
 d*x)*Sin[c + d*x] + 19200*a^7*b^3*(c + d*x)*Sin[c + d*x] - 15552*a^5*b^5*(
c + d*x)*Sin[c + d*x] + 4224*a^3*b^7*(c + d*x)*Sin[c + d*x] - 192*a*b^9*(c
+ d*x)*Sin[c + d*x] - 2880*a^8*b^2*Sin[2*(c + d*x)] + 6880*a^6*b^4*Sin[2*(c
 + d*x)] - 5182*a^4*b^6*Sin[2*(c + d*x)] + 1221*a^2*b^8*Sin[2*(c + d*x)] -
36*b^10*Sin[2*(c + d*x)] - 40*a^6*b^4*Sin[4*(c + d*x)] + 88*a^4*b^6*Sin[4*(
c + d*x)] - 56*a^2*b^8*Sin[4*(c + d*x)] + 8*b^10*Sin[4*(c + d*x)] + 2*a^4*b
^6*Sin[6*(c + d*x)] - 4*a^2*b^8*Sin[6*(c + d*x)] + 2*b^10*Sin[6*(c + d*x)])
/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2)))/(1024*b^7*d) - ((-60*a*(14336*a^1
0 - 49280*a^8*b^2 + 63360*a^6*b^4 - 36960*a^4*b^6 + 9240*a^2*b^8 - 693*b^10
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (43
0080*a^12*(c + d*x) - 1048320*a^10*b^2*(c + d*x) + 691200*a^8*b^4*(c + d*x)
 + 83040*a^6*b^6*(c + d*x) - 198000*a^4*b^8*(c + d*x) + 43200*a^2*b^10*(c +
 d*x) - 1200*b^12*(c + d*x) + 430080*a^11*b*Cos[c + d*x] - 1155840*a^9*b^3*
Cos[c + d*x] + 1042880*a^7*b^5*Cos[c + d*x] - 332800*a^5*b^7*Cos[c + d*x] +
 11060*a^3*b^9*Cos[c + d*x] + 4530*a*b^11*Cos[c + d*x] - 215040*a^10*b^2*(c
 + d*x)*Cos[2*(c + d*x)] + 631680*a^8*b^4*(c + d*x)*Cos[2*(c + d*x)] - 6614
40*a^6*b^6*(c + d*x)*Cos[2*(c + d*x)] + 289200*a^4*b^8*(c + d*x)*Cos[2*(c +
 d*x)] - 45600*a^2*b^10*(c + d*x)*Cos[2*(c + d*x)] + 1200*b^12*(c + d*x)*Co
s[2*(c + d*x)] - 35840*a^9*b^3*Cos[3*(c + d*x)] + 100800*a^7*b^5*Cos[3*(c +
 d*x)] - 98424*a^5*b^7*Cos[3*(c + d*x)] + 37808*a^3*b^9*Cos[3*(c + d*x)] -
4344*a*b^11*Cos[3*(c + d*x)] + 896*a^7*b^5*Cos[5*(c + d*x)] - 2184*a^5*b^7*
Cos[5*(c + d*x)] + 1680*a^3*b^9*Cos[5*(c + d*x)] - 392*a*b^11*Cos[5*(c + d*
x)] - 64*a^5*b^7*Cos[7*(c + d*x)] + 128*a^3*b^9*Cos[7*(c + d*x)] - 64*a*b^1
1*Cos[7*(c + d*x)] + 860160*a^11*b*(c + d*x)*Sin[c + d*x] - 2526720*a^9*b^3
*(c + d*x)*Sin[c + d*x] + 2645760*a^7*b^5*(c + d*x)*Sin[c + d*x] - 1156800*
a^5*b^7*(c + d*x)*Sin[c + d*x] + 182400*a^3*b^9*(c + d*x)*Sin[c + d*x] - 48
00*a*b^11*(c + d*x)*Sin[c + d*x] + 322560*a^10*b^2*Sin[2*(c + d*x)] - 91168
0*a^8*b^4*Sin[2*(c + d*x)] + 903680*a^6*b^6*Sin[2*(c + d*x)] - 362830*a^4*b
^8*Sin[2*(c + d*x)] + 49125*a^2*b^10*Sin[2*(c + d*x)] - 900*b^12*Sin[2*(c +
 d*x)] + 4480*a^8*b^4*Sin[4*(c + d*x)] - 11816*a^6*b^6*Sin[4*(c + d*x)] + 1
0392*a^4*b^8*Sin[4*(c + d*x)] - 3256*a^2*b^10*Sin[4*(c + d*x)] + 200*b^12*S
in[4*(c + d*x)] - 224*a^6*b^6*Sin[6*(c + d*x)] + 498*a^4*b^8*Sin[6*(c + d*x
)] - 324*a^2*b^10*Sin[6*(c + d*x)] + 50*b^12*Sin[6*(c + d*x)] + 20*a^4*b^8*
```

```
Sin[8*(c + d*x)] - 40*a^2*b^10*Sin[8*(c + d*x)] + 20*b^12*Sin[8*(c + d*x)])
/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2))/(15360*b^9*d)
```

Maple [B] time = 0.161, size = 2174, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 5/8/d/b^3*arctan(tan(1/2*d*x+1/2*c))-15/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^7+15/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*
c)^5-5/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+11/8/d/b^3/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)+75/d/b^7*arctan(tan(1/2*d*x+1
/2*c))*a^4-45/2/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2-46/5/d/b^4/(1+tan(1/2*
d*x+1/2*c)^2)^6*a+140/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*a^3-42/d/b^8/(1+ta
n(1/2*d*x+1/2*c)^2)^6*a^5-56/d/b^9*arctan(tan(1/2*d*x+1/2*c))*a^6-14/d*a^7/
b^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+19/d*a^5/b^6/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-5/d*a^3/b^4/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-11/8/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*ta
n(1/2*d*x+1/2*c)^11+5/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^9+1400/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^3-10/d*a
/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)
^2-43/d*a^6/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2
*d*x+1/2*c)+59/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^
2*tan(1/2*d*x+1/2*c)-16/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)+56/d*a^7/b^9/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-103/d*a^5/b^7/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+53/d*a^3/b^5/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d*a/b^3/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-420
/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^5-45/d/b^7/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^4+57/2/d/b^5/(1+tan(1/2*d*x+1/2*
c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^2+440/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1
/2*d*x+1/2*c)^4*a^3-15/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
11*a^4+27/2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^2-30/d
/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*a^4-420/d/b^8/(1+tan(1
/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^5-210/d/b^8/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)^8*a^5+260/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2
*d*x+1/2*c)^8*a^3-13/d*a^6/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2*tan(1/2*d*x+1/2*c)^3+17/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-4/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-14/d*a^7/b^8/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-9/d*a^5/b^6/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+33/d
*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)^2-92/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a+30/d/b^7/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^4-15/d/b^5/(1+tan(1/2*d*x
+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^2+45/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^3*a^4-57/2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^3*a^2-210/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^5+2
20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^3-186/5/d/b^4/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a+15/d/b^7/(1+tan(1/2*d*x+1/2
*c)^2)^6*tan(1/2*d*x+1/2*c)*a^4-27/2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1
/2*d*x+1/2*c)*a^2-42/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10
*a^5+60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^3-18/d/b^4
```

```
/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a+15/d/b^5/(1+tan(1/2*d*x
+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*a^2-84/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^4*a-54/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^8*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.9361, size = 2642, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/240*(64*a*b^7*cos(d*x + c)^7 - 4*(56*a^3*b^5 - 19*a*b^7)*cos(d*x + c)^5
 + 15*(448*a^6*b^2 - 600*a^4*b^4 + 180*a^2*b^6 - 5*b^8)*d*x*cos(d*x + c)^2
+ 10*(224*a^5*b^3 - 244*a^3*b^5 + 43*a*b^7)*cos(d*x + c)^3 - 15*(448*a^8 -
152*a^6*b^2 - 420*a^4*b^4 + 175*a^2*b^6 - 5*b^8)*d*x + 60*(56*a^7 + 9*a^5*b
^2 - 41*a^3*b^4 + 6*a*b^6 - (56*a^5*b^2 - 47*a^3*b^4 + 6*a*b^6)*cos(d*x + c
)^2 + 2*(56*a^6*b - 47*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*
log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 30*(224*a^7*b - 188*a^5*b^3 - 3
2*a^3*b^5 + 19*a*b^7)*cos(d*x + c) - (40*b^8*cos(d*x + c)^7 - 2*(56*a^2*b^6
 - 5*b^8)*cos(d*x + c)^5 + 5*(112*a^4*b^4 - 94*a^2*b^6 + 5*b^8)*cos(d*x + c
)^3 + 30*(448*a^7*b - 600*a^5*b^3 + 180*a^3*b^5 - 5*a*b^7)*d*x + 15*(672*a^
6*b^2 - 844*a^4*b^4 + 223*a^2*b^6 - 5*b^8)*cos(d*x + c))*sin(d*x + c))/(b^1
1*d*cos(d*x + c)^2 - 2*a*b^10*d*sin(d*x + c) - (a^2*b^9 + b^11)*d), -1/240*
(64*a*b^7*cos(d*x + c)^7 - 4*(56*a^3*b^5 - 19*a*b^7)*cos(d*x + c)^5 + 15*(4
48*a^6*b^2 - 600*a^4*b^4 + 180*a^2*b^6 - 5*b^8)*d*x*cos(d*x + c)^2 + 10*(22
4*a^5*b^3 - 244*a^3*b^5 + 43*a*b^7)*cos(d*x + c)^3 - 15*(448*a^8 - 152*a^6*
b^2 - 420*a^4*b^4 + 175*a^2*b^6 - 5*b^8)*d*x - 120*(56*a^7 + 9*a^5*b^2 - 41
*a^3*b^4 + 6*a*b^6 - (56*a^5*b^2 - 47*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2
*(56*a^6*b - 47*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 30*(224*a^7*b - 188*
a^5*b^3 - 32*a^3*b^5 + 19*a*b^7)*cos(d*x + c) - (40*b^8*cos(d*x + c)^7 - 2*
(56*a^2*b^6 - 5*b^8)*cos(d*x + c)^5 + 5*(112*a^4*b^4 - 94*a^2*b^6 + 5*b^8)*
cos(d*x + c)^3 + 30*(448*a^7*b - 600*a^5*b^3 + 180*a^3*b^5 - 5*a*b^7)*d*x +
 15*(672*a^6*b^2 - 844*a^4*b^4 + 223*a^2*b^6 - 5*b^8)*cos(d*x + c))*sin(d*x
 + c))/(b^11*d*cos(d*x + c)^2 - 2*a*b^10*d*sin(d*x + c) - (a^2*b^9 + b^11)*
d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34767, size = 1307, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(15*(448*a^6 - 600*a^4*b^2 + 180*a^2*b^4 - 5*b^6)*(d*x + c)/b^9 - 24
0*(56*a^7 - 103*a^5*b^2 + 53*a^3*b^4 - 6*a*b^6)*(pi*floor(1/2*(d*x + c)/pi
+ 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt
(a^2 - b^2)*b^9) + 240*(13*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 17*a^4*b^3*tan(1/
2*d*x + 1/2*c)^3 + 4*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 14*a^7*tan(1/2*d*x +
1/2*c)^2 + 9*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 33*a^3*b^4*tan(1/2*d*x + 1/2*
c)^2 + 10*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 43*a^6*b*tan(1/2*d*x + 1/2*c) - 59
*a^4*b^3*tan(1/2*d*x + 1/2*c) + 16*a^2*b^5*tan(1/2*d*x + 1/2*c) + 14*a^7 -
19*a^5*b^2 + 5*a^3*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*
c) + a)^2*b^8) + 2*(1800*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 1620*a^2*b^3*tan(1
/2*d*x + 1/2*c)^11 + 165*b^5*tan(1/2*d*x + 1/2*c)^11 + 5040*a^5*tan(1/2*d*x
 + 1/2*c)^10 - 7200*a^3*b^2*tan(1/2*d*x + 1/2*c)^10 + 2160*a*b^4*tan(1/2*d*
x + 1/2*c)^10 + 5400*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 3420*a^2*b^3*tan(1/2*d*
x + 1/2*c)^9 - 25*b^5*tan(1/2*d*x + 1/2*c)^9 + 25200*a^5*tan(1/2*d*x + 1/2*
c)^8 - 31200*a^3*b^2*tan(1/2*d*x + 1/2*c)^8 + 6480*a*b^4*tan(1/2*d*x + 1/2*
c)^8 + 3600*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 1800*a^2*b^3*tan(1/2*d*x + 1/2*c
)^7 + 450*b^5*tan(1/2*d*x + 1/2*c)^7 + 50400*a^5*tan(1/2*d*x + 1/2*c)^6 - 5
6000*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 11040*a*b^4*tan(1/2*d*x + 1/2*c)^6 -
3600*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1800*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 4
50*b^5*tan(1/2*d*x + 1/2*c)^5 + 50400*a^5*tan(1/2*d*x + 1/2*c)^4 - 52800*a^
3*b^2*tan(1/2*d*x + 1/2*c)^4 + 10080*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 5400*a^
4*b*tan(1/2*d*x + 1/2*c)^3 + 3420*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 25*b^5*t
an(1/2*d*x + 1/2*c)^3 + 25200*a^5*tan(1/2*d*x + 1/2*c)^2 - 26400*a^3*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 4464*a*b^4*tan(1/2*d*x + 1/2*c)^2 - 1800*a^4*b*tan(1
/2*d*x + 1/2*c) + 1620*a^2*b^3*tan(1/2*d*x + 1/2*c) - 165*b^5*tan(1/2*d*x +
 1/2*c) + 5040*a^5 - 5600*a^3*b^2 + 1104*a*b^4)/((tan(1/2*d*x + 1/2*c)^2 +
1)^6*b^8))/d
```

3.1267 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=485

$$\frac{\left(-645a^2b^2+630a^4+91b^4\right)\cos(c+dx)}{30b^7d}-\frac{\sqrt{a^2-b^2}\left(-29a^2b^2+42a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{\left(-60a^2b^2+63a^4\right.}{60a^2b^3d}$$

```
[Out] (a*(168*a^4 - 200*a^2*b^2 + 45*b^4)*x)/(8*b^8) - (Sqrt[a^2 - b^2]*(42*a^4 -
 29*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^8
*d) + ((630*a^4 - 645*a^2*b^2 + 91*b^4)*Cos[c + d*x])/(30*b^7*d) - ((84*a^4
 - 79*a^2*b^2 + 8*b^4)*Cos[c + d*x]*Sin[c + d*x])/(8*a*b^6*d) + ((210*a^4 -
 187*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(30*a^2*b^5*d) + (Cos[c
 + d*x]*Sin[c + d*x]^3)/(3*a*d*(a + b*Sin[c + d*x])^2) - (b*Cos[c + d*x]*Si
n[c + d*x]^4)/(12*a^2*d*(a + b*Sin[c + d*x])^2) - ((63*a^4 - 60*a^2*b^2 + 5
*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(60*a^2*b^3*d*(a + b*Sin[c + d*x])^2) -
(7*a*Cos[c + d*x]*Sin[c + d*x]^5)/(20*b^2*d*(a + b*Sin[c + d*x])^2) + (Cos[
c + d*x]*Sin[c + d*x]^6)/(5*b*d*(a + b*Sin[c + d*x])^2) - ((63*a^4 - 54*a^2
*b^2 + 4*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(12*a^2*b^4*d*(a + b*Sin[c + d*x
]))
```

Rubi [A] time = 1.71693, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2896, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{\left(-645a^2b^2+630a^4+91b^4\right)\cos(c+dx)}{30b^7d}-\frac{\sqrt{a^2-b^2}\left(-29a^2b^2+42a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{\left(-60a^2b^2+63a^4\right.}{60a^2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (a*(168*a^4 - 200*a^2*b^2 + 45*b^4)*x)/(8*b^8) - (Sqrt[a^2 - b^2]*(42*a^4 -
 29*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^8
*d) + ((630*a^4 - 645*a^2*b^2 + 91*b^4)*Cos[c + d*x])/(30*b^7*d) - ((84*a^4
 - 79*a^2*b^2 + 8*b^4)*Cos[c + d*x]*Sin[c + d*x])/(8*a*b^6*d) + ((210*a^4 -
 187*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(30*a^2*b^5*d) + (Cos[c
 + d*x]*Sin[c + d*x]^3)/(3*a*d*(a + b*Sin[c + d*x])^2) - (b*Cos[c + d*x]*Si
n[c + d*x]^4)/(12*a^2*d*(a + b*Sin[c + d*x])^2) - ((63*a^4 - 60*a^2*b^2 + 5
*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(60*a^2*b^3*d*(a + b*Sin[c + d*x])^2) -
(7*a*Cos[c + d*x]*Sin[c + d*x]^5)/(20*b^2*d*(a + b*Sin[c + d*x])^2) + (Cos[
c + d*x]*Sin[c + d*x]^6)/(5*b*d*(a + b*Sin[c + d*x])^2) - ((63*a^4 - 54*a^2
*b^2 + 4*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(12*a^2*b^4*d*(a + b*Sin[c + d*x
]))
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
```

```
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^4(c+dx)}{12a^2d(a+b\sin(c+dx))^2} - \frac{7a\cos(c+dx)\sin^5(c+dx)}{20b^2d(a+b\sin(c+dx))^2} +$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^4(c+dx)}{12a^2d(a+b\sin(c+dx))^2} - \frac{\left(63a^4-60a^2b^2+5b^4\right)\cos(c+}{60a^2b^3d(a+b\sin(c}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^4(c+dx)}{12a^2d(a+b\sin(c+dx))^2} - \frac{\left(63a^4-60a^2b^2+5b^4\right)\cos(c+}{60a^2b^3d(a+b\sin(c}$$

$$= \frac{\left(210a^4-187a^2b^2+15b^4\right)\cos(c+dx)\sin^2(c+dx)}{30a^2b^5d} + \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(}{12a^2d}$$

$$= -\frac{\left(84a^4-79a^2b^2+8b^4\right)\cos(c+dx)\sin(c+dx)}{8ab^6d} + \frac{\left(210a^4-187a^2b^2+15b^4\right)\cos(c+d}{30a^2b^5d}$$

$$= \frac{\left(630a^4-645a^2b^2+91b^4\right)\cos(c+dx)}{30b^7d} - \frac{\left(84a^4-79a^2b^2+8b^4\right)\cos(c+dx)\sin(c+dx)}{8ab^6d}$$

$$= \frac{a\left(168a^4-200a^2b^2+45b^4\right)x}{8b^8} + \frac{\left(630a^4-645a^2b^2+91b^4\right)\cos(c+dx)}{30b^7d} - \frac{\left(84a^4-79a^2\right.}{}$$

$$= \frac{a\left(168a^4-200a^2b^2+45b^4\right)x}{8b^8} + \frac{\left(630a^4-645a^2b^2+91b^4\right)\cos(c+dx)}{30b^7d} - \frac{\left(84a^4-79a^2\right.}{}$$

$$= \frac{a\left(168a^4-200a^2b^2+45b^4\right)x}{8b^8} + \frac{\left(630a^4-645a^2b^2+91b^4\right)\cos(c+dx)}{30b^7d} - \frac{\left(84a^4-79a^2\right.}{}$$

$$= \frac{a\left(168a^4-200a^2b^2+45b^4\right)x}{8b^8} - \frac{\sqrt{a^2-b^2}\left(42a^4-29a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{b^8d}$$

Mathematica [A] time = 12.8154, size = 517, normalized size = 1.07

$$\frac{\left(a^2-b^2\right)^2\left(30240a^5b^2\sin(2(c+dx))-96000a^4b^3c\sin(c+dx)-96000a^4b^3dx\sin(c+dx)-32640a^3b^4\sin(2(c+dx))+420a^3b^4\sin(4(c+dx))+21600a^2b^5c\sin(c+dx)+21600\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-1920*(a^2 - b^2)^(5/2)*(42*a^4 - 29*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]] + ((a^2 - b^2)^2*(40320*a^7*c - 27840*a^5*b^2*
c - 13200*a^3*b^4*c + 5400*a*b^6*c + 40320*a^7*d*x - 27840*a^5*b^2*d*x - 13
200*a^3*b^4*d*x + 5400*a*b^6*d*x + 10*b*(4032*a^6 - 3792*a^4*b^2 + 216*a^2*
b^4 + 59*b^6)*Cos[c + d*x] - 120*a*b^2*(168*a^4 - 200*a^2*b^2 + 45*b^4)*(c
+ d*x)*Cos[2*(c + d*x)] - 3360*a^4*b^3*Cos[3*(c + d*x)] + 3580*a^2*b^5*Cos[
3*(c + d*x)] - 526*b^7*Cos[3*(c + d*x)] + 84*a^2*b^5*Cos[5*(c + d*x)] - 58*
b^7*Cos[5*(c + d*x)] - 6*b^7*Cos[7*(c + d*x)] + 80640*a^6*b*c*Sin[c + d*x]
- 96000*a^4*b^3*c*Sin[c + d*x] + 21600*a^2*b^5*c*Sin[c + d*x] + 80640*a^6*b
*d*x*Sin[c + d*x] - 96000*a^4*b^3*d*x*Sin[c + d*x] + 21600*a^2*b^5*d*x*Sin[
c + d*x] + 30240*a^5*b^2*Sin[2*(c + d*x)] - 32640*a^3*b^4*Sin[2*(c + d*x)]
+ 5675*a*b^6*Sin[2*(c + d*x)] + 420*a^3*b^4*Sin[4*(c + d*x)] - 374*a*b^6*Si
n[4*(c + d*x)] - 21*a*b^6*Sin[6*(c + d*x)]))/(a + b*Sin[c + d*x])^2)/(1920*
(a - b)^2*b^8*(a + b)^2*d)
```

Maple [B] time = 0.171, size = 1676, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 120/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^4+10/d/b^6/(1+t
an(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a^3-27/4/d/b^4/(1+tan(1/2*d*x+1
/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a+30/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)^8*a^4+9/d/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)^2*a^4-27/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*a^2+37/d/b^6/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*a^5-47/d/b^4/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*a^3-50/d/b^6*arctan(
tan(1/2*d*x+1/2*c))*a^3+45/4/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a+6/d/b/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+12/d/b^
7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^6-15/d/b^5/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^4+3/d/b^3/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2+2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+42/d/b^8*arctan(tan(1/2*d*x+1/2*
c))*a^5+6/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8+12/d/b^3/(1
+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6+56/3/d/b^3/(1+tan(1/2*d*x+1/2
*c)^2)^5*tan(1/2*d*x+1/2*c)^4+28/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2
*d*x+1/2*c)^2+30/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^5*a^4-28/d/b^5/(1+tan(1/2*d
*x+1/2*c)^2)^5*a^2+46/15/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5+71/d/b^6/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^4-31/d/b
^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
*a^2-120/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^2+180/d/b^
7/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^4-160/d/b^5/(1+tan(1/2*
d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^2-20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^
5*tan(1/2*d*x+1/2*c)^3*a^3+15/2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*
x+1/2*c)^3*a+120/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^4+
11/d/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)^3*a^5-13/d/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)^3*a^3+2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*tan(1/2*d*x+1/2*c)^3*a+12/d/b^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*a^6-104/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)^2*a^2-10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x
```

```
+1/2*c)*a^3+27/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a-36/d
/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^2+20/d/b^6/(1+tan(1/
2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3-15/2/d/b^4/(1+tan(1/2*d*x+1/2*c)
^2)^5*tan(1/2*d*x+1/2*c)^7*a+10/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*a-42/d/b^8/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69394, size = 2326, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/120*(24*b^7*cos(d*x + c)^7 - 4*(21*a^2*b^5 - 4*b^7)*cos(d*x + c)^5 + 15*
(168*a^5*b^2 - 200*a^3*b^4 + 45*a*b^6)*d*x*cos(d*x + c)^2 + 10*(84*a^4*b^3
- 79*a^2*b^5 + 8*b^7)*cos(d*x + c)^3 - 15*(168*a^7 - 32*a^5*b^2 - 155*a^3*b
^4 + 45*a*b^6)*d*x - 30*(42*a^6 + 13*a^4*b^2 - 27*a^2*b^4 + 2*b^6 - (42*a^4
*b^2 - 29*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(42*a^5*b - 29*a^3*b^3 + 2*a*
b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)
) - 30*(84*a^6*b - 58*a^4*b^3 - 17*a^2*b^5 + 4*b^7)*cos(d*x + c) + (42*a*b^
6*cos(d*x + c)^5 - 5*(42*a^3*b^4 - 29*a*b^6)*cos(d*x + c)^3 - 30*(168*a^6*b
 - 200*a^4*b^3 + 45*a^2*b^5)*d*x - 15*(252*a^5*b^2 - 279*a^3*b^4 + 53*a*b^6
)*cos(d*x + c))*sin(d*x + c))/(b^10*d*cos(d*x + c)^2 - 2*a*b^9*d*sin(d*x +
c) - (a^2*b^8 + b^10)*d), 1/120*(24*b^7*cos(d*x + c)^7 - 4*(21*a^2*b^5 - 4*
b^7)*cos(d*x + c)^5 + 15*(168*a^5*b^2 - 200*a^3*b^4 + 45*a*b^6)*d*x*cos(d*x
 + c)^2 + 10*(84*a^4*b^3 - 79*a^2*b^5 + 8*b^7)*cos(d*x + c)^3 - 15*(168*a^7
 - 32*a^5*b^2 - 155*a^3*b^4 + 45*a*b^6)*d*x - 60*(42*a^6 + 13*a^4*b^2 - 27*
a^2*b^4 + 2*b^6 - (42*a^4*b^2 - 29*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(42*
a^5*b - 29*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(
d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 30*(84*a^6*b - 58*a^4*b^3 -
 17*a^2*b^5 + 4*b^7)*cos(d*x + c) + (42*a*b^6*cos(d*x + c)^5 - 5*(42*a^3*b^
4 - 29*a*b^6)*cos(d*x + c)^3 - 30*(168*a^6*b - 200*a^4*b^3 + 45*a^2*b^5)*d*
x - 15*(252*a^5*b^2 - 279*a^3*b^4 + 53*a*b^6)*cos(d*x + c))*sin(d*x + c))/(
b^10*d*cos(d*x + c)^2 - 2*a*b^9*d*sin(d*x + c) - (a^2*b^8 + b^10)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40237, size = 977, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(168*a^5 - 200*a^3*b^2 + 45*a*b^4)*(d*x + c)/b^8 - 120*(42*a^6 -
71*a^4*b^2 + 31*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
 arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^8
) + 120*(11*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 13*a^3*b^3*tan(1/2*d*x + 1/2*c)^
3 + 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 12*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*
b^2*tan(1/2*d*x + 1/2*c)^2 - 27*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 6*b^6*tan(
1/2*d*x + 1/2*c)^2 + 37*a^5*b*tan(1/2*d*x + 1/2*c) - 47*a^3*b^3*tan(1/2*d*x
 + 1/2*c) + 10*a*b^5*tan(1/2*d*x + 1/2*c) + 12*a^6 - 15*a^4*b^2 + 3*a^2*b^4
)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*b^7) + 2*(60
0*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 405*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 1800*a^
4*tan(1/2*d*x + 1/2*c)^8 - 2160*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*ta
n(1/2*d*x + 1/2*c)^8 + 1200*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 450*a*b^3*tan(1/
2*d*x + 1/2*c)^7 + 7200*a^4*tan(1/2*d*x + 1/2*c)^6 - 7200*a^2*b^2*tan(1/2*d
*x + 1/2*c)^6 + 720*b^4*tan(1/2*d*x + 1/2*c)^6 + 10800*a^4*tan(1/2*d*x + 1/
2*c)^4 - 9600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c
)^4 - 1200*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 450*a*b^3*tan(1/2*d*x + 1/2*c)^3
+ 7200*a^4*tan(1/2*d*x + 1/2*c)^2 - 6240*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 5
60*b^4*tan(1/2*d*x + 1/2*c)^2 - 600*a^3*b*tan(1/2*d*x + 1/2*c) + 405*a*b^3*
tan(1/2*d*x + 1/2*c) + 1800*a^4 - 1680*a^2*b^2 + 184*b^4)/((tan(1/2*d*x + 1
/2*c)^2 + 1)^5*b^7))/d
```

3.1268 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=237

$$\frac{15a\left(-3a^2b^2+2a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}+\frac{5\cos^3(c+dx)\left(4a^2+ab\sin(c+dx)-b^2\right)}{4b^4d(a+b\sin(c+dx))}-\frac{15\cos(c+dx)\left(4a\left(2a^2-\right.\right.}{}$$

```
[Out] (-15*(8*a^4 - 8*a^2*b^2 + b^4)*x)/(8*b^7) + (15*a*(2*a^4 - 3*a^2*b^2 + b^4)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*d)
+ (Cos[c + d*x]^5*(3*a + b*Sin[c + d*x]))/(4*b^2*d*(a + b*Sin[c + d*x])^2)
+ (5*Cos[c + d*x]^3*(4*a^2 - b^2 + a*b*Sin[c + d*x]))/(4*b^4*d*(a + b*Sin[c
 + d*x])) - (15*Cos[c + d*x]*(4*a*(2*a^2 - b^2) - b*(4*a^2 - b^2)*Sin[c + d
*x]))/(8*b^6*d)
```

Rubi [A] time = 0.465455, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2863, 2865, 2735, 2660, 618, 204}

$$\frac{15a\left(-3a^2b^2+2a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}+\frac{5\cos^3(c+dx)\left(4a^2+ab\sin(c+dx)-b^2\right)}{4b^4d(a+b\sin(c+dx))}-\frac{15\cos(c+dx)\left(4a\left(2a^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-15*(8*a^4 - 8*a^2*b^2 + b^4)*x)/(8*b^7) + (15*a*(2*a^4 - 3*a^2*b^2 + b^4)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*d)
+ (Cos[c + d*x]^5*(3*a + b*Sin[c + d*x]))/(4*b^2*d*(a + b*Sin[c + d*x])^2)
+ (5*Cos[c + d*x]^3*(4*a^2 - b^2 + a*b*Sin[c + d*x]))/(4*b^4*d*(a + b*Sin[c
 + d*x])) - (15*Cos[c + d*x]*(4*a*(2*a^2 - b^2) - b*(4*a^2 - b^2)*Sin[c + d
*x]))/(8*b^6*d)
```


Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
```

```
0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} - \frac{5\int \frac{\cos^4(c+dx)(-2b-6a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{8b^2}$$

$$= \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)\left(4a^2-b^2+ab\sin(c+dx)\right)}{4b^4d(a+b\sin(c+dx))} + \frac{5\int \cdots}{\cdots}$$

$$= \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)\left(4a^2-b^2+ab\sin(c+dx)\right)}{4b^4d(a+b\sin(c+dx))} - \frac{15\,\mathrm{c}\cdots}{\cdots}$$

$$= -\frac{15\left(8a^4-8a^2b^2+b^4\right)x}{8b^7} + \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)\left(4a^2-\cdots\right.}{4b^4d(a+b\,\mathrm{s}\cdots}$$

$$= -\frac{15\left(8a^4-8a^2b^2+b^4\right)x}{8b^7} + \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)\left(4a^2-\cdots\right.}{4b^4d(a+b\,\mathrm{s}\cdots}$$

$$= -\frac{15\left(8a^4-8a^2b^2+b^4\right)x}{8b^7} + \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)\left(4a^2-\cdots\right.}{4b^4d(a+b\,\mathrm{s}\cdots}$$

$$= -\frac{15\left(8a^4-8a^2b^2+b^4\right)x}{8b^7} + \frac{15a\left(2a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d} + \frac{\cos^5(c\cdots}{4b^2\cdots}$$

Mathematica [B] time = 8.28841, size = 1250, normalized size = 5.27

$$\frac{18\left(-8(c+dx)+\frac{2a\left(8a^4-20b^2a^2+15b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{3b\left(4a^4-7b^2a^2+2b^4\right)\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{ab\left(4a^2-3b^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}\right)}{b^3}-\frac{10\left(\frac{6ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\cos(c+dx)}{\cdots}\right.}{(a-b)^2(a+b)^{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((18*(-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*Cos
[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2*b
^2 + 2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))))/b^3
- (10*((6*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (Cos[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x]))/(a +
 b*Sin[c + d*x])^2))/((a - b)^2*(a + b)^2) + (10*(-24*(-8*a^2 + b^2)*(c + d
*x) - (6*a*(64*a^6 - 168*a^4*b^2 + 140*a^2*b^4 - 35*b^6)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 96*a*b*Cos[c + d*x] + (
a*b*(-16*a^4 + 20*a^2*b^2 - 5*b^4)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Si
n[c + d*x])^2) + (b*(112*a^6 - 220*a^4*b^2 + 115*a^2*b^4 - 10*b^6)*Cos[c +
d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])) - 8*b^2*Sin[2*(c + d*x)]))
/b^5 + ((12*a*(640*a^8 - 1920*a^6*b^2 + 2016*a^4*b^4 - 840*a^2*b^6 + 105*b^
8)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (-
3840*a^10*(c + d*x) + 7680*a^8*b^2*(c + d*x) - 2976*a^6*b^4*(c + d*x) - 177
6*a^4*b^6*(c + d*x) + 960*a^2*b^8*(c + d*x) - 48*b^10*(c + d*x) - 3840*a^9*
b*Cos[c + d*x] + 8640*a^7*b^3*Cos[c + d*x] - 5696*a^5*b^5*Cos[c + d*x] + 78
8*a^3*b^7*Cos[c + d*x] + 114*a*b^9*Cos[c + d*x] + 1920*a^8*b^2*(c + d*x)*Co
s[2*(c + d*x)] - 4800*a^6*b^4*(c + d*x)*Cos[2*(c + d*x)] + 3888*a^4*b^6*(c
+ d*x)*Cos[2*(c + d*x)] - 1056*a^2*b^8*(c + d*x)*Cos[2*(c + d*x)] + 48*b^10
*(c + d*x)*Cos[2*(c + d*x)] + 320*a^7*b^3*Cos[3*(c + d*x)] - 760*a^5*b^5*Co
s[3*(c + d*x)] + 560*a^3*b^7*Cos[3*(c + d*x)] - 120*a*b^9*Cos[3*(c + d*x)]
- 8*a^5*b^5*Cos[5*(c + d*x)] + 16*a^3*b^7*Cos[5*(c + d*x)] - 8*a*b^9*Cos[5*
(c + d*x)] - 7680*a^9*b*(c + d*x)*Sin[c + d*x] + 19200*a^7*b^3*(c + d*x)*Si
n[c + d*x] - 15552*a^5*b^5*(c + d*x)*Sin[c + d*x] + 4224*a^3*b^7*(c + d*x)*
Sin[c + d*x] - 192*a*b^9*(c + d*x)*Sin[c + d*x] - 2880*a^8*b^2*Sin[2*(c + d
*x)] + 6880*a^6*b^4*Sin[2*(c + d*x)] - 5182*a^4*b^6*Sin[2*(c + d*x)] + 1221
*a^2*b^8*Sin[2*(c + d*x)] - 36*b^10*Sin[2*(c + d*x)] - 40*a^6*b^4*Sin[4*(c
+ d*x)] + 88*a^4*b^6*Sin[4*(c + d*x)] - 56*a^2*b^8*Sin[4*(c + d*x)] + 8*b^1
0*Sin[4*(c + d*x)] + 2*a^4*b^6*Sin[6*(c + d*x)] - 4*a^2*b^8*Sin[6*(c + d*x)
] + 2*b^10*Sin[6*(c + d*x)])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2))/b^7)/(
256*d)
```

Maple [B] time = 0.157, size = 1325, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2-15/4/d/b^3*arc
tan(tan(1/2*d*x+1/2*c))-30/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^4+30/d/b^5*ar
ctan(tan(1/2*d*x+1/2*c))*a^2-10/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
```

```
*d*x+1/2*c)*b+a)^2+11/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c
)*b+a)^2-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*
d*x+1/2*c)^2-4/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(
1/2*d*x+1/2*c)-1/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*
a+9/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+1/4/d/b^3/(1+ta
n(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-1/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)^3-9/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+
1/2*c)-20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4*a^3+14/d/b^4/(1+tan(1/2*d*x+1/2*
c)^2)^4*a-60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3+42/d
/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a+6/d/b^5/(1+tan(1/2*d
*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a^2-60/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^4
*tan(1/2*d*x+1/2*c)^2*a^3+38/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^2*a+6/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a^2-6/d/b^5
/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2-20/d/b^6/(1+tan(1/2*d*
x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a^3+18/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*
tan(1/2*d*x+1/2*c)^6*a+21/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-31/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+35/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+30/d*a^5/b^7/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-45/d*a^3/b^5/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+15/
d*a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))-9/d*a^4/b^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1
/2*d*x+1/2*c)^3+9/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)^3-10/d*a^5/b^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-9/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.91075, size = 1864, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*b^5*cos(d*x + c)^5 - 15*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*d*x*cos(d*x
 + c)^2 - 10*(4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^3 + 15*(8*a^6 - 7*a^2*b^4 +
 b^6)*d*x + 30*(2*a^5 + a^3*b^2 - a*b^4 - (2*a^3*b^2 - a*b^4)*cos(d*x + c)^
2 + 2*(2*a^4*b - a^2*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)
*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*
x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2)) + 30*(4*a^5*b - 2*a^3*b^3 - a*b^5)*cos(d*x + c) - (2
*b^6*cos(d*x + c)^5 - 5*(2*a^2*b^4 - b^6)*cos(d*x + c)^3 - 30*(8*a^5*b - 8*
a^3*b^3 + a*b^5)*d*x - 15*(12*a^4*b^2 - 11*a^2*b^4 + b^6)*cos(d*x + c))*sin
```

```
(d*x + c))/(b^9*d*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x + c) - (a^2*b^7 + b^9)
*d), 1/8*(4*a*b^5*cos(d*x + c)^5 - 15*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*d*x*cos
(d*x + c)^2 - 10*(4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^3 + 15*(8*a^6 - 7*a^2*b
^4 + b^6)*d*x + 60*(2*a^5 + a^3*b^2 - a*b^4 - (2*a^3*b^2 - a*b^4)*cos(d*x +
 c)^2 + 2*(2*a^4*b - a^2*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(
d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(4*a^5*b - 2*a^3*b^3 - a
*b^5)*cos(d*x + c) - (2*b^6*cos(d*x + c)^5 - 5*(2*a^2*b^4 - b^6)*cos(d*x +
c)^3 - 30*(8*a^5*b - 8*a^3*b^3 + a*b^5)*d*x - 15*(12*a^4*b^2 - 11*a^2*b^4 +
 b^6)*cos(d*x + c))*sin(d*x + c))/(b^9*d*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x
 + c) - (a^2*b^7 + b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30803, size = 784, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(8*a^4 - 8*a^2*b^2 + b^4)*(d*x + c)/b^7 - 120*(2*a^5 - 3*a^3*b^2 +
 a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 8*(9*a^5*b*tan(1/2*d*
x + 1/2*c)^3 - 9*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 10*a^6*tan(1/2*d*x + 1/2*
c)^2 + 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^4*tan(1/2*d*x + 1/2*c)^2
 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 + 31*a^5*b*tan(1/2*d*x + 1/2*c) - 35*a^3*b^
3*tan(1/2*d*x + 1/2*c) + 4*a*b^5*tan(1/2*d*x + 1/2*c) + 10*a^6 - 11*a^4*b^2
 + a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a*
b^6) + 2*(24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 9*b^3*tan(1/2*d*x + 1/2*c)^7 +
80*a^3*tan(1/2*d*x + 1/2*c)^6 - 72*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*b*
tan(1/2*d*x + 1/2*c)^5 - b^3*tan(1/2*d*x + 1/2*c)^5 + 240*a^3*tan(1/2*d*x +
 1/2*c)^4 - 168*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*c
)^3 + b^3*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*tan(1/2*d*x + 1/2*c)^2 - 152*a*b
^2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b*tan(1/2*d*x + 1/2*c) + 9*b^3*tan(1/2*d
*x + 1/2*c) + 80*a^3 - 56*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^6))/d
```

3.1269 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=399

$$\frac{(2a^2+b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{2(5a^2+b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}+\frac{2(-9a^4b^2+10a^6-b^6)}{a^3b^5d\sqrt{}}$$

```
[Out] -x/(2*b^3) - (3*(2*a^2 - b^2)*x)/b^5 + (Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^5*d) - (2*Sqrt[a^2 - b^2]
*(5*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^5*d)
+ (2*(10*a^6 - 9*a^4*b^2 - b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(a^3*b^5*Sqrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) - (3*a*C
os[c + d*x])/(b^4*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b^3*d) + ((a^2 - b^2)
^2*Cos[c + d*x])/(2*a*b^4*d*(a + b*Sin[c + d*x])^2) + (3*(a^2 - b^2)*Cos[c
+ d*x])/(2*b^4*d*(a + b*Sin[c + d*x])) - ((a^2 - b^2)*(5*a^2 + b^2)*Cos[c +
 d*x])/(a^2*b^4*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.517162, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2897, 3770, 2638, 2635, 8, 2664, 2754, 12, 2660, 618, 204}

$$\frac{(2a^2+b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{2(5a^2+b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}+\frac{2(-9a^4b^2+10a^6-b^6)}{a^3b^5d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -x/(2*b^3) - (3*(2*a^2 - b^2)*x)/b^5 + (Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^5*d) - (2*Sqrt[a^2 - b^2]
*(5*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^5*d)
+ (2*(10*a^6 - 9*a^4*b^2 - b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(a^3*b^5*Sqrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) - (3*a*C
os[c + d*x])/(b^4*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b^3*d) + ((a^2 - b^2)
^2*Cos[c + d*x])/(2*a*b^4*d*(a + b*Sin[c + d*x])^2) + (3*(a^2 - b^2)*Cos[c
+ d*x])/(2*b^4*d*(a + b*Sin[c + d*x])) - ((a^2 - b^2)*(5*a^2 + b^2)*Cos[c +
 d*x])/(a^2*b^4*d*(a + b*Sin[c + d*x]))
```


Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}dx = \int \left(\frac{3(-2a^2+b^2)}{b^5} + \frac{\csc(c+dx)}{a^3} + \frac{3a\sin(c+dx)}{b^4} - \frac{\sin^2(c+dx)}{b^3} + \frac{(a^2-b^2)}{ab^5(a+b\sin(c\right.$$

$$= -\frac{3(2a^2-b^2)x}{b^5} + \frac{\int\csc(c+dx)\,dx}{a^3} + \frac{(3a)\int\sin(c+dx)\,dx}{b^4} - \frac{\int\sin^2(c+dx)\,dx}{b^3} +$$

$$= -\frac{3(2a^2-b^2)x}{b^5} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{3a\cos(c+dx)}{b^4d} + \frac{\cos(c+dx)\sin(c+dx)}{2b^3d}$$

$$= -\frac{x}{2b^3} - \frac{3(2a^2-b^2)x}{b^5} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{3a\cos(c+dx)}{b^4d} + \frac{\cos(c+dx)\sin(c}{2b^3d}$$

$$= -\frac{x}{2b^3} - \frac{3(2a^2-b^2)x}{b^5} + \frac{2(10a^6-9a^4b^2-b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^5\sqrt{a^2-b^2}d} - \frac{\tanh^{-1}(\cos}{a^3}$$

$$= -\frac{x}{2b^3} - \frac{3(2a^2-b^2)x}{b^5} + \frac{2(10a^6-9a^4b^2-b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^5\sqrt{a^2-b^2}d} - \frac{\tanh^{-1}(\cos}{a^3}$$

$$= -\frac{x}{2b^3} - \frac{3(2a^2-b^2)x}{b^5} - \frac{2\sqrt{a^2-b^2}(5a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^5d} + \frac{2(10a^6-9}{}$$

$$= -\frac{x}{2b^3} - \frac{3(2a^2-b^2)x}{b^5} + \frac{\sqrt{a^2-b^2}(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^5d} - \frac{2\sqrt{a^2-b^2}(5}{}$$

Mathematica [A] time = 1.9756, size = 243, normalized size = 0.61

$$\frac{\frac{2(5b^2-12a^2)(c+dx)}{b^5} + \frac{4(-11a^4b^2+a^2b^4+12a^6-2b^6)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^5\sqrt{a^2-b^2}} + \frac{2(a^2-b^2)^2\cos(c+dx)}{ab^4(a+b\sin(c+dx))^2} + \frac{2(5a^2b^2-7a^4+2b^4)\cos(c+dx)}{a^2b^4(a+b\sin(c+dx))} + \frac{4\log\left(\sin\left(\frac{1}{2}(c+\right.\right.}{a^3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*(-12*a^2 + 5*b^2)*(c + d*x))/b^5 + (4*(12*a^6 - 11*a^4*b^2 + a^2*b^4 -
2*b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^5*Sqrt[a^2
- b^2]) - (12*a*Cos[c + d*x])/b^4 - (4*Log[Cos[(c + d*x)/2]])/a^3 + (4*Log[
Sin[(c + d*x)/2]])/a^3 + (2*(a^2 - b^2)^2*Cos[c + d*x])/(a*b^4*(a + b*Sin[c
 + d*x])^2) + (2*(-7*a^4 + 5*a^2*b^2 + 2*b^4)*Cos[c + d*x])/(a^2*b^4*(a + b
*Sin[c + d*x])) + Sin[2*(c + d*x)]/b^3)/(4*d)
```

Maple [B] time = 0.204, size = 988, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*a-12/d/b^5*arctan(tan
(1/2*d*x+1/2*c))*a^2+5/d/b^3*arctan(tan(1/2*d*x+1/2*c))-5/d*a^2/b^3/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+1/d/b/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+4/d/a
^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)
^3-6/d*a^3/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*
d*x+1/2*c)^2-9/d*a/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*
tan(1/2*d*x+1/2*c)^2+9/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^
2/a*tan(1/2*d*x+1/2*c)^2+6/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-19/d*a^2/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*t
an(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+11/d/b/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+8/d/a^2*b/(tan(1/2*d*x+1/2*c
)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-6/d*a^3/b^4/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+3/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^2*a+3/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2+12/d*a^3/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))-11/d*a/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a^3*b/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a^3*ln(tan(1/2*d*x+1/2*c)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.84042, size = 2257, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(8*a^4*b^3*cos(d*x + c)^3 + 2*(12*a^5*b^2 - 5*a^3*b^4)*d*x*cos(d*x +
c)^2 - 2*(12*a^7 + 7*a^5*b^2 - 5*a^3*b^4)*d*x + (12*a^6 + 13*a^4*b^2 + 3*a^
2*b^4 + 2*b^6 - (12*a^4*b^2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(12*a^5*b
 + a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x +
 c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2)) - 2*(12*a^6*b + a^4*b^3 - 3*a^2*b^5)*cos(d*x + c) + 2*(
b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) - a^2*b^5 - b^7)*log(1/2*cos(d*x
+ c) + 1/2) - 2*(b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) - a^2*b^5 - b^7)
*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*b^4*cos(d*x + c)^3 + 2*(12*a^6*b - 5
*a^4*b^3)*d*x + 2*(9*a^5*b^2 - 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c
))/(a^3*b^7*d*cos(d*x + c)^2 - 2*a^4*b^6*d*sin(d*x + c) - (a^5*b^5 + a^3*b^
7)*d), -1/2*(4*a^4*b^3*cos(d*x + c)^3 + (12*a^5*b^2 - 5*a^3*b^4)*d*x*cos(d*
```

```
x + c)^2 - (12*a^7 + 7*a^5*b^2 - 5*a^3*b^4)*d*x - (12*a^6 + 13*a^4*b^2 + 3*
a^2*b^4 + 2*b^6 - (12*a^4*b^2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(12*a^5
*b + a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (12*a^6*b + a^4*b^3 - 3*a^2*b^5)*
cos(d*x + c) + (b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) - a^2*b^5 - b^7)*
log(1/2*cos(d*x + c) + 1/2) - (b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) -
a^2*b^5 - b^7)*log(-1/2*cos(d*x + c) + 1/2) - (a^3*b^4*cos(d*x + c)^3 + 2*(
12*a^6*b - 5*a^4*b^3)*d*x + 2*(9*a^5*b^2 - 3*a^3*b^4 - a*b^6)*cos(d*x + c))
*sin(d*x + c))/(a^3*b^7*d*cos(d*x + c)^2 - 2*a^4*b^6*d*sin(d*x + c) - (a^5*
b^5 + a^3*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28736, size = 857, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (12*a^2 - 5*b^2)*(d*x + c)/b^5
+ 2*(12*a^6 - 11*a^4*b^2 + a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/
2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2
 - b^2)*a^3*b^5) - 2*(6*a^5*b*tan(1/2*d*x + 1/2*c)^7 - a^3*b^3*tan(1/2*d*x
+ 1/2*c)^7 - 4*a*b^5*tan(1/2*d*x + 1/2*c)^7 + 12*a^6*tan(1/2*d*x + 1/2*c)^6
 + 13*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 - 9*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 - 6
*b^6*tan(1/2*d*x + 1/2*c)^6 + 54*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 9*a^3*b^3*t
an(1/2*d*x + 1/2*c)^5 - 16*a*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^6*tan(1/2*d*
x + 1/2*c)^4 + 39*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 - 21*a^2*b^4*tan(1/2*d*x +
 1/2*c)^4 - 12*b^6*tan(1/2*d*x + 1/2*c)^4 + 90*a^5*b*tan(1/2*d*x + 1/2*c)^3
 - 27*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 20*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 36
*a^6*tan(1/2*d*x + 1/2*c)^2 + 23*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^
4*tan(1/2*d*x + 1/2*c)^2 - 6*b^6*tan(1/2*d*x + 1/2*c)^2 + 42*a^5*b*tan(1/2*
d*x + 1/2*c) - 11*a^3*b^3*tan(1/2*d*x + 1/2*c) - 8*a*b^5*tan(1/2*d*x + 1/2*
c) + 12*a^6 - 3*a^4*b^2 - 3*a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1
/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) +
 a)^2*a^3*b^4))/d
```

3.1270 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=314

$$\frac{3\left(2a^2+b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}-\frac{6\left(-a^4b^2+2a^6-b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^4d\sqrt{a^2-b^2}}-\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{2a^2b^3d(a+b\sin(c+dx))^2}$$

```
[Out] (3*a*x)/b^4 + (3*Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]])/(a^2*b^4*d) - (6*(2*a^6 - a^4*b^2 - b^6)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*b^4*Sqrt[a^2 - b^2]*d) + (3*b*Ar
cTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b^3*d) - Cot[c + d*x]/(a^3*d)
- ((a^2 - b^2)^2*Cos[c + d*x])/(2*a^2*b^3*d*(a + b*Sin[c + d*x])^2) - (3*(a
^2 - b^2)*Cos[c + d*x])/(2*a*b^3*d*(a + b*Sin[c + d*x])) + (2*(a^2 - b^2)*(
2*a^2 + b^2)*Cos[c + d*x])/(a^3*b^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.4928, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 2638, 2664, 2754, 12, 2660, 618, 204}

$$\frac{3\left(2a^2+b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}-\frac{6\left(-a^4b^2+2a^6-b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^4d\sqrt{a^2-b^2}}-\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{2a^2b^3d(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```



```
[Out] (3*a*x)/b^4 + (3*Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]])/(a^2*b^4*d) - (6*(2*a^6 - a^4*b^2 - b^6)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*b^4*Sqrt[a^2 - b^2]*d) + (3*b*Ar
cTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b^3*d) - Cot[c + d*x]/(a^3*d)
- ((a^2 - b^2)^2*Cos[c + d*x])/(2*a^2*b^3*d*(a + b*Sin[c + d*x])^2) - (3*(a
^2 - b^2)*Cos[c + d*x])/(2*a*b^3*d*(a + b*Sin[c + d*x])) + (2*(a^2 - b^2)*(
2*a^2 + b^2)*Cos[c + d*x])/(a^3*b^3*d*(a + b*Sin[c + d*x]))
```


Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}dx = \int \left(\frac{3a}{b^4} - \frac{3b\csc(c+dx)}{a^4} + \frac{\csc^2(c+dx)}{a^3} - \frac{\sin(c+dx)}{b^3} - \frac{\left(a^2-b^2\right)^3}{a^2b^4(a+b\sin(c+dx))^3} + \frac{2}{a^3}\right.$$

$$= \frac{3ax}{b^4} + \frac{\int \csc^2(c+dx)\,dx}{a^3} - \frac{\int \sin(c+dx)\,dx}{b^3} - \frac{(3b)\int \csc(c+dx)\,dx}{a^4} - \frac{\left(a^2-b^2\right)^3 \int \frac{}{(a+}}{a^2b}$$

$$= \frac{3ax}{b^4} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^3d} - \frac{\left(a^2-b^2\right)^2\cos(c+dx)}{2a^2b^3d(a+b\sin(c+dx))^2} + \frac{2\left(a^2-b\right.}{a^3b}$$

$$= \frac{3ax}{b^4} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^3d} - \frac{\cot(c+dx)}{a^3d} - \frac{\left(a^2-b^2\right)^2\cos(c+dx}{2a^2b^3d(a+b\sin(c+dx}$$

$$= \frac{3ax}{b^4} - \frac{6\left(2a^6-a^4b^2-b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^4\sqrt{a^2-b^2}\,d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+}{b^3d}$$

$$= \frac{3ax}{b^4} - \frac{6\left(2a^6-a^4b^2-b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^4\sqrt{a^2-b^2}\,d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+}{b^3d}$$

$$= \frac{3ax}{b^4} - \frac{4\left(\frac{1}{a^2}-\frac{2a^2}{b^4}+\frac{1}{b^2}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{6\left(2a^6-a^4b^2-b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(\right.}{\sqrt{a^2-b}}\right.}{a^4b^4\sqrt{a^2-b^2}\,d}$$

$$= \frac{3ax}{b^4} - \frac{4\left(\frac{1}{a^2}-\frac{2a^2}{b^4}+\frac{1}{b^2}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{\sqrt{a^2-b^2}\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.}{a^2b^4d}$$

Mathematica [A] time = 6.20395, size = 332, normalized size = 1.06

$$\frac{-a^2b^2\cos(c+dx)+5a^4\cos(c+dx)-4b^4\cos(c+dx)}{2a^3b^3d(a+b\sin(c+dx))} + \frac{2a^2b^2\cos(c+dx)+a^4(-\cos(c+dx))-b^4\cos(c+dx)}{2a^2b^3d(a+b\sin(c+dx))^2} - \frac{3\left(-a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*(c + d*x))/(b^4*d) - (3*(2*a^6 - a^4*b^2 + a^2*b^4 - 2*b^6)*ArcTan[(Se
c[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])
/(a^4*b^4*Sqrt[a^2 - b^2]*d) + Cos[c + d*x]/(b^3*d) - Cot[(c + d*x)/2]/(2*a
^3*d) + (3*b*Log[Cos[(c + d*x)/2]])/(a^4*d) - (3*b*Log[Sin[(c + d*x)/2]])/(
a^4*d) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c + d*x])/
(2*a^2*b^3*d*(a + b*Sin[c + d*x])^2) + (5*a^4*Cos[c + d*x] - a^2*b^2*Cos[c
+ d*x] - 4*b^4*Cos[c + d*x])/(2*a^3*b^3*d*(a + b*Sin[c + d*x])) + Tan[(c +
d*x)/2]/(2*a^3*d)
```

Maple [B] time = 0.21, size = 903, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)+2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)+6/d/b^4*arcta
n(tan(1/2*d*x+1/2*c))*a+3/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c
)*b+a)^2*tan(1/2*d*x+1/2*c)^3*a+3/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1
/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2+4/d/b^3/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*a^2+9/d/b/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-3/d/a^2/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-10/d/a^
4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*
b^3+13/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*
x+1/2*c)*a+1/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/
2*d*x+1/2*c)-14/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*t
an(1/2*d*x+1/2*c)*b^2+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*a^2+1/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-5/d/a^
2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b-6/d/b^4/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+3/d/b^2/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/d
/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2
))+6/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2
)^(1/2))*b^2-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.01052, size = 2634, normalized size = 8.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(24*a^6*b*d*x*cos(d*x + c)^2 - 24*a^6*b*d*x + 2*(9*a^5*b^2 - a^3*b^4 -
 6*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b + 2*a^3*b^3 + 4*a*b^5 - 2*(2*a^5*b +
a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 + (2*a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 2*b^6
- (2*a^4*b^2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b
^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*
(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d
*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(3*a^5*b^2 - a^3*b^4 - 2*a
*b^6)*cos(d*x + c) + 6*(2*a*b^6*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos(d*x + c
)^2 - a^2*b^5 - b^7)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a*b^6
*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos(d*x + c)^2 - a^2*b^5 - b^7)*sin(d*x +
c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(6*a^5*b^2*d*x*cos(d*x + c)^2 + 2*a^4*
```

```
b^3*cos(d*x + c)^3 - 6*(a^7 + a^5*b^2)*d*x - 3*(2*a^6*b + a^4*b^3 - 3*a^2*b
^5)*cos(d*x + c))*sin(d*x + c))/(2*a^5*b^5*d*cos(d*x + c)^2 - 2*a^5*b^5*d +
 (a^4*b^6*d*cos(d*x + c)^2 - (a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), 1/2*(12*
a^6*b*d*x*cos(d*x + c)^2 - 12*a^6*b*d*x + (9*a^5*b^2 - a^3*b^4 - 6*a*b^6)*c
os(d*x + c)^3 - 3*(4*a^5*b + 2*a^3*b^3 + 4*a*b^5 - 2*(2*a^5*b + a^3*b^3 + 2
*a*b^5)*cos(d*x + c)^2 + (2*a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (2*a^4*b^
2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(3*a^5*b^2 - a^3*b
^4 - 2*a*b^6)*cos(d*x + c) + 3*(2*a*b^6*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos
(d*x + c)^2 - a^2*b^5 - b^7)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*
(2*a*b^6*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos(d*x + c)^2 - a^2*b^5 - b^7)*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + (6*a^5*b^2*d*x*cos(d*x + c)^2 +
2*a^4*b^3*cos(d*x + c)^3 - 6*(a^7 + a^5*b^2)*d*x - 3*(2*a^6*b + a^4*b^3 - 3
*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*a^5*b^5*d*cos(d*x + c)^2 - 2*a^5*b
^5*d + (a^4*b^6*d*cos(d*x + c)^2 - (a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32316, size = 622, normalized size = 1.98

$$\frac{6(dx+c)a}{b^4} - \frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4} + \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3} - \frac{6\left(2a^6-a^4b^2+a^2b^4-2b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^4b^4} + \frac{2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(d*x + c)*a/b^4 - 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + tan(1/2*d
*x + 1/2*c)/a^3 - 6*(2*a^6 - a^4*b^2 + a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^4*b^4) + (2*b^4*tan(1/2*d*x + 1/2*c)^3 - a*b^3*tan(1/
2*d*x + 1/2*c)^2 + 4*a^4*tan(1/2*d*x + 1/2*c) + 2*b^4*tan(1/2*d*x + 1/2*c)
- a*b^3)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^4*b^3) + 2*(3*a
^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^5*ta
n(1/2*d*x + 1/2*c)^3 + 4*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^2 - 3*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 10*b^6*tan(1/2*d*x + 1/2*c
)^2 + 13*a^5*b*tan(1/2*d*x + 1/2*c) + a^3*b^3*tan(1/2*d*x + 1/2*c) - 14*a*b
^5*tan(1/2*d*x + 1/2*c) + 4*a^6 + a^4*b^2 - 5*a^2*b^4)/((a*tan(1/2*d*x + 1/
2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^4*b^3))/d
```

3.1271 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=395

$$-\frac{6\left(a^2+b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{\left(2a^2+b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{6\left(a^2b^4+a^6-2b^6\right)\tan}{a^5b^3d\sqrt{a}}$$

```
[Out] -(x/b^3) - (6*Sqrt[a^2 - b^2]*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(a^3*b^3*d) + (Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a
*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^3*d) + (6*(a^6 + a^2*b^4 - 2*b^
6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*b^3*Sqrt[a^2 - b^
2]*d) - ArcTanh[Cos[c + d*x]]/(2*a^3*d) + (3*(a^2 - 2*b^2)*ArcTanh[Cos[c +
d*x]])/(a^5*d) + (3*b*Cot[c + d*x])/(a^4*d) - (Cot[c + d*x]*Csc[c + d*x])/(
2*a^3*d) + ((a^2 - b^2)^2*Cos[c + d*x])/(2*a^3*b^2*d*(a + b*Sin[c + d*x])^2
) + (3*(a^2 - b^2)*Cos[c + d*x])/(2*a^2*b^2*d*(a + b*Sin[c + d*x])) - (3*(a
^4 - b^4)*Cos[c + d*x])/(a^4*b^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.523451, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 3768, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{6\left(a^2+b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{\left(2a^2+b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{6\left(a^2b^4+a^6-2b^6\right)\tan}{a^5b^3d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -(x/b^3) - (6*Sqrt[a^2 - b^2]*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(a^3*b^3*d) + (Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a
*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^3*d) + (6*(a^6 + a^2*b^4 - 2*b^
6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*b^3*Sqrt[a^2 - b^
2]*d) - ArcTanh[Cos[c + d*x]]/(2*a^3*d) + (3*(a^2 - 2*b^2)*ArcTanh[Cos[c +
d*x]])/(a^5*d) + (3*b*Cot[c + d*x])/(a^4*d) - (Cot[c + d*x]*Csc[c + d*x])/(
2*a^3*d) + ((a^2 - b^2)^2*Cos[c + d*x])/(2*a^3*b^2*d*(a + b*Sin[c + d*x])^2
) + (3*(a^2 - b^2)*Cos[c + d*x])/(2*a^2*b^2*d*(a + b*Sin[c + d*x])) - (3*(a
^4 - b^4)*Cos[c + d*x])/(a^4*b^2*d*(a + b*Sin[c + d*x]))
```


Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{1}{b^3} - \frac{3\left(a^2-2b^2\right)\csc(c+dx)}{a^5} - \frac{3b\csc^2(c+dx)}{a^4} + \frac{\csc^3(c+dx)}{a^3} + \frac{\left(a^2 \cdots\right.}{a^3b^3(a+b \cdots}\right.$$

$$= -\frac{x}{b^3} + \frac{\int \csc^3(c+dx)\,dx}{a^3} - \frac{(3b)\int \csc^2(c+dx)\,dx}{a^4} - \frac{\left(3\left(a^2-2b^2\right)\right)\int \csc(c+dx)\,dx}{a^5} \cdots$$

$$= -\frac{x}{b^3} + \frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5 d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3 d} + \frac{\left(a^2-b^2\right)^2 \cdots}{2a^3b^2d(a+b \cdots}$$

$$= -\frac{x}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^3 d} + \frac{3\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5 d} + \frac{3b\cot(c+dx)}{a^4 d} \cdots$$

$$= -\frac{x}{b^3} + \frac{6\left(a^6+a^2b^4-2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^3\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^3 d} + \frac{3\left(a^2 \cdots\right.}{\cdots}$$

$$= -\frac{x}{b^3} + \frac{6\left(a^6+a^2b^4-2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^3\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^3 d} + \frac{3\left(a^2 \cdots\right.}{\cdots}$$

$$= -\frac{x}{b^3} - \frac{6\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{6\left(a^6+a^2b^4-2b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+\cdots\right.}{\sqrt{a^2-b^2}}\right.}{a^5b^3\sqrt{a^2-b^2}\,d} \cdots$$

$$= -\frac{x}{b^3} - \frac{6\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} + \frac{\sqrt{a^2-b^2}\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+\cdots\right.}{\sqrt{a^2-b^2}}\right.}{a^3b^3 d} \cdots$$

Mathematica [A] time = 6.25991, size = 384, normalized size = 0.97

$$\frac{\left(12b^2-5a^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5 d} + \frac{\left(5a^2-12b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5 d} - \frac{3\left(a^2b^2\cos(c+dx)+a^4\cos(c+dx)-2b^4 \cdots\right.}{2a^4b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((c + d*x)/(b^3*d)) + ((2*a^6 - a^4*b^2 + 11*a^2*b^4 - 12*b^6)*ArcTan[(Sec
[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/
(a^5*b^3*Sqrt[a^2 - b^2]*d) + (3*b*Cot[(c + d*x)/2])/(2*a^4*d) - Csc[(c + d
*x)/2]^2/(8*a^3*d) + ((5*a^2 - 12*b^2)*Log[Cos[(c + d*x)/2]])/(2*a^5*d) + (
(-5*a^2 + 12*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^5*d) + Sec[(c + d*x)/2]^2/(8*
a^3*d) + (a^4*Cos[c + d*x] - 2*a^2*b^2*Cos[c + d*x] + b^4*Cos[c + d*x])/(2*
a^3*b^2*d*(a + b*Sin[c + d*x])^2) - (3*(a^4*Cos[c + d*x] + a^2*b^2*Cos[c +
d*x] - 2*b^4*Cos[c + d*x]))/(2*a^4*b^2*d*(a + b*Sin[c + d*x])) - (3*b*Tan[(
c + d*x)/2])/(2*a^4*d)
```

Maple [B] time = 0.221, size = 943, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^4*tan(1/2*d*x+1/2*c)*b-2/d/b^3*arcta
n(tan(1/2*d*x+1/2*c))-1/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)^3-7/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+8/d/a^4*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-2/d*a/b^2/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-9/d/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2-3/d/a^3*b^2/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+14/d/a^5*
b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^
2-7/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2
*c)-13/d/a^2*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*
d*x+1/2*c)+20/d/a^4*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
*tan(1/2*d*x+1/2*c)-2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*a-5/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+7/d/a^3*b^
2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+2/d*a/b^3/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/d/a/b/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+11/d/a^
3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)-12/d/a^5*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))-1/8/d/a^3/tan(1/2*d*x+1/2*c)^2-5/2/d/a^3*ln(tan(1/2*d*x+1/2*c)
)+6/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^2+3/2/d*b/a^4/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.21889, size = 3688, normalized size = 9.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^5*b^2*d*x*cos(d*x + c)^4 - 4*(a^7 + 2*a^5*b^2)*d*x*cos(d*x + c)^
2 - 2*(2*a^6*b + 5*a^4*b^3 - 18*a^2*b^5)*cos(d*x + c)^3 + 4*(a^7 + a^5*b^2)
*d*x - (2*a^6 + 3*a^4*b^2 + 13*a^2*b^4 + 12*b^6 + (2*a^4*b^2 + a^2*b^4 + 12
*b^6)*cos(d*x + c)^4 - (2*a^6 + 5*a^4*b^2 + 14*a^2*b^4 + 24*b^6)*cos(d*x +
c)^2 + 2*(2*a^5*b + a^3*b^3 + 12*a*b^5 - (2*a^5*b + a^3*b^3 + 12*a*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 4*(a^6*b + 3*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c) - (5*a^4*b^3 -
7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos(d*x + c)^4 - (5*a^4*b^3 - 2*a
```

```
^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^4 - 12*a*b^6 - (5*a^3*b^4 - 12
*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (5*a^4*
b^3 - 7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos(d*x + c)^4 - (5*a^4*b^3
 - 2*a^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^4 - 12*a*b^6 - (5*a^3*b^
4 - 12*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) -
2*(4*a^6*b*d*x*cos(d*x + c)^2 - 4*a^6*b*d*x + 3*(a^5*b^2 + a^3*b^4 - 4*a*b^
6)*cos(d*x + c)^3 - (3*a^5*b^2 - a^3*b^4 - 12*a*b^6)*cos(d*x + c))*sin(d*x
+ c))/(a^5*b^5*d*cos(d*x + c)^4 - (a^7*b^3 + 2*a^5*b^5)*d*cos(d*x + c)^2 +
(a^7*b^3 + a^5*b^5)*d - 2*(a^6*b^4*d*cos(d*x + c)^2 - a^6*b^4*d)*sin(d*x +
c)), -1/4*(4*a^5*b^2*d*x*cos(d*x + c)^4 - 4*(a^7 + 2*a^5*b^2)*d*x*cos(d*x +
 c)^2 - 2*(2*a^6*b + 5*a^4*b^3 - 18*a^2*b^5)*cos(d*x + c)^3 + 4*(a^7 + a^5*
b^2)*d*x + 2*(2*a^6 + 3*a^4*b^2 + 13*a^2*b^4 + 12*b^6 + (2*a^4*b^2 + a^2*b^
4 + 12*b^6)*cos(d*x + c)^4 - (2*a^6 + 5*a^4*b^2 + 14*a^2*b^4 + 24*b^6)*cos(
d*x + c)^2 + 2*(2*a^5*b + a^3*b^3 + 12*a*b^5 - (2*a^5*b + a^3*b^3 + 12*a*b^
5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 4*(a^6*b + 3*a^4*b^3 - 9*a^2*b^5)*cos(
d*x + c) - (5*a^4*b^3 - 7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos(d*x +
 c)^4 - (5*a^4*b^3 - 2*a^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^4 - 12
*a*b^6 - (5*a^3*b^4 - 12*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) + (5*a^4*b^3 - 7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos
(d*x + c)^4 - (5*a^4*b^3 - 2*a^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^
4 - 12*a*b^6 - (5*a^3*b^4 - 12*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2) - 2*(4*a^6*b*d*x*cos(d*x + c)^2 - 4*a^6*b*d*x + 3*(a^
5*b^2 + a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (3*a^5*b^2 - a^3*b^4 - 12*a*b^6
)*cos(d*x + c))*sin(d*x + c))/(a^5*b^5*d*cos(d*x + c)^4 - (a^7*b^3 + 2*a^5*
b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + a^5*b^5)*d - 2*(a^6*b^4*d*cos(d*x + c)^2
 - a^6*b^4*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+b*sin(d*x+c))**3,x)

[Out] Timed out
```

Giac [A] time = 1.30663, size = 691, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/8*(8*(d*x + c)/b^3 + 4*(5*a^2 - 12*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a
^5 - (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6 - 8*(
2*a^6 - a^4*b^2 + 11*a^2*b^4 - 12*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^
2)*a^5*b^3) - (10*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^4*tan(1/2*d*x +
 1/2*c)^6 - 8*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 4*a^3*b^3*tan(1/2*d*x + 1/2*c)
^5 - 32*a*b^5*tan(1/2*d*x + 1/2*c)^5 - 16*a^6*tan(1/2*d*x + 1/2*c)^4 - 53*a
^4*b^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 + 16*b^6*
```

```
tan(1/2*d*x + 1/2*c)^4 - 56*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 44*a^3*b^3*tan(1
/2*d*x + 1/2*c)^3 + 112*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 16*a^6*tan(1/2*d*x +
 1/2*c)^2 - 32*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 76*a^2*b^4*tan(1/2*d*x + 1/
2*c)^2 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c) - a^4*b^2)/((a*tan(1/2*d*x + 1/2*c)
^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2*a^5*b^2))/d
```

3.1272 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=329

$$\frac{5\left(a^2-4b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(35a^2b^2+3a^4-60b^4\right)\cot(c+dx)}{6a^5b^2d}-\frac{5b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+d}{2a^6d}$$

```
[Out] (5*(a^2 - 4*b^2)*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^6*d) - (5*b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^6*d) + (
(3*a^4 + 35*a^2*b^2 - 60*b^4)*Cot[c + d*x])/(6*a^5*b^2*d) - Cos[c + d*x]/(b
*d*(a + b*Sin[c + d*x])^2) - (a*Cot[c + d*x])/(2*b^2*d*(a + b*Sin[c + d*x])
^2) - ((3*a^2 - 5*b^2)*Cot[c + d*x])/(3*a^3*d*(a + b*Sin[c + d*x])^2) + (5*
b*Cot[c + d*x]*Csc[c + d*x])/(6*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*
x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^2) - (5*(a^2 - 2*b^2)*Cot[c
+ d*x])/(2*a^4*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.32904, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{5\left(a^2-4b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(35a^2b^2+3a^4-60b^4\right)\cot(c+dx)}{6a^5b^2d}-\frac{5b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+d}{2a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*(a^2 - 4*b^2)*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^6*d) - (5*b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^6*d) + (
(3*a^4 + 35*a^2*b^2 - 60*b^4)*Cot[c + d*x])/(6*a^5*b^2*d) - Cos[c + d*x]/(b
*d*(a + b*Sin[c + d*x])^2) - (a*Cot[c + d*x])/(2*b^2*d*(a + b*Sin[c + d*x])
^2) - ((3*a^2 - 5*b^2)*Cot[c + d*x])/(3*a^3*d*(a + b*Sin[c + d*x])^2) + (5*
b*Cot[c + d*x]*Csc[c + d*x])/(6*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*
x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^2) - (5*(a^2 - 2*b^2)*Cot[c
+ d*x])/(2*a^4*d*(a + b*Sin[c + d*x]))
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
```

```
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2} + \frac{5b\cot(c+dx)\csc(c+dx)}{6a^2d(a+b\sin(c+dx))^2} -$$

$$= -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2} - \frac{(3a^2-5b^2)\cot(c+dx)}{3a^3d(a+b\sin(c+dx))^2} + \frac{5b}{6}$$

$$= -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2} - \frac{(3a^2-5b^2)\cot(c+dx)}{3a^3d(a+b\sin(c+dx))^2} + \frac{5b}{6}$$

$$= \frac{(3a^4+35a^2b^2-60b^4)\cot(c+dx)}{6a^5b^2d} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+d}$$

$$= \frac{(3a^4+35a^2b^2-60b^4)\cot(c+dx)}{6a^5b^2d} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+d}$$

$$= -\frac{5b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(3a^4+35a^2b^2-60b^4)\cot(c+dx)}{6a^5b^2d} - \frac{}{bd(a}$$

$$= -\frac{5b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(3a^4+35a^2b^2-60b^4)\cot(c+dx)}{6a^5b^2d} - \frac{}{bd(a}$$

$$= \frac{5(a^2-4b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} - \frac{5b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}$$

Mathematica [A] time = 6.26864, size = 490, normalized size = 1.49

$$\frac{5(3a^2b-4b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} - \frac{5(3a^2b-4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} + \frac{7a^2b^2\cos(c+dx)+a^4\cos(c+dx)-8b}{2a^5bd(a+b\sin(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] +
 a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((7*a^2*C
os[(c + d*x)/2] - 18*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^5*d) + (3
*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(2
4*a^3*d) - (5*(3*a^2*b - 4*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + (5*(3*a^
2*b - 4*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8
*a^4*d) + (Sec[(c + d*x)/2]*(-7*a^2*Sin[(c + d*x)/2] + 18*b^2*Sin[(c + d*x)
/2]))/(6*a^5*d) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c
 + d*x])/(2*a^4*b*d*(a + b*Sin[c + d*x])^2) + (a^4*Cos[c + d*x] + 7*a^2*b^2
*Cos[c + d*x] - 8*b^4*Cos[c + d*x])/(2*a^5*b*d*(a + b*Sin[c + d*x])) + (Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)
```

Maple [B] time = 0.229, size = 873, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b-9/8/d/a^3*
tan(1/2*d*x+1/2*c)+3/d/a^5*b^2*tan(1/2*d*x+1/2*c)-1/d/a/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+11/d/a^3/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-10/d/a^5/(
tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4
+9/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)^2*b+9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/
2*d*x+1/2*c)^2*b^3-18/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)^2*b^5+1/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+25/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2-26/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4+9/d/a^2/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b-9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2*b^3+5/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-25/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+20/d/a^6/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-1/24/d/a^3/tan(1/2*d*
x+1/2*c)^3+9/8/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^5/tan(1/2*d*x+1/2*c)*b^2+3/8/
d/a^4*b/tan(1/2*d*x+1/2*c)^2+15/2/d/a^4*b*ln(tan(1/2*d*x+1/2*c))-10/d/a^6*b
^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78155, size = 3528, normalized size = 10.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/12*(2*(3*a^5 + 35*a^3*b^2 - 60*a*b^4)*cos(d*x + c)^5 - 20*(2*a^5 + 5*a^3
*b^2 - 12*a*b^4)*cos(d*x + c)^3 - 15*(2*(a^3*b - 4*a*b^3)*cos(d*x + c)^4 +
2*a^3*b - 8*a*b^3 - 4*(a^3*b - 4*a*b^3)*cos(d*x + c)^2 + ((a^2*b^2 - 4*b^4)
*cos(d*x + c)^4 + a^4 - 3*a^2*b^2 - 4*b^4 - (a^4 - 2*a^2*b^2 - 8*b^4)*cos(d
*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2
 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(
d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
- b^2)) + 30*(a^5 + a^3*b^2 - 4*a*b^4)*cos(d*x + c) - 15*(6*a^3*b^2 - 8*a*b
^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d
*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4
 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(
```

```
d*x + c) + 1/2) + 15*(6*a^3*b^2 - 8*a*b^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b
^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 10*((11*a^4*b - 1
8*a^2*b^3)*cos(d*x + c)^3 - 6*(2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*sin(d*x +
 c))/(2*a^7*b*d*cos(d*x + c)^4 - 4*a^7*b*d*cos(d*x + c)^2 + 2*a^7*b*d + (a^
6*b^2*d*cos(d*x + c)^4 - (a^8 + 2*a^6*b^2)*d*cos(d*x + c)^2 + (a^8 + a^6*b^
2)*d)*sin(d*x + c)), 1/12*(2*(3*a^5 + 35*a^3*b^2 - 60*a*b^4)*cos(d*x + c)^5
 - 20*(2*a^5 + 5*a^3*b^2 - 12*a*b^4)*cos(d*x + c)^3 - 30*(2*(a^3*b - 4*a*b^
3)*cos(d*x + c)^4 + 2*a^3*b - 8*a*b^3 - 4*(a^3*b - 4*a*b^3)*cos(d*x + c)^2
+ ((a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + a^4 - 3*a^2*b^2 - 4*b^4 - (a^4 - 2*a^
2*b^2 - 8*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin
(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(a^5 + a^3*b^2 - 4*a*b^
4)*cos(d*x + c) - 15*(6*a^3*b^2 - 8*a*b^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b
^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(6*a^3*b^2 - 8*
a*b^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*co
s(d*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c
)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*
cos(d*x + c) + 1/2) - 10*((11*a^4*b - 18*a^2*b^3)*cos(d*x + c)^3 - 6*(2*a^4
*b - 3*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(2*a^7*b*d*cos(d*x + c)^4 - 4*a
^7*b*d*cos(d*x + c)^2 + 2*a^7*b*d + (a^6*b^2*d*cos(d*x + c)^4 - (a^8 + 2*a^
6*b^2)*d*cos(d*x + c)^2 + (a^8 + a^6*b^2)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+b*sin(d*x+c))**3,x)

[Out] Timed out
```

Giac [A] time = 1.28418, size = 645, normalized size = 1.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/24*(60*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 120*(a^4 -
5*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) - 24*(a^5*ta
n(1/2*d*x + 1/2*c)^3 - 11*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 10*a*b^4*tan(1/2
*d*x + 1/2*c)^3 - 9*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b^3*tan(1/2*d*x +
1/2*c)^2 + 18*b^5*tan(1/2*d*x + 1/2*c)^2 - a^5*tan(1/2*d*x + 1/2*c) - 25*a^
3*b^2*tan(1/2*d*x + 1/2*c) + 26*a*b^4*tan(1/2*d*x + 1/2*c) - 9*a^4*b + 9*a^
2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^6) +
(a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2*c)^2 - 27*a^6*tan(1
/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9 - (330*a^2*b*tan(1/2
*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*tan(1/2*d*x + 1/2
```

```
*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1/2*d*x + 1/2*c) + a^
3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

3.1273 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=355

$$-\frac{15b\left(a^2-2b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{\left(-25a^2b^2+a^4+30b^4\right)\cot(c+dx)}{2a^6bd}-\frac{15\left(-8a^2b^2+a^4+8b^4\right)\tanh}{8a^7d}$$

```
[Out] (-15*b*(a^2 - 2*b^2)*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/(a^7*d) - (15*(a^4 - 8*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/
(8*a^7*d) + ((a^4 - 25*a^2*b^2 + 30*b^4)*Cot[c + d*x])/(2*a^6*b*d) + (15*(3
*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^5*d) - Cot[c + d*x]/(2*b*d*(a
 + b*Sin[c + d*x])^2) - ((4*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(4*a^3*
d*(a + b*Sin[c + d*x])^2) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(2*a^2*d*(a + b
*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d*(a + b*Sin[c + d*x
])^2) - ((7*a^2 - 10*b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^4*d*(a + b*Sin[c
+ d*x]))
```

Rubi [A] time = 1.68169, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{15b\left(a^2-2b^2\right)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{\left(-25a^2b^2+a^4+30b^4\right)\cot(c+dx)}{2a^6bd}-\frac{15\left(-8a^2b^2+a^4+8b^4\right)\tanh}{8a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-15*b*(a^2 - 2*b^2)*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/(a^7*d) - (15*(a^4 - 8*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/
(8*a^7*d) + ((a^4 - 25*a^2*b^2 + 30*b^4)*Cot[c + d*x])/(2*a^6*b*d) + (15*(3
*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^5*d) - Cot[c + d*x]/(2*b*d*(a
 + b*Sin[c + d*x])^2) - ((4*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(4*a^3*
d*(a + b*Sin[c + d*x])^2) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(2*a^2*d*(a + b
*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d*(a + b*Sin[c + d*x
])^2) - ((7*a^2 - 10*b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^4*d*(a + b*Sin[c
+ d*x]))
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
```

```
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{b\cot(c+dx)\csc^2(c+dx)}{2a^2d(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))^2} +$$

$$= -\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{(4a^2-5b^2)\cot(c+dx)\csc(c+dx)}{4a^3d(a+b\sin(c+dx))^2} + \frac{b\cot(c+dx)\csc^2}{2a^2d(a+b\sin(c}$$

$$= -\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{(4a^2-5b^2)\cot(c+dx)\csc(c+dx)}{4a^3d(a+b\sin(c+dx))^2} + \frac{b\cot(c+dx)\csc^2}{2a^2d(a+b\sin(c}$$

$$= \frac{15(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{(4a^2-5b^2)\cot(}{4a^3d(a+b}$$

$$= \frac{(a^4-25a^2b^2+30b^4)\cot(c+dx)}{2a^6bd} + \frac{15(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{}{2bd(a}$$

$$= \frac{(a^4-25a^2b^2+30b^4)\cot(c+dx)}{2a^6bd} + \frac{15(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{}{2bd(a}$$

$$= -\frac{15(a^4-8a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d} + \frac{(a^4-25a^2b^2+30b^4)\cot(c+dx)}{2a^6bd} +$$

$$= -\frac{15(a^4-8a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d} + \frac{(a^4-25a^2b^2+30b^4)\cot(c+dx)}{2a^6bd} +$$

$$= -\frac{15b(a^2-2b^2)\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} - \frac{15(a^4-8a^2b^2+8b^4)\tanh^{-1}(\text{co}}{8a^7d}$$

Mathematica [A] time = 1.33923, size = 363, normalized size = 1.02

$$-\frac{1920b(-3a^2b^2+a^4+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 240(-8a^2b^2+a^4+8b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 240(-8a^2b^2+a^4+8b^4)\text{lo}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-1920*b*(a^4 - 3*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] - 240*(a^4 - 8*a^2*b^2 + 8*b^4)*Log[Cos[(c + d*x
)/2]] + 240*(a^4 - 8*a^2*b^2 + 8*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Cot[c +
d*x]*Csc[c + d*x]^5*(44*a^5 - 505*a^3*b^2 + 540*a*b^4 + (-68*a^5 + 660*a^3*
b^2 - 720*a*b^4)*Cos[2*(c + d*x)] + (8*a^5 - 155*a^3*b^2 + 180*a*b^4)*Cos[4
*(c + d*x)] - 176*a^4*b*Sin[c + d*x] - 260*a^2*b^3*Sin[c + d*x] + 600*b^5*S
in[c + d*x] + 66*a^4*b*Sin[3*(c + d*x)] + 170*a^2*b^3*Sin[3*(c + d*x)] - 30
0*b^5*Sin[3*(c + d*x)] + 2*a^4*b*Sin[5*(c + d*x)] - 50*a^2*b^3*Sin[5*(c + d
*x)] + 60*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x])^2)/(128*a^7*d)
```

Maple [B] time = 0.228, size = 1070, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/64/d/a^3*tan(1/2*d*x+1/2*c)^4-1/64/d/a^3/tan(1/2*d*x+1/2*c)^4+2/d/a/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+15/8/d/a^3*ln(tan(1/2*d*x+1/
2*c))+12/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*
d*x+1/2*c)^3*b^5-30/d/a^7*b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1
/2*c)+2*b)/(a^2-b^2)^(1/2))-15/d/a^4*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-15/d/a^5*b^4/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+3/d/a^2*b/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-9/d/a^3*b^2/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+5/d/a^2
*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-1
5/d/a^3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))+27/8/d/a^4*tan(1/2*d*x+1/2*c)*b-13/d/a^3*b^2/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)^2-15/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^2-27/8/d*b/
a^4/tan(1/2*d*x+1/2*c)+32/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c
)*b+a)^2*tan(1/2*d*x+1/2*c)*b^5+22/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^6+2/d/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2+1/4/d/a^3/tan(1/2*d*x+1/2*c)
^2-1/4/d/a^3*tan(1/2*d*x+1/2*c)^2-37/d/a^4*b^3/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+45/d/a^5*b^3/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/8/d/a^4*b/tan(1/2*d
*x+1/2*c)^3+5/d*b^3/a^6/tan(1/2*d*x+1/2*c)-1/8/d/a^4*tan(1/2*d*x+1/2*c)^3*b
+3/4/d/a^5*tan(1/2*d*x+1/2*c)^2*b^2-5/d/a^6*b^3*tan(1/2*d*x+1/2*c)+11/d/a^5
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4-3/4/d/a^5/tan(1/2*
d*x+1/2*c)^2*b^2+15/d/a^7*ln(tan(1/2*d*x+1/2*c))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.54943, size = 4537, normalized size = 12.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/16*(2*(8*a^6 - 155*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 10*(5*a^6 -
64*a^4*b^2 + 72*a^2*b^4)*cos(d*x + c)^3 + 60*((a^2*b^3 - 2*b^5)*cos(d*x + c
)^6 - a^4*b + a^2*b^3 + 2*b^5 - (a^4*b + a^2*b^3 - 6*b^5)*cos(d*x + c)^4 +
(2*a^4*b - a^2*b^3 - 6*b^5)*cos(d*x + c)^2 - 2*(a^3*b^2 - 2*a*b^4 + (a^3*b^
2 - 2*a*b^4)*cos(d*x + c)^4 - 2*(a^3*b^2 - 2*a*b^4)*cos(d*x + c)^2)*sin(d*x
```

```
 + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(a^6
 - 11*a^4*b^2 + 12*a^2*b^4)*cos(d*x + c) + 15*((a^4*b^2 - 8*a^2*b^4 + 8*b^6
)*cos(d*x + c)^6 - a^6 + 7*a^4*b^2 - 8*b^6 - (a^6 - 5*a^4*b^2 - 16*a^2*b^4
+ 24*b^6)*cos(d*x + c)^4 + (2*a^6 - 13*a^4*b^2 - 8*a^2*b^4 + 24*b^6)*cos(d*
x + c)^2 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5 + (a^5*b - 8*a^3*b^3 + 8*a*b^5)*c
os(d*x + c)^4 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((a^4*b^2 - 8*a^2*b^4 + 8*b^6)*cos(d*x
+ c)^6 - a^6 + 7*a^4*b^2 - 8*b^6 - (a^6 - 5*a^4*b^2 - 16*a^2*b^4 + 24*b^6)*
cos(d*x + c)^4 + (2*a^6 - 13*a^4*b^2 - 8*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 -
 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5 + (a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c
)^4 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2) + 4*(2*(a^5*b - 25*a^3*b^3 + 30*a*b^5)*cos(d*x + c)^5
 + 5*(3*a^5*b + 16*a^3*b^3 - 24*a*b^5)*cos(d*x + c)^3 - 15*(a^5*b + 2*a^3*b
^3 - 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9
+ 3*a^7*b^2)*d*cos(d*x + c)^4 + (2*a^9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9
 + a^7*b^2)*d - 2*(a^8*b*d*cos(d*x + c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*
b*d)*sin(d*x + c)), -1/16*(2*(8*a^6 - 155*a^4*b^2 + 180*a^2*b^4)*cos(d*x +
c)^5 - 10*(5*a^6 - 64*a^4*b^2 + 72*a^2*b^4)*cos(d*x + c)^3 - 120*((a^2*b^3
- 2*b^5)*cos(d*x + c)^6 - a^4*b + a^2*b^3 + 2*b^5 - (a^4*b + a^2*b^3 - 6*b^
5)*cos(d*x + c)^4 + (2*a^4*b - a^2*b^3 - 6*b^5)*cos(d*x + c)^2 - 2*(a^3*b^2
 - 2*a*b^4 + (a^3*b^2 - 2*a*b^4)*cos(d*x + c)^4 - 2*(a^3*b^2 - 2*a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c))) + 30*(a^6 - 11*a^4*b^2 + 12*a^2*b^4)*cos(d*x +
 c) + 15*((a^4*b^2 - 8*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - a^6 + 7*a^4*b^2 -
8*b^6 - (a^6 - 5*a^4*b^2 - 16*a^2*b^4 + 24*b^6)*cos(d*x + c)^4 + (2*a^6 - 1
3*a^4*b^2 - 8*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 8*a^3*b^3 + 8*a
*b^5 + (a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 8*a^3*b^3
+ 8*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(
(a^4*b^2 - 8*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - a^6 + 7*a^4*b^2 - 8*b^6 - (a
^6 - 5*a^4*b^2 - 16*a^2*b^4 + 24*b^6)*cos(d*x + c)^4 + (2*a^6 - 13*a^4*b^2
- 8*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5 + (a^
5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5)
*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(2*(a^5*b -
 25*a^3*b^3 + 30*a*b^5)*cos(d*x + c)^5 + 5*(3*a^5*b + 16*a^3*b^3 - 24*a*b^5
)*cos(d*x + c)^3 - 15*(a^5*b + 2*a^3*b^3 - 4*a*b^5)*cos(d*x + c))*sin(d*x +
 c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9 + 3*a^7*b^2)*d*cos(d*x + c)^4 + (2*a^
9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9 + a^7*b^2)*d - 2*(a^8*b*d*cos(d*x +
c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*b*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35957, size = 814, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(120*(a^4 - 8*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 - 96
0*(a^4*b - 3*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^7) +
 64*(3*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 1
2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*a^6*tan(1/2*d*x + 1/2*c)^2 - 9*a^4*b^2*t
an(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 22*b^6*tan(1/2*
d*x + 1/2*c)^2 + 5*a^5*b*tan(1/2*d*x + 1/2*c) - 37*a^3*b^3*tan(1/2*d*x + 1/
2*c) + 32*a*b^5*tan(1/2*d*x + 1/2*c) + 2*a^6 - 13*a^4*b^2 + 11*a^2*b^4)/((a
*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^7) - (250*a^4*t
an(1/2*d*x + 1/2*c)^4 - 2000*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 2000*b^4*tan(
1/2*d*x + 1/2*c)^4 + 216*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d
*x + 1/2*c)^3 - 16*a^4*tan(1/2*d*x + 1/2*c)^2 + 48*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 - 8*a^3*b*tan(1/2*d*x + 1/2*c) + a^4)/(a^7*tan(1/2*d*x + 1/2*c)^4) +
 (a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 16*a^9*tan(
1/2*d*x + 1/2*c)^2 + 48*a^7*b^2*tan(1/2*d*x + 1/2*c)^2 + 216*a^8*b*tan(1/2*
d*x + 1/2*c) - 320*a^6*b^3*tan(1/2*d*x + 1/2*c))/a^12)/d
```

3.1274 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=492

$$-\frac{\sqrt{a^2-b^2}\left(-29a^2b^2+2a^4+42b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{\left(-645a^2b^2+91a^4+630b^4\right)\cot(c+dx)}{30a^7d}+\frac{b\left(-200a^2b^2\right.}{}$$

```
[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcT
anh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*
x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/
(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^
2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((
5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a
+ b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*S
in[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])
^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b
^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 2.15507, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{\sqrt{a^2-b^2}\left(-29a^2b^2+2a^4+42b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{\left(-645a^2b^2+91a^4+630b^4\right)\cot(c+dx)}{30a^7d}+\frac{b\left(-200a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcT
anh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*
x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/
(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^
2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((
5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a
+ b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*S
in[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])
^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b
^2*d*(a + b*Sin[c + d*x]))
```


Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
```

```
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2} - \frac{\mathrm{c}}{5}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{\left(5a^4-60a^2b^2+63b^4\right)\cot(c+}{60a^3b^2d(a+b\sin(c+}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{\left(5a^4-60a^2b^2+63b^4\right)\cot(c+}{60a^3b^2d(a+b\sin(c+}$$

$$= -\frac{\left(15a^4-187a^2b^2+210b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c}{12b^2d(}$$

$$= \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c+dx)}{8a^6bd} - \frac{\left(15a^4-187a^2b^2+210b^4\right)\cot(c+dx)\mathrm{cs}}{30a^5b^2d}$$

$$= -\frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d} + \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c+dx)}{8a^6bd}$$

$$= -\frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d} + \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c+dx)}{8a^6bd}$$

$$= \frac{b\left(45a^4-200a^2b^2+168b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx}{30a^7d}$$

$$= \frac{b\left(45a^4-200a^2b^2+168b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx}{30a^7d}$$

$$= -\frac{\sqrt{a^2-b^2}\left(2a^4-29a^2b^2+42b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{b\left(45a^4-200a^2b^2+168b^4\right)\mathrm{t}}{8a^8d}$$

Mathematica [A] time = 1.71411, size = 448, normalized size = 0.91

$$-\frac{3840\left(-31a^4b^2+71a^2b^4+2a^6-42b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 480b\left(-200a^2b^2+45a^4+168b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 480b\left(-20\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-3840*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 480*b*(45*a^4 - 200*a^2*b^2 + 1
68*b^4)*Log[Cos[(c + d*x)/2]] - 480*b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*Log[
Sin[(c + d*x)/2]] + (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(-784*a^6 + 3256*a^4*b
^2 + 7860*a^2*b^4 - 12600*b^6 + 2*(384*a^6 - 2131*a^4*b^2 - 6315*a^2*b^4 +
9450*b^6)*Cos[2*(c + d*x)] + (-368*a^6 + 824*a^4*b^2 + 6060*a^2*b^4 - 7560*
b^6)*Cos[4*(c + d*x)] + 182*a^4*b^2*Cos[6*(c + d*x)] - 1290*a^2*b^4*Cos[6*(
c + d*x)] + 1260*b^6*Cos[6*(c + d*x)] - 8156*a^5*b*Sin[c + d*x] + 42270*a^3
*b^3*Sin[c + d*x] - 37800*a*b^5*Sin[c + d*x] + 3956*a^5*b*Sin[3*(c + d*x)]
- 20715*a^3*b^3*Sin[3*(c + d*x)] + 18900*a*b^5*Sin[3*(c + d*x)] - 608*a^5*b
```

```
*Sin[5*(c + d*x)] + 3975*a^3*b^3*Sin[5*(c + d*x)] - 3780*a*b^5*Sin[5*(c + d
*x)]))/(b + a*Csc[c + d*x])^2)/(3840*a^8*d)
```

Maple [B] time = 0.239, size = 1252, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 31/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^
(1/2))*b^2+11/16/d/a^3*tan(1/2*d*x+1/2*c)-11/16/d/a^3/tan(1/2*d*x+1/2*c)-21
/d/a^8*b^5*ln(tan(1/2*d*x+1/2*c))-13/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2*b^5-1/4/d/a^5/tan(1/2*d*x+1/2*c)^3*b^2-3/64/d/a^4*tan(1
/2*d*x+1/2*c)^4*b+1/4/d/a^5*tan(1/2*d*x+1/2*c)^3*b^2-5/4/d/a^6*tan(1/2*d*x+
1/2*c)^2*b^3+15/2/d/a^7*b^4*tan(1/2*d*x+1/2*c)-15/2/d/a^7/tan(1/2*d*x+1/2*c
)*b^4+3/64/d/a^4*b/tan(1/2*d*x+1/2*c)^4+5/4/d/a^6*b^3/tan(1/2*d*x+1/2*c)^2-
1/160/d/a^3/tan(1/2*d*x+1/2*c)^5+1/160/d/a^3*tan(1/2*d*x+1/2*c)^5-45/8/d/a^
4*b*ln(tan(1/2*d*x+1/2*c))-4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*b-2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*
b)/(a^2-b^2)^(1/2))+17/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)^2*b^3+27/4/d/a^5/tan(1/2*d*x+1/2*c)*b^2-3/4/d/a^4*b/tan(1/2*d*x+1/2*c)^
2+25/d/a^6*b^3*ln(tan(1/2*d*x+1/2*c))-5/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-38/d/a^7/(tan(1/2*d*x+1/2*c
)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^6+42/d/a^8/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6-14/d/a^
7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*
b^6-26/d/a^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*
x+1/2*c)^2*b^7+7/96/d/a^3/tan(1/2*d*x+1/2*c)^3+19/d/a^5/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4-7/96/d/a^3*tan(1/
2*d*x+1/2*c)^3+9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*
tan(1/2*d*x+1/2*c)^2*b^3-11/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2+3/4/d/a^4*tan(1/2*d*x+1/2*c)^2*b-27/4/d/a
^5*b^2*tan(1/2*d*x+1/2*c)-4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b+21/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5+49/d/a^5/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4-71/d/a^6/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.08991, size = 6091, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^
7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 3
7*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 60*(2*(2*a^5*b - 29*a
^3*b^3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*
a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 4
2*a*b^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6
 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*
b^4 + 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2)) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*cos(d*x + c)
 + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 +
 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d
*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a
^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168
*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(
d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^
2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4
 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*
(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200
*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5
 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*
b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^
4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)*cos(d*x + c)^
5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^3 + 15*(53*a^6
*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*a^9*b*d*cos(
d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c)^2 - 2*a^9*b*
d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x + c)^4 + (2*a^
10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*x + c)), -1/24
0*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^7 + 67
*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 37*a^5*
b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 120*(2*(2*a^5*b - 29*a^3*b^
3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*a^5*b
 - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b
^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6 - 2*
a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*b^4 +
 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*cos(d
*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt
(a^2 - b^2)*cos(d*x + c))) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6
)*cos(d*x + c) + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 -
 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 16
8*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x
+ c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200
*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 +
 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)
*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2
 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d
*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45
*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^
3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c
)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*
a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*
```

```
log(-1/2*cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)
*cos(d*x + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^
3 + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(
2*a^9*b*d*cos(d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c
)^2 - 2*a^9*b*d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x
+ c)^4 + (2*a^10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*
x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27661, size = 987, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/960*(120*(45*a^4*b - 200*a^2*b^3 + 168*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^8 + 960*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^8) + 960*(5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 19*a^3*b
^4*tan(1/2*d*x + 1/2*c)^3 + 14*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1
/2*d*x + 1/2*c)^2 - 9*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^5*tan(1/2*d
*x + 1/2*c)^2 + 26*b^7*tan(1/2*d*x + 1/2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/
2*c) - 49*a^3*b^4*tan(1/2*d*x + 1/2*c) + 38*a*b^6*tan(1/2*d*x + 1/2*c) + 4*
a^6*b - 17*a^4*b^3 + 13*a^2*b^5)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d
*x + 1/2*c) + a)^2*a^8) - (12330*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 54800*a^2*b
^3*tan(1/2*d*x + 1/2*c)^5 + 46032*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(
1/2*d*x + 1/2*c)^4 + 6480*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 7200*a*b^4*tan(1
/2*d*x + 1/2*c)^4 - 720*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1200*a^2*b^3*tan(1/2
*d*x + 1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 240*a^3*b^2*tan(1/2*d*x +
 1/2*c)^2 + 45*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^8*tan(1/2*d*x + 1/2*c
)^5) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^11*b*tan(1/2*d*x + 1/2*c)^4 -
70*a^12*tan(1/2*d*x + 1/2*c)^3 + 240*a^10*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*
a^11*b*tan(1/2*d*x + 1/2*c)^2 - 1200*a^9*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a
^12*tan(1/2*d*x + 1/2*c) - 6480*a^10*b^2*tan(1/2*d*x + 1/2*c) + 7200*a^8*b^
4*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.1275 $\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=600

$$-\frac{3b^2\sqrt{a^2-b^2}\left(-23a^2b^2+4a^4+24b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^{10}d}+\frac{\left(-889a^4b^2+3255a^2b^4+10a^6-2520b^6\right)\cot(c+dx)}{70a^9d}$$

```
[Out] (-3*b^2*Sqrt[a^2 - b^2]*(4*a^4 - 23*a^2*b^2 + 24*b^4)*ArcTan[(b + a*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]])/(a^10*d) - (3*b*(5*a^6 - 100*a^4*b^2 + 280*a^2
*b^4 - 192*b^6)*ArcTanh[Cos[c + d*x]])/(16*a^10*d) + ((10*a^6 - 889*a^4*b^2
 + 3255*a^2*b^4 - 2520*b^6)*Cot[c + d*x])/(70*a^9*d) + (3*b*(27*a^4 - 116*a
^2*b^2 + 96*b^4)*Cot[c + d*x]*Csc[c + d*x])/(16*a^8*d) - ((205*a^4 - 973*a^
2*b^2 + 840*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(70*a^7*d) + ((16*a^4 - 81*a^
2*b^2 + 72*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^6*b*d) - (3*(35*a^4 - 185
*a^2*b^2 + 168*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/(70*a^5*b^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^3)/(5*b*d*(a + b*Sin[c + d*x])^2) + (a*Cot[c + d*x]*Csc[c
 + d*x]^4)/(10*b^2*d*(a + b*Sin[c + d*x])^2) + ((7*a^4 - 35*a^2*b^2 + 30*b^
4)*Cot[c + d*x]*Csc[c + d*x]^4)/(35*a^3*b^2*d*(a + b*Sin[c + d*x])^2) + (3*
b*Cot[c + d*x]*Csc[c + d*x]^5)/(14*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c +
 d*x]*Csc[c + d*x]^6)/(7*a*d*(a + b*Sin[c + d*x])^2) + ((12*a^4 - 65*a^2*b^
2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/(10*a^4*b^2*d*(a + b*Sin[c + d*x])
)
```

Rubi [A] time = 3.22025, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{3b^2\sqrt{a^2-b^2}\left(-23a^2b^2+4a^4+24b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^{10}d}+\frac{\left(-889a^4b^2+3255a^2b^4+10a^6-2520b^6\right)\cot(c+dx)}{70a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-3*b^2*Sqrt[a^2 - b^2]*(4*a^4 - 23*a^2*b^2 + 24*b^4)*ArcTan[(b + a*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]])/(a^10*d) - (3*b*(5*a^6 - 100*a^4*b^2 + 280*a^2
*b^4 - 192*b^6)*ArcTanh[Cos[c + d*x]])/(16*a^10*d) + ((10*a^6 - 889*a^4*b^2
 + 3255*a^2*b^4 - 2520*b^6)*Cot[c + d*x])/(70*a^9*d) + (3*b*(27*a^4 - 116*a
^2*b^2 + 96*b^4)*Cot[c + d*x]*Csc[c + d*x])/(16*a^8*d) - ((205*a^4 - 973*a^
2*b^2 + 840*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(70*a^7*d) + ((16*a^4 - 81*a^
2*b^2 + 72*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^6*b*d) - (3*(35*a^4 - 185
*a^2*b^2 + 168*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/(70*a^5*b^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^3)/(5*b*d*(a + b*Sin[c + d*x])^2) + (a*Cot[c + d*x]*Csc[c
 + d*x]^4)/(10*b^2*d*(a + b*Sin[c + d*x])^2) + ((7*a^4 - 35*a^2*b^2 + 30*b^
4)*Cot[c + d*x]*Csc[c + d*x]^4)/(35*a^3*b^2*d*(a + b*Sin[c + d*x])^2) + (3*
b*Cot[c + d*x]*Csc[c + d*x]^5)/(14*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c +
 d*x]*Csc[c + d*x]^6)/(7*a*d*(a + b*Sin[c + d*x])^2) + ((12*a^4 - 65*a^2*b^
2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/(10*a^4*b^2*d*(a + b*Sin[c + d*x])
)
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
```

```
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2} + \frac{3b\cot(c+dx)\csc^5(c+d}{14a^2d(a+b\sin(c+dx))}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2} + \frac{\left(7a^4-35a^2b^2+30b^4\right)\mathrm{co}}{35a^3b^2d(a+b\,\mathrm{s}}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2} + \frac{\left(7a^4-35a^2b^2+30b^4\right)\mathrm{co}}{35a^3b^2d(a+b\,\mathrm{s}}$$

$$= -\frac{3\left(35a^4-185a^2b^2+168b^4\right)\cot(c+dx)\csc^4(c+dx)}{70a^5b^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} +$$

$$= \frac{\left(16a^4-81a^2b^2+72b^4\right)\cot(c+dx)\csc^3(c+dx)}{8a^6bd} - \frac{3\left(35a^4-185a^2b^2+168b^4\right)\cot(}{70a^5b^2d}$$

$$= -\frac{\left(205a^4-973a^2b^2+840b^4\right)\cot(c+dx)\csc^2(c+dx)}{70a^7d} + \frac{\left(16a^4-81a^2b^2+72b^4\right)\mathrm{co}}{8a^6bd}$$

$$= \frac{3b\left(27a^4-116a^2b^2+96b^4\right)\cot(c+dx)\csc(c+dx)}{16a^8d} - \frac{\left(205a^4-973a^2b^2+840b^4\right)\mathrm{c}}{70a^7d}$$

$$= \frac{\left(10a^6-889a^4b^2+3255a^2b^4-2520b^6\right)\cot(c+dx)}{70a^9d} + \frac{3b\left(27a^4-116a^2b^2+96b^4\right)\mathrm{c}}{16a^8d}$$

$$= \frac{\left(10a^6-889a^4b^2+3255a^2b^4-2520b^6\right)\cot(c+dx)}{70a^9d} + \frac{3b\left(27a^4-116a^2b^2+96b^4\right)\mathrm{c}}{16a^8d}$$

$$= -\frac{3b\left(5a^6-100a^4b^2+280a^2b^4-192b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^{10}d} + \frac{\left(10a^6-889a^4b^2+\right.}{}$$

$$= -\frac{3b\left(5a^6-100a^4b^2+280a^2b^4-192b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^{10}d} + \frac{\left(10a^6-889a^4b^2+\right.}{}$$

$$= -\frac{3b^2\sqrt{a^2-b^2}\left(4a^4-23a^2b^2+24b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^{10}d} - \frac{3b\left(5a^6-100a^4b^2+\right.}{}$$

Mathematica [A] time = 2.96027, size = 728, normalized size = 1.21

$$\frac{\frac{215040b^2\left(27a^4b^2-47a^2b^4-4a^6+24b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 13440b\left(-100a^4b^2+280a^2b^4+5a^6-192b^6\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((215040*b^2*(-4*a^6 + 27*a^4*b^2 - 47*a^2*b^4 + 24*b^6)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 13440*b*(-5*a^6 + 100*a^4
*b^2 - 280*a^2*b^4 + 192*b^6)*Log[Cos[(c + d*x)/2]] + 13440*b*(5*a^6 - 100*
a^4*b^2 + 280*a^2*b^4 - 192*b^6)*Log[Sin[(c + d*x)/2]] - (a*Csc[c + d*x]^9*
(28*(200*a^8 + 795*a^6*b^2 - 1218*a^4*b^4 - 4110*a^2*b^6 + 5040*b^8)*Cos[c
+ d*x] + 28*(120*a^8 - 1403*a^6*b^2 + 1952*a^4*b^4 + 8700*a^2*b^6 - 10080*b
^8)*Cos[3*(c + d*x)] + 1120*a^8*Cos[5*(c + d*x)] + 22948*a^6*b^2*Cos[5*(c +
 d*x)] - 18144*a^4*b^4*Cos[5*(c + d*x)] - 193200*a^2*b^6*Cos[5*(c + d*x)] +
 201600*b^8*Cos[5*(c + d*x)] + 160*a^8*Cos[7*(c + d*x)] - 5884*a^6*b^2*Cos[
7*(c + d*x)] - 5964*a^4*b^4*Cos[7*(c + d*x)] + 77700*a^2*b^6*Cos[7*(c + d*x
)] - 70560*b^8*Cos[7*(c + d*x)] - 40*a^6*b^2*Cos[9*(c + d*x)] + 3556*a^4*b^
4*Cos[9*(c + d*x)] - 13020*a^2*b^6*Cos[9*(c + d*x)] + 10080*b^8*Cos[9*(c +
d*x)] - 9660*a^7*b*Sin[2*(c + d*x)] + 194334*a^5*b^3*Sin[2*(c + d*x)] - 592
200*a^3*b^5*Sin[2*(c + d*x)] + 423360*a*b^7*Sin[2*(c + d*x)] + 6160*a^7*b*S
in[4*(c + d*x)] - 190582*a^5*b^3*Sin[4*(c + d*x)] + 585480*a^3*b^5*Sin[4*(c
 + d*x)] - 423360*a*b^7*Sin[4*(c + d*x)] - 3660*a^7*b*Sin[6*(c + d*x)] + 77
462*a^5*b^3*Sin[6*(c + d*x)] - 246120*a^3*b^5*Sin[6*(c + d*x)] + 181440*a*b
^7*Sin[6*(c + d*x)] + 160*a^7*b*Sin[8*(c + d*x)] - 11389*a^5*b^3*Sin[8*(c +
 d*x)] + 39900*a^3*b^5*Sin[8*(c + d*x)] - 30240*a*b^7*Sin[8*(c + d*x)]))/(b
 + a*Csc[c + d*x])^2)/(71680*a^10*d)
```

Maple [B] time = 0.252, size = 1576, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 14/d/a^9*b^6*tan(1/2*d*x+1/2*c)+5/32/d/a^6*b^3/tan(1/2*d*x+1/2*c)^4+21/8/d/
a^8*b^5/tan(1/2*d*x+1/2*c)^2-36/d/a^10*b^7*ln(tan(1/2*d*x+1/2*c))-17/d*b^7/
a^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-3/80/d/a^5/tan(1/2*
d*x+1/2*c)^5*b^2-5/8/d/a^7/tan(1/2*d*x+1/2*c)^3*b^4-14/d/a^9/tan(1/2*d*x+1/
2*c)*b^6+1/128/d/a^4*b/tan(1/2*d*x+1/2*c)^6-1/128/d/a^4*b*tan(1/2*d*x+1/2*c
)^6+3/80/d/a^5*tan(1/2*d*x+1/2*c)^5*b^2-5/32/d/a^6*tan(1/2*d*x+1/2*c)^4*b^3
+5/8/d/a^7*tan(1/2*d*x+1/2*c)^3*b^4-21/8/d/a^8*tan(1/2*d*x+1/2*c)^2*b^5-12/
d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2))*b^2-5/128/d/a^3*tan(1/2*d*x+1/2*c)+5/128/d/a^3/tan(1/2*d*x+1/2*c)+105/2
/d/a^8*b^5*ln(tan(1/2*d*x+1/2*c))+25/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2*b^5+7/16/d/a^5/tan(1/2*d*x+1/2*c)^3*b^2+9/128/d/a^4*tan
(1/2*d*x+1/2*c)^4*b-7/16/d/a^5*tan(1/2*d*x+1/2*c)^3*b^2+5/2/d/a^6*tan(1/2*d
*x+1/2*c)^2*b^3-135/8/d/a^7*b^4*tan(1/2*d*x+1/2*c)+135/8/d/a^7/tan(1/2*d*x+
1/2*c)*b^4-9/128/d/a^4*b/tan(1/2*d*x+1/2*c)^4-5/2/d/a^6*b^3/tan(1/2*d*x+1/2
*c)^2+1/896/d/a^3*tan(1/2*d*x+1/2*c)^7-1/896/d/a^3/tan(1/2*d*x+1/2*c)^7+1/1
28/d/a^3/tan(1/2*d*x+1/2*c)^5-1/128/d/a^3*tan(1/2*d*x+1/2*c)^5+15/16/d/a^4*
b*ln(tan(1/2*d*x+1/2*c))-18/d*b^8/a^9/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-34/d*b^9/a^10/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-50/d*b^8/a^9/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+72/d*b^8/a^10/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-8/d/a^
4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3-33/8/d/a^5/tan(1/
2*d*x+1/2*c)*b^2+45/128/d/a^4*b/tan(1/2*d*x+1/2*c)^2-75/4/d/a^6*b^3*ln(tan(
1/2*d*x+1/2*c))+73/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^
2*tan(1/2*d*x+1/2*c)*b^6-141/d/a^8/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6+27/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan
```

```
(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^6+33/d/a^8/(tan(1/2*d*x+1/2*c
)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^7-3/128/d/a^3/tan(
1/2*d*x+1/2*c)^3-9/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^
2*tan(1/2*d*x+1/2*c)^3*b^4+3/128/d/a^3*tan(1/2*d*x+1/2*c)^3-8/d/a^4/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3-45/12
8/d/a^4*tan(1/2*d*x+1/2*c)^2*b+33/8/d/a^5*b^2*tan(1/2*d*x+1/2*c)+9/d/a^6/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5-
23/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)*b^4+81/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(
a^2-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.90324, size = 8743, normalized size = 14.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/1120*(16*(10*a^7*b^2 - 889*a^5*b^4 + 3255*a^3*b^6 - 2520*a*b^8)*cos(d*x
+ c)^9 - 4*(40*a^9 - 1381*a^7*b^2 - 9492*a^5*b^4 + 48720*a^3*b^6 - 40320*a*
b^8)*cos(d*x + c)^7 - 28*(563*a^7*b^2 + 1068*a^5*b^4 - 9720*a^3*b^6 + 8640*
a*b^8)*cos(d*x + c)^5 + 140*(105*a^7*b^2 + 20*a^5*b^4 - 1200*a^3*b^6 + 1152
*a*b^8)*cos(d*x + c)^3 + 840*(2*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x
 + c)^8 + 8*a^5*b^3 - 46*a^3*b^5 + 48*a*b^7 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 2
4*a*b^7)*cos(d*x + c)^6 + 12*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x +
c)^4 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x + c)^2 + ((4*a^4*b^4 -
 23*a^2*b^6 + 24*b^8)*cos(d*x + c)^8 + 4*a^6*b^2 - 19*a^4*b^4 + a^2*b^6 + 2
4*b^8 - (4*a^6*b^2 - 7*a^4*b^4 - 68*a^2*b^6 + 96*b^8)*cos(d*x + c)^6 + 3*(4
*a^6*b^2 - 15*a^4*b^4 - 22*a^2*b^6 + 48*b^8)*cos(d*x + c)^4 - (12*a^6*b^2 -
 53*a^4*b^4 - 20*a^2*b^6 + 96*b^8)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 420*(11*a^7*b^2 - 8*a^5*b
^4 - 92*a^3*b^6 + 96*a*b^8)*cos(d*x + c) - 105*(10*a^7*b^2 - 200*a^5*b^4 +
560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*
b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)
*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*co
s(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*
x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a^2*b^7 - 192*b^9 + (5*
a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*x + c)^8 - (5*a^8*b -
80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos(d*x + c)^6 + 3*(5*a^8
*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)*cos(d*x + c)^4 - (15*
```

a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768*b^9)*cos(d*x + c)^2)*
sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 105*(10*a^7*b^2 - 200*a^5*b^4 +
 560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a
*b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8
)*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*c
os(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d
*x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a^2*b^7 - 192*b^9 + (5
*a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*x + c)^8 - (5*a^8*b -
 80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos(d*x + c)^6 + 3*(5*a^
8*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)*cos(d*x + c)^4 - (15
*a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768*b^9)*cos(d*x + c)^2)
*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((160*a^8*b - 11389*a^6*b^3
 + 39900*a^4*b^5 - 30240*a^2*b^7)*cos(d*x + c)^7 - 7*(165*a^8*b - 5207*a^6*
b^3 + 17340*a^4*b^5 - 12960*a^2*b^7)*cos(d*x + c)^5 + 35*(40*a^8*b - 1097*a
^6*b^3 + 3516*a^4*b^5 - 2592*a^2*b^7)*cos(d*x + c)^3 - 105*(5*a^8*b - 127*a
^6*b^3 + 396*a^4*b^5 - 288*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/(2*a^11*b*d
*cos(d*x + c)^8 - 8*a^11*b*d*cos(d*x + c)^6 + 12*a^11*b*d*cos(d*x + c)^4 -
8*a^11*b*d*cos(d*x + c)^2 + 2*a^11*b*d + (a^10*b^2*d*cos(d*x + c)^8 - (a^12
 + 4*a^10*b^2)*d*cos(d*x + c)^6 + 3*(a^12 + 2*a^10*b^2)*d*cos(d*x + c)^4 -
(3*a^12 + 4*a^10*b^2)*d*cos(d*x + c)^2 + (a^12 + a^10*b^2)*d)*sin(d*x + c))
, 1/1120*(16*(10*a^7*b^2 - 889*a^5*b^4 + 3255*a^3*b^6 - 2520*a*b^8)*cos(d*x
 + c)^9 - 4*(40*a^9 - 1381*a^7*b^2 - 9492*a^5*b^4 + 48720*a^3*b^6 - 40320*a
*b^8)*cos(d*x + c)^7 - 28*(563*a^7*b^2 + 1068*a^5*b^4 - 9720*a^3*b^6 + 8640
*a*b^8)*cos(d*x + c)^5 + 140*(105*a^7*b^2 + 20*a^5*b^4 - 1200*a^3*b^6 + 115
2*a*b^8)*cos(d*x + c)^3 + 1680*(2*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d
*x + c)^8 + 8*a^5*b^3 - 46*a^3*b^5 + 48*a*b^7 - 8*(4*a^5*b^3 - 23*a^3*b^5 +
 24*a*b^7)*cos(d*x + c)^6 + 12*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x
+ c)^4 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x + c)^2 + ((4*a^4*b^4
 - 23*a^2*b^6 + 24*b^8)*cos(d*x + c)^8 + 4*a^6*b^2 - 19*a^4*b^4 + a^2*b^6 +
 24*b^8 - (4*a^6*b^2 - 7*a^4*b^4 - 68*a^2*b^6 + 96*b^8)*cos(d*x + c)^6 + 3*
(4*a^6*b^2 - 15*a^4*b^4 - 22*a^2*b^6 + 48*b^8)*cos(d*x + c)^4 - (12*a^6*b^2
 - 53*a^4*b^4 - 20*a^2*b^6 + 96*b^8)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 420*
(11*a^7*b^2 - 8*a^5*b^4 - 92*a^3*b^6 + 96*a*b^8)*cos(d*x + c) - 105*(10*a^7
*b^2 - 200*a^5*b^4 + 560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4 +
 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280
*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^
3*b^6 - 192*a*b^8)*cos(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^
6 - 192*a*b^8)*cos(d*x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a^
2*b^7 - 192*b^9 + (5*a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*x
 + c)^8 - (5*a^8*b - 80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos(
d*x + c)^6 + 3*(5*a^8*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)*
cos(d*x + c)^4 - (15*a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768*
b^9)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 105*(10*a^
7*b^2 - 200*a^5*b^4 + 560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4
+ 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 28
0*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a
^3*b^6 - 192*a*b^8)*cos(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b
^6 - 192*a*b^8)*cos(d*x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a
^2*b^7 - 192*b^9 + (5*a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*
x + c)^8 - (5*a^8*b - 80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos
(d*x + c)^6 + 3*(5*a^8*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)
*cos(d*x + c)^4 - (15*a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768
*b^9)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((160*
a^8*b - 11389*a^6*b^3 + 39900*a^4*b^5 - 30240*a^2*b^7)*cos(d*x + c)^7 - 7*(
165*a^8*b - 5207*a^6*b^3 + 17340*a^4*b^5 - 12960*a^2*b^7)*cos(d*x + c)^5 +
35*(40*a^8*b - 1097*a^6*b^3 + 3516*a^4*b^5 - 2592*a^2*b^7)*cos(d*x + c)^3 -
 105*(5*a^8*b - 127*a^6*b^3 + 396*a^4*b^5 - 288*a^2*b^7)*cos(d*x + c))*sin(
d*x + c))/(2*a^11*b*d*cos(d*x + c)^8 - 8*a^11*b*d*cos(d*x + c)^6 + 12*a^11*

```
b*d*cos(d*x + c)^4 - 8*a^11*b*d*cos(d*x + c)^2 + 2*a^11*b*d + (a^10*b^2*d*c
os(d*x + c)^8 - (a^12 + 4*a^10*b^2)*d*cos(d*x + c)^6 + 3*(a^12 + 2*a^10*b^2
)*d*cos(d*x + c)^4 - (3*a^12 + 4*a^10*b^2)*d*cos(d*x + c)^2 + (a^12 + a^10*
b^2)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8/(a+b*sin(d*x+c))**3,x)

[Out] Timed out
```

Giac [A] time = 1.32264, size = 1374, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/4480*(840*(5*a^6*b - 100*a^4*b^3 + 280*a^2*b^5 - 192*b^7)*log(abs(tan(1/2
*d*x + 1/2*c)))/a^10 - 13440*(4*a^6*b^2 - 27*a^4*b^4 + 47*a^2*b^6 - 24*b^8)
*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^10) - 4480*(9*a^5*b^4*tan(1/2*d*x
 + 1/2*c)^3 - 27*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a*b^8*tan(1/2*d*x + 1/
2*c)^3 + 8*a^6*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^4*b^5*tan(1/2*d*x + 1/2*c)^
2 - 33*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 + 34*b^9*tan(1/2*d*x + 1/2*c)^2 + 23*
a^5*b^4*tan(1/2*d*x + 1/2*c) - 73*a^3*b^6*tan(1/2*d*x + 1/2*c) + 50*a*b^8*t
an(1/2*d*x + 1/2*c) + 8*a^6*b^3 - 25*a^4*b^5 + 17*a^2*b^7)/((a*tan(1/2*d*x
+ 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^10) - (10890*a^6*b*tan(1/2*d
*x + 1/2*c)^7 - 217800*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 609840*a^2*b^5*tan(
1/2*d*x + 1/2*c)^7 - 418176*b^7*tan(1/2*d*x + 1/2*c)^7 - 175*a^7*tan(1/2*d*
x + 1/2*c)^6 + 18480*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 - 75600*a^3*b^4*tan(1/2
*d*x + 1/2*c)^6 + 62720*a*b^6*tan(1/2*d*x + 1/2*c)^6 - 1575*a^6*b*tan(1/2*d
*x + 1/2*c)^5 + 11200*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 11760*a^2*b^5*tan(1/
2*d*x + 1/2*c)^5 + 105*a^7*tan(1/2*d*x + 1/2*c)^4 - 1960*a^5*b^2*tan(1/2*d*
x + 1/2*c)^4 + 2800*a^3*b^4*tan(1/2*d*x + 1/2*c)^4 + 315*a^6*b*tan(1/2*d*x
+ 1/2*c)^3 - 700*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 35*a^7*tan(1/2*d*x + 1/2*
c)^2 + 168*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 35*a^6*b*tan(1/2*d*x + 1/2*c) +
 5*a^7)/(a^10*tan(1/2*d*x + 1/2*c)^7) + (5*a^18*tan(1/2*d*x + 1/2*c)^7 - 35
*a^17*b*tan(1/2*d*x + 1/2*c)^6 - 35*a^18*tan(1/2*d*x + 1/2*c)^5 + 168*a^16*
b^2*tan(1/2*d*x + 1/2*c)^5 + 315*a^17*b*tan(1/2*d*x + 1/2*c)^4 - 700*a^15*b
^3*tan(1/2*d*x + 1/2*c)^4 + 105*a^18*tan(1/2*d*x + 1/2*c)^3 - 1960*a^16*b^2
*tan(1/2*d*x + 1/2*c)^3 + 2800*a^14*b^4*tan(1/2*d*x + 1/2*c)^3 - 1575*a^17*
b*tan(1/2*d*x + 1/2*c)^2 + 11200*a^15*b^3*tan(1/2*d*x + 1/2*c)^2 - 11760*a^
13*b^5*tan(1/2*d*x + 1/2*c)^2 - 175*a^18*tan(1/2*d*x + 1/2*c) + 18480*a^16*
b^2*tan(1/2*d*x + 1/2*c) - 75600*a^14*b^4*tan(1/2*d*x + 1/2*c) + 62720*a^12
*b^6*tan(1/2*d*x + 1/2*c))/a^21)/d
```

3.1276 $\int \frac{\cos^6(e+fx)}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=712

$$\frac{16b\left(-93a^2b^2+93a^4+32b^4\right)\cos(e+fx)}{693a^5f\left(a^2-b^2\right)^3\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}-\frac{8\left(-22a^4b^2+65a^2b^4+5a^6-32b^6\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{693a^5b^2df\left(a^2-b^2\right)^2(a+b\sin(e+fx))^{3/2}}-$$

```
[Out] (2*Cos[e + f*x]^5*Sqrt[d*Sin[e + f*x]])/(11*a*d*f*(a + b*Sin[e + f*x])^(11/
2)) - (20*(a^2 - b^2)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(99*a^2*b^2*d*f*(a
 + b*Sin[e + f*x])^(9/2)) + (80*(3*a^2 + 2*b^2)*Cos[e + f*x]*Sqrt[d*Sin[e +
 f*x]])/(693*a^3*b^2*d*f*(a + b*Sin[e + f*x])^(7/2)) - (4*(5*a^4 - 17*a^2*b
^2 + 16*b^4)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(231*a^4*b^2*(a^2 - b^2)*d*
f*(a + b*Sin[e + f*x])^(5/2)) - (8*(5*a^6 - 22*a^4*b^2 + 65*a^2*b^4 - 32*b^
6)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(693*a^5*b^2*(a^2 - b^2)^2*d*f*(a + b
*Sin[e + f*x])^(3/2)) + (16*b*(93*a^4 - 93*a^2*b^2 + 32*b^4)*Cos[e + f*x])/
(693*a^5*(a^2 - b^2)^3*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (
16*b*(93*a^4 - 93*a^2*b^2 + 32*b^4)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sq
rt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e
+ f*x])/(693*a^7*(a - b)^2*(a + b)^(5/2)*Sqrt[d]*f) - (16*(45*a^4 - 48*a^3*
b - 69*a^2*b^2 + 24*a*b^3 + 32*b^4)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sq
rt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e
+ f*x])/(693*a^6*(a - b)^2*(a + b)^(5/2)*Sqrt[d]*f)
```

Rubi [A] time = 2.64462, antiderivative size = 712, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2887, 2891, 3055, 2993, 2998, 2816, 2994}

$$\frac{16b\left(-93a^2b^2+93a^4+32b^4\right)\cos(e+fx)}{693a^5f\left(a^2-b^2\right)^3\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}-\frac{8\left(-22a^4b^2+65a^2b^4+5a^6-32b^6\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{693a^5b^2df\left(a^2-b^2\right)^2(a+b\sin(e+fx))^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(13/2)),x]
```


```
[Out] (2*Cos[e + f*x]^5*Sqrt[d*Sin[e + f*x]])/(11*a*d*f*(a + b*Sin[e + f*x])^(11/
2)) - (20*(a^2 - b^2)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(99*a^2*b^2*d*f*(a
 + b*Sin[e + f*x])^(9/2)) + (80*(3*a^2 + 2*b^2)*Cos[e + f*x]*Sqrt[d*Sin[e +
 f*x]])/(693*a^3*b^2*d*f*(a + b*Sin[e + f*x])^(7/2)) - (4*(5*a^4 - 17*a^2*b
^2 + 16*b^4)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(231*a^4*b^2*(a^2 - b^2)*d*
f*(a + b*Sin[e + f*x])^(5/2)) - (8*(5*a^6 - 22*a^4*b^2 + 65*a^2*b^4 - 32*b^
6)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(693*a^5*b^2*(a^2 - b^2)^2*d*f*(a + b
*Sin[e + f*x])^(3/2)) + (16*b*(93*a^4 - 93*a^2*b^2 + 32*b^4)*Cos[e + f*x])/
(693*a^5*(a^2 - b^2)^3*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (
16*b*(93*a^4 - 93*a^2*b^2 + 32*b^4)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sq
rt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e
+ f*x])/(693*a^7*(a - b)^2*(a + b)^(5/2)*Sqrt[d]*f) - (16*(45*a^4 - 48*a^3*
b - 69*a^2*b^2 + 24*a*b^3 + 32*b^4)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sq
rt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e
```

```
+ f*x])/(693*a^6*(a - b)^2*(a + b)^(5/2)*Sqrt[d]*f)
```

Rule 2887

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -Simp[(g*(g*C
os[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(m + 1))/(a*
d*f*(m + 1)), x] + Dist[(g^2*(2*m + 3))/(2*a*(m + 1)), Int[((g*Cos[e + f*x]
)^(p - 2)*(a + b*Sin[e + f*x])^(m + 1))/Sqrt[d*Sin[e + f*x]], x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p +
 1/2, 0]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{13/2}}\,dx = \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} + \frac{10\int \frac{\cos^4(e+fx)}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{11/2}}\,dx}{11a}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{99a^2b^2df(a+b\sin(e+fx))^{9/2}} +$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{99a^2b^2df(a+b\sin(e+fx))^{9/2}} +$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{99a^2b^2df(a+b\sin(e+fx))^{9/2}} +$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{99a^2b^2df(a+b\sin(e+fx))^{9/2}} +$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{99a^2b^2df(a+b\sin(e+fx))^{9/2}} +$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20\left(a^2-b^2\right)\cos(e+fx)\sqrt{d\sin(e+fx)}}{99a^2b^2df(a+b\sin(e+fx))^{9/2}} +$$

Mathematica [C] time = 6.93415, size = 1906, normalized size = 2.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(13/2))
,x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((2*(a^4*Cos[e + f*x] - 2*a^2*b^2*Co
s[e + f*x] + b^4*Cos[e + f*x]))/(11*a*b^4*(a + b*Sin[e + f*x])^6) - (4*(18*
a^4*Cos[e + f*x] - 13*a^2*b^2*Cos[e + f*x] - 5*b^4*Cos[e + f*x]))/(99*a^2*b
^4*(a + b*Sin[e + f*x])^5) + (4*(189*a^4*Cos[e + f*x] - 3*a^2*b^2*Cos[e + f
*x] + 40*b^4*Cos[e + f*x]))/(693*a^3*b^4*(a + b*Sin[e + f*x])^4) - (4*(42*a
^6*Cos[e + f*x] - 37*a^4*b^2*Cos[e + f*x] - 17*a^2*b^4*Cos[e + f*x] + 16*b^
6*Cos[e + f*x]))/(231*a^4*b^4*(a^2 - b^2)*(a + b*Sin[e + f*x])^3) + (2*(63*
a^8*Cos[e + f*x] - 146*a^6*b^2*Cos[e + f*x] + 151*a^4*b^4*Cos[e + f*x] - 26
0*a^2*b^6*Cos[e + f*x] + 128*b^8*Cos[e + f*x]))/(693*a^5*b^4*(a^2 - b^2)^2*
(a + b*Sin[e + f*x])^2) - (16*(93*a^4*b^2*Cos[e + f*x] - 93*a^2*b^4*Cos[e +
 f*x] + 32*b^6*Cos[e + f*x]))/(693*a^6*(a^2 - b^2)^3*(a + b*Sin[e + f*x])))
)/(f*Sqrt[d*Sin[e + f*x]]) + (8*Sqrt[Sin[e + f*x]]*((4*a*(45*a^6 - 114*a^4*
b^2 + 101*a^2*b^4 - 32*b^6)*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a +
 b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x])
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + P
i/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[
a + b*Sin[e + f*x]]) + 4*a*(-93*a^5*b + 93*a^3*b^3 - 32*a*b^5)*((Sqrt[((a +
 b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^
2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[((a + b)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(
-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
ec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x
)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) + 2*(93*a^4*b^2 -
 93*a^2*b^4 + 32*b^6)*((Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Sin[
e + f*x]]) + (I*Cos[(-e + Pi/2 - f*x)/2]*Csc[e + f*x]*EllipticE[I*ArcSinh[S
in[(-e + Pi/2 - f*x)/2]/Sqrt[Sin[e + f*x]]], (-2*a)/(-a - b)]*Sqrt[a + b*Si
n[e + f*x]])/(b*Sqrt[Cos[(-e + Pi/2 - f*x)/2]^2*Csc[e + f*x]]*Sqrt[(Csc[e +
 f*x]*(a + b*Sin[e + f*x]))/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(-e + Pi
/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2
*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/
a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqr
t[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (a*Sqrt[((a + b)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Si
n[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e +
 f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sq
rt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])))/b)))/(693*a^6*(a - b)^3*(a + b
)^3*f*Sqrt[d*Sin[e + f*x]])
```

Maple [B] time = 2.328, size = 58449, normalized size = 82.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{13}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/((b*sin(f*x + e) + a)^(13/2)*sqrt(d*sin(f*x + e)))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{}{b^7d\cos\left(fx+e\right)^8-\left(21\,a^2b^5+4\,b^7\right)d\cos\left(fx+e\right)^6+\left(35\,a^4b^3+63\,a^2b^5+6\,b^7\right)d\cos\left(fx+e\right)^4-\left(7\,a^6b+70\,a^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*cos(f*x + e)^6/(b^7*
d*cos(f*x + e)^8 - (21*a^2*b^5 + 4*b^7)*d*cos(f*x + e)^6 + (35*a^4*b^3 + 63
*a^2*b^5 + 6*b^7)*d*cos(f*x + e)^4 - (7*a^6*b + 70*a^4*b^3 + 63*a^2*b^5 + 4
*b^7)*d*cos(f*x + e)^2 + (7*a^6*b + 35*a^4*b^3 + 21*a^2*b^5 + b^7)*d - (7*a
*b^6*d*cos(f*x + e)^6 - 7*(5*a^3*b^4 + 3*a*b^6)*d*cos(f*x + e)^4 + 7*(3*a^5
*b^2 + 10*a^3*b^4 + 3*a*b^6)*d*cos(f*x + e)^2 - (a^7 + 21*a^5*b^2 + 35*a^3*
b^4 + 7*a*b^6)*d)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sin(f*x+e))**(13/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{13}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/((b*sin(f*x + e) + a)^(13/2)*sqrt(d*sin(f*x + e)))
, x)
```

3.1277 $\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=159

$$\frac{(2a^2-b^2)\sqrt{\sin(2e+2fx)}F\left(\frac{1}{4}(4e-\pi)+fx\middle|2\right)}{3fg^2\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}+\frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}}$$

```
[Out] (2*(a^2 + b^2)*Sqrt[d*Sin[e + f*x]])/(3*d*f*g*(g*Cos[e + f*x])^(3/2)) + (4*
a*b*(d*Sin[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2)) + ((2*a^2 -
b^2)*EllipticF[(4*e - Pi)/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*g^2*Sqrt
[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 0.427843, antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2911, 2563, 3202, 457, 329, 237, 335, 275, 232}

$$-\frac{2(2a^2-b^2)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+fx))^{3/2}F\left(\frac{1}{2}\csc^{-1}(\sin(e+fx))\middle|2\right)}{3d^2fg(g\cos(e+fx))^{3/2}}+\frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}+\frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]),x]
```

```
[Out] (2*(a^2 + b^2)*Sqrt[d*Sin[e + f*x]])/(3*d*f*g*(g*Cos[e + f*x])^(3/2)) + (4*
a*b*(d*Sin[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2)) - (2*(2*a^2
- b^2)*(1 - Csc[e + f*x]^2)^(3/4)*EllipticF[ArcCsc[Sin[e + f*x]]/2, 2]*(d*S
in[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 3202

```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*c^(2*IntPart[(m - 1)/2] + 1)*(c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2]))/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2
]), Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x],
 x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !Int
egerQ[m]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx = \frac{(2ab)\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{5/2}}\,dx}{d} + \int \frac{a^2+b^2\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$$

$$= \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} + \frac{\cos^2(e+fx)^{3/4}\operatorname{Subst}\left(\int \frac{a^2+b^2x^2}{\sqrt{dx}\left(1-x^2\right)^{7/4}}\,dx, x, \sin(e+f\right.}{fg(g\cos(e+fx))^{3/2}}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{\left(\left(-2a^2+b^2\right)\cos^2(e+\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{\left(2\left(-2a^2+b^2\right)\cos^2(e+\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{\left(2\left(-2a^2+b^2\right)\left(1-\csc\right.\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} + \frac{\left(2\left(-2a^2+b^2\right)\left(1-\csc\right.\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} + \frac{\left(\left(-2a^2+b^2\right)\left(1-\csc^2\right.\right.}{}$$

$$= \frac{2\left(a^2+b^2\right)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{2\left(2a^2-b^2\right)\left(1-\csc^2(e\right.}{}$$

Mathematica [C] time = 0.441046, size = 127, normalized size = 0.8

$$\frac{2\tan(e+fx)\left(15a^2\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\sin^2(e+fx)\right)+b\sin(e+fx)\left(10a+3b\sin(e+fx)\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{5}{4}\right.\right.\right.}{15fg^2\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(15*a^2*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[e +
f*x]^2] + b*Sin[e + f*x]*(10*a + 3*b*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2
F1[5/4, 7/4, 9/4, Sin[e + f*x]^2]*Sin[e + f*x]))*Tan[e + f*x])/(15*f*g^2*Sq
rt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Maple [B] time = 0.375, size = 371, normalized size = 2.3

$$\frac{\sqrt{2}\cos\left(fx+e\right)\sin\left(fx+e\right)}{3f\left(-1+\cos\left(fx+e\right)\right)}\left(-2\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\mathrm{si}}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*2^(1/2)*(-2*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))*a^2+cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1
/2*2^(1/2))*b^2+2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*a*b+cos(f*x+e)*2^(1/2)*a^2+
cos(f*x+e)*2^(1/2)*b^2-2*sin(f*x+e)*2^(1/2)*a*b-2^(1/2)*a^2-2^(1/2)*b^2)*co
s(f*x+e)*sin(f*x+e)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{d\sin\left(fx+e\right)}}{dg^3\cos\left(fx+e\right)^3\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(g*cos(
f*x + e))*sqrt(d*sin(f*x + e))/(d*g^3*cos(f*x + e)^3*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

3.1278 $\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{7/2}\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=193

$$\frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}}+\frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}}-\frac{8abE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{5dfg^4\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{d\sin(e+\cdots}}{5dfg(\cdots}$$

```
[Out] (8*a^2*Sqrt[d*Sin[e + f*x]])/(5*d*f*g^3*Sqrt[g*Cos[e + f*x]]) + (8*a*b*(d*S
in[e + f*x])^(3/2))/(5*d^2*f*g^3*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[d*Sin[e +
f*x]]*(a + b*Sin[e + f*x])^2)/(5*d*f*g*(g*Cos[e + f*x])^(5/2)) - (8*a*b*Sqr
t[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(5*d*f
*g^4*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 0.473507, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2888, 2838, 2563, 2571, 2572, 2639}

$$\frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}}+\frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}}-\frac{8abE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{5dfg^4\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{d\sin(e+\cdots}}{5dfg(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(7/2)*Sqrt[d*Sin[e + f*x]]),x]
```


```
[Out] (8*a^2*Sqrt[d*Sin[e + f*x]])/(5*d*f*g^3*Sqrt[g*Cos[e + f*x]]) + (8*a*b*(d*S
in[e + f*x])^(3/2))/(5*d^2*f*g^3*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[d*Sin[e +
f*x]]*(a + b*Sin[e + f*x])^2)/(5*d*f*g*(g*Cos[e + f*x])^(5/2)) - (8*a*b*Sqr
t[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(5*d*f
*g^4*Sqrt[Sin[2*e + 2*f*x]])
```


Rule 2888

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(g*Co
s[e + f*x])^(p + 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^m)/(d*f*g*(2*
m + 1)), x] + Dist[(2*a*m)/(g^2*(2*m + 1)), Int[((g*Cos[e + f*x])^(p + 2)*(
a + b*Sin[e + f*x])^(m - 1))/Sqrt[d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, e
, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && EqQ[m + p + 3/2, 0]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{7/2}\sqrt{d\sin(e+fx)}}\,dx = \frac{2\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^2}{5dfg(g\cos(e+fx))^{5/2}} + \frac{(4a)\int \frac{a+b\sin(e+fx)}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{5g^2}$$

$$= \frac{2\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^2}{5dfg(g\cos(e+fx))^{5/2}} + \frac{\left(4a^2\right)\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{5g^2} + \frac{(4a}{}$$

$$= \frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}} + \frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{d\sin(e+fx)}(a+b\sin(e}{5dfg(g\cos(e+fx))^{5/2}}$$

$$= \frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}} + \frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{d\sin(e+fx)}(a+b\sin(e}{5dfg(g\cos(e+fx))^{5/2}}$$

$$= \frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}} + \frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{d\sin(e+fx)}(a+b\sin(e}{5dfg(g\cos(e+fx))^{5/2}}$$

Mathematica [C] time = 0.652753, size = 105, normalized size = 0.54

$$\frac{2\tan(e+fx)\left(3\left(b^2-4a^2\right)\sin^2(e+fx)+15a^2+10ab\sin(e+fx)\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\sin^2(e+fx)\right)\right)}{15fg^2\sqrt{d\sin(e+fx)}(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(7/2)*Sqrt[d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(15*a^2 + 10*a*b*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[3/4, 9/4, 7/4,
 Sin[e + f*x]^2]*Sin[e + f*x] + 3*(-4*a^2 + b^2)*Sin[e + f*x]^2)*Tan[e + f*
x])/(15*f*g^2*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

Maple [B] time = 0.398, size = 616, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/5/f*2^(1/2)*(-8*cos(f*x+e)^3*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*a*b+4*cos(f*x+e)^3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b
-8*cos(f*x+e)^2*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/
2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b+4*cos(f
*x+e)^2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+4*2^(1/2)*cos(f
*x+e)^3*a*b-4*2^(1/2)*cos(f*x+e)^2*sin(f*x+e)*a^2+2^(1/2)*cos(f*x+e)^2*sin(
f*x+e)*b^2-2*2^(1/2)*cos(f*x+e)^2*a*b-2^(1/2)*sin(f*x+e)*a^2-2^(1/2)*sin(f*
x+e)*b^2-2*2^(1/2)*a*b)*cos(f*x+e)/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(g\cos\left(fx+e\right)\right)^{\frac{7}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(7/2)*sqrt(d*sin(f*x + e
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{d\sin\left(fx+e\right)}}{dg^4\cos\left(fx+e\right)^4\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(g*cos(
f*x + e))*sqrt(d*sin(f*x + e))/(d*g^4*cos(f*x + e)^4*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(g*cos(f*x+e))**(7/2)/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(g\cos\left(fx+e\right)\right)^{\frac{7}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(7/2)*sqrt(d*sin(f*x + e
))), x)
```

3.1279 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{a^2\sin(c+dx)}{b^3d}-\frac{a^3\log(a+b\sin(c+dx))}{b^4d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] -((a^3*Log[a + b*Sin[c + d*x]])/(b^4*d)) + (a^2*Sin[c + d*x])/(b^3*d) - (a*
Sin[c + d*x]^2)/(2*b^2*d) + Sin[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.0922599, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^2\sin(c+dx)}{b^3d}-\frac{a^3\log(a+b\sin(c+dx))}{b^4d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^3*Log[a + b*Sin[c + d*x]])/(b^4*d)) + (a^2*Sin[c + d*x])/(b^3*d) - (a*
Sin[c + d*x]^2)/(2*b^2*d) + Sin[c + d*x]^3/(3*b*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{b^3(a+x)}\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \frac{x^3}{a+x}\,dx,x,b\sin(c+dx)\right)}{b^4d}\\ &= \frac{\text{Subst}\left(\int \left(a^2-ax+x^2-\frac{a^3}{a+x}\right)dx,x,b\sin(c+dx)\right)}{b^4d}\\ &= -\frac{a^3\log(a+b\sin(c+dx))}{b^4d}+\frac{a^2\sin(c+dx)}{b^3d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\sin^3(c+dx)}{3bd}\end{aligned}$$

Mathematica [A] time = 0.201359, size = 66, normalized size = 0.87

$$\frac{6a^2b\sin(c+dx)-6a^3\log(a+b\sin(c+dx))-3ab^2\sin^2(c+dx)+2b^3\sin^3(c+dx)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-6*a^3*Log[a + b*Sin[c + d*x]] + 6*a^2*b*Sin[c + d*x] - 3*a*b^2*Sin[c + d*
x]^2 + 2*b^3*Sin[c + d*x]^3)/(6*b^4*d)
```

Maple [A] time = 0.027, size = 73, normalized size = 1.

$$-\frac{a^3\ln(a+b\sin(dx+c))}{b^4d}+\frac{a^2\sin(dx+c)}{b^3d}-\frac{(\sin(dx+c))^2a}{2b^2d}+\frac{(\sin(dx+c))^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -a^3*ln(a+b*sin(d*x+c))/b^4/d+a^2*sin(d*x+c)/b^3/d-1/2*a*sin(d*x+c)^2/b^2/d
+1/3*sin(d*x+c)^3/b/d
```

Maxima [A] time = 0.975544, size = 90, normalized size = 1.18

$$-\frac{\frac{6a^3\log(b\sin(dx+c)+a)}{b^4}-\frac{2b^2\sin(dx+c)^3-3ab\sin(dx+c)^2+6a^2\sin(dx+c)}{b^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^3*log(b*sin(d*x + c) + a)/b^4 - (2*b^2*sin(d*x + c)^3 - 3*a*b*sin
(d*x + c)^2 + 6*a^2*sin(d*x + c))/b^3)/d
```

Fricas [A] time = 1.46896, size = 167, normalized size = 2.2

$$\frac{3ab^2\cos(dx+c)^2-6a^3\log(b\sin(dx+c)+a)-2\left(b^3\cos(dx+c)^2-3a^2b-b^3\right)\sin(dx+c)}{6b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*b^2*cos(d*x + c)^2 - 6*a^3*log(b*sin(d*x + c) + a) - 2*(b^3*cos(d*
x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c))/(b^4*d)
```

Sympy [A] time = 2.56747, size = 128, normalized size = 1.68

$$\begin{cases} \frac{x\sin^3(c)\cos(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{x\sin^3(c)\cos(c)}{a+b\sin(c)} & \text{for } d = 0 \\ \frac{-\frac{\sin^2(c+dx)\cos^2(c+dx)}{2d} - \frac{\cos^4(c+dx)}{4d}}{a} & \text{for } b = 0 \\ -\frac{a^3\log\left(\frac{a}{b}+\sin(c+dx)\right)}{b^4d} + \frac{a^2\sin(c+dx)}{b^3d} + \frac{a\cos^2(c+dx)}{2b^2d} + \frac{\sin^3(c+dx)}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*sin(c)**3*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (x*sin(c)**3*cos(c)/
(a + b*sin(c)), Eq(d, 0)), ((-sin(c + d*x)**2*cos(c + d*x)**2/(2*d) - cos(c
 + d*x)**4/(4*d))/a, Eq(b, 0)), (-a**3*log(a/b + sin(c + d*x))/(b**4*d) + a
**2*sin(c + d*x)/(b**3*d) + a*cos(c + d*x)**2/(2*b**2*d) + sin(c + d*x)**3/
(3*b*d), True))
```

Giac [A] time = 1.1695, size = 92, normalized size = 1.21

$$-\frac{\frac{6\,a^3\log(|b\sin(dx+c)+a|)}{b^4} - \frac{2\,b^2\sin(dx+c)^3 - 3\,ab\sin(dx+c)^2 + 6\,a^2\sin(dx+c)}{b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^3*log(abs(b*sin(d*x + c) + a))/b^4 - (2*b^2*sin(d*x + c)^3 - 3*a*
b*sin(d*x + c)^2 + 6*a^2*sin(d*x + c))/b^3)/d
```

3.1280 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{a^2\log(a+b\sin(c+dx))}{b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] (a^2*Log[a + b*Sin[c + d*x]])/(b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c +
d*x]^2/(2*b*d)
```

Rubi [A] time = 0.0804525, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^2\log(a+b\sin(c+dx))}{b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a^2*Log[a + b*Sin[c + d*x]])/(b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c +
d*x]^2/(2*b*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{b^2(a+x)}\,dx,x,b\sin(c+dx)\right)}{bd}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{a+x}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= \frac{\text{Subst}\left(\int \left(-a+x+\frac{a^2}{a+x}\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= \frac{a^2\log(a+b\sin(c+dx))}{b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}
\end{aligned}$$

Mathematica [A] time = 0.103084, size = 49, normalized size = 0.89

$$\frac{2a^2\log(a+b\sin(c+dx))-2ab\sin(c+dx)+b^2\sin^2(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*Log[a + b*Sin[c + d*x]] - 2*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/(
2*b^3*d)
```

Maple [A] time = 0.027, size = 54, normalized size = 1.

$$\frac{\ln(a+b\sin(dx+c))a^2}{db^3}-\frac{a\sin(dx+c)}{b^2d}+\frac{(\sin(dx+c))^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b^3*ln(a+b*sin(d*x+c))*a^2-a*sin(d*x+c)/b^2/d+1/2*sin(d*x+c)^2/b/d
```

Maxima [A] time = 0.987241, size = 66, normalized size = 1.2

$$\frac{\frac{2a^2\log(b\sin(dx+c)+a)}{b^3}+\frac{b\sin(dx+c)^2-2a\sin(dx+c)}{b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*log(b*sin(d*x + c) + a)/b^3 + (b*sin(d*x + c)^2 - 2*a*sin(d*x +
c))/b^2)/d
```

Fricas [A] time = 1.51032, size = 119, normalized size = 2.16

$$-\frac{b^2\cos(dx+c)^2-2a^2\log(b\sin(dx+c)+a)+2ab\sin(dx+c)}{2b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^2 - 2*a^2*log(b*sin(d*x + c) + a) + 2*a*b*sin(d*x +
c))/(b^3*d)
```

Sympy [A] time = 1.50444, size = 87, normalized size = 1.58

$$\begin{cases} \frac{x\sin^2(c)\cos(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{x\sin^2(c)\cos(c)}{a+b\sin(c)} & \text{for } d=0 \\ \frac{\sin^3(c+dx)}{3ad} & \text{for } b=0 \\ \frac{a^2\log\left(\frac{a}{b}+\sin(c+dx)\right)}{b^3d} - \frac{a\sin(c+dx)}{b^2d} - \frac{\cos^2(c+dx)}{2bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*sin(c)**2*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (x*sin(c)**2*cos(c)/
(a + b*sin(c)), Eq(d, 0)), (sin(c + d*x)**3/(3*a*d), Eq(b, 0)), (a**2*log(a
/b + sin(c + d*x))/(b**3*d) - a*sin(c + d*x)/(b**2*d) - cos(c + d*x)**2/(2*
b*d), True))
```

Giac [A] time = 1.20535, size = 68, normalized size = 1.24

$$\frac{\frac{2a^2\log(|b\sin(dx+c)+a|)}{b^3} + \frac{b\sin(dx+c)^2-2a\sin(dx+c)}{b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(b*sin(d*x + c) + a))/b^3 + (b*sin(d*x + c)^2 - 2*a*sin(d
*x + c))/b^2)/d
```

3.1281 $\int \frac{\cos(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\sin(c+dx)}{bd}-\frac{a\log(a+b\sin(c+dx))}{b^2d}$$

```
[Out] -((a*Log[a + b*Sin[c + d*x]])/(b^2*d)) + Sin[c + d*x]/(b*d)
```

Rubi [A] time = 0.050181, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{\sin(c+dx)}{bd}-\frac{a\log(a+b\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[a + b*Sin[c + d*x]])/(b^2*d)) + Sin[c + d*x]/(b*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{b(a+x)}\,dx,x,b\sin(c+dx)\right)}{bd}\\
&= \frac{\text{Subst}\left(\int \frac{x}{a+x}\,dx,x,b\sin(c+dx)\right)}{b^2d}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a+x}\right)dx,x,b\sin(c+dx)\right)}{b^2d}\\
&= -\frac{a\log(a+b\sin(c+dx))}{b^2d}+\frac{\sin(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] time = 0.0229036, size = 33, normalized size = 0.97

$$-\frac{\frac{a\log(a+b\sin(c+dx))}{b^2}-\frac{\sin(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a*Log[a + b*Sin[c + d*x]])/b^2 - Sin[c + d*x]/b)/d)
```

Maple [A] time = 0.026, size = 35, normalized size = 1.

$$-\frac{a\ln(a+b\sin(dx+c))}{db^2}+\frac{\sin(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b^2*a*ln(a+b*sin(d*x+c))+sin(d*x+c)/b/d
```

Maxima [A] time = 0.988223, size = 45, normalized size = 1.32

$$-\frac{\frac{a\log(b\sin(dx+c)+a)}{b^2}-\frac{\sin(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(b*sin(d*x + c) + a)/b^2 - sin(d*x + c)/b)/d
```

Fricas [A] time = 1.52505, size = 74, normalized size = 2.18

$$-\frac{a\log(b\sin(dx+c)+a)-b\sin(dx+c)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*log(b*sin(d*x + c) + a) - b*sin(d*x + c))/(b^2*d)
```

Sympy [A] time = 0.869154, size = 66, normalized size = 1.94

$$\begin{cases} \frac{x\sin(c)\cos(c)}{a} & \text{for}\ b=0\wedge d=0\\ -\frac{\cos^2(c+dx)}{2ad} & \text{for}\ b=0\\ \frac{x\sin(c)\cos(c)}{a+b\sin(c)} & \text{for}\ d=0\\ -\frac{a\log\left(\frac{a}{b}+\sin(c+dx)\right)}{b^2d}+\frac{\sin(c+dx)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*sin(c)*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (-cos(c + d*x)**2/(2*a*
d), Eq(b, 0)), (x*sin(c)*cos(c)/(a + b*sin(c)), Eq(d, 0)), (-a*log(a/b + si
n(c + d*x))/(b**2*d) + sin(c + d*x)/(b*d), True))
```

Giac [A] time = 1.19328, size = 46, normalized size = 1.35

$$-\frac{\frac{a\log(|b\sin(dx+c)+a|)}{b^2}-\frac{\sin(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(b*sin(d*x + c) + a))/b^2 - sin(d*x + c)/b)/d
```

3.1282 $\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0414976, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sin(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{ad}\\
&= \frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0174214, size = 34, normalized size = 1.

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Maple [A] time = 0.036, size = 35, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{da}-\frac{\ln(a+b\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-1/d/a*ln(a+b*sin(d*x+c))
```

Maxima [A] time = 0.973546, size = 45, normalized size = 1.32

$$-\frac{\frac{\log(b\sin(dx+c)+a)}{a}-\frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(b*sin(d*x + c) + a)/a - log(sin(d*x + c))/a)/d
```

Fricas [A] time = 1.47085, size = 80, normalized size = 2.35

$$-\frac{\log(b\sin(dx+c)+a)-\log\left(-\frac{1}{2}\sin(dx+c)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(log(b*sin(d*x + c) + a) - log(-1/2*sin(d*x + c)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.17809, size = 47, normalized size = 1.38

$$-\frac{\frac{\log(|b\sin(dx+c)+a|)}{a}-\frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(b*sin(d*x + c) + a))/a - log(abs(sin(d*x + c)))/a)/d
```

3.1283 $\int \frac{\cot(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=50

$$-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + (b*Log[a + b*Sin[c
+ d*x]])/(a^2*d)
```

Rubi [A] time = 0.0724541, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 44}

$$-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + (b*Log[a + b*Sin[c
+ d*x]])/(a^2*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2}{x^2(a+x)}\,dx,x,b\sin(c+dx)\right)}{bd}\\
&= \frac{b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx,x,b\sin(c+dx)\right)}{d}\\
&= \frac{b\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx,x,b\sin(c+dx)\right)}{d}\\
&= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}
\end{aligned}$$

Mathematica [A] time = 0.0415074, size = 50, normalized size = 1.

$$-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + (b*Log[a + b*Sin[c
+ d*x]])/(a^2*d)
```

Maple [A] time = 0.037, size = 35, normalized size = 0.7

$$-\frac{\csc(dx+c)}{da}+\frac{b\ln(a\csc(dx+c)+b)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -csc(d*x+c)/d/a+1/d/a^2*b*ln(a*csc(d*x+c)+b)
```

Maxima [A] time = 0.971688, size = 63, normalized size = 1.26

$$\frac{\frac{b\log(b\sin(dx+c)+a)}{a^2}-\frac{b\log(\sin(dx+c))}{a^2}-\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(b*sin(d*x + c) + a)/a^2 - b*log(sin(d*x + c))/a^2 - 1/(a*sin(d*x + c
)))/d
```

Fricas [A] time = 1.37955, size = 143, normalized size = 2.86

$$\frac{b\log(b\sin(dx+c)+a)\sin(dx+c)-b\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-a}{a^2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b*log(b*sin(d*x + c) + a)*sin(d*x + c) - b*log(1/2*sin(d*x + c))*sin(d*x +
 c) - a)/(a^2*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.2312, size = 66, normalized size = 1.32

$$\frac{\frac{b\log(|b\sin(dx+c)+a|)}{a^2} - \frac{b\log(|\sin(dx+c)|)}{a^2} - \frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*sin(d*x + c) + a))/a^2 - b*log(abs(sin(d*x + c)))/a^2 - 1/(a*s
in(d*x + c)))/d
```

3.1284 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) + (b^2*Log[Sin[c + d*x]])
/(a^3*d) - (b^2*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.0913019, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) + (b^2*Log[Sin[c + d*x]])
/(a^3*d) - (b^2*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{b^3}{x^3(a+x)}\,dx,x,b\sin(c+dx)\right)}{bd}\\
&= \frac{b^2\,\text{Subst}\left(\int \frac{1}{x^3(a+x)}\,dx,x,b\sin(c+dx)\right)}{d}\\
&= \frac{b^2\,\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{a^2x^2}+\frac{1}{a^3x}-\frac{1}{a^3(a+x)}\right)dx,x,b\sin(c+dx)\right)}{d}\\
&= \frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}+\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}
\end{aligned}$$

Mathematica [A] time = 0.0466824, size = 72, normalized size = 1.

$$\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) + (b^2*Log[Sin[c + d*x]])
/(a^3*d) - (b^2*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Maple [A] time = 0.046, size = 73, normalized size = 1.

$$-\frac{b^2\ln(a+b\sin(dx+c))}{a^3d}-\frac{1}{2da(\sin(dx+c))^2}+\frac{b^2\ln(\sin(dx+c))}{a^3d}+\frac{b}{da^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -b^2*ln(a+b*sin(d*x+c))/a^3/d-1/2/d/a/sin(d*x+c)^2+b^2*ln(sin(d*x+c))/a^3/d
+1/d/a^2*b/sin(d*x+c)
```

Maxima [A] time = 0.977051, size = 89, normalized size = 1.24

$$-\frac{\frac{2b^2\log(b\sin(dx+c)+a)}{a^3}-\frac{2b^2\log(\sin(dx+c))}{a^3}-\frac{2b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(b*sin(d*x + c) + a)/a^3 - 2*b^2*log(sin(d*x + c))/a^3 - (2*
b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.59283, size = 234, normalized size = 3.25

$$-\frac{2ab\sin(dx+c)-a^2+2\left(b^2\cos(dx+c)^2-b^2\right)\log(b\sin(dx+c)+a)-2\left(b^2\cos(dx+c)^2-b^2\right)\log\left(-\frac{1}{2}\sin(dx+\right.}{2\left(a^3d\cos(dx+c)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) - a^2 + 2*(b^2*cos(d*x + c)^2 - b^2)*log(b*sin(d*x
 + c) + a) - 2*(b^2*cos(d*x + c)^2 - b^2)*log(-1/2*sin(d*x + c)))/(a^3*d*co
s(d*x + c)^2 - a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.19649, size = 96, normalized size = 1.33

$$-\frac{\frac{2\,b^2\log(|b\sin(dx+c)+a|)}{a^3}-\frac{2\,b^2\log(|\sin(dx+c)|)}{a^3}-\frac{2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(b*sin(d*x + c) + a))/a^3 - 2*b^2*log(abs(sin(d*x + c)))
/a^3 - (2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2))/d
```

3.1285 $\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=235

$$\frac{(-5a^2b^2+15a^4-2b^4)\cos(c+dx)}{15b^5d}-\frac{2a^4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{(5a^2-b^2)\sin^2(c+dx)\cos(c+dx)}{15b^3d}-\frac{a(\cdots}{\cdots}$$

```
[Out] (a*(8*a^4 - 4*a^2*b^2 - b^4)*x)/(8*b^6) - (2*a^4*Sqrt[a^2 - b^2]*ArcTan[(b
+ a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + ((15*a^4 - 5*a^2*b^2 - 2*
b^4)*Cos[c + d*x])/(15*b^5*d) - (a*(4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])
/(8*b^4*d) + ((5*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*b^3*d) - (a*Co
s[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(5*b*d
)
```

Rubi [A] time = 0.91264, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{(-5a^2b^2+15a^4-2b^4)\cos(c+dx)}{15b^5d}-\frac{2a^4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{(5a^2-b^2)\sin^2(c+dx)\cos(c+dx)}{15b^3d}-\frac{a(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (a*(8*a^4 - 4*a^2*b^2 - b^4)*x)/(8*b^6) - (2*a^4*Sqrt[a^2 - b^2]*ArcTan[(b
+ a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + ((15*a^4 - 5*a^2*b^2 - 2*
b^4)*Cos[c + d*x])/(15*b^5*d) - (a*(4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])
/(8*b^4*d) + ((5*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*b^3*d) - (a*Co
s[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(5*b*d
)
```


Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin^4(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{5bd} + \frac{\int \frac{\sin^3(c+dx)\left(-4a+b\sin(c+dx)+5a\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{5b}$$

$$= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} + \frac{\cos(c+dx)\sin^4(c+dx)}{5bd} + \frac{\int \frac{\sin^2(c+dx)\left(15a^2-ab\sin(c+dx\right.}{a+b\sin(c}}{20b^2}$$

$$= \frac{\left(5a^2-b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^3d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} + \frac{\cos(c+dx)\sin^4}{5bd}$$

$$= -\frac{a\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{\left(5a^2-b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^3d} - \frac{a\cos}{}$$

$$= \frac{\left(15a^4-5a^2b^2-2b^4\right)\cos(c+dx)}{15b^5d} - \frac{a\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{\left(5a^2-b^2\right.}{}$$

$$= \frac{a\left(8a^4-4a^2b^2-b^4\right)x}{8b^6} + \frac{\left(15a^4-5a^2b^2-2b^4\right)\cos(c+dx)}{15b^5d} - \frac{a\left(4a^2-b^2\right)\cos(c+d}{8b^4d}$$

$$= \frac{a\left(8a^4-4a^2b^2-b^4\right)x}{8b^6} + \frac{\left(15a^4-5a^2b^2-2b^4\right)\cos(c+dx)}{15b^5d} - \frac{a\left(4a^2-b^2\right)\cos(c+d}{8b^4d}$$

$$= \frac{a\left(8a^4-4a^2b^2-b^4\right)x}{8b^6} + \frac{\left(15a^4-5a^2b^2-2b^4\right)\cos(c+dx)}{15b^5d} - \frac{a\left(4a^2-b^2\right)\cos(c+d}{8b^4d}$$

$$= \frac{a\left(8a^4-4a^2b^2-b^4\right)x}{8b^6} - \frac{2a^4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} + \frac{\left(15a^4-5a^2b^2-2b\right.}{15b^5d}$$

Mathematica [A] time = 1.74478, size = 177, normalized size = 0.75

$$\frac{15a\left(4\left(-4a^2b^2+8a^4-b^4\right)(c+dx)-8a^2b^2\sin(2(c+dx))+b^4\sin(4(c+dx))\right)-60b\left(2a^2b^2-8a^4+b^4\right)\cos(c+dx)-}{480b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-960*a^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
- 60*b*(-8*a^4 + 2*a^2*b^2 + b^4)*Cos[c + d*x] - 10*(4*a^2*b^3 + b^5)*Cos[3
*(c + d*x)] + 6*b^5*Cos[5*(c + d*x)] + 15*a*(4*(8*a^4 - 4*a^2*b^2 - b^4)*(c
 + d*x) - 8*a^2*b^2*Sin[2*(c + d*x)] + b^4*Sin[4*(c + d*x)]))/(480*b^6*d)
```

Maple [B] time = 0.092, size = 871, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -8/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^2+2/d/b^4/(1+t
an(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3+3/2/d/b^2/(1+tan(1/2*d*x+1/
2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a-1/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)^9*a-1/4/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-1/d/b^4*arctan(tan
(1/2*d*x+1/2*c))*a^3+2/d/b^6*arctan(tan(1/2*d*x+1/2*c))*a^5-4/d/b/(1+tan(1/
2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6+4/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5*t
an(1/2*d*x+1/2*c)^4-4/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2
+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*a^4-2/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^
5*a^2+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^4-2/d/b^3/(
1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^2+8/d/b^5/(1+tan(1/2*d*x+1
/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^4-4/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*ta
n(1/2*d*x+1/2*c)^2*a^2+8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c
)^6*a^4-4/15/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2
)^5*tan(1/2*d*x+1/2*c)^6*a^2+1/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x
+1/2*c)^9*a^3+12/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^4-
2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a^3-3/2/d/b^2/(1+ta
n(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a+1/4/d/b^2/(1+tan(1/2*d*x+1/2*c
)^2)^5*tan(1/2*d*x+1/2*c)*a-2/d*a^4*(a^2-b^2)^(1/2)/b^6*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*ta
n(1/2*d*x+1/2*c)*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79199, size = 996, normalized size = 4.24

$$\left[\frac{24\,b^5\cos(dx+c)^5 + 120\,a^4 b\cos(dx+c) + 60\,\sqrt{-a^2+b^2}\,a^4\log\left(\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/120*(24*b^5*cos(d*x + c)^5 + 120*a^4*b*cos(d*x + c) + 60*sqrt(-a^2 + b^2
)*a^4*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos
(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 40*(a^2*b^3 + b^5)*cos(d*x
 + c)^3 + 15*(8*a^5 - 4*a^3*b^2 - a*b^4)*d*x + 15*(2*a*b^4*cos(d*x + c)^3 -
 (4*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d), 1/120*(24*b^5*cos
(d*x + c)^5 + 120*a^4*b*cos(d*x + c) + 120*sqrt(a^2 - b^2)*a^4*arctan(-(a*s
in(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 40*(a^2*b^3 + b^5)*cos(d
*x + c)^3 + 15*(8*a^5 - 4*a^3*b^2 - a*b^4)*d*x + 15*(2*a*b^4*cos(d*x + c)^3
 - (4*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24936, size = 630, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(8*a^5 - 4*a^3*b^2 - a*b^4)*(d*x + c)/b^6 - 240*(a^6 - a^4*b^2)*(
pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(60*a^3*b*tan(1/2*d*x + 1/2*
c)^9 - 15*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*x + 1/2*c)^8 - 1
20*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 90*a
*b^3*tan(1/2*d*x + 1/2*c)^7 + 480*a^4*tan(1/2*d*x + 1/2*c)^6 - 240*a^2*b^2*
tan(1/2*d*x + 1/2*c)^6 - 240*b^4*tan(1/2*d*x + 1/2*c)^6 + 720*a^4*tan(1/2*d
*x + 1/2*c)^4 - 160*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 80*b^4*tan(1/2*d*x + 1
/2*c)^4 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 90*a*b^3*tan(1/2*d*x + 1/2*c)^
3 + 480*a^4*tan(1/2*d*x + 1/2*c)^2 - 80*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 80
*b^4*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) + 15*a*b^3*tan(
1/2*d*x + 1/2*c) + 120*a^4 - 40*a^2*b^2 - 16*b^4)/((tan(1/2*d*x + 1/2*c)^2
+ 1)^5*b^5))/d
```

3.1286 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=191

$$-\frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d}+\frac{2a^3\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}+\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8b^3d}-\frac{x\left(-4a^2b^2+8a^4\right.}{8b^5}$$

```
[Out] -((8*a^4 - 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*Sqrt[a^2 - b^2]*ArcTan[(b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (a*(3*a^2 - b^2)*Cos[c + d
*x])/(3*b^4*d) + ((4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*b^3*d) - (a*C
os[c + d*x]*Sin[c + d*x]^2)/(3*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*
d)
```

Rubi [A] time = 0.663644, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d}+\frac{2a^3\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}+\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8b^3d}-\frac{x\left(-4a^2b^2+8a^4\right.}{8b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((8*a^4 - 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*Sqrt[a^2 - b^2]*ArcTan[(b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (a*(3*a^2 - b^2)*Cos[c + d
*x])/(3*b^4*d) + ((4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*b^3*d) - (a*C
os[c + d*x]*Sin[c + d*x]^2)/(3*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*
d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin^3(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{\int \frac{\sin^2(c+dx)\left(-3a+b\sin(c+dx)+4a\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{4b}$$

$$= -\frac{a\cos(c+dx)\sin^2(c+dx)}{3b^2d} + \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{\int \frac{\sin(c+dx)\left(8a^2-ab\sin(c+dx)-3(4a\right.}{a+b\sin(c+dx)}}{12b^2}$$

$$= \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos(c+dx)\sin^2(c+dx)}{3b^2d} + \frac{\cos(c+dx)\sin^3(c+}{4bd}$$

$$= -\frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d} + \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos(c+dx)\sin^2(c}{3b^2d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5} - \frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d} + \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5} - \frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d} + \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5} - \frac{a\left(3a^2-b^2\right)\cos(c+dx)}{3b^4d} + \frac{\left(4a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d}$$

$$= -\frac{\left(8a^4-4a^2b^2-b^4\right)x}{8b^5} + \frac{2a^3\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d} - \frac{a\left(3a^2-b^2\right)\cos(c+dx}{3b^4d}$$

Mathematica [A] time = 1.08552, size = 146, normalized size = 0.76

$$\frac{-12\left(-4a^2b^2+8a^4-b^4\right)(c+dx)+24a^2b^2\sin(2(c+dx))+24ab\left(b^2-4a^2\right)\cos(c+dx)+192a^3\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a^2-}}\right.}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-12*(8*a^4 - 4*a^2*b^2 - b^4)*(c + d*x) + 192*a^3*Sqrt[a^2 - b^2]*ArcTan[(
b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 24*a*b*(-4*a^2 + b^2)*Cos[c + d*
x] + 8*a*b^3*Cos[3*(c + d*x)] + 24*a^2*b^2*Sin[2*(c + d*x)] - 3*b^4*Sin[4*(
c + d*x)])/(96*b^5*d)
```

Maple [B] time = 0.087, size = 657, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2+1/4/d/b/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^6*a^3+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
```

```
/2*c)^6*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2-7/4/d
/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-6/d/b^4/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(
1/2*d*x+1/2*c)^4*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*
a^2+7/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3+2/3/d/b^2/(1+tan(1/2*d*x+1/2*c
)^2)^4*tan(1/2*d*x+1/2*c)^2*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*
x+1/2*c)*a^2-1/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-2/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^4*a^3+2/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*a-2/d/b
^5*arctan(tan(1/2*d*x+1/2*c))*a^4+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+1/
4/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d*a^3*(a^2-b^2)^(1/2)/b^5*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56439, size = 875, normalized size = 4.58

$$\left[\frac{8\,ab^3\cos(dx+c)^3 - 24\,a^3b\cos(dx+c) + 12\sqrt{-a^2+b^2}a^3\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right.}{24\,b^5d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(8*a*b^3*cos(d*x + c)^3 - 24*a^3*b*cos(d*x + c) + 12*sqrt(-a^2 + b^2)
*a^3*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 -
2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos
(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 3*(8*a^4 - 4*a^2*b^2 - b^4
)*d*x - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2 + b^4)*cos(d*x + c))*sin(d*x +
 c))/(b^5*d), 1/24*(8*a*b^3*cos(d*x + c)^3 - 24*a^3*b*cos(d*x + c) - 24*sqr
t(a^2 - b^2)*a^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)
)) - 3*(8*a^4 - 4*a^2*b^2 - b^4)*d*x - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2
 + b^4)*cos(d*x + c))*sin(d*x + c))/(b^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

[Out] Timed out

Giac [B] time = 1.18658, size = 494, normalized size = 2.59

$$-\frac{\frac{3\left(8a^4-4a^2b^2-b^4\right)(dx+c)}{b^5}-\frac{48\left(a^5-a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{2\left(12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24a^3\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(8*a^4 - 4*a^2*b^2 - b^4)*(d*x + c)/b^5 - 48*(a^5 - a^3*b^2)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/s
qrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) + 2*(12*a^2*b*tan(1/2*d*x + 1/2*c)^7
 - 3*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^3*tan(1/2*d*x + 1/2*c)^6 - 24*a*b^2*
tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 21*b^3*tan(1/2*d
*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 24*a*b^2*tan(1/2*d*x + 1/2*
c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 21*b^3*tan(1/2*d*x + 1/2*c)^3 + 72
*a^3*tan(1/2*d*x + 1/2*c)^2 - 8*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan
(1/2*d*x + 1/2*c) + 3*b^3*tan(1/2*d*x + 1/2*c) + 24*a^3 - 8*a*b^2)/((tan(1/
2*d*x + 1/2*c)^2 + 1)^4*b^4))/d
```

3.1287 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{(3a^2-b^2)\cos(c+dx)}{3b^3d}-\frac{2a^2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{ax\left(2a^2-b^2\right)}{2b^4}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\frac{\sin^2(c+\ldots}{\ldots}$$

```
[Out] (a*(2*a^2 - b^2)*x)/(2*b^4) - (2*a^2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + ((3*a^2 - b^2)*Cos[c + d*x])/(3*b^3*d
) - (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^2)
/(3*b*d)
```

Rubi [A] time = 0.458729, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{(3a^2-b^2)\cos(c+dx)}{3b^3d}-\frac{2a^2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{ax\left(2a^2-b^2\right)}{2b^4}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\frac{\sin^2(c+\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(2*a^2 - b^2)*x)/(2*b^4) - (2*a^2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + ((3*a^2 - b^2)*Cos[c + d*x])/(3*b^3*d
) - (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^2)
/(3*b*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin^2(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\cos(c+dx)\sin^2(c+dx)}{3bd} + \frac{\int \frac{\sin(c+dx)\left(-2a+b\sin(c+dx)+3a\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3b}$$
$$= -\frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c+dx)\sin^2(c+dx)}{3bd} + \frac{\int \frac{3a^2-ab\sin(c+dx)-2\left(3a^2-b^2\right)\text{si}}{a+b\sin(c+dx)}}{6b^2}$$
$$= \frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c+dx)\sin^2(c+dx)}{3bd} + \frac{\int}{}$$
$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c+dx)\,\text{s}}{3b}$$
$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c+dx)\,\text{s}}{3b}$$
$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c+dx)\,\text{s}}{3b}$$
$$= \frac{a\left(2a^2-b^2\right)x}{2b^4} - \frac{2a^2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d} + \frac{\left(3a^2-b^2\right)\cos(c+dx)}{3b^3d} - a$$

Mathematica [A] time = 0.25919, size = 130, normalized size = 0.88

$$-\frac{3b\left(b^2-4a^2\right)\cos(c+dx)+24a^2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)-12a^3c-12a^3dx+3ab^2\sin(2(c+dx))+6ab^2c+}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-12*a^3*c + 6*a*b^2*c - 12*a^3*d*x + 6*a*b^2*d*x + 24*a^2*Sqrt[a^2 - b^2]
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 3*b*(-4*a^2 + b^2)*Cos[
c + d*x] + b^3*Cos[3*(c + d*x)] + 3*a*b^2*Sin[2*(c + d*x)])/(12*b^4*d)
```

Maple [B] time = 0.082, size = 318, normalized size = 2.2

$$\frac{a}{db^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-3}+2\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^4a^2}{db^3\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)^3}-2\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^4}{bd\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5+2/d/b^3/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2-2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3
*tan(1/2*d*x+1/2*c)^4+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*a^2*tan(1/2*d*x+1/
2*c)^2-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)+2/d/b^3/(1+t
an(1/2*d*x+1/2*c)^2)^3*a^2-2/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3+2/d/b^4*arcta
```

```
n(tan(1/2*d*x+1/2*c))*a^3-1/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-2/d*a^2*(a^2
-b^2)^(1/2)/b^4*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60131, size = 736, normalized size = 4.97

$$\left[-\frac{2\,b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,a^2b\cos\left(dx+c\right)-3\,\sqrt{-a^2+b^2}a^2\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+\ldots}{b^2\cos(\ldots}\right.}{6\,b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 6*a^2*b*c
os(d*x + c) - 3*sqrt(-a^2 + b^2)*a^2*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
 c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
)) - 3*(2*a^3 - a*b^2)*d*x)/(b^4*d), -1/6*(2*b^3*cos(d*x + c)^3 + 3*a*b^2*c
os(d*x + c)*sin(d*x + c) - 6*a^2*b*cos(d*x + c) - 6*sqrt(a^2 - b^2)*a^2*arc
tan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(2*a^3 - a*b^
2)*d*x)/(b^4*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19669, size = 279, normalized size = 1.89

$$\frac{\frac{3\left(2\,a^3-ab^2\right)(dx+c)}{b^4}-\frac{12\left(a^4-a^2b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}b^4}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1\ldots}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)\ldots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - a*b^2)*(d*x + c)/b^4 - 12*(a^4 - a^2*b^2)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^
2)))/(sqrt(a^2 - b^2)*b^4) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/
2*d*x + 1/2*c)^4 - 6*b^2*tan(1/2*d*x + 1/2*c)^4 + 12*a^2*tan(1/2*d*x + 1/2*
c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 - 2*b^2)/((tan(1/2*d*x + 1/2*c)^2
 + 1)^3*b^3))/d
```

3.1288 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{2a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{x\left(2a^2-b^2\right)}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}$$

```
[Out] -((2*a^2 - b^2)*x)/(2*b^3) + (2*a*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(b^3*d) - (Cos[c + d*x]*(2*a - b*Sin[c + d*x]))/(2
*b^2*d)
```

Rubi [A] time = 0.165019, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2865, 2735, 2660, 618, 204}

$$\frac{2a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{x\left(2a^2-b^2\right)}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((2*a^2 - b^2)*x)/(2*b^3) + (2*a*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(b^3*d) - (Cos[c + d*x]*(2*a - b*Sin[c + d*x]))/(2
*b^2*d)
```


Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}+\frac{\int \frac{-ab-(2a^2-b^2)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b^2}\\ &= -\frac{(2a^2-b^2)x}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}+\frac{(a(a^2-b^2))\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^3}\\ &= -\frac{(2a^2-b^2)x}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}+\frac{(2a(a^2-b^2))\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\right.}{b^3d}\\ &= -\frac{(2a^2-b^2)x}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}-\frac{(4a(a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-}\right.}{b^3}\\ &= -\frac{(2a^2-b^2)x}{2b^3}+\frac{2a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{\cos(c+dx)(2a-b\sin(c+dx}{2b^2d}\end{aligned}$$

Mathematica [A] time = 0.23189, size = 104, normalized size = 1.04

$$\frac{8a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)-4a^2c-4a^2dx-4ab\cos(c+dx)+b^2\sin(2(c+dx))+2b^2c+2b^2dx}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*Sqrt[a^2 - b^2]*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 4*a*b*Cos[c + d*x] + b^2*Sin[2*(c
 + d*x)])/(4*b^3*d)
```

Maple [B] time = 0.072, size = 214, normalized size = 2.1

$$-\frac{1}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-2\,\frac{(\tan(1/2\,dx+c/2))^2\,a}{db^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+\frac{1}{bd}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/b^2/(1+tan(1/2*d
*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(
```

```
1/2*d*x+1/2*c)-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*a-2/d/b^3*arctan(tan(1/2*
d*x+1/2*c))*a^2+1/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d*a*(a^2-b^2)^(1/2)/b^3*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50194, size = 637, normalized size = 6.37

$$\left[\frac{b^2\cos(dx+c)\sin(dx+c) - \left(2a^2-b^2\right)dx - 2ab\cos(dx+c) + \sqrt{-a^2+b^2}\,a\log\left(-\frac{\left(2a^2-b^2\right)\cos(dx+c)^2 - 2ab\sin(dx+c) - a^2 - b^2 - 2(a}{b^2\cos(dx+c)^2 - 2ab\,s}\right.}{2b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b^2*cos(d*x + c)*sin(d*x + c) - (2*a^2 - b^2)*d*x - 2*a*b*cos(d*x + c
) + sqrt(-a^2 + b^2)*a*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^
2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(b^3*d),
1/2*(b^2*cos(d*x + c)*sin(d*x + c) - (2*a^2 - b^2)*d*x - 2*a*b*cos(d*x + c)
 - 2*sqrt(a^2 - b^2)*a*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*
x + c))))/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17071, size = 215, normalized size = 2.15

$$-\frac{\frac{\left(2a^2-b^2\right)(dx+c)}{b^3} - \frac{4\left(a^3-ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3} + \frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*(d*x + c)/b^3 - 4*(a^3 - a*b^2)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(
sqrt(a^2 - b^2)*b^3) + 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*
c)^2 - b*tan(1/2*d*x + 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/
d
```

3.1289 $\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{b}$$

```
[Out] -(x/b) + (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(a*b*d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rubi [A] time = 0.185381, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2889, 3058, 2660, 618, 204, 3770}

$$\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x/b) + (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(a*b*d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3058

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(C*x
)/(b*d), x] + (Dist[(A*b^2 + a^2*C)/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f
*x]), x], x] - Dist[(c^2*C + A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f
*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx\\
&= -\frac{x}{b}+\frac{\int \csc(c+dx)\,dx}{a}-\left(-\frac{a}{b}+\frac{b}{a}\right)\int \frac{1}{a+b\sin(c+dx)}\,dx\\
&= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\
&= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\left(4\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\
&= -\frac{x}{b}+\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}
\end{aligned}
$$

Mathematica [A] time = 0.0956503, size = 90, normalized size = 1.2

$$
-\frac{-2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+ac+adx-b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{abd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*c + a*d*x - 2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]] + b*Log[Cos[(c + d*x)/2]] - b*Log[Sin[(c + d*x)/2]])/(a*b*d))
```

Maple [A] time = 0.096, size = 137, normalized size = 1.8

$$
-2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{bd}+2\,\frac{a}{bd\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)+2\,b}{\sqrt{a^2-b^2}}\right)-2\,\frac{b}{da\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d/b*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a*b/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76127, size = 640, normalized size = 8.53

$$\left[-\frac{2\,adx + b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) - b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) - \sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-}{b^2\cos(dx+c)^2-2}\right.}{2\,abd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) - sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b*d),
 -1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(
d*x + c))))/(a*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2\left(c+dx\right)\csc\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.2187, size = 127, normalized size = 1.69

$$-\frac{\frac{dx+c}{b} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/b - log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*
sqrt(a^2 - b^2)/(a*b))/d
```

3.1290 $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] (-2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*
d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rubi [A] time = 0.252815, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*
d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{\cot(c+dx)}{ad}+\frac{\int \frac{\csc(c+dx)(-b-a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{a} \\
&= -\frac{\cot(c+dx)}{ad}-\frac{b\int \csc(c+dx)\,dx}{a^2}+\frac{\left(-a^2+b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2} \\
&= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{\left(2\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+d\right.\right.}{a^2d} \\
&= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}+\frac{\left(4\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\right.}{a^2d} \\
&= -\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.231628, size = 108, normalized size = 1.35

$$\frac{-4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+a\tan\left(\frac{1}{2}(c+dx)\right)-a\cot\left(\frac{1}{2}(c+dx)\right)-2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\log\left(\cos\left(\frac{1}{2}\right.\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - a*Co
t[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]] + a*
```

```
Tan[(c + d*x)/2])/(2*a^2*d)
```

Maple [B] time = 0.101, size = 155, normalized size = 1.9

$$\frac{1}{2\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{1}{d\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)+2\,b}{\sqrt{a^2-b^2}}\right)+2\,\frac{b^2}{da^2\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+}{\sqrt{a^2-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+
1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a^2*b
*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77877, size = 801, normalized size = 10.01

$$\left[\frac{b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\,s}{}\right.}{2\,a^2d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c)), 1/2*(b*log(1/2*
cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d
*x + c)))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.21378, size = 174, normalized size = 2.17

$$-\frac{\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{a^2}-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a + 4*(
pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/a^2 - (2*b*tan(1/2*d*x + 1/2*c) - a)/(
a^2*tan(1/2*d*x + 1/2*c)))/d
```

3.1291 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{2b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d}+\frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] (2*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3
*d) + ((a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (b*Cot[c + d*x])/(a
^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.456389, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d}+\frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3
*d) + ((a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (b*Cot[c + d*x])/(a
^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^3(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int \frac{\csc^2(c+dx)\left(-2b-a\sin(c+dx)+b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{2a}$$
$$= \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int \frac{\csc(c+dx)\left(-a^2+2b^2+ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{2a^2}$$
$$= \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\left(a^2-2b^2\right)\int \csc(c+dx)\,dx}{2a^3} + \frac{\left(b\left(a^2-b^2\right)\right)\int}{a}$$
$$= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\left(2b\left(a^2-b\right.\right.}{}$$
$$= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\left(4b\left(a^2-b\right.\right.}{}$$
$$= \frac{2b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3d} + \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\cot(c+dx)}{a^2d} -$$

Mathematica [A] time = 0.84715, size = 181, normalized size = 1.59

$$\frac{16b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + a^2\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right) + a^2\sec^2\left(\frac{1}{2}(c+dx)\right) - 4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^2\log}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (16*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 4*
a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 4*a^2*Log[Cos[(c + d*x)/2]]
 - 8*b^2*Log[Cos[(c + d*x)/2]] - 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log[Si
n[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 - 4*a*b*Tan[(c + d*x)/2])/(8*a^3*d
)
```

Maple [A] time = 0.108, size = 166, normalized size = 1.5

$$\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{b}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\frac{b\sqrt{a^2-b^2}}{da^3}\arctan\left(1/2\,\frac{2a\tan(1/2\,dx+c/2)+2b}{\sqrt{a^2-b^2}}\right) - \frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/2*d*x+1/2*c)*b+2/d*b*(a^2-b^2)
^(1/2)/a^3*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/8/d/a
/tan(1/2*d*x+1/2*c)^2-1/2/d/a*ln(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x
+1/2*c))*b^2+1/2/d/a^2*b/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94687, size = 1121, normalized size = 9.83

$$\left[-\frac{4\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^2\cos\left(dx+c\right)-2\left(b\cos\left(dx+c\right)^2-b\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+}{b^2\cos}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cos(d*x + c)*sin(d*x + c) - 2*a^2*cos(d*x + c) - 2*(b*cos(d*x
+ c)^2 - b)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sq
rt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - ((
a^2 - 2*b^2)*cos(d*x + c)^2 - a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((
a^2 - 2*b^2)*cos(d*x + c)^2 - a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a
^3*d*cos(d*x + c)^2 - a^3*d), -1/4*(4*a*b*cos(d*x + c)*sin(d*x + c) - 2*a^2
*cos(d*x + c) + 4*(b*cos(d*x + c)^2 - b)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - ((a^2 - 2*b^2)*cos(d*x + c)^2
- a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2 - 2*b^2)*cos(d*x + c)^2
- a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*d*cos(d*x + c)^2 - a^3*d)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2\left(c+dx\right)\csc^3\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.22197, size = 267, normalized size = 2.34

$$\frac{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{4\left(a^2-2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{16\left(a^2b-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3}+\frac{6\,a^2\tan\left(\frac{1}{2}\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 4*(a^2 - 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 16*(a^2*b - b^3)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*a^3) + (6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1
/2*d*x + 1/2*c)^2 + 4*a*b*tan(1/2*d*x + 1/2*c) - a^2)/(a^3*tan(1/2*d*x + 1/
2*c)^2))/d
```

3.1292 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=153

$$-\frac{2b^2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{(a^2-3b^2)\cot(c+dx)}{3a^3d}-\frac{b(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{b\cot(c+dx)\text{cs}}{2a^2d}$$

```
[Out] (-2*b^2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^4*d) - (b*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((a^2 - 3*b^2)
*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rubi [A] time = 0.673969, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{(a^2-3b^2)\cot(c+dx)}{3a^3d}-\frac{b(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{b\cot(c+dx)\text{cs}}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^4*d) - (b*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((a^2 - 3*b^2)
*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^4(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\int \frac{\csc^3(c+dx)\left(-3b-a\sin(c+dx)+2b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{3a}$$

$$= \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\int \frac{\csc^2(c+dx)\left(-2\left(a^2-3b^2\right)+ab\sin(c}{a+b\sin(c+d}}{6a^2}$$

$$= \frac{\left(a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \int$$

$$= \frac{\left(a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{(b}{}$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c-}{2a^2d}$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c-}{2a^2d}$$

$$= -\frac{2b^2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(a^2-3}{}$$

Mathematica [B] time = 6.23091, size = 351, normalized size = 2.29

$$\frac{\left(a^2b-2b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(2b^3-a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\right.}{6a^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*Sqrt[a^2 - b^2]*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Si
n[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((a^2*Cos[(c + d*x)/2] - 3*b^2
*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*
a^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-(a^2*b) + 2*b^
3)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2]
])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-(a^2*
Sin[(c + d*x)/2]) + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(24*a*d)
```

Maple [A] time = 0.112, size = 250, normalized size = 1.6

$$\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{b}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{1}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{b^2}{2\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 2\,\frac{b^2\sqrt{a^2-b^2}}{da^4}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a^2*tan(1/2*d*x+1/2*c)^2*b-1/8/d/a*tan(
1/2*d*x+1/2*c)+1/2/d/a^3*b^2*tan(1/2*d*x+1/2*c)-2/d*b^2*(a^2-b^2)^(1/2)/a^4
```

```
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/24/d/a/tan(1/2*
d*x+1/2*c)^3+1/8/d/a/tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)*b^2+1/
8/d/a^2*b/tan(1/2*d*x+1/2*c)^2+1/2/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-1/d/a^4*b
^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8975, size = 1420, normalized size = 9.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 12*a*b^2*cos(d*x + c) - 4*(a^3
- 3*a*b^2)*cos(d*x + c)^3 - 6*(b^2*cos(d*x + c)^2 - b^2)*sqrt(-a^2 + b^2)*l
og(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(a^2*b - 2*b^3 - (
a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3
*(a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c)), -1/12*(6*a
^2*b*cos(d*x + c)*sin(d*x + c) - 12*a*b^2*cos(d*x + c) - 4*(a^3 - 3*a*b^2)*
cos(d*x + c)^3 - 12*(b^2*cos(d*x + c)^2 - b^2)*sqrt(a^2 - b^2)*arctan(-(a*s
in(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 3*(a^2*b -
2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) + 3*(a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20101, size = 365, normalized size = 2.39

$$\frac{\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}+\frac{12\left(a^2b-2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}-\frac{48\left(a^2b^2-b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sg}\right.}{\sqrt{}}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*ta
n(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(a^2*b - 2*b^3)*
log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 48*(a^2*b^2 - b^4)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^4) - (22*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44*b^3*tan(1/
2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(1/2*d*x + 1/
2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x + 1/2*c)^3))/
d
```

3.1293 $\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{2b^3\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d}+\frac{\left(4a^2b^2+a^4-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d}+\frac{\left(a^2-4b^2\right)c}{}$$

```
[Out] (2*b^3*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^5*d) + ((a^4 + 4*a^2*b^2 - 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^5*d) - (b*(a
^2 - 3*b^2)*Cot[c + d*x])/(3*a^4*d) + ((a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(8*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) - (Cot[c + d*x]*
Csc[c + d*x]^3)/(4*a*d)
```

Rubi [A] time = 0.953255, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^3\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d}+\frac{\left(4a^2b^2+a^4-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d}+\frac{\left(a^2-4b^2\right)c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^5*d) + ((a^4 + 4*a^2*b^2 - 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^5*d) - (b*(a
^2 - 3*b^2)*Cot[c + d*x])/(3*a^4*d) + ((a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(8*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) - (Cot[c + d*x]*
Csc[c + d*x]^3)/(4*a*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^5(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$
$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{\int \frac{\csc^4(c+dx)\left(-4b-a\sin(c+dx)+3b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{4a}$$
$$= \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{\int \frac{\csc^3(c+dx)\left(-3\left(a^2-4b^2\right)+ab\sin(c+d\right.}{a+b\sin(c+dx)}}{12a^2}$$
$$= \frac{\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\csc^3(c+}{4ad}$$
$$= -\frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(c}{3a^2d}$$
$$= -\frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(c}{3a^2d}$$
$$= \frac{\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(a^2-4b^2\right)\cot(c}{8a}$$
$$= \frac{\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b\left(a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(a^2-4b^2\right)\cot(c}{8a}$$
$$= \frac{2b^3\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d} + \frac{\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b\left(a^2\right.}{}$$

Mathematica [B] time = 6.26106, size = 430, normalized size = 2.22

$$\frac{\left(a^2-4b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{\left(4b^2-a^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{\left(-4a^2b^2-a^4+8b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^5d} + \frac{\left(4a^2b^2+a^4-8\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*Sqrt[a^2 - b^2]*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin
[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*d) + ((-(a^2*b*Cos[(c + d*x)/2]) + 3
*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + ((a^2 - 4*b^2)*Csc[(c
+ d*x)/2]^2)/(32*a^3*d) + (b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^2*d
) - Csc[(c + d*x)/2]^4/(64*a*d) + ((a^4 + 4*a^2*b^2 - 8*b^4)*Log[Cos[(c + d
*x)/2]])/(8*a^5*d) + ((-a^4 - 4*a^2*b^2 + 8*b^4)*Log[Sin[(c + d*x)/2]])/(8*
a^5*d) + ((-a^2 + 4*b^2)*Sec[(c + d*x)/2]^2)/(32*a^3*d) + Sec[(c + d*x)/2]^
4/(64*a*d) + (Sec[(c + d*x)/2]*(a^2*b*Sin[(c + d*x)/2] - 3*b^3*Sin[(c + d*x
)/2]))/(6*a^4*d) - (b*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [A] time = 0.113, size = 315, normalized size = 1.6

$$\frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 - \frac{b}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{b^2}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + \frac{b}{8\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{b^3}{2\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a^2*tan(1/2*d*x+1/2*c)^3*b+1/8/d/a^3*t
an(1/2*d*x+1/2*c)^2*b^2+1/8/d/a^2*tan(1/2*d*x+1/2*c)*b-1/2/d/a^4*b^3*tan(1/
2*d*x+1/2*c)+2/d*b^3*(a^2-b^2)^(1/2)/a^5*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))-1/64/d/a/tan(1/2*d*x+1/2*c)^4-1/8/d/a*ln(tan(1/2*d*x
+1/2*c))-1/2/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/d/a^5*ln(tan(1/2*d*x+1/2*c)
)*b^4+1/24/d/a^2*b/tan(1/2*d*x+1/2*c)^3-1/8/d/a^3*b^2/tan(1/2*d*x+1/2*c)^2-
1/8/d/a^2*b/tan(1/2*d*x+1/2*c)+1/2/d*b^3/a^4/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.82981, size = 1901, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(6*(a^4 - 4*a^2*b^2)*cos(d*x + c)^3 - 24*(b^3*cos(d*x + c)^4 - 2*b^3
*cos(d*x + c)^2 + b^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 6*(a^4 + 4*a^2*b^2)*cos(d*x + c) - 3*((a^4 + 4*a^2*b^2 - 8*b^4)*co
s(d*x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4 - 2*(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*
x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*
x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4 - 2*(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x +
c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*(3*a*b^3*cos(d*x + c) + (a^3*b - 3*
a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*
x + c)^2 + a^5*d), -1/48*(6*(a^4 - 4*a^2*b^2)*cos(d*x + c)^3 + 48*(b^3*cos(
d*x + c)^4 - 2*b^3*cos(d*x + c)^2 + b^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(a^4 + 4*a^2*b^2)*cos(d*x +
c) - 3*((a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4
- 2*(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) +
 3*((a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4 - 2*
(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16
*(3*a*b^3*cos(d*x + c) + (a^3*b - 3*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a
^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x + c)^2 + a^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24553, size = 454, normalized size = 2.34

$$\frac{\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}-\frac{24\,\left(a^4+4\,a^2b^2-8\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*((3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 24*
a*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1
/2*d*x + 1/2*c))/a^4 - 24*(a^4 + 4*a^2*b^2 - 8*b^4)*log(abs(tan(1/2*d*x + 1
/2*c)))/a^5 + 384*(a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
+ arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^
5) + (50*a^4*tan(1/2*d*x + 1/2*c)^4 + 200*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 -
400*b^4*tan(1/2*d*x + 1/2*c)^4 - 24*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 96*a*b^3
*tan(1/2*d*x + 1/2*c)^3 - 24*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^3*b*tan(1
/2*d*x + 1/2*c) - 3*a^4)/(a^5*tan(1/2*d*x + 1/2*c)^4))/d
```

3.1294 $\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=238

$$-\frac{2b^4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(5a^2b^2+2a^4-15b^4\right)\cot(c+dx)}{15a^5d}-\frac{b\left(4a^2b^2+a^4-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d}$$

```
[Out] (-2*b^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^6*d) - (b*(a^4 + 4*a^2*b^2 - 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) + ((
2*a^4 + 5*a^2*b^2 - 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (b*(a^2 - 4*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(8*a^4*d) + ((a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x
]^2)/(15*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^4)/(5*a*d)
```

Rubi [A] time = 1.22683, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(5a^2b^2+2a^4-15b^4\right)\cot(c+dx)}{15a^5d}-\frac{b\left(4a^2b^2+a^4-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^6*d) - (b*(a^4 + 4*a^2*b^2 - 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) + ((
2*a^4 + 5*a^2*b^2 - 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (b*(a^2 - 4*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(8*a^4*d) + ((a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x
]^2)/(15*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^4)/(5*a*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^6(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)}{5ad} + \frac{\int \frac{\csc^5(c+dx)\left(-5b-a\sin(c+dx)+4b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{5a}$$

$$= \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad} + \frac{\int \frac{\csc^4(c+dx)\left(-4\left(a^2-5b^2\right)+ab\sin(\right.}{a+b\sin(c+}}{20a^2}$$

$$= \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{15a^3d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^4}{5ad}$$

$$= -\frac{b\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{15a^3d} + \frac{b\cot}{}$$

$$= \frac{\left(2a^4+5a^2b^2-15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(a^2-5b\right.}{}$$

$$= \frac{\left(2a^4+5a^2b^2-15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b\left(a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(a^2-5b\right.}{}$$

$$= -\frac{b\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{\left(2a^4+5a^2b^2-15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b}{}$$

$$= -\frac{b\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{\left(2a^4+5a^2b^2-15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b}{}$$

$$= -\frac{2b^4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} - \frac{b\left(a^4+4a^2b^2-8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} -$$

Mathematica [B] time = 1.81958, size = 506, normalized size = 2.13

$$\frac{-1920b^4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - 160a^3b^2\tan\left(\frac{1}{2}(c+dx)\right) + 32\left(5a^3b^2+2a^5-15ab^4\right)\cot\left(\frac{1}{2}(c+dx)\right) + 120}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-1920*b^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
 + 32*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*Cot[(c + d*x)/2] - 30*a^4*b*Csc[(c + d
*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4 - 1
20*a^4*b*Log[Cos[(c + d*x)/2]] - 480*a^2*b^3*Log[Cos[(c + d*x)/2]] + 960*b^
5*Log[Cos[(c + d*x)/2]] + 120*a^4*b*Log[Sin[(c + d*x)/2]] + 480*a^2*b^3*Log
[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 30*a^4*b*Sec[(c + d*x)
/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4 - 16*a
^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[(c + d*x)/
2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] - 64*a^5*Tan[(c +
 d*x)/2] - 160*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*x)/2] + 6*a^
5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

Maple [A] time = 0.119, size = 439, normalized size = 1.8

$$\frac{1}{160\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{b}{64\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}+\frac{1}{96\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{b^{2}}{24\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{b^{3}}{8\,da^{4}}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a^2*tan(1/2*d*x+1/2*c)^4*b+1/96/d/a*t
an(1/2*d*x+1/2*c)^3+1/24/d/a^3*tan(1/2*d*x+1/2*c)^3*b^2-1/8/d/a^4*tan(1/2*d
*x+1/2*c)^2*b^3-1/16/d/a*tan(1/2*d*x+1/2*c)-1/8/d/a^3*b^2*tan(1/2*d*x+1/2*c
)+1/2/d/a^5*b^4*tan(1/2*d*x+1/2*c)-2/d*b^4*(a^2-b^2)^(1/2)/a^6*arctan(1/2*(
2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/160/d/a/tan(1/2*d*x+1/2*c)^5
-1/96/d/a/tan(1/2*d*x+1/2*c)^3-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2+1/16/d/a
/tan(1/2*d*x+1/2*c)+1/8/d/a^3/tan(1/2*d*x+1/2*c)*b^2-1/2/d/a^5/tan(1/2*d*x+
1/2*c)*b^4+1/64/d/a^2*b/tan(1/2*d*x+1/2*c)^4+1/8/d/a^4*b^3/tan(1/2*d*x+1/2*
c)^2+1/8/d/a^2*b*ln(tan(1/2*d*x+1/2*c))+1/2/d/a^4*b^3*ln(tan(1/2*d*x+1/2*c)
)-1/d/a^6*b^5*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.80926, size = 2261, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(240*a*b^4*cos(d*x + c) - 16*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*cos(d*x
 + c)^5 + 80*(a^5 + a^3*b^2 - 6*a*b^4)*cos(d*x + c)^3 - 120*(b^4*cos(d*x +
c)^4 - 2*b^4*cos(d*x + c)^2 + b^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2))*sin(d*x + c) + 15*(a^4*b + 4*a^2*b^3 - 8*b^5 + (a^4*b + 4
*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x +
c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a^4*b + 4*a^2*b^3 - 8*
b^5 + (a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 4*a^2*b^3 - 8
*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*((a^4*
b - 4*a^2*b^3)*cos(d*x + c)^3 + (a^4*b + 4*a^2*b^3)*cos(d*x + c))*sin(d*x +
 c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
, -1/240*(240*a*b^4*cos(d*x + c) - 16*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*cos(d*
x + c)^5 + 80*(a^5 + a^3*b^2 - 6*a*b^4)*cos(d*x + c)^3 - 240*(b^4*cos(d*x +
 c)^4 - 2*b^4*cos(d*x + c)^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c)
 + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) + 15*(a^4*b + 4*a^2*b^3
```

```
- 8*b^5 + (a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 4*a^2*b^3
 - 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a^
4*b + 4*a^2*b^3 - 8*b^5 + (a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a
^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(
d*x + c) - 30*((a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 + (a^4*b + 4*a^2*b^3)*cos
(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 +
a^6*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21876, size = 599, normalized size = 2.52

$$\frac{\frac{6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-60\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-120\,a^2b^2\tan}{a^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 10
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b^3
*tan(1/2*d*x + 1/2*c)^2 - 60*a^4*tan(1/2*d*x + 1/2*c) - 120*a^2*b^2*tan(1/2
*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x + 1/2*c))/a^5 + 120*(a^4*b + 4*a^2*b^3
- 8*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 - 1920*(a^2*b^4 - b^6)*(pi*floo
r(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) - (274*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1
096*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*a
^5*tan(1/2*d*x + 1/2*c)^4 - 120*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 480*a*b^4*
tan(1/2*d*x + 1/2*c)^4 - 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 10*a^5*tan(1/
2*d*x + 1/2*c)^2 + 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*b*tan(1/2*d*x
 + 1/2*c) + 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

3.1295 $\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=149

$$-\frac{(a^2-b^2)\sin^3(c+dx)}{3b^3d}+\frac{a(a^2-b^2)\sin^2(c+dx)}{2b^4d}-\frac{a^2(a^2-b^2)\sin(c+dx)}{b^5d}+\frac{a^3(a^2-b^2)\log(a+b\sin(c+dx))}{b^6d}+\frac{a\sin}{}$$

```
[Out] (a^3*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^6*d) - (a^2*(a^2 - b^2)*Sin[c
+ d*x])/(b^5*d) + (a*(a^2 - b^2)*Sin[c + d*x]^2)/(2*b^4*d) - ((a^2 - b^2)*S
in[c + d*x]^3)/(3*b^3*d) + (a*Sin[c + d*x]^4)/(4*b^2*d) - Sin[c + d*x]^5/(5
*b*d)
```

Rubi [A] time = 0.198837, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{(a^2-b^2)\sin^3(c+dx)}{3b^3d}+\frac{a(a^2-b^2)\sin^2(c+dx)}{2b^4d}-\frac{a^2(a^2-b^2)\sin(c+dx)}{b^5d}+\frac{a^3(a^2-b^2)\log(a+b\sin(c+dx))}{b^6d}+\frac{a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a^3*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^6*d) - (a^2*(a^2 - b^2)*Sin[c
+ d*x])/(b^5*d) + (a*(a^2 - b^2)*Sin[c + d*x]^2)/(2*b^4*d) - ((a^2 - b^2)*S
in[c + d*x]^3)/(3*b^3*d) + (a*Sin[c + d*x]^4)/(4*b^2*d) - Sin[c + d*x]^5/(5
*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)}{b^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4\left(1-\frac{b^2}{a^2}\right)+a\left(a^2-b^2\right)x-\left(a^2-b^2\right)x^2+ax^3-x^4+\frac{a^5-a^3b^2}{a+x}\right)dx, x, b\,\text{s}\right.}{b^6d}$$
$$= \frac{a^3\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^6d} - \frac{a^2\left(a^2-b^2\right)\sin(c+dx)}{b^5d} + \frac{a\left(a^2-b^2\right)\sin^2(c+}{2b^4d}$$

Mathematica [A] time = 0.279319, size = 127, normalized size = 0.85

$$-\frac{\frac{60a^2(a-b)(a+b)\sin(c+dx)}{b^5}+\frac{60a^3(a-b)(a+b)\log(a+b\sin(c+dx))}{b^6}+\frac{15a\sin^4(c+dx)}{b^2}-\frac{20(a-b)(a+b)\sin^3(c+dx)}{b^3}+\frac{30a(a-b)(a+b)\sin^2(c+dx)}{b^4}-\frac{12\sin}{}}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((60*a^3*(a - b)*(a + b)*Log[a + b*Sin[c + d*x]])/b^6 - (60*a^2*(a - b)*(a
+ b)*Sin[c + d*x])/b^5 + (30*a*(a - b)*(a + b)*Sin[c + d*x]^2)/b^4 - (20*(a
 - b)*(a + b)*Sin[c + d*x]^3)/b^3 + (15*a*Sin[c + d*x]^4)/b^2 - (12*Sin[c +
 d*x]^5)/b)/(60*d)
```

Maple [A] time = 0.052, size = 182, normalized size = 1.2

$$-\frac{(\sin(dx+c))^5}{5\,bd}+\frac{a(\sin(dx+c))^4}{4\,b^2d}-\frac{a^2(\sin(dx+c))^3}{3\,db^3}+\frac{(\sin(dx+c))^3}{3\,bd}+\frac{(\sin(dx+c))^2a^3}{2\,db^4}-\frac{(\sin(dx+c))^2a}{2\,b^2d}-\frac{a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5*sin(d*x+c)^5/b/d+1/4*a*sin(d*x+c)^4/b^2/d-1/3/d/b^3*sin(d*x+c)^3*a^2+1
/3*sin(d*x+c)^3/b/d+1/2/d/b^4*sin(d*x+c)^2*a^3-1/2*a*sin(d*x+c)^2/b^2/d-1/d
/b^5*sin(d*x+c)*a^4+a^2*sin(d*x+c)/b^3/d+1/d*a^5/b^6*ln(a+b*sin(d*x+c))-a^3
*ln(a+b*sin(d*x+c))/b^4/d
```

Maxima [A] time = 1.0133, size = 177, normalized size = 1.19

$$-\frac{\frac{12\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+20\left(a^2b^2-b^4\right)\sin(dx+c)^3-30\left(a^3b-ab^3\right)\sin(dx+c)^2+60\left(a^4-a^2b^2\right)\sin(dx+c)}{b^5}-\frac{60\left(a^5-a^3b^2\right)\log(b\sin(dx+c)+a)}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*(a^2*b^2 - b^4
)*sin(d*x + c)^3 - 30*(a^3*b - a*b^3)*sin(d*x + c)^2 + 60*(a^4 - a^2*b^2)*s
```

```
in(d*x + c))/b^5 - 60*(a^5 - a^3*b^2)*log(b*sin(d*x + c) + a)/b^6)/d
```

Fricas [A] time = 1.62657, size = 297, normalized size = 1.99

$$\frac{15\,ab^4\cos\left(dx+c\right)^4-30\,a^3b^2\cos\left(dx+c\right)^2+60\left(a^5-a^3b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)-4\left(3\,b^5\cos\left(dx+c\right)^4+15\,a^4b-10\,a^2\right.}{60\,b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*b^4*cos(d*x + c)^4 - 30*a^3*b^2*cos(d*x + c)^2 + 60*(a^5 - a^3*b
^2)*log(b*sin(d*x + c) + a) - 4*(3*b^5*cos(d*x + c)^4 + 15*a^4*b - 10*a^2*b
^3 - 2*b^5 - (5*a^2*b^3 + b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17064, size = 201, normalized size = 1.35

$$-\frac{\frac{12\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+20\,a^2b^2\sin(dx+c)^3-20\,b^4\sin(dx+c)^3-30\,a^3b\sin(dx+c)^2+30\,ab^3\sin(dx+c)^2+60\,a^4\sin(dx+c)-60\,a^2b^2\sin(dx+c)}{b^5}-\frac{60\left(a^5\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*a^2*b^2*sin(d*
x + c)^3 - 20*b^4*sin(d*x + c)^3 - 30*a^3*b*sin(d*x + c)^2 + 30*a*b^3*sin(d
*x + c)^2 + 60*a^4*sin(d*x + c) - 60*a^2*b^2*sin(d*x + c))/b^5 - 60*(a^5 -
a^3*b^2)*log(abs(b*sin(d*x + c) + a))/b^6)/d
```

3.1296 $\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=119

$$-\frac{(a^2-b^2)\sin^2(c+dx)}{2b^3d}+\frac{a(a^2-b^2)\sin(c+dx)}{b^4d}-\frac{a^2(a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}+\frac{a\sin^3(c+dx)}{3b^2d}-\frac{\sin^4(c+dx)}{4bd}$$

```
[Out] -((a^2*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d)) + (a*(a^2 - b^2)*Sin[c
 + d*x])/(b^4*d) - ((a^2 - b^2)*Sin[c + d*x]^2)/(2*b^3*d) + (a*Sin[c + d*x]
^3)/(3*b^2*d) - Sin[c + d*x]^4/(4*b*d)
```

Rubi [A] time = 0.172154, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{(a^2-b^2)\sin^2(c+dx)}{2b^3d}+\frac{a(a^2-b^2)\sin(c+dx)}{b^4d}-\frac{a^2(a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}+\frac{a\sin^3(c+dx)}{3b^2d}-\frac{\sin^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^2*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d)) + (a*(a^2 - b^2)*Sin[c
 + d*x])/(b^4*d) - ((a^2 - b^2)*Sin[c + d*x]^2)/(2*b^3*d) + (a*Sin[c + d*x]
^3)/(3*b^2*d) - Sin[c + d*x]^4/(4*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)}{b^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3\left(1-\frac{b^2}{a^2}\right)-\left(a^2-b^2\right)x+ax^2-x^3+\frac{a^2(-a^2+b^2)}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{a^2\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5 d} + \frac{a\left(a^2-b^2\right)\sin(c+dx)}{b^4 d} - \frac{\left(a^2-b^2\right)\sin^2(c+dx)}{2b^3 d}$$

Mathematica [A] time = 0.347514, size = 104, normalized size = 0.87

$$\frac{6b^2\left(b^2-a^2\right)\sin^2(c+dx)+12ab\left(a^2-b^2\right)\sin(c+dx)+12a^2\left(b^2-a^2\right)\log(a+b\sin(c+dx))+4ab^3\sin^3(c+dx)-3b^4\text{si}}{12b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (12*a^2*(-a^2 + b^2)*Log[a + b*Sin[c + d*x]] + 12*a*b*(a^2 - b^2)*Sin[c + d
*x] + 6*b^2*(-a^2 + b^2)*Sin[c + d*x]^2 + 4*a*b^3*Sin[c + d*x]^3 - 3*b^4*Si
n[c + d*x]^4)/(12*b^5*d)
```

Maple [A] time = 0.049, size = 144, normalized size = 1.2

$$-\frac{(\sin(dx+c))^4}{4bd}+\frac{a(\sin(dx+c))^3}{3b^2d}-\frac{a^2(\sin(dx+c))^2}{2db^3}+\frac{(\sin(dx+c))^2}{2bd}+\frac{a^3\sin(dx+c)}{db^4}-\frac{a\sin(dx+c)}{b^2d}-\frac{a^4\ln(a+b}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4*sin(d*x+c)^4/b/d+1/3*a*sin(d*x+c)^3/b^2/d-1/2/d/b^3*a^2*sin(d*x+c)^2+1
/2*sin(d*x+c)^2/b/d+1/d/b^4*sin(d*x+c)*a^3-a*sin(d*x+c)/b^2/d-1/d*a^4/b^5*l
n(a+b*sin(d*x+c))+1/d/b^3*ln(a+b*sin(d*x+c))*a^2
```

Maxima [A] time = 0.980881, size = 142, normalized size = 1.19

$$-\frac{\frac{3b^3\sin(dx+c)^4-4ab^2\sin(dx+c)^3+6\left(a^2b-b^3\right)\sin(dx+c)^2-12\left(a^3-ab^2\right)\sin(dx+c)}{b^4}+\frac{12\left(a^4-a^2b^2\right)\log(b\sin(dx+c)+a)}{b^5}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*(a^2*b - b^3)*sin
(d*x + c)^2 - 12*(a^3 - a*b^2)*sin(d*x + c))/b^4 + 12*(a^4 - a^2*b^2)*log(b
```

```
*sin(d*x + c) + a)/b^5)/d
```

Fricas [A] time = 1.58068, size = 230, normalized size = 1.93

$$-\frac{3\,b^4\cos\left(dx+c\right)^4-6\,a^2b^2\cos\left(dx+c\right)^2+12\left(a^4-a^2b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)+4\left(ab^3\cos\left(dx+c\right)^2-3\,a^3b+2\,ab^3\right.}{12\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*cos(d*x + c)^4 - 6*a^2*b^2*cos(d*x + c)^2 + 12*(a^4 - a^2*b^2)
*log(b*sin(d*x + c) + a) + 4*(a*b^3*cos(d*x + c)^2 - 3*a^3*b + 2*a*b^3)*sin
(d*x + c))/(b^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22803, size = 158, normalized size = 1.33

$$-\frac{\frac{3\,b^3\sin(dx+c)^4-4\,ab^2\sin(dx+c)^3+6\,a^2b\sin(dx+c)^2-6\,b^3\sin(dx+c)^2-12\,a^3\sin(dx+c)+12\,ab^2\sin(dx+c)}{b^4}+\frac{12\left(a^4-a^2b^2\right)\log(|b\sin(dx+c)+a|)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*a^2*b*sin(d*x + c
)^2 - 6*b^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c) + 12*a*b^2*sin(d*x + c))/b
^4 + 12*(a^4 - a^2*b^2)*log(abs(b*sin(d*x + c) + a))/b^5)/d
```

3.1297 $\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{(a^2-b^2)\sin(c+dx)}{b^3d}+\frac{a(a^2-b^2)\log(a+b\sin(c+dx))}{b^4d}+\frac{a\sin^2(c+dx)}{2b^2d}-\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] (a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^4*d) - ((a^2 - b^2)*Sin[c + d*x]
)/(b^3*d) + (a*Sin[c + d*x]^2)/(2*b^2*d) - Sin[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.114042, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 772}

$$-\frac{(a^2-b^2)\sin(c+dx)}{b^3d}+\frac{a(a^2-b^2)\log(a+b\sin(c+dx))}{b^4d}+\frac{a\sin^2(c+dx)}{2b^2d}-\frac{\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^4*d) - ((a^2 - b^2)*Sin[c + d*x]
)/(b^3*d) + (a*Sin[c + d*x]^2)/(2*b^2*d) - Sin[c + d*x]^3/(3*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)}{b(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2\left(1-\frac{b^2}{a^2}\right)+ax-x^2+\frac{a^3-ab^2}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{a\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^4d} - \frac{\left(a^2-b^2\right)\sin(c+dx)}{b^3d} + \frac{a\sin^2(c+dx)}{2b^2d} - \frac{\sin^3(c}{3bc}$$

Mathematica [A] time = 0.200605, size = 79, normalized size = 0.89

$$\frac{6b\left(b^2-a^2\right)\sin(c+dx)+6a\left(a^2-b^2\right)\log(a+b\sin(c+dx))+3ab^2\sin^2(c+dx)-2b^3\sin^3(c+dx)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (6*a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]] + 6*b*(-a^2 + b^2)*Sin[c + d*x] +
3*a*b^2*Sin[c + d*x]^2 - 2*b^3*Sin[c + d*x]^3)/(6*b^4*d)
```

Maple [A] time = 0.046, size = 106, normalized size = 1.2

$$-\frac{(\sin(dx+c))^3}{3\,bd}+\frac{(\sin(dx+c))^2\,a}{2\,b^2d}-\frac{a^2\sin(dx+c)}{b^3d}+\frac{\sin(dx+c)}{bd}+\frac{a^3\ln(a+b\sin(dx+c))}{b^4d}-\frac{a\ln(a+b\sin(dx+}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3*sin(d*x+c)^3/b/d+1/2*a*sin(d*x+c)^2/b^2/d-a^2*sin(d*x+c)/b^3/d+sin(d*x
+c)/b/d+a^3*ln(a+b*sin(d*x+c))/b^4/d-1/d/b^2*a*ln(a+b*sin(d*x+c))
```

Maxima [A] time = 0.98536, size = 107, normalized size = 1.2

$$-\frac{\frac{2\,b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+6\left(a^2-b^2\right)\sin(dx+c)}{b^3}-\frac{6\left(a^3-ab^2\right)\log(b\sin(dx+c)+a)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^2 + 6*(a^2 - b^2)*sin(d*x
+ c))/b^3 - 6*(a^3 - a*b^2)*log(b*sin(d*x + c) + a)/b^4)/d
```

Fricas [A] time = 1.49913, size = 185, normalized size = 2.08

$$-\frac{3\,ab^2\cos\left(dx+c\right)^2-6\left(a^3-ab^2\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\left(b^3\cos\left(dx+c\right)^2-3\,a^2b+2\,b^3\right)\sin\left(dx+c\right)}{6\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a*b^2*cos(d*x + c)^2 - 6*(a^3 - a*b^2)*log(b*sin(d*x + c) + a) - 2*
(b^3*cos(d*x + c)^2 - 3*a^2*b + 2*b^3)*sin(d*x + c))/(b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17481, size = 115, normalized size = 1.29

$$-\frac{\frac{2\,b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+6\,a^2\sin(dx+c)-6\,b^2\sin(dx+c)}{b^3}-\frac{6\left(a^3-ab^2\right)\log(|b\sin(dx+c)+a|)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*((2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^2 + 6*a^2*sin(d*x + c) - 6
*b^2*sin(d*x + c))/b^3 - 6*(a^3 - a*b^2)*log(abs(b*sin(d*x + c) + a))/b^4)/
d
```

3.1298 $\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{bd}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a*b^2*d) -
 Sin[c + d*x]/(b*d)
```

Rubi [A] time = 0.117325, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a*b^2*d) -
 Sin[c + d*x]/(b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b(b^2-x^2)}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\log(\sin(c+dx))}{ad} + \frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d} - \frac{\sin(c+dx)}{bd}$$

Mathematica [A] time = 0.0688272, size = 53, normalized size = 0.9

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx)) - ab\sin(c+dx) + b^2\log(\sin(c+dx))}{ab^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^2*Log[Sin[c + d*x]] + (a^2 - b^2)*Log[a + b*Sin[c + d*x]] - a*b*Sin[c +
d*x])/(a*b^2*d)
```

Maple [A] time = 0.082, size = 68, normalized size = 1.2

$$-\frac{\sin(dx+c)}{bd} + \frac{a\ln(a+b\sin(dx+c))}{db^2} - \frac{\ln(a+b\sin(dx+c))}{da} + \frac{\ln(\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -sin(d*x+c)/b/d+1/d/b^2*a*ln(a+b*sin(d*x+c))-1/d/a*ln(a+b*sin(d*x+c))+ln(si
n(d*x+c))/a/d
```

Maxima [A] time = 0.992611, size = 73, normalized size = 1.24

$$\frac{\frac{\log(\sin(dx+c))}{a} - \frac{\sin(dx+c)}{b} + \frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{ab^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c))/a - sin(d*x + c)/b + (a^2 - b^2)*log(b*sin(d*x + c) + a)
/(a*b^2))/d
```

Fricas [A] time = 1.54545, size = 131, normalized size = 2.22

$$\frac{b^2\log\left(-\frac{1}{2}\sin\left(dx+c\right)\right)-ab\sin\left(dx+c\right)+\left(a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)}{ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b^2*log(-1/2*sin(d*x + c)) - a*b*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(a*b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22164, size = 76, normalized size = 1.29

$$\frac{\frac{\log\left(|\sin\left(dx+c\right)|\right)}{a}-\frac{\sin\left(dx+c\right)}{b}+\frac{\left(a^2-b^2\right)\log\left(|b\sin\left(dx+c\right)+a|\right)}{ab^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(sin(d*x + c)))/a - sin(d*x + c)/b + (a^2 - b^2)*log(abs(b*sin(d*x
+ c) + a))/(a*b^2))/d
```

3.1299 $\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) - ((1 - b^2/a^2)*Log[
a + b*Sin[c + d*x]])/(b*d)
```

Rubi [A] time = 0.12238, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) - ((1 - b^2/a^2)*Log[
a + b*Sin[c + d*x]])/(b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^2} - \frac{b^2}{a^2x} + \frac{-a^2+b^2}{a^2(a+x)}\right) dx, x, b\sin(c+dx)\right)}{bd}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{b\log(\sin(c+dx))}{a^2d} - \frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}$$

Mathematica [A] time = 0.0779329, size = 54, normalized size = 0.9

$$\frac{(b^2-a^2)\log(a+b\sin(c+dx)) - ab\csc(c+dx) + b^2(-\log(\sin(c+dx)))}{a^2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(a*b*Csc[c + d*x]) - b^2*Log[Sin[c + d*x]] + (-a^2 + b^2)*Log[a + b*Sin[c
 + d*x]])/(a^2*b*d)
```

Maple [A] time = 0.079, size = 72, normalized size = 1.2

$$-\frac{\ln(a+b\sin(dx+c))}{bd} + \frac{b\ln(a+b\sin(dx+c))}{da^2} - \frac{1}{da\sin(dx+c)} - \frac{b\ln(\sin(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -ln(a+b*sin(d*x+c))/b/d+1/d/a^2*b*ln(a+b*sin(d*x+c))-1/d/a/sin(d*x+c)-b*ln(
sin(d*x+c))/a^2/d
```

Maxima [A] time = 0.982208, size = 77, normalized size = 1.28

$$-\frac{\frac{b\log(\sin(dx+c))}{a^2} + \frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{a^2b} + \frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*log(sin(d*x + c))/a^2 + (a^2 - b^2)*log(b*sin(d*x + c) + a)/(a^2*b) + 1
/(a*sin(d*x + c)))/d
```

Fricas [A] time = 1.66308, size = 166, normalized size = 2.77

$$-\frac{b^2 \log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c) + \left(a^2-b^2\right)\log(b\sin(dx+c)+a)\sin(dx+c) + ab}{a^2bd\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b^2*log(1/2*sin(d*x + c))*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x + c) +
 a)*sin(d*x + c) + a*b)/(a^2*b*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19751, size = 80, normalized size = 1.33

$$-\frac{\frac{b\log(|\sin(dx+c)|)}{a^2} + \frac{\left(a^2-b^2\right)\log(|b\sin(dx+c)+a|)}{a^2b} + \frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(sin(d*x + c)))/a^2 + (a^2 - b^2)*log(abs(b*sin(d*x + c) + a))/(
a^2*b) + 1/(a*sin(d*x + c)))/d
```

3.1300 $\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c
+ d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.0918886, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c
+ d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx,x,b\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^3}-\frac{b^2}{a^2x^2}+\frac{-a^2+b^2}{a^3x}+\frac{a^2-b^2}{a^3(a+x)}\right)dx,x,b\sin(c+dx)\right)}{d}\\ &= \frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}\end{aligned}$$

Mathematica [A] time = 0.155569, size = 65, normalized size = 0.77

$$-\frac{2(a^2-b^2)(\log(\sin(c+dx))-\log(a+b\sin(c+dx)))+a^2\csc^2(c+dx)-2ab\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-2*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*(Log[Sin[c + d*x
]] - Log[a + b*Sin[c + d*x]]))/(2*a^3*d)
```

Maple [A] time = 0.088, size = 106, normalized size = 1.3

$$\frac{\ln(a+b\sin(dx+c))}{da}-\frac{b^2\ln(a+b\sin(dx+c))}{a^3d}-\frac{1}{2da(\sin(dx+c))^2}-\frac{\ln(\sin(dx+c))}{da}+\frac{b^2\ln(\sin(dx+c))}{a^3d}+\frac{}{da^2\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/a*ln(a+b*sin(d*x+c))-b^2*ln(a+b*sin(d*x+c))/a^3/d-1/2/d/a/sin(d*x+c)^2-
ln(sin(d*x+c))/a/d+b^2*ln(sin(d*x+c))/a^3/d+1/d/a^2*b/sin(d*x+c)
```

Maxima [A] time = 0.982888, size = 104, normalized size = 1.24

$$\frac{\frac{2\left(a^2-b^2\right)\log(b\sin(dx+c)+a)}{a^3}-\frac{2\left(a^2-b^2\right)\log(\sin(dx+c))}{a^3}+\frac{2b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^2 - b^2)*log(b*sin(d*x + c) + a)/a^3 - 2*(a^2 - b^2)*log(sin(d*x
+ c))/a^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.64487, size = 271, normalized size = 3.23

$$-\frac{2ab\sin(dx+c)-a^2-2\left(\left(a^2-b^2\right)\cos(dx+c)^2-a^2+b^2\right)\log(b\sin(dx+c)+a)+2\left(\left(a^2-b^2\right)\cos(dx+c)^2-a^2+b^2\right.}{2\left(a^3d\cos(dx+c)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) - a^2 - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)
*log(b*sin(d*x + c) + a) + 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(-
1/2*sin(d*x + c)))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20627, size = 119, normalized size = 1.42

$$-\frac{\frac{2\left(a^2-b^2\right)\log(|\sin(dx+c)|)}{a^3}-\frac{2\left(a^2b-b^3\right)\log(|b\sin(dx+c)+a|)}{a^3b}-\frac{2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^2*b - b^3)*log(abs(b*
sin(d*x + c) + a))/(a^3*b) - (2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2
))/d
```

3.1301 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=282

$$\frac{a\left(-20a^2b^2+15a^4+3b^4\right)\cos(c+dx)}{15b^6d}-\frac{2a^3\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}-\frac{\left(6a^2-7b^2\right)\sin^3(c+dx)\cos(c+dx)}{24b^3d}+$$

```
[Out] ((16*a^6 - 24*a^4*b^2 + 6*a^2*b^4 + b^6)*x)/(16*b^7) - (2*a^3*(a^2 - b^2)^(
3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) + (a*(15*a^4
 - 20*a^2*b^2 + 3*b^4)*Cos[c + d*x])/(15*b^6*d) - ((8*a^4 - 10*a^2*b^2 + b^
4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^5*d) + (a*(5*a^2 - 6*b^2)*Cos[c + d*x]*
Sin[c + d*x]^2)/(15*b^4*d) - ((6*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/
(24*b^3*d) + (a*Cos[c + d*x]*Sin[c + d*x]^4)/(5*b^2*d) - (Cos[c + d*x]*Sin[
c + d*x]^5)/(6*b*d)
```

Rubi [A] time = 1.00121, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2895, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{a\left(-20a^2b^2+15a^4+3b^4\right)\cos(c+dx)}{15b^6d}-\frac{2a^3\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}-\frac{\left(6a^2-7b^2\right)\sin^3(c+dx)\cos(c+dx)}{24b^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```


```
[Out] ((16*a^6 - 24*a^4*b^2 + 6*a^2*b^4 + b^6)*x)/(16*b^7) - (2*a^3*(a^2 - b^2)^(
3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) + (a*(15*a^4
 - 20*a^2*b^2 + 3*b^4)*Cos[c + d*x])/(15*b^6*d) - ((8*a^4 - 10*a^2*b^2 + b^
4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^5*d) + (a*(5*a^2 - 6*b^2)*Cos[c + d*x]*
Sin[c + d*x]^2)/(15*b^4*d) - ((6*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/
(24*b^3*d) + (a*Cos[c + d*x]*Sin[c + d*x]^4)/(5*b^2*d) - (Cos[c + d*x]*Sin[
c + d*x]^5)/(6*b*d)
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\cos(c+dx)\sin^5(c+dx)}{6bd} - \frac{\int \frac{\sin^3(c+dx)\left(6\left(4a^2-5b^2\right)-ab\sin(c+dx\right.}{a+b\sin(c+d}}{30b^2}$$

$$= -\frac{\left(6a^2-7b^2\right)\cos(c+dx)\sin^3(c+dx)}{24b^3d} + \frac{a\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\cos(c+dx)\sin^5(}{6bd}$$

$$= \frac{a\left(5a^2-6b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^4d} - \frac{\left(6a^2-7b^2\right)\cos(c+dx)\sin^3(c+dx)}{24b^3d} + \frac{a\cos(}{}$$

$$= -\frac{\left(8a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c+dx)}{16b^5d} + \frac{a\left(5a^2-6b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^4d}$$

$$= \frac{a\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^6d} - \frac{\left(8a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c+dx)}{16b^5d} + -$$

$$= \frac{\left(16a^6-24a^4b^2+6a^2b^4+b^6\right)x}{16b^7} + \frac{a\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^6d} - \frac{\left(8a^4-10a^2b\right.}{}$$

$$= \frac{\left(16a^6-24a^4b^2+6a^2b^4+b^6\right)x}{16b^7} + \frac{a\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^6d} - \frac{\left(8a^4-10a^2b\right.}{}$$

$$= \frac{\left(16a^6-24a^4b^2+6a^2b^4+b^6\right)x}{16b^7} + \frac{a\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^6d} - \frac{\left(8a^4-10a^2b\right.}{}$$

$$= \frac{\left(16a^6-24a^4b^2+6a^2b^4+b^6\right)x}{16b^7} - \frac{2a^3\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d} + \frac{a\left(15a^4-\right.}{}$$

Mathematica [A] time = 2.28009, size = 274, normalized size = 0.97

$$\frac{-240a^4b^2\sin(2(c+dx))+240a^2b^4\sin(2(c+dx))+30a^2b^4\sin(4(c+dx))+120ab\left(-10a^2b^2+8a^4+b^4\right)\cos(c+dx)+\left(6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (960*a^6*c - 1440*a^4*b^2*c + 360*a^2*b^4*c + 60*b^6*c + 960*a^6*d*x - 1440
*a^4*b^2*d*x + 360*a^2*b^4*d*x + 60*b^6*d*x - 1920*a^3*(a^2 - b^2)^(3/2)*Ar
cTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 120*a*b*(8*a^4 - 10*a^2*b^
2 + b^4)*Cos[c + d*x] + (-80*a^3*b^3 + 60*a*b^5)*Cos[3*(c + d*x)] + 12*a*b^
5*Cos[5*(c + d*x)] - 240*a^4*b^2*Sin[2*(c + d*x)] + 240*a^2*b^4*Sin[2*(c +
d*x)] + 15*b^6*Sin[2*(c + d*x)] + 30*a^2*b^4*Sin[4*(c + d*x)] - 15*b^6*Sin[
4*(c + d*x)] - 5*b^6*Sin[6*(c + d*x)])/(960*b^7*d)
```

Maple [B] time = 0.103, size = 1501, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/8/d/b*arctan(tan(1/2*d*x+1/2*c))-2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1
/2*d*x+1/2*c)^5*a^4+20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
4*a^5+5/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*a^2+2/d/b^6/(
1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^5+2/5/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a-1/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)*a^4+10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
2*a^5-13/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+47/24/d/b/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3-1/8/d/b/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)+2/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^6+2/5/d/b^2/(
1+tan(1/2*d*x+1/2*c)^2)^6*a-8/3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*a^3+2/d/b^
6/(1+tan(1/2*d*x+1/2*c)^2)^6*a^5+1/8/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2
*d*x+1/2*c)^11-24/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^4*a^3
-3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*a^4-3/d/b^5*arctan
(tan(1/2*d*x+1/2*c))*a^4+3/4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2-80/3/d/b^
4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^3+4/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a+1/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^5*a^2-47/24/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/
2*c)^9+13/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+10/d/b^6/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a^5+7/4/d/b^3/(1+tan(1/2*d*x+
1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*a^2-2/d*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-12/d/b^4/(1+tan(1/2*d*x+1/2*
c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^3+20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
2*d*x+1/2*c)^6*a^5+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*
a+3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^4-7/4/d/b^3/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^2+1/d/b^5/(1+tan(1/2*d*x+1/2
*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^4-5/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)^11*a^2+4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^4*a-16/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a^3-4/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^3+2/d/b^2/(1+tan(1/2*d*x
+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)^7*a^4-1/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*
c)^7*a^2+4/d*a^5/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b
)/(a^2-b^2)^(1/2))-2/d*a^7/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+
1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74563, size = 1204, normalized size = 4.27

$$\left[\frac{48\,ab^5\cos\left(dx+c\right)^5 - 80\,a^3b^3\cos\left(dx+c\right)^3 + 15\left(16\,a^6 - 24\,a^4b^2 + 6\,a^2b^4 + b^6\right)dx - 120\left(a^5 - a^3b^2\right)\sqrt{-a^2+b^2}\log\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/240*(48*a*b^5*cos(d*x + c)^5 - 80*a^3*b^3*cos(d*x + c)^3 + 15*(16*a^6 -
24*a^4*b^2 + 6*a^2*b^4 + b^6)*d*x - 120*(a^5 - a^3*b^2)*sqrt(-a^2 + b^2)*lo
g(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 240*(a^5*b - a^3*b^3)*cos(d*x + c
) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(8*a^4
*b^2 - 6*a^2*b^4 - b^6)*cos(d*x + c))*sin(d*x + c))/(b^7*d), 1/240*(48*a*b^
5*cos(d*x + c)^5 - 80*a^3*b^3*cos(d*x + c)^3 + 15*(16*a^6 - 24*a^4*b^2 + 6*
a^2*b^4 + b^6)*d*x + 240*(a^5 - a^3*b^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 240*(a^5*b - a^3*b^3)*cos(d*x
+ c) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(8*
a^4*b^2 - 6*a^2*b^4 - b^6)*cos(d*x + c))*sin(d*x + c))/(b^7*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18101, size = 980, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(15*(16*a^6 - 24*a^4*b^2 + 6*a^2*b^4 + b^6)*(d*x + c)/b^7 - 480*(a^7
- 2*a^5*b^2 + a^3*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 2*(120
*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 150*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 15*b
^5*tan(1/2*d*x + 1/2*c)^11 + 240*a^5*tan(1/2*d*x + 1/2*c)^10 - 480*a^3*b^2*
tan(1/2*d*x + 1/2*c)^10 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^10 + 360*a^4*b*tan
(1/2*d*x + 1/2*c)^9 - 210*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 - 235*b^5*tan(1/2*
d*x + 1/2*c)^9 + 1200*a^5*tan(1/2*d*x + 1/2*c)^8 - 1920*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^8 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^8 + 240*a^4*b*tan(1/2*d*x + 1/
2*c)^7 - 60*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 390*b^5*tan(1/2*d*x + 1/2*c)^7
 + 2400*a^5*tan(1/2*d*x + 1/2*c)^6 - 3200*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 +
480*a*b^4*tan(1/2*d*x + 1/2*c)^6 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 60*a^
2*b^3*tan(1/2*d*x + 1/2*c)^5 - 390*b^5*tan(1/2*d*x + 1/2*c)^5 + 2400*a^5*ta
n(1/2*d*x + 1/2*c)^4 - 2880*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 480*a*b^4*tan(
1/2*d*x + 1/2*c)^4 - 360*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 210*a^2*b^3*tan(1/2
*d*x + 1/2*c)^3 + 235*b^5*tan(1/2*d*x + 1/2*c)^3 + 1200*a^5*tan(1/2*d*x + 1
/2*c)^2 - 1440*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 48*a*b^4*tan(1/2*d*x + 1/2*
c)^2 - 120*a^4*b*tan(1/2*d*x + 1/2*c) + 150*a^2*b^3*tan(1/2*d*x + 1/2*c) -
15*b^5*tan(1/2*d*x + 1/2*c) + 240*a^5 - 320*a^3*b^2 + 48*a*b^4)/((tan(1/2*d
*x + 1/2*c)^2 + 1)^6*b^6))/d
```

3.1302 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=235

$$-\frac{\left(-20a^2b^2+15a^4+3b^4\right)\cos(c+dx)}{15b^5d}+\frac{2a^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{\left(5a^2-6b^2\right)\sin^2(c+dx)\cos(c+dx}{15b^3d}$$

```
[Out] -(a*(8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^6) + (2*a^2*(a^2 - b^2)^(3/2)*ArcT
an[(b + a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(b^6*d) - ((15*a^4 - 20*a^2*b
^2 + 3*b^4)*Cos[c + d*x])/(15*b^5*d) + (a*(4*a^2 - 5*b^2)*Cos[c + d*x]*Sin[
c + d*x])/(8*b^4*d) - ((5*a^2 - 6*b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*b^3
*d) + (a*Cos[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) - (Cos[c + d*x]*Sin[c + d*x
]^4)/(5*b*d)
```

Rubi [A] time = 0.716639, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2895, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{\left(-20a^2b^2+15a^4+3b^4\right)\cos(c+dx)}{15b^5d}+\frac{2a^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{\left(5a^2-6b^2\right)\sin^2(c+dx)\cos(c+dx}{15b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(a*(8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^6) + (2*a^2*(a^2 - b^2)^(3/2)*ArcT
an[(b + a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(b^6*d) - ((15*a^4 - 20*a^2*b
^2 + 3*b^4)*Cos[c + d*x])/(15*b^5*d) + (a*(4*a^2 - 5*b^2)*Cos[c + d*x]*Sin[
c + d*x])/(8*b^4*d) - ((5*a^2 - 6*b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*b^3
*d) + (a*Cos[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) - (Cos[c + d*x]*Sin[c + d*x
]^4)/(5*b*d)
```


Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
```

```
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{\cos(c+dx)\sin^4(c+dx)}{5bd} - \frac{\int \frac{\sin^2(c+dx)\left(5\left(3a^2-4b^2\right)-ab\sin(c\ldots}{a+b\sin(\ldots}}{20\ldots}$$

$$= -\frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^3d} + \frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{\cos(c+dx)\,\mathrm{si}\ldots}{5b\ldots}$$

$$= \frac{a\left(4a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin^2(c+dx)}{15b^3d} + \frac{a\,\mathrm{co}\ldots}{\ldots}$$

$$= -\frac{\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^5d} + \frac{a\left(4a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{8b^4d} - \frac{\left(5a^2\ldots\right.}{\ldots}$$

$$= -\frac{a\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} - \frac{\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^5d} + \frac{a\left(4a^2-5b^2\right)\cos\ldots}{8\ldots}$$

$$= -\frac{a\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} - \frac{\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^5d} + \frac{a\left(4a^2-5b^2\right)\cos\ldots}{8\ldots}$$

$$= -\frac{a\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} - \frac{\left(15a^4-20a^2b^2+3b^4\right)\cos(c+dx)}{15b^5d} + \frac{a\left(4a^2-5b^2\right)\cos\ldots}{8\ldots}$$

$$= -\frac{a\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{2a^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} - \frac{\left(15a^4-20a\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.98549, size = 186, normalized size = 0.79

$$\frac{-15a\left(4\left(-12a^2b^2+8a^4+3b^4\right)(c+dx)+\left(8b^4-8a^2b^2\right)\sin(2(c+dx))+b^4\sin(4(c+dx))\right)-60b\left(-10a^2b^2+8a^4+b^4\ldots\right.}{480b^6\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (960*a^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
 - 60*b*(8*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x] + 10*(4*a^2*b^3 - 3*b^5)*Co
s[3*(c + d*x)] - 6*b^5*Cos[5*(c + d*x)] - 15*a*(4*(8*a^4 - 12*a^2*b^2 + 3*b
^4)*(c + d*x) + (-8*a^2*b^2 + 8*b^4)*Sin[2*(c + d*x)] + b^4*Sin[4*(c + d*x)
]))/(480*b^6*d)
```

Maple [B] time = 0.096, size = 941, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3+5/4/d/b^2/(1+t
an(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9*a-3/4/d/b^2*a*arctan(tan(1/2*d*
x+1/2*c))-8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^4-2/d/b
/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8-2/d/b^6*arctan(tan(1/2*d*x
+1/2*c))*a^5-4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4-2/d/b^5/
```

```
(1+tan(1/2*d*x+1/2*c)^2)^5*a^4+8/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*a^2+2/d
/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2
))*a^2+3/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3+12/d/b^3/(1+tan(1/2*d*x+1/2*c
)^2)^5*tan(1/2*d*x+1/2*c)^6*a^2+28/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)^2*a^2-2/5/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5-2/d/b^5/(1+tan(1/2*d
*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8*a^4+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^8*a^2-8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2
*c)^6*a^4+2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a^3-1/2/d
/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a+2/d*a^6/b^6/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d*a^4/b^
4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-
5/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a+44/3/d/b^3/(1+tan
(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^2-1/d/b^4/(1+tan(1/2*d*x+1/2*c)
^2)^5*tan(1/2*d*x+1/2*c)^9*a^3-12/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*
d*x+1/2*c)^4*a^4+1/2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*
a+1/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90645, size = 1056, normalized size = 4.49

$$\left[-\frac{24\,b^{5}\cos\left(dx+c\right)^{5}-40\,a^{2}b^{3}\cos\left(dx+c\right)^{3}+15\left(8\,a^{5}-12\,a^{3}b^{2}+3\,ab^{4}\right)dx+60\left(a^{4}-a^{2}b^{2}\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{\left(2\,a^{2}-b^{2}\right)\cos(d\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/120*(24*b^5*cos(d*x + c)^5 - 40*a^2*b^3*cos(d*x + c)^3 + 15*(8*a^5 - 12
*a^3*b^2 + 3*a*b^4)*d*x + 60*(a^4 - a^2*b^2)*sqrt(-a^2 + b^2)*log(((2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*s
in(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2)) + 120*(a^4*b - a^2*b^3)*cos(d*x + c) + 15*(2*a*
b^4*cos(d*x + c)^3 - (4*a^3*b^2 - 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6
*d), -1/120*(24*b^5*cos(d*x + c)^5 - 40*a^2*b^3*cos(d*x + c)^3 + 15*(8*a^5
- 12*a^3*b^2 + 3*a*b^4)*d*x + 120*(a^4 - a^2*b^2)*sqrt(a^2 - b^2)*arctan(-(
a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 120*(a^4*b - a^2*b^3)
*cos(d*x + c) + 15*(2*a*b^4*cos(d*x + c)^3 - (4*a^3*b^2 - 3*a*b^4)*cos(d*x
+ c))*sin(d*x + c))/(b^6*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18468, size = 618, normalized size = 2.63

$$-\frac{\frac{15\left(8a^5-12a^3b^2+3ab^4\right)(dx+c)}{b^6}-\frac{240\left(a^6-2a^4b^2+a^2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{2\left(60a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-75ab^3\tan\left(\frac{1}{2}dx-\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(15*(8*a^5 - 12*a^3*b^2 + 3*a*b^4)*(d*x + c)/b^6 - 240*(a^6 - 2*a^4*
b^2 + a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2
*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(60*a^3*b*ta
n(1/2*d*x + 1/2*c)^9 - 75*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*
x + 1/2*c)^8 - 240*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 120*b^4*tan(1/2*d*x + 1
/2*c)^8 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 30*a*b^3*tan(1/2*d*x + 1/2*c)^
7 + 480*a^4*tan(1/2*d*x + 1/2*c)^6 - 720*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 7
20*a^4*tan(1/2*d*x + 1/2*c)^4 - 880*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*b^
4*tan(1/2*d*x + 1/2*c)^4 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 30*a*b^3*tan(
1/2*d*x + 1/2*c)^3 + 480*a^4*tan(1/2*d*x + 1/2*c)^2 - 560*a^2*b^2*tan(1/2*d
*x + 1/2*c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) + 75*a*b^3*tan(1/2*d*x + 1/2*
c) + 120*a^4 - 160*a^2*b^2 + 24*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*b^5))/
d
```

3.1303 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{2a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}+\frac{\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right)\sin(c+dx)\right)}{8b^4d}+\frac{x\left(-12a^2b^2+8a^4+3b\right.}{8b^5}$$

```
[Out] ((8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^5) - (2*a*(a^2 - b^2)^(3/2)*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (Cos[c + d*x]^3*(4*a - 3
*b*Sin[c + d*x]))/(12*b^2*d) + (Cos[c + d*x]*(8*a*(a^2 - b^2) - b*(4*a^2 -
3*b^2)*Sin[c + d*x]))/(8*b^4*d)
```

Rubi [A] time = 0.324701, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2865, 2735, 2660, 618, 204}

$$-\frac{2a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}+\frac{\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right)\sin(c+dx)\right)}{8b^4d}+\frac{x\left(-12a^2b^2+8a^4+3b\right.}{8b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^5) - (2*a*(a^2 - b^2)^(3/2)*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (Cos[c + d*x]^3*(4*a - 3
*b*Sin[c + d*x]))/(12*b^2*d) + (Cos[c + d*x]*(8*a*(a^2 - b^2) - b*(4*a^2 -
3*b^2)*Sin[c + d*x]))/(8*b^4*d)
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\int \frac{\cos^2(c+dx)\left(-ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{4b^2}$$

$$= -\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right)\sin(c\right.}{8b^4d}$$

$$= \frac{\left(8a^4-12a^2b^2+3b^4\right)x}{8b^5} - \frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\cos(c+dx)\left(8a\left(a^2-\right.\right.}{}$$

$$= \frac{\left(8a^4-12a^2b^2+3b^4\right)x}{8b^5} - \frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\cos(c+dx)\left(8a\left(a^2-\right.\right.}{}$$

$$= \frac{\left(8a^4-12a^2b^2+3b^4\right)x}{8b^5} - \frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\cos(c+dx)\left(8a\left(a^2-\right.\right.}{}$$

$$= \frac{\left(8a^4-12a^2b^2+3b^4\right)x}{8b^5} - \frac{2a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d} - \frac{\cos^3(c+dx)(4a-}{12b}$$

Mathematica [A] time = 1.0446, size = 155, normalized size = 0.97

$$\frac{3\left(4\left(-12a^2b^2+8a^4+3b^4\right)(c+dx)+\left(8b^4-8a^2b^2\right)\sin(2(c+dx))+b^4\sin(4(c+dx))\right)+24ab\left(4a^2-5b^2\right)\cos(c+dx}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-192*a*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
+ 24*a*b*(4*a^2 - 5*b^2)*Cos[c + d*x] - 8*a*b^3*Cos[3*(c + d*x)] + 3*(4*(8*
a^4 - 12*a^2*b^2 + 3*b^4)*(c + d*x) + (-8*a^2*b^2 + 8*b^4)*Sin[2*(c + d*x)]
 + b^4*Sin[4*(c + d*x)]))/(96*b^5*d)
```

Maple [B] time = 0.087, size = 760, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2-5/4/d/b/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4
*tan(1/2*d*x+1/2*c)^6*a^3-4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^6*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2+3/4/d/
b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5+6/d/b^4/(1+tan(1/2*d*x+1/
2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3-8/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)^4*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a
^2-3/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+6/d/b^4/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3-20/3/d/b^2/(1+tan(1/2*d*x+1/2*c
)^2)^4*tan(1/2*d*x+1/2*c)^2*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*
x+1/2*c)*a^2+5/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+2/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^4*a^3-8/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*a+2/d/b
^5*arctan(tan(1/2*d*x+1/2*c))*a^4-3/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+3/
4/d/b*arctan(tan(1/2*d*x+1/2*c))-2/d*a^5/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d*a^3/b^3/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/b*a/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64871, size = 945, normalized size = 5.94

$$\left[-\frac{8\,ab^3\cos\left(dx+c\right)^3 - 3\left(8\,a^4 - 12\,a^2b^2 + 3\,b^4\right)dx + 12\left(a^3 - ab^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2 - 2\,ab\sin(dx+c) - a^2 - b^2 - 2\,(a}{b^2\cos(dx+c)^2 - 2\,ab\sin}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/24*(8*a*b^3*cos(d*x + c)^3 - 3*(8*a^4 - 12*a^2*b^2 + 3*b^4)*d*x + 12*(a
^3 - a*b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sq
rt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 24
*(a^3*b - a*b^3)*cos(d*x + c) - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2 - 3*b^
4)*cos(d*x + c))*sin(d*x + c))/(b^5*d), -1/24*(8*a*b^3*cos(d*x + c)^3 - 3*(
8*a^4 - 12*a^2*b^2 + 3*b^4)*d*x - 24*(a^3 - a*b^2)*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 24*(a^3*b - a*b^3)*c
os(d*x + c) - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2 - 3*b^4)*cos(d*x + c))*s
in(d*x + c))/(b^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20264, size = 501, normalized size = 3.15

$$\frac{\frac{3\,(8\,a^4-12\,a^2b^2+3\,b^4)(dx+c)}{b^5}-\frac{48\,(a^5-2\,a^3b^2+ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{2\left(12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^4 - 12*a^2*b^2 + 3*b^4)*(d*x + c)/b^5 - 48*(a^5 - 2*a^3*b^2 +
a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) + 2*(12*a^2*b*tan(1/2*d*
x + 1/2*c)^7 - 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^3*tan(1/2*d*x + 1/2*c)^
6 - 48*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*b
^3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 96*a*b^2*tan(1/
2*d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 9*b^3*tan(1/2*d*x + 1/
2*c)^3 + 72*a^3*tan(1/2*d*x + 1/2*c)^2 - 80*a*b^2*tan(1/2*d*x + 1/2*c)^2 -
12*a^2*b*tan(1/2*d*x + 1/2*c) + 15*b^3*tan(1/2*d*x + 1/2*c) + 24*a^3 - 32*a
*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^4))/d
```

3.1304 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{x\left(2a^2-3b^2\right)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] ((2*a^2 - 3*b^2)*x)/(2*b^3) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (a*Cos[
c + d*x])/(b^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.288046, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2895, 3057, 2660, 618, 204, 3770}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{x\left(2a^2-3b^2\right)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 - 3*b^2)*x)/(2*b^3) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (a*Cos[
c + d*x])/(b^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 2895

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a*(n + 3)*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d*f*(m + n
+ 3)*(m + n + 4)), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(b*d^2*f*(m + n + 4)), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\int \frac{\csc(c+dx)\left(-2b^2-ab\sin(c+dx)-\left(2a^2-3b^2\right)\sin^2(c+dx)\right.}{a+b\sin(c+dx)}}{2b^2}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \csc(c+dx)\,dx}{a} - \frac{\left(a^2 - \right.}{}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\left(\right.}{}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\left(\right.}{}$$

$$= \frac{\left(2a^2-3b^2\right)x}{2b^3} - \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\,\mathrm{co}}{}$$

Mathematica [A] time = 0.262622, size = 143, normalized size = 1.15

$$-\frac{8\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - 4a^2b\cos(c+dx) - 4a^3c - 4a^3dx + ab^2\sin(2(c+dx)) + 6ab^2c + 6ab^2dx - 4b^3}{4ab^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-4*a^3*c + 6*a*b^2*c - 4*a^3*d*x + 6*a*b^2*d*x + 8*(a^2 - b^2)^(3/2)*ArcT
an[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 4*a^2*b*Cos[c + d*x] + 4*b^3
*Log[Cos[(c + d*x)/2]] - 4*b^3*Log[Sin[(c + d*x)/2]] + a*b^2*Sin[2*(c + d*x
)])/(4*a*b^3*d)
```

Maple [B] time = 0.109, size = 334, normalized size = 2.7

$$\frac{1}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}+2\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a}{db^{2}\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{2}}-\frac{1}{bd}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a-1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1
/2*d*x+1/2*c)+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*a+2/d/b^3*arctan(tan(1/2*d
*x+1/2*c))*a^2-3/d/b*arctan(tan(1/2*d*x+1/2*c))-2/d*a^3/b^3/(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/b*a/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a*b/(a^
2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a
*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28108, size = 853, normalized size = 6.88

$$\left[-\frac{ab^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^{2}b\cos\left(dx+c\right)+b^{3}\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-b^{3}\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(2\,a^{3}-\right.}{2\,ab^{3}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log(1/2
*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*b^2)
*d*x - (-a^2 + b^2)^(3/2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*
x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(
-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b^3
*d), -1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log
(1/2*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*
b^2)*d*x - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))))/(a*b^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{4}\left(c+dx\right)\csc\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.20589, size = 247, normalized size = 1.99

$$\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{(2a^2-3b^2)(dx+c)}{b^3}-\frac{4(a^4-2a^2b^2+b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}ab^3}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}\cdots\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a + (2*a^2 - 3*b^2)*(d*x + c)/b^3 - 4
*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b^3) + 2*
(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/
2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

3.1305 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{ax}{b^2}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

```
[Out] -((a*x)/b^2) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^2*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cos[c + d*x]/(
b*d) - Cot[c + d*x]/(a*d)
```

Rubi [A] time = 0.269578, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2894, 3057, 2660, 618, 204, 3770}

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{ax}{b^2}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^2*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cos[c + d*x]/(
b*d) - Cot[c + d*x]/(a*d)
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{\int \frac{\csc(c+dx)\left(b^2+2ab\sin(c+dx)+a^2\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{ab}$$

$$= -\frac{ax}{b^2} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{\left(a^2-b^2\right)^2 \int \frac{1}{a+b\sin(c+dx)}\,d}{a^2b^2}$$

$$= -\frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} + \frac{\left(2\left(a^2-b^2\right)^2\right)\text{Subst}}{}$$

$$= -\frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{\left(4\left(a^2-b^2\right)^2\right)\text{Subst}}{}$$

$$= -\frac{ax}{b^2} + \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx}{bd}$$

Mathematica [A] time = 0.757968, size = 146, normalized size = 1.4

$$-\frac{-4\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 2a^2b\cos(c+dx) + 2a^3c + 2a^3dx - ab^2\tan\left(\frac{1}{2}(c+dx)\right) + ab^2\cot\left(\frac{1}{2}(c+dx\right)}{2a^2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(2*a^3*c + 2*a^3*d*x - 4*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]] + 2*a^2*b*Cos[c + d*x] + a*b^2*Cot[(c + d*x)/2] - 2*b^3*L
og[Cos[(c + d*x)/2]] + 2*b^3*Log[Sin[(c + d*x)/2]] - a*b^2*Tan[(c + d*x)/2]
)/(2*a^2*b^2*d)
```

Maple [B] time = 0.112, size = 249, normalized size = 2.4

$$\frac{1}{2\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{1}{bd\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{a\arctan(\tan(1/2\,dx+c/2))}{db^2}+2\,\frac{a^2}{db^2\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\,\mathrm{t}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-2/d/b/(1+tan(1/2*d*x+1/2*c)^2)-2/d/b^2*a*arctan(
tan(1/2*d*x+1/2*c))+2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2))*a^2-4/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a^
2*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.52279, size = 981, normalized size = 9.43

$$\left[\frac{b^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-b^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,ab^2\cos(dx+c)-\left(a^2-b^2\right)\sqrt{-a^2}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 2*a*b^2*cos(d*x + c) - (a^2 - b^2)*sqrt(-a^2 + b^
2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 2*(a^3*d*x + a^2
*b*cos(d*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c)), 1/2*(b^3*log(1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 2*a*b^2*cos(d*x + c) - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) +
 b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 2*(a^3*d*x + a^2*b*cos(d
*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19248, size = 298, normalized size = 2.87

$$-\frac{\frac{6\,(dx+c)a}{b^2}+\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{12\left(a^4-2\,a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^2b^2}-\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a/b^2 + 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/
2*d*x + 1/2*c)/a - 12*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a
^2 - b^2)*a^2*b^2) - (2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/
2*c)^2 - 12*a^2*tan(1/2*d*x + 1/2*c) + 2*b^2*tan(1/2*d*x + 1/2*c) - 3*a*b)/
((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^2*b))/d
```

3.1306 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3bd}+\frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{x}{b}$$

```
[Out] x/b - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/(a^3*b*d) + ((3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (b*Cot[c
+ d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.301393, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2893, 3057, 2660, 618, 204, 3770}

$$-\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3bd}+\frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] x/b - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/(a^3*b*d) + ((3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (b*Cot[c
+ d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\int \frac{\csc(c+dx)\left(3a^2-2b^2-ab\sin(c+dx)-2a^2\sin^2(c+dx)\right)}{a+b\sin(c+dx)}}{2a^2}$$

$$= \frac{x}{b} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\left(3a^2-2b^2\right)\int\csc(c+dx)\,dx}{2a^3} - \frac{\left(a^2-\right.}{}$$

$$= \frac{x}{b} + \frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - $$

$$= \frac{x}{b} + \frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + $$

$$= \frac{x}{b} - \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3bd} + \frac{\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\,\mathrm{co}}{}$$

Mathematica [A] time = 1.53311, size = 204, normalized size = 1.66

$$-16\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - a^2b\csc^2\left(\frac{1}{2}(c+dx)\right) + a^2b\sec^2\left(\frac{1}{2}(c+dx)\right) - 12a^2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + $$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (8*a^3*c + 8*a^3*d*x - 16*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]] + 4*a*b^2*Cot[(c + d*x)/2] - a^2*b*Csc[(c + d*x)/2]^2 + 1
2*a^2*b*Log[Cos[(c + d*x)/2]] - 8*b^3*Log[Cos[(c + d*x)/2]] - 12*a^2*b*Log[
Sin[(c + d*x)/2]] + 8*b^3*Log[Sin[(c + d*x)/2]] + a^2*b*Sec[(c + d*x)/2]^2
- 4*a*b^2*Tan[(c + d*x)/2])/(8*a^3*b*d)
```

Maple [B] time = 0.12, size = 286, normalized size = 2.3

$$\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{b}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{bd}-2\,\frac{a}{bd\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan\left(1/2\,dx\right.}{\sqrt{a^2-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/2*d*x+1/2*c)*b+2/d/b*arctan(ta
n(1/2*d*x+1/2*c))-2/d/b*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))+4/d/a*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x
+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a^3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/8/d/a/tan(1/2*d*x+1/2*c)^2-3/2/d/
a*ln(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/2/d/a^2*b/tan
(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.65531, size = 1337, normalized size = 10.87

$$\left[\frac{4\,a^3dx\cos\left(dx+c\right)^2-4\,a^3dx-4\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)+2\,a^2b\cos\left(dx+c\right)-2\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^3*d*x*cos(d*x + c)^2 - 4*a^3*d*x - 4*a*b^2*cos(d*x + c)*sin(d*x +
 c) + 2*a^2*b*cos(d*x + c) - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqr
t(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))
/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - (3*a^2*b - 2*b^3
- (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^2*b
- 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(
a^3*b*d*cos(d*x + c)^2 - a^3*b*d), 1/4*(4*a^3*d*x*cos(d*x + c)^2 - 4*a^3*d*
x - 4*a*b^2*cos(d*x + c)*sin(d*x + c) + 2*a^2*b*cos(d*x + c) + 4*((a^2 - b^
2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)
/(sqrt(a^2 - b^2)*cos(d*x + c))) - (3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos
(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^2*b - 2*b^3 - (3*a^2*b - 2*
b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*b*d*cos(d*x + c)^2
- a^3*b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)

[Out] Timed out
```

Giac [A] time = 1.24938, size = 293, normalized size = 2.38

$$\frac{\frac{8\,(dx+c)}{b}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}-\frac{4\left(3\,a^{2}-2\,b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}-\frac{16\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\sqrt{a^{2}-b^{2}}}\right.\right.}{\sqrt{a^{2}-b^{2}}\,a^{3}b}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/8*(8*(d*x + c)/b + (a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/
a^2 - 4*(3*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 16*(a^4 - 2*a^
2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d
*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^3*b) + (18*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 + 4*a*b*tan(1/2*d*x + 1/2*
c) - a^2)/(a^3*tan(1/2*d*x + 1/2*c)^2))/d
```

3.1307 $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}$$

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2
)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rubi [A] time = 0.445549, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2725, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```


```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2
)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a*d)
```


Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \frac{\int \frac{\csc^2(c+dx)\left(2\left(4a^2-3b^2\right)-ab\sin(c+dx)-3\left(2a^2\right.\right.}{a+b\sin(c+dx)}}{6a^2}$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \int \frac{\csc(c+dx}{}$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\left(b\left(3a^2-\right.\right.}{}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}$$

$$= \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\text{co}}{3a^3d}$$

Mathematica [B] time = 6.12535, size = 350, normalized size = 2.27

$$\frac{\left(3a^2b-2b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(2b^3-3a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\right.}{6a^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((4*a^2*Cos[(c + d*x)/2] - 3*b^2*
Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*a
^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-3*a^2*b + 2*b^3
)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((3*a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2
]])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-4*a^
2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])/(24*a*d)
```

Maple [B] time = 0.121, size = 348, normalized size = 2.3

$$\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{b}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{5}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b^2}{2\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{1}{d\sqrt{a^2-b^2}}\arctan\left(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a^2*tan(1/2*d*x+1/2*c)^2*b-5/8/d/a*tan(
1/2*d*x+1/2*c)+1/2/d/a^3*b^2*tan(1/2*d*x+1/2*c)+2/d/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d/a^2/(a^2-b^2)^(1/2)*a
rctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+2/d/a^4/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-1/24/
d/a/tan(1/2*d*x+1/2*c)^3+5/8/d/a/tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1
/2*c)*b^2+1/8/d/a^2*b/tan(1/2*d*x+1/2*c)^2+3/2/d/a^2*b*ln(tan(1/2*d*x+1/2*c
))-1/d/a^4*b^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50799, size = 1507, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 - 3*a*b^2)*cos(d*x + c
)^3 + 6*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a^2 + b^2)*log(((2*a
```

```
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*
b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*
a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 12*(a^3 - a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 -
a^4*d)*sin(d*x + c)), -1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 -
 3*a*b^2)*cos(d*x + c)^3 + 12*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*si
n(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2
*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*
cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 12*(a^3 - a*b^2
)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20701, size = 369, normalized size = 2.4

$$\frac{\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}+\frac{12\left(3a^2b-2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}+\frac{48\left(a^4-2a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*t
an(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(3*a^2*b - 2*b^
3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*(pi*floo
r(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44
*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(
1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x +
 1/2*c)^3))/d
```

3.1308 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=198

$$-\frac{2b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d}-\frac{\left(-12a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d}+\frac{(5a$$

```
[Out] (-2*b*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^5*d) - ((3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^5*d) - (
b*(4*a^2 - 3*b^2)*Cot[c + d*x])/(3*a^4*d) + ((5*a^2 - 4*b^2)*Cot[c + d*x]*C
sc[c + d*x])/(8*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rubi [A] time = 0.760678, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2893, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d}-\frac{\left(-12a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d}+\frac{(5a$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^5*d) - ((3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^5*d) - (
b*(4*a^2 - 3*b^2)*Cot[c + d*x])/(3*a^4*d) + ((5*a^2 - 4*b^2)*Cot[c + d*x]*C
sc[c + d*x])/(8*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\int \frac{\csc^3(c+dx)\left(3\left(5a^2-4b^2\right)-ab\sin(c+dx)\right.}{a+b\sin(c+dx}}{12a^2}$$

$$= \frac{\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\csc^3(c+}{4ad}$$

$$= -\frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(}{3a^2d}$$

$$= -\frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(}{3a^2d}$$

$$= -\frac{\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(5a^2-4b^2\right)}{}$$

$$= -\frac{\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b\left(4a^2-3b^2\right)\cot(c+dx)}{3a^4d} + \frac{\left(5a^2-4b^2\right)}{}$$

$$= -\frac{2b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d} - \frac{\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d} -$$

Mathematica [B] time = 6.21264, size = 433, normalized size = 2.19

$$\frac{\left(5a^2-4b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{\left(4b^2-5a^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{\left(-12a^2b^2+3a^4+8b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^5d} + \frac{\left(12a^2b^2-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Si
n[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*d) + ((-4*a^2*b*Cos[(c + d*x)/2] +
3*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + ((5*a^2 - 4*b^2)*Csc[
(c + d*x)/2]^2)/(32*a^3*d) + (b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^
2*d) - Csc[(c + d*x)/2]^4/(64*a*d) + ((-3*a^4 + 12*a^2*b^2 - 8*b^4)*Log[Cos
[(c + d*x)/2]])/(8*a^5*d) + ((3*a^4 - 12*a^2*b^2 + 8*b^4)*Log[Sin[(c + d*x)
/2]])/(8*a^5*d) + ((-5*a^2 + 4*b^2)*Sec[(c + d*x)/2]^2)/(32*a^3*d) + Sec[(c
 + d*x)/2]^4/(64*a*d) + (Sec[(c + d*x)/2]*(4*a^2*b*Sin[(c + d*x)/2] - 3*b^3
*Sin[(c + d*x)/2]))/(6*a^4*d) - (b*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24
*a^2*d)
```

Maple [B] time = 0.128, size = 455, normalized size = 2.3

$$\frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 - \frac{b}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + \frac{b^2}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + \frac{5\,b}{8\,da^2}\tan\left(\frac{dx}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a^2*tan(1/2*d*x+1/2*c)^3*b-1/8/d/a*tan
(1/2*d*x+1/2*c)^2+1/8/d/a^3*tan(1/2*d*x+1/2*c)^2*b^2+5/8/d/a^2*tan(1/2*d*x+
```

```
1/2*c)*b-1/2/d/a^4*b^3*tan(1/2*d*x+1/2*c)-2/d/a*b/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3*b^3/(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d*b^5/a^5/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/64/d/
a/tan(1/2*d*x+1/2*c)^4+1/8/d/a/tan(1/2*d*x+1/2*c)^2-1/8/d/a^3*b^2/tan(1/2*d
*x+1/2*c)^2+3/8/d/a*ln(tan(1/2*d*x+1/2*c))-3/2/d/a^3*ln(tan(1/2*d*x+1/2*c))
*b^2+1/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^4+1/24/d/a^2*b/tan(1/2*d*x+1/2*c)^3-5
/8/d/a^2*b/tan(1/2*d*x+1/2*c)+1/2/d*b^3/a^4/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.22323, size = 2068, normalized size = 10.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(6*(5*a^4 - 4*a^2*b^2)*cos(d*x + c)^3 + 24*((a^2*b - b^3)*cos(d*x +
c)^4 + a^2*b - b^3 - 2*(a^2*b - b^3)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(3*a^4 - 4*a^2*b^2)*cos(d*x + c) +
 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 3*a^4 - 12*a^2*b^2 + 8*b^
4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1
/2) - 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 3*a^4 - 12*a^2*b^2 +
 8*b^4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
c) + 1/2) - 16*((4*a^3*b - 3*a*b^3)*cos(d*x + c)^3 - 3*(a^3*b - a*b^3)*cos(
d*x + c))*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x + c)^2 + a^
5*d), -1/48*(6*(5*a^4 - 4*a^2*b^2)*cos(d*x + c)^3 - 48*((a^2*b - b^3)*cos(d
*x + c)^4 + a^2*b - b^3 - 2*(a^2*b - b^3)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(3*a^4 - 4*
a^2*b^2)*cos(d*x + c) + 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 3*
a^4 - 12*a^2*b^2 + 8*b^4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^2)*l
og(1/2*cos(d*x + c) + 1/2) - 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4
 + 3*a^4 - 12*a^2*b^2 + 8*b^4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)
^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*((4*a^3*b - 3*a*b^3)*cos(d*x + c)^3 -
 3*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^
5*d*cos(d*x + c)^2 + a^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26145, size = 506, normalized size = 2.56

$$\frac{\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+120\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}+\frac{24\left(3\,a^4-12\,a^2b^2+\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*((3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 24*
a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*a^2*b*ta
n(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/a^4 + 24*(3*a^4 - 12*a^2*
b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 384*(a^4*b - 2*a^2*b^3 +
b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) - (150*a^4*tan(1/2*d*x + 1
/2*c)^4 - 600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 400*b^4*tan(1/2*d*x + 1/2*c)
^4 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 2
4*a^4*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*b*
tan(1/2*d*x + 1/2*c) + 3*a^4)/(a^5*tan(1/2*d*x + 1/2*c)^4))/d
```

3.1309 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=244

$$\frac{2b^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{\left(-20a^2b^2+3a^4+15b^4\right)\cot(c+dx)}{15a^5d}+\frac{b\left(-12a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\cos$$

```
[Out] (2*b^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^6*d) + (b*(3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d)
- ((3*a^4 - 20*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (b*(5*a^2 - 4*b
^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) + ((6*a^2 - 5*b^2)*Cot[c + d*x]*Cs
c[c + d*x]^2)/(15*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot
[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```

Rubi [A] time = 1.04989, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2893, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{\left(-20a^2b^2+3a^4+15b^4\right)\cot(c+dx)}{15a^5d}+\frac{b\left(-12a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\cos$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (2*b^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^6*d) + (b*(3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d)
- ((3*a^4 - 20*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (b*(5*a^2 - 4*b
^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) + ((6*a^2 - 5*b^2)*Cot[c + d*x]*Cs
c[c + d*x]^2)/(15*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot
[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```


Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
```

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad} - \frac{\int \frac{\csc^4(c+dx)\left(4\left(6a^2-5b^2\right)-ab\sin(\ldots}{a+b\sin\ldots}}{20\ldots}$$

$$= \frac{\left(6a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{15a^3d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc\ldots}{5ad}$$

$$= -\frac{b\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(6a^2-5b^2\right)\cot(c+dx)\csc^2(c+dx)}{15a^3d} + \frac{b\ldots}{\ldots}$$

$$= -\frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(6a^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{b\left(5a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{\left(6a^2\ldots\right.}{\ldots}$$

$$= \frac{b\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \ldots$$

$$= \frac{b\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(3a^4-20a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \ldots$$

$$= \frac{2b^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b\left(3a^4-12a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+d\ldots}{8a^6d}$$

Mathematica [B] time = 1.81488, size = 507, normalized size = 2.08

$$\frac{1920b^2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - 640a^3b^2\tan\left(\frac{1}{2}(c+dx)\right) - 32\left(-20a^3b^2+3a^5+15ab^4\right)\cot\left(\frac{1}{2}(c+dx)\right) + \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (1920*b^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
] - 32*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 150*a^4*b*Csc[(c
+ d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4
+ 360*a^4*b*Log[Cos[(c + d*x)/2]] - 1440*a^2*b^3*Log[Cos[(c + d*x)/2]] + 96
0*b^5*Log[Cos[(c + d*x)/2]] - 360*a^4*b*Log[Sin[(c + d*x)/2]] + 1440*a^2*b^
3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 150*a^4*b*Sec[(c
+ d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4
- 336*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 + 21*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[
(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 96*a^
5*Tan[(c + d*x)/2] - 640*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*x)
/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

Maple [B] time = 0.141, size = 583, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a^2*tan(1/2*d*x+1/2*c)^4*b-1/32/d/a*t
an(1/2*d*x+1/2*c)^3+1/24/d/a^3*tan(1/2*d*x+1/2*c)^3*b^2+1/8/d/a^2*tan(1/2*d
*x+1/2*c)^2*b-1/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b^3+1/16/d/a*tan(1/2*d*x+1/2*c
)-5/8/d/a^3*b^2*tan(1/2*d*x+1/2*c)+1/2/d/a^5*b^4*tan(1/2*d*x+1/2*c)+2/d/a^2
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b
^2-4/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2
)^(1/2))*b^4+2/d*b^6/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))-1/160/d/a/tan(1/2*d*x+1/2*c)^5+1/32/d/a/tan(1/2*d*x+
1/2*c)^3-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2-1/16/d/a/tan(1/2*d*x+1/2*c)+5/
8/d/a^3/tan(1/2*d*x+1/2*c)*b^2-1/2/d/a^5/tan(1/2*d*x+1/2*c)*b^4+1/64/d/a^2*
b/tan(1/2*d*x+1/2*c)^4-1/8/d/a^2*b/tan(1/2*d*x+1/2*c)^2+1/8/d/a^4*b^3/tan(1
/2*d*x+1/2*c)^2-3/8/d/a^2*b*ln(tan(1/2*d*x+1/2*c))+3/2/d/a^4*b^3*ln(tan(1/2
*d*x+1/2*c))-1/d/a^6*b^5*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.51621, size = 2437, normalized size = 9.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 + 80*(7*a^3*b^2
 - 6*a*b^4)*cos(d*x + c)^3 + 120*((a^2*b^2 - b^4)*cos(d*x + c)^4 + a^2*b^2 -
 b^4 - 2*(a^2*b^2 - b^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2
)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d
*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15*(3*a^4*b - 12*a^2*b^3 + 8*b^5 + (
3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*
b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(3*a^4*b
 - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(
3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 240*(a^3*b^2 - a*b^4)*cos(d*x + c) - 30*((5*a^4*b - 4*a^2*b^
3)*cos(d*x + c)^3 - (3*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6
*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(
16*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 + 80*(7*a^3*b^2 - 6*a*b^4
)*cos(d*x + c)^3 + 240*((a^2*b^2 - b^4)*cos(d*x + c)^4 + a^2*b^2 - b^4 - 2*
(a^2*b^2 - b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b
)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(3*a^4*b - 12*a^2*b^3 +
 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^
2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 1
5*(3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x +
```

```
c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) - 240*(a^3*b^2 - a*b^4)*cos(d*x + c) - 30*((5*a^4*b
- 4*a^2*b^3)*cos(d*x + c)^3 - (3*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x +
 c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25991, size = 653, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 30
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^3*b
*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 60*a^4*tan(1/2
*d*x + 1/2*c) - 600*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x + 1/
2*c))/a^5 - 120*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^6 + 1920*(a^4*b^2 - 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 -
 b^2)*a^6) + (822*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 3288*a^2*b^3*tan(1/2*d*x +
 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*tan(1/2*d*x + 1/2*c)^4
 + 600*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x + 1/2*c)^4 -
120*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 30*
a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*t
an(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

3.1310 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=212

$$\frac{\left(a^2-2b^2\right)\sin^5(c+dx)}{5b^3d}-\frac{a\left(a^2-2b^2\right)\sin^4(c+dx)}{4b^4d}+\frac{\left(a^2-b^2\right)^2\sin^3(c+dx)}{3b^5d}-\frac{a\left(a^2-b^2\right)^2\sin^2(c+dx)}{2b^6d}+\frac{a^2\left(a^2-b^2\right)^2 \mathrm{s}}{b^7d}$$

```
[Out] -((a^3*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^8*d)) + (a^2*(a^2 - b^2)^2
*Sin[c + d*x])/(b^7*d) - (a*(a^2 - b^2)^2*Sin[c + d*x]^2)/(2*b^6*d) + ((a^2
 - b^2)^2*Sin[c + d*x]^3)/(3*b^5*d) - (a*(a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b
^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^5)/(5*b^3*d) - (a*Sin[c + d*x]^6)/(6*b^
2*d) + Sin[c + d*x]^7/(7*b*d)
```

Rubi [A] time = 0.237986, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{\left(a^2-2b^2\right)\sin^5(c+dx)}{5b^3d}-\frac{a\left(a^2-2b^2\right)\sin^4(c+dx)}{4b^4d}+\frac{\left(a^2-b^2\right)^2\sin^3(c+dx)}{3b^5d}-\frac{a\left(a^2-b^2\right)^2\sin^2(c+dx)}{2b^6d}+\frac{a^2\left(a^2-b^2\right)^2 \mathrm{s}}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```



```
[Out] -((a^3*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^8*d)) + (a^2*(a^2 - b^2)^2
*Sin[c + d*x])/(b^7*d) - (a*(a^2 - b^2)^2*Sin[c + d*x]^2)/(2*b^6*d) + ((a^2
 - b^2)^2*Sin[c + d*x]^3)/(3*b^5*d) - (a*(a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b
^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^5)/(5*b^3*d) - (a*Sin[c + d*x]^6)/(6*b^
2*d) + Sin[c + d*x]^7/(7*b*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{b^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^8d}$$

$$= \frac{\text{Subst}\left(\int \left(\left(a^3-ab^2\right)^2 - a\left(a^2-b^2\right)^2 x + \left(a^2-b^2\right)^2 x^2 - a\left(a^2-2b^2\right)x^3 + \left(a^2-2b^2\right.\right.\right.}{b^8d}$$

$$= -\frac{a^3\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^8d} + \frac{a^2\left(a^2-b^2\right)^2\sin(c+dx)}{b^7d} - \frac{a\left(a^2-b^2\right)^2\sin^2}{2b^6d}$$

Mathematica [A] time = 1.24804, size = 180, normalized size = 0.85

$$\frac{84b^5\left(a^2-2b^2\right)\sin^5(c+dx) - 105ab^4\left(a^2-2b^2\right)\sin^4(c+dx) + 140b^3\left(a^2-b^2\right)^2\sin^3(c+dx) - 210ab^2\left(a^2-b^2\right)^2\sin^2(}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-420*a^3*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]] + 420*b*(a^3 - a*b^2)^2*Sin
[c + d*x] - 210*a*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 + 140*b^3*(a^2 - b^2)^2*
Sin[c + d*x]^3 - 105*a*b^4*(a^2 - 2*b^2)*Sin[c + d*x]^4 + 84*b^5*(a^2 - 2*b
^2)*Sin[c + d*x]^5 - 70*a*b^6*Sin[c + d*x]^6 + 60*b^7*Sin[c + d*x]^7)/(420*
b^8*d)
```

Maple [A] time = 0.052, size = 329, normalized size = 1.6

$$\frac{(\sin(dx+c))^7}{7\,bd} - \frac{a(\sin(dx+c))^6}{6\,b^2d} + \frac{(\sin(dx+c))^5a^2}{5\,db^3} - \frac{2\,(\sin(dx+c))^5}{5\,bd} - \frac{(\sin(dx+c))^4a^3}{4\,db^4} + \frac{a(\sin(dx+c))^4}{2\,b^2d} + \frac{a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/7*sin(d*x+c)^7/b/d-1/6*a*sin(d*x+c)^6/b^2/d+1/5/d/b^3*sin(d*x+c)^5*a^2-2/
5*sin(d*x+c)^5/b/d-1/4/d/b^4*sin(d*x+c)^4*a^3+1/2*a*sin(d*x+c)^4/b^2/d+1/3/
d/b^5*a^4*sin(d*x+c)^3-2/3/d/b^3*sin(d*x+c)^3*a^2+1/3*sin(d*x+c)^3/b/d-1/2/
d/b^6*sin(d*x+c)^2*a^5+1/d/b^4*sin(d*x+c)^2*a^3-1/2*a*sin(d*x+c)^2/b^2/d+1/
d/b^7*a^6*sin(d*x+c)-2/d/b^5*sin(d*x+c)*a^4+a^2*sin(d*x+c)/b^3/d-1/d*a^7/b^
8*ln(a+b*sin(d*x+c))+2/d*a^5/b^6*ln(a+b*sin(d*x+c))-a^3*ln(a+b*sin(d*x+c))/
b^4/d
```

Maxima [A] time = 0.988877, size = 277, normalized size = 1.31

$$\frac{\frac{60\,b^6\sin(dx+c)^7 - 70\,ab^5\sin(dx+c)^6 + 84\left(a^2b^4-2\,b^6\right)\sin(dx+c)^5 - 105\left(a^3b^3-2\,ab^5\right)\sin(dx+c)^4 + 140\left(a^4b^2-2\,a^2b^4+b^6\right)\sin(dx+c)^3 - 210\left(a^5b-2\,a^3b^3+ab^5\right)\sin(}{b^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*((60*b^6*sin(d*x + c)^7 - 70*a*b^5*sin(d*x + c)^6 + 84*(a^2*b^4 - 2*b
^6)*sin(d*x + c)^5 - 105*(a^3*b^3 - 2*a*b^5)*sin(d*x + c)^4 + 140*(a^4*b^2
- 2*a^2*b^4 + b^6)*sin(d*x + c)^3 - 210*(a^5*b - 2*a^3*b^3 + a*b^5)*sin(d*x
 + c)^2 + 420*(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x + c))/b^7 - 420*(a^7 - 2*
a^5*b^2 + a^3*b^4)*log(b*sin(d*x + c) + a)/b^8)/d
```

Fricas [A] time = 1.72782, size = 466, normalized size = 2.2

$$\frac{70\,ab^6\cos\left(dx+c\right)^6 - 105\,a^3b^4\cos\left(dx+c\right)^4 + 210\left(a^5b^2 - a^3b^4\right)\cos\left(dx+c\right)^2 - 420\left(a^7 - 2\,a^5b^2 + a^3b^4\right)\log\left(b\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*a*b^6*cos(d*x + c)^6 - 105*a^3*b^4*cos(d*x + c)^4 + 210*(a^5*b^2
- a^3*b^4)*cos(d*x + c)^2 - 420*(a^7 - 2*a^5*b^2 + a^3*b^4)*log(b*sin(d*x +
 c) + a) - 4*(15*b^7*cos(d*x + c)^6 - 105*a^6*b + 175*a^4*b^3 - 56*a^2*b^5
- 8*b^7 - 3*(7*a^2*b^5 + b^7)*cos(d*x + c)^4 + (35*a^4*b^3 - 28*a^2*b^5 - 4
*b^7)*cos(d*x + c)^2)*sin(d*x + c))/(b^8*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22209, size = 352, normalized size = 1.66

$$\frac{\frac{60\,b^6\sin(dx+c)^7 - 70\,ab^5\sin(dx+c)^6 + 84\,a^2b^4\sin(dx+c)^5 - 168\,b^6\sin(dx+c)^5 - 105\,a^3b^3\sin(dx+c)^4 + 210\,ab^5\sin(dx+c)^4 + 140\,a^4b^2\sin(dx+c)^3 - 280\,a^2b^4\sin(dx+c)^3}{b^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*((60*b^6*sin(d*x + c)^7 - 70*a*b^5*sin(d*x + c)^6 + 84*a^2*b^4*sin(d*
x + c)^5 - 168*b^6*sin(d*x + c)^5 - 105*a^3*b^3*sin(d*x + c)^4 + 210*a*b^5*
sin(d*x + c)^4 + 140*a^4*b^2*sin(d*x + c)^3 - 280*a^2*b^4*sin(d*x + c)^3 +
140*b^6*sin(d*x + c)^3 - 210*a^5*b*sin(d*x + c)^2 + 420*a^3*b^3*sin(d*x + c
)^2 - 210*a*b^5*sin(d*x + c)^2 + 420*a^6*sin(d*x + c) - 840*a^4*b^2*sin(d*x
 + c) + 420*a^2*b^4*sin(d*x + c))/b^7 - 420*(a^7 - 2*a^5*b^2 + a^3*b^4)*log
(abs(b*sin(d*x + c) + a))/b^8)/d
```

3.1311 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{(a^2-2b^2)\sin^4(c+dx)}{4b^3d}-\frac{a(a^2-2b^2)\sin^3(c+dx)}{3b^4d}+\frac{(a^2-b^2)^2\sin^2(c+dx)}{2b^5d}-\frac{a(a^2-b^2)^2\sin(c+dx)}{b^6d}+\frac{a^2(a^2-b^2)}{}$$

```
[Out] (a^2*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^7*d) - (a*(a^2 - b^2)^2*Sin[
c + d*x])/(b^6*d) + ((a^2 - b^2)^2*Sin[c + d*x]^2)/(2*b^5*d) - (a*(a^2 - 2*
b^2)*Sin[c + d*x]^3)/(3*b^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b^3*d) -
 (a*Sin[c + d*x]^5)/(5*b^2*d) + Sin[c + d*x]^6/(6*b*d)
```

Rubi [A] time = 0.207409, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 948}

$$\frac{(a^2-2b^2)\sin^4(c+dx)}{4b^3d}-\frac{a(a^2-2b^2)\sin^3(c+dx)}{3b^4d}+\frac{(a^2-b^2)^2\sin^2(c+dx)}{2b^5d}-\frac{a(a^2-b^2)^2\sin(c+dx)}{b^6d}+\frac{a^2(a^2-b^2)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a^2*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^7*d) - (a*(a^2 - b^2)^2*Sin[
c + d*x])/(b^6*d) + ((a^2 - b^2)^2*Sin[c + d*x]^2)/(2*b^5*d) - (a*(a^2 - 2*
b^2)*Sin[c + d*x]^3)/(3*b^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b^3*d) -
 (a*Sin[c + d*x]^5)/(5*b^2*d) + Sin[c + d*x]^6/(6*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)^2}{b^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a\left(a^2-b^2\right)^2 + \left(a^2-b^2\right)^2 x - a\left(a^2-2b^2\right)x^2 + \left(a^2-2b^2\right)x^3 - ax^4 + x^5 + \right.\right.}{b^7 d}$$
$$= \frac{a^2\left(a^2-b^2\right)^2 \log(a+b\sin(c+dx))}{b^7 d} - \frac{a\left(a^2-b^2\right)^2 \sin(c+dx)}{b^6 d} + \frac{\left(a^2-b^2\right)^2 \sin^2(c+dx}{2b^5 d}$$

Mathematica [A] time = 0.720677, size = 153, normalized size = 0.85

$$\frac{15b^4\left(a^2-2b^2\right)\sin^4(c+dx) - 20ab^3\left(a^2-2b^2\right)\sin^3(c+dx) + 30b^2\left(a^2-b^2\right)^2\sin^2(c+dx) - 60ab\left(a^2-b^2\right)^2\sin(c+dx) -}{60b^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*(a^3 - a*b^2)^2*Log[a + b*Sin[c + d*x]] - 60*a*b*(a^2 - b^2)^2*Sin[c +
d*x] + 30*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 - 20*a*b^3*(a^2 - 2*b^2)*Sin[c +
 d*x]^3 + 15*b^4*(a^2 - 2*b^2)*Sin[c + d*x]^4 - 12*a*b^5*Sin[c + d*x]^5 + 1
0*b^6*Sin[c + d*x]^6)/(60*b^7*d)
```

Maple [A] time = 0.049, size = 273, normalized size = 1.5

$$\frac{(\sin(dx+c))^6}{6bd} - \frac{a(\sin(dx+c))^5}{5b^2d} + \frac{(\sin(dx+c))^4 a^2}{4db^3} - \frac{(\sin(dx+c))^4}{2bd} - \frac{(\sin(dx+c))^3 a^3}{3db^4} + \frac{2a(\sin(dx+c))^3}{3b^2d} + \frac{(\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/6*sin(d*x+c)^6/b/d-1/5*a*sin(d*x+c)^5/b^2/d+1/4/d/b^3*sin(d*x+c)^4*a^2-1/
2*sin(d*x+c)^4/b/d-1/3/d/b^4*sin(d*x+c)^3*a^3+2/3*a*sin(d*x+c)^3/b^2/d+1/2/
d/b^5*sin(d*x+c)^2*a^4-1/d/b^3*a^2*sin(d*x+c)^2+1/2*sin(d*x+c)^2/b/d-1/d/b^
6*a^5*sin(d*x+c)+2/d/b^4*sin(d*x+c)*a^3-a*sin(d*x+c)/b^2/d+1/d*a^6/b^7*ln(a
+b*sin(d*x+c))-2/d*a^4/b^5*ln(a+b*sin(d*x+c))+1/d/b^3*ln(a+b*sin(d*x+c))*a^
2
```

Maxima [A] time = 0.982504, size = 232, normalized size = 1.29

$$\frac{\frac{10\,b^5\sin(dx+c)^6 - 12\,ab^4\sin(dx+c)^5 + 15\left(a^2b^3 - 2\,b^5\right)\sin(dx+c)^4 - 20\left(a^3b^2 - 2\,ab^4\right)\sin(dx+c)^3 + 30\left(a^4b - 2\,a^2b^3 + b^5\right)\sin(dx+c)^2 - 60\left(a^5 - 2\,a^3b^2 + ab^4\right)\sin(dx+c)}{b^6} + }{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*((10*b^5*sin(d*x + c)^6 - 12*a*b^4*sin(d*x + c)^5 + 15*(a^2*b^3 - 2*b^
5)*sin(d*x + c)^4 - 20*(a^3*b^2 - 2*a*b^4)*sin(d*x + c)^3 + 30*(a^4*b - 2*a
^2*b^3 + b^5)*sin(d*x + c)^2 - 60*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/b
^6 + 60*(a^6 - 2*a^4*b^2 + a^2*b^4)*log(b*sin(d*x + c) + a)/b^7)/d
```

Fricas [A] time = 1.63745, size = 377, normalized size = 2.09

$$-\frac{10\,b^6\cos\left(dx+c\right)^6-15\,a^2b^4\cos\left(dx+c\right)^4+30\left(a^4b^2-a^2b^4\right)\cos\left(dx+c\right)^2-60\left(a^6-2\,a^4b^2+a^2b^4\right)\log\left(b\sin\left(dx+c\right.\right.}{60\,b^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^6*cos(d*x + c)^6 - 15*a^2*b^4*cos(d*x + c)^4 + 30*(a^4*b^2 - a^
2*b^4)*cos(d*x + c)^2 - 60*(a^6 - 2*a^4*b^2 + a^2*b^4)*log(b*sin(d*x + c) +
 a) + 4*(3*a*b^5*cos(d*x + c)^4 + 15*a^5*b - 25*a^3*b^3 + 8*a*b^5 - (5*a^3*
b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^7*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20378, size = 288, normalized size = 1.6

$$\frac{\frac{10\,b^5\sin(dx+c)^6-12\,ab^4\sin(dx+c)^5+15\,a^2b^3\sin(dx+c)^4-30\,b^5\sin(dx+c)^4-20\,a^3b^2\sin(dx+c)^3+40\,ab^4\sin(dx+c)^3+30\,a^4b\sin(dx+c)^2-60\,a^2b^3\sin(dx+c)^2+30}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*((10*b^5*sin(d*x + c)^6 - 12*a*b^4*sin(d*x + c)^5 + 15*a^2*b^3*sin(d*x
 + c)^4 - 30*b^5*sin(d*x + c)^4 - 20*a^3*b^2*sin(d*x + c)^3 + 40*a*b^4*sin(
d*x + c)^3 + 30*a^4*b*sin(d*x + c)^2 - 60*a^2*b^3*sin(d*x + c)^2 + 30*b^5*s
in(d*x + c)^2 - 60*a^5*sin(d*x + c) + 120*a^3*b^2*sin(d*x + c) - 60*a*b^4*s
in(d*x + c))/b^6 + 60*(a^6 - 2*a^4*b^2 + a^2*b^4)*log(abs(b*sin(d*x + c) +
a))/b^7)/d
```

3.1312 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{\left(a^2-2b^2\right)\sin^3(c+dx)}{3b^3d}-\frac{a\left(a^2-2b^2\right)\sin^2(c+dx)}{2b^4d}+\frac{\left(a^2-b^2\right)^2\sin(c+dx)}{b^5d}-\frac{a\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^6d}-\frac{a\sin^4}{4}$$

```
[Out] -((a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^6*d)) + ((a^2 - b^2)^2*Sin[c
 + d*x])/(b^5*d) - (a*(a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^4*d) + ((a^2 - 2*b
^2)*Sin[c + d*x]^3)/(3*b^3*d) - (a*Sin[c + d*x]^4)/(4*b^2*d) + Sin[c + d*x]
^5/(5*b*d)
```

Rubi [A] time = 0.140724, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 772}

$$\frac{\left(a^2-2b^2\right)\sin^3(c+dx)}{3b^3d}-\frac{a\left(a^2-2b^2\right)\sin^2(c+dx)}{2b^4d}+\frac{\left(a^2-b^2\right)^2\sin(c+dx)}{b^5d}-\frac{a\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^6d}-\frac{a\sin^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^6*d)) + ((a^2 - b^2)^2*Sin[c
 + d*x])/(b^5*d) - (a*(a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^4*d) + ((a^2 - 2*b
^2)*Sin[c + d*x]^3)/(3*b^3*d) - (a*Sin[c + d*x]^4)/(4*b^2*d) + Sin[c + d*x]
^5/(5*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{b(a+x)}dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{a+x}dx, x, b\sin(c+dx)\right)}{b^6 d}$$
$$= \frac{\text{Subst}\left(\int \left((a^2-b^2)^2 - a(a^2-2b^2)x + (a^2-2b^2)x^2 - ax^3 + x^4 - \frac{a(a^2-b^2)^2}{a+x}\right)dx, x, b$$
$$= -\frac{a(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^6 d} + \frac{(a^2-b^2)^2\sin(c+dx)}{b^5 d} - \frac{a(a^2-2b^2)\sin^2(c+}{2b^4 d}$$

Mathematica [A] time = 0.611066, size = 128, normalized size = 0.86

$$\frac{20b^3(a^2-2b^2)\sin^3(c+dx) - 30ab^2(a^2-2b^2)\sin^2(c+dx) + 60b(a^2-b^2)^2\sin(c+dx) - 60a(a^2-b^2)^2\log(a+b\sin}{60b^6 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-60*a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]] + 60*b*(a^2 - b^2)^2*Sin[c + d
*x] - 30*a*b^2*(a^2 - 2*b^2)*Sin[c + d*x]^2 + 20*b^3*(a^2 - 2*b^2)*Sin[c +
d*x]^3 - 15*a*b^4*Sin[c + d*x]^4 + 12*b^5*Sin[c + d*x]^5)/(60*b^6*d)
```

Maple [A] time = 0.043, size = 215, normalized size = 1.5

$$\frac{(\sin(dx+c))^5}{5bd} - \frac{a(\sin(dx+c))^4}{4b^2d} + \frac{a^2(\sin(dx+c))^3}{3db^3} - \frac{2(\sin(dx+c))^3}{3bd} - \frac{(\sin(dx+c))^2a^3}{2db^4} + \frac{(\sin(dx+c))^2a}{b^2d} + \frac{a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/5*sin(d*x+c)^5/b/d-1/4*a*sin(d*x+c)^4/b^2/d+1/3/d/b^3*sin(d*x+c)^3*a^2-2/
3*sin(d*x+c)^3/b/d-1/2/d/b^4*sin(d*x+c)^2*a^3+a*sin(d*x+c)^2/b^2/d+1/d/b^5*
sin(d*x+c)*a^4-2*a^2*sin(d*x+c)/b^3/d+sin(d*x+c)/b/d-1/d*a^5/b^6*ln(a+b*sin
(d*x+c))+2*a^3*ln(a+b*sin(d*x+c))/b^4/d-1/d/b^2*a*ln(a+b*sin(d*x+c))
```

Maxima [A] time = 1.00154, size = 188, normalized size = 1.27

$$\frac{\frac{12b^4\sin(dx+c)^5 - 15ab^3\sin(dx+c)^4 + 20(a^2b^2-2b^4)\sin(dx+c)^3 - 30(a^3b-2ab^3)\sin(dx+c)^2 + 60(a^4-2a^2b^2+b^4)\sin(dx+c)}{b^5} - \frac{60(a^5-2a^3b^2+ab^4)\log(b\sin(d}{b^6}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*(a^2*b^2 - 2*b^
4)*sin(d*x + c)^3 - 30*(a^3*b - 2*a*b^3)*sin(d*x + c)^2 + 60*(a^4 - 2*a^2*b
```

```
^2 + b^4)*sin(d*x + c))/b^5 - 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(b*sin(d*x +
c) + a)/b^6)/d
```

Fricas [A] time = 1.50313, size = 328, normalized size = 2.22

$$-\frac{15\,ab^4\cos\left(dx+c\right)^4-30\left(a^3b^2-ab^4\right)\cos\left(dx+c\right)^2+60\left(a^5-2\,a^3b^2+ab^4\right)\log\left(b\sin\left(dx+c\right)+a\right)-4\left(3\,b^5\cos\left(dx+c\right.\right.}{60\,b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*a*b^4*cos(d*x + c)^4 - 30*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 + 60*(
a^5 - 2*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) + a) - 4*(3*b^5*cos(d*x + c)^4
+ 15*a^4*b - 25*a^2*b^3 + 8*b^5 - (5*a^2*b^3 - 4*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/(b^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17555, size = 223, normalized size = 1.51

$$\frac{\frac{12\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+20\,a^2b^2\sin(dx+c)^3-40\,b^4\sin(dx+c)^3-30\,a^3b\sin(dx+c)^2+60\,ab^3\sin(dx+c)^2+60\,a^4\sin(dx+c)-120\,a^2b^2\sin(dx+c)+60\,b^4\sin(\ldots}{b^5}\ldots}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*a^2*b^2*sin(d*x
 + c)^3 - 40*b^4*sin(d*x + c)^3 - 30*a^3*b*sin(d*x + c)^2 + 60*a*b^3*sin(d*
x + c)^2 + 60*a^4*sin(d*x + c) - 120*a^2*b^2*sin(d*x + c) + 60*b^4*sin(d*x
+ c))/b^5 - 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(abs(b*sin(d*x + c) + a))/b^6)/
d
```

3.1313 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{(a^2-2b^2)\sin(c+dx)}{b^3d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{ab^4d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\log(\sin(c+dx))}{ad}+\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a*b^4*d)
 + ((a^2 - 2*b^2)*Sin[c + d*x])/(b^3*d) - (a*Sin[c + d*x]^2)/(2*b^2*d) + Si
n[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.14031, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{(a^2-2b^2)\sin(c+dx)}{b^3d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{ab^4d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\log(\sin(c+dx))}{ad}+\frac{\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a*b^4*d)
 + ((a^2 - 2*b^2)*Sin[c + d*x])/(b^3*d) - (a*Sin[c + d*x]^2)/(2*b^2*d) + Si
n[c + d*x]^3/(3*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b(b^2-x^2)^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2-b^2)^2}{a(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{ab^4d} + \frac{(a^2-2b^2)\sin(c+dx)}{b^3d} - \frac{a\sin^2(c}{2b}$$

Mathematica [A] time = 0.139627, size = 101, normalized size = 0.94

$$\frac{-3a^2b^2\sin^2(c+dx)+6ab\left(a^2-2b^2\right)\sin(c+dx)+6\left(b^4\log(\sin(c+dx))-\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))\right)+2ab^3\sin^3(}{6ab^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (6*(b^4*Log[Sin[c + d*x]] - (a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]) + 6*a*b*
(a^2 - 2*b^2)*Sin[c + d*x] - 3*a^2*b^2*Sin[c + d*x]^2 + 2*a*b^3*Sin[c + d*x
]^3)/(6*a*b^4*d)
```

Maple [A] time = 0.075, size = 140, normalized size = 1.3

$$\frac{(\sin(dx+c))^3}{3bd} - \frac{(\sin(dx+c))^2 a}{2b^2d} + \frac{a^2\sin(dx+c)}{b^3d} - 2\frac{\sin(dx+c)}{bd} - \frac{a^3\ln(a+b\sin(dx+c))}{b^4d} + 2\frac{a\ln(a+b\sin(dx+c}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/3*sin(d*x+c)^3/b/d-1/2*a*sin(d*x+c)^2/b^2/d+a^2*sin(d*x+c)/b^3/d-2*sin(d*
x+c)/b/d-a^3*ln(a+b*sin(d*x+c))/b^4/d+2/d/b^2*a*ln(a+b*sin(d*x+c))-1/d/a*ln
(a+b*sin(d*x+c))+ln(sin(d*x+c))/a/d
```

Maxima [A] time = 0.989278, size = 134, normalized size = 1.25

$$\frac{\frac{6\log(\sin(dx+c))}{a} + \frac{2b^2\sin(dx+c)^3-3ab\sin(dx+c)^2+6\left(a^2-2b^2\right)\sin(dx+c)}{b^3} - \frac{6\left(a^4-2a^2b^2+b^4\right)\log(b\sin(dx+c)+a)}{ab^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(sin(d*x + c))/a + (2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^2 +
 6*(a^2 - 2*b^2)*sin(d*x + c))/b^3 - 6*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*
```

```
x + c) + a)/(a*b^4))/d
```

Fricas [A] time = 1.75831, size = 250, normalized size = 2.34

$$\frac{3\,a^2b^2\cos\left(dx+c\right)^2+6\,b^4\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-6\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\left(ab^3\cos\left(dx+c\right)^2-3\right.}{6\,ab^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*b^2*cos(d*x + c)^2 + 6*b^4*log(-1/2*sin(d*x + c)) - 6*(a^4 - 2*a
^2*b^2 + b^4)*log(b*sin(d*x + c) + a) - 2*(a*b^3*cos(d*x + c)^2 - 3*a^3*b +
 5*a*b^3)*sin(d*x + c))/(a*b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19394, size = 143, normalized size = 1.34

$$\frac{\frac{6\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}+\frac{2\,b^2\sin\left(dx+c\right)^3-3\,ab\sin\left(dx+c\right)^2+6\,a^2\sin\left(dx+c\right)-12\,b^2\sin\left(dx+c\right)}{b^3}-\frac{6\left(a^4-2\,a^2b^2+b^4\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{ab^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c)))/a + (2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c
)^2 + 6*a^2*sin(d*x + c) - 12*b^2*sin(d*x + c))/b^3 - 6*(a^4 - 2*a^2*b^2 +
b^4)*log(abs(b*sin(d*x + c) + a))/(a*b^4))/d
```

3.1314 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{a\sin(c+dx)}{b^2d}-\frac{\csc(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])/(a^2*b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c + d*x]^
2/(2*b*d)
```

Rubi [A] time = 0.155176, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{a\sin(c+dx)}{b^2d}-\frac{\csc(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])/(a^2*b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c + d*x]^
2/(2*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= \frac{\text{Subst}\left(\int \left(-a+\frac{b^4}{ax^2}-\frac{b^4}{a^2x}+x+\frac{(a^2-b^2)^2}{a^2(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{a\sin(c+a}{b^2d}$$

Mathematica [A] time = 0.175616, size = 86, normalized size = 0.9

$$\frac{\frac{2(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3}-\frac{2b\log(\sin(c+dx))}{a^2}-\frac{2a\sin(c+dx)}{b^2}-\frac{2\csc(c+dx)}{a}+\frac{\sin^2(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*Csc[c + d*x])/a - (2*b*Log[Sin[c + d*x]])/a^2 + (2*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(a^2*b^3) - (2*a*Sin[c + d*x])/b^2 + Sin[c + d*x]^2/b)/
(2*d)
```

Maple [A] time = 0.085, size = 124, normalized size = 1.3

$$\frac{(\sin(dx+c))^2}{2bd}-\frac{a\sin(dx+c)}{b^2d}+\frac{\ln(a+b\sin(dx+c))a^2}{db^3}-2\frac{\ln(a+b\sin(dx+c))}{bd}+\frac{b\ln(a+b\sin(dx+c))}{da^2}-\frac{}{da\,s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2*sin(d*x+c)^2/b/d-a*sin(d*x+c)/b^2/d+1/d/b^3*ln(a+b*sin(d*x+c))*a^2-2*ln
(a+b*sin(d*x+c))/b/d+1/d/a^2*b*ln(a+b*sin(d*x+c))-1/d/a/sin(d*x+c)-b*ln(sin
(d*x+c))/a^2/d
```

Maxima [A] time = 1.00935, size = 123, normalized size = 1.28

$$-\frac{\frac{2\,b\log(\sin(dx+c))}{a^2}-\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}+\frac{2}{a\sin(dx+c)}-\frac{2\left(a^4-2\,a^2b^2+b^4\right)\log(b\sin(dx+c)+a)}{a^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(sin(d*x + c))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2
 + 2/(a*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/(
```

```
a^2*b^3))/d
```

Fricas [A] time = 1.73027, size = 317, normalized size = 3.3

$$\frac{4\,a^{3}b\cos\left(dx+c\right)^{2}-4\,b^{4}\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-4\,a^{3}b-4\,ab^{3}+4\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)\sin}{4\,a^{2}b^{3}d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*b*cos(d*x + c)^2 - 4*b^4*log(1/2*sin(d*x + c))*sin(d*x + c) - 4*
a^3*b - 4*a*b^3 + 4*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)*sin(d*x
 + c) - (2*a^2*b^2*cos(d*x + c)^2 - a^2*b^2)*sin(d*x + c))/(a^2*b^3*d*sin(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19135, size = 142, normalized size = 1.48

$$-\frac{\frac{2\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{2}}-\frac{b\sin\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)}{b^{2}}-\frac{2\,\left(b\sin\left(dx+c\right)-a\right)}{a^{2}\sin\left(dx+c\right)}-\frac{2\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{2}b^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(sin(d*x + c)))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c)
)/b^2 - 2*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^
4)*log(abs(b*sin(d*x + c) + a))/(a^2*b^3))/d
```

3.1315 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^3b^2d}-\frac{\left(2a^2-b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{bd}$$

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((2*a^2 - b^2)*Log[Sin[
c + d*x]])/(a^3*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^3*b^2*d) +
Sin[c + d*x]/(b*d)
```

Rubi [A] time = 0.179845, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^3b^2d}-\frac{\left(2a^2-b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((2*a^2 - b^2)*Log[Sin[
c + d*x]])/(a^3*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^3*b^2*d) +
Sin[c + d*x]/(b*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^3(b^2-x^2)^2}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^4}{ax^3}-\frac{b^4}{a^2x^2}+\frac{-2a^2b^2+b^4}{a^3x}-\frac{(a^2-b^2)^2}{a^3(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^2 d}$$
$$= \frac{b\csc(c+dx)}{a^2 d} - \frac{\csc^2(c+dx)}{2ad} - \frac{(2a^2-b^2)\log(\sin(c+dx))}{a^3 d} - \frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^3b^2 d}$$

Mathematica [A] time = 0.3884, size = 97, normalized size = 0.92

$$\frac{\frac{\frac{2b^2(b^2-2a^2)\log(\sin(c+dx))-2(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^3}+2b\sin(c+dx)}{b^2}+\frac{2b\csc(c+dx)}{a^2}-\frac{\csc^2(c+dx)}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*b*Csc[c + d*x])/a^2 - Csc[c + d*x]^2/a + ((2*b^2*(-2*a^2 + b^2)*Log[Sin
[c + d*x]] - 2*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/a^3 + 2*b*Sin[c + d*x
])/b^2)/(2*d)
```

Maple [A] time = 0.086, size = 140, normalized size = 1.3

$$\frac{\sin(dx+c)}{bd} - \frac{a\ln(a+b\sin(dx+c))}{db^2} + 2\frac{\ln(a+b\sin(dx+c))}{da} - \frac{b^2\ln(a+b\sin(dx+c))}{a^3 d} - \frac{1}{2da(\sin(dx+c))^2} - 2\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] sin(d*x+c)/b/d-1/d/b^2*a*ln(a+b*sin(d*x+c))+2/d/a*ln(a+b*sin(d*x+c))-b^2*ln
(a+b*sin(d*x+c))/a^3/d-1/2/d/a/sin(d*x+c)^2-2*ln(sin(d*x+c))/a/d+b^2*ln(sin
(d*x+c))/a^3/d+1/d/a^2*b/sin(d*x+c)
```

Maxima [A] time = 1.00201, size = 134, normalized size = 1.28

$$\frac{\frac{2\sin(dx+c)}{b} - \frac{2(2a^2-b^2)\log(\sin(dx+c))}{a^3} - \frac{2(a^4-2a^2b^2+b^4)\log(b\sin(dx+c)+a)}{a^3b^2} + \frac{2b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sin(d*x + c)/b - 2*(2*a^2 - b^2)*log(sin(d*x + c))/a^3 - 2*(a^4 - 2*
a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/(a^3*b^2) + (2*b*sin(d*x + c) - a)/(
a^2*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.74236, size = 382, normalized size = 3.64

$$\frac{a^2b^2 + 2\left(a^4 - 2\,a^2b^2 + b^4 - \left(a^4 - 2\,a^2b^2 + b^4\right)\cos\left(dx + c\right)^2\right)\log\left(b\sin\left(dx + c\right) + a\right) + 2\left(2\,a^2b^2 - b^4 - \left(2\,a^2b^2 - b^4\right)\cos\ldots\right.}{2\left(a^3b^2d\cos\left(dx + c\right)^2 - a^3b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b^2 + 2*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x +
 c)^2)*log(b*sin(d*x + c) + a) + 2*(2*a^2*b^2 - b^4 - (2*a^2*b^2 - b^4)*cos
(d*x + c)^2)*log(-1/2*sin(d*x + c)) + 2*(a^3*b*cos(d*x + c)^2 - a^3*b - a*b
^3)*sin(d*x + c))/(a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21629, size = 176, normalized size = 1.68

$$\frac{\frac{2\,\sin(dx+c)}{b} - \frac{2\left(2\,a^2-b^2\right)\log(|\sin(dx+c)|)}{a^3} - \frac{2\left(a^4-2\,a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{a^3b^2} + \frac{6\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2+2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*sin(d*x + c)/b - 2*(2*a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^4
 - 2*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/(a^3*b^2) + (6*a^2*sin(d*x
 + c)^2 - 3*b^2*sin(d*x + c)^2 + 2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c
)^2))/d
```

3.1316 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{(2a^2-b^2)\csc(c+dx)}{a^3d}+\frac{b(2a^2-b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^4bd}+\frac{b\csc^2(c+dx)}{2a^2d}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] ((2*a^2 - b^2)*Csc[c + d*x])/(a^3*d) + (b*Csc[c + d*x]^2)/(2*a^2*d) - Csc[c
 + d*x]^3/(3*a*d) + (b*(2*a^2 - b^2)*Log[Sin[c + d*x]])/(a^4*d) + ((a^2 - b
^2)^2*Log[a + b*Sin[c + d*x]])/(a^4*b*d)
```

Rubi [A] time = 0.174388, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{(2a^2-b^2)\csc(c+dx)}{a^3d}+\frac{b(2a^2-b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^4bd}+\frac{b\csc^2(c+dx)}{2a^2d}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 - b^2)*Csc[c + d*x])/(a^3*d) + (b*Csc[c + d*x]^2)/(2*a^2*d) - Csc[c
 + d*x]^3/(3*a*d) + (b*(2*a^2 - b^2)*Log[Sin[c + d*x]])/(a^4*d) + ((a^2 - b
^2)^2*Log[a + b*Sin[c + d*x]])/(a^4*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^4(b^2-x^2)^2}{x^4(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^4(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^4} - \frac{b^4}{a^2x^3} + \frac{-2a^2b^2+b^4}{a^3x^2} + \frac{2a^2b^2-b^4}{a^4x} + \frac{(a^2-b^2)^2}{a^4(a+x)}\right) dx, x, b\sin(c+dx)\right)}{bd}$$

$$= \frac{(2a^2-b^2)\csc(c+dx)}{a^3 d} + \frac{b\csc^2(c+dx)}{2a^2 d} - \frac{\csc^3(c+dx)}{3ad} + \frac{b(2a^2-b^2)\log(\sin(c+d}{a^4 d}$$

Mathematica [A] time = 0.283179, size = 110, normalized size = 0.92

$$\frac{3a^2b^2\csc^2(c+dx) + 6ab(2a^2-b^2)\csc(c+dx) - 6b^2(b^2-2a^2)\log(\sin(c+dx)) + 6(a^2-b^2)^2\log(a+b\sin(c+dx))}{6a^4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (6*a*b*(2*a^2 - b^2)*Csc[c + d*x] + 3*a^2*b^2*Csc[c + d*x]^2 - 2*a^3*b*Csc[
c + d*x]^3 - 6*b^2*(-2*a^2 + b^2)*Log[Sin[c + d*x]] + 6*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(6*a^4*b*d)
```

Maple [A] time = 0.093, size = 163, normalized size = 1.4

$$\frac{\ln(a+b\sin(dx+c))}{bd} - 2\frac{b\ln(a+b\sin(dx+c))}{da^2} + \frac{b^3\ln(a+b\sin(dx+c))}{da^4} - \frac{1}{3\,da(\sin(dx+c))^3} + 2\frac{1}{da\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] ln(a+b*sin(d*x+c))/b/d-2/d/a^2*b*ln(a+b*sin(d*x+c))+1/d/a^4*b^3*ln(a+b*sin(
d*x+c))-1/3/d/a/sin(d*x+c)^3+2/d/a/sin(d*x+c)-1/d/a^3/sin(d*x+c)*b^2+2*b*ln
(sin(d*x+c))/a^2/d-1/d/a^4*b^3*ln(sin(d*x+c))+1/2/d/a^2*b/sin(d*x+c)^2
```

Maxima [A] time = 0.974852, size = 153, normalized size = 1.27

$$\frac{\frac{6(2a^2b-b^3)\log(\sin(dx+c))}{a^4} + \frac{6(a^4-2a^2b^2+b^4)\log(b\sin(dx+c)+a)}{a^4b} + \frac{3ab\sin(dx+c)+6(2a^2-b^2)\sin(dx+c)^2-2a^2}{a^3\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(2*a^2*b - b^3)*log(sin(d*x + c))/a^4 + 6*(a^4 - 2*a^2*b^2 + b^4)*lo
g(b*sin(d*x + c) + a)/(a^4*b) + (3*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*sin(d
```

```
*x + c)^2 - 2*a^2)/(a^3*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.69746, size = 455, normalized size = 3.79

$$-\frac{3\,a^2b^2\sin(dx+c)+10\,a^3b-6\,ab^3-6\left(2\,a^3b-ab^3\right)\cos(dx+c)^2+6\left(a^4-2\,a^2b^2+b^4-\left(a^4-2\,a^2b^2+b^4\right)\cos(dx+c)\right.}{6\left(a^4bd\cos(dx+c)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*b^2*sin(d*x + c) + 10*a^3*b - 6*a*b^3 - 6*(2*a^3*b - a*b^3)*cos
(d*x + c)^2 + 6*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x +
c)^2)*log(b*sin(d*x + c) + a)*sin(d*x + c) + 6*(2*a^2*b^2 - b^4 - (2*a^2*b^
2 - b^4)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))*sin(d*x + c))/((a^4*b*d*cos(
d*x + c)^2 - a^4*b*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21953, size = 204, normalized size = 1.7

$$\frac{\frac{6\left(2\,a^2b-b^3\right)\log(|\sin(dx+c)|)}{a^4}+\frac{6\left(a^4-2\,a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{a^4b}-\frac{22\,a^2b\sin(dx+c)^3-11\,b^3\sin(dx+c)^3-12\,a^3\sin(dx+c)^2+6\,ab^2\sin(dx+c)^2-3\,a^2b\sin(dx}{a^4\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(2*a^2*b - b^3)*log(abs(sin(d*x + c)))/a^4 + 6*(a^4 - 2*a^2*b^2 + b^
4)*log(abs(b*sin(d*x + c) + a))/(a^4*b) - (22*a^2*b*sin(d*x + c)^3 - 11*b^3
*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 6*a*b^2*sin(d*x + c)^2 - 3*a^2*b*
sin(d*x + c) + 2*a^3)/(a^4*sin(d*x + c)^3))/d
```

3.1317 $\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d}-\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d}+\frac{(a^2-b^2)^2\log(\sin(c+dx))}{a^5d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^5d}+$$

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 -
 b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]
)/(a^5*d)
```

Rubi [A] time = 0.133327, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d}-\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d}+\frac{(a^2-b^2)^2\log(\sin(c+dx))}{a^5d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^5d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 -
 b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]
)/(a^5*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^5}-\frac{b^4}{a^2x^4}+\frac{-2a^2b^2+b^4}{a^3x^3}+\frac{2a^2b^2-b^4}{a^4x^2}+\frac{(a^2-b^2)^2}{a^5x}-\frac{(a^2-b^2)^2}{a^5(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}\\ &= -\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d}+\frac{b\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4ad}+\frac{(a^2}{}\end{aligned}$$

Mathematica [A] time = 3.74642, size = 115, normalized size = 0.78

$$\frac{6a^2\left(2a^2-b^2\right)\csc^2(c+dx)+12ab\left(b^2-2a^2\right)\csc(c+dx)+12\left(a^2-b^2\right)^2\left(\log(\sin(c+dx))-\log(a+b\sin(c+dx))\right)+4a^3}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (12*a*b*(-2*a^2 + b^2)*Csc[c + d*x] + 6*a^2*(2*a^2 - b^2)*Csc[c + d*x]^2 +
4*a^3*b*Csc[c + d*x]^3 - 3*a^4*Csc[c + d*x]^4 + 12*(a^2 - b^2)^2*(Log[Sin[c
 + d*x]] - Log[a + b*Sin[c + d*x]]))/(12*a^5*d)
```

Maple [A] time = 0.093, size = 216, normalized size = 1.5

$$-\frac{\ln(a+b\sin(dx+c))}{da}+2\frac{b^2\ln(a+b\sin(dx+c))}{a^3d}-\frac{\ln(a+b\sin(dx+c))b^4}{da^5}-\frac{1}{4da(\sin(dx+c))^4}+\frac{1}{da(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/a*ln(a+b*sin(d*x+c))+2*b^2*ln(a+b*sin(d*x+c))/a^3/d-1/d/a^5*ln(a+b*sin
(d*x+c))*b^4-1/4/d/a/sin(d*x+c)^4+1/d/a/sin(d*x+c)^2-1/2/d/a^3/sin(d*x+c)^2
*b^2+ln(sin(d*x+c))/a/d-2*b^2*ln(sin(d*x+c))/a^3/d+1/d/a^5*ln(sin(d*x+c))*b
^4-2/d/a^2*b/sin(d*x+c)+1/d/a^4*b^3/sin(d*x+c)+1/3/d/a^2*b/sin(d*x+c)^3
```

Maxima [A] time = 0.998521, size = 188, normalized size = 1.27

$$-\frac{\frac{12\left(a^4-2a^2b^2+b^4\right)\log(b\sin(dx+c)+a)}{a^5}-\frac{12\left(a^4-2a^2b^2+b^4\right)\log(\sin(dx+c))}{a^5}-\frac{4a^2b\sin(dx+c)-12\left(2a^2b-b^3\right)\sin(dx+c)^3-3a^3+6\left(2a^3-ab^2\right)\sin(dx+c)^2}{a^4\sin(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/a^5 - 12*(a^4 - 2
*a^2*b^2 + b^4)*log(sin(d*x + c))/a^5 - (4*a^2*b*sin(d*x + c) - 12*(2*a^2*b
 - b^3)*sin(d*x + c)^3 - 3*a^3 + 6*(2*a^3 - a*b^2)*sin(d*x + c)^2)/(a^4*sin
(d*x + c)^4))/d
```

Fricas [A] time = 1.54603, size = 629, normalized size = 4.25

$$\frac{9a^4-6a^2b^2-6\left(2a^4-a^2b^2\right)\cos(dx+c)^2-12\left(\left(a^4-2a^2b^2+b^4\right)\cos(dx+c)^4+a^4-2a^2b^2+b^4-2\left(a^4-2a^2b^2+b^4\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(9*a^4 - 6*a^2*b^2 - 6*(2*a^4 - a^2*b^2)*cos(d*x + c)^2 - 12*((a^4 - 2
*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2
 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) + 12*((a^4 - 2*a^2*b^2 + b^
4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d
*x + c)^2)*log(-1/2*sin(d*x + c)) - 4*(5*a^3*b - 3*a*b^3 - 3*(2*a^3*b - a*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x +
 c)^2 + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27466, size = 271, normalized size = 1.83

$$\frac{\frac{12\left(a^4-2a^2b^2+b^4\right)\log(|\sin(dx+c)|)}{a^5}-\frac{12\left(a^4b-2a^2b^3+b^5\right)\log(|b\sin(dx+c)+a|)}{a^5b}-\frac{25a^4\sin(dx+c)^4-50a^2b^2\sin(dx+c)^4+25b^4\sin(dx+c)^4+24a^3b\sin(dx+c)\cdots}{a^5\,s\cdots}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(sin(d*x + c)))/a^5 - 12*(a^4*b - 2
*a^2*b^3 + b^5)*log(abs(b*sin(d*x + c) + a))/(a^5*b) - (25*a^4*sin(d*x + c)
^4 - 50*a^2*b^2*sin(d*x + c)^4 + 25*b^4*sin(d*x + c)^4 + 24*a^3*b*sin(d*x +
 c)^3 - 12*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 6*a^2*b^2*sin(d*x
 + c)^2 - 4*a^3*b*sin(d*x + c) + 3*a^4)/(a^5*sin(d*x + c)^4))/d
```

3.1318 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=179

$$\frac{(2a^2-b^2)\csc^3(c+dx)}{3a^3d}-\frac{b(2a^2-b^2)\csc^2(c+dx)}{2a^4d}-\frac{(a^2-b^2)^2\csc(c+dx)}{a^5d}-\frac{b(a^2-b^2)^2\log(\sin(c+dx))}{a^6d}+\frac{b(a^2-b^2)}{}$$

```
[Out] -(((a^2 - b^2)^2*Csc[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^3*d) + (b*Csc[c + d*x]^4)/(
4*a^2*d) - Csc[c + d*x]^5/(5*a*d) - (b*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^
6*d) + (b*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^6*d)
```

Rubi [A] time = 0.204636, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{(2a^2-b^2)\csc^3(c+dx)}{3a^3d}-\frac{b(2a^2-b^2)\csc^2(c+dx)}{2a^4d}-\frac{(a^2-b^2)^2\csc(c+dx)}{a^5d}-\frac{b(a^2-b^2)^2\log(\sin(c+dx))}{a^6d}+\frac{b(a^2-b^2)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Csc[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^3*d) + (b*Csc[c + d*x]^4)/(
4*a^2*d) - Csc[c + d*x]^5/(5*a*d) - (b*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^
6*d) + (b*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^6*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^6(b^2-x^2)^2}{x^6(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^6(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(\frac{b^4}{ax^6} - \frac{b^4}{a^2x^5} + \frac{-2a^2b^2+b^4}{a^3x^4} + \frac{2a^2b^2-b^4}{a^4x^3} + \frac{(a^2-b^2)^2}{a^5x^2} - \frac{(a^2-b^2)^2}{a^6x} + \frac{(a^2-b^2)^2}{a^6(a+x)}\right) dx, x, b\ldots}{d}$$

$$= -\frac{(a^2-b^2)^2\csc(c+dx)}{a^5 d} - \frac{b(2a^2-b^2)\csc^2(c+dx)}{2a^4 d} + \frac{(2a^2-b^2)\csc^3(c+dx)}{3a^3 d} + \frac{b\,\text{cs}\ldots}{\ldots}$$

Mathematica [A] time = 6.12829, size = 179, normalized size = 1.

$$\frac{(2a^2-b^2)\csc^3(c+dx)}{3a^3d} - \frac{b(2a^2-b^2)\csc^2(c+dx)}{2a^4d} - \frac{(a^2-b^2)^2\csc(c+dx)}{a^5d} - \frac{b(a^2-b^2)^2\log(\sin(c+dx))}{a^6d} + \frac{b(a^2-\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Csc[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^3*d) + (b*Csc[c + d*x]^4)/(
4*a^2*d) - Csc[c + d*x]^5/(5*a*d) - (b*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^
6*d) + (b*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^6*d)
```

Maple [A] time = 0.098, size = 274, normalized size = 1.5

$$\frac{b\ln(a+b\sin(dx+c))}{da^2} - 2\frac{b^3\ln(a+b\sin(dx+c))}{da^4} + \frac{b^5\ln(a+b\sin(dx+c))}{da^6} - \frac{1}{5da(\sin(dx+c))^5} + \frac{2}{3da(\sin(dx\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/a^2*b*ln(a+b*sin(d*x+c))-2/d/a^4*b^3*ln(a+b*sin(d*x+c))+1/d/a^6*b^5*ln(
a+b*sin(d*x+c))-1/5/d/a/sin(d*x+c)^5+2/3/d/a/sin(d*x+c)^3-1/3/d/a^3/sin(d*x
+c)^3*b^2-1/d/a/sin(d*x+c)+2/d/a^3/sin(d*x+c)*b^2-1/d/a^5/sin(d*x+c)*b^4-1/
d/a^2*b/sin(d*x+c)^2+1/2/d/a^4*b^3/sin(d*x+c)^2+1/4/d/a^2*b/sin(d*x+c)^4-b*
ln(sin(d*x+c))/a^2/d+2/d/a^4*b^3*ln(sin(d*x+c))-1/d/a^6*b^5*ln(sin(d*x+c))
```

Maxima [A] time = 1.00004, size = 230, normalized size = 1.28

$$\frac{\frac{60(a^4b-2a^2b^3+b^5)\log(b\sin(dx+c)+a)}{a^6} - \frac{60(a^4b-2a^2b^3+b^5)\log(\sin(dx+c))}{a^6} + \frac{15a^3b\sin(dx+c)-60(a^4-2a^2b^2+b^4)\sin(dx+c)^4-12a^4-30(2a^3b-ab^3)\sin\ldots}{a^5\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(a^4*b - 2*a^2*b^3 + b^5)*log(b*sin(d*x + c) + a)/a^6 - 60*(a^4*b
- 2*a^2*b^3 + b^5)*log(sin(d*x + c))/a^6 + (15*a^3*b*sin(d*x + c) - 60*(a^4
 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 - 12*a^4 - 30*(2*a^3*b - a*b^3)*sin(d*x
+ c)^3 + 20*(2*a^4 - a^2*b^2)*sin(d*x + c)^2)/(a^5*sin(d*x + c)^5))/d
```

Fricas [B] time = 1.52689, size = 807, normalized size = 4.51

$$-\frac{32\,a^5-100\,a^3b^2+60\,ab^4+60\left(a^5-2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)^4-20\left(4\,a^5-11\,a^3b^2+6\,ab^4\right)\cos\left(dx+c\right)^2-60\left(a^4b-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(32*a^5 - 100*a^3*b^2 + 60*a*b^4 + 60*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d
*x + c)^4 - 20*(4*a^5 - 11*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2 - 60*(a^4*b -
2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b - 2*a
^2*b^3 + b^5)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a)*sin(d*x + c) + 60*(a^
4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b
 - 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))*sin(d*x + c) + 15
*(3*a^4*b - 2*a^2*b^3 - 2*(2*a^4*b - a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))
/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20982, size = 339, normalized size = 1.89

$$-\frac{\frac{60\left(a^4b-2\,a^2b^3+b^5\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^6}-\frac{60\left(a^4b^2-2\,a^2b^4+b^6\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^6b}-\frac{137\,a^4b\sin\left(dx+c\right)^5-274\,a^2b^3\sin\left(dx+c\right)^5+137\,b^5\sin\left(dx+c\right)^5-60\,a^5\sin\left(dx\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*(a^4*b - 2*a^2*b^3 + b^5)*log(abs(sin(d*x + c)))/a^6 - 60*(a^4*b^
2 - 2*a^2*b^4 + b^6)*log(abs(b*sin(d*x + c) + a))/(a^6*b) - (137*a^4*b*sin(
d*x + c)^5 - 274*a^2*b^3*sin(d*x + c)^5 + 137*b^5*sin(d*x + c)^5 - 60*a^5*s
in(d*x + c)^4 + 120*a^3*b^2*sin(d*x + c)^4 - 60*a*b^4*sin(d*x + c)^4 - 60*a
^4*b*sin(d*x + c)^3 + 30*a^2*b^3*sin(d*x + c)^3 + 40*a^5*sin(d*x + c)^2 - 2
0*a^3*b^2*sin(d*x + c)^2 + 15*a^4*b*sin(d*x + c) - 12*a^5)/(a^6*sin(d*x + c
)^5))/d
```

3.1319 $\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=212

$$\frac{(2a^2-b^2)\csc^4(c+dx)}{4a^3d}-\frac{b(2a^2-b^2)\csc^3(c+dx)}{3a^4d}-\frac{(a^2-b^2)^2\csc^2(c+dx)}{2a^5d}+\frac{b(a^2-b^2)^2\csc(c+dx)}{a^6d}+\frac{b^2(a^2-b^2)}{}$$

```
[Out] (b*(a^2 - b^2)^2*Csc[c + d*x])/(a^6*d) - ((a^2 - b^2)^2*Csc[c + d*x]^2)/(2*
a^5*d) - (b*(2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^4*d) + ((2*a^2 - b^2)*Csc[c
+ d*x]^4)/(4*a^3*d) + (b*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(6*a*d)
 + (b^2*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^7*d) - (b^2*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(a^7*d)
```

Rubi [A] time = 0.240817, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{(2a^2-b^2)\csc^4(c+dx)}{4a^3d}-\frac{b(2a^2-b^2)\csc^3(c+dx)}{3a^4d}-\frac{(a^2-b^2)^2\csc^2(c+dx)}{2a^5d}+\frac{b(a^2-b^2)^2\csc(c+dx)}{a^6d}+\frac{b^2(a^2-b^2)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(a^2 - b^2)^2*Csc[c + d*x])/(a^6*d) - ((a^2 - b^2)^2*Csc[c + d*x]^2)/(2*
a^5*d) - (b*(2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^4*d) + ((2*a^2 - b^2)*Csc[c
+ d*x]^4)/(4*a^3*d) + (b*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(6*a*d)
 + (b^2*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^7*d) - (b^2*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(a^7*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^7(b^2-x^2)^2}{x^7(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^7(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \left(\frac{b^4}{ax^7} - \frac{b^4}{a^2x^6} + \frac{-2a^2b^2+b^4}{a^3x^5} + \frac{2a^2b^2-b^4}{a^4x^4} + \frac{(a^2-b^2)^2}{a^5x^3} - \frac{(a^2-b^2)^2}{a^6x^2} + \frac{(a^2-b^2)^2}{a^7x} - \frac{(a^2-b^2)^2}{a^7(a+x)}\right.\right.}{d}$$

$$= \frac{b(a^2-b^2)^2\csc(c+dx)}{a^6d} - \frac{(a^2-b^2)^2\csc^2(c+dx)}{2a^5d} - \frac{b(2a^2-b^2)\csc^3(c+dx)}{3a^4d} + \frac{(2a^2 -}{}$$

Mathematica [A] time = 2.86696, size = 165, normalized size = 0.78

$$\frac{15a^4(2a^2-b^2)\csc^4(c+dx) + 20a^3b(b^2-2a^2)\csc^3(c+dx) - 30a^2(a^2-b^2)^2\csc^2(c+dx) + 60ab(a^2-b^2)^2\csc(c+dx)}{60a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*a*b*(a^2 - b^2)^2*Csc[c + d*x] - 30*a^2*(a^2 - b^2)^2*Csc[c + d*x]^2 +
20*a^3*b*(-2*a^2 + b^2)*Csc[c + d*x]^3 + 15*a^4*(2*a^2 - b^2)*Csc[c + d*x]^
4 + 12*a^5*b*Csc[c + d*x]^5 - 10*a^6*Csc[c + d*x]^6 + 60*(-(a^2*b) + b^3)^2
*(Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]]))/(60*a^7*d)
```

Maple [A] time = 0.105, size = 330, normalized size = 1.6

$$-\frac{b^2\ln(a+b\sin(dx+c))}{a^3d} + 2\frac{\ln(a+b\sin(dx+c))b^4}{da^5} - \frac{b^6\ln(a+b\sin(dx+c))}{da^7} - \frac{1}{6da(\sin(dx+c))^6} + \frac{1}{2da(\sin(dx +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x)
```

```
[Out] -b^2*ln(a+b*sin(d*x+c))/a^3/d+2/d/a^5*ln(a+b*sin(d*x+c))*b^4-1/d/a^7*b^6*ln
(a+b*sin(d*x+c))-1/6/d/a/sin(d*x+c)^6+1/2/d/a/sin(d*x+c)^4-1/4/d/a^3/sin(d*
x+c)^4*b^2-1/2/d/a/sin(d*x+c)^2+1/d/a^3/sin(d*x+c)^2*b^2-1/2/d/a^5/sin(d*x+
c)^2*b^4-2/3/d/a^2*b/sin(d*x+c)^3+1/3/d/a^4*b^3/sin(d*x+c)^3+b^2*ln(sin(d*x
+c))/a^3/d-2/d/a^5*ln(sin(d*x+c))*b^4+1/d/a^7*b^6*ln(sin(d*x+c))+1/5/d/a^2*
b/sin(d*x+c)^5+1/d/a^2*b/sin(d*x+c)-2/d/a^4*b^3/sin(d*x+c)+1/d/a^6*b^5/sin(
d*x+c)
```

Maxima [A] time = 0.990868, size = 278, normalized size = 1.31

$$-\frac{\frac{60(a^4b^2-2a^2b^4+b^6)\log(b\sin(dx+c)+a)}{a^7} - \frac{60(a^4b^2-2a^2b^4+b^6)\log(\sin(dx+c))}{a^7} - \frac{12a^4b\sin(dx+c)+60(a^4b-2a^2b^3+b^5)\sin(dx+c)^5-10a^5-30(a^5-2a^3b^2+a}{a^6\sin(d}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*(a^4*b^2 - 2*a^2*b^4 + b^6)*log(b*sin(d*x + c) + a)/a^7 - 60*(a^4
*b^2 - 2*a^2*b^4 + b^6)*log(sin(d*x + c))/a^7 - (12*a^4*b*sin(d*x + c) + 60
*(a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^5 - 10*a^5 - 30*(a^5 - 2*a^3*b^2 +
a*b^4)*sin(d*x + c)^4 - 20*(2*a^4*b - a^2*b^3)*sin(d*x + c)^3 + 15*(2*a^5 -
 a^3*b^2)*sin(d*x + c)^2)/(a^6*sin(d*x + c)^6))/d
```

Fricas [B] time = 1.67498, size = 1031, normalized size = 4.86

$$\frac{10\,a^6 - 45\,a^4b^2 + 30\,a^2b^4 + 30\left(a^6 - 2\,a^4b^2 + a^2b^4\right)\cos\left(dx+c\right)^4 - 15\left(2\,a^6 - 7\,a^4b^2 + 4\,a^2b^4\right)\cos\left(dx+c\right)^2 - 60\left(\left(a^4b^2 \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(10*a^6 - 45*a^4*b^2 + 30*a^2*b^4 + 30*(a^6 - 2*a^4*b^2 + a^2*b^4)*cos
(d*x + c)^4 - 15*(2*a^6 - 7*a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^2 - 60*((a^4*
b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^6 - a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*
b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d
*x + c)^2)*log(b*sin(d*x + c) + a) + 60*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*
x + c)^6 - a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*
x + c)^4 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*log(-1/2*sin(d*x +
 c)) - 4*(8*a^5*b - 25*a^3*b^3 + 15*a*b^5 + 15*(a^5*b - 2*a^3*b^3 + a*b^5)*
cos(d*x + c)^4 - 5*(4*a^5*b - 11*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2)*sin(d*x
 + c))/(a^7*d*cos(d*x + c)^6 - 3*a^7*d*cos(d*x + c)^4 + 3*a^7*d*cos(d*x + c
)^2 - a^7*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35675, size = 406, normalized size = 1.92

$$\frac{\frac{60\left(a^4b^2-2\,a^2b^4+b^6\right)\log\left(|\sin(dx+c)|\right)}{a^7} - \frac{60\left(a^4b^3-2\,a^2b^5+b^7\right)\log\left(|b\sin(dx+c)+a|\right)}{a^7b} - \frac{147\,a^4b^2\sin(dx+c)^6-294\,a^2b^4\sin(dx+c)^6+147\,b^6\sin(dx+c)^6-60\,a^5b\,\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*(a^4*b^2 - 2*a^2*b^4 + b^6)*log(abs(sin(d*x + c)))/a^7 - 60*(a^4*b
^3 - 2*a^2*b^5 + b^7)*log(abs(b*sin(d*x + c) + a))/(a^7*b) - (147*a^4*b^2*s
```

```
in(d*x + c)^6 - 294*a^2*b^4*sin(d*x + c)^6 + 147*b^6*sin(d*x + c)^6 - 60*a^
5*b*sin(d*x + c)^5 + 120*a^3*b^3*sin(d*x + c)^5 - 60*a*b^5*sin(d*x + c)^5 +
 30*a^6*sin(d*x + c)^4 - 60*a^4*b^2*sin(d*x + c)^4 + 30*a^2*b^4*sin(d*x + c
)^4 + 40*a^5*b*sin(d*x + c)^3 - 20*a^3*b^3*sin(d*x + c)^3 - 30*a^6*sin(d*x
+ c)^2 + 15*a^4*b^2*sin(d*x + c)^2 - 12*a^5*b*sin(d*x + c) + 10*a^6)/(a^7*s
in(d*x + c)^6))/d
```

3.1320 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=467

$$-\frac{a\left(-245a^4b^2+161a^2b^4+105a^6-15b^6\right)\cos(c+dx)}{105b^8d}+\frac{2a^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}+\frac{\left(-85a^2b^2+40a^4+\right.}{}$$

```
[Out] -((128*a^8 - 320*a^6*b^2 + 240*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*x)/(128*b^9) +
 (2*a^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(b^9*d) - (a*(105*a^6 - 245*a^4*b^2 + 161*a^2*b^4 - 15*b^6)*Cos[c + d*x])/
(105*b^8*d) + ((64*a^6 - 144*a^4*b^2 + 88*a^2*b^4 - 5*b^6)*Cos[c + d*x]*Sin
[c + d*x])/(128*b^7*d) - (a*(35*a^4 - 77*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin
[c + d*x]^2)/(105*b^6*d) + ((48*a^4 - 104*a^2*b^2 + 59*b^4)*Cos[c + d*x]*Si
n[c + d*x]^3)/(192*b^5*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d) - ((28*a^
4 - 60*a^2*b^2 + 35*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(140*a*b^4*d) - (b*Co
s[c + d*x]*Sin[c + d*x]^5)/(5*a^2*d) + ((40*a^4 - 85*a^2*b^2 + 48*b^4)*Cos[
c + d*x]*Sin[c + d*x]^5)/(240*a^2*b^3*d) - (a*Cos[c + d*x]*Sin[c + d*x]^6)/
(7*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^7)/(8*b*d)
```

Rubi [A] time = 1.81436, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a\left(-245a^4b^2+161a^2b^4+105a^6-15b^6\right)\cos(c+dx)}{105b^8d}+\frac{2a^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}+\frac{\left(-85a^2b^2+40a^4+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((128*a^8 - 320*a^6*b^2 + 240*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*x)/(128*b^9) +
 (2*a^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(b^9*d) - (a*(105*a^6 - 245*a^4*b^2 + 161*a^2*b^4 - 15*b^6)*Cos[c + d*x])/
(105*b^8*d) + ((64*a^6 - 144*a^4*b^2 + 88*a^2*b^4 - 5*b^6)*Cos[c + d*x]*Sin
[c + d*x])/(128*b^7*d) - (a*(35*a^4 - 77*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin
[c + d*x]^2)/(105*b^6*d) + ((48*a^4 - 104*a^2*b^2 + 59*b^4)*Cos[c + d*x]*Si
n[c + d*x]^3)/(192*b^5*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d) - ((28*a^
4 - 60*a^2*b^2 + 35*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(140*a*b^4*d) - (b*Co
s[c + d*x]*Sin[c + d*x]^5)/(5*a^2*d) + ((40*a^4 - 85*a^2*b^2 + 48*b^4)*Cos[
c + d*x]*Sin[c + d*x]^5)/(240*a^2*b^3*d) - (a*Cos[c + d*x]*Sin[c + d*x]^6)/
(7*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^7)/(8*b*d)
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
```

```
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - \frac{b\cos(c+dx)\sin^5(c+dx)}{5a^2d} - \frac{a\cos(c+dx)\sin^6(c+dx)}{7b^2d}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - \frac{b\cos(c+dx)\sin^5(c+dx)}{5a^2d} + \frac{\left(40a^4-85a^2b^2+48b^4\right)\cos}{240a^2b^3}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - \frac{\left(28a^4-60a^2b^2+35b^4\right)\cos(c+dx)\sin^4(c+dx)}{140ab^4d} - b\cos$$

$$= \frac{\left(48a^4-104a^2b^2+59b^4\right)\cos(c+dx)\sin^3(c+dx)}{192b^5d} + \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - (28$$

$$= -\frac{a\left(35a^4-77a^2b^2+45b^4\right)\cos(c+dx)\sin^2(c+dx)}{105b^6d} + \frac{\left(48a^4-104a^2b^2+59b^4\right)\cos}{192b^5d}$$

$$= \frac{\left(64a^6-144a^4b^2+88a^2b^4-5b^6\right)\cos(c+dx)\sin(c+dx)}{128b^7d} - \frac{a\left(35a^4-77a^2b^2+45b\right.}{10}$$

$$= -\frac{a\left(105a^6-245a^4b^2+161a^2b^4-15b^6\right)\cos(c+dx)}{105b^8d} + \frac{\left(64a^6-144a^4b^2+88a^2b^4-\right.}{128}$$

$$= -\frac{\left(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8\right)x}{128b^9} - \frac{a\left(105a^6-245a^4b^2+161a^2b^4-\right.}{105b^8d}$$

$$= -\frac{\left(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8\right)x}{128b^9} - \frac{a\left(105a^6-245a^4b^2+161a^2b^4-\right.}{105b^8d}$$

$$= -\frac{\left(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8\right)x}{128b^9} - \frac{a\left(105a^6-245a^4b^2+161a^2b^4-\right.}{105b^8d}$$

$$= -\frac{\left(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8\right)x}{128b^9} + \frac{2a^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.}{b^9d}$$

Mathematica [A] time = 3.32049, size = 403, normalized size = 0.86

$$26880a^6b^2\sin(2(c+dx)) - 53760a^4b^4\sin(2(c+dx)) - 3360a^4b^4\sin(4(c+dx)) + 25200a^2b^6\sin(2(c+dx)) + 5040a^2b$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-107520*a^8*c + 268800*a^6*b^2*c - 201600*a^4*b^4*c + 33600*a^2*b^6*c + 42
00*b^8*c - 107520*a^8*d*x + 268800*a^6*b^2*d*x - 201600*a^4*b^4*d*x + 33600
*a^2*b^6*d*x + 4200*b^8*d*x + 215040*a^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 1680*a*b*(64*a^6 - 144*a^4*b^2 + 88*a^2*
b^4 - 5*b^6)*Cos[c + d*x] + 560*(16*a^5*b^3 - 28*a^3*b^5 + 9*a*b^7)*Cos[3*(
c + d*x)] - 1344*a^3*b^5*Cos[5*(c + d*x)] + 1680*a*b^7*Cos[5*(c + d*x)] + 2
40*a*b^7*Cos[7*(c + d*x)] + 26880*a^6*b^2*Sin[2*(c + d*x)] - 53760*a^4*b^4*
Sin[2*(c + d*x)] + 25200*a^2*b^6*Sin[2*(c + d*x)] + 1680*b^8*Sin[2*(c + d*x
)] - 3360*a^4*b^4*Sin[4*(c + d*x)] + 5040*a^2*b^6*Sin[4*(c + d*x)] - 840*b^
8*Sin[4*(c + d*x)] + 560*a^2*b^6*Sin[6*(c + d*x)] - 560*b^8*Sin[6*(c + d*x)
] - 105*b^8*Sin[8*(c + d*x)])/(107520*b^9*d)
```

Maple [B] time = 0.099, size = 2587, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^14*a+5/d/b^7/(1+tan(1
/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7*a^6-29/4/d/b^5/(1+tan(1/2*d*x+1/2*c
)^2)^8*tan(1/2*d*x+1/2*c)^7*a^4-2/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*
d*x+1/2*c)^14*a^7-322/3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)
^8*a^3+5/64/d/b*arctan(tan(1/2*d*x+1/2*c))-70/d/b^8/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^8*a^7+490/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2
*d*x+1/2*c)^8*a^5-30/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12
*a^3+1/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)*a^6+6/d/b^2/(1+t
an(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4*a+9/4/d/b^5/(1+tan(1/2*d*x+1/2*
c)^2)^8*tan(1/2*d*x+1/2*c)^15*a^4-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/
2*d*x+1/2*c)^14*a^3+2/d/b^9*a^9/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x
+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^6*a+5/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^6-2/d/b^8/(1+tan(1/2*d*x+
1/2*c)^2)^8*a^7+14/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*a^5-46/15/d/b^4/(1+ta
n(1/2*d*x+1/2*c)^2)^8*a^3+895/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^11-397/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13+5/
64/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-5/64/d/b/(1+tan(1/2
*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)+397/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*
tan(1/2*d*x+1/2*c)^3-895/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2
*c)^5-2/d/b^9*arctan(tan(1/2*d*x+1/2*c))*a^8+2/7/d/b^2/(1+tan(1/2*d*x+1/2*c
)^2)^8*a+1765/192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7-1765/
192/d/b/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9+2/d/b^2/(1+tan(1/2*
d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^12*a-14/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8
*tan(1/2*d*x+1/2*c)^12*a^7+38/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+
1/2*c)^12*a^5+2/7/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2*a+5
/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3*a^6-113/24/d/b^3/(1+
tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11*a^2-5/d/b^7/(1+tan(1/2*d*x+1/
2*c)^2)^8*tan(1/2*d*x+1/2*c)^13*a^6+37/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*t
an(1/2*d*x+1/2*c)^13*a^4-15/4/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^4+5/8/d/b^
3*arctan(tan(1/2*d*x+1/2*c))*a^2+61/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan
(1/2*d*x+1/2*c)^3*a^2+314/3/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/
2*c)^10*a^5+85/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^7*a^2
-1486/15/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6*a^3-2/d*a^3/
b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)-70/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6*a^7+470/3/d/b^6/
(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^6*a^5+29/4/d/b^5/(1+tan(1/2*d
*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9*a^4-11/8/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)
^8*tan(1/2*d*x+1/2*c)^15*a^2+10/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*
x+1/2*c)^8*a+10/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^10*a-9/
d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11*a^6+57/4/d/b^5/(1+ta
n(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11*a^4-85/24/d/b^3/(1+tan(1/2*d*x+
1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9*a^2-42/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*ta
n(1/2*d*x+1/2*c)^10*a^7-61/24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+
1/2*c)^13*a^2-1/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15*a^6-
218/3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^10*a^3+278/3/d/b^
6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4*a^5-838/15/d/b^4/(1+tan(1
/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4*a^3+6/d/b^6/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^14*a^5+6/d*a^5/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d*a^7/b^7/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-42/d/b^8/(1+tan(1/2*d*x+1
/2*c)^2)^8*tan(1/2*d*x+1/2*c)^4*a^7-9/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*ta
```

```
n(1/2*d*x+1/2*c)*a^4+11/8/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*
c)*a^2-14/d/b^8/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2*a^7+94/3/d/
b^6/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2*a^5-278/15/d/b^4/(1+tan
(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^2*a^3-5/d/b^7/(1+tan(1/2*d*x+1/2*c)
^2)^8*tan(1/2*d*x+1/2*c)^9*a^6-37/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/
2*d*x+1/2*c)^3*a^4+9/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5*
a^6-57/4/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5*a^4+113/24/d
/b^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00193, size = 1659, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] [1/13440*(1920*a*b^7*cos(d*x + c)^7 - 2688*a^3*b^5*cos(d*x + c)^5 + 4480*(a
^5*b^3 - a^3*b^5)*cos(d*x + c)^3 - 105*(128*a^8 - 320*a^6*b^2 + 240*a^4*b^4
 - 40*a^2*b^6 - 5*b^8)*d*x + 6720*(a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b
^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2
*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 13440*(a^7*b - 2*a^5*b^3 +
a^3*b^5)*cos(d*x + c) - 35*(48*b^8*cos(d*x + c)^7 - 8*(8*a^2*b^6 + b^8)*cos
(d*x + c)^5 + 2*(48*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*cos(d*x + c)^3 - 3*(64*a^
6*b^2 - 112*a^4*b^4 + 40*a^2*b^6 + 5*b^8)*cos(d*x + c))*sin(d*x + c))/(b^9*
d), 1/13440*(1920*a*b^7*cos(d*x + c)^7 - 2688*a^3*b^5*cos(d*x + c)^5 + 4480
*(a^5*b^3 - a^3*b^5)*cos(d*x + c)^3 - 105*(128*a^8 - 320*a^6*b^2 + 240*a^4*
b^4 - 40*a^2*b^6 - 5*b^8)*d*x - 13440*(a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2
- b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 13440
*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c) - 35*(48*b^8*cos(d*x + c)^7 - 8
*(8*a^2*b^6 + b^8)*cos(d*x + c)^5 + 2*(48*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*cos
(d*x + c)^3 - 3*(64*a^6*b^2 - 112*a^4*b^4 + 40*a^2*b^6 + 5*b^8)*cos(d*x + c
))*sin(d*x + c))/(b^9*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

[Out] Timed out

Giac [B] time = 1.24775, size = 1679, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13440*(105*(128*a^8 - 320*a^6*b^2 + 240*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*(d
*x + c)/b^9 - 26880*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*b^9) + 2*(6720*a^6*b*tan(1/2*d*x + 1/2*c)^15 - 1512
0*a^4*b^3*tan(1/2*d*x + 1/2*c)^15 + 9240*a^2*b^5*tan(1/2*d*x + 1/2*c)^15 -
525*b^7*tan(1/2*d*x + 1/2*c)^15 + 13440*a^7*tan(1/2*d*x + 1/2*c)^14 - 40320
*a^5*b^2*tan(1/2*d*x + 1/2*c)^14 + 40320*a^3*b^4*tan(1/2*d*x + 1/2*c)^14 -
13440*a*b^6*tan(1/2*d*x + 1/2*c)^14 + 33600*a^6*b*tan(1/2*d*x + 1/2*c)^13 -
 62160*a^4*b^3*tan(1/2*d*x + 1/2*c)^13 + 17080*a^2*b^5*tan(1/2*d*x + 1/2*c)
^13 + 13895*b^7*tan(1/2*d*x + 1/2*c)^13 + 94080*a^7*tan(1/2*d*x + 1/2*c)^12
 - 255360*a^5*b^2*tan(1/2*d*x + 1/2*c)^12 + 201600*a^3*b^4*tan(1/2*d*x + 1/
2*c)^12 - 13440*a*b^6*tan(1/2*d*x + 1/2*c)^12 + 60480*a^6*b*tan(1/2*d*x + 1
/2*c)^11 - 95760*a^4*b^3*tan(1/2*d*x + 1/2*c)^11 + 31640*a^2*b^5*tan(1/2*d*
x + 1/2*c)^11 - 31325*b^7*tan(1/2*d*x + 1/2*c)^11 + 282240*a^7*tan(1/2*d*x
+ 1/2*c)^10 - 703360*a^5*b^2*tan(1/2*d*x + 1/2*c)^10 + 488320*a^3*b^4*tan(1
/2*d*x + 1/2*c)^10 - 67200*a*b^6*tan(1/2*d*x + 1/2*c)^10 + 33600*a^6*b*tan(
1/2*d*x + 1/2*c)^9 - 48720*a^4*b^3*tan(1/2*d*x + 1/2*c)^9 + 23800*a^2*b^5*t
an(1/2*d*x + 1/2*c)^9 + 61775*b^7*tan(1/2*d*x + 1/2*c)^9 + 470400*a^7*tan(1
/2*d*x + 1/2*c)^8 - 1097600*a^5*b^2*tan(1/2*d*x + 1/2*c)^8 + 721280*a^3*b^4
*tan(1/2*d*x + 1/2*c)^8 - 67200*a*b^6*tan(1/2*d*x + 1/2*c)^8 - 33600*a^6*b*
tan(1/2*d*x + 1/2*c)^7 + 48720*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 23800*a^2*b
^5*tan(1/2*d*x + 1/2*c)^7 - 61775*b^7*tan(1/2*d*x + 1/2*c)^7 + 470400*a^7*t
an(1/2*d*x + 1/2*c)^6 - 1052800*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 + 665728*a^3
*b^4*tan(1/2*d*x + 1/2*c)^6 - 40320*a*b^6*tan(1/2*d*x + 1/2*c)^6 - 60480*a^
6*b*tan(1/2*d*x + 1/2*c)^5 + 95760*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 31640*a
^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 31325*b^7*tan(1/2*d*x + 1/2*c)^5 + 282240*a
^7*tan(1/2*d*x + 1/2*c)^4 - 622720*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 + 375424*
a^3*b^4*tan(1/2*d*x + 1/2*c)^4 - 40320*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 33600
*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 62160*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 1708
0*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 13895*b^7*tan(1/2*d*x + 1/2*c)^3 + 94080
*a^7*tan(1/2*d*x + 1/2*c)^2 - 210560*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 + 12454
4*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 - 1920*a*b^6*tan(1/2*d*x + 1/2*c)^2 - 6720
*a^6*b*tan(1/2*d*x + 1/2*c) + 15120*a^4*b^3*tan(1/2*d*x + 1/2*c) - 9240*a^2
*b^5*tan(1/2*d*x + 1/2*c) + 525*b^7*tan(1/2*d*x + 1/2*c) + 13440*a^7 - 3136
0*a^5*b^2 + 20608*a^3*b^4 - 1920*a*b^6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*b^8
))/d
```

3.1321 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=408

$$\frac{\left(-245a^4b^2+161a^2b^4+105a^6-15b^6\right)\cos(c+dx)}{105b^7d}-\frac{2a^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}+\frac{\left(-60a^2b^2+28a^4+35b\right.}{14}$$

```
[Out] (a*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*x)/(16*b^8) - (2*a^2*(a^2 - b
^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^8*d) + ((105
*a^6 - 245*a^4*b^2 + 161*a^2*b^4 - 15*b^6)*Cos[c + d*x])/(105*b^7*d) - (a*(
8*a^4 - 18*a^2*b^2 + 11*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^6*d) + ((35*a
^4 - 77*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(105*b^5*d) + (Cos[c
 + d*x]*Sin[c + d*x]^3)/(3*a*d) - ((6*a^4 - 13*a^2*b^2 + 8*b^4)*Cos[c + d*x
]*Sin[c + d*x]^3)/(24*a*b^4*d) - (b*Cos[c + d*x]*Sin[c + d*x]^4)/(4*a^2*d)
+ ((28*a^4 - 60*a^2*b^2 + 35*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(140*a^2*b^3
*d) - (a*Cos[c + d*x]*Sin[c + d*x]^5)/(6*b^2*d) + (Cos[c + d*x]*Sin[c + d*x
]^6)/(7*b*d)
```

Rubi [A] time = 1.45786, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{\left(-245a^4b^2+161a^2b^4+105a^6-15b^6\right)\cos(c+dx)}{105b^7d}-\frac{2a^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}+\frac{\left(-60a^2b^2+28a^4+35b\right.}{14}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (a*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*x)/(16*b^8) - (2*a^2*(a^2 - b
^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^8*d) + ((105
*a^6 - 245*a^4*b^2 + 161*a^2*b^4 - 15*b^6)*Cos[c + d*x])/(105*b^7*d) - (a*(
8*a^4 - 18*a^2*b^2 + 11*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^6*d) + ((35*a
^4 - 77*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(105*b^5*d) + (Cos[c
 + d*x]*Sin[c + d*x]^3)/(3*a*d) - ((6*a^4 - 13*a^2*b^2 + 8*b^4)*Cos[c + d*x
]*Sin[c + d*x]^3)/(24*a*b^4*d) - (b*Cos[c + d*x]*Sin[c + d*x]^4)/(4*a^2*d)
+ ((28*a^4 - 60*a^2*b^2 + 35*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(140*a^2*b^3
*d) - (a*Cos[c + d*x]*Sin[c + d*x]^5)/(6*b^2*d) + (Cos[c + d*x]*Sin[c + d*x
]^6)/(7*b*d)
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
```

```
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{b\cos(c+dx)\sin^4(c+dx)}{4a^2d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6b^2d}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{b\cos(c+dx)\sin^4(c+dx)}{4a^2d} + \frac{\left(28a^4-60a^2b^2+35b^4\right)\cos}{140a^2b}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{\left(6a^4-13a^2b^2+8b^4\right)\cos(c+dx)\sin^3(c+dx)}{24ab^4d} - \frac{b\cos(}{}$$

$$= \frac{\left(35a^4-77a^2b^2+45b^4\right)\cos(c+dx)\sin^2(c+dx)}{105b^5d} + \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{\left(6a^4\right.}{}$$

$$= -\frac{a\left(8a^4-18a^2b^2+11b^4\right)\cos(c+dx)\sin(c+dx)}{16b^6d} + \frac{\left(35a^4-77a^2b^2+45b^4\right)\cos(c+}{105b^5d}$$

$$= \frac{\left(105a^6-245a^4b^2+161a^2b^4-15b^6\right)\cos(c+dx)}{105b^7d} - \frac{a\left(8a^4-18a^2b^2+11b^4\right)\cos(c+}{16b^6d}$$

$$= \frac{a\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^8} + \frac{\left(105a^6-245a^4b^2+161a^2b^4-15b^6\right)\cos(c+}{105b^7d}$$

$$= \frac{a\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^8} + \frac{\left(105a^6-245a^4b^2+161a^2b^4-15b^6\right)\cos(c+}{105b^7d}$$

$$= \frac{a\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^8} + \frac{\left(105a^6-245a^4b^2+161a^2b^4-15b^6\right)\cos(c+}{105b^7d}$$

$$= \frac{a\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^8} - \frac{2a^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^8d} + \frac{(10}{}$$

Mathematica [A] time = 2.99382, size = 324, normalized size = 0.79

$$-\frac{1680a^5b^2\sin(2(c+dx)) - 3360a^3b^4\sin(2(c+dx)) - 210a^3b^4\sin(4(c+dx)) - 84a^2b^5\cos(5(c+dx)) + 105b\left(144a^4b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-6720*a^7*c + 16800*a^5*b^2*c - 12600*a^3*b^4*c + 2100*a*b^6*c - 6720*a^7
*d*x + 16800*a^5*b^2*d*x - 12600*a^3*b^4*d*x + 2100*a*b^6*d*x + 13440*a^2*(
a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 105*b*(
-64*a^6 + 144*a^4*b^2 - 88*a^2*b^4 + 5*b^6)*Cos[c + d*x] + 35*(16*a^4*b^3 -
 28*a^2*b^5 + 9*b^7)*Cos[3*(c + d*x)] - 84*a^2*b^5*Cos[5*(c + d*x)] + 105*b
^7*Cos[5*(c + d*x)] + 15*b^7*Cos[7*(c + d*x)] + 1680*a^5*b^2*Sin[2*(c + d*x
)] - 3360*a^3*b^4*Sin[2*(c + d*x)] + 1575*a*b^6*Sin[2*(c + d*x)] - 210*a^3*
b^4*Sin[4*(c + d*x)] + 315*a*b^6*Sin[4*(c + d*x)] + 35*a*b^6*Sin[6*(c + d*x
)])/(6720*b^8*d)
```

Maple [B] time = 0.102, size = 1808, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -66/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4*a^4-4/d/b^6/(1+ta
n(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3*a^5-5/8/d/b^2*a*arctan(tan(1/2*d
*x+1/2*c))+6/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^2+12/
d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^6-272/3/d/b^5/(1+t
an(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6*a^4+176/3/d/b^3/(1+tan(1/2*d*x+
1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6*a^2-5/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan
(1/2*d*x+1/2*c)^5*a^5-5/d/b^6*arctan(tan(1/2*d*x+1/2*c))*a^5+2/d/b^2/(a^2-b
^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+15/4
/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3+2/d/b^8*arctan(tan(1/2*d*x+1/2*c))*a^
7+2/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*a^6-6/d/b/(1+tan(1/2*d*x+1/2*c)^2)^7*t
an(1/2*d*x+1/2*c)^4-2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12-
10/d/b/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8-14/3/d/b^5/(1+tan(1/
2*d*x+1/2*c)^2)^7*a^4+46/15/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*a^2-2/7/d/b/(1
+tan(1/2*d*x+1/2*c)^2)^7-32/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/
2*c)^10*a^4+6/d*a^6/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2))-6/d*a^4/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d*a^8/b^8/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-85/24/d/b^2/(1+tan(1/2*d*x+1/2*c)
^2)^7*tan(1/2*d*x+1/2*c)^5*a+30/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*
x+1/2*c)^4*a^6+24/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^10*a^
2-218/3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a^4+146/3/d/b
^3/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a^2+40/d/b^7/(1+tan(1/2*
d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^6*a^6-7/6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)
^7*tan(1/2*d*x+1/2*c)^3*a+12/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1
/2*c)^2*a^6-80/3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^2*a^4+
30/d/b^7/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^8*a^6-6/d/b^5/(1+tan
(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^4+29/4/d/b^4/(1+tan(1/2*d*x+1/
2*c)^2)^7*tan(1/2*d*x+1/2*c)^5*a^3+232/15/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^7*
tan(1/2*d*x+1/2*c)^2*a^2-1/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2
*c)*a^5+9/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)*a^3-11/8/d/
b^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)*a+2/d/b^7/(1+tan(1/2*d*x+
1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^12*a^6+1/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*ta
n(1/2*d*x+1/2*c)^13*a^5-9/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/
2*c)^13*a^3+11/8/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^13*a+4
/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11*a^5-7/d/b^4/(1+tan(
1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^11*a^3+7/6/d/b^2/(1+tan(1/2*d*x+1/2*
c)^2)^7*tan(1/2*d*x+1/2*c)^11*a+5/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*
d*x+1/2*c)^9*a^5-29/4/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^9
*a^3+85/24/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^9*a+7/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^3*a^3+202/5/d/b^3/(1+tan(1/2*
d*x+1/2*c)^2)^7*tan(1/2*d*x+1/2*c)^4*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87915, size = 1436, normalized size = 3.52

$$\left[-\frac{240\,b^7\cos(dx+c)^7-336\,a^2b^5\cos(dx+c)^5+560\left(a^4b^3-a^2b^5\right)\cos(dx+c)^3-105\left(16\,a^7-40\,a^5b^2+30\,a^3b^4-5\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/1680*(240*b^7*cos(d*x + c)^7 - 336*a^2*b^5*cos(d*x + c)^5 + 560*(a^4*b^
3 - a^2*b^5)*cos(d*x + c)^3 - 105*(16*a^7 - 40*a^5*b^2 + 30*a^3*b^4 - 5*a*b
^6)*d*x - 840*(a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^
2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(
d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*si
n(d*x + c) - a^2 - b^2)) - 1680*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)
+ 35*(8*a*b^6*cos(d*x + c)^5 - 2*(6*a^3*b^4 - 5*a*b^6)*cos(d*x + c)^3 + 3*(
8*a^5*b^2 - 14*a^3*b^4 + 5*a*b^6)*cos(d*x + c))*sin(d*x + c))/(b^8*d), -1/1
680*(240*b^7*cos(d*x + c)^7 - 336*a^2*b^5*cos(d*x + c)^5 + 560*(a^4*b^3 - a
^2*b^5)*cos(d*x + c)^3 - 105*(16*a^7 - 40*a^5*b^2 + 30*a^3*b^4 - 5*a*b^6)*d
*x - 1680*(a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 1680*(a^6*b - 2*a^4*b^3 + a^2*b^5
)*cos(d*x + c) + 35*(8*a*b^6*cos(d*x + c)^5 - 2*(6*a^3*b^4 - 5*a*b^6)*cos(d
*x + c)^3 + 3*(8*a^5*b^2 - 14*a^3*b^4 + 5*a*b^6)*cos(d*x + c))*sin(d*x + c)
)/(b^8*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24234, size = 1165, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1680*(105*(16*a^7 - 40*a^5*b^2 + 30*a^3*b^4 - 5*a*b^6)*(d*x + c)/b^8 - 33
60*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2
- b^2)*b^8) + 2*(840*a^5*b*tan(1/2*d*x + 1/2*c)^13 - 1890*a^3*b^3*tan(1/2*d
*x + 1/2*c)^13 + 1155*a*b^5*tan(1/2*d*x + 1/2*c)^13 + 1680*a^6*tan(1/2*d*x
+ 1/2*c)^12 - 5040*a^4*b^2*tan(1/2*d*x + 1/2*c)^12 + 5040*a^2*b^4*tan(1/2*d
*x + 1/2*c)^12 - 1680*b^6*tan(1/2*d*x + 1/2*c)^12 + 3360*a^5*b*tan(1/2*d*x
```

```
+ 1/2*c)^11 - 5880*a^3*b^3*tan(1/2*d*x + 1/2*c)^11 + 980*a*b^5*tan(1/2*d*x
+ 1/2*c)^11 + 10080*a^6*tan(1/2*d*x + 1/2*c)^10 - 26880*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^10 + 20160*a^2*b^4*tan(1/2*d*x + 1/2*c)^10 + 4200*a^5*b*tan(1/2*d
*x + 1/2*c)^9 - 6090*a^3*b^3*tan(1/2*d*x + 1/2*c)^9 + 2975*a*b^5*tan(1/2*d*
x + 1/2*c)^9 + 25200*a^6*tan(1/2*d*x + 1/2*c)^8 - 61040*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^8 + 40880*a^2*b^4*tan(1/2*d*x + 1/2*c)^8 - 8400*b^6*tan(1/2*d*x +
 1/2*c)^8 + 33600*a^6*tan(1/2*d*x + 1/2*c)^6 - 76160*a^4*b^2*tan(1/2*d*x +
1/2*c)^6 + 49280*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 - 4200*a^5*b*tan(1/2*d*x +
1/2*c)^5 + 6090*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 - 2975*a*b^5*tan(1/2*d*x + 1
/2*c)^5 + 25200*a^6*tan(1/2*d*x + 1/2*c)^4 - 55440*a^4*b^2*tan(1/2*d*x + 1/
2*c)^4 + 33936*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 5040*b^6*tan(1/2*d*x + 1/2*
c)^4 - 3360*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5880*a^3*b^3*tan(1/2*d*x + 1/2*c
)^3 - 980*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 10080*a^6*tan(1/2*d*x + 1/2*c)^2 -
 22400*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 12992*a^2*b^4*tan(1/2*d*x + 1/2*c)^
2 - 840*a^5*b*tan(1/2*d*x + 1/2*c) + 1890*a^3*b^3*tan(1/2*d*x + 1/2*c) - 11
55*a*b^5*tan(1/2*d*x + 1/2*c) + 1680*a^6 - 3920*a^4*b^2 + 2576*a^2*b^4 - 24
0*b^6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*b^7))/d
```

3.1322 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}+\frac{\cos^3(c+dx)\left(8a\left(a^2-b^2\right)-b\left(6a^2-5b^2\right)\sin(c+dx)\right)}{24b^4d}-\frac{\cos(c+dx)\left(16a\left(a^2\right.\right.}{}$$

```
[Out] -((16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*x)/(16*b^7) + (2*a*(a^2 - b^2)
^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) - (Cos[c +
 d*x]^5*(6*a - 5*b*Sin[c + d*x]))/(30*b^2*d) + (Cos[c + d*x]^3*(8*a*(a^2 -
b^2) - b*(6*a^2 - 5*b^2)*Sin[c + d*x]))/(24*b^4*d) - (Cos[c + d*x]*(16*a*(a
^2 - b^2)^2 - b*(8*a^4 - 14*a^2*b^2 + 5*b^4)*Sin[c + d*x]))/(16*b^6*d)
```

Rubi [A] time = 0.515406, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2865, 2735, 2660, 618, 204}

$$\frac{2a\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}+\frac{\cos^3(c+dx)\left(8a\left(a^2-b^2\right)-b\left(6a^2-5b^2\right)\sin(c+dx)\right)}{24b^4d}-\frac{\cos(c+dx)\left(16a\left(a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*x)/(16*b^7) + (2*a*(a^2 - b^2)
^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) - (Cos[c +
 d*x]^5*(6*a - 5*b*Sin[c + d*x]))/(30*b^2*d) + (Cos[c + d*x]^3*(8*a*(a^2 -
b^2) - b*(6*a^2 - 5*b^2)*Sin[c + d*x]))/(24*b^4*d) - (Cos[c + d*x]*(16*a*(a
^2 - b^2)^2 - b*(8*a^4 - 14*a^2*b^2 + 5*b^4)*Sin[c + d*x]))/(16*b^6*d)
```


Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e, x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\int \frac{\cos^4(c+dx)\left(-ab-\left(6a^2-5b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{6b^2}$$

$$= -\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos^3(c+dx)\left(8a\left(a^2-b^2\right)-b\left(6a^2-5b^2\right)\sin(c+\right.}{24b^4d}$$

$$= -\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos^3(c+dx)\left(8a\left(a^2-b^2\right)-b\left(6a^2-5b^2\right)\sin(c+\right.}{24b^4d}$$

$$= -\frac{\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^7} - \frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos^3(c+dx)}{}$$

$$= -\frac{\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^7} - \frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos^3(c+dx)}{}$$

$$= -\frac{\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^7} - \frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos^3(c+dx)}{}$$

$$= -\frac{\left(16a^6-40a^4b^2+30a^2b^4-5b^6\right)x}{16b^7} + \frac{2a\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d} - \frac{\cos^5(c+}{}$$

Mathematica [A] time = 2.27283, size = 275, normalized size = 1.21

$$240a^4b^2\sin(2(c+dx)) - 480a^2b^4\sin(2(c+dx)) - 30a^2b^4\sin(4(c+dx)) - 120ab\left(-18a^2b^2+8a^4+11b^4\right)\cos(c+dx) + 2($$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-960*a^6*c + 2400*a^4*b^2*c - 1800*a^2*b^4*c + 300*b^6*c - 960*a^6*d*x + 2
400*a^4*b^2*d*x - 1800*a^2*b^4*d*x + 300*b^6*d*x + 1920*a*(a^2 - b^2)^(5/2)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 120*a*b*(8*a^4 - 18*a^2
*b^2 + 11*b^4)*Cos[c + d*x] + 20*(4*a^3*b^3 - 7*a*b^5)*Cos[3*(c + d*x)] - 1
2*a*b^5*Cos[5*(c + d*x)] + 240*a^4*b^2*Sin[2*(c + d*x)] - 480*a^2*b^4*Sin[2
*(c + d*x)] + 225*b^6*Sin[2*(c + d*x)] - 30*a^2*b^4*Sin[4*(c + d*x)] + 45*b
^6*Sin[4*(c + d*x)] + 5*b^6*Sin[6*(c + d*x)])/(960*b^7*d)
```

Maple [B] time = 0.094, size = 1551, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 5/8/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1
/2*d*x+1/2*c)^5*a^4-2/d/b*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2))-20/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^4*a^5-9/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*a^2-2/d
/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^5-62/5/d/b^2/(1+tan
(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a+1/d/b^5/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)*a^4-10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)^2*a^5+15/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-5/24/
d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+11/8/d/b/(1+tan(1/2*d*x
+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-2/d/b^7*arctan(tan(1/2*d*x+1/2*c))*a^6-46/1
5/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*a+14/3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*
a^3-2/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*a^5-11/8/d/b/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^11+44/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^4*a^3+3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*a^4+5/d
/b^5*arctan(tan(1/2*d*x+1/2*c))*a^4-15/4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a
^2+140/3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^3-92/3/d/b
^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a-5/2/d/b^3/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*a^2+5/24/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6
*tan(1/2*d*x+1/2*c)^9-15/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^7-10/d/b^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^8*a^5-19/4/d/b^3
/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*a^2+6/d*a^3/b^3/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+22/d/b^4/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2*a^3-20/d/b^6/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)^6*a^5-18/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(
1/2*d*x+1/2*c)^8*a-3/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*
a^4+19/4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*a^2-1/d/b^5/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^4+9/4/d/b^3/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*a^2-28/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^
6*tan(1/2*d*x+1/2*c)^4*a+26/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/
2*c)^8*a^3+6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a^3-6/d
/b^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10*a-2/d/b^5/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*a^4+5/2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)
^6*tan(1/2*d*x+1/2*c)^7*a^2-6/d*a^5/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d*a^7/b^7/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84988, size = 1310, normalized size = 5.75

$$\left[-\frac{48\,ab^5\cos\left(dx+c\right)^5-80\left(a^3b^3-ab^5\right)\cos\left(dx+c\right)^3+15\left(16\,a^6-40\,a^4b^2+30\,a^2b^4-5\,b^6\right)dx-120\left(a^5-2\,a^3b^2+ab^4\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(48*a*b^5*cos(d*x + c)^5 - 80*(a^3*b^3 - a*b^5)*cos(d*x + c)^3 + 15
*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*d*x - 120*(a^5 - 2*a^3*b^2 + a*
b^4)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x +
c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2
 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 240*(a^5*
b - 2*a^3*b^3 + a*b^5)*cos(d*x + c) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^
4 - 5*b^6)*cos(d*x + c)^3 + 3*(8*a^4*b^2 - 14*a^2*b^4 + 5*b^6)*cos(d*x + c)
)*sin(d*x + c))/(b^7*d), -1/240*(48*a*b^5*cos(d*x + c)^5 - 80*(a^3*b^3 - a*
b^5)*cos(d*x + c)^3 + 15*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*d*x + 2
40*(a^5 - 2*a^3*b^2 + a*b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(
sqrt(a^2 - b^2)*cos(d*x + c))) + 240*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x +
c) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^4 - 5*b^6)*cos(d*x + c)^3 + 3*(8*
a^4*b^2 - 14*a^2*b^4 + 5*b^6)*cos(d*x + c))*sin(d*x + c))/(b^7*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21778, size = 992, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*(d*x + c)/b^7 - 480*(
a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)
*b^7) + 2*(120*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 270*a^2*b^3*tan(1/2*d*x + 1/
2*c)^11 + 165*b^5*tan(1/2*d*x + 1/2*c)^11 + 240*a^5*tan(1/2*d*x + 1/2*c)^10
 - 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^10 + 720*a*b^4*tan(1/2*d*x + 1/2*c)^10
+ 360*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 570*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 - 2
5*b^5*tan(1/2*d*x + 1/2*c)^9 + 1200*a^5*tan(1/2*d*x + 1/2*c)^8 - 3120*a^3*b
^2*tan(1/2*d*x + 1/2*c)^8 + 2160*a*b^4*tan(1/2*d*x + 1/2*c)^8 + 240*a^4*b*t
an(1/2*d*x + 1/2*c)^7 - 300*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^5*tan(1/
2*d*x + 1/2*c)^7 + 2400*a^5*tan(1/2*d*x + 1/2*c)^6 - 5600*a^3*b^2*tan(1/2*d
```

```
*x + 1/2*c)^6 + 3680*a*b^4*tan(1/2*d*x + 1/2*c)^6 - 240*a^4*b*tan(1/2*d*x +
 1/2*c)^5 + 300*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 450*b^5*tan(1/2*d*x + 1/2*
c)^5 + 2400*a^5*tan(1/2*d*x + 1/2*c)^4 - 5280*a^3*b^2*tan(1/2*d*x + 1/2*c)^
4 + 3360*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 360*a^4*b*tan(1/2*d*x + 1/2*c)^3 +
570*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 25*b^5*tan(1/2*d*x + 1/2*c)^3 + 1200*a
^5*tan(1/2*d*x + 1/2*c)^2 - 2640*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 1488*a*b^
4*tan(1/2*d*x + 1/2*c)^2 - 120*a^4*b*tan(1/2*d*x + 1/2*c) + 270*a^2*b^3*tan
(1/2*d*x + 1/2*c) - 165*b^5*tan(1/2*d*x + 1/2*c) + 240*a^5 - 560*a^3*b^2 +
368*a*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*b^6))/d
```

3.1323 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=252

$$-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{b^4d}+\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}+\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2b^3d}-\frac{x\left(a^2-3b^2\right)}{2b^3}-$$

```
[Out] (-3*x)/(8*b) - ((a^2 - 3*b^2)*x)/(2*b^3) - ((a^4 - 3*a^2*b^2 + 3*b^4)*x)/b^
5 + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a*b^5*d) - ArcTanh[Cos[c + d*x]]/(a*d) - (a*Cos[c + d*x])/(b^2*d) - (a*(a^
2 - 3*b^2)*Cos[c + d*x])/(b^4*d) + (a*Cos[c + d*x]^3)/(3*b^2*d) + (3*Cos[c
+ d*x]*Sin[c + d*x])/(8*b*d) + ((a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2
*b^3*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

Rubi [A] time = 0.285735, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2897, 3770, 2638, 2635, 8, 2633, 2660, 618, 204}

$$-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{b^4d}+\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}+\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2b^3d}-\frac{x\left(a^2-3b^2\right)}{2b^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(8*b) - ((a^2 - 3*b^2)*x)/(2*b^3) - ((a^4 - 3*a^2*b^2 + 3*b^4)*x)/b^
5 + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a*b^5*d) - ArcTanh[Cos[c + d*x]]/(a*d) - (a*Cos[c + d*x])/(b^2*d) - (a*(a^
2 - 3*b^2)*Cos[c + d*x])/(b^4*d) + (a*Cos[c + d*x]^3)/(3*b^2*d) + (3*Cos[c
+ d*x]*Sin[c + d*x])/(8*b*d) + ((a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2
*b^3*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{-a^4+3a^2b^2-3b^4}{b^5} + \frac{\csc(c+dx)}{a} + \frac{a\left(a^2-3b^2\right)\sin(c+dx)}{b^4} + \frac{\left(-a^2+3b^2\right)\sin^2(\right.$$

$$= -\frac{\left(a^4-3a^2b^2+3b^4\right)x}{b^5} + \frac{\int \csc(c+dx)\,dx}{a} + \frac{a\int \sin^3(c+dx)\,dx}{b^2} - \frac{\int \sin^4(c+dx)\,dx}{b}$$

$$= -\frac{\left(a^4-3a^2b^2+3b^4\right)x}{b^5} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{a\left(a^2-3b^2\right)\cos(c+dx)}{b^4d} + \frac{\left(a^2-3b\right.}{}$$

$$= -\frac{\left(a^2-3b^2\right)x}{2b^3} - \frac{\left(a^4-3a^2b^2+3b^4\right)x}{b^5} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{a\cos(c+dx)}{b^2d} - \frac{a\left(a\right.}{}$$

$$= -\frac{3x}{8b} - \frac{\left(a^2-3b^2\right)x}{2b^3} - \frac{\left(a^4-3a^2b^2+3b^4\right)x}{b^5} + \frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^5d}$$

Mathematica [A] time = 0.542313, size = 220, normalized size = 0.87

$$-\frac{-24a^3b^2\sin(2(c+dx)) + 24a^2b\left(4a^2-9b^2\right)\cos(c+dx) - 8a^2b^3\cos(3(c+dx)) - 192\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(96*a^5*c - 240*a^3*b^2*c + 180*a*b^4*c + 96*a^5*d*x - 240*a^3*b^2*d*x + 1
80*a*b^4*d*x - 192*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]] + 24*a^2*b*(4*a^2 - 9*b^2)*Cos[c + d*x] - 8*a^2*b^3*Cos[3*(c + d
*x)] + 96*b^5*Log[Cos[(c + d*x)/2]] - 96*b^5*Log[Sin[(c + d*x)/2]] - 24*a^3
*b^2*Sin[2*(c + d*x)] + 48*a*b^4*Sin[2*(c + d*x)] + 3*a*b^4*Sin[4*(c + d*x)
])/(96*a*b^5*d)
```

Maple [B] time = 0.112, size = 827, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2+9/4/d/b/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^6*a^3+6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^6*a-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2+1/4/d
/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-6/d/b^4/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3+14/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan
(1/2*d*x+1/2*c)^4*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3
*a^2-1/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a^3+38/3/d/b^2/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*
d*x+1/2*c)*a^2-9/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-2/d/b^
4/(1+tan(1/2*d*x+1/2*c)^2)^4*a^3+14/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*a-2/
d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^4+5/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2
-15/4/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d*a^5/b^5/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d*a^3/b^3/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/b*a/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a*b/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/a*
ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.80253, size = 1212, normalized size = 4.81

$$\left[\frac{8\,a^2b^3\cos\left(dx+c\right)^3-12\,b^5\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+12\,b^5\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(8\,a^5-20\,a^3b^2+15\,ab^4\right)dx+}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(8*a^2*b^3*cos(d*x + c)^3 - 12*b^5*log(1/2*cos(d*x + c) + 1/2) + 12*b
^5*log(-1/2*cos(d*x + c) + 1/2) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*d*x + 1
2*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*co
s(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2)) - 24*(a^4*b - 2*a^2*b^3)*cos(d*x + c) - 3*(2*a*b^4*cos(d*x + c)^3
 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a*b^5*d), 1/24*(8*a^2
*b^3*cos(d*x + c)^3 - 12*b^5*log(1/2*cos(d*x + c) + 1/2) + 12*b^5*log(-1/2*
cos(d*x + c) + 1/2) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*d*x - 24*(a^4 - 2*a
^2*b^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)
*cos(d*x + c))) - 24*(a^4*b - 2*a^2*b^3)*cos(d*x + c) - 3*(2*a*b^4*cos(d*x
+ c)^3 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a*b^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27358, size = 537, normalized size = 2.13

$$\frac{\frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{3\left(8a^4-20a^2b^2+15b^4\right)(dx+c)}{b^5}+\frac{48\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}ab^5}-\frac{2\left(12a^2b\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)
*(d*x + c)/b^5 + 48*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a*b^5) - 2*(12*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 27*b^3*ta
n(1/2*d*x + 1/2*c)^7 + 24*a^3*tan(1/2*d*x + 1/2*c)^6 - 72*a*b^2*tan(1/2*d*x
 + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^
5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 168*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a
^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 + 72*a^3*tan(1/2
*d*x + 1/2*c)^2 - 152*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x +
 1/2*c) + 27*b^3*tan(1/2*d*x + 1/2*c) + 24*a^3 - 56*a*b^2)/((tan(1/2*d*x +
1/2*c)^2 + 1)^4*b^4))/d
```

3.1324 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=183

$$\frac{(a^2-3b^2)\cos(c+dx)}{b^3d}-\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}+\frac{ax(a^2-3b^2)}{b^4}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}$$

```
[Out] (a*x)/(2*b^2) + (a*(a^2 - 3*b^2)*x)/b^4 - (2*(a^2 - b^2)^(5/2)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^4*d) + (b*ArcTanh[Cos[c + d*x]
])/(a^2*d) + Cos[c + d*x]/(b*d) + ((a^2 - 3*b^2)*Cos[c + d*x])/(b^3*d) - Co
s[c + d*x]^3/(3*b*d) - Cot[c + d*x]/(a*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(
2*b^2*d)
```

Rubi [A] time = 0.255159, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2897, 3770, 3767, 8, 2638, 2635, 2633, 2660, 618, 204}

$$\frac{(a^2-3b^2)\cos(c+dx)}{b^3d}-\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}+\frac{ax(a^2-3b^2)}{b^4}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/(2*b^2) + (a*(a^2 - 3*b^2)*x)/b^4 - (2*(a^2 - b^2)^(5/2)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^4*d) + (b*ArcTanh[Cos[c + d*x]
])/(a^2*d) + Cos[c + d*x]/(b*d) + ((a^2 - 3*b^2)*Cos[c + d*x])/(b^3*d) - Co
s[c + d*x]^3/(3*b*d) - Cot[c + d*x]/(a*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(
2*b^2*d)
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{a^3-3ab^2}{b^4} - \frac{b\csc(c+dx)}{a^2} + \frac{\csc^2(c+dx)}{a} + \frac{\left(-a^2+3b^2\right)\sin(c+dx)}{b^3} + \frac{a\sin^2(c}{b^2}\right.$$

$$= \frac{a\left(a^2-3b^2\right)x}{b^4} + \frac{\int\csc^2(c+dx)\,dx}{a} + \frac{a\int\sin^2(c+dx)\,dx}{b^2} - \frac{\int\sin^3(c+dx)\,dx}{b} - \frac{b\int}{}$$

$$= \frac{a\left(a^2-3b^2\right)x}{b^4} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)}{b^3d} - \frac{a\cos(c+dx)\,\mathrm{si}}{2b^2d}$$

$$= \frac{ax}{2b^2} + \frac{a\left(a^2-3b^2\right)x}{b^4} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{bd} + \frac{\left(a^2-3b^2\right)\cos(c+}{b^3d}$$

$$= \frac{ax}{2b^2} + \frac{a\left(a^2-3b^2\right)x}{b^4} - \frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^4d} + \frac{b\tanh^{-1}(\cos(c+d}{a^2d}$$

Mathematica [A] time = 1.38042, size = 208, normalized size = 1.14

$$-\frac{3a^3b^2\sin(2(c+dx))-3a^2b\left(4a^2-9b^2\right)\cos(c+dx)+a^2b^3\cos(3(c+dx))+24\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+30a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-12*a^5*c + 30*a^3*b^2*c - 12*a^5*d*x + 30*a^3*b^2*d*x + 24*(a^2 - b^2)^(
5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 3*a^2*b*(4*a^2 - 9*
b^2)*Cos[c + d*x] + a^2*b^3*Cos[3*(c + d*x)] + 6*a*b^4*Cot[(c + d*x)/2] - 1
2*b^5*Log[Cos[(c + d*x)/2]] + 12*b^5*Log[Sin[(c + d*x)/2]] + 3*a^3*b^2*Sin[
2*(c + d*x)] - 6*a*b^4*Tan[(c + d*x)/2])/(12*a^2*b^4*d)
```

Maple [B] time = 0.119, size = 557, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)+1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x
+1/2*c)^5+2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2-6/d/b
/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+4/d/b^3/(1+tan(1/2*d*x+1/2
*c)^2)^3*a^2*tan(1/2*d*x+1/2*c)^2-8/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)^2-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)+2/d/b^
3/(1+tan(1/2*d*x+1/2*c)^2)^3*a^2-14/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3+2/d/b^
4*arctan(tan(1/2*d*x+1/2*c))*a^3-5/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-2/d*a
^4/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2))+6/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))*a^2-6/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))+2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))*b^2-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*
d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.04288, size = 1311, normalized size = 7.16

$$\left[\frac{3a^3b^2\cos(dx+c)^3+3b^5\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3b^5\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\left(a^4-2a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^3*b^2*cos(d*x + c)^3 + 3*b^5*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 3*b^5*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^4 - 2*a^2*b^2
 + b^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x +
 c) - 3*(a^3*b^2 + 2*a*b^4)*cos(d*x + c) - (2*a^2*b^3*cos(d*x + c)^3 - 3*(2
*a^5 - 5*a^3*b^2)*d*x - 6*(a^4*b - 2*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(
a^2*b^4*d*sin(d*x + c)), 1/6*(3*a^3*b^2*cos(d*x + c)^3 + 3*b^5*log(1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) - 3*b^5*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 6*(a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b
)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 3*(a^3*b^2 + 2*a*b^4)*cos(
d*x + c) - (2*a^2*b^3*cos(d*x + c)^3 - 3*(2*a^5 - 5*a^3*b^2)*d*x - 6*(a^4*b
 - 2*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^2*b^4*d*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25942, size = 408, normalized size = 2.23

$$-\frac{\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{3\left(2\,a^3-5\,ab^2\right)(dx+c)}{b^4}-\frac{3\left(2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{12\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\mathrm{ar}\ldots\right.}{\sqrt{a^2-b^2}\,a^2b^4}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/2*d*x + 1/2*c)/a - 3
*(2*a^3 - 5*a*b^2)*(d*x + c)/b^4 - 3*(2*b*tan(1/2*d*x + 1/2*c) - a)/(a^2*ta
n(1/2*d*x + 1/2*c)) + 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
 b^2)))/(sqrt(a^2 - b^2)*a^2*b^4) - 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2
*tan(1/2*d*x + 1/2*c)^4 - 18*b^2*tan(1/2*d*x + 1/2*c)^4 + 12*a^2*tan(1/2*d*
x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) +
 6*a^2 - 14*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

3.1325 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{x\left(2a^2-5b^2\right)}{2b^3}+\frac{b\cot(c+dx)}{a^2d}-\frac{a\cos(c+dx)}{b^2d}$$

```
[Out] -((2*a^2 - 5*b^2)*x)/(2*b^3) + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^3*d) + ((5*a^2 - 2*b^2)*ArcTanh[Cos[c + d
*x]])/(2*a^3*d) - (a*Cos[c + d*x])/(b^2*d) + (b*Cot[c + d*x])/(a^2*d) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.384695, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2896, 3057, 2660, 618, 204, 3770}

$$\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{x\left(2a^2-5b^2\right)}{2b^3}+\frac{b\cot(c+dx)}{a^2d}-\frac{a\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((2*a^2 - 5*b^2)*x)/(2*b^3) + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^3*d) + ((5*a^2 - 2*b^2)*ArcTanh[Cos[c + d
*x]])/(2*a^3*d) - (a*Cos[c + d*x])/(b^2*d) + (b*Cot[c + d*x])/(a^2*d) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
```

```
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{a\cos(c+dx)}{b^2d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\cos(c+dx)\sin(c+dx}{2bd}$$

$$= -\frac{\left(2a^2-5b^2\right)x}{2b^3}-\frac{a\cos(c+dx)}{b^2d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\cos(c}{}$$

$$= -\frac{\left(2a^2-5b^2\right)x}{2b^3}+\frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{a\cos(c+dx)}{b^2d}+\frac{b\cot(c+dx)}{a^2d}$$

$$= -\frac{\left(2a^2-5b^2\right)x}{2b^3}+\frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{a\cos(c+dx)}{b^2d}+\frac{b\cot(c+dx)}{a^2d}$$

$$= -\frac{\left(2a^2-5b^2\right)x}{2b^3}+\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{\left(5a^2-2b^2\right)\tanh^{-1}(\cos(c}{2a^3d}$$

Mathematica [A] time = 5.44156, size = 259, normalized size = 1.49

$$2a^3b^2\sin(2(c+dx))+16\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)-a^2b^3\csc^2\left(\frac{1}{2}(c+dx)\right)+a^2b^3\sec^2\left(\frac{1}{2}(c+dx)\right)-20a^2b^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-8*a^5*c + 20*a^3*b^2*c - 8*a^5*d*x + 20*a^3*b^2*d*x + 16*(a^2 - b^2)^(5/2
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 8*a^4*b*Cos[c + d*x] +
 4*a*b^4*Cot[(c + d*x)/2] - a^2*b^3*Csc[(c + d*x)/2]^2 + 20*a^2*b^3*Log[Cos
[(c + d*x)/2]] - 8*b^5*Log[Cos[(c + d*x)/2]] - 20*a^2*b^3*Log[Sin[(c + d*x)
/2]] + 8*b^5*Log[Sin[(c + d*x)/2]] + a^2*b^3*Sec[(c + d*x)/2]^2 + 2*a^3*b^2
*Sin[2*(c + d*x)] - 4*a*b^4*Tan[(c + d*x)/2])/(8*a^3*b^3*d)
```

Maple [B] time = 0.124, size = 483, normalized size = 2.8

$$\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{b}{2\,da^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}-2\,\frac{(\tan(1/2\,dx+c/2}{db^{2}\left(1+(\tan(1/2\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/2*d*x+1/2*c)*b-1/d/b/(1+tan(1/
2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)^2*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-2
/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*a-2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+
5/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d/b*a/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/a*b/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/a^3*b^3/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/8/d/a/ta
n(1/2*d*x+1/2*c)^2-5/2/d/a*ln(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/
2*c))*b^2+1/2/d/a^2*b/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.59221, size = 1739, normalized size = 9.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^4*b*cos(d*x + c)^3 + 2*(2*a^5 - 5*a^3*b^2)*d*x*cos(d*x + c)^2 -
2*(2*a^5 - 5*a^3*b^2)*d*x + 2*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b
^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
+ c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^
```

```
2)) - 2*(2*a^4*b + a^2*b^3)*cos(d*x + c) + (5*a^2*b^3 - 2*b^5 - (5*a^2*b^3
- 2*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - (5*a^2*b^3 - 2*b^5 -
 (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*
b^2*cos(d*x + c)^3 - (a^3*b^2 + 2*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a^3*b
^3*d*cos(d*x + c)^2 - a^3*b^3*d), -1/4*(4*a^4*b*cos(d*x + c)^3 + 2*(2*a^5 -
 5*a^3*b^2)*d*x*cos(d*x + c)^2 - 2*(2*a^5 - 5*a^3*b^2)*d*x - 4*(a^4 - 2*a^2
*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 2*(2*a^4*b + a^2*b
^3)*cos(d*x + c) + (5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)
*log(1/2*cos(d*x + c) + 1/2) - (5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos
(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*b^2*cos(d*x + c)^3 - (a^
3*b^2 + 2*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos(d*x + c)^2 - a^
3*b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24983, size = 582, normalized size = 3.34

$$\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}-\frac{4\left(2a^2-5b^2\right)(dx+c)}{b^3}-\frac{4\left(5a^2-2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{16\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\mathrm{arct}\right.}{\sqrt{a^2-b^2}a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 4*(2*a^2 -
 5*b^2)*(d*x + c)/b^3 - 4*(5*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^
3 + 16*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2
- b^2)*a^3*b^3) + (10*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 4*b^4*tan(1/2*d*x +
1/2*c)^6 - 8*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 4*a*b^3*tan(1/2*d*x + 1/2*c)^5
- 16*a^4*tan(1/2*d*x + 1/2*c)^4 + 19*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 8*b^4
*tan(1/2*d*x + 1/2*c)^4 + 8*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^3*tan(1/2*
d*x + 1/2*c)^3 - 16*a^4*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 - 4*b^4*tan(1/2*d*x + 1/2*c)^2 + 4*a*b^3*tan(1/2*d*x + 1/2*c) - a^2*
b^2)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2*a^3*b^2))/d
```

3.1326 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^2d}+\frac{\left(3a^2-b^2\right)\cot(c+dx)}{a^3d}-\frac{b\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{b\tanh^{-1}(\cos(c+\ldots}{2a^2d}$$

```
[Out] (a*x)/b^2 - (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^4*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(2*a^2*d) - (b*(3*a^2 - b^2
)*ArcTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b*d) - Cot[c + d*x]/(a*d)
+ ((3*a^2 - b^2)*Cot[c + d*x])/(a^3*d) - Cot[c + d*x]^3/(3*a*d) + (b*Cot[c
+ d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rubi [A] time = 0.275358, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2897, 3770, 3767, 8, 3768, 2638, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^2d}+\frac{\left(3a^2-b^2\right)\cot(c+dx)}{a^3d}-\frac{b\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{b\tanh^{-1}(\cos(c+\ldots}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/b^2 - (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^4*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(2*a^2*d) - (b*(3*a^2 - b^2
)*ArcTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b*d) - Cot[c + d*x]/(a*d)
+ ((3*a^2 - b^2)*Cot[c + d*x])/(a^3*d) - Cot[c + d*x]^3/(3*a*d) + (b*Cot[c
+ d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{a}{b^2}+\frac{\left(3a^2b-b^3\right)\csc(c+dx)}{a^4}+\frac{\left(-3a^2+b^2\right)\csc^2(c+dx)}{a^3}-\frac{b\csc^3(c+dx)}{a^2}+\text{cs}\ldots\right.$$

$$=\frac{ax}{b^2}+\frac{\int\csc^4(c+dx)\,dx}{a}-\frac{\int\sin(c+dx)\,dx}{b}-\frac{b\int\csc^3(c+dx)\,dx}{a^2}-\frac{\left(a^2-b^2\right)^3\int\frac{\ldots}{a+\ldots}}{a^4b^{\ldots}}$$

$$=\frac{ax}{b^2}-\frac{b\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{\cos(c+dx)}{bd}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\ldots$$

$$=\frac{ax}{b^2}+\frac{b\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{b\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{\cos(c+dx)}{bd}-\text{c}\ldots$$

$$=\frac{ax}{b^2}-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{b\left(3a^2-b^2\right)\ldots}{\ldots}$$

Mathematica [A] time = 6.19066, size = 379, normalized size = 1.92

$$\frac{\left(5a^2b-2b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d}+\frac{\left(2b^3-5a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d}+\frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(7a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\ldots\right.}{6a^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(b^2*d) - (2*(a^2 - b^2)^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Co
s[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*b^2*d) + Cos[c
 + d*x]/(b*d) + ((7*a^2*Cos[(c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c +
 d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*a^2*d) - (Cot[(c + d*x)/2]*
Csc[(c + d*x)/2]^2)/(24*a*d) + ((-5*a^2*b + 2*b^3)*Log[Cos[(c + d*x)/2]])/(
2*a^4*d) + ((5*a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^4*d) - (b*Sec[(c
+ d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-7*a^2*Sin[(c + d*x)/2] + 3*b^2
*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a
*d)
```

Maple [B] time = 0.124, size = 442, normalized size = 2.2

$$\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{b}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{9}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b^2}{2\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{1}{bd\left(1+(\tan(1/2\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a^2*tan(1/2*d*x+1/2*c)^2*b-9/8/d/a*tan(
1/2*d*x+1/2*c)+1/2/d/a^3*b^2*tan(1/2*d*x+1/2*c)+2/d/b/(1+tan(1/2*d*x+1/2*c)
^2)+2/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-2/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+6/d/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d/a^2/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+2/d/a^4/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-1/
24/d/a/tan(1/2*d*x+1/2*c)^3+9/8/d/a/tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*
x+1/2*c)*b^2+1/8/d/a^2*b/tan(1/2*d*x+1/2*c)^2+5/2/d/a^2*b*ln(tan(1/2*d*x+1/
2*c))-1/d/a^4*b^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.41925, size = 1855, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(7*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3 - 6*(a^4 - 2*a^2*b^2 + b^4 -
(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)
```

```
*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*
x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2))*sin(d*x + c) + 3*(5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2
*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(5*a^2*b
^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 12*(2*a^3*b^2 - a*b^4)*cos(d*x + c) + 6*(2*a^5*d*x*cos(d*
x + c)^2 + 2*a^4*b*cos(d*x + c)^3 - 2*a^5*d*x - (2*a^4*b + a^2*b^3)*cos(d*x
 + c))*sin(d*x + c))/((a^4*b^2*d*cos(d*x + c)^2 - a^4*b^2*d)*sin(d*x + c)),
 1/12*(4*(7*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3 - 12*(a^4 - 2*a^2*b^2 + b^4 -
 (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) + 3*(5*a^2*b^3 - 2*
b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 3*(5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(-1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 12*(2*a^3*b^2 - a*b^4)*cos(d*x + c) +
 6*(2*a^5*d*x*cos(d*x + c)^2 + 2*a^4*b*cos(d*x + c)^3 - 2*a^5*d*x - (2*a^4*
b + a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2*d*cos(d*x + c)^2 - a^4*b
^2*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22782, size = 428, normalized size = 2.17

$$\frac{\frac{24\,(dx+c)a}{b^2} + \frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 27\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3} + \frac{48}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b} + \frac{12\left(5\,a^2b-2\,b^3\right)\log\left(\left|\tan\right.\right.}{a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a/b^2 + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x
+ 1/2*c)^2 - 27*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3
 + 48/((tan(1/2*d*x + 1/2*c)^2 + 1)*b) + 12*(5*a^2*b - 2*b^3)*log(abs(tan(1
/2*d*x + 1/2*c)))/a^4 - 48*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2
 - b^2)))/(sqrt(a^2 - b^2)*a^4*b^2) - (110*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 4
4*b^3*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan
(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x
+ 1/2*c)^3))/d
```

3.1327 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=195

$$\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5bd}+\frac{b\left(b^2-2a^2\right)\cot(c+dx)}{a^4d}-\frac{\left(-20a^2b^2+15a^4+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^5d}+\frac{\left(7a^2-\right.}{}$$

```
[Out] -(x/b) + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^5*b*d) - ((15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a
^5*d) + (b*(-2*a^2 + b^2)*Cot[c + d*x])/(a^4*d) + (b*Cot[c + d*x]^3)/(3*a^2
*d) + ((7*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x]
^3*Csc[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.289505, antiderivative size = 275, normalized size of antiderivative = 1.41, number of steps used = 15, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2897, 3770, 3767, 8, 3768, 2660, 618, 204}

$$\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5bd}-\frac{b\left(3a^2-b^2\right)\cot(c+dx)}{a^4d}+\frac{\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\left(-3a^2b^2+3a^4+b^4\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x/b) + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^5*b*d) - (3*ArcTanh[Cos[c + d*x]])/(8*a*d) + ((3*a^2 - b^2)*ArcTanh
[Cos[c + d*x]])/(2*a^3*d) - ((3*a^4 - 3*a^2*b^2 + b^4)*ArcTanh[Cos[c + d*x]
])/(a^5*d) + (b*Cot[c + d*x])/(a^2*d) - (b*(3*a^2 - b^2)*Cot[c + d*x])/(a^4
*d) + (b*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d)
+ ((3*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (Cot[c + d*x]*Csc[c
 + d*x]^3)/(4*a*d)
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(-\frac{1}{b} + \frac{\left(3a^4-3a^2b^2+b^4\right)\csc(c+dx)}{a^5} + \frac{\left(3a^2b-b^3\right)\csc^2(c+dx)}{a^4} + \frac{\left(-3a^2+b^2\right)\cdots}{a^{\cdots}}\right.$$

$$= -\frac{x}{b} + \frac{\int \csc^5(c+dx)\,dx}{a} - \frac{b\int \csc^4(c+dx)\,dx}{a^2} + \frac{\left(a^2-b^2\right)^3 \int \frac{1}{a+b\sin(c+dx)}\,dx}{a^5b} - \frac{\left(3a^2\cdots\right.}{\cdots}$$

$$= -\frac{x}{b} - \frac{\left(3a^4-3a^2b^2+b^4\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(3a^2-b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3d} \cdots$$

$$= -\frac{x}{b} + \frac{\left(3a^2-b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{\left(3a^4-3a^2b^2+b^4\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \cdots$$

$$= -\frac{x}{b} + \frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5bd} - \frac{3\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{\left(3a^2-b^2\right)\mathrm{ta}\cdots}{\cdots}$$

Mathematica [B] time = 6.18269, size = 448, normalized size = 2.3

$$\frac{\left(9a^2-4b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{\left(4b^2-9a^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{\left(-20a^2b^2+15a^4+8b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^5d} + \frac{\left(20a^2\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((c + d*x)/(b*d)) + (2*(a^2 - b^2)^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(
c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*b*d) + ((-7*a^2*b
*Cos[(c + d*x)/2] + 3*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (
(9*a^2 - 4*b^2)*Csc[(c + d*x)/2]^2)/(32*a^3*d) + (b*Cot[(c + d*x)/2]*Csc[(c
 + d*x)/2]^2)/(24*a^2*d) - Csc[(c + d*x)/2]^4/(64*a*d) + ((-15*a^4 + 20*a^2
*b^2 - 8*b^4)*Log[Cos[(c + d*x)/2]])/(8*a^5*d) + ((15*a^4 - 20*a^2*b^2 + 8*
b^4)*Log[Sin[(c + d*x)/2]])/(8*a^5*d) + ((-9*a^2 + 4*b^2)*Sec[(c + d*x)/2]^
2)/(32*a^3*d) + Sec[(c + d*x)/2]^4/(64*a*d) + (Sec[(c + d*x)/2]*(7*a^2*b*Si
n[(c + d*x)/2] - 3*b^3*Sin[(c + d*x)/2]))/(6*a^4*d) - (b*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [B] time = 0.125, size = 523, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/24/d/a^2*tan(1/2*d*x+1/2*c)^3*b-1/4/d/a*tan
(1/2*d*x+1/2*c)^2+1/8/d/a^3*tan(1/2*d*x+1/2*c)^2*b^2+9/8/d/a^2*tan(1/2*d*x+
1/2*c)*b-1/2/d/a^4*b^3*tan(1/2*d*x+1/2*c)-2/d/b*arctan(tan(1/2*d*x+1/2*c))+
2/d/b*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))-6/d/a*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))+6/d/a^3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2))-2/d*b^5/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/64/d/a/tan(1/2*d*x+1/2*c)^4+1/4/d/a/tan(1
/2*d*x+1/2*c)^2-1/8/d/a^3*b^2/tan(1/2*d*x+1/2*c)^2+15/8/d/a*ln(tan(1/2*d*x+
1/2*c))-5/2/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/d/a^5*ln(tan(1/2*d*x+1/2*c))
*b^4+1/24/d/a^2*b/tan(1/2*d*x+1/2*c)^3-9/8/d/a^2*b/tan(1/2*d*x+1/2*c)+1/2/d
*b^3/a^4/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.56521, size = 2427, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(48*a^5*d*x*cos(d*x + c)^4 - 96*a^5*d*x*cos(d*x + c)^2 + 48*a^5*d*x
+ 6*(9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - 24*((a^4 - 2*a^2*b^2 + b^4)*cos(
```

```
d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)
^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2
+ b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(7*a^4*b
 - 4*a^2*b^3)*cos(d*x + c) + 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b -
 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos
(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5
 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^
3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*((7*a^3*b^2 -
3*a*b^4)*cos(d*x + c)^3 - 3*(2*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))
/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 + a^5*b*d), -1/48*(48*a
^5*d*x*cos(d*x + c)^4 - 96*a^5*d*x*cos(d*x + c)^2 + 48*a^5*d*x + 6*(9*a^4*b
 - 4*a^2*b^3)*cos(d*x + c)^3 + 48*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 +
 a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(7
*a^4*b - 4*a^2*b^3)*cos(d*x + c) + 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a
^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^
5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - 3*(15*a^4*b - 20*a^2*b^3 +
 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*
a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*((7*a^3*
b^2 - 3*a*b^4)*cos(d*x + c)^3 - 3*(2*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x
 + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 + a^5*b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26005, size = 535, normalized size = 2.74

$$-\frac{\frac{192\,(dx+c)}{b}-\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-48\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+216\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(192*(d*x + c)/b - (3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d
*x + 1/2*c)^3 - 48*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*
c)^2 + 216*a^2*b*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/a^4 -
24*(15*a^4 - 20*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 384*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a
^5*b) + (750*a^4*tan(1/2*d*x + 1/2*c)^4 - 1000*a^2*b^2*tan(1/2*d*x + 1/2*c)
^4 + 400*b^4*tan(1/2*d*x + 1/2*c)^4 + 216*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 96
*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 48*a^4*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b^2*
tan(1/2*d*x + 1/2*c)^2 - 8*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/(a^5*tan(1/2
```

```
*d*x + 1/2*c)^4))/d
```

3.1328 $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=241

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{\left(-35a^2b^2+23a^4+15b^4\right)\cot(c+dx)}{15a^5d}+\frac{b\left(-20a^2b^2+15a^4+8b^4\right)\tanh^{-1}(\text{co}}{8a^6d}$$

```
[Out] (-2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) -
((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) + (b*(-9*a^2 + 4*b
^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) + ((11*a^2 - 5*b^2)*Cot[c + d*x]*C
sc[c + d*x]^2)/(15*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Co
t[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```

Rubi [A] time = 1.11981, antiderivative size = 307, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{\left(-35a^2b^2+23a^4+15b^4\right)\cot(c+dx)}{15a^5d}+\frac{b\left(-20a^2b^2+15a^4+8b^4\right)\tanh^{-1}(\text{co}}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) -
((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(b*d) + ((8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x])
/(8*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*b^2*d) - ((15*a^4 - 22*a^
2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^3*b^2*d) + (b*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c}{}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} - \frac{\left(15a^4-22a^2b^2+10b^4\right)\cot(c+dx}{30a^3b^2d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{8a^4bd} + \frac{a\cot(c+dx)\csc}{2b^2d}$$

$$= -\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(}{8a^4bd}$$

$$= -\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(}{8a^4bd}$$

$$= \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c}{}$$

$$= \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c}{}$$

$$= -\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - (2$$

Mathematica [B] time = 1.36323, size = 504, normalized size = 2.09

$$\frac{-1920\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - 1120a^3b^2\tan\left(\frac{1}{2}(c+dx)\right) - 32\left(-35a^3b^2+23a^5+15ab^4\right)\cot\left(\frac{1}{2}(c+dx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-1920*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] -
 32*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 270*a^4*b*Csc[(c +
d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4 +
1800*a^4*b*Log[Cos[(c + d*x)/2]] - 2400*a^2*b^3*Log[Cos[(c + d*x)/2]] + 960
*b^5*Log[Cos[(c + d*x)/2]] - 1800*a^4*b*Log[Sin[(c + d*x)/2]] + 2400*a^2*b^
3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 270*a^4*b*Sec[(c
+ d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4
- 656*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 + 41*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[
(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 736*a
^5*Tan[(c + d*x)/2] - 1120*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*
x)/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

Maple [B] time = 0.126, size = 629, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] -7/96/d/a*tan(1/2*d*x+1/2*c)^3+7/96/d/a/tan(1/2*d*x+1/2*c)^3+1/4/d/a^2*tan(
1/2*d*x+1/2*c)^2*b-9/8/d/a^3*b^2*tan(1/2*d*x+1/2*c)+9/8/d/a^3/tan(1/2*d*x+1
/2*c)*b^2-1/4/d/a^2*b/tan(1/2*d*x+1/2*c)^2+5/2/d/a^4*b^3*ln(tan(1/2*d*x+1/2
*c))-6/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))*b^4-11/16/d/a/tan(1/2*d*x+1/2*c)+11/16/d/a*tan(1/2*d*x+1/2*c)-2/
d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+
1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/160/d/a/tan(1/2*d*x+1/2*c)^5-15/8/d/a^2*b*
ln(tan(1/2*d*x+1/2*c))+1/8/d/a^4*b^3/tan(1/2*d*x+1/2*c)^2-1/d/a^6*b^5*ln(ta
n(1/2*d*x+1/2*c))-1/64/d/a^2*tan(1/2*d*x+1/2*c)^4*b+1/24/d/a^3*tan(1/2*d*x+
1/2*c)^3*b^2-1/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b^3+1/2/d/a^5*b^4*tan(1/2*d*x+1
/2*c)-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2-1/2/d/a^5/tan(1/2*d*x+1/2*c)*b^4+
1/64/d/a^2*b/tan(1/2*d*x+1/2*c)^4+2/d*b^6/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+6/d/a^2/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.37732, size = 2566, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 1
3*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 120*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x
+ c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15
*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x +
 c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 2
0*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d
*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2
 + a*b^4)*cos(d*x + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*
b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d
*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(16*(23*a^5 - 35*a^3*b^2 + 1
5*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 13*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3
- 240*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(
a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
 c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(15*a^4*b - 20*a
^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4
```

```
*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5
)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x
 + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*b - 4*a^2*b^3)*co
s(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 +
 a^6*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26008, size = 662, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 70
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*b
*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^4*tan(1/
2*d*x + 1/2*c) - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x +
1/2*c))/a^5 - 120*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2
*c)))/a^6 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/
(sqrt(a^2 - b^2)*a^6) + (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 5480*a^2*b^3*t
an(1/2*d*x + 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(1/2*d
*x + 1/2*c)^4 + 1080*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x
 + 1/2*c)^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x +
1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^
2 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

3.1329 $\int \frac{\cot^6(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=363

$$\frac{2b\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{b\left(-35a^2b^2+23a^4+15b^4\right)\cot(c+dx)}{15a^6d}+\frac{\left(-30a^4b^2+40a^2b^4+5a^6-16b^6\right)\tan\text{l}}{16a^7d}$$

```
[Out] (2*b*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a
^7*d) + ((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*ArcTanh[Cos[c + d*x]])/
(16*a^7*d) + (b*(23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^6*d) - (
(11*a^4 - 18*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Csc[c + d*x])/(16*a^5*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(2*b*d) + ((15*a^4 - 22*a^2*b^2 + 10*b^4)*Cot[c +
d*x]*Csc[c + d*x]^2)/(30*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(3*b^2*
d) - ((8*a^4 - 13*a^2*b^2 + 6*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^3*b^2
*d) + (b*Cot[c + d*x]*Csc[c + d*x]^4)/(5*a^2*d) - (Cot[c + d*x]*Csc[c + d*x
]^5)/(6*a*d)
```

Rubi [A] time = 1.48389, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{b\left(-35a^2b^2+23a^4+15b^4\right)\cot(c+dx)}{15a^6d}+\frac{\left(-30a^4b^2+40a^2b^4+5a^6-16b^6\right)\tan\text{l}}{16a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] (2*b*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a
^7*d) + ((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*ArcTanh[Cos[c + d*x]])/
(16*a^7*d) + (b*(23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^6*d) - (
(11*a^4 - 18*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Csc[c + d*x])/(16*a^5*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(2*b*d) + ((15*a^4 - 22*a^2*b^2 + 10*b^4)*Cot[c +
d*x]*Csc[c + d*x]^2)/(30*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(3*b^2*
d) - ((8*a^4 - 13*a^2*b^2 + 6*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^3*b^2
*d) + (b*Cot[c + d*x]*Csc[c + d*x]^4)/(5*a^2*d) - (Cot[c + d*x]*Csc[c + d*x
]^5)/(6*a*d)
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
```

```
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{a\cot(c+dx)\csc^3(c+dx)}{3b^2d} + \frac{b\cot(c+dx)\csc^4(c+dx)}{5a^2d} -$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{a\cot(c+dx)\csc^3(c+dx)}{3b^2d} - \frac{(8a^4-13a^2b^2+6b^4)\cot(c+}{24a^3b^2d}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{(15a^4-22a^2b^2+10b^4)\cot(c+dx)\csc^2(c+dx)}{30a^4bd} + \frac{a\cot(c}{}$$

$$= -\frac{(11a^4-18a^2b^2+8b^4)\cot(c+dx)\csc(c+dx)}{16a^5d} - \frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{(15a^4-2}{}$$

$$= \frac{b(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^6d} - \frac{(11a^4-18a^2b^2+8b^4)\cot(c+dx)\csc(c+dx)}{16a^5d} -$$

$$= \frac{b(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^6d} - \frac{(11a^4-18a^2b^2+8b^4)\cot(c+dx)\csc(c+dx)}{16a^5d} -$$

$$= \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\tanh^{-1}(\cos(c+dx))}{16a^7d} + \frac{b(23a^4-35a^2b^2+15b^4)\cot(c}{15a^6d}$$

$$= \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\tanh^{-1}(\cos(c+dx))}{16a^7d} + \frac{b(23a^4-35a^2b^2+15b^4)\cot(c}{15a^6d}$$

$$= \frac{2b(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\tanh^{-1}(\cos(c}{16a^7d}$$

Mathematica [A] time = 1.50711, size = 356, normalized size = 0.98

$$\frac{7680b(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 240(30a^4b^2-40a^2b^4-5a^6+16b^6)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 240(-30a^4b^2+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (7680*b*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
+ 240*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*Log[Cos[(c + d*x)/2]] + 24
0*(-5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16*b^6)*Log[Sin[(c + d*x)/2]] + 2*a*C
ot[c + d*x]*Csc[c + d*x]^5*(-295*a^5 + 570*a^3*b^2 - 360*a*b^4 + 20*(7*a^5
- 42*a^3*b^2 + 24*a*b^4)*Cos[2*(c + d*x)] - 15*(11*a^5 - 18*a^3*b^2 + 8*a*b
^4)*Cos[4*(c + d*x)] + 1168*a^4*b*Sin[c + d*x] - 2320*a^2*b^3*Sin[c + d*x]
+ 1200*b^5*Sin[c + d*x] - 568*a^4*b*Sin[3*(c + d*x)] + 1240*a^2*b^3*Sin[3*(
c + d*x)] - 600*b^5*Sin[3*(c + d*x)] + 184*a^4*b*Sin[5*(c + d*x)] - 280*a^2
*b^3*Sin[5*(c + d*x)] + 120*b^5*Sin[5*(c + d*x)]))/(3840*a^7*d)
```

Maple [B] time = 0.127, size = 780, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/384/d/a*tan(1/2*d*x+1/2*c)^6-1/384/d/a/tan(1/2*d*x+1/2*c)^6+15/128/d/a*ta
n(1/2*d*x+1/2*c)^2-15/128/d/a/tan(1/2*d*x+1/2*c)^2-5/2/d/a^5*ln(tan(1/2*d*x
+1/2*c))*b^4-7/96/d/a^2*b/tan(1/2*d*x+1/2*c)^3-9/8/d*b^3/a^4/tan(1/2*d*x+1/
2*c)-5/16/d/a*ln(tan(1/2*d*x+1/2*c))-1/160/d/a^2*b*tan(1/2*d*x+1/2*c)^5+1/6
4/d/a^3*tan(1/2*d*x+1/2*c)^4*b^2+1/8/d/a^5*tan(1/2*d*x+1/2*c)^2*b^4-1/24/d/
a^4*tan(1/2*d*x+1/2*c)^3*b^3-1/64/d/a^3/tan(1/2*d*x+1/2*c)^4*b^2-1/8/d/a^5/
tan(1/2*d*x+1/2*c)^2*b^4+1/d/a^7*ln(tan(1/2*d*x+1/2*c))*b^6+1/160/d/a^2*b/t
an(1/2*d*x+1/2*c)^5+1/2/d*b^5/a^6/tan(1/2*d*x+1/2*c)-11/16/d/a^2*tan(1/2*d*
x+1/2*c)*b+15/8/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+11/16/d/a^2*b/tan(1/2*d*x+
1/2*c)-3/128/d/a*tan(1/2*d*x+1/2*c)^4+3/128/d/a/tan(1/2*d*x+1/2*c)^4-6/d/a^
3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2))-2/d*b^7/a^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))-1/4/d/a^3*tan(1/2*d*x+1/2*c)^2*b^2+9/8/d/a^4*b^3*tan(1/2*d*x+
1/2*c)+1/4/d/a^3*b^2/tan(1/2*d*x+1/2*c)^2+2/d/a*b/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/24/d/a^4*b^3/tan(1/2*d*x+
1/2*c)^3-1/2/d/a^6*b^5*tan(1/2*d*x+1/2*c)+6/d*b^5/a^5/(a^2-b^2)^(1/2)*arcta
n(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+7/96/d/a^2*tan(1/2*d*x+
1/2*c)^3*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.77411, size = 3362, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/480*(30*(11*a^6 - 18*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c)^5 - 80*(5*a^6 - 1
2*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 + 240*((a^4*b - 2*a^2*b^3 + b^5)*cos(
d*x + c)^6 - a^4*b + 2*a^2*b^3 - b^5 - 3*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x
+ c)^4 + 3*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(5*a^6 - 14*a^4*b^2 + 8*a^2*b^4)*
cos(d*x + c) + 15*((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^
6 - 5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16*b^6 - 3*(5*a^6 - 30*a^4*b^2 + 40*a
^2*b^4 - 16*b^6)*cos(d*x + c)^4 + 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b
^6)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - 15*((5*a^6 - 30*a^4*b^2 +
 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^6 - 5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16
*b^6 - 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^4 + 3*(5*a
^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c
) + 1/2) - 32*((23*a^5*b - 35*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^5 - 5*(7*a^5
```

```
*b - 13*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^3 + 15*(a^5*b - 2*a^3*b^3 + a*b^5)*
cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d*x + c)^6 - 3*a^7*d*cos(d*x + c)^4
+ 3*a^7*d*cos(d*x + c)^2 - a^7*d), 1/480*(30*(11*a^6 - 18*a^4*b^2 + 8*a^2*b
^4)*cos(d*x + c)^5 - 80*(5*a^6 - 12*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 - 4
80*((a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^6 - a^4*b + 2*a^2*b^3 - b^5 - 3*
(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + 3*(a^4*b - 2*a^2*b^3 + b^5)*cos(
d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*c
os(d*x + c))) + 30*(5*a^6 - 14*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c) + 15*((5*a
^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^6 - 5*a^6 + 30*a^4*b^2
- 40*a^2*b^4 + 16*b^6 - 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*
x + c)^4 + 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^2)*log
(1/2*cos(d*x + c) + 1/2) - 15*((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*c
os(d*x + c)^6 - 5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16*b^6 - 3*(5*a^6 - 30*a^
4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^4 + 3*(5*a^6 - 30*a^4*b^2 + 40*a^
2*b^4 - 16*b^6)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 32*((23*a^5*
b - 35*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^5 - 5*(7*a^5*b - 13*a^3*b^3 + 6*a*b
^5)*cos(d*x + c)^3 + 15*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x +
 c))/(a^7*d*cos(d*x + c)^6 - 3*a^7*d*cos(d*x + c)^4 + 3*a^7*d*cos(d*x + c)^
2 - a^7*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7/(a+b*sin(d*x+c)),x)

[Out] Timed out
```

Giac [A] time = 1.29279, size = 846, normalized size = 2.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/1920*((5*a^5*tan(1/2*d*x + 1/2*c)^6 - 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 4
5*a^5*tan(1/2*d*x + 1/2*c)^4 + 30*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 140*a^4*
b*tan(1/2*d*x + 1/2*c)^3 - 80*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^5*tan(
1/2*d*x + 1/2*c)^2 - 480*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*a*b^4*tan(1/2
*d*x + 1/2*c)^2 - 1320*a^4*b*tan(1/2*d*x + 1/2*c) + 2160*a^2*b^3*tan(1/2*d*
x + 1/2*c) - 960*b^5*tan(1/2*d*x + 1/2*c))/a^6 - 120*(5*a^6 - 30*a^4*b^2 +
40*a^2*b^4 - 16*b^6)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 + 3840*(a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^7) + (14
70*a^6*tan(1/2*d*x + 1/2*c)^6 - 8820*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 + 11760
*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 - 4704*b^6*tan(1/2*d*x + 1/2*c)^6 + 1320*a^
5*b*tan(1/2*d*x + 1/2*c)^5 - 2160*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 + 960*a*b^
5*tan(1/2*d*x + 1/2*c)^5 - 225*a^6*tan(1/2*d*x + 1/2*c)^4 + 480*a^4*b^2*tan
(1/2*d*x + 1/2*c)^4 - 240*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 140*a^5*b*tan(1/
2*d*x + 1/2*c)^3 + 80*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^6*tan(1/2*d*x +
 1/2*c)^2 - 30*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*a^5*b*tan(1/2*d*x + 1/2*
c) - 5*a^6)/(a^7*tan(1/2*d*x + 1/2*c)^6))/d
```

3.1330 $\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=417

$$-\frac{2b^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{\left(-161a^4b^2+245a^2b^4+15a^6-105b^6\right)\cot(c+dx)}{105a^7d}-\frac{b\left(-30a^4b^2+40a^2b^4\right.}{}$$

```
[Out] (-2*b^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(a^8*d) - (b*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*ArcTanh[Cos[c + d*
x]])/(16*a^8*d) + ((15*a^6 - 161*a^4*b^2 + 245*a^2*b^4 - 105*b^6)*Cot[c + d
*x])/(105*a^7*d) + (b*(11*a^4 - 18*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Csc[c + d*
x])/(16*a^6*d) - ((45*a^4 - 77*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*x]^
2)/(105*a^5*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(3*b*d) + ((8*a^4 - 13*a^2*b
^2 + 6*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^4*b*d) + (a*Cot[c + d*x]*Csc
[c + d*x]^4)/(4*b^2*d) - ((35*a^4 - 60*a^2*b^2 + 28*b^4)*Cot[c + d*x]*Csc[c
 + d*x]^4)/(140*a^3*b^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^2*d) - (C
ot[c + d*x]*Csc[c + d*x]^6)/(7*a*d)
```

Rubi [A] time = 1.83261, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{\left(-161a^4b^2+245a^2b^4+15a^6-105b^6\right)\cot(c+dx)}{105a^7d}-\frac{b\left(-30a^4b^2+40a^2b^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(a^8*d) - (b*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*ArcTanh[Cos[c + d*
x]])/(16*a^8*d) + ((15*a^6 - 161*a^4*b^2 + 245*a^2*b^4 - 105*b^6)*Cot[c + d
*x])/(105*a^7*d) + (b*(11*a^4 - 18*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Csc[c + d*
x])/(16*a^6*d) - ((45*a^4 - 77*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*x]^
2)/(105*a^5*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(3*b*d) + ((8*a^4 - 13*a^2*b
^2 + 6*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^4*b*d) + (a*Cot[c + d*x]*Csc
[c + d*x]^4)/(4*b^2*d) - ((35*a^4 - 60*a^2*b^2 + 28*b^4)*Cot[c + d*x]*Csc[c
 + d*x]^4)/(140*a^3*b^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^2*d) - (C
ot[c + d*x]*Csc[c + d*x]^6)/(7*a*d)
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
```

```
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc^3(c+dx)}{3bd} + \frac{a\cot(c+dx)\csc^4(c+dx)}{4b^2d} + \frac{b\cot(c+dx)\csc^5(c+dx}{6a^2d}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{3bd} + \frac{a\cot(c+dx)\csc^4(c+dx)}{4b^2d} - \frac{\left(35a^4-60a^2b^2+28b^4\right)\mathrm{c}}{140a^3}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{3bd} + \frac{\left(8a^4-13a^2b^2+6b^4\right)\cot(c+dx)\csc^3(c+dx)}{24a^4bd} + \frac{a\cot}{}$$

$$= -\frac{\left(45a^4-77a^2b^2+35b^4\right)\cot(c+dx)\csc^2(c+dx)}{105a^5d} - \frac{\cot(c+dx)\csc^3(c+dx)}{3bd} + \frac{(8}{}$$

$$= \frac{b\left(11a^4-18a^2b^2+8b^4\right)\cot(c+dx)\csc(c+dx)}{16a^6d} - \frac{\left(45a^4-77a^2b^2+35b^4\right)\cot(c+}{105a^5d}$$

$$= \frac{\left(15a^6-161a^4b^2+245a^2b^4-105b^6\right)\cot(c+dx)}{105a^7d} + \frac{b\left(11a^4-18a^2b^2+8b^4\right)\cot(c+}{16a^6d}$$

$$= \frac{\left(15a^6-161a^4b^2+245a^2b^4-105b^6\right)\cot(c+dx)}{105a^7d} + \frac{b\left(11a^4-18a^2b^2+8b^4\right)\cot(c+}{16a^6d}$$

$$= -\frac{b\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{\left(15a^6-161a^4b^2+245a}{105}$$

$$= -\frac{b\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{\left(15a^6-161a^4b^2+245a}{105}$$

$$= -\frac{2b^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} - \frac{b\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)\tanh}{16a^8d}$$

Mathematica [A] time = 1.97261, size = 442, normalized size = 1.06

$$\frac{-107520b^2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 3360b\left(-30a^4b^2+40a^2b^4+5a^6-16b^6\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 336}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-107520*b^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]] + 3360*(-5*a^6*b + 30*a^4*b^3 - 40*a^2*b^5 + 16*b^7)*Log[Cos[(c + d*x)
/2]] + 3360*b*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*Log[Sin[(c + d*x)/
2]] - 2*a*Cot[c + d*x]*Csc[c + d*x]^6*(1200*a^6 + 8176*a^4*b^2 - 16240*a^2*
b^4 + 8400*b^6 + 8*(225*a^6 - 1519*a^4*b^2 + 3115*a^2*b^4 - 1575*b^6)*Cos[2
*(c + d*x)] + 16*(45*a^6 + 329*a^4*b^2 - 665*a^2*b^4 + 315*b^6)*Cos[4*(c +
d*x)] + 120*a^6*Cos[6*(c + d*x)] - 1288*a^4*b^2*Cos[6*(c + d*x)] + 1960*a^2
*b^4*Cos[6*(c + d*x)] - 840*b^6*Cos[6*(c + d*x)] - 5110*a^5*b*Sin[c + d*x]
+ 13860*a^3*b^3*Sin[c + d*x] - 8400*a*b^5*Sin[c + d*x] + 2135*a^5*b*Sin[3*(
c + d*x)] - 7770*a^3*b^3*Sin[3*(c + d*x)] + 4200*a*b^5*Sin[3*(c + d*x)] - 1
155*a^5*b*Sin[5*(c + d*x)] + 1890*a^3*b^3*Sin[5*(c + d*x)] - 840*a*b^5*Sin[
5*(c + d*x)]))/(53760*a^8*d)
```

Maple [B] time = 0.129, size = 952, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/64/d/a^4*tan(1/2*d*x+1/2*c)^4*b^3+1/24/d/a^5*tan(1/2*d*x+1/2*c)^3*b^4+2/
d*b^8/a^8/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))+3/128/d/a*tan(1/2*d*x+1/2*c)^3-3/128/d/a/tan(1/2*d*x+1/2*c)^3-15/12
8/d/a^2*tan(1/2*d*x+1/2*c)^2*b+11/16/d/a^3*b^2*tan(1/2*d*x+1/2*c)-11/16/d/a
^3/tan(1/2*d*x+1/2*c)*b^2+15/128/d/a^2*b/tan(1/2*d*x+1/2*c)^2-15/8/d/a^4*b^
3*ln(tan(1/2*d*x+1/2*c))+6/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4+5/128/d/a/tan(1/2*d*x+1/2*c)-5/128/d/a*t
an(1/2*d*x+1/2*c)-1/896/d/a/tan(1/2*d*x+1/2*c)^7+1/896/d/a*tan(1/2*d*x+1/2*
c)^7-1/d/a^8*b^7*ln(tan(1/2*d*x+1/2*c))+1/2/d/a^7*b^6*tan(1/2*d*x+1/2*c)-1/
24/d/a^5/tan(1/2*d*x+1/2*c)^3*b^4-1/128/d/a*tan(1/2*d*x+1/2*c)^5+1/128/d/a/
tan(1/2*d*x+1/2*c)^5+5/16/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-1/2/d/a^7/tan(1/2*
d*x+1/2*c)*b^6-1/384/d/a^2*b*tan(1/2*d*x+1/2*c)^6-1/4/d/a^4*b^3/tan(1/2*d*x
+1/2*c)^2+5/2/d/a^6*b^5*ln(tan(1/2*d*x+1/2*c))+3/128/d/a^2*tan(1/2*d*x+1/2*
c)^4*b-7/96/d/a^3*tan(1/2*d*x+1/2*c)^3*b^2+1/4/d/a^4*tan(1/2*d*x+1/2*c)^2*b
^3-9/8/d/a^5*b^4*tan(1/2*d*x+1/2*c)+7/96/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2+9/8
/d/a^5/tan(1/2*d*x+1/2*c)*b^4-3/128/d/a^2*b/tan(1/2*d*x+1/2*c)^4-6/d*b^6/a^
6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-
1/8/d/a^6*tan(1/2*d*x+1/2*c)^2*b^5-2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+1/384/d/a^2*b/tan(1/2*d*x+1/2*
c)^6+1/64/d/a^4*b^3/tan(1/2*d*x+1/2*c)^4+1/8/d/a^6*b^5/tan(1/2*d*x+1/2*c)^2
-1/160/d/a^3/tan(1/2*d*x+1/2*c)^5*b^2+1/160/d/a^3*tan(1/2*d*x+1/2*c)^5*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.28469, size = 3822, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/3360*(32*(15*a^7 - 161*a^5*b^2 + 245*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^7
 + 224*(58*a^5*b^2 - 100*a^3*b^4 + 45*a*b^6)*cos(d*x + c)^5 - 1120*(10*a^5*
b^2 - 19*a^3*b^4 + 9*a*b^6)*cos(d*x + c)^3 + 1680*((a^4*b^2 - 2*a^2*b^4 + b
^6)*cos(d*x + c)^6 - a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*b^2 - 2*a^2*b^4 + b
^6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*sqrt(-a^
2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
```

```
 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) + 105*(5*a^6
*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5
- 16*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*c
os(d*x + c)^4 - 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)
^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(5*a^6*b - 30*a^4*b^3 +
40*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x
+ c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^4 - 3*
(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^2)*log(-1/2*cos(d
*x + c) + 1/2)*sin(d*x + c) + 3360*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x +
c) - 70*(3*(11*a^6*b - 18*a^4*b^3 + 8*a^2*b^5)*cos(d*x + c)^5 - 8*(5*a^6*b
- 12*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c)^3 + 3*(5*a^6*b - 14*a^4*b^3 + 8*a^2*
b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*d*cos(d*x + c)^6 - 3*a^8*d*cos(d*x +
 c)^4 + 3*a^8*d*cos(d*x + c)^2 - a^8*d)*sin(d*x + c)), 1/3360*(32*(15*a^7 -
 161*a^5*b^2 + 245*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^7 + 224*(58*a^5*b^2 -
100*a^3*b^4 + 45*a*b^6)*cos(d*x + c)^5 - 1120*(10*a^5*b^2 - 19*a^3*b^4 + 9*
a*b^6)*cos(d*x + c)^3 + 3360*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^6 -
a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^4 +
3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*si
n(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) + 105*(5*a^6*b
 - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 -
16*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos
(d*x + c)^4 - 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^2
)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(5*a^6*b - 30*a^4*b^3 + 40
*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x +
c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^4 - 3*(5
*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^2)*log(-1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) + 3360*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)
 - 70*(3*(11*a^6*b - 18*a^4*b^3 + 8*a^2*b^5)*cos(d*x + c)^5 - 8*(5*a^6*b -
12*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c)^3 + 3*(5*a^6*b - 14*a^4*b^3 + 8*a^2*b^
5)*cos(d*x + c))*sin(d*x + c))/((a^8*d*cos(d*x + c)^6 - 3*a^8*d*cos(d*x + c
)^4 + 3*a^8*d*cos(d*x + c)^2 - a^8*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23082, size = 1048, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*((15*a^6*tan(1/2*d*x + 1/2*c)^7 - 35*a^5*b*tan(1/2*d*x + 1/2*c)^6 -
 105*a^6*tan(1/2*d*x + 1/2*c)^5 + 84*a^4*b^2*tan(1/2*d*x + 1/2*c)^5 + 315*a
^5*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^3*b^3*tan(1/2*d*x + 1/2*c)^4 + 315*a^6*
```

tan(1/2*d*x + 1/2*c)^3 - 980*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 560*a^2*b^4*t
an(1/2*d*x + 1/2*c)^3 - 1575*a^5*b*tan(1/2*d*x + 1/2*c)^2 + 3360*a^3*b^3*ta
n(1/2*d*x + 1/2*c)^2 - 1680*a*b^5*tan(1/2*d*x + 1/2*c)^2 - 525*a^6*tan(1/2*
d*x + 1/2*c) + 9240*a^4*b^2*tan(1/2*d*x + 1/2*c) - 15120*a^2*b^4*tan(1/2*d*
x + 1/2*c) + 6720*b^6*tan(1/2*d*x + 1/2*c))/a^7 + 840*(5*a^6*b - 30*a^4*b^3
 + 40*a^2*b^5 - 16*b^7)*log(abs(tan(1/2*d*x + 1/2*c)))/a^8 - 26880*(a^6*b^2
 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^8)
 - (10890*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 65340*a^4*b^3*tan(1/2*d*x + 1/2*c)
^7 + 87120*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 34848*b^7*tan(1/2*d*x + 1/2*c)^
7 - 525*a^7*tan(1/2*d*x + 1/2*c)^6 + 9240*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 -
15120*a^3*b^4*tan(1/2*d*x + 1/2*c)^6 + 6720*a*b^6*tan(1/2*d*x + 1/2*c)^6 -
1575*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 3360*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 1
680*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 315*a^7*tan(1/2*d*x + 1/2*c)^4 - 980*a
^5*b^2*tan(1/2*d*x + 1/2*c)^4 + 560*a^3*b^4*tan(1/2*d*x + 1/2*c)^4 + 315*a^
6*b*tan(1/2*d*x + 1/2*c)^3 - 210*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 105*a^7*t
an(1/2*d*x + 1/2*c)^2 + 84*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 35*a^6*b*tan(1/
2*d*x + 1/2*c) + 15*a^7)/(a^8*tan(1/2*d*x + 1/2*c)^7))/d

3.1331 $\int \frac{\cot^6(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=476

$$\frac{2b^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^9d}-\frac{b\left(-161a^4b^2+245a^2b^4+15a^6-105b^6\right)\cot(c+dx)}{105a^8d}+\frac{\left(40a^6b^2-240a^4b^4+\right.}{}$$

```
[Out] (2*b^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^9*d) + ((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*ArcTa
nh[Cos[c + d*x]])/(128*a^9*d) - (b*(15*a^6 - 161*a^4*b^2 + 245*a^2*b^4 - 10
5*b^6)*Cot[c + d*x])/(105*a^8*d) + ((5*a^6 - 88*a^4*b^2 + 144*a^2*b^4 - 64*
b^6)*Cot[c + d*x]*Csc[c + d*x])/(128*a^7*d) + (b*(45*a^4 - 77*a^2*b^2 + 35*
b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(105*a^6*d) - ((59*a^4 - 104*a^2*b^2 + 48
*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(192*a^5*d) - (Cot[c + d*x]*Csc[c + d*x]
^4)/(4*b*d) + ((35*a^4 - 60*a^2*b^2 + 28*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/
(140*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(5*b^2*d) - ((48*a^4 - 85*a
^2*b^2 + 40*b^4)*Cot[c + d*x]*Csc[c + d*x]^5)/(240*a^3*b^2*d) + (b*Cot[c +
d*x]*Csc[c + d*x]^6)/(7*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^7)/(8*a*d)
```

Rubi [A] time = 2.21452, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2896, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^9d}-\frac{b\left(-161a^4b^2+245a^2b^4+15a^6-105b^6\right)\cot(c+dx)}{105a^8d}+\frac{\left(40a^6b^2-240a^4b^4+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (2*b^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^9*d) + ((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*ArcTa
nh[Cos[c + d*x]])/(128*a^9*d) - (b*(15*a^6 - 161*a^4*b^2 + 245*a^2*b^4 - 10
5*b^6)*Cot[c + d*x])/(105*a^8*d) + ((5*a^6 - 88*a^4*b^2 + 144*a^2*b^4 - 64*
b^6)*Cot[c + d*x]*Csc[c + d*x])/(128*a^7*d) + (b*(45*a^4 - 77*a^2*b^2 + 35*
b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(105*a^6*d) - ((59*a^4 - 104*a^2*b^2 + 48
*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(192*a^5*d) - (Cot[c + d*x]*Csc[c + d*x]
^4)/(4*b*d) + ((35*a^4 - 60*a^2*b^2 + 28*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/
(140*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(5*b^2*d) - ((48*a^4 - 85*a
^2*b^2 + 40*b^4)*Cot[c + d*x]*Csc[c + d*x]^5)/(240*a^3*b^2*d) + (b*Cot[c +
d*x]*Csc[c + d*x]^6)/(7*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^7)/(8*a*d)
```


Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
```

```
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{a\cot(c+dx)\csc^5(c+dx)}{5b^2d} + \frac{b\cot(c+dx)\csc^6(c+dx}{7a^2d}$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{a\cot(c+dx)\csc^5(c+dx)}{5b^2d} - \frac{\left(48a^4-85a^2b^2+40b^4\right)c}{240a^3}$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{\left(35a^4-60a^2b^2+28b^4\right)\cot(c+dx)\csc^4(c+dx)}{140a^4bd} + \frac{a\,c}{}$$

$$= -\frac{\left(59a^4-104a^2b^2+48b^4\right)\cot(c+dx)\csc^3(c+dx)}{192a^5d} - \frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{(3}{}$$

$$= \frac{b\left(45a^4-77a^2b^2+35b^4\right)\cot(c+dx)\csc^2(c+dx)}{105a^6d} - \frac{\left(59a^4-104a^2b^2+48b^4\right)\cot(c}{192a^5d}$$

$$= \frac{\left(5a^6-88a^4b^2+144a^2b^4-64b^6\right)\cot(c+dx)\csc(c+dx)}{128a^7d} + \frac{b\left(45a^4-77a^2b^2+35b\right.}{10}$$

$$= -\frac{b\left(15a^6-161a^4b^2+245a^2b^4-105b^6\right)\cot(c+dx)}{105a^8d} + \frac{\left(5a^6-88a^4b^2+144a^2b^4-6\right.}{128}$$

$$= -\frac{b\left(15a^6-161a^4b^2+245a^2b^4-105b^6\right)\cot(c+dx)}{105a^8d} + \frac{\left(5a^6-88a^4b^2+144a^2b^4-6\right.}{128}$$

$$= \frac{\left(5a^8+40a^6b^2-240a^4b^4+320a^2b^6-128b^8\right)\tanh^{-1}(\cos(c+dx))}{128a^9d} - \frac{b\left(15a^6-161a\right.}{}$$

$$= \frac{\left(5a^8+40a^6b^2-240a^4b^4+320a^2b^6-128b^8\right)\tanh^{-1}(\cos(c+dx))}{128a^9d} - \frac{b\left(15a^6-161a\right.}{}$$

$$= \frac{2b^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^9d} + \frac{\left(5a^8+40a^6b^2-240a^4b^4+320a^2b^6-12\right.}{128a^9d}$$

Mathematica [A] time = 3.57399, size = 593, normalized size = 1.25

$$\frac{1720320b^3\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) - 6720\left(40a^6b^2-240a^4b^4+320a^2b^6+5a^8-128b^8\right)\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (1720320*b^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]] + 6720*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*Log[
Cos[(c + d*x)/2]] - 6720*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 -
128*b^8)*Log[Sin[(c + d*x)/2]] + a*Csc[c + d*x]^8*(-35*a*(1765*a^6 + 680*a^
4*b^2 - 1392*a^2*b^4 + 960*b^6)*Cos[c + d*x] - 35*(895*a^7 - 904*a^5*b^2 +
2736*a^3*b^4 - 1728*a*b^6)*Cos[3*(c + d*x)] - 13895*a^7*Cos[5*(c + d*x)] -
17080*a^5*b^2*Cos[5*(c + d*x)] + 62160*a^3*b^4*Cos[5*(c + d*x)] - 33600*a*b
^6*Cos[5*(c + d*x)] - 525*a^7*Cos[7*(c + d*x)] + 9240*a^5*b^2*Cos[7*(c + d*
x)] - 15120*a^3*b^4*Cos[7*(c + d*x)] + 6720*a*b^6*Cos[7*(c + d*x)] + 13440*
a^6*b*Sin[2*(c + d*x)] + 88704*a^4*b^3*Sin[2*(c + d*x)] - 174720*a^2*b^5*Si
n[2*(c + d*x)] + 94080*b^7*Sin[2*(c + d*x)] + 13440*a^6*b*Sin[4*(c + d*x)]
- 86912*a^4*b^3*Sin[4*(c + d*x)] + 183680*a^2*b^5*Sin[4*(c + d*x)] - 94080*
b^7*Sin[4*(c + d*x)] + 5760*a^6*b*Sin[6*(c + d*x)] + 42112*a^4*b^3*Sin[6*(c
```

```
 + d*x)] - 85120*a^2*b^5*Sin[6*(c + d*x)] + 40320*b^7*Sin[6*(c + d*x)] + 96
0*a^6*b*Sin[8*(c + d*x)] - 10304*a^4*b^3*Sin[8*(c + d*x)] + 15680*a^2*b^5*S
in[8*(c + d*x)] - 6720*b^7*Sin[8*(c + d*x)]))/(860160*a^9*d)
```

Maple [B] time = 0.135, size = 1143, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/384/d/a*tan(1/2*d*x+1/2*c)^6+1/384/d/a/tan(1/2*d*x+1/2*c)^6-1/2048/d/a/t
an(1/2*d*x+1/2*c)^8+1/128/d/a*tan(1/2*d*x+1/2*c)^2-1/128/d/a/tan(1/2*d*x+1/
2*c)^2+15/8/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^4+3/128/d/a^2*b/tan(1/2*d*x+1/2*
c)^3+11/16/d*b^3/a^4/tan(1/2*d*x+1/2*c)-5/128/d/a*ln(tan(1/2*d*x+1/2*c))+1/
128/d/a^2*b*tan(1/2*d*x+1/2*c)^5-3/128/d/a^3*tan(1/2*d*x+1/2*c)^4*b^2-1/4/d
/a^5*tan(1/2*d*x+1/2*c)^2*b^4+7/96/d/a^4*tan(1/2*d*x+1/2*c)^3*b^3+3/128/d/a
^3/tan(1/2*d*x+1/2*c)^4*b^2+1/4/d/a^5/tan(1/2*d*x+1/2*c)^2*b^4-5/2/d/a^7*ln
(tan(1/2*d*x+1/2*c))*b^6-1/128/d/a^2*b/tan(1/2*d*x+1/2*c)^5-9/8/d*b^5/a^6/t
an(1/2*d*x+1/2*c)+5/128/d/a^2*tan(1/2*d*x+1/2*c)*b-5/16/d/a^3*ln(tan(1/2*d*
x+1/2*c))*b^2-5/128/d/a^2*b/tan(1/2*d*x+1/2*c)-2/d*b^9/a^9/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/256/d/a*tan(1/2*
d*x+1/2*c)^4-1/256/d/a/tan(1/2*d*x+1/2*c)^4+2/d/a^3*b^3/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/2048/d/a*tan(1/2*d*
x+1/2*c)^8-1/64/d/a^5/tan(1/2*d*x+1/2*c)^4*b^4-1/8/d/a^7/tan(1/2*d*x+1/2*c)
^2*b^6+1/2/d*b^7/a^8/tan(1/2*d*x+1/2*c)-1/384/d/a^3/tan(1/2*d*x+1/2*c)^6*b^
2+6/d*b^7/a^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))+1/160/d/a^4*b^3/tan(1/2*d*x+1/2*c)^5-1/2/d/a^8*b^7*tan(1/2*d*x+
1/2*c)-1/24/d/a^6*tan(1/2*d*x+1/2*c)^3*b^5+1/8/d/a^7*tan(1/2*d*x+1/2*c)^2*b
^6+1/64/d/a^5*tan(1/2*d*x+1/2*c)^4*b^4+1/24/d/a^6*b^5/tan(1/2*d*x+1/2*c)^3-
1/896/d/a^2*b*tan(1/2*d*x+1/2*c)^7+1/384/d/a^3*tan(1/2*d*x+1/2*c)^6*b^2-1/1
60/d/a^4*tan(1/2*d*x+1/2*c)^5*b^3+1/d/a^9*ln(tan(1/2*d*x+1/2*c))*b^8+1/896/
d/a^2*b/tan(1/2*d*x+1/2*c)^7+15/128/d/a^3*tan(1/2*d*x+1/2*c)^2*b^2-11/16/d/
a^4*b^3*tan(1/2*d*x+1/2*c)-15/128/d/a^3*b^2/tan(1/2*d*x+1/2*c)^2-7/96/d/a^4
*b^3/tan(1/2*d*x+1/2*c)^3+9/8/d/a^6*b^5*tan(1/2*d*x+1/2*c)-6/d*b^5/a^5/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/128/
d/a^2*tan(1/2*d*x+1/2*c)^3*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.86559, size = 4911, normalized size = 10.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/26880*(210*(5*a^8 - 88*a^6*b^2 + 144*a^4*b^4 - 64*a^2*b^6)*cos(d*x + c)
^7 + 70*(73*a^8 + 584*a^6*b^2 - 1200*a^4*b^4 + 576*a^2*b^6)*cos(d*x + c)^5
- 70*(55*a^8 + 440*a^6*b^2 - 1104*a^4*b^4 + 576*a^2*b^6)*cos(d*x + c)^3 - 1
3440*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^8 + a^4*b^3 - 2*a^2*b^5 + b^
7 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^6 + 6*(a^4*b^3 - 2*a^2*b^5 +
 b^7)*cos(d*x + c)^4 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sqrt(-
a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b
^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 210*(5*a^8 + 40*a^6*
b^2 - 112*a^4*b^4 + 64*a^2*b^6)*cos(d*x + c) - 105*((5*a^8 + 40*a^6*b^2 - 2
40*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^8 + 5*a^8 + 40*a^6*b^2 - 2
40*a^4*b^4 + 320*a^2*b^6 - 128*b^8 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 +
320*a^2*b^6 - 128*b^8)*cos(d*x + c)^6 + 6*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4
 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^4 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*
b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) +
105*((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c
)^8 + 5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8 - 4*(5*a^8 +
 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^6 + 6*(5*a^
8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^4 - 4*(5
*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^2)*lo
g(-1/2*cos(d*x + c) + 1/2) - 256*((15*a^7*b - 161*a^5*b^3 + 245*a^3*b^5 - 1
05*a*b^7)*cos(d*x + c)^7 + 7*(58*a^5*b^3 - 100*a^3*b^5 + 45*a*b^7)*cos(d*x
+ c)^5 - 35*(10*a^5*b^3 - 19*a^3*b^5 + 9*a*b^7)*cos(d*x + c)^3 + 105*(a^5*b
^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c)^8 -
 4*a^9*d*cos(d*x + c)^6 + 6*a^9*d*cos(d*x + c)^4 - 4*a^9*d*cos(d*x + c)^2 +
 a^9*d), -1/26880*(210*(5*a^8 - 88*a^6*b^2 + 144*a^4*b^4 - 64*a^2*b^6)*cos(
d*x + c)^7 + 70*(73*a^8 + 584*a^6*b^2 - 1200*a^4*b^4 + 576*a^2*b^6)*cos(d*x
 + c)^5 - 70*(55*a^8 + 440*a^6*b^2 - 1104*a^4*b^4 + 576*a^2*b^6)*cos(d*x +
c)^3 + 26880*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^8 + a^4*b^3 - 2*a^2*
b^5 + b^7 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^6 + 6*(a^4*b^3 - 2*a
^2*b^5 + b^7)*cos(d*x + c)^4 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2
)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c
))) + 210*(5*a^8 + 40*a^6*b^2 - 112*a^4*b^4 + 64*a^2*b^6)*cos(d*x + c) - 10
5*((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^
8 + 5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8 - 4*(5*a^8 + 4
0*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^6 + 6*(5*a^8
+ 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^4 - 4*(5*a
^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^2)*log(
1/2*cos(d*x + c) + 1/2) + 105*((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*
b^6 - 128*b^8)*cos(d*x + c)^8 + 5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*
b^6 - 128*b^8 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8
)*cos(d*x + c)^6 + 6*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*
b^8)*cos(d*x + c)^4 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 1
28*b^8)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 256*((15*a^7*b - 161
*a^5*b^3 + 245*a^3*b^5 - 105*a*b^7)*cos(d*x + c)^7 + 7*(58*a^5*b^3 - 100*a^
3*b^5 + 45*a*b^7)*cos(d*x + c)^5 - 35*(10*a^5*b^3 - 19*a^3*b^5 + 9*a*b^7)*c
os(d*x + c)^3 + 105*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c))*sin(d*x + c
))/(a^9*d*cos(d*x + c)^8 - 4*a^9*d*cos(d*x + c)^6 + 6*a^9*d*cos(d*x + c)^4
- 4*a^9*d*cos(d*x + c)^2 + a^9*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30762, size = 1280, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/215040*((105*a^7*tan(1/2*d*x + 1/2*c)^8 - 240*a^6*b*tan(1/2*d*x + 1/2*c)^
7 - 560*a^7*tan(1/2*d*x + 1/2*c)^6 + 560*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 + 1
680*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 1344*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 84
0*a^7*tan(1/2*d*x + 1/2*c)^4 - 5040*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 + 3360*a
^3*b^4*tan(1/2*d*x + 1/2*c)^4 - 5040*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 15680*a
^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 8960*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 1680*
a^7*tan(1/2*d*x + 1/2*c)^2 + 25200*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 53760*a
^3*b^4*tan(1/2*d*x + 1/2*c)^2 + 26880*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 8400*a
^6*b*tan(1/2*d*x + 1/2*c) - 147840*a^4*b^3*tan(1/2*d*x + 1/2*c) + 241920*a^
2*b^5*tan(1/2*d*x + 1/2*c) - 107520*b^7*tan(1/2*d*x + 1/2*c))/a^8 - 1680*(5
*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*log(abs(tan(1/2*d*
x + 1/2*c)))/a^9 + 430080*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(pi*floor
(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(
a^2 - b^2)))/(sqrt(a^2 - b^2)*a^9) + (22830*a^8*tan(1/2*d*x + 1/2*c)^8 + 18
2640*a^6*b^2*tan(1/2*d*x + 1/2*c)^8 - 1095840*a^4*b^4*tan(1/2*d*x + 1/2*c)^
8 + 1461120*a^2*b^6*tan(1/2*d*x + 1/2*c)^8 - 584448*b^8*tan(1/2*d*x + 1/2*c
)^8 - 8400*a^7*b*tan(1/2*d*x + 1/2*c)^7 + 147840*a^5*b^3*tan(1/2*d*x + 1/2*
c)^7 - 241920*a^3*b^5*tan(1/2*d*x + 1/2*c)^7 + 107520*a*b^7*tan(1/2*d*x + 1
/2*c)^7 - 1680*a^8*tan(1/2*d*x + 1/2*c)^6 - 25200*a^6*b^2*tan(1/2*d*x + 1/2
*c)^6 + 53760*a^4*b^4*tan(1/2*d*x + 1/2*c)^6 - 26880*a^2*b^6*tan(1/2*d*x +
1/2*c)^6 + 5040*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 15680*a^5*b^3*tan(1/2*d*x +
1/2*c)^5 + 8960*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 840*a^8*tan(1/2*d*x + 1/2*
c)^4 + 5040*a^6*b^2*tan(1/2*d*x + 1/2*c)^4 - 3360*a^4*b^4*tan(1/2*d*x + 1/2
*c)^4 - 1680*a^7*b*tan(1/2*d*x + 1/2*c)^3 + 1344*a^5*b^3*tan(1/2*d*x + 1/2*
c)^3 + 560*a^8*tan(1/2*d*x + 1/2*c)^2 - 560*a^6*b^2*tan(1/2*d*x + 1/2*c)^2
+ 240*a^7*b*tan(1/2*d*x + 1/2*c) - 105*a^8)/(a^9*tan(1/2*d*x + 1/2*c)^8))/d
```

3.1332 $\int \frac{\sin^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{a^3\log(a+b\sin(c+dx))}{b^2d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\sin(c+dx)}{bd}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
+ (a^3*Log[a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)*d) - Sin[c + d*x]/(b*d)
```

Rubi [A] time = 0.180854, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 1629}

$$\frac{a^3\log(a+b\sin(c+dx))}{b^2d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
+ (a^3*Log[a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)*d) - Sin[c + d*x]/(b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^3}{b^3(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{2(a+b)(b-x)}+\frac{a^3}{(a-b)(a+b)(a+x)}-\frac{b^2}{2(a-b)(b+x)}\right)dx, x, b\sin(c+dx)\right)}{b^2 d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} - \frac{\log(1+\sin(c+dx))}{2(a-b)d} + \frac{a^3\log(a+b\sin(c+dx))}{b^2(a^2-b^2)\,d} - \frac{\sin(c+dx)}{bd}$$

Mathematica [A] time = 0.210526, size = 83, normalized size = 0.89

$$-\frac{-\frac{2a^3\log(a+b\sin(c+dx))}{b^2(a^2-b^2)}+\frac{\log(1-\sin(c+dx))}{a+b}+\frac{\log(\sin(c+dx)+1)}{a-b}+\frac{2\sin(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Log[1 - Sin[c + d*x]]/(a + b) + Log[1 + Sin[c + d*x]]/(a - b) - (2*a^3*Lo
g[a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)) + (2*Sin[c + d*x])/b)/(2*d)
```

Maple [A] time = 0.064, size = 95, normalized size = 1.

$$-\frac{\sin(dx+c)}{bd}+\frac{a^3\ln(a+b\sin(dx+c))}{db^2(a+b)(a-b)}-\frac{\ln(\sin(dx+c)-1)}{d(2a+2b)}-\frac{\ln(1+\sin(dx+c))}{d(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -sin(d*x+c)/b/d+1/d/b^2*a^3/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/d/(2*a+2*b)*ln
(sin(d*x+c)-1)-1/d/(2*a-2*b)*ln(1+sin(d*x+c))
```

Maxima [A] time = 0.987271, size = 111, normalized size = 1.19

$$\frac{\frac{2a^3\log(b\sin(dx+c)+a)}{a^2b^2-b^4}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}-\frac{2\sin(dx+c)}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*log(b*sin(d*x + c) + a)/(a^2*b^2 - b^4) - log(sin(d*x + c) + 1)/
(a - b) - log(sin(d*x + c) - 1)/(a + b) - 2*sin(d*x + c)/b)/d
```

Fricas [A] time = 1.20683, size = 223, normalized size = 2.4

$$\frac{2\,a^3\log\left(b\sin\left(dx+c\right)+a\right)-\left(ab^2+b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(ab^2-b^3\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^2b-b^3\right)\sin\left(d\right.}{2\left(a^2b^2-b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*log(b*sin(d*x + c) + a) - (a*b^2 + b^3)*log(sin(d*x + c) + 1) -
(a*b^2 - b^3)*log(-sin(d*x + c) + 1) - 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*
b^2 - b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25744, size = 115, normalized size = 1.24

$$\frac{\frac{2\,a^3\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^2b^2-b^4}-\frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{a-b}-\frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{a+b}-\frac{2\,\sin\left(dx+c\right)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(b*sin(d*x + c) + a))/(a^2*b^2 - b^4) - log(abs(sin(d*x +
 c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b) - 2*sin(d*x + c)/b)/
d
```

3.1333 $\int \frac{\sin(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2\log(a+b\sin(c+dx))}{bd\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
- (a^2*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d)
```

Rubi [A] time = 0.155777, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2837, 12, 1629}

$$-\frac{a^2\log(a+b\sin(c+dx))}{bd\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
- (a^2*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^2}{b^2(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b}{2(a+b)(b-x)} - \frac{a^2}{(a-b)(a+b)(a+x)} + \frac{b}{2(a-b)(b+x)}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{a^2\log(a+b\sin(c+dx))}{b\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.0654118, size = 72, normalized size = 0.9

$$\frac{-2a^2\log(a+b\sin(c+dx)) - b(a-b)\log(1-\sin(c+dx)) + b(a+b)\log(\sin(c+dx)+1)}{2bd(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((a - b)*b*Log[1 - Sin[c + d*x]]) + b*(a + b)*Log[1 + Sin[c + d*x]] - 2*a
^2*Log[a + b*Sin[c + d*x]])/(2*(a - b)*b*(a + b)*d)
```

Maple [A] time = 0.062, size = 81, normalized size = 1.

$$-\frac{a^2\ln(a+b\sin(dx+c))}{d(a+b)(a-b)b} - \frac{\ln(\sin(dx+c)-1)}{d(2a+2b)} + \frac{\ln(1+\sin(dx+c))}{d(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a^2/(a+b)/(a-b)/b*ln(a+b*sin(d*x+c))-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)+1/
d/(2*a-2*b)*ln(1+sin(d*x+c))
```

Maxima [A] time = 0.988743, size = 92, normalized size = 1.15

$$-\frac{\frac{2a^2\log(b\sin(dx+c)+a)}{a^2b-b^3} - \frac{\log(\sin(dx+c)+1)}{a-b} + \frac{\log(\sin(dx+c)-1)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*log(b*sin(d*x + c) + a)/(a^2*b - b^3) - log(sin(d*x + c) + 1)/(
a - b) + log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A] time = 1.32669, size = 174, normalized size = 2.17

$$-\frac{2\,a^{2}\log\left(b\sin\left(dx+c\right)+a\right)-\left(ab+b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(ab-b^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^{2}b-b^{3}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*log(b*sin(d*x + c) + a) - (a*b + b^2)*log(sin(d*x + c) + 1) + (
a*b - b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin^{2}\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.21428, size = 96, normalized size = 1.2

$$-\frac{\frac{2\,a^{2}\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{2}b-b^{3}}-\frac{\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a-b}+\frac{\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x +
c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b))/d
```

3.1334 $\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{a\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
+ (a*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.0643939, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 801}

$$\frac{a\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
+ (a*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx&=\frac{\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx,x,b\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)}+\frac{a}{(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)(b+x)}\right)dx,x,b\sin(c+dx)\right)}{d}\\&=-\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{a\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.0791257, size = 87, normalized size = 1.18

$$\frac{a\log(a+b\sin(c+dx))+(b-a)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-(a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+d\right.\right.}{d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a + b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - (a + b)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + a*Log[a + b*Sin[c + d*x]])/((a - b)*(a + b)*d)
```

Maple [A] time = 0.055, size = 76, normalized size = 1.

$$\frac{a\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)\left(a-b\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{d\left(2a+2b\right)}-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)-1/d/(2*
a-2*b)*ln(1+sin(d*x+c))
```

Maxima [A] time = 0.997728, size = 88, normalized size = 1.19

$$\frac{\frac{2\,a\log\left(b\sin\left(dx+c\right)+a\right)}{a^2-b^2}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a-b}-\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a - b
) - log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A] time = 1.26761, size = 157, normalized size = 2.12

$$\frac{2\,a\log\left(b\sin\left(dx+c\right)+a\right)-\left(a+b\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a-b\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) - (a - b)*
log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.22567, size = 96, normalized size = 1.3

$$\frac{\frac{2\,ab\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^2b-b^3}-\frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{a-b}-\frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x + c
) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b))/d
```

3.1335 $\int \frac{\csc(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{b^2\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[Sin[c + d*x]]/(a*d) - Log[1 + Si
n[c + d*x]]/(2*(a - b)*d) + (b^2*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rubi [A] time = 0.145838, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2837, 12, 894}

$$\frac{b^2\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```


```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[Sin[c + d*x]]/(a*d) - Log[1 + Si
n[c + d*x]]/(2*(a - b)*d) + (b^2*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b}{x(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\,\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)} + \frac{1}{ab^2x} + \frac{1}{a(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)b^2(b+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{2(a-b)d} + \frac{b^2\log(a+b\sin(c}{a\left(a^2-b^2\right)c}$$

Mathematica [A] time = 0.106112, size = 84, normalized size = 0.9

$$-\frac{-\frac{2b^2\log(a+b\sin(c+dx))}{a\left(a^2-b^2\right)} + \frac{\log(1-\sin(c+dx))}{a+b} + \frac{\log(\sin(c+dx)+1)}{a-b} - \frac{2\log(\sin(c+dx))}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Log[1 - Sin[c + d*x]]/(a + b) - (2*Log[Sin[c + d*x]])/a + Log[1 + Sin[c +
 d*x]]/(a - b) - (2*b^2*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)))/(2*d)
```

Maple [A] time = 0.076, size = 95, normalized size = 1.

$$\frac{b^2\ln(a+b\sin(dx+c))}{da(a+b)(a-b)} - \frac{\ln(\sin(dx+c)-1)}{d(2a+2b)} - \frac{\ln(1+\sin(dx+c))}{d(2a-2b)} + \frac{\ln(\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^2/a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)-1/d
/(2*a-2*b)*ln(1+sin(d*x+c))+ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.01384, size = 108, normalized size = 1.16

$$\frac{\frac{2b^2\log(b\sin(dx+c)+a)}{a^3-ab^2} - \frac{\log(\sin(dx+c)+1)}{a-b} - \frac{\log(\sin(dx+c)-1)}{a+b} + \frac{2\log(\sin(dx+c))}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*log(b*sin(d*x + c) + a)/(a^3 - a*b^2) - log(sin(d*x + c) + 1)/(a
 - b) - log(sin(d*x + c) - 1)/(a + b) + 2*log(sin(d*x + c))/a)/d
```

Fricas [A] time = 1.72232, size = 225, normalized size = 2.42

$$\frac{2\,b^2\log\left(b\sin\left(dx+c\right)+a\right)+2\left(a^2-b^2\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(a^2+ab\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-ab\right)\log\left(-\sin\left(dx\right.\right.}{2\left(a^3-ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*log(b*sin(d*x + c) + a) + 2*(a^2 - b^2)*log(-1/2*sin(d*x + c)) -
 (a^2 + a*b)*log(sin(d*x + c) + 1) - (a^2 - a*b)*log(-sin(d*x + c) + 1))/((
a^3 - a*b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.23994, size = 116, normalized size = 1.25

$$\frac{\frac{2\,b^3\log\left(|b\sin(dx+c)+a|\right)}{a^3b-ab^3}-\frac{\log\left(|\sin(dx+c)+1|\right)}{a-b}-\frac{\log\left(|\sin(dx+c)-1|\right)}{a+b}+\frac{2\,\log\left(|\sin(dx+c)|\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(b*sin(d*x + c) + a))/(a^3*b - a*b^3) - log(abs(sin(d*x +
 c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b) + 2*log(abs(sin(d*x
+ c)))/a)/d
```

3.1336 $\int \frac{\csc^2(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{b^3\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - (b*Log[Sin[c
+ d*x]])/(a^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d) - (b^3*Log[a + b*Sin
[c + d*x]])/(a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.186549, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{b^3\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - (b*Log[Sin[c
+ d*x]])/(a^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d) - (b^3*Log[a + b*Sin
[c + d*x]])/(a^2*(a^2 - b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b^2}{x^2(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\,\text{Subst}\left(\int \frac{1}{x^2(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\,\text{Subst}\left(\int \left(\frac{1}{2b^3(a+b)(b-x)} + \frac{1}{ab^2x^2} - \frac{1}{a^2b^2x} - \frac{1}{a^2(a-b)(a+b)(a+x)} - \frac{1}{2b^3(-a+b)(b+x)}\right) dx, x, b\sin(c\right.}{d}$$

$$= -\frac{\csc(c+dx)}{ad} - \frac{\log(1-\sin(c+dx))}{2(a+b)d} - \frac{b\log(\sin(c+dx))}{a^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b^3 \text{l}}{}$$

Mathematica [A] time = 0.234847, size = 97, normalized size = 0.88

$$\frac{\frac{2b^3\log(a+b\sin(c+dx))}{a^2(b^2-a^2)} - \frac{2b\log(\sin(c+dx))}{a^2} - \frac{\log(1-\sin(c+dx))}{a+b} + \frac{\log(\sin(c+dx)+1)}{a-b} - \frac{2\csc(c+dx)}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*Csc[c + d*x])/a - Log[1 - Sin[c + d*x]]/(a + b) - (2*b*Log[Sin[c + d*x
]])/a^2 + Log[1 + Sin[c + d*x]]/(a - b) + (2*b^3*Log[a + b*Sin[c + d*x]])/(
a^2*(-a^2 + b^2)))/(2*d)
```

Maple [A] time = 0.082, size = 113, normalized size = 1.

$$-\frac{b^3\ln(a+b\sin(dx+c))}{d(a+b)(a-b)a^2} - \frac{\ln(\sin(dx+c)-1)}{d(2a+2b)} + \frac{\ln(1+\sin(dx+c))}{d(2a-2b)} - \frac{1}{da\sin(dx+c)} - \frac{b\ln(\sin(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*b^3/(a+b)/(a-b)/a^2*ln(a+b*sin(d*x+c))-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)+
1/d/(2*a-2*b)*ln(1+sin(d*x+c))-1/d/a/sin(d*x+c)-b*ln(sin(d*x+c))/a^2/d
```

Maxima [A] time = 0.982456, size = 128, normalized size = 1.16

$$-\frac{\frac{2b^3\log(b\sin(dx+c)+a)}{a^4-a^2b^2} - \frac{\log(\sin(dx+c)+1)}{a-b} + \frac{\log(\sin(dx+c)-1)}{a+b} + \frac{2b\log(\sin(dx+c))}{a^2} + \frac{2}{a\sin(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*log(b*sin(d*x + c) + a)/(a^4 - a^2*b^2) - log(sin(d*x + c) + 1)
/(a - b) + log(sin(d*x + c) - 1)/(a + b) + 2*b*log(sin(d*x + c))/a^2 + 2/(a
*sin(d*x + c)))/d
```

Fricas [A] time = 2.15817, size = 348, normalized size = 3.16

$$-\frac{2\,b^{3}\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)+2\,a^{3}-2\,ab^{2}+2\,\left(a^{2}b-b^{3}\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-\left(a^{3}+a^{2}b\right)\log}{2\,\left(a^{4}-a^{2}b^{2}\right)d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*log(b*sin(d*x + c) + a)*sin(d*x + c) + 2*a^3 - 2*a*b^2 + 2*(a^2
*b - b^3)*log(1/2*sin(d*x + c))*sin(d*x + c) - (a^3 + a^2*b)*log(sin(d*x +
c) + 1)*sin(d*x + c) + (a^3 - a^2*b)*log(-sin(d*x + c) + 1)*sin(d*x + c))/(
(a^4 - a^2*b^2)*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{2}\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.20944, size = 153, normalized size = 1.39

$$-\frac{\frac{2\,b^{4}\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^{4}b-a^{2}b^{3}}-\frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{a-b}+\frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{a+b}+\frac{2\,b\log\left(|\sin\left(dx+c\right)|\right)}{a^{2}}-\frac{2\,\left(b\sin\left(dx+c\right)-a\right)}{a^{2}\sin\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^4*log(abs(b*sin(d*x + c) + a))/(a^4*b - a^2*b^3) - log(abs(sin(d*
x + c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b) + 2*b*log(abs(sin
(d*x + c)))/a^2 - 2*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)))/d
```

3.1337 $\int \frac{\csc^3(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{b^4\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\text{cs}$$

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - Log[1 - Sin[c + d*x]]/(
2*(a + b)*d) + ((a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*d) - Log[1 + Sin[c + d*
x]]/(2*(a - b)*d) + (b^4*Log[a + b*Sin[c + d*x]])/(a^3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.202267, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{b^4\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\text{cs}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - Log[1 - Sin[c + d*x]]/(
2*(a + b)*d) + ((a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*d) - Log[1 + Sin[c + d*
x]]/(2*(a - b)*d) + (b^4*Log[a + b*Sin[c + d*x]])/(a^3*(a^2 - b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{b^3}{x^3(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\,\text{Subst}\left(\int \frac{1}{x^3(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\,\text{Subst}\left(\int \left(\frac{1}{2b^4(a+b)(b-x)} + \frac{1}{ab^2x^3} - \frac{1}{a^2b^2x^2} + \frac{a^2+b^2}{a^3b^4x} + \frac{1}{a^3(a-b)(a+b)(a+x)} + \frac{1}{2b^4(-a+b)(b+x)}\right) d\right.}{d}$$

$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{(a^2+b^2)\log(\sin(c+dx))}{a^3d} - \cdots$$

Mathematica [A] time = 0.534019, size = 132, normalized size = 1.

$$\frac{b^4\left(\frac{\csc(c+dx)}{a^2b^3} + \frac{(a^2+b^2)\log(\sin(c+dx))}{a^3b^4} + \frac{\log(a+b\sin(c+dx))}{a^3(a^2-b^2)} - \frac{\csc^2(c+dx)}{2ab^4} - \frac{\log(1-\sin(c+dx))}{2b^4(a+b)} - \frac{\log(\sin(c+dx)+1)}{2b^4(a-b)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^4*(Csc[c + d*x]/(a^2*b^3) - Csc[c + d*x]^2/(2*a*b^4) - Log[1 - Sin[c + d
*x]]/(2*b^4*(a + b)) + ((a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*b^4) - Log[1 +
Sin[c + d*x]]/(2*(a - b)*b^4) + Log[a + b*Sin[c + d*x]]/(a^3*(a^2 - b^2))))
/d
```

Maple [A] time = 0.092, size = 144, normalized size = 1.1

$$\frac{b^4\ln(a+b\sin(dx+c))}{d(a+b)(a-b)a^3} - \frac{\ln(\sin(dx+c)-1)}{d(2a+2b)} - \frac{\ln(1+\sin(dx+c))}{d(2a-2b)} - \frac{1}{2da(\sin(dx+c))^2} + \frac{\ln(\sin(dx+c))}{da} + \frac{b^2 \ln\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^4/(a+b)/(a-b)/a^3*ln(a+b*sin(d*x+c))-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)-1
/d/(2*a-2*b)*ln(1+sin(d*x+c))-1/2/d/a/sin(d*x+c)^2+ln(sin(d*x+c))/a/d+b^2*l
n(sin(d*x+c))/a^3/d+1/d/a^2*b/sin(d*x+c)
```

Maxima [A] time = 0.993531, size = 154, normalized size = 1.17

$$\frac{\frac{2b^4\log(b\sin(dx+c)+a)}{a^5-a^3b^2} - \frac{\log(\sin(dx+c)+1)}{a-b} - \frac{\log(\sin(dx+c)-1)}{a+b} + \frac{2(a^2+b^2)\log(\sin(dx+c))}{a^3} + \frac{2b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^4*log(b*sin(d*x + c) + a)/(a^5 - a^3*b^2) - log(sin(d*x + c) + 1)/
(a - b) - log(sin(d*x + c) - 1)/(a + b) + 2*(a^2 + b^2)*log(sin(d*x + c))/a
^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A] time = 3.16071, size = 501, normalized size = 3.8

$$\frac{a^4 - a^2b^2 + 2\left(b^4\cos\left(dx+c\right)^2 - b^4\right)\log\left(b\sin\left(dx+c\right)+a\right) - 2\left(a^4 - b^4 - \left(a^4 - b^4\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\sin\left(dx+c\right)\right) + }{2\left(\left(a^5 - \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^4 - a^2*b^2 + 2*(b^4*cos(d*x + c)^2 - b^4)*log(b*sin(d*x + c) + a) -
 2*(a^4 - b^4 - (a^4 - b^4)*cos(d*x + c)^2)*log(-1/2*sin(d*x + c)) + (a^4 +
 a^3*b - (a^4 + a^3*b)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + (a^4 - a^3*b
 - (a^4 - a^3*b)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^3*b - a*b^3)
*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2 - (a^5 - a^3*b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23968, size = 200, normalized size = 1.52

$$\frac{\frac{2\,b^5\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^5b-a^3b^3} - \frac{\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a-b} - \frac{\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a+b} + \frac{2\left(a^2+b^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^3} - \frac{3\,a^2\sin\left(dx+c\right)^2+3\,b^2\sin\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)+a^2}{a^3\sin\left(dx+c\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^5*log(abs(b*sin(d*x + c) + a))/(a^5*b - a^3*b^3) - log(abs(sin(d*x
 + c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b) + 2*(a^2 + b^2)*lo
g(abs(sin(d*x + c)))/a^3 - (3*a^2*sin(d*x + c)^2 + 3*b^2*sin(d*x + c)^2 - 2
*a*b*sin(d*x + c) + a^2)/(a^3*sin(d*x + c)^2))/d
```

3.1338 $\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=268

$$-\frac{a^3\cos(c+dx)}{b^2d\left(a^2-b^2\right)}+\frac{a\cos(c+dx)}{d\left(a^2-b^2\right)}+\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\left(a^2-b^2\right)^{3/2}}-\frac{3b\tan(c+dx)}{2d\left(a^2-b^2\right)}+\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\sin^2(c+dx)\tan(c+dx)}{2d\left(a^2-b^2\right)}$$

```
[Out] (3*b*x)/(2*(a^2 - b^2)) - (a^2*(2*a^2 + b^2)*x)/(2*b^3*(a^2 - b^2)) + (2*a^
5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*
d) + (a*Cos[c + d*x])/((a^2 - b^2)*d) - (a^3*Cos[c + d*x])/(b^2*(a^2 - b^2)
*d) + (a*Sec[c + d*x])/((a^2 - b^2)*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2
*b*(a^2 - b^2)*d) - (3*b*Tan[c + d*x])/(2*(a^2 - b^2)*d) + (b*Sin[c + d*x]^
2*Tan[c + d*x])/(2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.395946, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {2902, 2590, 14, 2591, 288, 321, 203, 2793, 3023, 2735, 2660, 618, 204}

$$-\frac{a^3\cos(c+dx)}{b^2d\left(a^2-b^2\right)}+\frac{a\cos(c+dx)}{d\left(a^2-b^2\right)}+\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\left(a^2-b^2\right)^{3/2}}-\frac{3b\tan(c+dx)}{2d\left(a^2-b^2\right)}+\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\sin^2(c+dx)\tan(c+dx)}{2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*x)/(2*(a^2 - b^2)) - (a^2*(2*a^2 + b^2)*x)/(2*b^3*(a^2 - b^2)) + (2*a^
5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*
d) + (a*Cos[c + d*x])/((a^2 - b^2)*d) - (a^3*Cos[c + d*x])/(b^2*(a^2 - b^2)
*d) + (a*Sec[c + d*x])/((a^2 - b^2)*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2
*b*(a^2 - b^2)*d) - (3*b*Tan[c + d*x])/(2*(a^2 - b^2)*d) + (b*Sin[c + d*x]^
2*Tan[c + d*x])/(2*(a^2 - b^2)*d)
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sin(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sin^2(c+dx)\tan^2(c+dx)(c+\cdots}{a^2-b^2}$$

$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d} - \frac{a^2\int \frac{a+b\sin(c+dx)-2a\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{2b\left(a^2-b^2\right)} - \frac{a\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,\cdots\right.}{\left(a^2-b^2\right)\cdots}$$

$$= -\frac{a^3\cos(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d} + \frac{b\sin^2(c+dx)\tan(c+dx)}{2\left(a^2-b^2\right)d} - \frac{a^2\int \cdots}{\cdots}$$

$$= -\frac{a^2\left(2a^2+b^2\right)x}{2b^3\left(a^2-b^2\right)} + \frac{a\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{a^3\cos(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a^2\cos(c+dx\cdots}{2b\left(a^2-\cdots\right.}$$

$$= \frac{3bx}{2\left(a^2-b^2\right)} - \frac{a^2\left(2a^2+b^2\right)x}{2b^3\left(a^2-b^2\right)} + \frac{a\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{a^3\cos(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} + \cdots$$

$$= \frac{3bx}{2\left(a^2-b^2\right)} - \frac{a^2\left(2a^2+b^2\right)x}{2b^3\left(a^2-b^2\right)} + \frac{a\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{a^3\cos(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} + \cdots$$

$$= \frac{3bx}{2\left(a^2-b^2\right)} - \frac{a^2\left(2a^2+b^2\right)x}{2b^3\left(a^2-b^2\right)} + \frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}d} + \frac{a\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{a^3\cos\cdots}{b^2\left(\cdots\right.}$$

Mathematica [A] time = 1.52499, size = 221, normalized size = 0.82

$$\frac{\frac{2a^2b^2(c+dx)+4a^4(c+dx)-4ab^3-6b^4(c+dx)}{b^5-a^2b^3} + \frac{8a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}} - \frac{4a\cos(c+dx)}{b^2} + \frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{4\sin\left(\frac{1}{2}\cdots\right.}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx\cdots\right.\right.}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-4*a*b^3 + 4*a^4*(c + d*x) + 2*a^2*b^2*(c + d*x) - 6*b^4*(c + d*x))/(-(a^
2*b^3) + b^5) + (8*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b
^3*(a^2 - b^2)^(3/2)) - (4*a*Cos[c + d*x])/b^2 + (4*Sin[(c + d*x)/2])/((a +
 b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (4*Sin[(c + d*x)/2])/((a - b)*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + Sin[2*(c + d*x)]/b)/(4*d)
```

Maple [A] time = 0.108, size = 283, normalized size = 1.1

$$-64\,\frac{1}{d\left(64\,a+64\,b\right)\left(\tan\left(1/2\,dx+c/2\right)-1\right)}-\frac{1}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}-2\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}}{db^{2}\left(1+\left(\tan\left(1/2\,dx+c\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] -64/d/(64*a+64*b)/(tan(1/2*d*x+1/2*c)-1)-1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)^3-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2
*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-2/d/b^2/(1+tan(1/2*d
*x+1/2*c)^2)^2*a-2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2-3/d/b*arctan(tan(1/
2*d*x+1/2*c))+2/d/(a-b)/(a+b)*a^5/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+64/d/(64*a-64*b)/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8168, size = 1138, normalized size = 4.25

$$\left[\frac{\sqrt{-a^2+b^2}a^5\cos\left(dx+c\right)\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)+2\,a^3b^3-2\,ab}{2\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^5*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^3*b^3 - 2*a*b^5 - (2*a^6 - a^4*b^2 - 4*a^2*b^4 + 3*b^6)*d*x*co
s(d*x + c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c)^2 - (2*a^2*b^4 - 2*
b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^4*b^3 -
 2*a^2*b^5 + b^7)*d*cos(d*x + c)), -1/2*(2*sqrt(a^2 - b^2)*a^5*arctan(-(a*s
in(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - 2*a^3*b^3 +
```

```
2*a*b^5 + (2*a^6 - a^4*b^2 - 4*a^2*b^4 + 3*b^6)*d*x*cos(d*x + c) + 2*(a^5*
b - 2*a^3*b^3 + a*b^5)*cos(d*x + c)^2 + (2*a^2*b^4 - 2*b^6 - (a^4*b^2 - 2*a
^2*b^4 + b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*
cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2004, size = 281, normalized size = 1.05

$$\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^5}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}+\frac{4\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\left(2a^2+3b^2\right)(dx+c)}{b^3}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 \cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^5/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) + 4*(b*t
an(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)) - (2*a^
2 + 3*b^2)*(d*x + c)/b^3 - 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^
2))/d
```

3.1339 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=183

$$\frac{a^2\cos(c+dx)}{bd(a^2-b^2)}-\frac{b\cos(c+dx)}{d(a^2-b^2)}-\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d(a^2-b^2)^{3/2}}+\frac{a\tan(c+dx)}{d(a^2-b^2)}-\frac{b\sec(c+dx)}{d(a^2-b^2)}+\frac{a^3x}{b^2(a^2-b^2)}-\frac{ax}{a^2-b^2}$$

```
[Out] -((a*x)/(a^2 - b^2)) + (a^3*x)/(b^2*(a^2 - b^2)) - (2*a^4*ArcTan[(b + a*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)*d) + (a^2*Cos[c + d
*x])/(b*(a^2 - b^2)*d) - (b*Cos[c + d*x])/((a^2 - b^2)*d) - (b*Sec[c + d*x]
)/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.2734, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2902, 3473, 8, 2590, 14, 2746, 12, 2735, 2660, 618, 204}

$$\frac{a^2\cos(c+dx)}{bd(a^2-b^2)}-\frac{b\cos(c+dx)}{d(a^2-b^2)}-\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d(a^2-b^2)^{3/2}}+\frac{a\tan(c+dx)}{d(a^2-b^2)}-\frac{b\sec(c+dx)}{d(a^2-b^2)}+\frac{a^3x}{b^2(a^2-b^2)}-\frac{ax}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/(a^2 - b^2)) + (a^3*x)/(b^2*(a^2 - b^2)) - (2*a^4*ArcTan[(b + a*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)*d) + (a^2*Cos[c + d
*x])/(b*(a^2 - b^2)*d) - (b*Cos[c + d*x])/((a^2 - b^2)*d) - (b*Sec[c + d*x]
)/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sin(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2}$$

$$= \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\int 1\,dx}{a^2-b^2} + \frac{a^2\int \frac{a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + \frac{b\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx\right.}{\left(a^2-b\right.}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a^3\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + \frac{b\,\text{Subst}\left(\int\left(-1+\right.\right.}{\left(a^2\right.}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan(c+}{\left(a^2-b^2\right.}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan(c+}{\left(a^2-b^2\right.}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan(c+}{\left(a^2-b^2\right.}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} - \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}d} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 1.03234, size = 186, normalized size = 1.02

$$\frac{\frac{a^3(-(c+dx))+ab^2(c+dx)+b^3}{b^4-a^2b^2} - \frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{\cos(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((b^3 - a^3*(c + d*x) + a*b^2*(c + d*x))/(-(a^2*b^2) + b^4) - (2*a^4*ArcTan
[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)) + Cos[c
 + d*x]/b + Sin[(c + d*x)/2]/((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) + Sin[(c + d*x)/2]/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/d
```

Maple [A] time = 0.102, size = 162, normalized size = 0.9

$$-32\,\frac{1}{d\,(32\,a+32\,b)\,(\tan(1/2\,dx+c/2)-1)} + 2\,\frac{1}{bd\left(1+(\tan(1/2\,dx+c/2))^2\right)} + 2\,\frac{a\arctan(\tan(1/2\,dx+c/2))}{db^2} - 2\,\frac{}{d\,(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -32/d/(32*a+32*b)/(tan(1/2*d*x+1/2*c)-1)+2/d/b/(1+tan(1/2*d*x+1/2*c)^2)+2/d
/b^2*a*arctan(tan(1/2*d*x+1/2*c))-2/d/(a-b)/(a+b)*a^4/b^2/(a^2-b^2)^(1/2)*a
rctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-32/d/(32*a-32*b)/(t
```

```
an(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6845, size = 954, normalized size = 5.21

$$\left[\frac{\sqrt{-a^2+b^2}a^4\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b^3+2b}{2(a^4b^2-2a^2b^4+b^6)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^4*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b^3 + 2*b^5 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) +
2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2 + 2*(a^3*b^2 - a*b^4)*sin(d*x +
c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^4*arct
an(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2
*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + (a^4*b - 2*a^2*b^
3 + b^5)*cos(d*x + c)^2 + (a^3*b^2 - a*b^4)*sin(d*x + c))/((a^4*b^2 - 2*a^2
*b^4 + b^6)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18588, size = 234, normalized size = 1.28

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{(a^2b^2-b^4)\sqrt{a^2-b^2}}-\frac{(dx+c)a}{b^2}+\frac{2\left(ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^2-2b^2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1\right)(a^2b-b^3)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^4/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) - (d*x + c)
*a/b^2 + 2*(a*b*tan(1/2*d*x + 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c)^2 + a*b*t
an(1/2*d*x + 1/2*c) + a^2 - 2*b^2)/((tan(1/2*d*x + 1/2*c)^4 - 1)*(a^2*b - b
^3)))/d
```

3.1340 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=133

$$\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{3/2}}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)}+\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)}-\frac{a^2x}{b\left(a^2-b^2\right)}+\frac{bx}{a^2-b^2}$$

```
[Out] -((a^2*x)/(b*(a^2 - b^2))) + (b*x)/(a^2 - b^2) + (2*a^3*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)*d) + (a*Sec[c + d*x])/(
(a^2 - b^2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.177964, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2902, 2606, 8, 3473, 2735, 2660, 618, 204}

$$\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{3/2}}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)}+\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)}-\frac{a^2x}{b\left(a^2-b^2\right)}+\frac{bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((a^2*x)/(b*(a^2 - b^2))) + (b*x)/(a^2 - b^2) + (2*a^3*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)*d) + (a*Sec[c + d*x])/(
(a^2 - b^2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d)
```


Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec(c+dx)\tan(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \tan^2(c+dx)\,dx}{a^2-b^2}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a^3\int \frac{1}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + \frac{b\int 1\,dx}{a^2-b^2} + \frac{a\,\text{Subst}(\int 1\,dx, x, \sec}{\left(a^2-b^2\right)d}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} + \frac{bx}{a^2-b^2} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,\right.}{b\left(a^2-b^2\right)}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} + \frac{bx}{a^2-b^2} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d} - \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,d\right.}{b\left(a^2\right.}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} + \frac{bx}{a^2-b^2} + \frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.769586, size = 152, normalized size = 1.14

$$\frac{\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b(a-b\sin(c+dx))-\left(a^2-b^2\right)(c+dx)\cos(c+dx)}{(a-b)(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)
 + (-((a^2 - b^2)*(c + d*x)*Cos[c + d*x]) + b*(a - b*Sin[c + d*x]))/((a - b
)*(a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])))/(b*d)
```

Maple [A] time = 0.093, size = 138, normalized size = 1.

$$-2\,{\frac {\arctan \left(\tan \left(1/2\,dx+c/2 \right) \right) }{bd}}+2\,{\frac {{a}^{3}}{d \left(a-b \right) \left(a+b \right) b\sqrt {{a}^{2}-{b}^{2}}}\arctan \left(1/2\,{\frac {2\,a\tan \left(1/2\,dx+c/2 \right) +2\,b}{\sqrt {{a}^{2}-{b}^{2}}}} \right) }+16\,{\frac {}{d \left(16\,a-1 \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/b*arctan(tan(1/2*d*x+1/2*c))+2/d/(a-b)/(a+b)*a^3/b/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+16/d/(16*a-16*b)/(tan
(1/2*d*x+1/2*c)+1)-16/d/(16*a+16*b)/(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62385, size = 822, normalized size = 6.18

$$\left[\frac{\sqrt{-a^{2} + b^{2}} a^{3} \cos\left(dx + c\right) \log\left(-\frac{{\left(2 \, a^{2} - b^{2}\right)} \cos\left(dx + c\right)^{2} - 2 \, a b \sin\left(dx + c\right) - a^{2} - b^{2} - 2 \, {\left(a \cos\left(dx + c\right) \sin\left(dx + c\right) + b \cos\left(dx + c\right)\right)} \sqrt{-a^{2} + b^{2}}}{b^{2} \cos\left(dx + c\right)^{2} - 2 \, a b \sin\left(dx + c\right) - a^{2} - b^{2}}\right) + 2 \, a^{3} b - 2 \,}{2 \, {\left(a^{4} b - 2 \, a^{2} b^{3} + b^{5}\right)} d \cos\left(dx + c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^3*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^3*b - 2*a*b^3 - 2*(a^4 - 2*a^2*b^2 + b^4)*d*x*cos(d*x + c) - 2
*(a^2*b^2 - b^4)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c)),
-(sqrt(a^2 - b^2)*a^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
 + c)))*cos(d*x + c) - a^3*b + a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*d*x*cos(d*x
+ c) + (a^2*b^2 - b^4)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x +
 c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19808, size = 177, normalized size = 1.33

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3}{\left(a^2b-b^3\right)\sqrt{a^2-b^2}}-\frac{dx+c}{b}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a^3/((a^2*b - b^3)*sqrt(a^2 - b^2)) - (d*x + c)/b
+ 2*(b*tan(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1))
)/d
```

3.1341 $\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.107109, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2727, 3767, 8, 2606, 2660, 618, 204}

$$-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan(c+dx)\,dx}{a^2-b^2}$$

$$= -\frac{a\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{\left(a^2-b^2\right)d} - \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d} - \frac{b\,\mathrm{Sub}}{}$$

$$= -\frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{\left(4a^2\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.198357, size = 152, normalized size = 1.58

$$\frac{\sqrt{a^2-b^2}(a\sin(c+dx)+b\cos(c+dx)-b)-2a^2\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a-b)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqr
t[a^2 - b^2]*(-b + b*Cos[c + d*x] + a*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[
a^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))
```

Maple [A] time = 0.088, size = 117, normalized size = 1.2

$$-2\,\frac{a^2}{d\,(a-b)\,(a+b)\,\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)+2\,b}{\sqrt{a^2-b^2}}\right)-8\,\frac{1}{d\,(8\,a-8\,b)\,(\tan(1/2\,dx+c/2)+1)}-8\,\frac{}{d\,(8\,a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d*a^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b
)/(a^2-b^2)^(1/2))-8/d/(8*a-8*b)/(tan(1/2*d*x+1/2*c)+1)-8/d/(8*a+8*b)/(tan(
1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53939, size = 684, normalized size = 7.12

$$\left[\frac{\sqrt{-a^2+b^2}a^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b+2b^3}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.18139, size = 144, normalized size = 1.5

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.1342 $\int \frac{\sec(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{2ab\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)}$$

```
[Out] (2*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)
*d) + (Sec[c + d*x]*(a - b*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Rubi [A] time = 0.0994956, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2866, 12, 2660, 618, 204}

$$\frac{2ab\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)
*d) + (Sec[c + d*x]*(a - b*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d} - \frac{\int \frac{ab}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d} + \frac{(ab)\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d} + \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}$$

$$= \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d} - \frac{(4ab)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx\right.\right.}{\left(a^2-b^2\right)d}$$

$$= \frac{2ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} + \frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.17441, size = 151, normalized size = 1.84

$$\frac{\sqrt{a^2-b^2}(a(-\cos(c+dx))+a-b\sin(c+dx))+2ab\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a-b)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(a - a*Cos[c + d*x] - b*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[a^
2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))
```

Maple [A] time = 0.082, size = 116, normalized size = 1.4

$$2\,\frac{ab}{d\,(a-b)\,(a+b)\,\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)+2\,b}{\sqrt{a^2-b^2}}\right)+4\,\frac{1}{d\,(4\,a-4\,b)\,(\tan(1/2\,dx+c/2)+1)}-4\,\frac{}{d\,(4\,a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 2/d*a*b/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))+4/d/(4*a-4*b)/(tan(1/2*d*x+1/2*c)+1)-4/d/(4*a+4*b)/(tan(1
/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59036, size = 687, normalized size = 8.38

$$\left[\frac{\sqrt{-a^2+b^2}ab\cos(dx+c)\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^3-2a}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a*b*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^3 - 2*a*b^2 - 2*(a^2*b - b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2
+ b^4)*d*cos(d*x + c)), -(sqrt(a^2 - b^2)*a*b*arctan(-(a*sin(d*x + c) + b)/
(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^3 + a*b^2 + (a^2*b - b^3)*
sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.19754, size = 143, normalized size = 1.74

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab}{(a^2-b^2)^{\frac{3}{2}}}+\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a*b/(a^2 - b^2)^(3/2) + (b*tan(1/2*d*x + 1/2*c) -
a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.1343 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{3/2}}+\frac{b\sec(c+dx)(b-a\sin(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(3/
2)*d) - ArcTanh[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + (b*Sec[c + d*x]*
(b - a*Sin[c + d*x]))/(a*(a^2 - b^2)*d)
```

Rubi [A] time = 0.230191, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2898, 2622, 321, 207, 2696, 12, 2660, 618, 204}

$$\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{3/2}}+\frac{b\sec(c+dx)(b-a\sin(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(3/
2)*d) - ArcTanh[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + (b*Sec[c + d*x]*
(b - a*Sin[c + d*x]))/(a*(a^2 - b^2)*d)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{\csc(c+dx)\sec^2(c+dx)}{a} - \frac{b\sec^2(c+dx)}{a(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \csc(c+dx)\sec^2(c+dx)\,dx}{a} - \frac{b\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a\left(a^2-b^2\right)d} + \frac{b\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+\right.}{ad}$$

$$= \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a\left(a^2-b^2\right)d} + \frac{b^3\int \frac{1}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{-1\ldots}\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a\left(a^2-b^2\right)d} + \frac{\left(2b^3\right)\text{Sub}}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a\left(a^2-b^2\right)d} - \frac{\left(4b^3\right)\text{Sub}}{}$$

$$= \frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{3/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b\ldots}{a\left(a^2-\ldots\right.}$$

Mathematica [A] time = 0.362104, size = 191, normalized size = 1.62

$$\frac{2b^3\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + \sqrt{a^2-b^2}\left(a(a-b\sin(c+dx)) - \left(a^2-b^2\right)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\ldots\right.\right.}{ad(a-b)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(-((a^2 - b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c
 + d*x)/2]])) + a*(a - b*Sin[c + d*x])))/(a*(a - b)*(a + b)*Sqrt[a^2 - b^2]
*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]))
```

Maple [A] time = 0.101, size = 130, normalized size = 1.1

$$2\,\frac{b^3}{d\,(a-b)\,(a+b)\,a\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)+2\,b}{\sqrt{a^2-b^2}}\right) + \frac{1}{d\,(a-b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} + \frac{1}{da}\ln\left(\tan\left(\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 2/d*b^3/(a-b)/(a+b)/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*
b)/(a^2-b^2)^(1/2))+1/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)+1/d/a*ln(tan(1/2*d*x+1
/2*c))-1/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.05825, size = 1084, normalized size = 9.19

$$\left[\frac{\sqrt{-a^2+b^2}b^3\cos(dx+c)\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^4-2a^2b^2}{2(a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^3*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)*log(1/2*c
os(d*x + c) + 1/2) + (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)*log(-1/2*cos(d*x
+ c) + 1/2) - 2*(a^3*b - a*b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*
cos(d*x + c)), -1/2*(2*sqrt(a^2 - b^2)*b^3*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - 2*a^4 + 2*a^2*b^2 + (a^4 - 2*a^
2*b^2 + b^4)*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - (a^4 - 2*a^2*b^2 +
b^4)*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2) + 2*(a^3*b - a*b^3)*sin(d*x
+ c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.18329, size = 182, normalized size = 1.54

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^3}{(a^3-ab^2)\sqrt{a^2-b^2}}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*b^3/((a^3 - a*b^2)*sqrt(a^2 - b^2)) + log(abs(tan(
1/2*d*x + 1/2*c)))/a + 2*(b*tan(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2
*d*x + 1/2*c)^2 - 1)))/d
```

3.1344 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=128

$$-\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] (-2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^
(3/2)*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d) - (b*Sec[
c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.288354, antiderivative size = 150, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2898, 2622, 321, 207, 2620, 14, 2696, 12, 2660, 618, 204}

$$-\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{3/2}}-\frac{b^2\sec(c+dx)(b-a\sin(c+dx))}{a^2d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^
(3/2)*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d) - (b*Sec[
c + d*x])/(a^2*d) - (b^2*Sec[c + d*x]*(b - a*Sin[c + d*x]))/(a^2*(a^2 - b^2
)*d) + Tan[c + d*x]/(a*d)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}dx = \int \left(-\frac{b\csc(c+dx)\sec^2(c+dx)}{a^2} + \frac{\csc^2(c+dx)\sec^2(c+dx)}{a} + \frac{b^2\sec^2(c+dx)}{a^2(a+b\sin(c+dx))}\right) d$$

$$= \frac{\int \csc^2(c+dx)\sec^2(c+dx)dx}{a} - \frac{b\int \csc(c+dx)\sec^2(c+dx)dx}{a^2} + \frac{b^2\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}dx}{a^2}$$

$$= -\frac{b^2\sec(c+dx)(b-a\sin(c+dx))}{a^2(a^2-b^2)d} - \frac{b^2\int \frac{b^2}{a+b\sin(c+dx)}dx}{a^2(a^2-b^2)} + \frac{\text{Subst}\left(\int \frac{1+x^2}{x^2}dx, x, \tan(c\right.}{ad}$$

$$= -\frac{b\sec(c+dx)}{a^2d} - \frac{b^2\sec(c+dx)(b-a\sin(c+dx))}{a^2(a^2-b^2)d} - \frac{b^4\int \frac{1}{a+b\sin(c+dx)}dx}{a^2(a^2-b^2)} + \frac{\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b^2\sec(c+dx)(b-a\sin(c+dx}{a^2(a^2-b^2)d}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b^2\sec(c+dx)(b-a\sin(c+dx}{a^2(a^2-b^2)d}$$

$$= -\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{3/2}d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} -$$

Mathematica [A] time = 0.933606, size = 205, normalized size = 1.6

$$\frac{-\frac{4b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{3/2}} - \frac{2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-4*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)
^(3/2)) - Cot[(c + d*x)/2]/a + (2*b*Log[Cos[(c + d*x)/2]])/a^2 - (2*b*Log[S
in[(c + d*x)/2]])/a^2 + (2*Sin[(c + d*x)/2])/((a + b)*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])) + Tan[(c + d*x)/2]/a)/(2*d)
```

Maple [A] time = 0.107, size = 169, normalized size = 1.3

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{d(a+b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} - 2\frac{b^4}{da^2(a-b)(a+b)\sqrt{a^2-b^2}}\arctan\left(1/2\frac{2a\tan(1/2dx+c/2)+}{\sqrt{a^2-b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-1/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)-2/d/a^2/(a-b)/(
a+b)*b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^
(1/2))-1/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a^2*
```

```
b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.92457, size = 1370, normalized size = 10.7

$$\left[\frac{\sqrt{-a^2+b^2}b^4\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)\sin(dx+c)+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^4*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2))*sin(d*x + c) + 2*a^5 - 2*a^3*b^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x
+ c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*c
os(d*x + c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*(2*a^5 - 3*a^3*b^
2 + a*b^4)*cos(d*x + c)^2 - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^6 - 2*a^4
*b^2 + a^2*b^4)*d*cos(d*x + c)*sin(d*x + c)), 1/2*(2*sqrt(a^2 - b^2)*b^4*ar
ctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)*sin
(d*x + c) + 2*a^5 - 2*a^3*b^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x +
c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*(2*a^5 - 3*a^3*b^2 + a*b^4
)*cos(d*x + c)^2 - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^6 - 2*a^4*b^2 + a^
2*b^4)*d*cos(d*x + c)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23745, size = 350, normalized size = 2.73

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^4}{\left(a^4-a^2b^2\right)\sqrt{a^2-b^2}}+\frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
 1/2*c) + b)/sqrt(a^2 - b^2)))*b^4/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + 6*b*
log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/2*d*x + 1/2*c)/a - (2*a^2*b*ta
n(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x +
1/2*c)^2 + 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 10*a^2*b*tan(1/2*d*x + 1/2*c) +
 2*b^3*tan(1/2*d*x + 1/2*c) + 3*a^3 - 3*a*b^2)/((a^4 - a^2*b^2)*(tan(1/2*d*
x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))))/d
```

3.1345 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)}+\frac{\left(3a^2-b^2\right)\sec(c+dx)}{2ad\left(a^2-b^2\right)}-\frac{\left(3a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+}{a^2d}$$

```
[Out] (2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(
3/2)*d) - ((3*a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (b*Cot[c + d*
x])/(a^2*d) + ((3*a^2 - b^2)*Sec[c + d*x])/(2*a*(a^2 - b^2)*d) - (Csc[c + d
*x]^2*Sec[c + d*x])/(2*a*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.359732, antiderivative size = 212, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2898, 2622, 321, 207, 2620, 14, 288, 2696, 12, 2660, 618, 204}

$$\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}+\frac{b^2\sec(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b^3\sec(c+dx)(b-a\sin(c+dx))}{a^3d\left(a^2-b^2\right)}-\frac{b\tan(c+}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(
3/2)*d) - (3*ArcTanh[Cos[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(
a^3*d) + (b*Cot[c + d*x])/(a^2*d) + (3*Sec[c + d*x])/(2*a*d) + (b^2*Sec[c +
 d*x])/(a^3*d) - (Csc[c + d*x]^2*Sec[c + d*x])/(2*a*d) + (b^3*Sec[c + d*x]*
(b - a*Sin[c + d*x]))/(a^3*(a^2 - b^2)*d) - (b*Tan[c + d*x])/(a^2*d)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{b^2\csc(c+dx)\sec^2(c+dx)}{a^3} - \frac{b\csc^2(c+dx)\sec^2(c+dx)}{a^2} + \frac{\csc^3(c+dx)\sec^2(c}{a}\right.$$

$$= \frac{\int \csc^3(c+dx)\sec^2(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\sec^2(c+dx)\,dx}{a^2} + \frac{b^2\int \csc(c+dx}{}$$

$$= \frac{b^3\sec(c+dx)(b-a\sin(c+dx))}{a^3\left(a^2-b^2\right)d} + \frac{b^3\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{a^3\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \text{s}\right.}{ad}$$

$$= \frac{b^2\sec(c+dx)}{a^3d} - \frac{\csc^2(c+dx)\sec(c+dx)}{2ad} + \frac{b^3\sec(c+dx)(b-a\sin(c+dx))}{a^3\left(a^2-b^2\right)d} + \frac{b^5}{}$$

$$= -\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{3\sec(c+dx)}{2ad} + \frac{b^2\sec(c+dx)}{a^3d} - \frac{\csc^2(}{}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{3\sec(c+dx)}{2ad}$$

$$= \frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}d} - \frac{3\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b}{}$$

Mathematica [A] time = 3.00455, size = 261, normalized size = 1.44

$$\frac{\frac{16b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}} + \frac{4\left(3a^2+2b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} - \frac{4\left(3a^2+2b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} - \frac{4b\tan\left(\frac{1}{2}(c+dx)\right)}{a^2} + \frac{4b\cot\left(\frac{1}{2}(c+dx)\right)}{a^2} + \frac{}{(a+b)(c}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((16*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)
^(3/2)) + (4*b*Cot[(c + d*x)/2])/a^2 - Csc[(c + d*x)/2]^2/a - (4*(3*a^2 + 2
*b^2)*Log[Cos[(c + d*x)/2]])/a^3 + (4*(3*a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]]
)/a^3 + Sec[(c + d*x)/2]^2/a + (8*Sin[(c + d*x)/2])/((a + b)*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])) - (8*Sin[(c + d*x)/2])/((a - b)*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])) - (4*b*Tan[(c + d*x)/2])/a^2)/(8*d)
```

Maple [A] time = 0.136, size = 227, normalized size = 1.3

$$\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{b}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{d(a+b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + 2\frac{b^5}{da^3(a-b)(a+b)\sqrt{a^2-b^2}}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/2*d*x+1/2*c)*b-1/d/(a+b)/(tan(
1/2*d*x+1/2*c)-1)+2/d/a^3/(a-b)/(a+b)*b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)-1/
8/d/a/tan(1/2*d*x+1/2*c)^2+3/2/d/a*ln(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/
2*d*x+1/2*c))*b^2+1/2/d/a^2*b/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.02943, size = 1931, normalized size = 10.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^6 - 4*a^4*b^2 - 2*(3*a^6 - 4*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 -
 2*(b^5*cos(d*x + c)^3 - b^5*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 -
b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*si
n(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*
sin(d*x + c) - a^2 - b^2)) + ((3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x
 + c)^3 - (3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) - ((3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^3 - (3
*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1
/2) - 4*(a^5*b - a^3*b^3 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^3 - (a^7 - 2*a^5*b^
2 + a^3*b^4)*d*cos(d*x + c)), -1/4*(4*a^6 - 4*a^4*b^2 - 2*(3*a^6 - 4*a^4*b^
2 + a^2*b^4)*cos(d*x + c)^2 + 4*(b^5*cos(d*x + c)^3 - b^5*cos(d*x + c))*sqr
t(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) +
 ((3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^3 - (3*a^6 - 4*a^4*b^2
 - a^2*b^4 + 2*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - ((3*a^6 - 4
*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^3 - (3*a^6 - 4*a^4*b^2 - a^2*b^4 +
 2*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 4*(a^5*b - a^3*b^3 - (
2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 2*a^5*b^
2 + a^3*b^4)*d*cos(d*x + c)^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c))
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19246, size = 331, normalized size = 1.83

$$\frac{\frac{16\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^5}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}+\frac{16\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}+\frac{4\left(3a^2+2b^2\right)\log\left(\left|\tan\right.\right.}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(16*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*b^5/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + 16*(b
*tan(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (a*
tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 + 4*(3*a^2 + 2*b^2)*
log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (18*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^
2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/(a^3*tan(1/2*d
*x + 1/2*c)^2))/d
```

3.1346 $\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{a^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+dx)+}{4d(a-b)^2}$$

```
[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d
) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.202472, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 801}

$$-\frac{a^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+dx)+}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d
) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(2a^2-b^2\right)x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2a-b)b^2}{2(a-b)^2(b+x)}\right) dx \ldots}{2b^2d}$$

$$= \frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{a^3\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2d} + \ldots$$

Mathematica [A] time = 0.504626, size = 117, normalized size = 0.93

$$\frac{-\frac{4a^3\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} - \frac{1}{(a+b)(\sin(c+dx)-1)} + \frac{1}{(a-b)(\sin(c+dx)+1)} + \frac{(2a+b)\log(1-\sin(c+dx))}{(a+b)^2} + \frac{(2a-b)\log(\sin(c+dx)+1)}{(a-b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((2*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^2 + ((2*a - b)*Log[1 + Sin[c + d
*x]])/(a - b)^2 - (4*a^3*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) - 1
/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)
```

Maple [A] time = 0.079, size = 164, normalized size = 1.3

$$-\frac{a^3\ln(a+b\sin(dx+c))}{d(a+b)^2(a-b)^2} - \frac{1}{d(4a+4b)(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)a}{2d(a+b)^2} + \frac{\ln(\sin(dx+c)-1)b}{4d(a+b)^2} + \frac{\ldots}{d(4a-\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-1/d/(4*a+4*b)/(sin(d*x+c)-1)+1/
2/d/(a+b)^2*ln(sin(d*x+c)-1)*a+1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*b+1/d/(4*a-4*
b)/(1+sin(d*x+c))+1/2*a*ln(1+sin(d*x+c))/(a-b)^2/d-1/4*b*ln(1+sin(d*x+c))/(
a-b)^2/d
```

Maxima [A] time = 0.974837, size = 192, normalized size = 1.52

$$-\frac{\frac{4\,a^3\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4} - \frac{(2\,a-b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2} - \frac{(2\,a+b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2} - \frac{2\,(b\sin(dx+c)-a)}{\left(a^2-b^2\right)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - (2*a - b)*log
(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (2*a + b)*log(sin(d*x + c) - 1)/(a
^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 - a^
2 + b^2))/d
```

Fricas [A] time = 2.00803, size = 367, normalized size = 2.91

$$-\frac{4\,a^3\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right)-\left(2\,a^3+3\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,a^3-3\,a^2b+b^3\right)\mathrm{cc}}{4\left(a^4-2\,a^2b^2+b^4\right)d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (2*a^3 + 3*a^2*b - b^3
)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a^2*b + b^3)*cos(d*x +
c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x + c
))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24064, size = 239, normalized size = 1.9

$$-\frac{\frac{4\,a^3b\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^4b-2\,a^2b^3+b^5}-\frac{\left(2\,a-b\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^2-2\,ab+b^2}-\frac{\left(2\,a+b\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^2+2\,ab+b^2}+\frac{2\left(a^3\sin\left(dx+c\right)^2-a^2b\sin\left(dx+c\right)+b^3\sin\left(dx+c\right)-ab^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin\left(dx+c\right)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*b*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - (2*a
 - b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (2*a + b)*log(abs(si
n(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(a^3*sin(d*x + c)^2 - a^2*b*sin(d*
x + c) + b^3*sin(d*x + c) - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2
 - 1)))/d
```

3.1347 $\int \frac{\sec(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{a^2b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{a\log(1-\sin(c+dx))}{4d(a+b)^2}-\frac{a\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

```
[Out] (a*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) - (a*Log[1 + Sin[c + d*x]])/(4*(a
 - b)^2*d) + (a^2*b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - (Sec[c + d
*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.230734, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 1647, 801}

$$\frac{a^2b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{a\log(1-\sin(c+dx))}{4d(a+b)^2}-\frac{a\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) - (a*Log[1 + Sin[c + d*x]])/(4*(a
 - b)^2*d) + (a^2*b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - (Sec[c + d
*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\mathrm{Subst}\left(\int \frac{x^2}{b^2(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{2d} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2-b^2}+\frac{ab^2x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2bd}$$

$$= -\frac{\sec^2(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{2d} + \frac{\mathrm{Subst}\left(\int \left(-\frac{ab}{2(a+b)^2(b-x)}+\frac{2a^2b^2}{(a-b)^2(a+b)^2(a+x)}-\frac{ab}{2(a-b)^2(b}\right.\right.}{2bd}$$

$$= \frac{a\log(1-\sin(c+dx))}{4(a+b)^2d} - \frac{a\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{a^2b\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2d} - \frac{\sec^2(c+}{}$$

Mathematica [A] time = 0.455436, size = 108, normalized size = 0.93

$$-\frac{-\frac{4a^2b\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}+\frac{1}{(a+b)(\sin(c+dx)-1)}+\frac{1}{(a-b)(\sin(c+dx)+1)}-\frac{a\log(1-\sin(c+dx))}{(a+b)^2}+\frac{a\log(\sin(c+dx)+1)}{(a-b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-((a*Log[1 - Sin[c + d*x]])/(a + b)^2) + (a*Log[1 + Sin[c + d*x]])/(a - b
)^2 - (4*a^2*b*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a + b)*
(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)
```

Maple [A] time = 0.075, size = 123, normalized size = 1.1

$$\frac{a^2b\ln(a+b\sin(dx+c))}{d(a+b)^2(a-b)^2} - \frac{1}{d(4a+4b)(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)a}{4d(a+b)^2} - \frac{1}{d(4a-4b)(1+\sin(dx+c))} - \frac{a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a^2/(a+b)^2*b/(a-b)^2*ln(a+b*sin(d*x+c))-1/d/(4*a+4*b)/(sin(d*x+c)-1)+1
/4/d/(a+b)^2*ln(sin(d*x+c)-1)*a-1/d/(4*a-4*b)/(1+sin(d*x+c))-1/4*a*ln(1+sin
(d*x+c))/(a-b)^2/d
```

Maxima [A] time = 0.980875, size = 178, normalized size = 1.53

$$\frac{\frac{4a^2b\log(b\sin(dx+c)+a)}{a^4-2a^2b^2+b^4} - \frac{a\log(\sin(dx+c)+1)}{a^2-2ab+b^2} + \frac{a\log(\sin(dx+c)-1)}{a^2+2ab+b^2} - \frac{2(a\sin(dx+c)-b)}{\left(a^2-b^2\right)\sin(dx+c)^2-a^2+b^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*b*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - a*log(sin(d*
x + c) + 1)/(a^2 - 2*a*b + b^2) + a*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^
2) - 2*(a*sin(d*x + c) - b)/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

Fricas [A] time = 2.09375, size = 369, normalized size = 3.18

$$\frac{4\,a^{2}b\cos\left(dx+c\right)^{2}\log\left(b\sin\left(dx+c\right)+a\right)-\left(a^{3}+2\,a^{2}b+ab^{2}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)+\left(a^{3}-2\,a^{2}b+ab^{2}\right)}{4\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*b*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (a^3 + 2*a^2*b + a*b^
2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^3 - 2*a^2*b + a*b^2)*cos(d*x +
 c)^2*log(-sin(d*x + c) + 1) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x +
c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{2}\left(c+dx\right)\sec^{3}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.23537, size = 227, normalized size = 1.96

$$\frac{\frac{4\,a^{2}b^{2}\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{4}b-2\,a^{2}b^{3}+b^{5}}-\frac{a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{2}-2\,ab+b^{2}}+\frac{a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{2}+2\,ab+b^{2}}+\frac{2\left(a^{2}b\sin\left(dx+c\right)^{2}-a^{3}\sin\left(dx+c\right)+ab^{2}\sin\left(dx+c\right)-b^{3}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\sin\left(dx+c\right)^{2}-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*b^2*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - a*l
og(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + a*log(abs(sin(d*x + c) - 1)
)/(a^2 + 2*a*b + b^2) + 2*(a^2*b*sin(d*x + c)^2 - a^3*sin(d*x + c) + a*b^2*
sin(d*x + c) - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

3.1348 $\int \frac{\sec^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{ab^2\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}-\frac{b\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{b\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

```
[Out] -(b*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + (b*Log[1 + Sin[c + d*x]])/(4*(
a - b)^2*d) - (a*b^2*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + (Sec[c +
d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.165122, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 823, 801}

$$-\frac{ab^2\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}-\frac{b\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{b\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + (b*Log[1 + Sin[c + d*x]])/(4*(
a - b)^2*d) - (a*b^2*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + (Sec[c +
d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{x}{b(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \frac{-ab^2+b^2x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \left(\frac{b(-a+b)}{2(a+b)(b-x)} + \frac{2ab^2}{(a-b)(a+b)(a+x)} - \frac{b(a+b)}{2(a-b)(b+x)}\right)\right.}{2\left(a^2-b^2\right)d}$$

$$= -\frac{b\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{b\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{ab^2\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2d} + \frac{\sec^2}{}$$

Mathematica [A] time = 0.374064, size = 162, normalized size = 1.38

$$\frac{-\frac{4ab^2\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2} + \frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{1}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^2} +}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^2 + (2*b*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^2 - (4*a*b^2*Log[a + b*Sin[c + d*x
]])/(a^2 - b^2)^2 + 1/((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + 1
/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(4*d)
```

Maple [A] time = 0.067, size = 123, normalized size = 1.1

$$-\frac{ab^2\ln(a+b\sin(dx+c))}{d(a+b)^2(a-b)^2} - \frac{1}{d(4a+4b)(\sin(dx+c)-1)} - \frac{\ln(\sin(dx+c)-1)b}{4d(a+b)^2} + \frac{1}{d(4a-4b)(1+\sin(dx+c))} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a*b^2/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-1/d/(4*a+4*b)/(sin(d*x+c)-1)-
1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*b+1/d/(4*a-4*b)/(1+sin(d*x+c))+1/4*b*ln(1+si
n(d*x+c))/(a-b)^2/d
```

Maxima [A] time = 0.993699, size = 178, normalized size = 1.52

$$-\frac{\frac{4\,ab^2\log\left(b\sin(dx+c)+a\right)}{a^4-2\,a^2b^2+b^4}-\frac{b\log\left(\sin(dx+c)+1\right)}{a^2-2\,ab+b^2}+\frac{b\log\left(\sin(dx+c)-1\right)}{a^2+2\,ab+b^2}-\frac{2\,(b\sin(dx+c)-a)}{\left(a^2-b^2\right)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a*b^2*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - b*log(sin(d
*x + c) + 1)/(a^2 - 2*a*b + b^2) + b*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b
^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

Fricas [A] time = 1.84681, size = 370, normalized size = 3.16

$$-\frac{4\,ab^2\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right)-\left(a^2b+2\,ab^2+b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2b-2\,ab^2+b^3\right)\cos\cdots}{4\left(a^4-2\,a^2b^2+b^4\right)d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*b^2*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (a^2*b + 2*a*b^2 + b
^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^2*b - 2*a*b^2 + b^3)*cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x +
 c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(c+dx\right)\sec^3\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.21551, size = 230, normalized size = 1.97

$$-\frac{\frac{4\,ab^3\log\left(|b\sin(dx+c)+a|\right)}{a^4b-2\,a^2b^3+b^5}-\frac{b\log\left(|\sin(dx+c)+1|\right)}{a^2-2\,ab+b^2}+\frac{b\log\left(|\sin(dx+c)-1|\right)}{a^2+2\,ab+b^2}+\frac{2\left(ab^2\sin(dx+c)^2-a^2b\sin(dx+c)+b^3\sin(dx+c)+a^3-2\,ab^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a*b^3*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - b*lo
g(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + b*log(abs(sin(d*x + c) - 1))
/(a^2 + 2*a*b + b^2) + 2*(a*b^2*sin(d*x + c)^2 - a^2*b*sin(d*x + c) + b^3*s
in(d*x + c) + a^3 - 2*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1))
)/d
```

3.1349 $\int \frac{\csc(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{b^4\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(2a+3b)\log(1-\sin(c+dx))}{4d(a+b)^2}$$

```
[Out] -((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + Log[Sin[c + d*x]]/(a
*d) - ((2*a - 3*b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (b^4*Log[a + b*
Sin[c + d*x]])/(a*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])) + 1
/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.241525, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{b^4\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(2a+3b)\log(1-\sin(c+dx))}{4d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + Log[Sin[c + d*x]]/(a
*d) - ((2*a - 3*b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (b^4*Log[a + b*
Sin[c + d*x]])/(a*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])) + 1
/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{b}{x(a+x)\left(b^2-x^2\right)^2}\,dx,x,b\sin(c+dx)\right)}{d}$$
$$= \frac{b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^2}\,dx,x,b\sin(c+dx)\right)}{d}$$
$$= \frac{b^4\,\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2}+\frac{2a+3b}{4b^4(a+b)^2(b-x)}+\frac{1}{ab^4x}-\frac{1}{a(a-b)^2(a+b)^2(a+x)}-\frac{1}{4(a-b)b^3(b+x)^2}+\frac{-}{4(a-l}\right.\right.}{d}$$
$$= -\frac{(2a+3b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{(2a-3b)\log(1+\sin(c+dx))}{4(a-b)^2d}-\frac{b}{}$$

Mathematica [A] time = 0.681881, size = 151, normalized size = 0.97

$$\frac{b^4\left(-\frac{1}{b^4(a+b)(\sin(c+dx)-1)}+\frac{1}{b^4(a-b)(\sin(c+dx)+1)}-\frac{(2a+3b)\log(1-\sin(c+dx))}{b^4(a+b)^2}+\frac{4\log(\sin(c+dx))}{ab^4}-\frac{(2a-3b)\log(\sin(c+dx)+1)}{b^4(a-b)^2}-\frac{4\log(a+b\sin(c+dx)}{a(a-b)^2(a+b)^2}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^4*(-(((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(b^4*(a + b)^2)) + (4*Log[Sin[c
 + d*x]])/(a*b^4) - ((2*a - 3*b)*Log[1 + Sin[c + d*x]])/((a - b)^2*b^4) - (
4*Log[a + b*Sin[c + d*x]])/(a*(a - b)^2*(a + b)^2) - 1/(b^4*(a + b)*(-1 + S
in[c + d*x])) + 1/((a - b)*b^4*(1 + Sin[c + d*x]))))/(4*d)
```

Maple [A] time = 0.089, size = 181, normalized size = 1.2

$$-\frac{b^4\ln(a+b\sin(dx+c))}{d(a+b)^2(a-b)^2a}-\frac{1}{d(4a+4b)(\sin(dx+c)-1)}-\frac{\ln(\sin(dx+c)-1)a}{2d(a+b)^2}-\frac{3\ln(\sin(dx+c)-1)b}{4d(a+b)^2}+\frac{}{d(4a-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*b^4/(a+b)^2/(a-b)^2/a*ln(a+b*sin(d*x+c))-1/d/(4*a+4*b)/(sin(d*x+c)-1)-
1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*a-3/4/d/(a+b)^2*ln(sin(d*x+c)-1)*b+1/d/(4*a-
4*b)/(1+sin(d*x+c))-1/2*a*ln(1+sin(d*x+c))/(a-b)^2/d+3/4*b*ln(1+sin(d*x+c))
/(a-b)^2/d+ln(sin(d*x+c))/a/d
```

Maxima [A] time = 0.999931, size = 211, normalized size = 1.35

$$-\frac{\frac{4b^4\log(b\sin(dx+c)+a)}{a^5-2a^3b^2+ab^4}+\frac{(2a-3b)\log(\sin(dx+c)+1)}{a^2-2ab+b^2}+\frac{(2a+3b)\log(\sin(dx+c)-1)}{a^2+2ab+b^2}-\frac{2(b\sin(dx+c)-a)}{\left(a^2-b^2\right)\sin(dx+c)^2-a^2+b^2}-\frac{4\log(\sin(dx+c))}{a}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^4*log(b*sin(d*x + c) + a)/(a^5 - 2*a^3*b^2 + a*b^4) + (2*a - 3*b)
*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(sin(d*x + c) -
 1)/(a^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^
2 - a^2 + b^2) - 4*log(sin(d*x + c))/a)/d
```

Fricas [A] time = 4.55277, size = 502, normalized size = 3.22

$$-\frac{4\,b^4\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right)-2\,a^4+2\,a^2b^2-4\left(a^4-2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)+(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^4*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - 2*a^4 + 2*a^2*b^2 - 4*
(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2*log(-1/2*sin(d*x + c)) + (2*a^4 + a^
3*b - 4*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a^4 -
a^3*b - 4*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^3
*b - a*b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24238, size = 284, normalized size = 1.82

$$-\frac{\frac{4\,b^5\log(|b\sin(dx+c)+a|)}{a^5b-2\,a^3b^3+ab^5}+\frac{(2\,a-3\,b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}+\frac{(2\,a+3\,b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}-\frac{4\,\log(|\sin(dx+c)|)}{a}-\frac{2\left(a^3\sin(dx+c)^2-2\,ab^2\sin(dx+c)^2+a^2b\right.}{\left(a^4-2\,a^2b^2+b^4\right)(}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*b^5*log(abs(b*sin(d*x + c) + a))/(a^5*b - 2*a^3*b^3 + a*b^5) + (2*a
 - 3*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(ab
s(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) - 4*log(abs(sin(d*x + c)))/a - 2*(
a^3*sin(d*x + c)^2 - 2*a*b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3*sin(
d*x + c) - 2*a^3 + 3*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))
/d
```

3.1350 $\int \frac{\csc^2(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=171

$$\frac{b^5\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)^2}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{1}{4d(a+b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(3a+4b)\log(1}{4d(a}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - ((3*a + 4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d)
 - (b*Log[Sin[c + d*x]])/(a^2*d) + ((3*a - 4*b)*Log[1 + Sin[c + d*x]])/(4*(
a - b)^2*d) + (b^5*Log[a + b*Sin[c + d*x]])/(a^2*(a^2 - b^2)^2*d) + 1/(4*(a
 + b)*d*(1 - Sin[c + d*x])) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.269614, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{b^5\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)^2}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{1}{4d(a+b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(3a+4b)\log(1}{4d(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(Csc[c + d*x]/(a*d)) - ((3*a + 4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d)
 - (b*Log[Sin[c + d*x]])/(a^2*d) + ((3*a - 4*b)*Log[1 + Sin[c + d*x]])/(4*(
a - b)^2*d) + (b^5*Log[a + b*Sin[c + d*x]])/(a^2*(a^2 - b^2)^2*d) + 1/(4*(a
 + b)*d*(1 - Sin[c + d*x])) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{b^2}{x^2(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^5\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{4b^4(a+b)(b-x)^2} + \frac{3a+4b}{4b^5(a+b)^2(b-x)} + \frac{1}{ab^4x^2} - \frac{1}{a^2b^4x} + \frac{1}{a^2(a-b)^2(a+b)^2(a+x)} + \frac{}{4(a-b)}\right.\right.}{d}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{(3a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} - \frac{b\log(\sin(c+dx))}{a^2d} + \frac{(3a-4b)\log(}{4(a}$$

Mathematica [A] time = 0.747822, size = 174, normalized size = 1.02

$$-\frac{\csc(c+dx)(a+b\sin(c+dx))\left(-\frac{4b^5\log(a+b\sin(c+dx))}{a^2(a-b)^2(a+b)^2} + \frac{4b\log(\sin(c+dx))}{a^2} + \frac{1}{(a+b)(\sin(c+dx)-1)} + \frac{1}{(a-b)(\sin(c+dx)+1)} + \frac{(3a+4b)\log(}{(a+}\right.}{4d(a\csc(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]*(a + b*Sin[c + d*x])*((4*Csc[c + d*x])/a + ((3*a + 4*b)*Log[
1 - Sin[c + d*x]])/(a + b)^2 + (4*b*Log[Sin[c + d*x]])/a^2 - ((3*a - 4*b)*L
og[1 + Sin[c + d*x]])/(a - b)^2 - (4*b^5*Log[a + b*Sin[c + d*x]])/(a^2*(a -
 b)^2*(a + b)^2) + 1/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c
+ d*x]))))/(4*d*(b + a*Csc[c + d*x]))
```

Maple [A] time = 0.099, size = 199, normalized size = 1.2

$$\frac{b^5\ln(a+b\sin(dx+c))}{d(a+b)^2(a-b)^2a^2} - \frac{1}{d(4a+4b)(\sin(dx+c)-1)} - \frac{3\ln(\sin(dx+c)-1)a}{4d(a+b)^2} - \frac{\ln(\sin(dx+c)-1)b}{d(a+b)^2} - \frac{}{d(4a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^5/(a+b)^2/(a-b)^2/a^2*ln(a+b*sin(d*x+c))-1/d/(4*a+4*b)/(sin(d*x+c)-1)
-3/4/d/(a+b)^2*ln(sin(d*x+c)-1)*a-1/d/(a+b)^2*ln(sin(d*x+c)-1)*b-1/d/(4*a-4
*b)/(1+sin(d*x+c))+3/4*a*ln(1+sin(d*x+c))/(a-b)^2/d-b*ln(1+sin(d*x+c))/(a-b
)^2/d-1/d/a/sin(d*x+c)-b*ln(sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.0573, size = 270, normalized size = 1.58

$$\frac{\frac{4b^5\log(b\sin(dx+c)+a)}{a^6-2a^4b^2+a^2b^4} + \frac{(3a-4b)\log(\sin(dx+c)+1)}{a^2-2ab+b^2} - \frac{(3a+4b)\log(\sin(dx+c)-1)}{a^2+2ab+b^2} + \frac{2\left(ab\sin(dx+c)-\left(3a^2-2b^2\right)\sin(dx+c)^2+2a^2-2b^2\right)}{\left(a^3-ab^2\right)\sin(dx+c)^3-\left(a^3-ab^2\right)\sin(dx+c)} - \frac{4b\log(\sin}{a^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^5*log(b*sin(d*x + c) + a)/(a^6 - 2*a^4*b^2 + a^2*b^4) + (3*a - 4*b
)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (3*a + 4*b)*log(sin(d*x + c)
- 1)/(a^2 + 2*a*b + b^2) + 2*(a*b*sin(d*x + c) - (3*a^2 - 2*b^2)*sin(d*x +
c)^2 + 2*a^2 - 2*b^2)/((a^3 - a*b^2)*sin(d*x + c)^3 - (a^3 - a*b^2)*sin(d*x
 + c)) - 4*b*log(sin(d*x + c))/a^2)/d
```

Fricas [A] time = 6.0214, size = 672, normalized size = 3.93

$$\frac{4\,b^5\cos(dx+c)^2\log(b\sin(dx+c)+a)\sin(dx+c)+2\,a^5-2\,a^3b^2-4\left(a^4b-2\,a^2b^3+b^5\right)\cos(dx+c)^2\log\left(\frac{1}{2}\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^5*cos(d*x + c)^2*log(b*sin(d*x + c) + a)*sin(d*x + c) + 2*a^5 - 2*
a^3*b^2 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2*log(1/2*sin(d*x + c))*
sin(d*x + c) + (3*a^5 + 2*a^4*b - 5*a^3*b^2 - 4*a^2*b^3)*cos(d*x + c)^2*log
(sin(d*x + c) + 1)*sin(d*x + c) - (3*a^5 - 2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) - 2*(3*a^5 - 5*a^3*b^2
+ 2*a*b^4)*cos(d*x + c)^2 - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^6 - 2*a^4
*b^2 + a^2*b^4)*d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29501, size = 377, normalized size = 2.2

$$\frac{\frac{12\,b^6\log(|b\sin(dx+c)+a|)}{a^6b-2\,a^4b^3+a^2b^5}+\frac{3\,(3\,a-4\,b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}-\frac{3\,(3\,a+4\,b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}-\frac{12\,b\log(|\sin(dx+c)|)}{a^2}+\frac{2\left(2\,b^5\sin(dx+c)^3-9\,a^5\sin(dx+c)^2+15\right.}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*b^6*log(abs(b*sin(d*x + c) + a))/(a^6*b - 2*a^4*b^3 + a^2*b^5) + 3
*(3*a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - 3*(3*a + 4*b)
*log(abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) - 12*b*log(abs(sin(d*x + c)
))/a^2 + 2*(2*b^5*sin(d*x + c)^3 - 9*a^5*sin(d*x + c)^2 + 15*a^3*b^2*sin(d*
x + c)^2 - 6*a*b^4*sin(d*x + c)^2 + 3*a^4*b*sin(d*x + c) - 3*a^2*b^3*sin(d*
x + c) - 2*b^5*sin(d*x + c) + 6*a^5 - 12*a^3*b^2 + 6*a*b^4)/((a^6 - 2*a^4*b
^2 + a^2*b^4)*(sin(d*x + c)^3 - sin(d*x + c))))/d
```

3.1351 $\int \frac{\csc^3(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$-\frac{b^6\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)^2}+\frac{\left(2a^2+b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(\sin}$$

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((4*a + 5*b)*Log[1 - Si
n[c + d*x]])/(4*(a + b)^2*d) + ((2*a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*d) -
((4*a - 5*b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (b^6*Log[a + b*Sin[c
+ d*x]])/(a^3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])) + 1/(4*
(a - b)*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.312379, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{b^6\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)^2}+\frac{\left(2a^2+b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((4*a + 5*b)*Log[1 - Si
n[c + d*x]])/(4*(a + b)^2*d) + ((2*a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*d) -
((4*a - 5*b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (b^6*Log[a + b*Sin[c
+ d*x]])/(a^3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])) + 1/(4*
(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{b^3}{x^3(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\,\text{Subst}\left(\int \frac{1}{x^3(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\,\text{Subst}\left(\int \left(\frac{1}{4b^5(a+b)(b-x)^2}+\frac{4a+5b}{4b^6(a+b)^2(b-x)}+\frac{1}{ab^4x^3}-\frac{1}{a^2b^4x^2}+\frac{2a^2+b^2}{a^3b^6x}-\frac{1}{a^3(a-b)^2(a+b)^2(a+x)}-\right.}{d}$$

$$= \frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{(4a+5b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{\left(2a^2+b^2\right)\log(\sin(c+}{a^3d}$$

Mathematica [A] time = 1.4216, size = 168, normalized size = 0.85

$$-\frac{\frac{4b^6\log(a+b\sin(c+dx))}{a^3(a-b)^2(a+b)^2}-\frac{4\left(2a^2+b^2\right)\log(\sin(c+dx))}{a^3}-\frac{4b\csc(c+dx)}{a^2}+\frac{1}{(a+b)(\sin(c+dx)-1)}-\frac{1}{(a-b)(\sin(c+dx)+1)}+\frac{(4a+5b)\log(1-\sin(c+dx))}{(a+b)^2}+\frac{(4a-5}{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((-4*b*Csc[c + d*x])/a^2 + (2*Csc[c + d*x]^2)/a + ((4*a + 5*b)*Log[1 - Sin
[c + d*x]])/(a + b)^2 - (4*(2*a^2 + b^2)*Log[Sin[c + d*x]])/a^3 + ((4*a - 5
*b)*Log[1 + Sin[c + d*x]])/(a - b)^2 + (4*b^6*Log[a + b*Sin[c + d*x]])/(a^3
*(a - b)^2*(a + b)^2) + 1/((a + b)*(-1 + Sin[c + d*x])) - 1/((a - b)*(1 + S
in[c + d*x])))/(4*d)
```

Maple [A] time = 0.111, size = 231, normalized size = 1.2

$$-\frac{b^6\ln(a+b\sin(dx+c))}{d(a+b)^2(a-b)^2a^3}-\frac{1}{d(4a+4b)(\sin(dx+c)-1)}-\frac{\ln(\sin(dx+c)-1)a}{d(a+b)^2}-\frac{5\ln(\sin(dx+c)-1)b}{4d(a+b)^2}+\frac{}{d(4a-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*b^6/(a+b)^2/(a-b)^2/a^3*ln(a+b*sin(d*x+c))-1/d/(4*a+4*b)/(sin(d*x+c)-1
)-1/d/(a+b)^2*ln(sin(d*x+c)-1)*a-5/4/d/(a+b)^2*ln(sin(d*x+c)-1)*b+1/d/(4*a-
4*b)/(1+sin(d*x+c))-a*ln(1+sin(d*x+c))/(a-b)^2/d+5/4*b*ln(1+sin(d*x+c))/(a-
b)^2/d-1/2/d/a/sin(d*x+c)^2+2*ln(sin(d*x+c))/a/d+b^2*ln(sin(d*x+c))/a^3/d+1
/d/a^2*b/sin(d*x+c)
```

Maxima [A] time = 1.02816, size = 329, normalized size = 1.67

$$-\frac{\frac{4b^6\log(b\sin(dx+c)+a)}{a^7-2a^5b^2+a^3b^4}+\frac{(4a-5b)\log(\sin(dx+c)+1)}{a^2-2ab+b^2}+\frac{(4a+5b)\log(\sin(dx+c)-1)}{a^2+2ab+b^2}-\frac{2\left(\left(3a^2b-2b^3\right)\sin(dx+c)^3+a^3-ab^2-\left(2a^3-ab^2\right)\sin(dx+c)^2-2\left(a^2b-\right.\right.}{\left(a^4-a^2b^2\right)\sin(dx+c)^4-\left(a^4-a^2b^2\right)\sin(dx+c)^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^6*log(b*sin(d*x + c) + a)/(a^7 - 2*a^5*b^2 + a^3*b^4) + (4*a - 5*
b)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (4*a + 5*b)*log(sin(d*x + c)
 - 1)/(a^2 + 2*a*b + b^2) - 2*((3*a^2*b - 2*b^3)*sin(d*x + c)^3 + a^3 - a*b
^2 - (2*a^3 - a*b^2)*sin(d*x + c)^2 - 2*(a^2*b - b^3)*sin(d*x + c))/((a^4 -
 a^2*b^2)*sin(d*x + c)^4 - (a^4 - a^2*b^2)*sin(d*x + c)^2) - 4*(2*a^2 + b^2
)*log(sin(d*x + c))/a^3)/d
```

Fricas [B] time = 8.90208, size = 973, normalized size = 4.94

$$-\frac{2\,a^{6} - 2\,a^{4}b^{2} - 2\left(2\,a^{6} - 3\,a^{4}b^{2} + a^{2}b^{4}\right)\cos\left(dx + c\right)^{2} + 4\left(b^{6}\cos\left(dx + c\right)^{4} - b^{6}\cos\left(dx + c\right)^{2}\right)\log\left(b\sin\left(dx + c\right) + a\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^6 - 2*a^4*b^2 - 2*(2*a^6 - 3*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 +
4*(b^6*cos(d*x + c)^4 - b^6*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) - 4*((2
*a^6 - 3*a^4*b^2 + b^6)*cos(d*x + c)^4 - (2*a^6 - 3*a^4*b^2 + b^6)*cos(d*x
+ c)^2)*log(-1/2*sin(d*x + c)) + ((4*a^6 + 3*a^5*b - 6*a^4*b^2 - 5*a^3*b^3)
*cos(d*x + c)^4 - (4*a^6 + 3*a^5*b - 6*a^4*b^2 - 5*a^3*b^3)*cos(d*x + c)^2)
*log(sin(d*x + c) + 1) + ((4*a^6 - 3*a^5*b - 6*a^4*b^2 + 5*a^3*b^3)*cos(d*x
 + c)^4 - (4*a^6 - 3*a^5*b - 6*a^4*b^2 + 5*a^3*b^3)*cos(d*x + c)^2)*log(-si
n(d*x + c) + 1) - 2*(a^5*b - a^3*b^3 - (3*a^5*b - 5*a^3*b^3 + 2*a*b^5)*cos(
d*x + c)^2)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^4 - (
a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29979, size = 371, normalized size = 1.88

$$-\frac{\frac{4\,b^{7}\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^{7}b-2\,a^{5}b^{3}+a^{3}b^{5}} + \frac{\left(4\,a-5\,b\right)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^{2}-2\,ab+b^{2}} + \frac{\left(4\,a+5\,b\right)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^{2}+2\,ab+b^{2}} - \frac{2\left(2\,a^{3}\sin\left(dx+c\right)^{2}-3\,ab^{2}\sin\left(dx+c\right)^{2}+a^{2}b\sin\left(dx+c\right)-b^{3}\sin(d}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\sin\left(dx+c\right)^{2}-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*b^7*log(abs(b*sin(d*x + c) + a))/(a^7*b - 2*a^5*b^3 + a^3*b^5) + (4
*a - 5*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + (4*a + 5*b)*log(
```

```
abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) - 2*(2*a^3*sin(d*x + c)^2 - 3*a*
b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3*sin(d*x + c) - 3*a^3 + 4*a*b^
2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)) - 4*(2*a^2 + b^2)*log(abs
(sin(d*x + c)))/a^3 + 2*(6*a^2*sin(d*x + c)^2 + 3*b^2*sin(d*x + c)^2 - 2*a*
b*sin(d*x + c) + a^2)/(a^3*sin(d*x + c)^2))/d
```

3.1352 $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^3\tan(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2b\sec(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2
)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^
2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.228666, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2660, 618, 204}

$$\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^3\tan(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2b\sec(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```


```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2
)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^
2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```


Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan^3(c+dx)\,dx}{a^2-b^2}$$

$$= -\frac{a^3\int \sec^2(c+dx)\,dx}{\left(a^2-b^2\right)^2} + \frac{a^4\int \frac{1}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b\right)\int \sec(c+dx)\tan(c+dx)\,dx}{\left(a^2-b^2\right)^2} + \frac{a\,\mathrm{Subst}\big(}{}$$

$$= \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^3\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{\left(a^2-b^2\right)^2 d} + \frac{\left(2a^4\right)\mathrm{S}}{}$$

$$= \frac{a^2b\sec(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^3\tan(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{\left(4a^4\right)\mathrm{Su}}{}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{a^2b\sec(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^3\tan(c+dx)}{\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 1.39924, size = 195, normalized size = 1.1

$$\frac{\frac{48a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{\sec^3(c+dx)\left(3b\left(11a^2-5b^2\right)\cos(c+dx)+12b\left(b^2-2a^2\right)\cos(2(c+dx))+11a^2b\cos(3(c+dx))-16a^2b+8a^3\sin(3(c+dx))+6ab^2\sin(c}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) - (Sec[c + d*x]^3*(-16*a^2*b + 4*b^3 + 3*b*(11*a^2 - 5*b^2)*Cos[c + d*x]
+ 12*b*(-2*a^2 + b^2)*Cos[2*(c + d*x)] + 11*a^2*b*Cos[3*(c + d*x)] - 5*b^3*
Cos[3*(c + d*x)] + 6*a*b^2*Sin[c + d*x] + 8*a^3*Sin[3*(c + d*x)] - 2*a*b^2*
Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

Maple [A] time = 0.111, size = 269, normalized size = 1.5

$$-\frac{32}{3\,d\,(32\,a+32\,b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-16\,\frac{1}{d\,(32\,a+32\,b)\,(\tan(1/2\,dx+c/2)-1)^2}+\frac{a}{d\,(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -32/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(32*a+32*b)-16/d/(32*a+32*b)/(tan(1/2*d*x+
1/2*c)-1)^2+1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a+1/2/d/(a+b)^2/(tan(1/2*d*x
+1/2*c)-1)*b+2/d*a^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-32/3/d/(tan(1/2*d*x+1/2*c)+1)^3/(32*a-32
*b)+16/d/(32*a-32*b)/(tan(1/2*d*x+1/2*c)+1)^2+1/d/(a-b)^2/(tan(1/2*d*x+1/2*
c)+1)*a-1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59453, size = 1054, normalized size = 5.95

$$\left[-\frac{3\sqrt{-a^2+b^2}a^4\cos(dx+c)^3\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^4b-}{6\left(a^6-3a^4b^2+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(2*a^4*b - 3*a^2*b^3 + b^5)*co
s(d*x + c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3), -1/3*(3*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
- b^2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(2*a^4*b
 - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*
a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19628, size = 325, normalized size = 1.84

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^2b\,\mathrm{t}}{\left(a^4-2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
+ (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*a^3*t
an(1/2*d*x + 1/2*c)^3 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d
*x + 1/2*c)^2 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) -
 5*a^2*b + 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))
/d
```

3.1353 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=142

$$-\frac{2a^3b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{b\tan^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a\sec^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)^2}$$

```
[Out] (-2*a^3*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (a*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]*(a - b*Si
n[c + d*x]))/((a^2 - b^2)^2*d) - (b*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.238903, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2902, 2606, 30, 2607, 2866, 12, 2660, 618, 204}

$$-\frac{2a^3b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{b\tan^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a\sec^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^3*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (a*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]*(a - b*Si
n[c + d*x]))/((a^2 - b^2)^2*d) - (b*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}dx = \frac{a\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sec(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}dx}{a^2-b^2} - \frac{b\int \sec^2(c+dx)\tan^2(}{a^2-b^2}$$

$$= -\frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} - \frac{a^2\int \frac{ab}{a+b\sin(c+dx)}dx}{\left(a^2-b^2\right)^2} + \frac{a\,\text{Subst}\left(\int x^2\,dx, x, \sec(\right.}{\left(a^2-b^2\right)d}$$

$$= \frac{a\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} - \frac{b\tan^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{\left(a^3b\right)\int \frac{1}{a+b\sin}}{\left(a^2-b^2\right.}$$

$$= \frac{a\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} - \frac{b\tan^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{\left(2a^3b\right)\text{Subst}}{}$$

$$= \frac{a\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} - \frac{b\tan^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\left(4a^3b\right)\text{Subst}}{}$$

$$= -\frac{2a^3b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{a\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} -$$

Mathematica [A] time = 1.41831, size = 184, normalized size = 1.3

$$\frac{\frac{\sec^3(c+dx)\left(3a\left(5a^2+b^2\right)\cos(c+dx)+8a^2b\sin(3(c+dx))-12a^3\cos(2(c+dx))+5a^3\cos(3(c+dx))-4a^3+ab^2\cos(3(c+dx))-8ab^2+6b^3\sin(c+dx)-2b^3\sin(3(c+dx))\right)}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-48*a^3*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
5/2) + (Sec[c + d*x]^3*(-4*a^3 - 8*a*b^2 + 3*a*(5*a^2 + b^2)*Cos[c + d*x] -
 12*a^3*Cos[2*(c + d*x)] + 5*a^3*Cos[3*(c + d*x)] + a*b^2*Cos[3*(c + d*x)]
+ 6*b^3*Sin[c + d*x] + 8*a^2*b*Sin[3*(c + d*x)] - 2*b^3*Sin[3*(c + d*x)]))/
((a - b)^2*(a + b)^2))/(24*d)
```

Maple [A] time = 0.105, size = 222, normalized size = 1.6

$$-\frac{16}{3d(16a+16b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - 8\frac{1}{d(16a+16b)(\tan(1/2dx+c/2)-1)^2} + \frac{a}{2d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -16/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/d/(16*a+16*b)/(tan(1/2*d*x+1
/2*c)-1)^2+1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-2/d*a^3*b/(a-b)^2/(a+b)^2
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-8
/d/(16*a-16*b)/(tan(1/2*d*x+1/2*c)+1)^2+16/3/d/(tan(1/2*d*x+1/2*c)+1)^3/(16
*a-16*b)-1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63637, size = 1027, normalized size = 7.23

$$\left[-\frac{3\sqrt{-a^2+b^2}a^3b\cos(dx+c)^3\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^5+4\cdots}{6\left(a^6-3a^4b^2+3a^2b^4-\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^3*b*cos(d*x + c)^3*log(-((2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2)) - 2*a^5 + 4*a^3*b^2 - 2*a*b^4 + 6*(a^5 - a^3*b^2)*cos(d*x + c
)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5 - (4*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c)
^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1
/3*(3*sqrt(a^2 - b^2)*a^3*b*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*c
os(d*x + c)))*cos(d*x + c)^3 + a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^5 - a^3*b^2)*
cos(d*x + c)^2 - (a^4*b - 2*a^2*b^3 + b^5 - (4*a^4*b - 5*a^2*b^3 + b^5)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22595, size = 306, normalized size = 2.15

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3b}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a\cdots}{\left(a^4-2a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}\cdots\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a^3*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2
)) + (3*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 3*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 10*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 4*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2
*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - 2*a^3 - a*b^2)/((a^4 - 2*a
^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

3.1354 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{2a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)^2}$$

```
[Out] (2*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(
5/2)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) + (a^2*Sec[c + d*x]*(b - a*S
in[c + d*x]))/((a^2 - b^2)^2*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d) + (a*Tan
[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.237724, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2902, 3767, 2606, 30, 2696, 12, 2660, 618, 204}

$$\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{2a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(
5/2)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) + (a^2*Sec[c + d*x]*(b - a*S
in[c + d*x]))/((a^2 - b^2)^2*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d) + (a*Tan
[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^4(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2-b^2}$$

$$= \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \frac{a^2\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int\left(1+x^2\right)dx,x,-\right.}{\left(a^2-b^2\right)d}$$

$$= -\frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d}$$

$$= -\frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d}$$

$$= -\frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d}$$

$$= \frac{2a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \ldots$$

Mathematica [A] time = 1.2621, size = 200, normalized size = 1.21

$$\frac{\frac{48a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{\sec^3(c+dx)\left(3b\left(5a^2+b^2\right)\cos(c+dx)-12a^2b\cos(2(c+dx))+5a^2b\cos(3(c+dx))-4a^2b-6a^3\sin(c+dx)+2a^3\sin(3(c+dx))+12ab^2\sin\ldots\right.}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(5/2) - (Sec[c + d*x]^3*(-4*a^2*b - 8*b^3 + 3*b*(5*a^2 + b^2)*Cos[c + d*x]
- 12*a^2*b*Cos[2*(c + d*x)] + 5*a^2*b*Cos[3*(c + d*x)] + b^3*Cos[3*(c + d*x
)] - 6*a^3*Sin[c + d*x] + 12*a*b^2*Sin[c + d*x] + 2*a^3*Sin[3*(c + d*x)] +
4*a*b^2*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

Maple [A] time = 0.102, size = 224, normalized size = 1.4

$$-\frac{8}{3d(8a+8b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - 4\frac{1}{d(8a+8b)(\tan(1/2dx+c/2)-1)^2} - \frac{b}{2d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + 2\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -8/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(8*a+8*b)-4/d/(8*a+8*b)/(tan(1/2*d*x+1/2*c)
-1)^2-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*b+2/d*a^2*b^2/(a-b)^2/(a+b)^2/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-8/3/
d/(tan(1/2*d*x+1/2*c)+1)^3/(8*a-8*b)+4/d/(8*a-8*b)/(tan(1/2*d*x+1/2*c)+1)^2
+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7465, size = 1033, normalized size = 6.26

$$\left[-\frac{3\sqrt{-a^2+b^2}a^2b^2\cos(dx+c)^3\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^4b}{6(a^6-3a^4b^2+3a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^2*b^2*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) +
 b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(a^4*b - a^2*b^3)*cos(d*x
+ c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3),
-1/3*(3*sqrt(a^2 - b^2)*a^2*b^2*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^
2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(a^4*b - a^2
*b^3)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (a^5 + a^3*b^2 - 2*a*b^4)
*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d
*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21032, size = 309, normalized size = 1.87

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2b^2}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-4a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^2*b^2/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^
2)) + (3*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^4 - 4*a^
3*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2
*d*x + 1/2*c)^2 + 3*a*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b - b^3)/((a^4 - 2*a
^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

3.1355 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{2ab^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}$$

```
[Out] (-2*a*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (Sec[c + d*x]^3*(a - b*Sin[c + d*x]))/(3*(a^2 - b^2)*d) - (Sec[c +
 d*x]*(3*a*b^2 - b*(a^2 + 2*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.223176, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2866, 12, 2660, 618, 204}

$$-\frac{2ab^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3d\left(a^2-b^2\right)}-\frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (Sec[c + d*x]^3*(a - b*Sin[c + d*x]))/(3*(a^2 - b^2)*d) - (Sec[c +
 d*x]*(3*a*b^2 - b*(a^2 + 2*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d)
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^2(c+dx)\left(-ab+2b^2\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\int -\frac{}{a-}}{3(}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} - \frac{\left(ab^3\right)}{}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} - \frac{\left(2ab^3\right.}{}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b\left(a^2+2b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\left(4ab^3\right.}{}$$

$$= -\frac{2ab^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2} d} + \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b\right.}{3\left(a^2\right.}$$

Mathematica [A] time = 1.26743, size = 203, normalized size = 1.47

$$\frac{\frac{\sec^3(c+dx)\left(-\frac{3}{2}a\left(a^2-7b^2\right)\cos(c+dx)-3a^2b\sin(c+dx)+a^2b\sin(3(c+dx))-\frac{1}{2}a^3\cos(3(c+dx))+4a^3-6ab^2\cos(2(c+dx))+\frac{7}{2}ab^2\cos(3(c+dx))-10ab^2+6b^3\sin(c+dx)+2b\right.}{(a-b)^2(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-24*a*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
5/2) + (Sec[c + d*x]^3*(4*a^3 - 10*a*b^2 - (3*a*(a^2 - 7*b^2)*Cos[c + d*x])
/2 - 6*a*b^2*Cos[2*(c + d*x)] - (a^3*Cos[3*(c + d*x)])/2 + (7*a*b^2*Cos[3*(
c + d*x)])/2 - 3*a^2*b*Sin[c + d*x] + 6*b^3*Sin[c + d*x] + a^2*b*Sin[3*(c +
 d*x)] + 2*b^3*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(12*d)
```

Maple [B] time = 0.095, size = 272, normalized size = 2.

$$-\frac{4}{3d(4a+4b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - 2\frac{1}{d(4a+4b)(\tan(1/2dx+c/2)-1)^2} - \frac{a}{2d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} - \frac{}{d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -4/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(4*a+4*b)-2/d/(4*a+4*b)/(tan(1/2*d*x+1/2*c)
-1)^2-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-1/d/(a+b)^2/(tan(1/2*d*x+1/2*c
)-1)*b-2/d*a*b^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/(4*a-4*b)/(tan(1/2*d*x+1/2*c)+1)^2+4/3/d
/(tan(1/2*d*x+1/2*c)+1)^3/(4*a-4*b)+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a-
1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62866, size = 1027, normalized size = 7.44

$$\left[-\frac{3\sqrt{-a^2+b^2}ab^3\cos(dx+c)^3\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^5}{6(a^6-3a^4b^2+3a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a*b^3*cos(d*x + c)^3*log(-((2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2)) - 2*a^5 + 4*a^3*b^2 - 2*a*b^4 + 6*(a^3*b^2 - a*b^4)*cos(d*x +
 c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)
^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1
/3*(3*sqrt(a^2 - b^2)*a*b^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*c
os(d*x + c)))*cos(d*x + c)^3 + a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^3*b^2 - a*b^4
)*cos(d*x + c)^2 - (a^4*b - 2*a^2*b^3 + b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.1955, size = 324, normalized size = 2.35

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab^3}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b^3\,\mathrm{ta}\cdots}{\left(a^4-2a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots\right.}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a*b^3/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2
)) + (3*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^4 - 6*a*b^2
*tan(1/2*d*x + 1/2*c)^4 - 4*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*
x + 1/2*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*b^3*tan(1/2*d*x + 1/2*c)
+ a^3 - 4*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/
d
```

3.1356 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}+\frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3ad\left(a^2-b^2\right)}-\frac{b\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3ad\left(a^2-b^2\right)^2}+$$

```
[Out] (-2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5
/2)*d) - ArcTanh[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/
(3*a*d) + (b*Sec[c + d*x]^3*(b - a*Sin[c + d*x]))/(3*a*(a^2 - b^2)*d) - (b*
Sec[c + d*x]*(3*b^3 + a*(2*a^2 - 5*b^2)*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*d
)
```

Rubi [A] time = 0.408696, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {2898, 2622, 302, 207, 2696, 2866, 12, 2660, 618, 204}

$$-\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}+\frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3ad\left(a^2-b^2\right)}-\frac{b\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3ad\left(a^2-b^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5
/2)*d) - ArcTanh[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/
(3*a*d) + (b*Sec[c + d*x]^3*(b - a*Sin[c + d*x]))/(3*a*(a^2 - b^2)*d) - (b*
Sec[c + d*x]*(3*b^3 + a*(2*a^2 - 5*b^2)*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*d
)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{\csc(c+dx)\sec^4(c+dx)}{a} - \frac{b\sec^4(c+dx)}{a(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a} - \frac{b\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3a\left(a^2-b^2\right)d} + \frac{b\int \frac{\sec^2(c+dx)\left(-2a^2+3b^2-2ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3a\left(a^2-b^2\right)} + \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3a\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3a\left(a^2-b^2\right)^2 d} -$$

$$= \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3a\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)\left(3b^3\right.}{3a}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{b\sec^3(c+dx)(b-a\sin(c+}{3a\left(a^2-b^2\right)d}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{b\sec^3(c+dx)(b-a\sin(c+}{3a\left(a^2-b^2\right)d}$$

$$= -\frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} +$$

Mathematica [A] time = 4.84422, size = 334, normalized size = 1.72

$$-\frac{24b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}} + \frac{2(7a+10b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2(10b-7a)\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-24*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^
(5/2)) - (12*Log[Cos[(c + d*x)/2]])/a + (12*Log[Sin[(c + d*x)/2]])/a + 1/((
a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (2*Sin[(c + d*x)/2])/((a
+ b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (2*(7*a + 10*b)*Sin[(c + d*
x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (2*Sin[(c + d*x)
/2])/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + 1/((a - b)*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*(-7*a + 10*b)*Sin[(c + d*x)/2])/((a
- b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*d)
```

Maple [A] time = 0.118, size = 279, normalized size = 1.4

$$-\frac{1}{3\,d\,(a+b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - \frac{1}{2\,d\,(a+b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2} - \frac{3\,a}{2\,d\,(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} - 2\,\frac{}{d\,(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^2-
3/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*b
-2/d*b^5/(a-b)^2/(a+b)^2/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))-1/2/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)^2+1/3/d/(a-b)/(
tan(1/2*d*x+1/2*c)+1)^3+3/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a-2/d/(a-b)^2/
(tan(1/2*d*x+1/2*c)+1)*b+1/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.34558, size = 1546, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*b^5*cos(d*x + c)^3*log(-((2*a^2 - b^2)*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*
cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2)) - 2*a^6 + 4*a^4*b^2 - 2*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cos(d*x + c)^3*log(-1/2*cos(d*x + c) + 1/2) - 6*(a^6 - 3*a
^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (2*a^5*
b - 7*a^3*b^3 + 5*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 3*a^5*b^2 +
3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^3), 1/6*(6*sqrt(a^2 - b^2)*b^5*arctan(-(a
*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^3 + 2*a^6 -
 4*a^4*b^2 + 2*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c)
^3*log(1/2*cos(d*x + c) + 1/2) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(
d*x + c)^3*log(-1/2*cos(d*x + c) + 1/2) + 6*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*c
os(d*x + c)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (2*a^5*b - 7*a^3*b^3 + 5*a*b
^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*
cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21228, size = 416, normalized size = 2.14

$$-\frac{\frac{6\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^5}{\left(a^5-2a^3b^2+ab^4\right)\sqrt{a^2-b^2}}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{2\left(3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4\cdots\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(6*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*b^5/((a^5 - 2*a^3*b^2 + a*b^4)*sqrt(a^2 - b^2
)) - 3*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(3*a^2*b*tan(1/2*d*x + 1/2*c)^5
 - 6*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a^3*tan(1/2*d*x + 1/2*c)^4 + 9*a*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 2*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 8*b^3*tan(1/2*d*x +
 1/2*c)^3 + 6*a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^2
+ 3*a^2*b*tan(1/2*d*x + 1/2*c) - 6*b^3*tan(1/2*d*x + 1/2*c) - 4*a^3 + 7*a*b
^2)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

3.1357 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{2b^6\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{\left(-10a^2b^2+6a^4+b^4\right)\tan(c+dx)}{3ad\left(a^2-b^2\right)^2}+\frac{b\left(2b^2-a^2\right)\sec(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\sec^3(c+dx)(b\sin(c+dx)}{3ad\left(a^2-b^2\right)}$$

```
[Out] (2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(
5/2)*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d) + (b*(-a^2
 + 2*b^2)*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x]^3*(-a + b*Sin[c
 + d*x]))/(3*a*(a^2 - b^2)*d) + ((6*a^4 - 10*a^2*b^2 + b^4)*Tan[c + d*x])/(
3*a*(a^2 - b^2)^2*d) + Tan[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.473599, antiderivative size = 247, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2898, 2622, 302, 207, 2620, 270, 2696, 2866, 12, 2660, 618, 204}

$$\frac{2b^6\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}-\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2d\left(a^2-b^2\right)}+\frac{b^2\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3a^2d\left(a^2-b^2\right)^2}-\frac{b\sec}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(
5/2)*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d) - (b*Sec[c
 + d*x])/(a^2*d) - (b*Sec[c + d*x]^3)/(3*a^2*d) - (b^2*Sec[c + d*x]^3*(b -
a*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]*(3*b^3 + a*(2*a^
2 - 5*b^2)*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*d) + (2*Tan[c + d*x])/(a*d)
+ Tan[c + d*x]^3/(3*a*d)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(-\frac{b\csc(c+dx)\sec^4(c+dx)}{a^2}+\frac{\csc^2(c+dx)\sec^4(c+dx)}{a}+\frac{b^2\sec^4(c+dx)}{a^2(a+b\sin(c+dx))}\right) d$$

$$= \frac{\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a}-\frac{b\int \csc(c+dx)\sec^4(c+dx)\,dx}{a^2}+\frac{b^2\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2}$$

$$= -\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2\left(a^2-b^2\right)d}-\frac{b^2\int \frac{\sec^2(c+dx)\left(-2a^2+3b^2-2ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3a^2\left(a^2-b^2\right)}+\frac{\text{Subst}\Big(}{}$$

$$= -\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2\left(a^2-b^2\right)d}+\frac{b^2\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\cot(c+dx)}{ad}-\frac{b\sec(c+dx)}{a^2d}-\frac{b\sec^3(c+dx)}{3a^2d}-\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2\left(a^2-b^2\right)d}+$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{b\sec(c+dx)}{a^2d}-\frac{b\sec^3(c+dx)}{3a^2d}-\frac{b^2\sec^3(c+d}{3a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{b\sec(c+dx)}{a^2d}-\frac{b\sec^3(c+dx)}{3a^2d}-\frac{b^2\sec^3(c+d}{3a^2}$$

$$= \frac{2b^6\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{b\sec(c+dx)}{a^2d}-$$

Mathematica [B] time = 6.44284, size = 450, normalized size = 2.05

$$\frac{2b^6\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(a\sin\left(\frac{1}{2}(c+dx)\right)+b\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}-\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}+\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}+\frac{\text{s}}{6d(a+b)\left(\cos\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^6*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/
Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(5/2)*d) - Cot[(c + d*x)/2]/(2*a*d) + (b
*Log[Cos[(c + d*x)/2]])/(a^2*d) - (b*Log[Sin[(c + d*x)/2]])/(a^2*d) + 1/(12
*(a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + Sin[(c + d*x)/2]/(6*(
a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + Sin[(c + d*x)/2]/(6*(a
- b)*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - 1/(12*(a - b)*d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^2) + (10*a*Sin[(c + d*x)/2] - 13*b*Sin[(c + d*
x)/2])/(6*(a - b)^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (10*a*Sin[(c
 + d*x)/2] + 13*b*Sin[(c + d*x)/2])/(6*(a + b)^2*d*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) + Tan[(c + d*x)/2]/(2*a*d)
```

Maple [A] time = 0.123, size = 317, normalized size = 1.4

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\frac{a}{d(a+b)^2(\tan(1/2dx+c/2)-1)}-\frac{5b}{2d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{1}{3d(a+b)}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-5/2/d/(a+b)
^2/(tan(1/2*d*x+1/2*c)-1)*b-1/3/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b
)/(tan(1/2*d*x+1/2*c)-1)^2+2/d/a^2/(a+b)^2/(a-b)^2*b^6/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/(a-b)^2/(tan(1/2*d
*x+1/2*c)+1)*a+5/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*b-1/3/d/(a-b)/(tan(1/2*
d*x+1/2*c)+1)^3+1/2/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)^2-1/2/d/a/tan(1/2*d*x+1/
2*c)-1/d/a^2*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.39312, size = 1891, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*b^6*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2))*sin(d*x + c) - 2*a^7 + 4*a^5*b^2 - 2*a^3*b^4 - 3*(a^6*b - 3*a^4*
b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^3*log(-1/2*cos(d
*x + c) + 1/2)*sin(d*x + c) + 2*(8*a^7 - 22*a^5*b^2 + 17*a^3*b^4 - 3*a*b^6)
*cos(d*x + c)^4 - 2*(4*a^7 - 11*a^5*b^2 + 7*a^3*b^4)*cos(d*x + c)^2 + 2*(a^
6*b - 2*a^4*b^3 + a^2*b^5 + 3*(a^6*b - 3*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^
2)*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^3*
sin(d*x + c)), -1/6*(6*sqrt(a^2 - b^2)*b^6*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^3*sin(d*x + c) - 2*a^7 + 4*a^5*b^
2 - 2*a^3*b^4 - 3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^3*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b
^7)*cos(d*x + c)^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*(8*a^7 - 2
2*a^5*b^2 + 17*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^4 - 2*(4*a^7 - 11*a^5*b^2 +
7*a^3*b^4)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5 + 3*(a^6*b - 3*a
```

```
^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4
*b^4 - a^2*b^6)*d*cos(d*x + c)^3*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25729, size = 482, normalized size = 2.19

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^6}{\left(a^6-2a^4b^2+a^2b^4\right)\sqrt{a^2-b^2}}-\frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{3\left(2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-\frac{4\left(6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*b^6/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(a^2 - b
^2)) - 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 3*tan(1/2*d*x + 1/2*c)/a +
3*(2*b*tan(1/2*d*x + 1/2*c) - a)/(a^2*tan(1/2*d*x + 1/2*c)) - 4*(6*a^3*tan(
1/2*d*x + 1/2*c)^5 - 9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*b*tan(1/2*d*x +
 1/2*c)^4 + 9*b^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 1
4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 12*b^3*ta
n(1/2*d*x + 1/2*c)^2 + 6*a^3*tan(1/2*d*x + 1/2*c) - 9*a*b^2*tan(1/2*d*x + 1
/2*c) - 4*a^2*b + 7*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 -
 1)^3))/d
```

3.1358 $\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=332

$$-\frac{2b^7\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{5/2}}+\frac{b^2\sec^3(c+dx)}{3a^3d}+\frac{b^2\sec(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b^3\sec^3(c+dx)(b-a\sin}{3a^3d\left(a^2-b^2\right)}$$

```
[Out] (-2*b^7*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^
(5/2)*d) - (5*ArcTanh[Cos[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/
(a^3*d) + (b*Cot[c + d*x])/(a^2*d) + (5*Sec[c + d*x])/(2*a*d) + (b^2*Sec[c
+ d*x])/(a^3*d) + (5*Sec[c + d*x]^3)/(6*a*d) + (b^2*Sec[c + d*x]^3)/(3*a^3*
d) - (Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*a*d) + (b^3*Sec[c + d*x]^3*(b - a*S
in[c + d*x]))/(3*a^3*(a^2 - b^2)*d) - (b^3*Sec[c + d*x]*(3*b^3 + a*(2*a^2 -
 5*b^2)*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*d) - (2*b*Tan[c + d*x])/(a^2*d)
 - (b*Tan[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.547913, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {2898, 2622, 302, 207, 2620, 270, 288, 2696, 2866, 12, 2660, 618, 204}

$$-\frac{2b^7\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{5/2}}+\frac{b^2\sec^3(c+dx)}{3a^3d}+\frac{b^2\sec(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b^3\sec^3(c+dx)(b-a\sin}{3a^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^7*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^
(5/2)*d) - (5*ArcTanh[Cos[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/
(a^3*d) + (b*Cot[c + d*x])/(a^2*d) + (5*Sec[c + d*x])/(2*a*d) + (b^2*Sec[c
+ d*x])/(a^3*d) + (5*Sec[c + d*x]^3)/(6*a*d) + (b^2*Sec[c + d*x]^3)/(3*a^3*
d) - (Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*a*d) + (b^3*Sec[c + d*x]^3*(b - a*S
in[c + d*x]))/(3*a^3*(a^2 - b^2)*d) - (b^3*Sec[c + d*x]*(3*b^3 + a*(2*a^2 -
 5*b^2)*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*d) - (2*b*Tan[c + d*x])/(a^2*d)
 - (b*Tan[c + d*x]^3)/(3*a^2*d)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
```

```
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{b^2\csc(c+dx)\sec^4(c+dx)}{a^3} - \frac{b\csc^2(c+dx)\sec^4(c+dx)}{a^2} + \frac{\csc^3(c+dx)\sec^4(c}{a}\right.$$

$$= \frac{\int \csc^3(c+dx)\sec^4(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a^2} + \frac{b^2\int \csc(c+dx}{}$$

$$= \frac{b^3\sec^3(c+dx)(b-a\sin(c+dx))}{3a^3\left(a^2-b^2\right)d} + \frac{b^3\int \frac{\sec^2(c+dx)\left(-2a^2+3b^2-2ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3a^3\left(a^2-b^2\right)} + \frac{\text{Subst}}{}$$

$$= -\frac{\csc^2(c+dx)\sec^3(c+dx)}{2ad} + \frac{b^3\sec^3(c+dx)(b-a\sin(c+dx))}{3a^3\left(a^2-b^2\right)d} - \frac{b^3\sec(c+dx)\left(3\right.}{3}$$

$$= \frac{b\cot(c+dx)}{a^2d} + \frac{b^2\sec(c+dx)}{a^3d} + \frac{b^2\sec^3(c+dx)}{3a^3d} - \frac{\csc^2(c+dx)\sec^3(c+dx)}{2ad} + \frac{b^3}{}$$

$$= -\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{5\sec(c+dx)}{2ad} + \frac{b^2\sec(c+dx)}{a^3d} + \frac{5\sec}{}$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{5\sec(c+dx)}{2ad}$$

$$= -\frac{2b^7\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d} - \frac{5\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} +$$

Mathematica [B] time = 6.23072, size = 947, normalized size = 2.85

$$16\left(-\frac{\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)\csc(c+dx)(a+b\sin(c+dx))b^7}{8a^3\left(a^2-b^2\right)^{5/2}d(b+a\csc(c+dx))} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)(a+}{32a^2d(b+a\csc(c+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] 16*((a*(13*a^2 - 19*b^2)*Csc[c + d*x]*(a + b*Sin[c + d*x]))/(96*(a^2 - b^2)
^2*d*(b + a*Csc[c + d*x])) - (b^7*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)
/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]]*Csc[c + d*x]*(a + b*Sin[c + d*x
]))/(8*a^3*(a^2 - b^2)^(5/2)*d*(b + a*Csc[c + d*x])) + (b*Cot[(c + d*x)/2]*
Csc[c + d*x]*(a + b*Sin[c + d*x]))/(32*a^2*d*(b + a*Csc[c + d*x])) - (Csc[(
c + d*x)/2]^2*Csc[c + d*x]*(a + b*Sin[c + d*x]))/(128*a*d*(b + a*Csc[c + d*
x])) + ((-5*a^2 - 2*b^2)*Csc[c + d*x]*Log[Cos[(c + d*x)/2]]*(a + b*Sin[c +
d*x]))/(32*a^3*d*(b + a*Csc[c + d*x])) + ((5*a^2 + 2*b^2)*Csc[c + d*x]*Log[
Sin[(c + d*x)/2]]*(a + b*Sin[c + d*x]))/(32*a^3*d*(b + a*Csc[c + d*x])) + (
Csc[c + d*x]*Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x]))/(128*a*d*(b + a*Csc[c
 + d*x])) + (Csc[c + d*x]*(a + b*Sin[c + d*x]))/(192*(a + b)*d*(b + a*Csc[c
 + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (Csc[c + d*x]*Sin[(c +
d*x)/2]*(a + b*Sin[c + d*x]))/(96*(a + b)*d*(b + a*Csc[c + d*x])*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^3) - (Csc[c + d*x]*Sin[(c + d*x)/2]*(a + b*Sin[
c + d*x]))/(96*(a - b)*d*(b + a*Csc[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3) + (Csc[c + d*x]*(a + b*Sin[c + d*x]))/(192*(a - b)*d*(b + a*Csc
[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Csc[c + d*x]*(-13*a*
Sin[(c + d*x)/2] + 16*b*Sin[(c + d*x)/2])*(a + b*Sin[c + d*x]))/(96*(a - b)
^2*d*(b + a*Csc[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (Csc[c +
 d*x]*(13*a*Sin[(c + d*x)/2] + 16*b*Sin[(c + d*x)/2])*(a + b*Sin[c + d*x]))
/(96*(a + b)^2*d*(b + a*Csc[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) - (b*Csc[c + d*x]*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2])/(32*a^2*d*(b + a
*Csc[c + d*x])))
```

Maple [A] time = 0.138, size = 376, normalized size = 1.1

$$\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{b}{2\,da^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{5\,a}{2\,d\,(a+b)^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-3\,\frac{b}{d\,(a+b)^{2}\,(\tan(1/2\,dx+c/2)-1)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/2*d*x+1/2*c)*b-5/2/d/(a+b)^2/(
tan(1/2*d*x+1/2*c)-1)*a-3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*b-1/3/d/(a+b)/(t
an(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)/(tan(1/2*d*x+1/2*c)-1)^2-2/d/a^3/(a+b)^2
/(a-b)^2*b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+5/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a-3/d/(a-b)^2/(tan(1/2*d*x+
1/2*c)+1)*b+1/3/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)^3-1/2/d/(a-b)/(tan(1/2*d*x+1
/2*c)+1)^2-1/8/d/a/tan(1/2*d*x+1/2*c)^2+5/2/d/a*ln(tan(1/2*d*x+1/2*c))+1/d/
a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/2/d/a^2*b/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.2822, size = 2596, normalized size = 7.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^8 - 8*a^6*b^2 + 4*a^4*b^4 - 6*(5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 -
 a^2*b^6)*cos(d*x + c)^4 + 4*(5*a^8 - 13*a^6*b^2 + 8*a^4*b^4)*cos(d*x + c)^
2 + 6*(b^7*cos(d*x + c)^5 - b^7*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(-((2*a
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x +
c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2)) + 3*((5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*
b^6 - 2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*b^6 - 2
*b^8)*cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) - 3*((5*a^8 - 13*a^6*b^2
+ 9*a^4*b^4 + a^2*b^6 - 2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4
*b^4 + a^2*b^6 - 2*b^8)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) - 4*(a
^7*b - 2*a^5*b^3 + a^3*b^5 - (8*a^7*b - 22*a^5*b^3 + 17*a^3*b^5 - 3*a*b^7)*
cos(d*x + c)^4 + (4*a^7*b - 11*a^5*b^3 + 7*a^3*b^5)*cos(d*x + c)^2)*sin(d*x
 + c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^5 - (a^9 - 3
*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3), -1/12*(4*a^8 - 8*a^6*b^2
 + 4*a^4*b^4 - 6*(5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 - a^2*b^6)*cos(d*x + c)^4
+ 4*(5*a^8 - 13*a^6*b^2 + 8*a^4*b^4)*cos(d*x + c)^2 - 12*(b^7*cos(d*x + c)^
5 - b^7*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(
a^2 - b^2)*cos(d*x + c))) + 3*((5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*b^6 -
2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*b^6 - 2*b^8)*
cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) - 3*((5*a^8 - 13*a^6*b^2 + 9*a^
4*b^4 + a^2*b^6 - 2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 +
 a^2*b^6 - 2*b^8)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) - 4*(a^7*b -
 2*a^5*b^3 + a^3*b^5 - (8*a^7*b - 22*a^5*b^3 + 17*a^3*b^5 - 3*a*b^7)*cos(d*
x + c)^4 + (4*a^7*b - 11*a^5*b^3 + 7*a^3*b^5)*cos(d*x + c)^2)*sin(d*x + c))
/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^5 - (a^9 - 3*a^7*b
^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30251, size = 563, normalized size = 1.7

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^7}{\left(a^7-2a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}-\frac{3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^2}-\frac{12\left(5a^2+2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{16\left(6a^2b\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))*b^7/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
 b^2)) - 3*(a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 12*(
5*a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 16*(6*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 - 9*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^3*tan(1/2*d*x + 1/2*c)^4 +
 12*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 14*b^3*
tan(1/2*d*x + 1/2*c)^3 + 12*a^3*tan(1/2*d*x + 1/2*c)^2 - 18*a*b^2*tan(1/2*d
*x + 1/2*c)^2 + 6*a^2*b*tan(1/2*d*x + 1/2*c) - 9*b^3*tan(1/2*d*x + 1/2*c) -
 7*a^3 + 10*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3)
 + 3*(30*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b
*tan(1/2*d*x + 1/2*c) + a^2)/(a^3*tan(1/2*d*x + 1/2*c)^2))/d
```

3.1359 $\int \frac{\sin^3(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=240

$$-\frac{a^8\log(a+b\sin(c+dx))}{b^3d\left(a^2-b^2\right)^3}-\frac{\left(35a^2+57ab+24b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(35a^2-57ab+24b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}$$

```
[Out] -((35*a^2 + 57*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((35
*a^2 - 57*a*b + 24*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (a^8*Log[
a + b*Sin[c + d*x]])/(b^3*(a^2 - b^2)^3*d) + (a*Sin[c + d*x])/(b^2*d) - Sin
[c + d*x]^2/(2*b*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*
d) + (Sec[c + d*x]^2*(4*b*(4*a^2 - 3*b^2) - a*(13*a^2 - 9*b^2)*Sin[c + d*x]
))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.617745, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1647, 1629}

$$-\frac{a^8\log(a+b\sin(c+dx))}{b^3d\left(a^2-b^2\right)^3}-\frac{\left(35a^2+57ab+24b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(35a^2-57ab+24b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((35*a^2 + 57*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((35
*a^2 - 57*a*b + 24*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (a^8*Log[
a + b*Sin[c + d*x]])/(b^3*(a^2 - b^2)^3*d) + (a*Sin[c + d*x])/(b^2*d) - Sin
[c + d*x]^2/(2*b*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*
d) + (Sec[c + d*x]^2*(4*b*(4*a^2 - 3*b^2) - a*(13*a^2 - 9*b^2)*Sin[c + d*x]
))/(8*(a^2 - b^2)^2*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{x^8}{b^8(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{x^8}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{a^2b^8}{a^2-b^2}+\frac{3ab^8x}{a^2-b^2}-4b^6x^2-4b^4x^4-4b^2x^6}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin\right.}{4b^5 d}$$

$$= \frac{\sec^2(c+dx)\left(4b\left(4a^2-3b^2\right)-a\left(13a^2-9b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin}{a^2}\right.}{4d}$$

$$= \frac{\sec^2(c+dx)\left(4b\left(4a^2-3b^2\right)-a\left(13a^2-9b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin}{a^2}\right.}{4d}$$

$$= -\frac{\left(35a^2+57ab+24b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3 d} + \frac{\left(35a^2-57ab+24b^2\right)\log(1+\sin(c+d\right.}{16(a-b)^3 d}$$

Mathematica [A] time = 3.03721, size = 212, normalized size = 0.88

$$\frac{-\frac{16a^8\log(a+b\sin(c+dx))}{b^3(a-b)^3(a+b)^3}-\frac{\left(35a^2+57ab+24b^2\right)\log(1-\sin(c+dx))}{(a+b)^3}+\frac{\left(35a^2-57ab+24b^2\right)\log(\sin(c+dx)+1)}{(a-b)^3}+\frac{16a\sin(c+dx)}{b^2}+\frac{13a+11b}{(a+b)^2(\sin(c+dx)-1)}+\frac{}{(a-}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((35*a^2 + 57*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) + ((35*a^2
 - 57*a*b + 24*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 - (16*a^8*Log[a + b*Si
n[c + d*x]])/((a - b)^3*b^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2)
+ (13*a + 11*b)/((a + b)^2*(-1 + Sin[c + d*x])) + (16*a*Sin[c + d*x])/b^2 -
 (8*Sin[c + d*x]^2)/b - 1/((a - b)*(1 + Sin[c + d*x])^2) + (13*a - 11*b)/((
a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

Maple [A] time = 0.095, size = 338, normalized size = 1.4

$$-\frac{(\sin(dx+c))^2}{2\,bd}+\frac{a\sin(dx+c)}{b^2 d}-\frac{a^8\ln(a+b\sin(dx+c))}{db^3(a+b)^3(a-b)^3}+\frac{1}{2\,d(8\,a+8\,b)(\sin(dx+c)-1)^2}+\frac{13\,a}{16\,d(a+b)^2(\sin(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2*sin(d*x+c)^2/b/d+a*sin(d*x+c)/b^2/d-1/d/b^3*a^8/(a+b)^3/(a-b)^3*ln(a+b
*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+13/16/d/(a+b)^2/(sin(d*x+c)-1
)*a+11/16/d/(a+b)^2/(sin(d*x+c)-1)*b-35/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-5
7/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-3/2/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2-1/2/
d/(8*a-8*b)/(1+sin(d*x+c))^2+13/16/d/(a-b)^2/(1+sin(d*x+c))*a-11/16/d/(a-b)
^2/(1+sin(d*x+c))*b+35/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2-57/16/d/(a-b)^3*ln
(1+sin(d*x+c))*a*b+3/2/d/(a-b)^3*ln(1+sin(d*x+c))*b^2
```

Maxima [A] time = 1.03944, size = 427, normalized size = 1.78

$$-\frac{\frac{16\,a^8\log(b\sin(dx+c)+a)}{a^6b^3-3\,a^4b^5+3\,a^2b^7-b^9}-\frac{\left(35\,a^2-57\,ab+24\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(35\,a^2+57\,ab+24\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(13\,a^3-9\,ab^2\right)\sin(dx+c)^3+14\,a^2b-10\cdots\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^8*log(b*sin(d*x + c) + a)/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^
9) - (35*a^2 - 57*a*b + 24*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) + (35*a^2 + 57*a*b + 24*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) - 2*((13*a^3 - 9*a*b^2)*sin(d*x + c)^3 + 14*a^2*b - 10*b
^3 - 4*(4*a^2*b - 3*b^3)*sin(d*x + c)^2 - (11*a^3 - 7*a*b^2)*sin(d*x + c))/
((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 -
2*a^2*b^2 + b^4)*sin(d*x + c)^2) + 8*(b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/
b^2)/d
```

Fricas [A] time = 3.75418, size = 946, normalized size = 3.94

$$-\frac{16\,a^8\cos(dx+c)^4\log(b\sin(dx+c)+a)+4\,a^4b^4-8\,a^2b^6+4\,b^8-8\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)\cos(dx+c)^6-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^8*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^4*b^4 - 8*a^2*b^
6 + 4*b^8 - 8*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^6 - (35*
a^5*b^3 + 48*a^4*b^4 - 42*a^3*b^5 - 64*a^2*b^6 + 15*a*b^7 + 24*b^8)*cos(d*x
 + c)^4*log(sin(d*x + c) + 1) + (35*a^5*b^3 - 48*a^4*b^4 - 42*a^3*b^5 + 64*
a^2*b^6 + 15*a*b^7 - 24*b^8)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*(a^6
*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^4 - 8*(4*a^4*b^4 - 7*a^2*b
^6 + 3*b^8)*cos(d*x + c)^2 - 2*(2*a^5*b^3 - 4*a^3*b^5 + 2*a*b^7 + 8*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)^4 - (13*a^5*b^3 - 22*a^3*b^5
+ 9*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
- b^9)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**8/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26324, size = 544, normalized size = 2.27

$$-\frac{\frac{16\,a^8\log(|b\sin(dx+c)+a|)}{a^6b^3-3\,a^4b^5+3\,a^2b^7-b^9}-\frac{\left(35\,a^2-57\,ab+24\,b^2\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(35\,a^2+57\,ab+24\,b^2\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{8\left(b\sin(dx+c)^2-2\,a\sin(dx+c)\right)}{b^2}+\frac{2\left(36\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^8*log(abs(b*sin(d*x + c) + a))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
 - b^9) - (35*a^2 - 57*a*b + 24*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^
2*b + 3*a*b^2 - b^3) + (35*a^2 + 57*a*b + 24*b^2)*log(abs(sin(d*x + c) - 1)
)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 8*(b*sin(d*x + c)^2 - 2*a*sin(d*x + c))
/b^2 + 2*(36*a^4*b*sin(d*x + c)^4 - 48*a^2*b^3*sin(d*x + c)^4 + 18*b^5*sin(
d*x + c)^4 - 13*a^5*sin(d*x + c)^3 + 22*a^3*b^2*sin(d*x + c)^3 - 9*a*b^4*si
n(d*x + c)^3 - 56*a^4*b*sin(d*x + c)^2 + 68*a^2*b^3*sin(d*x + c)^2 - 24*b^5
*sin(d*x + c)^2 + 11*a^5*sin(d*x + c) - 18*a^3*b^2*sin(d*x + c) + 7*a*b^4*s
in(d*x + c) + 22*a^4*b - 24*a^2*b^3 + 8*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

3.1360 $\int \frac{\sin^2(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=221

$$\frac{a^7\log(a+b\sin(c+dx))}{b^2d\left(a^2-b^2\right)^3}-\frac{\left(24a^2+37ab+15b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(24a^2-37ab+15b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+$$

```
[Out] -((24*a^2 + 37*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((24
*a^2 - 37*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^7*Log[
a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)^3*d) - Sin[c + d*x]/(b*d) + (Sec[c +
d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(3*a^
2 - 2*b^2) - b*(13*a^2 - 9*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.539119, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1647, 1629}

$$\frac{a^7\log(a+b\sin(c+dx))}{b^2d\left(a^2-b^2\right)^3}-\frac{\left(24a^2+37ab+15b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(24a^2-37ab+15b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((24*a^2 + 37*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((24
*a^2 - 37*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^7*Log[
a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)^3*d) - Sin[c + d*x]/(b*d) + (Sec[c +
d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(3*a^
2 - 2*b^2) - b*(13*a^2 - 9*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^7}{b^7(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^7}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^8}{a^2-b^2}-\frac{b^6\left(4a^2-b^2\right)x}{a^2-b^2}-4b^4x^3-4b^2x^5}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+a\right.}{4b^4d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(3a^2-2b^2\right)-b\left(13a^2-9b^2\right)\sin(c+\right.}{8\left(a^2-b^2\right)^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(3a^2-2b^2\right)-b\left(13a^2-9b^2\right)\sin(c+\right.}{8\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(24a^2+37ab+15b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{\left(24a^2-37ab+15b^2\right)\log(1+\sin(c+d\right.}{16(a-b)^3d}$$

Mathematica [A] time = 2.42689, size = 198, normalized size = 0.9

$$\frac{\frac{16a^7\log(a+b\sin(c+dx))}{b^2(a-b)^3(a+b)^3} - \frac{\left(24a^2+37ab+15b^2\right)\log(1-\sin(c+dx))}{(a+b)^3} - \frac{\left(24a^2-37ab+15b^2\right)\log(\sin(c+dx)+1)}{(a-b)^3} + \frac{11a+9b}{(a+b)^2(\sin(c+dx)-1)} + \frac{9b-11a}{(a-b)^2(\sin(c+dx)+1)} + \cdots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((24*a^2 + 37*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((24*a^2
 - 37*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^7*Log[a + b*Si
n[c + d*x]])/((a - b)^3*b^2*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2)
+ (11*a + 9*b)/((a + b)^2*(-1 + Sin[c + d*x])) - (16*Sin[c + d*x])/b + 1/((
a - b)*(1 + Sin[c + d*x])^2) + (-11*a + 9*b)/((a - b)^2*(1 + Sin[c + d*x]))
)/(16*d)
```

Maple [A] time = 0.095, size = 321, normalized size = 1.5

$$-\frac{\sin(dx+c)}{bd} + \frac{a^7\ln(a+b\sin(dx+c))}{db^2(a+b)^3(a-b)^3} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} + \frac{11a}{16d(a+b)^2(\sin(dx+c)-1)} + \frac{\cdots}{16d(a+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x)
```

```
[Out] -sin(d*x+c)/b/d+1/d/b^2*a^7/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8
*b)/(sin(d*x+c)-1)^2+11/16/d/(a+b)^2/(sin(d*x+c)-1)*a+9/16/d/(a+b)^2/(sin(d
*x+c)-1)*b-3/2/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-37/16/d/(a+b)^3*ln(sin(d*x+c)
-1)*a*b-15/16/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2+1/2/d/(8*a-8*b)/(1+sin(d*x+c))
^2-11/16/d/(a-b)^2/(1+sin(d*x+c))*a+9/16/d/(a-b)^2/(1+sin(d*x+c))*b-3/2/d/(
a-b)^3*ln(1+sin(d*x+c))*a^2+37/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-15/16/d/(a
-b)^3*ln(1+sin(d*x+c))*b^2
```

Maxima [A] time = 1.03794, size = 409, normalized size = 1.85

$$\frac{\frac{16\,a^7\log(b\sin(dx+c)+a)}{a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8} - \frac{\left(24\,a^2-37\,ab+15\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{\left(24\,a^2+37\,ab+15\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\left(\left(13\,a^2b-9\,b^3\right)\sin(dx+c)^3+10\,a^3-6\,ab^2-\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^7*log(b*sin(d*x + c) + a)/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8
) - (24*a^2 - 37*a*b + 15*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (24*a^2 + 37*a*b + 15*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3) - 2*((13*a^2*b - 9*b^3)*sin(d*x + c)^3 + 10*a^3 - 6*a*b^2
 - 4*(3*a^3 - 2*a*b^2)*sin(d*x + c)^2 - (11*a^2*b - 7*b^3)*sin(d*x + c))/((
a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*
a^2*b^2 + b^4)*sin(d*x + c)^2) - 16*sin(d*x + c)/b)/d
```

Fricas [A] time = 3.47397, size = 780, normalized size = 3.53

$$\frac{16\,a^7\cos(dx+c)^4\log(b\sin(dx+c)+a)+4\,a^5b^2-8\,a^3b^4+4\,ab^6-\left(24\,a^5b^2+35\,a^4b^3-24\,a^3b^4-42\,a^2b^5+8\,ab^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^7*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^5*b^2 - 8*a^3*b^4
 + 4*a*b^6 - (24*a^5*b^2 + 35*a^4*b^3 - 24*a^3*b^4 - 42*a^2*b^5 + 8*a*b^6 +
 15*b^7)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (24*a^5*b^2 - 35*a^4*b^3 -
24*a^3*b^4 + 42*a^2*b^5 + 8*a*b^6 - 15*b^7)*cos(d*x + c)^4*log(-sin(d*x + c
) + 1) - 8*(3*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^2 - 2*(2*a^4*b^3
- 4*a^2*b^5 + 2*b^7 + 8*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^
4 - (13*a^4*b^3 - 22*a^2*b^5 + 9*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**7/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26077, size = 518, normalized size = 2.34

$$\frac{16\,a^7\log(|b\sin(dx+c)+a|)}{a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8}-\frac{\left(24\,a^2-37\,ab+15\,b^2\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(24\,a^2+37\,ab+15\,b^2\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{16\,\sin(dx+c)}{b}+\frac{2\left(18\,a^5\sin(dx+c)^4-18\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^7*log(abs(b*sin(d*x + c) + a))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8) - (24*a^2 - 37*a*b + 15*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) - (24*a^2 + 37*a*b + 15*b^2)*log(abs(sin(d*x + c) - 1))
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 16*sin(d*x + c)/b + 2*(18*a^5*sin(d*x +
c)^4 - 18*a^3*b^2*sin(d*x + c)^4 + 6*a*b^4*sin(d*x + c)^4 - 13*a^4*b*sin(d*
x + c)^3 + 22*a^2*b^3*sin(d*x + c)^3 - 9*b^5*sin(d*x + c)^3 - 24*a^5*sin(d*
x + c)^2 + 16*a^3*b^2*sin(d*x + c)^2 - 4*a*b^4*sin(d*x + c)^2 + 11*a^4*b*si
n(d*x + c) - 18*a^2*b^3*sin(d*x + c) + 7*b^5*sin(d*x + c) + 8*a^5 - 2*a^3*b
^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

3.1361 $\int \frac{\sin(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=208

$$-\frac{a^6\log(a+b\sin(c+dx))}{bd\left(a^2-b^2\right)^3}-\frac{\left(15a^2+21ab+8b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(15a^2-21ab+8b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\text{s}$$

```
[Out] -((15*a^2 + 21*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((15*
a^2 - 21*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (a^6*Log[a
+ b*Sin[c + d*x]])/(b*(a^2 - b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x
]))/(4*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(4*b*(3*a^2 - 2*b^2) - a*(9*a^2 - 5
*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.512166, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 1647, 1629}

$$-\frac{a^6\log(a+b\sin(c+dx))}{bd\left(a^2-b^2\right)^3}-\frac{\left(15a^2+21ab+8b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(15a^2-21ab+8b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\text{s}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((15*a^2 + 21*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((15*
a^2 - 21*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (a^6*Log[a
+ b*Sin[c + d*x]])/(b*(a^2 - b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x
]))/(4*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(4*b*(3*a^2 - 2*b^2) - a*(9*a^2 - 5
*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^6}{b^6(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^6}{a^2-b^2}+\frac{3ab^6x}{a^2-b^2}-4b^4x^2-4b^2x^4}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx\right.}{4b^3d}$$

$$= \frac{\sec^2(c+dx)\left(4b\left(3a^2-2b^2\right)-a\left(9a^2-5b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+}{a^2-b}\right.}{4d}$$

$$= \frac{\sec^2(c+dx)\left(4b\left(3a^2-2b^2\right)-a\left(9a^2-5b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+}{a^2-b}\right.}{4d}$$

$$= -\frac{\left(15a^2+21ab+8b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{\left(15a^2-21ab+8b^2\right)\log(1+\sin(c+dx))}{16(a-b)^3d}$$

Mathematica [A] time = 1.73856, size = 187, normalized size = 0.9

$$\frac{-\frac{\left(15a^2+21ab+8b^2\right)\log(1-\sin(c+dx))}{(a+b)^3}+\frac{\left(15a^2-21ab+8b^2\right)\log(\sin(c+dx)+1)}{(a-b)^3}-\frac{16a^6\log(a+b\sin(c+dx))}{b(a-b)^3(a+b)^3}+\frac{9a+7b}{(a+b)^2(\sin(c+dx)-1)}+\frac{9a-7b}{(a-b)^2(\sin(c+dx)+1)}+}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((15*a^2 + 21*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) + ((15*a^2
- 21*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 - (16*a^6*Log[a + b*Sin[
c + d*x]])/((a - b)^3*b*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (9
*a + 7*b)/((a + b)^2*(-1 + Sin[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2
) + (9*a - 7*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

Maple [A] time = 0.089, size = 308, normalized size = 1.5

$$-\frac{a^6\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3b} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} + \frac{9a}{16d(a+b)^2(\sin(dx+c)-1)} + \frac{7b}{16d(a+b)^2(\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a^6/(a+b)^3/(a-b)^3/b*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-1
)^2+9/16/d/(a+b)^2/(sin(d*x+c)-1)*a+7/16/d/(a+b)^2/(sin(d*x+c)-1)*b-15/16/d
/(a+b)^3*ln(sin(d*x+c)-1)*a^2-21/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-1/2/d/(a
+b)^3*ln(sin(d*x+c)-1)*b^2-1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2+9/16/d/(a-b)^2/
(1+sin(d*x+c))*a-7/16/d/(a-b)^2/(1+sin(d*x+c))*b+15/16/d/(a-b)^3*ln(1+sin(d
*x+c))*a^2-21/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b+1/2/d/(a-b)^3*ln(1+sin(d*x+
c))*b^2
```

Maxima [A] time = 1.03065, size = 390, normalized size = 1.88

$$-\frac{\frac{16\,a^6\log(b\sin(dx+c)+a)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(15\,a^2-21\,ab+8\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(15\,a^2+21\,ab+8\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(9\,a^3-5\,ab^2\right)\sin(dx+c)^3+10\,a^2b-6\,b^3-4\cdots\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^6*log(b*sin(d*x + c) + a)/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
 - (15*a^2 - 21*a*b + 8*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) + (15*a^2 + 21*a*b + 8*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b +
3*a*b^2 + b^3) - 2*((9*a^3 - 5*a*b^2)*sin(d*x + c)^3 + 10*a^2*b - 6*b^3 - 4
*(3*a^2*b - 2*b^3)*sin(d*x + c)^2 - (7*a^3 - 3*a*b^2)*sin(d*x + c))/((a^4 -
 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b
^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A] time = 2.93716, size = 687, normalized size = 3.3

$$-\frac{16\,a^6\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)+4\,a^4b^2-8\,a^2b^4+4\,b^6-\left(15\,a^5b+24\,a^4b^2-10\,a^3b^3-24\,a^2b^4+3\,ab^5+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^6*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^4*b^2 - 8*a^2*b^
4 + 4*b^6 - (15*a^5*b + 24*a^4*b^2 - 10*a^3*b^3 - 24*a^2*b^4 + 3*a*b^5 + 8*
b^6)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (15*a^5*b - 24*a^4*b^2 - 10*a^3
*b^3 + 24*a^2*b^4 + 3*a*b^5 - 8*b^6)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)
- 8*(3*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 - 2*(2*a^5*b - 4*a^3*b^3
 + 2*a*b^5 - (9*a^5*b - 14*a^3*b^3 + 5*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))
/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24337, size = 501, normalized size = 2.41

$$-\frac{\frac{16\,a^6\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(15\,a^2-21\,ab+8\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(15\,a^2+21\,ab+8\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(18\,a^4b\sin\left(dx+c\right)^4-18\,a^2b^3\sin\left(dx+c\right)^4+6\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^6*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7) - (15*a^2 - 21*a*b + 8*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3) + (15*a^2 + 21*a*b + 8*b^2)*log(abs(sin(d*x + c) - 1))/(a
^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(18*a^4*b*sin(d*x + c)^4 - 18*a^2*b^3*sin
(d*x + c)^4 + 6*b^5*sin(d*x + c)^4 - 9*a^5*sin(d*x + c)^3 + 14*a^3*b^2*sin(
d*x + c)^3 - 5*a*b^4*sin(d*x + c)^3 - 24*a^4*b*sin(d*x + c)^2 + 16*a^2*b^3*
sin(d*x + c)^2 - 4*b^5*sin(d*x + c)^2 + 7*a^5*sin(d*x + c) - 10*a^3*b^2*sin
(d*x + c) + 3*a*b^4*sin(d*x + c) + 8*a^4*b - 2*a^2*b^3)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

3.1362 $\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{a^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c$$

```
[Out] -((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((8*a^2
 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a + b*
Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4
*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*Si
n[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.350526, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 801}

$$\frac{a^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((8*a^2
 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a + b*
Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4
*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*Si
n[c + d*x]))/(8*(a^2 - b^2)^2*d)
```


Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^6}{a^2-b^2} - \frac{b^4\left(4a^2-b^2\right)x}{a^2-b^2} - 4b^2x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(2a^2-b^2\right) - b\left(9a^2-5b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} + \text{Su}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(2a^2-b^2\right) - b\left(9a^2-5b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} + \text{Su}$$

$$= -\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{\left(8a^2-9ab+3b^2\right)\log(1+\sin(c+dx))}{16(a-b)^3d} + \frac{a^5\log(a+}{\left(a^2\right.}$$

Mathematica [A] time = 1.41459, size = 184, normalized size = 0.9

$$\frac{-\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{(a+b)^3} - \frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{(a-b)^3} + \frac{16a^5\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{7a+5b}{(a+b)^2(\sin(c+dx)-1)} + \frac{5b-7a}{(a-b)^2(\sin(c+dx)+1)} + \frac{}{(a+}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((8*a^2 - 9
*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^5*Log[a + b*Sin[c +
d*x]])/((a - b)^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (7*a + 5
*b)/((a + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-
7*a + 5*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

Maple [A] time = 0.088, size = 304, normalized size = 1.5

$$\frac{a^5\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} + \frac{7a}{16d(a+b)^2(\sin(dx+c)-1)} + \frac{5b}{16d(a+b)^2(\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a^5/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2
+7/16/d/(a+b)^2/(sin(d*x+c)-1)*a+5/16/d/(a+b)^2/(sin(d*x+c)-1)*b-1/2/d/(a+b
)^3*ln(sin(d*x+c)-1)*a^2-9/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-3/16/d/(a+b)^3
*ln(sin(d*x+c)-1)*b^2+1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2-7/16/d/(a-b)^2/(1+si
n(d*x+c))*a+5/16/d/(a-b)^2/(1+sin(d*x+c))*b-1/2/d/(a-b)^3*ln(1+sin(d*x+c))*
a^2+9/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-3/16/d/(a-b)^3*ln(1+sin(d*x+c))*b^2
```

Maxima [A] time = 1.02862, size = 389, normalized size = 1.91

$$\frac{\frac{16\,a^5\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(8\,a^2-9\,ab+3\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+9\,ab+3\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(9\,a^2b-5\,b^3\right)\sin(dx+c)^3+6\,a^3-2\,ab^2-4\left(2\,a^3-a\right.\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2\,a^2b^2+b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
(8*a^2 - 9*a*b + 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) - (8*a^2 + 9*a*b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2
 + b^3) - 2*((9*a^2*b - 5*b^3)*sin(d*x + c)^3 + 6*a^3 - 2*a*b^2 - 4*(2*a^3
- a*b^2)*sin(d*x + c)^2 - (7*a^2*b - 3*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2
 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*
sin(d*x + c)^2))/d
```

Fricas [A] time = 2.33264, size = 594, normalized size = 2.91

$$\frac{16\,a^5\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)-\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (8*a^5 + 15*a^4*b - 1
0*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^5 - 15*a^4*b
 + 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^
3*b^2 + 4*a*b^4 - 8*(2*a^5 - 3*a^3*b^2 + a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b
 - 4*a^2*b^3 + 2*b^5 - (9*a^4*b - 14*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29154, size = 463, normalized size = 2.27

$$\frac{\frac{16\,a^5b\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(8\,a^2-9\,ab+3\,b^2\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+9\,ab+3\,b^2\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,a^5\sin(dx+c)^4-9\,a^4b\sin(dx+c)^3+14\,a^2b^3\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^5*b*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7) - (8*a^2 - 9*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*a^5*sin(d*x + c)^4 - 9*a^4*b*sin(d*x + c)
^3 + 14*a^2*b^3*sin(d*x + c)^3 - 5*b^5*sin(d*x + c)^3 - 4*a^5*sin(d*x + c)^
2 - 12*a^3*b^2*sin(d*x + c)^2 + 4*a*b^4*sin(d*x + c)^2 + 7*a^4*b*sin(d*x +
c) - 10*a^2*b^3*sin(d*x + c) + 3*b^5*sin(d*x + c) + 8*a^3*b^2 - 2*a*b^4)/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

3.1363 $\int \frac{\sec(c+dx)\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=190

$$-\frac{a^4b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\left(a^2-b^2\right)}+\frac{\sec^2(c+dx)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}$$

```
[Out] -(a*(3*a + b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + (a*(3*a - b)*Log[1
+ Sin[c + d*x]])/(16*(a - b)^3*d) - (a^4*b*Log[a + b*Sin[c + d*x]])/((a^2 -
 b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d) + (Sec
[c + d*x]^2*(4*b*(2*a^2 - b^2) - a*(5*a^2 - b^2)*Sin[c + d*x]))/(8*(a^2 - b
^2)^2*d)
```

Rubi [A] time = 0.431329, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 1647, 801}

$$-\frac{a^4b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\left(a^2-b^2\right)}+\frac{\sec^2(c+dx)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*(3*a + b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + (a*(3*a - b)*Log[1
+ Sin[c + d*x]])/(16*(a - b)^3*d) - (a^4*b*Log[a + b*Sin[c + d*x]])/((a^2 -
 b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d) + (Sec
[c + d*x]^2*(4*b*(2*a^2 - b^2) - a*(5*a^2 - b^2)*Sin[c + d*x]))/(8*(a^2 - b
^2)^2*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^4}{b^4(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{x^4}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^4}{a^2-b^2}+\frac{3ab^4x}{a^2-b^2}-4b^2x^2}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4bd}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\sec^2(c+dx)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\sin(c+d\ldots}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\sec^2(c+dx)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\sin(c+d\ldots}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a(3a+b)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{a(3a-b)\log(1+\sin(c+dx))}{16(a-b)^3d} - \frac{a^4b\log(a+b\sin(c\ldots}{\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 1.59681, size = 169, normalized size = 0.89

$$\frac{-\frac{16a^4b\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{5a+3b}{(a+b)^2(\sin(c+dx)-1)}+\frac{5a-3b}{(a-b)^2(\sin(c+dx)+1)}+\frac{1}{(a+b)(\sin(c+dx)-1)^2}-\frac{1}{(a-b)(\sin(c+dx)+1)^2}-\frac{a(3a+b)\log(1-\sin(c+dx))}{(a+b)^3}\ldots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((a*(3*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^3) + (a*(3*a - b)*Log[1 + Si
n[c + d*x]])/(a - b)^3 - (16*a^4*b*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a +
 b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (5*a + 3*b)/((a + b)^2*(-1 + S
in[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (5*a - 3*b)/((a - b)^2*(
1 + Sin[c + d*x])))/(16*d)
```

Maple [A] time = 0.086, size = 260, normalized size = 1.4

$$-\frac{a^4b\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3}+\frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2}+\frac{5a}{16d(a+b)^2(\sin(dx+c)-1)}+\frac{3b}{16d(a+b)^2(\sin(dx+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a^4*b/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-1
)^2+5/16/d/(a+b)^2/(sin(d*x+c)-1)*a+3/16/d/(a+b)^2/(sin(d*x+c)-1)*b-3/16/d/
(a+b)^3*ln(sin(d*x+c)-1)*a^2-1/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-1/2/d/(8*a
-8*b)/(1+sin(d*x+c))^2+5/16/d/(a-b)^2/(1+sin(d*x+c))*a-3/16/d/(a-b)^2/(1+si
n(d*x+c))*b+3/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2-1/16/d/(a-b)^3*ln(1+sin(d*x
+c))*a*b
```

Maxima [A] time = 1.00613, size = 373, normalized size = 1.96

$$-\frac{\frac{16\,a^4b\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(3\,a^2-ab\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+ab\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(5\,a^3-ab^2\right)\sin(dx+c)^3+6\,a^2b-2\,b^3-4\left(2\,a^2b-b^3\right)\sin(dx+c}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\,a^2\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^4*b*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
 - (3*a^2 - a*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3
*a^2 + a*b)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*((5*a
^3 - a*b^2)*sin(d*x + c)^3 + 6*a^2*b - 2*b^3 - 4*(2*a^2*b - b^3)*sin(d*x +
c)^2 - (3*a^3 + a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^
4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A] time = 2.35374, size = 589, normalized size = 3.1

$$-\frac{16\,a^4b\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)-\left(3\,a^5+8\,a^4b+6\,a^3b^2-ab^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)+\left(3\,a^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^4*b*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (3*a^5 + 8*a^4*b +
 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (3*a^5 - 8*a^4*b
 + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^4*b - 8*a
^2*b^3 + 4*b^5 - 8*(2*a^4*b - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - 2*(2*a^5 -
4*a^3*b^2 + 2*a*b^4 - (5*a^5 - 6*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27666, size = 450, normalized size = 2.37

$$-\frac{\frac{16\,a^4b^2\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(3\,a^2-ab\right)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+ab\right)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,a^4b\sin\left(dx+c\right)^4-5\,a^5\sin\left(dx+c\right)^3+6\,a^3b^2\sin\left(dx+c\right)^3-ab^4\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^4*b^2*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b
^5 - b^7) - (3*a^2 - a*b)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) + (3*a^2 + a*b)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3) + 2*(6*a^4*b*sin(d*x + c)^4 - 5*a^5*sin(d*x + c)^3 + 6*a^3*b^2*si
n(d*x + c)^3 - a*b^4*sin(d*x + c)^3 - 4*a^4*b*sin(d*x + c)^2 - 12*a^2*b^3*s
in(d*x + c)^2 + 4*b^5*sin(d*x + c)^2 + 3*a^5*sin(d*x + c) - 2*a^3*b^2*sin(d
*x + c) - a*b^4*sin(d*x + c) + 8*a^2*b^3 - 2*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

3.1364 $\int \frac{\sec^2(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=182

$$\frac{a^3b^2\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\left(a^2-b^2\right)}-\frac{\sec^2(c+dx)\left(4a^3-b\left(5a^2-b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{b(3a+$$

```
[Out] (b*(3*a + b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((3*a - b)*b*Log[1 +
 Sin[c + d*x]])/(16*(a - b)^3*d) + (a^3*b^2*Log[a + b*Sin[c + d*x]])/((a^2
- b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Se
c[c + d*x]^2*(4*a^3 - b*(5*a^2 - b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.372706, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2837, 12, 1647, 823, 801}

$$\frac{a^3b^2\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\left(a^2-b^2\right)}-\frac{\sec^2(c+dx)\left(4a^3-b\left(5a^2-b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{b(3a+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(3*a + b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((3*a - b)*b*Log[1 +
 Sin[c + d*x]])/(16*(a - b)^3*d) + (a^3*b^2*Log[a + b*Sin[c + d*x]])/((a^2
- b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Se
c[c + d*x]^2*(4*a^3 - b*(5*a^2 - b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^3}{b^3(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(4a^2-b^2\right)x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(\frac{4a^3}{a^2-b^2}-\frac{b\left(5a^2-b^2\right)\sin(c+dx)}{a^2-b^2}\right)}{8\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(\frac{4a^3}{a^2-b^2}-\frac{b\left(5a^2-b^2\right)\sin(c+dx)}{a^2-b^2}\right)}{8\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \left(\cdots\right.\right.}{\cdots}$$

$$= \frac{b(3a+b)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{(3a-b)b\log(1+\sin(c+dx))}{16(a-b)^3d} + \frac{a^3b^2\log(a+b\sin(c\cdots}{\left(a^2-b^2\right)^3d}$$

Mathematica [A] time = 1.43084, size = 166, normalized size = 0.91

$$\frac{\frac{16a^3b^2\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{3a+b}{(a+b)^2(\sin(c+dx)-1)}+\frac{b-3a}{(a-b)^2(\sin(c+dx)+1)}+\frac{1}{(a+b)(\sin(c+dx)-1)^2}+\frac{1}{(a-b)(\sin(c+dx)+1)^2}+\frac{b(3a+b)\log(1-\sin(c+dx))}{(a+b)^3}-\cdots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((b*(3*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^3 - ((3*a - b)*b*Log[1 + Sin[c
 + d*x]])/(a - b)^3 + (16*a^3*b^2*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a +
b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (3*a + b)/((a + b)^2*(-1 + Sin[
c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-3*a + b)/((a - b)^2*(1 +
Sin[c + d*x])))/(16*d)
```

Maple [A] time = 0.082, size = 261, normalized size = 1.4

$$\frac{a^3b^2 \ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} + \frac{3a}{16d(a+b)^2(\sin(dx+c)-1)} + \frac{b}{16d(a+b)^2(\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a^3/(a+b)^3*b^2/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-
1)^2+3/16/d/(a+b)^2/(sin(d*x+c)-1)*a+1/16/d/(a+b)^2/(sin(d*x+c)-1)*b+3/16/d
/(a+b)^3*ln(sin(d*x+c)-1)*a*b+1/16/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2+1/2/d/(8*
a-8*b)/(1+sin(d*x+c))^2-3/16/d/(a-b)^2/(1+sin(d*x+c))*a+1/16/d/(a-b)^2/(1+s
in(d*x+c))*b-3/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b+1/16/d/(a-b)^3*ln(1+sin(d*
x+c))*b^2
```

Maxima [A] time = 1.00847, size = 360, normalized size = 1.98

$$\frac{\frac{16a^3b^2\log(b\sin(dx+c)+a)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{(3ab-b^2)\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3} + \frac{(3ab+b^2)\log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3} + \frac{2(4a^3\sin(dx+c)^2-(5a^2b-b^3)\sin(dx+c)^3-2a^3-2ab^2+(3a^2b}{(a^4-2a^2b^2+b^4)\sin(dx+c)^4+a^4-2a^2b^2+b^4-2(a^4-2a^2b^2+b}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*b^2*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
) - (3*a*b - b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (
3*a*b + b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a
^3*sin(d*x + c)^2 - (5*a^2*b - b^3)*sin(d*x + c)^3 - 2*a^3 - 2*a*b^2 + (3*a
^2*b + b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A] time = 2.41788, size = 579, normalized size = 3.18

$$16a^3b^2\cos(dx+c)^4\log(b\sin(dx+c)+a) - (3a^4b+8a^3b^2+6a^2b^3-b^5)\cos(dx+c)^4\log(\sin(dx+c)+1) + (3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^3*b^2*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (3*a^4*b + 8*a^3*
b^2 + 6*a^2*b^3 - b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (3*a^4*b - 8*
a^3*b^2 + 6*a^2*b^3 - b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 -
8*a^3*b^2 + 4*a*b^4 - 8*(a^5 - a^3*b^2)*cos(d*x + c)^2 - 2*(2*a^4*b - 4*a^2
*b^3 + 2*b^5 - (5*a^4*b - 6*a^2*b^3 + b^5)*cos(d*x + c)^2)*sin(d*x + c))/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25635, size = 440, normalized size = 2.42

$$\frac{\frac{16\,a^3b^3\log\left(|b\sin(dx+c)+a|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(3\,ab-b^2\right)\log\left(|\sin(dx+c)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,ab+b^2\right)\log\left(|\sin(dx+c)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,a^3b^2\sin(dx+c)^4-5\,a^4b\sin(dx+c)^3+6\,a^2b^3\sin(dx+c)^3-b^5\,\mathrm{s}\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^3*b^3*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^
5 - b^7) - (3*a*b - b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) + (3*a*b + b^2)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^
2 + b^3) + 2*(6*a^3*b^2*sin(d*x + c)^4 - 5*a^4*b*sin(d*x + c)^3 + 6*a^2*b^3
*sin(d*x + c)^3 - b^5*sin(d*x + c)^3 + 4*a^5*sin(d*x + c)^2 - 16*a^3*b^2*si
n(d*x + c)^2 + 3*a^4*b*sin(d*x + c) - 2*a^2*b^3*sin(d*x + c) - b^5*sin(d*x
+ c) - 2*a^5 + 6*a^3*b^2 + 2*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(s
in(d*x + c)^2 - 1)^2))/d
```

3.1365 $\int \frac{\sec^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=178

$$-\frac{a^2b^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\left(a^2-b^2\right)}+\frac{a\sec^2(c+dx)\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{a(a+$$

```
[Out] (a*(a + 3*b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - (a*(a - 3*b)*Log[1 +
 Sin[c + d*x]])/(16*(a - b)^3*d) - (a^2*b^3*Log[a + b*Sin[c + d*x]])/((a^2
- b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d) + (a*
Sec[c + d*x]^2*(4*a*b - (a^2 + 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.379682, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2837, 12, 1647, 823, 801}

$$-\frac{a^2b^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\left(a^2-b^2\right)}+\frac{a\sec^2(c+dx)\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{a(a+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] (a*(a + 3*b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - (a*(a - 3*b)*Log[1 +
 Sin[c + d*x]])/(16*(a - b)^3*d) - (a^2*b^3*Log[a + b*Sin[c + d*x]])/((a^2
- b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d) + (a*
Sec[c + d*x]^2*(4*a*b - (a^2 + 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^2}{b^2(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\,\text{Subst}\left(\int \frac{x^2}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{b\,\text{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2-b^2}+\frac{3ab^2x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{a\sec^2(c+dx)\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \cdots$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{a\sec^2(c+dx)\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \cdots$$

$$= \frac{a(a+3b)\log(1-\sin(c+dx))}{16(a+b)^3 d} - \frac{a(a-3b)\log(1+\sin(c+dx))}{16(a-b)^3 d} - \frac{a^2b^3\log(a+b\sin(c\cdots}{\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 1.22149, size = 163, normalized size = 0.92

$$\frac{-\frac{16a^2b^3\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{a-b}{(a+b)^2(\sin(c+dx)-1)}+\frac{a+b}{(a-b)^2(\sin(c+dx)+1)}+\frac{1}{(a+b)(\sin(c+dx)-1)^2}-\frac{1}{(a-b)(\sin(c+dx)+1)^2}+\frac{a(a+3b)\log(1-\sin(c+dx))}{(a+b)^3}\cdots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((a*(a + 3*b)*Log[1 - Sin[c + d*x]])/(a + b)^3 - (a*(a - 3*b)*Log[1 + Sin[c
 + d*x]])/(a - b)^3 - (16*a^2*b^3*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a +
b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (a - b)/((a + b)^2*(-1 + Sin[c
+ d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (a + b)/((a - b)^2*(1 + Sin[c
 + d*x]))))/(16*d)
```

Maple [A] time = 0.079, size = 262, normalized size = 1.5

$$-\frac{a^2b^3\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3}+\frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2}+\frac{a}{16d(a+b)^2(\sin(dx+c)-1)}-\frac{b}{16d(a+b)^2(\sin(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a^2*b^3/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)
-1)^2+1/16/d/(a+b)^2/(sin(d*x+c)-1)*a-1/16/d/(a+b)^2/(sin(d*x+c)-1)*b+1/16/
d/(a+b)^3*ln(sin(d*x+c)-1)*a^2+3/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-1/2/d/(8
*a-8*b)/(1+sin(d*x+c))^2+1/16/d/(a-b)^2/(1+sin(d*x+c))*a+1/16/d/(a-b)^2/(1+
sin(d*x+c))*b-1/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2+3/16/d/(a-b)^3*ln(1+sin(d
*x+c))*a*b
```

Maxima [A] time = 1.00567, size = 358, normalized size = 2.01

$$-\frac{\frac{16a^2b^3\log(b\sin(dx+c)+a)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(a^2-3ab)\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}-\frac{(a^2+3ab)\log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3}+\frac{2\left(4a^2b\sin(dx+c)^2-(a^3+3ab^2)\sin(dx+c)^3-2a^2b-2b^3-(a^3\right.}{(a^4-2a^2b^2+b^4)\sin(dx+c)^4+a^4-2a^2b^2+b^4-2(a^4-2a^2b^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^2*b^3*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6) + (a^2 - 3*a*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) -
(a^2 + 3*a*b)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*
a^2*b*sin(d*x + c)^2 - (a^3 + 3*a*b^2)*sin(d*x + c)^3 - 2*a^2*b - 2*b^3 - (
a^3 - 5*a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4
- 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A] time = 2.2791, size = 583, normalized size = 3.28

$$-\frac{16a^2b^3\cos(dx+c)^4\log(b\sin(dx+c)+a)+\left(a^5-6a^3b^2-8a^2b^3-3ab^4\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-\left(a^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^2*b^3*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + (a^5 - 6*a^3*b^2
 - 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^5 - 6*a^3
*b^2 + 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^4*b
 - 8*a^2*b^3 + 4*b^5 - 8*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 2*(2*a^5 - 4*a^
3*b^2 + 2*a*b^4 - (a^5 + 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25186, size = 439, normalized size = 2.47

$$-\frac{\frac{16\,a^2b^4\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}+\frac{\left(a^2-3\,ab\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(a^2+3\,ab\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,a^2b^3\sin(dx+c)^4-a^5\sin(dx+c)^3-2\,a^3b^2\sin(dx+c)^3+3\,ab\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^2*b^4*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b
^5 - b^7) + (a^2 - 3*a*b)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (a^2 + 3*a*b)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3) + 2*(6*a^2*b^3*sin(d*x + c)^4 - a^5*sin(d*x + c)^3 - 2*a^3*b^2*si
n(d*x + c)^3 + 3*a*b^4*sin(d*x + c)^3 + 4*a^4*b*sin(d*x + c)^2 - 16*a^2*b^3
*sin(d*x + c)^2 - a^5*sin(d*x + c) + 6*a^3*b^2*sin(d*x + c) - 5*a*b^4*sin(d
*x + c) - 2*a^4*b + 6*a^2*b^3 + 2*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(sin(d*x + c)^2 - 1)^2))/d
```

3.1366 $\int \frac{\sec^4(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{ab^4\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\left(a^2-b^2\right)}-\frac{\sec^2(c+dx)\left(4ab^2-b\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}-\frac{b(a+3$$

```
[Out] -(b*(a + 3*b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((a - 3*b)*b*Log[1
+ Sin[c + d*x]])/(16*(a - b)^3*d) + (a*b^4*Log[a + b*Sin[c + d*x]])/((a^2 -
 b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Sec
[c + d*x]^2*(4*a*b^2 - b*(a^2 + 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.24272, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 12, 823, 801}

$$\frac{ab^4\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\left(a^2-b^2\right)}-\frac{\sec^2(c+dx)\left(4ab^2-b\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}-\frac{b(a+3$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*(a + 3*b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((a - 3*b)*b*Log[1
+ Sin[c + d*x]])/(16*(a - b)^3*d) + (a*b^4*Log[a + b*Sin[c + d*x]])/((a^2 -
 b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Sec
[c + d*x]^2*(4*a*b^2 - b*(a^2 + 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{x}{b(a+x)\left(b^2-x^2\right)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\,\mathrm{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{b^2\,\mathrm{Subst}\left(\int \frac{-ab^2+3b^2x}{(a+x)\left(b^2-x^2\right)^2}\,dx,x,b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4ab^2-b\left(a^2+3b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} + \frac{\mathrm{Sub}}{}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4ab^2-b\left(a^2+3b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} + \frac{\mathrm{Sub}}{}$$

$$= -\frac{b(a+3b)\log(1-\sin(c+dx))}{16(a+b)^3 d} + \frac{(a-3b)b\log(1+\sin(c+dx))}{16(a-b)^3 d} + \frac{ab^4\log(a+b\sin(c}{\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 0.918303, size = 244, normalized size = 1.38

$$\frac{\frac{16ab^4\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^3} + \frac{a+3b}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{a-3b}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} + \cdots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*b*(a + 3*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^3 + (2*(
a - 3*b)*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^3 + (16*a*b^4*
Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^3 + 1/((a + b)*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^4) + (a + 3*b)/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^2) + 1/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + (a - 3*b)/((
a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(16*d)
```

Maple [A] time = 0.072, size = 259, normalized size = 1.5

$$\frac{ab^4\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} - \frac{a}{16d(a+b)^2(\sin(dx+c)-1)} - \frac{3b}{16d(a+b)^2(\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a*b^4/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-1)
^2-1/16/d/(a+b)^2/(sin(d*x+c)-1)*a-3/16/d/(a+b)^2/(sin(d*x+c)-1)*b-1/16/d/(
a+b)^3*ln(sin(d*x+c)-1)*a*b-3/16/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2+1/2/d/(8*a-
8*b)/(1+sin(d*x+c))^2+1/16/d/(a-b)^2/(1+sin(d*x+c))*a-3/16/d/(a-b)^2/(1+sin
(d*x+c))*b+1/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-3/16/d/(a-b)^3*ln(1+sin(d*x+
c))*b^2
```

Maxima [A] time = 1.0192, size = 360, normalized size = 2.03

$$\frac{\frac{16\,ab^4\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{\left(ab-3\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(ab+3\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,ab^2\sin(dx+c)^2-\left(a^2b+3\,b^3\right)\sin(dx+c)^3+2\,a^3-6\,ab^2-\left(a^2b-\right.\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\,a^2b^2+b\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a*b^4*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
+ (a*b - 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a*
b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a*b
^2*sin(d*x + c)^2 - (a^2*b + 3*b^3)*sin(d*x + c)^3 + 2*a^3 - 6*a*b^2 - (a^2
*b - 5*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A] time = 2.20101, size = 574, normalized size = 3.24

$$16\,ab^4\cos(dx+c)^4\log(b\sin(dx+c)+a)+\left(a^4b-6\,a^2b^3-8\,ab^4-3\,b^5\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-\left(a^4b-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*b^4*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + (a^4*b - 6*a^2*b^3
- 8*a*b^4 - 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^4*b - 6*a^2*b^
3 + 8*a*b^4 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^3*
b^2 + 4*a*b^4 - 8*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b - 4*a^2*b^3
 + 2*b^5 - (a^4*b + 2*a^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21719, size = 436, normalized size = 2.46

$$\frac{\frac{16\,ab^5\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}+\frac{(ab-3\,b^2)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(ab+3\,b^2)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,ab^4\sin(dx+c)^4-a^4b\sin(dx+c)^3-2\,a^2b^3\sin(dx+c)^3+3\,b^5\sin\cdots\right.}{\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a*b^5*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7) + (a*b - 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2
- b^3) - (a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2
+ b^3) + 2*(6*a*b^4*sin(d*x + c)^4 - a^4*b*sin(d*x + c)^3 - 2*a^2*b^3*sin(d
*x + c)^3 + 3*b^5*sin(d*x + c)^3 + 4*a^3*b^2*sin(d*x + c)^2 - 16*a*b^4*sin(
d*x + c)^2 - a^4*b*sin(d*x + c) + 6*a^2*b^3*sin(d*x + c) - 5*b^5*sin(d*x +
c) + 2*a^5 - 8*a^3*b^2 + 12*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(si
n(d*x + c)^2 - 1)^2))/d
```

3.1367 $\int \frac{\csc(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=233

$$\frac{b^6\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)^3}-\frac{\left(8a^2+21ab+15b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(8a^2-21ab+15b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{}{16}$$

```
[Out] -((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + Log[S
in[c + d*x]]/(a*d) - ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*
(a - b)^3*d) + (b^6*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)^3*d) + 1/(16*(a
 + b)*d*(1 - Sin[c + d*x])^2) + (5*a + 7*b)/(16*(a + b)^2*d*(1 - Sin[c + d*
x])) + 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) + (5*a - 7*b)/(16*(a - b)^2*d*
(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.365385, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{b^6\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)^3}-\frac{\left(8a^2+21ab+15b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(8a^2-21ab+15b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + Log[S
in[c + d*x]]/(a*d) - ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*
(a - b)^3*d) + (b^6*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)^3*d) + 1/(16*(a
 + b)*d*(1 - Sin[c + d*x])^2) + (5*a + 7*b)/(16*(a + b)^2*d*(1 - Sin[c + d*
x])) + 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) + (5*a - 7*b)/(16*(a - b)^2*d*
(1 + Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b}{x(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^6\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^6\,\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)(b-x)^3} + \frac{5a+7b}{16b^5(a+b)^2(b-x)^2} + \frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)} + \frac{1}{ab^6x} + \frac{1}{a(a-b)^3(a+b)^3(a+x)} + \frac{}{8}\right.\right.}{d}$$
$$= -\frac{\left(8a^2+21ab+15b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{\log(\sin(c+dx))}{ad} - \frac{\left(8a^2-21ab+15b^2\right)\text{lo}}{16(a-b}$$

Mathematica [A] time = 2.81411, size = 220, normalized size = 0.94

$$\frac{b^6\left(-\frac{\left(8a^2+21ab+15b^2\right)\log(1-\sin(c+dx))}{b^6(a+b)^3} - \frac{\left(8a^2-21ab+15b^2\right)\log(\sin(c+dx)+1)}{b^6(a-b)^3} + \frac{-5a-7b}{b^6(a+b)^2(\sin(c+dx)-1)} + \frac{5a-7b}{b^6(a-b)^2(\sin(c+dx)+1)} + \frac{1}{b^6(a+b)(\sin(c+dx}\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^6*(-(((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(b^6*(a + b)^3))
+ (16*Log[Sin[c + d*x]])/(a*b^6) - ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sin[c
 + d*x]])/((a - b)^3*b^6) + (16*Log[a + b*Sin[c + d*x]])/(a*(a - b)^3*(a +
b)^3) + 1/(b^6*(a + b)*(-1 + Sin[c + d*x])^2) + (-5*a - 7*b)/(b^6*(a + b)^2
*(-1 + Sin[c + d*x])) + 1/((a - b)*b^6*(1 + Sin[c + d*x])^2) + (5*a - 7*b)/
((a - b)^2*b^6*(1 + Sin[c + d*x]))))/(16*d)
```

Maple [A] time = 0.098, size = 321, normalized size = 1.4

$$\frac{b^6\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3a} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} - \frac{5a}{16d(a+b)^2(\sin(dx+c)-1)} - \frac{7b}{16d(a+b)^2(\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^6/(a+b)^3/(a-b)^3/a*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-1)
^2-5/16/d/(a+b)^2/(sin(d*x+c)-1)*a-7/16/d/(a+b)^2/(sin(d*x+c)-1)*b-1/2/d/(a
+b)^3*ln(sin(d*x+c)-1)*a^2-21/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-15/16/d/(a+
b)^3*ln(sin(d*x+c)-1)*b^2+1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2+5/16/d/(a-b)^2/(
1+sin(d*x+c))*a-7/16/d/(a-b)^2/(1+sin(d*x+c))*b-1/2/d/(a-b)^3*ln(1+sin(d*x+
c))*a^2+21/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-15/16/d/(a-b)^3*ln(1+sin(d*x+c
))*b^2+ln(sin(d*x+c))/a/d
```

Maxima [A] time = 1.02392, size = 404, normalized size = 1.73

$$\frac{\frac{16b^6\log(b\sin(dx+c)+a)}{a^7-3a^5b^2+3a^3b^4-ab^6} - \frac{\left(8a^2-21ab+15b^2\right)\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3} - \frac{\left(8a^2+21ab+15b^2\right)\log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(\left(3a^2b-7b^3\right)\sin(dx+c)^3+6a^3-10ab^2-4\left(a^3-2\right.\right.}{\left(a^4-2a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2a^2b^2+}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*b^6*log(b*sin(d*x + c) + a)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
- (8*a^2 - 21*a*b + 15*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
- b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3
*a*b^2 + b^3) + 2*((3*a^2*b - 7*b^3)*sin(d*x + c)^3 + 6*a^3 - 10*a*b^2 - 4*
(a^3 - 2*a*b^2)*sin(d*x + c)^2 - (5*a^2*b - 9*b^3)*sin(d*x + c))/((a^4 - 2*
a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2
+ b^4)*sin(d*x + c)^2) + 16*log(sin(d*x + c))/a)/d
```

Fricas [A] time = 9.77066, size = 790, normalized size = 3.39

$$\frac{16\, b^{6} \cos\left(dx+c\right)^{4} \log\left(b \sin\left(dx+c\right)+a\right)+4\, a^{6}-8\, a^{4} b^{2}+4\, a^{2} b^{4}+16 \left(a^{6}-3\, a^{4} b^{2}+3\, a^{2} b^{4}-b^{6}\right) \cos\left(dx+c\right)^{4} \log\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^6*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^6 - 8*a^4*b^2 + 4
*a^2*b^4 + 16*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c)^4*log(-1/2*s
in(d*x + c)) - (8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15
*a*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^6 - 3*a^5*b - 24*a^4*b^
2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) +
1) + 8*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 - 2*(2*a^5*b - 4*a^3*b^
3 + 2*a*b^5 + (3*a^5*b - 10*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2)*sin(d*x + c)
)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28314, size = 528, normalized size = 2.27

$$\frac{\frac{16\, b^{7} \log\left(\left|b \sin\left(dx+c\right)+a\right|\right)}{a^{7} b-3\, a^{5} b^{3}+3\, a^{3} b^{5}-a b^{7}}-\frac{\left(8\, a^{2}-21\, a b+15\, b^{2}\right) \log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{3}-3\, a^{2} b+3\, a b^{2}-b^{3}}-\frac{\left(8\, a^{2}+21\, a b+15\, b^{2}\right) \log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{3}+3\, a^{2} b+3\, a b^{2}+b^{3}}+\frac{16\, \log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}+\frac{2 \left(6\, a^{5} \sin\left(dx+c\right)^{4} \right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*b^7*log(abs(b*sin(d*x + c) + a))/(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
a*b^7) - (8*a^2 - 21*a*b + 15*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*
b + 3*a*b^2 - b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(abs(sin(d*x + c) - 1))/(
a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 16*log(abs(sin(d*x + c)))/a + 2*(6*a^5*sin
(d*x + c)^4 - 18*a^3*b^2*sin(d*x + c)^4 + 18*a*b^4*sin(d*x + c)^4 + 3*a^4*b
*sin(d*x + c)^3 - 10*a^2*b^3*sin(d*x + c)^3 + 7*b^5*sin(d*x + c)^3 - 16*a^5
*sin(d*x + c)^2 + 48*a^3*b^2*sin(d*x + c)^2 - 44*a*b^4*sin(d*x + c)^2 - 5*a
^4*b*sin(d*x + c) + 14*a^2*b^3*sin(d*x + c) - 9*b^5*sin(d*x + c) + 12*a^5 -
 34*a^3*b^2 + 28*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^
2 - 1)^2))/d
```

3.1368 $\int \frac{\csc^2(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=250

$$-\frac{b^7\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)^3}-\frac{\left(15a^2+37ab+24b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(15a^2-37ab+24b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}$$

```
[Out] -(Csc[c + d*x]/(a*d)) - ((15*a^2 + 37*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/
(16*(a + b)^3*d) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((15*a^2 - 37*a*b + 24*b
^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (b^7*Log[a + b*Sin[c + d*x]])
/(a^2*(a^2 - b^2)^3*d) + 1/(16*(a + b)*d*(1 - Sin[c + d*x])^2) + (7*a + 9*b
)/(16*(a + b)^2*d*(1 - Sin[c + d*x])) - 1/(16*(a - b)*d*(1 + Sin[c + d*x])^
2) - (7*a - 9*b)/(16*(a - b)^2*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.404545, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$-\frac{b^7\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)^3}-\frac{\left(15a^2+37ab+24b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(15a^2-37ab+24b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(Csc[c + d*x]/(a*d)) - ((15*a^2 + 37*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/
(16*(a + b)^3*d) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((15*a^2 - 37*a*b + 24*b
^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (b^7*Log[a + b*Sin[c + d*x]])
/(a^2*(a^2 - b^2)^3*d) + 1/(16*(a + b)*d*(1 - Sin[c + d*x])^2) + (7*a + 9*b
)/(16*(a + b)^2*d*(1 - Sin[c + d*x])) - 1/(16*(a - b)*d*(1 + Sin[c + d*x])^
2) - (7*a - 9*b)/(16*(a - b)^2*d*(1 + Sin[c + d*x]))
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5 \operatorname{Subst}\left(\int \frac{b^2}{x^2(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7 \operatorname{Subst}\left(\int \frac{1}{x^2(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7 \operatorname{Subst}\left(\int \left(\frac{1}{8b^5(a+b)(b-x)^3} + \frac{7a+9b}{16b^6(a+b)^2(b-x)^2} + \frac{15a^2+37ab+24b^2}{16b^7(a+b)^3(b-x)} + \frac{1}{ab^6x^2} - \frac{1}{a^2b^6x} - \frac{1}{a^2(a-b)^3(a+b)^3 \cdots}\right.\right.}{d}$$

$$= -\frac{\csc(c+dx)}{ad} - \frac{\left(15a^2+37ab+24b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{b\log(\sin(c+dx))}{a^2d} + \frac{(15a \cdots}{\cdots}$$

Mathematica [A] time = 6.20033, size = 257, normalized size = 1.03

$$\frac{b^7\left(-\frac{\left(15a^2+37ab+24b^2\right)\log(1-\sin(c+dx))}{16b^7(a+b)^3} - \frac{\log(\sin(c+dx))}{a^2b^6} + \frac{\left(15a^2-37ab+24b^2\right)\log(\sin(c+dx)+1)}{16b^7(a-b)^3} - \frac{\log(a+b\sin(c+dx))}{a^2(a-b)^3(a+b)^3} - \frac{7a-9b}{16b^6(a-b)^2(b\sin(c+dx)+b)} \cdots\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^7*(-(Csc[c + d*x]/(a*b^7)) - ((15*a^2 + 37*a*b + 24*b^2)*Log[1 - Sin[c +
 d*x]])/(16*b^7*(a + b)^3) - Log[Sin[c + d*x]]/(a^2*b^6) + ((15*a^2 - 37*a*
b + 24*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*b^7) - Log[a + b*Sin[c + d
*x]]/(a^2*(a - b)^3*(a + b)^3) + 1/(16*b^5*(a + b)*(b - b*Sin[c + d*x])^2)
+ (7*a + 9*b)/(16*b^6*(a + b)^2*(b - b*Sin[c + d*x])) - 1/(16*(a - b)*b^5*(
b + b*Sin[c + d*x])^2) - (7*a - 9*b)/(16*(a - b)^2*b^6*(b + b*Sin[c + d*x])
)))/d
```

Maple [A] time = 0.102, size = 340, normalized size = 1.4

$$-\frac{b^7\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3a^2} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} - \frac{7a}{16d(a+b)^2(\sin(dx+c)-1)} - \frac{9b}{16d(a+b)^2(\sin(dx+c \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*b^7/(a+b)^3/(a-b)^3/a^2*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)
-1)^2-7/16/d/(a+b)^2/(sin(d*x+c)-1)*a-9/16/d/(a+b)^2/(sin(d*x+c)-1)*b-15/16
/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-37/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-3/2/d/
(a+b)^3*ln(sin(d*x+c)-1)*b^2-1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2-7/16/d/(a-b)^
2/(1+sin(d*x+c))*a+9/16/d/(a-b)^2/(1+sin(d*x+c))*b+15/16/d/(a-b)^3*ln(1+sin
(d*x+c))*a^2-37/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b+3/2/d/(a-b)^3*ln(1+sin(d*
x+c))*b^2-1/d/a/sin(d*x+c)-b*ln(sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.02319, size = 487, normalized size = 1.95

$$-\frac{\frac{16b^7\log(b\sin(dx+c)+a)}{a^8-3a^6b^2+3a^4b^4-a^2b^6} - \frac{\left(15a^2-37ab+24b^2\right)\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3} + \frac{\left(15a^2+37ab+24b^2\right)\log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(\left(15a^4-27a^2b^2+8b^4\right)\sin(dx+c)^4+8a^4-1 \cdots\right)}{\left(a^5-2a^3b^2+ab^4 \cdots\right)}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^7*log(b*sin(d*x + c) + a)/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6) - (15*a^2 - 37*a*b + 24*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) + (15*a^2 + 37*a*b + 24*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) + 2*((15*a^4 - 27*a^2*b^2 + 8*b^4)*sin(d*x + c)^4 + 8*a^
4 - 16*a^2*b^2 + 8*b^4 - 4*(a^3*b - 2*a*b^3)*sin(d*x + c)^3 - (25*a^4 - 45*
a^2*b^2 + 16*b^4)*sin(d*x + c)^2 + 2*(3*a^3*b - 5*a*b^3)*sin(d*x + c))/((a^
5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c)^5 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x
 + c)^3 + (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c)) + 16*b*log(sin(d*x + c))/
a^2)/d
```

Fricas [A] time = 14.2377, size = 980, normalized size = 3.92

$$-\frac{16\,b^7\cos(dx+c)^4\log(b\sin(dx+c)+a)\sin(dx+c)-4\,a^7+8\,a^5b^2-4\,a^3b^4+16\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*b^7*cos(d*x + c)^4*log(b*sin(d*x + c) + a)*sin(d*x + c) - 4*a^7 +
 8*a^5*b^2 - 4*a^3*b^4 + 16*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x +
 c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) - (15*a^7 + 8*a^6*b - 42*a^5*b^2 -
 24*a^4*b^3 + 35*a^3*b^4 + 24*a^2*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
*sin(d*x + c) + (15*a^7 - 8*a^6*b - 42*a^5*b^2 + 24*a^4*b^3 + 35*a^3*b^4 -
24*a^2*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(15*a^7
- 42*a^5*b^2 + 35*a^3*b^4 - 8*a*b^6)*cos(d*x + c)^4 - 2*(5*a^7 - 14*a^5*b^2
 + 9*a^3*b^4)*cos(d*x + c)^2 + 4*(a^6*b - 2*a^4*b^3 + a^2*b^5 + 2*(a^6*b -
3*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*
a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^4*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29249, size = 564, normalized size = 2.26

$$-\frac{\frac{16\,b^8\log(|b\sin(dx+c)+a|)}{a^8b-3\,a^6b^3+3\,a^4b^5-a^2b^7}-\frac{\left(15\,a^2-37\,ab+24\,b^2\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(15\,a^2+37\,ab+24\,b^2\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{16\,b\log(|\sin(dx+c)|)}{a^2}+\frac{2\left(6\,a^4b\sin\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*b^8*log(abs(b*sin(d*x + c) + a))/(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 -
 a^2*b^7) - (15*a^2 - 37*a*b + 24*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*
a^2*b + 3*a*b^2 - b^3) + (15*a^2 + 37*a*b + 24*b^2)*log(abs(sin(d*x + c) -
1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 16*b*log(abs(sin(d*x + c)))/a^2 + 2*(
6*a^4*b*sin(d*x + c)^4 - 18*a^2*b^3*sin(d*x + c)^4 + 18*b^5*sin(d*x + c)^4
+ 7*a^5*sin(d*x + c)^3 - 18*a^3*b^2*sin(d*x + c)^3 + 11*a*b^4*sin(d*x + c)^
3 - 16*a^4*b*sin(d*x + c)^2 + 48*a^2*b^3*sin(d*x + c)^2 - 44*b^5*sin(d*x +
c)^2 - 9*a^5*sin(d*x + c) + 22*a^3*b^2*sin(d*x + c) - 13*a*b^4*sin(d*x + c)
 + 12*a^4*b - 34*a^2*b^3 + 28*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(si
n(d*x + c)^2 - 1)^2) - 16*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)))/d
```

3.1369 $\int \frac{\csc^3(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=274

$$\frac{b^8\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)^3}-\frac{\left(24a^2+57ab+35b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(3a^2+b^2\right)\log(\sin(c+dx))}{a^3d}-\frac{\left(24a^2-57ab+\right.}{}$$

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((24*a^2 + 57*a*b + 35*
b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((3*a^2 + b^2)*Log[Sin[c + d
*x]])/(a^3*d) - ((24*a^2 - 57*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*(a -
 b)^3*d) + (b^8*Log[a + b*Sin[c + d*x]])/(a^3*(a^2 - b^2)^3*d) + 1/(16*(a +
 b)*d*(1 - Sin[c + d*x])^2) + (9*a + 11*b)/(16*(a + b)^2*d*(1 - Sin[c + d*x
])) + 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) + (9*a - 11*b)/(16*(a - b)^2*d*
(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.473842, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{b^8\log(a+b\sin(c+dx))}{a^3d\left(a^2-b^2\right)^3}-\frac{\left(24a^2+57ab+35b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(3a^2+b^2\right)\log(\sin(c+dx))}{a^3d}-\frac{\left(24a^2-57ab+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((24*a^2 + 57*a*b + 35*
b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((3*a^2 + b^2)*Log[Sin[c + d
*x]])/(a^3*d) - ((24*a^2 - 57*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*(a -
 b)^3*d) + (b^8*Log[a + b*Sin[c + d*x]])/(a^3*(a^2 - b^2)^3*d) + 1/(16*(a +
 b)*d*(1 - Sin[c + d*x])^2) + (9*a + 11*b)/(16*(a + b)^2*d*(1 - Sin[c + d*x
])) + 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) + (9*a - 11*b)/(16*(a - b)^2*d*
(1 + Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^3}{x^3(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^8\,\text{Subst}\left(\int \frac{1}{x^3(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^8\,\text{Subst}\left(\int \left(\frac{1}{8b^6(a+b)(b-x)^3} + \frac{9a+11b}{16b^7(a+b)^2(b-x)^2} + \frac{24a^2+57ab+35b^2}{16b^8(a+b)^3(b-x)} + \frac{1}{ab^6x^3} - \frac{1}{a^2b^6x^2} + \frac{3a^2+b^2}{a^3b^8x} + \right.\right.}{}$$
$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{\left(24a^2+57ab+35b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{\left(3a^2+b^2\right.}{}$$

Mathematica [A] time = 6.24281, size = 281, normalized size = 1.03

$$\frac{b^8\left(\frac{\csc(c+dx)}{a^2b^7} - \frac{\left(24a^2+57ab+35b^2\right)\log(1-\sin(c+dx))}{16b^8(a+b)^3} + \frac{\left(3a^2+b^2\right)\log(\sin(c+dx))}{a^3b^8} - \frac{\left(24a^2-57ab+35b^2\right)\log(\sin(c+dx)+1)}{16b^8(a-b)^3} + \frac{\log(a+b\sin(c+dx))}{a^3(a-b)^3(a+b)^3} + \frac{}{16b^7(}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^8*(Csc[c + d*x]/(a^2*b^7) - Csc[c + d*x]^2/(2*a*b^8) - ((24*a^2 + 57*a*b
 + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*b^8*(a + b)^3) + ((3*a^2 + b^2)*Log[S
in[c + d*x]])/(a^3*b^8) - ((24*a^2 - 57*a*b + 35*b^2)*Log[1 + Sin[c + d*x]]
)/(16*(a - b)^3*b^8) + Log[a + b*Sin[c + d*x]]/(a^3*(a - b)^3*(a + b)^3) +
1/(16*b^6*(a + b)*(b - b*Sin[c + d*x])^2) + (9*a + 11*b)/(16*b^7*(a + b)^2*
(b - b*Sin[c + d*x])) + 1/(16*(a - b)*b^6*(b + b*Sin[c + d*x])^2) + (9*a -
11*b)/(16*(a - b)^2*b^7*(b + b*Sin[c + d*x]))))/d
```

Maple [A] time = 0.111, size = 371, normalized size = 1.4

$$\frac{b^8\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3a^3} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} - \frac{9a}{16d(a+b)^2(\sin(dx+c)-1)} - \frac{11b}{16d(a+b)^2(\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^8/(a+b)^3/(a-b)^3/a^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-
1)^2-9/16/d/(a+b)^2/(sin(d*x+c)-1)*a-11/16/d/(a+b)^2/(sin(d*x+c)-1)*b-3/2/d
/(a+b)^3*ln(sin(d*x+c)-1)*a^2-57/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-35/16/d/
(a+b)^3*ln(sin(d*x+c)-1)*b^2+1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2+9/16/d/(a-b)^
2/(1+sin(d*x+c))*a-11/16/d/(a-b)^2/(1+sin(d*x+c))*b-3/2/d/(a-b)^3*ln(1+sin(
d*x+c))*a^2+57/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-35/16/d/(a-b)^3*ln(1+sin(d
*x+c))*b^2-1/2/d/a/sin(d*x+c)^2+3*ln(sin(d*x+c))/a/d+b^2*ln(sin(d*x+c))/a^3
/d+1/d/a^2*b/sin(d*x+c)
```

Maxima [A] time = 1.07604, size = 570, normalized size = 2.08

$$\frac{16\,b^8\log(b\sin(dx+c)+a)}{a^9-3\,a^7b^2+3\,a^5b^4-a^3b^6}-\frac{\left(24\,a^2-57\,ab+35\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(24\,a^2+57\,ab+35\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(\left(15\,a^4b-27\,a^2b^3+8\,b^5\right)\sin(dx+c)^5-4\,a^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*b^8*log(b*sin(d*x + c) + a)/(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6
) - (24*a^2 - 57*a*b + 35*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (24*a^2 + 57*a*b + 35*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3) + 2*((15*a^4*b - 27*a^2*b^3 + 8*b^5)*sin(d*x + c)^5 - 4*a
^5 + 8*a^3*b^2 - 4*a*b^4 - 4*(3*a^5 - 5*a^3*b^2 + a*b^4)*sin(d*x + c)^4 - (
25*a^4*b - 45*a^2*b^3 + 16*b^5)*sin(d*x + c)^3 + 2*(9*a^5 - 15*a^3*b^2 + 4*
a*b^4)*sin(d*x + c)^2 + 8*(a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((a^6 - 2
*a^4*b^2 + a^2*b^4)*sin(d*x + c)^6 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x
+ c)^4 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^2) + 16*(3*a^2 + b^2)*log
(sin(d*x + c))/a^3)/d
```

Fricas [B] time = 21.0347, size = 1418, normalized size = 5.18

$$-\frac{4\,a^8-8\,a^6b^2+4\,a^4b^4-8\left(3\,a^8-8\,a^6b^2+6\,a^4b^4-a^2b^6\right)\cos\left(dx+c\right)^4+4\left(3\,a^8-8\,a^6b^2+5\,a^4b^4\right)\cos\left(dx+c\right)^2-16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^8 - 8*a^6*b^2 + 4*a^4*b^4 - 8*(3*a^8 - 8*a^6*b^2 + 6*a^4*b^4 - a
^2*b^6)*cos(d*x + c)^4 + 4*(3*a^8 - 8*a^6*b^2 + 5*a^4*b^4)*cos(d*x + c)^2 -
 16*(b^8*cos(d*x + c)^6 - b^8*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) - 16*
((3*a^8 - 8*a^6*b^2 + 6*a^4*b^4 - b^8)*cos(d*x + c)^6 - (3*a^8 - 8*a^6*b^2
+ 6*a^4*b^4 - b^8)*cos(d*x + c)^4)*log(-1/2*sin(d*x + c)) + ((24*a^8 + 15*a
^7*b - 64*a^6*b^2 - 42*a^5*b^3 + 48*a^4*b^4 + 35*a^3*b^5)*cos(d*x + c)^6 -
(24*a^8 + 15*a^7*b - 64*a^6*b^2 - 42*a^5*b^3 + 48*a^4*b^4 + 35*a^3*b^5)*cos
(d*x + c)^4)*log(sin(d*x + c) + 1) + ((24*a^8 - 15*a^7*b - 64*a^6*b^2 + 42*
a^5*b^3 + 48*a^4*b^4 - 35*a^3*b^5)*cos(d*x + c)^6 - (24*a^8 - 15*a^7*b - 64
*a^6*b^2 + 42*a^5*b^3 + 48*a^4*b^4 - 35*a^3*b^5)*cos(d*x + c)^4)*log(-sin(d
*x + c) + 1) - 2*(2*a^7*b - 4*a^5*b^3 + 2*a^3*b^5 - (15*a^7*b - 42*a^5*b^3
+ 35*a^3*b^5 - 8*a*b^7)*cos(d*x + c)^4 + (5*a^7*b - 14*a^5*b^3 + 9*a^3*b^5)
*cos(d*x + c)^2)*sin(d*x + c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*c
os(d*x + c)^6 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

[Out] Timed out

Giac [B] time = 1.28829, size = 795, normalized size = 2.9

$$\frac{16\,b^{9}\log(|b\sin(dx+c)+a|)}{a^{9}b-3\,a^{7}b^{3}+3\,a^{5}b^{5}-a^{3}b^{7}}-\frac{\left(24\,a^{2}-57\,ab+35\,b^{2}\right)\log(|\sin(dx+c)+1|)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{\left(24\,a^{2}+57\,ab+35\,b^{2}\right)\log(|\sin(dx+c)-1|)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{16\left(3\,a^{2}+b^{2}\right)\log(|\sin(dx+c)|)}{a^{3}}+\frac{2\left(4\,b^{8}\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*b^9*log(abs(b*sin(d*x + c) + a))/(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 -
a^3*b^7) - (24*a^2 - 57*a*b + 35*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a
^2*b + 3*a*b^2 - b^3) - (24*a^2 + 57*a*b + 35*b^2)*log(abs(sin(d*x + c) - 1
))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 16*(3*a^2 + b^2)*log(abs(sin(d*x + c))
)/a^3 + 2*(4*b^8*sin(d*x + c)^6 + 15*a^7*b*sin(d*x + c)^5 - 42*a^5*b^3*sin(
d*x + c)^5 + 35*a^3*b^5*sin(d*x + c)^5 - 8*a*b^7*sin(d*x + c)^5 - 12*a^8*si
n(d*x + c)^4 + 32*a^6*b^2*sin(d*x + c)^4 - 24*a^4*b^4*sin(d*x + c)^4 + 4*a^
2*b^6*sin(d*x + c)^4 - 8*b^8*sin(d*x + c)^4 - 25*a^7*b*sin(d*x + c)^3 + 70*
a^5*b^3*sin(d*x + c)^3 - 61*a^3*b^5*sin(d*x + c)^3 + 16*a*b^7*sin(d*x + c)^
3 + 18*a^8*sin(d*x + c)^2 - 48*a^6*b^2*sin(d*x + c)^2 + 38*a^4*b^4*sin(d*x
+ c)^2 - 8*a^2*b^6*sin(d*x + c)^2 + 4*b^8*sin(d*x + c)^2 + 8*a^7*b*sin(d*x
+ c) - 24*a^5*b^3*sin(d*x + c) + 24*a^3*b^5*sin(d*x + c) - 8*a*b^7*sin(d*x
+ c) - 4*a^8 + 12*a^6*b^2 - 12*a^4*b^4 + 4*a^2*b^6)/((a^9 - 3*a^7*b^2 + 3*a
^5*b^4 - a^3*b^6)*(sin(d*x + c)^3 - sin(d*x + c))^2))/d
```

3.1370 $\int \frac{\sqrt{g\cos(e+fx)}\sin^4(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=500

$$-\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}+\frac{a^4\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f\sqrt[4]{b^2-a^2}}-\frac{a^4\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f\sqrt[4]{b^2-a^2}}-\frac{2a^3E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e}}{b^4f\sqrt{\cos(e+fx)}}$$

```
[Out] (a^4*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(9/2)*(-a^2 + b^2)^(1/4)*f) - (a^4*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[
g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*(-a^2 + b^2)^(1/4)
*f) - (2*a^2*(g*Cos[e + f*x])^(3/2))/(3*b^3*f*g) - (2*(g*Cos[e + f*x])^(3/2
))/(3*b*f*g) + (2*(g*Cos[e + f*x])^(7/2))/(7*b*f*g^3) - (2*a^3*Sqrt[g*Cos[e
 + f*x]]*EllipticE[(e + f*x)/2, 2])/(b^4*f*Sqrt[Cos[e + f*x]]) - (4*a*Sqrt[
g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]) + (
a^5*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)
/2, 2])/(b^5*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^5*g*Sqrt[C
os[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5
*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (2*a*(g*Cos[e + f*x])^(3/
2)*Sin[e + f*x])/(5*b^2*f*g)
```

Rubi [A] time = 1.23742, antiderivative size = 500, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2898, 2640, 2639, 2565, 30, 2568, 14, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}+\frac{a^4\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f\sqrt[4]{b^2-a^2}}-\frac{a^4\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f\sqrt[4]{b^2-a^2}}-\frac{2a^3E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e}}{b^4f\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^4)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^4*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(9/2)*(-a^2 + b^2)^(1/4)*f) - (a^4*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[
g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*(-a^2 + b^2)^(1/4)
*f) - (2*a^2*(g*Cos[e + f*x])^(3/2))/(3*b^3*f*g) - (2*(g*Cos[e + f*x])^(3/2
))/(3*b*f*g) + (2*(g*Cos[e + f*x])^(7/2))/(7*b*f*g^3) - (2*a^3*Sqrt[g*Cos[e
 + f*x]]*EllipticE[(e + f*x)/2, 2])/(b^4*f*Sqrt[Cos[e + f*x]]) - (4*a*Sqrt[
g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]) + (
a^5*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)
/2, 2])/(b^5*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^5*g*Sqrt[C
os[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5
*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (2*a*(g*Cos[e + f*x])^(3/
2)*Sin[e + f*x])/(5*b^2*f*g)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\sin^4(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{a^3\sqrt{g\cos(e+fx)}}{b^4}+\frac{a^2\sqrt{g\cos(e+fx)}\sin(e+fx)}{b^3}-\frac{a\sqrt{g\cos(e+fx)}\sin^2(\right.$$

$$= -\frac{a^3\int\sqrt{g\cos(e+fx)}\,dx}{b^4}+\frac{a^4\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b^4}+\frac{a^2\int\sqrt{g\cos(e+fx)}\sin(e+f}{b^3}$$

$$= \frac{2a(g\cos(e+fx))^{3/2}\sin(e+fx)}{5b^2fg}-\frac{(2a)\int\sqrt{g\cos(e+fx)}\,dx}{5b^2}-\frac{a^2\,\mathrm{Subst}\left(\int\sqrt{x}\,d\right.}{b^3}$$

$$= -\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}-\frac{2a^3\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{\cos(e+fx)}}+\frac{2a(g\cos(e+fx)}{5b^2}$$

$$= -\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}-\frac{2(g\cos(e+fx))^{3/2}}{3bfg}+\frac{2(g\cos(e+fx))^{7/2}}{7bfg^3}-\frac{2a^3\sqrt{g\cos(}}{b^4}$$

$$= \frac{a^4\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}\sqrt[4]{-a^2+b^2}f}-\frac{a^4\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}\sqrt[4]{-a^2+b^2}f}-\frac{2a^2(g\cos(e+fx)}{3b^3fg}$$

Mathematica [C] time = 26.9448, size = 816, normalized size = 1.63

$$\frac{\sqrt{g\cos(e+fx)}\left(-\frac{(28a^2+19b^2)\cos(e+fx)}{42b^3}+\frac{\cos(3(e+fx))}{14b}+\frac{a\sin(2(e+fx))}{5b^2}\right)}{f}-\frac{a\sqrt{g\cos(e+fx)}\left(-\frac{(5a^2+2b^2)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},\ldots\right.\right.}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^4)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -(a*Sqrt[g*Cos[e + f*x]]*((-4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appe
llF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*C
os[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)
*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sq
rt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*
b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos
[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((5*a^2 + 2*b^2)*(a + b*Sqrt[1 - Cos[e
 + f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/
4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] -
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[
Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*
Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Cos[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2
)*(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(5*b^3*f*Sqrt[Cos[e + f*x]])
 + (Sqrt[g*Cos[e + f*x]]*(-((28*a^2 + 19*b^2)*Cos[e + f*x])/(42*b^3) + Cos[
3*(e + f*x)]/(14*b) + (a*Sin[2*(e + f*x)])/(5*b^2)))/f
```

Maple [C] time = 5.607, size = 1674, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 16/7/f/b*cos(1/2*f*x+1/2*e)^6*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-24/7/f/b*c
os(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/21/f/b*cos(1/2*f*x
+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/21/f/b*(2*cos(1/2*f*x+1/2*e)
^2*g-g)^(1/2)-4/3/f/b^3*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(
1/2)*a^2-4/3/f/b^3*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)*a^2+2/f/b^3*a^2*(g*(2
*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+1/2/f*g/b^3*a^4*sum((_R^6-_R^4*g-_R^2*g^2+g
^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2
*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=
RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^
2*g^4))+16/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*
g/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+
1/2*e)^5/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)-16/5/f*(g*
(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g/b^2/(-g*(2*sin(1
/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)^3/(g*(2*cos
```

```
(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+4/5/f*(g*(2*cos(1/2*f*x+1/2*
e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin
(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2)*cos(1/2*f*x+1/2*e)-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*a^3*g/b^4/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(
1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(cos(
1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^
2-1)^(1/2)-4/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*
a*g/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*
x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),
2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)-1/8/
f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a^3*g/b^6/(-g*(
2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(
2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/
2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alph
a^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha
^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)
^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(
g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2
*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)
/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+
12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2
*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))
*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2
*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2
+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*x+1/2*e)
^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sin\left(fx+e\right)^{4}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^4/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sin\left(fx+e\right)^4}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^4/(b*sin(f*x + e) + a), x)
```

3.1371 $\int \frac{\sqrt{g\cos(e+fx)}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=448

$$-\frac{a^3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt[4]{b^2-a^2}}+\frac{a^3\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt[4]{b^2-a^2}}+\frac{2a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^3f\sqrt{\cos(e+fx)}}-\frac{a^4g\sqrt{\cos(e+f}}{b^4f\left(b-\sqrt{\right.}}$$

```
[Out] -((a^3*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(b^(7/2)*(-a^2 + b^2)^(1/4)*f)) + (a^3*Sqrt[g]*ArcTanh[(Sqrt[b]*Sq
rt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*(-a^2 + b^2)^(1
/4)*f) + (2*a*(g*Cos[e + f*x])^(3/2))/(3*b^2*f*g) + (2*a^2*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(b^3*f*Sqrt[Cos[e + f*x]]) + (4*Sqrt[g*Cos[
e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*b*f*Sqrt[Cos[e + f*x]]) - (a^4*g*Sq
rt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/
(b^4*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^4*g*Sqrt[Cos[e + f
*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b + Sq
rt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*(g*Cos[e + f*x])^(3/2)*Sin[e +
 f*x])/(5*b*f*g)
```

Rubi [A] time = 0.960273, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {2898, 2640, 2639, 2565, 30, 2568, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a^3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt[4]{b^2-a^2}}+\frac{a^3\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt[4]{b^2-a^2}}+\frac{2a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^3f\sqrt{\cos(e+fx)}}-\frac{a^4g\sqrt{\cos(e+f}}{b^4f\left(b-\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```


```
[Out] -((a^3*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(b^(7/2)*(-a^2 + b^2)^(1/4)*f)) + (a^3*Sqrt[g]*ArcTanh[(Sqrt[b]*Sq
rt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*(-a^2 + b^2)^(1
/4)*f) + (2*a*(g*Cos[e + f*x])^(3/2))/(3*b^2*f*g) + (2*a^2*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(b^3*f*Sqrt[Cos[e + f*x]]) + (4*Sqrt[g*Cos[
e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*b*f*Sqrt[Cos[e + f*x]]) - (a^4*g*Sq
rt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/
(b^4*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^4*g*Sqrt[Cos[e + f
*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b + Sq
rt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*(g*Cos[e + f*x])^(3/2)*Sin[e +
 f*x])/(5*b*f*g)
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{a^2\sqrt{g\cos(e+fx)}}{b^3} - \frac{a\sqrt{g\cos(e+fx)}\sin(e+fx)}{b^2} + \frac{\sqrt{g\cos(e+fx)}\sin^2(e+\right.$$

$$= \frac{a^2\int\sqrt{g\cos(e+fx)}\,dx}{b^3} - \frac{a^3\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b^3} - \frac{a\int\sqrt{g\cos(e+fx)}\sin(e+fx)\,d}{b^2}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}\sin(e+fx)}{5bfg} + \frac{2\int\sqrt{g\cos(e+fx)}\,dx}{5b} + \frac{a\,\mathrm{Subst}\left(\int\sqrt{x}\,dx,x\right.}{b^2fg}$$

$$= \frac{2a(g\cos(e+fx))^{3/2}}{3b^2fg} + \frac{2a^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{\cos(e+fx)}} - \frac{2(g\cos(e+fx))^{3/2}}{5bfg}$$

$$= \frac{2a(g\cos(e+fx))^{3/2}}{3b^2fg} + \frac{2a^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{\cos(e+fx)}} + \frac{4\sqrt{g\cos(e+fx)}E\left(\right.}{5bf\sqrt{\cos(e}}$$

$$= -\frac{a^3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}\sqrt[4]{-a^2+b^2}f} + \frac{a^3\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}\sqrt[4]{-a^2+b^2}f} + \frac{2a(g\cos(e+fx}{3b^2fg}$$

Mathematica [C] time = 26.8593, size = 789, normalized size = 1.76

$$\frac{\sqrt{g\cos(e+fx)}\left(\frac{2a\cos(e+fx)}{3b^2} - \frac{\sin(2(e+fx))}{5b}\right)}{f} + \frac{\sqrt{g\cos(e+fx)}\left(-\frac{\left(5a^2+2b^2\right)\sin^2(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((-4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF
1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[
e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[
-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*C
os[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e
+ f*x]^2]*(a + b*Sin[e + f*x])) - ((5*a^2 + 2*b^2)*(a + b*Sqrt[1 - Cos[e +
f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*
(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*A
rcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqr
t[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos
[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[C
os[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(
1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(5*b^2*f*Sqrt[Cos[e + f*x]]) +
(Sqrt[g*Cos[e + f*x]]*((2*a*Cos[e + f*x])/(3*b^2) - Sin[2*(e + f*x)]/(5*b))
)/f
```

Maple [C] time = 6.549, size = 2363, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 4/3/f*a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/3/f*a
/b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-2/f*a/b^2*(g*(2*cos(1/2*f*x+1/2*e)^
2-1))^(1/2)-1/2/f*g*a^3/b^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5
*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^
2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^
2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+16/3/f*(g*(2
*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x
+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x
+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^5-16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-
1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+
1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*co
s(1/2*f*x+1/2*e)^5-8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^
(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*
f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3-4/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b^3/(-g*(2*sin(1/
2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/
2*f*x+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x
+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^2+4/3/f*(g*(2*cos(1/2*
f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2
-1))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/
2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(
1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^
2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE
(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1
/2*e)^2+1)^(1/2)+8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1
/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*
x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3+8/3/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*
```

```
f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*
f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+
1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*El
lipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/
2*f*x+1/2*e)^2+1)^(1/2)*a^2-4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/
2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/
2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2
+1)^(1/2)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b
/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e
)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2)
)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)-8/3/f*(g*(
2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*
x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*
x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)-1/4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b^5/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)*sum((2*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2-sin(1/2*f*x+1/2*e)
^2*a^2-2*b^2*_alpha^2+a^2)/_alpha/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-
2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^
(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha
^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1
)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(
2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-
3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2
*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g
*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)
^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^
(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=Roo
tOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sin\left(fx+e\right)^3}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

3.1372 $\int \frac{\sqrt{g\cos(e+fx)}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=369

$$\frac{a^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}-\frac{a^2\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}+\frac{a^3g\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^3g\sqrt{\cos}}{b^3f}$$

```
[Out] (a^2*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(5/2)*(-a^2 + b^2)^(1/4)*f) - (a^2*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[
g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(1/4)
*f) - (2*(g*Cos[e + f*x])^(3/2))/(3*b*f*g) - (2*a*Sqrt[g*Cos[e + f*x]]*Elli
pticE[(e + f*x)/2, 2])/(b^2*f*Sqrt[Cos[e + f*x]]) + (a^3*g*Sqrt[Cos[e + f*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(b - Sqrt
[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^3*g*Sqrt[Cos[e + f*x]]*EllipticP
i[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(b + Sqrt[-a^2 + b^2]
)*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 0.872078, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2898, 2640, 2639, 2565, 30, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{a^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}-\frac{a^2\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}+\frac{a^3g\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^3g\sqrt{\cos}}{b^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^2*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(5/2)*(-a^2 + b^2)^(1/4)*f) - (a^2*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[
g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(1/4)
*f) - (2*(g*Cos[e + f*x])^(3/2))/(3*b*f*g) - (2*a*Sqrt[g*Cos[e + f*x]]*Elli
pticE[(e + f*x)/2, 2])/(b^2*f*Sqrt[Cos[e + f*x]]) + (a^3*g*Sqrt[Cos[e + f*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(b - Sqrt
[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^3*g*Sqrt[Cos[e + f*x]]*EllipticP
i[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(b + Sqrt[-a^2 + b^2]
)*f*Sqrt[g*Cos[e + f*x]])
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{a\sqrt{g\cos(e+fx)}}{b^2}+\frac{\sqrt{g\cos(e+fx)}\sin(e+fx)}{b}+\frac{a^2\sqrt{g\cos(e+fx)}}{b^2(a+b\sin(e+fx))}\right)dx$$

$$= -\frac{a\int\sqrt{g\cos(e+fx)}\,dx}{b^2}+\frac{a^2\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b^2}+\frac{\int\sqrt{g\cos(e+fx)}\sin(e+fx)\,dx}{b}$$

$$= -\frac{\text{Subst}\left(\int\sqrt{x}\,dx,x,g\cos(e+fx)\right)}{bfg}-\frac{\left(a^3g\right)\int\frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2b^3}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3bfg}-\frac{2a\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}}+\frac{\left(2a^2g\right)\text{Subst}\left(\int\frac{}{(a^2}\right.}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3bfg}-\frac{2a\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}}+\frac{a^3g\sqrt{\cos(e+fx)}\Pi(}{b^3\left(b-\sqrt{-a^2+}\right.}$$

$$= \frac{a^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}\sqrt[4]{-a^2+b^2}f}-\frac{a^2\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}\sqrt[4]{-a^2+b^2}f}-\frac{2(g\cos(e+fx))^{3/}}{3bfg}$$

Mathematica [C] time = 17.6111, size = 372, normalized size = 1.01

$$\frac{\sqrt{g\cos(e+fx)}\left(-\frac{a\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(-\log\left(-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}\right.\right.\right.}{(a^2-b^2}\right.}{12b^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*(-8*b^(3/2)*Cos[e + f*x]^(3/2) - (a*(8*b^(5/2)*Appell
F1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Co
s[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f
*x]]))*(a + b*Sqrt[Sin[e + f*x]^2]))/((a^2 - b^2)*(a + b*Sin[e + f*x]))))/(
12*b^(5/2)*f*Sqrt[Cos[e + f*x]])
```

Maple [C] time = 5.386, size = 924, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -4/3/f/b*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/3/f/b*(2
*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+2/f/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
+1/2/f*g/b*a^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3
*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-
cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*
a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-2/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-si
n(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1)
)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*Elli
pticE(cos(1/2*f*x+1/2*e),2^(1/2))-1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1
/2*f*x+1/2*e)^2)^(1/2)*g*a/b^4/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^
2-1))^(1/2)*sum(1/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1
/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2
*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin
(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1
/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2
)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(
1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2
^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*
e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*
e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^
2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sin\left(fx+e\right)^2}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

3.1373 $\int \frac{\sqrt{g\cos(e+fx)}\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=341

$$-\frac{a\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt[4]{b^2-a^2}}+\frac{a\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt[4]{b^2-a^2}}-\frac{a^2g\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^2g\sqrt{\cos(e+}}{b^2f\left(\sqrt{b\right.}}$$

```
[Out] -((a*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(3/2)*(-a^2 + b^2)^(1/4)*f)) + (a*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g
*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(1/4)*
f) + (2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(b*f*Sqrt[Cos[e + f
*x]]) - (a^2*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(e + f*x)/2, 2])/(b^2*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^2
*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2,
 2])/(b^2*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 0.73471, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt[4]{b^2-a^2}}+\frac{a\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt[4]{b^2-a^2}}-\frac{a^2g\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^2g\sqrt{\cos(e+}}{b^2f\left(\sqrt{b\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(3/2)*(-a^2 + b^2)^(1/4)*f)) + (a*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g
*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(1/4)*
f) + (2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(b*f*Sqrt[Cos[e + f
*x]]) - (a^2*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(e + f*x)/2, 2])/(b^2*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^2
*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2,
 2])/(b^2*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\sin(e+fx)}{a+b\sin(e+fx)}\,dx = \frac{\int \sqrt{g\cos(e+fx)}\,dx}{b} - \frac{a\int \frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b}$$

$$= \frac{(a^2g)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2b^2} - \frac{(a^2g)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}+b\cos(e+fx)\right)}\,dx}{2b^2}$$

$$= \frac{2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{\cos(e+fx)}} - \frac{(2ag)\,\mathrm{Subst}\left(\int \frac{x^2}{(a^2-b^2)g^2+b^2x^4}\,dx, x, \sqrt{g\cos(e+fx)}\right.}{f}$$

$$= \frac{2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{\cos(e+fx)}} - \frac{a^2g\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2\left(b-\sqrt{-a^2+b^2}\right)f\sqrt{g\cos(e+fx)}} - \frac{a}{}$$

$$= -\frac{a\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\sqrt[4]{-a^2+b^2}f} + \frac{a\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\sqrt[4]{-a^2+b^2}f} + \frac{2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(\right.}{bf\sqrt{\cos(e+f}}$$

Mathematica [C] time = 20.0644, size = 351, normalized size = 1.03

$$-\frac{\sqrt{g\cos(e+fx)}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -(Sqrt[g*Cos[e + f*x]]*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2
 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2
)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1
/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e +
 f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^2])
)/(12*b^(3/2)*(-a^2 + b^2)*f*Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x]))
```

Maple [C] time = 6.25, size = 884, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/f*g*a*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2
*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(
1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*
g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^
2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*
x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*
EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(
```

```
1/2*f*x+1/2*e)^2+1)^(1/2)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*g/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/s
in(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticF(cos(1/2*f*
x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^
(1/2)+1/4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g/b^3
/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum((-2*sin(1/2*f*
x+1/2*e)^2*_alpha^2*b^2+sin(1/2*f*x+1/2*e)^2*a^2+2*b^2*_alpha^2-a^2)/_alpha
/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1
/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2
*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a
^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2
*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*E
llipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootO
f(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

3.1374 $\int \frac{\sqrt{g\cos(e+fx)}\csc(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=355

$$-\frac{\sqrt{b}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}+\frac{\sqrt{b}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}-\frac{g\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{g\sqrt{\cos(}}{f(}$$

```
[Out] (Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) - (Sqrt[b]*Sqrt[g]*Arc
Tan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2
+ b^2)^(1/4)*f) - (Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) + (
Sqrt[b]*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*
Sqrt[g])])/(a*(-a^2 + b^2)^(1/4)*f) - (g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b
)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((b - Sqrt[-a^2 + b^2])*f*Sqrt[g
*Cos[e + f*x]]) - (g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/((b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 0.832532, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2898, 2565, 329, 298, 203, 206, 2701, 2807, 2805, 205, 208}

$$-\frac{\sqrt{b}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}+\frac{\sqrt{b}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}-\frac{g\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{g\sqrt{\cos(}}{f(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```


```
[Out] (Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) - (Sqrt[b]*Sqrt[g]*Arc
Tan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2
+ b^2)^(1/4)*f) - (Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) + (
Sqrt[b]*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*
Sqrt[g])])/(a*(-a^2 + b^2)^(1/4)*f) - (g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b
)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((b - Sqrt[-a^2 + b^2])*f*Sqrt[g
*Cos[e + f*x]]) - (g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/((b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\csc(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(\frac{\sqrt{g\cos(e+fx)}\csc(e+fx)}{a} - \frac{b\sqrt{g\cos(e+fx)}}{a(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int\sqrt{g\cos(e+fx)}\csc(e+fx)\,dx}{a} - \frac{b\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx,x,g\cos(e+fx)\right)}{afg} + \frac{1}{2}g\int\frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos\right.}$$

$$= -\frac{2\,\text{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{g^2}}\,dx,x,\sqrt{g\cos(e+fx)}\right)}{afg} - \frac{\left(2b^2g\right)\text{Subst}\left(\int\frac{x^2}{\left(a^2-b^2\right)g^2+b^2x^4}\,dx,x,\right.}{af}$$

$$= -\frac{g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e+fx)\middle|2\right)}{\left(b-\sqrt{-a^2+b^2}\right)f\sqrt{g\cos(e+fx)}} - \frac{g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{-a^2+b^2}};\frac{1}{2}(e+\right.}{\left(b+\sqrt{-a^2+b^2}\right)f\sqrt{g\cos(e+}}$$

$$= \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{\sqrt{b}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\sqrt[4]{-a^2+b^2}f} - \frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right.}{af}$$

Mathematica [C] time = 14.9523, size = 534, normalized size = 1.5

$$\frac{\csc(e+fx)\sqrt{g\cos(e+fx)}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8ab\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+3\left(-\sqrt{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]*(8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*(2*Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]
])/(a^2 - b^2)^(1/4)] - 2*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*a^2*ArcTan[Sqrt[Cos
[e + f*x]]] - 4*b^2*ArcTan[Sqrt[Cos[e + f*x]]] + 2*a^2*Log[1 - Sqrt[Cos[e +
 f*x]]] - 2*b^2*Log[1 - Sqrt[Cos[e + f*x]]] - 2*a^2*Log[1 + Sqrt[Cos[e + f*
x]]] + 2*b^2*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4
)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]
] + b*Cos[e + f*x]] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]
 + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))
*(a + b*Sqrt[Sin[e + f*x]^2]))/(12*a*(a^2 - b^2)*f*Sqrt[Cos[e + f*x]]*(b +
a*Csc[e + f*x]))
```

Maple [A] time = 2.449, size = 188, normalized size = 0.5

$$-\frac{1}{2af}\sqrt{g}\ln\left(2\frac{\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx+e/2\right)\right)^2g+g}+2g\cos\left(1/2\,fx+e/2\right)-g}{-1+\cos\left(1/2\,fx+e/2\right)}\right)-\frac{1}{2af}\sqrt{g}\ln\left(2\frac{\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx\right.\right.}}{\mathrm{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/a/f*g^(1/2)*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*
e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))-1/2/a/f*g^(1/2)*ln(2/(cos(1/2*f*
x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/
2*e)-g))-1/a/(-g)^(1/2)/f*g*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)-g))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(e+fx\right)}\csc\left(e+fx\right)}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))*csc(e + f*x)/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

3.1375 $\int \frac{\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=433

$$\frac{b^{3/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt[4]{b^2-a^2}}-\frac{b^{3/2}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt[4]{b^2-a^2}}+\frac{bg\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{bg\sqrt{\cos(e}{af\left(\sqrt{\right.}}$$

```
[Out] -((b*Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f)) + (b^(3/2)*Sqrt
[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a
^2*(-a^2 + b^2)^(1/4)*f) + (b*Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]
)/(a^2*f) - (b^(3/2)*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(1/4)*f) - ((g*Cos[e + f*x])^(3/2
)*Csc[e + f*x])/(a*f*g) - (Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/
(a*f*Sqrt[Cos[e + f*x]]) + (b*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sq
rt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e
+ f*x]]) + (b*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
 (e + f*x)/2, 2])/(a*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 0.938672, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2898, 2565, 329, 298, 203, 206, 2570, 2640, 2639, 2701, 2807, 2805, 205, 208}

$$\frac{b^{3/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt[4]{b^2-a^2}}-\frac{b^{3/2}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt[4]{b^2-a^2}}+\frac{bg\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{bg\sqrt{\cos(e}{af\left(\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```


```
[Out] -((b*Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f)) + (b^(3/2)*Sqrt
[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a
^2*(-a^2 + b^2)^(1/4)*f) + (b*Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]
)/(a^2*f) - (b^(3/2)*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(1/4)*f) - ((g*Cos[e + f*x])^(3/2
)*Csc[e + f*x])/(a*f*g) - (Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/
(a*f*Sqrt[Cos[e + f*x]]) + (b*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sq
rt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e
+ f*x]]) + (b*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
 (e + f*x)/2, 2])/(a*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{b\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2}+\frac{\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{a}+\frac{b^2\sqrt{g\cos(e+\cdots}}{a^2(a+b\sin(e\cdots}\right.$$

$$= \frac{\int\sqrt{g\cos(e+fx)}\csc^2(e+fx)\,dx}{a}-\frac{b\int\sqrt{g\cos(e+fx)}\csc(e+fx)\,dx}{a^2}+\frac{b^2\int\frac{\sqrt{g\cos\cdots}}{a+b\sin\cdots}}{a^{\cdots}}$$

$$= -\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{afg}-\frac{\int\sqrt{g\cos(e+fx)}\,dx}{2a}+\frac{b\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx,x,g\cos\cdots\right.}{a^2fg}$$

$$= -\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{afg}+\frac{(2b)\,\mathrm{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{g^2}}\,dx,x,\sqrt{g\cos(e+fx)}\right)}{a^2fg}+\frac{(2b^3\cdots}{\cdots}$$

$$= -\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{afg}-\frac{\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{\cos(e+fx)}}+\frac{bg\sqrt{\cos(e+fx\cdots}}{a\left(b-\sqrt{-a\cdots}\right.}$$

$$= -\frac{b\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{b^{3/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2\sqrt[4]{-a^2+b^2}f}+\frac{b\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+\cdots}}{\sqrt{g}}\right.}{a^2f}$$

Mathematica [C] time = 27.0154, size = 1550, normalized size = 3.58

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[g*Cos[e + f*x]]*Cot[e + f*x])/(a*f)) + (Sqrt[g*Cos[e + f*x]]*((4*a*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x
]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2))
 + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 +
b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2
)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[C
os[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(
-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 +
 b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) + (5*b*(-1 +
 Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a
^2 - b^2)^(1/4)] - 6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*
Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt
[Cos[e + f*x]]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e +
f*x]]] - 6*b^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x
]]] + 6*b^2*Log[1 + Sqrt[Cos[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/
4)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x
]] + b*Cos[e + f*x]] + 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b
^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]
]))/(12*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - ((-1 + C
os[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*
x]*(-42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a
^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)
] + 84*b^(3/2)*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*Appell
F1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos
[e + f*x]^(3/2) + 48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b
^2)*Log[1 - Sqrt[Cos[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[
e + f*x]]] + 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] -
 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*Sqrt[b]
*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e +
f*x]))))/(4*a*f*Sqrt[Cos[e + f*x]])
```

Maple [C] time = 10.674, size = 1266, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/8*(4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin
(1/2*f*x+1/2*e)^2*g)^(3/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)*b*((-g)^(1/2
)*sum(1/_R/(_R^6*b^2-3*_R^4*b^2*g+8*_R^2*a^2*g^2-5*_R^2*b^2*g^2-b^2*g^3)*ln
((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R)
*(_R^6-_R^4*g-_R^2*g^2+g^3),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*
b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))*b^2*g+g^(1/2)*ln(2/(-1+cos(1/2*f*x+1
/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-
g))*(-g)^(1/2)+g^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x
+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)+2*g*ln(2/cos(1/2
*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g)))+(-8*(-g)^(
1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*(sin(1/2*f*x+
1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*
```

```
e),2^(1/2))*a*g-(-g)^(1/2)*g^3*sin(1/2*f*x+1/2*e)^4*(2*sin(1/2*f*x+1/2*e)^2
-1)^2/a*sum(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^
2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2
))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(
1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2
*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f
*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)
*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_
alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)
^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*
(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2
)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(
4*_Z^4*b^2-4*_Z^2*b^2+a^2)))*cos(1/2*f*x+1/2*e)-16*(-g)^(1/2)*(-2*sin(1/2*f
*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*a*g*sin(1/2*f*x+1/2*e)^4+16*(-g
)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*a*g*sin(1/
2*f*x+1/2*e)^2-4*(-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2
*g)^(3/2)*a*g)/a^2/(-g)^(1/2)/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g
+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/sin(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^2*
g+g)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(e+fx\right)}\csc^{2}\left(e+fx\right)}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))*csc(e + f*x)**2/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\csc\left(fx+e\right)^2}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

3.1376 $\int \frac{\sqrt{g\cos(e+fx)}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=544

$$-\frac{b^{5/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt[4]{b^2-a^2}}+\frac{b^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{5/2}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt[4]{b^2-a^2}}-\frac{b^2\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}$$

```
[Out] (Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) + (b^2*Sqrt[g]*ArcTa
n[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) - (b^(5/2)*Sqrt[g]*ArcTan[(Sqrt[b]
*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 + b^2)^(1/
4)*f) - (Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*Sqrt
[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(5/2)*Sqrt[g]*ArcTa
nh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2
 + b^2)^(1/4)*f) + (b*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a^2*f*g) - ((g*
Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(2*a*f*g) + (b*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(a^2*f*Sqrt[Cos[e + f*x]]) - (b^2*g*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(b - Sqr
t[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (b^2*g*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(b + Sqrt[-a^2 + b^2
])*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.05726, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2898, 2565, 329, 298, 203, 206, 2570, 2640, 2639, 290, 2701, 2807, 2805, 205, 208}

$$-\frac{b^{5/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt[4]{b^2-a^2}}+\frac{b^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{5/2}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt[4]{b^2-a^2}}-\frac{b^2\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```


```
[Out] (Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) + (b^2*Sqrt[g]*ArcTa
n[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) - (b^(5/2)*Sqrt[g]*ArcTan[(Sqrt[b]
*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 + b^2)^(1/
4)*f) - (Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*Sqrt
[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(5/2)*Sqrt[g]*ArcTa
nh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2
 + b^2)^(1/4)*f) + (b*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a^2*f*g) - ((g*
Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(2*a*f*g) + (b*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(a^2*f*Sqrt[Cos[e + f*x]]) - (b^2*g*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(b - Sqr
t[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (b^2*g*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(b + Sqrt[-a^2 + b^2
])*f*Sqrt[g*Cos[e + f*x]])
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
```

```
g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\csc^3(e+fx)}{a+b\sin(e+fx)}dx = \int \left(\frac{b^2\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^3} - \frac{b\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{a^2} + \frac{\sqrt{g\cos(e+\right.$$

$$= \frac{\int \sqrt{g\cos(e+fx)}\csc^3(e+fx)\,dx}{a} - \frac{b\int \sqrt{g\cos(e+fx)}\csc^2(e+fx)\,dx}{a^2} + \frac{b^2\int \sqrt{}}{}$$

$$= \frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2fg} + \frac{b\int \sqrt{g\cos(e+fx)}\,dx}{2a^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x^2}{g^2}\right)^2}dx,\right.}{afg}$$

$$= \frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2fg} - \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2afg} - \frac{\text{Subst}\left(\int \frac{\sqrt{}}{1-\frac{x}{g}}\right.}{}$$

$$= \frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2fg} - \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2afg} + \frac{b\sqrt{g\cos(e+}}{a^2f\sqrt{}}$$

$$= \frac{b^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} - \frac{b^{5/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^3\sqrt[4]{-a^2+b^2}f} - \frac{b^2\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos}}{\sqrt{}}\right.}{a^3f}$$

$$= \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} + \frac{b^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} - \frac{b^{5/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right.}{a^3\sqrt[4]{-a^2+b^2}f}$$

Mathematica [C] time = 29.0228, size = 1582, normalized size = 2.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((b*Cot[e + f*x])/a^2 - (Cot[e + f*x]*Csc[e + f*x])/(
2*a)))/f - (Sqrt[g*Cos[e + f*x]]*((6*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*(
(a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1
+ I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I
)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] -
(1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] +
 Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x
]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f
*x]^2]*(b + a*Csc[e + f*x])) - ((-a^2 - 5*b^2)*(-1 + Cos[e + f*x]^2)*(a + b
*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4
)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b
*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)]*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 -
Sqrt[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqr
t[Cos[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] +
3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(
```

```
a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(
1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (Sqrt[b]*(-1 + Cos[e + f*x]^2)*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]
*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f
*x]])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTa
n[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*
(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1,
 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2)
 + 48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqr
t[Cos[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21
*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^
2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*(a^3 - a*b^2)*(1 - Cos
[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Sqrt[
Cos[e + f*x]])
```

Maple [A] time = 2.48, size = 307, normalized size = 0.6

$$-\frac{1}{16\,af}\sqrt{-2\,\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}-\frac{1}{8\,af}\sqrt{g}\ln\left(\left(-4\,g\cos\left(1/2\,fx+e/2\right)+2\,\sqrt{g}\sqrt{-2\,\left(\sin\left(1\right.\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/16/f/a/(cos(1/2*f*x+1/2*e)+1)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/8/f*
g^(1/2)/a*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+
g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))+1/16/f/a/(-1+cos(1/2*f*x+1/2*e))*(-2*
sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/8/f*g^(1/2)/a*ln((4*g*cos(1/2*f*x+1/2*e)+
2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))
+1/8/f/a/cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-1/4/f*g/a/
(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/
2*f*x+1/2*e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

3.1377 $\int \frac{(g\cos(e+fx))^{3/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=621

$$\frac{a^3g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}+\frac{a^3g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}-\frac{2a^4g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\sqrt{g\cos(e+fx)}}+$$

```
[Out] (a^3*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*f) + (a^3*(-a^2 + b^2)^(1/4)*g^(3/2)*Arc
Tanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)
*f) - (2*a^3*g*Sqrt[g*Cos[e + f*x]])/(b^4*f) + (2*a*(g*Cos[e + f*x])^(5/2))
/(5*b^2*f*g) - (2*a^4*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b^
5*f*Sqrt[g*Cos[e + f*x]]) + (2*a^2*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*
x)/2, 2])/(3*b^3*f*Sqrt[g*Cos[e + f*x]]) + (4*g^2*Sqrt[Cos[e + f*x]]*Ellipt
icF[(e + f*x)/2, 2])/(21*b*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2 - b^2)*g^2*S
qrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])
/(b^5*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2
- b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e +
 f*x)/2, 2])/(b^5*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
+ (2*a^2*g*Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(3*b^3*f) + (4*g*Sqrt[g*Cos[e
 + f*x]]*Sin[e + f*x])/(21*b*f) - (2*(g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(
7*b*f*g)
```

Rubi [A] time = 1.54072, antiderivative size = 621, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {2898, 2635, 2642, 2641, 2565, 30, 2568, 2695, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{a^3g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}+\frac{a^3g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}-\frac{2a^4g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\sqrt{g\cos(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^3*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*f) + (a^3*(-a^2 + b^2)^(1/4)*g^(3/2)*Arc
Tanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)
*f) - (2*a^3*g*Sqrt[g*Cos[e + f*x]])/(b^4*f) + (2*a*(g*Cos[e + f*x])^(5/2))
/(5*b^2*f*g) - (2*a^4*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b^
5*f*Sqrt[g*Cos[e + f*x]]) + (2*a^2*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*
x)/2, 2])/(3*b^3*f*Sqrt[g*Cos[e + f*x]]) + (4*g^2*Sqrt[Cos[e + f*x]]*Ellipt
icF[(e + f*x)/2, 2])/(21*b*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2 - b^2)*g^2*S
qrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])
/(b^5*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2
- b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e +
 f*x)/2, 2])/(b^5*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
+ (2*a^2*g*Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(3*b^3*f) + (4*g*Sqrt[g*Cos[e
 + f*x]]*Sin[e + f*x])/(21*b*f) - (2*(g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(
7*b*f*g)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
```

```
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{a^2(g\cos(e+fx))^{3/2}}{b^3} - \frac{a(g\cos(e+fx))^{3/2}\sin(e+fx)}{b^2} + \frac{(g\cos(e+fx))^{3/2}}{b}\right.$$

$$= \frac{a^2\int (g\cos(e+fx))^{3/2}\,dx}{b^3} - \frac{a^3\int \frac{(g\cos(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{b^3} - \frac{a\int (g\cos(e+fx))^{3/2}\sin(e\right.}{b^2}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a^2g\sqrt{g\cos(e+fx)}\sin(e+fx)}{3b^3f} - \frac{2(g\cos(e+fx))^{5/2}}{7bfg}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} + \frac{2a^2g\sqrt{g\cos(e+fx)}\sin(e+f}{3b^3f}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} + \frac{2a^2g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+f\right.}{3b^3f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} - \frac{2a^4g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+f\right.}{b^5f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} - \frac{2a^4g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+f\right.}{b^5f\sqrt{g\cos(e+fx)}}$$

$$= \frac{a^3\sqrt[4]{-a^2+b^2}g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}f} + \frac{a^3\sqrt[4]{-a^2+b^2}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+}{\sqrt[4]{-a^2+b^2}\sqrt{}}\right.}{b^{9/2}f}$$

Mathematica [C] time = 27.5052, size = 1991, normalized size = 3.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((g*Cos[e + f*x])^(3/2)*((-2*(70*a^3 - 19*a*b^2)*(a + b*Sqrt[1 - Cos[e + f
*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*
(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b
^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)
*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sq
rt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*
b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^
2]*(a + b*Sin[e + f*x])) + ((210*a^3 - 21*a*b^2)*(a + b*Sqrt[1 - Cos[e + f*
x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4))
 - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x
]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e + f*x
]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*
```

```
Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4,
1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*
AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) +
((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 +
b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3
/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(
-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 +
b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*Cos[e + f*x]^2
)*(a + b*Sin[e + f*x])) - (2*(-98*a^2*b - 40*b^3)*(a + b*Sqrt[1 - Cos[e + f
*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*
Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/
((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2
+ b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos
[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e +
 f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a +
 b*Sin[e + f*x]))))/(420*b^3*f*Cos[e + f*x]^(3/2)) + ((g*Cos[e + f*x])^(3/2
)*Sec[e + f*x]*((a*Cos[2*(e + f*x)])/(5*b^2) + ((28*a^2 + 5*b^2)*Sin[e + f*
x])/(42*b^3) - Sin[3*(e + f*x)]/(14*b)))/f
```

Maple [C] time = 8.401, size = 3600, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 8/5/f*g*a/b^2*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/f
*g*a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/f*g*a/
b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-2/f*g*a^3/b^4*(g*(2*cos(1/2*f*x+1/2*
e)^2-1))^(1/2)+2/f*g*a/b^2*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2/f*g^3*a^5
/b^4*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2
-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1
/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^
4-4*b^2*g^3*_Z^2+b^2*g^4))-2/f*g^3*a^3/b^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R
^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e
)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*
b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-32/5/f*(g*
(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/
2*e)^7/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(
1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+272/15/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)^5/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g
)^(1/2)*cos(1/2*f*x+1/2*e)-16/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1
/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1
/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/
2*e)-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3*
sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2
*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1
```

```
/2*f*x+1/2*e)^2-1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*a^2+4/3/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1
/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1
/2*f*x+1/2*e)^2*g)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)
^2-1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))+4/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3*sin(1/2*f*x+1/2*e)/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g
)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*Ellip
ticE(cos(1/2*f*x+1/2*e),2^(1/2))*a^2-12/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*s
in(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e
)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin
(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2
*f*x+1/2*e),2^(1/2))+64/15/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*
e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(
-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+
4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3/sin(1
/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4
*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*
x+1/2*e)^2-1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*a^2-4/3/f*(g*(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*
x+1/2*e)^2*g)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)
^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*
x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/
2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(
cos(1/2*f*x+1/2*e),2^(1/2))*a^2+12/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)*g^2/b/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin(1/2*
f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2*f*x+
1/2*e),2^(1/2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)*g^2/b^5*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^4*su
m(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*a
rctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2
*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+
a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1
/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2
*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))
^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alp
ha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*s
in(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)*a^2*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a
^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*
f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/
2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin
(1/2*f*x+1/2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e
)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*si
n(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alp
ha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/2/f*(g*(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^5/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^4*sum(_alpha/(2*_alpha^2-1)*(2^(1/2
)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*
a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alph
a^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*si
n(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alpha^2
-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/
2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1
/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2
+a^2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/
```

```
b^3/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2*sum(_alpha/
(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/
2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*
f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2
)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a^
2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*
e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*El
lipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf
(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*sin(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

3.1378 $\int \frac{(g\cos(e+fx))^{3/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=514

$$-\frac{a^2g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}-\frac{a^2g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}+\frac{2a^3g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{g\cos(e+fx)}}-$$

```
[Out] -((a^2*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*f)) - (a^2*(-a^2 + b^2)^(1/4)*g^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7
/2)*f) + (2*a^2*g*Sqrt[g*Cos[e + f*x]])/(b^3*f) - (2*(g*Cos[e + f*x])^(5/2)
)/(5*b*f*g) + (2*a^3*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b^4
*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
2, 2])/(3*b^2*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^2*Sqrt[Cos[e + f
*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^2*Sq
rt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/
(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g*Sqrt
[g*Cos[e + f*x]]*Sin[e + f*x])/(3*b^2*f)
```

Rubi [A] time = 1.21448, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2898, 2635, 2642, 2641, 2565, 30, 2695, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{a^2g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}-\frac{a^2g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}+\frac{2a^3g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{g\cos(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*f)) - (a^2*(-a^2 + b^2)^(1/4)*g^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7
/2)*f) + (2*a^2*g*Sqrt[g*Cos[e + f*x]])/(b^3*f) - (2*(g*Cos[e + f*x])^(5/2)
)/(5*b*f*g) + (2*a^3*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b^4
*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
2, 2])/(3*b^2*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^2*Sqrt[Cos[e + f
*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^2*Sq
rt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/
(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g*Sqrt
[g*Cos[e + f*x]]*Sin[e + f*x])/(3*b^2*f)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(-\frac{a(g\cos(e+fx))^{3/2}}{b^2} + \frac{(g\cos(e+fx))^{3/2}\sin(e+fx)}{b} + \frac{a^2(g\cos(e+fx))^{3}}{b^2(a+b\sin(e+f}\right.$$

$$= -\frac{a\int (g\cos(e+fx))^{3/2}\,dx}{b^2} + \frac{a^2\int \frac{(g\cos(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{b^2} + \frac{\int (g\cos(e+fx))^{3/2}\sin(e}{b}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f} - \frac{2ag\sqrt{g\cos(e+fx)}\sin(e+fx)}{3b^2f} - \frac{\text{Subst}\left(\int x^{3/2}\,dx,x,g\right.}{bfg}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f} - \frac{2(g\cos(e+fx))^{5/2}}{5bfg} - \frac{2ag\sqrt{g\cos(e+fx)}\sin(e+fx)}{3b^2f} +$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f} - \frac{2(g\cos(e+fx))^{5/2}}{5bfg} - \frac{2ag^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right.}{3b^2f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f} - \frac{2(g\cos(e+fx))^{5/2}}{5bfg} + \frac{2a^3g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|\right.}{b^4f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f} - \frac{2(g\cos(e+fx))^{5/2}}{5bfg} + \frac{2a^3g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|\right.}{b^4f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{a^2\sqrt[4]{-a^2+b^2}g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}f} - \frac{a^2\sqrt[4]{-a^2+b^2}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e}}{\sqrt[4]{-a^2+b^2}}\right.}{b^{7/2}f}$$

Mathematica [C] time = 27.6361, size = 1953, normalized size = 3.8

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(-Cos[2*(e + f*x)]/(5*b) - (2*a*Sin[e
+ f*x])/(3*b^2)))/f + ((g*Cos[e + f*x])^(3/2)*((-2*(10*a^2 + 3*b^2)*(a + b*
Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1
- Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5
/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e
+ f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcT
an[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[
1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^
2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[
e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[
Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt
[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + ((30*a^2 - 3*b^2)*(a + b*Sqrt[
1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2
 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sq
rt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*
Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(
```

```
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
+ f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(
-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1
+ I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^
(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*
Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) + (28*a*b*(a + b*Sqrt[1 - Cos[e + f*x
]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((
-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x
]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 +
b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e
 + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f
*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b
*Sin[e + f*x]))))/(60*b^2*f*Cos[e + f*x]^(3/2))
```

Maple [C] time = 7.529, size = 1778, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] -8/5/f*g/b*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/5/f*g/
b*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/5/f*g/b*(2*cos(
1/2*f*x+1/2*e)^2*g-g)^(1/2)+2/f*g/b^3*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*
a^2-2/f*g/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2/f*g^3/b^3*a^4*sum((_R^4+
_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln
((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R)
,_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^
2+b^2*g^4))+2/f*g^3/b*a^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a
^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-co
s(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^
2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+8/3/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g^2*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)/b^2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)
^2*g)^(1/2)*cos(1/2*f*x+1/2*e)-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*a^3*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2)/b^4/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin(1/
2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticF(cos(1/2*f*
x+1/2*e),2^(1/2))+2/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)
^(1/2)*a*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^2/(-
2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(sin(1/2*f*x+1/2*e)^
2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1
/2))-4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g^2*
sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^2/(-2*sin(1/2*f*x
```

```
+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+1/8/f*(g*(2*co
s(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a^5*g^2/sin(1/2*f*x+1/2*e
)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^6*sum(1/_alpha/(2*_alpha^2-1)*(2^(
1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/
(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_a
lpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2
*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alph
a^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin
(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*
x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*
b^2+a^2))-1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a
^3*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^4*sum(1/_a
lpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arcta
nh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos
(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-
2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))
+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x
+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/
2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=R
ootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*sin(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

3.1379 $\int \frac{(g\cos(e+fx))^{3/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=426

$$\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}+\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}-\frac{2g^2\left(3a^2-b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+\right.}{3b^3f\sqrt{g\cos(e+fx)}}$$

```
[Out] (a*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*f) + (a*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh
[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*f)
- (2*(3*a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*b^3
*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b*(b - Sqrt[-
a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^2*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b
*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]
]*(3*a - b*Sin[e + f*x]))/(3*b^2*f)
```

Rubi [A] time = 0.983197, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}+\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}-\frac{2g^2\left(3a^2-b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+\right.}{3b^3f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```


```
[Out] (a*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*f) + (a*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh
[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*f)
- (2*(3*a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*b^3
*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b*(b - Sqrt[-
a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^2*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b
*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]
]*(3*a - b*Sin[e + f*x]))/(3*b^2*f)
```


Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx = -\frac{2g\sqrt{g\cos(e+fx)}(3a-b\sin(e+fx))}{3b^2f}+\frac{\left(2g^2\right)\int \frac{-ab-\frac{1}{2}\left(3a^2-b^2\right)\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{3b^2}$$

$$=-\frac{2g\sqrt{g\cos(e+fx)}(3a-b\sin(e+fx))}{3b^2f}+\frac{\left(a\left(a^2-b^2\right)g^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}(a+b\sin(e+}$$

$$=-\frac{2g\sqrt{g\cos(e+fx)}(3a-b\sin(e+fx))}{3b^2f}+\frac{\left(a^2\sqrt{-a^2+b^2}g^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+}\right.}}{2b^3}$$

$$=-\frac{2\left(3a^2-b^2\right)g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^3f\sqrt{g\cos(e+fx)}}-\frac{2g\sqrt{g\cos(e+fx)}(3a-b\sin(e}{3b^2f}$$

$$=-\frac{2\left(3a^2-b^2\right)g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^3f\sqrt{g\cos(e+fx)}}-\frac{a^2\sqrt{-a^2+b^2}g^2\sqrt{\cos(e+fx)}\Pi}{b^3\left(b-\sqrt{-a^2+b^2}\right)f}$$

$$=\frac{a\sqrt[4]{-a^2+b^2}g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}f}+\frac{a\sqrt[4]{-a^2+b^2}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}f}$$

Mathematica [C] time = 26.953, size = 1909, normalized size = 4.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((g*Cos[e + f*x])^(3/2)*((-2*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
- 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f
*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e +
 f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/
(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e +
f*x])) + (3*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/
2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b
^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcT
```

```
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(
-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1,
9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))
/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x
]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e
 + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e +
f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2
, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^
2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqr
t[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b
*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]
] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1
- Cos[e + f*x]^2]*(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) + (4*b*(a +
 b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4,
Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[
1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f
*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2,
 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*Appe
llF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*
Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[S
qrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*C
os[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 -
 Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(6*b*f*Cos[e + f*x]^(3/2)) + (2*(g
*Cos[e + f*x])^(3/2)*Tan[e + f*x])/(3*b*f)
```

Maple [C] time = 6.761, size = 2432, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] -2/f*g*a/b^2*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2/f*g^3*a^3/b^2*sum((_R^4
+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*l
n((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R
),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z
^2+b^2*g^4))-2/f*g^3*a*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*
g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1
/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g
^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+
1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)
^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(c
os(1/2*f*x+1/2*e),2^(1/2))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/
2*e)^2)^(1/2)*g^2/b*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(
1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),
2^(1/2))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/
b/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*
e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-s
in(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-4/f*(g*(2
```

```
*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b/sin(1/2*f*x+1/2*
e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*co
s(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^
2))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+1/2/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b^3*sin(1/2*f*x+1/2*e)/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)*a^2*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alp
ha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(
4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*
b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/
2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2
*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)
^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2
/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/2/f
*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/b*sin(1/2*f*
x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(_alpha/(2*_alpha^2-1)*(2^
(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)
/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_
alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(
2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alp
ha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-si
n(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f
*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2
*b^2+a^2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*
g^2/b^3/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2*sum(_al
pha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctan
h(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(
1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2
*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+
8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+
1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2
)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=Ro
otOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)*g^2/b/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^
2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_a
lpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/
2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*
(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1
/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(
1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

3.1380 $\int \frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=439

$$\frac{g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}f}+\frac{g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}f}+\frac{g^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\right.}{bf\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(}}$$

```
[Out] -((g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f)) + ((-a^2 + b^2)^(1/
4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g
])])/(a*Sqrt[b]*f) - (g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f)
+ ((-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[g])])/(a*Sqrt[b]*f) - (2*g^2*Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2])/(b*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^2*Sqrt[Cos[e
 + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2
- b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^2*Sqrt
[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b
*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.13782, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {2898, 2565, 321, 329, 212, 206, 203, 2695, 2867, 2642, 2641, 2702, 2807, 2805, 208, 205}

$$\frac{g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}f}+\frac{g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}f}+\frac{g^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\right.}{bf\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f)) + ((-a^2 + b^2)^(1/
4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g
])])/(a*Sqrt[b]*f) - (g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f)
+ ((-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[g])])/(a*Sqrt[b]*f) - (2*g^2*Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2])/(b*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^2*Sqrt[Cos[e
 + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2
- b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^2*Sqrt
[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b
*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{a} - \frac{b(g\cos(e+fx))^{3/2}}{a(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int (g\cos(e+fx))^{3/2}\csc(e+fx)\,dx}{a} - \frac{b\int \frac{(g\cos(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{a}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{afg} - \frac{g^2\int \frac{b+a\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin}}{a}$$

$$= -\frac{g\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{g^2}\right)}\,dx, x, g\cos(e+fx)\right)}{af} - \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{\left(\left(-a^2+b^2\right)g^2\right.}{}$$

$$= -\frac{(2g)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{af} + \frac{\left(\sqrt{-a^2+b^2}g^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}\right.}}{2b}$$

$$= -\frac{2g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}} - \frac{g^2\,\text{Subst}\left(\int \frac{1}{g-x^2}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{af} - \frac{g^2}{}$$

$$= -\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{2g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+\right.}{bf\sqrt{g\cos(e+fx)}}$$

$$= -\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} + \frac{\sqrt[4]{-a^2+b^2}g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\sqrt{b}f} - \frac{g^{3/2}\tanh^{-1}\left(\right.}{af}$$

Mathematica [C] time = 5.30359, size = 484, normalized size = 1.1

$$\frac{\csc(e+fx)(g\cos(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8ab^{3/2}\cos^{\frac{5}{2}}(e+fx)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-5\left(a^2-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]*(8*a*b^(3/2)*AppellF1[5/4, 1/2, 1, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2) - 5
*(a^2 - b^2)*(2*Sqrt[2]*(a^2 - b^2)^(1/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*Sqrt[2]*(a^2 - b^2)^(1/4)*ArcTan[1 +
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*Sqrt[b]*ArcTan[
Sqrt[Cos[e + f*x]]] - 2*Sqrt[b]*Log[1 - Sqrt[Cos[e + f*x]]] + 2*Sqrt[b]*Log
[1 + Sqrt[Cos[e + f*x]]] + Sqrt[2]*(a^2 - b^2)^(1/4)*Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - Sq
rt[2]*(a^2 - b^2)^(1/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^2]))/(
20*a*Sqrt[b]*(a^2 - b^2)*f*Cos[e + f*x]^(3/2)*(b + a*Csc[e + f*x]))
```

Maple [A] time = 2.729, size = 216, normalized size = 0.5

$$-\frac{1}{2af}g^{\frac{3}{2}}\ln\left(2\,\frac{\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}+2\,g\cos\left(1/2\,fx+e/2\right)-g}{-1+\cos\left(1/2\,fx+e/2\right)}\right)-\frac{1}{2af}g^{\frac{3}{2}}\ln\left(2\,\frac{\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx+\right.\right.}}{\mathrm{co}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/a/f*g^(3/2)*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*
e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))-1/2/a/f*g^(3/2)*ln(2/(cos(1/2*f*
x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/
2*e)-g))+2/a/f*g*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/a/(-g)^(1/2)/f*g^2*l
n(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

3.1381 $\int \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=469

$$-\frac{\sqrt{b}g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f}-\frac{\sqrt{b}g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f}-\frac{g^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{\cdots}}\right.}{af\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{\cdots}}$$

```
[Out] (b*g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f) - (Sqrt[b]*(-a^2 +
 b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[g])])/(a^2*f) + (b*g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(
a^2*f) - (Sqrt[b]*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*f) - (g*Sqrt[g*Cos[e + f*x]]*C
sc[e + f*x])/(a*f) + (g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(a*
f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2
*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*Ellipti
cPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.25723, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.515, Rules used = {2898, 2565, 321, 329, 212, 206, 203, 2567, 2642, 2641, 2695, 2867, 2702, 2807, 2805, 208, 205}

$$-\frac{\sqrt{b}g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f}-\frac{\sqrt{b}g^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f}-\frac{g^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{\cdots}}\right.}{af\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (b*g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f) - (Sqrt[b]*(-a^2 +
 b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[g])])/(a^2*f) + (b*g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(
a^2*f) - (Sqrt[b]*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*f) - (g*Sqrt[g*Cos[e + f*x]]*C
sc[e + f*x])/(a*f) + (g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(a*
f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2
*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*Ellipti
cPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a+b\sin(e+fx)}dx = \int\left(-\frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2} + \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a} + \frac{b^2(g\cos}{a^2(a+b}\right.$$

$$= \frac{\int(g\cos(e+fx))^{3/2}\csc^2(e+fx)\,dx}{a} - \frac{b\int(g\cos(e+fx))^{3/2}\csc(e+fx)\,dx}{a^2} + \frac{b^2\int}{}$$

$$= \frac{2bg\sqrt{g\cos(e+fx)}}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af} + \frac{b\,\text{Subst}\left(\int\frac{x^{3/2}}{1-\frac{x^2}{g^2}}dx, x, g\cos\right.}{a^2fg}$$

$$= -\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af} + \frac{(bg)\,\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x^2}{g^2}\right)}dx, x, g\cos(e+fx)\right)}{a^2f} + \frac{g}{}$$

$$= -\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af} - \frac{g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{g\cos(e+fx)}} + \frac{(2bg)\,\text{Subst}}{}$$

$$= -\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af} + \frac{g^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{g\cos(e+fx)}} + \frac{\left(bg^2\right)\text{Subst}}{}$$

$$= \frac{bg^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f} + \frac{bg^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\csc(e+}{af}$$

$$= \frac{bg^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f} - \frac{\sqrt{b}\sqrt[4]{-a^2+b^2}g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2f} + \frac{bg^{3/2}\tanh}{}$$

Mathematica [C] time = 27.4391, size = 2099, normalized size = 4.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((g*Cos[e + f*x])^(3/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
- 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f
*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e +
 f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/
(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b*(
-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[
e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e
 + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(
2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1
/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (
```

```
16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
+ b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4,
 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e
 + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/
4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5
/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2
 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1
- Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*
(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]
*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt
[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 -
Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (6*b*(-1 +
Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
+ b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*Appell
F1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2
*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]
^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])
/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Co
s[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f
*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/
4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2
]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2
)*(b + a*Csc[e + f*x]))))/(4*a*f*Cos[e + f*x]^(3/2)) - ((g*Cos[e + f*x])^(3
/2)*Csc[e + f*x]*Sec[e + f*x])/(a*f)
```

Maple [C] time = 11.012, size = 2324, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/2/f*g^(3/2)*b/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/
2*e)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))+1/2/f*g^(3/2)*b/a^2*ln((-4*
g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(co
s(1/2*f*x+1/2*e)+1))-1/f*g^2*b/a^2/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(
1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))-2/f*g^3*b*sum((_R^4+_R^2*g
)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*s
in(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=Ro
otOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*
g^4))+2/f*g^3*b^3/a^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g
^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/
2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^
2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^
2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x
+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2
)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticF(c
os(1/2*f*x+1/2*e),2^(1/2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+
1/2*e)^2)^(1/2)/a*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/
```

```
2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(2*sin(1/2*f*x+
1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e)
,2^(1/2))+1/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g
^2*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/cos(1/2*f*x+1/
2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)-1/2/f*(g*(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g^2*sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/
2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)-1/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*si
n(1/2*f*x+1/2*e)^2)^(1/2)/a*g/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2
-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*
x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*
sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g/sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*
e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+
sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/
2*f*x+1/2*e)^2)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e
)^2)^(1/2)/a*g/sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2
*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e
)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(
1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*g/si
n(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*
e)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*
x+1/2*e)+1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*
g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/b^2*sum(1/_alph
a/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(
1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/
2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/
a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/
2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*
EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=Root
Of(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)/a*g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2)*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^
2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_a
lpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/
2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*
(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1
/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(
1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*csc(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

3.1382 $\int \frac{(g\cos(e+fx))^{3/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=574

$$-\frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{3/2}g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f}-\frac{b^2g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{3/2}g^{3/2}\sqrt[4]{b^2-a^2}\ \mathrm{ta}}{}$$

```
[Out] (g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*g^(3/2)*ArcTa
n[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(3/2)*(-a^2 + b^2)^(1/4)*g^(3
/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a
^3*f) + (g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*g^(3
/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(3/2)*(-a^2 + b^2)^
(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(a^3*f) + (b*g*Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a^2*f) - (g*Sqr
t[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(2*a*f) - (b*g^2*Sqrt[Cos[e + f*x]]*Ellip
ticF[(e + f*x)/2, 2])/(a^2*f*Sqrt[g*Cos[e + f*x]]) + (b*(a^2 - b^2)*g^2*Sqr
t[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(
a^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (b*(a^2 - b^
2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x
)/2, 2])/(a^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.37029, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2898, 2565, 321, 329, 212, 206, 203, 2567, 2642, 2641, 288, 2695, 2867, 2702, 2807, 2805, 208, 205}

$$-\frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{3/2}g^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f}-\frac{b^2g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{3/2}g^{3/2}\sqrt[4]{b^2-a^2}\ \mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```


```
[Out] (g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*g^(3/2)*ArcTa
n[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(3/2)*(-a^2 + b^2)^(1/4)*g^(3
/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a
^3*f) + (g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*g^(3
/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(3/2)*(-a^2 + b^2)^
(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(a^3*f) + (b*g*Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a^2*f) - (g*Sqr
t[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(2*a*f) - (b*g^2*Sqrt[Cos[e + f*x]]*Ellip
ticF[(e + f*x)/2, 2])/(a^2*f*Sqrt[g*Cos[e + f*x]]) + (b*(a^2 - b^2)*g^2*Sqr
t[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(
a^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (b*(a^2 - b^
2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x
)/2, 2])/(a^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
```

```
g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{b^2(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^3} - \frac{b(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a^2} + \frac{(g\,\mathrm{co}}{}\right.$$

$$= \frac{\int (g\cos(e+fx))^{3/2}\csc^3(e+fx)\,dx}{a} - \frac{b\int (g\cos(e+fx))^{3/2}\csc^2(e+fx)\,dx}{a^2} +$$

$$= -\frac{2b^2g\sqrt{g\cos(e+fx)}}{a^3f} + \frac{bg\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2f} - \frac{\mathrm{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x^2}{g^2}\right)^2}dx,\right.}{afg}$$

$$= \frac{bg\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{2af} + \frac{g\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{bg\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{2af} + \frac{bg^2\sqrt{\cos(e+}}{a^2f\sqrt{}}$$

$$= \frac{bg\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{2af} - \frac{bg^2\sqrt{\cos(e+}}{a^2f\sqrt{}}$$

$$= \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} - \frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} + \frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+f}}{\sqrt{g}}\right.}{4af}$$

$$= \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} - \frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} + \frac{b^{3/2}\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan}{a^3f}$$

Mathematica [C] time = 30.6175, size = 2129, normalized size = 3.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((-2*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^
2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*Ap
pellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
 - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e +
f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f
*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 +
b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] +
(1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))
```

```
/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b^
2*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*C
sc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Co
s[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2
]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)
^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a
- (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Co
s[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF
1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-
a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log
[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[
2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt
[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*S
qrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1
 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (2*(-a^2
 + 3*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*
x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a
^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-
1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqr
t[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTa
n[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2
*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e
+ f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1
- Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Cos[e + f*x]^(3/2)) + ((
g*Cos[e + f*x])^(3/2)*((b*Csc[e + f*x])/a^2 - Csc[e + f*x]^2/(2*a))*Sec[e +
 f*x])/f
```

Maple [A] time = 2.707, size = 312, normalized size = 0.5

$$\frac{1}{8af}g^{\frac{3}{2}}\ln\left(\left(4g\cos\left(1/2\,fx+e/2\right)+2\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}-2g\right)\left(-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-1}\right)-\frac{g}{16af}\sqrt{-2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/8/f*g^(3/2)/a*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))-1/16/f*g/a/(cos(1/2*f*x+1/2*e)+
1)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/8/f*g^(3/2)/a*ln((-4*g*cos(1/2*f*x
+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2
*e)+1))-1/4/f*g^2/a/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^
2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))+1/16/f*g/a/(-1+cos(1/2*f*x+1/2*e))*(-2*si
n(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/8/f*g/a/cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x
+1/2*e)^2*g-g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}\csc(fx+e)^3}{b\sin(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*csc(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}\csc(fx+e)^3}{b\sin(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

3.1383 $\int \frac{(g\cos(e+fx))^{5/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=610

$$-\frac{a^3g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}f}+\frac{a^3g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}f}-\frac{2a^4g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e}}{b^5f\sqrt{\cos(e+fx)}}$$

```
[Out] -((a^3*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(11/2)*f)) + (a^3*(-a^2 + b^2)^(3/4)*g^(5/2)
*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(
11/2)*f) - (2*a^3*g*(g*Cos[e + f*x])^(3/2))/(3*b^4*f) + (2*a*(g*Cos[e + f*x
])^(7/2))/(7*b^2*f*g) - (2*a^4*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)
/2, 2])/(b^5*f*Sqrt[Cos[e + f*x]]) + (6*a^2*g^2*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*b^3*f*Sqrt[Cos[e + f*x]]) + (4*g^2*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(15*b*f*Sqrt[Cos[e + f*x]]) + (a^4*(a^2 - b
^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*
x)/2, 2])/(b^6*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2 -
 b^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e +
f*x)/2, 2])/(b^6*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (2*a^2*g*
(g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(5*b^3*f) + (4*g*(g*Cos[e + f*x])^(3/2
)*Sin[e + f*x])/(45*b*f) - (2*(g*Cos[e + f*x])^(7/2)*Sin[e + f*x])/(9*b*f*g
)
```

Rubi [A] time = 1.35914, antiderivative size = 610, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {2898, 2635, 2640, 2639, 2565, 30, 2568, 2695, 2867, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a^3g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}f}+\frac{a^3g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}f}-\frac{2a^4g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e}}{b^5f\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```


```
[Out] -((a^3*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(11/2)*f)) + (a^3*(-a^2 + b^2)^(3/4)*g^(5/2)
*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(
11/2)*f) - (2*a^3*g*(g*Cos[e + f*x])^(3/2))/(3*b^4*f) + (2*a*(g*Cos[e + f*x
])^(7/2))/(7*b^2*f*g) - (2*a^4*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)
/2, 2])/(b^5*f*Sqrt[Cos[e + f*x]]) + (6*a^2*g^2*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*b^3*f*Sqrt[Cos[e + f*x]]) + (4*g^2*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(15*b*f*Sqrt[Cos[e + f*x]]) + (a^4*(a^2 - b
^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*
x)/2, 2])/(b^6*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2 -
 b^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e +
f*x)/2, 2])/(b^6*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (2*a^2*g*
(g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(5*b^3*f) + (4*g*(g*Cos[e + f*x])^(3/2
)*Sin[e + f*x])/(45*b*f) - (2*(g*Cos[e + f*x])^(7/2)*Sin[e + f*x])/(9*b*f*g
)
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
```

```
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(\frac{a^2(g\cos(e+fx))^{5/2}}{b^3} - \frac{a(g\cos(e+fx))^{5/2}\sin(e+fx)}{b^2} + \frac{(g\cos(e+fx))^{5/2}}{b}\right.$$

$$= \frac{a^2\int(g\cos(e+fx))^{5/2}\,dx}{b^3} - \frac{a^3\int\frac{(g\cos(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx}{b^3} - \frac{a\int(g\cos(e+fx))^{5/2}\sin(e}{b^2}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a^2g(g\cos(e+fx))^{3/2}\sin(e+fx)}{5b^3f} - \frac{2(g\cos(e+fx}{9}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} + \frac{2a^2g(g\cos(e+fx))^{3/2}\sin(e}{5b^3f}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} + \frac{6a^2g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{5b^3f\sqrt{\cos(e+fx)}}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} - \frac{2a^4g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{b^5f\sqrt{\cos(e+fx)}}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} - \frac{2a^4g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e\right.}{b^5f\sqrt{\cos(e+fx)}}$$

$$= -\frac{a^3\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{11/2}f} + \frac{a^3\left(-a^2+b^2\right)^{3/4}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{b^{11/2}f}$$

Mathematica [C] time = 27.1559, size = 867, normalized size = 1.42

$$\frac{(g\cos(e+fx))^{5/2}\sec^2(e+fx)\left(-\frac{a\left(28a^2-9b^2\right)\cos(e+fx)}{42b^4} + \frac{a\cos(3(e+fx))}{14b^2} - \frac{\left(b^2-18a^2\right)\sin(2(e+fx))}{90b^3} - \frac{\sin(4(e+fx))}{36b}\right)}{f} - \frac{(g\cos(e+fx)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((g*Cos[e + f*x])^(5/2)*((-2*(6*a^3*b - 2*a*b^3)*(a + b*Sqrt[1 - Cos[e + f
*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan
[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1
+ ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2
+ b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e
+ f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Co
s[e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x
])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((15*a^4 - 9*a^2*b^2 -
 2*b^4)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1,
7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2)
+ 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e +
 f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x
]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^
```

```
(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*
x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))
/(15*b^4*f*Cos[e + f*x]^(5/2)) + ((g*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*(-(
a*(28*a^2 - 9*b^2)*Cos[e + f*x])/(42*b^4) + (a*Cos[3*(e + f*x)])/(14*b^2) -
 ((-18*a^2 + b^2)*Sin[2*(e + f*x)])/(90*b^3) - Sin[4*(e + f*x)]/(36*b)))/f
```

Maple [C] time = 7.449, size = 4548, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/2/f*g^3*a^5/b^4*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_
R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/
2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(
16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-4/3/f*g^2*a^3/b^4*(2*c
os(1/2*f*x+1/2*e)^2*g-g)^(1/2)+40/7/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2
*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2
))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*
x+1/2*e)^3+152/105/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1
/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*
f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)-152/105/
f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*si
n(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*co
s(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+64/7/f*(g*(2*cos(1/2*f*x+1/
2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin
(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2)*cos(1/2*f*x+1/2*e)^9-64/7/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^
(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1
/2*e)^9-576/35/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*
g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+
1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^7+576/35/f*(
g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1
/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1
/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^7+1216/105/f*(g*(2*cos(1/2*f*x
+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-
sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-
1))^(1/2)*cos(1/2*f*x+1/2*e)^5-1216/105/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin
(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*
e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/
2*f*x+1/2*e)^5-40/7/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2
*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3+20/21
/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*s
in(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*c
os(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+
1/2*e)^2+1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-12/5/f*(g*(2*cos(1/
2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*
e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*
e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/
2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-20/21/f*(g*(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f
*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
```

```
*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(c
os(1/2*f*x+1/2*e),2^(1/2))-16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))
^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+
1/2*e)^5*a^2+16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2
*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^5*a^2+8
/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2
*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2
*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3*a^2-8/f*(g*(2*cos(1/2*
f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*
e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*
e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3*a^2-8/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*
x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*
cos(1/2*f*x+1/2*e)*a^2+8/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*
e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2
)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e
)*a^2+12/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/
b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*
e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*co
s(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+16/7/f*g^
2*a/b^2*cos(1/2*f*x+1/2*e)^6*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/3/f*g^2*a
^3/b^4*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-24/7/f*g^2*a
/b^2*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+12/7/f*g^2*a/b
^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/f*(g*(2*cos(1/
2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^5/(-g*(2*sin(1/2*f*x+1/
2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/
2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(
1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*a^4-16/3/f*(g*(2*cos(1/2*f*x+1/2
*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-si
n(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1)
)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*Elli
pticF(cos(1/2*f*x+1/2*e),2^(1/2))*a^2+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin
(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/
2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin
(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/
2*f*x+1/2*e),2^(1/2))*a^2-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*g^3/b^5/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/
2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2
*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e)
,2^(1/2))*a^4+16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1
/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/
2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1
/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))
*a^2-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/
(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(
1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*a^2+12/7/f*
g^2*a/b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+1/4/f*(g*(2*cos(1/2*f*x+1/2*e)
^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^7/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*
f*x+1/2*e)^2-1))^(1/2)*sum((sin(1/2*f*x+1/2*e)^2*(2*_alpha^2*a^2*b^2-2*_alp
ha^2*b^4-a^4+a^2*b^2)-2*_alpha^2*a^2*b^2+2*_alpha^2*b^4+a^4-a^2*b^2)/_alpha
/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/
2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*
e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+
1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/
2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1
/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1
```

```
/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g
*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*
f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^
(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2
*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+2
/f*g^2*a^3/b^4*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2/f*g^2*a/b^2*(g*(2*cos
(1/2*f*x+1/2*e)^2-1))^(1/2)-1/2/f*g^3*a^3/b^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3
)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*s
in(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=Ro
otOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*
g^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*sin(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

3.1384 $\int \frac{(g\cos(e+fx))^{5/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=501

$$\frac{a^2g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}-\frac{a^2g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}+\frac{2a^3g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+}}{b^4f\sqrt{\cos(e+fx)}}$$

```
[Out] (a^2*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*f) - (a^2*(-a^2 + b^2)^(3/4)*g^(5/2)*Arc
Tanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)
*f) + (2*a^2*g*(g*Cos[e + f*x])^(3/2))/(3*b^3*f) - (2*(g*Cos[e + f*x])^(7/2
))/(7*b*f*g) + (2*a^3*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(
b^4*f*Sqrt[Cos[e + f*x]]) - (6*a*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*
x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^3*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5*(b - S
qrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^3*Sqrt[Cos[e
+ f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5*(b +
 Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g*(g*Cos[e + f*x])^(3/2)*
Sin[e + f*x])/(5*b^2*f)
```

Rubi [A] time = 1.19368, antiderivative size = 501, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2898, 2635, 2640, 2639, 2565, 30, 2695, 2867, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{a^2g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}-\frac{a^2g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}+\frac{2a^3g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+}}{b^4f\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^2*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*f) - (a^2*(-a^2 + b^2)^(3/4)*g^(5/2)*Arc
Tanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)
*f) + (2*a^2*g*(g*Cos[e + f*x])^(3/2))/(3*b^3*f) - (2*(g*Cos[e + f*x])^(7/2
))/(7*b*f*g) + (2*a^3*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(
b^4*f*Sqrt[Cos[e + f*x]]) - (6*a*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*
x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^3*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5*(b - S
qrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^3*Sqrt[Cos[e
+ f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5*(b +
 Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g*(g*Cos[e + f*x])^(3/2)*
Sin[e + f*x])/(5*b^2*f)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{a(g\cos(e+fx))^{5/2}}{b^2}+\frac{(g\cos(e+fx))^{5/2}\sin(e+fx)}{b}+\frac{a^2(g\cos(e+fx))^{5}}{b^2(a+b\sin(e+f}\right.$$

$$= -\frac{a\int(g\cos(e+fx))^{5/2}\,dx}{b^2}+\frac{a^2\int\frac{(g\cos(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx}{b^2}+\frac{\int(g\cos(e+fx))^{5/2}\sin(e}{b}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2ag(g\cos(e+fx))^{3/2}\sin(e+fx)}{5b^2f}-\frac{\text{Subst}\left(\int x^{5/2}\,dx\right.}{b}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2(g\cos(e+fx))^{7/2}}{7bfg}-\frac{2ag(g\cos(e+fx))^{3/2}\sin(e+f}{5b^2f}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2(g\cos(e+fx))^{7/2}}{7bfg}-\frac{6ag^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+f\right.}{5b^2f\sqrt{\cos(e+fx)}}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2(g\cos(e+fx))^{7/2}}{7bfg}+\frac{2a^3g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{b^4f\sqrt{\cos(e+fx)}}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2(g\cos(e+fx))^{7/2}}{7bfg}+\frac{2a^3g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{b^4f\sqrt{\cos(e+fx)}}$$

$$= \frac{a^2\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}f}-\frac{a^2\left(-a^2+b^2\right)^{3/4}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt[4]{}}\right.}{b^{9/2}f}$$

Mathematica [C] time = 27.1328, size = 824, normalized size = 1.64

$$a\left(-\frac{\left(5a^2-3b^2\right)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\cos^{\frac{3}{2}}(e+fx)b^{5/2}+3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-2\tan^{-1}\left(\sqrt{\right.\right.\right.}{12b^{3/2}\left(b^2-a^2\right)\left(1-\cos^2(e+\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(g*Cos[e + f*x])^(5/2)*((-4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*
Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I
*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Co
s[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((5*a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[
e + f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3
/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b
*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
```

```
rt[Cos[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^
2)*(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(5*b^3*f*Cos[e + f*x]^(5/2)
) + ((g*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*(-((-28*a^2 + 9*b^2)*Cos[e + f*x
])/(42*b^3) - Cos[3*(e + f*x)]/(14*b) - (a*Sin[2*(e + f*x)])/(5*b^2)))/f
```

Maple [C] time = 6.252, size = 1937, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] -16/7/f*g^2/b*cos(1/2*f*x+1/2*e)^6*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+24/7/
f*g^2/b*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-12/7/f*g^2/
b*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-12/7/f*g^2/b*(2*c
os(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/3/f*g^2/b^3*cos(1/2*f*x+1/2*e)^2*(2*cos(1/
2*f*x+1/2*e)^2*g-g)^(1/2)*a^2+4/3/f*g^2/b^3*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1
/2)*a^2-2/f*g^2/b^3*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2+2/f*g^2/b*(g*(
2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-1/2/f*g^3/b^3*a^4*sum((_R^6-_R^4*g-_R^2*g^
2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln(
(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),
_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2
+b^2*g^4))+1/2/f*g^3/b*a^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*
b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2
*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2
*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+16/5/f*(g*(2*
cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^2/(-g*(2*sin(1/
2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/
2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^7-32/5/f*(g*(2*cos(1/2*f*x+1/2*
e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-s
in(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*cos(1/2*f*x+1/2*e)^5+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)*g^3*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2
))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*
x+1/2*e)^3+2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^
3*a^3/b^4/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*
f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2
)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-6
/5/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^2/(-
g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(
g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(
sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)-4/5/f*(g*(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^2/(-g*(2*sin(1/2
*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2
*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a/b^6/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*
f*x+1/2*e)^2-1))^(1/2)*sum((a^2-b^2)/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2
)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)
*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^
2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/
2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_al
pha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4
*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alp
ha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*s
in(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*
(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
```

```
/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4
*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*sin(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

3.1385 $\int \frac{(g\cos(e+fx))^{5/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=413

$$-\frac{ag^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}+\frac{ag^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}-\frac{2g^2\left(5a^2-3b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^3f\sqrt{\cos(e+fx)}}$$

```
[Out] -((a*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*f)) + (a*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcT
anh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*
f) - (2*(5*a^2 - 3*b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
/(5*b^3*f*Sqrt[Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b - Sqrt[-a^2
+ b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b + Sqrt[-a^
2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*g*(g*Cos[e + f*x])^(3/2)*(5*a - 3*b*
Sin[e + f*x]))/(15*b^2*f)
```

Rubi [A] time = 0.972255, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{ag^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}+\frac{ag^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}-\frac{2g^2\left(5a^2-3b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^3f\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*f)) + (a*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcT
anh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*
f) - (2*(5*a^2 - 3*b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
/(5*b^3*f*Sqrt[Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b - Sqrt[-a^2
+ b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b + Sqrt[-a^
2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*g*(g*Cos[e + f*x])^(3/2)*(5*a - 3*b*
Sin[e + f*x]))/(15*b^2*f)
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx = -\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2f} + \frac{\left(2g^2\right)\int \frac{\sqrt{g\cos(e+fx)}\left(-ab-\frac{1}{2}\left(5a^2-3b^2\right)\sin(e}{a+b\sin(e+fx)}}{5b^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2f} - \frac{\left(\left(5a^2-3b^2\right)g^2\right)\int\sqrt{g\cos(e+fx)}\,dx}{5b^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2f} - \frac{\left(a^2\left(a^2-b^2\right)g^3\right)\int\frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}\right.}}{2b^4}$$

$$= -\frac{2\left(5a^2-3b^2\right)g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^3f\sqrt{\cos(e+fx)}} - \frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin}{15b^2f}$$

$$= -\frac{2\left(5a^2-3b^2\right)g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^3f\sqrt{\cos(e+fx)}} + \frac{a^2\left(a^2-b^2\right)g^3\sqrt{\cos(e+fx)}\Pi}{b^4\left(b-\sqrt{-a^2+b^2}\right)f}$$

$$= -\frac{a\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}f} + \frac{a\left(-a^2+b^2\right)^{3/4}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(}{\sqrt[4]{-a^2+b}\right.}}{b^{7/2}f}$$

Mathematica [C] time = 24.8467, size = 737, normalized size = 1.78

$$(g\cos(e+fx))^{5/2}\left(\frac{\left(5a^2-3b^2\right)\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(-\log\left(-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(}\right.\right.\right.}{12b^{7/2}(b}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*Cos[e + f*x]^(3/2)*(-5*a + 3*b*Sin[e + f*x]))/(
3*b^2) + ((5*a^2 - 3*b^2)*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*
x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(
a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 -
b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)
^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^
2]))/(12*b^(7/2)*(-a^2 + b^2)*(a + b*Sin[e + f*x])) + (4*a*((a*AppellF1[3/4
```

```
, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f
*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2
+ b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e
+ f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x]*(a + b*Sqrt[Sin[e + f*
x]^2]))/(b*Sqrt[Sin[e + f*x]^2]*(a + b*Sin[e + f*x]))))/(5*f*Cos[e + f*x]^(
5/2))
```

Maple [C] time = 6.952, size = 2612, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] -4/3/f*g^2*a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/
3/f*g^2*a/b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+2/f*g^2*a/b^2*(g*(2*cos(1/
2*f*x+1/2*e)^2-1))^(1/2)+1/2/f*g^3*a^3/b^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(
_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(
1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootO
f(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4
))-1/2/f*g^3*a*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3
*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-
cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*
a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-16/3/f*(g*(2*cos(1/2*f*x+
1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-s
in(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*cos(1/2*f*x+1/2*e)^5+16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2)
)^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x
+1/2*e)^5+8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3
/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2
*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3+4/f*(g*(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x
+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x
+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1
)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*a^2-16/3/f*(g*(2*cos(1/2*f*x+
1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-s
in(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*Ell
ipticF(cos(1/2*f*x+1/2*e),2^(1/2))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^
2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*
f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x
+1/2*e),2^(1/2))-8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1
/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*
f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3-8/3/f*
(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(
1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(
1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)-4/f*(g*(2*cos(1/2*f*x+1/2*e)^
2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/
2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1
/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*Elliptic
F(cos(1/2*f*x+1/2*e),2^(1/2))*a^2+16/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(
```

```
1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e
)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/
2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(cos(1/2*f
*x+1/2*e),2^(1/2))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^
(1/2)*g^3/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/
2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1
/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))
+8/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b/(-g*
(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*
(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+1/4/f*(g*(2*cos(1/2*f*
x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/b^5/a^2/sin(1/2*f*x+1/2*e)/(g
*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum((sin(1/2*f*x+1/2*e)^2*(2*_alpha^2*a^
2*b^2-2*_alpha^2*b^4-a^4+a^2*b^2)-2*_alpha^2*a^2*b^2+2*_alpha^2*b^4+a^4-a^2
*b^2)/_alpha/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(si
n(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(
1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(
sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(co
s(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*
b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*c
os(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2
)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e
)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1
/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1
/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2
*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

3.1386 $\int \frac{(g\cos(e+fx))^{5/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=425

$$-\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{ab^{3/2}f}+\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{ab^{3/2}f}+\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}} \ldots\right.}{b^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(\ldots}}$$

```
[Out] (g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) - ((-a^2 + b^2)^(3/4)*
g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*b^(3/2)*f) - (g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) + (
(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[g])])/(a*b^(3/2)*f) - (2*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE
[(e + f*x)/2, 2])/(b*f*Sqrt[Cos[e + f*x]]) + ((a^2 - b^2)*g^3*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(b - S
qrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(b + Sqr
t[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.14854, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {2898, 2565, 321, 329, 298, 203, 206, 2695, 2867, 2640, 2639, 2701, 2807, 2805, 205, 208}

$$-\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{ab^{3/2}f}+\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{ab^{3/2}f}+\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}} \ldots\right.}{b^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) - ((-a^2 + b^2)^(3/4)*
g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*b^(3/2)*f) - (g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) + (
(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[g])])/(a*b^(3/2)*f) - (2*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE
[(e + f*x)/2, 2])/(b*f*Sqrt[Cos[e + f*x]]) + ((a^2 - b^2)*g^3*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(b - S
qrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(b + Sqr
t[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{(g\cos(e+fx))^{5/2}\csc(e+fx)}{a} - \frac{b(g\cos(e+fx))^{5/2}}{a(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int (g\cos(e+fx))^{5/2}\csc(e+fx)\,dx}{a} - \frac{b\int \frac{(g\cos(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx}{a}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3af} - \frac{\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{afg} - \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}(}{a+b\sin}}{a}$$

$$= -\frac{g\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{af} - \frac{g^2\int \sqrt{g\cos(e+fx)}\,dx}{b} - \frac{\left(\left(-a^2+b^2\right)\right.}{}$$

$$= -\frac{(2g)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{af} - \frac{\left(\left(a^2-b^2\right)g^3\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+}\right.}}{2b^2}$$

$$= -\frac{2g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{\cos(e+fx)}} - \frac{g^3\,\text{Subst}\left(\int \frac{1}{g-x^2}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{af}$$

$$= \frac{g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{2g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(\right.}{bf\sqrt{\cos(e+fx}}$$

$$= \frac{g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{ab^{3/2}f} - \frac{g^{5/2}\tanh}{}$$

Mathematica [C] time = 24.5648, size = 484, normalized size = 1.14

$$\frac{\csc(e+fx)(g\cos(e+fx))^{5/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(8ab^{5/2}\cos^{\frac{7}{2}}(e+fx)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+7\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]*(8*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/
4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) +
7*(a^2 - b^2)*(-2*Sqrt[2]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqr
t[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*(a^2 - b^2)^(3/4)*ArcTan[1
+ (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*b^(3/2)*ArcTa
n[Sqrt[Cos[e + f*x]]] + 2*b^(3/2)*Log[1 - Sqrt[Cos[e + f*x]]] - 2*b^(3/2)*L
og[1 + Sqrt[Cos[e + f*x]]] + Sqrt[2]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] -
Sqrt[2]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^2]))
/(28*a*b^(3/2)*(a^2 - b^2)*f*Cos[e + f*x]^(5/2)*(b + a*Csc[e + f*x]))
```

Maple [A] time = 2.695, size = 259, normalized size = 0.6

$$-\frac{1}{2af}g^{\frac{5}{2}}\ln\left(2\frac{\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}-2g\cos\left(1/2\,fx+e/2\right)-g}{\cos\left(1/2\,fx+e/2\right)+1}\right)-\frac{1}{2af}g^{\frac{5}{2}}\ln\left(2\frac{\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx+e/2\right.\right.}}{-1+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/a/f*g^(5/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e
)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))-1/2/a/f*g^(5/2)*ln(2/(-1+cos(1/2*
f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/
2*e)-g))-4/3/a/f*g^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)*sin(1/2*f*x+1/2*e)
^2+2/3/a/f*g^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/a/(-g)^(1/2)/f*g^3*ln(
2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

3.1387 $\int \frac{(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=462

$$\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}f}-\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}f}-\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}}\right.}{abf\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e}}$$

```
[Out] -((b*g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f)) + ((-a^2 + b^2)
^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(a^2*Sqrt[b]*f) + (b*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]
)/(a^2*f) - ((-a^2 + b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x
]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*Sqrt[b]*f) - (g*(g*Cos[e + f*x])^(3
/2)*Csc[e + f*x])/(a*f) - (g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2,
2])/(a*f*Sqrt[Cos[e + f*x]]) - ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*b*(b - Sqrt[-a^2 + b^2
])*f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi
[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*b*(b + Sqrt[-a^2 + b^2])
*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.2614, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.515, Rules used = {2898, 2565, 321, 329, 298, 203, 206, 2567, 2640, 2639, 2695, 2867, 2701, 2807, 2805, 205, 208}

$$\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}f}-\frac{g^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}f}-\frac{g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}}\right.}{abf\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((b*g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f)) + ((-a^2 + b^2)
^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(a^2*Sqrt[b]*f) + (b*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]
)/(a^2*f) - ((-a^2 + b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x
]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*Sqrt[b]*f) - (g*(g*Cos[e + f*x])^(3
/2)*Csc[e + f*x])/(a*f) - (g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2,
2])/(a*f*Sqrt[Cos[e + f*x]]) - ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*b*(b - Sqrt[-a^2 + b^2
])*f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi
[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*b*(b + Sqrt[-a^2 + b^2])
*f*Sqrt[g*Cos[e + f*x]])
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int\left(-\frac{b(g\cos(e+fx))^{5/2}\csc(e+fx)}{a^2}+\frac{(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a}+\frac{b^2(g}{a^2(a}\right.$$

$$=\frac{\int(g\cos(e+fx))^{5/2}\csc^2(e+fx)\,dx}{a}-\frac{b\int(g\cos(e+fx))^{5/2}\csc(e+fx)\,dx}{a^2}+\frac{b}{}$$

$$=\frac{2bg(g\cos(e+fx))^{3/2}}{3a^2f}-\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af}+\frac{b\,\text{Subst}\left(\int\frac{x^{5/2}}{1-\frac{x^2}{g^2}}\,dx,\right.}{a^2f}$$

$$=-\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af}+\frac{(bg)\,\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx,x,g\cos(e+fx)\right)}{a^2f}+$$

$$=-\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af}-\frac{3g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{\cos(e+fx)}}+\frac{(2bg}{}$$

$$=-\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af}-\frac{g^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{\cos(e+fx)}}+\frac{\left(bg^3\right)}{}$$

$$=-\frac{bg^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{bg^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}-\frac{g(g\cos(e+fx))^{3/2}}{af}$$

$$=-\frac{bg^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2\sqrt{b}f}+\frac{bg^{5/2}}{}$$

Mathematica [C] time = 27.33, size = 1556, normalized size = 3.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((12*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF
1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[
e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[
-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*C
os[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^2]*(b
+ a*Csc[e + f*x])) + (5*b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x
]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4
)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b*AppellF1[3/4, 1/2, 1
, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2
) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 - Sqrt[Cos[e + f*x]]] -
 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqrt[Cos[e + f*x]]] - 3*
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + 3*Sqrt[2]*Sqrt[b]*(a^
```

```
2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt
[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(
b + a*Csc[e + f*x])) - ((-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^
2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b
^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 42
*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[C
os[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*(a^2 - b^2)*ArcTan[Sqrt[Cos[e
 + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 48*a*b^(5/2)*AppellF1[7/4,
 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f
*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqrt[Cos[e + f*x]]] + 42*b^(3/2)
*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2
*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Co
s[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*
Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ b*Cos[e + f*x]]))/(84*Sqrt[b]*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(-1 + 2*
Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a*f*Cos[e + f*x]^(5/2)) - ((g*Co
s[e + f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x])/(a*f)
```

Maple [C] time = 11.355, size = 1987, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/2/f*g^(5/2)*b/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/
2*e)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))+1/2/f*g^(5/2)*b/a^2*ln((-4*
g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(co
s(1/2*f*x+1/2*e)+1))+1/f*g^3*b/a^2/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(
1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))-1/2/f*g^3*b*sum((_R^6-_R^4
*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^
2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^
(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^
2*g^3*_Z^2+b^2*g^4))+1/2/f*g^3*b^3/a^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7
*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^
2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1
/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/a/sin(1/
2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*
g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f
*x+1/2*e)^2)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-1/2/f*(g*(2*cos(1/
2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/a/sin(1/2*f*x+1/2*e)/(g*(
2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2
*e)^2*g)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2
)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-1/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*s
in(1/2*f*x+1/2*e)^2)^(1/2)*g^3/a*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1
/2*e)^2*g)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^
(1/2)*g^3/a*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/cos(1/2
*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)-1/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/a/sin(1/2*f*x+1
/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*s
in(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)+1/2/
f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/a/sin(1/2*f
*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)
```

```
^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticF(c
os(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)+1/2/f*(g*(2*cos(1/2
*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/a/sin(1/2*f*x+1/2*e)^3/(g*
(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(
1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*
e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1
)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2/a/sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+
1/2*e)^2-1))^(1/2)/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+si
n(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)-1/8/f*(g*(2*cos(1/2*f*x+1/2*
e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3/a/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*
f*x+1/2*e)^2-1))^(1/2)/b^2*sum((-a^2+b^2)/_alpha*(2^(1/2)/(g*(2*_alpha^2*b^
2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1
/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^
(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-
sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/
2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2
*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_
alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*csc(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

3.1388 $\int \frac{(g\cos(e+fx))^{5/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=557

$$\frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{\sqrt{b}g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f}-\frac{b^2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{\sqrt{b}g^{5/2}\left(b^2-a\right.}{}$$

```
[Out] (-3*g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) + (b^2*g^(5/2)*Ar
cTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) - (Sqrt[b]*(-a^2 + b^2)^(3/4)*g
^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(a^3*f) + (3*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2
*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (Sqrt[b]*(-a^2 +
b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[g])])/(a^3*f) + (b*g*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a^2*f) -
 (g*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(2*a*f) + (b*g^2*Sqrt[g*Cos[e +
f*x]]*EllipticE[(e + f*x)/2, 2])/(a^2*f*Sqrt[Cos[e + f*x]]) + ((a^2 - b^2)*
g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2
, 2])/(a^2*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^
3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2,
2])/(a^2*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.34346, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2898, 2565, 321, 329, 298, 203, 206, 2567, 2640, 2639, 288, 2695, 2867, 2701, 2807, 2805, 205, 208}

$$\frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{\sqrt{b}g^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f}-\frac{b^2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{\sqrt{b}g^{5/2}\left(b^2-a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```


```
[Out] (-3*g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) + (b^2*g^(5/2)*Ar
cTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) - (Sqrt[b]*(-a^2 + b^2)^(3/4)*g
^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(a^3*f) + (3*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2
*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (Sqrt[b]*(-a^2 +
b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[g])])/(a^3*f) + (b*g*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a^2*f) -
 (g*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(2*a*f) + (b*g^2*Sqrt[g*Cos[e +
f*x]]*EllipticE[(e + f*x)/2, 2])/(a^2*f*Sqrt[Cos[e + f*x]]) + ((a^2 - b^2)*
g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2
, 2])/(a^2*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^
3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2,
2])/(a^2*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
```

```
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{b^2(g\cos(e+fx))^{5/2}\csc(e+fx)}{a^3} - \frac{b(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a^2} + \frac{(g\cos(e$$

$$= \frac{\int (g\cos(e+fx))^{5/2}\csc^3(e+fx)\,dx}{a} - \frac{b\int (g\cos(e+fx))^{5/2}\csc^2(e+fx)\,dx}{a^2} + \frac{b^2}{}$$

$$= -\frac{2b^2g(g\cos(e+fx))^{3/2}}{3a^3f} + \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(1-\frac{x^2}{g^2}\right)^2}\,dx\right.}{af}$$

$$= \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2af} + \frac{(3g)\,\text{Subst}}{}$$

$$= \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2af} + \frac{3bg^2\sqrt{g\cos}}{a^2}$$

$$= \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2af} + \frac{bg^2\sqrt{g\cos(}}{a^2}$$

$$= -\frac{3g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} + \frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} + \frac{3g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+}}{\sqrt{g}}\right.}{4af}$$

$$= -\frac{3g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} + \frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} - \frac{\sqrt{b}\left(-a^2+b^2\right)^{3/4}g^{5/2}\,\text{ta}}{a^3f}$$

Mathematica [C] time = 29.1487, size = 1590, normalized size = 2.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((g*Cos[e + f*x])^(5/2)*((6*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appel
lF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Co
s[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqr
t[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b
*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^2]*(
b + a*Csc[e + f*x])) - ((3*a^2 - 5*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1
 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTa
n[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a
```

```
^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b*Appell
F1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos
[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 - Sqrt[Co
s[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqrt[Cos[e
 + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + 3*Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(1 - Cos
[e + f*x]^2)*(b + a*Csc[e + f*x])) - (Sqrt[b]*(-1 + Cos[e + f*x]^2)*(a + b*
Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]*(a^2 -
 b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(
a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 + (
Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*(a^2 -
b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1, 7/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 48*a
*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqrt[Cos[e
 + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21*Sqrt[2
]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^2 - b^2
)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/
4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*(a^3 - a*b^2)*(1 - Cos[e + f*
x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Cos[e + f*x]
^(5/2)) + ((g*Cos[e + f*x])^(5/2)*((b*Cot[e + f*x])/a^2 - (Cot[e + f*x]*Csc
[e + f*x])/(2*a))*Sec[e + f*x]^2)/f
```

Maple [A] time = 2.873, size = 318, normalized size = 0.6

$$\frac{3}{8af} g^{\frac{5}{2}} \ln\left(\left(4g\cos\left(1/2\,fx+e/2\right)+2\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}-2g\right)\left(-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-1}\right)-\frac{g^{2}}{16af}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 3/8/f*g^(5/2)/a*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))-1/16/f*g^2/a/(cos(1/2*f*x+1/2*e
)+1)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+3/8/f*g^(5/2)/a*ln((-4*g*cos(1/2*f
*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1
/2*e)+1))+3/4/f*g^3/a/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e
)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))+1/16/f*g^2/a/(-1+cos(1/2*f*x+1/2*e))*(-
2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/8/f*g^2/a/cos(1/2*f*x+1/2*e)^2*(2*cos(1
/2*f*x+1/2*e)^2*g-g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*csc(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{3}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

3.1389 $\int \frac{\sin^4(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=509

$$-\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg}-\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{2a^3\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{g\cos(e+fx)}}+$$

```
[Out] -((a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(7/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g])) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*(-a^2 + b^2)^(3/4)*f*Sq
rt[g]) - (2*a^2*Sqrt[g*Cos[e + f*x]])/(b^3*f*g) - (2*Sqrt[g*Cos[e + f*x]])/
(b*f*g) + (2*(g*Cos[e + f*x])^(5/2))/(5*b*f*g^3) - (2*a^3*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2])/(b^4*f*Sqrt[g*Cos[e + f*x]]) - (4*a*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*b^2*f*Sqrt[g*Cos[e + f*x]]) + (a^5*S
qrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])
/(b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^5*Sqrt[
Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^
4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (2*a*Sqrt[g*Co
s[e + f*x]]*Sin[e + f*x])/(3*b^2*f*g)
```

Rubi [A] time = 1.50611, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2909, 2565, 14, 2568, 2642, 2641, 30, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg}-\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{2a^3\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{g\cos(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(7/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g])) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*(-a^2 + b^2)^(3/4)*f*Sq
rt[g]) - (2*a^2*Sqrt[g*Cos[e + f*x]])/(b^3*f*g) - (2*Sqrt[g*Cos[e + f*x]])/
(b*f*g) + (2*(g*Cos[e + f*x])^(5/2))/(5*b*f*g^3) - (2*a^3*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2])/(b^4*f*Sqrt[g*Cos[e + f*x]]) - (4*a*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*b^2*f*Sqrt[g*Cos[e + f*x]]) + (a^5*S
qrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])
/(b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^5*Sqrt[
Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^
4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (2*a*Sqrt[g*Co
s[e + f*x]]*Sin[e + f*x])/(3*b^2*f*g)
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
```

```
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}}dx}{b} - \frac{a\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}dx}{b}$$

$$= -\frac{a\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}}dx}{b^2} + \frac{a^2\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}dx}{b^2} - \frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{g^2}}{\sqrt{x}}dx, x, g\right.}{bfg}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}\sin(e+fx)}{3b^2fg} + \frac{a^2\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}}dx}{b^3} - \frac{a^3\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e}}{b^3}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx))^{5/2}}{5bfg^3} + \frac{2a\sqrt{g\cos(e+fx)}\sin(e+fx)}{3b^2fg} - \frac{a^3}{}$$

$$= -\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx))^{5/2}}{5bfg^3} - \frac{4a\sqrt{\cos(e+}}{3b^2f\sqrt{}}$$

$$= -\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx))^{5/2}}{5bfg^3} - \frac{2a^3\sqrt{\cos(e+}}{b^4f\sqrt{}}$$

$$= -\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx))^{5/2}}{5bfg^3} - \frac{2a^3\sqrt{\cos(e+}}{b^4f\sqrt{}}$$

$$= -\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg}$$

Mathematica [C] time = 27.2196, size = 1953, normalized size = 3.84

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]*(Cos[2*(e + f*x)]/(5*b) + (2*a*Sin[e + f*x])/(3*b^2)))/(f*Sqr
t[g*Cos[e + f*x]]) - (Sqrt[Cos[e + f*x]]*((-2*(10*a^2 - 27*b^2)*(a + b*Sqrt
[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e +
f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Co
s[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e
 + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4,
3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*
x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e +
f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 -
 Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + ((30*a^2 + 27*b^2)*(a + b*Sqrt[1 -
 Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 +
b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[
```

```
Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
- 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f
*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^
2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/
2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*Cos
[e + f*x]^2)*(a + b*Sin[e + f*x])) + (28*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2
])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*
(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2
*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e +
f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]
])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*
Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Si
n[e + f*x]))))/(60*b^2*f*Sqrt[g*Cos[e + f*x]])
```

Maple [C] time = 6.24, size = 1455, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 8/5/f/b*cos(1/2*f*x+1/2*e)^4/g*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/f/b*c
os(1/2*f*x+1/2*e)^2/g*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/f/b/g*(2*cos(1
/2*f*x+1/2*e)^2*g-g)^(1/2)-2/f/b^3*a^2/g*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/
2)+2/f*a^4/b^3*g*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*
_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x
+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*
b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-8/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*
sin(1/2*f*x+1/2*e)^2)^(1/2)*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1
/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*c
os(1/2*f*x+1/2*e)+4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)
^(1/2)*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1
/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+2/f*(
g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a^3/b^4/(-g*(2*sin
(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos
(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2
*e)^2-1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))+4/3/f*(g*(2*cos(1/2*f*
x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/b^2/(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2
-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*El
lipticF(cos(1/2*f*x+1/2*e),2^(1/2))-1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin
(1/2*f*x+1/2*e)^2)^(1/2)*a^3/b^6/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/
```

```
2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(
1/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*
e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(
1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(s
in(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(
1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-
2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/
2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b
^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^
2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2
*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2
*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(
1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=Root
Of(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e
)^2*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^4/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

3.1390 $\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=457

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}}-\frac{a^4\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}}\right.}{b^3f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)}$$

```
[Out] (a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
b^(5/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[
g]) + (2*a*Sqrt[g*Cos[e + f*x]])/(b^2*f*g) + (2*a^2*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2])/(b^3*f*Sqrt[g*Cos[e + f*x]]) + (4*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2])/(3*b*f*Sqrt[g*Cos[e + f*x]]) - (a^4*Sqrt[Cos[e
+ f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2
 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (a^4*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b
*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]]*
Sin[e + f*x])/(3*b*f*g)
```

Rubi [A] time = 1.17991, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2909, 2568, 2642, 2641, 2565, 30, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}}-\frac{a^4\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}}\right.}{b^3f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
b^(5/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[
g]) + (2*a*Sqrt[g*Cos[e + f*x]])/(b^2*f*g) + (2*a^2*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2])/(b^3*f*Sqrt[g*Cos[e + f*x]]) + (4*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2])/(3*b*f*Sqrt[g*Cos[e + f*x]]) - (a^4*Sqrt[Cos[e
+ f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2
 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (a^4*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b
*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]]*
Sin[e + f*x])/(3*b*f*g)
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}\sin(e+fx)}{3bfg} - \frac{a\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \frac{a^2\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+}}{b^2}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}\sin(e+fx)}{3bfg} + \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b^3} - \frac{a^3\int \frac{1}{\sqrt{g\cos(e+fx)}(a+b\sin(e}}{b^3}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} + \frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3bf\sqrt{g\cos(e+fx)}} - \frac{2\sqrt{g\cos(e+fx)}\sin(e}{3bfg}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} + \frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}} + \frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(\right.}{3bf\sqrt{g\cos(e+}}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} + \frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}} + \frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(\right.}{3bf\sqrt{g\cos(e+}}$$

$$= \frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} +$$

Mathematica [C] time = 26.9636, size = 1915, normalized size = 4.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Cos[e + f*x]*Sin[e + f*x])/(3*b*f*Sqrt[g*Cos[e + f*x]]) + (Sqrt[Cos[e +
 f*x]]*((-2*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Co
s[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF
1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-
a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 -
 I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^
2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^
(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos
[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a
^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)
)*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + (3*a*(a +
 b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*
ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/
2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4))
 + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) +
 (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e +
f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e +
f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1
+ Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b
^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^
2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*
x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*
(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) - (8*b*(a + b*Sqrt[1 - Cos[e
+ f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2
])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]
^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1,
9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a
^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[
e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
- b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + S
qrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(
a + b*Sin[e + f*x]))))/(6*b*f*Sqrt[g*Cos[e + f*x]])
```

Maple [C] time = 5.85, size = 2015, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 2/f*a/b^2/g*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2/f*a^3/b^2*g*sum((_R^4+_R
^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((
-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_
R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+
b^2*g^4))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b/(
-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/
(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/
2*f*x+1/2*e)^2-1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-4/f*(g*(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b/(-g*(2*sin(1/2*f*x+1/2*e
)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e
)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)
*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*si
n(1/2*f*x+1/2*e)^2)^(1/2)/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^
2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*
f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticF(cos(1/2*f*x+
1/2*e),2^(1/2))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)/b/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1
/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*
sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-1/2/f*(
g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b^3/(-g*(2*sin(1/2
*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2
*f*x+1/2*e)^2-1))^(1/2)*sum(1/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(
1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*_alpha^
4*b^2-8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*EllipticPi(cos(1/2*f*x+1/2
*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*si
n(1/2*f*x+1/2*e)^2-1)^(1/2)*_alpha^2*b^2+2^(1/2)*a^2*_alpha*arctanh(1/2/(-2
*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^
2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alph
a^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+
1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2
+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*
_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+
1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*
x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b^3/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x
+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*s
um(1/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*EllipticPi(
cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(sin(1/2*f*x+1/2*e)
^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*_alpha^4*b^2-8*(g*(2*_alpha^2*b^
2+a^2-2*b^2)/b^2)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^
2)/a^2,2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/
2)*_alpha^2*b^2+2^(1/2)*a^2*_alpha*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+s
in(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^
2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/
2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*
x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2
*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha
=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+
1/2*e)^2*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^3}{\sqrt{g\cos(fx+e)}\left(b\sin(fx+e)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^3}{\sqrt{g\cos(fx+e)}\left(b\sin(fx+e)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

3.1391 $\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=380

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a^3\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^3\sqrt{\cos(e+fx)}}{b^2f\left(a^2-b\left(\sqrt{b^2\right.\right.}}$$

```
[Out] -((a^2*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g])) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(3/4)*f*Sq
rt[g]) - (2*Sqrt[g*Cos[e + f*x]])/(b*f*g) - (2*a*Sqrt[Cos[e + f*x]]*Ellipti
cF[(e + f*x)/2, 2])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (a^3*Sqrt[Cos[e + f*x]]*
EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b*(b
- Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^3*Sqrt[Cos[e + f*x]]*Elli
pticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b*(b + Sq
rt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 0.930947, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {2909, 2565, 30, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a^3\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^3\sqrt{\cos(e+fx)}}{b^2f\left(a^2-b\left(\sqrt{b^2\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```


```
[Out] -((a^2*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g])) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(3/4)*f*Sq
rt[g]) - (2*Sqrt[g*Cos[e + f*x]])/(b*f*g) - (2*a*Sqrt[Cos[e + f*x]]*Ellipti
cF[(e + f*x)/2, 2])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (a^3*Sqrt[Cos[e + f*x]]*
EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b*(b
- Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^3*Sqrt[Cos[e + f*x]]*Elli
pticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b*(b + Sq
rt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```


Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{a\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{b^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, g\right.}{bfg}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} - \frac{a^3\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2b^2\sqrt{-a^2+b^2}} - \frac{a^3\int \frac{}{\sqrt{g\cos(e+fx)}(}}{2b^2}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} - \frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{g\cos(e+fx)}} + \frac{(2a^2g)\,\text{Subst}\left(\int \frac{}{(a^2-}\right.}{}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} - \frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{g\cos(e+fx)}} + \frac{a^3\sqrt{\cos(e+fx)}\Pi\left(\frac{}{b}\right.}{b^2\left(a^2-b\left(b-\sqrt{-a^2}\right.\right.}$$

$$= -\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} -$$

Mathematica [C] time = 25.5603, size = 1326, normalized size = 3.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*((-2*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*
b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))
) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])
```

```
/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-
a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 +
 b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x]))
- ((a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2
+ b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])
/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)
^(3/4)) + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 -
b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*
(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b
^2*(-1 + Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^
2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*
x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a
^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos
[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f
*x]^2]*(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(2*f*Sqrt[g*Cos[e +
f*x]])
```

Maple [C] time = 5.814, size = 855, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -2/f/b/g*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2/f*a^2/b*g*sum((_R^4+_R^2*g)
/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*si
n(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=Roo
tOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g
^4))+2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/b^2/(-
g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(
g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/
2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-1/8/f*(g*(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/b^4/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha/(2*_alpha^2-1)*(8*(g*(2*
_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2
*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1
),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1
/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2
-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1
/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2
*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-s
in(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2
+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))
^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^2}{\sqrt{g\cos(fx+e)}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^2}{\sqrt{g\cos(fx+e)}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

3.1392 $\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=352

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^2\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bf\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{a^2\sqrt{\cos(e+fx)}\Pi}{bf\left(a^2-b\left(\sqrt{b^2-\right.\right.}}$$

```
[Out] (a*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sq
rt[b]*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) + (a*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*
x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(3/4)*f*Sqrt[g])
+ (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b*f*Sqrt[g*Cos[e + f*x]
]) - (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e +
f*x)/2, 2])/(b*(a^2 - b^2 + b*Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (
a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2
, 2])/(b*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 0.72542, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^2\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bf\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{a^2\sqrt{\cos(e+fx)}\Pi}{bf\left(a^2-b\left(\sqrt{b^2-\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sq
rt[b]*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) + (a*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*
x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(3/4)*f*Sqrt[g])
+ (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b*f*Sqrt[g*Cos[e + f*x]
]) - (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e +
f*x)/2, 2])/(b*(a^2 - b^2 + b*Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (
a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2
, 2])/(b*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{b}$$

$$= \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2b\sqrt{-a^2+b^2}} + \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}+b\cos(e+fx)\right)}}{2b\sqrt{-a^2+b^2}}$$

$$= \frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}} - \frac{(2ag)\,\text{Subst}\left(\int \frac{1}{(a^2-b^2)g^2+b^2x^4}\,dx, x, \sqrt{g\cos}\right.}{f}$$

$$= \frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}} - \frac{a^2\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e+fx)\right|}{b\left(a^2-b^2+b\sqrt{-a^2+b^2}\right)f\sqrt{g\cos(e+f}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}\right.}{bf\sqrt{g\cos(e+}}$$

Mathematica [C] time = 6.28664, size = 546, normalized size = 1.55

$$-\frac{2\sqrt{\cos(e+fx)}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{5b(a^2-b^2)\sqrt{\sin^2(e+fx)}\sqrt{\cos(e+fx)}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\right.}{\left(a^2+b^2\cos^2(e+fx)-b^2\right)\left(2\cos^2(e+fx)\left(2b^2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)+(a^2-b^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4}\right.\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*(a + b*Sqrt[Sin[e + f*x]^2])*((a*(-2*ArcTan[1 - (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[S
qrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*C
os[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
*Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]^2])/((a^2 - b^2 + b^2*Cos[e + f*x]^2)
*(-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2))))/(f
*Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x]))
```

Maple [C] time = 5.057, size = 1181, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -2/f*a*g*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2
*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*
```

```
g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)
*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)/b*sum(_alpha/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2
*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(
1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^
3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)
^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2
*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3
)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*
_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*
(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^
2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(
1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=Root
Of(4*_Z^4*b^2-4*_Z^2*b^2+a^2))/a^2*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)/b*sum(_alpha/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2
)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi
(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_al
pha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*Elliptic
Pi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a
^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)
*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+
2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+
1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*
f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*
f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4
*_Z^2*b^2+a^2))/a^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sin(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sin(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

3.1393 $\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=369

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b\sqrt{\cos(e+fx)}\Pi(}{f\left(a^2-b\left(\sqrt{b^2-\right.\right.}}$$

```
[Out] -(ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*Sqrt[g])) + (b^(3/2)*ArcTan[(Sq
rt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^
(3/4)*f*Sqrt[g]) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*Sqrt[g]) + (b
^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) - (b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)
/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))
*f*Sqrt[g*Cos[e + f*x]]) - (b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*
Cos[e + f*x]])
```

Rubi [A] time = 0.818003, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2898, 2565, 329, 212, 206, 203, 2702, 2807, 2805, 208, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b\sqrt{\cos(e+fx)}\Pi(}{f\left(a^2-b\left(\sqrt{b^2-\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```


```
[Out] -(ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*Sqrt[g])) + (b^(3/2)*ArcTan[(Sq
rt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^
(3/4)*f*Sqrt[g]) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*Sqrt[g]) + (b
^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) - (b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)
/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))
*f*Sqrt[g*Cos[e + f*x]]) - (b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*
Cos[e + f*x]])
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx = \int \left(\frac{\csc(e+fx)}{a\sqrt{g\cos(e+fx)}} - \frac{b}{a\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{a}$$

$$= \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2\sqrt{-a^2+b^2}} + \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}+b\cos(e+fx)\right)}\,dx}{2\sqrt{-a^2+b^2}} - \cdots$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{afg} - \frac{\left(2b^2g\right)\text{Subst}\left(\int \frac{1}{(a^2-b^2)g^2+b^2x^4}\,dx, \cdots\right.}{af}$$

$$= -\frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)f\sqrt{g\cos(e+fx)}} - \frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{-a^2+b^2}};\frac{1}{2}\cdots\right.}{\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)f\sqrt{g\cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af\sqrt{g}} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af\sqrt{g}} + \cdots$$

Mathematica [C] time = 20.9325, size = 698, normalized size = 1.89

$$-\frac{2\sqrt{\cos(e+fx)}\left(\cos^2(e+fx)-1\right)\csc(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{\cdots}{\sqrt{1-\cos^2(e+fx)}\left(a^2+b^2\left(\cos^2(e+fx)-1\right)\right)\left(5\left(a^2-b^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4}\cdots\right.\right.}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2
])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2
, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e +
f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[
e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x
]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1
, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*
(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2
]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2
)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e
 + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/
2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x
]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^
(3/4))))/(f*Sqrt[g*Cos[e + f*x]]*(1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))
```

Maple [A] time = 2.322, size = 186, normalized size = 0.5

$$\frac{1}{af}\ln\left(2\,\frac{\sqrt{-g}\sqrt{-2\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}-g}{\cos\left(1/2\,fx+e/2\right)}\right)\frac{1}{\sqrt{-g}}-\frac{1}{2\,af}\ln\left(2\,\frac{\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}-2\,g\cos\left(1/\right.}{\cos\left(1/2\,fx+e/2\right)+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/a/(-g)^(1/2)/f*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-g))-1/2/a/g^(1/2)/f*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*
sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))-1/2/a/g^(1/2)/f*
ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2
*g*cos(1/2*f*x+1/2*e)-g))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(csc(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{g\cos\left(e+fx\right)}\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)}{\sqrt{g\cos(fx+e)}\,(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(csc(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

3.1394 $\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=448

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{b^2\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^2\sqrt{\cos(e+}}{af\left(a^2-b\left(\right.\right.}$$

```
[Out] (b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*Sqrt[g]) - (b^(5/2)*ArcTan[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 +
b^2)^(3/4)*f*Sqrt[g]) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*Sq
rt[g]) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4
)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) - (Sqrt[g*Cos[e + f*x]]*Csc
[e + f*x])/(a*f*g) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(a*f*Sq
rt[g*Cos[e + f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a
^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2 + b*Sqrt[-a^2 + b^2])*f*Sqrt[g*C
os[e + f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e +
 f*x]])
```

Rubi [A] time = 0.975404, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2898, 2565, 329, 212, 206, 203, 2570, 2642, 2641, 2702, 2807, 2805, 208, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{b^2\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^2\sqrt{\cos(e+}}{af\left(a^2-b\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*Sqrt[g]) - (b^(5/2)*ArcTan[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 +
b^2)^(3/4)*f*Sqrt[g]) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*Sq
rt[g]) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4
)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) - (Sqrt[g*Cos[e + f*x]]*Csc
[e + f*x])/(a*f*g) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(a*f*Sq
rt[g*Cos[e + f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a
^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2 + b*Sqrt[-a^2 + b^2])*f*Sqrt[g*C
os[e + f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e +
 f*x]])
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx &= \int \left(-\frac{b\csc(e+fx)}{a^2\sqrt{g\cos(e+fx)}}+\frac{\csc^2(e+fx)}{a\sqrt{g\cos(e+fx)}}+\frac{b^2}{a^2\sqrt{g\cos(e+fx)}(a+b\sin(e\right.\\
&= \frac{\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a}-\frac{b\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a^2}+\frac{b^2\int \frac{1}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{a^2}\\
&= -\frac{\sqrt{g\cos(e+fx)}\csc(e+fx)}{afg}+\frac{\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{2a}-\frac{b^2\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}\right.}}{2a\sqrt{-a^2+\right.}}\\
&= -\frac{\sqrt{g\cos(e+fx)}\csc(e+fx)}{afg}+\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{1-\frac{x^4}{g^2}}\,dx,x,\sqrt{g\cos(e+fx)}\right)}{a^2fg}\\
&= -\frac{\sqrt{g\cos(e+fx)}\csc(e+fx)}{afg}+\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{g\cos(e+fx)}}+\frac{b^2\sqrt{\cos(e}}{a\left(a^2-b^2\right.}\\
&= \frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f\sqrt{g}}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right.}{a^2f\sqrt{g}}
\end{aligned}$$

Mathematica [C] time = 30.1144, size = 2093, normalized size = 4.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -(Cot[e + f*x]/(a*f*Sqrt[g*Cos[e + f*x]])) - (Sqrt[Cos[e + f*x]]*((4*a*(a +
 b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt
[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]
^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF
1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos
[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*A
rcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*C
os[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e
 + f*x]^2]*(b + a*Csc[e + f*x])) - (b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 -
 Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)
*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqr
t[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))
- (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 +
b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*
(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b
^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1
 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*
Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] +
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a
*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]
^2)*(b + a*Csc[e + f*x])) - (6*b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[
e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1
- Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5
/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e
+ f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1
 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3
/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*
(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 -
Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqr
t[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[C
os[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a
^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a*f*Sqr
t[g*Cos[e + f*x]])
```

Maple [C] time = 10.609, size = 1217, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/8*(-4*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*g^(3/2)*(-
g)^(1/2)*a+4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^2
*g+g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b*(4*g
^(5/2)*(-g)^(1/2)*sum(1/(_R^6*b^2-3*_R^4*b^2*g+8*_R^2*a^2*g^2-5*_R^2*b^2*g^
2-b^2*g^3)*_R*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(
1/2)*2^(1/2)-_R)*(_R^2+g),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^
2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))*b^2-2*g^(3/2)*ln(2/cos(1/2*f*x+1/2*e)*
((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))+ln(2/(-1+cos(1/2*f*x+1/
2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g
))*(-g)^(1/2)*g+(-g)^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*g)+(-8*(-2*sin(1/2*f*x+
1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/
2))*g^(3/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(
-g)^(1/2)*a-g^(7/2)*sin(1/2*f*x+1/2*e)^4*(2*sin(1/2*f*x+1/2*e)^2-1)^2/a*sum
(1/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2
*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^
(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(
sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos
(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2
-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1
/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*
b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)
^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/
2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^
2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^
(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=Roo
tOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-g)^(1/2))*cos(1/2*f*x+1/2*e)+8*(-2*sin(1/
2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*g^(3/2)*(-g)^(1/2)*a*sin(1/2
*f*x+1/2*e)^2)/a^2/g^(3/2)/(-g)^(1/2)/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/
2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/sin(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{2}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^2}{\sqrt{g\cos(fx+e)}\,(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

3.1395 $\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=557

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}-\frac{b^3\sqrt{c}}{a^2f\left(a\right.}$$

```
[Out] (-3*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f*Sqrt[g]) - (b^2*ArcTan[Sqr
t[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f*Sqrt[g]) + (b^(7/2)*ArcTan[(Sqrt[b]*Sqrt
[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 + b^2)^(3/4)*f*
Sqrt[g]) - (3*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f*Sqrt[g]) - (b^2
*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f*Sqrt[g]) + (b^(7/2)*ArcTanh[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 +
b^2)^(3/4)*f*Sqrt[g]) + (b*Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a^2*f*g) - (
Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(2*a*f*g) - (b*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2])/(a^2*f*Sqrt[g*Cos[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(a^2 - b*
(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(a^2 - b*(b +
 Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.03165, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2898, 2565, 329, 212, 206, 203, 2570, 2642, 2641, 290, 2702, 2807, 2805, 208, 205}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}-\frac{b^3\sqrt{c}}{a^2f\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-3*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f*Sqrt[g]) - (b^2*ArcTan[Sqr
t[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f*Sqrt[g]) + (b^(7/2)*ArcTan[(Sqrt[b]*Sqrt
[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 + b^2)^(3/4)*f*
Sqrt[g]) - (3*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f*Sqrt[g]) - (b^2
*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f*Sqrt[g]) + (b^(7/2)*ArcTanh[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 +
b^2)^(3/4)*f*Sqrt[g]) + (b*Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a^2*f*g) - (
Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(2*a*f*g) - (b*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2])/(a^2*f*Sqrt[g*Cos[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(a^2 - b*
(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(a^2 - b*(b +
 Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
```

g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/2, 0])

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}dx = \int \left(\frac{b^2\csc(e+fx)}{a^3\sqrt{g\cos(e+fx)}} - \frac{b\csc^2(e+fx)}{a^2\sqrt{g\cos(e+fx)}} + \frac{\csc^3(e+fx)}{a\sqrt{g\cos(e+fx)}} - \frac{}{a^3\sqrt{g\cos(e+}}\right.$$

$$= \frac{\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}}dx}{a} - \frac{b\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}}dx}{a^2} + \frac{b^2\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}}dx}{a^3} - \frac{b^3\int \frac{}{\sqrt{g\cos(e+fx)}(}}{a^3}$$

$$= \frac{b\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2fg} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}}dx}{2a^2} + \frac{b^3\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\right.}}{2a^2\sqrt{-a^2+b^2}}$$

$$= \frac{b\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2fg} - \frac{\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{2afg} - \frac{3\,\mathrm{Subst}\left(\int \frac{}{\sqrt{x}(}\right.}{}$$

$$= \frac{b\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2fg} - \frac{\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{2afg} - \frac{b\sqrt{\cos(e+fx)}}{a^2f\sqrt{g\cos}}$$

$$= -\frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^3\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{b^2\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right.}{a^3f\sqrt{g}}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af\sqrt{g}} - \frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^3\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}$$

Mathematica [C] time = 30.5066, size = 2129, normalized size = 3.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]*((b*Csc[e + f*x])/a^2 - Csc[e + f*x]^2/(2*a)))/(f*Sqrt[g*Cos[
e + f*x]]) + (Sqrt[Cos[e + f*x]]*((-2*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*
((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2
- b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f
*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 +
 Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2
+ b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e
+ f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x
])) - (b^2*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e
+ f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (
10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a
^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f
*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
```

```
x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)
]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4
, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^
2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b
^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2)))
+ (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a -
 (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))
+ (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))
))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) -
 (2*(3*a^2 + 3*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*
Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x
]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2
, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^
2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*S
qrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1
+ (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(
3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e +
f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*
Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/
4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Sqrt[g*Cos[e +
 f*x]])
```

Maple [A] time = 3.082, size = 315, normalized size = 0.6

$$-\frac{3}{8af}\ln\left(\left(4g\cos\left(1/2\,fx+e/2\right)+2\sqrt{g}\sqrt{-2\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}-2g\right)\left(-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-1}\right)\frac{1}{\sqrt{g}}-\frac{1}{16afg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -3/8/f/a/g^(1/2)*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e
)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))-1/16/f/a/g/(cos(1/2*f*x+1/2*e)
+1)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-3/8/f/a/g^(1/2)*ln((-4*g*cos(1/2*f*
x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/
2*e)+1))+3/4/f/a/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g
-g)^(1/2))/cos(1/2*f*x+1/2*e))+1/16/f/a/g/(-1+cos(1/2*f*x+1/2*e))*(-2*sin(1
/2*f*x+1/2*e)^2*g+g)^(1/2)-1/8/f/a/g/cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/
2*e)^2*g-g)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3\left(e+fx\right)}{\sqrt{g\cos\left(e+fx\right)}\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

3.1396 $\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=584

$$\frac{2a^2(g\cos(e+fx))^{3/2}}{3bfg^3(a^2-b^2)}-\frac{2b(g\cos(e+fx))^{3/2}}{3fg^3(a^2-b^2)}+\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}fg^{3/2}(b^2-a^2)^{5/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}fg^{3/2}(b^2-a^2)^{5/4}}+\frac{2a^3E\left(\frac{1}{2}(e+f\right.}{b^2fg^2(a^2-}$$

```
[Out] (a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
b^(5/2)*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(5/4)*f*g^(3/
2)) - (2*b)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a^2*(g*Cos[e + f*x]
)^(3/2))/(3*b*(a^2 - b^2)*f*g^3) - (2*b*(g*Cos[e + f*x])^(3/2))/(3*(a^2 - b
^2)*f*g^3) - (4*a*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b
^2)*f*g^2*Sqrt[Cos[e + f*x]]) + (2*a^3*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
f*x)/2, 2])/(b^2*(a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 -
 b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 -
 b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*Sin[e + f*x])
/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.27586, antiderivative size = 584, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2902, 2566, 2640, 2639, 2565, 14, 2898, 30, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{2a^2(g\cos(e+fx))^{3/2}}{3bfg^3(a^2-b^2)}-\frac{2b(g\cos(e+fx))^{3/2}}{3fg^3(a^2-b^2)}+\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}fg^{3/2}(b^2-a^2)^{5/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}fg^{3/2}(b^2-a^2)^{5/4}}+\frac{2a^3E\left(\frac{1}{2}(e+f\right.}{b^2fg^2(a^2-}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
b^(5/2)*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(5/4)*f*g^(3/
2)) - (2*b)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a^2*(g*Cos[e + f*x]
)^(3/2))/(3*b*(a^2 - b^2)*f*g^3) - (2*b*(g*Cos[e + f*x])^(3/2))/(3*(a^2 - b
^2)*f*g^3) - (4*a*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b
^2)*f*g^2*Sqrt[Cos[e + f*x]]) + (2*a^3*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
f*x)/2, 2])/(b^2*(a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 -
 b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 -
 b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*Sin[e + f*x])
/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```


Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
```

```
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sqrt{g\cos(e+fx)}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx}{(a^2-b^2)\,g^2}$$

$$= \frac{2a\sin(e+fx)}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{(2a)\int \sqrt{g\cos(e+fx)}\,dx}{(a^2-b^2)\,g^2} - \frac{a^2\int \left(-\frac{a\sqrt{g\cos(e+f}}{b^2}\right.}{}$$

$$= \frac{2a\sin(e+fx)}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{a^3\int \sqrt{g\cos(e+fx)}\,dx}{b^2(a^2-b^2)\,g^2} - \frac{a^4\int \frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b^2(a^2-b^2)\,g^2}$$

$$= -\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2b(g\cos(e+fx))^{3/2}}{3(a^2-b^2)\,fg^3} - \frac{4a\sqrt{g\cos(e+fx)}E\left(\right.}{(a^2-b^2)\,fg^2\sqrt{\cos}}$$

$$= -\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2a^2(g\cos(e+fx))^{3/2}}{3b(a^2-b^2)\,fg^3} - \frac{2b(g\cos(e+fx))^{3/2}}{3(a^2-b^2)\,fg^3}$$

$$= -\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2a^2(g\cos(e+fx))^{3/2}}{3b(a^2-b^2)\,fg^3} - \frac{2b(g\cos(e+fx))^{3/2}}{3(a^2-b^2)\,fg^3}$$

$$= \frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}(-a^2+b^2)^{5/4}\,fg^{3/2}} - \frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}(-a^2+b^2)^{5/4}\,fg^{3/2}} - \frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos}}$$

Mathematica [C] time = 26.9525, size = 820, normalized size = 1.4

$$\frac{\left(\frac{2\cos(e+fx)}{3b} + \frac{2\sec(e+fx)(a\sin(e+fx)-b)}{a^2-b^2}\right)\cos^2(e+fx)}{f(g\cos(e+fx))^{3/2}} + \frac{a\left(\frac{4ab\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{aF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\cos^{\frac{3}{2}}(e+fx)}{3(a^2-b^2)} + \left(\frac{1}{8}+\frac{i}{8}\right)\left(2\tan\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]^2*((2*Cos[e + f*x])/(3*b) + (2*Sec[e + f*x]*(-b + a*Sin[e + f
*x]))/(a^2 - b^2)))/(f*(g*Cos[e + f*x])^(3/2)) + (a*Cos[e + f*x]^(3/2)*((4*
a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b
^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^
2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 +
 b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a
^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*
x])) - ((a^2 - 2*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b^(5/2)*AppellF1[
3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e
 + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b
```

```
]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b
^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]
]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[e + f*x]^2)*(a + b*Si
n[e + f*x]))))/((a - b)*b*(a + b)*f*(g*Cos[e + f*x])^(3/2))
```

Maple [C] time = 8.009, size = 1990, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 4/3/f/g^2/b*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/3/f/g
^2/b*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-2/f/g^2/b*(g*(2*cos(1/2*f*x+1/2*e)^
2-1))^(1/2)+1/2/f/g^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*
(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/2/f/g^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*
f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/2/f/g/b*a^4/(
a-b)/(a+b)*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2
*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(
1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*
g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+4/f/g*a/(a+b)/(a-b)/(-g*(2*si
n(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*e)/(g*(2*co
s(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^
2*g)^(1/2)*cos(1/2*f*x+1/2*e)+2/f/g*a^3/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*
e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/b^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+
1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)
*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(
1/2*f*x+1/2*e)^2-1)^(1/2)-4/f/g*a/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^4-s
in(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE
(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/
2*e)^2-1)^(1/2)-1/4/f*a^3/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f
*x+1/2*e)^2))^(1/2)/b^4*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2)*sum(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2
-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2)
)*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1
/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*
f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*
x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*
g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_a
lpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^
2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(
-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4
*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/8/f*a^3/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^
4-sin(1/2*f*x+1/2*e)^2))^(1/2)/b^4*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)*sum(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x
+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^
2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alp
ha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi
(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2
+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+s
in(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^
2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/
2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*
```

```
x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2
*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha
=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

3.1397 $\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=509

$$-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{bfg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{4bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{\ldots}}{fg^2\left(a^2-b^2\right)\sqrt{\ldots}}$$

```
[Out] -((a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(5/4)*f*g^(3/2))) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(5/4)*f*g^
(3/2)) + (2*a)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a^2*Sqrt[g*Cos[e
 + f*x]]*EllipticE[(e + f*x)/2, 2])/(b*(a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]
) + (4*b*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2
*Sqrt[Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*S
qrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-
a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*S
qrt[g*Cos[e + f*x]]) - (2*b*Sin[e + f*x])/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f
*x]])
```

Rubi [A] time = 1.06746, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2902, 2565, 30, 2566, 2640, 2639, 2867, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{bfg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{4bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{\ldots}}{fg^2\left(a^2-b^2\right)\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] -((a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(5/4)*f*g^(3/2))) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(5/4)*f*g^
(3/2)) + (2*a)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a^2*Sqrt[g*Cos[e
 + f*x]]*EllipticE[(e + f*x)/2, 2])/(b*(a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]
) + (4*b*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2
*Sqrt[Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*S
qrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-
a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*S
qrt[g*Cos[e + f*x]]) - (2*b*Sin[e + f*x])/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f
*x]])
```


Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
```

```
f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sqrt{g\cos(e+fx)}\sin(e+fx)}{a+b\sin(e+fx)}\,dx}{\left(a^2-b^2\right)g^2}$$

$$= -\frac{2b\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{a^2\int \sqrt{g\cos(e+fx)}\,dx}{b\left(a^2-b^2\right)g^2} + \frac{a^3\int \frac{\sqrt{g\cos(e+f}}{a+b\sin(e+f}}{b\left(a^2-b^2\right)}$$

$$= \frac{2a}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2b\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{a^4\int \frac{}{\sqrt{g\cos(}}}{}$$

$$= \frac{2a}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2a^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} + \frac{4b\sqrt{}}{}$$

$$= \frac{2a}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2a^2\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} + \frac{4b\sqrt{}}{}$$

$$= -\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} + \frac{2a}{\left(a^2-b^2\right)fg\sqrt{}}$$

Mathematica [C] time = 26.8789, size = 793, normalized size = 1.56

$$\frac{2\cos(e+fx)(a-b\sin(e+fx))}{f\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}-\frac{\cos^{\frac{3}{2}}(e+fx)\left(-\frac{\left(a^2-2b^2\right)\sin^2(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos}{b^2}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2)
) - (Cos[e + f*x]^(3/2)*((4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appell
F1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos
[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt
[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*S
qrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt
[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*
Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e
 + f*x]^2]*(a + b*Sin[e + f*x])) - ((a^2 - 2*b^2)*(a + b*Sqrt[1 - Cos[e + f
*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(
2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*Ar
cTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt
[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[
e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Co
s[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1
 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/((a - b)*(a + b)*f*(g*Cos[e + f*
x])^(3/2))
```

Maple [C] time = 9.466, size = 1613, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/f/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)+1/2/f/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-
1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/2/f/g*a^3/(a-b)/(a+b)*su
m((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b
^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e
)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^
2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)/b/g*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(
1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*
sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2
*e),2^(1/2))-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/
b/g/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/
2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-8/f*(g*
```

```
(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2*sin(1/2*f*x+1/
2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^2
-1)*cos(1/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(
1/2)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2/si
n(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*
f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4
*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+8/f*
(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2/sin(1/2*f*x
+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e
)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x
+1/2*e)-4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2
/sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin
(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2
*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))
-1/4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b^3/g/sin(
1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2/(a-b)/(a+b)*sum((-2
*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+sin(1/2*f*x+1/2*e)^2*a^2+2*b^2*_alpha^2-
a^2)/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2
)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*
b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b
^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))
^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(
1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-
1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_
alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{3}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

3.1398 $\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=453

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}$$

```
[Out] (a^2*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
Sqrt[b]*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(5/4)*f*g^(3/
2)) - (2*b)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*Sqrt[g*Cos[e + f*
x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]) - (a^
3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2,
2])/(b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) - (a^3*
Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2]
)/(b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*Si
n[e + f*x])/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 0.880679, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {2902, 2636, 2640, 2639, 2565, 30, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (a^2*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
Sqrt[b]*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(5/4)*f*g^(3/
2)) - (2*b)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*Sqrt[g*Cos[e + f*
x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]) - (a^
3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2,
2])/(b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) - (a^3*
Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2]
)/(b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*Si
n[e + f*x])/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```


Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{\left(a^2-b^2\right)g^2}$$

$$= \frac{2a\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{a\int \sqrt{g\cos(e+fx)}\,dx}{\left(a^2-b^2\right)g^2} + \frac{a^3\int \frac{}{\sqrt{g\cos(e+fx)}(}}{2b\,(a}$$

$$= -\frac{2b}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2a\sin(e+fx)}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{\left(2a^2b\right)\mathrm{Su}}{}$$

$$= -\frac{2b}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2a\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} - \frac{a^3}{b\,(a}$$

$$= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{5/4}fg^{3/2}} - \frac{2b}{\left(a^2-b^2\right)fg\sqrt{g}}$$

Mathematica [C] time = 16.6881, size = 785, normalized size = 1.73

$$\frac{2\cos(e+fx)(a\sin(e+fx)-b)}{f\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}} - \frac{a\cos^{\frac{3}{2}}(e+fx)\left(-\frac{\sin^2(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e}{b^2-a^2}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(-b + a*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2
)) - (a*Cos[e + f*x]^(3/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appe
```

```
llF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*C
os[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)
*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sq
rt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*
b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos
[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b
^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
 (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sq
rt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] +
Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 - Cos[e + f
*x]^2)*(a + b*Sin[e + f*x]))))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2))
```

Maple [C] time = 7.4, size = 1105, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/2/f/g^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)-1/2/f/g^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1
/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/2/f/g*b*a^2/(a-b)/(a+b)*s
um((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*
b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*
e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g
^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-4/f/g*a/(a+b)/(a-b)/sin(1/2*f*x+1/2*e)/(g
*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3-2/f/g*a/(a+b)/(a-b)
/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e
)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos
(1/2*f*x+1/2*e)^2+1)^(1/2)*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)
^2)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+4/f/g*a/(a+b)/(a-b)/sin(1/2
*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)+1/8/f/g
*a/b^2/(a+b)/(a-b)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*
sum(1/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e
)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),
-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2
*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e
),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*
e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2
*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x
+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/
2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*si
n(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(g*(
2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^2}{(g\cos(fx+e))^{\frac{3}{2}}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^2}{(g\cos(fx+e))^{\frac{3}{2}}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

3.1399 $\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=413

$$-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2(a-b\sin(e+f}{fg\left(a^2-b^2\right)\sqrt{g\cos(}$$

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/((-a^2 + b^2)^(5/4)*f*g^(3/2))) + (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[g
*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(5/4)*f*g^(3/2
)) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^
2*Sqrt[Cos[e + f*x]]) + (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[
-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt
[g*Cos[e + f*x]]) + (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*C
os[e + f*x]]) + (2*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*
x]])
```

Rubi [A] time = 0.953311, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2(a-b\sin(e+f}{fg\left(a^2-b^2\right)\sqrt{g\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/((-a^2 + b^2)^(5/4)*f*g^(3/2))) + (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[g
*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(5/4)*f*g^(3/2
)) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^
2*Sqrt[Cos[e + f*x]]) + (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[
-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt
[g*Cos[e + f*x]]) + (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*C
os[e + f*x]]) + (2*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*
x]])
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{2(a-b\sin(e+fx))}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2\int \frac{\sqrt{g\cos(e+fx)}\left(-ab-\frac{1}{2}b^2\sin(e+fx)\right)}{a+b\sin(e+fx)}\,dx}{\left(a^2-b^2\right)g^2}$$

$$= \frac{2(a-b\sin(e+fx))}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{b\int \sqrt{g\cos(e+fx)}\,dx}{\left(a^2-b^2\right)g^2} + \frac{(ab)\int \frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,d}{\left(a^2-b^2\right)g^2}$$

$$= \frac{2(a-b\sin(e+fx))}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2\left(a^2-b^2\right)g} + \frac{a^2\int}{}$$

$$= \frac{2b\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} + \frac{2(a-b\sin(e+fx))}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{\left(2ab^2\right)\cdots}{}$$

$$= \frac{2b\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} + \frac{a^2\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e+f\cdots\right.}{\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)fg\sqrt{g\cos(e\cdots}}$$

$$= -\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{5/4}fg^{3/2}} + \frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{5/4}fg^{3/2}} + \frac{2b\sqrt{g\cos(e+\cdots}}{\left(a^2-b^2\right)\cdots}$$

Mathematica [C] time = 16.3689, size = 783, normalized size = 1.9

$$\frac{2\cos(e+fx)(a-b\sin(e+fx))}{f\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}} + \frac{b\cos^{\frac{3}{2}}(e+fx)\left(-\frac{\sin^2(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(e+fx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right.\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2)
) + (b*Cos[e + f*x]^(3/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appel
lF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Co
s[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqr
t[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b
*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[
e + f*x]^2]*(a + b*Sin[e + f*x])) - ((a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b^
```

```
(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + L
og[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
 b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 - Cos[e + f*
x]^2)*(a + b*Sin[e + f*x]))))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2))
```

Maple [C] time = 8.036, size = 1938, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2/f/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)+1/2/f/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-
1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/2/f/g*a*b^2/(a-b)/(a+b)*
sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3
*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2
*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*
g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-8/f/g*b/(a+b)/(a-b)*sin(1/2*f*x+1/2*e)/(
g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3-4/f/g*b/(a+b)/(a-b
)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*sin(1/2*f*x+1/2*
e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*co
s(1/2*f*x+1/2*e)^2+1)^(1/2)*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e
)^2)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+8/f/g*b/(a+b)/(a-b)/sin(1/
2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos(1/2*f*x+1/2*e)^3+8/f/
g*b/(a+b)/(a-b)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*cos
(1/2*f*x+1/2*e)+4/f/g*b/(a+b)/(a-b)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x
+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(
sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*(g*(2*cos(1/2
*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2
^(1/2))-8/f/g*b/(a+b)/(a-b)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)*cos(1/2*f*x+1/2*e)+1/4/f/g/b/a^2/(a+b)/(a-b)*sin(1/2*f*x+1/2*e)/(g
*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*sum((2*_alpha^2*b^2-a^2)/_alpha/(2*_alpha^2-1)*(8*(g*(2*_
alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*
f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1)
,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/
2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-
1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/
2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*
f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2
)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-si
n(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+
a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^
(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/4/f/g/b/a^2/(a+b)/(a-b)/s
in(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum((2*_alpha^2*b^2-
a^2)/_alpha/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin
(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1
/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(s
in(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos
(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*co
```

```
s(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)
*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)
^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/
2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/
2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*
b^2+a^2))*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

3.1400 $\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=507

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2b(b-a\sin(e+f}{afg\left(a^2-b^2\right)\sqrt{g\cos(}$$

```
[Out] ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(3/2)) - (b^(5/2)*ArcTan[(Sqrt[
b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^(5/
4)*f*g^(3/2)) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(3/2)) + (b^(5
/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
a*(-a^2 + b^2)^(5/4)*f*g^(3/2)) + 2/(a*f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2*Sqrt[Cos[e
+ f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (e + f*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*
x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e
+ f*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]])
 + (2*b*(b - a*Sin[e + f*x]))/(a*(a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.3614, antiderivative size = 507, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {2898, 2565, 325, 329, 298, 203, 206, 2696, 2867, 2640, 2639, 2701, 2807, 2805, 205, 208}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2b(b-a\sin(e+f}{afg\left(a^2-b^2\right)\sqrt{g\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(3/2)) - (b^(5/2)*ArcTan[(Sqrt[
b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^(5/
4)*f*g^(3/2)) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(3/2)) + (b^(5
/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
a*(-a^2 + b^2)^(5/4)*f*g^(3/2)) + 2/(a*f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2*Sqrt[Cos[e
+ f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (e + f*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*
x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e
+ f*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]])
 + (2*b*(b - a*Sin[e + f*x]))/(a*(a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \int \left(\frac{\csc(e+fx)}{a(g\cos(e+fx))^{3/2}} - \frac{b}{a(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{1}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx}{a}$$

$$= \frac{2b(b-a\sin(e+fx))}{a\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{(2b)\int \frac{\sqrt{g\cos(e+fx)}\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{1}{2}ab\sin(e+fx)\right)}{a+b\sin(e+fx)}\,dx}{a\left(a^2-b^2\right)g^2}$$

$$= \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b(b-a\sin(e+fx))}{a\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx \ldots\right.}{af\ldots}$$

$$= \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b(b-a\sin(e+fx))}{a\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{g^2}}\,d\ldots\right.}{a\ldots}$$

$$= \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} + \frac{2b(b-a\ldots}{a\left(a^2-b^2\right)f\ldots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}} + \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b\ldots}{\ldots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\left(-a^2+b^2\right)^{5/4}fg^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}}\ldots$$

Mathematica [C] time = 27.5923, size = 1587, normalized size = 3.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -(Cos[e + f*x]^(3/2)*((8*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF1[
3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e
+ f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt
[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a
^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos
[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^2]*(b +
a*Csc[e + f*x])) - ((-2*a^2 + b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Co
s[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 -
 (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 -
b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b*AppellF1[3/
4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e +
f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 - Sqrt[Cos[e +
 f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqrt[Cos[e + f*
x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sq
```

```
rt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + 3*Sqrt[2]*Sq
rt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(1 - Cos[e +
f*x]^2)*(b + a*Csc[e + f*x])) - (Sqrt[b]*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[
1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]*(a^2 - b^2)
^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 -
 b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[
2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*(a^2 - b^2)*
ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1, 7/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 48*a*b^(5
/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
+ b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqrt[Cos[e + f*
x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21*Sqrt[2]*(a^
2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^2 - b^2)^(3/
4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)
*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(2*(a - b)*(a + b)*f*(g*Co
s[e + f*x])^(3/2)) + (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(
g*Cos[e + f*x])^(3/2))
```

Maple [A] time = 3.954, size = 425, normalized size = 0.8

$$\frac{1}{2\,af}\left(-\left(4\,\ln\left(2\,\frac{\sqrt{-g}\sqrt{-2\,\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}-g}{\cos\left(1/2\,fx+e/2\right)}\right)g^{5/2}+2\,\ln\left(2\,\frac{\sqrt{g}\sqrt{-2\,\left(\sin\left(1/2\,fx+e/2\right)\right)^{2}g+g}+2\,g\cos\left(1/\right.}{-1+\cos\left(1/2\,fx+e/2\right)}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/2*(-(4*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)-g))*g^(5/2)+2*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g^2+2*ln(2/(cos(1/
2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*
x+1/2*e)-g))*(-g)^(1/2)*g^2)*sin(1/2*f*x+1/2*e)^2+2*ln(2/cos(1/2*f*x+1/2*e)
*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))*g^(5/2)-4*(-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)*g^(3/2)*(-g)^(1/2)+ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^
(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(
1/2)*g^2+ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)
^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g^2)/g^(7/2)/(-g)^(1/2)/a/(2*s
in(1/2*f*x+1/2*e)^2-1)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

3.1401 $\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=627

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{afg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b^2(b-a\sin(e+f\dots}{a^2fg\left(a^2-b^2\right)\sqrt{g\cos(\dots}}$$

```
[Out] -((b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(3/2))) + (b^(7/2)*ArcT
an[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2
 + b^2)^(5/4)*f*g^(3/2)) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f
*g^(3/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(
1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (2*b)/(a^2*f*g*Sqrt[g*
Cos[e + f*x]]) - Csc[e + f*x]/(a*f*g*Sqrt[g*Cos[e + f*x]]) - (3*Sqrt[g*Cos[
e + f*x]]*EllipticE[(e + f*x)/2, 2])/(a*f*g^2*Sqrt[Cos[e + f*x]]) - (2*b^2*
Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(a*(a^2 - b^2)*f*g^2*Sqrt[C
os[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos
[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e
 + f*x]]) + (3*Sin[e + f*x])/(a*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b^2*(b - a*S
in[e + f*x]))/(a^2*(a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rubi [A] time = 1.49328, antiderivative size = 627, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2898, 2565, 325, 329, 298, 203, 206, 2570, 2636, 2640, 2639, 2696, 2867, 2701, 2807, 2805, 205, 208}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{afg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b^2(b-a\sin(e+f\dots}{a^2fg\left(a^2-b^2\right)\sqrt{g\cos(\dots}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] -((b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(3/2))) + (b^(7/2)*ArcT
an[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2
 + b^2)^(5/4)*f*g^(3/2)) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f
*g^(3/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(
1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (2*b)/(a^2*f*g*Sqrt[g*
Cos[e + f*x]]) - Csc[e + f*x]/(a*f*g*Sqrt[g*Cos[e + f*x]]) - (3*Sqrt[g*Cos[
e + f*x]]*EllipticE[(e + f*x)/2, 2])/(a*f*g^2*Sqrt[Cos[e + f*x]]) - (2*b^2*
Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(a*(a^2 - b^2)*f*g^2*Sqrt[C
os[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos
[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e
 + f*x]]) + (3*Sin[e + f*x])/(a*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b^2*(b - a*S
in[e + f*x]))/(a^2*(a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
```

```
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \int \left(-\frac{b\csc(e+fx)}{a^2(g\cos(e+fx))^{3/2}} + \frac{\csc^2(e+fx)}{a(g\cos(e+fx))^{3/2}} + \frac{b^2}{a^2(g\cos(e+fx))^{3/2}(a\right.$$

$$= \frac{\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2} + \frac{b^2\int \frac{1}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx)}}{a^2}$$

$$= -\frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}} - \frac{2b^2(b-a\sin(e+fx))}{a^2\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{3\int \frac{1}{(g\cos(e+fx))^{3/}}}{2a}$$

$$= -\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}} - \frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}} + \frac{3\sin(e+fx)}{afg\sqrt{g\cos(e+fx)}} - \frac{}{a^2}$$

$$= -\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}} - \frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}} + \frac{3\sin(e+fx)}{afg\sqrt{g\cos(e+fx)}} - \frac{}{a^2}$$

$$= -\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}} - \frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}} - \frac{3\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+\right.}{afg^2\sqrt{\cos(e+fx)}}$$

$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}} - \frac{2b}{a^2fg\sqrt{g\cos(e+fx)}}$$

$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2\left(-a^2+b^2\right)^{5/4}fg^{3/2}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e}}{\sqrt{g}}\right.}{a^2fg^{3/2}}$$

Mathematica [C] time = 28.3068, size = 1635, normalized size = 2.61

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -(Cos[e + f*x]^(3/2)*((-2*(6*a^3 + 2*a*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2]
)*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - (
(1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]
] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e +
f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e
```

```
+ f*x]^2]*(b + a*Csc[e + f*x])) - ((7*a^2*b - 5*b^3)*(-1 + Cos[e + f*x]^2)*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)]
- 6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[
e + f*x]])/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] +
 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)]*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Lo
g[1 - Sqrt[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1
 + Sqrt[Cos[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 -
 b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*
x]] + 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*
b^2)*(1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - ((-3*a^2*b + b^3)*(-1 + C
os[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*
x]*(-42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a
^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)
] + 84*b^(3/2)*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*Appell
F1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos
[e + f*x]^(3/2) + 48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b
^2)*Log[1 - Sqrt[Cos[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[
e + f*x]]] + 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] -
 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*b^(3/2)
*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e +
f*x]))))/(4*a*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)) + (Cos[e + f*x]^2*(
-(Cot[e + f*x]/a) + (2*Sec[e + f*x]*(-b + a*Sin[e + f*x]))/(a^2 - b^2)))/(f
*(g*Cos[e + f*x])^(3/2))
```

Maple [C] time = 14.586, size = 3469, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f/g^(3/2)*b/(2+2^(1/2))/(2^(1/2)-2)/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(
1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))-1/f/
g^2*b/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(
1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/f/g^(3/2)*b/(2+2^(1/2))/(2^(1/2
)-2)/a^2*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g
)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))+1/f/g*b/a^2/(-g)^(1/2)*ln((-2*g+2*(-g)
^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))+1/f/g^2*b/(2
+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-
2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/2/f/g*b^5/(a-b)/(a+b)/a^2*sum((_R^6-_R^
4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b
^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2
^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b
^2*g^3*_Z^2+b^2*g^4))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e
)^2)^(1/2)/g^3*a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)^5/(2*sin(1/2*f*x+1/2
*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1
/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3/a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)^5/
(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
```

*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b^2-6/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3*a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)+2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3/a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b^2+6/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3*a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)^3/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3/a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)^3/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b^2-3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3*a/sin(1/2*f*x+1/2*e)^5/(2*sin(1/2*f*x+1/2*e)^2-1)^(3/2)/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)+1/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3/a/sin(1/2*f*x+1/2*e)^5/(2*sin(1/2*f*x+1/2*e)^2-1)^(3/2)/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b^2+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g/a^3*sin(1/2*f*x+1/2*e)^3/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*b^2-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g/a^3*sin(1/2*f*x+1/2*e)/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*b^2+1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g/a^3/sin(1/2*f*x+1/2*e)/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^2-b^2)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/

```
2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*si
n(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alp
ha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*
a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/
2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2
*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*
b^2+a^2))*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^2/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

3.1402 $\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=601

$$\frac{2a^2\sqrt{g\cos(e+fx)}}{bfg^3(a^2-b^2)}-\frac{2b\sqrt{g\cos(e+fx)}}{fg^3(a^2-b^2)}-\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{5/2}(b^2-a^2)^{7/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{5/2}(b^2-a^2)^{7/4}}+\frac{2a^3\sqrt{\cos(e+fx)}F\left(\frac{1}{2}\right.}{b^2fg^2(a^2-b^2)\sqrt{g\,c}}$$

```
[Out] -((a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(7/4)*f*g^(5/2))) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(7/4)*f*g^
(5/2)) - (2*b)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) + (2*a^2*Sqrt[g*C
os[e + f*x]])/(b*(a^2 - b^2)*f*g^3) - (2*b*Sqrt[g*Cos[e + f*x]])/((a^2 - b^
2)*f*g^3) - (4*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^
2)*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*a^3*Sqrt[Cos[e + f*x]]*EllipticF[(e + f
*x)/2, 2])/(b^2*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
 f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2
- b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (a^5*
Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2]
)/(b^2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*
x]]) + (2*a*Sin[e + f*x])/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rubi [A] time = 1.39804, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {2902, 2566, 2642, 2641, 2565, 14, 2909, 30, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{2a^2\sqrt{g\cos(e+fx)}}{bfg^3(a^2-b^2)}-\frac{2b\sqrt{g\cos(e+fx)}}{fg^3(a^2-b^2)}-\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{5/2}(b^2-a^2)^{7/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{5/2}(b^2-a^2)^{7/4}}+\frac{2a^3\sqrt{\cos(e+fx)}F\left(\frac{1}{2}\right.}{b^2fg^2(a^2-b^2)\sqrt{g\,c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(7/4)*f*g^(5/2))) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(7/4)*f*g^
(5/2)) - (2*b)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) + (2*a^2*Sqrt[g*C
os[e + f*x]])/(b*(a^2 - b^2)*f*g^3) - (2*b*Sqrt[g*Cos[e + f*x]])/((a^2 - b^
2)*f*g^3) - (4*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^
2)*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*a^3*Sqrt[Cos[e + f*x]]*EllipticF[(e + f
*x)/2, 2])/(b^2*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
 f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2
- b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (a^5*
Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2]
)/(b^2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*
x]]) + (2*a*Sin[e + f*x])/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
```

```
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}dx = \frac{a\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}}dx}{a^2-b^2} - \frac{b\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{5/2}}dx}{a^2-b^2} - \frac{a^2\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}}{(a^2-b^2)g^2}$$

$$= \frac{2a\sin(e+fx)}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} - \frac{(2a)\int \frac{1}{\sqrt{g\cos(e+fx)}}dx}{3(a^2-b^2)g^2} - \frac{a^2\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+f}}}{b(a^2-b^2)g}$$

$$= \frac{2a\sin(e+fx)}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}}dx, x, g\cos(e+fx)\right)}{b(a^2-b^2)fg^3} +$$

$$= -\frac{2b}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{2a^2\sqrt{g\cos(e+fx)}}{b(a^2-b^2)fg^3} - \frac{2b\sqrt{g\cos(e+}}{(a^2-b^2)fg}$$

$$= -\frac{2b}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{2a^2\sqrt{g\cos(e+fx)}}{b(a^2-b^2)fg^3} - \frac{2b\sqrt{g\cos(e+}}{(a^2-b^2)fg}$$

$$= -\frac{2b}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{2a^2\sqrt{g\cos(e+fx)}}{b(a^2-b^2)fg^3} - \frac{2b\sqrt{g\cos(e+}}{(a^2-b^2)fg}$$

$$= -\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}(-a^2+b^2)^{7/4}fg^{5/2}} - \frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}(-a^2+b^2)^{7/4}fg^{5/2}} - \frac{}{3(a^2-b^2)fg(}$$

Mathematica [C] time = 27.2611, size = 1958, normalized size = 3.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(-b + a*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5
/2)) + (Cos[e + f*x]^(5/2)*((-2*(-7*a^2 + 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*
x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(
5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Co
s[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^
2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqr
t[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b
*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2
]*(a + b*Sin[e + f*x])) + ((3*a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])
*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*
Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1
/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-
a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e + f*x]])/b
- (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[
```

```
1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[C
os[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1
, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*Appell
F1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (
-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + ((1/4
- I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(
1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) -
 ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 +
 b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(
3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*Cos[e + f*x]^2)*(a +
 b*Sin[e + f*x])) - (4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^
2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] +
2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*
Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x
]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)
^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/
4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e +
f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(6*
(a - b)*(a + b)*f*(g*Cos[e + f*x])^(5/2))
```

Maple [C] time = 9.641, size = 1268, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f/g^3/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-1/12/f/g^3*b/(a^2-b^2)/(cos(
1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/f/g^
3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*
e)^2*g+g)^(1/2)-1/12/f/g^3*b/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))^2*(
-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/f/g^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*
f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2/f/g/b*a^4/(a-
b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2
*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*
g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)
*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*a/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(
1/2)/b^2*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g
*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x
+1/2*e),2^(1/2))+1/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^
(1/2)*a/g^3/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^
2)*cos(1/2*f*x+1/2*e)*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1
/2)/(cos(1/2*f*x+1/2*e)^2-1/2)^2-2/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)*a/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1
))^(1/2)/(a^2-b^2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)
^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(c
os(1/2*f*x+1/2*e),2^(1/2))+1/8/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+
1/2*e)^2)^(1/2)*a^5/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(
1/2)/(a-b)/(a+b)/b^4*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^
```

```
2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1
/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^
(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-
sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/
2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2
*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_
alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

3.1403 $\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=528

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bfg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{4b\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+\right.}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e}}$$

```
[Out] (a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
Sqrt[b]*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(7/4)*f*g^(5/
2)) + (2*a)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) - (2*a^2*Sqrt[Cos[e
+ f*x]]*EllipticF[(e + f*x)/2, 2])/(b*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]
]) + (4*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^
2*Sqrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqr
t[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b
)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2 - b^2)*(a^2 - b*(b + Sqr
t[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (2*b*Sin[e + f*x])/(3*(a^2 -
b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rubi [A] time = 1.08649, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {2902, 2565, 30, 2566, 2642, 2641, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bfg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{4b\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+\right.}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
Sqrt[b]*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(7/4)*f*g^(5/
2)) + (2*a)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) - (2*a^2*Sqrt[Cos[e
+ f*x]]*EllipticF[(e + f*x)/2, 2])/(b*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]
]) + (4*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^
2*Sqrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqr
t[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b
)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2 - b^2)*(a^2 - b*(b + Sqr
t[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (2*b*Sin[e + f*x])/(3*(a^2 -
b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```


Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
```

```
f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}}{\left(a^2-b^2\right)g^2}$$

$$= -\frac{2b\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b\left(a^2-b^2\right)g^2} + \frac{a^3\int \frac{}{\sqrt{g\cos(e+f}}}{b\left(a^2-\right.}$$

$$= \frac{2a}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2b\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{a^4\int}{}$$

$$= \frac{2a}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} + \frac{4b}{3}$$

$$= \frac{2a}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} + \frac{4b}{3}$$

$$= \frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{7/4}fg^{5/2}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{7/4}fg^{5/2}} + \frac{2}{3\left(a^2-b^2\right)fg(g}$$

Mathematica [C] time = 24.43, size = 1193, normalized size = 2.26

$$\frac{2\cos(e+fx)(a-b\sin(e+fx))}{3\left(a^2-b^2\right)f(g\cos(e+fx))^{5/2}}-\frac{\cos^{\frac{5}{2}}(e+fx)\left(\frac{4ab\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{5a(a^2-b^2\cdots}{\sqrt{1-\cos^2(e+fx)}\left(5(a^2-b^2)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-2\left(2F_1\left(\frac{5}{4};\cdots\right.\right.\right)}\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5/
2)) - (Cos[e + f*x]^(5/2)*((4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a
^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)
] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e +
 f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e
 + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e +
f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 +
 b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] +
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])
)/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e
+ f*x])) - (2*(3*a^2 - 2*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 -
 b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
 + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e +
f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b
^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^
(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e
 + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/
(3*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(5/2))
```

Maple [C] time = 12.217, size = 2331, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/12/f/g^3*a/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)-1/12/f/g^3*a*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2
^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/f/g^3*a/(a^2-b^2)/(cos(1/2
*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/f/g^3*a
*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^
```

```
2*g+g)^(1/2)+2/f/g*a^3/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g
+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)
^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z
^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-8/f*(g*(2*cos(1/2*
f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3*sin(1/2*f*x+1/2*e)/(g*(2*
cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^2-1)*cos(1/2
*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)+4/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*x+1
/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^
2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*
f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+8/f*(g*(2*cos(1
/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*x+1/2*e)/(g*
(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*
sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)-4/f
*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*
x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/
2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin
(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+2/3/f*(g*(
2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3*sin(1/2*f*x+1/2
*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*f*x+1/2*e)*(-g*(
2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e)^2-1
/2)^2-2/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3
/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*
f*x+1/2*e)*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/
2*f*x+1/2*e)^2-1/2)^2-4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e
)^2)^(1/2)*b/g^2*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a
^2-b^2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*
(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+
1/2*e),2^(1/2))+4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)*b/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2
)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin
(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e)
,2^(1/2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b
/g^2*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2/(a-b)/(a+b
)*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/
2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3
*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*
b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2)
)^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos
(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2
-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),
_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-
1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1
/2*e)^2-1))^(1/2)*a^2/(a-b)/(a+b)*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*
_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^
2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^
2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*
x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin
(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/
2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4
*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^3}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{3}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

3.1404 $\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=468

$$-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{a^3\sqrt{\cos(e+}}{fg^2\left(a^2-b^2\right)\left(a^2-\right.}$$

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/((-a^2 + b^2)^(7/4)*f*g^(5/2))) - (a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(7/4)*f*g^
(5/2)) - (2*b)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) + (2*a*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x
]]) - (a^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e +
 f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos
[e + f*x]]) - (a^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sq
rt[g*Cos[e + f*x]]) + (2*a*Sin[e + f*x])/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x]
)^(3/2))
```

Rubi [A] time = 0.900721, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {2902, 2636, 2642, 2641, 2565, 30, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{a^3\sqrt{\cos(e+}}{fg^2\left(a^2-b^2\right)\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/((-a^2 + b^2)^(7/4)*f*g^(5/2))) - (a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(7/4)*f*g^
(5/2)) - (2*b)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) + (2*a*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x
]]) - (a^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e +
 f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos
[e + f*x]]) - (a^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sq
rt[g*Cos[e + f*x]]) + (2*a*Sin[e + f*x])/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x]
)^(3/2))
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{1}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}}{\left(a^2-b^2\right)g^2}$$

$$= \frac{2a\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{3\left(a^2-b^2\right)g^2} - \frac{a^3\int \frac{}{\sqrt{g\cos(e+fx)}(}}{2\left(-a^2\right.}$$

$$= -\frac{2b}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{2a\sin(e+fx)}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{(2}{}$$

$$= -\frac{2b}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} + \frac{}{(}$$

$$= -\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}} - \frac{a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}} - \frac{}{3\left(a^2-\right.}$$

Mathematica [C] time = 24.039, size = 1184, normalized size = 2.53

$$\frac{2\cos(e+fx)(a\sin(e+fx)-b)}{3\left(a^2-b^2\right)f(g\cos(e+fx))^{5/2}} - \frac{a\cos^{\frac{5}{2}}(e+fx)\left(\frac{2b\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{5b\left(a^2-b^2\right)\sqrt{\cos(e}}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)b^2+\left(a^2-b^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};c\right.\right.\right.}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(-b + a*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5
/2)) - (a*Cos[e + f*x]^(5/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*
```

```
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
 + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos
[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2]
 + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]
]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[
e + f*x])) + (2*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sq
rt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*
AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) +
(a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] +
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b
*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))
*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(3*(a - b)*(
a + b)*f*(g*Cos[e + f*x])^(5/2))
```

Maple [C] time = 9.725, size = 1089, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/12/f/g^3*b/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-2*sin(1/2*f*x+
1/2*e)^2*g+g)^(1/2)+1/12/f/g^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*
2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/f/g^3*b/(a^2-b^2)/(cos(1/
2*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/f/g^3*
b*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-2/f/g*b*a^2/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^
2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g
+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g
*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/3/f*(g*(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/g^3/sin(1/2*f*x+1/2*e)/(
g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*f*x+1/2*e)*(-g*(2*sin
(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e)^2-1/2)^2
-2/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/g^2/sin(
1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*(sin(1/2*f*x+
1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)
^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))+1/8/f
*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a^3/g^2/sin(1/2*
f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a-b)/(a+b)/b^2*sum(1/_alph
a/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(
1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/
2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/
```

```
a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/
2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*
EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=Root
Of(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^2}{(g\cos(fx+e))^{\frac{5}{2}}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^2}{(g\cos(fx+e))^{\frac{5}{2}}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

3.1405 $\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=432

$$\frac{ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2b\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{a^2b\sqrt{\cos(e+fx)}\Pi}{fg^2\left(a^2-b^2\right)\left(a^2-b\left(b\right.\right.}$$

```
[Out] (a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g
])])/((-a^2 + b^2)^(7/4)*f*g^(5/2)) + (a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Co
s[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(7/4)*f*g^(5/2))
- (2*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*S
qrt[g*Cos[e + f*x]]) + (a^2*b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (a^2*b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)
/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-
a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*(a - b*Sin[e + f*x]))/(3*(a^2
 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rubi [A] time = 0.921283, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2866, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2b\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{a^2b\sqrt{\cos(e+fx)}\Pi}{fg^2\left(a^2-b^2\right)\left(a^2-b\left(b\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g
])])/((-a^2 + b^2)^(7/4)*f*g^(5/2)) + (a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Co
s[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(7/4)*f*g^(5/2))
- (2*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*S
qrt[g*Cos[e + f*x]]) + (a^2*b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (a^2*b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)
/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-
a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*(a - b*Sin[e + f*x]))/(3*(a^2
 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```


Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2\int \frac{-ab+\frac{1}{2}b^2\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{3\left(a^2-b^2\right)g^2}$$

$$= \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{3\left(a^2-b^2\right)g^2} + \frac{(ab)\int \frac{1}{\sqrt{g\cos(e+fx)}(a+b}}{\left(a^2-b^2\right)g}$$

$$= \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{\left(a^2b\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2\left(-a^2+b^2\right)^{3/2}g^2}$$

$$= -\frac{2b\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} + \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{(2ab}{}$$

$$= -\frac{2b\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} - \frac{a^2b\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e+\right.}{\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)fg^2\sqrt{g\,c}}$$

$$= \frac{ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}} + \frac{ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}} - \frac{2b\sqrt{\cos(e+f}}{3\left(a^2-b^2\right)f}$$

Mathematica [C] time = 23.9737, size = 1183, normalized size = 2.74

$$b\left(\frac{2b\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{5b\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\sqrt{1-\cos^2(e+fx)}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)b^2+\left(a^2-b^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\right)\cos^2(e+fx)-5\left(a^2-b^2\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2}{}\right.}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5/
2)) + (b*Cos[e + f*x]^(5/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
+ f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[
```

```
e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e +
 f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
+ b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2]
+ (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]
))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e
 + f*x])) + (2*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF
1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqr
t[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1
/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*A
ppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (
a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[
Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*
Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*
Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(3*(a - b)*(a
 + b)*f*(g*Cos[e + f*x])^(5/2))
```

Maple [C] time = 11.181, size = 2322, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/12/f/g^3*a/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)-1/12/f/g^3*a*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2
^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/f/g^3*a/(a^2-b^2)/(cos(1/2
*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/f/g^3*a
*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^
2*g+g)^(1/2)+2/f/g*a*b^2/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2
*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+
g)^(1/2)-cos(1/2*f*x+1/2*e)*g^(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*
_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-8/f*(g*(2*cos(1/
2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3*sin(1/2*f*x+1/2*e)/(g*(
2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e)^2-1)*cos(1
/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)+8/f*
(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*f*x
+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+1/2*e
)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x
+1/2*e)+4/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3
/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1
/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e
)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))-4
/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3/sin(1/2*
f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)/(2*sin(1/2*f*x+
1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+s
in(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+2/3/f*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^3*sin(1/2*f*x+1
/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*f*x+1/2*e)*(-g
*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e)^2
-1/2)^2-2/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g
^3/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b^2)*cos(1/
2*f*x+1/2*e)*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(
```

```
1/2*f*x+1/2*e)^2-1/2)^2-4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*b/g^2*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/
(a^2-b^2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-
g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*
x+1/2*e),2^(1/2))+4/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)
^(1/2)*b/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a^2-b
^2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*s
in(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*
e),2^(1/2))+1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)
*b/g^2*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)/(a-b)/(a+b)*
sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b
^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^
2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^
(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1
/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1
))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_a
lpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2
*e)^2-1))^(1/2)/(a-b)/(a+b)*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha
^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*
cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^
2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*
e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8/a^2*b^2*_alpha*(_alpha^2-1)*(sin(1/2*f
*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2
*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a
^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)}{(g\cos(fx+e))^{\frac{5}{2}}\,(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)}{(g\cos(fx+e))^{\frac{5}{2}}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

3.1406 $\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=527

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2b\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^3\sqrt{\cos(e+fx)}\Pi\left(\ldots\right.}{fg^2\left(a^2-b^2\right)\left(a^2-b\left(b-\ldots\right.\right.}$$

```
[Out] -(ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(5/2))) + (b^(7/2)*ArcTan[(Sq
rt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^
(7/4)*f*g^(5/2)) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(5/2)) + (b
^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + 2/(3*a*f*g*(g*Cos[e + f*x])^(3/2)) - (
2*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*Sqrt
[g*Cos[e + f*x]]) + (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2
 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*
g^2*Sqrt[g*Cos[e + f*x]]) + (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + S
qrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b
^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*b*(b - a*Sin[e + f*x]))/(3*a*(a^2 -
b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rubi [A] time = 1.31351, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {2898, 2565, 325, 329, 212, 206, 203, 2696, 2867, 2642, 2641, 2702, 2807, 2805, 208, 205}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2b\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^3\sqrt{\cos(e+fx)}\Pi\left(\ldots\right.}{fg^2\left(a^2-b^2\right)\left(a^2-b\left(b-\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] -(ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(5/2))) + (b^(7/2)*ArcTan[(Sq
rt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^
(7/4)*f*g^(5/2)) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(5/2)) + (b
^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + 2/(3*a*f*g*(g*Cos[e + f*x])^(3/2)) - (
2*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*Sqrt
[g*Cos[e + f*x]]) + (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2
 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*
g^2*Sqrt[g*Cos[e + f*x]]) + (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + S
qrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b
^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*b*(b - a*Sin[e + f*x]))/(3*a*(a^2 -
b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \int \left(\frac{\csc(e+fx)}{a(g\cos(e+fx))^{5/2}} - \frac{b}{a(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \frac{1}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx}{a}$$

$$= \frac{2b(b-a\sin(e+fx))}{3a\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{(2b)\int \frac{-\frac{a^2}{2}+\frac{3b^2}{2}-\frac{1}{2}ab\sin(e+fx)}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{3a\left(a^2-b^2\right)g^2} - \cdots$$

$$= \frac{2}{3afg(g\cos(e+fx))^{3/2}} + \frac{2b(b-a\sin(e+fx))}{3a\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{2}{3afg(g\cos(e+fx))^{3/2}} + \frac{2b(b-a\sin(e+fx))}{3a\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{2}{3afg(g\cos(e+fx))^{3/2}} - \frac{2b\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} + \frac{2b(b\cdots}{3a\left(a^2-b^2\right)\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} + \frac{2}{3afg(g\cos(e+fx))^{3/2}} - \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\left(-a^2+b^2\right)^{7/4}fg^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} \cdots$$

Mathematica [C] time = 30.1756, size = 2136, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]^(5/2)*((-8*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
+ b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*Appell
F1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2
*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]
^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]]
)/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a
^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b^2*(-
1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e
 + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2
*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/
```

```
4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (1
6*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4,
1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e
+ f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/
4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2
+ b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 -
 Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(
2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[C
os[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*
(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - C
os[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (2*(6*a^2 -
7*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*
((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2
- b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f
*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 +
 Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[C
os[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[S
qrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a
^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2
- b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f
*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1 - C
os[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(6*(a - b)*(a + b)*f*(g*Cos[e + f*x]
)^(5/2)) + (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e
+ f*x])^(5/2))
```

Maple [A] time = 4.605, size = 627, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/6*((24*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)-g))*g^(7/2)-12*(-g)^(1/2)*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*s
in(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*g^3-12*(-g)^(1/2)
*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2
*g*cos(1/2*f*x+1/2*e)-g))*g^3)*sin(1/2*f*x+1/2*e)^4+(-24*ln(2/cos(1/2*f*x+1
/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))*g^(7/2)+12*(-g)^(
1/2)*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1
/2)+2*g*cos(1/2*f*x+1/2*e)-g))*g^3+12*(-g)^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)+1
)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*g
^3)*sin(1/2*f*x+1/2*e)^2+6*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)-g))*g^(7/2)+4*(-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g
+g)^(1/2)*g^(5/2)-3*(-g)^(1/2)*ln(2/(-1+cos(1/2*f*x+1/2*e))*(g^(1/2)*(-2*si
n(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*g^3-3*(-g)^(1/2)*l
n(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g
*cos(1/2*f*x+1/2*e)-g))*g^3)/(-g)^(1/2)/g^(11/2)/a/(4*sin(1/2*f*x+1/2*e)^4-
```

```
4*sin(1/2*f*x+1/2*e)^2+1)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)}{(g\cos(fx+e))^{\frac{5}{2}}(b\sin(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

3.1407 $\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=651

$$-\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3afg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{b^4\sqrt{\cos(e+fx)}\Pi}{afg^2\left(a^2-b^2\right)\left(a^2-b\left(b\right.\right.}$$

[Out] (b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(5/2)) - (b^(9/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(5/2)) - (b^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(7/4)*f*g^(5/2)) - (2*b)/(3*a^2*f*g*(g*Cos[e + f*x])^(3/2)) - Csc[e + f*x]/(a*f*g*(g*Cos[e + f*x])^(3/2)) + (5*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*a*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*a*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]]) - (b^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (b^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (5*Sin[e + f*x])/(3*a*f*g*(g*Cos[e + f*x])^(3/2)) - (2*b^2*(b - a*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))

Rubi [A] time = 1.46433, antiderivative size = 651, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2898, 2565, 325, 329, 212, 206, 203, 2570, 2636, 2642, 2641, 2696, 2867, 2702, 2807, 2805, 208, 205}

$$-\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2b^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3afg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{b^4\sqrt{\cos(e+fx)}\Pi}{afg^2\left(a^2-b^2\right)\left(a^2-b\left(b\right.\right.}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]

[Out] (b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(5/2)) - (b^(9/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(5/2)) - (b^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(7/4)*f*g^(5/2)) - (2*b)/(3*a^2*f*g*(g*Cos[e + f*x])^(3/2)) - Csc[e + f*x]/(a*f*g*(g*Cos[e + f*x])^(3/2)) + (5*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*a*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*a*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]]) - (b^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (b^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (5*Sin[e + f*x])/(3*a*f*g*(g*Cos[e + f*x])^(3/2)) - (2*b^2*(b - a*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \int \left(-\frac{b\csc(e+fx)}{a^2(g\cos(e+fx))^{5/2}} + \frac{\csc^2(e+fx)}{a(g\cos(e+fx))^{5/2}} + \frac{b^2}{a^2(g\cos(e+fx))^{5/2}(a\right.$$

$$= \frac{\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2} + \frac{b^2\int \frac{1}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx)}}{a^2}$$

$$= -\frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}} - \frac{2b^2(b-a\sin(e+fx))}{3a^2\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{5\int \frac{}{(g\cos(}}{2}$$

$$= -\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}} - \frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}} + \frac{5\sin(e+fx)}{3afg(g\cos(e+fx)}$$

$$= -\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}} - \frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}} + \frac{5\sin(e+fx)}{3afg(g\cos(e+fx)}$$

$$= -\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}} - \frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}} + \frac{5\sqrt{\cos(e+fx)}F\left(\frac{1}{2}\right.}{3afg^2\sqrt{g\cos(e}}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}} - \frac{2b}{3a^2fg(g\cos(e+fx))^{3/}}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}} - \frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2\left(-a^2+b^2\right)^{7/4}fg^{5/2}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+f}}{\sqrt{g}}\right.}{a^2fg^{5/2}}$$

Mathematica [C] time = 27.8648, size = 2183, normalized size = 3.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]^(5/2)*((-2*(10*a^3 - 18*a*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2
])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a
^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e
```

```
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-
1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt
[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a
^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos
[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e +
f*x])) - ((-5*a^2*b + 3*b^3)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e +
f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1
 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2
 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt
[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*Arc
Tan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (2
00*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 -
 b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*
x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 +
Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[C
os[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(
a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*
(a^2 - b^2)^(3/4))))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b +
a*Csc[e + f*x])) - (2*(-7*a^2*b + 9*b^3)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[
1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/
(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2
, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*Ap
pellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
)*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt
[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/
4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*
Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]
]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2
] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])
/(8*a*(a^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(1
2*a*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(5/2)) + (Cos[e + f*x]^3*(-(Csc[e +
f*x]/a) + (2*Sec[e + f*x]^2*(-b + a*Sin[e + f*x]))/(3*(a^2 - b^2))))/(f*(g*
Cos[e + f*x])^(5/2))
```

Maple [C] time = 16.944, size = 2312, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f/g^(5/2)*b/(2+2^(1/2))^2/(2^(1/2)-2)^2/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*
g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(-1+cos(1/2*f*x+1/2*e)))+1
/6/f/g^3*b/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2
```

```
))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/6/f/g^3*b*2^(1/2)/(2+2^(1/2))/(2
^(1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)+2/f/g^(5/2)*b/(2+2^(1/2))^2/(2^(1/2)-2)^2/a^2*ln((-4*g*cos(1/
2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*
x+1/2*e)+1))-1/f/g^2*b/a^2/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+
1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))+1/6/f/g^3*b/(2+2^(1/2))/(2^(1/2)-2
)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g
)^(1/2)+1/6/f/g^3*b*2^(1/2)/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+
1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2/f/g*b^5/(a-b)/(a+
b)/a^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g
^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-cos(1/2*f*x+1/2*e)*g^
(1/2)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_
Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+5/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*a/g/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3
/2)/(a^2-b^2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(2*si
n(1/2*f*x+1/2*e)^2-1)^(3/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticF(cos(1/2*
f*x+1/2*e),2^(1/2))-1/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)
^(1/2)/a/g/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^2-b^
2)*sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(2*sin(1/2*f*x+1
/2*e)^2-1)^(3/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),
2^(1/2))*b^2-10/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)*a/g/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g
)^(3/2)/(a^2-b^2)*sin(1/2*f*x+1/2*e)^5/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)
+2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a/g/cos(1/2*
f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^2-b^
2)*sin(1/2*f*x+1/2*e)^5/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*b^2+1/8/f*(g*(
2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a^3/g^2/(a^2-b^2)/sin
(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*sum(1/_alpha/(2*_alpha
^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*Elli
pticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha
^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^
2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4
*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+
2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^
(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16
*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_
alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha
^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x
+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+
1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z
^2*b^2+a^2))*b^2+10/3/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)
^(1/2)*a/g/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)
^2*g)^(3/2)/(a^2-b^2)*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(
1/2)-2/f*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a/g/cos(
1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^
2-b^2)*sin(1/2*f*x+1/2*e)^3/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*b^2-1/2/f*
(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a/g/cos(1/2*f*x+1
/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^2-b^2)*si
n(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+1/2/f*(g*(2*cos(1/2*f
*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a/g/cos(1/2*f*x+1/2*e)/(-2*sin(1
/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/(a^2-b^2)*sin(1/2*f*x+1/2*e
)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^2/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

3.1408 $\int \frac{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=926

$$-\frac{2\sqrt{2}a^3\sqrt{g}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}d^3}{b^3\sqrt{b-a}\sqrt{a+b}f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}a^3\sqrt{g}\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{}}{b^3\sqrt{b-a}\sqrt{a+b}f\sqrt{d\sin(e+fx)}}$$

```
[Out] (a^2*d^(5/2)*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) + (d^(5/2)*Sqrt[g]*ArcTan[1 -
(Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(4*
Sqrt[2]*b*f) - (a^2*d^(5/2)*Sqrt[g]*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[
e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) - (d^(5/2)*Sqrt
[g]*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e
 + f*x]])])/(4*Sqrt[2]*b*f) - (a^2*d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Co
t[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/
(2*Sqrt[2]*b^3*f) - (d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (
Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sqrt[2]*b*f
) + (a^2*d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^3*f) + (d^(5/
2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos
[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sqrt[2]*b*f) - (2*Sqrt[2]*a^3*d^3*Sqr
t[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(S
qrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*Sqrt[-a + b]*
Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a^3*d^3*Sqrt[g]*EllipticPi
[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Si
n[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt
[d*Sin[e + f*x]]) - (d^2*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(2*b*
f*g) - (a*d^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[
e + f*x]])/(b^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 1.84226, antiderivative size = 926, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {2909, 2568, 2575, 297, 1162, 617, 204, 1165, 628, 2572, 2639, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}a^3\sqrt{g}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}d^3}{b^3\sqrt{b-a}\sqrt{a+b}f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}a^3\sqrt{g}\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{}}{b^3\sqrt{b-a}\sqrt{a+b}f\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2))/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^2*d^(5/2)*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) + (d^(5/2)*Sqrt[g]*ArcTan[1 -
(Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(4*
Sqrt[2]*b*f) - (a^2*d^(5/2)*Sqrt[g]*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[
e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) - (d^(5/2)*Sqrt
[g]*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e
 + f*x]])])/(4*Sqrt[2]*b*f) - (a^2*d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Co
t[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/
(2*Sqrt[2]*b^3*f) - (d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (
Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sqrt[2]*b*f
) + (a^2*d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt
```

[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^3*f) + (d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sqrt[2]*b*f) - (2*Sqrt[2]*a^3*d^3*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a^3*d^3*Sqrt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) - (d^2*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(2*b*f*g) - (a*d^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(b^2*f*Sqrt[Sin[2*e + 2*f*x]])

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1, p, 1] && GtQ[n, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1))/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] && LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx = \frac{d\int \sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{b}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} - \frac{\left(ad^2\right)\int \sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,d}{b^2}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} + \frac{\left(a^2d^3\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b^3} - \frac{\left(a^3d^3\right)\int \frac{}{\sqrt{d\,\text{si}}}}{}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} - \frac{ad^2\sqrt{g\cos(e+fx)}E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d}}{b^2f\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} - \frac{ad^2\sqrt{g\cos(e+fx)}E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d}}{b^2f\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{d^{5/2}\sqrt{g}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{8\sqrt{2}bf} + \frac{d^{5/2}\sqrt{g}\log\left(\sqrt{g}+\sqrt{g}\right.}{}$$

$$= \frac{d^{5/2}\sqrt{g}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}bf} - \frac{d^{5/2}\sqrt{g}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}bf} - \frac{a^2}{}$$

$$= \frac{a^2d^{5/2}\sqrt{g}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}b^3f} + \frac{d^{5/2}\sqrt{g}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}bf} - $$

Mathematica [C] time = 27.314, size = 1626, normalized size = 1.76

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2))/(a + b*Sin[e + f*x]
),x]
```

```
[Out] -(Sqrt[g*Cos[e + f*x]]*Cot[e + f*x]*Csc[e + f*x]*(d*Sin[e + f*x])^(5/2))/(2
*b*f) + (Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*((-2*b*(-(b*AppellF1[3
/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*A
ppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)
])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/
(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (Sqrt[Tan
[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[T
an[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^
2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x
]^2]))/(12*a*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1
+ Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e +
f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2,
```

```
1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3
/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2
 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(
3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 +
Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2
)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*Ar
cTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^
2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(
a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2
]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^
(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*L
og[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b
^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a
]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^
2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqr
t[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqr
t[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(
3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(42*a*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e +
 f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])
))/(4*b*f*Sqrt[Cos[e + f*x]]*Sin[e + f*x]^(5/2))
```

Maple [B] time = 0.391, size = 4649, normalized size = 5.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/4/f*2^(1/2)*a/b^3/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(
1/2)-a)*(a-b)*(4*I*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+I*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-4*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))
*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^3-4*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/
2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^3+4
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)
^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*a^3+2*cos(f*x+e)^2*2^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2-4*cos(f*x+e)^2*
2^(1/2)*(-a^2+b^2)^(1/2)*a*b+4*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a*b-cos(
f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2
)^(1/2)*b^2-4*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*(-a^2+b^2)^(1/2)*a^2-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
```

```
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+2*cos(f*x+e)*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+I*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-8*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+4*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-4*I*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-I*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+4*I*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^
2+4*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+
b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a^2-4*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b+4*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)
)*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2
)^(1/2)*a^2+4*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*a^2*b-4*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-8*cos(f*x+e)*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+4*cos(f*x+e)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-4*I*cos(f*x+
e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1
/2)*a^2-I*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1
/2))*(-a^2+b^2)^(1/2)*b^2+4*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^3+4*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
```

```
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a^2-4*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),
1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b+4*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(
1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(
-a^2+b^2)^(1/2)*a^2+4*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*a^2*b-4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+2*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2)*(d*sin(f*x+e))^(5/2)*(g*co
s(f*x+e))^(1/2)/cos(f*x+e)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(5/2)/(b*sin(f*x + e) + a),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(5/2)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(5/2)/(b*sin(f*x + e) + a),
 x)
```

3.1409 $\int \frac{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=578

$$\frac{2\sqrt{2}a^2d^2\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}a^2d^2\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+}}{\sqrt{g}\sqrt{\sin(e+}}\right.\right.}{b^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

```
[Out] -((a*d^(3/2)*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])]])/(Sqrt[2]*b^2*f)) + (a*d^(3/2)*Sqrt[g]*ArcTan[1
 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])]])/
(Sqrt[2]*b^2*f) + (a*d^(3/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (
Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2
*f) - (a*d^(3/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*f) + (2*Sqr
t[2]*a^2*d^2*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*
Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b
^2*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*a^2*d^2*Sq
rt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqr
t[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^2*Sqrt[-a + b]*Sq
rt[a + b]*f*Sqrt[d*Sin[e + f*x]]) + (d*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(b*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 1.13358, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {2909, 2572, 2639, 2575, 297, 1162, 617, 204, 1165, 628, 2906, 2905, 490, 1218}

$$\frac{2\sqrt{2}a^2d^2\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}a^2d^2\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+}}{\sqrt{g}\sqrt{\sin(e+}}\right.\right.}{b^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]),x]
```


```
[Out] -((a*d^(3/2)*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])]])/(Sqrt[2]*b^2*f)) + (a*d^(3/2)*Sqrt[g]*ArcTan[1
 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])]])/
(Sqrt[2]*b^2*f) + (a*d^(3/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (
Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2
*f) - (a*d^(3/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*f) + (2*Sqr
t[2]*a^2*d^2*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*
Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b
^2*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*a^2*d^2*Sq
rt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqr
t[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^2*Sqrt[-a + b]*Sq
rt[a + b]*f*Sqrt[d*Sin[e + f*x]]) + (d*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(b*f*Sqrt[Sin[2*e + 2*f*x]])
```


Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
```

```
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx = \frac{d\int \sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx}{b}$$

$$= -\frac{\left(ad^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b^2} + \frac{\left(a^2d^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx}{b^2} + \frac{\left(d\sqrt{g\cos(e+\right.}{}$$

$$= \frac{d\sqrt{g\cos(e+fx)}E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bf\sqrt{\sin(2e+2fx)}} + \frac{\left(2ad^3g\right)\text{Subst}\left(\int \frac{x^2}{g^2+d^2x^4}\right.}{b^2f}$$

$$= \frac{d\sqrt{g\cos(e+fx)}E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bf\sqrt{\sin(2e+2fx)}} - \frac{\left(ad^2g\right)\text{Subst}\left(\int \frac{g-dx^2}{g^2+d^2x^4}\,dx\right.}{b^2f}$$

$$= \frac{d\sqrt{g\cos(e+fx)}E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bf\sqrt{\sin(2e+2fx)}} + \frac{\left(ad^{3/2}\sqrt{g}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{g}}{\sqrt{d}}}{-\frac{g}{d}-\frac{\sqrt{2}}{}}\right.}{2\sqrt{2}b}$$

$$= \frac{ad^{3/2}\sqrt{g}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}b^2f} - \frac{ad^{3/2}\sqrt{g}\log\left(\sqrt{g}+\sqrt{g}\right.}{}$$

$$= -\frac{ad^{3/2}\sqrt{g}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}b^2f} + \frac{ad^{3/2}\sqrt{g}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}b^2f} -$$

Mathematica [C] time = 17.965, size = 176, normalized size = 0.3

$$\frac{2d\sqrt{d\sin(e+fx)}(g\cos(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(bF_1\left(\frac{3}{4};-\frac{3}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-aF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos\right.\right.}{3fg\left(a^2-b^2\right)\sqrt[4]{\sin^2(e+fx)}(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]
),x]
```

```
[Out] (2*d*(b*AppellF1[3/4, -3/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)] - a*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)])*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sqr
t[Sin[e + f*x]^2]))/(3*(a^2 - b^2)*f*g*(Sin[e + f*x]^2)^(1/4)*(a + b*Sin[e
+ f*x]))
```

Maple [B] time = 0.269, size = 3290, normalized size = 5.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*2^(1/2)*a/b^2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/
2)-a)*(a-b)*(I*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
```

```
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*cos(f*x+e)*a-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a+(-a^2+b^2)^(1/2)*2^(1/2)*co
s(f*x+e)^2*b-(-a^2+b^2)^(1/2)*2^(1/2)*cos(f*x+e)*b-(-a^2+b^2)^(1/2)*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b+(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+EllipticPi((-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b+EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*a^2-(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*a^2
-(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*a+(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2
+1/2*I,1/2*2^(1/2))*a-I*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)*a+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2+EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*a*b-(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*a*b-I
*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*a-(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*cos(f*x+e)*a+(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
```

```
+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)*a-(-a^2+b^2)^(1/2)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*
a-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+
e)*b+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos
(f*x+e)*b+I*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*a+(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*cos(f*x+e)*a)*(d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/cos
(f*x+e)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(3/2)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a),
 x)
```

3.1410 $\int \frac{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=509

$$-\frac{2\sqrt{2}ad\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}ad\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\cdots\right.}{bf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

```
[Out] (Sqrt[d]*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]
*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*f) - (Sqrt[d]*Sqrt[g]*ArcTan[1 + (Sqrt[
2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*
b*f) - (Sqrt[d]*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[
d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*f) + (Sqrt[d]*
Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e
+ f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*f) - (2*Sqrt[2]*a*d*Sqrt[g]*El
lipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*
Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*Sqrt[-a + b]*Sqrt[a +
b]*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a*d*Sqrt[g]*EllipticPi[Sqrt[-a + b]
/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])]
, -1]*Sqrt[Sin[e + f*x]])/(b*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]
])
```

Rubi [A] time = 0.834568, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2909, 2575, 297, 1162, 617, 204, 1165, 628, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}ad\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}ad\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\cdots\right.}{bf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[d]*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]
*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*f) - (Sqrt[d]*Sqrt[g]*ArcTan[1 + (Sqrt[
2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*
b*f) - (Sqrt[d]*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[
d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*f) + (Sqrt[d]*
Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e
+ f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*f) - (2*Sqrt[2]*a*d*Sqrt[g]*El
lipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*
Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*Sqrt[-a + b]*Sqrt[a +
b]*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a*d*Sqrt[g]*EllipticPi[Sqrt[-a + b]
/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])]
, -1]*Sqrt[Sin[e + f*x]])/(b*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]
])
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
```

```
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{d\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{\left(2d^2g\right)\operatorname{Subst}\left(\int \frac{x^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{bf} - \frac{\left(ad\sqrt{\sin(e+fx)}\right)\int \frac{\sqrt{g\cos(e+}}{\sqrt{\sin(e+fx)}(a+b\,\mathrm{s}}}{b\sqrt{d\sin(e+fx)}}$$

$$= \frac{(dg)\operatorname{Subst}\left(\int \frac{g-dx^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{bf} - \frac{(dg)\operatorname{Subst}\left(\int \frac{g+dx^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right.}{bf}$$

$$= -\frac{\left(\sqrt{d}\sqrt{g}\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{g}}{\sqrt{d}}+2x}{-\frac{g}{d}-\frac{\sqrt{2}\sqrt{g}x}{\sqrt{d}}-x^2}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}bf} - \frac{\left(\sqrt{d}\sqrt{g}\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{g}}{\sqrt{d}}-2}{-\frac{g}{d}+\frac{\sqrt{2}\sqrt{g}x}{\sqrt{d}}}\right.}{2\sqrt{2}bf}$$

$$= -\frac{\sqrt{d}\sqrt{g}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}bf} + \frac{\sqrt{d}\sqrt{g}\log\left(\sqrt{g}+\sqrt{g}\cot\right.}{2}$$

$$= \frac{\sqrt{d}\sqrt{g}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}bf} - \frac{\sqrt{d}\sqrt{g}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}bf} - \sqrt{d}\sqrt{g}$$

Mathematica [C] time = 6.89642, size = 178, normalized size = 0.35

$$\frac{2(d\sin(e+fx))^{3/2}(g\cos(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(bF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-aF_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};\mathrm{co}\right.\right.}{3dfg\left(a^2-b^2\right)\sin^2(e+fx)^{3/4}(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),
x]
```

```
[Out] (2*(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)] - a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)])*(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sqrt
[Sin[e + f*x]^2]))/(3*(a^2 - b^2)*d*f*g*(Sin[e + f*x]^2)^(3/4)*(a + b*Sin[e
 + f*x]))
```

Maple [A] time = 0.316, size = 744, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*2^(1/2)*a/b/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)
-a)*(a-b)*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^
2)^(1/2)+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)+a*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1
/2)),1/2*2^(1/2))+b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(
1/2)-a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^
2+b^2)^(1/2)-a),1/2*2^(1/2))-b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)))*(g*cos(f*x+e))^(1/2)*
sin(f*x+e)*(d*sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(e+fx\right)}\sqrt{g\cos\left(e+fx\right)}}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(1/2)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(d*sin(e + f*x))*sqrt(g*cos(e + f*x))/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a), x
)
```

3.1411 $\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=208

$$\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-\right.}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

```
[Out] (2*Sqrt[2]*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Co
s[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(Sqr
t[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[g]*Elliptic
Pi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 +
Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d
*Sin[e + f*x]])
```

Rubi [A] time = 0.414891, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2906, 2905, 490, 1218}

$$\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-\right.}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Co
s[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(Sqr
t[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[g]*Elliptic
Pi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 +
Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d
*Sin[e + f*x]])
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
```

```
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{\sqrt{\sin(e+fx)}\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{\sin(e+fx)}(a+b\sin(e+fx))}\,dx}{\sqrt{d\sin(e+fx)}}$$

$$= -\frac{\left(4\sqrt{2}g\sqrt{\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{\left((a+b)g^2+(a-b)x^4\right)\sqrt{1-\frac{x^4}{g^2}}}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{1+\sin(e+fx)}}\right)}{f\sqrt{d\sin(e+fx)}}$$

$$= -\frac{\left(2\sqrt{2}g\sqrt{\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a+b}g-\sqrt{-a+b}x^2\right)\sqrt{1-\frac{x^4}{g^2}}}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{1+\sin(e+fx)}}\right)}{\sqrt{-a+b}f\sqrt{d\sin(e+fx)}} + \cdots$$

$$= \frac{2\sqrt{2}\sqrt{g}\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{\sqrt{-a+b}\sqrt{a+b}f\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{2}\sqrt{g}\Pi\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 7.18232, size = 182, normalized size = 0.88

$$-\frac{4\sqrt{2}g\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)-1}}\tan^{\frac{3}{2}}\left(\frac{1}{2}(e+fx)\right)\left(\Pi\left(\frac{a}{\sqrt{b^2-a^2}-b};-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}}\right)\middle|-1\right)+\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};-\sin\cdots\right.\right.}{af\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (-4*Sqrt[2]*g*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(-1 + Cos[e + f*x])]*(El
lipticF[ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1] + EllipticPi[a/(-b + Sqrt[-a^
2 + b^2]), -ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1] + EllipticPi[-(a/(b + Sqr
t[-a^2 + b^2])), -ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1])*Tan[(e + f*x)/2]^(
3/2))/(a*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Maple [B] time = 0.303, size = 590, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*2^(1/2)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)
*(g*cos(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
(a-b)*(2*(-a^2+b^2)^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)-EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(
1/2))*(-a^2+b^2)^(1/2)+a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))+b*EllipticPi((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))-a*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2
)-a),1/2*2^(1/2))-b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)))*sin(f*x+e)^2/(d*sin(f*x+e))^(1/2
)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(e+fx\right)}}{\sqrt{d\sin\left(e+fx\right)}\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))/(sqrt(d*sin(e + f*x))*(a + b*sin(e + f*x))),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))),
 x)
```

3.1412 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=320

$$-\frac{2\sqrt{2}b\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{adf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}b\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right.\right.}{adf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(a*d*f*g*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b*S
qrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/
(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*Sqrt[-a + b]*
Sqrt[a + b]*d*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b*Sqrt[g]*EllipticPi[Sqr
t[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*Sqrt[-a + b]*Sqrt[a + b]*d*f*Sqrt[d*S
in[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d
*Sin[e + f*x]])/(a*d^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 0.747719, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {2910, 2570, 2572, 2639, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}b\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{adf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}b\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right.\right.}{adf\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(a*d*f*g*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b*S
qrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/
(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*Sqrt[-a + b]*
Sqrt[a + b]*d*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b*Sqrt[g]*EllipticPi[Sqr
t[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*Sqrt[-a + b]*Sqrt[a + b]*d*f*Sqrt[d*S
in[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d
*Sin[e + f*x]])/(a*d^2*f*Sqrt[Sin[2*e + 2*f*x]])
```


Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} - \frac{2\int \sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,dx}{ad^2} - \frac{\left(b\sqrt{\sin(\right.}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} + \frac{\left(4\sqrt{2}bg\sqrt{\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{\left((a+b)g^2+(a-b)x^4\right)}\right.}{adf\sqrt{d\sin(e+fx)}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{g\cos(e+fx)}E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx}}{ad^2f\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{2}b\sqrt{g}\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right.}{a\sqrt{-a+b}\sqrt{a+b}df\sqrt{d\sin(e+fx)}}$$

<u>Mathematica</u> [C] time = 24.5339, size = 1622, normalized size = 5.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x]))
),x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(a*f*(d*Sin[e + f*x])^(
3/2)) + (Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^(3/2)*((4*a*(-(b*AppellF1[3/4, -
1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*Appell
F1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Co
s[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a
^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (b*Sqrt[Tan[e
+ f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^
(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[
e + f*x]^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2
]))/(6*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 +
Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*
x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1,
 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2
) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 +
 b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/
2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcT
an[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)
^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*
a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3
```

```
/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log
[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2
]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
- b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[
2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[
a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/
2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f
*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))
/(a*f*Sqrt[Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))
```

Maple [B] time = 0.319, size = 2497, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*2^(1/2)/a/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a
)*(a-b)*(cos(f*x+e)*(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b-4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a+2*cos(f*x+e)*(-a^2+b^2)^(1/2)*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a-2*(-a^2+b^2)^(1/2)*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*b+cos(f*x+e)*(-a^2+b^2)
^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(
1/2))*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2
^(1/2))*cos(f*x+e)*a*b+cos(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*a*b-cos(f*x+e)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2+(-a^2+b^2
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a
^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*b-4*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*a+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
```

```
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a-2*
(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b+(-a^2+b^
2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2
^(1/2))*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2
*2^(1/2))*a*b+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*b^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*a*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^
(1/2)),1/2*2^(1/2))*b^2+2*cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a)*(g*cos(f*x
+e))^(1/2)*sin(f*x+e)/(d*sin(f*x+e))^(3/2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/2)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/2)
), x)
```

3.1413 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=366

$$\frac{2\sqrt{2}b^2\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}b^2\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right.\right.}{a^2d^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(3*a*d*f*g*(d*Sin[e + f*x])^(3/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(a^2*d^2*f*g*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt
[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sq
rt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*Sqrt[-a + b]*S
qrt[a + b]*d^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Sqrt[g]*EllipticPi[
Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin
[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*Sqrt[-a + b]*Sqrt[a + b]*d^2*f*S
qrt[d*Sin[e + f*x]]) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x,
2]*Sqrt[d*Sin[e + f*x]])/(a^2*d^3*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 1.02944, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2910, 2563, 2570, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{2\sqrt{2}b^2\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}b^2\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right.\right.}{a^2d^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(3*a*d*f*g*(d*Sin[e + f*x])^(3/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(a^2*d^2*f*g*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt
[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sq
rt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*Sqrt[-a + b]*S
qrt[a + b]*d^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Sqrt[g]*EllipticPi[
Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin
[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*Sqrt[-a + b]*Sqrt[a + b]*d^2*f*S
qrt[d*Sin[e + f*x]]) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x,
2]*Sqrt[d*Sin[e + f*x]])/(a^2*d^3*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{b^2\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx}{a^2d^2} - \frac{b\int \frac{\sqrt{g\cos(}}{(d\sin(e+}}{a^2c}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{(2b)\int\sqrt{g\cos(e+}}{a}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} - \frac{\left(4\sqrt{2}b^2g\sqrt{\sin(e+}\right.}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{2b\sqrt{g\cos(e+fx)}E}{a^2d^3}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{2\sqrt{2}b^2\sqrt{g}\Pi\left(-\frac{\sqrt{-a-}}{\sqrt{a+}}\right.}{a^2\sqrt{-}}$$

Mathematica [C] time = 21.953, size = 1648, normalized size = 4.5

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((2*b*Cot[e + f*x])/a^2 - (2*Cot[e + f*x]*Csc[e + f*x
])/(3*a))*Sin[e + f*x]^3)/(f*(d*Sin[e + f*x])^(5/2)) - (b*Sqrt[g*Cos[e + f*
x]]*Sin[e + f*x]^(5/2)*((4*a*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e
 + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sq
rt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x
]^2)^(3/4)*(a + b*Sin[e + f*x])) - (b*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2
)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] +
2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[
-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]
*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x
]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4,
1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(
3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(6*a^2*Cos[e + f*x]^(3
/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + (
Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*
x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-
a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*Appe
llF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]
*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt
[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]
- (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/
Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2
)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*Ar
```

```
cTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^
2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sq
rt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[T
an[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^
2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan
[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*
a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2
 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqr
t[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/
(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^
2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*
(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a^2*f*Sqrt[Cos[e + f*x]]
*(d*Sin[e + f*x])^(5/2))
```

Maple [B] time = 0.368, size = 2672, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/3/f*2^(1/2)/a^2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1
/2)-a)*(a-b)*(6*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/
2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*
sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b-6*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2+3*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*(-
a^2+b^2)^(1/2)*b^2-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^
(1/2)),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b^2-3*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*b^3+
3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)
)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2+3*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b^2+
3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)
)*cos(f*x+e)*sin(f*x+e)*b^3-12*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b+6*EllipticF((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/si
```

```
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b-6*EllipticF((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2+3*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*(-
a^2+b^2)^(1/2)*b^2-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^
(1/2)),1/2*2^(1/2))*sin(f*x+e)*a*b^2-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*b^3+3*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2
)^(1/2)*b^2+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a)
,1/2*2^(1/2))*sin(f*x+e)*a*b^2+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+
(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*b^3-12*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b-2*2^(1/2)*cos(f*x+e
)^2*(-a^2+b^2)^(1/2)*a^2+6*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a
*b)*(g*cos(f*x+e))^(1/2)*sin(f*x+e)/(d*sin(f*x+e))^(5/2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/2)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/2)
), x)
```

3.1414 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=513

$$-\frac{2b^2(g\cos(e+fx))^{3/2}}{a^3d^3fg\sqrt{d\sin(e+fx)}}-\frac{2b^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^3d^4f\sqrt{\sin(2e+2fx)}}-\frac{2\sqrt{2}b^3\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\mathrm{si}\right.}{a^3d^3f\sqrt{b-a}\sqrt{a+b}\sqrt{d}}$$

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(5*a*d*f*g*(d*Sin[e + f*x])^(5/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(3*a^2*d^2*f*g*(d*Sin[e + f*x])^(3/2)) - (4*(g*Cos[e + f
*x])^(3/2))/(5*a*d^3*f*g*Sqrt[d*Sin[e + f*x]]) - (2*b^2*(g*Cos[e + f*x])^(3
/2))/(a^3*d^3*f*g*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[g]*EllipticPi
[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 +
 Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^3*Sqrt[-a + b]*Sqrt[a + b]*d^3
*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*Sqrt[g]*EllipticPi[Sqrt[-a + b]/S
qrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])],
-1]*Sqrt[Sin[e + f*x]])/(a^3*Sqrt[-a + b]*Sqrt[a + b]*d^3*f*Sqrt[d*Sin[e +
f*x]]) - (4*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e
+ f*x]])/(5*a*d^4*f*Sqrt[Sin[2*e + 2*f*x]]) - (2*b^2*Sqrt[g*Cos[e + f*x]]*E
llipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^3*d^4*f*Sqrt[Sin[2*e +
 2*f*x]])
```

Rubi [A] time = 1.45355, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2910, 2570, 2572, 2639, 2563, 2906, 2905, 490, 1218}

$$-\frac{2b^2(g\cos(e+fx))^{3/2}}{a^3d^3fg\sqrt{d\sin(e+fx)}}-\frac{2b^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^3d^4f\sqrt{\sin(2e+2fx)}}-\frac{2\sqrt{2}b^3\sqrt{g}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\mathrm{si}\right.}{a^3d^3f\sqrt{b-a}\sqrt{a+b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(5*a*d*f*g*(d*Sin[e + f*x])^(5/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(3*a^2*d^2*f*g*(d*Sin[e + f*x])^(3/2)) - (4*(g*Cos[e + f
*x])^(3/2))/(5*a*d^3*f*g*Sqrt[d*Sin[e + f*x]]) - (2*b^2*(g*Cos[e + f*x])^(3
/2))/(a^3*d^3*f*g*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[g]*EllipticPi
[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 +
 Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^3*Sqrt[-a + b]*Sqrt[a + b]*d^3
*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*Sqrt[g]*EllipticPi[Sqrt[-a + b]/S
qrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])],
-1]*Sqrt[Sin[e + f*x]])/(a^3*Sqrt[-a + b]*Sqrt[a + b]*d^3*f*Sqrt[d*Sin[e +
f*x]]) - (4*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e
+ f*x]])/(5*a*d^4*f*Sqrt[Sin[2*e + 2*f*x]]) - (2*b^2*Sqrt[g*Cos[e + f*x]]*E
llipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^3*d^4*f*Sqrt[Sin[2*e +
 2*f*x]])
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
```

```
p, 1] && LtQ[n, 0]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}}\,dx}{5ad^2} + \frac{b^2\int \frac{\sqrt{g\cos(e+fx}}{(d\sin(e+fx))^{3/2}(a+b\,\mathrm{s}}}{a^2d^2}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e+f}{5ad^3fg\sqrt{d\sin(e}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e+f}{5ad^3fg\sqrt{d\sin(e}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e+f}{5ad^3fg\sqrt{d\sin(e}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e+f}{5ad^3fg\sqrt{d\sin(e}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(g\cos(e+f}{5ad^3fg\sqrt{d\sin(e}}$$

Mathematica [C] time = 23.4757, size = 1729, normalized size = 3.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((-2*(2*a^2*Cos[e + f*x] + 5*b^2*Cos[e + f*x])*Csc[e
+ f*x])/(5*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x])/(3*a^2) - (2*Cot[e + f*x]
*Csc[e + f*x]^2)/(5*a))*Sin[e + f*x]^4)/(f*(d*Sin[e + f*x])^(7/2)) - (Sqrt[
g*Cos[e + f*x]]*Sin[e + f*x]^(7/2)*((-2*(4*a^3 + 10*a*b^2)*(-(b*AppellF1[3/
4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*Ap
pellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
)*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(
3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^2*b
+ 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a
^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 -
 b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqr
t[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e +
 f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sq
rt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b - 5*b^3)*Cos[2*(e
 + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(
56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^
2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4
, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e +
```

```
f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e +
f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqr
t[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]]
)/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 +
(Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4)
- (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])
/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e
+ f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f
*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4)
+ (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x
]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[
a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*
Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b
^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(8
4*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 +
Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^3*f*Sqrt[Cos[e + f*x]]*(d*
Sin[e + f*x])^(7/2))
```

Maple [B] time = 0.342, size = 6120, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")

[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/2)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/2)
), x)
```

3.1415 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=598

$$\frac{2b^3(g\cos(e+fx))^{3/2}}{a^4d^4fg\sqrt{d\sin(e+fx)}}-\frac{2b^2(g\cos(e+fx))^{3/2}}{3a^3d^3fg(d\sin(e+fx))^{3/2}}+\frac{2b^3E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^4d^5f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}b^4\sqrt{g}\sqrt{\ldots}}{\ldots}$$

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(7*a*d*f*g*(d*Sin[e + f*x])^(7/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(5*a^2*d^2*f*g*(d*Sin[e + f*x])^(5/2)) - (8*(g*Cos[e + f
*x])^(3/2))/(21*a*d^3*f*g*(d*Sin[e + f*x])^(3/2)) - (2*b^2*(g*Cos[e + f*x])
^(3/2))/(3*a^3*d^3*f*g*(d*Sin[e + f*x])^(3/2)) + (4*b*(g*Cos[e + f*x])^(3/2
))/(5*a^2*d^4*f*g*Sqrt[d*Sin[e + f*x]]) + (2*b^3*(g*Cos[e + f*x])^(3/2))/(a
^4*d^4*f*g*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^4*Sqrt[g]*EllipticPi[-(Sqrt
[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^4*Sqrt[-a + b]*Sqrt[a + b]*d^4*f*Sqrt
[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^4*Sqrt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a +
 b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqr
t[Sin[e + f*x]])/(a^4*Sqrt[-a + b]*Sqrt[a + b]*d^4*f*Sqrt[d*Sin[e + f*x]])
+ (4*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x
]])/(5*a^2*d^5*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*b^3*Sqrt[g*Cos[e + f*x]]*Elli
pticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^4*d^5*f*Sqrt[Sin[2*e + 2*
f*x]])
```

Rubi [A] time = 1.82422, antiderivative size = 598, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2910, 2570, 2563, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{2b^3(g\cos(e+fx))^{3/2}}{a^4d^4fg\sqrt{d\sin(e+fx)}}-\frac{2b^2(g\cos(e+fx))^{3/2}}{3a^3d^3fg(d\sin(e+fx))^{3/2}}+\frac{2b^3E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^4d^5f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}b^4\sqrt{g}\sqrt{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(7*a*d*f*g*(d*Sin[e + f*x])^(7/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(5*a^2*d^2*f*g*(d*Sin[e + f*x])^(5/2)) - (8*(g*Cos[e + f
*x])^(3/2))/(21*a*d^3*f*g*(d*Sin[e + f*x])^(3/2)) - (2*b^2*(g*Cos[e + f*x])
^(3/2))/(3*a^3*d^3*f*g*(d*Sin[e + f*x])^(3/2)) + (4*b*(g*Cos[e + f*x])^(3/2
))/(5*a^2*d^4*f*g*Sqrt[d*Sin[e + f*x]]) + (2*b^3*(g*Cos[e + f*x])^(3/2))/(a
^4*d^4*f*g*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^4*Sqrt[g]*EllipticPi[-(Sqrt
[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^4*Sqrt[-a + b]*Sqrt[a + b]*d^4*f*Sqrt
[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^4*Sqrt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a +
 b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqr
t[Sin[e + f*x]])/(a^4*Sqrt[-a + b]*Sqrt[a + b]*d^4*f*Sqrt[d*Sin[e + f*x]])
+ (4*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x
]])/(5*a^2*d^5*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*b^3*Sqrt[g*Cos[e + f*x]]*Elli
pticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^4*d^5*f*Sqrt[Sin[2*e + 2*
f*x]])
```


Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
```

```
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
```

```
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{4\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}}\,dx}{7ad^2} + \frac{b^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e\cdots}}{a^2d^2}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\cos(e+fx)\cdots}{21ad^3fg(d\sin(e+\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\cos(e+fx)\cdots}{21ad^3fg(d\sin(e+\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\cos(e+fx)\cdots}{21ad^3fg(d\sin(e+\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\cos(e+fx)\cdots}{21ad^3fg(d\sin(e+\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\cos(e+fx)\cdots}{21ad^3fg(d\sin(e+\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{8(g\cos(e+fx)\cdots}{21ad^3fg(d\sin(e+\cdots}$$

Mathematica [C] time = 22.6319, size = 1771, normalized size = 2.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((2*(2*a^2*b*Cos[e + f*x] + 5*b^3*Cos[e + f*x])*Csc[e
 + f*x])/(5*a^4) - (2*(4*a^2*Cos[e + f*x] + 7*b^2*Cos[e + f*x])*Csc[e + f*x
]^2)/(21*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*a^2) - (2*Cot[e + f*x]
*Csc[e + f*x]^3)/(7*a))*Sin[e + f*x]^5)/(f*(d*Sin[e + f*x])^(9/2)) + (b*Sqr
t[g*Cos[e + f*x]]*Sin[e + f*x]^(9/2)*((-2*(4*a^3 + 10*a*b^2)*(-(b*AppellF1[
3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*
AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))
/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^2*
b + 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + S
qrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e
```

```
 + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]
^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*
Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a
 + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b - 5*b^3)*Cos[2*
(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])
*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 +
b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7
/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e
 + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e
 + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*S
qrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a
]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1
 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/
4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan
[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e +
 f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2
)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/
4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Lo
g[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2
]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(
a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 -
 b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/
(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1
+ Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^4*f*Sqrt[Cos[e + f*x]]*(
d*Sin[e + f*x])^(9/2))
```

Maple [B] time = 0.418, size = 6505, normalized size = 10.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/2)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\cos\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/2)
), x)
```

3.1416 $\int \frac{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=982

result too large to display

```
[Out] (3*d^(3/2)*g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[
d]*Sqrt[g*Cos[e + f*x]])])/(4*Sqrt[2]*b*f) + ((a^2 - b^2)*d^(3/2)*g^(3/2)*A
rcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*
x]])])/(Sqrt[2]*b^3*f) - (3*d^(3/2)*g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[g]*Sqr
t[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(4*Sqrt[2]*b*f) - ((a^2
 - b^2)*d^(3/2)*g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(
Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^3*f) + (2*Sqrt[2]*a*Sqrt[-a^2 +
b^2]*d^(3/2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])),
 ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^3*f
*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*a*Sqrt[-a^2 + b^2]*d^(3/2)*g^2*Sqrt[Cos
[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*
x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^3*f*Sqrt[g*Cos[e + f*x]]) -
(3*d^(3/2)*g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqr
t[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(8*Sqrt[2]*b*f) - ((a^2 - b^2)*d
^(3/2)*g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*
Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^3*f) + (3*d^(3/2)*g^(3/
2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]
] + Sqrt[d]*Tan[e + f*x]])/(8*Sqrt[2]*b*f) + ((a^2 - b^2)*d^(3/2)*g^(3/2)*L
og[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] +
Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^3*f) - (a*d*g*Sqrt[g*Cos[e + f*x]]*Sqrt
[d*Sin[e + f*x]])/(b^2*f) + (g*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))
/(2*b*f) + (a*d^2*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/
(2*b^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 1.69134, antiderivative size = 982, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {2901, 2838, 2568, 2573, 2641, 2574, 297, 1162, 617, 204, 1165, 628, 2909, 2908, 2907, 1218}

$$\frac{2\sqrt{2}a\sqrt{b^2-a^2}d^{3/2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)g^2}{b^3f\sqrt{g\cos(e+fx)}} - \frac{2\sqrt{2}a\sqrt{b^2-a^2}d^{3/2}\sqrt{\cos(e+fx)}\Pi\left(-\right.}{b^3f\sqrt{g\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]),x]
```


```
[Out] (3*d^(3/2)*g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[
d]*Sqrt[g*Cos[e + f*x]])])/(4*Sqrt[2]*b*f) + ((a^2 - b^2)*d^(3/2)*g^(3/2)*A
rcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*
x]])])/(Sqrt[2]*b^3*f) - (3*d^(3/2)*g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[g]*Sqr
t[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(4*Sqrt[2]*b*f) - ((a^2
 - b^2)*d^(3/2)*g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(
Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^3*f) + (2*Sqrt[2]*a*Sqrt[-a^2 +
b^2]*d^(3/2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])),
 ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^3*f
*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*a*Sqrt[-a^2 + b^2]*d^(3/2)*g^2*Sqrt[Cos
[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*
x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^3*f*Sqrt[g*Cos[e + f*x]]) -
(3*d^(3/2)*g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqr
```

```
t[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(8*Sqrt[2]*b*f) - ((a^2 - b^2)*d
^(3/2)*g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*
Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^3*f) + (3*d^(3/2)*g^(3/
2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]
] + Sqrt[d]*Tan[e + f*x]])/(8*Sqrt[2]*b*f) + ((a^2 - b^2)*d^(3/2)*g^(3/2)*L
og[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] +
Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^3*f) - (a*d*g*Sqrt[g*Cos[e + f*x]]*Sqrt
[d*Sin[e + f*x]])/(b^2*f) + (g*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))
/(2*b*f) + (a*d^2*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/
(2*b^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 2901

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[(g^2*(a^2 - b^2))/b^2, Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
```

```
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx = \frac{g^2\int \frac{(d\sin(e+fx))^{3/2}(a-b\sin(e+fx))}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{b^2}$$

$$= \frac{\left(ag^2\right)\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{g^2\int \frac{(d\sin(e+fx))^{5/2}}{\sqrt{g\cos(e+fx)}}\,dx}{bd} - \frac{\left(\left(a^2-b^2\right)dg^2\right)\int \frac{\sqrt{d\sin(e+f}}{\sqrt{g\cos(e+f}}}{b^3}$$

$$= -\frac{adg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{b^2f} + \frac{g\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}}{2bf} - \frac{(3}{}$$

$$= -\frac{adg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{b^2f} + \frac{g\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}}{2bf} + \frac{((}{}$$

$$= \frac{2\sqrt{2}a\sqrt{-a^2+b^2}d^{3/2}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\right|-}{b^3f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}a\sqrt{-a^2+b^2}d^{3/2}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\right|-}{b^3f\sqrt{g\cos(e+fx)}}$$

$$= \frac{\left(a^2-b^2\right)d^{3/2}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}b^3f} - \frac{\left(a^2-b^2\right)d^{3/2}g^{3/2}\tan^{-1}\left(1+\right.}{\sqrt{2}b^3f}$$

$$= \frac{3d^{3/2}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{4\sqrt{2}bf} + \frac{\left(a^2-b^2\right)d^{3/2}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin}}{\sqrt{d}\sqrt{g\cos}}\right.}{\sqrt{2}b^3f}$$

Mathematica [C] time = 28.7281, size = 1901, normalized size = 1.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*
x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(d*Sin[e + f*x])^(3/2))/(2*b*f) - ((g*
Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*((10*b*(a^2 - b^2)*Sqrt[Cos[e +
f*x]]*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((b*AppellF1[1/4, -3/4, 1, 5/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2])/(-
```

```
5*(a^2 - b^2)*AppellF1[1/4, -3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] + (4*b^2*AppellF1[5/4, -3/4, 2, 9/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 1/4, 1, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2) + (a*App
ellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
)/(5*(a^2 - b^2)*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, -1/4, 2, 9/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/4, 1, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2))*Sin[
e + f*x]^(5/2))/((1 - Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))*(a
+ b*Sin[e + f*x])) + (2*a*Sqrt[Sin[e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt
[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(
a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a
 + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*T
an[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4
, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(5/2))/(5*a^
2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a +
 b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*x]^2)^(3/2)) - (a*Cos[2*
(e + f*x)]*Sqrt[Sin[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])
*(-20*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] + 20*Sqrt[2]*a*ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + (10*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*ArcTa
n[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^
(3/4) - (10*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(
1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(3/4) + 10*Sqrt[2]*a*Log[1 -
 Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 10*Sqrt[2]*a*Log[1 + Sqrt[2]*
Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (5*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*Log[-
a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*
Tan[e + f*x]])/(a^2 - b^2)^(3/4) + (5*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*Log[a +
 Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan
[e + f*x]])/(a^2 - b^2)^(3/4) + 8*b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x
]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/2) + (40*b*Sqrt[Tan
[e + f*x]])/Sqrt[1 + Tan[e + f*x]^2] + (200*a^4*b*AppellF1[1/4, 1/2, 1, 5/4
, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sqrt[Tan[e + f*x]])/(
Sqrt[1 + Tan[e + f*x]^2]*(-5*a^2*AppellF1[1/4, 1/2, 1, 5/4, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2] + 2*(2*(a^2 - b^2)*AppellF1[5/4, 1/2, 2, 9
/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + a^2*AppellF1[5/4, 3/2
, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[e + f*x]^2)*
(-(b^2*Tan[e + f*x]^2) + a^2*(1 + Tan[e + f*x]^2)))))/(10*b^2*Cos[e + f*x]^
(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(-1 + Tan[e + f*x]^2)*Sqrt[1
+ Tan[e + f*x]^2])))/(4*b*f*Cos[e + f*x]^(3/2)*Sin[e + f*x]^(3/2))
```

Maple [B] time = 0.364, size = 2513, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/4/f*2^(1/2)*a/b^3/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^
(1/2)-a)*(a-b)*(-I*sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*b^2-4*I*sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e)
```

```
)/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2+I*sin(f*x+e)*(-a^2+b^2)^(1/2
)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2+4*I*sin(f*x+e)
*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^
2-4*sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1
/2*2^(1/2))*a^2+sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*b^2-4*sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2+sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2+4*sin(f*x+e)*(-a^2+b^
2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1
/2))*a^2+4*sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/(
b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b+4*sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2+4*sin(f
*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1
/2)),1/2*2^(1/2))*a*b-4*sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2*2^(1/2))*a*b-4*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b
+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^3+4*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2+4*sin(f*x+e)*((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^3-4*sin(f*x+
e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*
b^2+2*cos(f*x+e)^2*2^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2-2*sin(f*x+e)*cos
(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*b^2-4*cos(f*x+e)^2*2^(1/2)*(-a^2+b^2)^(1/2
)*a*b+4*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a*b)*(g*cos(f*x+e))^(3/2)*(d*si
n(f*x+e))^(3/2)/sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a
), x)
```

3.1417 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=611

$$-\frac{2\sqrt{2}\sqrt{d}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\sqrt{d}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}}\right.}{b^2f\sqrt{g\cos(e}}$$

```
[Out] -((a*Sqrt[d]*g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqr
t[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^2*f)) + (a*Sqrt[d]*g^(3/2)*ArcTan[1
 + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/
(Sqrt[2]*b^2*f) - (2*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[d]*g^2*Sqrt[Cos[e + f*x]
]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt
[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2
]*Sqrt[-a^2 + b^2]*Sqrt[d]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[
-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])
], -1])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (a*Sqrt[d]*g^(3/2)*Log[Sqrt[d] - (Sq
rt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e +
f*x]])/(2*Sqrt[2]*b^2*f) - (a*Sqrt[d]*g^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g
]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sq
rt[2]*b^2*f) + (g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(b*f) - (d*g^2
*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(2*b*f*Sqrt[g*Cos[e +
 f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 1.0299, antiderivative size = 611, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {2901, 2838, 2574, 297, 1162, 617, 204, 1165, 628, 2568, 2573, 2641, 2908, 2907, 1218}

$$-\frac{2\sqrt{2}\sqrt{d}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\sqrt{d}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}}\right.}{b^2f\sqrt{g\cos(e}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```


```
[Out] -((a*Sqrt[d]*g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqr
t[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^2*f)) + (a*Sqrt[d]*g^(3/2)*ArcTan[1
 + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/
(Sqrt[2]*b^2*f) - (2*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[d]*g^2*Sqrt[Cos[e + f*x]
]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt
[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2
]*Sqrt[-a^2 + b^2]*Sqrt[d]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[
-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])
], -1])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (a*Sqrt[d]*g^(3/2)*Log[Sqrt[d] - (Sq
rt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e +
f*x]])/(2*Sqrt[2]*b^2*f) - (a*Sqrt[d]*g^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g
]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sq
rt[2]*b^2*f) + (g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(b*f) - (d*g^2
*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(2*b*f*Sqrt[g*Cos[e +
 f*x]]*Sqrt[d*Sin[e + f*x]])
```


Rule 2901

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[(g^2*(a^2 - b^2))/b^2, Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{g^2\int \frac{\sqrt{d\sin(e+fx)}(a-b\sin(e+fx))}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{b^2}$$

$$= \frac{\left(ag^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{g^2\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}}\,dx}{bd} - \frac{\left(\left(a^2-b^2\right)g^2\sqrt{\cos(e+fx)}\right)}{b^2\sqrt{g\cos}}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{bf} - \frac{\left(dg^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}\,dx}{2b} + \frac{\left(2adg\right.}{}$$

$$= -\frac{2\sqrt{2}\sqrt{-a^2+b^2}\sqrt{d}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\right| -}{b^2f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\sqrt{-a^2+b^2}\sqrt{d}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\right| -}{b^2f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\sqrt{-a^2+b^2}\sqrt{d}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\right| -}{b^2f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{a\sqrt{d}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}b^2f} + \frac{a\sqrt{d}g^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}b^2f}$$

Mathematica [C] time = 21.5511, size = 604, normalized size = 0.99

$$-\frac{(d\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{2aF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{a^2-b^2} + \frac{\left(2a^2-b^2\right)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};\cos^2(e+fx\right.}{b^3-a^2b}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]
),x]
```

```
[Out] -((g*Cos[e + f*x])^(5/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sqrt[Sin[e + f*x]^2]
)*((2*a*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)])/(a^2 - b^2) + ((2*a^2 - b^2)*AppellF1[5/4, 3/4, 1, 9/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])/(-(a^2*b) + b^3) + (5*(-5*(a^2
 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*(a^2 - 2*b^2 + b^2*Cos[e + f*x]^2) + (-4*b^2*AppellF1[5/4, 3/4,
2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*
AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)])*Sin[e + f*x]^2*(a^2 - b^2*Sin[e + f*x]^2)))/(b*(-5*(a^2 - b^2)*AppellF1
[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (4*
b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)] + 3*(-a^2 + b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(Sin[e + f*x]^2)^(3/4)*(a^2 - b
^2*Sin[e + f*x]^2))))/(5*d*f*g*(Sin[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x]))
```

Maple [B] time = 0.264, size = 1926, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*2^(1/2)*a/b^2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1
/2)-a)*(a-b)*(-I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))
*sin(f*x+e)*(-a^2+b^2)^(1/2)*a+I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/
2*I,1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*a+(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*a-(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*(-a
^2+b^2)^(1/2)*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-
a),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*b+(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*a^2-(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*b^2
-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*
sin(f*x+e)*(-a^2+b^2)^(1/2)*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-
a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*b-(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*a^2+(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin
(f*x+e)*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*(-a^
2+b^2)^(1/2)*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*s
in(f*x+e)*(-a^2+b^2)^(1/2)*a+(-a^2+b^2)^(1/2)*2^(1/2)*cos(f*x+e)^2*b-(-a^2+
b^2)^(1/2)*2^(1/2)*cos(f*x+e)*b)*(g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/
(-1+cos(f*x+e))/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

3.1418 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=577

$$\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{ab\sqrt{d}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin\right.}{ab\sqrt{d}f\sqrt{g\cos(e+fx)}}$$

```
[Out] (g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*
Cos[e + f*x]])])/(Sqrt[2]*b*Sqrt[d]*f) - (g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[
g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b*Sqrt[d
]*f) + (2*Sqrt[2]*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b
 - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e
+ f*x]])], -1])/(a*b*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a^2
 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcS
in[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*b*Sqrt[d
]*f*Sqrt[g*Cos[e + f*x]]) - (g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*
Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*S
qrt[d]*f) + (g^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/S
qrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*Sqrt[d]*f) + (g^2
*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a*f*Sqrt[g*Cos[e + f
*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 0.972302, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {2900, 2838, 2573, 2641, 2574, 297, 1162, 617, 204, 1165, 628, 2908, 2907, 1218}

$$\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{ab\sqrt{d}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin\right.}{ab\sqrt{d}f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```


```
[Out] (g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*
Cos[e + f*x]])])/(Sqrt[2]*b*Sqrt[d]*f) - (g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[
g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b*Sqrt[d
]*f) + (2*Sqrt[2]*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b
 - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e
+ f*x]])], -1])/(a*b*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a^2
 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcS
in[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*b*Sqrt[d
]*f*Sqrt[g*Cos[e + f*x]]) - (g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*
Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*S
qrt[d]*f) + (g^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/S
qrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*Sqrt[d]*f) + (g^2
*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a*f*Sqrt[g*Cos[e + f
*x]]*Sqrt[d*Sin[e + f*x]])
```


Rule 2900

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/(a*b), I
```

```
nt[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(b - a*Sin[e + f*x]), x], x]
 + Dist[(g^2*(a^2 - b^2))/(a*b*d), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e +
 f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x
] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LtQ[n, -1] |
| (EqQ[p, 3/2] && EqQ[n, -2^(-1)]))
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{b-a\sin(e+fx)}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}\,dx}{ab} + \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{abd}$$

$$= \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}\,dx}{a} - \frac{g^2\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{bd} + \frac{\left(\left(a^2-b^2\right)g^2\sqrt{\cos(e}}{abc}$$

$$= -\frac{\left(2g^3\right)\text{Subst}\left(\int \frac{x^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\right)}{bf} + \frac{\left(2\sqrt{2}\left(a^2-b^2\right)\left(1-\frac{b}{\sqrt{-a^2+b^2}}\right)\right.}{}$$

$$= \frac{2\sqrt{2}\sqrt{-a^2+b^2}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{ab\sqrt{d}f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\sqrt{-a^2+b^2}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{ab\sqrt{d}f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\sqrt{-a^2+b^2}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{ab\sqrt{d}f\sqrt{g\cos(e+fx)}}$$

$$= \frac{g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}b\sqrt{d}f} - \frac{g^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}b\sqrt{d}f} + \frac{2\sqrt{2}}{}$$

Mathematica [C] time = 11.5558, size = 178, normalized size = 0.31

$$\frac{2\sqrt{d\sin(e+fx)}(g\cos(e+fx))^{5/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(bF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-aF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};\text{co}\right.\right.}{5dfg\left(a^2-b^2\right)\sqrt[4]{\sin^2(e+fx)}(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (2*(b*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)] - a*AppellF1[5/4, 3/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)])*(g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sqrt[Si
n[e + f*x]^2]))/(5*(a^2 - b^2)*d*f*g*(Sin[e + f*x]^2)^(1/4)*(a + b*Sin[e +
f*x]))
```

Maple [A] time = 0.273, size = 930, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*2^(1/2)/b/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a
)*(a-b)*(I*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-I*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+EllipticPi(((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(
1/2)*a+EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/
2*2^(1/2))*(-a^2+b^2)^(1/2)*a+2*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-EllipticPi(((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(
1/2)*a-EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2
+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-a^2*EllipticPi(((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))+Ellipti
cPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1
/2*2^(1/2))*b^2-EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-EllipticPi(((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*
(-a^2+b^2)^(1/2)*b+a^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))-EllipticPi(((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2)*((1-cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(3/2)*sin(f*x+e)
^2/(-1+cos(f*x+e))/(d*sin(f*x+e))^(1/2)/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

3.1419 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=321

$$-\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\mathrm{s}\right.}{a^2d^{3/2}f\sqrt{g\cos(e+f}}$$

```
[Out] (-2*Sqrt[2]*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqr
t[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]
])], -1])/(a^2*d^(3/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a^2 + b^2
]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqr
t[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*d^(3/2)*f*Sq
rt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(a*d*f*Sqrt[d*Sin[e + f*x]
]) - (b*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^2*d*f*S
qrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 0.701626, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2899, 2563, 2573, 2641, 2908, 2907, 1218}

$$-\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\mathrm{s}\right.}{a^2d^{3/2}f\sqrt{g\cos(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (-2*Sqrt[2]*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqr
t[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]
])], -1])/(a^2*d^(3/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a^2 + b^2
]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqr
t[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*d^(3/2)*f*Sq
rt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(a*d*f*Sqrt[d*Sin[e + f*x]
]) - (b*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^2*d*f*S
qrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```


Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\cdots}{a^2d^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{adf\sqrt{d\sin(e+fx)}} - \frac{\left(\left(a^2-b^2\right)g^2\sqrt{\cos(e+fx)}\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{\cos(e+fx)}(a+b\sin(e+f\cdots}}{a^2d^2\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{adf\sqrt{d\sin(e+fx)}} - \frac{bg^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{a^2df\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} - \frac{\left(2\sqrt{2}\left(a^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\sqrt{2}\sqrt{-a^2+b^2}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-\cdots\right)}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 20.37, size = 1092, normalized size = 3.4

$$-\frac{2\tan(e+fx)(g\cos(e+fx))^{3/2}}{af(d\sin(e+fx))^{3/2}}-\frac{\sin^{\frac{3}{2}}(e+fx)\left(\frac{2a\sqrt{\sin(e+fx)}\left(\sqrt{a}\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}+1\right)+\log\left(-a+\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x
]))),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2)*Tan[e + f*x])/(a*f*(d*Sin[e + f*x])^(3/2)) - ((g
*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(3/2)*((-2*b*(a + b*Sqrt[1 - Cos[e + f*x]
^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[
e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(
5*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*
Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Co
s[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^
2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*S
qrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*Ar
cTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos
[e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 +
Cos[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(
-1 + Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqr
t[-1 + Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f
*x]])/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin
[e + f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x])) + (2*a*Sqrt[S
in[e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*
x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(
1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 -
 b^2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2
)*Tan[e + f*x]^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 +
 Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*
x]]*(1 + Tan[e + f*x]^2)^(3/2))))/(a*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^
(3/2))
```

Maple [B] time = 0.281, size = 2587, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*2^(1/2)/a/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a
)*(a-b)*(2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
```

```
(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))
*cos(f*x+e)*b-(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*a-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b-(-a
^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a)
,1/2*2^(1/2))*cos(f*x+e)*a-cos(f*x+e)*(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)
)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b-EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2
))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*a^2+cos(
f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(
1/2))*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2
*2^(1/2))*cos(f*x+e)*a^2-cos(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a-(-a^2+b^2)^(1/2)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b-(-a^2
+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1
/2*2^(1/2))*a-(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*b-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(
a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2-EllipticPi((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2+2*cos(f*x+e)*(-a^2+b^2)^(1/2
)*2^(1/2)*a)*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/cos(f*x+e)^2/(d*sin(f*x+e))^(3
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

3.1420 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=435

$$-\frac{g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^3d^2f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}bg^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\right|}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}$$

```
[Out] (2*Sqrt[2]*b*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sq
rt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x
]])], -1])/(a^3*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[-a^2 +
b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[
Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*d^(5/2)*f
*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(3*a*d*f*(d*Sin[e + f*x
])^(3/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) +
 (2*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a*d^2*f*Sqr
t[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) - ((a^2 - b^2)*g^2*EllipticF[e - Pi
/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^3*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d
*Sin[e + f*x]])
```

Rubi [A] time = 1.01583, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2899, 2570, 2573, 2641, 2563, 2910, 2908, 2907, 1218}

$$-\frac{g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^3d^2f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}bg^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\right|}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (2*Sqrt[2]*b*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sq
rt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x
]])], -1])/(a^3*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[-a^2 +
b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[
Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*d^(5/2)*f
*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(3*a*d*f*(d*Sin[e + f*x
])^(3/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) +
 (2*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a*d^2*f*Sqr
t[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) - ((a^2 - b^2)*g^2*EllipticF[e - Pi
/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^3*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d
*Sin[e + f*x]])
```


Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}(a}}{a^2d^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(b\left(a^2-b^2\right)g^2\right)\int \frac{}{\sqrt{g\,c}}}{a^3}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(b\left(a^2-b^2\right)g^2\sqrt{\cos(e}\right.}{a^3d}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{2g^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)}{3ad^2f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}b\sqrt{-a^2+b^2}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-\right.}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 20.4419, size = 1135, normalized size = 2.61

$$\frac{(g\cos(e+fx))^{3/2}\left(\frac{2b\csc(e+fx)}{a^2}-\frac{2\csc^2(e+fx)}{3a}\right)\sin^2(e+fx)\tan(e+fx)}{f(d\sin(e+fx))^{5/2}} - \frac{(g\cos(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)\left(-\frac{2\left(a^2-3b^2\right)\left(a+b\sqrt{\right.}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((2*b*Csc[e + f*x])/a^2 - (2*Csc[e + f*x]^2)/(3*a))
*Sin[e + f*x]^2*Tan[e + f*x])/(f*(d*Sin[e + f*x])^(5/2)) - ((g*Cos[e + f*x]
)^(3/2)*Sin[e + f*x]^(5/2)*((-2*(a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^
2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5
*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2
*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sq
rt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[
e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + C
os[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-
1 + Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt
[-1 + Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*
x]])/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[
e + f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x])) - (4*a*b*Sqrt[
Sin[e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
```

```
e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2
- b^2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^
2)*Tan[e + f*x]^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1
+ Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f
*x]]*(1 + Tan[e + f*x]^2)^(3/2))))/(3*a^2*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f
*x])^(5/2))
```

Maple [B] time = 0.251, size = 3014, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/3/f*2^(1/2)/a^2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/
2)-a)*(a-b)*(3*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b+3*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*sin(f*
x+e)*(-a^2+b^2)^(1/2)*b^2+3*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b+3*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*co
s(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2+2*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(
f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^2-6*EllipticF((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^
2+3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*a^2*b-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b
+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*b^3-3*cos(f*x+e)*sin(
f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^
(1/2))*a^2*b+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a
),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*b^3+3*(-a^2+b^2)^(1/2)*sin(f*x+e)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/
2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b+3
```

```
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*
sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2+3*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b+3*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x
+e)*(-a^2+b^2)^(1/2)*b^2+2*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),1/2*2^(1/2))*a^2-6*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2+3*sin(f*x+e)*EllipticPi((-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b-3*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*b^3-3*
sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/
2*2^(1/2))*a^2*b+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/
2)-a),1/2*2^(1/2))*sin(f*x+e)*b^3-6*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2
)^(1/2)*a*b+2*(-a^2+b^2)^(1/2)*cos(f*x+e)*2^(1/2)*a^2)*(g*cos(f*x+e))^(3/2)
*sin(f*x+e)/cos(f*x+e)^2/(d*sin(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

3.1421 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=525

$$\frac{bg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^4d^3f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}b^2g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\right.}{a^4d^{7/2}f\sqrt{g\cos(e+fx)}}$$

```
[Out] (-2*Sqrt[2]*b^2*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b -
 Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e +
f*x]])], -1])/(a^4*d^(7/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[-a
^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), Ar
cSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^4*d^(7
/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(5*a*d*f*(d*Sin[e
+ f*x])^(5/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(3*a^2*d^2*f*(d*Sin[e + f*x])
^(3/2)) - (8*g*Sqrt[g*Cos[e + f*x]])/(5*a*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*
(a^2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(a^3*d^3*f*Sqrt[d*Sin[e + f*x]]) - (2*b
*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*d^3*f*Sqrt
[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (b*(a^2 - b^2)*g^2*EllipticF[e - P
i/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^4*d^3*f*Sqrt[g*Cos[e + f*x]]*Sqrt[
d*Sin[e + f*x]])
```

Rubi [A] time = 1.35288, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2899, 2570, 2563, 2573, 2641, 2910, 2908, 2907, 1218}

$$\frac{bg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^4d^3f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}b^2g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\right.}{a^4d^{7/2}f\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*b^2*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b -
 Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e +
f*x]])], -1])/(a^4*d^(7/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[-a
^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), Ar
cSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^4*d^(7
/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(5*a*d*f*(d*Sin[e
+ f*x])^(5/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(3*a^2*d^2*f*(d*Sin[e + f*x])
^(3/2)) - (8*g*Sqrt[g*Cos[e + f*x]])/(5*a*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*
(a^2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(a^3*d^3*f*Sqrt[d*Sin[e + f*x]]) - (2*b
*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*d^3*f*Sqrt
[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (b*(a^2 - b^2)*g^2*EllipticF[e - P
i/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^4*d^3*f*Sqrt[g*Cos[e + f*x]]*Sqrt[
d*Sin[e + f*x]])
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
```

```
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{7/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}}}{a^2d^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{\left(2bg^2\right)\int \frac{}{\sqrt{g\cos(e+f}}}{3a^2d}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{8g\sqrt{g\cos(e+fx}}{5ad^3f\sqrt{d\sin(e+}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{8g\sqrt{g\cos(e+fx}}{5ad^3f\sqrt{d\sin(e+}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{8g\sqrt{g\cos(e+fx}}{5ad^3f\sqrt{d\sin(e+}}$$

$$= -\frac{2\sqrt{2}b^2\sqrt{-a^2+b^2}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\right.}{a^4d^{7/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 20.498, size = 1162, normalized size = 2.21

$$b(g\cos(e+fx))^{3/2}\left(-\frac{2\left(a^2-3b^2\right)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\sqrt{\sin(e+fx)}\left(\frac{5a\left(a^2-b^2\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};\right.}{\left(1-\cos^2(e+fx)\right)^{3/4}\left(\left(3\left(a^2-b^2\right)F_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-4b^2F_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};\cos^2(\right.\right.\right.}}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((2*(a^2 - 5*b^2)*Csc[e + f*x])/(5*a^3) + (2*b*Csc[
e + f*x]^2)/(3*a^2) - (2*Csc[e + f*x]^3)/(5*a))*Sin[e + f*x]^3*Tan[e + f*x]
)/(f*(d*Sin[e + f*x])^(7/2)) + (b*(g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)
*((-2*(a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*Appe
llF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*S
qrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*AppellF1[1/4,
 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*
AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e +
f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) -
 ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 +
 b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*S
```

```
qrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] + Log[
Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] - ((1 + I)*
Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] + ((
1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)
^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[e + f*x]])/((1 - Cos[e + f
*x]^2)^(1/4)*(a + b*Sin[e + f*x])) - (4*a*b*Sqrt[Sin[e + f*x]]*((Sqrt[a]*(-
2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*Ar
cTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-a +
 Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan
[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] +
 Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*AppellF
1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e +
 f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(Cos[e
 + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*x]^2)^
(3/2))))/(3*a^3*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^(7/2))
```

Maple [B] time = 0.312, size = 5828, normalized size = 11.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e)+a)(d\sin(fx+e))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

3.1422 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=688

$$-\frac{b^2g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^5d^4f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}-\frac{2g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3a^3d^4f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}b^3g^2\sqrt{b^2-a^2}\sqrt{}}{}$$

```
[Out] (2*Sqrt[2]*b^3*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b -
Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f
*x]])], -1])/(a^5*d^(9/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[-a^
2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), Arc
Sin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^5*d^(9/
2)*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(7*a*d*f*(d*Sin[e +
 f*x])^(7/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(5*a^2*d^2*f*(d*Sin[e + f*x])^
(5/2)) - (4*g*Sqrt[g*Cos[e + f*x]])/(7*a*d^3*f*(d*Sin[e + f*x])^(3/2)) + (2
*(a^2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(3*a^3*d^3*f*(d*Sin[e + f*x])^(3/2)) +
 (8*b*g*Sqrt[g*Cos[e + f*x]])/(5*a^2*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*b*(a^
2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) + (4*g^2*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(7*a*d^4*f*Sqrt[g*Cos[
e + f*x]]*Sqrt[d*Sin[e + f*x]]) - (2*(a^2 - b^2)*g^2*EllipticF[e - Pi/4 + f
*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^3*d^4*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin
[e + f*x]]) - (b^2*(a^2 - b^2)*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*
e + 2*f*x]])/(a^5*d^4*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 1.77937, antiderivative size = 688, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2899, 2570, 2573, 2641, 2563, 2910, 2908, 2907, 1218}

$$-\frac{b^2g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^5d^4f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}-\frac{2g^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3a^3d^4f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}b^3g^2\sqrt{b^2-a^2}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (2*Sqrt[2]*b^3*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b -
Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f
*x]])], -1])/(a^5*d^(9/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[-a^
2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), Arc
Sin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^5*d^(9/
2)*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(7*a*d*f*(d*Sin[e +
 f*x])^(7/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(5*a^2*d^2*f*(d*Sin[e + f*x])^
(5/2)) - (4*g*Sqrt[g*Cos[e + f*x]])/(7*a*d^3*f*(d*Sin[e + f*x])^(3/2)) + (2
*(a^2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(3*a^3*d^3*f*(d*Sin[e + f*x])^(3/2)) +
 (8*b*g*Sqrt[g*Cos[e + f*x]])/(5*a^2*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*b*(a^
2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) + (4*g^2*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(7*a*d^4*f*Sqrt[g*Cos[
e + f*x]]*Sqrt[d*Sin[e + f*x]]) - (2*(a^2 - b^2)*g^2*EllipticF[e - Pi/4 + f
*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^3*d^4*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin
[e + f*x]]) - (b^2*(a^2 - b^2)*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*
e + 2*f*x]])/(a^5*d^4*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```


Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
```

```
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{9/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{5/2}(a+b}}{a^2d^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{\left(4bg^2\right)\int \frac{1}{\sqrt{g\cos(e+fx)}(d}}{5a^2d^3}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g\sqrt{g\cos(e+fx)}}{7ad^3f(d\sin(e+fx))^{3}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g\sqrt{g\cos(e+fx)}}{7ad^3f(d\sin(e+fx))^{3}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g\sqrt{g\cos(e+fx)}}{7ad^3f(d\sin(e+fx))^{3}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g\sqrt{g\cos(e+fx)}}{7ad^3f(d\sin(e+fx))^{3}}$$

$$= \frac{2\sqrt{2}b^3\sqrt{-a^2+b^2}g^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^5d^{9/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 21.1163, size = 1207, normalized size = 1.75

$$\frac{(g\cos(e+fx))^{3/2}\left(-\frac{2\csc^4(e+fx)}{7a}+\frac{2b\csc^3(e+fx)}{5a^2}+\frac{2\left(a^2-7b^2\right)\csc^2(e+fx)}{21a^3}-\frac{2b\left(a^2-5b^2\right)\csc(e+fx)}{5a^4}\right)\sin^4(e+fx)\tan(e+fx)}{f(d\sin(e+fx))^{9/2}} - \frac{(g\cos(e}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((-2*b*(a^2 - 5*b^2)*Csc[e + f*x])/(5*a^4) + (2*(a^
2 - 7*b^2)*Csc[e + f*x]^2)/(21*a^3) + (2*b*Csc[e + f*x]^3)/(5*a^2) - (2*Csc
[e + f*x]^4)/(7*a))*Sin[e + f*x]^4*Tan[e + f*x])/(f*(d*Sin[e + f*x])^(9/2))
```

```
 - ((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(9/2)*((-2*(2*a^4 + 7*a^2*b^2 - 21*
b^4)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4,
1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x
]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3
/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b
^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b
*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1
 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x
]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2
] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 +
 b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^
2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*
(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt
[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[e + f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a
 + b*Sin[e + f*x])) + (2*(2*a^3*b - 14*a*b^3)*Sqrt[Sin[e + f*x]]*((Sqrt[a]*
(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*
ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-a
 + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*T
an[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*Appel
lF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e
 + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(Cos
[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*x]^2
)^(3/2))))/(21*a^4*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^(9/2))
```

Maple [B] time = 0.402, size = 6707, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

3.1423 $\int \frac{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=936

$$-\frac{(a^2-b^2)\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)g^{5/2}}{\sqrt{2}b^3f}-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)g^{5/2}}{4\sqrt{2}bf}+\frac{(a^2-b^2)\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right.}{\sqrt{2}b^3f}$$

```
[Out] -(Sqrt[d]*g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g
]*Sqrt[d*Sin[e + f*x]])]))/(4*Sqrt[2]*b*f) - ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Ar
cTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x
]])]))/(Sqrt[2]*b^3*f) + (Sqrt[d]*g^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g
*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])]))/(4*Sqrt[2]*b*f) + ((a^2 -
b^2)*Sqrt[d]*g^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])]))/(Sqrt[2]*b^3*f) + (Sqrt[d]*g^(5/2)*Log[Sqrt[g]
 + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin
[e + f*x]]])/(8*Sqrt[2]*b*f) + ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Log[Sqrt[g] + S
qrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e +
 f*x]]])/(2*Sqrt[2]*b^3*f) - (Sqrt[d]*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e +
 f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sq
rt[2]*b*f) - ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x
] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2
]*b^3*f) - (2*Sqrt[2]*a*Sqrt[-a + b]*Sqrt[a + b]*d*g^(5/2)*EllipticPi[-(Sqr
t[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
 + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[
2]*a*Sqrt[-a + b]*Sqrt[a + b]*d*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b]
, ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[S
in[e + f*x]])/(b^3*f*Sqrt[d*Sin[e + f*x]]) + (g*(g*Cos[e + f*x])^(3/2)*Sqrt
[d*Sin[e + f*x]])/(2*b*f) + (a*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4
+ f*x, 2]*Sqrt[d*Sin[e + f*x]])/(b^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 1.60019, antiderivative size = 936, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 17, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.46, Rules used = {2901, 2838, 2572, 2639, 2568, 2575, 297, 1162, 617, 204, 1165, 628, 2909, 2906, 2905, 490, 1218}

$$-\frac{(a^2-b^2)\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)g^{5/2}}{\sqrt{2}b^3f}-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)g^{5/2}}{4\sqrt{2}bf}+\frac{(a^2-b^2)\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right.}{\sqrt{2}b^3f}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```


```
[Out] -(Sqrt[d]*g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g
]*Sqrt[d*Sin[e + f*x]])]))/(4*Sqrt[2]*b*f) - ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Ar
cTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x
]])]))/(Sqrt[2]*b^3*f) + (Sqrt[d]*g^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g
*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])]))/(4*Sqrt[2]*b*f) + ((a^2 -
b^2)*Sqrt[d]*g^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])]))/(Sqrt[2]*b^3*f) + (Sqrt[d]*g^(5/2)*Log[Sqrt[g]
 + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin
[e + f*x]]])/(8*Sqrt[2]*b*f) + ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Log[Sqrt[g] + S
qrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e +
 f*x]]])/(2*Sqrt[2]*b^3*f) - (Sqrt[d]*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e +
```

```
f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sq
rt[2]*b*f) - ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x
] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2
]*b^3*f) - (2*Sqrt[2]*a*Sqrt[-a + b]*Sqrt[a + b]*d*g^(5/2)*EllipticPi[-(Sqr
t[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
 + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[
2]*a*Sqrt[-a + b]*Sqrt[a + b]*d*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b]
, ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[S
in[e + f*x]])/(b^3*f*Sqrt[d*Sin[e + f*x]]) + (g*(g*Cos[e + f*x])^(3/2)*Sqrt
[d*Sin[e + f*x]])/(2*b*f) + (a*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4
+ f*x, 2]*Sqrt[d*Sin[e + f*x]])/(b^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2901

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[(g^2*(a^2 - b^2))/b^2, Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
```

```
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{g^2\int \sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}(a-b\sin(e+fx))\,dx}{b^2} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cdots}}{}}{b\cdots}$$

$$= \frac{\left(ag^2\right)\int \sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,dx}{b^2} - \frac{g^2\int \sqrt{g\cos(e+fx)}(d\sin(e+fx))\cdots}{bd}$$

$$= \frac{g(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bf} - \frac{\left(dg^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{4b} + \frac{\left(2\left(a^2-b^2\right)d^2g^3\right)\cdots}{}$$

$$= \frac{g(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bf} + \frac{ag^2\sqrt{g\cos(e+fx)}E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin\cdots}}{b^2f\sqrt{\sin(2e+2fx)}}$$

$$= \frac{g(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bf} + \frac{ag^2\sqrt{g\cos(e+fx)}E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin\cdots}}{b^2f\sqrt{\sin(2e+2fx)}}$$

$$= \frac{\left(a^2-b^2\right)\sqrt{d}g^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}b^3f} - \frac{\left(a^2-b^2\right)\sqrt{d}g\cdots}{}$$

$$= -\frac{\left(a^2-b^2\right)\sqrt{d}g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}b^3f} + \frac{\left(a^2-b^2\right)\sqrt{d}g^{5/2}\tan^{-1}\left(1+\frac{\sqrt{\cdots}}{\cdots}\right)}{\sqrt{2}b^3f}$$

$$= -\frac{\sqrt{d}g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}bf} - \frac{\left(a^2-b^2\right)\sqrt{d}g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e\cdots}}{\sqrt{g}\sqrt{d\sin(e+\cdots}}\right)}{\sqrt{2}b^3f}$$

Mathematica [C] time = 26.9864, size = 1618, normalized size = 1.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]
),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*Sec[e + f*x]*Sqrt[d*Sin[e + f*x]])/(2*b*f) - ((g*Co
s[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]*((2*b*(-(b*AppellF1[3/4, -1/4, 1, 7/
4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/
4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]
^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/((a^2 - b^2)*(1
 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (Sqrt[Tan[e + f*x]]*((3*Sq
rt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])
/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqr
t[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqr
t[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*Ap
pellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Ta
n[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a*Cos[
e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)
^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 +
 Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e +
f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2
+ b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f
*x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - S
qrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[
e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]
*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqr
t[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]]
)/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 -
 Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqr
t[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*S
qrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]]
)/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/
4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) +
(2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a +
Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[
e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Ta
n[e + f*x]^2])))/(42*a*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin
[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(4*b*f*Cos[e +
 f*x]^(5/2)*Sqrt[Sin[e + f*x]])
```

Maple [B] time = 0.404, size = 6219, normalized size = 6.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

3.1424 $\int \frac{(g\cos(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=572

$$\frac{ag^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}b^2\sqrt{d}f}-\frac{ag^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}+1\right)}{\sqrt{2}b^2\sqrt{d}f}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}}\right.}{b^2f\sqrt{d\sin(e+f}}$$

```
[Out] (a*g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[
d*Sin[e + f*x]])])/(Sqrt[2]*b^2*Sqrt[d]*f) - (a*g^(5/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^
2*Sqrt[d]*f) - (a*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqr
t[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*Sqrt[d]*f)
 + (a*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*
Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*Sqrt[d]*f) + (2*Sqrt[2
]*Sqrt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]),
ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin
[e + f*x]])/(b^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a +
 b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]
]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^2*f*Sqrt[d*
Sin[e + f*x]]) - (g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqr
t[d*Sin[e + f*x]])/(b*d*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 1.0231, antiderivative size = 572, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {2901, 2838, 2575, 297, 1162, 617, 204, 1165, 628, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{ag^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}b^2\sqrt{d}f}-\frac{ag^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}+1\right)}{\sqrt{2}b^2\sqrt{d}f}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}}\right.}{b^2f\sqrt{d\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[
d*Sin[e + f*x]])])/(Sqrt[2]*b^2*Sqrt[d]*f) - (a*g^(5/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^
2*Sqrt[d]*f) - (a*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqr
t[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*Sqrt[d]*f)
 + (a*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*
Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*Sqrt[d]*f) + (2*Sqrt[2
]*Sqrt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]),
ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin
[e + f*x]])/(b^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a +
 b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]
]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^2*f*Sqrt[d*
Sin[e + f*x]]) - (g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqr
t[d*Sin[e + f*x]])/(b*d*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2901

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
```

[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] - Dist[(g^2*(a^2 - b^2))/b^2, Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}(a-b\sin(e+fx))}{\sqrt{d\sin(e+fx)}}\,dx}{b^2} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx}{b^2}$$

$$= \frac{\left(ag^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b^2} - \frac{g^2\int \sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,dx}{bd} - \frac{\left(\left(a^2-b^2\right)g^2\right.}{}$$

$$= -\frac{\left(2adg^3\right)\mathrm{Subst}\left(\int \frac{x^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{b^2f} + \frac{\left(4\sqrt{2}\left(a^2-b^2\right)g^3\sqrt{\sin(e+fx}\right.}{}$$

$$= -\frac{g^2\sqrt{g\cos(e+fx)}E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bdf\sqrt{\sin(2e+2fx)}} + \frac{\left(ag^3\right)\mathrm{Subst}\left(\int \frac{g-dx^2}{g^2+d^2x^4}\right.}{b^2f}$$

$$= \frac{2\sqrt{2}\sqrt{-a+b}\sqrt{a+b}g^{5/2}\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{b^2f\sqrt{d\sin(e+fx)}}$$

$$= -\frac{ag^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}b^2\sqrt{d}f} + \frac{ag^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot\right.}{2\sqrt{}}$$

$$= \frac{ag^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}b^2\sqrt{d}f} - \frac{ag^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}b^2\sqrt{d}f} - \frac{ag^{5/2}}{}$$

Mathematica [C] time = 25.765, size = 1402, normalized size = 2.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*Sqrt[Sin[e + f*x]]*((Sqrt[Tan[e + f*x]]*((3*Sqrt[2]
*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt
[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]]
 - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2
 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF
1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e +
 f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e +
 f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3
/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Ta
n[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x
]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b
^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]
^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt
[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e +
 f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
 f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a
^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2
]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(
```

```
a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sq
rt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2
]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(
a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*
Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] +
Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqr
t[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e +
 f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e
 + f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[
e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(2*f*Cos[e + f*
x]^(5/2)*Sqrt[d*Sin[e + f*x]])
```

Maple [B] time = 0.303, size = 5148, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")

[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

3.1425 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=616

$$-\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{abdf\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\,\Pi\Big(}{abdf\sqrt{d\sin(}}$$

```
[Out] -((g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[
d*Sin[e + f*x]])])/(Sqrt[2]*b*d^(3/2)*f)) + (g^(5/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*d^(
3/2)*f) + (g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sq
rt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*d^(3/2)*f) - (g^(5/
2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x
]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*d^(3/2)*f) - (2*g*(g*Cos[e + f*x])^
(3/2))/(a*d*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g
^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/
(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*b*d*f*Sqrt[d*
Sin[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[Sqr
t[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*b*d*f*Sqrt[d*Sin[e + f*x]]) - (2*g^2*
Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a*
d^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 1.13606, antiderivative size = 616, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {2900, 2838, 2570, 2572, 2639, 2575, 297, 1162, 617, 204, 1165, 628, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{abdf\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\,\Pi\Big(}{abdf\sqrt{d\sin(}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[
d*Sin[e + f*x]])])/(Sqrt[2]*b*d^(3/2)*f)) + (g^(5/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*d^(
3/2)*f) + (g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sq
rt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*d^(3/2)*f) - (g^(5/
2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x
]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*d^(3/2)*f) - (2*g*(g*Cos[e + f*x])^
(3/2))/(a*d*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g
^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/
(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*b*d*f*Sqrt[d*
Sin[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[Sqr
t[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*b*d*f*Sqrt[d*Sin[e + f*x]]) - (2*g^2*
Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a*
d^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2900

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/(a*b), I
nt[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(b - a*Sin[e + f*x]), x], x]
 + Dist[(g^2*(a^2 - b^2))/(a*b*d), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e +
 f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x
] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LtQ[n, -1] |
| (EqQ[p, 3/2] && EqQ[n, -2^(-1)]))
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}(b-a\sin(e+fx))}{(d\sin(e+fx))^{3/2}}dx}{ab} + \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}dx}{abd}$$

$$= \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}}dx}{a} - \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}dx}{bd} + \frac{\left(\left(a^2-b^2\right)g^2\sqrt{\sin(e+fx)}\right)\int}{abd\sqrt{d\sin(e}}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}} - \frac{\left(2g^2\right)\int\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,dx}{ad^2} + \frac{\left(2g^3\right)}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}} - \frac{g^3\,\mathrm{Subst}\left(\int\frac{g-dx^2}{g^2+d^2x^4}dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{bdf} + \frac{g^3\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{2}\sqrt{-a+b}\sqrt{a+b}g^{5/2}\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e}}{\sqrt{g}\sqrt{1+\sin}}\right.\right.}{abdf\sqrt{d\sin(e+fx)}}$$

$$= \frac{g^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}bd^{3/2}f} - \frac{g^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot(e\right.}{2\sqrt{2}}$$

$$= -\frac{g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}bd^{3/2}f} + \frac{g^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}bd^{3/2}f} + \frac{g^{5/2}}{}$$

Mathematica [C] time = 26.797, size = 1614, normalized size = 2.62

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2)*Tan[e + f*x])/(a*f*(d*Sin[e + f*x])^(3/2)) + ((g
*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(3/2)*((2*a*(-(b*AppellF1[3/4, -1/4, 1, 7
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1
/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x
]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/((a^2 - b^2)*(
1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (b*Sqrt[Tan[e + f*x]]*((3
*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x
]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/
Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] -
Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b
*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(6*a^2*
Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x
]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt
[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[
e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-
a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
 + f*x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1
```

```
 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[
Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqr
t[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4
*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt
[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1
/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log
[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 +
 Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[
2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f
*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)
^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4
) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[
a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*
Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1
+ Tan[e + f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b
*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a*f*Cos[e
 + f*x]^(5/2)*(d*Sin[e + f*x])^(3/2))
```

Maple [B] time = 0.349, size = 5212, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{5}{2}}}{(b\sin(fx+e)+a)(d\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

3.1426 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=359

$$\frac{2bg^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^2d^3f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e}}{\sqrt{g}\sqrt{\sin(e+}}\right.\right.}{a^2d^2f\sqrt{d\sin(e+fx)}}$$

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(3*a*d*f*(d*Sin[e + f*x])^(3/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a
+ b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqr
t[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]]
)/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g^
(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqr
t[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*d^2*f*Sqrt[d*Si
n[e + f*x]]) + (2*b*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*S
qrt[d*Sin[e + f*x]])/(a^2*d^3*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 0.81217, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2899, 2563, 2570, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{2bg^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^2d^3f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e}}{\sqrt{g}\sqrt{\sin(e+}}\right.\right.}{a^2d^2f\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(3*a*d*f*(d*Sin[e + f*x])^(3/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a
+ b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqr
t[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]]
)/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g^
(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqr
t[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*d^2*f*Sqrt[d*Si
n[e + f*x]]) + (2*b*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*S
qrt[d*Sin[e + f*x]])/(a^2*d^3*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
```

```
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}}dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}dx}{a^2d^2} - \frac{(bg^2}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(2bg^2\right)\int\sqrt{g\cos(e+}}{a^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(4\sqrt{2}\left(a^2-b^2\right)g^3\sqrt{\mathrm{si}}\right.}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{2bg^2\sqrt{g\cos(e+fx)}E}{a^2d^3f}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{2\sqrt{2}\sqrt{-a+b}\sqrt{a+b}g}{}$$

<u>Mathematica</u> [C] time = 26.6989, size = 1659, normalized size = 4.62

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*b*Cot[e + f*x])/a^2 - (2*Cot[e + f*x]*Csc[e + f
*x])/(3*a))*Sin[e + f*x]*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x])^(5/2)) - ((g*C
os[e + f*x])^(5/2)*Sin[e + f*x]^(5/2)*((4*a*b*(-(b*AppellF1[3/4, -1/4, 1, 7
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1
/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x
]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)
*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((a^2 - 2*b^2)*Sqrt[Tan
[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[T
an[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^
2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x
]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(
1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e
+ f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2
, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^
(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a
^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a
^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*
ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 -
b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]
*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt
```

```
[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*
a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2
*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 -
 b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(
a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)
*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (S
qrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + S
qrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]
^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e +
 f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])
))/(a^2*f*Cos[e + f*x]^(5/2)*(d*Sin[e + f*x])^(5/2))
```

Maple [B] time = 0.264, size = 4668, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/3/f*2^(1/2)/a^2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/
2)-a)*(a-b)*(6*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2
*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*s
in(f*x+e)*(-a^2+b^2)^(1/2)*b^2-6*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^2-3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b+3*sin(f*x+e)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b+
3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))
*cos(f*x+e)*sin(f*x+e)*b^3-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^
2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*b^3+6*EllipticF((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2-3*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*(
-a^2+b^2)^(1/2)*b^2-6*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^2-3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2
^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b+3*cos(f*
x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
```

icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-
a),1/2*2^(1/2))*a^2*b+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^
2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*b^3+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*a*b^2-3*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*(-a^2+
b^2)^(1/2)*b^2-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)
-a),1/2*2^(1/2))*sin(f*x+e)*a*b^2-3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/
2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^3+3*cos(f*x+e)*s
in(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2
*2^(1/2))*a^3+3*sin(f*x+e)*(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2+3*sin(f*x+e)*(-a^2+b^2)^(1/2
)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)
)*a^2+3*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2+3*cos(f*x+e)*sin(f*x+e)*(
-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-
a),1/2*2^(1/2))*a^2+2*2^(1/2)*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*a^2-3*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)
*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a
-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b^2-3*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*
sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b
+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b^2-6*EllipticF((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b+12*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*
b-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/

```
2))*sin(f*x+e)*b^3-6*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b-3*s
in(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+
(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*a^3+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b
^2)^(1/2)-a),1/2*2^(1/2))*a^3-6*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b+12*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b)*(g*cos(
f*x+e))^(5/2)*sin(f*x+e)/cos(f*x+e)^3/(d*sin(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

3.1427 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=519

$$\frac{2g^2\left(a^2-b^2\right)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^3d^4f\sqrt{\sin(2e+2fx)}}+\frac{2g\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}{a^3d^3f\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}bg^{5/2}\sqrt{b-a}\sqrt{a+b}}{}$$

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(5*a*d*f*(d*Sin[e + f*x])^(5/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(3*a^2*d^2*f*(d*Sin[e + f*x])^(3/2)) - (4*g*(g*Cos[e +
 f*x])^(3/2))/(5*a*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*(a^2 - b^2)*g*(g*Cos[e
+ f*x])^(3/2))/(a^3*d^3*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[-a + b]
*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*
Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a
^3*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b*Sqrt[-a + b]*Sqrt[a + b]*g^(5
/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[
g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^3*d^3*f*Sqrt[d*Sin[
e + f*x]]) - (4*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
d*Sin[e + f*x]])/(5*a*d^4*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*(a^2 - b^2)*g^2*Sq
rt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^3*
d^4*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 1.20984, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {2899, 2570, 2572, 2639, 2563, 2910, 2906, 2905, 490, 1218}

$$\frac{2g^2\left(a^2-b^2\right)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^3d^4f\sqrt{\sin(2e+2fx)}}+\frac{2g\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}{a^3d^3f\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}bg^{5/2}\sqrt{b-a}\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(5*a*d*f*(d*Sin[e + f*x])^(5/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(3*a^2*d^2*f*(d*Sin[e + f*x])^(3/2)) - (4*g*(g*Cos[e +
 f*x])^(3/2))/(5*a*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*(a^2 - b^2)*g*(g*Cos[e
+ f*x])^(3/2))/(a^3*d^3*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[-a + b]
*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*
Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a
^3*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b*Sqrt[-a + b]*Sqrt[a + b]*g^(5
/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[
g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^3*d^3*f*Sqrt[d*Sin[
e + f*x]]) - (4*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
d*Sin[e + f*x]])/(5*a*d^4*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*(a^2 - b^2)*g^2*Sq
rt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^3*
d^4*f*Sqrt[Sin[2*e + 2*f*x]])
```


Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
```

```
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx}{a^2d^2} - \frac{\left(bg^2\right) \cdots}{\cdots}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} + \frac{\left(b\left(a^2-b^2\right)g^2\right)\int \cdots}{a^3 \cdots}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\cos(e+fx))^{3/2}}{5ad^3f\sqrt{d\sin(e+fx)}} \cdots$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\cos(e+fx))^{3/2}}{5ad^3f\sqrt{d\sin(e+fx)}} \cdots$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\cos(e+fx))^{3/2}}{5ad^3f\sqrt{d\sin(e+fx)}} \cdots$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\cos(e+fx))^{3/2}}{5ad^3f\sqrt{d\sin(e+fx)}} \cdots$$

Mathematica [C] time = 24.5224, size = 1737, normalized size = 3.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*(3*a^2*Cos[e + f*x] - 5*b^2*Cos[e + f*x])*Csc[e
 + f*x])/(5*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x])/(3*a^2) - (2*Cot[e + f*x
]*Csc[e + f*x]^2)/(5*a))*Sin[e + f*x]^2*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x])
^(7/2)) + ((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(7/2)*((-2*(6*a^3 - 10*a*b^2
)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x
]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin
[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f
*x])) + ((8*a^2*b - 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcT
an[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1
 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[
2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f
```

```
*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[
a^2 - b^2]*Tan[e + f*x]]])/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/
4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2))*(b*T
an[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[
Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-3*a^2*b
 + 5*b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 +
Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f
*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 +
 b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*
x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e
 + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt
[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])
/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 -
Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt
[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sq
rt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])
/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (
2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
+ Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + S
qrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e
 + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan
[e + f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Si
n[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^3*f*Cos[
e + f*x]^(5/2)*(d*Sin[e + f*x])^(7/2))
```

Maple [B] time = 0.339, size = 10138, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

3.1428 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=612

$$-\frac{2bg^2\left(a^2-b^2\right)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^4d^5f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}b^2g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{si}\right.}{a^4d^4f\sqrt{d\sin(e+fx)}}$$

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(7*a*d*f*(d*Sin[e + f*x])^(7/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(5*a^2*d^2*f*(d*Sin[e + f*x])^(5/2)) - (8*g*(g*Cos[e +
 f*x])^(3/2))/(21*a*d^3*f*(d*Sin[e + f*x])^(3/2)) + (2*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(3*a^3*d^3*f*(d*Sin[e + f*x])^(3/2)) + (4*b*g*(g*Cos[e +
f*x])^(3/2))/(5*a^2*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*b*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[-a
 + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sq
rt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]
])/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Sqrt[-a + b]*Sqrt[a +
b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]
/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^4*d^4*f*Sqrt
[d*Sin[e + f*x]]) + (4*b*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x,
 2]*Sqrt[d*Sin[e + f*x]])/(5*a^2*d^5*f*Sqrt[Sin[2*e + 2*f*x]]) - (2*b*(a^2
- b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e +
 f*x]])/(a^4*d^5*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 1.58397, antiderivative size = 612, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {2899, 2570, 2563, 2572, 2639, 2910, 2906, 2905, 490, 1218}

$$-\frac{2bg^2\left(a^2-b^2\right)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^4d^5f\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{2}b^2g^{5/2}\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(e+fx)}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{si}\right.}{a^4d^4f\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(7*a*d*f*(d*Sin[e + f*x])^(7/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(5*a^2*d^2*f*(d*Sin[e + f*x])^(5/2)) - (8*g*(g*Cos[e +
 f*x])^(3/2))/(21*a*d^3*f*(d*Sin[e + f*x])^(3/2)) + (2*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(3*a^3*d^3*f*(d*Sin[e + f*x])^(3/2)) + (4*b*g*(g*Cos[e +
f*x])^(3/2))/(5*a^2*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*b*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[-a
 + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sq
rt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]
])/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Sqrt[-a + b]*Sqrt[a +
b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]
/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^4*d^4*f*Sqrt
[d*Sin[e + f*x]]) + (4*b*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x,
 2]*Sqrt[d*Sin[e + f*x]])/(5*a^2*d^5*f*Sqrt[Sin[2*e + 2*f*x]]) - (2*b*(a^2
- b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e +
 f*x]])/(a^4*d^5*f*Sqrt[Sin[2*e + 2*f*x]])
```


Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
```

```
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx}{a^2d^2} - \frac{(b}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{\left(2bg^2\right)\int \frac{\sqrt{g\cos(e+}}{(d\sin(e+fx}}{5a^2d^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e+fx}{21ad^3f(d\sin(e+}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e+fx}{21ad^3f(d\sin(e+}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e+fx}{21ad^3f(d\sin(e+}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e+fx}{21ad^3f(d\sin(e+}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(g\cos(e+fx}{21ad^3f(d\sin(e+}$$

Mathematica [C] time = 24.0066, size = 1779, normalized size = 2.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((-2*(3*a^2*b*Cos[e + f*x] - 5*b^3*Cos[e + f*x])*Cs
c[e + f*x])/(5*a^4) + (2*(3*a^2*Cos[e + f*x] - 7*b^2*Cos[e + f*x])*Csc[e +
f*x]^2)/(21*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*a^2) - (2*Cot[e + f
*x]*Csc[e + f*x]^3)/(7*a))*Sin[e + f*x]^3*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x
])^(9/2)) - (b*(g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(9/2)*((-2*(6*a^3 - 10*a
*b^2)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e +
```

```
 f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])
*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e
 + f*x])) + ((8*a^2*b - 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*
ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcT
an[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + S
qrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e
 + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + S
qrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1
, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2))*
(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*S
qrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-3*a
^2*b + 5*b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[
1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e
 + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a
^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
+ f*x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1
- Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[T
an[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[T
an[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt
[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*
Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[
a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/
4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[
1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 +
Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2
]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*
x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4)
 + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*
x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a
 + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*T
an[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 +
 Tan[e + f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a +
b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^4*f*
Cos[e + f*x]^(5/2)*(d*Sin[e + f*x])^(9/2))
```

Maple [B] time = 0.438, size = 10704, normalized size = 17.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

3.1429 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{11/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=822

$$-\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,b^3}{a^5d^5f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\cdots\right.\right.}{a^5d^5f\sqrt{d\sin\cdots}}$$

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(9*a*d*f*(d*Sin[e + f*x])^(9/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(7*a^2*d^2*f*(d*Sin[e + f*x])^(7/2)) - (4*g*(g*Cos[e +
 f*x])^(3/2))/(15*a*d^3*f*(d*Sin[e + f*x])^(5/2)) + (2*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(5*a^3*d^3*f*(d*Sin[e + f*x])^(5/2)) + (8*b*g*(g*Cos[e +
f*x])^(3/2))/(21*a^2*d^4*f*(d*Sin[e + f*x])^(3/2)) - (2*b*(a^2 - b^2)*g*(g*
Cos[e + f*x])^(3/2))/(3*a^4*d^4*f*(d*Sin[e + f*x])^(3/2)) - (8*g*(g*Cos[e +
 f*x])^(3/2))/(15*a*d^5*f*Sqrt[d*Sin[e + f*x]]) + (4*(a^2 - b^2)*g*(g*Cos[e
 + f*x])^(3/2))/(5*a^3*d^5*f*Sqrt[d*Sin[e + f*x]]) + (2*b^2*(a^2 - b^2)*g*(
g*Cos[e + f*x])^(3/2))/(a^5*d^5*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^3*Sq
rt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcS
in[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e +
 f*x]])/(a^5*d^5*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*Sqrt[-a + b]*Sqrt
[a + b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e +
f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^5*d^5*f
*Sqrt[d*Sin[e + f*x]]) - (8*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f
*x, 2]*Sqrt[d*Sin[e + f*x]])/(15*a*d^6*f*Sqrt[Sin[2*e + 2*f*x]]) + (4*(a^2
- b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e +
 f*x]])/(5*a^3*d^6*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*b^2*(a^2 - b^2)*g^2*Sqrt[
g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^5*d^6
*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 2.1413, antiderivative size = 822, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {2899, 2570, 2572, 2639, 2563, 2910, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,b^3}{a^5d^5f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\cdots\right.\right.}{a^5d^5f\sqrt{d\sin\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(11/2)*(a + b*Sin[e + f*x])),x
]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(9*a*d*f*(d*Sin[e + f*x])^(9/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(7*a^2*d^2*f*(d*Sin[e + f*x])^(7/2)) - (4*g*(g*Cos[e +
 f*x])^(3/2))/(15*a*d^3*f*(d*Sin[e + f*x])^(5/2)) + (2*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(5*a^3*d^3*f*(d*Sin[e + f*x])^(5/2)) + (8*b*g*(g*Cos[e +
f*x])^(3/2))/(21*a^2*d^4*f*(d*Sin[e + f*x])^(3/2)) - (2*b*(a^2 - b^2)*g*(g*
Cos[e + f*x])^(3/2))/(3*a^4*d^4*f*(d*Sin[e + f*x])^(3/2)) - (8*g*(g*Cos[e +
 f*x])^(3/2))/(15*a*d^5*f*Sqrt[d*Sin[e + f*x]]) + (4*(a^2 - b^2)*g*(g*Cos[e
 + f*x])^(3/2))/(5*a^3*d^5*f*Sqrt[d*Sin[e + f*x]]) + (2*b^2*(a^2 - b^2)*g*(
g*Cos[e + f*x])^(3/2))/(a^5*d^5*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^3*Sq
rt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcS
in[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e +
 f*x]])/(a^5*d^5*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*Sqrt[-a + b]*Sqrt
[a + b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e +
```

```
f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^5*d^5*f
*Sqrt[d*Sin[e + f*x]]) - (8*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f
*x, 2]*Sqrt[d*Sin[e + f*x]])/(15*a*d^6*f*Sqrt[Sin[2*e + 2*f*x]]) + (4*(a^2
- b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e +
 f*x]])/(5*a^3*d^6*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*b^2*(a^2 - b^2)*g^2*Sqrt[
g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^5*d^6
*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2899

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[(b*g^2)/(a^2*d)
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[(g^2
*(a^2 - b^2))/(a^2*d^2), Int[((g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n
+ 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
```

```
b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{11/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{11/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx}{a^2d^2} - \frac{\left(bg^2\right.}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{\left(4bg^2\right)\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/}}}{7a^2d^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos(e+fx))^{3}}{15ad^3f(d\sin(e+fx}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos(e+fx))^{3}}{15ad^3f(d\sin(e+fx}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos(e+fx))^{3}}{15ad^3f(d\sin(e+fx}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos(e+fx))^{3}}{15ad^3f(d\sin(e+fx}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos(e+fx))^{3}}{15ad^3f(d\sin(e+fx}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(g\cos(e+fx))^{3}}{15ad^3f(d\sin(e+fx}$$

Mathematica [C] time = 24.9844, size = 1853, normalized size = 2.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(11/2)*(a + b*Sin[e + f*
x])),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*(2*a^4*Cos[e + f*x] + 9*a^2*b^2*Cos[e + f*x] -
15*b^4*Cos[e + f*x])*Csc[e + f*x])/(15*a^5) - (2*(3*a^2*b*Cos[e + f*x] - 7*
b^3*Cos[e + f*x])*Csc[e + f*x]^2)/(21*a^4) + (2*(a^2*Cos[e + f*x] - 3*b^2*C
os[e + f*x])*Csc[e + f*x]^3)/(15*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x]^3)/(
7*a^2) - (2*Cot[e + f*x]*Csc[e + f*x]^4)/(9*a))*Sin[e + f*x]^4*Tan[e + f*x]
^2)/(f*(d*Sin[e + f*x])^(11/2)) + ((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(11/
2)*((-2*(4*a^5 + 18*a^3*b^2 - 30*a*b^4)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Co
s[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1,
 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2
)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 -
Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^4*b + 24*a^2*b^3 - 30*b
^5)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(
1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*S
qrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]
))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 +
 Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin
[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^4*b - 9*a^2*b^3 + 15*b^5)*C
os[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x
]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a
^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*Appel
lF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*
Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[
Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] -
 (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/S
qrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)
^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*Arc
Tan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2
)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
 f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqr
t[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Ta
n[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2
)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a
^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2
- b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(
a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2
])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])
*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(15*a^5*f*Cos[e + f*x]^(
5/2)*(d*Sin[e + f*x])^(11/2))
```

Maple [B] time = 0.552, size = 17102, normalized size = 20.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x, al
gorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(11
/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(11/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x, al
gorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(11
/2)), x)
```

3.1430 $\int \frac{(d\sin(e+fx))^{5/2}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=616

$$-\frac{2\sqrt{2}a^2d^{5/2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}a^2d^{5/2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d}\cdots}{\sqrt{d}\sqrt{\cdots}}\right.\right.}{b^2f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

```
[Out] (a*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[
g*Cos[e + f*x]])])/(Sqrt[2]*b^2*f*Sqrt[g]) - (a*d^(5/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^
2*f*Sqrt[g]) - (2*Sqrt[2]*a^2*d^(5/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b
- Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e +
 f*x]])], -1])/(b^2*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*a
^2*d^(5/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSi
n[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^2*Sqrt[-a
^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) - (a*d^(5/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[
g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*S
qrt[2]*b^2*f*Sqrt[g]) + (a*d^(5/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Si
n[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^2*f
*Sqrt[g]) - (d^2*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(b*f*g) + (d^3*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(2*b*f*Sqrt[g*Cos[e +
f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 1.14104, antiderivative size = 616, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {2909, 2568, 2573, 2641, 2574, 297, 1162, 617, 204, 1165, 628, 2908, 2907, 1218}

$$-\frac{2\sqrt{2}a^2d^{5/2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}a^2d^{5/2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d}\cdots}{\sqrt{d}\sqrt{\cdots}}\right.\right.}{b^2f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(5/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[
g*Cos[e + f*x]])])/(Sqrt[2]*b^2*f*Sqrt[g]) - (a*d^(5/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^
2*f*Sqrt[g]) - (2*Sqrt[2]*a^2*d^(5/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b
- Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e +
 f*x]])], -1])/(b^2*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*a
^2*d^(5/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSi
n[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^2*Sqrt[-a
^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) - (a*d^(5/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[
g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*S
qrt[2]*b^2*f*Sqrt[g]) + (a*d^(5/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Si
n[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^2*f
*Sqrt[g]) - (d^2*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(b*f*g) + (d^3*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(2*b*f*Sqrt[g*Cos[e +
f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{5/2}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{d\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{(ad)\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{d^2\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{bfg} - \frac{\left(ad^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \frac{\left(a^2d^2\right)\int \frac{}{\sqrt{g}}}{}$$

$$= -\frac{d^2\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{bfg} - \frac{\left(2ad^3g\right)\text{Subst}\left(\int \frac{x^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(}}{\sqrt{g\cos(}}\right.}{b^2f}$$

$$= -\frac{d^2\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{bfg} + \frac{d^3F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{2bf\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} +$$

$$= -\frac{2\sqrt{2}a^2d^{5/2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{b^2\sqrt{-a^2+b^2}f\sqrt{g\cos(e+fx)}} + \frac{2}{}$$

$$= -\frac{2\sqrt{2}a^2d^{5/2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{b^2\sqrt{-a^2+b^2}f\sqrt{g\cos(e+fx)}} + \frac{2}{}$$

$$= \frac{ad^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}b^2f\sqrt{g}} - \frac{ad^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}b^2f\sqrt{g}} - \frac{2}{}$$

Mathematica [C] time = 27.2443, size = 1321, normalized size = 2.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(5/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*((2*Sqrt[Sin[e + f*x]]*((Sqrt[a]
*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2
*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-
a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*
Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*Appe
llF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[
e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(Co
s[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*x]^
2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Sin[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1
 + Tan[e + f*x]^2])*(-20*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] +
 20*Sqrt[2]*a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + (10*Sqrt[2]*Sqrt[a]*
(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqr
t[a]])/(a^2 - b^2)^(3/4) - (10*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*ArcTan[1 + (Sq
rt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(3/4) + 1
0*Sqrt[2]*a*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 10*Sqrt[2]
*a*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (5*Sqrt[2]*Sqrt[a]*
(2*a^2 - b^2)*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + (5*Sqrt[2]*Sqrt[a]*(2
*a^2 - b^2)*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] +
Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + 8*b*AppellF1[5/4, 1/2, 1
```

```
, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/
2) + (40*b*Sqrt[Tan[e + f*x]])/Sqrt[1 + Tan[e + f*x]^2] + (200*a^4*b*Appell
F1[1/4, 1/2, 1, 5/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sq
rt[Tan[e + f*x]])/(Sqrt[1 + Tan[e + f*x]^2]*(-5*a^2*AppellF1[1/4, 1/2, 1, 5
/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + 2*(2*(a^2 - b^2)*Appe
llF1[5/4, 1/2, 2, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + a^
2*AppellF1[5/4, 3/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2
])*Tan[e + f*x]^2)*(-(b^2*Tan[e + f*x]^2) + a^2*(1 + Tan[e + f*x]^2)))))/(2
0*b^2*Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(-1 + Tan[
e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(2*f*Sqrt[g*Cos[e + f*x]]*Sin[e + f
*x]^(5/2))
```

Maple [B] time = 0.307, size = 2313, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/f*2^(1/2)*a/b^2/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/
2)-a)*(I*sin(f*x+e)*(-a^2+b^2)^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*a^2+I*sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b-I*sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2-I*sin(f*x+e)*(-a^2+b^2)^(1
/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b+sin(f*x+e)*(
-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2-
sin(f*x+e)*(-a^2+b^2)^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*a*b+sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*a^2-sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b-sin(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2+sin(f*x+e)*
(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b-sin(f*x+e)
*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b^2-sin(f*x+e
)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
```

```
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2))
,1/2*2^(1/2))*a^2+sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^
2)^(1/2)-a),1/2*2^(1/2))*a^3-sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b-sin(f*x+e)*((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^3+sin(f*x+e)*((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2*b+cos(f*x+e)^
2*2^(1/2)*(-a^2+b^2)^(1/2)*a*b-(-a^2+b^2)^(1/2)*cos(f*x+e)^2*2^(1/2)*b^2-co
s(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a*b+(-a^2+b^2)^(1/2)*cos(f*x+e)*2^(1/2)*b
^2)*(d*sin(f*x+e))^(5/2)/(-1+cos(f*x+e))/sin(f*x+e)^2/(g*cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin \left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{g \cos \left(fx+e\right)}\left(b \sin \left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

3.1431 $\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=508

$$\frac{2\sqrt{2}ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\cdots}}{\sqrt{d}\sqrt{c\cdots}}\right.\right.}{bf\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[
g*Cos[e + f*x]])])/(Sqrt[2]*b*f*Sqrt[g])) + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b*f*S
qrt[g]) + (2*Sqrt[2]*a*d^(3/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[
-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])
], -1])/(b*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*a*d^(3/2)*
Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Si
n[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b*Sqrt[-a^2 + b^2]*f*S
qrt[g*Cos[e + f*x]]) + (d^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e
 + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*f*Sqrt
[g]) - (d^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g
*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*f*Sqrt[g])
```

Rubi [A] time = 0.772379, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {2909, 2574, 297, 1162, 617, 204, 1165, 628, 2908, 2907, 1218}

$$\frac{2\sqrt{2}ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\cdots}}{\sqrt{d}\sqrt{c\cdots}}\right.\right.}{bf\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(3/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[
g*Cos[e + f*x]])])/(Sqrt[2]*b*f*Sqrt[g])) + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b*f*S
qrt[g]) + (2*Sqrt[2]*a*d^(3/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[
-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])
], -1])/(b*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*a*d^(3/2)*
Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Si
n[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b*Sqrt[-a^2 + b^2]*f*S
qrt[g*Cos[e + f*x]]) + (d^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e
 + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*f*Sqrt
[g]) - (d^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g
*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*f*Sqrt[g])
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{d\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{b}$$

$$= \frac{\left(2d^2g\right)\text{Subst}\left(\int \frac{x^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\right)}{bf} - \frac{\left(ad\sqrt{\cos(e+fx)}\right)\int \frac{\sqrt{d}}{\sqrt{\cos(e+f}}}{b\sqrt{g\cos(e+f}}$$

$$= -\frac{d^2\,\text{Subst}\left(\int \frac{d-gx^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\right)}{bf} + \frac{d^2\,\text{Subst}\left(\int \frac{d+gx^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin}}{\sqrt{g\cos}}\right.}{bf}$$

$$= \frac{2\sqrt{2}ad^{3/2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{b\sqrt{-a^2+b^2}f\sqrt{g\cos(e+fx)}} - \frac{2\sqrt{2}a}{}$$

$$= \frac{2\sqrt{2}ad^{3/2}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{b\sqrt{-a^2+b^2}f\sqrt{g\cos(e+fx)}} - \frac{2\sqrt{2}a}{}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}bf\sqrt{g}} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}bf\sqrt{g}} + \frac{2\sqrt{}}{}$$

Mathematica [C] time = 16.5048, size = 518, normalized size = 1.02

$$\frac{10\left(a^2-b^2\right)\cot(e+fx)(d\sin(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{aF_1\left(\frac{1}{4};-\frac{1}{4},1;\right.}{\cos^2(e+fx)\left(\left(b^2-a^2\right)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-4b^2F_1\left(\frac{5}{4};\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(3/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (10*(a^2 - b^2)*Cot[e + f*x]*(d*Sin[e + f*x])^(3/2)*(a + b*Sqrt[Sin[e + f*x
]^2])*((a*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)])/(5*(a^2 - b^2)*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, -1/4, 2, 9/4, Cos
```

```
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4
, 3/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e +
f*x]^2) + (b*AppellF1[1/4, -3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)]*Sqrt[Sin[e + f*x]^2])/(-5*(a^2 - b^2)*AppellF1[1/4, -3/4,
1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (4*b^2*AppellF
1[5/4, -3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3
*(a^2 - b^2)*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)])*Cos[e + f*x]^2)))/(f*Sqrt[g*Cos[e + f*x]]*(-a + b*Sin[e +
f*x])*(a + b*Sin[e + f*x])^2)
```

Maple [B] time = 0.28, size = 941, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/f*2^(1/2)*a/b/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)
-a)*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*a*(-a^2+b^2)^(1/2)-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)-I*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*(-a^2+
b^2)^(1/2)+I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*(-a^2+b^2)^(1/2)+EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b*(-a^
2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*a*(-a^2+b^2)^(1/2)+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)+EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*
2^(1/2))*a^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a
-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*(-a^2+b^2)^(1/
2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^
2)^(1/2)-a),1/2*2^(1/2))*a^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b+EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)
)*a*(-a^2+b^2)^(1/2))*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
(d*sin(f*x+e))^(3/2)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{g \cos\left(fx+e\right)}\left(b \sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin \left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{g \cos \left(fx+e\right)}\left(b \sin \left(fx+e\right)+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

3.1432 $\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=209

$$\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx}}{\sqrt{d}\sqrt{\cos(e+fx)}}\right.\right.}{f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

```
[Out] (-2*Sqrt[2]*Sqrt[d]*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2]
)), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(Sq
rt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*Sqrt[d]*Sqrt[Cos[e + f*
x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sq
rt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x
]])
```

Rubi [A] time = 0.354131, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {2908, 2907, 1218}

$$\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx}}{\sqrt{d}\sqrt{\cos(e+fx)}}\right.\right.}{f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```


```
[Out] (-2*Sqrt[2]*Sqrt[d]*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2]
)), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(Sq
rt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*Sqrt[d]*Sqrt[Cos[e + f*
x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sq
rt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x
]])
```


Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
```

```
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{\sqrt{\cos(e+fx)}\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{\sqrt{g\cos(e+fx)}}$$

$$= \frac{\left(2\sqrt{2}\left(1-\frac{b}{\sqrt{-a^2+b^2}}\right)d\sqrt{\cos(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\left(b-\sqrt{-a^2+b^2}\right)d+ax^2\right)\sqrt{1-\frac{x^4}{d^2}}}\,dx, x \right.}{f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{\sqrt{-a^2+b^2}f\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{2}}{}$$

Mathematica [A] time = 4.20825, size = 170, normalized size = 0.81

$$\frac{2\sqrt{2}\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}\cot(e+fx)\sqrt{d\sin(e+fx)}\left(\Pi\left(\frac{a}{\sqrt{b^2-a^2}-b};-\sin^{-1}\left(\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};-\right.\right.}{f\sqrt{b^2-a^2}\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sin[e + f*x]]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (2*Sqrt[2]*Cot[e + f*x]*(EllipticPi[a/(-b + Sqrt[-a^2 + b^2]), -ArcSin[Sqrt
[Tan[(e + f*x)/2]]], -1] - EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), -ArcSin[
Sqrt[Tan[(e + f*x)/2]]], -1])*Sqrt[d*Sin[e + f*x]]*Sqrt[Tan[(e + f*x)/2]])/
(Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x
])])
```

Maple [B] time = 0.224, size = 520, normalized size = 2.5

$$-\frac{\sqrt{2}a\sin\left(fx+e\right)}{f\left(-1+\cos\left(fx+e\right)\right)}\sqrt{d\sin\left(fx+e\right)}\left(\sqrt{-a^2+b^2}\text{EllipticPi}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}, a\left(a-b+\sqrt{-a^2+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*a/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-
a)*(d*sin(f*x+e))^(1/2)*((-a^2+b^2)^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))+(-a^2+b^2)^(1/
2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)
^(1/2)-a),1/2*2^(1/2))+EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a-EllipticPi(((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b-EllipticPi
(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2
*2^(1/2))*a+EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(
```

```
-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*sin(f*x+e)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(e+fx\right)}}{\sqrt{g\cos\left(e+fx\right)}\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*sin(e + f*x))/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)}}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)),
 x)
```

3.1433 $\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=273

$$\frac{2\sqrt{2}b\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a\sqrt{d}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}b\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\right.}{a\sqrt{d}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

```
[Out] (2*Sqrt[2]*b*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), Arc
Sin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*Sqrt[-a
^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[Cos[e + f*x]]
*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[
d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*Sqrt[-a^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[
e + f*x]]) + (EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a*f*Sqr
t[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 0.586007, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2910, 2573, 2641, 2908, 2907, 1218}

$$\frac{2\sqrt{2}b\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a\sqrt{d}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}b\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\right.}{a\sqrt{d}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*b*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), Arc
Sin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*Sqrt[-a
^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[Cos[e + f*x]]
*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[
d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*Sqrt[-a^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[
e + f*x]]) + (EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a*f*Sqr
t[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}\,dx}{a} - \frac{b\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{\left(b\sqrt{\cos(e+fx)}\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{ad\sqrt{g\cos(e+fx)}} + \frac{\sqrt{\sin(2e+}}{a\sqrt{g\cos(e}}$$

$$= \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{af\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} - \frac{\left(2\sqrt{2}b\left(1-\frac{b}{\sqrt{-a^2+b^2}}\right)\sqrt{}\right.}{}$$

$$= \frac{2\sqrt{2}b\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-\right.}{a\sqrt{-a^2+b^2}\sqrt{d}f\sqrt{g\cos(e+fx)}}$$

Mathematica [A] time = 8.51983, size = 213, normalized size = 0.78

$$-\frac{4\sqrt{2}\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)-1}}\tan^{\frac{3}{2}}\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{b^2-a^2}F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}}\right)\middle|-1\right)+b\left(\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};-\mathrm{si}\right.\right.\right.}{af\sqrt{b^2-a^2}\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (-4*Sqrt[2]*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(-1 + Cos[e + f*x])]*(Sqrt
[-a^2 + b^2]*EllipticF[ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1] + b*(-Elliptic
Pi[a/(-b + Sqrt[-a^2 + b^2]), -ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1] + Elli
pticPi[-(a/(b + Sqrt[-a^2 + b^2])), -ArcSin[1/Sqrt[Tan[(e + f*x)/2]]], -1])
)*Tan[(e + f*x)/2]^(3/2))/(a*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d
*Sin[e + f*x]])
```

Maple [B] time = 0.269, size = 631, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/f*2^(1/2)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)*
(2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*
(-a^2+b^2)^(1/2)-2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a*b-EllipticPi((-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/
2))*b^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-
a^2+b^2)^(1/2)),1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b+E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(
1/2)-a),1/2*2^(1/2))*b^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b*(-a^2+b^2)^(1/2))*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)^2/(d*sin(f*x+e))^(1
/2)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sin\left(e+fx\right)}\sqrt{g\cos\left(e+fx\right)}\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*sin(e + f*x))*sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x)))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

3.1434 $\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=320

$$-\frac{2\sqrt{2}b^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}b^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx}}{\sqrt{d}\sqrt{\cos(e+fx)}}\right.\right.}{a^2d^{3/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

```
[Out] (-2*Sqrt[2]*b^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])),
ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sq
rt[-a^2 + b^2]*d^(3/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[Cos[e
+ f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]
/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqrt[-a^2 + b^2]*d^(3/2)*f*Sq
rt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]])/(a*d*f*g*Sqrt[d*Sin[e + f*x]
]) - (b*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^2*d*f*Sqrt[
g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 0.83484, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2910, 2563, 2573, 2641, 2908, 2907, 1218}

$$-\frac{2\sqrt{2}b^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}b^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx}}{\sqrt{d}\sqrt{\cos(e+fx)}}\right.\right.}{a^2d^{3/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*b^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])),
ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sq
rt[-a^2 + b^2]*d^(3/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[Cos[e
+ f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]
/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqrt[-a^2 + b^2]*d^(3/2)*f*Sq
rt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]])/(a*d*f*g*Sqrt[d*Sin[e + f*x]
]) - (b*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^2*d*f*Sqrt[
g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}(a}}{ad}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}} + \frac{b^2\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}(a+b\sin(e+fx))}\,dx}{a^2d^2} -$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}} + \frac{\left(b^2\sqrt{\cos(e+fx)}\right)\int \frac{\sqrt{d\sin(e}}{\sqrt{\cos(e+fx)}(a+}}{a^2d^2\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}} - \frac{bF\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2f}}{a^2df\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+f}}$$

$$= -\frac{2\sqrt{2}b^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx}}\right.\right.}{a^2\sqrt{-a^2+b^2}d^{3/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [A] time = 9.28494, size = 229, normalized size = 0.72

$$\frac{2\left(\frac{2\sqrt{2}b\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)-1}}\tan^{\frac{3}{2}}\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{b^2-a^2}F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}}\right)\middle|-1\right)+b\left(\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}}\right)\middle|-1\right)-\Pi\left(\frac{a}{\sqrt{b^2-a^2}-b};-\mathrm{s}\right.\right.\right.}{\sqrt{b^2-a^2}}}{a^2df\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x
]))，x]
```

```
[Out] (2*(-(a*Cos[e + f*x]) + (2*Sqrt[2]*b*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(
-1 + Cos[e + f*x])]*(Sqrt[-a^2 + b^2]*EllipticF[ArcSin[1/Sqrt[Tan[(e + f*x)
/2]]], -1] + b*(-EllipticPi[a/(-b + Sqrt[-a^2 + b^2]), -ArcSin[1/Sqrt[Tan[(
e + f*x)/2]]], -1] + EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), -ArcSin[1/Sqrt
[Tan[(e + f*x)/2]]], -1]))*Tan[(e + f*x)/2]^(3/2))/Sqrt[-a^2 + b^2]))/(a^2*
d*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Maple [B] time = 0.293, size = 2286, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] 1/f*2^(1/2)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-a)/
a*((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2)
)*cos(f*x+e)*(-a^2+b^2)^(1/2)*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-
b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*a*b^2-(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*b^3+EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/
2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*
(-a^2+b^2)^(1/2)*b^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*cos(f*x+e)*a*b^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*b^3+2*cos(f*x+e)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-2*cos(f*x+e)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+(-(-1+co
```

```
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2
)^(1/2)*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/
2*2^(1/2))*a*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2
)),1/2*2^(1/2))*b^3+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^2+EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^3+2*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-2*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+2*(-a^2+b^2)^(1/2)*cos(f*x+e)*2^
(1/2)*a^2-2*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a*b)*sin(f*x+e)/(d*sin(f*x+
e))^(3/2)/(g*cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

3.1435 $\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=424

$$\frac{b^2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^3d^2f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}b^3\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}b^3\sqrt{\cos(}{}$$

```
[Out] (2*Sqrt[2]*b^3*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), A
rcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*Sqr
t[-a^2 + b^2]*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[Cos[e +
 f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/
(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*Sqrt[-a^2 + b^2]*d^(5/2)*f*Sqr
t[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]])/(3*a*d*f*g*(d*Sin[e + f*x])^(
3/2)) + (2*b*Sqrt[g*Cos[e + f*x]])/(a^2*d^2*f*g*Sqrt[d*Sin[e + f*x]]) + (2*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a*d^2*f*Sqrt[g*Cos[
e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (b^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Si
n[2*e + 2*f*x]])/(a^3*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 1.16384, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {2910, 2570, 2573, 2641, 2563, 2908, 2907, 1218}

$$\frac{b^2\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^3d^2f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}b^3\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}b^3\sqrt{\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (2*Sqrt[2]*b^3*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), A
rcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*Sqr
t[-a^2 + b^2]*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[Cos[e +
 f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/
(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*Sqrt[-a^2 + b^2]*d^(5/2)*f*Sqr
t[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]])/(3*a*d*f*g*(d*Sin[e + f*x])^(
3/2)) + (2*b*Sqrt[g*Cos[e + f*x]])/(a^2*d^2*f*g*Sqrt[d*Sin[e + f*x]]) + (2*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a*d^2*f*Sqrt[g*Cos[
e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (b^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Si
n[2*e + 2*f*x]])/(a^3*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```


Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
```

```
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{5/2}}dx}{a} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}}}{ad}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}dx}{3ad^2}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b\sqrt{g\cos(e+fx)}}{a^2d^2fg\sqrt{d\sin(e+fx)}} - \frac{b^3\int}{}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b\sqrt{g\cos(e+fx)}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{2F}{3ad}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b\sqrt{g\cos(e+fx)}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{2F}{3ad}$$

$$= \frac{2\sqrt{2}b^3\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}}; \sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\right)}{a^3\sqrt{-a^2+b^2}d^{5/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] time = 20.3005, size = 1137, normalized size = 2.68

$$\frac{\cos(e+fx)\left(\frac{2b\csc(e+fx)}{a^2} - \frac{2\csc^2(e+fx)}{3a}\right)\sin^3(e+fx)}{f\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{5/2}} + \frac{\sqrt{\cos(e+fx)}\left(\frac{4ab\sqrt{\sin(e+fx)}\left(\sqrt{a}\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}\right)+2\tan^{-1}\left(\sqrt{2}\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (Cos[e + f*x]*((2*b*Csc[e + f*x])/a^2 - (2*Csc[e + f*x]^2)/(3*a))*Sin[e + f
*x]^3)/(f*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)) + (Sqrt[Cos[e + f*x]
]*Sin[e + f*x]^(5/2)*((-2*(2*a^2 + 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*
((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^
2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1
 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[C
os[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[
1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e +
f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e
 + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 +
Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1
+ Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])
/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[e +
f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x])) + (4*a*b*Sqrt[Sin[
```

```
e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^
2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Ta
n[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]
*(1 + Tan[e + f*x]^2)^(3/2))))/(3*a^2*f*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x
])^(5/2))
```

Maple [B] time = 0.331, size = 2987, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*2^(1/2)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2)-
a)/a^2*(4*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1
/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*sin(f*
x+e)*(-a^2+b^2)^(1/2)*a^3-4*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^2*b+6*EllipticF((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b^2-6*Ellipti
cF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(
1/2)*b^3+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2
*2^(1/2))*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^3+3*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)
*a*b^3-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2
^(1/2))*cos(f*x+e)*sin(f*x+e)*b^4+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*
b^3-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(
1/2))*cos(f*x+e)*sin(f*x+e)*a*b^3+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*b^4+4*EllipticF((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^3-4*Elli
```

```
pticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^2
*b+6*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^
(1/2)*a*b^2-6*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*
2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-
a^2+b^2)^(1/2)*b^3+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^
(1/2)),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^3+3*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*a*b^3-3*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(
f*x+e)*b^4+3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),
1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^3-3*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*a*b^3+3*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e
)*b^4+6*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*a^2*b-6*2^(1/2)*cos(
f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b^2-2*2^(1/2)*cos(f*x+e)*(-a^2+b^2)^(1
/2)*a^3+2*2^(1/2)*cos(f*x+e)*(-a^2+b^2)^(1/2)*a^2*b)*sin(f*x+e)/(d*sin(f*x+
e))^(5/2)/(g*cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{g\cos\left(fx+e\right)}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

3.1436 $\int \frac{(d\sin(e+fx))^{5/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=1064

result too large to display

```
[Out] -((a^2*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*S
qrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2))) + (b*d^(5/2)*ArcT
an[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]]
)])/(Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) + (a^2*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*(a^2
- b^2)*f*g^(3/2)) - (b*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f
*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) + (a
^2*d^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos
[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2)) - (b
*d^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e
 + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) - (a^2*d
^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e +
 f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2)) + (b*d^(
5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f
*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) - (2*Sqrt[2]
*a^3*d^3*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]
]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*(-a + b)^(3
/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a^3*d^3*Elli
pticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[
1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*(-a + b)^(3/2)*(a + b)^(3/2
)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*d^2*Sqrt[d*Sin[e + f*x]])/((a^2 -
b^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*d*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2
)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*d^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x
]])
```

Rubi [A] time = 1.62418, antiderivative size = 1064, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 17, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.46, Rules used = {2902, 2571, 2572, 2639, 2566, 2575, 297, 1162, 617, 204, 1165, 628, 2909, 2906, 2905, 490, 1218}

$$-\frac{2\sqrt{2}a^3\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}d^3}{b(b-a)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}a^3\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e}}{b(b-a)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(5/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^2*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*S
qrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2))) + (b*d^(5/2)*ArcT
an[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]]
)])/(Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) + (a^2*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*(a^2
- b^2)*f*g^(3/2)) - (b*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f
*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) + (a
^2*d^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos
[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2)) - (b
*d^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e
 + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) - (a^2*d
```

```
^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e +
 f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2)) + (b*d^(
5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f
*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) - (2*Sqrt[2]
*a^3*d^3*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]
]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*(-a + b)^(3
/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a^3*d^3*Elli
pticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[
1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*(-a + b)^(3/2)*(a + b)^(3/2
)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*d^2*Sqrt[d*Sin[e + f*x]])/((a^2 -
b^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*d*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2
)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*d^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x
]])
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2909

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[(a*d)/b, Int[((g*Co
s[e + f*x])^p*(d*Sin[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{5/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = -\frac{(bd)\int \frac{(d\sin(e+fx))^{3/2}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} + \frac{\left(ad^2\right)\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{\left(a^2d^2\right)\int \frac{\sqrt{g\cos(e+fx)}\sqrt{d}}{a+b\sin(e}}{\left(a^2-b^2\right)g}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{\left(2ad^2\right)\int \sqrt{g}}{}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{\left(2a^2d^4\right)\text{Subs}}{}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2ad^2\sqrt{g\cos(}}{(}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2ad^2\sqrt{g\cos(}}{(}$$

$$= \frac{a^2d^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}b\left(a^2-b^2\right)fg^{3/2}} - \frac{bd^{5/2}\log\left(\sqrt{g}+\sqrt{g}\,\mathrm{c}\right.}{2\sqrt{2}\,(}$$

$$= -\frac{a^2d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}b\left(a^2-b^2\right)fg^{3/2}} + \frac{bd^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\left(a^2-b^2\right)fg^{3/2}} + \cdots$$

Mathematica [C] time = 80.9139, size = 1296, normalized size = 1.22

$$\frac{2\cot(e+fx)\csc(e+fx)(d\sin(e+fx))^{5/2}(a\sin(e+fx)-b)}{\left(a^2-b^2\right)f(g\cos(e+fx))^{3/2}}-\frac{\cos^{\frac{3}{2}}(e+fx)(d\sin(e+fx))^{5/2}\left(-\frac{2\left(3a^2-b^2\right)\left(aF_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};\text{co}\ldots\right.\right.}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(5/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (2*Cot[e + f*x]*Csc[e + f*x]*(d*Sin[e + f*x])^(5/2)*(-b + a*Sin[e + f*x]))/
((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2)) - (Cos[e + f*x]^(3/2)*(d*Sin[e + f*x
])^(5/2)*((-2*(3*a^2 - b^2)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2
, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqr
t[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]
^2)^(3/4)*(a + b*Sin[e + f*x])) - (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*T
an[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4
, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e +
f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2,
 ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt
[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcT
an[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]
*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a
^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sq
rt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2
*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqr
t[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2
]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[
a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]
*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x
]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4)
 - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e +
 f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a*b*Cos[e + f*x]^(3/2)*Sqrt[Si
n[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x
]^2])))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(5/2))
```

Maple [B] time = 0.329, size = 4622, normalized size = 4.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*2^(1/2)*a/(a+b)/b/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)
^(1/2)-a)*(I*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
```

+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*(-a^2+b^2)^(1/2)*b^2+2*2^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2+Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/
2)),1/2*2^(1/2))*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*a^3+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+
(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*a^3-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(
b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*a^3+2*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a*b-2*2^(1/2)*(-a^2+
b^2)^(1/2)*a*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2
2^(1/2))(-a^2+b^2)^(1/2)*b^2+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-cos(f*x+e)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+2*cos(f*x+e)
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+I*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^
2-4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+2*
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-I*(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2
-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2
)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*(-a^2+b^2)^(1/2)*a^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b-EllipticPi((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f
x+e)(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)
*a^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+
b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*a^2*b+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-4*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+2*cos(f*x+e)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)

```
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-I*cos(f*x+e)*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-I*cos
(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^
2)^(1/2)*a^2+I*cos(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*(-a^2+b^2)^(1/2)*a^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^3-EllipticPi((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a^2+EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1
/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2*b-Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2
)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^
2+b^2)^(1/2)*a^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*a^2*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*(-a^2+b^2)^(1/2)*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1
/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/
2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+2*(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+I*EllipticPi((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a^2)*(d*sin(f*x+e))^(5/2)*
cos(f*x+e)/sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(5/2)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sin \left(fx + e\right)\right)^{\frac{5}{2}}}{\left(g \cos \left(fx + e\right)\right)^{\frac{3}{2}} \left(b \sin \left(fx + e\right) + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

3.1437 $\int \frac{(d\sin(e+fx))^{3/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=379

$$\frac{2bdE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2ad\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b(d\sin(e+fx))^{3/2}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+$$

```
[Out] (2*Sqrt[2]*a^2*d^2*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Co
s[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a
 + b)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*a^2*
d^2*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[
g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b
)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*a*d*Sqrt[d*Sin[e + f*x]])/((a^
2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b*(d*Sin[e + f*x])^(3/2))/((a^2 - b
^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*d*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x
]])
```

Rubi [A] time = 0.827231, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2902, 2563, 2571, 2572, 2639, 2906, 2905, 490, 1218}

$$\frac{2bdE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2ad\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b(d\sin(e+fx))^{3/2}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(3/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```


```
[Out] (2*Sqrt[2]*a^2*d^2*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Co
s[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a
 + b)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*a^2*
d^2*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[
g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b
)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*a*d*Sqrt[d*Sin[e + f*x]])/((a^
2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b*(d*Sin[e + f*x])^(3/2))/((a^2 - b
^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*d*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x
]])
```


Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
```

```
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{3/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = -\frac{(bd)\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} + \frac{\left(ad^2\right)\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} - \frac{\left(a^2d^2\right)\int}{}$$

$$= \frac{2ad\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{(2bd)\int \sqrt{g}}{}$$

$$= \frac{2ad\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{\left(4\sqrt{2}a^2d^2\right.}{}$$

$$= \frac{2ad\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2bd\sqrt{g\cos(}}{}$$

$$= \frac{2\sqrt{2}a^2d^2\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{2}a^2}{}$$

Mathematica [C] time = 23.9039, size = 1651, normalized size = 4.36

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(3/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (2*Cot[e + f*x]*(d*Sin[e + f*x])^(3/2)*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f
*(g*Cos[e + f*x])^(3/2)) - (Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^(3/2)*((4*a
*b*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Si
n[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e +
f*x])) + ((a^2 - b^2)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 -
 (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sq
rt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqr
t[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] +
 Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 -
b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2))*(b*Tan[e +
 f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e
+ f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)
]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-
3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[
e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2,
1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(
7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]
 - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^
2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2
- b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[
2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*S
qrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a
]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]
```

```
] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] +
Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqr
t[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sq
rt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqr
t[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e +
 f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2
)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/
4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*C
os[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x
]^2)*Sqrt[1 + Tan[e + f*x]^2])))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)*
Sin[e + f*x]^(3/2))
```

Maple [B] time = 0.305, size = 2540, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*2^(1/2)*a/(a+b)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^
(1/2)-a)*(2*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))*a+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^
(1/2))*cos(f*x+e)*b-4*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1
/2*2^(1/2))*cos(f*x+e)*b-(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*a+EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*a^2+Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2))
,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+
e)*a*b-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2
)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a
/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*a^2-(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*a*b+2*(-a^2+
b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a+2*(-a^2+b^2)^(
1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-4*(-a^2+b^2)^(1/2)*(
```

```
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-(-a^2+b^2)^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1
/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a+Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),
1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2+Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2
)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b-(-
a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a
),1/2*2^(1/2))*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)
-a),1/2*2^(1/2))*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(
1/2)-a),1/2*2^(1/2))*a*b+2*(-a^2+b^2)^(1/2)*2^(1/2)*cos(f*x+e)*b+2*2^(1/2)*
sin(f*x+e)*(-a^2+b^2)^(1/2)*a-2*2^(1/2)*(-a^2+b^2)^(1/2)*b)*(d*sin(f*x+e))^
(3/2)*cos(f*x+e)/(g*cos(f*x+e))^(3/2)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(3/2)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

3.1438 $\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=374

$$-\frac{2aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2a(d\sin(e+fx))^{3/2}}{dfg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-$$

```
[Out] (-2*Sqrt[2]*a*b*d*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos
[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a
+ b)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a*b*d
*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*
Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b)^(
3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*Sqrt[d*Sin[e + f*x]])/((a^2 - b
^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2)*d
*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 +
 f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 0.911256, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {2903, 2838, 2563, 2571, 2572, 2639, 2906, 2905, 490, 1218}

$$-\frac{2aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2a(d\sin(e+fx))^{3/2}}{dfg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sin[e + f*x]]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*a*b*d*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos
[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a
+ b)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a*b*d
*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*
Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b)^(
3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*Sqrt[d*Sin[e + f*x]])/((a^2 - b
^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2)*d
*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 +
 f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2903

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Dist[d/(a^2 - b^
2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1)*(b - a*Sin[e + f*x]), x
], x] + Dist[(a*b*d)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Si
n[e + f*x])^(n - 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f,
g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n,
0]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
```

```
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = -\frac{d\int \frac{b-a\sin(e+fx)}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} + \frac{(abd)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx}{\left(a^2-b^2\right)g^2}$$

$$= \frac{a\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{(bd)\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} + \frac{\left(abd\sqrt{\sin(e+\right.}{\left(a^2-\right.}$$

$$= -\frac{2b\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2a(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}} - \frac{(2a)\int\sqrt{}}{}$$

$$= -\frac{2b\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{2a(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}} - \frac{\left(2\sqrt{2}abd\right.}{}$$

$$= -\frac{2\sqrt{2}abd\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}} + \frac{2\sqrt{2}ab}{}$$

Mathematica [C] time = 23.0631, size = 1280, normalized size = 3.42

$$\frac{a\sqrt{d\sin(e+fx)}\left(\frac{4a\left(aF_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-bF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\right)\cos^{\frac{3}{2}}(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\sin^{\frac{3}{2}}(e+fx}{3\left(a^2-b^2\right)\left(1-\cos^2(e+fx)\right)^{3/4}(a+b\sin(e+fx))}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sin[e + f*x]]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (2*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]]*(-b + a*Sin[e + f*x]))/((a^2 - b^2)*f*
(g*Cos[e + f*x])^(3/2)) + (a*Cos[e + f*x]^(3/2)*Sqrt[d*Sin[e + f*x]]*((4*a*
(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e
 + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x
])) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan
[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]
^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^
2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^
2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[
2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e +
f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]
*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a
^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqr
t[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]
```

```
*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[
a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a
^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*S
qrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + S
qrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt
[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e +
f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e
+ f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e +
 f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/((a - b)*(a + b)*f
*(g*Cos[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

Maple [B] time = 0.344, size = 2539, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*2^(1/2)*a/(a+b)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(
1/2)-a)*(2*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1
/2*2^(1/2))*a+2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))*cos(f*x+e)*b-4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),1/2*2^(1/2))*a-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b+EllipticPi((-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*a*b+cos(f*x+
e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2)
)*b^2-cos(f*x+e)*(-a^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*a*b-cos(f*x+e)*EllipticPi((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*
2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2+2*(-a^2+b
^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a+2*(-a^2+b^2)^(1
/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-4*(-a^2+b^2)^(1/2)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
```

```
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a-(-a^2+b^2)^(1/2)*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b+EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1
/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b+(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-(-a
^2+b^2)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-
a),1/2*2^(1/2))*a*b-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*b^2+2*cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a+2*sin(f*x+e
)*2^(1/2)*(-a^2+b^2)^(1/2)*b-2*2^(1/2)*(-a^2+b^2)^(1/2)*a)*(d*sin(f*x+e))^(
1/2)*cos(f*x+e)/(g*cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(1/2)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sin\left(fx+e\right)}}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)
), x)
```

3.1439 $\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=380

$$-\frac{2b(d\sin(e+fx))^{3/2}}{d^2fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{dfg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2a\sqrt{d\sin(e+fx)}}{dfg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}$$

```
[Out] (2*Sqrt[2]*b^2*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e
+ f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b
)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Elli
pticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[
1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b)^(3/2)*
f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*a*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d
*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2)*d^2*
f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 +
f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 0.920007, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {2904, 2838, 2563, 2571, 2572, 2639, 2906, 2905, 490, 1218}

$$-\frac{2b(d\sin(e+fx))^{3/2}}{d^2fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{dfg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2a\sqrt{d\sin(e+fx)}}{dfg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```


```
[Out] (2*Sqrt[2]*b^2*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e
+ f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b
)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Elli
pticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[
1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b)^(3/2)*
f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*a*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d
*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2)*d^2*
f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 +
f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```


Rule 2904

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 - b^2
), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[b^2/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{a-b\sin(e+fx)}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} - \frac{b^2\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx)}}{\left(a^2-b^2\right)g^2}$$

$$= \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{\left(a^2-b^2\right)d} - \frac{(b}{}$$

$$= \frac{2a\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)d^2fg\sqrt{g\cos(e+}}$$

$$= \frac{2a\sqrt{d\sin(e+fx)}}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{\left(a^2-b^2\right)d^2fg\sqrt{g\cos(e+}}$$

$$= \frac{2\sqrt{2}b^2\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+f}}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}$$

Mathematica [C] time = 22.0951, size = 1285, normalized size = 3.38

$$b\sqrt{\sin(e+fx)}\left(\frac{4a\left(aF_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-bF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)\right)\cos^{\frac{3}{2}}(e+fx)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\sin^{\frac{3}{2}}(e+fx)}{3\left(a^2-b^2\right)\left(1-\cos^2(e+fx)\right)^{3/4}(a+b\sin(e+fx))} - \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (2*Cos[e + f*x]*Sin[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e
+ f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]) + (b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*
x]]*((4*a*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x
]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]
^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*S
in[e + f*x])) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sq
rt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Ta
n[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*
(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan
[e + f*x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan
[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqr
t[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (S
qrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) +
(4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sq
rt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*L
og[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1
```

```
 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqr
t[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e +
 f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^
2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1
/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
 f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Lo
g[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2
]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[
1 + Tan[e + f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2]))))/((-a + b
)*(a + b)*f*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

Maple [B] time = 0.3, size = 2523, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -1/f*2^(1/2)/(a+b)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1
/2)-a)*(2*cos(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^
(1/2))*a*b+2*cos(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2
*2^(1/2))*b^2-4*cos(f*x+e)*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))*a*b-cos(f*x+e)*(-a^2+b^2)^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2-cos(f*x+e)*(-a^2+b^2)^(1/2)*((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+cos
(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^
2+b^2)^(1/2)),1/2*2^(1/2))*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*a*b^2+cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
,a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*b^3-cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b
^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),
-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*(-a^2+b^2)^(1/2)*((1-cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+2*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2),1/2*2^(1/2))*b^2-4*(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
```

```
e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
),1/2*2^(1/2))*a*b-(-a^2+b^2)^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*b^2-(-a^2+b^2)^(1/2)*((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+EllipticPi(((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^2+EllipticPi(((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^3-((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*sin(f*x+e)*(-a^2+b^
2)^(1/2)*2^(1/2)*a^2+2*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a*b-2*2^(1/2)*(-
a^2+b^2)^(1/2)*a*b)*cos(f*x+e)/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
))), x)
```

3.1440 $\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=568

$$\frac{2b^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{ad^2fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}-\frac{4aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{d^2fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}-\frac{2b\sqrt{}}{d^2fg\left(a^2-\right.}$$

```
[Out] (-2*a)/((a^2 - b^2)*d*f*g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (2*b
^2*(g*Cos[e + f*x])^(3/2))/(a*(a^2 - b^2)*d*f*g^3*Sqrt[d*Sin[e + f*x]]) - (
2*Sqrt[2]*b^3*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e +
 f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*(-a +
b)^(3/2)*(a + b)^(3/2)*d*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*E
llipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sq
rt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*(-a + b)^(3/2)*(a + b)^(
3/2)*d*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*Sqrt[d*Sin[e + f*x]])/((a^2 -
 b^2)*d^2*f*g*Sqrt[g*Cos[e + f*x]]) + (4*a*(d*Sin[e + f*x])^(3/2))/((a^2 -
b^2)*d^3*f*g*Sqrt[g*Cos[e + f*x]]) - (4*a*Sqrt[g*Cos[e + f*x]]*EllipticE[e
- Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d^2*f*g^2*Sqrt[Sin[2*e
+ 2*f*x]]) + (2*b^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
d*Sin[e + f*x]])/(a*(a^2 - b^2)*d^2*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rubi [A] time = 1.41295, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2904, 2838, 2570, 2571, 2572, 2639, 2563, 2910, 2906, 2905, 490, 1218}

$$\frac{2b^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{ad^2fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}-\frac{4aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{d^2fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}-\frac{2b\sqrt{}}{d^2fg\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (-2*a)/((a^2 - b^2)*d*f*g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (2*b
^2*(g*Cos[e + f*x])^(3/2))/(a*(a^2 - b^2)*d*f*g^3*Sqrt[d*Sin[e + f*x]]) - (
2*Sqrt[2]*b^3*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e +
 f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*(-a +
b)^(3/2)*(a + b)^(3/2)*d*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*E
llipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sq
rt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*(-a + b)^(3/2)*(a + b)^(
3/2)*d*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*Sqrt[d*Sin[e + f*x]])/((a^2 -
 b^2)*d^2*f*g*Sqrt[g*Cos[e + f*x]]) + (4*a*(d*Sin[e + f*x])^(3/2))/((a^2 -
b^2)*d^3*f*g*Sqrt[g*Cos[e + f*x]]) - (4*a*Sqrt[g*Cos[e + f*x]]*EllipticE[e
- Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d^2*f*g^2*Sqrt[Sin[2*e
+ 2*f*x]]) + (2*b^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
d*Sin[e + f*x]])/(a*(a^2 - b^2)*d^2*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2904

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 - b^2
), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[b^2/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e + f*x])
```

```
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
```

```
0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{a-b\sin(e+fx)}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin\ldots}}{\left(a^2-b^2\right)g^2\ldots}$$

$$= \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin\ldots}}}{\left(a^2-b^2\right)d\ldots}$$

$$= -\frac{2a}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} + \frac{2\ldots}{a\left(a^2-\ldots\right.}$$

$$= -\frac{2a}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} + \frac{2\ldots}{a\left(a^2-\ldots\right.}$$

$$= -\frac{2a}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} + \frac{2\ldots}{a\left(a^2-\ldots\right.}$$

$$= -\frac{2a}{\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} + \frac{2\ldots}{a\left(a^2-\ldots\right.}$$

Mathematica [C] time = 24.4712, size = 1710, normalized size = 3.01

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f
*x])),x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x]^2*((-2*Cot[e + f*x])/a + (2*Sec[e + f*x]*(-b +
 a*Sin[e + f*x]))/(a^2 - b^2)))/(f*(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^
(3/2)) - (Cos[e + f*x]^(3/2)*Sin[e + f*x]^(3/2)*((-2*(4*a^3 - 2*a*b^2)*(-(b
*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b
^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f
*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x]))
+ ((2*a^2*b - 2*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 -
(Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqr
t[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] +
Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b
^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan
[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2))*(b*Tan[e +
f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e +
 f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b + b^3)
*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f
*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((
-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*App
ellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2
]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqr
t[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]
 - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])
/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^
2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*A
rcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b
^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*S
qrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[
Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b
^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]
*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^
2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sq
rt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])
/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]
^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x
])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a*(a - b)*(a + b)*f*(
g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2))
```

Maple [B] time = 0.323, size = 3104, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*2^(1/2)/(a+b)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/
2)-a)/a*(-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2
*2^(1/2))*(-a^2+b^2)^(1/2)*b^3-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
```

2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(
-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3+2*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3+4*(-a^2+b^2)^(1/2)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*a*b^2-2*(-a^2+b^2)^(1/2)*
2^(1/2)*a^3-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1
/2*2^(1/2))*(-a^2+b^2)^(1/2)*cos(f*x+e)*b^3-(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*cos(f*x+e)*b^3+
2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*cos(f*x+
e)*b^3-2*2^(1/2)*cos(f*x+e)*(-a^2+b^2)^(1/2)*a*b^2+4*2^(1/2)*cos(f*x+e)*(-a
^2+b^2)^(1/2)*a^3-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2
)-a),1/2*2^(1/2))*b^4+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)
^(1/2)),1/2*2^(1/2))*b^4-2*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),1/2*2^(1/2))*cos(f*x+e)*a*b^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*a*b^3-(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*a*b^3+4*(-a^2+
b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b^2-2*(-a^2+b^
2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b^2-8*(-a^2+b^2)
^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*a^3+4*(-a
^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
F((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*a^
3-8*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^3+4*
(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^3+Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)
),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^3-(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*

```
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a
*b^3+2*sin(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a^2*b-(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*b^4+(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*b
^4)*cos(f*x+e)*sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(g\cos(fx+e))^{\frac{3}{2}}(b\sin(fx+e)+a)(d\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(g\cos(fx+e))^{\frac{3}{2}}(b\sin(fx+e)+a)(d\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
3/2)), x)
```

3.1441 $\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=673

$$-\frac{2b^3(g\cos(e+fx))^{3/2}}{a^2d^2fg^3\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}}-\frac{2b^3E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^2d^3fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{4bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e}{d^3fg^2\left(a^2-b^2\right)\sqrt{\mathrm{si}}}$$

```
[Out] (-2*a)/(3*(a^2 - b^2)*d*f*g*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)) +
(2*b^2*(g*Cos[e + f*x])^(3/2))/(3*a*(a^2 - b^2)*d*f*g^3*(d*Sin[e + f*x])^(3
/2)) + (2*b)/((a^2 - b^2)*d^2*f*g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]
) - (2*b^3*(g*Cos[e + f*x])^(3/2))/(a^2*(a^2 - b^2)*d^2*f*g^3*Sqrt[d*Sin[e
+ f*x]]) + (2*Sqrt[2]*b^4*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sq
rt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]
])/(a^2*(-a + b)^(3/2)*(a + b)^(3/2)*d^2*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) -
(2*Sqrt[2]*b^4*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f
*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*(-a +
b)^(3/2)*(a + b)^(3/2)*d^2*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (8*a*Sqrt[d*Si
n[e + f*x]])/(3*(a^2 - b^2)*d^3*f*g*Sqrt[g*Cos[e + f*x]]) - (4*b*(d*Sin[e +
 f*x])^(3/2))/((a^2 - b^2)*d^4*f*g*Sqrt[g*Cos[e + f*x]]) + (4*b*Sqrt[g*Cos[
e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d
^3*f*g^2*Sqrt[Sin[2*e + 2*f*x]]) - (2*b^3*Sqrt[g*Cos[e + f*x]]*EllipticE[e
- Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^2*(a^2 - b^2)*d^3*f*g^2*Sqrt[Sin[
2*e + 2*f*x]])
```

Rubi [A] time = 1.81092, antiderivative size = 673, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2904, 2838, 2570, 2563, 2571, 2572, 2639, 2910, 2906, 2905, 490, 1218}

$$-\frac{2b^3(g\cos(e+fx))^{3/2}}{a^2d^2fg^3\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}}-\frac{2b^3E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^2d^3fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{4bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e}{d^3fg^2\left(a^2-b^2\right)\sqrt{\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),
x]
```


```
[Out] (-2*a)/(3*(a^2 - b^2)*d*f*g*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)) +
(2*b^2*(g*Cos[e + f*x])^(3/2))/(3*a*(a^2 - b^2)*d*f*g^3*(d*Sin[e + f*x])^(3
/2)) + (2*b)/((a^2 - b^2)*d^2*f*g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]
) - (2*b^3*(g*Cos[e + f*x])^(3/2))/(a^2*(a^2 - b^2)*d^2*f*g^3*Sqrt[d*Sin[e
+ f*x]]) + (2*Sqrt[2]*b^4*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sq
rt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]
])/(a^2*(-a + b)^(3/2)*(a + b)^(3/2)*d^2*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) -
(2*Sqrt[2]*b^4*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f
*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*(-a +
b)^(3/2)*(a + b)^(3/2)*d^2*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (8*a*Sqrt[d*Si
n[e + f*x]])/(3*(a^2 - b^2)*d^3*f*g*Sqrt[g*Cos[e + f*x]]) - (4*b*(d*Sin[e +
 f*x])^(3/2))/((a^2 - b^2)*d^4*f*g*Sqrt[g*Cos[e + f*x]]) + (4*b*Sqrt[g*Cos[
e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d
^3*f*g^2*Sqrt[Sin[2*e + 2*f*x]]) - (2*b^3*Sqrt[g*Cos[e + f*x]]*EllipticE[e
- Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^2*(a^2 - b^2)*d^3*f*g^2*Sqrt[Sin[
2*e + 2*f*x]])
```

Rule 2904

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 - b^2
), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[b^2/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e + f*x])
^n)/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{a-b\sin(e+fx)}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin}}{\left(a^2-b^2\right)g^2}$$

$$= \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin}}{\left(a^2-b^2\right)a}$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}} + \frac{}{3a}$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}} + \frac{}{3a}$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}} + \frac{}{3a}$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}} + \frac{}{3a}$$

$$= -\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}(d\sin(e+fx))^{3/2}} + \frac{}{3a}$$

Mathematica [C] time = 23.3683, size = 1730, normalized size = 2.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f
*x])),x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x]^3*((2*b*Cot[e + f*x])/a^2 - (2*Cot[e + f*x]*Cs
c[e + f*x])/(3*a) + (2*Sec[e + f*x]*(a - b*Sin[e + f*x]))/(a^2 - b^2)))/(f*
(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(5/2)) - (b*Cos[e + f*x]^(3/2)*Sin[
e + f*x]^(5/2)*((-2*(4*a^3 - 2*a*b^2)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Co
s[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^2*b - 2*b^3)*Sqrt[Tan[e +
 f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[T
an[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^
2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(
1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
 + f*x]^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]
))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 +
Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b + b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*
x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*Appe
llF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*
Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e +
```

```
 f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)
*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3
/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqr
t[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2
*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt
[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1
/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan
[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(
1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]
 - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (
2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a +
 Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan
[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^
2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b
*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(
3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 +
 Tan[e + f*x]^2])))/(a^2*(-a + b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)*(d*Sin[e
 + f*x])^(5/2))
```

Maple [B] time = 0.352, size = 3315, normalized size = 4.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/3/f*2^(1/2)/(a+b)/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(
1/2)-a)/a^2*(3*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a
/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*cos(f*x+e)*sin(f*x+e)*a*b^4+6*sin(f*x+e)*2^(1/2)*(-a^2+b^2)^
(1/2)*a^3*b-6*2^(1/2)*(-a^2+b^2)^(1/2)*a^4+3*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*b^5+6*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b^3+24*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^
3*b-12*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2
)^(1/2)*a*b^3+3*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-
a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^4+3*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*sin(
f*x+e)*(-a^2+b^2)^(1/2)*b^4-6*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
```

```
)^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2
))*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^4+8*cos(f*x+e)^2*2^(1/2)*(-a^2+
b^2)^(1/2)*a^4-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2
)),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b^4-12*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^3*b-12*sin(f*x+e)*cos(f*x
+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a^3*b+6*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(-a^2+b^
2)^(1/2)*a*b^3-6*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)
*(-a^2+b^2)^(1/2)*b^4-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^
2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*a*b^4+3*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*a*b^4-2*cos(f*x+e)^2*2
^(1/2)*(-a^2+b^2)^(1/2)*a^2*b^2+3*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)*sin(f*x+e)*b^5-3*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*cos(f*x+e)*sin
(f*x+e)*b^5+3*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^4+3*(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*
b^4-3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1
/2))*sin(f*x+e)*b^5-12*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(
f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^3*b+6*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b^3+24*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)
*a^3*b-12*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*sin(f*
x+e)*(-a^2+b^2)^(1/2)*a*b^3)*sin(f*x+e)*cos(f*x+e)/(g*cos(f*x+e))^(3/2)/(d*
sin(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
5/2)), x)
```

3.1442 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=331

$$\frac{\sqrt{2}bg^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2\sqrt{d}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{\sqrt{2}bg^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+f}}{\sqrt{d}\sqrt{\cos(e+f}}\right.\right.}{a^2\sqrt{d}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

```
[Out] (Sqrt[2]*b*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), A
rcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqr
t[-a^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (Sqrt[2]*b*g^2*Sqrt[Cos[e +
 f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/
(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqrt[-a^2 + b^2]*Sqrt[d]*f*Sqr
t[g*Cos[e + f*x]]) + (g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a*d*f*(
a + b*Sin[e + f*x])) + (g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f
*x]])/(2*a^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 0.793352, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2887, 2910, 2573, 2641, 2908, 2907, 1218}

$$\frac{\sqrt{2}bg^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2\sqrt{d}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{\sqrt{2}bg^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+f}}{\sqrt{d}\sqrt{\cos(e+f}}\right.\right.}{a^2\sqrt{d}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^2),x]
```

```
[Out] (Sqrt[2]*b*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), A
rcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqr
t[-a^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (Sqrt[2]*b*g^2*Sqrt[Cos[e +
 f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/
(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqrt[-a^2 + b^2]*Sqrt[d]*f*Sqr
t[g*Cos[e + f*x]]) + (g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a*d*f*(
a + b*Sin[e + f*x])) + (g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f
*x]])/(2*a^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 2887

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -Simp[(g*(g*C
os[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(m + 1))/(a*
d*f*(m + 1)), x] + Dist[(g^2*(2*m + 3))/(2*a*(m + 1)), Int[((g*Cos[e + f*x]
)^(p - 2)*(a + b*Sin[e + f*x])^(m + 1))/Sqrt[d*Sin[e + f*x]], x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p +
 1/2, 0]
```

Rule 2910

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[((g*Cos[e +
f*x])^p*(d*Sin[e + f*x])^(n + 1))/(a + b*Sin[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
```

```
p, 1] && LtQ[n, 0]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2908

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2907

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[(2*Sqrt[2]*d*(b + q))/(f*q), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[(2*Sqrt[2]*d*(b - q))/(f*q), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^2}\,dx = \frac{g\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} + \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))}\,dx}{2a}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} + \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}\,dx}{2a^2} - \frac{(bg^2)\int \frac{}{\sqrt{g}\ldots}}{}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} - \frac{\left(bg^2\sqrt{\cos(e+fx)}\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{\cos(e+fx)}(a+b\sin(e+fx))}}{2a^2d\sqrt{g\cos(e+fx)}}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} + \frac{g^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{2a^2f\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}} - \frac{\left(\sqrt{2}\ldots\right.}{}$$

$$= \frac{\sqrt{2}bg^2\sqrt{\cos(e+fx)}\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^2\sqrt{-a^2+b^2}\sqrt{d}f\sqrt{g\cos(e+fx)}} - \frac{\sqrt{2}bg^2\sqrt{\mathrm{co}\ldots}}{}$$

Mathematica [C] time = 16.0545, size = 717, normalized size = 2.17

$$\frac{\tan(e+fx)(g\cos(e+fx))^{3/2}}{af\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))} - \frac{\sin(e+fx)(g\cos(e+fx))^{3/2}\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{\cdots}{\left(1-\cos^2(e+fx)\right)^{3/4}\left(a^2+b^2(\cos\cdots\right.}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
^2),x]
```

```
[Out] -(((g*Cos[e + f*x])^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2
)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b
^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*AppellF
1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-
4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^
2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((
-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqr
t[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))]
+ Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] - ((1
 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^
(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2
] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f
*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(a*f*Cos[e + f*
x]^(3/2)*(1 - Cos[e + f*x]^2)^(1/4)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x
]))) + ((g*Cos[e + f*x])^(3/2)*Tan[e + f*x])/(a*f*Sqrt[d*Sin[e + f*x]]*(a +
 b*Sin[e + f*x]))
```

Maple [B] time = 0.422, size = 3490, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)/a/(-a^2+b^2)^(1/2)/(a-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2
)-a)*(-2*(-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2
*2^(1/2))*a^2-sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*a^2*b+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b-EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*b^3+(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
```

```
x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)^2
*b^3+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1
/2))*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)^2*a*b^2-2*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*b^2+Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),
1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e
)^2*(-a^2+b^2)^(1/2)*b^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*a*b^2-(-a^2+b^2)^(1/2)*sin(f*x+e)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/
2)),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b-(
-a^2+b^2)^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+
b^2)^(1/2)-a),1/2*2^(1/2))*a*b+2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*a*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*sin(f*x+e)*a*b^2-(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*a*b^2-2*
cos(f*x+e)^2*2^(1/2)*(-a^2+b^2)^(1/2)*a*b+2*cos(f*x+e)*2^(1/2)*(-a^2+b^2)^(
1/2)*a*b-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-
a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*b^3+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1
/2*2^(1/2))*b^3-2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2
)^(1/2)*a*b-2*(-a^2+b^2)^(1/2)*cos(f*x+e)*2^(1/2)*a^2+2*2^(1/2)*cos(f*x+e)^
2*(-a^2+b^2)^(1/2)*a^2+2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*si
n(f*x+e)*(-a^2+b^2)^(1/2)*a*b-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b^2+EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^2-(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^
2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
```

```
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(a-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))
*a*b^2+2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b
^2)*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e))/(-1+cos(f*x+e))/cos(f*
x+e)^2/(d*sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e)+a)^2\sqrt{d\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e
))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e))**2/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e)+a)^2\sqrt{d\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e
))), x)
```

3.1443 $\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=82

$$\frac{3a\tan(c+dx)}{2d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}-\frac{b\cos^3(c+dx)}{3d}+\frac{2b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] (-3*a*x)/2 + (2*b*Cos[c + d*x])/d - (b*Cos[c + d*x]^3)/(3*d) + (b*Sec[c + d
*x])/d + (3*a*Tan[c + d*x])/(2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.13668, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2591, 288, 321, 203, 2590, 270}

$$\frac{3a\tan(c+dx)}{2d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}-\frac{b\cos^3(c+dx)}{3d}+\frac{2b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*x)/2 + (2*b*Cos[c + d*x])/d - (b*Cos[c + d*x]^3)/(3*d) + (b*Sec[c + d
*x])/d + (3*a*Tan[c + d*x])/(2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin^2(c+dx)\tan^2(c+dx)\,dx + b\int \sin^3(c+dx)\tan^2(c+dx)\,dx$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, \cos\right.}{d}$$

$$= -\frac{a\sin^2(c+dx)\tan(c+dx)}{2d} + \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2d}$$

$$= \frac{2b\cos(c+dx)}{d} - \frac{b\cos^3(c+dx)}{3d} + \frac{b\sec(c+dx)}{d} + \frac{3a\tan(c+dx)}{2d} -$$

$$= -\frac{3ax}{2} + \frac{2b\cos(c+dx)}{d} - \frac{b\cos^3(c+dx)}{3d} + \frac{b\sec(c+dx)}{d} + \frac{3a\tan(c}{2d}$$

Mathematica [A] time = 0.408716, size = 82, normalized size = 1.

$$-\frac{3a(c+dx)}{2d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\tan(c+dx)}{d} + \frac{7b\cos(c+dx)}{4d} - \frac{b\cos(3(c+dx))}{12d} + \frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (7*b*Cos[c + d*x])/(4*d) - (b*Cos[3*(c + d*x)])/(1
2*d) + (b*Sec[c + d*x])/d + (a*Sin[2*(c + d*x)])/(4*d) + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.043, size = 104, normalized size = 1.3

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)} + \left((\sin(dx+c))^3 + \frac{3\sin(dx+c)}{2}\right)\cos(dx+c) - \frac{3dx}{2} - \frac{3c}{2}\right) + b\left(\frac{(\sin(dx+c))^6}{\cos(dx+c)} + \left(\frac{8}{3} + (\sin(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/
2*d*x-3/2*c)+b*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)
```

```
*cos(d*x+c)))
```

Maxima [A] time = 1.53752, size = 101, normalized size = 1.23

$$-\frac{3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a+2\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*
a + 2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b)/d
```

Fricas [A] time = 2.14495, size = 185, normalized size = 2.26

$$-\frac{2\,b\cos\left(dx+c\right)^4+9\,adx\cos\left(dx+c\right)-12\,b\cos\left(dx+c\right)^2-3\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)-6\,b}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*cos(d*x + c)^4 + 9*a*d*x*cos(d*x + c) - 12*b*cos(d*x + c)^2 - 3*(
a*cos(d*x + c)^2 + 2*a)*sin(d*x + c) - 6*b)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19943, size = 161, normalized size = 1.96

$$-\frac{9\left(dx+c\right)a+\frac{12\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-10\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a + 12*(a*tan(1/2*d*x + 1/2*c) + b)/(tan(1/2*d*x + 1/2*c)
^2 - 1) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 - 24*b
*tan(1/2*d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - 10*b)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^3)/d
```

3.1444 $\int \sin(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{3b\tan(c+dx)}{2d}-\frac{b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3bx}{2}$$

```
[Out] (-3*b*x)/2 + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (3*b*Tan[c + d*x])/(
2*d) - (b*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.107418, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2838, 2590, 14, 2591, 288, 321, 203}

$$\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{3b\tan(c+dx)}{2d}-\frac{b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*b*x)/2 + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (3*b*Tan[c + d*x])/(
2*d) - (b*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin(c+dx)\tan^2(c+dx)\,dx + b\int \sin^2(c+dx)\tan^2(c+dx)\,d$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \text{ta}\right.}{d}$$

$$= -\frac{b\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{a\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cos(c+\right.}{d}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3b\tan(c+dx)}{2d} - \frac{b\sin^2(c+dx)\,\text{ta}}{2d}$$

$$= -\frac{3bx}{2} + \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3b\tan(c+dx)}{2d} - \frac{b\sin^2(}{}$$

Mathematica [A] time = 0.185097, size = 63, normalized size = 0.97

$$\frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} - \frac{3b(c+dx)}{2d} + \frac{b\sin(2(c+dx))}{4d} + \frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*b*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (b*Sin[2
*(c + d*x)])/(4*d) + (b*Tan[c + d*x])/d
```

Maple [A] time = 0.04, size = 94, normalized size = 1.5

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)} + \left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right) + b\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)} + \left((\sin(dx+c))^3 + \frac{3\sin(dx+c)}{2}\right)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b*(sin(d*x+c)^
5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A] time = 1.4814, size = 84, normalized size = 1.29

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)b-2\,a\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*b
- 2*a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 2.17512, size = 153, normalized size = 2.35

$$-\frac{3\,bdx\cos\left(dx+c\right)-2\,a\cos\left(dx+c\right)^2-\left(b\cos\left(dx+c\right)^2+2\,b\right)\sin\left(dx+c\right)-2\,a}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(3*b*d*x*cos(d*x + c) - 2*a*cos(d*x + c)^2 - (b*cos(d*x + c)^2 + 2*b)*
sin(d*x + c) - 2*a)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23264, size = 140, normalized size = 2.15

$$-\frac{3\,(dx+c)b+\frac{4\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(3*(d*x + c)*b + 4*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)^
2 - 1) + 2*(b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1
/2*d*x + 1/2*c) - 2*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.1445 $\int (a + b\sin(c + dx))\tan^2(c + dx)\,dx$

Optimal. Leaf size=38

$$\frac{a\tan(c + dx)}{d} - ax + \frac{b\cos(c + dx)}{d} + \frac{b\sec(c + dx)}{d}$$

```
[Out] -(a*x) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0640617, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2722, 3473, 8, 2590, 14}

$$\frac{a\tan(c + dx)}{d} - ax + \frac{b\cos(c + dx)}{d} + \frac{b\sec(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx &= \int \left(a\tan^2(c+dx)+b\sin(c+dx)\tan^2(c+dx)\right)\,dx\\ &= a\int \tan^2(c+dx)\,dx + b\int \sin(c+dx)\tan^2(c+dx)\,dx\\ &= \frac{a\tan(c+dx)}{d} - a\int 1\,dx - \frac{b\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -ax + \frac{a\tan(c+dx)}{d} - \frac{b\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(c+dx)\right)}{d}\\ &= -ax + \frac{b\cos(c+dx)}{d} + \frac{b\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0303475, size = 47, normalized size = 1.24

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d} + \frac{a\tan(c+dx)}{d} + \frac{b\cos(c+dx)}{d} + \frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

Maple [A] time = 0.037, size = 59, normalized size = 1.6

$$\frac{1}{d}\left(a\left(\tan(dx+c)-dx-c\right)+b\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

Maxima [A] time = 1.53546, size = 53, normalized size = 1.39

$$-\frac{(dx+c-\tan(dx+c))a - b\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.70017, size = 108, normalized size = 2.84

$$-\frac{adx\cos(dx+c) - b\cos(dx+c)^2 - a\sin(dx+c) - b}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cos(d*x + c) - b*cos(d*x + c)^2 - a*sin(d*x + c) - b)/(d*cos(d*x +
c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23229, size = 78, normalized size = 2.05

$$-\frac{(dx+c)a + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 2b\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) + 2*b)
/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.1446 $\int \sec(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=27

$$\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}-bx$$

```
[Out] -(b*x) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Rubi [A] time = 0.0491544, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2838, 2606, 8, 3473}

$$\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(b*x) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx &= a\int \sec(c+dx)\tan(c+dx)\,dx+b\int \tan^2(c+dx)\,dx\\
&= \frac{b\tan(c+dx)}{d}-b\int 1\,dx+\frac{a\,\text{Subst}(\int 1\,dx,x,\sec(c+dx))}{d}\\
&= -bx+\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0155202, size = 36, normalized size = 1.33

$$\frac{a\sec(c+dx)}{d}-\frac{b\tan^{-1}(\tan(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((b*ArcTan[Tan[c + d*x]])/d) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Maple [A] time = 0.031, size = 32, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a}{\cos(dx+c)}+b(\tan(dx+c)-dx-c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a/cos(d*x+c)+b*(tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.51906, size = 43, normalized size = 1.59

$$-\frac{(dx+c-\tan(dx+c))b-\frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*b - a/cos(d*x + c))/d
```

Fricas [A] time = 1.5509, size = 82, normalized size = 3.04

$$-\frac{bdx\cos(dx+c)-b\sin(dx+c)-a}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b*d*x*cos(d*x + c) - b*sin(d*x + c) - a)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\sin(c+dx))\sin(c+dx)\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sin(c + d*x)*sec(c + d*x)**2, x)
```

Giac [A] time = 1.19254, size = 58, normalized size = 2.15

$$-\frac{(dx+c)b+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*b + 2*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)^2 - 1)
)/d
```

3.1447 $\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=36

$$\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Rubi [A] time = 0.0803195, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2838, 2622, 321, 207, 3767, 8}

$$\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx &= a\int \csc(c+dx)\sec^2(c+dx)\,dx + b\int \sec^2(c+dx)\,dx\\
&= \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} - \frac{b\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}\\
&= \frac{a\sec(c+dx)}{d} + \frac{b\tan(c+dx)}{d} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{b\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0352965, size = 56, normalized size = 1.56

$$\frac{a\sec(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[(c + d*x)/2]])/d) + (a*Log[Sin[(c + d*x)/2]])/d + (a*Sec[c + d
*x])/d + (b*Tan[c + d*x])/d
```

Maple [A] time = 0.051, size = 47, normalized size = 1.3

$$\frac{a}{d\cos(dx+c)} + \frac{a\ln(\csc(dx+c) - \cot(dx+c))}{d} + \frac{b\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a/cos(d*x+c)+1/d*a*ln(csc(d*x+c)-cot(d*x+c))+b*tan(d*x+c)/d
```

Maxima [A] time = 1.01055, size = 65, normalized size = 1.81

$$\frac{a\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) + 2\,b\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 2
*b*tan(d*x + c))/d
```

Fricas [A] time = 1.6692, size = 188, normalized size = 5.22

$$-\frac{a\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2b\sin(dx+c)-2a}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a*cos(d*x + c)*log(-1/2*
cos(d*x + c) + 1/2) - 2*b*sin(d*x + c) - 2*a)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19199, size = 65, normalized size = 1.81

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

3.1448 $\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=48

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{b\sec(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((b*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (b*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

Rubi [A] time = 0.111308, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2838, 2620, 14, 2622, 321, 207}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{b\sec(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (b*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^2(c+dx)\,dx + b\int \csc(c+dx)\sec^2(c+dx)\,d$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec\right.}{d}$$

$$= \frac{b\sec(c+dx)}{d} + \frac{a\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} + \frac{b\,\text{Subs}}{}$$

$$= -\frac{b\tanh^{-1}(\cos(c+dx))}{d} - \frac{a\cot(c+dx)}{d} + \frac{b\sec(c+dx)}{d} + \frac{a\tan(}{}$$

Mathematica [A] time = 0.0756153, size = 68, normalized size = 1.42

$$\frac{a\tan(c+dx)}{d} - \frac{a\cot(c+dx)}{d} + \frac{b\sec(c+dx)}{d} + \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/
2]])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.053, size = 69, normalized size = 1.4

$$\frac{a}{d\sin(dx+c)\cos(dx+c)} - 2\frac{\cot(dx+c)\,a}{d} + \frac{b}{d\cos(dx+c)} + \frac{b\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a/sin(d*x+c)/cos(d*x+c)-2*a*cot(d*x+c)/d+1/d*b/cos(d*x+c)+1/d*b*ln(csc(
d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.03981, size = 80, normalized size = 1.67

$$\frac{b\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 2\,a\left(\frac{1}{\tan(dx+c)} - \tan(dx+c)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 2
*a*(1/tan(d*x + c) - tan(d*x + c)))/d
```

Fricas [A] time = 1.66065, size = 269, normalized size = 5.6

$$-\frac{b\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-b\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+4\,a\cos(dx+c)^2-2\,b\sin(dx+c)-2\,a}{2\,d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*cos(d*x +
 c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 4*a*cos(d*x + c)^2 - 2*b*si
n(d*x + c) - 2*a)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21435, size = 139, normalized size = 2.9

$$\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a*tan(1/2*d*x + 1/2*c) - (2*b*t
an(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c)^2 + 10*b*tan(1/2*d*x + 1/
2*c) - 3*a)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

3.1449 $\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=75

$$\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{b\tan(c+dx)}{d}-\frac{b\cot(c+dx)}{d}$$

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x])/d + (3*a*Sec[c + d*x]
)/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (b*Tan[c + d*x])/d
```

Rubi [A] time = 0.136787, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2838, 2622, 288, 321, 207, 2620, 14}

$$\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{b\tan(c+dx)}{d}-\frac{b\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x])/d + (3*a*Sec[c + d*x]
)/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (b*Tan[c + d*x])/d
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^3(c+dx)\sec^2(c+dx)\,dx + b\int \csc^2(c+dx)\sec^2(c+dx)\,dx$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(\cdots\right.}{d}$$

$$= -\frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2d}\cdots$$

$$= -\frac{b\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d} - \frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{b\tan\cdots}{\cdots}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d} - \frac{a\csc^2(\cdots}{\cdots}$$

Mathematica [B] time = 0.337545, size = 172, normalized size = 2.29

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\sin\left(\frac{1}{2}(c+\cdots\right.}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\text{s}\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*Cot[2*(c + d*x)])/d - (a*Csc[(c + d*x)/2]^2)/(8*d) - (3*a*Log[Cos[(c
+ d*x)/2]])/(2*d) + (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]
^2)/(8*d) + (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
- (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.06, size = 93, normalized size = 1.2

$$-\frac{a}{2d(\sin(dx+c))^2\cos(dx+c)} + \frac{3a}{2d\cos(dx+c)} + \frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d} + \frac{b}{d\sin(dx+c)\cos(dx+c)} - 2\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2/cos(d*x+c)+3/2/d*a/cos(d*x+c)+3/2/d*a*ln(csc(d*x+c)-c
ot(d*x+c))+1/d*b/sin(d*x+c)/cos(d*x+c)-2*b*cot(d*x+c)/d
```

Maxima [A] time = 0.986827, size = 113, normalized size = 1.51

$$\frac{a\left(\frac{2\left(3\cos\left(dx+c\right)^{2}-2\right)}{\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-4b\left(\frac{1}{\tan\left(dx+c\right)}-\tan\left(dx+c\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*b*(1/tan(d*x + c) - tan(d*x
+ c)))/d
```

Fricas [A] time = 1.66725, size = 333, normalized size = 4.44

$$\frac{6\,a\cos\left(dx+c\right)^{2}-3\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\log}{4\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c)^3 - a*cos(d*x + c))*log(1/2*cos
(d*x + c) + 1/2) + 3*(a*cos(d*x + c)^3 - a*cos(d*x + c))*log(-1/2*cos(d*x +
 c) + 1/2) + 4*(2*b*cos(d*x + c)^2 - b)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)
^3 - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2304, size = 157, normalized size = 2.09

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{16\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-\frac{18\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,b\tan}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 12*a*log(abs(tan(1/2*d*x + 1/2*c))) + 4*b*t
an(1/2*d*x + 1/2*c) - 16*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)
^2 - 1) - (18*a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x + 1/2*c) + a)/tan(
1/2*d*x + 1/2*c)^2)/d
```

3.1450 $\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=94

$$\frac{(a^2+2b^2)\cos(c+dx)}{d}+\frac{(a^2+b^2)\sec(c+dx)}{d}+\frac{3ab\tan(c+dx)}{d}-\frac{ab\sin^2(c+dx)\tan(c+dx)}{d}-3abx-\frac{b^2\cos^3(c+$$

```
[Out] -3*a*b*x + ((a^2 + 2*b^2)*Cos[c + d*x])/d - (b^2*Cos[c + d*x]^3)/(3*d) + ((
a^2 + b^2)*Sec[c + d*x])/d + (3*a*b*Tan[c + d*x])/d - (a*b*Sin[c + d*x]^2*T
an[c + d*x])/d
```

Rubi [A] time = 0.18344, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2911, 2591, 288, 321, 203, 4357, 448}

$$\frac{(a^2+2b^2)\cos(c+dx)}{d}+\frac{(a^2+b^2)\sec(c+dx)}{d}+\frac{3ab\tan(c+dx)}{d}-\frac{ab\sin^2(c+dx)\tan(c+dx)}{d}-3abx-\frac{b^2\cos^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -3*a*b*x + ((a^2 + 2*b^2)*Cos[c + d*x])/d - (b^2*Cos[c + d*x]^3)/(3*d) + ((
a^2 + b^2)*Sec[c + d*x])/d + (3*a*b*Tan[c + d*x])/d - (a*b*Sin[c + d*x]^2*T
an[c + d*x])/d
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = (2ab)\int \sin^2(c+dx)\tan^2(c+dx)\,dx + \int \sin(c+dx)\left(a^2+b^2\sin^2(c\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(a^2+b^2-b^2x^2)}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}\left(\int \frac{}{(1-}\right.}{}$$

$$= -\frac{ab\sin^2(c+dx)\tan(c+dx)}{d} - \frac{\text{Subst}\left(\int \left(-a^2\left(1+\frac{2b^2}{a^2}\right)+\frac{a^2+b^2}{x^2}+b\right.\right.}{d}$$

$$= \frac{(a^2+2b^2)\cos(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{(a^2+b^2)\sec(c+dx)}{d} +$$

$$= -3abx + \frac{(a^2+2b^2)\cos(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{(a^2+b^2)\sec(c}{d}$$

Mathematica [A] time = 0.426966, size = 104, normalized size = 1.11

$$\frac{\sec(c+dx)\left(-24\cos(c+dx)\left(a^2+3ab(c+dx)+b^2\right)+4\left(3a^2+5b^2\right)\cos(2(c+dx))+36a^2+54ab\sin(c+dx)+6ab\sin(\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]*(36*a^2 + 45*b^2 - 24*(a^2 + b^2 + 3*a*b*(c + d*x))*Cos[c + d
*x] + 4*(3*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^2*Cos[4*(c + d*x)] + 54*a*b*Si
n[c + d*x] + 6*a*b*Sin[3*(c + d*x)]))/(24*d)
```

Maple [A] time = 0.05, size = 147, normalized size = 1.6

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+2\,ab\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\sin(dx+c))^3+3/2\,\sin(dx+c)\right)\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*a*b*(sin(d
*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+
b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)
))
```

Maxima [A] time = 1.54784, size = 131, normalized size = 1.39

$$-\frac{3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)ab+\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)b^2-3\,a^2\left(\frac{1}{\cos(dx+c)}+\cos\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*(3*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*
a*b + (cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b^2 - 3*a^2*(1/cos
(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.66871, size = 221, normalized size = 2.35

$$-\frac{b^2\cos\left(dx+c\right)^4+9\,abdx\cos\left(dx+c\right)-3\left(a^2+2\,b^2\right)\cos\left(dx+c\right)^2-3\,a^2-3\,b^2-3\left(ab\cos\left(dx+c\right)^2+2\,ab\right)\sin\left(dx+\right.}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(b^2*cos(d*x + c)^4 + 9*a*b*d*x*cos(d*x + c) - 3*(a^2 + 2*b^2)*cos(d*x
 + c)^2 - 3*a^2 - 3*b^2 - 3*(a*b*cos(d*x + c)^2 + 2*a*b)*sin(d*x + c))/(d*c
os(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19564, size = 232, normalized size = 2.47

$$-\frac{9\,(dx+c)ab+\frac{6\left(2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+b^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(9*(d*x + c)*a*b + 6*(2*a*b*tan(1/2*d*x + 1/2*c) + a^2 + b^2)/(tan(1/2
*d*x + 1/2*c)^2 - 1) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*tan(1/2*d*x
+ 1/2*c)^4 - 3*b^2*tan(1/2*d*x + 1/2*c)^4 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 -
12*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2 - 5*b^2)
/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.1451 $\int (a + b\sin(c + dx))^2 \tan^2(c + dx)\,dx$

Optimal. Leaf size=94

$$\frac{a^2\tan(c+dx)}{d} + a^2(-x) + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\sec(c+dx)}{d} + \frac{3b^2\tan(c+dx)}{2d} - \frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{3b^2x}{2}$$

```
[Out] -(a^2*x) - (3*b^2*x)/2 + (2*a*b*Cos[c + d*x])/d + (2*a*b*Sec[c + d*x])/d +
(a^2*Tan[c + d*x])/d + (3*b^2*Tan[c + d*x])/(2*d) - (b^2*Sin[c + d*x]^2*Tan
[c + d*x])/(2*d)
```

Rubi [A] time = 0.138516, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203}

$$\frac{a^2\tan(c+dx)}{d} + a^2(-x) + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\sec(c+dx)}{d} + \frac{3b^2\tan(c+dx)}{2d} - \frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{3b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (3*b^2*x)/2 + (2*a*b*Cos[c + d*x])/d + (2*a*b*Sec[c + d*x])/d +
(a^2*Tan[c + d*x])/d + (3*b^2*Tan[c + d*x])/(2*d) - (b^2*Sin[c + d*x]^2*Tan
[c + d*x])/(2*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx &= \int \left(a^2\tan^2(c+dx)+2ab\sin(c+dx)\tan^2(c+dx)+b^2\sin^2(c+dx)\tan^2(c+dx\right.\\
&= a^2\int \tan^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan^2(c+dx)\,dx+b^2\int \sin^2(c+dx)\\
&= \frac{a^2\tan(c+dx)}{d}-a^2\int 1\,dx-\frac{(2ab)\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}+\frac{b^2\operatorname{Subs}}{}\\
&= -a^2x+\frac{a^2\tan(c+dx)}{d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{(2ab)\operatorname{Subst}\left(\int\left(-1+\frac{1}{x^2}\right.\right.}{d}\\
&= -a^2x+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}\\
&= -a^2x-\frac{3b^2x}{2}+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{3b^2\tan(c}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.415523, size = 77, normalized size = 0.82

$$
\frac{-4\left(2a^2+3b^2\right)(c+dx)+\left(8a^2+9b^2\right)\tan(c+dx)+b\sec(c+dx)(8a\cos(2(c+dx))+24a+b\sin(3(c+dx)))}{8d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-4*(2*a^2 + 3*b^2)*(c + d*x) + b*Sec[c + d*x]*(24*a + 8*a*Cos[2*(c + d*x)]
 + b*Sin[3*(c + d*x)]) + (8*a^2 + 9*b^2)*Tan[c + d*x])/(8*d)
```

Maple [A] time = 0.048, size = 116, normalized size = 1.2

$$\frac{1}{d}\left(a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+b^2\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)
*cos(d*x+c))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos
(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A] time = 1.49602, size = 112, normalized size = 1.19

$$-\frac{2\,(dx+c-\tan(dx+c))a^2+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)b^2-4\,ab\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^2 + (3*d*x + 3*c - tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 2*tan(d*x + c))*b^2 - 4*a*b*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

Fricas [A] time = 1.63604, size = 190, normalized size = 2.02

$$-\frac{\left(2\,a^2+3\,b^2\right)dx\cos(dx+c)-4\,ab\cos(dx+c)^2-4\,ab-\left(b^2\cos(dx+c)^2+2\,a^2+2\,b^2\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*((2*a^2 + 3*b^2)*d*x*cos(d*x + c) - 4*a*b*cos(d*x + c)^2 - 4*a*b - (b^
2*cos(d*x + c)^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2057, size = 185, normalized size = 1.97

$$-\frac{\left(2\,a^{2}+3\,b^{2}\right)(dx+c)+\frac{4\left(a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{2\left(b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 + 3*b^2)*(d*x + c) + 4*(a^2*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2
*d*x + 1/2*c) + 2*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(b^2*tan(1/2*d*x +
1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.1452 $\int \sec(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=42

$$\frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}-2abx+\frac{2b^2\cos(c+dx)}{d}$$

```
[Out] -2*a*b*x + (2*b^2*Cos[c + d*x])/d + (Sec[c + d*x]*(a + b*Sin[c + d*x])^2)/d
```

Rubi [A] time = 0.0636757, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2861, 12, 2638}

$$\frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}-2abx+\frac{2b^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -2*a*b*x + (2*b^2*Cos[c + d*x])/d + (Sec[c + d*x]*(a + b*Sin[c + d*x])^2)/d
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx &= \frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}-\int 2b(a+b\sin(c+dx))\,dx\\
&= \frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}-(2b)\int (a+b\sin(c+dx))\,dx\\
&= -2abx+\frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}-\left(2b^2\right)\int \sin(c+dx)\,dx\\
&= -2abx+\frac{2b^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}
\end{aligned}$$

Mathematica [A] time = 0.307264, size = 66, normalized size = 1.57

$$\frac{\sec(c+dx)\left(2a^2+b^2\cos(2(c+dx))+3b^2\right)-2\left(a^2+2ab(c+dx)-2ab\tan(c+dx)+b^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] ((2*a^2 + 3*b^2 + b^2*Cos[2*(c + d*x)])*Sec[c + d*x] - 2*(a^2 + b^2 + 2*a*b
*(c + d*x) - 2*a*b*Tan[c + d*x]))/(2*d)
```

Maple [A] time = 0.042, size = 75, normalized size = 1.8

$$\frac{1}{d}\left(\frac{a^2}{\cos(dx+c)}+2\,ab\,(\tan(dx+c)-dx-c)+b^2\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2/cos(d*x+c)+2*a*b*(tan(d*x+c)-d*x-c)+b^2*(sin(d*x+c)^4/cos(d*x+c)+(
2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.48884, size = 76, normalized size = 1.81

$$-\frac{2\,(dx+c-\tan(dx+c))ab-b^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*x + c - tan(d*x + c))*a*b - b^2*(1/cos(d*x + c) + cos(d*x + c)) - a^
2/cos(d*x + c))/d
```

Fricas [A] time = 1.68545, size = 132, normalized size = 3.14

$$-\frac{2\,abdx\cos(dx+c)-b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*d*x*cos(d*x + c) - b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1957, size = 111, normalized size = 2.64

$$-\frac{2\left((dx+c)ab+\frac{2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+2\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((d*x + c)*a*b + (2*a*b*tan(1/2*d*x + 1/2*c)^3 + a^2*tan(1/2*d*x + 1/2*c
)^2 + 2*a*b*tan(1/2*d*x + 1/2*c) + a^2 + 2*b^2)/(tan(1/2*d*x + 1/2*c)^4 - 1
))/d
```

3.1453 $\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=46

$$\frac{(a^2+b^2)\sec(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\tan(c+dx)}{d}$$

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + ((a^2 + b^2)*Sec[c + d*x])/d + (2*a*b*Ta
n[c + d*x])/d
```

Rubi [A] time = 0.168135, antiderivative size = 70, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2911, 3767, 8, 3201, 446, 78, 63, 206}

$$\frac{(a^2+b^2)\sec(c+dx)}{d}-\frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)}{d}+\frac{2ab\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*Sec[c + d*x])/d - (a^2*ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[
c + d*x]^2]*Sec[c + d*x])/d + (2*a*b*Tan[c + d*x])/d
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3201

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \sec^2(c+dx)\,dx + \int \csc(c+dx)\sec^2(c+dx)\left(a^2+b^2\sin\right.$$

$$= -\frac{(2ab)\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} + \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+\right.}{}$$

$$= \frac{2ab\tan(c+dx)}{d} + \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\right)\text{Subst}\left(\int \frac{a^2+b^2x}{(1-x)^{3/2}x}\right.}{2d}$$

$$= \frac{\left(a^2+b^2\right)\sec(c+dx)}{d} + \frac{2ab\tan(c+dx)}{d} + \frac{\left(a^2\sqrt{\cos^2(c+dx)}\sec(\right.}{}$$

$$= \frac{\left(a^2+b^2\right)\sec(c+dx)}{d} + \frac{2ab\tan(c+dx)}{d} - \frac{\left(a^2\sqrt{\cos^2(c+dx)}\sec(\right.}{}$$

$$= \frac{\left(a^2+b^2\right)\sec(c+dx)}{d} - \frac{a^2\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)\sqrt{\cos^2(c+dx)}}{d}$$

Mathematica [A] time = 0.197143, size = 58, normalized size = 1.26

$$\frac{\left(a^2+b^2\right)\sec(c+dx)+a\left(a\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+2b\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*Sec[c + d*x] + a*(a*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x
)/2]]) + 2*b*Tan[c + d*x]))/d
```

Maple [A] time = 0.075, size = 68, normalized size = 1.5

$$\frac{a^{2}}{d\cos\left(dx+c\right)}+\frac{a^{2}\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}+2\frac{ab\tan\left(dx+c\right)}{d}+\frac{b^{2}}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2/cos(d*x+c)+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+2*a*b*tan(d*x+c)/d+1/d
*b^2/cos(d*x+c)
```

Maxima [A] time = 1.01197, size = 86, normalized size = 1.87

$$\frac{a^{2}\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+4\,ab\tan\left(dx+c\right)+\frac{2\,b^{2}}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) +
 4*a*b*tan(d*x + c) + 2*b^2/cos(d*x + c))/d
```

Fricas [A] time = 1.95302, size = 209, normalized size = 4.54

$$-\frac{a^{2}\cos\left(dx+c\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-a^{2}\cos\left(dx+c\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-4\,ab\sin\left(dx+c\right)-2\,a^{2}-2\,b^{2}}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a^2*cos(d*x + c)*log(-
1/2*cos(d*x + c) + 1/2) - 4*a*b*sin(d*x + c) - 2*a^2 - 2*b^2)/(d*cos(d*x +
c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21544, size = 77, normalized size = 1.67

$$\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+b^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(2*a*b*tan(1/2*d*x + 1/2*c) + a^2 +
 b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.1454 $\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=59

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] (-2*a*b*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d + (2*a*b*Sec[c + d*
x])/d + ((a^2 + b^2)*Tan[c + d*x])/d
```

Rubi [A] time = 0.281159, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2911, 2622, 321, 207, 3200, 14}

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d + (2*a*b*Sec[c + d*
x])/d + ((a^2 + b^2)*Tan[c + d*x])/d
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3200

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
```

```
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[(x^n*(a + (a + b)*ff^2*x^2)^p)/
(1 + ff^2*x^2)^((m + n)/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \csc(c+dx)\sec^2(c+dx)\,dx + \int \csc^2(c+dx)\sec^2(c+d$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+(a^2+b^2)x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\right.}{d}$$

$$= \frac{2ab\sec(c+dx)}{d} + \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right)+\frac{a^2}{x^2}\right)dx, x, \tan(c+dx\right.}{d}$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cot(c+dx)}{d} + \frac{2ab\sec(c+dx)}{d} +$$

Mathematica [A] time = 0.339447, size = 102, normalized size = 1.73

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\left(2a^2+b^2\right)\cos(2(c+dx))-b\left(4a\sin(c+dx)-2a\sin(2(c+dx))\right)\left(\log\left(c\right.\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sec[c + d*x]*((2*a^2 + b^2)*Cos[2*(c +
d*x)] - b*(b + 4*a*Sin[c + d*x] - 2*a*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c +
 d*x)/2]])*Sin[2*(c + d*x)])))/(4*d)
```

Maple [A] time = 0.071, size = 90, normalized size = 1.5

$$\frac{a^2}{d\sin(dx+c)\cos(dx+c)} - 2\frac{a^2\cot(dx+c)}{d} + 2\frac{ab}{d\cos(dx+c)} + 2\frac{ab\ln(\csc(dx+c)-\cot(dx+c))}{d} + \frac{b^2\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2/sin(d*x+c)/cos(d*x+c)-2*a^2*cot(d*x+c)/d+2/d*a*b/cos(d*x+c)+2/d*a*b
*ln(csc(d*x+c)-cot(d*x+c))+b^2*tan(d*x+c)/d
```

Maxima [A] time = 1.00398, size = 96, normalized size = 1.63

$$\frac{ab\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - a^2\left(\frac{1}{\tan(dx+c)} - \tan(dx+c)\right) + b^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (a*b*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - a^2
*(1/tan(d*x + c) - tan(d*x + c)) + b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.93319, size = 293, normalized size = 4.97

$$-\frac{ab\cos\left(dx+c\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-ab\cos\left(dx+c\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+\left(2a^{2}+b^{2}\cdots\right.}{d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(a*b*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*b*cos(d*x +
 c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 + b^2)*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17076, size = 173, normalized size = 2.93

$$\frac{12\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+20\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\cdots\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^2*tan(1/2*d*x + 1/2*c) - (
4*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1
/2*d*x + 1/2*c)^2 + 20*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/(tan(1/2*d*x + 1/2
*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

3.1455 $\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=100

$$\frac{(3a^2+2b^2)\sec(c+dx)}{2d}-\frac{(3a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{2ab\tan(c+dx)}{d}-\frac{2ab\,\mathrm{co}}{}$$

```
[Out] -((3*a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a*b*Cot[c + d*x])/d + (
(3*a^2 + 2*b^2)*Sec[c + d*x])/(2*d) - (a^2*Csc[c + d*x]^2*Sec[c + d*x])/(2*
d) + (2*a*b*Tan[c + d*x])/d
```

Rubi [A] time = 0.25142, antiderivative size = 124, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2911, 2620, 14, 3201, 446, 78, 51, 63, 206}

$$\frac{(3a^2+2b^2)\sec(c+dx)}{2d}-\frac{(3a^2+2b^2)\sqrt{\cos^2(c+dx)}\sec(c+dx)\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cot[c + d*x])/d + ((3*a^2 + 2*b^2)*Sec[c + d*x])/(2*d) - ((3*a^2 +
2*b^2)*ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x])/(2*
d) - (a^2*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (2*a*b*Tan[c + d*x])/d
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3201

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \csc^2(c+dx)\sec^2(c+dx)\,dx + \int \csc^3(c+dx)\sec^2(c+dx)$$

$$= \frac{(2ab)\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+d\right.}{}$$

$$= \frac{(2ab)\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} + \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c\right.}{}$$

$$= -\frac{2ab\cot(c+dx)}{d} - \frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{2ab\tan(c+dx)}{d} -$$

$$= -\frac{2ab\cot(c+dx)}{d} + \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2d} - \frac{a^2\csc^2(c+dx)\sec(c}{2d}$$

$$= -\frac{2ab\cot(c+dx)}{d} + \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2d} - \frac{a^2\csc^2(c+dx)\sec(c}{2d}$$

$$= -\frac{2ab\cot(c+dx)}{d} + \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2d} - \frac{\left(3a^2+2b^2\right)\tanh^{-1}\left(\right.}{}$$

Mathematica [B] time = 0.478155, size = 238, normalized size = 2.38

$$\frac{\csc^4(c+dx)\left(-2\left(3a^2+2b^2\right)\cos(2(c+dx))-\left(3a^2+2b^2\right)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^4*(2*a^2 + 4*b^2 - 2*(3*a^2 + 2*b^2)*Cos[2*(c + d*x)] + 3*a^2
*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 2*b^2*Cos[3*(c + d*x)]*Log[Cos[(c
 + d*x)/2]] - (3*a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin
[(c + d*x)/2]]) - 3*a^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 2*b^2*Cos[
3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 8*a*b*Sin[c + d*x] - 8*a*b*Sin[3*(c +
d*x)]))/(2*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

Maple [A] time = 0.089, size = 140, normalized size = 1.4

$$-\frac{a^2}{2\,d\left(\sin(dx+c)\right)^2\cos(dx+c)}+\frac{3\,a^2}{2\,d\cos(dx+c)}+\frac{3\,a^2\ln\left(\csc(dx+c)-\cot(dx+c)\right)}{2\,d}+2\,\frac{ab}{d\sin(dx+c)\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2/sin(d*x+c)^2/cos(d*x+c)+3/2/d*a^2/cos(d*x+c)+3/2/d*a^2*ln(csc(d*
x+c)-cot(d*x+c))+2/d*a*b/sin(d*x+c)/cos(d*x+c)-4*a*b*cot(d*x+c)/d+1/d*b^2/c
os(d*x+c)+1/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.0016, size = 166, normalized size = 1.66

$$\frac{a^2\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+2\,b^2\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\mathrm{l}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*b^2*(2/cos(d*x + c) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 8*a*b*(1/tan(d*x + c) - tan(d*
x + c)))/d
```

Fricas [A] time = 1.92053, size = 441, normalized size = 4.41

$$\frac{2\left(3\,a^2+2\,b^2\right)\cos(dx+c)^2-4\,a^2-4\,b^2-\left(\left(3\,a^2+2\,b^2\right)\cos(dx+c)^3-\left(3\,a^2+2\,b^2\right)\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)\right.}{4\left(d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(2*(3*a^2 + 2*b^2)*cos(d*x + c)^2 - 4*a^2 - 4*b^2 - ((3*a^2 + 2*b^2)*co
s(d*x + c)^3 - (3*a^2 + 2*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
((3*a^2 + 2*b^2)*cos(d*x + c)^3 - (3*a^2 + 2*b^2)*cos(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2) + 8*(2*a*b*cos(d*x + c)^2 - a*b)*sin(d*x + c))/(d*cos(d*x
 + c)^3 - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23266, size = 212, normalized size = 2.12

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\left(3\,a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{16\left(2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+b^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{18\,a^2\,\mathrm{ta}}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) + 4*(3*a^2 + 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*(2*a*b*tan(1/2*d*x + 1/2*c) + a^2
 + b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (18*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*
b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x
+ 1/2*c)^2)/d
```

3.1456 $\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=104

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{(2a^2+b^2)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{3ab\sec(c+dx)}{d}-\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-ab$$

```
[Out] (-3*a*b*ArcTanh[Cos[c + d*x]])/d - ((2*a^2 + b^2)*Cot[c + d*x])/d - (a^2*Co
t[c + d*x]^3)/(3*d) + (3*a*b*Sec[c + d*x])/d - (a*b*Csc[c + d*x]^2*Sec[c +
d*x])/d + ((a^2 + b^2)*Tan[c + d*x])/d
```

Rubi [A] time = 0.23094, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2911, 2622, 288, 321, 207, 3200, 448}

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{(2a^2+b^2)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{3ab\sec(c+dx)}{d}-\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-ab$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a*b*ArcTanh[Cos[c + d*x]])/d - ((2*a^2 + b^2)*Cot[c + d*x])/d - (a^2*Co
t[c + d*x]^3)/(3*d) + (3*a*b*Sec[c + d*x])/d - (a*b*Csc[c + d*x]^2*Sec[c +
d*x])/d + ((a^2 + b^2)*Tan[c + d*x])/d
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3200

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[(x^n*(a + (a + b)*ff^2*x^2)^p)/
(1 + ff^2*x^2)^((m + n)/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \csc^3(c+dx)\sec^2(c+dx)\,dx + \int \csc^4(c+dx)\sec^2(c+dx)$$

$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)\left(a^2+\left(a^2+b^2\right)x^2\right)}{x^4}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{ab\csc^2(c+dx)\sec(c+dx)}{d} + \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right)+\frac{a^2}{x^4}+\frac{2a^2+b^2}{x^2}\right.\right.}{d}$$

$$= -\frac{\left(2a^2+b^2\right)\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{3ab\sec(c+dx)}{d} - \frac{ab\,c}{}$$

$$= -\frac{3ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{\left(2a^2+b^2\right)\cot(c+dx)}{d} - \frac{a^2\cot^3(c+d}{3d}$$

Mathematica [A] time = 0.992715, size = 196, normalized size = 1.88

$$\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-4\left(4a^2+3b^2\right)\cos(2(c+dx))+\left(8a^2+6b^2\right)\cos(4(c+dx))+3b\left(10a\sin(c+dx)-6a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-4*(4*a^2 + 3*b^2)*Cos[2*(c + d*x)]
 + (8*a^2 + 6*b^2)*Cos[4*(c + d*x)] + 3*b*(2*b + 10*a*Sin[c + d*x] - 6*a*(L
og[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[2*(c + d*x)] - 6*a*Sin[3*
(c + d*x)] + 3*a*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] - 3*a*Log[Sin[(c +
d*x)/2]]*Sin[4*(c + d*x)])))/(192*d*(-1 + Cot[(c + d*x)/2]^2))
```

Maple [A] time = 0.082, size = 162, normalized size = 1.6

$$-\frac{a^2}{3d(\sin(dx+c))^3\cos(dx+c)}+\frac{4a^2}{3d\sin(dx+c)\cos(dx+c)}-\frac{8a^2\cot(dx+c)}{3d}-\frac{ab}{d(\sin(dx+c))^2\cos(dx+c)}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d*a^2/sin(d*x+c)^3/cos(d*x+c)+4/3/d*a^2/sin(d*x+c)/cos(d*x+c)-8/3*a^2*
cot(d*x+c)/d-1/d*a*b/sin(d*x+c)^2/cos(d*x+c)+3/d*a*b/cos(d*x+c)+3/d*a*b*ln(
csc(d*x+c)-cot(d*x+c))+1/d*b^2/sin(d*x+c)/cos(d*x+c)-2*b^2*cot(d*x+c)/d
```

Maxima [A] time = 1.01463, size = 166, normalized size = 1.6

$$\frac{3ab\left(\frac{2(3\cos(dx+c)^2-2)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-6b^2\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)-2a^2\left(\frac{6t}{}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/6*(3*a*b*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*lo
g(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 6*b^2*(1/tan(d*x + c) - ta
n(d*x + c)) - 2*a^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3*tan(d*x + c)
))/d
```

Fricas [A] time = 1.75016, size = 489, normalized size = 4.7

$$-\frac{4(4a^2+3b^2)\cos(dx+c)^4-6(4a^2+3b^2)\cos(dx+c)^2+9(ab\cos(dx+c)^3-ab\cos(dx+c))\log\left(\frac{1}{2}\cos(dx+c)\right.}{6(d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(4*(4*a^2 + 3*b^2)*cos(d*x + c)^4 - 6*(4*a^2 + 3*b^2)*cos(d*x + c)^2 +
 9*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)*sin(
d*x + c) - 9*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 6*a^2 + 6*b^2 - 6*(3*a*b*cos(d*x + c)^2 - 2*a*b)*sin(
d*x + c))/((d*cos(d*x + c)^3 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23387, size = 275, normalized size = 2.64

$$\frac{a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 6\,ab \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 72\,ab \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + 21\,a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 12\,b^2 \tan\left(\frac{1}{2}\,dx + \right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 + 72*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c))) + 21*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2
*d*x + 1/2*c) - 48*(a^2*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c) + 2
*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 21*a
^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 + 6*a*b*tan(1/2*d
*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.1457 $\int \sin(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=197

$$\frac{9a^2b\tan(c+dx)}{2d}-\frac{3a^2b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{9}{2}a^2bx+\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}+6a$$

```
[Out] (-9*a^2*b*x)/2 - (15*b^3*x)/8 + (a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x
])/d - (a*b^2*Cos[c + d*x]^3)/d + (a^3*Sec[c + d*x])/d + (3*a*b^2*Sec[c + d
*x])/d + (9*a^2*b*Tan[c + d*x])/(2*d) + (15*b^3*Tan[c + d*x])/(8*d) - (3*a^
2*b*Sin[c + d*x]^2*Tan[c + d*x])/(2*d) - (5*b^3*Sin[c + d*x]^2*Tan[c + d*x]
)/(8*d) - (b^3*Sin[c + d*x]^4*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.260817, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2912, 2590, 14, 2591, 288, 321, 203, 270}

$$\frac{9a^2b\tan(c+dx)}{2d}-\frac{3a^2b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{9}{2}a^2bx+\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}+6a$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```


```
[Out] (-9*a^2*b*x)/2 - (15*b^3*x)/8 + (a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x
])/d - (a*b^2*Cos[c + d*x]^3)/d + (a^3*Sec[c + d*x])/d + (3*a*b^2*Sec[c + d
*x])/d + (9*a^2*b*Tan[c + d*x])/(2*d) + (15*b^3*Tan[c + d*x])/(8*d) - (3*a^
2*b*Sin[c + d*x]^2*Tan[c + d*x])/(2*d) - (5*b^3*Sin[c + d*x]^2*Tan[c + d*x]
)/(8*d) - (b^3*Sin[c + d*x]^4*Tan[c + d*x])/(4*d)
```


Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx &= \int \left(a^3\sin(c+dx)\tan^2(c+dx)+3a^2b\sin^2(c+dx)\tan^2(c+dx)+3a \\
&= a^3\int \sin(c+dx)\tan^2(c+dx)\,dx+\left(3a^2b\right)\int \sin^2(c+dx)\tan^2(c+d \\
&= -\frac{a^3\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}+\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx,\right.}{d} \\
&= -\frac{3a^2b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{b^3\sin^4(c+dx)\tan(c+dx)}{4d}-\frac{a^3\,\mathrm{S}}{} \\
&= \frac{a^3\cos(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}+\frac{a^3\sec(c+d}{d} \\
&= -\frac{9}{2}a^2bx+\frac{a^3\cos(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}+\frac{a}{} \\
&= -\frac{9}{2}a^2bx-\frac{15b^3x}{8}+\frac{a^3\cos(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}-\frac{ab^2\cos^3(c+}{d}
\end{aligned}$$

Mathematica [A] time = 0.759607, size = 147, normalized size = 0.75

$$\frac{\sec(c+dx)\left(-24b\left(12a^2+5b^2\right)(c+dx)\cos(c+dx)+32\left(a^3+5ab^2\right)\cos(2(c+dx))+216a^2b\sin(c+dx)+24a^2b\sin(3(c}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]*(96*a^3 + 360*a*b^2 - 24*b*(12*a^2 + 5*b^2)*(c + d*x)*Cos[c +
 d*x] + 32*(a^3 + 5*a*b^2)*Cos[2*(c + d*x)] - 8*a*b^2*Cos[4*(c + d*x)] + 21
6*a^2*b*Sin[c + d*x] + 80*b^3*Sin[c + d*x] + 24*a^2*b*Sin[3*(c + d*x)] + 15
*b^3*Sin[3*(c + d*x)] - b^3*Sin[5*(c + d*x)]))/(64*d)
```

Maple [A] time = 0.059, size = 214, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+3a^2b\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\sin(dx+c))^3+3/2\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^2*b*(sin
(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c
)+3*a*b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(
d*x+c))+b^3*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*si
n(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c))
```

Maxima [A] time = 1.49203, size = 221, normalized size = 1.12

$$-\frac{12\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)a^2b+8\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\cos(dx+c)\right)ab^2+\left(15\,dx+15\,c-\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/8*(12*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))
*a^2*b + 8*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a*b^2 + (15*d
*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x
 + c)^2 + 1) - 8*tan(d*x + c))*b^3 - 8*a^3*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

Fricas [A] time = 1.67019, size = 319, normalized size = 1.62

$$-\frac{8\,ab^2\cos(dx+c)^4+3\left(12\,a^2b+5\,b^3\right)dx\cos(dx+c)-8\,a^3-24\,ab^2-8\left(a^3+6\,ab^2\right)\cos(dx+c)^2+\left(2\,b^3\cos(dx+c\right.}{8\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/8*(8*a*b^2*cos(d*x + c)^4 + 3*(12*a^2*b + 5*b^3)*d*x*cos(d*x + c) - 8*a^
3 - 24*a*b^2 - 8*(a^3 + 6*a*b^2)*cos(d*x + c)^2 + (2*b^3*cos(d*x + c)^4 - 2
4*a^2*b - 8*b^3 - 3*(4*a^2*b + 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22656, size = 454, normalized size = 2.3

$$-\frac{3\left(12\,a^2b+5\,b^3\right)(dx+c)+\frac{16\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3+3\,ab^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+7\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-8\,a^3\tan\left(\frac{1}{2}\,dx \cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(12*a^2*b + 5*b^3)*(d*x + c) + 16*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b
^3*tan(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(
12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 7*b^3*tan(1/2*d*x + 1/2*c)^7 - 8*a^3*tan(
1/2*d*x + 1/2*c)^6 - 24*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 + 15*b^3*tan(1/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4
 - 120*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 15*
b^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 - 136*a*b^2*tan(
1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) - 7*b^3*tan(1/2*d*x + 1/
2*c) - 8*a^3 - 40*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.1458 $\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=146

$$\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-$$

```
[Out] -(a^3*x) - (9*a*b^2*x)/2 + (3*a^2*b*Cos[c + d*x])/d + (2*b^3*Cos[c + d*x])/
d - (b^3*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Sec[c + d*x])/d + (b^3*Sec[c + d*
x])/d + (a^3*Tan[c + d*x])/d + (9*a*b^2*Tan[c + d*x])/(2*d) - (3*a*b^2*Sin[
c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.209068, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203, 270}

$$\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```


```
[Out] -(a^3*x) - (9*a*b^2*x)/2 + (3*a^2*b*Cos[c + d*x])/d + (2*b^3*Cos[c + d*x])/
d - (b^3*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Sec[c + d*x])/d + (b^3*Sec[c + d*
x])/d + (a^3*Tan[c + d*x])/d + (9*a*b^2*Tan[c + d*x])/(2*d) - (3*a*b^2*Sin[
c + d*x]^2*Tan[c + d*x])/(2*d)
```


Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx &= \int \left(a^3\tan^2(c+dx)+3a^2b\sin(c+dx)\tan^2(c+dx)+3ab^2\sin^2(c+dx)\tan^2(c+\right.\\
&= a^3\int \tan^2(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan^2(c+dx)\,dx+\left(3ab^2\right)\int \sin^2(c\\
&= \frac{a^3\tan(c+dx)}{d}-a^3\int 1\,dx-\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}+\frac{\left(3ab^2\right)}{}\\
&= -a^3x+\frac{a^3\tan(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \left(-1\right.\right.}{}\\
&= -a^3x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2b\sec(c+dx}{d}\\
&= -a^3x-\frac{9}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2b\,\mathrm{s}}{}
\end{aligned}$$

Mathematica [A] time = 0.726663, size = 113, normalized size = 0.77

$$\frac{3a\left(\left(8a^2+27b^2\right)\tan(c+dx)-4\left(2a^2+9b^2\right)(c+dx)\right)+b\sec(c+dx)\left(4\left(9a^2+5b^2\right)\cos(2(c+dx))+108a^2+9ab\sin(3(\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (b*Sec[c + d*x]*(108*a^2 + 45*b^2 + 4*(9*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^
2*Cos[4*(c + d*x)] + 9*a*b*Sin[3*(c + d*x)]) + 3*a*(-4*(2*a^2 + 9*b^2)*(c +
 d*x) + (8*a^2 + 27*b^2)*Tan[c + d*x]))/(24*d)
```

Maple [A] time = 0.056, size = 169, normalized size = 1.2

$$\frac{1}{d}\left(a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+3ab^2\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(tan(d*x+c)-d*x-c)+3*a^2*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^
2)*cos(d*x+c))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c
))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4
+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.5418, size = 161, normalized size = 1.1

$$-\frac{6(dx+c-\tan(dx+c))a^3+9\left(3dx+3c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan(dx+c)\right)ab^2+2\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(\ldots\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(6*(d*x + c - tan(d*x + c))*a^3 + 9*(3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*a*b^2 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c)
 - 6*cos(d*x + c))*b^3 - 18*a^2*b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.63957, size = 271, normalized size = 1.86

$$-\frac{2b^3\cos(dx+c)^4+3\left(2a^3+9ab^2\right)dx\cos(dx+c)-18a^2b-6b^3-6\left(3a^2b+2b^3\right)\cos(dx+c)^2-3\left(3ab^2\cos(dx+\ldots\right.}{6d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*b^3*cos(d*x + c)^4 + 3*(2*a^3 + 9*a*b^2)*d*x*cos(d*x + c) - 18*a^2*
b - 6*b^3 - 6*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 3*(3*a*b^2*cos(d*x + c)^2
+ 2*a^3 + 6*a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20582, size = 279, normalized size = 1.91

$$-\frac{3\left(2a^3+9ab^2\right)(dx+c)+\frac{12\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^2b+b^3\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}+\frac{2\left(9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-6b^3\tan\left(\frac{1}{2}dx\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 + 9*a*b^2)*(d*x + c) + 12*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^
2*tan(1/2*d*x + 1/2*c) + 3*a^2*b + b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(9
*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 6*b^3*tan
(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 24*b^3*tan(1/2*d*x
+ 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b - 10*b^3)/(tan(1/2*d*x
 + 1/2*c)^2 + 1)^3)/d
```

3.1459 $\int \sec(c+dx)(a+b\sin(c+dx))^3 \tan(c+dx)\,dx$

Optimal. Leaf size=75

$$-\frac{3}{2}bx\left(2a^2+b^2\right)+\frac{6ab^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d}+\frac{3b^3\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] (-3*b*(2*a^2 + b^2)*x)/2 + (6*a*b^2*Cos[c + d*x])/d + (3*b^3*Cos[c + d*x]*S
in[c + d*x])/(2*d) + (Sec[c + d*x]*(a + b*Sin[c + d*x])^3)/d
```

Rubi [A] time = 0.0720454, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2861, 12, 2644}

$$-\frac{3}{2}bx\left(2a^2+b^2\right)+\frac{6ab^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d}+\frac{3b^3\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-3*b*(2*a^2 + b^2)*x)/2 + (6*a*b^2*Cos[c + d*x])/d + (3*b^3*Cos[c + d*x]*S
in[c + d*x])/(2*d) + (Sec[c + d*x]*(a + b*Sin[c + d*x])^3)/d
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx &= \frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d}-\int 3b(a+b\sin(c+dx))^2\,dx\\ &= \frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d}-(3b)\int (a+b\sin(c+dx))^2\,dx\\ &= -\frac{3}{2}b\left(2a^2+b^2\right)x+\frac{6ab^2\cos(c+dx)}{d}+\frac{3b^3\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.536627, size = 91, normalized size = 1.21

$$\frac{3b\left(\left(8a^2+3b^2\right)\tan(c+dx)-4\left(2a^2+b^2\right)(c+dx)\right)+\sec(c+dx)\left(8a^3+12ab^2\cos(2(c+dx))+36ab^2+b^3\sin(3(c+dx\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (Sec[c + d*x]*(8*a^3 + 36*a*b^2 + 12*a*b^2*Cos[2*(c + d*x)] + b^3*Sin[3*(c
+ d*x)]) + 3*b*(-4*(2*a^2 + b^2)*(c + d*x) + (8*a^2 + 3*b^2)*Tan[c + d*x]))
/(8*d)
```

Maple [A] time = 0.049, size = 132, normalized size = 1.8

$$\frac{1}{d}\left(\frac{a^3}{\cos(dx+c)}+3a^2b\left(\tan(dx+c)-dx-c\right)+3ab^2\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+b^3\left(\frac{(\sin(dx\right.}{\cos(d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3/cos(d*x+c)+3*a^2*b*(tan(d*x+c)-d*x-c)+3*a*b^2*(sin(d*x+c)^4/cos(d*
x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^
3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A] time = 1.51166, size = 134, normalized size = 1.79

$$-\frac{6\left(dx+c-\tan(dx+c)\right)a^2b+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)b^3-6\,ab^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-\frac{2\,a^3}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*(d*x + c - tan(d*x + c))*a^2*b + (3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*b^3 - 6*a*b^2*(1/cos(d*x + c) + cos(d*x +
c)) - 2*a^3/cos(d*x + c))/d
```

Fricas [A] time = 1.63917, size = 211, normalized size = 2.81

$$\frac{6\,ab^2\cos(dx+c)^2-3\left(2\,a^2b+b^3\right)dx\cos(dx+c)+2\,a^3+6\,ab^2+\left(b^3\cos(dx+c)^2+6\,a^2b+2\,b^3\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*cos(d*x + c)^2 - 3*(2*a^2*b + b^3)*d*x*cos(d*x + c) + 2*a^3 +
6*a*b^2 + (b^3*cos(d*x + c)^2 + 6*a^2*b + 2*b^3)*sin(d*x + c))/(d*cos(d*x +
```

c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21742, size = 200, normalized size = 2.67

$$-\frac{3\left(2\,a^{2}b+b^{3}\right)(dx+c)+\frac{4\left(3\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}+3\,ab^{2}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{2\left(b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(2*a^2*b + b^3)*(d*x + c) + 4*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*t
an(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(b^3*
tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x +
 1/2*c) - 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.1460 $\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=78

$$\frac{3a^2b\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\tan(c+dx)}{d}-b^3x$$

```
[Out] -(b^3*x) - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Sec[c + d*x])/d + (3*a*b^2*
Sec[c + d*x])/d + (3*a^2*b*Tan[c + d*x])/d + (b^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.151567, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2912, 3767, 8, 2622, 321, 207, 2606, 3473}

$$\frac{3a^2b\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\tan(c+dx)}{d}-b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^3*x) - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Sec[c + d*x])/d + (3*a*b^2*
Sec[c + d*x])/d + (3*a^2*b*Tan[c + d*x])/d + (b^3*Tan[c + d*x])/d
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(3a^2b\sec^2(c+dx)+a^3\csc(c+dx)\sec^2(c+dx)+3ab^2\sec(c+$$

$$= a^3\int \csc(c+dx)\sec^2(c+dx)\,dx+\left(3a^2b\right)\int \sec^2(c+dx)\,dx+\left(3\right.$$

$$= \frac{b^3\tan(c+dx)}{d}-b^3\int 1\,dx+\frac{a^3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{d}$$

$$= -b^3x+\frac{a^3\sec(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{3a^2b\tan(c+dx)}{d}+$$

$$= -b^3x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\sec(c+dx)}{d}+\frac{3ab^2\sec(c+d}{d}$$

Mathematica [A] time = 0.298516, size = 83, normalized size = 1.06

$$\frac{b\left(3a^2+b^2\right)\tan(c+dx)+a\left(a^2+3b^2\right)\sec(c+dx)+a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^3\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)-b^3c-b}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-(b^3*c) - b^3*d*x - a^3*Log[Cos[(c + d*x)/2]] + a^3*Log[Sin[(c + d*x)/2]]
 + a*(a^2 + 3*b^2)*Sec[c + d*x] + b*(3*a^2 + b^2)*Tan[c + d*x])/d
```

Maple [A] time = 0.087, size = 100, normalized size = 1.3

$$\frac{a^3}{d\cos(dx+c)}+\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}+3\frac{a^2b\tan(dx+c)}{d}+3\frac{ab^2}{d\cos(dx+c)}-b^3x+\frac{b^3\tan(dx+c)}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3/cos(d*x+c)+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3*a^2*b*tan(d*x+c)/d+3
/d*a*b^2/cos(d*x+c)-b^3*x+b^3*tan(d*x+c)/d-1/d*b^3*c
```

Maxima [A] time = 1.49526, size = 116, normalized size = 1.49

$$-\frac{2\,(dx+c-\tan(dx+c))b^3 - a^3\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 6\,a^2b\tan(dx+c) - \frac{6\,ab^2}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*b^3 - a^3*(2/cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)) - 6*a^2*b*tan(d*x + c) - 6*a*b^2/cos(d*x +
 c))/d
```

Fricas [A] time = 1.71562, size = 262, normalized size = 3.36

$$-\frac{2\,b^3dx\cos(dx+c) + a^3\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - a^3\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 2\,a^3 - 6\,ab^2 - 2\,(3\,a^2b+b^3)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d*x*cos(d*x + c) + a^3*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)
 - a^3*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2) - 2*a^3 - 6*a*b^2 - 2*(3*a
^2*b + b^3)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23652, size = 116, normalized size = 1.49

$$-\frac{(dx+c)b^3 - a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + \frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3+3\,ab^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*b^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 2*(3*a^2*b*tan(1/2*d
*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*
c)^2 - 1))/d
```

3.1461 $\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=87

$$\frac{3a^2b\sec(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

```
[Out] (-3*a^2*b*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (3*a^2*b*Sec[c
+ d*x])/d + (b^3*Sec[c + d*x])/d + (a^3*Tan[c + d*x])/d + (3*a*b^2*Tan[c +
d*x])/d
```

Rubi [A] time = 0.195415, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2912, 3767, 8, 2622, 321, 207, 2620, 14, 2606}

$$\frac{3a^2b\sec(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (3*a^2*b*Sec[c
+ d*x])/d + (b^3*Sec[c + d*x])/d + (a^3*Tan[c + d*x])/d + (3*a*b^2*Tan[c +
d*x])/d
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \Big(3ab^2\sec^2(c+dx)+3a^2b\csc(c+dx)\sec^2(c+dx)+a^3\csc^2(c$$

$$= a^3\int \csc^2(c+dx)\sec^2(c+dx)\,dx+\left(3a^2b\right)\int \csc(c+dx)\sec^2(c$$

$$= \frac{a^3\,\mathrm{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(c+dx)\right)}{d}+\frac{\left(3a^2b\right)\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx\right.}{d}$$

$$= \frac{3a^2b\sec(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{a^3\,\mathrm{Subs}}{}$$

$$= -\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}$$

Mathematica [A] time = 0.389728, size = 114, normalized size = 1.31

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\Big(-2b\left(3a^2+b^2\right)\sin(c+dx)+\left(2a^3+3ab^2\right)\cos(2(c+dx))-3ab\big(a\sin(2(}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sec[c + d*x]*((2*a^3 + 3*a*b^2)*Cos[2*(
c + d*x)] - 2*b*(3*a^2 + b^2)*Sin[c + d*x] - 3*a*b*(b + a*(-Log[Cos[(c + d*
x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[2*(c + d*x)])))/(4*d)
```

Maple [A] time = 0.082, size = 111, normalized size = 1.3

$$\frac{a^3}{d\sin(dx+c)\cos(dx+c)} - 2\,\frac{a^3\cot(dx+c)}{d} + 3\,\frac{a^2b}{d\cos(dx+c)} + 3\,\frac{a^2b\ln(\csc(dx+c)-\cot(dx+c))}{d} + 3\,\frac{ab^2\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3/sin(d*x+c)/cos(d*x+c)-2*a^3*cot(d*x+c)/d+3/d*a^2*b/cos(d*x+c)+3/d*a
^2*b*ln(csc(d*x+c)-cot(d*x+c))+3*a*b^2*tan(d*x+c)/d+1/d*b^3/cos(d*x+c)
```

Maxima [A] time = 1.00241, size = 122, normalized size = 1.4

$$\frac{3\,a^2b\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 2\,a^3\left(\frac{1}{\tan(dx+c)} - \tan(dx+c)\right) + 6\,ab^2\tan(dx+c) + \frac{}{\cos}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(3*a^2*b*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1
)) - 2*a^3*(1/tan(d*x + c) - tan(d*x + c)) + 6*a*b^2*tan(d*x + c) + 2*b^3/c
os(d*x + c))/d
```

Fricas [A] time = 1.66043, size = 342, normalized size = 3.93

$$-\frac{3\,a^2b\cos(dx+c)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 3\,a^2b\cos(dx+c)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 2\,a^3}{2\,d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(3*a^2*b*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^2
*b*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*a^3 - 6*a*b^2
 + 2*(2*a^3 + 3*a*b^2)*cos(d*x + c)^2 - 2*(3*a^2*b + b^3)*sin(d*x + c))/(d*
cos(d*x + c)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2468, size = 200, normalized size = 2.3

$$\frac{6\,a^{2}b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+10\,a^{2}b\tan\left(\frac{1}{2}\,d\cdots\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^3*tan(1/2*d*x + 1/2*c) - (2
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan
(1/2*d*x + 1/2*c)^2 + 10*a^2*b*tan(1/2*d*x + 1/2*c) + 4*b^3*tan(1/2*d*x + 1
/2*c) - a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

3.1462 $\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=132

$$\frac{3a^2b\tan(c+dx)}{d}-\frac{3a^2b\cot(c+dx)}{d}+\frac{3a^3\sec(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{3ab^2}{}$$

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d -
(3*a^2*b*Cot[c + d*x])/d + (3*a^3*Sec[c + d*x])/(2*d) + (3*a*b^2*Sec[c + d*
x])/d - (a^3*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (3*a^2*b*Tan[c + d*x])/d
+ (b^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.227833, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {2912, 3767, 8, 2622, 321, 207, 2620, 14, 288}

$$\frac{3a^2b\tan(c+dx)}{d}-\frac{3a^2b\cot(c+dx)}{d}+\frac{3a^3\sec(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{3ab^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d -
(3*a^2*b*Cot[c + d*x])/d + (3*a^3*Sec[c + d*x])/(2*d) + (3*a*b^2*Sec[c + d*
x])/d - (a^3*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (3*a^2*b*Tan[c + d*x])/d
+ (b^3*Tan[c + d*x])/d
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\sec^2(c+dx)+3ab^2\csc(c+dx)\sec^2(c+dx)+3a^2b\csc^2(c\right.$$

$$= a^3\int \csc^3(c+dx)\sec^2(c+dx)\,dx+\left(3a^2b\right)\int \csc^2(c+dx)\sec^2($$

$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx,x,\sec(c+dx)\right)}{d}+\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{1+x^2}{x^2}\right.}{d}$$

$$= \frac{3ab^2\sec(c+dx)}{d}-\frac{a^3\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{b^3\tan(c+dx)}{d}$$

$$= -\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^2b\cot(c+dx)}{d}+\frac{3a^3\sec(c+dx}{2d}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^2b\,c}{}$$

Mathematica [B] time = 0.564102, size = 267, normalized size = 2.02

$$\frac{\csc^4(c+dx)\left(-6\left(a^3+2ab^2\right)\cos(2(c+dx))-3a\left(a^2+2b^2\right)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^4*(2*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*Cos[2*(c + d*x)] + 3*
a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 6*a*b^2*Cos[3*(c + d*x)]*Log[C
os[(c + d*x)/2]] - 3*a*(a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]] -
Log[Sin[(c + d*x)/2]]) - 3*a^3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 6*a
*b^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 12*a^2*b*Sin[c + d*x] + 6*b^3
*Sin[c + d*x] - 12*a^2*b*Sin[3*(c + d*x)] - 2*b^3*Sin[3*(c + d*x)]))/(2*d*(
Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

Maple [A] time = 0.098, size = 161, normalized size = 1.2

$$-\frac{a^3}{2d(\sin(dx+c))^2\cos(dx+c)}+\frac{3a^3}{2d\cos(dx+c)}+\frac{3a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}+3\frac{a^2b}{d\sin(dx+c)\cos(dx+c)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3/sin(d*x+c)^2/cos(d*x+c)+3/2/d*a^3/cos(d*x+c)+3/2/d*a^3*ln(csc(d*
x+c)-cot(d*x+c))+3/d*a^2*b/sin(d*x+c)/cos(d*x+c)-6*a^2*b*cot(d*x+c)/d+3/d*a
*b^2/cos(d*x+c)+3/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))+b^3*tan(d*x+c)/d
```

Maxima [A] time = 0.99946, size = 185, normalized size = 1.4

$$\frac{a^3\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+6ab^2\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^3*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a*b^2*(2/cos(d*x + c) - lo
g(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*a^2*b*(1/tan(d*x + c) - t
an(d*x + c)) + 4*b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.73089, size = 479, normalized size = 3.63

$$-\frac{4a^3+12ab^2-6\left(a^3+2ab^2\right)\cos(dx+c)^2+3\left(\left(a^3+2ab^2\right)\cos(dx+c)^3-\left(a^3+2ab^2\right)\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c\right.}{4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(4*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*cos(d*x + c)^2 + 3*((a^3 + 2*a*b
^2)*cos(d*x + c)^3 - (a^3 + 2*a*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) - 3*((a^3 + 2*a*b^2)*cos(d*x + c)^3 - (a^3 + 2*a*b^2)*cos(d*x + c))*log
(-1/2*cos(d*x + c) + 1/2) + 4*(3*a^2*b + b^3 - (6*a^2*b + b^3)*cos(d*x + c)
```

```
^2)*sin(d*x + c))/(d*cos(d*x + c)^3 - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27563, size = 242, normalized size = 1.83

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\left(a^3+2\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{16\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,c\right.\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) + 12*(a^3 +
 2*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*(3*a^2*b*tan(1/2*d*x + 1/2*c)
 + b^3*tan(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) -
 (18*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*
b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.1463 $\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=164

$$\frac{9a^2b\sec(c+dx)}{2d}-\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^2b\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-2a^3$$

```
[Out] (-9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTanh[Cos[c + d*x]])/d - (2
*a^3*Cot[c + d*x])/d - (3*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d
) + (9*a^2*b*Sec[c + d*x])/(2*d) + (b^3*Sec[c + d*x])/d - (3*a^2*b*Csc[c +
d*x]^2*Sec[c + d*x])/(2*d) + (a^3*Tan[c + d*x])/d + (3*a*b^2*Tan[c + d*x])/
d
```

Rubi [A] time = 0.261862, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2912, 2622, 321, 207, 2620, 14, 288, 270}

$$\frac{9a^2b\sec(c+dx)}{2d}-\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^2b\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-2a^3$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTanh[Cos[c + d*x]])/d - (2
*a^3*Cot[c + d*x])/d - (3*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d
) + (9*a^2*b*Sec[c + d*x])/(2*d) + (b^3*Sec[c + d*x])/d - (3*a^2*b*Csc[c +
d*x]^2*Sec[c + d*x])/(2*d) + (a^3*Tan[c + d*x])/d + (3*a*b^2*Tan[c + d*x])/
d
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx &= \int \left(b^3\csc(c+dx)\sec^2(c+dx)+3ab^2\csc^2(c+dx)\sec^2(c+dx)\right. \\
&= a^3\int \csc^4(c+dx)\sec^2(c+dx)\,dx+\left(3a^2b\right)\int \csc^3(c+dx)\sec^2(\\
&= \frac{a^3\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx,x,\tan(c+dx)\right)}{d}+\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \frac{x^4}{(-1+x^2}\right.}{d} \\
&= \frac{b^3\sec(c+dx)}{d}-\frac{3a^2b\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{a^3\operatorname{Subst}\left(\int \left(1\right.\right.}{} \\
&= -\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^3\cot(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d} \\
&= -\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^3\cot(}{d}
\end{aligned}$$

Mathematica [A] time = 1.34271, size = 287, normalized size = 1.75

$$\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-8\left(4a^3+9ab^2\right)\cos(2(c+dx))+4\left(4a^3+9ab^2\right)\cos(4(c+dx))+3b\left(6\left(5a^2+2b^2\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-8*(4*a^3 + 9*a*b^2)*Cos[2*(c + d*x
)] + 4*(4*a^3 + 9*a*b^2)*Cos[4*(c + d*x)] + 3*b*(12*a*b + 6*(5*a^2 + 2*b^2)
*Sin[c + d*x] - 2*(9*a^2 + 2*b^2)*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x
)/2]])*Sin[2*(c + d*x)] - 18*a^2*Sin[3*(c + d*x)] - 4*b^2*Sin[3*(c + d*x)]
+ 9*a^2*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] + 2*b^2*Log[Cos[(c + d*x)/2]
]*Sin[4*(c + d*x)] - 9*a^2*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] - 2*b^2*L
og[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)])))/(384*d*(-1 + Cot[(c + d*x)/2]^2))
```

Maple [A] time = 0.099, size = 209, normalized size = 1.3

$$-\frac{a^3}{3\,d\,(\sin(dx+c))^3\cos(dx+c)}+\frac{4\,a^3}{3\,d\sin(dx+c)\cos(dx+c)}-\frac{8\,a^3\cot(dx+c)}{3\,d}-\frac{3\,a^2b}{2\,d\,(\sin(dx+c))^2\cos(dx+c)}+\frac{}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d*a^3/sin(d*x+c)^3/cos(d*x+c)+4/3/d*a^3/sin(d*x+c)/cos(d*x+c)-8/3*a^3*
cot(d*x+c)/d-3/2/d*a^2*b/sin(d*x+c)^2/cos(d*x+c)+9/2/d*a^2*b/cos(d*x+c)+9/2
/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))+3/d*a*b^2/sin(d*x+c)/cos(d*x+c)-6*a*b^2*
cot(d*x+c)/d+1/d*b^3/cos(d*x+c)+1/d*b^3*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.00265, size = 219, normalized size = 1.34

$$\frac{9\,a^2b\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)+6\,b^3\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\mathrm{lo}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(9*a^2*b*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*b^3*(2/cos(d*x + c) -
 log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 36*a*b^2*(1/tan(d*x + c)
- tan(d*x + c)) - 4*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3*tan(d*x
+ c)))/d
```

Fricas [A] time = 1.69274, size = 608, normalized size = 3.71

$$-\frac{8\left(4\,a^3+9\,ab^2\right)\cos(dx+c)^4+12\,a^3+36\,ab^2-12\left(4\,a^3+9\,ab^2\right)\cos(dx+c)^2+3\left(\left(9\,a^2b+2\,b^3\right)\cos(dx+c)^3-\left(9\,a^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(8*(4*a^3 + 9*a*b^2)*cos(d*x + c)^4 + 12*a^3 + 36*a*b^2 - 12*(4*a^3 +
 9*a*b^2)*cos(d*x + c)^2 + 3*((9*a^2*b + 2*b^3)*cos(d*x + c)^3 - (9*a^2*b +
```

```
 2*b^3)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*((9*a^2*
b + 2*b^3)*cos(d*x + c)^3 - (9*a^2*b + 2*b^3)*cos(d*x + c))*log(-1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 6*(6*a^2*b + 2*b^3 - (9*a^2*b + 2*b^3)*cos(d*x
 + c)^2)*sin(d*x + c))/((d*cos(d*x + c)^3 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)

[Out] Timed out
```

Giac [A] time = 1.25716, size = 331, normalized size = 2.02

$$\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+21a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+36ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12\left(9a^2b+2b^3\right)\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 21*a^3*
tan(1/2*d*x + 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) + 12*(9*a^2*b + 2*b^3)
*log(abs(tan(1/2*d*x + 1/2*c))) - 48*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*ta
n(1/2*d*x + 1/2*c) + 3*a^2*b + b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (198*a^2
*b*tan(1/2*d*x + 1/2*c)^3 + 44*b^3*tan(1/2*d*x + 1/2*c)^3 + 21*a^3*tan(1/2*
d*x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*b*tan(1/2*d*x + 1/
2*c) + a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.1464 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=222

$$-\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}+\frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}-\frac{a^4\cos(c+dx)}{bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c}{2d(a+b)^2(1-}$$

```
[Out] -(x/b^2) - (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a
^2 - b^2)^(5/2)*d) + (4*a^3*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1
 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^4*
Cos[c + d*x])/(b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.364495, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2897, 2648, 2664, 12, 2660, 618, 204}

$$-\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}+\frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}-\frac{a^4\cos(c+dx)}{bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c}{2d(a+b)^2(1-}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(x/b^2) - (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a
^2 - b^2)^(5/2)*d) + (4*a^3*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1
 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^4*
Cos[c + d*x])/(b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \Bigg(-\frac{1}{b^2} - \frac{1}{2(a+b)^2(-1+\sin(c+dx))} + \frac{1}{2(a-b)^2(1+\sin(c+dx))} + \frac{}{b^2\left(-a^2+b^2\right.}$$

$$= -\frac{x}{b^2} + \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} + \frac{\left(2a^3\left(a^2-2b^2\right)\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2\left(a^2-b^2\right)^2} - \frac{a^4}{}$$

$$= -\frac{x}{b^2} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^4\cos(}{b\left(a^2-b^2\right)^2 d(a}$$

$$= -\frac{x}{b^2} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^4\cos(}{b\left(a^2-b^2\right)^2 d(a}$$

$$= -\frac{x}{b^2} + \frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{}{2(a-}$$

$$= -\frac{x}{b^2} + \frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{}{2(a-}$$

$$= -\frac{x}{b^2} - \frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{}{2(a+}$$

Mathematica [A] time = 2.03369, size = 236, normalized size = 1.06

$$\frac{-\frac{-2a^2b^2(c+dx)+a^4(c+dx)+2ab^3+b^4(c+dx)}{(b^3-a^2b)^2}+\frac{2a^3(a^2-4b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2(a^2-b^2)^{5/2}}-\frac{a^4\cos(c+dx)}{b(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-((2*a*b^3 + a^4*(c + d*x) - 2*a^2*b^2*(c + d*x) + b^4*(c + d*x))/(-(a^2*b
) + b^3)^2) + (2*a^3*(a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(b^2*(a^2 - b^2)^(5/2)) + Sin[(c + d*x)/2]/((a + b)^2*(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/((a - b)^2*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])) - (a^4*Cos[c + d*x])/((a - b)^2*b*(a + b)^2*(a + b*Si
n[c + d*x])))/d
```

Maple [A] time = 0.131, size = 303, normalized size = 1.4

$$-\frac{1}{d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-2\frac{\arctan(\tan(1/2\,dx+c/2))}{db^2}-2\frac{a^3\tan(1/2\,dx+c/2)}{d(a-b)^2(a+b)^2\left((\tan(1/2\,dx+c/2))^2a+2\tan(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-2/d/b^2*arctan(tan(1/2*d*x+1/2*c))-2/d*
a^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1
/2*d*x+1/2*c)-2/d*a^4/b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)+2/d*a^5/b^2/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-8/d*a^3/(a-b)^2/(a+b)^2/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/d/(a-b)^2/
(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01788, size = 1577, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*a^4*b^3 - 4*a^2*b^5 + 2*b^7 + 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b
^6)*d*x*cos(d*x + c) + 2*(a^6*b - b^7)*cos(d*x + c)^2 - ((a^5*b - 4*a^3*b^3
)*cos(d*x + c)*sin(d*x + c) + (a^6 - 4*a^4*b^2)*cos(d*x + c))*sqrt(-a^2 + b
^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2
*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^5*b^2 - 2*a^3*b^4 + a*
b^6 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*x*cos(d*x + c))*sin(d*x + c))
/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^
7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*cos(d*x + c)), -(a^4*b^3 - 2*a^2*b
^5 + b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b
- b^7)*cos(d*x + c)^2 + ((a^5*b - 4*a^3*b^3)*cos(d*x + c)*sin(d*x + c) + (a
^6 - 4*a^4*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/
(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^5*b^2 - 2*a^3*b^4 + a*b^6 - (a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*d*x*cos(d*x + c))*sin(d*x + c))/((a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^7*b^2 - 3*a^5*b^
4 + 3*a^3*b^6 - a*b^8)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.218, size = 356, normalized size = 1.6

$$\frac{\frac{2\left(a^5-4a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^2-2a^2b^4+b^6\right)\sqrt{a^2-b^2}}-\frac{2\left(2a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 \cdots\right.}{\left(a^4b-2a^2b^3+b^5\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}\cdots\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(a^5 - 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4*b^2 - 2*a^2*b^4 + b^6)*sqr
t(a^2 - b^2)) - 2*(2*a^3*b*tan(1/2*d*x + 1/2*c)^3 + a*b^3*tan(1/2*d*x + 1/2
*c)^3 + a^4*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3
*tan(1/2*d*x + 1/2*c) - a^4 - 2*a^2*b^2)/((a^4*b - 2*a^2*b^3 + b^5)*(a*tan(
1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)
- a)) - (d*x + c)/b^2)/d
```

3.1465 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=212

$$\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}-\frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}+\frac{a^3\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+ }{2d(a+b)^2(1-\mathrm{si} }$$

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(5/
2)*d) - (2*a^2*(a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(b*(a^2 - b^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x]
)) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) + (a^3*Cos[c + d*x])/(
(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.297868, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2897, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}-\frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}+\frac{a^3\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+ }{2d(a+b)^2(1-\mathrm{si} }$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(5/
2)*d) - (2*a^2*(a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(b*(a^2 - b^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x]
)) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) + (a^3*Cos[c + d*x])/(
(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))} - \frac{1}{2(a-b)^2(1+\sin(c+dx))} + \frac{a^3}{b\left(a^2-b^2\right)(a+b\sin\right.}\right.$$

$$= -\frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} - \frac{\left(a^2\left(a^2-3b^2\right)\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)^2} + \frac{a^3\int \frac{}{(a+b\,\mathrm{s}}}{b\left(a^2\right.}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} + \frac{a^3\cos(c+dx)}{\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} + \frac{a^3\cos(c+dx)}{\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= -\frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos}{2(a-b)^2d(}$$

$$= -\frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos}{2(a-b)^2d(}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} - \frac{2a^2\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c}{2(a+b)^2d(1}$$

Mathematica [A] time = 0.964558, size = 162, normalized size = 0.76

$$\frac{\frac{6a^2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{a^3\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{1}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}\right.\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((6*a^2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/
2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
- 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a^3*Cos[c + d*x])
/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [A] time = 0.122, size = 219, normalized size = 1.

$$-\frac{1}{d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\frac{a^2\tan(1/2\,dx+c/2)\,b}{d(a-b)^2(a+b)^2\left((\tan(1/2\,dx+c/2))^2a+2\tan(1/2\,dx+c/2)\,b+a\right)}+2\frac{}{d(a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)+2/d*a^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b+2/d*a^3/(a-b)^2/(a+
b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+6/d*a^2/(a-b)^2/(a+b
)^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2))+1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87343, size = 1203, normalized size = 5.67

$$\left[\frac{2a^5-4a^3b^2+2ab^4+2\left(a^5+a^3b^2-2ab^4\right)\cos(dx+c)^2-3\left(a^2b^2\cos(dx+c)\sin(dx+c)+a^3b\cos(dx+c)\right)\sqrt{-a^2+b}}{2\left(\left(a^6b-3a^4b^3+3a^2b^5-b^7\right)d\cos(dx+c)\sin(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(2*a^5 - 4*a^3*b^2 + 2*a*b^4 + 2*(a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c
)^2 - 3*(a^2*b^2*cos(d*x + c)*sin(d*x + c) + a^3*b*cos(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^4*b - 2*a^2*b^3 + b^
5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(
d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)), (a^5 - 2*
a^3*b^2 + a*b^4 + (a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c)^2 - 3*(a^2*b^2*cos
(d*x + c)*sin(d*x + c) + a^3*b*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin
(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^4*b - 2*a^2*b^3 + b^5)*
sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x
 + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19962, size = 300, normalized size = 1.42

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2b}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a^3-a}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)\left(a^4-2a^2b^2+b^4\right)}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*a^2*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
+ (3*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 - a^2*b*
tan(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c) - 2*a^3 - a*b^2)/((a*tan(
1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)
- a)*(a^4 - 2*a^2*b^2 + b^4)))/d
```

3.1466 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{4ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+d}$$

```
[Out] (-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*d) - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b
^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*
x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.308153, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2731, 2648, 2664, 12, 2660, 618, 204}

$$-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{4ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*d) - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b
^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*
x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]))
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))} + \frac{1}{2(a-b)^2(1+\sin(c+dx))} - \frac{a^2}{\left(a^2-b^2\right)(a+b\sin(c+$$

$$= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} - \frac{\left(2ab^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2} - \frac{a^2\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{a^2-b^2}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{\left(a^2-b^2\right)^2 d(a+b\sin(c+dx)}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{\left(a^2-b^2\right)^2 d(a+b\sin(c+dx)}$$

$$= -\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx)}$$

$$= -\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx)}$$

$$= -\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} - \frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx)}$$

Mathematica [A] time = 0.89359, size = 169, normalized size = 0.84

$$\frac{-\frac{2a\left(a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{a^2b\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{(a+b)^2\left(\cos\left(\frac{1}{2}(c\right.\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])) + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (a^2*b*
Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [A] time = 0.117, size = 282, normalized size = 1.4

$$-\frac{1}{d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-2\frac{ab^2\tan(1/2\,dx+c/2)}{d(a-b)^2(a+b)^2\left((\tan(1/2\,dx+c/2))^2a+2\tan(1/2\,dx+c/2)b+a\right)}-2\frac{}{d(a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-2/d*a/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)-2/d*a^2/(a-b)^2/(a+
b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b-2/d*a^3/(a-b)^2/(a
+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2))-4/d*a/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))*b^2-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87214, size = 1261, normalized size = 6.3

$$\left[-\frac{2a^4b-4a^2b^3+2b^5+2\left(2a^4b-a^2b^3-b^5\right)\cos(dx+c)^2+\left(\left(a^3b+2ab^3\right)\cos(dx+c)\sin(dx+c)+\left(a^4+2a^2b^2\right)\cos\right.}{2\left(\left(a^6b-3a^4b^3+3a^2b^5-b^7\right)d\cos(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x +
c)^2 + ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*cos
(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
```

```
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(
a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*co
s(d*x + c)), -(a^4*b - 2*a^2*b^3 + b^5 + (2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*
cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^
3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.21987, size = 339, normalized size = 1.7

$$-\frac{2\left(\frac{(a^3+2ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2
 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a
^2*b*tan(1/2*d*x + 1/2*c)^2 + 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*
x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*
c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*
a^2*b^2 + b^4)))/d
```

3.1467 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{2b\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\sec(c+dx)\left(2a^2-3ab\sin(c+dx)+b^2\right)}{d\left(a^2-b^2\right)^2}-\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}$$

```
[Out] (2*b*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2
- b^2)^(5/2)*d) - (a*Sec[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (
Sec[c + d*x]*(2*a^2 + b^2 - 3*a*b*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rubi [A] time = 0.210772, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2864, 2866, 12, 2660, 618, 204}

$$\frac{2b\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\sec(c+dx)\left(2a^2-3ab\sin(c+dx)+b^2\right)}{d\left(a^2-b^2\right)^2}-\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2
- b^2)^(5/2)*d) - (a*Sec[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (
Sec[c + d*x]*(2*a^2 + b^2 - 3*a*b*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{a\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx)(b-2a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= -\frac{a\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{\left(a^2-b^2\right)^2 d} + \frac{\int \frac{2a^2 \cdots}{a+b\sin \cdots}}{\left(a^2 - \cdots\right.}$$

$$= -\frac{a\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{\left(a^2-b^2\right)^2 d} + \frac{\left(b\left(2a^2 \cdots\right.\right.}{\cdots}$$

$$= -\frac{a\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{\left(a^2-b^2\right)^2 d} + \frac{\left(2b\left(2a^{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{a\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{\left(a^2-b^2\right)^2 d} - \frac{\left(4b\left(2a^{\cdots}\right.\right.}{\cdots}$$

$$= \frac{2b\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} - \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.84101, size = 169, normalized size = 1.27

$$\frac{\frac{2b\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{ab^2\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{\cdots}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+\cdots\right.\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*b*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
- b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c +
```

```
d*x)/2])) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a*b^2*C
os[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [B] time = 0.109, size = 280, normalized size = 2.1

$$-\frac{1}{d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\frac{b^3\tan(1/2\,dx+c/2)}{d(a-b)^2(a+b)^2\left((\tan(1/2\,dx+c/2))^2a+2\tan(1/2\,dx+c/2)\,b+a\right)}+2\frac{}{d(a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)+2/d*b^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*b^2/(a-b)^2/(a+b)
^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a+4/d*a^2/(a-b)^2/(a+b
)^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2))+2/d*b^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2))+1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86713, size = 1238, normalized size = 9.31

$$\left[\frac{2a^5-4a^3b^2+2ab^4+6\left(a^3b^2-ab^4\right)\cos(dx+c)^2-\left(\left(2a^2b^2+b^4\right)\cos(dx+c)\sin(dx+c)+\left(2a^3b+ab^3\right)\cos(dx+c)\right.}{2\left(\left(a^6b-3a^4b^3+3a^2b^5-b^7\right)d\cos(dx+c)\sin\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^5 - 4*a^3*b^2 + 2*a*b^4 + 6*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - ((
2*a^2*b^2 + b^4)*cos(d*x + c)*sin(d*x + c) + (2*a^3*b + a*b^3)*cos(d*x + c)
)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b
^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^4*b - 2*
a^2*b^3 + b^5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d
*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)
), (a^5 - 2*a^3*b^2 + a*b^4 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - ((2*a^2*
b^2 + b^4)*cos(d*x + c)*sin(d*x + c) + (2*a^3*b + a*b^3)*cos(d*x + c))*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) -
(a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b
```

```
^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*c
os(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.23368, size = 328, normalized size = 2.47

$$\frac{2\left(\frac{(2a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{2a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)(a^4-2\cdots}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2
- b^2)) + (2*a^2*b*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x + 1/2*c)^3 - a^
3*tan(1/2*d*x + 1/2*c)^2 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*b^3*tan(1/2*d
*x + 1/2*c) - a^3 - 2*a*b^2)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x +
 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*a^2*b^2 + b^4)))/d
```

3.1468 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=229

$$\frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{b^4\cos(c+dx)}{ad\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+}{a^2d}$$

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (2*b^3*(3*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/(a^2*(a^2 - b^2)^(5/2)*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/
(2*(a + b)^2*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c
 + d*x])) + (b^4*Cos[c + d*x])/(a*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.310499, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2897, 3770, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{b^4\cos(c+dx)}{ad\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (2*b^3*(3*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/(a^2*(a^2 - b^2)^(5/2)*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/
(2*(a + b)^2*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c
 + d*x])) + (b^4*Cos[c + d*x])/(a*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
```

```
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{\csc(c+dx)}{a^2} - \frac{1}{2(a+b)^2(-1+\sin(c+dx))} - \frac{1}{2(a-b)^2(1+\sin(c+dx))} - \frac{\cdots}{a\left(-a^2 \cdots\right.} \cdots\right.$$

$$= \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} + \frac{b^3\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{a\left(a^2-b^2\right)} + \frac{\left(b \cdots\right.}{\cdots}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\cos(c+dx)}{2(a+b)^2 d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2 d(1+\sin(c+dx))} + \cdots$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\cos(c+dx)}{2(a+b)^2 d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2 d(1+\sin(c+dx))} + \cdots$$

$$= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2} d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\cos(c+dx)}{2(a+b)^2 d(1-\sin(c \cdots}$$

$$= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2} d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\cos(c+dx)}{2(a+b)^2 d(1-\sin(c \cdots}$$

$$= \frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2} d} + \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2} d} - \frac{\tanh^{-1}(\cos(c \cdots}{a^2 d}$$

Mathematica [A] time = 2.21898, size = 203, normalized size = 0.89

$$\frac{-\frac{2(b^5-4a^2b^3)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{5/2}}+\frac{\frac{ab^4\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*(-4*a^2*b^3 + b^5)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2*(a^2 - b^2)^(5/2)) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))
+ (-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]] + (a*b^4*Cos[c + d*x])/((
a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/a^2)/d
```

Maple [A] time = 0.15, size = 304, normalized size = 1.3

$$-\frac{1}{d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\frac{b^5\tan(1/2\,dx+c/2)}{d(a-b)^2(a+b)^2a^2\left((\tan(1/2\,dx+c/2))^2a+2\tan(1/2\,dx+c/2)\,b+a\right)}+2\frac{}{d(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)+2/d*b^5/(a-b)^2/(a+b)^2/a^2/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d*b^4/(a-b)^2/(
a+b)^2/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+8/d*b^3/(a-b)^2/
(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))-2/d*b^5/(a-b)^2/(a+b)^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*
ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.80245, size = 2133, normalized size = 9.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^7 - 4*a^5*b^2 + 2*a^3*b^4 + 2*(2*a^5*b^2 - a^3*b^4 - a*b^6)*cos(d
*x + c)^2 + ((4*a^2*b^4 - b^6)*cos(d*x + c)*sin(d*x + c) + (4*a^3*b^3 - a*b
^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)
) - ((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + ((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2)
 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*sin(d*x + c))/((a^8*b - 3*a^6*b^3 + 3*a^
4*b^5 - a^2*b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^9 - 3*a^7*b^2 + 3*a^5*b^4
 - a^3*b^6)*d*cos(d*x + c)), 1/2*(2*a^7 - 4*a^5*b^2 + 2*a^3*b^4 + 2*(2*a^5*
b^2 - a^3*b^4 - a*b^6)*cos(d*x + c)^2 - 2*((4*a^2*b^4 - b^6)*cos(d*x + c)*s
in(d*x + c) + (4*a^3*b^3 - a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - ((a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + ((a^6*b - 3*a^4*b^3 + 3*
a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a
*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^6*b - 2*a^4*b^3 + a
^2*b^5)*sin(d*x + c))/((a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x
+ c)*sin(d*x + c) + (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c))
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2666, size = 424, normalized size = 1.85

$$\frac{\frac{2\left(4a^2b^3-b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-2a^4b^2+a^2b^4\right)\sqrt{a^2-b^2}}+\frac{2\left(2a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\left(a^6-2a^4b^2+a^2b^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqr
t(a^2 - b^2)) + 2*(2*a^4*b*tan(1/2*d*x + 1/2*c)^3 + b^5*tan(1/2*d*x + 1/2*c
)^3 - a^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + a*b^4
*tan(1/2*d*x + 1/2*c)^2 - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) - b^5*tan(1/2*d*x
+ 1/2*c) - a^5 - a^3*b^2 - a*b^4)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d
*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a))
 + log(abs(tan(1/2*d*x + 1/2*c)))/a^2)/d
```

3.1469 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=248

$$-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{5/2}}-\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}-\frac{b^5\cos(c+dx)}{a^2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{2b\tanh^{-1}(c\text{c}}{a^3c}$$

```
[Out] (-2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5
/2)*d) - (4*b^4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^3*(a^2 - b^2)^(5/2)*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[
c + d*x]/(a^2*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c
+ d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (b^5*Cos[c + d*x])/(a^2*(a^2 -
b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.372225, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2897, 3770, 3767, 8, 2648, 2664, 12, 2660, 618, 204}

$$-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{5/2}}-\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}-\frac{b^5\cos(c+dx)}{a^2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{2b\tanh^{-1}(c\text{c}}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5
/2)*d) - (4*b^4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^3*(a^2 - b^2)^(5/2)*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[
c + d*x]/(a^2*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c
+ d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (b^5*Cos[c + d*x])/(a^2*(a^2 -
b^2)^2*d*(a + b*Sin[c + d*x]))
```


Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{2b\csc(c+dx)}{a^3}+\frac{\csc^2(c+dx)}{a^2}-\frac{1}{2(a+b)^2(-1+\sin(c+dx))}+\frac{1}{2(a-b)^2(1+\sin(\ldots}\right.$$

$$=\frac{\int\csc^2(c+dx)\,dx}{a^2}+\frac{\int\frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2}-\frac{(2b)\int\csc(c+dx)\,dx}{a^3}-\frac{\int\frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2}-\frac{b^4\int\ldots}{\ldots}$$

$$=\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}-\ldots$$

$$=\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^2d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+\ldots}{2(a-b)^2d(1+\ldots}$$

$$=-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^2d}+\frac{\ldots}{2(a\ldots}$$

$$=-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^2d}+\frac{\ldots}{2(a\ldots}$$

$$=-\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}d}-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2b\tanh^{-1}(\cos\ldots}{a^3d}$$

Mathematica [A] time = 3.23369, size = 254, normalized size = 1.02

$$\frac{\frac{4b^4\left(2b^2-5a^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}}-\frac{2b^5\cos(c+dx)}{a^2(a-b)^2(a+b)^2(a+b\sin(c+dx))}-\frac{4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{a^2}-\frac{\cot\left(\frac{1}{2}(c+\ldots\right.}{a^2}\ldots}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((4*b^4*(-5*a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^3*(a^2 - b^2)^(5/2)) - Cot[(c + d*x)/2]/a^2 + (4*b*Log[Cos[(c + d*x)/2]]
)/a^3 - (4*b*Log[Sin[(c + d*x)/2]])/a^3 + (2*Sin[(c + d*x)/2])/((a + b)^2*(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)^2*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])) - (2*b^5*Cos[c + d*x])/(a^2*(a - b)^2*(
a + b)^2*(a + b*Sin[c + d*x])) + Tan[(c + d*x)/2]/a^2)/(2*d)
```

Maple [A] time = 0.159, size = 346, normalized size = 1.4

$$\frac{1}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{d\,(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-2\,\frac{b^6\tan(1/2\,dx+c/2)}{d\,(a-b)^2\,(a+b)^2\,a^3\left((\tan(1/2\,dx+c/2))^2\,a+2\,\tan(1/2\,dx+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-2/d*b^6/(a-
b)^2/(a+b)^2/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*
d*x+1/2*c)-2/d*b^5/(a-b)^2/(a+b)^2/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)-10/d*b^4/(a-b)^2/(a+b)^2/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+4/d*b^6/(a-b)^2/(a+b)^2/a^3/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/d/(a-b)^
2/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^3*b*ln(tan(1/2*
d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.63576, size = 2925, normalized size = 11.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*a^8 - 4*a^6*b^2 + 2*a^4*b^4 - 2*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 - a
^2*b^6)*cos(d*x + c)^2 - ((5*a^2*b^5 - 2*b^7)*cos(d*x + c)^3 - (5*a^3*b^4 -
 2*a*b^6)*cos(d*x + c)*sin(d*x + c) - (5*a^2*b^5 - 2*b^7)*cos(d*x + c))*sqr
t(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
- b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/
(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*((a^6*b^2 - 3*a^
4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^3 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
a*b^7)*cos(d*x + c)*sin(d*x + c) - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 2*((a^6*b^2 - 3*a^4*b^4 + 3*a^2
*b^6 - b^8)*cos(d*x + c)^3 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*
x + c)*sin(d*x + c) - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c))
*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5 + (2*a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - 2*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^9*b - 3
*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x + c)^3 - (a^10 - 3*a^8*b^2 + 3*a^
6*b^4 - a^4*b^6)*d*cos(d*x + c)*sin(d*x + c) - (a^9*b - 3*a^7*b^3 + 3*a^5*b
^5 - a^3*b^7)*d*cos(d*x + c)), -(a^8 - 2*a^6*b^2 + a^4*b^4 - (2*a^8 - 5*a^6
*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 - ((5*a^2*b^5 - 2*b^7)*cos(d*x +
 c)^3 - (5*a^3*b^4 - 2*a*b^6)*cos(d*x + c)*sin(d*x + c) - (5*a^2*b^5 - 2*b^
7)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b
^2)*cos(d*x + c))) - ((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^
3 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c) - (a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + ((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^3 - (a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c) - (a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - (a^7*b -
 2*a^5*b^3 + a^3*b^5 + (2*a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - 2*a*b^7)*cos(d*x
```

```
+ c)^2)*sin(d*x + c))/((a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x
+ c)^3 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)*sin(d*x +
c) - (a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32381, size = 706, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(20*(5*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3
*b^4)*sqrt(a^2 - b^2)) - (4*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 8*a^3*b^3*tan(1/
2*d*x + 1/2*c)^5 + 4*a*b^5*tan(1/2*d*x + 1/2*c)^5 - 25*a^6*tan(1/2*d*x + 1/
2*c)^4 - 2*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 - 21*a^2*b^4*tan(1/2*d*x + 1/2*c)
^4 - 12*b^6*tan(1/2*d*x + 1/2*c)^4 - 10*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 20*a
^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 30*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 20*a^6*ta
n(1/2*d*x + 1/2*c)^2 + 52*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a^2*b^4*tan(1
/2*d*x + 1/2*c)^2 + 12*b^6*tan(1/2*d*x + 1/2*c)^2 + 46*a^5*b*tan(1/2*d*x +
1/2*c) - 12*a^3*b^3*tan(1/2*d*x + 1/2*c) + 26*a*b^5*tan(1/2*d*x + 1/2*c) +
5*a^6 - 10*a^4*b^2 + 5*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x
 + 1/2*c)^5 + 2*b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a*t
an(1/2*d*x + 1/2*c))) + 20*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*tan(1/2
*d*x + 1/2*c)/a^2)/d
```

3.1470 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=295

$$\frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{5/2}}+\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{b^6\cos(c+dx)}{a^3d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{\left(a^2+3b^2\right)}{}$$

```
[Out] (2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(
5/2)*d) + (2*b^5*(5*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^4*(a^2 - b^2)^(5/2)*d) - ArcTanh[Cos[c + d*x]]/(2*a^2*d) - ((a^2
 + 3*b^2)*ArcTanh[Cos[c + d*x]])/(a^4*d) + (2*b*Cot[c + d*x])/(a^3*d) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a^2*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c
 + d*x])) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) + (b^6*Cos[c +
d*x])/(a^3*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.39499, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 3768, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{5/2}}+\frac{2b^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{b^6\cos(c+dx)}{a^3d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{\left(a^2+3b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(
5/2)*d) + (2*b^5*(5*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^4*(a^2 - b^2)^(5/2)*d) - ArcTanh[Cos[c + d*x]]/(2*a^2*d) - ((a^2
 + 3*b^2)*ArcTanh[Cos[c + d*x]])/(a^4*d) + (2*b*Cot[c + d*x])/(a^3*d) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a^2*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c
 + d*x])) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) + (b^6*Cos[c +
d*x])/(a^3*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{(a^2+3b^2)\csc(c+dx)}{a^4} - \frac{2b\csc^2(c+dx)}{a^3} + \frac{\csc^3(c+dx)}{a^2} - \frac{1}{2(a+b)^2(-1+\sin(c\right.$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^2} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{(2b)\int \csc^2(c+dx)\,dx}{a^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} +$$

$$= -\frac{(a^2+3b^2)\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{(a^2+3b^2)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{2b\cot(c+dx)}{a^3d} - \frac{\cot(c}{}$$

$$= \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{(a^2+3b^2)\tanh^{-1}}{a^4d}$$

$$= \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{(a^2+3b^2)\tanh^{-1}}{a^4d}$$

$$= \frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{5/2}d} + \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{5/2}d} - \frac{\tanh^{-1}(\cos}{2a^2}$$

Mathematica [A] time = 6.49093, size = 356, normalized size = 1.21

$$\frac{3(a^2+2b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} - \frac{3(a^2+2b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{b^6\cos(c+dx)}{a^3d(a-b)^2(a+b)^2(a+b\sin(c+dx))} + \frac{6b^5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6*b^5*(2*a^2 - b^2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^(5/2)*d) + (b*Cot[(c + d*x
)/2])/(a^3*d) - Csc[(c + d*x)/2]^2/(8*a^2*d) - (3*(a^2 + 2*b^2)*Log[Cos[(c
+ d*x)/2]])/(2*a^4*d) + (3*(a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^4*d) +
 Sec[(c + d*x)/2]^2/(8*a^2*d) + Sin[(c + d*x)/2]/((a + b)^2*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])) - Sin[(c + d*x)/2]/((a - b)^2*d*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])) + (b^6*Cos[c + d*x])/(a^3*(a - b)^2*(a + b)^2*d*(a +
b*Sin[c + d*x])) - (b*Tan[(c + d*x)/2])/(a^3*d)
```

Maple [A] time = 0.167, size = 404, normalized size = 1.4

$$\frac{1}{8da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{b}{da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + 2\frac{b^7\tan(}{da^4(a+b)^2(a-b)^2((\tan(1/2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*tan(1/2*d*x+1/2*c)^2-1/d/a^3*tan(1/2*d*x+1/2*c)*b-1/d/(a+b)^2/(ta
n(1/2*d*x+1/2*c)-1)+2/d/a^4*b^7/(a+b)^2/(a-b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*t
an(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)+2/d/a^3*b^6/(a+b)^2/(a-b)^2/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+12/d*b^5/(a-b)^2/(a+b)^2/a^2/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d
/a^4*b^7/(a+b)^2/(a-b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))+1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-1/8/d/a^2/tan(1/2
*d*x+1/2*c)^2+3/2/d/a^2*ln(tan(1/2*d*x+1/2*c))+3/d/a^4*ln(tan(1/2*d*x+1/2*c
))*b^2+1/d*b/a^3/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 14.6489, size = 3976, normalized size = 13.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^9 - 8*a^7*b^2 + 4*a^5*b^4 - 4*(4*a^7*b^2 - 8*a^5*b^4 + 7*a^3*b^6
 - 3*a*b^8)*cos(d*x + c)^4 - 6*(a^9 - 5*a^7*b^2 + 7*a^5*b^4 - 5*a^3*b^6 + 2
*a*b^8)*cos(d*x + c)^2 - 6*((2*a^3*b^5 - a*b^7)*cos(d*x + c)^3 - (2*a^3*b^5
 - a*b^7)*cos(d*x + c) + ((2*a^2*b^6 - b^8)*cos(d*x + c)^3 - (2*a^2*b^6 - b
^8)*cos(d*x + c))*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c)
+ b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2)) + 3*((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(
d*x + c)^3 - (a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c)
 + ((a^8*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c)^3 - (a^8
*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(d*x + c))*l
og(1/2*cos(d*x + c) + 1/2) - 3*((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*
a*b^8)*cos(d*x + c)^3 - (a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*c
os(d*x + c) + ((a^8*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x +
c)^3 - (a^8*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*a^8*b - 4*a^6*b^3 + 2*a^4*b^5
 - (5*a^8*b - 13*a^6*b^3 + 11*a^4*b^5 - 3*a^2*b^7)*cos(d*x + c)^2)*sin(d*x
+ c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^3 - (a^11 -
3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c) + ((a^10*b - 3*a^8*b^3 + 3*
a^6*b^5 - a^4*b^7)*d*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4
*b^7)*d*cos(d*x + c))*sin(d*x + c)), -1/4*(4*a^9 - 8*a^7*b^2 + 4*a^5*b^4 -
4*(4*a^7*b^2 - 8*a^5*b^4 + 7*a^3*b^6 - 3*a*b^8)*cos(d*x + c)^4 - 6*(a^9 - 5
```

```
*a^7*b^2 + 7*a^5*b^4 - 5*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2 + 12*((2*a^3*b^5
 - a*b^7)*cos(d*x + c)^3 - (2*a^3*b^5 - a*b^7)*cos(d*x + c) + ((2*a^2*b^6 -
 b^8)*cos(d*x + c)^3 - (2*a^2*b^6 - b^8)*cos(d*x + c))*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 3*
((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c)^3 - (a^9 -
a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c) + ((a^8*b - a^6*b^3
 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c)^3 - (a^8*b - a^6*b^3 - 3*a^4
*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c)
+ 1/2) - 3*((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c)^
3 - (a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c) + ((a^8*
b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c)^3 - (a^8*b - a^6*
b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/2*c
os(d*x + c) + 1/2) - 2*(2*a^8*b - 4*a^6*b^3 + 2*a^4*b^5 - (5*a^8*b - 13*a^6
*b^3 + 11*a^4*b^5 - 3*a^2*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 - 3*a^9
*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^3 - (a^11 - 3*a^9*b^2 + 3*a^7*b^
4 - a^5*b^6)*d*cos(d*x + c) + ((a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d
*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c)
)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25932, size = 571, normalized size = 1.94

$$\frac{\frac{48\left(2a^2b^5-b^7\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^8-2a^6b^2+a^4b^4\right)\sqrt{a^2-b^2}}+\frac{16\left(2a^6b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^5b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{\left(a^8-2a^6b^2+a^4b^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(48*(2*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4
)*sqrt(a^2 - b^2)) + 16*(2*a^6*b*tan(1/2*d*x + 1/2*c)^3 + b^7*tan(1/2*d*x +
 1/2*c)^3 - a^7*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 +
 a*b^6*tan(1/2*d*x + 1/2*c)^2 - 2*a^4*b^3*tan(1/2*d*x + 1/2*c) - b^7*tan(1/
2*d*x + 1/2*c) - a^7 - a^5*b^2 - a*b^6)/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan
(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)
 - a)) + 12*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + (a^2*tan(1/2
*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 - (18*a^2*tan(1/2*d*x + 1
/2*c)^2 + 36*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)
/(a^4*tan(1/2*d*x + 1/2*c)^2))/d
```

3.1471 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=388

$$-\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}+\frac{4a^4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}-\frac{2a^2\left(-3a^2b^2+a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(\right.}{\sqrt{a^2}}\right.}{b^2d\left(a^2-b^2\right)^{7/2}}$$

```
[Out] (4*a^4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2
*(a^2 - b^2)^(7/2)*d) - (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(7/2)*d) - (2*a^2*(a^4 - 3*a^2*b^2 + 6*b
^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(7/2
)*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a
 - b)^3*d*(1 + Sin[c + d*x])) - (a^4*Cos[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a
+ b*Sin[c + d*x])^2) - (3*a^5*Cos[c + d*x])/(2*b*(a^2 - b^2)^3*d*(a + b*Sin
[c + d*x])) + (2*a^3*(a^2 - 2*b^2)*Cos[c + d*x])/(b*(a^2 - b^2)^3*d*(a + b*
Sin[c + d*x]))
```

Rubi [A] time = 0.580089, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2897, 2648, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}+\frac{4a^4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}-\frac{2a^2\left(-3a^2b^2+a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(\right.}{\sqrt{a^2}}\right.}{b^2d\left(a^2-b^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (4*a^4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2
*(a^2 - b^2)^(7/2)*d) - (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(7/2)*d) - (2*a^2*(a^4 - 3*a^2*b^2 + 6*b
^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(7/2
)*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a
 - b)^3*d*(1 + Sin[c + d*x])) - (a^4*Cos[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a
+ b*Sin[c + d*x])^2) - (3*a^5*Cos[c + d*x])/(2*b*(a^2 - b^2)^3*d*(a + b*Sin
[c + d*x])) + (2*a^3*(a^2 - 2*b^2)*Cos[c + d*x])/(b*(a^2 - b^2)^3*d*(a + b*
Sin[c + d*x]))
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))}+\frac{1}{2(a-b)^3(1+\sin(c+dx))}+\frac{a^4}{b^2\left(-a^2+b^2\right)(a+b\sin\ldots}\right.$$

$$=\frac{\int\frac{1}{1+\sin(c+dx)}dx}{2(a-b)^3}-\frac{\int\frac{1}{-1+\sin(c+dx)}dx}{2(a+b)^3}+\frac{\left(2a^3\left(a^2-2b^2\right)\right)\int\frac{1}{(a+b\sin(c+dx))^2}dx}{b^2\left(a^2-b^2\right)^2}-\frac{a^4\int\frac{\ldots}{(a+b\sin\ldots}}{b^2\left(a^2\ldots\right.}$$

$$=\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))}-\frac{a^4\cos(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sin(c\ldots}$$

$$=\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))}-\frac{a^4\cos(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sin(c\ldots}$$

$$=-\frac{2a^2\left(a^4-3a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}+\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\ldots}{2(a-\ldots}$$

$$=-\frac{2a^2\left(a^4-3a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}+\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\ldots}{2(a-\ldots}$$

$$=\frac{4a^4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}-\frac{2a^2\left(a^4-3a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}\ldots$$

$$=\frac{4a^4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}-\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}-\frac{2a^2(a\ldots}{\ldots}$$

Mathematica [A] time = 3.16266, size = 195, normalized size = 0.5

$$\frac{-\frac{6a^2\left(a^2+4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}+\frac{a^3\cos(c+dx)\left(\left(a^2-8b^2\right)\sin(c+dx)-7ab\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a-b)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\ldots}{(a+b)^3(\ldots}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*a^2*(a^2 + 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a^3*
Cos[c + d*x]*(-7*a*b + (a^2 - 8*b^2)*Sin[c + d*x]))/((a - b)^3*(a + b)^3*(a
 + b*Sin[c + d*x])^2))/(2*d)
```

Maple [A] time = 0.145, size = 590, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-1/d*a^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-6/d*a^3/(a-b)^3/(
a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*
c)^3*b^2-7/d*a^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-14/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan(1/2*d*x+1/2*c)^2+1/d*a^5/(a-b)^3
/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)-22/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*tan(1/2*d*x+1/2*c)*b^2-7/d*a^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b-3/d*a^4/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-12/d*b^2/(a-b)^3/(a+
b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2
))*a^2-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13163, size = 2005, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(11*a^6*b - 5*a^4*b^3 -
 8*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + 3*((a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^
3 - 2*(a^5*b + 4*a^3*b^3)*cos(d*x + c)*sin(d*x + c) - (a^6 + 5*a^4*b^2 + 4*
a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + (a^7 - 11*a^5*b^2 + 4*
a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*
a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5
*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 +
 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), 1/2*(2*a^6*b -
6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (11*a^6*b - 5*a^4*b^3 - 8*a^2*b^5 + 2*b^7)*
cos(d*x + c)^2 + 3*((a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^3 - 2*(a^5*b + 4*a^3
*b^3)*cos(d*x + c)*sin(d*x + c) - (a^6 + 5*a^4*b^2 + 4*a^2*b^4)*cos(d*x + c
))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x +
c))) - (2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + (a^7 - 11*a^5*b^2 + 4*a^3
*b^4 + 6*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4
*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^
```

```
5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*
a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25279, size = 474, normalized size = 1.22

$$-\frac{\frac{3\left(a^{4}+4\,a^{2}b^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}+\frac{2\left(a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^{2}b-b^{3}\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}+\frac{a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,a^{3}b^{2}\tan\left(\cdots\right.}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(a^4 + 4*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*
tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x
+ 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x
 + 1/2*c)^2 - 1)) + (a^5*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*b^2*tan(1/2*d*x + 1
/2*c)^3 + 7*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 14*a^2*b^3*tan(1/2*d*x + 1/2*c)^
2 - a^5*tan(1/2*d*x + 1/2*c) + 22*a^3*b^2*tan(1/2*d*x + 1/2*c) + 7*a^4*b)/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2
*d*x + 1/2*c) + a)^2))/d
```

3.1472 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=366

$$\frac{a^3(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd(a^2-b^2)^{7/2}}-\frac{2a^3(a^2-3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd(a^2-b^2)^{7/2}}+\frac{2ab(a^2+3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}}$$

```
[Out] (-2*a^3*(a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*
(a^2 - b^2)^(7/2)*d) + (a^3*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(b*(a^2 - b^2)^(7/2)*d) + (2*a*b*(a^2 + 3*b^2)*ArcTan[(b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]
/(2*(a + b)^3*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[
c + d*x])) + (a^3*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2)
+ (3*a^4*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (a^2*(a^2
 - 3*b^2)*Cos[c + d*x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.490659, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2897, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{a^3(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd(a^2-b^2)^{7/2}}-\frac{2a^3(a^2-3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd(a^2-b^2)^{7/2}}+\frac{2ab(a^2+3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-2*a^3*(a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*
(a^2 - b^2)^(7/2)*d) + (a^3*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(b*(a^2 - b^2)^(7/2)*d) + (2*a*b*(a^2 + 3*b^2)*ArcTan[(b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]
/(2*(a + b)^3*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[
c + d*x])) + (a^3*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2)
+ (3*a^4*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (a^2*(a^2
 - 3*b^2)*Cos[c + d*x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```


Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} - \frac{1}{2(a-b)^3(1+\sin(c+dx))} + \frac{a^3}{b\left(a^2-b^2\right)(a+b\sin\cdots}\right.$$

$$= -\frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} - \frac{\left(a^2\left(a^2-3b^2\right)\right)\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{b\left(a^2-b^2\right)^2} + \frac{a^3\int \frac{\cdots}{(a+\cdots}}{b(\cdots}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3 d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3 d(1+\sin(c+dx))} + \frac{a^3\cos(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(\cdots}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3 d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3 d(1+\sin(c+dx))} + \frac{a^3\cos(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(\cdots}$$

$$= \frac{2ab\left(a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2} d} + \frac{\cos(c+dx)}{2(a+b)^3 d(1-\sin(c+dx))} + \frac{\cos(\cdots}{2(a-b)^3 d(1\cdots}$$

$$= \frac{2ab\left(a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2} d} + \frac{\cos(c+dx)}{2(a+b)^3 d(1-\sin(c+dx))} + \frac{\cos(\cdots}{2(a-b)^3 d(1\cdots}$$

$$= -\frac{2a^3\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2} d} + \frac{2ab\left(a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2} d} + \cdots$$

$$= -\frac{2a^3\left(a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2} d} + \frac{a^3\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2} d} + \frac{2a\cdots}{\cdots}$$

Mathematica [A] time = 3.26788, size = 204, normalized size = 0.56

$$\frac{\frac{6ab\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{a^2\cos(c+dx)\left(b\left(a^2+6b^2\right)\sin(c+dx)+2a^3+5ab^2\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}\cdots\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((6*a*b*(3*a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) - 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a^2
*Cos[c + d*x]*(2*a^3 + 5*a*b^2 + b*(a^2 + 6*b^2)*Sin[c + d*x]))/((a - b)^3*
(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.145, size = 702, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+3/d*a^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b+4/d*a^2/(a-b)^3
/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)^3*b^3+2/d*a^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+10/d*a/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^4+9/d*a^3/(a-b)^3/(
a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*
c)^2*b^2+5/d*a^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2*tan(1/2*d*x+1/2*c)*b+16/d*a^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^3+2/d*a^5/(a-b)^3/(a+
b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+5/d*a^3/(a-b)^3/(a
+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^2+9/d*a^3/(a-b)
^3/(a+b)^3*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+6/d*a/(a-b)^3/(a+b)^3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16709, size = 2030, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^7 - 12*a^5*b^2 + 12*a^3*b^4 - 4*a*b^6 + 2*(2*a^7 + 13*a^5*b^2 -
17*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^2 - 3*((3*a^3*b^3 + 2*a*b^5)*cos(d*x + c
)^3 - 2*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)*sin(d*x + c) - (3*a^5*b + 5*a^
3*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
 b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2)) - 2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (a^6*b + 11*a
^4*b^3 - 10*a^2*b^5 - 2*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^
6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b
^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3
*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), -1/2*
(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + (2*a^7 + 13*a^5*b^2 - 17*a^3*b^4
 + 2*a*b^6)*cos(d*x + c)^2 + 3*((3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^3 - 2*(3
*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)*sin(d*x + c) - (3*a^5*b + 5*a^3*b^3 + 2*
a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
 - b^2)*cos(d*x + c))) - (2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (a^6*b
```

```
+ 11*a^4*b^3 - 10*a^2*b^5 - 2*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2
- 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4
*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^
10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28636, size = 509, normalized size = 1.39

$$\frac{\frac{3\left(3a^3b+2ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^3-3ab^2\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{3a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^2b^3\cdots}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(3*a^3*b + 2*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*sqrt(a^2 - b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x
 + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*
x + 1/2*c)^2 - 1)) + (3*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 4*a^2*b^3*tan(1/2*d*
x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c)^2 + 9*a^3*b^2*tan(1/2*d*x + 1/2*c
)^2 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 5*a^4*b*tan(1/2*d*x + 1/2*c) + 16*a
^2*b^3*tan(1/2*d*x + 1/2*c) + 2*a^5 + 5*a^3*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2))/d
```

3.1473 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=350

$$-\frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\cdots}{2d\left(a^2\cdots\right.}$$

```
[Out] (-4*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^
(7/2)*d) - (a^2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/((a^2 - b^2)^(7/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(2*(a + b)^3*
d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) - (
a^2*b*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) - (3*a^3*b*C
os[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (2*a*b^3*Cos[c + d*
x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.56105, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2731, 2648, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\cdots}{2d\left(a^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^
(7/2)*d) - (a^2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/((a^2 - b^2)^(7/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(2*(a + b)^3*
d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) - (
a^2*b*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) - (3*a^3*b*C
os[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (2*a*b^3*Cos[c + d*
x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
```

```
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} + \frac{1}{2(a-b)^3(1+\sin(c+dx))} - \frac{a^2}{(a^2-b^2)(a+b\sin(c+dx)}\right.$$

$$= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} - \frac{(2ab^2)\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{(a^2-b^2)^2} - \frac{a^2\int \frac{1}{(a+b\sin(c+dx))^3}\,dx}{a^2-b^2} - \frac{(b}{}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{2(a^2-b^2)^2d(a+b\sin(c+dx))^2}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{2(a^2-b^2)^2d(a+b\sin(c+dx))^2}$$

$$= -\frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c}$$

$$= -\frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c}$$

$$= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}d} - \frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}d} + \frac{\cos(c+d}{2(a+b)^3d(1-\sin}$$

$$= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}d} - \frac{a^2(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}d} - \frac{2b^2(3a^2+b^2)\tan}{(a^2-b}$$

Mathematica [A] time = 3.23815, size = 212, normalized size = 0.61

$$\frac{-\frac{2(11a^2b^2+2a^4+2b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{ab\cos(c+dx)\left(b(3a^2+4b^2)\sin(c+dx)+4a^3+3ab^2\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a-b)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+d\right.\right.}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 + 11*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
))) - (a*b*Cos[c + d*x]*(4*a^3 + 3*a*b^2 + b*(3*a^2 + 4*b^2)*Sin[c + d*x]))
/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.142, size = 766, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-5/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-2/d*b^4/(a-b)
^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*
x+1/2*c)^3-4/d*a^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-11/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan(1/2*d*x+1/2*c)^2-6/d*b^5/(a-b)
^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+
1/2*c)^2-11/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2-10/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)-4/d*a^4/(a-b)^3/(a
+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b-3/d*b^3/(a-b)^3
/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2-2/d*a^4/(a
-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))-11/d*b^2/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2-2/d*b^4/(a-b)^3/(a+b)^3/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/d/(a-b)^3/(t
an(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.144, size = 2067, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(8*a^6*b + a^4*b^3 - 11
*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos(d*
x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) - (
2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log
(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 -
2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a
^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^
9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c
) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x
 + c)), 1/2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (8*a^6*b + a^4*b^3 -
 11*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos
(d*x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c)
```

```
- (2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^7 - 6*a^
5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^
2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*co
s(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(
d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b
^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27173, size = 518, normalized size = 1.48

$$-\frac{\frac{\left(2\,a^4+11\,a^2b^2+2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b-b^3\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,\cdots}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((2*a^4 + 11*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + a
rctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan
(1/2*d*x + 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(ta
n(1/2*d*x + 1/2*c)^2 - 1)) + (5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^4*ta
n(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 11*a^2*b^3*tan(1/2*
d*x + 1/2*c)^2 + 6*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 10*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b + 3*a^2*b^3)/((a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c)
+ a)^2))/d
```

3.1474 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=204

$$\frac{3ab\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{\sec(c+dx)\left(3a\left(2a^2+3b^2\right)-b\left(11a^2+4b^2\right)\sin(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{\left(3a^2+2b^2\right)}{2d\left(a^2-b^2\right)^2(a}$$

```
[Out] (3*a*b*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((
a^2 - b^2)^(7/2)*d) - (a*Sec[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]
)^2) - ((3*a^2 + 2*b^2)*Sec[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x
])) + (Sec[c + d*x]*(3*a*(2*a^2 + 3*b^2) - b*(11*a^2 + 4*b^2)*Sin[c + d*x])
)/(2*(a^2 - b^2)^3*d)
```

Rubi [A] time = 0.363555, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2864, 2866, 12, 2660, 618, 204}

$$\frac{3ab\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{\sec(c+dx)\left(3a\left(2a^2+3b^2\right)-b\left(11a^2+4b^2\right)\sin(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{\left(3a^2+2b^2\right)}{2d\left(a^2-b^2\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (3*a*b*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((
a^2 - b^2)^(7/2)*d) - (a*Sec[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]
)^2) - ((3*a^2 + 2*b^2)*Sec[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x
])) + (Sec[c + d*x]*(3*a*(2*a^2 + 3*b^2) - b*(11*a^2 + 4*b^2)*Sin[c + d*x])
)/(2*(a^2 - b^2)^3*d)
```


Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(2b-3a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx)\left(-5ab+\right.}{a+b\,\mathrm{s}}}{2\left(a\right.}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(3a\right(}{}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(3a\right(}{}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(3a\right(}{}$$

$$= -\frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+2b^2\right)\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(3a\right(}{}$$

$$= \frac{3ab\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{a\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\left(3a^2+\right.}{2\left(a^2-b^2\right)}$$

Mathematica [A] time = 2.97236, size = 206, normalized size = 1.01

$$\frac{\frac{6ab\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{b^2\cos(c+dx)\left(b\left(5a^2+2b^2\right)\sin(c+dx)+a\left(6a^2+b^2\right)\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((6*a*b*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) - 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (b^2
*Cos[c + d*x]*(a*(6*a^2 + b^2) + b*(5*a^2 + 2*b^2)*Sin[c + d*x]))/((a - b)^
3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.132, size = 643, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+7/d*a^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^3+6/d*a^3/(a-b)
^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+
1/2*c)^2*b^2+13/d*a/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^4+2/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2+17/d*a^2/(a-b
)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x
+1/2*c)*b^3+4/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+6/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^2+1/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a+6/d*a^3/(a-b)^3/(a+b)^3*b/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+9/d*a/(a-b)
^3/(a+b)^3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))+1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12024, size = 1979, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^7 - 12*a^5*b^2 + 12*a^3*b^4 - 4*a*b^6 + 2*(16*a^5*b^2 - 17*a^3*b
^4 + a*b^6)*cos(d*x + c)^2 - 3*((2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - 2*(2
*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)*sin(d*x + c) - (2*a^5*b + 5*a^3*b^3 + 3*
a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) - 2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (11*a^4*b^3 - 7*a^2*b^5
 - 4*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 -
 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*
a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^
4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), -1/2*(2*a^7 - 6*a^5*b^2
+ 6*a^3*b^4 - 2*a*b^6 + (16*a^5*b^2 - 17*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
3*((2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - 2*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x
 + c)*sin(d*x + c) - (2*a^5*b + 5*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a
^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos
(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b
^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6
 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.25063, size = 493, normalized size = 2.42

$$\frac{\frac{3\left(2a^3b+3ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^3-3ab^2\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{7a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^4b^2\tan\cdots}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(2*a^3*b + 3*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*sqrt(a^2 - b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x
 + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*
x + 1/2*c)^2 - 1)) + (7*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^4*b^2*tan(1/2*
d*x + 1/2*c)^2 + 13*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 2*b^6*tan(1/2*d*x + 1/
2*c)^2 + 17*a^3*b^3*tan(1/2*d*x + 1/2*c) + 4*a*b^5*tan(1/2*d*x + 1/2*c) + 6
*a^4*b^2 + a^2*b^4)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(a*tan(1/2*d*x +
 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2))/d
```

3.1475 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3} dx$

Optimal. Leaf size=402

$$\frac{2b^3(3a^2-b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad(a^2-b^2)^{7/2}}+\frac{b^3(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad(a^2-b^2)^{7/2}}+\frac{2b^3(-3a^2b^2+6a^4+b^4)\tan^{-1}\left(\frac{a\tan(}{\sqrt{}}\right.}{a^3d(a^2-b^2)^{7/2}}$$

```
[Out] (2*b^3*(3*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(
a^2 - b^2)^(7/2)*d) + (b^3*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/(a*(a^2 - b^2)^(7/2)*d) + (2*b^3*(6*a^4 - 3*a^2*b^2 + b^4)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d)
 - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c +
 d*x])) + Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) + (b^4*Cos[c + d*
x])/(2*a*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) + (3*b^4*Cos[c + d*x])/(2*
(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (b^4*(3*a^2 - b^2)*Cos[c + d*x])/(a
^2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.509691, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2897, 3770, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{2b^3(3a^2-b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad(a^2-b^2)^{7/2}}+\frac{b^3(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad(a^2-b^2)^{7/2}}+\frac{2b^3(-3a^2b^2+6a^4+b^4)\tan^{-1}\left(\frac{a\tan(}{\sqrt{}}\right.}{a^3d(a^2-b^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (2*b^3*(3*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(
a^2 - b^2)^(7/2)*d) + (b^3*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/(a*(a^2 - b^2)^(7/2)*d) + (2*b^3*(6*a^4 - 3*a^2*b^2 + b^4)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d)
 - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c +
 d*x])) + Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) + (b^4*Cos[c + d*
x])/(2*a*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) + (3*b^4*Cos[c + d*x])/(2*
(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (b^4*(3*a^2 - b^2)*Cos[c + d*x])/(a
^2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```


Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(\frac{\csc(c+dx)}{a^3} - \frac{1}{2(a+b)^3(-1+\sin(c+dx))} - \frac{1}{2(a-b)^3(1+\sin(c+dx))} - \frac{}{a\left(-a^2\right.}\right.$$

$$= \frac{\int\csc(c+dx)\,dx}{a^3} - \frac{\int\frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int\frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} + \frac{b^3\int\frac{1}{(a+b\sin(c+dx))^3}\,dx}{a\left(a^2-b^2\right)} + \frac{(b}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} +$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} +$$

$$= \frac{2b^3\left(6a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos}{2(a+b)^3d(}$$

$$= \frac{2b^3\left(6a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos}{2(a+b)^3d(}$$

$$= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d} + \frac{2b^3\left(6a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a^2-b^2}}\right.}{a^3\left(a^2-b^2\right)^{7/2}d}$$

$$= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d} + \frac{b^3\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d} + \frac{2b^3}{}$$

Mathematica [A] time = 6.58863, size = 322, normalized size = 0.8

$$\frac{9a^2b^4\cos(c+dx)-2b^6\cos(c+dx)}{2a^2d(a-b)^3(a+b)^3(a+b\sin(c+dx))} + \frac{b^3\left(-7a^2b^2+20a^4+2b^4\right)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(a\sin\left(\frac{1}{2}(c+dx)\right)+b\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{7/2}} + \frac{\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b^3*(20*a^4 - 7*a^2*b^2 + 2*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)
/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d) - Lo
g[Cos[(c + d*x)/2]]/(a^3*d) + Log[Sin[(c + d*x)/2]]/(a^3*d) + Sin[(c + d*x)
/2]/((a + b)^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - Sin[(c + d*x)/2]/
((a - b)^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^4*Cos[c + d*x])/(2
*a*(a - b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])^2) + (9*a^2*b^4*Cos[c + d*x]
- 2*b^6*Cos[c + d*x])/(2*a^2*(a - b)^3*(a + b)^3*d*(a + b*Sin[c + d*x]))
```

Maple [B] time = 0.178, size = 787, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+11/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1
/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-4/d*b^7/(a-b)^3/
(a+b)^3/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x
+1/2*c)^3+10/d*a/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^4+17/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^2-6/d*b^8/(a-b)^3
/(a+b)^3/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*
x+1/2*c)^2+29/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)-8/d*b^7/(a-b)^3/(a+b)^3/a^2/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+10/d*b^4/(a-b)^3/(a
+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a-3/d*b^6/(a-b)^3
/(a+b)^3/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+20/d*a/(a-b)
^3/(a+b)^3*b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))-7/d*b^5/(a-b)^3/(a+b)^3/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d*b^7/(a-b)^3/(a+b)^3/a^3/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/(a-b)^3/
(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 19.155, size = 3615, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^10 - 12*a^8*b^2 + 12*a^6*b^4 - 4*a^4*b^6 + 2*(10*a^8*b^2 - 2*a^6
*b^4 - 11*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^2 - ((20*a^4*b^5 - 7*a^2*b^7 +
2*b^9)*cos(d*x + c)^3 - 2*(20*a^5*b^4 - 7*a^3*b^6 + 2*a*b^8)*cos(d*x + c)*s
in(d*x + c) - (20*a^6*b^3 + 13*a^4*b^5 - 5*a^2*b^7 + 2*b^9)*cos(d*x + c))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*((a^8*b^2 - 4
*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*
b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*
a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*cos(d*x + c))*log(1/2*c
os(d*x + c) + 1/2) - 2*((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10
)*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*co
s(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2
*b^8 + b^10)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*a^9*b - 6*a^
```

```
7*b^3 + 6*a^5*b^5 - 2*a^3*b^7 - (6*a^7*b^3 + 5*a^5*b^5 - 13*a^3*b^7 + 2*a*b
^9)*cos(d*x + c)^2)*sin(d*x + c))/((a^11*b^2 - 4*a^9*b^4 + 6*a^7*b^6 - 4*a^
5*b^8 + a^3*b^10)*d*cos(d*x + c)^3 - 2*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4
*a^6*b^7 + a^4*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^13 - 3*a^11*b^2 + 2*a^
9*b^4 + 2*a^7*b^6 - 3*a^5*b^8 + a^3*b^10)*d*cos(d*x + c)), -1/2*(2*a^10 - 6
*a^8*b^2 + 6*a^6*b^4 - 2*a^4*b^6 + (10*a^8*b^2 - 2*a^6*b^4 - 11*a^4*b^6 + 3
*a^2*b^8)*cos(d*x + c)^2 + ((20*a^4*b^5 - 7*a^2*b^7 + 2*b^9)*cos(d*x + c)^3
 - 2*(20*a^5*b^4 - 7*a^3*b^6 + 2*a*b^8)*cos(d*x + c)*sin(d*x + c) - (20*a^6
*b^3 + 13*a^4*b^5 - 5*a^2*b^7 + 2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + ((a^8*b^2 - 4*a^6*
b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 +
 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b
^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*cos(d*x + c))*log(1/2*cos(d*
x + c) + 1/2) - ((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*cos(d
*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x +
 c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 +
b^10)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - (2*a^9*b - 6*a^7*b^3 + 6
*a^5*b^5 - 2*a^3*b^7 - (6*a^7*b^3 + 5*a^5*b^5 - 13*a^3*b^7 + 2*a*b^9)*cos(d
*x + c)^2)*sin(d*x + c))/((a^11*b^2 - 4*a^9*b^4 + 6*a^7*b^6 - 4*a^5*b^8 + a
^3*b^10)*d*cos(d*x + c)^3 - 2*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7
+ a^4*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^13 - 3*a^11*b^2 + 2*a^9*b^4 + 2
*a^7*b^6 - 3*a^5*b^8 + a^3*b^10)*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25187, size = 555, normalized size = 1.38

$$\frac{\left(20\,a^4b^3-7\,a^2b^5+2\,b^7\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^9-3\,a^7b^2+3\,a^5b^4-a^3b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3-3\,ab^2\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{11\,a^3b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((20*a^4*b^3 - 7*a^2*b^5 + 2*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
+ arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^9 - 3*a^7*b^2 +
 3*a^5*b^4 - a^3*b^6)*sqrt(a^2 - b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c) +
b^3*tan(1/2*d*x + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (11*a^3*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*
a*b^7*tan(1/2*d*x + 1/2*c)^3 + 10*a^4*b^4*tan(1/2*d*x + 1/2*c)^2 + 17*a^2*b
^6*tan(1/2*d*x + 1/2*c)^2 - 6*b^8*tan(1/2*d*x + 1/2*c)^2 + 29*a^3*b^5*tan(1
/2*d*x + 1/2*c) - 8*a*b^7*tan(1/2*d*x + 1/2*c) + 10*a^4*b^4 - 3*a^2*b^6)/((
a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(
1/2*d*x + 1/2*c) + a)^2) + log(abs(tan(1/2*d*x + 1/2*c)))/a^3)/d
```

3.1476 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=424

$$-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{7/2}}-\frac{b^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{7/2}}-\frac{2b^4\left(-9a^2b^2+10a^4+3b^4\right)\tan^{-1}\left(\frac{a\tan(}{}\right.}{a^4d\left(a^2-b^2\right)^{7/2}}$$

```
[Out] (-4*b^4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
2*(a^2 - b^2)^(7/2)*d) - (b^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(7/2)*d) - (2*b^4*(10*a^4 - 9*a^2*b^2 +
 3*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^
(7/2)*d) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^3*d) + Cos
[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*
(1 + Sin[c + d*x])) - (b^5*Cos[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sin[
c + d*x])^2) - (3*b^5*Cos[c + d*x])/(2*a*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x
])) - (2*b^5*(2*a^2 - b^2)*Cos[c + d*x])/(a^3*(a^2 - b^2)^3*d*(a + b*Sin[c
+ d*x]))
```

Rubi [A] time = 0.589752, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2897, 3770, 3767, 8, 2648, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{7/2}}-\frac{b^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{7/2}}-\frac{2b^4\left(-9a^2b^2+10a^4+3b^4\right)\tan^{-1}\left(\frac{a\tan(}{}\right.}{a^4d\left(a^2-b^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*b^4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
2*(a^2 - b^2)^(7/2)*d) - (b^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(7/2)*d) - (2*b^4*(10*a^4 - 9*a^2*b^2 +
 3*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^
(7/2)*d) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^3*d) + Cos
[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*
(1 + Sin[c + d*x])) - (b^5*Cos[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sin[
c + d*x])^2) - (3*b^5*Cos[c + d*x])/(2*a*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x
])) - (2*b^5*(2*a^2 - b^2)*Cos[c + d*x])/(a^3*(a^2 - b^2)^3*d*(a + b*Sin[c
+ d*x]))
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(-\frac{3b\csc(c+dx)}{a^4}+\frac{\csc^2(c+dx)}{a^3}-\frac{1}{2(a+b)^3(-1+\sin(c+dx))}+\frac{1}{2(a-b)^3(1+\sin(\right.$$

$$=\frac{\int\csc^2(c+dx)\,dx}{a^3}+\frac{\int\frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3}-\frac{(3b)\int\csc(c+dx)\,dx}{a^4}-\frac{\int\frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3}-\frac{b^4\int}{}$$

$$=\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))}-$$

$$=\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{\cot(c+dx)}{a^3d}+\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+}{2(a-b)^3d(1+}$$

$$=-\frac{2b^4\left(10a^4-9a^2b^2+3b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{7/2}d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{\cot(c+}{a^3c}$$

$$=-\frac{2b^4\left(10a^4-9a^2b^2+3b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{7/2}d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{\cot(c+}{a^3c}$$

$$=-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}d}-\frac{2b^4\left(10a^4-9a^2b^2+3b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+c}{\sqrt{a^2-b^2}}\right.}{a^4\left(a^2-b^2\right)^{7/2}d}$$

$$=-\frac{4b^4\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}d}-\frac{b^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}d}-\frac{2b^4(}{}$$

Mathematica [A] time = 6.34985, size = 379, normalized size = 0.89

$$4\left(-\frac{b^5\cos(c+dx)}{8a^2d(a-b)^2(a+b)^2(a+b\sin(c+dx))^2}+\frac{4b^7\cos(c+dx)-11a^2b^5\cos(c+dx)}{8a^3d(a-b)^3(a+b)^3(a+b\sin(c+dx))}-\frac{3b^4\left(-7a^2b^2+10a^4+2b^4\right)\tan^{-}}{4a^4c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] 4*((-3*b^4*(10*a^4 - 7*a^2*b^2 + 2*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c
+ d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(4*a^4*(a^2 - b^2)^(7/2)
*d) - Cot[(c + d*x)/2]/(8*a^3*d) + (3*b*Log[Cos[(c + d*x)/2]])/(4*a^4*d) -
(3*b*Log[Sin[(c + d*x)/2]])/(4*a^4*d) + Sin[(c + d*x)/2]/(4*(a + b)^3*d*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(4*(a - b)^3*d*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])) - (b^5*Cos[c + d*x])/(8*a^2*(a - b)^2*(a
+ b)^2*d*(a + b*Sin[c + d*x])^2) + (-11*a^2*b^5*Cos[c + d*x] + 4*b^7*Cos[c
+ d*x])/(8*a^3*(a - b)^3*(a + b)^3*d*(a + b*Sin[c + d*x])) + Tan[(c + d*x)/
2]/(8*a^3*d))
```

Maple [B] time = 0.194, size = 829, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-13/d*b^6/(a
-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2
*d*x+1/2*c)^3+6/d*b^8/(a-b)^3/(a+b)^3/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-12/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-19/d*b^7/(a-
b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a^2*tan(1/
2*d*x+1/2*c)^2+10/d*b^9/(a-b)^3/(a+b)^3/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1
/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-35/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)+14/d*b^8/(
a-b)^3/(a+b)^3/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(
1/2*d*x+1/2*c)-12/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2+5/d*b^7/(a-b)^3/(a+b)^3/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2-30/d*b^4/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+21/d*b^6/(a-b)^3/(a+b)^3/a^2/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-6/d
*b^8/(a-b)^3/(a+b)^3/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^3/tan(1/2
*d*x+1/2*c)-3/d/a^4*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 20.927, size = 4748, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^11 - 12*a^9*b^2 + 12*a^7*b^4 - 4*a^5*b^6 + 2*(4*a^9*b^2 - 4*a^7*
b^4 + 17*a^5*b^6 - 23*a^3*b^8 + 6*a*b^10)*cos(d*x + c)^4 - 2*(4*a^11 - 10*a
^9*b^2 + 14*a^7*b^4 + 7*a^5*b^6 - 21*a^3*b^8 + 6*a*b^10)*cos(d*x + c)^2 - 3
*(2*(10*a^5*b^5 - 7*a^3*b^7 + 2*a*b^9)*cos(d*x + c)^3 - 2*(10*a^5*b^5 - 7*a
^3*b^7 + 2*a*b^9)*cos(d*x + c) + ((10*a^4*b^6 - 7*a^2*b^8 + 2*b^10)*cos(d*x
 + c)^3 - (10*a^6*b^4 + 3*a^4*b^6 - 5*a^2*b^8 + 2*b^10)*cos(d*x + c))*sin(d
*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
```

```
 + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-
a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(2*(
a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^3 - 2*(a
^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c) + ((a^8*b
^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 - (a^10*b - 3
*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7 - 3*a^2*b^9 + b^11)*cos(d*x + c))*sin(d*x
+ c))*log(1/2*cos(d*x + c) + 1/2) + 6*(2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
 4*a^3*b^8 + a*b^10)*cos(d*x + c)^3 - 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
4*a^3*b^8 + a*b^10)*cos(d*x + c) + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^
2*b^9 + b^11)*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7
- 3*a^2*b^9 + b^11)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2
) - 2*(2*a^10*b - 6*a^8*b^3 + 6*a^6*b^5 - 2*a^4*b^7 + (8*a^10*b - 14*a^8*b^
3 + 28*a^6*b^5 - 31*a^4*b^7 + 9*a^2*b^9)*cos(d*x + c)^2)*sin(d*x + c))/(2*(
a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*a^7*b^7 + a^5*b^9)*d*cos(d*x + c)^3 - 2
*(a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*a^7*b^7 + a^5*b^9)*d*cos(d*x + c) + (
(a^12*b^2 - 4*a^10*b^4 + 6*a^8*b^6 - 4*a^6*b^8 + a^4*b^10)*d*cos(d*x + c)^3
 - (a^14 - 3*a^12*b^2 + 2*a^10*b^4 + 2*a^8*b^6 - 3*a^6*b^8 + a^4*b^10)*d*co
s(d*x + c))*sin(d*x + c)), -1/2*(2*a^11 - 6*a^9*b^2 + 6*a^7*b^4 - 2*a^5*b^6
 + (4*a^9*b^2 - 4*a^7*b^4 + 17*a^5*b^6 - 23*a^3*b^8 + 6*a*b^10)*cos(d*x + c
)^4 - (4*a^11 - 10*a^9*b^2 + 14*a^7*b^4 + 7*a^5*b^6 - 21*a^3*b^8 + 6*a*b^10
)*cos(d*x + c)^2 - 3*(2*(10*a^5*b^5 - 7*a^3*b^7 + 2*a*b^9)*cos(d*x + c)^3 -
 2*(10*a^5*b^5 - 7*a^3*b^7 + 2*a*b^9)*cos(d*x + c) + ((10*a^4*b^6 - 7*a^2*b
^8 + 2*b^10)*cos(d*x + c)^3 - (10*a^6*b^4 + 3*a^4*b^6 - 5*a^2*b^8 + 2*b^10)
*cos(d*x + c))*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(
sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*
a^3*b^8 + a*b^10)*cos(d*x + c)^3 - 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10)*cos(d*x + c) + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b
^9 + b^11)*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7 - 3
*a^2*b^9 + b^11)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
3*(2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^3
- 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c) + (
(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 - (a^10
*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7 - 3*a^2*b^9 + b^11)*cos(d*x + c))*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - (2*a^10*b - 6*a^8*b^3 + 6*a^6*b^
5 - 2*a^4*b^7 + (8*a^10*b - 14*a^8*b^3 + 28*a^6*b^5 - 31*a^4*b^7 + 9*a^2*b^
9)*cos(d*x + c)^2)*sin(d*x + c))/(2*(a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*a^
7*b^7 + a^5*b^9)*d*cos(d*x + c)^3 - 2*(a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*
a^7*b^7 + a^5*b^9)*d*cos(d*x + c) + ((a^12*b^2 - 4*a^10*b^4 + 6*a^8*b^6 - 4
*a^6*b^8 + a^4*b^10)*d*cos(d*x + c)^3 - (a^14 - 3*a^12*b^2 + 2*a^10*b^4 + 2
*a^8*b^6 - 3*a^6*b^8 + a^4*b^10)*d*cos(d*x + c))*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27455, size = 855, normalized size = 2.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*(10*a^4*b^4 - 7*a^2*b^6 + 2*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^10 - 3*a
^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) - (2*a^6*b*tan(1/2*d*x + 1/2
*c)^3 - 6*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^5*tan(1/2*d*x + 1/2*c)^3
 - 2*b^7*tan(1/2*d*x + 1/2*c)^3 - 5*a^7*tan(1/2*d*x + 1/2*c)^2 - 9*a^5*b^2*
tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 + a*b^6*tan(1/2*d
*x + 1/2*c)^2 + 10*a^6*b*tan(1/2*d*x + 1/2*c) + 10*a^4*b^3*tan(1/2*d*x + 1/
2*c) - 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 2*b^7*tan(1/2*d*x + 1/2*c) + a^7 -
3*a^5*b^2 + 3*a^3*b^4 - a*b^6)/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(t
an(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))) + 2*(13*a^3*b^6*tan(1/2*d*x
+ 1/2*c)^3 - 6*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*tan(1/2*d*x + 1/2*
c)^2 + 19*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 10*b^9*tan(1/2*d*x + 1/2*c)^2 +
35*a^3*b^6*tan(1/2*d*x + 1/2*c) - 14*a*b^8*tan(1/2*d*x + 1/2*c) + 12*a^4*b^
5 - 5*a^2*b^7)/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1
/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/2*d*x + 1/2*
c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3)/d
```

3.1477 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=470

$$\frac{b^5\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{7/2}}+\frac{2b^5\left(-17a^2b^2+15a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\left(a^2-b^2\right)^{7/2}}+\frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan}{}\right.}{a^3d\left(a^2-b^2\right)^{7/2}}$$

```
[Out] (2*b^5*(5*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^3*(a^2 - b^2)^(7/2)*d) + (b^5*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d) + (2*b^5*(15*a^4 - 17*a^2*b^2
 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2
)^(7/2)*d) - ArcTanh[Cos[c + d*x]]/(2*a^3*d) - ((a^2 + 6*b^2)*ArcTanh[Cos[c
 + d*x]])/(a^5*d) + (3*b*Cot[c + d*x])/(a^4*d) - (Cot[c + d*x]*Csc[c + d*x]
)/(2*a^3*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) + Cos[c + d*x
]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) + (b^6*Cos[c + d*x])/(2*a^3*(a^2 - b^2
)^2*d*(a + b*Sin[c + d*x])^2) + (3*b^6*Cos[c + d*x])/(2*a^2*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])) + (b^6*(5*a^2 - 3*b^2)*Cos[c + d*x])/(a^4*(a^2 - b^2
)^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.608277, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2897, 3770, 3767, 8, 3768, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{b^5\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{7/2}}+\frac{2b^5\left(-17a^2b^2+15a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\left(a^2-b^2\right)^{7/2}}+\frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan}{}\right.}{a^3d\left(a^2-b^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (2*b^5*(5*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^3*(a^2 - b^2)^(7/2)*d) + (b^5*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d) + (2*b^5*(15*a^4 - 17*a^2*b^2
 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2
)^(7/2)*d) - ArcTanh[Cos[c + d*x]]/(2*a^3*d) - ((a^2 + 6*b^2)*ArcTanh[Cos[c
 + d*x]])/(a^5*d) + (3*b*Cot[c + d*x])/(a^4*d) - (Cot[c + d*x]*Csc[c + d*x]
)/(2*a^3*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) + Cos[c + d*x
]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) + (b^6*Cos[c + d*x])/(2*a^3*(a^2 - b^2
)^2*d*(a + b*Sin[c + d*x])^2) + (3*b^6*Cos[c + d*x])/(2*a^2*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])) + (b^6*(5*a^2 - 3*b^2)*Cos[c + d*x])/(a^4*(a^2 - b^2
)^3*d*(a + b*Sin[c + d*x]))
```


Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(\frac{\left(a^2+6b^2\right)\csc(c+dx)}{a^5} - \frac{3b\csc^2(c+dx)}{a^4} + \frac{\csc^3(c+dx)}{a^3} - \frac{1}{2(a+b)^3(-1+\sin(c+\right.$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^3} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{(3b)\int \csc^2(c+dx)\,dx}{a^4} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} + \frac{(b^5}{}$$

$$= -\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+d}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{3b\cot(c+dx)}{a^4d} - \frac{\cot(c+d}{}$$

$$= \frac{2b^5\left(15a^4-17a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{\left(a^2+6b^2\right)}{}$$

$$= \frac{2b^5\left(15a^4-17a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{\left(a^2+6b^2\right)}{}$$

$$= \frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} + \frac{2b^5\left(15a^4-17a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a^2-b^2}}\right.}{a^5\left(a^2-b^2\right)^{7/2}d}$$

$$= \frac{2b^5\left(5a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} + \frac{b^5\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} + \frac{2b^5(}{}$$

Mathematica [A] time = 6.77968, size = 432, normalized size = 0.92

$$\frac{3\left(a^2+4b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} - \frac{3\left(a^2+4b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{b^6\cos(c+dx)}{2a^3d(a-b)^2(a+b)^2(a+b\sin(c+dx))^2} + \frac{13a}{2a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b^5*(14*a^4 - 13*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d
*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2)^(7/2)*d) +
 (3*b*Cot[(c + d*x)/2])/(2*a^4*d) - Csc[(c + d*x)/2]^2/(8*a^3*d) - (3*(a^2
```

```
+ 4*b^2)*Log[Cos[(c + d*x)/2]])/(2*a^5*d) + (3*(a^2 + 4*b^2)*Log[Sin[(c + d
*x)/2]])/(2*a^5*d) + Sec[(c + d*x)/2]^2/(8*a^3*d) + Sin[(c + d*x)/2]/((a +
b)^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - Sin[(c + d*x)/2]/((a - b)^3
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^6*Cos[c + d*x])/(2*a^3*(a -
b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])^2) + (13*a^2*b^6*Cos[c + d*x] - 6*b^8
*Cos[c + d*x])/(2*a^4*(a - b)^3*(a + b)^3*d*(a + b*Sin[c + d*x])) - (3*b*Ta
n[(c + d*x)/2])/(2*a^4*d)
```

Maple [B] time = 0.198, size = 897, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d/a^3*tan(1/2*d*x+1/2*c)^2-3/2/d/a^4*tan(1/2*d*x+1/2*c)*b-1/d/(a+b)^3/(
tan(1/2*d*x+1/2*c)-1)+15/d*b^7/(a-b)^3/(a+b)^3/a^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3-8/d*b^9/(a-b)^3/(a+b)^3/a^
4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3+
14/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^
2/a*tan(1/2*d*x+1/2*c)^2+21/d*b^8/(a-b)^3/(a+b)^3/a^3/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-14/d*b^10/(a-b)^3/(a+b)
^3/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*
c)^2+41/d*b^7/(a-b)^3/(a+b)^3/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)-20/d*b^9/(a-b)^3/(a+b)^3/a^4/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)+14/d*b^6/(a-b)^3/(a+
b)^3/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2-7/d*b^8/(a-b)^3/
(a+b)^3/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+42/d*b^5/(a
-b)^3/(a+b)^3/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))-39/d*b^7/(a-b)^3/(a+b)^3/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+12/d*b^9/(a-b)^3/(a+b)^3/a^5/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/d/(a-
b)^3/(tan(1/2*d*x+1/2*c)+1)-1/8/d/a^3/tan(1/2*d*x+1/2*c)^2+3/2/d/a^3*ln(tan
(1/2*d*x+1/2*c))+6/d/a^5*ln(tan(1/2*d*x+1/2*c))*b^2+3/2/d*b/a^4/tan(1/2*d*x
+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 34.9756, size = 6075, normalized size = 12.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")

[Out] [1/4*(4*a^12 - 12*a^10*b^2 + 12*a^8*b^4 - 4*a^6*b^6 - 2*(21*a^10*b^2 - 56*a
^8*b^4 + 82*a^6*b^6 - 65*a^4*b^8 + 18*a^2*b^10)*cos(d*x + c)^4 - 2*(3*a^12
- 31*a^10*b^2 + 68*a^8*b^4 - 88*a^6*b^6 + 66*a^4*b^8 - 18*a^2*b^10)*cos(d*x
 + c)^2 + 3*((14*a^4*b^7 - 13*a^2*b^9 + 4*b^11)*cos(d*x + c)^5 - (14*a^6*b^
5 + 15*a^4*b^7 - 22*a^2*b^9 + 8*b^11)*cos(d*x + c)^3 + (14*a^6*b^5 + a^4*b^
7 - 9*a^2*b^9 + 4*b^11)*cos(d*x + c) - 2*((14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^
10)*cos(d*x + c)^3 - (14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^10)*cos(d*x + c))*sin
(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 3*(
(a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*cos(d*x + c)^5
- (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 + 8*b^12)*cos(
d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 - 11*a^
2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a^5*b^7 - 15*a
^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*a^5*b^7 - 15*
a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + 3*((a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*cos(d*x +
 c)^5 - (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 + 8*b^12
)*cos(d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 -
 11*a^2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a^5*b^7
- 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*a^5*b^7
 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2) - 2*(2*a^11*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 + (12*a^9*b^3 - 2
8*a^7*b^5 + 47*a^5*b^7 - 43*a^3*b^9 + 12*a*b^11)*cos(d*x + c)^4 - (6*a^11*b
 - 10*a^9*b^3 + 2*a^7*b^5 + 29*a^5*b^7 - 39*a^3*b^9 + 12*a*b^11)*cos(d*x +
c)^2)*sin(d*x + c))/((a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b
^10)*d*cos(d*x + c)^5 - (a^15 - 2*a^13*b^2 - 2*a^11*b^4 + 8*a^9*b^6 - 7*a^7
*b^8 + 2*a^5*b^10)*d*cos(d*x + c)^3 + (a^15 - 3*a^13*b^2 + 2*a^11*b^4 + 2*a
^9*b^6 - 3*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) - 2*((a^14*b - 4*a^12*b^3 + 6
*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^3 - (a^14*b - 4*a^12*b^3 +
6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c))*sin(d*x + c)), 1/4*(4*a^1
2 - 12*a^10*b^2 + 12*a^8*b^4 - 4*a^6*b^6 - 2*(21*a^10*b^2 - 56*a^8*b^4 + 82
*a^6*b^6 - 65*a^4*b^8 + 18*a^2*b^10)*cos(d*x + c)^4 - 2*(3*a^12 - 31*a^10*b
^2 + 68*a^8*b^4 - 88*a^6*b^6 + 66*a^4*b^8 - 18*a^2*b^10)*cos(d*x + c)^2 - 6
*((14*a^4*b^7 - 13*a^2*b^9 + 4*b^11)*cos(d*x + c)^5 - (14*a^6*b^5 + 15*a^4*
b^7 - 22*a^2*b^9 + 8*b^11)*cos(d*x + c)^3 + (14*a^6*b^5 + a^4*b^7 - 9*a^2*b
^9 + 4*b^11)*cos(d*x + c) - 2*((14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^10)*cos(d*x
 + c)^3 - (14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^10)*cos(d*x + c))*sin(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))
) - 3*((a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*cos(d*x
+ c)^5 - (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 + 8*b^1
2)*cos(d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8
- 11*a^2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a^5*b^7
 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*a^5*b^
7 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c)
 + 1/2) + 3*((a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*co
s(d*x + c)^5 - (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 +
 8*b^12)*cos(d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^
4*b^8 - 11*a^2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a
^5*b^7 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*
a^5*b^7 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d
x + c) + 1/2) - 2(2*a^11*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 + (12*a^9*
b^3 - 28*a^7*b^5 + 47*a^5*b^7 - 43*a^3*b^9 + 12*a*b^11)*cos(d*x + c)^4 - (6
*a^11*b - 10*a^9*b^3 + 2*a^7*b^5 + 29*a^5*b^7 - 39*a^3*b^9 + 12*a*b^11)*cos
(d*x + c)^2)*sin(d*x + c))/((a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8
+ a^5*b^10)*d*cos(d*x + c)^5 - (a^15 - 2*a^13*b^2 - 2*a^11*b^4 + 8*a^9*b^6

```
- 7*a^7*b^8 + 2*a^5*b^10)*d*cos(d*x + c)^3 + (a^15 - 3*a^13*b^2 + 2*a^11*b^
4 + 2*a^9*b^6 - 3*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) - 2*((a^14*b - 4*a^12*
b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^3 - (a^14*b - 4*a^12
*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c))*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)

[Out] Timed out
```

Giac [B] time = 1.33171, size = 1215, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/8*(24*(14*a^4*b^5 - 13*a^2*b^7 + 4*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^11 - 3*
a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(a^2 - b^2)) + 16*(3*a^2*b*tan(1/2*d*x +
 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)) - (6*a^10*tan(1/2*d*x + 1/2*c)^
6 + 6*a^8*b^2*tan(1/2*d*x + 1/2*c)^6 - 54*a^6*b^4*tan(1/2*d*x + 1/2*c)^6 +
66*a^4*b^6*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^8*tan(1/2*d*x + 1/2*c)^6 + 12*
a^9*b*tan(1/2*d*x + 1/2*c)^5 + 60*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 252*a^5*
b^5*tan(1/2*d*x + 1/2*c)^5 + 156*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 32*a*b^9*
tan(1/2*d*x + 1/2*c)^5 + 13*a^10*tan(1/2*d*x + 1/2*c)^4 - 15*a^8*b^2*tan(1/
2*d*x + 1/2*c)^4 + 63*a^6*b^4*tan(1/2*d*x + 1/2*c)^4 - 341*a^4*b^6*tan(1/2*
d*x + 1/2*c)^4 + 96*a^2*b^8*tan(1/2*d*x + 1/2*c)^4 + 16*b^10*tan(1/2*d*x +
1/2*c)^4 + 4*a^9*b*tan(1/2*d*x + 1/2*c)^3 + 36*a^7*b^3*tan(1/2*d*x + 1/2*c)
^3 - 132*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 - 188*a^3*b^7*tan(1/2*d*x + 1/2*c)^
3 + 112*a*b^9*tan(1/2*d*x + 1/2*c)^3 + 8*a^10*tan(1/2*d*x + 1/2*c)^2 - 44*a
^8*b^2*tan(1/2*d*x + 1/2*c)^2 + 84*a^6*b^4*tan(1/2*d*x + 1/2*c)^2 - 180*a^4
*b^6*tan(1/2*d*x + 1/2*c)^2 + 76*a^2*b^8*tan(1/2*d*x + 1/2*c)^2 - 8*a^9*b*t
an(1/2*d*x + 1/2*c) + 24*a^7*b^3*tan(1/2*d*x + 1/2*c) - 24*a^5*b^5*tan(1/2*
d*x + 1/2*c) + 8*a^3*b^7*tan(1/2*d*x + 1/2*c) + a^10 - 3*a^8*b^2 + 3*a^6*b^
4 - a^4*b^6)/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2
*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2) + 12*(a^2 +
 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + (a^3*tan(1/2*d*x + 1/2*c)^2 -
12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.1478 $\int \frac{\sec^2(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=158

$$\frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{df}-\frac{\sqrt{a+b}\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx}}{\sqrt{a+b}\sqrt{d\sin(e+fx}}\right.\right.}{\sqrt{d}f}$$

```
[Out] (Sec[e + f*x]*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(d*f) - (Sqrt[
a + b]*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a
- b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt
[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(Sqrt[d]*f)
```

Rubi [A] time = 0.265491, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2888, 2816}

$$\frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{df}-\frac{\sqrt{a+b}\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx}}{\sqrt{a+b}\sqrt{d\sin(e+fx}}\right.\right.}{\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]])/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (Sec[e + f*x]*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(d*f) - (Sqrt[
a + b]*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a
- b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt
[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(Sqrt[d]*f)
```

Rule 2888

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(g*Co
s[e + f*x])^(p + 1)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^m)/(d*f*g*(2*
m + 1)), x] + Dist[(2*a*m)/(g^2*(2*m + 1)), Int[((g*Cos[e + f*x])^(p + 2)*(
a + b*Sin[e + f*x])^(m - 1))/Sqrt[d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, e
, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && EqQ[m + p + 3/2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx = \frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{df} + \frac{1}{2}a\int \frac{1}{\sqrt{d\sin(e+fx)}\sqrt{a+b}$$

$$= \frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{df} - \frac{\sqrt{a+b}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+c}{}}}{}$$

Mathematica [A] time = 6.18182, size = 198, normalized size = 1.25

$$\frac{4a^2\sin^4\left(\frac{1}{4}(2e+2fx-\pi)\right)\sec(e+fx)\sqrt{-\frac{(a+b)\sin(e+fx)(a+b\sin(e+fx))}{a^2(\sin(e+fx)-1)^2}}\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{a-b}}F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\sin(e+fx)}{a(\sin(e+fx)-1)}}\right.\right.}{f(a+b)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]])/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (4*a^2*Sqrt[-(((a + b)*Cot[(2*e - Pi + 2*f*x)/4]^2)/(a - b))]*EllipticF[Arc
Sin[Sqrt[-((a + b*Sin[e + f*x])/(a*(-1 + Sin[e + f*x])))]], (2*a)/(a - b)]*
Sec[e + f*x]*Sqrt[-(((a + b)*Sin[e + f*x]*(a + b*Sin[e + f*x]))/(a^2*(-1 +
Sin[e + f*x])^2))]*Sin[(2*e - Pi + 2*f*x)/4]^4 + (a + b)*(a + b*Sin[e + f*x
])*Tan[e + f*x])/((a + b)*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])
```

Maple [B] time = 0.481, size = 650, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)*((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/
(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(
f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*
a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-a^2+b^2)^(1/2)*sin(f
*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1
/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*sin(f*x+e)*cos(f*
x+e)*(-a^2+b^2)^(1/2)+(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)
*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-
b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-a^2+b^2)^(1/2)
*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(
1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*sin(f*x+e)*
cos(f*x+e)*b-sin(f*x+e)*cos(f*x+e)*2^(1/2)*b+sin(f*x+e)*2^(1/2)*b-cos(f*x+e
)*2^(1/2)*a+2^(1/2)*a)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)/(d*sin(f*x+e))
^(1/2)/(a+b*sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}\sec\left(fx+e\right)^2}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)}\sec\left(fx+e\right)^{2}}{d\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*sec(f*x + e)^2/(d*si
n(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e))**(1/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b\sin\left(fx+e\right)+a}\sec\left(fx+e\right)^{2}}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x)
```

3.1479 $\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=312

$$\frac{\sec(e+fx)(a\sin(e+fx)+b)\sqrt{a+b\sin(e+fx)}}{f\sqrt{d\sin(e+fx)}}-\frac{(a+b)^{3/2}\tan(e+fx)\sqrt{-\frac{a(\csc(e+fx)-1)}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{}}{\sqrt{a+}}\right.\right.}{\sqrt{d}f}$$

```
[Out] (Sec[e + f*x]*(b + a*Sin[e + f*x])*Sqrt[a + b*Sin[e + f*x]])/(f*Sqrt[d*Sin[
e + f*x]]) - ((a + b)^(3/2)*Sqrt[-((a*(-1 + Csc[e + f*x]))/(a + b))]*Sqrt[(
a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e +
 f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*
x])/(Sqrt[d]*f) - (b*(a + b)*Sqrt[-((a*(-1 + Csc[e + f*x]))/(a + b))]*Sqrt[
(b + a*Csc[e + f*x])/(-a + b)]*EllipticE[ArcSin[Sqrt[-((b + a*Csc[e + f*x])
/(a - b))]], (-a + b)/(a + b)]*(1 + Sin[e + f*x])*Tan[e + f*x])/(f*Sqrt[(a*
(1 + Csc[e + f*x]))/(a - b)]*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])
```

Rubi [F] time = 0.18996, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] Defer[Int][(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e + f*x]]
, x]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx=\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Mathematica [B] time = 26.654, size = 6059, normalized size = 19.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e + f*x]],
x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.381, size = 2313, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)/a*(2*(-a^2+b^2)^(1/2)*cos(f*x+e)^2*(((-a^2+b^2)^(1/2)*sin(f*
x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*((
(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*
EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a
^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^
2)^(1/2))^(1/2))*b^2-(-a^2+b^2)^(1/2)*cos(f*x+e)^2*(((-a^2+b^2)^(1/2)*sin(f
*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(
((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)
*EllipticF((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-
a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b
^2)^(1/2))^(1/2))*a^2-2*cos(f*x+e)^2*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*
x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1
/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE(((
-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2)
)/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/
2))*a^2*b+2*cos(f*x+e)^2*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x
+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+
e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1
/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e)
)^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^3+2*(-
a^2+b^2)^(1/2)*cos(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*
x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x
+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(
1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e
))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2-(-a
^2+b^2)^(1/2)*cos(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x
+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+
e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-a^2+b^2)^(1
/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e)
)^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2-2*co
s(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^
2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+
cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-
a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b
*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1
/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*b+2*cos(f*x+e)*(((-a
^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/
sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a
)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2)
)/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-co
s(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+
b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^3+cos(f*x+e)^2*2^(1/2)*a^2*b+cos(f*x
+e)*sin(f*x+e)*2^(1/2)*a*b^2+cos(f*x+e)*2^(1/2)*a^2*b-sin(f*x+e)*2^(1/2)*a^
3-sin(f*x+e)*2^(1/2)*a*b^2-2*2^(1/2)*a^2*b)/cos(f*x+e)/(d*sin(f*x+e))^(1/2)
/(a+b*sin(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{2}}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\sec\left(fx+e\right)^{2}\sin\left(fx+e\right)+a\sec\left(fx+e\right)^{2}\right)\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)}}{d\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2*sin(f*x + e) + a*sec(f*x + e)^2)*sqrt(b*sin(f*x
+ e) + a)*sqrt(d*sin(f*x + e))/(d*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e))**(3/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{2}}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x
)
```

3.1480 $\int \frac{\sec^4(e+fx)(a+b\sin(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=366

$$\frac{\sec^3(e+fx)\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{5/2}}{3df}+\frac{5a\sec(e+fx)(a\sin(e+fx)+b)\sqrt{a+b\sin(e+fx)}}{6f\sqrt{d\sin(e+fx)}}-\frac{5a(a+b)^{3/2}\text{ta}}{}$$

```
[Out] (5*a*Sec[e + f*x]*(b + a*Sin[e + f*x])*Sqrt[a + b*Sin[e + f*x]])/(6*f*Sqrt[
d*Sin[e + f*x]]) + (Sec[e + f*x]^3*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x]
)^(5/2))/(3*d*f) - (5*a*(a + b)^(3/2)*Sqrt[-((a*(-1 + Csc[e + f*x]))/(a + b
))]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a +
 b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*
Tan[e + f*x])/(6*Sqrt[d]*f) - (5*a*b*(a + b)*Sqrt[-((a*(-1 + Csc[e + f*x]))
/(a + b))]*Sqrt[(b + a*Csc[e + f*x])/(-a + b)]*EllipticE[ArcSin[Sqrt[-((b +
 a*Csc[e + f*x])/(a - b))]], (-a + b)/(a + b)]*(1 + Sin[e + f*x])*Tan[e + f
*x])/(6*f*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*Sqrt[d*Sin[e + f*x]]*Sqrt[a
+ b*Sin[e + f*x]])
```

Rubi [F] time = 0.391059, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^4(e+fx)(a+b\sin(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[e + f*x]^4*(a + b*Sin[e + f*x])^(5/2))/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (Sec[e + f*x]^3*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(5/2))/(3*d*f) +
(5*a*Defer[Int][(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e +
f*x]], x])/6
```

Rubi steps

$$\int \frac{\sec^4(e+fx)(a+b\sin(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}}\,dx=\frac{\sec^3(e+fx)\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{5/2}}{3df}+\frac{1}{6}(5a)\int \frac{\sec^2(e+fx)(a+}{\sqrt{d\sin(}}$$

Mathematica [B] time = 26.7563, size = 6142, normalized size = 16.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]^4*(a + b*Sin[e + f*x])^(5/2))/Sqrt[d*Sin[e + f*x]],
x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.457, size = 2426, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*2^(1/2)*(10*(-a^2+b^2)^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*
x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1
/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((
-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2)
)/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/
2))*cos(f*x+e)^4*b^2-5*(-a^2+b^2)^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin
(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)
^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF(
(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1
/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^
(1/2))*cos(f*x+e)^4*a^2-10*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f
*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*
x+e)-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^
(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+
e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*
x+e)^4*a^2*b+10*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/
(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(
f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*
a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f
*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1
/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)^4*b^3+
10*(-a^2+b^2)^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a
+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*
sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*s
in(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/
2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)^3*
b^2-5*(-a^2+b^2)^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e
)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)
-b*sin(f*x+e)+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))*a/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-a^2+b^2)^(1/2
)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^
(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)
^3*a^2-10*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a
^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)
+cos(f*x+e)*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(
-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+
b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(
1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)^3*a^2*b+10*(
((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/
2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)+cos(f*x+e)
*a-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))*a/(b+(-a^2+b^2)^(
1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e
)-cos(f*x+e)*a+a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-
a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*cos(f*x+e)^3*b^3+5*2^(1/2)*sin(f*x
+e)*cos(f*x+e)^3*a*b^2+5*2^(1/2)*cos(f*x+e)^4*a^2*b-2*2^(1/2)*cos(f*x+e)^4*
b^3-5*2^(1/2)*sin(f*x+e)*cos(f*x+e)^2*a^3+2^(1/2)*sin(f*x+e)*cos(f*x+e)^2*a
*b^2+5*2^(1/2)*cos(f*x+e)^3*a^2*b-4*cos(f*x+e)^2*2^(1/2)*a^2*b+4*2^(1/2)*co
```

```
s(f*x+e)^2*b^3-2*sin(f*x+e)*2^(1/2)*a^3-6*sin(f*x+e)*2^(1/2)*a*b^2-6*2^(1/2
)*a^2*b-2*2^(1/2)*b^3)/cos(f*x+e)^3/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sec\left(fx+e\right)^{4}}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sec(f*x + e)^4/sqrt(d*sin(f*x + e)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(2\,ab\sec\left(fx+e\right)^{4}\sin\left(fx+e\right)-\left(b^{2}\cos\left(fx+e\right)^{2}-a^{2}-b^{2}\right)\sec\left(fx+e\right)^{4}\right)\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)}}{d\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((2*a*b*sec(f*x + e)^4*sin(f*x + e) - (b^2*cos(f*x + e)^2 - a^2 - b
^2)*sec(f*x + e)^4)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))/(d*sin(f*
x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sec\left(fx+e\right)^{4}}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sec(f*x + e)^4/sqrt(d*sin(f*x + e)), x
)
```

3.1481 $\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=155

$$\frac{a\cos^2(c+dx)}{2d}+\frac{a\sec^4(c+dx)}{4d}-\frac{3a\sec^2(c+dx)}{2d}-\frac{3a\log(\cos(c+dx))}{d}-\frac{35b\sin^3(c+dx)}{24d}-\frac{35b\sin(c+dx)}{8d}+\frac{b\sin^3(c$$

```
[Out] (35*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Cos[c + d*x]^2)/(2*d) - (3*a*Log[Co
s[c + d*x]])/d - (3*a*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x]^4)/(4*d) - (3
5*b*Sin[c + d*x])/(8*d) - (35*b*Sin[c + d*x]^3)/(24*d) - (7*b*Sin[c + d*x]^
3*Tan[c + d*x]^2)/(8*d) + (b*Sin[c + d*x]^3*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.168642, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2834, 2590, 266, 43, 2592, 288, 302, 206}

$$\frac{a\cos^2(c+dx)}{2d}+\frac{a\sec^4(c+dx)}{4d}-\frac{3a\sec^2(c+dx)}{2d}-\frac{3a\log(\cos(c+dx))}{d}-\frac{35b\sin^3(c+dx)}{24d}-\frac{35b\sin(c+dx)}{8d}+\frac{b\sin^3(c$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (35*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Cos[c + d*x]^2)/(2*d) - (3*a*Log[Co
s[c + d*x]])/d - (3*a*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x]^4)/(4*d) - (3
5*b*Sin[c + d*x])/(8*d) - (35*b*Sin[c + d*x]^3)/(24*d) - (7*b*Sin[c + d*x]^
3*Tan[c + d*x]^2)/(8*d) + (b*Sin[c + d*x]^3*Tan[c + d*x]^4)/(4*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx = a\int \sin^2(c+dx)\tan^5(c+dx)\,dx + b\int \sin^3(c+dx)\tan^5(c+dx)$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^5}\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{x^8}{\left(1-x^2\right)^3}\,dx, x\right.}{d}$$

$$= \frac{b\sin^3(c+dx)\tan^4(c+dx)}{4d} - \frac{a\,\text{Subst}\left(\int \frac{(1-x)^3}{x^3}\,dx, x, \cos^2(c+dx\right.}{2d}$$

$$= -\frac{7b\sin^3(c+dx)\tan^2(c+dx)}{8d} + \frac{b\sin^3(c+dx)\tan^4(c+dx)}{4d} - -$$

$$= \frac{a\cos^2(c+dx)}{2d} - \frac{3a\log(\cos(c+dx))}{d} - \frac{3a\sec^2(c+dx)}{2d} + \frac{a\sec^4(}{4}$$

$$= \frac{a\cos^2(c+dx)}{2d} - \frac{3a\log(\cos(c+dx))}{d} - \frac{3a\sec^2(c+dx)}{2d} + \frac{a\sec^4(}{4}$$

$$= \frac{35b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\cos^2(c+dx)}{2d} - \frac{3a\log(\cos(c+dx))}{d}$$

Mathematica [A] time = 0.588935, size = 156, normalized size = 1.01

$$-\frac{a\left(2\sin^2(c+dx) - \sec^4(c+dx) + 6\sec^2(c+dx) + 12\log(\cos(c+dx))\right)}{4d} - \frac{b\sin^3(c+dx)\tan^4(c+dx)}{3d} - \frac{7b\left(8\sin(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -(a*(12*Log[Cos[c + d*x]] + 6*Sec[c + d*x]^2 - Sec[c + d*x]^4 + 2*Sin[c + d
*x]^2))/(4*d) - (b*Sin[c + d*x]^3*Tan[c + d*x]^4)/(3*d) - (7*b*(8*Sin[c + d
*x]*Tan[c + d*x]^4 + 5*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*Sec[c + d*x]*Tan[
c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))))/(24*d
)
```

Maple [A] time = 0.05, size = 219, normalized size = 1.4

$$\frac{a(\sin(dx+c))^8}{4d(\cos(dx+c))^4} - \frac{a(\sin(dx+c))^8}{2d(\cos(dx+c))^2} - \frac{a(\sin(dx+c))^6}{2d} - \frac{3a(\sin(dx+c))^4}{4d} - \frac{3(\sin(dx+c))^2a}{2d} - 3\frac{a\ln(\cos(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a*sin(d*x+c)^8/cos(d*x+c)^2-1/2*a*s
in(d*x+c)^6/d-3/4*a*sin(d*x+c)^4/d-3/2*a*sin(d*x+c)^2/d-3*a*ln(cos(d*x+c))/
d+1/4/d*b*sin(d*x+c)^9/cos(d*x+c)^4-5/8/d*b*sin(d*x+c)^9/cos(d*x+c)^2-5/8*b
*sin(d*x+c)^7/d-7/8*b*sin(d*x+c)^5/d-35/24*b*sin(d*x+c)^3/d-35/8*b*sin(d*x+
c)/d+35/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.981233, size = 178, normalized size = 1.15

$$-\frac{16b\sin(dx+c)^3 + 24a\sin(dx+c)^2 + 3(24a-35b)\log(\sin(dx+c)+1) + 3(24a+35b)\log(\sin(dx+c)-1) + 144}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/48*(16*b*sin(d*x + c)^3 + 24*a*sin(d*x + c)^2 + 3*(24*a - 35*b)*log(sin(
d*x + c) + 1) + 3*(24*a + 35*b)*log(sin(d*x + c) - 1) + 144*b*sin(d*x + c)
- 6*(13*b*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 11*b*sin(d*x + c) - 10*a)/
(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.10069, size = 401, normalized size = 2.59

$$\frac{24a\cos(dx+c)^6 - 3(24a-35b)\cos(dx+c)^4\log(\sin(dx+c)+1) - 3(24a+35b)\cos(dx+c)^4\log(-\sin(dx+c)+1}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(24*a*cos(d*x + c)^6 - 3*(24*a - 35*b)*cos(d*x + c)^4*log(sin(d*x + c)
 + 1) - 3*(24*a + 35*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 12*a*cos(d*
x + c)^4 - 72*a*cos(d*x + c)^2 + 2*(8*b*cos(d*x + c)^6 - 80*b*cos(d*x + c)^
4 - 39*b*cos(d*x + c)^2 + 6*b)*sin(d*x + c) + 12*a)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**7*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26742, size = 182, normalized size = 1.17

$$-\frac{16\,b\sin\left(dx+c\right)^{3}+24\,a\sin\left(dx+c\right)^{2}+3\,(24\,a-35\,b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,(24\,a+35\,b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/48*(16*b*sin(d*x + c)^3 + 24*a*sin(d*x + c)^2 + 3*(24*a - 35*b)*log(abs(
sin(d*x + c) + 1)) + 3*(24*a + 35*b)*log(abs(sin(d*x + c) - 1)) + 144*b*sin
(d*x + c) - 6*(18*a*sin(d*x + c)^4 + 13*b*sin(d*x + c)^3 - 24*a*sin(d*x + c
)^2 - 11*b*sin(d*x + c) + 8*a)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1482 $\int \sin(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=135

$$-\frac{15a\sin(c+dx)}{8d}+\frac{a\sin(c+dx)\tan^4(c+dx)}{4d}-\frac{5a\sin(c+dx)\tan^2(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\cos^2(c+dx)}{2d}$$

```
[Out] (15*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Cos[c + d*x]^2)/(2*d) - (3*b*Log[Co
s[c + d*x]])/d - (3*b*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^4)/(4*d) - (1
5*a*Sin[c + d*x])/(8*d) - (5*a*Sin[c + d*x]*Tan[c + d*x]^2)/(8*d) + (a*Sin[
c + d*x]*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.133346, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2834, 2592, 288, 321, 206, 2590, 266, 43}

$$-\frac{15a\sin(c+dx)}{8d}+\frac{a\sin(c+dx)\tan^4(c+dx)}{4d}-\frac{5a\sin(c+dx)\tan^2(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```


```
[Out] (15*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Cos[c + d*x]^2)/(2*d) - (3*b*Log[Co
s[c + d*x]])/d - (3*b*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^4)/(4*d) - (1
5*a*Sin[c + d*x])/(8*d) - (5*a*Sin[c + d*x]*Tan[c + d*x]^2)/(8*d) + (a*Sin[
c + d*x]*Tan[c + d*x]^4)/(4*d)
```


Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx &= a\int \sin(c+dx)\tan^5(c+dx)\,dx + b\int \sin^2(c+dx)\tan^5(c+dx)\,d\\ &= \frac{a\,\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^5}\,dx, x, \text{c}\right.}{d}\\ &= \frac{a\sin(c+dx)\tan^4(c+dx)}{4d} - \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \sin(c+d\right.}{4d}\\ &= -\frac{5a\sin(c+dx)\tan^2(c+dx)}{8d} + \frac{a\sin(c+dx)\tan^4(c+dx)}{4d} + \frac{(15a)}{}\\ &= \frac{b\cos^2(c+dx)}{2d} - \frac{3b\log(\cos(c+dx))}{d} - \frac{3b\sec^2(c+dx)}{2d} + \frac{b\sec^4(c}{4d}\\ &= \frac{15a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\cos^2(c+dx)}{2d} - \frac{3b\log(\cos(c+dx))}{d} - \end{aligned}$$

Mathematica [A] time = 0.352517, size = 133, normalized size = 0.99

$$-\frac{a\sin(c+dx)\tan^4(c+dx)}{d} - \frac{5a\left(6\tan(c+dx)\sec^3(c+dx) - 8\tan^3(c+dx)\sec(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx)) + \right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -(b*(12*Log[Cos[c + d*x]] + 6*Sec[c + d*x]^2 - Sec[c + d*x]^4 + 2*Sin[c + d
*x]^2))/(4*d) - (a*Sin[c + d*x]*Tan[c + d*x]^4)/d - (5*a*(6*Sec[c + d*x]^3*
Tan[c + d*x] - 8*Sec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x])))/(8*d)
```

Maple [A] time = 0.05, size = 205, normalized size = 1.5

$$\frac{a(\sin(dx+c))^7}{4d(\cos(dx+c))^4}-\frac{3a(\sin(dx+c))^7}{8d(\cos(dx+c))^2}-\frac{3a(\sin(dx+c))^5}{8d}-\frac{5a(\sin(dx+c))^3}{8d}-\frac{15a\sin(dx+c)}{8d}+\frac{15a\ln(\sec(dx+c}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a*sin(d*x+c)^7/cos(d*x+c)^2-3/8*a*s
in(d*x+c)^5/d-5/8*a*sin(d*x+c)^3/d-15/8*a*sin(d*x+c)/d+15/8/d*a*ln(sec(d*x+
c)+tan(d*x+c))+1/4/d*b*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*b*sin(d*x+c)^8/cos(d
*x+c)^2-1/2*b*sin(d*x+c)^6/d-3/4*b*sin(d*x+c)^4/d-3/2*b*sin(d*x+c)^2/d-3*b*
ln(cos(d*x+c))/d
```

Maxima [A] time = 0.996085, size = 163, normalized size = 1.21

$$-\frac{8b\sin(dx+c)^2-3(5a-8b)\log(\sin(dx+c)+1)+3(5a+8b)\log(\sin(dx+c)-1)+16a\sin(dx+c)-\frac{2(9a\sin(dx+}{s}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(8*b*sin(d*x + c)^2 - 3*(5*a - 8*b)*log(sin(d*x + c) + 1) + 3*(5*a +
8*b)*log(sin(d*x + c) - 1) + 16*a*sin(d*x + c) - 2*(9*a*sin(d*x + c)^3 + 12
*b*sin(d*x + c)^2 - 7*a*sin(d*x + c) - 10*b)/(sin(d*x + c)^4 - 2*sin(d*x +
c)^2 + 1))/d
```

Fricas [A] time = 2.06085, size = 360, normalized size = 2.67

$$\frac{8b\cos(dx+c)^6+3(5a-8b)\cos(dx+c)^4\log(\sin(dx+c)+1)-3(5a+8b)\cos(dx+c)^4\log(-\sin(dx+c)+1)-4b}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cos(d*x + c)^6 + 3*(5*a - 8*b)*cos(d*x + c)^4*log(sin(d*x + c) +
1) - 3*(5*a + 8*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 4*b*cos(d*x + c)
^4 - 24*b*cos(d*x + c)^2 - 2*(8*a*cos(d*x + c)^4 + 9*a*cos(d*x + c)^2 - 2*a
)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25774, size = 167, normalized size = 1.24

$$-\frac{8\,b\sin\left(dx+c\right)^{2}-3\,\left(5\,a-8\,b\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,\left(5\,a+8\,b\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+16\,a\sin\left(dx+c\right)-\frac{2\left(18\,b\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(8*b*sin(d*x + c)^2 - 3*(5*a - 8*b)*log(abs(sin(d*x + c) + 1)) + 3*(5
*a + 8*b)*log(abs(sin(d*x + c) - 1)) + 16*a*sin(d*x + c) - 2*(18*b*sin(d*x
+ c)^4 + 9*a*sin(d*x + c)^3 - 24*b*sin(d*x + c)^2 - 7*a*sin(d*x + c) + 8*b)
/(sin(d*x + c)^2 - 1)^2)/d
```

3.1483 $\int (a+b\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{(8a+15b)\log(1-\sin(c+dx))}{16d}-\frac{(8a-15b)\log(\sin(c+dx)+1)}{16d}+\frac{\tan^4(c+dx)(a+b\sin(c+dx))}{4d}-\frac{\tan^2(c+dx)(4a\cdots}{8\cdots}$$

```
[Out] -((8*a + 15*b)*Log[1 - Sin[c + d*x]])/(16*d) - ((8*a - 15*b)*Log[1 + Sin[c
+ d*x]])/(16*d) - (15*b*Sin[c + d*x])/(8*d) - ((4*a + 5*b*Sin[c + d*x])*Tan
[c + d*x]^2)/(8*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.108635, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2721, 819, 774, 633, 31}

$$-\frac{(8a+15b)\log(1-\sin(c+dx))}{16d}-\frac{(8a-15b)\log(\sin(c+dx)+1)}{16d}+\frac{\tan^4(c+dx)(a+b\sin(c+dx))}{4d}-\frac{\tan^2(c+dx)(4a\cdots}{8\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((8*a + 15*b)*Log[1 - Sin[c + d*x]])/(16*d) - ((8*a - 15*b)*Log[1 + Sin[c
+ d*x]])/(16*d) - (15*b*Sin[c + d*x])/(8*d) - ((4*a + 5*b*Sin[c + d*x])*Tan
[c + d*x]^2)/(8*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^4)/(4*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a+b\sin(c+dx))\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)}{(b^2-x^2)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{(a+b\sin(c+dx))\tan^4(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{x^3(4ab^2+5b^2x)}{(b^2-x^2)^2}\,dx,x,b\sin(c+dx)\right)}{4b^2d}$$

$$= -\frac{(4a+5b\sin(c+dx))\tan^2(c+dx)}{8d} + \frac{(a+b\sin(c+dx))\tan^4(c+dx)}{4d} + \frac{\text{Sub}}{}$$

$$= -\frac{15b\sin(c+dx)}{8d} - \frac{(4a+5b\sin(c+dx))\tan^2(c+dx)}{8d} + \frac{(a+b\sin(c+dx))\,\text{ta}}{4d}$$

$$= -\frac{15b\sin(c+dx)}{8d} - \frac{(4a+5b\sin(c+dx))\tan^2(c+dx)}{8d} + \frac{(a+b\sin(c+dx))\,\text{ta}}{4d}$$

$$= -\frac{(8a+15b)\log(1-\sin(c+dx))}{16d} - \frac{(8a-15b)\log(1+\sin(c+dx))}{16d} - \frac{15b\sin(}{8d}$$

Mathematica [A] time = 0.304658, size = 123, normalized size = 1.06

$$-\frac{a\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d} - \frac{b\sin(c+dx)\tan^4(c+dx)}{d} - \frac{5b\left(6\tan(c+dx)\sec^3(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((b*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*b*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*S
ec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(8*d)
```

Maple [A] time = 0.047, size = 147, normalized size = 1.3

$$\frac{a(\tan(dx+c))^4}{4d} - \frac{a(\tan(dx+c))^2}{2d} - \frac{a\ln(\cos(dx+c))}{d} + \frac{b(\sin(dx+c))^7}{4d(\cos(dx+c))^4} - \frac{3b(\sin(dx+c))^7}{8d(\cos(dx+c))^2} - \frac{3b(\sin(dx+c}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4*a*tan(d*x+c)^4/d-1/2*a*tan(d*x+c)^2/d-a*ln(cos(d*x+c))/d+1/4/d*b*sin(d*
x+c)^7/cos(d*x+c)^4-3/8/d*b*sin(d*x+c)^7/cos(d*x+c)^2-3/8*b*sin(d*x+c)^5/d-
5/8*b*sin(d*x+c)^3/d-15/8*b*sin(d*x+c)/d+15/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.967047, size = 146, normalized size = 1.26

$$-\frac{(8\,a-15\,b)\log\left(\sin\left(dx+c\right)+1\right)+(8\,a+15\,b)\log\left(\sin\left(dx+c\right)-1\right)+16\,b\sin\left(dx+c\right)-\frac{2\left(9\,b\sin(dx+c)^{3}+8\,a\sin(dx+c)^{2}-7\,b\sin\right.}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a - 15*b)*log(sin(d*x + c) + 1) + (8*a + 15*b)*log(sin(d*x + c) -
 1) + 16*b*sin(d*x + c) - 2*(9*b*sin(d*x + c)^3 + 8*a*sin(d*x + c)^2 - 7*b*
sin(d*x + c) - 6*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.09253, size = 302, normalized size = 2.6

$$-\frac{(8\,a-15\,b)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)+(8\,a+15\,b)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+16\,a\cos\left(dx+c\right)^{2}+}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*((8*a - 15*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (8*a + 15*b)*cos
(d*x + c)^4*log(-sin(d*x + c) + 1) + 16*a*cos(d*x + c)^2 + 2*(8*b*cos(d*x +
 c)^4 + 9*b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22834, size = 146, normalized size = 1.26

$$-\frac{(8\,a-15\,b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+(8\,a+15\,b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+16\,b\sin\left(dx+c\right)-\frac{2\left(6\,a\sin(dx+c)^{4}+9\,b\sin(dx+c)^{3}-4\,a\right.}{\left(\sin(dx+c)^{2}-\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*((8*a - 15*b)*log(abs(sin(d*x + c) + 1)) + (8*a + 15*b)*log(abs(sin(d
*x + c) - 1)) + 16*b*sin(d*x + c) - 2*(6*a*sin(d*x + c)^4 + 9*b*sin(d*x + c
)^3 - 4*a*sin(d*x + c)^2 - 7*b*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

3.1484 $\int \sec(c+dx)(a+b\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}-\frac{b\tan^2(c+dx)}{2d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - (b*Log[Cos[c + d*x]])/d - (3*a*Sec[c +
d*x]*Tan[c + d*x])/(8*d) - (b*Tan[c + d*x]^2)/(2*d) + (a*Sec[c + d*x]*Tan[c
 + d*x]^3)/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.120409, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2834, 2611, 3770, 3473, 3475}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}-\frac{b\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - (b*Log[Cos[c + d*x]])/d - (3*a*Sec[c +
d*x]*Tan[c + d*x])/(8*d) - (b*Tan[c + d*x]^2)/(2*d) + (a*Sec[c + d*x]*Tan[c
 + d*x]^3)/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))\tan^4(c+dx)\,dx = a\int \sec(c+dx)\tan^4(c+dx)\,dx + b\int \tan^5(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan^3(c+dx)}{4d} + \frac{b\tan^4(c+dx)}{4d} - \frac{1}{4}(3a)\int \sec(c+dx)\,\mathrm{ta}$$
$$= -\frac{3a\sec(c+dx)\tan(c+dx)}{8d} - \frac{b\tan^2(c+dx)}{2d} + \frac{a\sec(c+dx)\tan^3(c}{4d}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{b\log(\cos(c+dx))}{d} - \frac{3a\sec(c+dx)\tan(c+}{8d}$$

Mathematica [A] time = 0.309409, size = 106, normalized size = 1.03

$$\frac{a\tan^3(c+dx)\sec(c+dx)}{d} - \frac{a\left(6\tan(c+dx)\sec^3(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\sec(c+dx)\right)\right)}{8d} - \frac{b\left(-\mathrm{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*Sec[c + d*x]*Tan[c + d*x]^3)/d - (b*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x
]^2 - Tan[c + d*x]^4))/(4*d) - (a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 3*(ArcTa
nh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])))/(8*d)
```

Maple [A] time = 0.049, size = 133, normalized size = 1.3

$$\frac{a(\sin(dx+c))^5}{4d(\cos(dx+c))^4} - \frac{a(\sin(dx+c))^5}{8d(\cos(dx+c))^2} - \frac{a(\sin(dx+c))^3}{8d} - \frac{3a\sin(dx+c)}{8d} + \frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{b\,(\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a*sin(d*x+c)^5/cos(d*x+c)^2-1/8*a*s
in(d*x+c)^3/d-3/8*a*sin(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4*b*ta
n(d*x+c)^4/d-1/2*b*tan(d*x+c)^2/d-b*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.00186, size = 135, normalized size = 1.31

$$\frac{(3a-8b)\log(\sin(dx+c)+1) - (3a+8b)\log(\sin(dx+c)-1) + \frac{2\left(5a\sin(dx+c)^3 + 8b\sin(dx+c)^2 - 3a\sin(dx+c) - 6b\right)}{\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*((3*a - 8*b)*log(sin(d*x + c) + 1) - (3*a + 8*b)*log(sin(d*x + c) - 1)
 + 2*(5*a*sin(d*x + c)^3 + 8*b*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 6*b)/(si
```

```
n(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.98223, size = 270, normalized size = 2.62

$$\frac{(3a-8b)\cos(dx+c)^4\log(\sin(dx+c)+1)-(3a+8b)\cos(dx+c)^4\log(-\sin(dx+c)+1)-16b\cos(dx+c)^2-2}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*((3*a - 8*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a + 8*b)*cos(d*
x + c)^4*log(-sin(d*x + c) + 1) - 16*b*cos(d*x + c)^2 - 2*(5*a*cos(d*x + c)
^2 - 2*a)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2443, size = 135, normalized size = 1.31

$$\frac{(3a-8b)\log(|\sin(dx+c)+1|)-(3a+8b)\log(|\sin(dx+c)-1|)+\frac{2\left(6b\sin(dx+c)^4+5a\sin(dx+c)^3-4b\sin(dx+c)^2-3a\sin(dx+c)\right.}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*((3*a - 8*b)*log(abs(sin(d*x + c) + 1)) - (3*a + 8*b)*log(abs(sin(d*x
+ c) - 1)) + 2*(6*b*sin(d*x + c)^4 + 5*a*sin(d*x + c)^3 - 4*b*sin(d*x + c)^
2 - 3*a*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

3.1485 $\int \sec^2(c+dx)(a+b\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=74

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3b\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - (3*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) +
 (b*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d) + (a*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.132936, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2834, 2607, 30, 2611, 3770}

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - (3*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) +
 (b*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d) + (a*Tan[c + d*x]^4)/(4*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))\tan^3(c+dx)\,dx = a\int \sec^2(c+dx)\tan^3(c+dx)\,dx + b\int \sec(c+dx)\tan^4(c+dx)$$
$$= \frac{b\sec(c+dx)\tan^3(c+dx)}{4d} - \frac{1}{4}(3b)\int \sec(c+dx)\tan^2(c+dx)\,d$$
$$= -\frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec(c+dx)\tan^3(c+dx)}{4d} + \frac{a\tan}{}$$
$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} - \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec(c+}{}$$

Mathematica [A] time = 0.266137, size = 84, normalized size = 1.14

$$\frac{a\tan^4(c+dx)}{4d} + \frac{b\tan^3(c+dx)\sec(c+dx)}{d} - \frac{b\left(6\tan(c+dx)\sec^3(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\,\mathrm{se}\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (b*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a*Tan[c + d*x]^4)/(4*d) - (b*(6*Sec[c
+ d*x]^3*Tan[c + d*x] - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x
])))/(8*d)
```

Maple [A] time = 0.049, size = 114, normalized size = 1.5

$$\frac{a(\sin(dx+c))^4}{4d(\cos(dx+c))^4} + \frac{b(\sin(dx+c))^5}{4d(\cos(dx+c))^4} - \frac{b(\sin(dx+c))^5}{8d(\cos(dx+c))^2} - \frac{b(\sin(dx+c))^3}{8d} - \frac{3b\sin(dx+c)}{8d} + \frac{3b\ln(\sec(dx+c}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*b*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*b
*sin(d*x+c)^5/cos(d*x+c)^2-1/8*b*sin(d*x+c)^3/d-3/8*b*sin(d*x+c)/d+3/8/d*b*
ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.977327, size = 120, normalized size = 1.62

$$\frac{3b\log(\sin(dx+c)+1) - 3b\log(\sin(dx+c)-1) + \frac{2\left(5b\sin(dx+c)^3 + 4a\sin(dx+c)^2 - 3b\sin(dx+c) - 2a\right)}{\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*b*log(sin(d*x + c) + 1) - 3*b*log(sin(d*x + c) - 1) + 2*(5*b*sin(d*
x + c)^3 + 4*a*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/(sin(d*x + c)^4 - 2
*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.98, size = 247, normalized size = 3.34

$$\frac{3\,b\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,b\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-8\,a\cos\left(dx+c\right)^{2}-2\left(5\,b\cos\left(dx+c\right)^{2}-\right.}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(3*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*b*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) - 8*a*cos(d*x + c)^2 - 2*(5*b*cos(d*x + c)^2 - 2*b)*sin(d*x
 + c) + 4*a)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22193, size = 109, normalized size = 1.47

$$\frac{3\,b\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\,b\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{2\left(5\,b\sin\left(dx+c\right)^{3}+4\,a\sin\left(dx+c\right)^{2}-3\,b\sin\left(dx+c\right)-2\,a\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(3*b*log(abs(sin(d*x + c) + 1)) - 3*b*log(abs(sin(d*x + c) - 1)) + 2*(
5*b*sin(d*x + c)^3 + 4*a*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/(sin(d*x
+ c)^2 - 1)^2)/d
```

3.1486 $\int \sec^3(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=74

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}$$

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.139193, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2834, 2611, 3768, 3770, 2607, 30}

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sec^3(c+dx)\tan^2(c+dx)\,dx + b\int \sec^2(c+dx)\tan^3(c+dx)\,dx$$

$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}a\int \sec^3(c+dx)\,dx + \frac{b\,\mathrm{Subst}\left(\int x^3$$

$$= -\frac{a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{b\tan^4(c}{4d}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx}{4}$$

Mathematica [A] time = 0.0246122, size = 74, normalized size = 1.

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{a\tan(c+dx)\sec(c+dx)}{8d} + \frac{b\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.043, size = 100, normalized size = 1.4

$$\frac{a(\sin(dx+c))^3}{4d(\cos(dx+c))^4} + \frac{a(\sin(dx+c))^3}{8d(\cos(dx+c))^2} + \frac{a\sin(dx+c)}{8d} - \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{b(\sin(dx+c))^4}{4d(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a*sin(d*x+c)^3/cos(d*x+c)^2+1/8*a*s
in(d*x+c)/d-1/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b*sin(d*x+c)^4/cos(d*x+
c)^4
```

Maxima [A] time = 0.965348, size = 116, normalized size = 1.57

$$-\frac{a\log(\sin(dx+c)+1) - a\log(\sin(dx+c)-1) - \frac{2\left(a\sin(dx+c)^3+4b\sin(dx+c)^2+a\sin(dx+c)-2b\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c
)^3 + 4*b*sin(d*x + c)^2 + a*sin(d*x + c) - 2*b)/(sin(d*x + c)^4 - 2*sin(d*
x + c)^2 + 1))/d
```

Fricas [A] time = 2.02741, size = 240, normalized size = 3.24

$$-\frac{a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+8\,b\cos\left(dx+c\right)^{2}+2\,\left(a\cos\left(dx+c\right)^{2}\right.}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - a*cos(d*x + c)^4*log(-sin(d
*x + c) + 1) + 8*b*cos(d*x + c)^2 + 2*(a*cos(d*x + c)^2 - 2*a)*sin(d*x + c)
 - 4*b)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26667, size = 105, normalized size = 1.42

$$-\frac{a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(a\sin\left(dx+c\right)^{3}+4\,b\sin\left(dx+c\right)^{2}+a\sin\left(dx+c\right)-2\,b\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(a*log(abs(sin(d*x + c) + 1)) - a*log(abs(sin(d*x + c) - 1)) - 2*(a*s
in(d*x + c)^3 + 4*b*sin(d*x + c)^2 + a*sin(d*x + c) - 2*b)/(sin(d*x + c)^2
- 1)^2)/d
```

3.1487 $\int \sec^4(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=74

$$\frac{a\sec^4(c+dx)}{4d}-\frac{b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{b\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] -(b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Sec[c + d*x]^4)/(4*d) - (b*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.105266, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2834, 2606, 30, 2611, 3768, 3770}

$$\frac{a\sec^4(c+dx)}{4d}-\frac{b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Sec[c + d*x]^4)/(4*d) - (b*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx = a\int \sec^4(c+dx)\tan(c+dx)\,dx + b\int \sec^3(c+dx)\tan^2(c+dx)\,d$$

$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}b\int \sec^3(c+dx)\,dx + \frac{a\,\mathrm{Subst}\left(\int x\right.}{}$$

$$= \frac{a\sec^4(c+dx)}{4d} - \frac{b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c}{4d}$$

$$= -\frac{b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\sec^4(c+dx)}{4d} - \frac{b\sec(c+dx)\tan(c+}{8d}$$

Mathematica [A] time = 0.023758, size = 74, normalized size = 1.

$$\frac{a\sec^4(c+dx)}{4d} - \frac{b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Sec[c + d*x]^4)/(4*d) - (b*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.037, size = 92, normalized size = 1.2

$$\frac{a}{4\,d\left(\cos\left(dx+c\right)\right)^{4}} + \frac{b\left(\sin\left(dx+c\right)\right)^{3}}{4\,d\left(\cos\left(dx+c\right)\right)^{4}} + \frac{b\left(\sin\left(dx+c\right)\right)^{3}}{8\,d\left(\cos\left(dx+c\right)\right)^{2}} + \frac{b\sin\left(dx+c\right)}{8\,d} - \frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/cos(d*x+c)^4+1/4/d*b*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*b*sin(d*x+c)^3
/cos(d*x+c)^2+1/8*b*sin(d*x+c)/d-1/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.00289, size = 101, normalized size = 1.36

$$-\frac{b\log\left(\sin\left(dx+c\right)+1\right) - b\log\left(\sin\left(dx+c\right)-1\right) - \frac{2\left(b\sin\left(dx+c\right)^{3}+b\sin\left(dx+c\right)+2\,a\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(b*log(sin(d*x + c) + 1) - b*log(sin(d*x + c) - 1) - 2*(b*sin(d*x + c
)^3 + b*sin(d*x + c) + 2*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.98192, size = 212, normalized size = 2.86

$$-\frac{b\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-b\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(b\cos\left(dx+c\right)^{2}-2\,b\right)\sin\left(dx+c\right)-4\,a}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - b*cos(d*x + c)^4*log(-sin(d
*x + c) + 1) + 2*(b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - 4*a)/(d*cos(d*x +
c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23533, size = 90, normalized size = 1.22

$$-\frac{b\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-b\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(b\sin\left(dx+c\right)^{3}+b\sin\left(dx+c\right)+2\,a\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(b*log(abs(sin(d*x + c) + 1)) - b*log(abs(sin(d*x + c) - 1)) - 2*(b*s
in(d*x + c)^3 + b*sin(d*x + c) + 2*a)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1488 $\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=99

$$\frac{a\tan^4(c+dx)}{4d}+\frac{a\tan^2(c+dx)}{d}+\frac{a\log(\tan(c+dx))}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+3b\,\mathrm{ta}$$

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Log[Tan[c + d*x]])/d + (3*b*Sec[c +
d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c
 + d*x]^2)/d + (a*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.105285, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2834, 2620, 266, 43, 3768, 3770}

$$\frac{a\tan^4(c+dx)}{4d}+\frac{a\tan^2(c+dx)}{d}+\frac{a\log(\tan(c+dx))}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+3b\,\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Log[Tan[c + d*x]])/d + (3*b*Sec[c +
d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c
 + d*x]^2)/d + (a*Tan[c + d*x]^4)/(4*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc(c+dx)\sec^5(c+dx)\,dx + b\int \sec^5(c+dx)\,dx$$

$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3b)\int \sec^3(c+dx)\,dx + \frac{a\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{8}(3b)\int \text{se}\ldots$$

$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\ldots}{4\ldots}$$

$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\log(\tan(c+dx))}{d} + \frac{3b\sec(c+dx)\tan(c\ldots}{8d}$$

Mathematica [A] time = 1.25655, size = 99, normalized size = 1.

$$-\frac{a\left(-\sec^4(c+dx) - 2\sec^2(c+dx) - 4\log(\sin(c+dx)) + 4\log(\cos(c+dx))\right)}{4d} + \frac{b\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3b\left(\tanh^{-1}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec[c +
 d*x]^4))/(4*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*b*(ArcTanh[Sin
[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A] time = 0.059, size = 100, normalized size = 1.

$$\frac{a}{4d(\cos(dx+c))^4} + \frac{a}{2d(\cos(dx+c))^2} + \frac{a\ln(\tan(dx+c))}{d} + \frac{b(\sec(dx+c))^3\tan(dx+c)}{4d} + \frac{3b\sec(dx+c)\tan(dx+\ldots}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/cos(d*x+c)^4+1/2/d*a/cos(d*x+c)^2+a*ln(tan(d*x+c))/d+1/4*b*sec(d*x+
c)^3*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+3/8/d*b*ln(sec(d*x+c)+tan(d
*x+c))
```

Maxima [A] time = 0.999947, size = 147, normalized size = 1.48

$$-\frac{(8a-3b)\log(\sin(dx+c)+1)+(8a+3b)\log(\sin(dx+c)-1)-16a\log(\sin(dx+c))+\frac{2\left(3b\sin(dx+c)^3+4a\sin(dx+c)\cdots\right.}{\sin(dx+c)^4-2\sin(\cdots}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a - 3*b)*log(sin(d*x + c) + 1) + (8*a + 3*b)*log(sin(d*x + c) - 1
) - 16*a*log(sin(d*x + c)) + 2*(3*b*sin(d*x + c)^3 + 4*a*sin(d*x + c)^2 - 5
*b*sin(d*x + c) - 6*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.04968, size = 328, normalized size = 3.31

$$\frac{16a\cos(dx+c)^4\log\left(\frac{1}{2}\sin(dx+c)\right)-(8a-3b)\cos(dx+c)^4\log(\sin(dx+c)+1)-(8a+3b)\cos(dx+c)^4\log(-\cdots}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*cos(d*x + c)^4*log(1/2*sin(d*x + c)) - (8*a - 3*b)*cos(d*x + c)^
4*log(sin(d*x + c) + 1) - (8*a + 3*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)
 + 8*a*cos(d*x + c)^2 + 2*(3*b*cos(d*x + c)^2 + 2*b)*sin(d*x + c) + 4*a)/(d
*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.223, size = 153, normalized size = 1.55

$$-\frac{(8a-3b)\log(|\sin(dx+c)+1|)+(8a+3b)\log(|\sin(dx+c)-1|)-16a\log(|\sin(dx+c)|)-\frac{2\left(6a\sin(dx+c)^4-3b\sin(\cdots\right.}{\cdots}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*((8*a - 3*b)*log(abs(sin(d*x + c) + 1)) + (8*a + 3*b)*log(abs(sin(d*x
 + c) - 1)) - 16*a*log(abs(sin(d*x + c))) - 2*(6*a*sin(d*x + c)^4 - 3*b*sin
(d*x + c)^3 - 16*a*sin(d*x + c)^2 + 5*b*sin(d*x + c) + 12*a)/(sin(d*x + c)^
2 - 1)^2)/d
```

3.1489 $\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{15a\csc(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc(c+dx)\sec^4(c+dx)}{4d}+\frac{5a\csc(c+dx)\sec^2(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}$$

```
[Out] (15*a*ArcTanh[Sin[c + d*x]])/(8*d) - (15*a*Csc[c + d*x])/(8*d) + (b*Log[Tan
[c + d*x]])/d + (5*a*Csc[c + d*x]*Sec[c + d*x]^2)/(8*d) + (a*Csc[c + d*x]*S
ec[c + d*x]^4)/(4*d) + (b*Tan[c + d*x]^2)/d + (b*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.142762, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2834, 2621, 288, 321, 207, 2620, 266, 43}

$$-\frac{15a\csc(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc(c+dx)\sec^4(c+dx)}{4d}+\frac{5a\csc(c+dx)\sec^2(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (15*a*ArcTanh[Sin[c + d*x]])/(8*d) - (15*a*Csc[c + d*x])/(8*d) + (b*Log[Tan
[c + d*x]])/d + (5*a*Csc[c + d*x]*Sec[c + d*x]^2)/(8*d) + (a*Csc[c + d*x]*S
ec[c + d*x]^4)/(4*d) + (b*Tan[c + d*x]^2)/d + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx &= a\int \csc^2(c+dx)\sec^5(c+dx)\,dx + b\int \csc(c+dx)\sec^5(c+dx)\,d\\
&= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \csc(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x}\,dx,\right.}{d}\\
&= \frac{a\csc(c+dx)\sec^4(c+dx)}{4d} - \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \csc(c+\right.}{4d}\\
&= \frac{5a\csc(c+dx)\sec^2(c+dx)}{8d} + \frac{a\csc(c+dx)\sec^4(c+dx)}{4d} - \frac{(15a)\,\text{S}}{}\\
&= -\frac{15a\csc(c+dx)}{8d} + \frac{b\log(\tan(c+dx))}{d} + \frac{5a\csc(c+dx)\sec^2(c+}{8d}\\
&= \frac{15a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{15a\csc(c+dx)}{8d} + \frac{b\log(\tan(c+dx))}{d}
\end{aligned}$$

Mathematica [C] time = 0.377658, size = 76, normalized size = 0.66

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d} - \frac{b\left(-\sec^4(c+dx)-2\sec^2(c+dx)-4\log(\sin(c+dx))+4\log(\cos(c+dx\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (b*
(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec[c + d*x
]^4))/(4*d)
```

Maple [A] time = 0.062, size = 120, normalized size = 1.

$$\frac{a}{4\,d\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{4}}+\frac{5\,a}{8\,d\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{2}}-\frac{15\,a}{8\,d\sin\left(dx+c\right)}+\frac{15\,a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a/sin(d*x+c)/cos(d*x+c)^2-15/8/d*a/si
n(d*x+c)+15/8/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b/cos(d*x+c)^4+1/2/d*b/co
s(d*x+c)^2+b*ln(tan(d*x+c))/d
```

Maxima [A] time = 1.00408, size = 170, normalized size = 1.48

$$\frac{(15\,a-8\,b)\log\left(\sin\left(dx+c\right)+1\right)-(15\,a+8\,b)\log\left(\sin\left(dx+c\right)-1\right)+16\,b\log\left(\sin\left(dx+c\right)\right)-\frac{2\left(15\,a\sin(dx+c)^{4}+4\,b\sin(dx+c)^{3}-\right.}{\sin(dx+c)^{5}-2\sin(}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*((15*a - 8*b)*log(sin(d*x + c) + 1) - (15*a + 8*b)*log(sin(d*x + c) -
1) + 16*b*log(sin(d*x + c)) - 2*(15*a*sin(d*x + c)^4 + 4*b*sin(d*x + c)^3 -
 25*a*sin(d*x + c)^2 - 6*b*sin(d*x + c) + 8*a)/(sin(d*x + c)^5 - 2*sin(d*x
+ c)^3 + sin(d*x + c)))/d
```

Fricas [A] time = 2.14112, size = 429, normalized size = 3.73

$$\frac{16\,b\cos\left(dx+c\right)^{4}\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+(15\,a-8\,b)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-(15\,a+8}{16\,d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*b*cos(d*x + c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) + (15*a - 8*b)
*cos(d*x + c)^4*log(sin(d*x + c) + 1)*sin(d*x + c) - (15*a + 8*b)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1)*sin(d*x + c) - 30*a*cos(d*x + c)^4 + 10*a*cos(
d*x + c)^2 + 4*(2*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 4*a)/(d*cos(d*x + c)
^4*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25322, size = 181, normalized size = 1.57

$$\frac{(15\,a-8\,b)\log\left(|\sin\left(dx+c\right)+1|\right)-(15\,a+8\,b)\log\left(|\sin\left(dx+c\right)-1|\right)+16\,b\log\left(|\sin\left(dx+c\right)|\right)-\frac{16\,(b\sin(dx+c)+a)}{\sin(dx+c)}+\frac{2(6}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*((15*a - 8*b)*log(abs(sin(d*x + c) + 1)) - (15*a + 8*b)*log(abs(sin(d*
x + c) - 1)) + 16*b*log(abs(sin(d*x + c))) - 16*(b*sin(d*x + c) + a)/sin(d*
x + c) + 2*(6*b*sin(d*x + c)^4 - 7*a*sin(d*x + c)^3 - 16*b*sin(d*x + c)^2 +
 9*a*sin(d*x + c) + 12*b)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1490 $\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=135

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3a\tan^2(c+dx)}{2d}-\frac{a\cot^2(c+dx)}{2d}+\frac{3a\log(\tan(c+dx))}{d}-\frac{15b\csc(c+dx)}{8d}+\frac{15b\tanh^{-1}(\sin(c+dx))}{8d}-$$

```
[Out] (15*b*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Cot[c + d*x]^2)/(2*d) - (15*b*Csc[c
 + d*x])/(8*d) + (3*a*Log[Tan[c + d*x]])/d + (5*b*Csc[c + d*x]*Sec[c + d*x]
^2)/(8*d) + (b*Csc[c + d*x]*Sec[c + d*x]^4)/(4*d) + (3*a*Tan[c + d*x]^2)/(2
*d) + (a*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.156585, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2834, 2620, 266, 43, 2621, 288, 321, 207}

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3a\tan^2(c+dx)}{2d}-\frac{a\cot^2(c+dx)}{2d}+\frac{3a\log(\tan(c+dx))}{d}-\frac{15b\csc(c+dx)}{8d}+\frac{15b\tanh^{-1}(\sin(c+dx))}{8d}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```


```
[Out] (15*b*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Cot[c + d*x]^2)/(2*d) - (15*b*Csc[c
 + d*x])/(8*d) + (3*a*Log[Tan[c + d*x]])/d + (5*b*Csc[c + d*x]*Sec[c + d*x]
^2)/(8*d) + (b*Csc[c + d*x]*Sec[c + d*x]^4)/(4*d) + (3*a*Tan[c + d*x]^2)/(2
*d) + (a*Tan[c + d*x]^4)/(4*d)
```


Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^3(c+dx)\sec^5(c+dx)\,dx + b\int \csc^2(c+dx)\sec^5(c+dx)$$

$$= \frac{a\,\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, \tan(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x\right.}{d}$$

$$= \frac{b\csc(c+dx)\sec^4(c+dx)}{4d} + \frac{a\,\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, \tan^2(c+dx)\right)}{2d}$$

$$= \frac{5b\csc(c+dx)\sec^2(c+dx)}{8d} + \frac{b\csc(c+dx)\sec^4(c+dx)}{4d} + \frac{a\,\text{Sub}}{}$$

$$= -\frac{a\cot^2(c+dx)}{2d} - \frac{15b\csc(c+dx)}{8d} + \frac{3a\log(\tan(c+dx))}{d} + \frac{5b\,\text{cs}}{}$$

$$= \frac{15b\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\cot^2(c+dx)}{2d} - \frac{15b\csc(c+dx)}{8d} + \frac{3a}{}$$

Mathematica [C] time = 0.605704, size = 86, normalized size = 0.64

$$-\frac{a\left(2\csc^2(c+dx)-\sec^4(c+dx)-4\sec^2(c+dx)-12\log(\sin(c+dx))+12\log(\cos(c+dx))\right)}{4d} - \frac{b\csc(c+dx)\,{}_2F_1\left(-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (a*
(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Sec[c +
```

```
d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

Maple [A] time = 0.066, size = 151, normalized size = 1.1

$$\frac{a}{4d(\sin(dx+c))^2(\cos(dx+c))^4}+\frac{3a}{4d(\sin(dx+c))^2(\cos(dx+c))^2}-\frac{3a}{2d(\sin(dx+c))^2}+3\frac{a\ln(\tan(dx+c))}{d}+\frac{}{4d\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*a/sin(d*x+c)^2/cos(d*x+c)^2-3/2/d*a
/sin(d*x+c)^2+3*a*ln(tan(d*x+c))/d+1/4/d*b/sin(d*x+c)/cos(d*x+c)^4+5/8/d*b/
sin(d*x+c)/cos(d*x+c)^2-15/8/d*b/sin(d*x+c)+15/8/d*b*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 1.00901, size = 189, normalized size = 1.4

$$-\frac{3(8a-5b)\log(\sin(dx+c)+1)+3(8a+5b)\log(\sin(dx+c)-1)-48a\log(\sin(dx+c))+\frac{2\left(15b\sin(dx+c)^5+12a\sin(dx\right.}{\sin(d}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(3*(8*a - 5*b)*log(sin(d*x + c) + 1) + 3*(8*a + 5*b)*log(sin(d*x + c)
 - 1) - 48*a*log(sin(d*x + c)) + 2*(15*b*sin(d*x + c)^5 + 12*a*sin(d*x + c)
^4 - 25*b*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 8*b*sin(d*x + c) + 4*a)/(s
in(d*x + c)^6 - 2*sin(d*x + c)^4 + sin(d*x + c)^2))/d
```

Fricas [A] time = 2.17341, size = 532, normalized size = 3.94

$$\frac{24a\cos(dx+c)^4-12a\cos(dx+c)^2+48\left(a\cos(dx+c)^6-a\cos(dx+c)^4\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-3\left((8a-5b)\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(24*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 48*(a*cos(d*x + c)^6 - a*
cos(d*x + c)^4)*log(1/2*sin(d*x + c)) - 3*((8*a - 5*b)*cos(d*x + c)^6 - (8*
a - 5*b)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 3*((8*a + 5*b)*cos(d*x + c
)^6 - (8*a + 5*b)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(15*b*cos(d*x
+ c)^4 - 5*b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)^6 -
d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27938, size = 180, normalized size = 1.33

$$-\frac{3\,(8\,a-5\,b)\log\left(|\sin\left(dx+c\right)+1|\right)+3\,(8\,a+5\,b)\log\left(|\sin\left(dx+c\right)-1|\right)-48\,a\log\left(|\sin\left(dx+c\right)|\right)+\frac{2\left(15\,b\sin(dx+c)^{5}+12\ldots\right.}{\ldots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(3*(8*a - 5*b)*log(abs(sin(d*x + c) + 1)) + 3*(8*a + 5*b)*log(abs(sin
(d*x + c) - 1)) - 48*a*log(abs(sin(d*x + c))) + 2*(15*b*sin(d*x + c)^5 + 12
*a*sin(d*x + c)^4 - 25*b*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 8*b*sin(d*x
 + c) + 4*a)/(sin(d*x + c)^3 - sin(d*x + c))^2)/d
```

3.1491 $\int \csc^4(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=155

$$-\frac{35a\csc^3(c+dx)}{24d}-\frac{35a\csc(c+dx)}{8d}+\frac{35a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d}+\frac{7a\csc^3(c+dx)\sec^2(c+}{8d}$$

```
[Out] (35*a*ArcTanh[Sin[c + d*x]])/(8*d) - (b*Cot[c + d*x]^2)/(2*d) - (35*a*Csc[c
 + d*x])/(8*d) - (35*a*Csc[c + d*x]^3)/(24*d) + (3*b*Log[Tan[c + d*x]])/d +
 (7*a*Csc[c + d*x]^3*Sec[c + d*x]^2)/(8*d) + (a*Csc[c + d*x]^3*Sec[c + d*x]
^4)/(4*d) + (3*b*Tan[c + d*x]^2)/(2*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.160565, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2834, 2621, 288, 302, 207, 2620, 266, 43}

$$-\frac{35a\csc^3(c+dx)}{24d}-\frac{35a\csc(c+dx)}{8d}+\frac{35a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d}+\frac{7a\csc^3(c+dx)\sec^2(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (35*a*ArcTanh[Sin[c + d*x]])/(8*d) - (b*Cot[c + d*x]^2)/(2*d) - (35*a*Csc[c
 + d*x])/(8*d) - (35*a*Csc[c + d*x]^3)/(24*d) + (3*b*Log[Tan[c + d*x]])/d +
 (7*a*Csc[c + d*x]^3*Sec[c + d*x]^2)/(8*d) + (a*Csc[c + d*x]^3*Sec[c + d*x]
^4)/(4*d) + (3*b*Tan[c + d*x]^2)/(2*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \csc^4(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^4(c+dx)\sec^5(c+dx)\,dx + b\int \csc^3(c+dx)\sec^5(c+dx)$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \csc(c+dx)\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx,\right.}{d}$$

$$= \frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d} - \frac{(7a)\,\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(c+\right.}{4d}$$

$$= \frac{7a\csc^3(c+dx)\sec^2(c+dx)}{8d} + \frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d} - \frac{(35a}{}$$

$$= -\frac{b\cot^2(c+dx)}{2d} + \frac{3b\log(\tan(c+dx))}{d} + \frac{7a\csc^3(c+dx)\sec^2(c+}{8d}$$

$$= -\frac{b\cot^2(c+dx)}{2d} - \frac{35a\csc(c+dx)}{8d} - \frac{35a\csc^3(c+dx)}{24d} + \frac{3b\log(t}{}$$

$$= \frac{35a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{b\cot^2(c+dx)}{2d} - \frac{35a\csc(c+dx)}{8d} - \frac{35}{}$$

Mathematica [C] time = 0.859305, size = 90, normalized size = 0.58

$$-\frac{a\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(c+dx)\right)}{3d} - \frac{b\left(2\csc^2(c+dx) - \sec^4(c+dx) - 4\sec^2(c+dx) - 12\log(\sin(c+dx))\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*Csc[c + d*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[c + d*x]^2])/(3*d)
- (b*(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Se
c[c + d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

Maple [A] time = 0.068, size = 173, normalized size = 1.1

$$\frac{a}{4d(\sin(dx+c))^3(\cos(dx+c))^4} - \frac{7a}{12d(\sin(dx+c))^3(\cos(dx+c))^2} + \frac{35a}{24d\sin(dx+c)(\cos(dx+c))^2} - \frac{35a}{8d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/sin(d*x+c)^3/cos(d*x+c)^4-7/12/d*a/sin(d*x+c)^3/cos(d*x+c)^2+35/24/
d*a/sin(d*x+c)/cos(d*x+c)^2-35/8/d*a/sin(d*x+c)+35/8/d*a*ln(sec(d*x+c)+tan(
d*x+c))+1/4/d*b/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*b/sin(d*x+c)^2/cos(d*x+c)^2
-3/2/d*b/sin(d*x+c)^2+3*b*ln(tan(d*x+c))/d
```

Maxima [A] time = 0.978455, size = 204, normalized size = 1.32

$$\frac{3(35a-24b)\log(\sin(dx+c)+1) - 3(35a+24b)\log(\sin(dx+c)-1) + 144b\log(\sin(dx+c)) - \frac{2\left(105a\sin(dx+c)^6+36b\right.}{}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(35*a - 24*b)*log(sin(d*x + c) + 1) - 3*(35*a + 24*b)*log(sin(d*x +
 c) - 1) + 144*b*log(sin(d*x + c)) - 2*(105*a*sin(d*x + c)^6 + 36*b*sin(d*x
 + c)^5 - 175*a*sin(d*x + c)^4 - 54*b*sin(d*x + c)^3 + 56*a*sin(d*x + c)^2
+ 12*b*sin(d*x + c) + 8*a)/(sin(d*x + c)^7 - 2*sin(d*x + c)^5 + sin(d*x + c
)^3))/d
```

Fricas [A] time = 2.14723, size = 649, normalized size = 4.19

$$-\frac{210a\cos(dx+c)^6 - 280a\cos(dx+c)^4 + 42a\cos(dx+c)^2 - 144\left(b\cos(dx+c)^6 - b\cos(dx+c)^4\right)\log\left(\frac{1}{2}\sin(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(210*a*cos(d*x + c)^6 - 280*a*cos(d*x + c)^4 + 42*a*cos(d*x + c)^2 -
144*(b*cos(d*x + c)^6 - b*cos(d*x + c)^4)*log(1/2*sin(d*x + c))*sin(d*x + c
) - 3*((35*a - 24*b)*cos(d*x + c)^6 - (35*a - 24*b)*cos(d*x + c)^4)*log(sin
(d*x + c) + 1)*sin(d*x + c) + 3*((35*a + 24*b)*cos(d*x + c)^6 - (35*a + 24*
b)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1)*sin(d*x + c) - 12*(6*b*cos(d*x +
c)^4 - 3*b*cos(d*x + c)^2 - b)*sin(d*x + c) + 12*a)/((d*cos(d*x + c)^6 - d*
```

```
cos(d*x + c)^4)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27312, size = 216, normalized size = 1.39

$$\frac{3\,(35\,a-24\,b)\log\left(|\sin\left(dx+c\right)+1|\right)-3\,(35\,a+24\,b)\log\left(|\sin\left(dx+c\right)-1|\right)+144\,b\log\left(|\sin\left(dx+c\right)|\right)+\frac{6\left(18\,b\sin(dx+c)\right.}{}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/48*(3*(35*a - 24*b)*log(abs(sin(d*x + c) + 1)) - 3*(35*a + 24*b)*log(abs(
sin(d*x + c) - 1)) + 144*b*log(abs(sin(d*x + c))) + 6*(18*b*sin(d*x + c)^4
- 11*a*sin(d*x + c)^3 - 44*b*sin(d*x + c)^2 + 13*a*sin(d*x + c) + 28*b)/(si
n(d*x + c)^2 - 1)^2 - 8*(33*b*sin(d*x + c)^3 + 18*a*sin(d*x + c)^2 + 3*b*si
n(d*x + c) + 2*a)/sin(d*x + c)^3)/d
```

3.1492 $\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=189

$$-\frac{(a^2+3b^2)\sin(c+dx)}{d}-\frac{(15a^2+48ab+35b^2)\log(1-\sin(c+dx))}{16d}+\frac{(15a^2-48ab+35b^2)\log(\sin(c+dx)+1)}{16d}-\frac{ab\,\mathrm{s}}{}$$

```
[Out] -((15*a^2 + 48*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + ((15*a^2 - 48*
a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - ((a^2 + 3*b^2)*Sin[c + d*x])/
d - (a*b*Sin[c + d*x]^2)/d - (b^2*Sin[c + d*x]^3)/(3*d) - (Sec[c + d*x]^2*(
11*b + 9*a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a +
 b*Sin[c + d*x])^2*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.358873, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2837, 12, 1645, 1810, 633, 31}

$$-\frac{(a^2+3b^2)\sin(c+dx)}{d}-\frac{(15a^2+48ab+35b^2)\log(1-\sin(c+dx))}{16d}+\frac{(15a^2-48ab+35b^2)\log(\sin(c+dx)+1)}{16d}-\frac{ab\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((15*a^2 + 48*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + ((15*a^2 - 48*
a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - ((a^2 + 3*b^2)*Sin[c + d*x])/
d - (a*b*Sin[c + d*x]^2)/d - (b^2*Sin[c + d*x]^3)/(3*d) - (Sec[c + d*x]^2*(
11*b + 9*a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a +
 b*Sin[c + d*x])^2*Tan[c + d*x])/(4*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^6(a+x)^2}{b^6\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)\left(-ab\right.}{\ldots}\right.}{\ldots}$$

$$= -\frac{\sec^2(c+dx)(11b+9a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\sec^3(c\ldots}{}$$

$$= -\frac{\sec^2(c+dx)(11b+9a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\sec^3(c\ldots}{}$$

$$= -\frac{\left(a^2+3b^2\right)\sin(c+dx)}{d} - \frac{ab\sin^2(c+dx)}{d} - \frac{b^2\sin^3(c+dx)}{3d} - \frac{\text{se}\ldots}{}$$

$$= -\frac{\left(a^2+3b^2\right)\sin(c+dx)}{d} - \frac{ab\sin^2(c+dx)}{d} - \frac{b^2\sin^3(c+dx)}{3d} - \frac{\text{se}\ldots}{}$$

$$= -\frac{\left(15a^2+48ab+35b^2\right)\log(1-\sin(c+dx))}{16d} + \frac{\left(15a^2-48ab+35\ldots\right.}{}$$

Mathematica [A] time = 1.58653, size = 186, normalized size = 0.98

$$\frac{-48\left(a^2+3b^2\right)\sin(c+dx) - 3\left(15a^2+48ab+35b^2\right)\log(1-\sin(c+dx)) + 3\left(15a^2-48ab+35b^2\right)\log(\sin(c+dx)+1\ldots}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-3*(15*a^2 + 48*a*b + 35*b^2)*Log[1 - Sin[c + d*x]] + 3*(15*a^2 - 48*a*b +
 35*b^2)*Log[1 + Sin[c + d*x]] + (3*(a + b)^2)/(-1 + Sin[c + d*x])^2 + (3*(
a + b)*(9*a + 13*b))/(-1 + Sin[c + d*x]) - 48*(a^2 + 3*b^2)*Sin[c + d*x] -
48*a*b*Sin[c + d*x]^2 - 16*b^2*Sin[c + d*x]^3 - (3*(a - b)^2)/(1 + Sin[c +
```

```
d*x])^2 + (3*(9*a - 13*b)*(a - b))/(1 + Sin[c + d*x]))/(48*d)
```

Maple [A] time = 0.077, size = 355, normalized size = 1.9

$$\frac{a^2 (\sin(dx+c))^7}{4d (\cos(dx+c))^4} - \frac{3a^2 (\sin(dx+c))^7}{8d (\cos(dx+c))^2} - \frac{3 (\sin(dx+c))^5 a^2}{8d} - \frac{5a^2 (\sin(dx+c))^3}{8d} - \frac{15a^2 \sin(dx+c)}{8d} + \frac{15a^2 \ln(\sec(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a^2*sin(d*x+c)^7/cos(d*x+c)^2-3/8
*a^2*sin(d*x+c)^5/d-5/8*a^2*sin(d*x+c)^3/d-15/8*a^2*sin(d*x+c)/d+15/8/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b*sin(d*x+c)^8/cos(d*x+c)^4-1/d*a*b*sin(
d*x+c)^8/cos(d*x+c)^2-a*b*sin(d*x+c)^6/d-3/2*a*b*sin(d*x+c)^4/d-3*a*b*sin(d
*x+c)^2/d-6/d*a*b*ln(cos(d*x+c))+1/4/d*b^2*sin(d*x+c)^9/cos(d*x+c)^4-5/8/d*
b^2*sin(d*x+c)^9/cos(d*x+c)^2-5/8/d*b^2*sin(d*x+c)^7-7/8*b^2*sin(d*x+c)^5/d
-35/24*b^2*sin(d*x+c)^3/d-35/8*b^2*sin(d*x+c)/d+35/8/d*b^2*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 0.996051, size = 243, normalized size = 1.29

$$-\frac{16 b^2 \sin(dx+c)^3 + 48 ab \sin(dx+c)^2 - 3\left(15a^2 - 48ab + 35b^2\right)\log(\sin(dx+c)+1) + 3\left(15a^2 + 48ab + 35b^2\right)\log(\ldots}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(16*b^2*sin(d*x + c)^3 + 48*a*b*sin(d*x + c)^2 - 3*(15*a^2 - 48*a*b +
 35*b^2)*log(sin(d*x + c) + 1) + 3*(15*a^2 + 48*a*b + 35*b^2)*log(sin(d*x +
 c) - 1) + 48*(a^2 + 3*b^2)*sin(d*x + c) - 6*(24*a*b*sin(d*x + c)^2 + (9*a^
2 + 13*b^2)*sin(d*x + c)^3 - 20*a*b - (7*a^2 + 11*b^2)*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.52149, size = 502, normalized size = 2.66

$$\frac{48 ab \cos(dx+c)^6 - 24 ab \cos(dx+c)^4 + 3\left(15a^2 - 48ab + 35b^2\right)\cos(dx+c)^4 \log(\sin(dx+c)+1) - 3\left(15a^2 + 48ab \ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(48*a*b*cos(d*x + c)^6 - 24*a*b*cos(d*x + c)^4 + 3*(15*a^2 - 48*a*b +
35*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(15*a^2 + 48*a*b + 35*b^2)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 144*a*b*cos(d*x + c)^2 + 24*a*b +
2*(8*b^2*cos(d*x + c)^6 - 8*(3*a^2 + 10*b^2)*cos(d*x + c)^4 - 3*(9*a^2 + 13
```

```
*b^2)*cos(d*x + c)^2 + 6*a^2 + 6*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6*(a+b*sin(d*x+c))**2,x)

[Out] Timed out
```

Giac [A] time = 1.27115, size = 267, normalized size = 1.41

$$-\frac{16\,b^2\sin\left(dx+c\right)^3+48\,ab\sin\left(dx+c\right)^2+48\,a^2\sin\left(dx+c\right)+144\,b^2\sin\left(dx+c\right)-3\left(15\,a^2-48\,ab+35\,b^2\right)\log\left(|\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/48*(16*b^2*sin(d*x + c)^3 + 48*a*b*sin(d*x + c)^2 + 48*a^2*sin(d*x + c)
+ 144*b^2*sin(d*x + c) - 3*(15*a^2 - 48*a*b + 35*b^2)*log(abs(sin(d*x + c)
+ 1)) + 3*(15*a^2 + 48*a*b + 35*b^2)*log(abs(sin(d*x + c) - 1)) - 6*(36*a*b
*sin(d*x + c)^4 + 9*a^2*sin(d*x + c)^3 + 13*b^2*sin(d*x + c)^3 - 48*a*b*sin
(d*x + c)^2 - 7*a^2*sin(d*x + c) - 11*b^2*sin(d*x + c) + 16*a*b)/(sin(d*x +
 c)^2 - 1)^2)/d
```

3.1493 $\int (a+b\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=162

$$-\frac{\left(4a^2+15ab+12b^2\right)\log(1-\sin(c+dx))}{8d}+\frac{\left(15ab-4\left(a^2+3b^2\right)\right)\log(\sin(c+dx)+1)}{8d}-\frac{2ab\sin(c+dx)}{d}+\frac{\sec^4(c+dx)}{}$$

```
[Out] -((4*a^2 + 15*a*b + 12*b^2)*Log[1 - Sin[c + d*x]])/(8*d) + ((15*a*b - 4*(a^
2 + 3*b^2))*Log[1 + Sin[c + d*x]])/(8*d) - (2*a*b*Sin[c + d*x])/d - (b^2*Si
n[c + d*x]^2)/(2*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(4*d) - (Sec[
c + d*x]^2*(a + b*Sin[c + d*x])*(4*a + 5*b*Sin[c + d*x]))/(4*d)
```

Rubi [A] time = 0.27027, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2721, 1645, 1810, 633, 31}

$$-\frac{\left(4a^2+15ab+12b^2\right)\log(1-\sin(c+dx))}{8d}+\frac{\left(15ab-4\left(a^2+3b^2\right)\right)\log(\sin(c+dx)+1)}{8d}-\frac{2ab\sin(c+dx)}{d}+\frac{\sec^4(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((4*a^2 + 15*a*b + 12*b^2)*Log[1 - Sin[c + d*x]])/(8*d) + ((15*a*b - 4*(a^
2 + 3*b^2))*Log[1 + Sin[c + d*x]])/(8*d) - (2*a*b*Sin[c + d*x])/d - (b^2*Si
n[c + d*x]^2)/(2*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(4*d) - (Sec[
c + d*x]^2*(a + b*Sin[c + d*x])*(4*a + 5*b*Sin[c + d*x]))/(4*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)^2}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)\left(-2b^6-4ab^4x-4b^4x^2-4ab^2x^3-4b^2x^4\right.}{(b^2-x^2)^2}\right.}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))(4a+5b\sin}{4d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))(4a+5b\sin}{4d}$$

$$= -\frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \sec$$

$$= -\frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \sec$$

$$= -\frac{(4a^2+15ab+12b^2)\log(1-\sin(c+dx))}{8d} - \frac{(4a^2-15ab+12b^2)\log(1+\sin}{8d}$$

Mathematica [A] time = 2.15948, size = 164, normalized size = 1.01

$$\frac{-2\left(4a^2+15ab+12b^2\right)\log(1-\sin(c+dx))-2\left(4a^2-15ab+12b^2\right)\log(\sin(c+dx)+1)+\frac{(a-b)^2}{(\sin(c+dx)+1)^2}-\frac{(7a-11b)(a-b)}{\sin(c+dx)+1}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-2*(4*a^2 + 15*a*b + 12*b^2)*Log[1 - Sin[c + d*x]] - 2*(4*a^2 - 15*a*b + 1
2*b^2)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 + ((a + b)*(
7*a + 11*b))/(-1 + Sin[c + d*x]) - 32*a*b*Sin[c + d*x] - 8*b^2*Sin[c + d*x]
^2 + (a - b)^2/(1 + Sin[c + d*x])^2 - ((7*a - 11*b)*(a - b))/(1 + Sin[c + d
*x]))/(16*d)
```

Maple [A] time = 0.076, size = 270, normalized size = 1.7

$$\frac{a^2(\tan(dx+c))^4}{4d} - \frac{a^2(\tan(dx+c))^2}{2d} - \frac{a^2\ln(\cos(dx+c))}{d} + \frac{ab(\sin(dx+c))^7}{2d(\cos(dx+c))^4} - \frac{3ab(\sin(dx+c))^7}{4d(\cos(dx+c))^2} - \frac{3ab(\sin(d}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*tan(d*x+c)^2-1/d*a^2*ln(cos(d*x+c))+1/2/d*
a*b*sin(d*x+c)^7/cos(d*x+c)^4-3/4/d*a*b*sin(d*x+c)^7/cos(d*x+c)^2-3/4*a*b*s
in(d*x+c)^5/d-5/4*a*b*sin(d*x+c)^3/d-15/4*a*b*sin(d*x+c)/d+15/4/d*a*b*ln(se
c(d*x+c)+tan(d*x+c))+1/4/d*b^2*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*b^2*sin(d*x+
c)^8/cos(d*x+c)^2-1/2*b^2*sin(d*x+c)^6/d-3/4*b^2*sin(d*x+c)^4/d-3/2*b^2*sin
(d*x+c)^2/d-3/d*b^2*ln(cos(d*x+c))
```

Maxima [A] time = 0.993926, size = 212, normalized size = 1.31

$$-\frac{4\,b^2\sin\left(dx+c\right)^2+16\,ab\sin\left(dx+c\right)+\left(4\,a^2-15\,ab+12\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(4\,a^2+15\,ab+12\,b^2\right)\log\left(\sin\left(dx\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(4*b^2*sin(d*x + c)^2 + 16*a*b*sin(d*x + c) + (4*a^2 - 15*a*b + 12*b^2
)*log(sin(d*x + c) + 1) + (4*a^2 + 15*a*b + 12*b^2)*log(sin(d*x + c) - 1) -
 2*(9*a*b*sin(d*x + c)^3 - 7*a*b*sin(d*x + c) + 2*(2*a^2 + 3*b^2)*sin(d*x +
 c)^2 - 3*a^2 - 5*b^2)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.44838, size = 436, normalized size = 2.69

$$\frac{4\,b^2\cos\left(dx+c\right)^6-2\,b^2\cos\left(dx+c\right)^4-\left(4\,a^2-15\,ab+12\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(4\,a^2+15\,ab+12\,b^2\right)\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(4*b^2*cos(d*x + c)^6 - 2*b^2*cos(d*x + c)^4 - (4*a^2 - 15*a*b + 12*b^2
)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (4*a^2 + 15*a*b + 12*b^2)*cos(d*x
+ c)^4*log(-sin(d*x + c) + 1) - 4*(2*a^2 + 3*b^2)*cos(d*x + c)^2 + 2*a^2 +
2*b^2 - 2*(8*a*b*cos(d*x + c)^4 + 9*a*b*cos(d*x + c)^2 - 2*a*b)*sin(d*x + c
))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25318, size = 236, normalized size = 1.46

$$-\frac{4\,b^{2}\sin\left(dx+c\right)^{2}+16\,ab\sin\left(dx+c\right)+\left(4\,a^{2}-15\,ab+12\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(4\,a^{2}+15\,ab+12\,b^{2}\right)\log\left(\left|\sin\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*sin(d*x + c)^2 + 16*a*b*sin(d*x + c) + (4*a^2 - 15*a*b + 12*b^2
)*log(abs(sin(d*x + c) + 1)) + (4*a^2 + 15*a*b + 12*b^2)*log(abs(sin(d*x +
c) - 1)) - 2*(3*a^2*sin(d*x + c)^4 + 9*b^2*sin(d*x + c)^4 + 9*a*b*sin(d*x +
 c)^3 - 2*a^2*sin(d*x + c)^2 - 12*b^2*sin(d*x + c)^2 - 7*a*b*sin(d*x + c) +
 4*b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1494 $\int \sec(c+dx)(a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=150

$$-\frac{(3a^2+16ab+15b^2)\log(1-\sin(c+dx))}{16d}+\frac{(3a^2-16ab+15b^2)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)(5a\sin(c+dx)+7b}{8d}$$

```
[Out] -((3*a^2 + 16*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + ((3*a^2 - 16*a*
b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (b^2*Sin[c + d*x])/d - (Sec[c +
 d*x]^2*(7*b + 5*a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x
]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.284945, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2837, 12, 1645, 1810, 633, 31}

$$-\frac{(3a^2+16ab+15b^2)\log(1-\sin(c+dx))}{16d}+\frac{(3a^2-16ab+15b^2)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)(5a\sin(c+dx)+7b}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -((3*a^2 + 16*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + ((3*a^2 - 16*a*
b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (b^2*Sin[c + d*x])/d - (Sec[c +
 d*x]^2*(7*b + 5*a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x
]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x])/(4*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = \frac{b^5\operatorname{Subst}\left(\int \frac{x^4(a+x)^2}{b^4\left(b^2-x^2\right)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{Subst}\left(\int \frac{x^4(a+x)^2}{\left(b^2-x^2\right)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{(a+x)\left(-ab\right.}{}\right.}{}$$

$$= -\frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\sec^3(c+}{}$$

$$= -\frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\sec^3(c+}{}$$

$$= -\frac{b^2\sin(c+dx)}{d} - \frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\sin(c+}{8d}$$

$$= -\frac{b^2\sin(c+dx)}{d} - \frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\sin(c+}{8d}$$

$$= -\frac{\left(3a^2+16ab+15b^2\right)\log(1-\sin(c+dx))}{16d} - \frac{\left(16ab-3\left(a^2+5b^2\right.\right.}{1}$$

Mathematica [A] time = 1.03108, size = 151, normalized size = 1.01

$$\frac{-\left(3a^2+16ab+15b^2\right)\log(1-\sin(c+dx))+\left(3a^2-16ab+15b^2\right)\log(\sin(c+dx)+1)-\frac{(a-b)^2}{(\sin(c+dx)+1)^2}+\frac{(5a-9b)(a-b)}{\sin(c+dx)+1}+\frac{(}{s}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (-((3*a^2 + 16*a*b + 15*b^2)*Log[1 - Sin[c + d*x]]) + (3*a^2 - 16*a*b + 15*
b^2)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 + ((a + b)*(5*
a + 9*b))/(-1 + Sin[c + d*x]) - 16*b^2*Sin[c + d*x] - (a - b)^2/(1 + Sin[c
+ d*x])^2 + ((5*a - 9*b)*(a - b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A] time = 0.073, size = 262, normalized size = 1.8

$$\frac{(\sin(dx+c))^5 a^2}{4d(\cos(dx+c))^4} - \frac{(\sin(dx+c))^5 a^2}{8d(\cos(dx+c))^2} - \frac{a^2(\sin(dx+c))^3}{8d} - \frac{3a^2\sin(dx+c)}{8d} + \frac{3a^2\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2-1/8
*a^2*sin(d*x+c)^3/d-3/8*a^2*sin(d*x+c)/d+3/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/2/d*a*b*tan(d*x+c)^4-1/d*a*b*tan(d*x+c)^2-2/d*a*b*ln(cos(d*x+c))+1/4/d*
b^2*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*b^2*sin(d*x+c)^7/cos(d*x+c)^2-3/8*b^2*s
in(d*x+c)^5/d-5/8*b^2*sin(d*x+c)^3/d-15/8*b^2*sin(d*x+c)/d+15/8/d*b^2*ln(se
c(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.969534, size = 200, normalized size = 1.33

$$-\frac{16\,b^2\sin(dx+c) - \left(3\,a^2 - 16\,ab + 15\,b^2\right)\log(\sin(dx+c)+1) + \left(3\,a^2 + 16\,ab + 15\,b^2\right)\log(\sin(dx+c)-1) - \frac{2\left(16\,ab\sin\cdots\right.}{\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*b^2*sin(d*x + c) - (3*a^2 - 16*a*b + 15*b^2)*log(sin(d*x + c) + 1
) + (3*a^2 + 16*a*b + 15*b^2)*log(sin(d*x + c) - 1) - 2*(16*a*b*sin(d*x + c
)^2 + (5*a^2 + 9*b^2)*sin(d*x + c)^3 - 12*a*b - (3*a^2 + 7*b^2)*sin(d*x + c
))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.22936, size = 374, normalized size = 2.49

$$\frac{\left(3\,a^2 - 16\,ab + 15\,b^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1) - \left(3\,a^2 + 16\,ab + 15\,b^2\right)\cos(dx+c)^4\log(-\sin(dx+c)+1) - 3\cdots}{16\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/16*((3*a^2 - 16*a*b + 15*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a
^2 + 16*a*b + 15*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 32*a*b*cos(d*
x + c)^2 + 8*a*b - 2*(8*b^2*cos(d*x + c)^4 + (5*a^2 + 9*b^2)*cos(d*x + c)^2
 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23788, size = 212, normalized size = 1.41

$$-\frac{16\,b^2\sin\left(dx+c\right)-\left(3\,a^2-16\,ab+15\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(3\,a^2+16\,ab+15\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(12\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(16*b^2*sin(d*x + c) - (3*a^2 - 16*a*b + 15*b^2)*log(abs(sin(d*x + c)
 + 1)) + (3*a^2 + 16*a*b + 15*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(12*a*b*s
in(d*x + c)^4 + 5*a^2*sin(d*x + c)^3 + 9*b^2*sin(d*x + c)^3 - 8*a*b*sin(d*x
 + c)^2 - 3*a^2*sin(d*x + c) - 7*b^2*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/
d
```

3.1495 $\int \sec^2(c+dx)(a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{b(3a+4b)\log(1-\sin(c+dx))}{8d}+\frac{b(3a-4b)\log(\sin(c+dx)+1)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{\sec^2(c+dx)(a+}{}$$

```
[Out] -(b*(3*a + 4*b)*Log[1 - Sin[c + d*x]])/(8*d) + ((3*a - 4*b)*b*Log[1 + Sin[c
 + d*x]])/(8*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(4*d) - (Sec[c +
d*x]^2*(a + b*Sin[c + d*x])*(2*a + 3*b*Sin[c + d*x]))/(4*d)
```

Rubi [A] time = 0.223091, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2837, 12, 1645, 633, 31}

$$-\frac{b(3a+4b)\log(1-\sin(c+dx))}{8d}+\frac{b(3a-4b)\log(\sin(c+dx)+1)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{\sec^2(c+dx)(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] -(b*(3*a + 4*b)*Log[1 - Sin[c + d*x]])/(8*d) + ((3*a - 4*b)*b*Log[1 + Sin[c
 + d*x]])/(8*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(4*d) - (Sec[c +
d*x]^2*(a + b*Sin[c + d*x])*(2*a + 3*b*Sin[c + d*x]))/(4*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{b^5\operatorname{Subst}\left(\int \frac{x^3(a+x)^2}{b^3(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\operatorname{Subst}\left(\int \frac{x^3(a+x)^2}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} + \frac{\operatorname{Subst}\left(\int \frac{(a+x)(-2b^4-4ab^2x-4b^2x}{(b^2-x^2)^2}\right.}{4d}$$
$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx}{4d}$$
$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx}{4d}$$
$$= -\frac{b(3a+4b)\log(1-\sin(c+dx))}{8d} + \frac{(3a-4b)b\log(1+\sin(c+d}{8d}$$

Mathematica [A] time = 0.364141, size = 129, normalized size = 1.11

$$\frac{a^2\tan^4(c+dx)}{4d} + \frac{2ab\tan^3(c+dx)\sec(c+dx)}{d} - \frac{ab\left(6\tan(c+dx)\sec^3(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (2*a*b*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a^2*Tan[c + d*x]^4)/(4*d) - (b^2*(
4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d) - (a*b*(6*S
ec[c + d*x]^3*Tan[c + d*x] - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(4*d)
```

Maple [A] time = 0.067, size = 168, normalized size = 1.5

$$\frac{(\sin(dx+c))^4 a^2}{4d(\cos(dx+c))^4} + \frac{ab(\sin(dx+c))^5}{2d(\cos(dx+c))^4} - \frac{ab(\sin(dx+c))^5}{4d(\cos(dx+c))^2} - \frac{ab(\sin(dx+c))^3}{4d} - \frac{3ab\sin(dx+c)}{4d} + \frac{3ab\ln(\sec(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^4/cos(d*x+c)^4+1/2/d*a*b*sin(d*x+c)^5/cos(d*x+c)^4-1/4
/d*a*b*sin(d*x+c)^5/cos(d*x+c)^2-1/4*a*b*sin(d*x+c)^3/d-3/4*a*b*sin(d*x+c)/
d+3/4/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b^2*tan(d*x+c)^4-1/2/d*b^2*tan(
```

```
d*x+c)^2-1/d*b^2*ln(cos(d*x+c))
```

Maxima [A] time = 1.00683, size = 166, normalized size = 1.43

$$\frac{(3\,ab-4\,b^2)\log\left(\sin\left(dx+c\right)+1\right)-(3\,ab+4\,b^2)\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\,(5\,ab\sin(dx+c)^3-3\,ab\sin(dx+c)+2\,(a^2+2\,b^2)\sin(dx+c)^2-a^2-}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*((3*a*b - 4*b^2)*log(sin(d*x + c) + 1) - (3*a*b + 4*b^2)*log(sin(d*x +
c) - 1) + 2*(5*a*b*sin(d*x + c)^3 - 3*a*b*sin(d*x + c) + 2*(a^2 + 2*b^2)*si
n(d*x + c)^2 - a^2 - 3*b^2)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.1148, size = 313, normalized size = 2.7

$$\frac{(3\,ab-4\,b^2)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-(3\,ab+4\,b^2)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-4\,(a^2+2\,b^2)\cos\left(d\right.}{8\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*((3*a*b - 4*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a*b + 4*b^2)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 4*(a^2 + 2*b^2)*cos(d*x + c)^2 + 2
*a^2 + 2*b^2 - 2*(5*a*b*cos(d*x + c)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x +
c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23428, size = 176, normalized size = 1.52

$$\frac{(3\,ab-4\,b^2)\log\left(|\sin\left(dx+c\right)+1|\right)-(3\,ab+4\,b^2)\log\left(|\sin\left(dx+c\right)-1|\right)+\frac{2\,(3\,b^2\sin(dx+c)^4+5\,ab\sin(dx+c)^3+2\,a^2\sin(dx+c)^2-2\,b^2\sin}{(\sin(dx+c)^2-1)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a*b - 4*b^2)*log(abs(sin(d*x + c) + 1)) - (3*a*b + 4*b^2)*log(abs(s
in(d*x + c) - 1)) + 2*(3*b^2*sin(d*x + c)^4 + 5*a*b*sin(d*x + c)^3 + 2*a^2*
sin(d*x + c)^2 - 2*b^2*sin(d*x + c)^2 - 3*a*b*sin(d*x + c) - a^2)/(sin(d*x
+ c)^2 - 1)^2)/d
```

3.1496 $\int \sec^3(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=93

$$-\frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\sec^2(c+dx)\left((a^2+3b^2)\sin(c+dx)+4ab\right)}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^2}{4d}$$

```
[Out] -((a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - (Sec[c + d*x]^2*(4*a*b + (a^
2 + 3*b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Ta
n[c + d*x])/(4*d)
```

Rubi [A] time = 0.194086, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2837, 12, 1645, 778, 206}

$$-\frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\sec^2(c+dx)\left((a^2+3b^2)\sin(c+dx)+4ab\right)}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -((a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - (Sec[c + d*x]^2*(4*a*b + (a^
2 + 3*b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Ta
n[c + d*x])/(4*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
```

```
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^2(a+x)^2}{b^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^3\,\text{Subst}\left(\int \frac{x^2(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)}{4d} + \frac{b\,\text{Subst}\left(\int \frac{(a+x)(}{(b}\right.}{}$$
$$= -\frac{\sec^2(c+dx)\left(4ab+\left(a^2+3b^2\right)\sin(c+dx)\right)}{8d} + \frac{\sec^3(c+dx)(a+}{}$$
$$= -\frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{\sec^2(c+dx)\left(4ab+\left(a^2+3b\right.\right.}{8d}$$

Mathematica [A] time = 0.746953, size = 85, normalized size = 0.91

$$-\frac{\left(a^2-3b^2\right)\tanh^{-1}(\sin(c+dx)) + \frac{1}{4}\sec^4(c+dx)\left(2\sin(c+dx)\left(\left(a^2+5b^2\right)\cos(2(c+dx)) - 3a^2+b^2\right) + 16ab\cos(2(c\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -((a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]] + (Sec[c + d*x]^4*(16*a*b*Cos[2*(c +
d*x)] + 2*(-3*a^2 + b^2 + (a^2 + 5*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/4)
/(8*d)
```

Maple [B] time = 0.062, size = 209, normalized size = 2.3

$$\frac{a^2(\sin(dx+c))^3}{4d(\cos(dx+c))^4} + \frac{a^2(\sin(dx+c))^3}{8d(\cos(dx+c))^2} + \frac{a^2\sin(dx+c)}{8d} - \frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{ab(\sin(dx+c))^4}{2d(\cos(dx+c))^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8
*a^2*sin(d*x+c)/d-1/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b*sin(d*x+c)^
4/cos(d*x+c)^4+1/4/d*b^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*b^2*sin(d*x+c)^5/c
os(d*x+c)^2-1/8*b^2*sin(d*x+c)^3/d-3/8*b^2*sin(d*x+c)/d+3/8/d*b^2*ln(sec(d*
x+c)+tan(d*x+c))
```

Maxima [A] time = 0.976745, size = 162, normalized size = 1.74

$$-\frac{\left(a^2-3\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-3\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,\left(8\,ab\sin(dx+c)^2+\left(a^2+5\,b^2\right)\sin(dx+c)^3-4\,ab+\left(a^2-3\,b^2\right)\sin(dx+c\right.}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/16*((a^2 - 3*b^2)*log(sin(d*x + c) + 1) - (a^2 - 3*b^2)*log(sin(d*x + c)
 - 1) - 2*(8*a*b*sin(d*x + c)^2 + (a^2 + 5*b^2)*sin(d*x + c)^3 - 4*a*b + (a
^2 - 3*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.02974, size = 309, normalized size = 3.32

$$-\frac{\left(a^2-3\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-3\,b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+16\,ab\cos\left(dx+c\right)^2-8}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/16*((a^2 - 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^2 - 3*b^2)*c
os(d*x + c)^4*log(-sin(d*x + c) + 1) + 16*a*b*cos(d*x + c)^2 - 8*a*b + 2*((
a^2 + 5*b^2)*cos(d*x + c)^2 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^
4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27209, size = 167, normalized size = 1.8

$$-\frac{\left(a^2-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(a^2-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(a^2\sin(dx+c)^3+5\,b^2\sin(dx+c)^3+8\,ab\sin(dx+c)^2+a^2\sin(dx+c\right.}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*((a^2 - 3*b^2)*log(abs(sin(d*x + c) + 1)) - (a^2 - 3*b^2)*log(abs(sin
(d*x + c) - 1)) - 2*(a^2*sin(d*x + c)^3 + 5*b^2*sin(d*x + c)^3 + 8*a*b*sin(
d*x + c)^2 + a^2*sin(d*x + c) - 3*b^2*sin(d*x + c) - 4*a*b)/(sin(d*x + c)^2
 - 1)^2)/d
```

3.1497 $\int \sec^4(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=72

$$-\frac{\sec^2(c+dx)\left(ab\sin(c+dx)+b^2\right)}{4d}-\frac{ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}$$

```
[Out] -(a*b*ArcTanh[Sin[c + d*x]])/(4*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2
)/(4*d) - (Sec[c + d*x]^2*(b^2 + a*b*Sin[c + d*x]))/(4*d)
```

Rubi [A] time = 0.0988589, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2837, 12, 821, 639, 206}

$$-\frac{\sec^2(c+dx)\left(ab\sin(c+dx)+b^2\right)}{4d}-\frac{ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -(a*b*ArcTanh[Sin[c + d*x]])/(4*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2
)/(4*d) - (Sec[c + d*x]^2*(b^2 + a*b*Sin[c + d*x]))/(4*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x(a+x)^2}{b\left(b^2-x^2\right)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\,\text{Subst}\left(\int \frac{x(a+x)^2}{\left(b^2-x^2\right)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{b^2\,\text{Subst}\left(\int \frac{2b^2(a+x)}{\left(b^2-x^2\right)^2}\,dx,x,b\sin\ldots\right.}{4d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{b^4\,\text{Subst}\left(\int \frac{a+x}{\left(b^2-x^2\right)^2}\,dx,x,b\sin\ldots\right.}{2d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)\left(b^2+ab\sin(c+\ldots\right.}{4d}$$

$$= -\frac{ab\tanh^{-1}(\sin(c+dx))}{4d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \ldots$$

Mathematica [B] time = 2.7383, size = 215, normalized size = 2.99

$$\frac{2b^4\left(b^2-a^2\right)\tan^4(c+dx)+b\left(4a^2b^3-6a^4b\right)\tan^2(c+dx)+2a^4\left(a^2-b^2\right)\sec^4(c+dx)+2a^4b^2\sec^2(c+dx)+ab\left(a^2\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (a*b*(a^2 - b^2)^2*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^4*
b^2*Sec[c + d*x]^2 + 2*a^4*(a^2 - b^2)*Sec[c + d*x]^4 + 4*a^3*b*(a^2 - b^2)
*Sec[c + d*x]^3*Tan[c + d*x] + b*(-6*a^4*b + 4*a^2*b^3)*Tan[c + d*x]^2 + 2*
b^4*(-a^2 + b^2)*Tan[c + d*x]^4 - 2*a*b*(a^2 - b^2)*Sec[c + d*x]*Tan[c + d*
x]*(a^2 + b^2 + 2*b^2*Tan[c + d*x]^2))/(8*(a^2 - b^2)^2*d)
```

Maple [A] time = 0.053, size = 122, normalized size = 1.7

$$\frac{a^2}{4\,d\,(\cos(dx+c))^4} + \frac{ab\,(\sin(dx+c))^3}{2\,d\,(\cos(dx+c))^4} + \frac{ab\,(\sin(dx+c))^3}{4\,d\,(\cos(dx+c))^2} + \frac{ab\sin(dx+c)}{4\,d} - \frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{4\,d} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/cos(d*x+c)^4+1/2/d*a*b*sin(d*x+c)^3/cos(d*x+c)^4+1/4/d*a*b*sin(d*
x+c)^3/cos(d*x+c)^2+1/4*a*b*sin(d*x+c)/d-1/4/d*a*b*ln(sec(d*x+c)+tan(d*x+c)
)+1/4/d*b^2*sin(d*x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 0.989732, size = 131, normalized size = 1.82

$$-\frac{ab\log\left(\sin\left(dx+c\right)+1\right)-ab\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(ab\sin(dx+c)^3+2\,b^2\sin(dx+c)^2+ab\sin(dx+c)+a^2-b^2\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a*b*log(sin(d*x + c) + 1) - a*b*log(sin(d*x + c) - 1) - 2*(a*b*sin(d*
x + c)^3 + 2*b^2*sin(d*x + c)^2 + a*b*sin(d*x + c) + a^2 - b^2)/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.96922, size = 266, normalized size = 3.69

$$-\frac{ab\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-ab\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+4\,b^2\cos\left(dx+c\right)^2-2\,a^2-2\,b^2+2\left(ab\right.}{8\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(a*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 4*b^2*cos(d*x + c)^2 - 2*a^2 - 2*b^2 + 2*(a*b*cos(d*x + c
)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24516, size = 120, normalized size = 1.67

$$-\frac{ab\log\left(|\sin\left(dx+c\right)+1|\right)-ab\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(ab\sin(dx+c)^3+2\,b^2\sin(dx+c)^2+ab\sin(dx+c)+a^2-b^2\right)}{\left(\sin(dx+c)^2-1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(a*b*log(abs(sin(d*x + c) + 1)) - a*b*log(abs(sin(d*x + c) - 1)) - 2*(
a*b*sin(d*x + c)^3 + 2*b^2*sin(d*x + c)^2 + a*b*sin(d*x + c) + a^2 - b^2)/(
sin(d*x + c)^2 - 1)^2)/d
```

3.1498 $\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{\sec^4(c+dx)\left(a^2+2ab\sin(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a(4a+3b)\log(1-\sin(c+dx))}{8d}-\frac{a(4a-3b)\log(\sin}{8d}$$

```
[Out] -(a*(4*a + 3*b)*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[Sin[c + d*x]])/d -
(a*(4*a - 3*b)*Log[1 + Sin[c + d*x]])/(8*d) + (a*Sec[c + d*x]^2*(2*a + 3*b*
Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^4*(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(4
*d)
```

Rubi [A] time = 0.217093, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2837, 12, 1805, 823, 801}

$$\frac{\sec^4(c+dx)\left(a^2+2ab\sin(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a(4a+3b)\log(1-\sin(c+dx))}{8d}-\frac{a(4a-3b)\log(\sin}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*(4*a + 3*b)*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[Sin[c + d*x]])/d -
(a*(4*a - 3*b)*Log[1 + Sin[c + d*x]])/(8*d) + (a*Sec[c + d*x]^2*(2*a + 3*b*
Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^4*(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(4
*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b(a+x)^2}{x\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\,\text{Subst}\left(\int \frac{(a+x)^2}{x\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} - \frac{b^4\,\text{Subst}\left(\int \frac{-4a^2-6ax}{x\left(b^2-x^2\right)^2}\,dx, x\right.}{4d}$$

$$= \frac{a\sec^2(c+dx)(2a+3b\sin(c+dx))}{4d} + \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin\right.}{4d}$$

$$= \frac{a\sec^2(c+dx)(2a+3b\sin(c+dx))}{4d} + \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin\right.}{4d}$$

$$= -\frac{a(4a+3b)\log(1-\sin(c+dx))}{8d} + \frac{a^2\log(\sin(c+dx))}{d} - \frac{a(4a-3b)}{}$$

Mathematica [A] time = 0.917146, size = 137, normalized size = 1.09

$$\frac{16a^2\log(\sin(c+dx)) - \frac{(a+b)(5a+b)}{\sin(c+dx)-1} + \frac{(a-b)(5a-b)}{\sin(c+dx)+1} + \frac{(a+b)^2}{(\sin(c+dx)-1)^2} + \frac{(a-b)^2}{(\sin(c+dx)+1)^2} - 2a(4a+3b)\log(1-\sin(c+dx)) - 2a(4a}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*(4*a + 3*b)*Log[1 - Sin[c + d*x]] + 16*a^2*Log[Sin[c + d*x]] - 2*a*(4
*a - 3*b)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 - ((a + b
)*(5*a + b))/(-1 + Sin[c + d*x]) + (a - b)^2/(1 + Sin[c + d*x])^2 + ((a - b
)*(5*a - b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A] time = 0.089, size = 125, normalized size = 1.

$$\frac{a^2}{4d(\cos(dx+c))^4} + \frac{a^2}{2d(\cos(dx+c))^2} + \frac{a^2\ln(\tan(dx+c))}{d} + \frac{ab\tan(dx+c)(\sec(dx+c))^3}{2d} + \frac{3ab\tan(dx+c)\sec(d}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/cos(d*x+c)^4+1/2/d*a^2/cos(d*x+c)^2+1/d*a^2*ln(tan(d*x+c))+1/2/d*
a*b*tan(d*x+c)*sec(d*x+c)^3+3/4/d*a*b*tan(d*x+c)*sec(d*x+c)+3/4/d*a*b*ln(se
c(d*x+c)+tan(d*x+c))+1/4/d*b^2/cos(d*x+c)^4
```

Maxima [A] time = 0.982631, size = 176, normalized size = 1.4

$$\frac{8\,a^2\log\left(\sin\left(dx+c\right)\right)-\left(4\,a^2-3\,ab\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(4\,a^2+3\,ab\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,ab\sin(dx+c)^3+2\,a^2\sin\right.}{\sin(dx+c)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*a^2*log(sin(d*x + c)) - (4*a^2 - 3*a*b)*log(sin(d*x + c) + 1) - (4*a
^2 + 3*a*b)*log(sin(d*x + c) - 1) - 2*(3*a*b*sin(d*x + c)^3 + 2*a^2*sin(d*x
 + c)^2 - 5*a*b*sin(d*x + c) - 3*a^2 - b^2)/(sin(d*x + c)^4 - 2*sin(d*x + c
)^2 + 1))/d
```

Fricas [A] time = 2.02736, size = 360, normalized size = 2.86

$$\frac{8\,a^2\cos\left(dx+c\right)^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(4\,a^2-3\,ab\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(4\,a^2+3\,ab\right)\cos\left(dx+c\right)^4\,\mathrm{lo}}{8\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^2*cos(d*x + c)^4*log(1/2*sin(d*x + c)) - (4*a^2 - 3*a*b)*cos(d*x +
 c)^4*log(sin(d*x + c) + 1) - (4*a^2 + 3*a*b)*cos(d*x + c)^4*log(-sin(d*x +
 c) + 1) + 4*a^2*cos(d*x + c)^2 + 2*a^2 + 2*b^2 + 2*(3*a*b*cos(d*x + c)^2 +
 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23934, size = 181, normalized size = 1.44

$$\frac{8\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\left(4\,a^2-3\,ab\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(4\,a^2+3\,ab\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{2\left(3\,a^2\sin(dx+c)^4-3\,a\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*log(abs(sin(d*x + c))) - (4*a^2 - 3*a*b)*log(abs(sin(d*x + c) +
1)) - (4*a^2 + 3*a*b)*log(abs(sin(d*x + c) - 1)) + 2*(3*a^2*sin(d*x + c)^4
- 3*a*b*sin(d*x + c)^3 - 8*a^2*sin(d*x + c)^2 + 5*a*b*sin(d*x + c) + 6*a^2
+ b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1499 $\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$-\frac{(15a^2+16ab+3b^2)\log(1-\sin(c+dx))}{16d}+\frac{(15a^2-16ab+3b^2)\log(\sin(c+dx)+1)}{16d}+\frac{b\sec^4(c+dx)\left(\frac{(a^2+b^2)\sin(c+d}{b}\right.}{4d}$$

```
[Out] -((a^2*Csc[c + d*x])/d) - ((15*a^2 + 16*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])
/(16*d) + (2*a*b*Log[Sin[c + d*x]])/d + ((15*a^2 - 16*a*b + 3*b^2)*Log[1 +
Sin[c + d*x]])/(16*d) + (b*Sec[c + d*x]^2*(8*a + (3 + (7*a^2)/b^2)*b*Sin[c
+ d*x]))/(8*d) + (b*Sec[c + d*x]^4*(2*a + ((a^2 + b^2)*Sin[c + d*x])/b))/(4
*d)
```

Rubi [A] time = 0.353948, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1805, 1802}

$$-\frac{(15a^2+16ab+3b^2)\log(1-\sin(c+dx))}{16d}+\frac{(15a^2-16ab+3b^2)\log(\sin(c+dx)+1)}{16d}+\frac{b\sec^4(c+dx)\left(\frac{(a^2+b^2)\sin(c+d}{b}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) - ((15*a^2 + 16*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])
/(16*d) + (2*a*b*Log[Sin[c + d*x]])/d + ((15*a^2 - 16*a*b + 3*b^2)*Log[1 +
Sin[c + d*x]])/(16*d) + (b*Sec[c + d*x]^2*(8*a + (3 + (7*a^2)/b^2)*b*Sin[c
+ d*x]))/(8*d) + (b*Sec[c + d*x]^4*(2*a + ((a^2 + b^2)*Sin[c + d*x])/b))/(4
*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^2(a+x)^2}{x^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)^2}{x^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^4(c+dx)\left(2a+\frac{\left(a^2+b^2\right)\sin(c+dx)}{b}\right)}{4d} - \frac{b^5\,\text{Subst}\left(\int \frac{-4a^2-8ax-3\left(1+\frac{a^2}{b^2}\right)}{x^2\left(b^2-x^2\right)^2}\right.}{4d}$$

$$= \frac{b\sec^2(c+dx)\left(8a+\left(3+\frac{7a^2}{b^2}\right)b\sin(c+dx)\right)}{8d} + \frac{b\sec^4(c+dx)\left(2a+\right.}{4d}$$

$$= \frac{b\sec^2(c+dx)\left(8a+\left(3+\frac{7a^2}{b^2}\right)b\sin(c+dx)\right)}{8d} + \frac{b\sec^4(c+dx)\left(2a+\right.}{4d}$$

$$= -\frac{a^2\csc(c+dx)}{d} - \frac{\left(15a^2+16ab+3b^2\right)\log(1-\sin(c+dx))}{16d} + \frac{2ab\,l}{}$$

Mathematica [A] time = 2.87704, size = 162, normalized size = 0.96

$$-\frac{\left(15a^2+16ab+3b^2\right)\log(1-\sin(c+dx))-\left(15a^2-16ab+3b^2\right)\log(\sin(c+dx)+1)+16a^2\csc(c+dx)+\frac{(a+b)(7a+3b)}{\sin(c+dx)-1}+}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(16*a^2*Csc[c + d*x] + (15*a^2 + 16*a*b + 3*b^2)*Log[1 - Sin[c + d*x]] - 3
2*a*b*Log[Sin[c + d*x]] - (15*a^2 - 16*a*b + 3*b^2)*Log[1 + Sin[c + d*x]] -
 (a + b)^2/(-1 + Sin[c + d*x])^2 + ((a + b)*(7*a + 3*b))/(-1 + Sin[c + d*x]
) + (a - b)^2/(1 + Sin[c + d*x])^2 + ((7*a - 3*b)*(a - b))/(1 + Sin[c + d*x
]))/(16*d)
```

Maple [A] time = 0.088, size = 195, normalized size = 1.2

$$\frac{a^2}{4\,d\sin(dx+c)(\cos(dx+c))^4} + \frac{5\,a^2}{8\,d\sin(dx+c)(\cos(dx+c))^2} - \frac{15\,a^2}{8\,d\sin(dx+c)} + \frac{15\,a^2\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a^2/sin(d*x+c)/cos(d*x+c)^2-15/8/d*
a^2/sin(d*x+c)+15/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b/cos(d*x+c)^4+
1/d*a*b/cos(d*x+c)^2+2/d*a*b*ln(tan(d*x+c))+1/4/d*b^2*tan(d*x+c)*sec(d*x+c)
^3+3/8/d*b^2*tan(d*x+c)*sec(d*x+c)+3/8/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01052, size = 220, normalized size = 1.31

$$\frac{32\,ab\log\left(\sin\left(dx+c\right)\right)+\left(15\,a^2-16\,ab+3\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(15\,a^2+16\,ab+3\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/16*(32*a*b*log(sin(d*x + c)) + (15*a^2 - 16*a*b + 3*b^2)*log(sin(d*x + c)
 + 1) - (15*a^2 + 16*a*b + 3*b^2)*log(sin(d*x + c) - 1) - 2*(8*a*b*sin(d*x
+ c)^3 + 3*(5*a^2 + b^2)*sin(d*x + c)^4 - 12*a*b*sin(d*x + c) - 5*(5*a^2 +
b^2)*sin(d*x + c)^2 + 8*a^2)/(sin(d*x + c)^5 - 2*sin(d*x + c)^3 + sin(d*x +
 c)))/d
```

Fricas [A] time = 2.11247, size = 516, normalized size = 3.07

$$\frac{32\,ab\cos\left(dx+c\right)^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(15\,a^2-16\,ab+3\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/16*(32*a*b*cos(d*x + c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) + (15*a^2 -
16*a*b + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)*sin(d*x + c) - (15*a^2
 + 16*a*b + 3*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)*sin(d*x + c) - 6*(
5*a^2 + b^2)*cos(d*x + c)^4 + 2*(5*a^2 + b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^
2 + 8*(2*a*b*cos(d*x + c)^2 + a*b)*sin(d*x + c))/(d*cos(d*x + c)^4*sin(d*x
+ c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27147, size = 251, normalized size = 1.49

$$\frac{32\,ab\log\left(|\sin\left(dx+c\right)|\right)+\left(15\,a^2-16\,ab+3\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)-\left(15\,a^2+16\,ab+3\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)-\ldots}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(32*a*b*log(abs(sin(d*x + c))) + (15*a^2 - 16*a*b + 3*b^2)*log(abs(sin
(d*x + c) + 1)) - (15*a^2 + 16*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1)) - 16
*(2*a*b*sin(d*x + c) + a^2)/sin(d*x + c) + 2*(12*a*b*sin(d*x + c)^4 - 7*a^2
*sin(d*x + c)^3 - 3*b^2*sin(d*x + c)^3 - 32*a*b*sin(d*x + c)^2 + 9*a^2*sin(
d*x + c) + 5*b^2*sin(d*x + c) + 24*a*b)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1500 $\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=185

$$-\frac{(12a^2+15ab+4b^2)\log(1-\sin(c+dx))}{8d}+\frac{(3a^2+b^2)\log(\sin(c+dx))}{d}-\frac{(12a^2-15ab+4b^2)\log(\sin(c+dx)+1)}{8d}$$

```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((12*a^2 + 15*a*b +
4*b^2)*Log[1 - Sin[c + d*x]])/(8*d) + ((3*a^2 + b^2)*Log[Sin[c + d*x]])/d -
 ((12*a^2 - 15*a*b + 4*b^2)*Log[1 + Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*
(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*(2*(2*a^2 + b^2)
+ 7*a*b*Sin[c + d*x]))/(4*d)
```

Rubi [A] time = 0.38378, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1805, 1802}

$$-\frac{(12a^2+15ab+4b^2)\log(1-\sin(c+dx))}{8d}+\frac{(3a^2+b^2)\log(\sin(c+dx))}{d}-\frac{(12a^2-15ab+4b^2)\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((12*a^2 + 15*a*b +
4*b^2)*Log[1 - Sin[c + d*x]])/(8*d) + ((3*a^2 + b^2)*Log[Sin[c + d*x]])/d -
 ((12*a^2 - 15*a*b + 4*b^2)*Log[1 + Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*
(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*(2*(2*a^2 + b^2)
+ 7*a*b*Sin[c + d*x]))/(4*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^3(a+x)^2}{x^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\,\text{Subst}\left(\int \frac{(a+x)^2}{x^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} - \frac{b^6\,\text{Subst}\left(\int \frac{-4a^2-8ax-4(1\cdots}{x^3(b^2-\cdots}\right.}{\cdots}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} + \frac{\sec^2(c+dx)\left(2\left(2a^2+b^2\cdots\right.\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} + \frac{\sec^2(c+dx)\left(2\left(2a^2+b^2\cdots\right.\right.}{4d}$$

$$= -\frac{2ab\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{\left(12a^2+15ab+4b^2\right)\log(1-\text{s}\cdots}{8d}$$

Mathematica [A] time = 3.75378, size = 182, normalized size = 0.98

$$\frac{-2\left(12a^2+15ab+4b^2\right)\log(1-\sin(c+dx))+16\left(3a^2+b^2\right)\log(\sin(c+dx))-2\left(12a^2-15ab+4b^2\right)\log(\sin(c+dx)+1)\cdots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-32*a*b*Csc[c + d*x] - 8*a^2*Csc[c + d*x]^2 - 2*(12*a^2 + 15*a*b + 4*b^2)*
Log[1 - Sin[c + d*x]] + 16*(3*a^2 + b^2)*Log[Sin[c + d*x]] - 2*(12*a^2 - 15
*a*b + 4*b^2)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 - ((a
 + b)*(9*a + 5*b))/(-1 + Sin[c + d*x]) + (a - b)^2/(1 + Sin[c + d*x])^2 + (
(9*a - 5*b)*(a - b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A] time = 0.098, size = 209, normalized size = 1.1

$$\frac{a^2}{4d(\sin(dx+c))^2(\cos(dx+c))^4} + \frac{3a^2}{4d(\sin(dx+c))^2(\cos(dx+c))^2} - \frac{3a^2}{2d(\sin(dx+c))^2} + 3\frac{a^2\ln(\tan(dx+c))}{d} + \frac{\cdots}{2d\,\text{s}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*a^2/sin(d*x+c)^2/cos(d*x+c)^2-3/2
/d*a^2/sin(d*x+c)^2+3/d*a^2*ln(tan(d*x+c))+1/2/d*a*b/sin(d*x+c)/cos(d*x+c)^
4+5/4/d*a*b/sin(d*x+c)/cos(d*x+c)^2-15/4/d*a*b/sin(d*x+c)+15/4/d*a*b*ln(sec
(d*x+c)+tan(d*x+c))+1/4/d*b^2/cos(d*x+c)^4+1/2/d*b^2/cos(d*x+c)^2+1/d*b^2*l
```

```
n(tan(d*x+c))
```

Maxima [A] time = 0.992481, size = 247, normalized size = 1.34

$$-\frac{\left(12\,a^2-15\,ab+4\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(12\,a^2+15\,ab+4\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-8\left(3\,a^2+b^2\right)\log\left(\sin\left(dx\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*((12*a^2 - 15*a*b + 4*b^2)*log(sin(d*x + c) + 1) + (12*a^2 + 15*a*b +
4*b^2)*log(sin(d*x + c) - 1) - 8*(3*a^2 + b^2)*log(sin(d*x + c)) + 2*(15*a*
b*sin(d*x + c)^5 - 25*a*b*sin(d*x + c)^3 + 2*(3*a^2 + b^2)*sin(d*x + c)^4 +
 8*a*b*sin(d*x + c) - 3*(3*a^2 + b^2)*sin(d*x + c)^2 + 2*a^2)/(sin(d*x + c)
^6 - 2*sin(d*x + c)^4 + sin(d*x + c)^2))/d
```

Fricas [A] time = 2.25345, size = 683, normalized size = 3.69

$$\frac{4\left(3\,a^2+b^2\right)\cos\left(dx+c\right)^4-2\left(3\,a^2+b^2\right)\cos\left(dx+c\right)^2-2\,a^2-2\,b^2+8\left(\left(3\,a^2+b^2\right)\cos\left(dx+c\right)^6-\left(3\,a^2+b^2\right)\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(4*(3*a^2 + b^2)*cos(d*x + c)^4 - 2*(3*a^2 + b^2)*cos(d*x + c)^2 - 2*a^
2 - 2*b^2 + 8*((3*a^2 + b^2)*cos(d*x + c)^6 - (3*a^2 + b^2)*cos(d*x + c)^4)
*log(1/2*sin(d*x + c)) - ((12*a^2 - 15*a*b + 4*b^2)*cos(d*x + c)^6 - (12*a^
2 - 15*a*b + 4*b^2)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - ((12*a^2 + 15*a
*b + 4*b^2)*cos(d*x + c)^6 - (12*a^2 + 15*a*b + 4*b^2)*cos(d*x + c)^4)*log(
-sin(d*x + c) + 1) + 2*(15*a*b*cos(d*x + c)^4 - 5*a*b*cos(d*x + c)^2 - 2*a*
b)*sin(d*x + c))/(d*cos(d*x + c)^6 - d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27644, size = 257, normalized size = 1.39

$$-\frac{\left(12\,a^2-15\,ab+4\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)+\left(12\,a^2+15\,ab+4\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)-8\left(3\,a^2+b^2\right)\log\left(|\sin\left(dx+\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*((12*a^2 - 15*a*b + 4*b^2)*log(abs(sin(d*x + c) + 1)) + (12*a^2 + 15*a
*b + 4*b^2)*log(abs(sin(d*x + c) - 1)) - 8*(3*a^2 + b^2)*log(abs(sin(d*x +
c))) + 2*(15*a*b*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^4 + 2*b^2*sin(d*x + c)
^4 - 25*a*b*sin(d*x + c)^3 - 9*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 +
8*a*b*sin(d*x + c) + 2*a^2)/(sin(d*x + c)^3 - sin(d*x + c))^2)/d
```

3.1501 $\int (a+b\sin(c+dx))^3\tan^5(c+dx)\,dx$

Optimal. Leaf size=202

$$-\frac{b\left(24a^2+35b^2\right)\sin(c+dx)}{8d}-\frac{(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))}{16d}-\frac{(a-b)\left(8a^2-37ab+35b^2\right)\log(\sin$$

```
[Out] -((a + b)*(8*a^2 + 37*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*d) - ((a - b
)*(8*a^2 - 37*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (b*(24*a^2 + 35
*b^2)*Sin[c + d*x])/(8*d) - (3*a*b^2*Sin[c + d*x]^2)/(2*d) - (b^3*Sin[c + d
*x]^3)/(3*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(4*d) - (Sec[c + d*x
]^2*(a + b*Sin[c + d*x])^2*(8*a + 11*b*Sin[c + d*x]))/(8*d)
```

Rubi [A] time = 0.34084, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2721, 1645, 1629, 633, 31}

$$-\frac{b\left(24a^2+35b^2\right)\sin(c+dx)}{8d}-\frac{(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))}{16d}-\frac{(a-b)\left(8a^2-37ab+35b^2\right)\log(\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -((a + b)*(8*a^2 + 37*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*d) - ((a - b
)*(8*a^2 - 37*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (b*(24*a^2 + 35
*b^2)*Sin[c + d*x])/(8*d) - (3*a*b^2*Sin[c + d*x]^2)/(2*d) - (b^3*Sin[c + d
*x]^3)/(3*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(4*d) - (Sec[c + d*x
]^2*(a + b*Sin[c + d*x])^2*(8*a + 11*b*Sin[c + d*x]))/(8*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(-3b^6-4ab^4x-4b^4x^2-4ab^2x^3-4b^2x^4\right)}{\left(b^2-x^2\right)^2}\,d\ldots\right.}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2(8a+11b\sin\ldots}{8d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2(8a+11b\sin\ldots}{8d}$$

$$= -\frac{b\left(24a^2+35b^2\right)\sin(c+dx)}{8d} - \frac{3ab^2\sin^2(c+dx)}{2d} - \frac{b^3\sin^3(c+dx)}{3d} + \frac{\sec^4(c+\ldots}{}$$

$$= -\frac{b\left(24a^2+35b^2\right)\sin(c+dx)}{8d} - \frac{3ab^2\sin^2(c+dx)}{2d} - \frac{b^3\sin^3(c+dx)}{3d} + \frac{\sec^4(c+\ldots}{}$$

$$= -\frac{(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))}{16d} - \frac{(a-b)\left(8a^2-37ab+35b^2\right)\ldots}{16d}$$

Mathematica [A] time = 1.0589, size = 199, normalized size = 0.99

$$-\frac{144b\left(a^2+b^2\right)\sin(c+dx)+3\left(8a^2-37ab+35b^2\right)(a-b)\log(\sin(c+dx)+1)+3(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\text{s}\ldots}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -(3*(a + b)*(8*a^2 + 37*a*b + 35*b^2)*Log[1 - Sin[c + d*x]] + 3*(a - b)*(8*
a^2 - 37*a*b + 35*b^2)*Log[1 + Sin[c + d*x]] - (3*(a + b)^3)/(-1 + Sin[c +
d*x])^2 - (3*(a + b)^2*(7*a + 13*b))/(-1 + Sin[c + d*x]) + 144*b*(a^2 + b^2
)*Sin[c + d*x] + 72*a*b^2*Sin[c + d*x]^2 + 16*b^3*Sin[c + d*x]^3 - (3*(a -
b)^3)/(1 + Sin[c + d*x])^2 + (3*(7*a - 13*b)*(a - b)^2)/(1 + Sin[c + d*x]))
/(48*d)
```

Maple [B] time = 0.091, size = 420, normalized size = 2.1

$$\frac{a^3(\tan(dx+c))^4}{4d} - \frac{a^3(\tan(dx+c))^2}{2d} - \frac{a^3\ln(\cos(dx+c))}{d} + \frac{3a^2b(\sin(dx+c))^7}{4d(\cos(dx+c))^4} - \frac{9a^2b(\sin(dx+c))^7}{8d(\cos(dx+c))^2} - \frac{9a^2b(\sin(d\ldots}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)^4-1/2/d*a^3*tan(d*x+c)^2-1/d*a^3*ln(cos(d*x+c))+3/4/d*
a^2*b*sin(d*x+c)^7/cos(d*x+c)^4-9/8/d*a^2*b*sin(d*x+c)^7/cos(d*x+c)^2-9/8/d
*a^2*b*sin(d*x+c)^5-15/8/d*a^2*b*sin(d*x+c)^3-45/8/d*sin(d*x+c)*a^2*b+45/8/
d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a*b^2*sin(d*x+c)^8/cos(d*x+c)^4-3/2
/d*a*b^2*sin(d*x+c)^8/cos(d*x+c)^2-3/2/d*a*b^2*sin(d*x+c)^6-9/4/d*a*b^2*sin
(d*x+c)^4-9/2*a*b^2*sin(d*x+c)^2/d-9/d*a*b^2*ln(cos(d*x+c))+1/4/d*b^3*sin(d
*x+c)^9/cos(d*x+c)^4-5/8/d*b^3*sin(d*x+c)^9/cos(d*x+c)^2-5/8/d*b^3*sin(d*x+
c)^7-7/8/d*sin(d*x+c)^5*b^3-35/24*b^3*sin(d*x+c)^3/d-35/8*b^3*sin(d*x+c)/d+
35/8/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.00375, size = 293, normalized size = 1.45

$$-\frac{16\,b^3\sin\left(dx+c\right)^3+72\,ab^2\sin\left(dx+c\right)^2+3\left(8\,a^3-45\,a^2b+72\,ab^2-35\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(8\,a^3+45\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/48*(16*b^3*sin(d*x + c)^3 + 72*a*b^2*sin(d*x + c)^2 + 3*(8*a^3 - 45*a^2*
b + 72*a*b^2 - 35*b^3)*log(sin(d*x + c) + 1) + 3*(8*a^3 + 45*a^2*b + 72*a*b
^2 + 35*b^3)*log(sin(d*x + c) - 1) + 144*(a^2*b + b^3)*sin(d*x + c) - 6*((2
7*a^2*b + 13*b^3)*sin(d*x + c)^3 - 6*a^3 - 30*a*b^2 + 4*(2*a^3 + 9*a*b^2)*s
in(d*x + c)^2 - (21*a^2*b + 11*b^3)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1))/d
```

Fricas [A] time = 2.24481, size = 583, normalized size = 2.89

$$\frac{72\,ab^2\cos\left(dx+c\right)^6-36\,ab^2\cos\left(dx+c\right)^4-3\left(8\,a^3-45\,a^2b+72\,ab^2-35\,b^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/48*(72*a*b^2*cos(d*x + c)^6 - 36*a*b^2*cos(d*x + c)^4 - 3*(8*a^3 - 45*a^2
*b + 72*a*b^2 - 35*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*a^3 + 4
5*a^2*b + 72*a*b^2 + 35*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 12*a^3
 + 36*a*b^2 - 24*(2*a^3 + 9*a*b^2)*cos(d*x + c)^2 + 2*(8*b^3*cos(d*x + c)^6
 - 8*(9*a^2*b + 10*b^3)*cos(d*x + c)^4 + 18*a^2*b + 6*b^3 - 3*(27*a^2*b + 1
3*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24366, size = 339, normalized size = 1.68

$$-\frac{16\,b^{3}\sin\left(dx+c\right)^{3}+72\,ab^{2}\sin\left(dx+c\right)^{2}+144\,a^{2}b\sin\left(dx+c\right)+144\,b^{3}\sin\left(dx+c\right)+3\left(8\,a^{3}-45\,a^{2}b+72\,ab^{2}-35\,b^{3}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/48*(16*b^3*sin(d*x + c)^3 + 72*a*b^2*sin(d*x + c)^2 + 144*a^2*b*sin(d*x
+ c) + 144*b^3*sin(d*x + c) + 3*(8*a^3 - 45*a^2*b + 72*a*b^2 - 35*b^3)*log(
abs(sin(d*x + c) + 1)) + 3*(8*a^3 + 45*a^2*b + 72*a*b^2 + 35*b^3)*log(abs(s
in(d*x + c) - 1)) - 6*(6*a^3*sin(d*x + c)^4 + 54*a*b^2*sin(d*x + c)^4 + 27*
a^2*b*sin(d*x + c)^3 + 13*b^3*sin(d*x + c)^3 - 4*a^3*sin(d*x + c)^2 - 72*a*
b^2*sin(d*x + c)^2 - 21*a^2*b*sin(d*x + c) - 11*b^3*sin(d*x + c) + 24*a*b^2
)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1502 $\int \sec(c+dx)(a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=177

$$-\frac{3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3(a-b)\left(a^2-7ab+8b^2\right)\log(\sin(c+dx)+1)}{16d}-\frac{29ab^2\sin(c+dx)}{8d}$$

```
[Out] (-3*(a + b)*(a^2 + 7*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + (3*(a - b
)*(a^2 - 7*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (29*a*b^2*Sin[c + d
*x])/(8*d) - (b^3*Sin[c + d*x]^2)/(2*d) - (Sec[c + d*x]^2*(8*b + 5*a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^2)/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x]
)^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.367833, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2837, 12, 1645, 1629, 633, 31}

$$-\frac{3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3(a-b)\left(a^2-7ab+8b^2\right)\log(\sin(c+dx)+1)}{16d}-\frac{29ab^2\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (-3*(a + b)*(a^2 + 7*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + (3*(a - b
)*(a^2 - 7*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (29*a*b^2*Sin[c + d
*x])/(8*d) - (b^3*Sin[c + d*x]^2)/(2*d) - (Sec[c + d*x]^2*(8*b + 5*a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^2)/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x]
)^3*Tan[c + d*x])/(4*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^3\tan^4(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^4(a+x)^3}{b^4\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{x^4(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(-ab^4-\right.}{(}\right.}{}$$

$$= -\frac{\sec^2(c+dx)(8b+5a\sin(c+dx))(a+b\sin(c+dx))^2}{8d} + \frac{\sec^3(c+d}{}$$

$$= -\frac{\sec^2(c+dx)(8b+5a\sin(c+dx))(a+b\sin(c+dx))^2}{8d} + \frac{\sec^3(c+d}{}$$

$$= -\frac{29ab^2\sin(c+dx)}{8d} - \frac{b^3\sin^2(c+dx)}{2d} - \frac{\sec^2(c+dx)(8b+5a\sin(c+}{8d}$$

$$= -\frac{29ab^2\sin(c+dx)}{8d} - \frac{b^3\sin^2(c+dx)}{2d} - \frac{\sec^2(c+dx)(8b+5a\sin(c+}{8d}$$

$$= -\frac{3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))}{16d} + \frac{3(a-b)\left(a^2-7ab\right.}{}$$

Mathematica [A] time = 0.575711, size = 174, normalized size = 0.98

$$\frac{3\left(a^2-7ab+8b^2\right)(a-b)\log(\sin(c+dx)+1) - 3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx)) - 48ab^2\sin(c+dx) - \frac{(}{(\sin(}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (-3*(a + b)*(a^2 + 7*a*b + 8*b^2)*Log[1 - Sin[c + d*x]] + 3*(a - b)*(a^2 -
7*a*b + 8*b^2)*Log[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x])^2 + ((
a + b)^2*(5*a + 11*b))/(-1 + Sin[c + d*x]) - 48*a*b^2*Sin[c + d*x] - 8*b^3*
Sin[c + d*x]^2 - (a - b)^3/(1 + Sin[c + d*x])^2 + ((5*a - 11*b)*(a - b)^2)/
```

```
(1 + Sin[c + d*x]))/(16*d)
```

Maple [B] time = 0.086, size = 385, normalized size = 2.2

$$\frac{a^3 (\sin (dx+c))^5}{4 d (\cos (dx+c))^4}-\frac{a^3 (\sin (dx+c))^5}{8 d (\cos (dx+c))^2}-\frac{a^3 (\sin (dx+c))^3}{8 d}-\frac{3 a^3 \sin (dx+c)}{8 d}+\frac{3 a^3 \ln (\sec (dx+c)+\tan (dx+c))}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2-1/8
*a^3*sin(d*x+c)^3/d-3/8*a^3*sin(d*x+c)/d+3/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c)
)+3/4/d*a^2*b*tan(d*x+c)^4-3/2/d*a^2*b*tan(d*x+c)^2-3/d*a^2*b*ln(cos(d*x+c)
)+3/4/d*a*b^2*sin(d*x+c)^7/cos(d*x+c)^4-9/8/d*a*b^2*sin(d*x+c)^7/cos(d*x+c)
^2-9/8/d*a*b^2*sin(d*x+c)^5-15/8/d*a*b^2*sin(d*x+c)^3-45/8*a*b^2*sin(d*x+c)
/d+45/8/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b^3*sin(d*x+c)^8/cos(d*x+c)
^4-1/2/d*b^3*sin(d*x+c)^8/cos(d*x+c)^2-1/2/d*b^3*sin(d*x+c)^6-3/4/d*sin(d*x
+c)^4*b^3-3/2*b^3*sin(d*x+c)^2/d-3/d*b^3*ln(cos(d*x+c))
```

Maxima [A] time = 0.998339, size = 257, normalized size = 1.45

$$-\frac{8 b^3 \sin (dx+c)^2+48 ab^2 \sin (dx+c)-3 \left(a^3-8 a^2 b+15 ab^2-8 b^3\right) \log (\sin (dx+c)+1)+3 \left(a^3+8 a^2 b+15 ab^2\right.}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(8*b^3*sin(d*x + c)^2 + 48*a*b^2*sin(d*x + c) - 3*(a^3 - 8*a^2*b + 15
*a*b^2 - 8*b^3)*log(sin(d*x + c) + 1) + 3*(a^3 + 8*a^2*b + 15*a*b^2 + 8*b^3
)*log(sin(d*x + c) - 1) - 2*((5*a^3 + 27*a*b^2)*sin(d*x + c)^3 - 18*a^2*b -
 10*b^3 + 12*(2*a^2*b + b^3)*sin(d*x + c)^2 - 3*(a^3 + 7*a*b^2)*sin(d*x + c
))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.23466, size = 508, normalized size = 2.87

$$\frac{8 b^3 \cos (dx+c)^6-4 b^3 \cos (dx+c)^4+3 \left(a^3-8 a^2 b+15 ab^2-8 b^3\right) \cos (dx+c)^4 \log (\sin (dx+c)+1)-3 \left(a^3+8 a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(8*b^3*cos(d*x + c)^6 - 4*b^3*cos(d*x + c)^4 + 3*(a^3 - 8*a^2*b + 15*a
*b^2 - 8*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^3 + 8*a^2*b + 15*
a*b^2 + 8*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 12*a^2*b + 4*b^3 - 2
4*(2*a^2*b + b^3)*cos(d*x + c)^2 - 2*(24*a*b^2*cos(d*x + c)^4 - 2*a^3 - 6*a
```

```
*b^2 + (5*a^3 + 27*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28398, size = 298, normalized size = 1.68

$$-\frac{8\,b^3\sin\left(dx+c\right)^2+48\,ab^2\sin\left(dx+c\right)-3\left(a^3-8\,a^2b+15\,ab^2-8\,b^3\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\left(a^3+8\,a^2b+15\,ab^2+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(8*b^3*sin(d*x + c)^2 + 48*a*b^2*sin(d*x + c) - 3*(a^3 - 8*a^2*b + 15
*a*b^2 - 8*b^3)*log(abs(sin(d*x + c) + 1)) + 3*(a^3 + 8*a^2*b + 15*a*b^2 +
8*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(18*a^2*b*sin(d*x + c)^4 + 18*b^3*sin
(d*x + c)^4 + 5*a^3*sin(d*x + c)^3 + 27*a*b^2*sin(d*x + c)^3 - 12*a^2*b*sin
(d*x + c)^2 - 24*b^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - 21*a*b^2*sin(d*x
 + c) + 8*b^3)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1503 $\int \sec^2(c+dx)(a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=142

$$-\frac{3b(a+b)(3a+5b)\log(1-\sin(c+dx))}{16d}+\frac{3b(3a-5b)(a-b)\log(\sin(c+dx)+1)}{16d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}$$

```
[Out] (-3*b*(a + b)*(3*a + 5*b)*Log[1 - Sin[c + d*x]])/(16*d) + (3*(3*a - 5*b)*(a
 - b)*b*Log[1 + Sin[c + d*x]])/(16*d) - (15*b^3*Sin[c + d*x])/(8*d) + (Sec[
c + d*x]^4*(a + b*Sin[c + d*x])^3)/(4*d) - (Sec[c + d*x]^2*(a + b*Sin[c + d
*x])^2*(4*a + 7*b*Sin[c + d*x]))/(8*d)
```

Rubi [A] time = 0.303608, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2837, 12, 1645, 774, 633, 31}

$$-\frac{3b(a+b)(3a+5b)\log(1-\sin(c+dx))}{16d}+\frac{3b(3a-5b)(a-b)\log(\sin(c+dx)+1)}{16d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```



```
[Out] (-3*b*(a + b)*(3*a + 5*b)*Log[1 - Sin[c + d*x]])/(16*d) + (3*(3*a - 5*b)*(a
 - b)*b*Log[1 + Sin[c + d*x]])/(16*d) - (15*b^3*Sin[c + d*x])/(8*d) + (Sec[
c + d*x]^4*(a + b*Sin[c + d*x])^3)/(4*d) - (Sec[c + d*x]^2*(a + b*Sin[c + d
*x])^2*(4*a + 7*b*Sin[c + d*x]))/(8*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
```

```
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^3(a+x)^3}{b^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\,\text{Subst}\left(\int \frac{x^3(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(-3b^4-4ab^2x-4b^2x^2\right)}{\left(b^2-x^2\right)^2}\right.}{4d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2(}{8d}$$

$$= -\frac{15b^3\sin(c+dx)}{8d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx}{}$$

$$= -\frac{15b^3\sin(c+dx)}{8d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx}{}$$

$$= -\frac{3b(a+b)(3a+5b)\log(1-\sin(c+dx))}{16d} + \frac{3(3a-5b)(a-b)b\log(1}{16d}$$

Mathematica [A] time = 0.452349, size = 147, normalized size = 1.04

$$\frac{\frac{(a-b)^3}{(\sin(c+dx)+1)^2} - \frac{3(a-3b)(a-b)^2}{\sin(c+dx)+1} + \frac{3(a+b)^2(a+3b)}{\sin(c+dx)-1} + \frac{(a+b)^3}{(\sin(c+dx)-1)^2} + 3b(3a-5b)(a-b)\log(\sin(c+dx)+1) - 3b(a+b)(3a+5b)\log}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (-3*b*(a + b)*(3*a + 5*b)*Log[1 - Sin[c + d*x]] + 3*(3*a - 5*b)*(a - b)*b*L
og[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x])^2 + (3*(a + b)^2*(a +
3*b))/(-1 + Sin[c + d*x]) - 16*b^3*Sin[c + d*x] + (a - b)^3/(1 + Sin[c + d*
x])^2 - (3*(a - 3*b)*(a - b)^2)/(1 + Sin[c + d*x]))/(16*d)
```

Maple [B] time = 0.085, size = 297, normalized size = 2.1

$$\frac{a^3(\sin(dx+c))^4}{4d(\cos(dx+c))^4} + \frac{3a^2b(\sin(dx+c))^5}{4d(\cos(dx+c))^4} - \frac{3a^2b(\sin(dx+c))^5}{8d(\cos(dx+c))^2} - \frac{3a^2b(\sin(dx+c))^3}{8d} - \frac{9a^2b\sin(dx+c)}{8d} + \frac{9a^2b\ln(\text{se}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3/4/d*a^2*b*sin(d*x+c)^5/cos(d*x+c)^4-3
/8/d*a^2*b*sin(d*x+c)^5/cos(d*x+c)^2-3/8/d*a^2*b*sin(d*x+c)^3-9/8/d*sin(d*x
+c)*a^2*b+9/8/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a*b^2*tan(d*x+c)^4-3/
2/d*a*b^2*tan(d*x+c)^2-3/d*a*b^2*ln(cos(d*x+c))+1/4/d*b^3*sin(d*x+c)^7/cos(
d*x+c)^4-3/8/d*b^3*sin(d*x+c)^7/cos(d*x+c)^2-3/8/d*sin(d*x+c)^5*b^3-5/8*b^3
*sin(d*x+c)^3/d-15/8*b^3*sin(d*x+c)/d+15/8/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.990961, size = 234, normalized size = 1.65

$$-\frac{16\,b^3\sin(dx+c) - 3\left(3\,a^2b - 8\,ab^2 + 5\,b^3\right)\log(\sin(dx+c)+1) + 3\left(3\,a^2b + 8\,ab^2 + 5\,b^3\right)\log(\sin(dx+c)-1) - \frac{2(\cdots}{\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*b^3*sin(d*x + c) - 3*(3*a^2*b - 8*a*b^2 + 5*b^3)*log(sin(d*x + c)
 + 1) + 3*(3*a^2*b + 8*a*b^2 + 5*b^3)*log(sin(d*x + c) - 1) - 2*(3*(5*a^2*b
 + 3*b^3)*sin(d*x + c)^3 - 2*a^3 - 18*a*b^2 + 4*(a^3 + 6*a*b^2)*sin(d*x + c
)^2 - (9*a^2*b + 7*b^3)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
1))/d
```

Fricas [A] time = 2.14794, size = 423, normalized size = 2.98

$$\frac{3\left(3\,a^2b - 8\,ab^2 + 5\,b^3\right)\cos(dx+c)^4\log(\sin(dx+c)+1) - 3\left(3\,a^2b + 8\,ab^2 + 5\,b^3\right)\cos(dx+c)^4\log(-\sin(dx+c)+\cdots}{16\,d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(3*(3*a^2*b - 8*a*b^2 + 5*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
3*(3*a^2*b + 8*a*b^2 + 5*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^3
 + 12*a*b^2 - 8*(a^3 + 6*a*b^2)*cos(d*x + c)^2 - 2*(8*b^3*cos(d*x + c)^4 -
6*a^2*b - 2*b^3 + 3*(5*a^2*b + 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

[Out] Timed out

Giac [A] time = 1.26843, size = 254, normalized size = 1.79

$$-\frac{16\, b^3 \sin\left(dx+c\right) - 3\left(3\, a^2 b - 8\, a b^2 + 5\, b^3\right) \log\left(|\sin\left(dx+c\right)+1|\right) + 3\left(3\, a^2 b + 8\, a b^2 + 5\, b^3\right) \log\left(|\sin\left(dx+c\right)-1|\right) - \frac{2\,(18}{}}{16\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(16*b^3*sin(d*x + c) - 3*(3*a^2*b - 8*a*b^2 + 5*b^3)*log(abs(sin(d*x
+ c) + 1)) + 3*(3*a^2*b + 8*a*b^2 + 5*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(
18*a*b^2*sin(d*x + c)^4 + 15*a^2*b*sin(d*x + c)^3 + 9*b^3*sin(d*x + c)^3 +
4*a^3*sin(d*x + c)^2 - 12*a*b^2*sin(d*x + c)^2 - 9*a^2*b*sin(d*x + c) - 7*b
^3*sin(d*x + c) - 2*a^3)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1504 $\int \sec^3(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{(a^3-9ab^2-8b^3)\log(1-\sin(c+dx))}{16d}-\frac{(a^3-9ab^2+8b^3)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)\left((a^2+4b^2)\sin(c+dx\right.}{8d}$$

```
[Out] ((a^3 - 9*a*b^2 - 8*b^3)*Log[1 - Sin[c + d*x]])/(16*d) - ((a^3 - 9*a*b^2 +
8*b^3)*Log[1 + Sin[c + d*x]])/(16*d) - (Sec[c + d*x]^2*(a + b*Sin[c + d*x])
*(5*a*b + (a^2 + 4*b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c
 + d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.252769, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2837, 12, 1645, 819, 633, 31}

$$\frac{(a^3-9ab^2-8b^3)\log(1-\sin(c+dx))}{16d}-\frac{(a^3-9ab^2+8b^3)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)\left((a^2+4b^2)\sin(c+dx\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a^3 - 9*a*b^2 - 8*b^3)*Log[1 - Sin[c + d*x]])/(16*d) - ((a^3 - 9*a*b^2 +
8*b^3)*Log[1 + Sin[c + d*x]])/(16*d) - (Sec[c + d*x]^2*(a + b*Sin[c + d*x])
*(5*a*b + (a^2 + 4*b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c
 + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
```

```
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x^2(a+x)^3}{b^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\,\text{Subst}\left(\int \frac{x^2(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)}{4d} + \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2(-a\cdots}{(b^2-x\cdots}\right.}{\cdots}$$

$$= -\frac{\sec^2(c+dx)(a+b\sin(c+dx))\left(5ab+\left(a^2+4b^2\right)\sin(c+dx)\right)}{8d} + \cdots$$

$$= -\frac{\sec^2(c+dx)(a+b\sin(c+dx))\left(5ab+\left(a^2+4b^2\right)\sin(c+dx)\right)}{8d} + \cdots$$

$$= \frac{\left(a^3-9ab^2-8b^3\right)\log(1-\sin(c+dx))}{16d} - \frac{\left(a^3-9ab^2+8b^3\right)\log(1+\cdots}{16d}$$

Mathematica [A] time = 0.393818, size = 140, normalized size = 0.97

$$\frac{\left(a^3-9ab^2-8b^3\right)\log(1-\sin(c+dx)) - \left(a^3-9ab^2+8b^3\right)\log(\sin(c+dx)+1) - \frac{(a-b)^3}{(\sin(c+dx)+1)^2} + \frac{(a-7b)(a-b)^2}{\sin(c+dx)+1} + \frac{(a+b)^2(a+7b)}{\sin(c+dx)-1}\cdots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a^3 - 9*a*b^2 - 8*b^3)*Log[1 - Sin[c + d*x]] - (a^3 - 9*a*b^2 + 8*b^3)*Lo
g[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x])^2 + ((a + b)^2*(a + 7*b
))/(-1 + Sin[c + d*x]) - (a - b)^3/(1 + Sin[c + d*x])^2 + ((a - 7*b)*(a - b
)^2)/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A] time = 0.074, size = 263, normalized size = 1.8

$$\frac{a^3 (\sin (dx+c))^3}{4 d (\cos (dx+c))^4}+\frac{a^3 (\sin (dx+c))^3}{8 d (\cos (dx+c))^2}+\frac{a^3 \sin (dx+c)}{8 d}-\frac{a^3 \ln (\sec (dx+c)+\tan (dx+c))}{8 d}+\frac{3 a^2 b (\sin (dx+c))^4}{4 d (\cos (dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+1/8
*a^3*sin(d*x+c)/d-1/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^2*b*sin(d*x+c
)^4/cos(d*x+c)^4+3/4/d*a*b^2*sin(d*x+c)^5/cos(d*x+c)^4-3/8/d*a*b^2*sin(d*x+
c)^5/cos(d*x+c)^2-3/8/d*a*b^2*sin(d*x+c)^3-9/8*a*b^2*sin(d*x+c)/d+9/8/d*a*b
^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b^3*tan(d*x+c)^4-1/2/d*b^3*tan(d*x+c)^2-
1/d*b^3*ln(cos(d*x+c))
```

Maxima [A] time = 1.01877, size = 204, normalized size = 1.42

$$-\frac{\left(a^3-9 ab^2+8 b^3\right) \log (\sin (dx+c)+1)-\left(a^3-9 ab^2-8 b^3\right) \log (\sin (dx+c)-1)-\frac{2 \left(\left(a^3+15 ab^2\right) \sin(dx+c)^3-6 a^2 b-6 b^3+4 (}{\sin(dx+c)^4-}}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*((a^3 - 9*a*b^2 + 8*b^3)*log(sin(d*x + c) + 1) - (a^3 - 9*a*b^2 - 8*b
^3)*log(sin(d*x + c) - 1) - 2*((a^3 + 15*a*b^2)*sin(d*x + c)^3 - 6*a^2*b -
6*b^3 + 4*(3*a^2*b + 2*b^3)*sin(d*x + c)^2 + (a^3 - 9*a*b^2)*sin(d*x + c))/
(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.19204, size = 375, normalized size = 2.6

$$-\frac{\left(a^3-9 ab^2+8 b^3\right) \cos (dx+c)^4 \log (\sin (dx+c)+1)-\left(a^3-9 ab^2-8 b^3\right) \cos (dx+c)^4 \log (-\sin (dx+c)+1)-12}{16 d \cos (dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*((a^3 - 9*a*b^2 + 8*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^3
- 9*a*b^2 - 8*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 12*a^2*b - 4*b^3
 + 8*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 2*(2*a^3 + 6*a*b^2 - (a^3 + 15*a*b^
2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23294, size = 227, normalized size = 1.58

$$-\frac{\left(a^3-9\,ab^2+8\,b^3\right)\log\left(|\sin\left(dx+c\right)+1|\right)-\left(a^3-9\,ab^2-8\,b^3\right)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(6\,b^3\sin(dx+c)^4+a^3\sin(dx+c)^3+15\,ab^2\,\mathrm{si}\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*((a^3 - 9*a*b^2 + 8*b^3)*log(abs(sin(d*x + c) + 1)) - (a^3 - 9*a*b^2
- 8*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(6*b^3*sin(d*x + c)^4 + a^3*sin(d*x
 + c)^3 + 15*a*b^2*sin(d*x + c)^3 + 12*a^2*b*sin(d*x + c)^2 - 4*b^3*sin(d*x
 + c)^2 + a^3*sin(d*x + c) - 9*a*b^2*sin(d*x + c) - 6*a^2*b)/(sin(d*x + c)^
2 - 1)^2)/d
```

3.1505 $\int \sec^4(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=90

$$-\frac{3b\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3\sec^2(c+dx)(a+b\sin(c+dx))\left(ab\sin(c+dx)+b^2\right)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c}{4d}$$

```
[Out] (-3*b*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(a + b*Sin
[c + d*x])^3)/(4*d) - (3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])*(b^2 + a*b*Sin
[c + d*x]))/(8*d)
```

Rubi [A] time = 0.111033, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2837, 12, 805, 723, 206}

$$-\frac{3b\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3\sec^2(c+dx)(a+b\sin(c+dx))\left(ab\sin(c+dx)+b^2\right)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-3*b*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(a + b*Sin
[c + d*x])^3)/(4*d) - (3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])*(b^2 + a*b*Sin
[c + d*x]))/(8*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{x(a+x)^3}{b\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^4\,\text{Subst}\left(\int \frac{x(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\left(3b^4\right)\text{Subst}\left(\int \frac{(a+x)^2}{\left(b^2-x^2\right)^2}\,dx, x, b\sin\right.}{4d}$$
$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{3\sec^2(c+dx)(a+b\sin(c+dx))\,(}{8d}$$
$$= -\frac{3b\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))}{4d}$$

Mathematica [B] time = 1.45126, size = 370, normalized size = 4.11

$$\frac{\frac{1}{2}\left(10a^2b^3+5a^4b+b^5\right)\left(6b^2\left(6a^2+b^2\right)\sin(c+dx)+12ab^3\sin^2(c+dx)+3\left((a+b)^4\log(1-\sin(c+dx))-(a-b)^4\log(s\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (2*a*(a^2 - b^2)^2*Sec[c + d*x]^4*(a + b*Sin[c + d*x])^4 + 2*b*(a^2 - b^2)*
Sec[c + d*x]^4*(b - a*Sin[c + d*x])*(a + b*Sin[c + d*x])^5 + b*Sec[c + d*x]
^2*(a + b*Sin[c + d*x])^5*(5*a^2*b + b^3 - 3*a*(a^2 + b^2)*Sin[c + d*x]) +
((5*a^4*b + 10*a^2*b^3 + b^5)*(3*((a + b)^4*Log[1 - Sin[c + d*x]] - (a - b)
^4*Log[1 + Sin[c + d*x]]) + 6*b^2*(6*a^2 + b^2)*Sin[c + d*x] + 12*a*b^3*Sin
[c + d*x]^2 + 2*b^4*Sin[c + d*x]^3))/2 - a*b*(a^2 + b^2)*(6*(a + b)^5*Log[1
 - Sin[c + d*x]] - 6*(a - b)^5*Log[1 + Sin[c + d*x]] + 60*a*b^2*(2*a^2 + b^
2)*Sin[c + d*x] + 6*b^3*(10*a^2 + b^2)*Sin[c + d*x]^2 + 20*a*b^4*Sin[c + d*
x]^3 + 3*b^5*Sin[c + d*x]^4))/(8*(a^2 - b^2)^3*d)
```

Maple [B] time = 0.066, size = 231, normalized size = 2.6

$$\frac{a^3}{4d(\cos(dx+c))^4} + \frac{3a^2b(\sin(dx+c))^3}{4d(\cos(dx+c))^4} + \frac{3a^2b(\sin(dx+c))^3}{8d(\cos(dx+c))^2} + \frac{3a^2b\sin(dx+c)}{8d} - \frac{3a^2b\ln(\sec(dx+c)+\tan(dx+}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3/cos(d*x+c)^4+3/4/d*a^2*b*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*a^2*b*si
n(d*x+c)^3/cos(d*x+c)^2+3/8/d*sin(d*x+c)*a^2*b-3/8/d*a^2*b*ln(sec(d*x+c)+ta
```

```
n(d*x+c))+3/4/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)^4+1/4/d*b^3*sin(d*x+c)^5/cos(
d*x+c)^4-1/8/d*b^3*sin(d*x+c)^5/cos(d*x+c)^2-1/8*b^3*sin(d*x+c)^3/d-3/8*b^3
*sin(d*x+c)/d+3/8/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.993212, size = 189, normalized size = 2.1

$$-\frac{3\left(a^2b-b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^2b-b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(12\,ab^2\sin(dx+c)^2+\left(3\,a^2b+5\,b^3\right)\sin(dx+c)^3+2\,a^3-6\,ab^2+\right.}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a^2*b - b^3)*log(sin(d*x + c) + 1) - 3*(a^2*b - b^3)*log(sin(d*x
+ c) - 1) - 2*(12*a*b^2*sin(d*x + c)^2 + (3*a^2*b + 5*b^3)*sin(d*x + c)^3 +
 2*a^3 - 6*a*b^2 + 3*(a^2*b - b^3)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*
x + c)^2 + 1))/d
```

Fricas [A] time = 2.01338, size = 340, normalized size = 3.78

$$-\frac{3\left(a^2b-b^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^2b-b^3\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+24\,ab^2\cos\left(dx+\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(3*(a^2*b - b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^2*b - b^
3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 24*a*b^2*cos(d*x + c)^2 - 4*a^3
- 12*a*b^2 - 2*(6*a^2*b + 2*b^3 - (3*a^2*b + 5*b^3)*cos(d*x + c)^2)*sin(d*x
 + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36876, size = 192, normalized size = 2.13

$$-\frac{3\left(a^2b-b^3\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\left(a^2b-b^3\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,a^2b\sin(dx+c)^3+5\,b^3\sin(dx+c)^3+12\,ab^2\sin(dx+c)^2+\right.}{\left(\sin(dx+c)^2-1\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(3*(a^2*b - b^3)*log(abs(sin(d*x + c) + 1)) - 3*(a^2*b - b^3)*log(abs
(sin(d*x + c) - 1)) - 2*(3*a^2*b*sin(d*x + c)^3 + 5*b^3*sin(d*x + c)^3 + 12
*a*b^2*sin(d*x + c)^2 + 3*a^2*b*sin(d*x + c) - 3*b^3*sin(d*x + c) + 2*a^3 -
 6*a*b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1506 $\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=165

$$-\frac{(9a^2b+8a^3-b^3)\log(1-\sin(c+dx))}{16d}-\frac{(-9a^2b+8a^3+b^3)\log(\sin(c+dx)+1)}{16d}+\frac{\sec^4(c+dx)\left(b\left(3a^2+b^2\right)\sin(c\right.}{4d}$$

```
[Out] -((8*a^3 + 9*a^2*b - b^3)*Log[1 - Sin[c + d*x]])/(16*d) + (a^3*Log[Sin[c +
d*x]])/d - ((8*a^3 - 9*a^2*b + b^3)*Log[1 + Sin[c + d*x]])/(16*d) + (Sec[c
+ d*x]^2*(4*a^3 + b*(9*a^2 - b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^4*(a
*(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Sin[c + d*x]))/(4*d)
```

Rubi [A] time = 0.247341, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2837, 12, 1805, 823, 801}

$$-\frac{(9a^2b+8a^3-b^3)\log(1-\sin(c+dx))}{16d}-\frac{(-9a^2b+8a^3+b^3)\log(\sin(c+dx)+1)}{16d}+\frac{\sec^4(c+dx)\left(b\left(3a^2+b^2\right)\sin(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -((8*a^3 + 9*a^2*b - b^3)*Log[1 - Sin[c + d*x]])/(16*d) + (a^3*Log[Sin[c +
d*x]])/d - ((8*a^3 - 9*a^2*b + b^3)*Log[1 + Sin[c + d*x]])/(16*d) + (Sec[c
+ d*x]^2*(4*a^3 + b*(9*a^2 - b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^4*(a
*(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Sin[c + d*x]))/(4*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b(a+x)^3}{x\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\,\text{Subst}\left(\int \frac{(a+x)^3}{x\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4d} - \frac{b^4\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sec^2(c+dx)\left(4a^3+b\left(9a^2-b^2\right)\sin(c+dx)\right)}{8d} + \frac{\sec^4(c+dx)\left(a\left(a^2+\right.\right.}{}$$

$$= \frac{\sec^2(c+dx)\left(4a^3+b\left(9a^2-b^2\right)\sin(c+dx)\right)}{8d} + \frac{\sec^4(c+dx)\left(a\left(a^2+\right.\right.}{}$$

$$= -\frac{\left(8a^3+9a^2b-b^3\right)\log(1-\sin(c+dx))}{16d} + \frac{a^3\log(\sin(c+dx))}{d} - \frac{\left(8a\right.}{}$$

Mathematica [A] time = 0.571409, size = 157, normalized size = 0.95

$$\frac{-\left(9a^2b+8a^3-b^3\right)\log(1-\sin(c+dx))-\left(-9a^2b+8a^3+b^3\right)\log(\sin(c+dx)+1)+16a^3\log(\sin(c+dx))-\frac{(5a-b)(a+b)^2}{\sin(c+dx)-1}+}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-((8*a^3 + 9*a^2*b - b^3)*Log[1 - Sin[c + d*x]]) + 16*a^3*Log[Sin[c + d*x]
] - (8*a^3 - 9*a^2*b + b^3)*Log[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c +
 d*x])^2 - ((5*a - b)*(a + b)^2)/(-1 + Sin[c + d*x]) + (a - b)^3/(1 + Sin[c
 + d*x])^2 + ((a - b)^2*(5*a + b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A] time = 0.105, size = 216, normalized size = 1.3

$$\frac{a^3}{4d(\cos(dx+c))^4} + \frac{a^3}{2d(\cos(dx+c))^2} + \frac{a^3\ln(\tan(dx+c))}{d} + \frac{3a^2b\tan(dx+c)(\sec(dx+c))^3}{4d} + \frac{9a^2b\tan(dx+c)\,\text{se}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3/cos(d*x+c)^4+1/2/d*a^3/cos(d*x+c)^2+1/d*a^3*ln(tan(d*x+c))+3/4/d*
a^2*b*tan(d*x+c)*sec(d*x+c)^3+9/8/d*a^2*b*tan(d*x+c)*sec(d*x+c)+9/8/d*a^2*b
*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a*b^2/cos(d*x+c)^4+1/4/d*b^3*sin(d*x+c)^3/
cos(d*x+c)^4+1/8/d*b^3*sin(d*x+c)^3/cos(d*x+c)^2+1/8*b^3*sin(d*x+c)/d-1/8/d
*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.999918, size = 216, normalized size = 1.31

$$\frac{16\,a^3\log\left(\sin\left(dx+c\right)\right)-\left(8\,a^3-9\,a^2b+b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^3+9\,a^2b-b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(4\,a^3\sin\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*log(sin(d*x + c)) - (8*a^3 - 9*a^2*b + b^3)*log(sin(d*x + c) +
 1) - (8*a^3 + 9*a^2*b - b^3)*log(sin(d*x + c) - 1) - 2*(4*a^3*sin(d*x + c)
^2 + (9*a^2*b - b^3)*sin(d*x + c)^3 - 6*a^3 - 6*a*b^2 - (15*a^2*b + b^3)*si
n(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.18695, size = 416, normalized size = 2.52

$$\frac{16\,a^3\cos\left(dx+c\right)^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(8\,a^3-9\,a^2b+b^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^3+9\,a^2b-b^3\right)\mathrm{co}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^3*cos(d*x + c)^4*log(1/2*sin(d*x + c)) - (8*a^3 - 9*a^2*b + b^3)
*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^3 + 9*a^2*b - b^3)*cos(d*x + c
)^4*log(-sin(d*x + c) + 1) + 8*a^3*cos(d*x + c)^2 + 4*a^3 + 12*a*b^2 + 2*(6
*a^2*b + 2*b^3 + (9*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
 c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31428, size = 236, normalized size = 1.43

$$\frac{16\,a^{3}\log\left(|\sin\left(dx+c\right)|\right)-\left(8\,a^{3}-9\,a^{2}b+b^{3}\right)\log\left(|\sin\left(dx+c\right)+1|\right)-\left(8\,a^{3}+9\,a^{2}b-b^{3}\right)\log\left(|\sin\left(dx+c\right)-1|\right)+\frac{2\left(6\,a^{3}\sin\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(16*a^3*log(abs(sin(d*x + c))) - (8*a^3 - 9*a^2*b + b^3)*log(abs(sin(d
*x + c) + 1)) - (8*a^3 + 9*a^2*b - b^3)*log(abs(sin(d*x + c) - 1)) + 2*(6*a
^3*sin(d*x + c)^4 - 9*a^2*b*sin(d*x + c)^3 + b^3*sin(d*x + c)^3 - 16*a^3*si
n(d*x + c)^2 + 15*a^2*b*sin(d*x + c) + b^3*sin(d*x + c) + 12*a^3 + 6*a*b^2)
/(sin(d*x + c)^2 - 1)^2)/d
```

3.1507 $\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=171

$$\frac{b\sec^4(c+dx)\left(ab\left(\frac{a^2}{b^2}+3\right)\sin(c+dx)+3a^2+b^2\right)}{4d}+\frac{ab\sec^2(c+dx)\left(b\left(\frac{7a^2}{b^2}+9\right)\sin(c+dx)+12a\right)}{8d}+\frac{3a^2b\log(\sin(c}{d}$$

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a*(a + b)*(5*a + 3*b)*Log[1 - Sin[c + d*x]])/(
16*d) + (3*a^2*b*Log[Sin[c + d*x]])/d + (3*a*(5*a - 3*b)*(a - b)*Log[1 + Si
n[c + d*x]])/(16*d) + (b*Sec[c + d*x]^4*(3*a^2 + b^2 + a*(3 + a^2/b^2)*b*Si
n[c + d*x]))/(4*d) + (a*b*Sec[c + d*x]^2*(12*a + (9 + (7*a^2)/b^2)*b*Sin[c
+ d*x]))/(8*d)
```

Rubi [A] time = 0.370669, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1805, 1802}

$$\frac{b\sec^4(c+dx)\left(ab\left(\frac{a^2}{b^2}+3\right)\sin(c+dx)+3a^2+b^2\right)}{4d}+\frac{ab\sec^2(c+dx)\left(b\left(\frac{7a^2}{b^2}+9\right)\sin(c+dx)+12a\right)}{8d}+\frac{3a^2b\log(\sin(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a*(a + b)*(5*a + 3*b)*Log[1 - Sin[c + d*x]])/(
16*d) + (3*a^2*b*Log[Sin[c + d*x]])/d + (3*a*(5*a - 3*b)*(a - b)*Log[1 + Si
n[c + d*x]])/(16*d) + (b*Sec[c + d*x]^4*(3*a^2 + b^2 + a*(3 + a^2/b^2)*b*Si
n[c + d*x]))/(4*d) + (a*b*Sec[c + d*x]^2*(12*a + (9 + (7*a^2)/b^2)*b*Sin[c
+ d*x]))/(8*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^2(a+x)^3}{x^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)^3}{x^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^4(c+dx)\left(3a^2+b^2+a\left(3+\frac{a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} - \frac{b^5\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{b\sec^4(c+dx)\left(3a^2+b^2+a\left(3+\frac{a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} + \frac{ab\sec^2(c+d\cdots}{\cdots}$$

$$= \frac{b\sec^4(c+dx)\left(3a^2+b^2+a\left(3+\frac{a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} + \frac{ab\sec^2(c+d\cdots}{\cdots}$$

$$= -\frac{a^3\csc(c+dx)}{d} - \frac{3a(a+b)(5a+3b)\log(1-\sin(c+dx))}{16d} + \frac{3a^2b\,\text{lo}\cdots}{\cdots}$$

Mathematica [A] time = 1.24584, size = 161, normalized size = 0.94

$$-\frac{-48a^2b\log(\sin(c+dx))+16a^3\csc(c+dx)+\frac{(a+b)^2(7a+b)}{\sin(c+dx)-1}+\frac{(a-b)^2(7a-b)}{\sin(c+dx)+1}-\frac{(a+b)^3}{(\sin(c+dx)-1)^2}+\frac{(a-b)^3}{(\sin(c+dx)+1)^2}+3a(a+b)(5a+3b\cdots}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(16*a^3*Csc[c + d*x] + 3*a*(a + b)*(5*a + 3*b)*Log[1 - Sin[c + d*x]] - 48*
a^2*b*Log[Sin[c + d*x]] - 3*a*(5*a - 3*b)*(a - b)*Log[1 + Sin[c + d*x]] - (
a + b)^3/(-1 + Sin[c + d*x])^2 + ((a + b)^2*(7*a + b))/(-1 + Sin[c + d*x])
+ (a - b)^3/(1 + Sin[c + d*x])^2 + ((a - b)^2*(7*a - b))/(1 + Sin[c + d*x])
)/(16*d)
```

Maple [A] time = 0.105, size = 221, normalized size = 1.3

$$\frac{a^3}{4d\sin(dx+c)(\cos(dx+c))^4} + \frac{5a^3}{8d\sin(dx+c)(\cos(dx+c))^2} - \frac{15a^3}{8d\sin(dx+c)} + \frac{15a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3/sin(d*x+c)/cos(d*x+c)^4+5/8/d*a^3/sin(d*x+c)/cos(d*x+c)^2-15/8/d*
a^3/sin(d*x+c)+15/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^2*b/cos(d*x+c)^
```

```
4+3/2/d*a^2*b/cos(d*x+c)^2+3/d*a^2*b*ln(tan(d*x+c))+3/4/d*a*b^2*tan(d*x+c)*
sec(d*x+c)^3+9/8/d*a*b^2*tan(d*x+c)*sec(d*x+c)+9/8/d*a*b^2*ln(sec(d*x+c)+ta
n(d*x+c))+1/4/d*b^3/cos(d*x+c)^4
```

Maxima [A] time = 1.01105, size = 254, normalized size = 1.49

$$\frac{48\,a^{2}b\log\left(\sin\left(dx+c\right)\right)+3\left(5\,a^{3}-8\,a^{2}b+3\,ab^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(5\,a^{3}+8\,a^{2}b+3\,ab^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/16*(48*a^2*b*log(sin(d*x + c)) + 3*(5*a^3 - 8*a^2*b + 3*a*b^2)*log(sin(d*
x + c) + 1) - 3*(5*a^3 + 8*a^2*b + 3*a*b^2)*log(sin(d*x + c) - 1) - 2*(12*a
^2*b*sin(d*x + c)^3 + 3*(5*a^3 + 3*a*b^2)*sin(d*x + c)^4 + 8*a^3 - 5*(5*a^3
 + 3*a*b^2)*sin(d*x + c)^2 - 2*(9*a^2*b + b^3)*sin(d*x + c))/(sin(d*x + c)^
5 - 2*sin(d*x + c)^3 + sin(d*x + c)))/d
```

Fricas [A] time = 2.18192, size = 560, normalized size = 3.27

$$\frac{48\,a^{2}b\cos\left(dx+c\right)^{4}\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+3\left(5\,a^{3}-8\,a^{2}b+3\,ab^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)\sin\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(48*a^2*b*cos(d*x + c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(5*a^3
 - 8*a^2*b + 3*a*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)*sin(d*x + c) - 3
*(5*a^3 + 8*a^2*b + 3*a*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)*sin(d*x
+ c) - 6*(5*a^3 + 3*a*b^2)*cos(d*x + c)^4 + 4*a^3 + 12*a*b^2 + 2*(5*a^3 + 3
*a*b^2)*cos(d*x + c)^2 + 4*(6*a^2*b*cos(d*x + c)^2 + 3*a^2*b + b^3)*sin(d*x
 + c))/(d*cos(d*x + c)^4*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35581, size = 284, normalized size = 1.66

$$\frac{48\,a^{2}b\log\left(\left|\sin\left(dx+c\right)\right|\right)+3\left(5\,a^{3}-8\,a^{2}b+3\,ab^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\left(5\,a^{3}+8\,a^{2}b+3\,ab^{2}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(48*a^2*b*log(abs(sin(d*x + c))) + 3*(5*a^3 - 8*a^2*b + 3*a*b^2)*log(a
bs(sin(d*x + c) + 1)) - 3*(5*a^3 + 8*a^2*b + 3*a*b^2)*log(abs(sin(d*x + c)
- 1)) - 16*(3*a^2*b*sin(d*x + c) + a^3)/sin(d*x + c) + 2*(18*a^2*b*sin(d*x
+ c)^4 - 7*a^3*sin(d*x + c)^3 - 9*a*b^2*sin(d*x + c)^3 - 48*a^2*b*sin(d*x +
 c)^2 + 9*a^3*sin(d*x + c) + 15*a*b^2*sin(d*x + c) + 36*a^2*b + 2*b^3)/(sin
(d*x + c)^2 - 1)^2)/d
```

3.1508 $\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=221

$$-\frac{3(a+b)\left(8a^2+7ab+b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3a\left(a^2+b^2\right)\log(\sin(c+dx))}{d}-\frac{3(a-b)\left(8a^2-7ab+b^2\right)\log(\sin(c$$

```
[Out] (-3*a^2*b*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (3*(a + b)*(8*a^2
+ 7*a*b + b^2)*Log[1 - Sin[c + d*x]])/(16*d) + (3*a*(a^2 + b^2)*Log[Sin[c +
 d*x]])/d - (3*(a - b)*(8*a^2 - 7*a*b + b^2)*Log[1 + Sin[c + d*x]])/(16*d)
+ (b^2*Sec[c + d*x]^4*(a*(3 + a^2/b^2) + (1 + (3*a^2)/b^2)*b*Sin[c + d*x]))
/(4*d) + (b^2*Sec[c + d*x]^2*(4*a*(3 + (2*a^2)/b^2) + 3*(1 + (7*a^2)/b^2)*b
*Sin[c + d*x]))/(8*d)
```

Rubi [A] time = 0.434452, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 12, 1805, 1802}

$$-\frac{3(a+b)\left(8a^2+7ab+b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3a\left(a^2+b^2\right)\log(\sin(c+dx))}{d}-\frac{3(a-b)\left(8a^2-7ab+b^2\right)\log(\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (3*(a + b)*(8*a^2
+ 7*a*b + b^2)*Log[1 - Sin[c + d*x]])/(16*d) + (3*a*(a^2 + b^2)*Log[Sin[c +
 d*x]])/d - (3*(a - b)*(8*a^2 - 7*a*b + b^2)*Log[1 + Sin[c + d*x]])/(16*d)
+ (b^2*Sec[c + d*x]^4*(a*(3 + a^2/b^2) + (1 + (3*a^2)/b^2)*b*Sin[c + d*x]))
/(4*d) + (b^2*Sec[c + d*x]^2*(4*a*(3 + (2*a^2)/b^2) + 3*(1 + (7*a^2)/b^2)*b
*Sin[c + d*x]))/(8*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{b^3(a+x)^3}{x^3(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\,\text{Subst}\left(\int \frac{(a+x)^3}{x^3(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\sec^4(c+dx)\left(a\left(3+\frac{a^2}{b^2}\right)+\left(1+\frac{3a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} - \frac{b^6\,\text{Subst}\Big(\int$$

$$= \frac{b^2\sec^4(c+dx)\left(a\left(3+\frac{a^2}{b^2}\right)+\left(1+\frac{3a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} + \frac{b^2\sec^2(c+$$

$$= \frac{b^2\sec^4(c+dx)\left(a\left(3+\frac{a^2}{b^2}\right)+\left(1+\frac{3a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} + \frac{b^2\sec^2(c+$$

$$= -\frac{3a^2b\csc(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d} - \frac{3(a+b)\left(8a^2+7ab+b^2\right)\log}{16d}$$

Mathematica [A] time = 3.32417, size = 190, normalized size = 0.86

$$\frac{48a\left(a^2+b^2\right)\log(\sin(c+dx)) - 3(a+b)\left(8a^2+7ab+b^2\right)\log(1-\sin(c+dx)) - 3(a-b)\left(8a^2-7ab+b^2\right)\log(\sin(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-48*a^2*b*Csc[c + d*x] - 8*a^3*Csc[c + d*x]^2 - 3*(a + b)*(8*a^2 + 7*a*b +
 b^2)*Log[1 - Sin[c + d*x]] + 48*a*(a^2 + b^2)*Log[Sin[c + d*x]] - 3*(a - b
)*(8*a^2 - 7*a*b + b^2)*Log[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x
])^2 - (3*(a + b)^2*(3*a + b))/(-1 + Sin[c + d*x]) + (a - b)^3/(1 + Sin[c +
 d*x])^2 + (3*(a - b)^2*(3*a - b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A] time = 0.12, size = 285, normalized size = 1.3

$$\frac{a^3}{4d(\sin(dx+c))^2(\cos(dx+c))^4} + \frac{3a^3}{4d(\sin(dx+c))^2(\cos(dx+c))^2} - \frac{3a^3}{2d(\sin(dx+c))^2} + 3\frac{a^3\ln(\tan(dx+c))}{d} + \frac{}{4d\,\text{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3/sin(d*x+c)^2/cos(d*x+c)^4+3/4/d*a^3/sin(d*x+c)^2/cos(d*x+c)^2-3/2
/d*a^3/sin(d*x+c)^2+3/d*a^3*ln(tan(d*x+c))+3/4/d*a^2*b/sin(d*x+c)/cos(d*x+c
)^4+15/8/d*a^2*b/sin(d*x+c)/cos(d*x+c)^2-45/8/d*a^2*b/sin(d*x+c)+45/8/d*a^2
*b*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a*b^2/cos(d*x+c)^4+3/2/d*a*b^2/cos(d*x+c
)^2+3/d*a*b^2*ln(tan(d*x+c))+1/4/d*b^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*b^3*ta
n(d*x+c)*sec(d*x+c)+3/8/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.00325, size = 293, normalized size = 1.33

$$-\frac{3\left(8a^3-15a^2b+8ab^2-b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(8a^3+15a^2b+8ab^2+b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-48\left(a^3+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(3*(8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*log(sin(d*x + c) + 1) + 3*(8*a^
3 + 15*a^2*b + 8*a*b^2 + b^3)*log(sin(d*x + c) - 1) - 48*(a^3 + a*b^2)*log(
sin(d*x + c)) + 2*(3*(15*a^2*b + b^3)*sin(d*x + c)^5 + 12*(a^3 + a*b^2)*sin
(d*x + c)^4 + 24*a^2*b*sin(d*x + c) - 5*(15*a^2*b + b^3)*sin(d*x + c)^3 + 4
*a^3 - 18*(a^3 + a*b^2)*sin(d*x + c)^2)/(sin(d*x + c)^6 - 2*sin(d*x + c)^4
+ sin(d*x + c)^2))/d
```

Fricas [A] time = 2.2876, size = 791, normalized size = 3.58

$$\frac{24\left(a^3+ab^2\right)\cos\left(dx+c\right)^4-4a^3-12ab^2-12\left(a^3+ab^2\right)\cos\left(dx+c\right)^2+48\left(\left(a^3+ab^2\right)\cos\left(dx+c\right)^6-\left(a^3+ab^2\right)\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(24*(a^3 + a*b^2)*cos(d*x + c)^4 - 4*a^3 - 12*a*b^2 - 12*(a^3 + a*b^2)
*cos(d*x + c)^2 + 48*((a^3 + a*b^2)*cos(d*x + c)^6 - (a^3 + a*b^2)*cos(d*x
+ c)^4)*log(1/2*sin(d*x + c)) - 3*((8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*cos(d
*x + c)^6 - (8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*cos(d*x + c)^4)*log(sin(d*x
+ c) + 1) - 3*((8*a^3 + 15*a^2*b + 8*a*b^2 + b^3)*cos(d*x + c)^6 - (8*a^3 +
 15*a^2*b + 8*a*b^2 + b^3)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(3*(1
5*a^2*b + b^3)*cos(d*x + c)^4 - 6*a^2*b - 2*b^3 - (15*a^2*b + b^3)*cos(d*x
+ c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6 - d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

[Out] Timed out

Giac [A] time = 1.38735, size = 324, normalized size = 1.47

$$-\frac{3\left(8a^3-15a^2b+8ab^2-b^3\right)\log\left(|\sin\left(dx+c\right)+1|\right)+3\left(8a^3+15a^2b+8ab^2+b^3\right)\log\left(|\sin\left(dx+c\right)-1|\right)-48\left(a^3+ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(3*(8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*log(abs(sin(d*x + c) + 1)) + 3*
(8*a^3 + 15*a^2*b + 8*a*b^2 + b^3)*log(abs(sin(d*x + c) - 1)) - 48*(a^3 + a
*b^2)*log(abs(sin(d*x + c))) + 2*(45*a^2*b*sin(d*x + c)^5 + 3*b^3*sin(d*x +
 c)^5 + 12*a^3*sin(d*x + c)^4 + 12*a*b^2*sin(d*x + c)^4 - 75*a^2*b*sin(d*x
+ c)^3 - 5*b^3*sin(d*x + c)^3 - 18*a^3*sin(d*x + c)^2 - 18*a*b^2*sin(d*x +
c)^2 + 24*a^2*b*sin(d*x + c) + 4*a^3)/(sin(d*x + c)^3 - sin(d*x + c))^2)/d
```

3.1509 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=295

$$-\frac{\left(6a^2b^2\left(1-n^2\right)+a^4\left(-\left(n^2-4n+3\right)\right)-b^4\left(n^2+4n+3\right)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{8d(n+1)}-\frac{abn\left(a^2(2\right.}{}$$

```
[Out] -((6*a^2*b^2*(1 - n^2) - a^4*(3 - 4*n + n^2) - b^4*(3 + 4*n + n^2))*Hyperge
ometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(
8*d*(1 + n)) - (a*b*n*(a^2*(2 - n) - b^2*(2 + n))*Hypergeometric2F1[1, (2 +
 n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(2*d*(2 + n)) + (Se
c[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a^4 + 6*a^2*b^2 + b^4 + 4*a*b*(a^2 + b^2
)*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*Sin[c + d*x]^(1 + n)*(a^4*(3 - n)
- 6*a^2*b^2*(1 + n) - b^4*(5 + n) + 4*a*b*(a^2*(2 - n) - b^2*(2 + n))*Sin[c
 + d*x]))/(8*d)
```

Rubi [A] time = 0.530405, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 1806, 808, 364}

$$-\frac{\left(6a^2b^2\left(1-n^2\right)+a^4\left(-\left(n^2-4n+3\right)\right)-b^4\left(n^2+4n+3\right)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{8d(n+1)}-\frac{abn\left(a^2(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^4,x]
```



```
[Out] -((6*a^2*b^2*(1 - n^2) - a^4*(3 - 4*n + n^2) - b^4*(3 + 4*n + n^2))*Hyperge
ometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(
8*d*(1 + n)) - (a*b*n*(a^2*(2 - n) - b^2*(2 + n))*Hypergeometric2F1[1, (2 +
 n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(2*d*(2 + n)) + (Se
c[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a^4 + 6*a^2*b^2 + b^4 + 4*a*b*(a^2 + b^2
)*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*Sin[c + d*x]^(1 + n)*(a^4*(3 - n)
- 6*a^2*b^2*(1 + n) - b^4*(5 + n) + 4*a*b*(a^2*(2 - n) - b^2*(2 + n))*Sin[c
 + d*x]))/(8*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^4\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^4}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^4+6a^2b^2+b^4+4ab\left(a^2+b^2\right)\sin(c+\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^4+6a^2b^2+b^4+4ab\left(a^2+b^2\right)\sin(c+\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^4+6a^2b^2+b^4+4ab\left(a^2+b^2\right)\sin(c+\right.}{4d}$$

$$= -\frac{\left(6a^2b^2\left(1-n^2\right)-a^4\left(3-4n+n^2\right)-b^4\left(3+4n+n^2\right)\right)\,{}_2F_1\left(1,\frac{1+n}{2},\right.}{8d(1+n)}$$

Mathematica [A] time = 0.196805, size = 164, normalized size = 0.56

$$\frac{\sin^{n+1}(c+dx)\left(6\left(a^2-b^2\right)^2\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+2(a-b)^4\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))+(3a+5b)(a-b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((6*(a^2 - b^2)^2*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2
] + (a - b)^3*(3*a + 5*b)*Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]
 + (3*a - 5*b)*(a + b)^3*Hypergeometric2F1[2, 1 + n, 2 + n, Sin[c + d*x]] +
 2*(a - b)^4*Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]] + 2*(a + b)^
4*Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]])*Sin[c + d*x]^(1 + n))/(
16*d*(1 + n))
```

Maple [F] time = 1.954, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^5(\sin(dx+c))^n(a+b\sin(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(-\left(4\left(ab^3\cos\left(dx+c\right)^2-a^3b-ab^3\right)\sec\left(dx+c\right)^5\sin\left(dx+c\right)-\left(b^4\cos\left(dx+c\right)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] integral(-(4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sec(d*x + c)^5*sin(d*x
+ c) - (b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*co
s(d*x + c)^2)*sec(d*x + c)^5)*sin(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(dx+c\right)+a\right)^4\sin\left(dx+c\right)^n\sec\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

3.1510 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=186

$$\frac{a\left(a^2(3-n)-3b^2(n+1)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{b\left(3a^2(2-n)-b^2(n+2)\right)\sin^{n+2}(c+dx)\,{}_2F_1\left(2\right.}{4d(n+2)}$$

```
[Out] (a*(a^2*(3 - n) - 3*b^2*(1 + n))*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2,
 Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(4*d*(1 + n)) + (b*(3*a^2*(2 - n) -
b^2*(2 + n))*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin
[c + d*x]^(2 + n))/(4*d*(2 + n)) + (Sec[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a*
(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Sin[c + d*x]))/(4*d)
```

Rubi [A] time = 0.296152, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 1806, 808, 364}

$$\frac{a\left(a^2(3-n)-3b^2(n+1)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{b\left(3a^2(2-n)-b^2(n+2)\right)\sin^{n+2}(c+dx)\,{}_2F_1\left(2\right.}{4d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (a*(a^2*(3 - n) - 3*b^2*(1 + n))*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2,
 Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(4*d*(1 + n)) + (b*(3*a^2*(2 - n) -
b^2*(2 + n))*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin
[c + d*x]^(2 + n))/(4*d*(2 + n)) + (Sec[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a*
(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Sin[c + d*x]))/(4*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] && !GtQ[m, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\sin(c+c\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\sin(c+c\right.}{4d}$$

$$= \frac{a\left(a^2(3-n)-3b^2(1+n)\right)\,{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(}{4d(1+n)}$$

Mathematica [A] time = 0.161711, size = 158, normalized size = 0.85

$$\frac{\sin^{n+1}(c+dx)\left(6a(a+b)(a-b)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+2(a-b)^3\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))+3(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((6*a*(a - b)*(a + b)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*
x]^2] + 3*(a - b)^2*(a + b)*Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x
]] + 3*(a - b)*(a + b)^2*Hypergeometric2F1[2, 1 + n, 2 + n, Sin[c + d*x]] +
 2*(a - b)^3*Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]] + 2*(a + b)^
3*Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]])*Sin[c + d*x]^(1 + n))/(
16*d*(1 + n))
```

Maple [F] time = 1.816, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^5 (\sin(dx+c))^n (a+b\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^3\sin(dx+c)^n\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\left(b^3\cos(dx+c)^2-3a^2b-b^3\right)\sec(dx+c)^5\sin(dx+c)+\left(3ab^2\cos(dx+c)^2-a^3-3ab^2\right)\sec(dx+c)^5\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-((b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sec(d*x + c)^5*sin(d*x + c)
 + (3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2)*sec(d*x + c)^5)*sin(d*x + c)^n,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^3\sin(dx+c)^n\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

3.1511 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{\left(a^2(3-n)-b^2(n+1)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{\sec^4(c+dx)\sin^{n+1}(c+dx)\left(a^2+2ab\sin(c+a\right.}{4d}$$

```
[Out] ((a^2*(3 - n) - b^2*(1 + n))*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(4*d*(1 + n)) + (a*b*(2 - n)*Hypergeomet
ric2F1[2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(2*d*
(2 + n)) + (Sec[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a^2 + b^2 + 2*a*b*Sin[c +
d*x]))/(4*d)
```

Rubi [A] time = 0.258059, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 1806, 808, 364}

$$\frac{\left(a^2(3-n)-b^2(n+1)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{\sec^4(c+dx)\sin^{n+1}(c+dx)\left(a^2+2ab\sin(c+a\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] ((a^2*(3 - n) - b^2*(1 + n))*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(4*d*(1 + n)) + (a*b*(2 - n)*Hypergeomet
ric2F1[2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(2*d*
(2 + n)) + (Sec[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a^2 + b^2 + 2*a*b*Sin[c +
d*x]))/(4*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} - \frac{b^3\,\text{Subst}}{}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} + \frac{\left(ab^4(2-\right.}{}$$

$$= \frac{\left(a^2(3-n)-b^2(1+n)\right)\,{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx}{4d(1+n)}$$

Mathematica [A] time = 0.218732, size = 158, normalized size = 0.99

$$\frac{\sin^{n+1}(c+dx)\left(2\left(3a^2-b^2\right)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+2(a-b)^2\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))+(3a+b)(a-b\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*(3*a^2 - b^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2
] + (a - b)*(3*a + b)*Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]] + (
3*a - b)*(a + b)*Hypergeometric2F1[2, 1 + n, 2 + n, Sin[c + d*x]] + 2*(a -
b)^2*Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]] + 2*(a + b)^2*Hyperg
eometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]])*Sin[c + d*x]^(1 + n))/(16*d*(1
+ n))
```

Maple [F] time = 1.642, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^5(\sin(dx+c))^n(a+b\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^2\sin(dx+c)^n\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2\,ab\sec\left(dx+c\right)^{5}\sin\left(dx+c\right)-\left(b^{2}\cos\left(dx+c\right)^{2}-a^{2}-b^{2}\right)\sec\left(dx+c\right)^{5}\right)\sin\left(dx+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral((2*a*b*sec(d*x + c)^5*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^5)*sin(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)+a\right)^{2}\sin\left(dx+c\right)^{n}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

3.1512 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{a\sin^{n+1}(c+dx)\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)}+\frac{b\sin^{n+2}(c+dx)\,{}_2F_1\left(3,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)}$$

```
[Out] (a*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^
(1 + n))/(d*(1 + n)) + (b*Hypergeometric2F1[3, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n))
```

Rubi [A] time = 0.113608, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 808, 364}

$$\frac{a\sin^{n+1}(c+dx)\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)}+\frac{b\sin^{n+2}(c+dx)\,{}_2F_1\left(3,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^
(1 + n))/(d*(1 + n)) + (b*Hypergeometric2F1[3, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\left(ab^5\right)\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d} + \frac{b^6\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^{1+n}}{\left(b^2-x^2\right)}\right.}{}$$
$$= \frac{a\,{}_2F_1\left(3, \frac{1+n}{2}; \frac{3+n}{2}; \sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)} + \frac{b\,{}_2F_1\left(3, \frac{2+n}{2}; \frac{4+}{2}\right.}{}$$

Mathematica [A] time = 0.1109, size = 89, normalized size = 1.

$$\frac{\sin^{n+1}(c+dx)\left(a(n+2)\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)+b(n+1)\sin(c+dx)\,{}_2F_1\left(3,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)\right)}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(a*(2 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2,
 Sin[c + d*x]^2] + b*(1 + n)*Hypergeometric2F1[3, (2 + n)/2, (4 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x]))/(d*(1 + n)*(2 + n))
```

Maple [F] time = 0.885, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^5 (\sin(dx+c))^n (a+b\sin(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)\sin(dx+c)^n\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec(dx+c)^5\sin(dx+c)+a\sec(dx+c)^5\right)\sin(dx+c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5*sin(d*x + c) + a*sec(d*x + c)^5)*sin(d*x + c)^n,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)\sin(dx+c)^n\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

3.1513 $\int \frac{\sec^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=360

$$\frac{(3a^2-9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{16d(n+1)(a-b)^3}+\frac{(3a^2+9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n$$

```
[Out] ((3*a^2 - 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*
Sin[c + d*x]^(1 + n))/(16*(a - b)^3*d*(1 + n)) + ((3*a^2 + 9*a*b + 8*b^2)*H
ypergeometric2F1[1, 1 + n, 2 + n, Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(16*(
a + b)^3*d*(1 + n)) - (b^6*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c +
d*x])/a)]*Sin[c + d*x]^(1 + n))/(a*(a^2 - b^2)^3*d*(1 + n)) + ((3*a - 5*b)*
Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(16
*(a - b)^2*d*(1 + n)) + ((3*a + 5*b)*Hypergeometric2F1[2, 1 + n, 2 + n, Sin
[c + d*x]]*Sin[c + d*x]^(1 + n))/(16*(a + b)^2*d*(1 + n)) + (Hypergeometric
2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(8*(a - b)*d*(1 +
 n)) + (Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]]*Sin[c + d*x]^(1 +
n))/(8*(a + b)*d*(1 + n))
```

Rubi [A] time = 0.541221, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 961, 64}

$$\frac{(3a^2-9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{16d(n+1)(a-b)^3}+\frac{(3a^2+9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```


```
[Out] ((3*a^2 - 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*
Sin[c + d*x]^(1 + n))/(16*(a - b)^3*d*(1 + n)) + ((3*a^2 + 9*a*b + 8*b^2)*H
ypergeometric2F1[1, 1 + n, 2 + n, Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(16*(
a + b)^3*d*(1 + n)) - (b^6*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c +
d*x])/a)]*Sin[c + d*x]^(1 + n))/(a*(a^2 - b^2)^3*d*(1 + n)) + ((3*a - 5*b)*
Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(16
*(a - b)^2*d*(1 + n)) + ((3*a + 5*b)*Hypergeometric2F1[2, 1 + n, 2 + n, Sin
[c + d*x]]*Sin[c + d*x]^(1 + n))/(16*(a + b)^2*d*(1 + n)) + (Hypergeometric
2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(8*(a - b)*d*(1 +
 n)) + (Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]]*Sin[c + d*x]^(1 +
n))/(8*(a + b)*d*(1 + n))
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{\sec^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^5\,\text{Subst}\left(\int \left(\frac{\left(\frac{x}{b}\right)^n}{8b^3(a+b)(b-x)^3} + \frac{(3a+5b)\left(\frac{x}{b}\right)^n}{16b^4(a+b)^2(b-x)^2} + \frac{\left(3a^2+9ab+8b^2\right)\left(\frac{x}{b}\right)^n}{16b^5(a+b)^3(b-x)} - \frac{\left(\frac{x}{b}\right)^n}{(a-b)^3(a+b)^3(a+x)} - \frac{(}{8b^3(-a}\right.\right.}{d}$$

$$= \frac{((3a-5b)b)\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(b+x)^2}\,dx, x, b\sin(c+dx)\right)}{16(a-b)^2d} + \frac{b^2\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(b+x)^3}\,dx, x, b\sin(c+d\right.}{8(a-b)d}$$

$$= \frac{\left(3a^2-9ab+8b^2\right)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{16(a-b)^3d(1+n)} + \frac{\left(3a^2+9ab+8b^2\right.}{}$$

Mathematica [A] time = 0.451814, size = 241, normalized size = 0.67

$$\frac{\sin^{n+1}(c+dx)\left(\frac{\left(3a^2-9ab+8b^2\right){}_2F_1(1,n+1;n+2;-\sin(c+dx))}{(a-b)^3} + \frac{\left(3a^2+9ab+8b^2\right){}_2F_1(1,n+1;n+2;\sin(c+dx))}{(a+b)^3} - \frac{16b^6\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a(a-b)^3(a+b)^3} + \frac{(3a-5}{}\right.}{16d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((((3*a^2 - 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]
])/(a - b)^3 + ((3*a^2 + 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n,
Sin[c + d*x]])/(a + b)^3 - (16*b^6*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*
Sin[c + d*x])/a)])/(a*(a - b)^3*(a + b)^3) + ((3*a - 5*b)*Hypergeometric2F1
[2, 1 + n, 2 + n, -Sin[c + d*x]])/(a - b)^2 + ((3*a + 5*b)*Hypergeometric2F
1[2, 1 + n, 2 + n, Sin[c + d*x]])/(a + b)^2 + (2*Hypergeometric2F1[3, 1 + n
, 2 + n, -Sin[c + d*x]])/(a - b) + (2*Hypergeometric2F1[3, 1 + n, 2 + n, Si
n[c + d*x]])/(a + b))*Sin[c + d*x]^(1 + n))/(16*d*(1 + n))
```

Maple [F] time = 1.071, size = 0, normalized size = 0.

$$\int \frac{(\sec(dx+c))^5 (\sin(dx+c))^n}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(dx+c\right)^{n}\sec\left(dx+c\right)^{5}}{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*sec(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)^{n}\sec\left(dx+c\right)^{5}}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sec(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

3.1514 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=487

$$\frac{3\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,1;n+2;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)}{16d(n+1)}+\frac{3\sin^{n+1}(c+dx}{}$$

```
[Out] (3*AppellF1[1 + n, -p, 1, 2 + n, -((b*Sin[c + d*x])/a), -Sin[c + d*x]]*Sin[
c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(1 + (b*Sin[c + d*x]
)/a)^p) + (3*AppellF1[1 + n, -p, 1, 2 + n, -((b*Sin[c + d*x])/a), Sin[c + d
*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(1 + (b*Sin
[c + d*x])/a)^p) + (3*AppellF1[1 + n, -p, 2, 2 + n, -((b*Sin[c + d*x])/a),
-Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(
1 + (b*Sin[c + d*x])/a)^p) + (3*AppellF1[1 + n, -p, 2, 2 + n, -((b*Sin[c +
d*x])/a), Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*
(1 + n)*(1 + (b*Sin[c + d*x])/a)^p) + (AppellF1[1 + n, -p, 3, 2 + n, -((b*S
in[c + d*x])/a), -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p
)/(8*d*(1 + n)*(1 + (b*Sin[c + d*x])/a)^p) + (AppellF1[1 + n, -p, 3, 2 + n,
 -((b*Sin[c + d*x])/a), Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d
*x])^p)/(8*d*(1 + n)*(1 + (b*Sin[c + d*x])/a)^p)
```

Rubi [A] time = 0.566236, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2837, 961, 135, 133, 912}

$$\frac{3\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,1;n+2;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)}{16d(n+1)}+\frac{3\sin^{n+1}(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
```


```
[Out] (3*AppellF1[1 + n, -p, 1, 2 + n, -((b*Sin[c + d*x])/a), -Sin[c + d*x]]*Sin[
c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(1 + (b*Sin[c + d*x]
)/a)^p) + (3*AppellF1[1 + n, -p, 1, 2 + n, -((b*Sin[c + d*x])/a), Sin[c + d
*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(1 + (b*Sin
[c + d*x])/a)^p) + (3*AppellF1[1 + n, -p, 2, 2 + n, -((b*Sin[c + d*x])/a),
-Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(
1 + (b*Sin[c + d*x])/a)^p) + (3*AppellF1[1 + n, -p, 2, 2 + n, -((b*Sin[c +
d*x])/a), Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*
(1 + n)*(1 + (b*Sin[c + d*x])/a)^p) + (AppellF1[1 + n, -p, 3, 2 + n, -((b*S
in[c + d*x])/a), -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p
)/(8*d*(1 + n)*(1 + (b*Sin[c + d*x])/a)^p) + (AppellF1[1 + n, -p, 3, 2 + n,
 -((b*Sin[c + d*x])/a), Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d
*x])^p)/(8*d*(1 + n)*(1 + (b*Sin[c + d*x])/a)^p)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^5\,\text{Subst}\left(\int \left(\frac{\left(\frac{x}{b}\right)^n(a+x)^p}{8b^3(b-x)^3} + \frac{3\left(\frac{x}{b}\right)^n(a+x)^p}{16b^4(b-x)^2} + \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{8b^3(b+x)^3} + \frac{3\left(\frac{x}{b}\right)^n(a+x)^p}{16b^4(b+x)^2} + \cdots\right.\right.}{d}$$

$$= \frac{(3b)\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{(b-x)^2}\,dx, x, b\sin(c+dx)\right)}{16d} + \frac{(3b)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(3b)\,\text{Subst}\left(\int \left(\frac{\left(\frac{x}{b}\right)^n(a+x)^p}{2b(b-x)} + \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{2b(b+x)}\right)dx, x, b\sin(c+dx)\right)}{8d} + \cdots$$

$$= \frac{3F_1\left(1+n;-p,2;2+n;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)\sin^{1+n}(c+dx\cdots}{16d(1+n)}$$

$$= \frac{3F_1\left(1+n;-p,2;2+n;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)\sin^{1+n}(c+dx\cdots}{16d(1+n)}$$

$$= \frac{3F_1\left(1+n;-p,1;2+n;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)\sin^{1+n}(c+dx\cdots}{16d(1+n)}$$

Mathematica [F] time = 11.9553, size = 0, normalized size = 0.

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
```

```
[Out] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

Maple [F] time = 1.485, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{5} (\sin (dx+c))^{n} (a+b\sin (dx+c))^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="maxima
")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sin (dx+c)+a)^{p} \sin (dx+c)^{n} \sec (dx+c)^{5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="fricas
")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^p \sin(dx+c)^n \sec(dx+c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

3.1515 $\int \frac{\sec^6(e+fx)(a+b\sin(e+fx))^{9/2}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=502

$$-\frac{3ab\left(b^2-2a^2\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{5f\sqrt{d\sin(e+fx)}}-\frac{3a\sec^3(e+fx)\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}\left(\left(8a^2b-4b^3\right)\sin^3(e+\right.}{2}$$

```
[Out] (-3*a*b*(-2*a^2 + b^2)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(5*f*Sqrt[d*S
in[e + f*x]]) + (Sec[e + f*x]^5*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(
9/2))/(5*d*f) - (3*a*Sec[e + f*x]^3*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]]*(-(a*(7*a^2 + b^2)) + 2*b*(-7*a^2 + b^2)*Sin[e + f*x] + 5*a*(a^2 - b
^2)*Sin[e + f*x]^2 + (8*a^2*b - 4*b^3)*Sin[e + f*x]^3))/(20*d*f) - (3*a*(a
+ b)^(3/2)*(5*a^2 + 3*a*b - 4*b^2)*Sqrt[-((a*(-1 + Csc[e + f*x]))/(a + b))]
*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*
Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan
[e + f*x])/(20*Sqrt[d]*f) - (3*b*(2*a^4 - 3*a^2*b^2 + b^4)*Sqrt[-((a*(-1 +
Csc[e + f*x]))/(a + b))]*EllipticE[ArcSin[Sqrt[-((b + a*Csc[e + f*x])/(a -
b))]], 1 - (2*a)/(a + b)]*Sqrt[d*Sin[e + f*x]]*Sqrt[-((a*Csc[e + f*x]^2*(1
+ Sin[e + f*x])*(a + b*Sin[e + f*x]))/(a - b)^2)]*Tan[e + f*x])/(5*d*f*Sqrt
[a + b*Sin[e + f*x]])
```

Rubi [F] time = 0.370213, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec^6(e+fx)(a+b\sin(e+fx))^{9/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[e + f*x]^6*(a + b*Sin[e + f*x])^(9/2))/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (Sec[e + f*x]^5*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(9/2))/(5*d*f) +
(9*a*Defer[Int][(Sec[e + f*x]^4*(a + b*Sin[e + f*x])^(7/2))/Sqrt[d*Sin[e +
f*x]], x])/10
```

Rubi steps

$$\int \frac{\sec^6(e+fx)(a+b\sin(e+fx))^{9/2}}{\sqrt{d\sin(e+fx)}}\,dx=\frac{\sec^5(e+fx)\sqrt{d\sin(e+fx)}(a+b\sin(e+fx))^{9/2}}{5df}+\frac{1}{10}(9a)\int \frac{\sec^4(e+fx)(a+}{\sqrt{d\sin}}$$

Mathematica [C] time = 23.9746, size = 1600, normalized size = 3.19

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^6*(a + b*Sin[e + f*x])^(9/2))/Sqrt[d*Sin[e + f*x]],
x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((Sec[e + f*x]*(15*a^4 - 15*a^2*b^2
+ 4*b^4 + 24*a^3*b*Sin[e + f*x] - 12*a*b^3*Sin[e + f*x]))/20 + (Sec[e + f*x
]^3*(3*a^4 - 3*a^2*b^2 - 4*b^4 + 9*a^3*b*Sin[e + f*x] - 5*a*b^3*Sin[e + f*x
]))/10 + (Sec[e + f*x]^5*(a^4 + 6*a^2*b^2 + b^4 + 4*a^3*b*Sin[e + f*x] + 4*
a*b^3*Sin[e + f*x]))/5))/(f*Sqrt[d*Sin[e + f*x]]) + (3*a*Sqrt[Sin[e + f*x]]
*((4*a*(5*a^4 - 9*a^2*b^2 + 4*b^4)*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
+ f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2
]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[
(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]
]*Sqrt[a + b*Sin[e + f*x]]) + 4*a*(-8*a^3*b + 4*a*b^3)*((Sqrt[((a + b)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f
*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*S
in[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e
+ f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a
+ b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[((a + b)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f
*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*S
in[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])
/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) + 2*(8*a^2*b^2 - 4*b^4)*(
(Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Sin[e + f*x]]) + (I*Cos[(-e
 + Pi/2 - f*x)/2]*Csc[e + f*x]*EllipticE[I*ArcSinh[Sin[(-e + Pi/2 - f*x)/2]
/Sqrt[Sin[e + f*x]]], (-2*a)/(-a - b)]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Co
s[(-e + Pi/2 - f*x)/2]^2*Csc[e + f*x]]*Sqrt[(Csc[e + f*x]*(a + b*Sin[e + f*
x]))/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b
)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[
-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a
+ b*Sin[e + f*x]]) - (a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]
*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-
e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a
 + b*Sin[e + f*x]])))/b)))/(40*f*Sqrt[d*Sin[e + f*x]])
```

Maple [B] time = 0.783, size = 5578, normalized size = 11.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{9}{2}}\sec\left(fx+e\right)^{6}}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(9/2)*sec(f*x + e)^6/sqrt(d*sin(f*x + e)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(4\left(ab^3\cos\left(fx+e\right)^2 - a^3b - ab^3\right)\sec\left(fx+e\right)^6\sin\left(fx+e\right) - \left(b^4\cos\left(fx+e\right)^4 + a^4 + 6\,a^2b^2 + b^4 - 2\left(3\,a^2b^2\right.\right.\right.}{d\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(4*(a*b^3*cos(f*x + e)^2 - a^3*b - a*b^3)*sec(f*x + e)^6*sin(f*x
+ e) - (b^4*cos(f*x + e)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*co
s(f*x + e)^2)*sec(f*x + e)^6)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
/(d*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sin(f*x+e))**(9/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{9}{2}}\sec\left(fx+e\right)^6}{\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(9/2)*sec(f*x + e)^6/sqrt(d*sin(f*x + e)), x
)
```

3.1516 $\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=458

$$-\frac{3(c+d)^2\left(208a^2cd^2-64abd\left(3c^2-5d^2\right)+b^2c\left(54c^2+d^2\right)\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e\right.}{1040\sqrt{2}d^4f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (-9*(64*a*b*c*d - 26*a^2*d^2 - b^2*(18*c^2 - 13*d^2))*Cos[e + f*x]*(c + d*S
in[e + f*x])^(7/3))/(2080*d^3*f) - (9*b*(3*b*c - 2*a*d)*Cos[e + f*x]*Sin[e
+ f*x]*(c + d*Sin[e + f*x])^(7/3))/(208*d^2*f) + (3*Cos[e + f*x]*(a + b*Sin
[e + f*x])^2*(c + d*Sin[e + f*x])^(7/3))/(16*d*f) - (3*(c + d)^2*(208*a^2*c
*d^2 - 64*a*b*d*(3*c^2 - 5*d^2) + b^2*c*(54*c^2 + d^2))*AppellF1[1/2, 1/2,
-7/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*
x]*(c + d*Sin[e + f*x])^(1/3))/(1040*Sqrt[2]*d^4*f*Sqrt[1 + Sin[e + f*x]]*(
(c + d*Sin[e + f*x])/(c + d))^(1/3)) - (3*(c - d)*(c + d)^2*(192*a*b*c*d -
208*a^2*d^2 - b^2*(54*c^2 + 91*d^2))*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sin
[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(1/3))/(1040*Sqrt[2]*d^4*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x]
)/(c + d))^(1/3))
```

Rubi [A] time = 1.24225, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2922, 3050, 3033, 3023, 2756, 2665, 139, 138}

$$-\frac{3(c+d)^2\left(208a^2cd^2-64abd\left(3c^2-5d^2\right)+b^2c\left(54c^2+d^2\right)\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e\right.}{1040\sqrt{2}d^4f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-9*(64*a*b*c*d - 26*a^2*d^2 - b^2*(18*c^2 - 13*d^2))*Cos[e + f*x]*(c + d*S
in[e + f*x])^(7/3))/(2080*d^3*f) - (9*b*(3*b*c - 2*a*d)*Cos[e + f*x]*Sin[e
+ f*x]*(c + d*Sin[e + f*x])^(7/3))/(208*d^2*f) + (3*Cos[e + f*x]*(a + b*Sin
[e + f*x])^2*(c + d*Sin[e + f*x])^(7/3))/(16*d*f) - (3*(c + d)^2*(208*a^2*c
*d^2 - 64*a*b*d*(3*c^2 - 5*d^2) + b^2*c*(54*c^2 + d^2))*AppellF1[1/2, 1/2,
-7/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*
x]*(c + d*Sin[e + f*x])^(1/3))/(1040*Sqrt[2]*d^4*f*Sqrt[1 + Sin[e + f*x]]*(
(c + d*Sin[e + f*x])/(c + d))^(1/3)) - (3*(c - d)*(c + d)^2*(192*a*b*c*d -
208*a^2*d^2 - b^2*(54*c^2 + 91*d^2))*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sin
[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(1/3))/(1040*Sqrt[2]*d^4*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x]
)/(c + d))^(1/3))
```

Rule 2922

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{4/3}\,dx = \int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{4/3}\left(1-\sin^2(e+\right.$$

$$= \frac{3\cos(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{7/3}}{16df} +$$

$$= -\frac{9b(3bc-2ad)\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx}{208d^2f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+fx)(c}{2080d^3f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+fx)(c}{2080d^3f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+fx)(c}{2080d^3f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+fx)(c}{2080d^3f}$$

$$= -\frac{9\left(64abcd-26a^2d^2-b^2\left(18c^2-13d^2\right)\right)\cos(e+fx)(c}{2080d^3f}$$

Mathematica [B] time = 7.04558, size = 3522, normalized size = 7.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(4/3),
x]
```

```
[Out] (513*a*b*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d
)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e
 + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(
1/3))/(455*f) + (81*b^2*c^4*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f
*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*S
qrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c +
d*Sin[e + f*x])^(1/3))/(7280*d^3*f) - (18*a*b*c^3*AppellF1[1/3, 1/2, 1/2, 4
/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/
d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e
+ f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/(455*d^2*f) + (54*a^2*c^2*Appe
llF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Si
n[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d
)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/(35*d*f)
```

+ (5211*b^2*c^2*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/(14560*d*f) + (9*a^2*d*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/(40*f) + (63*b^2*d*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/(640*f) + (9*a*b*c^2*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(65*f) + (81*b^2*c^5*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(7280*d^3*f) - (18*a*b*c^4*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(455*d^2*f) + (3*a^2*c^3*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(70*d*f) - (21*b^2*c^3*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(1040*d*f) + (153*a^2*c*d*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(280*f) + (9603*b^2*c*d*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2, 1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(58240*f) + (24*a*b*d^2*((-3*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1/3))/d^2 + (3*AppellF1[4/3, 1/2,

```
1/2, 7/3, -((c + d*Sin[e + f*x])/((1 - c/d)*d)), -((c + d*Sin[e + f*x])/((
-1 - c/d)*d))]*Sec[e + f*x]*Sqrt[(-d - d*Sin[e + f*x])/(c - d)]*Sqrt[(d - d
*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(4/3))/(4*d^2)))/(91*f) + ((c
+ d*Sin[e + f*x])^(1/3)*((-3*(-216*b^2*c^4 + 768*a*b*c^3*d - 832*a^2*c^2*d^
2 + 332*b^2*c^2*d^2 + 7232*a*b*c*d^3 + 2912*a^2*d^4 + 1729*b^2*d^4)*Cos[e +
 f*x])/(58240*d^3) - (3*(8*b^2*c^2 + 896*a*b*c*d + 416*a^2*d^2 + 117*b^2*d^
2)*Cos[3*(e + f*x)])/(16640*d) + (3*b^2*d*Cos[5*(e + f*x)])/256 + (3*(-18*b
^2*c^3 + 64*a*b*c^2*d + 1144*a^2*c*d^2 + 23*b^2*c*d^2 + 80*a*b*d^3)*Sin[2*(
e + f*x)])/(14560*d^2) - (3*b*(17*b*c + 32*a*d)*Sin[4*(e + f*x)])/1664))/f
```

Maple [F] time = 0.376, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2}\left(a+b\sin\left(fx+e\right)\right)^{2}\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\left(b^{2}c+2\,abd\right)\cos\left(fx+e\right)^{4}-\left(2\,abd+\left(a^{2}+b^{2}\right)c\right)\cos\left(fx+e\right)^{2}+\left(b^{2}d\cos\left(fx+e\right)^{4}-\left(2\,abc+\left(a^{2}+b^{2}\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="fricas")
```

```
[Out] integral(-((b^2*c + 2*a*b*d)*cos(f*x + e)^4 - (2*a*b*d + (a^2 + b^2)*c)*cos
(f*x + e)^2 + (b^2*d*cos(f*x + e)^4 - (2*a*b*c + (a^2 + b^2)*d)*cos(f*x + e
)^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2,
 x)
```

3.1517 $\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=341

$$\frac{3(c+d)^2\left(-13acd+6bc^2-10bd^2\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{65\sqrt{2}d^3f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}-\ldots$$

```
[Out] (-3*(6*b*c - 13*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(130*d^2*f) +
 (3*b*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(13*d*f) + (3*(
c + d)^2*(6*b*c^2 - 13*a*c*d - 10*b*d^2)*AppellF1[1/2, 1/2, -7/3, 3/2, (1 -
 Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e
 + f*x])^(1/3))/(65*Sqrt[2]*d^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^(1/3)) - (3*(c - d)*(c + d)^2*(6*b*c - 13*a*d)*AppellF1[1/2, 1
/2, -4/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^(1/3))/(65*Sqrt[2]*d^3*f*Sqrt[1 + Sin[e + f*x]]
*((c + d*Sin[e + f*x])/(c + d))^(1/3))
```

Rubi [A] time = 0.651351, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2922, 3034, 3023, 2756, 2665, 139, 138}

$$\frac{3(c+d)^2\left(-13acd+6bc^2-10bd^2\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{65\sqrt{2}d^3f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-3*(6*b*c - 13*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(130*d^2*f) +
 (3*b*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(13*d*f) + (3*(
c + d)^2*(6*b*c^2 - 13*a*c*d - 10*b*d^2)*AppellF1[1/2, 1/2, -7/3, 3/2, (1 -
 Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e
 + f*x])^(1/3))/(65*Sqrt[2]*d^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^(1/3)) - (3*(c - d)*(c + d)^2*(6*b*c - 13*a*d)*AppellF1[1/2, 1
/2, -4/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^(1/3))/(65*Sqrt[2]*d^3*f*Sqrt[1 + Sin[e + f*x]]
*((c + d*Sin[e + f*x])/(c + d))^(1/3))
```

Rule 2922

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
```

```
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{4/3}\,dx = \int (a+b\sin(e+fx))(c+d\sin(e+fx))^{4/3}\left(1-\sin^2(e+f\right.$$

$$= \frac{3b\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx))^{7/3}}{13df} + \frac{3\int(c}{}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + \frac{3b\,\mathrm{co}}{}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + \frac{3b\,\mathrm{co}}{}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + \frac{3b\,\mathrm{co}}{}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + \frac{3b\,\mathrm{co}}{}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f} + \frac{3b\,\mathrm{co}}{}$$

Mathematica [A] time = 5.06916, size = 398, normalized size = 1.17

$$\frac{3\sec(e+fx)\sqrt[3]{c+d\sin(e+fx)}\left(3\left(52ac^3d+663acd^3+84bc^2d^2-24bc^4+160bd^4\right)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{-\frac{d(\sin(e+fx)+1)}{c-d}}(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (3*Sec[e + f*x]*(c + d*Sin[e + f*x])^(1/3)*(12*(-c^2 + d^2)*(-24*b*c^3 + 52
*a*c^2*d + 68*b*c*d^2 + 91*a*d^3)*AppellF1[1/3, 1/2, 1/2, 4/3, (c + d*Sin[e
 + f*x])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-1 + Sin[e + f*x
]))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))] + 3*(-24*b*c^4 + 52*a
*c^3*d + 84*b*c^2*d^2 + 663*a*c*d^3 + 160*b*d^4)*AppellF1[4/3, 1/2, 1/2, 7/
3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-
1 + Sin[e + f*x]))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))]*(c + d
*Sin[e + f*x]) - 4*d^2*Cos[e + f*x]^2*(24*b*c^3 - 52*a*c^2*d + 128*b*c*d^2
+ 91*a*d^3 + 14*d^2*(14*b*c + 13*a*d)*Cos[2*(e + f*x)] - 2*d*(8*b*c^2 + 286
*a*c*d + 45*b*d^2)*Sin[e + f*x] + 70*b*d^3*Sin[3*(e + f*x)])))/(14560*d^4*f
)
```

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(bd\cos\left(fx+e\right)^{4}-(bc+ad)\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-(ac+bd)\cos\left(fx+e\right)^{2}\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral(-(b*d*cos(f*x + e)^4 - (b*c + a*d)*cos(f*x + e)^2*sin(f*x + e) - (
a*c + b*d)*cos(f*x + e)^2)*(d*sin(f*x + e) + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x
)
```

3.1518 $\int \cos^2(e+fx)(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=125

$$\frac{3\cos(e+fx)(c+d\sin(e+fx))^{7/3}F_1\left(\frac{7}{3};-\frac{1}{2},-\frac{1}{2};\frac{10}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{7df\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (3*AppellF1[7/3, -1/2, -1/2, 10/3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin
[e + f*x])/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(7*d*f*Sqrt[1
- (c + d*Sin[e + f*x])/(c - d)]*Sqrt[1 - (c + d*Sin[e + f*x])/(c + d)])
```

Rubi [A] time = 0.129703, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2704, 138}

$$\frac{3\cos(e+fx)(c+d\sin(e+fx))^{7/3}F_1\left(\frac{7}{3};-\frac{1}{2},-\frac{1}{2};\frac{10}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{7df\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (3*AppellF1[7/3, -1/2, -1/2, 10/3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin
[e + f*x])/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(7*d*f*Sqrt[1
- (c + d*Sin[e + f*x])/(c - d)]*Sqrt[1 - (c + d*Sin[e + f*x])/(c + d)])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(c+d\sin(e+fx))^{4/3}\,dx = \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int (c+dx)^{4/3}\sqrt{-\frac{d}{c-d}-\frac{dx}{c-d}}\sqrt{\frac{d}{c+d}-\frac{dx}{c+d}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$
$$= \frac{3F_1\left(\frac{7}{3}; -\frac{1}{2}, -\frac{1}{2}; \frac{10}{3}; \frac{c+d\sin(e+fx)}{c-d}, \frac{c+d\sin(e+fx)}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{7df\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [B] time = 2.1045, size = 301, normalized size = 2.41

$$-\frac{3\sec(e+fx)\sqrt[3]{c+d\sin(e+fx)}\left(-3c\left(4c^2+51d^2\right)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{-\frac{d(\sin(e+fx)+1)}{c-d}}(c+d\sin(e+fx))F_1\left(\frac{4}{3}; \frac{1}{2}, \frac{1}{2}; \frac{7}{3}; \frac{c+d\sin\ldots}{\ldots}\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-3*Sec[e + f*x]*(c + d*Sin[e + f*x])^(1/3)*(12*(4*c^4 + 3*c^2*d^2 - 7*d^4)
*AppellF1[1/3, 1/2, 1/2, 4/3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[e +
f*x])/(c + d)]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[-((d*(1 + Sin[
e + f*x]))/(c - d))] - 3*c*(4*c^2 + 51*d^2)*AppellF1[4/3, 1/2, 1/2, 7/3, (c
 + d*Sin[e + f*x])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-1 + S
in[e + f*x]))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))]*(c + d*Sin[
e + f*x]) + 4*d^2*Cos[e + f*x]^2*(-4*c^2 + 7*d^2 + 14*d^2*Cos[2*(e + f*x)]
- 44*c*d*Sin[e + f*x])))/(1120*d^3*f)
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2 \left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+c\cos\left(fx+e\right)^{2}\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e)^2*sin(f*x + e) + c*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x)
```

3.1519 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)},x\right)$$

```
[Out] Unintegrable[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x
]), x]
```

Rubi [A] time = 0.210811, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x]),x]
```

```
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])
, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$$

Mathematica [A] time = 60.8395, size = 0, normalized size = 0.

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x]),
x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x]),
 x]
```

Maple [A] time = 0.24, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2}{a+b\sin\left(fx+e\right)}\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**(4/3)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a), x
)
```

3.1520 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x
])^2, x]
```

Rubi [A] time = 0.207739, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])
^2, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 45.2223, size = 0, normalized size = 0.

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])^
2,x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])^
2, x]
```

Maple [A] time = 0.517, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2}{\left(a+b\sin\left(fx+e\right)\right)^2}\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\sin(fx+e)+c)^{\frac{4}{3}}\cos(fx+e)^2}{(b\sin(fx+e)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2,
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**(4/3)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\sin(fx+e)+c)^{\frac{4}{3}}\cos(fx+e)^2}{(b\sin(fx+e)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2,
 x)
```

3.1521 $\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=35

$$\text{Unintegrable}\left(\cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,
x]
```

Rubi [A] time = 0.130924, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Mathematica [A] time = 7.07023, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

Maple [A] time = 0.384, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+b\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] Timed out
```

3.1522 $\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/
3), x]
```

Rubi [A] time = 0.195611, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3)
, x]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx = \int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$$

Mathematica [A] time = 35.8489, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3),
x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3),
 x]
```

Maple [A] time = 0.256, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+b\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\left(b\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*(b*sin(f*x + e) + a)^m*cos(f*x + e)^2,
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+c\cos\left(fx+e\right)^{2}\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}}\left(b\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="fricas")
```

```
[Out] integral((d*cos(f*x + e)^2*sin(f*x + e) + c*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^(1/3)*(b*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}\left(b\sin\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*(b*sin(f*x + e) + a)^m*cos(f*x + e)^2,
 x)
```

3.1523 $\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=552

$$-\frac{\sqrt{2}(c+d)\cos(e+fx)\left(a^2cd^2\left(n^2+7n+12\right)-2abd(n+4)\left(2c^2-d^2(n+2)\right)+b^2c\left(6c^2-d^2\left(-n^2-n+3\right)\right)\right)(c+d\sin(e}{d^4f(n+2)(n+3)(n+4)\sqrt{\sin(e+fx)+1}}$$

```
[Out] ((2*a^2*d^2*(3 + n) - 4*a*b*c*d*(4 + n) + b^2*(6*c^2 - d^2*(3 + n)))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^3*f*(2 + n)*(3 + n)*(4 + n)) - (b*(
3*b*c - 2*a*d)*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2
*f*(3 + n)*(4 + n)) + (Cos[e + f*x]*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f
*x])^(1 + n))/(d*f*(4 + n)) - (Sqrt[2]*(c + d)*(a^2*c*d^2*(12 + 7*n + n^2)
- 2*a*b*d*(4 + n)*(2*c^2 - d^2*(2 + n)) + b^2*c*(6*c^2 - d^2*(3 - n - n^2))
)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^4*f*(2 + n)*(3 + n)*(4
 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (Sqrt[2]*(
c^2 - d^2)*(4*a*b*c*d*(4 + n) - a^2*d^2*(12 + 7*n + n^2) - b^2*(6*c^2 + d^2
*(3 + 4*n + n^2)))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
- Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^4*f*(2 +
n)*(3 + n)*(4 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 1.51228, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2922, 3050, 3033, 3023, 2756, 2665, 139, 138}

$$-\frac{\sqrt{2}(c+d)\cos(e+fx)\left(a^2cd^2\left(n^2+7n+12\right)-2abd(n+4)\left(2c^2-d^2(n+2)\right)+b^2\left(6c^3-cd^2\left(-n^2-n+3\right)\right)\right)(c+d\sin(e}{d^4f(n+2)(n+3)(n+4)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] ((2*a^2*d^2*(3 + n) - 4*a*b*c*d*(4 + n) + b^2*(6*c^2 - d^2*(3 + n)))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^3*f*(2 + n)*(3 + n)*(4 + n)) - (b*(
3*b*c - 2*a*d)*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2
*f*(3 + n)*(4 + n)) + (Cos[e + f*x]*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f
*x])^(1 + n))/(d*f*(4 + n)) - (Sqrt[2]*(c + d)*(a^2*c*d^2*(12 + 7*n + n^2)
- 2*a*b*d*(4 + n)*(2*c^2 - d^2*(2 + n)) + b^2*(6*c^3 - c*d^2*(3 - n - n^2))
)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^4*f*(2 + n)*(3 + n)*(4
 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (Sqrt[2]*(
c^2 - d^2)*(4*a*b*c*d*(4 + n) - a^2*d^2*(12 + 7*n + n^2) - b^2*(6*c^2 + d^2
*(3 + 4*n + n^2)))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
- Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^4*f*(2 +
n)*(3 + n)*(4 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2922

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\left(1-\sin^2(e+fx)\right)$$

$$= \frac{\cos(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{1+n}}{df(4+n)} + \frac{\int (a}{}$$

$$= -\frac{b(3bc-2ad)\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+n)(4+n)}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right)\cos(e +}{d^3f(2+n)(3+n)(4+n)}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right)\cos(e +}{d^3f(2+n)(3+n)(4+n)}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right)\cos(e +}{d^3f(2+n)(3+n)(4+n)}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right)\cos(e +}{d^3f(2+n)(3+n)(4+n)}$$

$$= \frac{\left(2a^2d^2(3+n)-4abcd(4+n)+b^2\left(6c^2-d^2(3+n)\right)\right)\cos(e +}{d^3f(2+n)(3+n)(4+n)}$$

Mathematica [F] time = 8.32011, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.464, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+b\sin\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\left(b^2\cos\left(fx+e\right)^4-2\,ab\cos\left(fx+e\right)^2\sin\left(fx+e\right)-\left(a^2+b^2\right)\cos\left(fx+e\right)^2\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^4 - 2*a*b*cos(f*x + e)^2*sin(f*x + e) - (a^2 +
b^2)*cos(f*x + e)^2)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

3.1524 $\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=375

$$-\frac{\sqrt{2}(c+d)\cos(e+fx)\left(acd(n+3)-b\left(2c^2-d^2(n+2)\right)\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2};\frac{1}{2}(1-\text{si}\right.}{d^3f(n+2)(n+3)\sqrt{\sin(e+fx)+1}}$$

```
[Out] -(((2*b*c - a*d*(3 + n))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2*f*
(2 + n)*(3 + n))) + (b*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(1 +
n))/(d*f*(3 + n)) - (Sqrt[2]*(c + d)*(a*c*d*(3 + n) - b*(2*c^2 - d^2*(2 + n
)))*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f
*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^3*f*(2 + n)*(3 + n)*
Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (Sqrt[2]*(c^2 -
d^2)*(2*b*c - a*d*(3 + n))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2
, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^3
*f*(2 + n)*(3 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.635151, antiderivative size = 373, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2922, 3034, 3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2}(c+d)\cos(e+fx)\left(-acd(n+3)+2bc^2-bd^2(n+2)\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2};\frac{1}{2}(1-\text{sin}\right.}{d^3f(n+2)(n+3)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -(((2*b*c - a*d*(3 + n))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2*f*
(2 + n)*(3 + n))) + (b*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(1 +
n))/(d*f*(3 + n)) + (Sqrt[2]*(c + d)*(2*b*c^2 - b*d^2*(2 + n) - a*c*d*(3 +
n))*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f
*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^3*f*(2 + n)*(3 + n)*
Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (Sqrt[2]*(c^2 -
d^2)*(2*b*c - a*d*(3 + n))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2
, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^3
*f*(2 + n)*(3 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2922

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
```

```
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))(c+d\sin(e+fx))^n\left(1-\sin^2(e+fx)\right)\,d$$

$$= \frac{b\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+n)} + \frac{\int(c+d\sin}{}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(2+n)(3+n)} + \frac{b\cos}{}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(2+n)(3+n)} + \frac{b\cos}{}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(2+n)(3+n)} + \frac{b\cos}{}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(2+n)(3+n)} + \frac{b\cos}{}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(2+n)(3+n)} + \frac{b\cos}{}$$

Mathematica [F] time = 3.30566, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+a\cos\left(fx+e\right)^{2}\right)\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((b*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

3.1525 $\int \cos^2(e+fx)(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=127

$$\frac{\cos(e+fx)(c+d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},-\frac{1}{2};n+2;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{df(n+1)\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] (AppellF1[1 + n, -1/2, -1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Si
n[e + f*x])/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n
)*Sqrt[1 - (c + d*Sin[e + f*x])/(c - d)]*Sqrt[1 - (c + d*Sin[e + f*x])/(c +
 d)])
```

Rubi [A] time = 0.0930213, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2704, 138}

$$\frac{\cos(e+fx)(c+d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},-\frac{1}{2};n+2;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{df(n+1)\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, -1/2, -1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Si
n[e + f*x])/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n
)*Sqrt[1 - (c + d*Sin[e + f*x])/(c - d)]*Sqrt[1 - (c + d*Sin[e + f*x])/(c +
 d)])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(e+fx)(c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int (c+dx)^n\sqrt{-\frac{d}{c-d}-\frac{dx}{c-d}}\sqrt{\frac{d}{c+d}-\frac{dx}{c+d}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$
$$= \frac{F_1\left(1+n;-\frac{1}{2},-\frac{1}{2};2+n;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\cos(e+fx)(c+d\sin(e+f}{df(1+n)\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [F] time = 0.310351, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n, x]
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\sin\left(fx+e\right)+c\right)^n\cos\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

3.1526 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=35

$$\text{Unintegrable}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)},x\right)$$

```
[Out] Unintegrable[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]),
x]
```

Rubi [A] time = 0.140571, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]),x]
```

```
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$$

Mathematica [A] time = 3.03357, size = 0, normalized size = 0.

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]),x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]), x]
```

Maple [A] time = 0.697, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

3.1527 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=35

$$\text{Unintegrable}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2
, x]
```

Rubi [A] time = 0.13335, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2,
x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 4.45675, size = 0, normalized size = 0.

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2, x
]
```

Maple [A] time = 0.872, size = 0, normalized size = 0.

$$\int \frac{\left(\cos\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+b\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b^2*cos(f*x + e)^2 - 2*a*b
*sin(f*x + e) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2, x)
```

3.1528 $\int \cos^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=188

$$-\frac{(aB+Ab)\sin^8(c+dx)}{8d}-\frac{(aA-3bB)\sin^7(c+dx)}{7d}+\frac{(aB+Ab)\sin^6(c+dx)}{2d}+\frac{3(aA-bB)\sin^5(c+dx)}{5d}-\frac{3(aB+A}{}$$

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((3*a*A - b*B)*
Sin[c + d*x]^3)/(3*d) - (3*(A*b + a*B)*Sin[c + d*x]^4)/(4*d) + (3*(a*A - b*
B)*Sin[c + d*x]^5)/(5*d) + ((A*b + a*B)*Sin[c + d*x]^6)/(2*d) - ((a*A - 3*b
*B)*Sin[c + d*x]^7)/(7*d) - ((A*b + a*B)*Sin[c + d*x]^8)/(8*d) - (b*B*Sin[c
 + d*x]^9)/(9*d)
```

Rubi [A] time = 0.241527, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 772}

$$-\frac{(aB+Ab)\sin^8(c+dx)}{8d}-\frac{(aA-3bB)\sin^7(c+dx)}{7d}+\frac{(aB+Ab)\sin^6(c+dx)}{2d}+\frac{3(aA-bB)\sin^5(c+dx)}{5d}-\frac{3(aB+A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((3*a*A - b*B)*
Sin[c + d*x]^3)/(3*d) - (3*(A*b + a*B)*Sin[c + d*x]^4)/(4*d) + (3*(a*A - b*
B)*Sin[c + d*x]^5)/(5*d) + ((A*b + a*B)*Sin[c + d*x]^6)/(2*d) - ((a*A - 3*b
*B)*Sin[c + d*x]^7)/(7*d) - ((A*b + a*B)*Sin[c + d*x]^8)/(8*d) - (b*B*Sin[c
 + d*x]^9)/(9*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3\,dx,x,b\sin(c+dx)\right)}{b^7d}\\ &= \frac{\text{Subst}\left(\int \left(aAb^6+b^5(Ab+aB)x+b^4(-3aA+bB)x^2-3b^3\right.\right.}{}\\ &= \frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}-\frac{(3aA-bB)\sin}{3d}\end{aligned}$$

Mathematica [A] time = 0.774268, size = 151, normalized size = 0.8

$$\frac{\sin(c+dx)\left(-315(aB+Ab)\sin^7(c+dx)-360(aA-3bB)\sin^6(c+dx)+1260(aB+Ab)\sin^5(c+dx)+1512(aA-bB)\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(2520*a*A + 1260*(A*b + a*B)*Sin[c + d*x] - 840*(3*a*A - b*B)
*Sin[c + d*x]^2 - 1890*(A*b + a*B)*Sin[c + d*x]^3 + 1512*(a*A - b*B)*Sin[c
+ d*x]^4 + 1260*(A*b + a*B)*Sin[c + d*x]^5 - 360*(a*A - 3*b*B)*Sin[c + d*x]
^6 - 315*(A*b + a*B)*Sin[c + d*x]^7 - 280*b*B*Sin[c + d*x]^8))/(2520*d)
```

Maple [A] time = 0.088, size = 128, normalized size = 0.7

$$\frac{1}{d}\left(Bb\left(-\frac{(\cos(dx+c))^8\sin(dx+c)}{9}+\frac{\sin(dx+c)}{63}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))^4}{5}+\frac{8(\cos(dx+c))^2}{5}\right)\right)-\frac{Ab}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(B*b*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+
c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*A*b*cos(d*x+c)^8-1/8*a*B*cos(d*x+c)^
8+1/7*a*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.01799, size = 204, normalized size = 1.09

$$-\frac{280\,Bb\sin(dx+c)^9+315\,(Ba+Ab)\sin(dx+c)^8+360\,(Aa-3\,Bb)\sin(dx+c)^7-1260\,(Ba+Ab)\sin(dx+c)^6-1512}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2520*(280*B*b*sin(d*x + c)^9 + 315*(B*a + A*b)*sin(d*x + c)^8 + 360*(A*a
 - 3*B*b)*sin(d*x + c)^7 - 1260*(B*a + A*b)*sin(d*x + c)^6 - 1512*(A*a - B*
b)*sin(d*x + c)^5 + 1890*(B*a + A*b)*sin(d*x + c)^4 + 840*(3*A*a - B*b)*sin
(d*x + c)^3 - 2520*A*a*sin(d*x + c) - 1260*(B*a + A*b)*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.53072, size = 274, normalized size = 1.46

$$-\frac{315\,(Ba+Ab)\cos(dx+c)^8+8\left(35\,Bb\cos(dx+c)^8-5\,(9\,Aa+Bb)\cos(dx+c)^6-6\,(9\,Aa+Bb)\cos(dx+c)^4-8\,(9\,A\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2520*(315*(B*a + A*b)*cos(d*x + c)^8 + 8*(35*B*b*cos(d*x + c)^8 - 5*(9*A
*a + B*b)*cos(d*x + c)^6 - 6*(9*A*a + B*b)*cos(d*x + c)^4 - 8*(9*A*a + B*b)
*cos(d*x + c)^2 - 144*A*a - 16*B*b)*sin(d*x + c))/d
```

Sympy [A] time = 21.9177, size = 228, normalized size = 1.21

$$\begin{cases} \frac{16Aa\sin^7(c+dx)}{35d} + \frac{8Aa\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2Aa\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{Aa\sin(c+dx)\cos^6(c+dx)}{d} - \frac{Ab\cos^8(c+dx)}{8d} - \frac{Ba\cos^8(c+dx)}{8d} + \\ x\left(A + B\sin(c)\right)\left(a + b\sin(c)\right)\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a*sin(c + d*x)**7/(35*d) + 8*A*a*sin(c + d*x)**5*cos(c + d*
x)**2/(5*d) + 2*A*a*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a*sin(c + d*x)*co
s(c + d*x)**6/d - A*b*cos(c + d*x)**8/(8*d) - B*a*cos(c + d*x)**8/(8*d) + 1
6*B*b*sin(c + d*x)**9/(315*d) + 8*B*b*sin(c + d*x)**7*cos(c + d*x)**2/(35*d
) + 2*B*b*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + B*b*sin(c + d*x)**3*cos(c
 + d*x)**6/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))*cos(c)**7, Tr
ue))
```

Giac [A] time = 1.24839, size = 246, normalized size = 1.31

$$-\frac{Bb\sin(9\,dx + 9\,c)}{2304\,d} + \frac{7\,Aa\sin(3\,dx + 3\,c)}{64\,d} - \frac{(Ba + Ab)\cos(8\,dx + 8\,c)}{1024\,d} - \frac{(Ba + Ab)\cos(6\,dx + 6\,c)}{128\,d} - \frac{7\,(Ba + Ab)\,\mathrm{c}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2304*B*b*sin(9*d*x + 9*c)/d + 7/64*A*a*sin(3*d*x + 3*c)/d - 1/1024*(B*a
+ A*b)*cos(8*d*x + 8*c)/d - 1/128*(B*a + A*b)*cos(6*d*x + 6*c)/d - 7/256*(B
*a + A*b)*cos(4*d*x + 4*c)/d - 7/128*(B*a + A*b)*cos(2*d*x + 2*c)/d + 1/179
2*(4*A*a - 5*B*b)*sin(7*d*x + 7*c)/d + 1/320*(7*A*a - 2*B*b)*sin(5*d*x + 5*
c)/d + 7/128*(10*A*a + B*b)*sin(d*x + c)/d
```

3.1529 $\int \cos^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{(aB+Ab)\sin^6(c+dx)}{6d}+\frac{(aA-2bB)\sin^5(c+dx)}{5d}-\frac{(aB+Ab)\sin^4(c+dx)}{2d}-\frac{(2aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin^2}{2d}$$

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((2*a*A - b*B)*
Sin[c + d*x]^3)/(3*d) - ((A*b + a*B)*Sin[c + d*x]^4)/(2*d) + ((a*A - 2*b*B)
*Sin[c + d*x]^5)/(5*d) + ((A*b + a*B)*Sin[c + d*x]^6)/(6*d) + (b*B*Sin[c +
d*x]^7)/(7*d)
```

Rubi [A] time = 0.170796, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 772}

$$\frac{(aB+Ab)\sin^6(c+dx)}{6d}+\frac{(aA-2bB)\sin^5(c+dx)}{5d}-\frac{(aB+Ab)\sin^4(c+dx)}{2d}-\frac{(2aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((2*a*A - b*B)*
Sin[c + d*x]^3)/(3*d) - ((A*b + a*B)*Sin[c + d*x]^4)/(2*d) + ((a*A - 2*b*B)
*Sin[c + d*x]^5)/(5*d) + ((A*b + a*B)*Sin[c + d*x]^6)/(6*d) + (b*B*Sin[c +
d*x]^7)/(7*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}\\ &= \frac{\text{Subst}\left(\int \left(aAb^4+b^3(Ab+aB)x+b^2(-2aA+bB)x^2-2b(Ab\right.\right.}{}\\ &= \frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}-\frac{(2aA-bB)\sin^3(c}{3d}\end{aligned}$$

Mathematica [A] time = 0.291036, size = 116, normalized size = 0.81

$$\frac{\sin(c+dx)\left(35(aB+Ab)\sin^5(c+dx)+42(aA-2bB)\sin^4(c+dx)-105(aB+Ab)\sin^3(c+dx)-70(2aA-bB)\sin^2(\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(210*a*A + 105*(A*b + a*B)*Sin[c + d*x] - 70*(2*a*A - b*B)*Si
n[c + d*x]^2 - 105*(A*b + a*B)*Sin[c + d*x]^3 + 42*(a*A - 2*b*B)*Sin[c + d*
x]^4 + 35*(A*b + a*B)*Sin[c + d*x]^5 + 30*b*B*Sin[c + d*x]^6))/(210*d)
```

Maple [A] time = 0.063, size = 108, normalized size = 0.8

$$\frac{1}{d}\left(Bb\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)\right)-\frac{Ab(\cos(dx+c))^6}{6}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(B*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/6*A*b*cos(d*x+c)^6-1/6*a*B*cos(d*x+c)^6+1/5*a*A*(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.00513, size = 157, normalized size = 1.1

$$\frac{30\,Bb\sin(dx+c)^7+35\,(Ba+Ab)\sin(dx+c)^6+42\,(Aa-2\,Bb)\sin(dx+c)^5-105\,(Ba+Ab)\sin(dx+c)^4-70\,(2\,A}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/210*(30*B*b*sin(d*x + c)^7 + 35*(B*a + A*b)*sin(d*x + c)^6 + 42*(A*a - 2*
B*b)*sin(d*x + c)^5 - 105*(B*a + A*b)*sin(d*x + c)^4 - 70*(2*A*a - B*b)*sin
(d*x + c)^3 + 210*A*a*sin(d*x + c) + 105*(B*a + A*b)*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.34837, size = 224, normalized size = 1.57

$$-\frac{35\,(Ba+Ab)\cos(dx+c)^6+2\left(15\,Bb\cos(dx+c)^6-3\,(7\,Aa+Bb)\cos(dx+c)^4-4\,(7\,Aa+Bb)\cos(dx+c)^2-56\,A\right.}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/210*(35*(B*a + A*b)*cos(d*x + c)^6 + 2*(15*B*b*cos(d*x + c)^6 - 3*(7*A*a
 + B*b)*cos(d*x + c)^4 - 4*(7*A*a + B*b)*cos(d*x + c)^2 - 56*A*a - 8*B*b)*s
```

```
in(d*x + c))/d
```

Sympy [A] time = 7.3374, size = 178, normalized size = 1.24

$$\begin{cases} \frac{8Aa\sin^5(c+dx)}{15d} + \frac{4Aa\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Aa\sin(c+dx)\cos^4(c+dx)}{d} - \frac{Ab\cos^6(c+dx)}{6d} - \frac{Ba\cos^6(c+dx)}{6d} + \frac{8Bb\sin^7(c+dx)}{105d} + \frac{4Bb\sin^5(c+dx)\text{c}}{15d} \\ x(A+B\sin(c))(a+b\sin(c))\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a*sin(c + d*x)**5/(15*d) + 4*A*a*sin(c + d*x)**3*cos(c + d*x
)**2/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**4/d - A*b*cos(c + d*x)**6/(6*d)
 - B*a*cos(c + d*x)**6/(6*d) + 8*B*b*sin(c + d*x)**7/(105*d) + 4*B*b*sin(c
+ d*x)**5*cos(c + d*x)**2/(15*d) + B*b*sin(c + d*x)**3*cos(c + d*x)**4/(3*d
), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))*cos(c)**5, True))
```

Giac [A] time = 1.23733, size = 196, normalized size = 1.37

$$-\frac{Bb\sin(7dx+7c)}{448d} - \frac{(Ba+Ab)\cos(6dx+6c)}{192d} - \frac{(Ba+Ab)\cos(4dx+4c)}{32d} - \frac{5(Ba+Ab)\cos(2dx+2c)}{64d} + \frac{(4Aa-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/448*B*b*sin(7*d*x + 7*c)/d - 1/192*(B*a + A*b)*cos(6*d*x + 6*c)/d - 1/32
*(B*a + A*b)*cos(4*d*x + 4*c)/d - 5/64*(B*a + A*b)*cos(2*d*x + 2*c)/d + 1/3
20*(4*A*a - 3*B*b)*sin(5*d*x + 5*c)/d + 1/192*(20*A*a - B*b)*sin(3*d*x + 3*
c)/d + 5/64*(8*A*a + B*b)*sin(d*x + c)/d
```

3.1530 $\int \cos^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{(aB+Ab)\sin^4(c+dx)}{4d}-\frac{(aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}-\frac{bB\sin^5(c+dx)}{5d}$$

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((a*A - b*B)*Si
n[c + d*x]^3)/(3*d) - ((A*b + a*B)*Sin[c + d*x]^4)/(4*d) - (b*B*Sin[c + d*x
]^5)/(5*d)
```

Rubi [A] time = 0.115297, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 772}

$$-\frac{(aB+Ab)\sin^4(c+dx)}{4d}-\frac{(aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}-\frac{bB\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((a*A - b*B)*Si
n[c + d*x]^3)/(3*d) - ((A*b + a*B)*Sin[c + d*x]^4)/(4*d) - (b*B*Sin[c + d*x
]^5)/(5*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx=\frac{\text{Subst}\left(\int(a+x)\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}$$
$$=\frac{\text{Subst}\left(\int\left(aAb^2+b(Ab+aB)x-(aA-bB)x^2-\frac{(Ab+aB)x^3}{b}\right.\right.}{b^3d}$$
$$=\frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}-\frac{(aA-bB)\sin^3}{3d}$$

Mathematica [A] time = 0.250193, size = 80, normalized size = 0.82

$$\frac{\sin(c+dx)\left(-15(aB+Ab)\sin^3(c+dx)-20(aA-bB)\sin^2(c+dx)+30(aB+Ab)\sin(c+dx)+60aA-12bB\sin^4(c\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(60*a*A + 30*(A*b + a*B)*Sin[c + d*x] - 20*(a*A - b*B)*Sin[c
+ d*x]^2 - 15*(A*b + a*B)*Sin[c + d*x]^3 - 12*b*B*Sin[c + d*x]^4))/(60*d)
```

Maple [A] time = 0.059, size = 88, normalized size = 0.9

$$\frac{1}{d}\left(Bb\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)-\frac{Ab(\cos(dx+c))^4}{4}-\frac{aB(\cos(dx+c))^4}{4}+\frac{aA}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(B*b*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
4*A*b*cos(d*x+c)^4-1/4*a*B*cos(d*x+c)^4+1/3*a*A*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

Maxima [A] time = 0.989765, size = 108, normalized size = 1.11

$$-\frac{12\,Bb\sin(dx+c)^5+15\,(Ba+Ab)\sin(dx+c)^4+20\,(Aa-Bb)\sin(dx+c)^3-60\,Aa\sin(dx+c)-30\,(Ba+Ab)\sin(dx}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/60*(12*B*b*sin(d*x + c)^5 + 15*(B*a + A*b)*sin(d*x + c)^4 + 20*(A*a - B*
b)*sin(d*x + c)^3 - 60*A*a*sin(d*x + c) - 30*(B*a + A*b)*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.41141, size = 174, normalized size = 1.79

$$-\frac{15\,(Ba+Ab)\cos(dx+c)^4+4\left(3\,Bb\cos(dx+c)^4-(5\,Aa+Bb)\cos(dx+c)^2-10\,Aa-2\,Bb\right)\sin(dx+c)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/60*(15*(B*a + A*b)*cos(d*x + c)^4 + 4*(3*B*b*cos(d*x + c)^4 - (5*A*a + B
*b)*cos(d*x + c)^2 - 10*A*a - 2*B*b)*sin(d*x + c))/d
```

Sympy [A] time = 2.23447, size = 128, normalized size = 1.32

$$\begin{cases} \frac{2Aa\sin^3(c+dx)}{3d}+\frac{Aa\sin(c+dx)\cos^2(c+dx)}{d}-\frac{Ab\cos^4(c+dx)}{4d}-\frac{Ba\cos^4(c+dx)}{4d}+\frac{2Bb\sin^5(c+dx)}{15d}+\frac{Bb\sin^3(c+dx)\cos^2(c+dx)}{3d} & \text{for } d\neq 0 \\ x\,(A+B\sin(c))\,(a+b\sin(c))\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a*sin(c + d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d
 - A*b*cos(c + d*x)**4/(4*d) - B*a*cos(c + d*x)**4/(4*d) + 2*B*b*sin(c + d*
x)**5/(15*d) + B*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d), Ne(d, 0)), (x*(A
+ B*sin(c))*(a + b*sin(c))*cos(c)**3, True))
```

Giac [A] time = 1.27918, size = 135, normalized size = 1.39

$$-\frac{12\,Bb\sin\left(dx+c\right)^{5}+15\,Ba\sin\left(dx+c\right)^{4}+15\,Ab\sin\left(dx+c\right)^{4}+20\,Aa\sin\left(dx+c\right)^{3}-20\,Bb\sin\left(dx+c\right)^{3}-30\,Ba\sin}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/60*(12*B*b*sin(d*x + c)^5 + 15*B*a*sin(d*x + c)^4 + 15*A*b*sin(d*x + c)^
4 + 20*A*a*sin(d*x + c)^3 - 20*B*b*sin(d*x + c)^3 - 30*B*a*sin(d*x + c)^2 -
 30*A*b*sin(d*x + c)^2 - 60*A*a*sin(d*x + c))/d
```

3.1531 $\int \cos(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{bB\sin^3(c+dx)}{3d}$$

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) + (b*B*Sin[c + d*
x]^3)/(3*d)
```

Rubi [A] time = 0.0537233, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2833, 43}

$$\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{bB\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) + (b*B*Sin[c + d*
x]^3)/(3*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(aA+\frac{(Ab+aB)x}{b}+\frac{Bx^2}{b}\right)dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}+\frac{bB\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0802957, size = 45, normalized size = 0.87

$$\frac{\sin(c+dx)\left(3(aB+Ab)\sin(c+dx)+6aA+2bB\sin^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(6*a*A + 3*(A*b + a*B)*Sin[c + d*x] + 2*b*B*Sin[c + d*x]^2))/
(6*d)
```

Maple [A] time = 0.029, size = 44, normalized size = 0.9

$$\frac{1}{d}\left(\frac{B\left(\sin\left(dx+c\right)\right)^{3}b}{3}+\frac{\left(Ab+aB\right)\left(\sin\left(dx+c\right)\right)^{2}}{2}+A\sin\left(dx+c\right)a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*B*sin(d*x+c)^3*b+1/2*(A*b+B*a)*sin(d*x+c)^2+A*sin(d*x+c)*a)
```

Maxima [A] time = 0.970237, size = 61, normalized size = 1.17

$$\frac{2\,Bb\sin\left(dx+c\right)^{3}+6\,Aa\sin\left(dx+c\right)+3\,\left(Ba+Ab\right)\sin\left(dx+c\right)^{2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/6*(2*B*b*sin(d*x + c)^3 + 6*A*a*sin(d*x + c) + 3*(B*a + A*b)*sin(d*x + c)
^2)/d
```

Fricas [A] time = 1.38161, size = 123, normalized size = 2.37

$$-\frac{3\,\left(Ba+Ab\right)\cos\left(dx+c\right)^{2}+2\,\left(Bb\cos\left(dx+c\right)^{2}-3\,Aa-Bb\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -1/6*(3*(B*a + A*b)*cos(d*x + c)^2 + 2*(B*b*cos(d*x + c)^2 - 3*A*a - B*b)*s
in(d*x + c))/d
```

Sympy [A] time = 0.600874, size = 75, normalized size = 1.44

$$\begin{cases}\frac{Aa\sin\left(c+dx\right)}{d}-\frac{Ab\cos^{2}\left(c+dx\right)}{2d}-\frac{Ba\cos^{2}\left(c+dx\right)}{2d}+\frac{Bb\sin^{3}\left(c+dx\right)}{3d} & \text{for}\: d\neq 0\\x\left(A+B\sin\left(c\right)\right)\left(a+b\sin\left(c\right)\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)/d - A*b*cos(c + d*x)**2/(2*d) - B*a*cos(c + d*x
)**2/(2*d) + B*b*sin(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b
*sin(c))*cos(c), True))
```

Giac [A] time = 1.2267, size = 70, normalized size = 1.35

$$\frac{2\,Bb\sin\left(dx+c\right)^{3}+3\,Ba\sin\left(dx+c\right)^{2}+3\,Ab\sin\left(dx+c\right)^{2}+6\,Aa\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*B*b*sin(d*x + c)^3 + 3*B*a*sin(d*x + c)^2 + 3*A*b*sin(d*x + c)^2 + 6
*A*a*sin(d*x + c))/d
```

3.1532 $\int \sec(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=64

$$-\frac{(a+b)(A+B)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{bB\sin(c+dx)}{d}$$

```
[Out] -((a + b)*(A + B)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)*(A - B)*Log[1 + S
in[c + d*x]])/(2*d) - (b*B*Sin[c + d*x])/d
```

Rubi [A] time = 0.108234, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2837, 774, 633, 31}

$$-\frac{(a+b)(A+B)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{bB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a + b)*(A + B)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)*(A - B)*Log[1 + S
in[c + d*x]])/(2*d) - (b*B*Sin[c + d*x])/d
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{b^2-x^2}dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{bB\sin(c+dx)}{d} - \frac{b\,\text{Subst}\left(\int \frac{-aA-bB-\left(A+\frac{aB}{b}\right)x}{b^2-x^2}dx, x, b\sin(c+d$$
$$= -\frac{bB\sin(c+dx)}{d} - \frac{((a-b)(A-B))\,\text{Subst}\left(\int \frac{1}{-b-x}dx, x, b\sin(c$$
$$= -\frac{(a+b)(A+B)\log(1-\sin(c+dx))}{2d} + \frac{(a-b)(A-B)\log(1+$$

Mathematica [A] time = 0.0294744, size = 68, normalized size = 1.06

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d} - \frac{aB\log(\cos(c+dx))}{d} - \frac{Ab\log(\cos(c+dx))}{d} - \frac{bB\sin(c+dx)}{d} + \frac{bB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*ArcTanh[Sin[c + d*x]])/d - (A*b*Log[Co
s[c + d*x]])/d - (a*B*Log[Cos[c + d*x]])/d - (b*B*Sin[c + d*x])/d
```

Maple [A] time = 0.063, size = 83, normalized size = 1.3

$$-\frac{Ab\ln(\cos(dx+c))}{d} + \frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{Bb\sin(dx+c)}{d} - \frac{aB\ln(\cos(dx+c))}{d} + \frac{Bb\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] -1/d*A*b*ln(cos(d*x+c))+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))-b*B*sin(d*x+c)/d-
1/d*a*B*ln(cos(d*x+c))+1/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.981998, size = 86, normalized size = 1.34

$$-\frac{2\,Bb\sin(dx+c) - ((A-B)a-(A-B)b)\log(\sin(dx+c)+1) + ((A+B)a+(A+B)b)\log(\sin(dx+c)-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -1/2*(2*B*b*sin(d*x + c) - ((A - B)*a - (A - B)*b)*log(sin(d*x + c) + 1) +
((A + B)*a + (A + B)*b)*log(sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.50181, size = 170, normalized size = 2.66

$$-\frac{2\,Bb\sin\left(dx+c\right)-\left((A-B)a-(A-B)b\right)\log\left(\sin\left(dx+c\right)+1\right)+\left((A+B)a+(A+B)b\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -1/2*(2*B*b*sin(d*x + c) - ((A - B)*a - (A - B)*b)*log(sin(d*x + c) + 1) +
((A + B)*a + (A + B)*b)*log(-sin(d*x + c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A+B\sin\left(c+dx\right)\right)\left(a+b\sin\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))*sec(c + d*x), x)
```

Giac [A] time = 1.28562, size = 90, normalized size = 1.41

$$-\frac{2\,Bb\sin\left(dx+c\right)-\left(Aa-Ba-Ab+Bb\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(Aa+Ba+Ab+Bb\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*B*b*sin(d*x + c) - (A*a - B*a - A*b + B*b)*log(abs(sin(d*x + c) + 1
)) + (A*a + B*a + A*b + B*b)*log(abs(sin(d*x + c) - 1)))/d
```

3.1533 $\int \sec^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=59

$$\frac{(aA-bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{2d}$$

```
[Out] ((a*A - b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(A*b + a*B + (a
*A + b*B)*Sin[c + d*x]))/(2*d)
```

Rubi [A] time = 0.0757744, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 778, 206}

$$\frac{(aA-bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a*A - b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(A*b + a*B + (a
*A + b*B)*Sin[c + d*x]))/(2*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b^3\operatorname{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^2}dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{2d} + \frac{(b(aA-$$

$$= \frac{(aA-bB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec^2(c+dx)(Ab+aB+}{2d}$$

Mathematica [A] time = 0.209759, size = 54, normalized size = 0.92

$$\frac{(aA-bB)\tanh^{-1}(\sin(c+dx))+\sec^2(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a*A - b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]^2*(A*b + a*B + (a*A + b*B
)*Sin[c + d*x]))/(2*d)
```

Maple [B] time = 0.077, size = 129, normalized size = 2.2

$$\frac{aA\sec(dx+c)\tan(dx+c)}{2d} + \frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{aB}{2d(\cos(dx+c))^2} + \frac{Ab}{2d(\cos(dx+c))^2} + \frac{B(\sin}{2d(\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a*A*sec(d*x+c)*tan(d*x+c)+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a
*B/cos(d*x+c)^2+1/2/d*A*b/cos(d*x+c)^2+1/2/d*B*b*sin(d*x+c)^3/cos(d*x+c)^2+
1/2*b*B*sin(d*x+c)/d-1/2/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.989002, size = 105, normalized size = 1.78

$$\frac{(Aa-Bb)\log(\sin(dx+c)+1)-(Aa-Bb)\log(\sin(dx+c)-1)-\frac{2(Ba+Ab+(Aa+Bb)\sin(dx+c))}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((A*a - B*b)*log(sin(d*x + c) + 1) - (A*a - B*b)*log(sin(d*x + c) - 1)
- 2*(B*a + A*b + (A*a + B*b)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.43386, size = 232, normalized size = 3.93

$$\frac{(Aa-Bb)\cos(dx+c)^2\log(\sin(dx+c)+1)-(Aa-Bb)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2Ba+2Ab+2(Aa+B}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*((A*a - B*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a - B*b)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*B*a + 2*A*b + 2*(A*a + B*b)*sin(d*x + c)
)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30888, size = 113, normalized size = 1.92

$$\frac{(Aa-Bb)\log(|\sin(dx+c)+1|)-(Aa-Bb)\log(|\sin(dx+c)-1|)-\frac{2(Aa\sin(dx+c)+Bb\sin(dx+c)+Ba+Ab)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*((A*a - B*b)*log(abs(sin(d*x + c) + 1)) - (A*a - B*b)*log(abs(sin(d*x +
 c) - 1)) - 2*(A*a*sin(d*x + c) + B*b*sin(d*x + c) + B*a + A*b)/(sin(d*x +
c)^2 - 1))/d
```

3.1534 $\int \sec^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=88

$$\frac{(3aA-bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{4d}+\frac{(3aA-bB)\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] ((3*a*A - b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(A*b + a*B +
(a*A + b*B)*Sin[c + d*x]))/(4*d) + ((3*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x]
)/(8*d)
```

Rubi [A] time = 0.0897351, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 778, 199, 206}

$$\frac{(3aA-bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{4d}+\frac{(3aA-bB)\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((3*a*A - b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(A*b + a*B +
(a*A + b*B)*Sin[c + d*x]))/(4*d) + ((3*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x]
)/(8*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{4d} + \frac{\left(b^3(3aA-\right.}{}$$

$$= \frac{\sec^4(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{4d} + \frac{(3aA-bB)}{}$$

$$= \frac{(3aA-bB)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(Ab+aB+(a}{4d}$$

Mathematica [A] time = 0.55725, size = 82, normalized size = 0.93

$$\frac{\sec^4(c+dx)\left((bB-3aA)\sin^3(c+dx)+(5aA+bB)\sin(c+dx)+(3aA-bB)\cos^4(c+dx)\tanh^{-1}(\sin(c+dx))+2(aB+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^4*(2*(A*b + a*B) + (3*a*A - b*B)*ArcTanh[Sin[c + d*x]]*Cos[c
+ d*x]^4 + (5*a*A + b*B)*Sin[c + d*x] + (-3*a*A + b*B)*Sin[c + d*x]^3))/(8*
d)
```

Maple [B] time = 0.08, size = 173, normalized size = 2.

$$\frac{A\tan(dx+c)\,a\,(\sec(dx+c))^3}{4d} + \frac{3aA\sec(dx+c)\tan(dx+c)}{8d} + \frac{3aA\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{aB}{4d(\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*A*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a*A*sec(d*x+c)*tan(d*x+c)+3/8/d*a*A
*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a*B/cos(d*x+c)^4+1/4/d*A*b/cos(d*x+c)^4+1/
4/d*B*b*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*B*b*sin(d*x+c)^3/cos(d*x+c)^2+1/8*b
*B*sin(d*x+c)/d-1/8/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.985599, size = 151, normalized size = 1.72

$$\frac{(3Aa-Bb)\log(\sin(dx+c)+1)-(3Aa-Bb)\log(\sin(dx+c)-1)-\frac{2\left((3Aa-Bb)\sin(dx+c)^3-2Ba-2Ab-(5Aa+Bb)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*((3*A*a - B*b)*log(sin(d*x + c) + 1) - (3*A*a - B*b)*log(sin(d*x + c)
- 1) - 2*((3*A*a - B*b)*sin(d*x + c)^3 - 2*B*a - 2*A*b - (5*A*a + B*b)*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.49354, size = 286, normalized size = 3.25

$$\frac{(3\,Aa - Bb)\cos\left(dx + c\right)^4\log\left(\sin\left(dx + c\right) + 1\right) - (3\,Aa - Bb)\cos\left(dx + c\right)^4\log\left(-\sin\left(dx + c\right) + 1\right) + 4\,Ba + 4\,Ab + 2\,\big(}{16\,d\cos\left(dx + c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/16*((3*A*a - B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A*a - B*b)*co
s(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*B*a + 4*A*b + 2*((3*A*a - B*b)*cos(
d*x + c)^2 + 2*A*a + 2*B*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27742, size = 154, normalized size = 1.75

$$\frac{(3\,Aa - Bb)\log\left(|\sin\left(dx + c\right) + 1|\right) - (3\,Aa - Bb)\log\left(|\sin\left(dx + c\right) - 1|\right) - \frac{2\left(3\,Aa\sin(dx+c)^3 - Bb\sin(dx+c)^3 - 5\,Aa\sin(dx+c) - Bb\sin(}{\left(\sin(dx+c)^2 - 1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/16*((3*A*a - B*b)*log(abs(sin(d*x + c) + 1)) - (3*A*a - B*b)*log(abs(sin(
d*x + c) - 1)) - 2*(3*A*a*sin(d*x + c)^3 - B*b*sin(d*x + c)^3 - 5*A*a*sin(d
*x + c) - B*b*sin(d*x + c) - 2*B*a - 2*A*b)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1535 $\int \sec^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$\frac{(5aA-bB)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^6(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{6d}+\frac{(5aA-bB)\tan(c+dx)\sec^3(c+dx)}{24d}$$

```
[Out] ((5*a*A - b*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^6*(A*b + a*B +
 (a*A + b*B)*Sin[c + d*x]))/(6*d) + ((5*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x
])/(16*d) + ((5*a*A - b*B)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d)
```

Rubi [A] time = 0.105369, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 778, 199, 206}

$$\frac{(5aA-bB)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^6(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{6d}+\frac{(5aA-bB)\tan(c+dx)\sec^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((5*a*A - b*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^6*(A*b + a*B +
 (a*A + b*B)*Sin[c + d*x]))/(6*d) + ((5*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x
])/(16*d) + ((5*a*A - b*B)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^4}dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d} + \frac{\left(b^5(5aA\right.}{}$$

$$= \frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d} + \frac{(5aA-}{}$$

$$= \frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d} + \frac{(5aA-}{}$$

$$= \frac{(5aA-bB)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{\sec^6(c+dx)(Ab+aB+}{6}$$

Mathematica [A] time = 0.871246, size = 104, normalized size = 0.88

$$-\frac{\sec^6(c+dx)\left((3bB-15aA)\sin^5(c+dx)+8(5aA-bB)\sin^3(c+dx)-3(11aA+bB)\sin(c+dx)-3(5aA-bB)\cos^6\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^6*(-8*(A*b + a*B) - 3*(5*a*A - b*B)*ArcTanh[Sin[c + d*x]]*Co
s[c + d*x]^6 - 3*(11*a*A + b*B)*Sin[c + d*x] + 8*(5*a*A - b*B)*Sin[c + d*x]
^3 + (-15*a*A + 3*b*B)*Sin[c + d*x]^5))/(48*d)
```

Maple [A] time = 0.082, size = 217, normalized size = 1.8

$$\frac{A\tan(dx+c)\,a\,(\sec(dx+c))^5}{6d} + \frac{5A\tan(dx+c)\,a\,(\sec(dx+c))^3}{24d} + \frac{5aA\sec(dx+c)\tan(dx+c)}{16d} + \frac{5aA\ln(\sec(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/6/d*a*A*tan(d*x+c)*sec(d*x+c)^5+5/24/d*a*A*tan(d*x+c)*sec(d*x+c)^3+5/16/d
*a*A*sec(d*x+c)*tan(d*x+c)+5/16/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*a*B/c
os(d*x+c)^6+1/6/d*A*b/cos(d*x+c)^6+1/6/d*B*b*sin(d*x+c)^3/cos(d*x+c)^6+1/8/
d*B*b*sin(d*x+c)^3/cos(d*x+c)^4+1/16/d*B*b*sin(d*x+c)^3/cos(d*x+c)^2+1/16*b
*B*sin(d*x+c)/d-1/16/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.21774, size = 193, normalized size = 1.64

$$\frac{3(5Aa-Bb)\log(\sin(dx+c)+1)-3(5Aa-Bb)\log(\sin(dx+c)-1)-\frac{2\left(3(5Aa-Bb)\sin(dx+c)^5-8(5Aa-Bb)\sin(dx+c)^3+8Ba\right.}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(5*A*a - B*b)*log(sin(d*x + c) + 1) - 3*(5*A*a - B*b)*log(sin(d*x +
 c) - 1) - 2*(3*(5*A*a - B*b)*sin(d*x + c)^5 - 8*(5*A*a - B*b)*sin(d*x + c)
^3 + 8*B*a + 8*A*b + 3*(11*A*a + B*b)*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin
(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.57303, size = 342, normalized size = 2.9

$$\frac{3\,(5\,Aa-Bb)\cos{(dx+c)}^{6}\log{(\sin{(dx+c)}+1)}-3\,(5\,Aa-Bb)\cos{(dx+c)}^{6}\log{(-\sin{(dx+c)}+1)}+16\,Ba+16\,Ab+2}{96\,d\cos{(dx+c)}^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/96*(3*(5*A*a - B*b)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A*a - B*b
)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 16*B*a + 16*A*b + 2*(3*(5*A*a - B
*b)*cos(d*x + c)^4 + 2*(5*A*a - B*b)*cos(d*x + c)^2 + 8*A*a + 8*B*b)*sin(d*
x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28641, size = 188, normalized size = 1.59

$$\frac{3\,(5\,Aa-Bb)\log{(|\sin{(dx+c)}+1|)}-3\,(5\,Aa-Bb)\log{(|\sin{(dx+c)}-1|)}-\frac{2\left(15\,Aa\sin(dx+c)^{5}-3\,Bb\sin(dx+c)^{5}-40\,Aa\sin(dx+c)^{3}+8\right.}{\left(\sin(dx\right.}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/96*(3*(5*A*a - B*b)*log(abs(sin(d*x + c) + 1)) - 3*(5*A*a - B*b)*log(abs(
sin(d*x + c) - 1)) - 2*(15*A*a*sin(d*x + c)^5 - 3*B*b*sin(d*x + c)^5 - 40*A
*a*sin(d*x + c)^3 + 8*B*b*sin(d*x + c)^3 + 33*A*a*sin(d*x + c) + 3*B*b*sin(
d*x + c) + 8*B*a + 8*A*b)/(sin(d*x + c)^2 - 1)^3)/d
```

3.1536 $\int \cos^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=349

$$\frac{3\left(-7a^2B+2aAb+b^2B\right)(a+b\sin(c+dx))^8}{8b^8d}-\frac{\left(15a^2Ab-35a^3B+15ab^2B-3Ab^3\right)(a+b\sin(c+dx))^7}{7b^8d}+\frac{\left(20a^3Ab+\right.}{}$$

```
[Out] -((a^2 - b^2)^3*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^8*d) + ((a^2 - b^2
)^2*(6*a*A*b - 7*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^8*d) - (3*(a^2
 - b^2)*(5*a^2*A*b - A*b^3 - 7*a^3*B + 3*a*b^2*B)*(a + b*Sin[c + d*x])^5)/(
5*b^8*d) + ((20*a^3*A*b - 12*a*A*b^3 - 35*a^4*B + 30*a^2*b^2*B - 3*b^4*B)*(
a + b*Sin[c + d*x])^6)/(6*b^8*d) - ((15*a^2*A*b - 3*A*b^3 - 35*a^3*B + 15*a
*b^2*B)*(a + b*Sin[c + d*x])^7)/(7*b^8*d) + (3*(2*a*A*b - 7*a^2*B + b^2*B)*
(a + b*Sin[c + d*x])^8)/(8*b^8*d) - ((A*b - 7*a*B)*(a + b*Sin[c + d*x])^9)/
(9*b^8*d) - (B*(a + b*Sin[c + d*x])^10)/(10*b^8*d)
```

Rubi [A] time = 0.392181, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{3\left(-7a^2B+2aAb+b^2B\right)(a+b\sin(c+dx))^8}{8b^8d}-\frac{\left(15a^2Ab-35a^3B+15ab^2B-3Ab^3\right)(a+b\sin(c+dx))^7}{7b^8d}+\frac{\left(20a^3Ab+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a^2 - b^2)^3*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^8*d) + ((a^2 - b^2
)^2*(6*a*A*b - 7*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^8*d) - (3*(a^2
 - b^2)*(5*a^2*A*b - A*b^3 - 7*a^3*B + 3*a*b^2*B)*(a + b*Sin[c + d*x])^5)/(
5*b^8*d) + ((20*a^3*A*b - 12*a*A*b^3 - 35*a^4*B + 30*a^2*b^2*B - 3*b^4*B)*(
a + b*Sin[c + d*x])^6)/(6*b^8*d) - ((15*a^2*A*b - 3*A*b^3 - 35*a^3*B + 15*a
*b^2*B)*(a + b*Sin[c + d*x])^7)/(7*b^8*d) + (3*(2*a*A*b - 7*a^2*B + b^2*B)*
(a + b*Sin[c + d*x])^8)/(8*b^8*d) - ((A*b - 7*a*B)*(a + b*Sin[c + d*x])^9)/
(9*b^8*d) - (B*(a + b*Sin[c + d*x])^10)/(10*b^8*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3 dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{\left(-a^2+b^2\right)^3(Ab-aB)(a+x)^2}{b}+\frac{\left(-a^2+b^2\right)^2\left(6aAb-7a^2B+b^2B\right)(a+x)}{b}\right.\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)^3(Ab-aB)(a+b\sin(c+dx))^3}{3b^8d}+\frac{\left(a^2-b^2\right)^2\left(6aA\right.}{}$$

Mathematica [A] time = 1.46784, size = 295, normalized size = 0.85

$$\frac{-315b^8\left(a^2B+2aAb-3b^2B\right)\sin^8(c+dx)+360b^8\left(a^2(-A)+6abB+3Ab^2\right)\sin^7(c+dx)+1260b^8\left(a^2B+2aAb-b^2B\right)\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-3*a^4*(a^6 - 9*a^4*b^2 + 42*a^2*b^4 - 210*b^6)*B + 2520*a^2*A*b^8*Sin[c +
 d*x] + 1260*a*b^8*(2*A*b + a*B)*Sin[c + d*x]^2 + 840*b^8*(-3*a^2*A + A*b^2
 + 2*a*b*B)*Sin[c + d*x]^3 + 630*b^8*(-6*a*A*b - 3*a^2*B + b^2*B)*Sin[c + d
*x]^4 - 1512*b^8*(-(a^2*A) + A*b^2 + 2*a*b*B)*Sin[c + d*x]^5 + 1260*b^8*(2*
a*A*b + a^2*B - b^2*B)*Sin[c + d*x]^6 + 360*b^8*(-(a^2*A) + 3*A*b^2 + 6*a*b
*B)*Sin[c + d*x]^7 - 315*b^8*(2*a*A*b + a^2*B - 3*b^2*B)*Sin[c + d*x]^8 - 2
80*b^9*(A*b + 2*a*B)*Sin[c + d*x]^9 - 252*b^10*B*Sin[c + d*x]^10)/(2520*b^8
*d)
```

Maple [A] time = 0.078, size = 229, normalized size = 0.7

$$\frac{1}{d}\left(\frac{a^2A\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))^4}{5}+\frac{8(\cos(dx+c))^2}{5}\right)-\frac{Ba^2(\cos(dx+c))^8}{8}-\frac{Aab(\cos(dx\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/7*a^2*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)-1/8*B*a^2*cos(d*x+c)^8-1/4*A*a*b*cos(d*x+c)^8+2*B*a*b*(-1/9*cos(d*x+c)
^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*si
n(d*x+c))+A*b^2*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+B*b^2*(-1/10*sin(d*x+c)^2*cos(d*x
+c)^8-1/40*cos(d*x+c)^8))
```

Maxima [A] time = 0.999976, size = 321, normalized size = 0.92

$$-\frac{252\,Bb^2\sin(dx+c)^{10}+280\left(2\,Bab+Ab^2\right)\sin(dx+c)^9+315\left(Ba^2+2\,Aab-3\,Bb^2\right)\sin(dx+c)^8+360\left(Aa^2-6\,Bab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2520*(252*B*b^2*sin(d*x + c)^10 + 280*(2*B*a*b + A*b^2)*sin(d*x + c)^9 +
 315*(B*a^2 + 2*A*a*b - 3*B*b^2)*sin(d*x + c)^8 + 360*(A*a^2 - 6*B*a*b - 3*
A*b^2)*sin(d*x + c)^7 - 1260*(B*a^2 + 2*A*a*b - B*b^2)*sin(d*x + c)^6 - 151
2*(A*a^2 - 2*B*a*b - A*b^2)*sin(d*x + c)^5 + 630*(3*B*a^2 + 6*A*a*b - B*b^2
)*sin(d*x + c)^4 - 2520*A*a^2*sin(d*x + c) + 840*(3*A*a^2 - 2*B*a*b - A*b^2
)*sin(d*x + c)^3 - 1260*(B*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.73448, size = 425, normalized size = 1.22

$$\frac{252\,Bb^2\cos(dx+c)^{10} - 315\left(Ba^2 + 2\,Aab + Bb^2\right)\cos(dx+c)^8 - 8\left(35\left(2\,Bab + Ab^2\right)\cos(dx+c)^8 - 5\left(9\,Aa^2 + 2\,Bal\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2520*(252*B*b^2*cos(d*x + c)^10 - 315*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x +
 c)^8 - 8*(35*(2*B*a*b + A*b^2)*cos(d*x + c)^8 - 5*(9*A*a^2 + 2*B*a*b + A*b
^2)*cos(d*x + c)^6 - 6*(9*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^4 - 144*A*a
^2 - 32*B*a*b - 16*A*b^2 - 8*(9*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2)*si
n(d*x + c))/d
```

Sympy [A] time = 36.3224, size = 389, normalized size = 1.11

$$\begin{cases} \frac{16Aa^2\sin^7(c+dx)}{35d} + \frac{8Aa^2\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2Aa^2\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{Aa^2\sin(c+dx)\cos^6(c+dx)}{d} - \frac{Aab\cos^8(c+dx)}{4d} + \frac{16Ab^2\sin^9(}{315d} \\ x\left(A + B\sin(c)\right)\left(a + b\sin(c)\right)^2\cos^7(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a**2*sin(c + d*x)**7/(35*d) + 8*A*a**2*sin(c + d*x)**5*cos(
c + d*x)**2/(5*d) + 2*A*a**2*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a**2*sin
(c + d*x)*cos(c + d*x)**6/d - A*a*b*cos(c + d*x)**8/(4*d) + 16*A*b**2*sin(c
 + d*x)**9/(315*d) + 8*A*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 2*A*
b**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + A*b**2*sin(c + d*x)**3*cos(c +
 d*x)**6/(3*d) - B*a**2*cos(c + d*x)**8/(8*d) + 32*B*a*b*sin(c + d*x)**9/(3
15*d) + 16*B*a*b*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 4*B*a*b*sin(c + d
*x)**5*cos(c + d*x)**4/(5*d) + 2*B*a*b*sin(c + d*x)**3*cos(c + d*x)**6/(3*d
) - B*b**2*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - B*b**2*cos(c + d*x)**10/
(40*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))**2*cos(c)**7, True))
```

Giac [A] time = 1.38306, size = 377, normalized size = 1.08

$$\frac{Bb^2\cos(10\,dx + 10\,c)}{5120\,d} + \frac{7\,Aa^2\sin(3\,dx + 3\,c)}{64\,d} - \frac{\left(Ba^2 + 2\,Aab - Bb^2\right)\cos(8\,dx + 8\,c)}{1024\,d} - \frac{\left(8\,Ba^2 + 16\,Aab - Bb^2\right)\cos(}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/5120*B*b^2*cos(10*d*x + 10*c)/d + 7/64*A*a^2*sin(3*d*x + 3*c)/d - 1/1024*
(B*a^2 + 2*A*a*b - B*b^2)*cos(8*d*x + 8*c)/d - 1/1024*(8*B*a^2 + 16*A*a*b -
 B*b^2)*cos(6*d*x + 6*c)/d - 1/256*(7*B*a^2 + 14*A*a*b + B*b^2)*cos(4*d*x +
 4*c)/d - 7/512*(4*B*a^2 + 8*A*a*b + B*b^2)*cos(2*d*x + 2*c)/d - 1/2304*(2*
B*a*b + A*b^2)*sin(9*d*x + 9*c)/d + 1/1792*(4*A*a^2 - 10*B*a*b - 5*A*b^2)*s
in(7*d*x + 7*c)/d + 1/320*(7*A*a^2 - 4*B*a*b - 2*A*b^2)*sin(5*d*x + 5*c)/d
+ 7/128*(10*A*a^2 + 2*B*a*b + A*b^2)*sin(d*x + c)/d
```

3.1537 $\int \cos^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=231

$$-\frac{(-5a^2B+2aAb+b^2B)(a+b\sin(c+dx))^6}{3b^6d}+\frac{2(3a^2Ab-5a^3B+3ab^2B-Ab^3)(a+b\sin(c+dx))^5}{5b^6d}-\frac{(a^2-b^2)(-5\ldots}{}$$

```
[Out] ((a^2 - b^2)^2*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^6*d) - ((a^2 - b^2)
*(4*a*A*b - 5*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^6*d) + (2*(3*a^2*
A*b - A*b^3 - 5*a^3*B + 3*a*b^2*B)*(a + b*Sin[c + d*x])^5)/(5*b^6*d) - ((2*
a*A*b - 5*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^6)/(3*b^6*d) + ((A*b - 5*a*B)
*(a + b*Sin[c + d*x])^7)/(7*b^6*d) + (B*(a + b*Sin[c + d*x])^8)/(8*b^6*d)
```

Rubi [A] time = 0.259003, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$-\frac{(-5a^2B+2aAb+b^2B)(a+b\sin(c+dx))^6}{3b^6d}+\frac{2(3a^2Ab-5a^3B+3ab^2B-Ab^3)(a+b\sin(c+dx))^5}{5b^6d}-\frac{(a^2-b^2)(-5\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```



```
[Out] ((a^2 - b^2)^2*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^6*d) - ((a^2 - b^2)
*(4*a*A*b - 5*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^6*d) + (2*(3*a^2*
A*b - A*b^3 - 5*a^3*B + 3*a*b^2*B)*(a + b*Sin[c + d*x])^5)/(5*b^6*d) - ((2*
a*A*b - 5*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^6)/(3*b^6*d) + ((A*b - 5*a*B)
*(a + b*Sin[c + d*x])^7)/(7*b^6*d) + (B*(a + b*Sin[c + d*x])^8)/(8*b^6*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^2 dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{\left(-a^2+b^2\right)^2(Ab-aB)(a+x)^2}{b}+\frac{\left(-a^2+b^2\right)\left(4aAb-5a^2B+b^2B\right)(a+x)^3}{b}\right.\right.}{}$$

$$= \frac{\left(a^2-b^2\right)^2(Ab-aB)(a+b\sin(c+dx))^3}{3b^6d}-\frac{\left(a^2-b^2\right)\left(4aAb-\right.}{}$$

Mathematica [A] time = 0.49881, size = 227, normalized size = 0.98

$$\frac{140b^6\left(a^2B+2aAb-2b^2B\right)\sin^6(c+dx)+168b^6\left(a^2A-4abB-2Ab^2\right)\sin^5(c+dx)+210b^6\left(-2a^2B-4aAb+b^2B\right)\sin^4(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^4*(3*a^4 - 28*a^2*b^2 + 210*b^4)*B + 840*a^2*A*b^6*Sin[c + d*x] + 420*a*
b^6*(2*A*b + a*B)*Sin[c + d*x]^2 + 280*b^6*(-2*a^2*A + A*b^2 + 2*a*b*B)*Sin
[c + d*x]^3 + 210*b^6*(-4*a*A*b - 2*a^2*B + b^2*B)*Sin[c + d*x]^4 + 168*b^6
*(a^2*A - 2*A*b^2 - 4*a*b*B)*Sin[c + d*x]^5 + 140*b^6*(2*a*A*b + a^2*B - 2*
b^2*B)*Sin[c + d*x]^6 + 120*b^7*(A*b + 2*a*B)*Sin[c + d*x]^7 + 105*b^8*B*Si
n[c + d*x]^8)/(840*b^6*d)
```

Maple [A] time = 0.074, size = 199, normalized size = 0.9

$$\frac{1}{d}\left(\frac{a^2A\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)-\frac{Ba^2(\cos(dx+c))^6}{6}-\frac{Aab(\cos(dx+c))^6}{3}+2Bab\left(-1/7\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*B*a^2*cos
(d*x+c)^6-1/3*A*a*b*cos(d*x+c)^6+2*B*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+A*b^2*(-1/7*sin(d*x+c)*cos
(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+B*b^2*(-1/8*
sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

Maxima [A] time = 0.987959, size = 248, normalized size = 1.07

$$\frac{105\,Bb^2\sin(dx+c)^8+120\left(2\,Bab+Ab^2\right)\sin(dx+c)^7+140\left(Ba^2+2\,Aab-2\,Bb^2\right)\sin(dx+c)^6+168\left(Aa^2-4\,Bab-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/840*(105*B*b^2*sin(d*x + c)^8 + 120*(2*B*a*b + A*b^2)*sin(d*x + c)^7 + 14
0*(B*a^2 + 2*A*a*b - 2*B*b^2)*sin(d*x + c)^6 + 168*(A*a^2 - 4*B*a*b - 2*A*b
^2)*sin(d*x + c)^5 - 210*(2*B*a^2 + 4*A*a*b - B*b^2)*sin(d*x + c)^4 + 840*A
*a^2*sin(d*x + c) - 280*(2*A*a^2 - 2*B*a*b - A*b^2)*sin(d*x + c)^3 + 420*(B
*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.64761, size = 356, normalized size = 1.54

$$\frac{105\,Bb^2\cos\left(dx+c\right)^8 - 140\left(Ba^2 + 2\,Aab + Bb^2\right)\cos\left(dx+c\right)^6 - 8\left(15\left(2\,Bab + Ab^2\right)\cos\left(dx+c\right)^6 - 3\left(7\,Aa^2 + 2\,Bab\right.\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/840*(105*B*b^2*cos(d*x + c)^8 - 140*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x + c
)^6 - 8*(15*(2*B*a*b + A*b^2)*cos(d*x + c)^6 - 3*(7*A*a^2 + 2*B*a*b + A*b^2
)*cos(d*x + c)^4 - 56*A*a^2 - 16*B*a*b - 8*A*b^2 - 4*(7*A*a^2 + 2*B*a*b + A
*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 13.3417, size = 335, normalized size = 1.45

$$\begin{cases} \frac{8Aa^2\sin^5(c+dx)}{15d} + \frac{4Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)\cos^4(c+dx)}{d} - \frac{Aab\cos^6(c+dx)}{3d} + \frac{8Ab^2\sin^7(c+dx)}{105d} + \frac{4Ab^2\sin^5(c+dx)\cos^2(c+}{15d} \\ x\left(A + B\sin(c)\right)\left(a + b\sin(c)\right)^2\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a**2*sin(c + d*x)**5/(15*d) + 4*A*a**2*sin(c + d*x)**3*cos(c
 + d*x)**2/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x)**4/d - A*a*b*cos(c + d*
x)**6/(3*d) + 8*A*b**2*sin(c + d*x)**7/(105*d) + 4*A*b**2*sin(c + d*x)**5*c
os(c + d*x)**2/(15*d) + A*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - B*a*
*2*cos(c + d*x)**6/(6*d) + 16*B*a*b*sin(c + d*x)**7/(105*d) + 8*B*a*b*sin(c
 + d*x)**5*cos(c + d*x)**2/(15*d) + 2*B*a*b*sin(c + d*x)**3*cos(c + d*x)**4
/(3*d) + B*b**2*sin(c + d*x)**8/(24*d) + B*b**2*sin(c + d*x)**6*cos(c + d*x
)**2/(6*d) + B*b**2*sin(c + d*x)**4*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(A
 + B*sin(c))*(a + b*sin(c))**2*cos(c)**5, True))
```

Giac [A] time = 1.28887, size = 312, normalized size = 1.35

$$\frac{Bb^2\cos\left(8\,dx+8\,c\right)}{1024\,d} - \frac{\left(2\,Ba^2 + 4\,Aab - Bb^2\right)\cos\left(6\,dx+6\,c\right)}{384\,d} - \frac{\left(8\,Ba^2 + 16\,Aab + Bb^2\right)\cos\left(4\,dx+4\,c\right)}{256\,d} - \frac{\left(10\,Ba^2 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1024*B*b^2*cos(8*d*x + 8*c)/d - 1/384*(2*B*a^2 + 4*A*a*b - B*b^2)*cos(6*d
*x + 6*c)/d - 1/256*(8*B*a^2 + 16*A*a*b + B*b^2)*cos(4*d*x + 4*c)/d - 1/128
```

```
*(10*B*a^2 + 20*A*a*b + 3*B*b^2)*cos(2*d*x + 2*c)/d - 1/448*(2*B*a*b + A*b^
2)*sin(7*d*x + 7*c)/d + 1/320*(4*A*a^2 - 6*B*a*b - 3*A*b^2)*sin(5*d*x + 5*c
)/d + 1/192*(20*A*a^2 - 2*B*a*b - A*b^2)*sin(3*d*x + 3*c)/d + 5/64*(8*A*a^2
 + 2*B*a*b + A*b^2)*sin(d*x + c)/d
```

3.1538 $\int \cos^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{(-3a^2B+2aAb+b^2B)(a+b\sin(c+dx))^4}{4b^4d}-\frac{(a^2-b^2)(Ab-aB)(a+b\sin(c+dx))^3}{3b^4d}-\frac{(Ab-3aB)(a+b\sin(c+dx))}{5b^4d}$$

```
[Out] -((a^2 - b^2)*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^4*d) + ((2*a*A*b - 3
*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^4*d) - ((A*b - 3*a*B)*(a + b*S
in[c + d*x])^5)/(5*b^4*d) - (B*(a + b*Sin[c + d*x])^6)/(6*b^4*d)
```

Rubi [A] time = 0.170227, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{(-3a^2B+2aAb+b^2B)(a+b\sin(c+dx))^4}{4b^4d}-\frac{(a^2-b^2)(Ab-aB)(a+b\sin(c+dx))^3}{3b^4d}-\frac{(Ab-3aB)(a+b\sin(c+dx))}{5b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a^2 - b^2)*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^4*d) + ((2*a*A*b - 3
*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^4*d) - ((A*b - 3*a*B)*(a + b*S
in[c + d*x])^5)/(5*b^4*d) - (B*(a + b*Sin[c + d*x])^6)/(6*b^4*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int\left(\frac{(-a^2+b^2)(Ab-aB)(a+x)^2}{b}+\frac{(2aAb-3a^2B+b^2B)(a+x)^3}{b}+\frac{(-}{}\right.\right.}{b^3d}\\ &= -\frac{(a^2-b^2)(Ab-aB)(a+b\sin(c+dx))^3}{3b^4d}+\frac{(2aAb-3a^2B}{}\end{aligned}$$

Mathematica [A] time = 0.245859, size = 111, normalized size = 0.84

$$\frac{(a+b\sin(c+dx))^3\left(3b\left(a^2(-B)+2aAb+5b^2B\right)\sin(c+dx)-2a^2Ab+a^3B-6b^2(2Ab-aB)\sin^2(c+dx)-5ab^2B+20A\right.}{60b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a + b*Sin[c + d*x])^3*(-2*a^2*A*b + 20*A*b^3 + a^3*B - 5*a*b^2*B + 3*b*(2
*a*A*b - a^2*B + 5*b^2*B)*Sin[c + d*x] - 6*b^2*(2*A*b - a*B)*Sin[c + d*x]^2
 - 10*b^3*B*Sin[c + d*x]^3))/(60*b^4*d)
```

Maple [A] time = 0.078, size = 169, normalized size = 1.3

$$\frac{1}{d}\left(\frac{a^2A\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}-\frac{Ba^2(\cos(dx+c))^4}{4}-\frac{Aab(\cos(dx+c))^4}{2}+2\,Bab\left(-1/5\,\sin(dx+c)(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)-1/4*B*a^2*cos(d*x+c)^4-1/2*A*a*b
*cos(d*x+c)^4+2*B*a*b*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*s
in(d*x+c))+A*b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*
x+c))+B*b^2*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4))
```

Maxima [A] time = 0.992413, size = 173, normalized size = 1.31

$$-\frac{10\,Bb^2\sin(dx+c)^6+12\left(2\,Bab+Ab^2\right)\sin(dx+c)^5+15\left(Ba^2+2\,Aab-Bb^2\right)\sin(dx+c)^4-60\,Aa^2\sin(dx+c)+20}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/60*(10*B*b^2*sin(d*x + c)^6 + 12*(2*B*a*b + A*b^2)*sin(d*x + c)^5 + 15*(
B*a^2 + 2*A*a*b - B*b^2)*sin(d*x + c)^4 - 60*A*a^2*sin(d*x + c) + 20*(A*a^2
 - 2*B*a*b - A*b^2)*sin(d*x + c)^3 - 30*(B*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.47332, size = 284, normalized size = 2.15

$$\frac{10\,Bb^2\cos(dx+c)^6-15\left(Ba^2+2\,Aab+Bb^2\right)\cos(dx+c)^4-4\left(3\left(2\,Bab+Ab^2\right)\cos(dx+c)^4-10\,Aa^2-4\,Bab-2\,Ab^2\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/60*(10*B*b^2*cos(d*x + c)^6 - 15*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x + c)^4
 - 4*(3*(2*B*a*b + A*b^2)*cos(d*x + c)^4 - 10*A*a^2 - 4*B*a*b - 2*A*b^2 - (
5*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 4.73452, size = 228, normalized size = 1.73

$$\begin{cases} \frac{2Aa^2\sin^3(c+dx)}{3d} + \frac{Aa^2\sin(c+dx)\cos^2(c+dx)}{d} - \frac{Aab\cos^4(c+dx)}{2d} + \frac{2Ab^2\sin^5(c+dx)}{15d} + \frac{Ab^2\sin^3(c+dx)\cos^2(c+dx)}{3d} - \frac{Ba^2\cos^4(c+dx)}{4d} + \frac{4Bab\sin \ldots}{1\ldots} \\ x(A+B\sin(c))(a+b\sin(c))^2\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x
)**2/d - A*a*b*cos(c + d*x)**4/(2*d) + 2*A*b**2*sin(c + d*x)**5/(15*d) + A*
b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - B*a**2*cos(c + d*x)**4/(4*d) +
 4*B*a*b*sin(c + d*x)**5/(15*d) + 2*B*a*b*sin(c + d*x)**3*cos(c + d*x)**2/(
3*d) + B*b**2*sin(c + d*x)**6/(12*d) + B*b**2*sin(c + d*x)**4*cos(c + d*x)*
*2/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))**2*cos(c)**3, True))
```

Giac [A] time = 1.28835, size = 227, normalized size = 1.72

$$-\frac{10\,Bb^2\sin(dx+c)^6 + 24\,Bab\sin(dx+c)^5 + 12\,Ab^2\sin(dx+c)^5 + 15\,Ba^2\sin(dx+c)^4 + 30\,Aab\sin(dx+c)^4 - 15\,B\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/60*(10*B*b^2*sin(d*x + c)^6 + 24*B*a*b*sin(d*x + c)^5 + 12*A*b^2*sin(d*x
 + c)^5 + 15*B*a^2*sin(d*x + c)^4 + 30*A*a*b*sin(d*x + c)^4 - 15*B*b^2*sin(
d*x + c)^4 + 20*A*a^2*sin(d*x + c)^3 - 40*B*a*b*sin(d*x + c)^3 - 20*A*b^2*s
in(d*x + c)^3 - 30*B*a^2*sin(d*x + c)^2 - 60*A*a*b*sin(d*x + c)^2 - 60*A*a^
2*sin(d*x + c))/d
```

3.1539 $\int \cos(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$\frac{(Ab-aB)(a+b\sin(c+dx))^3}{3b^2d}+\frac{B(a+b\sin(c+dx))^4}{4b^2d}$$

```
[Out] ((A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^2*d) + (B*(a + b*Sin[c + d*x])^4)
/(4*b^2*d)
```

Rubi [A] time = 0.07638, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{(Ab-aB)(a+b\sin(c+dx))^3}{3b^2d}+\frac{B(a+b\sin(c+dx))^4}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^2*d) + (B*(a + b*Sin[c + d*x])^4)
/(4*b^2*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+x)^2}{b}+\frac{B(a+x)^3}{b}\right)dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{(Ab-aB)(a+b\sin(c+dx))^3}{3b^2d}+\frac{B(a+b\sin(c+dx))^4}{4b^2d}\end{aligned}$$

Mathematica [A] time = 0.0670486, size = 41, normalized size = 0.76

$$\frac{(a+b\sin(c+dx))^3(-aB+4Ab+3bB\sin(c+dx))}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a + b*Sin[c + d*x])^3*(4*A*b - a*B + 3*b*B*Sin[c + d*x]))/(12*b^2*d)
```

Maple [A] time = 0.033, size = 73, normalized size = 1.4

$$\frac{1}{d}\left(\frac{Bb^2(\sin(dx+c))^4}{4}+\frac{\left(Ab^2+2Bab\right)(\sin(dx+c))^3}{3}+\frac{\left(2Aab+Ba^2\right)(\sin(dx+c))^2}{2}+a^2A\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/4*B*b^2*sin(d*x+c)^4+1/3*(A*b^2+2*B*a*b)*sin(d*x+c)^3+1/2*(2*A*a*b+B
*a^2)*sin(d*x+c)^2+a^2*A*sin(d*x+c))
```

Maxima [A] time = 0.976322, size = 100, normalized size = 1.85

$$\frac{3Bb^2\sin(dx+c)^4+12Aa^2\sin(dx+c)+4\left(2Bab+Ab^2\right)\sin(dx+c)^3+6\left(Ba^2+2Aab\right)\sin(dx+c)^2}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*B*b^2*sin(d*x + c)^4 + 12*A*a^2*sin(d*x + c) + 4*(2*B*a*b + A*b^2)*
sin(d*x + c)^3 + 6*(B*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

Fricas [A] time = 1.38124, size = 213, normalized size = 3.94

$$\frac{3Bb^2\cos(dx+c)^4-6\left(Ba^2+2Aab+Bb^2\right)\cos(dx+c)^2+4\left(3Aa^2+2Bab+Ab^2-\left(2Bab+Ab^2\right)\cos(dx+c)^2\right)\sin}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*B*b^2*cos(d*x + c)^4 - 6*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x + c)^2 +
 4*(3*A*a^2 + 2*B*a*b + A*b^2 - (2*B*a*b + A*b^2)*cos(d*x + c)^2)*sin(d*x +
 c))/d
```

Sympy [A] time = 1.30888, size = 143, normalized size = 2.65

$$\begin{cases}\frac{Aa^2\sin(c+dx)}{d}-\frac{Aab\cos^2(c+dx)}{d}+\frac{Ab^2\sin^3(c+dx)}{3d}-\frac{Ba^2\cos^2(c+dx)}{2d}+\frac{2Bab\sin^3(c+dx)}{3d}-\frac{Bb^2\sin^2(c+dx)\cos^2(c+dx)}{2d}-\frac{Bb^2\cos^4(c+dx)}{4d} & \text{fo} \\ x(A+B\sin(c))(a+b\sin(c))^2\cos(c) & \text{o}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)/d - A*a*b*cos(c + d*x)**2/d + A*b**2*sin(c +
 d*x)**3/(3*d) - B*a**2*cos(c + d*x)**2/(2*d) + 2*B*a*b*sin(c + d*x)**3/(3*
d) - B*b**2*sin(c + d*x)**2*cos(c + d*x)**2/(2*d) - B*b**2*cos(c + d*x)**4/
(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))**2*cos(c), True))
```

Giac [A] time = 1.23297, size = 116, normalized size = 2.15

$$\frac{3\,Bb^2\sin\left(dx+c\right)^4+8\,Bab\sin\left(dx+c\right)^3+4\,Ab^2\sin\left(dx+c\right)^3+6\,Ba^2\sin\left(dx+c\right)^2+12\,Aab\sin\left(dx+c\right)^2+12\,Aa^2\sin\left(d\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/12*(3*B*b^2*sin(d*x + c)^4 + 8*B*a*b*sin(d*x + c)^3 + 4*A*b^2*sin(d*x + c
)^3 + 6*B*a^2*sin(d*x + c)^2 + 12*A*a*b*sin(d*x + c)^2 + 12*A*a^2*sin(d*x +
 c))/d
```

3.1540 $\int \sec(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{b(2aB+Ab)\sin(c+dx)}{d}+\frac{(a-b)^2(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))}{2d}-\frac{b^2B\sin^2(c}{2d}$$

```
[Out] -((a + b)^2*(A + B)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^2*(A - B)*Log[1
 + Sin[c + d*x]])/(2*d) - (b*(A*b + 2*a*B)*Sin[c + d*x])/d - (b^2*B*Sin[c +
 d*x]^2)/(2*d)
```

Rubi [A] time = 0.173667, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2837, 801, 633, 31}

$$-\frac{b(2aB+Ab)\sin(c+dx)}{d}+\frac{(a-b)^2(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))}{2d}-\frac{b^2B\sin^2(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a + b)^2*(A + B)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^2*(A - B)*Log[1
 + Sin[c + d*x]])/(2*d) - (b*(A*b + 2*a*B)*Sin[c + d*x])/d - (b^2*B*Sin[c +
 d*x]^2)/(2*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{b^2-x^2}dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(-A-\frac{2aB}{b}-\frac{Bx}{b}+\frac{b\left(a^2A+Ab^2+2abB\right)+\left(2aAb+a^2B+b^2B\right)x}{b\left(b^2-x^2\right)}\right)\right.}{d}$$

$$= -\frac{b(Ab+2aB)\sin(c+dx)}{d} - \frac{b^2B\sin^2(c+dx)}{2d} + \frac{\text{Subst}\left(\int \frac{b(a^2}{}\right.}{}$$

$$= -\frac{b(Ab+2aB)\sin(c+dx)}{d} - \frac{b^2B\sin^2(c+dx)}{2d} - \frac{\left((a-b)^2(A-\right.}{}$$

$$= -\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))}{2d} + \frac{(a-b)^2(A-B)\log(1}{2d}$$

Mathematica [A] time = 0.197923, size = 81, normalized size = 0.86

$$-\frac{2b(2aB+Ab)\sin(c+dx)+(a-b)^2(-(A-B))\log(\sin(c+dx)+1)+(a+b)^2(A+B)\log(1-\sin(c+dx))+b^2B\sin^2(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -((a + b)^2*(A + B)*Log[1 - Sin[c + d*x]] - (a - b)^2*(A - B)*Log[1 + Sin[c
 + d*x]] + 2*b*(A*b + 2*a*B)*Sin[c + d*x] + b^2*B*Sin[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.079, size = 161, normalized size = 1.7

$$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{Ba^2\ln(\cos(dx+c))}{d} - 2\frac{Aab\ln(\cos(dx+c))}{d} - 2\frac{Bab\sin(dx+c)}{d} + 2\frac{Bab\ln(\text{sec}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))-1/d*B*a^2*ln(cos(d*x+c))-2/d*A*a*b*ln(c
os(d*x+c))-2/d*B*a*b*sin(d*x+c)+2/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))-1/d*A*b
^2*sin(d*x+c)+1/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))-1/2*b^2*B*sin(d*x+c)^2/d-
1/d*B*b^2*ln(cos(d*x+c))
```

Maxima [A] time = 0.987584, size = 147, normalized size = 1.56

$$-\frac{Bb^2\sin(dx+c)^2-\left((A-B)a^2-2(A-B)ab+(A-B)b^2\right)\log(\sin(dx+c)+1)+\left((A+B)a^2+2(A+B)ab+(A+B)b\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(B*b^2*sin(d*x + c)^2 - ((A - B)*a^2 - 2*(A - B)*a*b + (A - B)*b^2)*lo
g(sin(d*x + c) + 1) + ((A + B)*a^2 + 2*(A + B)*a*b + (A + B)*b^2)*log(sin(d
*x + c) - 1) + 2*(2*B*a*b + A*b^2)*sin(d*x + c))/d
```

Fricas [A] time = 1.54315, size = 273, normalized size = 2.9

$$\frac{Bb^2\cos(dx+c)^2 + \left((A-B)a^2 - 2(A-B)ab + (A-B)b^2\right)\log(\sin(dx+c)+1) - \left((A+B)a^2 + 2(A+B)ab + (A+B\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(B*b^2*cos(d*x + c)^2 + ((A - B)*a^2 - 2*(A - B)*a*b + (A - B)*b^2)*log
(sin(d*x + c) + 1) - ((A + B)*a^2 + 2*(A + B)*a*b + (A + B)*b^2)*log(-sin(d
*x + c) + 1) - 2*(2*B*a*b + A*b^2)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sin(c+dx))(a + b\sin(c+dx))^2 \sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))**2*sec(c + d*x), x)
```

Giac [A] time = 1.37102, size = 174, normalized size = 1.85

$$-\frac{Bb^2\sin(dx+c)^2 + 4Bab\sin(dx+c) + 2Ab^2\sin(dx+c) - \left(Aa^2 - Ba^2 - 2Aab + 2Bab + Ab^2 - Bb^2\right)\log(|\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*(B*b^2*sin(d*x + c)^2 + 4*B*a*b*sin(d*x + c) + 2*A*b^2*sin(d*x + c) -
(A*a^2 - B*a^2 - 2*A*a*b + 2*B*a*b + A*b^2 - B*b^2)*log(abs(sin(d*x + c) +
1)) + (A*a^2 + B*a^2 + 2*A*a*b + 2*B*a*b + A*b^2 + B*b^2)*log(abs(sin(d*x +
 c) - 1)))/d
```

3.1541 $\int \sec^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=112

$$-\frac{(a+b)(aA-b(A+2B))\log(1-\sin(c+dx))}{4d}+\frac{(a-b)(aA+b(A-2B))\log(\sin(c+dx)+1)}{4d}+\frac{\sec^2(c+dx)(a+b\sin(c+dx)}{}$$

```
[Out] -((a + b)*(a*A - b*(A + 2*B))*Log[1 - Sin[c + d*x]])/(4*d) + ((a - b)*(a*A
+ b*(A - 2*B))*Log[1 + Sin[c + d*x]])/(4*d) + (Sec[c + d*x]^2*(a + b*Sin[c
+ d*x])*(A*b + a*B + (a*A + b*B)*Sin[c + d*x]))/(2*d)
```

Rubi [A] time = 0.179982, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2837, 819, 633, 31}

$$-\frac{(a+b)(aA-b(A+2B))\log(1-\sin(c+dx))}{4d}+\frac{(a-b)(aA+b(A-2B))\log(\sin(c+dx)+1)}{4d}+\frac{\sec^2(c+dx)(a+b\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```


```
[Out] -((a + b)*(a*A - b*(A + 2*B))*Log[1 - Sin[c + d*x]])/(4*d) + ((a - b)*(a*A
+ b*(A - 2*B))*Log[1 + Sin[c + d*x]])/(4*d) + (Sec[c + d*x]^2*(a + b*Sin[c
+ d*x])*(A*b + a*B + (a*A + b*B)*Sin[c + d*x]))/(2*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^2}dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))(Ab+aB+(aA+bB)\sin(}{2d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))(Ab+aB+(aA+bB)\sin(}{2d}$$

$$= -\frac{(a+b)(aA-b(A+2B))\log(1-\sin(c+dx))}{4d} + \frac{(a-b)(}{}$$

Mathematica [A] time = 1.46034, size = 174, normalized size = 1.55

$$\frac{\left(-6a^3Ab+4aAb^3+2b^4B\right)\tan^2(c+dx)+\left(a^2-b^2\right)((a+b)(aA-b(A+2B))\log(1-\sin(c+dx))-(a-b)(aA+b(A}{4d\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)*((a + b)*(a*A - b*(A + 2*B))*Log[1 - Sin[c + d*x]] - (a - b)*(
a*A + b*(A - 2*B))*Log[1 + Sin[c + d*x]]) - 2*a^3*(-(A*b) + a*B)*Sec[c + d*
x]^2 - 2*(a^2 - b^2)*(a^2*A + A*b^2 + 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x] +
(-6*a^3*A*b + 4*a*A*b^3 + 2*b^4*B)*Tan[c + d*x]^2)/(4*(-a^2 + b^2)*d)
```

Maple [B] time = 0.095, size = 231, normalized size = 2.1

$$\frac{a^2A\sec(dx+c)\tan(dx+c)}{2d} + \frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{Ba^2}{2d(\cos(dx+c))^2} + \frac{Aab}{d(\cos(dx+c))^2} + \frac{Bab(s}{d(co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/2
/d*B*a^2/cos(d*x+c)^2+1/d*A*a*b/cos(d*x+c)^2+1/d*B*a*b*sin(d*x+c)^3/cos(d*x
+c)^2+1/d*B*a*b*sin(d*x+c)-1/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*A*b^2*
sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*A*b^2*sin(d*x+c)-1/2/d*A*b^2*ln(sec(d*x+c)+
tan(d*x+c))+1/2/d*B*b^2*tan(d*x+c)^2+1/d*B*b^2*ln(cos(d*x+c))
```

Maxima [A] time = 0.98654, size = 165, normalized size = 1.47

$$\frac{\left(Aa^2-2\,Bab-(A-2\,B)b^2\right)\log(\sin(dx+c)+1)-\left(Aa^2-2\,Bab-(A+2\,B)b^2\right)\log(\sin(dx+c)-1)-\frac{2\left(Ba^2+2\,Aab+B\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((A*a^2 - 2*B*a*b - (A - 2*B)*b^2)*log(sin(d*x + c) + 1) - (A*a^2 - 2*B
*a*b - (A + 2*B)*b^2)*log(sin(d*x + c) - 1) - 2*(B*a^2 + 2*A*a*b + B*b^2 +
(A*a^2 + 2*B*a*b + A*b^2)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.55613, size = 329, normalized size = 2.94

$$\frac{\left(Aa^2 - 2\,Bab - (A - 2\,B)b^2\right)\cos\left(dx + c\right)^2\log\left(\sin\left(dx + c\right) + 1\right) - \left(Aa^2 - 2\,Bab - (A + 2\,B)b^2\right)\cos\left(dx + c\right)^2\log\left(-\sin\left(dx\right.\right.}{4\,d\cos\left(dx + c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*((A*a^2 - 2*B*a*b - (A - 2*B)*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1)
 - (A*a^2 - 2*B*a*b - (A + 2*B)*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
+ 2*B*a^2 + 4*A*a*b + 2*B*b^2 + 2*(A*a^2 + 2*B*a*b + A*b^2)*sin(d*x + c))/(
d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31049, size = 197, normalized size = 1.76

$$\frac{\left(Aa^2 - 2\,Bab - Ab^2 + 2\,Bb^2\right)\log\left(|\sin\left(dx + c\right) + 1|\right) - \left(Aa^2 - 2\,Bab - Ab^2 - 2\,Bb^2\right)\log\left(|\sin\left(dx + c\right) - 1|\right) - \frac{2\left(Bb^2\sin(dx+c)\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/4*((A*a^2 - 2*B*a*b - A*b^2 + 2*B*b^2)*log(abs(sin(d*x + c) + 1)) - (A*a^
2 - 2*B*a*b - A*b^2 - 2*B*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(B*b^2*sin(d*
x + c)^2 + A*a^2*sin(d*x + c) + 2*B*a*b*sin(d*x + c) + A*b^2*sin(d*x + c) +
 B*a^2 + 2*A*a*b)/(sin(d*x + c)^2 - 1))/d
```

3.1542 $\int \sec^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$\frac{(3a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2A-2abB+Ab^2)\sin(c+dx)+2b(2aA-bB)\right)}{8d}+\frac{\sec^4}{}$$

```
[Out] ((3*a^2*A - A*b^2 - 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4
*(B + A*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(4*d) + (Sec[c + d*x]^2*(2*b*
(2*a*A - b*B) + (3*a^2*A + A*b^2 - 2*a*b*B)*Sin[c + d*x]))/(8*d)
```

Rubi [A] time = 0.155705, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2837, 821, 778, 206}

$$\frac{(3a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2A-2abB+Ab^2)\sin(c+dx)+2b(2aA-bB)\right)}{8d}+\frac{\sec^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((3*a^2*A - A*b^2 - 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4
*(B + A*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(4*d) + (Sec[c + d*x]^2*(2*b*
(2*a*A - b*B) + (3*a^2*A + A*b^2 - 2*a*b*B)*Sin[c + d*x]))/(8*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^3}dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{4d} - \frac{b^3\,\text{Sub}}{}$$

$$= \frac{\sec^4(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{4d} + \frac{\sec^2(c}{}$$

$$= \frac{\left(3a^2A-Ab^2-2abB\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(B}{}$$

Mathematica [A] time = 1.72069, size = 186, normalized size = 1.52

$$\frac{4\left(b^2-a^2\right)\sec^4(c+dx)(a+b\sin(c+dx))^3((bB-aA)\sin(c+dx)-aB+Ab)+\left(-3a^2A+2abB+Ab^2\right)\left(\left(4ab^3-6a^3b\right)\text{ta}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (4*(-a^2 + b^2)*Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*(A*b - a*B + (-(a*A)
+ b*B)*Sin[c + d*x]) + (-3*a^2*A + A*b^2 + 2*a*b*B)*((a^2 - b^2)^2*(Log[1 -
 Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^3*b*Sec[c + d*x]^2 - 2*(a^4 -
 b^4)*Sec[c + d*x]*Tan[c + d*x] + (-6*a^3*b + 4*a*b^3)*Tan[c + d*x]^2))/(16
*(a^2 - b^2)^2*d)
```

Maple [B] time = 0.097, size = 299, normalized size = 2.5

$$\frac{a^2A\tan(dx+c)(\sec(dx+c))^3}{4d} + \frac{3a^2A\sec(dx+c)\tan(dx+c)}{8d} + \frac{3a^2A\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{Ba^2}{4d(\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a^2*A*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^2*A*sec(d*x+c)*tan(d*x+c)+3/8/d
*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*B*a^2/cos(d*x+c)^4+1/2/d*A*a*b/cos(d
*x+c)^4+1/2/d*B*a*b*sin(d*x+c)^3/cos(d*x+c)^4+1/4/d*B*a*b*sin(d*x+c)^3/cos(
d*x+c)^2+1/4/d*B*a*b*sin(d*x+c)-1/4/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/4/d
*A*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*A*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8/
d*A*b^2*sin(d*x+c)-1/8/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*B*b^2*sin(d*
x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 0.99136, size = 231, normalized size = 1.89

$$\frac{\left(3\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(4\,Bb^2\sin(dx+c)^2-\left(3\,Aa^2-2\,B\ldots\right.\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/16*((3*A*a^2 - 2*B*a*b - A*b^2)*log(sin(d*x + c) + 1) - (3*A*a^2 - 2*B*a*
b - A*b^2)*log(sin(d*x + c) - 1) + 2*(4*B*b^2*sin(d*x + c)^2 - (3*A*a^2 - 2
*B*a*b - A*b^2)*sin(d*x + c)^3 + 2*B*a^2 + 4*A*a*b - 2*B*b^2 + (5*A*a^2 + 2
*B*a*b + A*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.52496, size = 414, normalized size = 3.39

$$\frac{\left(3\,Aa^2-2\,Bab-Ab^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,Aa^2-2\,Bab-Ab^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\ldots\right.}{16\,d\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/16*((3*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3
*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 8*B*b^2*c
os(d*x + c)^2 + 4*B*a^2 + 8*A*a*b + 4*B*b^2 + 2*(2*A*a^2 + 4*B*a*b + 2*A*b^
2 + (3*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40568, size = 252, normalized size = 2.07

$$\frac{\left(3\,Aa^2-2\,Bab-Ab^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(3\,Aa^2-2\,Bab-Ab^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,Aa^2\sin(dx+c)^3-2\,Bab\sin\ldots\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/16*((3*A*a^2 - 2*B*a*b - A*b^2)*log(abs(sin(d*x + c) + 1)) - (3*A*a^2 - 2
*B*a*b - A*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(3*A*a^2*sin(d*x + c)^3 - 2*
B*a*b*sin(d*x + c)^3 - A*b^2*sin(d*x + c)^3 - 4*B*b^2*sin(d*x + c)^2 - 5*A*
a^2*sin(d*x + c) - 2*B*a*b*sin(d*x + c) - A*b^2*sin(d*x + c) - 2*B*a^2 - 4*
A*a*b + 2*B*b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

3.1543 $\int \sec^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{(5a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^4(c+dx)\left((5a^2A-2abB+3Ab^2)\sin(c+dx)+2b(4aA-bB)\right)}{24d}+\frac{(5a$$

```
[Out] ((5*a^2*A - A*b^2 - 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^
6*(B + A*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(6*d) + (Sec[c + d*x]^4*(2*b
*(4*a*A - b*B) + (5*a^2*A + 3*A*b^2 - 2*a*b*B)*Sin[c + d*x]))/(24*d) + ((5*
a^2*A - A*b^2 - 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d)
```

Rubi [A] time = 0.205196, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2837, 821, 778, 199, 206}

$$\frac{(5a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^4(c+dx)\left((5a^2A-2abB+3Ab^2)\sin(c+dx)+2b(4aA-bB)\right)}{24d}+\frac{(5a$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((5*a^2*A - A*b^2 - 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^
6*(B + A*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(6*d) + (Sec[c + d*x]^4*(2*b
*(4*a*A - b*B) + (5*a^2*A + 3*A*b^2 - 2*a*b*B)*Sin[c + d*x]))/(24*d) + ((5*
a^2*A - A*b^2 - 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{\left(b^2-x^2\right)^4}dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{6d} - \frac{b^5\,\text{Sub}}{}$$

$$= \frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{6d} + \frac{\sec^4(c}{}$$

$$= \frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{6d} + \frac{\sec^4(c}{}$$

$$= \frac{\left(5a^2A-Ab^2-2abB\right)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{\sec^6(c+dx)(B}{}$$

Mathematica [A] time = 1.48697, size = 242, normalized size = 1.51

$$-\frac{\frac{3b\left(-5a^2A+2abB+Ab^2\right)\left(\left(4ab^3-6a^3b\right)\tan^2(c+dx)+\left(a^2-b^2\right)^2(\log(1-\sin(c+dx))-\log(\sin(c+dx)+1))-2\left(a^4-b^4\right)\tan(c+dx)\sec(c+dx)+2a^3b\sec^2(c+dx)\right)}{16(a-b)(a+b)} + b\,\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6*(a + b*Sin[c + d*x])^3*(A*b - a*B + (-(a*A) + b*B)*Sin[c
+ d*x]) + (b*Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*(3*A*b + (-5*a*A + 2*b*B
)*Sin[c + d*x]))/4 - (3*b*(-5*a^2*A + A*b^2 + 2*a*b*B)*((a^2 - b^2)^2*(Log[
1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^3*b*Sec[c + d*x]^2 - 2*(a^
4 - b^4)*Sec[c + d*x]*Tan[c + d*x] + (-6*a^3*b + 4*a*b^3)*Tan[c + d*x]^2))/
(16*(a - b)*(a + b)))/(6*b*(-a^2 + b^2)*d)
```

Maple [B] time = 0.097, size = 396, normalized size = 2.5

$$\frac{a^2A\tan(dx+c)(\sec(dx+c))^5}{6d} + \frac{5a^2A\tan(dx+c)(\sec(dx+c))^3}{24d} + \frac{5a^2A\sec(dx+c)\tan(dx+c)}{16d} + \frac{5a^2A\ln(\sec(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x)
```

```
[Out] 1/6/d*a^2*A*tan(d*x+c)*sec(d*x+c)^5+5/24/d*a^2*A*tan(d*x+c)*sec(d*x+c)^3+5/
16/d*a^2*A*sec(d*x+c)*tan(d*x+c)+5/16/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/6
/d*B*a^2/cos(d*x+c)^6+1/3/d*A*a*b/cos(d*x+c)^6+1/3/d*B*a*b*sin(d*x+c)^3/cos
(d*x+c)^6+1/4/d*B*a*b*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*B*a*b*sin(d*x+c)^3/co
s(d*x+c)^2+1/8/d*B*a*b*sin(d*x+c)-1/8/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/6
/d*A*b^2*sin(d*x+c)^3/cos(d*x+c)^6+1/8/d*A*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/
16/d*A*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/16/d*A*b^2*sin(d*x+c)-1/16/d*A*b^2*l
n(sec(d*x+c)+tan(d*x+c))+1/6/d*B*b^2*sin(d*x+c)^4/cos(d*x+c)^6+1/12/d*B*b^2
*sin(d*x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 1.02698, size = 285, normalized size = 1.78

$$\frac{3\left(5\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(5\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\left(5\,Aa^2-2\,Bab-Ab^2\right)\sin\cdots\right.}{\cdots}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*log(sin(d*x + c) + 1) - 3*(5*A*a^2 - 2*
B*a*b - A*b^2)*log(sin(d*x + c) - 1) - 2*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*sin
(d*x + c)^5 + 12*B*b^2*sin(d*x + c)^2 - 8*(5*A*a^2 - 2*B*a*b - A*b^2)*sin(d
*x + c)^3 + 8*B*a^2 + 16*A*a*b - 4*B*b^2 + 3*(11*A*a^2 + 2*B*a*b + A*b^2)*s
in(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.56532, size = 493, normalized size = 3.08

$$\frac{3\left(5\,Aa^2-2\,Bab-Ab^2\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-3\left(5\,Aa^2-2\,Bab-Ab^2\right)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/96*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) -
3*(5*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 24*B*
b^2*cos(d*x + c)^2 + 16*B*a^2 + 32*A*a*b + 16*B*b^2 + 2*(3*(5*A*a^2 - 2*B*a
*b - A*b^2)*cos(d*x + c)^4 + 8*A*a^2 + 16*B*a*b + 8*A*b^2 + 2*(5*A*a^2 - 2*
B*a*b - A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4389, size = 309, normalized size = 1.93

$$3\left(5\,Aa^2 - 2\,Bab - Ab^2\right)\log\left(|\sin\left(dx+c\right)+1|\right) - 3\left(5\,Aa^2 - 2\,Bab - Ab^2\right)\log\left(|\sin\left(dx+c\right)-1|\right) - \frac{2\left(15\,Aa^2\sin(dx+c)^5 - 6\,Bab\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/96*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*log(abs(sin(d*x + c) + 1)) - 3*(5*A*a^2
 - 2*B*a*b - A*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(15*A*a^2*sin(d*x + c)^5
 - 6*B*a*b*sin(d*x + c)^5 - 3*A*b^2*sin(d*x + c)^5 - 40*A*a^2*sin(d*x + c)^
3 + 16*B*a*b*sin(d*x + c)^3 + 8*A*b^2*sin(d*x + c)^3 + 12*B*b^2*sin(d*x + c
)^2 + 33*A*a^2*sin(d*x + c) + 6*B*a*b*sin(d*x + c) + 3*A*b^2*sin(d*x + c) +
 8*B*a^2 + 16*A*a*b - 4*B*b^2)/(sin(d*x + c)^2 - 1)^3)/d
```

3.1544 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=315

$$\frac{\left(a^2(-B)+aAb+3b^2B\right)\sin^5(c+dx)}{5b^3d}-\frac{\left(a^2-3b^2\right)(Ab-aB)\sin^4(c+dx)}{4b^4d}+\frac{\left(a^3Ab+3a^2b^2B+a^4(-B)-3aAb^3-3b^4\right.}{3b^5d}$$

```
[Out] -(((a^2 - b^2)^3*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^8*d)) + ((a^5*A*b
 - 3*a^3*A*b^3 + 3*a*A*b^5 - a^6*B + 3*a^4*b^2*B - 3*a^2*b^4*B + b^6*B)*Sin[
c + d*x])/(b^7*d) - ((a^4 - 3*a^2*b^2 + 3*b^4)*(A*b - a*B)*Sin[c + d*x]^2)/
(2*b^6*d) + ((a^3*A*b - 3*a*A*b^3 - a^4*B + 3*a^2*b^2*B - 3*b^4*B)*Sin[c +
d*x]^3)/(3*b^5*d) - ((a^2 - 3*b^2)*(A*b - a*B)*Sin[c + d*x]^4)/(4*b^4*d) +
((a*A*b - a^2*B + 3*b^2*B)*Sin[c + d*x]^5)/(5*b^3*d) - ((A*b - a*B)*Sin[c +
 d*x]^6)/(6*b^2*d) - (B*Sin[c + d*x]^7)/(7*b*d)
```

Rubi [A] time = 0.356067, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{\left(a^2(-B)+aAb+3b^2B\right)\sin^5(c+dx)}{5b^3d}-\frac{\left(a^2-3b^2\right)(Ab-aB)\sin^4(c+dx)}{4b^4d}+\frac{\left(a^3Ab+3a^2b^2B+a^4(-B)-3aAb^3-3b^4\right.}{3b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^3*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^8*d)) + ((a^5*A*b
 - 3*a^3*A*b^3 + 3*a*A*b^5 - a^6*B + 3*a^4*b^2*B - 3*a^2*b^4*B + b^6*B)*Sin[
c + d*x])/(b^7*d) - ((a^4 - 3*a^2*b^2 + 3*b^4)*(A*b - a*B)*Sin[c + d*x]^2)/
(2*b^6*d) + ((a^3*A*b - 3*a*A*b^3 - a^4*B + 3*a^2*b^2*B - 3*b^4*B)*Sin[c +
d*x]^3)/(3*b^5*d) - ((a^2 - 3*b^2)*(A*b - a*B)*Sin[c + d*x]^4)/(4*b^4*d) +
((a*A*b - a^2*B + 3*b^2*B)*Sin[c + d*x]^5)/(5*b^3*d) - ((A*b - a*B)*Sin[c +
 d*x]^6)/(6*b^2*d) - (B*Sin[c + d*x]^7)/(7*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{a^5Ab-3a^3Ab^3+3aAb^5-a^6B+3a^4b^2B-3a^2b^4B+b^6B}{b} - \frac{\left(a^4-3a^2b^2+3b^4\right)(Ab-aB)x}{b} - \frac{(-a^3A}{}\right.\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)^3(Ab-aB)\log(a+b\sin(c+dx))}{b^8 d} + \frac{\left(a^5Ab-3a^3Ab^3+3aAb^5-a^6B\right.}{}$$

Mathematica [A] time = 0.875837, size = 218, normalized size = 0.69

$$\frac{\frac{(Ab-aB)\left(20ab^3\left(a^2-3b^2\right)\sin^3(c+dx)-30b^2\left(a^2-b^2\right)^2\sin^2(c+dx)+60ab\left(-3a^2b^2+a^4+3b^4\right)\sin(c+dx)+15b^4\left(b^2-a^2\right)\cos^4(c+dx)-60\left(a^2-b^2\right)^3\log(a+b\sin(c+dx))+12a\right.}{60b}}{b^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((A*b - a*B)*(15*b^4*(-a^2 + b^2)*Cos[c + d*x]^4 + 10*b^6*Cos[c + d*x]^6 -
 60*(a^2 - b^2)^3*Log[a + b*Sin[c + d*x]] + 60*a*b*(a^4 - 3*a^2*b^2 + 3*b^4
)*Sin[c + d*x] - 30*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 + 20*a*b^3*(a^2 - 3*b^
2)*Sin[c + d*x]^3 + 12*a*b^5*Sin[c + d*x]^5))/(60*b) + (b^6*B*(1225*Sin[c +
 d*x] + 245*Sin[3*(c + d*x)] + 49*Sin[5*(c + d*x)] + 5*Sin[7*(c + d*x)]))/2
240)/(b^7*d)
```

Maple [B] time = 0.075, size = 689, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] 3/d/b^2*A*a*sin(d*x+c)-1/d/b^2*A*sin(d*x+c)^3*a-1/d/b^7*B*a^6*sin(d*x+c)+1/
5/d/b^2*A*sin(d*x+c)^5*a-1/5/d/b^3*B*sin(d*x+c)^5*a^2-3/2/d/b^4*B*sin(d*x+c
)^2*a^3-3/4/d/b^2*B*sin(d*x+c)^4*a-3/d/b^3*ln(a+b*sin(d*x+c))*A*a^2-3/d/b^4
*A*a^3*sin(d*x+c)+3/4/d/b*A*sin(d*x+c)^4+1/d/b*ln(a+b*sin(d*x+c))*A-3/2/d/b
*A*sin(d*x+c)^2-1/6/d/b*A*sin(d*x+c)^6+3/2/d/b^3*A*sin(d*x+c)^2*a^2-1/d/b^2
*ln(a+b*sin(d*x+c))*B*a-1/d/b^7*ln(a+b*sin(d*x+c))*A*a^6+3/d/b^5*ln(a+b*sin
(d*x+c))*A*a^4+3/d/b^5*B*a^4*sin(d*x+c)-3/d/b^3*B*a^2*sin(d*x+c)+1/6/d/b^2*
B*sin(d*x+c)^6*a+1/d/b^8*ln(a+b*sin(d*x+c))*B*a^7+1/3/d/b^4*A*sin(d*x+c)^3*
a^3+3/d/b^4*ln(a+b*sin(d*x+c))*B*a^3+1/d/b^6*A*a^5*sin(d*x+c)+1/2/d/b^6*B*s
in(d*x+c)^2*a^5-1/4/d/b^3*A*sin(d*x+c)^4*a^2-1/2/d/b^5*A*sin(d*x+c)^2*a^4+1
/d/b^3*B*sin(d*x+c)^3*a^2-3/d/b^6*ln(a+b*sin(d*x+c))*B*a^5-1/3/d/b^5*B*sin(
d*x+c)^3*a^4+1/4/d/b^4*B*sin(d*x+c)^4*a^3+3/2/d/b^2*B*sin(d*x+c)^2*a+B*sin(
d*x+c)/b/d-B*sin(d*x+c)^3/b/d-1/7*B*sin(d*x+c)^7/b/d+3/5*B*sin(d*x+c)^5/b/d
```

Maxima [A] time = 1.01263, size = 494, normalized size = 1.57

$$-\frac{60\,Bb^6\sin(dx+c)^7-70\left(Bab^5-Ab^6\right)\sin(dx+c)^6+84\left(Ba^2b^4-Aab^5-3\,Bb^6\right)\sin(dx+c)^5-105\left(Ba^3b^3-Aa^2b^4-3\,Bab^5+3\,Ab^6\right)\sin(dx+c)^4+140\left(Ba^4b^2-Aa^3b^3-3\,Ba^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/420*((60*B*b^6*sin(d*x + c)^7 - 70*(B*a*b^5 - A*b^6)*sin(d*x + c)^6 + 84
*(B*a^2*b^4 - A*a*b^5 - 3*B*b^6)*sin(d*x + c)^5 - 105*(B*a^3*b^3 - A*a^2*b^
4 - 3*B*a*b^5 + 3*A*b^6)*sin(d*x + c)^4 + 140*(B*a^4*b^2 - A*a^3*b^3 - 3*B*
a^2*b^4 + 3*A*a*b^5 + 3*B*b^6)*sin(d*x + c)^3 - 210*(B*a^5*b - A*a^4*b^2 -
3*B*a^3*b^3 + 3*A*a^2*b^4 + 3*B*a*b^5 - 3*A*b^6)*sin(d*x + c)^2 + 420*(B*a^
6 - A*a^5*b - 3*B*a^4*b^2 + 3*A*a^3*b^3 + 3*B*a^2*b^4 - 3*A*a*b^5 - B*b^6)*
sin(d*x + c))/b^7 - 420*(B*a^7 - A*a^6*b - 3*B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*
a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + A*b^7)*log(b*sin(d*x + c) + a)/b^8)/d
```

Fricas [A] time = 1.90702, size = 833, normalized size = 2.64

$$-\frac{70\left(Bab^6 - Ab^7\right)\cos\left(dx+c\right)^6 - 105\left(Ba^3b^4 - Aa^2b^5 - Bab^6 + Ab^7\right)\cos\left(dx+c\right)^4 + 210\left(Ba^5b^2 - Aa^4b^3 - 2\,Ba^3b^4 + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/420*(70*(B*a*b^6 - A*b^7)*cos(d*x + c)^6 - 105*(B*a^3*b^4 - A*a^2*b^5 -
B*a*b^6 + A*b^7)*cos(d*x + c)^4 + 210*(B*a^5*b^2 - A*a^4*b^3 - 2*B*a^3*b^4
+ 2*A*a^2*b^5 + B*a*b^6 - A*b^7)*cos(d*x + c)^2 - 420*(B*a^7 - A*a^6*b - 3*
B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + A*b^7)*log(
b*sin(d*x + c) + a) - 4*(15*B*b^7*cos(d*x + c)^6 - 105*B*a^6*b + 105*A*a^5*
b^2 + 280*B*a^4*b^3 - 280*A*a^3*b^4 - 231*B*a^2*b^5 + 231*A*a*b^6 + 48*B*b^
7 - 3*(7*B*a^2*b^5 - 7*A*a*b^6 - 6*B*b^7)*cos(d*x + c)^4 + (35*B*a^4*b^3 -
35*A*a^3*b^4 - 63*B*a^2*b^5 + 63*A*a*b^6 + 24*B*b^7)*cos(d*x + c)^2)*sin(d*
x + c))/(b^8*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30407, size = 690, normalized size = 2.19

$$-\frac{60\,Bb^6\sin(dx+c)^7 - 70\,Bab^5\sin(dx+c)^6 + 70\,Ab^6\sin(dx+c)^6 + 84\,Ba^2b^4\sin(dx+c)^5 - 84\,Aab^5\sin(dx+c)^5 - 252\,Bb^6\sin(dx+c)^5 - 105\,Ba^3b^3\sin(dx+c)^4 + 105\,Aa^2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/420*((60*B*b^6*sin(d*x + c)^7 - 70*B*a*b^5*sin(d*x + c)^6 + 70*A*b^6*sin
(d*x + c)^6 + 84*B*a^2*b^4*sin(d*x + c)^5 - 84*A*a*b^5*sin(d*x + c)^5 - 252
*B*b^6*sin(d*x + c)^5 - 105*B*a^3*b^3*sin(d*x + c)^4 + 105*A*a^2*b^4*sin(d*
x + c)^4 + 315*B*a*b^5*sin(d*x + c)^4 - 315*A*b^6*sin(d*x + c)^4 + 140*B*a^
4*b^2*sin(d*x + c)^3 - 140*A*a^3*b^3*sin(d*x + c)^3 - 420*B*a^2*b^4*sin(d*x
 + c)^3 + 420*A*a*b^5*sin(d*x + c)^3 + 420*B*b^6*sin(d*x + c)^3 - 210*B*a^5
*b*sin(d*x + c)^2 + 210*A*a^4*b^2*sin(d*x + c)^2 + 630*B*a^3*b^3*sin(d*x +
c)^2 - 630*A*a^2*b^4*sin(d*x + c)^2 - 630*B*a*b^5*sin(d*x + c)^2 + 630*A*b^
6*sin(d*x + c)^2 + 420*B*a^6*sin(d*x + c) - 420*A*a^5*b*sin(d*x + c) - 1260
*B*a^4*b^2*sin(d*x + c) + 1260*A*a^3*b^3*sin(d*x + c) + 1260*B*a^2*b^4*sin(
d*x + c) - 1260*A*a*b^5*sin(d*x + c) - 420*B*b^6*sin(d*x + c))/b^7 - 420*(B
*a^7 - A*a^6*b - 3*B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*a^3*b^4 - 3*A*a^2*b^5 - B*
a*b^6 + A*b^7)*log(abs(b*sin(d*x + c) + a))/b^8)/d
```

3.1545 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=202

$$-\frac{\left(a^2(-B)+aAb+2b^2B\right)\sin^3(c+dx)}{3b^3d}+\frac{\left(a^2-2b^2\right)(Ab-aB)\sin^2(c+dx)}{2b^4d}-\frac{\left(a^3Ab+2a^2b^2B+a^4(-B)-2aAb^3-b^4\right.}{b^5d}$$

```
[Out] ((a^2 - b^2)^2*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^6*d) - ((a^3*A*b - 2
*a*A*b^3 - a^4*B + 2*a^2*b^2*B - b^4*B)*Sin[c + d*x])/(b^5*d) + ((a^2 - 2*b
^2)*(A*b - a*B)*Sin[c + d*x]^2)/(2*b^4*d) - ((a*A*b - a^2*B + 2*b^2*B)*Sin[
c + d*x]^3)/(3*b^3*d) + ((A*b - a*B)*Sin[c + d*x]^4)/(4*b^2*d) + (B*Sin[c +
 d*x]^5)/(5*b*d)
```

Rubi [A] time = 0.247878, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$-\frac{\left(a^2(-B)+aAb+2b^2B\right)\sin^3(c+dx)}{3b^3d}+\frac{\left(a^2-2b^2\right)(Ab-aB)\sin^2(c+dx)}{2b^4d}-\frac{\left(a^3Ab+2a^2b^2B+a^4(-B)-2aAb^3-b^4\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)^2*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^6*d) - ((a^3*A*b - 2
*a*A*b^3 - a^4*B + 2*a^2*b^2*B - b^4*B)*Sin[c + d*x])/(b^5*d) + ((a^2 - 2*b
^2)*(A*b - a*B)*Sin[c + d*x]^2)/(2*b^4*d) - ((a*A*b - a^2*B + 2*b^2*B)*Sin[
c + d*x]^3)/(3*b^3*d) + ((A*b - a*B)*Sin[c + d*x]^4)/(4*b^2*d) + (B*Sin[c +
 d*x]^5)/(5*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)(b^2-x^2)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a^3Ab+2aAb^3+a^4B-2a^2b^2B+b^4B}{b} - \frac{(-a^2+2b^2)(Ab-aB)x}{b} - \frac{(aAb-a^2B+2b^2B)x^2}{b} + \frac{(Al}{}\right.\right.}{b^5d}$$
$$= \frac{(a^2-b^2)^2(Ab-aB)\log(a+b\sin(c+dx))}{b^6d} - \frac{(a^3Ab-2aAb^3-a^4B+2a^2b^2B-}{b^5d}$$

Mathematica [A] time = 0.450887, size = 148, normalized size = 0.73

$$\frac{20(Ab-aB)\left(6b^2\left(a^2-b^2\right)\sin^2(c+dx)-12ab\left(a^2-2b^2\right)\sin(c+dx)+12\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))-4ab^3\sin^3(c}{240b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (20*(A*b - a*B)*(3*b^4*Cos[c + d*x]^4 + 12*(a^2 - b^2)^2*Log[a + b*Sin[c +
d*x]] - 12*a*b*(a^2 - 2*b^2)*Sin[c + d*x] + 6*b^2*(a^2 - b^2)*Sin[c + d*x]^
2 - 4*a*b^3*Sin[c + d*x]^3) + b^5*B*(150*Sin[c + d*x] + 25*Sin[3*(c + d*x)]
 + 3*Sin[5*(c + d*x)]))/(240*b^6*d)
```

Maple [B] time = 0.072, size = 397, normalized size = 2.

$$\frac{B(\sin(dx+c))^5}{5bd} + \frac{A(\sin(dx+c))^4}{4bd} - \frac{B(\sin(dx+c))^4a}{4b^2d} - \frac{A(\sin(dx+c))^3a}{3b^2d} + \frac{B(\sin(dx+c))^3a^2}{3db^3} - \frac{2B(\sin(dx+c))^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/5*B*sin(d*x+c)^5/b/d+1/4/d/b*A*sin(d*x+c)^4-1/4/d/b^2*B*sin(d*x+c)^4*a-1/
3/d/b^2*A*sin(d*x+c)^3*a+1/3/d/b^3*B*sin(d*x+c)^3*a^2-2/3*B*sin(d*x+c)^3/b/
d+1/2/d/b^3*A*sin(d*x+c)^2*a^2-1/d/b*A*sin(d*x+c)^2-1/2/d/b^4*B*sin(d*x+c)^
2*a^3+1/d/b^2*B*sin(d*x+c)^2*a-1/d/b^4*A*a^3*sin(d*x+c)+2/d/b^2*A*a*sin(d*x
+c)+1/d/b^5*B*a^4*sin(d*x+c)-2/d/b^3*B*a^2*sin(d*x+c)+B*sin(d*x+c)/b/d+1/d/
b^5*ln(a+b*sin(d*x+c))*A*a^4-2/d/b^3*ln(a+b*sin(d*x+c))*A*a^2+1/d/b*ln(a+b*
sin(d*x+c))*A-1/d/b^6*ln(a+b*sin(d*x+c))*B*a^5+2/d/b^4*ln(a+b*sin(d*x+c))*B
*a^3-1/d/b^2*ln(a+b*sin(d*x+c))*B*a
```

Maxima [A] time = 0.997679, size = 297, normalized size = 1.47

$$\frac{\frac{12Bb^4\sin(dx+c)^5-15\left(Bab^3-Ab^4\right)\sin(dx+c)^4+20\left(Ba^2b^2-Aab^3-2Bb^4\right)\sin(dx+c)^3-30\left(Ba^3b-Aa^2b^2-2Bab^3+2Ab^4\right)\sin(dx+c)^2+60\left(Ba^4-Aa^3b-2Ba^2b^2+2Aal\right.}{b^5}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/60*((12*B*b^4*sin(d*x + c)^5 - 15*(B*a*b^3 - A*b^4)*sin(d*x + c)^4 + 20*(
B*a^2*b^2 - A*a*b^3 - 2*B*b^4)*sin(d*x + c)^3 - 30*(B*a^3*b - A*a^2*b^2 - 2
*B*a*b^3 + 2*A*b^4)*sin(d*x + c)^2 + 60*(B*a^4 - A*a^3*b - 2*B*a^2*b^2 + 2*
A*a*b^3 + B*b^4)*sin(d*x + c))/b^5 - 60*(B*a^5 - A*a^4*b - 2*B*a^3*b^2 + 2*
A*a^2*b^3 + B*a*b^4 - A*b^5)*log(b*sin(d*x + c) + a)/b^6)/d
```

Fricas [A] time = 1.69814, size = 498, normalized size = 2.47

$$-\frac{15\left(Bab^4 - Ab^5\right)\cos\left(dx+c\right)^4 - 30\left(Ba^3b^2 - Aa^2b^3 - Bab^4 + Ab^5\right)\cos\left(dx+c\right)^2 + 60\left(Ba^5 - Aa^4b - 2\,Ba^3b^2 + 2\,Aa^2 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/60*(15*(B*a*b^4 - A*b^5)*cos(d*x + c)^4 - 30*(B*a^3*b^2 - A*a^2*b^3 - B*
a*b^4 + A*b^5)*cos(d*x + c)^2 + 60*(B*a^5 - A*a^4*b - 2*B*a^3*b^2 + 2*A*a^2
*b^3 + B*a*b^4 - A*b^5)*log(b*sin(d*x + c) + a) - 4*(3*B*b^5*cos(d*x + c)^4
 + 15*B*a^4*b - 15*A*a^3*b^2 - 25*B*a^2*b^3 + 25*A*a*b^4 + 8*B*b^5 - (5*B*a
^2*b^3 - 5*A*a*b^4 - 4*B*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33364, size = 386, normalized size = 1.91

$$\frac{\frac{12\,Bb^4\sin(dx+c)^5 - 15\,Bab^3\sin(dx+c)^4 + 15\,Ab^4\sin(dx+c)^4 + 20\,Ba^2b^2\sin(dx+c)^3 - 20\,Aab^3\sin(dx+c)^3 - 40\,Bb^4\sin(dx+c)^3 - 30\,Ba^3b\sin(dx+c)^2 + 30\,Aa^2b^2\sin \cdots}{b^5}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/60*((12*B*b^4*sin(d*x + c)^5 - 15*B*a*b^3*sin(d*x + c)^4 + 15*A*b^4*sin(d
*x + c)^4 + 20*B*a^2*b^2*sin(d*x + c)^3 - 20*A*a*b^3*sin(d*x + c)^3 - 40*B*
b^4*sin(d*x + c)^3 - 30*B*a^3*b*sin(d*x + c)^2 + 30*A*a^2*b^2*sin(d*x + c)^
2 + 60*B*a*b^3*sin(d*x + c)^2 - 60*A*b^4*sin(d*x + c)^2 + 60*B*a^4*sin(d*x
+ c) - 60*A*a^3*b*sin(d*x + c) - 120*B*a^2*b^2*sin(d*x + c) + 120*A*a*b^3*s
in(d*x + c) + 60*B*b^4*sin(d*x + c))/b^5 - 60*(B*a^5 - A*a^4*b - 2*B*a^3*b^
2 + 2*A*a^2*b^3 + B*a*b^4 - A*b^5)*log(abs(b*sin(d*x + c) + a))/b^6)/d
```

3.1546 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{\left(a^2(-B)+aAb+b^2B\right)\sin(c+dx)}{b^3d}-\frac{\left(a^2-b^2\right)(Ab-aB)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-aB)\sin^2(c+dx)}{2b^2d}-\frac{B\sin^3(c+dx)}{3bd}$$

```
[Out] -(((a^2 - b^2)*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^4*d)) + ((a*A*b - a^
2*B + b^2*B)*Sin[c + d*x])/(b^3*d) - ((A*b - a*B)*Sin[c + d*x]^2)/(2*b^2*d)
 - (B*Sin[c + d*x]^3)/(3*b*d)
```

Rubi [A] time = 0.162742, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{\left(a^2(-B)+aAb+b^2B\right)\sin(c+dx)}{b^3d}-\frac{\left(a^2-b^2\right)(Ab-aB)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-aB)\sin^2(c+dx)}{2b^2d}-\frac{B\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^4*d)) + ((a*A*b - a^
2*B + b^2*B)*Sin[c + d*x])/(b^3*d) - ((A*b - a*B)*Sin[c + d*x]^2)/(2*b^2*d)
 - (B*Sin[c + d*x]^3)/(3*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)}{a+x}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{aAb-a^2B+b^2B}{b}+\frac{(-Ab+aB)x}{b}-\frac{Bx^2}{b}+\frac{\left(-a^2+b^2\right)(Ab-aB)}{b(a+x)}\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= -\frac{\left(a^2-b^2\right)(Ab-aB)\log(a+b\sin(c+dx))}{b^4d}+\frac{\left(aAb-a^2B+b^2B\right)\sin(c+dx)}{b^3d}-
\end{aligned}$$

Mathematica [A] time = 0.37608, size = 89, normalized size = 0.8

$$\frac{\left(A-\frac{aB}{b}\right)\left(\left(b^2-a^2\right)\log(a+b\sin(c+dx))+ab\sin(c+dx)-\frac{1}{2}b^2\sin^2(c+dx)\right)+\frac{1}{12}b^2B(9\sin(c+dx)+\sin(3(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((A - (a*B)/b)*((-a^2 + b^2)*Log[a + b*Sin[c + d*x]] + a*b*Sin[c + d*x] - (
b^2*Sin[c + d*x]^2)/2) + (b^2*B*(9*Sin[c + d*x] + Sin[3*(c + d*x)]))/12)/(b
^3*d)
```

Maple [A] time = 0.066, size = 186, normalized size = 1.7

$$-\frac{B(\sin(dx+c))^3}{3bd}-\frac{A(\sin(dx+c))^2}{2bd}+\frac{B(\sin(dx+c))^2a}{2b^2d}+\frac{A\sin(dx+c)a}{b^2d}-\frac{Ba^2\sin(dx+c)}{db^3}+\frac{B\sin(dx+c)}{bd}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3*B*sin(d*x+c)^3/b/d-1/2/d/b*A*sin(d*x+c)^2+1/2/d/b^2*B*sin(d*x+c)^2*a+1
/d/b^2*A*a*sin(d*x+c)-1/d/b^3*B*a^2*sin(d*x+c)+B*sin(d*x+c)/b/d-1/d/b^3*ln(
a+b*sin(d*x+c))*A*a^2+1/d/b*ln(a+b*sin(d*x+c))*A+1/d/b^4*ln(a+b*sin(d*x+c))
*B*a^3-1/d/b^2*ln(a+b*sin(d*x+c))*B*a
```

Maxima [A] time = 0.984976, size = 151, normalized size = 1.36

$$-\frac{\frac{2Bb^2\sin(dx+c)^3-3\left(Bab-Ab^2\right)\sin(dx+c)^2+6\left(Ba^2-Aab-Bb^2\right)\sin(dx+c)}{b^3}-\frac{6\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)\log(b\sin(dx+c)+a)}{b^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/6*((2*B*b^2*sin(d*x + c)^3 - 3*(B*a*b - A*b^2)*sin(d*x + c)^2 + 6*(B*a^2
 - A*a*b - B*b^2)*sin(d*x + c))/b^3 - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)
*log(b*sin(d*x + c) + a)/b^4)/d
```

Fricas [A] time = 1.53238, size = 255, normalized size = 2.3

$$-\frac{3\left(Bab^2-Ab^3\right)\cos(dx+c)^2-6\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)\log(b\sin(dx+c)+a)-2\left(Bb^3\cos(dx+c)^2-3Ba^2b\right.}{6b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/6*(3*(B*a*b^2 - A*b^3)*cos(d*x + c)^2 - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A
*b^3)*log(b*sin(d*x + c) + a) - 2*(B*b^3*cos(d*x + c)^2 - 3*B*a^2*b + 3*A*a
*b^2 + 2*B*b^3)*sin(d*x + c))/(b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21751, size = 174, normalized size = 1.57

$$-\frac{\frac{2\,Bb^2\sin(dx+c)^3-3\,Bab\sin(dx+c)^2+3\,Ab^2\sin(dx+c)^2+6\,Ba^2\sin(dx+c)-6\,Aab\sin(dx+c)-6\,Bb^2\sin(dx+c)}{b^3}-\frac{6\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)\log(|b\sin(dx+c)+a|)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*((2*B*b^2*sin(d*x + c)^3 - 3*B*a*b*sin(d*x + c)^2 + 3*A*b^2*sin(d*x +
c)^2 + 6*B*a^2*sin(d*x + c) - 6*A*a*b*sin(d*x + c) - 6*B*b^2*sin(d*x + c))/
b^3 - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*log(abs(b*sin(d*x + c) + a))/b^
4)/d
```

3.1547 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=41

$$\frac{(Ab-aB)\log(a+b\sin(c+dx))}{b^2d}+\frac{B\sin(c+dx)}{bd}$$

```
[Out] ((A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^2*d) + (B*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.0701739, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{(Ab-aB)\log(a+b\sin(c+dx))}{b^2d}+\frac{B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^2*d) + (B*Sin[c + d*x])/(b*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{a+x}\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+x)}\right)dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{(Ab-aB)\log(a+b\sin(c+dx))}{b^2d}+\frac{B\sin(c+dx)}{bd}\end{aligned}$$

Mathematica [A] time = 0.0450554, size = 39, normalized size = 0.95

$$\frac{\frac{(Ab-aB)\log(a+b\sin(c+dx))}{b}+B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((A*b - a*B)*Log[a + b*Sin[c + d*x]])/b + B*Sin[c + d*x])/(b*d)
```

Maple [A] time = 0.034, size = 56, normalized size = 1.4

$$\frac{B\sin(dx+c)}{bd}+\frac{\ln(a+b\sin(dx+c))\,A}{bd}-\frac{\ln(a+b\sin(dx+c))\,Ba}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] B*sin(d*x+c)/b/d+1/d/b*ln(a+b*sin(d*x+c))*A-1/d/b^2*ln(a+b*sin(d*x+c))*B*a
```

Maxima [A] time = 0.971681, size = 54, normalized size = 1.32

$$\frac{\frac{B\sin(dx+c)}{b}-\frac{(Ba-Ab)\log(b\sin(dx+c)+a)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] (B*sin(d*x + c)/b - (B*a - A*b)*log(b*sin(d*x + c) + a)/b^2)/d
```

Fricas [A] time = 1.48445, size = 89, normalized size = 2.17

$$\frac{Bb\sin(dx+c)-(Ba-Ab)\log(b\sin(dx+c)+a)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] (B*b*sin(d*x + c) - (B*a - A*b)*log(b*sin(d*x + c) + a))/(b^2*d)
```

Sympy [A] time = 0.818668, size = 104, normalized size = 2.54

$$\begin{cases}\frac{x(A+B\sin(c))\cos(c)}{a} & \text{for } b=0\wedge d=0\\ \frac{\frac{A\sin(c+dx)}{d}-\frac{B\cos^2(c+dx)}{2d}}{a} & \text{for } b=0\\ \frac{x(A+B\sin(c))\cos(c)}{a+b\sin(c)} & \text{for } d=0\\ \frac{A\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd}-\frac{Ba\log\left(\frac{a}{b}+\sin(c+dx)\right)}{b^2d}+\frac{B\sin(c+dx)}{bd} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(A + B*sin(c))*cos(c)/a, Eq(b, 0) & Eq(d, 0)), ((A*sin(c + d*x
)/d - B*cos(c + d*x)**2/(2*d))/a, Eq(b, 0)), (x*(A + B*sin(c))*cos(c)/(a +
b*sin(c)), Eq(d, 0)), (A*log(a/b + sin(c + d*x))/(b*d) - B*a*log(a/b + sin(
c + d*x))/(b**2*d) + B*sin(c + d*x)/(b*d), True))
```

Giac [A] time = 1.22548, size = 55, normalized size = 1.34

$$\frac{\frac{B\sin(dx+c)}{b} - \frac{(Ba-Ab)\log(|b\sin(dx+c)+a|)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*sin(d*x + c)/b - (B*a - A*b)*log(abs(b*sin(d*x + c) + a))/b^2)/d
```

3.1548 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)}+\frac{(A-B)\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -((A + B)*Log[1 - Sin[c + d*x]])/(2*(a + b)*d) + ((A - B)*Log[1 + Sin[c + d
*x]])/(2*(a - b)*d) - ((A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.14837, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 801}

$$-\frac{(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)}+\frac{(A-B)\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((A + B)*Log[1 - Sin[c + d*x]])/(2*(a + b)*d) + ((A - B)*Log[1 + Sin[c + d
*x]])/(2*(a - b)*d) - ((A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)\left(b^2-x^2\right)}\,dx,x,b\sin(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{A+B}{2b(a+b)(b-x)}+\frac{-Ab+aB}{(a-b)b(a+b)(a+x)}+\frac{A-B}{2(a-b)b(b+x)}\right)dx,x,b\sin(c+dx)\right)}{d}\\ &= -\frac{(A+B)\log(1-\sin(c+dx))}{2(a+b)d}+\frac{(A-B)\log(1+\sin(c+dx))}{2(a-b)d}-\frac{(Ab-aB)\log(a}{\left(a^2-\right.}\end{aligned}$$

Mathematica [A] time = 0.177182, size = 99, normalized size = 1.1

$$\frac{\frac{(aB-Ab)\log(a+b\sin(c+dx))+(a+b)(A-B)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b}-(A+B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((A + B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]) + ((a + b)*(A - B)*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (-(A*b) + a*B)*Log[a + b*Sin[c + d*
x]])/(a - b))/((a + b)*d)
```

Maple [A] time = 0.076, size = 156, normalized size = 1.7

$$-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)Ab}{d\left(a+b\right)\left(a-b\right)}+\frac{\ln\left(a+b\sin\left(dx+c\right)\right)aB}{d\left(a+b\right)\left(a-b\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)A}{d\left(2a+2b\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)B}{d\left(2a+2b\right)}+\frac{\ln\left(1+\sin\right.}{d\left(2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(a+b)/(a-b)*ln(a+b*sin(d*x+c))*A*b+1/d/(a+b)/(a-b)*ln(a+b*sin(d*x+c))*
a*B-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)*A-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)*B+1/d/(
2*a-2*b)*ln(1+sin(d*x+c))*A-1/d/(2*a-2*b)*ln(1+sin(d*x+c))*B
```

Maxima [A] time = 0.966279, size = 107, normalized size = 1.19

$$\frac{\frac{2\,(Ba-Ab)\log(b\sin(dx+c)+a)}{a^2-b^2}+\frac{(A-B)\log(\sin(dx+c)+1)}{a-b}-\frac{(A+B)\log(\sin(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/2*(2*(B*a - A*b)*log(b*sin(d*x + c) + a)/(a^2 - b^2) + (A - B)*log(sin(d*
x + c) + 1)/(a - b) - (A + B)*log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A] time = 1.87924, size = 213, normalized size = 2.37

$$\frac{2\,(Ba-Ab)\log\left(b\sin\left(dx+c\right)+a\right)+((A-B)a+(A-B)b)\log\left(\sin\left(dx+c\right)+1\right)-((A+B)a-(A+B)b)\log\left(-\sin\left(\right.\right.}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(B*a - A*b)*log(b*sin(d*x + c) + a) + ((A - B)*a + (A - B)*b)*log(si
n(d*x + c) + 1) - ((A + B)*a - (A + B)*b)*log(-sin(d*x + c) + 1))/((a^2 - b
^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sin(c + dx))\sec(c + dx)}{a + b\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.1869, size = 117, normalized size = 1.3

$$\frac{\frac{2\left(Bab-Ab^2\right)\log(|b\sin(dx+c)+a|)}{a^2b-b^3} + \frac{(A-B)\log(|\sin(dx+c)+1|)}{a-b} - \frac{(A+B)\log(|\sin(dx+c)-1|)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(B*a*b - A*b^2)*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) + (A - B)
*log(abs(sin(d*x + c) + 1))/(a - b) - (A + B)*log(abs(sin(d*x + c) - 1))/(a
 + b))/d
```

3.1549 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{b^2(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{2d\left(a^2-b^2\right)}-\frac{(aA+b(2A+B))\log(1-\sin}{4d(a+b)^2}$$

```
[Out] -((a*A + b*(2*A + B))*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a*A - b*(2
*A - B))*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) + (b^2*(A*b - a*B)*Log[a +
b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(A*b - a*B - (a*A - b*
B)*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.289702, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$\frac{b^2(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{2d\left(a^2-b^2\right)}-\frac{(aA+b(2A+B))\log(1-\sin}{4d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*A + b*(2*A + B))*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a*A - b*(2
*A - B))*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) + (b^2*(A*b - a*B)*Log[a +
b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(A*b - a*B - (a*A - b*
B)*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d} - \frac{b\,\text{Subst}\left(\int \frac{-a^2A+2Ab^2-abB-(a}{(a+x)\left(b^2-x^2\right)}\right.}{2\left(a^2-\right.}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d} - \frac{b\,\text{Subst}\left(\int\left(\frac{(a-b)(-aA-b(2A+B}{2b(a+b)(b-x)}\right.\right.}{}$$

$$= -\frac{(aA+b(2A+B))\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(aA-b(2A-B))\log(1+\sin(c+dx}{4(a-b)^2d}$$

Mathematica [A] time = 0.759357, size = 197, normalized size = 1.24

$$\frac{\frac{4b^2(Ab-aB)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2} + \frac{A+B}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{B-A}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{2(aA+b(2A+B))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}{(a+b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(a*A + b*(2*A + B))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^
2 + (2*(a*A + b*(-2*A + B))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a -
b)^2 + (4*b^2*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^2 + (A + B)/
((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (-A + B)/((a - b)*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(4*d)
```

Maple [A] time = 0.1, size = 297, normalized size = 1.9

$$\frac{b^3\ln(a+b\sin(dx+c))A}{d(a+b)^2(a-b)^2} - \frac{b^2\ln(a+b\sin(dx+c))aB}{d(a+b)^2(a-b)^2} - \frac{A}{d(4a+4b)(\sin(dx+c)-1)} - \frac{B}{d(4a+4b)(\sin(dx+c)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*A-1/d*b^2/(a+b)^2/(a-b)^2*ln(a+b
*sin(d*x+c))*a*B-1/d/(4*a+4*b)/(sin(d*x+c)-1)*A-1/d/(4*a+4*b)/(sin(d*x+c)-1
)*B-1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*a*A-1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*A*b-1
/4/d/(a+b)^2*ln(sin(d*x+c)-1)*B*b-1/d/(4*a-4*b)/(1+sin(d*x+c))*A+1/d/(4*a-4
*b)/(1+sin(d*x+c))*B+1/4/d/(a-b)^2*ln(1+sin(d*x+c))*a*A-1/2/d/(a-b)^2*ln(1+
sin(d*x+c))*A*b+1/4/d/(a-b)^2*ln(1+sin(d*x+c))*B*b
```

Maxima [A] time = 1.00238, size = 236, normalized size = 1.48

$$-\frac{\frac{4\left(Bab^2-Ab^3\right)\log(b\sin(dx+c)+a)}{a^4-2a^2b^2+b^4} - \frac{(Aa-(2A-B)b)\log(\sin(dx+c)+1)}{a^2-2ab+b^2} + \frac{(Aa+(2A+B)b)\log(\sin(dx+c)-1)}{a^2+2ab+b^2} + \frac{2(Ba-Ab+(Aa-Bb)\sin(dx+c))}{\left(a^2-b^2\right)\sin(dx+c)^2-a^2+b^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/4*(4*(B*a*b^2 - A*b^3)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) -
 (A*a - (2*A - B)*b)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (A*a + (2*
A + B)*b)*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2) + 2*(B*a - A*b + (A*a -
 B*b)*sin(d*x + c))/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

Fricas [A] time = 3.51595, size = 531, normalized size = 3.34

$$\frac{2\,Ba^3 - 2\,Aa^2b - 2\,Bab^2 + 2\,Ab^3 - 4\left(Bab^2 - Ab^3\right)\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right) + \left(Aa^3 + Ba^2b - (3\,A - 2\,B)ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(2*B*a^3 - 2*A*a^2*b - 2*B*a*b^2 + 2*A*b^3 - 4*(B*a*b^2 - A*b^3)*cos(d*
x + c)^2*log(b*sin(d*x + c) + a) + (A*a^3 + B*a^2*b - (3*A - 2*B)*a*b^2 - (
2*A - B)*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^3 + B*a^2*b - (3*
A + 2*B)*a*b^2 + (2*A + B)*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(
A*a^3 - B*a^2*b - A*a*b^2 + B*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d
*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sin(c+dx))\sec^3(c+dx)}{a + b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.24874, size = 351, normalized size = 2.21

$$-\frac{\frac{4\left(Bab^3-Ab^4\right)\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5} + \frac{(Aa+2\,Ab+Bb)\log(|-\sin(dx+c)+1|)}{a^2+2\,ab+b^2} - \frac{(Aa-2\,Ab+Bb)\log(|-\sin(dx+c)-1|)}{a^2-2\,ab+b^2} + \frac{2\left(Bab^2\sin(dx+c)^2-Ab^3\sin(dx+c)\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/4*(4*(B*a*b^3 - A*b^4)*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 +
 b^5) + (A*a + 2*A*b + B*b)*log(abs(-sin(d*x + c) + 1))/(a^2 + 2*a*b + b^2)
```

```
 - (A*a - 2*A*b + B*b)*log(abs(-sin(d*x + c) - 1))/(a^2 - 2*a*b + b^2) + 2*
(B*a*b^2*sin(d*x + c)^2 - A*b^3*sin(d*x + c)^2 + A*a^3*sin(d*x + c) - B*a^2
*b*sin(d*x + c) - A*a*b^2*sin(d*x + c) + B*b^3*sin(d*x + c) + B*a^3 - A*a^2
*b - 2*B*a*b^2 + 2*A*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

3.1550 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=263

$$-\frac{b^4(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(3a^2A+ab(9A+B)+b^2(8A+3B)\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(3a^2A-ab(9A-\right.}{}$$

```
[Out] -((3*a^2*A + a*b*(9*A + B) + b^2*(8*A + 3*B))*Log[1 - Sin[c + d*x]])/(16*(a
 + b)^3*d) + ((3*a^2*A + b^2*(8*A - 3*B) - a*b*(9*A - B))*Log[1 + Sin[c + d
*x]])/(16*(a - b)^3*d) - (b^4*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 -
b^2)^3*d) - (Sec[c + d*x]^4*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(4*(a^2
 - b^2)*d) + (Sec[c + d*x]^2*(4*b^2*(A*b - a*B) + (3*a^3*A - 7*a*A*b^2 + a^
2*b*B + 3*b^3*B)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.447782, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$-\frac{b^4(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(3a^2A+ab(9A+B)+b^2(8A+3B)\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(3a^2A-ab(9A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((3*a^2*A + a*b*(9*A + B) + b^2*(8*A + 3*B))*Log[1 - Sin[c + d*x]])/(16*(a
 + b)^3*d) + ((3*a^2*A + b^2*(8*A - 3*B) - a*b*(9*A - B))*Log[1 + Sin[c + d
*x]])/(16*(a - b)^3*d) - (b^4*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 -
b^2)^3*d) - (Sec[c + d*x]^4*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(4*(a^2
 - b^2)*d) + (Sec[c + d*x]^2*(4*b^2*(A*b - a*B) + (3*a^3*A - 7*a*A*b^2 + a^
2*b*B + 3*b^3*B)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{b^3\,\text{Subst}\left(\int \frac{-3a^2A+4Ab^2-abB-}{(a+x)\left(b^2-x^2\right.}\right.}{4\left(a^2-\right.}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\left(4b^2(Ab-aB)\right.}{}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\left(4b^2(Ab-aB)\right.}{}$$

$$= -\frac{\left(3a^2A+ab(9A+B)+b^2(8A+3B)\right)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{\left(3a^2A+b^2(8A-\right.}{}$$

Mathematica [A] time = 1.27897, size = 321, normalized size = 1.22

$$\frac{\frac{16b^4(Ab-aB)\log(a+b\sin(c+dx))}{\left(b^2-a^2\right)^3} - \frac{2\left(3a^2A+ab(9A+B)+b^2(8A+3B)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3} + \frac{2\left(3a^2A+ab(B-9A)+b^2(8A-3B)\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right.\right.}{(a-b)^3}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(3*a^2*A + a*b*(9*A + B) + b^2*(8*A + 3*B))*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/(a + b)^3 + (2*(3*a^2*A + b^2*(8*A - 3*B) + a*b*(-9*A + B))
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^3 + (16*b^4*(A*b - a*B)*
Log[a + b*Sin[c + d*x]])/(-a^2 + b^2)^3 + (A + B)/((a + b)*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^4) + (3*a*A + 5*A*b + a*B + 3*b*B)/((a + b)^2*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^2) + (-A + B)/((a - b)*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^4) + (-3*a*A + 5*A*b + a*B - 3*b*B)/((a - b)^2*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^2))/(16*d)
```

Maple [B] time = 0.106, size = 586, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*b^5/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*A+1/d*b^4/(a+b)^3/(a-b)^3*ln(a+
b*sin(d*x+c))*a*B+1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2*A+1/2/d/(8*a+8*b)/(sin(d
*x+c)-1)^2*B-3/16/d/(a+b)^2/(sin(d*x+c)-1)*a*A-5/16/d/(a+b)^2/(sin(d*x+c)-1
```

```
)*A*b-1/16/d/(a+b)^2/(sin(d*x+c)-1)*a*B-3/16/d/(a+b)^2/(sin(d*x+c)-1)*B*b-3
/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2*A-9/16/d/(a+b)^3*ln(sin(d*x+c)-1)*A*a*b-
1/2/d/(a+b)^3*ln(sin(d*x+c)-1)*A*b^2-1/16/d/(a+b)^3*ln(sin(d*x+c)-1)*B*a*b-
3/16/d/(a+b)^3*ln(sin(d*x+c)-1)*B*b^2-1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2*A+1/
2/d/(8*a-8*b)/(1+sin(d*x+c))^2*B-3/16/d/(a-b)^2/(1+sin(d*x+c))*a*A+5/16/d/(
a-b)^2/(1+sin(d*x+c))*A*b+1/16/d/(a-b)^2/(1+sin(d*x+c))*a*B-3/16/d/(a-b)^2/
(1+sin(d*x+c))*B*b+3/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2*A-9/16/d/(a-b)^3*ln(
1+sin(d*x+c))*A*a*b+1/2/d/(a-b)^3*ln(1+sin(d*x+c))*A*b^2+1/16/d/(a-b)^3*ln(
1+sin(d*x+c))*B*a*b-3/16/d/(a-b)^3*ln(1+sin(d*x+c))*B*b^2
```

Maxima [A] time = 1.01621, size = 495, normalized size = 1.88

$$\frac{\frac{16\left(Bab^4-Ab^5\right)\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{\left(3\,Aa^2-(9\,A-B)ab+(8\,A-3\,B)b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(3\,Aa^2+(9\,A+B)ab+(8\,A+3\,B)b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(2\,B\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*(16*(B*a*b^4 - A*b^5)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6) + (3*A*a^2 - (9*A - B)*a*b + (8*A - 3*B)*b^2)*log(sin(d*x + c)
+ 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*A*a^2 + (9*A + B)*a*b + (8*A + 3*
B)*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(2*B*a^3
- 2*A*a^2*b - 6*B*a*b^2 + 6*A*b^3 - (3*A*a^3 + B*a^2*b - 7*A*a*b^2 + 3*B*b^
3)*sin(d*x + c)^3 + 4*(B*a*b^2 - A*b^3)*sin(d*x + c)^2 + (5*A*a^3 - B*a^2*b
 - 9*A*a*b^2 + 5*B*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)
^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A] time = 9.24453, size = 929, normalized size = 3.53

$$\frac{4\,Ba^5-4\,Aa^4b-8\,Ba^3b^2+8\,Aa^2b^3+4\,Bab^4-4\,Ab^5+16\left(Bab^4-Ab^5\right)\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)+\left(3\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/16*(4*B*a^5 - 4*A*a^4*b - 8*B*a^3*b^2 + 8*A*a^2*b^3 + 4*B*a*b^4 - 4*A*b^5
 + 16*(B*a*b^4 - A*b^5)*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + (3*A*a^5 +
 B*a^4*b - 10*A*a^3*b^2 - 6*B*a^2*b^3 + (15*A - 8*B)*a*b^4 + (8*A - 3*B)*b^
5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A*a^5 + B*a^4*b - 10*A*a^3*b^2
 - 6*B*a^2*b^3 + (15*A + 8*B)*a*b^4 - (8*A + 3*B)*b^5)*cos(d*x + c)^4*log(-
sin(d*x + c) + 1) - 8*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c
)^2 + 2*(2*A*a^5 - 2*B*a^4*b - 4*A*a^3*b^2 + 4*B*a^2*b^3 + 2*A*a*b^4 - 2*B*
b^5 + (3*A*a^5 + B*a^4*b - 10*A*a^3*b^2 + 2*B*a^2*b^3 + 7*A*a*b^4 - 3*B*b^5
)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(
d*x + c)^4)
```

<u>**Sympy**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[B]** time = 1.29397, size = 728, normalized size = 2.77

$$\frac{16\left(Bab^5-Ab^6\right)\log(|b\sin(dx+c)+a|)}{a^6b-3a^4b^3+3a^2b^5-b^7}-\frac{\left(3Aa^2+9Aab+Bab+8Ab^2+3Bb^2\right)\log(|-\sin(dx+c)+1|)}{a^3+3a^2b+3ab^2+b^3}+\frac{\left(3Aa^2-9Aab+Bab+8Ab^2-3Bb^2\right)\log(|-\sin(dx+c)-1|)}{a^3-3a^2b+3ab^2-b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/16*(16*(B*a*b^5 - A*b^6)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3
+ 3*a^2*b^5 - b^7) - (3*A*a^2 + 9*A*a*b + B*a*b + 8*A*b^2 + 3*B*b^2)*log(ab
s(-sin(d*x + c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (3*A*a^2 - 9*A*a*b
+ B*a*b + 8*A*b^2 - 3*B*b^2)*log(abs(-sin(d*x + c) - 1))/(a^3 - 3*a^2*b + 3
*a*b^2 - b^3) + 2*(6*B*a*b^4*sin(d*x + c)^4 - 6*A*b^5*sin(d*x + c)^4 - 3*A*
a^5*sin(d*x + c)^3 - B*a^4*b*sin(d*x + c)^3 + 10*A*a^3*b^2*sin(d*x + c)^3 -
 2*B*a^2*b^3*sin(d*x + c)^3 - 7*A*a*b^4*sin(d*x + c)^3 + 3*B*b^5*sin(d*x +
c)^3 + 4*B*a^3*b^2*sin(d*x + c)^2 - 4*A*a^2*b^3*sin(d*x + c)^2 - 16*B*a*b^4
*sin(d*x + c)^2 + 16*A*b^5*sin(d*x + c)^2 + 5*A*a^5*sin(d*x + c) - B*a^4*b*
sin(d*x + c) - 14*A*a^3*b^2*sin(d*x + c) + 6*B*a^2*b^3*sin(d*x + c) + 9*A*a
*b^4*sin(d*x + c) - 5*B*b^5*sin(d*x + c) + 2*B*a^5 - 2*A*a^4*b - 8*B*a^3*b^
2 + 8*A*a^2*b^3 + 12*B*a*b^4 - 12*A*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*(sin(d*x + c)^2 - 1)^2))/d
```

3.1551 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=383

$$\frac{b^6(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{\left(a^2b(20A+B)+5a^3A+ab^2(29A+4B)+b^3(16A+5B)\right)\log(1-\sin(c+dx))}{32d(a+b)^4}+$$

```
[Out] -((5*a^3*A + a^2*b*(20*A + B) + a*b^2*(29*A + 4*B) + b^3*(16*A + 5*B))*Log[
1 - Sin[c + d*x]])/(32*(a + b)^4*d) + ((5*a^3*A - b^3*(16*A - 5*B) + a*b^2*
(29*A - 4*B) - a^2*b*(20*A - B))*Log[1 + Sin[c + d*x]])/(32*(a - b)^4*d) +
(b^6*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - (Sec[c + d*x]
^6*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(6*(a^2 - b^2)*d) + (Sec[c + d*x
]^4*(6*b^2*(A*b - a*B) + (5*a^3*A - 11*a*A*b^2 + a^2*b*B + 5*b^3*B)*Sin[c +
 d*x]))/(24*(a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(8*b^4*(A*b - a*B) - (5*a^5*
A - 16*a^3*A*b^2 + 19*a*A*b^4 + a^4*b*B - 4*a^2*b^3*B - 5*b^5*B)*Sin[c + d*
x]))/(16*(a^2 - b^2)^3*d)
```

Rubi [A] time = 0.680853, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$\frac{b^6(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{\left(a^2b(20A+B)+5a^3A+ab^2(29A+4B)+b^3(16A+5B)\right)\log(1-\sin(c+dx))}{32d(a+b)^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((5*a^3*A + a^2*b*(20*A + B) + a*b^2*(29*A + 4*B) + b^3*(16*A + 5*B))*Log[
1 - Sin[c + d*x]])/(32*(a + b)^4*d) + ((5*a^3*A - b^3*(16*A - 5*B) + a*b^2*
(29*A - 4*B) - a^2*b*(20*A - B))*Log[1 + Sin[c + d*x]])/(32*(a - b)^4*d) +
(b^6*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - (Sec[c + d*x]
^6*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(6*(a^2 - b^2)*d) + (Sec[c + d*x
]^4*(6*b^2*(A*b - a*B) + (5*a^3*A - 11*a*A*b^2 + a^2*b*B + 5*b^3*B)*Sin[c +
 d*x]))/(24*(a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(8*b^4*(A*b - a*B) - (5*a^5*
A - 16*a^3*A*b^2 + 19*a*A*b^4 + a^4*b*B - 4*a^2*b^3*B - 5*b^5*B)*Sin[c + d*
x]))/(16*(a^2 - b^2)^3*d)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b^7 \operatorname{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)\left(b^2-x^2\right)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d} - \frac{b^5 \operatorname{Subst}\left(\int \frac{-5a^2A+6Ab^2-abB-\ldots}{(a+x)\left(b^2-x^2\ldots\right.}\right.}{6\left(a^2-\ldots\right.}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d} + \frac{\sec^4(c+dx)\left(6b^2(Ab-aB)\ldots\right.}{\ldots}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d} + \frac{\sec^4(c+dx)\left(6b^2(Ab-aB)\ldots\right.}{\ldots}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d} + \frac{\sec^4(c+dx)\left(6b^2(Ab-aB)\ldots\right.}{\ldots}$$

$$= -\frac{\left(5a^3A+a^2b(20A+B)+ab^2(29A+4B)+b^3(16A+5B)\right)\log(1-\sin(c+dx))\ldots}{32(a+b)^4d}$$

Mathematica [A] time = 2.59314, size = 565, normalized size = 1.48

$$\frac{\frac{768b^6(Ab-aB)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^4} - \frac{48\left(a^2b(20A+B)+5a^3A+ab^2(29A+4B)+b^3(16A+5B)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^4} + \frac{48\left(a^2b(B-20A)+5a^3A+ab^2(2\ldots\right.}{\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-48*(5*a^3*A + a^2*b*(20*A + B) + a*b^2*(29*A + 4*B) + b^3*(16*A + 5*B))*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^4 + (48*(5*a^3*A + a*b^2*
(29*A - 4*B) + a^2*b*(-20*A + B) + b^3*(-16*A + 5*B))*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]])/(a - b)^4 + (768*b^6*(A*b - a*B)*Log[a + b*Sin[c + d*x
]])/(a^2 - b^2)^4 + (Sec[c + d*x]^6*(-128*a^4*A*b + 352*a^2*A*b^3 - 368*A*b
^5 + 128*a^5*B - 352*a^3*b^2*B + 368*a*b^4*B - 96*b^2*(a^2 - 3*b^2)*(-(A*b)
 + a*B)*Cos[2*(c + d*x)] - 48*b^4*(A*b - a*B)*Cos[4*(c + d*x)] + 198*a^5*A*
Sin[c + d*x] - 480*a^3*A*b^2*Sin[c + d*x] + 330*a*A*b^4*Sin[c + d*x] - 114*
a^4*b*B*Sin[c + d*x] + 264*a^2*b^3*B*Sin[c + d*x] - 198*b^5*B*Sin[c + d*x]
+ 85*a^5*A*Sin[3*(c + d*x)] - 272*a^3*A*b^2*Sin[3*(c + d*x)] + 259*a*A*b^4*
Sin[3*(c + d*x)] + 17*a^4*b*B*Sin[3*(c + d*x)] - 4*a^2*b^3*B*Sin[3*(c + d*x
)] - 85*b^5*B*Sin[3*(c + d*x)] + 15*a^5*A*Sin[5*(c + d*x)] - 48*a^3*A*b^2*S
in[5*(c + d*x)] + 57*a*A*b^4*Sin[5*(c + d*x)] + 3*a^4*b*B*Sin[5*(c + d*x)]
```

```
- 12*a^2*b^3*B*Sin[5*(c + d*x)] - 15*b^5*B*Sin[5*(c + d*x)]))/(a^2 - b^2)^3
)/(768*d)
```

Maple [B] time = 0.109, size = 990, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/32/d/(a-b)^3/(1+sin(d*x+c))*B*a^2+5/32/d/(a-b)^3/(1+sin(d*x+c))*B*b^2+5/3
2/d/(a-b)^4*ln(1+sin(d*x+c))*a^3*A-1/2/d/(a-b)^4*ln(1+sin(d*x+c))*A*b^3+5/3
2/d/(a-b)^4*ln(1+sin(d*x+c))*B*b^3-5/32/d/(a+b)^3/(sin(d*x+c)-1)*a^2*A-11/3
2/d/(a+b)^3/(sin(d*x+c)-1)*A*b^2-1/32/d/(a+b)^3/(sin(d*x+c)-1)*B*a^2-5/32/d
/(a+b)^4*ln(sin(d*x+c)-1)*B*b^3-5/32/d/(a+b)^4*ln(sin(d*x+c)-1)*a^3*A-1/16/
d/(a-b)^2/(1+sin(d*x+c))^2*B*b-1/2/d/(a+b)^4*ln(sin(d*x+c)-1)*A*b^3-5/32/d/
(a+b)^3/(sin(d*x+c)-1)*B*b^2+1/32/d/(a-b)^2/(1+sin(d*x+c))^2*a*B-1/32/d/(a+
b)^4*ln(sin(d*x+c)-1)*B*a^2*b-1/8/d/(a+b)^4*ln(sin(d*x+c)-1)*B*a*b^2+1/d*b^
7/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*A-1/8/d/(a-b)^3/(1+sin(d*x+c))*B*a*b-5
/8/d/(a-b)^4*ln(1+sin(d*x+c))*A*a^2*b+29/32/d/(a-b)^4*ln(1+sin(d*x+c))*A*a*
b^2+1/32/d/(a-b)^4*ln(1+sin(d*x+c))*B*a^2*b+3/32/d/(a-b)^2/(1+sin(d*x+c))^2
*A*b-1/3/d/(16*a-16*b)/(1+sin(d*x+c))^3*A+1/3/d/(16*a-16*b)/(1+sin(d*x+c))^
3*B-1/3/d/(16*a+16*b)/(sin(d*x+c)-1)^3*A-1/3/d/(16*a+16*b)/(sin(d*x+c)-1)^3
*B-1/8/d/(a-b)^4*ln(1+sin(d*x+c))*B*a*b^2-1/8/d/(a+b)^3/(sin(d*x+c)-1)*B*a*
b-7/16/d/(a+b)^3/(sin(d*x+c)-1)*A*a*b+1/16/d/(a+b)^2/(sin(d*x+c)-1)^2*a*A+3
/32/d/(a+b)^2/(sin(d*x+c)-1)^2*A*b+1/32/d/(a+b)^2/(sin(d*x+c)-1)^2*a*B+1/16
/d/(a+b)^2/(sin(d*x+c)-1)^2*B*b-1/16/d/(a-b)^2/(1+sin(d*x+c))^2*a*A-5/32/d/
(a-b)^3/(1+sin(d*x+c))*a^2*A-11/32/d/(a-b)^3/(1+sin(d*x+c))*A*b^2-1/d*b^6/(
a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*a*B-29/32/d/(a+b)^4*ln(sin(d*x+c)-1)*A*a*
b^2-5/8/d/(a+b)^4*ln(sin(d*x+c)-1)*A*a^2*b+7/16/d/(a-b)^3/(1+sin(d*x+c))*A*
a*b
```

Maxima [A] time = 1.11085, size = 853, normalized size = 2.23

$$-\frac{\frac{96\left(Bab^6-Ab^7\right)\log(b\sin(dx+c)+a)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{3\left(5Aa^3-(20A-B)a^2b+(29A-4B)ab^2-(16A-5B)b^3\right)\log(\sin(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{3\left(5Aa^3+(20A+B)a^2b+(29A+4B)ab^2 \cdots\right.}{a^4+4a^3b+6a^2b^2 \cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(96*(B*a*b^6 - A*b^7)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8) - 3*(5*A*a^3 - (20*A - B)*a^2*b + (29*A - 4*B)*a*b
^2 - (16*A - 5*B)*b^3)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4
*a*b^3 + b^4) + 3*(5*A*a^3 + (20*A + B)*a^2*b + (29*A + 4*B)*a*b^2 + (16*A
+ 5*B)*b^3)*log(sin(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4) + 2*(8*B*a^5 - 8*A*a^4*b - 28*B*a^3*b^2 + 28*A*a^2*b^3 + 44*B*a*b^4 - 44
*A*b^5 + 3*(5*A*a^5 + B*a^4*b - 16*A*a^3*b^2 - 4*B*a^2*b^3 + 19*A*a*b^4 - 5
*B*b^5)*sin(d*x + c)^5 + 24*(B*a*b^4 - A*b^5)*sin(d*x + c)^4 - 8*(5*A*a^5 +
 B*a^4*b - 16*A*a^3*b^2 - 2*B*a^2*b^3 + 17*A*a*b^4 - 5*B*b^5)*sin(d*x + c)^
3 + 12*(B*a^3*b^2 - A*a^2*b^3 - 5*B*a*b^4 + 5*A*b^5)*sin(d*x + c)^2 + 3*(11
*A*a^5 - B*a^4*b - 32*A*a^3*b^2 + 4*B*a^2*b^3 + 29*A*a*b^4 - 11*B*b^5)*sin(
```

```
d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sin(d*x + c)^6 - a^6 + 3*a^4
*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sin(d*x + c)
^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sin(d*x + c)^2))/d
```

Fricas [A] time = 23.3608, size = 1454, normalized size = 3.8

$$\frac{16\,Ba^7 - 16\,Aa^6b - 48\,Ba^5b^2 + 48\,Aa^4b^3 + 48\,Ba^3b^4 - 48\,Aa^2b^5 - 16\,Bab^6 + 16\,Ab^7 - 96\left(Bab^6 - Ab^7\right)\cos\left(dx + c\right)^6 \log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/96*(16*B*a^7 - 16*A*a^6*b - 48*B*a^5*b^2 + 48*A*a^4*b^3 + 48*B*a^3*b^4 -
48*A*a^2*b^5 - 16*B*a*b^6 + 16*A*b^7 - 96*(B*a*b^6 - A*b^7)*cos(d*x + c)^6*
log(b*sin(d*x + c) + a) + 3*(5*A*a^7 + B*a^6*b - 21*A*a^5*b^2 - 5*B*a^4*b^3
 + 35*A*a^3*b^4 + 15*B*a^2*b^5 - (35*A - 16*B)*a*b^6 - (16*A - 5*B)*b^7)*co
s(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A*a^7 + B*a^6*b - 21*A*a^5*b^2 -
5*B*a^4*b^3 + 35*A*a^3*b^4 + 15*B*a^2*b^5 - (35*A + 16*B)*a*b^6 + (16*A + 5
*B)*b^7)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 48*(B*a^3*b^4 - A*a^2*b^5
- B*a*b^6 + A*b^7)*cos(d*x + c)^4 - 24*(B*a^5*b^2 - A*a^4*b^3 - 2*B*a^3*b^4
 + 2*A*a^2*b^5 + B*a*b^6 - A*b^7)*cos(d*x + c)^2 + 2*(8*A*a^7 - 8*B*a^6*b -
 24*A*a^5*b^2 + 24*B*a^4*b^3 + 24*A*a^3*b^4 - 24*B*a^2*b^5 - 8*A*a*b^6 + 8*
B*b^7 + 3*(5*A*a^7 + B*a^6*b - 21*A*a^5*b^2 - 5*B*a^4*b^3 + 35*A*a^3*b^4 -
B*a^2*b^5 - 19*A*a*b^6 + 5*B*b^7)*cos(d*x + c)^4 + 2*(5*A*a^7 + B*a^6*b - 2
1*A*a^5*b^2 + 3*B*a^4*b^3 + 27*A*a^3*b^4 - 9*B*a^2*b^5 - 11*A*a*b^6 + 5*B*b
^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6
 + b^8)*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45496, size = 1224, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/96*(96*(B*a*b^7 - A*b^8)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3
 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) + 3*(5*A*a^3 + 20*A*a^2*b + B*a^2*b + 29*A*
```

```
a*b^2 + 4*B*a*b^2 + 16*A*b^3 + 5*B*b^3)*log(abs(-sin(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 3*(5*A*a^3 - 20*A*a^2*b + B*a^2*b +
29*A*a*b^2 - 4*B*a*b^2 - 16*A*b^3 + 5*B*b^3)*log(abs(-sin(d*x + c) - 1))/(a
^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 2*(44*B*a*b^6*sin(d*x + c)^6 -
44*A*b^7*sin(d*x + c)^6 + 15*A*a^7*sin(d*x + c)^5 + 3*B*a^6*b*sin(d*x + c)^
5 - 63*A*a^5*b^2*sin(d*x + c)^5 - 15*B*a^4*b^3*sin(d*x + c)^5 + 105*A*a^3*b
^4*sin(d*x + c)^5 - 3*B*a^2*b^5*sin(d*x + c)^5 - 57*A*a*b^6*sin(d*x + c)^5
+ 15*B*b^7*sin(d*x + c)^5 + 24*B*a^3*b^4*sin(d*x + c)^4 - 24*A*a^2*b^5*sin(
d*x + c)^4 - 156*B*a*b^6*sin(d*x + c)^4 + 156*A*b^7*sin(d*x + c)^4 - 40*A*a
^7*sin(d*x + c)^3 - 8*B*a^6*b*sin(d*x + c)^3 + 168*A*a^5*b^2*sin(d*x + c)^3
 + 24*B*a^4*b^3*sin(d*x + c)^3 - 264*A*a^3*b^4*sin(d*x + c)^3 + 24*B*a^2*b^
5*sin(d*x + c)^3 + 136*A*a*b^6*sin(d*x + c)^3 - 40*B*b^7*sin(d*x + c)^3 + 1
2*B*a^5*b^2*sin(d*x + c)^2 - 12*A*a^4*b^3*sin(d*x + c)^2 - 72*B*a^3*b^4*sin
(d*x + c)^2 + 72*A*a^2*b^5*sin(d*x + c)^2 + 192*B*a*b^6*sin(d*x + c)^2 - 19
2*A*b^7*sin(d*x + c)^2 + 33*A*a^7*sin(d*x + c) - 3*B*a^6*b*sin(d*x + c) - 1
29*A*a^5*b^2*sin(d*x + c) + 15*B*a^4*b^3*sin(d*x + c) + 183*A*a^3*b^4*sin(d
*x + c) - 45*B*a^2*b^5*sin(d*x + c) - 87*A*a*b^6*sin(d*x + c) + 33*B*b^7*si
n(d*x + c) + 8*B*a^7 - 8*A*a^6*b - 36*B*a^5*b^2 + 36*A*a^4*b^3 + 72*B*a^3*b
^4 - 72*A*a^2*b^5 - 88*B*a*b^6 + 88*A*b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 -
4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^3))/d
```

3.1552 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=324

$$\frac{\left(-3a^2B+2aAb+3b^2B\right)\sin^4(c+dx)}{4b^4d}-\frac{\left(3a^2Ab-4a^3B+6ab^2B-3Ab^3\right)\sin^3(c+dx)}{3b^5d}+\frac{\left(4a^3Ab+9a^2b^2B-5a^4B-6aA\right.}{2b^6d}$$

```
[Out] ((a^2 - b^2)^2*(6*a*A*b - 7*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^8*d)
 - ((5*a^4*A*b - 9*a^2*A*b^3 + 3*A*b^5 - 6*a^5*B + 12*a^3*b^2*B - 6*a*b^4*B
)*Sin[c + d*x])/(b^7*d) + ((4*a^3*A*b - 6*a*A*b^3 - 5*a^4*B + 9*a^2*b^2*B -
 3*b^4*B)*Sin[c + d*x]^2)/(2*b^6*d) - ((3*a^2*A*b - 3*A*b^3 - 4*a^3*B + 6*a
*b^2*B)*Sin[c + d*x]^3)/(3*b^5*d) + ((2*a*A*b - 3*a^2*B + 3*b^2*B)*Sin[c +
d*x]^4)/(4*b^4*d) - ((A*b - 2*a*B)*Sin[c + d*x]^5)/(5*b^3*d) - (B*Sin[c + d
*x]^6)/(6*b^2*d) + ((a^2 - b^2)^3*(A*b - a*B))/(b^8*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.399274, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{\left(-3a^2B+2aAb+3b^2B\right)\sin^4(c+dx)}{4b^4d}-\frac{\left(3a^2Ab-4a^3B+6ab^2B-3Ab^3\right)\sin^3(c+dx)}{3b^5d}+\frac{\left(4a^3Ab+9a^2b^2B-5a^4B-6aA\right.}{2b^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] ((a^2 - b^2)^2*(6*a*A*b - 7*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^8*d)
 - ((5*a^4*A*b - 9*a^2*A*b^3 + 3*A*b^5 - 6*a^5*B + 12*a^3*b^2*B - 6*a*b^4*B
)*Sin[c + d*x])/(b^7*d) + ((4*a^3*A*b - 6*a*A*b^3 - 5*a^4*B + 9*a^2*b^2*B -
 3*b^4*B)*Sin[c + d*x]^2)/(2*b^6*d) - ((3*a^2*A*b - 3*A*b^3 - 4*a^3*B + 6*a
*b^2*B)*Sin[c + d*x]^3)/(3*b^5*d) + ((2*a*A*b - 3*a^2*B + 3*b^2*B)*Sin[c +
d*x]^4)/(4*b^4*d) - ((A*b - 2*a*B)*Sin[c + d*x]^5)/(5*b^3*d) - (B*Sin[c + d
*x]^6)/(6*b^2*d) + ((a^2 - b^2)^3*(A*b - a*B))/(b^8*d*(a + b*Sin[c + d*x]))
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3}{(a+x)^2}dx, x, b\sin(c+dx)\right)}{b^7d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-5a^4Ab+9a^2Ab^3-3Ab^5+6a^5B-12a^3b^2B+6ab^4B}{b} - \frac{\left(-4a^3Ab+6aAb^3+5a^4B-9a^2b^2B+3\right.}{b}\right.\right.}{}$$

$$= \frac{\left(a^2-b^2\right)^2\left(6aAb-7a^2B+b^2B\right)\log(a+b\sin(c+dx))}{b^8d} - \frac{\left(5a^4Ab-9a^2Ab^3+\right.}{}$$

Mathematica [A] time = 1.66063, size = 396, normalized size = 1.22

$$\frac{6(Ab-aB)\left(-4a^2b^4\sin^4(c+dx)+2ab^3\left(5a^2-7b^2\right)\sin^3(c+dx)-2b^2\left(-29a^2b^2+15a^4+8b^4\right)\sin^2(c+dx)+4\left(a^2-b^2\right)^2\left(15a^2\log(a+b\sin(c+dx))+4a^2-4b^2\right)+4ab\sin(c+\right.}{a+b\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*(15*b^4*(-a^2 + b^2)*Cos[c + d*x]^4 + 10*b^6*Cos[c + d*x]^6 - 60*(a^2 -
b^2)^3*Log[a + b*Sin[c + d*x]] + 60*a*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Sin[c + d
*x] - 30*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 + 20*a*b^3*(a^2 - 3*b^2)*Sin[c +
d*x]^3 + 12*a*b^5*Sin[c + d*x]^5) + (6*(A*b - a*B)*(2*b^6*Cos[c + d*x]^6 +
4*(a^2 - b^2)^2*(4*a^2 - 4*b^2 + 15*a^2*Log[a + b*Sin[c + d*x]]) + 4*a*b*(-
11*a^4 + 18*a^2*b^2 - 4*b^4 + 15*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])*Sin
[c + d*x] - 2*b^2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*Sin[c + d*x]^2 + 2*a*b^3*(5
*a^2 - 7*b^2)*Sin[c + d*x]^3 - 4*a^2*b^4*Sin[c + d*x]^4 + b^4*Cos[c + d*x]^
4*(-a^2 + 4*b^2 + 3*a*b*Sin[c + d*x])))/(a + b*Sin[c + d*x]))/(60*b^8*d)
```

Maple [B] time = 0.126, size = 721, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] B*ln(a+b*sin(d*x+c))/b^2/d+3/d/b^6/(a+b*sin(d*x+c))*B*a^5-3/d/b^4/(a+b*sin(
d*x+c))*B*a^3+1/d/b^2/(a+b*sin(d*x+c))*B*a-1/5/d/b^2*A*sin(d*x+c)^5+1/d/b^2
*A*sin(d*x+c)^3-3/d/b^2*A*sin(d*x+c)-1/d/b/(a+b*sin(d*x+c))*A-5/d/b^6*A*a^4
*sin(d*x+c)+9/d/b^4*A*a^2*sin(d*x+c)+6/d/b^7*B*a^5*sin(d*x+c)-12/d/b^5*B*a^
3*sin(d*x+c)+6/d/b^3*B*a*sin(d*x+c)+15/d/b^6*ln(a+b*sin(d*x+c))*B*a^4-9/d/b
^4*ln(a+b*sin(d*x+c))*B*a^2+1/d/b^7/(a+b*sin(d*x+c))*A*a^6-3/d/b^5/(a+b*sin
(d*x+c))*A*a^4+3/d/b^3/(a+b*sin(d*x+c))*A*a^2+6/d/b^7*ln(a+b*sin(d*x+c))*A*
a^5-12/d/b^5*ln(a+b*sin(d*x+c))*A*a^3+6/d/b^3*ln(a+b*sin(d*x+c))*A*a-7/d/b^
8*ln(a+b*sin(d*x+c))*B*a^6-1/d/b^8/(a+b*sin(d*x+c))*B*a^7+9/2/d/b^4*B*sin(d
*x+c)^2*a^2-3/4/d/b^4*B*sin(d*x+c)^4*a^2+2/5/d/b^3*B*sin(d*x+c)^5*a+1/2/d/b
^3*A*sin(d*x+c)^4*a+2/d/b^5*A*sin(d*x+c)^2*a^3-3/d/b^3*A*sin(d*x+c)^2*a-1/d
/b^4*A*sin(d*x+c)^3*a^2+4/3/d/b^5*B*sin(d*x+c)^3*a^3-2/d/b^3*B*sin(d*x+c)^3
*a-5/2/d/b^6*B*sin(d*x+c)^2*a^4-1/6*B*sin(d*x+c)^6/b^2/d+3/4*B*sin(d*x+c)^4
/b^2/d-3/2*B*sin(d*x+c)^2/b^2/d
```

Maxima [A] time = 0.98725, size = 509, normalized size = 1.57

$$-\frac{\frac{60\left(Ba^7-Aa^6b-3\,Ba^5b^2+3\,Aa^4b^3+3\,Ba^3b^4-3\,Aa^2b^5-Bab^6+Ab^7\right)}{b^9\sin(dx+c)+ab^8}+\frac{10\,Bb^5\sin(dx+c)^6-12\left(2\,Bab^4-Ab^5\right)\sin(dx+c)^5+15\left(3\,Ba^2b^3-2\,Aab^4-3\,Bb^5\right)\sin(dx+c)^4-}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/60*(60*(B*a^7 - A*a^6*b - 3*B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*a^3*b^4 - 3*A*
a^2*b^5 - B*a*b^6 + A*b^7)/(b^9*sin(d*x + c) + a*b^8) + (10*B*b^5*sin(d*x +
 c)^6 - 12*(2*B*a*b^4 - A*b^5)*sin(d*x + c)^5 + 15*(3*B*a^2*b^3 - 2*A*a*b^4
 - 3*B*b^5)*sin(d*x + c)^4 - 20*(4*B*a^3*b^2 - 3*A*a^2*b^3 - 6*B*a*b^4 + 3*
A*b^5)*sin(d*x + c)^3 + 30*(5*B*a^4*b - 4*A*a^3*b^2 - 9*B*a^2*b^3 + 6*A*a*b
^4 + 3*B*b^5)*sin(d*x + c)^2 - 60*(6*B*a^5 - 5*A*a^4*b - 12*B*a^3*b^2 + 9*A
*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*sin(d*x + c))/b^7 + 60*(7*B*a^6 - 6*A*a^5*b
 - 15*B*a^4*b^2 + 12*A*a^3*b^3 + 9*B*a^2*b^4 - 6*A*a*b^5 - B*b^6)*log(b*sin
(d*x + c) + a)/b^8)/d
```

Fricas [A] time = 2.13177, size = 1220, normalized size = 3.77

$$-\frac{480\,Ba^7-480\,Aa^6b-3720\,Ba^5b^2+3360\,Aa^4b^3+5705\,Ba^3b^4-4710\,Aa^2b^5-2402\,Bab^6+1536\,Ab^7+16\left(7\,Bab^6-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/480*(480*B*a^7 - 480*A*a^6*b - 3720*B*a^5*b^2 + 3360*A*a^4*b^3 + 5705*B*
a^3*b^4 - 4710*A*a^2*b^5 - 2402*B*a*b^6 + 1536*A*b^7 + 16*(7*B*a*b^6 - 6*A*
b^7)*cos(d*x + c)^6 - 8*(35*B*a^3*b^4 - 30*A*a^2*b^5 - 33*B*a*b^6 + 24*A*b^
7)*cos(d*x + c)^4 + 16*(105*B*a^5*b^2 - 90*A*a^4*b^3 - 190*B*a^3*b^4 + 150*
A*a^2*b^5 + 81*B*a*b^6 - 48*A*b^7)*cos(d*x + c)^2 + 480*(7*B*a^7 - 6*A*a^6*
b - 15*B*a^5*b^2 + 12*A*a^4*b^3 + 9*B*a^3*b^4 - 6*A*a^2*b^5 - B*a*b^6 + (7*
B*a^6*b - 6*A*a^5*b^2 - 15*B*a^4*b^3 + 12*A*a^3*b^4 + 9*B*a^2*b^5 - 6*A*a*b
^6 - B*b^7)*sin(d*x + c))*log(b*sin(d*x + c) + a) - (80*B*b^7*cos(d*x + c)^
6 + 2880*B*a^6*b - 2400*A*a^5*b^2 - 5720*B*a^4*b^3 + 4320*A*a^3*b^4 + 2967*
B*a^2*b^5 - 1626*A*a*b^6 - 190*B*b^7 - 24*(7*B*a^2*b^5 - 6*A*a*b^6 - 5*B*b^
7)*cos(d*x + c)^4 + 16*(35*B*a^4*b^3 - 30*A*a^3*b^4 - 54*B*a^2*b^5 + 42*A*a
*b^6 + 15*B*b^7)*cos(d*x + c)^2)*sin(d*x + c))/(b^9*d*sin(d*x + c) + a*b^8*
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27108, size = 770, normalized size = 2.38

$$-\frac{\frac{60\left(7 Ba^6-6 Aa^5b-15 Ba^4b^2+12 Aa^3b^3+9 Ba^2b^4-6 Aab^5-Bb^6\right)\log(|b\sin(dx+c)+a|)}{b^8}-\frac{60\left(7 Ba^6b\sin(dx+c)-6 Aa^5b^2\sin(dx+c)-15 Ba^4b^3\sin(dx+c)+12 A\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(7*B*a^6 - 6*A*a^5*b - 15*B*a^4*b^2 + 12*A*a^3*b^3 + 9*B*a^2*b^4
- 6*A*a*b^5 - B*b^6)*log(abs(b*sin(d*x + c) + a))/b^8 - 60*(7*B*a^6*b*sin(d
*x + c) - 6*A*a^5*b^2*sin(d*x + c) - 15*B*a^4*b^3*sin(d*x + c) + 12*A*a^3*b
^4*sin(d*x + c) + 9*B*a^2*b^5*sin(d*x + c) - 6*A*a*b^6*sin(d*x + c) - B*b^7
*sin(d*x + c) + 6*B*a^7 - 5*A*a^6*b - 12*B*a^5*b^2 + 9*A*a^4*b^3 + 6*B*a^3*
b^4 - 3*A*a^2*b^5 - A*b^7)/((b*sin(d*x + c) + a)*b^8) + (10*B*b^10*sin(d*x
+ c)^6 - 24*B*a*b^9*sin(d*x + c)^5 + 12*A*b^10*sin(d*x + c)^5 + 45*B*a^2*b^
8*sin(d*x + c)^4 - 30*A*a*b^9*sin(d*x + c)^4 - 45*B*b^10*sin(d*x + c)^4 - 8
0*B*a^3*b^7*sin(d*x + c)^3 + 60*A*a^2*b^8*sin(d*x + c)^3 + 120*B*a*b^9*sin(
d*x + c)^3 - 60*A*b^10*sin(d*x + c)^3 + 150*B*a^4*b^6*sin(d*x + c)^2 - 120*
A*a^3*b^7*sin(d*x + c)^2 - 270*B*a^2*b^8*sin(d*x + c)^2 + 180*A*a*b^9*sin(d
*x + c)^2 + 90*B*b^10*sin(d*x + c)^2 - 360*B*a^5*b^5*sin(d*x + c) + 300*A*a
^4*b^6*sin(d*x + c) + 720*B*a^3*b^7*sin(d*x + c) - 540*A*a^2*b^8*sin(d*x +
c) - 360*B*a*b^9*sin(d*x + c) + 180*A*b^10*sin(d*x + c))/b^12)/d
```

3.1553 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=206

$$-\frac{\left(-3a^2B+2aAb+2b^2B\right)\sin^2(c+dx)}{2b^4d}+\frac{\left(3a^2Ab-4a^3B+4ab^2B-2Ab^3\right)\sin(c+dx)}{b^5d}-\frac{\left(a^2-b^2\right)^2(Ab-aB)}{b^6d(a+b\sin(c+dx))}-\frac{(a^2-b\ldots}{}$$

```
[Out] -(((a^2 - b^2)*(4*a*A*b - 5*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^6*d)
) + ((3*a^2*A*b - 2*A*b^3 - 4*a^3*B + 4*a*b^2*B)*Sin[c + d*x])/(b^5*d) - ((
2*a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x]^2)/(2*b^4*d) + ((A*b - 2*a*B)*Sin
[c + d*x]^3)/(3*b^3*d) + (B*Sin[c + d*x]^4)/(4*b^2*d) - ((a^2 - b^2)^2*(A*b
 - a*B))/(b^6*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.271905, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$-\frac{\left(-3a^2B+2aAb+2b^2B\right)\sin^2(c+dx)}{2b^4d}+\frac{\left(3a^2Ab-4a^3B+4ab^2B-2Ab^3\right)\sin(c+dx)}{b^5d}-\frac{\left(a^2-b^2\right)^2(Ab-aB)}{b^6d(a+b\sin(c+dx))}-\frac{(a^2-b\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^2 - b^2)*(4*a*A*b - 5*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^6*d)
) + ((3*a^2*A*b - 2*A*b^3 - 4*a^3*B + 4*a*b^2*B)*Sin[c + d*x])/(b^5*d) - ((
2*a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x]^2)/(2*b^4*d) + ((A*b - 2*a*B)*Sin
[c + d*x]^3)/(3*b^3*d) + (B*Sin[c + d*x]^4)/(4*b^2*d) - ((a^2 - b^2)^2*(A*b
 - a*B))/(b^6*d*(a + b*Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^2}{(a+x)^2}dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{3a^2Ab-2Ab^3-4a^3B+4ab^2B}{b} + \frac{\left(-2aAb+3a^2B-2b^2B\right)x}{b} + \frac{(Ab-2aB)x^2}{b} + \frac{Bx^3}{b} + \frac{(-\ldots}{}\right.\right.}{b^5 d}$$

$$= -\frac{\left(a^2-b^2\right)\left(4aAb-5a^2B+b^2B\right)\log(a+b\sin(c+dx))}{b^6 d} + \frac{\left(3a^2Ab-2Ab^3-\ldots\right.}{}$$

Mathematica [A] time = 2.10966, size = 234, normalized size = 1.14

$$\frac{4\left(A-\frac{aB}{b}\right)\left(\left(8a^2b-4b^3\right)\sin(c+dx) + \frac{b^4\cos^4(c+dx)-4\left(a^2-b^2\right)\left(3a^2\log(a+b\sin(c+dx))+a^2+3ab\sin(c+dx)\log(a+b\sin(c+dx))-b^2\right)}{a+b\sin(c+dx)} - 2ab^2\,\text{s}\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*(3*b^3*Cos[c + d*x]^4 + (12*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/b - 1
2*a*(a^2 - 2*b^2)*Sin[c + d*x] + 6*b*(a^2 - b^2)*Sin[c + d*x]^2 - 4*a*b^2*S
in[c + d*x]^3) + 4*(A - (a*B)/b)*((8*a^2*b - 4*b^3)*Sin[c + d*x] - 2*a*b^2*
Sin[c + d*x]^2 + (b^4*Cos[c + d*x]^4 - 4*(a^2 - b^2)*(a^2 - b^2 + 3*a^2*Log
[a + b*Sin[c + d*x]] + 3*a*b*Log[a + b*Sin[c + d*x]]*Sin[c + d*x]))/(a + b*
Sin[c + d*x])))/(12*b^5*d)
```

Maple [B] time = 0.125, size = 422, normalized size = 2.1

$$\frac{B(\sin(dx+c))^4}{4b^2d} + \frac{A(\sin(dx+c))^3}{3b^2d} - \frac{2B(\sin(dx+c))^3 a}{3db^3} - \frac{A(\sin(dx+c))^2 a}{db^3} + \frac{3B(\sin(dx+c))^2 a^2}{2db^4} - \frac{B(\sin(dx\ldots}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4*B*sin(d*x+c)^4/b^2/d+1/3/d/b^2*A*sin(d*x+c)^3-2/3/d/b^3*B*sin(d*x+c)^3*
a-1/d/b^3*A*sin(d*x+c)^2*a+3/2/d/b^4*B*sin(d*x+c)^2*a^2-B*sin(d*x+c)^2/b^2/
d+3/d/b^4*A*a^2*sin(d*x+c)-2/d/b^2*A*sin(d*x+c)-4/d/b^5*B*a^3*sin(d*x+c)+4/
d/b^3*B*a*sin(d*x+c)-4/d/b^5*ln(a+b*sin(d*x+c))*A*a^3+4/d/b^3*ln(a+b*sin(d*
x+c))*A*a+5/d/b^6*ln(a+b*sin(d*x+c))*B*a^4-6/d/b^4*ln(a+b*sin(d*x+c))*B*a^2
+B*ln(a+b*sin(d*x+c))/b^2/d-1/d/b^5/(a+b*sin(d*x+c))*A*a^4+2/d/b^3/(a+b*sin
(d*x+c))*A*a^2-1/d/b/(a+b*sin(d*x+c))*A+1/d/b^6/(a+b*sin(d*x+c))*B*a^5-2/d/
b^4/(a+b*sin(d*x+c))*B*a^3+1/d/b^2/(a+b*sin(d*x+c))*B*a
```

Maxima [A] time = 0.973817, size = 309, normalized size = 1.5

$$\frac{\frac{12\left(Ba^5-Aa^4b-2Ba^3b^2+2Aa^2b^3+Bab^4-Ab^5\right)}{b^7\sin(dx+c)+ab^6} + \frac{3Bb^3\sin(dx+c)^4-4\left(2Bab^2-Ab^3\right)\sin(dx+c)^3+6\left(3Ba^2b-2Aab^2-2Bb^3\right)\sin(dx+c)^2-12\left(4Ba^3-3Aa^2b-4\ldots\right.}{b^5}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(12*(B*a^5 - A*a^4*b - 2*B*a^3*b^2 + 2*A*a^2*b^3 + B*a*b^4 - A*b^5)/(b
^7*sin(d*x + c) + a*b^6) + (3*B*b^3*sin(d*x + c)^4 - 4*(2*B*a*b^2 - A*b^3)*
sin(d*x + c)^3 + 6*(3*B*a^2*b - 2*A*a*b^2 - 2*B*b^3)*sin(d*x + c)^2 - 12*(4
*B*a^3 - 3*A*a^2*b - 4*B*a*b^2 + 2*A*b^3)*sin(d*x + c))/b^5 + 12*(5*B*a^4 -
 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(b*sin(d*x + c) + a)/b^6)/
d
```

Fricas [A] time = 1.87003, size = 763, normalized size = 3.7

$$\frac{96\,Ba^5 - 96\,Aa^4b - 504\,Ba^3b^2 + 432\,Aa^2b^3 + 383\,Bab^4 - 256\,Ab^5 - 8\left(5\,Bab^4 - 4\,Ab^5\right)\cos\left(dx+c\right)^4 + 16\left(15\,Ba^3b^2 - 12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/96*(96*B*a^5 - 96*A*a^4*b - 504*B*a^3*b^2 + 432*A*a^2*b^3 + 383*B*a*b^4 -
 256*A*b^5 - 8*(5*B*a*b^4 - 4*A*b^5)*cos(d*x + c)^4 + 16*(15*B*a^3*b^2 - 12
*A*a^2*b^3 - 13*B*a*b^4 + 8*A*b^5)*cos(d*x + c)^2 + 96*(5*B*a^5 - 4*A*a^4*b
 - 6*B*a^3*b^2 + 4*A*a^2*b^3 + B*a*b^4 + (5*B*a^4*b - 4*A*a^3*b^2 - 6*B*a^2
*b^3 + 4*A*a*b^4 + B*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (24*B*b^5
*cos(d*x + c)^4 - 384*B*a^4*b + 288*A*a^3*b^2 + 392*B*a^2*b^3 - 208*A*a*b^4
 - 33*B*b^5 - 16*(5*B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cos(d*x + c)^2)*sin(d*
x + c))/(b^7*d*sin(d*x + c) + a*b^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.53147, size = 443, normalized size = 2.15

$$\frac{\frac{12\left(5\,Ba^4 - 4\,Aa^3b - 6\,Ba^2b^2 + 4\,Aab^3 + Bb^4\right)\log\left(|b\sin\left(dx+c\right)+a|\right)}{b^6} - \frac{12\left(5\,Ba^4b\sin\left(dx+c\right) - 4\,Aa^3b^2\sin\left(dx+c\right) - 6\,Ba^2b^3\sin\left(dx+c\right) + 4\,Aab^4\sin\left(dx+c\right) + Bb^5\sin\left(dx+c\right)\right.}{\left(b\sin\left(dx+c\right)+a\right)b^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/12*(12*(5*B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(abs(b*
sin(d*x + c) + a))/b^6 - 12*(5*B*a^4*b*sin(d*x + c) - 4*A*a^3*b^2*sin(d*x +
```

```
 c) - 6*B*a^2*b^3*sin(d*x + c) + 4*A*a*b^4*sin(d*x + c) + B*b^5*sin(d*x + c
) + 4*B*a^5 - 3*A*a^4*b - 4*B*a^3*b^2 + 2*A*a^2*b^3 + A*b^5)/((b*sin(d*x +
c) + a)*b^6) + (3*B*b^6*sin(d*x + c)^4 - 8*B*a*b^5*sin(d*x + c)^3 + 4*A*b^6
*sin(d*x + c)^3 + 18*B*a^2*b^4*sin(d*x + c)^2 - 12*A*a*b^5*sin(d*x + c)^2 -
 12*B*b^6*sin(d*x + c)^2 - 48*B*a^3*b^3*sin(d*x + c) + 36*A*a^2*b^4*sin(d*x
 + c) + 48*B*a*b^5*sin(d*x + c) - 24*A*b^6*sin(d*x + c))/b^8)/d
```

3.1554 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=113

$$\frac{(a^2-b^2)(Ab-aB)}{b^4d(a+b\sin(c+dx))}+\frac{(-3a^2B+2aAb+b^2B)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-2aB)\sin(c+dx)}{b^3d}-\frac{B\sin^2(c+dx)}{2b^2d}$$

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^4*d) - ((A*b - 2*a
*B)*Sin[c + d*x])/(b^3*d) - (B*Sin[c + d*x]^2)/(2*b^2*d) + ((a^2 - b^2)*(A*
b - a*B))/(b^4*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.168508, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2837, 772}

$$\frac{(a^2-b^2)(Ab-aB)}{b^4d(a+b\sin(c+dx))}+\frac{(-3a^2B+2aAb+b^2B)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-2aB)\sin(c+dx)}{b^3d}-\frac{B\sin^2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^4*d) - ((A*b - 2*a
*B)*Sin[c + d*x])/(b^3*d) - (B*Sin[c + d*x]^2)/(2*b^2*d) + ((a^2 - b^2)*(A*
b - a*B))/(b^4*d*(a + b*Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)(b^2-x^2)}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^3d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{-Ab+2aB}{b}-\frac{Bx}{b}+\frac{(-a^2+b^2)(Ab-aB)}{b(a+x)^2}+\frac{2aAb-3a^2B+b^2B}{b(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^3d}\\
&= \frac{(2aAb-3a^2B+b^2B)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-2aB)\sin(c+dx)}{b^3d}-\frac{B\sin^2(}{2b}
\end{aligned}$$

Mathematica [A] time = 0.504945, size = 111, normalized size = 0.98

$$\frac{\frac{B(b^2-a^2)\log(a+b\sin(c+dx))}{b}+\left(A-\frac{aB}{b}\right)\left(\frac{(a-b)(a+b)}{a+b\sin(c+dx)}+2a\log(a+b\sin(c+dx))-b\sin(c+dx)\right)+aB\sin(c+dx)-\frac{1}{2}bB\sin}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (((-a^2 + b^2)*B*Log[a + b*Sin[c + d*x]])/b + a*B*Sin[c + d*x] - (b*B*Sin[c
 + d*x]^2)/2 + (A - (a*B)/b)*(2*a*Log[a + b*Sin[c + d*x]] - b*Sin[c + d*x]
+ ((a - b)*(a + b))/(a + b*Sin[c + d*x])))/(b^3*d)
```

Maple [A] time = 0.121, size = 202, normalized size = 1.8

$$-\frac{B(\sin(dx+c))^2}{2b^2d}-\frac{A\sin(dx+c)}{b^2d}+2\frac{aB\sin(dx+c)}{db^3}+2\frac{\ln(a+b\sin(dx+c))Aa}{db^3}-3\frac{\ln(a+b\sin(dx+c))Ba^2}{db^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2*B*sin(d*x+c)^2/b^2/d-1/d/b^2*A*sin(d*x+c)+2/d/b^3*B*a*sin(d*x+c)+2/d/b
^3*ln(a+b*sin(d*x+c))*A*a-3/d/b^4*ln(a+b*sin(d*x+c))*B*a^2+B*ln(a+b*sin(d*x
+c))/b^2/d+1/d/b^3/(a+b*sin(d*x+c))*A*a^2-1/d/b/(a+b*sin(d*x+c))*A-1/d/b^4/
(a+b*sin(d*x+c))*B*a^3+1/d/b^2/(a+b*sin(d*x+c))*B*a
```

Maxima [A] time = 0.976019, size = 159, normalized size = 1.41

$$-\frac{\frac{2(Ba^3-Aa^2b-Bab^2+Ab^3)}{b^5\sin(dx+c)+ab^4}+\frac{Bb\sin(dx+c)^2-2(2Ba-Ab)\sin(dx+c)}{b^3}+\frac{2(3Ba^2-2Aab-Bb^2)\log(b\sin(dx+c)+a)}{b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)/(b^5*sin(d*x + c) + a*b^4) + (B
*b*sin(d*x + c)^2 - 2*(2*B*a - A*b)*sin(d*x + c))/b^3 + 2*(3*B*a^2 - 2*A*a*
b - B*b^2)*log(b*sin(d*x + c) + a)/b^4)/d
```

Fricas [A] time = 1.55436, size = 408, normalized size = 3.61

$$-\frac{4Ba^3-4Aa^2b-11Bab^2+8Ab^3+2(3Bab^2-2Ab^3)\cos(dx+c)^2+4(3Ba^3-2Aa^2b-Bab^2+(3Ba^2b-2Aab^2}{4(b^5d\sin(dx+c)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/4*(4*B*a^3 - 4*A*a^2*b - 11*B*a*b^2 + 8*A*b^3 + 2*(3*B*a*b^2 - 2*A*b^3)*
cos(d*x + c)^2 + 4*(3*B*a^3 - 2*A*a^2*b - B*a*b^2 + (3*B*a^2*b - 2*A*a*b^2
- B*b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a) - (2*B*b^3*cos(d*x + c)^2 +
8*B*a^2*b - 4*A*a*b^2 - B*b^3)*sin(d*x + c))/(b^5*d*sin(d*x + c) + a*b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18877, size = 254, normalized size = 2.25

$$-\frac{\frac{(b\sin(dx+c)+a)^2\left(B-\frac{2\left(3\,Bab-Ab^2\right)}{(b\sin(dx+c)+a)b}\right)}{b^4}-\frac{2\left(3\,Ba^2-2\,Aab-Bb^2\right)\log\left(\frac{|b\sin(dx+c)+a|}{(b\sin(dx+c)+a)^2|b|}\right)}{b^4}+\frac{2\left(\frac{Ba^3b^2}{b\sin(dx+c)+a}-\frac{Aa^2b^3}{b\sin(dx+c)+a}-\frac{Bab^4}{b\sin(dx+c)+a}+\frac{Ab^5}{b\sin(dx+c)+a}\right)}{b^6}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*((b*sin(d*x + c) + a)^2*(B - 2*(3*B*a*b - A*b^2)/((b*sin(d*x + c) + a)
*b))/b^4 - 2*(3*B*a^2 - 2*A*a*b - B*b^2)*log(abs(b*sin(d*x + c) + a)/((b*si
n(d*x + c) + a)^2*abs(b)))/b^4 + 2*(B*a^3*b^2/(b*sin(d*x + c) + a) - A*a^2*
b^3/(b*sin(d*x + c) + a) - B*a*b^4/(b*sin(d*x + c) + a) + A*b^5/(b*sin(d*x
+ c) + a))/b^6)/d
```

3.1555 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$\frac{B\log(a+b\sin(c+dx))}{b^2d}-\frac{Ab-aB}{b^2d(a+b\sin(c+dx))}$$

```
[Out] (B*Log[a + b*Sin[c + d*x]])/(b^2*d) - (A*b - a*B)/(b^2*d*(a + b*Sin[c + d*x
]))
```

Rubi [A] time = 0.0767375, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2833, 43}

$$\frac{B\log(a+b\sin(c+dx))}{b^2d}-\frac{Ab-aB}{b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*Log[a + b*Sin[c + d*x]])/(b^2*d) - (A*b - a*B)/(b^2*d*(a + b*Sin[c + d*x
]))
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2}\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+x)^2}+\frac{B}{b(a+x)}\right)dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{B\log(a+b\sin(c+dx))}{b^2d}-\frac{Ab-aB}{b^2d(a+b\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0868795, size = 42, normalized size = 0.88

$$\frac{\frac{aB-Ab}{a+b\sin(c+dx)}+B\log(a+b\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*Log[a + b*Sin[c + d*x]] + (-(A*b) + a*B)/(a + b*Sin[c + d*x]))/(b^2*d)
```

Maple [A] time = 0.056, size = 63, normalized size = 1.3

$$\frac{B\ln\left(a+b\sin\left(dx+c\right)\right)}{b^{2}d}-\frac{A}{bd\left(a+b\sin\left(dx+c\right)\right)}+\frac{aB}{b^{2}d\left(a+b\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] B*ln(a+b*sin(d*x+c))/b^2/d-1/d/b/(a+b*sin(d*x+c))*A+1/d/b^2/(a+b*sin(d*x+c)
)*B*a
```

Maxima [A] time = 0.992207, size = 65, normalized size = 1.35

$$\frac{\frac{Ba-Ab}{b^{3}\sin(dx+c)+ab^{2}}+\frac{B\log(b\sin(dx+c)+a)}{b^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] ((B*a - A*b)/(b^3*sin(d*x + c) + a*b^2) + B*log(b*sin(d*x + c) + a)/b^2)/d
```

Fricas [A] time = 1.42007, size = 128, normalized size = 2.67

$$\frac{Ba-Ab+\left(Bb\sin\left(dx+c\right)+Ba\right)\log\left(b\sin\left(dx+c\right)+a\right)}{b^{3}d\sin\left(dx+c\right)+ab^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] (B*a - A*b + (B*b*sin(d*x + c) + B*a)*log(b*sin(d*x + c) + a))/(b^3*d*sin(d
*x + c) + a*b^2*d)
```

Sympy [A] time = 1.27253, size = 178, normalized size = 3.71

$$\begin{cases}\frac{x(A+B\sin(c))\cos(c)}{a^{2}} & \text{for}\ b=0\wedge d=0\\ \frac{\frac{A\sin(c+dx)}{d}-\frac{B\cos^{2}(c+dx)}{2d}}{a^{2}} & \text{for}\ b=0\\ \frac{x(A+B\sin(c))\cos(c)}{(a+b\sin(c))^{2}} & \text{for}\ d=0\\ -\frac{Ab}{ab^{2}d+b^{3}d\sin(c+dx)}+\frac{Ba\log\left(\frac{a}{b}+\sin(c+dx)\right)}{ab^{2}d+b^{3}d\sin(c+dx)}+\frac{Ba}{ab^{2}d+b^{3}d\sin(c+dx)}+\frac{Bb\log\left(\frac{a}{b}+\sin(c+dx)\right)\sin(c+dx)}{ab^{2}d+b^{3}d\sin(c+dx)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(A + B*sin(c))*cos(c)/a**2, Eq(b, 0) & Eq(d, 0)), ((A*sin(c +
d*x)/d - B*cos(c + d*x)**2/(2*d))/a**2, Eq(b, 0)), (x*(A + B*sin(c))*cos(c)
/(a + b*sin(c))**2, Eq(d, 0)), (-A*b/(a*b**2*d + b**3*d*sin(c + d*x)) + B*a
*log(a/b + sin(c + d*x))/(a*b**2*d + b**3*d*sin(c + d*x)) + B*a/(a*b**2*d +
 b**3*d*sin(c + d*x)) + B*b*log(a/b + sin(c + d*x))*sin(c + d*x)/(a*b**2*d
+ b**3*d*sin(c + d*x)), True))
```

Giac [A] time = 1.35983, size = 108, normalized size = 2.25

$$-\frac{\frac{B\left(\frac{\log\left(\frac{|b\sin(dx+c)+a|}{(b\sin(dx+c)+a)^2|b|}\right)}{b}-\frac{a}{(b\sin(dx+c)+a)b}\right)}{b}+\frac{A}{(b\sin(dx+c)+a)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -(B*(log(abs(b*sin(d*x + c) + a)/((b*sin(d*x + c) + a)^2*abs(b)))/b - a/((b
*sin(d*x + c) + a)*b))/b + A/((b*sin(d*x + c) + a)*b))/d
```

3.1556 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=135

$$\frac{Ab-aB}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{(A-B)}{}$$

```
[Out] -((A + B)*Log[1 - Sin[c + d*x]])/(2*(a + b)^2*d) + ((A - B)*Log[1 + Sin[c +
 d*x]])/(2*(a - b)^2*d) - ((2*a*A*b - a^2*B - b^2*B)*Log[a + b*Sin[c + d*x]
])/((a^2 - b^2)^2*d) + (A*b - a*B)/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.193716, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2837, 801}

$$\frac{Ab-aB}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{(A-B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((A + B)*Log[1 - Sin[c + d*x]])/(2*(a + b)^2*d) + ((A - B)*Log[1 + Sin[c +
 d*x]])/(2*(a - b)^2*d) - ((2*a*A*b - a^2*B - b^2*B)*Log[a + b*Sin[c + d*x]
])/((a^2 - b^2)^2*d) + (A*b - a*B)/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2\left(b^2-x^2\right)}\,dx,x,b\sin(c+dx)\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \left(\frac{A+B}{2b(a+b)^2(b-x)}+\frac{-Ab+aB}{(a-b)b(a+b)(a+x)^2}+\frac{-2aAb+a^2B+b^2B}{(a-b)^2b(a+b)^2(a+x)}+\frac{A-B}{2(a-b)^2b(b+x)}\right)dx,x\right.}{d}\\ &= -\frac{(A+B)\log(1-\sin(c+dx))}{2(a+b)^2d}+\frac{(A-B)\log(1+\sin(c+dx))}{2(a-b)^2d}-\frac{\left(2aAb-a^2B-b\right.}{\left(\right.}\end{aligned}$$

Mathematica [A] time = 1.27208, size = 178, normalized size = 1.32

$$\frac{b\left(A-\frac{aB}{b}\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2b(a-b)^2}-\frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right)-\frac{B((b-a)\log(1-\sin(c+dx))+(a+b)\log(\sin}{2b(b-a)(}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-(B*((-a + b)*Log[1 - Sin[c + d*x]] + (a + b)*Log[1 + Sin[c + d*x]] - 2*b*
Log[a + b*Sin[c + d*x]]))/(2*b*(-a + b)*(a + b)) + b*(A - (a*B)/b)*(-Log[1
- Sin[c + d*x]]/(2*b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (
2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*
Sin[c + d*x]))))/d
```

Maple [A] time = 0.135, size = 240, normalized size = 1.8

$$\frac{Ab}{d\,(a+b)\,(a-b)\,(a+b\sin(dx+c))}-\frac{aB}{d\,(a+b)\,(a-b)\,(a+b\sin(dx+c))}-2\,\frac{\ln(a+b\sin(dx+c))\,Aab}{d\,(a+b)^2\,(a-b)^2}+\frac{\ln(a+b\,\mathrm{si}}{d\,(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d/(a+b)/(a-b)/(a+b*sin(d*x+c))*A*b-1/d/(a+b)/(a-b)/(a+b*sin(d*x+c))*a*B-2
/d/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*A*a*b+1/d/(a+b)^2/(a-b)^2*ln(a+b*sin(
d*x+c))*B*a^2+1/d/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*B*b^2-1/2/d/(a+b)^2*ln
(sin(d*x+c)-1)*A-1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*B+1/2/d/(a-b)^2*ln(1+sin(d*
x+c))*A-1/2/d/(a-b)^2*ln(1+sin(d*x+c))*B
```

Maxima [A] time = 0.99445, size = 198, normalized size = 1.47

$$\frac{\frac{2\left(Ba^2-2\,Aab+Bb^2\right)\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}+\frac{(A-B)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{(A+B)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(Ba-Ab)}{a^3-ab^2+\left(a^2b-b^3\right)\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(B*a^2 - 2*A*a*b + B*b^2)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 +
 b^4) + (A - B)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (A + B)*log(sin
(d*x + c) - 1)/(a^2 + 2*a*b + b^2) - 2*(B*a - A*b)/(a^3 - a*b^2 + (a^2*b -
b^3)*sin(d*x + c)))/d
```

Fricas [B] time = 3.37979, size = 671, normalized size = 4.97

$$-\frac{2\,Ba^3-2\,Aa^2b-2\,Bab^2+2\,Ab^3-2\left(Ba^3-2\,Aa^2b+Bab^2+\left(Ba^2b-2\,Aab^2+Bb^3\right)\sin(dx+c)\right)\log(b\sin(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] -1/2*(2*B*a^3 - 2*A*a^2*b - 2*B*a*b^2 + 2*A*b^3 - 2*(B*a^3 - 2*A*a^2*b + B*
a*b^2 + (B*a^2*b - 2*A*a*b^2 + B*b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a)
 - ((A - B)*a^3 + 2*(A - B)*a^2*b + (A - B)*a*b^2 + ((A - B)*a^2*b + 2*(A -
 B)*a*b^2 + (A - B)*b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((A + B)*a^3
 - 2*(A + B)*a^2*b + (A + B)*a*b^2 + ((A + B)*a^2*b - 2*(A + B)*a*b^2 + (A
+ B)*b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*
d*sin(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sin(c + dx))\sec(c + dx)}{(a + b\sin(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.33868, size = 277, normalized size = 2.05

$$\frac{\frac{2\left(Ba^2b-2\,Aab^2+Bb^3\right)\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}-\frac{(A+B)\log(|-\sin(dx+c)+1|)}{a^2+2\,ab+b^2}+\frac{(A-B)\log(|-\sin(dx+c)-1|)}{a^2-2\,ab+b^2}-\frac{2\left(Ba^2b\sin(dx+c)-2\,Aab^2\sin(dx+c)+Bb^3\sin(dx+\right.}{\left(a^4-2\,a^2b^2+b^4\right)(b\sin(dx+c)+}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/2*(2*(B*a^2*b - 2*A*a*b^2 + B*b^3)*log(abs(b*sin(d*x + c) + a))/(a^4*b -
2*a^2*b^3 + b^5) - (A + B)*log(abs(-sin(d*x + c) + 1))/(a^2 + 2*a*b + b^2)
+ (A - B)*log(abs(-sin(d*x + c) - 1))/(a^2 - 2*a*b + b^2) - 2*(B*a^2*b*sin(
d*x + c) - 2*A*a*b^2*sin(d*x + c) + B*b^3*sin(d*x + c) + 2*B*a^3 - 3*A*a^2*
b + A*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

3.1557 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=228

$$-\frac{b\left(a^2A-4abB+3Ab^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b^2\left(-3a^2B+4aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^2(c+dx)(-(aA-bB)\sin(c}{2d\left(a^2-b^2\right)(a+b\sin}$$

```
[Out] -((a*A + 3*A*b + 2*b*B)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a*A - 3*
A*b + 2*b*B)*Log[1 + Sin[c + d*x]])/(4*(a - b)^3*d) + (b^2*(4*a*A*b - 3*a^2
*B - b^2*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) - (b*(a^2*A + 3*A*b^
2 - 4*a*b*B))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[c + d*x]^2*(A
*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
)
```

Rubi [A] time = 0.326292, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$-\frac{b\left(a^2A-4abB+3Ab^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b^2\left(-3a^2B+4aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^2(c+dx)(-(aA-bB)\sin(c}{2d\left(a^2-b^2\right)(a+b\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -((a*A + 3*A*b + 2*b*B)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a*A - 3*
A*b + 2*b*B)*Log[1 + Sin[c + d*x]])/(4*(a - b)^3*d) + (b^2*(4*a*A*b - 3*a^2
*B - b^2*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) - (b*(a^2*A + 3*A*b^
2 - 4*a*b*B))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[c + d*x]^2*(A
*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
)
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b^3\,\mathrm{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b\,\mathrm{Subst}\left(\int \frac{-a^2A+3Ab^2-2abB-2}{(a+x)^2\left(b^2-x^2\right.}\right.}{2\left(a^2-\right.}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b\,\mathrm{Subst}\left(\int\left(-\frac{(a-b)(aA+3Ab+2b}{2b(a+b)^2(b-x)}\right.\right.}{}$$

$$= -\frac{(aA+3Ab+2bB)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(aA-3Ab+2bB)\log(1+\sin(c+d}{4(a-b)^3d}$$

Mathematica [A] time = 1.64412, size = 246, normalized size = 1.08

$$\frac{b\left(a^2A-4abB+3Ab^2\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2b(a-b)^2}-\frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right)+\frac{(aA-bB)((a-b)\log(1-\sin(c}{}}{2d\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (((a*A - b*B)*((a - b)*Log[1 - Sin[c + d*x]] - (a + b)*Log[1 + Sin[c + d*x]
] + 2*b*Log[a + b*Sin[c + d*x]]))/((a - b)*(a + b)) + (Sec[c + d*x]^2*(A*b
- a*B + (-(a*A) + b*B)*Sin[c + d*x]))/(a + b*Sin[c + d*x]) + b*(a^2*A + 3*A
*b^2 - 4*a*b*B)*(-Log[1 - Sin[c + d*x]]/(2*b*(a + b)^2) + Log[1 + Sin[c + d
*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2)
+ 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/(2*(-a^2 + b^2)*d)
```

Maple [A] time = 0.17, size = 388, normalized size = 1.7

$$-\frac{Ab^3}{d(a+b)^2(a-b)^2(a+b\sin(dx+c))}+\frac{ab^2B}{d(a+b)^2(a-b)^2(a+b\sin(dx+c))}+4\frac{b^3\ln(a+b\sin(dx+c))Aa}{d(a+b)^3(a-b)^3}-3\frac{b^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*b^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))*A+1/d*b^2/(a+b)^2/(a-b)^2/(a+b*si
n(d*x+c))*a*B+4/d*b^3/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*A*a-3/d*b^2/(a+b)^
3/(a-b)^3*ln(a+b*sin(d*x+c))*B*a^2-1/d*b^4/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c
))*B-1/4/d/(a+b)^2/(sin(d*x+c)-1)*A-1/4/d/(a+b)^2/(sin(d*x+c)-1)*B-1/4/d/(a
+b)^3*ln(sin(d*x+c)-1)*a*A-3/4/d/(a+b)^3*ln(sin(d*x+c)-1)*A*b-1/2/d/(a+b)^3
*ln(sin(d*x+c)-1)*B*b-1/4/d/(a-b)^2/(1+sin(d*x+c))*A+1/4/d/(a-b)^2/(1+sin(d
*x+c))*B+1/4/d/(a-b)^3*ln(1+sin(d*x+c))*a*A-3/4/d/(a-b)^3*ln(1+sin(d*x+c))*
A*b+1/2/d/(a-b)^3*ln(1+sin(d*x+c))*B*b
```

Maxima [A] time = 1.00418, size = 467, normalized size = 2.05

$$-\frac{\frac{4\left(3\,Ba^2b^2-4\,Aab^3+Bb^4\right)\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{(Aa-(3\,A-2\,B)b)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(Aa+(3\,A+2\,B)b)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(Ba^3-2\,Aa^2b+3\,Bab^2-2\,\ldots\right.}{a^5-2\,a^3b^2+ab^4-\left(a^4b-2\,a^2b\ldots\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/4*(4*(3*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(b*sin(d*x + c) + a)/(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6) - (A*a - (3*A - 2*B)*b)*log(sin(d*x + c) + 1)/(a
^3 - 3*a^2*b + 3*a*b^2 - b^3) + (A*a + (3*A + 2*B)*b)*log(sin(d*x + c) - 1)
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(B*a^3 - 2*A*a^2*b + 3*B*a*b^2 - 2*A*b
^3 + (A*a^2*b - 4*B*a*b^2 + 3*A*b^3)*sin(d*x + c)^2 + (A*a^3 - B*a^2*b - A*
a*b^2 + B*b^3)*sin(d*x + c))/(a^5 - 2*a^3*b^2 + a*b^4 - (a^4*b - 2*a^2*b^3
+ b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c)^2 + (a^4*b -
 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

Fricas [B] time = 7.8936, size = 1337, normalized size = 5.86

$$\frac{2\,Ba^5-2\,Aa^4b-4\,Ba^3b^2+4\,Aa^2b^3+2\,Bab^4-2\,Ab^5-2\left(Aa^4b-4\,Ba^3b^2+2\,Aa^2b^3+4\,Bab^4-3\,Ab^5\right)\cos\left(dx+c\right)^2\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*B*a^5 - 2*A*a^4*b - 4*B*a^3*b^2 + 4*A*a^2*b^3 + 2*B*a*b^4 - 2*A*b^5
- 2*(A*a^4*b - 4*B*a^3*b^2 + 2*A*a^2*b^3 + 4*B*a*b^4 - 3*A*b^5)*cos(d*x + c
)^2 - 4*((3*B*a^2*b^3 - 4*A*a*b^4 + B*b^5)*cos(d*x + c)^2*sin(d*x + c) + (3
*B*a^3*b^2 - 4*A*a^2*b^3 + B*a*b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a)
 + ((A*a^4*b + 2*B*a^3*b^2 - 6*(A - B)*a^2*b^3 - 2*(4*A - 3*B)*a*b^4 - (3*A
 - 2*B)*b^5)*cos(d*x + c)^2*sin(d*x + c) + (A*a^5 + 2*B*a^4*b - 6*(A - B)*a
^3*b^2 - 2*(4*A - 3*B)*a^2*b^3 - (3*A - 2*B)*a*b^4)*cos(d*x + c)^2)*log(sin
(d*x + c) + 1) - ((A*a^4*b + 2*B*a^3*b^2 - 6*(A + B)*a^2*b^3 + 2*(4*A + 3*B
)*a*b^4 - (3*A + 2*B)*b^5)*cos(d*x + c)^2*sin(d*x + c) + (A*a^5 + 2*B*a^4*b
 - 6*(A + B)*a^3*b^2 + 2*(4*A + 3*B)*a^2*b^3 - (3*A + 2*B)*a*b^4)*cos(d*x +
 c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^5 - B*a^4*b - 2*A*a^3*b^2 + 2*B*a^2*
b^3 + A*a*b^4 - B*b^5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
*d*cos(d*x + c)^2*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*co
s(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32555, size = 452, normalized size = 1.98

$$-\frac{\frac{4\left(3\,Ba^{2}b^{3}-4\,Aab^{4}+Bb^{5}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}}-\frac{\left(Aa-3\,Ab+2\,Bb\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(Aa+3\,Ab+2\,Bb\right)\log\left(\left|-\sin\left(dx+c\right)+1\right|\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\left(Aa^{2}b\sin\left(dx+c\right)^{2}-4\,Bab^{2}\cdots\right.}{\cdots}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/4*(4*(3*B*a^2*b^3 - 4*A*a*b^4 + B*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6
*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7) - (A*a - 3*A*b + 2*B*b)*log(abs(sin(d*x +
 c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (A*a + 3*A*b + 2*B*b)*log(abs(-
sin(d*x + c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(A*a^2*b*sin(d*x + c
)^2 - 4*B*a*b^2*sin(d*x + c)^2 + 3*A*b^3*sin(d*x + c)^2 + A*a^3*sin(d*x + c
) - B*a^2*b*sin(d*x + c) - A*a*b^2*sin(d*x + c) + B*b^3*sin(d*x + c) + B*a^
3 - 2*A*a^2*b + 3*B*a*b^2 - 2*A*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x +
c)^3 + a*sin(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

3.1558 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=372

$$-\frac{b\left(-12a^2Ab^2+3a^4A+2a^3bB+22ab^3B-15Ab^4\right)}{8d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{b^4\left(-5a^2B+6aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{(3a^2A+2al}{}$$

```
[Out] -((3*a^2*A + 2*a*b*(6*A + B) + b^2*(15*A + 8*B))*Log[1 - Sin[c + d*x]])/(16
*(a + b)^4*d) + ((3*a^2*A + b^2*(15*A - 8*B) - 2*a*b*(6*A - B))*Log[1 + Sin
[c + d*x]])/(16*(a - b)^4*d) - (b^4*(6*a*A*b - 5*a^2*B - b^2*B)*Log[a + b*S
in[c + d*x]])/((a^2 - b^2)^4*d) - (b*(3*a^4*A - 12*a^2*A*b^2 - 15*A*b^4 + 2
*a^3*b*B + 22*a*b^3*B))/(8*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (Sec[c +
 d*x]^4*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin
[c + d*x])) + (Sec[c + d*x]^2*(b*(a^2*A + 5*A*b^2 - 6*a*b*B) + (3*a^3*A - 9
*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d*(a + b*Si
n[c + d*x]))
```

Rubi [A] time = 0.564163, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$-\frac{b\left(-12a^2Ab^2+3a^4A+2a^3bB+22ab^3B-15Ab^4\right)}{8d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{b^4\left(-5a^2B+6aAb-b^2B\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{(3a^2A+2al}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -((3*a^2*A + 2*a*b*(6*A + B) + b^2*(15*A + 8*B))*Log[1 - Sin[c + d*x]])/(16
*(a + b)^4*d) + ((3*a^2*A + b^2*(15*A - 8*B) - 2*a*b*(6*A - B))*Log[1 + Sin
[c + d*x]])/(16*(a - b)^4*d) - (b^4*(6*a*A*b - 5*a^2*B - b^2*B)*Log[a + b*S
in[c + d*x]])/((a^2 - b^2)^4*d) - (b*(3*a^4*A - 12*a^2*A*b^2 - 15*A*b^4 + 2
*a^3*b*B + 22*a*b^3*B))/(8*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (Sec[c +
 d*x]^4*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin
[c + d*x])) + (Sec[c + d*x]^2*(b*(a^2*A + 5*A*b^2 - 6*a*b*B) + (3*a^3*A - 9
*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d*(a + b*Si
n[c + d*x]))
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b^3\,\text{Subst}\left(\int \frac{-3a^2A+5Ab^2-2abB\cdots}{(a+x)^2\left(b^2-x\cdots\right.}\cdots\right.}{4\left(a^2\cdots\right.}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b\left(a^2A+5Ab^2\cdots\right.\right.}{8\cdots}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b\left(a^2A+5Ab^2\cdots\right.\right.}{8\cdots}$$

$$= -\frac{\left(3a^2A+2ab(6A+B)+b^2(15A+8B)\right)\log(1-\sin(c+dx))}{16(a+b)^4d} + \frac{\left(3a^2A+b^2(15\cdots\right.}{\cdots}$$

Mathematica [A] time = 4.12523, size = 370, normalized size = 0.99

$$-\frac{\left(3a^3A+2a^2bB-9aAb^2+4b^3B\right)((a-b)\log(1-\sin(c+dx))-(a+b)\log(\sin(c+dx)+1)+2b\log(a+b\sin(c+dx)))}{(a-b)(a+b)} + b\left(12a^2Ab^2-3a^4A-2a^3bB-22ab^3B\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-((((3*a^3*A - 9*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*((a - b)*Log[1 - Sin[c + d*
x]] - (a + b)*Log[1 + Sin[c + d*x]] + 2*b*Log[a + b*Sin[c + d*x]]))/((a - b
)*(a + b))) + (2*(-a^2 + b^2)*Sec[c + d*x]^4*(A*b - a*B + (-(a*A) + b*B)*Si
n[c + d*x]))/(a + b*Sin[c + d*x]) + (Sec[c + d*x]^2*(b*(a^2*A + 5*A*b^2 - 6
*a*b*B) + (3*a^3*A - 9*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*Sin[c + d*x]))/(a + b
*Sin[c + d*x]) + b*(-3*a^4*A + 12*a^2*A*b^2 + 15*A*b^4 - 2*a^3*b*B - 22*a*b
^3*B)*(-Log[1 - Sin[c + d*x]]/(2*b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a
 - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2
- b^2)*(a + b*Sin[c + d*x]))))/(8*(a^2 - b^2)^2*d)
```

Maple [A] time = 0.175, size = 675, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -6/d*b^5/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*A*a+5/d*b^4/(a+b)^4/(a-b)^4*ln(
a+b*sin(d*x+c))*B*a^2-1/d*b^4/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))*a*B+1/d*b^5/
(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))*A-3/4/d/(a+b)^4*ln(sin(d*x+c)-1)*A*a*b-1/8
/d/(a+b)^4*ln(sin(d*x+c)-1)*B*a*b-3/4/d/(a-b)^4*ln(1+sin(d*x+c))*A*a*b+1/8/
d/(a-b)^4*ln(1+sin(d*x+c))*B*a*b+1/16/d/(a-b)^2/(1+sin(d*x+c))^2*B-1/16/d/(
a-b)^2/(1+sin(d*x+c))^2*A+1/16/d/(a+b)^2/(sin(d*x+c)-1)^2*A+1/16/d/(a+b)^2/
(sin(d*x+c)-1)^2*B-1/16/d/(a+b)^3/(sin(d*x+c)-1)*a*B-5/16/d/(a+b)^3/(sin(d*
x+c)-1)*B*b-3/16/d/(a+b)^4*ln(sin(d*x+c)-1)*a^2*A-15/16/d/(a+b)^4*ln(sin(d*
x+c)-1)*A*b^2-1/2/d/(a+b)^4*ln(sin(d*x+c)-1)*B*b^2-3/16/d/(a-b)^3/(1+sin(d*
x+c))*a*A+7/16/d/(a-b)^3/(1+sin(d*x+c))*A*b+1/16/d/(a-b)^3/(1+sin(d*x+c))*a
*B-5/16/d/(a-b)^3/(1+sin(d*x+c))*B*b+3/16/d/(a-b)^4*ln(1+sin(d*x+c))*a^2*A+
15/16/d/(a-b)^4*ln(1+sin(d*x+c))*A*b^2-1/2/d/(a-b)^4*ln(1+sin(d*x+c))*B*b^2
-7/16/d/(a+b)^3/(sin(d*x+c)-1)*A*b+1/d*b^6/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c
))*B-3/16/d/(a+b)^3/(sin(d*x+c)-1)*a*A
```

Maxima [A] time = 1.07899, size = 890, normalized size = 2.39

$$\frac{\frac{16\left(5\,Ba^{2}b^{4}-6\,Aab^{5}+Bb^{6}\right)\log(b\sin(dx+c)+a)}{a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}}+\frac{\left(3\,Aa^{2}-2\,(6\,A-B)ab+(15\,A-8\,B)b^{2}\right)\log(\sin(dx+c)+1)}{a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}}-\frac{\left(3\,Aa^{2}+2\,(6\,A+B)ab+(15\,A+8\,B)b^{2}\right)\log(\sin(dx-}{a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/16*(16*(5*B*a^2*b^4 - 6*A*a*b^5 + B*b^6)*log(b*sin(d*x + c) + a)/(a^8 - 4
*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (3*A*a^2 - 2*(6*A - B)*a*b + (15*
A - 8*B)*b^2)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4) - (3*A*a^2 + 2*(6*A + B)*a*b + (15*A + 8*B)*b^2)*log(sin(d*x + c) - 1)
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(2*B*a^5 - 4*A*a^4*b - 12*
B*a^3*b^2 + 20*A*a^2*b^3 - 14*B*a*b^4 + 8*A*b^5 - (3*A*a^4*b + 2*B*a^3*b^2
- 12*A*a^2*b^3 + 22*B*a*b^4 - 15*A*b^5)*sin(d*x + c)^4 - (3*A*a^5 + 2*B*a^4
*b - 12*A*a^3*b^2 + 2*B*a^2*b^3 + 9*A*a*b^4 - 4*B*b^5)*sin(d*x + c)^3 + (5*
A*a^4*b + 10*B*a^3*b^2 - 28*A*a^2*b^3 + 38*B*a*b^4 - 25*A*b^5)*sin(d*x + c)
^2 + (5*A*a^5 - 16*A*a^3*b^2 + 6*B*a^2*b^3 + 11*A*a*b^4 - 6*B*b^5)*sin(d*x
+ c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5
 - b^7)*sin(d*x + c)^5 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)
^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^3 - 2*(a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^2 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7)*sin(d*x + c)))/d
```

Fricas [B] time = 21.0228, size = 1997, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/16*(4*B*a^7 - 4*A*a^6*b - 12*B*a^5*b^2 + 12*A*a^4*b^3 + 12*B*a^3*b^4 - 12
*A*a^2*b^5 - 4*B*a*b^6 + 4*A*b^7 - 2*(3*A*a^6*b + 2*B*a^5*b^2 - 15*A*a^4*b^
```

```
3 + 20*B*a^3*b^4 - 3*A*a^2*b^5 - 22*B*a*b^6 + 15*A*b^7)*cos(d*x + c)^4 + 2*
(A*a^6*b - 6*B*a^5*b^2 + 3*A*a^4*b^3 + 12*B*a^3*b^4 - 9*A*a^2*b^5 - 6*B*a*b
^6 + 5*A*b^7)*cos(d*x + c)^2 + 16*((5*B*a^2*b^5 - 6*A*a*b^6 + B*b^7)*cos(d*
x + c)^4*sin(d*x + c) + (5*B*a^3*b^4 - 6*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)^
4)*log(b*sin(d*x + c) + a) + ((3*A*a^6*b + 2*B*a^5*b^2 - 15*A*a^4*b^3 - 20*
B*a^3*b^4 + 5*(9*A - 8*B)*a^2*b^5 + 6*(8*A - 5*B)*a*b^6 + (15*A - 8*B)*b^7)
*cos(d*x + c)^4*sin(d*x + c) + (3*A*a^7 + 2*B*a^6*b - 15*A*a^5*b^2 - 20*B*a
^4*b^3 + 5*(9*A - 8*B)*a^3*b^4 + 6*(8*A - 5*B)*a^2*b^5 + (15*A - 8*B)*a*b^6
)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - ((3*A*a^6*b + 2*B*a^5*b^2 - 15*A*
a^4*b^3 - 20*B*a^3*b^4 + 5*(9*A + 8*B)*a^2*b^5 - 6*(8*A + 5*B)*a*b^6 + (15*
A + 8*B)*b^7)*cos(d*x + c)^4*sin(d*x + c) + (3*A*a^7 + 2*B*a^6*b - 15*A*a^5
*b^2 - 20*B*a^4*b^3 + 5*(9*A + 8*B)*a^3*b^4 - 6*(8*A + 5*B)*a^2*b^5 + (15*A
 + 8*B)*a*b^6)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(2*A*a^7 - 2*B*a^
6*b - 6*A*a^5*b^2 + 6*B*a^4*b^3 + 6*A*a^3*b^4 - 6*B*a^2*b^5 - 2*A*a*b^6 + 2
*B*b^7 + (3*A*a^7 + 2*B*a^6*b - 15*A*a^5*b^2 + 21*A*a^3*b^4 - 6*B*a^2*b^5 -
 9*A*a*b^6 + 4*B*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6
*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4*sin(d*x + c) + (a^9 - 4*a^7*b^
2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35944, size = 1027, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/16*(16*(5*B*a^2*b^5 - 6*A*a*b^6 + B*b^7)*log(abs(b*sin(d*x + c) + a))/(a^
8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (3*A*a^2 + 12*A*a*b + 2*B*
a*b + 15*A*b^2 + 8*B*b^2)*log(abs(-sin(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4) + (3*A*a^2 - 12*A*a*b + 2*B*a*b + 15*A*b^2 - 8*B*b^2
)*log(abs(-sin(d*x + c) - 1))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) -
 16*(5*B*a^2*b^5*sin(d*x + c) - 6*A*a*b^6*sin(d*x + c) + B*b^7*sin(d*x + c)
 + 6*B*a^3*b^4 - 7*A*a^2*b^5 + A*b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8)*(b*sin(d*x + c) + a)) + 2*(30*B*a^2*b^4*sin(d*x + c)^4 - 36*A*a
*b^5*sin(d*x + c)^4 + 6*B*b^6*sin(d*x + c)^4 - 3*A*a^6*sin(d*x + c)^3 - 2*B
*a^5*b*sin(d*x + c)^3 + 15*A*a^4*b^2*sin(d*x + c)^3 - 12*B*a^3*b^3*sin(d*x
+ c)^3 - 5*A*a^2*b^4*sin(d*x + c)^3 + 14*B*a*b^5*sin(d*x + c)^3 - 7*A*b^6*s
in(d*x + c)^3 + 12*B*a^4*b^2*sin(d*x + c)^2 - 16*A*a^3*b^3*sin(d*x + c)^2 -
 68*B*a^2*b^4*sin(d*x + c)^2 + 88*A*a*b^5*sin(d*x + c)^2 - 16*B*b^6*sin(d*x
 + c)^2 + 5*A*a^6*sin(d*x + c) - 2*B*a^5*b*sin(d*x + c) - 17*A*a^4*b^2*sin(
d*x + c) + 20*B*a^3*b^3*sin(d*x + c) + 3*A*a^2*b^4*sin(d*x + c) - 18*B*a*b^
5*sin(d*x + c) + 9*A*b^6*sin(d*x + c) + 2*B*a^6 - 4*A*a^5*b - 14*B*a^4*b^2
```

```
+ 24*A*a^3*b^3 + 36*B*a^2*b^4 - 56*A*a*b^5 + 12*B*b^6)/((a^8 - 4*a^6*b^2 +
6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2))/d
```

3.1559 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2} dx$

Optimal. Leaf size=550

$$-\frac{b\left(-23a^4Ab^2+47a^2Ab^4+5a^6A-12a^3b^3B+2a^5bB-54ab^5B+35Ab^6\right)}{16d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{b^6\left(-7a^2B+8aAb-b^2B\right)\log(a+b\sin(c+$$

```
[Out] -((5*a^3*A + a^2*b*(25*A + 2*B) + a*b^2*(47*A + 10*B) + b^3*(35*A + 16*B))*
Log[1 - Sin[c + d*x]])/(32*(a + b)^5*d) + ((5*a^3*A - b^3*(35*A - 16*B) + a
*b^2*(47*A - 10*B) - a^2*(25*A*b - 2*b*B))*Log[1 + Sin[c + d*x]])/(32*(a -
b)^5*d) + (b^6*(8*a*A*b - 7*a^2*B - b^2*B)*Log[a + b*Sin[c + d*x]])/((a^2 -
 b^2)^5*d) - (b*(5*a^6*A - 23*a^4*A*b^2 + 47*a^2*A*b^4 + 35*A*b^6 + 2*a^5*b
*B - 12*a^3*b^3*B - 54*a*b^5*B))/(16*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
- (Sec[c + d*x]^6*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(6*(a^2 - b^2)*d*
(a + b*Sin[c + d*x])) + (Sec[c + d*x]^4*(b*(a^2*A + 7*A*b^2 - 8*a*b*B) + (5
*a^3*A - 13*a*A*b^2 + 2*a^2*b*B + 6*b^3*B)*Sin[c + d*x]))/(24*(a^2 - b^2)^2
*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(b*(5*a^4*A - 18*a^2*A*b^2 - 35*
A*b^4 + 2*a^3*b*B + 46*a*b^3*B) + 3*(5*a^5*A - 18*a^3*A*b^2 + 29*a*A*b^4 +
2*a^4*b*B - 10*a^2*b^3*B - 8*b^5*B)*Sin[c + d*x]))/(48*(a^2 - b^2)^3*d*(a +
 b*Sin[c + d*x]))
```

Rubi [A] time = 0.928159, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2837, 823, 801}

$$-\frac{b\left(-23a^4Ab^2+47a^2Ab^4+5a^6A-12a^3b^3B+2a^5bB-54ab^5B+35Ab^6\right)}{16d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{b^6\left(-7a^2B+8aAb-b^2B\right)\log(a+b\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -((5*a^3*A + a^2*b*(25*A + 2*B) + a*b^2*(47*A + 10*B) + b^3*(35*A + 16*B))*
Log[1 - Sin[c + d*x]])/(32*(a + b)^5*d) + ((5*a^3*A - b^3*(35*A - 16*B) + a
*b^2*(47*A - 10*B) - a^2*(25*A*b - 2*b*B))*Log[1 + Sin[c + d*x]])/(32*(a -
b)^5*d) + (b^6*(8*a*A*b - 7*a^2*B - b^2*B)*Log[a + b*Sin[c + d*x]])/((a^2 -
 b^2)^5*d) - (b*(5*a^6*A - 23*a^4*A*b^2 + 47*a^2*A*b^4 + 35*A*b^6 + 2*a^5*b
*B - 12*a^3*b^3*B - 54*a*b^5*B))/(16*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
- (Sec[c + d*x]^6*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(6*(a^2 - b^2)*d*
(a + b*Sin[c + d*x])) + (Sec[c + d*x]^4*(b*(a^2*A + 7*A*b^2 - 8*a*b*B) + (5
*a^3*A - 13*a*A*b^2 + 2*a^2*b*B + 6*b^3*B)*Sin[c + d*x]))/(24*(a^2 - b^2)^2
*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(b*(5*a^4*A - 18*a^2*A*b^2 - 35*
A*b^4 + 2*a^3*b*B + 46*a*b^3*B) + 3*(5*a^5*A - 18*a^3*A*b^2 + 29*a*A*b^4 +
2*a^4*b*B - 10*a^2*b^3*B - 8*b^5*B)*Sin[c + d*x]))/(48*(a^2 - b^2)^3*d*(a +
 b*Sin[c + d*x]))
```


Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2\left(b^2-x^2\right)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b^5\,\text{Subst}\left(\int \frac{-5a^2A+7Ab^2-2\cdots}{(a+x)^2(\cdots}\cdots\right.}{6(\cdots}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)\left(b\left(a^2A+7\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)\left(b\left(a^2A+7\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)\left(b\left(a^2A+7\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(5a^3A+a^2b(25A+2B)+ab^2(47A+10B)+b^3(35A+16B)\right)\log(1-\sin(c\cdots}{32(a+b)^5d}$$

Mathematica [A] time = 6.20188, size = 766, normalized size = 1.39

$$b^7\left(\frac{\frac{\left(6a\left(-18a^3Ab^2+5a^5A-10a^2b^3B+2a^4bB+29aAb^4-8b^5B\right)-3\left(-13a^4Ab^2+11a^2Ab^4+5a^6A-8a^3b^3B+2a^5bB+38ab^5B-35Ab^6\right)\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log(\sin\cdots}{2b(\cdots}\cdots\right.}{2b^2\left(b^2-a^2\right)}}{\cdots}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (b^7*(-(Sec[c + d*x]^6*(-(A*b^2) + a*b*B - b*(-(a*A) + b*B)*Sin[c + d*x]))/
(6*b^8*(-a^2 + b^2)*(a + b*Sin[c + d*x])) + (-(Sec[c + d*x]^4*(-6*a*b^2*(a*
A - b*B) - b^2*(-5*a^2*A + 7*A*b^2 - 2*a*b*B) - b*(-6*b^2*(a*A - b*B) - a*(
-5*a^2*A + 7*A*b^2 - 2*a*b*B))*Sin[c + d*x]))/(4*b^6*(-a^2 + b^2)*(a + b*Si
n[c + d*x])) + (-(Sec[c + d*x]^2*(4*a*b^2*(5*a^3*A - 13*a*A*b^2 + 2*a^2*b*B
 + 6*b^3*B) - b^2*(15*a^4*A - 34*a^2*A*b^2 + 35*A*b^4 + 6*a^3*b*B - 22*a*b^
3*B) - b*(4*b^2*(5*a^3*A - 13*a*A*b^2 + 2*a^2*b*B + 6*b^3*B) - a*(15*a^4*A
- 34*a^2*A*b^2 + 35*A*b^4 + 6*a^3*b*B - 22*a*b^3*B))*Sin[c + d*x]))/(2*b^4*
(-a^2 + b^2)*(a + b*Sin[c + d*x])) + (-6*(5*a^5*A - 18*a^3*A*b^2 + 29*a*A*b
^4 + 2*a^4*b*B - 10*a^2*b^3*B - 8*b^5*B)*(-Log[1 - Sin[c + d*x]]/(2*b*(a +
b)) + Log[1 + Sin[c + d*x]]/(2*(a - b)*b) - Log[a + b*Sin[c + d*x]]/(a^2 -
b^2)) + (6*a*(5*a^5*A - 18*a^3*A*b^2 + 29*a*A*b^4 + 2*a^4*b*B - 10*a^2*b^3*
B - 8*b^5*B) - 3*(5*a^6*A - 13*a^4*A*b^2 + 11*a^2*A*b^4 - 35*A*b^6 + 2*a^5*
b*B - 8*a^3*b^3*B + 38*a*b^5*B))*(-Log[1 - Sin[c + d*x]]/(2*b*(a + b)^2) +
Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a -
 b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/(2*b^2*(-a^2 + b^
2)))/(4*b^2*(-a^2 + b^2)))/(6*b^2*(-a^2 + b^2))))/d
```

Maple [B] time = 0.184, size = 1080, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*b^6/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))*a*B+8/d*b^7/(a+b)^5/(a-b)^5*ln(a+b
*sin(d*x+c))*A*a-7/d*b^6/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))*B*a^2-1/48/d/(a
-b)^2/(1+sin(d*x+c))^3*A+1/48/d/(a-b)^2/(1+sin(d*x+c))^3*B-1/48/d/(a+b)^2/(
sin(d*x+c)-1)^3*B-1/48/d/(a+b)^2/(sin(d*x+c)-1)^3*A-25/32/d/(a+b)^5*ln(sin(
d*x+c)-1)*A*a^2*b+9/16/d/(a-b)^4/(1+sin(d*x+c))*A*a*b-19/32/d/(a+b)^4/(sin(
d*x+c)-1)*A*b^2-1/32/d/(a+b)^4/(sin(d*x+c)-1)*B*a^2-11/32/d/(a+b)^4/(sin(d*
x+c)-1)*B*b^2-5/32/d/(a+b)^5*ln(sin(d*x+c)-1)*a^3*A-35/32/d/(a+b)^5*ln(sin(
d*x+c)-1)*b^3*A-1/2/d/(a+b)^5*ln(sin(d*x+c)-1)*B*b^3-1/16/d/(a-b)^3/(1+sin(
d*x+c))^2*a*A+1/8/d/(a-b)^3/(1+sin(d*x+c))^2*A*b+1/32/d/(a-b)^3/(1+sin(d*x+
c))^2*a*B-3/32/d/(a-b)^3/(1+sin(d*x+c))^2*B*b+1/16/d/(a+b)^3/(sin(d*x+c)-1)
^2*a*A+1/8/d/(a+b)^3/(sin(d*x+c)-1)^2*A*b-5/32/d/(a-b)^4/(1+sin(d*x+c))*a^2
*A-19/32/d/(a-b)^4/(1+sin(d*x+c))*A*b^2+1/32/d/(a-b)^4/(1+sin(d*x+c))*B*a^2
+1/16/d/(a-b)^5*ln(1+sin(d*x+c))*B*a^2*b-1/d*b^8/(a+b)^5/(a-b)^5*ln(a+b*sin
(d*x+c))*B-5/16/d/(a-b)^5*ln(1+sin(d*x+c))*B*a*b^2-1/d*b^7/(a+b)^4/(a-b)^4/
(a+b*sin(d*x+c))*A-9/16/d/(a+b)^4/(sin(d*x+c)-1)*A*a*b+11/32/d/(a-b)^4/(1+s
in(d*x+c))*B*b^2+5/32/d/(a-b)^5*ln(1+sin(d*x+c))*a^3*A-35/32/d/(a-b)^5*ln(1
+sin(d*x+c))*b^3*A+1/2/d/(a-b)^5*ln(1+sin(d*x+c))*B*b^3+1/32/d/(a+b)^3/(sin
(d*x+c)-1)^2*a*B+3/32/d/(a+b)^3/(sin(d*x+c)-1)^2*B*b-47/32/d/(a+b)^5*ln(sin
(d*x+c)-1)*A*a*b^2-5/32/d/(a+b)^4/(sin(d*x+c)-1)*a^2*A-3/16/d/(a+b)^4/(sin(
d*x+c)-1)*B*a*b-25/32/d/(a-b)^5*ln(1+sin(d*x+c))*A*a^2*b+47/32/d/(a-b)^5*ln
(1+sin(d*x+c))*A*a*b^2-1/16/d/(a+b)^5*ln(sin(d*x+c)-1)*B*a^2*b-5/16/d/(a+b)
^5*ln(sin(d*x+c)-1)*B*a*b^2-3/16/d/(a-b)^4/(1+sin(d*x+c))*B*a*b
```

Maxima [B] time = 1.18321, size = 1462, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/96*(96*(7*B*a^2*b^6 - 8*A*a*b^7 + B*b^8)*log(b*sin(d*x + c) + a)/(a^10 -
 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(5*A*a^3 - (25
*A - 2*B)*a^2*b + (47*A - 10*B)*a*b^2 - (35*A - 16*B)*b^3)*log(sin(d*x + c)
 + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) + 3*(5*A*a^
3 + (25*A + 2*B)*a^2*b + (47*A + 10*B)*a*b^2 + (35*A + 16*B)*b^3)*log(sin(d
*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*
(8*B*a^7 - 16*A*a^6*b - 44*B*a^5*b^2 + 80*A*a^4*b^3 + 136*B*a^3*b^4 - 208*A
*a^2*b^5 + 92*B*a*b^6 - 48*A*b^7 + 3*(5*A*a^6*b + 2*B*a^5*b^2 - 23*A*a^4*b^
3 - 12*B*a^3*b^4 + 47*A*a^2*b^5 - 54*B*a*b^6 + 35*A*b^7)*sin(d*x + c)^6 + 3
*(5*A*a^7 + 2*B*a^6*b - 23*A*a^5*b^2 - 12*B*a^4*b^3 + 47*A*a^3*b^4 + 2*B*a^
2*b^5 - 29*A*a*b^6 + 8*B*b^7)*sin(d*x + c)^5 - 8*(5*A*a^6*b + 2*B*a^5*b^2 -
 23*A*a^4*b^3 - 19*B*a^3*b^4 + 55*A*a^2*b^5 - 55*B*a*b^6 + 35*A*b^7)*sin(d*
x + c)^4 - 4*(10*A*a^7 + 4*B*a^6*b - 46*A*a^5*b^2 - 17*B*a^4*b^3 + 86*A*a^3
*b^4 - 2*B*a^2*b^5 - 50*A*a*b^6 + 15*B*b^7)*sin(d*x + c)^3 + 3*(11*A*a^6*b
+ 10*B*a^5*b^2 - 57*A*a^4*b^3 - 76*B*a^3*b^4 + 161*A*a^2*b^5 - 126*B*a*b^6
+ 77*A*b^7)*sin(d*x + c)^2 + (33*A*a^7 + 2*B*a^6*b - 139*A*a^5*b^2 - 8*B*a^
4*b^3 + 227*A*a^3*b^4 - 38*B*a^2*b^5 - 121*A*a*b^6 + 44*B*b^7)*sin(d*x + c)
)/(a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8 - (a^8*b - 4*a^6*b^3 + 6
*a^4*b^5 - 4*a^2*b^7 + b^9)*sin(d*x + c)^7 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 -
 4*a^3*b^6 + a*b^8)*sin(d*x + c)^6 + 3*(a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a
^2*b^7 + b^9)*sin(d*x + c)^5 + 3*(a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 +
 a*b^8)*sin(d*x + c)^4 - 3*(a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9
)*sin(d*x + c)^3 - 3*(a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*sin(
d*x + c)^2 + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*sin(d*x + c)
))/d
```

Fricas [B] time = 56.1148, size = 2850, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/96*(16*B*a^9 - 16*A*a^8*b - 64*B*a^7*b^2 + 64*A*a^6*b^3 + 96*B*a^5*b^4 -
96*A*a^4*b^5 - 64*B*a^3*b^6 + 64*A*a^2*b^7 + 16*B*a*b^8 - 16*A*b^9 - 6*(5*A
*a^8*b + 2*B*a^7*b^2 - 28*A*a^6*b^3 - 14*B*a^5*b^4 + 70*A*a^4*b^5 - 42*B*a^
3*b^6 - 12*A*a^2*b^7 + 54*B*a*b^8 - 35*A*b^9)*cos(d*x + c)^6 + 2*(5*A*a^8*b
 + 2*B*a^7*b^2 - 28*A*a^6*b^3 + 42*B*a^5*b^4 + 6*A*a^4*b^5 - 90*B*a^3*b^6 +
 52*A*a^2*b^7 + 46*B*a*b^8 - 35*A*b^9)*cos(d*x + c)^4 + 4*(A*a^8*b - 8*B*a^
7*b^2 + 4*A*a^6*b^3 + 24*B*a^5*b^4 - 18*A*a^4*b^5 - 24*B*a^3*b^6 + 20*A*a^2
*b^7 + 8*B*a*b^8 - 7*A*b^9)*cos(d*x + c)^2 - 96*((7*B*a^2*b^7 - 8*A*a*b^8 +
 B*b^9)*cos(d*x + c)^6*sin(d*x + c) + (7*B*a^3*b^6 - 8*A*a^2*b^7 + B*a*b^8)
*cos(d*x + c)^6)*log(b*sin(d*x + c) + a) + 3*((5*A*a^8*b + 2*B*a^7*b^2 - 28
*A*a^6*b^3 - 14*B*a^5*b^4 + 70*A*a^4*b^5 + 70*B*a^3*b^6 - 28*(5*A - 4*B)*a^
2*b^7 - 2*(64*A - 35*B)*a*b^8 - (35*A - 16*B)*b^9)*cos(d*x + c)^6*sin(d*x +
 c) + (5*A*a^9 + 2*B*a^8*b - 28*A*a^7*b^2 - 14*B*a^6*b^3 + 70*A*a^5*b^4 + 7
0*B*a^4*b^5 - 28*(5*A - 4*B)*a^3*b^6 - 2*(64*A - 35*B)*a^2*b^7 - (35*A - 16
*B)*a*b^8)*cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 3*((5*A*a^8*b + 2*B*a^7*
b^2 - 28*A*a^6*b^3 - 14*B*a^5*b^4 + 70*A*a^4*b^5 + 70*B*a^3*b^6 - 28*(5*A +
 4*B)*a^2*b^7 + 2*(64*A + 35*B)*a*b^8 - (35*A + 16*B)*b^9)*cos(d*x + c)^6*s
in(d*x + c) + (5*A*a^9 + 2*B*a^8*b - 28*A*a^7*b^2 - 14*B*a^6*b^3 + 70*A*a^5
*b^4 + 70*B*a^4*b^5 - 28*(5*A + 4*B)*a^3*b^6 + 2*(64*A + 35*B)*a^2*b^7 - (3
```

```
5*A + 16*B)*a*b^8)*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(8*A*a^9 - 8*
B*a^8*b - 32*A*a^7*b^2 + 32*B*a^6*b^3 + 48*A*a^5*b^4 - 48*B*a^4*b^5 - 32*A*
a^3*b^6 + 32*B*a^2*b^7 + 8*A*a*b^8 - 8*B*b^9 + 3*(5*A*a^9 + 2*B*a^8*b - 28*
A*a^7*b^2 - 14*B*a^6*b^3 + 70*A*a^5*b^4 + 14*B*a^4*b^5 - 76*A*a^3*b^6 + 6*B
*a^2*b^7 + 29*A*a*b^8 - 8*B*b^9)*cos(d*x + c)^4 + 2*(5*A*a^9 + 2*B*a^8*b -
28*A*a^7*b^2 + 54*A*a^5*b^4 - 12*B*a^4*b^5 - 44*A*a^3*b^6 + 16*B*a^2*b^7 +
13*A*a*b^8 - 6*B*b^9)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b - 5*a^8*b^3 +
10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11)*d*cos(d*x + c)^6*sin(d*x + c) +
 (a^11 - 5*a^9*b^2 + 10*a^7*b^4 - 10*a^5*b^6 + 5*a^3*b^8 - a*b^10)*d*cos(d*
x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38827, size = 1600, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/96*(96*(7*B*a^2*b^7 - 8*A*a*b^8 + B*b^9)*log(abs(b*sin(d*x + c) + a))/(a
^10*b - 5*a^8*b^3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) + 3*(5*A*a^
3 + 25*A*a^2*b + 2*B*a^2*b + 47*A*a*b^2 + 10*B*a*b^2 + 35*A*b^3 + 16*B*b^3)
*log(abs(-sin(d*x + c) + 1))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5) - 3*(5*A*a^3 - 25*A*a^2*b + 2*B*a^2*b + 47*A*a*b^2 - 10*B*a*b^2
 - 35*A*b^3 + 16*B*b^3)*log(abs(-sin(d*x + c) - 1))/(a^5 - 5*a^4*b + 10*a^3
*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) - 96*(7*B*a^2*b^7*sin(d*x + c) - 8*A*a*b
^8*sin(d*x + c) + B*b^9*sin(d*x + c) + 8*B*a^3*b^6 - 9*A*a^2*b^7 + A*b^9)/(
(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*(b*sin(d*x
+ c) + a)) + 2*(308*B*a^2*b^6*sin(d*x + c)^6 - 352*A*a*b^7*sin(d*x + c)^6 +
 44*B*b^8*sin(d*x + c)^6 + 15*A*a^8*sin(d*x + c)^5 + 6*B*a^7*b*sin(d*x + c)
^5 - 84*A*a^6*b^2*sin(d*x + c)^5 - 42*B*a^5*b^3*sin(d*x + c)^5 + 210*A*a^4*
b^4*sin(d*x + c)^5 - 78*B*a^3*b^5*sin(d*x + c)^5 - 84*A*a^2*b^6*sin(d*x + c
)^5 + 114*B*a*b^7*sin(d*x + c)^5 - 57*A*b^8*sin(d*x + c)^5 + 120*B*a^4*b^4*
sin(d*x + c)^4 - 144*A*a^3*b^5*sin(d*x + c)^4 - 1020*B*a^2*b^6*sin(d*x + c)
^4 + 1200*A*a*b^7*sin(d*x + c)^4 - 156*B*b^8*sin(d*x + c)^4 - 40*A*a^8*sin(
d*x + c)^3 - 16*B*a^7*b*sin(d*x + c)^3 + 224*A*a^6*b^2*sin(d*x + c)^3 + 48*
B*a^5*b^3*sin(d*x + c)^3 - 480*A*a^4*b^4*sin(d*x + c)^3 + 240*B*a^3*b^5*sin
(d*x + c)^3 + 160*A*a^2*b^6*sin(d*x + c)^3 - 272*B*a*b^7*sin(d*x + c)^3 + 1
36*A*b^8*sin(d*x + c)^3 + 36*B*a^6*b^2*sin(d*x + c)^2 - 48*A*a^5*b^3*sin(d*
x + c)^2 - 300*B*a^4*b^4*sin(d*x + c)^2 + 384*A*a^3*b^5*sin(d*x + c)^2 + 11
28*B*a^2*b^6*sin(d*x + c)^2 - 1392*A*a*b^7*sin(d*x + c)^2 + 192*B*b^8*sin(d
*x + c)^2 + 33*A*a^8*sin(d*x + c) - 6*B*a^7*b*sin(d*x + c) - 156*A*a^6*b^2*
sin(d*x + c) + 42*B*a^5*b^3*sin(d*x + c) + 270*A*a^4*b^4*sin(d*x + c) - 210
```

```
*B*a^3*b^5*sin(d*x + c) - 60*A*a^2*b^6*sin(d*x + c) + 174*B*a*b^7*sin(d*x +
 c) - 87*A*b^8*sin(d*x + c) + 8*B*a^8 - 16*A*a^7*b - 52*B*a^6*b^2 + 96*A*a^
5*b^3 + 180*B*a^4*b^4 - 288*A*a^3*b^5 - 400*B*a^2*b^6 + 560*A*a*b^7 - 88*B*
b^8)/((a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*(sin(
d*x + c)^2 - 1)^3))/d
```

3.1560 $\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=39

$$\text{Unintegrable}\left((A+B\sin(e+fx))(g\cos(e+fx))^{-m-1}(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e
+ f*x]), x]
```

Rubi [A] time = 0.0973966, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x
]
```

```
[Out] Defer[Int][(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e +
f*x]), x]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin($$

Mathematica [A] time = 4.73579, size = 0, normalized size = 0.

$$\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f
*x]),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f
*x]), x]
```

Maple [A] time = 0.451, size = 0, normalized size = 0.

$$\int \left(g\cos\left(fx+e\right)\right)^{-1-m}\left(a+b\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(g\cos(fx+e))^{-m-1}(b\sin(fx+e)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^(-m - 1)*(b*sin(f*x + e) +
a)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\sin(fx+e)+A)(g\cos(fx+e))^{-m-1}(b\sin(fx+e)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algo
rithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^(-m - 1)*(b*sin(f*x + e) + a
)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m)*(a+b*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algo
rithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.1561 $\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=330

$$\frac{g(g\cos(e+fx))^{p-1}\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(1-p)(bc-ad)}-\frac{g(g\cos(e+\cdots}{\cdots}$$

```
[Out] -((g*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[e + f*
x]), (a - b)/(a + b*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((b*(1 - Sin
[e + f*x]))/(a + b*Sin[e + f*x])))^((1 - p)/2)*((b*(1 + Sin[e + f*x]))/(a +
 b*Sin[e + f*x]))^((1 - p)/2))/((b*c - a*d)*f*(1 - p))) + (g*AppellF1[1 - p
, (1 - p)/2, (1 - p)/2, 2 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c + d
*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(c + d*
Sin[e + f*x])))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(
(1 - p)/2))/((b*c - a*d)*f*(1 - p))
```

Rubi [A] time = 0.415848, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2924, 2703}

$$\frac{g(g\cos(e+fx))^{p-1}\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(1-p)(bc-ad)}-\frac{g(g\cos(e+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```


```
[Out] -((g*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[e + f*
x]), (a - b)/(a + b*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((b*(1 - Sin
[e + f*x]))/(a + b*Sin[e + f*x])))^((1 - p)/2)*((b*(1 + Sin[e + f*x]))/(a +
 b*Sin[e + f*x]))^((1 - p)/2))/((b*c - a*d)*f*(1 - p))) + (g*AppellF1[1 - p
, (1 - p)/2, (1 - p)/2, 2 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c + d
*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(c + d*
Sin[e + f*x])))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(
(1 - p)/2))/((b*c - a*d)*f*(1 - p))
```


Rule 2924

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && Integer
sQ[2*m, 2*n]
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx = \int \left(\frac{b(g\cos(e+fx))^p}{(bc-ad)(a+b\sin(e+fx))} - \frac{d(g\cos(e+fx))^p}{(bc-ad)(c+d\sin(e+fx))}\right)dx$$
$$= \frac{b\int \frac{(g\cos(e+fx))^p}{a+b\sin(e+fx)}\,dx}{bc-ad} - \frac{d\int \frac{(g\cos(e+fx))^p}{c+d\sin(e+fx)}\,dx}{bc-ad}$$
$$= -\frac{gF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)(g\cos(e+fx))^{-1+}}{(bc-ad)f(1-p)}$$

Mathematica [B] time = 34.3345, size = 5087, normalized size = 15.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.72, size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p}{\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(g\cos\left(fx+e\right)\right)^p}{bd\cos\left(fx+e\right)^2-ac-bd-(bc+ad)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(g*cos(f*x + e))^p/(b*d*cos(f*x + e)^2 - a*c - b*d - (b*c + a*d)*
sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g\cos(fx+e))^p}{(b\sin(fx+e)+a)(d\sin(fx+e)+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

3.1562 $\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=508

$$-\frac{bg(g\cos(e+fx))^{p-1}\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(e+fx)+1)}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)}{f(1-p)(bc-ad)^2}+\frac{bg(g}{}$$

```
[Out] -((b*g*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[e +
f*x]), (a - b)/(a + b*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((b*(1 - S
in[e + f*x]))/(a + b*Sin[e + f*x])))^((1 - p)/2)*((b*(1 + Sin[e + f*x]))/(a
 + b*Sin[e + f*x]))^((1 - p)/2))/((b*c - a*d)^2*f*(1 - p))) + (b*g*AppellF1
[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/
(c + d*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(
c + d*Sin[e + f*x])))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*
x]))^((1 - p)/2))/((b*c - a*d)^2*f*(1 - p)) + (g*AppellF1[2 - p, (1 - p)/2,
 (1 - p)/2, 3 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c + d*Sin[e + f*x
])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(c + d*Sin[e + f*x]
)))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^((1 - p)/2))/
((b*c - a*d)*f*(2 - p)*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.524493, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2924, 2703}

$$-\frac{bg(g\cos(e+fx))^{p-1}\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(e+fx)+1)}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)}{f(1-p)(bc-ad)^2}+\frac{bg(g}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```


```
[Out] -((b*g*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[e +
f*x]), (a - b)/(a + b*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((b*(1 - S
in[e + f*x]))/(a + b*Sin[e + f*x])))^((1 - p)/2)*((b*(1 + Sin[e + f*x]))/(a
 + b*Sin[e + f*x]))^((1 - p)/2))/((b*c - a*d)^2*f*(1 - p))) + (b*g*AppellF1
[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/
(c + d*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(
c + d*Sin[e + f*x])))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*
x]))^((1 - p)/2))/((b*c - a*d)^2*f*(1 - p)) + (g*AppellF1[2 - p, (1 - p)/2,
 (1 - p)/2, 3 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c + d*Sin[e + f*x
])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(c + d*Sin[e + f*x]
)))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^((1 - p)/2))/
((b*c - a*d)*f*(2 - p)*(c + d*Sin[e + f*x]))
```


Rule 2924

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && Integer
sQ[2*m, 2*n]
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] && !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \int \left(\frac{b^2(g\cos(e+fx))^p}{(bc-ad)^2(a+b\sin(e+fx))} - \frac{d(g\cos(e+fx))^p}{(bc-ad)(c+d\sin(e+fx))^2} - \frac{b\ldots}{(bc-\ldots}\right.$$

$$= \frac{b^2\int \frac{(g\cos(e+fx))^p}{a+b\sin(e+fx)}\,dx}{(bc-ad)^2} - \frac{(bd)\int \frac{(g\cos(e+fx))^p}{c+d\sin(e+fx)}\,dx}{(bc-ad)^2} - \frac{d\int \frac{(g\cos(e+fx))^p}{(c+d\sin(e+fx))^2}\,dx}{bc-ad}$$

$$= -\frac{bgF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)(g\cos(e+fx))^{-1+p}\ldots}{(bc-ad)^2f(1-p)\ldots}$$

Mathematica [B] time = 35.8543, size = 38759, normalized size = 76.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),
x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.034, size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^p}{\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(g\cos\left(fx+e\right)\right)^{p}}{ac^{2}+2\,bcd+ad^{2}-\left(2\,bcd+ad^{2}\right)\cos\left(fx+e\right)^{2}-\left(bd^{2}\cos\left(fx+e\right)^{2}-bc^{2}-2\,acd-bd^{2}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p/(a*c^2 + 2*b*c*d + a*d^2 - (2*b*c*d + a*d^2)*co
s(f*x + e)^2 - (b*d^2*cos(f*x + e)^2 - b*c^2 - 2*a*c*d - b*d^2)*sin(f*x + e
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\cos\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^2),
 x)
```

3.1563 $\int \frac{(g\sec(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=308

$$\frac{\sec(e+fx)(g\sec(e+fx))^p\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}F_1\left(p+1;\frac{p+1}{2},\frac{p+1}{2};p+2;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(p+1)(bc-ad)} - \text{se}\ldots$$

```
[Out] -((AppellF1[1 + p, (1 + p)/2, (1 + p)/2, 2 + p, (a + b)/(a + b*Sin[e + f*x]
), (a - b)/(a + b*Sin[e + f*x])]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(-((b*(1 -
 Sin[e + f*x]))/(a + b*Sin[e + f*x])))^((1 + p)/2)*((b*(1 + Sin[e + f*x]))/
(a + b*Sin[e + f*x]))^((1 + p)/2))/((b*c - a*d)*f*(1 + p))) + (AppellF1[1 +
 p, (1 + p)/2, (1 + p)/2, 2 + p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c +
 d*Sin[e + f*x])]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(-((d*(1 - Sin[e + f*x]))
/(c + d*Sin[e + f*x])))^((1 + p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e +
f*x]))^((1 + p)/2))/((b*c - a*d)*f*(1 + p))
```

Rubi [A] time = 0.615735, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2926, 2924, 2703}

$$\frac{\sec(e+fx)(g\sec(e+fx))^p\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}F_1\left(p+1;\frac{p+1}{2},\frac{p+1}{2};p+2;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(p+1)(bc-ad)} - \text{se}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((AppellF1[1 + p, (1 + p)/2, (1 + p)/2, 2 + p, (a + b)/(a + b*Sin[e + f*x]
), (a - b)/(a + b*Sin[e + f*x])]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(-((b*(1 -
 Sin[e + f*x]))/(a + b*Sin[e + f*x])))^((1 + p)/2)*((b*(1 + Sin[e + f*x]))/
(a + b*Sin[e + f*x]))^((1 + p)/2))/((b*c - a*d)*f*(1 + p))) + (AppellF1[1 +
 p, (1 + p)/2, (1 + p)/2, 2 + p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c +
 d*Sin[e + f*x])]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(-((d*(1 - Sin[e + f*x]))
/(c + d*Sin[e + f*x])))^((1 + p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e +
f*x]))^((1 + p)/2))/((b*c - a*d)*f*(1 + p))
```

Rule 2926

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[((a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(g*Cos[e + f*x])^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] &&  !IntegerQ[p]
```

Rule 2924

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && Integer
sQ[2*m, 2*n]
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] && !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx = \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int \frac{(g\cos(e+fx))^{-p}}{(a+b\sin(e+fx))(c+d\sin(e+f$$
$$= \left((g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int \left(\frac{b(g\cos(e+fx))^{-p}}{(bc-ad)(a+b\sin(e+fx))} - \frac{}{(bc$$
$$= \frac{\left(b(g\cos(e+fx))^p(g\sec(e+fx))^p\right)\int \frac{(g\cos(e+fx))^{-p}}{a+b\sin(e+fx)}\,dx}{bc-ad} - \frac{\left(d(g\cos(e+fx\right.}{}$$
$$= -\frac{F_1\left(1+p;\frac{1+p}{2},\frac{1+p}{2};2+p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)\sec(e+fx)(g\sec(e}{(bc-ad)f(1+p)}$$

Mathematica [B] time = 28.9841, size = 5101, normalized size = 16.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.807, size = 0, normalized size = 0.

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^p}{\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(g\sec\left(fx+e\right)\right)^{p}}{bd\cos\left(fx+e\right)^{2}-ac-bd-(bc+ad)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(g*sec(f*x + e))^p/(b*d*cos(f*x + e)^2 - a*c - b*d - (b*c + a*d)*
sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\sec\left(e+fx\right)\right)^{p}}{\left(a+b\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((g*sec(e + f*x))**p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(g\sec\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*sec(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```